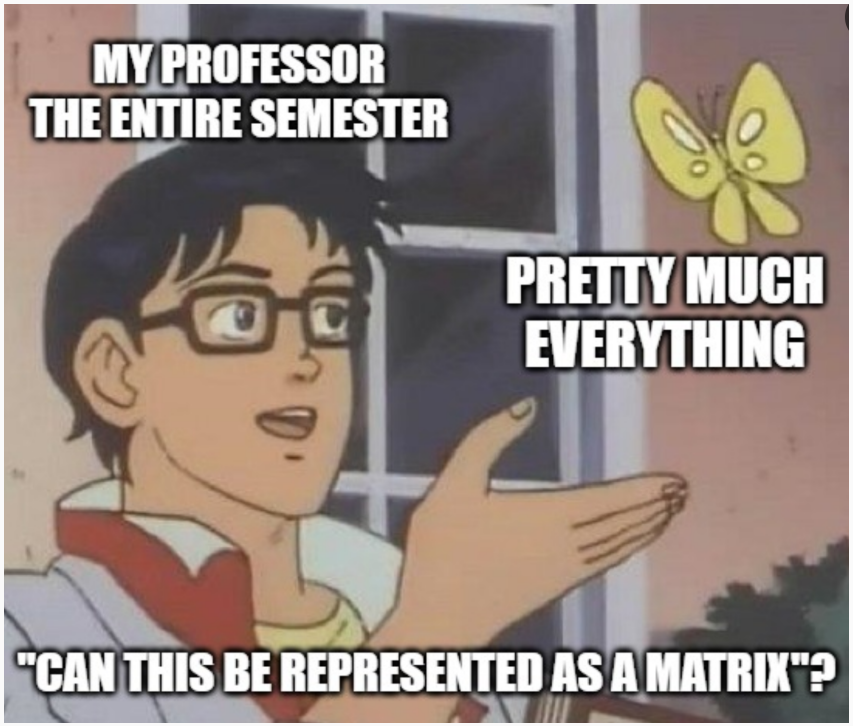
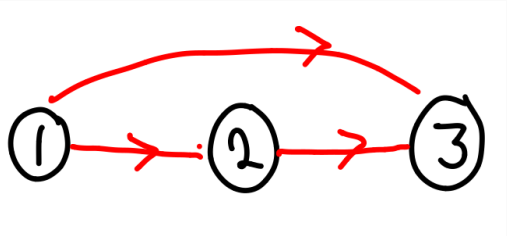
Games, graphs, and machines



August 14, 2024

Warm up

Find the adjacency matrix A and its powers A^2, A^3, A^4, $\cdots$ for the following graph.



Why does A^k count length k paths?

Theorem
The (i, j) entry if A^k is the number of paths from vertex i to vertex j.

Suppose $n = 3$.

$$A^2_{i,j} = A_{i,1} \cdot A_{1,j} + A_{i,2} \cdot A_{2,j} + A_{i,3} \cdot A_{3,j}$$

Why does A^k count length k paths?

$$A^3_{i,j} = A^2_{i,1} \cdot A_{1,j} + A^2_{i,2} \cdot A_{2,j} + A^2_{i,3} \cdot A_{3,j}$$

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Sum of powers

What do the entries of $A + A^2 + A^3 + A^4$ represent?

Acyclic graphs

We say that G is *acyclic* if it has no (directed) cycle. Suppose G is acyclic and has 100 vertices. What can you say about A^{100}?

Longest path

Let G be a graph with adjacency matrix A. Using A, how will you find the longest possible path in G?