Architecture and Parallel Computers

Chris Kauffman

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Logistics

Reading: Grama Ch 2

- **Focus on 2.3-5**, material pertaining to **distributed memory**
  - We will return to shared memory arch later in the course
  - Cache Coherence, PRAM models, False Sharing, Memory Bus are all shared memory topics we'll discuss later
- Sections 2.1 and 2.2 optional, deeper architectures
- Sections 2.6 and 2.7 encouraged, deeper on networks

SISD, SIMD, MIMD, SPAM, and other 4-letter words

- Traditional CPU, Single Instruction Single Data (SISD)

```
ADD r1, r2        # add int in r2 to r1
```

- Most computers now have cpu instructions to add multiple

```
PHADD mm1, mm2  # add two ints in mm2 to ints in mm1
```

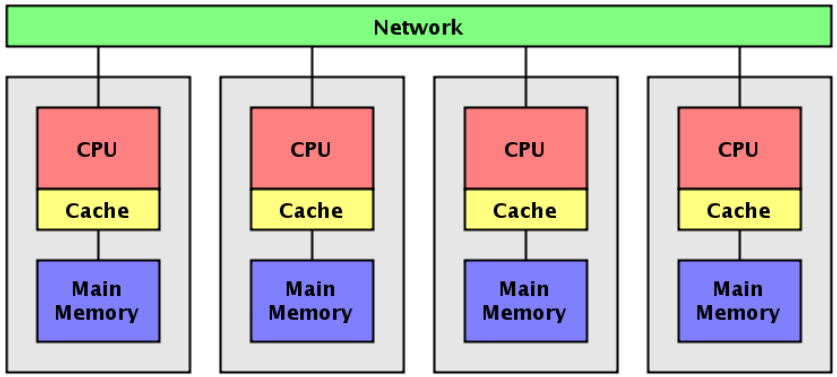
- Smart compilers will select **SIMD / Vector instructions** when appropriate architecture support is available
- Explicit hardware parallelism is good for multimedia stuff (graphics, games, images, sound, videos)
- Flynn's taxonomy of Parallel Architecture includes

I: Instruction	SISD	SIMD	SPMD	P: Program
D: Data	MISD	MIMD	MPMD	

- Some parallel programs exist as Multiple Program Mulitple Data (MPMD) like client server models (`client.c` and `server.c` are separate programs)
- Our focus and the most common type of parallel program: **Single Program Multiple Data (SPMD)**: *Write one program which processes different hunks of data in parallel*

Distributed vs Shared Memory Architectures

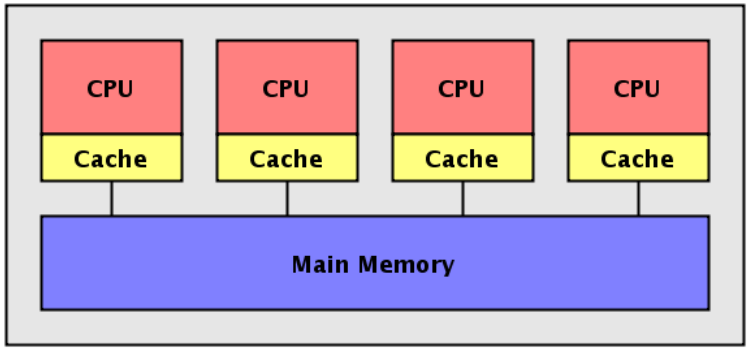
Distributed Memory



Source: Kaminsky/Parallel Java

- Far more scalable/cost effective
- Sharing information requires explicit send/receive commands between processors
- Communication requires more care/more expensive

Shared Memory



Source: Kaminsky/Parallel Java

- Convenience: no explicit send/receive, write shared memory address
- Requires coordination to prevent corrupting memory
- Communication cost is low but requires discipline

Modeling Distributed Memory Parallel Computers

Will spend a some time discussing networks used in parallel computing. These have consequences for algorithms, but unless you're building your own machine (min cost $1M) you're stuck with what you get. Examples:

1. You can test programs on your laptop which likely has 4 cores and no interconnect; MPI models distributed memory as individual processes but scale is limited...
2. We will use the Zaratan cluster for MPI programming which uses an Infiniband Interconnect comprised of switches / processors / storage in a Switched Fabric Topology, balances cost and scale reasonably
3. If you have a chance to work on the #7 Super Computer in the World, Fugaku, in Japan, it is reported to have a Multi-dimensional Torus Topology dubbed "Tofu"

Communication protocols on individual machines vary in details BUT coding via a library like MPI will utilize the best available underlying tech possible

Static Networks for Distributed Machines

- String up a bunch of **Processing Elements (PEs)**
- Decide which PE is connected to which other PE
- Live with the effects on cost of communication

Communication Cost Factors

When sending a message of size m words of memory

- t_s: Startup time, incurred once
- t_h: Per-hop time, overhead incurred for each link between source and destination
- t_w: Per-word transfer time between two nodes, takes $t_w \times M$ time for each link between source and destination
- L: number of links to traverse
- M: number of words being sent
- Typical model for communication time w/ packet routing

$$t_{comm} = t_s + Lt_h + t_w M$$

Basics of Network Design : Cost vs Communication

- Balance number of links / connection pattern complexity
- VS "Distance" between PEs + Contention

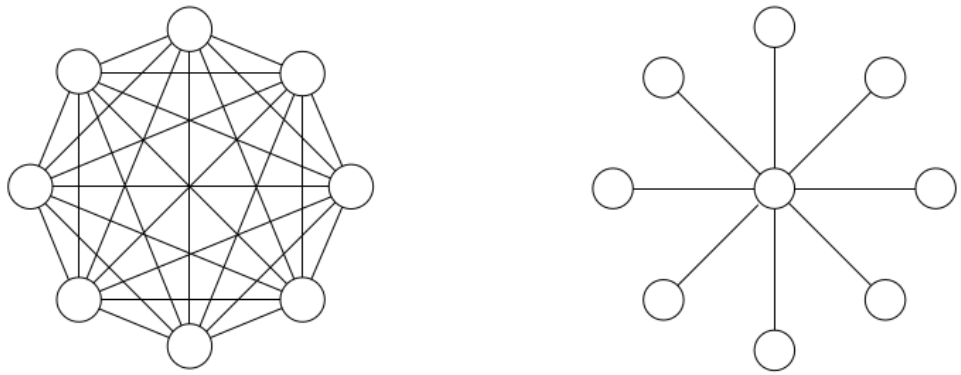


Figure 2.14 (a) A completely-connected network of eight nodes; (b) a star connected network of nine nodes.

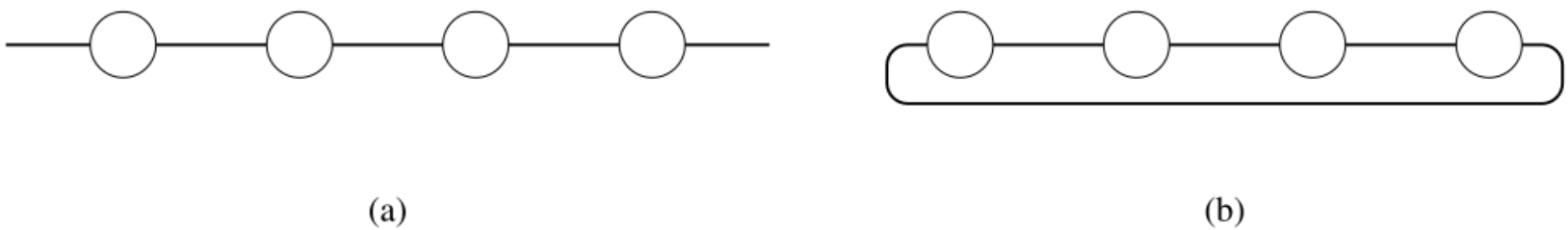


Figure 2.15 Linear arrays: (a) with no wraparound links; (b) with wraparound link.

Source: Grama, Sec 2.4.3

Grid and Torus

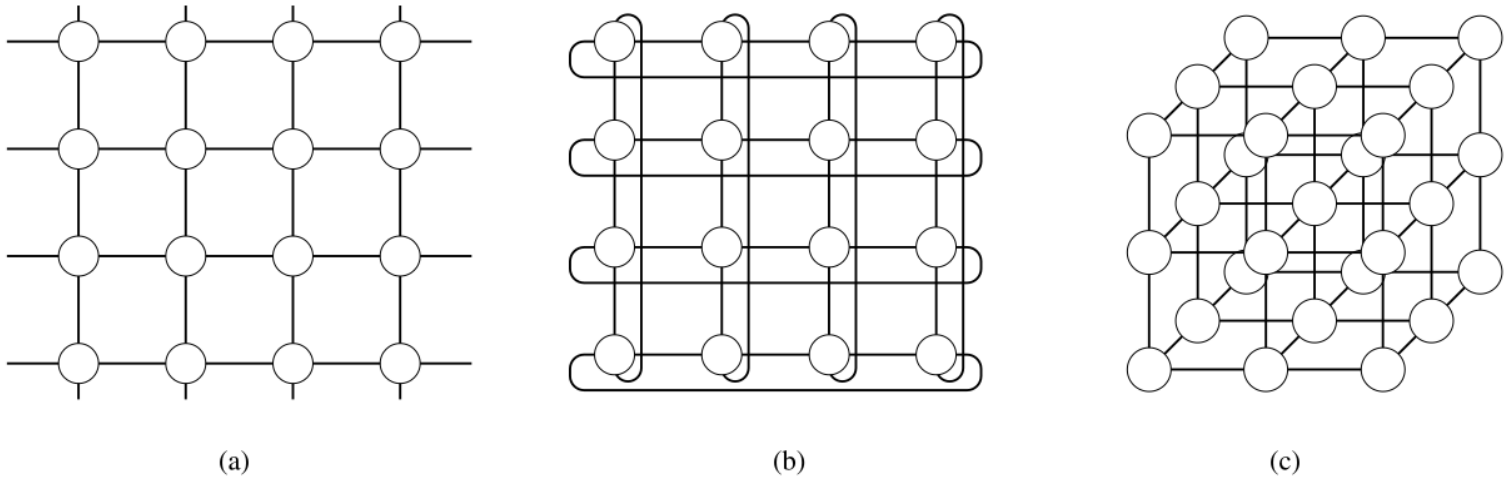


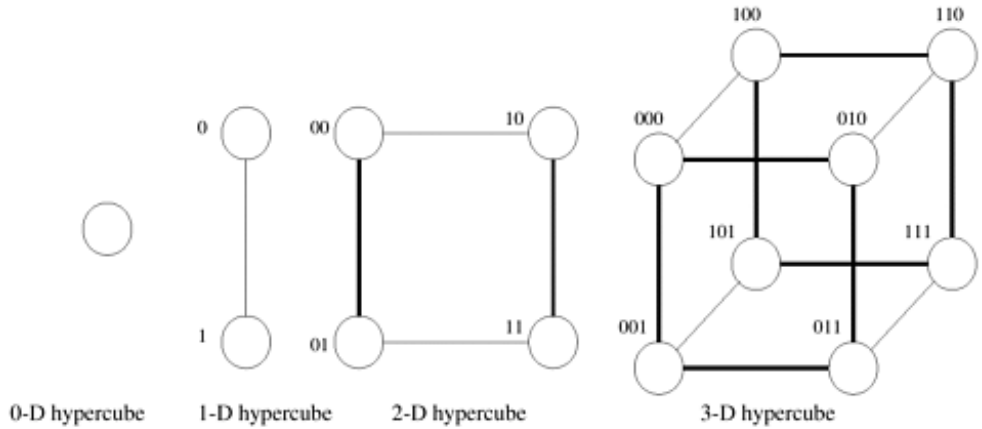
Figure 2.16 Two and three dimensional meshes: (a) 2-D mesh with no wraparound; (b) 2-D mesh with wraparound link (2-D torus); and (c) a 3-D mesh with no wraparound.

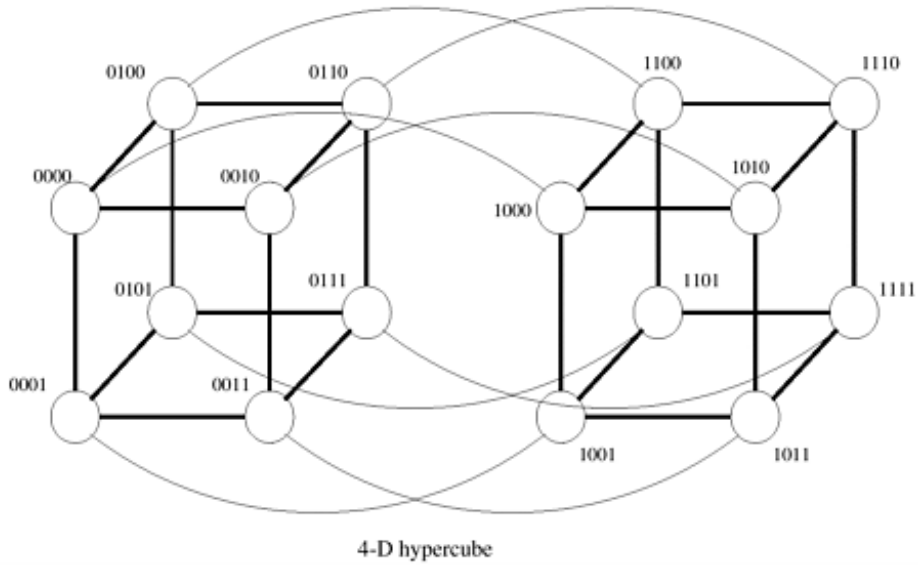
Source: Grama, Sec 2.4.3

- Common arrangement of links between PEs
- Each PE node connected to neighbors
- When wrapping around, grid becomes a torus
- For a 2D torus with p nodes, how many links are required?
- *Hint: surprisingly simple, think of each processor "owning" down and right links*
- How many links in a 3D torus?

Exercise: HyperCube

- D-dimension hypercube: connect two $(D-1)$ dimension hypercubes, link corresponding nodes
- How many nodes and links in a D-dimension hypercube?
- *Hint: Nodes are easy, links are tricky, try Grama textbook...*





Answers: HyperCube

D-dimensional Hypercube has

- 2^D Processors
- $2^D \times D/2$ links

Can show this via Proof by Induction but that's not our focus

That's a lot of Links

- Many communication patterns have excellent performance on a hypercube
- Building one requires wiring processors together in a highly complex manner[1]
- Ex: 10-dimensional hypercube with 1024 Processors each with 10 links to a unique set of other processors
- Hypercubes are a favorite theoretical topology and useful in some cases for algorithm analyses but ...
- Too expensive/complex for large-scale machines

[1] Academic papers that describe new network architectures sometimes include *wiring algorithms* to show their complex network is actually practical to construct in reality: example

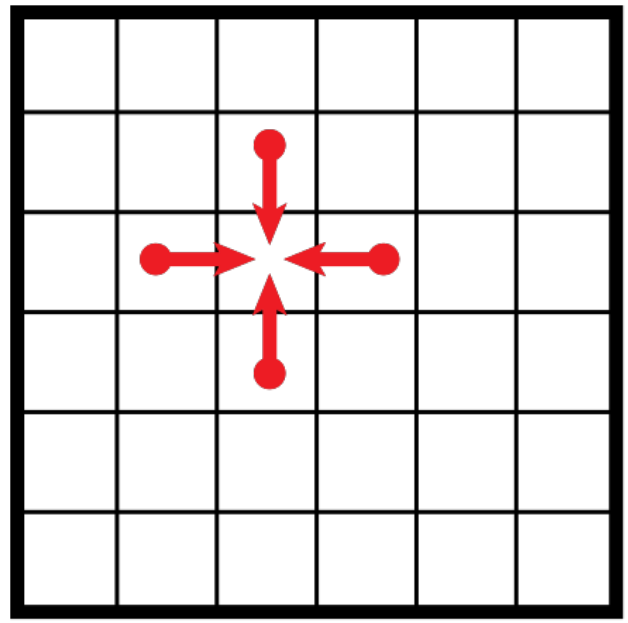
Exercise: Compare Networks on Parallel Stencil

- P processors
- Network 1: 2D-Mesh: around $2P$ links
- Network 2: $\log_2(P)$ dim. Hypercube w/ $(P \log_2(P)/2)$ links
- **Discuss** advantages/disadvantages of Mesh vs Hypercube arrangement for this application
- Outline an algorithm, estimate cost-effectiveness of code+hardware

Image "blurring"

- A large image is distributed across the P processors
- Each proc holds a 2D hunk of the image
- To blur the entire image, must assign RGB values which are average of "neighborhood"

Stencil



Answers: Compare Networks on Parallel Stencil

- Divide image into 2D hunks
- PEs must communicate with other PEs that have neighboring hunks of the image

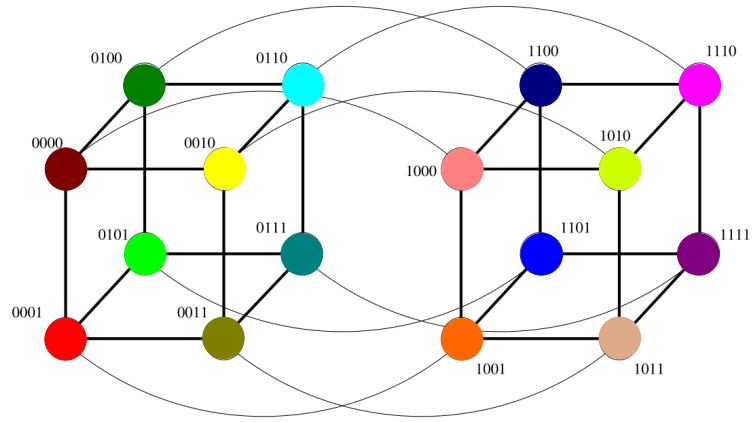
2D Mesh

- Maps VERY easily onto a 2D Mesh / Grid / Torus
- PEs locally blur own portion of image
- Exchange boundary pixels with 4 neighbors except for outer edge PEs

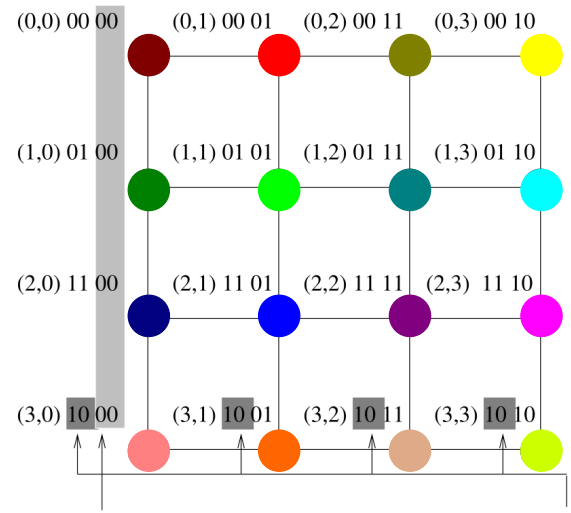
Answers: Compare Networks on Parallel Stencil

Hypercube

- Intuition: have many more links than in the 2D Torus, should be possible to place neighboring pixel hunks on neighboring procs
- *Embed 2D-Mesh into a Hypercube*: discussed in Grama 2.7.1, uses Gray Codes for Proc Numbering and is beyond in-class / exam questions (perhaps an assignment problem)
- After embedding Mesh in Hypercub, use Mesh algorithm



4-D hypercube



Processors in a column have identical two least–significant bits

Processors in a row have identical two most–significant bits

Network Embedding

- Some algorithms work well in a particular network
- When running them on another network, look for an **embedding** that replicates (as much as possible) features of the original network
- Embedding (informally):
  - Assignment of PEs in network A to PEs in network B
  - Assignment of links network A to links in network B
- Assignments lead to consequences
  - Ex: PE4 and PE5 are connected by a single link in Network A but are 2 links apart in Network B
  - Ex: In network A, parts of links X,Y,Z are all mapped onto Link W in network B
- Metrics like **Dilation** and **Congestion** evaluate the quality of different embedding choices
- Previous example found it is possible embed 2D Mesh in a Hypercube with Dilation / Congestion of 1

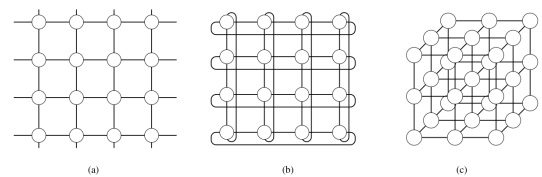
Exercise: Compare Networks on Parallel Sum

- P processors (assume P is a power of 2)
- Network 1: 2D-torus: $2P$ links
- Network 2: $\log_2(P)$ dim. Hypercube w/ $(P \log_2(P)/2)$ links
- **Discuss** advantages/disadvantages of torus vs hypercube arrangement for this application
- Outline an algorithm, estimate cost-effectiveness of code+hardware

Sum Array of Numbers

- Each proc holds a hunk of the data array
- Want total sum on root PE0 at end of algorithm
- **State your algorithm**: Try to minimize communication at each step, exploit as much parallelism as possible

Networks



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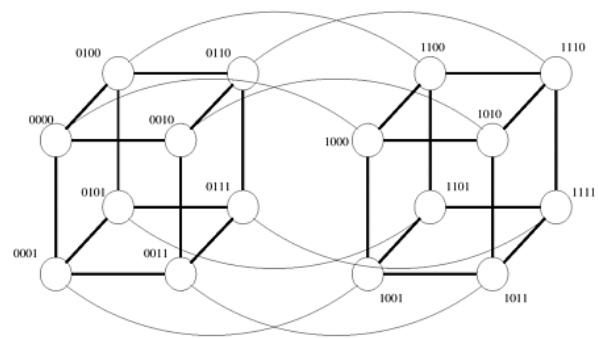


4-D hypercube

Answers: Compare Networks: Parallel Sum

Goal: Get sum on Proc 0

First, each Proc sums its own chunk of numbers then...

2D Torus: N by N Square

- Send values UP rows then LEFT across columns
  - 2*N Communication steps, always neighbors
  - Many Procs **Idle** during communication
- Other Communication steps will result in multi-hop communication with non-neighbor procs - will revisit this later

N-dimensional HyperCube

- Each Proc has a binary address: ex: 100110
- Starting with bit $i = (N - 1)$ while $i > 0$
  - Each Proc with bit $i == 1$ sends to $i == 0$
  - Decrement i, repeat
- Takes N communication steps

Communication Patterns Later

- We will talk more about Parallel Sum later
- Parallel Sum is an example of a **reduction** - general communication pattern that recurs often in Parallel Computing
- Covered in more detail in Section 6.6
- Parallel Sum is discussed in Lecture notes by Susan Hayes

Characteristics of Various Networks

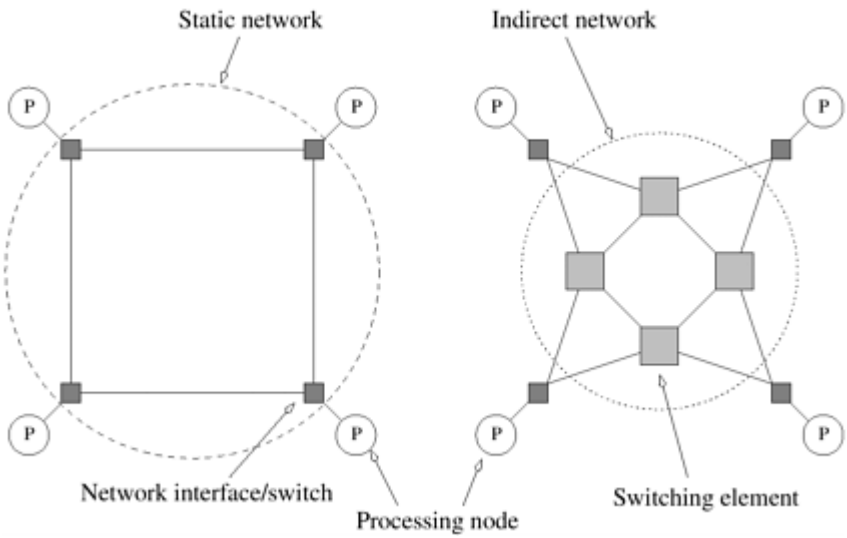
Table 2.1 A summary of the characteristics of various static network topologies connecting p nodes.

Network	Diameter	Bisection Width	Arc Connectivity	Cost (No. of links)
Completely-connected	1	$p^2/4$	$p-1$	$p(p-1)/2$
Star	2	1	1	$p-1$
Complete binary tree	$2\log((p+1)/2)$	1	1	$p-1$
Linear array	$p-1$	1	1	$p-1$
2-D mesh, no wraparound	$2(\sqrt{p}-1)$	$\sqrt{p}$	2	$2(p-\sqrt{p})$
2-D wraparound mesh	$2\lfloor\sqrt{p}/2\rfloor$	$2\sqrt{p}$	4	$2p$
Hypercube	$\log p$	$p/2$	$\log p$	$(p\log p)/2$
Wraparound k-ary d-cube	$d\lfloor k/2\rfloor$	$2k^{d-1}$	$2d$	dp

Several metrics described in textbook

- *Diameter*: max hops away any two procs can be
- *Bisection width*: remove N links to get 2 networks, equal size
- *Arc Connectivity*: remove N links to get 2 networks, any size
- *Cost*: can correspond to number of links

Dynamic Networks



- In a static network, connections are fixed
- Dynamic networks use switches: send data into network with destination, may alter a connection to point in a different direction
- Akin to the internet: packet switching network
- **Most modern HPC networks are Dynamic**: balances cost and scale better than the older static networks

Fat Trees

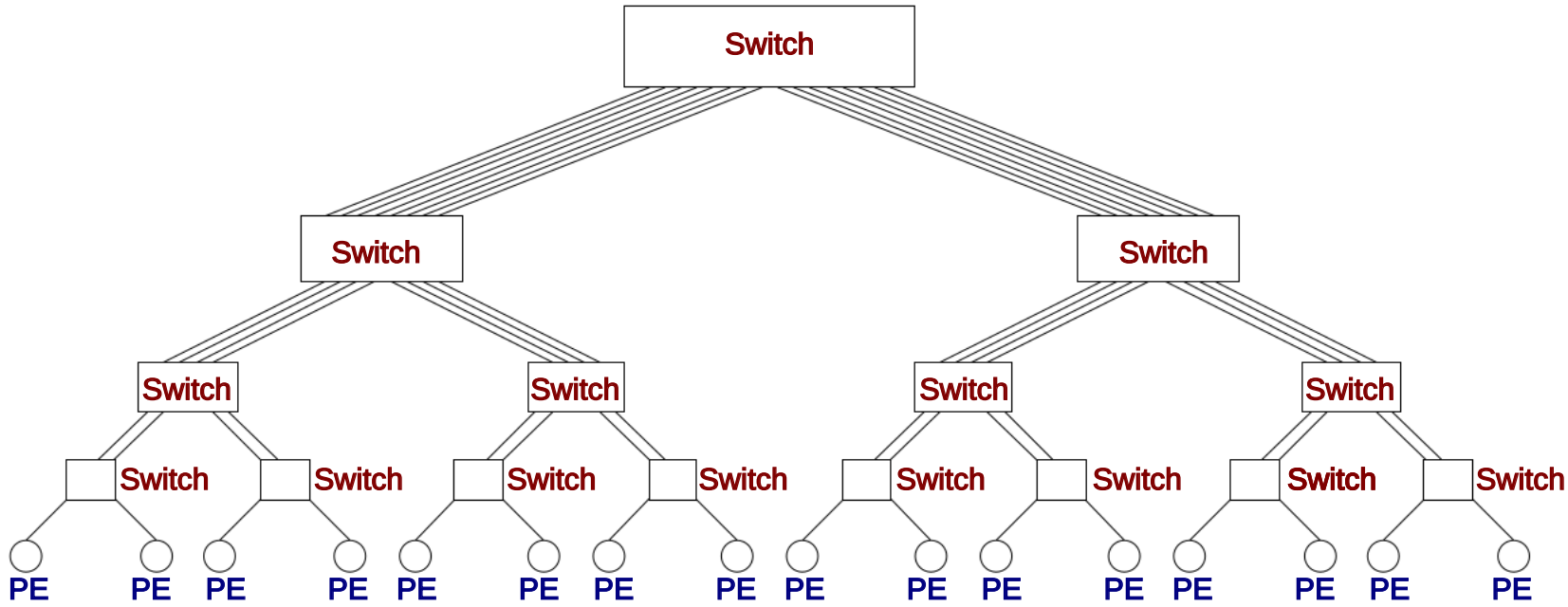
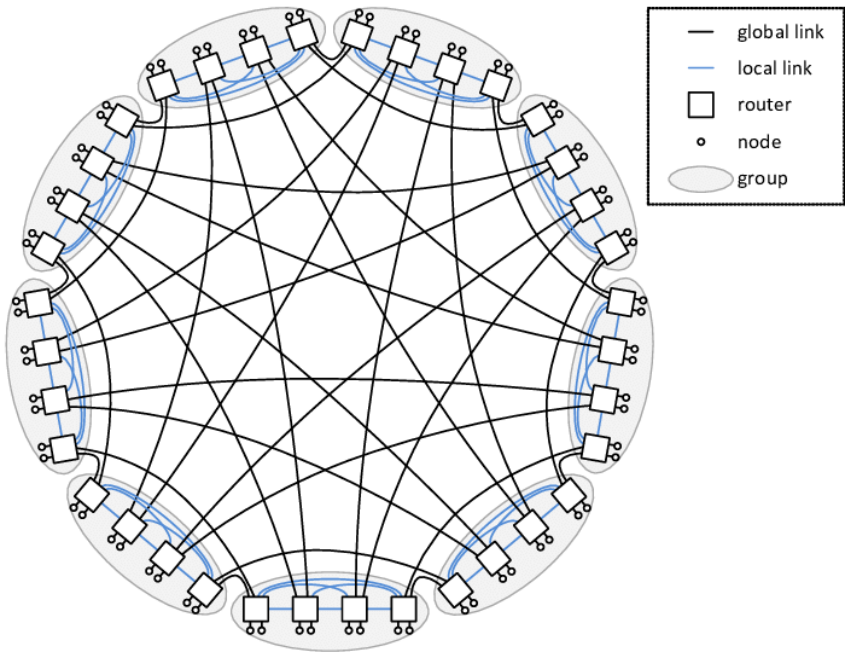


Figure 2.19 A fat tree network of 16 processing nodes.

Often used as network switches are inexpensive and widely available while still providing good communication speeds

Dragonfly Networks



Source: Garcia et al. International Conference on Parallel Processing 2012

- Some years ago a new HPC network topology coined "Dragonfly Networks" were introduced
- Proved influential due to...
- Low diameter: limits number of hops between any nodes (processors)
- "Groups" are fully interconnected
- Low Cost due to use of available commodity switches

Routing: Store/Forward Packet and Packet Switching

When sending messages, intermediate nodes must decide what to do with a message: **Routing protocol/scheme**

Store and Forward

- Accumulate the whole message (all M words), store it until it can be forwarded to next hop
- Easy to build but requires large-ish internal buffers and generally has bad performance

Standard Packet Switching

- Break message into chunks (packets)
- Use packet header to carry error-correction info, routing info
- Optimized for the unreliable internet: go around overloaded / dead nodes, adjust to faster paths if found
- Better but incurs robustness overhead isn't necessary present in most reliable HPC machine networks

Routing: Cut-Through Communications

- Standard in HPC network design to optimize communication protocol: sacrifice some robustness to improve speed
- Cut-through Routing is an abstract version of this
- Similar to packet switching: break message into chunks
- Send a *tracer* from source to destination to determine route - all packets then follow that route
- Send message in *flits* (packets) along tracer route - reduces latency over Store/Forward
- Minimize data in packet for error correction, re-routing, etc. - reduced overhead vs Standard Packet Switching
- Comm time dominated by initial path determination $t_h L$ and total message size $t_w M$

Our Approach

Algorithm + Specific Network

Assume Cut-Through Routing, account for hops between PEs

$$t_{comm} = t_s + Lt_h + t_w M$$

- Simplified model for Comm but reasonable enough to guide algorithm decisions on how to utilize specific network
- Minimize L between communicating PEs in algorithms
- L changes with topology: e.g. Hypercube needs fewer communication steps than a Torus due to more abundant links

Algorithm + Arbitrary Network

Will ignore network topology, congestion, number of hops

$$t_{comm} = t_s + t_w M$$

- Abstracted away from specific network features which will vary
- Ignores path lengths, unrealistic but understandable when network structure is unknown
- Still accounts for number and size of communications in algorithm