1)

~~$f'(x) = 2x^3$~~

$f(x) = \frac{1}{2}x^4 + \frac{2}{3}x^3 - \frac{1}{2}x^2 + \frac{1}{4}$

$\Rightarrow \left(4 \cdot \frac{1}{2} \cdot x^{4-1}\right) + \left(3 \cdot \frac{2}{3} \cdot x^{3-1}\right) - \left(2 \cdot \frac{1}{2} \cdot x^{2-1}\right) + 0$

$\Rightarrow (2x^3) + (2x^2) - (x)$

2) $f(x) = x^2 + \sqrt{x}$

$f(x) = x^2 + (x)^{\frac{1}{2}}$

$f'(x) = 2 \cdot x^{2-1} + \frac{1}{2} \cdot x^{\frac{1}{2}-1} \Rightarrow (2x) + \frac{1}{2}x^{-\frac{1}{2}} \Rightarrow (2x) + \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}}$

$\Rightarrow (2x) + \left(\frac{1}{2\sqrt{x}}\right)$

3) $f(x) = x^3 \cdot \cos(x)$

$f(x) = a(x) \cdot b(x) \Rightarrow f'(x) = a'(x)b(x) + a(x)b'(x)$

$\Rightarrow a(x) = x^3 \quad b(x) = \cos(x)$

$a'(x) = 3x^2 \quad b'(x) = -\operatorname{sen}(x)$

$\Rightarrow f'(x) = 3x^2\cos(x) + \left(-x^3 \cdot \operatorname{sen}(x)\right) = 3x^2\cos(x) - x^3\operatorname{sen}(x)$

4) $f(x) = x^3(2x^2 - 3x)$

$F(x) = a(x) + b(x) \Rightarrow f'(x) = a'(x)b(x) + a(x)b'(x)$

$\Rightarrow a(x) = x^3 \quad b(x) = (2x^2 - 3x)$

$a'(x) = 3x^2 \quad b'(x) = 4x - 3$

$\Rightarrow f'(x) = 3x^2(2x^2 - 3x) + x^3(4x - 3) \Rightarrow \underline{(6x^4 - 9x^3) + (4x^4 - 3x^3)}$

5) $f(x) = \frac{2x+5}{4x}$

$F(x) = \frac{a(x)}{b(x)} \Rightarrow f'(x) = \frac{a'(x)b(x) - a(x)b'(x)}{b^2(x)}$

$\Rightarrow \quad a(x) = 2x + 5 \quad b(x) = 4x \quad b^2(x) = 16x^2$

$a'(x) = 2 \quad b'(x) = 4$

$\Rightarrow \frac{(2)(4x) - (2x+5)(4)}{(16x^2)} = \frac{(8x) - (8x + 20)}{16x^2} = -\frac{20}{16x^2}$

$\Rightarrow -\frac{(4.5)}{(4.4)x^2} = -\frac{5}{4x^2}$

6) $f(x) = \left(\frac{8}{5}\right)^{x^3}$

$\Rightarrow f'(x) = x' \cdot \ln(a) \cdot a^x \Rightarrow a = \frac{8}{5}$, $x = x^3 \rightsquigarrow x' = 3x^2$, $\ln(a) = \ln\left(\frac{8}{5}\right)$

$\Rightarrow f'(x) = (3x^2)\left(\ln\left(\frac{8}{5}\right)\right) \cdot \left(\frac{8}{5}\right)^{x^3}$

7) $f(x) = 2^{3x-1}$

$\Rightarrow f'(x) = x' \cdot \ln(a) \cdot a^x \rightsquigarrow x = 3x-1 \rightsquigarrow x' = 3$, $a = 2 \rightsquigarrow \ln(a) = \ln(2)$

$\Rightarrow f'(x) = (3)(\ln(2)) \cdot 2^{3x-1}$

8) $f(x) = 3^x$

$f(x) = a^x \Rightarrow f'(x) = x' \cdot a^x \cdot \ln(a)$

$\Rightarrow f(x) = 3^x \Rightarrow f'(x) = 1 \cdot 3^x \cdot \ln(3) = 3^x \ln(3)$

9) $f(x) = \ln(x^2)$

$[F(g(x))]' = f'(g(x)) \cdot g'(x)$ ↱ $f(x) = g(h(x))$, $f'(x) = g'(h(x)) \cdot h'(x)$

$\Rightarrow f(g(x)) = \ln(x^2)$ $\quad g(x) = x^2$

$f'(g(x)) = \cos(x^2)$ $\quad g'(x) = 2x$

$\Rightarrow \cos(x^2) \cdot 2x \Rightarrow 2x\cos(x^2)$

10) $f(x) = \cos\left(\frac{1}{x}\right)$

$f(x) = \cos(x^{-1})$

$f(x) = g(h(x)) \Rightarrow f'(x) = g'(h(x)) \cdot h'(x)$

$g(h(x)) = \cos\left(\frac{1}{x}\right)$ $\quad h(x) = x^{-1}$

$g'(h(x)) = -\operatorname{sen}\left(\frac{1}{x}\right)$ $\quad h'(x) = -1 \cdot x^{-2} = -1 \cdot \frac{1}{x^2} = -\frac{1}{x^2}$

$f'(x) = \left(-\operatorname{sen}\left(\frac{1}{x}\right)\right) \cdot \left(-\frac{1}{x^2}\right) = \frac{\operatorname{sen}\left(\frac{1}{x}\right)}{x^2}$

11) $(x^2+5x+2)^7$

$f(x) = g(h(x)) \Rightarrow f'(x) = g'(h(x)) \cdot h'(x)$

$g(h(x)) = (x^2+5x+2)^7$ $\quad h(x) = x^2+5x+2$

$g'(h(x)) = 7(x^2+5x+2)^6$ $\quad h'(x) = 2x+5$

$f'(x) = 7(x^2+5x+2)^6 \cdot (2x+5) = (14x+35)(x^2+5x+2)^6$

12) $f(x) = \left(\frac{3x+2}{2x+1}\right)^5$

$f'(x) = 5\left(\frac{3x+2}{2x+1}\right)^4 \cdot h'(x)$

$h(x) = \frac{3x+2}{2x+1} \Rightarrow h(x) = \frac{i(x)}{g(x)} \Rightarrow h'(x) = \frac{i'(x)\,g(x) - i(x)\,g'(x)}{g^2(x)}$

continuação na próxima

$i(x) = 3x+2$ $\quad g(x) = 2x+1$ $\quad g^2(x) = 4x^2+4x+1$ ← $(2x+1)(2x+1)$

$i'(x) = 3$ $\quad g'(x) = 2$

$\Rightarrow h'(x) = \dfrac{3(2x+1) - (3x+2)(2)}{4x^2+4x+1} = \dfrac{(6x+3) - (6x+4)}{4x^2+4x+1}$

$f'(x) = 5\left(\dfrac{3x+2}{2x+1}\right)^4 \cdot \dfrac{(6x+3) - (6x+4)}{4x^2+4x+1}$

13) $f(x) = \dfrac{1}{3}(2x^5 + 6x^{-3})^5$

$f'(x) = \dfrac{5}{3}(2x^5+6x^{-3})^4 \cdot h'(x)$

$\Rightarrow h(x) = 2x^5 + 6x^{-3}$

$h'(x) = 10x^4 - 18x^{-4}$

$f'(x) = \dfrac{5}{3}(2x^5+6x^{-3})^4 (10x^4 - 18x^{-4})$

14) $f(x) = \dfrac{1}{\sqrt[5]{x^3-1}}$

$f(x) = \left(\sqrt[5]{x^3-1}\right)^{-1}$

$f'(x) = (-1)\left(\sqrt[5]{x^3-1}\right)^{-2} \cdot h'(x)$

$h(x) = \sqrt[5]{x^3-1} = (x^3-1)^{\frac{1}{5}}$

$\rightarrow h'(x) = \frac{1}{5}(x^3-1)^{-\frac{4}{5}}$

$h'(x) = \dfrac{1}{5\sqrt[5]{(x^3-1)^4}}$

$f'(x) = \dfrac{-1}{\left(\sqrt[5]{x^3-1}\right)^2} \cdot \dfrac{1}{5\sqrt[5]{(x^3-1)^4}}$

15) $f(x) = \cos(x^3 - 4)$

$f'(x) = -\operatorname{sen}(x^3 - 4) \cdot h'(x)$

$h(x) = x^3 - 4$
$h'(x) = 3x^2$

$f'(x) = -3x^2 \operatorname{sen}(x^3 - 4)$

16) $f(x) = (x^3 - 6)^5$

$f'(x) = 5(x^3 - 6)^4 \cdot h'(x)$

$h(x) = x^3 - 6$
$h'(x) = 3x^2$

$f'(x) = 5(x^3 - 6)^4 (3x^2) = 15x^2 (x^3 - 6)^4$

17) $f(x) = \dfrac{2}{(2 + \ln(x))^2}$

$f(x) = 2(2 + \ln(x))^{-2}$

$f'(x) = -4(2 + \ln(x))^{-3} \cdot h'(x) = -\dfrac{4(2 + \ln(x))^{-3}}{x}$

$h(x) = \ln(x)$
$h'(x) = \frac{1}{x}$

18) $F(x) = 3x^2 + 5$

$F'(x) = 6x$