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# Experiment 3 Part 2

Objective: To find the frequency of an RLC circuit for different values of resistance (and thus zeta)

Theory: For the given series RLC circuit, we can obtain the same circuit in Laplace domain by making $R \to R$, $L \to Ls$, $C \to \frac{1}{Cs}$.

Then, we have $\mathcal{L}\{V_o(t)\} = I \cdot \frac{1}{Cs}$

$$\mathcal{L}\{V_o(t)\} = \frac{\mathcal{L}\{V_{in}(t)\}}{R + Ls + \frac{1}{Cs}} \cdot \frac{1}{Cs}$$

$$\Rightarrow \frac{\mathcal{L}\{V_o(t)\}}{\mathcal{L}\{V_{in}(t)\}} = \frac{1}{RCs + LCs^2 + 1}$$

Now $Y(s) = H(s) \cdot X(s)$

where $Y(s)$ is Output, $X(s)$ is Input and $H(s)$ is transfer function

$$\therefore H(s) = \frac{Y(s)}{X(s)}$$

$$\therefore \text{Our } H(s) = \frac{1}{LCs^2 + RCs + 1} \quad -①$$

Now, the standard Laplace domain transfer function is given by. (for series RLC)

$$H(s) = \frac{1}{\left(\frac{1}{\omega_n^2}\right)s^2 + \left(\frac{2\xi_p}{\omega_n}\right)s + 1} \quad -②$$

$\therefore$ On comparing ① and ②, we get,

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\frac{2\xi_p}{\omega_n} = RC \Rightarrow \xi_p = \frac{RC\,\omega_n}{2} = \frac{R}{2} C \cdot \frac{1}{\sqrt{LC}} = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$\Rightarrow R = 2\xi_p\sqrt{\frac{L}{C}}$, where $L = 0.1\text{ H}$ and $C = 0.1\,\mu F$

$$\therefore R = 2\xi_p\sqrt{\frac{0.1}{0.1\times10^{-6}}} = 2\xi_p \times 10^3$$

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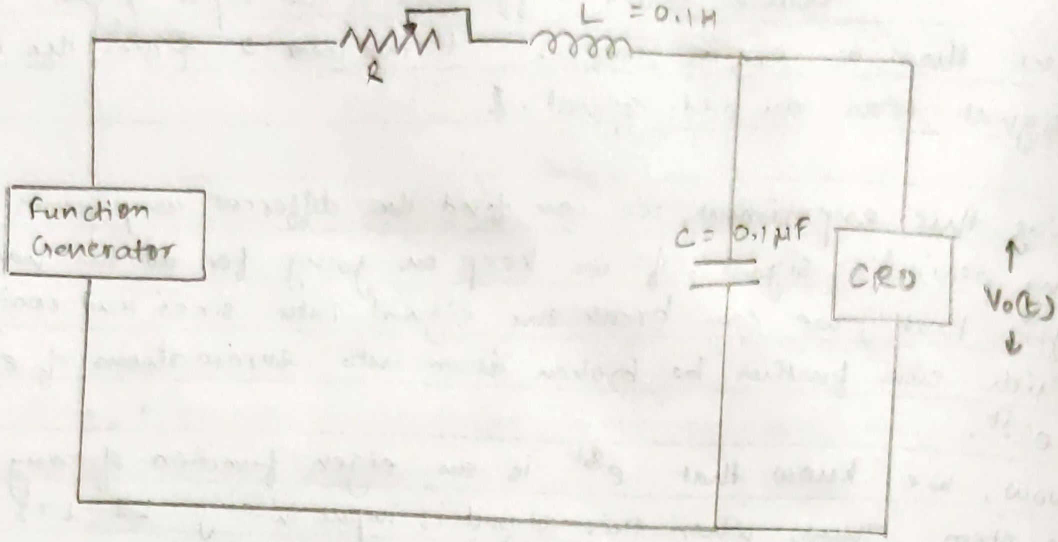

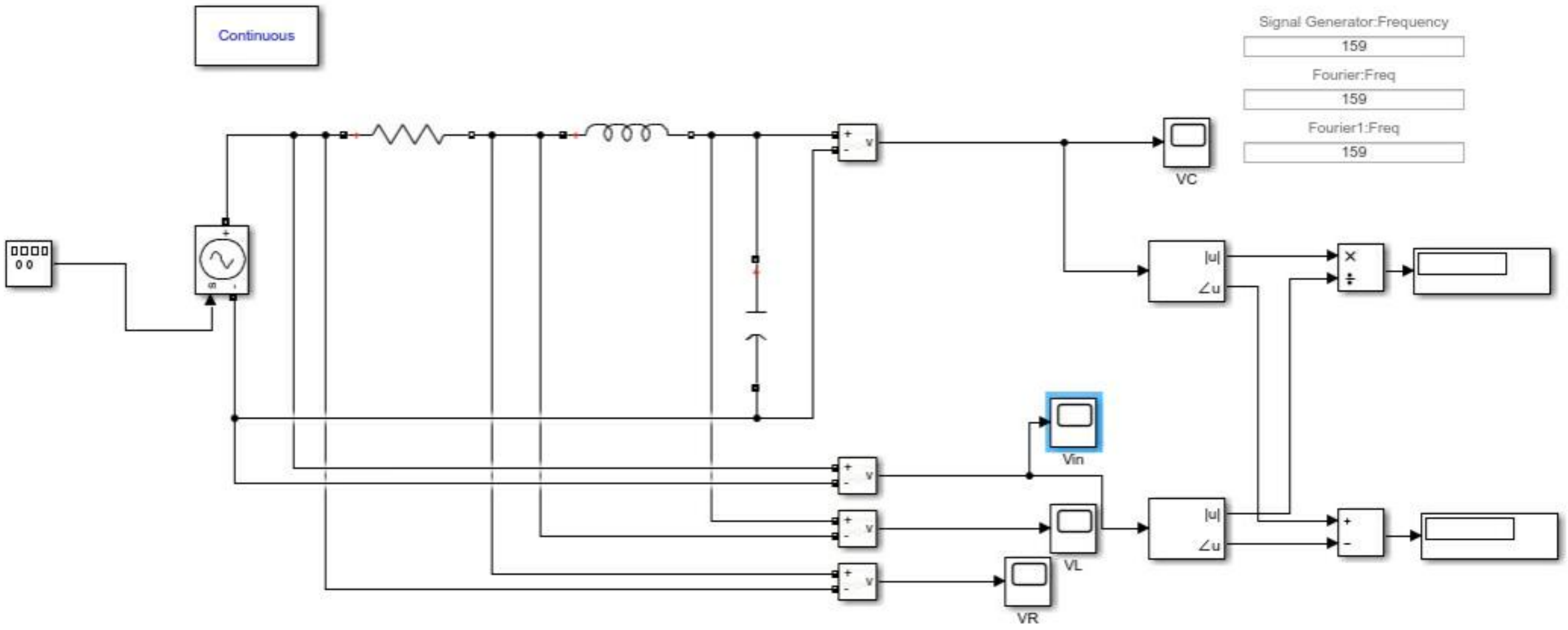
Continuous
Signal Generator:Frequency
159
Fourier:Freq
159
Fourier1:Freq
159
VC
Vin
VL
VR
|u|
∠u
|u|
∠u

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$\therefore R = 2000\ \Omega$

Equipment (Simulink Blocks) Needed:

- Controlled Voltage Source
- Signal Generator
- Series RLC Branch (Made into Resistor)
- Series RLC Branch (Made into Inductor)
- Series RLC Branch (Made into capacitor)
- Powergui (Continuous)
- 4x Voltage Measurement
- 4x Scope
- 2x Fourier
- Add
- Divide
- 2x Display
- 3x Edit

Simulink Procedure for Running the experiment:

→ Change the solver to ode23t this time.

→ Set Amplitude to 3V in the Signal Generator (Sine Wave)

→ Set ~~R~~ L = 0.1 H and C = 0.1 μF and then vary R according to zeta values

→ Click Run and measure Voltage across R, C, L and then find Vout/Vin and Phase.

→ Perform this experiment for different frequencies to obtain a range of values.

→ Plot the magnitude response

→ Plot the phase response.

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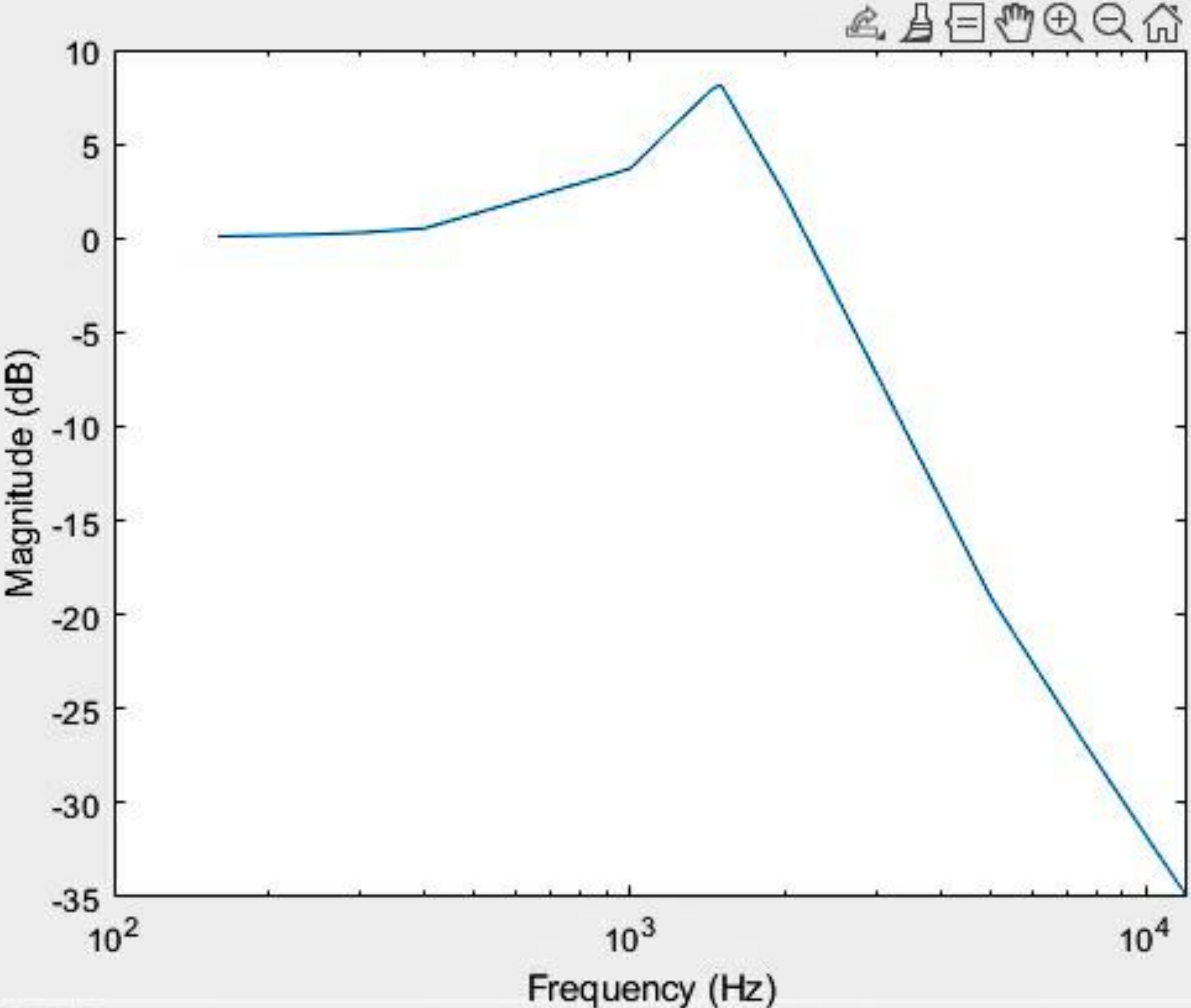
Observation Table:

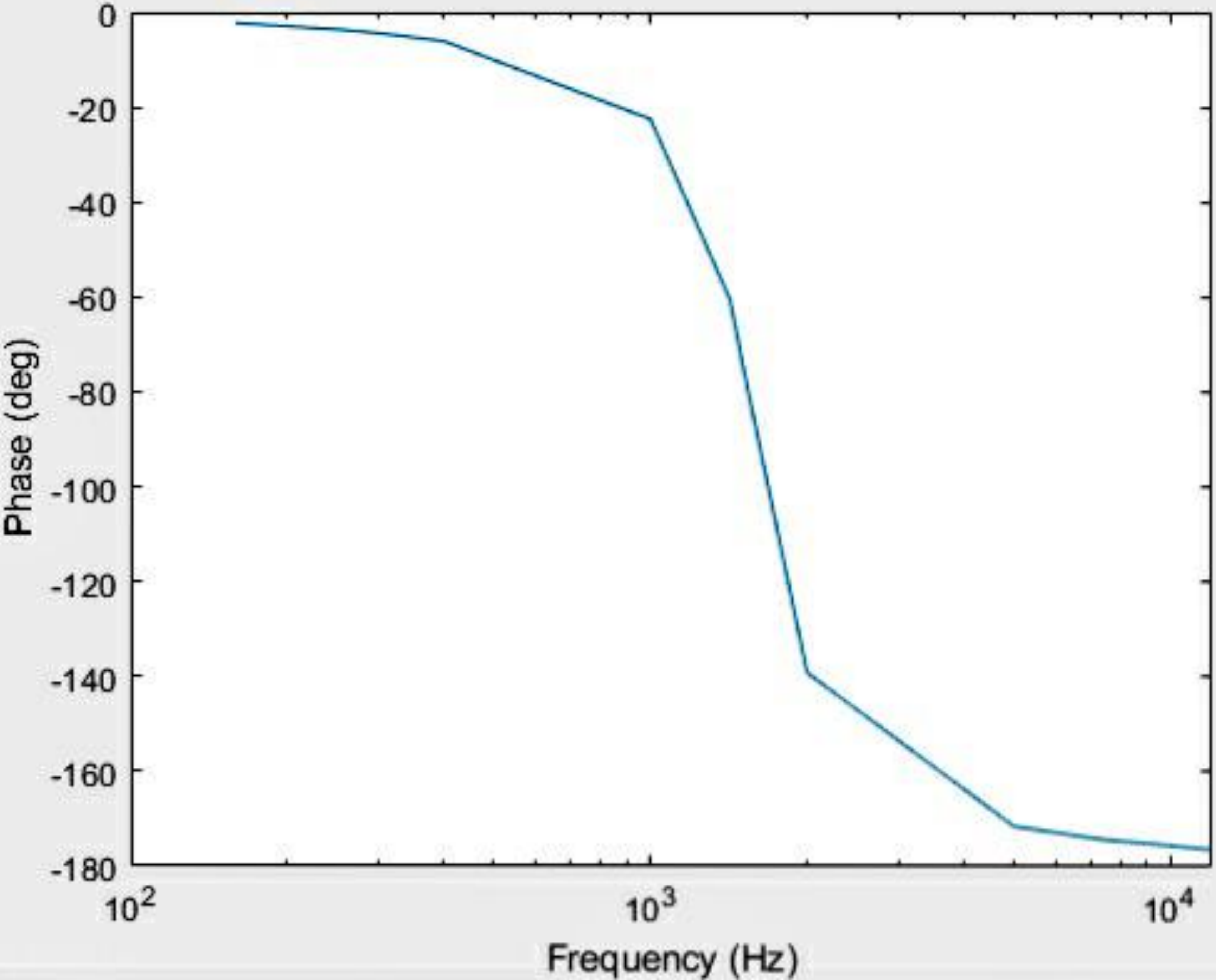
(i) $\xi = 0.2 \Rightarrow R = 2000 \times 0.2 = 400\ \Omega$

for input ($V_{in}$) of 3V Amplitude sine wave,

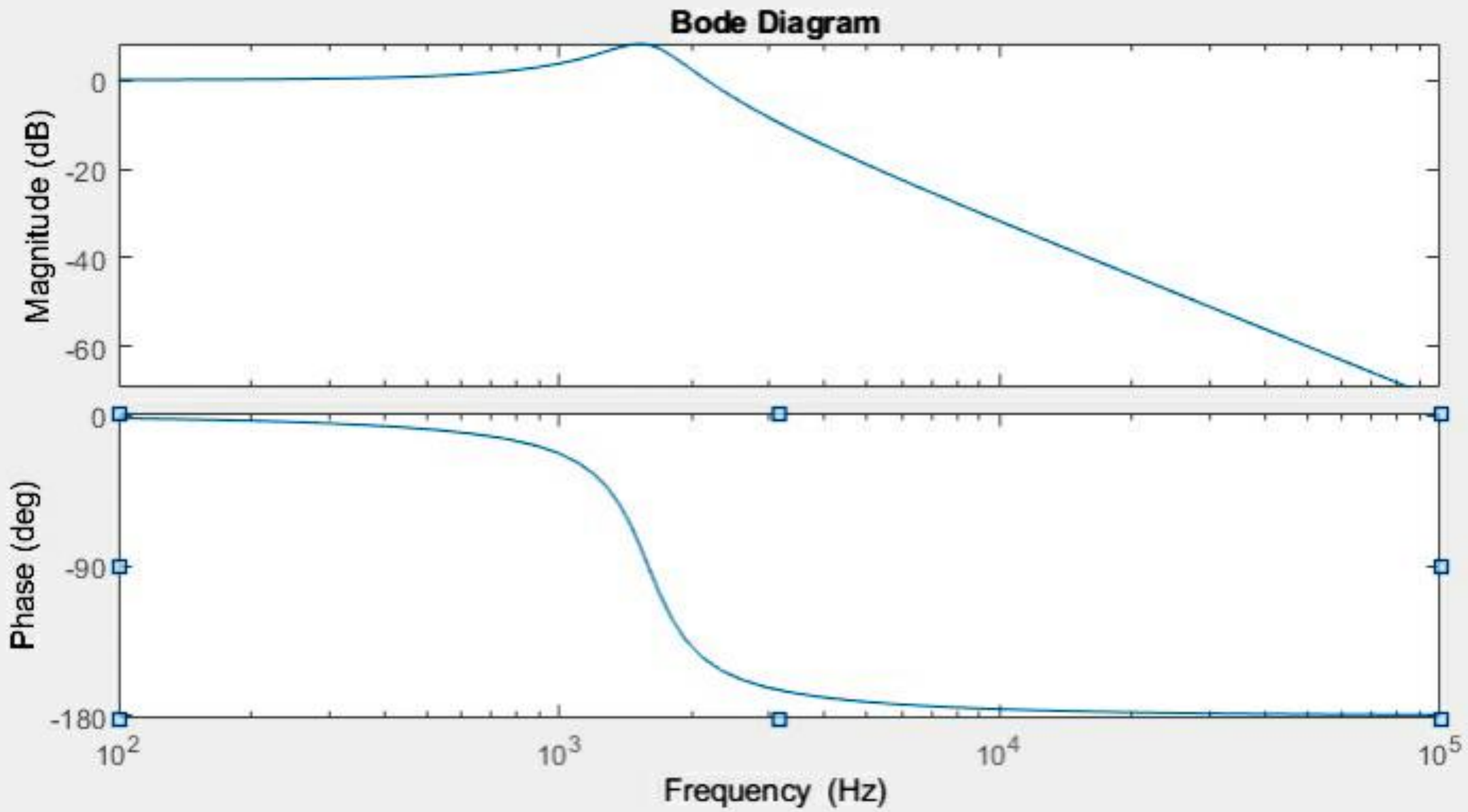
| frequency (f) in Hz | $V_c$ (in V) | $V_L$ (in V) | $V_R$ (in V) | Gain ($V_c/V_{in}$) | Phase (Degrees) |
|---|---|---|---|---|---|
| 159 | 3.025 | 0.030 | 0.121 | 1.010 | -2.232 |
| 250 | 3.070 | 0.076 | 0.193 | 1.024 | -3.612 |
| 300 | 3.101 | 0.110 | 0.234 | 1.034 | -4.378 |
| 400 | 3.183 | 0.201 | 0.320 | 1.062 | -6.050 |
| 1000 | 4.586 | 1.817 | 1.154 | 1.529 | -22.500 |
| 1420 | 7.314 | 5.843 | 2.615 | 2.441 | -60.408 |
| 1441 | 7.428 | 6.114 | 2.696 | 2.480 | -63.721 |
| 1460 | 7.518 | 6.348 | 2.764 | 2.509 | -66.847 |
| 1500 | 7.632 | 6.810 | 2.884 | 2.549 | -73.730 |
| 2000 | 3.878 | 6.156 | 1.954 | 1.294 | -139.120 |
| 5000 | 0.332 | 3.303 | 0.419 | 0.111 | -171.480 |
| 7500 | 0.140 | 3.128 | 0.264 | 0.047 | -174.310 |
| 10000 | 0.077 | 3.071 | 0.195 | 0.026 | -175.490 |
| 12000 | 0.053 | 3.043 | 0.161 | 0.018 | -176.390 |



10
5
0
-5
-10
-15
-20
-25
-30
-35
Magnitude (dB)
$10^2$
$10^3$
$10^4$
Frequency (Hz)



0
-20
-40
-60
-80
-100
-120
-140
-160
-180
Phase (deg)
$10^2$
$10^3$
$10^4$
Frequency (Hz)


Bode Diagram
Magnitude (dB)
0
-20
-40
-60
Phase (deg)
0
-90
-180
10^2
10^3
10^4
10^5
Frequency (Hz)

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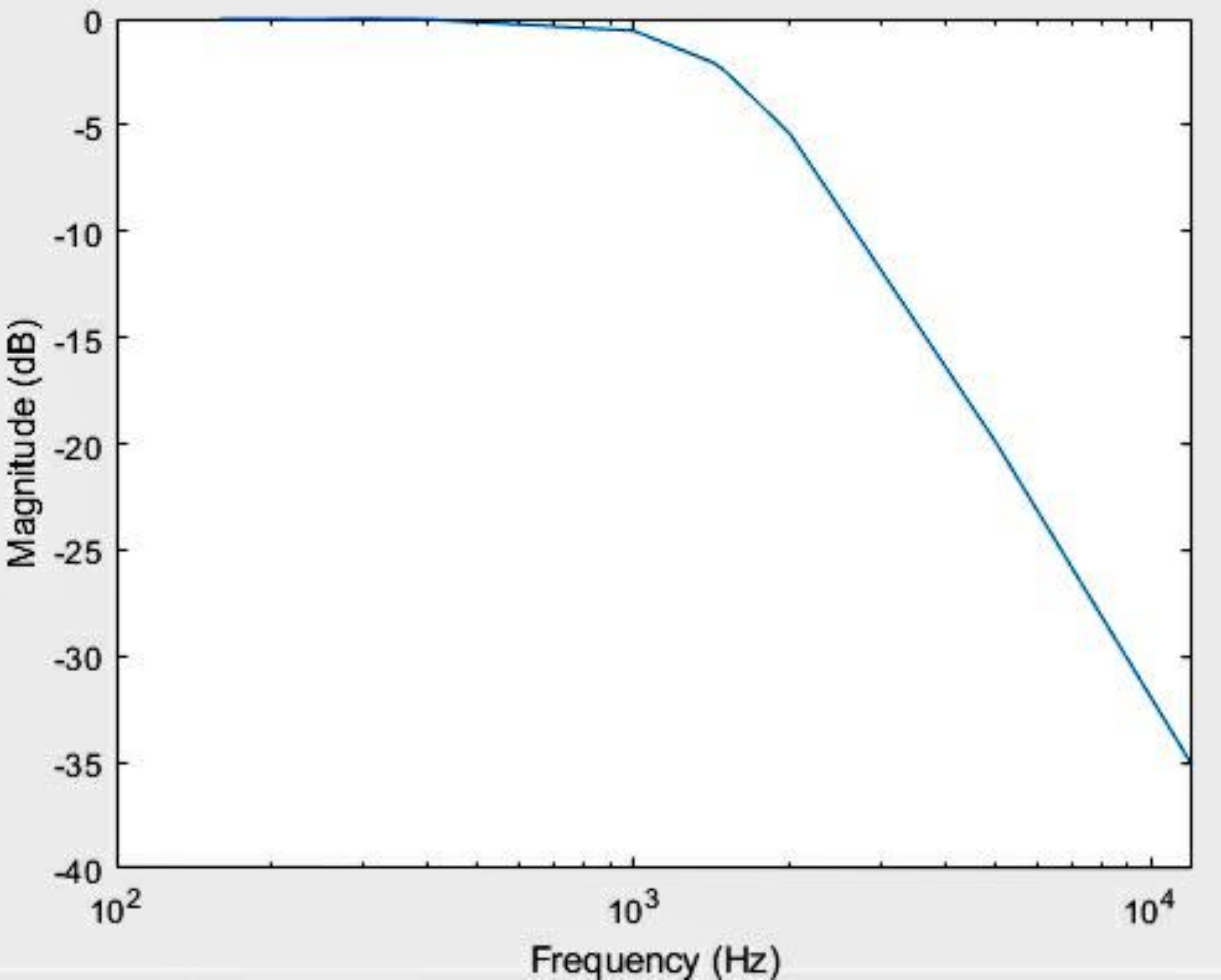
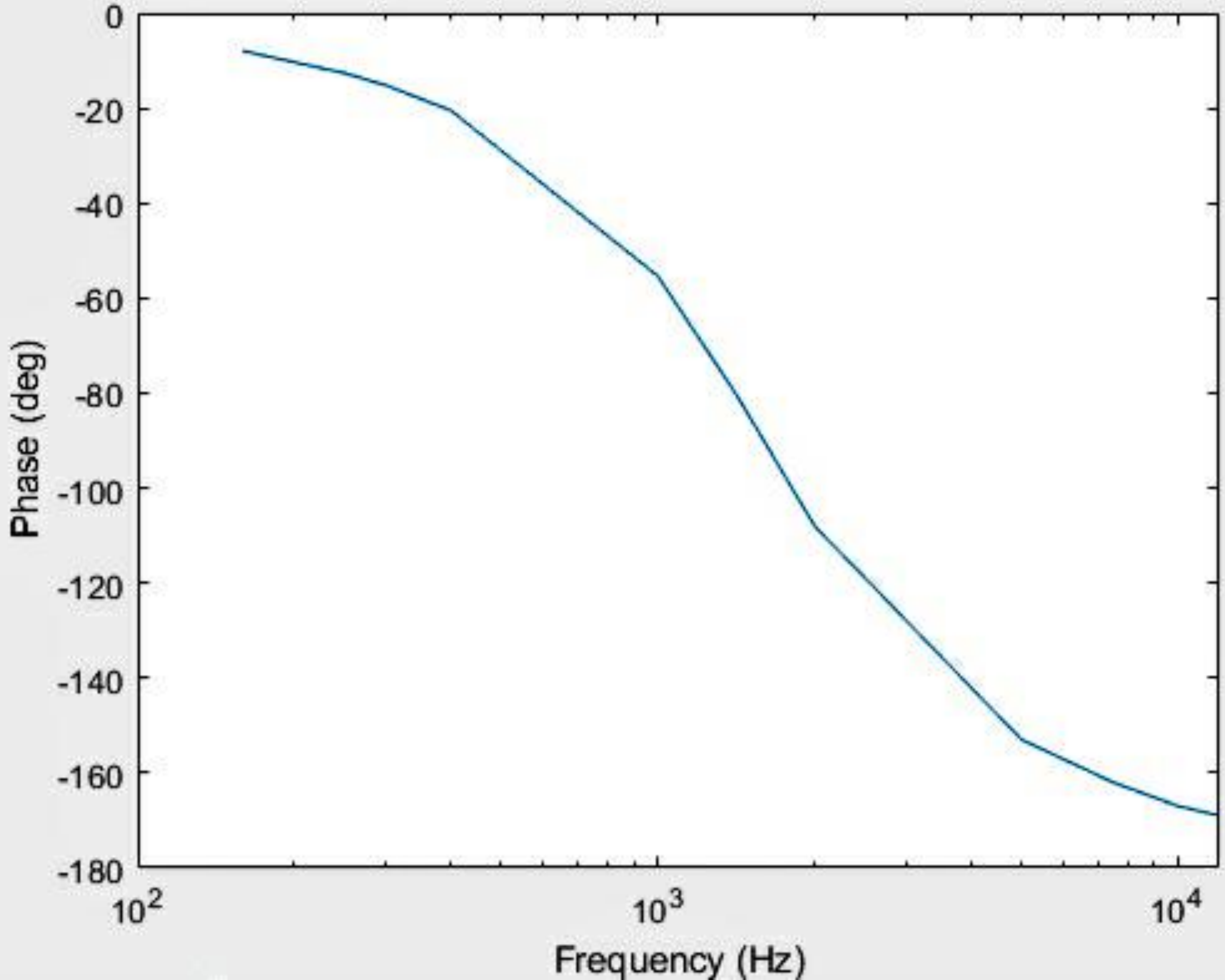
(ii) $\xi = 0.7 \Rightarrow R = 2000 \times 0.7 = 1400\,\Omega$

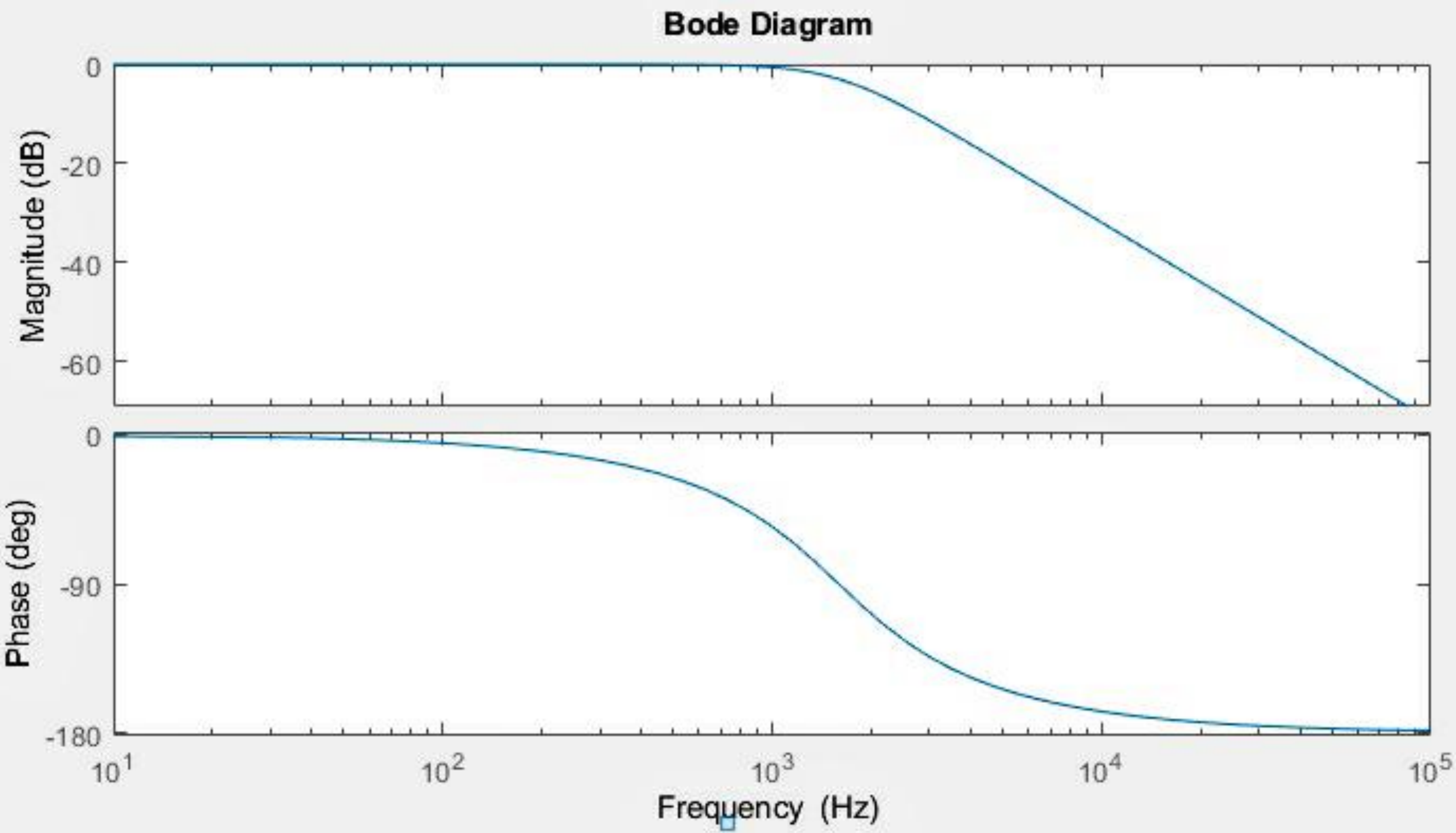
For input (Vin) = 3V(Amplitude) Sine wave.

| Frequency (f) in Hz | $V_c$ (in V) | $V_L$ (in V) | $V_R$ (in V) | Gain ($V_c$/ Vin) | Phase (Degree) |
|---|---|---|---|---|---|
| 159 | 2.995 | 0.030 | 0.419 | 0.998 | -7.950 |
| 250 | 2.992 | 0.074 | 0.661 | 1.000 | -12.644 |
| 300 | 3.000 | 0.107 | 0.793 | 0.998 | -15.224 |
| 400 | 2.990 | 0.190 | 1.054 | 0.997 | -20.500 |
| 1000 | 2.809 | 1.112 | 2.474 | 0.938 | -55.420 |
| 1420 | 2.367 | 1.891 | 2.962 | 0.790 | -80.631 |
| 1441 | 2.339 | 1.925 | 2.971 | 0.781 | -81.846 |
| 1460 | 2.315 | 1.955 | 2.978 | 0.772 | -82.842 |
| 1500 | 2.261 | 2.017 | 2.990 | 0.755 | -85.010 |
| 2000 | 1.614 | 2.561 | 2.846 | 0.538 | -108.120 |
| 5000 | 0.301 | 2.991 | 1.328 | 0.101 | -153.220 |
| 7500 | 0.134 | 2.999 | 0.888 | 0.045 | -162.000 |
| 10000 | 0.075 | 3.001 | 0.666 | 0.025 | -166.610 |
| 12000 | 0.052 | 3.001 | 0.555 | 0.018 | -168.690 |



0
-5
-10
-15
-20
-25
-30
-35
-40
Magnitude (dB)
$10^2$
$10^3$
$10^4$
Frequency (Hz)


0
-20
-40
-60
-80
-100
-120
-140
-160
-180
Phase (deg)
$10^2$
$10^3$
$10^4$
Frequency (Hz)



Bode Diagram
Magnitude (dB)
0
-20
-40
-60
Phase (deg)
0
-90
-180
$10^1$
$10^2$
$10^3$
$10^4$
$10^5$
Frequency (Hz)

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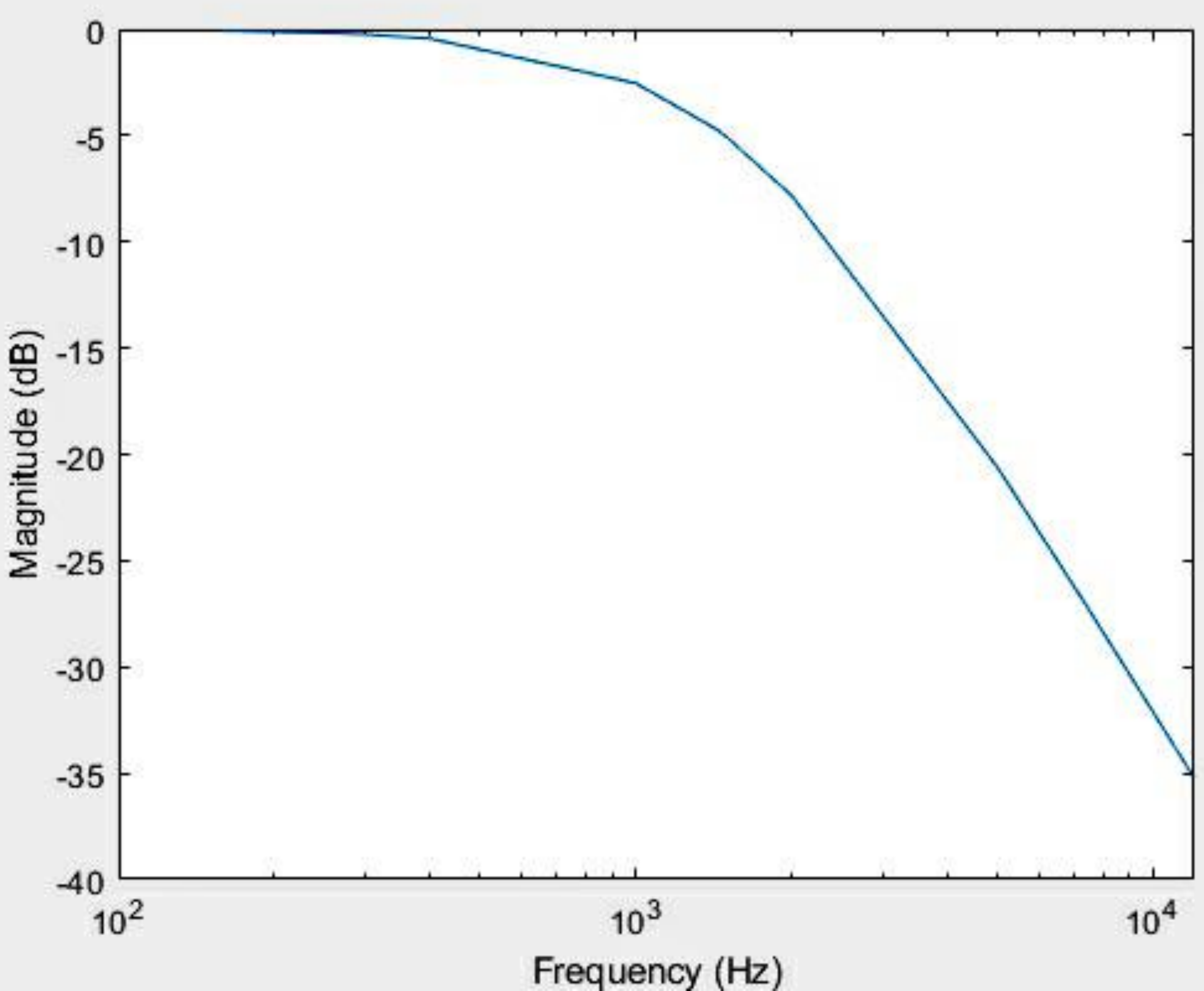
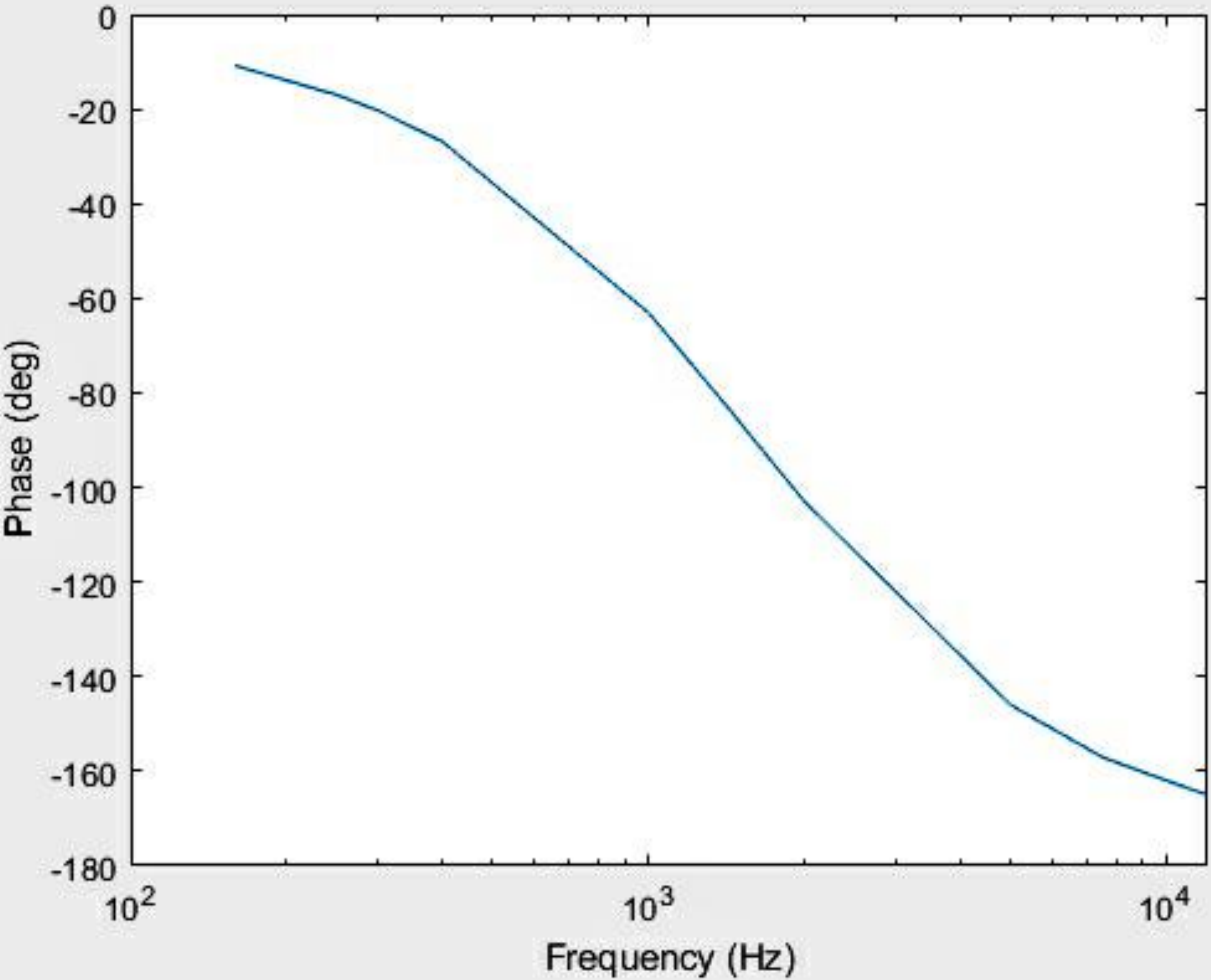
(iii) $\xi_f = 0.95$ $\Rightarrow$ $R = 2000 \times 0.95 = 1900\ \Omega$

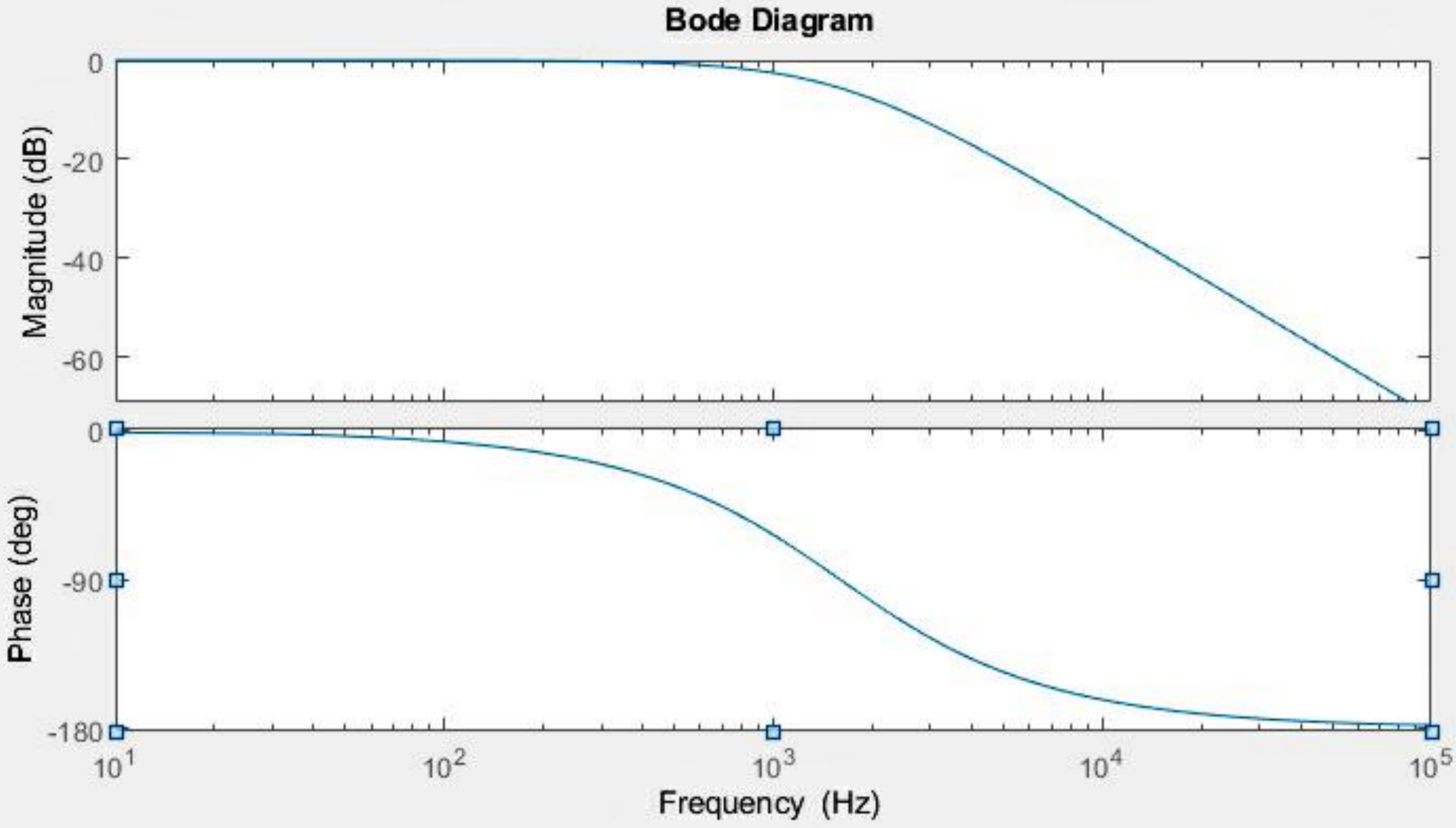
for input ($V_{in}$) = 3V (Amplitude) as sine wave.

| Frequency (f) in Hz | $V_C$ (in V) | $V_L$ (in V) | $V_R$ (in V) | Gain ($V_C/V_{in}$) | Phase (Degrees) |
|---|---|---|---|---|---|
| 159 | 2.970 | 0.030 | 0.535 | 0.993 | -10.790 |
| 250 | 2.936 | 0.073 | 0.877 | 0.981 | -17.010 |
| 300 | 2.913 | 0.104 | 1.046 | 0.973 | -20.270 |
| 400 | 2.852 | 0.181 | 1.364 | 0.951 | -26.940 |
| 1000 | 2.239 | 0.887 | 2.676 | 0.747 | -63.050 |
| 1420 | 1.754 | 1.402 | 2.979 | 0.585 | -83.020 |
| 1441 | 1.732 | 1.425 | 2.984 | 0.578 | -83.840 |
| 1460 | 1.711 | 1.445 | 2.988 | 0.571 | -84.640 |
| 1500 | 1.669 | 1.489 | 2.995 | 0.557 | -86.290 |
| 2000 | 1.218 | 1.931 | 2.914 | 0.406 | -103.490 |
| 5000 | 0.279 | 2.771 | 1.670 | 0.093 | -145.610 |
| 7500 | 0.129 | 2.895 | 1.163 | 0.043 | -156.700 |
| 10000 | 0.074 | 2.940 | 0.886 | 0.025 | -162.300 |
| 12000 | 0.052 | 2.959 | 0.742 | 0.017 | -165.000 |



0
-5
-10
-15
-20
-25
-30
-35
-40
Magnitude (dB)
$10^2$
$10^3$
$10^4$
Frequency (Hz)


0
-20
-40
-60
-80
-100
-120
-140
-160
-180
Phase (deg)
$10^2$
$10^3$
$10^4$
Frequency (Hz)


Bode Diagram
0
-20
-40
-60
Magnitude (dB)
0
-90
-180
Phase (deg)
$10^1$
$10^2$
$10^3$
$10^4$
$10^5$
Frequency (Hz)

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(iv) $\xi_f = 1.5$ $\Rightarrow R = 2000 \times 1.5 = 3000\ \Omega$

For input ($V_{in}$) = 3V (Amplitude) as sine waves

| Frequency (f) in Hz | $V_C$ (in V) | $V_L$ (in V) | $V_R$ (in V) | Gain ($V_C/V_{in}$) | Phase Degree. |
|---|---|---|---|---|---|
| 159 | 2.897 | 0.029 | 0.869 | 0.967 | −16.780 |
| 250 | 2.767 | 0.069 | 1.306 | 0.924 | −25.710 |
| 300 | 2.683 | 0.096 | 1.520 | 0.895 | −30.300 |
| 400 | 2.488 | 0.158 | 1.883 | 0.832 | −38.760 |
| 1000 | 1.513 | 0.599 | 2.857 | 0.505 | −72.100 |
| 1420 | 1.115 | 0.892 | 2.992 | 0.372 | −85.440 |
| 1441 | 1.100 | 0.905 | 2.993 | 0.367 | −86.020 |
| 1460 | 1.086 | 0.918 | 2.995 | 0.363 | −86.480 |
| 1500 | 1.058 | 0.944 | 2.998 | 0.353 | −87.540 |
| 2000 | 0.785 | 1.244 | 2.965 | 0.262 | −98.480 |
| 5000 | 0.231 | 2.291 | 2.181 | 0.077 | −133.000 |
| 7500 | 0.117 | 2.616 | 1.660 | 0.039 | −146.000 |
| 10000 | 0.069 | 2.766 | 1.315 | 0.023 | −153.400 |
| 12000 | 0.049 | 2.831 | 1.122 | 0.017 | −157.400 |

Calculations :

for (i), The maximum $\text{gain}_{\text{observed}} = 2.549$ occurs at $f = 1500$ Hz

$\therefore f_{m,\text{observed}} = 1500$ Hz.

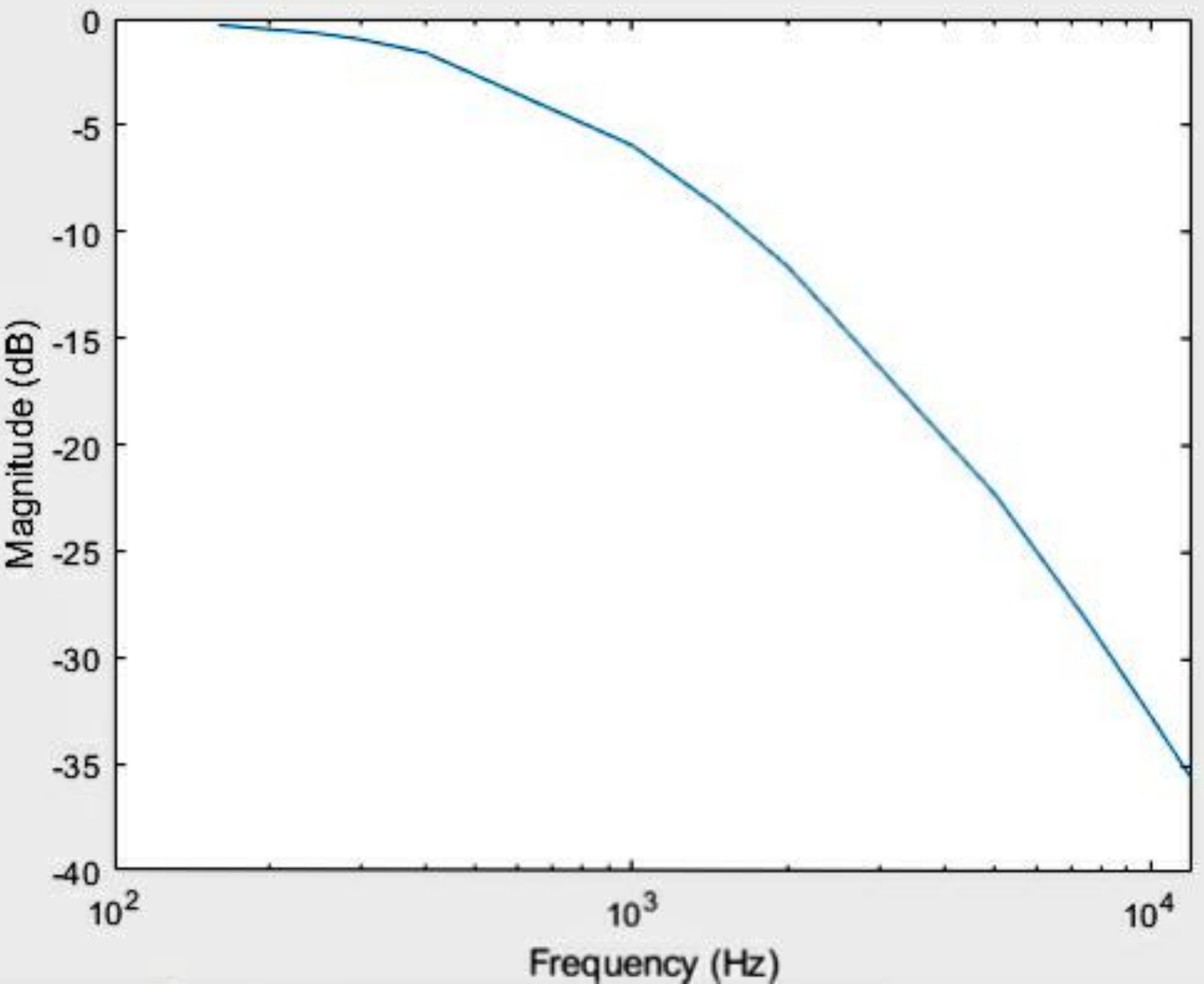
The maximum gain, theoretical $= \dfrac{1}{2\xi\sqrt{1-\xi^2}} = \dfrac{1}{2\times0.2\sqrt{1-0.04}} = 2.552$

$f_m$, theoretical $= \dfrac{\omega_n}{2\pi} \times \sqrt{1-2\xi^2} = 1527$ Hz.

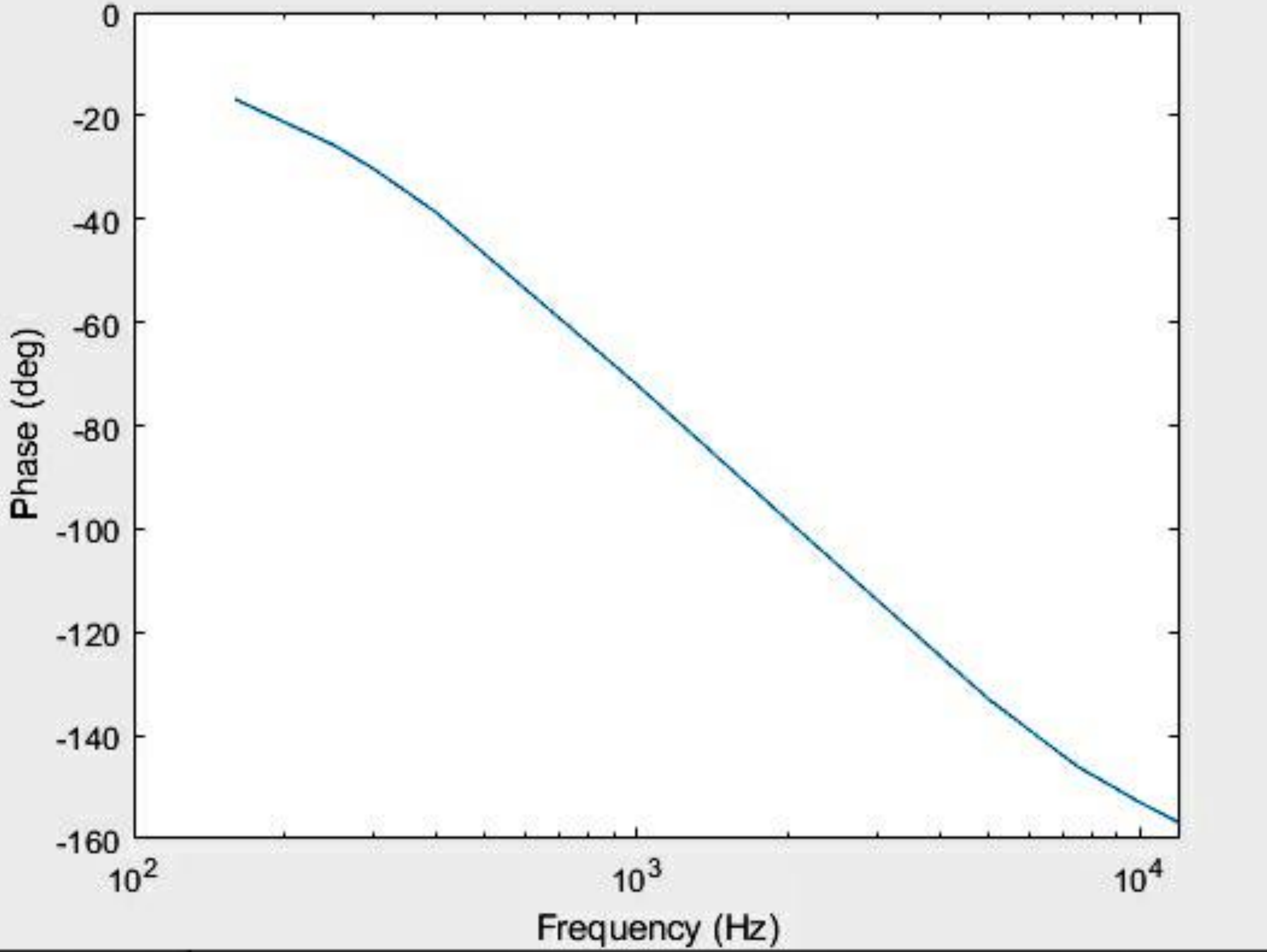
( Since $\xi = 0.2$ and $\omega_n = 10^4$ rad/s)

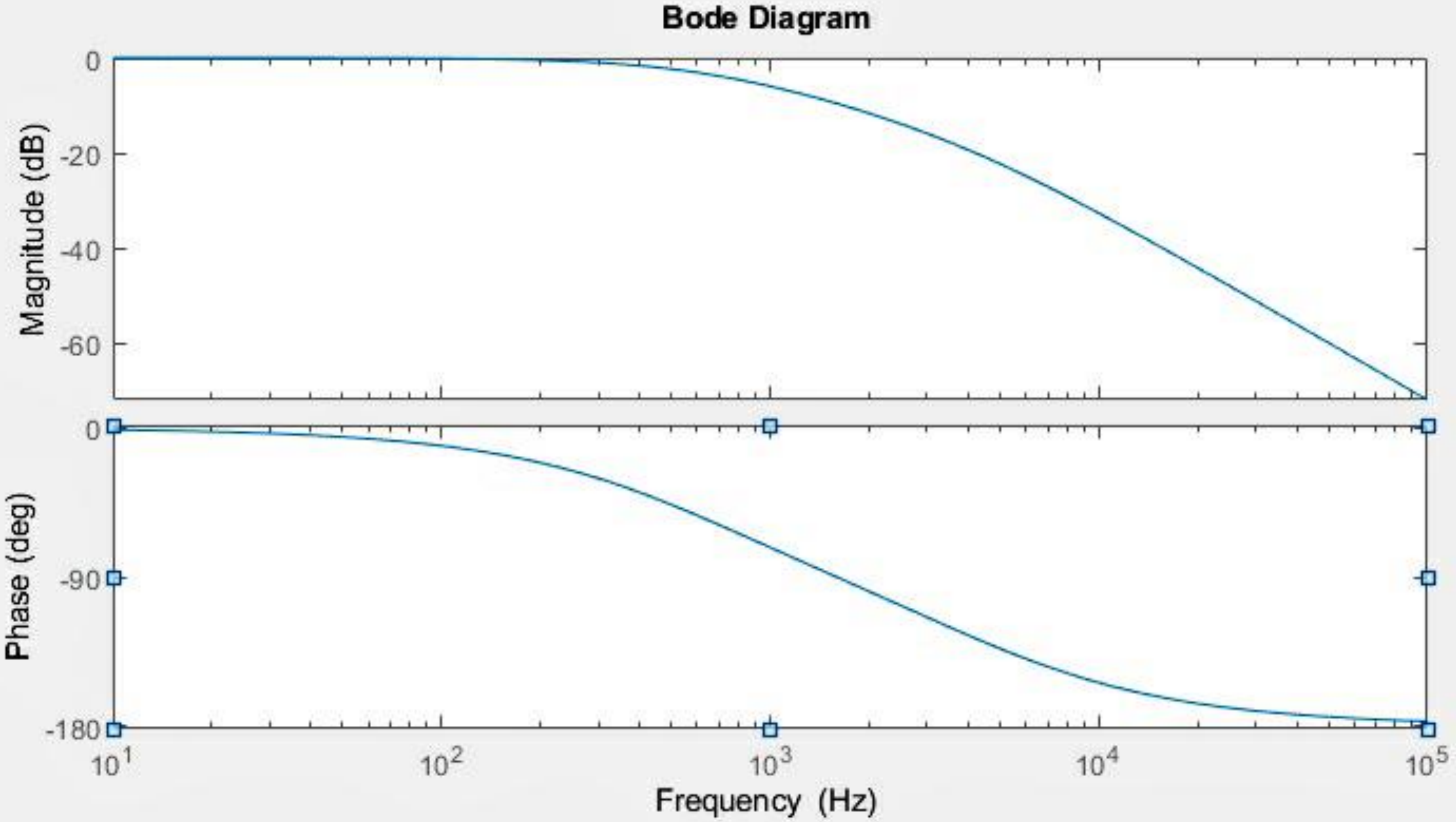
$\therefore$ Percentage error in Maximum Gain $= \dfrac{|2.552 - 2.549|}{2.552} \times 100$

$= 0.12\%$



0
-5
-10
-15
-20
-25
-30
-35
-40
Magnitude (dB)
$10^2$
$10^3$
$10^4$
Frequency (Hz)


Phase (deg)
0
-20
-40
-60
-80
-100
-120
-140
-160
$10^2$
$10^3$
$10^4$
Frequency (Hz)


Bode Diagram
Magnitude (dB)
0
-20
-40
-60
Phase (deg)
0
-90
-180
10^1
10^2
10^3
10^4
10^5
Frequency (Hz)

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Percentage Error in ~~Phase~~ $f_m = \frac{|1527 - 1500|}{1527} \times 100\%$

$= 1.77\%$

for (ii), The maximum gain, observed is 1.000 occuring at $f = 300$ Hz.

$\therefore f_m$, observed $= 250$ Hz

The maximum gain, theoretical $= \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{2\times 0.7\sqrt{1-0.49}} = 1.000$

$f_m$, theoretical $= \frac{\omega_n}{2\pi}\sqrt{1-2\xi^2} = 225$ Hz.

$\therefore$ Percentage Error in Maximum Gain $= \frac{|1.000 - 1.000|}{1.000} \times 100\%$

$= 0\%$

Percentage Error in ~~Phase~~ $f_m = \frac{|250 - 225|}{225} \times 100$

$= 11.11\%$

for (iii) and (iv), there is no maximum gain, since they are monotonically decreasing gains.
Thus, they also do not have any $f_m$.

Discussions: This experiment enables us to define what are called filters. Since we do the experiment for an entire range of frequencies, we find that for the given circuit, some set of frequencies have a high gain (they pass through the circuit), while some set of frequencies have a low gain (they do not pass through the circuit). This leads to the coining of several terms, like

- Low Pass Filter: only low frequencies pass through
- High Pass Filter : only high frequencies pass through

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- Band Pass Filter: Only the frequencies that lie within a certain band, pass through.
- Band Reject Filter: All those frequencies that lie outside a certain band, pass through.

When we perform the experiment in real life, in the lab, we will expect a much higher error percentage, ~~bec~~ because MATLAB ignores a lot of the non idealities of the real world, like extra resistances, capacitances and inductances, losses and many other factors. Also, the devices used to measure the voltages may give some error and thus overall error increases in a real life experiment and its accuracy decreases