INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Computer Science and Engineering

Switching Circuits and Logic Design (CS21002)

Assignment – 1 (Spring)

Group: 20 *Marks:* 30

Answer ALL the questions using xournal or similar software to edit the PDF

Q1: Given that $(16)_{10}$ = $(100)_b$, determine the value of b. 4

$$16 = 0 * b^0 + 0 * b^1 + 1 * b^2$$

$$16 = b^2 \implies b = 4$$

Q2: The n-bit fixed-point representation of an unsigned real number X uses f bits for the fraction part. What is the range of decimal values for X in this representation? 5

For unsigned real number the minimum value is 0, when all bits are turned off. The maximum value occurs when all bits are turned on, i.e.,

$$X = 2^{n-f-1} + \ldots + 2^1 + 2^0 + 2^{-1} + \ldots + 2^{-f}$$

$$X = \frac{2^{n-f-1}(1-(\frac{1}{2})^n)}{1-\frac{1}{2}}$$

$$X = \frac{2^n - 1}{2^f}$$

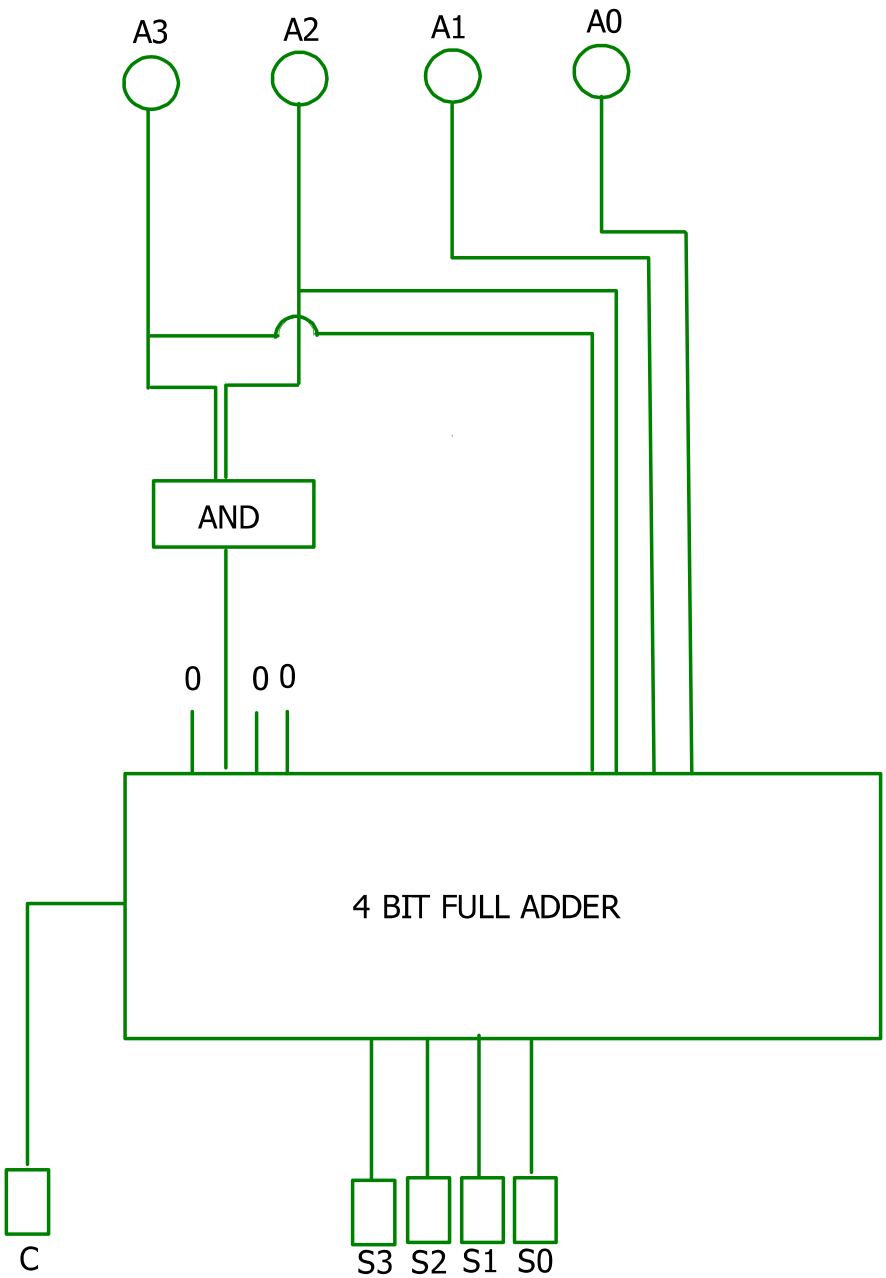
Therefore, we have,

$$0 \leq X \leq \frac{2^n - 1}{2^f}$$

Q3: Encode each of the ten decimal digits 0, 1, . . . , 9 by means of the weighted binary code 7 3 2 -1. 5

	7	3	2	-1
0	0	0	0	0
1	0	0	1	1
2	0	0	1	0
3	0	1	0	0
4	0	1	1	1
5	0	1	1	0
6	1	0	0	1
7	1	0	0	0
8	1	0	1	1
9	1	1	0	1

Q4: Design a circuit which converts a four bit input binary number to a five bit output representing the radix-12 representation of the input number and a carry-out bit. You may use a 4-bit binary adder block and basic logic gates. 8



The answer is: 0 0 0 C S3 S2 S1 S0

Q5: Prove that the Hamming distance satisfies the triangle inequality. That is, show that $HD(x,y)+HD(y,z) \geq H(x,z)$. 8

Hamming distance between x and y and can be written as a sum over n digits

$$HD(x,y) = \sum_{i=1}^{n} HD(x_i, y_i)$$

Now consider x_i,y_i and z_i and the distances $HD(x_i, y_i)$,$HD(y_i, z_i)$ and $HD(z_i, x_i)$

- Case 1: $x_i = y_i$
  - Case 1a $x_i = z_i$

$$\implies HD(x_i, y_i) = 0, HD(y_i, z_i) = 0, HD(z_i, x_i) = 0$$

$$\therefore HD(x_i, z_i) \leq HD(y_i, z_i) + HD(y_i, x_i)$$

  - Case 1b $x_i \neq z_i$

$$\implies HD(x_i, y_i) = 0, HD(y_i, z_i) = 1, HD(z_i, x_i) = 1$$

$$\therefore HD(x_i, z_i) \leq HD(y_i, z_i) + HD(y_i, x_i)$$

- Case 2: $x_i \neq y_i$
  - Case 2a $x_i = z_i$

$$\implies HD(x_i, y_i) = 1, HD(y_i, z_i) = 1, HD(z_i, x_i) = 0$$

$$\therefore HD(x_i, z_i) \leq HD(y_i, z_i) + HD(y_i, x_i)$$

  - Case 2b $x_i \neq z_i$

$$\implies HD(x_i, y_i) = 1, HD(y_i, z_i) = 1, HD(z_i, x_i) = 0$$

$$\therefore HD(x_i, z_i) \leq HD(y_i, z_i) + HD(y_i, x_i)$$

$\therefore$ In each case we have

$$HD(x_i, z_i) \leq HD(y_i, z_i) + HD(y_i, x_i)$$

Summing over i, we get

$$\sum_{i=1}^{n} (HD(x_i, z_i) \leq HD(y_i, z_i) + HD(y_i, x_i))$$

$$\implies \sum_{i=1}^{n} HD(x_i, z_i) \leq \sum_{i=1}^{n} HD(y_i, z_i) + \sum_{i=1}^{n} HD(y_i, x_i)$$

$$\implies HD(x, z) \leq HD(x, y) + HD(y, z)$$