Damped Harmonic Oscillator

The damped harmonic oscillator is a classic model in physics that describes the motion of a particle under the influence of both a restoring force (harmonic) and a damping force. It is commonly encountered in various physical systems, such as mechanical vibrations and electrical circuits.

Equation of Motion

The equation of motion for a damped harmonic oscillator is given by the second-order linear differential equation:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0 \tag{1}$$

where:

- m is the mass of the particle,
- c is the damping coefficient,
- k is the spring constant,
- x is the displacement of the particle as a function of time t.

This differential equation can also be expressed using the angular frequency ω_0 and the damping ratio ζ as follows:

$$\frac{d^2x}{dt^2} + 2\zeta\omega_0\frac{dx}{dt} + \omega_0^2 x = 0 \tag{2}$$

where:

- $\omega_0 = \sqrt{\frac{k}{m}}$ is the natural angular frequency,
- $\zeta = \frac{c}{2m\omega_0}$ is the damping ratio.

Solution

A possible solution to the damped harmonic oscillator equation is given by the following exponential decay form:

$$x(t) = e^{-\zeta\omega_0 t}\left(A\cos(\omega_d t) + B\sin(\omega_d t)\right) \tag{3}$$

where:

- $\omega_d = \omega_0\sqrt{1-\zeta^2}$ is the damped angular frequency,
- A and B are constants determined by initial conditions.

This solution captures the decay of the oscillations due to the damping effect, and the oscillatory behavior is modulated by the cosine and sine terms. As shown in Figure 1, the displacement over time follows an exponential decay.

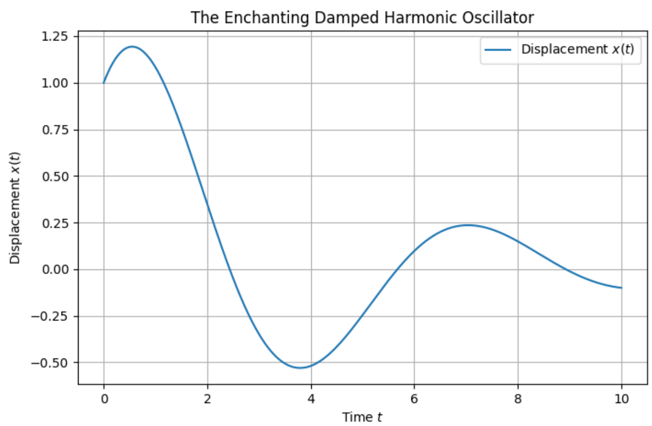

Figure 1: Damped Harmonic Oscillator: Displacement over Time (in this image m=1; c=0.5; k=1; A=1 and B=1)