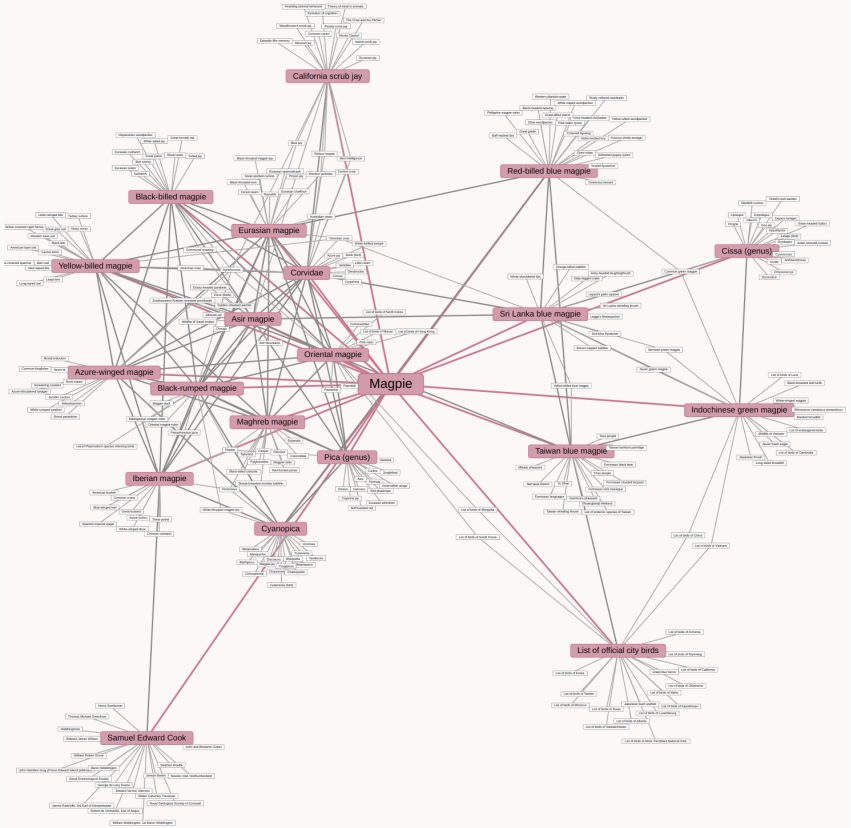
Games, graphs, and machines

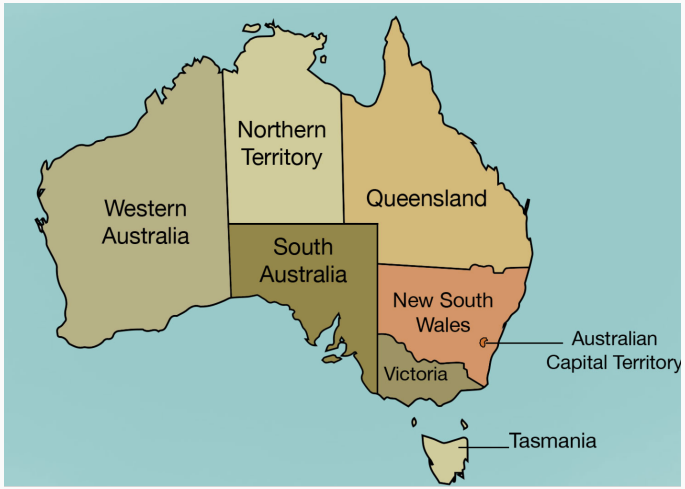


August 13, 2024

Neighbour graph

Draw the graph whose

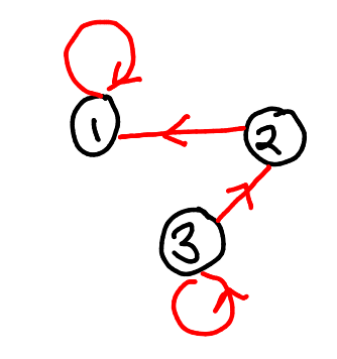
- vertices are the states or territories of Australia,
- two vertices are joined by an edge if they share a border.



Write the adjacency matrix.

Another adjacency matrix

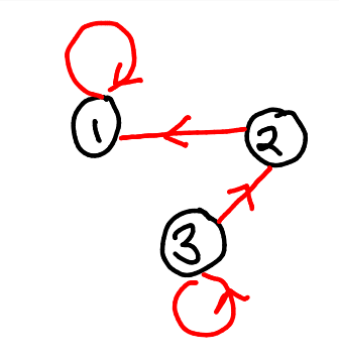
Write the adjacency matrix of the following directed graph.



Degree of a vertex

- The *out-degree* of a vertex is the number of edges going out of it.
- The *in-degree* of a vertex is the number of edges coming into it.

1. Find the incoming and outgoing degrees in the previous graph.



2. How are you read off the degrees from the adjacency matrix?

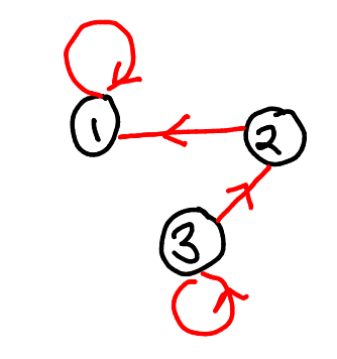
Matrix multiplication

Multiply the following matrices

$$\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 3 & 1 \end{pmatrix}.$$

Powers of the adjacency matrix

Let A be the adjacency matrix of



Calculate A^2 and A^3.

Why does A^k count length k paths?

Theorem
The (i, j) entry if A^k is the number of paths from vertex i to vertex j.

Suppose $n = 3$.

$$A^2_{i,j} = A_{i,1} \cdot A_{1,j} + A_{i,2} \cdot A_{2,j} + A_{i,3} \cdot A_{3,j}$$

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