Simulation Exercise with Exponential distribution

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Overview

In this exercise, there will be a demonstration of a simulation of a distribution of a small sample to approximate the population parameter. And the demonstration will use an exponential distribution.

Simulation

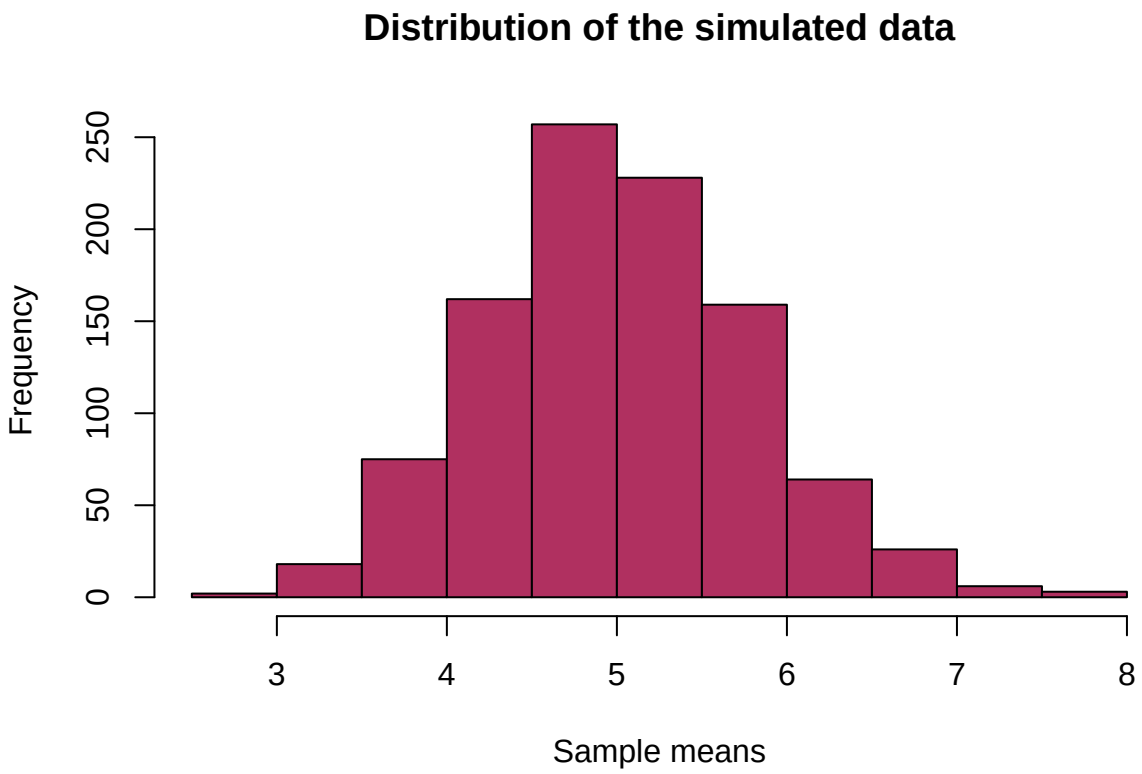
Here, we will let the lambda (rate parameter of the distribution) be 0.2. For an exponential distribution, *mean = Standard deviation = 1/lambda*. Let's set the lambda = 0.2 for all the simulation and we will simulate the average of 40 exponential samples a thousand time.

```
lambda <- 0.2
n <- 40
set.seed(123) # for reproducibility
sim_vec <- NULL # A NULL vector to loop the simulation
for (i in 1:1000) { # loop for thousand time
    sim_vec <- c(sim_vec, mean(rexp(n, rate = lambda))) # average of exponential samples
}
head(sim_vec) # see the simulation
```

```
## [1] 4.811212 5.360077 4.592871 4.900051 5.516619 5.612835
```

The distribution of simulated data of 1000 are shown in the following histogram.

```
hist(sim_vec,
     main = "Distribution of the simulated data",
     xlab = "Sample means",
     col = "maroon")
```

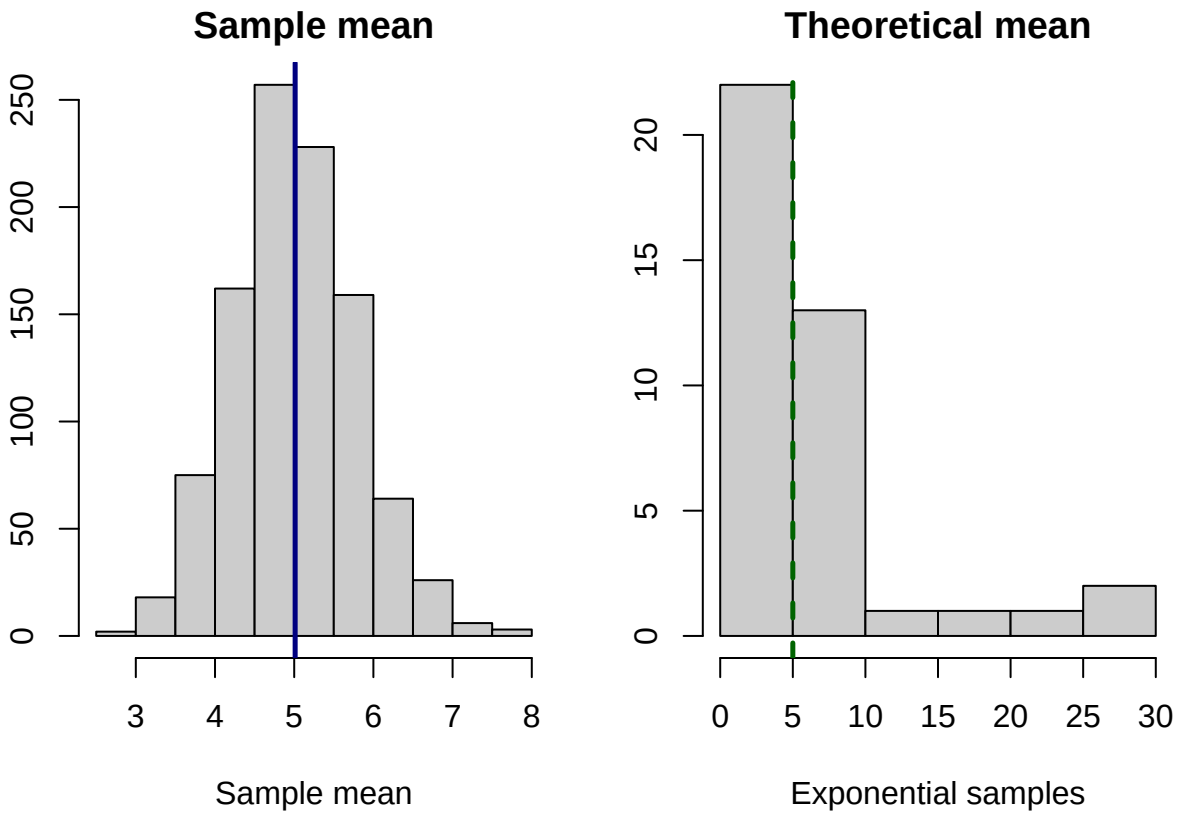


Sample mean versus Theorectical mean

```
## Calculate the means
sample_mean <- mean(sim_vec)
theoretical_mean <- 1/lambda
```

Here:
- The sample mean from average of 1000 simulation is 5
- Theoretical mean = 5.
You can see that the sample mean is equal to theoretical mean.

```
par(mfrow = c(1,2), mar = c(5,2,2,2))
hist(sim_vec, main = "Sample mean", xlab = "Sample mean", col = "grey80")
abline(v = sample_mean, lwd = 2.5, lty= 1, col = "navy")
hist(rexp(40,0.2), main = "Theoretical mean", xlab = "Exponential samples", col = "grey80")
abline(v = theoretical_mean, lwd = 2.5, lty= 2, col = "darkgreen")
```



Sample Variance vs Theoretical Variance

```
sample_variance <- var(sim_vec) # sample standard deviation
theoretical_variance <- 1/lambda^2/n # theoretical standard deviation
```

- Sample variance = 0.6004928
- Theoretical variance = 0.625
  Both the sample variance and theoretical variance are approximately 0.6

Distribution

We will see if the distribution of simulated sample means are normal through visualization.

```
x <- seq(min(sim_vec), max(sim_vec), length.out = 100)
y <- dnorm(x, mean(sim_vec), sd(sim_vec))
hist(sim_vec, breaks = 100, freq = FALSE, axes = FALSE,
     main = "Probability density funtion of the sample means",
     xlab = "Sample mean")
lines(x,y, col = "blue", lwd = 2)
```

The figure shows that the sample means are distributed normally.

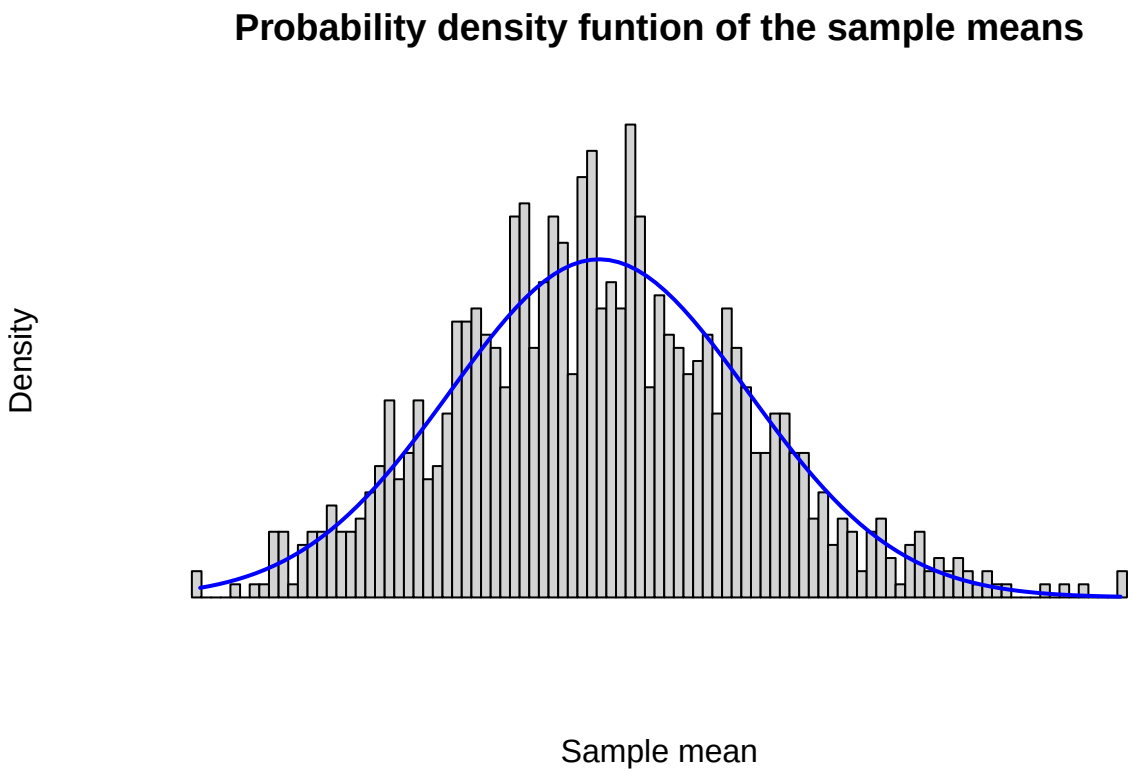


Figure 1: Probabilty density function of the sample means with normal density curve