

Principles of Parallel Algorithm Design

Chris Kauffman

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Logistics

Reading: Grama Ch 2 + 3

- ▶ **Ch 2.3-5** is most important for Ch 2
- ▶ **Ch 3 all**

Assignment 1

- ▶ Up now, Due Thu 12-Feb
- ▶ Analysis + serial coding
- ▶ Single or Pair Submission
- ▶ Free Collaboration allowed (see syllabus)
- ▶ **Questions?**

This Week

- ▶ Finish Parallel architecture (A1: #1-2)
- ▶ Parallel Algorithm Decomposition (A1: #3,4,5,6)

Dependency Graphs

- ▶ Relation of tasks to one another
- ▶ Vertices: tasks, often labeled with time to complete
- ▶ Edges: indicate what must happen first
- ▶ Should be a DAG: Directed Acyclic Graph
(If not, you're in trouble)

Features of Dependency Graphs

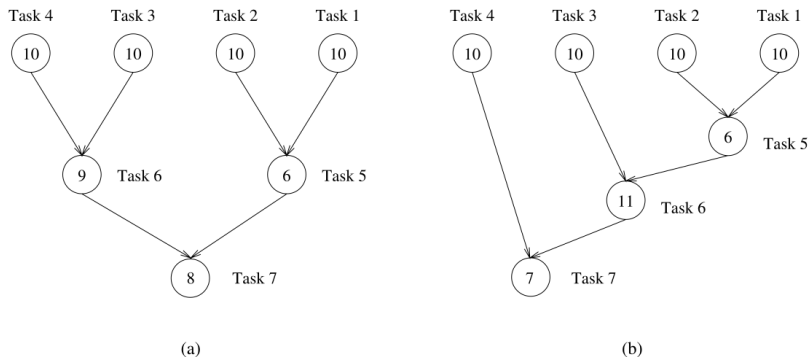


Figure 3.5 Abstractions of the task graphs of Figures 3.2 and 3.3, respectively.

- ▶ Critical Path Length = Sum of longest path
- ▶ Max. Degree of Concurrency = # of task in “widest” section
- ▶ Avg. Degree of Concurrency =

$$\frac{\text{Sum of all vertices}}{\text{Critical Path Length}}$$

Computing Features of Dependency Graphs

Maximum Degree of Concurrency

- ▶ (a) 4
- ▶ (b) 4

Total Task Work

- ▶ (a) 63
- ▶ (b) 64

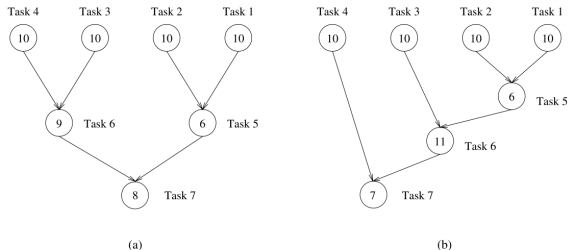


Figure 3.5 Abstractions of the task graphs of Figures 3.2 and 3.3, respectively.

Critical Path Length

- ▶ (a) 27 (leftmost path)
- ▶ (b) 34 (rightmost)

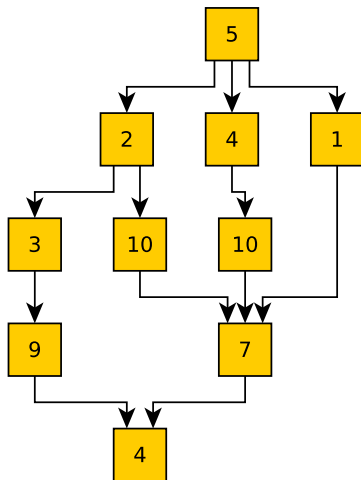
Average Degree of Concurrency

- ▶ (a) $63 / 27 = 2.33$
- ▶ (b) $64 / 34 = 1.88$

Exercise: Compute Features of Dependency Graph

Compute

- ▶ Total Work
- ▶ Maximum degree of concurrency
- ▶ Critical Path Length
- ▶ Average Degree of Concurrency



Answers: Compute Features of Dependency Graph

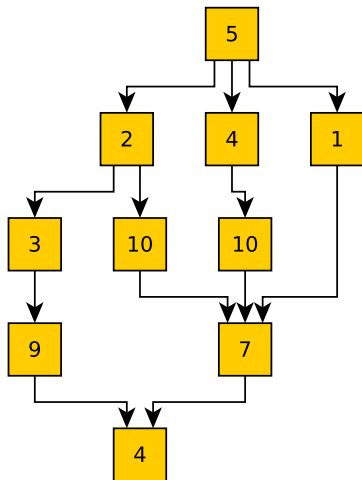
Compute

- ▶ Total Work: 55
- ▶ Max deg of Concur.:
3 (or 4)^a
- ▶ Critical Path Length: 30
- ▶ Average Deg. of Concur.:
 $55/30 = 1.83$

Note

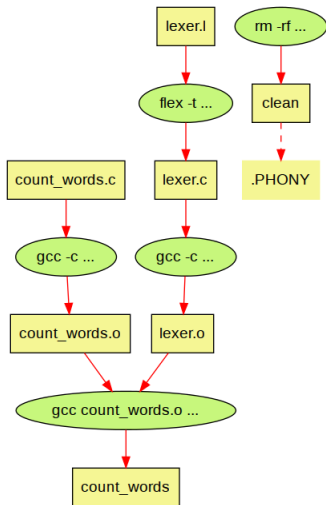
Calculations are easier if each task node has same “work” associated; this is the case in A1

^aWould accept 3 but technically could delay the rightmost 1-unit work to get 4 degrees of concurrency though of little use in practice



Makefiles

- ▶ Most build systems for programs calculate task graphs
- ▶ Makefiles describe DAGs to build projects with make



Source: Luke Luo

```
count_words: count_words.o lexer.o
gcc count_words.o lexer.o -lfl \
-o count_words
```

```
count_words.o: count_words.c
gcc -c count_words.c
```

```
lexer.o: lexer.c
gcc -c lexer.c
```

```
lexer.c: lexer.l
flex -t lexer.l > lexer.c
```

```
.PHONY: clean
```

```
clean:
rm -rf *.o lexer.c count_words
```

Look up make -j 4 option: use 4 processors for concurrency

Identifying Tasks for Parallel Programs

- ▶ This is the tricky part
- ▶ Several techniques surveyed in the text that we'll overview
- ▶ Two general paradigms for creating parallel programs

Parallelize a Serial Code

- ▶ Already have a solution to the problem
- ▶ Identify tasks within solution
- ▶ Construct a task graph and parallelize based on it
- ▶ We'll spend most of our time on this as it is more common

Redesign for Parallelism

- ▶ Best serial code may not parallelize well
- ▶ Change the approach entirely to exploit parallelism
- ▶ Usually harder, more special purpose, we will spend less time on it

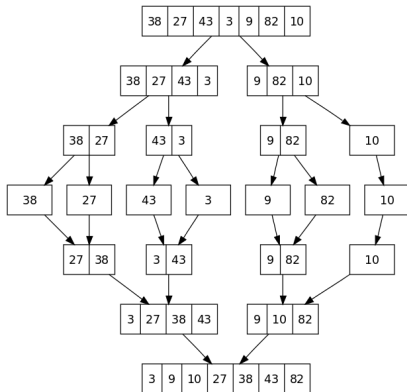
Recursion Provides Parallelism

Algorithms which use *multiple* recursive calls provide easy opportunities for parallelism BUT this often has limited practical benefit in speeding up algorithms

Multiple Recursive Call Algs

- ▶ Fibonacci calculations
- ▶ Quicksort
- ▶ Graph searches
- ▶ **Mergesort**

Examine the nearby Mergesort diagram: branches are potential independent tasks; could do them in parallel BUT it's pointless to assign 7 Procs single numbers only to force serial merges next



Source: [Wikipedia Mergesort](#)

Reformulation As Recursive Algorithms

- ▶ Can sometimes reformulate an iterative algorithm as a recursive one:

Redesign for parallelism

- ▶ **Show task graph** for `RECURSIVE_MIN` on array

$A = \{4, 9, 1, 7, 8, 11, 2, 12\}$, $n = 8$

```
procedure SERIAL_MIN (A, n)
begin
min = A[0];
for i := 1 to n - 1 do
    if (A[i] < min) then
        min := A[i];
    endif
endfor;
return min;
end SERIAL_MIN
```

Specifics of how `RECURSIVE_MIN()` should share data/work among Procs to make it parallel is nontrivial. Dividing up the data in `A` and running `SERIAL_MIN()` on each is straight-forward.

```
procedure RECURSIVE_MIN (A, n)
begin
if (n = 1) then
    min := A[0];
else
    lmin := RECURSIVE_MIN (A, n/2);
    rmin := RECURSIVE_MIN (&(A[n/2]),
                           n - n/2);
    if (lmin < rmin) then
        min := lmin;
    else
        min := rmin;
    endelse;
endelse;
return min;
end RECURSIVE_MIN
```

Data Decomposition: the Goto Design Technique

Identifying parallel tasks based on nature of input or output data is often more straight-forward than an algorithmic/recursive approach

Output Partitioning

- ▶ Among algorithm Output Data...
- ▶ Determine if tasks to compute output are (relatively) independent
- ▶ Parallelize by assigning tasks to Procs based on Output that will be on the Proc

Input Partitioning

- ▶ Output tasks not easily independent
- ▶ Can build up output via independent tasks on input
- ▶ Requires a way to combine results from different sections of input
- ▶ Parallelize by assigning tasks to chunks of input then combining

Combinations of Input/Output partitioning are common so don't expect examples to be clearly ONLY one or the other

Exercise: Matrix-Vector Multiplication

- ▶ Input: matrix A, vector x
- ▶ Output: vector b

$$A * x = b$$

A(1,1)	A(1,2)	A(1,j)	A(1,n)	x(1)	b(1)
A(2,1)	A(2,2)	A(2,j)	A(2,n)	x(2)	b(2)
				x(3)	b(3)
A(i,1)	A(i,2)	A(i,j)	A(i,n)	x(j)	b(i)
A(m,1)	A(m,2)	A(m,j)	A(m,n)	x(n)	b(m)

Output Partitioning

- ▶ What tasks are required to compute each element of output b?
- ▶ What data must each processor hold to perform those tasks?

Answers: Output Partitioning of Mat-Vec Mult

- ▶ Must perform a series of multiply adds of a row of the matrix by the vector
- ▶ If an individual proc holds in memory
 - ▶ Whole Matrix $A[]$ OR
 - ▶ Whole Rows of Matrix $A[b:e, :]$ AND
 - ▶ Vector x

computing elements of $b[]$ are independent tasks spread across procs

- ▶ Output vector $b[]$ would be spread across the procs at the end of the computation

Exercise: Matrix-Vector Multiplication

- ▶ Input: matrix A , vector x
- ▶ Output: vector b

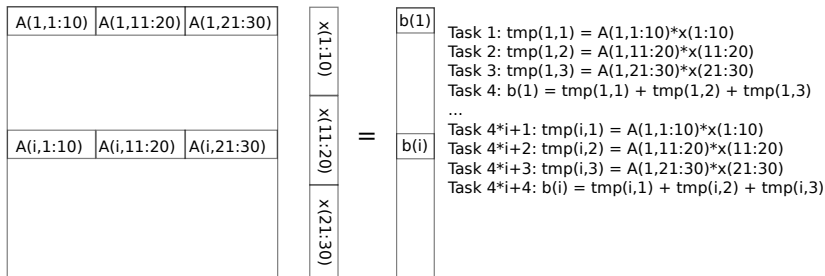
$$A * x = b$$

$A(1,1)$	$A(1,2)$	$A(1,j)$	$A(1,n)$	$x(1)$	$b(1)$
$A(2,1)$	$A(2,2)$	$A(2,j)$	$A(2,n)$	$x(2)$	$b(2)$
				$x(3)$	$b(3)$
$A(i,1)$	$A(i,2)$	$A(i,j)$	$A(i,n)$	$x(j)$	$b(i)$
$A(m,1)$	$A(m,2)$	$A(m,j)$	$A(m,n)$	$x(n)$	$b(m)$

Input Partitioning

- ▶ **Constraint:** Processors have little memory, can't hold whole rows of A and all of x
- ▶ **Propose** an input partitioning: chunks of A and x , do some computation, combine results to form elements of b

Answers: Input Partitioning for Mat-Vec Mult



- ▶ Most Tasks: multiply part of a row of A with part of x
- ▶ Some Tasks: combine partial sums to produce single element of output b
- ▶ Note: Computing chunks of b now requires communication

Exercise: Item Set Frequency Calculation

Typical data mining task: count how many times items {D, E} were bought together in a database of transactions

- ▶ Input: database + itemsets of interest
- ▶ Output: frequency of itemsets of interest

Describe tasks for...

- ▶ Input partitioning
- ▶ Output partitioning
- ▶ Combined partitioning

Database Transactions	A, B, C, E, G, H	Itemsets	A, B, C	Itemset Frequency
	B, D, E, F, K, L		D, E	
	A, B, F, H, L		C, F, G	
	D, E, F, H		A, E	
	F, G, H, K,		C, D	
	A, E, F, K, L		D, K	
	B, C, D, G, H, L		B, C, F	
	G, H, L		C, D, K	
	D, E, F, K, L			
	F, G, H, L			

Answers: Item Set Frequency Calculation

Output Partitioning

- ▶ Whole Database fits on each Proc
- ▶ Divide up Itemsets among Procs
- ▶ Each Proc scans whole DB counting its Itemsets

Input Partitioning

- ▶ DB spread across Procs, each has Partial DB
- ▶ Assume each Proc can hold all Itemsets
- ▶ Each Proc scans its DB portion, counts all Itemsets
- ▶ Procs communicate to Sum all itemsets (Reduction)

Combined Partitioning

- ▶ DB and Itemsets Spread Across Procs
- ▶ Follow Input Partitioning except...
- ▶ Procs only communicate in Groups based on Itemsets

More Details in Grama 3.2

Exploratory Decomposition

Problem Formulations

- ▶ Graph Breadth-first and Depth-first search
- ▶ Path finding in discrete environments
- ▶ Combinatorial search (15-puzzle)
- ▶ Find a good move in a game (Chess, Go)

Algorithms

- ▶ Similar to recursive decomposition
- ▶ Each step has several possibilities to explore
- ▶ Serial algorithm must try one, then unwind
- ▶ Parallel algorithm may explore multiple paths simultaneously
- ▶ Involve “discovering” tasks that are mostly independent, processing them in parallel

Fifteen Puzzle via Exploratory Decomposition

1	2	3	4
5	6	7	8
9	10	7	11
13	14	15	12

(a)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	12

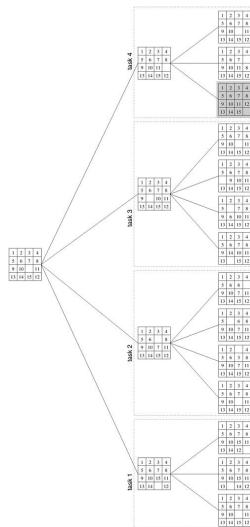
(b)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	12

(c)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

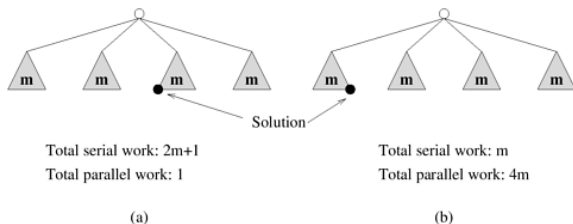
(d)



Source: Grama Fig 3.17

Features of Exploratory Decomposition

- ▶ Data duplication may be necessary so each PE can change its own data (puzzle state)
- ▶ Redundancy may occur: two PEs arrive at the same state
 - ▶ Detect duplication requires programming/communication
 - ▶ Ignoring duplication wastes PE time
- ▶ Termination is trickier: once a solution is found, must signal to all active PEs that they can quit or move on
- ▶ Can lead to “super-linear” speedups over serial algorithms by getting lucky on a search path



Static and Dynamic Task Generation

Static Task Generation

- ▶ All tasks known ahead of time
- ▶ Easier to plan and distribute data
- ▶ Examples abound: matrix operations, sorting (mostly), data analysis, image processing, simple physical simulations

Dynamic task Generation

- ▶ Tasks are “discovered” during the program run
- ▶ Tougher to deal with scheduling, data distribution, coordination and termination
- ▶ Difficulty with message passing paradigm
- ▶ Examples: exploratory decomposition (game trees, graph traversal, recursive search)

We will focus on Static Task Generation

Most complex simulations involve both but we are learning

Static and Dynamic Scheduling (Mapping)

- ▶ Given tasks and dependencies, must schedule them to run on actual processors
- ▶ Problems to solve include Load imbalance (unequal work), Communication overhead, Data distribution as work changes

Static Mapping/Scheduling

- ▶ Specify which tasks happen on which processes ahead of time
- ▶ Usually baked into the code/algorithm
- ▶ Works well for message passing/distributed paradigm

Dynamic Mapping/Scheduling

- ▶ Figure out where tasks get run as you go
- ▶ More or less required if tasks are “discovered”
- ▶ Centralized Scheduling Schemes: manager tracks tasks in a data structure, doles out to workers
- ▶ Distributed scheduling schemes: workers share tasks directly

Reducing the Overhead of Parallelism

Parallel algorithms always introduce overhead: work that doesn't exist in a serial computation. Reducing overhead usually comes in three flavors.

1. Make tasks as independent as possible
2. Minimize data transfers
3. Overlap communication with computation

#1 and #2 are often in tension: **why?**

Broad Categories of Parallel Program Designs

Related to parallel Algorithm design, must also select a Program Design / Software Architecture for how a parallel program will be constructed. Broad categories include the following.

Data-parallel

Every processors gets data, computes similar things, syncs data with group, repeats;
Example: matrix multiplication

Task Graph

Explicitly account for Task Graph, Every proc assigned some tasks and associated data, compute then sync, Example: parallel quicksort (later)

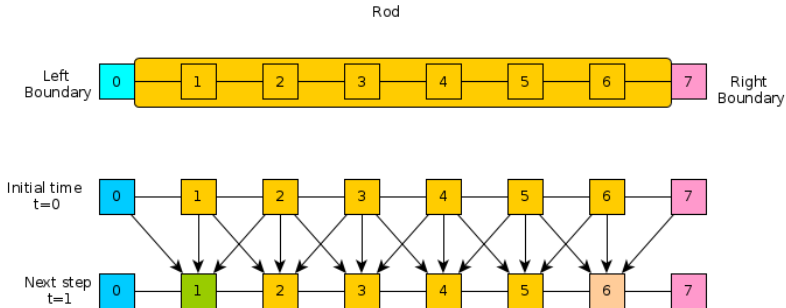
Work-pool + Manager

Initial tasks go into “pool”, doled out to workers by manager, discover new tasks, go into pool, distributed to workers....
Example: web server

Stream/Pipeline/Map-Reduce

Raw data goes in, comp1 done to it, fed to comp2, then to comp3, etc. Example: Frequency counts of all documents, LU factorization

Exercise: A1's Heat Problem



- ▶ What are the tasks? How does the task graph look?
- ▶ What kind of scheduling seems like it will work?
- ▶ How should the data be distributed?
- ▶ What broad category of approach seems to fit?
Data parallel, Task graph distribution,
Work-pool/Manager-worker, Stream/Pipeline

Answers: A1's Heat Problem

Well, it wouldn't be much of an assignment if I gave you my answers...