Problem Set 02

[15] 1. O & N: Chapter 3: Homework Problems: 1
Find $\bar{v}$ and the generation time Λ for thermal neutrons. Calculate $\bar{v}$ as the spectrum average for a normalized Maxwell spectrum:

$$\phi(E)dE = \frac{E}{kT}e^{-\frac{E}{kT}}\frac{dE}{kT}$$

Use T = 900 K and $\nu\Sigma_f = 0.3$/cm.

Solution:

We trying to find $\langle v \rangle$. Since the kinetic energy of a particle is $E = \frac{m_n v^2}{2}$, we get $v = \sqrt{\frac{2}{m}}\sqrt{E}$. So, to get $\langle v \rangle$ we need $\sqrt{\frac{2}{m}}\langle\sqrt{E}\rangle$. We are given $P(E)\,dE = \frac{E}{kT}e^{-\frac{E}{kT}}\frac{dE}{kT}$, so $\langle\sqrt{E}\rangle = \int_0^\infty \sqrt{E}\,P(E)\,dE$. This yields an equation for the expected velocity:

$$\langle v \rangle = \sqrt{\frac{2}{m}}\int_0^\infty \frac{E^{\frac{3}{2}}}{kT}e^{-\frac{E}{kT}}\frac{dE}{kT}$$

Numerically, we get $\langle v \rangle = 5120\,[\frac{m}{s}]$, which when used with the given value of $\nu\Sigma_f = 0.3$/cm, gives $\Lambda = 6.51$ [μs].

2. O & N: Chapter 3: Homework Problems: 2

Find $\bar{v}$ and $\overline{\left(\frac{1}{v}\right)}$ for a two group representation of a thermal reactor spectrum, composed for simplicity of a Maxwellian and a $\frac{1}{E}$ spectrum:

$$\begin{aligned} \phi_1(E) &= \frac{a}{E} \qquad , \quad for\ 0.2\,\mathbf{eV} \le E \le 2\,\mathbf{MeV} \\ \phi_2(E) &= \frac{b\,E}{(k\,T)^2}\,e^{-\frac{E}{k\,T}} \qquad , \quad for\ 0 \le E \le \infty \end{aligned}$$

[5] (a) Find a and b such that the two components of the normalized $\phi(E)$ provide equal contributions to the energy integral.

Solution:

$$\begin{aligned} 1 &= \int_{0.2\,eV}^{2.0\,MeV} \phi_1\,dE + \int_0^\infty \phi_2\,dE \\ 1 &= a\,7\,ln|10| + b \\ 1 &= a\,14\,ln|10| \\ a &= \frac{1}{14\,ln|10|} \\ a &= 0.031 \end{aligned}$$

$$\begin{aligned} \int_0^\infty \phi_2\,dE &= \int_{0.2\,eV}^{2.0\,MeV} \phi_1\,dE \\ b &= a\,7\,ln|10| \\ b &= \frac{7\,ln|10|}{14\,ln|10|} \\ b &= 0.5 \end{aligned}$$

[4] (b) Find the average velocities for both groups ($\bar{v}_1$ and $\bar{v}_2$). If necessary, leave as a function of temperature, T, in [K].

Solution:

$$\begin{aligned} \langle v_1 \rangle &= \sqrt{\frac{2}{m_n}}\frac{\int_{0.2\,eV}^{2.0\,MeV} \sqrt{E}\phi_1(E)\,dE}{\int_{0.2\,eV}^{2.0\,MeV} \phi_1(E)\,dE} \\ &= 2.428 \cdot 10^6\,[\frac{m}{s}] \end{aligned}$$

$$\begin{aligned} \langle v_2 \rangle(T) &= \sqrt{\frac{2}{m_n}}\frac{\int_0^\infty \sqrt{E}\phi_2(E)\,dE}{\int_0^\infty \phi_2(E)\,dE} \\ &= 170.685\,\sqrt{T}\,[\frac{m}{s}] \end{aligned}$$

[4] (c) Express the two group definitions of $\bar{v}$ and $\overline{\left(\frac{1}{v}\right)}$ as functions of temperature, T, in [K].

Solution:

$$
\begin{aligned}
\langle v\rangle(T) &= \sqrt{\frac{2}{m_n}}\frac{\int\limits_{0.2\,eV}^{2.0\,MeV}\sqrt{E}\phi_1(E)\,dE+\int\limits_0^\infty\sqrt{E}\phi_2(E)\,dE}{\int\limits_{0.2\,eV}^{2.0\,MeV}\phi_1(E)\,dE+\int\limits_0^\infty\phi_2(E)\,dE}\\
&= 1.2\cdot 10^6+85.3\,\sqrt{T}\,[\frac{m}{s}]
\end{aligned}
$$

$$
\begin{aligned}
\langle\frac{1}{v}\rangle(T) &= \sqrt{\frac{m_n}{2}}\frac{\int\limits_{0.2\,eV}^{2.0\,MeV}\frac{1}{\sqrt{E}}\phi_1(E)\,dE+\int\limits_0^\infty\frac{1}{\sqrt{E}}\phi_2(E)\,dE}{\int\limits_{0.2\,eV}^{2.0\,MeV}\phi_1(E)\,dE+\int\limits_0^\infty\phi_2(E)\,dE}\\
&= 9.983\cdot 10^{-6}+\frac{0.00345109}{\sqrt{T}}\,[\frac{m}{s}]
\end{aligned}
$$

[2] (d) Find the corresponding numerical values for T = 900 [K].

Solution:

$$
\begin{aligned}
\langle v\rangle(900) &= 1.22\cdot 10^6\,[\frac{m}{s}]\\
\langle\frac{1}{v}\rangle(900) &= 1.24\cdot 10^{-4}\,[\frac{s}{m}]
\end{aligned}
$$

[5] 3. O & N: Chapter 3: Review Questions: 6
Give two equivalent differential equations for the power of a nuclear reactor. Have one equation use k & l and the other use ρ & Λ. Treat **all** neutrons as prompt and neglect external sources.

Solution:

$$\begin{aligned}\frac{\partial P}{\partial t} &= \frac{\rho}{\Lambda}P \\ \frac{\partial P}{\partial t} &= \frac{k-1}{l}P\end{aligned}$$

4. Using the answer from Question 3 that contains ρ & Λ, account for the presence of a constant external source of neutrons in the reactor by introducing S_o, expressed in [J], into the differential equation.

[1] (a) What is the differential equation describing this new system? Be sure to provide the initial condition.

Solution:

$$\frac{\partial P}{\partial t} = \frac{\rho}{\Lambda}P + S_o$$

[5] (b) Solve the differential equation for a constant ρ, such that $k \neq 1$.

Solution:

$$\begin{aligned}
\frac{\partial P}{\partial t} &= \frac{\rho}{\Lambda}P + S_o \\
\frac{\partial P}{\partial t} - \frac{\rho}{\Lambda}P &= S_o \\
\frac{\partial P}{\partial t}e^{-\frac{\rho}{\Lambda}t} - \frac{\rho}{\Lambda}e^{-\frac{\rho}{\Lambda}t}P &= S_o\, e^{-\frac{\rho}{\Lambda}t} \\
\frac{\partial\left(P\, e^{-\frac{\rho}{\Lambda}t}\right)}{\partial t} &= S_o\, e^{-\frac{\rho}{\Lambda}t} \\
\int_0^t \frac{\partial\left(P(t')\, e^{-\frac{\rho}{\Lambda}t'}\right)}{\partial t'}dt' &= \int_0^t S_o\, e^{-\frac{\rho}{\Lambda}t'}dt' \\
\int_{P(0)}^{P(t)e^{-\frac{\rho}{\Lambda}t}} df &= \left[-\frac{S_o\Lambda}{\rho}e^{-\frac{\rho}{\Lambda}t}\right]_0^t \\
P(t)e^{-\frac{\rho}{\Lambda}t} - P(0) &= -\frac{S_o\Lambda}{\rho}e^{-\frac{\rho}{\Lambda}t} + \frac{S_o\Lambda}{\rho} \\
P(t) &= P(0)e^{\frac{\rho}{\Lambda}t} + \frac{S_o\Lambda}{\rho}\left(e^{\frac{\rho}{\Lambda}t} - 1\right)
\end{aligned}$$

[5] (c) Solve the differential equation for k = 1.

Solution:

$$\begin{aligned}
\frac{\partial P}{\partial t} &= \frac{\rho}{\Lambda}P + S_o \\
\frac{\partial P}{\partial t} &= S_o \\
\int_0^t \frac{\partial P(t')}{\partial t'}dt' &= S_o \int_0^t dt' \\
\int_{P(0)}^{P(t)} df &= S_o \left[t'\right]_0^t \\
P(t) - P(0) &= S_o\, t \\
P(t) &= P(0) + S_o\, t
\end{aligned}$$

[2] (d) What is the value of k that renders a steady state solution for a source S_o?

Solution:

$$\begin{aligned}
\frac{\partial P}{\partial t} &= \frac{\rho}{\Lambda}P + S_o \\
0 &= \frac{k-1}{l}P + S_o \\
\frac{k-1}{l}P_o &= -S_o \\
k-1 &= -\frac{S_o l}{P_o} \\
k &= 1 - \frac{S_o l}{P_o}
\end{aligned}$$

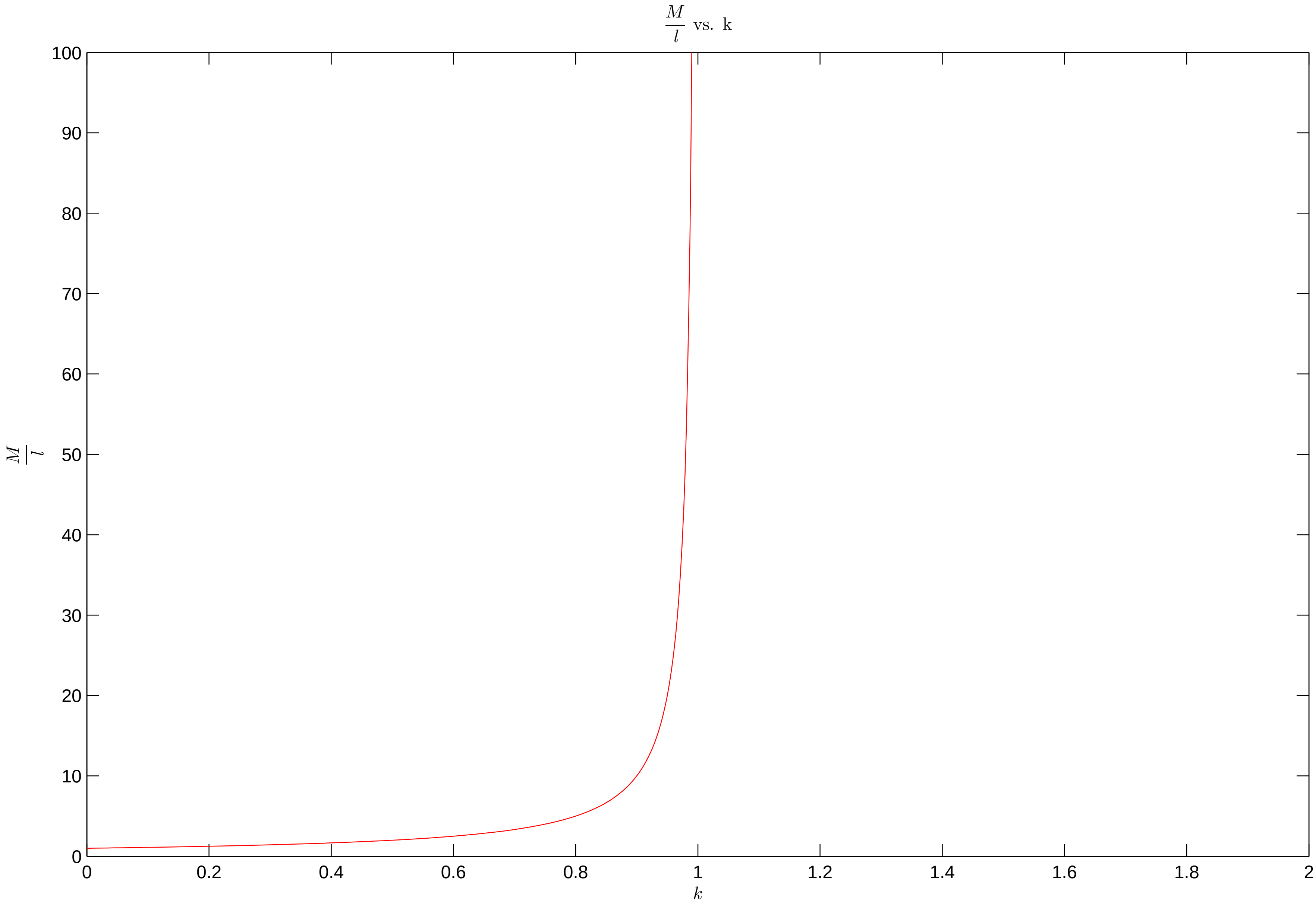
[1] (e) What is the steady state multiplication factor, M, for the reactor in terms of k?
Note: $\mathrm{M} = \dfrac{P_o}{S_o}$

Solution:

$$\begin{aligned}
\frac{\partial P}{\partial t} &= \frac{\rho}{\Lambda} P + S_o \\
0 &= \frac{\rho}{\Lambda} P_o + S_o \\
\frac{P_o}{S_o} &= -\frac{\Lambda}{\rho} \\
M &= \frac{l}{1-k}
\end{aligned}$$

[1] (f) Plot $\frac{M}{l}$ vs. k.

Solution:
See next page for graph.



$\frac{M}{l}$ vs. k
$\frac{M}{l}$
k
0
10
20
30
40
50
60
70
80
90
100
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2