# CSCI 2021: Binary, Integers, Arithmetic

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### Logisitcs

#### Reading

- C references: whole language:
- ► Bryant/O'Hallaron Ch 2.1-3

#### Goals

- Binary Representations / Notation
- Integers in binary
- Arithmetic operations

### Assignments: Questions?

- ▶ P1 Ongoing
- ► Lab03 on File Input
- ► HW03 on Binary Ints

Date		Event
Fri	09/24	Binary Ints/Chars
Mon	09/24	Binary Ints/Chars
Wed	09/29	Lec Practice Exam
		Lab Review
Fri	10/01	Exam 1
Mon	10/04	Project 1 Due

# Exam 1 Logistics

- ▶ In-person in class on Fri 10/01
- Exam starts at 2:30pm, ends at 3:20pm
- Expect 3 sides of paper (front, back, front)

### **Open Resource Exam**

## Open Resource Exam Rules

- ► Sign the sheet on turning in your exam to show attendence
- Silence your devices and keep screens visible to instructor
- Protect your work from theft
- You may be asked to show your Student ID

### Can Use, physical or electronic

- ► Notes, Slides, Dictionary
- Your own previous Exams (on Gradescope okay)
- ► Textbook(s) (online ok)
- ► Editor, Compiler, Vole, SSH
- ➤ Your code / Instructor code
- Locally stored webpages
- Online Manual Pages http://man.he.net/ ex: search for ascii

#### Cannot Use

- ► General Internet
- Piazza Discussion
- Online calculators, converters, tables
- ► Chat, Texting, IM, etc.
- Communication with anyone but Instructor/Proctor

If you aren't sure of something, ask

# Unsigned Integers: Decimal and Binary

Unsigned integers are always positive:

```
unsigned int i = 12345;
```

To understand binary, recall how decimal numbers "work"

### Decimal: Base 10 Example

Each digit adds on a power 10

### Binary: Base 2 Example

Each digit adds on a power 2

So,  $11001_2 = 25_{10}$ 

# Exercise: Convert Binary to Decimal

### Base 2 Example:

So,  $11001_2 = 25_{10}$ 

$$11001 = 1 \times 2^{0} + 1 \\
0 \times 2^{1} + 0 \\
0 \times 2^{2} + 0 \\
1 \times 2^{3} + 8 \\
1 \times 2^{4} + 16 \\
= 1 + 8 + 16 = 25$$

### Try With a Pal

Convert the following two numbers from base 2 (binary) to base 10 (decimal)

- **111**
- **11010**
- **>** 01100001

# **Answers**: Convert Binary to Decimal

$$\begin{aligned} 111_2 =& 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ =& 1 \times 4 + 1 \times 2 + 1 \times 1 \\ =& 7_{10} \\ 11010_2 =& 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ =& 1 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 0 \times 1 \\ =& 26_{10} \\ 01100001_2 =& 0 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 \\ & + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ =& 0 \times 128 + \times 64 + 1 \times 32 + 0 \times 16 \\ & + 0 \times 8 + 0 \times 4 + 0 \times 2 + 1 \times 1 \\ =& 97_{10} \end{aligned}$$

Note: last example ignores leading 0's

### The Other Direction: Base 10 to Base 2

Converting a number from base 10 to base 2 is easily done using repeated division by 2; keep track of **remainders** 

#### Convert 124 to base 2:

$$124 \div 2 = 62$$
 rem 0  
 $62 \div 2 = 31$  rem 0  
 $31 \div 2 = 15$  rem 1  
 $15 \div 2 = 7$  rem 1  
 $7 \div 2 = 3$  rem 1  
 $3 \div 2 = 1$  rem 1  
 $1 \div 2 = 0$  rem 1

- Last step got 0 so we're done.
- ▶ Binary digits are in remainders in reverse
- Answer: 1111100
- ► Check:

$$0 + 0 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 = 4 + 8 + 16 + 32 + 64 = 124$$

# Decimal, Hexadecimal, Octal, Binary

- Numbers exist independent of any writing system
- Can write the same number in a variety of bases
- C provides syntax for most common bases used in computing

	Decimal	Binary	Hexadecimal	Octal
Base	10	2	16	8
Mathematical	125	$1111101_2$	7D <sub>16</sub>	175 <sub>8</sub>
C Prefix	None	0b	0x	0
C Example	125	0b1111101	0x7D	0175

- ► **Hexadecimal** often used to express long-ish byte sequences Larger than base 10 so for 10-15 uses letters A-F
- Examine number\_writing.c and table.c for patterns
- ► **Expectation**: Gain familiarity with doing conversions between bases as it will be useful in practice

### Hexadecimal: Base 16

- Hex: compact way to write bit sequences
- ▶ One byte is 8 bits
- Each hex character represents 4 bits
- ► Each Byte is 2 hex digits

   Byte	Hex	Dec
0101 0111	57 = 5*16 + 7	87
0011 1100   3 C=12	3C = 3*16 + 12	60
1110 0010     E=14 2	E2 = 14*16 + 2	226

### Hex to 4 bit equivalence

Dec	Bits	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	Α
11	1011	В
12	1100	C
13	1101	D
14	1110	Ε
15	1111	F

### Exercise: Conversion Tricks for Hex and Octal

Examples shown in this week's HW, What tricks are illustrated?

Decimal	Byte = 8bits	Byte by 4	Hexadecimal
87	01010111 	bin: 0101 0111   hex: 5 7	57 = 5*16 + 7   hex dec
60		bin: 0011 1100 hex: 3 C=12	
226	11100010	bin: 1110 0010 hex: E=14 2	E2 = 14*16 + 2   hex dec
Decimal	Byte = 8bits	Byte by 3	Octal
87	01010111 	bin: 01 010 111   oct: 1 2 7	127 = 1*8^2 + 2*8 + 7   oct dec
60	00111100 		074 = 0*8^2 + 7*8 + 4   oct dec
226	11100010	bin: 11 100 010     oct: 3 4 2	342 = 3*8^2 + 4*8 + 2

### **Answers**: Conversion Tricks for Hex and Octal

Converting between Binary and Hexadecimal is easiest when grouping bits by 4: each 4 bits corresponds to one hexadecimal digit

```
bin: 0101 0111 bin: 1110 0010 hex: 5 7 hex: E=14 2
```

Converting between Binary and Octal is easiest when grouping bits by 3: each 3 bits corresponds to one octal digit

```
bin: 01 010 111 bin: 11 100 010 oct: 1 2 7 oct: 3 4 2
```

#### Unix Permissions with Octal

- Octal arises associated with Unix file permissions
- ▶ Every file has 3 permissions for 3 entities
- Permissions are true/false so a single bit will suffice
- ▶ 1s -1: long list files, shows permissions
- chmod 665 somefile.txt: change permissions of somefile.txt to those shown to the right
- chmod 777 x.txt: read /
  write / exec for everyone
- chmod also honors letter versions like r and w
- chmod u+x script.sh #
  make file executable

```
binary octal

110110101 = 665

rw-rw-r-x somefile.txt

U G O

S R T

E O H

R U E

P R
```

Readable chmod version: chmod u=rw,g=rw,o=rx somefile.txt

# **Character Coding Conventions**

- Would be hard for people to share words if they interpretted bits as letters differently
- ► **ASCII**: American Standard Code for Information Interchange An old standard for bit/character correspondence
- ▶ 7 bits per character, includs upper, lower case, punctuation

Dec	Hex	Binary	Char	Dec	Hex	Binary	Char
65	41	01000001	Α	78	4E	01001110	N
66	42	01000010	В	79	4F	01001111	0
67	43	01000011	C	80	50	01010000	P
68	44	01000100	D	81	51	01010001	Q
69	45	01000101	E	82	52	01010010	Ř
70	46	01000110	F	83	53	01010011	S
71	47	01000111	G	84	54	01010100	Т
72	48	01001000	Н	85	55	01010101	U
73	49	01001001	1	86	56	01010110	V
74	4A	01001010	J	87	57	01010111	W
75	4B	01001011	K	88	58	01011000	Χ
76	4C	01001100	L	89	59	01011001	Υ
77	4D	01001101	M	90	5A	01011010	Z
91	5B	01011101	[	97	61	01100001	а
92	5C	01011110	\	98	62	01100010	b

#### Unicode

► World: why can't I write

人

in my code/web address/email?

- America: ASCII has 128 chars. Deal with it.
- World: Seriously?
- America: We invented computers. 'Merica!
- ► World:



- America: ... Unicode?
- World: But my language takes more bytes than American.
- America: Deal with it. 'Merica!

- ► ASCII Uses 7 bits per char, limited to 128 characters
- ► UTF-8 uses **1-4 bytes per character** to represent **many**more characters
  (1,112,064 *codepoints*)
- Uses 8th bit in a byte to indicate extension to more than a single byte
- Requires software to understand coding convention allowing broader language support
- ASCII is a proper subset of UTF-8 making UTF-8 backwards compatible and increasingly popular

# Binary Integer Addition/Subtraction

Adding/subtracting in binary works the same as with decimal EXCEPT that carries occur on values of 2 rather than 10

```
ADDITION #1
                         SUBTRACTION #1
   1 11 <-carries
                                   ? <-carries
  0100 \ 1010 = 74
                           0111 \ 1001 = 121
+ 0101 1001 = 89
                         - 0001 0011 = 19
   1010\ 0011 = 163
                            VVVVVVVVVVVVVV
                            VVVVVVVVVVVVV
ADDITION #2
                            VVVVVVVVVVVVV
   1111 1 <-carries
                                 x12 <-carries
  0110 1101 = 109
                          0111 0001 = 119
+ 0111 1001 = 121
                         - 0001 0011 = 19
  1110\ 0110 = 230
                         0110 \ 0110 = 102
```

# Two's Complement Integers: Representing Negative Values

- ► To represent negative integers, must choose a coding system
- ▶ Two's complement is the most common for this
- Alternatives exist
  - ▶ Signed magnitude: leading bit indicates pos (0) or neg (1)
  - ▶ One's complement: invert bits to go between positive negative
- Great advantage of two's complement: signed and unsigned arithmetic are identical
- ► Hardware folks only need to make one set of units for both unsigned and signed arithmetic

# Summary of Two's Complement

Short explanation: most significant bit is associated with a negative power of two.

```
UNSTGNED BINARY
                     TWO's COMPLEMENT (signed)
-----
7654 3210 : position 7654 3210 : position
ABCD EFGH: 8 bits ABCD EFGH: 8-bits
A: 0/1 * + (2^7) *POS* A: 0/1 * -(2^7) *NEG*
B: 0/1 * + (2^6) B: 0/1 * + (2^6)
C: 0/1 * + (2^5) C: 0/1 * + (2^5)
H: 0/1 * +(2^0)
                     H: 0/1 * +(2^0)
UNSTGNED BINARY
                     TWO's COMPLEMENT (signed)
7654 3210 : position 7654 3210 : position
1000\ 0000 = +128
                     1000\ 0000 = -128
1000 0001 = +129
                    1000\ 0001 = -127 = -128+1
1000 0011 = +131
                    1000\ 0011 = -125 = -128 + 1 + 2
1111 1111 = +255
                     1111 \ 1111 = -1 = -128 + 1 + 2 + 4 + \ldots + 64
                     0000 \ 0000 = 0
0000 \ 0001 = +1
                     0000 \ 0001 = +1
0000 0101 = +5
                   0000\ 0101 = +5
0111 \ 1111 = +127
                    0111 \ 1111 = +127
```

## Two's Complement Notes

- Leading 1 indicates negative, 0 indicates positive
- ► All 0's = Zero
- Positive numbers are identical to unsigned

#### Conversion Trick

Positive -> Negative

► Invert bits, Add 1

Negative -> Positive

► Invert bits, Add 1

Same trick works both ways, implemented in hardware for the **unary minus** operator as in . .

```
int y = -x;
```

```
~ 1001 1000 = negative, invert
  0110 \ 0111 = +103 \ inverted
  0110\ 1000 = +104\ (original = -104)
~ 0110 1000 pos to neg
  1001 0111
             inverted
  1001 \ 1000 = -104
original bits
```

### Add Pos/Neg should give 0

```
1 1111 <-carries
0110 1000 = +104
+ 1001 1000 = -104
------
x 0000 0000 = zero
```

#### Overflow

- Sums that exceed the representation of the bits associated with the integral type overflow
- Excess significant bits are dropped
- Addition can result in a sum smaller than the summands, even for two positive numbers (!?)
- ▶ Integer arithmetic in fixed bits is a mathematical ring

### Examples of Overflow in 8 bits

ADDITION #3 OVERFLOW	ADDITION #4 OVERFLOW
1 1111 111 <-carries	1 1 <-carries
1111 1111 = 255	1010 1001 = 169
+ 0000 0001 = 1	+ 1100 0001 = 193
1 0000 0000 = 256	1 0110 1010 = 362
x drop 9th bit	x drop 9th bit
$0000\ 0000 = 0$	0110 1010 = 106

#### **Underflow**

- Underflow occurs in unsigned arithmetic when values go below 0 (no longer positive)
- Pretend that there is an extra significant bit to carry out subtraction
- Subtracting a positive integer from a positive integer may result in a larger positive integer (?!?)
- Integer arithmetic in fixed bits is a mathematical ring

### Examples of 8-bit Underflow

```
SUBTRACTIION #2 UNDERFLOW
           ?<-carries
   0000 0000 =
  0000 0001 =
           ?<-carries
1\ 0000\ 0000 = 256\ (pretend)
- 0000 0001 =
           2<-carries
0 1111 1110 = 256
  0000 0001 =
   1111 \ 1111 = 255
```

# Overflow and Underflow In C Programs

- See over\_under\_flow.c for demonstrations in a C program.
- ▶ No runtime errors for under/overflow
- Good for hashing and cryptography
- ▶ Bad for most other applications: system critical operations should use checks for over-/under-flow
- See textbook Arianne Rocket crash which was due to overflow of an integer converting from a floating point value
- At assembly level, there are condition codes indicating that overflow has occurred

### Endinaness: Byte ordering in Memory

- Single bytes like ASCII characters lay out sequentially in memory in increasing address
- Multi-byte entities like 4-byte ints require decisions on byte ordering
- ▶ We think of a 32-bit int like this

Decimal: 6377

- But need to assign memory addresses to each byte
  - Little Endian: least significant byte early
  - Big Endian: most significant byte early
- Example: Integer starts at address #1024

#### Address

LittleEnd:	#1027	7	#1026	3	#102	5	#1024	1
Binary:	0000	0000	0000	0000	0001	1000	1110	1001
	0	0	0	0	1	8	E	9
BigEnd:	#1024	1	#1025	5	#1026	3	#1027	7
	Addra	200						

## Little Endian vs. Big Endian

- Most modern machines use little endian by default
- Processor may actually support big endian
- Both Big and Little Endian have engineering trade-offs
- At one time debated hotly among hardware folks: a la Gulliver's Travels conflicts
- Intel chips were little endian and "won" so set the basis for most modern use
- Big endian byte order shows up in network programming: sending bytes over the network is done in big endian ordering
- Examine show\_endianness.c to see C code to print bytes in order
- ➤ Since most machines are little endian, will see bytes print in the revers order usually think of them

## Output of show\_endianness.c

```
1 > cat show endianness.c
 2 // Show endiannes layout of a binary number in memory Most machines
 3 // are little endian so bytes will print leas signficant earlier.
   #include <stdio.h>
   int main(){
      int bin = 0b000000000000000001100011101001:
                                                      // 6377
     //
                                                  // show decimal/hex of binary
10
     printf("%d\n%x\n",bin,bin);
     unsigned char *ptr = (unsigned char *) &bin; // pointer to beginning of bin
11
12
     for(int i=0: i<4: i++){
                                                  // print bytes of bin from low
       printf("%x ", ptr[i]);
                                                  // to high memory address
13
14
15
     printf("\n"):
16
     return 0:
17
   > gcc show endianness.c
19
20 > ./a.out
21 6377
22 18e9
23 e9 18 0 0
```

**Notice:** num prints with value 18e9 but bytes appear in reverse order e9 18 when looking at memory

## Integer Ops and Speed

- Along with Addition and Subtraction, Multiplication and Division can also be done in binary
- Algorithms are the same as base 10 but more painful to do by hand
- This pain is reflected in hardware speed of these operations
- The Arithmetic and Logic Unit (ALU) does integer ops in the machine
- A clock ticks in the machine at some rate like 3Ghz (3 billion times per second)

Under ideal circumstances, typical ALU Op speeds are

Operation	Cycles
Addition	1
Logical	1
Shifts	1
Subtraction	1
Multiplication	3
Division	>30

- Due to disparity, it is worth knowing about relation between multiply/divide and bitwise operations
- Compiler often uses such tricks: shift rather than multiply/divide

# Mangling bits puts hair on your chest

Below contrasts difference between logical and bitwise operations.

- ▶ Bitwise ops evaluate on a per-bit level
- 32 bits for int, 4 bits shown

Bitwise OR	Bitwise AND	Bitwise XOR	Bitwise NOT
1100 = 12	1100 = 12	1100 = 12	
1010 = 10	& 1010 = 10	^ 1010 = 10	~ 1100 = 12
1110 = 14	1000 = 8	0110 = 6	0011 = 3

#### Bitwise Shifts

- ▶ **Shift** operations move bits within a field of bits
- Shift operations are

```
x = y \ll k; // left shift y by k bits, store in x x = y \gg k; // right shift y by k bits, store in x
```

- ▶ All integral types can use shifts: long, int, short, char
- Not applicable to pointers or floating point
- Examples in 8 bits

```
// 76543210
char x = 0b00010111; // 23
char y = x << 2; // left shift by 2
// y = 0b01011100; // 92
// x = 0b00010111; // not changed
char z = x >> 3; // right shift by 3
// z = 0b00000010; // 2
// x = 0b00010111; // not changed
char n = 0b10000000; // -128, signed
char s = n >> 4; // right shift by 4
// s = 0b11111000; // -8, sign extension
// right shift >> is "arithmetic"
```

### Shifty Arithmetic Tricks

- Shifts with add/subtract can be used instead of multiplication and division
- ► Turn on optimization: gcc -03 code.c
- Compiler automatically does this if it thinks it will save cycles
- Sometimes programmers should do this but better to convince compiler to do it for you, comment if doing manually

### Multiplication

#### Division

```
76543210
       76543210
                             char y = 0b01101110; // 110
char x = 0b00001010; // 10
                             char y2 = y >> 1; // 110/2
char x2 = x << 1; // 10*2
                             // y2 = 0b00110111; // 55
// x2 = 0b00010100; // 20
                             char y4 = y >> 2; // 110/4
char x4 = x << 2; // 10*4
                             // y4 = 0b00011011; // 27
// x4 = 0b00101000; // 40
                             char z = 0b10101100: // -84
char x7 = (x << 3)-x; // 10*7
                             char z2 = z \gg 2; // -84/4
// x7 = (x * 8)-x; // 10*7
                             // z2 = 0b11101011; // -21
// x7 = 0b01000110: // 70
                                 right shift sign extension
          76543210
//
```

# Exercise: Checking / Setting Bits

Use a combination of bit shift / bitwise logic operations to...

- 1. Check if bit i of int x is set (has value 1)
- 2. Clear bit i (set bit at index i to value 0)

Show C code for this

# **Answers:** Checking / Setting Bits

1. Check if bit i of int x is set (has value 1)

2. Clear bit i (set bit at index i to value 0)

```
int x = ...;
int mask = 1; // or 0b0001 or 0x01 ...
int shifted = mask << i; // shifted 0b00...010..00
int inverted = ~shifted; // inverted 0b11...101..11
x = x & inverted; // x & 0b10...010..01
... // 0b10...000..01</pre>
```

# Showing Bits

printf() capabilities:

```
%d as Decimal
%x as Hexadecimal
%o as Octal
%c as Character
```

- ► No specifier for binary
- Can construct such with bitwise operations
- Code pack contains two codes to do this
  - printbits.c: single args printed as 32 bits
  - showbits.c: multiple args printed in binary, hex, decimal

- Showing bits usually involves shifting and bitwise AND &
- Example from showbits.c

```
#define INT_BITS 32
// print bits for x to screen
void showbits(int x){
 for(int i=INT_BITS-1; i>=0; i--){
    int mask = 1 << i;
    if(mask & x){
      printf("1");
    } else {
     printf("0");
```

### Bit Masking

- Semi-common for functions to accept bit patterns which indicate true/false options
- Frequently makes use of bit masks which are constants associated with specific bits
- Example from earlier: Unix permissions might be...

```
#define S_IRUSR 0b100000000 // User Read
#define S_IWUSR 0b010000000 // User Write
#define S_IXUSR 0b001000000 // User Execute
#define S_IRGRP 0b000100000 // Group Read
...
#define S_IWOTH 0b000000010 // Others Write
#define S_IXOTH 0b000000001 // Others Execute
```

Use them to create options to C functions like
int permissions = S\_IRUSR|S\_IWUSR|S\_RGRP;
chmod("/home/kauffman/solution.zip",permissions);