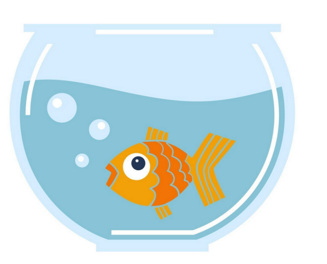
Games, graphs, and machines



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Distinguishable strings

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Say that L *distinguishes* x and y if there exists a z such that exactly one of xz or yz is in L.

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Example

Let $L = 01^*0|10^*1$.

1. Does L distinguish 0 and 1?
2. What about 01 and 10?
3. What about 010 and 101?

Indistinguishable strings

Say that $x \sim_L y$ if L cannot distinguish x and y.

Proposition: $\sim_L$ is an equivalence relation.

How do we know $x \sim y$?

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Proposition: If x and y end at the same state in M, then $x \sim_L y$.

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Proposition: The number of $\sim_L$ equivalence classes is at most the number of states of M.

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Consequence: There is no DFA for L.

The Myhill-Nerode Theorem

Theorem: L is regular if and only if $\sim_L$ has finitely many equivalence classes.