

(5) Let  $(S, \preceq)$  be a finite poset. Let  $G$  be the directed graph of the relation  $\preceq$  and let  $A$  be the adjacency matrix of  $G$ . Let  $I$  be the identity matrix of the same size as  $A$ . True or false: some positive power of  $A - I$  must be zero. If true, justify it. Otherwise give an example where no positive power of  $A - I$  is zero.

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subtracting any adjacency matrix  $A$  by  $I$  is simply the act of removing the reflexive edges from  $G$ , which can be treated as a conversion of  $(S, \preceq)$  to  $(S, \prec)$ . Since the graph of  $(S, \prec)$  is finite and asymmetric, there is a maximum length path which exists. Therefore after a certain power  $A - I$  is zero.