(2)

Let R and T be relations on S. Decide if the following are true or false. Justify your answers.

(a) If R and T are symmetric then $R \cup T$ is symmetric

$(a,b) \in R \cup T \implies (a,b) \in R \vee (a,b) \in T$.
Case 1: $(a,b) \in R \implies (b,a) \in R$ (as R is symmetric).
since $R \subset R \cup T, (a,b) \in R \implies (b,a) \in R \cup T$
Case 2: $(a,b) \in T \implies (b,a) \in T$ (as T is symmetric)
since $T \subset R \cup T, (a,b) \in T \implies (b,a) \in R \cup T$
$\therefore (a,b) \in R \cup T \implies (b,a) \in R \cup T$
$\therefore$ the statement is true.

(b) If R and T are transitive then $R \cup T$ is transitive

Take the example where R is the relation $<$ and T is the relation $>$.
R is transitive as $\forall a,b,c \in S, a < b, b < c \implies a < c$.
T is transitive as $\forall a,b,c \in S, a > b, b > c \implies a > c$.
Take the case where $a,b,c \in S, a = c > c$, in this case $a > b, b < c \in R \cup T$ however $(a,c) \notin R \cup T$.
$\therefore$ by counterexample the statement is false.