Lista 4

1

① $y' = 3x^2y$

$\frac{dy}{dx} = 3x^2y \Rightarrow \frac{dy}{y} = 3x^2dx \Rightarrow \frac{1}{y}dy = 3x^2dx \Rightarrow \int \frac{1}{y}dy = \int 3x^2dx \Rightarrow$

$\Rightarrow \ln|y| = 3\int \frac{x^2}{\cancel{2}}dx \Rightarrow \ln|y| = \cancel{3}.\frac{x^3}{\cancel{3}} \Rightarrow e^{\ln|y|} = e^{x^3} \Rightarrow \boxed{y = e^{x^3} + C}$

② $y' = \frac{(2x+1)}{y} \Rightarrow \frac{dy}{dx}.y = (2x+1) \Rightarrow y\,dy = (2x+1)dx \Rightarrow \int y\,dy = \int 2x\,dx + \int 1\,dx$

$\Rightarrow \frac{y^2}{2} = \cancel{2}.\frac{x^2}{\cancel{2}} + x \Rightarrow y^2 = 2x^2 + 2x \Rightarrow \boxed{y = -\sqrt{2x^2+2x}}$

③ $y' = \frac{(2x+y)}{x}$

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③ $y' = \frac{(2x+y)}{x} \Rightarrow y' = \frac{2x}{x} + \frac{y}{x} \Rightarrow y' = 2 + \frac{y}{x} \Rightarrow y' - \frac{1}{x} \cdot y = 2$

$\Rightarrow y' - \frac{1}{x} y = 2 \Rightarrow$ Pelo 4º caso de integração $\Rightarrow P(x) = -\frac{1}{x}$ $\Rightarrow \mu(x) = e^{\int -\frac{1}{x} dx}$

$f(x) = 2$

$\Rightarrow \mu = e^{-1\int \frac{1}{x} dx} \Rightarrow \mu = e^{-1 \ln(x)} \Rightarrow \mu = e^{\ln x^{-1}} \Rightarrow \mu = e^{\ln \frac{1}{x}} \Rightarrow \boxed{\mu = \frac{1}{x}}$

$\Rightarrow$ Fator de integração: $\mu = \frac{1}{x} \Rightarrow$ Aplicando μ na fórmula da solução geral $\Rightarrow y = \frac{1}{\mu} \cdot \int \mu \cdot f(x) dx \Rightarrow y = \frac{1}{\frac{1}{x}} \cdot \int \frac{1}{x} \cdot 2 \cdot dx \Rightarrow$

$\Rightarrow y = 1 \cdot \frac{x}{1} \cdot 2 \int \frac{1}{x} \cdot dx \Rightarrow \boxed{y = 2x \cdot \ln|x| + C}$

(2)

(4) $\int \frac{1}{y^2}dy = \cancel{4}^{2} \cdot \frac{x^2}{\cancel{2}} \Rightarrow \int y^{-2} dy = 2x^2 \Rightarrow \frac{y^{-2+1}}{-2+1} = 2x^2 \Rightarrow \frac{y^{-1}}{-1} = 2x^2 \Rightarrow$

$\Rightarrow -\frac{1}{y} = 2x^2 \Rightarrow -\frac{1}{2x^2} = y \Rightarrow \boxed{y = -\frac{1}{2x^2}}$

(5) $y' = \frac{y}{2x-1} \Rightarrow \frac{dy}{dx} = \frac{y}{2x-1} \Rightarrow \frac{1}{y}dy = \frac{1}{2x-1}dx \Rightarrow \int \frac{1}{y}dy = \int \frac{1}{2x-1}dx \Rightarrow$

$\Rightarrow \ln|y| = \ldots \Rightarrow$ $u = 2x-1$, $du = 2dx$, $\hookrightarrow dx = \frac{du}{2}$ $\Rightarrow \ln|y| = \int \frac{1}{u} \cdot \frac{du}{2} \Rightarrow \ln|y| = \frac{1}{2} \cdot \ln|2x-1|$

$\Rightarrow \ln|y| = \ln|2x-1|^{\frac{1}{2}} \Rightarrow e^{\ln|y|} = e^{\ln|2x-1|^{\frac{1}{2}}} \Rightarrow \boxed{y = e^{\ln|\sqrt{2x-1}|}}$

(6) $y' - \frac{(1-y^2)}{x} = 0$

(7) $y' = \frac{2xy}{1+x^2} \Rightarrow \frac{dy}{dx} = \frac{2xy}{1+x^2} \Rightarrow \frac{1}{y}dy = \frac{2x}{1+x^2}dx \Rightarrow \int \frac{1}{y}dy = \int \frac{2x}{1+x^2}dx$

$\Rightarrow \ln|y| = 2\int \frac{x}{1+x^2}dx \Rightarrow$ $u = 1+x^2$, $du = 2x\,dx$, $\hookrightarrow \frac{du}{2} = x\,dx$ $\Rightarrow 2\int \frac{1}{u} \cdot \frac{du}{2} \Rightarrow \frac{\cancel{2}}{\cancel{2}}\int \frac{1}{u}du \Rightarrow \ln|1+x^2|$

$\Rightarrow \ln|y| = \ln|1+x^2| \Rightarrow e^{\ln|y|} = e^{\ln|1+x^2|} \Rightarrow \boxed{y = 1+x^2}$

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(8) $y' = \frac{4x}{y^2+1}$

$\Rightarrow \frac{dy}{dx}\cdot(y^2+1) = 4x \Rightarrow (y^2+1)\,dy = 4x\,dx \Rightarrow \int y^2+1\, dy = 4\int x\,dx \Rightarrow$

$\Rightarrow \int y^2 dy + \int 1\, dy = \cancel{4}^{2}\cdot\frac{x^2}{\cancel{2}} \Rightarrow \frac{y^3}{3} + y = 2x^2$

$\Rightarrow y' = \frac{4x}{y^2+1} \Rightarrow$ Multiplicando por y $\Rightarrow y\cdot y' = \frac{4xy}{y^2+1} \Rightarrow y\,dy = \frac{4xy}{y^2+1}\,dx$

$\Rightarrow \int y\,dy = \int \frac{4xy}{y^2+1}\,dx \Rightarrow$ $u = y^2+1$, ↳ $y = \sqrt{u-1}$ $\Rightarrow \frac{du}{2} = \frac{4x\sqrt{u-1}}{u}\,dx \Rightarrow$

$du = 2y\,dy$

↳ $y\,dy = \frac{du}{2}$

$\Rightarrow \frac{du}{2} = \frac{4x\sqrt{u-1}}{\sqrt{u^2}}\,dx \Rightarrow \frac{du}{2} = 4x\cdot dx\cdot\sqrt{\frac{u-1}{u^2}} \Rightarrow \frac{du}{2}\cdot\frac{1}{\sqrt{\frac{u-1}{u^2}}} = 4x\,dx$

$\Rightarrow \int \frac{du}{\cancel{2}}\cdot\frac{1}{\left(\frac{u-1}{u^2}\right)^{\frac12}} = \int 8x\,dx \Rightarrow \int \frac{1}{\left(\frac{u-1}{u^2}\right)^{\frac12}}\,du = \cancel{8}^{4}\cdot\frac{x^2}{\cancel{2}} \Rightarrow \left(\frac{u-1}{u^2}\right)^{\frac12} = (a)^{\frac12}$

$\Rightarrow \int \frac{1}{a^{\frac12}}\,da = 4x^2 \Rightarrow \frac{a^{-\frac12+1}}{-\frac12+1} = 4x^2 \Rightarrow \frac{a^{\frac12}}{\frac12} = 4x^2 \Rightarrow \sqrt{a} = 2x^2$

$\Rightarrow \sqrt{\frac{u-1}{u^2}} = 2x^2 \Rightarrow \frac{u-1}{u^2} = 4x^4 \Rightarrow \frac{(y^2+\cancel{1})-\cancel{1}}{y^4+2y^2+1} = 4x^4 \Rightarrow \frac{y^2}{(y^2+1)^2} = 4x^4$

$\Rightarrow \frac{y}{y^2+1} = 2x^2 \Rightarrow \frac{\cancel{y}(1)}{\cancel{y}(y+\frac{1}{y})} = 2x^2 \Rightarrow \frac{1}{y+\frac{1}{y}} = 2x^2 \Rightarrow \frac{y}{y^2+1} = 2x^2 \Rightarrow \boxed{y = 2x^2y^2 + 2x^2}$

$\Rightarrow -$

9) $y' = \frac{(1+y^2)}{x} \Rightarrow \frac{dy}{dx} = \frac{1+y^2}{x} \Rightarrow \int \frac{1}{1+y^2}\,dy = \int \frac{1}{x}\,dx \Rightarrow y = \tan\theta$

$dy = \sec^2\theta\, d\theta$

$\Rightarrow \int \frac{1}{1+\tan^2\theta}\sec^2\theta\, d\theta \Rightarrow \int \frac{1}{\cancel{\sec^2\theta}}\cancel{\sec^2\theta}\, d\theta \Rightarrow \frac{\overset{\tan^2\theta}{\cancel{\sin^2\theta}}}{\cancel{\cos^2\theta}} + \frac{\overset{1}{\cancel{\cos^2\theta}}}{\cancel{\cos^2\theta}} = \frac{1}{\cancel{\cos^2\theta}}\ \sec^2\theta$

$\Rightarrow \int 1\, d\theta = \theta = \arctan(y) \Rightarrow \arctan(y) = \ln|x| \Rightarrow \boxed{y = \tan(\ln|x|) + c}$

10) $y'(x+1) - 2y = 0 \Rightarrow \frac{dy}{dx}(x+1) = 2y \Rightarrow \frac{dy}{dx} = \frac{2y}{(x+1)} \Rightarrow \frac{1}{2y}\,dy = \frac{1}{x+1}\,dx$

$\Rightarrow \frac{1}{2}\int \frac{1}{y}\,dy = \int \frac{1}{x+1}\,dx \Rightarrow \frac{1}{2}\cdot \ln|y| = \begin{matrix} u = x+1 \\ du = dx \end{matrix} \Rightarrow \int \frac{1}{u}\,du \Rightarrow \ln|x+1|$

$\Rightarrow \frac{1}{2}\ln|y| = \ln|x+1| \Rightarrow e^{\ln|y|} = e^{2\ln|x+1|} \Rightarrow \boxed{y = e^{2\ln|x+1|}}$

11) $y' - x^2y = 0 \Rightarrow \frac{dy}{dx} = x^2y \Rightarrow \int \frac{1}{y}\,dy = \int x^2\,dx \Rightarrow \ln|y| = \frac{x^3}{3} \Rightarrow \boxed{y = e^{\frac{x^3}{3}}}$

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12) $y' = x^2 e^{-y} \Rightarrow \frac{dy}{dx} = x^2 e^{-y} \Rightarrow \frac{1}{e^{-y}}\, dy = x^2 dx \Rightarrow e^y dy = x^2 dx \Rightarrow \cancel{y = \ln x^2}$

$\int e^y dy = \int x^2 dx \Rightarrow e^y = \frac{x^3}{3} \Rightarrow y = \ln \frac{x^3}{3} \Rightarrow y = \ln x^3 - \ln 3 \Rightarrow \boxed{y = 3\ln x - \ln 3 + C}$

13) $y' y^2 - 3x^2 = 0 \Rightarrow \frac{dy}{dx} \cdot y^2 = 3x^2 \Rightarrow y^2 dy = 3x^2 dx \Rightarrow \int y^2 dy = 3\int x^2 dx$

$\Rightarrow \frac{y^3}{3} = \cancel{3} \cdot \frac{x^3}{\cancel{3}} \Rightarrow y^3 = 3x^3 \Rightarrow y = \sqrt[3]{3x^3} \Rightarrow \boxed{y = \sqrt[3]{3} \cdot x + C}$

14) $y' = \frac{(y+1)}{(x+1)^2} \Rightarrow \frac{dy}{dx} = \frac{(y+1)}{(x+1)^2} \Rightarrow \frac{1}{(y+1)} dy = \frac{1}{(x+1)^2} dx \Rightarrow \int \frac{1}{y+1} dy = \int \frac{1}{(x+1)^2} dx$

$\Rightarrow \begin{matrix} u = x+1 \\ du = dx \end{matrix} \Rightarrow \int \frac{1}{u^2} du \Rightarrow \int u^{-2} du \Rightarrow \frac{u^{-2+1}}{-2+1} = \frac{(x+1)^{-1}}{-1} \Rightarrow -\frac{1}{(x+1)}$

$\Rightarrow \begin{matrix} u = y+1 \\ du = dy \end{matrix} \Rightarrow \int \frac{1}{u} du = \ln|u| = \ln|y| \Rightarrow \ln|y| = -\frac{1}{(x+1)} \Rightarrow$

$\Rightarrow \boxed{y = e^{-\frac{1}{(x+1)}} + C}$

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15) $y' - 2xy\ln(x) = 0 \Rightarrow \frac{dy}{dx} = 2xy\ln(x) \Rightarrow \frac{1}{y}\,dy = 2x\ln(x)\,dx \Rightarrow$

$\int \frac{1}{y}\,dy = 2\int x\ln(x)\,dx \Rightarrow$ $u = \ln(x)$ $\quad dv = x\,dx$

$du = \frac{1}{x}\,dx \quad v = \frac{x^2}{2}$ $\Rightarrow \int x\ln(x)\,dx = uv - \int v\,du$

$\Rightarrow \frac{x^2\ln(x)}{2} - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx \Rightarrow \frac{x^2\ln(x)}{2} - \int \frac{x}{2}\,dx \Rightarrow \frac{x^2\ln(x)}{2} - \frac{1}{2}\int x\,dx$

$\Rightarrow \frac{x^2\ln(x)}{2} - \frac{1}{2}\cdot\frac{x^2}{2} \Rightarrow \frac{x^2\ln(x)}{2} - \frac{x^2}{4} \Rightarrow \ln|y| = 2\left(\frac{x^2\ln(x)}{2} - \frac{x^2}{4}\right)$

$\Rightarrow \ln|y| = \frac{x^2\ln(x)}{\cancel{1}_2} - \frac{x^2}{\cancel{2}_1} \Rightarrow \ln|y| = \frac{2x^2\ln(x) - x^2}{2} \Rightarrow \boxed{y = e^{\frac{2x^2\ln(x) - x^2}{2}} + C}$

Lista 4

16) $y'+2y'x+y'x^2-2x-3x^2=1 \Rightarrow y'x^2+2xy'+y'=3x^2+2x+1$

$\Rightarrow \cancel{x} \; y'(x^2+2x+1)=3x^2+2x+1 \Rightarrow y'=\frac{3x^2+2x+1}{x^2+2x+1} \Rightarrow dy=\frac{3x^2+2x+1}{x^2+2x+1}dx$

$\Rightarrow \int dy = \int \frac{3x^2}{x^2+2x+1}dx + \int \frac{2x}{x^2+2x+1}dx + \int \frac{1}{x^2+2x+1}dx \Rightarrow 3\int \frac{x^2}{x^2+2x+1}dx + 2\int \frac{x}{x^2+2x+1}dx + \int \frac{1}{x^2+2x+1}d$

$\Rightarrow \Delta=(2)^2-4(1)(1)=0 \Rightarrow x=\frac{-2\pm 0}{2}=-\frac{2}{2}=-1 \Rightarrow (x+1)(x+1)$

$\Rightarrow I=3\int \frac{x^2}{(x+1)^2}dx \; , \; II=2\int \frac{x}{(x+1)^2}dx + III=\int \frac{1}{(x+1)^2}dx$

$I-\; u=x+1 \to x=u-1$
$du=dx$
$\Rightarrow 3\int \frac{(u-1)^2}{u^2}du \Rightarrow 3\int \frac{u^2-2u+1}{u^2}du \Rightarrow 3\int \frac{u^2}{u^2}du - 6\int \frac{u\,du}{u^2} + \int$

$\Rightarrow 3\int 1\,du - 6\int \frac{1}{u}du + 3\int \frac{1}{u^2}du \Rightarrow \; \hookrightarrow \int u^{-2}du = \frac{u^{-2+1}}{-2+1} = \frac{u^{-1}}{-1} = -\frac{1}{u}$

$\Rightarrow 3u - 6\ln|u| + 3\left(-\frac{1}{u}\right) \Rightarrow 3u - 6\ln|u| - \frac{3}{u}$

$\Rightarrow$ Verificando $\Rightarrow 3\int \frac{u^2-2u+1}{u^2}du = 3\left(\int 1\,du - 2\int \frac{1}{u}du + \int \frac{1}{u^2}du\right)$ ✓

$I - \frac{3(x+1)}{\cancel{1}(x+1)} - \frac{6\ln|x+1|}{\cancel{1}(x+1)} - \frac{3}{\cancel{(x+1)}\,1} = \frac{3(x+1)^2-6(x+1)\ln|x+1|-3}{(x+1)} = \frac{3(x+1)^2-6x\ln|x+1|-6\ln|x+1|-3}{(x+1)}$

Lista 4

→ continuação da 16

$$II - 2\int \frac{x}{(x+1)^2}dx \Rightarrow \begin{matrix} u = x+1 \\ du = dx \\ x = u-1 \end{matrix} \Rightarrow 2\int \frac{u-1}{u^2}du \Rightarrow 2\left(\int \frac{u}{u^2}du - \int \frac{1}{u^2}du\right)$$

$$\Rightarrow 2\int \frac{1}{u}du - 2\int \frac{1}{u^2}du \Rightarrow 2\ln|u| - 2\left(-\frac{1}{u}\right) \Rightarrow \frac{2\ln|u|}{1/u} + \frac{2}{\cancel{u}/1} = \frac{2u\ln|u|+2}{u}$$

$$\Rightarrow \frac{2u\ln|u|+2}{u} \Rightarrow \frac{2(x+1)\ln|x+1|+2}{(x+1)} = \frac{2x\ln|x+1|+2\ln|x+1|+2}{(x+1)}$$

$$III - \int \frac{1}{(x+1)^2}dx \Rightarrow \begin{matrix} u = x+1 \quad du = dx \\ 1 = u-x \end{matrix} \Rightarrow \int \frac{u-x}{u^2}du = \int \frac{u}{u^2}du - x\int \frac{1}{u^2}du$$

$$\Rightarrow \int \frac{1}{u}du - x\int \frac{1}{u^2}du \Rightarrow \ln|u| - x\left(-\frac{1}{u}\right) \Rightarrow \frac{\ln|u|}{1/u} + \frac{x}{\cancel{u}/1} = \frac{u\ln|u|+x}{u} \Rightarrow$$

$$\Rightarrow \frac{(x+1)\ln|x+1|+x}{(x+1)} = \frac{x\ln|x+1|+\ln|x+1|+x}{(x+1)}$$

$$I + II + III \Rightarrow \frac{3(x+1)^2 - 6x\ln|x+1| - 6\ln|x+1| - 3}{(x+1)} + \frac{2x\ln|x+1| + 2\ln|x+1| + 2}{(x+1)} + \frac{x\ln|x+1| + \ln|x+1| + x}{(x+1)}$$

$$\Rightarrow \frac{3(x+1)^2 - 3x\ln|x+1| - 3\ln|x+1| - 1 + x}{(x+1)} = \frac{3x^2+6x+3 - 3x\ln|x+1| - 3\ln|x+1| - 1 + x}{(x+1)}$$

$$\Rightarrow \frac{3x^2+7x-2 - 3x\ln|x+1| - 3\ln|x+1|}{(x+1)} \Rightarrow \boxed{\frac{3x^2+7x-2-3\ln|x+1|(x+1)}{(x+1)} + C}$$

$$\boxed{y = \frac{3x^2+7x-2-3\ln|x+1|(x+1)}{(x+1)} + C}$$

Lista 4

17) $y'' = \dfrac{sen(x)y'}{(1+x^2)^{-1}} \Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{sen(x)}{(1+x^2)^{-1}} \cdot \dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{sen(x)}{(1+x^2)^{-1}}$

$\Rightarrow dy = (1+x^2)sen(x)dx \Rightarrow \int dy = (1+x^2)sen(x)\,dx \Rightarrow y = \int (1+x^2)sen(x)dx$

$\Rightarrow u = (1+x^2) \quad dv = sen(x)$
$du = 2x \quad v = (-cos(x)) \Rightarrow \int u\,dv = uv - \int v\,du \Rightarrow (1+x^2)(-cos(x)) - \int 2x(-cos(x))$

$\Rightarrow -x^2cos(x) - cos(x) - \int 2x(-cos(x))\,dx \Rightarrow -x^2cos(x) - cos(x) - (-2)\int x\,cos(x)dx$

$\Rightarrow -x^2cos(x) - cos(x) + 2\int x\,cos(x)dx \rightarrow u = x \quad dv = cos(x)$
$du = dx \quad v = sen(x) \Rightarrow uv - \int v\,du \Rightarrow$

$\Rightarrow -x^2cos(x) - cos(x) + 2\left(x\,sen(x) - \int sen(x)dx\right) \Rightarrow -x^2cos(x) - cos(x) + 2(x\,sen(x) - \ldots$

$\Rightarrow -x^2cos(x) - cos(x) + 2(x\,sen(x) \cancel{+} (-cos(x))) \Rightarrow -x^2cos(x) - cos(x) + 2(x\,sen(x) + cos(x))$

$\Rightarrow -x^2cos(x) - cos(x) + 2x\,sen(x) + 2cos(x) \Rightarrow \boxed{y = -x^2cos(x) + 2x\,sen(x) + cos(x) + C}$

Lista 04

(18) $\frac{1}{y'} = \frac{(x-1)^3}{3x^2-5x+2} \Rightarrow y' = \frac{3x^2-5x+2}{(x-1)^3} \Rightarrow \Delta = (-5)^2 - 4(3)(2) = 25-24 = 1$

$\Rightarrow x = \frac{5\pm 1}{6} \Rightarrow x_1 = \frac{6}{6} = 1$

$x_2 = \frac{4}{6} = \frac{2}{3}$

$\Rightarrow y' = \frac{(x-1)(x-\frac{2}{3})}{(x-1)^3} \Rightarrow y' = \frac{(x-\frac{2}{3})}{(x-1)^2} \Rightarrow dy = \frac{(x-\frac{2}{3})}{(x-1)^2}dx \Rightarrow \int dy = \int \frac{(x-\frac{2}{3})}{(x-1)^2}dx$

I II

$\Rightarrow y = \int \frac{x}{(x-1)^2}dx - \int \frac{\frac{2}{3}}{(x-1)^2}dx \Rightarrow y = \int \frac{x}{(x-1)^2}dx - \frac{2}{3}\int \frac{1}{(x-1)^2}dx$

I $u = x-1$, $du = dx$, $x = u+1$ $\Rightarrow \int \frac{u+1}{u^2}du \Rightarrow \int \frac{u}{u^2}du + \int \frac{1}{u^2}du \Rightarrow \int \frac{1}{u}du + \int \frac{1}{u^2}du$

$\Rightarrow \ln|u| - \frac{u^{-1}}{1} \Rightarrow \frac{\ln|u|}{1/u} - \frac{1}{u} = \frac{u\ln|u|-1}{u} \Rightarrow \boxed{\frac{(x-1)\ln|x-1|-1}{x-1}}$

II $-\frac{2}{3}\int \frac{1}{(x-1)^2}dx$ $u = x-1$, $du = dx$, $1 = x-u$ $\Rightarrow -\frac{2}{3}\int \frac{x-u}{u^2}du \Rightarrow -\frac{2}{3}\left(\int \frac{x}{u^2}du - \int \frac{u}{u^2}du\right) \Rightarrow$

$\Rightarrow -\frac{2}{3}\left(x\int \frac{1}{u^2}du - \int \frac{1}{u}du\right) \Rightarrow -\frac{2}{3}\left(x\left(-\frac{1}{u}\right) - (\ln|u|)\right) \Rightarrow \frac{2x}{3u} + \frac{2\ln|u|}{3}$

$\Rightarrow \frac{2x+2u\ln|u|}{3u} \Rightarrow \frac{2x+2(x-1)\ln|x-1|}{3(x-1)} = \boxed{\frac{2x+2x\ln|x-1|-2\ln|x-1|}{3(x-1)}}$

$\Rightarrow$ I + II $= \frac{(x-1)\ln|x-1|-1}{(x-1)} + \frac{2x+2x\ln|x-1|-2\ln|x-1|}{3(x-1)}$

$\Rightarrow \frac{3x\ln|x-1|-3\ln|x-1|-3+2x+2x\ln|x-1|+2\ln|x-1|}{3(x-1)} = \boxed{\frac{x\ln|x-1|-\ln|x-1|-2x-3}{3(x-1)} + C}$