① Given: Random Sample $(x_1, x_2, \ldots x_n)$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(-x_i-\mu)^2}{2\sigma^2}}$$

Taking natural log of likelihood func

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{-(x_i-\mu)^2}{2\sigma^2} - \frac{1}{2} \ln(2\pi\sigma^2) \right)$$

to find MLE, diff log likelihood w.r.t θ_1, θ_2

$$\frac{\partial}{\partial\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{x_i-\mu}{\sigma^2} \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i - n\mu = 0$$

$$\frac{\theta_1}{\mu} = \frac{1}{n} \sum x_i$$

for θ_2

$$\frac{\partial}{\partial\theta_2} \ln L(\theta_1, \theta_2) = \sum \left(-\frac{(x_i-\theta_1)^2}{2\theta_2^2} + \frac{1}{2\theta_2} \right) = 0$$

$$\Rightarrow \sum \left(\frac{(x_i-\theta_1)^2}{\theta_2} \right) = \frac{n}{\theta_2} = 0$$

$$\frac{\theta_2^2}{\theta_2} = \frac{1}{n} \sum (x_i-\theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum (x_i-\theta_1)^2$$

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(2) $L(\theta) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$

taking ln

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left(\ln\binom{m}{x_i} + x_i \ln(\theta) + (m-x_i)\ln(1-\theta)\right)$$

$$\frac{\partial}{\partial\theta} \ln(L(\theta)) = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta}\right) = 0$$

Solving for θ

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \sum \frac{m-x_i}{1-\theta}$$

$$\sum x_i (1-\theta) = \sum (m-x_i)\theta$$

$$\theta \sum x_i = m \sum \theta$$

$$\theta = \frac{1}{m} \sum x_i$$

$\therefore$ MLE of θ is sample mean of observations.