Survival Package Functions

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1 Introduction

> Let us change or traditional attitude to the construction of programs. Instead of imagining that our main task is to instruct a *computer* what to do, let us concentrate rather on explaining to *humans* what we want the computer to do. (Donald E. Knuth, 1984).

This is the definition of a coding style called *literate programming*. I first made use of it in the *coxme* library and have become a full convert. For the survival library only selected objects are documented in this way; as I make updates and changes I am slowly converting the source code. The first motivation for this is to make the code easier for me, both to create and to maintain. As to maintinance, I have found that whenver I need to update code I spend a lot of time in the "what was I doing in these x lines?" stage. The code never has enough documentation, even for the author. (The survival library is already better than the majority of packages in R, whose comment level is abysmal. In the pre-noweb source code about 1 line in 6 has a comment, for the noweb document the documentation/code ratio is 2:1.) I also find it helps in creating new code to have the real documentation of intent — formulas with integrals and such — closely integrated. The second motivation is to leave code that is well enough explained that someone else can take it over.

The source code is structured using *noweb*, one of the simpler literate programming environments. The source code files look remakably like Sweave, and the .Rnw mode of emacs works perfectly for them. This is not too surprising since Sweave was also based on noweb. Sweave is not sufficient to process the files, however, since it has a different intention. The noweb.R file contains functions that can tangle the code (extract a given R function), but the creation of the pdf still requires the noweb exectuable itself. I am working towards correcting that.

2 Cox Models

2.1 Coxph

The `coxph` routine is the underlying basis for all the models. The source was converted to noweb when adding time-transform terms.

The call starts out with the basic building of a model frame and proceeds from there.

⟨*coxph*⟩=
```
 #tt <- function(x) x
 coxph <- function(formula, data, weights, subset, na.action,
         init, control, ties= c("efron", "breslow", "exact"),
         singular.ok =TRUE, robust=FALSE,
         model=FALSE, x=FALSE, y=TRUE,  tt, method=ties, ...) {

     ties <- match.arg(ties)
     Call <- match.call()
     ⟨coxph-getdata⟩

     Y <- model.extract(mf, "response")
```

```
if (!inherits(Y, "Surv")) stop("Response must be a survival object")
type <- attr(Y, "type")
if (type!='right' && type!='counting')
    stop(gettextf("Cox model doesn't support \"%s\" survival data", type))
data.n <- nrow(Y)   #remember this before any time transforms

⟨coxph-bothsides⟩

# The time transform will expand the data frame mf.  To do this
#  it needs Y and the strata.  Everything else (cluster, offset, weights)
#  should be extracted after the transform
#
strats <- attr(Terms, "specials")$strata
if (length(strats)) {
    stemp <- untangle.specials(Terms, 'strata', 1)
    if (length(stemp$vars)==1) strata.keep <- mf[[stemp$vars]]
    else strata.keep <- strata(mf[,stemp$vars], shortlabel=TRUE)
    strats <- as.numeric(strata.keep)
    }

timetrans <- attr(Terms, "specials")$tt
if (missing(tt)) tt <- NULL
if (length(timetrans)) {
     ⟨coxph-transform⟩
     }

cluster<- attr(Terms, "specials")$cluster
if (length(cluster)) {
    robust <- TRUE  #flag to later compute a robust variance
    tempc <- untangle.specials(Terms, 'cluster', 1:10)
    ord <- attr(Terms, 'order')[tempc$terms]
    if (any(ord>1)) stop("Cluster can not be used in an interaction")
    cluster <- strata(mf[,tempc$vars], shortlabel=TRUE)  #allow multiples
    dropterms <- tempc$terms  #we won't want this in the X matrix
    # Save away xlevels after removing cluster (we don't want to save upteen
    #  levels of that variable, which we will never need).
    xlevels <- .getXlevels(Terms[-tempc$terms], mf)
}
else {
    dropterms <- NULL
    if (missing(robust)) robust <- FALSE
    xlevels <- .getXlevels(Terms, mf)
}

contrast.arg <- NULL  #due to shared code with model.matrix.coxph
⟨coxph-make-X⟩
```

⟨*coxph-setup*⟩
⟨*coxph-penal*⟩
⟨*coxph-compute*⟩
⟨*coxph-finish*⟩
}

Standard code to grab the data.

```
⟨coxph-getdata⟩=
 # create a call to model.frame() that contains the formula (required)
 #  and any other of the relevant optional arguments
 # then evaluate it in the proper frame
 indx <- match(c("formula", "data", "weights", "subset", "na.action"),
               names(Call), nomatch=0)
 if (indx[1] ==0) stop("A formula argument is required")
 temp <- Call[c(1,indx)]  # only keep the arguments we wanted
 temp[[1L]] <- quote(stats::model.frame)  # change the function called

 special <- c("strata", "cluster", "tt")
 temp$formula <- if(missing(data)) terms(formula, special)
                 else              terms(formula, special, data=data)
 # Make "tt" visible for coxph formulas, without making it visible elsewhere
 if (!is.null(attr(temp$formula, "specials")$tt)) {
     coxenv <- new.env(parent= environment(formula))
     assign("tt", function(x) x, env=coxenv)
     environment(temp$formula) <- coxenv
 }

 mf <- eval(temp, parent.frame())
 if (nrow(mf) ==0) stop("no (non-missing) observations")
 Terms <- terms(mf)


 ## We want to pass any ... args to coxph.control, but not pass things
 ##  like "dats=mydata" where someone just made a typo.  The use of ...
 ##  is simply to allow things like "eps=1e6" with easier typing
 extraArgs <- list(...)
 if (length(extraArgs)) {
     controlargs <- names(formals(coxph.control)) #legal arg names
     indx <- pmatch(names(extraArgs), controlargs, nomatch=0L)
     if (any(indx==0L))
         stop(gettextf("argument %s was not matched", names(extraArgs)[indx==0L]), domain = "R-sur
 }
 if (missing(control)) control <- coxph.control(...)
```

An increasingly common error is for users to put the time variable on both sides of the formula, in the mistaken idea that this will deal with a failure of proportional hazards. Add a

test for such models, and bail out. The `variables` attribute of the Terms object is the expression form of a list that contains the response variable followed by the predictors. Subscripting this, element 1 is the call to "list" itself so we always retain it. My `terms.inner` function works only with formula objects.

⟨*coxph-bothsides*⟩=
```
 if (length(attr(Terms, 'variables')) > 2) { # a ~1 formula has length 2
     ytemp <- terms.inner(formula[1:2])
     xtemp <- terms.inner(formula[-2])
     if (any(!is.na(match(xtemp, ytemp))))
         warning("a variable appears on both the left and right sides of the formula")
 }
```

At this point we deal with any time transforms. The model frame is expanded to a "fake" data set that has a separate stratum for each unique event-time/strata combination, and any tt() terms in the formula are processed. The first step is to create the index vector `tindex` and new strata `.strata.`. This last is included in a model.frame call (for others to use), internally the code simply replaces the `strats` variable. A (modestly) fast C-routine first counts up and indexes the observations. We start out with error checks; since the computation can be slow we want to complain early.

⟨*coxph-transform*⟩=
```
 timetrans <- untangle.specials(Terms, 'tt')
 ntrans <- length(timetrans$terms)

 if (is.null(tt)) {
     tt <- function(x, time, riskset, weights){ #default to O'Brien's logit rank
         obrien <- function(x) {
             r <- rank(x)
             (r-.5)/(.5+length(r)-r)
         }
         unlist(tapply(x, riskset, obrien))
     }
 }
 if (is.function(tt)) tt <- list(tt)  #single function becomes a list

 if (is.list(tt)) {
     if (any(!sapply(tt, is.function)))
         stop(gettextf("'%s' argument must contain a function or list of functions", "tt"))
     if (length(tt) != ntrans) {
         if (length(tt) ==1) {
             temp <- vector("list", ntrans)
             for (i in seq_len(ntrans)) temp[[i]] <- tt[[1]]
             tt <- temp
         }
         else stop(gettextf("wrong length for '%s' argument", "tt"))
     }
```

```
}
else stop(gettextf("'%s' argument must contain a function or list of functions", "tt"))

if (ncol(Y)==2) {
    if (length(strats)==0) {
        sorted <- order(-Y[,1], Y[,2])
        newstrat <- rep.int(0L, nrow(Y))
        newstrat[1] <- 1L
        }
    else {
        sorted <- order(strats, -Y[,1], Y[,2])
        #newstrat marks the first obs of each strata
        newstrat <-  as.integer(c(1, 1*(diff(strats[sorted])!=0)))
        }
    if (storage.mode(Y) != "double") storage.mode(Y) <- "double"
    counts <- .Call(Ccoxcount1, Y[sorted,],
                    as.integer(newstrat))
    tindex <- sorted[counts$index]
}
else {
    if (length(strats)==0) {
        sort.end  <- order(-Y[,2], Y[,3])
        sort.start<- order(-Y[,1])
        newstrat  <- c(1L, rep(0, nrow(Y) -1))
    }
    else {
        sort.end  <- order(strats, -Y[,2], Y[,3])
        sort.start<- order(strats, -Y[,1])
        newstrat  <- c(1L, as.integer(diff(strats[sort.end])!=0))
    }
    if (storage.mode(Y) != "double") storage.mode(Y) <- "double"
    counts <- .Call(Ccoxcount2, Y,
                    as.integer(sort.start -1L),
                    as.integer(sort.end -1L),
                    as.integer(newstrat))
    tindex <- counts$index
}
```

The C routine has returned a list with 4 elements

nrisk a vector containing the number at risk at each event time

time the vector of event times

status a vector of status values

index a vector containing the set of subjects at risk for event time 1, followed by those at risk at event time 2, those at risk at event time 3, etc.

The new data frame is then a simple creation. The subtle part below is a desire to retain transformation information so that a downstream call to `termplot` will work. The tt function supplied by the user often finishes with a call to `pspline` or `ns`. If the returned value of the `tt` call has a class for which a `makepredictcall` method exists then we need to do 2 things:

1. Construct a fake call, e.g., "pspline(age)", then feed it and the result of tt as arguments to `makepredictcall`
2. Replace that componenent in the predvars attribute of the terms.

The `timetrans$terms` value is a count of the right hand side of the formula. Some objects in the terms structure are unevaluated calls that include y, this adds 2 to the count (the call to "list" and the response).

⟨*coxph-transform*⟩=
```
mf <- mf[tindex,]
Y <- Surv(rep(counts$time, counts$nrisk), counts$status)
type <- 'right'  # new Y is right censored, even if the old was (start, stop]
strats <- rep(1:length(counts$nrisk), counts$nrisk)
weights <- model.weights(mf)
if (!is.null(weights) && any(!is.finite(weights)))
    stop("weights must be finite")

tcall <- attr(Terms, 'variables')[timetrans$terms+2]
pvars <- attr(Terms, 'predvars')
pmethod <- sub("makepredictcall.", "", as.vector(methods("makepredictcall")))
for (i in seq_len(ntrans)) {
    newtt <- (tt[[i]])(mf[[timetrans$var[i]]], Y[,1], strats, weights)
    mf[[timetrans$var[i]]] <- newtt
    nclass <- class(newtt)
    if (any(nclass %in% pmethod)) { # It has a makepredictcall method
        dummy <- as.call(list(as.name(class(newtt)[1]), tcall[[i]][[2]]))
        ptemp <- makepredictcall(newtt, dummy)
        pvars[[timetrans$terms[i]+2]] <- ptemp
    }
}
attr(Terms, "predvars") <- pvars
```

This is the C code for time-transformation. For the first case it expects y to contain time and status sorted from longest time to shortest, and strata=1 for the first observation of each strata.

⟨*coxcount1*⟩=
```
#include "survS.h"
/*
** Count up risk sets and identify who is in each
*/
SEXP coxcount1(SEXP y2, SEXP strat2) {
```

```
int ntime, nrow;
int i, j, n;
int stratastart=0;  /* start row for this strata */
int nrisk=0;  /* number at risk (=0 to stop -Wall complaint)*/
double *time, *status;
int *strata;
double dtime;
SEXP rlist, rlistnames, rtime, rn, rindex, rstatus;
int *rrindex, *rrstatus;

n = nrows(y2);
time = REAL(y2);
status = time +n;
strata = INTEGER(strat2);

/*
** First pass: count the total number of death times (risk sets)
**  and the total number of rows in the new data set.
*/
ntime=0; nrow=0;
for (i=0; i<n; i++) {
    if (strata[i] ==1) nrisk =0;
    nrisk++;
    if (status[i] ==1) {
        ntime++;
        dtime = time[i];
        /* walk across tied times, if any */
        for (j=i+1; j<n && time[j]==dtime && status[j]==1 && strata[j]==0;
             j++) nrisk++;
        i = j-1;
        nrow += nrisk;
    }
}
⟨coxcount-alloc-memory⟩

/*
** Pass 2, fill them in
*/
ntime=0;
for (i=0; i<n; i++) {
    if (strata[i] ==1) stratastart =i;
    if (status[i]==1) {
        dtime = time[i];
        for (j=stratastart; j<i; j++) *rrstatus++=0; /*non-deaths */
        *rrstatus++ =1; /* this death */
        /* tied deaths */
```

```
            for(j= i+1; j<n && status[j]==1 && time[j]==dtime  && strata[j]==0;
                j++) *rrstatus++ =1;
            i = j-1;

            REAL(rtime)[ntime] = dtime;
            INTEGER(rn)[ntime] = i +1 -stratastart;
            ntime++;
            for (j=stratastart; j<=i; j++) *rrindex++ = j+1;
            }
    }
    ⟨coxcount-list-return⟩
}
```

The start-stop case is a bit more work. The set of subjects still at risk is an arbitrary set so we have to keep an index vector `atrisk`. At each new death time we write out the set of those at risk, with the deaths last. I toyed with the idea of a binary tree then realized it was not useful: at each death we need to list out all the subjects at risk into the index vector which is an $O(n)$ process, tree or not.

⟨*coxcount1*⟩=

```
#include "survS.h"
/* count up risk sets and identify who is in each, (start,stop] version */
SEXP coxcount2(SEXP y2, SEXP isort1, SEXP isort2, SEXP strat2) {
    int ntime, nrow;
    int i, j, istart, n;
    int nrisk=0, *atrisk;
    double *time1, *time2, *status;
    int *strata;
    double dtime;
    int iptr, jptr;

    SEXP rlist, rlistnames, rtime, rn, rindex, rstatus;
    int *rrindex, *rrstatus;
    int *sort1, *sort2;

    n = nrows(y2);
    time1 = REAL(y2);
    time2 =  time1+n;
    status = time2 +n;
    strata = INTEGER(strat2);
    sort1 = INTEGER(isort1);
    sort2 = INTEGER(isort2);

    /*
    ** First pass: count the total number of death times (risk sets)
    **  and the total number of rows in the new data set
    */
```

```
ntime=0; nrow=0;
istart =0;  /* walks along the sort1 vector (start times) */
    for (i=0; i<n; i++) {
    iptr = sort2[i];
    if (strata[i]==1) nrisk=0;
    nrisk++;
    if (status[iptr] ==1) {
        ntime++;
        dtime = time2[iptr];
        for (; istart <i && time1[sort1[istart]] >= dtime; istart++)
                    nrisk--;
        for(j= i+1; j<n; j++) {
            jptr = sort2[j];
            if (status[jptr]==1 && time2[jptr]==dtime && strata[jptr]==0)
                nrisk++;
            else break;
            }
        i= j-1;
        nrow += nrisk;
        }
    }

⟨coxcount-alloc-memory⟩
atrisk = (int *)R_alloc(n, sizeof(int)); /* marks who is at risk */

/*
** Pass 2, fill them in
*/
ntime=0; nrisk=0;
j=0;  /* pointer to time1 */;
istart=0;
for (i=0; i<n; ) {
    iptr = sort2[i];
    if (strata[i] ==1) {
        nrisk=0;
        for (j=0; j<n; j++) atrisk[j] =0;
        }
    nrisk++;
    if (status[iptr]==1) {
        dtime = time2[iptr];
        for (; istart<i && time1[sort1[istart]] >=dtime; istart++) {
            atrisk[sort1[istart]]=0;
            nrisk--;
            }
        for (j=1; j<nrisk; j++) *rrstatus++ =0;
        for (j=0; j<n; j++) if (atrisk[j]) *rrindex++ = j+1;
```

```
            atrisk[iptr] =1;
            *rrstatus++ =1;
            *rrindex++ = iptr +1;
            for (j=i+1; j<n; j++) {
                jptr = sort2[j];
                if (time2[jptr]==dtime && status[jptr]==1 && strata[jptr]==0){
                    atrisk[jptr] =1;
                    *rrstatus++ =1;
                    *rrindex++ = jptr +1;
                    nrisk++;
                    }
                else break;
                }
            i = j;
            REAL(rtime)[ntime] = dtime;
            INTEGER(rn)[ntime] = nrisk;
            ntime++;
        }
        else {
            atrisk[iptr] =1;
            i++;
        }
    }
    ⟨coxcount-list-return⟩
}
```

⟨*coxcount-alloc-memory*⟩=

```
/*
**  Allocate memory
*/
PROTECT(rtime = allocVector(REALSXP, ntime));
PROTECT(rn = allocVector(INTSXP, ntime));
PROTECT(rindex=allocVector(INTSXP, nrow));
PROTECT(rstatus=allocVector(INTSXP,nrow));
rrindex = INTEGER(rindex);
rrstatus= INTEGER(rstatus);
```

⟨*coxcount-list-return*⟩=

```
/* return the list */
PROTECT(rlist = allocVector(VECSXP, 4));
SET_VECTOR_ELT(rlist, 0, rn);
SET_VECTOR_ELT(rlist, 1, rtime);
SET_VECTOR_ELT(rlist, 2, rindex);
SET_VECTOR_ELT(rlist, 3, rstatus);
PROTECT(rlistnames = allocVector(STRSXP, 4));
```

```
SET_STRING_ELT(rlistnames, 0, mkChar("nrisk"));
SET_STRING_ELT(rlistnames, 1, mkChar("time"));
SET_STRING_ELT(rlistnames, 2, mkChar("index"));
SET_STRING_ELT(rlistnames, 3, mkChar("status"));
setAttrib(rlist, R_NamesSymbol, rlistnames);

unprotect(6);
return(rlist);
```

We now return to the original thread of the program, though perhaps with new data, and build the X matrix. Creation of the X matrix for a Cox model requires just a bit of trickery. The baseline hazard for a Cox model plays the role of an intercept, but does not appear in the X matrix. However, to create the columns of X for factor variables correctly, we need to call the model.matrix routine in such a way that it *thinks* there is an intercept. If there are strata the proper X matrix is constructed as though there were one intercept per strata. One simple way to handle this is to call model.matrix on the original formula and then remove the terms we don't need. However,

1. The cluster() term, if any, could lead to thousands of extraneous "intercept" columns which are never needed.
2. Likewise, nested case-control models can have thousands of strata, again leading many intercepts we never need.
3. If there are strata by factor interactions in the model, the dummy intercepts-per-strata columns are necessary information for the model.matrix routine to correctly compute other columns of X.

For reasons 1 and 2 above the usual plan is to remove cluster and strata terms from the "Terms" object *before* calling model.matrix, unless there are strata by covariate interactions in which case we remove them after. For the first strategy the `assign` attribute of the resulting model matrix then needs to be fixed up, since we want it to index into the original formula. For example imagine the right hand side of `age + strata(sex) + trt` where trt is a factor with 3 levels. The assign attribute from the modified formula will be (0,1,2,2) corresponding to the intercept, age, and treatment columns. The final X matrix has no intercept, and a proper assign attribute of (1,3,3) since trt is the third variable in the original formula.

The dropterms variable contains terms to drop before creation of the X matrix. It was initialized far above in the code when we dealt with cluster terms.

⟨*coxph-make-X*⟩=
```
attr(Terms, "intercept") <- 1
adrop <- 0  #levels of "assign" to be dropped; 0= intercept
stemp <- untangle.specials(Terms, 'strata', 1)
if (length(stemp$vars) > 0) {  #if there is a strata statement
    hasinteractions <- FALSE
    for (i in stemp$vars) {  #multiple strata terms are allowed
        # The factors attr has one row for each variable in the frame, one
        #   col for each term in the model.  Pick rows for each strata
```

```
        #   var, and find if it participates in any interactions.
        if (any(attr(Terms, 'order')[attr(Terms, "factors")[i,] >0] >1))
            hasinteractions <- TRUE
        }
    if (!hasinteractions)
        dropterms <- c(dropterms, stemp$terms)
    else adrop <- c(0, match(stemp$var, colnames(attr(Terms, 'factors'))))
}

if (length(dropterms)) {
    temppred <- attr(terms, "predvars")
    Terms2 <- Terms[ -dropterms]
    if (!is.null(temppred)) {
        # subscripting a Terms object currently drops predvars, in error
        attr(Terms2, "predvars") <- temppred[-(1+dropterms)] # "Call" object
    }
    X <- model.matrix(Terms2, mf, constrasts=contrast.arg)
    # we want to number the terms wrt the original model matrix
    # Do not forget the intercept, which will be a zero
    renumber <- match(colnames(attr(Terms2, "factors")),
                      colnames(attr(Terms,  "factors")))
    attr(X, "assign") <- c(0, renumber)[1+attr(X, "assign")]
}
else X <- model.matrix(Terms, mf, contrasts=contrast.arg)

# drop the intercept after the fact, and also drop strata if necessary
Xatt <- attributes(X)
xdrop <- Xatt$assign %in% adrop  #columns to drop (always the intercept)
X <- X[, !xdrop, drop=FALSE]
attr(X, "assign") <- Xatt$assign[!xdrop]
#if (any(adrop>0)) attr(X, "contrasts") <- Xatt$contrasts[-adrop]
#else attr(X, "contrasts") <- Xatt$contrasts
attr(X, "contrasts") <- Xatt$contrasts
```

Finish the setup. If someone includes and init statement, make sure that it does not lead to instant code failure due to overflow/underflow.

⟨*coxph-setup*⟩=

```
offset <- model.offset(mf)
if (is.null(offset) | all(offset==0)) offset <- rep(0., nrow(mf))
else if (any(!is.finite(offset))) stop("offsets must be finite")

weights <- model.weights(mf)
if (!is.null(weights) && any(!is.finite(weights)))
    stop("weights must be finite")

assign <- attrassign(X, Terms)
```

```
 contr.save <- attr(X, "contrasts")
 if (missing(init)) init <- NULL
 else {
     if (length(init) != ncol(X)) stop(gettextf("wrong length for '%s' argument", "init"))
     temp <- X %*% init - sum(colMeans(X) * init)
     if (any(temp < .Machine$double.min.exp | temp > .Machine$double.max.exp))
         stop("initial values lead to overflow or underflow of the exp function")
 }
```

Check for penalized terms in the model, and set up infrastructure for the fitting routines to deal with them.

⟨*coxph-penal*⟩=

```
 pterms <- sapply(mf, inherits, 'coxph.penalty')
 if (any(pterms)) {
     pattr <- lapply(mf[pterms], attributes)
     pname <- names(pterms)[pterms]
     #
     # Check the order of any penalty terms
     ord <- attr(Terms, "order")[match(pname, attr(Terms, 'term.labels'))]
     if (any(ord>1)) stop('Penalty terms cannot be in an interaction')
     pcols <- assign[match(pname, names(assign))]

     fit <- coxpenal.fit(X, Y, strats, offset, init=init,
                         control,
                         weights=weights, method=method,
                         row.names(mf), pcols, pattr, assign)
 }
```

⟨*coxph-compute*⟩=

```
 else {
     if( method=="breslow" || method =="efron") {
         if (type== 'right')  fitter <- get("coxph.fit")
         else                 fitter <- get("agreg.fit")
     }
     else if (method=='exact') {
         if (type== "right")  fitter <- get("coxexact.fit")
         else  fitter <- get("agexact.fit")
     }
     else stop(gettextf("unknown '%s' method", method))

     fit <- fitter(X, Y, strats, offset, init, control, weights=weights,
                   method=method, row.names(mf))
 }
```

⟨*coxph-finish*⟩=

```
 if (is.character(fit)) {
```

```
    fit <- list(fail=fit)
    class(fit) <- 'coxph'
}
else {
    if (!is.null(fit$coefficients) && any(is.na(fit$coefficients))) {
       vars <- seq_along(fit$coefficients)[is.na(fit$coefficients)]
       msg <- gettextf("X matrix deemed to be singular; variable %s", paste(vars, collapse=" "))
       if (singular.ok) warning(msg)
       else             stop(msg)
    }
    fit$n <- data.n
    fit$nevent <- sum(Y[,ncol(Y)])
    fit$terms <- Terms
    fit$assign <- assign
    class(fit) <- fit$method

    if (robust) {
        fit$naive.var <- fit$var
        fit$method    <- method
        # a little sneaky here: by calling resid before adding the
        #   na.action method, I avoid having missings re-inserted
        # I also make sure that it doesn't have to reconstruct X and Y
        fit2 <- c(fit, list(x=X, y=Y, weights=weights))
        if (length(strats)) fit2$strata <- strats
        if (length(cluster)) {
            temp <- residuals.coxph(fit2, type='dfbeta', collapse=cluster,
                                    weighted=TRUE)
            # get score for null model
            if (is.null(init))
                    fit2$linear.predictors <- 0*fit$linear.predictors
            else fit2$linear.predictors <- c(X %*% init)
            temp0 <- residuals.coxph(fit2, type='score', collapse=cluster,
                                     weighted=TRUE)
    }
        else {
            temp <- residuals.coxph(fit2, type='dfbeta', weighted=TRUE)
            fit2$linear.predictors <- 0*fit$linear.predictors
            temp0 <- residuals.coxph(fit2, type='score', weighted=TRUE)
    }
        fit$var <- t(temp) %*% temp
        u <- apply(as.matrix(temp0), 2, sum)
        fit$rscore <- coxph.wtest(t(temp0)%*%temp0, u, control$toler.chol)$test
    }
    #Wald test
    if (length(fit$coefficients) && is.null(fit$wald.test)) {
        #not for intercept only models, or if test is already done
```

```
        nabeta <- !is.na(fit$coefficients)
        # The init vector might be longer than the betas, for a sparse term
        if (is.null(init)) temp <- fit$coefficients[nabeta]
        else temp <- (fit$coefficients -
                      init[1:length(fit$coefficients)])[nabeta]
        fit$wald.test <-  coxph.wtest(fit$var[nabeta,nabeta], temp,
                                      control$toler.chol)$test
    }
    na.action <- attr(mf, "na.action")
    if (length(na.action)) fit$na.action <- na.action
    if (model) {
        if (length(timetrans)) {
            # Fix up the model frame -- still in the thinking stage
            mf[[".surv."]]   <- Y
            mf[[".strata."]] <- strats
            stop("Time transform + model frame: code incomplete")
        }
        fit$model <- mf
    }
    if (x)  {
        fit$x <- X
        if (length(strats)) {
            if (length(timetrans)) fit$strata <- strats
            else     fit$strata <- strata.keep
        }
    }
    if (y)     fit$y <- Y
}
```

If any of the weights were not 1, save the results. Add names to the means component, which are occassionally useful to survfit.coxph. Other objects below are used when we need to recreate a model frame.

⟨*coxph-finish*⟩=

```
if (!is.null(weights) && any(weights!=1)) fit$weights <- weights
names(fit$means) <- names(fit$coefficients)

fit$formula <- formula(Terms)
if (length(xlevels) >0) fit$xlevels <- xlevels
fit$contrasts <- contr.save
if (any(offset !=0)) fit$offset <- offset
fit$call <- Call
fit$method <- method
fit
```

The model.matrix and model.frame routines are called after a Cox model to reconstruct those portions. Much of their code is shared with the coxph routine.

⟨*model.matrix.coxph*⟩=
```
 # In internal use "data" will often be an already derived model frame.
 #  We detect this via it having a terms attribute.
 model.matrix.coxph <- function(object, data=NULL,
                                contrast.arg=object$contrasts, ...) {
     #
     # If the object has an "x" component, return it, unless a new
     #   data set is given
     if (is.null(data) && !is.null(object[['x']]))
         return(object[['x']]) #don't match "xlevels"

     Terms <- delete.response(object$terms)
     if (is.null(data)) mf <- stats::model.frame(object)
     else {
         if (is.null(attr(data, "terms")))
             mf <- stats::model.frame(Terms, data, xlev=object$xlevels)
         else mf <- data  #assume "data" is already a model frame
     }

     cluster <- attr(Terms, "specials")$cluster
     if (length(cluster)) {
         temp <- untangle.specials(Terms, "cluster")
         dropterms <- temp$terms
     }
     else dropterms <- NULL

     ⟨coxph-make-X⟩
     X
 }
```

In parallel is the model.frame routine, which reconstructs the model frame. This routine currently doesn't do all that we want. To wit, the following code fails:

```
> tfun <- function(formula, ndata) {
      fit <- coxph(formula, data=ndata)
      model.frame(fit)
      }
> tfun(Surv(time, status) ~ age, lung)
Error: ndata not found
```

The genesis of this problem is hard to unearth, but has to do with non standard evaluation rules used by model.frame.default. In essence it pays attention to the environment of the formula, but the enclos argument of eval appears to be ignored. I've not yet found a solution.

⟨*model.matrix.coxph*⟩=
```
 model.frame.coxph <- function(formula, ...) {
     dots <- list(...)
```

```
nargs <- dots[match(c("data", "na.action", "subset", "weights"),
                    names(dots), 0)]
# If nothing has changed and the coxph object had a model component,
#   simply return it.
if (length(nargs) ==0  && !is.null(formula$model)) return(formula$model)
else {
    # Rebuild the original call to model.frame
    Terms <- terms(formula)
    fcall <- formula$call
    indx <- match(c("formula", "data", "weights", "subset", "na.action"),
              names(fcall), nomatch=0)
    if (indx[1] ==0) stop("The coxph call is missing a formula!")

    temp <- fcall[c(1,indx)]  # only keep the arguments we wanted
    temp[[1]] <- quote(stats::model.frame)  # change the function called
    temp$xlev <- formula$xlevels
    temp$formula <- Terms   #keep the predvars attribute
    # Now, any arguments that were on this call overtake the ones that
    #  were in the original call.
    if (length(nargs) >0)
        temp[names(nargs)] <- nargs

    # The documentation for model.frame implies that the environment arg
    #  to eval will be ignored, but if we omit it there is a problem.
    if (is.null(environment(formula$terms)))
        mf <- eval(temp, parent.frame())
    else mf <- eval(temp, environment(formula$terms), parent.frame())

    if (!is.null(attr(formula$terms, "dataClasses")))
        .checkMFClasses(attr(formula$terms, "dataClasses"), mf)

   if (!is.null(attr(Terms, "specials")$tt)) {
      # Do time transform
      tt <- eval(formula$call$tt)
      Y <- model.response(mf)
      strats <- attr(Terms, "specials")$strata
      if (length(strats)) {
          stemp <- untangle.specials(Terms, 'strata', 1)
          if (length(stemp$vars)==1) strata.keep <- mf[[stemp$vars]]
          else strata.keep <- strata(mf[,stemp$vars], shortlabel=TRUE)
          strats <- as.numeric(strata.keep)
      }

      ⟨coxph-transform⟩
      mf[[".strata."]] <- strats
   }
```

```
        mf
    }
}
```

3 Exact partial likelihood

Let $r_i = \exp(X_i\beta)$ be the risk score for observation i. For one of the time points assume that there that there are d tied deaths among n subjects at risk. For convenience we will index them as $i = 1, \ldots, d$ in the n at risk. Then for the exact parial likelihood, the contribution at this time point is

$$\begin{aligned} L &= \sum_{i=1}^{d} \log(r_i) - \log(D) \\ \frac{\partial L}{\partial \beta_j} &= x_{ij} - (1/D)\frac{\partial D}{\partial \beta_j} \\ \frac{\partial^2 L}{\partial \beta_j \partial \beta_k} &= (1/D^2)\left[D\frac{\partial^2 D}{\partial \beta_j \partial \beta_k} - \frac{\partial D}{\partial \beta_j}\frac{\partial D}{\partial \beta_k}\right] \end{aligned}$$

The hard part of this computation is D, which is a sum

$$D = \sum_{S(d,n)} r_{s_1} r_{s_2} \ldots r_{s_d}$$

where $S(d, n)$ is the set of all possible subsets of size d from n objects, and $s_1, s_2, \ldots$ indexes the current selection. So if $n = 6$ and $d = 2$ we would have the 15 pairs 12, 13, ... 56; for $n = 5$ and $d = 3$ there would be 10 triples 123, 124, 125, ..., 345.

The brute force computation of all subsets can take a very long time. Gail et al [**?**] show simple recursion formulas that speed this up considerably. Let $D(d, n)$ be the denominator with d deaths and n subjects. Then

$$D(d,n) = r_n D(d-1, n-1) + D(d, n-1) \tag{1}$$

$$\frac{\partial D(d,n)}{\partial \beta_j} = \frac{\partial D(d,n-1)}{\partial \beta_j} + r_n \frac{\partial D(d-1,n-1)}{\partial \beta_j} + x_{nj} r_n D(d-1, n-1) \tag{2}$$

$$\begin{aligned} \frac{\partial^2 D(d,n}{\partial \beta_j \partial \beta_k} = \frac{\partial^2 D(d,n-1)}{\partial \beta_j \partial \beta_k} + r_n \frac{\partial^2 D(d-1,n-1)}{\partial \beta_j \partial \beta_k} + x_{nj} r_n \frac{\partial D(d-1,n-1)}{\partial \beta_k} + \\ x_{nk} r_n \frac{\partial D(d-1,n-1)}{\partial \beta_j} + x_{nj} x_{nk} r_n D(d-1, n-1) \end{aligned} \tag{3}$$

The above recursion is captured in the three routines below. The first calculates D. It is called with d, n, an array that will contain all the values of $D(d, n)$ computed so far, and the the first dimension of the array. The intial condition $D(0, n) = 1$ is important to all three routines.

⟨*excox-recur*⟩=

```
double coxd0(int d, int n, double *score, double *dmat,
```

```
             int dmax) {
    double *dn;

    if (d==0) return(1.0);
    dn = dmat + (n-1)*dmax + d -1;  /* pointer to dmat[d,n] */

    if (*dn ==0) {  /* still to be computed */
        *dn = score[n-1]* coxd0(d-1, n-1, score, dmat, dmax);
            if (d<n) *dn += coxd0(d, n-1, score, dmat, dmax);
    }
    return(*dn);
}
```

The next routine calculates the derivative with respect to a particular coefficient. It will be called once for each covariate with d1 pointing to the work array for that covariate. The second derivative calculation is per pair of variables; the `d1j` and `d1k` arrays are the appropriate first derivative arrays of saved values. It is possible for the first derivative to be exactly 0 (if all values of the covariate are 0 for instance) in which case we may recalculate the derivative for a particular (d,n) case multiple times unnecessarily, since we are using value=0 as a marker for "not yet computed". This case is essentially nonexistent in real data, however.

⟨*excox-recur*⟩=

```
double coxd1(int d, int n, double *score, double *dmat, double *d1,
             double *covar, int dmax) {
    int indx;

    indx = (n-1)*dmax + d -1;  /*index to the current array member d1[d.n]*/
    if (d1[indx] ==0) { /* still to be computed */
        d1[indx] = score[n-1]* covar[n-1]* coxd0(d-1, n-1, score, dmat, dmax);
        if (d<n) d1[indx] += coxd1(d, n-1, score, dmat, d1, covar, dmax);
        if (d>1) d1[indx] += score[n-1]*
                         coxd1(d-1, n-1, score, dmat, d1, covar, dmax);
    }
    return(d1[indx]);
}

double coxd2(int d, int n, double *score, double *dmat, double *d1j,
             double *d1k, double *d2, double *covarj, double *covark,
             int dmax) {
    int indx;

    indx = (n-1)*dmax + d -1;  /*index to the current array member d1[d,n]*/
    if (d2[indx] ==0) { /*still to be computed */
        d2[indx] = coxd0(d-1, n-1, score, dmat, dmax)*score[n-1] *
            covarj[n-1]* covark[n-1];
        if (d<n) d2[indx] += coxd2(d, n-1, score, dmat, d1j, d1k, d2, covarj,
```

```
                                covark, dmax);
        if (d>1) d2[indx] += score[n-1] * (
            coxd2(d-1, n-1, score, dmat, d1j, d1k, d2, covarj, covark, dmax) +
            covarj[n-1] * coxd1(d-1, n-1, score, dmat, d1k, covark, dmax) +
            covark[n-1] * coxd1(d-1, n-1, score, dmat, d1j, covarj, dmax));
        }
    return(d2[indx]);
}
```

Now for the main body. Start with the dull part of the code: declarations. I use `maxiter2` for the S structure and `maxiter` for the variable within it, and etc for the other input arguments. All the input arguments except strata are read-only. The output beta vector starts as a copy of ibeta.

⟨*coxexact*⟩=

```
#include <math.h>
#include "survS.h"
#include "survproto.h"
#include <R_ext/Utils.h>

⟨excox-recur⟩

SEXP coxexact(SEXP maxiter2,  SEXP y2,
              SEXP covar2,    SEXP offset2, SEXP strata2,
              SEXP ibeta,     SEXP eps2,    SEXP toler2) {
    int i,j,k;
    int     iter;

    double **covar, **imat;  /*ragged arrays */
    double *time, *status;   /* input data */
    double *offset;
    int    *strata;
    int    sstart;   /* starting obs of current strata */
    double *score;
    double *oldbeta;
    double  zbeta;
    double  newlk=0;
    double  temp;
    int     halving;    /*are we doing step halving at the moment? */
    int     nrisk;   /* number of subjects in the current risk set */
    int dsize,       /* memory needed for one coxc0, coxc1, or coxd2 array */
        dmemtot,     /* amount needed for all arrays */
        ndeath;      /* number of deaths at the current time point */
    double maxdeath;    /* max tied deaths within a strata */

    double dtime;    /* time value under current examiniation */
    double *dmem0, **dmem1, *dmem2; /* pointers to memory */
```

```
    double *dtemp;   /* used for zeroing the memory */
    double *d1;     /* current first derivatives from coxd1 */
    double d0;      /* global sum from coxc0 */

    /* copies of scalar input arguments */
    int     nused, nvar, maxiter;
    double  eps, toler;

    /* returned objects */
    SEXP imat2, beta2, u2, loglik2;
    double *beta, *u, *loglik;
    SEXP rlist, rlistnames;
    int nprotect;  /* number of protect calls I have issued */

    ⟨excox-setup⟩
    ⟨excox-strata⟩
    ⟨excox-iter0⟩
    ⟨excox-iter⟩
    }
```

Setup is ordinary. Grab S objects and assign others. I use `R_alloc` for temporary ones since it is released automatically on return.

```
⟨excox-setup⟩=
 nused = LENGTH(offset2);
 nvar  = ncols(covar2);
 maxiter = asInteger(maxiter2);
 eps  = asReal(eps2);     /* convergence criteria */
 toler = asReal(toler2);  /* tolerance for cholesky */

 /*
 **  Set up the ragged array pointer to the X matrix,
 **    and pointers to time and status
 */
 covar= dmatrix(REAL(covar2), nused, nvar);
 time = REAL(y2);
 status = time +nused;
 strata = INTEGER(PROTECT(duplicate(strata2)));
 offset = REAL(offset2);

 /* temporary vectors */
 score = (double *) R_alloc(nused+nvar, sizeof(double));
 oldbeta = score + nused;

 /*
 ** create output variables
 */
```

```
PROTECT(beta2 = duplicate(ibeta));
beta = REAL(beta2);
PROTECT(u2 = allocVector(REALSXP, nvar));
u = REAL(u2);
PROTECT(imat2 = allocVector(REALSXP, nvar*nvar));
imat = dmatrix(REAL(imat2),  nvar, nvar);
PROTECT(loglik2 = allocVector(REALSXP, 5)); /* loglik, sctest, flag,maxiter*/
loglik = REAL(loglik2);
nprotect = 5;
```

The data passed to us has been sorted by strata, and reverse time within strata (longest subject first). The variable `strata` will be 1 at the start of each new strata. Separate strata are completely separate computations: time 10 in one strata and time 10 in another are not comingled. Compute the largest product (size of strata)* (max tied deaths in strata) for allocating scratch space. When computing D it is advantageous to create all the intermediate values of $D(d, n)$ in an array since they will be used in the derivative calculation. Likewise, the first derivatives are used in calculating the second. Even more importantly, say we have a large data set. It will be sorted with the longest times first. If there is a death with 30 at risk and another with 40 at risk, the intermediate sums we computed for the n=30 case are part of the computation for n=40. To make this work we need to index our matrices, within any strata, by the maximum number of tied deaths in the strata. We save this in the strata variable: first obs of a new strata has the number of events. And what if a strata had 0 events? We mark it with a 1.

Note that the maxdeath variable is floating point. I had someone call this routine with a data set that gives an integer overflow in that situation. We now keep track of this further below and fail with a message. Such a run would take longer than forever to complete even if integer subscripts did not overflow.

⟨*excox-strata*⟩=
```
strata[0] =1;  /* in case the parent forgot */
temp = 0;      /* temp variable for dsize */

maxdeath =0;
j=0;   /* start of the strata */
for (i=0; i<nused;) {
  if (strata[i]==1) { /* first obs of a new strata */
      if (i>0) {
          /* If maxdeath <2 leave the strata alone at it's current value of 1 */
          if (maxdeath >1) strata[j] = maxdeath;
          j = i;
          if (maxdeath*nrisk > temp) temp = maxdeath*nrisk;
          }
      maxdeath =0;  /* max tied deaths at any time in this strata */
      nrisk=0;
      ndeath =0;
      }
  dtime = time[i];
```

```
  ndeath =0;  /*number tied here */
  while (time[i] ==dtime) {
      nrisk++;
      ndeath += status[i];
      i++;
      if (i>=nused || strata[i] >0) break;  /*tied deaths don't cross strata */
      }
  if (ndeath > maxdeath) maxdeath=ndeath;
  }
if (maxdeath*nrisk > temp) temp = maxdeath*nrisk;
if (maxdeath >1) strata[j] = maxdeath;

/* Now allocate memory for the scratch arrays
   Each per-variable slice is of size dsize
*/
dsize = temp;
temp    = temp * ((nvar*(nvar+1))/2 + nvar + 1);
dmemtot = dsize * ((nvar*(nvar+1))/2 + nvar + 1);
if (temp != dmemtot) { /* the subscripts will overflow */
    error("(number at risk) * (number tied deaths) is too large");
}
dmem0 = (double *) R_alloc(dmemtot, sizeof(double)); /*pointer to memory */
dmem1 = (double **) R_alloc(nvar, sizeof(double*));
dmem1[0] = dmem0 + dsize; /*points to the first derivative memory */
for (i=1; i<nvar; i++) dmem1[i] = dmem1[i-1] + dsize;
d1 = (double *) R_alloc(nvar, sizeof(double)); /*first deriv results */
```

Here is a standard iteration step. Walk forward to a new time, then through all the ties with that time. If there are any deaths, the contributions to the loglikilihood, first, and second derivatives at this time point are

$$L = \left(\sum_{i \in deaths} X_i \beta \right) - \log(D) \tag{4}$$

$$\frac{\partial L}{\partial \beta_j} = \left(\sum_{i \in deaths} X_{ij} \right) - \frac{\partial D(d,n)}{\partial \beta_j} D^{-1}(d,n) \tag{5}$$

$$\frac{\partial^2 L}{\partial \beta_j \partial \beta_k} = \frac{\partial^2 D(d,n)}{\partial \beta_j \partial \beta_k} D^{-1}(d,n) - \frac{\partial D(d,n)}{\partial \beta_j} \frac{\partial D(d,n)}{\partial \beta_k} D^{-2}(d,n) \tag{6}$$

Even the efficient calculation can be compuatationally intense, so check for user interrupt requests on a regular basis.

⟨*excox-addup*⟩=
```
 for (i=0; i<nused; ) {
     if (strata[i] >0) { /* first obs of a new strata */
         maxdeath= strata[i];
```

```
        dtemp = dmem0;
        for (j=0; j<dmemtot; j++) *dtemp++ =0.0;
        sstart =i;
        nrisk =0;
    }

    dtime = time[i];  /*current unique time */
    ndeath =0;
    while (time[i] == dtime) {
        zbeta= offset[i];
        for (j=0; j<nvar; j++) zbeta += covar[j][i] * beta[j];
        score[i] = exp(zbeta);
        if (status[i]==1) {
            newlk += zbeta;
            for (j=0; j<nvar; j++) u[j] += covar[j][i];
            ndeath++;
        }
        nrisk++;
        i++;
        if (i>=nused || strata[i] >0) break;
    }

    /* We have added up over the death time, now process it */
    if (ndeath >0) { /* Add to the loglik */
        d0 = coxd0(ndeath, nrisk, score+sstart, dmem0, maxdeath);
        R_CheckUserInterrupt();
        newlk -= log(d0);
        dmem2 = dmem0 + (nvar+1)*dsize;  /*start for the second deriv memory */
        for (j=0; j<nvar; j++) { /* for each covariate */
            d1[j] = coxd1(ndeath, nrisk, score+sstart, dmem0, dmem1[j],
                          covar[j]+sstart, maxdeath) / d0;
            if (ndeath > 3) R_CheckUserInterrupt();
            u[j] -= d1[j];
            for (k=0; k<= j; k++) {  /* second derivative*/
                temp = coxd2(ndeath, nrisk, score+sstart, dmem0, dmem1[j],
                             dmem1[k], dmem2, covar[j] + sstart,
                             covar[k] + sstart, maxdeath);
                if (ndeath > 5) R_CheckUserInterrupt();
                imat[k][j] += temp/d0 - d1[j]*d1[k];
                dmem2 += dsize;
            }
        }
    }
}
```

Do the first iteration of the solution. The first iteration is different in 3 ways: it is used to

set the initial log-likelihood, to compute the score test, and we pay no attention to convergence criteria or diagnositics. (I expect it not to converge in one iteration).

⟨*excox-iter0*⟩=
```
 /*
 ** do the initial iteration step
 */
 newlk =0;
 for (i=0; i<nvar; i++) {
     u[i] =0;
     for (j=0; j<nvar; j++)
         imat[i][j] =0 ;
     }
```
⟨*excox-addup*⟩

```
 loglik[0] = newlk;   /* save the loglik for iteration zero  */
 loglik[1] = newlk;  /* and it is our current best guess */
 /*
 **   update the betas and compute the score test
 */
 for (i=0; i<nvar; i++) /*use 'd1' as a temp to save u0, for the score test*/
     d1[i] = u[i];

 loglik[3] = cholesky2(imat, nvar, toler);
 chsolve2(imat,nvar, u);        /* u replaced by  u *inverse(imat) */

 loglik[2] =0;                  /* score test stored here */
 for (i=0; i<nvar; i++)
     loglik[2] +=  u[i]*d1[i];

 if (maxiter==0) {
     iter =0;  /*number of iterations */
     ⟨excox-finish⟩
     }

 /*
 **  Never, never complain about convergence on the first step.  That way,
 **  if someone has to they can force one iter at a time.
 */
 for (i=0; i<nvar; i++) {
     oldbeta[i] = beta[i];
     beta[i] = beta[i] + u[i];
     }
```

Now the main loop. This has code for convergence and step halving. Be careful about order. For our current guess at the solution beta:

1. Compute the loglik, first, and second derivatives
2. If the loglik has converged, return beta and information just computed for this beta (loglik, derivatives, etc). Don't update beta.
3. If not converged
   - If The loglik got worse try beta= (beta + oldbeta)/2
   - Otherwise update beta

```
⟨excox-iter⟩=
 halving =0 ;              /* =1 when in the midst of "step halving" */
 for (iter=1; iter<=maxiter; iter++) {
     newlk =0;
     for (i=0; i<nvar; i++) {
         u[i] =0;
         for (j=0; j<nvar; j++)
                 imat[i][j] =0;
         }
     ⟨excox-addup⟩

     /* am I done?
     **   update the betas and test for convergence
     */
     loglik[3] = cholesky2(imat, nvar, toler);

     if (fabs(1-(loglik[1]/newlk))<= eps && halving==0) { /* all done */
         loglik[1] = newlk;
        ⟨excox-finish⟩
         }

     if (iter==maxiter) break;  /*skip the step halving and etc */

     if (newlk < loglik[1])   {    /*it is not converging ! */
             halving =1;
             for (i=0; i<nvar; i++)
                 beta[i] = (oldbeta[i] + beta[i]) /2; /*half of old increment */
             }
     else {
             halving=0;
             loglik[1] = newlk;
             chsolve2(imat,nvar,u);

             for (i=0; i<nvar; i++) {
                 oldbeta[i] = beta[i];
                 beta[i] = beta[i] +  u[i];
                 }
```

```
            }
    }   /* return for another iteration */


 /*
 ** Ran out of iterations
 */
 loglik[1] = newlk;
 loglik[3] = 1000;  /* signal no convergence */
 ⟨excox-finish⟩
```

The common code for finishing. Invert the information matrix, copy it to be symmetric, and put together the output structure.

```
⟨excox-finish⟩=
 loglik[4] = iter;
 chinv2(imat, nvar);
 for (i=1; i<nvar; i++)
     for (j=0; j<i; j++)  imat[i][j] = imat[j][i];

 /* assemble the return objects as a list */
 PROTECT(rlist= allocVector(VECSXP, 4));
 SET_VECTOR_ELT(rlist, 0, beta2);
 SET_VECTOR_ELT(rlist, 1, u2);
 SET_VECTOR_ELT(rlist, 2, imat2);
 SET_VECTOR_ELT(rlist, 3, loglik2);

 /* add names to the list elements */
 PROTECT(rlistnames = allocVector(STRSXP, 4));
 SET_STRING_ELT(rlistnames, 0, mkChar("coef"));
 SET_STRING_ELT(rlistnames, 1, mkChar("u"));
 SET_STRING_ELT(rlistnames, 2, mkChar("imat"));
 SET_STRING_ELT(rlistnames, 3, mkChar("loglik"));
 setAttrib(rlist, R_NamesSymbol, rlistnames);

 unprotect(nprotect+2);
 return(rlist);
```

3.1 Anderson-Gill fits

When the survival data set has (start, stop] data a couple of computational issues are added. A primary one is how to do this compuation efficiently. At each event time we need to compute 3 quantities, each of them added up over the current risk set.

- The weighted sum of the risk scores $\sum w_i r_i$ where $r_i = \exp(\eta_i)$ and $\eta_i = x_{i1}\beta_1 + x_{i2}\beta_2 + \ldots$ is the current linear predictor.

- The weighted mean of the covariates x, with weight $w_i r_i$.
- The weighted variance-covariance matrix of x.

The current risk set at some event time t is the set of all (start, stop] intervals that overlap t, and are part of the same strata. The round/square brackets in the prior sentence are important: for an event time $t = 20$ the interval $(5, 20]$ is considered to overlap t and the interval $(20, 55]$ does not overlap t.

Our routine for the simple right censored Cox model computes these efficiently by keeping a cumulative sum. Starting with the longest survival move backwards through time, adding and subtracting subject from the sum as we go. The code below creates two sort indices, one orders the data by reverse stop time and the other by reverse start time, each within strata. For the first events are sorted before censors for a computational reason detailed later.

The fit routine is called by the coxph function with arguments

x matrix of covariates

y three column matrix containing the start time, stop time, and event for each observation

strata for stratified fits, the strata of each subject

offset the offset, usually a vector of zeros

init initial estimate for the coefficients

control results of the coxph.control function

weights case weights, often a vector of ones.

method how ties are handled: 1=Breslow, 2=Efron

rownames used to label the residuals

⟨*agreg.fit*⟩=

```
agreg.fit <- function(x, y, strata, offset, init, control,
                      weights, method, rownames)
    {
    n <- nrow(y)
    nvar <- ncol(x)
    start <- y[,1]
    stopp <- y[,2]
    event <- y[,3]
    if (all(event==0)) stop("Can't fit a Cox model with 0 failures")

    # Sort the data (or rather, get a list of sorted indices)
    #  For both stop and start times, the indices go from last to first
    if (length(strata)==0) {
        sort.end  <- order(-stopp) -1L #indices start at 0 for C code
        sort.start<- order(-start) -1L
        newstrat  <- n
```

```
    }
else {
    sort.end  <- order(strata, -stopp) -1L
    sort.start<- order(strata, -start) -1L
    newstrat  <- cumsum(table(strata))
    }
if (missing(offset) || is.null(offset)) offset <- rep(0.0, n)
if (missing(weights)|| is.null(weights))weights<- rep(1.0, n)
else if (any(weights<=0)) stop("Invalid weights, must be >0")
else weights <- as.vector(weights)

if (is.null(nvar) || nvar==0) {
    # A special case: Null model.  Just return obvious stuff
    #  To keep the C code to a small set, we call the usual routines, but
    #  with a dummy X matrix and 0 iterations
    nvar <- 1
    x <- matrix(as.double(1:n), ncol=1)  #keep the .C call happy
    maxiter <- 0
    nullmodel <- TRUE
    if (length(init) !=0) stop("Wrong length for inital values")
    init <- 0.0  #dummy value to keep a .C call happy (doesn't like 0 length)
    }
else {
    nullmodel <- FALSE
    maxiter <- control$iter.max

    if (is.null(init)) init <- rep(0., nvar)
    if (length(init) != nvar) stop("Wrong length for inital values")
    }

# the returned value of agfit$coef starts as a copy of init, so make sure
#  is is a vector and not a matrix; as.double does so.
# Solidify the storage mode of other arguments
storage.mode(y) <- storage.mode(x) <- "double"
storage.mode(offset) <- storage.mode(weights) <- "double"
storage.mode(newstrat) <- "integer"
agfit <- .Call(Cagfit4,
               y, x, newstrat, weights,
               offset,
               as.double(init),
               sort.start, sort.end,
               as.integer(method=="efron"),
               as.integer(maxiter),
               as.double(control$eps),
               as.double(control$toler.chol),
               as.integer(1)) # internally rescale
```

```
    ⟨agreg-fixup⟩
    ⟨agreg-finish⟩
}
```

Upon return we need to clean up two simple things. The first is that if any of the covariates were redudant then this will be marked by zeros on the diagonal of the variance matrix. Replace these coefficients and their variances with NA. The second is to post a warning message about possible infinite coefficients. The algorithm for determining this is unreliable, unfortunately. Sometimes coefficients are marked as infinite when the solution is not tending to infinity (usually associated with a very skewed covariate), and sometimes one that is tending to infinity is not marked. Que sera sera.

```
⟨agreg-fixup⟩=
 var <- matrix(agfit$imat,nvar,nvar)
 coef <- agfit$coef
 if (agfit$flag[1] < nvar) which.sing <- diag(var)==0
 else which.sing <- rep(FALSE,nvar)

 infs <- abs(agfit$u %*% var)
 if (maxiter >1) {
     if (agfit$iter > maxiter)
         warning("Ran out of iterations and did not converge")
     else {
         infs <- ((infs > control$eps) &
                  infs > control$toler.inf*abs(coef))
         if (any(infs))
             warning(gettextf("Loglik converged before variable %s; beta may be infinite.",
                         paste(seq_len(nvar)[infs], collapse = ", ")))
     }
 }
```

The last of the code is very standard. Compute residuals and package up the results.

```
⟨agreg-finish⟩=
 lp  <- as.vector(x %*% coef + offset - sum(coef * colMeans(x)))
 score <- as.double(exp(lp))
 resid <- .Call(Cagmart3,
                y, score, weights,
                newstrat,
                cbind(sort.end, sort.start),
                as.integer(method=='efron'))
 names(resid) <- rownames
 if (nullmodel) {
     list(loglik=agfit$loglik[2],
          linear.predictors = offset,
          residuals = resid,
```

```
        method= c("coxph.null", 'coxph') )
}
else {
    names(coef) <- dimnames(x)[[2]]
    if (maxiter > 0) coef[which.sing] <- NA  # always leave iter=0 alone
    flag <- agfit$flag
    names(flag) <- c("rank", "rescale", "step halving")

    concordance <- survConcordance.fit(y, lp, strata, weights)
    list(coefficients  = coef,
         var    = var,
         loglik = agfit$loglik,
         score  = agfit$sctest,
         iter   = agfit$iter,
         linear.predictors = as.vector(lp),
         residuals = resid,
         means = colMeans(x),
         concordance = concordance,
         first = agfit$u,
         info = flag,
         method= 'coxph')
}
```

The details of the C code contain the more challenging part of the computations. It starts with the usual dull stuff. My standard coding style for a variable zed to to use `zed2` as the variable name for the R object, and `zed` for the pointer to the contents of the object, i.e., what the C code will manipulate. For the matrix objects I make use of ragged arrays, this allows for reference to the i,j element as `cmat[i][j]` and makes for more readable code.

⟨*agfit4*⟩=
```
#include <math.h>
#include "survS.h"
#include "survproto.h"

SEXP agfit4(SEXP surv2,      SEXP covar2,    SEXP strata2,
            SEXP weights2,   SEXP offset2,   SEXP ibeta2,
            SEXP sort12,     SEXP sort22,    SEXP method2,
            SEXP maxiter2,   SEXP  eps2,     SEXP tolerance2,
            SEXP doscale2) {

    int i,j,k, person;
    int indx1, istrat, p, p1;
    int nrisk;
    int nused, nvar;
    int rank, rank2, fail;

    double **covar, **cmat, **imat;  /*ragged array versions*/
```

```
double *a, *oldbeta;
double *scale;
double *a2, **cmat2;
double *eta;
double  denom, zbeta, risk;
double  dtime;
double  temp, temp2;
double  newlk =0;
int     halving;    /*are we doing step halving at the moment? */
double  tol_chol, eps;
double  meanwt;
int deaths;
double denom2, etasum;
int *keep;               /* marker for useless obs */

/* inputs */
double *start, *tstop, *event;
double *weights, *offset;
int *sort1, *sort2, maxiter;
int *strata, nstrat;
double method;  /* saving this as double forces some double arithmetic */
int doscale;

/* returned objects */
SEXP imat2, beta2, u2, loglik2;
double *beta, *u, *loglik;
SEXP sctest2, flag2, iter2;
double *sctest;
int *flag, *iter;
SEXP rlist;
static const char *outnames[]={"coef", "u", "imat", "loglik",
                               "sctest", "flag", "iter", ""};
int nprotect;  /* number of protect calls I have issued */

/* get sizes and constants */
nused = nrows(covar2);
nvar  = ncols(covar2);
method= asInteger(method2);
eps   = asReal(eps2);
tol_chol = asReal(tolerance2);
maxiter = asInteger(maxiter2);
doscale = asInteger(doscale2);
nstrat = LENGTH(strata2);

/* input arguments */
start = REAL(surv2);
```

```
tstop  = start + nused;
event = tstop + nused;
weights = REAL(weights2);
offset = REAL(offset2);
sort1  = INTEGER(sort12);
sort2  = INTEGER(sort22);
strata = INTEGER(strata2);

/*
** scratch space
**  nvar: a, a2, oldbeta, scale
**  nvar*nvar: cmat, cmat2
**  nused:  eta, keep
*/
eta = (double *) R_alloc(nused + 4*nvar + 2*nvar*nvar, sizeof(double));
a = eta + nused;
a2= a + nvar;
scale  = a2 + nvar;
oldbeta = scale + nvar;
keep = (int *) R_alloc(nused, sizeof(int));

/*
**  Set up the ragged arrays
**  covar2 might not need to be duplicated, even though
**  we are going to modify it, due to the way this routine was
**  was called.  In this case NAMED(covar2) will =0
*/
PROTECT(imat2 = allocVector(REALSXP, nvar*nvar));
nprotect =1;
if (NAMED(covar2)>0) {
    PROTECT(covar2 = duplicate(covar2));
    nprotect++;
    }
covar= dmatrix(REAL(covar2), nused, nvar);
imat = dmatrix(REAL(imat2),  nvar, nvar);
cmat = dmatrix(oldbeta+ nvar,   nvar, nvar);
cmat2= dmatrix(oldbeta+ nvar + nvar*nvar, nvar, nvar);

/*
** create the output structures
*/
PROTECT(rlist = mkNamed(VECSXP, outnames));
nprotect++;
beta2 = SET_VECTOR_ELT(rlist, 0, duplicate(ibeta2));
beta  = REAL(beta2);
u2 =    SET_VECTOR_ELT(rlist, 1, allocVector(REALSXP, nvar));
```

```
u = REAL(u2);

SET_VECTOR_ELT(rlist, 2, imat2);
loglik2 = SET_VECTOR_ELT(rlist, 3, allocVector(REALSXP, 2));
loglik  = REAL(loglik2);

sctest2 = SET_VECTOR_ELT(rlist, 4, allocVector(REALSXP, 1));
sctest =  REAL(sctest2);
flag2  =  SET_VECTOR_ELT(rlist, 5, allocVector(INTSXP, 3));
flag   =  INTEGER(flag2);
for (i=0; i<3; i++) flag[i]=0;

iter2  =  SET_VECTOR_ELT(rlist, 6, allocVector(INTSXP, 1));
iter = INTEGER(iter2);

/*
** Subtract the mean from each covar, as this makes the variance
**  computation much more stable.  The mean is taken per stratum,
**  the scaling is overall.
*/
if (nvar==1) doscale =0;  /* scaling has no impact, so skip it */
for (i=0; i<nvar; i++) {
    person=0;
    for (istrat=0; istrat<nstrat; istrat++) {
        temp=0;
        temp2 =0;
        for (k=person; k<strata[istrat]; k++) {
            j = sort2[k];
            temp += weights[j] * covar[i][j];
            temp2 += weights[j];
        }
        temp /= temp2;   /* mean for this covariate, this strata */
        for (; person< strata[istrat]; person++) {
            j = sort2[person];
            covar[i][j] -=temp;
        }
    }
    if (doscale ==1) { /* also scale the regression */
        /* this cannot be done per stratum */
        temp =0;
        temp2 =0;
        for (person=0; person<nused; person++) {
            temp += weights[person] * fabs(covar[i][person]);
            temp2 += weights[person];
            }
        if (temp >0) temp = temp2/temp;  /* 1/scale */
```

```
            else temp = 1.0;  /* rare case of a constant covariate */
            scale[i] = temp;
            for (person=0; person<nused; person++) {
                covar[i][person] *= temp;
            }
        }
    }

    if (doscale ==1) {
        for (i=0; i<nvar; i++) beta[i] /= scale[i]; /* rescale initial betas */
        }
    else {for (i=0; i<nvar; i++) scale[i] = 1.0;}
    ⟨agfit4-toss⟩
    ⟨agfit4-iter⟩
    ⟨agfit4-finish⟩
}
```

As we walk through the risk sets observations are both added and removed from a set of running totals. We have 6 running totals:

- sum of the weights, denom = $\sum w_i r_i$
- totals for each covariate a[j] = $\sum w_i r_i x_{ij}$
- totals for each covariate pair cmat[j,k]= $\sum w_i r_i x_{ij} x_{ik}$
- the same three quantities, but only for times that are exactly tied with the current death time, named denom2, a2, cmat2. This allows for easy compuatation of the Efron approximation for ties.

We have to be careful to never subtract out an observation before it is added in, as the 'number at risk' counter could become zero when it really should not be so; certain subtotals would then be inappropriately zeroed. The algorithm moves forward to the next unique ending time, removes old observations, and then adds new ones. Observations that are not part of any risk set add unnecessary noise since they will be added and then subtracted from all the totals, but the intermediate values are never used. If said observation had a large risk score this could be exceptionally bad. We do a first pass to mark them in the `keep` vector. For most sensible input all the elements of `keep` will be 1= true, but survSplit can create observations that are not used.

⟨*agfit4-toss*⟩=
```
 indx1 =0;
 person =0;
 for (k=0; k<nused; k++) keep[k] =1;
 for (istrat=0; istrat<nstrat; istrat++) {
    while(person < strata[istrat]) {
        /* find the next death */
        for (k=person; k< strata[istrat]; k++) {
```

```
            p = sort2[k];
            if (event[p] ==1) {
                dtime = tstop[p];
                break;
            }
        }
        if (k== strata[istrat]) {
            /* no more deaths in this strata */
            person = k;
            indx1 =k;  /* we can move on */
        }

        for (; indx1 < strata[istrat]; indx1++) {
            p1 = sort1[indx1];
            if (start[p1] < dtime) break;
            keep[p1]--;
        }
        for (; person < strata[istrat]; person++) {
            p = sort2[person];
            if (tstop[p] < dtime) break;
            if (keep[p] ==1) keep[p] =2;
        }
    }
}
```

At one point I spent a lot of time worrying about r_i values that are too large, but it turns out that the overall scale of the weights does not really matter since they always appear as a ratio. (Assuming we avoid exponential overflow and underflow, of course.) What does get the code in trouble is when there are large and small weights and we get an update of (large + small) - large. For example suppose a data set has a time dependent covariate which grows with time and the data has values like below:

time1	time2	status	x
0	90	1	1
0	105	0	2
100	120	1	50
100	124	0	51

The code moves from large times to small, so the first risk set has subjects 3 and 4, the second has 1 and 2. The original code would do removals only when necessary, i.e., at the event times of 120 and 90, and additions as they came along. This leads to adding in subjects 1 and 2 before the update at time 90 when observations 3 and 4 are removed; for a coefficient greater than about .6 this leads to a loss of all of the significant digits. The defense is to remove subjects from the risk set as early as possible, and defer additions for as long as possible. Every time we hit a new (unique) death time, and only then, update the totals: first remove any old observations no longer in the risk set and then add any new ones.

The three primary quantities for the Cox model are the log-likelihood L, the score vector U and the Hessian matrix H.

$$\begin{aligned}
L &= \sum_i w_i \delta_i \left[\eta_i - \log(d(t))\right] \\
d(t) &= \sum_j w_j r_j Y_j(t) \\
U_k &= \sum_i w_i \delta_i \left[(X_{ik} - \mu_k(t_i))\right] \\
\mu_k(t) &= \frac{\sum_j w_j r_j Y_j(t) X_{jk}}{d(t)} \\
H_{kl} &= \sum_i w_i \delta_i V_{kl}(t_i) \\
V_{kl}(t) &= \frac{\sum_j w_j r_j Y_j(t)[X_{jk} - \mu_k(t)][X_{jl} - \mu_l(t)]}{d(t)} \\
&= \frac{\sum_j w_j r_j Y_j(t) X_{jk} X_{jl}}{d(t)} - d(t)\mu_k(t)\mu_l(t)
\end{aligned}$$

In the above $\delta_i = 1$ for an event and 0 otherwise, w_i is the per subject weight, η_i is the current linear predictor $X\beta$ for the subject, $r_i = \exp(\eta_i)$ is the risk score and $Y_i(t)$ is 1 if observation i is at risk at time t. The vector $\mu(t)$ is the weighted mean of the covariates at time t using a weight of $wrY(t)$ for each subject, and $V(t)$ is the weighted variance matrix of X at time t.

Tied deaths and the Efron approximation add a small complication to the formula. Say there are three tied deaths at some particular time t. When calculating the denominator $d(t)$, mean $\mu(t)$ and variance $V(t)$ at that time the inclusion value $Y_i(t)$ is 0 or 1 for all other subjects, as usual, but for the three tied deaths Y(t) is taken to be 1 for the first death, 2/3 for the second, and 1/3 for the third. The idea is that if the tied death times were randomly broken by adding a small random amount then each of these three would be in the first risk set, have 2/3 chance of being in the second, and 1/3 chance of being in the risk set for the third death. In the code this means that at a death time we add the `denom2`, `a2` and `c2` portions in a little at at time: for three tied death the code will add in 1/3, update totals, add in another 1/3, update totals, then the last 1/3, and update totals.

The variance formula is stable if μ is small relative to the total variance. This is guarranteed by having a working estimate m of the mean along with the formula:

$$\begin{aligned}
(1/n)\sum w_i r_i (x_i - \mu)^2 &= (1/n)\sum w_i r_i (x - m)^2 - (\mu - m)^2 \\
\mu &= (1/n)\sum w_i r_i (x_i - m) \\
n &= \sum w_i r_i
\end{aligned}$$

A refinement of this is to scale the covariates, since the Cholesky decomposition can lose precision when variables are on vastly different scales. We do this centering and scaling once at the beginning of the calculation. Centering is done per strata — what if someone had two strata and a covariate with mean 0 in the first but mean one million in the second? (Users do amazing

things). Scaling is required to be a single value for each covariate, however. For a univariate model scaling does not add any precision.

Weighted sums can still be unstable if the weights get out of hand. Because of the exponential $r_i = exp(\eta_i)$ the original centering of the X matrix may not be enough. A particular example was a data set on hospital adverse events with "number of nurse shift changes to date" as a time dependent covariate. At any particular time point the covariate varied only by ± 3 between subjects (weekends often use 12 hour nurse shifts instead of 8 hour). The regression coefficient was around 1 and the data duration was 11 weeks (about 200 shifts) so that *eta* values could be over 100 even after centering. We keep a time dependent average of η and renorm the weights as necessary. This should be very rare.

The last numerical problem is when one or more coefficients gets too large, leading to a huge weight exp(eta). This usually happens when a coefficient is tending to infinity, but can also be due to a bad step in the intermediate Newton-Raphson path. In the infinite coefficient case the log-likelihood trends to an asymptote and there is a race between three conditions: convergence of the loglik, singularity of the variance matrix, or an invalid log-likelihood. The first of these wins the race most of the time, especially if the data set is small, and is the simplest case. The last occurs when the denominator becomes < 0 due to round off so that log(denom) is undefined, the second when extreme weights cause the second derivative to lose precision. In all 3 we revert to step halving, since a bad Newton-Raphson step can cause the same issues to arise.

The next section of code adds up the totals for a given iteration. This is the workhorse. For a given death time all of the events tied at that time must be handled together, hence the main loop below proceeds in batches:

1. Find the time of the next death. Whenever crossing a stratum boundary, zero cetain intermediate sums.

2. Remove all observations in the stratum with time1 ¿ dtime. When survSplit was used to create a data set, this will often remove all. If so we can rezero temporaries and regain precision.

3. Add new observations to the risk set and to the death counts.

⟨*agfit4-addup*⟩=
```
 for (person=0; person<nused; person++) {
     zbeta = 0;      /* form the term beta*z   (vector mult) */
     for (i=0; i<nvar; i++)
         zbeta += beta[i]*covar[i][person];
     eta[person] = zbeta + offset[person];
 }

 /*
 **  'person' walks through the the data from 1 to n,
 **     sort1[0] points to the largest stop time, sort1[1] the next, ...
 **  'dtime' is a scratch variable holding the time of current interest
 **  'indx1' walks through the start times.
 */
 newlk =0;
```

```
for (i=0; i<nvar; i++) {
    u[i] =0;
    for (j=0; j<nvar; j++) imat[i][j] =0;
}
person =0;
indx1 =0;
istrat =0;

/* this next set is rezeroed at the start of each stratum */
denom=0;
nrisk=0;
etasum =0;
for (i=0; i<nvar; i++) {
    a[i] =0;
    for (j=0; j<nvar; j++) cmat[i][j] =0;
}
/* end of the per-stratum set */

while (person < nused) {
    /* find the next death time */
    for (k=person; k< nused; k++) {
        if (k == strata[istrat]) {
            /* hit a new stratum; reset temporary sums */
            istrat++;
            denom = 0;
            nrisk = 0;
            etasum =0;
            for (i=0; i<nvar; i++) {
                a[i] =0;
                for (j=0; j<nvar; j++) cmat[i][j] =0;
            }
            person =k;  /* skip to end of stratum */
            indx1  =k;
        }
        p = sort2[k];
        if (event[p] == 1) {
            dtime = tstop[p];
            break;
        }
    }
    if (k == nused) person =k;  /* no more deaths to be processed */
    else {
        /* remove any subjects no longer at risk */
        ⟨agreg-remove⟩

        /*
```

```
    ** add any new subjects who are at risk
    ** denom2, a2, cmat2, meanwt and deaths count only the deaths
    */
    denom2= 0;
    meanwt =0;
    deaths=0;
    for (i=0; i<nvar; i++) {
        a2[i]=0;
        for (j=0; j<nvar; j++) {
            cmat2[i][j]=0;
        }
    }

    for (; person<strata[istrat]; person++) {
        p = sort2[person];
        if (tstop[p] < dtime) break; /* no more to add */
        risk = exp(eta[p]) * weights[p];

        if (event[p] ==1 ){
            nrisk++;
            etasum += eta[p];
            deaths++;
            denom2 += risk*event[p];
            meanwt += weights[p];
            newlk += weights[p]* eta[p];
            for (i=0; i<nvar; i++) {
                u[i] += weights[p] * covar[i][p];
                a2[i]+= risk*covar[i][p];
                for (j=0; j<=i; j++)
                    cmat2[i][j] += risk*covar[i][p]*covar[j][p];
            }
        }
        else if (keep[p] >0) {
            nrisk++;
            etasum += eta[p];
            denom += risk;
            for (i=0; i<nvar; i++) {
                a[i] += risk*covar[i][p];
                for (j=0; j<=i; j++)
                    cmat[i][j] += risk*covar[i][p]*covar[j][p];
            }
        }
    }
    ⟨breslow-efron⟩
    ⟨fixeta⟩
}
```

```
 }   /* end  of accumulation loop */
```

The last step in the above loop adds terms to the loglik, score and information matrices. Assume that there were 3 tied deaths. The difference between the Efron and Breslow approximations is that for the Efron the three tied subjects are given a weight of 1/3 for the first, 2/3 for the second, and 3/3 for the third death; for the Breslow they get 3/3 for all of them. Note that `imat` is symmetric, and that the cholesky routine will utilize the upper triangle of the matrix as input, using the lower part for its own purposes. The inverse from `chinv` is also in the upper triangle.

⟨*breslow-efron*⟩=
```
 /*
 ** Add results into u and imat for all events at this time point
 */
 if (method==0 || deaths ==1) { /*Breslow */
     denom += denom2;
     newlk -= meanwt*log(denom);  /* sum of death weights*/
     for (i=0; i<nvar; i++) {
         a[i] += a2[i];
         temp = a[i]/denom;   /*mean covariate at this time */
         u[i] -= meanwt*temp;
         for (j=0; j<=i; j++) {
             cmat[i][j] += cmat2[i][j];
             imat[j][i] += meanwt*((cmat[i][j]- temp*a[j])/denom);
         }
     }
 }
 else {
     meanwt /= deaths;
     for (k=0; k<deaths; k++) {
         denom += denom2/deaths;
         newlk -= meanwt*log(denom);
         for (i=0; i<nvar; i++) {
             a[i] += a2[i]/deaths;
             temp = a[i]/denom;
             u[i] -= meanwt*temp;
             for (j=0; j<=i; j++) {
                 cmat[i][j] += cmat2[i][j]/deaths;
                 imat[j][i] += meanwt*((cmat[i][j]- temp*a[j])/denom);
             }
             }
     }
 }
```

Code to process the removals:

⟨*agreg-remove*⟩=
```
 /*
```

```
** subtract out the subjects whose start time is to the right
** If everyone is removed reset the totals to zero.  (This happens when
** the survSplit function is used, so it is worth checking).
*/
for (; indx1<strata[istrat]; indx1++) {
    p1 = sort1[indx1];
    if (start[p1] < dtime) break;
    if (keep[p1] == 0) continue;  /* skip any never-at-risk rows */
    nrisk--;
    if (nrisk ==0) {
        etasum =0;
        denom =0;
        for (i=0; i<nvar; i++) {
            a[i] =0;
            for (j=0; j<=i; j++) cmat[i][j] =0;
        }
    }
    else {
        etasum -= eta[p1];
        risk = exp(eta[p1]) * weights[p1];
        denom -= risk;
        for (i=0; i<nvar; i++) {
            a[i] -= risk*covar[i][p1];
            for (j=0; j<=i; j++)
                cmat[i][j] -= risk*covar[i][p1]*covar[j][p1];
        }
    }
    ⟨fixeta⟩
}
```

The next bit of code exists for the sake of rather rare data sets. Assume that there is a time dependent covariate that rapidly climbs in such a way that the eta gets large but the range of eta stays modest. An example would be something like "payments made to date" for a portfolio of loans. Then even though the data has been centered and the global mean is fine, the current values of eta are outrageous with respect to the exp function. Since replacing eta with (eta -c) for any c does not change the likelihood, do it. Unfortunately, we can't do this once and for all: this is a step that will occur at least twice per iteration for those rare cases, e.g., eta is too small at early times and too large at late ones. I've seen this issue in about 1 data set per decade, by the way.

⟨*fixeta*⟩=

```
 /*
 ** We must avoid overflow in the exp function (~750 on Intel)
 ** and want to act well before that, but not take action very often.
 ** One of the case-cohort papers suggests an offset of -100 meaning
 ** that etas of 50-100 can occur in "ok" data, so make it larger
 ** than this.
```

```
** If the range of eta is more then log(1e16) = 37 then the data is
**  hopeless: some observations will have effectively 0 weight.  Keeping
**  the mean sensible suffices to keep the max in check for all other
*   data sets.
*/
if (fabs(etasum/nrisk) > 200) {
    flag[1]++;  /* a count, for debugging/profiling purposes */
    temp = etasum/nrisk;
    for (i=0; i<nused; i++) eta[i] -= temp;
    temp = exp(-temp);
    denom *= temp;
    for (i=0; i<nvar; i++) {
        a[i] *= temp;
        for (j=0; j<nvar; j++) {
            cmat[i][j]*= temp;
        }
    }
    etasum =0;
}
```

Now, I'm finally to the actual iteration steps. We first update beta, and then calculate the log-likelihood, first and second derviative for that value of beta.

```
⟨agfit4-iter⟩=
 /* First iteration, which has different ending criteria */
 ⟨agfit4-addup⟩
 loglik[0] = newlk;   /* save the loglik for iteration zero  */
 loglik[1] = newlk;

 /* Calculate the score test */
 for (i=0; i<nvar; i++) /*use 'a' as a temp to save u0, for the score test*/
     a[i] = u[i];
 rank = cholesky2(imat, nvar, tol_chol);
 chsolve2(imat,nvar,a);        /* a replaced by  a *inverse(i) */
 *sctest=0;
 for (i=0; i<nvar; i++)
     *sctest +=  u[i]*a[i];
```

The Cox model calculation rarely gets into numerical difficulty, and when it does step halving has always been sufficient. Let $\beta^{(0)}$, $\beta^{(1)}$, etc be the iteration steps in the search for the maximum likelihood solution $\hat{\beta}$. The flow of the algorithm is

1. For the kth iteration, start with the new trial estimate $\beta^{(k)}$. This new estimate is `beta` in the code and the most recent successful estimate is `oldbeta`.

2. For this new trial estimate, compute the log-likelihood, and the first and second derivatives.

3. Test if the log-likelihood if finite, has converged *and* the last estimate was not generated by step-halving. In the latter case the algorithm may *appear* to have converged but the solution is not sure. An infinite loglik is very rare, it arises when denom ¡0 due to catastrophic loss of significant digits when range(eta) is too large.

   - if converged return beta and the the other information
   - if this was the last iteration, return the best beta found so far (perhaps beta, more likely oldbeta), the other information, and a warning flag.
   - otherwise, compute the next guess and return to the top
     - if our latest trial guess `beta` made things worse use step halving: $\beta^{(k+1)}$ = oldbeta + (beta-oldbeta)/2. The assumption is that the current trial step was in the right direction, it just went too far.
     - otherwise take a Newton-Raphson step

I am particularly careful not to make a mistake that I have seen in several other Cox model programs. All the hard work is to calculate the first and second derivatives U (u) and H (imat), once we have them the next Newton-Rhapson update UH^{-1} is just a little bit more. Many programs succumb to the temptation of this "one more for free" idea, and as a consequence return $\beta^{(k+1)}$ along with the log-likelihood and variance matrix for $\beta^{(k)}$. If a user has specified for instance only 1 or 2 iterations the answers can be seriously out of joint. If iteration has gone to completion they will differ by only a gnat's eyelash (so what's the point of an update).

⟨*agfit4-iter*⟩=
```
 /* main loop */
 halving =0 ;             /* =1 when in the midst of "step halving" */
 fail =0;                 /* iteration 1 is never marked as a failure */
 for (*iter=1; *iter<= maxiter; (*iter)++) {
     R_CheckUserInterrupt();  /* be polite -- did the user hit cntrl-C? */
     if (*iter >1) {
         /* on iteration 1 the cholesky has already been done */
         rank2 = cholesky2(imat, nvar, tol_chol);
         /* Are we done? */
         fail = isnan(loglik[1]) + isinf(loglik[1]) + (rank-rank2);
         if (fail ==0 && halving ==0 &&
             fabs(1-(loglik[1]/newlk)) <= eps) break;
     }

     /* Update coefficients */
     if (fail >0 || newlk < loglik[1]) { /*never true on iteration 1 */
         /*
         ** The routine has not made progress past the last good value.
         */
         halving =1; flag[2]++;
         for (i=0; i<nvar; i++)
             beta[i] = (oldbeta[i] + beta[i]) /2; /*half of old increment */
     }
```

```
    else {
         halving=0;
         loglik[1] = newlk;
         chsolve2(imat,nvar,u);

         for (i=0; i<nvar; i++) {
             oldbeta[i] = beta[i];
             beta[i] = beta[i] +  u[i];
         }
    }
```

⟨ *agfit4-addup* ⟩

```
} /*return for another iteration */
```

⟨ *agfit4-finish* ⟩

Save away the final bits, compute the inverse of imat and symmetrize it, release memory and return.

⟨ *agfit4-finish* ⟩=

```
 (*iter)--;  /* the loop index is always 1 beyond where it finished */
 flag[0] = rank;
 loglik[1] = newlk;
 chinv2(imat, nvar);
 for (i=0; i<nvar; i++) {
     beta[i] *= scale[i];  /* return to original scale */
     u[i] /= scale[i];
     imat[i][i] *= scale[i] * scale[i];
     for (j=0; j<i; j++) {
         imat[j][i] *= scale[i] * scale[j];
         imat[i][j] = imat[j][i];
     }
 }
 UNPROTECT(nprotect);
 return(rlist);
```

4 Cox models

4.1 Predicted survival

The `survfit` method for a Cox model produces individual survival curves. As might be expected these have much in common with ordinary survival curves, and share many of the same methods. The primary differences are first that a predicted curve always refers to a particular set of covariate values. It is often the case that a user wants multiple values at once, in which case

the result will be a matrix of survival curves with a row for each time and a column for each covariate set. The second is that the computations are somewhat more difficult.

The input arguments are

formula a fitted object of class 'coxph'. The argument name of 'formula' is historic, from when the survfit function was not a generic and only did Kaplan-Meier type curves.

newdata contains the data values for which curves should be produced, one per row

se.fit TRUE/FALSE, should standard errors be computed.

individual a particular option for time-dependent covariates

type computation type for the survival curve

vartype computation type for the variance

censor if FALSE, remove any times that have no events from the output. This is for backwards compatability with older versions of the code.

id replacement and extension for the individual argument

All the other arguments are common to all the methods, refer to the help pages.

```
⟨survfit.coxph⟩=
 survfit.coxph <-
   function(formula, newdata, se.fit=TRUE, conf.int=.95, individual=FALSE,
            type, vartype,
            conf.type=c("log", "log-log", "plain", "none"),
            censor=TRUE, id,
            na.action=na.pass, ...) {

     Call <- match.call()
     Call[[1]] <- as.name("survfit")  #nicer output for the user
     object <- formula     #'formula' because it has to match survfit
     ⟨survfit.coxph-setup⟩
     ⟨survfit.coxph-result⟩
     ⟨survfit.coxph-finish⟩
     }
```

The third line `as.name('survfit')` causes the printout to say 'survfit' instead of 'survfit.coxph'.

The setup for the routine is fairly pedestrian. If the newdata argument is missing we use `object$means` as the default value. This choice has lots of statistical shortcomings, particularly in a stratified model, but is common in other packages and a historic option here. If the type or vartype are missing we use the appropriate one for the method in the Cox model. That is, the `coxph` computation used for `method=''exact''` is the same approximation used in the Kalbfleish-Prentice estimate, that for the Breslow method matches the Aalen survival estimate, and the Efron approximation the Efron survival estimate. The other two rows of labels in `temp1` are historical; we include them for backwards compatability but they don't appear in the documentation.

One particular special case (that gave me fits for a while) is when there are non-heirarchical models, for example `~ age + age:sex`. The fit of such a model will *not* be the same using the variable `age2 <- age-50`; I originally thought it a flaw induced by my subtraction. The routine simply cannot give a sensible curve for a model like this. The issue continued to surprise me each time I rediscovered it, leading to an error message for my own protection. I'm not convinced at this time that there is a sensible survival curve that *could* be calculated for such a model. A model with `age + age:strata(sex)` will be ok, because the coxph routine treats this last term as though it had a * in it, i.e., fits a stratified model.

⟨*survfit.coxph-setup*⟩=
```
if (!is.null(attr(object$terms, "specials")$tt))
    stop("The survfit function can not yet process coxph models with a tt term")

if (missing(type)) {
    # Use the appropriate one from the model
    temp1 <- c("exact", "breslow", "efron")
    survtype <- match(object$method, temp1)
}
else {
    temp1 <- c("kalbfleisch-prentice", "aalen", "efron",
               "kaplan-meier", "breslow", "fleming-harrington",
               "greenwood", "tsiatis", "exact")
    survtype <- match(match.arg(type, temp1), temp1)
    survtype <- c(1,2,3,1,2,3,2,2,1)[survtype]
    }
if (missing(vartype)) {
    vartype <- survtype
    }
else {
    temp2 <- c("greenwood", "aalen", "efron", "tsiatis")
    vartype <- match(match.arg(vartype, temp2), temp2)
    if (vartype==4) vartype<- 2
    }

if (!se.fit) conf.type <- "none"
else conf.type <- match.arg(conf.type)

tfac <- attr(terms(object), 'factors')
temp <- attr(terms(object), 'specials')$strata
has.strata <- !is.null(temp)
if (has.strata) {
    # Toss out strata terms in tfac before doing the test 1 line below, as
    #  strata end up in the model with age:strat(grp) terms or *strata() terms
    #  (There might be more than one strata term)
    for (i in temp) tfac <- tfac[,tfac[i,] ==0]  # toss out strata terms
    }
```

```
 if (any(tfac >1))
     stop("not able to create a curve for models that contain an interaction without the lower orde
```

I need to retrieve a copy of the original data. We always need the X matrix and y, both of which may be found in the data object. If the original call included either strata, offset, or weights, or if either x or y are missing from the `coxph` object, then the model frame will need to be reconstructed. We have to use `object['x']` instead of `object$x` since the latter will pick off the `xlevels` component if the `x` component is missing (which is the default).

⟨*survfit.coxph-setup*⟩=
```
 if (is.null(object$y) || is.null(object[['x']]) ||
     !is.null(object$call$weights) ||
     (has.strata && is.null(object$strata)) ||
     !is.null(attr(object$terms, 'offset'))) {

     mf <- stats::model.frame(object)
     }
 else mf <- NULL  #useful for if statements later
```

If a model frame was created, then it is trivial to grab `y` from the new frame and compare it to `object$y` from the original one. This is to avoid nonsense results that arise when someone changes the data set under our feet. For instance

⟨*dummy*⟩=
```
   fit <- coxph(Surv(time,status) ~ age, data=lung)
   lung <- lung[1:100,]
   survfit(fit)
```

⟨*survfit.coxph-setup*⟩=
```
 if (is.null(mf)) y <- object[['y']]
 else {
     y <- model.response(mf)
     y2 <- object[['y']]
     if (!is.null(y2) && any(as.matrix(y2) != as.matrix(y)))
         stop("Could not reconstruct the y vector")
     }

 if (is.null(object[['x']])) x <- model.matrix.coxph(object, data=mf)
 else x <- object[['x']]

 n <- nrow(y)
 if (n != object$n[1] || nrow(x) !=n)
     stop("Failed to reconstruct the original data set")

 if (is.null(mf)) wt <- rep(1., n)
 else {
     wt <- model.weights(mf)
```

```
    if (is.null(wt)) wt <- rep(1.0, n)
    }

type <- attr(y, 'type')
if (type != 'right' && type != 'counting')
    stop(gettextf("Cannot handle \"%s\" type survival data", type))
missid <- missing(id) # I need this later, and setting id below makes
                      # "missing(id)" always false
if (!missid) individual <- TRUE
else if (missid && individual) id <- rep(0,n)  #dummy value
else id <- NULL

if (individual && missing(newdata)) {
    stop("the id and/or individual options only make sense with new data")
}

if (individual && type!= 'counting')
    stop("The individual option is  only valid for start-stop data")

if (is.null(mf)) offset <- 0
else {
    offset <- model.offset(mf)
    if (is.null(offset)) offset <- 0
    }

Terms <- object$terms
if (!has.strata)  strata <- rep(0L,n)
else {
    stangle <- untangle.specials(Terms, 'strata') # used multiple times
    strata <- object$strata #try this first
    if (is.null(strata)){
        if (length(stangle$vars) ==1) strata <- mf[[stangle$vars]]
        else strata <- strata(mf[, stangle$vars], shortlabel=TRUE)
    }
}
```

In two places below we need to know if there are strata by covariate interactions, which requires looking at attributes of the terms object. The factors attribute will have a row for the strata variable, or maybe more than one (multiple strata terms are legal). If it has a 1 in a column that corresponds to something of order 2 or greater, that is a strata by covariate interaction.

⟨*survfit.coxph-setup*⟩=

```
if (has.strata) {
    temp <- attr(Terms, "specials")$strata
    factors <- attr(Terms, "factors")[temp,]
    strata.interaction <- any(t(factors)*attr(Terms, "order") >1)
}
```

If a variable is deemed redundant the `coxph` routine will have set its coefficient to NA as a marker. We want to ignore that coefficient: treating it as a zero has the desired effect. Another special case is a null model, having either 1 or only an offset on the right hand side. In that case we create a dummy covariate to allow the rest of the code to work without special if/else. The last special case is a model with a sparse frailty term. We treat the frailty coefficients as 0 variance (in essence as an offset). The frailty is removed from the model variables but kept in the risk score. This isn't statistically very defensible, but it is backwards compatatble. A non-sparse frailty does not need special code and works out like any other variable.

We also remove the means from each column of the X matrix. The reason for this is to avoid huge values when calculating $\exp(X\beta)$; this would happen if someone had a variable with a mean of 1000 and a variance of 1. Any constant can be subtracted, mathematically the results are identical as long as the same values are subtracted from the old and new X data. The mean is used because it is handy, we just need to get $X\beta$ in the neighborhood of zero.

⟨*survfit.coxph-setup*⟩=
```
 if (is.null(x) || ncol(x)==0) { # a model with ~1 on the right hand side
     # Give it a dummy x so the rest of the code goes through
     #  (This case is really rare)
     x <- matrix(0., nrow=n)
     coef <- 0.0
     varmat <- matrix(0.0,1,1)
     risk <- rep(exp(offset- mean(offset)), length=n)
     }
 else {
     varmat <- object$var
     coef <- ifelse(is.na(object$coefficients), 0, object$coefficients)
     xcenter <- object$means
     if (is.null(object$frail)) {
         x <- scale(x, center=xcenter, scale=FALSE)
         risk <- c(exp(x%*% coef + offset - mean(offset)))
         }
    else {
        keep <- !is.na(match(dimnames(x)[[2]], names(coef)))
        x <- x[,keep, drop=F]
#        varmat <- varmat[keep,keep]  #coxph already has trimmed it
        risk <- exp(object$linear.predictor)
        x <- scale(x, center=xcenter, scale=FALSE)
        }
     }
```

The `risk` vector and `x` matrix come from the original data, and are the raw data for the survival curve and its variance. We also need the risk score $\exp(X\beta)$ for the target subject(s).

- For predictions with time-dependent covariates the user will have either included an `id` statement (newer style) or specified the `individual=TRUE` option. If the latter, then `newdata` is presumed to contain only a single indivual represented by multiple rows. If

the former then the `id` variable marks separate individuals. In either case we need to retrieve the covariates, strata, and repsonse from the new data set.

- For ordinary predictions only the covariates are needed.
- If newdata is not present we assume that this is the ordinary case, and use the value of `object$means` as the default covariate set. This is not ideal statistically since many users view this as an "average" survival curve, which it is not.

When grabbing [newdata] we want to use model.frame processing, both to handle missing values correctly and, perhaps more importantly, to correctly map any factor variables between the original fit and the new data. (The new data will often have only one of the original levels represented.) Also, we want to correctly handle data-dependent nonlinear terms such as ns and pspline. However, the simple call found in predict.lm, say, `model.frame(Terms, data=newdata, ..` isn't used here for a few reasons. The first is a decision on our part that the user should not have to include unused terms in the model. The second is that if there are strata, the user may or may not have included strata variables in their data set and we need to act accordingly. The third is that we might have an `id` statement in this call, which is another variable to be fetched. Last, there is no ability to use sparse frailties and newdata together; it is a hard case and so rare as to not be worth it.

First, remove unnecessary terms from the orginal model formula. Any `cluster` terms can be deleted, If `individual` is false then the repsonse variable can go.

The dataClasses and predvars attributes, if present, have elements in the same order as the first dimension of the "factors" attribute of the terms. Subscripting the terms argument does not preserve dataClasses or predvars, however. Use the pre and post subscripting factors attribute to determine what elements of them to keep. The predvars component is a call objects with one element for each term in the formula, so y age + ns(height) would lead to a predvars of length 4, element 1 is the call itself, 2 would be y, etc. The dataClasses object is a simple list.

⟨*survfit.coxph-setup*⟩=

```
 subterms <- function(tt, i) {
     dataClasses <- attr(tt, "dataClasses")
     predvars <- attr(tt, "predvars")
     oldnames <-  dimnames(attr(tt, 'factors'))[[1]]
     tt <- tt[i]
     index <- match(dimnames(attr(tt, 'factors'))[[1]], oldnames)
     if (length(index) >0) {
         if (!is.null(predvars))
             attr(tt, "predvars") <- predvars[c(1, index+1)]
         if (!is.null(dataClasses))
             attr(tt, "dataClasses") <- dataClasses[index]
         }
     tt
 }
 temp <- untangle.specials(Terms, 'cluster')
 if (length(temp$terms))
     Terms <- subterms(Terms, -temp$terms)
```

```
if (missing(newdata)) {
    mf2 <- as.list(object$means)   #create a dummy newdata
    names(mf2) <- names(object$coefficients)
    mf2 <- as.data.frame(mf2)
    found.strata <- FALSE
}
else {
    if (!is.null(object$frail))
        stop("Newdata cannot be used when a model has frailty terms")

    Terms2 <- Terms
    if (!individual)  Terms2 <- delete.response(Terms)
    ⟨survfit.coxph-newdata2⟩
    }
```

For backwards compatability, I allow someone to give an ordinary vector instead of a data frame (when only one curve is required). In this case I also need to verify that the elements have a name. Then turn it into a data frame, like it should have been from the beginning. (Documentation of this ability has been suppressed, however. I'm hoping people forget it ever existed.)

⟨*survfit.coxph-newdata2*⟩=

```
if (is.vector(newdata, "numeric")) {
    if (individual) stop("'newdata' argument must be a data frame")
    if (is.null(names(newdata))) {
        stop("'newdata' argument must be a data frame")
    }
    newdata <- data.frame(as.list(newdata))
}
```

Finally get my new model frame mf2. There are two cases. If the call does not has an "id" argument then we use the semantics of top-level functions like coxph: get a copy of the call, keep what we need, change the called function's name to "model.fram" and evalutate it. then we If all is particularly simple we can use a simple call. Otherwise get an abbreviated form of the original call that has only the calling function, na.action, and id. The calling function is always element 1, the others are found by name. Now manipulate it: add the formula, data and xlev components (the last might be NULL), and then change the name of the call. If the original call was `survfit(fit1, newdata=mydat, conf.int=.9)` the result is `model.frame(data= copy of newdat, formula=Terms2, xlev=myxlev)`. If there is no id argument we use a simple call, except that we allow the user to leave out any strata() variables if they so desire, *if* there are no strata by covariate interactions.

How does one check if the strata variables are or are not available in the call? My first attempt at this was to wrap the call in a try() construct and see if it failed. This doesn't work.

- What if there is no strata variable in newdata, but they do have, by bad luck, a variable of the same name in their main directory?

- It would seem like changing the environment to NULL would be wise, so that we don't find variables anywhere but in the data argument, a sort of sandboxing. Not wise: you then won't find functions like "log".
- We don't dare modify the environment of the formula at all. It is needed for the sneaky caller who uses his own function inside the formula, 'mycosine' say, and that function can only be found if we retain the environment.

One way out of this is to evaluate each of the strata terms (there can be more than one) one at a time, in an environment that knows nothing except "list" and a fake definition of "strata", and newdata. Variables that are part of the global environment won't be found. I even watch out for the case of either "strata" or "list" is the name of the stratification variable, which causes my fake strata function to return a function when said variable is not in newdata.

⟨*survfit.coxph-newdata2*⟩=
```
 if (missid) {
     if (has.strata && !strata.interaction) {
         found.strata <- TRUE
         tempenv <- new.env(, parent=emptyenv())
         assign("strata", function(..., na.group, shortlabel, sep)
                        list(...), envir=tempenv)
         assign("list", list, envir=tempenv)
         for (svar in stangle$vars) {
             temp <- try(eval(parse(text=svar), newdata, tempenv),
                                  silent=TRUE)
             if (!is.list(temp) ||
                 any(unlist(lapply(temp, class))== "function"))
                 found.strata <- FALSE
         }

         if (found.strata) mf2 <- stats::model.frame(Terms2, data=newdata,
                                  na.action=na.action, xlev=object$xlevels)
         else {
             Terms2 <- subterms(Terms2, -attr(Terms2, 'specials')$strata)
             if (!is.null(object$xlevels)) {
                 myxlev <- object$xlevels[match(attr(Terms2, "term.labels"),
                                     names(object$xlevels), nomatch=0)]
                 if (length(myxlev)==0) myxlev <- NULL
             }
             else myxlev <- NULL
             mf2 <- stats::model.frame(Terms2, data=newdata, na.action=na.action,
                                  xlev=myxlev)
             }
         }
     else {
         mf2 <- stats::model.frame(Terms2, data=newdata, na.action=na.action,
                               xlev=object$xlevels)
```

```
        found.strata <- has.strata  #would have failed otherwise
        }
    }
else {
    tcall <- Call[c(1, match(c('id', "na.action"),
                             names(Call), nomatch=0))]
    tcall$data <- newdata
    tcall$formula <- Terms2
    tcall$xlev <- object$xlevels
    tcall[[1L]] <- quote(stats::model.frame)
    mf2 <- eval(tcall)
    found.strata <- has.strata # would have failed otherwise
}
```

Now, finally, extract the x2 matrix from the just-created frame.

⟨*survfit.coxph-result*⟩=

```
if (has.strata && found.strata) { #pull them off
    temp <- untangle.specials(Terms2, 'strata')
    strata2 <- strata(mf2[temp$vars], shortlabel=TRUE)
    strata2 <- factor(strata2, levels=levels(strata))
    if (any(is.na(strata2)))
        stop("New data set has strata levels not found in the original")
    # An expression like age:strata(sex) will have temp$vars= "strata(sex)"
    #  and temp$terms = integer(0).  This does not work as a subscript
    if (length(temp$terms) >0) Terms2 <- Terms2[-temp$terms]
}
else strata2 <- factor(rep(0, nrow(mf2)))

if (individual) {
    if (missing(newdata))
        stop("The newdata argument must be present when 'individual=TRUE'")
    if (!missid) {  #grab the id variable
        id <- model.extract(mf2, "id")
        if (is.null(id)) stop("'id=NULL' is an invalid argument")
        }
    else id <- rep(1, nrow(mf2))

    x2 <- model.matrix(Terms2, mf2)[,-1, drop=FALSE]  #no intercept
    if (length(x2)==0) stop("Individual survival but no variables")
    x2 <- scale(x2, center=xcenter, scale=FALSE)

    offset2 <- model.offset(mf2)
    if (length(offset2) >0) offset2 <- offset2 - mean(offset)
    else offset2 <- 0

    y2 <- model.extract(mf2, 'response')
```

```
    if (attr(y2,'type') != type)
        stop("Survival type of newdata does not match the fitted model")
    if (attr(y2, "type") != "counting")
        stop("'individual=TRUE' is only valid for counting process data")
    y2 <- y2[,1:2, drop=F]  #throw away status, it's never used

    newrisk <- exp(c(x2 %*% coef) + offset2)
    result <- survfitcoxph.fit(y, x, wt, x2, risk, newrisk, strata,
                               se.fit, survtype, vartype, varmat,
                               id, y2, strata2)
   }
```

If there is no newdata argument, the centering means that we need to predict for x2=0. The second the most common call to the routine.

⟨*survfit.coxph-result*⟩=

```
 else {
    if (missing(newdata)) {
        if (has.strata && strata.interaction)
            stop("Models with strata by covariate interaction terms require newdata")
        x2 <- matrix(0.0, nrow=1, ncol=ncol(x))
        offset2 <- 0
    }
    else {
       offset2 <- model.offset(mf2)
       if (length(offset2) >0) offset2 <- offset2 - mean(offset)
       else offset2 <- 0
       x2 <- model.matrix(Terms2, mf2)[,-1, drop=FALSE]  #no intercept
       x2 <- scale(x2, center=xcenter, scale=FALSE)
   }

    newrisk <- exp(c(x2 %*% coef) + offset2)
    result <- survfitcoxph.fit(y, x, wt, x2, risk, newrisk, strata,
                               se.fit, survtype, vartype, varmat)
    if (has.strata && found.strata) {
        if (is.matrix(result$surv)) {
            ⟨newstrata-fixup⟩
            }
    }
 }
```

The final bit of work. If the newdata arg contained strata then the user should not get a matrix of survival curves containing every newdata obs * strata combination, but rather a vector of curves, each one with the appropriate strata. It was faster to compute them all, however, than to use the individual=T logic. So now pick off the bits we want. The names of the curves will be the rownames of the newdata arg, if they exist.

⟨*newstrata-fixup*⟩=

```
nr <- nrow(result$surv)  #a vector if newdata had only 1 row
indx1 <- split(1:nr, rep(1:length(result$strata), result$strata))
rows <- indx1[as.numeric(strata2)]  #the rows for each curve

indx2 <- unlist(rows)  #index for time, n.risk, n.event, n.censor
indx3 <- as.integer(strata2) #index for n and strata

for(i in 2:length(rows)) rows[[i]] <- rows[[i]]+ (i-1)*nr #linear subscript
indx4 <- unlist(rows)   #index for surv and std.err
temp <- result$strata[indx3]
names(temp) <- row.names(mf2)
new <- list(n = result$n[indx3],
            time= result$time[indx2],
            n.risk= result$n.risk[indx2],
            n.event=result$n.event[indx2],
            n.censor=result$n.censor[indx2],
            strata = temp,
            surv= result$surv[indx4],
            cumhaz = result$cumhaz[indx4])
if (se.fit) new$std.err <- result$std.err[indx4]
result <- new
```

Finally, the last (somewhat boring) part of the code. First, if given the argument `censor=FALSE` we need to remove all the time points from the output at which there was only censoring activity. This action is mostly for backwards compatability with older releases that never returned censoring times. Second, add in the variance and the confidence intervals to the result. The code is nearly identical to that in survfitKM.

⟨*survfit.coxph-finish*⟩=

```
if (!censor) {
    kfun <- function(x, keep){ if (is.matrix(x)) x[keep,,drop=F]
                               else if (length(x)==length(keep)) x[keep]
                               else x}
    keep <- (result$n.event > 0)
    if (!is.null(result$strata)) {
        temp <- factor(rep(names(result$strata), result$strata),
                       levels=names(result$strata))
        result$strata <- c(table(temp[keep]))
        }
    result <- lapply(result, kfun, keep)
    }

if (se.fit) {
    zval <- qnorm(1- (1-conf.int)/2, 0,1)
    if (conf.type=='plain') {
        temp1 <- result$surv + zval* result$std.err * result$surv
        temp2 <- result$surv - zval* result$std.err * result$surv
```

```
        result <- c(result, list(upper=pmin(temp1,1), lower=pmax(temp2,0),
                        conf.type='plain', conf.int=conf.int))
        }
    if (conf.type=='log') {
        xx <- ifelse(result$surv==0,1,result$surv)  #avoid some "log(0)" messages
        temp1 <- ifelse(result$surv==0, 0*result$std.err,
                        exp(log(xx) + zval* result$std.err))
        temp2 <- ifelse(result$surv==0, 0*result$std.err,
                        exp(log(xx) - zval* result$std.err))
        result <- c(result, list(upper=pmin(temp1,1), lower=temp2,
                        conf.type='log', conf.int=conf.int))
        }
    if (conf.type=='log-log') {
        who <- (result$surv==0 | result$surv==1) #special cases
        xx <- ifelse(who, .1,result$surv)  #avoid some "log(0)" messages
        temp1 <- exp(-exp(log(-log(xx)) + zval*result$std.err/log(xx)))
        temp1 <- ifelse(who, result$surv + 0*result$std.err, temp1)
        temp2 <- exp(-exp(log(-log(xx)) - zval*result$std.err/log(xx)))
        temp2 <- ifelse(who, result$surv + 0*result$std.err, temp2)
        result <- c(result, list(upper=temp1, lower=temp2,
                        conf.type='log-log', conf.int=conf.int))
        }
    }

result$call <- Call

# The "type" component is in the middle -- match history
indx <- match('surv', names(result))
result <- c(result[1:indx], type=attr(y, 'type'), result[-(1:indx)])
if (is.R()) class(result) <- c('survfit.cox', 'survfit')
else        oldClass(result) <- 'survfit.cox'
result
```

Now, we're ready to do the main compuation. Before this revision (the one documented here using noweb) there were three C routines used in calculating survival after a Cox model

1. agsurv1 creates a single curve, but for the most general case of a *covariate path*. It is used for time dependent covariates.

2. agsurv2 creates a set of curves. These curves are for a fixed covariate set, although (start, stop] data is supported. If there were 3 strata in the fit and 4 covariate sets are given, the result will be 12 curves.

3. agsurv3 is used to create population survival curves. The result is average survival curve (for 3 different definitions of 'average'). If there were 3 strata and 100 subjects, the first curve returned would be the average for those 100 individual curves in strata 1, the second for strata 2, and the third for strata 3.

In June 2010 the first two were re-written in (mostly) R, in the process of adding functionality and repairing some flaws in the computation of a weighted variance. In effect, the changes are similar to the rewrite of the survfitKM function a few years ago.

Computations are separate for each strata, and each strata will have a different number of time points in the result. Thus we can't preallocate a matrix. Instead we generate an empty list, one per strata, and then populate it with the survival curves. At the end we unlist the individual components one by one. This is memory efficient, the number of curves is usually small enough that the "for" loop is no great cost, and it's easier to see what's going on than C code.

First, compute the baseline survival curves for each strata. If the strata was a factor we want to leave it in the same order, otherwise sort it. This fitting routine was set out as a separate function for the sake of the rms package. They want to utilize the computation, but have a diffferent recreation process for the x and y data.

⟨*survfitcoxph.fit*⟩=
```
survfitcoxph.fit <- function(y, x, wt, x2, risk, newrisk, strata, se.fit,
                             survtype, vartype, varmat, id, y2, strata2,
                             unlist=TRUE) {
    if (is.factor(strata)) ustrata <- levels(strata)
    else                   ustrata <- sort(unique(strata))
    nstrata <- length(ustrata)
    survlist <- vector('list', nstrata)
    names(survlist) <- ustrata

    for (i in 1:nstrata) {
        indx <- which(strata== ustrata[i])
        survlist[[i]] <- agsurv(y[indx,,drop=F], x[indx,,drop=F],
                                wt[indx], risk[indx],
                                survtype, vartype)
        }

    ⟨survfit.coxph-compute⟩

    if (unlist) {
        if (length(result)==1) { # the no strata case
            if (se.fit)
                result[[1]][c("n", "time", "n.risk", "n.event", "n.censor",
                              "surv", "cumhaz", "std.err")]
            else result[[1]][c("n", "time", "n.risk", "n.event", "n.censor",
                               "surv", "cumhaz")]
        }
        else {
            temp <-list(n   =    unlist(lapply(result, function(x) x$n),
                                        use.names=FALSE),
                        time=    unlist(lapply(result, function(x) x$time),
                                        use.names=FALSE),
                        n.risk=  unlist(lapply(result, function(x) x$n.risk),
```

```
                                    use.names=FALSE),
                    n.event= unlist(lapply(result, function(x) x$n.event),
                                    use.names=FALSE),
                    n.censor=unlist(lapply(result, function(x) x$n.censor),
                                    use.names=FALSE),
                    strata = sapply(result, function(x) length(x$time)))
            names(temp$strata) <- names(result)

            if ((missing(id) || is.null(id)) && nrow(x2)>1) {
                 temp$surv <- t(matrix(unlist(lapply(result,
                                  function(x) t(x$surv)), use.names=FALSE),
                                      nrow= nrow(x2)))
                 dimnames(temp$surv) <- list(NULL, row.names(x2))
                 temp$cumhaz <- t(matrix(unlist(lapply(result,
                                  function(x) t(x$cumhaz)), use.names=FALSE),
                                      nrow= nrow(x2)))
                 if (se.fit)
                     temp$std.err <- t(matrix(unlist(lapply(result,
                                      function(x) t(x$std.err)), use.names=FALSE),
                                              nrow= nrow(x2)))
                 }
            else {
                temp$surv <- unlist(lapply(result, function(x) x$surv),
                                    use.names=FALSE)
                temp$cumhaz <- unlist(lapply(result, function(x) x$cumhaz),
                                    use.names=FALSE)
                if (se.fit)
                    temp$std.err <- unlist(lapply(result,
                                    function(x) x$std.err), use.names=FALSE)
                }
            temp
            }
        }
    else {
        names(result) <- ustrata
        result
        }
    }
```

In an ordinary survival curve object with multiple strata, as produced by `survfitKM`, the time, survival and etc components are each a single vector that contains the results for strata 1, followed by strata 2, ... The strata compontent is a vector of integers, one per strata, that gives the number of elements belonging to each stratum. The reason is that each strata will have a different number of observations, so that a matrix form was not viable, and the underlying C routines were not capable of handling lists (the code predates the .Call function by a decade). The underlying computation of `survfitcoxph.fit` naturally creates the list form, we unlist it

to `survfit` form as our last action unless the caller requests otherwise.

For `individual=FALSE` we have a second dimension, namely each of the target covariate sets (if there are multiples). Each of these generates a unique set of survival and variance(survival) values, but all of the same size since each uses all the strata. The final output structure in this case has single vectors for the time, number of events, number censored, and number at risk values since they are common to all the curves, and a marix of survival and variance estimates, one column for each of the distinct target values. If Λ_0 is the baseline cumulative hazard from the above calculation, then $r_i\Lambda_0$ is the cumulative hazard for the ith new risk score r_i. The variance has two parts, the first of which is $r_i^2 H_1$ where H_1 is returned from the `agsurv` routine, and the second is

$$H_2(t) = d'(t) V d(t)$$
$$d(t) = \int_0^t [z - \overline{x}(s)] d\Lambda(s)$$

V is the variance matrix for β from the fitted Cox model, and $d(t)$ is the distance between the target covariate z and the mean of the original data, summed up over the interval from 0 to t. Essentially the variance in $\hat{\beta}$ has a larger influence when prediction is far from the mean. The function below takes the basic curve from the list and multiplies it out to matrix form.

```
⟨survfit.coxph-compute⟩=
 expand <- function(fit, x2, varmat, se.fit) {
     if (survtype==1)
         surv <- cumprod(fit$surv)
     else surv <- exp(-fit$cumhaz)

     if (is.matrix(x2) && nrow(x2) >1) {  #more than 1 row in newdata
         fit$surv <- outer(surv, newrisk, '^')
         dimnames(fit$surv) <- list(NULL, row.names(x2))
         if (se.fit) {
             varh <- matrix(0., nrow=length(fit$varhaz), ncol=nrow(x2))
             for (i in 1:nrow(x2)) {
                 dt <- outer(fit$cumhaz, x2[i,], '*') - fit$xbar
                 varh[,i] <- (cumsum(fit$varhaz) + rowSums((dt %*% varmat)* dt))*
                     newrisk[i]^2
                 }
             fit$std.err <- sqrt(varh)
             }
         fit$cumhaz <- outer(fit$cumhaz, newrisk, '*')
         }
     else {
         fit$surv <- surv^newrisk
         if (se.fit) {
             dt <-  outer(fit$cumhaz, c(x2)) - fit$xbar
             varh <- (cumsum(fit$varhaz) + rowSums((dt %*% varmat)* dt)) *
                 newrisk^2
```

```
            fit$std.err <- sqrt(varh)
            }
        fit$cumhaz <- fit$cumhaz * newrisk
        }
    fit
    }
```

In the lines just above: I have a matrix `dt` with one row per death time and one column per variable. For each row d_i separately we want the quadratic form $d_i V d_i'$. The first matrix product can be done for all rows at once: found in the inner parenthesis. Ordinary (not matrix) multiplication followed by rowsums does the rest in one fell swoop.

Now, if `id` is missing we can simply apply the `expand` function to each strata. For the case with `id` not missing, we create a single survival curve for each unique id (subject). A subject will spend blocks of time with different covariate sets, sometimes even jumping between strata. Retrieve each one and save it into a list, and then sew them together end to end. The `n` component is the number of observations in the strata — but this subject might visit several. We report the first one they were in for printout. The `time` component will be cumulative on this subject's scale. Counting this is a bit trickier than I first thought. Say that the subject's first interval goes from 1 to 10, with observed time points in that interval at 2, 5, and 7, and a second interval from 12 to 20 with observed time points in the data of 15 and 18. On the subject's time scale things happen at days 1, 4, 6, 12 and 15. The deltas saved below are 2-1, 5-2, 7-5, 3+ 14-12, 17-14. Note the 3+ part, kept in the `timeforward` variable. Why all this "adding up" nuisance? If the subject spent time in two strata, the second one might be on an internal time scale of 'time since entering the strata'. The two intervals in newdata could be 0–10 followed by 0–20. Time for the subject can't go backwards though: the change between internal/external time scales is a bit like following someone who was stepping back and forth over the international date line.

In the code the `indx` variable points to the set of times that the subject was present, for this row of the new data. Note the $>$ on one end and $\leq$ on the other. If someone's interval 1 was 0–10 and interval 2 was 10–20, and there happened to be a jump in the baseline survival curve at exactly time 10 (someone else died), that jump is counted only in the first interval.

⟨*survfit.coxph-compute*⟩=

```
 if (missing(id) || is.null(id))
     result <- lapply(survlist, expand, x2, varmat, se.fit)
 else {
     onecurve <- function(slist, x2, y2, strata2,  newrisk, se.fit) {
         ntarget <- nrow(x2)  #number of different time intervals
         surv <- vector('list', ntarget)
         n.event <- n.risk <- n.censor <- varh1 <- varh2 <-  time <- surv
         hazard  <- vector('list', ntarget)
         stemp <- as.integer(strata2)
         timeforward <- 0
         for (i in 1:ntarget) {
             slist <- survlist[[stemp[i]]]
             indx <- which(slist$time > y2[i,1] & slist$time <= y2[i,2])
             if (length(indx)==0) {
```

```
            timeforward <- timeforward + y2[i,2] - y2[i,1]
            # No deaths or censors in user interval.  Possible
            # user error, but not uncommon at the tail of the curve.
            }
        else {
            time[[i]] <- diff(c(y2[i,1], slist$time[indx])) #time increments
            time[[i]][1] <- time[[i]][1] + timeforward
            timeforward <- y2[i,2] - max(slist$time[indx])

            hazard[[i]] <- slist$hazard[indx]*newrisk[i]
            if (survtype==1) surv[[i]] <- slist$surv[indx]^newrisk[i]

            n.event[[i]] <- slist$n.event[indx]
            n.risk[[i]]  <- slist$n.risk[indx]
            n.censor[[i]]<- slist$n.censor[indx]
            dt <-  outer(slist$cumhaz[indx], x2[i,]) - slist$xbar[indx,,drop=F]
            varh1[[i]] <- slist$varhaz[indx] *newrisk[i]^2
            varh2[[i]] <- rowSums((dt %*% varmat)* dt) * newrisk[i]^2
            }
        }

    cumhaz <- cumsum(unlist(hazard))
    if (survtype==1) surv <- cumprod(unlist(surv))  #increments (K-M)
    else surv <- exp(-cumhaz)

    if (se.fit)
        list(n=as.vector(table(strata)[stemp[1]]),
               time=cumsum(unlist(time)),
               n.risk = unlist(n.risk),
               n.event= unlist(n.event),
               n.censor= unlist(n.censor),
               surv = surv,
               cumhaz= cumhaz,
               std.err = sqrt(cumsum(unlist(varh1)) + unlist(varh2)))
    else list(n=as.vector(table(strata)[stemp[1]]),
               time=cumsum(unlist(time)),
               n.risk = unlist(n.risk),
               n.event= unlist(n.event),
               n.censor= unlist(n.censor),
               surv = surv,
               cumhaz= cumhaz)
    }

if (all(id ==id[1])) {
    result <- list(onecurve(survlist, x2, y2, strata2, newrisk, se.fit))
    }
```

```
else {
    uid <- unique(id)
    result <- vector('list', length=length(uid))
    for (i in 1:length(uid)) {
        indx <- which(id==uid[i])
        result[[i]] <- onecurve(survlist, x2[indx,,drop=FALSE],
                                y2[indx,,drop=FALSE],
                                strata2[indx],  newrisk[indx], se.fit)
        }
    names(result) <- uid
    }
}
```

Next is the code for the `agsurv` function, which actually does the work. The estimates of survival are the Kalbfleisch-Prentice (KP), Breslow, and Efron. Each has an increment at each unique death time. First a bit of notation: $Y_i(t)$ is 1 if bservation i is "at risk" at time t and 0 otherwise. For a simple surivival (`ncol(y)==2`) a subject is at risk until the time of censoring or death (first column of `y`). For (start, stop] data (`ncol(y)==3`) a subject becomes a part of the risk set at start+0 and stays through stop. $dN_i(t)$ will be 1 if subject i had an event at time t. The risk score for each subject is $r_i = \exp(X_i\beta)$.

The Breslow increment at time t is $\sum w_i dN_i(t) / \sum w_i r_i Y_i(t)$, the number of events at time t over the number at risk at time t. The final survival is `exp(-cumsum(increment))`.

The Kalbfleish-Prentice increment is a multiplicative term z which is the solution to the equation

$$\sum w_i r_i Y_i(t) = \sum dN_i(t) w_i \frac{r_i}{1 - z(t)^{r_i}}$$

The left hand side is the weighted number at risk at time t, the right hand side is a sum over the tied events at that time. If there is only one event the equation has a closed form solution. If not, and knowing the solution must lie between 0 and 1, we do 35 steps of bisection to get a solution within 1e-8. An alternative is to use the -log of the Breslow estimate as a starting estimate, which is faster but requires a more sophisticated iteration logic. The final curve is $\prod_t z(t)^{r_c}$ where r_c is the risk score for the target subject.

The Efron estimate can be viewed as a modified Breslow estimate under the assumption that tied deaths are not really tied – we just don't know the order. So if there are 3 subjects who die at some time t we will have three psuedo-terms for t, $t + \epsilon$, and $t + 2\epsilon$. All 3 subjects are present for the denominator of the first term, 2/3 of each for the second, and 1/3 for the third terms denominator. All contribute 1/3 of the weight to each numerator (1/3 chance they were the one to die there). The formulas will require $\sum w_i dN_i(t)$, $\sum w_i r_i dN_i(t)$, and $\sum w_i X_i dN_i(t)$, i.e., the sums only over the deaths.

For simple survival data the risk sum $\sum w_i r_i Y_i(t)$ for all the unique death times t is fast to compute as a cumulative sum, starting at the longest followup time and summing towards the shortest. There are two algorithms for (start, stop] data.

- Do a separate sum at each death time. The problem is for very large data sets. For each death time the selection `who <- (start<t & stop>=t)` is $O(n)$ and can take more time then all the remaining calculations together.

- Use the difference of two cumulative sums, one ordered by start time and one ordered by stop time. This is $O(2n)$ for the intial sums. The problem here is potential round off error if the sums get large, which can happen if the time scale were very, very finely divided. This issue is mostly precluded by subtracting means first.

We compute the extended number still at risk — all whose stop time is $\geq$ each unique death time — in the vector `xin`. From this we have to subtract all those who haven't actually entered yet found in `xout`. Remember that (3,20] enters at time 3+. The total at risk at any time is the difference between them. Output is only for the stop times; a call to approx is used to reconcile the two time sets. The `irisk` vector is for the printout, it is a sum of weighted counts rather than weighted risk scores.

⟨*agsurv*⟩=
```
 agsurv <- function(y, x, wt, risk, survtype, vartype) {
     nvar <- ncol(as.matrix(x))
     status <- y[,ncol(y)]
     dtime <- y[,ncol(y) -1]
     death <- (status==1)

     time <- sort(unique(dtime))
     nevent <- as.vector(rowsum(wt*death, dtime))
     ncens  <- as.vector(rowsum(wt*(!death), dtime))
     wrisk <- wt*risk
     rcumsum <- function(x) rev(cumsum(rev(x))) # sum from last to first
     nrisk <- rcumsum(rowsum(wrisk, dtime))
     irisk <- rcumsum(rowsum(wt, dtime))
     if (ncol(y) ==2) {
         temp2  <- rowsum(wrisk*x, dtime)
         xsum   <- apply(temp2, 2, rcumsum)
         }
     else {
         delta <- min(diff(time))/2
         etime <- c(sort(unique(y[,1])), max(y[,1])+delta)  #unique entry times
         indx  <- approx(etime, 1:length(etime), time, method='constant',
                         rule=2, f=1)$y
         esum <- rcumsum(rowsum(wrisk, y[,1]))  #not yet entered
         nrisk <- nrisk - c(esum,0)[indx]
         irisk <- irisk - c(rcumsum(rowsum(wt, y[,1])),0)[indx]
         xout   <- apply(rowsum(wrisk*x, y[,1]), 2, rcumsum) #not yet entered
         xin  <- apply(rowsum(wrisk*x, dtime), 2, rcumsum) # dtime or alive
         xsum  <- xin - (rbind(xout,0))[indx,,drop=F]
         }

     ndeath <- rowsum(status, dtime)  #unweighted death count
```

The KP estimate requires a short C routine to do the iteration efficiently, and the Efron estimate needs a second C routine to efficiently compute the partial sums.

⟨*agsurv*⟩=

```
    ntime  <- length(time)
    if (survtype ==1) {  #Kalbfleisch-Prentice
        indx <- (which(status==1))[order(dtime[status==1])] #deaths
        km <- .C(Cagsurv4,
             as.integer(ndeath),
             as.double(risk[indx]),
             as.double(wt[indx]),
             as.integer(ntime),
             as.double(nrisk),
             inc = double(ntime))
        }

    if (survtype==3 || vartype==3) {  # Efron approx
        xsum2 <- rowsum((wrisk*death) *x, dtime)
        erisk <- rowsum(wrisk*death, dtime)  #risk score sums at each death
        tsum  <- .C(Cagsurv5,
                    as.integer(length(nevent)),
                    as.integer(nvar),
                    as.integer(ndeath),
                    as.double(nrisk),
                    as.double(erisk),
                    as.double(xsum),
                    as.double(xsum2),
                    sum1 = double(length(nevent)),
                    sum2 = double(length(nevent)),
                    xbar = matrix(0., length(nevent), nvar))
        }
    haz <- switch(survtype,
                   nevent/nrisk,
                   nevent/nrisk,
                   nevent* tsum$sum1)
    varhaz <- switch(vartype,
                   nevent/(nrisk *
                             ifelse(nevent>=nrisk, nrisk, nrisk-nevent)),
                   nevent/nrisk^2,
                   nevent* tsum$sum2)
    xbar <- switch(vartype,
                  (xsum/nrisk)*haz,
                  (xsum/nrisk)*haz,
                  nevent * tsum$xbar)

    result <- list(n= nrow(y), time=time, n.event=nevent, n.risk=irisk,
                  n.censor=ncens, hazard=haz,
                  cumhaz=cumsum(haz), varhaz=varhaz, ndeath=ndeath,
                  xbar=apply(matrix(xbar, ncol=nvar),2, cumsum))
```

```
    if (survtype==1) result$surv <- km$inc
    result
    }
```

The arguments to this function are the number of unique times n, which is the length of the vectors ndeath (number at each time), denom, and the returned vector km. The risk and wt vectors contain individual values for the subjects with an event. Their length will be equal to sum(ndeath).

⟨*agsurv4*⟩=

```
#include "survS.h"
#include "survproto.h"

void agsurv4(Sint   *ndeath,   double *risk,    double *wt,
             Sint   *sn,        double *denom,   double *km)
{
    int i,j,k, l;
    int n;  /* number of unique death times */
    double sumt, guess, inc;

    n = *sn;
    j =0;
    for (i=0; i<n; i++) {
        if (ndeath[i] ==0) km[i] =1;
        else if (ndeath[i] ==1) { /* not a tied death */
            km[i] = pow(1- wt[j]*risk[j]/denom[i], 1/risk[j]);
            }
        else { /* biscection solution */
            guess = .5;
            inc = .25;
            for (l=0; l<35; l++) { /* bisect it to death */
                sumt =0;
                for (k=j; k<(j+ndeath[i]); k++) {
                    sumt +=  wt[k]*risk[k]/(1-pow(guess, risk[k]));
                }
            if (sumt < denom[i])  guess += inc;
            else          guess -= inc;
            inc = inc/2;
            }
            km[i] = guess;
        }
        j += ndeath[i];
    }
}
```

Do a computation which is slow in R, needed for the Efron approximation. Input arguments are

n number of observations (unique death times)

d number of deaths at that time

nvar number of covariates

x1 weighted number at risk at the time

x2 sum of weights for the deaths

xsum matrix containing the cumulative sum of x values

xsum2 matrix of sums, only for the deaths

On output the values are

- d=0: the outputs are unchanged (they initialize at 0)
- d=1

  sum1 `1/x1`

  sum2 `1/x1^2`

  xbar `xsum/x1^2`

- d=2

  sum1 (1/2) `( 1/x1 + 1/(x1 - x2/2))`

  sum2 (1/2) (same terms, squared)

  xbar (1/2) `(xsum/x1^2 + (xsum - 1/2 x3)/(x1- x2/2)^2)`

- d=3

  sum1 (1/3) `(1/x1 + 1/(x1 - x2/3 + 1/(x1 - 2*x2/3))`

  sum2 (1/3) (same terms, squared)

  xbar (1/3)
  ```
  (xsum/x1^2 + (xsum - 1/3 xsum2)/(x1- x2/3)^2 +
      (xsum - 2/3 xsum2)/(x1- 2/3 x3)^2)
  ```

- etc

Sum1 will be the increment to the hazard, sum2 the increment to the first term of the variance, and xbar the increment in the hazard times the mean of x at this point.

⟨*agsurv5*⟩=
```
 #include "survS.h"
 void agsurv5(Sint *n2,     Sint *nvar2,  Sint *dd, double *x1,
              double *x2,   double *xsum, double *xsum2,
              double *sum1, double *sum2, double *xbar) {
     double temp;
     int i,j, k, kk;
     double d;
```

```
int n, nvar;

n = n2[0];
nvar = nvar2[0];

for (i=0; i< n; i++) {
    d = dd[i];
    if (d==1){
        temp = 1/x1[i];
        sum1[i] = temp;
        sum2[i] = temp*temp;
        for (k=0; k< nvar; k++)
            xbar[i+ n*k] = xsum[i + n*k] * temp*temp;
        }
    else {
        temp = 1/x1[i];
        for (j=0; j<d; j++) {
            temp = 1/(x1[i] - x2[i]*j/d);
            sum1[i] += temp/d;
            sum2[i] += temp*temp/d;
            for (k=0; k< nvar; k++){
                kk = i + n*k;
                xbar[kk] += ((xsum[kk] - xsum2[kk]*j/d) * temp*temp)/d;
                }
            }
        }
    }
}
```

5 The Fine-Gray model

For competing risks with ending states 1, 2, ... k, the Fine-Gray approach turns these into a set of simple 2-state Cox models:

- (not yet in state 1) $\longrightarrow$ state 1
- (not yet in state 2) $\longrightarrow$ state 2
- ...

Each of these is now a simple Cox model, assuming that we are willing to make a proportional hazards assumption. There is one added complication: when estimating the first model, one wants to use the data set that would have occured if the subjects being followed for state 1 had not had an artificial censoring, that is, had been clinically followed as though they were still in the denominator of subjects who are at risk of transition to state 1. Sometimes this can be filled in directly, e.g., if we knew the enrollment dates for each subject along with the date that follow-up for the study was terminated, and there was no lost to follow-up (only administrative

censoring.) In practice what is done is to estimate the overall censoring distribution and give subjects artificial follow-up.

The function below creates a data set that can then be used with coxph.

⟨*finegray*⟩=

```
 finegray <- function(formula, data, subset, na.action= na.pass,
                      etype, prefix="fg", count="", id) {
     Call <- match.call()
     indx <- match(c("formula", "data", "subset", "id"),
               names(Call), nomatch=0)
     if (indx[1] ==0) stop(gettextf("'%s' argument is required", "formula"))
     temp <- Call[c(1,indx)]  # only keep the arguments we wanted
     temp$na.action <- na.action
     temp[[1L]] <- quote(stats::model.frame)  # change the function called

     special <- c("strata", "cluster")
     temp$formula <- if(missing(data)) terms(formula, special)
     else              terms(formula, special, data=data)

     mf <- eval(temp, parent.frame())
     if (nrow(mf) ==0) stop("No (non-missing) observations")
     Terms <- terms(mf)

     Y <- model.extract(mf, "response")
     if (!inherits(Y, "Surv")) stop("Response must be a survival object")
     type <- attr(Y, "type")
     if (type!='mright' && type!='mcounting')
         stop("Fine-Gray model requires a multi-state survival")
     nY <- ncol(Y)
     states <- attr(Y, "states")

     strats <- attr(Terms, "specials")$strata
     if (length(strats)) {
         stemp <- untangle.specials(Terms, 'strata', 1)
         if (length(stemp$vars)==1) strata <- mf[[stemp$vars]]
         else strata <- survival::strata(mf[,stemp$vars], shortlabel=TRUE)
         istrat <- as.numeric(strata)
         mf[stemp$vars] <- NULL
         }
     else istrat <- rep(1, nrow(mf))

     id <- model.extract(mf, "id")
     if (!is.null(id)) mf["(id)"] <- NULL  # don't leave it in result

     cluster<- attr(Terms, "specials")$cluster
     if (length(cluster)) {
```

```
    if (!is.null(id)) stop("'id' argument and a 'cluster()' term are redundant")
    tempc <- untangle.specials(Terms, 'cluster', 1)
    id <- strata(mf[,tempc$vars], shortlabel=TRUE)  #allow multiples
    mf[tempc$vars] <- NULL
}

# If there is start-stop data, then there needs to be a cluster()
#  argument or an id argument, and we check that this is indeed
#  a competing risks form of data.
# Mark the first and last obs of each subject, as we need it later.
#  Observations may not be in time order within a subject
delay <- FALSE  # is there delayed entry?
if (type=="mcounting") {
    if (is.null(id)) stop("(start, stop] data requires a subject id")
    else {
        index <- order(id, Y[,2]) # by time within id
        sorty <- Y[index,]
        first <- which(!duplicated(id[index]))
        last  <- c(first[-1] -1, length(id))
        if (any(sorty[-last, 3]) != 0)
            stop("a subject has a transition before their last time point")
        delta <- c(sorty[-1,1], 0) - sorty[,2]
        if (any(delta[-last] !=0))
            stop("a subject has gaps in time")
        if (any(Y[first,1] > min(Y[,2]))) delay <- TRUE
        temp1 <- temp2 <- rep(FALSE, nrow(mf))
        temp1[index[first]] <- TRUE
        temp2[index[last]]  <- TRUE
        first <- temp1  #used later
        last <-  temp2
     }
} else last <- rep(TRUE, nrow(mf))

if (missing(etype)) enum <- 1  #generate a data set for which endpoint?
else {
    index <- match(etype, states)
    if (any(is.na(index)))
        stop("'etype' argument has a state that is not in the data")
    enum <- index[1]
    if (length(index) > 1) warning("only the first endpoint was used")
}

# make sure count, if present is syntactically valid
if (!missing(count)) count <- make.names(count) else count <- NULL
oname <- paste0(prefix, c("start", "stop", "status", "wt"))
```

⟨*finegray-censor*⟩
⟨*finegray-build*⟩
}

The censoring and truncation distributions are

$$G(t) = \prod_{s \le t} \left(1 - \frac{c(s)}{r_c(s)}\right)$$
$$H(t) = \prod_{s > t} \left(1 - \frac{e(s)}{r_e(s)}\right)$$

where $c(t)$ is the number of subjects censored at time t, $e(t)$ is the number who enter at time t, and r is the size of the relevant risk set. These are equations 5 and 6 of Geskus (Biometrics 2011). Note that both G and H are right continuous functions. For tied times the assumption is that event < censor < entry. For G we use a modified Kapan-Meier where any events at censoring time t are removed from the risk set just before time t. To avoid issues with times that are nearly identical (but not quite) we first convert to an integer time scale, and then move events backwards by .2. Since this is a competing risks data set any non-censored observation for a subject is their last, so this time shift does not goof up the alignment of start, stop data. For the truncation distribution it is the subjects with times at or before time t that are in the risk set $r_e(t)$ for truncation at (or before) t. H can be calculated using an ordinary KM on the reverse time scale.

⟨*finegray-censor*⟩=
```
find2 <- function(x, vec, left.open=FALSE, ...) {
    if (!left.open) findInterval(x, vec, ...)
    else {
        # the left.open arg is a recent addition to findInterval, and I want
        #  this to work in 3.2.0 (institutional installs).  In another cycle or
        #  so we can drop this workaround and call findInterval directly
        #
        length(vec) - findInterval(-x, rev(-vec), ...)
    }
}

if (ncol(Y) ==2) {
    temp <- min(Y[,1], na.rm=TRUE)
    if (temp >0) zero <- 0
    else zero <- 2*temp -1  # a value less than any observed y
    Y <- cbind(zero, Y)  # add a start column
}
utime <- sort(unique(c(Y[,1:2])))  # all the unique times
newtime <- matrix(findInterval(Y[,1:2], utime), ncol=2)
status <- Y[,3]

newtime[status !=0, 2] <- newtime[status !=0,2] - .2
```

```
Gsurv <- survfit(Surv(newtime[,1], newtime[,2], status==0) ~ istrat,
                 se.fit=FALSE)
```

The calculation for H is also done on the integer scale. Otherwise we will someday be clobbered by times that differ only in round off error. The only nuisance is the status variable, which is 1 for the first row of each subject, since the data set may not be in sorted order. The offset of .2 used above is not needed, but due to the underlying integer scale it doesn't harm anything either. Reversal of the time scale leads to a left continuous function which we fix up later.

⟨*finegray-censor*⟩=
```
if (delay)
    Hsurv <- survfit(Surv(-newtime[,2], -newtime[,1], first) ~ istrat,
                     se.fit =FALSE)
```

Consider the following data set:

- Events of type 1 at times 1, 4, 5, 10
- Events of type 2 at times 2, 5, 8
- Censors at times 3, 4, 4, 6, 8, 9, 12

The censoring distribution will have the following shape:

interval	(0,3]	(3,4]	(4,6]	(6,8]	(8,12]	12+
C(t)	1	11/12	(11/12)(8/10)	(11/15)(5/6)	(11/15)(5/6)(3/4)	0
	1.0000	.9167	.7333	.6111	.4583	

Notice that the event at time 4 is not counted in the risk set at time 4, so the jump is 8/10 rather than 8/11. Likewise at time 8 the risk set has 4 instead of 5: censors occur after deaths.

When creating the data set for event type 1, subjects who have an event of type 2 get extended out using this censoring distribution. The event at time 2, for instance, appears as a censored observation with time dependent weights of $G(t)$. The type 2 event at time 5 has weight 1 up through time 5, then weights of $G(t)/C(5)$ for the remainder. This means a weight of 1 over (5,6], 5/6 over (6,8], (5/6)(3/4) over (9,12] and etc.

Though there are 6 unique censoring intervals, in the created data set for event type 1 we only need to know case weights at times 1, 4, 5, and 10; the information from the (4,6] and (6,8] intervals will never be used. To create a minimal sized data set we can leave those intervals out. $G(t)$ only drops to zero if the largest time(s) are censored observations, so by definition no events lie in an interval with $G(t) = 0$.

If there is delayed entry, then the set of intervals is larger due to a merge with the jumps in Hsurv. The truncation distribution Hsurv (H) will become 0 at the first entry time; it is a left continuous function whereas Gsurv (G) is right continuous. We can slide H one point to the left and merge them at the jump points.

⟨*finegray-build*⟩=
```
status <- Y[, 3]
maxtime <- max(Y[,2])
```

```
# Do computations separately for each stratum
stratfun <- function(i) {
    keep <- (istrat ==i)
    times <- sort(unique(Y[keep & status == enum, 2])) #unique event times
    if (length(times)==0) return(NULL)  #no events in this stratum
    tdata <- mf[keep, -1, drop=FALSE]

    if (dim(Gsurv)==1) {
        # keep only the event times, and convert back to the original time units
        if (delay) {
            dtime <- rev(-Hsurv$time[Hsurv$n.event > 0])
            dprob <- c(rev(Hsurv$surv[Hsurv$n.event > 0])[-1], 1)
            ctime <- Gsurv$time[Gsurv$n.event > 0]
            cprob <- c(1, Gsurv$surv[Gsurv$n.event > 0])
            temp <- sort(unique(c(dtime, ctime))) # these will all be integers
            index1 <- findInterval(temp, dtime)
            index2 <- findInterval(temp, ctime)
            ctime <- utime[temp]
            cprob <- dprob[index1] * cprob[index2+1]  # G(t)H(t), eq 11 Geskus
        }
        else {
            ctime <- utime[Gsurv$time[Gsurv$n.event > 0]]
            cprob <- Gsurv$surv[Gsurv$n.event > 0]
        }
    } else {
        Gtemp <- Gsurv[i]
        if (delay) {
            Htemp <- Hsurv[i]
            dtime <- rev(-Htemp$time[Htemp$n.event > 0])
            dprob <- c(rev(Htemp$surv[Htemp$n.event > 0])[-1], 1)
            ctime <- Gtemp$time[Gtemp$n.event > 0]
            cprob <- c(1, Gtemp$surv[Gtemp$n.event > 0])
            temp <- sort(unique(c(dtime, ctime))) # these will all be integers
            index1 <- findInterval(temp, dtime)
            index2 <- findInterval(temp, ctime)
            ctime <- utime[temp]
            cprob <- dprob[index1] * cprob[index2+1]  # G(t)H(t), eq 11 Geskus
        }
        else {
            ctime <- utime[Gtemp$time[Gtemp$n.event > 0]]
            cprob <- Gtemp$surv[Gtemp$n.event > 0]
        }
    }

    ct2 <- c(ctime, maxtime)
```

```
    cp2 <- c(1.0, cprob)
    index <- find2(times, ct2, left.open=TRUE)
    index <- sort(unique(index))  # the intervals that were actually seen
    # times before the first ctime get index 0, those between 1 and 2 get 1
    ckeep <- rep(FALSE, length(ct2))
    ckeep[index] <- TRUE
    expand <- (Y[keep, 3] !=0 & Y[keep,3] != enum & last[keep]) #which rows to expand
    split <- .Call("finegray", Y[keep,1], Y[keep,2], ct2, cp2, expand,
                   c(TRUE, ckeep))
    tdata <- tdata[split$row,,drop=FALSE]
    tstat <- ifelse(status[split$row]== enum, 1, 0)

    tdata[[oname[1]]] <- split$start
    tdata[[oname[2]]] <- split$end
    tdata[[oname[3]]] <- tstat
    tdata[[oname[4]]] <- split$wt
    if (!is.null(count)) tdata[[count]] <- split$add
    tdata
}

if (max(istrat) ==1) result <- stratfun(1)
else {
    tlist <- lapply(seq_len(max(istrat)), stratfun)
    result <- do.call("rbind", tlist)
}

rownames(result) <- NULL   #remove all the odd labels that R adds
attr(result, "event") <- states[enum]
result
```

5.1 The predict method

The `predict.coxph` function produces various types of predicted values from a Cox model. The arguments are

obejct The result of a call to `coxph`.

newdata Optionally, a new data set for which prediction is desired. If this is absent predictions are for the observations used fit the model.

type The type of prediction

- lp = the linear predictor for each observation
- risk = the risk score $exp(lp)$ for each observation
- expected = the expected number of events
- terms = a matrix with one row per subject and one column for each term in the model.

se.fit Whether or not to return standard errors of the predictions.

na.action What to do with missing values *if* there is new data.

terms The terms that are desired. This option is almost never used, so rarely in fact that it's hard to justify keeping it.

collapse An optional vector of subject identifiers, over which to sum or 'collapse' the results

reference the reference context for centering the results

... All predict methods need to have a ... argument; we make no use of it however.

The first task of the routine is to reconsruct necessary data elements that were not saved as a part of the `coxph` fit. We will need the following components:

- for type='expected' residuals we need the orignal survival y. This is saved in coxph objects by default so will only need to be fetched in the highly unusual case that a user specfied `y=FALSE` in the orignal call.
- for any call with either newdata, standard errors, or type='terms' the original X matrix, weights, strata, and offset. When checking for the existence of a saved X matrix we can't use `object$x` since that will also match the `xlevels` component.
- the new data matrix, if any

```
⟨predict.coxph⟩=
 predict.coxph <- function(object, newdata,
                        type=c("lp", "risk", "expected", "terms"),
                        se.fit=FALSE, na.action=na.pass,
                        terms=names(object$assign), collapse,
                        reference=c("strata", "sample"), ...) {
     ⟨pcoxph-init⟩
     ⟨pcoxph-getdata⟩
     if (type=="expected") {
         ⟨pcoxph-expected⟩
         }
     else {
         ⟨pcoxph-simple⟩
         ⟨pcoxph-terms⟩
         }
     ⟨pcoxph-finish⟩
     }
```

We start of course with basic argument checking. Then retrieve the model parameters: does it have a strata statement, offset, etc. The `Terms2` object is a model statement without the strata or cluster terms, appropriate for recreating the matrix of covariates X. For type=expected the response variable needs to be kept, if not we remove it as well since the user's newdata might not contain one.

⟨*pcoxph-init*⟩=
```
 if (!inherits(object, 'coxph'))
     stop("Primary argument much be a coxph object")

 Call <- match.call()
 type <-match.arg(type)
 n <- object$n
 Terms <-  object$terms

 if (!missing(terms)) {
     if (is.numeric(terms)) {
         if (any(terms != floor(terms) |
                 terms > length(object$assign) |
                 terms <1)) stop("Invalid terms argument")
         }
     else if (any(is.na(match(terms, names(object$assign)))))
        stop("a name given in the terms argument not found in the model")
     }

 # I will never need the cluster argument, if present delete it.
 #  Terms2 are terms I need for the newdata (if present), y is only
 #  needed there if type == 'expected'
 if (length(attr(Terms, 'specials')$cluster)) {
     temp <- untangle.specials(Terms, 'cluster', 1)
     Terms  <- object$terms[-temp$terms]
     }
 else Terms <- object$terms

 if (type != 'expected') Terms2 <- delete.response(Terms)
 else Terms2 <- Terms

 has.strata <- !is.null(attr(Terms, 'specials')$strata)
 has.offset <- !is.null(attr(Terms, 'offset'))
 has.weights <- any(names(object$call) == 'weights')
 na.action.used <- object$na.action
 n <- length(object$residuals)

 if (missing(reference) && type=="terms") reference <- "sample"
 else reference <- match.arg(reference)
```

The next task of the routine is to reconsruct necessary data elements that were not saved as a part of the `coxph` fit. We will need the following components:

- for type='expected' residuals we need the orignal survival y. This is saved in coxph objects by default so will only need to be fetched in the highly unusual case that a user specfied `y=FALSE` in the orignal call. We also need the strata in this case. Grabbing it is the same amount of work as grabbing X, so gets lumped with that case in the code.

- for any call with either standard errors, reference strata, or type='terms' the original X matrix, weights, strata, and offset. When checking for the existence of a saved X matrix we can't use `object$x` since that will also match the `xlevels` component.
- the new data matrix, if present, along with offset and strata.

For the case that none of the above are needed, we can use the `linear.predictors` component of the fit. The variable `use.x` signals this case, which takes up almost none of the code but is common in usage.

The check below that nrow(mf)==n is to avoid data sets that change under our feet. A fit was based on data set "x", and when we reconstruct the data frame it is a different size! This means someone changed the data between the model fit and the extraction of residuals. One other non-obvious case is that coxph treats the model `age:strata(grp)` as though it were `age:strata(grp) + strata(grp)`. The untangle.specials function will return `vars= strata(grp), terms=integer(0)`; the first shows a strata to extract and the second that there is nothing to remove from the terms structure.

⟨*pcoxph-getdata*⟩=
```
 have.mf <- FALSE
 if (type == 'expected') {
     y <- object[['y']]
     if (is.null(y)) {  # very rare case
         mf <- stats::model.frame(object)
         y <-  model.extract(mf, 'response')
         have.mf <- TRUE  #for the logic a few lines below, avoid double work
         }
     }

 if (se.fit || type=='terms' || (!missing(newdata) && type=="expected") ||
     (has.strata && (reference=="strata") || type=="expected")) {
     use.x <- TRUE
     if (is.null(object[['x']]) || has.weights || has.offset ||
          (has.strata && is.null(object$strata))) {
         # I need the original model frame
         if (!have.mf) mf <- stats::model.frame(object)
         if (nrow(mf) != n)
             stop("Data is not the same size as it was in the original fit")
         x <- model.matrix(object, data=mf)
         if (has.strata) {
             if (!is.null(object$strata)) oldstrat <- object$strata
             else {
                 stemp <- untangle.specials(Terms, 'strata')
                 if (length(stemp$vars)==1) oldstrat <- mf[[stemp$vars]]
                 else oldstrat <- strata(mf[,stemp$vars], shortlabel=TRUE)
               }
         }
         else oldstrat <- rep(0L, n)
```

```
        weights <- model.weights(mf)
        if (is.null(weights)) weights <- rep(1.0, n)
        offset <- model.offset(mf)
        if (is.null(offset))  offset  <- 0
    }
    else {
        x <- object[['x']]
        if (has.strata) oldstrat <- object$strata
        else oldstrat <- rep(0L, n)
        weights <-  rep(1.,n)
        offset <-   0
    }
}
else {
    # I won't need strata in this case either
    if (has.strata) {
        stemp <- untangle.specials(Terms, 'strata', 1)
        Terms2  <- Terms2[-stemp$terms]
        has.strata <- FALSE  #remaining routine never needs to look
    }
    oldstrat <- rep(0L, n)
    offset <- 0
    use.x <- FALSE
}
```

Now grab data from the new data set. We want to use model.frame processing, in order to correctly expand factors and such. We don't need weights, however, and don't want to make the user include them in their new dataset. Thus we build the call up the way it is done in coxph itself, but only keeping the newdata argument. Note that terms2 may have fewer variables than the original model: no cluster and if type!= expected no response. If the original model had a strata, but newdata does not, we need to remove the strata from xlev to stop a spurious warning message.

⟨*pcoxph-getdata*⟩=

```
if (!missing(newdata)) {
    use.x <- TRUE  #we do use an X matrix later
    tcall <- Call[c(1, match(c("newdata", "collapse"), names(Call), nomatch=0))]
    names(tcall)[2] <- 'data'  #rename newdata to data
    tcall$formula <- Terms2  #version with no response
    tcall$na.action <- na.action #always present, since there is a default
    tcall[[1L]] <- quote(stats::model.frame)  # change the function called

    if (!is.null(attr(Terms, "specials")$strata) && !has.strata) {
       temp.lev <- object$xlevels
       temp.lev[[stemp$vars]] <- NULL
```

```
        tcall$xlev <- temp.lev
    }
    else tcall$xlev <- object$xlevels
    mf2 <- eval(tcall, parent.frame())

    collapse <- model.extract(mf2, "collapse")
    n2 <- nrow(mf2)

    if (has.strata) {
        if (length(stemp$vars)==1) newstrat <- mf2[[stemp$vars]]
        else newstrat <- strata(mf2[,stemp$vars], shortlabel=TRUE)
        if (any(is.na(match(newstrat, oldstrat))))
            stop("New data has a strata not found in the original model")
        else newstrat <- factor(newstrat, levels=levels(oldstrat)) #give it all
        if (length(stemp$terms))
            newx <- model.matrix(Terms2[-stemp$terms], mf2,
                        contr=object$contrasts)[,-1,drop=FALSE]
        else newx <- model.matrix(Terms2, mf2,
                        contr=object$contrasts)[,-1,drop=FALSE]
         }
    else {
        newx <- model.matrix(Terms2, mf2,
                        contr=object$contrasts)[,-1,drop=FALSE]
        newstrat <- rep(0L, nrow(mf2))
        }

    newoffset <- model.offset(mf2)
    if (is.null(newoffset)) newoffset <- 0
    if (type== 'expected') {
        newy <- model.response(mf2)
        if (attr(newy, 'type') != attr(y, 'type'))
            stop("New data has a different survival type than the model")
        }
    na.action.used <- attr(mf2, 'na.action')
    }
else n2 <- n
```

When we do not need standard errors the computation of expected hazard is very simple since the martingale residual is defined as status - expected. The 0/1 status is saved as the last column of y.

⟨*pcoxph-expected*⟩=

```
if (missing(newdata))
    pred <- y[,ncol(y)] - object$residuals
if (!missing(newdata) || se.fit) {
    ⟨pcoxph-expected2⟩
    }
```

The more general case makes use of the [agsurv] routine to calculate a survival curve for each strata. The routine is defined in the section on individual Cox survival curves. The code here closely matches that. The routine only returns values at the death times, so we need approx to get a complete index.

One non-obvious, but careful choice is to use the residuals for the predicted value instead of the compuation below, whenever operating on the original data set. This is a consequence of the Efron approx. When someone in a new data set has exactly the same time as one of the death times in the original data set, the code below implicitly makes them the "last" death in the set of tied times. The Efron approx puts a tie somewhere in the middle of the pack. This is way too hard to work out in the code below, but thankfully the original Cox model already did it. However, it does mean that a different answer will arise if you set newdata = the original coxph data set. Standard errors have the same issue, but 1. they are hardly used and 2. the original coxph doesn't do that calculation. So we do what's easiest.

⟨*pcoxph-expected2*⟩=
```
 ustrata <- unique(oldstrat)
 risk <- exp(object$linear.predictors)
 x <- x - rep(object$means, each=nrow(x))  #subtract from each column
 if (missing(newdata)) #se.fit must be true
     se <- double(n)
 else {
     pred <- se <- double(nrow(mf2))
     newx <- newx - rep(object$means, each=nrow(newx))
     newrisk <- c(exp(newx %*% object$coef))
     }

 survtype<- ifelse(object$method=='efron', 3,2)
 for (i in ustrata) {
     indx <- which(oldstrat == i)
     afit <- agsurv(y[indx,,drop=F], x[indx,,drop=F],
                                   weights[indx], risk[indx],
                                   survtype, survtype)
     afit.n <- length(afit$time)
     if (missing(newdata)) {
         # In this case we need se.fit, nothing else
         j1 <- approx(afit$time, 1:afit.n, y[indx,1], method='constant',
                      f=0, yleft=0, yright=afit.n)$y
         chaz <- c(0, afit$cumhaz)[j1 +1]
         varh <- c(0, cumsum(afit$varhaz))[j1 +1]
         xbar <- rbind(0, afit$xbar)[j1+1,,drop=F]
         if (ncol(y)==2) {
             dt <- (chaz * x[indx,]) - xbar
             se[indx] <- sqrt(varh + rowSums((dt %*% object$var) *dt)) *
                 risk[indx]
             }
         else {
```

```
        j2 <- approx(afit$time, 1:afit.n, y[indx,2], method='constant',
                 f=0, yleft=0, yright=afit.n)$y
        chaz2 <- c(0, afit$cumhaz)[j2 +1]
        varh2 <- c(0, cumsum(afit$varhaz))[j2 +1]
        xbar2 <- rbind(0, afit$xbar)[j2+1,,drop=F]
        dt <- (chaz * x[indx,]) - xbar
        v1 <- varh +  rowSums((dt %*% object$var) *dt)
        dt2 <- (chaz2 * x[indx,]) - xbar2
        v2 <- varh2 + rowSums((dt2 %*% object$var) *dt2)
        se[indx] <- sqrt(v2-v1)* risk[indx]
        }
    }

else {
    #there is new data
    use.x <- TRUE
    indx2 <- which(newstrat == i)
    j1 <- approx(afit$time, 1:afit.n, newy[indx2,1],
                 method='constant', f=0, yleft=0, yright=afit.n)$y
    chaz <-c(0, afit$cumhaz)[j1+1]
    pred[indx2] <- chaz * newrisk[indx2]
    if (se.fit) {
        varh <- c(0, cumsum(afit$varhaz))[j1+1]
        xbar <- rbind(0, afit$xbar)[j1+1,,drop=F]
        }
    if (ncol(y)==2) {
        if (se.fit) {
            dt <- (chaz * newx[indx2,]) - xbar
            se[indx2] <- sqrt(varh + rowSums((dt %*% object$var) *dt)) *
                newrisk[indx2]
            }
        }
    else {
        j2 <- approx(afit$time, 1:afit.n, newy[indx2,2],
                 method='constant', f=0, yleft=0, yright=afit.n)$y
                    chaz2 <- approx(-afit$time, afit$cumhaz, -newy[indx2,2],
                   method="constant", rule=2, f=0)$y
        chaz2 <-c(0, afit$cumhaz)[j2+1]
        pred[indx2] <- (chaz2 - chaz) * newrisk[indx2]

        if (se.fit) {
            varh2 <- c(0, cumsum(afit$varhaz))[j1+1]
            xbar2 <- rbind(0, afit$xbar)[j1+1,,drop=F]
            dt <- (chaz * newx[indx2,]) - xbar
            dt2 <- (chaz2 * newx[indx2,]) - xbar2
```

```
                v2 <- varh2 + rowSums((dt2 %*% object$var) *dt2)
                v1 <- varh +  rowSums((dt %*% object$var) *dt)
                se[indx2] <- sqrt(v2-v1)* risk[indx2]
                }
            }
        }
    }
```

For these three options what is returned is a *relative* prediction which compares each observation to the average for the data set. Partly this is practical. Say for instance that a treatment covariate was coded as 0=control and 1=treatment. If the model were refit using a new coding of 3=control 4=treatment, the results of the Cox model would be exactly the same with respect to coefficients, variance, tests, etc. The raw linear predictor $X\beta$ however would change, increasing by a value of 3β. The relative predictor

$$\eta_i = X_i\beta - (1/n)\sum_j X_j\beta \tag{7}$$

will stay the same. The second reason for doing this is that the Cox model is a relative risks model rather than an absolute risks model, and thus relative predictions are almost certainly what the user was thinking of.

When the fit was for a stratified Cox model more care is needed. For instance assume that we had a fit that was stratified by sex with covaritate x, and a second data set were created where for the females x is replaced by $x + 3$. The Cox model results will be unchanged for the two models, but the 'normalized' linear predictors $(x - \overline{x})'\beta$ will not be the same. This reflects a more fundamental issue that the for a stratified Cox model relative risks are well defined only *within* a stratum, i.e. for subject pairs that share a common baseline hazard. The example above is artificial, but the problem arises naturally whenever the model includes a strata by covariate interaction. So for a stratified Cox model the predictions should be forced to sum to zero within each stratum, or equivalently be made relative to the weighted mean of the stratum. Unfortunately, this important issue was not realized until late in 2009 when a puzzling query was sent to the author involving the results from such an interaction. Note that this issue did not arise with type='expected', which has a natural scaling.

An offset variable, if specified, is treated like any other covariate with respect to centering. The logic for this choice is not as compelling, but it seemed the best that I could do. Note that offsets play no role whatever in predicted terms, only in the lp and risk.

Start with the simple ones

⟨*pcoxph-simple*⟩=
```
 if (is.null(object$coefficients))
     coef<-numeric(0)
 else {
     # Replace any NA coefs with 0, to stop NA in the linear predictor
     coef <- ifelse(is.na(object$coefficients), 0, object$coefficients)
     }

 if (missing(newdata)) {
```

```
    offset <- offset - mean(offset)
    if (has.strata && reference=="strata") {
        # We can't use as.integer(oldstrat) as an index, if oldstrat is
        #   a factor variable with unrepresented levels as.integer could
        #   give 1,2,5 for instance.
        xmeans <- rowsum(x*weights, oldstrat)/c(rowsum(weights, oldstrat))
        newx <- x - xmeans[match(oldstrat,row.names(xmeans)),]
        }
    else if (use.x) newx <- x - rep(object$means, each=nrow(x))
    }
else {
    offset <- newoffset - mean(offset)
    if (has.strata && reference=="strata") {
        xmeans <- rowsum(x*weights, oldstrat)/c(rowsum(weights, oldstrat))
        newx <- newx - xmeans[match(newstrat, row.names(xmeans)),]
        }
    else newx <- newx - rep(object$means, each=nrow(newx))
    }

if (type=='lp' || type=='risk') {
    if (use.x) pred <- drop(newx %*% coef) + offset
    else pred <- object$linear.predictors
    if (se.fit) se <- sqrt(rowSums((newx %*% object$var) *newx))

    if (type=='risk') {
        pred <- exp(pred)
        if (se.fit) se <- se * sqrt(pred)  # standard Taylor series approx
        }
    }
```

The type=terms residuals are a bit more work. In Splus this code used the Build.terms function, which was essentially the code from predict.lm extracted out as a separate function. As of March 2010 (today) a check of the Splus function and the R code for predict.lm revealed no important differences. A lot of the bookkeeping in both is to work around any possible NA coefficients resulting from a singularity. The basic formula is to

1. If the model has an intercept, then sweep the column means out of the X matrix. We've already done this.
2. For each term separately, get the list of coefficients that belong to that term; call this list `tt`.
3. Restrict X, β and V (the variance matrix) to that subset, then the linear predictor is $X\beta$ with variance matrix XVX'. The standard errors are the square root of the diagonal of this latter matrix. This can be computed, as colSums((X

Note that the `assign` component of a coxph object is the same as that found in Splus models (a list), most R models retain a numeric vector which contains the same information but it is not

as easily used. The first first part of predict.lm in R rebuilds the list form as its `asgn` variable. I can skip this part since it is already done.

⟨*pcoxph-terms*⟩=
```
 else if (type=='terms') {
     asgn <- object$assign
     nterms<-length(asgn)
     pred<-matrix(ncol=nterms,nrow=NROW(newx))
     dimnames(pred) <- list(rownames(newx), names(asgn))
     if (se.fit) se <- pred

     for (i in 1:nterms) {
         tt <- asgn[[i]]
         tt <- tt[!is.na(object$coefficients[tt])]
         xtt <- newx[,tt, drop=F]
         pred[,i] <- xtt %*% object$coefficient[tt]
         if (se.fit)
             se[,i] <- sqrt(rowSums((xtt %*% object$var[tt,tt]) *xtt))
         }
     pred <- pred[,terms, drop=F]
     if (se.fit) se <- se[,terms, drop=F]

     attr(pred, 'constant') <- sum(object$coefficients*object$means, na.rm=T)
     }
```

To finish up we need to first expand out any missings in the result based on the na.action, and optionally collapse the results within a subject. What should we do about the standard errors when collapse is specified? We assume that the individual pieces are independent and thus var(sum) = sum(variances). The statistical justification of this is quite solid for the linear predictor, risk and terms type of prediction due to independent increments in a martingale. For expecteds the individual terms are positively correlated so the se will be too small. One solution would be to refuse to return an se in this case, but the the bias should usually be small, and besides it would be unkind to the user.

Prediction of type='terms' is expected to always return a matrix, or the R termplot() function gets unhappy.

⟨*pcoxph-finish*⟩=
```
 if (type != 'terms') {
     pred <- drop(pred)
     if (se.fit) se <- drop(se)
     }

 if (!is.null(na.action.used)) {
     pred <- napredict(na.action.used, pred)
     if (is.matrix(pred)) n <- nrow(pred)
     else                 n <- length(pred)
     if(se.fit) se <- napredict(na.action.used, se)
```

```
    }

if (!missing(collapse) && !is.null(collapse)) {
    if (length(collapse) != n2) stop("Collapse vector is the wrong length")
    pred <- rowsum(pred, collapse)  # in R, rowsum is a matrix, always
    if (se.fit) se <- sqrt(rowsum(se^2, collapse))
    if (type != 'terms') {
        pred <- drop(pred)
        if (se.fit) se <- drop(se)
        }
    }

if (se.fit) list(fit=pred, se.fit=se)
else pred
```

5.2 Concordance

The concordance statistic is gaining popularity as a measure of goodness-of-fit in survival models. Consider all pairs of subjects with (r_i, r_j) as the two risk scores for each pair and (s_i, s_j) the corresponding survival times. The c-statistic is defined by dividing these sets into four groups.

- Concordant pairs: for a Cox model this will be pairs where a shorter survival is paired with a larger risk score, e.g. $r_i > r_j$ and $s_i < s_j$
- Discordant pairs: the lower risk score has a shorter survival
- Tied pairs: there are three common choices
  - Kendall's tau: any pair where $r_i = r_j$ or $s_i = s_j$ is considered tied.
  - AUC: pairs with $r_i = r_j$ are tied; those with $s_i = s_j$ are considered incomparable. This is the definition of the AUC in logisitic regression, and has become the most common choice for Cox models as well.
  - Somer's D: All ties are treated as incomparable.
- Incomparable pairs: For survival this always includes pairs where the survival times cannot be ranked with certainty. For instance s_i is censored at time 10 and s_j is an event (or censor) at time 20. Subject i may or may not survive longer than subject j. Note that if s_i is censored at time 10 and s_j is an event at time 10 then $s_i > s_j$. Add onto this those ties that are treated as incomparable.
  Observations that are in different strata are also incomparable, since the Cox model only compares within strata.

Then the concordance statistic is defined as $(C + T/2)/(C + D + T)$. The denominator is the number of comparable pairs.

The program creates 4 variables, which are the number of concordant pairs, discordant, tied on time, and tied on x but not on time. The default concordance is based on the AUC definition, but all 4 values are reported back so that a user can recreate the others if desired.

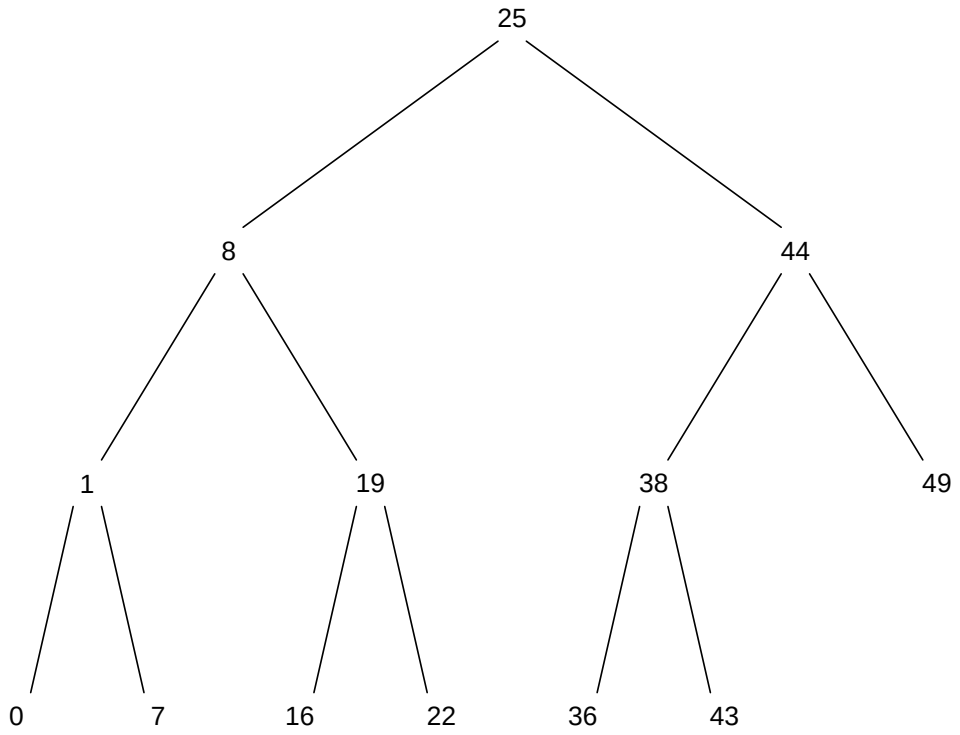

Figure 1: A balanced tree of 13 nodes.

The primary compuational questions is how to do this efficiently, i.e., better that the naive $O(n^2)$ algorithm that loops across all $n(n-1)/2$ possible pairs. There are two key ideas.

1. Rearrange the counting so that we do it by death times. For each death we count the number of other subjects in the risk set whose score is higher, lower, or tied and add it into the totals. This also neatly solves the question of time-dependent covariates.

2. To count the number higher and lower we need to rank the subjects in the risk set by their scores r_i. This can be done in $O(\log n)$ time if the data is kept in a binary tree.

Figure 1 shows a balanced binary tree containing 13 risk scores. For each node the left child and all its descendants have a smaller value than the parent, the right child and all its descendents have a larger value. Each node in figure 1 is also annotated with the total weight of observations in that node and the weight for all its children (not shown on graph). Assume that the tree shown represents all of the subjects still alive at the time a particular subject "Smith" expires, and that Smith has the risk score 2.1. The concordant pairs are all of those with a risk score greater than 2.1, which can be found by traversing the tree from the top down, adding the (parent - child) value each time we branch left (5-3 at the 2.6 node), with a last addition of the right hand child when we find the node with Smith's value (1). There are 3 concordant and

12-3=9 discordant pairs. This takes a little less than $\log_2(n)$ steps on average, as compared to an average of $n/2$ for the naive method. The difference can matter when n is large since this traversal must be done for each event. (In the code below we start at Smith's node and walk up.)

The classic way to store trees is as a linked list. There are several algorithms for adding and subtracting nodes from a tree while maintaining the balance (red-black trees, AA trees, etc) but we take a different approach. Since we need to deal with case weights in the model and we know all the risk score at the outset, the full set of risk scores is organised into a tree at the beginning and node counts are changed to zero as observations are removed.

If we index the nodes of the tree as 1 for the top, 2–3 for the next horizontal row, 4–7 for the next, ... then the parent-child traversal becomes particularly easy. The parent of node i is $i/2$ (integer arithmetic) and the children of node i are $2i$ and $2i+1$. In C code the indices start at 0 and the children are $2i+1$ and $2i+2$ and the parent is $(i-1)/2$. The following bit of code returns the indices of a sorted list when placed into such a tree. The basic idea is that the rows of the tree start at indices 1, 2, 4, ... For the above tree, the last row will contains the 1st, 3rd, ..., 11th smallest ranks. The next row above contains every other value of the ranks *not yet assigned*, and etc to the top of the tree. There is some care to make sure the result is an integer.

⟨*btree*⟩=
```
 btree <- function(n) {
     ranks <- rep(0L, n)  #will be overwritten
     yet.to.do <- 1:n
     depth <- floor(logb(n,2))
     start <- as.integer(2^depth)
     lastrow.length <- 1+n-start
     indx <- seq(1L, by=2L, length= lastrow.length)
     ranks[yet.to.do[indx]] <- start + 0:(length(indx)-1L)
     yet.to.do <- yet.to.do[-indx]

     while (start >1) {
         start <- as.integer(start/2)
         indx <- seq(1L, by=2L, length=start)
         ranks[yet.to.do[indx]] <- start + 0:(start-1L)
         yet.to.do <- yet.to.do[-indx]
     }
     ranks
 }
```

Referring again to figure 1, `btree(13)` yields the vector `8  4  9  2 10  5 11  1 12  6 13  3  7` meaning that the smallest element will be in position 8 of the tree, the next smallest in position 4, etc.

Here is a shorter recursive version. It knows the form of trees with 1, 2, or 3 nodes; and builds the others from them. The maximum depth of recursion is $\log_2(n) - 1$. It is more clever but a bit slower. (Not that it matters as both take less than 5 seconds for a million elements.)

⟨*btree2*⟩=
```
 btree <- function(n) {
```

```
tfun <- function(n, id, power) {
    if (n==1) id
    else if (n==2) c(2L *id, id)
    else if (n==3) c(2L*id, id, 2L*id +1L)
    else {
        nleft <- if (n== power*2) power  else min(power-1, n-power/2)
        c(tfun(nleft, 2L *id, power/2), id,
          tfun(n-(nleft+1), 2L*id +1L, power/2))
        }
    }
tfun(n, 1L, 2^(floor(logb(n-1,2))))
}
```

¡¡¡¡¡¡¡ local A second question is how to compute the variance of the result. ======= The next question is how to compute a variance for the result. One approach is to compute an infinitesimal jackknife (IJ) estimate, for which we need derivatives with respect to the weights. For each event the two numbers of interest are the numerator $C - D$ and the denominator $C + D + T$, which can be written directly in terms of the weights.

$$C - D = \sum_i w_i \delta_i \sum_{t_j > t_i} w_j \text{sign}(\text{r}_\text{i} - \text{r}_\text{j})$$
$$C + D + T = \sum_i w_i \delta_i \sum_{t_j > t_i} w_j$$

The first derivatives of these quantities with respect to an arbitrary subject k are

$$\frac{\partial C - D}{\partial w_k} = \sum_{t_j > t_k} w_j \delta_k \text{sign}(\text{r}_\text{k} - \text{r}_\text{j}) - \sum_{\text{t}_\text{j} < \text{t}_\text{k}} \text{w}_\text{j} \delta_\text{j} \text{sign}(\text{r}_\text{k} - \text{r}_\text{j}) \tag{8}$$
$$\frac{\partial C + D + T}{\partial w_k} = \sum_{t_j > t_k} w_j \delta_k + \sum_{t_j < t_k} w_j \delta_j \tag{9}$$
$$\tag{10}$$

We need to keep two vectors of derivatives each of length n, the number of observations. The parent routine in R will worry about data sets with multiple rows per subject, for which the person has to be added back together.

I had originally avoided this approach because it appears to be an $O(nd)$ computation, adding a bit to each at-risk subject each time there is an event. A key insight due to David Watson is that we can avoid this. Within the tree we keep the sum of weights for each node and its left and right children, along with a parallel triple that totals only the deaths. The tree is updated as each subject is added or deleted, containing the current cumulative totals. We only need to update derivative values for each subject when the subject enters and when they leave. (For ordinary Cox models everyone enters at 0, or rather leaves at zero since our code will go from longest to shortest time.)

A second variance estimate is based on the Cox model. ¿¿¿¿¿¿¿ other The insight used here is to consider a Cox model with time dependent covariates, where the covariate x at each death

time has been transformed into rank(x). ¡¡¡¡¡¡¡ local It is easy to show that the Cox score statistic contribution at each death is $(D - C)/2$ where C and D are the number of concordant and discordant pairs contributed at that death time (for a Cox fit using the Breslow approximation). ======= One can show that the Cox score statistic contribution of $r_i - \overline{r}$ at each death time is equal to $(C - D)/2$ where C and D are the number of concordant and discordant pairs comparing that death to all those at risk, and using the Breslow approximation for ties. ¿¿¿¿¿¿¿ other The contribution to the variance of the score statistic is $V(t) = \sum(r_i - \overline{r})^2/n$, the r_i being the ranks at that time point and n the number at risk. How can we update this sum using an update formula? First remember the identity

$$\sum w_i(x_i - \overline{x})^2 = \sum w_i(x_i - c)^2 - \sum w_i(c - \overline{x})^2$$

true for any set of values x and centering constant c. For weighted data define the rank of an observation with risk score r_k as

$$\text{rank} = \sum_{r_i < r_k} w_i + (1/2) \sum_{r_i = r_k} w_i$$

These correspond to the midpoints of the rise on an empirical CDF, and for unweighted data without ties gives ranks of .5, 1.5, ..., $n - .5$.

Assume we have just added obs k to the tree. Since the mean rank = $\mu_g = \sum(w_i)/2$ the grand mean increases by $w_k/2$. ¡¡¡¡¡¡¡ local =======

We can divide the subjects currently in the tree into 3 groups.

1. Those with risk score lower than the new addition. Their rank will not change.
2. Those tied with the new addition. Their rank will increase by $w_k/2$.
3. Those above the new addition. Their rank increases by w_k.

¿¿¿¿¿¿¿ other Let μ_ℓ be the mean rank for all observations currently in the tree of rank lower than r_k, the item we are about to add, μ_u be the mean for all those above in rank (after the addition), μ_g the grand mean, and μ_n the new grand mean after adding in subject k. We have

$$\mu_\ell = \sum_{r_i < r_k} w_i/2$$
$$\mu_u = \sum_{r_i <= r_k} w_i + \sum_{r_i \geq r_k} w_i/2$$

For items of lower rank than r_k, none of their ranks will change with the addition of this new observation. This leads to the update formula on the third line below. (I'm using $i < k$ as

shorthand for $r_i < r_k$ below)

$$\begin{aligned}
\sum_{i<k} w_i(r_i - \mu_g)^2 &= \sum_{i<k} w_i(r_i - \mu_\ell)^2 + (\sum_{i<k} w_i)(\mu_\ell - \mu_g)^2 \\
\sum_{i<k} w_i(r_i - \mu_n)^2 &= \sum_{i<k} w_i(r_i - \mu_\ell)^2 + (\sum_{i<k} w_i)(\mu_\ell - \mu_n)^2 \\
\sum_{i<k} w_i(r_i - \mu_n)^2 - \sum_{i<k} w_i(r_i - \mu_g)^2 &= (\sum_{i<k} w_i)[(\mu_\ell - \mu_n)^2 - (\mu_\ell - \mu_g)^2] \\
&= (\sum_{i<k} w_i)(\mu_n + \mu_g - 2\mu_\ell)(\mu_n - \mu_g) \qquad (11) \\
&= (\sum_{i<k} w_i)(\mu_n + \mu_g - 2\mu_\ell)w_k/2 \qquad (12)
\end{aligned}$$

For items of larger rank than r_k, all of the ranks increase by w_k when we add the new item and μ_u increases by w_k, thus the sum of squares within the group is unchanged. The same derivation as above gives an update of

$$\begin{aligned}
\sum_{i>k} w_i(r_i - \mu_n)^2 - \sum_{i>k} w_i(r_i - \mu_g)^2 &= (\sum_{i>k} w_i)[(\mu_u - \mu_n)^2 - ((\mu_u - w_k) - \mu_g)^2] \\
&= (\sum_{i>k} w_i)(\mu_n + z - 2\mu_u)(\mu_n - z) \qquad (13) \\
&= (\sum_{i>k} w_i)(\mu_n + z - 2\mu_u)(-w_k/2) \qquad (14) \\
z &\equiv \mu_g + w_k
\end{aligned}$$

For items of tied rank, their rank increases by the same amount as the overall mean, and so their contribution to the total SS is unchanged. The final part of the update step is to add in the SS contributed by the new observation.

An observation is removed from the tree whenver the current time becomes less than the (start, stop] interval of the datum. The ranks for observations of lower risk are unchanged by the removal so equation (11) applies just as before, but with the new mean smaller than the old so the last term in equation (12) changes sign. For the observations of higher risk both the mean and the ranks change by w_k and equation (13) holds but with $z = \mu_0 - w_k$.

We can now define the C-routine that does the bulk of the work. First we give the outline shell of the code and then discuss the parts one by one. This routine is for ordinary survival data, and will be called once per stratum. Input variables are

n the number of observations

y matrix containing the time and status, data is sorted by ascending time, with deaths preceding censorings.

indx the tree node at which this observation's risk score resides

wt case weight for the observation

sum scratch space, weights for each node of the tree: 3 values are for the node, all left children, and all right children

count the returned counts of concordant, discordant, tied on x, tied on time, and the variance

⟨*concordance1*⟩=
```
 #include "survS.h"
 SEXP concordance1(SEXP y, SEXP wt2,  SEXP indx2, SEXP ntree2) {
     int i, j, k, index;
     int child, parent;
     int n, ntree;
     double *time, *status;
     double *twt, *nwt, *count;
     double vss, myrank, wsum1, wsum2, wsum3; /*sum of wts below, tied, above*/
     double lmean, umean, oldmean, newmean;

     double ndeath;   /* weighted number of deaths at this point */

     SEXP count2;
     double *wt;
     int    *indx;

     n = nrows(y);
     ntree = asInteger(ntree2);
     wt = REAL(wt2);
     indx = INTEGER(indx2);

     time = REAL(y);
     status = time + n;
     PROTECT(count2 = allocVector(REALSXP, 5));
     count = REAL(count2);  /* count5 contains the information matrix */
     twt = (double *) R_alloc(2*ntree, sizeof(double));
     nwt = twt + ntree;
     for (i=0; i< 2*ntree; i++) twt[i] =0.0;
     for (i=0; i<5; i++) count[i]=0.0;
     vss=0;

     ⟨concordance1-work⟩

     UNPROTECT(1);
     return(count2);
 }
```

The key part of our computation is to update the vectors of weights. We don't actually pass the risk score values r into the routine, it is enough for each observation to point to the appropriate tree node. The tree contains the for everyone whose survival is larger than the time currently under review, so starts with all weights equal to zero. For any pair of observations i, j we need to

add `wt[i]*wt[j]` to the appropriate count. Starting at the largest time (which is sorted last), walk through the tree.

- If it is a death time, we need to process all the deaths tied at this time.
  1. Add `wt[i] * wt[j]` to the tied-on-time total, for all pairs i, j of tied times.
  2. The addition to tied-on-r will be the weight of this observation times the sum of weights for all others with the same risk score and a a greater time, i.e., the weight found at `indx[i]` in the tree.
  3. Similarly for those with smaller or larger risk scores. First add in the children of this node. The left child will be smaller risk scores (and longer times) adding to the concordant pairs, the right child discordant. Then walk up the tree to the root. At each step up we add in data for the 'not me' branch. If we were the right branch (even number node) of a parent then when moving up we add in the left branch counts, and vice-versa.
- Now add this set of subject weights into the tree. The weight for a node is `nwt` and for the node and all its children is `twt`.

⟨*concordance1-work*⟩=
```
 for (i=n-1; i>=0; ) {
     ndeath =0;
     if (status[i]==1) { /* process all tied deaths at this point */
         for (j=i; j>=0 && status[j]==1 && time[j]==time[i]; j--) {
             ndeath += wt[j];
             index = indx[j];
             for (k=i; k>j; k--) count[3] += wt[j]*wt[k]; /* tied on time */
             count[2] += wt[j] * nwt[index];              /* tied on x */
             child = (2*index) +1;  /* left child */
             if (child < ntree)
                 count[0] += wt[j] * twt[child];  /*left children */
             child++;
             if (child < ntree)
                 count[1] += wt[j] * twt[child]; /*right children */

             while (index >0) {  /* walk up the tree  */
                 parent = (index-1)/2;
                 if (index & 1)   /* I am the left child */
                     count[1] += wt[j] * (twt[parent] - twt[index]);
                 else count[0] += wt[j] * (twt[parent] - twt[index]);
                 index = parent;
                 }
             }
         }
     else j = i-1;
```

```
    /* Add the weights for these obs into the tree and update variance*/
    for (; i>j; i--) {
        wsum1=0;
        oldmean = twt[0]/2;
        index = indx[i];
        nwt[index] += wt[i];
        twt[index] += wt[i];
        wsum2 = nwt[index];
        child = 2*index +1;  /* left child */
        if (child < ntree) wsum1 += twt[child];

        while (index >0) {
            parent = (index-1)/2;
            twt[parent] += wt[i];
            if (!(index&1)) /* I am a right child */
                wsum1 += (twt[parent] - twt[index]);
            index=parent;
            }
        wsum3 = twt[0] - (wsum1 + wsum2); /* sum of weights above */
        lmean = wsum1/2;
        umean = wsum1 + wsum2 + wsum3/2;  /* new upper mean */
        newmean = twt[0]/2;
        myrank = wsum1 + wsum2/2;
        vss += wsum1*(newmean+ oldmean - 2*lmean) * (newmean - oldmean);
        vss += wsum3*(newmean+ oldmean+ wt[i]- 2*umean) *(oldmean-newmean);
        vss += wt[i]* (myrank -newmean)*(myrank -newmean);
        }
    count[4] += ndeath * vss/twt[0];
    }
```

The code for [start, stop) data is quite similar. As in the agreg routines there are two sort indices, the first indexes the data by stop time, longest to earliest, and the second by start time. The `y` variable now has three columns.

⟨*concordance1*⟩=

```
 SEXP concordance2(SEXP y,     SEXP wt2,  SEXP indx2, SEXP ntree2,
                   SEXP sortstop, SEXP sortstart) {
    int i, j, k, index;
    int child, parent;
    int n, ntree;
    int istart, iptr, jptr;
    double *time1, *time2, *status, dtime;
    double *twt, *nwt, *count;
    int *sort1, *sort2;
    double vss, myrank;
    double wsum1, wsum2, wsum3; /*sum of wts below, tied, above*/
    double lmean, umean, oldmean, newmean;
```

```
    double ndeath;
    SEXP count2;
    double *wt;
    int    *indx;

    n = nrows(y);
    ntree = asInteger(ntree2);
    wt = REAL(wt2);
    indx = INTEGER(indx2);
    sort2 = INTEGER(sortstop);
    sort1 = INTEGER(sortstart);

    time1 = REAL(y);
    time2 = time1 + n;
    status= time2 + n;
    PROTECT(count2 = allocVector(REALSXP, 5));
    count = REAL(count2);
    twt = (double *) R_alloc(2*ntree, sizeof(double));
    nwt = twt + ntree;
    for (i=0; i< 2*ntree; i++) twt[i] =0.0;
    for (i=0; i<5; i++) count[i]=0.0;
    vss =0;
    ⟨concordance2-work⟩

    UNPROTECT(1);
    return(count2);
}
```

The processing changes in 2 ways

- The loops go from 0 to $n-1$ instead of $n-1$ to 0. We need to use `sort1[i]` instead of `i` as the subscript for the time2 and wt vectors. (The sort vectors go backwards in time.) This happens enough that we use a temporary variables `iptr` and `jptr` to avoid the double subscript.
- As we move from the longest time to the shortest observations are added into the tree of weights whenever we encounter their stop time. This is just as before. Weights now also need to be removed from the tree whenever we encounter an observation's start time. It is convenient "catch up" on this second task whenever we encounter a death.

```
⟨concordance2-work⟩=
 istart = 0;  /* where we are with start times */
 for (i=0; i<n; ) {
     iptr = sort2[i];  /* In  reverse death time order */
     ndeath =0;
     if (status[iptr]==1) {
         /* Toss people out of the tree  and update variance */
```

```
dtime = time2[iptr];
for (; istart < n && time1[sort1[istart]] >= dtime; istart++) {
    wsum1 =0;
    oldmean = twt[0]/2;
    jptr = sort1[istart];
    index = indx[jptr];
    nwt[index] -= wt[jptr];
    twt[index] -= wt[jptr];
    wsum2 = nwt[index];
    child = 2*index +1;  /* left child */
    if (child < ntree) wsum1 += twt[child];
    while (index >0) {
        parent = (index-1)/2;
        twt[parent] -= wt[jptr];
        if (!(index&1)) /* I am a right child */
            wsum1 += (twt[parent] - twt[index]);
        index=parent;
        }
    wsum3 = twt[0] - (wsum1 + wsum2);
    lmean = wsum1/2;
    umean = wsum1 + wsum2 + wsum3/2;  /* new upper mean */
    newmean = twt[0]/2;
    myrank = wsum1 + wsum2/2;
    vss += wsum1*(newmean+ oldmean - 2*lmean) * (newmean-oldmean);
    oldmean -= wt[jptr];  /* the z in equations above */
    vss += wsum3*(newmean+ oldmean -2*umean) * (newmean-oldmean);
    vss -= wt[jptr]* (myrank -newmean)*(myrank -newmean);
    }

/* Process deaths */
for (j=i; j <n && status[sort2[j]]==1 && time2[sort2[j]]==dtime; j++) {
    jptr =  sort2[j];
    ndeath += wt[jptr];
    index = indx[jptr];
    for (k=i; k<j; k++) count[3] += wt[jptr]*wt[sort2[k]];
    count[2] += wt[jptr] * nwt[index];            /* tied on x */
    child = (2*index) +1;   /* left child */
    if (child < ntree) count[0] += wt[jptr] * twt[child];
    child++;
    if (child < ntree) count[1] += wt[jptr] * twt[child];

    while (index >0) {  /* walk up the tree  */
        parent = (index-1)/2;
        if (index &1)   /* I am the left child */
             count[1] += wt[jptr] * (twt[parent] - twt[index]);
        else count[0] += wt[jptr] * (twt[parent] - twt[index]);
```

```
            index = parent;
            }
        }
    }
else j = i+1;

/* Add the weights for these obs into the tree and compute variance */
for (; i<j; i++) {
    wsum1 =0;
    oldmean = twt[0]/2;
    iptr = sort2[i];
    index = indx[iptr];
    nwt[index] += wt[iptr];
    twt[index] += wt[iptr];
    wsum2 = nwt[index];
    child = 2*index +1;  /* left child */
    if (child < ntree) wsum1 += twt[child];
    while (index >0) {
        parent = (index-1)/2;
        twt[parent] += wt[iptr];
        if (!(index&1)) /* I am a right child */
            wsum1 += (twt[parent] - twt[index]);
        index=parent;
        }
    wsum3 = twt[0] - (wsum1 + wsum2);
    lmean = wsum1/2;
    umean = wsum1 + wsum2 + wsum3/2;  /* new upper mean */
    newmean = twt[0]/2;
    myrank = wsum1 + wsum2/2;
    vss += wsum1*(newmean+ oldmean - 2*lmean) * (newmean-oldmean);
    vss += wsum3*(newmean+ oldmean +wt[iptr] - 2*umean) * (oldmean-newmean);
    vss += wt[iptr]* (myrank -newmean)*(myrank -newmean);
    }
count[4] += ndeath * vss/twt[0];
}
```

One last wrinkle is tied risk scores: they are all set to point to the same node of the tree. Here is the main routine.

⟨*survConcordance*⟩=

```
 survConcordance <- function(formula, data,
                             weights, subset, na.action) {
    Call <- match.call()  # save a copy of of the call, as documentation

    m <- match.call(expand.dots=FALSE)
    m[[1L]] <-  quote(stats::model.frame)
    m$formula <- if(missing(data)) terms(formula, "strata")
```

```
                else              terms(formula, "strata", data=data)
    m <- eval(m, sys.parent())
    Terms <- attr(m, 'terms')

    Y <- model.extract(m, "response")
    if (!inherits(Y, "Surv")) {
        if (is.numeric(Y) && is.vector(Y))  Y <- Surv(Y)
        else stop("left hand side of the formula  must be a numeric vector or a surival")
    }
    n <- nrow(Y)

    wt <- model.extract(m, 'weights')
    offset<- attr(Terms, "offset")
    if (length(offset)>0) stop("Offset terms not allowed")

    stemp <- untangle.specials(Terms, 'strata')
    if (length(stemp$vars)) {
        if (length(stemp$vars)==1) strat <- m[[stemp$vars]]
        else strat <- strata(m[,stemp$vars], shortlabel=TRUE)
        Terms <- Terms[-stemp$terms]
    }
    else strat <- NULL

    x <- model.matrix(Terms, m)[,-1, drop=FALSE]  #remove the intercept
    if (ncol(x) > 1) stop("Only one predictor variable allowed")

    count <- survConcordance.fit(Y, x, strat, wt)
    if (is.null(strat)) {
        concordance <- (count[1] + count[3]/2)/sum(count[1:3])
        std.err <- count[5]/(2* sum(count[1:3]))
        }
    else {
        temp <- colSums(count)
        concordance <- (temp[1] + temp[3]/2)/ sum(temp[1:3])
        std.err <- temp[5]/(2*sum(temp[1:3]))
        }

    fit <- list(concordance= concordance, stats=count, n=n,
                std.err=std.err, call=Call)
    na.action <- attr(m, "na.action")
    if (length(na.action)) fit$na.action <- na.action

    oldClass(fit) <- 'survConcordance'
    fit
}
```

```
print.survConcordance <- function(x, ...) {
    if(!is.null(cl <- x$call)) {
        cat("Call:\n")
        dput(cl)
        cat("\n")
        }
    omit <- x$na.action
    if(length(omit))
        cat("  n=", x$n, " (", naprint(omit), ")\n", sep = "")
    else cat("  n=", x$n, "\n")
    cat("Concordance= ", format(x$concordance), " se= ", format(x$std.err),
        '\n', sep='')
    print(x$stats)

    invisible(x)
    }
```

This part of the compuation is a separate function, since it is also called by the coxph routines. Although we are very careful to create integers and/or doubles for the arguments to .Call I still wrap them in the appropriate as.xxx construction: "belt and suspenders". Also, referring to the the mathematics many paragraphs ago, the C routine returns the variance of $(C - D)/2$ and we return the standard deviation of $(C - D)$. If this routine is called with all the x values identical, then C and D will both be zero, but the calculated variance of $C - D$ can be a nonzero tiny number due to round off error. Since this can cause a warning message from the sqrt function we check and correct this.

⟨*survConcordance.fit*⟩=

```
survConcordance.fit <- function(y, x, strata, weight) {
    # The coxph program may occassionally fail, and this will kill the C
    #  routine below
    if (any(is.na(x)) || any(is.na(y))) return(NULL)
    ⟨btree⟩

    docount <- function(stime, risk, wts) {
        if (attr(stime, 'type') == 'right') {
            ord <- order(stime[,1], -stime[,2])
            ux <- sort(unique(risk))
            n2 <- length(ux)
            index <- btree(n2)[match(risk[ord], ux)] - 1L
             .Call(Cconcordance1, stime[ord,],
                   as.double(wts[ord]),
                   as.integer(index),
                   as.integer(length(ux)))
        }
        else if (attr(stime, 'type') == "counting") {
            sort.stop <- order(-stime[,2], stime[,3])
            sort.start <- order(-stime[,1])
```

```
            ux <- sort(unique(risk))
            n2 <- length(ux)
            index <- btree(n2)[match(risk, ux)] - 1L

            .Call(Cconcordance2, stime,
                  as.double(wts),
                  as.integer(index),
                  as.integer(length(ux)),
                  as.integer(sort.stop-1L),
                  as.integer(sort.start-1L))
        }
        else stop("Invalid survival type for concordance")
    }

    if (missing(weight) || length(weight)==0)
        weight <- rep(1.0, length(x))
    storage.mode(y) <- "double"

    if (missing(strata) || length(strata)==0) {
        count <- docount(y, x, weight)
        if (count[1]==0 && count[2]==0) count[5]<-0
        else count[5] <- 2*sqrt(count[5])
        names(count) <- c("concordant", "discordant", "tied.risk", "tied.time",
                          "std(c-d)")
    }
    else {
        strata <- as.factor(strata)
        ustrat <- levels(strata)[table(strata) >0]  #some strata may have 0 obs
        count <- matrix(0., nrow=length(ustrat), ncol=5)
        for (i in seq_along(ustrat)) {
            keep <- which(strata == ustrat[i])
            count[i,] <- docount(y[keep,,drop=F], x[keep], weight[keep])
        }

        count[,5] <- 2*sqrt(ifelse(count[,1]+count[,2]==0, 0, count[,5]))
        dimnames(count) <- list(ustrat,  c("concordant", "discordant",
                                           "tied.risk", "tied.time",
                                           "std(c-d)"))
    }
    count
}
```

6 Expected Survival

The expected survival routine creates the overall survival curve for a *group* of people. It is possible to take the set of expected survival curves for each individual and average them, which is the `Ederer` method below, but this is not always the wisest choice: the Hakulinen and conditional methods average in anothers ways, both of which are more sophisticated ways to deal with censoring. The individual curves are dervived either from population rate tables such as the US annual life tables from the National Center for Health Statistics or the larger multi-national collection at mortality.org, or by using a previously fitted Cox model as the table.

The arguments for `survexp` are

formula The model formula. The right hand side consists of grouping variables, identically to `survfit` and an optional `ratetable` directive. The "response" varies by method:

- for the Hakulinen method it is a vector of censoring times. This is the actual censoring time for censored subjecs, and is what the censoring time 'would have been' for each subject who died.
- for the conditional method it is the usual Surv(time, status) code
- for the Ederer method no response is needed

data, weights, subset, na.action as usual

rmap an optional mapping for rate table variables, see more below.

times An optional vector of time points at which to compute the response. For the Hakulinen and conditional methods the program uses the vector of unique y values if this is missing. For the Ederer the component is not optional.

method The method used for the calculation. Choices are individual survival, or the Ederer, Hakulinen, or conditional methods for cohort survival.

cohort, conditional Older arguments that were used to select the method.

ratetable the population rate table to use as a reference. This can either be a ratetable object or a previously fitted Cox model

scale Scale the resulting output times, e.g., 365.25 to turn days into years.

se.fit This has been deprecated.

model, x, y usual

The output of survexp contains a subset of the elements in a `survfit` object, so many of the survfit methods can be applied. The result has a class of `c('survexp', 'survfit')`.

⟨*survexp*⟩=
```
 survexp <- function(formula, data,
         weights, subset, na.action, rmap, times,
         method=c("ederer", "hakulinen", "conditional", "individual.h",
                  "individual.s"),
```

```
        cohort=TRUE,  conditional=FALSE,
        ratetable=survival::survexp.us, scale=1, se.fit,
        model=FALSE, x=FALSE, y=FALSE) {
    ⟨survexp-setup⟩
    ⟨survexp-compute⟩
    ⟨survexp-format⟩
    ⟨survexp-finish⟩
}
```

The first few lines are standard. Keep a copy of the call, then manufacture a call to `model.frame` that contains only the arguments relevant to that function.

```
⟨survexp-setup⟩=
 call <- match.call()
 m <- match.call(expand.dots=FALSE)

 # keep the first element (the call), and the following selected arguments
 m <- m[c(1, match(c('formula', 'data', 'weights', 'subset', 'na.action'),
                   names(m), nomatch=0))]
 m[[1L]] <- quote(stats::model.frame)

 Terms <- if(missing(data)) terms(formula, 'ratetable')
          else              terms(formula, 'ratetable',data=data)
```

The function works with two data sets, the user's data on an actual set of subjects and the reference ratetable. This leads to a particular nuisance, that the variable names in the data set may not match those in the ratetable. For instance the United States overall death rate table `survexp.us` expects 3 variables, as shown by `summary(survexp.us)`

- age = age in days for each subject at the start of follow-up
- sex = sex of the subject, "male" or "female" (the routine accepts any unique abbreviation and is case insensitive)
- year = date of the start of follow-up

Up until the most recent revision, the formula contained any necessary mapping between the variables in the data set and the ratetable. For instance

```
survexp( ~ sex + ratetable(age=age*365.25, sex=sex,
                           year=entry.dt),
           data=mydata, ratetable=survexp.us)
```

In this case the user's data set has a variable 'age' containing age in years, along with sex and an entry date. This had to be changed for two reasons. The primary one is that the data in a `ratetable` call had to be converted into a matrix in order to "pass through" the model.frame logic. With the recent updates to coxph so that it remembers factor codings correctly in new data sets, it is advantageous to keep factors as factors. The second is that a coxph model with a large number of covariates induces a very long ratetable clause; at about 40 variable it caused

one of the R internal routines to fail due to a long expression. A third reason, perhaps the most pressing in reality, is that I've always felt that the prior code was confusing since it used the same term 'ratetable' for two different tasks.

The new process adds the `rmap` argument, an example would be `rmap=list(age =age*365.25, year=entry.dt)`. Any variables in the ratetable that are not found in `rmap` are assumed to not need a mapping, this would be `sex` in the above example. For backwards compatability we allow the old style argument, converting it into the new style.

The `rmap` argument needs to be examined without evaluating it; we then add the appropriate extra variables into a temporary formula so that the model frame has all that is required. The ratetable variables then can be retrieved from the model frame. The `pyears` routine uses the same rmap argument; this segment of the code is given its own name so that it can be included there as well.

⟨*survexp-setup*⟩=
```
 rate <- attr(Terms, "specials")$ratetable
 if(length(rate) > 1)
         stop("Can have only 1 'ratetable()' call in a formula")
 ⟨survexp-setup-rmap⟩

 m <- eval(m, parent.frame())
```

⟨*survexp-setup-rmap*⟩=
```
 if(length(rate) == 1) {
     if (!missing(rmap))
         stop("The 'ratetable()' call in a formula is depreciated")

     stemp <- untangle.specials(Terms, 'ratetable')
     rcall <- as.call(parse(text=stemp$var)[[1]])   # as a call object
     rcall[[1]] <- as.name('list')                  # make it a call to list(..
     Terms <- Terms[-stemp$terms]                   # remove from the formula
     }
 else if (!missing(rmap)) {
     rcall <- substitute(rmap)
     if (!is.call(rcall) || rcall[[1]] != as.name('list'))
         stop(gettextf("invalid '%s' argument", "rcall"))
     }
 else rcall <- NULL   # A ratetable, but not rcall argument

 # Check that there are no illegal names in rcall, then expand it
 #  to include all the names in the ratetable
 if(is.ratetable(ratetable))   varlist <- attr(ratetable, "dimid")
 else if(inherits(ratetable, "coxph")) {
     ## Remove "log" and such things, to get just the list of
     #   variable names
     varlist <- all.vars(delete.response(ratetable$terms))
     }
 else stop("Invalid rate table")
```

```
temp <- match(names(rcall)[-1], varlist) # 2,3,... are the argument names
if (any(is.na(temp)))
    stop(gettextf("Variable not found in the ratetable: %s", (names(rcall))[is.na(temp)]))

if (any(!(varlist %in% names(rcall)))) {
    to.add <- varlist[!(varlist %in% names(rcall))]
    temp1 <- paste(text=paste(to.add, to.add, sep='='), collapse=',')
    if (is.null(rcall)) rcall <- parse(text=paste("list(", temp1, ")"))[[1]]
    else {
        temp2 <- deparse(rcall)
        rcall <- parse(text=paste("c(", temp2, ",list(", temp1, "))"))[[1]]
        }
    }
```

The formula below is used only in the call to `model.frame` to ensure that the frame has both the formula and the ratetable variables. We don't want to modify the original formula, since we use it to create the X matrix and the response variable. The non-obvious bit of code is the addition of an environment to the formula. The `model.matrix` routine has a non-standard evaluation - it uses the frame of the formula, rather than the parent.frame() argument below, along with the `data` to look up variables. If a formula is long enough deparse() will give two lines, hence the extra paste call to re-collapse it into one.

⟨*survexp-setup-rmap*⟩=
```
# Create a temporary formula, used only in the call to model.frame
newvar <- all.vars(rcall)
if (length(newvar) > 0) {
    tform <- paste(paste(deparse(Terms), collapse=""),
                   paste(newvar, collapse='+'), sep='+')
    m$formula <- as.formula(tform, environment(Terms))
    }
```

If the user data has 0 rows, e.g. from a mistaken `subset` statement that eliminated all subjects, we need to stop early. Otherwise the .C code fails in a nasty way.

⟨*survexp-setup*⟩=
```
n <- nrow(m)
if (n==0) stop("data set has 0 rows")
if (!missing(se.fit) && se.fit)
    warning("'se.fit' value ignored")

weights <- model.extract(m, 'weights')
if (length(weights) ==0) weights <- rep(1.0, n)
if (class(ratetable)=='ratetable' && any(weights !=1))
    warning("weights ignored")

if (any(attr(Terms, 'order') >1))
```

```
        stop("Survexp cannot have interaction terms")
if (!missing(times)) {
    if (any(times<0)) stop("Invalid time point requested")
    if (length(times) >1 )
        if (any(diff(times)<0)) stop("Times must be in increasing order")
    }
```

If a response variable was given, we only need the times and not the status. To be correct, computations need to be done for each of the times given in the `times` argument as well as for each of the unique y values. This ends up as the vector `newtime`. If a `times` argument was given we will subset down to only those values at the end. For a population rate table and the Ederer method the times argument is required.

⟨*survexp-setup*⟩=

```
Y <- model.extract(m, 'response')
no.Y <- is.null(Y)
if (no.Y) {
    if (missing(times)) {
        if (is.ratetable(ratetable))
            stop("either a times argument or a response is needed")
        }
    else newtime <- times
    }
else {
    if (is.matrix(Y)) {
        if (is.Surv(Y) && attr(Y, 'type')=='right') Y <- Y[,1]
        else stop("Illegal response value")
        }
    if (any(Y<0)) stop("Negative follow up time")
#    if (missing(npoints)) temp <- unique(Y)
#    else                  temp <- seq(min(Y), max(Y), length=npoints)
    temp <- unique(Y)
    if (missing(times)) newtime <- sort(temp)
    else  newtime <- sort(unique(c(times, temp[temp<max(times)])))
    }

if (!missing(method)) method <- match.arg(method)
else {
    # the historical defaults and older arguments
    if (!missing(conditional) && conditional) method <- "conditional"
    else {
        if (no.Y) method <- "ederer"
        else method <- "hakulinen"
        }
    if (!missing(cohort) && !cohort) method <- "individual.s"
    }
if (no.Y && (method!="ederer"))
```

```
    stop("a response is required in the formula unless method='ederer'")
```

The next step is to check out the ratetable. For a population rate table a set of consistency checks is done by the `match.ratetable` function, giving a set of sanitized indices `R`. This function wants characters turned to factors. For a Cox model `R` will be a model matix whose covariates are coded in exactly the same way that variables were coded in the original Cox model. We call the model.matrix.coxph function to avoid repeating the steps found there (remove cluster statements, etc). We also need to use the `mf` argument of the function, otherwise it will call model.frame internally and fail when it can't find the response variable (which we don't need).

Note that for a population rate table the standard error of the expected is by definition 0 (the population rate table is based on a huge sample). For a Cox model rate table, an se formula is currently only available for the Ederer method.

⟨*survexp-compute*⟩=
```
 ovars <- attr(Terms, 'term.labels')
 # rdata contains the variables matching the ratetable
 rdata <- data.frame(eval(rcall, m), stringsAsFactors=TRUE)
 if (is.ratetable(ratetable)) {
     israte <- TRUE
     if (no.Y) {
         Y <- rep(max(times), n)
         }
     rtemp <- match.ratetable(rdata, ratetable)
     R <- rtemp$R
     }
 else if (inherits(ratetable, 'coxph')) {
     israte <- FALSE
     Terms <- ratetable$terms
 #    if (!is.null(attr(Terms, 'offset')))
 #        stop("Cannot deal with models that contain an offset")
 #    strats <- attr(Terms, "specials")$strata
 #    if (length(strats))
 #        stop("survexp cannot handle stratified Cox models")
 #
     if (any(names(m[,rate]) !=  attr(ratetable$terms, 'term.labels')))
         stop("Unable to match new data to old formula")
     }
 else stop("Invalid ratetable")
```

Now for some calculation. If cohort is false, then any covariates on the right hand side (other than the rate table) are irrelevant, the function returns a vector of expected values rather than survival curves.

⟨*survexp-compute*⟩=
```
 if (substring(method, 1, 10) == "individual") { #individual survival
     if (no.Y) stop("for individual survival an observation time must be given")
     if (israte)
```

```
     temp <- survexp.fit (1:n, R, Y, max(Y), TRUE, ratetable)
else {
    rmatch <- match(names(data), names(rdata))
    if (any(is.na(rmatch))) rdata <- cbind(rdata, data[,is.na(rmatch)])
    temp <- survexp.cfit(1:n, rdata, Y, 'individual', ratetable)
}
if (method == "individual.s") xx <- temp$surv
else xx <- -log(temp$surv)
names(xx) <- row.names(m)
na.action <- attr(m, "na.action")
if (length(na.action)) return(naresid(na.action, xx))
else return(xx)
}
```

Now for the more commonly used case: returning a survival curve. First see if there are any grouping variables. The results of the `tcut` function are often used in person-years analysis, which is somewhat related to expected survival. However tcut results aren't relevant here and we put in a check for the confused user. The strata command creates a single factor incorporating all the variables.

⟨*survexp-compute*⟩=
```
if (length(ovars)==0)  X <- rep(1,n)  #no categories
else {
    odim <- length(ovars)
    for (i in 1:odim) {
        temp <- m[[ovars[i]]]
        ctemp <- class(temp)
        if (!is.null(ctemp) && ctemp=='tcut')
            stop("Can't use tcut variables in expected survival")
        }
    X <- strata(m[ovars])
    }

#do the work
if (israte)
    temp <- survexp.fit(as.numeric(X), R, Y, newtime,
                        method=="conditional", ratetable)
else {
    temp <- survexp.cfit(as.numeric(X), rdata, Y, method, ratetable, weights)
    newtime <- temp$time
    }
```

Now we need to package up the curves properly All the results can be returned as a single matrix of survivals with a common vector of times. If there was a times argument we need to subset to selected rows of the computation.

⟨*survexp-format*⟩=
```
if (missing(times)) {
```

```
    n.risk <- temp$n
    surv <- temp$surv
    }
else {
    if (israte) keep <- match(times, newtime)
    else {
        # The result is from a Cox model, and it's list of
        #  times won't match the list requested in the user's call
        # Interpolate the step function, giving survival of 1
        #  for requested points that precede the Cox fit's
        #  first downward step.  The code is like summary.survfit.
        n <- length(temp$time)
        keep <- approx(temp$time, 1:n, xout=times, yleft=0,
                       method='constant', f=0, rule=2)$y
        }

    if (is.matrix(temp$surv)) {
        surv <- (rbind(1,temp$surv))[keep+1,,drop=FALSE]
        n.risk <- temp$n[pmax(1,keep),,drop=FALSE]
         }
    else {
        surv <- (c(1,temp$surv))[keep+1]
        n.risk <- temp$n[pmax(1,keep)]
        }
    newtime <- times
    }
newtime <- newtime/scale
if (is.matrix(surv)) {
    dimnames(surv) <- list(NULL, levels(X))
    out <- list(call=call, surv= drop(surv), n.risk=drop(n.risk),
                    time=newtime)
    }
else {
     out <- list(call=call, surv=c(surv), n.risk=c(n.risk),
                   time=newtime)
     }
```

Last do the standard things: add the model, x, or y components to the output if the user asked for them. (For this particular routine I can't think of a reason they every would.) Copy across summary information from the rate table computation if present, and add the method and class to the output.

⟨*survexp-finish*⟩=

```
if (model) out$model <- m
else {
    if (x) out$x <- X
    if (y) out$y <- Y
```

```
    }
if (israte && !is.null(rtemp$summ)) out$summ <- rtemp$summ
if (no.Y) out$method <- 'Ederer'
else if (conditional) out$method <- 'conditional'
else                  out$method <- 'cohort'
class(out) <- c('survexp', 'survfit')
out
```

7 Person years

The person years routine and the expected survival code are the two parts of the survival package that make use of external rate tables, of which the United States mortality tables `survexp.us` and `survexp.usr` are examples contained in the package. The arguments for pyears are

formula The model formula. The right hand side consists of grouping variables and is essentially identical to `survfit`, the result of the model will be a table of results with dimensions determined from the right hand variables. The formula can include an optional `ratetable` directive; but this style has been superseded by the `rmap` argument.

data, weights, subset, na.action as usual

rmap an optional mapping for rate table variables, see more below.

ratetable the population rate table to use as a reference. This can either be a ratetable object or a previously fitted Cox model

scale Scale the resulting output times, e.g., 365.25 to turn days into years.

expect Should the output table include the expected number of events, or the expected number of person-years of observation?

model, x, y as usual

data.frame if true the result is returned as a data frame, if false as a set of tables.

```
⟨pyears⟩=
 pyears <- function(formula, data,
         weights, subset, na.action, rmap,
         ratetable, scale=365.25,  expect=c('event', 'pyears'),
         model=FALSE, x=FALSE, y=FALSE, data.frame=FALSE) {

     ⟨pyears-setup⟩
     ⟨pyears-compute⟩
     ⟨pyears-finish⟩
     }
```

Start out with the standard model processing, which involves making a copy of the input call, but keeping only the arguments we want. We then process the special argument `rmap`. This is discussed in the section on the `survexp` function so we need not repeat the explantation here.

⟨*pyears-setup*⟩=
```
 expect <- match.arg(expect)
 call <- match.call()
 m <- match.call(expand.dots=FALSE)
 m <- m[c(1, match(c('formula', 'data', 'weights', 'subset', 'na.action'),
                   names(m), nomatch=0))]
 m[[1L]] <- quote(stats::model.frame)

 Terms <- if(missing(data)) terms(formula, 'ratetable')
          else              terms(formula, 'ratetable',data=data)
 if (any(attr(Terms, 'order') >1))
         stop("Pyears cannot have interaction terms")

 rate <- attr(Terms, "specials")$ratetable
 if (length(rate) >0 || !missing(rmap) || !missing(ratetable)) {
     has.ratetable <- TRUE
     if(length(rate) > 1)
         stop("Can have only 1 ratetable() call in a formula")
     if (missing(ratetable)) stop("No rate table specified")

     ⟨survexp-setup-rmap⟩
     }
 else has.ratetable <- FALSE

 if (is.R())  m <- eval(m, parent.frame())
 else         m <- eval(m, sys.parent())

 Y <- model.extract(m, 'response')
 if (is.null(Y)) stop("Follow-up time must appear in the formula")
 if (!is.Surv(Y)){
     if (any(Y <0)) stop("Negative follow up time")
     Y <- as.matrix(Y)
     if (ncol(Y) >2) stop("Y has too many columns")
     }
 else {
     stype <- attr(Y, 'type')
     if (stype == 'right') {
         if (any(Y[,1] <0)) stop("Negative survival time")
         nzero <- sum(Y[,1]==0 & Y[,2] ==1)
         if (nzero >0)
             warning(gettextf("%d observations with an event and 0 follow-up time, any rate calcula
         }
     else if (stype != 'counting')
         stop("Only right-censored and counting process survival types are supported")
     }
```

```
n <- nrow(Y)
if (is.null(n) || n==0) stop("data set has 0 observations")

weights <- model.extract(m, 'weights')
if (is.null(weights)) weights <- rep(1.0, n)
```

The next step is to check out the ratetable. For a population rate table a set of consistency checks is done by the `match.ratetable` function, giving a set of sanitized indices `R`. This function wants characters turned to factors. For a Cox model `R` will be a model matix whose covariates are coded in exactly the same way that variables were coded in the original Cox model. We call the model.matrix.coxph function so as not to have to repeat the steps found there (remove cluster statements, etc).

⟨*pyears-setup*⟩=
```
# rdata contains the variables matching the ratetable
if (has.ratetable) {
    rdata <- data.frame(eval(rcall, m), stringsAsFactors=TRUE)
    if (is.ratetable(ratetable)) {
        israte <- TRUE
        rtemp <- match.ratetable(rdata, ratetable)
        R <- rtemp$R
        }
    else if (inherits(ratetable, 'coxph')) {
        israte <- FALSE
        Terms <- ratetable$terms
        if (!is.null(attr(Terms, 'offset')))
            stop("Cannot deal with models that contain an offset")
        strats <- attr(Terms, "specials")$strata
        if (length(strats))
            stop("pyears cannot handle stratified Cox models")

        if (any(names(m[,rate]) !=  attr(ratetable$terms, 'term.labels')))
             stop("Unable to match new data to old formula")
        R <- model.matrix.coxph(ratetable, data=rdata)
        }
    else stop("Invalid ratetable")
    }
```

Now we process the non-ratetable variables. Those of class `tcut` set up time-dependent classes. For these the cutpoints attribute sets the intervals, if there were 4 cutpoints of 1, 5,6, and 10 the 3 intervals will be 1-5, 5-6 and 6-10, and odims will be 3. All other variables are treated as factors.

⟨*pyears-setup*⟩=
```
ovars <- attr(Terms, 'term.labels')
if (length(ovars)==0)  {
    # no categories!
```

```
    X <- rep(1,n)
    ofac <- odim <- odims <- ocut <- 1
    }
else {
    odim <- length(ovars)
    ocut <- NULL
    odims <- ofac <- double(odim)
    X <- matrix(0, n, odim)
    outdname <- vector("list", odim)
    names(outdname) <- attr(Terms, 'term.labels')
    for (i in 1:odim) {
        temp <- m[[ovars[i]]]
        if (inherits(temp, 'tcut')) {
            X[,i] <- temp
            temp2 <- attr(temp, 'cutpoints')
            odims[i] <- length(temp2) -1
            ocut <- c(ocut, temp2)
            ofac[i] <- 0
            outdname[[i]] <- attr(temp, 'labels')
            }
        else {
            temp2 <- as.factor(temp)
            X[,i] <- temp2
            temp3 <- levels(temp2)
            odims[i] <- length(temp3)
            ofac[i] <- 1
            outdname[[i]] <- temp3
            }
    }
}
```

Now do the computations. The code above has separated out the variables into 3 groups:

- The variables in the rate table. These determine where we *start* in the rate table with respect to retrieving the relevant death rates. For the US table `survexp.us` this will be the date of study entry, age (in days) at study entry, and sex of each subject.
- The variables on the right hand side of the model. These are interpreted almost identically to a call to `table`, with special treatment for those of class *tcut*.
- The response variable, which tells the number of days of follow-up and optionally the status at the end of follow-up.

Start with the rate table variables. There is an oddity about US rate tables: the entry for age (year=1970, age=55) contains the daily rate for anyone who turns 55 in that year, from their birthday forward for 365 days. So if your birthday is on Oct 2, the 1970 table applies from 2Oct 1970 to 1Oct 1971. The underlying C code wants to make the 1970 rate table apply from 1Jan 1970 to 31Dec 1970. The easiest way to finess this is to fudge everyone's enter-the-study date.

If you were born in March but entered in April, make it look like you entered in Febuary; that way you get the first 11 months at the entry year's rates, etc. The birth date is entry date - age in days (based on 1/1/1960).

The other aspect of the rate tables is that "older style" tables, those that have the factor attribute, contained only decennial data which the C code would interpolate on the fly. The value of `atts$factor` was 10 indicating that there are 10 years in the interpolation interval. The newer tables do not do this and the C code is passed a 0/1 for continuous (age and year) versus discrete (sex, race).

⟨*pyears-compute*⟩=
```
 ocut <-c(ocut,0)   #just in case it were of length 0
 osize <- prod(odims)
 if (has.ratetable) {  #include expected
     atts <- attributes(ratetable)
     cuts <- atts$cutpoints
     if (is.null(atts$type)) {
         #old stlye table
         rfac <- atts$factor
         us.special <- (rfac >1)
         }
     else {
         rfac <- 1*(atts$type ==1)
         us.special <- (atts$type==4)
         }
     if (any(us.special)) {  #special handling for US pop tables
         # Now, the 'entry' date on a US rate table is the number of days
         #  since 1/1/1960, and the user data has been aligned to the
         #  same system by match.ratetable and marked as "year".
         # The birth date is entry date - age in days (based on 1/1/1960).
         # I don't much care which date functions I use to do the arithmetic
         #  below.  Unfortunately R and Splus don't share one.  My "date"
         #  class is simple, but is also one of the earlier date class
         #  attempts, has less features than others, and will one day fade
         #  away; so I don't want to depend on it alone.
         #
         cols <- match(c("age", "year"), atts$dimid)
               if (any(is.na(cols)))
              stop("Ratetable does not have expected shape")
         if (exists("as.Date")) {  # true for modern version of R
             bdate <- as.Date('1960/1/1') + (R[,cols[2]] - R[,cols[1]])
             byear <- format(bdate, "%Y")
             offset <- bdate - as.Date(paste(byear, "01/01", sep='/'),
                                       origin="1960/01/01")
             }
         #else if (exists('month.day.year')) { # Splus, usually
         #    bdate <- R[,cols[2]] - R[,cols[1]]
```

```
    #    byear <- month.day.year(bdate)$year
    #    offset <- bdate - julian(1,1,byear)
    #    }
    #else if (exists('date.mdy')) { # Therneau's date class is available
    #    bdate <- as.date(R[,cols[2]] - R[,cols[1]])
    #    byear <- date.mdy(bdate)$year
    #    offset <- bdate - mdy.date(1,1,byear)
    #    }
    else stop("Can't find an appropriate date class")
    R[,cols[2]] <- R[,cols[2]] - offset

    # Doctor up "cutpoints" - only needed for old style rate tables
    #  for which the C code does interpolation on the fly
    if (any(rfac >1)) {
        temp <-  which(us.special)
        nyear <- length(cuts[[temp]])
        nint <- rfac[temp]       #intervals to interpolate over
        cuts[[temp]] <- round(approx(nint*(1:nyear), cuts[[temp]],
                                     nint:(nint*nyear))$y - .0001)
        }
    }
docount <- is.Surv(Y)
temp <- .C(Cpyears1,
                as.integer(n),
                as.integer(ncol(Y)),
                as.integer(is.Surv(Y)),
                as.double(Y),
                as.double(weights),
                as.integer(length(atts$dim)),
                as.integer(rfac),
                as.integer(atts$dim),
                as.double(unlist(cuts)),
                as.double(ratetable),
                as.double(R),
                as.integer(odim),
                as.integer(ofac),
                as.integer(odims),
                as.double(ocut),
                as.integer(expect=='event'),
                as.double(X),
                pyears=double(osize),
                pn    =double(osize),
                pcount=double(if(docount) osize else 1),
                pexpect=double(osize),
                offtable=double(1))[18:22]
}
```

```
 else {   #no expected
     docount <- as.integer(ncol(Y) >1)
     temp <- .C(Cpyears2,
                     as.integer(n),
                     as.integer(ncol(Y)),
                     as.integer(docount),
                     as.double(Y),
                     as.double(weights),
                     as.integer(odim),
                     as.integer(ofac),
                     as.integer(odims),
                     as.double(ocut),
                     as.double(X),
                     pyears=double(osize),
                     pn    =double(osize),
                     pcount=double(if (docount) osize else 1),
                     offtable=double(1)) [11:14]
     }
```

Create the output object.

⟨*pyears-finish*⟩=

```
 if (data.frame) {
     # Create a data frame as the output, rather than a set of
     #  rate tables
     keep <- (temp$pyears >0)  # what rows to keep in the output
     names(outdname) <- ovars
     if (length(outdname) ==1) {
         # if there is only one variable, the call to "do.call" loses
         #  the variable name, since expand.grid returns a factor
         df <- data.frame((outdname[[1]])[keep],
                          pyears= temp$pyears[keep]/scale,
                          n = temp$pn[keep])
         names(df) <- c(names(outdname), 'pyears', 'n')
         }
     else {
         df <- cbind(do.call("expand.grid", outdname)[keep,],
                          pyears= temp$pyears[keep]/scale,
                          n = temp$pn[keep])
         }
     row.names(df) <- 1:nrow(df)
     if (has.ratetable) df$expected <- temp$pexpect[keep]
     if (expect=='pyears') df$expected <- df$expected/scale
     if (docount) df$event <- temp$pcount[keep]

     out <- list(call=call,
                 data= df, offtable=temp$offtable/scale)
```

```
    if (has.ratetable && !is.null(rtemp$summ))
        out$summary <- rtemp$summ
    }

else if (prod(odims) ==1) {  #don't make it an array
    out <- list(call=call, pyears=temp$pyears/scale, n=temp$pn,
                offtable=temp$offtable/scale)
    if (has.ratetable) {
        out$expected <- temp$pexpect
        if (expect=='pyears') out$expected <- out$expected/scale
        if (!is.null(rtemp$summ)) out$summary <- rtemp$summ
        }
    if (docount) out$event <- temp$pcount
    }
else {
    out <- list(call = call,
            pyears= array(temp$pyears/scale, dim=odims, dimnames=outdname),
            n     = array(temp$pn,     dim=odims, dimnames=outdname),
            offtable = temp$offtable/scale)
    if (has.ratetable) {
        out$expected <- array(temp$pexpect, dim=odims, dimnames=outdname)
        if (expect=='pyears') out$expected <- out$expected/scale
        if (!is.null(rtemp$summ)) out$summary <- rtemp$summ
        }
    if (docount)
            out$event <- array(temp$pcount, dim=odims, dimnames=outdname)
    }
out$observations <- nrow(m)
out$terms <- Terms
na.action <- attr(m, "na.action")
if (length(na.action))  out$na.action <- na.action
if (model) out$model <- m
else {
    if (x) out$x <- X
    if (y) out$y <- Y
    }
oldClass(out) <- 'pyears'
out
```

8 Accelerated Failure Time models

The `survreg` function fits parametric failure time models. This includes accerated failure time models, the Weibull, log-normal, and log-logistic models. It also fits as well as censored linear regression; with left censoring this is referred to in economics *Tobit* regression.

8.1 Residuals

The residuals for a `survreg` model are one of several types

response residual `y` value on the scale of the original data

deviance an approximate deviance residual. A very bad idea statistically, retained for the sake of backwards compatability.

dfbeta a matrix with one row per observation and one column per parameter showing the approximate influence of each observation on the final parameter value

dfbetas the dfbeta residuals scaled by the standard error of each coefficient

working residuals on the scale of the linear predictor

ldcase likelihood displacement wrt case weights

ldresp likelihood displacement wrt response changes

ldshape likelihood displacement wrt changes in shape

matrix matrix of derivatives of the log-likelihood wrt paramters

The other parameters are

rsigma whether the scale parameters should be included in the result for dfbeta results. I can think of no reason why one would not want them — unless of course the scale was fixed by the user, in which case there is no parameter.

collapse optional vector of subject identifiers. This is for the case where a subject has multiple observations in a data set, and one wants to have residuals per subject rather than residuals per observation.

weighted whether the residuals should be multiplied by the case weights. The sum of weighted residuals will be zero.

The routine starts with standard stuff, checking arguments for validity and etc. The two cases of response or working residuals require a lot less computation. and are the most common calls, so they are taken care of first.

⟨*residuals.survreg*⟩=

```
 # $Id$
 #
 #  Residuals for survreg objects
 residuals.survreg <- function(object, type=c('response', 'deviance',
                       'dfbeta', 'dfbetas', 'working', 'ldcase',
                       'ldresp', 'ldshape', 'matrix'),
                       rsigma =TRUE, collapse=FALSE, weighted=FALSE, ...) {
     type <-match.arg(type)
     n <- length(object$linear.predictors)
```

```
    Terms <- object$terms
    if(!inherits(Terms, "terms"))
            stop("invalid terms component of  object")

    # If the variance wasn't estimated then it has no error
    if (nrow(object$var) == length(object$coefficients)) rsigma <- FALSE

    # If there was a cluster directive in the model statment then remove
    #  it.  It does not correspond to a coefficient, and would just confuse
    #  things later in the code.
    cluster <- untangle.specials(Terms,"cluster")$terms
    if (length(cluster) >0 )
        Terms <- Terms[-cluster]

    strata <- attr(Terms, 'specials')$strata
    coef <- object$coefficients
    intercept <- attr(Terms, "intercept")
    response  <- attr(Terms, "response")
    weights <- object$weights
    if (is.null(weights)) weighted <- FALSE

    ⟨rsr-dist⟩
    ⟨rsr-data⟩
    ⟨rsr-resid⟩
    ⟨rsr-finish⟩
    }
```

First retrieve the distribution, which is used multiple times. The common case is a character string pointing to some element of `survreg.distributions`, but the other is a user supplied list of the form contained there. Some distributions are defined as the transform of another in which case we need to set `itrans` and `dtrans` and follow the link, otherwise the transformation and its inverse are the identity.

```
⟨rsr-dist⟩=
 if (is.character(object$dist))
            dd <- survreg.distributions[[object$dist]]
 else dd <- object$dist
 if (is.null(dd$itrans)) {
         itrans <- dtrans <-function(x)x
         }
 else {
         itrans <- dd$itrans
         dtrans <- dd$dtrans
         }
 if (!is.null(dd$dist))  dd <- survreg.distributions[[dd$dist]]
 deviance <- dd$deviance
 dens <- dd$density
```

The next task is to decide what data we need. The response is always needed, but is normally saved as a part of the model. If it is a transformed distribution such as the Weibull (a transform of the extreme value) the saved object `y` is the transformed data, so we need to replicate that part of the survreg() code. (Why did I even allow for y=F in survreg? Because I was mimicing the lm function — oh the long, long consequences of a design decision.)

The covariate matrix `x` will be needed for all but response, deviance, and working residuals. If the model included a strata() term then there will be multiple scales, and the strata variable needs to be recovered. The variable `sigma` is set to a scalar if there are no strata, but otherwise to a vector with `n` elements containing the appropriate scale for each subject.

The leverage type residuals all need the second derivative matrix. If there was a `cluster` statement in the model this will be found in `naive.var`, otherwise in the `var` component.

⟨*rsr-data*⟩=
```
 if (is.null(object$naive.var)) vv <- object$var
 else                           vv <- object$naive.var

 need.x <- is.na(match(type, c('response', 'deviance', 'working')))
 if (is.null(object$y) || !is.null(strata) || (need.x & is.null(object[['x']])))
     mf <- stats::model.frame(object)

 y <- object$y
 if (is.null(y)) {
     y <- model.extract(mf, 'response')
     if (!is.null(dd$trans)) {
         tranfun <- dd$trans
         exactsurv <- y[,ncol(y)] ==1
         if (any(exactsurv)) logcorrect <-sum(log(dd$dtrans(y[exactsurv,1])))

         if (type=='interval') {
             if (any(y[,3]==3))
                     y <- cbind(tranfun(y[,1:2]), y[,3])
             else y <- cbind(tranfun(y[,1]), y[,3])
             }
         else if (type=='left')
              y <- cbind(tranfun(y[,1]), 2-y[,2])
         else     y <- cbind(tranfun(y[,1]), y[,2])
         }
     else {
         if (type=='left') y[,2] <- 2- y[,2]
         else if (type=='interval' && all(y[,3]<3)) y <- y[,c(1,3)]
         }
     }

 if (!is.null(strata)) {
     temp <- untangle.specials(Terms, 'strata', 1)
     Terms2 <- Terms[-temp$terms]
```

```
    if (length(temp$vars)==1) strata.keep <- mf[[temp$vars]]
    else strata.keep <- strata(mf[,temp$vars], shortlabel=TRUE)
    strata <- as.numeric(strata.keep)
    nstrata <- max(strata)
    sigma <- object$scale[strata]
    }
else {
    Terms2 <- Terms
    nstrata <- 1
    sigma <- object$scale
    }

if (need.x) {
   x <- object[['x']]  #don't grab xlevels component
   if (is.null(x))
        x <- model.matrix(Terms2, mf, contrasts.arg=object$contrasts)
    }
```

The most common residual is type response, which requires almost no more work, for the others we need to create the matrix of derivatives before proceeding. We use the `center` component from the deviance function for the distribution, which returns the data point `y` itself for an exact, left, or right censored observation, and an appropriate midpoint for interval censored ones.

⟨*rsr-resid*⟩=
```
if (type=='response') {
    yhat0 <- deviance(y, sigma, object$parms)
    rr <-  itrans(yhat0$center) - itrans(object$linear.predictor)
    }
else {
    ⟨rtr-deriv⟩
    ⟨rtr-resid2⟩
    }
```

The matrix of derviatives is used in all of the other cases. The starting point is the `density` function of the distribtion which return a matrix with columns of $F(x)$, $1-F(x)$, $f(x)$, $f'(x)/f(x)$ and $f''(x)/f(x)$. The matrix type residual contains columns for each of

$$L_i \quad \frac{\partial L_i}{\partial \eta_i} \quad \frac{\partial^2 L_i}{\partial \eta_i^2} \quad \frac{\partial L_i}{\partial \log(\sigma)} \quad \frac{\partial L_i}{\partial \log(\sigma)^2} \quad \frac{\partial^2 L_i}{\partial \eta \partial \log(\sigma)}$$

where L_i is the contribution to the log-likelihood from each individual. Note that if there are multiple scales, i.e. a strata() term in the model, then terms 3–6 are the derivatives for that subject with respect to their *particular* scale factor; derivatives with respect to all the other scales are zero for that subject.

The log-likelihood can be written as

$$\begin{aligned} L &= \sum_{exact} [\log(f(z_i)) - \log(\sigma_i)] + \sum_{censored} \log\left(\int_{z_i^l}^{z_i^u} f(u)du\right) \\ &\equiv \sum_{exact} [g_1(z_i) - \log(\sigma_i)] + \sum_{censored} \log(g_2(z_i^l, z_i^u)) \\ z_i &= (y_i - \eta_i)/\sigma_i \end{aligned}$$

For the interval censored observations we have a z defined at both the lower and upper endpoints. The linear predictor is $\eta = X\beta$.

The derivatives are shown below. Note that $f(-\infty) = f(\infty) = F(-\infty) = 0$, $F(\infty) = 1$, $z^u = \infty$ for a right censored observation and $z^l = -\infty$ for a left censored one.

$$\begin{aligned} \frac{\partial g_1}{\partial \eta} &= -\frac{1}{\sigma}\left[\frac{f'(z)}{f(z)}\right] \\ \frac{\partial g_2}{\partial \eta} &= -\frac{1}{\sigma}\left[\frac{f(z^u) - f(z^l)}{F(z^u) - F(z^l)}\right] \\ \frac{\partial^2 g_1}{\partial \eta^2} &= \frac{1}{\sigma^2}\left[\frac{f''(z)}{f(z)}\right] - (\partial g_1/\partial\eta)^2 \\ \frac{\partial^2 g_2}{\partial \eta^2} &= \frac{1}{\sigma^2}\left[\frac{f'(z^u) - f'(z^l)}{F(z^u) - F(z^l)}\right] - (\partial g_2/\partial\eta)^2 \\ \frac{\partial g_1}{\partial \log\sigma} &= -\left[\frac{zf'(z)}{f(z)}\right] \\ \frac{\partial g_2}{\partial \log\sigma} &= -\left[\frac{z^u f(z^u) - z^l f(z^l)}{F(z^u) - F(z^l)}\right] \\ \frac{\partial^2 g_1}{\partial (\log\sigma)^2} &= \left[\frac{z^2 f''(z) + zf'(z)}{f(z)}\right] - (\partial g_1/\partial\log\sigma)^2 \\ \frac{\partial^2 g_2}{\partial (\log\sigma)^2} &= \left[\frac{(z^u)^2 f'(z^u) - (z^l)^2 f'(z_l)}{F(z^u) - F(z^l)}\right] - \partial g_1/\partial\log\sigma(1 + \partial g_1/\partial\log\sigma) \\ \frac{\partial^2 g_1}{\partial\eta\partial\log\sigma} &= \frac{zf''(z)}{\sigma f(z)} - \partial g_1/\partial\eta(1 + \partial g_1/\partial\log\sigma) \\ \frac{\partial^2 g_2}{\partial\eta\partial\log\sigma} &= \frac{z^u f'(z^u) - z^l f'(z^l)}{\sigma[F(z^u) - F(z^l)]} - \partial g_2/\partial\eta(1 + \partial g_2/\partial\log\sigma) \end{aligned}$$

In the code `z` is the relevant point for exact, left, or right censored data, and `z2` the upper endpoint for an interval censored one. The variable `tdenom` contains the denominator for each subject (which is the same for all derivatives for that subject). For an interval censored observation we try to avoid numeric cancellation by using the appropriate tail of the distribution. For instance with $(z^l, z^u) = (12, 15)$ the value of $F(x)$ will be very near 1 and it is better to subtract two upper tail values $(1 - F)$ than two lower tail ones F.

⟨*rtr-deriv*⟩=

```
status <- y[,ncol(y)]
eta <- object$linear.predictors
z <- (y[,1] - eta)/sigma
dmat <- dens(z, object$parms)
dtemp<- dmat[,3] * dmat[,4]    #f'
if (any(status==3)) {
    z2 <- (y[,2] - eta)/sigma
    dmat2 <- dens(z2, object$parms)
    }
else {
    dmat2 <- dmat   #dummy values
    z2 <- 0
    }

tdenom <- ((status==0) * dmat[,2]) +  #right censored
          ((status==1) * 1 )       +  #exact
          ((status==2) * dmat[,1]) +  #left
          ((status==3) * ifelse(z>0, dmat[,2]-dmat2[,2],
                                     dmat2[,1] - dmat[,1])) #interval
g <- log(ifelse(status==1, dmat[,3]/sigma, tdenom))  #loglik
tdenom <- 1/tdenom
dg <- -(tdenom/sigma) *(((status==0) * (0-dmat[,3])) +    #dg/ eta
                        ((status==1) * dmat[,4]) +
                        ((status==2) * dmat[,3]) +
                        ((status==3) * (dmat2[,3]- dmat[,3])))

ddg <- (tdenom/sigma^2) *(((status==0) * (0- dtemp)) +  #ddg/eta^2
                          ((status==1) * dmat[,5]) +
                          ((status==2) * dtemp) +
                          ((status==3) * (dmat2[,3]*dmat2[,4] - dtemp)))

ds  <- ifelse(status<3, dg * sigma * z,
                        tdenom*(z2*dmat2[,3] - z*dmat[,3]))
dds <- ifelse(status<3, ddg* (sigma*z)^2,
                        tdenom*(z2*z2*dmat2[,3]*dmat2[,4] -
                                z * z*dmat[,3] * dmat[,4]))
dsg <- ifelse(status<3, ddg* sigma*z,
              tdenom *(z2*dmat2[,3]*dmat2[,4] - z*dtemp))
deriv <- cbind(g, dg, ddg=ddg- dg^2,
               ds = ifelse(status==1, ds-1, ds),
               dds=dds - ds*(1+ds),
               dsg=dsg - dg*(1+ds))
```

Now, we can calcultate the actual residuals case by case. For the dfbetas there will be one column per coefficient, so if there are strata column 4 of the deriv matrix needs to be *un*collapsed

into a matrix with nstrata columns. The same manipulation is needed for the ld residuals.

⟨*rtr-resid2*⟩=
```
 if (type=='deviance') {
     yhat0 <- deviance(y, sigma, object$parms)
     rr <- (-1)*deriv[,2]/deriv[,3]  #working residuals
     rr <- sign(rr)* sqrt(2*(yhat0$loglik - deriv[,1]))
     }

 else if (type=='working') rr <- (-1)*deriv[,2]/deriv[,3]

 else if (type=='dfbeta' || type== 'dfbetas' || type=='ldcase') {
     score <- deriv[,2] * x  # score residuals
     if (rsigma) {
         if (nstrata > 1) {
             d4 <- matrix(0., nrow=n, ncol=nstrata)
             d4[cbind(1:n, strata)] <- deriv[,4]
             score <- cbind(score, d4)
             }
         else score <- cbind(score, deriv[,4])
         }
     rr <- score %*% vv
     if (type=='dfbetas') rr <- rr %*% diag(1/sqrt(diag(vv)))
     if (type=='ldcase')  rr<- rowSums(rr*score)
     }

 else if (type=='ldresp') {
     rscore <-  deriv[,3] *  (x * sigma)
     if (rsigma) {
         if (nstrata >1) {
             d6 <- matrix(0., nrow=n, ncol=nstrata)
             d6[cbind(1:n, strata)] <- deriv[,6]*sigma
             rscore <- cbind(rscore, d6)
             }
         else rscore <- cbind(rscore, deriv[,6] * sigma)
         }
     temp <-  rscore %*% vv
     rr <- rowSums(rscore * temp)
     }

 else if (type=='ldshape') {
     sscore <- deriv[,6] *x
     if (rsigma) {
         if (nstrata >1) {
             d5 <- matrix(0., nrow=n, ncol=nstrata)
             d5[cbind(1:n, strata)] <- deriv[,5]
```

```
            sscore <- cbind(sscore, d5)
            }
        else sscore <- cbind(sscore, deriv[,5])
        }
    temp <- sscore %*% vv
    rr <- rowSums(sscore * temp)
    }

else {  #type = matrix
    rr <- deriv
    }
```

Finally the two optional steps of adding case weights and collapsing over subject id.

⟨*rsr-finish*⟩=

```
#case weights
if (weighted) rr <- rr * weights

#Expand out the missing values in the result
if (!is.null(object$na.action)) {
    rr <- naresid(object$na.action, rr)
    if (is.matrix(rr)) n <- nrow(rr)
    else               n <- length(rr)
    }

# Collapse if desired
if (!missing(collapse)) {
    if (length(collapse) !=n) stop("Wrong length for 'collapse'")
    rr <- drop(rowsum(rr, collapse))
    }

rr
```

9 Survival curves

The survfit function was set up as a method so that we could apply the function to both formulas (to compute the Kaplan-Meier) and to coxph objects. The downside to this is that the manual pages get a little odd, but from a programming perspective it was a good idea. At one time, long long ago, we allowed the function to be called with "Surv(time, status)" as the formula, i.e., without a tilde. That was a bad idea, now abandoned.

A note on times: one of the things that drove me nuts was the problem of "tied but not quite tied" times. As an example consider two values of $24173 = 23805 + 368$. These are values from an actual study with times in days. However, the user chose to use age in years, and saved those values out in a CSV file, resulting in values for the above of 66.18206708000000 and 66.18206708000001. The R phrase `unique(x)` sees these two values as distinct but `table(x)` and `tapply` see it as a single value since they first apply `factor` to the values, and that in turn

uses `as.character`. The survfit routines use `table` and then turn the labels back into a vector of times, in order to ensure a consistent treatment of near ties.

⟨*survfit*⟩=
```
survfit <- function(formula, ...) {
    UseMethod("survfit", formula)
}

⟨survfit-subscript⟩
⟨survfit-formula⟩
⟨survfit-Surv⟩
```

The result of a survival curve can have a `surv` component that is a vector or a matrix, and an optional strata component. A dual subscript to a survfit object always associates the first subscript with the strata and the second with the matrix. When a survfit object has only one or the other of the two, we allow a single subscript to be used and map it appropriately.

⟨*survfit-subscript*⟩=
```
dim.survfit <- function(x) {
    if (is.null(x$strata)) {
        if (is.matrix(x$surv)) ncol(x$surv)
        else 1
    }
    else {
        nr <- length(x$strata)
        if (is.matrix(x$surv)) c(nr, ncol(x$surv))
        else if (is.matrix(x$prev)) c(nr, ncol(x$prev))
        else nr
    }
}

"[.survfit" <- function(x, ..., drop=TRUE) {
    nmatch <- function(indx, target) {
        # This function lets R worry about character, negative, or logical subscripts
        #  It always returns a set of positive integer indices
        temp <- 1:length(target)
        names(temp) <- target
        temp[indx]
    }

    if (missing(..1)) i<- NULL  else i <- ..1
    if (missing(..2)) j<- NULL  else j <- ..2
    if (is.null(i) && is.null(j)) return (x) #no subscripts present!
    if (!is.matrix(x$surv) && !is.null(j))
        stop("survfit object does not have 2 dimensions")

    if (is.null(x$strata)) {
```

```
    if (is.matrix(x$surv)) {
        if (is.null(j) && !is.null(i)) j <- i #special case noted above
        x$surv <- x$surv[,j,drop=drop]
        if (!is.null(x$std.err)) x$std.err <- x$std.err[,j,drop=drop]
        if (!is.null(x$upper)) x$upper <- x$upper[,j,drop=drop]
        if (!is.null(x$lower)) x$lower <- x$lower[,j,drop=drop]
        if (!is.null(x$cumhaz)) x$cumhaz <- x$cumhaz[,j,drop=drop]
    }
    else warning("survfit object has only a single survival curve")
}
else {
    if (is.null(i)) keep <- seq(along.with=x$time)
    else {
        indx <- nmatch(i, names(x$strata)) #strata to keep
        if (any(is.na(indx)))
            stop(gettextf("strata %s not matched", paste(i[is.na(indx)], collapse=' ')))

        # Now, indx may not be in order: some can use curve[3:2] to reorder
        #  The list/unlist construct will reorder the data
        temp <- rep(1:length(x$strata), x$strata)
        keep <- unlist(lapply(indx, function(x) which(temp==x)))

        if (length(indx) <=1 && drop) x$strata <- NULL
        else               x$strata  <- x$strata[i]

        x$n       <- x$n[indx]
        x$time    <- x$time[keep]
        x$n.risk  <- x$n.risk[keep]
        x$n.event <- x$n.event[keep]
        x$n.censor<- x$n.censor[keep]
        if (!is.null(x$enter)) x$enter <- x$enter[keep]
    }
    if (is.matrix(x$surv)) {
        # If the curve has been selected by strata and keep has only
        #  one row, we don't want to lose the second subscript too
        if (!is.null(i) && (is.null(j) ||length(j) >1)) drop <- FALSE
        if (is.null(j)) {
            x$surv <- x$surv[keep,,drop=drop]
            if (!is.null(x$std.err))
                    x$std.err <- x$std.err[keep,,drop=drop]
            if (!is.null(x$upper)) x$upper <-x$upper[keep,,drop=drop]
            if (!is.null(x$lower)) x$lower <-x$lower[keep,,drop=drop]
            if (!is.null(x$cumhaz)) x$cumhaz <-x$cumhaz[keep,,drop=drop]
        }
        else {
            x$surv <- x$surv[keep,j, drop=drop]
```

```
                if (!is.null(x$std.err))
                    x$std.err <- x$std.err[keep,j, drop=drop]
                if (!is.null(x$upper)) x$upper <- x$upper[keep,j, drop=drop]
                if (!is.null(x$lower)) x$lower <- x$lower[keep,j, drop=drop]
                if (!is.null(x$cumhaz)) x$cumhaz <- x$cumhaz[keep,j, drop=drop]
                }
        }
        else {
            x$surv <- x$surv[keep]
            if (!is.null(x$std.err)) x$std.err <- x$std.err[keep]
            if (!is.null(x$upper)) x$upper <- x$upper[keep]
            if (!is.null(x$lower)) x$lower <- x$lower[keep]
            if (!is.null(x$cumhaz)) x$cumhaz <- x$cumhaz[keep]
        }
    }
    x
}
```

9.1 Kaplan-Meier

The most common use of the survfit function is with a formula as the first argument, and the most common outcome of such a call is a Kaplan-Meier curve.

The id argument is from an older version of the competing risks code; most people will use `cluster(id)` in the formula instead. The istate argument only applies to competing risks, but don't print an error message if it is accidentally there.

⟨*survfit-formula*⟩=

```
survfit.formula <- function(formula, data, weights, subset,
                            na.action, etype, id, istate, ...) {
    Call <- match.call()
    Call[[1]] <- as.name('survfit')  #make nicer printout for the user
    # create a copy of the call that has only the arguments we want,
    #  and use it to call model.frame()
    indx <- match(c('formula', 'data', 'weights', 'subset','na.action',
                    'istate', 'id', "etype"), names(Call), nomatch=0)
    #It's very hard to get the next error message other than malice
    #  eg survfit(wt=Surv(time, status) ~1)
    if (indx[1]==0) stop(gettextf("'%s' argument is required", "formula"))
    temp <- Call[c(1, indx)]
    temp[[1L]] <- quote(stats::model.frame)
    m <- eval.parent(temp)

    Terms <- terms(formula, c("strata", "cluster"))
    ord <- attr(Terms, 'order')
    if (length(ord) & any(ord !=1))
            stop("Interaction terms are not valid for this function")
```

```
n <- nrow(m)
Y <- model.extract(m, 'response')
if (!is.Surv(Y)) stop("Response must be a survival object")

casewt <- model.extract(m, "weights")
if (is.null(casewt)) casewt <- rep(1,n)

if (!is.null(attr(Terms, 'offset'))) warning("Offset term ignored")

id    <- model.extract(m, 'id')
istate <- model.extract(m,"istate")
temp <- untangle.specials(Terms, "cluster")
if (length(temp$vars)>0) {
    if (length(temp$vars) > 1) stop("can not have two cluster terms")
    if (!is.null(id)) stop("can not have both a cluster term and an id variable")
    id <- m[[temp$vars]]
    Terms <- Terms[-temp$terms]
}

ll <- attr(Terms, 'term.labels')
if (length(ll) == 0) X <- factor(rep(1,n))  # ~1 on the right
else X <- strata(m[ll])

if (!is.Surv(Y)) stop(gettextf("'%s' argument is not an object of class %s", "y", dQuote("Surv

# Backwards support for the now-depreciated etype argument
etype <- model.extract(m, "etype")
if (!is.null(etype)) {
    if (attr(Y, "type") == "mcounting" ||
        attr(Y, "type") == "mright")
        stop("cannot use both the 'etype' argument and 'mstate' survival type")
    if (length(istate))
        stop("cannot use both the 'etype' and 'istate' arguments")
    status <- Y[,ncol(Y)]
    etype <- as.factor(etype)
    temp <- table(etype, status==0)

    if (all(rowSums(temp==0) ==1)) {
        # The user had a unique level of etype for the censors
        newlev <- levels(etype)[order(-temp[,2])] #censors first
    }
    else newlev <- c(" ", levels(etype)[temp[,1] >0])
    status <- factor(ifelse(status==0,0, as.numeric(etype)),
                         labels=newlev)
```

```
    if (attr(Y, 'type') == "right")
        Y <- Surv(Y[,1], status, type="mstate")
    else if (attr(Y, "type") == "counting")
        Y <- Surv(Y[,1], Y[,2], status, type="mstate")
    else stop("'etype' argument incompatable with survival type")
}

# At one point there were lines here to round the survival
# times to a certain number of digits.  This approach worked
# almost all the time, but only almost.  The better logic is
# now in the individual compuation routines
if (attr(Y, 'type') == 'left' || attr(Y, 'type') == 'interval')
    temp <-  survfitTurnbull(X, Y, casewt, ...)
else if (attr(Y, 'type') == "right" || attr(Y, 'type')== "counting")
    temp <- survfitKM(X, Y, casewt, ...)
else if (attr(Y, 'type') == "mright" || attr(Y, "type")== "mcounting")
    temp <- survfitCI(X, Y, weights=casewt, id=id,  istate=istate, ...)
else {
    # This should never happen
    stop("unrecognized survival type")
}
if (is.null(temp$states)) class(temp) <- 'survfit'
else class(temp) <- c("survfitms", "survfit")

if (!is.null(attr(m, 'na.action')))
        temp$na.action <- attr(m, 'na.action')

temp$call <- Call
temp
}
```

Once upon a time I allowed survfit to be called without the '~1' portion of the formula. This was a mistake for multiple reasons, but the biggest problem is timing. If the subject has a data statement but the first argument is not a formula, R needs to evaluate Surv(t,s) to know that it is a survival object, but it also needs to know that this is a survival object before evaluation in order to dispatch the correct method. The method below helps give a useful error message in some cases.

⟨*survfit-Surv*⟩=
```
 survfit.Surv <- function(formula, ...)
    stop("the survfit function requires a formula as its first argument")
```

9.2 Competing risks

The competing risks routine is very general, allowing subjects to enter or exit states multiple times. For this reason I prefer the label *current prevalence* estimate, since it estimates what fraction of the subjects are in any given state across time. However the word "prevalence" is

likely to generate confusion whenever death is one of the states, due to its historic use as the fraction of living subjects who have a particular condition. We will use the phrase *probability in state* or simply P from this point forward.

The easiest way to understand the estimate is to consider first the case of no censoring. In that setting the estimate of $F_k(t) = 1 - S_k(t)$ for all states is obtained from a simple table of the current state at time t of the subjects, divided by n, the original sample size. When there is censoring the conceptually simple way to extend this is via the redistribute-to-the-right algorithm, which allocates the case weight for a censored subject evenly to all the others in the same state at the time of censoring.

The literature refers to these as "cumulative incidence" curves, which is confusing since prevalence is not the integral of incidence, but the routine name survfitCI endures. The cannonical call is

```
fit <- survfit(Surv(time, status, type='mstate') ~ sex, data=mine)
```

Optionally, there can be an id statement or cluster term to indicate a data set with multiple transitions per subject. A multi-state survival fit has a status variable with multiple levels, the first of which by default is censoring, and others indicating the type of transition that occured. The result will be a matrix of survival curves, one for each event type. In no initial state is specified then subjects are assumed to start in a "null" state, which gets listed last and by default will not be printed or plotted. (But it is present, with a name of '');

The first part of the code is standard, parsing out options and checking the data.

⟨*survfitCI*⟩=

```
survfitCI <- function(X, Y, weights, id, istate,
                      type=c('kaplan-meier', 'fleming-harrington', 'fh2'),
                      se.fit=TRUE,
                      conf.int= .95,
                      conf.type=c('log',  'log-log',  'plain', 'none'),
                      conf.lower=c('usual', 'peto', 'modified')){

    method <- match.arg(type)
#    error <- match.arg(error)
#    if (error != "inf")
#        warning("Only the infinetesimal jackknife error is supported for CI curves")
    conf.type <- match.arg(conf.type)
    conf.lower<- match.arg(conf.lower)
    if (is.logical(conf.int)) {
        # A common error is for users to use "conf.int = FALSE"
        #  it's illegal per documentation, but be kind
        if (!conf.int) conf.type <- "none"
        conf.int <- .95
    }

    type <- attr(Y, "type")
    # This line should be unreachable, unless they call "surfitCI"
    if (type !='mright' && type!='mcounting')
```

```
     stop(gettextf("multi-state computation doesn't support \"%s\" survival data", type))

n <- nrow(Y)
status <- Y[,ncol(Y)]
ncurve <- length(levels(X))

state.names <- attr(Y, "states")
nstate <- length(state.names)
has.istate <- !missing(istate)
if (missing(istate) || is.null(istate)) {
    istate <- rep(nstate+ 1L, n)
    state.names <- c(state.names, "")
    }
else if (is.factor(istate) || is.character(istate)) {
    # Match levels with the survival variable
    temp <- as.factor(istate)
    # append any starting states not found in Y, but remember that
    #  if istate was a factor then not all its levels might appear
    appear <- (levels(temp))[unique(as.numeric(temp))]
    state.names <- unique(c(attr(Y, "states"), appear))
    istate <- as.numeric(factor(as.character(temp), levels=state.names))
}
else {
    if (!is.numeric(istate) || any(istate != floor(istate)) ||
         any(istate < 1))
    stop("'istate' argument should be a vector of integers or a factor")
    if (max(istate) > nstate)
        state.names <- c(state.names, (1+nstate):max(istate))
    }

if (length(id) ==0) id <- 1:n
# these next two lines should be impossible, since istate came from
#   the data frame
if (length(istate) ==1) istate <- rep(istate,n)
if (length(istate) !=n) stop(gettextf("wrong length for '%s' argument", "istate"))

# The states of the status variable are the first columns in the output
states <- unique(c(seq_len(nstate), istate))
⟨survfitCI-compute⟩
```

To make it easier to keep track of things in the computational kernel that does all the real work, we have ensured that any ending states (ones you can reach) are 1, 2, 3 .... The status vector will have values of 0 for censored.

⟨*survfitCI*⟩=

```
    curves <- vector("list", ncurve)
    names(curves) <- levels(X)
```

```
    if (ncol(Y)==2) {  # 1 transition per subject
        indx <- which(status == istate & status!=0)
        if (length(indx)) {
            warning("an observation transitions to it's starting state, transition ignored")
            status[indx] <- 0
        }
        if (length(id) && any(duplicated(id)))
            stop("Cannot have duplicate id values with (time, status) data")

        # make a table of transitions.  The from can range across
        #  all of the states, to can only have nstate categories
        nst <- length(state.names)
        transitions <- table(factor(istate, 1:nst), factor(Y[,2], 1:nstate))
        dimnames(transitions) <-list(from=state.names, to=state.names[1:nstate])

        # dummy entry time that is < any event time
        entry <- rep(min(-1, 2*min(Y[,1])-1), n)
        for (i in levels(X)) {
            indx <- which(X==i)
#            temp  <- docurve1(entry[indx], Y[indx,1], status[indx],
#                              istate[indx], weights[indx], states,
#                              id[indx])
            curves[[i]] <- docurve2(entry[indx], Y[indx,1], status[indx],
                                    istate[indx], weights[indx], states,
                                    id[indx], se.fit)
         }
    }
    else {
        ⟨survfitCI-idcheck⟩
        ⟨survfitCI-startstop⟩
    }

    ⟨survfitCI-finish⟩
}
```

In the multi-state case we can calculate the current prevalence vector $p(t)$ using the product-limit form

$$\begin{aligned} p(t) &= p(0) \prod_{s<=t} [I + dA(s)] \\ &= p(0) \prod_{s<=t} H(s) \end{aligned}$$

Where p is a row vector and H is the multi-state hazard matrix. At each event time we define

the off diagonal elements of H by

$$H_{jk}(t) = \sum_i w_i dN_{ijk}(t) / \sum w_i Y_{ij}(t)$$

where N_{ijk} counts the number of observed trasitions between state j and state k for subject i, $Y_{ij}(t)$ is 1 if subject i is in state j at time t, w_i is the weight for subject i, and 0/0 is treated as 0. Row j of $H(t)$ describes the fate of those subjects in state j, going from time $t-0$ to time t. The diagonal elements of H are set so that each row of H sums to 1 (everyone has to go somewhere). This formula collapses to the Kaplan-Meier in the simple case where $p(t)$ is a vector of length 2 with state 1 = alive and state 2 = dead.

A robust variance for the product-limit estimate is based on the chain rule. Consider the n by k matrix of per subject influence values

$$\begin{aligned} U_{ik}(t) &= \frac{\partial p_k(t)}{\partial w_i} \\ &= \frac{\partial [p(t-)H_{.k}(t)]}{\partial w_i} \qquad (15) \\ &= U_{i.}(t-)H_{.k}(t) + p(t-)\frac{\partial H_{.k}(t)}{\partial w_i} \qquad (16) \end{aligned}$$

$$\frac{\partial H_{jk}(t)}{\partial w_i} = \begin{cases} \left(dN_{ijk}(t) - Y_{ij}(t)H_{jk}\right)/n_j(t) & j \neq k \\ -\left(dN_{ij.}(t) - Y_{ij}(t)(1-H_{jj})\right)/n_j(t) & j = k \end{cases} \qquad (17)$$

where $H_{.k}$ is the kth column of H amd $n_j(t) = \sum_i Y_{ij}(t)w_i$ is the weighted number of subjects in state j. Equation (15) replaces $p(t)$ with the last step of the compuation that created it. The next writes this out carefully using the chain rule, leading to an recursive equation. The first term of (16) is the formula for ordinary matrix multiplication. In equation (17) the derivative of H with respect to subject i will be a matrix which is non-zero only for the row corresponding to the current state of the subject. (I've skipped some intermediate steps in the derivation, they are left as an exercise for the reader). Since each row of H sums to a constant, each row of the deviative must sum to zero.

The weighted sum of each column of U must zero (if computed correctly) and the weighted sum of squares for each column will be the infinitesimal jackknife estimate of variance for the elements of p. The entire variance-covariance matrix for the states is $U'WU$ where W is a diagonal matrix of weights, but we currently don't report that back. Note that this is for sampling weights. If one has real case weights, where an integer weight of 2 means 2 observations that were collapsed in to one row of data to save space, then the formula is $U'W^2U$. Case weights were somewhat common in my youth, due to small computer memory, but I haven't seen such data in 20 years.

Below is the function for a single curve. For the status variable a value if 0 is "no event". One nuisance in the function is that we need to ensure the tapply command gives totals for all states, not just the ones present in the data — a call using the `subset` argument might not have all the states — which leads to using factor commands. Another more confusing one is for multiple rows per subject data, where the cstate and U objects have only one row per subject; any given subject is only in one state at a time. This leads to indices of `atrisk` for the set of rows in the risk set but `aindx` for the subjects in the risk set, `death` for the rows that have an event as some given time and `dindx` for the corresponding subjects.

⟨*survfitCI-compute-old*⟩=

```
 docurve1 <- function(entry, etime, status, istate, wt, states, id) {
     #
     # round off error can cause trouble: if two times are within machine
     #  precision then "unique(time)" and the "table" command may differ
     # solve this by using creating a factor
     ftime <- factor(etime)
     ntime <- length(levels(ftime))
     # If someone has chosen to set the OutSep option to ',' (France) the simple
     #  as.numeric(levels(ftime)) will fail
     timeset <- type.convert(levels(ftime), as.is=TRUE, dec=getOption("OutDec"))
     ftime <- as.numeric(ftime)

     nstate <- length(states)
     Pmat <- matrix(0., nrow= ntime, ncol=nstate)
     vP <- Pmat  #variance
     A <- array(0., dim=c(nstate, nstate, ntime))
     uid <- sort(unique(id))
     U <- matrix(0., length(uid), nstate)  #one row per subject
     P <- as.vector(tapply(wt, factor(istate, levels=states), sum) / sum(wt))
     P <- Pmat[1,] <- ifelse(is.na(P), 0, P)
     cstate <- istate[match(uid, id)]   #current state for each observation

     nrisk <- integer(ntime)  #to be returned
     wrisk <- double(ntime)  #weighted number at risk
     nevent <- table(ftime, status>0)
     for (i in 1:ntime) {
         atrisk <- (ftime >=i & timeset[i] > entry)
         nrisk[i] <- sum(atrisk)
         wrisk[i] <- sum(wt[atrisk])
         tiedtime <- (ftime==i)
         if (nevent[i,2] ==0)  { # all censored here
             Pmat[i,] <- P
             if (i>1) {
                 A[,,i] <- A[,,i-1]
                 vP[i,] <- vP[i-1,]
                 }
          }
         else {
             # do real work
             #  A bit of nuisance is to force tapply to give totals for all states
             aindx <- match(id[atrisk], uid)   #the id pointer for those at risk
             ns <- as.vector(tapply(wt[atrisk], factor(cstate[aindx], levels=states),sum))
             dead <- which(tiedtime & status >0)  #the events at this time
             dindx <- match(id[dead], uid)
             nevent[i] <- length(dead)
```

```
            H <- tapply(wt[dead], list(factor(cstate[dindx], levels=states),
                                       factor(status[dead], levels=states)),sum)/ns
            H <- ifelse(is.na(H), 0, H) # H has NA for combinations with no representatives
            diag(H) <- 1- rowSums(H)

            H2 <- H
            diag(H2) <- diag(H2) -1  #version of H needed for U and A, rows sum to 0
            if (i==1) A[,,1] <- H2
            else      A[,,i] <- A[,,i-1] + H2
            newstate <- status[dead]    # where the transitions go, will never be 0
            oldstate <- cstate[dindx]   # where they came from
            U <- U%*%H   #first part of update

            U[aindx,] <- U[aindx,] - (P*H2/ns)[cstate[aindx], ]
            temp <- P[oldstate]/ns[oldstate]    #the extra update for the events
            U[cbind(dindx, oldstate)] <- U[cbind(dindx, oldstate)] - temp
            U[cbind(dindx, newstate)] <- U[cbind(dindx, newstate)] + temp
            cstate[dindx] <- newstate
            P <- Pmat[i,] <- c(P %*% H)
            vP[i,] <- colSums((wt[match(uid, id)] *U)^2)
        }
    }
    list(time =as.vector(timeset), pmat=Pmat, std=sqrt(vP),
         n.event= table(ftime, status)[,-1], n.risk= as.vector(nrisk),
         w.risk=wrisk, cumhaz=A)
}
```

The above function was used to work through all of my test cases, but is too slow in large data sets. Rewrite it using underlying C-code, but retain the former one for debugging purposes. The C code appears at the end of this chapter.

The setup for (start, stop] data is a bit more work. We want to ensure that a subject's weight is fixed, that they have a continuous period of observation, and that they don't transfer from a state to itself. The last is not strictly an error, so only warn.

⟨*survfitCI-idcheck*⟩=

```
if (missing(id) || is.null(id))
    stop("the id argument is required for start:stop data")

indx <- order(id, Y[,2])  #ordered event times within subject
indx1 <- indx[-length(indx)]  #a pair of lagged indices
indx2 <- indx[-1]
#if indx1[5] == index2[5] that means that the 5th and 6th are the same id
same <- (id[indx1] == id[indx2])
if (any(same & X[indx1] != X[indx2])) {
    who <- min(which(same & X[indx1] != X[indx2]))
```

```
    stop(gettextf("subject is in two different groups, id %s", id[indx1[who]]))
}
if (any(same & Y[indx1,2] != Y[indx2,1])) {
    who <- min(which(same & Y[indx1,2] != Y[indx2,1]))
    stop(gettextf("gap in follow-up, id %s", id[indx1[who]]))
}
if (any(Y[,1] == Y[,2]))
    stop("cannot have start time == stop time")

if (any(same & (Y[indx1,3] == Y[indx2,3]) & (Y[indx1,3] !=0))) {
    who <-  min(which(same & (Y[indx1,3] == Y[indx2,3]) & (Y[indx1,3] !=0)))
    warning(gettextf("subject changes to the same state, id %s", id[indx1[who]]))
}
if (any(same & weights[indx1] != weights[indx2])) {
    who <-  min(which(same & weights[indx1] != weights[indx2]))
    stop(gettextf("subject changes case weights, id %s", id[indx1[who]]))
}

# Make the table of transitions
nst <- length(state.names)
first <- indx[!duplicated(id[indx])]
transitions <- table(factor(istate[first], 1:nst),
                     factor(Y[first,3], 1:nstate))
if (any(same))
    transitions <- transitions + table(factor(Y[indx1[same],3], 1:nst),
                                       factor(Y[indx2[same],3], 1:nstate))
dimnames(transitions) = list(from=state.names, to=state.names[1:nstate])
```

⟨*survfitCI-startstop*⟩=

```
# We only want to pay attention to the istate variable for the very first
#  observation of any given subject, but the program logic does better with
#  a full one.  So construct one that will do this
indx <- order(Y[,2])
uid <- unique(id)
temp <- (istate[indx])[match(uid, id[indx])]  #first istate for each subject
istate <- temp[match(id, uid)]  #replicate it to full length

# Now to work
for (i in levels(X)) {
    indx <- which(X==i)
#    temp <- docurve1(Y[indx,1], Y[indx,2], status[indx],
#                           istate[indx], weights[indx], states, id[indx])
    curves[[i]] <- docurve2(Y[indx,1], Y[indx,2], status[indx],
                          istate[indx], weights[indx], states, id[indx], se.fit)
}
```

⟨*survfitCI-finish*⟩=

```
# Turn the result into a survfit type object
grabit <- function(clist, element) {
    temp <-(clist[[1]][[element]])
    if (is.matrix(temp)) {
        nc <- ncol(temp)
        matrix(unlist(lapply(clist, function(x) t(x[[element]]))),
                       byrow=T, ncol=nc)
        }
    else {
        xx <- as.vector(unlist(lapply(clist, function(x) x[element])))
        if (class(temp)=="table") matrix(xx, byrow=T, ncol=length(temp))
        else xx
    }
}
kfit <- list(n =      as.vector(table(X)),
             time =   grabit(curves, "time"),
             n.risk=  grabit(curves, "n.risk"),
             n.event= grabit(curves, "n.event"),
             n.censor=grabit(curves, "n.censor"),
             prev   = grabit(curves, "prev"),
             p0  = grabit(curves, "p0"),
             transitions = transitions)
nstate <- length(states)
kfit$cumhaz <- array(unlist(lapply(curves, function(x) x$cumhaz)),
                           dim=c(nstate, nstate, length(kfit$time)))
if (length(curves) >1)
    kfit$strata <- unlist(lapply(curves, function(x) length(x$time)))
if (se.fit) kfit$std.err <- grabit(curves, "std")
kfit$istate <- has.istate  # used later in plots
```

Add the confidence bands. The idea is modeled on survfitKM but with the important differences that we are dealing with P instead of S, and the "modified lower limit" logic does not apply. We make the assumption that $\log(1-P)$ will have better CI behavior than P, with standard error of $rmse(P)/(1-P)$.

⟨*survfitCI-finish*⟩=

```
#
# Last bit: add in the confidence bands:
#   modeled on survfit.km, though for P instead of S
#
#
if (se.fit) {
    std.err <- kfit$std.err
    zval <- qnorm(1- (1-conf.int)/2, 0,1)
    surv <- 1-kfit$prev
```

```
    if (conf.type=='plain') {
        temp <- zval* std.err
        kfit <- c(kfit, list(lower =pmax(kfit$prev-temp, 0),
                             upper=pmin(kfit$prev+temp, 1),
                         conf.type='plain', conf.int=conf.int))
        }

    if (conf.type=='log') {
        #avoid some "log(0)" messages
        xx <- ifelse(kfit$prev==1, 1, 1- kfit$prev)

        temp1 <- ifelse(surv==0, NA, exp(log(xx) + zval* std.err/xx))
        temp2 <- ifelse(surv==0, NA, exp(log(xx) - zval* std.err/xx))
        kfit <- c(kfit, list(lower=pmax(1-temp1,0), upper= 1- temp2,
                         conf.type='log', conf.int=conf.int))
        }

    if (conf.type=='log-log') {
        who <- (surv==0 | surv==1) #special cases
        temp3 <- ifelse(surv==0, NA, 1)
        xx <- ifelse(who, .1,kfit$surv)  #avoid some "log(0)" messages
        temp1 <- exp(-exp(log(-log(xx)) + zval*std.err/(xx*log(xx))))
        temp1 <- ifelse(who, temp3, temp1)
        temp2 <- exp(-exp(log(-log(xx)) - zval*std.err/(xx*log(xx))))
        temp2 <- ifelse(who, temp3, temp2)
        kfit <- c(kfit, list(lower=1-temp1, upper=1-temp2,
                         conf.type='log-log', conf.int=conf.int))
        }
    }

 kfit$states <- state.names
 kfit$type   <- attr(Y, "type")
 kfit
```

The updated docurve function is here

⟨*survfitCI-compute*⟩=

```
 docurve2 <- function(entry, etime, status, istate, wt, states, id, se.fit) {
    #
    # round off error can cause trouble, if two times are within machine
    #  precsion
    # solve this by creating a factor
    ftime <- factor(c(entry,etime))
    ltime <- levels(ftime)
    ftime <- matrix(as.integer(ftime), ncol=2)
    timeset <- as.numeric(ltime[sort(unique(ftime[,2]))]) #unique event times
```

```
    nstate <- length(states)
    uid <- sort(unique(id))
    P <- as.vector(tapply(wt, factor(istate, levels=states), sum) / sum(wt))
    P <- ifelse(is.na(P), 0, P) # initial probability distribution
    cstate <- istate[match(uid, id)]   #initial state for each observation

    storage.mode(wt) <- "double" # just in case someone had integer weights
    storage.mode(cstate) <- "integer"
    storage.mode(status) <- "integer"
    # C code has 0 based subscripts
    fit <- .Call(Csurvfitci, ftime,
                 order(ftime[,1]) - 1L,
                 order(ftime[,2]) - 1L,
                 length(timeset),
                 status,
                 cstate - 1L,
                 wt,
                 match(id, uid) -1L,
                 P, as.integer(se.fit))
    p0 <- table(factor(cstate, levels=states), exclude=NA)/length(cstate)
    n.event <- table(ftime[,2], factor(status,c(0,states)))[,-1]
    if (se.fit)
        list(time=timeset, prev=t(fit$p), std=sqrt(t(fit$var)),
             n.risk = t(fit$nrisk),
             n.event = n.event,
             n.censor=fit$ncensor, p0 = p0,
             cumhaz=array(fit$cumhaz, dim=c(nstate,nstate, length(timeset))))
    else list(time=timeset, prev=t(fit$p),
             n.risk = t(fit$nrisk),
             n.event = n.event,
             n.censor=fit$ncensor,  p0=p0,
             cumhaz=array(fit$cumhaz, dim=c(nstate,nstate, length(timeset))))
}
```

9.2.1 C-code

(This is set up as a separate file in the source code directory since it is easier to make the code stay in C-mode if the file has a .nw extension.)

⟨*survfitci*⟩=

```
#include "survS.h"
⟨survfitci-dmatrix⟩

SEXP survfitci(SEXP ftime2,  SEXP sort12,  SEXP sort22, SEXP ntime2,
                   SEXP status2, SEXP cstate2, SEXP wt2,  SEXP id2,
                   SEXP p2,  SEXP sefit2) {
```

```
    ⟨survfitci-declare⟩
    ⟨survfitci-compute⟩
    ⟨survfitci-return⟩
}
```

Arguments to the routine are the following. For an R object "zed" I use the convention of `zed2` to refer to the object and `zed` to the contents of the object.

ftime A two column matrix containing the entry and exit times for each subject.

sort1 Order vector for the entry times. The first element of sort1 points to the first entry time, etc.

sort2 Order vector for the event times.

ntime Number of unique event time values. This fixes the size of the output arrays.

status Status for each observation. 0= censored

cstate The initial state for each subject, which will be updated during computation to always be the current state.

wt Case weight for each observation.

id The subject id for each observation.

p The initial distribution of states. This will be updated during computation to be the current distribution.

sefit If 1 then do the se compuatation, otherwise forget it.

The local dmatrix2 function makes it easier to declare ragged arrays, which allows for the nice `x[i][j]` notation for arrays.

⟨*survfitci-dmatrix*⟩=

```
/* allocate a ragged array of a given number of rows and columns */
static double **dmatrix2(int nrow, int ncol) {
    int i;
    double **mat;
    double *d;

    mat = (double **) R_alloc(nrow, sizeof(double *));
    d   = (double *) R_alloc(nrow*ncol, sizeof(double));
    for (i=0; i<nrow; i++) {
        mat[i] = d;
        d += ncol;
        }
    return(mat);
    }
```

Declare all of the variables.

⟨*survfitci-declare*⟩=
```
 int i, j, k, kk;   /* generic loop indices */
 int ck, itime, eptr; /*specific indices */
 int ctime;      /*current time of interest, in the main loop */
 int nprotect;   /* number of protect calls issued */
 int oldstate, newstate; /*when changing state */

 double temp, *temp2;  /* scratch */
 double *p;         /* current prevalence vector */
 double **hmat;      /* hazard matrix at this time point */
 double **umat;     /* per subject leverage at this time point */
 int *atrisk;       /* 1 if the subject is currently at risk */
 int   *ns;         /* number curently in each state */
 double *ws;        /* weighted count of number state */
 double *wtp;       /* case weights indexed by subject */
 double wevent;     /* weighted number of events at current time */
 int nstate;        /* number of states */
 int n, nperson;    /*number of obs, subjects*/
 double **chaz;     /* cumulative hazard matrix */

 /* pointers to the R variables */
 int *sort1, *sort2;  /*sort index for entry time, event time */
 int *entry,* etime;  /*entry time, event time */
 int ntime;          /* number of unique event time values */
 int *status;        /*0=censored, 1,2,... new states */
 int *cstate;        /* current state for each subject */
 double *wt;         /* weight for each observation */
 int *id;            /* for each obs, which subject is it */
 int sefit;

 /* returned objects */
 SEXP rlist;         /* the returned list and variable names of same */
 const char *rnames[]= {"nrisk","nevent","ncensor", "p",
                        "cumhaz", "var", ""};
 SEXP pmat2, vmat2, cumhaz2;  /*list components */
 SEXP nevent2, ncensor2, nrisk2;
 double *pmat, *vmat, *cumhaz;
 int  *ncensor, *nrisk, *nevent;
```

Now set up pointers for all of the R objects sent to us. The two that will be updated need to be replaced by duplicates.

⟨*survfitci-declare*⟩=
```
 ntime= asInteger(ntime2);
 nperson = LENGTH(cstate2);
 n   = LENGTH(sort12);
 PROTECT(cstate2 = duplicate(cstate2));
```

```
cstate  = INTEGER(cstate2);
entry= INTEGER(ftime2);
etime= entry + n;
sort1= INTEGER(sort12);
sort2= INTEGER(sort22);
status= INTEGER(status2);
wt = REAL(wt2);
id = INTEGER(id2);
PROTECT(p2 = duplicate(p2));  /*copy of initial prevalence */
p = REAL(p2);
nstate = LENGTH(p2);  /* number of states */
sefit = asInteger(sefit2);

/* allocate space for the output objects */
PROTECT(pmat2 = allocMatrix(REALSXP, nstate, ntime));
pmat = REAL(pmat2);
if (sefit >0)
    PROTECT(vmat2 = allocMatrix(REALSXP, nstate, ntime));
else PROTECT(vmat2 = allocMatrix(REALSXP, 1, 1)); /* dummy object */
vmat = REAL(vmat2);
PROTECT(nevent2 = allocVector(INTSXP, ntime));
nevent = INTEGER(nevent2);
PROTECT(ncensor2= allocVector(INTSXP, ntime));
ncensor = INTEGER(ncensor2);
PROTECT(nrisk2 = allocMatrix(INTSXP, nstate, ntime));
nrisk = INTEGER(nrisk2);
PROTECT(cumhaz2= allocVector(REALSXP, nstate*nstate*ntime));
cumhaz = REAL(cumhaz2);
nprotect = 8;

/* allocate space for scratch vectors */
ws = (double *) R_alloc(2*nstate, sizeof(double));
temp2 = ws + nstate;
ns  = (int *) R_alloc(nstate, sizeof(int));
atrisk = (int *) R_alloc(nperson, sizeof(int));
wtp = (double *) R_alloc(nperson, sizeof(double));
hmat = (double**) dmatrix2(nstate, nstate);
if (sefit >0) umat = (double**) dmatrix2(nperson, nstate);
chaz = (double**) dmatrix2(nstate, nstate);

/* R_alloc does not zero allocated memory */
for (i=0; i<nstate; i++) {
    ws[i] =0;
    ns[i] =0;
    for (j=0; j<nstate; j++) {
            hmat[i][j] =0;
```

```
            chaz[i][j] =0;
        }
        if (sefit) {for (j=0; j<nperson; j++) umat[j][i]=0;}
    }
    for (i=0; i<nperson; i++) atrisk[i] =0;
```

The primary loop of the program walks along the `sort2` vector, with one pass through the loop for each unique event time. Observations are at risk in the interval (entry, event], note the round and square brackets, so we need `entry < ctime <= event`, where `ctime` is the unique event time of current interest. The basic loop is to add new subjects to the risk set, compute, save results, then remove expired ones from the risk set. The `ns` and `ws` vectors keep track of the number of subjects currently in each state and the weighted number currently in each state. There are four indexing patterns in play which may be confusing.

- The output matrices, which index by unique event time `itime`
- The `n` observations (variables entry, event, sort1, sort2, status, wt, id)
- The `nperson` individual subjects (variables cstate, atrisk)
- The `nstate` states (variables hmat, p)

⟨*survfitci-compute*⟩=

```
 itime =0; /*current time index, for output arrays */
 eptr  = 0; /*index to sort1, the entry times */
 for (i=0; i<n; ) {
     ck = sort2[i];
     ctime = etime[ck];  /* current time value of interest */

     /* Add subjects whose entry time is < ctime into the counts */
     for (; eptr<n; eptr++) {
         k = sort1[eptr];
         if (entry[k] < ctime) {
             kk = cstate[id[k]];  /*current state of the addition */
             ns[kk]++;
             ws[kk] += wt[k];
             wtp[id[k]] = wt[k];
             atrisk[id[k]] =1;   /* mark them as being at risk */
         }
         else break;
     }

     ⟨survfitci-compute-matrices⟩
     ⟨survfitci-compute-update⟩

     /* Take the current events and censors out of the risk set */
     for (; i<n; i++) {
         j= sort2[i];
```

```
        if (etime[j] == ctime) {
            oldstate = cstate[id[j]]; /*current state */
            ns[oldstate]--;
            ws[oldstate] -= wt[j];
            if (status[j] >0) cstate[id[j]] = status[j]-1; /*new state */
            atrisk[id[j]] =0;
        }
        else break;
    }
    itime++;
}
```

The key variables for the computation are the matrix H and the current prevalence vector P. H is created anew at each unique time point. Row j of H concerns everyone in state j just before the time point, and contains the transitions at that time point. So the jk element is the (weighted) fraction who change from state j to state k, and the jj element the fraction who stay put. Each row of H by definition sums to 1. If no one is in the state then the jj element is set to 1. A second version which we call H2 has 1 subtracted from each diagonal and so that the row sums are 0, we go back and forth depending on which is needed at the moment. If there are no events at this time point P and U do not update.

⟨*survfitci-compute-matrices*⟩=

```
for (j=0; j<nstate; j++) {
    for (k=0; k<nstate; k++) {
        hmat[j][k] =0;
    }
 }

/* Count up the number of events and censored at this time point */
nevent[itime] =0;
ncensor[itime] =0;
wevent =0;
for (j=i; j<n; j++) {
    k = sort2[j];
    if (etime[k] == ctime) {
        if (status[k] >0) {
            newstate = status[k] -1;  /* 0 based subscripts */
            oldstate = cstate[id[k]];
            nevent[itime]++;
            wevent += wt[k];
            hmat[oldstate][newstate] += wt[k];
        }
        else ncensor[itime]++;
    }
    else break;
 }
```

```
if (nevent[itime]> 0) {
    /* finish computing H */
    for (j=0; j<nstate; j++) {
        if (ns[j] >0) {
            temp =0;
            for (k=0; k<nstate; k++) {
                temp += hmat[j][k];
                hmat[j][k] /= ws[j]; /* events/n */
            }
            hmat[j][j] =1 -temp/ws[j]; /*rows sum to one */
        }
        else hmat[j][j] =1.0;

    }
    if (sefit >0) {
        ⟨survfitci-compute-U⟩
    }
    ⟨survfitci-compute-P⟩
}
```

The most complicated part of the code is the update of the per subject influence matrix U, which has nperson rows and nstate columns. It has 3 steps. Refer to equation (17) for the mathematical details.

1. The entire matrix is multiplied by H.
2. Consider the scaled matrix J whose kth row is the matrix H2 scaled by the value p[k]/ws[k]. (Probability of being in the state divided by the weighted number in the state). If subject i is currently at risk and currently in state k, then row k of J is subtracted from U[i,].
3. For each subject i who had an event at this time and went from state j to state k, U[i,j] will decrease by p[j]/ws[j] and U[i,k] will increase by the same amount.

If standard errors are not needed we can skip this calculation, which speeds up the code considerably.

⟨*survfitci-compute-U*⟩=
```
 /* Update U, part 1  U = U %*% H -- matrix multiplication */
 for (j=0; j<nperson; j++) { /* row of U */
     for (k=0; k<nstate; k++) { /* column of U */
         temp2[k]=0;
         for (kk=0; kk<nstate; kk++)
                 temp2[k] += umat[j][kk] * hmat[kk][k];
     }
     for (k=0; k<nstate; k++) umat[j][k] = temp2[k];
  }

 /* Update U, part 2, subtract from everyone at risk
```

```
       For this I need H2 */
 for (j=0; j<nstate; j++) hmat[j][j] -= 1;
 for (j=0; j<nperson; j++) {
     if (atrisk[j]==1) {
         kk = cstate[j];
         for (k=0; k<nstate; k++)
             umat[j][k] -= (p[kk]/ws[kk])* hmat[kk][k];
     }
  }

 /* Update U, part 3.  An addition for each event */
 for (j=i; j<n; j++) {
     k = sort2[j];
     if (etime[k] == ctime) {
         if (status[k] >0) {
             kk = id[k];  /* row number in U */
             oldstate= cstate[kk];
             newstate= status[k] -1;
             umat[kk][oldstate] -= p[oldstate]/ws[oldstate];
             umat[kk][newstate] += p[oldstate]/ws[oldstate];
         }
     }
     else break;
  }
```

Now update the cumulative hazard by adding H2 to it, and update p to pH. If sefit is 1 then H has already been transformed to H2 form.

⟨*survfitci-compute-P*⟩=
```
 /* Finally, update chaz and p.  */
 for (j=0; j<nstate; j++) {
     if (sefit ==0) hmat[j][j] -= 1;  /* conversion to H2*/
     for (k=0; k<nstate; k++) chaz[j][k] += hmat[j][k];

     hmat[j][j] +=1;  /* change from H2 to H */
     temp2[j] =0;
     for (k=0; k<nstate; k++)
         temp2[j] += p[k] * hmat[k][j];
  }
 for (j=0; j<nstate; j++) p[j] = temp2[j];
```

⟨*survfitci-compute-update*⟩=
```
 /* store into the matrices that will be passed back */
 for (j=0; j<nstate; j++) {
     *pmat++ = p[j];
     *nrisk++ = ns[j];
     for (k=0; k<nstate; k++) *cumhaz++ = chaz[k][j];
```

```
    temp=0;
    if (sefit >0) {
        for (k=0; k<nperson; k++)
            temp += wtp[k]* umat[k][j]*umat[k][j];
        *vmat++ = temp;
    }
  }
```

⟨*survfitci-return*⟩=
```
 /* return a list */
 PROTECT(rlist=mkNamed(VECSXP, rnames));
 SET_VECTOR_ELT(rlist, 0, nrisk2);
 SET_VECTOR_ELT(rlist, 1, nevent2);
 SET_VECTOR_ELT(rlist, 2, ncensor2);
 SET_VECTOR_ELT(rlist, 3, pmat2);
 SET_VECTOR_ELT(rlist, 4, cumhaz2);
 SET_VECTOR_ELT(rlist, 5, vmat2);
 UNPROTECT(nprotect +1);
 return(rlist);
```

9.2.2 Printing and plotting

The `survfitms` class differs from a `survfit`, but many of the same methods nearly apply.

⟨*survfitms*⟩=
```
 # Methods for survfitms objects
 ⟨survfitms-summary⟩
 ⟨survfitms-subscript⟩
```

The subscript method is a near copy of that for survfit objects, but with a slightly different set of components. The object could have strata and will almost always have multiple columns. If there is only one subscript it is preferentially associated with the strata, if there is no strata argument `i` will associate with the columns. If there are two subscripts the first goes with the strata. The little `nmatch` function allow the user to use either names or integer indices.

⟨*survfitms-subscript*⟩=
```
 "[.survfitms" <- function(x, ..., drop=TRUE) {
     nmatch <- function(indx, target) {
         # This function lets R worry about character, negative, or logical subscripts
         #  It always returns a set of positive integer indices
         temp <- seq_along(target)
         names(temp) <- target
         temp[indx]
     }

     if (missing(..1)) i<- NULL  else i <- sort(..1)
     if (missing(..2)) j<- NULL  else j <- ..2
```

```
n <- length(x$time)

if (is.null(x$strata) && is.matrix(x$prev)) {
    # No strata, but a matrix of prevalence values
    #  In this case, allow them to use a single i subscript as well
    if (is.null(j) && !is.null(i)) {
        j <- i
        i <- NULL
    }
}
if (is.null(i)) {
    i2 <- seq_len(n)
    if (is.null(strata)) i <- 1
    else i <- seq(along=strata)
}
else {
    if (is.null(x$strata) && (length(i) > 1 || i != 1))
        stop("subscript is out of bounds")
    indx <- nmatch(i, names(x$strata)) #strata to keep
    if (any(is.na(indx)))
        stop(gettextf("strata %s not matched", paste(i[is.na(indx)], collapse=' ')))
    # Now, i may not be in order: a user has curve[3:2] to reorder
    #  a plot.  Hence the "unlist(lapply(" construct which will reorder
    #  the data in the curves
    temp <- rep(seq_along(x$strata), x$strata)
    keep <- unlist(lapply(i, function(x) which(temp==x)))

    if (length(i) <=1 && drop) x$strata <- NULL
    else               x$strata  <- x$strata[indx]
    i2 <- keep
}

if (!is.null(j)) {
    indx <- nmatch(j, x$states)
    if (any(is.na(indx)))
        stop("subscript is out of bounds")
    else j <- as.vector(indx)
}

if (length(i2) ==1 && !is.null(j) && missing(drop)) drop <- FALSE

# all the elements that can have "nstate" elements or columns
#  The n.event variable can have fewer
temp <- c("states", "n.risk", "n.event", "n.censor", "prev",
          "cumhaz", "std.err", "lower", "upper")
sfun <- function(z) {
```

```
        if (is.null(j)) {
            if (is.array(z)) {
                if (length(dim(z)) > 2) z[,,i2, drop=drop]
                else z[i2,,drop=drop]
            }
            else z
        }
        else {
            if (is.array(z)) {
                if (length(dim(z)) > 2) z[j,j,i2, drop=drop]
                else z[i2,j, drop=drop]
            }
            else z[j]
        }
    }
    for (k in temp) x[[k]] <- sfun(x[[k]])
    x$n <- x$n[i]
    x$time <- x$time[i2]
    x$transitions <- NULL  # this is incorrect after subscripting

    if (is.null(j)) x$p0<- x$p0[i,]
    else x$p0 <- x$p0[i,j]

    x
}
```

The summary.survfit and summary.survfitms functions share a large amount of code. One part of the code that once was subtle is dealing with intermediate time points; the findInterval function in base R has made that much easier.

The key computational idea is to create a pair of variables `indx1` and `indx2`, which point to the appropriate rows of the input data. When there is a `times` argument is when this gets interesting. Say that the data has values at time 5, 10, 15, 20 ..., and a user asks for `times=c(7, 15, 20)`. In the input object `n.risk` refers to the number at risk just before time 5, 10, ...; it is a left-continuous function. The survival is a right-continuous function. So at time 7 we want to take the survival from time 5 and number at risk from time 10; `indx1` will be the right-continuous index and `indx2` the left continuous one. For counts of events, censoring, and entry we want to know the total number that happened during the intervals of 0-7, 7-15, and 15-20. Technically censorings at time 15 happen just after time 15 so would go into the third line of the report. However, this would lead to terrible confusion for the user since using `times=c(5, 10, 15, 20)` would lead to different counts than a call that did not contain the times argument, so all 3 of the intermediates are computed using indx1.

⟨*survfitms-summary*⟩=
```
 summary.survfit <- function(object, times, censored=FALSE,
                             scale=1, extend=FALSE,
                             rmean=getOption('survfit.rmean'),
```

```
                        ...) {
fit <- object
if (!inherits(fit, 'survfit'))
        stop("'summary.survfit()' function can only be used for objects of class \"survfit\"")

# The print.rmean option is depreciated, it is still listened
#   to in print.survfit, but ignored here
if (is.null(rmean)) rmean <- "common"
if (is.numeric(rmean)) {
    if (is.null(object$start.time)) {
        if (rmean < min(object$time))
            stop("Truncation point for the mean is < smallest survival")
    }
    else if (rmean < object$start.time)
        stop("Truncation point for the mean is < smallest survival")
}
else {
    rmean <- match.arg(rmean, c('none', 'common', 'individual'))
    if (length(rmean)==0) stop("Invalid value for 'rmean' option")
}

temp <- survmean(fit, scale=scale, rmean)
table <- temp$matrix  #for inclusion in the output list
rmean.endtime <- temp$end.time

if (!missing(times)) {
    if (!is.numeric(times)) stop(gettextf("'%s' argument must be numeric", "times"))
    times <- sort(times)
}

# The fit$surv object is sometimes a vector and sometimes a
#  matrix.  We calculate row indices first, and then deal
#  with the cases at the end.
nsurv <- length(fit$time)
if (is.null(fit$strata)) {
    nstrat <- 1
    stemp <- rep(1L, nsurv)
    strata.names <- ""
    }
else   {
    nstrat <- length(fit$strata)
    stemp <- rep(seq_len(nstrat), fit$strata)
    strata.names <- names(fit$strata)
}
```

⟨*survsum-findrows*⟩

```
# Create an output structure
temp <- object
temp$table <- table
if (length(rmean.endtime)>0  && !is.na(rmean.endtime))
        temp$rmean.endtime <- rmean.endtime
if (length(indx1)==length(fit$time) && all(indx1 == seq(along=fit$time))) {
    temp$time <- temp$time/scale
    if (!is.null(temp$strata))
        temp$strata <- factor(stemp, labels=strata.names)

}
else if (missing(times)) {  #default censor=FALSE case
    temp$time <- temp$time[indx1]/scale
    for (j in c("n.risk", "n.event", "n.censor", "n.enter", "prev",
                "surv", "std.err", "cumhaz", "lower", "upper")) {
        zed <- temp[[j]]
        if (!is.null(zed)) {
            if (is.matrix(zed)) temp[[j]] <- zed[indx1,,drop=FALSE]
            else temp[[j]] <- zed[indx1]
        }
    }
    if (!is.null(temp$strata))
        temp$strata <- factor(stemp[indx1], levels=seq_len(nstrat),
                              labels=strata.names)
}
else { #times argument was given
    temp$time <- unlist(newtime)/scale
    tfun <- function(x, init=0, index=indx1) {
         if (is.matrix(x))
            rbind(rep(init, ncol(x)), x)[1+index,,drop=FALSE]
         else c(init, x)[1 + index]
    }
    tfun2 <- function(x, end=0, index=indx2) {
         if (is.matrix(x))
            rbind(x, rep(end, ncol(x)))[1+index,,drop=FALSE]
         else c(x,end)[1 + index]
    }
    temp$surv  <- tfun(temp$surv, 1)
    temp$n.risk <- tfun2(temp$n.risk)
    for (j in c("std.err", "cumhaz", "lower", "upper")) {
        if (!is.null(temp[[j]])) temp[[j]] <- tfun(temp[[j]])
    }
    for (j in c("n.event", "n.censor", "n.enter")){
        zed <- temp[[j]]
        if (!is.null(zed)) temp[[j]] <- cfun(zed)
```

```
        }

        if (!is.null(fit$strata)) {
            scount <- unlist(lapply(ilist1, length))
            temp$strata <- factor(rep(seq_len(nstrat), scount), levels=seq_len(nstrat),
                                  labels=strata.names)
        }
    }

    # An ordinary survfit object contain std(cum hazard), change scales
    if (!is.null(temp$std.err)) temp$std.err <- temp$std.err * temp$surv
    class(temp) <- 'summary.survfit'
    temp
}
```

Grab rows: if there is no `times` argument it is easy. At the end `indx1` is the right continuous index and `indx2` the left continuous one. If a curve has jumps at 1 and 2, at time 1.5 the survival is the same as that at time 1 (indx1), the number at risk the same as time 2 (indx2).

⟨*survsum-findrows*⟩=
```
if (missing(times)) {
    # just pick off the appropriate rows of the output
    # For a survfitms object n.event is a matrix, pick off all rows with an
    #  event for some endpoint.
    if (censored) indx1 <- seq(along=fit$time)
    else indx1 <- which(rowSums(as.matrix(fit$n.event)) >0)
    indx2 <- indx1
}
```

This second case is actual work, since may involve "in between" points in the curves. In R version 3.2.? findInterval gains an argument so as to return both left and right continuous results, but I currently need to integrate with an older version.

⟨*survsum-findrows*⟩=
```
else {
    find2 <- function(x, vec, left.open=FALSE, ...) {
        if (!left.open) findInterval(x, vec, ...)
        else length(vec) - findInterval(-x, rev(-vec), ...)
    }
    # Process the curves one at a time, adding them to the two lists
    ilist1 <- ilist2 <- ilist3 <- vector('list', nstrat)
    newtime <- ilist1
    n <- length(stemp)
    for (i in seq_len(nstrat)) {
        who <- seq_len(n)[stemp==i]  # the rows of the object for this strata
        stime <- fit$time[who]
```

```
    # First, toss any printing times that are outside our range
    if (is.null(fit$start.time)) mintime <- min(stime, 0)
    else                         mintime <- fit$start.time
    ptimes <- times[times >= mintime]

    if (!extend) {
        maxtime <- max(stime)
        ptimes <- ptimes[ptimes <= maxtime]
        }
    j <- find2(ptimes, stime)
    ilist1[[i]] <- c(0, who)[1+ j]
    ilist2[[i]] <- c(0, who)[1+ find2(ptimes, stime, left.open=TRUE)]
    ilist3[[i]] <- j  #index within a group
    newtime[[i]] <- ptimes
}
indx1 <- unlist(ilist1)
indx2 <- unlist(ilist2)

# All of the indices (ilist1, indx1, ...) contain 0 for a time point that
#  is prior to the first observed time in the curve.  Times that
#  are >= to the last observed time will point to that last observed
#  time.  Variable ilist3 contains indices that are relative to the
#  start of a curve, all other indices point to row numbers in the
#  entire object.
#
cfun <- function(x, init=0) {  #cumulative counts over a time interval
    tlist <- vector("list", nstrat)
    if (is.matrix(x)) {
        for (i in seq_len(nstrat)) {
            # stemp is 1,1,1,....2,2,2,,.. to mark curves
            x2 <- x[stemp==i,]  # all those in the group
            j  <- c(0, ilist3[[i]])
            tlist[[i]] <- apply(rbind(0, x2), 2, function(z) {
                diff(cumsum(z)[1+j])})
        }
        matrix(unlist(lapply(tlist, t)), byrow=T, ncol=ncol(x))
    }
    else {
        for (i in seq_len(nstrat)) {
            x2 <- x[stemp==i]
            j  <- c(0, ilist3[[i]])
            tlist[[i]] <- diff(cumsum(c(0,x2))[1+j])
        }
        unlist(tlist)
    }
}
```

}

Repeat the code for survfitms objects. The only real difference is the preservation of `prev` and `cumhaz` instead of `surv`, use of survmean2, and use of p0 for initial states.

⟨*survfitms-summary*⟩=

```
summary.survfitms <- function(object, times, censored=FALSE,
                              scale=1, extend=FALSE,
                              rmean= getOption("survfit.rmean"),
                              ...) {
    fit <- object
    if (!inherits(fit, 'survfitms'))
            stop("'summary.survfitms()' can only be used for objects of class \"survfitms\"")

    # The print.rmean option is depreciated, it is still listened
    #   to in print.survfit, but ignored here
    if (is.null(rmean)) rmean <- "common"
    if (is.numeric(rmean)) {
        if (is.null(object$start.time)) {
            if (rmean < min(object$time))
                stop("Truncation point for the mean is < smallest survival")
        }
        else if (rmean < object$start.time)
            stop("Truncation point for the mean is < smallest survival")
    }
    else {
        rmean <- match.arg(rmean, c('none', 'common', 'individual'))
        if (length(rmean)==0) stop("Invalid value for 'rmean' option")
    }

    temp <- survmean2(fit, scale=scale, rmean)
    table <- temp$matrix  #for inclusion in the output list
    rmean.endtime <- temp$end.time

    if (!missing(times)) {
        if (!is.numeric(times)) stop(gettextf("'%s' argument must be numeric", "times"))
        times <- sort(times)
    }

    # The fit$prev object is sometimes a vector and sometimes a
    #  matrix.  We calculate row indices first, and then deal
    #  with the cases at the end.
    nsurv <- length(fit$time)
    if (is.null(fit$strata)) {
        nstrat <- 1
        stemp <- rep(1L, nsurv)
```

```
    strata.names <- ""
    }
else   {
    nstrat <- length(fit$strata)
    stemp <- rep(seq_len(nstrat), fit$strata)
    strata.names <- names(fit$strata)
}
```

⟨*survsum-findrows*⟩

```
# Create an output structure
temp <- object
temp$table <- table
if (length(rmean.endtime)>0  && !is.na(rmean.endtime))
        temp$rmean.endtime <- rmean.endtime
if (length(indx1)==length(fit$time) && all(indx1 == seq(along=fit$time))) {
    temp$time <- temp$time/scale
    if (!is.null(temp$strata))
        temp$strata <- factor(stemp, labels=strata.names)

}
else if (missing(times)) {  #default censor=FALSE case
    temp$time <- temp$time[indx1]/scale
    for (j in c("n.risk", "n.event", "n.censor", "n.enter",
                "prev", "std.err", "cumhaz", "lower", "upper")) {
        zed <- temp[[j]]
        if (!is.null(zed)) {
            if (is.matrix(zed)) temp[[j]] <- zed[indx1,,drop=FALSE]
            else temp[[j]] <- zed[indx1]
        }
    }
    if (!is.null(temp$strata))
        temp$strata <- factor(stemp[indx1], levels=seq_len(nstrat),
                              labels=strata.names)
}
else { #times argument was given
    temp$time <- unlist(newtime)/scale
    tfun <- function(x, init=0, index= indx1) {
        if (is.matrix(x))
            rbind(rep(init, ncol(x)), x)[1+index,,drop=FALSE]
        else c(init, x)[1 + index]
    }
    tfun2 <- function(x, end=0, index=indx2) {
         if (is.matrix(x))
            rbind(x, rep(end, ncol(x)))[1+index,,drop=FALSE]
         else c(x,end)[1 + index]
```

```
        }
        temp$prev <- tfun(temp$prev, 0)
        # fix up the initial states
        if (any(indx1==0)) {
            if (nstrat==1) temp$prev[indx1==0,] <- temp$p0
            else {
                ninit <- sapply(ilist1, function(x) sum(x==0))
                zz <- rep(seq_len(nstrat), ninit)
                temp$prev[indx1==0,] <- temp$p0[zz,]
            }
        }
        temp$n.risk <- tfun2(temp$n.risk)
        for (j in c("std.err", "cumhaz", "lower", "upper")) {
            if (!is.null(temp[[j]])) temp[[j]] <- tfun(temp[[j]])
        }
        for (j in c("n.event", "n.censor", "n.enter")){
            zed <- temp[[j]]
            if (!is.null(zed)) temp[[j]] <- cfun(zed)
        }

        if (!is.null(fit$strata)) {
            scount <- unlist(lapply(ilist1, length))
            temp$strata <- factor(rep(seq_len(nstrat), scount), levels=seq_len(nstrat),
                                  labels=strata.names)
        }
    }
    class(temp) <- "summary.survfitms"
    temp
}

⟨printms⟩
⟨survmean2⟩
```

Printing for a survfitms object is different than for a survfit one. The big difference is that I don't have an estimate of the median, or any other quantile for that matter. Mean time in state makes sense, but I don't have a standard error for it at the moment. The other is that there is usually a mismatch between the n.event matrix and the n.risk matrix. The latter has all the states that were possible whereas the former only has states with an arrow pointing in. We need to manufacture the 0 events for the other states.

```
⟨printms⟩=
print.survfitms <- function(x, scale=1,
                            rmean = getOption("survfit.rmean"), ...) {
    if (!is.null(cl<- x$call)) {
        cat(gettext("Call:", domain = "R-survival"))
        dput(cl)
```

```
        cat("\n")
        }
    omit <- x$na.action
    if (length(omit)) cat("  ", naprint(omit), "\n")

    if (is.null(rmean)) rmean <- "common"
    if (is.numeric(rmean)) {
        if (is.null(x$start.time)) {
            if (rmean < min(x$time))
                stop("Truncation point for the mean is < smallest survival")
        }
        else if (rmean < x$start.time)
            stop("Truncation point for the mean is < smallest survival")
    }
    else {
        rmean <- match.arg(rmean, c('none', 'common', 'individual'))
        if (length(rmean)==0) stop("Invalid value for 'rmean' option")
    }

    temp <- survmean2(x, scale=scale, rmean)
    if (is.null(temp$end.time)) print(temp$matrix, ...)
    else {
        etime <- temp$end.time
        dd <- dimnames(temp$matrix)
        cname <- dd[[2]]
        cname[length(cname)] <- paste0(cname[length(cname)], '*')
        dd[[2]] <- cname
        dimnames(temp$matrix) <- dd
        print(temp$matrix, ...)
        if (length(etime) ==1)
             cat(gettextf("   *mean time in state, restricted (max time = %s)\n", format(etime, .
        else cat(gettextf("   *mean time in state, restricted (per curve cutoff)\n", domain = "R-s
    }
    invisible(x)
}
```

This part of the computation is set out separately since it is called by both print and summary.

⟨*survmean2*⟩=

```
survmean2 <- function(x, scale, rmean) {
    nstate <- length(x$states)  #there will always be at least 1 state
    ngrp   <- max(1, length(x$strata))
    if (ngrp >1)  {
        igrp <- rep(seq_len(ngrp), x$strata)
        rname <- names(x$strata)
        }
    else {
```

```
    igrp <- rep(1, length(x$time))
    rname <- NULL
    }

# The n.event matrix may not have nstate columms.  Its
#  colnames are the first elements of states, however
if (is.matrix(x$n.event)) {
    nc <- ncol(x$n.event)
    nevent <- tapply(x$n.event, list(rep(igrp, nc), col(x$n.event)), sum)
    dimnames(nevent) <- list(rname, x$states[seq_len(nc)])
    }
else {
    nevent <- tapply(x$n.event, igrp, sum)
    names(nevent) <- rname
    }

outmat <- matrix(0., nrow=nstate*ngrp , ncol=2)
outmat[,1] <- rep(x$n, nstate)
outmat[seq_along(nevent), 2] <- c(nevent)

if (ngrp >1)
    rowname <- c(outer(rname, x$states, paste, sep=", "))
else rowname <- x$states

# Caculate the mean time in each state
if (rmean != "none") {
    if (is.numeric(rmean)) maxtime <- rep(rmean, ngrp)
    else if (rmean=="common") maxtime <- rep(max(x$time), ngrp)
    else maxtime <- tapply(x$time, igrp, max)

    meantime <- matrix(0., ngrp, nstate)
    p0 <- matrix(x$p0, nrow=ngrp)  #in case there is only one row
    for (i in seq_len(ngrp)) {
        if (is.matrix(x$prev))
            temp <- rbind(p0[i,], x$prev[igrp==i,, drop=FALSE])
        else temp <- matrix(c(p0[i,], x$prev[igrp==i]), ncol=1)

        if (is.null(x$start.time)) tt <- c(0, x$time[igrp==i])
        else tt <- c(x$start.time, x$time[igrp==i])

        # Now cut it off at maxtime
        delta <- diff(c(tt[tt<maxtime[i]], maxtime[i]))
        if (length(delta) > nrow(temp)) delta <- delta[seq_len(nrow(temp))]
        if (length(delta) < nrow(temp))
            delta <- c(delta, rep(0, nrow(temp) - length(delta)))
        meantime[i,] <- colSums(delta*temp)
```

```
        }

        outmat <- cbind(outmat, c(meantime)/scale)
        cname <- c("n", "nevent", "mean")
        # report back a single time, if there is only one
        if (all(maxtime == maxtime[1])) maxtime <- maxtime[1]
    }
    else cname <- c("n", "nevent")
    dimnames(outmat) <- list(rowname, cname)

    if (rmean=='none') list(matrix=outmat)
    else list(matrix=outmat, end.time=maxtime/scale)
}
```

10 Plotting survival curves

I found a problem where plot.survfit, lines.survfit, and points.survfit sometimes did different things. This is due to copied code that later changed in one function but not another. Since they have so much code in common, this section of the noweb code consolodates them so as to restore order by using common code blocks. First define the top level routines.

```
⟨plot.survfit⟩=
 plot.survfit<- function(x, conf.int,  mark.time=FALSE,
                         mark=3, col=1,lty=1, lwd=1,
                         cex=1, log=FALSE,
                         xscale=1, yscale=1,
                         firstx=0, firsty=1,
                         xmax, ymin=0,
                         fun, xlab="", ylab="", xaxs='S',
                         conf.times, conf.cap=.005, conf.offset=.012, ...) {

    dotnames <- names(list(...))
    if (any(dotnames=='type'))
        stop("The graphical argument 'type' is not allowed")
    if (missing(mark.time) & !missing(mark)) mark.time <- TRUE

    ⟨plot-transform-ms⟩
    if (missing(firsty) && !is.null(x$p0)) firsty <- 1-x$p0
    ⟨plot-plot-setup1⟩
    ⟨plot-common-args⟩
    ⟨plot-firstx⟩
    ⟨plot-plot-setup2⟩
    ⟨plot-functions⟩
    plot.surv <- TRUE
    type <- 's'
```

```
    ⟨plot-draw⟩
}

lines.survfit <- function(x, type='s',
                          mark=3, col=1, lty=1, lwd=1,
                          cex=1,
                          mark.time=FALSE, xscale=1,
                          firstx=0, firsty=1, xmax,
                          fun,  conf.int=FALSE,
                          conf.times, conf.cap=.005, conf.offset=.012, ...) {
    xlog <- par("xlog")
   if (missing(mark.time) & !missing(mark)) mark.time <- TRUE

    ⟨plot-transform-ms⟩
    if (missing(firsty) && !is.null(x$p0)) firsty <- 1-x$p0
    ⟨plot-common-args⟩
    ⟨plot-firstx⟩
    ⟨plot-functions⟩
    ⟨plot-draw⟩
}

points.survfit <- function(x, xscale=1,
                           xmax, fun, ...) {
    ⟨plot-transform-ms⟩
    firstx <- NA  # flag used in the common args
    conf.int <- FALSE
    ⟨plot-common-args⟩
    if (ncol(ssurv)==1) points(stime, ssurv, ...)
    else  matpoints(stime, ssurv, ...)
}
```

Block of code to transform components of a `survfitms` object so that the standard plotting methods work.

```
⟨plot-transform-ms⟩=
 if (inherits(x, "survfitms")) {
     x$surv <- 1- x$prev
     if (is.matrix(x$surv)) {
         dimnames(x$surv) <- list(NULL, x$states)
         if (ncol(x$surv) > 1 && any(x$states == '')) {
             x$surv <- x$surv[, x$states != '']
             if (is.matrix(x$p0)) x$p0 <- x$p0[, x$states != '']
             else x$p0 <- x$p0[x$states != '']
         }
     }

     if (!is.null(x$lower)) {
```

```
        x$lower <- 1- x$lower
        x$upper <- 1- x$upper
    }
    if (missing(fun)) fun <- "event"
 }
```

⟨*plot-common-args*⟩=

```
 ssurv <- as.matrix(x$surv)
 stime <- x$time
 if( !is.null(x$upper)) {
     supper <- as.matrix(x$upper)
     slower <- as.matrix(x$lower)
 }
 else {
     conf.int <- FALSE
     supper <- NULL  #marker for later code
 }

 # set up strata
 if (is.null(x$strata)) {
     nstrat <- 1
     stemp <- rep(1, length(x$time)) # same length as stime
 }
 else {
     nstrat <- length(x$strata)
     stemp <- rep(1:nstrat, x$strata) # same length as stime
 }
 ncurve <- nstrat * ncol(ssurv)
 firsty <- matrix(firsty, nrow=nstrat, ncol=ncol(ssurv))
```

The xmax argument is used to prune back the survival curve to a small set of time points. This is a bit of bother since we have to do our own clipping of the data to prevent warning messages from the underlying plot routines. A further special case is when we are drawing lines and a curve got pruned so severely that only a horizontal segment from the curve start remains. In this case I need to reference the firsty arg.

⟨*plot-common-args*⟩=

```
 if (!missing(xmax) && any(x$time>xmax)) {
     # prune back the survival curves
     # I need to replace x's over the limit with xmax, and y's over the
     #  limit with either the prior y value  or firsty
     keepx <- keepy <- NULL  # lines to keep
     tempn <- table(stemp)
     offset <- cumsum(c(0, tempn))
     for (i in 1:nstrat) {
         ttime <-stime[stemp==i]
         if (all(ttime <= xmax)) {
```

```
            keepx <- c(keepx, 1:tempn[i] + offset[i])
            keepy <- c(keepy, 1:tempn[i] + offset[i])
        }
        else {
            bad <- min((1:tempn[i])[ttime>xmax])
            if (bad==1)  {  #lost them all
                if (!is.na(firstx)) { # and we are plotting lines
                    keepy <- c(keepy, 1+offset[i])
                    ssurv[1+offset[i],] <- firsty[i,]
                }
                }
            else  keepy<- c(keepy, c(1:(bad-1), bad-1) + offset[i])
            keepx <- c(keepx, (1:bad)+offset[i])
            stime[bad+offset[i]] <- xmax
            x$n.event[bad+offset[i]] <- 1   #don't plot a tick mark
        }
    }

    # ok, now actually prune it
    stime <- stime[keepx]
    stemp <- stemp[keepx]
    x$n.event <- x$n.event[keepx]
    if (!is.null(x$n.censor)) x$n.censor <- x$n.censor[keepx]
    ssurv <- ssurv[keepy,,drop=FALSE]
    if (!is.null(supper)) {
        supper <- supper[keepy,,drop=FALSE]
        slower <- slower[keepy,,drop=FALSE]
    }
}
#stime <- stime/xscale  #scaling is deferred until xmax processing is done

if (!missing(fun)) {
    if (is.character(fun)) {
        tfun <- switch(fun,
                       'log' = function(x) x,
                       'event'=function(x) 1-x,
                       'cumhaz'=function(x) -log(x),
                       'cloglog'=function(x) log(-log(x)),
                       'pct' = function(x) x*100,
                       'logpct'= function(x) 100*x,  #special case further below
                   'identity'= function(x) x,
                       stop("Unrecognized function argument")
                       )
    }
    else if (is.function(fun)) tfun <- fun
    else stop("Invalid 'fun' argument")
```

```
    ssurv <- tfun(ssurv )
    if (!is.null(supper)) {
        supper <- tfun(supper)
        slower <- tfun(slower)
        }
    firsty <- tfun(firsty)
}
```

The data structure for a survival plot does not include the first plot point. Those routines start their computation at the first endpoint, and leave it to here to decide on a starting location. The points routine doesn't have to deal with this nuisance.

- The initial time value `firstx` is the first of
  1. a value given to `firstx` by the user
  2. `start.time`, if present in the surv object
  3. if a logarithmic axis is specified, the smallest time ¿0 in the object
  4. the smaller of the minimum time or 0

⟨*plot-firstx*⟩=

```
if (missing(firstx)) {
    if (!is.null(x$start.time))
        firstx <- x$start.time
    else {
        if (xlog) firstx <- min(stime[stime>0])
        else      firstx <- min(0, stime)
    }
}

# The default for plot and lines is to add confidence limits
#  if there is only one curve
if (missing(conf.int) && missing(conf.times))  conf.int <- (ncurve==1)
if (missing(conf.times)) conf.times <- NULL
else {
    if (!is.numeric(conf.times)) stop(gettextf("'%s' argument must be numeric", "conf.times"))
    if (missing(conf.int)) conf.int <- TRUE
}

if (is.logical(conf.int)) plot.surv <- TRUE
else {
    temp <- match.arg(conf.int, c("both", "only", "none"))
    if (is.na(temp)) stop("invalid value for 'conf.int' argument")
    if (temp=="none") conf.int <- FALSE  else conf.int <- TRUE
    if (temp=="only") plot.surv <- FALSE  else plot.surv <- TRUE
}
⟨plot-setup-marks⟩
```

⟨*plot-setup-marks*⟩=
```
 # Marks are not placed on confidence bands
 mark <- rep(mark, length.out=ncurve)
 mcol <- rep(col,  length.out=ncurve)
 if (is.numeric(mark.time)) mark.time <- sort(mark.time)

 # The actual number of curves is ncurve*3 if there are confidence bands,
 #  unless conf.times has been given.  Colors and line types in the latter
 #  match the curves
 # If the number of line types is 1 and lty is an integer, then use lty
 #    for the curve and lty+1 for the CI
 # If the length(lty) <= length(ncurve), use the same color for curve and CI
 #   otherwise assume the user knows what they are about and has given a full
 #   vector of line types.
 # Colors and line widths work like line types, excluding the +1 rule.
 if (conf.int & is.null(conf.times)) {
     if (length(lty)==1 && is.numeric(lty))
         lty <- rep(c(lty, lty+1, lty+1), ncurve)
     else if (length(lty) <= ncurve)
         lty <- rep(rep(lty, each=3), length.out=(ncurve*3))
     else lty <- rep(lty, length.out= ncurve*3)

     if (length(col) <= ncurve) col <- rep(rep(col, each=3), length.out=3*ncurve)
     else col <- rep(col, length.out=3*ncurve)

     if (length(lwd) <= ncurve) lwd <- rep(rep(lwd, each=3), length.out=3*ncurve)
     else lwd <- rep(lwd, length.out=3*ncurve)
 }
 else {
     col  <- rep(col, length.out=ncurve)
     lty  <- rep(lty, length.out=ncurve)
     lwd  <- rep(lwd, length.out=ncurve)
 }
```

Here is the rest of the setup for the plot routine, mostly having to do with setting up axes. The `xlog` and `ylog` variables are internal reminders of the choice, and `logax` is what will be passed to the plot function

⟨*plot-plot-setup1*⟩=
```
 if (is.logical(log)) {
     ylog <- log
     xlog <- FALSE
     if (ylog) logax <- 'y'
     else      logax <- ""
 }
 else {
     ylog <- (log=='y' || log=='xy')
```

```
    xlog <- (log=='x' || log=='xy')
    logax  <- log
}

if (!missing(fun)) {
    if (is.character(fun)) {
        if (fun=='log'|| fun=='logpct') ylog <- TRUE
        if (fun=='cloglog') {
            xlog <- TRUE
            if (ylog) logax <- 'xy'
            else logax <- 'x'
        }
    }
}

# The special x axis style only applies when firstx is not given
if (missing(xaxs) && (firstx!=0 || !missing(fun) ||
                      (missing(fun) && inherits(x, "survfitms"))))
    xaxs <- par("xaxs")  #use the default
```

⟨*plot-plot-setup2*⟩=

```
#axis setting parmaters that depend on the fun argument
if (!missing(fun)) {
    ymin <- tfun(ymin)  #lines routine doesn't have it
}

# Do axis range computations
if (xaxs=='S') {
    #special x- axis style for survival curves
    xaxs <- 'i'  #what S thinks
    tempx <- max(stime) * 1.04
}
else tempx <- max(stime)
tempx <- c(firstx, tempx, firstx)

if (ylog) {
    tempy <-  range(ssurv[is.finite(ssurv)& ssurv>0])
    if (tempy[2]==1) tempy[2] <- .99
    if (any(ssurv==0)) {
        tempy[1] <- tempy[1]*.8
        ssurv[ssurv==0] <- tempy[1]
        if (!is.null(supper)) {
            supper[supper==0] <- tempy[1]
            slower[slower==0] <- tempy[1]
        }
    }
```

```
    tempy <- c(tempy, firsty)
}
else tempy <- range(ssurv, firsty, finite=TRUE, na.rm=TRUE)

if (missing(fun)) {
    tempx <- c(tempx, firstx)
    if (!ylog) tempy <- c(tempy, ymin)
}

#
# Draw the basic box
#
plot(range(tempx, finite=TRUE, na.rm=TRUE)/xscale,
     range(tempy, finite=TRUE, na.rm=TRUE)*yscale,
     type='n', log=logax, xlab=xlab, ylab=ylab, xaxs=xaxs,...)

if(yscale != 1) {
    if (ylog) par(usr =par("usr") -c(0, 0, log10(yscale), log10(yscale)))
    else par(usr =par("usr")/c(1, 1, yscale, yscale))
}
if (xscale !=1) {
    if (xlog) par(usr =par("usr") -c(log10(xscale), log10(xscale), 0,0))
    else par(usr =par("usr")*c(xscale, xscale, 1, 1))
}
```

The use of `par(usr)` just above is a bit sneaky. I want the lines and points routines to be able to add to the plot, *without* passing them a global parameter that determines the y-scale or forcing the user to repeat it. The xscale argument was added before yscale, and before I saw this trick. By then there were hundreds of lines of code that have an xscale argument to lines() so the other change was to ignore the argument there.

The next functions do the actual drawing.

⟨*plot-functions*⟩=

```
# Create a step function, removing redundancies that sometimes occur in
#  curves with lots of censoring.
dostep <- function(x,y) {
    keep <- is.finite(x) & is.finite(y)
    if (!any(keep)) return()  #all points were infinite or NA
    if (!all(keep)) {
        # these won't plot anyway, so simplify (CI values are often NA)
        x <- x[keep]
        y <- y[keep]
    }
    n <- length(x)
    if (n==1)       list(x=x, y=y)
    else if (n==2)  list(x=x[c(1,2,2)], y=y[c(1,1,2)])
    else {
```

```
        # replace verbose horizonal sequences like
        # (1, .2), (1.4, .2), (1.8, .2), (2.3, .2), (2.9, .2), (3, .1)
        # with (1, .2), (.3, .2),(3, .1).
        #  They are slow, and can smear the looks of the line type.
        temp <- rle(y)$lengths
        drops <- 1 + cumsum(temp[-length(temp)])  # points where the curve drops

        #create a step function
        if (n %in% drops) {  #the last point is a drop
            xrep <- c(x[1], rep(x[drops], each=2))
            yrep <- rep(y[c(1,drops)], c(rep(2, length(drops)), 1))
        }
        else {
            xrep <- c(x[1], rep(x[drops], each=2), x[n])
            yrep <- c(rep(y[c(1,drops)], each=2))
        }
        list(x=xrep, y=yrep)
    }
}

drawmark <- function(x, y, mark.time, censor, cex, ...) {
    if (!is.numeric(mark.time)) {
        xx <- x[censor]
        yy <- y[censor]
    }
    else { #interpolate
        xx <- mark.time
        yy <- approx(x, y, xx, method="constant", f=0)$y
    }
    points(xx, yy, cex=cex, ...)
}
```

The code to actually draw curves for the plot.
The code to draw the lines and confidence bands.

⟨*plot-draw*⟩=

```
c1 <- 1  # keeps track of the curve number
c2 <- 1  # keeps track of the lty, col, etc
xend <- yend <- double(ncurve)
if (length(conf.offset) ==1)
    temp.offset <- (1:ncurve - (ncurve-1)/2)* conf.offset* diff(par("usr")[1:2])
else temp.offset <- rep(conf.offset, length=ncurve) *  diff(par("usr")[1:2])
temp.cap    <-  conf.cap    * diff(par("usr")[1:2])

for (j in 1:ncol(ssurv)) {
    for (i in unique(stemp)) {  #for each strata
        who <- which(stemp==i)
```

```
censor <- if (is.null(x$n.censor))
    (x$n.event[who] ==0)  else (x$n.censor[who] >0) #censoring ties
xx <- c(firstx, stime[who])
censor <- c(FALSE, censor)  #no mark at firstx

yy <- c(firsty[i,j], ssurv[who,j])
if (plot.surv) {
    if (type=='s')
        lines(dostep(xx, yy), lty=lty[c2], col=col[c2], lwd=lwd[c2])
    else lines(xx, yy, type=type, lty=lty[c2], col=col[c2], lwd=lwd[c2])
    if (is.numeric(mark.time) || mark.time)
        drawmark(xx, yy, mark.time, censor, pch=mark[c1], col=mcol[c1],
                 cex=cex)
}
xend[c1] <- max(xx)
yend[c1] <- yy[length(yy)]

if (conf.int && !is.null(conf.times)) {
    # add vertical bars at the specified times
    x2 <- conf.times + temp.offset[c1]
    templow <- approx(xx, c(firsty[i,j], slower[who,j]), x2,
                      method='constant', f=1)$y
    temphigh<- approx(xx, c(firsty[i,j], supper[who,j]), x2,
                      method='constant', f=1)$y
    segments(x2, templow, x2, temphigh,
             lty=lty[c2], col=col[c2], lwd=lwd[c2])
    if (conf.cap>0) {
        segments(x2-temp.cap, templow, x2+temp.cap, templow,
                 lty=lty[c2], col=col[c2], lwd=lwd[c2] )
        segments(x2-temp.cap, temphigh, x2+temp.cap, temphigh,
                  lty=lty[c2], col=col[c2], lwd=lwd[c2])
    }

}
c1 <- c1 +1
c2 <- c2 +1

if (conf.int && is.null(conf.times)) {
    if (type == 's') {
        lines(dostep(xx, c(firsty[i,j], slower[who,j])), lty=lty[c2],
              col=col[c2],lwd=lwd[c2])
        c2 <- c2 +1
        lines(dostep(xx, c(firsty[i,j], supper[who,j])), lty=lty[c2],
              col=col[c2], lwd= lwd[c2])
        c2 <- c2 + 1
    }
```

```
            else {
                lines(xx, c(firsty[i,j], slower[who,j]), lty=lty[c2],
                      col=col[c2],lwd=lwd[c2], type=type)
                c2 <- c2 +1
                lines(xx, c(firsty[i,j], supper[who,j]), lty=lty[c2],
                      col=col[c2], lwd= lwd[c2], type= type)
                c2 <- c2 + 1
            }
        }

    }
}
invisible(list(x=xend, y=yend))
```

11 tmerge

The tmerge function was designed around a set of specific problems. The idea is to build up a time dependent data set one endpoint at at time. The primary arguments are

- data1: the base data set that will be added onto
- data2: the source for new information
- id: the subject identifier in the new data
- ...: additional arguments that add variables to the data set
- tstart, tstop: used to set the time range for each subject
- options

The created data set has three new variables (at least), which are `id`, `tstart` and `tstop`.

The key part of the call are the "..." arguments which each can be one of four types: tdc() and cumtdc() add a time dependent variable, event() and cumevent() add a new endpoint. In the survival routines time intervals are open on the left and closed on the right, i.e., (tstart, tstop]. Time dependent covariates apply from the start of an interval and events occur at the end of an interval. If a data set already had intervals of (0,10] and (10, 14] a new time dependent covariate or event at time 8 would lead to three intervals of (0,8], (8,10], and (10,14]; the new time-dependent covariate value would be added to the second interval, a new event would be added to the first one.

A typical call would be

```
newdata <- tmerge(newdata, old, id=clinic, diabetes=tdc(diab.time))
```

which would add a new time dependent covariate `diabetes` to the data set.

⟨*tmerge*⟩=

```
tmerge <- function(data1, data2, id, ..., tstart, tstop, options) {
    Call <- match.call()
```

```
    # The function wants to recognize special keywords in the
    #  arguments, so define a set of functions which will be used to
    #  mark objects
    new <- new.env(parent=parent.frame())
    assign("tdc", function(time, value=NULL) {
        x <- list(time=time, value=value);
        class(x) <- "tdc"; x},
           envir=new)
    assign("cumtdc", function(time, value=NULL) {
        x <- list(time=time, value=value);
        class(x) <-"cumtdc"; x},
           envir=new)
    assign("event", function(time, value=NULL, censor=NULL) {
        x <- list(time=time, value=value, censor=censor);
        class(x) <-"event"; x},
           envir=new)
    assign("cumevent", function(time, value=NULL, censor=NULL) {
        x <- list(time=time, value=value, censor=censor);
        class(x) <-"cumevent"; x},
           envir=new)

    if (missing(data1) || missing(data2) || missing(id))
        stop("'data1', 'data2', and 'id' arguments are required")
    if (!inherits(data1, "data.frame")) stop(gettextf("'%s' argument must be a data frame", "data

    ⟨tmerge-setup⟩
    ⟨tmerge-addvar⟩
    ⟨tmerge-finish⟩
}
```

The program can't use formulas because the ... arguments need to be named. This results in a bit of evaluation magic to correctly assess arguments. The routine below could have been set out as a separate top-level routine, the argument is where we want to document it: within the tmerge page or not. I decided on the former.

⟨*tmerge-setup*⟩=

```
tmerge.control <- function(idname="id", tstartname="tstart", tstopname="tstop",
                           delay =0, na.rm=TRUE, ...) {
    extras <- list(...)
    if (length(extras) > 0)
        stop(sprintf(ngettext(length(extras), "unrecognized option: %s", "unrecognized options: %s
    if (length(idname) != 1 || make.names(idname) != idname)
        stop(gettextf("'%s' argument must be a variable name", "idname"))
    if (!is.null(tstartname) &&
        (length(tstartname) !=1 || make.names(tstartname) != tstartname))
        stop(gettextf("'%s' argument must be NULL or a variable name", "tstartname"))
    if (length(tstopname) != 1 || make.names(tstopname) != tstopname)
```

```
        stop(gettextf("'%s' argument must be a variable name", "tstopname"))
    if (length(delay) !=1 || !is.numeric(delay) || delay < 0)
        stop(gettextf("'%s' argument must be a number >= 0", "delay"))
    if (length(na.rm) !=1 || ! is.logical(na.rm))
        stop(gettextf("'%s' argument must be TRUE or FALSE", "na.rm"))
   list(idname=idname, tstartname=tstartname, tstopname=tstopname, delay=delay, na.rm=na.rm)
}

tname <- attr(data1, "tname")
firstcall <- is.null(tname)  #first call to the function
if (!firstcall && any(is.null(match(unlist(tname), names(data1)))))
    stop("'data1' does not match its own 'tname' attribute")

if (!missing(options)) {
    if (!is.list(options)) stop("options must be a list")
    if (!is.null(tname)) {
        # If an option name matches one already in tname, don't confuse
        #  the tmerge.control routine with duplicate arguments
        temp <- match(names(options), names(tname), nomatch=0)
        topt <- do.call(tmerge.control, c(options, tname[temp==0]))
        if (any(temp >0)) {
            # A variable name is changing midstream, update the
            # variable names in data1
            varname <- tname[c("idname", "tstartname", "tstopname")]
            temp2 <- match(varname, names(data1))
            names(data1)[temp2] <- varname
        }
    }
    else topt <- do.call(tmerge.control, options)
}
else if (length(tname)) topt <- do.call(tmerge.control, tname)
else  topt <- tmerge.control()

# id, tstart, tstop are found in data2
if (missing(id)) stop(gettextf("'%s' argument is required", "id"))
if (missing(data1) || missing(data2))
    stop("two data sets are required")
id <- eval(Call[["id"]], data2, enclos=emptyenv()) #don't find it elsewhere
if (is.null(id)) stop("the id variable is null")

if (firstcall) {
    if (!missing(tstop)) {
         tstop <-  eval(Call[["tstop"]],  data2)
         if (length(tstop) != length(id))
             stop(gettextf("'%s' and '%s' arguments must be the same length", "tstop", "id"))
         # The neardate routine will check for legal tstop data type
```

```
      }
    if (!missing(tstart)) {
        tstart <- eval(Call[["tstart"]], data2)
        if (length(tstart)==1) tstart <- rep(tstart, length(id))
        if  (length(tstart) != length(id))
             stop(gettextf("'%s' and '%s' arguments must be the same length", "tstart", "id"))
        if (any(tstart >= tstop))
            stop("'tstart' must be < 'tstop'")
         }
}
else {
    if (!missing(tstart) || !missing(tstop))
        stop("'tstart' and 'tstop' arguments only apply to the first call")
}
```

Get the ... arguments. They are evaluated in a special frame, set up earlier, so that the definitions of the functions tdc, cumtdc, event, and cumevent are local to tmerge. Check that they are all legal: each argument is named, and is one of the four allowed types.

⟨*tmerge-setup*⟩=
```
# grab the... arguments
notdot <- c("data1", "data2", "id", "tstart", "tstop", "options")
dotarg <- Call[is.na(match(names(Call), notdot))]
dotarg[[1]] <- as.name("list")  # The as-yet dotarg arguments
if (missing(data2)) args <- eval(dotarg, envir=new)
else  args <- eval(dotarg, data2, enclos=new)

argclass <- sapply(args, function(x) (class(x))[1])
argname <- names(args)
if (any(argname== "")) stop("all additional arguments must have a name")

check <- match(argclass, c("tdc", "cumtdc", "event", "cumevent"))
if (any(is.na(check)))
    stop(gettextf("argument(s) %s not a recognized type", argname[is.na(check)]))
```

The tcount matrix keeps track of what we have done, and is added to the final object at the end. This is useful to the user for debugging what may have gone right or wrong in their usage.

⟨*tmerge-setup*⟩=
```
# The tcount matrix is useful for debugging
tcount <- matrix(0L, length(argname), 8)
dimnames(tcount) <- list(argname, c("early","late", "gap", "within",
                                    "boundary", "leading", "trailing",
                                    "tied"))
tevent <- attr(data1, "tevent") # event type variables
tcens  <- attr(data1, "tcensor")# censor code for variables
if (is.null(tcens)) tcens <- vector('list', 0)
```

The very first call to the routine is special, since this is when the range of legal times is set. We also apply an initial sort to the data if necessary so that times are in order. There are 2 cases:

1. Adding a time range: tstop comes from data2, optional tstart, and the id can be simply matched, by which we mean no duplicates in data1.

2. The more common case: there is no tstop, one observation per subject, and the first optional argument is an event or cumevent. We then use its time as the range.

One thing we could add, but didn't, was to warn if any of the three new variables will stomp on ones already in data1.

⟨*tmerge-setup*⟩=
```
 newdata <- data1 #make a copy
 if (firstcall) {
     # The line below finds id, tstop, and tstart variables in data1
     indx <- match(c(topt$idname, topt$tstartname, topt$tstopname), names(data1),
                   nomatch=0)
     if (any(indx[2:3]>0) && FALSE) {  # warning currently turned off. Be chatty?
         overwrite <- c(topt$tstartname, topt$tstopname)[indx[2:3]]
         warning(gettextf("overwriting data1 variables %s", paste(overwrite, collapse = " ")))
         }

     if (indx[1] == 0) {
         # the topt$id variable name is not in data1.  Deal with the
         # fairly common case that data1 == data2.  If data1 has the
         # variable used as 'id' in data2, add it to data1 under the
         # new name
         temp <- as.character(Call[["id"]])
         if (is.name(Call[["id"]]) && !is.na(match(temp, names(data1)))) {
             data1[[topt$idname]] <- data1[[temp]]
             baseid <- data1[[temp]]
             }
         else stop("id variable not found in data1")
     }
     else baseid <- data1[[indx[1]]]

     if (any(duplicated(baseid)))
         stop("for the first call (that establishes the time range) data1 must have no duplicat ide

     if (length(baseid)== length(id) && all(baseid == id)) newdata <- data1
     else {
         indx2 <- match(id, baseid)
         if (any(is.na(indx2)))
             stop("'id' has values not in data1")
         newdata <- data1[indx2,]
```

```
        }
    if (missing(tstop)) { # case 2
        if (length(argclass)==0 || argclass[1] != "event")
            stop("neither a tstop argument nor an initial event argument was found")
        tstop <- args[[1]][[1]]
        }

    # at this point newdata and data2 are in the same order, same # rows
    if (any(is.na(tstop)))
        stop("missing time value, when that variable defines the span")
    if (missing(tstart)) tstart <- rep(0, length(id))
    if (any(tstart >= tstop))
        stop("'tstart' must be > 'tstop'")
    newdata[[topt$tstartname]] <- tstart
    newdata[[topt$tstopname]] <- tstop
    if (any(duplicated(id))) {
        # sort by time within id
        indx1 <- match(id, unique(id))
        newdata <- newdata[order(indx1, tstop),]
    }
    n <- nrow(newdata)
    temp <- newdata[[topt$idname]]
    if (any(tstart >= tstop)) stop("'tstart' must be < 'tstop'")
    if (any(newdata$tstart[-n] > newdata$tstop[-1] &
            temp[-n] == temp[-1]))
        stop("there are overlapping time intervals")
}
else { #not a first call
    if (any(is.na(match(id, data1[[topt$idname]]))))
        stop("id values were found in data2 which are not in data1")
}
```

Now for the real work. For each additional argument we first match the id/time pairs of the new data to the current data set, and categorize each into a type. If the time value in data2 is NA, then that addition is skipped (or if the argument is NA). This is a convenience for the user, who will often be merging in a variable like "day of first diabetes diagnosis" which is missing for those who never had that outcome occur. If the na.rm option is true, we do the same for the variable argument of the call.

⟨*tmerge-addvar*⟩=

```
saveid <- id
for (ii in seq(along.with=args)) {
    argi <- args[[ii]]
    baseid <- newdata[[topt$idname]]
    dstart <- newdata[[topt$tstartname]]
    dstop  <- newdata[[topt$tstopname]]
    argcen <- argi$censor
```

```
 # if an event time is missing then skip that obs
 etime <- argi$time
 if (length(etime) != length(saveid))
     stop(gettextf("'%s' argument is not the same length as '%s' argument", argname[ii], "id"))
if (!is.null(argi$value)) {
    if (length(argi$value) != length(saveid))
         stop(gettextf("'%s' argument is not the same length as '%s' argument", argname[ii], "i
     if (topt$na.rm) keep <- !(is.na(etime) | is.na(argi$value))
     else keep <- !is.na(etime)
     if (!all(keep)) {
         etime <- etime[keep]
         argi$value <- argi$value[keep]
         }
     }
 else {
   keep <- !is.na(etime)
   etime <- etime[keep]
   }
 id <- saveid[keep]

 # For an event or cumevent, one of the later steps becomes much
 #  easier if we sort the new data by id and time
 indx <- order(id, etime)
 id <- id[indx]
 etime <- etime[indx]
 if (!is.null(argi$value))
     yinc <- argi$value[indx]

 # indx1 points to the closest start time in the baseline data (data1)
 #  that is <= etime.  indx2 to the closest end time that is >=etime.
 # If etime falls into a (tstart, tstop) interval, indx1 and indx2
 #   will match
 # If the "delay" argument is set and this event is of type tdc, then
 #   move any etime that is after the entry time for a subject.
 if (topt$delay >0 && argclass[ii] %in% c("tdc", "cumtdc")) {
     mintime <- tapply(dstart, baseid, min)
     index <- match(id, names(mintime))
     etime <- ifelse(etime <= mintime[index], etime, etime+ topt$delay)
 }

 indx1 <- neardate(id, baseid, etime, dstart, best="prior")
 indx2 <- neardate(id, baseid, etime, dstop, best="after")

 # The event times fall into one of 5 categories
 #   1. Before the first interval
```

```
    #   2. After the last interval
    #   3. Outside any interval but with time span, i.e, it falls into
    #       a gap in follow-up
    #   4. Strictly inside an interval (does't touch either end)
    #   5. Inside an interval, but touching.
    itype <- ifelse(is.na(indx1), 1,
                    ifelse(is.na(indx2), 2,
                           ifelse(indx2 > indx1, 3,
                                  ifelse(etime== dstart[indx1] |
                                         etime== dstop[indx2], 5, 4))))

    # Subdivide the events that touch on a boundary
    #  1: intervals of (a,b] (b,d], new count at b  "tied edge"
    #  2: intervals of (a,b] (c,d] with c>b, new count at c, "front edge"
    #  3: intervals of (a,b] (c,d] with c>b, new count at b, "back edge"
    #
    subtype <- ifelse(itype!=5, 0,
                      ifelse(indx1 == indx2+1, 1,
                             ifelse(etime==dstart[indx1], 2, 3)))
    tcount[ii,1:7] <- table(factor(itype+subtype, levels=c(1:4, 6:8)))

    # count ties.  id and etime are not necessarily sorted
    tcount[ii,8] <- sum(tapply(etime, id, function(x) sum(duplicated(x))))
    ⟨tmerge-addin2⟩
}
```

An argument of `tdc(etime)` causes a time-dependent covariate value of 1, one of `tdc(etime, x)` causes the created time dependent variable to have a value of x.

⟨*tmerge-addin2*⟩=
```
# Look to see if this term has one or two arguments.  If one arg
#  then the increment is 1, else it is the second arg.
if (!is.null(argi$value)) yinc <- argi$value
else yinc <- rep(1.0, length(etime))
```

A `tdc` or `cumtdc` operator defines a new time-dependent variable which applies to all future times. Say that we had the following scenario for one subject

current		addition	
tstart	tstop	time	x
2	5	1	20.2
6	7	7	11
7	15	8	17.3
15	30		

The resulting data set will have intervals of (2,5), (6,7), (7,8) and (8,15) with covariate values of 20.2, 20.2, 11, and 17.3. Only a covariate change that occurs within an interval causes a new

data row. Covariate changes that happen after the last interval are ignored, i.e. at change at time ≥ 30 in the above example.

If instead this had been events at times 1, 7, and 8, the first event would be ignored since it happens outside of any interval, so would an event at exactly time 2. The event at time 7 would be recorded in the (6,7) interval and the one at time 8 in the (7,8) interval: events happen at the ends of intervals. In both cases new rows are only generated for new time values that fall strictly within one of the old intervals.

When a subject has two increments on the same day they get summed. This is odd but possible for events, likely an error for time-dependent covariates. We report back the number of ties so that the user can deal with it.

Where are we now with the variables?

itype	class	indx1	indx2
1	before	NA	next interval
2	after	prior interval	NA
3	in a gap	prior interval	next interval
4	within interval	containing interval	containing interval
5-1	on a join	next interval	prior interval
5-2	front edge	containing	containing
5-3	back edge	containing	containing

If there are any itype 4, start by expanding the data set to add new cut points, which will turn all the 4's into 5-1 types. When expanding, all the event type variables turn into "censor" at the newly added times and other variables stay the same. A subject could have more than one new cutpoint added within an interval so we have to count each. In newdata all the rows for a given subject are contiguous and in time order, though the data set may not be in subject order.

⟨*tmerge-addin2*⟩=
```
 indx4 <- which(itype==4)
 n4 <- length(indx4)
 if (n4 > 0) {
     icount <- tapply(etime[indx4], indx1[indx4], function(x) sort(unique(x)))
     n.add <- sapply(icount, length)  #number of rows to add

     # expand the data
     irep <- rep.int(1L, nrow(newdata))
     erow <- unique(indx1[indx4])   # which rows in newdata to be expanded
     irep[erow] <- 1+ n.add # number of rows in new data
     jrep <- rep(seq_len(nrow(newdata)), irep)  #stutter the duplicated rows
     newdata <- newdata[jrep,]  #expand it out
     dstart <- dstart[jrep]
     dstop <-  dstop[jrep]

     #fix up times
     nfix <- length(erow)
     temp <- vector("list", nfix)
     iend <- (cumsum(irep))[irep >1]  #end row of each duplication set
```

```
    for (j in seq_len(nfix)) temp[[j]] <-  -(seq(n.add[j] -1, 0)) + iend[j]
    newrows <- unlist(temp)
    dstart[newrows] <- dstop[newrows-1] <- unlist(icount)
    newdata[[topt$tstartname]] <- dstart
    newdata[[topt$tstopname]]  <- dstop
    for (ename in tevent) newdata[newrows-1, ename] <- tcens[[ename]]

    # refresh indices
    baseid <- newdata[[topt$idname]]
    indx1 <- neardate(id, baseid, etime, dstart, best="prior")
    indx2 <- neardate(id, baseid, etime, dstop, best="after")
    subtype[itype==4] <- 1  #all the "insides" are now on a tied edge
    itype[itype==4]   <- 5
}
```

Now we can add the new variable. Events and cumevents are easy because each affects only one interval. Counts are more work and for this we use a C routine.

⟨*tmerge-addin2*⟩=

```
# add it in
if (argclass[ii] %in% c("cumtdc", "cumevent")) {
    if (!is.numeric(yinc)) stop("invalid increment for cumtdc or cumevent")
    yinc <- unlist(tapply(yinc, id, cumsum))
    }

newvar <- newdata[[argname[ii]]]  #does the variable exist?
if (argclass[ii] %in% c("event", "cumevent")) {
    if (is.null(newvar)) {
        if (is.factor(yinc))
            newvar <- factor(rep(levels(yinc)[1], nrow(newdata)),
                             levels(yinc))
        else if (is.numeric(yinc)) newvar <- rep(0L, nrow(newdata))
        else stop("invalid value for a status variable")
    }
    keep <- (subtype==1 | subtype==3) # all other events are thrown away
    newvar[indx2[keep]] <- yinc[keep]

    if (!(argname[ii] %in% tevent)) {
        tevent <- c(tevent, argname[[ii]])
        if (is.factor(yinc)) tcens <- c(tcens, levels(yinc)[1])
        else tcens <- c(tcens, 0)
        names(tcens) <- tevent
        }
}
else {
    keep <- itype != 2  # changes after the last interval are ignored
    indx <- ifelse(subtype==1, indx1,
```

```
                    ifelse(subtype==3, indx2+1L, indx2))

    if (is.null(newvar)) {
        if (is.null(argi$value)) newvar <- rep(0.0, nrow(newdata))
        else newvar <- rep(NA_real_, nrow(newdata))
        }
    # id can be any data type; feed integers to the C routine
    storage.mode(yinc) <- storage.mode(dstop) <- "double"
    storage.mode(newvar) <- storage.mode(etime) <- "double"
    newvar <- .Call("tmerge", match(baseid, baseid), dstop, newvar,
                    match(id, baseid)[keep], etime[keep],
                    yinc[keep], indx[keep])
}

newdata[[argname[ii]]] <- newvar
```

Finish up by adding the attributes and the class

⟨*tmerge-finish*⟩=
```
attr(newdata, "tname") <- topt[c("idname", "tstartname", "tstopname")]
attr(newdata, "tcount") <- rbind(attr(data1, "tcount"), tcount)
if (length(tevent)) {
    attr(newdata, "tevent") <- tevent
    attr(newdata, "tcensor" ) <- tcens
    }
row.names(newdata) <- NULL
class(newdata) <- c("data.frame")
newdata
```

The print routine is for checking: it simply prints out the attributes.

⟨*tmerge-print*⟩=
```
print.tmerge <- function(x, ...) {
    print(attr(x, "tcount"))
}
"[.tmerge" <- function(x, ..., drop=TRUE){
    class(x) <- "data.frame"
    NextMethod(,x)
    }
```