(b) Find $\lim_{k\to\infty} A^k$

$$A = \begin{bmatrix} 0.4 & 0.6 \\ 0.5 & 0.5 \end{bmatrix}$$

$$\text{Let } D = E^{-1}AE \tag{1}$$

$$(\lambda I - A)v = 0 \tag{2}$$

$$\det(\lambda I - A) = 0$$

$$\det\begin{pmatrix} \lambda - 0.4 & -0.6 \\ -0.5 & \lambda - 0.5 \end{pmatrix} = 0$$

$$(\lambda - 0.4)(\lambda - 0.5) - (-0.5)(-0.6) = 0$$

$$\lambda^2 - 0.9\lambda + 0.2 - 0.3 = 0$$

$$\lambda^2 - 0.9\lambda - 0.1 = 0$$

$$(\lambda + 0.1)(\lambda - 1) = 0$$

$$\lambda = -0.1, 1$$

$$D = \begin{bmatrix} -0.1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} \lambda - 0.4 & -0.6 \\ -0.5 & \lambda - 0.5 \end{bmatrix} v = 0 \qquad \text{from (2)}$$

$$\begin{bmatrix} -0.1 - 0.4 & -0.6 \\ -0.5 & -0.1 - 0.5 \end{bmatrix} v = 0 \qquad \begin{bmatrix} 1 - 0.4 & -0.6 \\ -0.5 & 1 - 0.5 \end{bmatrix} v = 0$$

$$\begin{bmatrix} -0.5 & -0.6 \\ -0.5 & -0.6 \end{bmatrix} v = 0 \qquad \begin{bmatrix} 0.6 & -0.6 \\ -0.5 & 0.5 \end{bmatrix} v = 0$$

$$\begin{bmatrix} -0.5 & -0.6 \\ -0.5 & -0.6 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} = 0 \qquad \begin{bmatrix} 0.6 & -0.6 \\ -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} = 0$$

$$\begin{bmatrix} -0.5 & -0.6 \\ -0.5 & -0.6 \end{bmatrix} \begin{bmatrix} -6 \\ 5 \end{bmatrix} = 0 \qquad \begin{bmatrix} 0.6 & -0.6 \\ -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$

$$
\begin{aligned}
v &= \begin{bmatrix} -6 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
E &= \begin{bmatrix} -6 & 1 \\ 5 & 1 \end{bmatrix} && (4) \\
\begin{bmatrix} -0.1 & 0 \\ 0 & 1 \end{bmatrix} &= \begin{bmatrix} -6 & 1 \\ 5 & 1 \end{bmatrix}^{-1} A \begin{bmatrix} -6 & 1 \\ 5 & 1 \end{bmatrix} && \text{sub (3),(4) into (1)} \\
A &= \begin{bmatrix} -6 & 1 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} -0.1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -6 & 1 \\ 5 & 1 \end{bmatrix}^{-1} \\
\lim_{k\to\infty} A^k &= \lim_{k\to\infty} \left(\begin{bmatrix} -6 & 1 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} -0.1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -6 & 1 \\ 5 & 1 \end{bmatrix}^{-1} \right)^k \\
&= \lim_{k\to\infty} \begin{bmatrix} -6 & 1 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} -0.1^k & 0 \\ 0 & 1^k \end{bmatrix} \begin{bmatrix} -6 & 1 \\ 5 & 1 \end{bmatrix}^{-1} \\
&= \begin{bmatrix} -6 & 1 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -6 & 1 \\ 5 & 1 \end{bmatrix}^{-1} \\
&= \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -6 & 1 \\ 5 & 1 \end{bmatrix}^{-1} \\
& \left[\begin{array}{cc|cc} -6 & 1 & 1 & 0 \\ 5 & 1 & 0 & 1 \end{array}\right] && \text{compute inverse} \\
& \left[\begin{array}{cc|cc} -11 & 0 & 1 & -1 \\ 5 & 1 & 0 & 1 \end{array}\right] && \text{(1)-(2)} \\
& \left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{11} & \frac{1}{11} \\ 5 & 1 & 0 & 1 \end{array}\right] && \frac{(1)}{-11} \\
& \left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{11} & \frac{1}{11} \\ 0 & 1 & \frac{5}{11} & \frac{6}{11} \end{array}\right] && \text{(2)-5(1)} \\
\lim_{k\to\infty} A^k &= \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{11} & \frac{1}{11} \\ \frac{5}{11} & \frac{6}{11} \end{bmatrix} \\
&= \begin{bmatrix} \frac{5}{11} & \frac{6}{11} \\ \frac{5}{11} & \frac{6}{11} \end{bmatrix}
\end{aligned}
$$