Lista 08 - EDO - Página 1

Questão ① - $y'' + 4y = 0$

$R^2 + 4R^0 = 0 \Rightarrow R^2 + 0R + 4R^0 = 0 \Rightarrow a = 1 \; b = 0 \; c = 4$

$\Delta = (0)^2 - 4(1)(4) \Rightarrow \Delta = -16$

$R = \dfrac{-(0) \pm \sqrt{-16}}{2(1)} \Rightarrow R = \pm \dfrac{\sqrt{16}\sqrt{-1}}{2} \Rightarrow R = \pm \dfrac{4i}{2} \Rightarrow \begin{array}{l} R_1 = 2i \\ R_2 = -2i \end{array}$

* Caso 2 : $Y(x) = C_1 e^{-bx} \cos(ax) + C_2 e^{-bx} \sin(ax)$, em que $R = -b \pm ai$

$-b = 0 \therefore \boxed{b = 0} \; ; \; ai = 2i \therefore \boxed{a = 2}$

$Y(x) = C_1 e^{-(0)x} \cos(2x) + C_2 e^{-(0)x} \sin(2x) \therefore \boxed{y(x) = C_1 \cos(2x) + C_2 \sin(2x)}$

Questão 2 - $y'' - 9y = 0 \; ; \; Y(0) = 1 \; e \; Y'(0) = 0$

$Y'' + 0Y' - 9Y = 0 \therefore R^2 + 0R^1 - 9 = 0 \therefore a = 1, b = 0, c = -9$

$\Delta = (0)^2 - 4(1)(-9) \therefore \Delta = 36$

$R = \dfrac{-(0) \pm \sqrt{36}}{2(1)} = \dfrac{\pm 6}{2} = \begin{array}{l} R_1 = 3 \\ R_2 = -3 \end{array}$

* Caso 1 : $Y(x) = C_1 e^{R_1 x} + C_2 e^{R_2 x}$

$Y(x) = C_1 e^{3x} + C_2 e^{-3x}$

$Y(0) = 1 \therefore Y(0) = C_1 e^{3(0)} + C_2 e^{-3(0)} = 1 \therefore \boxed{C_1 + C_2 = 1}$

$y'(x) = \left[\dfrac{d}{du}(e^u) \cdot \dfrac{d}{dx}(3x) \right] \cdot C_1 + \left[\dfrac{d}{du}(e^u) \cdot \dfrac{d}{dx}(-3x) \right] \cdot C_2$

$y'(x) = 3C_1 e^{3x} - 3C_2 e^{-3x}$

$y'(0) = 0 \therefore 3C_1 e^{3(0)} - 3C_2 e^{-3(0)} = 0 \therefore 3C_1 - 3C_2 = 0 \Rightarrow \boxed{C_1 = C_2}$

Lista 08 - EDO - Página 2

Questão 2 - continuação

$$\begin{cases} C_1 + C_2 = 1 \\ C_1 = C_2 \end{cases}$$

$$2C_1 = 1 \quad \therefore \quad C_1 = \frac{1}{2} \quad \therefore \quad C_2 = \frac{1}{2}$$

$$\boxed{y(x) = \frac{1}{2}e^{3x} + \frac{1}{2}e^{-3x}}$$

Questão 3 - $y'' - y = 0$

$$R^2 - R^1 = 0 \quad \therefore \quad a=1 \quad b=-1 \quad c=0$$

$$\Delta = (-1)^2 - 4(1)(0) \quad \therefore \quad \Delta = 1$$

$$R = \frac{1 \pm \sqrt{1}}{2(1)} \quad \therefore \quad R_1 = \frac{1+1}{2} \quad \therefore \quad \boxed{R_1 = 1} \quad ; \quad R_2 = \frac{1-1}{2} \quad \therefore \quad \boxed{R_2 = 0}$$

$*$ Caso 1 : $y(x) = C_1 e^{(1)x} + C_2 e^{(0)x} \quad \therefore \quad \boxed{y(x) = C_1 e^x}$

Questão 4 - $y'' + 6y' + 9y = 0$

$$R^2 + 6R^1 + 9R^0 = 0 \quad \therefore \quad a=1 \quad b=6 \quad c=9$$

$$\Delta = (6)^2 - 4(1)(9) = 36 - 36 \quad \therefore \quad \Delta = 0$$

$$R = \frac{-(6) \pm \sqrt{0}}{2(1)} \quad \therefore \quad R_1 = -\frac{6}{2} \quad \therefore \quad \boxed{R_1 = -3} \quad ; \quad R_2 = -\frac{6}{2} \quad \therefore \quad \boxed{R_2 = -3}$$

$*$ 3° caso : $y(x) = C_1 e^{R_1 x} + x C_2 e^{R_2 x}$

$$\boxed{y(x) = C_1 e^{-3x} + x C_2 e^{-3x}}$$

Lista 06 - EDO - Página 3

Questão 5 - $y'' + 4y' + 4y = 0$, $y(0) = 0$ e $y'(0) = 2$

$R^2 + 4R^1 + 4R^0 = 0$ ∴ $a = 1$ $b = 4$ $c = 4$

$\Delta = 0$ ∴ $\sqrt{\Delta} = 0$

$R = \dfrac{-4 \pm 0}{2(1)}$ ∴ $\boxed{R_1 = -2}$ $\boxed{R_2 = -2}$

$y(x) = C_1 e^{-2x} + x C_2 e^{-2x}$

$y'(x) = -2C_1 e^{-2x} - 2x C_2 e^{-2x} + C_2 e^{-2x}$

$y(0) = C_1 e^{-2(0)} + (0)C_2 e^{-2(0)} = \boxed{C_1 = 0}$

$y'(0) = -2C_1 e^{-2(0)} - 2(0)C_2 e^{-2(0)} + C_2 e^{-2(0)}$ ∴ $-2C_1 + C_2 = 2$ ∴ $\boxed{C_2 = 2}$

$y(x) = (0)e^{-2x} + x \cdot (2)e^{-2x}$

$\boxed{y(x) = 2x e^{-2x}}$

Lista 8 - EDO - Página 4

Questão 6 - $y'' - 5y' + 6y = 0$

$R^2 - 5R^1 + 6R^0 = 0$ ∴ $a = 1$ $b = -5$ $c = 6$

$\Delta = 25 - 24$ ∴ $\Delta = 1$

$R = \frac{5 \pm 1}{2}$ ∴ $\boxed{R_1 = 3}$, $\boxed{R_2 = 2}$

✳ Caso 1

$$\boxed{y(x) = C_1 e^{3x} + C_2 e^{2x}}$$

Questão 7 - $y'' - 2y' + y = 0$

$R^2 - 2R^1 + 1R^0 = 0$ ∴ $a = 1$ $b = -2$ $c = 1$

$\Delta = 4 - 4$ ∴ $\Delta = 0$

$R = \frac{2 \pm 0}{2}$ ∴ $\boxed{R_1 = 1}$, $\boxed{R_2 = 1}$

$$\boxed{y(x) = C_1 e^{(1)x} + x C_2 e^{(1)x}}$$

Questão 8 - $y'' + 9y = 0$, $y(0) = 2$, $y'(0) = -1$

$R^2 + 0R^1 + 9R^0 = 0$ ∴ $a = 1$ $b = 0$ $c = 9$

$\Delta = (0)^2 - 4(1)(9)$ ∴ $\Delta = -36$ ∴ $\sqrt{\Delta} = 6i$

$R = \frac{\pm 6i}{2(1)}$ ∴ $R_1 = 3i$, $R_2 = -3i$ ∴ $b = 0$ $a = 3$

$y(x) = C_1 e^{-bx} \cos(ax) + C_2 e^{-bx} \sen(ax)$ ∴ $y(x) = C_1 e^{(0)x} \cos(3x) + C_2 e^{(0)x} \sen(3x)$

$y(x) = C_1 \cos(3x) + C_2 \sen(3x)$

$y'(x) = -3C_1 \sen(3x) + 3C_2 \cos(3x)$

$y(0) = C_1 \cos(3(0)) + C_2 \sen(3(0))$ ∴ $\boxed{C_1 = 2}$

$y'(0) = -3(2) \sen(3(0)) + 3C_2 \cos(3(0))$ ∴ $\boxed{C_2 = -\frac{1}{3}}$

$$\boxed{y(x) = 2\cos(3x) - \frac{1}{3}\sen(3x)}$$

Lista 08 - EDO - Página 5

Questão 9 - $y'' + 3y' + 2y = 0$

$R^2 + 3R^1 + 2R^0 = 0$ ∴ $a=1$ $b=3$ $c=2$

$\Delta = 9-8$ ∴ $\Delta = 1$ ∴ $\sqrt{\Delta} = 1$

$R = \dfrac{-3 \pm 1}{2(1)}$ ∴ $\boxed{R_1 = -1}$ $\boxed{R_2 = -2}$

$\boxed{y(x) = C_1 e^{-x} + C_2 e^{-2x}}$

Questão 10 - $y'' - 7y' + 12y = 0$

$R^2 - 7R^1 + 12R^0 = 0$ ∴ $a=1$ $b=-7$ $c=12$

$\Delta = 49-48$ ∴ $\Delta = 1$ ∴ $\sqrt{\Delta} = 1$

$R = \dfrac{7 \pm 1}{2(1)}$ ∴ $\boxed{R_1 = 4}$ $\boxed{R_2 = 3}$

$\boxed{y(x) = C_1 e^{4x} + C_2 e^{3x}}$

Questão 11 - $y'' - 3y' - 4y = 0$; $Y(0)=1$; $Y'(0)=-1$

$R^2 - 3R^1 - 4R^0 = 0$ ∴ $a=1$ $b=-3$ $c=-4$

$\Delta = 9+16$ ∴ $\Delta = 25$ ∴ $\sqrt{\Delta} = 5$

$R = \dfrac{3 \pm 5}{2(1)}$ ∴ $\boxed{R_1 = 4}$ $\boxed{R_2 = -1}$

$Y(x) = C_1 e^{4x} + C_2 e^{-x}$

$Y'(x) = 4C_1 e^{4x} - C_2 e^{-x}$

$Y(0) = C_1 e^{4(0)} + C_2 e^{-(0)}$ ∴ $C_1 + C_2 = 1$ ∴ $C_2 = 1 - C_1$

$Y'(0) = 4C_1 e^{4(0)} - C_2 e^{-(0)}$ ∴ $4C_1 - C_2 = -1$ ∴ $4(C_1) - (1 - C_1) = -1$

$4C_1 + C_1 - 1 = -1$ ∴ $5C_1 = 0$ ∴ $\boxed{C_1 = 0}$ $\boxed{C_2 = 1}$

$Y(x) = (0) e^{4x} + (1) e^{-x}$ ∴ $\boxed{Y(x) = e^{-x}}$

Lista 08 - EDO - Rafaguar 6

Questão 12 - $y'' - 2y' - 3y = 0$

$R^2 - 2R^1 - 3R^0 = 0$ ∴ $a = 1$ $b = -2$ $c = -3$

$\Delta = 4 + 12$ ∴ $\Delta = 16$ ∴ $\sqrt{\Delta} = 4$

$R = \dfrac{2 \pm 4}{2}$ ∴ $\boxed{R_1 = 3}$ $\boxed{R_2 = -1}$

$$\boxed{y(x) = C_1 e^{3x} + C_2 e^{-x}}$$

Questão 13 - $y'' + 5y' + 6y = 0$

$R^2 + 5R^1 + 6R^0 = 0$ ∴ $a = 1$ $b = 5$ $c = 6$

$\Delta = 25 - 24$ ∴ $\Delta = 1$ ∴ $\sqrt{\Delta} = 1$

$R = \dfrac{-5 \pm 1}{2(1)}$ ∴ $\boxed{R_1 = -2}$ $\boxed{R_2 = -3}$

$$\boxed{y(x) = C_1 e^{-2x} + C_2 e^{-3x}}$$

Questão 14 - $y'' + 4y' + 13y = 0$, $y(0) = 0$ e $y'(0) = 1$

$R^2 + 4R^1 + 13R^0 = 0$ ∴ $a = 1$ $b = 4$ $c = 13$

$\Delta = 16 - 52$ ∴ $\Delta = -36$ ∴ $\sqrt{\Delta} = 6i$

$R = \dfrac{-4 \pm 6i}{2(1)}$ ∴ $R = -2 \pm 3i$ ∴ $b = -2$ $a = 3$

$y(x) = C_1 e^{-2x} \cos(3x) + C_2 e^{-2x} \sin(3x)$

$y'(x) = C_1(-2e^{-2x}\cos(3x) - 3e^{-2x}\sin(3x)) + C_2(-2e^{-2x}\sin(3x) + 3e^{-2x}\cos(3x))$

$y(0) = C_1 e^{-2(0)} \cos(3(0)) + C_2 e^{-2(0)} \sin(3(0))$ ∴ $\boxed{C_1 = 0}$

$y'(0) = (0)(\cdots) + C_2(-2e^{-2(0)}\sin(3(0)) + 3e^{-2(0)}\cos(3(0)))$ ∴ $3C_2 = 1$ ∴ $\boxed{C_2 = \frac{1}{3}}$

$$\boxed{y(x) = \frac{1}{3}e^{-2x}\sin(3x)}$$

Lista 08 - EDO - Página 7

Questão 15 - $Y'' - 4Y' + 4Y = 0$

$R^2 - 4R^1 + 4R^0 = 0$ ∴ $a=1$ $b=-4$ $C=4$

$\Delta = 16 - 16$ ∴ $\Delta = 0$ ∴ $\sqrt{\Delta} = 0$

$R = \dfrac{4}{2(1)}$ ∴ $\boxed{R_1 = 2}$ $\boxed{R_2 = 2}$

$\boxed{Y(x) = C_1 e^{2x} + x C_2 e^{2x}}$

Questão 16 - $Y'' + 6Y' + 25Y = 0$

$R^2 + 6R^1 + 25R^0 = 0$ ∴ $a=1$ $b=6$ $C=25$

$\Delta = 36 - 100$ ∴ $\Delta = -64$ ∴ $\sqrt{\Delta} = 8i$

$R = \dfrac{-6 \pm 8i}{2(1)}$ ∴ $R = -3 \pm 4i$ ∴ $b=-3$ $a=4$

$\boxed{Y(x) = C_1 e^{-3x} \cos(4x) + C_2 e^{-3x} \operatorname{sen}(4x)}$

Questão 17 - $Y'' - 2Y' - 8Y = 0$, $Y(0) = 1$ e $Y'(0) = 2$

$R^2 - 2R^1 - 8R^0 = 0$ ∴ $a=1$ $b=-2$ $C=-8$

$\Delta = 4 + 32$ ∴ $\Delta = 36$ ∴ $\sqrt{\Delta} = 6$

$R = \dfrac{2 \pm 6}{2(1)}$ ∴ $\boxed{R_1 = 4}$ $\boxed{R_2 = -2}$

$Y(x) = C_1 e^{4x} + C_2 e^{-2x}$

$Y'(x) = 4C_1 e^{4x} - 2C_2 e^{-2x}$

$Y(0) = C_1 e^{4(0)} + C_2 e^{-2(0)}$ ∴ $Y(0) = C_1 + C_2 = 1$ ∴ $C_2 = 1 - C_1$

$Y'(0) = 4C_1 e^{4(0)} - 2C_2 e^{-2(0)}$ ∴ $4C_1 - 2C_2 = 2$ ∴ $4C_1 - 2 + 2C_1 = 2$ ∴ $6C_1 = 4$

$\boxed{C_1 = \dfrac{2}{3}}$ $\boxed{C_2 = \dfrac{1}{3}}$ ∴ $\boxed{Y(x) = \dfrac{2}{3} e^{4x} + \dfrac{1}{3} e^{-2x}}$

Lista 08 - EDO - Página 8

Questão 18 - $Y'' + 2Y' - 3Y = 0$

$R^2 + 2R^1 - 3R^0 = 0 \therefore a=1 \ b=2 \ c=-3 \quad \therefore \Delta = 16 \quad \therefore \sqrt{\Delta} = 4$

$R = \dfrac{-2 \pm 4}{2(1)} \quad \therefore \boxed{R_1 = 1} \ \boxed{R_2 = -3}$

$\boxed{Y(x) = C_1 e^{1x} + C_2 e^{-3x}}$

Questão 19 - $Y'' + 6Y' + 8Y = 0 \ , \ Y(0) = 1 \ e \ Y'(0) = -2$

$R^2 + 6R^1 + 8R^0 = 0 \therefore a=1 \ b=6 \ c=8 \quad \therefore \Delta = 4 \quad \therefore \sqrt{\Delta} = 2$

$R = \dfrac{-6 \pm 2}{2(1)} \quad \therefore \boxed{R_1 = -2} \ \boxed{R_2 = -4} \quad \therefore Y(x) = C_1 e^{-2x} + C_2 e^{-4x}$

$Y'(x) = -2C_1 e^{-2x} - 4C_2 e^{-4x}$

$Y(0) = C_1 e^{-2(0)} + C_2 e^{-4(0)} \quad \therefore C_1 + C_2 = 1 \quad \therefore C_2 = 1 - C_1$

$Y'(0) = -2(C_2) e^{-2(0)} - 4(1-C_1) e^{-4(0)} \quad \therefore Y'(0) = -2C_1 - 4 + 4C_1 = -2$

$2C_1 = 2 \therefore \boxed{C_1 = 1} \ \boxed{C_2 = 0} \quad \boxed{Y(x) = e^{-2x}}$

Questão 20 - $Y'' - 3Y' - 10Y = 0$

$R^2 - 3R^1 - 10R^0 = 0 \therefore a=1 \ b=-3 \ c=-10 \quad \therefore \Delta = 49 \therefore \sqrt{\Delta} = 7$

$R = \dfrac{3 \pm 7}{2(1)} \quad \therefore \boxed{R_1 = 5} \ \boxed{R_2 = -2}$

$\boxed{Y(x) = C_1 e^{5x} + C_2 e^{-2x}}$