Games, graphs, and machines

Modular arithmetic

August 2, 2024

A cautionary tale

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Is this an equivalence relation?

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What are the equivalence classes?

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Let $\overline{R}$ be the equivalence classes. Define $+$ on equivalence classes by the rule

$$[a] + [b] = [a + b].$$

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Let $\overline{R}$ be the equivalence classes. Define $\times$ on equivalence classes by the rule

$$[a] \times [b] = [a + b].$$

Is this well-defined?

Linear equations

Find all $x \in \mathbb{Z}/5\mathbb{Z}$ such that

$$\overline{2} \cdot x + \overline{7} = 0.$$

More equations

Find all $x \in \mathbb{Z}/8\mathbb{Z}$ such that

$$\overline{x}^2 = 1.$$

Exponentiation

What is 2^{2024} modulo 7?