Homework 1

Kaushik Manivannan (km6329@nyu.edu)

Question 1:

A.1) 10011011_2

$= 1.2^0 + 1.2^1 + 0.2^2 + 1.2^3 + 1.2^4 + 0.2^5 + 0.2^6 + 1.2^7$
$= 2^0 + 2^1 + 2^3 + 2^4 + 2^7$
$= 1 + 2 + 8 + 16 + 128$
$= \mathbf{155_{10}}$

A.2) 456_7

$= 6.7^0 + 5.7^1 + 4.7^2$
$= 6 + 35 + 196$
$= \mathbf{237_{10}}$

A.3) $38A_{16}$

$= A.16^0 + 8.16^1 + 3.16^2$
$= A + 128 + 768$
$= 10 + 128 + 768$ (Since the decimal representation of A is 10)
$= \mathbf{906_{10}}$

A.4) 2214_5

$= 4.5^0 + 1.5^1 + 2.5^2 + 2.5^3$
$= 4 + 5 + 50 + 250$
$= \mathbf{309_{10}}$

B.1) 69_{10}

Bits:	1	0	0	0	1	0	1
Bit Values:	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	||	||	||	||	||	||	||
	64	32	16	8	4	2	1

Hence, the binary representation of 69_{10} is $\mathbf{1000101_2}$.

B.2) 485_{10}

Bits:	1	1	1	1	0	0	1	0	1
Bit Values:	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	||	||	||	||	||	||	||	||	||
	256	128	64	32	16	8	4	2	1

Hence, the binary representation of 485_{10} is $\mathbf{111100101_2}$.

B.3) $6D1A_{16}$

Hexadecimal Digit	4-Bit Binary Representation
6	0110
D	1101
1	0001
A	1010

Hence, the binary representation of $6D1A_{16}$ is $\mathbf{110110100011010_2}$.

C.1) 1101011_2

4-Bit Binary Representation:	0110	1011
	||	||
Corresponding Hex Digits:	6	B

Hence, the corresponding hexadecimal representation is **$6B_{16}$**.

C.2) 895_{10}

Hex Digits:	<u>3</u>	<u>7</u>	<u>F</u>
Hex Values:	16^2	16^1	16^0
	||	||	||
	256	16	1

Hence, the corresponding hexadecimal representation is **$37F_{16}$**.

<u>Question 2:</u>

1) $7566_8 + 4515_8$

$$\begin{array}{r} {\scriptstyle 1\,1\,1\,1} \\ 7566_8 \\ +\ 4515_8 \\ \hline 14303_8 \\ \hline \end{array}$$

$\therefore\ 7566_8 + 4515_8 =$ $\mathbf{14303_8}$.

---

2) $10110011_2 + 1101_2$

$$\begin{array}{r} {\scriptstyle 1\,1\,1\,1\,1\,1} \\ 10110011_2 \\ +\qquad 1101_2 \\ \hline 11000000_2 \\ \hline \end{array}$$

$\therefore\ 10110011_2 + 1101_2 =$ $\mathbf{11000000_2}$.

---

3) $7A66_{16} + 45C5_{16}$

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 7A66_{16} \\ +\ 45C5_{16} \\ \hline C02B_{16} \\ \hline \end{array}$$

$\therefore\ 7A66_{16} + 45C5_{16} =$ $\mathbf{C02B_{16}}$.

---

4) $3022_5 - 2433_5$

$$\begin{array}{r} {\scriptstyle 2\ 4\ 11\ 12} \\ \cancel{3}\cancel{0}\cancel{2}\cancel{2}_5 \\ -\ 2433_5 \\ \hline 0034_5 \\ \hline \end{array}$$

$\therefore\ 3022_5 - 2433_5 = \mathbf{34_5}.$

<u>Question 3:</u>

A.1) 124_{10}

Bits:	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>0</u>	<u>0</u>
Bit Values:	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	||	||	||	||	||	||	||
	64	32	16	8	4	2	1

Since 0 must be padded to the left of the unsigned binary representation for a positive integer, the 8-bit 2's complement representation of 124_{10} is **$01111100_{\text{8-bit 2's Complement}}$.**

A.2) -124_{10}

<u>Step 1:</u> The decimal number 124 must be converted to its equivalent 8-bit binary representation.

Bits:	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>0</u>	<u>0</u>
Bit Values:	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	||	||	||	||	||	||	||
	64	32	16	8	4	2	1

Thus, the 8-bit binary representation of 124_{10} is 01111100_2.

<u>Step 2:</u> Each bit in the 8-bit binary representation must be flipped.

Bits:	<u>0</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>0</u>	<u>0</u>
	||	||	||	||	||	||	||	||
Flipped Bits:	<u>1</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>1</u>	<u>1</u>

Step 3: Finally, 1 is added to the resultant flipped binary number.

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 10000011_2 \\ +\quad\quad\quad\; 1_2 \\ \hline 10000100_2 \\ \hline \end{array}$$

Hence, the 8-bit 2's complement representation of -124_{10} is **10000100**$_{\textbf{8-bit 2's Complement}}$.

A.3) 109_{10}

Bits:	<u>1</u>	<u>1</u>	<u>0</u>	<u>1</u>	<u>1</u>	<u>0</u>	<u>1</u>
Bit Values:	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	||	||	||	||	||	||	||
	64	32	16	8	4	2	1

Since 0 must be padded to the left of the unsigned binary representation for a positive integer, the 8-bit 2's complement representation of 109_{10} is **01101101**$_{\textbf{8-bit 2's Complement}}$.

A.4) -79_{10}

The decimal number 79 is converted to its equivalent 8-bit binary representation.

Bits:	<u>1</u>	<u>0</u>	<u>0</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>
Bit Values:	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	||	||	||	||	||	||	||
	64	32	16	8	4	2	1

Thus, the 8-bit binary representation of 79_{10} is 01001111_2.

Since we know that the sum of a number and its additive inverse is 2^k, we can use this to find the 8-bit 2's complement representation of -79_{10},

$$
\begin{array}{rl}
{\scriptstyle 1\,1\,1\,1\,1\,1\,1\,1} & \\
01001111_2 & \leftarrow 79_{10} \\
+\ 10110001_2 & \leftarrow -79_{10} \\
\hline
100000000_2 & \leftarrow 2^8 = 128 \\
\hline
\end{array}
$$

Hence, the 8-bit 2's complement representation of -79_{10} is $\mathbf{10110001}_{\textbf{8-bit 2's Complement}}$.

B.1) $00011110_{\text{8-bit 2's comp}}$

Since the Most Significant Bit (MSB) of the binary number is 0, we know that this is a positive number. Moreover, the subsequent bits following the MSB represent the decimal number.

Bits:	0	0	1	1	1	1	0
Bit Values:	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	||	||	||	||	||	||	||
	64	32	16	8	4	2	1

$= 1.2^4 + 1.2^3 + 1.2^2 + 1.2^1$
$= 16 + 8 + 4 + 2$
$= 30_{10}$

Hence, the decimal representation of $00011110_{\text{8-bit 2's comp}}$ is $\mathbf{30_{10}}$.

B.2) $11100110_{\text{8-bit 2's comp}}$

Since the Most Significant Bit (MSB) of the binary number is 1, we know that this is a negative number. In addition, since we know that the sum of a number and its additive inverse is 2^k,

$$
\begin{array}{rl}
{\scriptstyle 1\,1\,1\,1\,1\,1\,1} & \\
11100110_2 & \leftarrow \text{Negative Binary Representation of Number} \\
+\ 00011010_2 & \leftarrow \text{Positive Binary Representation of Number} \\
\hline
100000000_2 & \leftarrow 2^8
\end{array}
$$

Converting the positive binary representation (00011010_2) of the number back to its decimal equivalent, we get,

Bits:	0	0	0	1	1	0	1	0
Bit Values:	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$= 1.2^4 + 1.2^3 + 1.2^1$
$= 16 + 8 + 2$
$= 26_{10}$

Hence, the decimal representation of $11100110_{\text{8-bit 2's comp}}$ is **-26_{10}**.

B.3) $00101101_{\text{8-bit 2's comp}}$

Since the Most Significant Bit (MSB) of the binary number is 0, we know that this is a positive number. Moreover, the subsequent bits following the MSB represent the decimal number.

Bits:	0	1	0	1	1	0	1
Bit Values:	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	||	||	||	||	||	||	||
	64	32	16	8	4	2	1

$= 1.2^5 + 1.2^3 + 1.2^2 + 1.2^0$
$= 32 + 8 + 4 + 1$
$= 45_{10}$

Hence, the decimal representation of $00101101_{\text{8-bit 2's comp}}$ is $\mathbf{45_{10}}$.

B.4) $10011110_{\text{8-bit 2's comp}}$

Since the Most Significant Bit (MSB) of the binary number is 1, we know that this is a negative number. In addition, since we know that the sum of a number and its additive inverse is 2^k,

$$
\begin{array}{rl}
{\scriptstyle 1\,1\,1\,1\,1\,1\,1} & \\
10011110_2 & \leftarrow \text{Negative Binary Representation of Number} \\
+\ 01100010_2 & \leftarrow \text{Positive Binary Representation of Number} \\
\hline
100000000_2 & \leftarrow 2^8 = 128 \\
\hline
\end{array}
$$

Converting the positive binary representation (01100010_2) of the number back to its decimal equivalent, we get,

Bits:	<u>0</u>	<u>1</u>	<u>1</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>1</u>	<u>0</u>
Bit Values:	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	||	||	||	||	||	||	||	||
	128	64	32	16	8	4	2	1

$= 1.2^6 + 1.2^5 + 1.2^1$
$= 64 + 32 + 2$
$= 98_{10}$

Hence, the decimal representation of $10011110_{\text{8-bit 2's comp}}$ is **-98_{10}**.

Question 4:

1) Exercise 1.2.4

(b) $\neg(\boldsymbol{p} \vee \boldsymbol{q})$

p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	**F**
T	F	T	**F**
F	T	T	**F**
F	F	T	**T**

(c) $\boldsymbol{r} \vee (\boldsymbol{p} \wedge \neg \boldsymbol{q})$

p	q	r	$\neg q$	$p \wedge \neg q$	$r \vee (p \wedge \neg q)$
T	T	T	F	F	**T**
T	T	F	F	F	**F**
T	F	T	T	T	**T**
T	F	F	T	T	**T**
F	T	T	F	F	**T**
F	T	F	F	F	**F**
F	F	T	T	F	**T**
F	F	F	T	F	**F**

---

2) Exercise 1.3.4

(b) $(\boldsymbol{p} \rightarrow \boldsymbol{q}) \rightarrow (\boldsymbol{q} \rightarrow \boldsymbol{p})$

$\boldsymbol{p}$	$\boldsymbol{q}$	$\boldsymbol{p} \rightarrow \boldsymbol{q}$	$\boldsymbol{q} \rightarrow \boldsymbol{p}$	$(\boldsymbol{p} \rightarrow \boldsymbol{q}) \rightarrow (\boldsymbol{q} \rightarrow \boldsymbol{p})$
T	T	T	T	**T**
T	F	F	T	**T**
F	T	T	F	**F**
F	F	T	T	**T**

(d) $(\boldsymbol{p} \leftrightarrow \boldsymbol{q}) \oplus (\boldsymbol{p} \leftrightarrow \neg \boldsymbol{q})$

$\boldsymbol{p}$	$\boldsymbol{q}$	$\neg \boldsymbol{q}$	$\boldsymbol{p} \leftrightarrow \boldsymbol{q}$	$\boldsymbol{p} \leftrightarrow \neg \boldsymbol{q}$	$(\boldsymbol{p} \leftrightarrow \boldsymbol{q}) \oplus (\boldsymbol{p} \leftrightarrow \neg \boldsymbol{q})$
T	T	F	T	F	**T**
T	F	T	F	T	**T**
F	T	F	F	T	**T**
F	F	T	T	F	**T**

Question 5:

1) Exercise 1.2.7

(b) The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

$$(\boldsymbol{B} \wedge \boldsymbol{D}) \vee (\boldsymbol{D} \wedge \boldsymbol{M}) \vee (\boldsymbol{M} \wedge \boldsymbol{B}) \vee (\boldsymbol{B} \wedge \boldsymbol{D} \wedge \boldsymbol{M})$$

(c) Applicant must present either a birth certificate or both a driver's license and a marriage license.

$$\boldsymbol{B} \vee (\boldsymbol{D} \wedge \boldsymbol{M})$$

---

2) Exercise 1.3.7

(b) A person can park in the school parking lot if they are a senior or at least seventeen years of age.

$$(\boldsymbol{s} \vee \boldsymbol{y}) \rightarrow \boldsymbol{p}$$

(c) Being 17 years of age is a necessary condition for being able to park in the school parking lot.

$$\boldsymbol{p} \rightarrow \boldsymbol{y}$$

(d) A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

$$\boldsymbol{p} \leftrightarrow (\boldsymbol{s} \wedge \boldsymbol{y})$$

(e) Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

$$\boldsymbol{p} \rightarrow (\boldsymbol{s} \vee \boldsymbol{y})$$

---

3) Exercise 1.3.9

(c) The applicant can enroll in the course only if the applicant has parental permission.

$$\boldsymbol{c \rightarrow p}$$

(d) Having parental permission is a necessary condition for enrolling in the course.

$$\boldsymbol{c \rightarrow p}$$

<u>Question 6:</u>

1) Exercise 1.3.6

(b) Maintaining a B average is necessary for Joe to be eligible for the honors program.

If Joe is to be eligible for the honors program, then he must maintain a B average.

(c) Rajiv can go on the roller coaster only if he is at least four feet tall.

If Rajiv can go on the roller coaster, then he has to be at least four feet tall.

(d) Rajiv can go on the roller coaster if he is at least four feet tall.

If Rajiv is at least four feet tall, then he can go on the roller coaster.

---

2) Exercise 1.3.10

(c) $(p \lor r) \leftrightarrow (q \land r)$

Replacing the propositional variables with its respective truth values,

$$(T \lor r) \leftrightarrow (F \land r)$$

Despite the unknown value of r, $(T \lor r)$ evaluates to true due to the disjunction operation while $(F \land r)$ evaluates to false because of the conjunction operation.

$$T \leftrightarrow F$$

Hence, the logical expression evaluates to <u>False</u>.

(d) $(p \wedge r) \leftrightarrow (q \wedge r)$

Replacing the propositional variables with its respective truth values,

$$(T \wedge r) \leftrightarrow (F \wedge r)$$

Now, $(F \wedge r)$ evaluates to false because of the conjunction operation.

$$(T \wedge r) \leftrightarrow F$$

Since the value of r is unknown, $(T \wedge r)$ could evaluate to either true or false depending on the value of r. Moreover, if $(T \wedge r)$ evaluates to false, the logical expression would also be false whereas if it evaluates to true, the logical expression would be true.

Hence, the truth value of the logical expression is Unknown.

(e) $p \rightarrow (r \vee q)$

Replacing the propositional variables with its respective truth values,

$$T \rightarrow (r \vee F)$$

Since the value of r is unknown, $(r \vee F)$ could evaluate to either true or false depending on the value of r. Moreover, if $(r \vee F)$ evaluates to false, the logical expression would also be false whereas if it evaluates to true, the logical expression would be true.

Hence, the truth value of the logical expression is Unknown.

(f) $(p \wedge q) \rightarrow r$

Replacing the propositional variables with its respective truth values,

$$(T \wedge F) \rightarrow r$$

Here, $(T \wedge F)$ evaluates to false due to the conjunction operation.

$$F \rightarrow r$$

Since the hypothesis of this conditional expression is false, then the statement is true regardless of the truth value of the conclusion.

Hence, the truth value of the logical expression is True.

Question 7:

(b) If Sally did not get the job, then she was late for her interview or did not update her resume.
If Sally updated her resume and was not late for her interview, then she got the job.

j	l	r	$\neg j$	$\neg l$	$\neg r$	$l \vee \neg r$	$r \wedge \neg l$	$\neg j \to (l \vee \neg r)$	$(r \wedge \neg l) \to j$
T	T	T	F	F	F	T	F	**T**	**T**
T	T	F	F	F	T	T	F	**T**	**T**
T	F	T	F	T	F	F	T	**T**	**T**
T	F	F	F	T	T	T	F	**T**	**T**
F	T	T	T	F	F	T	F	**T**	**T**
F	T	F	T	F	T	T	F	**T**	**T**
F	F	T	T	T	F	F	T	**F**	**F**
F	F	F	T	T	T	T	F	**T**	**T**

From the table, $\neg j \to (l \vee \neg r) \equiv (r \wedge \neg l) \to j$ since they have the same truth values regardless of the truth values of their individual propositions.

(c) If Sally got the job then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.

j	l	r	$\neg j$	$\neg l$	$j \to \neg l$	$\neg j \to l$
T	T	T	F	F	**T**	**T**
T	T	F	F	F	**F**	**T**
T	F	T	F	T	**T**	**T**
T	F	F	F	T	**T**	**T**
F	T	T	T	F	**T**	**T**
F	T	F	T	F	**T**	**T**
F	F	T	T	T	**T**	**F**
F	F	F	T	T	**T**	**F**

From the table, $\boldsymbol{j \to \neg l \not\equiv \neg j \to l}$ since $(j \to \neg l) \leftrightarrow (\neg j \to l)$ is not a tautology.

(d) If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.

$\boldsymbol{j}$	$\boldsymbol{l}$	$\boldsymbol{r}$	$\boldsymbol{\neg j}$	$\boldsymbol{\neg l}$	$\boldsymbol{\neg r}$	$\boldsymbol{r \vee \neg l}$	$\boldsymbol{r \wedge \neg l}$	$\boldsymbol{(r \vee \neg l) \to j}$	$\boldsymbol{j \to (r \wedge \neg l)}$
T	T	T	F	F	F	T	F	T	F
T	T	F	F	F	T	F	F	T	F
T	F	T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	F	T	F
F	T	T	T	F	F	T	F	F	T
F	T	F	T	F	T	F	F	T	T
F	F	T	T	T	F	T	T	F	T
F	F	F	T	T	T	T	F	F	T

From the table, $\boldsymbol{(r \vee \neg l) \to j \not\equiv j \to (r \wedge \neg l)}$ since they do not have the same truth values. Moreover, $((r \vee \neg l) \to j) \leftrightarrow (j \to (r \wedge \neg l))$ is not a tautology.

Question 8:

1) Exercise 1.5.2

(c) $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$(p \to q) \land (p \to r)$	**Laws of Propositional Logic**
$(\neg p \lor q) \land (\neg p \lor r)$	Conditional Identity
$\neg p \lor (q \land r)$	Distributive Law
$p \to (q \land r)$	Conditional Identity

(f) $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$\neg(p \lor (\neg p \land q))$	**Laws of Propositional Logic**
$\neg p \land \neg (\neg p \land q)$	De Morgan's Law
$\neg p \land (\neg\neg p \lor \neg q)$	De Morgan's Law
$\neg p \land (p \lor \neg q)$	Double Negation Law
$(\neg p \land p) \lor (\neg p \land \neg q)$	Distributive Law
$(p \land \neg p) \lor (\neg p \land \neg q)$	Commutative Law
$F \lor (\neg p \land \neg q)$	Complement Law
$(\neg p \land \neg q) \lor F$	Commutative Law
$(\neg p \land \neg q)$	Identity Law

(i) $(p \land q) \rightarrow r \equiv (p \land \neg r) \rightarrow \neg q$

$(p \land q) \rightarrow r$	**Laws of Propositional Logic**
$\neg(p \land q) \lor r$	Conditional Identity
$(\neg p \lor \neg q) \lor r$	De Morgan's Law
$(\neg p \lor r) \lor \neg q$	Associative Law
$\neg(p \land \neg r) \lor \neg q$	De Morgan's Law
$(p \land \neg r) \rightarrow \neg q$	Conditional Identity

---

2) Exercise 1.5.3

(c) $\neg r \lor (\neg r \rightarrow p)$

$\neg r \lor (\neg r \rightarrow p)$	**Laws of Propositional Logic**
$\neg r \lor (\neg\neg r \lor p)$	Conditional Identity
$\neg r \lor (r \lor p)$	Double Negation Law
$(\neg r \lor r) \lor p$	Associative Law
$(r \lor \neg r) \lor p$	Commutative Law
$T \lor p$	Complement Law
$p \lor T$	Commutative Law
T	Domination Law

Hence, the above statement is a **tautology** since $\neg r \lor (\neg r \rightarrow p) \equiv T$.

(d) $\neg(p \rightarrow q) \rightarrow \neg q$

$\neg(p \rightarrow q) \rightarrow \neg q$	**Laws of Propositional Logic**
$\neg\neg(p \rightarrow q) \vee \neg q$	Conditional Identity
$(p \rightarrow q) \vee \neg q$	Double Negation Law
$(\neg p \vee q) \vee \neg q$	Conditional Identity
$\neg p \vee (q \vee \neg q)$	Associative Law
$\neg p \vee T$	Complement Law
T	Domination Law

Hence, the above statement is a **tautology** since $\neg(p \rightarrow q) \rightarrow \neg q \equiv T$.

Question 9:

1) Exercise 1.6.3

(c) There is a number that is equal to its square.

$$\exists x(x = x^2)$$

(d) Every number is less than or equal to its square plus 1.

$$\forall x(x \leq x^2 + 1)$$

---

2) Exercise 1.7.4

(b) Everyone was well and went to work yesterday.

$$\forall x(\neg S(x) \wedge W(x))$$

(c) Everyone who was sick yesterday did not go to work.

$$\forall x(S(x) \rightarrow \neg W(x))$$

(d) Yesterday someone was sick and went to work.

$$\exists x(S(x) \wedge W(x))$$

Question 10:

1) Exercise 1.7.9

(c) $\exists x((x = c) \to P(x))$

If $x = c$, the statement $(c = c) \to P(c)$ becomes $T \to F$. Hence, the quantified expression evaluates to **False**.

(d) $\exists x(Q(x) \wedge R(x))$

When $x = e$, the expression becomes $Q(e) \wedge R(e)$ i.e. $T \wedge T$. Therefore, the quantified expression evaluates to **True**.

(e) $Q(a) \wedge P(d)$

Here, $Q(a) = T$ and $P(d) = T$. As a result, $Q(a) \wedge P(d)$ evaluates to **True**.

(f) $\forall x\ ((x \neq b) \to Q(x))$

For all values a, c, d and e, $(x \neq b)$ evaluates to true and $Q(x)$ is also true. Hence, the quantified expression evaluates to **True**.

(g) $\forall x\ (P(x) \vee R(x))$

Counter example is when $x = c$, the expression $P(x) \vee R(x)$ evaluates to false. Hence, the quantified expression evaluates to **False**.

(h) $\forall x\ (R(x) \to P(x))$

For all values of a, b, c, d and e, the expression $R(x) \to P(x)$ evaluates to true. Hence, the quantified expression evaluates to **True**.

(i) $\exists x(Q(x) \lor R(x))$

For values a, c, d and e, the expression $Q(x) \lor R(x)$ evaluates to true. Hence, the quantified expression evaluates to **True**.

---

2) Exercise 1.9.2

(b) $\exists x \; \forall y \; Q(x, y)$

True. When $x = 2$, it is true for all y.

(c) $\exists y \; \forall x \; P(x, y)$

True. For all x, there exists a column of y which makes $P(x, y)$ true.

(d) $\exists x \; \exists y \; S(x, y)$

False. There is no value of x or y which makes $S(x, y)$ true.

(e) $\forall x \; \exists y \; Q(x, y)$

False. When $x = 1$, there is no y that makes $Q(x, y)$ true.

(f) $\forall x \; \exists y \; P(x, y)$

True. For every value of x, there exists a column of y that makes $P(x, y)$ true.

(g) $\forall x \; \forall y \; P(x, y)$

False. Counter example is when $x = 2$ and $y = 2$, $P(x, y)$ evaluates to false.

(h) $\exists x\ \exists y\ Q(x, y)$

True. When $x = 2$ and $y = 2$, $Q(x, y)$ evaluates to true. Hence, the quantified statement evaluates to true.

(i) $\forall x\ \forall y\ \neg S(x, y)$

True. For all possible combinations of x and y, $\neg S(x, y)$ evaluates to true.

Question 11:

1) Exercise 1.10.4

(c) There are two numbers whose sum is equal to their product.

$$\exists x \exists y (x + y = xy)$$

(d) The ratio of every two positive numbers is also positive.

$$\forall x \forall y \big(((x > 0) \wedge (y > 0)) \rightarrow ((x/y) > 0)\big)$$

(e) The reciprocal of every positive number less than one is greater than one.

$$\forall x(((x > 0) \wedge (x < 1)) \rightarrow ((1/x) > 1)$$

(f) There is no smallest number.

$$\neg \exists x \forall y (x < y)$$

(g) Every number other than 0 has a multiplicative inverse.

$$\forall x \exists y \big((x \neq 0) \rightarrow (xy = 1)\big)$$

---

2) Exercise 1.10.7

(c) There is at least one new employee who missed the deadline.

$$\exists x \big(N(x) \wedge D(x)\big)$$

(d) Sam knows the phone number of everyone who missed the deadline.

$$\forall \boldsymbol{y}\big(\boldsymbol{D}(\boldsymbol{y}) \to \boldsymbol{P}(\boldsymbol{Sam},\boldsymbol{y})\big)$$

(e) There is a new employee who knows everyone's phone number.

$$\exists \boldsymbol{x}\forall \boldsymbol{y}\big(\boldsymbol{N}(\boldsymbol{x}) \wedge \boldsymbol{P}(\boldsymbol{x},\boldsymbol{y})\big)$$

(f) Exactly one new employee missed the deadline.

$$\exists \boldsymbol{x}\forall \boldsymbol{y}((\boldsymbol{N}(\boldsymbol{x}) \wedge \boldsymbol{D}(\boldsymbol{x})) \wedge (((\boldsymbol{x} \neq \boldsymbol{y}) \wedge \boldsymbol{N}(\boldsymbol{y})) \to \neg \boldsymbol{D}(\boldsymbol{y}))$$

---

3) Exercise 1.10.10

(c) Every student has taken at least one class other than Math 101.

$$\forall \boldsymbol{x}\exists \boldsymbol{y}\big(\boldsymbol{T}(\boldsymbol{x},\boldsymbol{y}) \wedge (\boldsymbol{y} \neq \boldsymbol{Math101})\big)$$

(d) There is a student who has taken every math class other than Math 101.

$$\exists \boldsymbol{x}\forall \boldsymbol{y}\big((\boldsymbol{y} \neq \boldsymbol{Math101}) \to \boldsymbol{T}(\boldsymbol{x},\boldsymbol{y})\big)$$

(e) Everyone other than Sam has taken at least two different math classes.

$$\forall \boldsymbol{x}\exists \boldsymbol{y}\exists \boldsymbol{z}(((\boldsymbol{x} \neq \boldsymbol{Sam})) \to ((\boldsymbol{y} \neq \boldsymbol{z}) \wedge \boldsymbol{T}(\boldsymbol{x},\boldsymbol{y}) \wedge \boldsymbol{T}(\boldsymbol{x},\boldsymbol{z}))$$

(f) Sam has taken exactly two math classes.

$$\exists \boldsymbol{y}\exists \boldsymbol{z}\forall \boldsymbol{w}\Big(\big((\boldsymbol{y} \neq \boldsymbol{z}) \wedge \boldsymbol{T}(\boldsymbol{Sam},\boldsymbol{y}) \wedge \boldsymbol{T}(\boldsymbol{Sam},\boldsymbol{z})\big) \wedge \Big(\big((\boldsymbol{w} \neq \boldsymbol{y}) \wedge (\boldsymbol{w} \neq \boldsymbol{z})\big) \to \neg \boldsymbol{T}(\boldsymbol{Sam},\boldsymbol{w})\Big)\Big)$$

Question 12:

1) Exercise 1.8.2

P(x): x was given the placebo
D(x): x was given the medication
M(x): x had migraines

(b) Every patient was given the medication or the placebo or both.

The logical expression of the above English statement is,

$$\forall x(D(x) \vee P(x))$$

Adding the negation operation yields,

$$\neg\forall x(D(x) \vee P(x))$$

Applying De Morgan's Law to the quantified expression,

$$\exists x\ \neg(D(x) \vee P(x))$$

Applying De Morgan's Law to the compound proposition, we get,

$$\exists x\ \neg D(x) \wedge \neg P(x))$$

Translating the logical expression back to English yields,

There is a patient who was not given the medication and not given the placebo.

(c) There is a patient who took the medication and had migraines.

The logical expression of the above English statement is,

$$\exists x(D(x) \wedge M(x))$$

Adding the negation operation yields,

$$\neg\exists x(D(x) \wedge M(x))$$

Applying De Morgan's Law to the quantified expression,

$$\forall x\ \neg(D(x) \wedge M(x))$$

Applying De Morgan's Law to the compound proposition, we get,

$$\forall x \neg D(x) \vee \neg M(x))$$

Translating the logical expression back to English yields,

Every patient was either not given the medication or did not have migraines (or both).

(d) Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, p → q ≡ ¬p ∨ q.)

The logical expression of the above English statement is,

$$\forall x(\mathrm{P}(x) \rightarrow M(x))$$

Adding the negation operation yields,

$$\neg\forall x(\mathrm{P}(x) \rightarrow M(x))$$

$\neg\forall \mathrm{x}(P(\mathrm{x}) \rightarrow \mathrm{M}(\mathrm{x}))$	**Laws of Propositional Logic**
$\exists \mathrm{x} \neg(P(\mathrm{x}) \rightarrow \mathrm{M}(\mathrm{x}))$	De Morgan's Law
$\exists \mathrm{x} \neg(\neg P(\mathrm{x}) \vee \mathrm{M}(\mathrm{x}))$	Conditional Identity
$\exists \mathrm{x} \neg\neg P(\mathrm{x}) \wedge \neg\mathrm{M}(\mathrm{x}))$	De Morgan's Law
$\exists \mathrm{x}\, P(\mathrm{x}) \wedge \neg\mathrm{M}(\mathrm{x})$	Double Negation Law

Translating the logical expression back to English yields,

There is a patient who was given the placebo and did not have migraines.

(e) There is a patient who had migraines and was given the placebo.

The logical expression of the above English statement is,

$$\exists x(M(x) \wedge P(x))$$

Adding the negation operation yields,

$$\boldsymbol{\neg\exists x(M(x) \wedge P(x))}$$

$\neg\exists x(M(x) \wedge P(x))$	**Laws of Propositional Logic**
$\forall x\, \neg(M(x) \wedge P(x))$	De Morgan's Law
$\forall x\, \neg M(x) \vee \neg P(x)$	De Morgan's Law

Translating the logical expression back to English yields,

Every patient either did not have migraines or was not given the placebo (or both).

---

2) Exercise 1.9.4

(c) $\exists x \forall y\big(P(x,y) \to Q(x,y)\big)$

Applying negation to the nested quantified expression yields,

$$\neg\exists x \forall y\big(P(x,y) \to Q(x,y)\big)$$

$\neg\exists x \forall y\big(P(x,y) \to Q(x,y)\big)$	**Laws of Propositional Logic**
$\forall x \exists y \neg\big(P(x,y) \to Q(x,y)\big)$	De Morgan's Law
$\forall x \exists y \neg\big(\neg P(x,y) \vee Q(x,y)\big)$	Conditional Identity
$\forall x \exists y\big(\neg\neg P(x,y) \wedge \neg Q(x,y)\big)$	De Morgan's Law
$\forall x \exists y\big(P(x,y) \wedge \neg Q(x,y)\big)$	Double Negation Law

Hence, the final quantified expression is $\boldsymbol{\forall x \exists y\big(P(x,y) \wedge \neg Q(x,y)\big)}$.

(d) $\exists x \forall y \big(P(x,y) \leftrightarrow P(y,x)\big)$

Applying negation to the nested quantified expression yields,

$$\neg \exists x \forall y \big(P(x,y) \leftrightarrow P(y,x)\big)$$

$\neg \exists x \forall y \big(P(x,y) \leftrightarrow P(y,x)\big)$	**Laws of Propositional Logic**
$\forall x \exists y \neg \big(P(x,y) \leftrightarrow P(y,x)\big)$	De Morgan's Law
$\forall x \exists y \neg ((P(x,y) \rightarrow P(y,x)) \wedge (P(y,x) \rightarrow P(x,y)))$	Conditional Identity
$\forall x \exists y \neg ((\neg P(x,y) \vee P(y,x)) \wedge (\neg P(y,x) \vee P(x,y)))$	Conditional Identity
$\forall x \exists y \neg ((\neg P(x,y) \vee P(y,x)) \wedge (\neg P(y,x) \vee P(x,y)))$	De Morgan's Law
$\forall x \exists y (\neg(\neg P(x,y) \vee P(y,x)) \vee \neg(\neg P(y,x) \vee P(x,y)))$	De Morgan's Law
$\forall x \exists y ((\neg\neg P(x,y) \wedge \neg P(y,x)) \vee (\neg\neg P(y,x) \wedge \neg P(x,y)))$	De Morgan's Law
$\forall x \exists y ((P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y)))$	Double Negation Law

Hence, the final quantified expression is,

$$\boldsymbol{\forall x \exists y ((P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y)))}$$

(e) $\exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y)$

Applying negation to the nested quantified expression yields,

$$\neg(\exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y))$$

$\neg(\exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y))$	**Laws of Propositional Logic**
$\neg \exists x \exists y P(x,y) \vee \neg \forall x \forall y Q(x,y)$	De Morgan's Law
$\forall x \forall y \neg P(x,y) \vee \exists x \exists y \neg Q(x,y)$	De Morgan's Law

Hence, the final quantified expression is,

$$\boldsymbol{\forall x \forall y \neg P(x,y) \vee \exists x \exists y \neg Q(x,y)}$$
