Lista - EDO - Página 1

(1) $y'' + 3y' + 2y = 2e^t$

I - Homogênea: $y'' + 3y' + 2y = 0$

$a = 1 \quad b = 3 \quad c = 2$

$\Delta = 9 - 8 \therefore \Delta = 1$

$R = \dfrac{-3 \pm 1}{2(1)} \therefore R_1 = -1 \; ; \; R_2 = -2$

$y_c(t) = C_1 e^{-x} + C_2 e^{-2x}$

II - Particular

$$\left.\begin{aligned} y_p(t) &= Ce^t \\ y_p'(t) &= Ce^t \\ y_p''(t) &= Ce^t \end{aligned}\right\} \quad \begin{aligned} &(Ce^t) + 3(Ce^t) + 2(Ce^t) = 2e^t \therefore 6Ce^t = 2e^t \therefore C = 1/3 \\ &\Rightarrow y_p(t) = \frac{1}{3}e^t \end{aligned}$$

III - Solução Geral

$y(t) = y_c(t) + y_p(t)$

$$\boxed{y(t) = C_1 e^{-t} + C_2 e^{-2t} + \frac{1}{3}e^t}$$



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Lista 1 - EDO - Página 2

(2) $y'' - 2y' - 3y = t^2 + 1$

I - Homogênea

$y'' - 2y' - 3y = 0 \therefore a = 1 \; b = -2 \; c = -3 \therefore \Delta = 16 \therefore \sqrt{\Delta} = 4 \therefore R = \frac{2 \pm 4}{2(1)}$

$R_1 = 3 \quad R_2 = -1 \therefore \boxed{y_c(t) = C_1 e^{3t} + C_2 e^{-t}}$

II - Particular

Para $Dx^2$, a particular é $Ax^2 + Bx + C$

$y_p(t) = At^2 + Bt + C$

$y_p'(t) = 2At + B$

$y_p''(t) = 2A$

$(2A) - 2(2At + B) - 3(At^2 + Bt + C) = t^2 + 1$

$(2A) + (-4At - 2B) + (-3At^2 - 3Bt - 3C) = t^2 + 1$

$\Rightarrow -3At^2 = t^2 \therefore -3A = 1 \therefore \boxed{A = -\frac{1}{3}}$

$\Rightarrow -4At - 3Bt = 0t \therefore -4 \cdot \left(-\frac{1}{3}\right) - 3B = 0 \therefore \frac{4}{3} = 3B \therefore \boxed{B = \frac{4}{9}}$

$\Rightarrow 2A - 2B - 3C = 1 \therefore 2\left(-\frac{1}{3}\right) - 2\left(\frac{4}{9}\right) - 3C = 1 \therefore -\frac{2}{3} - \frac{8}{9} - \frac{3C}{1} = 1$

$\to$ ~~$\times(9) \therefore 6 - 8 - 27C = 1$~~ $\Rightarrow \frac{-6 - 8 - 27C}{9} = 1 \therefore -14 - 27C = 9$

$\to -27C = 23 \therefore \boxed{C = -\frac{23}{27}}$

$$\boxed{y_p(t) = -\frac{1}{3}t^2 + \frac{4}{9}t - \frac{23}{27}}$$

III - Solução Geral

$y(t) = y_c(t) + y_p(t) \therefore \boxed{y(t) = C_1 e^{3t} + C_2 e^{-t} - \frac{1}{3}t^2 + \frac{4}{9}t - \frac{23}{29}}$

Lista ~~1~~ - EDO - Página 3

③ $y'' + 5y' + 6y = 3\cos(t)$

I- Homogênea

$y'' + 5y' + 6y = 0 \therefore a = 1 \; b = 5 \; c = 6 \therefore \Delta = 1 \therefore \sqrt{\Delta} = 1 \therefore R = \frac{-5 \pm 1}{2(1)}$

$R_1 = -2 \quad R_2 = -3 \therefore \boxed{y_c(t) = C_1 e^{-2t} + C_2 e^{-3t}}$

II- Particular

$y_p(t) = A\,\text{sen}(t) + B\cos(t)$

$y_p'(t) = A\cos(t) - B\,\text{sen}(t)$ $\left(-A\,\text{sen}(t) - B\cos(t)\right) + 5A\cos(t) - 5B\,\text{sen}(t) + 6A\,\text{sen}(t) + 6B\cos(t)$

$y_p''(t) = -A\,\text{sen}(t) - B\cos(t)$

$\therefore -A\,\text{sen}(t) - B\cos(t) + 5A\cos(t) - 5B\,\text{sen}(t) + 6A\,\text{sen}(t) + 6B\cos(t) = 3\cos(t)$

$-A\,\text{sen}(t) - 5B\,\text{sen}(t) + 6A\,\text{sen}(t) = 0\,\text{sen}(t) \therefore 5A\,\text{sen}(t) - 5B\,\text{sen}(t) = 0\,\text{sen}(t)$

$5A - 5B = 0 \Rightarrow \boxed{A = B}$

$-B\cos(t) + 5A\cos(t) + 6B\cos(t) = 3\cos(t) \therefore 5A\cos(t) + 5B\cos(t) = 3\cos(t)$

$10A = 3 \therefore A = \frac{3}{10}, \; B = \frac{3}{10}$

$y_p(t) = \frac{3}{10}\,\text{sen}(t) + \frac{3}{10}\cos(t)$

III - Solução geral

$$\boxed{y(t) = C_1 e^{-2t} + C_2 e^{-3t} + \frac{3}{10}\,\text{sen}(t) + \frac{3}{10}\cos(t)}$$

Lista 1 - EDO - Página 4

4) $y'' - 4y' + 4y = 4e^{t}$

I - Homogênea

$y'' - 4y' + 4y = 0 \therefore a = 1\ b = -4\ c = 4 \therefore \Delta = 0 \therefore R = \frac{4 \pm 0}{2(1)} \therefore \boxed{R_1 = 2}\ \boxed{R_2 = 2}$

$\boxed{y_c(t) = C_1 e^{2t} + x C_2 e^{2t}}$

II - Particular

$y_p(t) = Ce^{t}$

$y_p'(t) = Ce^{t}$

$y_p''(t) = Ce^{t}$

$(Ce^{t}) - \cancel{4Ce^{t}} + \cancel{4Ce^{t}} = 4e^{t} \therefore C = 4 \therefore \boxed{y_p(t) = 4e^{t}}$

III - Solução geral

$y(t) = y_c(t) + y_p(t) \therefore \boxed{y(t) = C_1 e^{2t} + x C_2 e^{2t} + 4e^{t}}$

5) $y'' + 2y' + y = \operatorname{sen}(t)$

I - Homogênea

$y'' + 2y' + y = 0 \therefore a = 1\ b = 2\ c = 1 \therefore \Delta = 0 \therefore R = \frac{-2 \pm 0}{2(1)} \therefore \boxed{R_1 = -1}\ \boxed{R_2 = -1}$

$\boxed{y_c(t) = C_1 e^{-t} + x C_2 e^{-t}}$

II - Particular

$y_p(t) = D\operatorname{sen}(t) + E\cos(t)$

$y_p'(t) = D\cos(t) - E\operatorname{sen}(t)$

$y_p''(t) = -D\operatorname{sen}(t) - E\cos(t)$

$(-D\operatorname{sen}(t) - E\cos(t)) + (2D\cos(t) - 2E\operatorname{sen}(t)) + (D\operatorname{sen}(t) + E\cos(t)) = \operatorname{sen}(t)$

$-E\cancel{\cos(t)} + 2D\cos(t) + E\cancel{\cos(t)} = 0\cos(t) \therefore 2D = 0 \therefore \boxed{D = 0}$

$\Rightarrow -(0)\cancel{\operatorname{sen}(t)}^{0} - 2E\operatorname{sen}(t) + (0)\cancel{\operatorname{sen}(t)}^{0} = \operatorname{sen}(t) \therefore -2E = 1 \therefore \boxed{E = -\frac{1}{2}}$

III - Solução Geral: $\boxed{y(x) = C_1 e^{-t} + x C_2 e^{-t} - \frac{1}{2}\cos(t)}$

Lista - EDO - Página 5

6) $y'' - 3y' + 2y = 5\,sen(t)$

I - Homogênea

$y'' - 3y' + 2y = 0 \therefore a = 1\ b = -3\ c = 2 \therefore \Delta = 1 \therefore \sqrt{\Delta} = 1 \therefore R = \frac{3 \pm 1}{2(1)} \therefore \boxed{R_1 = 2}\ \boxed{R_2 = 1}$

$$\boxed{Y_c(t) = c_1 e^{2t} + c_2 e^{t}}$$

II - Particular

$Y_p(t) = D\,sen(t) + E\cos(t)$

$Y_p'(t) = D\cos(t) - E\,sen(t)$

$Y_p''(t) = -D\,sen(t) - E\cos(t)$

$\Rightarrow -D\,sen(t) - E\cos(t) - 3D\cos(t) + 3E\,sen(t) + 2D\,sen(t) + 2E\cos(t) = 5\,sen\,t$

* $D\,sen(t) + E\cos(t) - 3D\cos(t) + 3E\,sen(t) = 5\,sen(t)$

$E\cos(t) - 3D\cos(t) = 0\cos(t) \therefore E - 3D = 0 \therefore \boxed{E = 3D}$

$\Rightarrow D\,sen(t) + 3E\,sen(t) = 5\,sen(t) \therefore D\,sen(t) + 3(3D)\,sen(t) = 5\,sen(t) \therefore 10D = 5 \therefore \boxed{D = \frac{1}{2}}$

$E = 3\left(\frac{1}{2}\right) \therefore \boxed{E = \frac{3}{2}}$

III - Solução Geral

$$\boxed{y(x) = c_1 e^{2t} + c_2 e^{t} + \frac{1}{2}\,sen(t) + \frac{3}{2}\cos(t)}$$

Lista 9 - EDO - Página 6

(7) $y'' + 4y' + 4y = 2e^{t} + \cos(t)$

I- Homogênea

$\Delta = 16 - 16 = 0 \therefore R = \frac{-4}{2(1)} \therefore \boxed{R_1 = -2}\ \boxed{R_2 = -2} \therefore \boxed{y_c(t) = C_1 e^{-2t} + xC_2 e^{-2t}}$

II- Particular

$ce^{t} \to Ce^{t}$

$\cos(t) \to D\,\text{sen}(t) + E\cos(t)$

$y_p(t) = Ce^{t} + D\,\text{sen}(t) + E\cos(t)$

$y_p'(t) = Ce^{t} + D\cos(t) - E\,\text{sen}(t)$

$y_p''(t) = Ce^{t} - D\,\text{sen}(t) - E\cos(t)$

$\to (Ce^{t} - D\,\text{sen}(t) - E\cos(t)) + (4Ce^{t} + 4D\cos(t) - 4E\,\text{sen}(t)) + (4Ce^{t} + 4D\,\text{sen}(t) + 4E\cos(t))$

$9Ce^{t} = 2e^{t} \therefore \boxed{C = \frac{2}{9}}$

$\Rightarrow -D\,\text{sen}(t) - 4E\,\text{sen}(t) + 4D\,\text{sen}(t) = 0\,\text{sen}(t) \therefore -D - 4E + 4D = 0 \therefore -4E + 3D = 0 \therefore 3D = 4E \therefore \boxed{D = \frac{4E}{3}}$

$\Rightarrow -E\cos(t) + 4D\cos(t) + 4E\cos(t) = \cos(t) \therefore 4D + 3E = 1 \therefore 4\left(\frac{4E}{3}\right) + 3E = 1 \therefore \frac{16E}{3} + 3E = 1$

$\Rightarrow x(3) \therefore 16E + 9E = 3 \therefore 25E = 3 \therefore \boxed{E = \frac{3}{25}}\quad D = \frac{4}{\cancel{3}}\left(\frac{\cancel{3}}{25}\right) \therefore \boxed{D = \frac{4}{25}}$

III- Solução Geral

$$\boxed{Y(x) = C_1 e^{-2t} + xC_2 e^{-2t} + \frac{2}{9}e^{t} + \frac{4}{25}\,\text{sen}(t) + \frac{3}{25}\cos(t)}$$

Lista 9 - EDO - Página 7

(8) $y'' - 2y' + y = t^2 + 3t + 1$

I - Homogênea

$\Delta = 4 - 4 = 0 \therefore R = \frac{2}{2(1)} \therefore \boxed{R_1 = 1}\ \boxed{R_2 = 1} \therefore \boxed{y_c(t) = c_1 e^t + x c_2 e^t}$

II - Particular

$y_p = A\cancel{x}^2 + Bt + C$

$y_p' = 2At + B$

$y_p'' = 2A$

$\Rightarrow (2A) + (-4At - 2B) + (At^2 + Bt + C) = t^2 + 3t + 1$

$At^2 = t^2 \therefore \boxed{A = 1}$

$-4At + Bt = 3t \therefore -4A + B = 3 \therefore -4 + B = 3 \therefore \boxed{B = 7}$

$\Rightarrow 2A - 2B + C = 1 \therefore 2 - 14 + C = 1 \therefore \boxed{C = 13}$

III - Solução geral

$$\boxed{y(x) = c_1 e^t + x c_2 e^t + 1t^2 + 7t + 13}$$

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9) $y''+3y'+2y=e^{-t}\operatorname{sen}(t)$

$a=1 \quad b=3 \quad c=2$

$\Delta=1$

$R=\frac{-3\pm1}{2(1)}$

$R_1=-1 \quad R_2=-2$

$\boxed{y_p(t)=\cancel{A}e^{-t}B\operatorname{sen}(t)+\cancel{A}e^{-t}C\cos(t)}$

$y_p'(t)=\left(-e^{-t}\operatorname{sen}(t)+e^{-t}\cos(t)\right)B+C\left(-e^{-t}\cos(t)-e^{-t}\operatorname{sen}(t)\right)$

$\boxed{y_c(x)=C_1e^{-x}+C_2e^{-2x}}$

$y_p(x)=Ae^{-x}\cdot\left[B\operatorname{sen}(x)+C\cos(x)\right]$

$B\ \frac{d}{dt}\left(e^{-t}\operatorname{sen}(t)\right)=-e^{-t}\operatorname{sen}(t)+e^{-t}\cos(t)$

$C\ \frac{d}{dt}\left(e^{-t}\cos(t)\right)=-e^{-t}\cos(t)-e^{-t}\operatorname{sen}(t)$

$\boxed{y'_p(t)=-Be^{-t}\operatorname{sen}(t)+Be^{-t}\cos(t)-Ce^{-t}\cos(t)-Ce^{-t}\operatorname{sen}(t)}$

$y_p''(t)=-B\left(-e^{-t}\operatorname{sen}(t)+e^{-t}\cos(t)\right)+B\left(-e^{-t}\cos(t)-e^{-t}\operatorname{sen}(t)\right)-C\left(-e^{-t}\cos(t)-e^{-t}\operatorname{sen}(t)\right)-C\left(-e^{-t}\operatorname{sen}(t)+e^{-t}\cos(t)\right)$

$y_p''(t)=\cancel{Be^{-t}\operatorname{sen}(t)}-Be^{-t}\cos(t)-Be^{-t}\cos(t)-\cancel{Be^{-t}\operatorname{sen}(t)}+\cancel{Ce^{-t}\cos(t)}+Ce^{-t}\operatorname{sen}(t)+Ce^{-t}\operatorname{sen}(t)-\cancel{Ce^{-t}\cos(t)}$

$\boxed{y_p''(t)=-2Be^{-t}\cos(t)+2Ce^{-t}\operatorname{sen}(t)}$

$\boxed{3y'_p(t)=-3Be^{-t}\operatorname{sen}(t)+3Be^{-t}\cos(t)-3Ce^{-t}\cos(t)-3Ce^{-t}\operatorname{sen}(t)}$

$\boxed{2y_p(t)=Be^{-t}\operatorname{sen}(t)+Ce^{-t}\cos(t)}$

$=-2Be^{-t}\cos(t)+2Ce^{-t}\operatorname{sen}(t)-3Be^{-t}\operatorname{sen}(t)+3Be^{-t}\cos(t)-3Ce^{-t}\cos(t)-3Ce^{-t}\operatorname{sen}(t)+Be^{-t}\operatorname{sen}(t)+Ce^{-t}\cos(t)$

$\Rightarrow Be^{-t}\cos(t)-Ce^{-t}\operatorname{sen}(t)-2Be^{-t}\operatorname{sen}(t)-2Ce^{-t}\cos(t)=e^{-t}\operatorname{sen}(t)$

$\Rightarrow\left[\cos(t)\left(Be^{-t}-2Ce^{-t}\right)\right]+\left[\operatorname{sen}(t)\left(-Ce^{-t}-2Be^{-t}\right)\right]=e^{-t}\operatorname{sen}(t)+(0)\cos(t)$

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- $Be^{-t}-2Ce^{-t}=0 \therefore B\cancel{e^{-t}}=2C\cancel{e^{-t}} \therefore \boxed{B=2C}$
- $-Ce^{-t}-2Be^{-t}=e^{-t} \therefore \cancel{e^{-t}}(-C-2B)=\cancel{e^{-t}} \therefore -C-2B=1 \therefore -C-4C=1 \therefore \boxed{C=-\frac{1}{5}}$

$\Rightarrow B=2\left(-\frac{1}{5}\right) \therefore \boxed{B=-\frac{2}{5}}$

$\Rightarrow \boxed{y_p(t)=-\frac{2}{5}e^{-t}\operatorname{sen}(t)-\frac{1}{5}e^{-t}\cos(t)} \rightarrow \div(e^{-t})(?) \rightarrow \times t \quad \boxed{y_p(t)=-\frac{2}{5}te^{-t}-\frac{1}{5}e^{-t}\cos(t)}$

III - Solução geral: $\boxed{y(x)=C_1e^{-t}+C_2e^{-2t}-\frac{2}{5}te^{-t}-\frac{1}{5}e^{-t}\cos(t)}$

$\textcircled{10}$ $y'' - 5y' + 6y = t^3 - t^2 + t - 1 \Rightarrow \boxed{y(x) = C_1e^{3t} + C_2e^{2t} + \frac{1}{6}t^3 + \frac{1}{4}t^2 + \frac{5}{12}t + \frac{7}{72}}$

I- Homogênea

$\Delta = 1 \therefore \sqrt{1} = 1 \therefore R = \frac{5 \pm 1}{2(1)} \therefore \boxed{R_1 = 3}\ \boxed{R_2 = 2} \therefore \boxed{y_c(t) = C_1e^{3t} + C_2e^{2t}}$

II- Particular

$y_p(t) = At^3 + Bt^2 + Ct + D$

$y_p'(t) = 3At^2 + 2Bt + C$

$y_p''(t) = 6At + 2B$

$\overset{6At+2B}{\cancel{(At^3+Bt^2+Ct+D)}} - 5(3At^2 + 2Bt + C) + 6(At^3 + Bt^2 + Ct + D) = t^3 - t^2 + t - 1$

$6At + 2B - 15At^2 - 10Bt - 5C + 6At^3 + 6Bt^2 + 6Ct + 6D = t^3 - t^2 + t - 1$

$6At^3 = t^3 \therefore \boxed{A = \frac{1}{6}}$

$\Rightarrow -15At^2 + 6Bt^2 = -t^2 \therefore -15A + 6B = -1 \therefore -15\left(\frac{1}{6}\right) + 6B = -1 \therefore -\frac{5}{2} + 6B = -1$

$\Rightarrow 6B = -1 + \frac{5}{2} \therefore 6B = \frac{-2+5}{2} \therefore 12B = 3 \therefore \boxed{B = \frac{1}{4}}$

$\Rightarrow 6At - 10Bt + 6Ct = t \therefore 6A - 10B + 6C = 1 \therefore 6\left(\frac{1}{6}\right) - 10\left(\frac{1}{4}\right) + 6C = 1 \therefore 1 - \frac{5}{2} + 6C = 1$

$\Rightarrow -\frac{5}{2} + 6C = 0 \therefore 6C = \frac{5}{2} \therefore \boxed{C = \frac{5}{12}}$ $\quad 2B - 5C + 6D = -1 \therefore 2\left(\frac{1}{4}\right) - 5\left(\frac{5}{12}\right) + 6D = -1$

$\Rightarrow \frac{1}{2} - \frac{25}{12} + \frac{6D}{1} = -1 \therefore \frac{6 - 25 + 72D}{12} = -1 \therefore 72D = -12 - 6 + 25 \therefore 72D = 7 \Rightarrow \boxed{D = \frac{7}{72}}$

Lista 09 - EDO - Página 8

(11) $y'' + 4y' + 3y = 4t^3 + 2t^2 + 3t + 1$

I - Homogênea

$y'' + 4y' + 3y = 0 \therefore \Delta = 4 \therefore \sqrt{\Delta} = 2 \therefore R = \frac{-4 \pm 2}{2(1)} \therefore \boxed{R_1 = -1} \boxed{R_2 = -3} \therefore \boxed{y_c(t) = C_1 e^{-t} + C_2 e^{-3t}}$

II - Particular

$y_p(t) = At^3 + Bt^2 + Ct + D$

$y_p'(t) = 3At^2 + 2Bt + C$

$y_p''(t) = 6At + 2B$

$\Rightarrow (6At + 2B) + 4(3At^2 + 2Bt + C) + 3(At^3 + Bt^2 + Ct + D) = 4t^3 + 2t^2 + 3t + 1$

$* (6At + 2B) + (12At^2 + 8Bt + 4C) + (3At^3 + 3Bt^2 + 3Ct + 3D) = 4t^3 + 2t^2 + 3t + 1$

$\Rightarrow 3At^3 = 4t^3 \therefore 3A = 4 \therefore \boxed{A = \frac{4}{3}}$

$\Rightarrow 12At^2 + 3Bt^2 = 2t^2 \therefore 12A + 3B = 2 \therefore 12\left(\frac{4}{3}\right) + 3B = 2 \therefore 16 + 3B = 2 \therefore \boxed{B = -\frac{14}{3}}$

$\Rightarrow 6At + 8Bt + 3Ct = 3t \therefore 6A + 8B + 3C = 3 \therefore 6\left(\frac{4}{3}\right) + 8\left(-\frac{14}{3}\right) + 3C = 3 \therefore 8 - \frac{112}{3} + 3C = 3$

$\Rightarrow 3C = \frac{3}{1} - \frac{8}{1} + \frac{112}{3} \therefore 3C = \frac{9 - 24 + 112}{3} \therefore 9C = 97 \therefore \boxed{C = \frac{97}{9}}$

$\Rightarrow 2B + 4C + 3D = 1 \therefore 2\left(-\frac{14}{3}\right) + 4\left(\frac{97}{9}\right) + 3D = 1 \therefore \boxed{D = -\frac{295}{27}}$

III - Solução geral

$$\boxed{y(x) = C_1 e^{-t} + C_2 e^{-3t} + \frac{4}{3}t^3 - \frac{14}{3}t^2 + \frac{97}{9}t - \frac{295}{27}}$$

Rascunho

(b2)

$Y'' - 2Y' + 2Y = e^t \cos(t)$

I - Homog.

$Y'' - 2Y' + 2Y = 0 \therefore \Delta = 0 \therefore R = \frac{2}{2} \therefore R_1 = 1 \; R_2 = 1 \therefore \boxed{Y(x) = C_1 e^x + x C_2 e^x}$

II - Particular

$e^t (A \operatorname{sen}(t) + B \cos(t)) \Rightarrow \boxed{Y_p = A e^t \operatorname{sen}(t) + B e^t \cos(t)}$

$\boxed{Y_p'(t) = A(e^t \operatorname{sen}(t) + e^t \cos(t)) + B(e^t \cos(t) - e^t \operatorname{sen}(t))}$

$Y_p''(t) = A[((e^t \operatorname{sen}(t)) + (e^t \cos(t))) + ((e^t \cos(t)) - (e^t \operatorname{sen}(t)))] + B[((e^t \cos(t)) - (e^t \operatorname{sen}(t))) - ((e^t \operatorname{sen}(t)) + (e^t \cos(t)))]$

$Y_p''(t) = \cancel{A e^t \operatorname{sen}(t)} + A e^t \cos(t) + A e^t \cos(t) - \cancel{A e^t \operatorname{sen}(t)} + \cancel{B e^t \cos(t)} - B e^t \operatorname{sen}(t) - B e^t \operatorname{sen}(t) - \cancel{B e^t \cos(t)}$

$\boxed{Y_p''(t) = 2A e^t \cos(t) - 2B e^t \operatorname{sen}(t)}$

$\boxed{-2Y_p'(t) = -2A e^t \operatorname{sen}(t) - 2A e^t \cos(t) - 2B e^t \cos(t) + 2B e^t \operatorname{sen}(t)}$

$\boxed{+2Y_p(t) = A e^t \operatorname{sen}(t) + B e^t \cos(t)}$

$\Rightarrow = \cancel{2A e^t \cos(t)} - \cancel{2B e^t \operatorname{sen}(t)} - 2A e^t \operatorname{sen}(t) - \cancel{2A e^t \cos(t)} - 2B e^t \cos(t) + \cancel{2B e^t \operatorname{sen}(t)} + A e^t \operatorname{sen}(t) + B e^t \cos(t)$

$= -A e^t \operatorname{sen}(t) - B e^t \cos(t) \Rightarrow -A e^t \operatorname{sen}(t) - B e^t \cos(t) = e^t \cos(t) + (0) e^t \operatorname{sen}(t)$

$\Rightarrow -\cancel{A e^t \operatorname{sen}(t)} = \cancel{0 e^t \operatorname{sen}(t)} \therefore \boxed{A = 0}$

$\Rightarrow -\cancel{B e^t \cos(t)} = \cancel{e^t \cos(t)} \therefore -B = 1 \therefore \boxed{B = 1}$

$Y_p(t) = e^t \cos(t) \rightsquigarrow$ L.D $\rightsquigarrow x(t)$

$\Rightarrow \boxed{Y_p(t) = t^2 e^t \cos(t)}$

$\Rightarrow \boxed{Y(x) = C_1 e^t + t e^t C_2 + t^2 e^t \cos(t)}$

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~~Lista 9 - EDO - Página 9~~ Rascunho

(13) $y'' + 6y' + 9y = e^t(3\cos(t) - \operatorname{sen}(t))$

$y'' + 6y' + 9y = 0$

$\Delta = 36 - 36 = 0$

$R = -\frac{6}{2} \Rightarrow \boxed{R_1 = -3}, \boxed{R_2 = -3}$

$\boxed{y_c(t) = C_1 e^{-3t} + tC_2 e^{-3t}}$

II - Particular

$y_p(t) = Ae^t(B\operatorname{sen}(t) + C\cos(t)) = \underline{e^t(D\operatorname{sen}(t) + E\cos(t))}$

$y_p'(t) = e^t(D\operatorname{sen}(t) + E\cos(t)) + e^t(D\cos(t) - E\operatorname{sen}(t)) = \underline{De^t\operatorname{sen}(t) + Ee^t\cos(t) + De^t\cos(t) - Ee^t\operatorname{sen}(t)}$

$y_p''(t) = D(e^t\operatorname{sen}(t) + e^t\cos(t)) + E(e^t\cos(t) - e^t\operatorname{sen}(t)) + D(e^t\cos(t) - e^t\operatorname{sen}(t)) - E(e^t\operatorname{sen}(t) + e^t\cos(t))$

$y_p''(t) = \cancel{De^t\operatorname{sen}(t)} + De^t\cos(t) + \cancel{Ee^t\cos(t)} - Ee^t\operatorname{sen}(t) + De^t\cos(t) - \cancel{De^t\operatorname{sen}(t)} - Ee^t\operatorname{sen}(t) - \cancel{Ee^t\cos(t)}$

$\underline{y_p''(t) = 2De^t\cos(t) - 2Ee^t\operatorname{sen}(t)}$

$\underline{6y_p'(t) = 6De^t\operatorname{sen}(t) + 6Ee^t\cos(t) + 6De^t\cos(t) - 6Ee^t\operatorname{sen}(t)}$

$\underline{9y_p(t) = 9De^t\operatorname{sen}(t) + 9Ee^t\cos(t)}$

(+)

$2De^t\cos(t) - 2Ee^t\operatorname{sen}(t) + 6De^t\operatorname{sen}(t) + 6Ee^t\cos(t) + 6De^t\cos(t) - 6Ee^t\operatorname{sen}(t) + 9De^t\operatorname{sen}(t) + 9Ee^t\cos(t)$

$\Rightarrow 8De^t\cos(t) - 8Ee^t\operatorname{sen}(t) + 15De^t\operatorname{sen}(t) + 15Ee^t\cos(t) = e^t$

$\Rightarrow \cancel{e^t}(8D\cos(t) - 8E\operatorname{sen}(t) + 15D\operatorname{sen}(t) + 15E\cos(t)) = \cancel{e^t}(1)$

$\Rightarrow \operatorname{sen}(t)(15D - 8E) + \cos(t)(15E + 8D) = 1$ ∴ Lembrando que: $\operatorname{sen}^2 t + \cos^2 t = 1$

$\hookrightarrow 15D - 8E = \operatorname{sen}(t), \; 15E + 8D = \cos(t) \Rightarrow \boxed{D = \frac{\operatorname{sen}(t) + 8E}{15}}$

$\Rightarrow 15E + 8\left(\frac{\operatorname{sen}(t) + 8E}{15}\right) = \cos(t)$ ∴ $225E + 8\operatorname{sen}(t) + 64E = \cos(t)$

(14) $y'' - 4y' + 8y = t^2 e^t$

I- Homogênea

$\Delta = 16 - 32 = -16$

$\sqrt{\Delta} = 4i$

$R = \frac{4 \pm 4i}{2} \therefore R = 2 \pm 2i \therefore b = 2,\ a = 2 \therefore \boxed{y_c(t) = C_1 e^{2t} sen(2t) + C_2 e^{2t} \cos(2t)}$

II- Particular

$y_p(t) = Ae^t(Bt^2 + Ct + D) \therefore \boxed{y_p(t) = e^t(Et^2 + ft + G)}$

$y_p'(t) = e^t(Et^2 + ft + G) + e^t(2Et + f)$

$y_p''(t) = [e^t(Et^2+ft+G) + e^t(2Et+f)] + [e^t(2Et+f) + e^t(2E)]$

$y_p''(t) = e^t(Et^2+ft+G) + 2e^t(2Et+f) + e^t(2E)$

$-4y_p'(t) = -4e^t(Et^2+ft+G) - 4e^t(2Et+f)$

$8y_p(t) = 8e^t(Et^2+ft+G)$

$+$

$e^t(Et^2+ft+G) + 2e^t(2Et+f) + e^t(2E) - 4e^t(Et^2+ft+G) - 4e^t(2Et+f) + 8e^t(Et^2+ft+G)$

$e^t(Et^2+ft+G) + e^t(4Et+2f) + e^t(2E) + e^t(-4E\cancel{e^t}t^2 - 4f\cancel{e^t}t - 4G\cancel{e^t}) + e^t(-8Et - 4f) + e^t(8Et^2+8ft+8G)$

$\Rightarrow e^t(Et^2 + ft + G + 4Et + 2f + 2E - 4Et^2 - 4ft - 4G - 8Et - 4f + 8Et^2 + 8ft + 8G)$

$\Rightarrow \cancel{5Et^2}\ 5E\cancel{t^2}(\cancel{e^t}) = \cancel{t^2}(\cancel{e^t}) \Rightarrow \boxed{E = \frac{1}{5}}$

$\cancel{\Rightarrow 5ft = (0)t \therefore \boxed{f=0}} \Rightarrow \cancel{t}(f + 4E - 4f - 8E + 8f) = (0)\cancel{t} \therefore 5f - 4\left(\frac{1}{5}\right) = 0 \Rightarrow 5f = \frac{4}{5} \Rightarrow \boxed{f = \frac{4}{25}}$

$\rightarrow 5G - 2f + 2E = 0 \therefore 5G - 2\left(\frac{4}{25}\right) + 2\left(\frac{1}{5}\right) = 0 \therefore 5G - \frac{8}{25} + \frac{2}{5} = 0 \quad \times(25) \therefore 100G - 8 + 10 = 0$

$\rightarrow 100G = -2 \therefore \boxed{G = -\frac{1}{50}}$

III- Sol. Geral

$$\boxed{y(x) = C_1 e^{2t} sen(2t) + C_2 e^{2t} \cos(2t) + e^t\left(\frac{1}{5}t^2 + \frac{4}{25}t + -\frac{1}{50}\right)}$$

16) $y'' - 5y' + 6y = (t^2+1)e^{2t}$

I- Homogênea

$\Delta = 1 \therefore R = \frac{5 \pm 1}{2(1)} \therefore R_1 = 3 \quad R_2 = 2 \quad \therefore \boxed{y_c(t) = C_1e^{3t} + C_2e^{2t}}$

II- Particular

$y_p(t) = (At^2 + Bt + C)e^{2t} \cdot D \quad \therefore y_p(t) = (Et^2 + ft + G)e^{2t}$

* Por essa incluir a solução da homogênea, ou seja, ser linearmente dependente, multiplica-se por $t$.

$y_p(t) = (te^{2t})(Et^2 + ft + G)$

$y_p'(t) = [(e^{2t} + 2te^{2t})(Et^2 + ft + G)] + [(te^{2t})(2Et + f)]$

$y_p''(t) = [(2te^{2t} + 2e^{2t} + 4te^{2t})(Et^2 + ft + G) + (e^{2t} + 2te^{2t})(2Et + f)] + [(e^{2t} + 2te^{2t})(2Et + f) + (te^{2t})(2E)]$

$y_p''(t) = (6te^{2t} + 2e^{2t})(Et^2 + ft + G) + (e^{2t} + 2te^{2t})(2Et + f) + (e^{2t} + 2te^{2t})(2Et + f) + (t^2e^{2t})(2E)$

$y_p''(t) = (6te^{2t} + 2e^{2t})(Et^2 + ft + G) + 2(e^{2t} + 2te^{2t})(2Et + f) + (t^2e^{2t})(2E)$

$y_p''(t) = 6Ee^{2t}t^3 + 6fe^{2t}t^2 + 6Ge^{2t} + 2Ee^{2t}t^2 + 2fe^{2t}t + 2Ge^{2t} + (2e^{2t} + 4te^{2t})(2Et + f) + 2Et^2e^{2t}$

$y_p''(t) = 6(Ee^{2t}t^3 + fe^{2t}t^2 + Ge^{2t}) + 2e^{2t}(2Et^2 + ft + G) + 2e^{2t}[(1+2t)(2Et + f) + Et^2]$

$y_p''(t) = 6e^{2t}(Et^3 + ft^2 + G) + 2e^{2t}[(2Et^2 + ft + G) + (2Et + 4Et^2 + f + 2ft) + Et^2]$

$y_p''(t) = 6e^{2t}(Et^3 + ft^2 + G) + 2e^{2t}[7Et^2 + 3ft + 2Et + f + G]$

$y_p''(t) = e^{2t}[6(Et^3 + ft^2 + G) + 2(7Et^2 + 3ft + 2Et + f + G)]$

$y_p''(t) = \cancel{e^{2t}}[6Et^3 + 6ft^2 + 6G + 14Et^2 + 6ft + 4Et + 2f + 2G] = \cancel{e^{2t}}(t^2+1)$

$6Et^3 + 6ft^2 + 14Et^2 + 4Et + 6ft + 2f + 8G = t^2 + 1$

$6f\cancel{t^2} + 14E\cancel{t^2} = 1\cancel{t^2} \therefore 6f + 14E = 1 \therefore \boxed{f = \frac{1}{6}}$

$6Et^3 = 0t^3 \therefore \boxed{E = 0}$

$\cancel{t(4E + 6f) = 0t} \Rightarrow 2f + 8G = 1 \Rightarrow \frac{1}{3} + 8G = 1 \therefore 8G = \frac{2}{3} \therefore \boxed{G = \frac{1}{12}}$

$$\boxed{y(t) = C_1e^{3t} + C_2e^{2t} + (te^{2t})\left(\frac{1}{6}t + \frac{1}{12}\right)}$$

(17) $y'' - 2y' + 2y = t^3 e^{-t}$

I- Homogênea

$\Delta = 4 - 8 = -4 \therefore \sqrt{\Delta} = 2i \therefore R = \dfrac{2 \pm 2i}{2(1)} \therefore R = 1 \pm 1i \therefore \boxed{y(x) = C_1 e^{1t} \cos(t) + C_2 e^{t} \operatorname{sen}(t)}$ $a = 1$ $b = 1$

II- Particular

$Y_p(t) = (At^3 + Bt^2 + Ct + D)(e^{-t})$

$Y_p(t) = Ae^{-t}.t^3 + Be^{-t}.t^2 + Ce^{-t}.t + De^{-t}$

$Y_p'(t) = (-Ae^{-t}.t^3 + 3Ae^{-t}t^2) + (-Be^{-t}t^2 + 2Be^{-t}t) + (-Ce^{-t}.t + Ce^{-t}) + (-De^{-t})$

$Y_p''(t) = [-A((-1)e^{-t}t^3 + 3e^{-t}t^2) + 3A((-1)e^{-t}.t^2 + 2e^{-t}t)] + [-B((-1)e^{-t}t^2 + 2e^{-t}.t) + 2B((-1)(e^{-t}.t) + e^{-t})] +$

$+ [C((-1)e^{-t}.t + e^{-t}) + C((-1)e^{-t})] + [-D((-1)e^{-t})]$

$Y_p''(t) = [(Ae^{-t}.t^3 - 3Ae^{-t}.t^2)] + [-3Ae^{-t}.t^2 + 6Ae^{-t}.t] + [(Be^{-t}t^2 - 2Be^{-t}.t)] + [(-2Be^{-t}t + 2Be^{-t})]$

$+ [(-Ce^{-t}.t + \cancel{Ce^{-t}})] + [\cancel{-Ce^{-t}}] + [De^{-t}]$

$Y_p''(t) = Ae^{-t}.t^3 - 6Ae^{-t}.t^2 + Be^{-t}.t^2 + 6Ae^{-t}.t - 4Be^{-t}.t - Ce^{-t}.t + 2Be^{-t} + De^{-t}$

$Y_p''(t) = t^3(Ae^{-t}) + t^2(-6Ae^{-t} + Be^{-t}) + t(6Ae^{-t} - 4Be^{-t} - Ce^{-t}) + (2Be^{-t} + De^{-t})$

$Y_p''(t) = t^3e^{-t}(A) + t^2e^{-t}(-6A + B) + te^{-t}(6A - 4B - C) + e^{-t}(2B + D)$

$-2Y_p'(t) = t^3e^{-t}(2A) + t^2e^{-t}(-6A + 2B) + te^{-t}(-4B + 2C) + e^{-t}(-2C + 2D)$

$2Y_p(t) = t^3e^{-t}(2A) + t^2e^{-t}(2B) + te^{-t}(2C) + e^{-t}(2D)$

(+)

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$t^3e^{-t}(5A) + t^2e^{-t}(-12A + 5B) + te^{-t}(6A - 8B + 3C) + e^{-t}(2B - 2C + 5D) = t^3e^{-t}(1)$

$\Rightarrow 5A = 1 \therefore \boxed{A = \frac{1}{5}}$ | $-12\left(\frac{1}{5}\right) + 5B = 0 \therefore -12 + 25B = 0$ | $25B = 12 \therefore \boxed{B = \frac{12}{25}}$ | $6\left(\frac{1}{5}\right) - 8\left(\frac{12}{25}\right) + 3C = 0$

$\Rightarrow x(25) = 30 - 96 + 3C = 0 \therefore 3C = 66 \therefore \boxed{C = 22}$ | $2\left(\frac{12}{25}\right) - 2(22) + 5D = 0$ $x(25)$ $\therefore$

$\Rightarrow 24 - 1100 + 125D = 0 \therefore 125D = 1076 \therefore \boxed{D = \frac{1076}{125}}$

$$\boxed{y(x) = C_1 e^{t} \cos(t) + C_2 e^{t} \operatorname{sen}(t) + \left(\frac{1}{5}t^3 + \frac{12}{25}t^2 + 22t + \frac{1076}{125}\right) e^{-t}}$$

18) $y'' + 6y' + 13y = e^t \cos(3t)$

I- Homogênea

$\Delta = 36 - 52 = -16 \therefore \sqrt{\Delta} = 4i \therefore R = \dfrac{-6 \pm 4i}{2(1)} \therefore R = -3 \pm 2i \therefore \boxed{b = -3}\ \boxed{a = 2} \therefore \boxed{y_c(t) = c_1 e^{-3t}\cos(2t) + c_2 e^{-3t}\operatorname{sen}(2t)}$

II- Particular

$y_p(t) = e^t(A\operatorname{sen}(3t) + B\cos(3t))$

$y_p(t) = Ae^t\operatorname{sen}(3t) + Be^t\cos(3t)$

$y_p'(t) = [Ae^t\operatorname{sen}(3t) + 3Ae^t\cos(3t)] + [Be^t\cos(3t) - 3Be^t\operatorname{sen}(3t)] \therefore y_p'(t) = Ae^t\operatorname{sen}(3t) + 3Ae^t\cos(3t) + Be^t\cos(3t) - 3Be^t\operatorname{sen}(3t)$

$y_p''(t) = [Ae^t\operatorname{sen}(3t) + Ae^t\cos(3t)] + [-3Ae^t\operatorname{sen}(3t) + 3Ae^t\cos(3t)] + [Be^t\cos(3t) - 3Be^t\operatorname{sen}(t)] + [-3Be^t\operatorname{sen}(3t) - 9Be^t\cos(3t)]$

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$y_p''(t) = [Ae^t(\operatorname{sen}(3t) + \cos(3t))] + [3Ae^t(-\operatorname{sen}(3t) + \cos(3t))] + [Be^t(-\operatorname{sen}(t) + \cos(3t))] + 3Be^t(-\operatorname{sen}(3t) - 3\cos(3t))$

$6y_p'(t) = 6Ae^t\operatorname{sen}(3t) + 18Ae^t\cos(3t) + 6Be^t\cos(3t) - 18Be^t\operatorname{sen}(3t)$

$13y_p(t) = 13Ae^t\operatorname{sen}(3t) + 13Be^t\cos(3t)$

$y_p''(t) = \operatorname{sen}(3t)\left(Ae^t - 3Ae^t - Be^t - 3Be^t\right) + \cos(3t)e^t\left(A + 3A + B - 9B\right)$

$y_p''(t) = \operatorname{sen}(3t)e^t(A - 3A - B - 3B) + \cos(3t)e^t(A + 3A + B - 9B)$

$y_p''(t) = \operatorname{sen}(3t)e^t(-2A - 4B) + \cos(3t)e^t(4A - 8B)$

$6y_p'(t) = \operatorname{sen}(3t)e^t(6A - 18B) + \cos(3t)e^t(18A + 6B)$

$13y_p(t) = \operatorname{sen}(3t)e^t(13A) + \cos(3t)e^t(13B)$ (+)

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$\operatorname{sen}(3t)e^t(17A - 22B) + \cos(3t)e^t(22A + 11B) = e^t\cos(3t)$

$\begin{cases} 22A + 11B = 1 \therefore 11B = 1 - 22A \\ 17A - 22B = 0 \therefore 17A - 2(1 - 22A) = 0 \therefore 17A - 2 + 44A = 0 \therefore \boxed{A = \frac{2}{61}} \end{cases}$

$\Rightarrow 11B = 1 - 22\left(\frac{2}{61}\right) \therefore 11B = 1 - \frac{44}{61} \therefore \boxed{B = \frac{17}{671}}$

$\Rightarrow \boxed{y(x) = c_1 e^{-3t}\cos(2t) + c_2 e^{-3t}(2t) + \frac{2}{61}e^t\operatorname{sen}(3t) + \frac{17}{671}e^t\cos(3t)}$

(19) $y'' - 4y' + 13y = e^{t}\cos(t) + e^{-2t}\operatorname{sen}(3t)$

I- Homogênea

$\Delta = 16 - 4(13) \therefore 16 - 52 \therefore -36 \therefore \sqrt{\Delta} = 6i \therefore R = \frac{4 \pm 6i}{2(1)} \therefore 2 \pm 3i \therefore \boxed{y_c(t) = C_1 e^{2t}\cos(3t) + C_2 e^{2t}\operatorname{sen}(3t)}$

II- Particular

$y_p(t) = e^{t}(A\operatorname{sen}(t) + B\cos(t)) + e^{-2t}(C\operatorname{sen}(3t) + D\cos(3t))$

$y_p'(t) = [e^{t}(A\operatorname{sen}(t) + B\cos(t)) + e^{t}(A\cos(t) - B\operatorname{sen}(t))] + [-2e^{-2t}(C\operatorname{sen}(3t) + D\cos(3t)) + e^{-2t}(3C\cos(3t) - 3D\operatorname{sen}(3t))]$

• $y_p''(t) = \{[e^{t}(A\operatorname{sen}(t) + B\cos(t)) + e^{t}(A\cos(t) - B\operatorname{sen}(t))] + [e^{t}(A\cos(t) - B\operatorname{sen}(t)) + e^{t}(-A\operatorname{sen}(t) - B\cos(t))]\}$
$+ \{[4e^{-2t}(C\operatorname{sen}(3t) + D\cos(3t)) - 2e^{-2t}(3C\cos(3t) - 3D\operatorname{sen}(3t))] + [-2e^{-2t}(3C\cos(3t) - 3D\operatorname{sen}(3t))$
$+ e^{-2t}(-9C\operatorname{sen}(3t) - 9D\cos(3t))]\}$

• $-4y_p'(t) = [-4e^{t}(A\operatorname{sen}(t) + B\cos(t))] + [-4e^{t}(A\cos(t) - B\operatorname{sen}(t))] + [8e^{-2t}(C\operatorname{sen}(3t) + D\cos(3t))]$
$+ [-4e^{-2t}(3C\cos(3t) - 3D\operatorname{sen}(3t))]$

• $13y_p(t) = 13e^{t}(A\operatorname{sen}(t) + B\cos(t)) + 13e^{-2t}(C\operatorname{sen}(3t) + D\cos(3t))$

$\Rightarrow \cancel{e^{t}\cos(t)}[\cancel{B} + A + A - \cancel{B} - 4B - 4A + 13B] = \cancel{e^{t}\cos(t)}(1) \therefore -2A + 9B = 1$

$\Rightarrow \cancel{e^{-2t}\operatorname{sen}(3t)}[4C + 6D + 6D - 9C + 8C + 12D] = \cancel{e^{-2t}\operatorname{sen}(3t)}(1) \therefore 3C + 24D = 1$

$\Rightarrow \cancel{e^{t}\operatorname{sen}(t)}[\cancel{A} - B - B - \cancel{A} - 4A + 4B + 13A] = \cancel{e^{t}\operatorname{sen}(t)}(0) \therefore 9A + 2B = 0$

$\Rightarrow \cancel{e^{-2t}\cos(3t)}[4D - 6C - 6C - 9D + 8D - 12C + 13D] = \cancel{e^{-2t}\cos(3t)}(0) \therefore -24C + 16D = 0$

$\Rightarrow B = \frac{1 + 2A}{9} \therefore 9A + 2\left(\frac{1 + 2A}{9}\right) = 0 \therefore (\times 9) \therefore 81A + 2 + 4A = 0 \therefore 85A = -2 \therefore \boxed{A = -\frac{2}{85}}$

$\Rightarrow -2\left(-\frac{2}{85}\right) + 9B = 1 \therefore \times(85) \therefore 4 + 765B = 1 \therefore \boxed{B = -\frac{3}{765}}$ | $D = \frac{1 - 3C}{24} \therefore -24C + 16\left(\frac{1 - 3C}{24}\right) = 0$

$\times(24) \therefore -576C + 16 - 48C = 0 \therefore \boxed{C = \frac{16}{624}}$ | $-24\left(\frac{16}{624}\right) + 16D = 0 \therefore D = \frac{8}{13 \cdot 16} \therefore \boxed{D = \frac{1}{26}}$

$$\boxed{y(x) = C_1 e^{2t}\cos(3t) + C_2 e^{2t}\operatorname{sen}(3t) + e^{t}\left(-\frac{2}{85}\operatorname{sen}(t) - \frac{3}{765}\cos(t)\right) + e^{-2t}\left(\frac{16}{624}\operatorname{sen}(3t) + \frac{1}{26}\cos(3t)\right)}$$