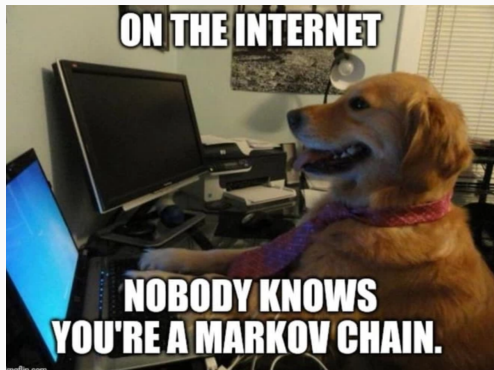


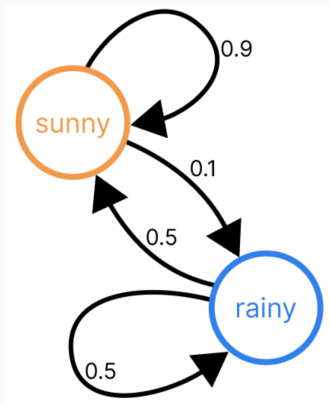
Games, graphs, and machines



August 27, 2024

Warm up

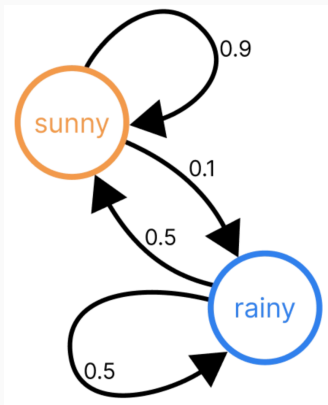
Write the transition matrix A for the following Markov chain.



Warm up

Write the transition matrix A for the following Markov chain.

Calculate A^2 . What do entries of A^2 represent?



Why does k th power represent k -step probabilities?

$$A_{i,j}^2 = A_{i,1} \cdot A_{1,j} + A_{i,2} \cdot A_{2,j}.$$

Large powers

We have $A = EDE^{-1}$, where

$$E = \begin{pmatrix} 1 & 1 \\ 1 & -5 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 0 \\ 0 & 0.4 \end{pmatrix} \quad E^{-1} = \begin{pmatrix} 5/6 & 1/6 \\ 1/6 & -1/6 \end{pmatrix}.$$

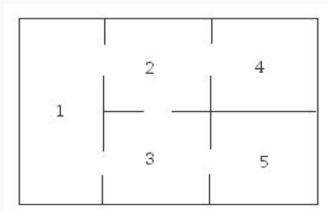
Find (approximate) A^k for large k . What do the entries represent?

When do large powers converge?

Suppose $A = EDE^{-1}$, where D is diagonal. When will A^k converge (to a matrix with finite entries) as k grows?

A maze

A maze used for training rats has the following shape.



Suppose that at every stage, the rat picks a door at random (with equal probability) and goes through that door. Write the Markov chain and the transition matrix.

Powers of the maze

The matrix A is diagonalisable with eigenvalues $[-1/3, 1, 1, -0.43, 0.76]$.

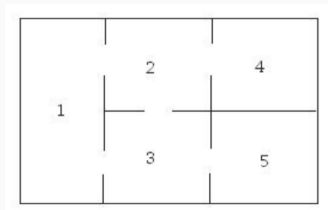
Will powers of A converge?

Powers of the maze

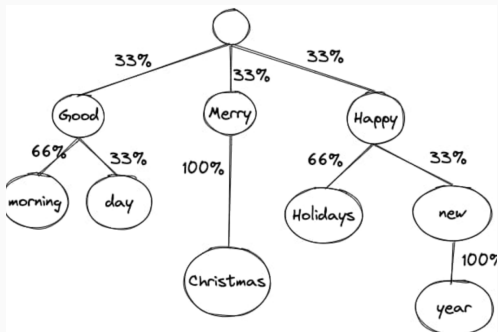
The powers of A converge to

$$\begin{pmatrix} 0 & 0 & 0 & 0.50 & 0.50 \\ 0 & 0 & 0 & 0.62 & 0.38 \\ 0 & 0 & 0 & 0.38 & 0.62 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Can you interpret the entries?



A text generator



Let A be the corresponding transition matrix. The powers of A stabilise. When do they stabilise? What is the first row of A^{100} ?