

# Lista 10 - EDO - Resposta Única (Respostas)

$$\textcircled{1} \quad x(t) = C_1 e^{3t} \cos(\sqrt{47}t) + C_2 e^{3t} \sin(\sqrt{47}t) ; y(t) = C_1 e^{3t} \cos(\sqrt{47}t) + C_2 e^{3t} \sin(\sqrt{47}t)$$

$$\textcircled{2} \quad x(t) = C_1 e^{(-1+\sqrt{6})t} + C_2 e^{(-1-\sqrt{6})t} ; y(t) = C_1 e^{(-1+\sqrt{6})t} + C_2 e^{(-1-\sqrt{6})t}$$

$$\textcircled{3} \quad x(t) = C_1 e^{(-\frac{1}{2}+\sqrt{6})t} + C_2 e^{(-\frac{1}{2}-\sqrt{6})t} ; y(t) = C_1 e^{(-1+\sqrt{10})t} + C_2 e^{(-1-\sqrt{10})t}$$

$$\textcircled{4} \quad x(t) = C_1 e^{\left(\frac{-1+\sqrt{41}}{2}\right)t} + C_2 e^{\left(\frac{-1-\sqrt{41}}{2}\right)t} ; y(t) = C_1 e^{\left(\frac{-1+\sqrt{41}}{2}\right)t} + C_2 e^{\left(\frac{-1-\sqrt{41}}{2}\right)t}$$

$$\textcircled{5} \quad x(t) = C_1 e^{\sqrt{7}t} + C_2 e^{-\sqrt{7}t} ; y(t) = C_1 e^{\sqrt{7}t} + C_2 e^{-\sqrt{7}t}$$

$$\textcircled{6} \quad x(t) = C_1 e^{\left(\frac{5+\sqrt{33}}{2}\right)t} + C_2 e^{\left(\frac{5-\sqrt{33}}{2}\right)t} ; y(t) = C_1 e^{\left(\frac{5+\sqrt{33}}{2}\right)t} + C_2 e^{\left(\frac{5-\sqrt{33}}{2}\right)t}$$

$$\textcircled{7} \quad x(t) = C_1 e^{\frac{5}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) + C_2 e^{\frac{5}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right) ; y(t) = C_1 e^{\frac{5}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) + C_2 e^{\frac{5}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)$$

$$\textcircled{8} \quad x(t) = C_1 e^{\left(\frac{-3+\sqrt{5}}{2}\right)t} + C_2 e^{\left(\frac{-3-\sqrt{5}}{2}\right)t} ; y(t) = C_1 e^{\left(\frac{-3+\sqrt{5}}{2}\right)t} + C_2 e^{\left(\frac{-3-\sqrt{5}}{2}\right)t}$$

$$\textcircled{9} \quad x(t) = C_1 e^t \cos(\sqrt{6}t) + C_2 e^t \sin(\sqrt{6}t) ; y(t) = C_1 e^t \cos(\sqrt{6}t) + C_2 e^t \sin(\sqrt{6}t)$$

$$\textcircled{10} \quad x(t) = C_1 e^{-1t} + C_2 e^{-5t} ; y(t) = C_1 e^{-1t} + C_2 e^{-5t}$$

Observação : todos os desmembramentos se encontram nas páginas posteriores.

$$\textcircled{1} \begin{cases} \frac{dx}{dt} = 2x - 3y \\ \frac{dy}{dt} = 4x + y \end{cases} \Rightarrow \begin{cases} Dx - 2x + 3y = 0 \\ Dy - y - 4x = 0 \end{cases} \Rightarrow \begin{cases} X(D-2) + 3Y = 0 \\ Y(D-1) - 4X = 0 \end{cases} \Rightarrow \text{Eliminar } Y \Rightarrow \begin{matrix} * \\ X(D-1) \\ X(-3) \end{matrix}$$

$$\Rightarrow \begin{cases} X(D-2)(D-1) + 3Y(D-1) = 0 \\ -3Y(D-1) + 12X = 0 \end{cases} \Rightarrow X(D^2 - 3D + 2) + 12X = D^2X - 3DX + 14X$$

$$\Rightarrow \Delta = -47 \Rightarrow \sqrt{\Delta} = \sqrt{47}i \Rightarrow R = \frac{3 \pm \sqrt{47}i}{2(1)} \Rightarrow R_1 = \frac{3 + \sqrt{47}i}{2} \Rightarrow b=3 \quad a=\sqrt{47}$$

$$R_2 = \frac{3 - \sqrt{47}i}{2}$$

$$\Rightarrow \boxed{x(t) = C_1 e^{3t} \cos(\sqrt{47}t) + C_2 e^{3t} \sin(\sqrt{47}t)}$$

$$\Rightarrow \text{Eliminar } X \Rightarrow \begin{matrix} * \\ x(4) \\ x(D-2) \end{matrix} = \begin{cases} 4x(D-2) + 12y = 0 \\ Y(D-1)(D-2) - 4x(D-2) = 0 \end{cases} = Y(D^2 - 3D + 2) + 12y$$

$$\Rightarrow \boxed{y(t) = C_1 e^{3t} \cos(\sqrt{47}t) + C_2 e^{3t} \sin(\sqrt{47}t)}$$

Observação: \* é referente ao sistema linear desordenado no topo de todas as resoluções.

$$\textcircled{2} \begin{cases} \frac{dx}{dt} = -x + 2y \\ \frac{dy}{dt} = 3x - y \end{cases} \Rightarrow \begin{cases} Dx + x - 2y = 0 \\ Dy + y - 3x = 0 \end{cases} \Rightarrow \begin{cases} x(D+1) - 2y = 0 & x(D+1) \\ y(D+1) - 3x = 0 & x(2) \end{cases}$$

$$\begin{array}{c|c} 24 & 2 \\ 12 & 2 \\ 0 & 2 \\ 3 & 3 \end{array} \quad 2\sqrt{6}$$

$$\Rightarrow \begin{cases} x(D+1)^2 - 2y(D+1) = 0 \\ 2y(D+1) - 6x = 0 \end{cases} \Rightarrow \begin{cases} x(D^2 + 2D + 1) - 6x = 0 \Rightarrow D^2x + 2Dx + x - 6x = 0 \\ D^2x + 2Dx - 5x = 0 \Rightarrow \Delta = 4 + 20 \Rightarrow \Delta = 24 \end{cases}$$

$$\Rightarrow R = \frac{-2 \pm \sqrt{24}}{2(1)} = \frac{-2 \pm 2\sqrt{6}}{2} = -1 \pm \sqrt{6} \Rightarrow \begin{cases} R_1 = -1 + \sqrt{6} \\ R_2 = -1 - \sqrt{6} \end{cases} \Rightarrow x(t) = C_1 e^{(-1+\sqrt{6})t} + C_2 e^{(-1-\sqrt{6})t}$$

$$\Rightarrow \begin{cases} Dx + x - 2y = 0 \\ Dy + y - 3x = 0 \end{cases} \Rightarrow \begin{cases} x(D+1) - 2y = 0 & x(3) \\ y(D+1) - 3x = 0 & x(D+1) \end{cases} \Rightarrow \begin{cases} 3x(D+1) - 6y = 0 \\ y(D+1)^2 - 3x(D+1) = 0 \end{cases}$$

$$\Rightarrow y(D^2 + 2D + 1) - 6y \Rightarrow D^2y + 2Dy + y - 6y = 0 \Rightarrow D^2 + 2Dy - 5y = 0$$

$$\Rightarrow y(x) = C_3 e^{(-1+\sqrt{6})t} + C_4 e^{(-1-\sqrt{6})t}$$

$$\textcircled{3} \begin{cases} \frac{dx}{dt} = x + y \\ \frac{dy}{dt} = 2x - 3y \end{cases} \Rightarrow \begin{cases} Dx - x - y = 0 \\ Dy + 3y - 2x = 0 \end{cases} \Rightarrow \begin{cases} x(D-1) - y = 0 \\ y(D+3) - 2x = 0 \end{cases} \quad * \text{Eliminate } y \Rightarrow$$

$$\Rightarrow \begin{matrix} x(D+3) \\ x(1) \end{matrix} \Rightarrow \begin{cases} x(D-1)(D+3) - y(D+3) = 0 \\ y(D+3) - 2x = 0 \end{cases} \Rightarrow \begin{cases} x(D^2 + 3D - D - 3) - 2x = 0 \\ = 0 \end{cases}$$

$$\Rightarrow D^2x + 2Dx - 5x = 0 \Rightarrow a=1 \quad b=2 \quad c=-5 \Rightarrow \Delta = 4 + 20 \Rightarrow \Delta = 24$$

$$\Rightarrow \sqrt{\Delta} = -1 \pm 2\sqrt{6} \Rightarrow \begin{cases} R_1 = -\frac{1}{2} + \sqrt{6} \\ R_2 = -\frac{1}{2} - \sqrt{6} \end{cases}$$

$$\Rightarrow x(t) = C_1 e^{(-\frac{1}{2} + \sqrt{6})t} + C_2 e^{(-\frac{1}{2} - \sqrt{6})t}$$



Continuação 3

$$\begin{array}{c|cc} 40 & 2 & 2\sqrt{10} \\ 20 & 2 & \\ 10 & 2 & \\ 5 & 2 & \\ 1 & 2 & \end{array}$$

$$\Rightarrow x(t) = C_1 e^{(-\frac{1}{2} + \sqrt{10})t} + C_2 e^{(-\frac{1}{2} - \sqrt{10})t}$$

$$\Rightarrow \begin{cases} x(D-1)-2y=0 \\ y(D+3)-3x=0 \end{cases} \quad \text{* Eliminar } x \Rightarrow \begin{cases} x(3) \\ x(D-1) \end{cases} \Rightarrow \begin{cases} 3x(D-1)-6y=0 \\ y(D+3)(D-1)-3x(D-1)=0 \end{cases}$$

$$\Rightarrow y(D^2-D+3D-3)-6y=0 \Rightarrow D^2y+2Dy-9y=0 \Rightarrow a=1 \quad b=2 \quad c=-9$$

$$\Rightarrow \Delta = 4+36 \Rightarrow \Delta=40 \Rightarrow \sqrt{40} = 2\sqrt{10} \Rightarrow R = \frac{-2 \pm 2\sqrt{10}}{2(1)} \Rightarrow R = -1 \pm \sqrt{10}$$

$$\Rightarrow \boxed{R_1 = -1 + \sqrt{10}} \quad \boxed{R_2 = -1 - \sqrt{10}} \Rightarrow y(x) = C_1 e^{(-1+\sqrt{10})t} + C_2 e^{(-1-\sqrt{10})t}$$

$$\textcircled{4} \begin{cases} \frac{dx}{dt} = 3x+y \\ \frac{dy}{dt} = -2x-4y \end{cases} \Rightarrow \begin{cases} Dx-3x-y=0 \\ Dy+4y+2x=0 \end{cases} \Rightarrow \begin{cases} x(D-3)-y=0 \\ y(D+4)+2x=0 \end{cases} \Rightarrow \text{* Eliminar } y$$

$$\Rightarrow \begin{matrix} x(D+4) \\ x(1) \end{matrix} \Rightarrow \begin{cases} x(D-3)(D+4)-y(D+4)=0 \\ y(D+4)+2x=0 \end{cases} \Rightarrow \begin{matrix} x(D^2+4D-3D-12)+2x \\ =0 \end{matrix}$$

$$\Rightarrow D^2x + Dx - 10x = 0 \Rightarrow \Delta = 41 \Rightarrow \sqrt{\Delta} = \sqrt{41} \Rightarrow R = \frac{-1 \pm \sqrt{41}}{2(1)} \Rightarrow \boxed{R_1 = \frac{-1 + \sqrt{41}}{2}} \quad \boxed{R_2 = \frac{-1 - \sqrt{41}}{2}}$$

$$\Rightarrow x(t) = C_1 e^{\left(\frac{-1+\sqrt{41}}{2}\right)t} + C_2 e^{\left(\frac{-1-\sqrt{41}}{2}\right)t}$$

$$\Rightarrow \text{* Eliminar } x \Rightarrow \begin{cases} x(D-3)-y=0 \quad \times (-2) \\ y(D+4)+2x=0 \quad \times (D-3) \end{cases} \Rightarrow \begin{cases} -2x(D-3)+2y=0 \\ y(D+4)(D-3)+2x(D-3)=0 \end{cases}$$

$$\Rightarrow y(D^2-3D+4D-12)+2y \Rightarrow D^2y+Dy-10y=0 \Rightarrow \boxed{R_1 = \frac{-1+\sqrt{41}}{2}} \quad \boxed{R_2 = \frac{-1-\sqrt{41}}{2}}$$

$$y(t) = C_1 e^{\left(\frac{-1+\sqrt{41}}{2}\right)t} + C_2 e^{\left(\frac{-1-\sqrt{41}}{2}\right)t}$$

$$\begin{aligned}
 \textcircled{5} \quad & \begin{cases} \frac{dx}{dt} = 2x + 3y \\ \frac{dy}{dt} = -x - 2y \end{cases} \Rightarrow \begin{cases} DX - 2x + 3y = 0 \\ DY + 2y + x = 0 \end{cases} \Rightarrow \begin{cases} x(D-2) + 3y = 0 \\ y(D+2) + x = 0 \end{cases} \Rightarrow \text{Eliminar } y \\
 & \begin{array}{r|l} 28 & 2 \\ 14 & 7 \\ 2 & 2 \\ 1 & \end{array} \quad \textcircled{2\sqrt{7}}
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow * \quad & \begin{cases} x(D+2) = \begin{cases} x(D-2)(D+2) + 3y(D+2) = 0 \\ -3y(D+2) - 3x = 0 \end{cases} \\ x(-3) \end{cases} \Rightarrow x(D^2 - 2^2) - 3x \Rightarrow D^2 x - 4x
 \end{aligned}$$

$$\Rightarrow a=1 \quad b=0 \quad c=-7 \Rightarrow \Delta=28 \Rightarrow \sqrt{\Delta}=2\sqrt{7} \Rightarrow R = \frac{\pm 2\sqrt{7}}{2(1)} \Rightarrow \boxed{R_1 = \sqrt{7}} \quad \boxed{R_2 = -\sqrt{7}}$$

$$\Rightarrow \boxed{x(t) = C_1 e^{\sqrt{7}t} + C_2 e^{-\sqrt{7}t}}$$

Continuação da 5

Eliminar  $x \Rightarrow$  
$$\begin{aligned} x(-1) &= \begin{cases} -x(D-2) - 3y = 0 \\ y(D+2)(D-2) + x(D-2) = 0 \end{cases} = y(D^2-4) - 3y = D^2y - 7y \\ x(D-2) & \end{aligned}$$

$$\Rightarrow y(t) = C_1 e^{\sqrt{7}t} + C_2 e^{-\sqrt{7}t}$$

6) 
$$\begin{cases} \frac{dx}{dt} = x - 2y \\ \frac{dy}{dt} = -3x + 4y \end{cases} \Rightarrow \begin{cases} Dx - x + 2y = 0 \\ Dy - 4y + 3x = 0 \end{cases} \Rightarrow \begin{cases} x(D-1) + 2y = 0 \\ y(D-4) + 3x = 0 \end{cases} \Rightarrow \text{Eliminar } y \Rightarrow$$

$$\begin{aligned} x(D-4) &= \begin{cases} x(D-1)(D-4) + 2y(D-4) = 0 \\ -2y(D-4) - 6x = 0 \end{cases} = x(D^2-4D-1D+4) - 6x = 0 \Rightarrow \\ x(-2) & \end{aligned}$$

$$\Rightarrow D^2x - 5D - 2x = 0 \Rightarrow D = 25 + 8 \Rightarrow D = 33 \Rightarrow \sqrt{D} = \sqrt{33} \Rightarrow$$

$$\Rightarrow R = \frac{5 \pm \sqrt{33}}{2(1)} \Rightarrow \begin{aligned} R_1 &= \frac{(5+\sqrt{33})}{2} \\ R_2 &= \frac{(5-\sqrt{33})}{2} \end{aligned} \Rightarrow x(t) = C_1 e^{\frac{(5+\sqrt{33})}{2}t} + C_2 e^{\frac{(5-\sqrt{33})}{2}t}$$

Eliminar  $x \Rightarrow$  
$$\begin{aligned} x(-3) &= \begin{cases} -3x(D-1) - 6y = 0 \\ y(D-4)(D-1) + 3x(D-1) = 0 \end{cases} \\ x(D-1) & \end{aligned} \Rightarrow y(D^2-5D+4) - 6y = 0$$

$$\Rightarrow y(t) = C_1 e^{\frac{(5+\sqrt{33})}{2}t} + C_2 e^{\frac{(5-\sqrt{33})}{2}t}$$



$$⑦ \begin{cases} \frac{dx}{dt} = 2x - y \\ \frac{dy}{dt} = x + 3y \end{cases} \Rightarrow \begin{cases} Dx - 2x + y = 0 \\ Dy - 3y - x = 0 \end{cases} \Rightarrow \begin{cases} x(D-2) + y = 0 \\ y(D-3) - x = 0 \end{cases} \Rightarrow \text{Eliminar } y \Rightarrow \begin{matrix} * \\ x(D-3) \\ x(-1) \end{matrix}$$

$$\Rightarrow \begin{cases} x(D-2)(D-3) + y(D-3) = 0 \\ -y(D-3) + x = 0 \end{cases} \Rightarrow x(D^2 - 3D - 2D + 6) + x = 0 \Rightarrow D^2x - 5Dx + 7x = 0$$

$$\Rightarrow \Delta = 25 - 28 \Rightarrow \Delta = -3 \Rightarrow \sqrt{\Delta} = \sqrt{3}i \Rightarrow R = \frac{5 \pm \sqrt{3}i}{2} \Rightarrow R = \frac{5}{2} \pm \frac{\sqrt{3}i}{2}$$

$$\Rightarrow b = \frac{5}{2} \quad a = \frac{\sqrt{3}}{2} \Rightarrow \boxed{x(t) = C_1 e^{\frac{5}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) + C_2 e^{\frac{5}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)}$$

$$\Rightarrow \text{Eliminar } x \Rightarrow \begin{matrix} * \\ x(1) \\ x(D-2) \end{matrix} = \begin{cases} x(D-2) + y = 0 \\ y(D-3)(D-2) - x(D-2) = 0 \end{cases} \Rightarrow y(D^2 - 3D - 2D + 6) + y = 0$$

$$\Rightarrow \boxed{y(t) = C_1 e^{\frac{5}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) + C_2 e^{\frac{5}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)}$$

$$⑧ \begin{cases} \frac{dx}{dt} = -x + y \\ \frac{dy}{dt} = 2x - 2y \end{cases} \Rightarrow \begin{cases} Dx + x - y = 0 \\ Dy + 2y - 2x = 0 \end{cases} \Rightarrow \begin{cases} x(D+1) - y = 0 \\ y(D+2) - 2x = 0 \end{cases} \Rightarrow \text{Eliminar } y \Rightarrow \begin{matrix} * \\ x(D+2) \\ x(1) \end{matrix}$$

$$\Rightarrow \begin{cases} x(D+1)(D+2) - y(D+2) = 0 \\ y(D+2) - 2x = 0 \end{cases} \Rightarrow x(D^2 + 3D + 3) - 2x = D^2x + 3Dx + x$$

$$\Rightarrow \Delta = 9 - 4 \Rightarrow \Delta = 5 \Rightarrow \sqrt{\Delta} = \sqrt{5} \Rightarrow R = \frac{-3 \pm \sqrt{5}}{2} \Rightarrow \boxed{\begin{matrix} R_1 = \frac{-3 + \sqrt{5}}{2} \\ R_2 = \frac{-3 - \sqrt{5}}{2} \end{matrix}}$$

$$\Rightarrow \boxed{x(t) = C_1 e^{\left(\frac{-3 + \sqrt{5}}{2}\right)t} + C_2 e^{\left(\frac{-3 - \sqrt{5}}{2}\right)t}}$$

$$\Rightarrow \text{Eliminar } x \Rightarrow \begin{matrix} * \\ x(2) \\ x(D+1) \end{matrix} \Rightarrow \begin{cases} 2x(D+1) - 2y = 0 \\ y(D+2)(D+1) - 2x(D+1) = 0 \end{cases} \Rightarrow y(D^2 + 3D + 3) - 2y$$

$$\Rightarrow \boxed{y(t) = C_1 e^{\left(\frac{-3 + \sqrt{5}}{2}\right)t} + C_2 e^{\left(\frac{-3 - \sqrt{5}}{2}\right)t}}$$

$$\textcircled{9} \begin{cases} \frac{dx}{dt} = x + 2y \\ \frac{dy}{dt} = -3x + y \end{cases} \Rightarrow \begin{cases} Dx - x - 2y = 0 \\ Dy - y + 3x = 0 \end{cases} \Rightarrow \begin{cases} x(D-1) - 2y = 0 \\ y(D-1) + 3x = 0 \end{cases} \Rightarrow \text{Eliminar } y \Rightarrow \begin{matrix} x(D-1) \\ x(2) \end{matrix} \Rightarrow$$

$$\Rightarrow \begin{cases} x(D-1)^2 - 2y(D-1) = 0 \\ 2y(D-1) + 6x = 0 \end{cases} \Rightarrow x(D^2 - 2D + 1) + 6x = D^2x - 2Dx + 7x \Rightarrow \Delta = 4 - 28 \Rightarrow \Delta = -24$$

$$\Rightarrow \sqrt{\Delta} = \sqrt{24}i = \begin{matrix} 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \end{matrix} = 2\sqrt{6} \Rightarrow \sqrt{\Delta} = 2\sqrt{6}i \Rightarrow R = \frac{2 \pm 2\sqrt{6}i}{2(1)} \Rightarrow \begin{matrix} R_1 = 1 + \sqrt{6}i \\ R_2 = 1 - \sqrt{6}i \end{matrix}$$

$$\Rightarrow a = 1 \quad \alpha = \sqrt{6} \Rightarrow \boxed{x(t) = C_1 e^* \cos(\sqrt{6}t) + C_2 e^* \sin(\sqrt{6}t)}$$

$$\Rightarrow \text{Eliminar } x \Rightarrow \begin{matrix} * \\ x(-3) \\ x(D-1) \end{matrix} = \begin{cases} -3x(D-1) + 6y = 0 \\ y(D-1)^2 + 3x(D-1) = 0 \end{cases} = y(D^2 - 2D + 1) + 6y \Rightarrow$$

$$\Rightarrow \boxed{y(t) = C_1 e^* \cos(\sqrt{6}t) + C_2 e^* \sin(\sqrt{6}t)}$$

$$\textcircled{10} \begin{cases} \frac{dx}{dt} = -2x + 3y \\ \frac{dy}{dt} = x - 4y \end{cases} \Rightarrow \begin{cases} Dx + 2x - 3y = 0 \\ Dy + 4y - x = 0 \end{cases} \Rightarrow \begin{cases} x(D+2) - 3y = 0 \\ y(D+4) - x = 0 \end{cases} \Rightarrow \text{Eliminar } y \Rightarrow \begin{matrix} x(D+4) \\ x(3) \end{matrix}$$

$$= \begin{cases} x(D+2)(D+4) - 3y(D+4) = 0 \\ 3y(D+4) - 3x = 0 \end{cases} = x(D^2 + 6D + 8) - 3x = D^2x + 6Dx + 5x \Rightarrow \Delta = 16$$

$$\Rightarrow \sqrt{\Delta} = 4 \Rightarrow R = \frac{-6 \pm 4}{2} \Rightarrow \begin{matrix} R_1 = -1 \\ R_2 = -5 \end{matrix} \Rightarrow \boxed{x(t) = C_1 e^{-t} + C_2 e^{-5t}} \Rightarrow \text{Eliminar } x$$

$$\Rightarrow \begin{matrix} * \\ x(1) \\ x(D+2) \end{matrix} = \begin{cases} x(D+2) - 3y = 0 \\ y(D+4)(D+2) - x(D+2) = 0 \end{cases} = y(D^2 + 6D + 8) - 3y \Rightarrow \boxed{y(t) = C_1 e^{-t} + C_2 e^{-5t}}$$