Lista 11 - Página 1

1) $L[sen(t)]$

$f(t) = sen(t) \therefore f'(t) = cos(t) \therefore f''(t) = -sen(t)$

$L[f''(t)] = ? \Rightarrow \lim_{b \to \infty} \int_0^b f''(t) e^{-st} dt \Rightarrow$ $u = e^{-st}$ $dv = f''(t)$ ; $du = -se^{-st} dt$ $v = f'(t)$

$\Rightarrow f'(t)e^{-st} + s \int f'(t) e^{-st} dt$

$\Rightarrow \int f'(t) e^{-st} dt =$ $u = e^{-st}$ $dv = f'(t)$ ; $du = -se^{-st}$ $v = f(t)$ $\Rightarrow f(t)e^{-st}\Big|_0^{\infty} + s \underbrace{\int f(t) e^{-st} dt}_{F(s)}$

$\Rightarrow \cancel{f(\infty) e^{-s(\infty)}}^{0} - f(0)\cancel{e^{-s(0)}}^{1} + sF(s) \Rightarrow \boxed{sF(s) - f(0)}$

$\Rightarrow f'(t)e^{-st}\Big|_0^{\infty} + s\left(sF(s) - f(0)\right) \Rightarrow \cancel{f'(\infty) e^{-s(\infty)}}^{0} - f'(0)\cancel{e^{-s(0)}}^{1} + \ldots$

$\Rightarrow \boxed{L[f''(t)] = s^2F(s) - sf(0) - f'(0)}$

$\Rightarrow L[-sen(t)] = L[f''(t)] \Rightarrow s^2F(s) - s(\cancel{sen(0)}^{0}) - (\cancel{cos(0)}^{1})$

$\Rightarrow L[f''(t)] = s^2F(s) - 1 \Rightarrow \boxed{F(s) = \frac{L[f''(t)] + 1}{s^2}}$

Lista 11 - Página 2

② $L[\cos(2t)] \Rightarrow L[\cos(at)]$

$$\int_0^{\infty} \cos(at)e^{-st}dt \Rightarrow \begin{matrix} u = \cos(at) & dv = e^{-st}dt \\ du = -a\,\text{sen}(at)dt & v = -\frac{1}{s}e^{-st} \end{matrix} \Rightarrow$$

$$\Rightarrow -\frac{\cos(at)e^{-st}}{s} - \frac{a}{s}\int \text{sen}(at)e^{-st}dt$$

$$\Rightarrow \int \text{sen}(at)e^{-st}dt \Rightarrow \begin{matrix} u = \text{sen}(at) & dv = e^{-st}dt \\ du = a\cos(at)dt & v = -\frac{1}{s}e^{-st} \end{matrix} \Rightarrow$$

$$\Rightarrow = -\frac{\cos(at)e^{-st}}{s}\Bigg|_0^{\infty} - \frac{a}{s}\left[-\frac{\text{sen}(at)e^{-st}}{s} + \frac{a}{s}\int \cos(at)e^{-st}dt\right]$$

$$\Rightarrow = -\frac{\cos(at)e^{-st}}{s}\Bigg|_0^{\infty} - \frac{a}{s}\left[-\frac{\text{sen}(at)e^{-st}}{s}\Bigg|_0^{\infty} + \frac{a}{s}\cdot L[\cos(at)]\right]$$

$$\Rightarrow \left[-\cancel{\frac{\cos(a(\infty))}{s}e^{-s(\infty)}}^{0} - \left(\cancel{-\frac{\cos(0)e^0}{s}}\right)^{+1}\right] + \left[-\frac{a}{s}\left[\cancel{-\frac{\text{sen}(a(\infty))e^{-s(\infty)}}{s}}^{0} - \left(\cancel{-\frac{\text{sen}(0)e^0}{s}}\right)^{0}\right]\ldots\right.$$

$$\Rightarrow \frac{1}{s} - \frac{a}{s}\left[0 + \frac{a}{s}\cdot L[\cos(at)]\right] \Rightarrow \frac{1}{s} - \frac{a^2}{s^2}L[\cos(at)]$$

$$\Rightarrow L[\cos(at)] = \frac{1}{s} - \frac{a^2}{s^2}L[\cos(at)] \Rightarrow L[\cos(at)]\left(1 + \frac{a^2}{s^2}\right) = \frac{1}{s}$$

$$\Rightarrow L[\cos(at)] = \frac{\frac{1}{s}}{\frac{1}{1} + \frac{a^2}{s^2}} = \frac{\frac{1}{s}}{\frac{s^2+a^2}{s^2}} \Rightarrow \frac{1}{\cancel{s}}\cdot\frac{s^{\cancel{2}}}{s^2+a^2} = \frac{s}{s^2+a^2}$$

$$\Rightarrow \boxed{L[\cos(at)] = \frac{s}{s^2+a^2}}$$

Lista 11 - Página 3

3) $L[3t^3+2t^2+t]$

$3.L[t^3]+2.L[t^2]+L[t]$

$L[t]=\int_0^\infty te^{-st}dt \Rightarrow \begin{matrix} u=t & dv=e^{-st}dt \\ du=dt & v=-\frac{1}{s}e^{-st}\end{matrix} \Rightarrow -\frac{t}{s}e^{-st}+\frac{1}{s}\int e^{-st}dt$

$\Rightarrow -\frac{t}{s}e^{-st}+\frac{1}{s}\left(-\frac{1}{s}e^{-st}\right) \Rightarrow -\frac{t}{s}e^{-st}-\frac{1}{s^2}e^{-st}\Big|_0^\infty \Rightarrow$

$\Rightarrow \cancel{-\frac{(\infty)e^{-s(\infty)}}{s}}^{0} \cancel{-\frac{1}{s^2}e^{-s(\infty)}}^{0} + \cancel{\frac{(0)}{s}e^{-s(0)}}^{0} + \frac{1}{s^2}\cancel{e^{-s(0)}}^{1} \Rightarrow \boxed{L[t]=\frac{1}{s^2}}$

$\Rightarrow L[t^2]=\int_0^\infty t^2e^{-st}dt \Rightarrow \begin{matrix} u=t^2 & dv=e^{-st}dt \\ du=2tdt & v=-\frac{1}{s}e^{-st}\end{matrix} \Rightarrow -\frac{t^2}{s}e^{-st}+\frac{2}{s}\int_0^\infty te^{-st}dt$

~~$\Rightarrow -\frac{t^2}{s}e^{-st}+\frac{2}{s}\left(\frac{1}{s^2}\right) \Rightarrow -\frac{t^2}{s}e^{-st}+\frac{2}{s^3}\Big|_0^\infty$~~

~~$\Rightarrow -\frac{(\infty)^2e^{-s(\infty)}}{s}+\frac{2}{s^3}+\frac{(0)^2e^{-s(0)}}{s}-\frac{2}{s^3}$~~

$\Rightarrow -\frac{t^2}{s}e^{-st}+\frac{2}{s}\left[-\frac{t}{s}e^{-st}-\frac{1}{s^2}e^{-st}\right] \Rightarrow \left(-\frac{t^2}{s}-\frac{2t}{s^2}-\frac{2}{s^3}\right)(e^{-st})\Big|_0^\infty$

$\Rightarrow \cancel{\left(-\frac{(\infty)^2}{s}-\frac{2(\infty)}{s^2}-\frac{2}{s^3}\right)(e^{-s(\infty)})}^{0} + \left(\cancel{\frac{(0)^2}{s}}^{0}+\cancel{\frac{2(0)}{s^2}}^{0}+\frac{2}{s^3}\right)\cancel{(e^{-s(0)})}^{1} = \frac{2}{s^3}$

$\Rightarrow \boxed{L[t^2]=\frac{2}{s^3}}$

Lista III - Página 4

Continuação da 3

$$L[t^3] = \int_0^\infty t^3 e^{-st}\,dt \Rightarrow \begin{matrix} u = t^3 & dv = e^{-st}dt \\ du = 3t^2 dt & v = -\frac{1}{s}e^{-st} \end{matrix} \Rightarrow$$

$$\Rightarrow -\frac{t^3}{s}e^{-st} + \frac{3}{s}\int t^2 e^{-st}\,dt \Rightarrow -\frac{t^3}{s} + \frac{3}{s}\left[-\frac{t^2}{s}e^{-st} - \frac{2t}{s^2}e^{-st} - \frac{2}{s^3}e^{-st}\right]$$

$$\Rightarrow -\frac{e^{-st}t^3}{s} - \frac{3t^2e^{-st}}{s^2} - \frac{6te^{-st}}{s^3} - \frac{6e^{-st}}{s^4}\Bigg|_0^\infty \Rightarrow \dots$$

$$\Rightarrow -\cancel{\frac{e^{-s(\infty)}(\infty)^3}{s}}^{\,0} - \cancel{\frac{3(\infty)^2e^{-s(\infty)}}{s^2}}^{\,0} - \cancel{\frac{6(\infty)e^{-s(\infty)}}{s^3}}^{\,0} - \cancel{\frac{6e^{-s(\infty)}}{s^4}}^{\,0} + \cancel{\frac{(0)^3}{s}}^{\,0} + \cancel{\frac{3(0)^2e^{-s(0)}}{s^2}}^{\,0} + \dots$$

$$= \frac{6\cancel{e^{-s(0)}}^{\,1}}{s^4} = \frac{6}{s^4} \Rightarrow \boxed{L[t^3] = \frac{6}{s^4}}$$

$$3L[t^3] + 2L[t^2] + L[t] = 3\left(\frac{6}{s^4}\right) + 2\left(\frac{2}{s^3}\right) + \left(\frac{1}{s^2}\right) = \boxed{\frac{18}{s^4} + \frac{4}{s^3} + \frac{1}{s^2}}$$

$$\boxed{L[3t^3 + 2t^2 + t] = \frac{18}{s^4} + \frac{4}{s^3} + \frac{1}{s^2}}$$

Lista 11 - Página 5

④ $L[5e^{4t}]$

$$5.L[e^{4t}] \Rightarrow 5.\int_0^\infty e^{4t}.e^{-st}dt \Rightarrow 5\int_0^\infty e^{t(4-s)}dt \Rightarrow \begin{matrix} u = t(4-s) \\ du = (4-s)dt \\ dt = \frac{du}{4-s} \end{matrix} \Rightarrow$$

$$\Rightarrow 5.\int_0^\infty e^{u}.\frac{du}{4-s} \Rightarrow \frac{5}{4-s}\int_0^\infty e^{u}du \Rightarrow \frac{5}{4-s}e^{t(4-s)}\Big|_0^\infty \Rightarrow$$

$$\Rightarrow \frac{5}{4-s}e^{(\infty)(4-s)} - \frac{5}{4-s}e^{(0)(4-s)} \Rightarrow \text{Condição: } 4-S<0 \Rightarrow \boxed{S>4}$$

$$\Rightarrow \text{Assim: } 0 - \frac{5}{4-s} \Rightarrow \boxed{\frac{5}{S-4}} \Rightarrow \boxed{L[5e^{4t}] = \frac{5}{S-4}}$$

⑤ $L[5t^2-4t+3]$

$$5L[t^2] - 4L[t] + L[3]$$

$$L[3] = \int_0^\infty 3e^{-st}dt \Rightarrow \begin{matrix} u = -St \\ du = -Sdt \\ dt = -\frac{du}{S} \end{matrix} \Rightarrow -\frac{3}{S}\int_0^\infty e^{u}du = -\frac{3}{S}e^{-St}\Big|_0^\infty$$

$$\Rightarrow -\frac{3}{S}\cancel{e^{-S(\infty)}}^{0} - \left(-\frac{3}{S}e^{-S(0)}\right) = \frac{3}{S} \Rightarrow \boxed{L[3] = \frac{3}{S}}$$

Lembrando que: $L[t] = -\frac{t}{S}e^{-st} - \frac{1}{S^2}e^{-st}$

$$L[t^2] = -\frac{t^2}{S}e^{-st} - \frac{2t}{S^2}e^{-st} - \frac{2}{S^3}e^{-st}$$

$$\text{Então: } 5.\left[(e^{-st})\left(-\frac{t^2}{S} - \frac{2t}{S^2} - \frac{2}{S^3}\right)\right]_0^\infty - 4\left[(e^{-st})\left(-\frac{t}{S} - \frac{1}{S^2}\right)\right]_0^\infty + \left(\frac{3}{S}\right)$$

$$\Rightarrow 5.\left[\cancel{e^{-S(\infty)}\left(-\frac{(\infty)^2}{S} - \frac{2(\infty)}{S^2} - \frac{2}{S^3}\right)}^{0} + \left(\cancel{e^{-S(0)}}^{1}\left(\cancel{\frac{(0)^2}{S}}^{0} + \cancel{\frac{2(0)}{S^2}}^{0} + \frac{2}{S^3}\right)\right)\right] = \boxed{\frac{10}{S^3}}$$

Lista 11 - Página 6

continuação da 5

$$\boxed{L[5t^2] = \frac{10}{s^3}} \qquad \boxed{L[3] = \frac{3}{s}}$$

$$-4L[t] = -4\left[e^{-st}\left(-\frac{t}{s} - \frac{1}{s^2}\right)\right]_0^\infty = -4\left[\cancel{e^{-s(\infty)}\left(-\frac{(\infty)}{s} - \frac{1}{s^2}\right)} + e^{-s(0)}\left(\frac{\cancel{(0)}}{\cancel{s}} + \frac{1}{s^2}\right)\right]$$

$$-4\cdot\left(\frac{1}{s^2}\right) \Rightarrow \boxed{L[-4t] = -\frac{4}{s^2}}$$

$$\boxed{L[y(t)] = \frac{10}{s^3} - \frac{4}{s^2} + \frac{3}{s}}$$

6) $L[f'''(t)] =$

$$\int_0^\infty f'''(t)e^{-st}dt \Rightarrow \begin{matrix} u = e^{-st} & dv = f'''(t)dt \\ du = -\frac{1}{s}e^{-st}dt & v = f''(t) \end{matrix} \Rightarrow f''(t)e^{-st}\Big|_0^\infty + \frac{1}{s}\int_0^\infty f''(t)e^{-st}dt$$

$\Rightarrow$ Lembrando que: $L[f''(t)] = s^2F(s) - sf(0) - f'(0)$

$\Rightarrow$ Então: $f''(t)e^{-st}\Big|_0^\infty + \frac{1}{s}\left[s^2F(s) - sf(0) - f'(0)\right]$

$$\Rightarrow \cancel{f''(\infty)e^{-s(\infty)}} - f''(0)e^{-s(0)} + \frac{s\cancel{^2}F(s)}{\cancel{s}} - f(0) - \frac{f'(0)}{s}$$

$$\Rightarrow \boxed{L[f'''(t)] = sF(s) - f''(0) - \frac{f'(0)}{s} - f(0)}$$

Lista 11 - Página 7

(7) $L[e^{2t} sen(t)]$

$$= \int_0^\infty sen(t) e^{(2-s)t} dt \Rightarrow \begin{matrix} u = sen(t) & dv = e^{t(2-s)} dt \\ du = cos(t)dt & v = \frac{1}{2-s} e^{(2-s)t} \end{matrix} \Rightarrow \frac{sen(t)}{2-s} e^{(2-s)t} \Big|_0^\infty - \dots$$

$$\Rightarrow \frac{sen(t)}{2-s} e^{(2-s)t} \Big|_0^\infty - \frac{1}{2-s} \int cos(t) e^{(2-s)t} dt \Rightarrow \begin{matrix} u = cos(t) & dv = e^{t(2-s)} dt \\ du = -sen(t)dt & v = \frac{1}{2-s} e^{(2-s)t} \end{matrix}$$

$$\Rightarrow \frac{sen(t)}{2-s} e^{(2-s)t} \Big|_0^\infty - \left[ \frac{cos(t)}{2-s} e^{(2-s)t} \Big|_0^\infty + \frac{1}{2-s} \int_0^\infty sen(t) e^{(2-s)t} dt \right]$$

$\Rightarrow$ Condição : $(2-s) < 0 \Rightarrow \boxed{s > 2}$

$$\Rightarrow \frac{sen(\infty) e^{\cancel{-(2-s)(\infty)}^{0}}}{\cancel{2-s}} - \frac{sen(0) e^{-(2-s)(0)^{0}}}{\cancel{2-s}} - \left[ \frac{cos(\infty) e^{\cancel{-(2-s)(\infty)}^{0}}}{\cancel{2-s}} - \frac{cos(0)}{2-s} e^{-(2-s)(0)} + \frac{F(s)}{2-s} \right]$$

$$\Rightarrow -\left[ -\frac{1}{2-s} + \frac{F(s)}{2-s} \right] \Rightarrow \frac{1}{2-s} - \frac{F(s)}{2-s} \Rightarrow \boxed{L[e^{2t} sen(t)] = \frac{1-F(s)}{2-s}}$$

(8) $L[\frac{d}{dt}(3e^t)]$

Anteriormente: $L[f'(t)] = sF(s) - f(0) \Rightarrow sF(s) - (3e^{(0)}) \Rightarrow sF(s) - 3$

$$\boxed{L[k'(t)] = sF(s) - 3}$$

Lista 11 - Página 8

9) $L[\operatorname{sen}(4t)] + L[\cos(2t)]$

$L[\operatorname{sen}(4t)] = \int_0^\infty \operatorname{sen}(4t)e^{-st}dt \Rightarrow$ $u = \operatorname{sen}(4t)$, $du = 4\cos(4t)dt$; $dv = e^{-st}dt$, $v = -\frac{1}{s}e^{-st}dt$ $\Rightarrow -\frac{\operatorname{sen}(4t)}{s}e^{-st}dt \ldots$

$\Rightarrow -\frac{\operatorname{sen}(4t)}{s}e^{-st}dt + \frac{4}{s}\int_0^\infty \cos(4t)e^{-st}dt \Rightarrow$ $u = \cos(4t)$, $du = -4\operatorname{sen}(4t)dt$; $dv = e^{-st}dt$, $v = -\frac{1}{s}e^{-st}$ $\Rightarrow$

$\Rightarrow -\frac{\operatorname{sen}(4t)}{s}e^{-st}\Big|_0^\infty + \frac{4}{s}\left[-\frac{\cos(4t)}{s}e^{-st}\Big|_0^\infty - \frac{4}{s}\int_0^\infty \operatorname{sen}(t)e^{-st}dt\right]$

$\Rightarrow \underbrace{-\frac{\operatorname{sen}(4t)}{s}e^{-st}\Big|_0^\infty}_{\Rightarrow -0+0} \underbrace{- \frac{4\cos(4t)}{s^2}e^{-st}\Big|_0^\infty}_{-0+\frac{4}{s^2}} - \frac{16}{s^2}\cdot F(s) = F(s)$

$-0 + \frac{4}{s^2} - \frac{16}{s^2}F(s) = F(s) \Rightarrow \frac{4}{s^2} - \frac{4^2}{s^2}F(s) = F(s)$

$\Rightarrow \frac{4}{s^2} = F(s) + \frac{4^2}{s^2}F(s) \xrightarrow{\times(s^2)} 4 = s^2F(s) + 4^2F(s) \Rightarrow F(s)(s^2+4^2) = 4$

$\Rightarrow \boxed{L[\operatorname{sen}(4t)] = \frac{4}{s^2+4^2}}$ $\Rightarrow$ De forma análoga: $\boxed{L[\operatorname{sen}(at)] = \frac{a}{s^2+a^2}}$

$L[\cos(2t)] \Rightarrow \int_0^\infty \cos(2t)e^{-st}dt \Rightarrow$ $u = \cos(2t)$, $du = -2\operatorname{sen}(2t)dt$; $dv = e^{-st}dt$, $v = -\frac{1}{s}e^{-st}$ $\Rightarrow$

$\Rightarrow -\frac{\cos(2t)}{s}e^{-st}\Big|_0^\infty - \frac{2}{s}\int_0^\infty \operatorname{sen}(2t)e^{-st}dt \Rightarrow$ $u = \operatorname{sen}(2t)$, $du = 2\cos(2t)dt$; $dv = e^{-st}dt$, $v = -\frac{1}{s}e^{-st}$ $\Rightarrow$

$\Rightarrow -\frac{\cos(2t)}{s}e^{-st}\Big|_0^\infty - \frac{2}{s}\left[-\frac{\operatorname{sen}(2t)}{s}e^{-st}\Big|_0^\infty + \frac{2}{s}\int_0^\infty \cos(2t)e^{-st}dt\right] \Rightarrow$

$\Rightarrow -\frac{\cos(2t)}{s}e^{-st}\Big|_0^\infty + \frac{2\operatorname{sen}(2t)}{s^2}e^{-st}\Big|_0^\infty - \frac{2^2}{s^2}F(s) = F(s) \Rightarrow$

$\Rightarrow \left[(-0) + \left(\frac{1}{s}\right)\right] + \left[(0) - (0)\right] - \frac{2^2}{s^2}F(s) = F(s) \Rightarrow \frac{1}{s} - \frac{2^2}{s^2}F(s) = F(s)$

$\Rightarrow \frac{1}{s} = F(s)\left(1 + \frac{2^2}{s^2}\right) \Rightarrow \ldots = \boxed{\frac{s}{s^2+2^2}}$

Lista 11 - Página 9

$$\boxed{L[m(t)] = \frac{4}{s^2+4^2} + \frac{s}{s^2+2^2}}$$

1) $L[\pi]$

$$= \int_0^\infty \pi e^{-st}dt \Rightarrow \pi\int_0^\infty e^{-st}dt \Rightarrow \begin{matrix} u=-st \\ du=-s\,dt \\ dt=-\frac{du}{s} \end{matrix} \Rightarrow \pi\int_0^\infty e^{u}\cdot\left(-\frac{du}{s}\right) \Rightarrow -\frac{\pi}{s}\int_0^\infty e^{u}du$$

$$\Rightarrow -\frac{\pi}{s}e^{-st}\Big|_0^\infty \Rightarrow -\frac{\pi}{s}e^{-s(\infty)} - \left(-\frac{\pi}{s}e^{-s(0)}\right) \Rightarrow \cancel{-\frac{\pi}{s}e^{-s(\infty)}}^{0} + \frac{\pi}{s}\cancel{e^{-s(0)}}^{1} = \boxed{\frac{\pi}{s}}$$

$$\boxed{L[\pi] = \frac{\pi}{s}}$$

13) $L[e^{2t}]$

$$= \int_0^\infty e^{t(2-s)}dt \Rightarrow \begin{matrix} u=t(2-s) \\ du=dt(2-s) \\ dt=\frac{du}{2-s} \end{matrix} \Rightarrow \int_0^\infty e^{u}\cdot\frac{du}{2-s} \Rightarrow \frac{1}{2-s}e^{t(2-s)}\Big|_0^\infty \Rightarrow \text{Condição: } s>2$$

$\Rightarrow$ Para que o expoente seja negativo $\Rightarrow$

$$\cancel{\frac{1}{2-s}e^{\infty(2-s)}}^{0} - \frac{1}{2-s}e^{(0)(2-s)} = -\frac{1}{2-s} = \boxed{\frac{1}{s-2}}$$

$$\boxed{L[e^{2t}] = \frac{1}{s-2}}$$

5) $L[H'(t)]$

$$H(t) = 3t^3+2t^2+t \Rightarrow H'(t) = 9t^2+4t+1 \Rightarrow L[9t^2+4t+1]$$

$$\Rightarrow L[9t^2] = 9\cdot\left(\frac{2}{s^3}\right) = \boxed{\frac{18}{s^3}} \quad , \quad L[4t] = 4\left(\frac{1}{s^2}\right) = \boxed{\frac{4}{s^2}} \quad , \quad L[1] = \boxed{\frac{1}{s}}$$

$$\boxed{L[H'(t)] = \frac{18}{s^3} + \frac{4}{s^2} + \frac{1}{s}}$$

Lista 11 - Página 10

(17) $L[\operatorname{sen}^2(t)]$

Sabendo que: $\operatorname{sen}^2(t) = \frac{1-\cos(2t)}{2} \Rightarrow L\left[\frac{1-\cos(2t)}{2}\right] \Rightarrow \frac{1}{2}(1-\cos(2t)) \Rightarrow$

$\Rightarrow \frac{1}{2}\Big[L[1] - L[\cos(2t)]\Big] \Rightarrow \frac{1}{2}\left[\frac{1}{\cancel{s}} - \frac{\cancel{s}}{\cancel{s^2+2^2}}\right] \Rightarrow \frac{1}{2}\left[\frac{\cancel{s}+\frac{2^2}{s}-\cancel{s}}{s^2+2^2}\right] \Rightarrow \begin{matrix} x(s) \\ x(s) \end{matrix}$

$s+\frac{2^2}{s}$ $\quad 1$

$\Rightarrow \frac{1}{\cancel{2}}\left[\frac{2^{\cancel{2}}}{s^3+2^2s}\right] \Rightarrow \boxed{L[\operatorname{sen}^2(t)] = \frac{2}{s^3+2^2s}}$

(18) $L[\cos^2(t)]$

Sabendo que: $\cos^2(t) = \frac{1+\cos(2t)}{2} \Rightarrow \frac{1}{2}(1+\cos(2t)) \Rightarrow \frac{1}{2}\Big[L[1] + L[\cos(2t)]\Big]$

$\Rightarrow \frac{1}{2}\left[\frac{1}{\cancel{s}} + \frac{s}{\cancel{s^2+2^2}}\right] \Rightarrow \frac{1}{2}\left[\frac{s+\frac{2^2}{s}+s}{s^2+2^2}\right] \begin{matrix} x(s) \\ = \\ x(s) \end{matrix} \frac{1}{2}\left[\frac{2s^2+2^2}{s^3+2^2s}\right] \Rightarrow \left[\frac{s^2+2}{s^3+2^2s}\right]$

$s+\frac{2^2}{s}$ $\quad 1$

$\boxed{L[\cos^2(t)] = \frac{s^2+2}{s^3+2^2s}}$

(19) $L\left[\frac{d^2}{dt^2}(2t^3)\right]$

$2t^3 \xrightarrow{\frac{d}{dt}} 6t^2 \xrightarrow{\frac{d}{dt}} 12t \Rightarrow L[12t] \Rightarrow 12 \cdot L[t] \Rightarrow 12 \cdot \frac{1}{s^2}$

$\boxed{L\left[\frac{d^2}{dt^2}(2t^3)\right] = \frac{12}{s^2}}$ ou $\boxed{L\left[\frac{d^2}{dt^2}(\cancel{2t}(t))\right] = s^2F(s)}$

(20) $L\left[\frac{d^2}{dt^2}(4t^2+3t+1)\right]$

$4t^2+3t+1 \xrightarrow{\frac{d}{dt}} 8t+3 \xrightarrow{\frac{d}{dt}} 8 \Rightarrow \boxed{L[8] = \frac{8}{s}}$ ou $\boxed{L[w(t)] = s^2F(s) - s - 3}$

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Lista 11 - Página 11

10) $L[\ln(t+1)]$

$$\ln(t+1) \xrightarrow{\frac{d}{dt}} \frac{1}{t+1} \Rightarrow L[\ln(t+1)'] = \frac{L[1]}{L[t]+L[1]} \Rightarrow sF(s) - \cancel{(\ln(1))}^{0} = \frac{\frac{1}{s}}{\frac{1}{\cancel{s^2}_1} + \frac{1}{\cancel{s}_s}}$$

$$\Rightarrow \frac{\frac{1}{s}}{\frac{1+s}{s^2}} \Rightarrow \frac{1}{\cancel{s}} \cdot \frac{s^{\cancel{2}}}{1+s} \Rightarrow \frac{s}{1+s} \Rightarrow \cancel{s}F(s) = \frac{\cancel{s}}{1+s} \Rightarrow \boxed{F(s) = \frac{1}{1+s}}$$

21) $L[\ln(2t)]$

$$\ln(2t) = \cancel{\ln(2)}^{0{,}3} + \ln(t) \Rightarrow L[0{,}3] + L[\ln(t)]$$

$$= \int_0^\infty \ln(t) e^{-st} dt \Rightarrow \begin{aligned} u &= st \\ du &= s\,dt \\ dt &= \frac{du}{s} \\ t &= \frac{u}{s} \end{aligned} \Rightarrow \frac{1}{s}\int_0^\infty \ln\left(\frac{u}{s}\right) e^{-u} du$$

$$\Rightarrow \frac{1}{s}\left[\int_0^\infty \ln(u) e^{-u} du - \int_0^\infty \ln(s) e^{-u} du\right] \Rightarrow \frac{1}{s}\left[(-\gamma) - \left(-e^{-st}\right)\Big|_0^\infty\right]$$

$$\Rightarrow \frac{1}{s}\left[(-\gamma) + (-\ln(s))\right] \Rightarrow \frac{-\gamma - \ln(s)}{s} \Rightarrow \boxed{L[\ln(t)] = -\frac{\gamma + \ln(s)}{s}}$$

Para $\gamma = \int_0^\infty \ln(u) e^{-u} du$ e $s \neq 0$

$$\Rightarrow L[0{,}3] + L[\ln(t)] \Rightarrow \frac{0{,}3}{s} - \frac{\gamma + \ln(s)}{s} \Rightarrow \boxed{L(x(t)) = \frac{0{,}3 - \gamma - \ln(s)}{s}}$$

Lista 11 - Página 12

(22) $L\left[\frac{d}{dt}(e^t-1)\right]$

$$L[y'] = SF(S) - (\cancel{e^0}^{\,1} - 1) \Rightarrow \boxed{L[y'] = SF(S)}$$

$$L\left[\frac{d}{dt}(e^t-1)\right] = L[e^t] = \int_0^\infty e^{t(1-S)}dt \Rightarrow \begin{matrix} u = t(1-S) \\ du = dt(1-S) \\ dt = \frac{du}{1-S} \end{matrix} \Rightarrow \frac{1}{1-S}\int_0^\infty e^u du \Rightarrow$$

$$\Rightarrow \frac{1}{1-S} e^{t(1-S)}\Big|_0^\infty \Rightarrow \text{Condição: } S>1 \Rightarrow -\frac{1}{1-S}e^{-t(1-S)}\Big|_0^\infty \Rightarrow$$

$$\Rightarrow \cancel{-\frac{1}{1-S}e^{-(\infty)(1-S)}}^{\,0} - \left(-\frac{1}{1-S}\cancel{e^{-(0)(1-S)}}^{\,1}\right) = \boxed{\frac{1}{1-S}}$$

$$SF(S) = \frac{1}{1-S} \Rightarrow \boxed{F(S) = \frac{1}{S-S^2}}$$

(23) $L[\pi e^{2t}]$

$$= \int_0^\infty \pi e^{t(2-S)}dt = \pi\int_0^\infty e^{t(2-S)}dt \Rightarrow \begin{matrix} u = t(2-S) \\ du = dt(2-S) \\ dt = \frac{du}{2-S} \end{matrix} \Rightarrow \frac{\pi}{2-S}\int_0^\infty e^u du \Rightarrow$$

$$\Rightarrow \frac{\pi}{2-S}e^{t(2-S)}\Big|_0^\infty \Rightarrow \text{Condição: } S>2 \Rightarrow -\frac{\pi}{2-S}e^{-t(2-S)}\Big|_0^\infty \Rightarrow$$

$$\Rightarrow \cancel{-\frac{\pi}{2-S}e^{-(\infty)(2-S)}}^{\,0} + \frac{\pi}{2-S}\cancel{e^{-(0)(2-S)}}^{\,1} \Rightarrow \boxed{L[\pi e^{2t}] = \frac{\pi}{2-S}}$$

(26) $L[g''(t)]$

$$L[f''] = SF(S) - Sf(0) - f'(0) \Rightarrow g(t) = 5e^{4t},\ g'(t) = 20e^{4t},\ g''(t) = 80e^{4t} \Rightarrow \boxed{SF(S) - 5S - 20}$$

$$L[80e^{4t}] = 80\int_0^\infty e^{t(4-S)}dt \Rightarrow \dots \Rightarrow \frac{80}{4-S}e^{t(4-S)}\Big|_0^\infty \Rightarrow \text{Condição: } (S) > 4$$

$$\Rightarrow -\frac{80}{4-S}e^{-t(4-S)}\Big|_0^\infty \Rightarrow \dots \Rightarrow \boxed{\frac{80}{4-S}} \quad \Rightarrow SF(S) - 5S - 20 = \frac{80}{4-S} \Rightarrow \dots \Rightarrow$$

$$\Rightarrow \boxed{L[g''(t)] = \frac{80}{4S-S^2} + 5 + \frac{20}{S}}$$

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(12) $L[t^2e^{-t}]$

$$\Rightarrow \int_0^{\infty} t^2 e^{t(-1-s)}\,dt \Rightarrow \quad a = t(-1-s) \quad da = dt(-1-s) \quad dt = \frac{da}{(-1-s)} \quad t = \frac{a}{(-1-s)} \Rightarrow \int_0^{\infty} \frac{a^2}{(-1-s)^2} \cdot e^a \cdot \frac{da}{(-1-s)} \Rightarrow$$

$$\Rightarrow \frac{1}{(-1-s)^3} \cdot \int_0^{\infty} a^2 e^a\,da \Rightarrow \quad b = a^2 \quad db = 2a\,da \quad dv = e^a da \quad v = e^a \Rightarrow a^2e^a - 2\int_0^{\infty} e^a . a . da$$

$$\Rightarrow \frac{1}{(-1-s)^3}\left[a^2e^a\Big|_0^{\infty} - 2\int_0^{\infty} a e^a\,da\right] \Rightarrow \quad c = a \quad dc = da \quad dv = e^a da \quad v = e^a \Rightarrow$$

$$\Rightarrow \frac{1}{(-1-s)^3}\left[a^2e^a\Big|_0^{\infty} - 2\left[ae^a\Big|_0^{\infty} - \int e^a da\right]\right] \Rightarrow \frac{1}{(-1-s)^3}\left[a^2e^a\Big|_0^{\infty} - 2\left(ae^a\Big|_0^{\infty} - e^a\Big|_0^{\infty}\right)\right]$$

$$\Rightarrow \frac{a^2e^a}{(-1-s)^3}\Big|_0^{\infty} - \frac{2ae^a}{(-1-s)^3}\Big|_0^{\infty} + \frac{2e^a}{(-1-3)^3}\Big|_0^{\infty} \Rightarrow \text{Substituindo } a = t(-1-s)$$

$$\Rightarrow \frac{(t(-1-s))^2 e^{(t)(-1-s)}}{(-1-s)^3}\Big|_0^{\infty} - \frac{2(t)(-1-s)e^{(t)(-1-s)}}{(-1-s)^3}\Big|_0^{\infty} + \frac{2e^{(t)(-1-s)}}{(-1-s)^3}\Big|_0^{\infty}$$

$\Rightarrow$ Condição : $s > -1$

$$\Rightarrow \frac{((t)(-1-s))^2 e^{-(t)(-1-s)}}{(-1-s)^3}\Big|_0^{\infty} + \frac{2(t)(-1-s)e^{-(t)(-1-s)}}{(-1-s)^3} + \frac{2e^{-(t)(-1-s)}}{(-1-s)^3}\Big|_0^{\infty}$$

$$= [(0)-(0)] + [(0)-(0)] + \cancel{\frac{2e^{-(\infty)(-1-s)}}{(-1-s)^3}}^{\,0} - \frac{2\cancel{e^{-(0)(-1-s)}}^{\,1}}{(-1-s)^3} \Rightarrow \boxed{L[t^2e^{-t}] = \frac{2}{(-1-s)^3}}$$

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1) $L[senh(3t)]$

$\Rightarrow senh(at) = \frac{e^{at} - e^{-at}}{2} \Rightarrow L\left[\frac{e^{at}}{2}\right] - L\left[\frac{e^{-at}}{2}\right] \Rightarrow \frac{1}{2}\left(L[e^{at}] - L[e^{-at}]\right)$

$\Rightarrow L[e^{at}] = \frac{1}{s-a}$ e $L[e^{-at}] = \frac{1}{s+a} \Rightarrow \frac{1}{2}\left(\frac{1}{s-a} - \frac{1}{s+a}\right) \Rightarrow \frac{1}{2}\left(\frac{\cancel{s}+a+\cancel{s}+a}{s^2-a^2}\right)$

$s+a \quad s-a$

$\Rightarrow \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}a}{s^2-a^2} \Rightarrow \boxed{L[senh(at)] = \frac{a}{s^2-a^2}} \Rightarrow \boxed{L[senh(3t)] = \frac{3}{s^2-3^2}}$

16) $L[cosh(2t)]$

$\Rightarrow Cosh(at) = \frac{e^{at} + e^{-at}}{2} \Rightarrow \frac{1}{2} \cdot \left(L[e^{at}] + L[e^{-at}]\right) \Rightarrow \frac{1}{2}\left(\frac{1}{s-a} + \frac{1}{s+a}\right)$

$s+a \quad s-a$

$\Rightarrow \frac{1}{2}\left(\frac{s+\cancel{a}+s-\cancel{a}}{s^2-a^2}\right) \Rightarrow \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}s}{s^2-a^2} \Rightarrow \boxed{L[cosh(at)] = \frac{s}{s^2-a^2}} \Rightarrow \boxed{L[cosh(2t)] = \frac{s}{s^2-2^2}}$

25) $L[cosh^2(t)]$

$Cosh^2(t) = \frac{1 + cosh(2t)}{2} \Rightarrow \frac{1}{2}\left[L[1] + L[cosh(2t)]\right] = \frac{1}{2}\left(\frac{1}{s} + \frac{s}{s^2-2^2}\right) = \frac{1}{2}\left(\frac{s - \frac{2^2}{s} + s}{s^2-2^2}\right)$

$s - \frac{2^2}{s} \quad 1$

$\Rightarrow \frac{1}{2}\left(\frac{2s - \frac{2^2}{s}}{s^2-2^2}\right) \Rightarrow \frac{s - \frac{2}{s}}{s^2-2^2} \frac{\times(s)}{\times(s)} \Rightarrow \frac{s^2-2}{s(s^2-2^2)} = \boxed{\frac{s^2-2}{s^3-2^2s} = L[cosh^2(t)]}$

24) $L[senh^2(t)]$

$senh^2(t) = cosh^2 t - 1 \Rightarrow L[cosh^2(t)] - L[1] = \frac{s^2-2}{s^3-2^2s} - \frac{1}{s} = \frac{\cancel{s^2}-2-\cancel{s^2}+2^2}{s^3-2^2s} = \boxed{\frac{2}{s^3-2^2s}}$

$1 \quad s^2-2^2$

$\boxed{L[senh^2(t)] = \frac{2}{s^3-2^2s}}$