Algorithms Tutorial 2 (Worksheet)

Page No.
Date :

1.1 For first loop, ~~complexity~~ loops runs $\log_2 n$ times.
For second loop, loop runs $\log_2 n$ times per iteration of first loop
⇒ So total running time is $(\log_2 n) * (\log_2 n)$
For third, loop runs $n/2$ times per iteration of second loop.
⇒ So total running time is $(\log_2 n \times \log_2 n) \times n/2$

$\therefore W(n) = \frac{n}{2}(\log_2 n)^2 =$

worst case time complexity ⇒ $W(n) = O(n(\log n)^2)$

1.2 For each iteration of the first loop,
For $j = \underbrace{1, 2, 4, 8, \ldots, n}_{(\log_2 n) + 1 \text{ terms}}$

The innermost loop runs $\underbrace{1 + 2 + 4 + 8 \ldots n}_{(\log_2 n + 1) \text{ terms}}$ times

$\Rightarrow \sum_{i=0}^{\log_2 n} 2^i$ (term = time per iteration of first loop)

$\Rightarrow 2^0 \frac{(2^{\log_2 n + 1} - 1)}{2 - 1} = 2 \cdot 2^{\log_2 n} - 1$

$= 2 \cdot n - 1$ times per each iteration of first loop

$\therefore$ For n iterations of first loop, total running time,

$W(n) = n(2n - 1)$

$= 2^m (2 \cdot 2^m - 1)$

$= 2 \cdot 2^{2m} - 2^m$

$\Rightarrow W(m) = O(2^{2m})$ or $W(n) = O(n^2)$

1.3 For each iteration of the first loop, i.e,

for temp = $\overset{1}{\downarrow}, \overset{2}{\downarrow}, \overset{4}{\downarrow}, \overset{8}{\downarrow} \cdots \overset{n}{\downarrow}$,

i runs for $1, 2, 4, 8 \cdots n$ times respectively.

Now if j loop is added too,

The total running time becomes

$$1\left(\frac{n}{2} + \log_2 n\right) + 2\left(\frac{n}{2} + \log_2 n\right) + 4\left(\frac{n}{2} + \log_2 n\right) \cdots + n\left(\frac{n}{2} + \log_2 n\right)$$

($\log_2 n + 1$) terms

$$\Rightarrow \sum_{i=0}^{\log_2 n} 2^i \left(\frac{n}{2} + \log_2 n\right)$$

$$\Rightarrow \left(\frac{n}{2} + \log_2 n\right)\left(\sum_{i=0}^{\log_2 n} 2^i\right)$$

$$\Rightarrow \left(\frac{n}{2} + \log_2 n\right)\left(\frac{2^0\left(2^{\log_2 n + 1} - 1\right)}{2 - 1}\right)$$

$$\Rightarrow \left(\frac{n}{2} + \log_2 n\right)\left(2 \cdot 2^{\log_2 n} - 1\right)$$

$$\Rightarrow \left(\frac{n}{2} + \log_2 n\right)\left(2n - 1\right)$$

$$\Rightarrow n^2 - \frac{n}{2} + 2n \log_2 n - \log_2 n$$

$\therefore W(n) = n^2 - \frac{n}{2} + 2n \log_2 n - \log_2 n$

$W(n) = O(n^2)$

$W(n) = \Theta(n^2)$