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Experiment 3 Part 1

Objective : Experimental verification of Fourier Coefficients of a Square wave signal using Passive Network

Theory: Any periodic signal can be broken down into sine and cosine waves, in the form of

$$x(t) = a_0 + (a_1 \cos \omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t \ldots) + (b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t \ldots)$$

where $x(t)$ is the periodic signal and ω is the fundamental angular frequency.

$$\Rightarrow x(t) = a_0 + (a_1 \cos \omega t + b_1 \sin \omega t) + (a_2 \cos 2\omega t + b_2 \cos 2\omega t) \ldots$$

$$= a_0 + \sqrt{a_1^2 + b_1^2}\left[\frac{a_1 \cos \omega t}{\sqrt{a_1^2+b_1^2}} + \frac{b_1 \sin \omega t}{\sqrt{a_1^2+b_1^2}}\right] + \sqrt{a_2^2+b_2^2}\left[\frac{a_2 \cos 2\omega t}{\sqrt{a_2^2+b_2^2}} + \frac{b_2 \sin 2\omega t}{\sqrt{a_2^2+b_2^2}}\right] \ldots$$

$$= a_0 + \sqrt{a_1^2+b_1^2}\, \sin(\omega t + \phi_1) + \sqrt{a_2^2+b_2^2}\, \sin(2\omega t + \phi_2) \ldots$$

$$\text{where } \tan \phi_n = \frac{a_n}{b_n}$$

$$\therefore x(t) = a_0 + \sum_{n=1}^{\infty} \sqrt{a_n^2 + b_n^2}\, \sin(n\omega t + \phi_n)$$

Equipment (Simulink Blocks) Needed:

- Controlled voltage source
- Signal Generator
- 2 x Voltage Measurement
- 2 x Scope
- Series RLC Branch (Made into Resistor)
- Series RLC Branch (Made into Inductor)
- Series RLC Branch (Made into Capacitor)
- 2 x Fourier (One for Input, other for Output)
- Add

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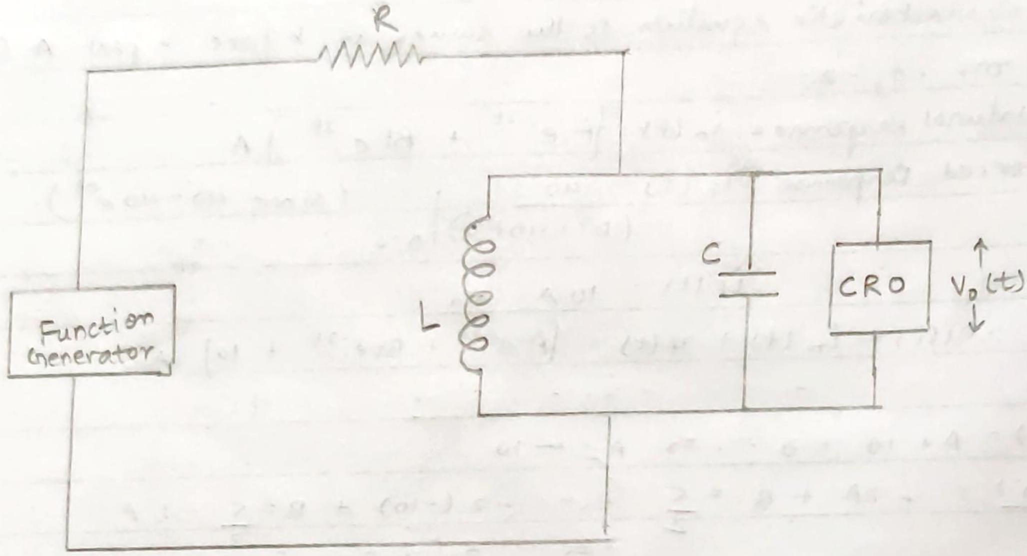

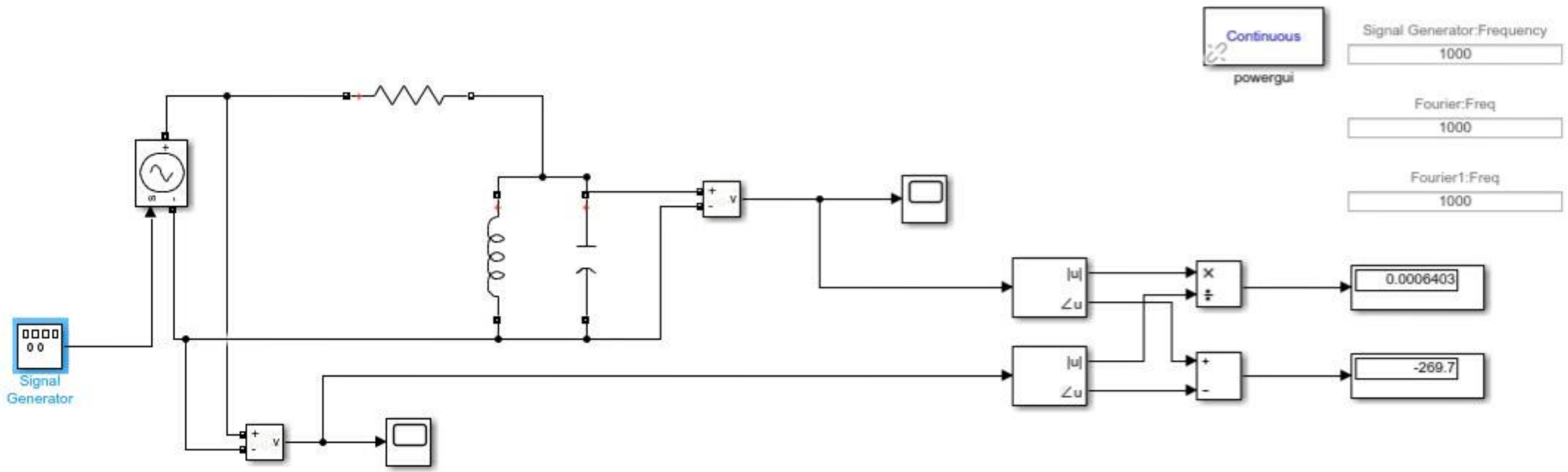

Continuous
powergui
Signal Generator:Frequency
1000
Fourier:Freq
1000
Fourier1:Freq
1000
|u|
∠u
0.0006403
-269.7
Signal Generator

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- Divide
- 2x Display
- Power gui (continuous)
- 3x Edit

Part I

Simulink Procedure for running the experiment:

→ Right Click in the Background and choose Model Configuration Parameters. In the menu that opens, we choose the solver that has the name 'ode 23t (mod. stiff / Trapezoidal)

→ Set the stop time to 5s at the top of the window

→ Then click Run. after taking 3V and 100Hz sine wave in ~~volt~~ signal Generator

→ Vary frequency after each run to obtain the table below.

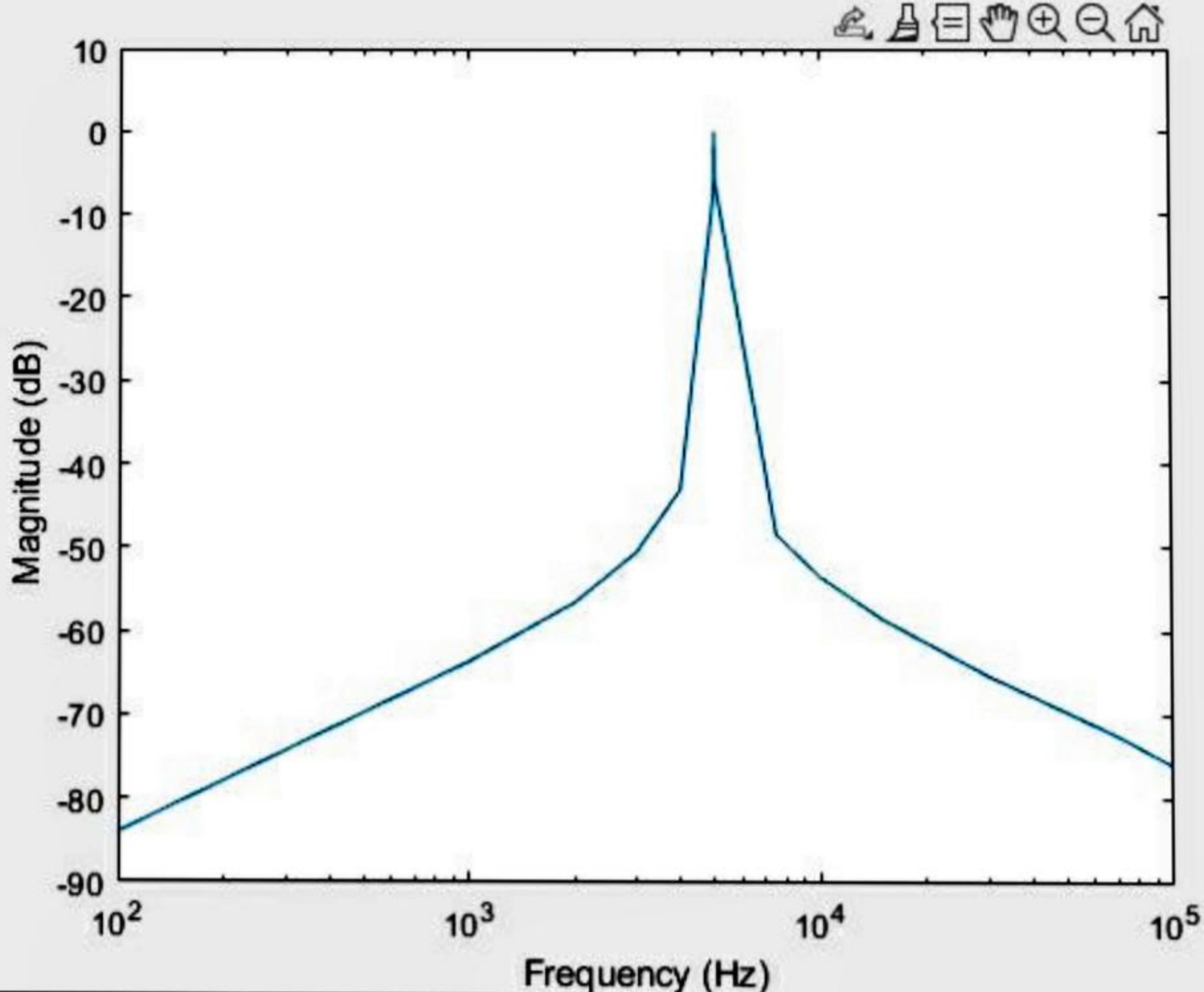
Observation Table: for Vin = 3V (Amplitude) for sine wave

Frequency (f) in Hz	Vout (in V)	Gain
100	1.895×10^{-4}	6.317×10^{-5}
1000	1.968×10^{-3}	6.56×10^{-4}
2000	4.437×10^{-3}	1.479×10^{-3}
3000	8.898×10^{-3}	2.966×10^{-3}
4000	2.1×10^{-2}	7×10^{-3}
5000	1.283	4.276×10^{-1}
5016	2.996	9.985×10^{-1}
5017	3.003	1.001
5018	2.965	9.883×10^{-1}
5030	1.529	5.098×10^{-1}
7500	1.137×10^{-2}	3.789×10^{-3}
10000	6.333×10^{-3}	2.111×10^{-3}
15000	3.549×10^{-3}	1.183×10^{-3}
30000	1.629×10^{-3}	5.431×10^{-4}
70000	7.125×10^{-4}	2.375×10^{-4}
100000	4.749×10^{-4}	1.583×10^{-4}

$R = 100 k\Omega$

$L = 10 mH$

$C = 100 nF$



10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
Magnitude (dB)
10^2
10^3
10^4
10^5
Frequency (Hz)

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Converting the circuit into Laplace domain, we get

$R \rightarrow R$

$L \rightarrow Ls$

$C \rightarrow \frac{1}{Cs}$

$\therefore$ Now $Y(s) = G(s) \cdot X(s)$

Transfer function

$$\Rightarrow G(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

$$G(s) = \frac{I \cdot \left[Ls \cdot \frac{1}{Cs} \right]}{\left[Ls + \frac{1}{Cs} \right]} \cdot \frac{1}{V_{in}(s)}$$

$$G(s) = \frac{V_{in}(s)}{\left[R + \frac{Ls \cdot \frac{1}{Cs}}{Ls + \frac{1}{Cs}} \right]} \cdot \left[\frac{Ls \cdot \frac{1}{Cs}}{Ls + \frac{1}{Cs}} \right] \cdot \frac{1}{V_{in}(s)}$$

$$\therefore G(s) = \frac{\frac{L}{C}}{\frac{LCs^2+1}{Cs}} \cdot \frac{1}{R + \frac{\frac{L}{C}}{\frac{LCs^2+1}{Cs}}}$$

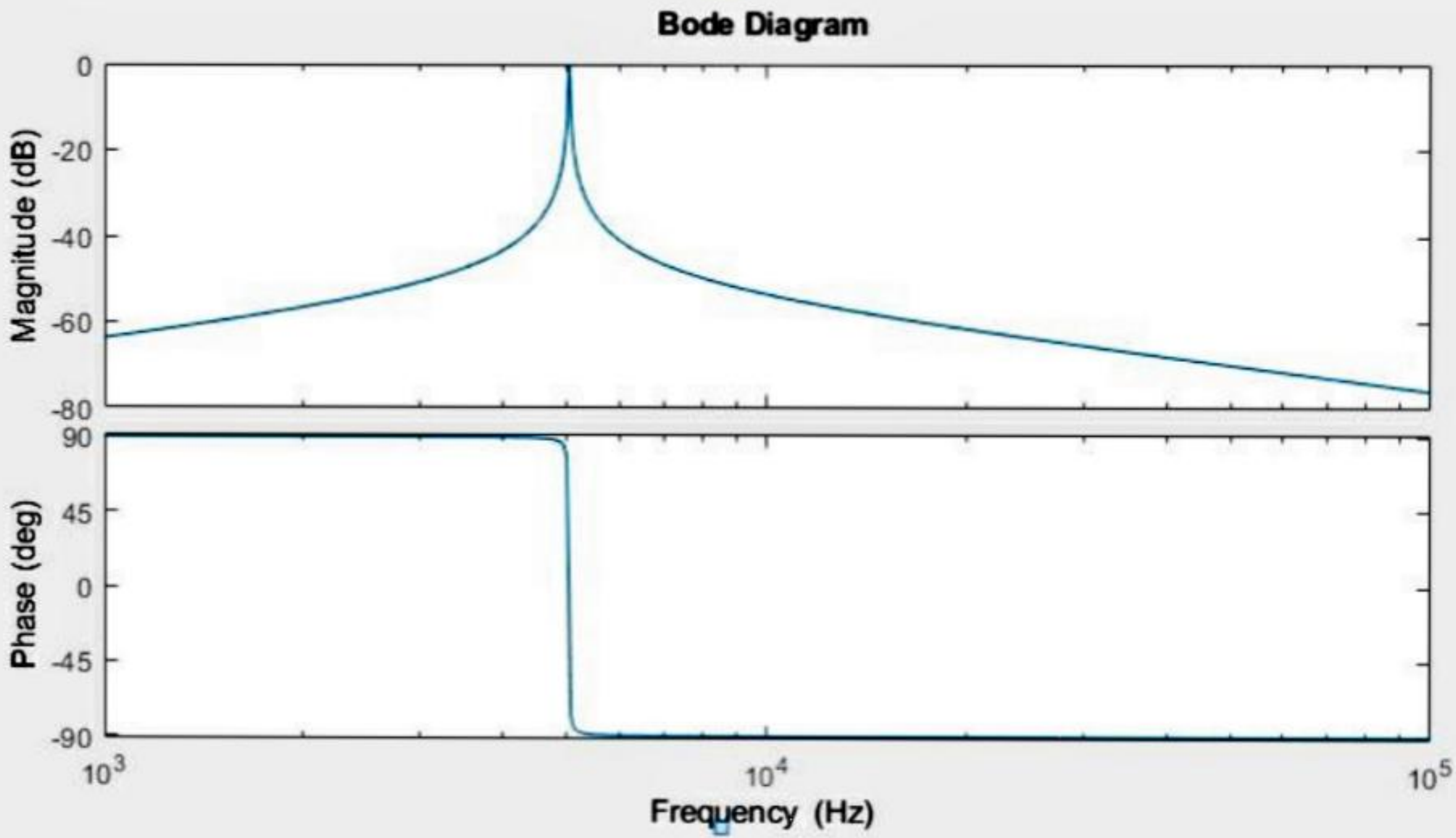
$$= \frac{Ls}{LCs^2+1} \cdot \frac{1}{R + \frac{Ls}{LCs^2+1}}$$

$$= \frac{1}{1 + \frac{R}{Ls}(LCs^2+1)}$$

$$G(s) = \frac{Ls}{Ls + R(LCs^2+1)} = \frac{10 \times 10^{-3} s}{10 \times 10^{-3} s + 100 \times 10^{3} (10 \times 10^{-3} \times 100 \times 10^{-9} s^2 + 1)}$$

$$G(s) = \frac{10^{-2} s}{10^{-2} s + 10^{5} (10^{-9} s^2 + 1)} = \frac{s}{s + 10^{7} (10^{-9} s^2 + 1)}$$

$$\boxed{G(s) = \frac{s}{s + \frac{s^2}{100} + 10^7}}$$


Bode Diagram
Magnitude (dB)
0
-20
-40
-60
-80
Phase (deg)
90
45
0
-45
-90
10^3
10^4
10^5
Frequency (Hz)

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Discussion: The observational peak of the magnitude graph (in dB) comes at 5017 Hz, whereas, the theoretical peak comes at 5033 Hz.

Thus, the error percentage $= \left|\frac{5033 - 5017}{5033}\right| \times 100\%$

$= \boxed{0.318\%}$

Using this part, we found the fundamental frequency.

Part II

Simulink Procedure for Running the experiment:

→ Click Run

→ Vary frequency after ~~each~~ taking 3 V and 5017 Hz square wave in signal Generator.

→ Vary frequency by 1 Hz each time. (fine tuning)

Observation Table: for Vin = 3 V for square wave

Frequency (f) in Hz	Vout (in V)	Gain
5017	3.530	1.177
5018	3.574	1.191
5019	3.631	1.210
5020	3.756	1.252
5021	3.82	1.273
5022	3.815	1.272
5023	3.764	1.255

R = 100 k Ω
L = 10 mH
C = 100 nF

Gain and Vout are maximum for f = 5021 Hz (After fine tuning)
Thus, Vout = 3.82 V is the fundamental fourier series coefficient.
Since the square wave signal is an odd function, thus we have all a_i = 0 (i = 0, 1, 2, 3, ---)
Thus, this Vout = 3.82 V is the term b_1

∴ $b_{1,observed}$ = 3.82 V

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Now, b_1, theoretical $= \frac{2}{T}\int_0^T x(t)\sin(\omega t)\,dt$

$$= 2(5021)\int_0^{T/2}(3)(\sin((2\pi)(5021)t))\,dt + 2(5021)\int_{T/2}^{T}-(3)(\sin((2\pi)(5021)t))\,dt$$

$$= 3.8197 \text{ V}.$$

Discussion: The observed b_1 is 3.82 V, while the theoretical b_1 is 3.8197 V.

$\therefore$ Error percentage $= \frac{|3.8197 - 3.82|}{3.8197} \times 100\%$

$= 0.00785\ \%$

This percentage is very small and can be ignored as approximation error.

In this part, we found the fundamental fourier coefficient of $x(t)$.

Part iii

Simulink Procedure for Running the experiment

→ Change frequency to $\frac{1}{3}$rd of 5021 Hz $\approx$ 1674 Hz, and voltage = 3V Square wave in signal Generator

→ Vary frequency by 1Hz each time (fine tunings)

→ Click Run

Observation Table (for Vin = 3V for square waves)

Frequency (f) in Hz	Vout (in V)	Gain
1672	1.185	0.395
1673	1.258	0.419
1674	1.281	0.427
1675	1.229	0.409
1676	0.8779	0.293

R = 100k Ω

L = 10mH

C = 100nF

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Gain and Vout are maximum for $f = 1674$ Hz (After fine tuning)
Thus, $V_{out} = 1.281$ V is the 3rd Harmonic Fourier Series Coefficient
$\therefore b_{3,\text{ observed}} = 1.281$ V.

Now, $b_{3,\text{ theoretical}} = \frac{2}{T} \int_0^T x(t) \sin(3\omega t)\, dt$

$$= 2(5021) \int_0^{T/2} 3 \sin(3 \cdot 2\pi \cdot 5021 t)\, dt$$

$$+$$

$$2(5021) \int_{T/2}^{T} (-3) \sin(3 \cdot 2\pi \cdot 5021 t)\, dt$$

$$= 1.2732 \text{ V}.$$

Discussion: The observed b_3 is 1.281 V and the theoretical b_3 is 1.2732 V. $\therefore$ The percentage error is $\frac{|1.281 - 1.2732|}{1.2732} \times 100\%$

$$= 0.61\%$$

This is a small error which may be caused due to division, ~~[illegible]~~ since we use $f = 1674$ Hz, which is not exactly ~~[illegible]~~ 5021 Hz divided by 3.

Simulink Procedure for Running the Experiment:

- change frequency to $\frac{1}{5}$th of 5021 Hz $\approx$ 1004 Hz. and voltage = 3V square wave in signal generator.
- vary frequency by 1 Hz each time (fine tuning)
- Click Run.

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Observation Table: for $V_{in} = 3V$, for Square wave

frequency (f) in Hz	V_{out} (in V)	Gain.
1002	0.463	0.154
1003	0.696	0.232
1004	0.769	0.256
1005	0.740	0.247
1006	0.393	0.131

Gain and Vout are maximum for f = 1004 Hz (After fine tuning).
Thus, Vout = 0.769 V is the 5^{th} Harmonic Fourier series coefficient.

$\therefore b_5$, observed $= 0.769$ V.

Now b_5, theoretical $= \frac{2}{T}\int_0^T x(t) \sin(5\omega t)\, dt$

$$\cancel{\frac{2}{T}\int_0^{T/2}} = 2(5021)\int_0^{T/2} 3 \sin(5 \cdot 2\pi \cdot 5021 t) dt$$

$$+$$

$$2(5021)\int_{T/2}^{T} (-3) \sin(5 \cdot 2\pi \cdot 5021\, t)\, dt$$

$$= 0.7639 \text{ V.}$$

Discussion: The observed b_5 is 0.769 V while the theoretical b_5 is 0.7639. $\therefore$ Percentage Error $= \frac{|0.769 - 0.7639|}{0.769} \times 100\%$

$= 0.66\%$

This is a small error which may be caused due to division, since we use f = 1004 Hz, which is not exactly 5021 Hz divided by 5.

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Discussion : In this experiment, we found out the different fourier series coefficients of the input signal. Here the $a_0, a_1, a_2, a_3 \ldots$ terms were 0 since the input signal was an odd signal. &

Using this experiment, we can find the different components of any periodic signal, if we keep on going for all the harmonics. After that, we can break the signal into sines and cosines, which can further be broken down into summations of e^{jt} and e^{-jt}.

Now, we know that e^{st} is an eigen function of any LTI system. Thus, when this signal is input in any LTI system, we get an output of $H(s) \cdot e^{st}$ where $H(s)$ is the transfer function of that system. Now this $H(s)$ is a constant w.r.t. time t.

Thus, when e^{st} is input in any LTI system, we get a ~~funct~~ signal of the similar.

This information can also be used in the following way →

→ Given a certain response.

→ Break into its corresponding sines and cosines and then e^{st}.

→ Find each exponential's input by getting $\frac{e^{st}}{H(s)}$.

→ sum all these input.

→ Get the input signal for the output signal.

Thus, if we have the ability to breakdown any signal into its corresponding sines and cosines, and ~~corr~~ thus, e^{st}, we can then find the response for any input and also the input for any response for any LTI system, given its transfer function.

This experiment provides us with this ability to break down periodic signals into their corresponding sines and cosines and thus e^{st}.