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(5) Given $Ax = b$ and $A \in \mathbb{R}^{n \times n}$ and invertible, $b \in \mathbb{R}^n$.

If A is orthogonal, we get the following advantages

↳ $A \cdot A^T = I$

⇒ Magnification: Since A is orthogonal, then $\|A\|_2 = 1$

$\therefore$ maxmag(A) = minmag(A) = 1

Thus, the unit sphere after transformation will still remain a unit sphere

⇒ QR Decomposition: $Q = A$ and $R = I_{n \times n}$ as A is already orthogonal

⇒ Inverse Calculation: $A^{-1} = A^T$ for any orthogonal matrix.

This is useful, as A^{-1} is replaced by A^T and thus, the complexity is reduced, since calculating A^T is computationally less expensive than calculating A^{-1}. (This is because $A \cdot A^T = I$, $\therefore A^T = A^{-1}$)

⇒ Solving an equation: For $Ax = b$

$\Rightarrow x = A^{-1} \cdot b$

Since A is orthogonal, $A^{-1} = A^T$

$\Rightarrow x = A^T \cdot b$

Thus $Ax = b$ can be easily solved for x.

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⇒ Condition Number: Orthogonal matrice have a condition number of 1. This is the best possible condition number.
Thus for small perturbations in b, ~~x~~ x doesn't change by a huge amount. This gives stable solutions.