MATH2301 Assignment 2

(1) Let S be the set of squares on a standard 8×8. Consider the following relations on S and determine how many equivalence classes there are.

(a) $B = (s, t) \in S \times S | t$ is reachable from s by a sequence of zero or more bishop moves.

There are 2 equivalence classes - one for the black and one for the white tiles.

(b) $R = (s, t) \in S \times S | t$ is reachable from s by a sequence of zero or more rook moves.

There is 1 equivalence class containing the whole board.

(c) $K = (s, t) \in S \times S | t$ is reachable from s by a sequence of zero or more knight moves.

There is 1 equivalence class containing the whole board.

(2) Fix a modulus d. Recall that $\mathbb{Z}/d\mathbb{Z}$ is the set of equivalence classes of integers modulo d. Let us try to define the operation of exponentiation on $\mathbb{Z}/d\mathbb{Z}$ as follows. Given $A, B \in \mathbb{Z}/d\mathbb{Z}$, we pick an $a \in A$, a positive $b \in B$, and declare $A^B = [a^b]$. Is this operation well-defined? Justify your answer.

Choose $d = 4$, $A = [2] = [6]$, $B = [1] = [5]$

$2^1 \mod 4 = 2$

$6^5 \mod 4 = 0$

Since $[2] \neq [0]$, the result depends on the choice of representatives. By counterexample, the operation of exponentiation is not well-defined on $\mathbb{Z}/d\mathbb{Z}$

(3) Consider modular arithmetic with the modulus $d = 10$. For each equivalence class $[x] \in \mathbb{Z}/d\mathbb{Z}$ determine whether or not $[x]$ has the multiplicative inverse, and if yes, find the inverse. That is, figure out whether there is some $[y]$ such that $[x] \cdot [y] = [1]$.

$[0]$ has no inverse.

$[1] \cdot [1] = [1]$

$[2]$ has no inverse.

$[3] \cdot [7] = [21] = [1]$

$[4]$ has no inverse.

$[5]$ has no inverse.

$[6]$ has no inverse.

$[7] \cdot [3] = [21] = [1]$

$[8]$ has no inverse.

$[9] \cdot [9] = [81] = [1]$

(4) Take the modulus to be $d = 7$. Show that if $[3x] = 5 \mod 7$ then $x = 4$

$[3] \cdot [x] = [5]$
$[5] \cdot [3] \cdot [x] = [5] \cdot [5]$
$[14] \cdot [x] = [25]$
$[1] \cdot [x] = [4]$
$[x] = [4]$