MATH2301 Assignment 1

(1)

Let $S = \mathbb{R} - 0$
Define $R \subset S \times S$ as $(x, y)|xy = 3$ and R is the I/O relation of f.
Find $f(1)$ and $f(3)$. Justify your answers.

R is the I/O relation of $f(x) = \frac{3}{x}$, this is true because
$\forall x \in S, \exists y \in S | y = \frac{3}{x}$
$\therefore f(1) = 3, f(3) = 1$

(2)

Let R and T be relations on S. Decide if the following are true or false. Justify your answers.

(a) If R and T are symmetric then $R \cup T$ is symmetric

$(a, b) \in R \cup T \implies (a, b) \in R \vee (a, b) \in T$.
Case 1: $(a, b) \in R \implies (b, a) \in R$ (as R is symmetric).
since $R \subset R \cup T, (a, b) \in R \implies (b, a) \in R \cup T$
Case 2: $(a, b) \in T \implies (b, a) \in T$ (as T is symmetric)
since $T \subset R \cup T, (a, b) \in T \implies (b, a) \in R \cup T$
$\therefore (a, b) \in R \cup T \implies (b, a) \in R \cup T$
$\therefore$ the statement is true.

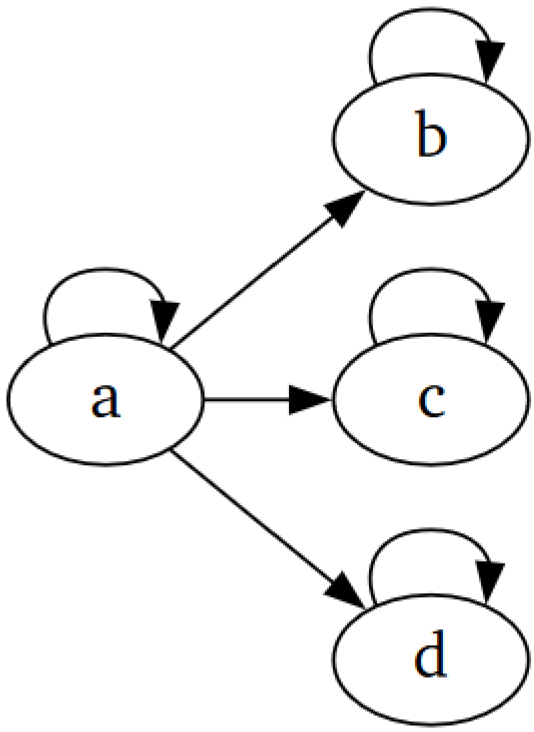
(b) If R and T are transitive then $R \cup T$ is transitive

Take the example where R is the relation $<$ and T is the relation $>$.
R is transitive as $\forall a, b, c \in S, a < b, b < c \implies a < c$.
T is transitive as $\forall a, b, c \in S, a > b, b > c \implies a > c$.
Take the case where $a, b, c \in S, a = c > c$, in this case $a > b, b < c \in R \cup T$ however $(a, c) \notin R \cup T$.
$\therefore$ by counterexample the statement is false.

(3)

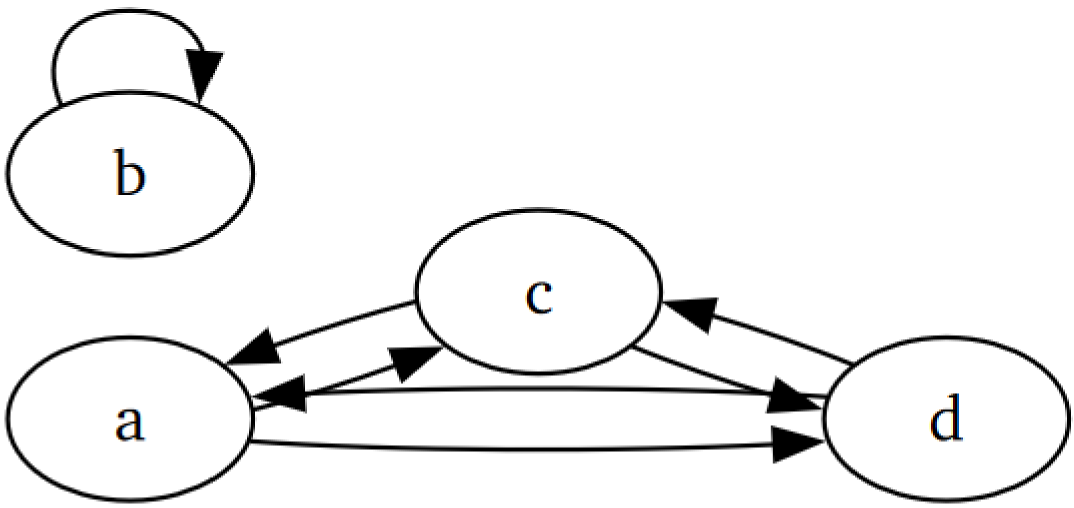
Consider the following graphs, for each one write down which of the following properties are satisfied by the graph: reflexivity, symmetry, transivity, being the I/O of a function.

(a)



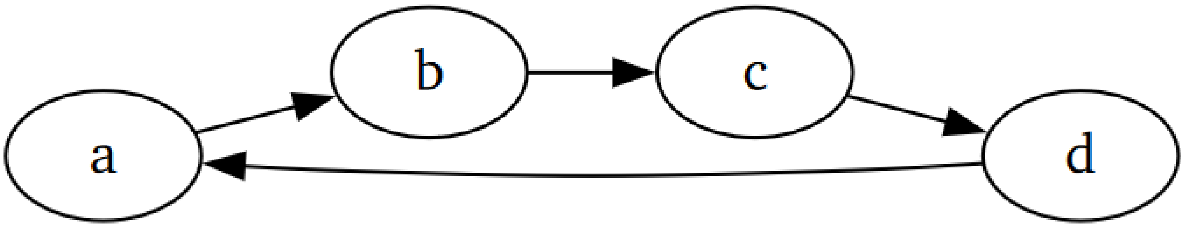
This graph is reflexive and transitive.

(b)



This graph is symmetric and transitive.

(c)



This graph is the I/O of a function.

(4)

Let $S = \mathbb{R} \times \mathbb{R}$. Define a relation R on S as follows:
$R = (a, b), (c, d) | a + b = c + d$.

(a) prove R is an equivalence relation

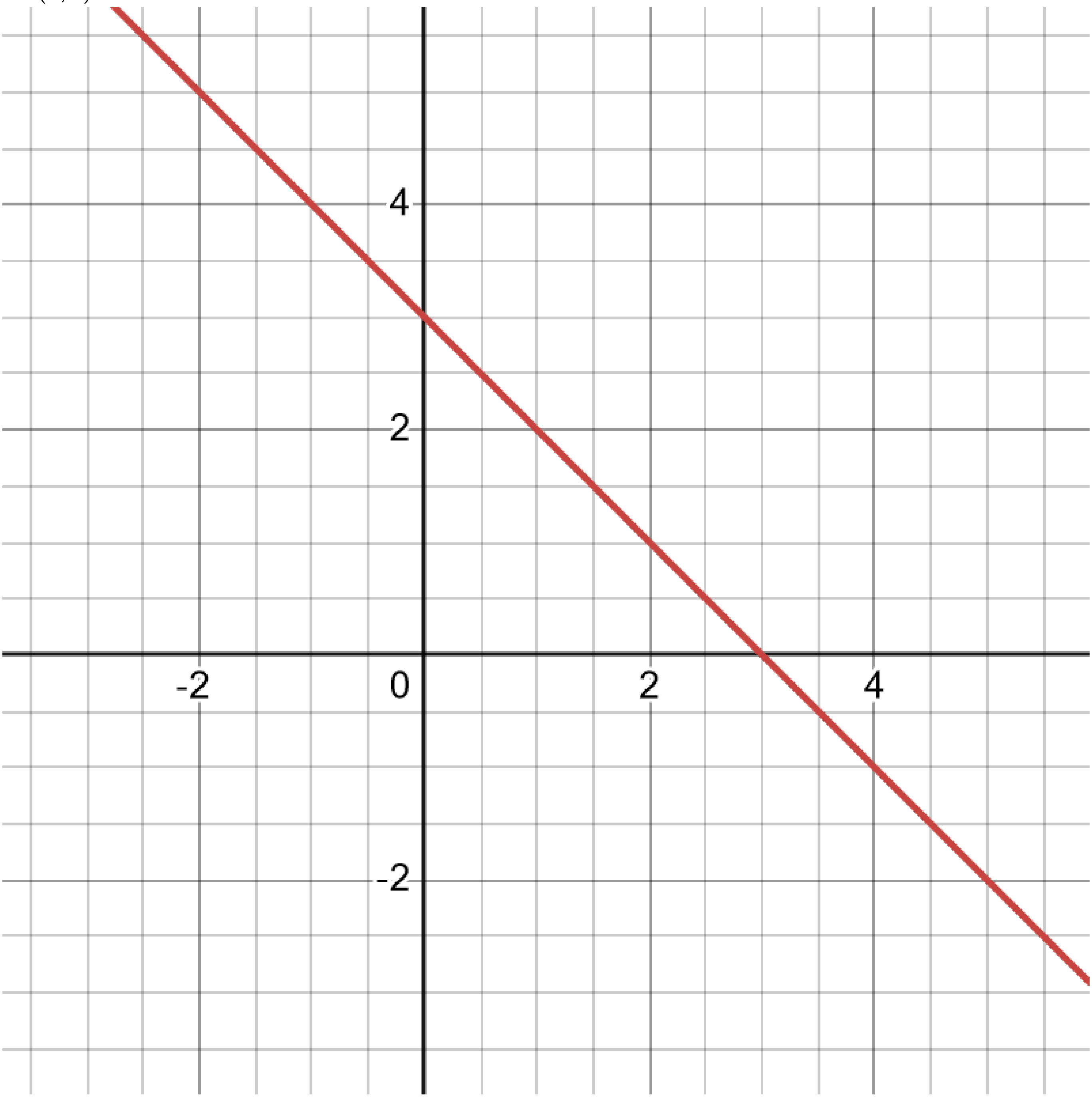
$\forall (a, b) \in S, a + b = a + b$ so R is reflexive.
$\forall (a, b) \in S, a + b = c + d \implies c + d = a + b$ so R is symmetric.
$\forall ((a, b), (c, d), (e, f)) \in S, a + b = c + d, c + d = e + f \implies a + b = e + f$ so R is transitive.

Since R is reflexive, symmetric and transitive, R is an equivalence relation.

(b) Describe the equivalence classes in words and draw sketches in $\mathbb{R}^2$ of the equivalance class of $(1, 2)$ and of $(0, 0)$.

$[x]_R = (a, b) \in \mathbb{R}^2 | a + b = x$

for $(1, 2)$



for $(0, 0)$