MATH2301 Assignment 3

(1) The units digit of a perfect square can only be $0, 1, 4, 5, 6$ or 9. What are the possible units digits of

(a) perfect cubes?

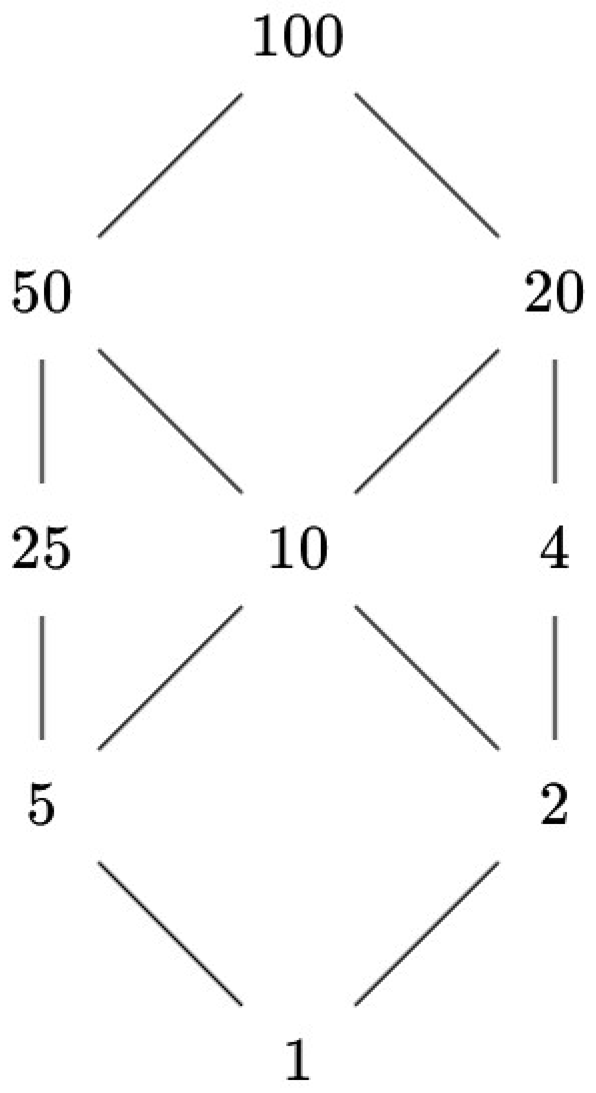
$0^3 = 0$
$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$
So all of the digits are possible.

(b) perfect fourth powers?

$0^4 = 0$
$1^4 = 1$
$2^4 = 16$
$3^4 = 81$
$4^4 = 256$
$5^4 = 625$
$6^4 = 1296$
$7^4 = 2401$
$8^4 = 4096$
$9^4 = 6561$
So the possible digits are $0, 1, 5, 6$
This is because when considering the units digit of a power you are effectively working with modulo 10 so it is sufficient to check only the results for $[0, 9]$.

(2) Let S be the divisor poset of 100. Suppose $f: S \to Z$ is a rank function such that $f(1) = 0$.

(a) Find $f(4)$ and $f(100)$



$f(4) = 2$
$f(10) = 2$

(b) Find all $d \in S$ such that $f(d) = 3$

$f^{-1}(3) = 20, 50$

(3) Let $S = (a, b) \in \mathbb{Z}^2 | a < b$. Define $\preccurlyeq$ on S by the rule $(a, b) \preccurlyeq (c, d)$ if $c \le a$ and $b \le d$

(a) Recall the notion of a locally finite poset from the workshop. Is $(S, \preccurlyeq)$ locally finite?

Yes because its on $\mathbb{Z}^2$

(b) Recall the notion of a maximal chain from Friday's lecture. All maximal chains ending at

$[0, 10]$ have the same length, what is this length?

Example of a maximum chain: $[0, 1], [0, 2], [0, 3], \ldots, [0, 10]$. This chain has length 10.

(c) Every element of S has the same number of immediate successors. How many?

$[a, b]$ has immediate successors $[a-1, b]$ and $[a, b+1]$ so each element has 2 immediate successors.

(4) Let $S = 1, 2, 3, 4$. Find all partial orders on S in which 1 is the minimum and 4 is maximal and $2 \preccurlyeq 3$.