19CS10039
Kaushal Banthia
(1)

(b) $f(P_{ij}) = F_{ij}$

Also, $f(u, v) = \theta_1 + \theta_2 u + \theta_3 v + \theta uv$

$\Rightarrow F_{ij} = f(P_{ij})$

$= f(x_i, y_j)$

$= \theta_1 + \theta_2 x_i + \theta_3 y_j + \theta_4 x_i y_j$

(a) Given $A\theta = b$ and $b = \begin{bmatrix} F_{11} \\ F_{12} \\ \vdots \\ F_{1N} \\ F_{21} \\ \vdots \\ F_{MN} \end{bmatrix}_{MN \times 1}$ and $\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}_{4 \times 1}$

19CS10039
Kaushal Banthia
(2)

$$\therefore A = \begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_1 & y_2 & x_1 y_2 \\ \vdots & & & \\ 1 & x_1 & y_N & x_1 y_N \\ 1 & x_2 & y_1 & x_2 y_1 \\ \vdots & & & \\ 1 & x_2 & y_N & x_2 y_N \\ \vdots & & & \\ 1 & x_M & y_N & x_M y_N \end{bmatrix}_{MN \times 4}$$

(b) For a unique solution to the system of equations $A\theta = b$, A must tall or A must be square.

$\therefore MN \geq 4$

For minimum value of MN, $MN = 4$.

$\therefore$ A is $MN \times 4$ matrix

$\Rightarrow$ A is 4×4 matrix

Now, $M = 1$ or $N = 1$ not allowed, as they give a line, instead of a grid (since x remains constant).

Thus, $M > 1$ and $N > 1$

$\Rightarrow$ Minimum values of M, N $\Rightarrow M = 2$ and $N = 2$

$\Rightarrow$ For unique solution, $M_{min} = 2$, $N_{min} = 2$