(1)

Construct an NFA recognising the following languages. Justifications not required.

(a)

$L = \{w | w \text{ ends with } 00\}$

Let NFA M recognise $L = L(M)$, M has:

- states $Q = \{q_0, q_1, q_2\}$
- start state $q_0 \in Q$
- accept state $A = \{q_2\}$
- transition function δ :

Input State	**Letter**	**Output State**
q_0	$0, a \in \Sigma \neq 0$	q_0
q_0	0	q_1
q_1	0	q_2

(b)

The language $L = L(1^*0^*1^*)$

Let NFA M recognise $L = L(M)$, M has:

- states $Q = \{q_0, q_1, q_2, q_3\}$
- start state $q_0 \in Q$
- accept states $A = \{q_0, q_3\}$
- transition function δ :

Input State	Letter	Output State
q_0	1	q_1
q_0	0	q_2
q_1	1	q_1
q_1	0	q_2
q_1	ϵ	q_3
q_2	0	q_2
q_2	1	q_3
q_3	1	q_3

2

Convert the following regular expressions to equivalent NFAs. (In each case, break down the given regex into manageable pieces such that you can directly construct a DFA/NFA for each "piece". Then combine the pieces using the procedures we discussed in class.)

(a)

$r = (0|1)^*000(0|1)^*$

Let $r_0 = (0|1)^*, r_1 = 000$

Let NFA M_0 recognise $L(r_0) = L(M_0), M_0$ has:

- state $P = \{p_0\}$
- start state $p_0 \in P$
- accept state $A = \{p_0\}$
- transition function δ:

Input State	Letter	Output State
p_0	0	p_0
p_0	1	p_0

Let NFA M_1 recognise $L(r_1) = L(M_1), M_0$ has:

- state $Q = \{q_0, q_1, q_2, q_3\}$
- start state $q_0 \in Q$
- accept state $A = \{q_3\}$
- transition function δ:

Input State	Letter	Output State
q_0	0	q_1
q_1	0	q_2
q_2	0	q_3

Combining the 3 parts: Let NFA M recognise $L(r) = L(M), M$ has:

- states $Q = \{p_0, q_0, q_1, q_2, q_3, r_0\}$
- start state $p_0 \in Q$
- accept state $A = \{r_0\}$
- transition function δ:

Input State	Letter	Output State
p_0	0	p_0

Input State	Letter	Output State
p_0	1	p_0
p_0	ϵ	q_0
q_0	0	q_1
q_1	0	q_2
q_2	0	q_3
q_0	ϵ	r_0
r_0	0	r_0
r_0	1	r_0

(b)

$r = (((00)^*(11))|01)^*$

Let $r_0 = (00)^*, r_1 = 11, r_2 = 01$

Let NFA M_0 recognise $L(r_0) = L(M_0), M_0$ has:

- states $P = \{p_0, p_1\}$
- start state $p_0 \in P$
- accept state $A = \{p_0\}$
- transition function δ:

Input State	Letter	Output State
p_0	0	p_1
p_1	0	p_0

Let NFA M_1 recognise $L(r_1) = L(M_1), M_1$ has:

- states $Q = \{q_0, q_1, q_2\}$
- start state $q_0 \in Q$
- accept state $A = \{q_2\}$
- transition function δ:

Input State	Letter	Output State
q_0	1	q_1
q_1	1	q_2

Let NFA M_2 recognise $L(r_2) = L(M_2), M_2$ has:

- states $O = \{o_0, o_1, o_2\}$
- start state $o_0 \in O$
- accept state $A = \{o_2\}$
- transition function δ:

Input State	Letter	Output State
o_0	0	o_1
o_1	1	o_2

Let $r_3 = ((00)^*(11))|01$,
Combining M_0, M_1, M_2.
Let NFA M_3 recognise $L(r_3) = L(M_3), M_3$ has:

- states $Q = \{s, o_0, o_1, o_2, p_0, p_1, q_0, q_1, q_2\}$
- start state $s \in Q$
- accept states $A = \{o_2, q_2\}$

- transition function δ:

Input State	Letter	Output State
s	ϵ	o_0
s	ϵ	p_0
p_0	0	p_1
p_1	0	p_0
p_0	ϵ	q_0
q_0	1	q_1
q_1	1	q_2
o_0	0	o_1
o_1	1	o_2

Now constructing the final NFA for r:
Let NFA M recognise $L(r) = L(M), M$ has:

- states $Q = \{s, o_0, o_1, o_2, p_0, p_1, q_0, q_1, q_2\}$
- start state $s \in Q$
- accept state $A = \{s\}$
- transition function δ:

Input State	Letter	Output State
s	ϵ	o_0
s	ϵ	p_0
p_0	0	p_1
p_1	0	p_0
p_0	ϵ	q_0
q_0	1	q_1
q_1	1	q_2
o_0	0	o_1
o_1	1	o_2
o_2	ϵ	s
q_2	ϵ	s

3

(a)

Convert the following NFA into an equivalent DFA.
M has:

- states $Q = \{q_0, q_1\}$
- start state $q_0 \in Q$
- accept states $A = \{q_0, q_1\}$
- transition function δ:

Input State	Letter	Output State
q_0	0	q_0
q_0	$0, 1$	q_1
q_1	1	q_1

Let N be the DFA equal to M.
N has:

- states $P(Q) = \{\emptyset, \{q_0\}, \{q_1\}, \{q_0, q_1\}\}$
- start state $\{q_0\} \in P(Q)$
- accept states $A = \{\{q_0\}, \{q_1\}, \{q_0, q_1\}\}$
- transition function δ:

Input State	Letter	Output State
$\{q_0\}$	0	$\{q_0, q_1\}$
$\{q_0\}$	1	$\{q_1\}$
$\{q_1\}$	0	$\emptyset$
$\{q_1\}$	1	$\{q_1\}$
$\{q_0, q_1\}$	0	$\{q_0, q_1\}$
$\{q_0, q_1\}$	1	$\{q_1\}$
$\emptyset$	0	$\emptyset$
$\emptyset$	1	$\emptyset$

(b)

Let L be the language of the DFA above. How many equivalence classes does $\sim_L$ have? Justify your answer.

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from proposition 62 of the book:
Suppose M is a DFA whose language is L. if x and y end at the same state of M, then $x \sim_L y$.
In the above DFA, $\{q_0\}$ and $\{q_0, q_1\}$ end in the same states of N so $\{q_0\} \sim_L \{q_0, q_1\}$. The other states have unique output sets so are in their own equivalence classes.
$\therefore$ the number of equivalence classes in $\sim_L$ is 3.

(4)

Let L be the language

$$L = \{w| \text{ Read in binary, the number } w \text{ is divisible by } 3\}.$$

Is L recognised by an automaton? If yes, draw an DFA/NFA for L. Otherwise justify why an automaton does not exist.

We make the convetion that leading 0s do not affect the number. So the number 00011 is the same as the number 11, which is the number three.