

(2) Fix a modulus d . Recall that $\mathbb{Z}/d\mathbb{Z}$ is the set of equivalence classes of integers modulo d . Let us try to define the operation of exponentiation on $\mathbb{Z}/d\mathbb{Z}$ as follows. Given $A, B \in \mathbb{Z}/d\mathbb{Z}$, we pick an $a \in A$, a positive $b \in B$, and declare $A^B = [a^b]$. Is this operation well-defined? Justify your answer.

Choose $d = 4$, $A = [2] = [6]$, $B = [1] = [5]$

$$2^1 \bmod 4 = 2$$

$$6^5 \bmod 4 = 0$$

Since $[2] \neq [0]$, the result depends on the choice of representatives. By counterexample, the operation of exponentiation is not well-defined on $\mathbb{Z}/d\mathbb{Z}$.