

Min Graph Coloring Approximation Notes

Explain Strategy:

The approximate solution used uses a greedy algorithm.

- This greedy solution visits the vertices one by one.
- At each vertex, the algorithm assigns the smallest color number not used by its colored neighbors.
- This creates a valid coloring of the graph, but most likely not the best solution.

This solution's answer solely depends on the order the vertices are visited, so I added an element of randomness to it.

- I repeat the coloring algorithm a set number of times.
- Each time randomizing the order of the vertices I visit.
- The program keeps the best solution after all the iterations.

Analytical Runtime:

n = number of vertices

m = number of edges

T = number of iterations (right now I use 100)

1. Reading the graph (read_graph):
 - Reading in m edge lines and doing dictionary lookups costs $O(m)$ amortized because the dictionary operations are $O(1)$.
 - Building the adjacency list costs $O(n)$, which makes the total cost $O(n + m)$.
2. One iteration of greedy_coloring_random_order:
 - First it randomly shuffles the visiting order which costs $O(n)$.
 - Loop through vertices in random order. For each vertex:
 - Get neighbors colors: costs $O(\text{degree}(v))$
 - $O(\text{degree}(v)) = O(2m) = O(m)$
 - Full Cost: $O(n + m)$
3. Repeating for T iterations:
 - Each iteration cost $O(n + m)$
 - Total with iterations $O(T * (n + m))$
4. Final printing: $O(n)$

Overall Runtime: $O(T * (n + m))$