The Welfare Impact of Reemployment Bonuses

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April 19, 2022

Abstract

This paper studies the role of reemployment bonuses in the design of unemployment insurance. I derive a sufficient statistics formula from a simple framework of unemployment insurance and reemployment bonuses and show that the addition of reemployment bonuses can alleviate incentive costs of UI benefits. I show how to identify all of the parameters in the formula using simple empirical analogues. I find that reemployment bonuses lead to a sizable welfare increase and, in some cases, the policy pays for itself.

Keywords: Reemployment bonus; Sufficient statistic approach; Unemployment insurance; Consumption smoothing; Moral hazard; MVPF

JEL classification: H20, J64, J65

1 Introduction

A well-known downside of unemployment insurance (UI) is that it discourages unemployed people from searching for a job by increasing the value of unemployment. For example, Schmieder and von Wachter (2017) report that the mean estimate of the elasticity of unemployment duration with respect to UI benefits in the literature is about 0.5, implying that a 10 percent increase in UI benefits increases unemployment spells by about 5 percent. Inefficiently long unemployment duration due to UI is problematic since it increases government spending on UI and also decreases income tax revenues in such a way that is not internalized in the optimization problem of unemployed people.

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I would like to thank Naoki Aizawa, Jesse Gregory, Emanuel Hansen, John Kennan, Corina Mommaerts, Shuhei Otani, Jeffery Smith, Christopher Taber, Matthew Wiswall and Hoyoung Yoo for helpful comments and suggestions.

Accordingly, a policy that can fix this distortion while retaining the insurance value of UI could be welfare enhancing.

This paper considers reemployment bonuses as a way to fix the distortion of UI. The idea dates back at least to the 1980s when several field experiments were conducted to assess the labor supply impact of reemployment bonuses (Meyer, 1995). Although these field experiments did not directly lead to actual policies, recent shortages of workers due to the Covid-19 pandemic led some states to offer reemployment bonuses. For example, Arizona state offers a $2,000 bonus to eligible individuals who return to a full-time job and $1,000 to those who return to a part-time job.[1]

This paper examines the role of reemployment bonuses in the design of unemployment insurance. Following the literature on optimal UI exploiting the so-called sufficient statistics approach (Baily 1978, Chetty 2006), I first construct a standard dynamic model of consumption, saving, and job search to derive a formula for the welfare impact of providing reemployment bonuses. The formula depends on a couple of reduced-form parameters that can be estimated in a reduced-form way. I calibrate the formula using the parameter estimates that are common in the literature and find that reemployment bonuses lead to a sizable welfare increase.

Importantly, providing reemployment bonuses, unlike reducing UI benefits, does not hinder consumption smoothing of unemployed people, and taxes required for providing subsidies are partly offset by the reduction in UI spending. The policy can pay for itself in a case where labor supply is so elastic to reemployment bonuses that all the spending on reemployment bonuses is covered by the decrease in UI spending. Even if labor supply is not so elastic and therefore additional taxes are needed, the policy still leads to a welfare gain because (i) consumption of unemployed people is unaffected and (ii) consumption of employed people increases since taxes they need to pay to finance the subsidies are less than proportional to the subsidies because of the decrease in the spending on UI.

A key parameter for the welfare evaluation is the response of unemployment duration to reemployment bonuses. If I find an exogenous variation in reemployment bonuses, I could pin down the parameter. Instead, I propose a novel approach that does not require a variation in reemployment bonuses. Specifically, I use the first-order condition with respect to the search effort to derive estimable bounds on the parameter where the bounds depend on the labor supply elasticity with respect to UI benefits, not reemployment bonuses .[2] An upside to this approach is that I can rely on the large literature on labor supply that has a consensus on the impacts of UI generosity on un-

[1] See https://azgovernor.gov/governor/news/2021/05/governor-ducey-announces-arizona-back-work

[2] Several field experiments were conducted in the U.S. between 1984-1989 in which people in the treated group were allowed to get a lump-sum reemployment bonus if they found a job within a certain period after filing for UI. The results are summarized in Meyer (1995). How bonuses/subsidies were provided slightly differed across experiments, which makes it difficult to map the results of those experiments to a sufficient statistic in my welfare characterization.

employment duration (Krueger and Meyer 2002, Chetty and Finkelstein 2013). Although I specify the decision problem of unemployed people, this approach does not require a specific functional form of the job search cost and hence provides a robust bound on the parameter.

I then evaluate the welfare impact of reemployment bonuses using reduced-form parameters common in the literature. To compare results in my paper to the welfare impacts of other existent government programs, I compute the Marginal Value of Public Funds (MVPF) introduced by Hendren and Sprung-Keyser (2020). They define the MVPF of a policy as the aggregated willingness to pay for the policy divided by the net cost of the policy. My results show that the MVPF of providing reemployment bonuses exceeds 2.0. This is greater than the MVPFs of most of the government programs targeted at adults considered in Hendren and Sprung-Keyser (2020).

In addition to studying the impact of the marginal change in reemployment bonuses for getting a job, I also study non-marginal policy changes by specifying detailed parts of the model that do not need to be specified in the sufficient-statistics analysis and extrapolating from the local policy change. Current UI programs in the U.S. typically provide about 50 percent of previous earnings as UI benefits. Given the current UI, the model-implied optimal level of reemployment bonuses is about 50 percent of previous earnings. Yet, naturally, the optimal UI generosity itself also depends on the level of reemployment bonuses. I examine how the optimal level of UI benefits depends on the level of reemployment bonuses and find that providing reemployment bonuses makes the optimal UI more generous, i.e., reemployment bonuses and UI generosity are complementary. This result implies that providing reemployment bonuses makes it possible for the government to choose a more generous UI as an optimal policy, thereby enabling the unemployed to experience smaller consumption declines than currently experienced.

Related Literature. The idea of providing UI recipients with financial incentives to reduce spending on UI was investigated in the 1980s by conducting several field experiments. Those experiments show that providing lump-sum reemployment bonuses indeed shortened unemployment duration as summarized by Meyer (1995). Various aspects of reemployment bonuses have been studied so far including, for instance, displacement effects on people not offered bonuses (Davidson and Woodbury, 1993), recall from former employers (Anderson, 1992), and take-up rates (Meyer, 1995). Although some papers do simple cost-benefit analysis (e.g. Anderson, Corson and Paul 1991, Woodbury and Spiegelman 1987), no papers have evaluated the welfare implication using a rigorous economic model. Based on recent developments in the literature on optimal UI, I contribute to this literature by providing an empirical evaluation of the welfare consequences of providing financial incentives to find a job. Furthermore, I study reemployment bonuses and UI *jointly*, which is important since, as shown in the analysis of this paper, UI benefits and reemployment bonuses are complementary in the sense that providing reemployment bonuses increases the optimal level of UI benefits.

My paper mainly relates to the literature studying optimal UI. There are several theoretical papers examining the optimal UI benefits profile using principal-agent models (Shavell and Weiss 1979, Hopenhayn and Nicolini 1997, Werning 2002). In particular, Hopenhayn and Nicolini (1997) show that when a planner can use UI benefits and taxes after reemployment as policy instruments, the optimal policy has two properties; (i) UI benefits decline with unemployment spells, and (ii) taxes after reemployment increase with unemployment spells. The first property is, albeit imperfectly, embedded in UI programs in many countries in the sense that typically unemployed people can get constant benefits for a certain period, and then they can get no benefits after that. On the other hand, the second property is not necessarily widely observed. The provision of reemployment bonuses after unemployment can be regarded as reducing taxes for people finding a job within a certain period. In this sense, this paper provides an empirical evaluation of the second property of the optimal policy in Hopenhayn and Nicolini (1997) although the way reemployment bonuses are provided in my paper is not exactly the same as negative taxes in their paper.[3] In terms of methodology, this paper is an application of the sufficient-statistics approach developed by Baily (1978) and Chetty (2006). Many researchers apply the same approach to study, for example, the optimal level of UI benefits (Gruber 1997, Chetty 2008, Landais 2015, Kroft and Notowidigdo 2016), the optimal UI duration (Ganong and Noel 2019), and the optimal path of UI benefits (Kolsrud, Landais, Nilsson and Spinnewijn 2018).[4]

One drawback of this approach is that I cannot extrapolate results to the case where a policy change is not marginal because (i) the envelope theorem used to characterize the welfare impact is not valid for non-marginal policy changes, and (ii) the formula is characterized by reduced-form parameters that are not necessarily policy-invariant. Papers such as Lentz (2009), Michelacci and Ruffo (2015) and Lawson (2017) estimate structural parameters to compute the globally optimal levels of UI benefits by specifying detailed parts of a model such as utility functions or wage processes, which do not need to be specified in the sufficient-statistics approach. In addition to the sufficient statistics analysis, I also take the approach of these papers to study non-marginal policy changes.

[3]To characterize the optimal profile of UI benefits and taxes, Hopenhayn and Nicolini (1997) consider a simplified model that does not allow several important features such as savings and heterogeneity. My model allows savings, heterogeneity, and other features that are not taken into account by Hopenhayn and Nicolini (1997) by restricting the space of policy instruments, i.e., focusing only on the level of benefits and taxes.

[4]Chetty and Finkelstein (2013) and Schmieder and Von Wachter (2016) provide a survey of this literature.

Roadmap. Section 2 provides a stylized model of consumption, saving, and job search, and derives a formula for welfare evaluation of a local policy change. Section 3 evaluates the formula derived in Section 2. Section 4 imposes additional assumptions on the baseline model in Section 2 and studies non-marginal policy changes. Section 5 gives concluding remarks.

2 Model

In this section, I set up a dynamic model of consumption, saving, and job search mostly based on Chetty (2008). I characterize the welfare impact of marginal changes in reemployment bonuses with a couple of reduced-form parameters. An advantage of taking this approach is that I do not need to fully specify the model. For instance, I assume that the search cost function is convex but I do not specify an exact functional form for it. A disadvantage is that I cannot compute the welfare impact of large policy changes and therefore cannot understand the globally optimal UI generosity and reemployment bonuses. In Section 4, I impose additional assumptions on the baseline model in this section and calibrate parameters to study non-marginal policy changes.

2.1 Environment

People in the model start unemployed at $t = 1$, and, once getting employed they continue employed until the end of the model T as in standard models in the literature.[5] While unemployed, they get UI benefits b_u for a maximum of T_B periods. Once they find a job, they get reemployment bonuses b_e. I define $b = (b_u, b_e)$. I assume that people finding a job before period T_B can get b_e until period T_B. For example, suppose that a person finds a job at a period $\bar{t} < T_B$. Then, he receives UI benefits b_u during periods $t = 1, \ldots, \bar{t}$ and gets reemployment bonuses b_e during periods $t = \bar{t} + 1, \ldots, T_B$. Throughout the paper, I focus on the case where b_u and b_e are constant during T_B periods, and T_B is also fixed. The government collects taxes τ from each worker to pay for UI benefits and the government budget must be satisfied. I denote the intensity of search effort at time t by e_t and normalize it so that effort e_t is equal to the probability of finding a job. Then, the survival probability, i.e., the probability of staying unemployed after searching for a job in a period t is given by $S_t = \prod_{j=1}^{t}(1 - e_j)$. Expected unemployment duration is given by $D = \sum_{t=1}^{T} S_t$ and UI-compensated expected unemployment duration is defined as $D_B = \sum_{t=1}^{T_B} S_t$.

[5] For example, Chetty (2008), Landais (2015), and Kolsrud et al. (2018) adopts the same assumption to get a welfare formula.

2.2 Individual Problem

For simplicity, I assume that the interest rate is zero and there is no discounting. In each period t, an unemployed individual first searches for a job and then decides how much to consume.

Optimal Consumption Choice of Employed Individuals. If individuals are employed at a period t with asset A_t, their value function is given by

$$V^E_{s,t}(A_t;b,\tau) = \max_{A_{t+1}\geq L} U(A_t - A_{t+1} + y_{e,s,t}) + V^E_{s,t+1}(A_{t+1};b,\tau), \tag{1}$$

where s is the period he found a job and therefore I always have $s \leq t$. A_t is the amount of asset at the start of period t and L is the liquidity constraint. $y_{e,s,t}$ is income at period t of people who found a job at period s and it is given by $y_{e,s,t} = w_{s,t} - \tau + b_e$ if $t \leq T_B$ and $y_{e,t} = w_{s,t} - \tau$ if $t > T_B$ where $w_{s,t}$ is earnings. I allow earnings $w_{s,t}$ to depend on s to capture the possibility that unemployment duration affects future wages. For example, human capital depreciation would imply $w_{s,t} \leq w_{\tilde{s},t}$ for $s \geq \tilde{s}$. The model is flexible in the sense that I do not assume a specific wage process. All the relevant information required for the welfare analysis later turns out to be captured by marginal utilities of consumption. I also assume that $w_{s,t}$ is fixed in the sense that wage offers have no uncertainty and assume that there are no equilibrium effects on wages although I discuss the potential impacts of relaxing these assumptions at the end of the next section.[6] $U(\cdot)$ is the utility function of consumption and I assume that it is twice-differentiable, strictly increasing and strictly concave on $\mathbb{R}_+$. Since I assume that employment is an absorbing state, employed people face no uncertainty.

Optimal Consumption Choice of Unemployed Individuals. If individuals with asset A_t does not find a job at the beginning of a period t, their value function is given by

$$V^U_t(A_t;b,\tau) = \max_{A_{t+1}\geq L} U(A_t - A_{t+1} + y_{u,t}) + V_{t+1}(A_{t+1};b,\tau), \tag{2}$$

where $y_{u,t}$ is income for unemployed individuals and given by $y_{u,t} = \bar{y} + b_u$ if $t \leq T_B$ while $y_{u,t} = \bar{y}$ otherwise. $\bar{y}$ is income other than unemployment insurance benefits and it can include anything they can get. For instance, it can include income of other family members, informal transfers from their parents and other public transfers such as food stamps. As long as my goal is to compute the welfare impact of marginal policy changes, I do not need to specify what exactly $\bar{y}$ is because,

[6] Landais, Michaillat and Saez (2018b) considers optimal UI in matching models and extends the Baily-Chetty formula to incorporate equilibrium effects.

as it turns out later, consumption declines after job loss contain sufficient information to infer the welfare impacts of UI and reemployment bonuses. The value function $V_{t+1}(\cdot)$ in the second term is the expected value at the start of period $t+1$ before searching a job.

Optimal Search Choice. The value function of unemployed people with asset A_t at the beginning of each period before searching for a job is given by

$$V_t(A_t;b,\tau) = \max_{e_t \in [0,1]} e_t V^E_{t,t}(A_t;b,\tau) + (1-e_t)V^U_t(A_t;b,\tau) - \psi(e_t), \tag{3}$$

where $\psi(\cdot)$ is the utility cost of search effort. Note that the two subscripts of the value function $V^E_{t,t}(\cdot)$ in the first term are both t because this is the value at period t (2nd subscript) of people finding a job at period t (1st subscript). I impose a standard assumption that $\psi(\cdot)$ is twice-differentiable, strictly increasing and strictly convex on $[0,1]$. At the start of each period, an unemployed individual solves this problem to get an optimal level of search effort e_t. Note that $V^U_t(\cdot)$ might not be concave because of $V_{t+1}(\cdot)$ in the second term in (2). I simply assume that $V^U_t(\cdot)$ is concave under plausible parameters as assumed in Chetty (2008).

Henceforth, I suppress b and τ in the value functions for notational brevity in most cases. Assuming an interior solution, the optimal level of search effort is given by the solution to the following first-order condition:

$$\psi'(e_t) = V^E_{t,t}(A_t) - V^U_t(A_t). \tag{4}$$

Since $\psi(\cdot)$ is convex, the left-hand side of the equation is increasing in e_t. This implies that people exert more search effort as the value of employment increases and exert less search effort as the value of unemployment increases. Since the increase in UI benefits b_u makes staying unemployed more valuable, more generous UI leads to the lower search effort. Conversely, more generous reemployment bonuses b_e induce larger search effort by increasing the value of being employed. Let $t \leq T_B$. Then I have

$$\frac{\partial e_t}{\partial b_u} = \frac{-\frac{\partial V^U_t(A_t;b,\tau)}{\partial b_u}}{\psi''(e_t)}, \qquad \frac{\partial e_t}{\partial b_e} = \frac{\frac{\partial V^E_{t,t}(A_t;b,\tau)}{\partial b_e} - \frac{\partial V^U_t(A_t;b,\tau)}{\partial b_e}}{\psi''(e_t)}. \tag{5}$$

The envelope conditions from the individual optimization problem imply

$$\frac{\partial V_t^U(A_t;b,\tau)}{\partial b_u} = U'(c_{u,t}) + \sum_{k=t+1}^{T_B} S_{k|t+1} U'(c_{u,k}),$$
$$\frac{\partial V_{t,t}^E(A_t;b,\tau)}{\partial b_e} = \sum_{j=t}^{T_B} U'(c_{e,t,j}),$$
$$\frac{\partial V_t^U(A_t;b,\tau)}{\partial b_e} = e_{t+1}\frac{\partial V_{t+1,t+1}^E(A_{t+1};b,\tau)}{\partial b_e} + (1-e_{t+1})\frac{\partial V_{t+1}^U(A_{t+1};b,\tau)}{\partial b_e}$$

where $c_{u,k}$ is consumption of unemployed people at period k, $c_{e,t,k}$ is consumption of employed people at period k who find a job at period t, and $S_{k|t+1} = \prod_{j=t+1}^{k}(1-e_j)$ is the probability of being unemployed at the end of period k conditional on being unemployed at the beginning of period $t+1$. Since $U'(c) > 0$, the first equation in (5) is negative, implying that search effort is indeed decreasing in UI benefits b_u. In Appendix A.1, I show that $\frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \frac{\partial V_t^U(A_t)}{\partial b_e} > 0$ under an additional assumption that $c_{e,s,t} = c_e$ for $s,t \leq T_B$, which is an approximation assumption I make in Section 3 when I evaluate the welfare impact of reemployment bonuses. I discuss this assumption later in Section 3. Then, the second equation in (5) is positive, implying that search effort is increasing in reemployment bonuses b_e.

2.3 Government Problem

In this subsection, I derive a formula for evaluating the welfare impact of a marginal change in b_e. Let $W(b)$ where $b = (b_u, b_e)$ denote the value of unemployed people at the beginning of the model as a function of UI benefits b_u, reemployment bonuses b_e, given the budget constraint. Formally, given the assets at the beginning of the model A_1, I define

$$W(b) = V_1(A_1;b,\tau(b)) \quad \text{where} \quad \tau(b) = \frac{D_B b_u + (T_B - D_B) b_e}{T-D}.$$

Note that $\tau(b)$ is the budget-balancing level of taxes as a function of UI benefits b_u and reemployment bonuses b_e. The government problem is to choose UI benefits b_u and reemployment bonuses b_e that maximize the welfare $W(b)$. In this section, I do not consider globally optimal policies by solving this problem. Instead, I characterize the welfare impact of the marginal increase in b_e given UI benefits b_u. Importantly, I assume that there are no fixed administrative costs of running the system of reemployment bonuses.

The government budget constraint is given by

$$[T - D(b,\tau)]\tau(b) = D_B(b,\tau)b_u + [T_B - D_B(b,\tau)]b_e.$$

The impact of providing reemployment bonuses is given by

$$\frac{\partial \tau(b)}{\partial b_e} = \underbrace{\frac{T_B - D_B(b,\tau(b))}{T - D(b,\tau(b))}}_{\text{Mechanical Impact}} + \underbrace{\frac{1}{T - D(b,\tau(b))}\left[\frac{dD_B(b,\tau(b))}{db_e}(b_u - b_e) + \frac{dD(b,\tau(b))}{db_e}\tau(b)\right]}_{\text{Behavioral Impact}}, \quad (6)$$

where $\frac{dD_B(b,\tau(b))}{db_e}$ and $\frac{dD(b,\tau(b))}{db_e}$ are the response of D_B and D to a budget-balance increase in b_e, which includes the effect of the change in τ needed to finance the increase in b_e. The first term represents the increase in taxes that are needed if there is no response by unemployed people, which I refer to as the mechanical impact. In addition to the mechanical impact, there is an additional effect that arises because of the response of unemployed people, which I refer to as the behavioral impact. If a budget-balance increase in reemployment bonuses induces shorter unemployment duration (i.e., $\frac{dD_B}{db_e} < 0$ and $\frac{dD}{db_e} < 0$), the increase in taxes is less than the mechanical cost of the policy change, which turns out to lead to the welfare improvement. The following proposition characterizes the welfare impact of the marginal change in reemployment bonuses b_e given UI benefits b_u. In the proposition, I normalize the welfare impact $\frac{\partial W(b)}{\partial b_e}$ by the expected welfare gain from increasing wages $\frac{\partial V_1}{\partial w}$.

Proposition 1. *The welfare impact of the marginal increase in reemployment bonuses b_e is given by*

$$\frac{\partial W(b)}{\partial b_e} \Big/ \frac{\partial V_1}{\partial w} = \frac{T_B - D_B}{T - D}\left[\frac{MU_{e,T_B} - MU_{e,T}}{MU_{e,T}} - \frac{dD_B}{db_e}\frac{b_u - b_e}{T_B - D_B} - \frac{dD}{db_e}\frac{\tau(b)}{T_B - D_B}\right], \quad (7)$$

where $D = D(b,\tau(b))$, $D_B = D_B(b,\tau(b))$, and $MU_{e,t}$ is given by

$$MU_{e,t} = \sum_{k=1}^{t} \frac{1 - S_k}{t - D_B}\mathbb{E}_s[U'(c_{e,s,k})] \quad \textit{for} \quad t = T_B, T.$$

Proof. See Appendix A.2. □

To get a money metric for the welfare impact, I normalize it here by the welfare gain from a permanent increase in wages. $MU_{e,t}$ is the weighted average of marginal utilities of consumption during employment in which the weights are proportional to the probability that a person is employed at period t. The expectation is taken over all the possible histories leading to the state where a person is employed at period t.[7]

The interpretation of the formula (7) is as follows. The first term in the bracket is the consumption smoothing gain from the policy change. There is no corresponding term in the static case. This term arises because of intertemporal transfers associated with the increase in reemployment bonuses b_e and the corresponding increase in taxes τ.[8] This term is positive if consumption levels just after getting reemployed are smaller than those long after getting reemployed. The second term and the third term are the welfare gain coming from the behavioral impact of reemployment bonuses that corresponds to the second term in equation (6).

Note that the first term disappears in the special case where $c_{e,s,t} = c_e$ for all s,t. This would be a valid approximation if unemployment has only a negligible impact on lifetime income. Then the welfare impact is simply given by

$$\frac{\partial W(b)}{\partial b_e} = -\frac{dD_B}{db_e}\frac{b_u - b_e}{T-D} - \frac{dD}{db_e}\frac{\tau}{T-D}.$$

Importantly, the mechanical budgetary impact represented by the first term in equation (6) does not appear in this equation. In the absence of behavioral responses, reemployment bonuses are just transfers from employed people to employed people, so there is almost no impact on welfare.[9] Behavioral responses matter because those who are induced to get a job by marginal changes in reemployment bonuses are people who are indifferent between getting a job and not getting a job by the envelope condition.

How does the optimal level of reemployment bonuses depend on the level of UI benefits? Denote the optimal level of reemployment bonuses given UI benefits b_u by $b_e^*(b_u)$ that maximizes the welfare given b_u. Assuming again that the first term in (7) is negligible, the optimal reemployment bonuses $b_e^*(b_u)$ given UI benefits b_u is simply given by

$$b_e^*(b_u) = b_u + \frac{\frac{dD}{db_e}}{\frac{dD_B}{db_e}}\tau(b_u, b_e^*(b_u)). \tag{8}$$

The first term implies that reemployment bonuses should increase in proportion to UI benefits.[10] Intuitively, when UI benefits are large, the government can save more money by inducing unem-

[7]The precise expression of $\mathbb{E}_s[U'(c_{e,s,k})]$ is given in equation (22) in Appendix A.2.

[8]Note that people can get b_e only for a maximum of T_B periods whereas they have to pay taxes until the end of the model T.

[9]More precisely, there is welfare change coming from the difference between consumption before and after T_B which makes $MU_{e,T_B} - MU_{e,T} \neq 0$ in the first term of equation (7).

[10]This is not an explicit characterization of the optimal level of reemployment bonuses $b_e^*(b_u)$ since $\frac{dD}{db_e}$, $\frac{dD_B}{db_e}$ and τ depend on b_e.

ployed people to get a job and therefore optimally provide larger reemployment bonuses. Note that I cannot get $b_e^*(b_u)$ just by solving this equation using estimates of reduced-form parameters $\frac{dD}{db_e}$ and $\frac{dD_B}{db_e}$ since solving it involves large changes in b_e from the current policy $b_e = 0$ whereas the reduced-form parameters should not be used for extrapolation. I return to the problem of optimal levels of reemployment bonuses b_e given UI benefits b_u in Section 4 where I impose additional assumptions on the baseline model and numerically solve it to explore the welfare implications of large policy changes.

3 Empirical Implementation

In this section, I evaluate the formula for the welfare impact of the marginal increase in reemployment bonuses b_e that is derived in the previous section. A key parameter for the evaluation is the response of unemployment duration to reemployment bonuses. To get the parameter, I use the first-order conditions of the individuals' optimization problem to derive bounds on the parameter. An advantage of taking this approach is that I can get the parameter based on existing estimates of the response of unemployment duration to UI benefits on which the literature has a consensus (Krueger and Meyer 2002, Chetty and Finkelstein 2013). In this sense, I believe this approach would bring some external validity to this calibration exercise.

3.1 Approximation Assumptions

I begin this section by providing assumptions on consumption that are used for the empirical evaluation of the welfare function.

Assumption. (A.1) *consumption after finding a job does not depend on s and t, i.e.,* $c_{e,s,t} = c_e$*,*

(A.2) *consumption does not decrease by finding a job, i.e.,* $c_e \geq c_{u,t}$*,*

(A.3) *consumption during unemployment does not increase over time, i.e.,* $c_{u,t} \leq c_{u,t-1}$*,*

(A.4) *consumption after finding a job is not greater than consumption before job loss, i.e.,* $c_0 \geq c_e$ *where* c_0 *is consumption before job loss.*

Assumption (A.1) requires consumption after reemployment to depend neither on when a person finds a job s or when consumption is measured t. This would be a reasonable approximation if the impact of unemployment on lifetime income is negligible. Chetty (2008) also adopts this assumption to simplify the welfare formula and provides a simulation result that shows this approximation has negligible impact on the evaluated welfare. Assumption (A.2) says that finding a job does not decrease consumption, which would be true unless UI is too generous. Assumption (A.3) says consumption does not increase over time during unemployment. Assumption (A.4) says that consumption after unemployment is not greater than consumption before job loss, which is empirically supported by, for example, Stephens Jr (2001) that uses PSID to study the impact of job displacement on food consumption. Since I do not specify the wage process $w_{s,t}$ in equation (1) and the income during unemployment $y_{u,t}$ in equation in (2), solving the model does not directly imply those consumption patterns, but I believe those assumptions are reasonable given the reasons mentioned above.

3.2 Inferring the Response to reemployment bonuses from the Response to UI benefits

Recall that the formula in Proposition 1 requires a reduced-form parameter about how unemployment duration responds to reemployment bonuses b_e. It is difficult to estimate that parameter since the provision of reemployment bonuses considered here is not an actual policy but rather a counterfactual policy. There are no variations in b_e in real life that can be used to estimate the parameter. To deal with this issue, I develop a way to infer the parameter I need from a labor supply response to UI benefits, which can be estimated.

Recall that the first-order condition for search effort is given by

$$\psi'(e_t) = V^E_{t,t}(A_t) - V^U_t(A_t).$$

For $t \leq T_B$, differentiating both sides of the equation with respect to either b_u or b_e yields

$$\frac{\partial e_t}{\partial b_u} = \frac{-\frac{\partial V^U_t(A_t)}{\partial b_u}}{\psi''(e_t)}, \qquad \frac{\partial e_t}{\partial b_e} = \frac{\frac{\partial V^E_{t,t}(A_t)}{\partial b_e} - \frac{\partial V^U_t(A_t)}{\partial b_e}}{\psi''(e_t)}. \tag{5}$$

Note that the response of search effort to UI benefits b_u and reemployment bonuses b_e both depend on $\psi''(\cdot)$, the curvature of search cost function. Hence, I obtain

$$\frac{\partial e_t}{\partial b_e} = \frac{\partial e_t}{\partial b_u} \times \frac{\frac{\partial V^E_{t,t}(A_t)}{\partial b_e} - \frac{\partial V^U_t(A_t)}{\partial b_e}}{-\frac{\partial V^U_t(A_t)}{\partial b_u}} \tag{9}$$

The left-hand side is the response of search efforts to reemployment bonuses. The right-hand side is the response of search effort to UI benefits multiplied by the ratio of the marginal values of reemployment bonuses and UI benefits. This expression implies that one can obtain the response to reemployment bonuses by adjusting the response to UI benefits by multiplying the ratio of how much each policy changes the value functions. The following proposition formalizes this idea and provides a bound on the response of UI-compensated unemployment duration to changes in reemployment bonuses b_e.

Proposition 2. *Suppose assumptions (A.1)-(A.3) are satisfied. Then, the response of the expected length of the UI-compensated unemployment spell D_B to the change in reemployment bonuses b_e is bounded as follows:*

$$\underline{M}_{T_B} \frac{\partial D_B(b,\tau)}{\partial b_u} \leq -\frac{\partial D_B(b,\tau)}{\partial b_e} \leq \overline{M}_{T_B} \frac{\partial D_B(b,\tau)}{\partial b_u}, \tag{10}$$

where

$$\overline{M}_{T_B} = \frac{U'(c_e)}{U'(c_{u,1})}, \quad \underline{M}_{T_B} = \frac{U'(c_e)}{U'(c_{u,T_B})}.$$

Proof. See Appendix A.3. □

The proposition states that labor supply response to reemployment bonuses, which is difficult to estimate, is bounded by labor supply response to UI and the marginal rate of substitution between consumption for the employed and the unemployed, both of which are easy to obtain. The intuition is as follows. How much unemployed people respond to reemployment bonuses or UI depends on two things. First, it depends on the convexity of the search cost function as one can see in the denominators of equations (5). To the extent that $\psi''(e_t)$ is smaller, unemployed people respond more to a change in reemployment bonuses or UI. Second, it depends on how much a change in reemployment bonuses or UI benefits increases the value of unemployed people as one can see in the numerators of equations (5). For example, if an unemployed person does not value an increase in UI benefits at all and hence $\frac{\partial V_t^U(A_t)}{\partial b_u} = 0$, then that person does not respond to the increase in UI benefits at all. Since both labor supply response to reemployment bonuses and UI benefits depend on the convexity of $\psi(e_t)$, the response to UI benefits provides some information on the response to reemployment bonuses. But the response to reemployment bonuses and UI benefits diverge from each other to the extent that the marginal value of reemployment bonuses and UI benefits are different from each other. I can adjust that divergence by multiplying the ratio of the marginal utilities of consumption during employment and unemployment as the marginal utility of consumption during employment and unemployment represent how much individuals value reemployment bonuses and UI benefits, respectively.

Remark. The bound derived in Proposition 2 is based on the partial derivatives that do not take into account the impact of the changes in taxes associated with the changes in reemployment

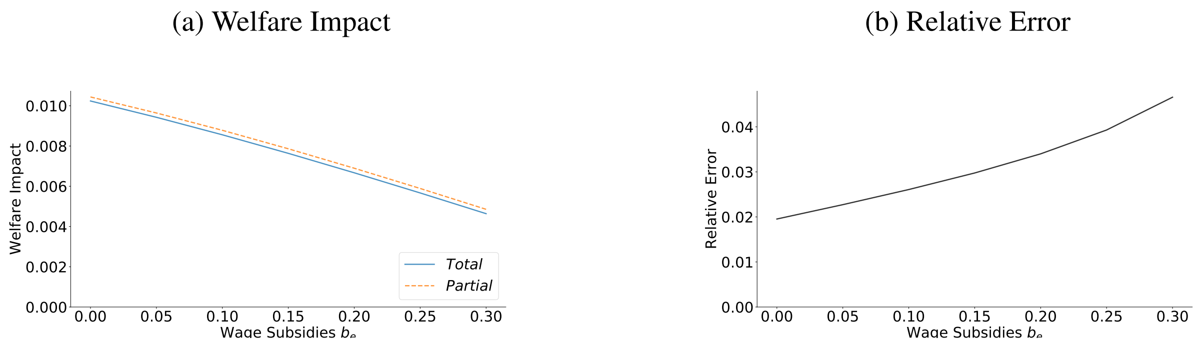

Figure 1: Total Impact vs. Partial Impact

Note: This figure shows how much ignoring the effect of the change in taxes associated with the increase in reemployment bonuses affect the calculation of the welfare impact based on a numerical simulation using a calibrated model in Section 4. In both panels, the horizontal line represents reemployment bonuses as a fraction of previous wages. In the left panel, the vertical line represents the welfare impact of the marginal increase in reemployment bonuses $\frac{\partial W}{\partial b_e} / \frac{\partial V}{\partial w}$. The blue solid line is based on the original formula (7). The red dashed line is based on the same equation except that the total derivatives $\frac{dD_B}{db_e}$ and $\frac{dD}{db_e}$ are replaced by the partial derivatives $\frac{\partial D_B}{\partial b_e}$ and $\frac{\partial D}{\partial b_e}$. In the right panel, the relative error defined by $\frac{\text{Total Impact}-\text{Partial Impact}}{\text{Total Impact}}$ is taken on the vertical axis.

bonuses whereas the welfare formula (7) depends on the total derivatives. Note that

$$\frac{dD_B(b,\tau(b))}{db_e} = \frac{\partial D_B(b,\tau(b))}{\partial b_e} + \left.\frac{\partial D_B(b,\tau)}{\partial \tau}\right|_{\tau=\tau(b)} \times \frac{\partial \tau(b)}{\partial b_e}, \tag{11}$$

$$\frac{dD(b,\tau(b))}{db_e} = \frac{\partial D(b,\tau(b))}{\partial b_e} + \left.\frac{\partial D(b,\tau)}{\partial \tau}\right|_{\tau=\tau(b)} \times \frac{\partial \tau(b)}{\partial b_e}. \tag{12}$$

To the extent that the increase in taxes is large, the difference between the total derivatives and the partial derivatives is large. However, this term is likely to be small because reemployment bonuses are paid to a small fraction of total employed people. Following Chetty (2008) and Kolsrud et al. (2018), I ignore the difference between the partial and the total impact of reemployment bonuses on expected unemployment duration, but I check how large the difference would be using a simulation based on a calibrated model in Section 4. I refer to Section 4 for the details of functional forms and model parameters used in the simulation.

Figure 1a shows how much the welfare effect of the marginal increase in reemployment bonuses is affected by ignoring the labor supply impact of the change in taxes needed to finance the increase in reemployment bonuses. The welfare impact (not the level of the welfare) based on the exact formula (blue solid line) is below the welfare impact that does not include the labor supply impact of the tax changes (red dashed line) and therefore using the partial derivatives leads to the overevaluation of the welfare impact, but the difference is very small. Indeed, Figure 1b shows the

Table 1: Parameters Used for the Welfare Evaluation of the Marginal Change in b_e

Parameter	Description	Value
u	Fraction of periods being unemployed	0.054
D	Expected unemployment duration (weeks)	24.3
D_B	Expected UI-compensated unemployment duration (weeks)	15.8
η_{D_B,b_u}	Elasticity of D_B with respect to UI benefits	0.53

Note: D, D_B, and η_{D_B,b_u} are directly taken from Chetty (2008). The paper also reports $\sigma = \frac{T-D}{T} = 0.946$, which implies that $u = 1 - \sigma = 0.054$.

relative error in the welfare impact is about two percent when $b_e = 0$. I use $\frac{\partial D}{\partial b_e}$ and $\frac{\partial D_B}{\partial b_e}$ as proxies for $\frac{dD}{db_e}$ and $\frac{dD_B}{db_e}$ throughout this section.

3.3 Evaluating Welfare Impact

Proposition 1 in the previous section gives the welfare impact of providing reemployment bonuses. Replacing the response of unemployment duration to reemployment bonuses using Proposition 2, I obtain a lower bound on the welfare impact of the provision of reemployment bonuses.

Corollary 1. *Suppose that assumptions (A.1)-(A.3) are satisfied. Then the welfare impact of the marginal change in reemployment bonuses b_e is given by*

$$\frac{\partial W(b)}{\partial b_e} \Big/ \frac{\partial V_1}{\partial w} \geq \frac{u}{1-u}\frac{D_B}{D}\underline{M}_{T_B}\eta_{D_B,b_u}, \quad \textit{where} \quad \eta_{D_B,b_u} = \frac{dD_B}{db_u}\frac{b_u}{D_B} \textit{ and } u = \frac{D}{T}. \tag{13}$$

Proof. See Appendix A.4. □

Note that η_{D_B,b_u} is the total elasticity of expected insured duration with respect to UI benefits, and u is the fraction of periods in which a person is expected to be unemployed. Under the assumption that consumption is constant once a person gets a job, the first term in equation (7) disappears, so the policy change has no consumption smoothing gain.

There are four parameters $(\sigma, D, D_B, \underline{M}_{T_B}, \eta_{D_B,b_u})$ that have to be determined to evaluate the right-hand side of equation (13). For parameters other than $\underline{M}_{T_B}$, I use values reported in Chetty (2008) summarized in Table 1.[11]

The remaining parameter to be determined is $\underline{M}_{T_B}$. Assuming CRRA utility function $U(c) =$

[11]There are many papers other than Chetty (2008) that estimate the labor supply response to UI benefits. In Appendix C.1, I examine how results in this section change when I use parameters taken from other studies.

Table 2: Welfare Impact of the Marginal Increase in b_e

c_u/c_e	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$
0.85	0.017	0.014	0.012
0.90	0.018	0.016	0.014
0.95	0.019	0.018	0.017

Note: This table reports the welfare impact of the marginal increase in reemployment bonuses b_e from zero. The first column reports consumption ratio $c_{u,T_B}/c_e$ used in the evaluation. The second to the fourth columns report $\frac{\partial W(b)}{\partial b_e} / \frac{\partial V_1}{\partial w}$ under different values of CRRA parameters γ. The unit of measure is the marginal value of a permanent increase in wages.

$\frac{c^{1-\gamma}}{1-\gamma}$, I have

$$\underline{M}_{T_B} = \left(\frac{c_{u,T_B}}{c_e} \right)^{\gamma}. \tag{14}$$

Evaluating this equation requires high-frequency consumption data together with employment information. Because I do not have access to such data, I first just try several possible values of $c_{u,T_B}/c_e$. I use a range of values of CRRA parameter γ as well because the literature has little consensus on this parameter.

Table 2 shows the welfare impact of the marginal change in financial work incentives b_e. The first column shows the consumption ratio $c_{u,T_B}/c_e$ used in the evaluation. The second to the fourth columns show the welfare gain. The unit is the marginal gain from a permanent increase in the wage $\frac{\partial V}{\partial w}$. The table shows that the welfare impact is increasing in the consumption ratio c_u/c_e and decreasing in CRRA parameter γ. This is because given other parameters fixed, the larger consumption ratio or the lower CRRA parameter implies the larger $\underline{M}_{T_B}$ in equation (14), which in turn implies the larger labor supply response as can be seen in equation (10). In the most conservative case where $c_{u,T_B}/c_e = 0.85$ and $\gamma = 3$, the welfare impact of a \$1 balanced-budget increase in b_e is equivalent to that of a 1.2 cent permanent increase in weekly earnings. If I aggregate this over lifetime by assuming that $T = 450$, which can be backed out from $u = \frac{D}{T}$ where $u = 0.054$ and $D = 24.3$ given in Table 1, then the result implies that \$1 balanced-budget increase in b_e is equivalent to $0.012 \times 450 = 5.4$ dollar increase in lifetime income per person.

In the welfare evaluation above, the consumption ratio $c_{u,T_B}/c_e$ plays a key role and one might wonder whether the assumption $c_{u,T_B}/c_e = 0.85$ is exaggerating the welfare impact. To get a rough sense of what is an appropriate value of consumption ratio $c_{u,T_B}/c_e$, I use information on high-frequency consumption reported in Ganong and Noel (2019).[12] Assumption (A.4) implies

$$\underline{M}_{T_B} \geq \left(\frac{c_{u,T_B}}{c_0}\right)^{\gamma},$$

where c_0 is consumption before job loss. The right-hand side variable is consumption decline upon unemployment. The event study plot in Figure 1 of Ganong and Noel (2019) shows that the ratio of consumption before job loss and consumption at UI exhaustion exceeds 0.85. Therefore, $c_{u,T_B}/c_e = 0.85$ in Table 2 would not exaggerate the welfare impact of the proposed policy.

3.4 Comparison to Historical Policy Changes in the U.S.

To compare the welfare gain of this policy to the welfare impacts of historical policy changes in the U.S., I compute the Marginal Value of Public Funds (MVPF) introduced by Hendren and Sprung-Keyser (2020). MVPF of a policy is defined as the aggregated willingness to pay for the policy divided by the net cost to the government.[13] They introduce this notion to provide a unified framework to assess the impacts of government policies on social welfare. As in the previous subsection, I impose the approximation assumptions (i) $c_{e,s,t} = c_e$ and (ii) $\underline{M}_T = 0$. These assumptions do not lead to the overestimation of the welfare impact as mentioned earlier. The willingness to pay for \$1 increase in reemployment bonuses b_e is given by

$$WTP = T_B - D_B.$$

The net cost of it is given by

$$\begin{aligned} Net\ Cost &= (T-D)\frac{\partial \tau(b)}{\partial b_e} \\ &= (T_B - D_B) + \frac{dD_B}{db_e}(b_u - b_e) + \frac{dD}{db_e}\tau \\ &= (T_B - D_B)(1 + FE_e), \end{aligned}$$

[12]Ganong and Noel (2019) use monthly consumption data from JPMorgan Chase Institute and conduct several descriptive analyses.

[13]The model in this paper has no heterogeneity and therefore there is no difference between the aggregated willingness to pay and that of an individual.

Table 3: MVPFs of the Marginal Increase in b_e

c_u/c_e	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$
0.85	3.31	2.46	2.02
0.90	3.83	2.99	2.49
0.95	4.54	3.86	3.38

Note: This table reports the Marginal Value of Public Funds (MVPF) of the marginal increase in reemployment bonuses b_e from zero. The first column reports consumption ratio $c_{u,T_B}/c_e$ used in the evaluation. The second to the forth columns report the MVPFs under different values of CRRA parameters γ.

where $FE_e = \frac{1}{T_B - D_B}\left[\frac{dD_B}{db_e}(b_u - b_e) + \frac{dD}{db_e}\right]$ represents the fiscal externality due to the behavioral response of unemployed people. Evaluating FE_e at the point $b_e = 0$ yields

$$FE_e = \frac{u_B}{1-u_B}\frac{dD_B}{db_e}\frac{b_u}{D_B} + \frac{u_B}{1-u_B}\frac{u}{1-u}\frac{dD}{db_e}\frac{b_u}{D},$$

where $u_B = \frac{D_B}{T_B}$. Using Proposition 2 and the assumption that $\underline{M}_T = 0$, I get

$$FE_e \leq -\frac{u_B}{1-u_B}\underline{M}_{T_B}\eta_{D_B,b_u}.$$

Note that FE_e is negative, implying that the net cost is less than 1. The lower bound on the MVPF is given by

$$MVPF = \frac{WTP}{Net\ Cost} \geq \frac{1}{1 - \frac{u_B}{1-u_B}\underline{M}_{T_B}\eta_{D_B,b_u}}. \tag{15}$$

The last column of Table 3 shows that the MVPF of the proposed policy change exceeds 2.0 even in a conservative case where $c_{u,T_B}/c_e = 0.85$ and $\gamma = 3$. This result is remarkable because Hendren and Sprung-Keyser (2020) reports MVPFs of 133 historical policy changes in the U.S. and most policies targeting adults have MVPFs below 2.

I show the MVPF of UI derived from my model. I provide detailed derivation in Appendix A.5. The MVPF of the marginal increase in UI benefits b_u is given by

$$MVPF \approx \frac{1 + \gamma\frac{c_e - c_u}{c_e}}{1 + \frac{1}{1-u}\eta_{D_B,b_u}}.$$

This expression can be interpreted in the following way. The willingness to pay for an additional dollar of UI benefits in the numerator depends on the relative difference between the marginal utility of consumption during unemployment and employment, which, in the case of CRRA utility, can be approximated to the consumption drop upon unemployment multiplied by the risk aversion parameter γ as in the numerator in the equation above. To the extent that consumption declines more upon unemployment, individuals find greater value in an additional UI. On the other hand, the net cost of an additional dollar of UI benefits depends on the moral hazard impact of UI, which can be captured by the elasticity of insured duration with respect to UI benefits. Suppose that $\gamma = 2.0$ and the consumption drop upon unemployment is given by 0.068 as documented by Gruber (1997). In addition, suppose that u and η_{D_B,b_u} are given as in Table 1. Then, the MVPF of the marginal increase in UI benefits is given by

$$MVPF \approx \frac{1+2\times 0.068}{1+\frac{1}{1-0.054}\times 0.053} \approx 0.73,$$

which is within a range of the MVPFs of UI reported in Hendren and Sprung-Keyser (2020).

This illustrates that the large MVPFs of the marginal increase in reemployment bonuses reported in Table 3 is the flip side of the low MVPF of the marginal increase in UI benefits described above. The MVPF of additional UI benefits is small because the moral hazard cost of UI is large compared to the consumption smoothing gain. Note that this large moral hazard cost is due to the large response of unemployment spells to UI generosity. As shown in Proposition 2, this large labor supply response to UI generosity translates into the large labor supply response to reemployment bonuses. This enables the government to decrease a large amount of spending on UI by encouraging job search through reemployment bonuses , which results in a welfare gain through the reduction in taxes.

3.5 Discussion

The baseline model analyzed so far is highly stylized and one might think that the model lacks many important features that exist in reality. It turns out that my results are robust to some extensions while the model also has some limitations.

State Dependencies of Marginal Utility of Consumption. Marginal utilities $U'(c_{u,t})$ and $U'(c_{e,t,t})$ play an important role in connecting what is estimable $\frac{\partial D_B}{\partial b_u}$ to what needs to be obtained $\frac{\partial D_B}{\partial b_e}$. In the welfare evaluation so far, I assume that marginal utilities of consumption do not directly depend on employment status. However, there are several reasons that marginal utilities of consumption might be different depending on whether the person is working or not. For example, if consumption and leisure are not separable, then the true marginal rate of substitution should be different from what I use in the proposition above. Let $U_u(c)$ be utility from consumption while unemployed and $U_e(c)$ be utility from consumption while employed. If consumption and hours of work are Frisch complements as is empirically shown by Ziliak and Kniesner (2005), then the marginal utility of consumption is larger while working, i.e., $U_u'(c) < U_e'(c)$. Under this condition, using the proposition above leads to the underestimation of the true response since $\frac{U_u'(c_{e,t,t})}{U_u'(c_{u,t})} < \frac{U_e'(c_{e,t,t})}{U_u'(c_{u,t})}$.

Another possibility is that unemployed people have more time to spend on shopping and home production, and they can achieve the same level of utility with a lower level of *expenditure* (Aguiar and Hurst, 2007).[14] For example, suppose that a person is busy while employed and often eats out to save time. If this person gets unemployed, he has more free time and he can spend it on cooking to save money. Denote expenditure by c and consumption (as the output of time and expenditure) by $\tilde{c}$. Then, it is likely that $\frac{c_e}{c_u} > \frac{\tilde{c}_e}{\tilde{c}_u}$. This implies that the marginal rate of substitution between *consumption* while employed and while unemployed would be larger than the marginal rate of substitution between *expenditure* while employed and while unemployed, which is used in the proposition. Again, the bound derived in the proposition is an underestimate of the true response in this case.

How do these affect the welfare evaluation? The behavioral response $\frac{dD_B}{db_e}$ represents how shorter the unemployment duration becomes as a result of the increase in reemployment bonuses. The larger response is beneficial since a shorter unemployment duration leads to smaller spending on UI. Given this relationship, underestimating the behavioral response by ignoring the state dependencies of marginal utilities of consumption leads to a conservative estimate of the welfare impact of the proposed policy.

Heterogeneity. The model in this section does not assume heterogeneity among unemployed people while data show that unemployed people are heterogeneous, especially in terms of their liquidity (Chetty, 2008). Yet, it turns out that incorporating heterogeneity does not affect the representation of the formula. If I redefine welfare as $W(b) = \int V_1(A_1(\omega), \omega; b, \tau(b)) dF(\omega)$ where $V_1(A_1(\omega), \omega; b, \tau(b))$ is the value of an individual and $F(\omega)$ is the distribution of individuals, then I can get a formula that is the same as in equation (7) except that I need to integrate the marginal utilities with respect to $F(\omega)$. Appendix B.1 derives the formula in the presence of heterogeneity in detail.

Uncertainty in Wage Offers. Many papers analyzing job search behavior of unemployed people assume that stochastic wage offers are drawn from a distribution (McCall, 1970). Although this adds a reservation wage R_t to the model as an additional choice variable, the envelope theorem implies that the response of reservation wages to reemployment bonuses has no first-order impact

[14] Here, consumption and expenditure are different objects. Consumption is thought of as the output produced by using time and market goods (expenditure) as inputs. See Becker (1965) and Aguiar and Hurst (2005).

on individual values, and the effect of the response of reservation wages on the government budget is captured by $\frac{dD}{db_e}$ in equation (7). As a result, uncertainty in wage offers does not change the welfare formula. I refer to Appendix B.2 for a more rigorous treatment of this issue.

Ex-ante Savings Response. The analysis so far focuses on the problem after a job loss and ignores the possibility that individuals respond to the change in policies *before* losing a job. For example, a more generous UI would lead to less precautionary savings (Engen and Gruber, 2001). However, as long as the policy change is marginal, the ex-ante response does not affect the analysis in this section because of the envelope condition. It is true that changes in reemployment bonuses b_e would affect the asset choice before job loss, but what matters for the welfare calculation is the direct impact of reemployment bonuses b_e on utility and the externality through the government budget constraint. Other effects through individual choices cancel out because of the first-order conditions of the agent's optimization problem.

Inflow into Unemployment. Wage subsidies might increase the inflow into unemployment. For example, suppose that a match of a job and a worker produces a joint surplus of

$$S = V^W + V^F - V_o^W - V_o^F,$$

where V^W and V^F are job values for workers and firms, and V_o^W and V_o^F are values of outside options for workers and firms, respectively. If S is positive, a pair of a firm and a worker split the surplus and if it is negative, the match resolves. Wage subsidies, like UI, put downward pressure on the joint surplus S by increasing the value of the worker's outside option V_o^W. How much UI and reemployment bonuses affect unemployment inflow depends on how much UI and reemployment bonuses increase the value of the worker's outside option. Note that the model in the previous section implies that

$$\frac{\partial V_o^W}{\partial b_u} = D_B U'(c_u), \quad \frac{\partial V_o^W}{\partial b_e} = (T_B - D_B) U'(c_e),$$

where I assume for simplicity that consumption during unemployment and that after getting a job are constant and given by c_u and c_e with $c_u < c_e$. The parameters in Table 1 show that $T_B - D_B > D_B$, and the concavity of the utility function $U(\cdot)$ implies $U'(c_u) > U'(c_e)$. This means that a \$1 increase in reemployment bonuses increases the value of the worker's outside option *less than* a \$1 increase in UI benefits. Therefore, although the provision of reemployment bonuses might be associated with unemployment inflow, the impact of providing reemployment bonuses on unemployment inflow would be *smaller* than the impact of the increase in UI benefits on it.

Due to the lack of empirical evidence on the impact of UI on job separation, I do not incorporate the effect on unemployment inflow into the welfare evaluation. Instead, I ask how much unemployment inflow do we need to offset the beneficial impact of reemployment bonuses calculated earlier in this section. There is a unit mass of unemployed people at the start of the model. Suppose that \$1 increase in b_e increases the size of unemployed people by α. I compute α that satisfies

$$\underbrace{-\frac{dD_B}{db_e}(b_u - b_e)}_{\text{Behavioral Impact}} = \underbrace{\alpha\left[\left(D_B + \frac{dD_B}{db_e}\right)b_u + \left(T_B - D_B - \frac{dD_B}{db_e}\right)b_e\right]}_{\text{Spending for Unemployment Inflow}}. \tag{16}$$

The left-hand side represents how much the government can save due to the shorter unemployment duration caused by reemployment bonuses. The right-hand side represents how much the government needs to spend on UI benefits and reemployment bonuses for those who are separated from their job due to the policy change. Evaluating this at $b_e = 0$ and arranging terms, I obtain

$$\alpha = -\frac{\frac{dD_B}{db_e}/D_B}{\frac{dD_B}{db_e}/D_B + 1}.$$

Using the bounding approach used earlier in this section, I have

$$\alpha \geq \frac{\underline{M}_B \frac{\eta_{D_B,b_u}}{b_u}}{1 - \underline{M}_B \frac{\eta_{D_B,b_u}}{b_u}}.$$

As in the conservative welfare evaluation earlier in this section, suppose that $c_{u,T_B}/c_e = 0.85$ and $\gamma = 3$. In addition, let the weekly UI benefits be $b_u = \$166$ which is the average weakly UI benefits reported in Chetty (2008). Then, I obtain

$$\alpha \geq 0.002.$$

This implies that reemployment bonuses improve welfare as long as the unemployment inflow associated with \$100 of reemployment bonuses per week is less than 20 percent of the total number of unemployed people.

Equilibrium Effects. Wage subsidies affect labor supply decisions. My model assumes that wages are fixed, but in reality, wages should be determined in equilibrium. In the standard matching models in which wages are determined through Nash bargaining (Pissarides, 2000), reemployment bonuses, like UI, would improve the outside options of workers by increasing the value of unemployment, thereby increasing bargained wages. The increase in wages in turn decreases job creation in the equilibrium, offsetting a part of the beneficial effect of reemployment bonuses. Another possibility is that the increase in employment of people to whom reemployment bonuses are offered might displace those who are not offered reemployment bonuses if vacancies are not adjusted quickly. There are several papers reporting that the displacement effect partly offsets the effects of labor market policies (Davidson and Woodbury 1993, Lise, Seitz and Smith 2004, Crépon, Duflo, Gurgand, Rathelot and Zamora 2013). Taken together, a part of the welfare gain computed in this section would be offset by the increase in unemployment due to equilibrium effects. How much equilibrium effects offset the impact of reemployment bonuses is an important question. In the context of UI, Landais et al. (2018b) and Landais, Michaillat and Saez (2018a) extend the Baily-Chetty formula so that it can take into account equilibrium effects. In their formula, one important parameter is the *macro* elasticity of unemployment with respect to UI generosity that takes into account equilibrium effects. Since it is difficult to get the corresponding macro elasticity of unemployment with respect to reemployment bonuses, I do not address this issue in the current paper.

Additional Fiscal Externalities. In the context of UI, fiscal externalities refer to the additional spending on UI and the decline in tax revenue due to longer unemployment spells. As in standard models in the literature, the government in my model spends tax income only on UI and reemployment bonuses. In reality, taxes collected from employed people are also spent on various other public programs as well. In that case, the decrease in tax revenue due to longer unemployment duration leads to added welfare costs. Lawson (2017) considers this additional fiscal externality and shows that when UI is only a small part of total government spending, the Baily-Chetty formula overestimates the value of marginally increasing UI benefits b_u. Meanwhile, the same intuition implies that my formula for the marginal change in reemployment bonuses b_e ignoring the added fiscal externality *underestimates* the welfare impact of the policy change because providing reemployment bonuses leads to shorter unemployment duration and hence a larger tax base, which can be used to finance non-UI spending. In this respect, the calibration result in the next section would be a conservative estimate of the welfare impact of reemployment bonuses.

4 Toward the Global Optimum

Based on the sufficient statistic approach, the analysis in the previous section finds room for welfare improvement in the current UI system. In particular, I find that the marginal increase in reemployment bonuses from zero has a large impact in terms of cost-effectiveness measured by MVPF. In this section, I consider the following two questions. First, what is the optimal level of reemploy-

Table 4: Parameters Selected Externally

Parameter	Description	Value
γ	Coefficient of relative risk aversion	2.0
T	Time length of the model (weeks)	450
L	Liquidity constraint	0.0
β	Discount factor	1.0
r	Interest rate	0.0
A_1	Initial asset	5.16
$\bar{y}$	Income other than UI	0.5
b_u	UI replacement rate	0.5
b_e	reemployment bonuses	0.0

Note: For the choice of T, I back it out from $u = \frac{D}{T}$ where $u = 0.054$ and $D = 24.3$ given in Table 1. Using SIPP, Chetty (2008) reports median annual wage of unemployed people before job loss is \$17,780 and median liquid wealth is \$1,763. Since weekly wages are normalized to 1 in the model, I choose initial asset $A_1 = \$17,780/(\$1,763/52\text{weeks}) \approx 5.16$. $\bar{y} = 0.5$ is chosen to generate consumption declines upon unemployment around 10 percent.

ment bonuses? Also, how does providing reemployment bonuses affect the optimal level of UI benefits? To answer these questions involving non-marginal policy changes, I impose additional assumptions on the baseline model and numerically solve the model.

4.1 Parametric Assumptions

I impose several functional form assumptions to the model considered in section 3, mainly following Kolsrud et al. (2018). The per-period utility is given by $U(c_t) - \psi(e_t)$ where

$$U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}, \quad \psi(e_t) = \theta \frac{e_t^{1+\xi}}{1+\xi}, \tag{17}$$

where γ represents the coefficient of relative risk aversion, θ is the weight on the utility cost of job search, and ξ is the parameter governing the convexity of the utility cost of job search. Note that $1/\xi$ is the elasticity of search effort with respect to the net value of being employed. I normalize wages to be 1, so the levels of UI benefits b_u can be regarded as a replacement rate. I choose parameters of the search cost function (θ, ξ) to match the simulated moments to actual moments given other parameters that are chosen externally. The parameters other than (θ, ξ) are summarized in Table 4.[15]

When determining (θ, ξ), I match (i) average unemployment duration D and (ii) the elasticity

[15] In Appendix C.2, I alter some parameters in Table 4 to examine how results are sensitive to those parameters.

Table 5: Calibrated Parameters

Parameter	Description	Value
θ	Utility cost of search: weight	390.7
ξ	Utility cost of search: convexity	0.028

Note: The search cost function is given by $\psi(e) = \theta \frac{e^{1+\xi}}{1+\xi}$. These parameters are chosen to match $D = 24.3$ and $\eta_{D,b_u} = 0.53$ reported in Chetty (2008).

of unemployment duration with respect to UI benefits η_{D,b_u} in the model and data. Note that matching unemployment duration D in the model and data helps to determine θ while matching the elasticity of unemployment duration with respect to UI benefits η_{D,b_u} helps to determine ξ. The tax rate τ is chosen so that the government budget constraint is satisfied. Therefore, τ is chosen each time when I change benefit levels b_u and b_e in the simulation.

Table 5 shows the calibrated parameters.

4.2 Globally Optimal reemployment bonuses b_e given the Current UI

Once I calibrate the model, I can simulate the model under different values of reemployment bonuses b_e and see how welfare changes. To highlight trade-offs associated with policy changes, I define the consumption smoothing gain

$$CS_e = \frac{MU_{e,T_B} - MU_{e,T}}{MU_{e,T}}, \quad \text{where} \quad MU_{e,t} = \sum_{k=1}^{t} \frac{1-S_k}{t-D_B} \mathbb{E}_s[U'(c_{e,s,k})] \quad \text{for} \quad t = T_B, T. \tag{18}$$

and the fiscal externality cost

$$FE_e = \frac{1}{T_B - D_B} \left[\frac{dD_B}{db_e}(b_u - b_e) + \frac{dD}{db_e}\tau \right] \tag{19}$$

associated with the increase in reemployment bonuses b_e. Note that $1 + CS_e$ can be regarded as the marginal willingness to pay (WTP) for the policy change while $1 + FE_e$ can be regarded as the marginal cost of the policy change. Appendix A.6 shows that given UI benefits b_u fixed, the optimal reemployment bonuses b_e satisfy $CS_e = FE_e$.

In Figure 2, I plot the marginal WTP $1 + CS_e$ and the marginal cost $1 + FE_e$ at each b_e. As reemployment bonuses b_e increase, consumption upon reemployment increases, which is costly because consumption upon reemployment gets too large; it is not worth increasing taxes to increase consumption of people just after they get a job because their marginal utility of consumption gets

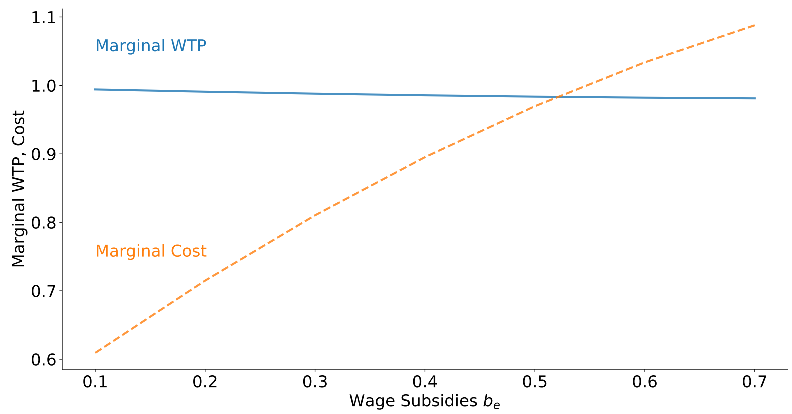

Figure 2: Welfare Impact of Wage Subsidies

Note: This figure shows the welfare impact of the marginal change in reemployment bonuses b_e at each point of b_e given UI benefits $b_u = 0.5$. The horizontal axis represents reemployment bonuses b_e as a fraction of wages. The solid line represents the marginal willingness to pay $1+CS_e$ where CS_e is given in equation (18) and the dashed line represents the marginal cost $1+FE_e$ where FE_e is given in equation (19).

lower. Consequently, the marginal WTP $1+CS_e$ goes down. On the other hand, the marginal cost $1+FE_e$ increases. This is because the reduction in unemployment spells reduces the government spending in proportion to $b_u - b_e$ and, as reemployment bonuses b_e increases, the government has to pay more to people getting a job. Figure 2 shows that the two lines intersect around $b_e = 0.5$ which is close to exogenously given UI benefits $b_u = 0.5$.

4.3 How does Optimal Level of UI Benefits b_u Depend on reemployment bonuses b_e?

How the provision of reemployment bonuses affects the optimal level of UI benefits depends on the shape of the welfare function. Specifically, what matters is the cross derivative of the welfare function:

$$\frac{\partial^2 W(b)}{\partial b_u \partial b_e} = \frac{\partial^2 V_1}{\partial b_u \partial b_e} + \frac{\partial^2 V_1}{\partial b_u \partial \tau} + \frac{\partial V_1}{\partial \tau}\frac{\partial^2 \tau(b)}{\partial b_e \partial b_u}.$$

It is difficult to analytically determine the sign of this. For example, consider the first term in the right-hand side. I have $\frac{\partial V_1}{\partial b_e} = \sum_{t=1}^{T_B}(1-S_t)\mathbb{E}_s[U'(c_{e,s,t})]$ and it is ambiguous whether this is increasing in b_u since a budget-balance increase in b_u would increase the survival probability S_t while the increase in taxes associated with the increase in b_u would increase the marginal utility $\mathbb{E}_s[U'(c_{e,s,t})]$ by decreasing consumption of the employed $c_{e,s,t}$. Simulating the calibrated model

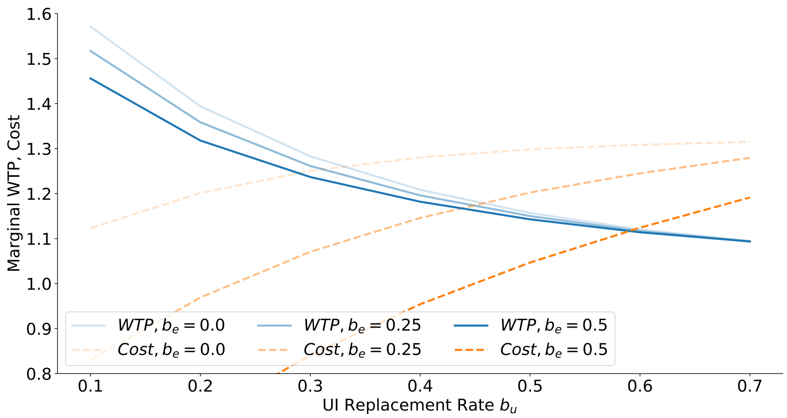

Figure 3: Welfare Impact of UI Benefits under Different Wage Subsidies

Note: This figure shows the welfare impact of the marginal change in UI benefits b_u at each point of b_u under different values of reemployment bonuses b_e. The horizontal axis represents UI benefits b_u as a fraction of wages. The solid lines represent the marginal willingness to pay $1+CS_u$ where CS_u is given in equation (20) and the dashed lines represent the marginal cost $1+FE_u$ where FE_u is given in equation (21).

allows me to study the dependence of the optimal level of UI benefits on reemployment bonuses.

I define the consumption smoothing gain of additional UI benefits as

$$CS_u = \frac{MU_{u,T_B} - MU_{e,T}}{MU_{e,T}}, \qquad \text{where} \qquad MU_{u,T_B} = \sum_{t=1}^{T_B} \frac{S_t}{D_B} U'(c_{u,t}). \tag{20}$$

and the fiscal externality cost as

$$FE_u = \frac{1}{D_B} \left[\frac{dD_B}{db_u}(b_u - b_e) + \frac{dD}{db_u}\tau \right] \tag{21}$$

associated with the increase in UI benefits b_u. Again, $1+CS_u$ can be regarded as the marginal WTP for the policy change while $1+FE_u$ can be regarded as the marginal cost of the policy change. Appendix A.6 shows that given b_e the optimal UI benefits b_u satisfies $CS_u = FE_u$. Since generous UI leads to longer unemployment spells (i.e. $\frac{dD_B}{db_u} > 0$), the marginal cost $1+FE_u$ is decreasing in b_e. This is because the longer unemployment spells increase the government's spending in proportion to $b_u - b_e$ and, as reemployment bonuses b_e increases, the additional government's spending due to longer unemployment spells decreases.

Figure 3 shows how the welfare impact of UI interacts with reemployment bonuses. The solid lines in the figure represent the marginal WTP $1+CS_u$ while the dashed lines show the marginal cost $1+FE_u$. The horizontal axis represents UI benefits b_u and each line corresponds to a different level of reemployment bonuses b_e. The lightest color corresponds to the case where $b_e = 0$ while the darkest color corresponds to the case where $b_e = 0.5$. As UI benefits b_u increase, the consumption smoothing gain declines since the gap between consumption during unemployment c_u and consumption during employment c_e shrinks. Meanwhile, the fiscal externality cost increases since larger b_u implies that added costs due to longer unemployment spells are larger. Each intersection point represents the optimal UI generosity given each b_e. Note that the optimal UI generosity is increasing in reemployment bonuses b_e. This is because of the following two factors; (i) the consumption smoothing gains from UI depend on reemployment bonuses b_e but only slightly (i.e. the solid lines are close to each other) and (ii) the moral hazard costs of UI highly depend on reemployment bonuses b_e (i.e. the dashed lines spread out). In the absence of reemployment bonuses, the optimal UI replacement rate is about 0.3. As reemployment bonuses b_e increase, the optimal UI becomes more generous. When $b_e = 0.5$, the optimal UI replacement rate is given by the intersection of the darkest lines and it exceeds 0.5. Hence, providing reemployment bonuses enables the social government to offer more generous UI.

5 Conclusion

This paper studies the role of reemployment bonuses in the design of a UI system. Specifically, I construct a dynamic model of consumption, saving, and job search, and derive a formula for the welfare impact of a marginal increase in reemployment bonuses that can be evaluated with several reduced form parameters. Taking parameter values from the literature, I find that providing reemployment bonuses for reemployment brings about a large welfare impact by reducing the cost of UI and thereby increasing the consumption of employed people. Specifically, the policy outperforms many historical policy changes targeting adults considered in Hendren and Sprung-Keyser (2020) in terms of cost-effectiveness. I also calibrate parameters in the model and show that providing reemployment bonuses makes the optimal UI more generous, which means that reemployment bonuses enable the government to provide more generous protection against consumption declines during unemployment as an optimal policy than currently provided.

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Online Appendix (Not for Publication)

Appendix A Proofs

A.1 Proof of $\frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \frac{\partial V_t^U(A_t)}{\partial b_e} > 0$

Proof. First, let $S_{K|k} = \prod_{j=k}^{K}(1-e_j)$ denote the probability of staying unemployed at the end of period K conditional on being unemployed at the beginning of period k. Note that

$$\begin{aligned}
\frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \frac{\partial V_t^U(A_t)}{\partial b_e} &= \frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \left[e_{t+1}\frac{\partial V_{t+1,t+1}^E(A_{t+1})}{\partial b_e} + (1-e_{t+1})\frac{\partial V_{t+1}^U(A_{t+1})}{\partial b_e}\right] \\
&= \frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \frac{\partial V_{t+1,t+1}^E(A_{t+1})}{\partial b_e} + (1-e_{t+1})\left[\frac{\partial V_{t+1,t+1}^E(A_{t+1})}{\partial b_e} - \frac{\partial V_{t+1}^U(A_{t+1})}{\partial b_e}\right].
\end{aligned}$$

Exploiting the envelope condition, I have

$$\frac{\partial V_{t,t}^E(A_t)}{\partial b_e} = \sum_{j=t}^{T_B} U'(c_{e,t,j}).$$

Therefore, the first two terms are

$$\frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \frac{\partial V_{t+1,t+1}^E(A_{t+1})}{\partial b_e} = U'(c_{e,t,t}) + \sum_{j=t+1}^{T_B}[U'(c_{e,t,j}) - U'(c_{e,t+1,j})].$$

Now I impose an approximation assumption that $c_{e,s,t} = c_e$ for $s,t \leq T_B$. Then the expression reduces down to

$$\frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \frac{\partial V_t^U(A_t)}{\partial b_e} = U'(c_e) + \sum_{k=t+1}^{T_B} S_{k|t+1}U'(c_e) > 0.$$

□

A.2 Proof of Proposotion 1

Proof. Remember that the welfare function $W(b)$ is defined as $W(b) = V_1(A_1; b, \tau(b))$ given A_1 where $\tau(b)$ is the budget-balancing taxes. Differentiating this with respect to wage subsidies b_e

yields

$$\frac{\partial W(b)}{\partial b_e} = \frac{\partial V_1}{\partial b_e} + \frac{\partial V_1}{\partial \tau}\frac{\partial \tau(b)}{\partial b_e}.$$

Exploiting the envelope conditions from individual's optimization problem, I have

$$\begin{aligned}
\frac{\partial V_1}{\partial b_e} &= e_1 \frac{\partial V_1^E}{\partial b_e} + (1-e_1)\frac{\partial V_1^U}{\partial b_e} \\
&= e_1 \sum_{t=1}^{T_B} U'(c_{e,1,t}) + S_1 e_2 \sum_{t=2}^{T_B} U'(c_{e,2,t}) + \cdots + S_{T_B-1} e_{T_B} U'(c_{e,T_B,T_B}) \\
&= e_1 U'(c_{e,1,1}) + \sum_{t=1}^{2} S_{t-1} e_t U'(c_{e,t,T_B}) + \cdots + \sum_{t=1}^{T_B} S_{t-1} e_t U'(c_{e,t,T_B}),
\end{aligned}$$

where $S_0 = 1$. Note that $\sum_{t=1}^{j} S_{t-1} e_t = 1 - S_j$. I obtain

$$\frac{\partial V_1}{\partial b_e} = \sum_{t=1}^{T_B} (1-S_t)\mathbb{E}_s[U'(c_{e,s,t})],$$

where

$$\mathbb{E}_s[U'(c_{e,s,t})] = \sum_{s=1}^{t} \kappa_s U'(c_{e,s,t}) \quad \text{where} \quad \kappa_s = \frac{S_{s-1} e_s}{1-S_t}. \tag{22}$$

Similarly, I obtain

$$\frac{\partial V_1}{\partial \tau} = -\sum_{t=1}^{T} (1-S_t)\mathbb{E}_s[U'(c_{e,s,t})].$$

The government budget constraint is given by

$$(T-D)\tau(b) = D_B b_u + (T_B - D_B) b_e.$$

Differentiating this with respect to b_e yields

$$(T-D)\frac{\partial \tau(b)}{\partial b_e} - \frac{dD}{db_e}\tau(b) = T_B - D_B + \frac{dD_B}{db_e}(b_u - b_e).$$

Arranging terms, I obtain

$$\frac{\partial \tau(b)}{\partial b_e} = \frac{T_B - D_B}{T-D}\left[1 + \frac{dD_B}{db_e}\frac{b_u - b_e}{T_B - D_B} + \frac{dD}{db_e}\frac{\tau(b)}{T_B - D_B}\right].$$

To get a money metric of the welfare impact of wage subsidies, I normalize $\frac{\partial W(b)}{\partial b_e}$ by the welfare gain from permanently increasing wages by \$1, i.e., $\frac{\partial V_1}{\partial w}$. Note that $\frac{\partial V_1}{\partial w} = -\frac{\partial V_1}{\partial \tau}$. Then, I have

$$\begin{aligned}\frac{\partial W(b)}{\partial b_e} \Big/ \frac{\partial V_1}{\partial w} &= \frac{\frac{\partial V_1}{\partial b_e}}{-\frac{\partial V_1}{\partial \tau}} - \frac{\partial \tau(b)}{\partial b_e} \\ &= \frac{T_B - D_B}{T - D}\left[\frac{MU_{e,T_B} - MU_{e,T}}{MU_{e,T}} - \frac{dD_B}{db_e}\frac{b_u - b_e}{T_B - D_B} - \frac{dD}{db_e}\frac{\tau(b)}{T_B - D_B}\right],\end{aligned}$$

where $MU_{e,t}$ is given by

$$MU_{e,t} = \sum_{k=1}^{t} \frac{1 - S_k}{t - D_B}\mathbb{E}_s[U'(c_{e,s,k})] \quad \text{for} \quad t = T_B, T.$$

$\square$

A.3 Proof of Proposition 2

Proof. Let $t \leq T_B$. Remember that

$$\frac{\partial e_t}{\partial b_u} = \frac{-\frac{\partial V_t^U(A_t)}{\partial b_u}}{\psi''(e_t)}, \qquad \frac{\partial e_t}{\partial b_e} = \frac{\frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \frac{\partial V_t^U(A_t)}{\partial b_e}}{\psi''(e_t)}.$$

Exploiting the envelope condition, I obtain

$$\begin{aligned}\frac{\partial V_t^U(A_t)}{\partial b_u} &= U'(c_{u,t}) + \sum_{k=t+1}^{T_B} S_{k|t+1}U'(c_{u,k}), \\ \frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \frac{\partial V_t^U(A_t)}{\partial b_e} &= U'(c_e) + \sum_{k=t+1}^{T_B} S_{k|t+1}U'(c_e),\end{aligned}$$

where the second equation is derived in Appendix A.1 under assumption (A.1). For expositional brevity, define $\mu_{e,t} = U'(c_e) + \sum_{k=t+1}^{T_B} S_{k|t+1}U'(c_e)$ and $\mu_{u,t} = U'(c_{u,t}) + \sum_{k=t+1}^{T_B} S_{k|t+1}U'(c_{u,k})$. Then I obtain

$$\frac{\partial e_t}{\partial b_e} = -\frac{\mu_{e,t}}{\mu_{u,t}}\frac{\partial e_t}{\partial b_u}.$$

This can be rewritten as

$$\frac{\partial(1 - e_t)}{\partial b_e} = -\frac{\mu_{e,t}}{\mu_{u,t}}\frac{\partial(1 - e_t)}{\partial b_u}. \tag{23}$$

First, I compute $\frac{\partial S_t}{\partial b_e}$. Remember that $S_t = \prod_{j=1}^{t}(1-e_j)$. Using the equation above, I have

$$
\begin{aligned}
\frac{\partial S_t}{\partial b_e} &= \frac{\partial}{\partial b_e}\prod_{j=1}^{t}(1-e_j) \\
&= \sum_{k=1}^{t}\frac{\partial(1-e_k)}{\partial b_e}\prod_{j\neq k}(1-e_j) \\
&= -\sum_{k=1}^{t}\frac{\mu_{e,k}}{\mu_{u,k}}\frac{\partial(1-e_k)}{\partial b_u}\prod_{\tilde{t}\neq k}(1-e_j)
\end{aligned}
$$

where the last equality comes from equation (23). Compensated unemployment duration D_B is defined as $D_B = \sum_{t=1}^{T_B} S_t$. Therefore, I have

$$
\begin{aligned}
\frac{\partial D_B}{\partial b_e} &= \sum_{t=1}^{T_B}\frac{\partial S_t}{\partial b_e} \\
&= -\sum_{t=1}^{T_B}\left[\sum_{k=1}^{t}\frac{\mu_{e,k}}{\mu_{u,k}}\frac{\partial(1-e_k)}{\partial b_u}\prod_{\tilde{t}\neq k}(1-e_j)\right].
\end{aligned}
$$

Now, I compute a bound on $\frac{\mu_{e,t}}{\mu_{u,t}}$. Using assumption (A.3), i.e., $c_{u,t} \leq c_{u,t-1}$, I obtain

$$
\begin{aligned}
\frac{\mu_{e,t}}{\mu_{u,t}} &= \frac{U'(c_e)+\sum_{k=t+1}^{T_B} S_{k|t+1}U'(c_e)}{U'(c_{u,t})+\sum_{k=t+1}^{T_B} S_{k|t+1}U'(c_{u,k})} \\
&\geq \frac{[1+\sum_{k=t+1}^{T_B} S_{k|t+1}]U'(c_e)}{[1+\sum_{k=t+1}^{T_B} S_{k|t+1}]U'(c_{u,T_B})} \\
&= \frac{U'(c_e)}{U'(c_{u,T_B})}.
\end{aligned}
$$

Using the assumptions (A.2) $c_{u,t} \leq c_e$ and (A.3) $c_{u,t} \geq c_{u,t+1}$, I also have

$$
\begin{aligned}
\frac{\mu_{e,t}}{\mu_{u,t}} &= \frac{U'(c_e)+\sum_{k=t+1}^{T_B} S_{k|t+1}U'(c_e)}{U'(c_{u,t})+\sum_{k=t+1}^{T_B} S_{k|t+1}U'(c_{u,k})} \\
&\leq \frac{U'(c_e)}{U'(c_{u,t})} \\
&\leq \frac{U'(c_e)}{U'(c_{u,1})}.
\end{aligned}
$$

Define

$$\overline{M}_{T_B} = \frac{U'(c_e)}{U'(c_{u,1})} \quad \underline{M}_{T_B} = \frac{U'(c_e)}{U'(c_{u,T_B})}.$$

Then, I obtain

$$\begin{aligned}
-\frac{\partial D_B}{\partial b_e} &= \sum_{t=1}^{T_B}\left[\sum_{k=1}^{t}\frac{\mu_{e,k}}{\mu_{u,k}}\frac{\partial(1-e_k)}{\partial b_u}\prod_{\tilde{t}\neq k}(1-e_j)\right] \\
&\leq \sum_{t=1}^{T_B}\left[\sum_{k=1}^{t}\overline{M}_{T_B}\frac{\partial(1-e_k)}{\partial b_u}\prod_{\tilde{t}\neq k}(1-e_j)\right] \\
&= \overline{M}_{T_B}\sum_{t=1}^{T_B}\left[\sum_{k=1}^{t}\frac{\partial(1-e_k)}{\partial b_u}\prod_{\tilde{t}\neq k}(1-e_j)\right] \\
&= \overline{M}_{T_B}\sum_{t=1}^{T_B}\frac{\partial S_t}{\partial b_u} \\
&= \overline{M}_{T_B}\frac{\partial D_B}{\partial b_u}.
\end{aligned}$$

In the same way, I can show that

$$-\frac{\partial D_B}{\partial b_e} \geq \underline{M}_{T_B}\frac{\partial D_B}{\partial b_u}.$$

Combining these, I obtain

$$\underline{M}_{T_B}\frac{\partial D_B}{\partial b_u} \leq -\frac{\partial D_B}{\partial b_e} \leq \overline{M}_{T_B}\frac{\partial D_B}{\partial b_u}.$$

□

A.4 Proof of Corollary 1

Proof. From Appendix A.2, remember that

$$\frac{\partial W(b)}{\partial b_e}\Big/\frac{\partial V_1}{\partial w} = \frac{T_B - D_B}{T - D}\left[\frac{MU_{e,T_B} - MU_{e,T}}{MU_{e,T}} - \frac{dD_B}{db_e}\frac{b_u - b_e}{T_B - D_B} - \frac{dD}{db_e}\frac{\tau(b)}{T_B - D_B}\right].$$

Evaluating this at $b_e = 0$ yields

$$\frac{\partial W(b)}{\partial b_e}\Big/\frac{\partial V_1}{\partial w} = \frac{T_B - D_B}{T - D}\left[\frac{MU_{e,T_B} - MU_{e,T}}{MU_{e,T}} - \frac{dD_B}{db_e}\frac{b_u}{T_B - D_B} - \frac{dD}{db_e}\frac{D_B b_u}{(T_B - D_B)^2}\right].$$

Assumption (A.1) implies $MU_{e,T_B} = MU_{e,T}$, so the first term in the bracket disappears. Using

Proposition 2, which requires assumptions (A.1)-(A.3), I obtain

$$\begin{aligned}
\frac{\partial W(b)}{\partial b_e} \bigg/ \frac{\partial V_1}{\partial w} &= -\frac{T_B - D_B}{T - D}\left[\frac{dD_B}{db_e}\frac{b_u}{T_B - D_B} + \frac{dD}{db_e}\frac{D_B b_u}{(T_B - D_B)^2}\right] \\
&\geq \frac{T_B - D_B}{T - D}\left[\frac{dD_B}{db_u}\frac{b_u}{D_B}\frac{D_B}{T_B - D_B}\underline{M}_{T_B} + \frac{dD}{db_u}\frac{b_u}{D}\frac{D_B D}{(T_B - D_B)^2}\underline{M}_T\right] \\
&= \frac{T_B - D_B}{T - D}\left[\eta_{D_B,b_u}\frac{D_B}{T_B - D_B}\underline{M}_{T_B} + \eta_{D,b_u}\frac{D_B D}{(T_B - D_B)^2}\underline{M}_T\right].
\end{aligned}$$

Since $\underline{M}_T = U'(c_e)/U'(c_{u,T}) \geq 0$ and $\eta_{D,b_u} \geq 0$, arranging terms yields

$$\frac{\partial W(b)}{\partial b_e} \bigg/ \frac{\partial V_1}{\partial w} \geq \frac{u}{1-u}\frac{D_B}{D}\underline{M}_{T_B}\eta_{D_B,b_u},$$

where $u = \frac{D}{T}$ is the fraction of periods during which a person is expected to be unemployed. □

A.5 Derivation of MVPF of the Marginal Increase in b_u

The willingness to pay for a marginal increase in UI benefits b_u in terms of dollars during employment is given by

$$\begin{aligned}
WTP &= \frac{\partial V_1}{\partial b_u} \bigg/ \frac{\partial V_1}{\partial w} \\
&= D_B \frac{MU_{u,T_B} - MU_{e,T}}{MU_{e,T}},
\end{aligned}$$

where MU_{u,T_B} and $MU_{e,T}$ are the marginal utilities of consumption averaged over time:

$$MU_{u,T_B} = \sum_{t=1}^{T_B} \frac{S_t}{D_B} U'(c_{u,t}), \quad MU_{e,T} = \sum_{t=1}^{T} \frac{1 - S_t}{T - D_B} \mathbb{E}_s[U'(c_{e,s,t})].$$

Since it is difficult to track individual-level consumption at a week/month frequency over time, many papers just assume that $c_{u,t}$ and $c_{e,s,t}$ do not depend on t or s (Chetty 2009, Hendren and Sprung-Keyser 2020). Rigorously computing the willingness to pay for UI is beyond the scope of paper, and I simply let $c_{u,t} = c_u$ and $c_{e,s,t} = c_e$. Assuming CRRA utility $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$, I obtain

$$WTP = D_B \frac{c_u^{-\gamma} - c_e^{-\gamma}}{c_e^{-\gamma}} \approx D_B\left(1 + \gamma\frac{c_e - c_u}{c_e}\right),$$

where I take the first-order approximation with respect to c_e around c_u.

The net cost of a marginal increase in UI benefits b_u is given by

$$\begin{aligned} Net\ Cost &= (T-D)\frac{\partial \tau(b)}{\partial b_u} \\ &= D_B(1+FE_u), \end{aligned}$$

where $FE_u = \frac{1}{D_B}\left[\frac{dD_B}{db_u}(b_u - b_e) + \frac{dD}{db_e}\tau\right]$ represents the fiscal externality due to the behavioral response of unemployment people. Evaluating FE_u at the point $b_e = 0$ yields

$$\begin{aligned} FE_u &= \frac{1}{D_B}\left[\frac{dD_B}{db_u}b_u + \frac{dD}{db_u}\tau\right] \\ &= \eta_{D_B,b_u} + \eta_{D,b_u}\frac{u}{1-u}. \end{aligned}$$

Following Chetty (2008), I assume that $\eta_{D_B,b_u} = \eta_{D,b_u}$. Then, I obtain $FE_u = \frac{1}{1-u}\eta_{D_B,b_u}$. Combining these, I obtain

$$MVPF \approx \frac{1+\gamma\frac{c_e - c_u}{c_e}}{1+\frac{1}{1-u}\eta_{D_B,b_u}}.$$

A.6 Marginal Gains and Costs of b_e and b_u

In Section 4, I study non-marginal policy changes but it is useful to characterize the welfare impact of the marginal changes in the policy instrument to better understand mechanisms behind simulation exercises. Specifically, I characterizes the welfare impacts $\frac{\partial W(b)}{\partial b_e}$ and $\frac{\partial W(b)}{\partial b_u}$ as in Section 3 but this time I impose neither $b_e = 0$ nor $b_u = 0.5w$. As for $\frac{\partial W}{\partial b_e}$, the same argument as in the proof of Proposition 1 implies that

$$\frac{\partial W(b)}{\partial b_e} \Big/ \frac{\partial V_1}{\partial w} = \frac{T_B - D_B}{T-D}\left[\frac{MU_{e,T_B} - MU_{e,T}}{MU_{e,T}} + \frac{dD_B}{db_e}\frac{1}{T_B - D_B}(b_u - b_e) + \frac{dD}{db_e}\frac{\tau(b)}{T_B - D_B}\right], \tag{24}$$

I define

$$\begin{aligned} CS_e &= \frac{MU_{e,T_B} - MU_{e,T}}{MU_{e,T}} \\ FE_e &= \frac{1}{T_B - D_B}\left[\frac{dD_B}{db_e}(b_u - b_e) + \frac{dD}{db_e}\tau(b)\right]. \end{aligned}$$

equation (24) implies that the optimal b_e should satisfy the first-order condition $CS_e = FE_e$.

In the case of the welfare impact of UI benefits $\frac{\partial W}{\partial b_u}$, first compute the change in the value at the beginning of the period with respect to b_u. Exploiting the envelope conditions, I obtain

$$\frac{\partial W(b)}{\partial b_u} = \frac{\partial V_1}{\partial b_u} + \frac{\partial V_1}{\partial \tau}\frac{\partial \tau(b)}{\partial b_u},$$

where

$$\frac{\partial V_1}{\partial b_u} = \sum_{t=1}^{T_B} S_t U'(c_{u,t}) \quad \text{and} \quad \frac{\partial V_1}{\partial \tau} = \sum_{t=1}^{T}(1-S_t)\mathbb{E}_s[U'(c_{e,s,t})].$$

Differentiating the government budget constraint with respect to b_u yields

$$\frac{\partial \tau(b)}{\partial b_u} = \frac{D_B}{T-D}\left[1 + \frac{dD_B}{db_u}\frac{1}{D_B}(b_u - b_e) + \frac{dD}{db_u}\frac{\tau(b)}{D_B}\right].$$

I obtain

$$\frac{\partial W(b)}{\partial b_u} \Big/ \frac{\partial V_1}{\partial w} = \frac{D_B}{T-D}\left[\frac{MU_{u,T_B} - MU_{e,T}}{MU_{e,T}} - \frac{dD_B}{db_u}\frac{b_u - b_e}{D_B} - \frac{dD}{db_u}\frac{\tau(b)}{D_B}\right] \tag{25}$$

where

$$MU_{u,T_B} = \sum_{t=1}^{T_B}\frac{S_t}{D_B}U'(c_{u,t})$$

I define

$$CS_u = \frac{MU_{u,T_B} - MU_{e,T}}{MU_{e,T}}$$
$$FE_u = \frac{1}{D_B}\left[\frac{dD_B}{db_u}(b_u - b_e) + \frac{dD}{db_u}\tau(b)\right].$$

equation (25) implies that the optimal b_e should satisfy the first-order condition $CS_e = FE_e$.

Appendix B Extensions

B.1 Heterogeneity

The model in the main text assumes homogeneous individuals. In this subsection, I assume that individuals are heterogeneous in many respects including utility functions, income processes, initial assets, and liquidity constraints. I show that the welfare formula is almost unchanged in the presence of heterogeneity although additional conditions are required.

I begin by rewriting individual problems that account for individual heterogeneity. I assume that individuals are heterogeneous and each individual is indexed by $\theta \in \Theta$. If individual θ is employed at a period t with asset A_t, the value function is given by

$$V^E_{s,t}(A_t,\theta) = \max_{A_{t+1}\geq L(\theta)} U(A_t - A_{t+1} + y_{e,s,t}(\theta);\theta) + V^E_{s,t+1}(A_{t+1},\theta),$$

where the utility function U directly depends on θ. The liquidity constraint L and income y also depend on θ. More precisely, income is given by

$$y_{e,s,t}(\theta) = \begin{cases} w_{e,s,t}(\theta) + b_e & \text{if } t \leq T_B, \\ w_{e,s,t}(\theta) & \text{if } t > T_B, \end{cases}$$

where $w_{e,s,t}(\theta)$ is non-stochastic earnings for individual θ.

If individual θ with asset A_t does not find a job at the beginning of a period t, the value function is given by

$$V^U_t(A_t,\theta) = \max_{A_{t+1}\geq L(\theta)} U(A_t - A_{t+1} + y_{u,t}(\theta);\theta) + V_{t+1}(A_{t+1},\theta),$$

where income is given by

$$y_{u,t}(\theta) = \begin{cases} y_{u,t}(\theta) + b_e & \text{if } t \leq T_B, \\ y_{u,t}(\theta) & \text{if } t > T_B, \end{cases}$$

where $y_{u,t}(\theta)$ is non-stochastic non-labor income for individual θ. The value function of unemployed individual θ with asset A_t at the beginning of each period before searching for a job is given by

$$V_t(A_t,\theta) = \max_{e_t\in[0,1]} e_t V^E_{t,t}(A_t,\theta) + (1-e_t)V^U_t(A_t,\theta) - \psi(e_t;\theta),$$

where the disutility of job search ψ directly depends on θ. Let $e_t(\theta)$ be the search effort of individual θ at time t. I define a survival probability, expected unemployed duration, and expected insured duration for each individual as follows:

$$S_t(\theta) = \prod_{s=1}^{t}(1-e_s(\theta)),$$

$$D(\theta) = \sum_{t=1}^{T} S_t(\theta),$$

$$D_B(\theta) = \sum_{t=1}^{T_B} S_t(\theta).$$

The government budget constraint is given by

$$(T-\overline{D})\tau = \overline{D}_B b_u + (T_B - \overline{D}_B) b_e,$$

where

$$\overline{D} = \int_{\theta\in\Theta} D(\theta)dF(\theta),$$
$$\overline{D}_B = \int_{\theta\in\Theta} D_B(\theta)dF(\theta).$$

Let $\tau(b)$ be the tax as a function of UI benefits and wage subsidies b implied by the budget constraint above. I define the welfare function $W(b)$ as

$$W(b) = \int_{\theta\in\Theta} V_1(A_1(\theta),\theta;b,\tau(b))dF(\theta),$$

where F is the distribution of θ in the population and $A_1(\theta)$ is the initial asset exogenously given to individual θ. Here I make the value function explicitly depend on the government policy (b,τ). My objective here is to characterize $\frac{\partial W(b)}{\partial b_e}$.

Differentiating the welfare function with respect to b_e yields

$$\frac{\partial W(b)}{\partial b_e} = \int_{\theta\in\Theta}\left[\frac{\partial V_1(A_1(\theta),\theta;b,\tau)}{\partial b_e} + \frac{\partial V_1(A_1(\theta),\theta;b,\tau)}{\partial \tau}\frac{\partial \tau(b)}{\partial b_e}\right]dF(\theta).$$

The same calculation as in the proof of Proposition 1 implies

$$\frac{\partial V_1(A_1(\theta),\theta;b,\tau)}{\partial b_e} = \sum_{t=1}^{T_B}(1-S_t(\theta))\mathbb{E}_{s,\theta}[U'(c_{e,s,t}(\theta);\theta)],$$
$$\frac{\partial V_1(A_1(\theta),\theta;b,\tau)}{\partial \tau} = -\sum_{t=1}^{T}(1-S_t(\theta))\mathbb{E}_{s,\theta}[U'(c_{e,s,t}(\theta);\theta)],$$

where

$$\mathbb{E}_{s,\theta}[U'(c_{e,s,t}(\theta);\theta)] = \sum_{s=1}^{t}\frac{S_{s-1}(\theta)e_s(\theta)}{1-S_t(\theta)}U'(c_{e,s,t}(\theta);\theta).$$

The budgetary impact of wage subsidies is given by

$$\frac{\partial \tau(b)}{\partial b_e} = \frac{T_B-\overline{D}_B}{T-\overline{D}}\left[1+\frac{d\overline{D}_B}{db_e}\frac{b_u-b_e}{T_B-\overline{D}_B}+\frac{d\overline{D}}{db_e}\frac{\tau(b)}{T_B-\overline{D}_B}\right].$$

To get a money metric of the welfare impact of wage subsidies, I normalize $\frac{\partial W(b)}{\partial b_e}$ by the average welfare gain from permanently increasing wages by \$1, i.e., $\int \frac{\partial V_1}{\partial w} dF(\theta)$. Note that $\frac{\partial V_1}{\partial w} = -\frac{\partial V_1}{\partial \tau}$. Then, I have

$$\frac{\frac{\partial W(b)}{\partial b_e}}{\int_{\theta\in\Theta} \frac{\partial V_1}{\partial w} dF(\theta)} = \frac{\int_{\theta\in\Theta} \frac{\partial V_1(A_1(\theta),\theta;b,\tau)}{\partial b_e} dF(\theta)}{-\int_{\theta\in\Theta} \frac{\partial V_1(A_1(\theta),\theta;b,\tau)}{\partial \tau} dF(\theta)} - \frac{\partial \tau(b)}{\partial b_e}$$
$$= \frac{T_B - \overline{D}_B}{T - \overline{D}} \left[\frac{\overline{MU}_{e,T_B} - \overline{MU}_{e,T}}{\overline{MU}_{e,T}} - \frac{d\overline{D}_B}{db_e} \frac{b_u - b_e}{T_B - \overline{D}_B} - \frac{d\overline{D}}{db_e} \frac{\tau(b)}{T_B - \overline{D}_B} \right],$$

where $\overline{MU}_{e,t}$ is given by

$$\overline{MU}_{e,t} = \int_{\theta\in\Theta} \sum_{k=1}^{t} \frac{1 - S_k(\theta)}{t - \overline{D}_B} \mathbb{E}_{s,\theta}[U'(c_{e,s,k}(\theta);\theta)] dF(\theta) \quad \text{for} \quad t = T_B, T.$$

How is this formula in the presence of heterogeneity different from the formula (7) that is derived under the assumption that individuals are homogeneous? One difference is that the average marginal utility $MU_{e,t}$ in equation (7) is now replaced by $\overline{MU}_{e,t}$ which is $MU_{e,t}$ averaged over individuals. Another difference is that the expected unemployment duration D and the expected insured duration D_B are also replaced by the ones averaged over individuals $\overline{D}$ and $\overline{D}_B$.

Now I evaluate the formula at $b_e = 0$, and in addition, I impose the assumption (A.1). Then, I obtain

$$\frac{\frac{\partial W(b)}{\partial b_e}}{\int_{\theta\in\Theta} \frac{\partial V_1}{\partial w} dF(\theta)} = -\frac{T_B - \overline{D}_B}{T - \overline{D}} \left[\frac{d\overline{D}_B}{db_e} \frac{b_u}{T_B - \overline{D}_B} + \frac{d\overline{D}}{db_e} \frac{\overline{D}_B b_u}{(T_B - \overline{D}_B)^2} \right].$$

Proposition 2 cannot be directly applied to $\overline{D}_B$. Instead, I have

$$\frac{\partial \overline{D}_B}{\partial b_e} = \int_{\theta\in\Theta} \frac{\partial D_B(\theta)}{\partial b_e} dF(\theta),$$

which implies

$$\int_{\theta\in\Theta} \underline{M}_{T_B}(\theta) \frac{\partial D_B(\theta)}{\partial b_u} dF(\theta) \leq -\frac{\partial \overline{D}_B}{\partial b_e} \leq \int_{\theta\in\Theta} \overline{M}_{T_B}(\theta) \frac{\partial D_B(\theta)}{\partial b_u} dF(\theta),$$

where

$$\overline{M}_{T_B}(\theta) = \frac{U'(c_e(\theta);\theta)}{U'(c_{u,1}(\theta);\theta)}, \quad \underline{M}_{T_B}(\theta) = \frac{U'(c_e(\theta);\theta)}{U'(c_{u,T_B}(\theta);\theta)}.$$

If the heterogeneity in the population is such that $\underline{M}_{T_B}(\theta)$ and $\overline{M}_{T_B}(\theta)$ are independent of

$\frac{\partial D_B(\theta)}{\partial b_u}$, then the inequality simplifies to

$$\int_{\theta\in\Theta} \underline{M}_{T_B}(\theta)dF(\theta)\frac{\partial \overline{D}_B(\theta)}{\partial b_u} \leq -\frac{\partial \overline{D}_B}{\partial b_e} \leq \int_{\theta\in\Theta} \overline{M}_{T_B}(\theta)dF(\theta)\frac{\partial \overline{D}_B(\theta)}{\partial b_u}.$$

As in Corollary 1, let $\underline{M}_T = 0$. Then, arranging terms yields

$$\frac{\frac{\partial W(b)}{\partial b_e}}{\int_{\theta\in\Theta}\frac{\partial V_1}{\partial w}dF(\theta)} \geq \frac{u}{1-u}\frac{\overline{D}_B}{\overline{D}}\eta_{\overline{D}_B,b_u}\int_{\theta\in\Theta}\underline{M}_{T_B}dF(\theta),$$

where $u = \frac{\overline{D}}{T}$ is the fraction of periods during which a person is expected to be unemployed, and $\eta_{\overline{D}_B,b_u}$ is the elasticity of insured duration with respect to UI benefits. For the parameters $(u, \overline{D}_B, \overline{D}, \eta_{\overline{D}_B,b_u})$, the same parameter values can be used as in Table 1. The remaining thing to be determined is $\int_{\theta\in\Theta}\underline{M}_{T_B}dF(\theta)$. If the utility for consumption is homogeneous (i.e., $U(c;\theta) = U(c)$) and is given by CRRA function, then

$$\int_{\theta\in\Theta}\underline{M}_{T_B}dF(\theta) = \int_{\theta\in\Theta}\left(\frac{c_{u,T_B}(\theta)}{c_e(\theta)}\right)^{\gamma}dF(\theta).$$

If CRRA parameter is $\gamma \geq 1$, which is a standard assumption, then Jensen's inequality implies

$$\int_{\theta\in\Theta}\underline{M}_{T_B}dF(\theta) \geq \left[\int_{\theta\in\Theta}\frac{c_{u,T_B}(\theta)}{c_e(\theta)}dF(\theta)\right]^{\gamma},$$

where the expression inside the bracket is the average consumption due to unemployment. Together with assumption (A.4), I obtain

$$\begin{aligned}\frac{\frac{\partial W(b)}{\partial b_e}}{\int_{\theta\in\Theta}\frac{\partial V_1}{\partial w}dF(\theta)} &\geq \frac{u}{1-u}\frac{\overline{D}_B}{\overline{D}}\eta_{\overline{D}_B,b_u}\left[\int_{\theta\in\Theta}\frac{c_{u,T_B}(\theta)}{c_0(\theta)}dF(\theta)\right]^{\gamma} \\ &\approx \frac{u}{1-u}\frac{\overline{D}_B}{\overline{D}}\eta_{\overline{D}_B,b_u}\left[\frac{\int_\theta c_{u,T_B}(\theta)dF(\theta)}{\int_\theta c_0(\theta)dF(\theta)}\right]^{\gamma},\end{aligned}$$

where the second line is based on the first-order approximation around the averages of $c_{u,T_B}(\theta)$ and $c_e(\theta)$ in the population. As in Section 3, the last term can be computed based on high-frequency average consumption profile reported in, for example, Ganong and Noel (2019). The resulting welfare impact is the same as in Section 3.

B.2 Stochastic Wage Offers

The discussion here mostly follows Chetty (2008) except that I differentiate the welfare function with respect to wage subsidies instead of UI benefits. As before, individuals exert search effort e_t but this time I assume that conditional on getting a job offer, the wage offer is stochastic and follows a distribution F_w. If individuals are employed at a period t with asset A_t and wage w, their value function is given by

$$V^E_{s,t}(A_t, w) = \max_{A_{t+1} \geq L} U(A_t - A_{t+1} + y_{e,s,t}) + V^E_{s,t+1}(A_{t+1}, w),$$

where income is given by

$$y_{e,s,t} = \begin{cases} w + b_e & \text{if } t \leq T_B, \\ w & \text{if } t > T_B. \end{cases}$$

If individuals with asset A_t does not find a job at the beginning of a period t, their value function is given by

$$V^U_t(A_t) = \max_{A_{t+1} \geq L} U(A_t - A_{t+1} + y_{u,t}) + V_{t+1}(A_{t+1}),$$

where $y_{u,t}$ is the same as in the main text.

Now consider the search decision of individuals. Individuals follow a reservation wage policy. Letting R_t be the reservation wage at t, the value function of unemployed people with asset A_t at the beginning of each period before searching for a job is given by

$$V_t(A_t) = \max_{e_t \in [0,1], R_t} e_t \Pr(w \geq R_t) \mathbb{E}[V^E_t(A_t, w) | w \geq R_t] + (1 - e_t \Pr(w \geq R_t)) V^U_t(A_t; b, \tau) - \psi(e_t).$$

This optimization problem is different from the one in the main text in that individuals now can choose the reservation wage R_t in addition to the search effort e_t.

As in the main text, I define the welfare function as $W(b) = V_1(A_1; b, \tau(b))$ given A_1. Differentiating this with respect to wage subsidies b_e yields

$$\frac{\partial W(b)}{\partial b_e} = \frac{\partial V_1}{\partial b_e} + \frac{\partial V_1}{\partial \tau} \frac{\partial \tau(b)}{\partial b_e}.$$

Exploiting the envelope conditions from individual's optimization problem, I have

$$\frac{\partial V_1}{\partial b_e} = e_1 \Pr(w \geq R_1) \frac{\partial \mathbb{E} V^E_1}{\partial b_e} + (1 - e_1 \Pr(w \geq R_1)) \frac{\partial V^U_1}{\partial b_e}.$$

By defining a survival probability as $S_t = \prod_{j=1}^{t}[1 - e_j \Pr(w \geq R_j)]$, the same calculation as in Appendix A.2 yields

$$\frac{\partial V_1}{\partial b_e} = \sum_{t=1}^{T_B}(1 - S_t)\mathbb{E}_s[U'(c_{e,s,t})], \quad \frac{\partial V_1}{\partial \tau} = -\sum_{t=1}^{T}(1 - S_t)\mathbb{E}_s[U'(c_{e,s,t})].$$

This is exactly the same as the case where wage offers are not stochastic except that the survival probability is now redefined so that it reflects the probability that an offered wage is above the reservation wage. Introducing stochastic wage offers into the model does not change the welfare formula because the additional variable R_t shows up in the formula only through the survival probability S_t. Since the government budget constraint is not affected by the introduction of stochastic wage offers, the formula for the welfare impact of the marginal change in b_e in the presence of stochastic wage offers is the same as the one in the main text.

Appendix C Sensitivity Analysis

C.1 Sensitivity of MVPF to η_{D_B,b_u}

In the main text, I compute the welfare impact using the reduced form parameters mostly taken from Chetty (2008). One key parameter is the elasticity of expected insured duration with respect to UI benefits η_{D_B,b_u}. There are many papers that estimate the elasticity, but since each of them uses a different identification strategy and a different sample to estimate the parameter, estimated values for η_{D_B,b_u} are different across papers. To understand how the welfare impact computed in the main text is sensitive to the elasticity parameter, I compute the MVPF of wage subsidies based on estimation results in several other papers.

Table 6 summarizes the estimated elasticities for a set of studies using U.S. data surveyed in Schmieder and Von Wachter (2016) and the MVPFs of wage subsidies computed using those elasticities. In the case such as Solon (1985) where the estimated elasticities are small, the MVPF takes 1.13-1.16 depending on the risk aversion parameter γ. This is much smaller than the numbers shown in the main text, but it is still above 1, implying that the policy provides more than $1 of benefits per dollar of government spending. In some cases where the estimated elasticities are large, the denominator of the MVPF is negative and I define the MVPF in this case as infinity following Hendren and Sprung-Keyser (2020). This means that the labor supply response to wage subsidies are so large that the large reduction in the spending on UI benefits enables the policy to pay for itself.

Table 6: Sensitivity of MVPF to η_{D_B,b_u}

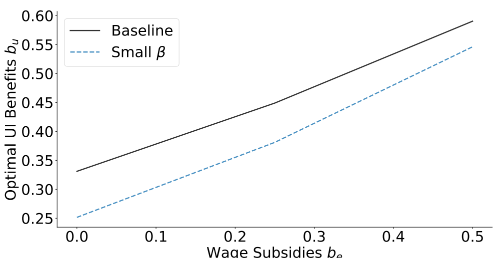
Study	η_{D_B,b_u}	MVPF $\gamma=1$	MVPF $\gamma=2$	MVPF $\gamma=3$
Moffitt (1985)	0.36	2.01	1.82	1.68
Solon (1985)	0.10	1.16	1.14	1.13
Katz and Meyer (1990)	0.80	∞	∞	10.35
Meyer and Mok (2007)	0.12-0.60	1.2-6.12	1.18-4.95	1.16-3.10
Landais (2015)	0.29	1.68	1.57	1.49
Kroft and Notowidigdo (2016)	0.63	8.22	4.77	3.47
Card, Johnston, Leung, Mas and Pei (2015)	0.38-1.21	2.13-∞	1.91-∞	1.75-∞

Note: This table shows the estimated elasticity of insured duration with respect to UI benefits for a set of studies surveyed in Schmieder and Von Wachter (2016) and shows the MVPFs computed based on those elasticities. γ is the risk aversion parameter in the utility function $u(c)=\frac{c^{1-\gamma}}{1-\gamma}$. The MVPFs are calculated using equation (15). In some cases where η_{D_B,b_u} is large, the denominator of the MVPF is negative. In that case, I define the MVPF to be infinity following Hendren and Sprung-Keyser (2020).

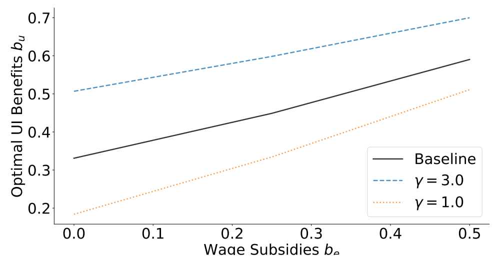
C.2 Sensitivity of Simulation Results to Externally Chosen Parameters

In Section 4, I choose the parameters in the utility cost of job search $\psi(e)=\theta\frac{e^{1+\xi}}{1+\xi}$ by matching moments in simulated data and actual data given other parameters that are externally chosen. I examine how my simulation results change when I alter some parameters that are externally chosen. Specifically, I examine the sensitivity of the main finding in the simulation section that the complementarity of UI benefits and wage subsidies. Figure 4 shows the optimal UI benefits b_u as a function of wage subsidies b_e under a different set of parameters. I check whether the optimal UI benefit level is increasing function of wage subsidies b_e. In panel (a), the baseline case is represented by the black solid line in which the discount factor is $\beta=1$ while the blue dashed line is the case where $\beta=\left(\frac{1}{1+0.0105}\right)^{-1/52}<1$ which is taken from Lawson (2017). Although smaller β makes the optimal UI benefits less generous, it is still an increasing function of wage subsidies. The same patterns can be found in panel (b), in which I change the risk aversion parameter γ, and panel (c), in which I change the initial asset level. Although the optimal level of UI benefits is affected by the choice of those parameters, the qualitative pattern is robust to them.

(a) Different β



(b) Different γ



(c) Different A_1

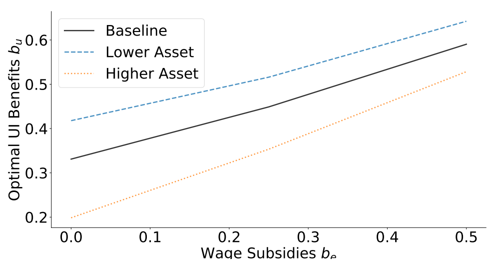


Figure 4: Sensitivity to Externally Chosen Parameters

Note: These graphs plot optimal UI benefits b_u as a function of wage subsidies b_e in which each line corresponds to a different set of parameters. In panel (a), the solid line represents the baseline case where $\beta = 1$ while the dashed line is the case where $\beta = \left(\frac{1}{1+0.0105}\right)^{-1/52}$ which is taken from Lawson (2017). In panel (b), the solid line represents the baseline case where $\gamma = 2.0$ while the dashed line and the dotted line are the case where $\gamma = 3.0$ and $\gamma = 1.0$, respectively. In panel (b), the solid line is the baseline case where the initial asset is $A_1 = 5.16 \times (\text{weekly wage})$ while the dashed line is the case where A_1 is 0.5 times the baseline case and the dotted line is the case where A_1 is 2.0 times the baseline case.