# Homework 3

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## Question 7:

### A. Exercise 3.1.1

(a) $27 \in A$ : **True.** Since 27 is an integer multiple of 3, it is an element of A.

(b) $27 \in B$ : **False.** Since 27 is not a perfect square, it is not an element of B.

(c) $100 \in B$ **: True.** Since 100 is a perfect square, it is an element of B.

(d) $E \subseteq C \; or \; C \subseteq E$ : **False.** E is not a subset of C and C is not a subset of E.

(e) $E \subseteq A$ : **True.** The set { 3, 6, 9 } is a subset of A.

(f) $A \subset E$ : **False.** A is not a proper subset of E.

(g) $E \in A$ : **False.** The set { 3, 6, 9 } is not an element of A.

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### B. Exercise 3.1.2

(a) $15 \subset A$ : **False.** The element 15 is not a proper subset of A.

(b) $\{15\} \subset A$ : **True.** The set { 15 } is a proper subset of A.

(c) $\emptyset \subset A$ : **True.** The empty set ∅ is a proper subset of A.

(d) $A \subseteq A$ : **True.** The set A is a subset of itself.

(e) $\emptyset \in B$ : **False.** The empty set ∅ is not an element of B.

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## C. Exercise 3.1.5

(b) { 3, 6, 9, 12, .... }

Let $A = \{ 3,\ 6,\ 9,\ 12,\ \ldots. \}$

$A = \{ x \in \mathbb{Z}^+$ : x is an integer multiple of 3 }

Here, A is an infinite set.

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(d) { 0, 10, 20, 30, ...., 1000 }

Let B = { 0, 10, 20, 30, ...., 1000 }

$B = \{ x \in \mathbb{N}$ : x is an integer multiple of 10 and x ≤ 1000 }

Here, B is a finite set and $|B| = 101$.

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## D. Exercise 3.2.1

(a) $2 \in X$ : **True.** 2 is an element of $X$.

(b) $\{2\} \subseteq X$ : **True.** $2 \in X$, so the set $\{2\}$ is a subset of $X$.

(c) $\{2\} \in X$ : **False.** The set $\{2\}$ is not an element of $X$.

(d) $3 \in X$ : **False.** 3 is not an element of $X$.

(e) $\{1,\ 2\} \in X$ : **True.** The set $\{1, 2\}$ is an element of $X$.

(f) $\{1,\ 2\} \subseteq X$ : **True.** $\{1,\ 2\} \in X$, so the set $\{1,\ 2\}$ is a subset of $X$.

(g) $\{2,\ 4\} \subseteq X$ : **True.** $2 \in X$ and $4 \in X$, so the set $\{2, 4\}$ is a subset of $X$.

(h) $\{2,\ 4\} \in X$ : **False.** The set $\{2, 4\}$ is not an element of $X$.

(i) $\{2,\ 3\} \subseteq X$ : **False.** $2 \in X$ and $3 \notin X$, so the set $\{2, 3\}$ is not a subset of $X$.

(j) $\{2,\ 3\} \in X$ : **False.** The set $\{2, 3\}$ is not an element of $X$.

(k) $|X| = 7$ : **False.** Since $X$ contains 6 distinct elements, $|X| = 6$.

# Question 8:

## Exercise 3.2.4

(b) Let A = {1, 2, 3}. What is $\{ X \in P(A) \colon 2 \in X \}$?

Let B = $\{ X \in P(A) \colon 2 \in X \}$

The power set of A, denoted by P(A), is the set of all the subsets of A. Moreover, P(A) contains $2^3 = 8$ elements. From the given set A,

$$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{3, 1\}, \{1, 2, 3\}\}$$

From the set builder notation, the condition provided is $2 \in X$. Hence, the subsets that contain element 2 are retained within the set B.

Therefore, **B = {{2}, {1, 2}, {2, 3}, {1, 2, 3}}**.

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## Question 9:

### A. Exercise 3.3.1

(c) $A \cap C$

$$A \cap C = \{ -3, 1, 17 \}$$

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(d) $A \cup (B \cap C)$

$$B \cap C = \{ -5, 1 \}$$

$$A \cup (B \cap C) = \{ -5, -3, 0, 1, 4, 17 \}$$

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(e) $A \cap B \cap C$

$$A \cap B = \{ 1, 4 \}$$

$$(A \cap B) \cap C = \{ 1 \}$$

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## B. Exercise 3.3.3

(a) $\bigcap_{i=2}^{5} A_i$

$$A_2 = \{ 1, 2, 4 \}$$
$$A_3 = \{ 1, 3, 9 \}$$
$$A_4 = \{ 1, 4, 16 \}$$
$$A_5 = \{ 1, 5, 25 \}$$

$$\bigcap_{i=2}^{5} A_i = A_2 \cap A_3 \cap A_4 \cap A_5 = \{1, 2, 4\} \cap \{1, 3, 9\} \cap \{1, 4, 16\} \cap \{1, 5, 25\}$$

Therefore, $\bigcap_{i=2}^{5} A_i = \{1\}$

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(b) $\bigcup_{i=2}^{5} A_i$

$$A_2 = \{ 1, 2, 4 \}$$
$$A_3 = \{ 1, 3, 9 \}$$
$$A_4 = \{ 1, 4, 16 \}$$
$$A_5 = \{ 1, 5, 25 \}$$

$$\bigcup_{i=2}^{5} A_i = A_2 \cup A_3 \cup A_4 \cup A_5 = \{1, 2, 4\} \cup \{1, 3, 9\} \cup \{1, 4, 16\} \cup \{1, 5, 25\}$$

Therefore, $\bigcup_{i=2}^{5} A_i = \{1, 2, 3, 4, 5, 9, 16, 25\}$

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(e) $\bigcap_{i=1}^{100} C_i$

$$\bigcap_{i=1}^{100} C_i = C_1 \cap C_2 \cap C_3 \cap \ldots\ldots \cap C_{100}$$

$$C_1 = \{\, x \in \mathbb{R} : \; -\ 1 \le x \le 1 \,\}$$
$$C_2 = \{\, x \in \mathbb{R} : \; -\ 1/2 \le x \le 1/2 \,\}$$
$$C_3 = \{\, x \in \mathbb{R} : \; -\ 1/3 \le x \le 1/3 \,\}$$
$$\vdots$$
$$C_{100} = \{\, x \in \mathbb{R} : \; -\ 1/100 \le x \le 1/100 \,\}$$

Since the $x$ values between $\frac{-1}{100}$ and $\frac{1}{100}$ are present in all the possible sets of C i.e. $C_1$, $C_2$, $C_3$, …. , $C_{100}$,

$$C_1 \cap C_2 \cap C_3 \cap \ldots\ldots \cap C_{100} = \{\, x \in \mathbb{R} : -\ 1/100 \le x \le 1/100 \,\}$$

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(f) $\bigcup_{i=1}^{100} C_i$

$$\bigcup_{i=1}^{100} C_i = C_1 \cup C_2 \cup C_3 \cup \ldots\ldots \cup C_{100}$$

$$C_1 = \{\, x \in \mathbb{R} : \; -\ 1 \le x \le 1 \,\}$$
$$C_2 = \{\, x \in \mathbb{R} : \; -\ 1/2 \le x \le 1/2 \,\}$$
$$C_3 = \{\, x \in \mathbb{R} : \; -\ 1/3 \le x \le 1/3 \,\}$$
$$\vdots$$
$$C_{100} = \{\, x \in \mathbb{R} : \; -\ 1/100 \le x \le 1/100 \,\}$$

Since the $x$ values between -1 and 1 contain all the possible values represented by the sets $C_1$, $C_2$, $C_3$, …. , $C_{100}$,

$$C_1 \cup C_2 \cup C_3 \cup \ldots\ldots \cup C_{100} = \{\, x \in \mathbb{R} : -\ 1 \le x \le 1 \,\}$$

## C. Exercise 3.3.4

(b) $P(A \cup B)$

$$A \cup B = \{ \text{a, b, c} \}$$

Here $A \cup B$ contains 3 elements. Therefore, $P(A \cup B)$ contains $2^3 = 8$ elements.

$$\therefore P(A \cup B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$$

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(d) $P(A) \cup P(B)$

$$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

$$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$$

$$\therefore P(A) \cup P(B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$$

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# Question 10:

## A. Exercise 3.5.1

(b) $B \times A \times C = $ (foam, tall, non-fat)

(c) $B \times C = \{$(foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole)$\}$

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## B. Exercise 3.5.3

(b) $\mathbb{Z}^2 \subseteq \mathbb{R}^2$

**True.** Since the set of pairs of integers is a subset of a set of pairs of real numbers, $\mathbb{Z}^2$ is a subset of $\mathbb{R}^2$.

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(c) $\mathbb{Z}^2 \cap \mathbb{Z}^3 = \emptyset$

**True.** Since the elements of $\mathbb{Z}^3$ is $\{ (a,\ b,\ c) :\ a,\ b,\ c \in \mathbb{Z} \}$ and the elements of $\mathbb{Z}^2$ is $\{ (a,\ b) :\ a,\ b \in \mathbb{Z} \}$, the elements of $\mathbb{Z}^2$ are not members in $\mathbb{Z}^3$.

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(e) For any three sets, A, B, and C, if $A \subseteq B$, then $A \times C \subseteq B \times C$.

**True.** Since $A \subseteq B$, it means that every element in A is also present in B. Hence, every element in $A \times C$ is also an element of $B \times C$.

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## C. Exercise 3.5.6

(d) $\{xy: \text{where } x \in \{0\} \cup \{0\}^2 \text{ and } y \in \{1\} \cup \{1\}^2\}$

$$\{0\} \cup \{0\}^2 = \{\ 0, 00\ \}$$

$$\{1\} \cup \{1\}^2 = \{\ 1, 11\ \}$$

$$\text{Let } A = \{xy: \text{where } x \in \{0\} \cup \{0\}^2 \text{ and } y \in \{1\} \cup \{1\}^2\}$$

$$\therefore A = \{\ 01, 011, 001, 0011\ \}$$

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(e) $\{xy: x \in \{aa, ab\} \text{ and } y \in \{a\} \cup \{a\}^2\}$

$$x \in \{aa, ab\}$$

$$y \in \{a\} \cup \{a\}^2 = \{\ a, aa\ \}$$

$$\text{Let } B = \{xy: x \in \{aa, ab\} \text{ and } y \in \{a\} \cup \{a\}^2\}$$

$$\therefore B = \{\ aaa, aaaa, aba, abaa\ \}$$

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## D. Exercise 3.5.7

(c) $(A \times B) \cup (A \times C)$

$$(A \times B) = \{\ ab, ac\ \}$$

$$(A \times C) = \{\ aa, ab, ad\ \}$$

$$\therefore (A \times B) \cup (A \times C) = \{\ aa, ab, ac, ad\ \}$$

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(f) $P(A \times B)$

$$(A \times B) = \{\ ab, ac\ \}$$

$$\therefore P(A \times B) = \{\ \emptyset, \{ab\}, \{ac\}, \{ab, ac\}\ \}$$

(g) $P(A) \times P(B)$

$$P(A) = \{ \emptyset, \{a\} \}$$

$$P(B) = \{ \emptyset, \{b\}, \{c\}, \{b, c\}\}$$

$$\therefore P(A) \times P(B) = \{(\emptyset, \emptyset), (\emptyset, \{b\}), (\emptyset, \{c\}), (\emptyset, \{b, c\}), (\{a\}, \emptyset), (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{b, c\})\}$$

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# Question 11:

## A. Exercise 3.6.2

(b) $(B \cup A) \cap (\overline{B} \cup A) \;=\; A$

| | |
|---|---|
| $(B \cup A) \cap (\overline{B} \cup A)$ | |
| $(A \cup B) \cap (A \cup \overline{B})$ | Commutative Law |
| $A \cup (B \cap \overline{B})$ | Distributive Law |
| $A \cup \emptyset$ | Complement Law |
| $A$ | Identity Law |

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(c) $\overline{A \cap \overline{B}} = \overline{A} \cup B$

| | |
|---|---|
| $\overline{A \cap \overline{B}}$ | |
| $\overline{A} \cup \overline{\overline{B}}$ | De Morgan's Law |
| $\overline{A} \cup B$ | Double Complement Law |

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## B. Exercise 3.6.3

(b) $A - (B \cap A) = A$

$$\text{Let } A = \{ 0, 1, 2, 3 \} \text{ and } B = \{ 0 \}$$

$$B \cap A = \{ 0 \}$$

$$A - (B \cap A) = \{ 1, 2, 3 \}$$

$$\text{In this case, } A - (B \cap A) = \{ 1, 2, 3 \} \text{ and } A = \{ 0, 1, 2, 3 \}$$

$$\therefore A - (B \cap A) \neq A$$

Hence, $A - (B \cap A) = A$ is not a set identity.

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(d) $(B - A) \cup A = A$

$$\text{Let } A = \{ 1, 2, 3, 4 \} \text{ and } B = \{ 2, 3, 4, 5 \}$$

$$B - A = \{ 5 \}$$

$$(B - A) \cup A = \{ 1, 2, 3, 4, 5 \}$$

$$\text{In this case, } (B - A) \cup A = \{ 1, 2, 3, 4, 5 \} \text{ and } A = \{ 1, 2, 3, 4 \}$$

$$\therefore (B - A) \cup A \neq A$$

Hence, $(B - A) \cup A = A$ is not a set identity.

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## C. Exercise 3.6.4

(b) $A \cap (B - A) = \emptyset$

| | |
|---|---|
| $A \cap (B - A)$ | |
| $A \cap (B \cap \overline{A})$ | Set Subtraction Law |
| $A \cap (\overline{A} \cap B)$ | Commutative Law |
| $(A \cap \overline{A}) \cap B)$ | Associative Law |
| $\emptyset \cap B$ | Complement Law |
| $B \cap \emptyset$ | Commutative Law |
| $\emptyset$ | Domination Law |

(c) A ∪ (B - A) = A ∪ B

| | |
|---|---|
| $A \cup (B - A)$ | |
| $A \cup (B \cap \overline{A})$ | Set Subtraction Law |
| $(A \cup B) \cap (A \cup \overline{A})$ | Distributive Law |
| $(A \cup B) \cap U$ | Complement Law |
| $A \cup B$ | Identity Law |