Lista 5 - Página 1

① $(x+3y)-xy' = 0$

$$-xy'+3y = -x \xrightarrow{\div(-x)} y' - \frac{3}{x}\cdot y = 1 \Rightarrow p(x) = -\frac{3}{x} \;, f(x) = 1$$

$$\Rightarrow \mu(x) = e^{\int p(x) dx} \Rightarrow \mu(x) = e^{\int -\frac{3}{x}\cdot dx} \Rightarrow e^{-3\int \frac{1}{x} dx} \Rightarrow$$

$$\Rightarrow e^{-3\ln|x|+c} \Rightarrow k e^{\ln|x|^{(-3)}} \Rightarrow kx^{(-3)} = \mu(x)$$

$$\Rightarrow y = \frac{1}{\mu(x)}\cdot \int \mu(x) f(x)\, dx \Rightarrow y = \frac{1}{kx^{(-3)}}\cdot \int kx^{(-3)}\cdot 1. dx \Rightarrow y = \frac{x^3}{\cancel{k}}\cdot \cancel{k}\int x^{-3} dx$$

$$\Rightarrow y = x^3 \cdot \frac{x^{-2}}{-2} \Rightarrow y = -\frac{x^{3-2}}{2} \Rightarrow \boxed{y = -\frac{x}{2} + c}$$

② $y' = 2y + e^x$

$$y' - 2y = e^x \Rightarrow p(x) = -2 \;, f(x) = e^x \Rightarrow \mu = e^{\int -2 dx} = e^{-2\int dx} = e^{-2x+}$$

$$\Rightarrow e^{-2x+c} = ke^{-2x} = \mu$$

Lista 5 - Página 2

$\Rightarrow \mu = ke^{-2x} \Rightarrow y = \frac{1}{ke^{-2x}} \cdot \int ke^{-2x} \cdot e^{x} \cdot dx \Rightarrow y = \frac{e^{2x}}{\cancel{k}} \cdot \cancel{k} \int e^{-2x+x} dx$

$\Rightarrow y = e^{2x} \cdot \int e^{-x} dx \Rightarrow u = (-x),\ du = -dx \Rightarrow dx = -du \Rightarrow \int e^{u} \cdot (-du)$

$\Rightarrow -1 \int e^{u} du \Rightarrow -1 \cdot e^{-x} \Rightarrow \boxed{y = e^{2x} - e^{-x} + C}$

(3) $y' - 2xy = x$

$p(x) = -2x,\ f(x) = x \Rightarrow \mu = e^{\int -2x dx} = e^{-2\int x dx} = e^{-\cancel{2}\frac{x^2}{\cancel{2}} + c} = ke^{-x^2}$

$\Rightarrow \mu = ke^{-x^2} \Rightarrow y = \frac{1}{ke^{-x^2}} \cdot \int ke^{-x^2} \cdot x \cdot dx = \frac{e^{x^2}}{\cancel{k}} \cdot \cancel{k} \int e^{-x^2} \cdot x \cdot dx$

$\Rightarrow y = e^{x^2} \cdot \int x e^{-x^2} dx \Rightarrow$ I. por substituição $\Rightarrow u = -x^2,\ du = -2x dx$

$\Rightarrow dx = -\frac{du}{2x} \Rightarrow \int \cancel{x} e^{u} \left(-\frac{du}{2\cancel{x}}\right) = -\frac{1}{2} \int e^{u} du = -\frac{1}{2} e^{-x^2} \Rightarrow$

$\Rightarrow \boxed{y = e^{x^2} - \frac{1}{2} e^{-x^2} + C}$

Lista 5 - Página 3

(4) $y' + 3y = x + e^{-2x}$

$p(x) = 3\ ,\ f(x) = x + e^{-2x} \Rightarrow \mu = e^{\int 3dx} = e^{3\int dx} = e^{3x+C} \Rightarrow \mu = ke^{3x}$

$\Rightarrow y = \frac{1}{ke^{3x}} \cdot \int ke^{3x} \cdot (x + e^{-2x}) dx \Rightarrow y = \frac{1}{\cancel{k}e^{3x}} \cdot \cancel{k} \int e^{3x}(x) + e^{3x}(e^{-2x})\, dx$

$\Rightarrow y = \frac{1}{e^{3x}} \cdot \underbrace{\int xe^{3x} dx}_{\mathrm{I}} + \underbrace{\int e^{x} dx}_{\mathrm{II}} \Rightarrow$ (I) $u = 3x\ ,\ du = 3dx\ ,\ dx = \frac{du}{3}$

$\Rightarrow \int xe^{u} \cdot \frac{du}{3} = \frac{x}{3} \int e^{u} du = \frac{x}{3} \cdot e^{3x} \Rightarrow \int xe^{3x} dx = \frac{xe^{3x}}{3}$

$\Rightarrow y = \frac{1}{e^{3x}} \cdot \left(\frac{xe^{3x}}{3_1} + \frac{e^{x}}{1_3} \right) \Rightarrow y = \frac{1}{e^{3x}} \cdot \left(\frac{xe^{3x} + 3e^{x}}{3} \right) \Rightarrow y = \frac{1}{e^{3x}} \cdot \frac{xe^{3x} + 3e^{x}}{3}$

$\Rightarrow y = \frac{xe^{3x} + 3e^{x}}{3e^{3x}} = \frac{x\cancel{e^{3x}}}{3\cancel{e^{3x}}} + \frac{\cancel{3}e^{x}}{\cancel{3}e^{3x}} \Rightarrow \boxed{y = \frac{x}{3} + e^{-2x} + C}$

(5) $y' - 2y = x^2 e^{2x}$

$p(x) = -2\ ,\ f(x) = x^2 e^{2x} \Rightarrow \mu = e^{\int -2dx} = e^{-2\int dx} = e^{-2x+C} = ke^{-2x}$

$\Rightarrow \mu = ke^{-2x} \Rightarrow y = \frac{e^{2x}}{\cancel{k}} \cdot \cancel{k} \int e^{-2x} x^2 e^{2x} dx \Rightarrow y = e^{2x} \int x^2 e^{2x-2x} dx$

$\Rightarrow y = e^{2x} \int x^2 dx = e^{2x} \cdot \frac{x^3}{3} + C \Rightarrow \boxed{\frac{x^3 e^{2x}}{3} + C = y}$

Lista 5 - Página 4

⑦ $xy' + y = 3x\cos(2x) \xrightarrow{\div(x)} y' + \frac{1}{x}\cdot y = 3\cos(2x) \Rightarrow$

$\Rightarrow p(x) = \frac{1}{x}$, $f(x) = 3\cos(2x) \Rightarrow \mu = e^{\int \frac{1}{x}dx} = e^{\ln|x|+C} \Rightarrow \mu = kx$

$\Rightarrow y = \frac{1}{kx}\int kx \cdot 3\cos(2x)dx = \frac{3}{\cancel{k}x}\cdot \cancel{k}\int x\cos(2x)dx \Rightarrow u = 2x,\ du = 2dx$

$\Rightarrow dx = \frac{du}{2} \Rightarrow \int x\cos(u)\left(\frac{du}{2}\right) = \frac{x}{2}\int \cos(u)du = \frac{x}{2}\cdot \text{sen}(2x)$

$\Rightarrow y = \frac{3}{\cancel{x}}\cdot\left(\frac{\cancel{x}}{2}\text{sen}(2x)\right) \Rightarrow \boxed{y = \frac{3}{2}\cdot \text{sen}(2x) + C}$

⑧ $y' - y = 2e^x \Rightarrow p(x) = -1,\ f(x) = 2e^x$

$p(x) = -1,\ f(x) = 2e^x \Rightarrow \mu = e^{\int -1dx} = e^{-1\int dx} = e^{-x+C} = ke^{-x} = \mu$

$\Rightarrow y = \frac{e^x}{\cancel{k}}\cdot \cancel{k}\int e^{-x}\cdot 2e^x\cdot dx \Rightarrow y = e^x\cdot\int 2\cancel{e^{x-x}}^{1}dx \Rightarrow y = e^x\int 2dx$

$\Rightarrow y = e^x\cdot 2\int dx \Rightarrow y = e^x\cdot 2x + C \Rightarrow \boxed{y = 2xe^x + C}$

Lista 5 - Página 5

(9) $xy' + 2y = \text{sen}(x) \overset{\div(x)}{\Rightarrow} y' + \frac{2}{x}y = \frac{\text{sen}(x)}{x}$

$y' + \frac{2}{x}y = \frac{\text{sen}(x)}{x} \Rightarrow P(x) = \frac{2}{x}, \; f(x) = \frac{\text{sen}(x)}{x} \Rightarrow \mu = e^{\int \frac{2}{x}dx} \Rightarrow$

$\Rightarrow \mu = e^{2\int \frac{1}{x}dx} = e^{2\ln|x|+C} = ke^{\ln|x|^2} \Rightarrow \boxed{\mu(x) = kx^2}$

$\Rightarrow y = \frac{1}{kx^2} \cdot \int k\cancel{x^2} \cdot \frac{\text{sen}(x)}{\cancel{x}} \cdot dx = \frac{1}{\cancel{k}x^2} \cdot \cancel{k} \int x\,\text{sen}(x)dx \Rightarrow$ I. por partes

$\Rightarrow u = x, \; du = dx, \; dv = \text{sen}(x)dx, \; v = -\cos(x) + C \Rightarrow uv - \int v\,du$

$\Rightarrow -x\cos(x) - \int -\cos(x)dx \Rightarrow -x\cos(x) - (-1)\int \cos(x)dx \Rightarrow$

$\Rightarrow -x\cos(x) + \text{sen}(x) + C \Rightarrow y = \frac{1}{\cancel{k}x^2} \cdot (-x\cos(x) + \text{sen}(x))$

$\Rightarrow y = \frac{\text{sen}(x)}{\cancel{k}x^2} - \frac{\cancel{x}\cos(x)}{x^{\cancel{2}}} \Rightarrow \boxed{y = \frac{\text{sen}(x)}{x^2} - \frac{\cos(x)}{x} + C}$

(10) $y' - 2y = e^{2x}$

$P(x) = -2, \; f(x) = e^{2x} \Rightarrow \mu = e^{\int -2dx} \Rightarrow \boxed{\mu = ke^{-2x}}$

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Lista 5 - Página 6

$\Rightarrow \mu = ke^{-2x} \Rightarrow y = \frac{e^{2x}}{k} \cdot \int ke^{-2x} \cdot e^{2x} \cdot dx = \frac{e^{2x}}{\cancel{k}} \cdot \cancel{k} \int \cancel{e^{2x-2x}} dx$

$\Rightarrow \boxed{y = e^{2x} \cdot x + C}$

(11) $y' + \frac{1}{x} y = x^2$

$p(x) = \frac{1}{x}, \ f(x) = x^2 \Rightarrow \mu = e^{\int \frac{1}{x} dx} = e^{\ln|x| + C} \Rightarrow \mu = ke^{\ln|x|} = kx$

$\Rightarrow y = \frac{1}{\cancel{k}x} \cancel{k} \int x \cdot x^2 dx = y = \frac{1}{x} \int x^3 dx = \frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{4}3}}{4} \Rightarrow \boxed{y = \frac{x^3}{4} + C}$

(12) $xy' + 2y = \text{sen}(x)$

$\div (x)$

$\Rightarrow y' + \frac{2}{x} y = \frac{\text{sen}(x)}{x} \Rightarrow P(x) = \frac{2}{x}, \ f(x) = \frac{\text{sen}(x)}{x} \Rightarrow \mu = e^{2\int \frac{1}{x} dx}$

$\Rightarrow \mu = e^{2\ln|x| + C} = ke^{\ln|x|^2} \Rightarrow \mu = kx^2 \Rightarrow y = \frac{1}{\cancel{k}x^2} \cancel{k} \int x^{\cancel{2}} \cdot \frac{\text{sen}(x)}{\cancel{x}} dx$

$\Rightarrow y = \frac{1}{x^2} \int x\,\text{sen}(x) dx \Rightarrow$ feito na questão 9 $\Rightarrow y = \frac{1}{x^2} \cdot (\text{sen}(x) - x\cos(x))$

$\Rightarrow y = \frac{\text{sen}(x)}{x^2} - \frac{\cancel{x}\cos(x)}{x^{\cancel{2}}} + C \Rightarrow \boxed{y = \frac{\text{sen}(x)}{x^2} - \frac{\cos(x)}{x} + C}$

Lista 5 - Página 7

(13) $x^2y' + 2xy = \cos(x)$

$\div(x^2)$

$\Rightarrow y' + \frac{2}{x}y = \frac{\cos(x)}{x^2} \Rightarrow p(x) = \frac{2}{x}, f(x) = \frac{\cos(x)}{x^2} \Rightarrow \mu = e^{2\int \frac{1}{x}dx} \Rightarrow$

$\Rightarrow \mu = ke^{\ln|x|^2} \Rightarrow \mu = kx^2 \Rightarrow y = \frac{1}{\cancel{k}x^2} \cdot \cancel{k}\int \cancel{kx^2} \cdot \frac{\cos(x)}{\cancel{x^2}} dx \Rightarrow y = \frac{1}{x^2} \cdot \int \cos(x) dx$

$\Rightarrow \boxed{y = \frac{1}{x^2} \cdot \operatorname{sen}(x) + c}$

(14) $\operatorname{sen}(x)y' + \cos(x)y = 1$

$\div(\operatorname{sen}(x))$

$\Rightarrow y' + \frac{\cos(x)}{\operatorname{sen}(x)} = \frac{1}{\operatorname{sen}(x)} \Rightarrow p(x) = \frac{\cos(x)}{\operatorname{sen}(x)}, f(x) = \frac{1}{\operatorname{sen}(x)} \Rightarrow \mu = e^{\int \frac{\cos(x)}{\operatorname{sen}(x)}dx}$

$\Rightarrow u = \operatorname{sen}(x), du = \cos(x)dx \Rightarrow \int \frac{1}{u} \cdot du = \ln|u| = \ln|\operatorname{sen}(x)| = u$

$\Rightarrow \mu = ke^{\ln|\operatorname{sen}(x)|} = k\operatorname{sen}(x) = u$

$\Rightarrow y = \frac{1}{\cancel{k}\operatorname{sen}(x)} \cdot \cancel{k} \int \cancel{\operatorname{sen}(x)} \cdot \frac{1}{\cancel{\operatorname{sen}(x)}} \cdot dx \Rightarrow y = \frac{1}{\operatorname{sen}(x)} \int dx \Rightarrow \boxed{y = \frac{x}{\operatorname{sen}(x)} + c}$

Lista 5 - Página 8

15 $(1+x^2)y' + xy = -(1+x^2)^{\frac{5}{2}}$

$\div(1+x^2)$

$\Rightarrow y' + \frac{x}{(1+x^2)}y = (-1)\frac{(1+x^2)^{\frac{5}{2}}}{(1+x^2)^1} \Rightarrow y' + \frac{x}{(1+x^2)}y = (-1)(1+x^2)^{\frac{3}{2}}$

$\Rightarrow p(x) = \frac{x}{(1+x^2)}$, $f(x) = -(1+x^2)^{\frac{3}{2}} \Rightarrow \mu = e^{\int \frac{x}{(1+x^2)}dx} \Rightarrow u = 1+x^2 \Rightarrow$

$\Rightarrow du = 2x\,dx$, $dx = \frac{du}{2x} \Rightarrow e^{\int \cancel{x} \cdot \frac{1}{u} \cdot \frac{du}{2\cancel{x}}} = e^{\frac{1}{2}\int \frac{1}{u}du} = ke^{\frac{1}{2}\ln|1+x^2|}$

$\Rightarrow ke^{\ln|1+x^2|^{\frac{1}{2}}} \Rightarrow \mu = k(1+x^2)^{\frac{1}{2}} \Rightarrow y = \frac{1}{\cancel{k}(1+x^2)^{\frac{1}{2}}} \cdot \cancel{k}\int (1+x^2)^{\frac{1}{2}}(-1)(1+x^2)^{\frac{3}{2}}dx$

$\Rightarrow y = (1+x^2)^{-\frac{1}{2}}(-1)\int (1+x^2)^2 dx \Rightarrow u = (1+x^2)$, $du = 2x\,dx$, $dx = \frac{du}{2x} \Rightarrow$

$$\Rightarrow \int u^2 \cdot \frac{du}{2x} = \frac{1}{2x} \cdot \frac{u^3}{3} = \frac{1}{6x}(1+x^2)^3 \Rightarrow y = (1+x^2)^{-\frac{1}{2}}(-1)\frac{1}{6x}(1+x^2)^3$$

$$\Rightarrow \boxed{y = -\frac{(1+x^2)^{\frac{5}{2}}}{6x}}$$

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Lista 5 - Página 9

(16) $y' = y \cdot \ln(x)$

$y' - \ln(x)y = 0 \Rightarrow p(x) = -\ln(x),\ f(x) = 0 \Rightarrow \mu = e^{\int -\ln(x)dx} = e^{-1\int \ln(x)dx}$

$\Rightarrow$ I. por partes $\Rightarrow u = \ln(x),\ du = \frac{1}{x}dx,\ dv = dx,\ v = x, \Rightarrow uv - \int v du$

$\Rightarrow x\ln(x) - \int x \cdot \frac{1}{x}dx \Rightarrow x\ln(x) - x \Rightarrow \mu = e^{x\ln(x)-x} = e^{x(\ln(x)-1)}$

$\Rightarrow \mu = e^x \cdot e^{\ln(x)-1} = \frac{e^x e^{\ln(x)}}{e^1} \Rightarrow \mu = ke^x\left(\frac{e^{\ln(x)}}{e}\right)$

$\Rightarrow y = \frac{1}{\cancel{k}e^x\left(\frac{e^{\ln(x)}}{e}\right)} \cancel{k}\int \cancel{k}e^x\left(\frac{e^{\ln(x)}}{e}\right)(0)\,dx \Rightarrow y = \frac{1}{e^x\left(\frac{\cancel{x}}{e}\right)} \cdot \int 0\,dx$

$\Rightarrow \boxed{y = \frac{c}{e^x\left(\frac{x}{e}\right)}}$

(17) $xy' = y$

$\Rightarrow xy' - y = 0 \xRightarrow{\div(x)} \cancel{p(x)} - \frac{1}{x} \cdot y = 0 \Rightarrow p(x) = -\frac{1}{x},\ f(x) = 0$

$\Rightarrow \mu = e^{-1\int \frac{1}{x}dx} \Rightarrow \mu = e^{-\ln|x|+c} \Rightarrow k(-x) \Rightarrow \mu(x) = -kx$

$\Rightarrow y = -\frac{1}{\cancel{k}x} \cdot (-1)(\cancel{k})\int x \cdot 0 \cdot dx \Rightarrow y = \frac{1}{x} \cdot \int 0\,dx \Rightarrow \boxed{y = \frac{1}{x} \cdot c}$

Lista 5 - Página 10

(18) $3x^2y' = 2y(y-3)$

$\Rightarrow 3x^2y' = 2y^2 - 6y$

$\hookrightarrow$ Equação Diferencial Ordinária de Segunda Ordem