Lista de Integrais

1) $\int (3x^2+5x-1)\,dx$

$\Rightarrow \int 3x^2 dx + \int 5x\,dx - \int 1dx$

$\Rightarrow 3\int x^2 dx + 5\int x\,dx - \int 1dx$

$\Rightarrow 3\left(\frac{x^3}{3}+C_1\right) + 5\left(\frac{x^2}{2}+C_2\right) - \left(x+C_3\right)$

$\Rightarrow \boxed{x^3 + \frac{5x^2}{2} - x + C_4}$

↳ Em que $C_1+C_2+C_3 = C_4$

2) $\int \cos(x) - \operatorname{sen}(x)\,dx$

$\Rightarrow \int \cos(x)dx - \int \operatorname{sen}(x)dx$

⇝ Sendo

- $f(x) = \operatorname{sen}(x) \rightsquigarrow f'(x) = \cos(x)dx$
- $f(x) = -\cos(x) \rightsquigarrow f'(x) = \operatorname{sen}(x)dx$

⇝ Então:

$\Rightarrow \operatorname{sen}(x) + C_1 - (-\cos(x) + C_2)$

$\Rightarrow \boxed{\operatorname{sen}(x) + \cos(x) + C_3}$

3) $\int e^x - \frac{1}{x}\,dx$

$\Rightarrow \int e^x dx - \int \frac{1}{x}dx$

⇝ Sendo

- $f(x) = e^x \rightsquigarrow f'(x) = e^x dx$
- $f(x) = \ln|x| \rightsquigarrow f'(x) = \frac{1}{x}dx$

⇝ Então:

$\Rightarrow (e^x + C_1) - (\ln|x| + C_2)$

$\Rightarrow \boxed{e^x - \ln|x| + C_3}$

4) $\int 2\sec^2(x)dx$

$\Rightarrow 2\int \sec^2(x)dx$

⇝ Sendo

- $f(x) = \tan(x) \rightsquigarrow f'(x) = \sec^2(x)dx$

⇝ Então:

$\Rightarrow 2\int \sec^2(x)dx \Rightarrow \boxed{2\tan(x) + C_1}$

5) $\int \ln(x)+\frac{1}{x}\,dx$

$\Rightarrow \int \ln(x)dx + \int \frac{1}{x}dx$

↳ Por partes

$\to du = \frac{1}{x}dx$

$u = \ln(x)$

$dv = dx$

$v = x$

$\Rightarrow \int u\,dv = uv - \int v\,du$

$\Rightarrow \int \ln(x)dx = (\ln(x))(x) - \int \cancel{x}\cdot\frac{1}{\cancel{x}}dx$

$\Rightarrow \int \ln(x)dx = x\ln(x) - \int 1\,dx$

$\Rightarrow \int \ln(x)dx = x\ln(x) - x + C_1$

$\Rightarrow \int \ln(x)dx + \int \frac{1}{x}dx$

$\Rightarrow (x\ln(x) - x + C_1) + \int \frac{1}{x}dx$

↝ Sendo

- $f(x) = \ln|x| \rightsquigarrow f'(x) = \frac{1}{x}dx$

↝ Então:

$\Rightarrow (x\ln(x) - x + C_1) + (\ln|x| + C_2)$

$\Rightarrow \boxed{x\ln(x) - x + \ln|x| + C_3}$

6) $\int (4x^3 - 2x^2 + 7x)dx$

$\Rightarrow \int 4x^3dx - \int 2x^2dx + \int 7x\,dx$

$\Rightarrow 4\int x^3dx - 2\int x^2dx + 7\int x\,dx$

$\Rightarrow 4\left(\frac{x^4}{4} + C_1\right) - 2\left(\frac{x^3}{3} + C_2\right) + 7\left(\frac{x^2}{2}\right)$

$\Rightarrow \frac{\cancel{4}x^4}{\cancel{4}} + C_1 - \frac{2x^3}{3} + C_2 + \frac{7x^2}{2} + C_3$

$\Rightarrow \boxed{x^4 - \frac{2x^3}{3} + \frac{7x^2}{2} + C_4}$

7) $\int (2\cos(x) + 3\,\text{sen}(x))dx$

$\Rightarrow \int 2\cos(x)dx + \int 3\,\text{sen}(x)dx$

$\Rightarrow 2\int \cos(x)dx + 3\int \text{sen}(x)dx$

$\Rightarrow 2(\text{sen}(x) + C_1) + 3(-\cos(x) + C_2)$

$\Rightarrow \boxed{2\,\text{sen}(x) - 3\cos(x) + C_3}$

8) $\int e^{2x} + \frac{1}{x^2}\, dx$

$\Rightarrow \underbrace{\int e^{2x}dx} + \int \frac{1}{x^2}dx$ * substituição

$\hookrightarrow \int e^{2x}dx \Rightarrow u = 2x$, $du = 2dx$ $\to dx = \frac{du}{2}$

$\Rightarrow \int e^{2x}dx \to \int e^u \cdot \frac{du}{2} \to \frac{1}{2}\int e^u du$

$\Rightarrow \frac{e^u}{2} + C \Rightarrow \frac{e^{2x}}{2} + C_1$

$\Rightarrow \int \frac{1}{x^2}dx \Rightarrow \int x^{-2}dx \Rightarrow \frac{x^{-2+1}}{-2+1} + C_2$

$\Rightarrow \frac{x^{-1}}{-1} + C_2 \Rightarrow -\frac{1}{x} + C_2$

$\Rightarrow \int e^{2x}dx + \int \frac{1}{x^2}dx = \frac{e^{2x}}{2} + \left(-\frac{1}{x}\right) + C_3$

$\Rightarrow \boxed{\frac{e^{2x}}{2} - \frac{1}{x} + C_3}$

9) $\int 3x^2 - 2x + 1\, dx \Rightarrow \int 3x^2dx - \int 2xdx + \int 1dx$

$\Rightarrow \int 3x^2dx \Rightarrow 3\int x^2dx \Rightarrow 3 \cdot \frac{x^3}{3} \Rightarrow x^3 + C_1$

$\Rightarrow \int 2xdx \Rightarrow 2\int xdx \Rightarrow 2 \cdot \frac{x^2}{2} \to x^2 + C_2$

$\Rightarrow \int 1dx \Rightarrow x + C_3$

$\Rightarrow \int 3x^2 - 2x + 1dx \Rightarrow \boxed{x^3 - x^2 + x + C_4}$

10) $\int 3\sec(x)\tan(x)dx$

$\Rightarrow 3\underbrace{\int \sec(x)\tan(x)dx}$

$\hookrightarrow$ Considerando que a derivada da função $\sec(x)$ é igual a $\sec(x)\tan(x)dx$, então:

$\Rightarrow \frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)dx$

Assim, o inverso também pode ser escrito:

$\Rightarrow \int \sec(x)\tan(x)dx = \sec(x)$

$\Rightarrow$ Então:

$\Rightarrow 3\int \sec(x)\tan(x)dx = \boxed{3\sec(x) + C_1}$

11) $\int x\ln(x)dx$

* integração por partes
$\hookrightarrow$ x é mais fácil de integrar
$\hookrightarrow$ ln(x) é mais fácil de derivar

$\Rightarrow u = \ln(x)$ $dv = x\,dx$

$du = \frac{1}{x}dx$ $v = \frac{x^2}{2}$

$\Rightarrow \int u dv = uv - \int v du$

$\Rightarrow \int x\ln(x)dx = \frac{x^2\ln(x)}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} \cdot dx$

$\Rightarrow = \frac{x^2\ln(x)}{2} - \frac{1}{2}\int x dx$

$\Rightarrow \boxed{= \frac{x^2\ln(x)}{2} - \frac{x^2}{4} + C_1}$

12) $\int x^2 + 1\,dx \Rightarrow \int x^2 dx + \int 1 dx = \frac{x^3}{3} + C_1 + x + C_2$

$\Rightarrow \boxed{\frac{x^3}{3} + x + C_3}$

13) $\int \frac{\ln(x)}{x} dx$

* Por substituição

$\Rightarrow u = \ln(x)$
$du = \frac{1}{x} dx$
$\Rightarrow \int \underbrace{\ln(x)}_{u} \cdot \underbrace{\frac{dx}{x}}_{du} \Rightarrow \int u\,du$

$\Rightarrow \frac{u^2}{2} + C_1 \Rightarrow \frac{(\ln(x))^2}{2} + C_1 \Rightarrow \boxed{\frac{\ln^2(x)}{2} + C_1}$

14) $\int x^2 \cos(x) dx$

* Por partes

$\Rightarrow u = x^2$ $\quad dv = \cos(x) dx$
$du = 2x\,dx$ $\quad v = \operatorname{sen}(x)$ $\Rightarrow \int u\,dv = uv - \int v\,du$

$\Rightarrow = x^2 \operatorname{sen}(x) - \int 2x \operatorname{sen}(x) dx$

$\Rightarrow = x^2 \operatorname{sen}(x) - 2\int x \operatorname{sen}(x) dx$

* Por partes

$\Rightarrow u = x$ $\quad dv = \operatorname{sen}(x) dx$
$du = dx$ $\quad v = -\cos(x)$ $\Rightarrow \int u\,dv = uv - \int v\,du$

$\Rightarrow = -x\cos(x) - \int -\cos(x) dx$

$\Rightarrow = -x\cos(x) - (-\operatorname{sen}(x))$

$\Rightarrow = -x\cos(x) + \operatorname{sen}(x) + C$

$\Rightarrow x^2 \operatorname{sen}(x) - 2(-x\cos(x) + \operatorname{sen}(x))$

$\Rightarrow \boxed{x^2 \operatorname{sen}(x) + 2x\cos(x) - 2\operatorname{sen}(x) + C}$

15) $\int x^2 \ln(x) dx$

* Por partes

$u = \ln(x)$ $\quad dv = x^2 dx$
$du = \frac{1}{x} dx$ $\quad v = \frac{x^3}{3} + C$ $\Rightarrow \int u\,dv = uv - \int v\,du$

$\Rightarrow = \left(\frac{x^3}{3}\right)(\ln(x)) - \int \left(\frac{x^3}{3}\right)\left(\frac{1}{x}\right) dx$

$\Rightarrow \frac{x^3 \ln(x)}{3} - \int \frac{x^2}{3} dx$

$\Rightarrow \frac{x^3 \ln(x)}{3} - \frac{1}{3}\int x^2 dx$

$\Rightarrow \frac{x^3 \ln(x)}{3} - \frac{1}{3}\left(\frac{x^3}{3} + C\right)$

$\Rightarrow \boxed{\frac{x^3 \ln(x)}{3} - \frac{x^3}{9} + C}$

16) $\int (x^2+1) \operatorname{sen}(x) dx$

$\Rightarrow \int x^2 \operatorname{sen}(x) + \operatorname{sen}(x) dx$

$\Rightarrow \int x^2 \operatorname{sen}(x) \cancel{dx} + \int \operatorname{sen}(x) dx$

$\Rightarrow \int x^2 \operatorname{sen}(x) dx - \cos(x) + C$

$\Rightarrow u = x^2$ $\quad dv = \operatorname{sen}(x) dx$
$du = 2x\,dx$ $\quad v = -\cos(x)$ $\Rightarrow = uv - \int v\,du$

$\Rightarrow (x^2)(-\cos(x)) - \int (-\cos(x))(2x\,dx)$

$\Rightarrow -x^2 \cos(x) - \int -2x\cos(x) dx$

$\Rightarrow -x^2 \cos(x) + 2\int x\cos(x) dx$

$\Rightarrow$ Continuação

$\Rightarrow$ continuação da 16

$\Rightarrow -x^2\cos(x) + 2\int x\cos(x)dx$

* Por partes

$u = x \quad dv = \cos(x)dx \Rightarrow = uv - \int v du$
$du = dx \quad v = sen(x) + C$

$\Rightarrow (x)(sen(x)) - \int (sen(x))(dx)$

$\Rightarrow x\,sen(x) - \int sen(x)dx$

$\Rightarrow x\,sen(x) - (-\cos(x) + C)$

$\Rightarrow x\,sen(x) + \cos(x) + C$

...

$\Rightarrow -x^2\cos(x) + 2(x\,sen(x) + \cos(x))$

$\Rightarrow \boxed{-x^2\cos(x) + 2x\,sen(x) + 2\cos(x) + C}$

17) Questão anulada pelo professor

18) $\int e^x . x^3 . dx$

* Por partes

$\Rightarrow u = x^3 \quad dv = e^x dx \Rightarrow = uv - \int v du$
$du = 3x^2 dx \quad v = e^x + C$

$\Rightarrow x^3e^x - \int 3x^2e^x \Rightarrow x^3e^x - 3\int x^2e^x dx$

* Por partes

$\Rightarrow u = x^2 \quad dv = e^x dx \Rightarrow = uv - \int v du$
$du = 2x dx \quad v = e^x + C$

$\Rightarrow x^2e^x - \int 2xe^x dx$

$\Rightarrow x^2e^x - 2\int xe^x dx$

* Por partes

$\Rightarrow u = x \quad dv = e^x dx \Rightarrow = uv - \int v du$
$du = dx \quad v = e^x + C$

$\Rightarrow xe^x - \int e^x dx \Rightarrow xe^x - e^x$

...

$\Rightarrow x^2e^x - 2(xe^x - e^x)$

$\Rightarrow x^2e^x - 2xe^x + 2e^x$

...

$\Rightarrow x^3e^x - 3(x^2e^x - 2xe^x + 2e^x)$

$\Rightarrow \boxed{x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C}$

19) $\int x \ln^2(x) dx$

* Por partes

$\Rightarrow u = \ln^2(x) \quad dv = x dx$
$du = * \quad v = \frac{x^2}{2} + C$

$\frac{d}{dx}(\ln^2(x)) = \frac{d}{dx}(\ln(x))^2$

* Regra da Cadeia

$[f(g(x))]' = f'(g(x)) \cdot g'(x)$

$\hookrightarrow \frac{1}{x}$

$\hookrightarrow 2\ln(x)$

$\Rightarrow \frac{2\ln(x)}{x} = du$

$\Rightarrow$ Continuação da 19

$u = \ln^2(x)$ $\quad dv = x\,dx$

$du = \dfrac{2\ln(x)}{x}dx$ $\quad v = \dfrac{x^2}{2} + C$

$\Rightarrow = uv - \int v\,du$

$$\Rightarrow \frac{x^2\ln^2(x)}{2} - \int \frac{2x^2\ln(x)}{2x}dx$$

$$\Rightarrow \frac{x^2\ln^2(x)}{2} - \int x\ln(x)dx$$

* Por partes

↳ De acordo com o resultado da questão 11

$$\Rightarrow \frac{x^2\ln^2(x)}{2} - \left(\frac{x^2\ln(x)}{2} - \frac{x^2}{4} + C_1\right)$$

$$\Rightarrow \boxed{\frac{x^2\ln^2(x)}{2} - \frac{x^2\ln(x)}{2} + \frac{x^2}{4} + C_2}$$

20) Questão anulada pelo professor

21) $\displaystyle\int \frac{2x-1}{x^2+x-2}dx$

* Ver o passo a passo no rascunho

$\Rightarrow x^2+x-2 \Rightarrow \Delta = (1)^2 - 4(1)(-2) = 9$

$\Rightarrow x = \dfrac{-(1)\pm 3}{2(1)} \Rightarrow x_1 = 1$, $x_2 = -2$

$\Rightarrow x^2+x-2 = (x-1)(x+2)$

$$\Rightarrow \int \frac{2x-1}{(x-1)(x+2)}dx \Rightarrow$$ * Frações parciais

$$\Rightarrow \int \frac{2x-1}{(x-1)(x+2)}dx = \int \frac{A}{(x-1)} + \frac{B}{(x+2)}\,dx$$

$$\Rightarrow \frac{2x-1}{(x-1)(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x+2)}$$

$(x+2)$ $\quad (x-1)$

$\Rightarrow 2x-1 = A(x+2) + B(x-1)$

$\Rightarrow 2x-1 = Ax + 2A + Bx - B$

$\Rightarrow \begin{cases} A+B = 2 \\ 2A - B = -1 \end{cases} \Rightarrow$ Soma de equações

$\Rightarrow 3A + 0B = 1 \Rightarrow 3A = 1 \Rightarrow A = \frac{1}{3}$

$\Rightarrow \frac{1}{3} + B = 2 \Rightarrow \frac{1}{3} + B = \frac{6}{3} \Rightarrow B = \frac{5}{3}$

$$\Rightarrow \int \frac{2x-1}{(x-1)(x+2)}dx = \int \frac{\frac{1}{3}}{(x-1)}dx + \int \frac{\frac{5}{3}}{(x+2)}dx$$

$$\Rightarrow = \frac{1}{3}\int \frac{1}{(x-1)}dx + \frac{5}{3}\int \frac{1}{(x+2)}dx$$

$$\Rightarrow = \frac{1}{3}\cdot\ln|x-1| + C_1 + \frac{5}{3}\cdot\ln|x+2| + C_2$$

$$\Rightarrow \boxed{\int \frac{2x-1}{x^2+x-2} = \frac{\ln|x-1|}{3} + \frac{5\ln|x+2|}{3} + C_3}$$

* A partir do número 22, em decorrência do tamanho dos cálculos, seu passo a passo poderá ser visualizado <u>somente</u> no rascunho.

22) $\int \frac{x^2+5x-6}{x^3+3x^2+2x}\,dx = -3\ln|x| + 10\ln|x+1| - 6\ln|x+2|$

$\Rightarrow \boxed{-3\ln|x| + 10\ln|x+1| - 6\ln|x+2| + C_4}$

23) $\int \frac{2x+1}{x^2-4x+3}\,dx$

* Relembrando que todo o desenvolvimento da questão pode ser visto no rascunho, da questão 22 em diante.

$\Rightarrow \boxed{\frac{7\ln|x-3|}{2} - \frac{3\ln|x-1|}{2} + C_3}$

24) $\int \frac{x+1}{x^3-x^2-2x}\,dx$

$\Rightarrow \boxed{-\frac{\ln|x|}{2} + \frac{\ln|x-2|}{2} + C_3}$

25) $\int \frac{x^2+3x+2}{x^3+3x^2+2x}\,dx \Rightarrow \int \frac{(x^2+3x+2)}{x(x^2+3x+2)}\,dx$

$\Rightarrow \int \frac{1}{x}\,dx \Rightarrow \boxed{\ln|x| + C_1}$

26) $\int \sqrt{16-x^2}\,dx$

$\Rightarrow \boxed{8\left(\operatorname{arcsen}\left(\frac{x}{4}\right) + \frac{\operatorname{sen}\left(2\operatorname{arcsen}\left(\frac{x}{4}\right)\right)}{2}\right) + C}$

27) $\int \frac{x^2+1}{\sqrt{x^2+1}}\,dx$

$\Rightarrow \boxed{\frac{x\sqrt{1+x^2}}{2} + \frac{\ln|x+\sqrt{1+x^2}|}{2} + C}$

28) $\int \frac{x^2+1}{\sqrt{x^2-1}}\,dx$

$\Rightarrow \boxed{\frac{x\sqrt{x^2-1}}{2} + \frac{3\ln|x+\sqrt{x^2-1}|}{2} + C}$

29) $\int \frac{\sqrt{9-x^2}}{x}\,dx$

$\Rightarrow \boxed{3\ln\left|\tan\left(\frac{\operatorname{arcsen}\left(\frac{x}{3}\right)}{2}\right)\right| + \sqrt{9-x^2}}$

30) $\int \frac{\sqrt{x^2-4}}{x^2}\,dx$

$\Rightarrow \boxed{\ln\left|\frac{x+\sqrt{x^2-4}}{2}\right| + \frac{3\sqrt{x^2-4}}{x} - \frac{\sqrt{(x^2-4)^3}}{3x^3} + C}$

Rascunho

8) $\int e^{2x} + \frac{1}{x^2} \Rightarrow \int e^{2x}dx + \int \frac{1}{x^2}dx \Rightarrow \int e^{2x} dx \Rightarrow \begin{matrix} u = 2x \\ du = 2\,dx \end{matrix} \Rightarrow \int e^{2x}dx \Rightarrow \int e^u \frac{du}{2}$.

$\Rightarrow \frac{1}{2}\int e^u\,du \Rightarrow \boxed{\frac{1}{2} e^{2x} + C}$ ⇒ Por substituição

$\int \frac{1}{x^2}dx \Rightarrow \begin{matrix} u = 2x \\ du = 2\,dx \end{matrix} \Rightarrow dx = \frac{du}{2} \Rightarrow \int \frac{1}{x^2} = \int \frac{1}{u} \cdot \frac{du}{2} \Rightarrow \frac{1}{2}\int \frac{du}{u}$

$\int x^{-2}dx \Rightarrow \frac{x^{-1}}{-1}dx \Rightarrow \boxed{-\frac{1}{x} + C_2}$

$\Rightarrow \int e^{2x}dx + \int \frac{1}{x^2}dx \Rightarrow \begin{matrix} u = 2x \\ du = 2dx \\ \hookrightarrow dx = \frac{du}{2} \end{matrix} \Rightarrow \int e^{2x} dx \Rightarrow \int e^u \cdot \frac{du}{2} \Rightarrow \frac{1}{2}\int e^u\,du$

$\Rightarrow \frac{1}{2} \cdot e^u \Rightarrow \boxed{\frac{1}{2} \cdot e^{2x} + C_1}$

$\Rightarrow \int \frac{1}{x^2}dx \Rightarrow \int x^{-2}dx \Rightarrow \frac{x^{-1}}{-1} + C \Rightarrow \boxed{-\frac{1}{x^1} + C_2}$

$\Rightarrow \boxed{\frac{e^{2x}}{2} - \frac{1}{x} + C_3}$

9) $\int 3x^2dx - \int 2x\,dx + \int 1\,dy = x^3 - x^2 + x + C_4$

10) $\int 3\sec(x)\tan(x) \Rightarrow 3\int \sec(x)\tan(x)dx \Rightarrow \frac{1}{\cos(x)} \cdot \frac{\operatorname{sen}(x)}{\cos(x)} - \int \frac{\operatorname{sen}(x)}{\cos^2(x)}dx$

$$\begin{tikzcd} \tan(x) \arrow[r, bend left, "\frac{d}{dx}"] & \sec(x)\tan(x) \arrow[l, bend left, "\int dx"] \end{tikzcd}$$

⇒ o

11) $\int x \ln(x)\,dx$ $\qquad \int u\,dv = uv - \int v\,du$

$u = x$ $\quad dv = \ln(x)$ $\Rightarrow$ $u = \ln(x)$ $\quad dv = x\,dx$

$du = dx$ $\quad v = ?$ $\quad du = \frac{\ln(x)}{x}dx$ $\quad v = \frac{x^2}{2}$

$= \ln(x) \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x}\,dx \Rightarrow \ln(x) \cdot \frac{x^2}{2} - \frac{1}{2}\int x\,dx \Rightarrow \frac{x^2 \ln(x)}{2} - \frac{x^2}{4} + C_3$

12) $\int x^2 + 1\,dx \Rightarrow \int x^2 dx + \int 1\,dx \Rightarrow \frac{x^3}{3} + \frac{x}{\cancel{2}} + C_3$

13) $\int \frac{\ln(x)}{x}\,dx \Rightarrow \int \ln(x) \cdot \frac{dx}{x} \Rightarrow \int u\,du \Rightarrow \frac{u^2}{2} + C$

$u = \ln(x)$ $\quad \to x\,du = dx$

$du = \frac{1}{x}\,dx$

$\Rightarrow \frac{(\ln(x))^2}{2} + C \Rightarrow \frac{\ln^2(x)}{2} + C_1$

14) $\int x^2 \cos(x)\,dx$

$u = x^2$ $\quad dv = \cos(x)\,dx$

$du = 2x\,dx$ $\quad v = \operatorname{sen}(x) + C_1 \Rightarrow \int u\,dv = uv - \int v\,du \Rightarrow x^2 \operatorname{sen}(x) - \int 2x \operatorname{sen}(x)\,dx$

$\Rightarrow \int x^2 \cos(x)\,dx = x^2 \operatorname{sen}(x) - 2\int x \operatorname{sen}(x)\,dx$

$u = x$ $\quad dv = \operatorname{sen}(x)\,dx$ $\quad \Rightarrow -x\cos(x) + C_2 - \int -\cos(x)\,dx$

$du = dx$ $\quad v = -\cos(x) + C_2$

$\Rightarrow -x\cos(x) + C_2 - (-\operatorname{sen}(x) + C_3) \Rightarrow -x\cos(x) + C_2 + \operatorname{sen}(x) + C_3$

$\Rightarrow x^2 \operatorname{sen}(x) - 2\left((-x\cos(x)) + (\operatorname{sen}(x)) + C_4\right)$

$\Rightarrow x^2 \operatorname{sen}(x) + 2x\cos(x) - 2\operatorname{sen}(x) + C$

Rascunho

15) $\int x^2 \ln(x)\,dx$

$u = \ln(x)$, $dv = x^2\,dx$

$du = \frac{1}{x}$ $\quad v = \frac{x^3}{3}$ $\Rightarrow uv - \int v\,du \Rightarrow \frac{x^3 \ln(x)}{3} - \int \frac{x^3}{3} \cdot \left(\frac{1}{x}\right) dx$

$\Rightarrow \frac{x^3 \ln(x)}{3} - \int \frac{x^2}{3}\,dx \Rightarrow \frac{x^3 \ln(x)}{3} - \frac{1}{3}\int x^2\,dx \Rightarrow \frac{x^3 \ln(x)}{3} - \frac{1}{3} \cdot \frac{x^3}{3} + c$

$\Rightarrow \boxed{\frac{x^3 \ln(x)}{3} - \frac{x^3}{9} + C}$

16) $\int (x^2+1)\,\mathrm{sen}(x)\,dx$

$\Rightarrow \int x^2 \mathrm{sen}(x) + \mathrm{sen}(x)\,dx \Rightarrow \int x^2 \mathrm{sen}(x)\,dx + \int \mathrm{sen}(x)\,dx$

$\Rightarrow \cancel{u = \mathrm{sen}(x)} \quad \cancel{dv = x}$

$\cancel{du = \cos(x)\,dx}$

$\Rightarrow u = x^2$ $\quad dv = \mathrm{sen}(x)\,dx$

$du = 2x\,dx$ $\quad v = -\cos(x) + C$ $\Rightarrow = uv - \int v\,du \Rightarrow -x^2\cos(x) - \int -2x\cos(x)$

$\Rightarrow -x^2\cos(x) + 2\int x\cos(x)\,dx$

$\Rightarrow u = x$ $\quad dv = \cos(x)\,dx$

$du = dx$ $\quad v = \mathrm{sen}(x) + C$ $\Rightarrow = uv - \int v\,du \Rightarrow x\,\mathrm{sen}(x) - \int \mathrm{sen}(x)\,dx$

$\Rightarrow x\,\mathrm{sen}(x) - (-\cos(x)) \Rightarrow x\,\mathrm{sen}(x) + \cos(x) + C$

$-x^2\cos(x) + 2x\,\mathrm{sen}(x) + 2\cos(x) + C$

Rascunho

18) $\int e^x \cdot x^3 \cdot dx$

$u = x^3 \quad dv = e^x dx \Rightarrow = uv - \int v du \Rightarrow x^3 e^x - \int 3x^2 e^x dx$

$du = 3x^2 dx \quad v = e^x + C$

$\Rightarrow x^3 e^x - 3\int e^x x^2 dx \Rightarrow u = x^2 \quad dv = e^x dx$

$du = 2x dx \quad v = e^x + C \Rightarrow uv - \int v du$

$\Rightarrow x^2 e^x - \int 2x e^x dx \Rightarrow x^2 e^x - 2\int x e^x dx$

$\Rightarrow u = x \quad dv = e^x dx$

$du = dx \quad v = e^x + C \Rightarrow = uv - \int v du \Rightarrow x e^x - \int e^x du \Rightarrow x e^x - e^x$

...

$\Rightarrow x^2 e^x - 2(x e^x - e^x) \Rightarrow x^2 e^x - 2x e^x + 2e^x$

...

$\Rightarrow x^3 e^x - 3\left(x^2 e^x - 2x e^x + 2e^x\right) \Rightarrow \boxed{x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x}$

19) $\int x \ln^2(x) dx$

$u = \ln^2(x) \quad dv = x\,dx$

$du = * \quad v = \frac{x^2}{2} + C \Rightarrow * \frac{d}{dx}(\ln(x)\ln(x)) = \frac{d}{dx}(\ln(x))^2$

* Regra da Cadeia

$[f(g(x))]' = f'(g(x)) \cdot g'(x) = 2\ln(x) \cdot \frac{1}{x} = \frac{2\ln(x)}{x}$

$u = \ln^2(x) \quad dv = x\,dx$

$du = \frac{2\ln(x)}{x} \quad v = \frac{x^2}{2} + C \Rightarrow uv - \int v du \Rightarrow \frac{x^2 \ln^2(x)}{2} - \int \frac{\cancel{2} x^{\cancel{2}} \ln(x)}{\cancel{2x}} dx$

$\Rightarrow \frac{x^2 \ln^2(x)}{2} - \int x \ln(x) dx \Rightarrow \frac{x^2 \ln^2(x)}{2} - \left(\frac{x^2 \ln(x)}{2} - \frac{x^2}{4}\right) = \frac{x^2 \ln^2(x)}{2} - \frac{x^2 \ln(x)}{2} + \frac{x^2}{4}$

Rascunho

21) $\int \frac{2x-1}{x^2+x-2} \Rightarrow x^2+x-2 \Rightarrow \Delta = b^2-4ac \Rightarrow (1)^2-4(1)(-2) \Rightarrow 1+8 \Rightarrow \Delta = 9$

$\Rightarrow x = \frac{-1\pm\sqrt{9}}{2(1)} \Rightarrow x = \frac{-1\pm 3}{2} \Rightarrow \begin{cases} x_1 = \frac{-1+3}{2} = \frac{2}{2} = 1 \\ x_2 = \frac{-1-3}{2} = -\frac{4}{2} = -2 \end{cases}$

$\Rightarrow x^2+x-2 = (x-r_1)(x-r_2) \Rightarrow x^2+x-2 = (x-1)(x+2)$

$\Rightarrow \int \frac{2x-1}{x^2+x-2} = \int \frac{2x-1}{(x-1)(x+2)} \Rightarrow \int \frac{2x-1}{(x-1)(x+2)}dx = \int \frac{A}{(x-1)}dx + \int \frac{B}{(x+2)}dx$

$\Rightarrow \frac{A}{(x-1)} + \frac{B}{(x+2)} = \frac{2x-1}{(x-1)(x+2)} \Rightarrow A(x+2) + B(x-1) = 2x-1$

$\Rightarrow Ax+2A+Bx-B = 2x-1 \Rightarrow Ax+Bx+2A-B = 2x-1 \Rightarrow x(A+B)+2A-B = 2x-1$

$\Rightarrow A+B=2$

$+\ 2A-B=-1$

$3A+0B=1 \Rightarrow 3A=1 \Rightarrow \boxed{A=\frac{1}{3}} \Rightarrow \frac{1}{3}+B=2 \Rightarrow \frac{1}{3}+B=\frac{6}{3}$

$\Rightarrow B = \frac{6}{3}-\frac{1}{3} \Rightarrow \boxed{B=\frac{5}{3}}$

$\Rightarrow \int \frac{2x-1}{(x-1)(x+2)}dx = \int \frac{\frac{1}{3}}{(x-1)}dx + \int \frac{\frac{5}{3}}{(x+2)}dx \Rightarrow \frac{1}{3}\int \frac{1}{x-1}dx + \frac{5}{3}\int \frac{1}{x+2}dx$

$\Rightarrow \boxed{\int \frac{2x-1}{(x-1)(x+2)}dx = \frac{1}{3}\ln|x-1| + \frac{5}{3}\ln|x+2| + C}$

Rascunho

22) $\int \frac{x^2+5x-6}{x^3+3x^2+2x}\,dx$

$\int \frac{x^2+5x-6}{x^3+3x^2+2x}\,dx = \int \frac{x^2+5x-6}{x(x^2+3x+2)}\,dx \Rightarrow x^2+3x+2 \Rightarrow \Delta = (3)^2-4(1)(2) = 9-8=1$

$\Rightarrow \sqrt{\Delta} \Rightarrow \sqrt{1} = 1 \Rightarrow x = \frac{-(3)\pm 1}{2(1)} \Rightarrow$ $x_1 = \frac{-3+1}{2} = -\frac{2}{2} = -1$

$x_2 = \frac{-3-1}{2} = -\frac{4}{2} = -2$

$\Rightarrow x^2+3x+2 = (x+1)(x+2) \Rightarrow \int \frac{x^2+5x-6}{x((x+1)(x+2))}\,dx \Rightarrow \int \frac{x^2+5x-6}{(x)(x+1)(x+2)}\,dx$

$\Rightarrow$ Frações Parciais $\Rightarrow \int \frac{x^2+5x-6}{(x)(x+1)(x+2)}\,dx = \int \frac{A}{(x)} + \frac{B}{(x+1)} + \frac{C}{(x+2)}\,dx$

$\Rightarrow \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2} = \frac{x^2+5x-6}{(x)(x+1)(x+2)} \Rightarrow \frac{A(x+1)(x+2)+B(x)(x+2)+C(x)(x+1)}{(x)(x+1)(x+2)}$

$(x+1)(x+2)$ $(x)(x+2)$ $(x)(x+1)$

$\Rightarrow A(x+1)(x+2)+B(x)(x+2)+C(x)(x+1) = x^2+5x-6$

$\Rightarrow A(x^2+3x+2)+B(x^2+2x)+C(x^2+x) = x^2+5x-6$

$\Rightarrow (Ax^2+3Ax+2A)+(Bx^2+2Bx)+(Cx^2+Cx) = x^2+5x-6$

$\Rightarrow (Ax^2+Bx^2+Cx^2)+(3Ax+2Bx+Cx)+(2A) = (1)x^2+(5)x+(-6)$

$\Rightarrow x^2\underbrace{(A+B+C)}_{=1}+x\underbrace{(3A+2B+C)}_{=5}+\underbrace{(2A)}_{=-6} \Rightarrow \begin{cases} A+B+C=1 \\ 3A+2B+C=5 \\ 2A=-6 \end{cases} \Rightarrow A = -\frac{6}{2} = -3$

$\boxed{A=-3}$

$\boxed{B=10}$

$\boxed{C=-6}$

$\Rightarrow \begin{cases} (-3)+B+C=1 \\ 3(-3)+2B+C=5 \end{cases} \Rightarrow \begin{cases} B+C=4 \\ 2B+C=14 \end{cases} \Rightarrow \begin{array}{r} 2B+C=14 \\ -B-C=-4 \\ \hline B=10 \end{array}$

Rascunho

* Continuação da 22

A = -3
B = 10
C = -6

$$\Rightarrow \int \frac{A}{(x)} + \frac{B}{(x+1)} + \frac{C}{(x+2)}\,dx \Rightarrow \int \frac{-3}{x}\,dx + \int \frac{10}{(x+1)}\,dx + \int \frac{-6}{(x+2)}\,dx$$

$$\Rightarrow -3\int \frac{1}{x}\,dx + 10\int \frac{1}{(x+1)}\,dx - 6\int \frac{1}{x+2}\,dx \Rightarrow -3\ln|x| + C_1 + 10\ln|x+1| + C_2 - 6\ln|x+2| + C_3$$

$$\Rightarrow \boxed{\int \frac{x^2+5x-6}{x^3+3x^2+2x}\,dx = -3\ln|x| + 10\ln|x+1| - 6\ln|x+2| + C_4}$$

Rascunho

B) $\displaystyle\int \frac{2x+1}{x^2-4x+3}\,dx \Rightarrow x^2-4x+3$

$\hookrightarrow \Delta = (-4)^2 - 4(1)(3) \Rightarrow \Delta = 16 - 12 \Rightarrow \Delta = 4 \Rightarrow \sqrt{\Delta} = 2$

$$\Rightarrow x = \frac{-(-4)\pm 2}{2(1)} \Rightarrow \frac{4\pm 2}{2} \Rightarrow \begin{array}{l} x_1 = \frac{6}{2} = 3 \\ x_2 = \frac{2}{2} = 1 \end{array} \Rightarrow x^2-4x+3 = (x-3)(x-1)$$

$$\Rightarrow \int \frac{2x+1}{(x-3)(x-1)}\,dx \Rightarrow \text{Frações Parciais} \Rightarrow \int \frac{2x+1}{(x-3)(x-1)}\,dx = \int \frac{A}{(x-3)} + \frac{B}{(x-1)}\,dx$$

$$\Rightarrow \frac{A}{(x-3)} \cdot (x-1) + \frac{B}{(x-1)} \cdot (x-3) = \frac{2x+1}{(x-3)(x-1)} \Rightarrow \frac{A(x-1)+B(x-3)}{(x-3)(x-1)} = \frac{2x+1}{(x-3)(x-1)}$$

$$\Rightarrow (Ax-A)+(Bx-3B) = 2x+1 \Rightarrow (Ax+Bx)+(-A-3B) = (2)x+(+1) \Rightarrow$$

$$\Rightarrow x\underbrace{(A+B)}_{2} + \underbrace{(-A-3B)}_{+1} = (2)x+(+1) \Rightarrow \begin{cases} A+B=2 \\ -A-3B=+1 \end{cases} \Rightarrow \begin{array}{r} \cancel{A}+B=2 \\ \oplus\ -\cancel{A}-3B=+1 \\ \hline -2B=3 \end{array}$$

$\Rightarrow A+B=2$

$$\Rightarrow \boxed{B=-\frac{3}{2}} \Rightarrow A-\frac{3}{2} = \frac{4}{2} \Rightarrow A = \frac{4}{2}+\frac{3}{2} \Rightarrow \boxed{A=\frac{7}{2}}$$

$$\Rightarrow \int \frac{\frac{7}{2}}{(x-3)}\,dx + \int \frac{-\frac{3}{2}}{(x-1)}\,dx \Rightarrow \frac{7}{2}\int \frac{1}{(x-3)}\,dx - \frac{3}{2}\int \frac{1}{(x-1)}\,dx$$

$$\Rightarrow \frac{7}{2}\ln|x-3| + C_1 - \frac{3}{2}\ln|x-1| + C_2 \Rightarrow \boxed{\frac{7\ln|x-3|}{2} - \frac{3\ln|x-1|}{2} + C_3}$$

Rascunho

$$2\#1) \int \frac{x+1}{x^3-x^2-2x}\,dx$$

$$\Rightarrow \int \frac{x+1}{x^3-x^2-2x}\,dx \Rightarrow \int \frac{x+1}{x(x^2-x-2)}\,dx \Rightarrow x^2-x-2 \Rightarrow \Delta = (-1)^2-4(1)(-2) = 1+8 \Rightarrow \Delta = 9$$

$$\Rightarrow x = \frac{-(-1)\pm 3}{2(1)} \Rightarrow \frac{1\pm 3}{2} \Rightarrow \begin{matrix} x_1 = \frac{4}{2} = 2 \\ x_2 = -\frac{2}{2} = -1 \end{matrix} \Rightarrow x^2-x-2 = (x-2)(x+1)$$

$$\Rightarrow \int \frac{\cancel{(x+1)}}{(x)\cancel{(x+1)}(x-2)}\,dx \Rightarrow \int \frac{1}{(x)(x-2)}\,dx \Rightarrow \text{Frações Parciais} \Rightarrow$$

$$\int \frac{1}{(x)(x-2)}\,dx = \int \frac{A}{(x)} + \frac{B}{(x-2)}\,dx \Rightarrow \int \frac{A}{x}\,dx + \int \frac{B}{(x-2)}\,dx \Rightarrow \frac{A}{(x)} + \frac{B}{(x-2)} = \frac{1}{(x)(x-2)}$$

$(x-2)$ (x)

$$\Rightarrow \frac{A(x-2)+B(x)}{(x)(x-2)} = \frac{1}{(x)(x-2)} \Rightarrow Ax-2A+Bx = 1 \Rightarrow x\underbrace{(A+B)}_{0}\underbrace{-2A}_{1} = 1$$

$$\Rightarrow \begin{cases} A+B=0 \\ -2A=1 \end{cases} \Rightarrow \boxed{A = -\frac{1}{2}} \Rightarrow -\frac{1}{2}+B = 0 \Rightarrow \boxed{B = \frac{1}{2}}$$

$$\Rightarrow \int \frac{A}{x}\,dx + \int \frac{B}{(x-2)}\,dx \Rightarrow \int \frac{-\frac{1}{2}}{x}\,dx + \int \frac{\frac{1}{2}}{(x-2)}\,dx \Rightarrow -\frac{1}{2}\int \frac{1}{x}\,dx + \frac{1}{2}\int \frac{1}{(x-2)}\,dx$$

$$\Rightarrow -\frac{1}{2}\cdot \ln|x| + C_1 + \frac{1}{2}\ln|x-2| + C_2 \Rightarrow \boxed{-\frac{\ln|x|}{2} + \frac{\ln|x-2|}{2} + C_3}$$

Rascunho

25) Desenvolvimento na página de soluções

26) $\int \sqrt{16-x^2}\,dx$

$\Rightarrow \int \sqrt{4^2-x^2}\,dx \Rightarrow$ Substituição trigonométrica $\Rightarrow$ $x = a\,sen\,\theta$, $dx = a\cos\theta\,d\theta$ $\Rightarrow$ $x = 4\,sen\,\theta$, $dx = 4\cos\theta\,d\theta$ $\Rightarrow \int \sqrt{4^2-16\,sen^2\theta}\,(4\cos\theta)d\theta$

$\Rightarrow \int \sqrt{16-16\,sen^2\theta}\,(4\cos\theta)d\theta \Rightarrow$ Relação fundamental da trigonometria $\Rightarrow sen^2\theta + \cos^2\theta = 1 \xrightarrow{\times 16}$

$\Rightarrow 16\,sen^2\theta + 16\cos^2\theta = 16 \Rightarrow 16\cos^2\theta = 16 - 16\,sen^2\theta \Rightarrow \int \sqrt{16\cos^2\theta}\,(4\cos\theta)d\theta \Rightarrow$

$\Rightarrow \int (4\cos\theta)(4\cos\theta)\,d\theta \Rightarrow \int 16\cos^2\theta \Rightarrow 16\int \cos^2\theta \Rightarrow$ transformação trigonométrica $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

$\Rightarrow 16\int \dfrac{1+\cos(2\theta)}{2}d\theta \Rightarrow 16\cdot\dfrac{1}{2}\int 1+\cos(2\theta)\,d\theta \Rightarrow 8\left(\int 1\,d\theta + \int \cos(2\theta)d\theta\right)$

$\Rightarrow u = 2\theta$
$du = 2d\theta$
$\hookrightarrow \dfrac{du}{2} = d\theta$

$\Rightarrow \int \cos(u)\cdot\dfrac{du}{2} \Rightarrow \dfrac{1}{2}\int \cos(u)du \Rightarrow \dfrac{1}{2}sen(2\theta)+C \Rightarrow$

$\Rightarrow 8\left(\theta + \dfrac{sen(2\theta)}{2}\right) + C_3 \Rightarrow x = 4\,sen\,\theta \Rightarrow \dfrac{x}{4} = sen\,\theta \Rightarrow arcsen\left(\dfrac{x}{4}\right) = \theta$

$\Rightarrow \boxed{8\left(arcsen\left(\dfrac{x}{4}\right) + \dfrac{sen\left(arcsen\left(\frac{x}{4}\right)\right)}{2}\right) + C_3}$

Rascunho

27) $\int \frac{x^2+1}{\sqrt{x^2+1}}\,dx \Rightarrow \int \frac{x^2+1}{(x^2+1)^{\frac{1}{2}}}\,dx \Rightarrow \int (x^2+1)^1 \cdot (x^2+1)^{-\frac{1}{2}}\,dx \Rightarrow \int (x^2+1)^{\frac{1}{2}}\,dx \Rightarrow \int \sqrt{1^2+x^2}\,dx \Rightarrow$

$\Rightarrow$ Substituição trigonométrica $\Rightarrow$ $x = a\tan\theta$, $dx = a\sec^2\theta\,d\theta$ $\Rightarrow$ $x = \tan\theta$, $dx = \sec^2\theta\,d\theta$ $\Rightarrow \int \sqrt{1^2+\tan^2\theta}\,(\sec^2\theta)\,d\theta \Rightarrow \int \sqrt{\frac{1}{1}+\frac{\sin^2\theta}{\cos^2\theta}}\,(\sec^2\theta)\,d\theta$

$\cos^2\theta$ $\quad 1$

$\Rightarrow \int \sqrt{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}\,(\sec^2\theta)\,d\theta \Rightarrow$ Relação fundamental da trigonometria $\sin^2\theta + \cos^2\theta = 1$ $\Rightarrow \int \sqrt{\frac{1}{\cos^2\theta}}\,(\sec^2\theta)\,d\theta$

$\Rightarrow \sec^2\theta = \frac{1}{\cos^2\theta} \Rightarrow \int \sqrt{\sec^2\theta}\,(\sec^2\theta)\,d\theta \Rightarrow \int (\sec\theta)(\sec^2\theta)\,d\theta \Rightarrow \int \sec^3\theta\,d\theta \Rightarrow$

$\Rightarrow \int \sec\theta \cdot \sec^2\theta\,d\theta \Rightarrow$ Integração por partes $\Rightarrow$ $u = \sec\theta$, $dv = \sec^2\theta\,d\theta$; $du = \sec\theta\tan\theta\,d\theta$, $v = \tan\theta$ $\Rightarrow \int u\,dv = uv - \int v\,du$

$\Rightarrow = \sec\theta\tan\theta - \int \sec\theta\tan^2\theta\,d\theta \Rightarrow$ R. F. da trigonometria $\Rightarrow \sin^2\theta + \cos^2\theta = 1 \xrightarrow{\div\cos^2\theta} \tan^2\theta + 1 = \sec^2\theta$

$\Rightarrow \tan^2\theta + 1 = \sec^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1 \Rightarrow \sec\theta\tan\theta - \int \sec\theta(\sec^2\theta - 1)\,d\theta \Rightarrow$

$\Rightarrow \sec\theta\tan\theta - \int \sec^3\theta - \sec\theta\,d\theta \Rightarrow \int \sec^3\theta\,d\theta = \sec\theta\tan\theta - \int \sec^3\theta\,d\theta + \int \sec\theta\,d\theta$

$\Rightarrow 2\int \sec^3\theta\,d\theta = \sec\theta\tan\theta + \int \sec\theta\,d\theta^{*} \Rightarrow {*}\int \sec\theta\,d\theta = \int \sec\theta \cdot \frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}\,d\theta$

$\Rightarrow \int \frac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta+\tan\theta}\,d\theta \Rightarrow$ Integração por substituição $\Rightarrow$ $u = \sec\theta + \tan\theta$, $du = \sec\theta\tan\theta + \sec^2\theta\,d\theta$ $\Rightarrow \int \frac{du}{u}$

$\Rightarrow = \ln|u| + C_1 \Rightarrow = \ln|\sec(\theta) + \tan(\theta)| + C_1$

$\Rightarrow \cancel{2}\int \sec^3\theta = \frac{\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta| + C_1}{2} \Rightarrow x = \tan\theta \Rightarrow \theta = \arctan x$

$\Rightarrow \int \sec^3(\operatorname{arctg} x)\,dx = \frac{\sec(\operatorname{arctg} x)\tan(\operatorname{arctg} x)}{2} + \frac{\ln|\sec(\operatorname{arctg} x) + \tan(\operatorname{arctg} x)|}{2} + C_1$

$\Rightarrow \tan(\operatorname{arctg} x) = x$; $\sec(\operatorname{arctg} x) = \sqrt{1+x^2} \Rightarrow = \boxed{\frac{x\sqrt{1+x^2}}{2} + \frac{\ln|\sqrt{1+x^2} + x|}{2} + C_1}$

Rascunho

28) $\int \frac{x^2+1}{\sqrt{x^2-1}}dx \Rightarrow \int \frac{x^2}{\sqrt{x^2-1}}dx + \int \frac{1}{\sqrt{x^2-1}}dx$ (I) (II) $\Rightarrow$ (I) $\int \frac{x^2}{\sqrt{x^2-1^2}}dx \Rightarrow$ $x = \sec\theta$, $dx = \sec\theta\tan\theta\,d\theta$

$\Rightarrow \int \frac{\sec^2\theta}{\sqrt{\sec^2\theta - 1}}(\sec\theta\tan\theta)d\theta \Rightarrow$ R. f. da trigo. $\Rightarrow \sin^2\theta + \cos^2\theta = 1 \Rightarrow$ $\div\cos^2\theta$ $\frac{\sin^2\theta}{\cos^2\theta} + \ldots \Rightarrow$

$\Rightarrow \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \Rightarrow \tan^2\theta + 1 = \sec^2\theta$ * $\Rightarrow \int \frac{\sec^2\theta}{\sqrt{\tan^2\theta + \cancel{1} - \cancel{1}}}(\sec\theta\tan\theta)d\theta \Rightarrow \int \frac{\sec^2\theta}{\tan\theta}\sec\theta t$

$\Rightarrow \int \frac{\sec^2\theta}{\cancel{\tan\theta}} \cdot \sec\theta \cancel{\tan\theta}\, d\theta \Rightarrow \int \sec^3\theta\, d\theta \Rightarrow$ Como visto anteriormente na resolução da questão 27 $\Rightarrow$

$\Rightarrow \int \sec^3\theta\, d\theta = \frac{\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|}{2} + C \Rightarrow \theta = \text{arcsec}\, x \Rightarrow$

$\Rightarrow \frac{\sec(\text{arcsec}\, x)\tan(\text{arcsec}\, x) + \ln|\sec(\text{arcsec}\, x) + \tan(\text{arcsec}\, x)|}{2} + C \Rightarrow$

$\Rightarrow \sec(\text{arcsec}\, x) = x$; $\tan(\text{arcsec}\, x) = \sqrt{x^2-1} \Rightarrow \frac{x\sqrt{x^2-1}}{2} + \frac{\ln|x+\sqrt{x^2-1}|}{2}$

II) $\Rightarrow \int \frac{1}{\sqrt{x^2-1}}dx \Rightarrow$ $x = \sec\theta$, $dx = \sec\theta\tan\theta\, d\theta$ $\Rightarrow \int \frac{1}{\sqrt{\sec^2\theta - 1}}\sec\theta\tan\theta\, d\theta \Rightarrow \int \frac{1}{\sqrt{\tan^2\theta}}$ * $\sec\theta\tan\theta\, d\theta$

$\Rightarrow \int \frac{1}{\cancel{\tan\theta}} \cdot \sec\theta \cancel{\tan\theta}\, d\theta \Rightarrow \int \sec\theta\, d\theta \Rightarrow$ Como visto na resolução da Questão 27 $\Rightarrow$

$\Rightarrow \int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C \Rightarrow \theta = \text{arcsec}\, x \Rightarrow \ln|\sec(\text{arcsec}\, x) + \tan(\text{arcsec}\, x)| +$

$\Rightarrow \ln|x + \sqrt{x^2-1}| + C \Rightarrow$ I + II $\Rightarrow \frac{x\sqrt{x^2-1}}{2} + \frac{\ln|x+\sqrt{x^2-1}|}{2} + \ln|x+\sqrt{x^2-1}| \Rightarrow$

$\Rightarrow \boxed{\frac{x\sqrt{x^2-1}}{2} + \frac{3\ln|x+\sqrt{x^2-1}|}{2} + C}$

Rascunho

2) $\int \frac{\sqrt{9-x^2}}{x}dx \Rightarrow \int \frac{\sqrt{3^2-x^2}}{x}dx \Rightarrow \begin{matrix} x = 3\,\text{sen}\,\theta \\ dx = 3\cos\theta\, d\theta \end{matrix} \Rightarrow \int \frac{\sqrt{9-9\,\text{sen}^2\theta}}{\cancel{3}\,\text{sen}\,\theta}\cdot \cancel{3}\cos\theta\, d\theta \Rightarrow$ R.F. da trigo

$\Rightarrow \text{sen}^2\theta + \cos^2\theta = 1 \Rightarrow {}^{*}\cos^2\theta = 1 - \text{sen}^2\theta \overset{\times 9}{\Rightarrow} 9\cos^2\theta = 9 - 9\,\text{sen}^2\theta \Rightarrow \int \frac{\sqrt{9\cos^2\theta}}{\text{sen}\,\theta}\cdot\cos\theta\, d\theta \Rightarrow$

$\Rightarrow \int \frac{3\cos\theta}{\text{sen}\,\theta}\cdot\cos\theta\, d\theta \Rightarrow 3\int\frac{\cos^2\theta}{\text{sen}\,\theta}d\theta \overset{*}{\Rightarrow} 3\int\frac{1-\text{sen}^2\theta}{\text{sen}\,\theta}d\theta \Rightarrow 3\left(\overset{\text{I}}{\int\frac{1}{\text{sen}\,\theta}d\theta} - \overset{\text{II}}{\int \text{sen}\,\theta\, d\theta}\right)$

$\Rightarrow$ I) $\int \frac{1}{\text{sen}\,\theta}d\theta \Rightarrow$ Sabendo que $\text{sen}(a+b) = \text{sen}(a)\cos(b) + \text{sen}(b)\cos(a) \Rightarrow \begin{matrix}\theta = 2u \\ d\theta = 2du\end{matrix} \Rightarrow$

$\Rightarrow \int\frac{1}{\text{sen}(2u)}2du \Rightarrow \int\frac{2du}{\text{sen}(u)\cos(u) + \text{sen}(u)\cos(u)} \Rightarrow \int \frac{\cancel{2}du}{\cancel{2}\,\text{sen}(u)\cos(u)} \Rightarrow \int\frac{1}{\text{sen}(u)\cos(u)}du \Rightarrow$

$\Rightarrow \int \frac{\text{sen}^2u + \cos^2u}{\text{sen}\,u\cos u}du \Rightarrow \overset{\alpha}{\int\frac{\text{sen}^2u}{\text{sen}\,u\cos u}du} + \overset{\beta}{\int\frac{\cos^2u}{\text{sen}\,u\cos u}du} \Rightarrow \alpha)\int\frac{\text{sen}^2u}{\text{sen}\,u\cos u}du$

$\Rightarrow \int\frac{\text{sen}\,u}{\cos u}du \Rightarrow \begin{matrix} t = \cos u \\ dt = -\text{sen}\,u\,du \\ \hookrightarrow du = -\frac{dt}{\text{sen}\,u}\end{matrix} \Rightarrow \int \frac{\cancel{\text{sen}\,u}}{t}\cdot\left(-\frac{dt}{\cancel{\text{sen}\,u}}\right) \Rightarrow \int -\frac{dt}{t} \Rightarrow -1\int\frac{dt}{t}$

$\Rightarrow -\ln|\cos u|$ | $\beta) \int\frac{\cos^2u}{\text{sen}\,u\cos u}du \Rightarrow \int\frac{\cos u}{\text{sen}\,u}du \Rightarrow \begin{matrix} k = \text{sen}\,u \\ dk = \cos u\,du \\ \hookrightarrow du = \frac{dk}{\cos u}\end{matrix} \Rightarrow$

$\Rightarrow \int\frac{dk}{k} = \ln|k| \Rightarrow \ln|\text{sen}\,u|$ | $\alpha + \beta \Rightarrow -\ln|\cos u| + \ln|\text{sen}\,u| + C$

$\Rightarrow \theta = 2u \Rightarrow u = \frac{\theta}{2} \Rightarrow -\ln\left|\cos\frac{\theta}{2}\right| + \ln\left|\text{sen}\,\frac{\theta}{2}\right| + C \Rightarrow$ Propriedade de logaritmo $\Rightarrow \frac{\log a}{\log b} = \log a + \ldots$

$\Rightarrow \ln\left|\tan\frac{\theta}{2}\right| + C \Rightarrow \int\frac{1}{\text{sen}\,\theta}d\theta = \ln\left|\tan\frac{\theta}{2}\right| + C \Rightarrow 3\left(\underset{\text{I}}{\ln\left|\tan\frac{\theta}{2}\right|} + \underset{\text{II}}{\cos\theta}\right) \Rightarrow \theta = \text{arcsen}\left(\frac{x}{3}\right)$

$\Rightarrow 3\ln\left|\tan\left(\frac{\text{arcsen}\left(\frac{x}{3}\right)}{2}\right)\right| + 3\cos\left(\text{arcsen}\left(\frac{x}{3}\right)\right) \Rightarrow 3\ln\left|\tan\left(\frac{\text{arcsen}\left(\frac{x}{3}\right)}{2}\right)\right| + \cancel{3}\cdot\frac{\sqrt{9-x^2}}{\cancel{3}}$

$\Rightarrow \boxed{3\ln\left|\tan\left(\frac{\text{arcsen}\left(\frac{x}{3}\right)}{2}\right)\right| + \sqrt{9-x^2}}$

30)

Rascunho

$$\int \frac{\sqrt{x^2-2^2}}{x^2}\,dx \Rightarrow \begin{array}{l} x = 2\sec\theta \\ dx = 2\sec\theta\tan\theta\, d\theta \end{array} \Rightarrow \int \frac{\sqrt{4\sec^2\theta - 4}}{\cancel{4}_2\sec^2\theta}\,\cancel{2}^1\sec\theta\tan\theta\, d\theta \Rightarrow$$

$$\Rightarrow \frac{\operatorname{sen}^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \Rightarrow \tan^2\theta + 1 = \sec^2\theta \Rightarrow 4\tan^2\theta = 4\sec^2\theta - 4 \Rightarrow \int \frac{\sqrt{4\tan^2\theta}}{2\sec^2\theta} \Rightarrow$$

$$\Rightarrow \int \frac{\sqrt{4\tan^2\theta}}{2\sec^2\theta}\sec\theta\tan\theta\, d\theta \Rightarrow \int \frac{\cancel{2}\tan\theta}{\cancel{2}\sec\theta}\tan\theta\, d\theta \Rightarrow \int \frac{\tan^2\theta}{\sec\theta}\,d\theta \Rightarrow \int \frac{\cancel{2}\tan\theta}{\cancel{4}\sec^2\theta}\cdot\cancel{2\sec\theta\tan\theta}$$

$$\Rightarrow \int \frac{\tan^2\theta}{\sec\theta} \Rightarrow \int \tan^2\theta\cos\theta \Rightarrow \int \frac{\operatorname{sen}^2\theta}{\cos^{\cancel{2}}\theta}\cdot\cancel{\cos\theta} \Rightarrow \int \frac{\operatorname{sen}\theta}{\cos\theta}\cdot\operatorname{sen}\theta\, d\theta \Rightarrow \int \tan\theta\operatorname{sen}\theta\, d\theta$$

$$\Rightarrow \operatorname{sen}^2\theta = 1-\cos^2\theta \Rightarrow \int\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\cdot(1-\cos^2\theta)\,d\theta \Rightarrow \int \frac{1-2\cos^2\theta+\cos^4\theta}{\cos\theta}\,d\theta$$

$$\Rightarrow \int \frac{1}{\cos\theta} - \frac{2\cos^2\theta}{\cos\theta} + \frac{\cos^4\theta}{\cos\theta}\,d\theta \Rightarrow \overset{\mathrm{I}}{\int \frac{1}{\cos\theta}\,d\theta} - 2\overset{\mathrm{II}}{\int \frac{\cos^2\theta}{\cos\theta}\,d\theta} + \overset{\mathrm{III}}{\int \frac{\cos^4\theta}{\cos\theta}\,d\theta}$$

$\Rightarrow$ I) $\displaystyle\int \frac{1}{\cos\theta}\,d\theta \Rightarrow$ Como visto anteriormente $\Rightarrow \ln|\tan\theta + \sec\theta| + C \Rightarrow$

$\Rightarrow$ II) $\displaystyle 2\int \frac{\cos^2\theta}{\cos\theta}\,d\theta \Rightarrow 2\int \cos\theta\, d\theta \Rightarrow 2\operatorname{sen}\theta + C$ | III) $\displaystyle\int \frac{\cos^4\theta}{\cos\theta}\,d\theta - \int \cos^3\theta\, d\theta \Rightarrow$

$$\Rightarrow \int \cos^3\theta\, d\theta \Rightarrow \int (1-\operatorname{sen}^2\theta)\cos\theta\, d\theta \Rightarrow \begin{array}{l} u = \operatorname{sen}\theta \\ du = \cos\theta\, d\theta \end{array} \Rightarrow \int (1-u^2)\,du \Rightarrow \int 1\,du - \int u^2\,du \Rightarrow$$

$\Rightarrow \displaystyle = u - \frac{u^3}{3} + C \Rightarrow \operatorname{sen}\theta - \frac{\operatorname{sen}^3\theta}{3} + C$ | $\mathrm{I} + \mathrm{II} + \mathrm{III} \Rightarrow \ln|\tan\theta + \sec\theta| + 2\operatorname{sen}\theta + \operatorname{sen}\theta - \frac{\operatorname{sen}^3\theta}{3}$

$$\Rightarrow \ln|\tan\theta+\sec\theta| + 2\operatorname{sen}\theta + \operatorname{sen}\theta - \frac{\operatorname{sen}^3\theta}{3} \Rightarrow \frac{\ln|\tan\theta+\sec\theta|}{1/3} + \frac{3\operatorname{sen}\theta}{1/3} - \frac{\operatorname{sen}^3\theta}{3/1} \Rightarrow$$

$$\Rightarrow \frac{3\ln|\tan\theta+\sec\theta| + 9\operatorname{sen}\theta - \operatorname{sen}^3\theta}{3} \Rightarrow \theta = \operatorname{arcsec}\left(\frac{x}{2}\right) \Rightarrow \frac{3\ln\left|\frac{\sqrt{x^2-4}}{2}+\frac{x}{2}\right| + \frac{9\sqrt{x^2-4}}{x} - \frac{\sqrt{(x^2-4)^3}}{x^3}}{3}$$

$$\Rightarrow \frac{\cancel{3}\ln\left|\frac{x+\sqrt{x^2-4}}{2}\right|}{\cancel{3}_1} + \frac{\cancel{9}^3\sqrt{x^2-4}}{\cancel{3}x} - \frac{\sqrt{(x^2-4)^3}}{3x^3} = \boxed{\ln\left|\frac{x+\sqrt{x^2-4}}{2}\right| + \frac{3\sqrt{x^2-4}}{x} - \frac{\sqrt{(x^2-4)^3}}{3x^3} + C}$$