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(a) For $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$ s.t. $Ax = b$.

→ For existence of solution : $b \in$ ~~cols~~ colspan(A)

$\Rightarrow b \in \text{span}\{a_1, a_2, \ldots, a_n\}$

where $A = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix}$

→ For uniqueness of solution ; If columns of A form basis for colspan(A), then $\exists$ a <u>unique</u> x st. $Ax = b$.

An equivalent statement would, if the columns of A were linearly independent, then $\exists$ a <u>unique</u> x, s.t. $A.x = b$

This is because Ax can be written as $a_1 x_1 + a_2 x_2 + \cdots + a_n x$

where $A = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix}$ and $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$a_i \in \mathbb{R}^m$ and $x_i \in \mathbb{R}$

Thus, if columns of A (a_i) are linearly independent, then b can be written uniquely as their linear combination with coefficients x_i.