CSCI 2021: Binary, Integers, Arithmetic

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Logisitcs

Reading

Bryant/O'Hallaron Ch 2.1-2.3

Goals

- Binary Representations / Notation
- Integers in binary
- Arithmetic operations

Assignments

- P1 Ongoing
- Lab03 on File Input
- ► HW03 on Binary Ints
- ► Prev Lab/HW Posted

| Date | | Event |
|------|--------|-----------------------------|
| Mon | 19-Sep | Heap / Valgrind |
| Wed | 21-Sep | Structs / File IO |
| | | Lab03 |
| Fri | 23-Sep | Binary Ints/Chars |
| Mon | 26-Sep | Binary Ints/Chars |
| Tue | 27-Sep | Unified Office Hours |
| Wed | 28-Sep | Lec: Practice Exam |
| | | Lab04: Review |
| | | P1 Due |
| Fri | 30-Sep | Exam 1 |
| | | P1 Late Submission |
| | | |

Questions??

Announcements: None

Exam 1 Logistics

- ► In-person in class on Fri 30-Sep
- Exam runs lecture period: 50min
- Expect 3 sides of paper (front, back, front)

Open Resource Exam

Open Resource Exam Rules

- Sign the log on turning in your exam to show attendence
- ▶ Silence your devices and keep screens visible to instructor
- Protect your work from theft
- You may be asked to show your Student ID

Can Use Your Own Physical, Digital, or Online

- ► Notes, Slides, Dictionary
- Your own previous Exams
- Textbook(s) (online ok)
- ► Editor, Compiler, Vole, SSH
- Your code / Instructor code
- Locally stored webpages
- Online Manual Pages http://man.he.net/ ex: search for ascii

Cannot Use

- ► General Internet
- Piazza Discussion
- Online calculators, converters, tables
- Chat, Texting, IM, etc.
- Communication with anyone but Instructor/Proctor

If you aren't sure of something, ask

Unsigned Integers: Decimal and Binary

Unsigned integers are always positive:

unsigned int i = 12345;

► To understand binary, recall how decimal numbers "work"

Decimal: Base 10 Example

Each digit adds on a power 10

Binary: Base 2 Example

Each digit adds on a power 2

So,
$$11001_2 = 25_{10}$$

Exercise: Convert Binary to Decimal

Base 2 Example:

$$11001 = 1 \times 2^{0} + 1$$

$$0 \times 2^{1} + 0$$

$$0 \times 2^{2} + 0$$

$$1 \times 2^{3} + 8$$

$$1 \times 2^{4} + 16$$

$$= 1 + 8 + 16 = 25$$

So, $11001_2 = 25_{10}$

Try With a Neighbor

Convert the following two numbers from base 2 (binary) to base 10 (decimal)

- **111**
- **11010**
- **>** 01100001

Answers: Convert Binary to Decimal

$$\begin{aligned} 111_2 =& 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ =& 1 \times 4 + 1 \times 2 + 1 \times 1 \\ =& 7_{10} \\ 11010_2 =& 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ =& 1 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 0 \times 1 \\ =& 26_{10} \\ 01100001_2 =& 0 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 \\ & + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ =& 0 \times 128 + \times 64 + 1 \times 32 + 0 \times 16 \\ & + 0 \times 8 + 0 \times 4 + 0 \times 2 + 1 \times 1 \\ =& 97_{10} \end{aligned}$$

Note: last example ignores leading 0's

The Other Direction: Base 10 to Base 2

Converting a number from base 10 to base 2 is easily done using repeated division by 2; keep track of **remainders**

Convert 124 to base 2:

| $124 \div 2 = 62$ | rem 0 |
|-------------------|-------|
| $62 \div 2 = 31$ | rem 0 |
| $31 \div 2 = 15$ | rem 1 |
| $15 \div 2 = 7$ | rem 1 |
| $7 \div 2 = 3$ | rem 1 |
| $3 \div 2 = 1$ | rem 1 |
| $1 \div 2 = 0$ | rem 1 |

- Last step got 0 quotient so we're done.
- ▶ Binary digits are in remainders in reverse
- Answer: 1111100
- ► Check:

$$0 + 0 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 = 4 + 8 + 16 + 32 + 64 = 124$$

Decimal, Hexadecimal, Octal, Binary

- Numbers exist independent of any writing system
- Can write the same number in a variety of bases
- ▶ C provides syntax for most common bases used in computing

| | Decimal | Binary | Hexadecimal | Octal |
|--------------|---------|-------------|------------------|------------------|
| Base | 10 | 2 | 16 | 8 |
| Mathematical | 125 | 1111101_2 | 7D ₁₆ | 175 ₈ |
| C Prefix | None | 0b | 0x | 0 |
| C Example | 125 | 0b1111101 | 0x7D | 0175 |

- ► **Hexadecimal** often used to express long-ish byte sequences Larger than base 10 so for 10-15 uses letters A-F
- Examine number_writing.c and table.c for patterns
- ► **Expectation**: Gain familiarity with doing conversions between bases as it will be useful in practice

Hexadecimal: Base 16

- Hex: compact way to write bit sequences
- ▶ One byte is 8 bits
- Each Hex character represents 4 bits
- ► Each Byte is 2 Hex Digits

| Byte | Hex | Dec |
|-------------------------|----------------|--------------|
| 0101 0111 5 7 | 57 = 5*16 + 7 | 87 87 |
| 0011 1100 3 | 3C = 3*16 + 12 | 60 |
| 1110 0010 E=14 2 | E2 = 14*16 + 2 | 226 |

Hex to 4 bit equivalence

| Dec | Bits | Hex |
|-----|------|-----|
| 0 | 0000 | 0 |
| 1 | 0001 | 1 |
| 2 | 0010 | 2 |
| 3 | 0011 | 3 |
| 4 | 0100 | 4 |
| 5 | 0101 | 5 |
| 6 | 0110 | 6 |
| 7 | 0111 | 7 |
| 8 | 1000 | 8 |
| 9 | 1001 | 9 |
| 10 | 1010 | Α |
| 11 | 1011 | В |
| 12 | 1100 | C |
| 13 | 1101 | D |
| 14 | 1110 | Ε |
| 15 | 1111 | F |

Exercise: Conversion Tricks for Hex and Octal

Examples shown in this week's HW, What tricks are illustrated?

| Decimal | + Byte = 8bits | Byte by 4 | + Hexadecimal |
|---------------|---------------------|----------------------------|--------------------------------------|
| 87 | 01010111 | bin: 0101 0111 hex: 5 7 | 57 = 5*16 + 7 |
| 60 | 00111100 | | 3C = 3*16 + 12 |
| 226 | 11100010 | | E2 = 14*16 + 2 |
| Decimal | Byte = 8bits | Byte by 3 | Octal |
| 87 | 01010111 | | 127 = 1*8^2 + 2*8 + 7 oct dec |
| 60 | 00111100 | | 074 = 0*8^2 + 7*8 + 4 oct dec |
| 226 | 11100010 | | 342 = 3*8^2 + 4*8 + 2 oct dec |

Answers: Conversion Tricks for Hex and Octal

Converting between Binary and Hexadecimal is easiest when grouping bits by 4: each 4 bits corresponds to one hexadecimal digit

```
bin: 0101 0111 bin: 1110 0010 hex: 5 7 hex: E=14 2
```

Converting between Binary and Octal is easiest when grouping bits by 3: each 3 bits corresponds to one octal digit

```
bin: 01 010 111 bin: 11 100 010 oct: 1 2 7 oct: 3 4 2
```

Character Coding Conventions

- Would be hard for people to share words if they interpretted bits as letters differently
- ► **ASCII**: American Standard Code for Information Interchange An old standard for bit/character correspondence
- ▶ 7 bits per character, includs upper, lower case, punctuation

| Dec | Hex | Binary | Char | Dec | Hex | Binary | Char |
|-----|-----|----------|------|-----|-----|----------|------|
| 65 | 41 | 01000001 | Α | 78 | 4E | 01001110 | N |
| 66 | 42 | 01000010 | В | 79 | 4F | 01001111 | 0 |
| 67 | 43 | 01000011 | C | 80 | 50 | 01010000 | Р |
| 68 | 44 | 01000100 | D | 81 | 51 | 01010001 | Q |
| 69 | 45 | 01000101 | E | 82 | 52 | 01010010 | R |
| 70 | 46 | 01000110 | F | 83 | 53 | 01010011 | S |
| 71 | 47 | 01000111 | G | 84 | 54 | 01010100 | Т |
| 72 | 48 | 01001000 | Н | 85 | 55 | 01010101 | U |
| 73 | 49 | 01001001 | 1 | 86 | 56 | 01010110 | V |
| 74 | 4A | 01001010 | J | 87 | 57 | 01010111 | W |
| 75 | 4B | 01001011 | K | 88 | 58 | 01011000 | Χ |
| 76 | 4C | 01001100 | L | 89 | 59 | 01011001 | Υ |
| 77 | 4D | 01001101 | M | 90 | 5A | 01011010 | Z |
| 91 | 5B | 01011101 | [| 97 | 61 | 01100001 | а |
| 92 | 5C | 01011110 | \ | 98 | 62 | 01100010 | b |
| | | | | | | | |

Unicode

▶ World: why can't I write 컴퓨터

in my code/web address/email?

- America: ASCII has 128 chars. Deal with it.
- World: Seriously?
- America: We invented computers. 'Merica!
- ► World:



- America: ··· Unicode?
- World: But my language takes more bytes than American.
- America: Deal with it. 'Merica!

- ► ASCII Uses 7 bits per char, limited to 128 characters
- ► UTF-8 uses **1-4 bytes per character** to represent **many**more characters
 (1,112,064 *codepoints*)
- Uses 8th bit in a byte to indicate extension to more than a single byte
- Requires software to understand coding convention allowing broader language support
- ASCII is a proper subset of UTF-8 making UTF-8 backwards compatible and increasingly popular

Binary Integer Addition/Subtraction

Adding/subtracting in binary works the same as with decimal EXCEPT that carries occur on values of 2 rather than 10

```
ADDITION #1
                         SUBTRACTION #1
   1 11 <-carries
                                   ? <-carries
  0100 \ 1010 = 74
                           0111 \ 1001 = 121
+ 0101 1001 = 89
                         - 0001 0011 = 19
   1010\ 0011 = 163
                            VVVVVVVVVVVVVVV
                            VVVVVVVVVVVVV
ADDITION #2
                            VVVVVVVVVVVVV
   1111 1 <-carries
                                 x12 <-carries
  0110 1101 = 109
                          0111 0001 = 119
+ 0111 1001 = 121
                         - 0001 0011 = 19
  1110\ 0110 = 230
                         0110 \ 0110 = 102
```

Two's Complement Integers: Representing Negative Values

- ► To represent negative integers, must choose a coding system
- ▶ Two's complement is the most common for this
- Alternatives exist
 - ► Signed magnitude: leading bit indicates pos (0) or neg (1)
 - One's complement: invert bits to go between positive negative
- Great advantage of two's complement: signed and unsigned arithmetic are identical
- ► Hardware folks only need to make one set of units for both unsigned and signed arithmetic

Summary of Two's Complement

Short explanation: most significant bit is associated with a negative power of two.

```
UNSTGNED BINARY
                     TWO's COMPLEMENT (signed)
-----
7654 3210 : position 7654 3210 : position
ABCD EFGH: 8 bits ABCD EFGH: 8-bits
A: 0/1 * + (2^7) *POS* A: 0/1 * -(2^7) *NEG*
B: 0/1 * + (2^6) B: 0/1 * + (2^6)
C: 0/1 * + (2^5) C: 0/1 * + (2^5)
H: 0/1 * +(2^0)
                     H: 0/1 * +(2^0)
UNSTGNED BINARY
                     TWO's COMPLEMENT (signed)
7654 3210 : position 7654 3210 : position
1000\ 0000 = +128
                     1000\ 0000 = -128
1000 0001 = +129
                     1000\ 0001 = -127 = -128+1
1000 0011 = +131
                     1000\ 0011 = -125 = -128 + 1 + 2
1111 1111 = +255
                     1111 \ 1111 = -1 = -128 + 1 + 2 + 4 + \ldots + 64
                     0000 \ 0000 = 0
0000 \ 0001 = +1
                     0000\ 0001 = +1
0000 0101 = +5
                   0000\ 0101 = +5
0111 \ 1111 = +127
                    0111 \ 1111 = +127
```

Two's Complement Notes

- Leading 1 indicates negative, 0 indicates positive
- ► All 0's = Zero
- Positive numbers are identical to unsigned

Conversion Trick

Positive → Negative

► Invert bits, Add 1

Negative → Positive

► Invert bits, Add 1

Same trick works both ways, implemented in hardware for the **unary minus** operator as in

int
$$y = -x$$
;

Add Pos/Neg should give 0

Overflow

- Sums that exceed the representation of the bits associated with the integral type overflow
- Excess significant bits are dropped
- Addition can result in a sum smaller than the summands, even for two positive numbers (!?)
- Integer arithmetic in fixed bits is a mathematical ring

Examples of Overflow in 8 bits

| ADDITION #3 OVERFLOW | ADDITION #4 OVERFLOW |
|----------------------|----------------------|
| 1 1111 111 <-carries | 1 1 <-carries |
| 1111 1111 = 255 | 1010 1001 = 169 |
| + 0000 0001 = 1 | + 1100 0001 = 193 |
| | |
| 1 0000 0000 = 256 | 1 0110 1010 = 362 |
| x drop 9th bit | x drop 9th bit |
| | |
| $0000\ 0000 = 0$ | 0110 1010 = 106 |

Underflow

- Underflow occurs in unsigned arithmetic when values go below 0 (no longer positive)
- Pretend that there is an extra significant bit to carry out subtraction
- Subtracting a positive integer from a positive integer may result in a larger positive integer (?!?)
- Integer arithmetic in fixed bits is a mathematical ring

Examples of 8-bit Underflow

```
SUBTRACTION #2 UNDERFLOW
           ?<-carries
   0000 0000 =
  0000 0001 =
           ?<-carries
1\ 0000\ 0000 = 256\ (pretend)
- 0000 0001 =
           2<-carries
0 1111 1110 = 256
  0000 0001 =
   1111 \ 1111 = 255
```

Overflow and Underflow In C Programs

- See over_under_flow.c for demonstrations in a C program.
- No runtime errors for under/overflow
- Good for hashing and cryptography
- ▶ Bad for most other applications: system critical operations should use checks for over-/under-flow
- See textbook Ariane Rocket Crash which was due to overflow of an integer converted from a floating point value
- ► At the assembly level, there are condition codes indicating that overflow has occurred

Endinaness: Byte ordering in Memory

- Single bytes like ASCII characters lay out sequentially in memory in increasing address
- Multi-byte entities like 4-byte ints require decisions on byte ordering
- ▶ We think of a 32-bit int like this

Decimal: 6377

- But need to assign memory addresses to each byte
 - Little Endian: least significant byte early
 - Big Endian: most significant byte early
- Example: Integer starts at address #1024

Address

| LittleEnd: | #1027 | 7 | #1026 | 3 | #102 | 5 | #1024 | 1 |
|------------|-------|------|-------|------|-------|------|-------|------|
| Binary: | 0000 | 0000 | 0000 | 0000 | 0001 | 1000 | 1110 | 1001 |
| | 0 | 0 | 0 | 0 | 1 | 8 | E | 9 |
| BigEnd: | #1024 | 1 | #1025 | 5 | #1026 | 3 | #1027 | 7 |
| | Addra | 200 | | | | | | |

Address

Little Endian vs. Big Endian

- Most modern machines use little endian by default
- Processor may actually support big endian
- ▶ Both Big and Little Endian have engineering trade-offs
- At one time debated hotly among hardware folks: a la Gulliver's Travels conflicts
- Intel chips were little endian and "won" so set the basis for most modern use
- Big endian byte order shows up in network programming: sending bytes over the network is done in big endian ordering
- Examine show_endianness.c to see C code to print bytes in order
- ► Since most machines are little endian, will see bytes print in the revers order usually think of them

Output of show_endianness.c

```
1 > cat show endianness.c
 2 // Show endiannes layout of a binary number in memory Most machines
 3 // are little endian so bytes will print leas signficant earlier.
   #include <stdio.h>
   int main(){
      int bin = 0b000000000000000001100011101001:
                                                      // 6377
     //
10
     printf("%d\n%x\n",bin,bin);
                                                  // show decimal/hex of binary
     unsigned char *ptr = (unsigned char *) &bin; // pointer to beginning of bin
11
12
     for(int i=0: i<4: i++){
                                                  // print bytes of bin from low to high
       printf("%hhx ", ptr[i]);
13
                                                  // memory address
                                                  // '%hhx' : 1-byte char in hex
14
15
     printf("\n"):
                                                  // '%hx' : 2-byte short in hex
16
     return 0:
                                                  // '%x' : 4-byte int in hex
17 F
   > gcc show endianness.c
19
20 > ./a.out
21 6377
22 18e9
23 e9 18 0 0
```

Notice: num prints with value 18e9 but bytes appear in reverse order e9 18 when looking at memory

Integer Ops and Speed

- Along with Addition and Subtraction, Multiplication and Division can also be done in binary
- Algorithms are the same as base 10 but more painful to do by hand
- This pain is reflected in hardware speed of these operations
- The Arithmetic and Logic Unit (ALU) does integer ops in the machine
- ➤ A **clock** ticks in the machine at some rate like 3Ghz (3 billion times per second)

Under ideal circumstances, typical ALU Op speeds are

| Operation | Cycles |
|----------------|--------|
| Addition | 1 |
| Logical | 1 |
| Shifts | 1 |
| Subtraction | 1 |
| Multiplication | 3 |
| Division | >30 |

- Due to disparity, it is worth knowing about relation between multiply/divide and bitwise operations
- Compiler often uses such tricks: shift rather than multiply/divide

Mangling bits puts hair on your chest

Below contrasts difference between logical and bitwise operations.

- Bitwise ops evaluate on a per-bit level
- 32 bits for int, 4 bits shown

| Bitwise OR | Bitwise AND | Bitwise XOR | Bitwise NOT |
|------------|-------------|-------------|-------------|
| 1100 = 12 | 1100 = 12 | 1100 = 12 | |
| 1010 = 10 | & 1010 = 10 | ^ 1010 = 10 | ~ 1100 = 12 |
| | | | |
| 1110 = 14 | 1000 = 8 | 0110 = 6 | 0011 = 3 |

Bitwise Shifts

- ▶ **Shift** operations move bits within a field of bits
- Shift operations are

```
x = y \ll k; // left shift y by k bits, store in x x = y \gg k; // right shift y by k bits, store in x
```

- ▶ All integral types can use shifts: long, int, short, char
- Not applicable to pointers or floating point
- Examples in 8 bits

```
// 76543210
char x = 0b00010111; // 23
char y = x << 2; // left shift by 2
// y = 0b01011100; // 92
// x = 0b00010111; // not changed
char z = x >> 3; // right shift by 3
// z = 0b00000010; // 2
// x = 0b00010111; // not changed
char n = 0b10000000; // -128, signed
char s = n >> 4; // right shift by 4
// s = 0b11111000; // -8, sign extension
// right shift >> is "arithmetic"
```

Shifty Arithmetic Tricks

- Shifts with add/subtract can be used instead of multiplication and division
- ► Turn on optimization: gcc -03 code.c
- Compiler automatically does this if it thinks it will save cycles
- Sometimes programmers should do this but better to convince compiler to do it for you, comment if doing manually

Multiplication

Division

```
76543210
       76543210
                             char y = 0b01101110; // 110
char x = 0b00001010; // 10
                             char y2 = y >> 1; // 110/2
char x2 = x << 1; // 10*2
                             // y2 = 0b00110111; // 55
// x2 = 0b00010100; // 20
                             char y4 = y >> 2; // 110/4
char x4 = x << 2; // 10*4
                             // y4 = 0b00011011; // 27
// x4 = 0b00101000; // 40
                             char z = 0b10101100: // -84
char x7 = (x << 3)-x; // 10*7
                             char z2 = z \gg 2; // -84/4
// x7 = (x * 8)-x; // 10*7
                             // z2 = 0b11101011; // -21
// x7 = 0b01000110: // 70
                                 right shift sign extension
          76543210
//
```

Exercise: Checking / Setting Bits

Use a combination of bit shift / bitwise logic operations to…

- 1. Check if bit i of int x is set (has value 1)
- 2. Clear bit i (set bit at index i to value 0)

Show C code for this

```
{
  int x = ...;
  int i = ...;
  if( ??? ) { // ith bit of x is set
    printf("set!\n");
  }
  i = ...;
  ???;
  printf("ith bith of x now cleared to 0\n");
}
```

Answers: Checking / Setting Bits

1. Check if bit i of int x is set (has value 1)

2. Clear bit i (set bit at index i to value 0)

```
int x = ...;
int mask = 1; // or 0b0001 or 0x01 ...
int shifted = mask << i; // shifted 0b00...010..00
int inverted = ~shifted; // inverted 0b11...101..11
x = x & inverted; // x & 0b10...010..01
... // 0b10...000..01</pre>
```

Showing Bits

printf() capabilities:

```
%d as Decimal
%x as Hexadecimal
%o as Octal
%c as Character
```

- ► No specifier for binary
- Can construct such with bitwise operations
- Code pack contains two codes to do this
 - printbits.c: single args printed as 32 bits
 - showbits.c: multiple args printed in binary, hex, decimal

- Showing bits usually involves shifting and bitwise AND &
- Example from showbits.c

```
#define INT_BITS 32
// print bits for x to screen
void showbits(int x){
 for(int i=INT_BITS-1; i>=0; i--){
    int mask = 1 << i;
    if(mask & x){
      printf("1");
    } else {
     printf("0");
```

Bit Masking

- Semi-common for functions to accept bit patterns which indicate true/false options
- Frequently makes use of bit masks which are constants associated with specific bits
- Example from earlier: Unix permissions might be…

```
#define S_IRUSR 0b100000000 // User Read
#define S_IWUSR 0b010000000 // User Write
#define S_IXUSR 0b001000000 // User Execute
#define S_IRGRP 0b000100000 // Group Read
...
#define S_IWOTH 0b000000010 // Others Write
#define S_IXOTH 0b000000001 // Others Execute
```

Use them to create options to C functions like
int permissions = S_IRUSR|S_IWUSR|S_RGRP;
chmod("/home/kauffman/solution.zip",permissions);

Unix Permissions with Octal

- Octal arises associated with Unix file permissions
- ▶ Every file has 3 permissions for 3 entities
- Permissions are true/false so a single bit will suffice
- ▶ 1s -1: long list files, shows permissions
- chmod 665 somefile.txt: change permissions of somefile.txt to those shown to the right
- chmod 777 x.txt: read /
 write / exec for everyone
- chmod also honors letter versions like r and w
- chmod u+x script.sh #
 make file executable

Readable chmod version: chmod u=rw,g=rw,o=rx somefile.txt