Games, graphs, and machines

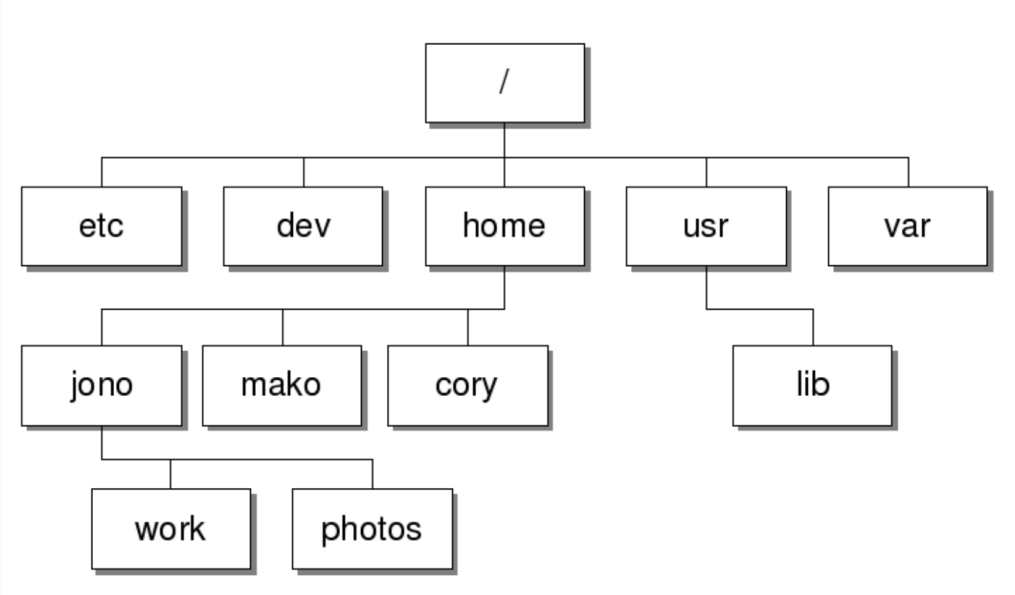
Partial orders

August 6, 2024

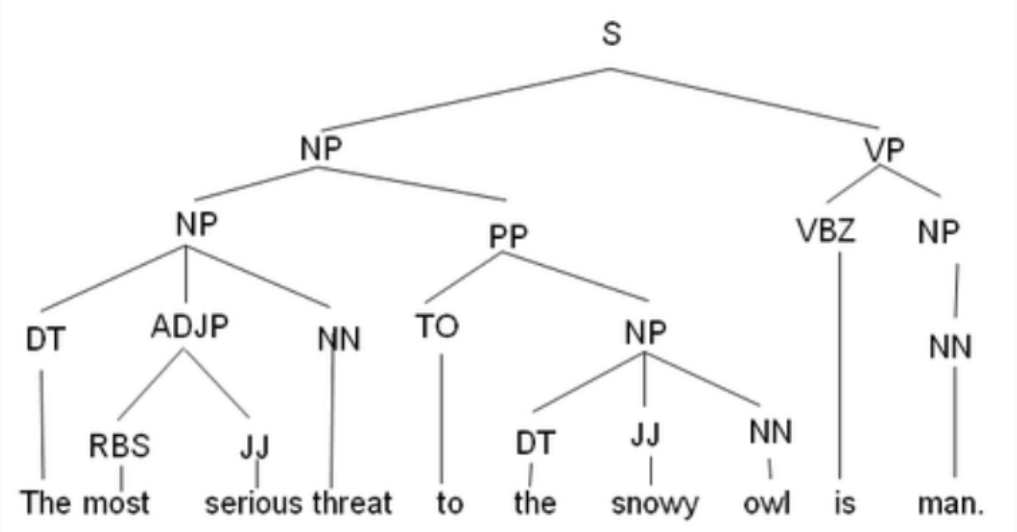
A partial order is

1. Reflexive
2. Anti-symmetric
3. Transitive

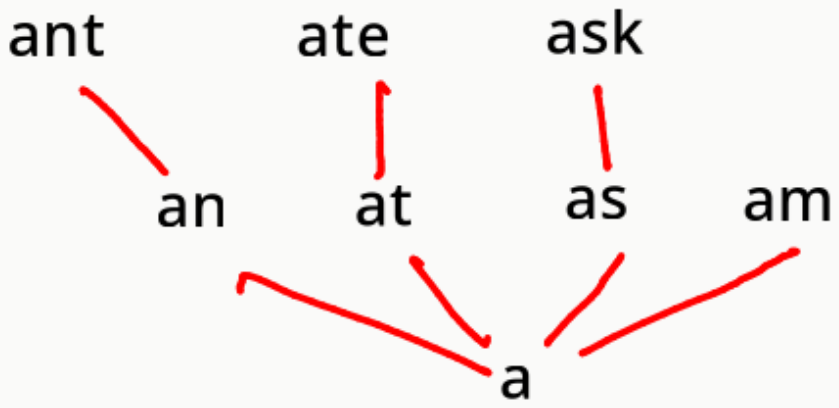
Partial orders from real life



Partial orders from real life



Partial orders from real life



Partial orders from real life

Paper, Scissors, Rock: Partial order?

- Paper $\preceq$ Scissors
- Scissors $\preceq$ Rock
- Rock $\preceq$ Paper

Divisor poset

Let $S = \{1, 2, 3, 4, 6, 12\}$.

Say $a \preceq b$ if a divides b.

What is the Hasse diagram?

Subset poset

Let $S = \mathrm{Pow}(\{1, 2, 3\})$.

Say $A \preceq B$ if $A \subset B$.

What is the Hasse diagram?

Product poset

Let $\leq$ be the usual order on $\mathbb{R}$. Define $\preceq$ on $\mathbb{R} \times \mathbb{R}$ by

$$(a, b) \preceq (c, d) \text{ if } a \leq b \text{ and } c \leq d.$$

1. Give an example of two incomparable elements under $\preceq$.
2. Plot all elements that are $\preceq (2, 3)$.
3. Plot all elements (x, y) with $(1, 1) \preceq (x, y) \preceq (2, 3)$.

Max/min

In all the examples so far, identify

- the maximum (if it exists)
- the minimum (if it exists)
- all maximal elements
- all minimal elements