Assisgnment 1, TDT4171

Martin Bjerke

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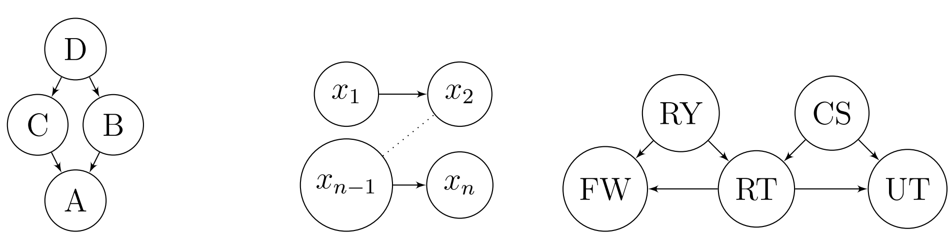
1 5-card Poker Hands

(a) The total number of possible 5-card hands are $\binom{52}{5} = 2598960$ (number of atomic events)

(b) Since the dealer is fair, each atomic event has an equal probability, $P = \frac{1}{2598960} = 3.85 \cdot 10^{-7}$

(c) As there are only 4 possible combinations of a royal straight flush-suit(one for each suit) $P(RSF) = 4 \cdot \frac{1}{2598960} = 1.54 \cdot 10^{-6}$ and there are 13 possible four of a kind, with the last card(one of 48) does not matter $P(FOK) = 13 \cdot \cdot 48 \frac{1}{2598960} = 2.40 \cdot 10^{-4}$.

2 Bayesian Network Construction



(a) Problem 1 (b) Problem 2 (c) Problem 3

2.1 Problem 1

The complete network would have a CPT with $2^4 - 1 = 15$ entries, but using the indepence structure this can be reduced to only $2^0 + 2^1 + 2^1 + 2^2 = 9$ entries.

P(D)
0.9

D	P(C)
t	0.6
f	0.5

D	P(B)
t	0.5
f	0.6

B	C	P(A)
t	t	0.1
t	f	0.2
f	t	0.3
f	f	0.4

2.2 Problem 2

The complete network would have a CPT with $2^n - 1$ entries, but using the indepence structure this can be reduced to only $2^0 + 2^1 + 2^1 + ... + 2^1 = 2^0 + (n-1)2^1$ entries.

P(X_1)
0.9

X_1	P(X_2)
t	0.8
f	0.7

X_2	P(X_3)
t	0.8
f	0.7

...

X_{n-1}	P(X_n)
t	0.1
f	0.2

2.3 Problem 3

The complete network would have a CPT with $2^5 - 1 = 31$ entries, but using the indepence structure this can be reduced to only $2^0 + 2^0 + 2^2 + 2^2 + 2^2 = 14$ entries.

P(RY)
0.9

P(CS)
0.8

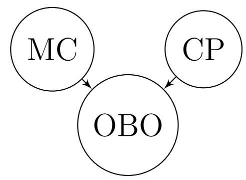
RY	RT	P(FW)
t	t	0.1
t	f	0.2
f	t	0.3
f	f	0.4

CS	RY	P(RT)
t	t	0.2
t	f	0.3
f	t	0.5
f	f	0.7

CS	RT	P(UT)
t	t	0.15
t	f	0.22
f	t	0.36
f	f	0.48

3 Bayesian Network Application

Door	P(MC)
1	1/3
2	1/3
3	1/3



Door	P(CP)
1	1/3
2	1/3
3	1/3

P(MC)	P(CP)	P(OBO)		
		1	2	3
1	1	0	1/2	1/2
1	2	0	0	1
1	3	0	1	0
2	1	0	0	1
2	2	1/2	0	1/2
2	3	1	0	0
3	1	0	1	0
3	2	1	0	0
3	3	1/2	1/2	0

Figure 5: MC - MyChoice, CP - ContainsPrize, OBO - OpenedByOfficial

Throughout the game the probabilities change in the following way:

1. First I choose a door with the probability of a prize beeing behind it P(prize) = 1/3.

2. Then the official chooses a door with no prize behind it. The probability of which door he chooses is shown in the above CPT. Summarising, 1/3 of the time he can choose between 2 doors, each beeing equally possible, and the other 2/3 of the time he can only choose one door.

3. Now the probability of the prize beeing behind the door I choose first is 1/3 and the other door has a probability of 2/3 of having a prize behind it.

The above CPT for P(OBO) show all the probabilities of which doors the official opens, given the door I choose and the door the money price is behind. From the table it is easy to see that in only 1/3 of the possible outcomes the prize is behind the door I first choose. In the other 2/3 of the possible outcomes I will have choosen a door without a prize behind it. The official will then remove another door with no prize behind it and I will be guarnteed a prize if I change to the last door. Totaling in a probability of 2/3 of me getting a prize if I change to the other door after the official opens one door.