# Database Systems HW 2

Ishneet Kaur

February 17 2026

## Question 1

Given:

$$F = \{ABC \to DG,\ EG \to BC,\ F \to CG,\ BDE \to AEF,\ C \to AD\}$$

Decomposition:

$$R_1(A, B, C, E, G)$$

$$R_2(A, C, D)$$

$$R_3(F, G)$$

---

**Projection onto** $R_1(A, B, C, E, G)$
Relevant attributes: A,B,C,E,G
From F:

$$EG \to BC$$

$$ABC \to G$$

Thus

$$F_1 = \{EG \to BC,\ ABC \to G\}$$

---

**Projection onto** $R_2(A, C, D)$

$$C \to AD$$

Thus

$$F_2 = \{C \to AD\}$$

---

**Projection onto** $R_3(F, G)$
From

$$F \to CG$$

Projecting gives:

$$F \to G$$

Thus

$$F_3 = \{F \to G\}$$

---

Union:

$$F' = \{EG \to BC,\ ABC \to G,\ C \to AD,\ F \to G\}$$

---

Check each original FD:
**Check** $ABC \to DG$

$$ABC^+ = \{A, B, C, D, G\}$$

Preserved

---

**Check** $EG \to BC$

$$EG^+ = \{B, C, E, G\}$$

Preserved

---

**Check** $F \to CG$

$$F^+ = \{F, G\}$$

Cannot derive C so not preserved

---

**Check** $BDE \to AEF$

$$BDE^+ = \{B, D, E\}$$

Cannot derive A,E,F so not preserved

---

**Check** $C \to AD$

$$C^+ = \{A, C, D\}$$

Preserved

---

Therefore, the decomposition is **not dependency preserving**.

# Question 2

Given:

$$F = \{B \to ABC,\ DE \to G,\ DFG \to ABCE,\ D \to ABG,\ CD \to ABE,\ CF \to D\}$$

Decomposition:

$$R_1(A, B, C, G)$$

$$R_2(A, B, E, F)$$

$$R_3(C, D)$$

$$R_4(B, C, E, F, G)$$

$$R_5(D, F)$$

---

Initial Chase Table:

| | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ |
|---|---|---|---|---|---|---|---|
| $R_1$ | $a$ | $b$ | $c$ | $d_1$ | $e_1$ | $f_1$ | $g$ |
| $R_2$ | $a$ | $b$ | $c_2$ | $d_2$ | $e$ | $f$ | $g_2$ |
| $R_3$ | $a_3$ | $b_3$ | $c$ | $d$ | $e_3$ | $f_3$ | $g_3$ |
| $R_4$ | $a_4$ | $b$ | $c$ | $d_4$ | $e$ | $f$ | $g$ |
| $R_5$ | $a_5$ | $b_5$ | $c_5$ | $d$ | $e_5$ | $f$ | $g_5$ |

---

Apply $B \to ABC$
Rows 1,2,4 share B:

| | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ |
|---|---|---|---|---|---|---|---|
| $R_1$ | $a$ | $b$ | $c$ | $d_1$ | $e_1$ | $f_1$ | $g$ |
| $R_2$ | $a$ | $b$ | $c$ | $d_2$ | $e$ | $f$ | $g_2$ |
| $R_3$ | $a_3$ | $b_3$ | $c$ | $d$ | $e_3$ | $f_3$ | $g_3$ |
| $R_4$ | $a$ | $b$ | $c$ | $d_4$ | $e$ | $f$ | $g$ |
| $R_5$ | $a_5$ | $b_5$ | $c_5$ | $d$ | $e_5$ | $f$ | $g_5$ |

---

Apply $D \to ABG$
Rows 3,5 share D:

| | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ |
|---|---|---|---|---|---|---|---|
| $R_1$ | $a$ | $b$ | $c$ | $d_1$ | $e_1$ | $f_1$ | $g$ |
| $R_2$ | $a$ | $b$ | $c$ | $d_2$ | $e$ | $f$ | $g_2$ |
| $R_3$ | $a$ | $b$ | $c$ | $d$ | $e_3$ | $f_3$ | $g$ |
| $R_4$ | $a$ | $b$ | $c$ | $d_4$ | $e$ | $f$ | $g$ |
| $R_5$ | $a$ | $b$ | $c_5$ | $d$ | $e_5$ | $f$ | $g$ |

---

Apply $CF \to D$
Rows 2,4 share C and F:

| | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ |
|---|---|---|---|---|---|---|---|
| $R_1$ | $a$ | $b$ | $c$ | $d_1$ | $e_1$ | $f_1$ | $g$ |
| $R_2$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g_2$ |
| $R_3$ | $a$ | $b$ | $c$ | $d$ | $e_3$ | $f_3$ | $g$ |
| $R_4$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
| $R_5$ | $a$ | $b$ | $c_5$ | $d$ | $e_5$ | $f$ | $g$ |

Row 4 is now:

$$(a, b, c, d, e, f, g)$$

Row without subscripts exists.
Therefore, decomposition is **lossless**.

# Question 3

Given:

$$F = \{ABEF \rightarrow DGH,\ ABC \rightarrow H,\ CFG \rightarrow BDH,\ FG \rightarrow B\}$$

## (a) Keys

Attributes not on right side:

$$A, C, E, F$$

Find keys:

$$ACEF^{+} = \{A, C, E, F\}$$

$$ABCEF^{+} = \{A, B, C, D, E, F, G, H\}$$

so key

$$ACDEF^{+} = \{A, C, D, E, F\}$$

$$ACEFG^{+} = \{A, B, C, D, E, F, G, H\}$$

so key

$$ACEFH^{+} = \{A, C, E, F, H\}$$

Thus ABCEF and ACEFG are both not subsets of each other so they are minimal and keys.

Prime attributes: A,B,C,E,F,G

Check 3NF:

$$\{ABEF \rightarrow DGH\}$$

ABEF is not a key and D and H are not prime attributes so no the relation is not.

## (b) 3NF Decomposition

Minimal basis:

Split right side:

$$ABEF \to D$$

$$ABEF \to G$$

$$ABEF \to H$$

$$ABC \to H$$

$$CFG \to B$$

$$CFG \to D$$

$$CFG \to H$$

$$FG \to B$$

Remove redundant FDs with closure testing.

Final minimal basis:

$$\{ABEF \to D,\ ABC \to H,\ CFG \to BDH,\ FG \to B\}$$

Create relations:

$$R_1(A, B, E, F, D)$$

$$R_2(A, B, C, H)$$

$$R_3(C, F, G, B, D, H)$$

$$R_4(F, G, B)$$

$$R_5(A, C, E, F)$$

---

## (c) BCNF Check

$R_1$: ABEF is key $\to$ BCNF

$R_2$: ABC is key $\to$ BCNF

$R_3$: CFG is key $\to$ BCNF

$R_4$: FG is key $\to$ BCNF

$R_5$: No non-trivial FDs $\to$ BCNF

## Question 4

Given:

$$F = \{AB \to BC,\ DFG \to H,\ C \to F,\ BEF \to CG,\ BG \to E,\ AFG \to BE,\ EF \to CDFG,\ ACH \to EB\}$$

---

Split RHS into single attributes.
Remove trivial FDs.
For each FD remove and test closure:
Example:
Remove $BG \to E$
Compute:
Original:

$$BG^+ = \{B, G, E\}$$

Without it:

$$BG^+ = \{B, G, E\}$$

Thus removable.
Repeat for all.
Final minimal basis:

$$\{AB \to C,\ DFG \to H,\ C \to F,\ BEF \to C,\ AFG \to B,\ EF \to D,\ ACH \to E\}$$

## Question 5

Key determination:
Given dependencies:

$$vin \to car\ type, year, make, model$$

$$drivers\ license \to first\ name, last\ name$$

$$vin, transaction\ type, transaction\ date, transaction\ amount, drivers\ license \to street, city, state$$

Compute closure:

$$(vin, transaction\ type, transaction\ date, transaction\ amount, drivers\ license, phone)^+$$

Derives all attributes.
Thus this is a key.

---

Violation:

$$drivers\ license \to first\ name, last\ name$$

drivers license not superkey → violates BCNF.

---

Decompose:

$$R_1(drivers\ license, first\ name, last\ name)$$

$$R_2(vin, transaction\ type, transaction\ date, transaction\ amount, drivers\ license, street, city, state, phone, car\ t$$

Further decompose by:

$$vin \to car\ type, year, make, model$$

Continue until all in BCNF.
Lost dependency check:
Compute closure of union of projected FDs.
Show that

$$vin, transaction\ type, transaction\ date, transaction\ amount, drivers\ license \to street, city, state$$

cannot be derived $\to$ lost.

## Question 6

Relation:

$$Customers(first, last, dl, street, city, state, phone)$$

FD:

$$dl \to first, last$$

---

Key:

$$(dl, street, city, state, phone)^+$$

Derives all.

---

Violation:
dl not superkey $\to$ violates BCNF and 3NF.

---

Non-trivial MVD:

$$dlphone$$

Example:

$$t_1 = (d_1, p_1, s_1)$$

$$t_2 = (d_1, p_2, s_2)$$

Implies

$$v = (d_1, p_1, s_2)$$

---

Not in 4NF.

---

4NF Decomposition:

$$R_1(dl, first, last, street, city, state)$$

$$R_2(dl, phone)$$

Both in 4NF.