(1)

Let $\Sigma = \{0, 1\}$. For each language L described below, write down a regular expresion r such that $L(r) = L$. That is the strings of Σ^* that match r are exactly the strings of L. Be careful to make sure that nothing else matches the regular expression you write down! Justification is not required.

(a) $L = \{w \in \Sigma^* | w \text{ starts with a } 1\}$

$r = 1(1|0)^*$

(b) $L = \{w \in \Sigma^* | w \text{ any ones in } w \text{ are next to eachother in a single block}\}$

$r = (0)^*(1)^*(0)^*$

(c) $L = \{w \in \Sigma^* | w \text{ contains an even number of zeroes}\}$

$r = ((01^*0)|1)^*$

(2)

Let $\Sigma = \{a, b, c\}$. For each regular expression r written below, describe in words the language $L(r)$. Justification not required.

(a) $r = (\epsilon|bc|c)(abc)^*(\epsilon|a|ab)$

$L = \{w \in \Sigma^* | \text{ every letter in } w \text{ is always followed by the next letter in the order of } \Sigma\}$

(b) $r = ((b|c|\epsilon)^* a(b|c|\epsilon)^* a(b|c|\epsilon)^*)^*$

$L = \{w \in \Sigma^* | \text{ every non-}a \text{ letter in } w \text{ is always followed by } a \text{ unless it is the final letter}\}$

(3)

Let $\Sigma = \{0,1\}$ and L be the language

$$L = w|\text{ the number of occurences of 01 in } w \text{ is equal to the number of occurences of 10}$$

For example, the word 010 is in L because it has one occurences of 01 and one of 10. The word 01101 is not in L because it has 2 occurences of 01 but only one of 10. Does there exist a regular expression r such that $L = L(r)$? If yes, find one. If not, explain why not.

$r = (10(0^*)1|01(1^*)0)^*$

(4)

Let $L \subseteq \Sigma^*$ be a language. The compliment of L, denoted L^c, is the compliment of L in Σ^*. That is, for every $w \in \Sigma^*$, we have $w \in L^c$ if and only if $w \notin L$

(a)

Given a DFA M recognising a language $L = L(M)$, explain how to construct a DFA M' such that $L(M') = L^c$

Let M have:

- states Q
- start state $q_0 \in Q$
- accept states $A \subseteq Q$ and,
- transition function $\delta : Q \times \Sigma \to Q\backslash$

then M' must have:

- states Q
- start state q_0
- accept states $Q \setminus A$ and,
- transition function δ

(b)

Construct a DFA recognising the following language:

$$L = \{w \in \Sigma^* |\text{ every odd position of } w \text{ is } 1\}$$

Justification not required.

Let DFA M recognise $L = L(M)$, M has:

- states $Q = \{q_0, q_1, q_2, q_3\}$
- start state $q_0 \in Q$
- accept states $A = \{q_0, q_1, q_2\}$ and,
- transition function δ :

Input State	Letter	Output State
q_0	1	q_1
q_0	$a \in \Sigma, a \neq 1$	q_3
q_1	$a \in \Sigma$	q_2
q_2	1	q_1
q_2	$a \in \Sigma, a \neq 1$	q_3
q_3	$a \in \Sigma$	q_3

(c)

Now use your method from the first part to draw a DFA for the complement of the language L above. Justification not required.