Assignment 3: Single bit ECC with display

Group 20

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1 Error Detection For More Than 1-Bit

Single bit Hamming Code can be extended to 2 bit error detection (but not correction) by adding a single parity bit check over the entire code. For an 11 bit message we can use 4 bits for finding 1 bit errors and the 16^{th} bit for the 2 bit errors.
This can be done as follows

1. Consider a 11 bit message in a 16 bit block, let the bits in positions $1, 2, 4, 8$ (c_1, c_2, c_4, c_8) and the bit in position 0 (c_0) be used as parity checks.

2. Let $a[i]$ be the state of the i^{th} bit.

3. Positions $3, 5, 6, 7, 9, 10, 11, 12, 13, 14$ and 15 will be used to transmit information.

4. The bit at c_{2^i} $i = 0, 1, 2, 3$ will store the xor of the values of the positions whose $(i+1)^{th}$ bit is 1.

5. For example c_1 will store the $\oplus$ of the values of the positions who first bit is turned on, that is
$$a[1] = a[3] \oplus a[5] \oplus a[7] \oplus a[9] \oplus a[11] \oplus a[13] \oplus a[15]$$

6. Similarly,
$$a[2] = a[3] \oplus a[6] \oplus a[7] \oplus a[10] \oplus a[11] \oplus a[14] \oplus a[15]$$
$$a[4] = a[5] \oplus a[6] \oplus a[7] \oplus a[12] \oplus a[13] \oplus a[14] \oplus a[15]$$
$$a[8] = a[9] \oplus a[10] \oplus a[11] \oplus a[12] \oplus a[13] \oplus a[14] \oplus a[15]$$

7. Now $a[2^i]$ stores parity of the relevant positions, so if there is a 1-bit error then we can find the exact position of the flipped bit using the four parity bits.

8. Observe that this can be done by taking $\lambda = \oplus_{i=1}^{15}(i * \dot{a}[i])$ where $\dot{a}[i]$ is the received bit. This quantity is the xor of the positions which are turned on in the received message. This would be 0 if there are no errors, if this is $\neq 0$ then that is the position of the flipped bit.

9. To extend this scheme to detecting (but not correcting) 2-bit errors, we can utilise the bit at c_0.

10. By setting $a[0] = \oplus_{i=1}^{15} a[i]$ (total parity of the original message now becomes 0). We can detect errors by comparing $\oplus_{i=0}^{15} \dot{a}[i]$ (parity of received bits) to λ (same as above) .
    (a) Now if $\oplus_{i=0}^{15} \dot{a}[i] = 1$ then we have a 1-bit error, whose position can be determined using λ.
    (b) If $\oplus_{i=0}^{15} \dot{a}[i] = 0$
        i. $\lambda \neq 0$, this means that we have a 2 bit error (parity continues to be correct but there is some change)
        ii. $\lambda = 0$, this means that we have no error.

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