Problem Set 04

1. This problem deals with the following differential equation:

$$\frac{du(t)}{dt} = \lambda\left(u(t) - cos(t)\right) - sin(t),\ u(t_o) = \eta$$

The analytical solution to this differential equation given by:

$$u(t) = e^{\lambda\,(t-t_o)}(\eta - cos(t_o)) + cos(t)$$

[10] (a) Let λ, t_o, and η be -2100, 0 and 1.0, respectively. What is the maximum time step possible to achieve a stable algorithm for Forward Euler? Answer the question analytically following the procedure presented in class.

(b) Let $\lambda = -10^6$, $t_o = 0$ and $\eta = 1.0$ be the data for the following subparts.

[10] i. Use the Backward Euler method to numerically solve the differential equation out to $t = 4.0$ [s]. Use the following five time step sizes: $k = (0.5, 0.1, 0.05, 0.01, 0.005)$. Report the difference between the numerical solution and the analytic, $abs\left|U^N - u\right|$, solution at $t = 4.0$ [s] for each time step size.

[10] ii. Use the Trapezoidal method to numerically solve the differential equation out to $t = 4.0$ [s] with the following five time step sizes: $k = (0.5, 0.1, 0.05, 0.01, 0.005)$. Report the difference between the numerical solution and the analytic, $abs\left|U^N - u\right|$, solution at $t = 4.0$ [s] for each time step size.

(c) Let $\lambda = -10^6$, $t_o = 0$ and $\eta = 1.5$ be the data for the following subparts.

[10] i. Use the Backward Euler method to numerically solve the differential equation out to $t = 4.0$ [s] with the following five time step sizes: $k = (0.5, 0.1, 0.05, 0.01, 0.005)$. Report the difference between the numerical solution and the analytic, $abs\left|U^N - u\right|$, solution at $t = 4.0$ [s] for each time step size.

[10] ii. Use the Trapezoidal method to numerically solve the differential equation out to $t = 4.0$ [s] with the following five time step sizes: $k = (0.5, 0.1, 0.05, 0.01, 0.005)$. Report the difference between the numerical solution and the analytic, $abs\left|U^N - u\right|$, solution at $t = 4.0$ [s] for each time step size.