The Welfare Impact of Reemployment Bonuses

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Abstract

Worker moral hazard is inherent in an unemployment insurance (UI) program. This paper studies reemployment bonuses, which offer monetary incentives to workers who secure employment, as a way to mitigate worker moral hazard in UI without disrupting consumption smoothing. Using a dynamic job search model with the sufficient statistic method, I find a large positive impact of reemployment bonuses on welfare. Furthermore, reemployment bonuses allow the government to optimally provide more generous UI benefits by alleviating the moral hazard cost of UI. In the calibrated model, the current UI benefit level in the U.S. is too generous, but it is nearly optimal once reemployment bonuses are optimally incorporated.

Keywords: Reemployment bonus; Sufficient statistic approach; Unemployment insurance; Consumption smoothing; Moral hazard; MVPF

JEL classification: H20, J64, J65

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1 Introduction

Unemployment insurance (UI) is a crucial element of social insurance programs in developed countries, designed to protect workers from income loss during periods of unemployment. However, it is widely acknowledged that UI can present a challenge by discouraging job search among the unemployed, which causes a fiscal externality by increasing government expenditure on UI benefits and reducing income tax revenues. This fiscal burden that arises from worker moral hazard is a concern not only for economists but also for policymakers as well. For example, only 20 states in the U.S. have sufficient reserves to sustain a year's worth of UI benefit payments if a recession occurs (White House, 2016). In response to this concern, the Obama administration proposed a reform of the UI policy with a key objective of ensuring the financial stability of state UI trust funds (O'Leary and Wandner, 2018).[1]

One potential way to address the issue of worker moral hazard is reemployment bonuses. Reemployment bonuses are rewards given to unemployed people if they manage to find a job within a certain period. This policy is intended to encourage unemployed individuals to look for jobs more actively, which could help to alleviate the burden on the UI budget. This approach has been tested in the U.S. and implemented in a few other countries.[2] While there have been several studies examining the effects of these bonuses, there has not been much research quantifying their welfare impact or how the government should set reemployment bonuses jointly with UI benefits.

This paper investigates the welfare implications of reemployment bonuses by considering a dynamic model of consumption, saving, and job search. In the first part, I adopt the sufficient statistic approach and develop a formula to quantify the welfare impact of the first dollar of reemployment bonuses. This formula incorporates several reduced-form parameters and does not need a fully calibrated model. An essential parameter in the formula is the response of unemployment duration to changes in reemployment bonuses. Instead of directly examining exogenous variations in reemployment bonuses that are not widely available, I theoretically establish a link between the labor supply response to reemployment bonuses and the response to UI benefits. This connection allows me to leverage the vast empirical research on the labor supply effects of UI benefits (e.g. Krueger and Meyer 2002, Schmieder and Von Wachter 2016).

I find that the first dollar of reemployment bonuses has a positive welfare impact, which is equivalent to a five-dollar increase in lifetime income per person. To facilitate a comparison of this finding with the welfare effects of other existing government programs, I use the concept of the Marginal Value of Public Funds (MVPF) as introduced by Hendren and Sprung-Keyser (2020).

[1]The proposal did not pass the Congress.

[2]Refer to Meyer (1995) for the field experiments in the U.S. Actual implementation of bonus programs in South Korea and Taiwan can be found in Ahn (2018) and Huang and Yang (2021), respectively.

The MVPF of a policy is defined as the aggregate willingness to pay for the policy divided by its net cost. I find that the MVPF of the first dollar of reemployment bonuses exceeds two. This value exceeds the MVPFs observed for many government programs targeted at adults such as unemployment insurance, disability insurance, job training, earned income tax credits, and many others, as documented in Hendren and Sprung-Keyser (2020). This underscores the potential for large welfare improvement through the implementation of reemployment bonuses.

In the second part of the paper, I proceed to fully calibrate the model to examine non-marginal policy changes. I select the parameters of the job search disutility function to align with the actual duration of unemployment and the elasticity of unemployment duration to UI benefits as observed in the data. Using the fully calibrated model, I first solve for the optimal level of UI benefits in the absence of reemployment bonuses. I find that optimal UI benefits equate to 18 percent of wages, significantly below the baseline replacement rate of 50 percent, yet still within the range of existing estimates. This reduction in UI benefits leads to a decrease in the unemployment rate by 2.4 p.p. while increasing the magnitude of consumption decline upon job loss by 18 p.p. These findings suggest that the moral hazard cost associated with UI in the calibrated economy is substantial, and to maximize welfare, it is necessary to reduce the UI benefit level as a way to reduce the moral hazard, which is consistent with, for instance, Hansen and Imrohoroğlu (1992) and Gruber (1997).

Next, I solve for the optimal combination of UI benefits and reemployment bonuses. I find that setting the UI benefit replacement rate at 49 percent, together with reemployment bonuses equivalent to 56 percent of wages, maximizes welfare. Despite the UI benefit level being nearly identical to that in the baseline policy, implementing the jointly optimal policy decreases the unemployment rate by 2.0 p.p. due to the effect of reemployment bonuses. At the same time, the policy change increases consumption decline following job loss by only 4 percentage points from the baseline. Notably, the impact on consumption decline is significantly less than the case in which the government chooses the optimal UI benefit level without reemployment bonuses. Hence, reemployment bonuses allow the government to counteract worker moral hazard without significantly disrupting consumption smoothing. In terms of welfare, measured via consumption equivalence, optimally selecting UI benefits alone results in a 0.25 percent improvement in welfare. In contrast, the jointly optimal policy yields a 0.39 percent improvement in welfare.

Related Literature. The concept of offering financial incentives to UI recipients in order to reduce reliance on UI benefits was explored through field experiments conducted in the 1980s. These experiments revealed that providing lump-sum reemployment bonuses indeed resulted in shorter durations of unemployment, as summarized by Meyer (1995). Research on reemployment bonuses has investigated various aspects, such as the displacement effects on individuals not offered bonuses (Davidson and Woodbury, 1993), recall rates from former employers (Anderson, 1992), and take-up rates (Meyer, 1995). Some studies have conducted simple cost-benefit analyses (Anderson, Corson and Paul 1991, Woodbury and Spiegelman 1987). While these studies mostly consider a lump-sum payment to a worker who finds a job within a specified period, I examine bonuses paid periodically until a certain point. In this latter case, the worker receives more benefits to the extent that he/she finds a job more quickly, thus it provides a stronger incentive for job search. Furthermore, I utilize a dynamic job search model to assess the effect of reemployment bonuses on worker welfare.

The papers mentioned above rely on field experiments conducted in the U.S. More recently, Ahn (2018) and Huang and Yang (2021) have evaluated the impact of reemployment bonuses on unemployment duration in a quasi-experimental environment, making use of actual policy changes. Ahn (2018) exploits the eligibility discontinuity in South Korea's reemployment bonus provision and finds a reduction in unemployment duration. Similarly, Huang and Yang (2021) leverage the introduction of reemployment bonuses in Taiwan and apply a regression kink design to find a positive impact of reemployment bonuses on unemployed workers' job finding. They also study the welfare impact using the sufficient statistic approach. Although the policy design they study aligns closely with my study, due to the specific policy variation they use for the estimation, their welfare evaluation requires a specific simultaneous extension of both UI benefit duration and reemployment bonus durations. In contrast, my approach allows for the examination of the welfare impact of the reemployment bonus level while keeping other UI aspects constant. In addition, by complementing the sufficient statistic approach with a fully calibrated model, I am able to study non-marginal adjustments in reemployment bonuses and calculate the optimal mix of UI benefits and reemployment bonuses.

There are papers theoretically examine the optimal UI benefits profile in principal-agent models. Notably, Hopenhayn and Nicolini (1997) demonstrate that when a planner can utilize UI benefits and taxes after reemployment as policy instruments, the optimal policy exhibits two key properties: (i) UI benefits decrease as unemployment spells increase, and (ii) taxes after reemployment rise with unemployment spells. While the first property is partially reflected in UI programs implemented in many countries, where unemployed individuals typically receive fixed benefits for a certain period and then no benefits thereafter, the second property is not widely observed. The provision of reemployment bonuses can be considered to be a form of reduced taxes for individuals who find a job within a specified timeframe. In this sense, this paper empirically evaluates the second property of the optimal policy proposed by Hopenhayn and Nicolini (1997). Yet, it is worth noting that their main contribution is to theoretically characterize the optimal profile of UI benefits and taxes. In contrast, I quantify the welfare impact of the policy change based on reasonable reduced-form parameters plugged into the sufficient statistic formula. In this sense, I view my contribution as complementary to Hopenhayn and Nicolini (1997).

Methodologically, this paper employs the sufficient statistic approach, pioneered by Baily (1978) and Chetty (2006). This approach has been extensively utilized by researchers in examining various aspects of UI. For instance, it has been used to study the optimal level of UI benefits (Gruber 1997, Chetty 2008, Landais 2015, Kroft and Notowidigdo 2016), the optimal duration of UI (Ganong and Noel 2019), and the optimal trajectory of UI benefits (Kolsrud, Landais, Nilsson and Spinnewijn 2018).[3] This approach of welfare evaluation is useful as it requires only a subset of model parameters to be identified. But the results from the sufficient statistic approach cannot be extended to cases where policy changes are not marginal due to two reasons: (i) the envelope theorem, used to characterize the welfare impact, does not hold for non-marginal policy changes, and (ii) the formula depends on reduced-form parameters that may not be policy-invariant. I supplement the results by investigating non-marginal changes in reemployment bonuses through the full calibration of the model. In this vein, Lentz (2009), Michelacci and Ruffo (2015), and Lawson (2017) estimate a partial-equilibrium dynamic model of consumption, saving, and job search to study an optimal UI benefit design. While these papers focus on the UI benefit level, I study reemployment bonuses and the joint design of UI benefits and reemployment bonuses.

2 Model

In this section, I present a dynamic model of consumption, saving, and job search, building upon the framework in Chetty (2008). The primary objective is to analyze the welfare implications of marginal adjustments in reemployment bonuses, relying on a set of reduced-form parameters. This approach offers certain advantages as it does not need a complete specification of the model. For instance, while assuming convexity of the search cost function, I do not specify an exact functional form for it. However, a drawback of this approach is that I cannot quantify the welfare impact of non-marginal policy changes, thereby limiting insights into the globally optimal levels of UI benefits and reemployment bonuses. Later in Section 4, I fully calibrate the baseline model presented in this section to study non-marginal policy changes.

2.1 Environment

Workers in the model are initially unemployed in $t = 1$. Upon becoming employed, they remain employed until the end of the model at period T, which is a standard assumption in the literature and greatly simplifies the problem.[4]

[3] Chetty and Finkelstein (2013) and Schmieder and Von Wachter (2016) offer comprehensive overviews of this literature.

[4] For example, Chetty (2008), Landais (2015), and Kolsrud et al. (2018) adopts the same assumption to get a welfare formula.

While unemployed, workers receive UI benefits denoted as b_u for a maximum of T_B periods. Upon finding a job, workers receive reemployment bonuses denoted as b_e. I define the vector $\mathbf{b} = (b_u, b_e)$ to represent these benefit levels. I assume that individuals who find a job before period T_B are eligible to receive b_e until period T_B. For instance, if a worker finds a job in period $\bar{t} < T_B$, the worker receives UI benefits b_u during periods $t = 1, \ldots, \bar{t}$ and reemployment bonuses b_e during periods $t = \bar{t} + 1, \ldots, T_B$.

In this analysis, I focus on the case where b_u and b_e remain constant over the T_B periods, and T_B is fixed. The government finances UI benefits through taxes denoted as τ, which are collected from each employed worker. The search effort of individuals at time t is denoted as e_t, and I normalize it such that the level of effort e_t is equal to the probability of finding a job. Consequently, the survival probability, which represents the probability of remaining unemployed after searching for a job in period t, is given by $S_t = \prod_{j=1}^{t}(1 - e_j)$. The expected unemployment duration is defined as $D = \sum_{t=1}^{T} S_t$, and the expected duration of UI compensation is denoted as $D_B = \sum_{t=1}^{T_B} S_t$.

2.2 Individual Problem

To simplify the analysis, I assume a zero interest rate and no discounting. In each period t, an unemployed individual engages in job search and subsequently determines their consumption level.

Optimal consumption of employed workers. If a worker is employed in period t with assets A_t, the value function is defined as

$$V_{s,t}^{E}(A_t; \mathbf{b}, \tau) = \max_{A_{t+1} \geq L} U(A_t - A_{t+1} + y_{e,s,t}) + V_{s,t+1}^{E}(A_{t+1}; \mathbf{b}, \tau). \tag{1}$$

Here, s represents the period in which the worker finds the job, A_t denotes the asset level at the start of period t, and L represents the exogenous liquidity constraint. The income of workers who find a job in period s, denoted as $y_{e,s,t}$, is given by $y_{e,s,t} = w_{s,t} - \tau + b_e$ for $t \leq T_B$, and $y_{e,t} = w_{s,t} - \tau$ for $t > T_B$. The wage $w_{s,t}$ may depend on s to capture the potential impact of unemployment duration on future wages. I assume that wages are non-random and deterministic, but allowing for stochastic wages does not change results as I discuss later. The utility function $U(\cdot)$ is assumed to be twice-differentiable, strictly increasing, and strictly concave on $\mathbb{R}_+$.

Optimal consumption of unemployed workers. If an unemployed worker with asset A_t does not find a job at the start of period t, the value function is given by

$$V_t^{U}(A_t; \mathbf{b}, \tau) = \max_{A_{t+1} \geq L} U(A_t - A_{t+1} + y_{u,t}) + V_{t+1}(A_{t+1}; \mathbf{b}, \tau), \tag{2}$$

where $y_{u,t}$ represents the income for unemployed workers, which is determined by $y_{u,t} = \bar{y} + b_u$ if $t \leq T_B$, and $y_{u,t} = \bar{y}$ otherwise. The parameter $\bar{y}$ represents an additional income source that does not impact the government budget. For instance, it can account for the income of other family members or informal transfers from parents. The term $V_{t+1}(\cdot)$ in the second part of the equation denotes the expected value at the beginning of period $t+1$ before initiating a job search.

Optimal search effort. The value function of an unemployed worker with asset level A_t at the beginning of each period, before initiating a job search, is given by

$$V_t(A_t;\mathbf{b},\tau) = \max_{e_t \in [0,1]} e_t V_{t,t}^E(A_t;\mathbf{b},\tau) + (1-e_t) V_t^U(A_t;\mathbf{b},\tau) - \psi(e_t), \tag{3}$$

where $\psi(\cdot)$ represents the disutility of search effort. The function $\psi(\cdot)$ is twice-differentiable, strictly increasing, and strictly convex on the interval $(0,1)$. I impose the conditions $\psi'(0) = 0$ and $\lim_{e \to 1} \psi(e) = +\infty$ to ensure an interior solution. $V_t^U(\cdot)$ may not be concave due to the presence of $V_{t+1}(\cdot)$ in the second term of Equation (2). I assume that $V_t^U(\cdot)$ is concave under plausible parameter values, as assumed in Chetty (2008).

The optimal level of search effort is determined by solving the following first-order condition

$$\psi'(e_t(A_t;\mathbf{b},\tau)) = V_{t,t}^E(A_t;\mathbf{b},\tau) - V_t^U(A_t;\mathbf{b},\tau). \tag{4}$$

Given that $\psi(\cdot)$ is a convex function, the left-hand side of the equation is increasing in e_t. This implies that individuals exert greater search effort when the value of employment is higher and exert less search effort when the value of unemployment is higher. Consequently, an increase in UI benefits b_u, which makes remaining unemployed more valuable, leads to a decrease in search effort. Conversely, more generous reemployment bonuses b_e incentivize higher search effort by increasing the value of being employed.

2.3 Government Problem

In this subsection, I derive a formula for assessing the welfare impact of a marginal change in b_e. To evaluate this impact, let $W(\mathbf{b})$ represent the value of unemployed individuals at the beginning of the model as a function of UI benefits b_u and reemployment bonuses b_e subject to the budget constraint. Specifically, given the initial assets A_1, I define

$$W(\mathbf{b}) = V_1(A_1;\mathbf{b},\tau(\mathbf{b})) \quad \text{where} \quad \tau(\mathbf{b}) = \frac{D_B b_u + (T_B - D_B) b_e}{T - D}. \tag{5}$$

Here, $\tau(\mathbf{b})$ represents the budget-balancing level of taxes as a function of UI benefits b_u and reemployment bonuses b_e. The government's objective is to determine the optimal UI benefits b_u and reemployment bonuses b_e that maximize the welfare measure $W(\mathbf{b})$. In this section, I do not solve the problem to obtain a globally optimal policy. Instead, I analyze the welfare impact of a marginal increase in b_e from the current level of $b_e = 0$.

The government budget constraint can be expressed as

$$[T - D(\mathbf{b}, \tau)]\tau(\mathbf{b}) = D_B(\mathbf{b}, \tau)b_u + [T_B - D_B(\mathbf{b}, \tau)]b_e. \tag{6}$$

The impact of providing reemployment bonuses is given by

$$\begin{aligned}\frac{\partial \tau(\mathbf{b})}{\partial b_e} = & \underbrace{\frac{T_B - D_B(\mathbf{b}, \tau(\mathbf{b}))}{T - D(\mathbf{b}, \tau(\mathbf{b}))}}_{\text{Mechanical Impact}} \\ & \underbrace{+ \frac{1}{T - D(\mathbf{b}, \tau(\mathbf{b}))}\left[\frac{dD_B(\mathbf{b}, \tau(\mathbf{b}))}{db_e}(b_u - b_e) + \frac{dD(\mathbf{b}, \tau(\mathbf{b}))}{db_e}\tau(\mathbf{b})\right]}_{\text{Behavioral Impact}},\end{aligned} \tag{7}$$

where $\frac{dD_B(\mathbf{b},\tau(\mathbf{b}))}{db_e}$ and $\frac{dD(\mathbf{b},\tau(\mathbf{b}))}{db_e}$ are the response of D_B and D to a balanced-budget increase in b_e, which includes the effect of the change in τ needed to finance the increase in b_e. The first term captures the increase in taxes required to fund reemployment bonuses in the absence of any response from unemployed workers, referred to as the mechanical impact. The second term shows that the response of unemployed workers has an impact on the budget through a change in unemployment duration, which I refer to as the behavioral impact. If a budget-balance increase in reemployment bonuses induces shorter unemployment duration (i.e., $\frac{dD_B}{db_e} < 0$ and $\frac{dD}{db_e} < 0$), the increase in taxes is less than the mechanical cost of the policy change.

Reemployment bonuses also affect consumption after reemployment. Taking that into account as well, the following proposition characterizes the welfare impact of the marginal change in reemployment bonuses b_e given UI benefits b_u.

Proposition 1. *The welfare impact of the marginal increase in reemployment bonuses b_e is given by*

$$\frac{\partial W(\mathbf{b})}{\partial b_e} \Big/ \frac{\partial V_1}{\partial w} = \frac{T_B - D_B}{T - D}\left[\frac{MU_{e,T_B} - MU_{e,T}}{MU_{e,T}} - \frac{dD_B}{db_e}\frac{b_u - b_e}{T_B - D_B} - \frac{dD}{db_e}\frac{\tau(\mathbf{b})}{T_B - D_B}\right], \tag{8}$$

where $D = D(b, \tau(b))$, $D_B = D_B(\mathbf{b}, \tau(\mathbf{b}))$, and $MU_{e,t}$ is given by

$$MU_{e,t} = \sum_{k=1}^{t} \frac{1 - S_k}{t - D_B}\mathbb{E}_s[U'(c_{e,s,k})] \quad \textit{for} \quad t = T_B, T. \tag{9}$$

Proof. See Appendix A.2. □

To obtain a money metric for assessing the welfare impact, I normalize $\frac{\partial W(\mathbf{b})}{\partial b_e}$ by the welfare gain resulting from a permanent increase in wages $\frac{\partial V_1}{\partial w}$. $MU_{e,t}$ is the weighted average of marginal utilities of consumption during employment, where the weights are proportional to the probability that an individual is employed in period t. This expectation is computed by considering all possible histories that lead to the state where a person is employed in period t.[5]

The interpretation of formula (8) is as follows. The first term in the bracket captures the redistributive effect of the policy change, which arises from intertemporal transfers associated with the increase in reemployment bonuses b_e and the corresponding rise in taxes τ. It is worth noting that individuals can receive b_e for a maximum of T_B periods, while they are required to pay taxes until the end of the model period T. This difference generates transfers across periods. The second and third terms represent the welfare gain resulting from the behavioral impact of reemployment bonuses, corresponding to the second term in equation (7).

The sign of the first term can be theoretically positive or negative, depending on the underlying income processes. On the one hand, since $MU_{e,T}$ assigns weights to workers who find a job later, $MU_{e,T}$ should be larger than MU_{e,T_B} to the extent that human capital losses during unemployment lead to consumption losses after reemployment. On the other hand, $MU_{e,T}$ also assigns weights to workers who have been employed for a longer period after reemployment. If these workers accumulate human capital and experience increased consumption, $MU_{e,T}$ should be smaller than MU_{e,T_B}.

3 Empirical Implementation

3.1 Inferring the Response to Reemployment Bonuses from the Response to UI benefits

One crucial parameter in the formula is the response of unemployment duration to reemployment bonuses. Instead of attempting to directly estimate this parameter by seeking exogenous policy variations, I theoretically establish a connection between the labor supply response to UI benefits and the labor supply response to reemployment bonuses. This connection allows me to leverage the vast empirical literature on the labor supply response to UI, as documented in Krueger and Meyer (2002) and Chetty and Finkelstein (2013).

[5] The precise expression of $\mathbb{E}_s[U'(c_{e,s,k})]$ is provided in equation (35) in Appendix A.2.

Recall that the first-order condition for search effort is given by:

$$\psi'(e_t(A_t;\mathbf{b},\tau)) = V^E_{t,t}(A_t;\mathbf{b},\tau) - V^U_t(A_t;\mathbf{b},\tau). \tag{4}$$

By differentiating both sides of the equation with respect to either b_u or b_e for $t \le T_B$, I obtain:

$$\frac{\partial e_t}{\partial b_u} = \frac{-\frac{\partial V^U_t(A_t;\mathbf{b},\tau)}{\partial b_u}}{\psi''(e_t)}, \tag{10}$$

$$\frac{\partial e_t}{\partial b_e} = \frac{\frac{\partial V^E_{t,t}(A_t;\mathbf{b},\tau)}{\partial b_e} - \frac{\partial V^U_t(A_t;\mathbf{b},\tau)}{\partial b_e}}{\psi''(e_t)}. \tag{11}$$

It is important to note that the response of search effort to UI benefits b_u and reemployment bonuses b_e both depend on the curvature of the search cost function $\psi''(\cdot)$. By rearranging the equations, I derive the following relationship:

$$\frac{\partial e_t}{\partial b_e} = \frac{\partial e_t}{\partial b_u} \times \frac{\frac{\partial V^E_{t,t}(A_t;\mathbf{b},\tau)}{\partial b_e} - \frac{\partial V^U_t(A_t;\mathbf{b},\tau)}{\partial b_u}}{-\frac{\partial V^U_t(A_t;\mathbf{b},\tau)}{\partial b_u}}. \tag{12}$$

The left-hand side represents the response of search effort to reemployment bonuses, while the right-hand side represents the response of search effort to UI benefits multiplied by a correction term. The correction term accounts for the ratio of the marginal values of reemployment bonuses and UI benefits, which depend on the marginal utility of consumption.

To obtain a tractable expression for the correction term, I need to impose several assumptions regarding the evolution of consumption. These assumptions are listed below:

Assumption. (A.1) *Consumption after finding a job does not depend on s and t, i.e.,* $c_{e,s,t} = c_e$.

(A.2) *Finding a job does not decrease consumption, i.e.,* $c_e \ge c_{u,t}$.

(A.3) *Consumption during unemployment does not increase over time, i.e.,* $c_{u,t} \le c_{u,t-1}$.

It is worth emphasizing that all of these assumptions are either common or empirically supported. Assumption (A.1) is reasonable for an approximation when the impact of unemployment on lifetime income is negligible. This assumption is commonly made in research that derives a welfare formula for UI benefits (e.g. Chetty 2008, Kroft and Notowidigdo 2016). Moreover, Chetty (2008) provides simulation results demonstrating that this approximation has a negligible impact on the evaluated welfare. The remaining assumptions (A.2)-(A.3), while intuitively plausible, also gain empirical support from studies examining consumption patterns such as Ganong and Noel (2019).

With these assumptions, I am now able to use the relationship between the response of *search effort* to UI benefits and reemployment benefits in equation (12) to establish the link between the response of *unemployment duration* (not search effort) to UI benefits and reemployment bonuses.

Proposition 2. *Suppose assumptions (A.1)-(A.3) are satisfied. Then, the response of the expected UI-compensated unemployment spells D_B to a change in reemployment bonuses b_e is bounded as follows:*

$$\underline{M}_{T_B}\frac{\partial D_B(\mathbf{b},\tau)}{\partial b_u} \leq -\frac{\partial D_B(\mathbf{b},\tau)}{\partial b_e} \leq \overline{M}_{T_B}\frac{\partial D_B(\mathbf{b},\tau)}{\partial b_u}, \tag{13}$$

where

$$\overline{M}_{T_B} = \frac{U'(c_e)}{U'(c_{u,1})}, \quad \underline{M}_{T_B} = \frac{U'(c_e)}{U'(c_{u,T_B})}.$$

Proof. See Appendix A.3. □

The proposition establishes the upper and lower bounds for the labor supply response to reemployment bonuses. The bounds depend on the response to UI benefits and the marginal rate of substitution between consumption during periods of employment and unemployment. As previously discussed, the labor supply response to either policy is influenced by the curvature of the job-search disutility function, making the response to UI benefits a useful quantity in inferring the response to reemployment bonuses. However, certain modifications are needed given that UI benefits influence consumption during unemployment, while reemployment bonuses impact consumption during employment. For instance, if consumption during employment significantly exceeds consumption during unemployment, the concave nature of the utility function suggests that offering reemployment bonuses exerts a smaller influence on the net value of employment than UI benefits.

Remark 1. The bounds established in Proposition 2 are derived based on partial derivatives and do not account for the effects of changes in taxes associated with changes in reemployment bonuses. However, the welfare formula (8) relies on total derivatives. It is important to note that:

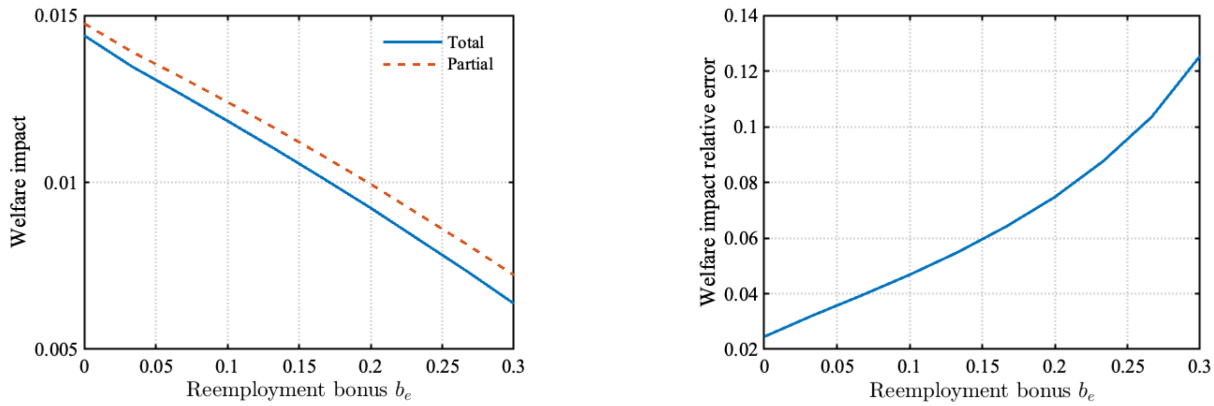
$$\frac{dD_B(\mathbf{b},\tau(\mathbf{b}))}{db_e} = \frac{\partial D_B(\mathbf{b},\tau(\mathbf{b}))}{\partial b_e} + \left.\frac{\partial D_B(\mathbf{b},\tau)}{\partial \tau}\right|_{\tau=\tau(\mathbf{b})} \times \frac{\partial \tau(\mathbf{b})}{\partial b_e}, \tag{14}$$

$$\frac{dD(\mathbf{b},\tau(\mathbf{b}))}{db_e} = \frac{\partial D(\mathbf{b},\tau(\mathbf{b}))}{\partial b_e} + \left.\frac{\partial D(\mathbf{b},\tau)}{\partial \tau}\right|_{\tau=\tau(\mathbf{b})} \times \frac{\partial \tau(\mathbf{b})}{\partial b_e}. \tag{15}$$

When the tax changes associated with reemployment bonuses are large, the difference between the total derivatives and the partial derivatives becomes significant. However, in practice, this term is likely to be small due to the fact that reemployment bonuses are typically provided to a small fraction of the overall employed population. In line with the approach taken by Chetty (2008) and

Figure 1: Total Impact vs. Partial Impact

(a) Welfare Impact (b) Relative Error



Note: In both panels, the horizontal axis represents reemployment bonuses as a fraction of previous wages. In the left panel, the vertical axis represents the welfare impact of the marginal increase in reemployment bonuses $\frac{\partial W}{\partial b_e} / \frac{\partial V}{\partial w}$. The blue solid line is based on the original formula (8). The red dashed line is based on the same equation except that the total derivatives are replaced by the partial derivatives. In the right panel, the relative error defined by $\log \text{Partial Impact} - \log \text{Partial Impact}$ is taken on the vertical axis.

Kolsrud et al. (2018), I ignore the difference between the partial and total impact of reemployment bonuses on expected unemployment duration. Nevertheless, I conduct a simulation in Section 4 using a calibrated model to examine the potential magnitude of this difference. Further details regarding the functional forms and model parameters employed in the simulation are provided in Section 4.

Figure 1a illustrates the impact of ignoring the labor supply effects arising from changes in taxes necessary to finance an increase in reemployment bonuses on the welfare effect of a marginal increase in these bonuses. The blue solid line represents the welfare impact calculated using the exact formula, while the red dashed line depicts the welfare impact when the impact of tax changes is disregarded. It is evident that the welfare impact based on the partial derivatives overestimates the true welfare impact, albeit by a small margin.

To further quantify this discrepancy, Figure 1b presents the relative error across a wide range of reemployment bonuses. At the current policy level ($b_e = 0$), the relative error is approximately two percent. Throughout this section, I use $\frac{\partial D}{\partial b_e}$ and $\frac{\partial D_B}{\partial b_e}$ as approximations for $\frac{dD}{db_e}$ and $\frac{dD_B}{db_e}$, respectively.

Remark 2. In this remaining part of the section, I use the lower limit outlined in Proposition 2 to obtain the minimum potential impact on welfare by reemployment bonuses. Notably, even using the lower bound, I turn out to find a substantial improvement in welfare. However, it is

Table 1: Parameters for the Welfare Evaluation of the Marginal Change in b_e

Parameter	Description	Value
u	Fraction of periods being unemployed	0.054
D	Expected unemployment duration (weeks)	24.3
D_B	Expected UI-compensated unemployment duration (weeks)	15.8
η_{D_B,b_u}	Elasticity of D_B with respect to UI benefits	0.53

Note: D, D_B, and η_{D_B,b_u} are directly taken from Chetty (2008). The paper also reports $\sigma = \frac{T-D}{T} = 0.946$, which implies that $u = 1 - \sigma = 0.054$.

worth examining the extent of the disparity between the lower and upper bounds. To illustrate this, recognize that the only difference between these bounds comes from the difference between $\underline{M}_{T_B}$ and $\overline{M}_{T_B}$. By computing their ratio, I obtain

$$\frac{\overline{M}_{T_B}}{\underline{M}_{T_B}} = \frac{U'(c_{u,T_B})}{U'(c_{u,1})}. \tag{16}$$

Assuming a CRRA utility function with a risk aversion parameter of γ, this simplifies to

$$\frac{\overline{M}_{T_B}}{\underline{M}_{T_B}} = \left(\frac{c_{u,1}}{c_{u,T_B}}\right)^{\gamma}. \tag{17}$$

The right-hand side can be obtained by comparing the consumption level immediately after a job loss, $c_{u,1}$, and that of T_B weeks into unemployment—typically a 26-week period. Ganong and Noel (2019) document unemployed workers' consumption trajectory on a monthly basis. They find a sharp decline in consumption following the onset of unemployment, which then keeps decreasing only gradually until the expiration of UI benefits. Consequently, $c_{u,1}$ is not significantly higher than c_{u,T_B}. Indeed, a comparison of consumption during the first month of UI benefit receipt and the final month, as illustrated in Figure 2 of Ganong and Noel (2019), suggests that $c_{u,1}$ surpasses c_{u,T_B} by approximately five percent. Considering this consumption pattern, equation 17 implies that the upper boundary exceeds the lower one by a factor of $(1.05)^{\gamma}$. If I consider a plausible parameter choice of $\gamma = 2$, for instance, the upper bound is approximately 10 percent higher than the lower bound.

3.2 Evaluating the Welfare Impact

Proposition 1 in the previous section gives the welfare impact of providing reemployment bonuses. I replace the response to reemployment bonuses with the response to UI benefits using Proposition 2, and evaluate it at $b_e = 0$. Then, I obtain a lower bound on the welfare impact of the provision of

Table 2: Welfare Impact of the Marginal Increase in b_e

c_u/c_e	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$
0.85	0.017	0.014	0.012
0.90	0.018	0.016	0.014
0.95	0.019	0.018	0.017

Note: This table reports the welfare impact of the marginal increase in reemployment bonuses b_e from zero. The first column reports the consumption ratio $c_{u,T_B}/c_e$ used in the evaluation. The second to the fourth columns report $\frac{\partial W(b)}{\partial b_e} \Big/ \frac{\partial V_1}{\partial w}$ under different values of CRRA parameters γ. The unit of measure is the marginal value of a permanent increase in wages.

reemployment bonuses that can be evaluated by readily available reduced-form parameters.

Corollary 1. *Suppose that assumptions (A.1)-(A.3) are satisfied. Then the welfare impact of the marginal increase in reemployment bonuses b_e from zero is given by*

$$\frac{\partial W(\mathbf{b})}{\partial b_e} \Big/ \frac{\partial V_1}{\partial w} \geq \frac{u}{1-u}\frac{D_B}{D}\underline{M}_{T_B}\eta_{D_B,b_u}, \quad \textit{where} \quad \eta_{D_B,b_u} = \frac{dD_B}{db_u}\frac{b_u}{D_B} \textit{ and } u = \frac{D}{T}. \tag{18}$$

Proof. See Appendix A.4. □

Note that η_{D_B,b_u} represents the elasticity of expected UI-compensated duration with respect to UI benefits, while u corresponds to the fraction of periods during which an individual is expected to be unemployed.

The evaluation of the formula requires the following five parameters: u, D, D_B, $\underline{M}_{T_B}$, and η_{D_B,b_u}. As for the parameters other than $\underline{M}_{T_B}$, I use the values reported in Chetty (2008), as summarized in Table 1. The remaining parameter to be determined is $\underline{M}_{T_B}$. Assuming a constant relative risk aversion (CRRA) utility function of the form $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$, I obtain

$$\underline{M}_{T_B} = \left(\frac{c_{u,T_B}}{c_e}\right)^{\gamma}. \tag{19}$$

I can pin it down by comparing consumption without job loss with consumption of T_B weeks into unemployment. Again, I rely on Ganong and Noel (2019) to determine this consumption ratio. They document that consumption just before UI expiration is about 10 percent lower than consumption before job loss. Assuming that consumption would be constant if job loss does not happen. Then, $c_{u,T_B}/c_e \approx 0.9$.

In Table 2, I report the welfare impact of reemployment bonuses for various values for $c_{u,T_B}/c_e$ including the one corresponding to the estimate in Ganong and Noel (2019). I also choose a range of values for the CRRA parameter γ due to the lack of consensus in the literature regarding its precise value. The first column displays the consumption ratio ($c_{u,T_B}/c_e$) used in the evaluation.

Table 3: MVPFs of the Marginal Increase in b_e

c_u/c_e	$\gamma=1$	$\gamma=2$	$\gamma=3$
0.85	3.31	2.46	2.02
0.90	3.83	2.99	2.49
0.95	4.54	3.86	3.38

Note: This table presents the Marginal Value of Public Funds (MVPF) of the marginal increase in reemployment bonuses b_e from zero. The first column indicates the consumption ratio $c_{u,T_B}/c_e$ used in the evaluation. The second to fourth columns report the MVPFs under different values of CRRA parameter γ.

The second to fourth columns show the welfare gain measured in units of the marginal gain from a permanent increase in wages ($\frac{\partial V}{\partial w}$). The table reveals that the welfare impact increases with the consumption ratio (c_u/c_e) and decreases with the CRRA parameter (γ). This pattern arises because, with other parameters held constant, a larger consumption ratio or a lower CRRA parameter results in a larger value of $\underline{M}_{T_B}$ in equation (19). Consequently, this implies a larger labor supply response, as evident in equation (13).

Even in the most conservative case where $c_{u,T_B}/c_e = 0.85$ and $\gamma = 3$, the first dollar of reemployment bonuses yields a welfare impact equivalent to a permanent increase of 1.2 cents in weekly earnings. To aggregate this impact over a lifetime, I assume $T = 450$ based on the calculation $u = \frac{D}{T}$, where $u = 0.054$ and $D = 24.3$ from Table 1. Under this assumption, the welfare impact is equivalent to a $0.012 \times (450 - 24.3) = 5.1$ dollar increase in lifetime income per person.

3.3 Comparison to Historical Policy Changes in the U.S.

To facilitate a comparison of the finding above with the welfare effects of other existing government programs, I calculate the Marginal Value of Public Funds (MVPF) as introduced by Hendren and Sprung-Keyser (2020). The MVPF of a policy is defined as the aggregated willingness to pay for the policy divided by the net cost to the government.[6] This notion provides a unified framework for assessing the impacts of government policies on social welfare. I keep making the same assumptions (A.1)-(A.3) as before. The willingness to pay for a \$1 increase in reemployment bonuses b_e is given by

$$WTP_e = T_B - D_B. \tag{20}$$

[6] Since the model in this paper assumes no ex-ante heterogeneity, there is no distinction between the aggregated willingness to pay and that of an individual.

The net cost of this policy change is given by

$$
\begin{aligned}
Net\ Cost_e &= (T-D)\frac{\partial \tau(\mathbf{b})}{\partial b_e} \\
&= (T_B - D_B)(1+FE_e),
\end{aligned} \tag{21}
$$

where

$$
FE_e = \frac{1}{T_B - D_B}\left[\frac{dD_B}{db_e}(b_u - b_e) + \frac{dD}{db_e}\tau\right]. \tag{22}
$$

Here, FE_e represents the fiscal externality due to the behavioral response of unemployed people. To quantify the welfare impact of the first dollar of reemployment bonuses, evaluating FE_e at the point $b_e = 0$ yields

$$
FE_e = \frac{u_B}{1-u_B}\frac{dD_B}{db_e}\frac{b_u}{D_B} + \frac{u_B}{1-u_B}\frac{u}{1-u}\frac{dD}{db_e}\frac{b_u}{D}, \tag{23}
$$

where $u_B = \frac{D_B}{T_B}$. Using Proposition 2 and the fact that the second term is positive, I get

$$
FE_e \leq -\frac{u_B}{1-u_B}\underline{M}_{T_B}\eta_{D_B,b_u}. \tag{24}
$$

Note that FE_e is negative, implying that the net cost is less than 1. The lower bound on the MVPF is given by

$$
MVPF_e = \frac{WTP_e}{Net\ Cost_e} \geq \frac{1}{1-\frac{u_B}{1-u_B}\underline{M}_{T_B}\eta_{D_B,b_u}}. \tag{25}
$$

The last column of Table 3 shows that the MVPF of the proposed policy change exceeds two even in a conservative case where $c_{u,T_B}/c_e = 0.85$ and $\gamma = 3$. This result is remarkable as Hendren and Sprung-Keyser (2020) reports MVPFs of 133 historical policy changes in the U.S. and most policies targeting adults have MVPFs below two.

For comparison purposes, I also calculate the MVPF of unemployment insurance (UI) derived from my model. A detailed derivation can be found in Appendix A.5. The MVPF of the marginal increase in UI benefits b_u is approximately given by

$$
MVPF_u \approx \frac{1+\gamma\frac{c_e - c_u}{c_e}}{1+\frac{1}{1-u}\eta_{D_B,b_u}}. \tag{26}
$$

This expression can be interpreted as follows: The willingness to pay for an additional dollar of UI benefits in the numerator depends on the relative difference between the marginal utility of consumption during unemployment and employment, which, in the case of CRRA utility, can be approximated by the consumption drop upon unemployment multiplied by the risk aversion parameter γ as shown in the numerator above. If consumption declines more significantly upon unemployment, individuals place a higher value on additional UI benefits. On the other hand, the net cost of an additional dollar of UI benefits depends on the moral hazard impact of UI, which can be captured by the elasticity of insured duration with respect to UI benefits.

Suppose I have $\gamma = 2.0$ and a consumption drop upon unemployment of 0.068, as documented by Gruber (1997). Furthermore, let's assume that the values of u and η_{D_B,b_u} are as provided in Table 1. Then, the MVPF of the marginal increase in UI benefits is approximately given by

$$MVPF_u \approx \frac{1+2\times 0.068}{1+\frac{1}{1-0.054}\times 0.053} \approx 0.73, \tag{27}$$

which falls within the range of MVPFs of UI reported in Hendren and Sprung-Keyser (2020).

This observation highlights the inverse relationship between the large MVPFs of the marginal increase in reemployment bonuses reported in Table 3 and the low MVPF of the marginal increase in UI benefits above. The small MVPF of UI benefits is primarily driven by the significant moral hazard cost associated with UI, which outweighs the consumption smoothing gain. It is important to note that this substantial moral hazard cost arises from the considerable response of unemployment spells to the generosity of UI benefits. As demonstrated in Proposition 2, this large labor supply response to UI generosity translates into a corresponding labor supply response to reemployment bonuses to the extent that it is driven by the curvature of the search disutility function. Consequently, the government can achieve substantial reductions in UI spending by encouraging job search through reemployment bonuses, leading to a welfare gain through tax reductions.

3.4 Discussion

The baseline model analyzed in the preceding sections is undeniably highly stylized, and it is important to acknowledge that the model may not capture some important realism. In this subsection, I aim to provide a comprehensive discussion on the robustness of the analysis as well as its limitations.

Sensitivity to the elasticity parameter. A key upside of my approach, which connects the response of unemployment duration to reemployment bonuses and the response to UI benefits, is that I can utilize the extensive research on labor supply responses to UI generosity. In this section, I calculate the MVPFs for reemployment bonuses again, applying estimation results from several other studies.

Table 4 outlines the estimated elasticities from a collection of studies using U.S. data, reviewed in Schmieder and Von Wachter (2016). It also shows the MVPFs of reemployment bonuses deter-

Table 4: Sensitivity of MVPF to η_{D_B,b_u}

Study	η_{D_B,b_u}	MVPF $\gamma = 1$	$\gamma = 2$	$\gamma = 3$
Moffitt (1985)	0.36	2.01	1.82	1.68
Solon (1985)	0.10	1.16	1.14	1.13
Katz and Meyer (1990)	0.80	∞	∞	10.35
Meyer and Mok (2007)	0.12-0.60	1.2-6.12	1.18-4.95	1.16-3.10
Landais (2015)	0.29	1.68	1.57	1.49
Kroft and Notowidigdo (2016)	0.63	8.22	4.77	3.47
Card, Johnston, Leung, Mas and Pei (2015)	0.38-1.21	2.13-∞	1.91-∞	1.75-∞

Note: This table shows the estimated elasticity of insured duration with respect to UI benefits for a collection of studies surveyed in Schmieder and Von Wachter (2016) and shows the MVPFs computed based on those elasticities.

mined using these elasticities. For studies like Solon (1985) with smaller estimated elasticities, the MVPF ranges from 1.13-1.16 depending on the risk aversion parameter γ. While this is significantly smaller than the figures discussed earlier in this section, it still exceeds 1, suggesting that the policy provides welfare benefits greater than one dollar for each dollar of government expenditure. Conversely, for studies with larger estimated elasticities, the MVPF's denominator turns negative. In these cases, I define the MVPF as infinity, following Hendren and Sprung-Keyser (2020). An infinite MVPF indicates that the labor supply response to reemployment bonuses is so substantial that the significant reduction in UI benefit spending allows the reemployment bonus policy to fund itself.

State dependencies of marginal utility of consumption. In Proposition 2, the marginal utilities $U'(c_{u,t})$ and $U'(c_{e,t,t})$ play a crucial role in connecting the estimable quantity $\frac{\partial D_B}{\partial b_u}$ to what needs to be obtained $\frac{\partial D_B}{\partial b_e}$. So far, I have assumed that the marginal utilities of consumption do not directly depend on employment status as commonly assumed. However, they might depend on employment status for a few reasons. For instance, if consumption and leisure are not separable, the true marginal rate of substitution should deviate from what I have used in Proposition 2. Let $U_u(c)$ denote utility from consumption while unemployed and $U_e(c)$ denote utility from consumption while employed. Empirical evidence, such as that presented by Ziliak and Kniesner (2005), suggests that consumption and hours of work are Frisch complements, and hence the marginal utility of consumption is higher during periods of employment, i.e., $U_u'(c) < U_e'(c)$. Importantly, under this condition, Proposition 2 provides a conservative estimate of the lower bound since I have $\frac{U_u'(c_{e,t,t})}{U_u'(c_{u,t})} < \frac{U_e'(c_{e,t,t})}{U_u'(c_{u,t})}$, which in turn leads to a conservative estimate of the welfare impact of reemployment bonuses.

Another possibility is that unemployed individuals have more time to devote to shopping and home production, allowing them to achieve the same level of utility with a lower level of *expen-*

diture, as discussed by (Aguiar and Hurst, 2007).[7] For example, suppose a person is busy while employed and often dines out to save time. If this person becomes unemployed, they have more free time and can spend it on cooking to save money. Let c represent expenditure and $\tilde{c}$ represent consumption (as the output of time and expenditure). It is likely that $\frac{c_e}{c_u} > \frac{\tilde{c}_e}{\tilde{c}_u}$. This implies that the marginal rate of substitution between *consumption* while employed and while unemployed would be greater than the marginal rate of substitution between *expenditure* while employed and while unemployed, which is used in the proposition. Once again, this leads to a conservative estimate of the lower bound for the response of unemployment duration, which in turn leads to a conservative estimate of the welfare impact of reemployment bonuses.

Heterogeneity. While the model presented in this section does not assume heterogeneity among unemployed individuals, unemployed people in reality would exhibit heterogeneity, particularly in terms of their liquidity, as discussed by (Chetty, 2008). However, incorporating heterogeneity into the model does not affect the formulation of the formula. By redefining welfare as $W(\mathbf{b}) = \int V_1(A_1(\omega), \omega; b, \tau(\mathbf{b}))dF(\omega)$, where $V_1(A_1(\omega), \omega; \mathbf{b}, \tau(\mathbf{b}))$ represents the value of an individual and $F(\omega)$ denotes the distribution of individuals, the resulting formula remains the same as equation (8), except that the marginal utilities need to be integrated with respect to $F(\omega)$. A detailed derivation of the formula in the presence of heterogeneity is in Appendix B.1.

Uncertainty in wage offers. In many studies that analyze the job search behavior of unemployed individuals, it is commonly assumed that stochastic wage offers are drawn from a distribution, as discussed by (McCall, 1970). While incorporating this assumption introduces a reservation wage (R_t) as an additional choice variable in the model, the envelope theorem suggests that the response of reservation wages to reemployment bonuses does not have a first-order impact on individual values. Furthermore, the effect of the response of reservation wages on the government budget is captured by $\frac{dD}{db_e}$ in equation (8). Consequently, the presence of uncertainty in wage offers does not alter the welfare formula. A more rigorous treatment of this issue is in Appendix B.2.

Ex-ante Savings Response. The preceding analysis has focused on the problem that arises after individuals experience a job loss, disregarding the possibility that individuals may respond to policy changes even before losing their jobs. For instance, a more generous unemployment insurance (UI) policy could lead to reduced precautionary savings, as discussed by Engen et al. (Engen and Gruber, 2001). However, given that the policy change is marginal, the ex-ante response does not have an impact on the analysis presented in this section due to the envelope condition. Although

[7] Here, consumption and expenditure are distinct concepts. Consumption is considered to be the output produced by utilizing time and market goods (expenditure) as inputs. See Becker (1965) and Aguiar and Hurst (2005).

changes in reemployment bonuses (b_e) may influence pre-job loss asset choices, what is crucial for the welfare calculation is the direct impact of reemployment bonuses (b_e) on utility and the externality through the government budget constraint. Other effects resulting from individual choices cancel out due to the first-order conditions of the agent's optimization problem.

Additional fiscal externalities. In the context of UI, fiscal externalities stem from heightened expenditure on UI benefits and a contraction in tax revenue due to extended unemployment durations caused by worker moral hazard. In my model, I assume that all the taxes derived from the employed workers are spent on UI benefits for unemployed individuals. However, in reality, the government uses taxes collected from employed workers for other programs, such as Social Security and Medicare. Consequently, the reduction in tax revenue provoked by UI-induced extended unemployment results in additional welfare costs (Lawson, 2017). The same argument suggests that, when accounting for such additional fiscal externalities, reemployment bonuses could generate a larger welfare gain by facilitating the government's collection of taxes for other programs.

4 Optimal Policy in the Calibrated Economy

Based on the sufficient statistic approach, the analysis in the previous section reveals the potential for welfare improvement within the current UI system. Given the large welfare gain from the first dollar of reemployment bonuses, it would be natural to ask what is the globally optimal mix of reemployment bonuses and UI benefits. By fully calibrating the model, I numerically solve for the optimal combination of UI benefits and reemployment bonuses that maximizes welfare.

4.1 Quantitative Specification

The per-period utility is given by $U(c_t) - \psi(e_t)$ where

$$U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}, \quad \psi(e_t) = \theta \frac{e_t^{1+\xi}}{1+\xi}, \tag{28}$$

where γ represents the coefficient of relative risk aversion, θ is the weight on the utility cost of job search, and ξ is the parameter governing the convexity of the utility cost of job search.

I use $\gamma = 2.0$ for the coefficient of relative risk aversion, which is a standard value in the literature (e.g. Michelacci and Ruffo 2015). As before, I keep assuming $r = 0$ and $\beta = 1$. I normalize wages to be 1. I set the initial asset A_1 to the median asset in the Survey of Income and Program Participation (SIPP) reported by Chetty (2008).[8] Time length of the model is set to

[8] Using the SIPP, Chetty (2008) reports the median annual wage of unemployed people before job loss is \$17,780

Table 5: List of Parameters

Parameter	Description	Value	Target
	Externally set		
γ	Coefficient of relative risk aversion	2.0	Michelacci and Ruffo (2015)
r	Interest rate	0	
β	Discount factor	1	
A_1	Initial asset (relative to weekly wage)	5.16	Median asset in the SIPP (Chetty, 2008)
T	Time length of the model (weeks)	450	Unemployment rate of 5.4% (Chetty, 2008)
L	Liquidity constraint	0.0	
	Internally estimated		
θ	Utility cost of search: weight	378.6	Avg. unemployment duration of 24.3 weeks
ξ	Utility cost of search: convexity	0.020	Elasticity of duration w.r.t. UI of 0.53
$\bar{y}$	Nonlabor income	0.512	Consumption loss upon job loss of 10%

$T = 450$ weeks so that the unemployment rate in the model $u = \frac{D}{T}$ is 5.4% where $D = 24.3$ is targeted in the estimation. The borrowing limit is set to $L = 0$. UI benefits and reemployment bonuses in the baseline are set to $b_u = 0.5$ and $b_e = 0$. Since wages are normalized to 1, $b_u = 0.5$ represents the replacement rate of 50%.

The remaining parameters $(\theta, \xi, \bar{y})$ are determined by matching several data moments. The weight on job search disutility θ directly influences the amount of effort an unemployed worker puts forth. I target the average unemployment duration $D = 24.3$ to pin down θ. The curvature ξ is informed by the responsiveness of a worker's job search effort to a change in the relative value of employment versus unemployment. In particular, I determine ξ by targeting the elasticity of compensated unemployment duration with respect to UI benefits $\eta_{D_B,b_u} = 0.53$. Lastly, $\bar{y}$ is determined by targeting a 10 percent consumption loss upon unemployment. Specifically, I calculate the consumption drop by $(c_{u,T_B} - c_e)/c_e$ where c_{u,T_B} is the consumption level of an unemployed worker after $T_B = 26$ weeks while c_e is the consumption level that realizes if a worker never loses a job.[9] Table 5 summarizes all the parameters.

4.2 Optimal Joint Design of UI Benefits and Reemployment Bonuses

With the calibrated model, I numerically solve the following optimization problem.

$$\max_{b_u, b_e} W(\mathbf{b}) = V_1(A_1; \mathbf{b}, \tau(\mathbf{b})) \quad \text{where} \quad \tau(\mathbf{b}) = \frac{D_B b_u + (T_B - D_B) b_e}{T - D}. \tag{29}$$

and median liquid wealth is \$1,763. Since weekly wages are normalized to 1 in the model, I choose initial asset $A_1 = (\$17,780/52\text{weeks})/\$1,763 \approx 5.16$.

[9] Here, c_e is constant because $(1+r)\beta = 1$ and there is no uncertainty.

Table 6: Comparison of Baseline Economy with Economy under Optimal Policy

	Baseline	Optimal UI	Optimal Joint
Unemployment rate (%)	5.38	2.94	3.41
Consumption drop (%)	10.03	27.76	14.18
UI spending	8.69	2.23	5.17
RB spending	0.00	0.00	8.61
Welfare gain (%)	0.00	0.25	0.39
Policy parameters:			
UI benefit	0.50	0.18	0.49
Reemployment bonus	0.00	0.00	0.56

Note: UI refers to unemployment insurance. RB refers to reemployment bonuses. Welfare gain is measured in a welfare-equivalent increase in consumption.

The objective function is the value function in the initial period where the tax $\tau(\mathbf{b})$ satisfies the government budget constraint.

Table 6 summarizes the simulation results. First, I discuss the case where the government can choose only UI benefits. The first column represents the baseline economy while the second column corresponds to the case where b_e is fixed at zero, and only b_u is chosen to maximize welfare. The optimal UI benefit level in the calibrated economy is 18% of weekly wages. Although this value is lower than what other papers suggest, it still falls within the range of existing estimates (e.g., Hansen and Imrohoroğlu 1992, Gruber 1997). Consequently, the unemployment rate decreases from 5.38% in the baseline case to 2.94% in this counterfactual, resulting in reduced government spending on UI benefits. In the baseline economy, each unemployed worker receives UI benefits equivalent to 8.69 weeks of wages on average, whereas, in the counterfactual economy, this number decreases to 2.23. Additionally, unemployed workers experience a much larger decline in consumption of 28% compared to 10% in the baseline. Overall, workers in the counterfactual economy experience a 0.25% increase in welfare in terms of consumption equivalence. In other words, workers would be indifferent between the baseline economy and the counterfactual economy if their consumption in the baseline economy increased by 0.25%.

The third column of the same table presents the outcomes in an economy where both b_u and b_e are optimally chosen. The optimal combination of UI benefits and reemployment bonuses is determined to be $(b_u, b_e) = (0.49, 0.56)$. This finding suggests that the baseline UI benefit level of 0.5 in the economy happens to be close to the optimal level when the government also provides reemployment bonuses equivalent to 56% of weekly wages. It is worth noting that the optimal policy sets b_u close to b_e, resulting in a situation where an unemployed worker receives almost the same amount of benefits regardless of whether the worker finds a job or not. This effectively addresses the moral hazard associated with the provision of UI benefits. In this counterfactual scenario, the unemployment rate decreases significantly to 3.41%, compared to the 5.38% observed in the baseline economy. Despite the low unemployment rate, the decline in consumption upon job loss is 14%, only slightly lower than the 10% experienced in the baseline economy. Welfare increases by 0.39%, suggesting that incorporating reemployment bonuses into the UI system has a quantitatively large impact on welfare.

5 Conclusion

This paper studies the welfare impact of reemployment bonuses as a means to address worker moral hazard inherent in the UI system. By employing a dynamic model of consumption, saving, and job search, I derived a formula that quantifies the welfare impact of the first dollar of reemployment bonuses. I evaluate the formula levaraging the existing estimates of the response of unemployment duration to changes in UI benefits, I found that this initial one-dollar increase in reemployment bonuses leads to a welfare improvement equivalent to a lifetime income increase of $5.1. When evaluating the welfare impact using the Marginal Value of Public Funds (MVPF), I find that this impact exceeds many historical policy changes documented in the work of Hendren and Sprung-Keyser (2020).

Additionally, by fully calibrating the model, I extended the analysis to examine non-marginal policy changes. Notably, my findings indicate that the current UI benefit level, amounting to 50 percent of wages, is nearly optimal when complemented by the provision of reemployment bonuses equivalent to 56 percent of wages. This combination effectively mitigates worker moral hazard while ensuring consumption smoothing.

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Online Appendix (Not for Publication)

Appendix A Proofs

A.1 Properties of Job Search Effort

Here, I discuss how unemployed workers respond to a change in UI benefits or reemployment bonuses through their search effort. The optimal level of search effort is determined by solving the following first-order condition

$$\psi'(e_t(A_t;\mathbf{b},\tau)) = V^E_{t,t}(A_t;\mathbf{b},\tau) - V^U_t(A_t;\mathbf{b},\tau). \tag{4}$$

For $t \leq T_B$, I have

$$\frac{\partial e_t}{\partial b_u} = \frac{-\frac{\partial V^U_t(A_t;\mathbf{b},\tau)}{\partial b_u}}{\psi''(e_t)}, \tag{30}$$

$$\frac{\partial e_t}{\partial b_e} = \frac{\frac{\partial V^E_{t,t}(A_t;\mathbf{b},\tau)}{\partial b_e} - \frac{\partial V^U_t(A_t;\mathbf{b},\tau)}{\partial b_e}}{\psi''(e_t)}. \tag{31}$$

The envelope conditions from the individual optimization problem imply

$$\frac{\partial V^U_t(A_t;\mathbf{b},\tau)}{\partial b_u} = U'(c_{u,t}) + \sum_{k=t+1}^{T_B} S_{k|t+1} U'(c_{u,k}), \tag{32}$$

$$\frac{\partial V^E_{t,t}(A_t;\mathbf{b},\tau)}{\partial b_e} = \sum_{j=t}^{T_B} U'(c_{e,t,j}), \tag{33}$$

$$\frac{\partial V^U_t(A_t;\mathbf{b},\tau)}{\partial b_e} = e_{t+1}\frac{\partial V^E_{t+1,t+1}(A_{t+1};\mathbf{b},\tau)}{\partial b_e} + (1-e_{t+1})\frac{\partial V^U_{t+1}(A_{t+1};\mathbf{b},\tau)}{\partial b_e} \tag{34}$$

where $c_{u,k}$ represents the consumption of unemployed individuals in period k, $c_{e,t,k}$ represents the consumption of employed individuals in period k who find a job in period t, and $S_{k|t+1} = \prod_{j=t+1}^{k}(1-e_j)$ denotes the probability of remaining unemployed at the end of period k, given that the individual was unemployed at the beginning of period $t+1$.

The right-hand side of equation (30) is negative, implying that search effort is decreasing in UI benefits b_u. For search effort to be increasing in reemployment bonuses, I need to show $\frac{\partial V^E_{t,t}(A_t;\mathbf{b},\tau)}{\partial b_e} - \frac{\partial V^U_t(A_t;\mathbf{b},\tau)}{\partial b_e} > 0$. I prove it below under an additional assumption that $c_{e,s,t} = c_e$ for $s,t \leq T_B$. This assumption is reasonable unless the worker is constrained by liquidity and experiences rapid wage growth after reemployment. Based on this, equation (31) implies that the partial derivative $\frac{\partial e_t}{\partial b_e}$ is positive. Therefore, search effort is expected to increase with higher reemployment bonuses b_e.

Now, I show that $\frac{\partial V_{t,t}^E(A_t;\mathbf{b},\tau)}{\partial b_e} - \frac{\partial V_t^U(A_t;\mathbf{b},\tau)}{\partial b_e} > 0$. First, let $S_{K|k} = \prod_{j=k}^{K}(1-e_j)$ denote the probability of staying unemployed at the end of period K conditional on being unemployed at the beginning of period k. Note that

$$\begin{aligned}\frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \frac{\partial V_t^U(A_t)}{\partial b_e} &= \frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \left[e_{t+1}\frac{\partial V_{t+1,t+1}^E(A_{t+1})}{\partial b_e} + (1-e_{t+1})\frac{\partial V_{t+1}^U(A_{t+1})}{\partial b_e}\right] \\ &= \frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \frac{\partial V_{t+1,t+1}^E(A_{t+1})}{\partial b_e} + (1-e_{t+1})\left[\frac{\partial V_{t+1,t+1}^E(A_{t+1})}{\partial b_e} - \frac{\partial V_{t+1}^U(A_{t+1})}{\partial b_e}\right].\end{aligned}$$

Exploiting the envelope condition, I have

$$\frac{\partial V_{t,t}^E(A_t)}{\partial b_e} = \sum_{j=t}^{T_B} U'(c_{e,t,j}).$$

Therefore, the first two terms are

$$\frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \frac{\partial V_{t+1,t+1}^E(A_{t+1})}{\partial b_e} = U'(c_{e,t,t}) + \sum_{j=t+1}^{T_B}[U'(c_{e,t,j}) - U'(c_{e,t+1,j})].$$

Now I impose an approximation assumption that $c_{e,s,t} = c_e$ for $s,t \leq T_B$. Then the expression reduces down to

$$\frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \frac{\partial V_t^U(A_t)}{\partial b_e} = U'(c_e) + \sum_{k=t+1}^{T_B} S_{k|t+1}U'(c_e),$$

which is positive since the marginal utility is always positive.

A.2 Proof of Proposotion 1

Proof. Remember that the welfare function $W(\mathbf{b})$ is defined as $W(\mathbf{b}) = V_1(A_1;b,\tau(\mathbf{b}))$ given A_1 where $\tau(\mathbf{b})$ is the budget-balancing taxes. Differentiating this with respect to wage subsidies b_e yields

$$\frac{\partial W(\mathbf{b})}{\partial b_e} = \frac{\partial V_1}{\partial b_e} + \frac{\partial V_1}{\partial \tau}\frac{\partial \tau(\mathbf{b})}{\partial b_e}.$$

Exploiting the envelope conditions from individual's optimization problem, I have

$$\begin{aligned}
\frac{\partial V_1}{\partial b_e} &= e_1 \frac{\partial V_1^E}{\partial b_e} + (1-e_1)\frac{\partial V_1^U}{\partial b_e} \\
&= e_1 \sum_{t=1}^{T_B} U'(c_{e,1,t}) + S_1 e_2 \sum_{t=2}^{T_B} U'(c_{e,2,t}) + \cdots + S_{T_B-1} e_{T_B} U'(c_{e,T_B,T_B}) \\
&= e_1 U'(c_{e,1,1}) + \sum_{t=1}^{2} S_{t-1} e_t U'(c_{e,t,T_B}) + \cdots + \sum_{t=1}^{T_B} S_{t-1} e_t U'(c_{e,t,T_B}),
\end{aligned}$$

where $S_0 = 1$. Note that $\sum_{t=1}^{j} S_{t-1} e_t = 1 - S_j$. I obtain

$$\frac{\partial V_1}{\partial b_e} = \sum_{t=1}^{T_B} (1-S_t)\mathbb{E}_s[U'(c_{e,s,t})],$$

where

$$\mathbb{E}_s[U'(c_{e,s,t})] = \sum_{s=1}^{t} \kappa_s U'(c_{e,s,t}) \quad \text{where} \quad \kappa_s = \frac{S_{s-1}e_s}{1-S_t}. \tag{35}$$

Similarly, I obtain

$$\frac{\partial V_1}{\partial \tau} = -\sum_{t=1}^{T} (1-S_t)\mathbb{E}_s[U'(c_{e,s,t})].$$

The government budget constraint is given by

$$(T-D)\tau(\mathbf{b}) = D_B b_u + (T_B - D_B) b_e.$$

Differentiating this with respect to b_e yields

$$(T-D)\frac{\partial \tau(\mathbf{b})}{\partial b_e} - \frac{dD}{db_e}\tau(\mathbf{b}) = T_B - D_B + \frac{dD_B}{db_e}(b_u - b_e).$$

Arranging terms, I obtain

$$\frac{\partial \tau(\mathbf{b})}{\partial b_e} = \frac{T_B - D_B}{T-D}\left[1 + \frac{dD_B}{db_e}\frac{b_u - b_e}{T_B - D_B} + \frac{dD}{db_e}\frac{\tau(\mathbf{b})}{T_B - D_B}\right].$$

To get a money metric of the welfare impact of wage subsidies, I normalize $\frac{\partial W(\mathbf{b})}{\partial b_e}$ by the welfare gain from permanently increasing wages by \$1, i.e., $\frac{\partial V_1}{\partial w}$. Note that $\frac{\partial V_1}{\partial w} = -\frac{\partial V_1}{\partial \tau}$. Then, I have

$$\frac{\partial W(\mathbf{b})}{\partial b_e} \bigg/ \frac{\partial V_1}{\partial w} = \frac{\frac{\partial V_1}{\partial b_e}}{-\frac{\partial V_1}{\partial \tau}} - \frac{\partial \tau(\mathbf{b})}{\partial b_e}$$
$$= \frac{T_B - D_B}{T - D} \left[\frac{MU_{e,T_B} - MU_{e,T}}{MU_{e,T}} - \frac{dD_B}{db_e} \frac{b_u - b_e}{T_B - D_B} - \frac{dD}{db_e} \frac{\tau(\mathbf{b})}{T_B - D_B} \right],$$

where $MU_{e,t}$ is given by

$$MU_{e,t} = \sum_{k=1}^{t} \frac{1 - S_k}{t - D_B} \mathbb{E}_s[U'(c_{e,s,k})] \quad \text{for} \quad t = T_B, T.$$

□

A.3 Proof of Proposition 2

Proof. Let $t \leq T_B$. Remember that

$$\frac{\partial e_t}{\partial b_u} = \frac{-\frac{\partial V_t^U(A_t)}{\partial b_u}}{\psi''(e_t)}, \qquad \frac{\partial e_t}{\partial b_e} = \frac{\frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \frac{\partial V_t^U(A_t)}{\partial b_e}}{\psi''(e_t)}.$$

Exploiting the envelope condition, I obtain

$$\frac{\partial V_t^U(A_t)}{\partial b_u} = U'(c_{u,t}) + \sum_{k=t+1}^{T_B} S_{k|t+1} U'(c_{u,k}),$$
$$\frac{\partial V_{t,t}^E(A_t)}{\partial b_e} - \frac{\partial V_t^U(A_t)}{\partial b_e} = U'(c_e) + \sum_{k=t+1}^{T_B} S_{k|t+1} U'(c_e),$$

where the second equation is derived in Appendix A.1 under assumption (A.1). For expositional brevity, define $\mu_{e,t} = U'(c_e) + \sum_{k=t+1}^{T_B} S_{k|t+1} U'(c_e)$ and $\mu_{u,t} = U'(c_{u,t}) + \sum_{k=t+1}^{T_B} S_{k|t+1} U'(c_{u,k})$. Then I obtain

$$\frac{\partial e_t}{\partial b_e} = -\frac{\mu_{e,t}}{\mu_{u,t}} \frac{\partial e_t}{\partial b_u}.$$

This can be rewritten as

$$\frac{\partial(1 - e_t)}{\partial b_e} = -\frac{\mu_{e,t}}{\mu_{u,t}} \frac{\partial(1 - e_t)}{\partial b_u}. \tag{36}$$

First, I compute $\frac{\partial S_t}{\partial b_e}$. Remember that $S_t = \prod_{j=1}^{t}(1-e_j)$. Using the equation above, I have

$$\begin{aligned}
\frac{\partial S_t}{\partial b_e} &= \frac{\partial}{\partial b_e}\prod_{j=1}^{t}(1-e_j) \\
&= \sum_{k=1}^{t}\frac{\partial(1-e_k)}{\partial b_e}\prod_{j\neq k}(1-e_j) \\
&= -\sum_{k=1}^{t}\frac{\mu_{e,k}}{\mu_{u,k}}\frac{\partial(1-e_k)}{\partial b_u}\prod_{\tilde{t}\neq k}(1-e_j)
\end{aligned}$$

where the last equality comes from equation (36). Compensated unemployment duration D_B is defined as $D_B = \sum_{t=1}^{T_B} S_t$. Therefore, I have

$$\begin{aligned}
\frac{\partial D_B}{\partial b_e} &= \sum_{t=1}^{T_B}\frac{\partial S_t}{\partial b_e} \\
&= -\sum_{t=1}^{T_B}\left[\sum_{k=1}^{t}\frac{\mu_{e,k}}{\mu_{u,k}}\frac{\partial(1-e_k)}{\partial b_u}\prod_{\tilde{t}\neq k}(1-e_j)\right].
\end{aligned}$$

Now, I compute a bound on $\frac{\mu_{e,t}}{\mu_{u,t}}$. Using assumption (A.3), i.e., $c_{u,t} \leq c_{u,t-1}$, I obtain

$$\begin{aligned}
\frac{\mu_{e,t}}{\mu_{u,t}} &= \frac{U'(c_e) + \sum_{k=t+1}^{T_B} S_{k|t+1}U'(c_e)}{U'(c_{u,t}) + \sum_{k=t+1}^{T_B} S_{k|t+1}U'(c_{u,k})} \\
&\geq \frac{[1+\sum_{k=t+1}^{T_B} S_{k|t+1}]U'(c_e)}{[1+\sum_{k=t+1}^{T_B} S_{k|t+1}]U'(c_{u,T_B})} \\
&= \frac{U'(c_e)}{U'(c_{u,T_B})}.
\end{aligned}$$

Using the assumptions (A.2) $c_{u,t} \leq c_e$ and (A.3) $c_{u,t} \geq c_{u,t+1}$, I also have

$$\begin{aligned}
\frac{\mu_{e,t}}{\mu_{u,t}} &= \frac{U'(c_e) + \sum_{k=t+1}^{T_B} S_{k|t+1}U'(c_e)}{U'(c_{u,t}) + \sum_{k=t+1}^{T_B} S_{k|t+1}U'(c_{u,k})} \\
&\leq \frac{U'(c_e)}{U'(c_{u,t})} \\
&\leq \frac{U'(c_e)}{U'(c_{u,1})}.
\end{aligned}$$

Define

$$\overline{M}_{T_B} = \frac{U'(c_e)}{U'(c_{u,1})} \quad \underline{M}_{T_B} = \frac{U'(c_e)}{U'(c_{u,T_B})}.$$

Then, I obtain

$$
\begin{aligned}
-\frac{\partial D_B}{\partial b_e} &= \sum_{t=1}^{T_B}\left[\sum_{k=1}^{t}\frac{\mu_{e,k}}{\mu_{u,k}}\frac{\partial(1-e_k)}{\partial b_u}\prod_{\tilde{t}\neq k}(1-e_j)\right] \\
&\leq \sum_{t=1}^{T_B}\left[\sum_{k=1}^{t}\overline{M}_{T_B}\frac{\partial(1-e_k)}{\partial b_u}\prod_{\tilde{t}\neq k}(1-e_j)\right] \\
&= \overline{M}_{T_B}\sum_{t=1}^{T_B}\left[\sum_{k=1}^{t}\frac{\partial(1-e_k)}{\partial b_u}\prod_{\tilde{t}\neq k}(1-e_j)\right] \\
&= \overline{M}_{T_B}\sum_{t=1}^{T_B}\frac{\partial S_t}{\partial b_u} \\
&= \overline{M}_{T_B}\frac{\partial D_B}{\partial b_u}.
\end{aligned}
$$

In the same way, I can show that

$$
-\frac{\partial D_B}{\partial b_e} \geq \underline{M}_{T_B}\frac{\partial D_B}{\partial b_u}.
$$

Combining these, I obtain

$$
\underline{M}_{T_B}\frac{\partial D_B}{\partial b_u} \leq -\frac{\partial D_B}{\partial b_e} \leq \overline{M}_{T_B}\frac{\partial D_B}{\partial b_u}.
$$

□

A.4 Proof of Corollary 1

Proof. From Appendix A.2, remember that

$$
\frac{\partial W(\mathbf{b})}{\partial b_e}\bigg/\frac{\partial V_1}{\partial w} = \frac{T_B-D_B}{T-D}\left[\frac{MU_{e,T_B}-MU_{e,T}}{MU_{e,T}} - \frac{dD_B}{db_e}\frac{b_u-b_e}{T_B-D_B} - \frac{dD}{db_e}\frac{\tau(\mathbf{b})}{T_B-D_B}\right].
$$

Evaluating this at $b_e = 0$ yields

$$
\frac{\partial W(\mathbf{b})}{\partial b_e}\bigg/\frac{\partial V_1}{\partial w} = \frac{T_B-D_B}{T-D}\left[\frac{MU_{e,T_B}-MU_{e,T}}{MU_{e,T}} - \frac{dD_B}{db_e}\frac{b_u}{T_B-D_B} - \frac{dD}{db_e}\frac{D_B b_u}{(T_B-D_B)^2}\right].
$$

Assumption (A.1) implies $MU_{e,T_B} = MU_{e,T}$, so the first term in the bracket disappears. Using

Proposition 2, which requires assumptions (A.1)-(A.3), I obtain

$$\begin{aligned}\frac{\partial W(\mathbf{b})}{\partial b_e} \bigg/ \frac{\partial V_1}{\partial w} &= -\frac{T_B - D_B}{T - D}\left[\frac{dD_B}{db_e}\frac{b_u}{T_B - D_B} + \frac{dD}{db_e}\frac{D_B b_u}{(T_B - D_B)^2}\right] \\ &\geq \frac{T_B - D_B}{T - D}\left[\frac{dD_B}{db_u}\frac{b_u}{D_B}\frac{D_B}{T_B - D_B}\underline{M}_{T_B} + \frac{dD}{db_u}\frac{b_u}{D}\frac{D_B D}{(T_B - D_B)^2}\underline{M}_T\right] \\ &= \frac{T_B - D_B}{T - D}\left[\eta_{D_B, b_u}\frac{D_B}{T_B - D_B}\underline{M}_{T_B} + \eta_{D, b_u}\frac{D_B D}{(T_B - D_B)^2}\underline{M}_T\right].\end{aligned}$$

Since $\underline{M}_T = U'(c_e)/U'(c_{u,T}) \geq 0$ and $\eta_{D,b_u} \geq 0$, arranging terms yields

$$\frac{\partial W(\mathbf{b})}{\partial b_e} \bigg/ \frac{\partial V_1}{\partial w} \geq \frac{u}{1-u}\frac{D_B}{D}\underline{M}_{T_B}\eta_{D_B, b_u},$$

where $u = \frac{D}{T}$ is the fraction of periods during which a person is expected to be unemployed. $\square$

A.5 Derivation of MVPF of the Marginal Increase in b_u

The willingness to pay for a marginal increase in UI benefits b_u in terms of dollars during employment is given by

$$\begin{aligned}WTP &= \frac{\partial V_1}{\partial b_u} \bigg/ \frac{\partial V_1}{\partial w} \\ &= D_B \frac{MU_{u,T_B} - MU_{e,T}}{MU_{e,T}},\end{aligned}$$

where MU_{u,T_B} and $MU_{e,T}$ are the marginal utilities of consumption averaged over time:

$$MU_{u,T_B} = \sum_{t=1}^{T_B} \frac{S_t}{D_B} U'(c_{u,t}), \quad MU_{e,T} = \sum_{t=1}^{T} \frac{1 - S_t}{T - D_B} \mathbb{E}_s[U'(c_{e,s,t})].$$

Since it is difficult to track individual-level consumption at a week/month frequency over time, many papers just assume that $c_{u,t}$ and $c_{e,s,t}$ do not depend on t or s (Chetty 2009, Hendren and Sprung-Keyser 2020). Rigorously computing the willingness to pay for UI is beyond the scope of paper, and I simply let $c_{u,t} = c_u$ and $c_{e,s,t} = c_e$. Assuming CRRA utility $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$, I obtain

$$WTP = D_B \frac{c_u^{-\gamma} - c_e^{-\gamma}}{c_e^{-\gamma}} \approx D_B \left(1 + \gamma \frac{c_e - c_u}{c_e}\right),$$

where I take the first-order approximation with respect to c_e around c_u.

The net cost of a marginal increase in UI benefits b_u is given by

$$\begin{aligned} Net\ Cost &= (T-D)\frac{\partial \tau(\mathbf{b})}{\partial b_u} \\ &= D_B(1+FE_u), \end{aligned}$$

where $FE_u = \frac{1}{D_B}\left[\frac{dD_B}{db_u}(b_u - b_e) + \frac{dD}{db_e}\tau\right]$ represents the fiscal externality due to the behavioral response of unemployment people. Evaluating FE_u at the point $b_e = 0$ yields

$$\begin{aligned} FE_u &= \frac{1}{D_B}\left[\frac{dD_B}{db_u}b_u + \frac{dD}{db_u}\tau\right] \\ &= \eta_{D_B,b_u} + \eta_{D,b_u}\frac{u}{1-u}. \end{aligned}$$

Following Chetty (2008), I assume that $\eta_{D_B,b_u} = \eta_{D,b_u}$. Then, I obtain $FE_u = \frac{1}{1-u}\eta_{D_B,b_u}$. Combining these, I obtain

$$MVPF \approx \frac{1+\gamma\frac{c_e - c_u}{c_e}}{1+\frac{1}{1-u}\eta_{D_B,b_u}}.$$

Appendix B Extensions

B.1 Heterogeneity

The model in the main text assumes homogeneous individuals. In this subsection, I assume that individuals are heterogeneous in many respects including utility functions, income processes, initial assets, and liquidity constraints. I show that the welfare formula is almost unchanged in the presence of heterogeneity although additional conditions are required.

I begin by rewriting individual problems that account for individual heterogeneity. I assume that individuals are heterogeneous and each individual is indexed by $\theta \in \Theta$. If individual θ is employed at a period t with asset A_t, the value function is given by

$$V^E_{s,t}(A_t,\theta) = \max_{A_{t+1}\geq L(\theta)} U(A_t - A_{t+1} + y_{e,s,t}(\theta);\theta) + V^E_{s,t+1}(A_{t+1},\theta),$$

where the utility function U directly depends on θ. The liquidity constraint L and income y also depend on θ. More precisely, income is given by

$$y_{e,s,t}(\theta) = \begin{cases} w_{e,s,t}(\theta) + b_e & \text{if } t \leq T_B, \\ w_{e,s,t}(\theta) & \text{if } t > T_B, \end{cases}$$

where $w_{e,s,t}(\theta)$ is non-stochastic earnings for individual θ.

If individual θ with asset A_t does not find a job at the beginning of a period t, the value function is given by

$$V_t^U(A_t,\theta) = \max_{A_{t+1} \geq L(\theta)} U(A_t - A_{t+1} + y_{u,t}(\theta);\theta) + V_{t+1}(A_{t+1},\theta),$$

where income is given by

$$y_{u,t}(\theta) = \begin{cases} y_{u,t}(\theta) + b_e & \text{if } t \leq T_B, \\ y_{u,t}(\theta) & \text{if } t > T_B, \end{cases}$$

where $y_{u,t}(\theta)$ is non-stochastic non-labor income for individual θ. The value function of unemployed individual θ with asset A_t at the beginning of each period before searching for a job is given by

$$V_t(A_t,\theta) = \max_{e_t \in [0,1]} e_t V_{t,t}^E(A_t,\theta) + (1-e_t)V_t^U(A_t,\theta) - \psi(e_t;\theta),$$

where the disutility of job search ψ directly depends on θ. Let $e_t(\theta)$ be the search effort of individual θ at time t. I define a survival probability, expected unemployed duration, and expected insured duration for each individual as follows:

$$S_t(\theta) = \prod_{s=1}^{t}(1 - e_s(\theta)),$$
$$D(\theta) = \sum_{t=1}^{T} S_t(\theta),$$
$$D_B(\theta) = \sum_{t=1}^{T_B} S_t(\theta).$$

The government budget constraint is given by

$$(T - \overline{D})\tau = \overline{D}_B b_u + (T_B - \overline{D}_B) b_e,$$

where

$$\overline{D} = \int_{\theta \in \Theta} D(\theta) dF(\theta),$$
$$\overline{D}_B = \int_{\theta \in \Theta} D_B(\theta) dF(\theta).$$

Let $\tau(\mathbf{b})$ be the tax as a function of UI benefits and wage subsidies b implied by the budget constraint above. I define the welfare function $W(\mathbf{b})$ as

$$W(\mathbf{b}) = \int_{\theta \in \Theta} V_1(A_1(\theta),\theta; b, \tau(\mathbf{b})) dF(\theta),$$

where F is the distribution of θ in the population and $A_1(\theta)$ is the initial asset exogenously given to individual θ. Here I make the value function explicitly depend on the government policy (b,τ). My objective here is to characterize $\frac{\partial W(\mathbf{b})}{\partial b_e}$.

Differentiating the welfare function with respect to b_e yields

$$\frac{\partial W(\mathbf{b})}{\partial b_e} = \int_{\theta\in\Theta}\left[\frac{\partial V_1(A_1(\theta),\theta;b,\tau)}{\partial b_e} + \frac{\partial V_1(A_1(\theta),\theta;b,\tau)}{\partial \tau}\frac{\partial \tau(\mathbf{b})}{\partial b_e}\right] dF(\theta).$$

The same calculation as in the proof of Proposition 1 implies

$$\begin{aligned}\frac{\partial V_1(A_1(\theta),\theta;b,\tau)}{\partial b_e} &= \sum_{t=1}^{T_B}(1-S_t(\theta))\mathbb{E}_{s,\theta}[U'(c_{e,s,t}(\theta);\theta)],\\ \frac{\partial V_1(A_1(\theta),\theta;b,\tau)}{\partial \tau} &= -\sum_{t=1}^{T}(1-S_t(\theta))\mathbb{E}_{s,\theta}[U'(c_{e,s,t}(\theta);\theta)],\end{aligned}$$

where

$$\mathbb{E}_{s,\theta}[U'(c_{e,s,t}(\theta);\theta)] = \sum_{s=1}^{t}\frac{S_{s-1}(\theta)e_s(\theta)}{1-S_t(\theta)}U'(c_{e,s,t}(\theta);\theta).$$

The budgetary impact of wage subsidies is given by

$$\frac{\partial \tau(\mathbf{b})}{\partial b_e} = \frac{T_B-\overline{D}_B}{T-\overline{D}}\left[1+\frac{d\overline{D}_B}{db_e}\frac{b_u-b_e}{T_B-\overline{D}_B}+\frac{d\overline{D}}{db_e}\frac{\tau(\mathbf{b})}{T_B-\overline{D}_B}\right].$$

To get a money metric of the welfare impact of wage subsidies, I normalize $\frac{\partial W(\mathbf{b})}{\partial b_e}$ by the average welfare gain from permanently increasing wages by \$1, i.e., $\int \frac{\partial V_1}{\partial w}dF(\theta)$. Note that $\frac{\partial V_1}{\partial w} = -\frac{\partial V_1}{\partial \tau}$. Then, I have

$$\begin{aligned}\frac{\frac{\partial W(\mathbf{b})}{\partial b_e}}{\int_{\theta\in\Theta}\frac{\partial V_1}{\partial w}dF(\theta)} &= \frac{\int_{\theta\in\Theta}\frac{\partial V_1(A_1(\theta),\theta;b,\tau)}{\partial b_e}dF(\theta)}{-\int_{\theta\in\Theta}\frac{\partial V_1(A_1(\theta),\theta;b,\tau)}{\partial \tau}dF(\theta)} - \frac{\partial \tau(\mathbf{b})}{\partial b_e}\\ &= \frac{T_B-\overline{D}_B}{T-\overline{D}}\left[\frac{\overline{MU}_{e,T_B}-\overline{MU}_{e,T}}{\overline{MU}_{e,T}} - \frac{d\overline{D}_B}{db_e}\frac{b_u-b_e}{T_B-\overline{D}_B} - \frac{d\overline{D}}{db_e}\frac{\tau(\mathbf{b})}{T_B-\overline{D}_B}\right],\end{aligned}$$

where $\overline{MU}_{e,t}$ is given by

$$\overline{MU}_{e,t} = \int_{\theta\in\Theta}\sum_{k=1}^{t}\frac{1-S_k(\theta)}{t-\overline{D}_B}\mathbb{E}_{s,\theta}[U'(c_{e,s,k}(\theta);\theta)]dF(\theta) \quad \text{for} \quad t=T_B,T.$$

How is this formula in the presence of heterogeneity different from the formula (8) that is derived under the assumption that individuals are homogeneous? One difference is that the average marginal utility $MU_{e,t}$ in equation (8) is now replaced by $\overline{MU}_{e,t}$ which is $MU_{e,t}$ averaged over individuals. Another difference is that the expected unemployment duration D and the expected insured duration D_B are also replaced by the ones averaged over individuals $\overline{D}$ and $\overline{D}_B$.

Now I evaluate the formula at $b_e = 0$, and in addition, I impose the assumption (A.1). Then, I obtain

$$\frac{\frac{\partial W(\mathbf{b})}{\partial b_e}}{\int_{\theta\in\Theta}\frac{\partial V_1}{\partial w}dF(\theta)} = -\frac{T_B-\overline{D}_B}{T-\overline{D}}\left[\frac{d\overline{D}_B}{db_e}\frac{b_u}{T_B-\overline{D}_B}+\frac{d\overline{D}}{db_e}\frac{\overline{D}_B b_u}{(T_B-\overline{D}_B)^2}\right].$$

Proposition 2 cannot be directly applied to $\overline{D}_B$. Instead, I have

$$\frac{\partial \overline{D}_B}{\partial b_e} = \int_{\theta\in\Theta}\frac{\partial D_B(\theta)}{\partial b_e}dF(\theta),$$

which implies

$$\int_{\theta\in\Theta}\underline{M}_{T_B}(\theta)\frac{\partial D_B(\theta)}{\partial b_u}dF(\theta) \leq -\frac{\partial \overline{D}_B}{\partial b_e} \leq \int_{\theta\in\Theta}\overline{M}_{T_B}(\theta)\frac{\partial D_B(\theta)}{\partial b_u}dF(\theta),$$

where

$$\overline{M}_{T_B}(\theta) = \frac{U'(c_e(\theta);\theta)}{U'(c_{u,1}(\theta);\theta)}, \quad \underline{M}_{T_B}(\theta) = \frac{U'(c_e(\theta);\theta)}{U'(c_{u,T_B}(\theta);\theta)}.$$

If the heterogeneity in the population is such that $\underline{M}_{T_B}(\theta)$ and $\overline{M}_{T_B}(\theta)$ are independent of $\frac{\partial D_B(\theta)}{\partial b_u}$, then the inequality simplifies to

$$\int_{\theta\in\Theta}\underline{M}_{T_B}(\theta)dF(\theta)\frac{\partial \overline{D}_B(\theta)}{\partial b_u} \leq -\frac{\partial \overline{D}_B}{\partial b_e} \leq \int_{\theta\in\Theta}\overline{M}_{T_B}(\theta)dF(\theta)\frac{\partial \overline{D}_B(\theta)}{\partial b_u}.$$

As in Corollary 1, let $\underline{M}_T = 0$. Then, arranging terms yields

$$\frac{\frac{\partial W(\mathbf{b})}{\partial b_e}}{\int_{\theta\in\Theta}\frac{\partial V_1}{\partial w}dF(\theta)} \geq \frac{u}{1-u}\frac{\overline{D}_B}{\overline{D}}\eta_{\overline{D}_B,b_u}\int_{\theta\in\Theta}\underline{M}_{T_B}dF(\theta),$$

where $u = \frac{\overline{D}}{T}$ is the fraction of periods during which a person is expected to be unemployed, and $\eta_{\overline{D}_B,b_u}$ is the elasticity of insured duration with respect to UI benefits. For the parameters $(u,\overline{D}_B,\overline{D},\eta_{\overline{D}_B,b_u})$, the same parameter values can be used as in Table 1. The remaining thing to be determined is $\int_{\theta\in\Theta}\underline{M}_{T_B}dF(\theta)$. If the utility for consumption is homogeneous (i.e., $U(c;\theta)=U(c)$) and is given by CRRA function, then

$$\int_{\theta\in\Theta}\underline{M}_{T_B}dF(\theta) = \int_{\theta\in\Theta}\left(\frac{c_{u,T_B}(\theta)}{c_e(\theta)}\right)^{\gamma}dF(\theta).$$

If CRRA parameter is $\gamma \geq 1$, which is a standard assumption, then Jensen's inequality implies

$$\int_{\theta\in\Theta} \underline{M}_{T_B} dF(\theta) \geq \left[\int_{\theta\in\Theta} \frac{c_{u,T_B}(\theta)}{c_e(\theta)} dF(\theta)\right]^{\gamma},$$

where the expression inside the bracket is the average consumption due to unemployment. Together with assumption (A.4), I obtain

$$\begin{aligned} \frac{\frac{\partial W(\mathbf{b})}{\partial b_e}}{\int_{\theta\in\Theta} \frac{\partial V_1}{\partial w} dF(\theta)} &\geq \frac{u}{1-u} \frac{\overline{D}_B}{\overline{D}} \eta_{\overline{D}_B, b_u} \left[\int_{\theta\in\Theta} \frac{c_{u,T_B}(\theta)}{c_0(\theta)} dF(\theta)\right]^{\gamma} \\ &\approx \frac{u}{1-u} \frac{\overline{D}_B}{\overline{D}} \eta_{\overline{D}_B, b_u} \left[\frac{\int_\theta c_{u,T_B}(\theta) dF(\theta)}{\int_\theta c_0(\theta) dF(\theta)}\right]^{\gamma}, \end{aligned}$$

where the second line is based on the first-order approximation around the averages of $c_{u,T_B}(\theta)$ and $c_e(\theta)$ in the population. As in Section 3, the last term can be computed based on high-frequency average consumption profile reported in, for example, Ganong and Noel (2019). The resulting welfare impact is the same as in Section 3.

B.2 Stochastic Wage Offers

The discussion here mostly follows Chetty (2008) except that I differentiate the welfare function with respect to wage subsidies instead of UI benefits. As before, individuals exert search effort e_t but this time I assume that conditional on getting a job offer, the wage offer is stochastic and follows a distribution F_w. If individuals are employed at a period t with asset A_t and wage w, their value function is given by

$$V^E_{s,t}(A_t, w) = \max_{A_{t+1}\geq L} U(A_t - A_{t+1} + y_{e,s,t}) + V^E_{s,t+1}(A_{t+1}, w),$$

where income is given by

$$y_{e,s,t} = \begin{cases} w + b_e & \text{if } t \leq T_B, \\ w & \text{if } t > T_B. \end{cases}$$

If individuals with asset A_t does not find a job at the beginning of a period t, their value function is given by

$$V^U_t(A_t) = \max_{A_{t+1}\geq L} U(A_t - A_{t+1} + y_{u,t}) + V_{t+1}(A_{t+1}),$$

where $y_{u,t}$ is the same as in the main text.

Now consider the search decision of individuals. Individuals follow a reservation wage policy. Letting R_t be the reservation wage at t, the value function of unemployed people with asset A_t at the beginning of each period before searching for a job is given by

$$V_t(A_t) = \max_{e_t \in [0,1], R_t} e_t \Pr(w \geq R_t) \mathbb{E}[V_t^E(A_t, w) | w \geq R_t] + (1 - e_t \Pr(w \geq R_t)) V_t^U(A_t; b, \tau) - \psi(e_t).$$

This optimization problem is different from the one in the main text in that individuals now can choose the reservation wage R_t in addition to the search effort e_t.

As in the main text, I define the welfare function as $W(\mathbf{b}) = V_1(A_1; b, \tau(\mathbf{b}))$ given A_1. Differentiating this with respect to wage subsidies b_e yields

$$\frac{\partial W(\mathbf{b})}{\partial b_e} = \frac{\partial V_1}{\partial b_e} + \frac{\partial V_1}{\partial \tau} \frac{\partial \tau(\mathbf{b})}{\partial b_e}.$$

Exploiting the envelope conditions from individual's optimization problem, I have

$$\frac{\partial V_1}{\partial b_e} = e_1 \Pr(w \geq R_1) \frac{\partial \mathbb{E} V_1^E}{\partial b_e} + (1 - e_1 \Pr(w \geq R_1)) \frac{\partial V_1^U}{\partial b_e}.$$

By defining a survival probability as $S_t = \prod_{j=1}^{t} [1 - e_j \Pr(w \geq R_j)]$, the same calculation as in Appendix A.2 yields

$$\frac{\partial V_1}{\partial b_e} = \sum_{t=1}^{T_B} (1 - S_t) \mathbb{E}_s[U'(c_{e,s,t})], \quad \frac{\partial V_1}{\partial \tau} = -\sum_{t=1}^{T} (1 - S_t) \mathbb{E}_s[U'(c_{e,s,t})].$$

This is exactly the same as the case where wage offers are not stochastic except that the survival probability is now redefined so that it reflects the probability that an offered wage is above the reservation wage. Introducing stochastic wage offers into the model does not change the welfare formula because the additional variable R_t shows up in the formula only through the survival probability S_t. Since the government budget constraint is not affected by the introduction of stochastic wage offers, the formula for the welfare impact of the marginal change in b_e in the presence of stochastic wage offers is the same as the one in the main text.