CMSC216: Binary, Integers, Arithmetic

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Logistics

Reading

Finish up C References

Bryant/O'Hallaron Ch 2.1-2.3

- Number Systems
- Binary Encoding of Data
- Signed/Unsigned Integers
- Character Data

After Exam 1, Ch 2.4 (Floats)

Goals

- Wrap C discussion
- Integers/characters in binary
- Arithmetic operations, Negative numbers in binary

Assignments

- Lab04 / HW04: No New material, practice exercises for Project 1 and Exam 1
- Project 1: Due Mon 24-Feb-2025
- Exam Review: Tue 25-Feb
- Exam 1: Thu 27-Feb

Announcements

Office Hours with TAs: Whiteboard Queue

When you enter AVW 4166 (TA room) and see folks waiting for help **add yourself to the whiteboard under Kauffman OH Queue**; TAs will help students by order of arrival; make sure you talk to a TA from Kauffman sections (1xx/2xx) as Prof. Herman's TAs (3xx/4xx) know less about P1 than you do

Block Commenting

Learn how your editor can comment/uncomment blocks of code quickly

- VS Code: highlight some code via Mouse, press Ctrl-/ OR Cmd-/ to comment it; press again to uncomment
- VIM / Emacs: been doing this since 1980, RTFM
- Others: If your editor can't do this, find a better editor

Exam 1 Logistics

Practice + Review

- Practice Exam 1A will be posted Mon 24-Feb-2025
- Practice Exam 1B and Review in class Tue 25-Feb-2025
- Solutions to practice exam will be posted for students

Exam 1

- In-person in class on Thu 27-Sep-2025
- Exam runs lecture period: 75min
- Expect 2.5 pages front/back
- **Open Resource Exam**: review rules for this posted at bottom of course schedule (beneath slides)
  Questions on Open Resource Exam boundaries?

Unsigned Integers: Decimal and Binary

- Unsigned integers are always positive:

  `unsigned int i = 12345;`
- To understand binary, recall how decimal numbers "work"

Decimal: Base 10 Example

Each digit adds on a power 10

$80,345 =$	$5 \times 10^0 +$	5 ones
	$4 \times 10^1 +$	40 tens
	$3 \times 10^2 +$	300 hundreds
	$0 \times 10^3 +$	0 thousands
	8×10^4	80 tens of thousands
$5 + 40 + 300 + 80,000$		

Binary: Base 2 Example

Each digit adds on a power 2

$11001_2 =$	$1 \times 2^0 +$	1 ones
	$0 \times 2^1 +$	0 twos
	$0 \times 2^2 +$	0 fours
	$1 \times 2^3 +$	8 eights
	$1 \times 2^4 +$	16 sixteens
	$= 1 + 8 + 16$	$= 25$

So, $11001_2 = 25_{10}$

Exercise: Convert Binary to Decimal

Base 2 Example:

$$
\begin{aligned}
11001 = & 1 \times 2^0 + & 1 \\
& 0 \times 2^1 + & 0 \\
& 0 \times 2^2 + & 0 \\
& 1 \times 2^3 + & 8 \\
& 1 \times 2^4 + & 16 \\
= & 1 + 8 + 16 & = 25
\end{aligned}
$$

So, $11001_2 = 25_{10}$

Try With a Neighbor

Convert the following two numbers from base 2 (binary) to base 10 (decimal)

- 111
- 11010
- 01100001

Answers: Convert Binary to Decimal

$$
\begin{aligned}
111_2 =& 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\
=& 1 \times 4 + 1 \times 2 + 1 \times 1 \\
=& 7_{10} \\
11010_2 =& 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\
=& 1 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 0 \times 1 \\
=& 26_{10} \\
01100001_2 =& 0 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 \\
& + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\
=& 0 \times 128 + \times 64 + 1 \times 32 + 0 \times 16 \\
& + 0 \times 8 + 0 \times 4 + 0 \times 2 + 1 \times 1 \\
=& 97_{10}
\end{aligned}
$$

Note: last example ignores leading 0's

The Other Direction: Decimal to Binary

Converting a number from base 10 to base 2 is easily done using repeated division by 2; keep track of **remainders**

Convert 124 to base 2:

$$
\begin{aligned}
124 \div 2 = 62 &\quad \text{rem } 0 \\
62 \div 2 = 31 &\quad \text{rem } 0 \\
31 \div 2 = 15 &\quad \text{rem } 1 \\
15 \div 2 = 7 &\quad \text{rem } 1 \\
7 \div 2 = 3 &\quad \text{rem } 1 \\
3 \div 2 = 1 &\quad \text{rem } 1 \\
1 \div 2 = 0 &\quad \text{rem } 1
\end{aligned}
$$

- Last step got 0 quotient so we're done.
- Binary digits are in **remainders in reverse**
- Answer: `1111100`
- Check:
  $0 + 0 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 = 4 + 8 + 16 + 32 + 64 = 124$

Decimal, Hexadecimal, Octal, Binary Notation

- Numbers exist independent of any writing system
- Can write the same number in a variety of bases
- C provides syntax for most common bases used in computing

	Decimal	Binary	Hex	Octal
Base	10	2	16	8
Mathematical	125	1111101_2	$7D_{16}$	175_8
C Prefix	None	`0b...`	`0x..`	`0...`
C Example	125	`0b1111101`	`0x7D`	`0175`
`printf()`	`"%d"`	N/A	`"%x"`	`"%o"`

- **Hexadecimal** often used to express long-ish byte sequences
  Larger than base 10 so for 10-15 uses letters A-F
- **Examine** `number_writing.c` and `table.c` for patterns
- **Expectation**: Gain familiarity with doing conversions between bases as it will be useful in practice

Hexadecimal: Base 16

- Hex: compact way to write bit sequences
- One byte is 8 bits
- Each Hex character represents 4 bits
- **Each Byte is 2 Hex Digits**

```
|-----------+----------------+-----|
| Byte      | Hex            | Dec |
|-----------+----------------+-----|
| 0101 0111 | 57 = 5*16 + 7  |  87 |
| 5    7    |                |     |
| 0011 1100 | 3C = 3*16 + 12 |  60 |
| 3    C=12 |                |     |
| 1110 0010 | E2 = 14*16 + 2 | 226 |
| E=14 2    |                |     |
|-----------+----------------+-----|
```

Hex to 4 bit equivalence

Dec	Bits	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Exercise: Conversion Tricks for Hex and Octal

Examples shown in this week's HW, What tricks are illustrated?

```
|---------+-------------+-----------------+----------------------|
| Decimal | Byte = 8bits | Byte by 4      | Hexadecimal          |
|---------+-------------+-----------------+----------------------|
|      87 |    01010111 | bin: 0101 0111  | 57 = 5*16 + 7        |
|         |             | hex: 5    7     | hex  dec             |
|         |             |                 |                      |
|      60 |    00111100 | bin: 0011 1100  | 3C = 3*16 + 12       |
|         |             | hex: 3    C=12  | hex  dec             |
|         |             |                 |                      |
|     226 |    11100010 | bin: 1110 0010  | E2 = 14*16 + 2       |
|         |             | hex: E=14 2     | hex  dec             |
|---------+-------------+-----------------+----------------------|
| Decimal | Byte = 8bits | Byte by 3      | Octal                |
|---------+-------------+-----------------+----------------------|
|      87 |    01010111 | bin: 01 010 111 | 127 = 1*8^2 + 2*8 + 7 |
|         |             | oct: 1  2   7   | oct   dec            |
|         |             |                 |                      |
|      60 |    00111100 | bin: 00 111 100 | 074 = 0*8^2 + 7*8 + 4 |
|         |             | oct: 0  7   4   | oct   dec            |
|         |             |                 |                      |
|     226 |    11100010 | bin: 11 100 010 | 342 = 3*8^2 + 4*8 + 2 |
|         |             | oct: 3  4   2   | oct   dec            |
|---------+-------------+-----------------+----------------------|
```

Answers: Conversion Tricks for Hex and Octal

- Converting between Binary and Hexadecimal is easiest when grouping bits by 4: each 4 bits corresponds to one hexadecimal digit

```
bin: 0101 0111    bin: 1110 0010
hex: 5    7       hex: E=14 2
```

- Converting between Binary and Octal is easiest when grouping bits by 3: each 3 bits corresponds to one octal digit

```
bin: 01 010 111  bin: 11 100 010
oct: 1  2   7    oct: 3  4   2
```

Character Coding Conventions

- Would be hard for people to share words if they interpretted bits as letters differently
- **ASCII**: American Standard Code for Information Interchange
  An old standard for bit/character correspondence
- 7 bits per character, includes upper, lower case, punctuation

Dec	Hex	Binary	Char	Dec	Hex	Binary	Char
65	41	01000001	A	78	4E	01001110	N
66	42	01000010	B	79	4F	01001111	O
67	43	01000011	C	80	50	01010000	P
68	44	01000100	D	81	51	01010001	Q
69	45	01000101	E	82	52	01010010	R
70	46	01000110	F	83	53	01010011	S
71	47	01000111	G	84	54	01010100	T
72	48	01001000	H	85	55	01010101	U
73	49	01001001	I	86	56	01010110	V
74	4A	01001010	J	87	57	01010111	W
75	4B	01001011	K	88	58	01011000	X
76	4C	01001100	L	89	59	01011001	Y
77	4D	01001101	M	90	5A	01011010	Z
91	5B	01011101	[	97	61	01100001	a
92	5C	01011110	\	98	62	01100010	b

Partial Table of ASCII Codes / Values, try `man 7 ascii` in a terminal for a full table

Exercise: Characters vs Numbers

Explain the following program and its output

```c
// char_ints.c:
#include <stdio.h>
#include <string.h>
int main(){
  ...
  char nums[64] = {
    72, 101, 108, 108, 111, 32,
    87, 111, 114, 108, 100, 33,
    0
  };
  printf("%s\n",nums);
  len = strlen(nums);
  for(int i=0; i<len; i++){
    printf("[%2d] %c %3d %02X\n",
           i,nums[i],nums[i],nums[i]);
  }
  return 0;
}
```

```
>> gcc char_ints.c
>> ./a.out
...
Hello World!
[ 0] H  72 48
[ 1] e 101 65
[ 2] l 108 6C
[ 3] l 108 6C
[ 4] o 111 6F
[ 5]    32 20
[ 6] W  87 57
[ 7] o 111 6F
[ 8] r 114 72
[ 9] l 108 6C
[10] d 100 64
[11] !  33 21
```

Answers: Characters vs Numbers

The Whole Array

```
char nums[64] = {
  72, 101, 108, 108, 111, 32,
  87, 111, 114, 108, 100, 33,
  0
};
```

Lays out a bit pattern at each spot the array; bit pattern is specified with decimal numbers

```
printf("%s\n",nums);
```

Print the array as though it were "string": an array of characters that is null terminated

Elements of the Array

```
printf("[%2d] %c %3d %02X\n",
       i,nums[i],nums[i],nums[i]);
```

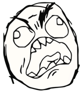
Print a single element of the array as

- `%c` : a character (ASCII table lookup for the glyph to draw)
- `%3d` : a decimal number (padding to width 3)
- `%02X` : as a hexadecimal number (with leading 0's if needed and padded with width 2 - *noice!*)

Unicode

- World: Why can't I write

  컴퓨터

  in my code/web address/email?
- America: ASCII has 128 chars. Deal with it.
- World: Seriously?
- America: We invented computers. 'Merica!
- World: 
- America: ··· Unicode?
- World: But my language takes more bytes than American.
- America: Deal with it. 'Merica!

- ASCII Uses 7 bits per char, limited to 128 characters
- UTF-8 uses **1-4 bytes per character** to represent **many** more characters (1,112,064 *codepoints*)
- Uses 8th bit in a byte to indicate extension to more than a single byte
- Requires software to understand coding convention allowing broader language support
- ASCII is a proper subset of UTF-8 making UTF-8 backwards compatible and wildly popular

Binary Integer Addition/Subtraction

Adding/subtracting in binary works the same as with decimal EXCEPT that carries occur on values of 2 rather than 10

```
ADDITION #1                 SUBTRACTION #1
   1 11     <-carries                 ? <-carries
   0100 1010 = 74              0111 1001 = 121
 + 0101 1001 = 89            - 0001 0011 =  19
------------                ------------
   1010 0011 = 163             VVVVVVVVVVVVV
                               VVVVVVVVVVVVV
ADDITION #2                    VVVVVVVVVVVVV
   1111   1 <-carries               x12 <-carries
   0110 1101 = 109             0111 0001 = 119
 + 0111 1001 = 121           - 0001 0011 =  19
------------                ------------
   1110 0110 = 230             0110 0110 = 102
```

When 0/1 is represented as Low/High Voltage, one can design digital circuits that implement arithmetic

Two's Complement Integers: Representing Negative Values

- To represent negative integers, must choose a **different** coding system than for positive-only integers
- The **Two's Complement Encoding** is the most common coding system for signed numbers so we will study it
- Alternatives exist
  - Signed magnitude: leading bit indicates pos (0) or neg (1)
  - One's complement: invert bits to go between positive negative
- Great advantage of two's complement: **signed and unsigned arithmetic are identical**
- Hardware folks only need to make one set of units for both unsigned and signed arithmetic

Summary of Two's Complement

TL;DR: *Most significant bit is a negative power of two.*

```
UNSIGNED BINARY          TWO's COMPLEMENT (signed)
---------------          -------------------------
7654 3210 : position     7654 3210 : position
ABCD EFGH : 8 bits       ABCD EFGH : 8-bits
A: 0/1 * +(2^7) *POS*    A: 0/1 * -(2^7) *NEG*
B: 0/1 * +(2^6)          B: 0/1 * +(2^6)
C: 0/1 * +(2^5)          C: 0/1 * +(2^5)
...                      ...
H: 0/1 * +(2^0)          H: 0/1 * +(2^0)

UNSIGNED BINARY          TWO's COMPLEMENT (signed)
---------                ---------
7654 3210 : position     7654 3210 : position
1000 0000 = +128         1000 0000 = -128
1000 0001 = +129         1000 0001 = -127 = -128+1
1000 0011 = +131         1000 0011 = -125 = -128+1+2
1111 1111 = +255         1111 1111 = -1   = -128+1+2+4+..+64
0000 0000 = 0            0000 0000 =  0         [   +127    ]
0000 0001 = +1           0000 0001 = +1
0000 0101 = +5           0000 0101 = +5
0111 1111 = +127         0111 1111 = +127
```

Two's Complement Notes

Unsigned/Signed Equivalents

Unsigned	`1000 0110 = 134`
Signed	`1000 0110 = -121`
	`= 134 - 256`
Unsigned	`1111 0001 = 241`
Signed	`1111 0001 = -15`
	`=241-256`
Unsigned	`0011 0011 = 51`
Signed	`0011 0011 = 51`

When/Why `X-256`?

- Leading (leftmost) bit is 1
- Counted as 128 in Unsigned
- Counts as -128 in Signed
- Take -256 to compensate

Negation in Two's Complement

```
int y = -x;
```

- Unary Minus operator
- **Invert bits, Add 1**
- Works for both Pos→Neg and Neg→Pos

```
~ 0110 1000  +104 : negate
-----------
  1001 0111  inverted
+         1
-----------
  1001 1000 = -104

~ 1001 1000 = -104 : negate
-----------
  0110 0111 = +103 inverted
+         1
-----------
  0110 1000 = +104
```

Exercise: Two's Complement Conversions

	Bits		Hex		Decimal Unsigned		Decimal Signed
	1111 1111	A:	____	B:	___	C:	___
	1001 0110		0x96	D:	___	E:	___
F:	____ ____		0x3E	G:	___	H:	___
	0010 0011	I:	____		35	J:	___
K:	____ ____	L:	____	M:	___		-35

Answers: Two's Complement Conversions

	Bits		Hex	Decimal Unsigned	Decimal Signed
	1111 1111	A:	0xFF	B: 255	C: -1
	1001 0110		0x96	D: 150	E: -106
F:	0011 1110		0x3E	G: 62	H: 62
	0010 0011	I:	0x23	35	J: 35
K:	1101 1101	L:	0xBB	M: 221	-35

Converting 35 to −35 can be done via +255 OR via Invert Bits + 1

Overflow

- Sums that exceed the representation of the bits associated with the integral type **overflow**
- Excess significant bits are **dropped**
- Addition can result in a sum smaller than the summands, even for two positive numbers (!?)
- Integer arithmetic in fixed bits is a mathematical **ring**

Examples of Overflow in 8 bits

```
ADDITION #3 OVERFLOW
 1 1111 111 <-carries
   1111 1111 = 255
 + 0000 0001 =   1
------------
 1 0000 0000 = 256
 x drop 9th bit
------------
   0000 0000 = 0
```

```
ADDITION #4 OVERFLOW
 1         1 <-carries
   1010 1001 = 169
 + 1100 0001 = 193
------------
 1 0110 1010 = 362
 x drop 9th bit
------------
   0110 1010 = 106
```

Underflow

- **Underflow** occurs in unsigned arithmetic when values go below 0 (no longer positive)
- Pretend that there is an extra significant bit to carry out subtraction
- Subtracting a positive integer from a positive integer may result in a **larger** positive integer (?!?)
- Integer arithmetic in fixed bits is a mathematical **ring**

Examples of 8-bit Underflow

```
SUBTRACTIION #2 UNDERFLOW
          ?<-carries
  0000 0000 =   0
- 0000 0001 =   1
------------
vvvvvvvvvvvvv
          ?<-carries
1 0000 0000 = 256 (pretend)
- 0000 0001 =   1
------------
vvvvvvvvvvvvv
x         2<-carries
0 1111 1110 = 256
- 0000 0001 =   1
------------
  1111 1111 = 255
```

Overflow and Underflow In C Programs

- See `over_under_flow.c` for demonstrations in a C program.
- **No runtime errors** for under/overflow
- Good for hashing and cryptography
- Bad for most other applications: system critical operations should use checks for overflow / underflow
- Textbook mentions the Ariane Rocket Crash which was due to overflow of an integer converted from a floating point value

  > *The Ariane explosion is an instructive example for several reasons. (1) Software re-use caused the problem subverting the usual wisdom of relying on tested software; hardware changes ALWAYS trump software. (2) Sometimes computer science IS rocket science*

- Assembly provides condition codes indicating when overflow occurs but checking in C is tricky and painful[1]

---

[1] Many compilers like GCC can generate assembly instructions that will detect overflow and abort programs. See the demo program `overflow_detect.c` and GCCs `-ftrapv` option.

Interlude: Brief Introduction to GDB, The GNU Debugger

- P2 will include a "debugging problem" called `puzzlebox`
- Easiest to solve this problem using GDB (or some other debugger)
- You may benefit from using GDB to complete P1 as well
- **Debuggers allow one to stop time in a program**, inspect variables, pause execution at certain points and skip forwards
- If you've added tons of `printf()`'s to your code and still can't figure out what's going on, a Debugger is your next option
- Basic mechanics demonstrated by solving first phase of the upcoming `puzzlebox`
- Associated Reading: 2021 Quick Guide to GDB

Endinaness: Byte ordering in Memory

- Single bytes like ASCII characters lay out sequentially in memory in increasing address
- Multi-byte entities like 4-byte `ints` require decisions on byte ordering
- We think of a 32-bit int like this

```
          Most Signifcant <------> Least Significant
Binary:   0000 0000  0000 0000  0001 1000  1110 1001
          0    0     0    0     1    8     E    9
Hex   :   000018E9
Decimal:  6377
```

- There are 2 Options to for ordering multi-byte data in memory
  - **Little Endian**: Least Significant byte at low address
  - **Big Endian**: Most Significant Byte at low address
- Example: Integer starts at address #1024

```
           Address
LittleEnd: #1027      #1026      #1025      #1024
Binary:    0000 0000  0000 0000  0001 1000  1110 1001
           0    0     0    0     1    8     E    9
BigEnd:    #1024      #1025      #1026      #1027
           Address
```

Little Endian vs. Big Endian

- Most modern machines use **Little Endian** ordering by default
- Some processor (ARM) support both Little / Big Endian BUT and one is chosen at startup and used until turned off
- Both Big and Little Endian have (minor) engineering trade-offs
- At one time debated hotly among hardware folks: a la Gulliver's Travels conflicts
- **Intel Chips** were little endian and have dominated computing for several decades, set the precedent for modern platforms
- Big endian byte order shows up in **network programming**: sending bytes over the network is done in big endian ordering
- **Examine** `show_endianness.c` : uses C code to print bytes in order, reveals whether a machine is Little or Big Endian

Output of `show_endianness.c`

```c
// show_endianness.c: Shows endiannes layout of a binary number in
// memory. Intel machines and some ARM machines (Apple M1) are little
// endian so bytes will print least signficant earlier.
#include <stdio.h>

int main(){
  int bin = 0b00000000000000000001100011101001;    // 6377
  //           |   |   |   |   |   |   |   |
  //           0   0   0   0   1   8   e   9
  printf("%d\n%08x\n",bin,bin);                    // show decimal and hex representation of bi
  char *ptr = (char *) &bin;                       // pointer to beginning of bin
  for(int i=0; i<4; i++){                          // print bytes of bin from low to high
    printf("%hhx ", ptr[i]);                       // memory address
  }                                                // '%hhx' : 1-byte char in hex
  printf("\n");                                    // '%hx'  : 2-byte short in hex
  return 0;                                        // '%x'   : 4-byte int in hex
}
```

```
>> gcc show_endianness.c
>> ./a.out
6377
000018e9
e9 18 0 0
```

Notice: `num` prints with value `18e9` but bytes appear in reverse order `e9 18` when run on a Little Endian machine: the "littlest" byte appears earliest in memory

Integer Ops and Speed

- Along with Addition and Subtraction, **Multiplication and Division** can also be done in binary
- Algorithms are the same as base 10 but more painful to do by hand
- This pain is reflected in hardware speed of these operations
- The **Arithmetic and Logic Unit (ALU)** does integer ops in the machine
- A **clock** ticks in the machine at some rate like 3Ghz (3 billion times per second)
- Under ideal circumstances, typical ALU Op speeds are

Operation	Cycles
Addition	1
Logical	1
Shifts	1
Subtraction	1
Multiplication	3
Division	>30

- Due to disparity, it is worth knowing about **relation** between multiply/divide and **bitwise** operations
- Compiler often uses such tricks: shift rather than multiply/divide

Mangling Bits Puts Muscle on Your Bones

Below illustrates difference between logical and bitwise operations.

```c
int xl = 12 || 10; // truthy (Logical OR)
int xb = 12 |  10; // 14     (Bitwise OR)
int yl = 12 && 10; // truthy (Logical AND)
int yb = 12 &  10; // 8      (Bitwise AND)
int zb = 12 ^  10; // 6      (Bitwise XOR)
int wl = !12;      // falsey (Logical NOT)
int wb = ~12;      // 3      (Bitwise NOT/INVERT)
```

- Bitwise ops evaluate on a per-bit level
- 32 bits for `int`, 4 bits shown

```
Bitwise OR     Bitwise AND    Bitwise XOR    Bitwise NOT
  1100 = 12      1100 = 12      1100 = 12
| 1010 = 10    & 1010 = 10    ^ 1010 = 10    ~ 1100 = 12
-----------    -----------    -----------    -----------
  1110 = 14      1000 =  8      0110 =  6      0011 =  3
```

Bitwise Shifts

- **Shift** operations move bits within a field of bits
- Shift operations are

```
x = y << k; // left  shift y by k bits, store in x
x = y >> k; // right shift y by k bits, store in x
```

- All integral types can use shifts: `long`, `int`, `short`, `char`
- **Not applicable** to pointers or floating point
- Examples in 8 bits

```
//          76543210
char x = 0b00010111; // 23
char y = x << 2;     // left shift by 2
//   y = 0b01011100; // 92
//   x = 0b00010111; // not changed
char z = x >> 3;     // right shift by 3
//   z = 0b00000010; // 2
//   x = 0b00010111; // not changed
char n = 0b10000000; // -128, signed
char s = n >> 4;     // right shift by 4
//   s = 0b11111000; // -8, sign extension
//   right shift >> is "arithmetic"
```

Shifty Arithmetic Tricks

- Shifts with add/subtract can be used instead of multiplication and division
- Turn on optimization: `gcc -O3 code.c`
- Compiler automatically does this if it thinks it will save cycles
- *Sometimes* programmers should do this but better to convince compiler to do it for you, **comment** if doing manually

Multiplication

```c
//          76543210
char  x = 0b00001010;  // 10
char x2 = x << 1;      // 10*2
//   x2 = 0b00010100;  // 20
char x4 = x << 2;      // 10*4
//   x4 = 0b00101000;  // 40
char x7 = (x << 3)-x;  // 10*7
//   x7 = (x * 8)-x;   // 10*7
//   x7 = 0b01000110;  // 70
//          76543210
```

Division

```c
//          76543210
char y  = 0b01101110;  // 110
char y2 = y >> 1;      // 110/2
//   y2 = 0b00110111;  // 55
char y4 = y >> 2;      // 110/4
//   y4 = 0b00011011;  // 27
char z  = 0b10101100;  // -84
char z2 = z >> 2;      // -84/4
//   z2 = 0b11101011;  // -21
//   right shift sign extension
```

Exercise: Checking / Setting Bits

Use a combination of bit shift / bitwise logic operations to⋯

1. Check if bit `i` of `int x` is set (has value 1)
2. Clear bit `i` (set bit at index `i` to value 0)

Show C code for this

```
{
  int x = ...;
  int i = ...;
  if( ??? ) { // ith bit of x is set
    printf("set!\n");
  }

  i = ...;
  ???;
  printf("ith bit of x now cleared to 0\n");
}
```

Answers: Checking / Setting Bits

1. Check if bit `i` of `int x` is set (has value 1)

```
int x = ...;
int mask = 1; // or 0b0001 or 0x01 ...
int shifted  = mask << i; // shifted   0b00...010..00
if(x & shifted){          //       x & 0b10...010..01
  ...                     //       ------------------
}                         //           0b00...010..00
```

2. Clear bit `i` (set bit at index `i` to value 0)

```
int x = ...;
int mask = 1; // or 0b0001 or 0x01 ...
int shifted    = mask << i;  // shifted   0b00...010..00
int inverted   = ~shifted;   // inverted  0b11...101..11
x = x & inverted;            //       x & 0b10...010..01
  ...                        //       ------------------
                             //           0b10...000..01
```

Showing Bits

- `printf()` capabilities:

`%d`	as Decimal
`%x`	as Hexadecimal
`%o`	as Octal
`%c`	as Character

- No specifier for binary
- Can construct such with bitwise operations
- Code pack contains two codes to do this
  - `printbits.c`: single args printed as 32 bits
  - `showbits.c`: multiple args printed in binary, hex, decimal

- Showing bits usually involves shifting and bitwise AND `&`
- Example from `showbits.c`

```
#define INT_BITS 32

// print bits for x to screen
void showbits(int x){
  for(int i=INT_BITS-1; i>=0; i--){
    int mask = 1 << i;
    if(mask & x){
      printf("1");
    } else {
      printf("0");
    }
  }
}
```

Bit Masking

- Semi-common for functions to accept bit patterns which indicate true/false options
- Frequently makes use of bit **masks** which are constants associated with specific bits
- Example from earlier: Unix permissions might be···

```
#define S_IRUSR 0b100000000 // User   Read
#define S_IWUSR 0b010000000 // User   Write
#define S_IXUSR 0b001000000 // User   Execute
#define S_IRGRP 0b000100000 // Group  Read
...
#define S_IWOTH 0b000000010 // Others Write
#define S_IXOTH 0b000000001 // Others Execute
```

- Use them to create options to C functions like

```
int permissions = S_IRUSR|S_IWUSR|S_RGRP;
chmod("/home/kauffman/solution.zip",permissions);
```

Unix Permissions with Octal

- Octal arises associated with **Unix file permissions**
- Every file has 3 permissions for 3 entities
- Permissions are true/false so a single bit will suffice

- `ls -l`: long list files, shows permissions
- `chmod 665 somefile.txt`: change permissions of `somefile.txt` to those shown to the right
- `chmod 777 x.txt`: read / write / exec for everyone
- chmod also honors letter versions like `r` and `w`
- `chmod u+x script.sh # make file executable`

```
binary      octal
110110101 = 665
rw-rw-r-x somefile.txt
U  G  O
S  R  T
E  O  H
R  U  E
   P  R

Readable chmod version:
chmod u=rw,g=rw,o=rx somefile.txt
```