

# Games, graphs, and machines

Equivalence relations

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July 30, 2024

# Reminders

1. Office hours: Tue 10:30-11:30 and Fri 11:30-12:30 (HN 4.56).
2. Need two class representatives.

# Equivalence relation or not?

Is  $\sim$  an equivalence relation on  $S$ ?

1.  $S = \mathbb{R}$  and  $a \sim b$  if  $|a - b| < 1$ .
2.  $S = \text{States of a chess-board}$  and  $a \sim b$  if we can reach  $b$  from  $a$  by a sequence of legal moves.

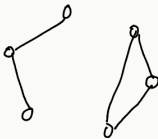
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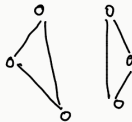
1.  $S = \text{Pow}(A)$  and  $A \sim B$  if  $|A| = |B|$ .
2.  $S = \text{Pow}(A)$  and  $A \sim B$  if  $A \subseteq B$ .

# Graph of an equivalence relation

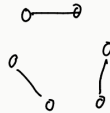
Which of the following is the graph of an equivalence relation?  
(Self-loops and arrows omitted)



①



②



③

# Equivalence classes 1

Let  $\sim$  be the relation on  $\mathbb{R}^2$  defined by

$$(a, b) \sim (c, d) \text{ if } ab = cd.$$

Describe the equivalence classes.

## Equivalence classes 2

Let  $\sim$  be the relation on  $\mathbb{Z}$  defined by

$$(a, b) \sim (c, d) \text{ if } ab \text{ is a square.}$$

Describe the equivalence classes.

# Equivalence classes and partitions

Let  $\sim$  be an equivalence relations on  $S$ . Convince yourself that the equivalence classes *partition*  $S$ :

- any two distinct equivalence classes are disjoint
- the union of all equivalence classes is  $S$ .



## Equivalence relation defined by a function

Let  $f: S \rightarrow T$  be a function. Say  $a \sim b$  if  $f(a) = f(b)$ .

1. Is this an equivalence relation?
2. Describe the equivalence classes for  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  defined by  $f(a, b) = ab$ .