# IVCV vs TSLS Bias

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## Data Generating Process

- Set $n_{per}$ – the number of observations per treatment group
- Assign each of our $N$ observations to $K$ treatment groups, each group with $n_{per}$ observations
  - For $i = 1, \ldots, N$
    - Let $i_k$ represent unit $i$'s treatment group
    - $U_i \sim \mathcal{N}(1, 25)$, the unobserved confounder
    - $\begin{pmatrix} X_{i,1} \\ X_{i,2} \end{pmatrix} \sim \mathcal{MN}\left(\begin{pmatrix} \gamma_k + 5U_i \\ \gamma_k \end{pmatrix}, \begin{pmatrix} 1 & 0.64 \\ 0.64 & 1 \end{pmatrix}\right)$, where $\gamma_k$ is a group-level fixed effect
    - $Y_i = X_{i,1} + X_{i,2} + 5 \cdot U_i + \varepsilon_i, \varepsilon_i \sim \mathcal{N}(0, 1)$

Suppose we have $P$ experiments, each with two treatment assignments (i.e., treated or control). Then, we would have $K = 2^P$ treatment groups. We assume that $\gamma_k$ is the number of experiments that a unit belongs to the treatment group for. In other words, being assigned to the treatment group for an additional experiment would increase $\mathbb{E}[X_{i,1}]$ and $\mathbb{E}[X_{i,2}]$ by 1. Suppose unit $i$ is treated in 10 experiments; then the fixed effect for unit $i$'s treatment group is 10: $\gamma_{i_k} = 10$.

Additionally, $X_1$ is the endogeneous feature whose causal effect we are interested in. The treatment effect is 1, i.e., a 1 unit increase in $X_1$ will cause a 1 unit increase in $Y$, in expectation.

## Experiments

We considered two different experimental setups: one in which we set $n_{per}$ and increase $K$ by increasing the number of experiments; and another in which we fix $K$ and increase $n_{per}$. We consider $n_{per} \in \{1000, 2000, \ldots, 10000\}$ and $K \in \{2^1, 2^2, , \ldots, 2^{10}\}$. For each combination of $n_{per}$ and $K$, we compare the results using the Instrumental Variables Cross Validation (IVCV) procedure and traditional Two Stage Least Squares (TSLS) using the squared error:

$$SE = \left(\hat{\beta} - 1\right)^2.$$

## Results

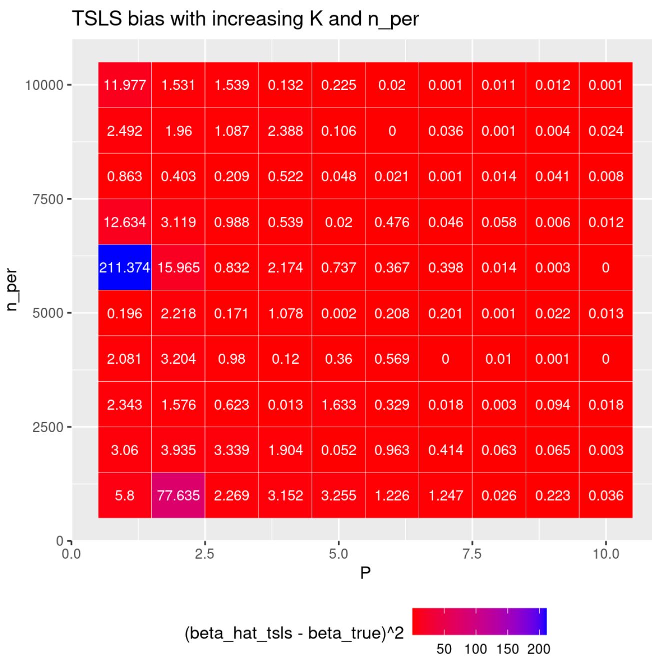


Figure 1: We increase the number of experiments $P$ with binary treatments from 1 to 20 on the x-axis and increase $n_per$ from 1000 to 10000 on the y-axis. For each $(P, n_{per})$ tuple, we display the squared error of the Two Stage Least Squares estimate, represented by color and shown in text. Brighter red blocks indicate less biased estimates while darker, blue blocks represent more biased estimates.

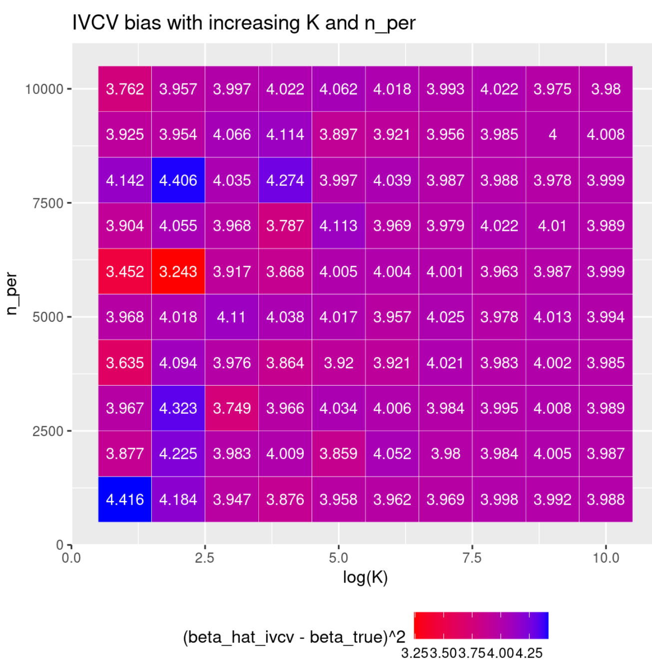


Figure 2: We increase the number of experiments $P$ with binary treatments from 1 to 20 on the x-axis and increase $n_per$ from 1000 to 10000 on the y-axis. For each $(P, n_{per})$ tuple, we display the squared error of the Instrumental Variables Cross Validation procedure estimate, represented by color and shown in text. Brighter red blocks indicate less biased estimates while darker, blue blocks represent more biased estimates.