Supplemental material to: "Line-graph qubit routing: from kagome to heavy-hex and more"

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Abstract

We showcase and benchmark line-graph qubit routing by routing random circuits (with fixed virtual graph) and circuits needed for the quantum simulation of the Heisenberg antiferromagnet on various graphs. The code implementing line-routing itself is found in line_graph_routing.py and maps Qiskit quantum circuits to Qiskit quantum circuits. (This can be altered to other circuit libraries with little effort.) We benchmark our results against other methods. The interactive version of this document is line_graph_routing.ipynb.

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1 Requirements

This notebook should typically run after installing the following packages with pip (or conda). In a terminal, run

pip install qiskit[visualization]

or

pip install 'qiskit[visualization]'

and

pip install netket networkx tabulate

Note Netket currently needs Python 3.9 (and SciPy >= 1.9.3). Netket is only used to generate patches of the kagome lattice as graphs and not for line-graph routing itself. This notebook was tested with a pip environment that can be recreated with requirements.txt by running pip install-requirements.txt (after creating a new environment).

The file line_graph_routing.py should be placed in the same folder as the current notebook.

```
import line_graph_routing as lgr
import networkx as nx
```

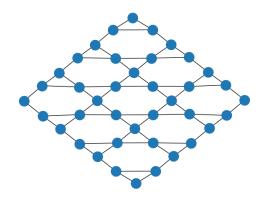
2 Kagome to heavy-hex

2.1 Random

Create a random circuit on a patch of the kagome lattice of 3×3 unit cells and show the circuit's coupling graph. With probability 2/5, a CNOT is placed along an edge of the connectivity graph. With a probability 3/5 a gate from $\{H,S,T\}$ is chosen uniformly at random and placed at a random node.

```
lg = lgr.kagome(3, 3)
qc = lgr.random_circuit(lg, 10**4)
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
print(qc.depth())
```

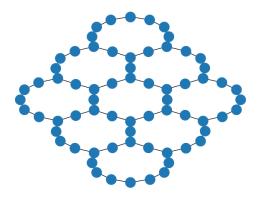
755



Route the circuit to a circuit with heavy-hex coupling graph.

```
qc = lgr.line_graph_route(qc)
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
print(qc.depth())
```

1337



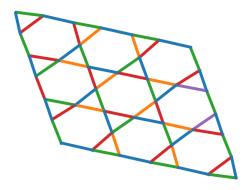
2.2 Quantum simulation

We base our circuits on edge colorings of the (kagome) lattice by identifying every color with a layer of HEIS gates. One of the colors (color '0') doubles as a color specifying the initial state by indicating along which edges singlet states are placed. The entire circuit is repeated p times, excluding initial state preparation. Every HEIS gets its own parameter. These parameters can later be bound to specific values to obtain circuits for dynamical quantum simulation or for simulated adiabatic state preparation.

First, create and show an edge coloring of the kagome lattice.

```
lg = lgr.edge_coloring(lg)
lgr.draw_edge_coloring(lg)
```

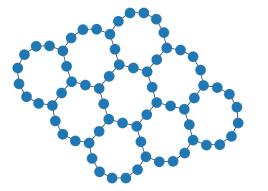
Matching is perfect Edge coloring is not minimal



Create the associated circuit, route it to heavy-hex hardware and show the coupling graph of the routed circuit.

```
p = 1
qc = lgr.heis_circuit(lg, p)
print(qc.depth())
qc = lgr.line_graph_route(qc)
print(qc.depth())
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
```

6 15

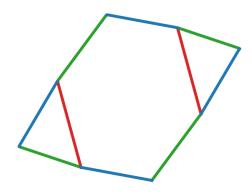


Since the above circuit has the connectivity of a 2D graph, the corresponding quantum circuit diagram will not be very insightful. However, for a single unit cell of the kagome lattice, the routed quantum circuit becomes a circuit on a circle, which allows for a clear representation as a quantum circuit diagram. We first

create an edge coloring of the unit cell patch.

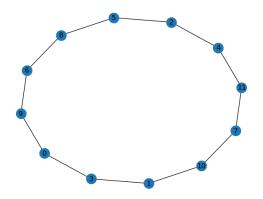
```
lg = lgr.kagome(1, 1)
lg = lgr.edge_coloring(lg)
lgr.draw_edge_coloring(lg)
```

Matching is perfect
Edge coloring is not minimal



Create the HEIS circuit ased on this coloring, map it to heavy-hex hardware, and show the coupling graph of the routed circuit.

7



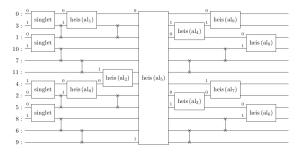
Show the circuit diagram of the routed circuit, with parameters al_i.

```
wo = [0, 3, 1, 10, 7, 11, 4, 2, 5, 8, 6, 

→9] # In the circuit diagram, place 

→ qubits in this order.

qc.draw('latex', wire_order = wo)
```



The fircuit depth can be reduced further by replacement of the initial and final SWAP gates between qubits (10,7) and (8,6) by a relabeling of those qubits.

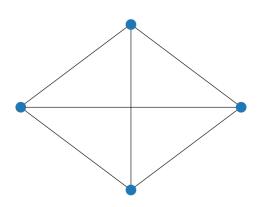
3 Complete graph to star graph

3.1 Random

Create a random circuit on the complete graph of four nodes and show the circuit's coupling graph.

```
n = 5
lg = nx.complete_graph(4)
qc = lgr.random_circuit(lg, 10**2)
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
print(qc.depth())
```

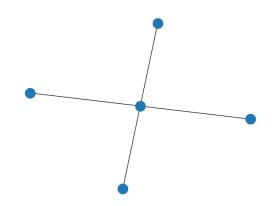
56



Route the circuit to a circuit with star-graph connectivity.

```
qc = lgr.line_graph_route(qc)
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
```

108



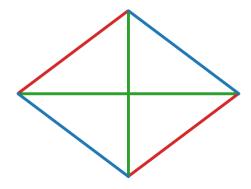
3.2 Quantum simulation

As before, circuits are defined by identifying every color with a layer of HEIS-gates. For more details, see the kagome to heavy-hex section.

Create and show an edge coloring of the complete graph.

```
lg = lgr.edge_coloring(lg)
lgr.draw_edge_coloring(lg)
```

Matching is perfect
Edge coloring is not minimal

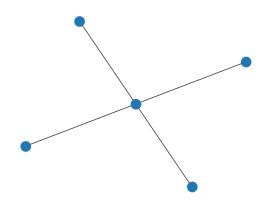


Create the associated circuit, route it to heavy-hex hardware, and show the coupling graph of the routed circuit.

```
p = 1
qc = lgr.heis_circuit(lg, p)
print(qc.depth())
qc = lgr.line_graph_route(qc)
```

```
print(qc.depth())
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
```

4 20



We do not show the circuit diagram in this case because the routed circuit is not a circuit on a line.

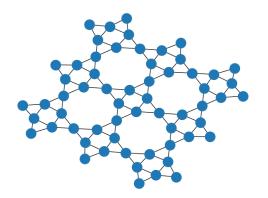
4 Shuriken to heavy squareoctagon

4.1 Random

Create a random circuit on a patch of the shuriken lattice of 3×3 unit cells.

```
n = 5
lg = lgr.shuriken(3, 3)
qc = lgr.random_circuit(lg, 10**4)
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
print(qc.depth())
```

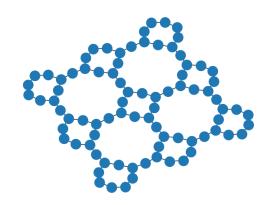
517



Route the circuit to a circuit with heavy-squareoctagon connectivity.

```
qc = lgr.line_graph_route(qc)
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
print(qc.depth())
```

877



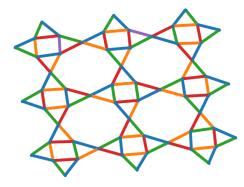
4.2 Quantum simulation

As before, circuits are defined by identifying every color with a layer of HEIS-gates. For more details, see the kagome to heavy-hex section.

Create and show an edge coloring of the shuriken lattice.

```
lg = lgr.edge_coloring(lg)
lgr.draw_edge_coloring(lg)
```

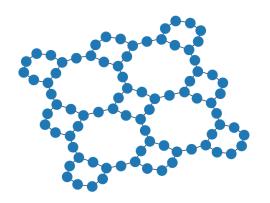
Matching is perfect Edge coloring is not minimal

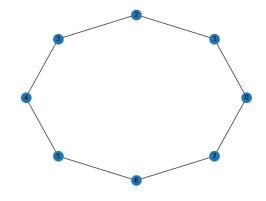


Create the associated circuit, route it to heavy-square-octagon hardware, and show the coupling graph of the routed circuit.

```
p = 1
qc = lgr.heis_circuit(lg, p)
print(qc.depth())
qc = lgr.line_graph_route(qc)
print(qc.depth())
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
```

6 12



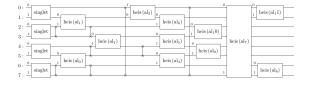


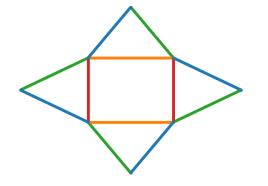
Again, the resulting circuit diagram will not be very insightful, but it will be for a single-unit cell patch of the shuriken lattice.

```
lg = lgr.shuriken(1, 1)
lg = lgr.edge_coloring(lg)
lgr.draw_edge_coloring(lg)
```

Matching is perfect Edge coloring is minimal







```
p = 1
qc = lgr.heis_circuit(lg, p)
print(qc.depth())
qc = lgr.line_graph_route(qc)
```

5 Checkerboard to heavysquare

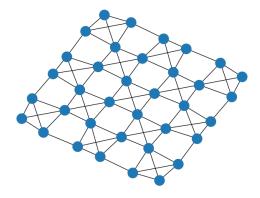
```
m = 2.5 # For the checkerboard lattice, 

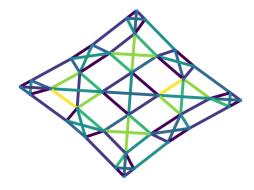
specify dimentions in nodes by nodes.

lg = lgr.checkerboard(m, m)
qc = lgr.random_circuit(lg, 10**4)
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
print(qc.depth())
```

872

5

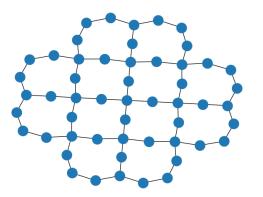




Route the circuit to a circuit with a heavy-square coupling graph.

```
qc = lgr.line_graph_route(qc)
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
print(qc.depth())
```

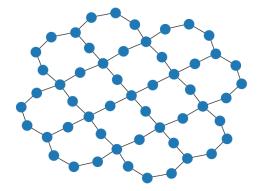
1622



Create the associated circuit, route it to heavy-square-octagon hardware, and show the coupling graph of the routed circuit.

```
p = 1
qc = lgr.heis_circuit(lg, p)
print(qc.depth())
qc = lgr.line_graph_route(qc)
print(qc.depth())
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
```

8 33



5.1 Quantum simulation

As before, circuits are defined by identifying every color with a layer of HEIS-gates. For more details, see the kagome to heavy-hex section.

Create and show an edge coloring of the checkerboard lattice

```
lg = lgr.edge_coloring(lg)
lgr.draw_edge_coloring(lg, spectral=True)

→# Use spactral method to find location

→ of nodes.
```

Matching is perfect Edge coloring is not minimal

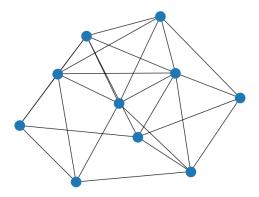
6 Random line graph to random heavy graph

6.1 Random

Create a random circuit on a random graph with 6 nodes and show the circuit's coupling graph. For details on radom_line_graph generation, see its function definition in line_graph_routing.py.

```
n = 6
lg = lgr.random_line_graph(6)
qc = lgr.random_circuit(lg, 10**3)
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
print(qc.depth())
```

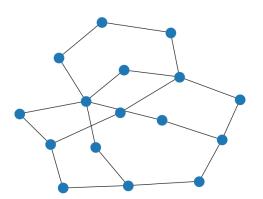
287



Route the circuit to a circuit with the associated heavy connectivity.

```
qc = lgr.line_graph_route(qc)
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
print(qc.depth())
```

576



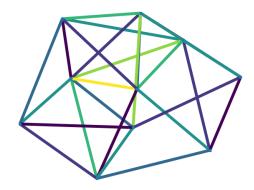
6.2 Quantum simulation

As before, circuits are defined by identifying every color with a layer of HEIS-gates. For more details, see the kagome to heavy-hex section.

Create and show an edge coloring of random graph. The method we use to find a perfect matching (needed for initial state preparation) is limited and may not find a perfect matching even if it exists. If a perfect matching is not found, try to create another random line graph (i.e., evaluate the two cells above) or use more sophisticated (or brute-force) methods to find a perfect matching.

```
lg = lgr.edge_coloring(lg)
lgr.draw_edge_coloring(lg)
```

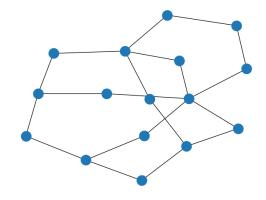
Matching is perfect
Edge coloring is not minimal



Create the associated circuit, route it to heavy-hex hardware and show the coupling graph of the routed circuit.

```
p = 1
qc = lgr.heis_circuit(lg, p)
print(qc.depth())
qc = lgr.line_graph_route(qc)
print(qc.depth())
cg = lgr.coupling_graph(qc)
nx.draw_kamada_kawai(cg)
```

8 32



We do not show the circuit diagram in this case because the routed circuit is not a circuit on a line.

7 Benchmarking

We benchmark line graph routing by performing the above routing tasks (but for larger unit cells) using both line-graph qubit routing and all methods available in qiskit. These methods are 'basic', 'lookahead', 'stochastic', and 'sabre' [1].

The benchmarking settings are specified by the following options: - name The name of the virtual graph, either kagome, shuriken or complete. - size. In case of kagome and shuriken: the size of the patch in unit cells by unit cells. In case of complete: the number of nodes of the complete graph. - circuit type. Either quantum simulation or random, as presented in this notebook. - p In case of kagome and shuriken: the number of cycles of the circuit. In case of complete: the number of random gates from the set H,T,S,CNOT. - repetitions. The number of runs for the methods sabre and stochastic. The methods line-graph and basic are deterministic and hence only run once. Correspondingly, the reported total time pertains to the time taken for this single run in case of the latter two methods. - optimization_level. Either 0, 1, 2, or 3. This specifies the optimization level used for the routing methods implemented in qiskit [2]. This parameter is passed directly to Qiskit's transpiler [1]. - methods. The methods to benchmark line graph transpilation against. Must be a list containing elements from ['sabre', 'basic', 'lookahead', stochastic]. These methods are passed directly to Qiskit's transpiler [1].

The methods sabre and stochastic are probabilistic, achieving a different routing each time they are run, and hence achieve different performance characteristics with each run. We therefore run these methods repetitions times and report the average, confidence interval, and best performance out of these runs. Error bars on the data show the (symmetrized) 95% confidence interval and are obtained by bootstrapping the data. The error interval for num_qubits is sometimes given by nan because in those cases the number of qubits was equal for all runs. The routing methodsline-graph and basic are deterministic and for these we enforce repetitions=1.

We consider the following performance characteristics. - method The routing method. - av. n_swaps The average number of swaps obtained among the repetitions runs of the routing method. - min n_swap The number of swaps of the run that achieved the lowest depth. - av. depth The average depth of the routed circuits among the repetitions runs of the routing method. We focus on the performance of routing so none of the gates in any of the routing methods are compiled into hardware native gates. That is, for the purposes of assessing routing performance, we assume the gate set SWAP, HEIS, H, X, Z CNOT for the quantum simulation circuits. For the random circuits we assume SWAP, CNOT, H, S, T. - min depth The minimum depth among the repetitions runs of the routing method. - av. n_qubits The average number obtained among the repetitions runs of the routing method. - av. time The total wall clock time (in seconds) needed to perform all repetitions runs of the routing method. - av. time The average (minimum) wall clock time of the repetitions routing runs. - min. time The number wall clock run time of the run that achieved the lowest depth.

 $[1] \ Qiskit \ 0.43.0 \ documentation, \ https://qiskit.org/documentation/stubs/qiskit.compiler.transpile.html, \ accessed \ 11h \ May \ 2023.$

[2] https://github.com/Qiskit/qiskit-terra/tree/main/qiskit/transpiler/preset passmanagers

7.0.1 Quantum simulation, kagome and shuriken, agianst SABRE

```
'circuit_type': 'quantum_simulation',
                     'p': p,
                     'repetitions' : 16,
                     'optimization_level' : optimization_level,
                     'methods' : ['sabre']
                settings.append(setting)
## Uncomment to rerun benchmarks. This takes a couple of hours.
#results=[]
#for setting in settings:
    result=lgr.benchmark(**setting)
    results.append(result)
     lgr.print_benchmark(result)
#
#with open('benchmark_data/kagome_shuriken.pkl','wb') as f:
    pickle.dump(results,f)
#Load previously obtained results from disk and show them.
import pickle
with open('benchmark_data/kagome_shuriken.pkl','rb') as f:
    results=pickle.load(f)
for result in results:
    lgr.print_benchmark(result)
```

```
name = kagome, size = (1, 1), circuit_type = quantum_simulation, p = 1, repetitions = 16, optimization_level = 0
                                      min. depth av. n_qubits min. qubits total time (s) av. time (s) min. time (s)
method av. n_swaps min. n_swap av. depth
line-graph 12 ± 0.0
                         12 7 ± 0.0
                                             7 12 ± 0.0
                                                                  12
                                                                              0.08 0.08 ± 0.0
     16 ± 2.06
                         6 12.38 ± 2.09
                                             6 10.25 ± 1.0
                                                                  12
                                                                              0.7 0.04 ± 0.06
name = kagome, size = (1, 1), circuit_type = quantum_simulation, p = 1, repetitions = 16, optimization_level = 1
       av. n_swaps min. n_swap av. depth min. depth av. n_qubits
                                                            min. qubits total time (s) av. time (s) min. time (s)
line-graph 12 ± 0.0
                         12 7 ± 0.0
                                              7 12 ± 0.0
                                                                  12
                                                                             0.08 0.08 ± 0.0
                          6 6.44 ± 0.34
                                            6 8.75 ± 0.75
      10 ± 0.81
                                                                             0.32 0.02 ± nan
name = kagome, size = (1, 1), circuit_type = quantum_simulation, p = 1, repetitions = 16, optimization_level = 2
method av. n_swaps min. n_swap av. depth min. depth av. n_qubits min. qubits total time (s) av. time (s) min. time (s)
                                                                  12
                                                                             0.08 0.08 ± 0.0
line-graph 12 ± 0.0
                        12 7 ± 0.0
                                              7 12 ± 0.0
      6 ± nan
                          6 6.0 ± nan
                                            6 8.0 ± nan
                                                                             0.33 0.02 ± 0.0
                                                                  8
                                                                                                      0.02
```

name = kago	ome, size = (1,	1), circuit_typ	e = quantum_si	mulation, p =	1, repetitions	= 16, optimizati	on_level = 3		
method	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s)
line-graph	12 ± 0.0	12	7 ± 0.0	7	12 ± 0.0	12	0.08	0.08 ± 0.0	0.08
sabre	6 ± nan		6.0 ± nan		8.0 ± nan	8		0.05 ± 0.02	0.04
name = kago method line-graph sabre	av. n_swaps	min. n_swap		min. depth	8, repetitions av. n_qubits	= 16, optimization min. qubits	total time (s)	av. time (s)	min. time (s)
name = kago	av. n_swaps	min. n_swap		min. depth	8, repetitions av. n_qubits		total time (s)	av. time (s)	min. time (s)
sabre	48 ± 8.62	48	42.88 ± 3.28	41	8.5 ± 0.75	8	1.67	0.1 ± 0.02	0.09
name = kago method		 circuit_typ min. n_swap 	e = quantum_si	mulation, p =	8, repetitions av. n_qubits	= 16, optimization	on_level = 2 total time (s)		min. time (s)
name = kago method	ome, size = (1, av. n_swaps	1), circuit_typ	e = quantum_si	mulation, p =	8, repetitions av. n_qubits	= 16, optimization	on_level = 2 total time (s)		min. time (s)
name = kago	ome, size = (1, av. n_swaps	1), circuit_typ min. n_swap	e = quantum_si av. depth	min. depth	8, repetitions av. n_qubits	min. qubits	on_level = 2 total time (s)	av. time (s)	min. time (s)
name = kage method line-graph sabre name = kage method	ome, size = (1, av. n_swaps	1), circuit_typ min. n_swap 96 48 1), circuit_typ min. n_swap 96	ee = quantum_si av. depth	mulation, p = min. depth 49 41	8, repetitions = av. n_qubits	= 16, optimizati. min. qubits 12 8 = 16, optimizati. min. qubits	on_level = 2 total time (s) 0.64 2.07	av. time (s) 0.64 ± 0.0 0.13 ± 0.02 av. time (s)	min. time (s) 0.64
name = kage method line-graph sabre name = kage method	ome, size = (1, av. n_swaps	1), circuit_typ min. n_swap 96 48	e = quantum_si av. depth 	mulation, p = min. depth 49 41 mulation, p = min. depth 49 41	8, repetitions av. n_qubits 12 ± 0.0 8.25 ± 0.62 8, repetitions av. n_qubits 12 ± 0.0 8.0 ± nan	= 16, optimizati min. qubits 12 8 = 16, optimizati min. qubits	on_level = 2 total time (s) 0.64 2.07	av. time (s) 0.64 ± 0.0 0.13 ± 0.02 av. time (s) 0.64 ± 0.0 0.32 ± 0.02	min. time (s) 0.64 0.11
name = kago method line-graph sabre name = kago method line-graph sabre line-graph	ome, size = (1, av. n_swaps	1), circuit_typ min. n_swap	e = quantum_si av. depth 49 ± 0.0 41.56 ± 1.41 e = quantum_si av. depth 49 ± 0.0 41.0 ± nan e = quantum_si av. depth 97 ± 0.0	mulation, p = min. depth 49 41 mulation, p = min. depth 49 41 mulation, p = min. depth 97	8, repetitions av. n_qubits 12 ± 0.0 8.25 ± 0.62 8, repetitions av. n_qubits 12 ± 0.0 8.0 ± nan	= 16, optimization min. qubits 12 8 = 16, optimization min. qubits 12 8 = 16, optimization min. qubits	on_level = 2 total time (s) 0.64 2.07 on_level = 3 total time (s) 0.64 5.07	av. time (s) 0.64 ± 0.0 0.13 ± 0.02 av. time (s) 0.64 ± 0.0 0.32 ± 0.02	min. time (s) min. time (s) 0.64 0.29 min. time (s)

method	ome, size = (1, av. n_swaps	min. n_swap	av denth	min denth	av namhite	min auhite	total time (s)	au tima (c)	min. time (s
	av. n_swaps	ши. и_зwap	av. depth		av. n_qubics	min. quoits		av. time (3)	min. time (
Line-graph	192 ± 0.0	192	97 ± 0.0	97	12 ± 0.0	12	1.31	1.31 ± 0.0	1.:
sabre	96 ± 17.62	96	86.69 ± 8.53	81	8.5 ± 0.75	8	3.18	0.2 ± 0.03	0.1
name = kage	ome, size = (1,	1), circuit_typ	e = quantum_si	mulation, p =	16, repetitions	= 16, optimizat	ion_level = 2		
nethod	av. n_swaps	min. n_swap			av. n_qubits		total time (s)		min. time (s)
	192 ± 0.0		97 ± 0.0		12 ± 0.0	12	1.3	1.3 ± 0.0	1.3
sabre	96 ± nan	96	81.0 ± nan	81	8.0 ± nan	8	3.74	0.23 ± 0.02	0.21
name = kago	ome, size = (1,	1), circuit_typ	e = quantum_si	mulation, p =		= 16, optimizat			
nethod	av. n_swaps	min. n_swap			av. n_qubits		total time (s)	av. time (s)	min. time (s)
line-graph	192 ± 0.0	192	97 ± 0.0	97	12 ± 0.0	12	1.29	1.29 ± 0.0	1.2
sabre	96 ± nan	06	81.0 ± nan	01		8	40.00		0.67
			01.0 # nan	01	8.0 ± nan	•	10.26	0.64 ± 0.02	
			e = quantum_si	mulation, p =		= 16, optimizati			min. time (s)
method	ome, Size = (3,	3), circuit_typ	e = quantum_si	mulation, p =	1, repetitions	= 16, optimizati	on_level = 0 total time (s)		min. time (s)
method	ome, size = (3, av. n_swaps	3), circuit_typ min. n_swap	e = quantum_si av. depth	mulation, p = min. depth	1, repetitions av. n_qubits	= 16, optimization	on_level = 0 total time (s)	av. time (s)	min. time (s)
nethod	ome, size = (3, av. n_swaps 	3), circuit_typ min. n_swap	e = quantum_si av. depth 	mulation, p = min. depth	1, repetitions av. n_qubits	= 16, optimizati min. qubits	on_level = 0 total time (s)	av. time (s) 	min. time (s)
nethod Line-graph sabre	ome, size = (3, av. n_swaps 	3), circuit_typ min. n_swap	e = quantum_si av. depth 15 ± 0.0 72.62 ± 3.5	mulation, p = min. depth	1, repetitions av. n_qubits 68 ± 0.0 55.0 ± 1.66	= 16, optimizati min. qubits	on_level = 0 total time (s) 0.6 1.83	av. time (s) 	min. time (s)
nethod Line-graph sabre amme = kago	ome, size = (3, av. n_swaps 112 ± 0.0 307 ± 3.44	3), circuit_typ min. n_swap 112 286 3), circuit_typ min. n_swap	ne = quantum_si av. depth 	mulation, p = min. depth 15 63 mulation, p = min. depth	1, repetitions av. n_qubits 68 ± 0.0 55.0 ± 1.66	= 16, optimization min. qubits 68 55	on_level = 0 total time (s) 0.6 1.83 on_level = 1 total time (s)	av. time (s) 0.6 ± 0.0 0.11 ± 0.02	min. time (s). 0.6 0.09
nethod line-graph sabre name = kago	ome, size = (3, av. n_swaps 112 ± 0.0 307 ± 3.44	3), circuit_typ min. n_swap 112 286 3), circuit_typ min. n_swap	ne = quantum_si av. depth 	mulation, p = min. depth 15 63 mulation, p = min. depth	1, repetitions av. n_qubits 68 ± 0.0 55.0 ± 1.66	= 16, optimizati: min. qubits 68 55	on_level = 0 total time (s) 0.6 1.83 on_level = 1 total time (s)	av. time (s) 0.6 ± 0.0 0.11 ± 0.02	min. time (s) 0.6 0.09
nethodine-graph tabre name = kago	ome, size = (3, av. n_swaps 	3), circuit_typ min. n_swap 112 286 3), circuit_typ min. n_swap	e = quantum_si av. depth 15 ± 0.0 72.62 ± 3.5 e = quantum_si av. depth 15 ± 0.0 27.38 ± 1.34	mulation, p = min. depth 15 63 mulation, p = min. depth	1, repetitions av. n_qubits	= 16, optimization min. qubits 68 55 = 16, optimization min. qubits 68 52	on_level = 0 total time (s) 0.6 1.83 on_level = 1 total time (s)	av. time (s) 0.6 ± 0.0 av. time (s) av. time (s) 0.6 ± 0.0 0.11 ± 0.02	min. time (s) 0.6 0.0)
nethod ine-graph sabre aname = kago nethod ine-graph	ome, size = (3, av. n_swaps 112 ± 0.0 307 ± 3.44 ome, size = (3, av. n_swaps 112 ± 0.0 85 ± 3.56	3), circuit_typ min. n_swap 112 286 3), circuit_typ min. n_swap 112 85	ne = quantum_si av. depth 15 ± 0.0 72.62 ± 3.5 15 ± 0.0 27.38 ± 1.34	mulation, p = min. depth 63 mulation, p = min. depth	1, repetitions av. n_qubits 68 ± 0.0 55.0 ± 1.66 1, repetitions av. n_qubits 68 ± 0.0 45.25 ± 1.66	= 16, optimization min. qubits 68 55 = 16, optimization min. qubits 68 52	on_level = 0 total time (s) 0.6 1.83 on_level = 1 total time (s)	av. time (s) 0.6 ± 0.0 0.11 ± 0.02 av. time (s) 0.6 ± 0.0 0.11 ± 0.02	min. time (s. 0.6 0.0!
nethod line-graph sabre name = kago nethod line-graph sabre	ome, size = (3, av. n_swaps 112 ± 0.0 307 ± 3.44 ome, size = (3, av. n_swaps 112 ± 0.0 85 ± 3.56	3), circuit_typ min. n_swap 112 286 3), circuit_typ min. n_swap 112 85	e = quantum_si av. depth 15 ± 0.0 72.62 ± 3.5 e = quantum_si av. depth 27.38 ± 1.34	mulation, p = min. depth 15 63 mulation, p = min. depth 22 mulation, p =	1, repetitions av. n_qubits 68 ± 0.0 55.0 ± 1.66 1, repetitions av. n_qubits 68 ± 0.0 45.25 ± 1.66	= 16, optimizati: min. qubits 68 55 = 16, optimizati: min. qubits 68 52	on_level = 0 total time (s) 0.6 1.83 on_level = 1 total time (s) 0.6 1.8	av. time (s) 0.6 ± 0.0 0.11 ± 0.02 av. time (s) 0.6 ± 0.0 0.11 ± 0.02	min. time (s) 0.6 0.09
nethod Line-graph sabre name = kago nethod Line-graph sabre	ome, size = (3, av. n_swaps 112 ± 0.0 307 ± 3.44 ome, size = (3, av. n_swaps 112 ± 0.0 85 ± 3.56 ome, size = (3, av. n_swaps 112 ± 0.0 85 ± 3.56	3), circuit_typ min. n_swap 112 286 3), circuit_typ min. n_swap 112 85	e = quantum_si av. depth 15 ± 0.0 72.62 ± 3.5 e = quantum_si av. depth 27.38 ± 1.34 e = quantum_si av. depth	mulation, p = min. depth 15 63 mulation, p = min. depth 22 mulation, p = min. depth	1, repetitions av. n_qubits 68 ± 0.0 55.0 ± 1.66 1, repetitions av. n_qubits 68 ± 0.0 45.25 ± 1.66	= 16, optimization min. qubits 68 55 = 16, optimization min. qubits 68 52	on_level = 0 total time (s) 0.6 1.83 on_level = 1 total time (s) 0.6 1.8	av. time (s) 0.6 ± 0.0 0.11 ± 0.02 av. time (s) 0.6 ± 0.0 0.11 ± 0.02	min. time (s. 0.6 0.05 min. time (c. 0.05 min. time
nethod Line-graph sabre Line-graph sabre Line-graph sabre Line-graph sabre	ome, size = (3, av. n_swaps 112 ± 0.0 307 ± 3.44 ome, size = (3, av. n_swaps 112 ± 0.0 85 ± 3.56 ome, size = (3, av. n_swaps 112 ± 0.0 85 ± 3.56	3), circuit_typ min. n_swap 112 286 3), circuit_typ min. n_swap 112 85 3), circuit_typ min. n_swap	e = quantum_si av. depth 15 ± 0.0 72.62 ± 3.5 e = quantum_si av. depth 27.38 ± 1.34 e = quantum_si av. depth	mulation, p = min. depth 15 63 mulation, p = min. depth 22 mulation, p = min. depth	1, repetitions av. n_qubits 68 ± 0.0 55.0 ± 1.66 1, repetitions av. n_qubits 68 ± 0.0 45.25 ± 1.66	= 16, optimization min. qubits 68 55 = 16, optimization min. qubits 68 52	on_level = 0 total time (s) 0.6 1.83 on_level = 1 total time (s) 1.8	av. time (s) 0.6 ± 0.0 0.11 ± 0.02 av. time (s) 0.6 ± 0.0 0.11 ± 0.02	min. time (s 0.6 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

name = kag nethod	n	min	ou dor+1	min 3	or nantite	min	+o+o1 +: ()	ov +im- (-)	min +: / '
ethod	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s)
ine-graph	112 ± 0.0	112	15 ± 0.0	15	68 ± 0.0	68	0.5	0.5 ± 0.0	0.5
abre	64 ± 2.34	67	21.44 ± 1.94	14	46.0 ± 1.75	47	7.77	0.49 ± 0.02	0.5
ame = kag	ome, size = (3,	3), circuit_typ	e = quantum_sim	ulation, p = :	8, repetitions	16, optimization	_level = 0		
ethod	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s
ine-graph	896 ± 0.0	896	113 ± 0.0	113	68 ± 0.0	68	4.25	4.25 ± 0.0	4.2
abre	1361 ± 33.51	1203	312.5 ± 9.31	287	59.5 ± 1.75	51	10.83	0.68 ± 0.03	0.69
ame = kag	ome, size = (3,	3), circuit_typ	e = quantum_sim	ulation, p = 3	8, repetitions =	= 16, optimization	_level = 1		
ethod	av. n_swaps	min. n_swap	av. depth		av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (
ine-graph	896 ± 0.0	896	113 ± 0.0	113	68 ± 0.0	68	4.39	4.39 ± 0.0	4.
abre	625 ± 19.67	240							0
			192.38 ± 14.0		45.19 ± 1.28	44		0.69 ± 0.04	
ame = kag		3), circuit_typ	e = quantum_sim	ulation, p = ;	8, repetitions = av. n_qubits	= 16, optimization		av. time (s)	min. time (
ame = kag ethod	ome, size = (3, av. n_swaps	3), circuit_typ	e = quantum_sim	ulation, p = 3	8, repetitions = av. n_qubits	16, optimization	t_level = 2 total time (s)	av. time (s)	min. time (
ame = kag ethod ine-graph	ome, size = (3, av. n_swaps	3), circuit_typ min. n_swap	e = quantum_sim	ulation, p = : min. depth	8, repetitions = av. n_qubits	* 16, optimization min. qubits	total time (s)	av. time (s)	min. time (
ame = kag	ome, size = (3, av. n_swaps 	3), circuit_typ min. n_swap 896 621	e = quantum_sim av. depth	ulation, p = : min. depth	av. n_qubits 68 ± 0.0 43.56 ± 1.4	= 16, optimization min. qubits 68 44	1_level = 2 total time (s) 4.42 14.53	av. time (s)	min. time (
ame = kag	ome, size = (3, av. n_swaps 896 ± 0.0 621 ± 9.75 ome, size = (3, av. n_swaps	3), circuit_typ min. n_swap 896 621 3), circuit_typ min. n_swap	e = quantum_sim av. depth 113 * 0.0 182.06 * 9.79 e = quantum_sim av. depth	min. depth 113 152 ulation, p = 1	8, repetitions av. n_qubits 68 ± 0.0 43.56 ± 1.4	= 16, optimization min. qubits 68 44 = 16, optimization min. qubits	total time (s) 4.42 14.53 1.level = 3 total time (s)	av. time (s) 4.42 ± 0.0 0.91 ± 0.04 av. time (s)	min. time (
ame = kag	ome, size = (3, av. n_swaps 896 ± 0.0 621 ± 9.75 ome, size = (3, av. n_swaps	3), circuit_typ min. n_swap 896 621 3), circuit_typ min. n_swap	e = quantum_sim av. depth 113 * 0.0 182.06 * 9.79 e = quantum_sim av. depth	min. depth 113 152 ulation, p = 3 min. depth	8, repetitions av. n_qubits 68 ± 0.0 43.56 ± 1.4	= 16, optimization min. qubits 68 44	total time (s) 4.42 14.53 Level = 3 total time (s)	av. time (s) 4.42 ± 0.0 0.91 ± 0.04 av. time (s)	min. time (
me = kag	ome, size = (3, av. n_swaps	3), circuit_typ min. n_swap 896 621 3), circuit_typ min. n_swap	e = quantum_sim av. depth 113 ± 0.0 182.06 ± 9.79 e = quantum_sim av. depth	ulation, p = ; min. depth 113 152 ulation, p = ; min. depth	8, repetitions av. n_qubits 68 ± 0.0 43.56 ± 1.4 8, repetitions av. n_qubits	16, optimization min. qubits 68 44	1_level = 2 total time (s) 4.42 14.53 1_level = 3 total time (s)	av. time (s)	min. time 4
ame = kag	ome, size = (3, av. n_swaps 	3), circuit_typ min. n_swap 896 621 3), circuit_typ min. n_swap 896 555	e = quantum_sim av. depth 113 ± 0.0 182.06 ± 9.79 ie = quantum_sim av. depth 113 ± 0.0 176.25 ± 7.62	ulation, p = ; min. depth 113 152 ulation, p = ; min. depth 113 149	8, repetitions av. n_qubits 68 ± 0.0 43.56 ± 1.4 8, repetitions av. n_qubits 68 ± 0.0 43.44 ± 0.72	# 16, optimization min. qubits 68 44 # 16, optimization min. qubits 68 43	1.level = 2 total time (s) 4.42 14.53 1.level = 3 total time (s) 4.23 45.11	av. time (s) 4.42 ± 0.0 0.91 ± 0.04 av. time (s) 4.23 ± 0.0 2.82 ± 0.04	min. time (
ame = kag	ome, size = (3, av. n_swaps	3), circuit_typ min. n_swap 896 621 3), circuit_typ min. n_swap 555	e = quantum_sim av. depth 113 ± 0.0 182.06 ± 9.79 e = quantum_sim av. depth 113 ± 0.0 176.25 ± 7.62	min. depth 113 152 ulation, p = ; min. depth 113 149	8, repetitions av. n_qubits 68 ± 0.0 43.56 ± 1.4 8, repetitions av. n_qubits 68 ± 0.0 43.44 ± 0.72	# 16, optimization min. qubits 68 44 # 16, optimization min. qubits 68 43	1.level = 2 total time (s) 4.42 14.53 1.level = 3 total time (s) 4.23 45.11	av. time (s) 4.42 ± 0.0 0.91 ± 0.04 av. time (s) 4.23 ± 0.0 2.82 ± 0.04	min. time (
ame = kag	ome, size = (3, av. n_swaps	3), circuit_typ min. n_swap 896 621 3), circuit_typ min. n_swap 555 3), circuit_typ min. n_swap	e = quantum_sim av. depth 113 ± 0.0 182.06 ± 9.79 20 = quantum_sim av. depth 113 ± 0.0 176.25 ± 7.62	min. depth 113 152 ulation, p = 1 min. depth 113 149 ulation, p = 1 min. depth	8, repetitions av. n_qubits 68 ± 0.0 43.56 ± 1.4 8, repetitions av. n_qubits 68 ± 0.0 43.44 ± 0.72	# 16, optimization min. qubits 68 44 # 16, optimization min. qubits 68 43	1_level = 2 total time (s) 4.42 14.53 1_level = 3 total time (s) 4.23 45.11	av. time (s) 4.42 ± 0.0 0.91 ± 0.04 av. time (s) 4.23 ± 0.0 2.82 ± 0.04	min. time (4. 0. min. time (4. 2.
ame = kag.	ome, size = (3, av. n_swaps	3), circuit_typ min. n_swap 896 621 3), circuit_typ min. n_swap 555 3), circuit_typ min. n_swap	e = quantum_sim av. depth 113 ± 0.0 182.06 ± 9.79 20 = quantum_sim av. depth 113 ± 0.0 176.25 ± 7.62	min. depth 113 152 ulation, p = 1 min. depth 113 149 ulation, p = min. depth	8, repetitions av. n_qubits 68 ± 0.0 43.56 ± 1.4 8, repetitions av. n_qubits 68 ± 0.0 43.44 ± 0.72	16, optimization min. qubits 68 44 16, optimization min. qubits 68 43 16, optimization min. qubits	1.level = 2 total time (s) 4.42 14.53 1.level = 3 total time (s) 4.23 45.11	av. time (s) 4.42 ± 0.0 0.91 ± 0.04 av. time (s) 4.23 ± 0.0 2.82 ± 0.04	min. time (

name = kago									
method	av. n_swaps	min. n_swap		min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s
line-graph	1792 ± 0.0	1792	225 ± 0.0	225	68 ± 0.0	68	8.87	8.87 ± 0.0	8.8
sabre	1213 ± 28.0	1213	388.12 ± 18.66	318	43.75 ± 1.16	44	20.66	1.29 ± 0.05	1.3
name = kage	ome, size = (3,	3), circuit_typ	e = quantum_simu	lation, p = 16	6, repetitions	= 16, optimization	n_level = 2		
method	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s
line-graph	1792 ± 0.0	1792	225 ± 0.0	225	68 ± 0.0	68	8.4	8.4 ± 0.0	8.4
sabre	1215 ± 14.41	1142	377.69 ± 19.52	311	44.06 ± 1.19	49	28.01	1.75 ± 0.01	1.7
name = kago	ome, size = (3,	3), circuit_typ	e = quantum_simu	ulation, p = 16	6, repetitions =	= 16, optimization	n_level = 3		
method	av. n_swaps	min. n_swap		min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s
line-graph	1792 ± 0.0	1792	225 ± 0.0	225	68 ± 0.0	68	8.57	8.57 ± 0.0	8.5
sabre	1172 ± 15.38		342.25 ± 13.38		43.56 ± 1.0	43		5.4 ± 0.05	
			e = quantum_simu	ulation, p = 1	, repetitions =		level = 0		
name = kago	ome, size = (5, av. n_swaps	5), circuit_typ	e = quantum_simu	min. depth	, repetitions = av. n_qubits	16, optimization min. qubits	level = 0 total time (s)	av. time (s)	min. time (s)
name = kago	ome, size = (5, av. n_swaps	5), circuit_typ min. n_swap	e = quantum_simu av. depth	min. depth	, repetitions = av. n_qubits	16, optimization,	level = 0 total time (s)	av. time (s)	min. time (s)
name = kago method line-graph	ome, size = (5, av. n_swaps 	5), circuit_typ min. n_swap	e = quantum_simu av. depth	min. depth	av. n_qubits	16, optimization min. qubits	level = 0 total time (s)	av. time (s)	min. time (s)
name = kago method line-graph sabre	ome, size = (5, av. n_swaps 270 ± 0.0 1247 ± 9.78	5), circuit_typ min. n_swap 270 1247	e = quantum_simu av. depth 	min. depth	av. n_qubits 164 ± 0.0 128.31 ± 1.72	16, optimization min. qubits	level = 0 total time (s) 1.48 5.12	av. time (s)	min. time (s)
name = kage method line-graph sabre name = kage method	ome, size = (5, av. n_swaps	55), circuit_typ min. n_swap 270 1247 55), circuit_typ min. n_swap	e = quantum_simu av. depth 14 ± 0.0 132.81 ± 3.09 e = quantum_simu av. depth	min. depth 14 123 12alation, p = 1, min. depth	, repetitions = av. n_qubits 164 ± 0.0 128.31 ± 1.72 , repetitions = av. n_qubits	16, optimization, min. qubits 164 125	level = 0 total time (s) 1.48 5.12 level = 1 total time (s)	av. time (s) 1.48 ± 0.0 0.32 ± 0.03 av. time (s)	min. time (s) 1.48 0.37
name = kagemethod line-graph sabre name = kagemethod	ome, size = (5, av. n_swaps	5), circuit_typ min. n_swap 270 1247 5), circuit_typ min. n_swap	e = quantum_simu av. depth 14 ± 0.0 132.81 ± 3.09 e = quantum_simu av. depth	min. depth 14 123 slation, p = 1, min. depth = 1, min. dept	, repetitions = av. n_qubits 164 ± 0.0 128.31 ± 1.72 , repetitions = av. n_qubits	16, optimization, min. qubits 164 125	level = 0 total time (s) 1.48 5.12 level = 1 total time (s)	av. time (s) 1.48 ± 0.0 0.32 ± 0.03 av. time (s)	min. time (s) 1.48 0.37
name = kago method line-graph sabre mame = kago method	ome, size = (5, av. n_swaps 270 ± 0.0 1247 ± 9.78 ome, size = (5, av. n_swaps	5), circuit_typ min. n_swap 270 1247 5), circuit_typ min. n_swap	e = quantum_simu av. depth 14 ± 0.0 132.81 ± 3.09 e = quantum_simu av. depth	min. depth 14 123 llation, p = 1, min. depth 14 123	, repetitions = av. n_qubits 164 ± 0.0 128.31 ± 1.72 , repetitions = av. n_qubits	16, optimization min. qubits 164 125 16, optimization min. qubits	level = 0 total time (s) 1.48 5.12 level = 1 total time (s)	av. time (s) 1.48 ± 0.0 0.32 ± 0.03 av. time (s)	min. time (s) 1.48 0.37
name = kago	ome, size = (5, av. n_swaps 270 ± 0.0 1247 ± 9.78 ome, size = (5, av. n_swaps 270 ± 0.0 338 ± 14.72	5), circuit_typ min. n_swap 270 1247 5), circuit_typ min. n_swap 270 345	e = quantum_simu av. depth 14 ± 0.0 132.81 ± 3.09 e = quantum_simu av. depth 14 ± 0.0 50.94 ± 3.44	lation, p = 1, min. depth 14 123 lation, p = 1, min. depth 3 lation, p = 1, alation, p = 1,	, repetitions = av. n_qubits 164 ± 0.0 128.31 ± 1.72 , repetitions = av. n_qubits 164 ± 0.0 110.69 ± 1.34	16, optimization min. qubits 164 125 16, optimization min. qubits 164 109	level = 0 total time (s) 1.48 5.12	av. time (s) 1.48 ± 0.0 0.32 ± 0.03 av. time (s) 1.47 ± 0.0 0.34 ± 0.02	min. time (s) 1.48 0.37 min. time (s) 1.47 0.3
name = kagomethod line-graph sabre line-graph sabre line-graph sabre	me, size = (5, av. n_swaps	5), circuit_typ min. n_swap 270 1247 5), circuit_typ min. n_swap 270 345	e = quantum_simu av. depth 14 ± 0.0 132.81 ± 3.09 e = quantum_simu av. depth 14 ± 0.0 50.94 ± 3.44	lation, p = 1, min. depth 14 123 lation, p = 1, min. depth a 38 :	, repetitions = av. n_qubits 164 ± 0.0 128.31 ± 1.72 , repetitions = av. n_qubits 164 ± 0.0 110.69 ± 1.34 , repetitions = av. n_qubits	16, optimization, min. qubits 164 125 16, optimization, min. qubits	level = 0 total time (s) 1.48 5.12 total time (s) 1.47 5.47 level = 2 total time (s)	av. time (s) 1.48 ± 0.0 0.32 ± 0.03 av. time (s) 1.47 ± 0.0 0.34 ± 0.02 av. time (s)	min. time (s) 1.48 0.37 min. time (s) 1.47 0.3
name = kago method line-graph sabre method line-graph sabre	me, size = (5, av. n_swaps	5), circuit_typ min. n_swap 270 1247 5), circuit_typ min. n_swap 270 345 5), circuit_typ min. n_swap	e = quantum_simu av. depth 14 ± 0.0 132.81 ± 3.09 e = quantum_simu av. depth 14 ± 0.0 50.94 ± 3.44	lation, p = 1, min. depth 14 123 lation, p = 1, min. depth a 38 :	, repetitions = av. n_qubits 164 ± 0.0 128.31 ± 1.72 , repetitions = av. n_qubits 164 ± 0.0 110.69 ± 1.34 , repetitions = av. n_qubits	16, optimization min. qubits 164 125 16, optimization, min. qubits 164 109	level = 0 total time (s) 1.48 5.12 total time (s) 1.47 5.47 1.47	av. time (s) 1.48 ± 0.0 0.32 ± 0.03 av. time (s) 1.47 ± 0.0 0.34 ± 0.02 av. time (s)	min. time (s) 1.48 0.37 min. time (s) 1.47 0.3

ine-graph	av. n_swaps 		av. depth		av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s)
			14 ± 0.0	1.4					
abre	274 ± 9.62	269		1-1	164 ± 0.0	164	1.47	1.47 ± 0.0	1.47
		203	37.44 ± 3.25	30	106.25 ± 1.62	108	36.54	2.28 ± 0.03	2.33
				olation n = 0		16, optimization			
	av. n_swaps	min. n_swap			av. n_qubits		total time (s)	av. time (s)	min. time (s)
ine-graph	2160 ± 0.0	2160	98 ± 0.0	98	164 ± 0.0	164	10.96	10.96 ± 0.0	10.96
abre	6068 ± 87.86	5579	557.62 ± 12.5	511	134.88 ± 2.03	134	27.49	1.72 ± 0.06	1.58
ame = kagon	ne, size = (5, 5), circuit_typ	e = quantum_simu		3, repetitions =	16, optimization			
	av. n_swaps	min. n_swap	-		n av. n_qubits		total time (s)) av. time (s)	min. time (s
	2160 ± 0.0		98 ± 0.0		3 164 ± 0.0	164	11.01	1 11.01 ± 0.0	11.0
bre	2124 ± 45.1	2056	341.69 ± 14.66	286	5 106.5 ± 2.38	118	30.7	7 1.92 ± 0.05	1.
	ne, size = (5, 5 av. n_swaps), circuit_typ			<pre>a, repetitions = n av. n_qubits</pre>	16, optimization min. qubits	total time (s)) av. time (s)	min. time (
	2160 ± 0.0		98 ± 0.0		3 164 ± 0.0	164		3 10.93 ± 0.0	10.9
abre	1894 ± 32.99	1953	324.31 ± 22.53	268	5 106.62 ± 2.47	118	44.89	9 2.81 ± 0.03	2.
ame = kagon	ne, size = (5, 5), circuit_typ	e = quantum_simu	alation, p = 8	3, repetitions =	16, optimization			
		min. n_swap				min. qubits			
	2160 ± 0.0		98 ± 0.0		3 164 ± 0.0	164		3 10.88 ± 0.0	10.
abre	1869 ± 23.79	1758	282.88 ± 13.62	236	5 104.12 ± 1.91	109	158.7	9.92 ± 0.08	9.
me = kagon			e = quantum_simu av. depth			= 16, optimization min. qubits		s) av. time (s)	min. time
ethod									
ame = kagon						= 16, optimization		s) av. time (s)	min. ti

min. time (: 21. 3.: min. time (: 21. 5.
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nin. time (s
21.5
18.2
nin. time (s
2.8
0.7
in. time (s)
in. time (s)
2.86
2.86

name = kago									
ethod	av. n_swaps	min. n_swap	av. depth	min. depth	-	min. qubits	total time (s)	av. time (s)	min. time (s)
ine-graph	498 ± 0.0	498	16 ± 0.0	16	300 ± 0.0	300	2.87	2.87 ± 0.0	2.87
abre	868 ± 28.66	737	66.25 ± 3.94	53	194.75 ± 2.84	189	135.51	8.47 ± 0.05	8.39
ame = kago	ome, size = (7, 7	7), circuit_typ	e = quantum_simu	ulation, p = 8	, repetitions =	16, optimization	_level = 0		
ethod	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s
	3984 ± 0.0		114 ± 0.0	114	300 ± 0.0	300	21.51	21.51 ± 0.0	21.5
abre	13056 ± 225.0	13056	766.81 ± 18.31	686	247.75 ± 2.97	251	61.25	5 3.83 ± 0.12	4.0
 me = kage	ome, size = (7, 7	'), circuit_typ	e = quantum_simu	ulation, p = 8	, repetitions =	16, optimization			
ethod	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)		min. time (s)
	3984 ± 0.0		114 ± 0.0	114	300 ± 0.0	300	20.92	20.92 ± 0.0	20.92
ibre									
	5532 ± 133.17		569.0 ± 30.89		199.12 ± 3.19	207	68.32	4.27 ± 0.11	
			e = quantum_simu	ulation, p = 8		16, optimization	_level = 2		
ame = kago	ome, size = (7, 7	r), circuit_typ	e = quantum_simu	min. depth	, repetitions = av. n_qubits	16, optimization min. qubits	_level = 2 total time (s)	av. time (s)	min. time (s
ame = kago	ome, size = (7, 7 av. n_swaps	'), circuit_typ min. n_swap	e = quantum_simu	min. depth	, repetitions = av. n_qubits	16, optimization	_level = 2 total time (s)	av. time (s)	min. time (s
ame = kago	ome, size = (7, 7 av. n_swaps 	"), circuit_typ min. n_swap 3984 4644	e = quantum_sim av. depth 114 ± 0.0 511.88 ± 33.91	min. depth	, repetitions = av. n_qubits 300 ± 0.0 194.75 ± 3.38	16, optimization min. qubits 300 199	_level = 2 total time (s) 	av. time (s)	min. time (s
ame = kago	ome, size = (7, 7 av. n_swaps 3984 ± 0.0 4758 ± 150.83 ome, size = (7, 7 av. n_swaps	nin. n_swap 3984 4644 '), circuit_typ min. n_swap	e = quantum_simu av. depth 114 ± 0.0 511.88 ± 33.91 e = quantum_simu av. depth	min. depth 114 429 alation, p = 8 min. depth	, repetitions = av. n_qubits 300 ± 0.0 194.75 ± 3.38 , repetitions = av. n_qubits	16, optimization min. qubits 300 199 16, optimization min. qubits	_level = 2 total time (s)	20.7 ± 0.0 3 7.4 ± 0.07	min. time (s
ame = kago	ome, size = (7, 7 av. n_swaps 	"), circuit_typ min. n_swap 3984 4644 "), circuit_typ min. n_swap	e = quantum_sim av. depth 114 ± 0.0 511.88 ± 33.91	min. depth 114 429 alation, p = 8	, repetitions = av. n_qubits	16, optimization min. qubits 300 199	_level = 2 total time (s)	20.7 ± 0.0 3 7.4 ± 0.07	min. time (s
me = kago	ome, size = (7, 7 av. n_swaps 3984 ± 0.0 4758 ± 150.83 ome, size = (7, 7 av. n_swaps	min. n_swap 3984 4644 //), circuit_typ min. n_swap min. n_swap	e = quantum_simu av. depth 114 * 0.0 511.88 * 33.91 e = quantum_simu av. depth	min. depth 114 429 min. depth 114 114 114	, repetitions = av. n_qubits 300 ± 0.0 194.75 ± 3.38 , repetitions = av. n_qubits	16, optimization min. qubits 300 199 16, optimization min. qubits	_level = 2total time (s)	20.7 ± 0.0 3 7.4 ± 0.07	min. time (;
ame = kago	ome, size = (7, 7 av. n_swaps 3984 ± 0.0 4758 ± 150.83 ome, size = (7, 7 av. n_swaps 3984 ± 0.0 4702 ± 106.77	7), circuit_typ min. n_swap 3984 4644 7), circuit_typ min. n_swap 3984 4300	e = quantum_simu av. depth 114 * 0.0 511.88 * 33.91 e = quantum_simu av. depth 114 * 0.0 460.69 * 27.72	min. depth 114 429 ulation, p = 8 min. depth	, repetitions = av. n_qubits 300 ± 0.0 194.75 ± 3.38 , repetitions = av. n_qubits 300 ± 0.0 191.88 ± 2.75	16, optimization min. qubits 300 199 16, optimization min. qubits 300 200	_level = 2 total time (s)	20.7 ± 0.0 3 7.4 ± 0.07 av. time (s) 1 20.74 ± 0.0 3 1.53 ± 0.21	min. time (s
ame = kago	ome, size = (7, 7 av. n_swaps 3984 ± 0.0 4758 ± 150.83 ome, size = (7, 7 av. n_swaps 3984 ± 0.0 4702 ± 106.77	"), circuit_typ min. n_swap 3984 4644 "), circuit_typ min. n_swap 3984 4300	e = quantum_simu av. depth 114 * 0.0 511.88 * 33.91 e = quantum_simu av. depth 114 * 0.0 460.69 * 27.72	min. depth 114 429 alation, p = 8 min. depth 114 375	, repetitions = av. n_qubits 300 ± 0.0 194.75 ± 3.38 , repetitions = av. n_qubits 300 ± 0.0 191.88 ± 2.75	16, optimization min. qubits 300 199 16, optimization min. qubits 300 200	_level = 2 total time (s)	20.7 ± 0.0 3 7.4 ± 0.07	min. time (s 20.7 7.3 min. time (s 20.7 31.3
ame = kago	ome, size = (7, 7 av. n_swaps 3984 ± 0.0 4758 ± 150.83 ome, size = (7, 7 av. n_swaps 3984 ± 0.0 4702 ± 106.77 ome, size = (7, 7 av. n_swaps	min. n_swap 3984 4644 7), circuit_typ min. n_swap 3984 4300	e = quantum_sim av. depth 114 ± 0.0 511.88 ± 33.91 e = quantum_sim av. depth 114 ± 0.0 460.69 ± 27.72	min. depth 114 429 min. depth 114 375	, repetitions = av. n_qubits 300 ± 0.0 194.75 ± 3.38 , repetitions = av. n_qubits 300 ± 0.0 191.88 ± 2.75	16, optimization min. qubits 300 199 16, optimization min. qubits 300 200 16, optimization min. qubits	_level = 2 total time (s)	20.7 ± 0.0 3 7.4 ± 0.07 av. time (s) 20.7 ± 0.0 3 1.4 ± 0.07	min. time (s

name = kago									
method	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s	s) av. time (s)	min. time
line-graph	7968 ± 0.0	7968	226 ± 0.0	226	300 ± 0.0	300	41.0	09 41.09 ± 0.0	41
sabre	9497 ± 345.67	9370	1049.75 ± 55.28	888	194.06 ± 3.81	198	134.0	06 8.38 ± 0.28	8.
name = kage	ome, size = (7, 7	7), circuit_typ	e = quantum_simu	nlation, p = 16	, repetitions =	16, optimization	n_level = 2		
nethod	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)) av. time (s)	min. time (s
line-graph	7968 ± 0.0	7968	226 ± 0.0	226	300 ± 0.0	300	41.59	9 41.59 ± 0.0	41.5
sabre	9742 ± 181.92	9337	978.25 ± 31.91	856	197.69 ± 3.94	206	227.53	3 14.22 ± 0.41	15.8
		7				16			
mame = kago	ome, size = (7, 7	min. n_swap			av. n_qubits		total time (s)	av. time (s)	min. time (s)
	7968 ± 0.0		226 ± 0.0	226	300 ± 0.0	300	42 57	42.57 ± 0.0	42.57
sabre	8452 ± 100.74		899.0 ± 24.78			200		55.38 ± 1.31	60.7
					192.25 ± 2.62				
name = shur			ype = quantum_si		1, repetitions s	= 16, optimizatio			min. time (s)
name = shum	riken, size = (1,	, 1), circuit_t min. n_swap	ype = quantum_si av. depth	mulation, p =	1, repetitions s	= 16, optimizatio	on_level = 0		min. time (s)
name = shum	riken, size = (1,	, 1), circuit_t min. n_swap	ype = quantum_si av. depth	mulation, p =	1, repetitions	= 16, optimizatio	on_level = 0 total time (s) a	av. time (s)	min. time (s)
name = shumethod	riken, size = (1,	, 1), circuit_t min. n_swap	ype = quantum_si av. depth	mulation, p = min. depth av	1, repetitions	= 16, optimizatio	on_level = 0 total time (s) a	av. time (s)	min. time (s)
ame = shur method ine-graph	riken, size = (1, av. n_swaps 	, 1), circuit_t min. n_swap	ype = quantum_si av. depth 	mulation, p = min. depth av	1, repetitions . n_qubits	= 16, optimization min. qubits	on_level = 0 total time (s) a	av. time (s)	min. time (s)
name = shuname =	riken, size = (1, av. n_swaps	8 8 8	ype = quantum_si av. depth 	mulation, p = min. depth av 9 8 8 8.	1, repetitions to a n_qubits ± 0.0 0 ± nan	= 16, optimization min. qubits 19 8 8 8	on_level = 0 total time (s) a 0.09 (0.32 (av. time (s) 	min. time (s) 0.09 0.02
name = shunethod	riken, size = (1, av. n_swaps 8 ± 0.0 8 ± nan	, 1), circuit_t min. n_swap 8 8 8 , 1), circuit_t min. n_swap	ype = quantum_si av. depth 9 ± 0.0 8.88 ± 0.19 ype = quantum_si av. depth	mulation, p = min. depth av 9 8 8 8.	1, repetitions n_qubits ± 0.0 0 ± nan 1, repetitions n_qubits	= 16, optimization min. qubits 18 8 8 8	0.09 (0.32 (0.1000 1	av. time (s) 0.09 ± 0.0 0.02 ± nan	min. time (s) 0.09 0.02 min. time (s)
name = shunethod	riken, size = (1, av. n_swaps 8 ± 0.0 8 ± nan riken, size = (1, av. n_swaps	, 1), circuit_t min. n_swap 8 8 , 1), circuit_t min. n_swap	ype = quantum_si av. depth 9 ± 0.0 8.88 ± 0.19 ype = quantum_si av. depth	mulation, p = min. depth av 9 8 8 8.	to n_qubits ± 0.0 0 * nan 1, repetitions * 1, repetitions * 1, republits	= 16, optimization min. qubits 18 8 8 8	0.09 (0.32 (0.1 time (s) a conclusion conclusion (s) a	av. time (s) 0.09 ± 0.0 0.02 ± nan	min. time (s) 0.09 0.02 min. time (s)
ame = shunding-graph abre ame = shunding-graph ine-graph abre	riken, size = (1, av. n_swaps	8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8	ype = quantum_si av. depth 9 ± 0.0 8.88 ± 0.19 ype = quantum_si av. depth 9 ± 0.0 7.5 ± 0.25	mulation, p = min. depth av 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8	1, repetitions : . n_qubits ± 0.0 0 ± nan 1, repetitions : . n_qubits ± 0.0 0 ± nan	= 16, optimization min. qubits 18 8 8 = 16, optimization min. qubits 18 8 8	on_level = 0 total time (s) a 0.09 (0.32 (on_level = 1 total time (s) a 0.09 (0.32 (av. time (s) 0.09 ± 0.0 0.02 ± nan av. time (s) 0.09 ± 0.0 0.02 ± nan	min. time (s) 0.09 0.02 min. time (s) 0.09 0.02
ine-graph tabre ame = shut tabre ine-graph tabre ine-graph tabre	riken, size = (1, av. n_swaps	8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8	ype = quantum_si av. depth 9 ± 0.0 8.88 ± 0.19 ype = quantum_si av. depth 9 ± 0.0 7.5 ± 0.25	mulation, p = min. depth av 9 8 8 8.	1, repetitions 1 n_qubits 1 0.0 0 ± nan 1, repetitions 1 n_qubits ± 0.0 0 ± nan	= 16, optimization min. qubits state	0.09 (0.32 (0.09 (0.32 (0.09 (0.32 (0.09 (0.32 (0.32 (av. time (s) 0.09 ± 0.0 0.02 ± nan av. time (s)	min. time (s) 0.09 0.02 min. time (s) 0.09
name = shunethod	riken, size = (1, av. n_swaps	8 8 8 1, 1), circuit_t	ype = quantum_si av. depth 9 ± 0.0 8.88 ± 0.19 ype = quantum_si av. depth 9 ± 0.0 7.5 ± 0.25	mulation, p = min. depth av 9 8 8 8. mulation, p = min. depth av 9 8 7 8.	1, repetitions	= 16, optimization min. qubits statement state	on_level = 0 total time (s) a 0.09 (0.32 (on_level = 1 total time (s) a 0.09 (0.32 (av. time (s) 0.09 ± 0.0 0.02 ± nan av. time (s)	min. time (s) 0.09 0.02 min. time (s) 0.09 0.02
name = shunathod Line-graph name = shunathod Line-graph sabre	riken, size = (1, av. n_swaps 8 ± 0.0 8 ± nan riken, size = (1, av. n_swaps 8 ± 0.0 6 ± nan	, 1), circuit_t min. n_swap 8 8 8 . , 1), circuit_t min. n_swap 6 . , 1), circuit_t min. n_swap	ype = quantum_si av. depth 9 ± 0.0 8.88 ± 0.19 ype = quantum_si av. depth 7.5 ± 0.25 ype = quantum_si av. depth	mulation, p = min. depth av	1, repetitions ± 0.0 0 ± nan 1, repetitions ± 0.0 0 ± nan 1, repetitions n_qubits	= 16, optimization min. qubits	on_level = 0 total time (s) a 0.09 (0.32 (0.09 (av. time (s) 0.09 ± 0.0 0.02 ± nan av. time (s) 0.09 ± 0.0 0.09 ± 0.0 0.02 ± nan	min. time (s) 0.09 0.02 min. time (s) 0.09 0.02
name = shuname =	riken, size = (1, av. n_swaps 8 ± 0.0 8 ± nan riken, size = (1, av. n_swaps 8 ± 0.0 6 ± nan	, 1), circuit_t min. n_swap 8 8	ype = quantum_si av. depth 9 ± 0.0 8.88 ± 0.19 ype = quantum_si av. depth 7.5 ± 0.25 ype = quantum_si av. depth	mulation, p = min. depth av	1, repetitions ± 0.0 0 ± nan 1, repetitions ± 0.0 0 ± nan 1, repetitions 1, repetitions 1, repetitions	= 16, optimization min. qubits	0.09 (0.32 (0.09 (0.32 (0.09 (0.09 (0.09 (av. time (s) 0.09 ± 0.0 0.02 ± nan av. time (s) 0.09 ± 0.0 0.09 ± 0.0 0.02 ± nan	min. time (s) 0.09 0.02 min. time (s) 0.09 0.02

name = shun nethod		min n arre-	au danth	min don+1	av. n_qubits	min auhita	total time (s)	av time (a)	min +ima /-
	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s)
ine-graph	8 ± 0.0	8	9 ± 0.0	9	8 ± 0.0	8	0.09	0.09 ± 0.0	0.09
abre	6 ± nan	6	7.62 ± 0.22	7	8.0 ± nan	8	0.64	0.04 ± nan	0.04
ame = shur	riken, size = (1,	1), circuit_t	ype = quantum_	simulation, p	= 8, repetition	ns = 16, optimizat	tion_level = 0		
ethod	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s
ine-graph	64 ± 0.0	64	65 ± 0.0	65	8 ± 0.0	8	0.74	0.74 ± 0.0	0.7
abre	74 ± 1.56	64	67.94 ± 2.31	61	8.0 ± nan	8	1.66	0.1 ± 0.02	0.0
me = shu	riken, size = (1,	1), circuit_t	ype = quantum_	simulation, p	= 8, repetition	ns = 16, optimizat	tion_level = 1		
ethod	av. n_swaps	min. n_swap		min. depth		min. qubits	total time (s)	av. time (s)	min. time (
ine-graph	64 ± 0.0	64	65 ± 0.0	65	8 ± 0.0	8	0.73	0.73 ± 0.0	0.
abre	62 ± nan		60.75 ± 0.53		8.0 ± nan	8		0.11 ± 0.02	
			ype = quantum_	simulation, p	= 8, repetition				min. time (
ame = shur	riken, size = (1,	1), circuit_t	ype = quantum_ av. depth	simulation, p	= 8, repetition av. n_qubits	ns = 16, optimizat min. qubits	tion_level = 2 total time (s)	av. time (s)	min. time (
mme = shun sthod e-graph	av. n_swaps	1), circuit_t min. n_swap	ype = quantum_ av. depth	simulation, p	= 8, repetition av. n_qubits 8 ± 0.0	ns = 16, optimizat min. qubits	tion_level = 2 total time (s)	av. time (s)	min. time (
me = shunthod	riken, size = (1,	1), circuit_t min. n_swap	ype = quantum_ av. depth 	simulation, p	= 8, repetition av. n_qubits	ns = 16, optimizat min. qubits	tion_level = 2 total time (s)	av. time (s)	min. time (
me = shur thod .ne-graph .bre	av. n_swaps 64 ± 0.0 62 ± nan	1), circuit_t min. n_swap 64 62	ype = quantum_ av. depth 	simulation, p min. depth 65	= 8, repetition av. n_qubits 8 ± 0.0 8.0 ± nan	ns = 16, optimizat min. qubits	tion_level = 2 total time (s) 0.77 2.04	av. time (s)	min. time (
ame = shur sthod ane-graph abre	fiken, size = (1, av. n_swaps 64 * 0.0 62 * nan fiken, size = (1, av. n_swaps	1), circuit_t min. n_swap 64 62 1), circuit_t min. n_swap	ype = quantum_ av. depth 65 ± 0.0 60.19 ± 0.69 ype = quantum_ av. depth	simulation, p min. depth 65 58 simulation, p	= 8, repetition av. n_qubits 8 ± 0.0 8.0 ± nan	ns = 16, optimizat min. qubits 8	tion_level = 2 total time (s) 0.77 2.04 tion_level = 3 total time (s)	av. time (s) 0.77 ± 0.0 0.13 ± 0.02 av. time (s)	min. time (
ume = shur sthod une-graph abre	fiken, size = (1, av. n_swaps 64 * 0.0 62 * nan fiken, size = (1, av. n_swaps	1), circuit_t min. n_swap 64 62 1), circuit_t min. n_swap	ype = quantum_ av. depth 65 ± 0.0 60.19 ± 0.69 ype = quantum_ av. depth	simulation, p min. depth 65 58	= 8, repetition av. n_qubits 8 ± 0.0 8.0 ± nan	ns = 16, optimizat min. qubits 8 8 8	tion_level = 2 total time (s) 0.77 2.04 tion_level = 3 total time (s)	av. time (s) 0.77 ± 0.0 0.13 ± 0.02 av. time (s)	min. time (
me = shur thod 	riken, size = (1, av. n_swaps	1), circuit_t min. n_swap 64 62 1), circuit_t min. n_swap 64	ype = quantum_ av. depth	simulation, p min. depth 65 58 simulation, p min. depth	= 8, repetition av. n_qubits 8 ± 0.0 8.0 ± nan = 8, repetition av. n_qubits	min. qubits 8 8 8 8 s = 16, optimizat min. qubits	tion_level = 2 total time (s) 0.77 2.04 tion_level = 3 total time (s)	av. time (s) 0.77 ± 0.0 0.13 ± 0.02 av. time (s) 0.75 ± 0.0 0.36 ± 0.03	min. time (
ume = shur sthod une-graph abre shur sthod une-graph	riken, size = (1, av. n_swaps 64 ± 0.0 62 ± nan riken, size = (1, av. n_swaps 64 ± 0.0 62 ± nan	1), circuit_t min. n_swap 64 62 1), circuit_t min. n_svap 64 62	ype = quantum_ av. depth 65 ± 0.0 60.19 ± 0.69 ype = quantum_ av. depth 65 ± 0.0 60.69 ± 0.66	simulation, p min. depth 65 58 simulation, p min. depth 65 58	= 8, repetition av. n_qubits 8 ± 0.0 8.0 ± nan = 8, repetition av. n_qubits	ns = 16, optimizat min. qubits 8 8 8	tion_level = 2 total time (s) 0.77 2.04 tion_level = 3 total time (s) 0.75 5.7	av. time (s) 0.77 ± 0.0 0.13 ± 0.02 av. time (s) 0.75 ± 0.0 0.36 ± 0.03	min. time (
ume = shur sthod une-graph abre sme = shur sthod une-graph abre	riken, size = (1, av. n_swaps 64 ± 0.0 62 ± nan riken, size = (1, av. n_swaps 64 ± 0.0 62 ± nan	1), circuit_t min. n_swap 64 62 1), circuit_t min. n_svap 64 62	ype = quantum_ av. depth 65 ± 0.0 60.19 ± 0.69 ype = quantum_ av. depth 65 ± 0.0 60.69 ± 0.66	simulation, p min. depth 58 simulation, p min. depth 58	= 8, repetition av. n_qubits	min. qubits 8 8 8 16, optimizat min. qubits 11, optimizat min. qubits 8 8	tion_level = 2 total time (s) 0.77 2.04 tion_level = 3 total time (s) 0.75 5.7	av. time (s) 0.77 ± 0.0 0.13 ± 0.02 av. time (s) 0.75 ± 0.0 0.36 ± 0.03	min. time (0. 0. min. time (0. 0.
ume = shundine-graph white = shundine-graph white = shundine-graph white = shundine-graph white = shundine-graph	riken, size = (1, av. n_swaps 64 * 0.0 62 * nan riken, size = (1, av. n_swaps 64 * 0.0 62 * nan	1), circuit_t min. n_swap 64 62 1), circuit_t min. n_swap 64 62 1), circuit_t min. n_swap	ype = quantum_ av. depth 65 ± 0.0 60.19 ± 0.69 ype = quantum_ av. depth 65 ± 0.0 60.69 ± 0.66	simulation, p min. depth 65 58 simulation, p min. depth 65 58	= 8, repetition av. n_qubits 8 ± 0.0 8.0 ± nan = 8, repetition av. n_qubits 8 ± 0.0 8.0 ± nan	ns = 16, optimizat min. qubits 8 8 8 s= 16, optimizat min. qubits 8 8	tion_level = 2 total time (s) 0.77 2.04 tion_level = 3 total time (s) 0.75 5.7	av. time (s) 0.77 ± 0.0 0.13 ± 0.02 av. time (s) 0.75 ± 0.0 0.36 ± 0.03	min. time (0. 0. min. time (0. 0. min. time (

method	av. n_swaps	min. n_swap	av. depth	min. depth	n av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (
ine-graph	128 ± 0.0	128	129 ± 0.0	129	8 ± 0.0	8	1.53	3 1.53 ± 0.0	1.
sabre	126 ± nan	126	121.12 ± 1.06	119	9 8.0 ± nan	8	3.57	0.22 ± 0.02	0.
name = shur	riken, size = (1	., 1), circuit_t	type = quantum_s	simulation, p	= 16, repetition	s = 16, optimiza	tion_level = 2		
	av. n_swaps	min. n_swap	av. depth				total time (s)		min. time
ine-graph	128 ± 0.0	128	129 ± 0.0	129	8 ± 0.0	8	1.58	3 1.58 ± 0.0	1
abre	126 ± nan		121.06 ± 0.88	118	8 8.0 ± nan	8	4.24	1 0.26 ± 0.03	0.
ame = shu	riken, size = (1	., 1), circuit_t		simulation, p	= 16, repetition	s = 16, optimiza	tion_level = 3		
	av. n_swaps	min. n_swap	-		n av. n_qubits		total time (s)		
ine-graph	128 ± 0.0	128	129 ± 0.0	129	8 ± 0.0	8	1.5	1.5 ± 0.0	1
abre	126 ± nan	126	120.81 ± 0.94	116	8.0 ± nan	8	11.06	0.69 ± 0.02	0
ethod	riken, size = (3 av. n_swaps	3, 3), circuit_t min. n_swap			= 1, repetitions av. n_qubits		ion_level = 0 total time (s)		
ethod	av. n_swaps	min. n_swap		min. depth			total time (s)		
ethod ine-graph	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)		1
ethod ine-graph abre	av. n_swaps 	min. n_swap 	av. depth	min. depth	av. n_qubits 	min. qubits 	1 2.68	1.0 ± 0.0	1
ethod ine-graph abre ame = shu	av. n_swaps 130 ± 0.0 469 ± 8.24 riken, size = (3 av. n_swaps	min. n_swap 130 460 3, 3), circuit_t min. n_swap	av. depth 12 ± 0.0 67.94 ± 2.41	min. depth 12 60 simulation, p min. depth	av. n_qubits 84 * 0.0 83.75 * 0.22 = 1, repetitions av. n_qubits	min. qubits 84 84 = 16, optimizat min. qubits	total time (s) 1 2.68 ion_level = 1 total time (s)	1.0 ± 0.0 0.17 ± 0.02	1 0.
ethod ine-graph abre ame = shu	av. n_swaps	min. n_swap 130 460 3, 3), circuit_t min. n_swap	av. depth 12 ± 0.0 67.94 ± 2.41 Eype = quantum_c av. depth	min. depth 12 60 simulation, p min. depth	av. n_qubits 84 ± 0.0 83.75 ± 0.22 = 1, repetitions av. n_qubits	min. qubits 84 84 84 = 16, optimizat min. qubits	total time (s) 1 2.68 ion_level = 1	1.0 ± 0.0 0.17 ± 0.02	1 0.
ine-graph abre ame = shui	av. n_swaps 130 ± 0.0 469 ± 8.24 riken, size = (3 av. n_swaps 130 ± 0.0	min. n_swap 130 460 3, 3), circuit_t min. n_swap	av. depth 12 * 0.0 67.94 * 2.41	min. depth 12 60 simulation, p min. depth	av. n_qubits 84 ± 0.0 83.75 ± 0.22 = 1, repetitions av. n_qubits	min. qubits 84 84 84 = 16, optimizat min. qubits	total time (s) 1 2.68 ion_level = 1 total time (s)	1.0 ± 0.0 0.17 ± 0.02	1 0.
ethod ine-graph abre shutethod ine-graph	av. n_swaps 130 ± 0.0 469 ± 8.24 riken, size = (3 av. n_swaps 130 ± 0.0 177 ± 6.53	min. n_swap 130 460 3, 3), circuit_t min. n_swap	av. depth 12 ± 0.0 67.94 ± 2.41 2.41 2.41 2.42 av. depth 12 ± 0.0 34.5 ± 2.94	min. depth 12 60 simulation, p min. depth 12 27	av. n_qubits 	min. qubits 84 84 84 = 16, optimizat min. qubits 84 67	total time (s) 1 2.68 ion_level = 1 total time (s)	1.0 ± 0.0 0.17 ± 0.02 av. time (s) 	min. time (s
ethod ine-graph abre ame = shu ethod ine-graph abre ame = shu	av. n_swaps 130 ± 0.0 469 ± 8.24 riken, size = (3 av. n_swaps 130 ± 0.0 177 ± 6.53	min. n_swap 130 460 3, 3), circuit_t min. n_swap 130 187	av. depth 12 ± 0.0 67.94 ± 2.41 Eype = quantum_s av. depth 12 ± 0.0 34.5 ± 2.94	min. depth 12 60 simulation, p min. depth 12 27	av. n_qubits 84 * 0.0 83.75 * 0.22 = 1, repetitions av. n_qubits 84 * 0.0 64.88 * 1.19	min. qubits 84 84 = 16, optimizat min. qubits 84 67	total time (s) 1 2.68 ion_level = 1 total time (s) 0.89 2.79	1.0 ± 0.0 0.17 ± 0.02 av. time (s) 0.89 ± 0.0 0.17 ± 0.02	min. time (s
ame = shuu ethod ine-graph abre ine-graph ame = shuu ethod abre ame = shuu ethod	av. n_swaps 130 ± 0.0 469 ± 8.24 riken, size = (3 av. n_swaps 130 ± 0.0 177 ± 6.53 av. n_swaps	min. n_swap 130 460 3, 3), circuit_t min. n_swap 130 187	av. depth 12 ± 0.0 67.94 ± 2.41 Expe = quantum_4 av. depth 12 ± 0.0 34.5 ± 2.94 Expe = quantum_4 av. depth	min. depth 12 60 min. depth 12 27 simulation, p min. depth	av. n_qubits	min. qubits 84 84 84 = 16, optimizat min. qubits 84 67	total time (s) 1 2.68 ion_level = 1 total time (s) 0.89 2.79 ion_level = 2 total time (s)	av. time (s) av. time (s) av. time (s) av. time (s)	0.8 0.8 0.1
nethodine-graph name = shun nethodine-graph name = shun	av. n_swaps 130 ± 0.0 469 ± 8.24 riken, size = (3 av. n_swaps 177 ± 6.53 av. n_swaps 130 ± 0.0 177 ± 6.53	min. n_swap 130 460 3, 3), circuit_t min. n_swap 130 187	av. depth 12 ± 0.0 67.94 ± 2.41	min. depth 12 60 simulation, p min. depth 27 simulation, p min. depth	av. n_qubits	min. qubits 84 84 84 = 16, optimizat min. qubits 84 67 = 16, optimizat min. qubits	total time (s) 1 2.68 ion_level = 1 total time (s) 0.89 2.79 ion_level = 2 total time (s)	av. time (s) av. time (s) av. time (s) 0.89 ± 0.0 av. time (s)	1 0. min. time (s

method	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s)
	130 ± 0.0		12 ± 0.0		84 ± 0.0	84	1	1.0 ± 0.0	1
sabre	138 ± 6.41			20		63		0.89 ± 0.03	0.94
name = shur	riken, size = (3	. 3), circuit t		imulation, p =	8, repetitions	= 16. optimizati	on level = 0		
nethod	av. n_swaps	min. n_swap	av. depth	min. depth		min. qubits	total time (s)		min. time (s
	1040 ± 0.0		89 ± 0.0		84 ± 0.0	84		7.06 ± 0.0	7.0
sabre	2778 ± 36.62	2575	400.94 ± 8.12	362	83.81 ± 0.19	84	18.11	1.13 ± 0.03	1.1
name = shur nethod	av. n_swaps	, 3), circuit_t min. n_swap			8, repetitions av. n_qubits		on_level = 1 total time (s) av. time (s)	min. time (
	1040 ± 0.0		89 ± 0.0		84 ± 0.0	84	6 9	2 6.92 ± 0.0	6.
abre	1268 ± 32.28		254.25 ± 18.57		64.94 ± 1.41	63		3 1.0 ± 0.04	0.
nethod	av. n_swaps	min. n_swap	ype = quantum_s: av. depth	imulation, p =	8, repetitions	min. qubits	total time (s) av. time (s)	
nethod	av. n_swaps	min. n_swap	ype = quantum_s: av. depth	imulation, p =	8, repetitions	min. qubits	total time (s) av. time (s)	
nethod	av. n_swaps	min. n_swap	ype = quantum_s: av. depth	imulation, p = min. depth	8, repetitions	min. qubits	total time (s) av. time (s)	6.
ethod ine-graph abre	av. n_swaps 	min. n_swap 	ype = quantum_s: av. depth 	min. depth	8, repetitions av. n_qubits 84 ± 0.0 64.69 ± 1.41	min. qubits	total time (s.) av. time (s)	6.
ethod ine-graph abre	av. n_swaps 1040 ± 0.0 1051 ± 26.45 riken, size = (3 av. n_swaps	min. n_swap 1040 926 , 3), circuit_t min. n_swap	ype = quantum_s: av. depth 89 * 0.0 215.75 * 15.27 ype = quantum_s: av. depth	min. depth 88 170 imulation, p = min. depth	8, repetitions av. n_qubits 84 ± 0.0 64.69 ± 1.41	min. qubits 84 65 16, optimizati min. qubits	total time (s. 6.9; 22.5 on_level = 3 total time (s)	av. time (s) 2 6.92 ± 0.0 1.41 ± 0.05	6. 1. min. time (s
nethod ine-graph sabre amame = shur	av. n_swaps 1040 ± 0.0 1051 ± 26.45 riken, size = (3 av. n_swaps	min. n_swap 1040 926 , 3), circuit_t min. n_swap	ype = quantum_s: av. depth 89 ± 0.0 215.75 ± 15.27 ype = quantum_s: av. depth	min. depth 89 170 imulation, p = min. depth	8, repetitions av. n_qubits 84 ± 0.0 64.69 ± 1.41	min. qubits 84 65 = 16, optimizati min. qubits	total time (s)	av. time (s) 2 6.92 ± 0.0 1.41 ± 0.05	6. 1. min. time (s
athod ine-graph abre shure sthod ine-graph	av. n_swaps 1040 ± 0.0 1051 ± 26.45 Tiken, size = (3 av. n_swaps 1040 ± 0.0 966 ± 14.37	min. n_swap 1040 926 , 3), circuit_t min. n_swap 1040 892	ype = quantum_s: av. depth 89 ± 0.0 215.75 ± 15.27 ype = quantum_s: av. depth 89 ± 0.0 192.5 ± 13.59	imulation, p = min. depth	8, repetitions 84 ± 0.0 64.69 ± 1.41 8, repetitions av. n_qubits	min. qubits	on_level = 3 total time (s) 7.06	av. time (s) 2 6.92 ± 0.0 1.41 ± 0.05 av. time (s) 7.06 ± 0.0 4.94 ± 0.07	min. time (
ethod ine-graph abre ame = shur ethod ine-graph	av. n_swaps 1040 ± 0.0 1051 ± 26.45 Tiken, size = (3 av. n_swaps 1040 ± 0.0 966 ± 14.37	min. n_swap 1040 926 , 3), circuit_t min. n_swap	ype = quantum_s: av. depth 89 ± 0.0 215.75 ± 15.27 ype = quantum_s: av. depth 89 ± 0.0 192.5 ± 13.59	min. depth 89 170 imulation, p = min. depth	8, repetitions av. n_qubits 84 ± 0.0 64.69 ± 1.41 8, repetitions av. n_qubits	min. qubits	6.9: 22.5 0n_level = 3 total time (s)	av. time (s) 1.41 ± 0.05 av. time (s) 7.06 ± 0.0 4.94 ± 0.07	6. 1. min. time (s
nethod ine-graph sabre same = shun ethod ine-graph sabre	av. n_swaps 1040 ± 0.0 1051 ± 26.45 riken, size = (3 av. n_swaps 1040 ± 0.0 966 ± 14.37	min. n_swap 1040 926 , 3), circuit_t min. n_swap 1040 892	ype = quantum_s: av. depth 89 ± 0.0 215.75 ± 15.27 ype = quantum_s: av. depth 89 ± 0.0 192.5 ± 13.59	imulation, p = min. depth 89 170 imulation, p = min. depth 145	8, repetitions av. n_qubits 84 ± 0.0 64.69 ± 1.41 8, repetitions av. n_qubits 44 ± 0.0 64.44 ± 1.09	min. qubits 84 65 = 16, optimizati min. qubits 84 68	total time (s) 6.9 22.5 on_level = 3 total time (s) 7.06 79.12	av. time (s) 2 6.92 ± 0.0 1.41 ± 0.05 av. time (s) 7.06 ± 0.0 4.94 ± 0.07	6. 1. min. time (s
nethod Line-graph sabre Line-graph sabre Line-graph sabre Line-graph sabre	av. n_swaps 1040 ± 0.0 1051 ± 26.45 Tiken, size = (3 av. n_swaps 1040 ± 0.0 966 ± 14.37	min. n_swap 1040 926 , 3), circuit_t min. n_swap 1040 892	ype = quantum_s: av. depth 89 ± 0.0 215.75 ± 15.27 ype = quantum_s: av. depth 192.5 ± 13.59 ype = quantum_s: av. depth	imulation, p = min. depth min. depth min. depth 145 imulation, p = min. depth	8, repetitions av. n_qubits 84 ± 0.0 64.69 ± 1.41 8, repetitions av. n_qubits 84 ± 0.0 64.44 ± 1.09	min. qubits 16, optimizati min. qubits 16, optimizati min. qubits 16, optimizati	on_level = 3 total time (s) 7.06 79.12 ion_level = 0 total time (s)	av. time (s) 2 6.92 ± 0.0 1.41 ± 0.05 av. time (s) 7.06 ± 0.0 4.94 ± 0.07	fin. time (s
method line-graph sabre line-graph line-graph sabre name = shun method	av. n_swaps 1040 ± 0.0 1051 ± 26.45 Tiken, size = (3 av. n_swaps 1040 ± 0.0 966 ± 14.37	min. n_swap 1040 926 , 3), circuit_t min. n_swap 1040 892	ype = quantum_s: av. depth 89 ± 0.0 215.75 ± 15.27 ype = quantum_s: av. depth 192.5 ± 13.59 ype = quantum_s: av. depth	imulation, p = min. depth min. depth min. depth 145 imulation, p = min. depth	8, repetitions av. n_qubits 84 ± 0.0 64.69 ± 1.41 8, repetitions av. n_qubits 84 ± 0.0 64.44 ± 1.09	min. qubits 16, optimizati min. qubits 16, optimizati min. qubits 16, optimizati	on_level = 3 total time (s) 7.06 79.12 ion_level = 0 total time (s)	av. time (s) 2 6.92 ± 0.0 1.41 ± 0.05 av. time (s) 7.06 ± 0.0 4.94 ± 0.07	min. time

	riken, size = (3,								
nethod	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s
line-graph	2080 ± 0.0	2080	177 ± 0.0	177	84 ± 0.0	84	14.13	14.13 ± 0.0	14.1
sabre	2035 ± 50.28	1861	450.12 ± 38.78	302	65.19 ± 1.28	66	35.28	2.2 ± 0.06	2.1
name = shur	riken, size = (3,	3), circuit_t	ype = quantum_si	mulation, p =	16, repetitions	= 16, optimizati	on_level = 2		
nethod	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s
line-graph	2080 ± 0.0	2080	177 ± 0.0	177	84 ± 0.0	84	13.71	13.71 ± 0.0	13.7
sabre	2080 ± 41.66	1890	398.25 ± 19.59	334	65.38 ± 0.94	63	44.87	2.8 ± 0.06	2.8
name = shur	riken, size = (3,	3), circuit_t	ype = quantum_si	mulation, p =	16, repetitions	= 16, optimizati	on_level = 3		
nethod	av. n_swaps	min. n_swap		min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s
line-graph	2080 ± 0.0	2080	177 ± 0.0	177	84 ± 0.0	84	13.89	13.89 ± 0.0	13.8
,									
sabre	1826 ± 35.68		373.75 ± 26.89		64.44 ± 1.53	61		9.42 ± 0.09	
			ype = quantum_si	imulation, p =	1, repetitions		n_level = 0		
name = shur	riken, size = (5, av. n_swaps	5), circuit_t	ype = quantum_si av. depth	mulation, p =	1, repetitions av. n_qubits	= 16, optimizatio	n_level = 0 total time (s)	av. time (s)	min. time (s)
name = shur	riken, size = (5, av. n_swaps	5), circuit_t min. n_swap	ype = quantum_si av. depth	mulation, p =	1, repetitions av. n_qubits	= 16, optimizatio	n_level = 0 total time (s)	av. time (s)	min. time (s)
name = shur nethod Line-graph	riken, size = (5, av. n_swaps 	5), circuit_t min. n_swap	ype = quantum_si av. depth 	mulation, p =	1, repetitions av. n_qubits	= 16, optimizatic min. qubits	n_level = 0 total time (s)	av. time (s) 2.76 ± 0.0	min. time (s)
name = shun nethod Line-graph sabre	av. n_swaps 	5), circuit_t min. n_swap 418 1706	ype = quantum_si av. depth 14 * 0.0 117.56 * 4.03	min. depth	1, repetitions av. n_qubits 240 ± 0.0 237.69 ± 0.81	= 16, optimizatic min. qubits	n_level = 0 total time (s) 2.76 7.59	av. time (s) 2.76 ± 0.0	min. time (s)
name = shun nethod Line-graph sabre	riken, size = (5, av. n_swaps	5), circuit_t min. n_swap 418 1706 5), circuit_t min. n_swap	ype = quantum_si av. depth 14 ± 0.0 117.56 ± 4.03 ype = quantum_si av. depth	min. depth 14 100 min. depth imulation, p =	1, repetitions av. n_qubits 240 ± 0.0 237.69 ± 0.81	= 16, optimization min. qubits 240 239 = 16, optimization min. qubits	n_level = 0 total time (s) 2.76 7.59 n_level = 1 total time (s)	av. time (s) 2.76 ± 0.0 0.47 ± 0.03 av. time (s)	min. time (s) 2.76 0.43
name = shun method line-graph sabre name = shun method	riken, size = (5, av. n_swaps	5), circuit_t min. n_swap 418 1706 5), circuit_t min. n_swap	ype = quantum_si av. depth 14 ± 0.0 117.56 ± 4.03 ype = quantum_si av. depth	min. depth 14 100 min. depth 1au 1au 1bu 1bu 1cu 1cu 1cu 1cu 1cu 1c	1, repetitions av. n_qubits 240 ± 0.0 237.69 ± 0.81	= 16, optimization min. qubits 240 239	n_level = 0 total time (s) 2.76 7.59 n_level = 1 total time (s)	av. time (s) 2.76 ± 0.0 0.47 ± 0.03 av. time (s)	min. time (s) 2.76 0.43
name = shunethod	riken, size = (5, av. n_swaps	5), circuit_t min. n_swap 418 1706 5), circuit_t min. n_swap 418	ype = quantum_si av. depth 14 ± 0.0 117.56 ± 4.03 ype = quantum_si av. depth	min. depth 14 100 min. depth imulation, p =	1, repetitions av. n_qubits 240 ± 0.0 237.69 ± 0.81 1, repetitions av. n_qubits	= 16, optimization min. qubits 240 239 = 16, optimization min. qubits	n_level = 0 total time (s) 2.76 7.59 n_level = 1 total time (s)	av. time (s) 2.76 ± 0.0 0.47 ± 0.03 av. time (s)	min. time (s)
name = shun nethod line-graph sabre name = shun nethod line-graph sabre	riken, size = (5, av. n_swaps	5), circuit_t min. n_swap 418 1706 5), circuit_t min. n_swap 418 918	ype = quantum_si av. depth 14 ± 0.0 117.56 ± 4.03 ype = quantum_si av. depth 14 ± 0.0 85.12 ± 4.62	min. depth 14 100 min. depth 14 170 min. depth 18 mulation, p =	1, repetitions av. n_qubits 240 ± 0.0 237.69 ± 0.81 1, repetitions av. n_qubits 240 ± 0.0 179.31 ± 2.32	= 16, optimization min. qubits 240 239 = 16, optimization min. qubits 240 182	n_level = 0 total time (s) 2.76 7.59 n_level = 1 total time (s) 2.61 11.16	av. time (s) 2.76 ± 0.0 0.47 ± 0.03 av. time (s) 2.61 ± 0.0 0.7 ± 0.03	min. time (s) 2.76 0.43 min. time (s) 2.61 0.8
name = shunethod	riken, size = (5, av. n_swaps 418 ± 0.0 1722 ± 13.92 riken, size = (5, av. n_swaps 418 ± 0.0 993 ± 25.43	5), circuit_t min. n_swap 418 1706 5), circuit_t min. n_swap 418 918 5), circuit_t min. n_swap	ype = quantum_si av. depth 14 * 0.0 117.56 * 4.03 ype = quantum_si av. depth 14 * 0.0 85.12 * 4.62 ype = quantum_si av. depth	mulation, p = min. depth 14 100 mulation, p = min. depth 70 mulation, p = min. depth	1, repetitions av. n_qubits 240 ± 0.0 237.69 ± 0.81 1, repetitions av. n_qubits 240 ± 0.0 179.31 ± 2.32 1, repetitions av. n_qubits	= 16, optimization min. qubits 240 239 = 16, optimization min. qubits	n_level = 0 total time (s) 2.76 7.59 n_level = 1 total time (s) 2.61 11.16 n_level = 2 total time (s)	av. time (s) 2.76 ± 0.0 0.47 ± 0.03 av. time (s) 2.61 ± 0.0 0.7 ± 0.03	min. time (s) 2.76 0.43 min. time (s) 2.61 0.8
name = shunethod	riken, size = (5, av. n_swaps 418 ± 0.0 1722 ± 13.92 riken, size = (5, av. n_swaps 418 ± 0.0 993 ± 25.43	5), circuit_t min. n_swap 418 1706 5), circuit_t min. n_swap 418 918 5), circuit_t min. n_swap	ype = quantum_si av. depth 14 * 0.0 117.56 * 4.03 ype = quantum_si av. depth 14 * 0.0 85.12 * 4.62 ype = quantum_si av. depth	mulation, p = min. depth 14 100 mulation, p = min. depth 70 mulation, p = min. depth	1, repetitions av. n_qubits 240 ± 0.0 237.69 ± 0.81 1, repetitions av. n_qubits 240 ± 0.0 179.31 ± 2.32 1, repetitions av. n_qubits	= 16, optimization min. qubits 240 239 = 16, optimization min. qubits 240 182	n_level = 0 total time (s) 2.76 7.59	av. time (s) 2.76 ± 0.0 0.47 ± 0.03 av. time (s) 2.61 ± 0.0 0.7 ± 0.03	min. time (s) 2.76 0.43 min. time (s) 2.61 0.8

nethod	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s)
	419 + 0 0		14 + 0 0		240 + 0 0	240			
abre	418 ± 0.0 702 ± 15.83		14 ± 0.0 66.81 ± 6.6		240 ± 0.0 176.12 ± 2.69	183		2.6 ± 0.0 6.6 ± 0.04	2.6 6.58
ame = shu	riken, size = (5,	5), circuit_t	ype = quantum_s	imulation, p	= 8, repetitions	= 16, optimizat	ion_level = 0		
ethod	av. n_swaps	min. n_swap	av. depth	min. dep	th av. n_qubits	min. qubits	total time (s	av. time (s)	min. time (s
ine-graph	3344 ± 0.0	3344	105 ± 0.0	1	05 240 ± 0.0	240	19.2	1 19.21 ± 0.0	19.2
abre	10085 ± 62.18	10397	654.81 ± 12.53		19 238.31 ± 0.59	238	53.6	1 3.35 ± 0.12	3.6
me = shu	riken, size = (5,	5), circuit_t			= 8, repetitions	= 16, optimizat		av. time (s)	min. time (s
		-	-		-	-			
ne-graph	3344 ± 0.0	3344	105 ± 0.0	10	5 240 ± 0.0	240	19.42	19.42 ± 0.0	19.4
bre	4949 ± 172.47	4147	581.88 ± 36.5	46	1 178.88 ± 2.91	173	60.32	3.77 ± 0.09	3.72
			ype = quantum_s	imulation, p	= 8, repetitions	= 16, optimizat	ion_level = 2) av. time (s)	
ame = shu	riken, size = (5, av. n_swaps	5), circuit_t min. n_swap	<pre>gype = quantum_s av. depth</pre>	imulation, p	= 8, repetitions	= 16, optimizat min. qubits	ion_level = 2 total time (s	av. time (s)	min. time (s
ethod	riken, size = (5, av. n_swaps	5), circuit_t min. n_swap	sype = quantum_s av. depth	imulation, p min. dep	= 8, repetitions th av. n_qubits	= 16, optimizat min. qubits	total time (s	av. time (s)	min. time (s
thod	av. n_swaps 	5), circuit_t min. n_swap	av. depth 	imulation, p min. dep	= 8, repetitions th av. n_qubits	= 16, optimizat min. qubits	total time (s	av. time (s) 5 19.65 ± 0.0	min. time (:
ame = shu	riken, size = (5, av. n_swaps 	5), circuit_t min. n_swap 3344 3792	ype = quantum_s av. depth 	imulation, p min. dep 11 4	= 8, repetitions th av. n_qubits 	= 16, optimizat min. qubits	ion_level = 2 total time (s 19.6 109.0) av. time (s) 	min. time (:
ame = shu: athod ine-graph abre ame = shu: athod	riken, size = (5, av. n_swaps 3344 ± 0.0 4321 ± 153.74 riken, size = (5, av. n_swaps	5), circuit_t min. n_swap 3344 3792 5), circuit_t min. n_swap	ype = quantum_s av. depth	imulation, p min. dep in 4 imulation, p min. dep	= 8, repetitions th av. n_qubits	= 16, optimizat min. qubits 240 171 = 16, optimizat min. qubits	total time (s) av. time (s) 5 19.65 ± 0.0 2 6.81 ± 0.22	min. time (a
when the shure and the shure are shure as the shure as the standard and the shure as the standard are shure as the standar	riken, size = (5, av. n_swaps 3344 ± 0.0 4321 ± 153.74 riken, size = (5, av. n_swaps	5), circuit_t min. n_swap 3344 3792 5), circuit_t min. n_swap	ype = quantum_s av. depth	imulation, p min. dep in in dep in dep in dep min. dep	= 8, repetitions th av. n_qubits	= 16, optimizat min. qubits 240 171 = 16, optimizat min. qubits	total time (s) av. time (s) 5 19.65 ± 0.0 2 6.81 ± 0.22	min. time (:
me = shu	riken, size = (5, av. n_swaps 3344 ± 0.0 4321 ± 153.74 riken, size = (5, av. n_swaps 3344 ± 0.0 3405 ± 65.66	5), circuit_t min. n_swap 3344 3792 5), circuit_t min. n_swap 3344 3312	ype = quantum_s av. depth	imulation, p min. dep 4 imulation, p min. dep	= 8, repetitions th av. n_qubits 05 240 ± 0.0 14 173.81 ± 2.09 = 8, repetitions th av. n_qubits 05 240 ± 0.0 43 170.62 ± 2.41	= 16, optimizat min. qubits 240 171 = 16, optimizat min. qubits 240 165	ion_level = 2 total time (s 19.6 109.0 ion_level = 3 total time (s) av. time (s) 5 19.65 * 0.0 2 6.81 * 0.22) av. time (s) 3 19.33 * 0.0 8 26.84 * 0.98	min. time (
me = shu: thode-graph threme = shu: thod	riken, size = (5, av. n_swaps 	5), circuit_t min. n_swap 3344 3792 5), circuit_t min. n_swap 3344 3312	ype = quantum_s av. depth	imulation, p min. dep 1 4 imulation, p min. dep imulation, p min. dep	= 8, repetitions th av. n_qubits	= 16, optimizat min. qubits 240 171 = 16, optimizat min. qubits	ion_level = 2 total time (s 19.6 109.0 ion_level = 3 total time (s 19.3 429.4) av. time (s) 5 19.65 ± 0.0 2 6.81 ± 0.22) av. time (s) 3 19.33 ± 0.0 8 26.84 ± 0.98	min. time (1
ine-graph thod ine-graph thre ine-graph thre ine-graph thre ine-graph	riken, size = (5, av. n_swaps 	5), circuit_t min. n_swap 3344 3792 5), circuit_t min. n_swap 3344 3312	ype = quantum_s av. depth	imulation, p min. dep 11 4	= 8, repetitions th av. n_qubits	= 16, optimizat min. qubits 240 171 = 16, optimizat min. qubits 240 165	ion_level = 2 total time (s 19.6 109.0 ion_level = 3 total time (s 19.3 429.4) av. time (s) 5 19.65 ± 0.0 2 6.81 ± 0.22) av. time (s) 3 19.33 ± 0.0 8 26.84 ± 0.98	min. time (s
me = shu: thod ne-graph thre thod ne-graph thre me = shu: thod	riken, size = (5, av. n_swaps 3344 ± 0.0 4321 ± 153.74 riken, size = (5, av. n_swaps 3344 ± 0.0 3405 ± 65.66	5), circuit_t min. n_swap 3344 3792 5), circuit_t min. n_swap 3344 3312 5), circuit_t min. n_swap	ype = quantum_s av. depth 105 * 0.0 566.06 * 43.95 ype = quantum_s av. depth 105 * 0.0 408.12 * 21.62	imulation, p min. dep d imulation, p min. dep	= 8, repetitions th av. n_qubits 05	= 16, optimizat min. qubits 240 171 = 16, optimizat min. qubits 240 165	ion_level = 2 total time (s 19.6 109.0 ion_level = 3 total time (s 429.4 tion_level = 0 ts total time) av. time (s) 5 19.65 ± 0.0 2 6.81 ± 0.22) av. time (s) 5 19.33 ± 0.0 8 26.84 ± 0.98	min. time (s
when = shu	riken, size = (5, av. n_swaps 3344 ± 0.0 4321 ± 153.74 riken, size = (5, av. n_swaps 3344 ± 0.0 3405 ± 65.66	5), circuit_t min. n_swap 3344 3792 5), circuit_t min. n_swap 3344 3312 5), circuit_t min. n_swap	ype = quantum_s av. depth	imulation, p min. dep d imulation, p min. dep imulation, p min. dep	= 8, repetitions th av. n_qubits 05	= 16, optimizat min. qubits 240 171 = 16, optimizat min. qubits 240 165 3 = 16, optimizas 3 min. qubi	ion_level = 2 total time (s 19.6 109.0 ion_level = 3 total time (s 429.4 tion_level = 0 ts total time) av. time (s) 5 19.65 ± 0.0 2 6.81 ± 0.22) av. time (s) 3 19.33 ± 0.0 8 26.84 ± 0.98	min. time (19. 7. min. time (19. 25.

) min. time (:
6.8
min. time (s
38.4
12.2
min. time (s
38.1
43.4
min. time (s)
5.19
5.19
5.19 1.13
5.19
5.19 1.13
5.19 1.13
5.19 1.13 min. time (s) 5.2 2.78

name = shu:	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s)
	av. n_swaps								min. cime (3)
ine-graph	850 ± 0.0	850	14 ± 0.0	14	476 ± 0.0	476	5.22	5.22 ± 0.0	5.22
abre	2079 ± 57.9	1891	124.12 ± 8.31	93	342.06 ± 3.19	354	628.32	39.27 ± 0.78	37.63
ame = shu	riken, size = (7,	7), circuit_ty	ype = quantum_sin	ulation, p =	8, repetitions =	16, optimizatio	n_level = 0		
ethod	av. n_swaps	min. n_swap	av. depth	min. deptl	n av. n_qubits	min. qubits	total time (s) av. time (s)	min. time (s
ine-graph	6800 ± 0.0	6800	105 ± 0.0	10	5 476 ± 0.0	476	38.5	8 38.58 ± 0.0	38.8
abre	25487 ± 146.92	25288	875.19 ± 21.34	78	5 474.06 ± 1.12	475	111.3	6.96 ± 0.17	6.8
me = shu	riken, size = (7,	7), circuit_tj	ype = quantum_sim	ulation, p =	8, repetitions =	16, optimizatio	n_level = 1		
ethod	av. n_swaps	min. n_swap	-	min. deptl	n av. n_qubits	min. qubits	total time (s) av. time (s)	min. time (s
	6800 ± 0.0		105 ± 0.0		5 476 ± 0.0	476		2 38.22 ± 0.0	38.:
abre	14038 ± 501.85	11570		024		345	407.0		10.
			1110.0 ± 72.38		5 349.44 ± 5.69			6 10.49 ± 0.26	
			ype = quantum_sin	ulation, p =			n_level = 2		
ame = shu	riken, size = (7, av. n_swaps	7), circuit_tj	ype = quantum_sim	min. deptl	8, repetitions =	16, optimizatio	n_level = 2 total time (s) av. time (s)	min. time (
ethod	riken, size = (7, av. n_swaps	7), circuit_t; min. n_swap	ype = quantum_sim av. depth	min. depth	8, repetitions =	16, optimizatio	n_level = 2 total time (s) av. time (s)	min. time (
ame = shu	riken, size = (7, av. n_swaps 	7), circuit_t; min. n_swap	ype = quantum_sim av. depth 105 ± 0.0	min. depth	8, repetitions = n av. n_qubits 5 476 ± 0.0	16, optimizatio	n_level = 2 total time (s) av. time (s)	min. time (
ame = shu	riken, size = (7, av. n_swaps 	7), circuit_t; min. n_swap 6800 12192	ype = quantum_sim av. depth 105 ± 0.0 1092.25 ± 51.37	min. depth	8, repetitions = n av. n_qubits	16, optimizatio	n_level = 2 total time (s 37.9 352.1:	av. time (s) 4 37.94 ± 0.0 3 22.01 ± 0.34	min. time (
ame = shu: athod ine-graph abre ame = shu: athod	riken, size = (7, av. n_swaps 6800 ± 0.0 11889 ± 326.3	7), circuit_t; min. n_swap 6800 12192 7), circuit_t; min. n_swap	ype = quantum_sim av. depth 105 ± 0.0 1092.25 ± 51.37 ype = quantum_sim av. depth	mulation, p = min. depth	8, repetitions = n av. n_qubits 5 476 ± 0.0 2 343.62 ± 5.27 8, repetitions = av. n_qubits	16, optimizatio min. qubits 476 347	n_level = 2 total time (s.) 37.9 352.1: n_level = 3 total time (s)	av. time (s) 4 37.94 ± 0.0 3 22.01 ± 0.34 av. time (s)	min. time (
ame = shu: sthod ine-graph abre ame = shu:	riken, size = (7, av. n_swaps 	7), circuit_t; min. n_swap 6800 12192 7), circuit_t; min. n_swap	ype = quantum_sim av. depth 105 ± 0.0 1092.25 ± 51.37 ype = quantum_sim av. depth	min. depti	8, repetitions = n av. n_qubits 5 476 ± 0.0 2 343.62 ± 5.27 8, repetitions = av. n_qubits	16, optimizatio min. qubits 476 347	n_level = 2 total time (s 37.9 352.1: n_level = 3 total time (s)	av. time (s) 4 37.94 ± 0.0 3 22.01 ± 0.34 av. time (s)	min. time (
me = shu	riken, size = (7, av. n_swaps 	7), circuit_t; min. n_swap 6800 12192 7), circuit_t; min. n_swap 6800 9200	ype = quantum_sim av. depth 105 ± 0.0 1092.25 ± 51.37 ype = quantum_sim av. depth 105 ± 0.0 791.69 ± 50.65	min. depth 101 922 min. depth 102 103 104 105 589	8, repetitions = n av. n_qubits 5 476 ± 0.0 2 343.62 ± 5.27 8, repetitions = av. n_qubits 476 ± 0.0 336.75 ± 4.19	16, optimizatio min. qubits 476 347 16, optimizatio min. qubits 476 333	n_level = 2 total time (s. 37.9 352.1: n_level = 3 total time (s) 38.25 1612.88	av. time (s) av. time (s) 2.01 ± 0.34 av. time (s) 38.25 ± 0.0 100.8 ± 1.84	min. time (
the shu	riken, size = (7, av. n_swaps 	7), circuit_t; min. n_swap 6800 12192 7), circuit_t; min. n_swap 6800 9200	ype = quantum_sim av. depth 105 ± 0.0 1092.25 ± 51.37 ype = quantum_sim av. depth 105 ± 0.0 791.69 ± 50.65	mulation, p = min. depth 92:	8, repetitions = n av. n_qubits 5 476 ± 0.0 2 343.62 ± 5.27 8, repetitions = av. n_qubits 476 ± 0.0 336.75 ± 4.19	16, optimizatio min. qubits 476 347 16, optimizatio min. qubits 476 333	n_level = 2 total time (s 37.9 352.1: n_level = 3 total time (s) 38.25 1612.88	av. time (s) 3 7.94 ± 0.0 3 22.01 ± 0.34 av. time (s) 38.25 ± 0.0 100.8 ± 1.84	min. time (
ame = shu	riken, size = (7, av. n_swaps 6800 ± 0.0 11889 ± 326.3 riken, size = (7, av. n_swaps 6800 ± 0.0 9431 ± 295.31	7), circuit_t; min. n_swap 6800 12192 7), circuit_t; min. n_swap 6800 9200 7), circuit_t; min. n_swap	ype = quantum_sim av. depth 105 ± 0.0 1092.25 ± 51.37 ype = quantum_sim av. depth 105 ± 0.0 791.69 ± 50.65	min. depth min. depth min. depth 105 589 mulation, p = min. depth	8, repetitions = av. n_qubits 5 476 ± 0.0 2 343.62 ± 5.27 8, repetitions = av. n_qubits 476 ± 0.0 336.75 ± 4.19	16, optimizatio min. qubits 476 347 16, optimizatio min. qubits 476 333	n_level = 2 total time (s) 37.9 352.1: n_level = 3 total time (s) 38.25 1612.88	av. time (s) 4 37.94 ± 0.0 3 22.01 ± 0.34 av. time (s) 38.25 ± 0.0 100.8 ± 1.84	min. time (s min. time (s 38.2 99.3
ame = shu: sthod ine-graph abre sthod ine-graph abre ame = shu: sthod	riken, size = (7, av. n_swaps 6800 ± 0.0 11889 ± 326.3 riken, size = (7, av. n_swaps 6800 ± 0.0 9431 ± 295.31	7), circuit_t; min. n_swap 6800 12192 7), circuit_t; min. n_swap 9200 7), circuit_t; min. n_swap	ype = quantum_sim av. depth 105 ± 0.0 1092.25 ± 51.37 ype = quantum_sim av. depth 105 ± 0.0 791.69 ± 50.65	mulation, p = min. depth 104 922 mulation, p = min. depth 105 589	8, repetitions = av. n_qubits 5 476 ± 0.0 2 343.62 ± 5.27 8, repetitions = av. n_qubits 476 ± 0.0 336.75 ± 4.19	16, optimizatio min. qubits 476 347 16, optimizatio min. qubits 476 333	n_level = 2 total time (s) 37.9 352.1: n_level = 3 total time (s) 38.25 1612.88	av. time (s) 4 37.94 ± 0.0 3 22.01 ± 0.34 av. time (s) 38.25 ± 0.0 100.8 ± 1.84	min. time (s min. time (s 38.2 99.3

```
name = shuriken, size = (7, 7), circuit_type = quantum_simulation, p = 16, repetitions = 16, optimization_level = 1
      av. n_swaps
                 min. n_swap av. depth
                                    min. depth av. n_qubits
                                                     min. qubits total time (s) av. time (s) min. time (s)
209 476 ± 0.0
                   19213 1943.62 ± 179.76
                                       1252 342.62 ± 4.44
                                                          334
                                                                   295.31 18.46 ± 0.33
      24677 ± 1008.11
                                                                                        19.18
name = shuriken, size = (7, 7), circuit_type = quantum_simulation, p = 16, repetitions = 16, optimization_level = 2
     av. n_swaps
               min. n_swap av. depth
                                   min. depth av. n_qubits min. qubits total time (s) av. time (s)
13600 209 ± 0.0
                                      209 476 ± 0.0
                                                        476
                                                                  75.21 75.21 ± 0.0
line-graph 13600 ± 0.0
                                                                                      75.21
      22096 ± 699.86
                    19115 1676.12 ± 96.85
                                       1293 348.44 ± 4.94
                                                         347
                                                                   573.52 35.84 ± 0.93
name = shuriken, size = (7, 7), circuit_type = quantum_simulation, p = 16, repetitions = 16, optimization_level = 3
     av. n_swaps min. n_swap av. depth
                                   min. depth av. n_qubits min. qubits total time (s) av. time (s) min. time (s)
476
                                                         323
                   17064 1369.19 ± 66.21
                                      1099 336.31 ± 4.62
      18148 ± 348.43
                                                                  2519.55 157.47 ± 3.75
                                                                                      146.54
```

7.0.2 Quantum simulation, checkerboard, agianst SABRE

```
import line_graph_routing as lgr # Loading these makes these cells stand-alone
import pickle
settings=[]
for name in ['checkerboard']:
    for side in [i+0.5 \text{ for } i \text{ in } range(1,9,2)]:
        for p in [1,8,16]:
            for optimization_level in range(4):
                setting={'name':name,
                      'size': (side,side),
                      'circuit_type': 'quantum_simulation',
                      'p': p,
                      'repetitions' : 16,
                      'optimization_level' : optimization_level,
                      'methods' : ['sabre']
                settings.append(setting)
## Uncomment to rerun benchmarks. This takes a couple of hours.
#results=[]
#for setting in settings:
  result=lqr.benchmark(**setting)
```

```
# results.append(result)
# lgr.print_benchmark(result)
#
#with open('benchmark_data/checkerboard.pkl','wb') as f:
# pickle.dump(results,f)

#Load previously obtained results from disk and show them.
import pickle
with open('benchmark_data/checkerboard.pkl','rb') as f:
    results=pickle.load(f)

for result in results:
    lgr.print_benchmark(result)
```

```
name = checkerboard, size = (1.5, 1.5), circuit_type = quantum_simulation, p = 1, repetitions = 16, optimization_level = 0
      av. n_swaps min. n_swap av. depth min. depth av. n_qubits min. qubits total time (s) av. time (s) min. time (s)
42 27 ± 0.0
                                                27 21 ± 0.0
line-graph 42 ± 0.0
                                                                      21
                                                                                  0.17 \quad 0.17 \pm 0.0
                          47 32.44 ± 1.97
                                              27 19.06 ± 0.66
                                                                     21
                                                                                  0.99 0.06 ± 0.03
name = checkerboard, size = (1.5, 1.5), circuit_type = quantum_simulation, p = 1, repetitions = 16, optimization_level = 1
                                        min. depth av. n_qubits
       av. n_swaps min. n_swap av. depth
                                                              min. qubits total time (s) av. time (s)
line-graph 42 ± 0.0
                          42 27 ± 0.0
                                              27 21 ± 0.0
                                                                     21
                                                                                 0.37 0.37 ± 0.0
      32 ± 1.44
                         22 26.56 ± 2.0
                                             19 17.69 ± 0.66
                                                                     16
                                                                                0.67 0.04 ± 0.03
name = checkerboard, size = (1.5, 1.5), circuit_type = quantum_simulation, p = 1, repetitions = 16, optimization_level = 2
                                        min. depth av. n_qubits min. qubits total time (s) av. time (s) min. time (s)
       av. n_swaps min. n_swap av. depth
                          42 27 ± 0.0
                                               27 21 ± 0.0
line-graph 42 ± 0.0
                                                                     21
                                                                                  0.35 0.35 ± 0.0
                         31 23.81 ± 1.81
                                              19 17.94 ± 0.87
        28 ± 0.84
                                                                     21
                                                                                  1.34 0.08 ± 0.04
name = checkerboard, size = (1.5, 1.5), circuit_type = quantum_simulation, p = 1, repetitions = 16, optimization_level = 3
method av. n_swaps min. n_swap av. depth min. depth av. n_qubits min. qubits total time (s) av. time (s) min. time (s)
                                                                     21
line-graph 42 ± 0.0
                          42 27 ± 0.0
                                              27 21 ± 0.0
                                                                                  0.17 0.17 ± 0.0
      28 ± 0.78
                         25 25.31 ± 2.03
                                              18 16.81 ± 0.41
                                                                      18
                                                                                 2.56 0.16 ± 0.04
                                                                                                           0.11
```

method	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s)
line-graph	322 ± 0.0	322	188 ± 0.0	188	21 ± 0.0	21	1.89	1.89 ± 0.0	1.89
sabre	362 ± 7.38	343	235.0 ± 6.03	214	21.0 ± nan	21	4.87	0.3 ± 0.06	0.18
							otimization_level		
ethod	av. n_swaps	min. n_swap	-		av. n_qubits		total time (s)		min. time (s
.ine-graph	322 ± 0.0	322	188 ± 0.0	188	21 ± 0.0	21	1.98	1.98 ± 0.0	1.9
abre	239 ± 3.71	240	197.25 ± 7.75	175	17.94 ± 0.84	16	4.99	0.31 ± 0.06	0.2
ame = chec	ckerboard, size	= (1.5, 1.5), c	ircuit_type = q	uantum_simula	tion, p = 8, rep	etitions = 16, o	otimization_level	= 2	
nethod	av. n_swaps	min. n_swap	-		av. n_qubits		total time (s)		min. time (s)
ine-graph	322 ± 0.0	322	188 ± 0.0	188	21 ± 0.0	21	1.94	1.94 ± 0.0	1.94
						16	5.04		0.28
sabre	237 ± 2.75		183.5 ± 7.31		17.12 ± 0.5			0.36 ± 0.05	
name = chec	ckerboard, size		ircuit_type = q	uantum_simula min. depth		etitions = 16, o	otimization_level	= 3	min. time (s)
name = chec	ckerboard, size	= (1.5, 1.5), c	ircuit_type = q	uantum_simula min. depth	tion, p = 8, rep av. n_qubits	etitions = 16, o	ptimization_level total time (s)	= 3 av. time (s)	min. time (s)
name = chec	ckerboard, size av. n_swaps	= (1.5, 1.5), c min. n_swap	ircuit_type = q	uantum_simula min. depth	tion, p = 8, repa	etitions = 16, o	otimization_level total time (s)	= 3 av. time (s)	min. time (s)
mame = checkethod	av. n_swaps 	= (1.5, 1.5), c min. n_swap 	ircuit_type = q av. depth	uantum_simula min. depth 188 175	tion, p = 8, rep. av. n_qubits	etitions = 16, o min. qubits 21	otimization_level total time (s) 1.95 17.58	= 3 av. time (s) 	min. time (s)
ame = checeethod	av. n_swaps 322 * 0.0 217 * 2.41	= (1.5, 1.5), c	ircuit_type = q av. depth	uantum_simula min. depth 188 175	tion, p = 8, rep. av. n_qubits	etitions = 16, o min. qubits 21 16	otimization_level total time (s)	= 3 av. time (s) 	min. time (s) 1.98
ine-graph	av. n_swaps 322 ± 0.0 217 ± 2.41 ckerboard, size av. n_swaps	322 221 = (1.5, 1.5), c min. n_swap	ircuit_type = q av. depth 188 * 0.0 187.5 * 4.34 ircuit_type = q av. depth	uantum_simula min. depth 188 175	tion, p = 8, rep. av. n_qubits 21 ± 0.0 16.62 ± 0.44 tion, p = 16, rep. h av. n_qubits	etitions = 16, opinin. qubits 21 16 petitions = 16, opinin. qubits	total time (s) 1.95 17.58 optimization_leve	= 3 av. time (s) 1.95 ± 0.0 1.1 ± 0.01	min. time (s) 1.95 1.08
ame = checosthod ine-graph abre ame = checosthod	22 ± 0.0 217 ± 2.41 240.0 217 ± 2.41 241 241 241 241 241 241 241 2	322 221 = (1.5, 1.5), c min. n_swap = 322 221	av. depth 188 ± 0.0 187.5 ± 4.34 ircuit_type = q av. depth 372 ± 0.0	uantum_simula min. depth 188 175	tion, p = 8, rep av. n_qubits 21 ± 0.0 16.62 ± 0.44 tion, p = 16, rep h av. n_qubits	etitions = 16, opmin. qubits 21 16 petitions = 16, opmin. qubits min. qubits	total time (s) 1.95 17.58 poptimization_leve	= 3 av. time (s)	min. time (s) 1.98 1.00 min. time (
ame = chec ethod 	av. n_swaps 322 ± 0.0 217 ± 2.41 Ekerboard, size av. n_swaps 642 ± 0.0 763 ± 20.84	= (1.5, 1.5), c min. n_swap 322 221 = (1.5, 1.5), c min. n_swap 642 657	ircuit_type = q av. depth	uantum_simula min. depth 188 175 uantum_simula min. dept	tion, p = 8, rep. av. n_qubits 21 ± 0.0 16.62 ± 0.44 tion, p = 16, rep. h av. n_qubits 2 21 ± 0.0 7 21.0 ± nan	petitions = 16, or min. qubits 21 16 petitions = 16, or min. qubits	total time (s) 1.95 17.58 poptimization_leve	= 3 av. time (s) 1.95 ± 0.0 1.1 ± 0.01 1 = 0 1 av. time (s) 5 3.85 ± 0.0 7 0.5 ± 0.05	min. time (s) 1.95 1.08 min. time (
name = checkethod ine-graph sabre ame = checkethod ine-graph sabre	22 ± 0.0 217 ± 2.41 240.0 217 ± 2.41 241 241 242 241 242 243 244 244 245 246 247 247 248 248 248 248 248 248 248 248	= (1.5, 1.5), c min. n_swap 322 221 = (1.5, 1.5), c min. n_swap 642 657	ircuit_type = q av. depth 188 ± 0.0 187.5 ± 4.34 ircuit_type = q av. depth 372 ± 0.0 485.38 ± 12.94	uantum_simula min. depth 188 175 uantum_simula min. dept	tion, p = 8, rep av. n_qubits 21 ± 0.0 16.62 ± 0.44 tion, p = 16, rep h av. n_qubits 2 21 ± 0.0 7 21.0 ± nan	petitions = 16, or min. qubits 21 16 petitions = 16, or min. qubits 21 21 21	total time (s) 1.95 17.58 optimization_leve total time (s) 3.8 8.0	= 3 av. time (s) 1.95 ± 0.0 1.1 ± 0.01 1 = 0) av. time (s) 5 3.85 ± 0.0 7 0.5 ± 0.05	min. time (s) 1.95 1.08 min. time (
name = checkethod	22 ± 0.0 217 ± 2.41 22 ± 0.0 217 ± 2.41 24 ± 2.41 25 ± 2.41 26 ± 2.41 27 ± 2.41 28 ± 2.41 29 ± 2.41 20 ± 2.41 20 ± 2.41 20 ± 2.41 21 ± 2.41 22 ± 0.0 23 ± 2.84	= (1.5, 1.5), c min. n_swap 322 221 = (1.5, 1.5), c min. n_swap 642 657	ircuit_type = q av. depth	uantum_simula min. depth 188 175 uantum_simula min. dept 37 41 uantum_simula min. dept	tion, p = 8, rep av. n_qubits 21 ± 0.0 16.62 ± 0.44 tion, p = 16, rep h av. n_qubits 2 21 ± 0.0 7 21.0 ± nan tion, p = 16, rep h av. n_qubits	petitions = 16, or min. qubits 21 16 min. qubits 21 22 21 21 petitions = 16, or min. qubits	total time (s) 1.95 17.58 poptimization_leve total time (s) 3.8	= 3 av. time (s) 1.95 ± 0.0 1.1 ± 0.01 1 = 0 1 av. time (s) 7 0.5 ± 0.05 1 = 1 1 av. time (s)	min. time (s) 1.95 1.08 min. time (min. time (
name = checkethod	22 ± 0.0 217 ± 2.41 22 ± 0.0 217 ± 2.41 24 ± 2.41 25 ± 2.41 26 ± 2.41 27 ± 2.41 28 ± 2.41 29 ± 2.41 20 ± 2.41 20 ± 2.41 20 ± 2.41 21 ± 2.41 22 ± 0.0 23 ± 2.84	= (1.5, 1.5), c min. n_swap = (1.5, 1.5), c min. n_svap 642 657	ircuit_type = q av. depth	uantum_simula min. depth 188 175 uantum_simula min. dept 37 41 uantum_simula min. dept	tion, p = 8, rep av. n_qubits 21 ± 0.0 16.62 ± 0.44 tion, p = 16, rep h av. n_qubits 2 21 ± 0.0 7 21.0 ± nan	petitions = 16, or min. qubits 21 16 min. qubits 21 22 21 21 petitions = 16, or min. qubits	total time (s) 1.95 17.58 poptimization_leve total time (s) 3.8 8.0	= 3 av. time (s) 1.95 ± 0.0 1.1 ± 0.01 1 = 0 1 av. time (s) 7 0.5 ± 0.05 1 = 1 1 av. time (s)	min. time (s) 1.95 1.08 min. time (3. 0.

method	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s)
line-graph	642 ± 0.0	642	372 ± 0.0	372	21 ± 0.0	21	3.22	3.22 ± 0.0	3.2
sabre	463 ± 5.31	466	376.0 ± 8.98	346	17.5 ± 0.81	21	10.29	0.64 ± 0.04	0.52
name = chec	ckerboard, size	= (1.5, 1.5), c			tion, p = 16, rep	petitions = 16, o	ptimization_leve		min. time
	- •								
line-graph	642 ± 0.0	642	372 ± 0.0	37	2 21 ± 0.0	21	3.2	3 3.23 ± 0.0	3.
sabre	454 ± 2.81	447	353.69 ± 12.16	32	1 16.88 ± 0.5	16	29.5	1.84 ± 0.03	1.
						etitions = 16, op			
	av. n_swaps				av. n_qubits	min. qubits	total time (s)		
line-graph	271 ± 0.0	271	30 ± 0.0	30	97 ± 0.0	97	0.92	0.92 ± 0.0	0.9
sabre	550 ± 9.03	548	104.56 ± 3.78	90	88.5 ± 1.56	92	2.56	0.16 ± 0.02	0.1
nethod	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits		total time (s)	av. time (s)	
nethod	av. n_swaps 	min. n_swap	av. depth	min. depth	av. n_qubits 97 ± 0.0	min. qubits	total time (s)	av. time (s) 1.01 ± 0.0	1.0
nethod	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	1.0
nethod Line-graph sabre name = check	av. n_swaps 	min. n_swap 271 233	av. depth 30 ± 0.0 63.81 ± 4.25	min. depth 30 48	av. n_qubits 	97 68 stitions = 16, op	total time (s) 1.01 3.03 timization_level	av. time (s) 1.01 ± 0.0 0.19 ± 0.02	1.0
method line-graph sabre name = ched method	av. n_swaps 271 * 0.0 257 * 7.69 ckerboard, size av. n_swaps	min. n_swap 271 233 = (3.5, 3.5), c min. n_swap	av. depth 30 ± 0.0 63.81 ± 4.26 ircuit_type = q av. depth	min. depth 30 48 uantum_simula	av. n_qubits 97 ± 0.0 69.0 ± 1.38 tion, p = 1, repeav. n_qubits	min. qubits 97 68	total time (s) 1.01 3.03 timization_level total time (s)	av. time (s) 1.01 ± 0.0 0.19 ± 0.02 = 2 av. time (s)	1.0 0.1
method line-graph sabre name = chec	av. n_swaps 271 * 0.0 257 * 7.69 ckerboard, size av. n_swaps	min. n_swap 271 233 = (3.5, 3.5), c min. n_swap	av. depth 30 ± 0.0 63.81 ± 4.26 ircuit_type = q av. depth	min. depth 30 48 uantum_simula min. depth	av. n_qubits 97 ± 0.0 69.0 ± 1.38 tion, p = 1, repeav. n_qubits	min. qubits 97 68 estitions = 16, or	total time (s) 1.01 3.03 timization_level total time (s)	av. time (s) 1.01 ± 0.0 0.19 ± 0.02 = 2 av. time (s)	1.0 0.1
nethod line-graph sabre name = chec nethod	av. n_swaps 271 ± 0.0 257 ± 7.69 ckerboard, size av. n_swaps 271 ± 0.0 228 ± 6.72	min. n_swap 271 233 = (3.5, 3.5), c min. n_swap 271 234	av. depth 30 * 0.0 63.81 * 4.25 circuit_type = q av. depth 30 * 0.0 57.94 * 3.75	min. depth 30 48 uantum_simular min. depth 30 48	av. n_qubits 97 ± 0.0 69.0 ± 1.38 tion, p = 1, repe av. n_qubits 97 ± 0.0 70.56 ± 1.25	### ##################################	timization_level total time (s) 1.01 3.03 timization_level total time (s) 1.01 4.87	av. time (s) 1.01 ± 0.0 0.19 ± 0.02 = 2 av. time (s) 1.01 ± 0.0 0.3 ± 0.03	1.0 0.1 min. time (s
method line-graph sabre mame = checken line-graph sabre	av. n_swaps 	min. n_swap 271 233 = (3.5, 3.5), c min. n_swap 271 234	av. depth 30 ± 0.0 63.81 ± 4.25 ircuit_type = q av. depth 30 ± 0.0 57.94 ± 3.75	min. depth 30 48 uantum_simula min. depth 30 48	av. n_qubits 97 ± 0.0 69.0 ± 1.38 tion, p = 1, repeav. n_qubits 97 ± 0.0 70.56 ± 1.25	min. qubits 97 68 stitions = 16, or min. qubits 97 71	timization_level	av. time (s) 1.01 ± 0.0 0.19 ± 0.02 = 2 av. time (s) 1.01 ± 0.0 0.3 ± 0.03	1.0 0.1
method line-graph sabre method line-graph sabre	av. n_swaps 271 ± 0.0 257 ± 7.69 ckerboard, size av. n_swaps 271 ± 0.0 228 ± 6.72 ckerboard, size av. n_swaps	min. n_swap 271 233 = (3.5, 3.5), c min. n_swap 271 234 = (3.5, 3.5), c min. n_swap	av. depth 30 ± 0.0 63.81 ± 4.25 circuit_type = q av. depth 30 ± 0.0 57.94 ± 3.75 circuit_type = q av. depth	min. depth 30 48 uantum_simula min. depth 48 uantum_simula min. depth	av. n_qubits 97 ± 0.0 69.0 ± 1.38 tion, p = 1, repe av. n_qubits 97 ± 0.0 70.56 ± 1.25 tion, p = 1, repe av. n_qubits	min. qubits 97 68 etitions = 16, or min. qubits 97 71	timization_level total time (s)	av. time (s) 1.01 ± 0.0 0.19 ± 0.02 = 2 av. time (s) 1.01 ± 0.0 0.3 ± 0.03	1.0: 0.1: min. time (s) 0.20 min. time (s)
method line-graph sabre method line-graph sabre	av. n_swaps 271 ± 0.0 257 ± 7.69 ckerboard, size av. n_swaps 271 ± 0.0 228 ± 6.72 ckerboard, size av. n_swaps	min. n_swap 271 233 = (3.5, 3.5), c min. n_swap 271 234 = (3.5, 3.5), c min. n_swap	av. depth 30 ± 0.0 63.81 ± 4.25 circuit_type = q av. depth 30 ± 0.0 57.94 ± 3.75 circuit_type = q av. depth	min. depth 30 48 uantum_simular min. depth 48 uuantum_simular	av. n_qubits 97 ± 0.0 69.0 ± 1.38 tion, p = 1, repe av. n_qubits 97 ± 0.0 70.56 ± 1.25 tion, p = 1, repe av. n_qubits	min. qubits 97 68 etitions = 16, or min. qubits 97 71 etitions = 16, or min. qubits	timization_level total time (s) 1.01 3.03 timization_level total time (s) 1.01 4.87	av. time (s) 1.01 ± 0.0 0.19 ± 0.02 = 2 av. time (s) 1.01 ± 0.0 0.3 ± 0.03	1.01 0.17 min. time (s) 1.01 0.26

method	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s
line-graph	2077 ± 0.0	2077	198 ± 0.0	198	97 ± 0.0	97	7.69	7.69 ± 0.0	7.6
sabre	3416 ± 49.08	3159	597.62 ± 15.12	544	92.62 ± 0.75	92	18.07	1.13 ± 0.05	1.1
ame = chec	ckerboard, size	= (3.5, 3.5), c			on, p = 8, reperator		imization_level total time (s)		min. time (
	- •	-	av. depth	-	-		total time (s)	av. time (s)	min. time (
line-graph	2077 ± 0.0	2077	198 ± 0.0	198	97 ± 0.0	97	7.68	7.68 ± 0.0	7.6
sabre	1870 ± 26.84	1894	481.25 ± 11.75	444	70.19 ± 1.84		20.33	1.27 ± 0.05	1.3
							imization_level		
nethod	av. n_swaps	min. n_swap	av. depth				total time (s)		min. time (s)
.ine-graph	2077 ± 0.0	2077	198 ± 0.0	198	97 ± 0.0	97	7.91	7.91 ± 0.0	7.9
sabre	1785 ± 15.44	1756	456.5 ± 14.31	404	69.56 ± 1.16	73	29.02	1.81 ± 0.05	1.86
		(0.5)							
name = chec	ckerboard, size	min. n_swap		min. depth	on, p = 8, repe av. n_qubits	min. qubits	rimization_level (min. time (:
nethod		min. n_swap	av. depth	min. depth	on, p = 8, repe av. n_qubits	min. qubits	total time (s)		
ethod ine-graph	av. n_swaps	min. n_swap	av. depth	min. depth	on, p = 8, reperary.	min. qubits	total time (s)	av. time (s)	7.
ethod ine-graph abre	av. n_swaps 	min. n_swap 	av. depth 	min. depth	on, p = 8, repe av. n_qubits 	min. qubits	7.73	av. time (s) 	7.
ethodine-graph sabre	av. n_swaps 	min. n_swap 	av. depth 	min. depth 198 387	on, p = 8, repe av. n_qubits 	min. qubits 97 76	total time (s)	av. time (s) 	7. 6.
ethod ine-graph abre amme = checkethod	av. n_swaps 2077 ± 0.0 1740 ± 16.03 ckerboard, size av. n_swaps	min. n_swap 2077 1731 = (3.5, 3.5), c min. n_swap	av. depth 198 ± 0.0 446.75 ± 14.74 ircuit_type = qua av. depth	min. depth 198 387 antum_simulat:	on, p = 8, repe av. n_qubits 97 ± 0.0 69.12 ± 2.0 on, p = 16, rep av. n_qubits	min. qubits 97 76 etitions = 16, or	7.73 104.9 ctimization_level total time (s)	av. time (s) 7.73 ± 0.0 6.56 ± 0.05	7. 6. min. time (
ethod ine-graph abre ame = chece ethod	av. n_swaps 2077 ± 0.0 1740 ± 16.03 	2077 1731 = (3.5, 3.5), c min. n_swap	av. depth 198 ± 0.0 446.75 ± 14.74 ircuit_type = qua av. depth	min. depth 198 387 antum_simulat: min. depth	on, p = 8, repe av. n_qubits 97 ± 0.0 69.12 ± 2.0 	min. qubits 97 76 etitions = 16, or min. qubits	total time (s) 7.73 104.9 timization_level total time (s)	av. time (s) 7.73 ± 0.0 6.56 ± 0.05 = 0 av. time (s)	7. 6
ine-graph abre ame = checkethod ine-graph	av. n_swaps 2077 ± 0.0 1740 ± 16.03 **Ckerboard, size av. n_swaps 4141 ± 0.0 6142 ± 72.56	min. n_swap 2077 1731 = (3.5, 3.5), c min. n_swap 4141 5875	av. depth 198 ± 0.0 446.75 ± 14.74 ircuit_type = quantum av. depth 390 ± 0.0 1117.75 ± 21.0	min. depth 198 387 antum_simulat: min. depth 390 1021	on, p = 8, repe av. n_qubits 97 ± 0.0 69.12 ± 2.0 on, p = 16, rep av. n_qubits 97 ± 0.0 94.0 ± nan	min. qubits 97 76 etitions = 16, or min. qubits	total time (s) 7.73 104.9 timization_level total time (s)	av. time (s) 7.73 ± 0.0 6.56 ± 0.05 = 0 av. time (s) 14.99 ± 0.0 2.29 ± 0.13	min. time (:
ine-graph sabre ame = chec ethod ine-graph abre	av. n_swaps 2077 ± 0.0 1740 ± 16.03 ckerboard, size av. n_swaps 4141 ± 0.0 6142 ± 72.56	min. n_swap 2077 1731 = (3.5, 3.5), c min. n_swap 4141 5875	av. depth 198 ± 0.0 446.75 ± 14.74 ircuit_type = qua av. depth 390 ± 0.0 1117.75 ± 21.0 ircuit_type = qua	min. depth 198 387 antum_simulat: min. depth 390 1021	on, p = 8, repe av. n_qubits 97 ± 0.0 69.12 ± 2.0 on, p = 16, rep av. n_qubits 97 ± 0.0 94.0 ± nan	min. qubits 97 76 etitions = 16, or min. qubits 97 94	total time (s) 7.73 104.9 ctimization_level total time (s) 36.66	av. time (s) 7.73 ± 0.0 6.56 ± 0.05 av. time (s) 14.99 ± 0.0 2.29 ± 0.13	7. 6. min. time (
nethodine-graph sabreame = checkethodine-graph sabreame = checkethod	av. n_swaps 2077 ± 0.0 1740 ± 16.03 ckerboard, size av. n_swaps 4141 ± 0.0 6142 ± 72.56 ckerboard, size av. n_swaps	min. n_swap 2077 1731 = (3.5, 3.5), c min. n_swap 4141 5875 = (3.5, 3.5), c min. n_swap	av. depth 198 ± 0.0 446.75 ± 14.74 ircuit_type = qua av. depth 390 ± 0.0 1117.75 ± 21.0 ircuit_type = qua av. depth	min. depth 198 387 antum_simulati min. depth 1021 antum_simulati min. depth	on, p = 8, repe av. n_qubits 97 ± 0.0 69.12 ± 2.0 on, p = 16, rep av. n_qubits 97 ± 0.0 94.0 ± nan on, p = 16, rep av. n_qubits	min. qubits 97 76 etitions = 16, or min. qubits 97 94 etitions = 16, or min. qubits	total time (s) 7.73 104.9 timization_level total time (s) 14.99 36.66	av. time (s) 7.73 ± 0.0 6.56 ± 0.05 = 0 av. time (s) 14.99 ± 0.0 2.29 ± 0.13 = 1 av. time (s)	14.5 2.7 min. time (s
nethod Line-graph sabre name = chec nethod name = chec name = chec	av. n_swaps 2077 ± 0.0 1740 ± 16.03 Ekerboard, size av. n_swaps 4141 ± 0.0 6142 ± 72.56 av. n_swaps	min. n_swap 2077 1731 = (3.5, 3.5), c min. n_swap 4141 5875 = (3.5, 3.5), c min. n_swap	av. depth 198 ± 0.0 446.75 ± 14.74 ircuit_type = qua av. depth 390 ± 0.0 1117.75 ± 21.0 ircuit_type = qua av. depth	min. depth antum_simulat; min. depth antum_simulat; min. depth	on, p = 8, repe av. n_qubits 97 ± 0.0 69.12 ± 2.0 on, p = 16, rep av. n_qubits 97 ± 0.0 94.0 ± nan on, p = 16, rep av. n_qubits	min. qubits 97 76 etitions = 16, or min. qubits 97 94 etitions = 16, or min. qubits	total time (s) 7.73 104.9 timization_level total time (s) 36.66	av. time (s) 7.73 ± 0.0 6.56 ± 0.05 = 0 av. time (s) 14.99 ± 0.0 2.29 ± 0.13 = 1 av. time (s)	min. time (s

method	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s
line-graph	4141 ± 0.0	4141	390 ± 0.0	390	97 ± 0.0	97	15.66	15.66 ± 0.0	15.
abre	3684 ± 25.79	3608	949.5 ± 19.69	893	70.06 ± 2.69	71	57.58	3.6 ± 0.1	3.
ame = chec	ckerboard, size					petitions = 16, o			min. time
	av. n_swaps	min. n_swap				min. qubits	total time (s)		min. time
ine-graph	4141 ± 0.0	4141	390 ± 0.0	390	97 ± 0.0	97	15.64	1 15.64 ± 0.0	15
abre	3580 ± 20.27	3621	924.56 ± 17.07	854	67.0 ± 1.47	65	207.53	3 12.97 ± 0.12	12
ethod	ckerboard, size	min. n_swap	av. depth	min. depth	av. n_qubits	etitions = 16, op		av. time (s)	
	682 ± 0.0		34 ± 0.0		221 ± 0.0	221		2.48 ± 0.0	2.
abre	1770 ± 17.16	1791	138.56 ± 4.66	121	211.31 ± 1.47	213	6.81	0.43 ± 0.04	0.
ethod	ckerboard, size	= (5.5, 5.5), c			cion, p = 1, repe av. n_qubits		timization_level		min. time (
ethod	av. n_swaps	min. n_swap		min. depth	av. n_qubits	min. qubits	total time (s)		
ethod	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	2.
ethod ine-graph abre ame = checeethod	682 ± 0.0 678 ± 30.68	min. n_swap 682 678 = (5.5, 5.5), c min. n_swap	av. depth 34 ± 0.0 107.06 ± 7.62 ircuit_type = qr av. depth	min. depth 34 77 uantum_simulat min. depth	av. n_qubits 	min. qubits 221 160 stitions = 16, op	total time (s) 2.45 9.63 timization_level total time (s)	av. time (s) 2.45 ± 0.0 0.6 ± 0.04 = 2 av. time (s)	2. 0. min. time (s
ethod ine-graph abre ame = chece	682 ± 0.0 678 ± 30.68	min. n_swap 682 678 = (5.5, 5.5), c min. n_swap	av. depth 34 ± 0.0 107.06 ± 7.62 ircuit_type = qr av. depth	min. depth 34 77 uantum_simulat min. depth	av. n_qubits 	min. qubits 221 160 stitions = 16, op	total time (s) 2.45 9.63 timization_level total time (s)	av. time (s) 2.45 ± 0.0 0.6 ± 0.04 = 2 av. time (s)	2. 0.
ethod ine-graph abre ame = chec ethod ine-graph abre	av. n_swaps 	min. n_swap 682 678 = (5.5, 5.5), c min. n_swap 682 773	av. depth 34 ± 0.0 107.06 ± 7.62 circuit_type = quav. depth 34 ± 0.0 102.81 ± 7.0	min. depth uantum_simulat min. depth 34 82	av. n_qubits 221 ± 0.0 160.38 ± 1.34 201.20 201.20 201.20 202.20 203.20 204.20 205.20 205.20 206.20	min. qubits 221 160 stitions = 16, op min. qubits 221 160	total time (s) 2.45 9.63 timization_level total time (s) 2.47 18.67	av. time (s) 2.45 ± 0.0 0.6 ± 0.04 = 2 av. time (s) 2.47 ± 0.0 1.17 ± 0.03	2 0 min. time (1 2.4
ethod ine-graph abre ame = chec ethod ine-graph abre ame = chec	av. n_swaps 682 ± 0.0 678 ± 30.68 ckerboard, size av. n_swaps 682 ± 0.0 808 ± 21.59	min. n_swap 682 678 = (5.5, 5.5), c min. n_swap 682 773	av. depth	min. depth 34 77 uantum_simulat min. depth 34 82	av. n_qubits 221 ± 0.0 160.38 ± 1.34 210, p = 1, repe av. n_qubits 221 ± 0.0 159.62 ± 2.76	min. qubits 221 160 stitions = 16, op min. qubits 221 160	total time (s) 2.45 9.63 timization_level total time (s) 2.47 18.67	av. time (s) 2.45 ± 0.0 0.6 ± 0.04 = 2 av. time (s) 2.47 ± 0.0 1.17 ± 0.03	2. din. time (s
ame = checethod	av. n_swaps 682 ± 0.0 678 ± 30.68 ckerboard, size av. n_swaps 682 ± 0.0 808 ± 21.59 ckerboard, size av. n_swaps	min. n_swap 682 678 = (5.5, 5.5), c min. n_swap 682 773 = (5.5, 5.5), c min. n_swap	av. depth 34 ± 0.0 107.06 ± 7.62 circuit_type = qu av. depth 34 ± 0.0 102.81 ± 7.0 circuit_type = qu av. depth	min. depth 34 77 uantum_simulat min. depth 32 uantum_simulat min. depth	av. n_qubits 221 ± 0.0 160.38 ± 1.34 201.20 201.20 201.20 201.20 202.20 203.20 203.20 204.20 205.20 205.20 206.20	min. qubits 221 160 stitions = 16, op min. qubits 221 160	total time (s) 2.45 9.63 timization_level total time (s) 2.47 18.67 timization_level total time (s)	av. time (s) 2.45 ± 0.0 0.6 ± 0.04 = 2 av. time (s) 1.17 ± 0.03 = 3 av. time (s)	2. 0. min. time (s
ine-graph sabre ame = checkethod ine-graph sabre	av. n_swaps 682 ± 0.0 678 ± 30.68 ckerboard, size av. n_swaps 682 ± 0.0 808 ± 21.59 ckerboard, size av. n_swaps	min. n_swap 682 678 = (5.5, 5.5), c min. n_swap 773 = (5.5, 5.5), c min. n_swap	av. depth 34 ± 0.0 107.06 ± 7.62 circuit_type = qu av. depth 34 ± 0.0 102.81 ± 7.0 circuit_type = qu av. depth	min. depth auantum_simulat min. depth 82 uantum_simulat min. depth	av. n_qubits 221 ± 0.0 160.38 ± 1.34 201.20 201.20 201.20 201.20 202.20 203.20 203.20 204.20 205.20 205.20 206.20	min. qubits 221 160 221 24titions = 16, op min. qubits 221 160 2titions = 16, op min. qubits	total time (s) 2.45 9.63 timization_level total time (s) 2.47 18.67	av. time (s) 2.45 ± 0.0 0.6 ± 0.04 = 2 av. time (s) 1.17 ± 0.03 = 3 av. time (s)	2. do

method	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s
	5148 ± 0.0		223 ± 0.0		221 ± 0.0	221		18.16 ± 0.0	18.
sabre	10367 ± 118.27	10404	823.12 ± 18.63	746	217.38 ± 0.62	219	46.53	2.91 ± 0.16	2.8
name = chec	kerboard. size =	(5.5. 5.5). ci	ircuit_type = qua	ntum simulati	on. p = 8. repet:	itions = 16. opti	imization level =	1	
method	av. n_swaps	min. n_swap			av. n_qubits		total time (s)		min. time (s)
line-graph	5148 ± 0.0	5148	223 ± 0.0	223	221 ± 0.0	221	18.72	18.72 ± 0.0	18.72
sabre	5644 ± 120.68		800.31 ± 38.98		159.44 ± 2.66	166		3.29 ± 0.1	3.38
	ckerboard, size =		ircuit_type = qua		-	-			
method	av. n_swaps	min. n_swap	av. depth		av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s)
ine-graph	5148 ± 0.0	5148	223 ± 0.0	223	221 ± 0.0	221	18.19	18.19 ± 0.0	18.19
sabre	5241 ± 43.19	5091	724 05 + 00 05			474			F 00
			734.25 ± 22.25		157.81 ± 3.06	171		5.59 ± 0.12	
			ircuit_type = quai	ntum_simulati		itions = 16, opti		3	5.86
name = chec	ekerboard, size =	(5.5, 5.5), ci	ircuit_type = quan	ntum_simulati min. depth	on, p = 8, repet: av. n_qubits	itions = 16, opt; min. qubits	imization_level =	3 av. time (s)	min. time (s)
nethod	ckerboard, size =	(5.5, 5.5), ci min. n_swap	ircuit_type = quai	ntum_simulati min. depth	on, p = 8, repet: av. n_qubits	itions = 16, opti	imization_level = total time (s)	3	min. time (s)
nethod	av. n_swaps 5148 ± 0.0	(5.5, 5.5), ci min. n_swap	ircuit_type = quai av. depth 	ntum_simulati min. depth	on, p = 8, repet: av. n_qubits 	itions = 16, opti	imization_level = total time (s)	3 av. time (s)	min. time (s)
name = checkethod	av. n_swaps 	(5.5, 5.5), ci min. n_swap 5148 4839	ircuit_type = quai av. depth 	ntum_simulati min. depth 223 656	on, p = 8, repet: av. n_qubits 	min. qubits 221 159	total time (s) 18.3 363.83	3 av. time (s) 	min. time (s)
name = checked	av. n_swaps 	(5.5, 5.5), ci min. n_swap 5148 4839 (5.5, 5.5), ci min. n_swap	ircuit_type = quar av. depth 223 ± 0.0 723.25 ± 16.98 ircuit_type = quar av. depth	ntum_simulati min. depth 223 656	on, p = 8, repet: av. n_qubits 221 ± 0.0 157.69 ± 2.69 on, p = 16, repet av. n_qubits	itions = 16, opti min. qubits 221 159 titions = 16, opti min. qubits	timization_level = total time (s) 18.3 363.83	3 av. time (s)	min. time (s) 18.3 23.19
name = checanethod	av. n_swaps 	(5.5, 5.5), ci min. n_swap 5148 4839 (5.5, 5.5), ci min. n_swap	ircuit_type = quantum av. depth	ntum_simulati min. depth 223 656	on, p = 8, repet: av. n_qubits 221 ± 0.0 157.69 ± 2.69 on, p = 16, repet av. n_qubits	itions = 16, opti min. qubits 221 159 titions = 16, opti min. qubits	total time (s) 18.3 363.83 cimization_level = total time (s)	3 av. time (s)	min. time (s) 18.3 23.19
ame = checkethodine-graph abreame = checkethodine-graph	av. n_swaps 	(5.5, 5.5), ci min. n_swap 5148 4839 (5.5, 5.5), ci min. n_swap	ircuit_type = quar av. depth 223 ± 0.0 723.25 ± 16.98 ircuit_type = quar av. depth	ntum_simulati min. depth 223 656 ntum_simulati min. depth	on, p = 8, repet: av. n_qubits 221 ± 0.0 157.69 ± 2.69 on, p = 16, repet av. n_qubits	min. qubits 221 159 titions = 16, opti	total time (s) 18.3 363.83 timization_level total time (s)	3 av. time (s)	min. time (s
name = checkers and a sabre	av. n_swaps 	(5.5, 5.5), ci min. n_swap 5148 4839 (5.5, 5.5), ci min. n_swap 10252 18281	ircuit_type = quar av. depth 223 ± 0.0 723.25 ± 16.98 ircuit_type = quar av. depth	ntum_simulati min. depth 223 656 ntum_simulati min. depth 439 1464	on, p = 8, repet: av. n_qubits 221 ± 0.0 157.69 ± 2.69 on, p = 16, repet av. n_qubits 221 ± 0.0 217.38 ± 0.78	itions = 16, optimin. qubits 221 159 titions = 16, optimin. qubits 221 212 218	timization_level = total time (s) 18.3 363.83 timization_level total time (s) 36.56 98.11	3 av. time (s)	min. time (s
name = checkers abre sabre sab	av. n_swaps 	(5.5, 5.5), ci min. n_swap 5148 4839 (5.5, 5.5), ci min. n_swap 10252 18281	ircuit_type = quar av. depth 223 ± 0.0 723.25 ± 16.98 ircuit_type = quar av. depth 439 ± 0.0 1546.5 ± 24.0	ntum_simulati min. depth 223 656 ntum_simulati min. depth 439 1464	on, p = 8, repet: av. n_qubits 221 ± 0.0 157.69 ± 2.69 on, p = 16, repet av. n_qubits 221 ± 0.0 217.38 ± 0.78	titions = 16, optimin. qubits 221 159 titions = 16, optimin. qubits 221 218	timization_level = total time (s) 18.3 363.83 timization_level total time (s) 36.56 98.11	3 av. time (s) 18.3 ± 0.0 22.74 ± 0.2 = 0 av. time (s) 36.56 ± 0.0 6.13 ± 0.21	min. time (s) 18.3 23.19 min. time (s) 36.56
name = checkethod ine-graphame = checkethodine-graphame = checkethod	av. n_swaps 5148 ± 0.0 5054 ± 30.28 ckerboard, size = av. n_swaps 10252 ± 0.0 18950 ± 206.58	(5.5, 5.5), ci min. n_swap 5148 4839 (5.5, 5.5), ci min. n_swap 10252 18281	ircuit_type = quar av. depth 223 ± 0.0 723.25 ± 16.98 ircuit_type = quar av. depth 439 ± 0.0 1546.5 ± 24.0	ntum_simulati min. depth 223 656 ntum_simulati min. depth 439 1464 ntum_simulati min. depth	on, p = 8, repet: av. n_qubits 221 ± 0.0 157.69 ± 2.69 on, p = 16, repet av. n_qubits 221 ± 0.0 217.38 ± 0.78 on, p = 16, repet av. n_qubits	titions = 16, optimin. qubits 221 159 titions = 16, optimin. qubits 221 218	total time (s) 18.3 363.83 timization_level total time (s) 36.56 98.11	3 av. time (s) 18.3 ± 0.0 22.74 ± 0.2 = 0 av. time (a) 36.56 ± 0.0 6.13 ± 0.21	min. time (s) 18.3 23.19 min. time (s) 36.56
name = checkethod	av. n_swaps 5148 ± 0.0 5054 ± 30.28 ckerboard, size = av. n_swaps 10252 ± 0.0 18950 ± 206.58	(5.5, 5.5), ci min. n_swap 5148 4839 (5.5, 5.5), ci min. n_swap 10252 18281	ircuit_type = quai av. depth 223 ± 0.0 723.25 ± 16.98 ircuit_type = quai av. depth 439 ± 0.0 1546.5 ± 24.0 ircuit_type = quai av. depth	ntum_simulati min. depth 223 656 ntum_simulati min. depth 439 1464	on, p = 8, repet: av. n_qubits 221 ± 0.0 157.69 ± 2.69 on, p = 16, repet av. n_qubits 221 ± 0.0 217.38 ± 0.78 on, p = 16, repet av. n_qubits	titions = 16, optimin. qubits 221 159 titions = 16, optimin. qubits 221 218	total time (s) 18.3 363.83 timization_level total time (s) 36.56 98.11	3 av. time (s) 18.3 ± 0.0 22.74 ± 0.2 = 0 av. time (a) 36.56 ± 0.0 6.13 ± 0.21	min. time (s) 18.3 23.19 min. time (s) 36.56

name = chec	av. n_swaps	min. n_swap	av denth	min denth	av. n_qubits	min. qubits	total time (s)	au time (e)	min. time (s
		min. n_swap						uv. time (3)	min. cime (2
ine-graph	10252 ± 0.0	10252	439 ± 0.0	439	221 ± 0.0	221	36.69	36.69 ± 0.0	36.6
sabre	10265 ± 67.73	10411	1565.5 ± 31.69	1469	162.69 ± 4.19	163	171.93	10.75 ± 0.22	10.2
ame = che	ckerboard, size =	(5.5, 5.5), c	ircuit_type = qua	ntum_simulati	on, p = 16, repe	titions = 16, op	timization_level	= 3	
ethod	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (
ine-graph	10252 ± 0.0	10252	439 ± 0.0	439	221 ± 0.0	221	37.43	3 37.43 ± 0.0	37.
abre	10373 ± 43.89	10384	1537.19 ± 30.88	1390	155.31 ± 2.59	158	681.17	7 42.57 ± 0.25	42.
	skawhoowd gigo s	- (7 5 7 5)		ntum gimulati	on n = 1 ronot	itions = 16 ont	imigation level	- 0	
ethod	av. n_swaps	min. n_swap	<pre>ircuit_type = qua av. depth</pre>		av. n_qubits		total time (s)		min. time (s)
ine-graph	1211 ± 0.0		30 ± 0.0	30	393 ± 0.0	393		4.76 ± 0.0	4.70
abre			198 25 ± 3 97		350 88 ± 3 41				
	4053 ± 23.25		198.25 ± 3.97		350.88 ± 3.41	355		0.88 ± 0.04	
ame = chec	ckerboard, size = av. n_swaps	(7.5, 7.5), c	ircuit_type = qua	ntum_simulati min. depth	on, p = 1, repet av. n_qubits	itions = 16, opt		1	min. time (
ame = chec	ckerboard, size = av. n_swaps	* (7.5, 7.5), c	ircuit_type = qua	ntum_simulati	on, p = 1, repet	itions = 16, opt	imization_level =	1	min. time (
ame = chece ethod ine-graph	ckerboard, size = av. n_swaps	(7.5, 7.5), c min. n_swap	ircuit_type = qua av. depth	ntum_simulati min. depth	on, p = 1, repet av. n_qubits	itions = 16, opt. min. qubits	imization_level = total time (s)	av. time (s)	min. time (
ame = checethod	av. n_swaps 1211 ± 0.0 2459 ± 56.57	(7.5, 7.5), c min. n_swap	av. depth 30 ± 0.0 186.38 ± 11.72	ntum_simulati min. depth 30 152	on, p = 1, repet av. n_qubits 	itions = 16, opt min. qubits 393 289	total time (s) 4.49 25.29	av. time (s) 4.49 ± 0.0 1.58 ± 0.05	min. time (
ame = chec	av. n_swaps 1211 ± 0.0 2459 ± 56.57	1211 2107	ircuit_type = qua av. depth 	ntum_simulati min. depth 30 152	on, p = 1, repet av. n_qubits 	itions = 16, opt min. qubits 393 289	total time (s) 4.49 25.29	1 av. time (s) 4.49 ± 0.0 1.58 ± 0.05	min. time (:
ame = checeethod	av. n_swaps 1211 ± 0.0 2459 ± 56.57 ckerboard, size = av. n_swaps	(7.5, 7.5), c min. n_swap 1211 2107 (7.5, 7.5), c min. n_swap	ircuit_type = qua av. depth 	ntum_simulati min. depth 30 152	on, p = 1, repet av. n_qubits 393 ± 0.0 284.44 ± 2.91 on, p = 1, repet av. n_qubits	itions = 16, opt min. qubits 393 289 itions = 16, opt. min. qubits	total time (s) 4.49 25.29 imization_level = total time (s)	1 av. time (s) 4.49 ± 0.0 1.58 ± 0.05	min. time (
ame = checkethodine-graph abreame = checkethodame = checkethod	av. n_swaps 1211 ± 0.0 2459 ± 56.57 ckerboard, size = av. n_swaps	1211 2107	ircuit_type = qua av. depth 30 ± 0.0 186.38 ± 11.72 ircuit_type = qua av. depth 30 ± 0.0	ntum_simulati min. depth 30 152	on, p = 1, repet av. n_qubits 393 ± 0.0 284.44 ± 2.91 on, p = 1, repet av. n_qubits 393 ± 0.0	itions = 16, opt. min. qubits 393 289 itions = 16, opt. min. qubits	total time (s) 4.49 25.29 imization_level = total time (s)	1 av. time (s) 4.49 ± 0.0 1.58 ± 0.05	min. time (1
ame = chec	av. n_swaps 1211 ± 0.0 2459 ± 56.57 	1211 2107 * (7.5, 7.5), c min. n_swap 	ircuit_type = qua av. depth 	ntum_simulati min. depth 30 152 ntum_simulati min. depth 30 138	on, p = 1, repet av. n_qubits 393 ± 0.0 284.44 ± 2.91 on, p = 1, repet av. n_qubits 393 ± 0.0 286.19 ± 3.31	itions = 16, opt. min. qubits 393 289 itions = 16, opt. min. qubits 393 293	total time (s) 4.49 25.29 imization_level = total time (s) 4.58 57.96	* 1 av. time (s) 4.49 ± 0.0 1.58 ± 0.05 * 2 av. time (s) 4.58 ± 0.0 3.62 ± 0.05	min. time (4. 1. min. time (4. 3.
ame = chec	av. n_swaps 1211 ± 0.0 2459 ± 56.57 ckerboard, size = av. n_swaps 1211 ± 0.0 2189 ± 48.72	(7.5, 7.5), c min. n_swap 1211 2107 (7.5, 7.5), c min. n_swap	ircuit_type = qua av. depth 30 ± 0.0 186.38 ± 11.72 ircuit_type = qua av. depth 30 ± 0.0	ntum_simulati min. depth 30 152 ntum_simulati min. depth 30 138	on, p = 1, repet av. n_qubits 393 ± 0.0 284.44 ± 2.91 on, p = 1, repet av. n_qubits 393 ± 0.0 286.19 ± 3.31	itions = 16, opt. min. qubits 393 289 itions = 16, opt. min. qubits 393 293	total time (s) 4.49 25.29 imization_level = total time (s) 4.58	av. time (s) 4.49 ± 0.0 1.58 ± 0.05	min. time (4.4.1.6)
ame = chece ethod ine-graph abre ame = chece ethod	av. n_swaps 1211 ± 0.0 2459 ± 56.57 ckerboard, size = av. n_swaps 1211 ± 0.0 2189 ± 48.72	(7.5, 7.5), c min. n_swap 1211 2107 (7.5, 7.5), c min. n_swap 1211 1989	ircuit_type = qua av. depth 30 ± 0.0 186.38 ± 11.72 ircuit_type = qua av. depth 30 ± 0.0 167.81 ± 10.59	ntum_simulati min. depth 30 152	on, p = 1, repet av. n_qubits 393 ± 0.0 284.44 ± 2.91 on, p = 1, repet av. n_qubits 393 ± 0.0 286.19 ± 3.31	itions = 16, opt min. qubits 393 289 itions = 16, opt. min. qubits 393 293	total time (s) 4.49 25.29 imization_level = total time (s) 4.58 57.96	1 av. time (s) 4.49 ± 0.0 1.58 ± 0.05 2 av. time (s) 4.58 ± 0.0 3.62 ± 0.05	min. time (4.4.1.6)
ame = checkethod	av. n_swaps 1211 ± 0.0 2459 ± 56.57 ckerboard, size = av. n_swaps 1211 ± 0.0 2189 ± 48.72	(7.5, 7.5), c min. n_swap 1211 2107 (7.5, 7.5), c min. n_swap 1211 1989	ircuit_type = qua av. depth 30 ± 0.0 186.38 ± 11.72 ircuit_type = qua av. depth 30 ± 0.0 167.81 ± 10.59	ntum_simulati min. depth 30 152 ntum_simulati min. depth 30 138	on, p = 1, repet av. n_qubits 393 ± 0.0 284.44 ± 2.91 on, p = 1, repet av. n_qubits 393 ± 0.0 286.19 ± 3.31 on, p = 1, repet	itions = 16, opt. min. qubits 393 289 itions = 16, opt. min. qubits 393 293	total time (s) 4.49 25.29 imization_level = total time (s) 4.58 57.96	1 av. time (s) 4.49 ± 0.0 1.58 ± 0.05 2 av. time (s) 4.58 ± 0.0 3.62 ± 0.05	min. time (s
ame = checeethod ame = checeethod ine-graph abre ame = checeethod ame = checeethod	av. n_swaps 1211 ± 0.0 2459 ± 56.57 ckerboard, size = av. n_swaps 1211 ± 0.0 2189 ± 48.72	(7.5, 7.5), c min. n_swap 1211 2107 (7.5, 7.5), c min. n_swap 1211 1989	ircuit_type = qua av. depth 30 ± 0.0 186.38 ± 11.72 ircuit_type = qua av. depth 30 ± 0.0 167.81 ± 10.59 ircuit_type = qua av. depth	ntum_simulati min. depth 30 152 ntum_simulati min. depth 330 138	on, p = 1, repet av. n_qubits 393 ± 0.0 284.44 ± 2.91 on, p = 1, repet av. n_qubits 393 ± 0.0 286.19 ± 3.31 on, p = 1, repet av. n_qubits	itions = 16, opt. min. qubits 393 289 itions = 16, opt. min. qubits 393 293 itions = 16, opt. min. qubits	total time (s) 4.49 25.29 imization_level = total time (s)	1 av. time (s) 4.49 ± 0.0 1.58 ± 0.05 2 av. time (s) 4.58 ± 0.0 3.62 ± 0.05	min. time (s
ame = checethod ame = checethod ine-graph ame = checethod ine-graph ame = checethod	av. n_swaps 1211 ± 0.0 2459 ± 56.57 2459 ± 56.57 2487 2487 2111 ± 0.0 2189 ± 48.72 2487	(7.5, 7.5), c min. n_swap 1211 2107 (7.5, 7.5), c min. n_swap 1211 1989	ircuit_type = qua av. depth 30 ± 0.0 186.38 ± 11.72 ircuit_type = qua av. depth 30 ± 0.0 167.81 ± 10.59 ircuit_type = qua av. depth	ntum_simulati min. depth 30 152 ntum_simulati min. depth 3138 ntum_simulati min. depth 320 330	on, p = 1, repet av. n_qubits 393 ± 0.0 284.44 ± 2.91 on, p = 1, repet av. n_qubits 393 ± 0.0 286.19 ± 3.31 on, p = 1, repet av. n_qubits 393 ± 0.0	itions = 16, opt. min. qubits 393 289 itions = 16, opt. min. qubits 393 293 itions = 16, opt. min. qubits	total time (s) 4.49 25.29 imization_level = total time (s) 4.58 57.96	*1 av. time (s) 4.49 ± 0.0 1.58 ± 0.05 *2 av. time (s) 4.58 ± 0.0 3.62 ± 0.05	min. time (4. 1. min. time (4. 3.

9289 ± 0.0 9289 219 ± 0.0 219 393 ± 0.0 393 34.14 34.14 ± 0.0 22164 ± 192.21 21331 1027.31 ± 17.21 951 361.69 ± 3.69 377 93.75 5.86 ± 0.21 kerboard, size = (7.5, 7.5), circuit_type = quantum_simulation, p = 8, repetitions = 16, optimization_level = 1		34.14 ± 0.0		393	393 ± 0.0					
22164 ± 192.21 21331 1027.31 ± 17.21 951 361.69 ± 3.69 377 93.75 5.86 ± 0.21						219	219 ± 0.0			
		5.86 ± 0.21	93.75 5	377	361.69 ± 3.69			9289	9289 ± 0.0	line-graph
kerboard, size = (7.5, 7.5), circuit_type = quantum_simulation, p = 8, repetitions = 16, optimization_level = 1						951	1027.31 ± 17.21	21331	22164 ± 192.21	sabre
kerboard, size = (7.5, 7.5), circuit_type = quantum_simulation, p = 8, repetitions = 16, optimization_level = 1										
kerboard, size = (7.5, 7.5), circuit_type = quantum_simulation, p = 8, repetitions = 16, optimization_level = 1										
$ \hbox{av. $n_swaps} \qquad \hbox{min. $n_swap} \hbox{av. $depth} \qquad \hbox{min. depth} \hbox{av. $n_qubits} \qquad \hbox{min. qubits} \hbox{total time (s)} \hbox{av. time (s)} \hbox{min.} $) :									
9289 ± 0.0 9289 219 ± 0.0 219 393 ± 0.0 393 34.49 34.49 ± 0.0		34.49 ± 0.0	34.49 3	393	393 ± 0.0	219	219 ± 0.0	9289	9289 ± 0.0	line-graph
12118 ± 289.99 11694 1276.06 ± 66.48 1103 285.69 ± 3.28 293 118.35 7.4 ± 0.19			118.35 7	293						
kerboard, size = (7.5, 7.5), circuit_type = quantum_simulation, p = 8, repetitions = 16, optimization_level = 2			cion_level = 2	, optimi:	p = 8, repetition	imulation	rcuit_type = quantu	(7.5, 7.5), ci:	kerboard, size =	name = chec
av. n_swaps min. n_swap av. depth min. depth av. n_qubits min. qubits total time (s) av. time (s) min.) 1									
9289 ± 0.0 9289 219 ± 0.0 219 393 ± 0.0 393 33.66 33.66 ± 0.0 12194 ± 132.53 11396 1219.62 ± 31.86 1093 287.31 ± 3.31 286 222.24 13.89 ± 0.23										
kerboard, size = (7.5, 7.5), circuit_type = quantum_simulation, p = 8, repetitions = 16, optimization_level = 3										
av. n_swaps min. n_swap av. depth min. depth av. n_qubits min. qubits total time (s) av. time (s) min.										nethod
9289 ± 0.0 9289 219 ± 0.0 219 393 ± 0.0 393 33.34 33.34 ± 0.0		33.34 ± 0.0	33.34 3	393	393 ± 0.0	219	219 ± 0.0	9289	9289 ± 0.0	line-graph
11481 ± 103.84		60.92 ± 0.4	974.77 6	278	279.0 ± 3.91	1052	1151.81 ± 30.17	11276	11481 ± 103.84	sabre
kerboard, size = (7.5, 7.5), circuit_type = quantum_simulation, p = 16, repetitions = 16, optimization_level = 0		0	ation_level = 0	.6, optim	, p = 16, repetition	imulation	rcuit_type = quantu	(7.5, 7.5), ci	kerboard, size =	name = chec
av. n_swaps min. n_swap av. depth min. depth av. n_qubits min. qubits total time (s) av. time (s) min.										
18521 ± 0.0 18521 435 ± 0.0 435 393 ± 0.0 393 68.34 ± 0.0										
		10.61 ± 0.29	169.73 1	363	359.75 ± 3.12	1805	1884.31 ± 37.54	38867	39936 ± 352.99	sabre
		68.34 ± 0.0	68.34 6	393 363	393 ± 0.0 359.75 ± 3.12	435 1805	435 ± 0.0	18521 38867	18521 ± 0.0 39936 ± 352.99	line-graph

```
name = checkerboard, size = (7.5, 7.5), circuit_type = quantum_simulation, p = 16, repetitions = 16, optimization_level = 2
      av. n_swaps min. n_swap av. depth
                                        min. depth av. n_qubits
                                                            min. qubits total time (s) av. time (s) min. time (s)
2243 288.56 ± 5.34
                                                                280
     23286 ± 127.03
                      22930 2395.75 ± 51.28
                                                                           389.15 24.32 ± 0.38
                                                                                                  24.34
name = checkerboard, size = (7.5, 7.5), circuit_type = quantum_simulation, p = 16, repetitions = 16, optimization_level = 3
      av. n_swaps min. n_swap av. depth min. depth av. n_qubits min. qubits total time (s) av. time (s) min. time (s)
                    18521 435 ± 0.0 435 393 ± 0.0 393 67.2 67.2 ± 0.0
                                                                                                  67.2
line-graph 18521 ± 0.0
       22170 ± 140.89
                       23060 2352.75 ± 72.43
                                            2121 281.62 ± 4.06
                                                                 282
                                                                            1750.38 109.4 ± 0.67
```

7.0.3 Random circuit, kagome and shuriken, against SABRE

```
import line_graph_routing as lgr # Loading these makes these cells stand-alone
import pickle
settings=[]
for name in ['kagome','shuriken']:
    for side in range(1,7,2):
        for p in [side**2*500]:
            for optimization_level in [1]:
                setting={'name':name,
                     'size': (side, side),
                     'circuit_type': 'random',
                     'p': p,
                     'repetitions' : 16,
                     'optimization_level' : optimization_level,
                     'methods' : ['sabre']
                settings.append(setting)
## Uncomment to rerun benchmarks.
#results=[]
#for setting in settings:
   result=lgr.benchmark(**setting)
  results.append(result)
#
    lgr.print_benchmark(result)
#with open('benchmark_data/random.pkl','wb') as f:
  pickle.dump(results,f)
#Load previously obtained results from disk and show them.
import pickle
with open('benchmark_data/random.pkl','rb') as f:
```

results=pickle.load(f) for result in results:

lgr.print_benchmark(result)

method	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s)
line-graph	200 ± 0.0	200	249 ± 0.0	249	12 ± 0.0	12	0.37	0.37 ± 0.0	0.37
sabre	96 ± 0.62	96	207.5 ± 1.19	205	12.0 ± nan	12	1.17	0.07 ± 0.03	0.06
name = kag	ome, size = (3,	3), circuit_typ	pe = random, p =	4500, repeti	tions = 16, optim	ization_level =	1		
nethod	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s
line-graph	2611 ± 0.0	2611	568 ± 0.0	568	68 ± 0.0	68	1.14	1.14 ± 0.0	1.1
sabre	1444 ± 25.94	1450	838.88 ± 35.0	720	44.31 ± 1.75	54	10.24	0.64 ± 0.03	0.6
name = kago	ome, size = (5,	5), circuit_typ	oe = random, p =		itions = 16, opti		1		
method	av. n_swaps	min. n_swap	av. depth		th av. n_qubits	min. qubits		s) av. time (s)	
							3.1	02 3.02 ± 0.0	
ine-graph		7592		7		164	3.1		
ine-graph	7592 ± 0.0 5637 ± 82.31	7592 5452	735 ± 0.0 1774.44 ± 54.79	7 15 15 = 500, repet	35 164 ± 0.0 90 108.69 ± 3.56	164 107 mization_level =	3	02 3.02 ± 0.0 72 2.04 ± 0.04	:
ine-graph	7592 ± 0.0 5637 ± 82.31	7592 5452	735 ± 0.0 1774.44 ± 54.79	7 15 15 = 500, repet	35 164 ± 0.0 90 108.69 ± 3.56	164 107 mization_level =	3.	02 3.02 ± 0.0 72 2.04 ± 0.04	
ine-graph abre abre ame = shunethod	7592 ± 0.0 5637 ± 82.31	7592 5452 , 1), circuit_t min. n_swap	735 ± 0.0 1774.44 ± 54.79	7 9 15	35 164 ± 0.0 90 108.69 ± 3.56	164 107 mization_level =	3. 32. 1 1 total time (s)	02 3.02 ± 0.0 72 2.04 ± 0.04	2
ine-graph abre amme = shur sthod	7592 ± 0.0 5637 ± 82.31 riken, size = (1 av. n_swaps	7592 5452 , 1), circuit_t min. n_swap	735 ± 0.0 1774.44 ± 54.78 Expe = random, p av. depth	500, repet	35 164 * 0.0 90 108.69 * 3.56 	164 107 107 mization_level = min. qubits	3. 32. 32. 1 total time (s) 0.07	02 3.02 ± 0.0 72 2.04 ± 0.04 av. time (s)	min. time (
ine-graph abre ame = shu ethod ine-graph	7592 ± 0.0 5637 ± 82.31 riken, size = (1 av. n_swaps	7592 5452 , 1), circuit_t min. n_swap	735 ± 0.0 1774.44 ± 54.79 Eype = random, p av. depth 247 ± 0.0	500, repet	35 164 ± 0.0 90 108.69 ± 3.56 itions = 16, opti av. n_qubits 	164 107 mization_level = min. qubits	3. 32. 32. 1 total time (s) 0.07	av. time (s)	min. time (
ine-graph sabre mame = shun sethod ine-graph sabre	7592 ± 0.0 5637 ± 82.31 riken, size = (1 av. n_swaps 122 ± 0.0 85 ± nan	7592 5452 , 1), circuit_t min. n_swap	735 ± 0.0 1774.44 ± 54.75 Expe = random, p av. depth 247 ± 0.0 231.12 ± 0.31	500, repet min. depth 247 231	35 164 * 0.0 90 108.69 * 3.56 itions = 16, opti av. n_qubits 8 * 0.0 8.0 * nan	mization_level = min. qubits 8 8	3 32 1 total time (s) 0.07 0.84	av. time (s) 0.07 ± 0.0 0.05 ± 0.0	min. time (
ine-graph abre ame = shun ethod ine-graph abre ame = shun	7592 ± 0.0 5637 ± 82.31 riken, size = (1 av. n_swaps 122 ± 0.0 85 ± nan	7592 5452 , 1), circuit_t min. n_svap	735 ± 0.0 1774.44 ± 54.75 Expe = random, p av. depth 247 ± 0.0 231.12 ± 0.31	500, repet min. depth 247 231	35 164 ± 0.0 90 108.69 ± 3.56 itions = 16, opti av. n_qubits 8 ± 0.0 8.0 ± nan	mization_level = min. qubits 8 8	3. 32. 32. 1 total time (s) 0.07 0.84	av. time (s) 0.07 ± 0.0 0.05 ± 0.0	min. time (
ine-graph abre ame = shu uethod ine-graph abre ame = shu	7592 ± 0.0 5637 ± 82.31 riken, size = (1 av. n_swaps 122 ± 0.0 85 ± nan	7592 5452 , 1), circuit_t min. n_swap 122 85	735 ± 0.0 1774.44 ± 54.75 Expe = random, p av. depth 247 ± 0.0 231.12 ± 0.31	247 231 = 4500, repet min. depth	35 164 * 0.0 90 108.69 * 3.56 itions = 16, opti av. n_qubits 8 * 0.0 8.0 * nan	mization_level = min. qubits 8 8	1 total time (s) 0.07 0.84	av. time (s) 0.07 ± 0.0 0.05 ± 0.0	min. time (

```
name = shuriken, size = (5, 5), circuit_type = random, p = 12500, repetitions = 16, optimization_level = 1

method av. n_swaps min. n_swap av. depth min. depth av. n_qubits min. qubits total time (s) av. time (s) min. time (s)

line-graph 6828 ± 0.0 6828 405 ± 0.0 405 240 ± 0.0 240 2.83 2.83 ± 0.0 2.83

sabre 6811 ± 192.15 5969 1403.31 ± 94.74 1094 177.31 ± 2.94 176 38.36 2.4 ± 0.04 2.38
```

7.0.4 Random circuit, complete graph, against SABRE

```
import line_graph_routing as lgr
import pickle
settings=[]
for name in ['complete']:
    for side in range(3,10,2):
        for p in [side*100]:
            for optimization_level in range(2):
                setting={'name':name,
                         'size': side,
                         'circuit_type': 'random',
                          'p': p,
                          'repetitions' : 16,
                         'optimization_level' : optimization_level,
                          'methods' : ['sabre']
                settings.append(setting)
## Uncomment to rerun benchmarks
#results=[]
#for setting in settings:
   result=lqr.benchmark(**setting)
    results.append(result)
    lgr.print_benchmark(result)
#with open('benchmark_data/complete.pkl','wb') as f:
    pickle.dump(results,f)
with open('benchmark_data/complete.pkl','rb') as f:
   results=pickle.load(f)
for result in results:
    lgr.print_benchmark(result)
```

name = comp	olete, size = 3,								
method	av. n_swaps	min. n_swap	av. depth	min. depth	av. n_qubits	min. qubits	total time (s)	av. time (s)	min. time (s
	154 ± 0.0		334 ± 0.0	224	4 ± 0.0	4	0.13	0.12 ± 0.0	0.1
sabre	40 ± 0.97		267.88 ± 2.34		3.0 ± nan	3		0.06 ± 0.06	0.04
	plete, size = 3,				16				
method	av. n_swaps	min. n_swap		min. depth a			total time (s)	av. time (s)	min. time (s)
line-graph	140 ± 0.0	140	313 ± 0.0	313	4 ± 0.0	4	0.11	0.11 ± 0.0	0.11
sabre	34 ± nan		220.0 ± nan		3.0 ± nan	3		0.06 ± 0.0	0.07
-	plete, size = 5,		-	-	-				
method	av. n_swaps	min. n_swap		min. depth		min. qubits	total time (s)		min. time (s)
line-graph	314 ± 0.0	314	548 ± 0.0	548	6 ± 0.0	6	0.19	0.19 ± 0.0	0.19
sabre	88 ± 1.25		458.06 ± 4.0		5.0 ± nan	5		0.06 ± nan	0.06
			random, p = 50	00, repetitions	s = 16, optimiza	tion_level = 1	0.96		
name = comp	plete, size = 5, av. n_swaps	circuit_type =	random, p = 50	min. depth	s = 16, optimiza av. n_qubits	tion_level = 1 min. qubits	total time (s)	av. time (s)	min. time (s.
name = comp method line-graph	olete, size = 5, av. n_swaps 	circuit_type = min. n_swap	random, p = 50 av. depth 	min. depth	s = 16, optimiza av. n_qubits 6 ± 0.0	tion_level = 1 min. qubits	total time (s)	av. time (s)	min. time (s)
name = comp method	plete, size = 5, av. n_swaps	circuit_type = min. n_swap	random, p = 50	min. depth	s = 16, optimiza av. n_qubits	tion_level = 1 min. qubits	total time (s)	av. time (s)	min. time (s,
name = comp method line-graph sabre	olete, size = 5, av. n_swaps 	circuit_type = min. n_swap	random, p = 50 av. depth 	00, repetition min. depth 503 386	s = 16, optimizs av. n_qubits 	tion_level = 1 min. qubits 6 5	total time (s)	av. time (s)	min. time (s)
name = comp method	olete, size = 5, av. n_swaps 286 ± 0.0 72 ± nan olete, size = 7, av. n_swaps	circuit_type = min. n_swap 286 72 circuit_type = min. n_swap	random, p = 50 av. depth 503 * 0.0 387.25 * 0.31 random, p = 70 av. depth	min. depth 503 386	s = 16, optimiza av. n_qubits 6 * 0.0 5.0 * nan s = 16, optimiza av. n_qubits	tion_level = 1 min. qubits 6 5 tion_level = 0 min. qubits	total time (s) 0.18 1.44 total time (s)	av. time (s) 0.18 ± 0.0 0.09 ± nan av. time (s)	min. time (s
name = comp method line-graph sabre	olete, size = 5, av. n_swaps 286 ± 0.0 72 ± nan olete, size = 7, av. n_swaps	circuit_type = min. n_swap 286 72 circuit_type = min. n_swap	random, p = 50 av. depth 503 * 0.0 387.25 * 0.31 random, p = 70 av. depth	min. depth 503 386	s = 16, optimiza av. n_qubits 6 * 0.0 5.0 * nan s = 16, optimiza av. n_qubits	tion_level = 1 min. qubits 6 5 tion_level = 0 min. qubits	total time (s) 0.18 1.44 total time (s)	av. time (s) 0.18 ± 0.0 0.09 ± nan av. time (s)	min. time (s.
name = commethod	286 ± 0.0 72 ± nan plete, size = 7, av. n_swaps	circuit_type = min. n_swap 286 72 circuit_type = min. n_swap 436	random, p = 50 av. depth 503 ± 0.0 387.25 ± 0.31 random, p = 70 av. depth	min. depth 503 386	s = 16, optimiza av. n_qubits 6 ± 0.0 5.0 ± nan s = 16, optimiza av. n_qubits	tion_level = 1 min. qubits 6 5 tion_level = 0 min. qubits	total time (s) 0.18 1.44 total time (s)	av. time (s)	min. time (s)
name = comp method line-graph sabre line-graph sabre	286 ± 0.0 72 ± nan plete, size = 7, av. n_swaps 436 ± 0.0 115 ± 1.59	circuit_type = min. n_swap 286 72 circuit_type = min. n_swap 436 115	random, p = 50 av. depth 503 ± 0.0 387.25 ± 0.31 random, p = 70 av. depth 723 ± 0.0 609.25 ± 6.01	00, repetitions min. depth 503 386 00, repetitions 723 586	s = 16, optimiza av. n_qubits 6 ± 0.0 5.0 ± nan s = 16, optimiza av. n_qubits 7.0 ± nan	tion_level = 1 min. qubits 6 5 tion_level = 0 min. qubits 7	total time (s) 0.18 1.44 total time (s)	av. time (s)	min. time (s
name = comp method line-graph sabre line-graph sabre line-graph sabre	olete, size = 5, av. n_swaps 286 * 0.0 72 * nan olete, size = 7, av. n_swaps 436 * 0.0 115 * 1.59 olete, size = 7, av. n_swaps	circuit_type = min. n_swap 286 72 circuit_type = min. n_swap 115 circuit_type = min. n_swap	random, p = 50 av. depth 503 ± 0.0 387.25 ± 0.31 random, p = 70 av. depth 723 ± 0.0 609.25 ± 6.01 random, p = 70 av. depth	00, repetitions min. depth 386	s = 16, optimiza av. n_qubits 6 ± 0.0 5.0 ± nan s = 16, optimiza av. n_qubits 7.0 ± nan	tion_level = 1 min. qubits 6 5 tion_level = 0 min. qubits 7 tion_level = 1 min. qubits	total time (s) 0.18 1.44 total time (s) 0.25	av. time (s) 10.18 ± 0.0 10.09 ± nan 20.25 ± 0.0 20.10 ± 0.02 20.25 ± 0.00 20.25 ± 0.00 20.25 ± 0.00	min. time (s. 0.1)
name = comp nethod line-graph sabre line-graph sabre name = comp nethod	olete, size = 5, av. n_swaps 286 * 0.0 72 * nan olete, size = 7, av. n_swaps 436 * 0.0 115 * 1.59 olete, size = 7, av. n_swaps	circuit_type = min. n_swap 286 72 circuit_type = min. n_swap 436 115 circuit_type =	random, p = 50 av. depth 503 ± 0.0 387.25 ± 0.31 random, p = 70 av. depth 723 ± 0.0 609.25 ± 6.01 random, p = 70 av. depth	00, repetitions min. depth 503 386 500, repetitions min. depth 586 586 500, repetitions min. depth 586 586 586 586 586 586 586 586 586 586	s = 16, optimiza av. n_qubits 6 ± 0.0 5.0 ± nan s = 16, optimiza av. n_qubits 7.0 ± nan	tion_level = 1 min. qubits 6 5 tion_level = 0 min. qubits 7 tion_level = 1 min. qubits	total time (s) 0.18 1.44 total time (s) 0.25 1.65	av. time (s) 10.18 ± 0.0 10.09 ± nan 20.25 ± 0.0 20.10 ± 0.02 20.25 ± 0.00 20.25 ± 0.00 20.25 ± 0.00	min. time (s

```
name = complete, size = 9, circuit_type = random, p = 900, repetitions = 16, optimization_level = 0
                                  min. depth av. n_qubits min. qubits total time (s) av. time (s) min. time (s)
      av. n_swaps min. n_swap av. depth
946 10 ± 0.0
                                                          10
                     143 770.0 ± 6.84
                                                           9
     151 ± 2.0
                                       730 9.0 ± nan
                                                                                          0.1
                                                                      1.6 0.1 ± nan
name = complete, size = 9, circuit_type = random, p = 900, repetitions = 16, optimization_level = 1
     av. n_swaps min. n_swap av. depth min. depth av. n_qubits min. qubits total time (s) av. time (s) min. time (s)
line-graph 650 ± 0.0 650 1036 ± 0.0 1036 10 ± 0.0 10
                                                                    0.33 0.33 ± 0.0
      149 ± 0.59
                     146 726.5 ± 1.59
                                       720 9.0 ± nan
                                                            9
                                                                      2.74 0.17 ± 0.02
```

7.0.5 Against other routing methods

Above, we only ran SABRE because it outperforms the other methods available in Qiskit by default. The standard methods available are

```
from qiskit import transpiler
transpiler.preset_passmanagers.plugin.list_stage_plugins('routing')
```

```
['basic', 'lookahead', 'none', 'sabre', 'stochastic']
```

```
import line_graph_routing as lgr
import pickle
settings=[]
for name in ['kagome','shuriken']:
    for side in [1]:
        for p in [1]:
            for optimization_level in [3]:
                setting={'name':name,
                         'size': (side,side),
                         'circuit_type': 'quantum_simulation',
                         'p': p,
                         'repetitions' : 16,
                         'optimization_level' : optimization_level,
                         'methods' : ['basic', 'lookahead', 'sabre', 'stochastic']
                settings.append(setting)
## Uncomment to rerun benchmarks
#results=[]
#for setting in settings:
   result=lqr.benchmark(**setting)
#
  results.append(result)
    lgr.print_benchmark(result)
```

```
#with open('benchmark_data/other_methods_1x1.pkl','wb') as f:
# pickle.dump(results,f)

with open('benchmark_data/other_methods_1x1.pkl','rb') as f:
    results=pickle.load(f)

for result in results:
    lgr.print_benchmark(result)
```

```
name = kagome, size = (1, 1), circuit_type = quantum_simulation, p = 1, repetitions = 16, optimization_level = 3
     av. n_swaps min. n_swap av. depth min. depth av. n_qubits
                                                     min. qubits total time (s) av. time (s)
12 7 ± 0.0
      14 ± 0.0
                     14 17.0 ± 0.0
                                       17 8.0 ± 0.0
                                                          8
                                                                    0.05 0.05 ± 0.0
                                                                                         0.05
                                                          8
                                                               19.55 1.22 ± 0.04
lookahead 8 ± nan
                       8 8.81 ± 0.19
                                        8 8.0 ± nan
                                                          8
sabre 6 ± nan
                      6 6.0 ± nan
                                       6 8.0 ± nan
                                                                   0.49 0.03 ± 0.0
                                                                                          0.03
                      6 6.06 ± 0.16
                                        6 8.0 ± nan
                                                                    0.62 0.04 ± 0.0
stochastic 6 ± 0.62
                                                                                          0.03
name = shuriken, size = (1, 1), circuit_type = quantum_simulation, p = 1, repetitions = 16, optimization_level = 3
      av. n_swaps min. n_swap av. depth min. depth av. n_qubits min. qubits total time (s) av. time (s)
line-graph 8 ± 0.0
                      8 9 ± 0.0
                                      9 8 ± 0.0 8
                                                                    0.05 0.05 ± 0.0
                                                                    0.04 0.04 ± 0.0
     16 ± 0.0
                     16 19.0 ± 0.0
                                      19 8.0 ± 0.0
                                                          8
                                                                                         0.04
basic
                     6 8.0 ± nan
                                                          8
                                                                   14.22 0.89 ± 0.01
lookahead 6 ± nan
                                       8 8.0 ± nan
                                       7 8.0 ± nan
                                                          8
                                                                   0.5 0.03 ± 0.0
sabre 6 ± nan
                      6 7.62 ± 0.22
                      6 7.0 ± nan
                                       7 8.0 ± nan
                                                                    0.58 0.04 ± 0.0
stochastic 6 ± nan
```

The method 'lookahead' takes an impractical amount of time, so we exclude it when running benchmarks for larger/deeper circuits.

```
'methods' : ['basic', 'sabre', 'stochastic']
}
settings.append(setting)

## Uncomment to rerun benchmarks
#results=[]
#for setting in settings:
# result=lgr.benchmark(**setting)
# results.append(result)
# lgr.print_benchmark(result)
#
#with open('benchmark_data/other_methods_3x3.pkl','wb') as f:
# pickle.dump(results,f)

with open('benchmark_data/other_methods_3x3.pkl','rb') as f:
    results=pickle.load(f)

for result in results:
    lgr.print_benchmark(result)
```

```
name = kagome, size = (3, 3), circuit_type = quantum_simulation, p = 3, repetitions = 16, optimization_level = 3
       av. n_swaps min. n_swap av. depth
                                    min. depth av. n_qubits min. qubits total time (s) av. time (s) min. time (s)
43 68 ± 0.0
line-graph 336 ± 0.0
                      336 43 ± 0.0
                                                             68
                                                                        1.33 1.33 ± 0.0
                                         287 45.0 ± 0.0
                                                             45
                                                                        1.21 1.21 ± 0.0
                       475 287.0 ± 0.0
                     203 63.25 ± 3.78
                                        50 43.94 ± 0.97
                                                            45
       215 ± 3.56
                                                                       12.37 0.77 ± 0.04
                                                                                              0.93
name = shuriken, size = (3, 3), circuit_type = quantum_simulation, p = 3, repetitions = 16, optimization_level = 3
                                   min. depth av. n_qubits min. qubits total time (s) av. time (s) min. time (s)
       av. n_swaps min. n_swap av. depth
                   390 34 ± 0.0
                                      34 84 ± 0.0
                                                            84
      1195 ± 0.0
                     1195 491.0 ± 0.0
                                       491 71.0 ± 0.0
                                                            71
                                                                        2.14 2.14 ± 0.0
                                                                                             2.14
basic
                                                             67
                                                                       24.94 1.56 ± 0.12
                     344 78.19 ± 4.42
       380 ± 10.03
                                        66 64.19 ± 0.94
                                                             71
                       609 104.0 ± 7.88
                                          76 66.56 ± 1.44
                                                                       62.25 3.89 ± 0.09
stochastic 813 ± 32.04
```

We see SABRE outperforms the other methods available by default in Qiskit, but not line-graph routing for the circuits considered.

7.0.6 Wall-clock time of line-graph routing

Create a random graph, construct the line graph, create a circuit on the line graph, and put this circuit into line-graph routing

```
from time import time
side=25
lg = lgr.kagome(side, side)
print('number of nodes =',lg.number_of_nodes())
```

```
La=10**5
print('number of gates =',La)
qc = lgr.random_circuit(lg, La)
begin=time()
qc = lgr.line_graph_route(qc)
end=time()
print('wall clock time =',end-begin,'(s)')

number of nodes = 1976
number of gates = 100000
```

wall clock time = 25.544434070587158 (s)