# CMSC216: Binary, Integers, Arithmetic

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# Logistics

## Reading

- C References
- ► Bryant/O'Hallaron Ch 2.1-2.3

## Assignments

- ► Lab03/HW03: due Mon 19-Feb on Gradescope
- Project 1: due 20-Feb
  - Bug fixes to font\_alternate.txt
  - Bug fixes to associated Problem 2 tests

#### Goals

- Wrap C discussion
- Binary Notation and Relations
- Integers in binary
- Arithmetic operations

# Announcements: None

# Exam 1 Logistics

- ▶ In-person in class on Thu 22-Feb
- Exam runs lecture period: 75min
- Expect 2 pages front/back
- ▶ Open Resource Exam: examine rules for this posted at bottom of course schedule (beneath slides)

#### Practice + Review

- Practice Exam 1A will be posted Fri 16-Feb
- Practice Exam 1B and Review in class Tue 20-Feb
- Solutions to practice exam will be posted for students

# Unsigned Integers: Decimal and Binary

Unsigned integers are always positive:

```
unsigned int i = 12345;
```

► To understand binary, recall how decimal numbers "work"

## Decimal: Base 10 Example

Each digit adds on a power 10

## Binary: Base 2 Example

Each digit adds on a power 2

So, 
$$11001_2 = 25_{10}$$

# Exercise: Convert Binary to Decimal

## Base 2 Example:

$$11001 = 1 \times 2^{0} + 1$$

$$0 \times 2^{1} + 0$$

$$0 \times 2^{2} + 0$$

$$1 \times 2^{3} + 8$$

$$1 \times 2^{4} + 16$$

$$= 1 + 8 + 16 = 25$$

So, 
$$11001_2 = 25_{10}$$

## Try With a Neighbor

Convert the following two numbers from base 2 (binary) to base 10 (decimal)

- **111**
- **11010**
- **>** 01100001

# **Answers**: Convert Binary to Decimal

$$\begin{aligned} 111_2 &= 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= 1 \times 4 + 1 \times 2 + 1 \times 1 \\ &= 7_{10} \\ 11010_2 &= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ &= 1 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 0 \times 1 \\ &= 26_{10} \\ 01100001_2 &= 0 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 \\ &+ 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 0 \times 128 + \times 64 + 1 \times 32 + 0 \times 16 \\ &+ 0 \times 8 + 0 \times 4 + 0 \times 2 + 1 \times 1 \\ &= 97_{10} \end{aligned}$$

Note: last example ignores leading 0's

## The Other Direction: Base 10 to Base 2

Converting a number from base 10 to base 2 is easily done using repeated division by 2; keep track of **remainders** 

#### Convert 124 to base 2:

$124 \div 2 = 62$	rem 0
$62 \div 2 = 31$	rem 0
$31 \div 2 = 15$	rem 1
$15 \div 2 = 7$	rem 1
$7 \div 2 = 3$	rem 1
$3 \div 2 = 1$	rem 1
$1 \div 2 = 0$	rem 1

- Last step got 0 quotient so we're done.
- ▶ Binary digits are in remainders in reverse
- Answer: 1111100
- ► Check:

$$0 + 0 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 = 4 + 8 + 16 + 32 + 64 = 124$$

# Decimal, Hexadecimal, Octal, Binary

- Numbers exist independent of any writing system
- Can write the same number in a variety of bases
- C provides syntax for most common bases used in computing

	Decimal	Binary	Hexadecimal	Octal
Base	10	2	16	8
Mathematical	125	$1111101_2$	7D <sub>16</sub>	175 <sub>8</sub>
C Prefix	None	0b	0x	0
C Example	125	0b1111101	0x7D	0175

- ► **Hexadecimal** often used to express long-ish byte sequences Larger than base 10 so for 10-15 uses letters A-F
- Examine number\_writing.c and table.c for patterns
- ► **Expectation**: Gain familiarity with doing conversions between bases as it will be useful in practice

## Hexadecimal: Base 16

- Hex: compact way to write bit sequences
- ▶ One byte is 8 bits
- ► Each Hex character represents 4 bits
- ► Each Byte is 2 Hex Digits

Byte	Hex	Dec
0101 0111	57 = 5*16 + 7	
0011 1100     3	3C = 3*16 + 12	60
1110 0010     E=14 2	E2 = 14*16 + 2	226

## Hex to 4 bit equivalence

Dec	Bits	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	Α
11	1011	В
12	1100	C
13	1101	D
14	1110	E
15	1111	F

## Exercise: Conversion Tricks for Hex and Octal

Examples shown in this week's HW, What tricks are illustrated?

Decimal	Byte = 8bits	Byte by 4	Hexadecimal
87	01010111	bin: 0101 0111     hex: 5 7	57 = 5*16 + 7   hex dec
60	00111100	bin: 0011 1100     hex: 3	
226	11100010	bin: 1110 0010     hex: E=14 2	·
Decimal	Byte = 8bits	Byte by 3	Octal
87	01010111	bin: 01 010 111     oct: 1 2 7	127 = 1*8^2 + 2*8 + 7   oct dec
60	00111100	bin: 00 111 100     oct: 0 7 4	074 = 0*8^2 + 7*8 + 4 oct dec
226	11100010	   him, 11 100 010	   342 = 3*8^2 + 4*8 + 2

## **Answers**: Conversion Tricks for Hex and Octal

Converting between Binary and Hexadecimal is easiest when grouping bits by 4: each 4 bits corresponds to one hexadecimal digit

```
bin: 0101 0111 bin: 1110 0010 hex: 5 7 hex: E=14 2
```

► Converting between Binary and Octal is easiest when grouping bits by 3: each 3 bits corresponds to one octal digit

```
bin: 01 010 111 bin: 11 100 010 oct: 1 2 7 oct: 3 4 2
```

# **Character Coding Conventions**

- Would be hard for people to share words if they interpretted bits as letters differently
- ► **ASCII**: American Standard Code for Information Interchange An old standard for bit/character correspondence
- ▶ 7 bits per character, includes upper, lower case, punctuation

Dec	Hex	Binary	Char	Dec	Hex	Binary	Char
65	41	01000001	Α	78	4E	01001110	N
66	42	01000010	В	79	4F	01001111	0
67	43	01000011	C	80	50	01010000	Р
68	44	01000100	D	81	51	01010001	Q
69	45	01000101	E	82	52	01010010	R
70	46	01000110	F	83	53	01010011	S
71	47	01000111	G	84	54	01010100	Т
72	48	01001000	Н	85	55	01010101	U
73	49	01001001	1	86	56	01010110	V
74	4A	01001010	J	87	57	01010111	W
75	4B	01001011	K	88	58	01011000	Χ
76	4C	01001100	L	89	59	01011001	Υ
77	4D	01001101	M	90	5A	01011010	Z
91	5B	01011101	[	97	61	01100001	а
92	5C	01011110	\	98	62	01100010	b

## Exercise: Characters vs Numbers

#### Explain the following program and its output

```
1 // char ints.c:
2 #include <stdio.h>
                                                    >> ./a.out
3 #include <string.h>
  int main(){
                                                    Hello World!
 5
                                                      0] H 72 48
    char nums[64] = {
                                                      1] e 101 65
7
      72, 101, 108, 108, 111, 32,
                                                      2] 1 108 6C
8
       87, 111, 114, 108, 100, 33,
                                                      3] 1 108 6C
9
       0
                                                      4] o 111 6F
10
    };
                                                            32 20
   printf("%s\n",nums);
11
                                                      61 W 87 57
     len = strlen(nums);
12
                                                      7] o 111 6F
13
     for(int i=0; i<len; i++){</pre>
                                                      87 r 114 72
      printf("[%2d] %c %3d %02X\n",
14
                                                         1 108 6C
              i,nums[i],nums[i],nums[i]);
15
                                                    [10] d 100 64
     }
16
                                                    [11] ! 33 21
17
     return 0;
18 }
```

#### **Answers**: Characters vs Numbers

## The Whole Array

```
char nums[64] = {
  72, 101, 108, 108, 111, 32,
  87, 111, 114, 108, 100, 33,
  0
};
```

Lays out a bit pattern at each spot the array; bit pattern is specified with decimal numbers

```
printf("%s\n",nums);
```

Print the array as though it were "string": an array of characters that is null terminated

## Elements of the Array

Print a single element of the array as

- %c : a character (ASCII table lookup for the glyph to draw)
- %3d : a decimal number (padding to width 3)
- %02X: as a hexadecimal number (with leading 0's if needed and padded with width 2)

#### Unicode

▶ World: Why can't I write 컴퓨터

in my code/web address/email?

- America: ASCII has 128 chars. Deal with it.
- World: Seriously?
- America: We invented computers. 'Merica!



- ► World:
- ► America: · · · Unicode?
- ► World: But my language takes more bytes than American.
- America: Deal with it. 'Merica!

- ► ASCII Uses 7 bits per char, limited to 128 characters
- ► UTF-8 uses **1-4 bytes per character** to represent **many**more characters
  (1,112,064 *codepoints*)
- Uses 8th bit in a byte to indicate extension to more than a single byte
- Requires software to understand coding convention allowing broader language support
- ASCII is a proper subset of UTF-8 making UTF-8 backwards compatible and wildly popular

# Binary Integer Addition/Subtraction

Adding/subtracting in binary works the same as with decimal EXCEPT that carries occur on values of 2 rather than 10

```
ADDITION #1
                         SUBTRACTION #1
   1 11 <-carries
                                  ? <-carries
  0100 \ 1010 = 74
                          0111 \ 1001 = 121
+ 0101 1001 = 89
                         - 0001 0011 = 19
   1010\ 0011 = 163
                         VVVVVVVVVVVVV
                           VVVVVVVVVVVVV
ADDITION #2
                           VVVVVVVVVVVVV
   1111 1 <-carries
                                x12 <-carries
  0110 \ 1101 = 109
                         0111 \ 0001 = 119
                          -00010011 = 19
+ 0111 1001 = 121
  1110\ 0110 = 230
                           0110 \ 0110 = 102
```

# Two's Complement Integers: Representing Negative Values

- ► To represent negative integers, must choose a coding system
- ▶ Two's complement is the most common for this
- Alternatives exist
  - ► Signed magnitude: leading bit indicates pos (0) or neg (1)
  - One's complement: invert bits to go between positive negative
- Great advantage of two's complement: signed and unsigned arithmetic are identical
- ► Hardware folks only need to make one set of units for both unsigned and signed arithmetic

# Summary of Two's Complement

Short explanation: most significant bit is associated with a negative power of two.

```
UNSIGNED BINARY
                     TWO's COMPLEMENT (signed)
7654 3210 : position 7654 3210 : position
ABCD EFGH: 8 bits ABCD EFGH: 8-bits
A: 0/1 * +(2^7) *POS* A: 0/1 * -(2^7) *NEG*
B: 0/1 * + (2^6) B: 0/1 * + (2^6)
C: 0/1 * + (2^5) C: 0/1 * + (2^5)
H: 0/1 * +(2^0) H: 0/1 * +(2^0)
UNSIGNED BINARY
                    TWO's COMPLEMENT (signed)
7654 3210 : position 7654 3210 : position
1000\ 0000 = +128 1000\ 0000 = -128
1000\ 0001 = +129 1000\ 0001 = -127 = -128+1
1000\ 0011 = +131 1000\ 0011 = -125 = -128+1+2
1111 1111 = +255
                    1111 \ 1111 = -1 = -128 + 1 + 2 + 4 + \ldots + 64
0000 \ 0000 = 0 0000 \ 0000 = 0 0000 \ 127
0000 0001 = +1 0000 0001 = +1
0000 0101 = +5 0000 0101 = +5
0111 \ 1111 = +127
                    0111 \ 1111 = +127
```

# Two's Complement Notes

- Leading 1 indicates negative, 0 indicates positive
- ► All 0's = Zero
- Positive numbers are identical to unsigned

#### Conversion Trick

Positive → Negative

► Invert bits, Add 1

Negative → Positive

► Invert bits, Add 1

Same trick works both ways, implemented in hardware for the **unary minus** operator as in

int 
$$y = -x$$
;

```
~ 0110 1000 +104 : negate
  1001 0111
             inverted
  1001 \ 1000 = -104
\sim 1001 1000 = -104 : negate
  0110 \ 0111 = +103 \ inverted
  0110\ 1000 = +104
Add Pos/Neg should give 0
           <-carries
  0110\ 1000 = +104
+ 1001 1000 = -104
x 0000 0000 = zero
```

#### Overflow

- Sums that exceed the representation of the bits associated with the integral type overflow
- Excess significant bits are dropped
- Addition can result in a sum smaller than the summands, even for two positive numbers (!?)
- Integer arithmetic in fixed bits is a mathematical ring

## Examples of Overflow in 8 bits

```
ADDITION #3 OVERFLOW
1 1111 111 <-carries
1111 1111 = 255
1010 1001 = 169
+ 0000 0001 = 1
1 0000 0000 = 256
x drop 9th bit

0000 0000 = 0
1 0110 1010 = 362
x drop 9th bit

0110 1010 = 106
```

## **Underflow**

- Underflow occurs in unsigned arithmetic when values go below 0 (no longer positive)
- Pretend that there is an extra significant bit to carry out subtraction
- Subtracting a positive integer from a positive integer may result in a larger positive integer (?!?)
- Integer arithmetic in fixed bits is a mathematical ring

## Examples of 8-bit Underflow

```
SUBTRACTITON #2 UNDERFLOW
           ?<-carries
   0000 0000 =
- 0000 0001 =
           ?<-carries
1\ 0000\ 0000 = 256\ (pretend)
- 0000 0001 =
VVVVVVVVVVVVVV
           2<-carries
  1111 1110 = 256
 - 0000 0001 =
```

# Overflow and Underflow In C Programs

- See over\_under\_flow.c for demonstrations in a C program.
- ▶ No runtime errors for under/overflow
- Good for hashing and cryptography
- ▶ Bad for most other applications: system critical operations should use checks for over-/under-flow
- See textbook Ariane Rocket Crash which was due to overflow of an integer converted from a floating point value
- At the assembly level, there are condition codes indicating that overflow has occurred but there is not a universal method to check for this in C<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>Many compilers like GCC can generate assembly instructions that will detect overflow and abort programs. See the demo program overflow\_detect.c and GCCs -ftrapv option.

## Endinaness: Byte ordering in Memory

- Single bytes like ASCII characters lay out sequentially in memory in increasing address
- Multi-byte entities like 4-byte ints require decisions on byte ordering
- ▶ We think of a 32-bit int like this

Decimal: 6377

- But need to assign memory addresses to each byte
  - Little Endian: least significant byte early
  - Big Endian: most significant byte early
- Example: Integer starts at address #1024

#### Address

LittleEnd:	#1027	7	#1026	3	#102	5	#1024	1
Binary:	0000	0000	0000	0000	0001	1000	1110	1001
	0	0	0	0	1	8	E	9
BigEnd:	#1024	1	#1025	5	#1026	3	#1027	7
	Addre	ess						

## Little Endian vs. Big Endian

- Most modern machines use little endian by default
- Processor may actually support big endian
- Both Big and Little Endian have engineering trade-offs
- At one time debated hotly among hardware folks: a la Gulliver's Travels conflicts
- ► Intel Chips were little endian and have dominated computing for several decades, set the precedent for modern platforms
- Big endian byte order shows up in network programming: sending bytes over the network is done in big endian ordering
- ► Examine show\_endianness.c : uses C code to print bytes in order, reveals whether a machine is Little or Big Endian

# Output of show\_endianness.c

```
1 > cat show endianness.c
 2 // Show endiannes layout of a binary number in memory. Most machines
 3 // are little endian so bytes will print leas signficant earlier.
4 #include <stdio.h>
 5
   int main(){
     int bin = 0b000000000000000001100011101001;
                                                     // 6377
 8
    //
10
    printf("%d\n%x\n".bin.bin):
                                                  // show decimal/hex of binary
    unsigned char *ptr = (unsigned char *) &bin; // pointer to beginning of bin
11
    for(int i=0; i<4; i++){
12
                                                  // print bytes of bin from low to high
       printf("%hhx ", ptr[i]);
                                                  // memory address
13
14
                                                  // '%hhx' : 1-byte char in hex
                                                  // '%hx' : 2-byte short in hex
15
     printf("\n");
     return 0:
                                                  // '%x' : 4-bvte int in hex
16
17 }
18
   > gcc show endianness.c
20 > ./a.out
21 6377
22 18e9
23 e9 18 0 0
```

**Notice:** num prints with value 18e9 but bytes appear in reverse order e9 18 when looking at memory

# Integer Ops and Speed

- Along with Addition and Subtraction, Multiplication and Division can also be done in binary
- Algorithms are the same as base 10 but more painful to do by hand
- This pain is reflected in hardware speed of these operations
- The Arithmetic and Logic Unit (ALU) does integer ops in the machine
- ➤ A **clock** ticks in the machine at some rate like 3Ghz (3 billion times per second)

 Under ideal circumstances, typical ALU Op speeds are

Operation	Cycles
Addition	1
Logical	1
Shifts	1
Subtraction	1
Multiplication	3
Division	>30

- Due to disparity, it is worth knowing about relation between multiply/divide and bitwise operations
- ► Compiler often uses such tricks: shift rather than multiply/divide

## Mangling Bits Puts Muscle on Your Bones

Below illustrates difference between logical and bitwise operations.

- Bitwise ops evaluate on a per-bit level
- ▶ 32 bits for int, 4 bits shown

Bitwise OR	Bitwise AND	Bitwise XOR	Bitwise NOT
1100 = 12	1100 = 12	1100 = 12	
1010 = 10	& 1010 = 10	^ 1010 = 10	~ 1100 = 12
1110 = 14	1000 = 8	0110 = 6	0011 = 3

#### Bitwise Shifts

- ▶ **Shift** operations move bits within a field of bits
- Shift operations are

```
x = y \ll k; // left shift y by k bits, store in x x = y \gg k; // right shift y by k bits, store in x
```

- ▶ All integral types can use shifts: long, int, short, char
- Not applicable to pointers or floating point
- Examples in 8 bits

```
// 76543210

char x = 0b00010111; // 23

char y = x << 2; // left shift by 2

// y = 0b01011100; // 92

// x = 0b00010111; // not changed

char z = x >> 3; // right shift by 3

// z = 0b00000010; // 2

// x = 0b00010111; // not changed

char n = 0b10000000; // -128, signed

char s = n >> 4; // right shift by 4

// s = 0b11111000; // -8, sign extension

// right shift >> is "arithmetic"
```

## Shifty Arithmetic Tricks

- Shifts with add/subtract can be used instead of multiplication and division
- ► Turn on optimization: gcc -03 code.c
- ► Compiler automatically does this if it thinks it will save cycles
- Sometimes programmers should do this but better to convince compiler to do it for you, comment if doing manually

## Multiplication

# // 76543210 char x = 0b00001010; // 10 char x2 = x << 1; // 10\*2 // x2 = 0b00010100; // 20 char x4 = x << 2; // 10\*4 // x4 = 0b00101000; // 40 char x7 = (x << 3)-x; // 10\*7 // x7 = (x \* 8)-x; // 10\*7 // x7 = 0b01000110; // 70 // 76543210

#### Division

```
// 76543210

char y = 0b01101110; // 110

char y2 = y >> 1; // 110/2

// y2 = 0b00110111; // 55

char y4 = y >> 2; // 110/4

// y4 = 0b00011011; // 27

char z = 0b10101100; // -84

char z2 = z >> 2; // -84/4

// z2 = 0b11101011; // -21

// right shift sign extension
```

## Exercise: Checking / Setting Bits

Use a combination of bit shift / bitwise logic operations to…

- 1. Check if bit i of int x is set (has value 1)
- 2. Clear bit i (set bit at index i to value 0)

```
Show C code for this
{
   int x = ...;
   int i = ...;
   if( ??? ) { // ith bit of x is set
      printf("set!\n");
   }

   i = ...;
   ???;
   printf("ith bith of x now cleared to 0\n");
}
```

# **Answers:** Checking / Setting Bits

1. Check if bit i of int x is set (has value 1)

2. Clear bit i (set bit at index i to value 0)

```
int x = ...;
int mask = 1; // or 0b0001 or 0x01 ...
int shifted = mask << i; // shifted 0b00...010..00
int inverted = ~shifted; // inverted 0b11...101..11
x = x & inverted; // x & 0b10...010..01
... // 0b10...000..01</pre>
```

# Showing Bits

printf() capabilities:

```
%d as Decimal
%x as Hexadecimal
%o as Octal
%c as Character
```

- ► No specifier for binary
- Can construct such with bitwise operations
- Code pack contains two codes to do this
  - printbits.c: single args printed as 32 bits
  - showbits.c: multiple args printed in binary, hex, decimal

- Showing bits usually involves shifting and bitwise AND &
- Example from showbits.c

```
#define INT BITS 32
// print bits for x to screen
void showbits(int x){
  for(int i=INT_BITS-1; i>=0; i--){
    int mask = 1 << i;</pre>
    if(mask & x){
      printf("1");
    } else {
      printf("0");
```

## Bit Masking

- Semi-common for functions to accept bit patterns which indicate true/false options
- Frequently makes use of bit masks which are constants associated with specific bits
- Example from earlier: Unix permissions might be…

```
#define S_IRUSR 0b100000000 // User Read
#define S_IWUSR 0b010000000 // User Write
#define S_IXUSR 0b001000000 // User Execute
#define S_IRGRP 0b000100000 // Group Read
...
#define S_IWOTH 0b000000010 // Others Write
#define S_IXOTH 0b000000001 // Others Execute
```

Use them to create options to C functions like
int permissions = S\_IRUSR|S\_IWUSR|S\_RGRP;
chmod("/home/kauffman/solution.zip",permissions);

## Unix Permissions with Octal

- Octal arises associated with Unix file permissions
- ▶ Every file has 3 permissions for 3 entities
- Permissions are true/false so a single bit will suffice
- ▶ 1s -1: long list files, shows permissions
- chmod 665 somefile.txt: change permissions of somefile.txt to those shown to the right
- chmod 777 x.txt: read /
  write / exec for everyone
- chmod also honors letter versions like r and w
- chmod u+x script.sh #
  make file executable

Readable chmod version: chmod u=rw,g=rw,o=rx somefile.txt