

(3) Let A be the boolean adjacency matrix of G . Prove that G is transitive iff $A = A + A^{*2}$.

The adjacency matrix of any G^+ with n nodes is $\sum_{i=1}^{n-1} A^{*i}$

if G is transitive then $G = G^+$

$\implies A = A + \sum_{i=2}^{n-1} A^{*i} = A + A^{*2}$ (If G is transitive, the matrix A already includes paths of length 2. Thus, A^{*2} and beyond contribute nothing more to the sum.)

$\therefore G = G^+ \implies A = A + A^{*2}$

$A = A + A^{*2}$ implies every path of length 2 (or any further length by recursiveness) between (i, k) is already represented by a direct path. So all transitive connections are already in A which means G must be transitive.

$\therefore G = G^+ \iff A = A + A^{*2}$