CMSC216: Makefiles, Binary, Integers, Arithmetic

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Logistics

Reading

Bryant/O'Hallaron Ch 2.1-2.3

- Number Systems
- Binary Encoding of Data
- Signed/Unsigned Integers
- Character Data
- Optional: Ch 2.4 Floats

Makefiles: consult GNU Make Manual if you want details

Assignments

- **Lab04/HW04:** Due Sun 28-Sep, relevant to P1 and Exam 1 review
- **Project 1:** Due Mon 29-Sep, late submission with penalties
- **Exam 1**:
  - Review: Tue 30-Sep
  - Execute: Thu 02-Oct

Goals

- Wrap C discussion
- Brief discussion of make and Makefiles
- Integers/characters in binary
- Arithmetic operations, Negative numbers in binary

Announcements

Exam Week Discussions

- No new HW or Labs during Week05
- Discussion will meet and feature
  1. A review activity worth bonus dots
  2. Time on Mon/Wed to give students help on P1 / Exam Review
- If you want help on P1, **go to your discussion section**

Discussion Review Activity for Bonus Dots

- On Mon/Wed, staff will conduct a short review activity
- Students that participate will be eligible for bonus dots
- **Higher attendance at your discussion section means a higher potential bonus for everyone in the section**: slightly cooperative
- The section that "does the best" will get additional bonus dots: slightly competitive

Exam 1 Logistics

Practice + Review

- Practice Exam 1A will be posted Mon 29-Sep
- Practice Exam 1B and Review in class Tue 30-Sep
- Solutions to practice exam will be posted for students

Exam 1

- In-person in class on Thu 02-Oct
- Exam runs lecture period: 75min
- Expect 2.5 pages front/back
- **Open Resource Exam**: review rules for this posted at bottom of course schedule (beneath slides)
  Questions on Open Resource Exam boundaries?

Compilation to Object Files and Executables

- Compilation is actually a Series of 7 or 8 Phases we will discuss over semester including conversion to assembly, conversion to binary, optimization, etc.
- Two phases are important here:
  - **Compilation** (proper) converts C code to a lower form
  - **Linking** melds many lower forms into an executable
- GCC can be instructed to compile ONLY (not Link) using commands like `gcc -c file.c`
- Results in an **Object File**[1] (`file.o`) in a binary format
- A piece of a program but not yet an executable due to being incomplete and unlinked
- This allows **Separate Compilation** of C source files; most compiled languages do likewise (Java, OCaml, Rust, etc.) which enables...
  - Generating a function call without the function definition
  - Programming Libraries of compiled code
  - More efficient project builds

[1] https://en.wikipedia.org/wiki/Object_file

Sample Session of Separate Compilation

```
>> gcc -o myexec main_func.c              # can't make a program, incomplete
/usr/bin/ld: /tmp/ccQgKONk.o: in function 'main':
main_func.c:(.text+0x10): undefined reference to 'func_01'
collect2: error: ld returned 1 exit status

>> gcc -c main_func.c                     # create a .o object file
>> file main_func.o
main_func.o: ELF 64-bit LSB relocatable, x86-64, version 1 (SYSV), not stripped

>> ./main_func.o                          # not an exectuable format
bash: ./main_func.o: cannot execute binary file: Exec format error

>> gcc -c func_01.c                       # compile missing function
>> gcc -o myexec main_func.o func_01.o    # link to produce a program
>> file myexec
myexec: ELF 64-bit LSB pie executable, x86-64, version 1 (SYSV), dynamically li

>> ./myexec                               # executable program
Calling function func_01
```

Exercise: Separate Compilation

```
# COMPILATION 1
>> gcc -c func_01.c
>> gcc -c main_func.c
>> gcc -o main_func main_func.o func_01.o

# COMPILATION 2
>> gcc -o main_func main_func.c func_01.c
```

- Describe differences between compilations above
- What is the result in each case?
- How are they different: any *artifacts* created in one but not the other?
- Any advantages/disadvantages to them?

Answers: Separate Compilation

```
# COMPILATION 1
>> gcc -c func_01.c                          # creates func_01.o
>> gcc -c main_func.c                        # creates main_func.o
>> gcc -o main_func main_func.o func_01.o    # links to create main_func

# COMPILATION 2
>> gcc -o main_func main_func.c func_01.c    # only creates main_func
```

Compilation 1: Separate Compilation

- Separately compile `func_01.c` and `main_func.c` to binary
- Results in 2 `.o` **object files**
- Final step is to **link** two objects together to create an executable

Compilation 2: "Together" Compilation

- Compile all the C files at once to produce an executable
- Still does separate compilation BUT `.o` files are temporary and immediately deleted after producing the executable

Why bother with separate compilation if it is more typing?

Exercise: Separate Compilation Time

- Mack is building a large application
- Has a `main_func.c` and `func_01.c, func_02.c ...` that define application, up to `func_20.c`
- During build process notices that it takes about 10s for to compile each C file and 20s to link the C files
- Mack frequently edits `func_08.c` to add features then compiles the project with the command

  ```
  >> gcc -o main_func *.c
  ```
- **Estimate** his typical build time in seconds
- **Suggest** a way that he might reduce his build time if he has edited only a small number of files

Answers: Separate Compilation Time

Total Build Time `gcc -o main_func *.c`

Item	Example	Build	Tot
Library C files	`func_01.c`	20 x 10s	200s
Main C file	`main_func.c`	1 x 10s	10s
Linking	all `.o` files	1 x 20s	20s
Total Time	$\sim$ 4min	22 steps	230s

- Explicitly recompiling all C files to object code despite many not changing
- Spends valuable human time waiting to redo the same task as has been done many before

Answers: Separate Compilation Time

Exploit Separate Compilation

- Assume already compiled all files, have `func_01.o`, `func_02.o`
- Edit `func_08.c` to add a new feature
- **Don't** recompile C files that haven't changed
- Compile like this

```
> gcc -c func_08.c
> gcc -o main_func *.o
```

Item	Example	Build	Time
Library `.o` files	`func_01.o`	19 x 0s	0s
Main `.o` file	`main_func.o`	1 x 0s	0s
Changed `.c` files	`func_08.c`	1 x 10s	10s
Linking	all `.o` files	1 x 20s	20s
Total Time	$\sim$ 30 seconds	2 steps	30s

Build Systems Exploit Separate Compilation

- Build Systems like `make` exploit separate compilation
- Establishes a dependency structure via a `Makefile`
  - **Targets** are usually files to create
  - **Dependencies** are files/targets used to create a given target
  - **Commands** are executed to create a Target from Dependencies after all Depndencies are created
- Only rebuild a Target if a Dependency **changes**

```
# Typical Makefile gives targets, dependencies,
# commands to create target using dependencies
# TARGET : DEPENDENCIES
#       COMMANDS / ACTIONS

main_func : main_func.o func_01.o func_02.o
        gcc -o main_funcs main_func.o func_01.o func_02.o

main_func.o : main_func.c
        gcc -c main_funcs.c

func_01.o : func_01.c
        gcc -c funcs_01.c
```

Example Builds from big-compile/

```
>> make clean
rm -f *.o main_func

# first compiles, no object files built, build everything
>> make main_func
gcc -c main_func.c
gcc -c func_01.c
gcc -c func_02.c
...
gcc -c func_20.c
gcc -o main_func main_func.o func_01.o func_02.o...

# check primary target
>> file main_func
main_func: ELF 64-bit LSB pie executable, x86-64, version 1 (SYSV)

# edit func_08.c

# 1 file changed, recompile it and re-link
>> make main_func
gcc -c func_08.c    # ONLY NEED TO RECOMPILE THIS
gcc -o main_func main_func.o func_01.o func_02.o...

# no edits, no need to rebuild
>> make main_func
make: Nothing to be done for 'main_func'.
```

Makefile Format Notes

- `make` is a program, typing `make` will by default use data in `Makefile`
- `Makefiles` are comprised of **Rules**: Target/Dependencies/Commands
- Targets are always required for a Rule; only one of Dependencies or Commands need be present
- Commands require a **Tab** indentation to work (not spaces); historically a source of errors and oft repeated lessons in software dev
- The first Target in `Makefile` is the default but any target can be named on the command line:

```
>> make                        # build the default target
>> make func_01.o              # build this specific target
>> make func_02.o main_func    # build these targets
```

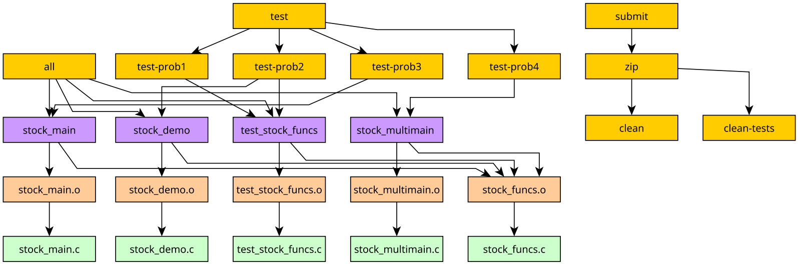
- `Makefiles` have many conventions that commonly appear BUT must be written by the author

```
>> make all                    # build everything
>> make clean                  # remove some compiled artifacts
>> make install                # install compiled software on system
```

- `Makefiles` have LOADS of tricks like built-in variables, user-defined variables, conditions, functions of a sort, etc.; see `Makefile-shortcuts` in codepack and inspect `Makefile` provided in Projects and Labs
- `make / Makefiles` are NOT just for C: allow compilation and automation of whatever commands you want (this slide was posted via `make push`)
- `make` is among the oldest Build System still in use but younger systems like `CMake`, `Maven`, `Ant`, `Cargo`, `Dune`, `Gradle`, `Ninja`, `Leiningen` and dozens of other are popular with the youths...

Makefiles Create a Directed Acyclic Graph (DAG)

- DAGs are a useful data structure for representing dependencies, associated with Topological Sorting
- More complex than Trees: "multiple parents" (shown), edges may cut across layers (not shown)



DAG representing some targets in a typical Project 1 Makefile

In some later projects, students will need to create some or all of Lab/Project `Makefiles` so take Lab04 as practice

Unsigned Integers: Decimal and Binary

- Unsigned integers are always positive:

```c
unsigned int i = 12345;
```

- To understand their binary encoding, first recall how decimal numbers "work" to encode quantities

Decimal: Base 10 Example

Each digit adds on a power 10

$$
\begin{aligned}
80,345 = & 5 \times 10^0 + && \text{5 ones} \\
& 4 \times 10^1 + && \text{40 tens} \\
& 3 \times 10^2 + && \text{300 hundreds} \\
& 0 \times 10^3 + && \text{0 thousands} \\
& 8 \times 10^4 && \text{80 tens of thousands} \\
& 5 + 40 + 300 + 80,000
\end{aligned}
$$

Binary: Base 2 Example

Each digit adds on a power 2

$$
\begin{aligned}
11001_2 = & 1 \times 2^0 + && \text{1 ones} \\
& 0 \times 2^1 + && \text{0 twos} \\
& 0 \times 2^2 + && \text{0 fours} \\
& 1 \times 2^3 + && \text{8 eights} \\
& 1 \times 2^4 + && \text{16 sixteens} \\
= & 1 + 8 + 16 && = 25
\end{aligned}
$$

So, $11001_2 = 25_{10}$

Exercise: Convert Binary to Decimal

Base 2 Example:

$$
\begin{aligned}
11001 = & 1 \times 2^0 + & 1 \\
& 0 \times 2^1 + & 0 \\
& 0 \times 2^2 + & 0 \\
& 1 \times 2^3 + & 8 \\
& 1 \times 2^4 + & 16 \\
= & 1 + 8 + 16 & = 25
\end{aligned}
$$

So, $11001_2 = 25_{10}$

Try With a Neighbor

Convert the following two numbers from base 2 (binary) to base 10 (decimal)

- 111
- 11010
- 01100001

Answers: Convert Binary to Decimal

$$
\begin{aligned}
111_2 =& 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\
=& 1 \times 4 + 1 \times 2 + 1 \times 1 \\
=& 7_{10} \\
11010_2 =& 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\
=& 1 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 0 \times 1 \\
=& 26_{10} \\
01100001_2 =& 0 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 \\
& + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\
=& 0 \times 128 + \times 64 + 1 \times 32 + 0 \times 16 \\
& + 0 \times 8 + 0 \times 4 + 0 \times 2 + 1 \times 1 \\
=& 97_{10}
\end{aligned}
$$

Note: last example ignores leading 0's

The Other Direction: Decimal to Binary

Converting a number from base 10 to base 2 is easily done using repeated division by 2; keep track of **remainders**

Convert 124 to base 2:

$$\begin{aligned}
124 \div 2 = 62 &\quad \text{rem } 0 \\
62 \div 2 = 31 &\quad \text{rem } 0 \\
31 \div 2 = 15 &\quad \text{rem } 1 \\
15 \div 2 = 7 &\quad \text{rem } 1 \\
7 \div 2 = 3 &\quad \text{rem } 1 \\
3 \div 2 = 1 &\quad \text{rem } 1 \\
1 \div 2 = 0 &\quad \text{rem } 1
\end{aligned}$$

- Last step got 0 quotient so we're done.
- Binary digits are in **remainders in reverse**
- Answer: `1111100`
- Check:
  $0 + 0 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 = 4 + 8 + 16 + 32 + 64 = 124$

Decimal, Hexadecimal, Octal, Binary Notation

- Numbers exist independent of any writing system
- Can write the same number in a variety of bases
- C provides syntax for most common bases used in computing

	Decimal	Binary	Hex	Octal
Base	10	2	16	8
Mathematical	125	1111101_2	$7D_{16}$	175_8
C Prefix	None	`0b...`	`0x..`	`0...`
C Example	125	`0b1111101`	`0x7D`	`0175`
`printf()`	`"%d"`	N/A	`"%x"`	`"%o"`

- **Hexadecimal** often used to express long-ish byte sequences
  Larger than base 10 so for 10-15 uses letters A-F
- **Examine** `number_writing.c` and `table.c` for patterns
- **Expectation**: Gain familiarity with doing conversions between bases as it will be useful in practice

Hexadecimal: Base 16

- Hex: compact way to write bit sequences
- One byte is 8 bits
- Each Hex character represents 4 bits
- **Each Byte is 2 Hex Digits**

```
|-----------+----------------+-----|
| Byte      | Hex            | Dec |
|-----------+----------------+-----|
| 0101 0111 | 57 = 5*16 + 7  |  87 |
| 5    7    |                |     |
| 0011 1100 | 3C = 3*16 + 12 |  60 |
| 3    C=12 |                |     |
| 1110 0010 | E2 = 14*16 + 2 | 226 |
| E=14 2    |                |     |
|-----------+----------------+-----|
```

Hex to 4 bit equivalence

Dec	Bits	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Exercise: Conversion Tricks for Hex and Octal

Examples shown in this week's HW, What tricks are illustrated?

```
|---------+--------------+-----------------+-----------------------|
| Decimal | Byte = 8bits | Byte by 4       | Hexadecimal           |
|---------+--------------+-----------------+-----------------------|
|      87 |     01010111 | bin: 0101 0111  | 57 = 5*16 + 7         |
|         |              | hex: 5    7     | hex  dec              |
|         |              |                 |                       |
|      60 |     00111100 | bin: 0011 1100  | 3C = 3*16 + 12        |
|         |              | hex: 3    C=12  | hex  dec              |
|         |              |                 |                       |
|     226 |     11100010 | bin: 1110 0010  | E2 = 14*16 + 2        |
|         |              | hex: E=14 2     | hex  dec              |
|---------+--------------+-----------------+-----------------------|
| Decimal | Byte = 8bits | Byte by 3       | Octal                 |
|---------+--------------+-----------------+-----------------------|
|      87 |     01010111 | bin: 01 010 111 | 127 = 1*8^2 + 2*8 + 7 |
|         |              | oct: 1  2   7   | oct   dec             |
|         |              |                 |                       |
|      60 |     00111100 | bin: 00 111 100 | 074 = 0*8^2 + 7*8 + 4 |
|         |              | oct: 0  7   4   | oct   dec             |
|         |              |                 |                       |
|     226 |     11100010 | bin: 11 100 010 | 342 = 3*8^2 + 4*8 + 2 |
|         |              | oct: 3  4   2   | oct   dec             |
|---------+--------------+-----------------+-----------------------|
```

Answers: Conversion Tricks for Hex and Octal

- Converting between Binary and Hexadecimal is easiest when grouping bits by 4: each 4 bits corresponds to one hexadecimal digit

```
bin: 0101 0111    bin: 1110 0010
hex: 5    7       hex: E=14 2
```

- Converting between Binary and Octal is easiest when grouping bits by 3: each 3 bits corresponds to one octal digit

```
bin: 01 010 111  bin: 11 100 010
oct: 1  2   7    oct: 3  4   2
```

Character Coding Conventions

- Would be hard for people to share words if they interpretted bits as letters differently
- **ASCII**: American Standard Code for Information Interchange
  An old standard for bit/character correspondence
- 7 bits per character, includes upper, lower case, punctuation

Dec	Hex	Binary	Char	Dec	Hex	Binary	Char
65	41	01000001	A	78	4E	01001110	N
66	42	01000010	B	79	4F	01001111	O
67	43	01000011	C	80	50	01010000	P
68	44	01000100	D	81	51	01010001	Q
69	45	01000101	E	82	52	01010010	R
70	46	01000110	F	83	53	01010011	S
71	47	01000111	G	84	54	01010100	T
72	48	01001000	H	85	55	01010101	U
73	49	01001001	I	86	56	01010110	V
74	4A	01001010	J	87	57	01010111	W
75	4B	01001011	K	88	58	01011000	X
76	4C	01001100	L	89	59	01011001	Y
77	4D	01001101	M	90	5A	01011010	Z
91	5B	01011101	[	97	61	01100001	a
92	5C	01011110	\	98	62	01100010	b

Partial Table of ASCII Codes / Values, try `man 7 ascii` in a terminal for a full table

Exercise: Characters vs Numbers

Explain the following program and its output

```c
// char_ints.c:
#include <stdio.h>
#include <string.h>
int main(){
  ...
  char nums[64] = {
    72, 101, 108, 108, 111, 32,
    87, 111, 114, 108, 100, 33,
    0
  };
  printf("%s\n",nums);
  len = strlen(nums);
  for(int i=0; i<len; i++){
    printf("[%2d] %c %3d %02X\n",
           i,nums[i],nums[i],nums[i]);
  }
  return 0;
}
```

```
>> gcc char_ints.c
>> ./a.out
...
Hello World!
[ 0] H  72 48
[ 1] e 101 65
[ 2] l 108 6C
[ 3] l 108 6C
[ 4] o 111 6F
[ 5]    32 20
[ 6] W  87 57
[ 7] o 111 6F
[ 8] r 114 72
[ 9] l 108 6C
[10] d 100 64
[11] !  33 21
```

Answers: Characters vs Numbers

The Whole Array

```
char nums[64] = {
  72, 101, 108, 108, 111, 32,
  87, 111, 114, 108, 100, 33,
  0
};
```

Lays out a bit pattern at each spot the array; bit pattern is specified with decimal numbers

```
printf("%s\n",nums);
```

Print the array as though it were "string": an array of characters that is null terminated

Elements of the Array

```
printf("[%2d] %c %3d %02X\n",
       i,nums[i],nums[i],nums[i]);
```

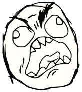
Print a single element of the array as

- `%c` : a character (ASCII table lookup for the glyph to draw)
- `%3d` : a decimal number (padding to width 3)
- `%02X` : as a hexadecimal number (with leading 0's if needed and padded with width 2 - *noice!*)

Unicode

- *World*: Why can't I write

  컴퓨터

  in my code/web address/email?
- *America*: ASCII has 128 chars. Deal with it.
- *World*: Seriously?
- *America*: We invented computers. 'Merica!
- *World*: 
- *America*: ... Unicode?
- *World*: But my language takes more bytes than American.
- *America*: Deal with it. 'Merica!

- ASCII Uses 7 bits per char, limited to 128 characters
- UTF-8 uses **1-4 bytes per character** to represent **many** more characters (1,112,064 *codepoints*)
- Uses 8th bit in a byte (**high-order bit**) to indicate extension to multiple bytes
- Requires software to understand coding convention allowing broader language support
- ASCII is a proper subset of UTF-8 making UTF-8 backwards compatible and wildly popular

Binary Integer Addition/Subtraction

Adding/subtracting in binary works the same as with decimal EXCEPT that carries occur on values of 2 rather than 10

```
ADDITION #1
   1 11     <-carries
   0100 1010 = 74
 + 0101 1001 = 89
------------
   1010 0011 = 163

ADDITION #2
   1111   1 <-carries
   0110 1101 = 109
 + 0111 1001 = 121
------------
   1110 0110 = 230
```

```
SUBTRACTION #1
          ? <-carries
   0111 1001 = 121
 - 0001 0011 =  19
------------
   VVVVVVVVVVVVV
   VVVVVVVVVVVVV
   VVVVVVVVVVVVV
        x12 <-carries
   0111 0001 = 119
 - 0001 0011 =  19
------------
   0110 0110 = 102
```

When 0/1 is represented as Low/High Voltage, one can design digital circuits that implement arithmetic

Two's Complement Integers: Representing Negative Values

- To represent negative integers, must choose a **different** coding system than for positive-only integers
- The **Two's Complement Encoding** is the most common coding system for signed numbers so we will study it
- Alternatives exist
  - Signed magnitude: leading bit indicates pos (0) or neg (1)
  - One's complement: invert bits to go between positive negative
- Great advantage of two's complement: **signed and unsigned arithmetic are identical**
- Hardware folks only need to make one set of units for both unsigned and signed arithmetic

Summary of Two's Complement

TL;DR: *Most significant bit is a negative power of two.*

```
UNSIGNED BINARY              TWO's COMPLEMENT (signed)
---------------              -------------------------
7654 3210 : position         7654 3210 : position
ABCD EFGH : 8 bits           ABCD EFGH : 8-bits
A: 0/1 * +(2^7) *POS*        A: 0/1 * -(2^7) *NEG*
B: 0/1 * +(2^6)              B: 0/1 * +(2^6)
C: 0/1 * +(2^5)              C: 0/1 * +(2^5)
...                          ...
H: 0/1 * +(2^0)              H: 0/1 * +(2^0)

UNSIGNED BINARY              TWO's COMPLEMENT (signed)
---------                    ---------
7654 3210 : position         7654 3210 : position
1000 0000 = +128             1000 0000 = -128
1000 0001 = +129             1000 0001 = -127 = -128+1
1000 0011 = +131             1000 0011 = -125 = -128+1+2
1111 1111 = +255             1111 1111 = -1   = -128+1+2+4+..+64
0000 0000 = 0                0000 0000 =  0        [   +127    ]
0000 0001 = +1               0000 0001 = +1
0000 0101 = +5               0000 0101 = +5
0111 1111 = +127             0111 1111 = +127
```

Two's Complement Notes

Unsigned/Signed Equivalents

Unsigned	`1000 0110 = 134`
Signed	`1000 0110 = -121`
	`= 134 - 256`
Unsigned	`1111 0001 = 241`
Signed	`1111 0001 = -15`
	`=241-256`
Unsigned	`0011 0011 = 51`
Signed	`0011 0011 = 51`

When/Why `X-256`?

- Leading (leftmost) bit is 1
- Counted as 128 in Unsigned
- Counts as -128 in Signed
- Take -256 to compensate

Negation in Two's Complement

```
int y = -x;
```

- Unary Minus operator
- **Invert bits, Add 1**
- Works for both Pos→Neg and Neg→Pos

```
~ 0110 1000   +104 : negate
-----------
  1001 0111   inverted
+         1
-----------
  1001 1000 = -104

~ 1001 1000 = -104 : negate
-----------
  0110 0111 = +103 inverted
+         1
-----------
  0110 1000 = +104
```

Exercise: Two's Complement Conversions

- Fill in the missing entries in the following table
- **Very similar** to an upcoming exam problem

Bits	Hex	Decimal Unsigned	Decimal Signed
1111 1111	A: ____	B: ___	C: ___
1001 0110	0x96	D: ___	E: ___
F: ____ ____	0x3E	G: ___	H: ___
0010 0011	I: ____	35	J: ___
K: ____ ____	L: ____	M: ___	-35

Answers: Two's Complement Conversions

	Bits	Hex	Decimal Unsigned	Decimal Signed
	1111 1111	A: 0xFF	B: 255	C: -1
	1001 0110	0x96	D: 150	E: -106
F:	0011 1110	0x3E	G: 62	H: 62
	0010 0011	I: 0x23	35	J: 35
K:	1101 1101	L: 0xDD	M: 221	-35

`K / L / M:` Converting 35 to −35 decimal/bits can be done via (-35+255) AND/OR via Invert Bits + 1

Overflow

- Sums that exceed the representation of the bits associated with the integral type **overflow**
- Excess significant bits are **dropped**
- Addition can result in a sum smaller than the summands, even for two positive numbers (!?)
- Integer arithmetic in fixed bits is a mathematical **ring**

Examples of Overflow in 8 bits

```
ADDITION #3 OVERFLOW
 1 1111 111 <-carries
   1111 1111 = 255
 + 0000 0001 =   1
------------
 1 0000 0000 = 256
 x drop 9th bit
------------
   0000 0000 = 0
```

```
ADDITION #4 OVERFLOW
 1         1 <-carries
   1010 1001 = 169
 + 1100 0001 = 193
------------
 1 0110 1010 = 362
 x drop 9th bit
------------
   0110 1010 = 106
```

Underflow

- **Underflow** occurs in unsigned arithmetic when values go below 0 (no longer positive)
- Pretend that there is an extra significant bit to carry out subtraction
- Subtracting a positive integer from a positive integer may result in a **larger** positive integer (?!?)
- Integer arithmetic in fixed bits is a mathematical **ring**

Examples of 8-bit Underflow

```
SUBTRACTIION #2 UNDERFLOW
           ?<-carries
   0000 0000 =   0
 - 0000 0001 =   1
------------
VVVVVVVVVVVVV
           ?<-carries
 1 0000 0000 = 256 (pretend)
 - 0000 0001 =   1
------------
VVVVVVVVVVVVV
 x         2<-carries
 0 1111 1110 = 256
 - 0000 0001 =   1
------------
   1111 1111 = 255
```

Overflow and Underflow In C Programs

- See `over_under_flow.c` for demonstrations in a C program.
- **No runtime errors** for under/overflow
- Good for hashing and cryptography
- Bad for most other applications: system critical operations should use checks for overflow / underflow
- Textbook mentions the Ariane Rocket Crash which was due to overflow of an integer converted from a floating point value

  > *The Ariane explosion is an instructive example for several reasons.*
  > (*1*) *Software re-use caused the problem subverting the usual wisdom of relying on tested software; hardware changes ALWAYS trump software.*
  > (*2*) *Sometimes computer science IS rocket science*

- Assembly provides condition codes indicating when overflow occurs but checking in C is tricky and painful[2]

---

[2] Many compilers like GCC can generate assembly instructions that will detect overflow and abort programs. See the demo program `overflow_detect.c` and GCCs `-ftrapv` option.

======== END EXAM 1 CONTENT ========

Content which follows will be covered after Exam 1

Integer Ops and Speed

- Along with Addition and Subtraction, **Multiplication and Division** can also be done in binary
- Algorithms are the same as base 10 but more painful to do by hand
- This pain is reflected in hardware speed of these operations
- The **Arithmetic and Logic Unit (ALU)** does integer ops in the machine
- A **clock** ticks in the machine at some rate like 3Ghz (3 billion times per second)
- Under ideal circumstances, typical ALU Op speeds are

Operation	Cycles
Addition	1
Logical	1
Shifts	1
Subtraction	1
Multiplication	3
Division	>30

- Due to disparity, it is worth knowing about **relation** between multiply/divide and **bitwise** operations
- Compiler often uses such tricks: shift rather than multiply/divide

Mangling Bits Puts Muscle on Your Bones

Below illustrates difference between logical and bitwise operations.

```c
int xl = 12 || 10; // truthy (Logical OR)
int xb = 12 |  10; // 14     (Bitwise OR)
int yl = 12 && 10; // truthy (Logical AND)
int yb = 12 &  10; // 8      (Bitwise AND)
int zb = 12 ^  10; // 6      (Bitwise XOR)
int wl = !12;      // falsey (Logical NOT)
int wb = ~12;      // 3      (Bitwise NOT/INVERT)
```

- Bitwise ops evaluate on a per-bit level
- 32 bits for `int`, 4 bits shown

```
Bitwise OR     Bitwise AND    Bitwise XOR    Bitwise NOT
  1100 = 12      1100 = 12      1100 = 12
| 1010 = 10    & 1010 = 10    ^ 1010 = 10    ~ 1100 = 12
-----------    -----------    -----------    -----------
  1110 = 14      1000 =  8      0110 =  6      0011 =  3
```

Bitwise Shifts

- **Shift** operations move bits within a field of bits
- Shift operations are

```
x = y << k; // left  shift y by k bits, store in x
x = y >> k; // right shift y by k bits, store in x
```

- All integral types can use shifts: `long`, `int`, `short`, `char`
- **Not applicable** to pointers or floating point
- Examples in 8 bits

```
//          76543210
char x = 0b00010111; // 23
char y = x << 2;     // left shift by 2
//   y = 0b01011100; // 92
//   x = 0b00010111; // not changed
char z = x >> 3;     // right shift by 3
//   z = 0b00000010; // 2
//   x = 0b00010111; // not changed
char n = 0b10000000; // -128, signed
char s = n >> 4;     // right shift by 4
//   s = 0b11111000; // -8, sign extension
//   right shift >> is "arithmetic"
```

Shifty Arithmetic Tricks

- Shifts with add/subtract can be used instead of multiplication and division
- Turn on optimization: `gcc -O3 code.c`
- Compiler automatically does this if it thinks it will save cycles
- *Sometimes* programmers should do this but better to convince compiler to do it for you, **comment** if doing manually

Multiplication

```c
//           76543210
char  x = 0b00001010;   // 10
char x2 = x << 1;       // 10*2
//   x2 = 0b00010100;   // 20
char x4 = x << 2;       // 10*4
//   x4 = 0b00101000;   // 40
char x7 = (x << 3)-x;   // 10*7
//   x7 = (x * 8)-x;    // 10*7
//   x7 = 0b01000110;   // 70
//           76543210
```

Division

```c
//           76543210
char y  = 0b01101110;  // 110
char y2 = y >> 1;      // 110/2
//   y2 = 0b00110111;  // 55
char y4 = y >> 2;      // 110/4
//   y4 = 0b00011011;  // 27
char z  = 0b10101100;  // -84
char z2 = z >> 2;      // -84/4
//   z2 = 0b11101011;  // -21
//   right shift sign extension
```

Exercise: Checking / Setting Bits

Use a combination of bit shift / bitwise logic operations to...

1. Check if bit `i` of `int x` is set (has value 1)
2. Clear bit `i` (set bit at index `i` to value 0)

Show C code for this

```
{
  int x = ...;
  int i = ...;
  if( ??? ) { // ith bit of x is set
    printf("set!\n");
  }

  i = ...;
  ???;
  printf("ith bit of x now cleared to 0\n");
}
```

Answers: Checking / Setting Bits

1. Check if bit `i` of `int x` is set (has value 1)

```
int x = ...;
int mask = 1; // or 0b0001 or 0x01 ...
int shifted  = mask << i; // shifted   0b00...010..00
if(x & shifted){          //       x & 0b10...010..01
  ...                     //       ------------------
}                         //           0b00...010..00
```

2. Clear bit `i` (set bit at index `i` to value 0)

```
int x = ...;
int mask = 1; // or 0b0001 or 0x01 ...
int shifted   = mask << i;  // shifted   0b00...010..00
int inverted  = ~shifted;   // inverted  0b11...101..11
x = x & inverted;           //       x & 0b10...010..01
  ...                       //       ------------------
                            //           0b10...000..01
```

Showing Bits

- `printf()` capabilities:

%d	as Decimal
%x	as Hexadecimal
%o	as Octal
%c	as Character

- No specifier for binary
- Can construct such with bitwise operations
- Code pack contains two codes to do this
  - `printbits.c`: single args printed as 32 bits
  - `showbits.c`: multiple args printed in binary, hex, decimal

- Showing bits usually involves shifting and bitwise AND &
- Example from `showbits.c`

```c
#define INT_BITS 32

// print bits for x to screen
void showbits(int x){
  for(int i=INT_BITS-1; i>=0; i--){
    int mask = 1 << i;
    if(mask & x){
      printf("1");
    } else {
      printf("0");
    }
  }
}
```

Bit Masking

- Semi-common for functions to accept bit patterns which indicate true/false options
- Frequently makes use of bit **masks** which are constants associated with specific bits
- Example from earlier: Unix permissions might be...

```
#define S_IRUSR 0b100000000 // User   Read
#define S_IWUSR 0b010000000 // User   Write
#define S_IXUSR 0b001000000 // User   Execute
#define S_IRGRP 0b000100000 // Group  Read
...
#define S_IWOTH 0b000000010 // Others Write
#define S_IXOTH 0b000000001 // Others Execute
```

- Use them to create options to C functions like

```
int permissions = S_IRUSR|S_IWUSR|S_RGRP;
chmod("/home/kauffman/solution.zip",permissions);
```

Endinaness: Byte ordering in Memory

- Single bytes like ASCII characters lay out sequentially in memory in increasing address
- Multi-byte entities like 4-byte `ints` require decisions on byte ordering
- We think of a 32-bit int like this

```
          Most Signifcant <------> Least Significant
Binary:   0000 0000  0000 0000  0001 1000  1110 1001
          0    0     0    0     1    8     E    9
Hex    :  000018E9
Decimal:  6377
```

- There are 2 Options to for ordering multi-byte data in memory
  - **Little Endian**: Least Significant byte at low address
  - **Big Endian**: Most Significant Byte at low address
- Example: Integer starts at address #1024

```
           Address
LittleEnd: #1027      #1026      #1025      #1024
Binary:    0000 0000  0000 0000  0001 1000  1110 1001
           0    0     0    0     1    8     E    9
BigEnd:    #1024      #1025      #1026      #1027
           Address
```

Little Endian vs. Big Endian

- Most modern machines use **Little Endian** ordering by default
- Some processor (ARM) support both Little / Big Endian BUT and one is chosen at startup and used until turned off
- Both Big and Little Endian have (minor) engineering trade-offs
- At one time debated hotly among hardware folks: a la Gulliver's Travels conflicts
- **Intel Chips** were little endian and have dominated computing for several decades, set the precedent for modern platforms
- Big endian byte order shows up in **network programming**: sending bytes over the network is done in big endian ordering
- **Examine** `show_endianness.c` : uses C code to print bytes in order, reveals whether a machine is Little or Big Endian

Output of `show_endianness.c`

```c
// show_endianness.c: Shows endiannes layout of a binary number in
// memory. Intel machines and some ARM machines (Apple M1) are little
// endian so bytes will print least signficant earlier.
#include <stdio.h>

int main(){
  int bin = 0b00000000000000000001100011101001;    // 6377
  //          |   |   |   |   |   |   |   |
  //          0   0   0   0   1   8   e   9
  printf("%d\n%08x\n",bin,bin);                  // show decimal and hex representation of bi
  char *ptr = (char *) &bin;                     // pointer to beginning of bin
  for(int i=0; i<4; i++){                        // print bytes of bin from low to high
    printf("%hhx ", ptr[i]);                     // memory address
  }                                              // '%hhx' : 1-byte char in hex
  printf("\n");                                  // '%hx'  : 2-byte short in hex
  return 0;                                      // '%x'   : 4-byte int in hex
}
```

```
>> gcc show_endianness.c
>> ./a.out
6377
000018e9
e9 18 0 0
```

Notice: `num` prints with value 18e9 but bytes appear in reverse order e9 18 when run on a Little Endian machine: the "littlest" byte appears earliest in memory

Unix Permissions with Octal

- Octal arises associated with **Unix file permissions**
- Every file has 3 permissions for 3 entities
- Permissions are true/false so a single bit will suffice

- `ls -l`: long list files, shows permissions
- `chmod 665 somefile.txt`: change permissions of `somefile.txt` to those shown to the right
- `chmod 777 x.txt`: read / write / exec for everyone
- `chmod` also honors letter versions like `r` and `w`
- `chmod u+x script.sh # make file executable`

```
binary      octal
110110101 = 665
rw-rw-r-x somefile.txt
U  G  O
S  R  T
E  O  H
R  U  E
   P  R

Readable chmod version:
chmod u=rw,g=rw,o=rx somefile.txt
```