

CMSC216: Binary, Integers, Arithmetic

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Logistics

Reading

Finish up C References

Bryant/O'Hallaron Ch 2.1-2.3

- ▶ Number Systems
- ▶ Binary Encoding of Data
- ▶ Signed/Unsigned Integers
- ▶ Character Data

After Exam 1, Ch 2.4 (Floats)

Goals

- ▶ Wrap C discussion
- ▶ Integers/characters in binary
- ▶ Arithmetic operations, Negative numbers in binary

Assignments

- ▶ Lab04 / HW04: No New material, practice exercises for Project 1 and Exam 1
- ▶ Project 1: Due Mon 24-Feb-2025
- ▶ Exam Review: Tue 25-Feb
- ▶ **Exam 1**: Thu 27-Feb

Announcements

Freeing Nested Data Structures

- ▶ Staff advised students frequently find P1's `maze_free()` to be challenging, get memory leaks reported in test cases
- ▶ Involves deallocating all blocks associated with a Maze
bottom up
- ▶ Examine previous examples of `free()` for inspiration
 - ▶ Lab02: `list_clear()`
 - ▶ Lab03: `treasuremap_free()` and `treasuremap_print()`
- ▶ `maze_free()` is slightly more complex than these as is fitting for a project; my solution is only 15 lines w/ closing curlyes so code is not long

Exam 1 Logistics

Practice + Review

- ▶ Practice Exam 1A will be posted Mon 24-Feb-2025
- ▶ Practice Exam 1B and Review in class Tue 25-Feb-2025
- ▶ Solutions to practice exam will be posted for students

Exam 1

- ▶ In-person in class on Thu 27-Sep-2025
- ▶ Exam runs lecture period: 75min
- ▶ Expect 2.5 pages front/back
- ▶ **Open Resource Exam:** review rules for this posted at bottom of course schedule (beneath slides)

Questions on Open Resource Exam boundaries?

Unsigned Integers: Decimal and Binary

- ▶ Unsigned integers are always positive:
`unsigned int i = 12345;`
- ▶ To understand binary, recall how decimal numbers “work”

Decimal: Base 10 Example

Each digit adds on a power 10

$$\begin{array}{rcl} 80,345 = & 5 \times 10^0 + & 5 \text{ ones} \\ & 4 \times 10^1 + & 40 \text{ tens} \\ & 3 \times 10^2 + & 300 \text{ hundreds} \\ & 0 \times 10^3 + & 0 \text{ thousands} \\ & 8 \times 10^4 & 80 \text{ tens of thousands} \\ & & 5 + 40 + 300 + 80,000 \end{array}$$

Binary: Base 2 Example

Each digit adds on a power 2

$$\begin{array}{rcl} 11001_2 = & 1 \times 2^0 + & 1 \text{ ones} \\ & 0 \times 2^1 + & 0 \text{ twos} \\ & 0 \times 2^2 + & 0 \text{ fours} \\ & 1 \times 2^3 + & 8 \text{ eights} \\ & 1 \times 2^4 + & 16 \text{ sixteens} \\ & & = 1 + 8 + 16 = 25 \end{array}$$

$$\text{So, } 11001_2 = 25_{10}$$

Exercise: Convert Binary to Decimal

Base 2 Example:

$$\begin{aligned} 11001 &= 1 \times 2^0 + & 1 \\ &0 \times 2^1 + & 0 \\ &0 \times 2^2 + & 0 \\ &1 \times 2^3 + & 8 \\ &1 \times 2^4 + & 16 \\ &= 1 + 8 + 16 &= 25 \end{aligned}$$

So, $11001_2 = 25_{10}$

Try With a Neighbor

Convert the following two numbers from base 2 (binary) to base 10 (decimal)

► 111

► 11010

► 01100001

Answers: Convert Binary to Decimal

$$\begin{aligned}111_2 &= 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\&= 1 \times 4 + 1 \times 2 + 1 \times 1 \\&= 7_{10}\end{aligned}$$

$$\begin{aligned}11010_2 &= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\&= 1 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 0 \times 1 \\&= 26_{10}\end{aligned}$$

$$\begin{aligned}01100001_2 &= 0 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 \\&\quad + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\&= 0 \times 128 + 1 \times 64 + 1 \times 32 + 0 \times 16 \\&\quad + 0 \times 8 + 0 \times 4 + 0 \times 2 + 1 \times 1 \\&= 97_{10}\end{aligned}$$

Note: last example ignores leading 0's

The Other Direction: Decimal to Binary

Converting a number from base 10 to base 2 is easily done using repeated division by 2; keep track of **remainders**

Convert 124 to base 2:

$$124 \div 2 = 62 \text{ rem } 0$$

$$62 \div 2 = 31 \text{ rem } 0$$

$$31 \div 2 = 15 \text{ rem } 1$$

$$15 \div 2 = 7 \text{ rem } 1$$

$$7 \div 2 = 3 \text{ rem } 1$$

$$3 \div 2 = 1 \text{ rem } 1$$

$$1 \div 2 = 0 \text{ rem } 1$$

- ▶ Last step got 0 quotient so we're done.
- ▶ Binary digits are in **remainders in reverse**
- ▶ Answer: 1111100
- ▶ Check:

$$0 + 0 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 = 4 + 8 + 16 + 32 + 64 = 124$$

Decimal, Hexadecimal, Octal, Binary Notation

- ▶ Numbers exist independent of any writing system
- ▶ Can write the same number in a variety of bases
- ▶ C provides syntax for most common bases used in computing

	Decimal	Binary	Hex	Octal
Base	10	2	16	8
Mathematical	125	1111101 ₂	7D ₁₆	175 ₈
C Prefix	None	0b...	0x..	0...
C Example	125	0b1111101	0x7D	0175
printf()	"%d"	N/A	"%x"	"%o"

- ▶ **Hexadecimal** often used to express long-ish byte sequences
Larger than base 10 so for 10-15 uses letters A-F
- ▶ **Examine** `number_writing.c` and `table.c` for patterns
- ▶ **Expectation:** Gain familiarity with doing conversions between bases as it will be useful in practice

Hexadecimal: Base 16

- ▶ Hex: compact way to write bit sequences
- ▶ One byte is 8 bits
- ▶ Each Hex character represents 4 bits
- ▶ **Each Byte is 2 Hex Digits**

Byte	Hex	Dec
0101 0111	57 = $5 \cdot 16 + 7$	87
5 7		
0011 1100	3C = $3 \cdot 16 + 12$	60
3 C=12		
1110 0010	E2 = $14 \cdot 16 + 2$	226
E=14 2		

Hex to 4 bit equivalence

Dec	Bits	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Exercise: Conversion Tricks for Hex and Octal

Examples shown in this week's HW, **What tricks are illustrated?**

Decimal	Byte = 8bits	Byte by 4	Hexadecimal
87	01010111	bin: 0101 0111 hex: 5 7	57 = $5 \cdot 16 + 7$ hex dec
60	00111100	bin: 0011 1100 hex: 3 C=12	3C = $3 \cdot 16 + 12$ hex dec
226	11100010	bin: 1110 0010 hex: E=14 2	E2 = $14 \cdot 16 + 2$ hex dec
Decimal	Byte = 8bits	Byte by 3	Octal
87	01010111	bin: 01 010 111 oct: 1 2 7	127 = $1 \cdot 8^2 + 2 \cdot 8 + 7$ oct dec
60	00111100	bin: 00 111 100 oct: 0 7 4	074 = $0 \cdot 8^2 + 7 \cdot 8 + 4$ oct dec
226	11100010	bin: 11 100 010 oct: 3 4 2	342 = $3 \cdot 8^2 + 4 \cdot 8 + 2$ oct dec

Answers: Conversion Tricks for Hex and Octal

- ▶ Converting between Binary and Hexadecimal is easiest when grouping bits by 4: each 4 bits corresponds to one hexadecimal digit

bin: 0101 0111	bin: 1110 0010
hex: 5 7	hex: E=14 2

- ▶ Converting between Binary and Octal is easiest when grouping bits by 3: each 3 bits corresponds to one octal digit

bin: 01 010 111	bin: 11 100 010
oct: 1 2 7	oct: 3 4 2

Character Coding Conventions

- ▶ Would be hard for people to share words if they interpreted bits as letters differently
- ▶ **ASCII**: American Standard Code for Information Interchange
An old standard for bit/character correspondence
- ▶ 7 bits per character, includes upper, lower case, punctuation

Dec	Hex	Binary	Char	Dec	Hex	Binary	Char
65	41	01000001	A	78	4E	01001110	N
66	42	01000010	B	79	4F	01001111	O
67	43	01000011	C	80	50	01010000	P
68	44	01000100	D	81	51	01010001	Q
69	45	01000101	E	82	52	01010010	R
70	46	01000110	F	83	53	01010011	S
71	47	01000111	G	84	54	01010100	T
72	48	01001000	H	85	55	01010101	U
73	49	01001001	I	86	56	01010110	V
74	4A	01001010	J	87	57	01010111	W
75	4B	01001011	K	88	58	01011000	X
76	4C	01001100	L	89	59	01011001	Y
77	4D	01001101	M	90	5A	01011010	Z
91	5B	01011101	[97	61	01100001	a
92	5C	01011110	\	98	62	01100010	b

Partial Table of ASCII Codes / Values, try `man 7 ascii` in a terminal for a full table

Exercise: Characters vs Numbers

Explain the following program and its output

```
1 // char_ints.c:
2 #include <stdio.h>
3 #include <string.h>
4 int main(){
5     ...
6     char nums[64] = {
7         72, 101, 108, 108, 111, 32,
8         87, 111, 114, 108, 100, 33,
9         0
10    };
11    printf("%s\n",nums);
12    len = strlen(nums);
13    for(int i=0; i<len; i++){
14        printf("[%2d] %c %3d %02X\n",
15            i,nums[i],nums[i],nums[i]);
16    }
17    return 0;
18 }
```

```
>> gcc char_ints.c
>> ./a.out
...
Hello World!
[ 0] H  72 48
[ 1] e 101 65
[ 2] l 108 6C
[ 3] l 108 6C
[ 4] o 111 6F
[ 5]   32 20
[ 6] W  87 57
[ 7] o 111 6F
[ 8] r 114 72
[ 9] l 108 6C
[10] d 100 64
[11] !  33 21
```

Answers: Characters vs Numbers

The Whole Array

```
char nums[64] = {  
    72, 101, 108, 108, 111, 32,  
    87, 111, 114, 108, 100, 33,  
    0  
};
```

Lays out a bit pattern at each spot the array; bit pattern is specified with decimal numbers

```
printf("%s\n",nums);
```

Print the array as though it were “string”: an array of characters that is null terminated

Elements of the Array

```
printf("[%2d] %c %3d %02X\n",  
        i,nums[i],nums[i],nums[i]);
```

Print a single element of the array as

- ▶ %c : a character (ASCII table lookup for the glyph to draw)
- ▶ %3d : a decimal number (padding to width 3)
- ▶ %02X : as a hexadecimal number (with leading 0's if needed and padded with width 2 - *noice!*)

Unicode

- ▶ World: Why can't I write
컴퓨터
in my code/web address/email?
- ▶ America: ASCII has 128 chars.
Deal with it.
- ▶ World: Seriously?
- ▶ America: We invented
computers. 'Merica!



- ▶ World:
- ▶ America: ... Unicode?
- ▶ World: But my language takes
more bytes than American.
- ▶ America: Deal with it. 'Merica!

- ▶ ASCII Uses 7 bits per char,
limited to 128 characters
- ▶ UTF-8 uses **1-4 bytes per character** to represent **many**
more characters
(1,112,064 *codepoints*)
- ▶ Uses 8th bit in a byte to
indicate extension to more than
a single byte
- ▶ Requires software to understand
coding convention allowing
broader language support
- ▶ ASCII is a proper subset of
UTF-8 making UTF-8
backwards compatible and
wildly popular

Binary Integer Addition/Subtraction

Adding/subtracting in binary works the same as with decimal
EXCEPT that carries occur on values of 2 rather than 10

ADDITION #1

```
  1 11    <-carries
  0100 1010 = 74
+ 0101 1001 = 89
-----
  1010 0011 = 163
```

ADDITION #2

```
  1111    1 <-carries
  0110 1101 = 109
+ 0111 1001 = 121
-----
  1110 0110 = 230
```

SUBTRACTION #1

```
      ? <-carries
  0111 1001 = 121
- 0001 0011 =  19
-----
```

```
VVVVVVVVVVVVVV
VVVVVVVVVVVVVV
VVVVVVVVVVVVVV
```

```
      x12 <-carries
  0111 0001 = 119
- 0001 0011 =  19
-----
  0110 0110 = 102
```

When 0/1 is represented as Low/High Voltage, one can design digital circuits that implement arithmetic

Two's Complement Integers: Representing Negative Values

- ▶ To represent negative integers, must choose a **different** coding system than for positive-only integers
- ▶ The **Two's Complement Encoding** is the most common coding system for signed numbers so we will study it
- ▶ Alternatives exist
 - ▶ Signed magnitude: leading bit indicates pos (0) or neg (1)
 - ▶ One's complement: invert bits to go between positive negative
- ▶ Great advantage of two's complement: **signed and unsigned arithmetic are identical**
- ▶ Hardware folks only need to make one set of units for both unsigned and signed arithmetic

Summary of Two's Complement

TL;DR: *Most significant bit is a negative power of two.*

UNSIGNED BINARY

7654 3210 : position
ABCD EFGH : 8 bits
A: $0/1 * +(2^7) * POS*$
B: $0/1 * +(2^6)$
C: $0/1 * +(2^5)$
...
H: $0/1 * +(2^0)$

TWO's COMPLEMENT (signed)

7654 3210 : position
ABCD EFGH : 8-bits
A: $0/1 * -(2^7) * NEG*$
B: $0/1 * +(2^6)$
C: $0/1 * +(2^5)$
...
H: $0/1 * +(2^0)$

UNSIGNED BINARY

7654 3210 : position
1000 0000 = +128
1000 0001 = +129
1000 0011 = +131
1111 1111 = +255
0000 0000 = 0
0000 0001 = +1
0000 0101 = +5
0111 1111 = +127

TWO's COMPLEMENT (signed)

7654 3210 : position
1000 0000 = -128
1000 0001 = -127 = -128+1
1000 0011 = -125 = -128+1+2
1111 1111 = -1 = -128+1+2+4+...+64
0000 0000 = 0 [+127]
0000 0001 = +1
0000 0101 = +5
0111 1111 = +127

Two's Complement Notes

Unsigned/Signed Equivalents

Unsigned	1000 0110 = 134
Signed	1000 0110 = -121
	= 134 - 256
Unsigned	1111 0001 = 241
Signed	1111 0001 = -15
	= 241 - 256
Unsigned	0011 0011 = 51
Signed	0011 0011 = 51

When/Why X-256?

- ▶ Leading (leftmost) bit is 1
- ▶ Counted as 128 in Unsigned
- ▶ Counts as -128 in Signed
- ▶ Take -256 to compensate

Negation in Two's Complement

int y = -x;

- ▶ Unary Minus operator
- ▶ **Invert bits, Add 1**
- ▶ Works for both Pos→Neg and Neg→Pos

```
~ 0110 1000 +104 : negate
-----
  1001 0111 inverted
+           1
-----
  1001 1000 = -104
```

```
~ 1001 1000 = -104 : negate
-----
  0110 0111 = +103 inverted
+           1
-----
  0110 1000 = +104
```

Exercise: Two's Complement Conversions

- ▶ **Fill in** the missing entries in the following table
- ▶ **Very similar** to an upcoming exam problem

Bits		Hex	Decimal Unsigned	Decimal Signed
1111 1111	A:	----	B: ---	C: ---
1001 0110		0x96	D: ---	E: ---
F: ----		0x3E	G: ---	H: ---
0010 0011	I:	----	35	J: ---
K: ----	L:	----	M: ---	-35

Answers: Two's Complement Conversions

	Bits	Hex	Decimal Unsigned	Decimal Signed
	1111 1111	A: 0xFF	B: 255	C: -1
	1001 0110	0x96	D: 150	E: -106
F:	0011 1110	0x3E	G: 62	H: 62
	0010 0011	I: 0x23	35	J: 35
K:	1101 1101	L: 0xDD	M: 221	-35

K / L / M: Converting 35 to -35 decimal/bits can be done via $(-35+255)$ AND/OR via Invert Bits + 1

Overflow

- ▶ Sums that exceed the representation of the bits associated with the integral type **overflow**
- ▶ Excess significant bits are **dropped**
- ▶ Addition can result in a sum smaller than the summands, even for two positive numbers (!?)
- ▶ Integer arithmetic in fixed bits is a mathematical **ring**

Examples of Overflow in 8 bits

ADDITION #3 OVERFLOW

```
1 1111 111 <-carries
  1111 1111 = 255
+ 0000 0001 =   1
```

```
1 0000 0000 = 256
x drop 9th bit
```

```
0000 0000 = 0
```

ADDITION #4 OVERFLOW

```
1          1 <-carries
  1010 1001 = 169
+ 1100 0001 = 193
```

```
1 0110 1010 = 362
x drop 9th bit
```

```
0110 1010 = 106
```

Underflow

- ▶ **Underflow** occurs in unsigned arithmetic when values go below 0 (no longer positive)
- ▶ Pretend that there is an extra significant bit to carry out subtraction
- ▶ Subtracting a positive integer from a positive integer may result in a **larger** positive integer (!?)
- ▶ Integer arithmetic in fixed bits is a mathematical **ring**

Examples of 8-bit Underflow

SUBTRACTIION #2 UNDERFLOW

?<-carries

0000 0000 = 0

- 0000 0001 = 1

VVVVVVVVVVVVVV

?<-carries

1 0000 0000 = 256 (pretend)

- 0000 0001 = 1

VVVVVVVVVVVVVV

x 2<-carries

0 1111 1110 = 256

- 0000 0001 = 1

1111 1111 = 255

Overflow and Underflow In C Programs

- ▶ See `over_under_flow.c` for demonstrations in a C program.
- ▶ **No runtime errors** for under/overflow
- ▶ Good for hashing and cryptography
- ▶ Bad for most other applications: system critical operations should use checks for overflow / underflow
- ▶ Textbook mentions the [Ariane Rocket Crash](#) which was due to overflow of an integer converted from a floating point value

The Ariane explosion is an instructive example for several reasons.
(1) Software re-use caused the problem subverting the usual wisdom of relying on tested software; hardware changes ALWAYS trump software.
(2) Sometimes computer science IS rocket science
- ▶ Assembly provides condition codes indicating when overflow occurs but checking in C is tricky and painful¹

¹Many compilers like GCC can generate assembly instructions that will detect overflow and abort programs. See the demo program `overflow_detect.c` and GCC's `-ftrapv` option.

Interlude: Brief Introduction to GDB, The GNU Debugger

- ▶ P2 will include a “debugging problem” called `puzzlebox`
- ▶ Easiest to solve this problem using GDB (or some other debugger)
- ▶ You may benefit from using GDB to complete P1 as well
- ▶ **Debuggers allow one to stop time in a program**, inspect variables, pause execution at certain points and skip forwards
- ▶ If you’ve added tons of `printf()`’s to your code and still can’t figure out what’s going on, a Debugger is your next option
- ▶ Basic mechanics demonstrated on Lab04: set a break point, run program, inspect values, step
- ▶ Associated Reading: [Quick Guide to GDB](#)

Endianness: Byte ordering in Memory

- ▶ Single bytes like ASCII characters lay out sequentially in memory in increasing address
- ▶ Multi-byte entities like 4-byte ints require decisions on byte ordering
- ▶ We think of a 32-bit int like this

	Most Significant	<----->				Least Significant			
Binary:	0000	0000	0000	0000	0001	1000	1110	1001	
	0	0	0	0	1	8	E	9	
Hex :	000018E9								
Decimal:	6377								

- ▶ There are 2 Options to for ordering multi-byte data in memory
 - ▶ **Little Endian:** Least Significant byte at low address
 - ▶ **Big Endian:** Most Significant Byte at low address
- ▶ Example: Integer starts at address #1024

	Address							
LittleEnd:	#1027		#1026		#1025		#1024	
Binary:	0000	0000	0000	0000	0001	1000	1110	1001
	0	0	0	0	1	8	E	9
BigEnd:	#1024		#1025		#1026		#1027	
	Address							

Little Endian vs. Big Endian

- ▶ Most modern machines use **Little Endian** ordering by default
- ▶ Some processor (ARM) support both Little / Big Endian BUT one is chosen at startup and used until turned off
- ▶ Both Big and Little Endian have (minor) engineering trade-offs
- ▶ At one time debated hotly among hardware folks: a la *Gulliver's Travels* conflicts
- ▶ **Intel Chips** were little endian and have dominated computing for several decades, set the precedent for modern platforms
- ▶ Big endian byte order shows up in **network programming**: sending bytes over the network is done in big endian ordering
- ▶ **Examine** `show_endianness.c` : uses C code to print bytes in order, reveals whether a machine is Little or Big Endian

Output of show_endianness.c

```

1 // show_endianness.c: Shows endianness layout of a binary number in
2 // memory. Intel machines and some ARM machines (Apple M1) are little
3 // endian so bytes will print least significant earlier.
4 #include <stdio.h>
5
6 int main(){
7     int bin = 0b0000000000000000000000001100011101001;    // 6377
8     //           |   |   |   |   |   |   |   |
9     //           0   0   0   0   1   8   e   9
10    printf("%d\n%08x\n",bin,bin);                             // show decimal and hex representation of bin
11    char *ptr = (char *) &bin;                                // pointer to beginning of bin
12    for(int i=0; i<4; i++){                                    // print bytes of bin from low to high
13        printf("%hhx ", ptr[i]);                              // memory address
14    }                                                           // '%hhx' : 1-byte char in hex
15    printf("\n");                                              // '%hx'  : 2-byte short in hex
16    return 0;                                                  // '%x'   : 4-byte int in hex
17 }

```

```

>> gcc show_endianness.c
>> ./a.out
6377
000018e9
e9 18 0 0

```

Notice: num prints with value 18e9 but bytes appear in reverse order e9 18 when run on a Little Endian machine: the “littlest” byte appears earliest in memory

Integer Ops and Speed

- ▶ Along with Addition and Subtraction, **Multiplication and Division** can also be done in binary
- ▶ Algorithms are the same as base 10 but more painful to do by hand
- ▶ This pain is reflected in hardware speed of these operations
- ▶ The **Arithmetic and Logic Unit (ALU)** does integer ops in the machine
- ▶ A **clock** ticks in the machine at some rate like 3Ghz (3 billion times per second)

- ▶ Under ideal circumstances, typical ALU Op speeds are

Operation	Cycles
Addition	1
Logical	1
Shifts	1
Subtraction	1
Multiplication	3
Division	>30

- ▶ Due to disparity, it is worth knowing about **relation** between multiply/divide and **bitwise** operations
- ▶ Compiler often uses such tricks: shift rather than multiply/divide

Mangling Bits Puts Muscle on Your Bones

Below illustrates difference between logical and bitwise operations.

```
int x1 = 12 || 10; // truthy (Logical OR)
int xb = 12 | 10;  // 14      (Bitwise OR)
int y1 = 12 && 10; // truthy (Logical AND)
int yb = 12 & 10;  // 8       (Bitwise AND)
int zb = 12 ^ 10;  // 6       (Bitwise XOR)
int w1 = !12;      // falsey  (Logical NOT)
int wb = ~12;      // 3       (Bitwise NOT/INVERT)
```

- ▶ Bitwise ops evaluate on a per-bit level
- ▶ 32 bits for int, 4 bits shown

Bitwise OR	Bitwise AND	Bitwise XOR	Bitwise NOT
1100 = 12	1100 = 12	1100 = 12	
1010 = 10	& 1010 = 10	^ 1010 = 10	~ 1100 = 12
-----	-----	-----	-----
1110 = 14	1000 = 8	0110 = 6	0011 = 3

Bitwise Shifts

- ▶ **Shift** operations move bits within a field of bits

- ▶ Shift operations are

```
x = y << k; // left shift y by k bits, store in x
```

```
x = y >> k; // right shift y by k bits, store in x
```

- ▶ All integral types can use shifts: long, int, short, char

- ▶ **Not applicable** to pointers or floating point

- ▶ Examples in 8 bits

```
//          76543210
```

```
char x = 0b00010111; // 23
```

```
char y = x << 2;      // left shift by 2
```

```
// y = 0b01011100; // 92
```

```
// x = 0b00010111; // not changed
```

```
char z = x >> 3;      // right shift by 3
```

```
// z = 0b00000010; // 2
```

```
// x = 0b00010111; // not changed
```

```
char n = 0b10000000; // -128, signed
```

```
char s = n >> 4;      // right shift by 4
```

```
// s = 0b11111000; // -8, sign extension
```

```
// right shift >> is "arithmetic"
```


Shifty Arithmetic Tricks

- ▶ Shifts with add/subtract can be used instead of multiplication and division
- ▶ Turn on optimization: `gcc -O3 code.c`
- ▶ Compiler automatically does this if it thinks it will save cycles
- ▶ *Sometimes* programmers should do this but better to convince compiler to do it for you, **comment** if doing manually

Multiplication

```
//          76543210
char x = 0b00001010; // 10
char x2 = x << 1;      // 10*2
// x2 = 0b00010100;   // 20
char x4 = x << 2;      // 10*4
// x4 = 0b00101000;   // 40
char x7 = (x << 3)-x;  // 10*7
// x7 = (x * 8)-x;    // 10*7
// x7 = 0b01000110;   // 70
//          76543210
```

Division

```
//          76543210
char y = 0b01101110; // 110
char y2 = y >> 1;     // 110/2
// y2 = 0b00110111;  // 55
char y4 = y >> 2;     // 110/4
// y4 = 0b00011011;  // 27
char z = 0b10101100;  // -84
char z2 = z >> 2;     // -84/4
// z2 = 0b11101011;  // -21
// right shift sign extension
```

Exercise: Checking / Setting Bits

Use a combination of bit shift / bitwise logic operations to...

1. Check if bit *i* of `int x` is set (has value 1)
2. Clear bit *i* (set bit at index *i* to value 0)

Show C code for this

```
{  
    int x = ...;  
    int i = ...;  
    if( ??? ) { // ith bit of x is set  
        printf("set!\n");  
    }  
  
    i = ...;  
    ???;  
    printf("ith bit of x now cleared to 0\n");  
}
```

Answers: Checking / Setting Bits

1. Check if bit *i* of int *x* is set (has value 1)

```
int x = ...;
int mask = 1; // or 0b0001 or 0x01 ...
int shifted = mask << i; // shifted  0b00...010..00
if(x & shifted){          //          x & 0b10...010..01
    ...                  //          -----
}                          //          0b00...010..00
```

2. Clear bit *i* (set bit at index *i* to value 0)

```
int x = ...;
int mask = 1; // or 0b0001 or 0x01 ...
int shifted = mask << i; // shifted  0b00...010..00
int inverted = ~shifted; // inverted 0b11...101..11
x = x & inverted;         //          x & 0b10...010..01
    ...                  //          -----
                          //          0b10...000..01
```

Showing Bits

- ▶ `printf()` capabilities:
 - `%d` as Decimal
 - `%x` as Hexadecimal
 - `%o` as Octal
 - `%c` as Character
- ▶ No specifier for binary
- ▶ Can construct such with bitwise operations
- ▶ Code pack contains two codes to do this
 - ▶ `printbits.c`: single args printed as 32 bits
 - ▶ `showbits.c`: multiple args printed in binary, hex, decimal

- ▶ Showing bits usually involves shifting and bitwise AND &

- ▶ Example from `showbits.c`

```
#define INT_BITS 32

// print bits for x to screen
void showbits(int x){
    for(int i=INT_BITS-1; i>=0; i--){
        int mask = 1 << i;
        if(mask & x){
            printf("1");
        } else {
            printf("0");
        }
    }
}
```

Bit Masking

- ▶ Semi-common for functions to accept bit patterns which indicate true/false options
- ▶ Frequently makes use of bit **masks** which are constants associated with specific bits
- ▶ Example from earlier: Unix permissions might be...

```
#define S_IRUSR 0b100000000 // User   Read
#define S_IWUSR 0b010000000 // User   Write
#define S_IXUSR 0b001000000 // User   Execute
#define S_IRGRP 0b000100000 // Group  Read
...
#define S_IWOTH 0b000000010 // Others Write
#define S_IXOTH 0b000000001 // Others Execute
```

- ▶ Use them to create options to C functions like

```
int permissions = S_IRUSR|S_IWUSR|S_RGRP;
chmod("/home/kauffman/solution.zip",permissions);
```

Unix Permissions with Octal

- ▶ Octal arises associated with **Unix file permissions**
- ▶ Every file has 3 permissions for 3 entities
- ▶ Permissions are true/false so a single bit will suffice

- ▶ `ls -l`: long list files, shows permissions

- ▶ `chmod 665 somefile.txt`:
change permissions of
`somefile.txt` to those
shown to the right

binary	octal	
110110101	= 665	
rw-rw-r-x	<code>somefile.txt</code>	
U	G	O
S	R	T
E	O	H
R	U	E
	P	R

- ▶ `chmod 777 x.txt`: read /
write / exec for everyone

- ▶ `chmod` also honors letter
versions like `r` and `w`

Readable `chmod` version:
`chmod u=rw,g=rw,o=rx somefile.txt`

- ▶ `chmod u+x script.sh` #
make file executable