(b) Let A be the transition matrix. Use the Perron-Frobenius theroem to find $\lim_{k\to\infty} A^k$

$$A = \begin{bmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 \end{bmatrix}$$

$$vA = v$$

$$\begin{bmatrix} v_1 & v_2 & v_3 & v_4 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 \end{bmatrix} = \begin{bmatrix} v_1 & v_2 & v_3 & v_4 \end{bmatrix} \text{where } \sum_{i=1}^{4} v_i = 1$$

$$\begin{cases} 0.5v_1 + v_3 = v_1 \\ 0.5v_4 = v_2 \\ 0.5v_4 = v_3 \\ 0.5v_1 + v_2 = v_4 \\ v_1 + v_2 + v_3 + v_4 = 1 \end{cases} = \begin{cases} 0.5v_1 = v_3 \\ v_2 = 0.5v_4 \\ v_3 = 0.5v_4 \\ v_1 + v_2 + v_3 + v_4 = 1 \end{cases}$$

$$= \begin{cases} v_1 = v_4 \\ v_2 = 0.5v_4 \\ v_3 = 0.5v_4 \\ v_1 + v_2 + v_3 + v_4 = 1 \end{cases}$$

$$= \begin{cases} v_1 = v_4 \\ v_2 = 0.5v_4 \\ v_3 = 0.5v_4 \\ v_4 + 0.5v_4 + 0.5v_4 + v_4 = 1 \end{cases}$$

$$= \begin{cases} v_1 = \frac{1}{3} \\ v_2 = \frac{1}{6} \\ v_3 = \frac{1}{6} \\ v_4 = \frac{1}{3} \end{cases}$$

$$\implies \lim_{k\to\infty} A^k = \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} \end{bmatrix}$$