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Stock valuation in dynamic economies ☆

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Abstract

This article develops and empirically implements a stock valuation model. The model makes three assumptions: (i) dividend equals a fixed fraction of net earnings-per-share plus noise, (ii) the economy's pricing kernel is consistent with the Vasicek term structure of interest rates, and (iii) the expected earnings growth rate follows a mean-reverting stochastic process. The resulting stock valuation formula has three variables as input: net earnings-per-share, expected earnings growth, and interest rate. Using a sample of stocks, our empirical exercise shows that the derived valuation formula produces significantly lower pricing errors than existing models, both in- and out-of-sample. Modeling earnings growth dynamics properly is the most crucial

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for achieving better performance, while modeling the discounting dynamics properly also makes a significant difference.

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0. Introduction

Valuation models for derivative and fixed-income securities have changed risk management and investment practice in significant ways. Examples include Black and Scholes (1973), Cox et al. (1979) binomial option-pricing model, the Vasicek (1977), the Brennan and Schwartz (1979), Cox et al. (1985), and Heath et al. (1992) bond valuation models. In contrast, the stock valuation literature has not made as much progress. Granted that stocks are intrinsically more difficult to value, it is also true that asset pricing research has focused largely on expected-return models, but not on stock valuation per se. In the absence of a parameterized structure for future cashflows, an expected-return characterization is insufficient for solving the stock valuation problem. Observe that existing equity valuation models are primarily dividend-based and developed mostly for the purpose of valuing aggregate market indices (e.g., Bakshi and Chen, 1996, 1997; Bekaert and Grenadier, 2000; Campbell and Shiller, 1988). For this reason, many such models may not be applicable to individual stocks, and they typically fail to address the stochastic nature of interest rates and expected earnings growth.

The purpose of this paper is to develop a stock valuation model based on earnings instead of dividends. While our model is equally applicable to market indices, our focus is on valuing individual stocks. The approach taken is partial-equilibrium, that is, we assume from the outset that there is a stochastic earnings stream made possible by the firm's existing production plan and financial policy, and that a fixed proportion of the earnings (plus some noise) will be paid out as dividends to shareholders. The goal of model development is to derive a stock valuation formula that explicitly relates the stock's fair value to currently observable fundamental variables.

The model makes two additional assumptions. One, we assume a pricing-kernel process that is consistent with a single-factor Vasicek (1977) term structure of interest rates. This pricing kernel serves as the economy-wide valuation standard for future payoffs, and it implies a single-beta equation for determining expected returns. Next, we assume that for the firm under valuation, its earnings-per-share (EPS) obeys a proportional process and grows at a stochastic rate, with its expected growth following a mean-reverting process. This earnings structure captures the firm's growth cycle with three parameters: the long-run EPS growth rate, the speed at which the current expected EPS growth reverts to its long-run mean, and the volatility of expected EPS growth. This parameterization not only helps distinguish long-run growth from transitory growth, but also measures the length and stages of the firm's growth cycle. It leads to a dynamic term structure of expected earnings growth rates.

Compared with the classic Gordon (1962) model, our framework offers two modeling innovations. First, the discounting function of future risky payoffs is based on a stochastic term structure of interest rates, whereas the Gordon model is based on a constant, flat yield curve. This aspect will be referred to as the "stochastic interest-rates" feature. Second, the (conditional) expected EPS growth follows a stochastic process and is hence dynamically changing over time, whereas growth is constant and perpetual in the Gordon model. As our empirical exercise will show, this "stochastic expected growth" feature is important to the success of the pricing model.

To study its empirical performance, we apply our stock valuation model and its special cases to price the S&P 500 index, the 30 stocks in the Dow Jones Industrial Average, and a sample of 20 technology stocks. Our empirical findings are summarized below:

- In pricing the stocks and the S&P 500, our main model's performance is significantly better than any nested special case according to the average absolute pricing error, mean pricing error, and pricing-error standard deviation. Among the sample stocks, the main model's average absolute pricing errors range from 8.17% to 23.87% out-of-sample.
- The stochastic earnings growth feature contributes the most to our model's pricing performance, while the stochastic interest-rates feature is also significant. Overall, our empirical tests indicate that modeling the EPS growth process properly is of the first-order importance.
- High-tech growth stocks such as Intel are generally harder to price than traditional blue-chip stocks (e.g., GE and Exxon). The dispersion in model pricing-errors is generally much higher for the former than the latter. Thus, a higher-dimensional earnings-growth structure may be necessary to further improve the performance for technology firms.
- The model's out-of-sample pricing errors are highly persistent within several months, and they are correlated across the stocks, suggesting the existence of systematic factors that are important in the market's valuation but are missing from the stock valuation model.
- The pricing performance results are robust with respect to the estimation methods used. Our empirical exercise shows that alternative proxies for the interest rate and the expected EPS growth rate do not significantly change the model pricing errors.

Among other potential applications, our valuation model can be employed to recover, from the stock price, the market's expectations about the firm's future EPS growth. According to the efficient market hypothesis (EMH), a security's price should reflect all information relevant for determining its future value. If the EMH is true and if our model is empirically well-specified, one should be able to use it to extract the market's expectations. This can be done in the same way that the

Black-Scholes model is applied to back out the volatility from option prices. For example, if one believes that the market provides a better assessment of the firm's future than analysts, one can then use the market-implied expectations to price the same stock out-of-sample. When our model is implemented using the market-implied expectations, a substantial improvement is observed in pricing performance.

Residual-income based valuation methods have also shown promise (e.g., Ohlson, 1990, 1995; Fetham and Ohlson, 1999; Lee et al., 1999; Ang and Liu, 2001 and references therein). There are, however, several fundamental differences between the accounting residual-income approach and our framework. First, the basic building block for the residual-income models is the clean surplus relation, which internalizes dividends, earnings and book value. In contrast, our setting is partial equilibrium and takes an exogenous earnings process as a given (and leaves out dividend and other corporate policy issues). Second, the empirical implementation methods differ. For instance, Lee et al. (1999) rely on a multi-stage residual-income discounting formula, and they calculate the model price as the sum of (i) the current book value, (ii) the discounted value of multi-year residual-income forecasts, and (iii) the discounted value of the terminal stock price at the end of the forecasting horizon. In their case, forecasts of the terminal stock price and future multi-year residual-income are crucial inputs. Instead of depending on such forecasts, our stock valuation model posits a joint Markov structure for earnings, expected earnings growth and interest rate, so that the conditional expectations are analytically solved as functions of currently observable variables. Consequently, for empirical implementations of our model, there is no need to estimate a terminal stock price or forecast future residual-incomes.

The paper proceeds as follows. Section 1 outlines our assumptions and develops the main stock valuation model. In Section 2, we examine several extensions, including the existence of jumps in the earnings processes. Section 3 describes the data and the choice of empirical proxies. The parameter estimates and empirical methods are discussed in Section 4. Section 5 focuses on the empirical pricing performance and robustness issues. Section 6 examines the market-implied earnings growth expectations. Concluding remarks are offered in Section 7.

1. A dynamic stock valuation model

Consider a continuous-time, infinite-horizon economy whose underlying valuation standard for all securities is represented by some pricing-kernel process, denoted by $M(t)$. Take any generic firm in the economy, and assume that each equity share of the firm entitles its holder to an infinite dividend stream $\{D(t) : t \geqslant 0\}$. Our goal is to determine the time-t per-share value, $S(t)$, for each time $t \geqslant 0$. Since the pricing kernel applies to every security, we have by a standard argument

$$S(t) = \int_t^{\infty} \mathrm{E}_t\left[\frac{M(\tau)}{M(t)}\, D(\tau)\right] \mathrm{d}\tau, \tag{1}$$

where $E_t[\cdot]$ is the time-t conditional expectation operator with respect to the objective probability measure. Throughout this paper, all variables are given in nominal terms.

Given Eq. (1), the valuation task amounts to two steps: (i) specify "appropriate" processes for $M(t)$ and $D(t)$, and (ii) solve the conditional expectations and the integral in (1). As in the term structure and derivatives literature, the search for appropriate specifications is usually subject to three considerations. First, the specified structure for each variable should be consistent with as many known empirical properties as possible. Second, the choice should be such that the valuation problem in (1) is technically tractable. Parsimony and implementability are clearly desirable. Third, the resulting solution to the problem in (1) should represent an empirically well-performing stock valuation model.

1.1. The assumptions

As in Constantinides (1992), we assume from the outset that $M(t)$ follows an Ito process satisfying

$$\frac{\mathrm{d}M(t)}{M(t)} = -R(t)\,\mathrm{d}t - \sigma_m\,\mathrm{d}W_m(t), \tag{2}$$

for a constant σ_m, where the instantaneous interest rate, $R(t)$, follows an Ornstein-Uhlenbeck mean-reverting process:

$$\mathrm{d}\,R(t) = \kappa_r(\mu_r^0 - R(t))\,\mathrm{d}t + \sigma_r\,\mathrm{d}W_r(t), \tag{3}$$

for constants κ_r, μ_r^0 and σ_r. This pricing kernel leads to a single-factor Vasicek (1977) term structure of interest rates. As in every single-factor term structure model, bond yields are perfectly correlated and hence perfectly substitutable, a counterfactual feature. This limitation can however be relaxed by adopting multi-factor models.

Assume that the firm under valuation has a constant dividend-payout ratio (plus noise), δ (with $1 \geqslant \delta \geqslant 0$), that is,

$$D(t)\,\mathrm{d}t = \delta\,Y(t)\,\mathrm{d}t + \mathrm{d}\,Z(t), \tag{4}$$

where $Y(t)$ is the firm's earnings-per-share (EPS) flow at t (hence $Y(t)\,\mathrm{d}t$ is the total EPS over the interval from t to $t+\mathrm{d}t$), and $\mathrm{d}\,Z(t)$ is the increment to a martingale process with zero mean. Economically, $\mathrm{d}\,Z(t)$ cannot have a non-zero mean, because the firm would otherwise be assumed to pay an average non-zero dividend stream even if $Y(t)$ is zero at each t. The existence of $\mathrm{d}\,Z(t)$ allows the firm's dividend to deviate randomly from the fixed proportion of its EPS, and it makes $D(t)$ and $Y(t)$ not perfectly substitutable. The constant dividend-payout-ratio assumption is commonly used in accounting and equity valuation (e.g., Lee et al., 1999).

Note that the firm's dividend policy is exogenous to our model. It is understood that the firm's production plan, operating revenues and expenses, and target payout-ratio are all fixed outside our model, and the net earnings process, $Y(t)$, is determined accordingly. Any deviation from the fixed exogenous structure will lead to a change in the $Y(t)$ process. For example, changing δ will lead to a new process for $Y(t)$. Thus, taking δ and its associated $Y(t)$ process as given, our goal is simply to value the cashflow stream given in (4). In doing so, we are also abstracting from the firm's investment policy and growth opportunities, with the understanding that the $Y(t)$ process should indirectly incorporate these aspects.

The assumption embedded in (4) is only an approximation of economic reality. Strictly speaking, many firms do not pay cash dividends and hence the notion of dividend payout does not apply. But it is a crucial assumption in our model development. First, it allows us to link stock price directly to the firm's earnings, instead of dividends. This is an important feature because (i) dividend-based stock valuation models (e.g., the Gordon model and its variants) have not succeeded empirically and (ii) investors are far more interested in the earnings potential of a stock rather than in its dividend. Also, precisely due to assumption (4), we can now value stocks even if they do not pay any cash dividends. In recent years, more and more firms (especially technology firms) have chosen to pay no cash dividends, but instead use their earnings to repurchase outstanding shares or simply reinvest in new projects (Fama and French, 2001).

Second, net earnings should in general be more informative than dividends because, even if a firm pays cash dividends, the dividend process is usually managed and artificially smoothed (and thus less informative of the firm's current conditions). For firms that pay no dividends, earnings will be the most effective signal for valuation. Therefore, $Y(t)$ should be a more efficient summary of the firm's financial and operating conditions. Linking a stock valuation formula to earnings (instead of dividends) should make the model more likely to succeed empirically.

Assume that under the objective probability measure, $Y(t)$ follows a process given below:

$$\frac{\mathrm{d}\,Y(t)}{Y(t)} = G(t)\,\mathrm{d}t + \sigma_y\,\mathrm{d}W_y(t) \tag{5}$$

$$\mathrm{d}\,G(t) = \kappa_g(\mu_g^0 - G(t))\,\mathrm{d}t + \sigma_g\,\mathrm{d}W_g(t), \tag{6}$$

for constants σ_y, κ_g, μ_g^0 and σ_g. The long-run mean for both $G(t)$ and actual EPS growth $\mathrm{d}\,Y(t)/Y(t)$ is μ_g^0, and the speed at which $G(t)$ adjusts to μ_g^0 is reflected by κ_g. Volatility for both earnings growth and changes in $G(t)$ is time-invariant. Shocks to expected growth, W_g, may be correlated with both systematic shocks W_m and interest rate shocks W_r, with their respective correlation coefficients denoted by $\rho_{g,m}$ and $\rho_{g,r}$. In addition, the correlations of W_y with W_g, W_m and W_r are respectively denoted by $\rho_{g,y}$, $\rho_{m,y}$ and $\rho_{r,y}$. Thus, both actual and expected EPS growth shocks are priced risk factors. The noise process $\mathrm{d}Z(t)$ in (4) is, however, assumed to be uncorrelated with $G(t)$, $M(t)$, $R(t)$ and $Y(t)$, and hence it is not a priced risk factor.

The earnings process as parameterized in (5) offers enough modeling flexibility of a firm. First, both actual and expected earnings growth can take either positive or negative values, capturing the fact that a firm may experience transition stages in its growth cycle. Second, expected EPS growth $G(t)$ is mean-reverting and has both a permanent component (reflected by μ_g^0) and a transitory component, so that $G(t)$ can be high or low relative to its long-run mean μ_g^0. As shown later, this separate parameterization of the EPS growth process is useful for understanding both time-series and cross-sectional variations in equity valuation. Finally, since $Y(t)$ is observable and $G(t)$ can be obtained from analyst estimates, the resulting equity valuation formula will be implementable.

The EPS process in (5) has one undesirable feature, however. Under the assumed structure, $Y(t)$ is positive with probability one. Yet, in practice, firms have negative earnings from time to time. For instance, about 7.08% of the observations in the I/B/E/S database have negative EPS from 1976 to 1998. Therefore, even with this limitation, our theoretical framework is applicable to a majority of publicly traded stocks. With suitable modifications to (5), earnings can take both positive and negative values (see, for example, the extension in Dong, 2000).

1.2. The valuation formula

Substituting the above assumptions into (1), we see that $S(t)$ must be a function of $G(t)$, $R(t)$ and $Y(t)$. Next, we apply Ito's lemma to $S(t)$ and substitute the resulting expression into a risk-return equation, we arrive at a partial differential equation (PDE) for $S(t)$:

$$\begin{aligned}\frac{1}{2}\sigma_y^2 Y^2\frac{\partial^2 S}{\partial Y^2} &+ (G-\lambda_y)Y\frac{\partial S}{\partial Y} + \rho_{g,y}\sigma_y\sigma_g Y\frac{\partial^2 S}{\partial Y\partial G} + \rho_{r,y}\sigma_y\sigma_r Y\frac{\partial^2 S}{\partial Y\partial R}\\ &+ \rho_{g,r}\sigma_g\sigma_r\frac{\partial^2 S}{\partial G\partial R} + \frac{1}{2}\sigma_r^2\frac{\partial^2 S}{\partial R^2} + \kappa_r(\mu_r - R)\frac{\partial S}{\partial R} + \frac{1}{2}\sigma_g^2\frac{\partial^2 S}{\partial G^2}\\ &+ \kappa_g(\mu_g - G)\frac{\partial S}{\partial G} - RS + \delta Y = 0, \end{aligned} \tag{7}$$

subject to $S(t) < \infty$, where $\lambda_y \equiv \sigma_m\sigma_y\rho_{m,y}$ is the risk premium for the firm's earnings shocks, and $\mu_g \equiv \mu_g^0 - \sigma_m\sigma_g\rho_{g,m}/\kappa_g$ and $\mu_r \equiv \mu_r^0 - \sigma_m\sigma_r\rho_{m,r}/\kappa_r$ are, respectively, the long-run means of $G(t)$ and $R(t)$ under the risk-neutral probability measure defined by the pricing kernel $M(t)$, with $\rho_{m,r}$ being the correlation between $W_m(t)$ and $W_r(t)$. We can show that the solution to this PDE is

$$S(t) = \delta\int_0^\infty s(t,\tau;G,R,Y)\,\mathrm{d}\tau, \tag{8}$$

where $s(t,\tau;G,R,Y)$ is the time-t price of a claim that pays $Y(t+\tau)$ at a future date $t+\tau$:

$$s(t,\tau;G,R,Y) = Y(t)\exp[\varphi(\tau) - \varrho(\tau)R(t) + \vartheta(\tau)G(t)], \tag{9}$$

where

$$\begin{aligned}\varphi(\tau) = &-\lambda_y\tau + \frac{1}{2}\frac{\sigma_r^2}{\kappa_r^2}\left(\tau + \frac{1-\mathrm{e}^{-2\kappa_r\tau}}{2\kappa_r} - \frac{2(1-\mathrm{e}^{-\kappa_r\tau})}{\kappa_r}\right) - \frac{\kappa_r\mu_r + \sigma_y\sigma_r\rho_{r,y}}{\kappa_r}\\ &\times\left(\tau - \frac{1-\mathrm{e}^{-\kappa_r\tau}}{\kappa_r}\right) + \frac{1}{2}\frac{\sigma_g^2}{\kappa_g^2}\left(\tau + \frac{1-\mathrm{e}^{-2\kappa_g\tau}}{2\kappa_g} - \frac{2}{\kappa_g}(1-\mathrm{e}^{-\kappa_g\tau})\right)\end{aligned}$$

$$+\frac{\kappa_g\mu_g+\sigma_y\sigma_g\rho_{g,y}}{\kappa_g}\left(\tau-\frac{1-\mathrm{e}^{-\kappa_g\tau}}{\kappa_g}\right)-\frac{\sigma_r\sigma_g\rho_{g,r}}{\kappa_r\kappa_g}$$
$$\times\left(\tau-\frac{1}{\kappa_r}(1-\mathrm{e}^{-\kappa_r\tau})-\frac{1}{\kappa_g}(1-\mathrm{e}^{-\kappa_g\tau})+\frac{1-\mathrm{e}^{-(\kappa_r+\kappa_g)\tau}}{\kappa_r+\kappa_g}\right), \tag{10}$$

$$\varrho(\tau)=\frac{1-\mathrm{e}^{-\kappa_r\tau}}{\kappa_r}, \tag{11}$$

$$\vartheta(\tau)=\frac{1-\mathrm{e}^{-\kappa_g\tau}}{\kappa_g}, \tag{12}$$

subject to the transversality condition that

$$\mu_r-\mu_g>\frac{\sigma_r^2}{2\,\kappa_r^2}-\frac{\sigma_r\sigma_y\rho_{r,y}}{\kappa_r}+\frac{\sigma_g^2}{2\kappa_g^2}+\frac{\sigma_g\sigma_y\rho_{g,y}}{\kappa_g}-\frac{\sigma_g\sigma_r\rho_{g,r}}{\kappa_g\kappa_r}-\lambda_y. \tag{13}$$

Thus, $S(t)$ is the summed value of a continuum of claims that each pay in the future an amount respectively determined by the earnings process. The presence of an integral in (8) should not hamper the applicability of the model. For applications, one can numerically compute the integral in the same way as numerical techniques are applied to solve the integral in option-pricing models.

Note that the valuation formula in (8) is solely a function of time-t variables. As a result, implementation does not require estimating multiple future cashflows or the end-of-horizon stock value (as is commonly done for multi-stage discounted cashflow or residual-income models). This convenient feature is due to the fact that with the parameterized Markov structure for $Y(t)$, $G(t)$ and $R(t)$, all the conditional expectations in (1) are solved in terms of functions of time-t variables.

In deriving the valuation formula, we relied on a CAPM-like risk-return relation to arrive at the PDE in (7). In this sense, our model is consistent with and built upon developments in the risk-return literature. But, as can be seen, a risk-return equation alone is not sufficient to determine a fair value for a stock since assumptions on the earnings/cashflow processes are also needed.

Three special cases of the *main model* in (8) are worth noting. The first case is obtained by setting $R(t)$ to be constant over time, but maintaining the stochastic processes for $Y(t)$ and $G(t)$ as in (5) and (6). We refer to the resulting case as the *SG model*, since it allows for stochastic expected EPS growth. For the second case, we set $G(t)$ to be constant over time, but allow interest rate $R(t)$ to be stochastic as given in (3). We call this case the *SI model* (i.e., the stochastic-interest-rate model). The corresponding valuation formulas are provided in Appendix A for the SG and the SI models. The third special case is an extended version of the *Gordon model*, in which both $G(t)$ and $R(t)$ are constant over time: $G(t)=g$ and $R(t)=r$, for constants g and r. Consequently, both $M(t)$ and $Y(t)$ follow a geometric Brownian motion. In this case, we obtain

$$S(t)=\frac{\delta\,Y(t)}{r+\lambda_y-g}, \tag{14}$$

provided that $r + \lambda_y - g > 0$. In the empirical analysis to follow, we will compare the relative pricing performance by our main model and its nested variants.

In equity valuation, a common relative-price yardstick is the price/earnings (P/E) ratio, partly because P/E is comparable both across different times for the same firm and among distinct firms. For this reason, we can re-express (8) below:

$$P(t) \equiv \frac{S(t)}{Y(t)} = \delta \int_0^{\infty} \exp[\varphi(\tau) - \varrho(\tau)R(t) + \vartheta(\tau)G(t)]\,\mathrm{d}\tau. \tag{15}$$

Thus, the P/E ratio, $P(t)$, is a function of the interest rate, the expected EPS growth, the firm's required risk premiums, and parameters governing the earnings and interest rate processes.

2. Extensions

The models analyzed so far extend the Gordon framework in several important directions, but arguably they are still a simplified version of economic reality. For instance, the single-factor Vasicek term-structure assumption is hardly realistic as it requires all interest rates and forward rates to be perfectly correlated. Therefore, one direction to extend the main model is to allow for a multi-factor term structure. Since these extensions are relatively straightforward given the vast term-structure literature, this section focuses on different specifications of the earnings process. In addition, as our empirical exercise will demonstrate, modeling the earnings process properly is far more crucial, especially for pricing individual stocks.

Consider Model 1 in which the pricing kernel and the interest rate are respectively as given in (2)–(3) and the earnings growth as given in (5) except that $G(t)$ follows the process below:

$$\mathrm{d}\,G(t) = \kappa_g[A(t) - G(t)]\,\mathrm{d}t + \sigma_g\,\mathrm{d}W_g(t) \tag{16}$$

$$A(t) = \eta \int_0^t \mathrm{e}^{-\eta(t-u)} G(u)\,\mathrm{d}u, \tag{17}$$

where $\eta \geqslant 0$ and $A(t)$ is the "moving target" to which the expected growth rate $G(t)$ reverts at a speed reflected by κ_g. That is, the long-run mean growth rate in net earnings is no longer a constant, but is determined by a weighted average of past expected growth rates. In determining $A(t)$, more recent growth rates have more weight. Under Model 1, the stock price becomes

$$S(t) = \delta\,Y(t) \int_0^{\infty} \exp[\varphi(\tau) - \varrho(\tau)R(t) + \vartheta(\tau)\,G(t) + \varsigma(\tau)\,A(t)]\,\mathrm{d}\tau, \tag{18}$$

where $\varrho(\tau)$ is as in (11) and $\vartheta(\tau)$, $\varsigma(\tau)$, and $\varphi(\tau)$ are defined in (43)–(45) of Appendix A.

In Model 2, we again keep the same pricing kernel and interest rate processes, but add a jump component to the expected earnings growth process:

$$\frac{\mathrm{d}Y(t)}{Y(t)} = G(t)\,\mathrm{d}t + \sigma_y\,\mathrm{d}W_y(t), \tag{19}$$

$$\mathrm{d}G(t) = \kappa_g[\mu_g - G(t)]\,\mathrm{d}t + \sigma_g\,\mathrm{d}W_g(t) + J(t)\,\mathrm{d}q(t), \tag{20}$$

where $J(t)$ (the jump amplitude) is i.i.d. and follows an exponential distribution with parameter v and $q(t)$ is a Poisson jump counter (i.e., 1 with probability λ_J dt and zero otherwise). Then

$$S(t) = \delta\,Y(t)\int_0^\infty \exp[\varphi(\tau) + \xi(\tau) - \varrho(\tau)R(t) + \vartheta(\tau)G(t)]\,\mathrm{d}\tau, \tag{21}$$

where $\varphi(\tau)$, $\varrho(\tau)$, $\vartheta(\tau)$are given in (10)–(12) and $\xi(\tau) \equiv \lambda_J([1/v\,(\kappa_g - 1)])\log[(v\,\kappa_g - 1 + \mathrm{e}^{-\kappa_g\tau})/\kappa_g\mathrm{e}^{-\kappa_g\tau}] - \tau)$. The transversality condition becomes

$$\mu_r - \mu_g > \frac{\sigma_r^2}{2\kappa_r^2} - \frac{\sigma_r\sigma_y\rho_{r,y}}{\kappa_r} + \frac{\sigma_g^2}{2\kappa_g^2} + \frac{\sigma_g\sigma_y\rho_{g,y}}{\kappa_g} - \frac{\sigma_g\sigma_r\rho_{g,r}}{\kappa_g\kappa_r} - \lambda_y - \lambda_J.$$

Finally, in Model 3, we allow for a two-factor model of the expected earnings growth. That is, let

$$G(t) = X_1(t) + X_2(t), \tag{22}$$

where the unobservable factors $X_1(t)$ and $X_2(t)$ follow the dynamics below:

$$\mathrm{d}X_\ell(t) = \kappa_{x_\ell}[\mu_{x_\ell} - X_\ell(t)]\,\mathrm{d}t + \sigma_{x_\ell}\,\mathrm{d}W_{x_\ell} \tag{23}$$

for $\ell = 1, 2$. Assume that $X_1(t)$ and $X_2(t)$ are independent for all t. After this modification to the main model, the resulting term structure of expected earnings growth has $X_1(t)$ and $X_2(t)$ as the two driving factors. It should thus fit the term structure of actual earnings forecasts better. Furthermore, for any two distinct forecasting horizons τ_1 and τ_2, the expected τ_1- and τ_2-period-ahead earnings growth rates, $G(t,\tau_1)$ and $G(t,\tau_2)$, are both linear in $X_1(t)$ and $X_2(t)$:

$$G(t,\tau_1) = a_{10} + a_{11}X_1(t) + a_{12}X_2(t), \tag{24}$$

$$G(t,\tau_2) = a_{20} + a_{21}X_1(t) + a_{22}X_2(t), \tag{25}$$

where $a_{\ell 0}$, $a_{\ell 1}$ and $a_{\ell 2}$ for $\ell = 1, 2$, are given in (46)–(51) of Appendix A. For instance, $G(t,\tau_1)$ can be the expected 1-year-ahead growth rate and $G(t,\tau_2)$ can be the expected 2-year-ahead earnings growth. Solving the stock valuation problem and replacing the two unobservable factors $X_1(t)$ and $X_2(t)$ by $G(t,\tau_1)$ and $G(t,\tau_2)$, we have

$$S(t) = \delta\,Y(t)\int_0^\infty \exp[\varphi(\tau) - \varrho(\tau)R(t) + \vartheta(\tau)\,G(t,\tau_1) + \theta(\tau)G(t,\tau_2)]\,\mathrm{d}\tau \tag{26}$$

where $\varrho(\tau)$ is as defined in (11) and $\vartheta(\tau)$, $\theta(\tau)$ and $\varphi(\tau)$ are displayed in (52)–(54) of Appendix A.

3. Data description and implementation considerations

To evaluate the empirical performance of our main model, we need to choose a sample of stocks and address several empirical issues. For stock price and earnings data, we use the I/B/E/S US History File, which contains *mid-month observations* on reported actual EPS and consensus analyst forecasts of future earnings, plus the contemporaneous stock prices. Since the available stock universe is rather large, we focus on three representative sets of stocks/indices. In the first set, we include all 30 stocks comprising the Dow Jones Industrial Average (hereafter, the Dow).

The second set contains 20 technology stocks, including firms under tickers ADBE, ALTR, AMAT, CMPQ, COMS, CSC, CSCO, DELL, INTC, KEAN, MOT, MSFT, NNCX, NT, ORCL, QNTM, STK, SUNW, TXN and WDC. As technology firms face greater uncertainty and exhibit more earnings volatility, valuing them has been challenging both in theory and in practice. Finally, we include the S&P 500 index in our valuation exercise.

For the S&P 500, I/B/E/S did not collect analyst EPS estimates until January 1982. The I/B/E/S coverage starting date is January 1976 for some individual stocks and later for others. In the original sample, there are 7664 observations for all the Dow component stocks, and 3464 observations for all of the 20 technology stocks.

To construct the required data, we need three fundamental variables: current EPS $Y(t)$, expected EPS growth $G(t)$ and interest rate $R(t)$. First, we use the I/B/E/S consensus analyst estimate for current-year (FY1) EPS as a proxy for $Y(t)$ (provided $Y(t)>0$). In any given month, the FY1 estimate may contain actual quarterly EPS numbers (for the passed quarters of the fiscal year), with the EPS numbers for the remaining quarters being consensus analyst forecasts. Note that for a typical firm, while actual earnings occur continuously over time, accounting earnings are reported only quarterly. Furthermore, most firms' earnings exhibit seasonalities. For this reason, the total EPS over a fiscal year should be a natural proxy for $Y(t)$.

For our empirical work, we use the analyst-expected EPS growth from the current (FY1) to the next fiscal-year (FY2) as a proxy for $G(t)$. This proxy choice is reasonable since the year-over-year EPS growth has been the conventional calculation method in the industry. For instance, quarter-over-quarter and month-over-month (if available) EPS growth rates would not be better proxies for $G(t)$, as they would be subject to seasonal biases in earnings and revenue. We can also back out the instantaneous expected EPS growth $G(t)$ from forecasts for longer-horizon EPS growth rates using (6):

$$G(t) = \frac{\kappa_g \tau}{1 - e^{-\kappa_g \tau}} \left(G(t,\tau) - \mu_g^0 + \frac{1}{2}\sigma_y^2 + \frac{\mu_g^0 (1 - e^{-\kappa_g \tau})}{\kappa_g \tau} \right), \tag{27}$$

where for $G(t,\tau)$ we can use the expected one-year-forward growth rate. In Section 5.3, we will implement this alternative and compare the resulting performance.

As there is no established benchmark for the spot rate (Chapman et al., 1999), we employ the 30-year Treasury yield as a proxy. The reasons are as follows. First, in the Vasicek term structure, the yields of different maturities are perfectly correlated with each other and hence interchangeable. Second, empirically, the 30-year yield is much more closely followed by stock market participants than short-term rates, as the long-term rate is believed to be more relevant for equity valuation (see Lee et al., 1999).[1] In addition, we have also experimented with the 3-month Treasury bill rate and the 10-year Treasury yield, a topic to be examined in Section 5.3.

Table 1 presents summary statistics for the stock samples and four individual stocks. The average expected EPS growth over each stock's sample period ranges from 7.81% for Exxon, 10.13% for S&P 500, to 51.59% for Intel. The average expected growth rate for the Dow stocks is 21.07%, compared to 49.23% for the 20 technology stocks. Among the four individual stocks, GE has the lowest growth volatility, while Intel's growth is by far the most volatile. Intel has the highest average market P/E, whereas the slowest-growth Exxon has the lowest average P/E. The average P/E is 14.22 for the Dow stocks versus 22.56 for the 20 technology stocks.

4. Parameter estimation

In this section we use the main model to illustrate our empirical methods. We then discuss the parameter estimates. Two approaches are considered for estimating the parameters in (8):

1. Estimate the parameters directly from the individual time-series of $G(t)$, $Y(t)$ and $R(t)$. While the parameters so estimated serve as a useful benchmark, such a procedure ignores how the stock has been valued by the market in the past.
2. Estimate all the parameters from historical data (including the stock's past prices), in the same way as one recovers volatility from observed option prices (Bakshi et al., 1997). This approach is at the center of our empirical estimation, for two reasons. First, parameters so estimated reflect the historical valuation standard applied to the stock by the market. Second, it permits the joint estimation of the risk-neutralized parameters and the earnings risk premium.

To reduce the estimation burden, we preset two parameters: $\rho_{g,y} = 1$, and $\rho \equiv \rho_{g,r} = \rho_{r,y}$. That is, the actual and expected earnings growth rates are subject to the same random shocks. For the main model, there are 10 parameters remaining: $\Phi \equiv \{\mu_g, \kappa_g, \sigma_g, \sigma_y, \lambda_y, \rho, \delta, \mu_r, \kappa_r, \sigma_r\}$. The SG and the SI models each have 7 structural parameters to be estimated.

[1]To substantiate this point we regress the earnings yield of the S&P 500 index on the 30-year Treasury yield (denoted "Long") and the 3-month Treasury bill rate (denoted "Short"): $Y(t)/S(t) = -0.007 + 0.34\text{Short}(t) + 0.67\text{Long}(t) + \tilde{\omega}(t)$. The t-statistics for both coefficients are significant, with an adjusted R^2 of 89.4%. When the 3-month rate is the sole independent variable, the adjusted R^2 is 79.09%; when the 30-year yield is the explanatory variable, the adjusted R^2 is 86.55%. Our exclusion tests show that the 30-year yield is more significant in explaining the earnings yield of the S&P 500. Similar conclusions hold when we replace the S&P 500 by the Dow stocks.

Table 1
Summary statistics for the stock sample

We adopt two exclusion criteria in constructing the stock sample. First, if the earnings data is missing for over 3 months, the entire year is omitted for that stock. There are 7664 observations across all the 30 Dow stocks, and 3464 observations across the 20 technology stocks. Second, we delete observations with negative earnings. Therefore, 68 observations (0.89%) in the Dow, and 78 observations (2.25%) in the technology stock sample are deleted. This leaves a final sample of 7596 and 3336 monthly observations respectively for the Dow and tech stocks. The expected EPS growth, $G(t)$, is the consensus EPS forecast for FY2 divided by that for FY1, minus 1; the P/E ratio is the current price normalized by FY1 EPS. We report the average, the standard deviation, the maximum, and the minimum for each variable. The 20 technology stocks in the sample are: ADBE, ALTR, AMAT, CMPQ, COMS, CSC, CSCO, DELL, INTC, KEAN, MOT, MSFT, NNCX, NT, ORCL, QNTM, STK, SUNW, TXN and WDC. For most DOW stocks, I/B/E/S started reporting monthly analyst EPS forecasts in January 1976. The S&P 500 index sample begins in January 1982. The final month for the sample is July 1998. The summary statistics are reported for S&P 500, GE (General Electric), XON (Exxon), INTC (Intel), and MOT (Motorola).

		S&P 500	GE	XON	INTC	MOT	All 30 Dow stocks	All 20 Tech stocks
P/E ratio, $P(t)$	NOBS	199	271	271	271	271	7596	3386
	Average	15.10	13.62	11.27	23.17	18.72	14.22	22.56
	Std. dev.	4.13	4.89	4.81	18.10	10.59	3.24	5.86
	Max.	26.48	33.47	25.05	121.78	157.33	23.47	34.77
	Min.	7.24	7.25	4.23	8.34	8.17	10.09	12.28
Expected earnings growth, $G(t)$	Average	0.1013	0.1218	0.0781	0.5159	0.3092	0.2107	0.4923
	Std. dev.	0.0531	0.0311	0.0483	0.7958	0.3908	0.1093	0.3129
	Max.	0.2616	0.2321	0.2514	5.0000	5.1667	0.5547	1.2594
	Min.	0.0009	0.0227	−0.0722	−0.9500	−0.0131	0.0781	0.1856
Stock return	Average	0.0126	0.0141	0.0104	0.0248	0.0141	0.0122	0.0337
	Std. dev.	0.0403	0.0563	0.0448	0.1099	0.0922	0.0053	0.0139
	Max.	0.1671	0.2423	0.1460	0.3275	0.3303	0.0283	0.0665
	Min.	−0.1955	−0.2014	−0.1192	−0.3821	−0.3491	0.0004	0.0139

4.1. Parameter estimates independent of stock prices

We first estimate the parameters under the objective probability measure. To explain the maximum-likelihood method, take the dynamics of $G(t)$ given in (6) as an example. By solving the backward equation for the transition density of $G(t)$, we obtain the maximum-likelihood criterion function:

$$\mathscr{L} \equiv \max_{\kappa_g,\mu_g,\sigma_g} \frac{1}{T}\sum_{t=1}^{T} \log\left(\frac{1}{\pi}\int_0^\infty \mathrm{Re}[F(t,\tau,G(t);u)\times\exp(-\mathrm{i}uG(t+\tau))]\,\mathrm{d}u\right), \tag{28}$$

where $\mathrm{i}=\sqrt{-1}$, T is the number of months in the sample, and $F(t,\tau,G(t);u)$ is the characteristic function for the density given by $\exp[\mathrm{i}u\mu_g^0(1-\mathrm{e}^{-\kappa_g\tau})-u^2\sigma_g^2/4\kappa_g(1-\mathrm{e}^{-2\kappa_g\tau})+\mathrm{i}u\mathrm{e}^{-\kappa_g\tau}G(t)]$.

Based on the monthly time-series for $G(t)$, we set $\tau = \frac{1}{12}$ and report the maximum-likelihood parameter estimates in Table 2. Several observations are in order. First, according to the κ_g estimates, the S&P 500 has the longest growth cycle (as its κ_g is the lowest) while Intel's has the shortest. Second, consistent with Table 1, the point estimate of long-run growth rate, μ_g^0, is higher for the technology stocks than for the Dow stocks. Third, the volatility of expected EPS growth (σ_g) is about twice as high for the technology stocks as for the Dow stocks. Finally, the mean-reverting process in (6) fits the expected EPS growth process for the Dow stocks better than for the technology stocks (based on the maximized $\mathscr{L}$). Therefore, for technology firms, richer dynamics for $G(t)$ may be necessary to improve the structural fit.

Using a similar maximum-likelihood procedure, we estimate the interest rate parameters from the 30-year yield time-series. The long-run interest rate, μ_r^0, is 7.94% and its volatility, σ_r, is 1.18%, both of which are consistent with their historical counterparts. The estimate for κ_r is 0.107. These estimates are comparable to those reported in Chan et al. (1992).

In addition, Table 2 presents (i) the sample correlation between interest rate changes and changes in expected EPS growth, ρ, and (ii) the dividend-payout ratio, δ. The estimate for δ is obtained by regressing dividend yield on earnings yield (with zero intercept). The average δ across the technology stocks is 3.84%. Moreover, the average ρ across the Dow (technology) stocks is 0.02 (−0.02). The earnings risk-premium, λ_y, cannot be estimated from the realizations of earnings and expected earnings growth, and it can only be backed out from stock prices.

4.2. Parameter estimates implied by stock prices

Like option pricing formulas, a stock valuation model such as (8) is not a set of moment restrictions on asset prices. Rather, it is an exact restriction on the equity price in relation to the contemporaneous EPS, the expected EPS growth, and the interest rate. For this reason, the generalized method of moments and related econometric techniques may not be applicable.

We follow two estimation methods, one correcting, and the other not correcting, for the serial correlation of the model errors. To describe the first method, let $\overline{P}(t)$ be its month-t observed P/E ratio for a stock and define $\varepsilon(t) \equiv \overline{P}(t) - P(t)$, where $P(t)$ is the model P/E determined in (15). Then, the *SSE estimation* procedure tries to find a Φ to solve

$$\text{RMSE} \equiv \min_{\Phi} \sqrt{\frac{1}{T}\sum_{t=1}^{T} |\varepsilon(t)|^2}, \tag{29}$$

subject to the transversality condition in (13). This method seeks to minimize the sum of squared errors between each observed P/E and the model-determined P/E. P/E serves as a normalized price that is comparable across time periods for the same firm.

The optimized objective function value from (29), RMSE, is zero only if the obtained Φ estimate leads to a perfect fit of each market P/E by the model. In general, the average in-sample P/E-pricing error (i.e., the mean value of $\varepsilon(t)$) will not

Table 2
Parameter estimates under the objective probability measure

The reported structural parameters for the *expected growth rate* process are based on the maximum-likelihood estimation of the transition density function:

$$\mathscr{L} \equiv \max_{\kappa_g,\mu_g^0,\sigma_g} \frac{1}{T}\sum_{t=1}^{T} \log\left(\frac{1}{\pi}\int_0^\infty \mathrm{Re}[\exp(-\mathrm{i}uG(t+\tau)) \times F(t,\tau,G(t);u)]\,\mathrm{d}\phi\right),$$

where $\mathrm{i} = \sqrt{-1}$, $\tau = \frac{1}{12}$ (for monthly sampled observations) and the characteristic function, denoted $F(t,\tau,G(t);u)$, is

$$F(t,\tau,G(t);u) = \exp\left[\mathrm{i}u\mu_g^0(1-\mathrm{e}^{-\kappa_g\tau}) - \frac{u^2\sigma_g^2}{4\kappa_g}(1-\mathrm{e}^{-2\kappa_g\tau}) + \mathrm{i}u\mathrm{e}^{-\kappa_g\tau}G(t)\right].$$

The asymptotic p-values are recorded in square brackets and the (cross-sectional) standard errors in parentheses. The maximum likelihood estimation of the interest rate process leads to $\mu_r^0 = 0.0794$, $\kappa_r = 0.107$ and $\sigma_r = 0.0118$. The reported ρ is the correlation coefficient between $\mathrm{d}R(t)$ and $\mathrm{d}G(t)$; and δ is obtained by regressing dividend yield on the earnings yield (without a constant). Average dividend divided by average net-earnings per share yields a similar δ. Among the technology stocks, only six firms paid any cash dividends (ADBE, CMPQ, INTC, MOT NT, and TXN).

Parameter	S&P 500	GE	XON	INTC	MOT	All 30 Dow stocks	All 20 Tech stocks
μ_g^0	0.084	0.115	0.074	0.227	0.298	0.169	0.296
	[0.000]	[0.000]	[0.050]	[0.000]	[0.040]	(0.022)	(0.044)
κ_g	1.110	2.120	2.438	3.284	1.508	2.613	2.688
	[0.330]	[0.000]	[0.040]	[0.000]	[0.050]	(0.771)	(0.485)
σ_g	0.053	0.056	0.098	1.367	0.163	0.200	0.425
	[0.000]	[0.000]	[0.020]	[0.000]	[0.040]	(0.049)	(0.083)
$\mathscr{L}$	2.388	2.554	2.195	−0.206	1.318	1.629	0.776
						(0.138)	(0.156)
ρ	0.02	0.09	0.16	−0.04	0.00	0.02	−0.02
						(0.01)	(0.02)
δ	44.29%	41.48%	61.79%	3.39%	18.07%	36.31%	3.84%
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	(2.34)	(1.57)

be zero because the objective in (29) is to minimize the sum of squared errors, but not the average pricing errors.

One possible drawback of the least-squares method just outlined is that it fails to account for serial correlations of the model pricing errors. To incorporate this feature into our procedure, we assume a first-order autoregressive process for the error: $\varepsilon(t) = \phi\varepsilon(t-1) + \eta(t)$, where $\eta(t) \sim \mathscr{N}(0,\sigma_\eta^2)$. This procedure, referred to as the *AR-1 estimation*, involves the function

$$-\mathscr{L}^* \equiv \max_{\Phi,\phi,\sigma_\eta} -\log\left(\sqrt{2\pi\sigma_\eta^2}\right) - \frac{1}{2\sigma_\eta^2 T}\sum_{t=2}^{T} |\varepsilon(t) - \phi\varepsilon(t-1)|^2, \tag{30}$$

and the procedure is applied to re-estimate all the models.

Based on the full sample, we report in Panel A of Table 3 the parameter estimates for the main model. First, note that in most cases, the market-implied parameter values in Table 3 are consistent with their respective independent estimates in Table 2. For example, in the case of σ_g the relative estimates are similar between Tables 2 and 3, in that the σ_g for the technology stocks is significantly higher than for the Dow stocks. However, some differences exist in the κ_g estimate between the two tables. While the κ_g estimate is comparable for the technology stocks, the market-implied κ_g for the Dow stocks is lower than its counterpart in Table 2. Second, the parameter estimates are generally comparable between the SSE estimation and the AR1 estimation methods. For example, according to the market-implied μ_g, the Dow stocks have an average long-run growth of 14.6% under the SSE and 14.7% under the AR1 estimation method.

The most significant inconsistency between Tables 2 and 3 concerns the correlation ρ: while the historical sample averages suggest a positive (or slightly negative) correlation between interest rates and expected EPS growth, the market-implied ρ is significantly negative for all stocks. Another inconsistency lies in the

Table 3
Market-implied parameter estimates

For each stock/index, the structural parameters are estimated either using the full sample (reported under "estimates based on full sample") or the 3-year subsamples (reported under "estimates based on 3-year rolling subsamples"). For the former, the reported parameters are based separately on the SSE and the AR-1 estimation methods (as described in the text), while for the latter only the SSE estimation results are reported. Wherever applicable, the standard error for the parameter is in parentheses. The square root of the minimized sum of squared errors between observed and model P/E ratios is, after being normalized by the number of observations, given under RMSE and the average log-likelihood by $\mathscr{L}^*$. For the 3-year rolling subsamples, the reported coefficient estimates are averages across the respective samples.

SSE and AR1

		S&P 500	GE	XON	INTC	MOT	Dow 30	Tech 20
Panel A: *Estimates based on full sample*								
μ_g	SSE	0.065	0.082	0.057	0.460	0.116	0.146	0.371
	AR1	0.049	0.106	0.038	0.627	0.077	0.147	0.428
κ_g	SSE	0.757	0.144	3.860	1.757	1.976	1.381	2.443
	AR1	0.661	0.187	2.573	2.636	2.197	1.391	2.481
σ_g	SSE	0.328	0.010	0.800	1.315	0.790	0.259	0.327
	AR1	0.399	0.013	0.533	1.901	0.533	0.260	0.371
σ_y	SSE	0.011	0.013	0.849	0.850	0.012	0.274	0.345
	AR1	0.012	0.014	0.566	1.193	0.010	0.282	0.394
ρ	SSE	−0.022	−0.751	−0.930	−0.920	−0.883	−0.409	−0.468
	AR1	−0.010	−0.950	0.010	−0.880	−0.020	−0.381	−0.475
δ	SSE	0.620	0.663	0.661	0.019	0.087	0.452	0.204
	AR1	0.930	0.900	0.794	0.021	0.130	0.609	0.260
λ_y	SSE	0.104	0.069	0.257	1.323	0.118	0.256	0.420
	AR1	0.193	0.114	0.153	1.687	0.008	0.267	0.515
μ_r	SSE	0.120	0.135	0.133	0.101	0.128	0.114	0.102

Table 3 (*continued*)

SSE and AR1

		S&P 500	GE	XON	INTC	MOT	Dow 30	Tech 20
	AR1	0.104	0.103	0.160	0.107	0.110	0.123	0.113
κ_r	SSE	0.029	0.015	0.017	0.500	0.085	0.056	0.153
	AR1	0.026	0.016	0.011	0.750	0.070	0.057	0.202
σ_r	SSE	0.004	0.006	0.002	0.015	0.008	0.004	0.005
	AR1	0.002	0.006	0.005	0.018	0.001	0.005	0.006
RMSE	SSE	1.660	2.444	2.522	7.123	4.816	3.088	6.033
$\mathscr{L}^*$	AR1	1.163	1.660	1.132	3.205	2.412	1.876	3.107
ϕ	AR1	0.750	0.734	0.880	0.830	0.733	0.745	0.746
σ_η	AR1	0.774	1.272	0.751	3.500	2.500	1.542	2.634
SSE only								
Panel B: Estimates based on 3-year rolling subsamples								
μ_g	SSE	0.050	0.063	0.044	0.443	0.109	0.132	0.364
	ARI	(0.001)	(0.001)	(0.001)	(0.020)	(0.005)	(0.015)	(0.085)
κ_g	SSE	0.690	0.168	3.931	1.700	1.865	1.288	2.211
	ARI	(0.035)	(0.003)	(0.165)	(0.081)	(0.088)	(0.209)	(0.339)
σ_g	SSE	0.262	0.009	0.619	1.192	0.729	0.232	0.302
	ARI	(0.010)	(0.001)	(0.004)	(0.048)	(0.034)	(0.050)	(0.077)
	AR1	0.008	0.012	0.686	0.821	0.012	0.261	0.336
σ_y	SSE	(0.001)	(0.001)	(0.020)	(0.038)	(0.001)	(0.052)	(0.075)
	AR1	0.084	−0.148	−0.759	−0.582	−0.448	−0.244	−0.283
ρ	SSE	(0.015)	(0.087)	(0.026)	(0.069)	(0.067)	(0.038)	(0.043)
	AR1	0.660	0.717	0.721	0.016	0.077	0.447	0.192
δ	SSE	(0.013)	(0.012)	(0.007)	(0.001)	(0.003)	(0.042)	(0.025)
	AR1	0.066	0.050	0.174	1.219	0.109	0.232	0.406
λ_y	SSE	(0.006)	(0.002)	(0.006)	(0.049)	(0.022)	(0.052)	(0.101)
	AR1	0.140	0.149	0.128	0.089	0.128	0.119	0.102
μ_r	SSE	(0.003)	(0.005)	(0.005)	(0.003)	(0.005)	(0.005)	(0.005)
	ARI	0.035	0.013	0.014	0.513	0.086	0.057	0.151
κ_r	SSE	(0.000)	(0.001)	(0.001)	(0.002)	(0.004)	(0.007)	(0.033)
	AR1	0.004	0.006	0.002	0.016	0.008	0.004	0.005
σ_r	AR1	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)	(0.001)
RMSE	SSE	1.058	1.514	1.248	5.615	3.046	1.934	4.409
$\mathscr{L}^*$	AR1	(0.131)	(0.149)	(0.172)	(0.920)	(0.292)	(0.133)	(0.428)
ϕ	AR1							
σ_η	AR1							

dividend-payout ratio δ: the historical average payout ratios in Table 2 are generally lower than their market-implied counterparts in Table 3. These differences are indicative of how the market has priced the stocks in the past, that is, the estimates in Table 3 reflects the parameter values under the risk-neutralized probability.

Table 3 also gives the market-implied earnings risk premium, λ_y, and it differs across the stocks. The average λ_y is 0.256 (0.420) for the Dow (technology) stocks. Therefore, the Dow stocks generally have lower risk premium than the technology stocks.

Note that the error-term's serial correlation ϕ is strongly positive for both the Dow stocks and the technology stocks. Despite the positive serial correlation in the P/E pricing errors, the parameters are not overly sensitive to the existence of the AR1 correction term. This implies that the parameters based on the SSE method are relatively robust.

In Panel B of Table 3, we report the parameter estimates based on *rolling 3-year subsamples*. Take Intel as an example. We first apply the least-squares procedure (the SSE method) to the 1976–1978 subperiod (36 monthly observations on Intel and interest rates), to estimate the ten parameters in Φ. Next, we use the 1977–1979 subperiod as input to re-estimate the parameters in Φ. This rolling process continues for each of the 20 years until 1997, to create a time series of estimates for each parameter. As it does not assume the constancy of the parameters over long intervals, the rolling procedure is attractive from an empirical perspective and these subsample-based estimates will be used in our out-of-sample pricing exercise. Panel B reports both the mean and standard error (in parentheses) of each parameter's time series. We do not report any AR1 estimation results for the subsamples as Panel A has established the similarity between the SSE and the AR1 methods.

The 3-year-subsamples based estimates in Panel B of Table 3 are close to their full-sample counterparts in Panel A. This is especially true for κ_g, σ_g, δ and all the interest-rate parameters. Since the risk premia are allowed to vary over time, the subsample λ_y estimates appear slightly lower than their full-sample counterparts. As it is easier to fit the shorter subsamples than the full sample, the average RMSE fitting errors are lowered from 1.66 to 1.058 for the S&P 500 index, and from 3.088 to 1.934 for the Dow stocks. Doing the estimations more frequently improves the in-sample fit of the main model considerably.

In summary, the market-implied parameter values are mostly consistent with their historical counterparts, except for a few parameter estimates. These exceptions are indeed challenging from the perspective of model development. If we accept the assumption that the market prices are correct, then the inconsistencies between Tables 2 and 3 suggest that the model is misspecified (even though to a lesser degree than the Gordon model and other alternatives, as discussed later). But, economically, how significant is our main model's misspecification? This is the question that we want to address next.

5. Empirical pricing performance

While the consistency of market-implied parameter values with their independently estimated counterpart is an important concern for model development, an economic yardstick may be whether a given model provides a "good enough" approximation of the market's implicit valuation process. In this section, we study the in- and out-of-sample pricing performance of our main model and its three special cases. Our task is to address the following points: (i) the relative performance of the models, (ii) the contribution and importance of each modeling feature, (iii) the time-series and cross-sectional properties of model pricing errors, and (iv) sensitivity to the empirical proxies.

A common research issue on stock valuation concerns what constitutes a "good" model. In this regard, we follow Lee et al. (1999) to propose the following perspective. Suppose that at time t both the market and the model try to approximate a stock's unobservable true value process by giving $\overline{S}(t)$ and $S(t)$, respectively. Let $\overline{\varepsilon}(t)$ and $\varepsilon^*(t)$ be their respective approximation errors. The difference in errors is

$$e(t) \equiv \overline{\varepsilon}(t) - \varepsilon^*(t) = \overline{S}(t) - S(t). \tag{31}$$

We can then evaluate the model by examining the pricing-error process $\{e(t)\colon t \geqslant 0\}$, or its market-price-normalized counterpart, $\{\varepsilon(t)\colon t \geqslant 0\}$, where $\varepsilon(t) \equiv (\overline{S}(t) - S(t))/\overline{S}(t)$. For a "good" valuation model, the percentage pricing error $\varepsilon(t)$ should first have a zero mean and a low standard deviation over time. Even if the mean of $\varepsilon(t)$ is non-zero, the valuation model will still be empirically acceptable, so long as $\varepsilon(t)$ has little or no variation over time. If the mean and standard deviation of $\varepsilon(t)$ are both zero, the model captures the market valuation mechanism perfectly (though the market could still be wrong in approximating the true value). On the other hand, if $\varepsilon(t)$ is large in magnitude, either the model or the market is wrong. For our discussions to follow, we assume that the market fairly prices stocks and we use the statistics of $\varepsilon(t)$ to draw inferences about model misspecifications.

5.1. In-sample pricing

We start with in-sample pricing by the four cases: (i) our main model, (ii) the SG model (allowing for stochastic $G(t)$ but a constant yield curve), (iii) the SI model (accounting for a stochastic yield curve but constant expected EPS growth), and (iv) the extended Gordon model. For the in-sample pricing error in each month, the parameter values estimated from the stock's entire sample period are used as input to compute the model price of the stock. That is, in the case of formula (8), the Φ estimate is applied to price the stock in the same time period as the parameter-estimation sample. This calculation is separately done for both the SSE and the AR1 estimation methods.

In Table 4, we present three percentage pricing-error measures, computed by dividing the market-to-model price difference by the market price: (i) the absolute percentage pricing error, denoted by *APE*, (ii) the mean percentage pricing error, denoted by *MPE*, and (iii) the standard deviation of the pricing-error time series, denoted by *STD*. The MPE reflects the *average* pricing performance, while the APE reflects the magnitude of the pricing errors as negative and positive errors do not cancel out each other.[2]

[2]Throughout this subsection, the interest-rate parameters are separately estimated and applied for each stock. In theory, the same set of interest-rate parameters should apply to every stock in the economy. For this reason, we have applied the interest-rate parameters estimated from the S&P 500 data to all other stocks/indices and found the pricing performance to be only marginally worse. We thus choose to report the pricing results based on separately estimated interest-rate parameters.

First, let us examine the performance by our main model based on the SSE parameter estimates. According to the pricing-error measures, the model's fit is reasonable across the stocks/indices. The average MPE and average STD are respectively −4.14% and 20.37% for the Dow stocks, and −5.71% and 27.90% for the technology stocks. Given the negative sign of the MPE, the model price is on average higher than the market price. The APE ranges from the S&P 500's 7.78%, to 16.53% for the Dow stocks, and 22.61% for the technology stocks. All three measures indicate worse performance in pricing the technology stocks than the Dow stocks.

Second, note from Table 4 that the pricing performance is slightly worse when the main model is implemented using the AR1 estimation than the SSE method. For the S&P 500, the APE is 8.23% under the AR1 estimation and 7.78% under the SSE estimation. Similarly, the APEs are respectively 17.85% and 16.53% for the Dow stocks, and 22.84% and 22.61% for the technology stocks. While it is theoretically desirable to account for the serial correlation of the pricing errors in parameter estimation, the pricing performance is slightly worse. For this reason, our remaining discussions will rely on the results based on the SSE estimation method.

Compared to the main model, the SG, the SI and the extended Gordon models each provide a much poorer fit. In implementing each of these models, we allow both $R(t)$ and $G(t)$ to take their respective currently observed values (even when one or both of them are assumed to be constant in a given model), but restrict each model's parameters according to its assumptions. For example, in the case of the Gordon model, the month-t values used for r and g are still the 30-year yield and the firm's expected EPS growth as of month t (thus, they both vary over time), but there are only two parameters to be estimated (see Eq. (14)): δ and λ_y. Consequently, this *ad hoc* treatment should give each special case a favorable bias.

It should be mentioned that several constraints are imposed on the estimation procedure of each model. First, we require $0 \leqslant \delta \leqslant 99.0\%$, ensuring that the firm's dividend does not exceed its earnings. Second, in each specific case, the corresponding transversality condition usually puts severe restrictions on its feasible parameter values. For example, in the extended Gordon model, λ_y has to satisfy $\lambda_y > \max\{G(t) - R(t): t = 1, \ldots, T\}$, so that the denominator in (14) would not be zero or negative. For technology stocks such as Intel, this restriction means that λ_y has to take extremely high values (because of the frequently high expected EPS growth rates). Consequently, the Gordon model prices for such stocks are usually extremely low, resulting in extremely high pricing errors for the model. To save space, we do not report the parameter estimates for these special-case models. However, we should point out that for all these special cases and for each stock/index, the estimated δ always hits the upper bound at 99.0%. That is, for these models to achieve their pricing performance shown in Table 4, the average dividend-payout ratio has to be as high as 99.0%, which is at odds with the dividend-payout ratios given in Tables 2 and 3. Furthermore, under the extended Gordon model, for example, the estimate for λ_y is 500% for Intel. Therefore, the SG, the SI and the Gordon models are significantly more misspecified than our main model.

Table 4
In-sample percentage pricing errors of each model

For each stock/index, its full sample is used to estimate the structural parameters. For the SSE method, all parameters for each model are searched together to minimize the sum of squared differences between the stock's observed and model P/E ratios. In the AR-1 estimation method, we correct for the serial correlation of the model errors (see the procedure described in the text). Using the respective parameter values so estimated as input, we compute for each stock and for every month the model price (i.e., $\delta \int_0^\infty s(t,\tau; G(t), R(t), Y(t), \Phi)\, d\tau$). Our Main model and its three special cases are compared: the stochastic-expected-earnings-growth model with constant interest rates (the SG), the stochastic-interest-rates model with constant expected earnings growth (the SI), and the extended Gordon model. For each model and every stock, we generate three pricing-error measures (i) the average absolute percentage pricing error (denoted APE), (ii) the standard deviation of the percentage pricing errors (denoted STD), and (iii) the average percentage pricing error (denoted MPE). The sample period is 1976 through July 1998.

		S&P 500	GE	XON	INTC	MOT	Dow 30	Tech 20
Based on SSE estimation								
Main model	APE	7.78%	13.51%	19.97%	29.63%	22.03%	16.53%	22.61%
	STD	9.78%	16.76%	25.18%	33.93%	28.55%	20.37%	27.90%
	MPE	−0.86%	0.54%	−5.48%	−13.39%	−7.57%	−4.14%	−5.71%
SG model	APE	31.32	20.77	24.45	80.67	67.41	43.82	60.64
	STD	18.92	15.76	27.53	16.11	10.09	17.35	23.01
	MPE	29.32	18.42	−0.80	80.32	67.27	40.70	57.91
SI model	APE	50.76	27.55	43.92	87.51	89.01	69.42	84.67
	STD	30.24	35.41	60.38	16.33	8.58	20.79	14.73
	MPE	44.82	12.49	26.10	87.42	89.00	65.60	83.86
Gordon model	APE	60.40	31.18	44.95	98.07	97.64	76.32	92.09
	STD	19.99	30.94	52.92	6.00	8.38	17.97	13.51
	MPE	59.00	23.83	31.53	98.07	97.59	73.96	91.39
Based on AR-1 estimation								
Main model	APE	8.23%	12.30%	26.02%	32.23%	24.45%	17.85%	22.84%
	STD	10.56%	15.14%	29.88%	37.63%	31.27%	20.76%	27.95%
	MPE	−2.06%	2.84%	−10.47%	−11.22%	−10.63%	−3.85%	−6.53%
SG Model	APE	29.05	27.35	26.97	76.22	48.52	40.15	55.17
	STD	15.51	12.85	30.53	9.08	13.10	14.93	17.90
	MPE	28.39	26.98	−11.69	76.22	48.52	36.75	53.47
SI model	APE	74.40	67.76	57.25	84.76	85.32	84.86	72.91
	STD	16.43	10.85	44.32	20.61	9.52	12.27	22.93
	MPE	73.82	67.54	50.27	82.84	85.31	84.12	69.03
Gordon model	APE	73.42	81.31	77.62	99.96	98.88	87.64	95.02
	STD	7.86	4.86	9.66	0.02	4.81	5.09	4.58
	MPE	73.42	81.31	77.62	99.96	98.88	87.64	95.02

According to the percentage pricing errors in Table 4, our main model is ranked the best, followed by the SG, the SI, and the Gordon model. This relative performance ranking holds regardless of the stock/index being priced and irrespective of the sample period (based on the full sample or the unreported subsamples). The fact that the SI model performs better than the Gordon model means that allowing for stochastic interest rates improves the discounting of future cashflow by the model. The percentage improvement in the APE is the highest for the S&P 500. For example, in comparison to the Gordon model, the SI model reduces the pricing errors of the S&P 500 to 50.76%. Thus, going from the Gordon to the SI model, the pricing fit improves for every stock, especially for the S&P 500.

From the Gordon to the SG, the in-sample pricing-error reduction is even more dramatic. Therefore, modeling the expected EPS growth, $G(t)$, properly is crucial for stock valuation. Recall that in implementing Gordon and SI models the g parameter is allowed to assume the current expected EPS growth value (and is hence time-varying). But even given such a favorable treatment for these two models, the SG model can still improve upon them significantly, implying that allowing the growth rate to vary over time is not enough and, more importantly, one should parameterize the firm's growth process so that growth-cycle aspects of the firm are separately captured.

While modeling stochastic expected EPS growth is of the first importance, adding the stochastic-interest-rate feature leads to further improvement. This conclusion can be drawn from the lower pricing errors achieved by the main model relative to the SG model. For example, the APE for the Dow stocks is 16.53% by the main model versus 43.82% by the SG model. Similarly, the APE and MPE for the S&P 500 are respectively 31.32% and 29.32% by the SG model, but reduce to 7.78% and −0.86% by the main model. In summary, the proper parameterization of both the discounting structure and the earnings process is key to the improved performance by the main model. However, even under the main model, the magnitude of the APE, the MPE and the pricing-error standard deviation is still quite high, especially for the technology stocks. This means that further improvement is warranted.

It is worth noting that under the extended Gordon model, both the APE and MPE are 98.07% for Intel. As noted in the preceding section, this high level of pricing error is mostly due to the transversality condition that λ_y must be high enough to ensure $r + \lambda_y - g > 0$. Therefore, it is the model's internal parameterization that causes the high pricing errors.

Finally, we can examine relative performance from the following regression specification:

$$\overline{P}(t) = \beta_0 + \beta_1 P(t) + \tilde{\omega}(t), \tag{32}$$

where $\overline{P}(t)$ is the market P/E in month t and $P(t)$ the counterpart determined by the model under consideration. If a given model perfectly fits a stock's P/E variations over time, we should obtain $\beta_1 = 1$, with a regression R^2 of 100%. The regression results are presented in Table 5 for each pricing model. According to the R^2, the SI model is not uniformly better than the extended Gordon model. Judged on both the β_1 and R^2 values, the main model performs by far the best: it can account for 83.35%

of the S&P 500's P/E fluctuations and 59.53% of Intel's P/E variations. For the main model, all the β_1 estimates are close to one. The SG model's fit is significantly better than both the Gordon and the SI model, re-affirming our conclusion that allowing the expected EPS growth to be stochastic is the most important for equity valuation. The relative pricing results in Table 5 demonstrate that having more parameters in a stock valuation model does not guarantee better performance: it is the structural fit provided by the model that is more crucial.

5.2. Out-of-sample pricing

For out-of-sample pricing, suppose that we intend to price the stock for each month of year t. First, we take as input the parameter values estimated from the 3 years prior to year t, and apply them to each month of year t. Next, we substitute these estimates and the current-month values of $G(t)$, $R(t)$ and $Y(t)$ into the formula to determine the current-month model price. Then, we proceed to year $t+1$ to get a new trailing 3-year subsample, and apply the re-estimated parameters to each month of year $t+1$. This procedure continues until 1998. Since the initial 3 years of data (from January 1976:01 to December 1978) is required to determine the first set of out-of-sample pricing errors, all out-of-sample results are based on the January

Table 5
Assessing the quantitative fit of each model (based on the SSE estimation)

All reported results are based on the following regression ($\tilde{\omega}_n(t)$ is regression disturbance):

$$\overline{P}(t) = \beta_0 + \beta_1 P(t) + \tilde{\omega}(t),$$

where $\overline{P}(t)$ is the market P/E ratio observed in month t and $P(t)$ the model counterpart. Each model's P/E is based on parameters estimated in-sample and using the full sample. Reported in square brackets are the p-values when the standard errors are computed using the Newey-West estimator. For the Dow 30 and Tech 20, the standard deviation is reported in curly brackets. The intercept coefficients were not significantly different from zero and omitted to save on space.

	S&P 500		GE		XON		INTC		MOT		Dow 30		Tech 20	
	β_1	R^2	β_1	R^2	β_1	R^2	β_1	R^2	β_1	R^2	β_1	R^2	β_1	R^2
Main model	1.00	83.35	0.91	75.76	1.00	72.40	0.86	59.53	1.02	79.59	0.98	68.74	1.09	58.18
	[0.00]		[0.00]		[0.00]		[0.00]		[0.00]		{0.04}	{14.59}	{0.43}	{20.95}
SG model	0.64	39.38	1.18	64.83	1.83	70.77	1.18	35.67	0.92	65.17	0.87	49.60	0.64	40.33
	[0.00]		[0.00]		[0.00]		[0.00]		[0.00]		{0.34}	{24.04}	{0.33}	{28.25}
SI model	0.20	5.61	0.45	14.19	0.05	0.01	0.77	8.63	0.83	43.94	0.46	19.74	0.53	22.37
	[0.00]		[0.00]		[0.32]		[0.00]		[0.00]		{0.36}	{23.73}	{0.36}	{22.46}
Gordon model	0.31	8.44	0.53	18.25	0.16	0.01	0.81	9.30	0.82	43.94	0.50	19.86	0.55	20.51
	[0.00]		[0.00]		[0.12]		[0.00]		[0.00]		{0.35}	{22.62}	{0.38}	{19.55}

1979:01 to July 1998 sample period. Thus, there are at most 235 observations of pricing errors for any stock. Table 6 consolidates the out-of-sample results for percentage pricing error series for all the models. In the discussion below, we concentrate on the main model for two reasons. First, the relative performance ranking among the main model and its special cases remains unchanged, whether it is based on in- or out-of-sample pricing results. Second, it allows us to conduct a detailed examination and gauge their sensitivity to empirical proxies.

Consider first the statistics for out-of-sample percentage pricing errors: the average absolute value (APE), the mean (MPE), and the standard deviation (STD). Typically, the out-of-sample errors are slightly worse than their in-sample counterpart in Table 4. For example, the S&P 500's APE is now 8.17%, compared to its in-sample counterparts of 7.78%. The average APEs across the Dow stocks and the technology stocks are respectively 17.00% and 22.91% out-of-sample, versus their in-sample counterparts of 16.53% and 22.61%. Given that the APEs are generally higher for the technology stocks, our model is considerably more misspecified when applied to growth stocks.

Based on the MPE statistic, the main model underprices the S&P 500, while it overprices the Dow stocks and the technology stocks on average. The average MPE is -1.15% for the Dow stocks, and -2.58% for the technology stocks. Despite the low MPEs, the pricing-error standard deviations are large: the average STD is 21.38% across the Dow stocks and 28.26% across the technology stocks. Therefore, there is substantial variation, both over time and across stocks, in the ability of the model to track stock prices. In general, there is a high correlation between the in-sample and out-of-sample error statistics.

Although not reported we also computed average absolute dollar pricing error and the mean dollar pricing error. The average absolute dollar pricing error is \$4.75 for the Dow stocks versus \$3.89 for the technology stocks. The average absolute dollar pricing error and mean dollar pricing error are respectively \$41.99 and \$22.78 for the S&P 500. We should note, however, that most firms' EPS and their stock prices tend to go up over time, so that even if a firm's percentage pricing error stays within a range over time, the dollar pricing errors for more recent months can dominate earlier errors. Since more expensive stocks are likely to have higher dollar errors, the dollar pricing errors are not directly comparable across stocks.

We can also analyze the model mispricing patterns by combining Table 6 with Fig. 1, where we plot separately for the S&P 500 and Intel: (i) the actual versus the out-of-sample model price path, and (ii) the percentage pricing-error path. For the most part, the main model tracks the actual S&P 500 index reasonably well, except for a few short periods. For instance, in the case of the S&P 500 the pricing error was more than 10% several times (in early 1985, early through October 1987, early 1991, September 1996 through December 1997, and most recently in 1998). These past periods were each time followed by a downward movement in the S&P 500. As of July 1998, the model pricing error was 17.06%. On the other hand, there have also been periods where the market price was lower than the model price. This is true for much of 1986 and 1988, from mid-1993 to mid-1994, and lastly in early 1996. As the graph demonstrates, these periods were followed by a substantial rise in

Table 6
Out-of-sample pricing performance using rolling parameter estimates

For each stock/index and at the beginning of each year, the most recent 3-year subsample is used to estimate the stock's structural parameters by minimizing the sum of squared differences between the stock's observed and model P/E ratios; we then apply these parameter estimates to the model to value the stock for each month of the year, which results in a model-price series of the stock for the year. Next, we move to the following year and use the new recent 3-year subsample to estimate the parameters, which are then applied to price the stock for each month of the following year. This rolling process continues until December 1997. Two out-of-sample pricing-error series are generated for each stock: (i) dollar pricing errors (market stock price minus model price), and (i) percentage pricing errors (the dollar pricing error normalized by the market stock price). The average percentage pricing error (denoted MPE), the average absolute percentage pricing error (denoted APE), the average dollar pricing error (denoted MDE), are reported for each stock. The standard deviation of the percentage pricing-error time series is denoted as STD.

		Percentage pricing errors						
		S&P 500	GE	XON	INTC	MOT	Dow 30	Tech 20
Main model	APE	8.17	11.65	11.41	23.87	21.47	17.00	22.91
	STD	9.80	14.43	14.01	29.16	26.57	21.38	28.26
	MPE	3.40	3.21	4.49	−0.97	−2.80	−1.15	−2.58
SG model	APE	33.46	18.54	26.23	50.13	51.28	41.13	47.22
	STD	19.13	18.22	19.37	38.78	35.81	24.34	31.11
	MPE	27.62	11.31	−10.98	40.25	34.47	30.37	44.35
SI model	APE	48.11	30.85	47.34	56.44	54.09	49.43	59.68
	STD	29.77	36.22	48.33	39.55	38.07	26.12	29.76
	MPE	39.39	17.38	18.48	57.26	44.50	36.35	50.85
Gordon model	APE	57.37	33.59	46.17	70.07	77.36	59.68	65.33
	STD	28.64	34.11	53.32	16.00	18.64	18.34	19.65
	MPE	44.33	20.19	29.93	68.07	69.39	47.94	60.15

the S&P 500, giving a mean-reversion character to the out-of-sample percentage pricing errors.

In the case of Intel, the model's pricing-error path is highly volatile and it has three major periods. From January 1979 through October 1987, the pricing errors were mostly positive, with an average error of 21.7%. Early in this period, Intel underwent major product transitions with its EPS ranging between \$0.03 and \$0.20, while its stock was trading between \$2.00 and \$4.00. This made Intel's P/E ratio both high and volatile, leading to large model fitting errors. For this period the pricing-error standard deviation was 30.23%. For the next major period from November 1988 to April 1995, the model pricing errors were mostly negative, with an average of −20.1%. In fact, this period was associated with the two most severe pricing errors, of −110.7% and −93.9%, respectively in November and December 1988. In

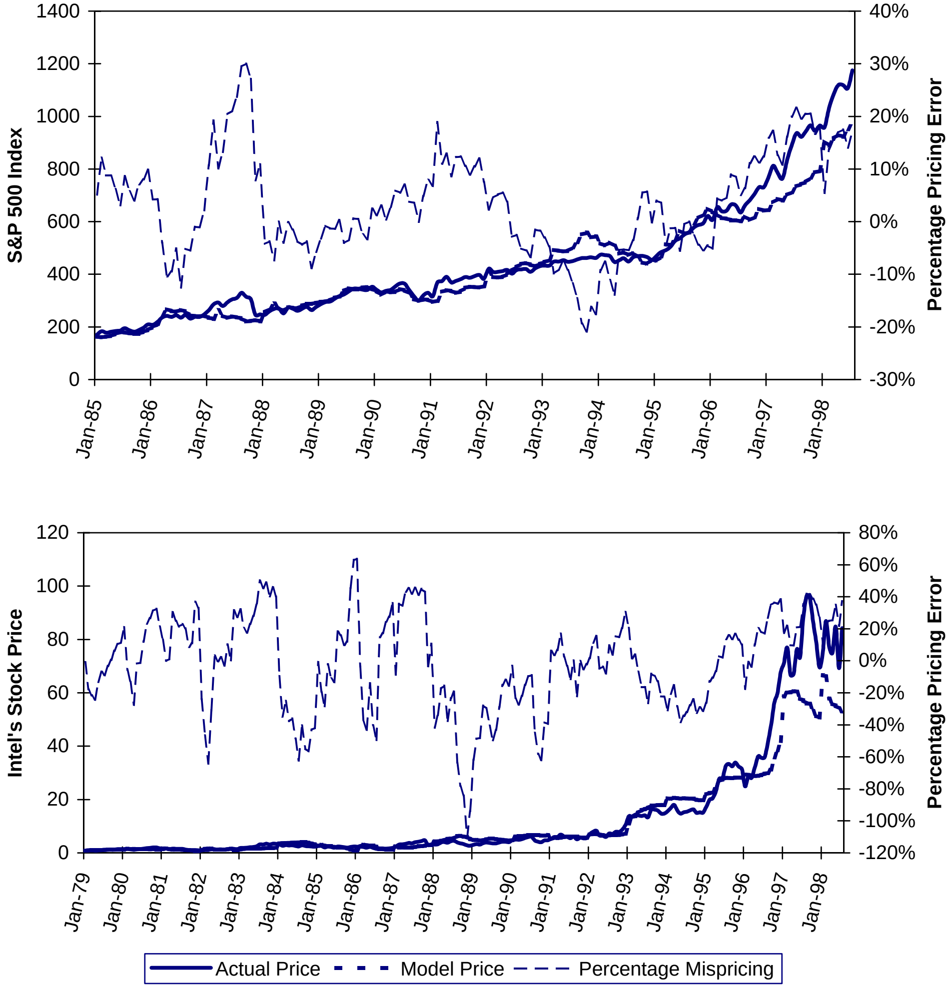

Fig. 1. Out-of-sample percentage pricing errors.

November 1988, the market price of Intel was \$2.84, whereas the model price was \$5.98. Note that the market price was relatively depressed, even though Intel's EPS increased from \$0.23 to \$1.98. Finally, during the remaining years in the sample after May 1995, the pricing errors were mostly positive again with an average of 20.6%. During this period, Intel's EPS was more than doubled from \$2.09 to \$4.47, while its stock price appreciated from \$14.7 to a high of \$96.38.

The fact that the model pricing errors would go through periods of high and low levels (especially for stocks like Intel) reinforces our earlier conclusion that for high-tech firms, the single-factor earnings dynamics in (5) and (6) may be too simple and not rich enough. To reduce the extent of such systematically recurring pricing errors, one may include other firm-specific growth factors that are important for the market's valuation.

To further study the model's properties, we can look at both the autocorrelation and cross-stock correlations of the out-of-sample pricing errors. Table 7 shows the autocorrelations at lags of up to 36 months. At the 1-month lag, the autocorrelations range from 0.73 to 0.94 for the Dow stocks, and from 0.50 to 0.90 for the technology firms. At the 12-month lag, they go down to about 0.20. As the lag increases to 24 months and 36 months, more firms in our sample start to have negative autocorrelations. The Ljung-Box Q-statistic also suggests that the null hypothesis for the absence of autocorrelation is overwhelmingly rejected (see $Q(24)$ and the corresponding p-values for the χ^2 test). Thus, the pricing errors are highly persistent within several months but mean-revert at longer horizons, particularly for the blue-chip Dow stocks. For the technology stocks, the persistence level is slightly lower as these stocks are generally more volatile.

Table 8 presents the levels of covariation in pricing errors across stocks. Specifically, we run the time-series regression below:

$$\varepsilon_n(t) = a_n + b_n \varepsilon^*(t) + \tilde{\omega}_n(t), \quad n = 1, \ldots, N, \tag{33}$$

where $\varepsilon_n(t)$ is the percentage pricing error of stock n in month t and $\varepsilon^*(t)$ is the percentage pricing error of either the S&P 500 or Motorola (arbitrarily selected for comparison). We report the b coefficient, its associated p-value and adjusted R^2. For

Table 7
Pricing-error autocorrelations for individual stocks

Reported below are the autocorrelations of (out-of-sample) percentage pricing errors. For S&P 500, the autocorrelation at lag 1, 12, 24, and 36 are respectively 0.90, 0.05, −0.08, and −0.33. For the Dow stocks, the respective average autocorrelations are: 0.85, 0.11, −0.02, and −0.13. Similarly for the 20 technology stocks, we have the corresponding average autocorrelations at 0.83, 0.10,−0.03, −0.10. We also report the Box-Pierce Q statistic at 24 lags and the corresponding p-value for the χ^2 test. Under the null hypothesis of no serial correlation, $Q(24)$ is χ^2-distributed with 24 degrees of freedom.

No.	Stock	Autocorrelations					
		Lag 1	Lag 12	Lag 24	Lag 36	$Q(24)$	p-Val
1	TRV	0.77	0.13	−0.00	−0.05	235.97	0.00
2	GE	0.78	0.22	0.04	0.04	477.13	0.00
3	XON	0.89	−0.13	0.06	0.03	649.24	0.00
4	CHV	0.86	−0.24	0.19	−0.21	659.27	0.00
5	CAT	0.87	0.13	0.17	0.04	744.12	0.00
6	KO	0.91	0.18	−0.09	−0.29	1006.57	0.00
7	DIS	0.85	0.26	0.10	0.08	735.67	0.00
8	DD	0.87	−0.12	−0.04	−0.05	692.92	0.00
9	EK	0.90	0.19	0.05	−0.08	950.72	0.00
10	GM	0.82	0.34	0.08	−0.19	865.25	0.00
11	GT	0.83	0.37	0.07	−0.14	1003.73	0.00
12	IBM	0.86	0.02	−0.30	−0.09	950.86	0.00
13	MCD	0.90	0.19	−0.07	−0.04	818.92	0.00
14	MRK	0.85	0.17	−0.30	−0.22	952.78	0.00
15	UK	0.87	−0.05	−0.02	−0.06	572.62	0.00

Table 7 (*continued*)

No.	Stock	Autocorrelations					
		Lag 1	Lag 12	Lag 24	Lag 36	$Q(24)$	p-Val
16	AXP	0.88	0.29	0.14	−0.05	1100.24	0.00
17	HWP	0.68	−0.02	−0.09	−0.22	400.34	0.00
18	IP	0.85	−0.13	0.16	−0.15	620.79	0.00
19	JNJ	0.89	0.19	−0.25	−0.32	1009.55	0.00
20	JPM	0.81	0.07	−0.13	−0.29	375.46	0.00
21	MO	0.89	0.17	−0.15	−0.17	779.67	0.00
22	S	0.91	0.25	−0.02	−0.32	974.02	0.00
23	UTX	0.85	0.10	0.15	0.06	629.43	0.00
24	WMT	0.92	0.37	−0.13	−0.32	1267.76	0.00
25	ALD	0.89	−0.19	−0.03	−0.07	606.59	0.00
26	AA	0.83	0.15	0.10	−0.12	680.11	0.00
27	MMM	0.88	0.10	−0.06	−0.03	786.31	0.00
28	PG	0.88	0.09	0.07	−0.10	630.59	0.00
29	BA	0.92	0.37	−0.18	−0.34	1161.37	0.00
30	T	0.91	0.28	−0.27	−0.47	1012.09	0.00
31	ADBE	0.76	0.10	0.00	0.06	170.93	0.00
32	ALTR	0.71	−0.32	0.28	−0.12	146.26	0.00
33	AMAT	0.88	0.20	−0.20	−0.19	710.76	0.00
34	CMPQ	0.82	0.04	−0.13	−0.05	403.99	0.00
35	COMS	0.84	0.09	−0.13	0.03	436.59	0.00
36	CSC	0.83	−0.08	0.14	0.04	441.00	0.00
37	CSCO	0.87	−0.06	0.08	−0.10	492.59	0.00
38	DELL	0.83	0.16	−0.01	−0.23	532.24	0.00
39	INTC	0.85	0.03	0.03	−0.20	553.95	0.00
40	KEAN	0.86	0.04	−0.05	−0.11	636.20	0.00
41	MOT	0.79	0.22	0.17	−0.08	752.60	0.00
42	MSFT	0.90	0.22	−0.16	−0.29	869.66	0.00
43	NNCX	0.88	0.27	−0.21	−0.17	960.47	0.00
44	NT	0.88	0.09	−0.11	−0.10	812.64	0.00
45	ORCL	0.84	−0.04	−0.04	−0.04	466.57	0.00
46	QNTM	0.85	0.27	−0.05	0.02	790.62	0.00
47	STK	0.76	0.15	−0.09	−0.12	533.23	0.00
48	SUNW	0.76	0.18	0.07	0.09	464.35	0.00
49	TXN	0.85	0.13	−0.16	−0.10	775.04	0.00
50	WDC	0.86	0.14	0.01	0.11	620.46	0.00

most of the stocks, the b estimates are significant, with R^2 values as high as 80.41% (when the S&P 500's pricing error is the explanatory variable) and 43.79% (when MOT is the independent variable). Based on this 50 stock sample, the pricing errors appear to be significantly positively correlated across stocks. For instance, with the S&P 500, the average slope coefficient, b, is 0.997 across the Dow stocks and 1.004 across the technology stocks. Therefore, a 1% model pricing error in the S&P 500 from one month to the next causes the model pricing errors for an average stock to rise or fall in the same proportion.

Table 8
Common variations in mispricing of individual stocks

We examine the link between model pricing errors of stocks by running the following regression: $\varepsilon_n(t) = a + b\,\varepsilon^*(t) + \tilde{\omega}_n(t)$, where $\varepsilon^*(t)$ is taken to be either the time-t (out-of-sample) percentage pricing error of the S&P 500 index or that of Motorola (MOT); and $\varepsilon_i(t)$ denotes, for $n = 1, \ldots, 50$, the time-t percentage pricing error for a given stock. Reported are the slope coefficient, the p-value (based on Newey-West estimator), and the regression R^2.

No.	With S&P 500				With MOT		
	Stock	b	p-Val. for b	R^2 (%)	b	p-Val. for b	R^2 (%)
1	TRV	0.82	0.00	21.11	0.33	0.00	9.55
2	GE	0.82	0.00	32.22	0.17	0.00	11.99
3	XON	0.96	0.00	38.74	0.04	0.10	0.71
4	CHV	0.74	0.00	32.44	0.03	0.26	0.34
5	CAT	1.51	0.00	24.06	0.71	0.00	39.43
6	KO	1.82	0.00	54.79	0.19	0.00	5.89
7	DIS	0.89	0.00	26.65	0.37	0.00	27.58
8	DD	1.15	0.00	45.84	0.28	0.00	25.73
9	EK	0.10	0.22	0.37	0.27	0.00	18.34
10	GM	1.61	0.00	27.62	0.12	0.05	1.48
11	GT	1.86	0.00	47.66	0.13	0.02	2.11
12	IBM	1.44	0.00	19.99	0.30	0.00	11.64
13	MCD	0.37	0.00	11.23	0.27	0.00	21.29
14	MRK	1.54	0.00	45.29	0.24	0.00	11.28
15	UK	1.62	0.00	24.35	0.44	0.00	17.86
16	AXP	0.86	0.00	17.86	0.42	0.00	27.27
17	HWP	1.05	0.00	21.61	0.41	0.00	26.45
18	IP	0.47	0.03	2.13	0.40	0.00	16.90
19	JNJ	1.42	0.00	53.47	0.19	0.00	8.53
20	JPM	1.17	0.00	14.63	−0.19	0.00	4.63
21	MO	1.01	0.00	33.37	0.13	0.00	5.20
22	S	0.48	0.00	5.84	0.49	0.00	39.38
23	UTX	0.45	0.00	6.22	0.35	0.00	32.74
24	WMT	1.54	0.00	37.17	0.30	0.00	10.95
25	ALD	1.53	0.00	43.99	0.25	0.00	10.96
26	AA	0.96	0.00	9.82	0.70	0.00	40.18
27	MMM	0.93	0.00	65.68	0.21	0.00	22.08
28	PG	1.27	0.00	63.81	0.17	0.00	10.98
29	BA	−0.46	0.04	1.97	0.33	0.00	9.09
30	T	0.00	0.50	0.00	0.15	0.00	6.95
31	ADBE	0.19	0.24	0.56	0.23	0.08	2.17
32	ALTR	0.47	0.05	3.45	0.38	0.01	6.59
33	AMAT	0.56	0.05	1.99	1.06	0.00	43.79
34	CMPQ	0.92	0.00	14.22	0.44	0.00	23.16
35	COMS	−0.34	0.09	1.45	0.22	0.01	4.28
36	CSC	0.76	0.00	10.31	0.28	0.00	12.82
37	CSCO	1.26	0.00	26.19	0.04	0.41	0.09
38	DELL	3.71	0.00	80.41	0.98	0.00	18.77
39	INTC	1.80	0.00	38.51	0.71	0.00	41.37
40	KEAN	1.81	0.00	49.41	0.36	0.03	5.37
41	MOT	0.89	0.00	12.30	NA	NA	NA
42	MSFT	2.00	0.00	64.71	−0.01	0.47	0.01
43	NNCX	−0.03	0.47	0.01	0.89	0.00	22.67

Table 8 (*continued*)

No.	With S&P 500				With MOT		
	Stock	b	p-Val. for b	R^2 (%)	b	p-Val. for b	R^2 (%)
44	NT	1.22	0.00	20.81	0.55	0.00	34.04
45	ORCL	−0.52	0.01	5.76	0.56	0.00	19.64
46	QNTM	1.62	0.00	22.01	0.11	0.19	0.66
47	STK	0.57	0.02	6.13	0.66	0.00	27.52
48	SUNW	0.75	0.00	12.17	0.02	0.42	0.04
49	TXN	1.39	0.00	20.95	0.77	0.00	48.81
50	WDC	1.06	0.00	10.10	0.46	0.00	9.13

These results on pricing-error persistence and correlations indicate that there must be market-wide factors missing from the model. First, although the model is separately estimated for each stock, the pricing errors are contemporaneously correlated across stocks, implying the existence of some "systematic state variable" that is important in the market's valuation but missing from our model. Second, since the out-of-sample model price for a stock is based on the parameters estimated from the stock's recent 3-year history, the model valuation represents "where the stock should be traded today if the market would price the stock in the same way as in the past 3 years," that is, the model price captures the market's recent valuation standard for the stock. When the pricing errors persist, it means that the model is persistently behind and cannot "catch up" with the changing market. Again, there may be state variables (possibly firm-specific) that are absent in our model.[3]

In light of the above discussion, we can examine whether our model pricing errors are related to four known systematic factors: (1) the default spread between Baa-rated and Aaa-rated corporate bonds (DEF), (2) the term spread between the 10-year Treasury yield and the 1-month T-bill rate (SLOPE), (3) the Fama and French (1996) size premium (SMB), and (4) the Fama-French value premium (HML). For each stock/index, we perform the following regression:

$$\begin{aligned}\varepsilon(t) = c_0 + c_1\,\mathrm{HML}(t-1) + c_2\,\mathrm{SMB}(t-1) + c_3\,\mathrm{DEF}(t-1)\\ + c_4\,\mathrm{SLOPE}(t-1) + \tilde{\omega}(t),\end{aligned} \tag{34}$$

where $\varepsilon(t)$ represents the month-t percentage pricing error of a stock. Since the market and model prices are mid-month values, we use the prior-month value of each factor to avoid any look-ahead bias. Several conclusions emerge for our sample of stocks. First, the average adjusted-R^2 for the technology stocks is 15.4% versus

[3]To reduce the level of persistence, the parameter estimation procedure can further correct for autocorrelations in the pricing errors, as is done in our AR1 estimation method. But, as we learned in the previous section, such a statistical correction may produce worse pricing performance. Thus, a more desirable alternative may be to search for better model specifications (rather than more statistical corrections).

8.6% for the Dow stocks, implying that the pricing errors of the technology stocks are more predictable using these factors. Second, the pricing errors are unrelated to the Fama-French HML factor: the coefficient on the HML is statistically insignificant for 50 out of the 51 stocks (based on the Newey-West estimator). Third, the size premium SMB has explanatory power for the pricing errors of the technology stocks, but not for the Dow stocks. In particular, the SMB coefficient is positive and significant for a majority of the technology stocks. That is, periods of high size premium coincide with high pricing errors, and vice versa. Third, the interest-rate factors are potentially important. For example, the DEF coefficient is significant for 30 stocks, whereas the SLOPE coefficient is so for 21 stocks. This exercise supports our earlier assertion that a multi-factor term structure model or a pricing kernel with multiple risk factors may be required to reduce model misspecification.

5.3. Robustness and sensitivity to empirical proxies

To investigate the robustness and sensitivity of our results to empirical proxies for $G(t)$ and $R(t)$, we use the 30 Dow stocks as the focus, with the understanding that similar conclusions hold for the other stocks. Furthermore, we confine the discussion to out-of-sample pricing errors.

First, in Panel A of Table 9, we divide the full sample into the January 1979 to December 1989 and the January 1990 to July 1998 subsamples. An important point to note is that the performance of the main model is roughly consistent across the two subsamples. For instance, the average out-of-sample APE and STD for the Dow stocks are only slightly worse in the second than in the first subsample. The difference in the average MPE is more significant: the average MPE is −2.39% in the 1990's versus −0.28% in the 1980s. These numbers are also comparable to their counterpart in Table 6.

Up until now, we have employed the analyst-expected EPS growth from FY1 to FY2 as a proxy for $G(t)$. To investigate an alternative proxy, we analytically substitute the $G(t)$ expression from (27) into the valuation model (8)–(9), where $G(t,\tau)$ is taken to be the analyst-expected 1-year forward EPS growth and $\tau = 1$. Following the same procedure as in (29), we re-estimate the parameters on a rolling basis by fitting the model P/E to the market P/E. In each estimation, we internally recover $G(t)$ from the expected 1-year-ahead EPS growth as described in (27). The resulting out-of-sample pricing errors are reported in Panel B of Table 9. They show that this substitution technique does not mitigate the model's misspecifications and it actually makes the pricing errors larger. For example, comparing Tables 6 and 9, we see that for the full sample the average APE for the Dow stocks is now 18.67% versus the previous 17%, and the average STD is now 22.70% versus the previous 21.38%. The average MPE experiences the largest increase in magnitude. Therefore, while our conclusions based on the APE and STD are largely intact, this alternative choice for $G(t)$ makes the overall pricing fit worse.

Next, we assess the sensitivity of out-of-sample pricing errors to the proxy choice of $R(t)$. In this experiment, we re-estimate the parameters as before, except that we

Table 9
Robustness analysis for the main model

In the robustness checks performed below, only the main model is considered and all reported results are for the 30 stocks in the Dow. First, reported under the heading "sub-sample results" are out-of-sample percentage and dollar pricing-error measures for two subsamples (i) 1979:01 through 1989:12 and (ii) 1990:01 through 1998:07. Full refers to the entire 1979:01-1998:07 sample period. In the second robustness exercise, we substitute the $G(t)$ expression from Eq. (18) into the pricing formula (11) and re-estimate the parameters on a rolling 3-year-subsample basis. For the instantaneous growth rate so obtained, the corresponding out-of-sample percentage pricing errors are recorded under "instantaneous growth rate." Third, we re-estimate the parameters using the 3-month Treasury bill rate (instead of the 30-year Treasury yield), with the out-of-sample pricing results reported under "short-term interest rate." In the final robustness exercise, for all firms, we preset the three interest-rate parameters to those implied by the S&P 500 index (i.e., the values of κ_r, μ_r, and σ_r are fixed based on the first panel of Table 3). The percentage pricing errors are recorded under "pricing kernel fixed." The average percentage pricing error (denoted MPE), the average absolute percentage pricing error (denoted APE), the average dollar pricing error (denoted MDE), and the average absolute dollar pricing error (denoted ADE) are reported for each case. The standard deviation of the percentage (dollar) measure is denoted by STD (STDD).

		Panel A: Sub-samples		Panel B: Instantaneous growth			Panel C: Short-rate			Panel D: Pricing kernel fixed		
		79:01–89:12	90:01–98:07	79:01–89:12	90:01–98:07	Full	79:01–89:12	90:01–98:07	Full	79:01–89:12	90:01–98:07	Full
Percentage pricing errors (in %)	APE	16.69	17.43	17.34	20.34	18.67	20.56	27.80	24.02	16.83	19.06	17.83
	STD	20.54	21.50	20.30	22.71	22.70	14.08	16.51	16.14	20.30	22.71	23.16
	MPE	−0.28	−2.39	3.90	7.40	5.33	18.12	23.48	20.80	−1.85	0.17	−0.67
Dollar pricing errors (in \$)	ADE	2.94	6.88	3.05	8.16	5.37	3.56	11.22	7.19	3.17	7.63	5.25
	STDD	3.94	8.79	3.77	9.31	7.46	3.03	9.05	7.73	3.77	9.31	7.18
	MDE	−0.19	−0.01	0.74	4.03	2.19	3.15	9.72	6.33	−0.39	1.18	0.46

replace the 30-year Treasury yield by the 3-month Treasury bill rate. Panel C of Table 9 presents the pricing-error statistics for the full sample and the two subsamples. With this interest rate proxy, the average APE for the Dow stocks is 24.02%, over 7% higher than its counterpart in Table 6, while the average MPE is now 20.80% (versus the previous −1.15%). Similarly substantial increases in both APE and MPE occur for the two subsamples. The worse pricing fit is also true based on the dollar pricing errors. Therefore, the 3-month Treasury rate is a worse proxy than the 30-year Treasury yield. As we discussed before, a possible reason is that the 30-year yield is better for discounting equity shares because stocks have an infinite-maturity date. For equity pricing considerations, our exercise thus supports the choice of the 30-year yield.

In our last robustness check, we hold the interest rate parameters constant across individual stocks. As noted from (2) and (3), this amounts to fixing the pricing kernel for all the stocks. There are two steps in the implementation process. First, we back out the interest rate parameters from the S&P 500 data (jointly with the 30-year yield data). Then, pre-fixing the interest rate parameters according to the S&P 500-based estimates, we re-estimate the firm-specific parameters on a rolling basis using the trailing 3 years of data (as for Table 6, re-estimate once every year from 1979 through 1997). The out-of-sample pricing errors so obtained for the Dow stocks are displayed in Panel D of Table 9. This modification only increases the average APE to 17.83% for the full sample period, from the previous 17.00% in Table 6. Therefore, the said implementation does not affect our results in any noticeable way. Our conclusion above is still robust.

6. Market-implied earnings expectations

In the options literature, implied volatility is often viewed as capturing the market's assessment of the implicit future uncertainty in the underlying stock. In stock valuation, we can ask a similar question: given a stock's historical valuation, what is the implicit expected EPS growth necessary to support the current stock price? This question is relevant not only because the market-implied EPS growth provides a distinct way of assessing the stock's price, but also because security analysts are sometimes slower than the market in responding to new information. As a result, the prevailing stock price may convey more up-to-date information about the growth potential of a stock. That information can be recovered from the stock price via a stock valuation model.

6.1. Implied expected earnings growth

Based on our main model, the theoretical price is monotonically increasing in $G(t)$. Thus, it allows us to invert the value of expected EPS growth from the stock's market price, $\overline{S}(t)$. Denote the implied EPS growth rate by $\widehat{G}(t)$. That is, substituting the observed $\overline{S}(t)$, $R(t)$, $Y(t)$ and the Φ estimates from Panel B of Table 3 (the rolling parameter estimates), we numerically solve the following equation for $\widehat{G}(t)$ (one for each month t):

$$0 = \overline{S}(t) - \delta Y(t) \int_0^\infty \exp[\varphi(\tau) - \varrho(\tau)R(t) + \vartheta(\tau)\widehat{G}(t)]\, d\tau, \tag{35}$$

where $\varphi(\tau)$, $\varrho(\tau)$ and $\vartheta(\tau)$ are as given in (10)–(12).

Fig. 2 plots the implied expected EPS growth against the consensus analyst growth forecast separately for the S&P 500 and Intel. In both cases, the average market-implied growth is higher than the analyst estimates: they are respectively 0.1310 versus 0.0939 for the S&P 500 and 0.5619 versus 0.5243 for Intel. On the other hand, the analyst estimates are far less volatile than the market-implied forecasts. In the case of the S&P 500, the standard deviation is 0.1014 for the $\widehat{G}(t)$ series and 0.0493 for the analyst expected $G(t)$ series. The respective standard deviations are 0.9286 and 0.6530 for Intel. Comparing Figs. 1 and 2, we can also note that periods of positive (negative) pricing errors generally coincide with periods when market-implied forecasts are higher (lower) than analyst estimates. Therefore, pricing errors and the relative position of the market-implied versus analyst forecasts convey related information.

To further investigate the relationship, we can regress the market-implied $\widehat{G}(t)$ on the analyst estimate $G(t)$. For example, for the S&P 500 index and Intel, we obtain the following:

$$\text{S\&P500:} \quad \widehat{G}(t) = 0.04 + 0.97G(t) + \tilde{\omega}(t), \quad R^2 = 22\%,$$

$$\text{Intel:} \quad \widehat{G}(t) = -0.02 + 1.12G(t) + \tilde{\omega}(t), \quad R^2 = 62\%.$$

The slope coefficients are near unity, so the market-implied and analyst estimates co-move strongly (especially for the S&P 500). Although not shown, the divergence between the market-implied and the analyst estimates is larger for both the Dow stocks and the technology stocks, with their respective average slope coefficients at 1.44 and 1.22. Based on the Newey-West procedure, most of the slope coefficients are also statistically significant. Overall, the fit of the above regressions is reasonable with an average adjusted-R^2 of about 32%.

It should be cautioned that the usefulness of the market-implied EPS forecast depends on the model's extent of misspecification. If the model is completely misspecified, the implied forecast may reflect the model noise more than any true information embedded in the market. Since every valuation model is expected to be misspecified, the resulting market-implied EPS forecast will, like the Black-Scholes implied volatility, be a combination of noise and true information.

6.2. *Pricing stocks using market-implied earnings expectations*

In this subsection, we give an example in which the market-implied forecast is applied to price the stock out of sample. If the market is faster in incorporating new information than analysts, we should then expect the market-implied forecast to improve the model's pricing performance. For this exercise, we again focus on our main model. Specifically, for each stock/index and at each month t we first back out $\widehat{G}(t)$, as outlined in (35). Next, we use this market-implied $\widehat{G}(t)$, together with the parameter estimates and the observed $R(t+1)$ and $Y(t+1)$, to price the stock in month $(t+1)$. This valuation process continues until the end of the sample period. The resulting out-of-sample percentage and dollar pricing errors are reported in Table 10.

Comparing Tables 6 and 10, we see that the market-implied forecasts lead to consistently lower pricing errors than the analyst estimates. This is true for every stock/index and according to every pricing-error metric. For instance, the APE, STD and MPE for the S&P 500 are respectively 8.17%, 9.8% and 3.4% based on the analyst estimates, whereas they are 3.81%, 5.66% and 0.05% based on the market-implied. The ADE, STDD and MDE for Intel's dollar pricing errors are \$3.33, \$7.38 and \$1.93 based on the analyst estimates, but they are only \$1.38, \$3.67 and \$0.09 based on the market-implied, respectively. Thus, incorporating market-implied

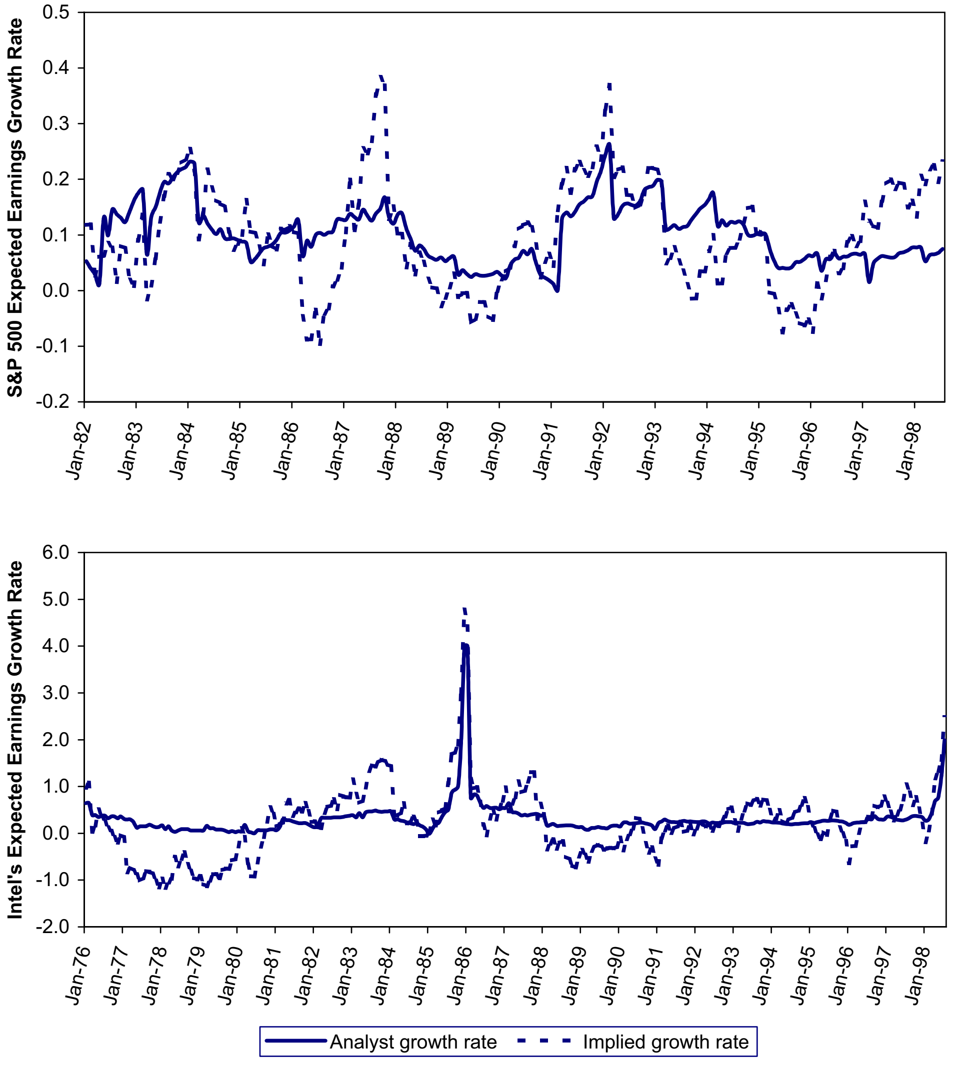

Fig. 2. Implied versus analyst expected earnings growth.

growth rates into the model reduces both the magnitude and dispersion of the pricing errors.

This pricing improvement can be interpreted in two ways. First, as documented in Table 7, the pricing errors for a given stock are highly persistent from month to month. As the market-implied forecasts and pricing errors are closely related, the expected EPS growth implied by this month's market price should then fit next month's stock price better than the analyst estimates. The second interpretation is that analysts may indeed perform worse than the market in forecasting future EPS.

Table 10
Out-of-sample pricing performance using the market-implied earnings growth

Only the main model is considered here, with all the parameter values, Φ, the same as used for calculating the SSE out-of-sample pricing errors (i.e., Panel B of Table 3). For each stock/index and at each month t, we first back out the expected earnings growth $\widehat{G}(t)$ by numerically solving

$$\overline{S}(t) - \delta Y(t) \int_0^{\infty} \exp[\varphi(\tau) - \varrho(\tau)R(t) + \vartheta(\tau)\widehat{G}(t)]\,\mathrm{d}\tau = 0,$$

where $\varphi(\tau)$, $\varrho(\tau)$ and $\vartheta(\tau)$ are as given in (12)–(14), and $\overline{S}(t)$ is the market price of the stock, Next, we use the month-t market-implied expected growth, $\widehat{G}(t)$, together with the parameter vector Φ and the observed $R(t+1)$ and $Y(t+1)$ to compute the month-$(t+1)$ model price for the stock. This valuation process continues for the full sample of the stock/index (1979:01 through 1998:07). As before, two out-of-sample pricing-error series are generated for each stock: (i) dollar pricing errors (market stock price minus model price), and (ii) percentage pricing errors (the dollar pricing error normalized by the market stock price). The average percentage pricing error (denoted MPE), the average absolute percentage pricing error (denoted APE), the average dollar pricing error (denoted MDE), and the average absolute dollar pricing error (denoted ADE) values are reported below for each series. The standard deviation of the percentage pricing error series is represented by STD, and that of the dollar pricing errors is represented by STDD.

		S&P 500	GE	XON	INTC	MOT	Dow 30	Tech 20
Percentage pricing	APE	3.81	4.76	5.32	12.12	10.50	7.79	14.27
errors (in %)	STD	5.66	6.36	6.63	19.30	23.07	16.30	26.63
	MPE	0.05	0.08	−0.09	−1.35	−0.86	−0.92	−2.34
Dollar pricing	ADE	16.23	0.86	1.30	1.38	2.37	2.06	2.25
errors (in $)	STDD	24.03	1.44	1.91	3.67	4.40	4.70	4.41
	MDE	1.24	0.08	0.00	0.09	0.03	−0.21	−0.14

As a result, we should pay more attention to the market-implied EPS forecasts. However, which of the two interpretations is closer to truth can only be determined through further research.

7. Concluding remarks

In this paper, we separate the stock valuation problem from the firm's production, dividend and financial policies. While this partial equilibrium approach leaves out important corporate issues, it does afford us a more focused problem: that is, we only need to value an exogenous stochastic cashflow stream. Under this framework, the task is to search for the appropriate specifications of the pricing-kernel and the earnings processes. In our main model, the parameterization of these processes embeds a stochastic term structure for both interest rates and expected EPS growth.

Our empirical work demonstrates that modeling the earnings growth properly has a first-order impact: omitting the stochastic EPS growth feature considerably worsens the pricing performance of the valuation model. Adding a stochastic yield curve to the framework further improves the pricing fit. The performance of our valuation model is significantly better than its variants, with its average (out-of-sample) absolute pricing errors ranging from 8.17% to 23.87%. Our pricing-error metrics reveal worse model performance for growth-oriented technology stocks than for blue-chip stocks. We also show that the pricing errors are serially correlated and often experience long cycles of high/low errors, suggesting missing state variables from the model's earnings dynamics. Furthermore, within our sample of stocks the pricing errors are highly correlated across stocks, implying the existence of systematic factors that are important in the market's valuation but excluded from our model.

The empirical findings suggest several research directions. First, one can introduce richer earnings dynamics that lead to a multi-factor term structure of expected earnings growth. For technology stocks, additional variables may be required to match the short and the long-end of the expected-growth curve. Second, one can consider earnings processes that can take both negative and positive values (e.g., Dong, 2000). Similarly, it may be empirically desirable to examine jump-diffusion processes for earnings and/or expected earnings growth rates. Third, we can look for other specifications for the pricing kernel and the term structure of interest rates. Lastly, one can compare the empirical performance of our model with discounted cashflow models (such as the residual-income model) and evaluate their relative performance.

Many applications can be pursued with our valuation approach. For example, Chen and Dong (1999) adapt our model to study how the model valuation can be combined with traditional stock selection methods to produce better investment strategies. Chen and Jindra (2000) apply this model to re-examine stock-market seasonalities, while Brown and Cliff (2005) use our model to shed new light on seasoned equity offerings and investor sentiment, respectively. With a better specified stock valuation model, we can clearly ask new questions and re-address old ones.

Appendix A. Proof of the equity pricing models

A.1. The main model in (8)

Conjecture that the solution to the PDE (7) is of the form (8). Solving the resulting valuation equation and the associated Ricatti equations subject to the boundary condition $s(t+\tau;0) = Y(t+\tau)$ yields (8) and (9).

A special case of the main model is obtained by setting $\kappa_r = \mu_r = \sigma_r = 0$ in (7). This parametric case has constant interest rate r, but stochastic expected earnings growth $G(t)$. In this case, we get

$$s(t,\tau;G,Y) = Y(t)\exp[\varphi(\tau) + \vartheta(\tau)G(t)], \tag{36}$$

where

$$\varphi(\tau) = -\lambda_y\tau - r\tau + \frac{1}{2}\frac{\sigma_g^2}{\kappa_g^2}\left(\tau + \frac{1-\mathrm{e}^{-2\kappa_g\tau}}{2\kappa_g} - \frac{2}{\kappa_g}(1-\mathrm{e}^{-\kappa_g\tau})\right) + \frac{\kappa_g\mu_g + \sigma_y\sigma_g\rho_{g,y}}{\kappa_g}\left(\tau - \frac{1-\mathrm{e}^{-\kappa_g\tau}}{\kappa_g}\right),$$

$$\vartheta(\tau) = \frac{1-\mathrm{e}^{-\kappa_g\tau}}{\kappa_g},$$

subject to $r - \mu_g > \sigma_g^2/2\kappa_g^2 + \sigma_g\sigma_y\rho_{g,y}/2\kappa_g - \lambda_y$.We refer to this special case as the SG model.

Next, let $\kappa_g = \mu_g = \sigma_g = 0$ in (7), which leads to a constant expected earnings growth g. Solving this model results in

$$s(t,\tau; R, Y) = Y(t)\exp[\varphi(\tau) - \varrho(\tau)R(t)] \tag{37}$$

with

$$\varphi(\tau) = -\lambda_y\tau + g\tau + \frac{1}{2}\frac{\sigma_r^2}{\kappa_r^2}\left(\tau + \frac{1-\mathrm{e}^{-2\kappa_r\tau}}{2\kappa_r} - \frac{2(1-\mathrm{e}^{-\kappa_r\tau})}{\kappa_r}\right) - \frac{\kappa_r\mu_r + \sigma_y\sigma_r\rho_{r,y}}{\kappa_r}\left(\tau - \frac{1-\mathrm{e}^{-\kappa_r\tau}}{\kappa_r}\right),$$

$$\varrho(\tau) = \frac{1-\mathrm{e}^{-\kappa_r\tau}}{\kappa_r},$$

subject to $\mu_r - g > \sigma_r^2/2\kappa_r^2 - \sigma_r\sigma_y\rho_{r,y}/\kappa_r - \lambda_y$. We call this special case the SI model.

A.2. Extension models

Proof of the Equity formula in (18). Using Leibnitz's rule, one can express Eq. (17) as

$$\mathrm{d}A(t) = \eta[G(t) - A(t)]\,\mathrm{d}t. \tag{38}$$

Write the stock price as $S(R, G, Y, A)$. Then $S(t)$ must solve

$$\frac{1}{2}\sigma_y^2 Y^2\frac{\partial^2 S}{\partial Y^2} + [G-\lambda_y]Y\frac{\partial S}{\partial Y} + \rho_{g,y}\sigma_y\sigma_g Y\frac{\partial^2 S}{\partial Y\partial G} + \rho_{r,y}\sigma_y\sigma_r Y\frac{\partial^2 S}{\partial Y\partial R} + \rho_{g,r}\sigma_g\sigma_r\frac{\partial^2 S}{\partial G\partial R} + \frac{1}{2}\sigma_r^2\frac{\partial^2 S}{\partial R^2} + \kappa_r[\mu_r - R]\frac{\partial S}{\partial R} + \frac{1}{2}\sigma_g^2\frac{\partial^2 S}{\partial G^2} + \kappa_g[A-G]\frac{\partial S}{\partial G} + \eta[G-A]\frac{\partial S}{\partial A} - RS + \delta Y = 0 \tag{39}$$

subject to the standard transversality condition. Let the candidate solution be

$$S(t) = \int_0^\infty \delta Y(t)\exp[\varphi(\tau) - \varrho(\tau)R(t) + \vartheta(\tau)G(t) + \varsigma(\tau)A(t)]\,\mathrm{d}\tau.$$

Inserting this conjecture into (39) and rearranging, we get $\varrho(\tau)$ as in (11) and

$$\frac{\partial \vartheta}{\partial \tau} = 1 + \eta\varsigma(\tau) - \kappa_g\,\vartheta(\tau), \tag{40}$$

$$\frac{\partial \varsigma}{\partial \tau} = \kappa_g\vartheta(\tau) - \eta\varsigma(\tau). \tag{41}$$

From the set of linear ordinary differential equations inn (40) and (41), we get

$$\frac{\partial^2 \vartheta}{\partial \tau^2} + (\eta + \kappa_g)\frac{\partial \vartheta}{\partial \tau} - \eta = 0. \tag{42}$$

Solving this second-order ordinary differential equation subject to $(\partial\vartheta/\partial\tau)(0) = 1$ and $\vartheta(0) = 0$ yields

$$\vartheta(\tau) = \frac{\eta\tau}{\eta + \kappa_g} + \frac{\kappa_g}{(\eta + \kappa_g)^2}[1 - \mathrm{e}^{-(\kappa_g+\eta)\tau}], \tag{43}$$

and using (41) and (43), we obtain

$$\varsigma(\tau) = -\frac{\kappa_g}{\eta(\eta + \kappa_g)} + \frac{\kappa_g\tau}{\eta + \kappa_g} + \frac{\kappa_g \mathrm{e}^{-(\kappa_g+\eta)\tau}}{\eta(\eta + \kappa_g)} + \frac{\kappa_g^2[1 - \mathrm{e}^{-(\kappa_g+\eta)\tau}]}{\eta(\eta + \kappa_g)^2}. \tag{44}$$

Finally, we have

$$\begin{aligned}\varphi(\tau) = {} & \frac{1}{2}\,\sigma_r^2 \int_0^\tau \varrho^2(u)\,\mathrm{d}u - \kappa_r\mu_r \int_0^\tau \varrho(u)\,\mathrm{d}u + \frac{1}{2}\,\sigma_g^2 \int_0^\tau \vartheta^2(u)\,\mathrm{d}u \\ & - \rho_{r,g}\sigma_r\sigma_g \int_0^\tau \varrho(u)\,\vartheta(u)\,\mathrm{d}u. \qquad \square\end{aligned} \tag{45}$$

Proof of Model 3 in (26). Regarding the problem at hand, we first note that

$$a_{11} \equiv \frac{1 - \mathrm{e}^{-\kappa_{x_1}\tau_1}}{\tau_1\kappa_{x_1}}, \tag{46}$$

$$a_{12} \equiv \frac{1 - \mathrm{e}^{-\kappa_{x_2}\tau_1}}{\tau_1\kappa_{x_2}}, \tag{47}$$

$$a_{21} \equiv \frac{1 - \mathrm{e}^{-\kappa_{x_1}\tau_2}}{\tau_2\kappa_{x_1}}, \tag{48}$$

$$a_{22} \equiv \frac{1 - \mathrm{e}^{-\kappa_{x_2}\tau_2}}{\tau_2\kappa_{x_2}} \tag{49}$$

$$a_{10} \equiv \mu_{x_1} + \mu_{x_2} - \tfrac{1}{2}\sigma_y^2 - \mu_{x_1}a_{11} - \mu_{x_2}a_{12}, \tag{50}$$

$$a_{20} \equiv \mu_{x_1} + \mu_{x_2} - \tfrac{1}{2}\sigma_y^2 - \mu_{x_1}a_{21} - \mu_{x_2}a_{22}. \tag{51}$$

Following the same steps as in the proof of the main model, we obtain the equity valuation formula for Model 3 with

$$\vartheta(\tau) = \frac{a_{22}}{a_{22}a_{11} - a_{12}a_{21}}\left[\frac{1 - \mathrm{e}^{-\kappa_{x_1}\tau}}{\kappa_{x_1}}\right] - \frac{a_{21}}{a_{22}a_{11} - a_{12}a_{21}}\left[\frac{1 - \mathrm{e}^{-\kappa_{x_2}\tau}}{\kappa_{x_2}}\right], \tag{52}$$

$$\theta(\tau) = \frac{a_{11}}{a_{22}a_{11} - a_{12}a_{21}}\left[\frac{1 - \mathrm{e}^{-\kappa_{x_2}\tau}}{\kappa_{x_2}}\right] - \frac{a_{12}}{a_{22}a_{11} - a_{12}a_{21}}\left[\frac{1 - \mathrm{e}^{-\kappa_{x_1}\tau}}{\kappa_{x_1}}\right] \tag{53}$$

and

$$\begin{aligned}
\varphi(\tau) = & -\lambda_y\tau + \frac{1}{2}\frac{\sigma_r^2}{\kappa_r^2}\left[\tau + \frac{1 - \mathrm{e}^{-2\kappa_r\tau}}{2\kappa_r} - \frac{2(1 - \mathrm{e}^{-\kappa_r\tau})}{\kappa_r}\right] - \frac{\kappa_r\mu_r + \sigma_y\sigma_r\rho_{r,y}}{\kappa_r} \\
& \times\left[\tau - \frac{1 - \mathrm{e}^{-\kappa_r\tau}}{\kappa_r}\right] + \frac{1}{2}\frac{\sigma_{x_1}^2}{\kappa_{x_1}^2}\left[\tau + \frac{1 - \mathrm{e}^{-2\kappa_{x_1}\tau}}{2\kappa_{x_1}} - \frac{2}{\kappa_{x_1}}(1 - \mathrm{e}^{-\kappa_{x_1}\tau})\right] \\
& + \frac{\kappa_{x_1}\mu_{x_1} + \sigma_y\sigma_{x_1}\rho_{x_1,y}}{\kappa_{x_1}}\left[\tau - \frac{1 - \mathrm{e}^{-\kappa_{x_1}\tau}}{\kappa_{x_1}}\right] + \frac{1}{2}\frac{\sigma_{x_2}^2}{\kappa_{x_2}^2}\left[\tau + \frac{1 - \mathrm{e}^{-2\kappa_{x_2}\tau}}{2\kappa_{x_2}}\right. \\
& \left. - \frac{2}{\kappa_{x_2}}(1 - \mathrm{e}^{-\kappa_{x_2}\tau})\right] + \frac{\kappa_{x_2}\mu_{x_2} + \sigma_y\sigma_{x_2}\rho_{x_2,y}}{\kappa_{x_2}}\left[\tau - \frac{1 - \mathrm{e}^{-\kappa_{x_2}\tau}}{\kappa_{x_2}}\right] \\
& - \frac{\sigma_r\sigma_{x_1}\rho_{x_1,r}}{\kappa_r\kappa_{x_1}}\left\{\tau - \frac{1}{\kappa_r}(1 - \mathrm{e}^{-\kappa_r\tau}) - \frac{1}{\kappa_{x_1}}(1 - \mathrm{e}^{-\kappa_{x_1}\tau}) + \frac{1 - \mathrm{e}^{-(\kappa_r + \kappa_{x_1})\tau}}{\kappa_r + \kappa_{x_1}}\right\} \\
& - \frac{\sigma_r\sigma_{x_2}\rho_{x_2,r}}{\kappa_r\kappa_{x_2}}\left\{\tau - \frac{1}{\kappa_r}(1 - \mathrm{e}^{-\kappa_r\tau}) - \frac{1}{\kappa_{x_2}}(1 - \mathrm{e}^{-\kappa_{x_2}\tau}) + \frac{1 - \mathrm{e}^{-(\kappa_r + \kappa_{x_2})\tau}}{\kappa_r + \kappa_{x_2}}\right\} \\
& + \frac{a_{22}a_{10} - a_{12}a_{20}}{a_{22}a_{11} - a_{12}a_{21}}\left[\frac{1 - \mathrm{e}^{-\kappa_{x_1}\tau}}{\kappa_{x_1}}\right] + \frac{a_{21}a_{10} - a_{20}a_{11}}{a_{22}a_{11} - a_{12}a_{21}}\left[\frac{1 - \mathrm{e}^{-\kappa_{x_2}\tau}}{\kappa_{x_2}}\right].
\end{aligned} \tag{54}$$

The transversality condition for this model is

$$\begin{aligned}
\mu_r - \mu_{x_1} - \mu_{x_2} > & \frac{1}{2}\frac{\sigma_r^2}{\kappa_r^2} + \frac{1}{2}\frac{\sigma_{x_1}^2}{\kappa_{x_1}^2} + \frac{1}{2}\frac{\sigma_{x_2}^2}{\kappa_{x_2}^2} - \frac{\sigma_y\sigma_r\rho_{r,y}}{\kappa_r} + \frac{\sigma_y\sigma_{x_1}\rho_{x_1,y}}{\kappa_{x_1}} + \frac{\sigma_y\sigma_{x_2}\rho_{x_2,y}}{\kappa_{x_2}} \\
& - \frac{\sigma_r\sigma_{x_1}\rho_{x_1,r}}{\kappa_r\kappa_{x_1}} - \frac{\sigma_r\sigma_{x_2}\rho_{x_2,r}}{\kappa_r\kappa_{x_2}} - \lambda_y. \qquad \square
\end{aligned}$$

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