Database Systems HW 1 for Question 4

Ishneet Kaur

February 3 2026

1 Question 4

You are given the following set F of functional dependencies for a relation $R(A, B, C, D, E, F)$.

$$F = \{A \to D,\ C \to BD,\ BD \to E,\ E \to CDF\}$$

Is $AC \to CEF$ implied by F? In other words, is $AC \to CEF \in F^+$? If yes, use the inference rules to show how you can obtain this functional dependency from the dependencies in F.

Solution

Yes, $AC \to CEF$ is implied by F. This can be shown by computing the closure of AC and by using Armstrong's inference rules.

Part 1: Compute $(AC)^+$

With $A \to D$, we can add D:

$$(AC)^+ = \{A, C, D\}$$

With C now being in the closure, we can use the functional dependency $C \to BD$ to add B:

$$(AC)^+ = \{A, B, C, D\}$$

With both attributes B and D in the closure, we can use the functional dependency $BD \to E$ to add E:

$$(AC)^+ = \{A, B, C, D, E\}$$

With this updated closure, we can now use the functional dependency $E \to CDF$ to add F:

$$(AC)^+ = \{A, B, C, D, E, F\}$$

Since all attributes are included in $(AC)^+$ including CEF, we conclude:

$$AC \to CEF \in F^+$$

Part 2: Inference Rules

We will now examine if the inference rules confirm $AC \rightarrow$ CEF

We will start with a given functional dependency.

Given: $A \rightarrow$ D

Next, we can use augmentation to add C to both sides as we want the left side to represent AC.

Augmentation: $AC \rightarrow$ CD

We can now decompose this to consolidate to the variables we are looking for on the right side.

Splitting: $AC \rightarrow$ C

With C now on the right side alone, we can use another given functional dependency.

Given: $C \rightarrow$ BD

We can now apply transivity.

Transivity: $AC \rightarrow$ C and $C \rightarrow$ BD so $AC \rightarrow$ BD

This now gives us another given functional dependency on the right.

Given: $BD \rightarrow$ E

We can apply transivity again.

Transivity: $AC \rightarrow$ BD and $BD \rightarrow$ E so $AC \rightarrow$ E

Now with all of the implications we have seen for AC, we can combine them to the right side.

Combining: $AC \rightarrow$ C and $AC \rightarrow$ BD and $AC \rightarrow$ E so $AC \rightarrow$ BCDE

To get the last attribute, F, we can use another given functional dependency. For this, we can split to get the E which is on the left side of the functional dependency that will give us F.

Splitting: $AC \rightarrow$ E

Given: $E \rightarrow$ CDF

Transivity: $AC \rightarrow$ E and $E \rightarrow$ CDF so $AC \rightarrow$ CDF

We can now combine everything together and see everything that AC implies.

Combining: $AC \rightarrow$ BCDE and $AC \rightarrow$ CDF so $AC \rightarrow$ BCDEF

To confirm the final answer, we can use splitting to get the exact attributes we are trying to validate.

Splitting: $AC \rightarrow$ CEF

Conclusion

Therefore, $AC \rightarrow CEF$ is implied by the given set of functional dependencies so

$$AC \rightarrow CEF \in F^+$$