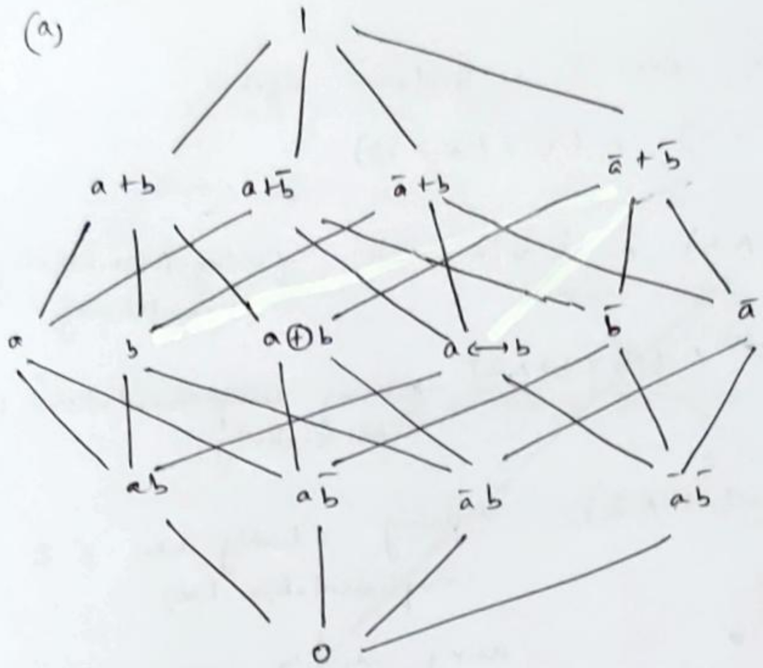
Kaushal Banthia
19CS10039

Q1 (a)



(b) Here $(a \oplus b) + (\bar{a}+b)$ indicates $(a \oplus b)$ join $(\bar{a}+b)$

$\Rightarrow$ The upper bound of $a \oplus b = \{a+b, 1, \bar{a}+\bar{b}, a \oplus b\}$

The upper bound of $\bar{a}+b = \{\bar{a}+b, 1\}$

The upper bound of $a \oplus b$ and $\bar{a}+b = \{1\}$

$\therefore$ The lowest upper bound of $a \oplus b$ and $\bar{a}+b = \{1\}$

$\therefore (a \oplus b) + (\bar{a}+b) = 1$

(c) Here $(a \oplus b) \cdot (\bar{a}+b)$ indicates $(a \oplus b)$ meet $(\bar{a}+b)$

$\Rightarrow$ The lower bound of $a \oplus b = \{a\bar{b}, 0, \bar{a}b, a \oplus b\}$

The lower bound of $\bar{a}+b = \{\bar{a}+b, b, ab, \bar{a}b, 0, a \leftrightarrow b, \bar{a}\bar{b}, \bar{a}\}$

The ~~lower~~ greatest lower bound of $a \oplus b$ and $\bar{a}+b = \{\bar{a}b\}$

$\therefore (a \oplus b) \cdot (\bar{a}+b) = \bar{a}b.$

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Q2

(a) For $(a \wedge \bar{b}) \vee (\bar{a} \wedge b) = 0$,

LHS $= (a \wedge \bar{b}) \vee (\bar{a} \wedge b)$

using distributivity, (since its Boolean algebra, distributivity holds)

$= (a \vee (\bar{a} \wedge b)) \wedge (\bar{b} \vee (\bar{a} \wedge b))$

~~$= ((a \vee \bar{a}) \wedge b) \wedge (\bar{b} \vee (b \wedge \bar{a}))$ (using associativity & commutativity)~~

~~$= (1 \wedge b) \wedge ((\bar{b} \vee b) \wedge \bar{a})$~~ (using complementation law & ~~Associativity~~)

~~$= b \wedge (1 \wedge a$~~

$= [(a \vee \bar{a}) \wedge (a \vee b)] \wedge [(\bar{b} \vee \bar{a}) \wedge (\bar{b} \vee b)]$

$= [1 \wedge (a \vee b)] \wedge [(\bar{b} \vee \bar{a}) \wedge 1]$ (complementation Law)

$= (a \vee b) \wedge (\bar{b} \vee \bar{a})$ (Identity Law)

For this to be 0, either $a \vee b = 0$ or $\bar{b} \vee \bar{a} = 0$

Case 1 $\rightarrow a \vee b = 0$

This implies that both a and b are 0.

Case 2 $\rightarrow \bar{b} \vee \bar{a} = 0$

This implies that both $\bar{b}$ and $\bar{a}$ are 0.

This implies that both b and a are 1

Thus, the LHS = 0 iff $a = b = 0$ or $a = b = 1$
For other values of a and b, $a \neq b$ and LHS = 1.
Thus, we can say that

$a = b$ If and only if $(a \wedge \bar{b}) \vee (\bar{a} \wedge b) = 0$

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(b) For $(a \wedge \bar{b}) \vee (\bar{a} \wedge b) = b$,

LHS $= (a \wedge \bar{b}) \vee (\bar{a} \wedge b)$

~~For from part (a)~~,

from part (a), we know that

LHS $= (a \wedge \bar{b}) \vee (\bar{a} \wedge b)$ ~~$= (a \vee b) \wedge (a \vee \bar{b})$~~

$= (a \vee b) \wedge (\bar{b} \vee \bar{a})$

Here if $a = 0$, LHS $= (0 \vee b) \wedge (\bar{b} \vee \bar{0})$

$= (b) \wedge (\bar{b} \vee 1)$ (Using Identity and Complementation Laws)

$= (b) \wedge (1)$ (Using Identity Laws)

$= b$ (Using Identity Laws)

if $a = 1$, LHS $= (1 \vee b) \wedge (\bar{b} \vee \bar{1})$

$= 1 \wedge (\bar{b} \vee 0)$ (Using Identity & Complementation Laws)

$= (\bar{b})$ (Using Identity Laws)

Since a can take only 2 values, either 0 or 1, this was exhaustive and we ~~cay~~ say that when $a = 0$, LHS $= b$.

Also, when LHS $= b$, $a = 0$ (as the LHS can also have only 2 values, b or its complement).

$\therefore$ $a = 0$ if and only if $(a \wedge \bar{b}) \vee (\bar{a} \wedge b) = b$. $\forall b \in B$.