

(4)

Let $S = \mathbb{R} \times \mathbb{R}$. Define a relation R on S as follows:

$$R = \{(a, b), (c, d) \mid a + b = c + d\}.$$

(a) prove R is an equivalence relation

$\forall (a, b) \in S, a + b = a + b$ so R is reflexive.

$\forall (a, b) \in S, a + b = c + d \implies c + d = a + b$ so R is symmetric.

$\forall ((a, b), (c, d), (e, f)) \in S, a + b = c + d, c + d = e + f \implies a + b = e + f$ so R is transitive.

Since R is reflexive, symmetric and transitive, R is an equivalence relation.