Parameter estimation

① Mean = θ_1

Variance = θ_2

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

Take log

$$\log L(\theta_1, \theta_2) = -\frac{n}{2}\log(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

for θ_1 diff $\log(L(\theta_1,\theta_2))$ wrt θ_1

$$\frac{d\log(L)}{d\theta_1} = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i-\theta_1) = 0$$

$$\cancel{\theta_1 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2}$$

$$\theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)$$

② $B(m,\theta)$ $\theta \in (0,1)$

m → no. of trials

$$L_\theta = \prod_{i=1}^{n} f(x_i, m, \theta)$$

$$\text{PMF} = {}^{m}C_{x}\, \theta^{m}(1-\theta)$$

$$L(\theta) = \prod_{i=1}^{n} ({}^{m}C_{x_i})(\theta^{x_i})(1-\theta)^{m-x_i}$$

Take log.

$$\log(L(\theta)) = \sum_{i=1}^{n}\log({}^{m}C_{x_i}) + \sum_{i=1}^{n} x_i \log(\theta) + \sum(m-x_i)\log(1-$$

Multiply both sides by $(1-\theta)$

$$(1-\theta)\sum_{i=1}^{n} x_i = \theta\sum_{i=1}^{n}(m-x_i)$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{m}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$