# Games, graphs, and machines

## matrix (n.)

late 14c., *matris*, *matrice*, "uterus, womb," from Old French *matrice* "womb, uterus" and directly from Latin *mātrix* (genitive *mātricis*) "pregnant animal," in Late Latin "womb," also "source, origin," from *māter* (genitive *mātris*) "mother" (see **mother** (n.1)).

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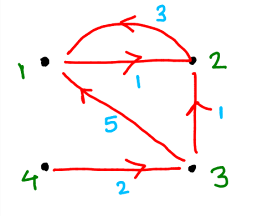
# Warm up

Remember that $\oplus = \min$ and $\odot = +$. Find

$$2 \odot (3 \oplus 1) \oplus 3 \odot (\infty \oplus 2).$$

# Weighted adjacency matrix

Write the min/plus weighted adjacency matrix of the graph. Assume that the loops have weight 0 (not shown).



# Min/plus powers

Find the min/plus square and cube of the adjacency matrix. What do its entries represent?

# Why do min/plus powers give shortest paths?

For example, the third power:

$$\begin{aligned} A^3_{i,j} &= (A^2_{i,1} \odot A_{1,j}) \oplus (A^2_{i,2} \odot A_{2,j}) \oplus (A^3_{i,3} \odot A_{3,j}) \oplus (A^2_{i,4} \odot A_{4,j}) \\ &= \min(A^2_{i,1} + A_{1,j}, A^2_{i,2} + A_{2,j}, A^3_{i,3} + A_{3,j}, A^2_{i,4} + A_{4,j}) \end{aligned}$$

# When do we stop?

Assume:

1. we have all loops with weight 0,
2. all weights are non-negative.

**Theorem**
*Let $n$ be the number of vertices. Then* $A^{\odot(n-1)} = A^{\odot n} = A^{\odot(n+1)} = \cdots$.