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Experiment 2 Part 2

Q2

Aim: To solve an RLC circuit (with initial conditions) and find the voltage across the capacitor and the current across the inductor as a function of time

(A) Here the initial conditions are again relaxed, with $v(0^-)=0$ and $i(0^-)=0$. Also, since $L\frac{di(t)}{dt} = v(t)$ $\therefore v(0^-) = \frac{di}{dt}(0^-)=0$

Using Kirchhoff's Voltage Law, we get

$$20 - R(i_R(t)) - L\frac{di(t)}{dt} = 0 \quad — (1)$$

and

$$L\frac{di(t)}{dt} - v(t) = 0 \Rightarrow L\frac{di(t)}{dt} = v(t) \Rightarrow L\frac{d^2 i(t)}{dt^2} = \frac{dv(t)}{dt} \quad — (2)$$

From Kirchhoff's current Law, we get,

$$i_R(t) = i(t) + i_c(t). \quad — (3)$$

Also, we have

$$i_c(t) = C\frac{dv(t)}{dt} \quad — (4)$$

From (1) and (3)

$$\Rightarrow 20 - R\left[i(t) + i_c(t)\right] - L\frac{di}{dt}(t) = 0 \quad — (5)$$

From (4) and (5),

$$20 - R\left[i(t) + \left(C\frac{dv(t)}{dt}\right)\right] - L\frac{di(t)}{dt} = 0 \quad — (6)$$

From (2) and (6)

$$20 - R\left[i(t) + C\cdot L\frac{d^2 i(t)}{dt^2}\right] - L\frac{di(t)}{dt} = 0$$

$$\Rightarrow 20 - 2\left[i(t) + C(2)\frac{d^2 i(t)}{dt^2}\right] - 2\frac{di(t)}{dt} = 0$$

$$\Rightarrow 4C\frac{d^2 i(t)}{dt^2} + 2\frac{di(t)}{dt} + 2i(t) = 20 \Rightarrow \frac{d^2 i(t)}{dt^2} + \frac{1}{2C}\frac{di(t)}{dt} + \frac{1}{2C}i(t) = \frac{5}{C} \quad — (7)$$

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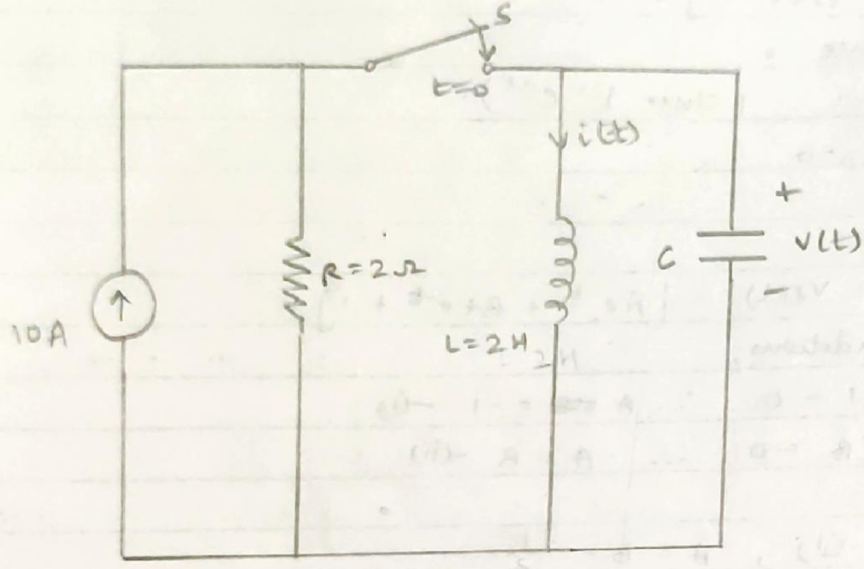
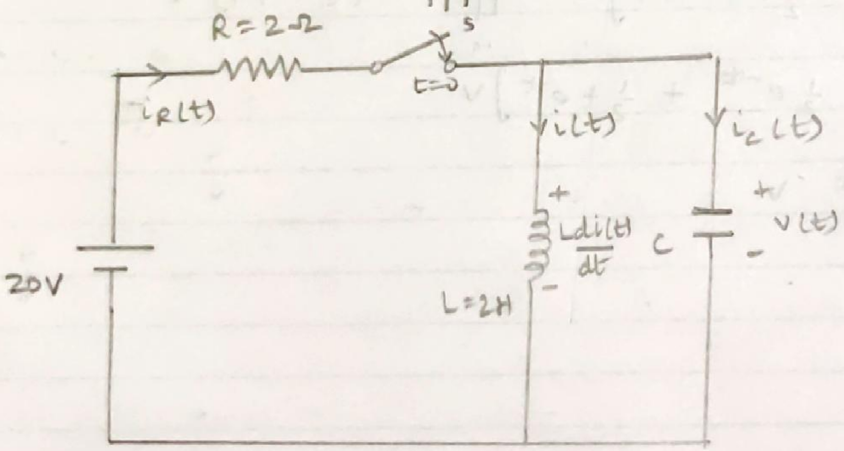


≡ (Current Source → Voltage Source)
Transformed to



$v(0^-)=0$

$i(0^-)=0$

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→ For (i), $C = \frac{1}{9}$ F

∴ From (a), we get

$$\boxed{\frac{d^2 i(t)}{dt^2} + \frac{9}{2}\frac{di(t)}{dt} + \frac{9}{2} i(t) = 45} \Rightarrow \left(D^2 + \frac{9}{2}D + \frac{9}{2}\right) i(t) = 45$$

Characteristic equation ⇒ $m^2 + \frac{9}{2}m + \frac{9}{2} = 0$

$\Rightarrow (m+3)\left(m + \frac{3}{2}\right) = 0$

$\therefore m = -3, -\frac{3}{2}$

The roots are real and distinct

∴ The complementary solution (natural response) is

$i_n(t) = \left[A e^{-3t} + B e^{-\frac{3}{2}t}\right]$ A

The forced response is

$i_f(t) = \left.\frac{45}{\left(D^2 + \frac{9}{2}D + \frac{9}{2}\right)}\right|_{D=0}$ (since $40 = 40e^{0t}$)

$i_f(t) = \frac{45}{9/2}$ A $= 10$ A

$\therefore i(t) = i_n(t) + i_f(t) = \left[A e^{-3t} + B e^{-\frac{3}{2}t} + 10\right]$ A

Using initial conditions,

$i(0^-) = A + B + 10 = 0$ —(i)

$\frac{di(0)}{dt} = -3A - \frac{3}{2}B = 0 \Rightarrow 2A + B = 0$ —(ii)

from (i) and (ii), $A = 10$ ∴ $B = -20$

$\therefore i(t) = \left[10e^{-3t} - 20e^{-\frac{3}{2}t} + 10\right]$ A $= 10\left[e^{-3t} - 2e^{-\frac{3}{2}t} + 1\right]$ A

$= 10\left[e^{-\frac{3}{2}t} - 1\right]^2$ A

$v(t) = L\frac{di(t)}{dt} = 2\left[10 \cdot 2\left(e^{-\frac{3}{2}t} - 1\right) \cdot \left(-\frac{3}{2}\right) e^{-\frac{3}{2}t}\right]$ V.

$v(t) = -60\left(e^{-3t} - e^{-\frac{3}{2}t}\right)$ V

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for (ii), $c = \frac{1}{4}F$

From (9), we get,

$$\boxed{\frac{d^2 i(t)}{dt^2} + 2\frac{di(t)}{dt} + 2i(t) = 20} \Rightarrow (D^2 + 2D + 2)\, i(t) = 20$$

Characteristic equation $\Rightarrow m^2 + 2m + 2 = 0$

$$\Rightarrow (m+1)^2 = -1$$

$$\therefore m = -1 \pm j$$

The roots are complex and distinct.

The complementary solution (natural response)

$$i_n(t) = [A e^{(-1+j)t} + B e^{(-1-j)t}]\ A$$

The forced response is

$$i_p(t) = \frac{20}{(D^2+2D+2)}\Big|_{D=0} \quad (\text{Since } 20 = 20e^{0t})$$

$$i_f(t) = 10\ A$$

$$\therefore i(t) = i_n(t) + i_f(t) = [A\,e^{(-1+j)t} + Be^{(-1-j)t} + 10]\ A$$

Using Initial conditions,

$$i(0^-) = A + B + 10 = 0 \Rightarrow A = -10 - B \quad \text{— (i)}$$

$$\frac{di(0^-)}{dt} = (-1+j)A + (-1-j)B = 0 \quad \text{— (ii)}$$

From (i) and (ii),

$$(-1+j)(-10-B) - (1+j)B = 0$$

$$\Rightarrow \cancel{10(j-1)}\ 10(1-j) + B(1-j) - (1+j)B = 0$$

$$\Rightarrow 10(1-j) = 2jB$$

$$\therefore B = \frac{10(1-j)}{2j} = 5\left(\frac{1-j}{j}\right) = 5(-j-1)$$

$$\therefore A = -10 - B = -10 - 5(-j-1) = -10 + 5(j+1)$$

$$= -5 + 5j$$

$$\therefore A = 5(j-1)$$

$$\therefore i(t) = [5(j-1)\,e^{(-1+j)t} + 5(-j-1)\,e^{(-1-j)t} + 10]\ A$$

$$= 5[e^{-t}(je^{jt} - e^{jt} - je^{-jt} - e^{-jt}) + 2]\ A$$

$$= 5[e^{-t}(-e^{jt} - e^{-jt} + je^{jt} - je^{-jt}) + 2]\ A$$

$$= 10\left[-e^{-t}\left\{\left(\frac{e^{jt} + e^{-jt}}{2}\right) + \left(\frac{e^{jt} - e^{-jt}}{2j}\right)\right\} + 1\right] A$$

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$\therefore i(t) = 10\left[-e^{-t}(\cos t + \sin t) + 1\right] = -10e^{-t}\left[\sin t + \cos t - e^{t}\right]$

~~$\therefore v(t) = L\frac{di(t)}{dt} = 2\left[10.\left(e^{-t}\cos t\right.\right.$~~

~~$\therefore v(t) = \ldots\left(\sin(t) + \cos(t)\right)$~~

$\therefore v(t) = L\frac{di(t)}{dt} = 2(-10)\left[-e^{-t}(\sin t + \cos t - e^{t}) + e^{-t}(\cos t - \sin t - e^{t})\right]$

$= -20\left[-2e^{-t}\sin t\right]$

$= 40e^{-t}\sin t$

→ For (iii), $c = \frac{1}{8}F$

from (α), we get

$\boxed{\frac{d^2 i(t)}{dt^2} + 4\frac{di(t)}{dt} + 4i(t) = 40} \Rightarrow (D^2 + 4D + 4)\,i(t) = 40$

characteristic Equation is $m^2 + 4m + 4 = 0$

$\Rightarrow (m+2)^2 = 0$

$\therefore m = -2, -2$

The roots are real and equal.

$\therefore$ The complementary solution (natural response)

$i_n(t) = \left(Ae^{-2t} + Bte^{-2t}\right)A$

The forced response is

$i_f(t) = \left.\frac{40}{(D^2+4D+4)}\right|_{D=0}$ (Since $40 = 40e^{0t}$)

$\therefore i_f(t) = \frac{40}{4} = 10\ A$

$\therefore i(t) = i_n(t) + i_f(t) = \left[Ae^{-2t} + Bte^{-2t} + 10\right]A$

Using initial conditions

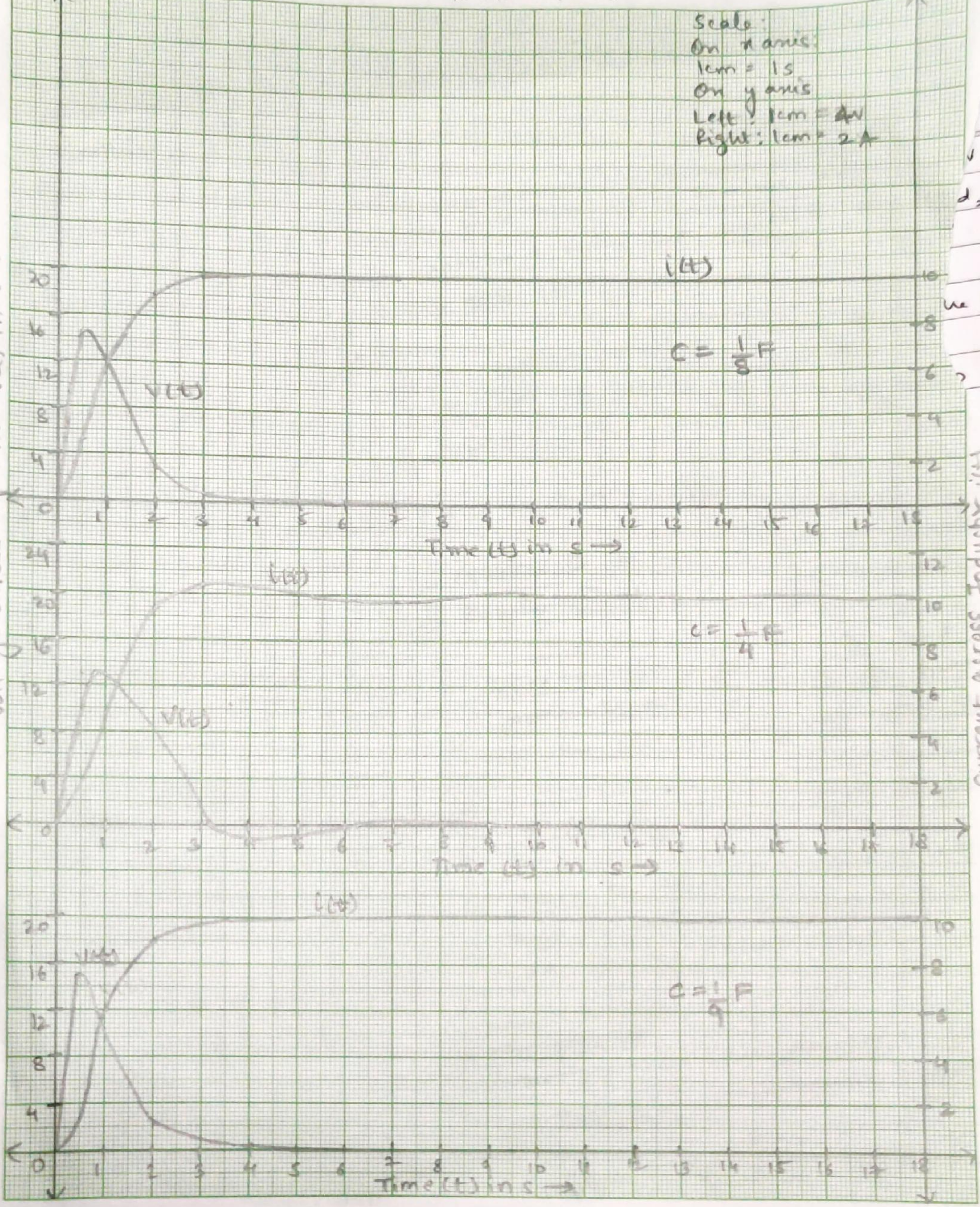
$i(0^-) = A + 10 = 0 \quad \therefore A = -10$

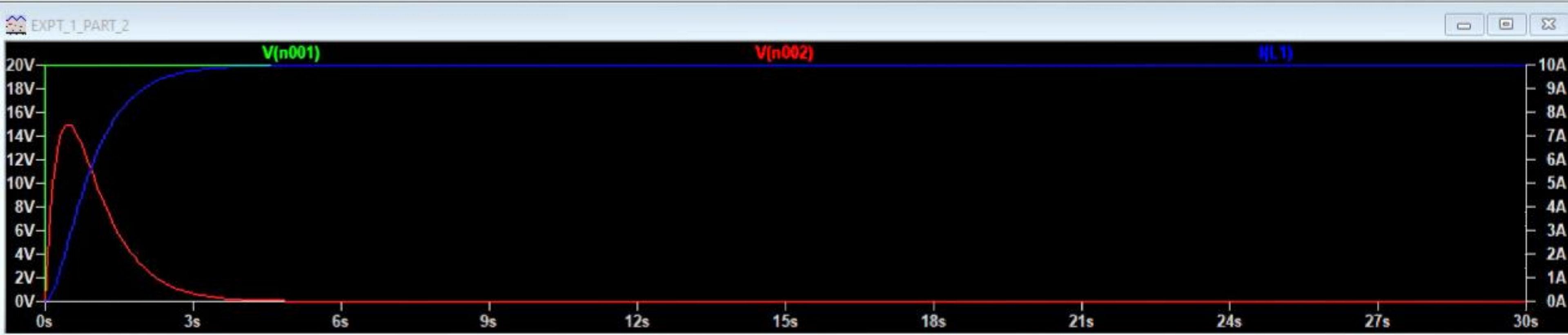
$\frac{di(0^-)}{dt} = -2A + B = 0 \quad \therefore 2A = B \quad \therefore B = -20$

$\therefore i(t) = \left[-10e^{-2t} - 20te^{-2t} + 10\right]A = -10\left[e^{-2t} + 2te^{-2t} - 1\right]A$

$\therefore v(t) = L\frac{di(t)}{dt} = 2\left[-10\left(-2e^{-2t} + 2e^{-2t} - \cancel{4}te^{-2t}\right)\right] = 80te^{-2t}\ V$

v(t) and i(t) v/s t for Q2 (A) Kaushal Banthia 19CS10039

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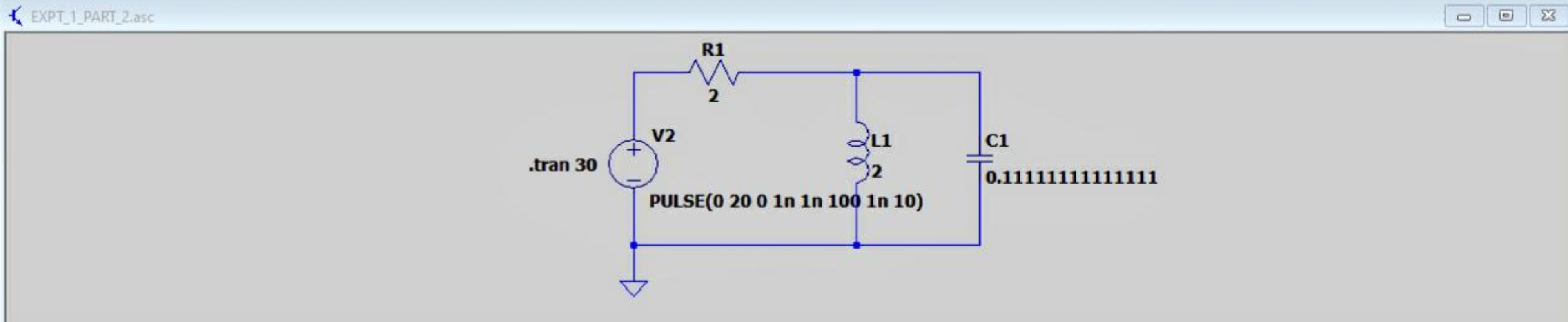
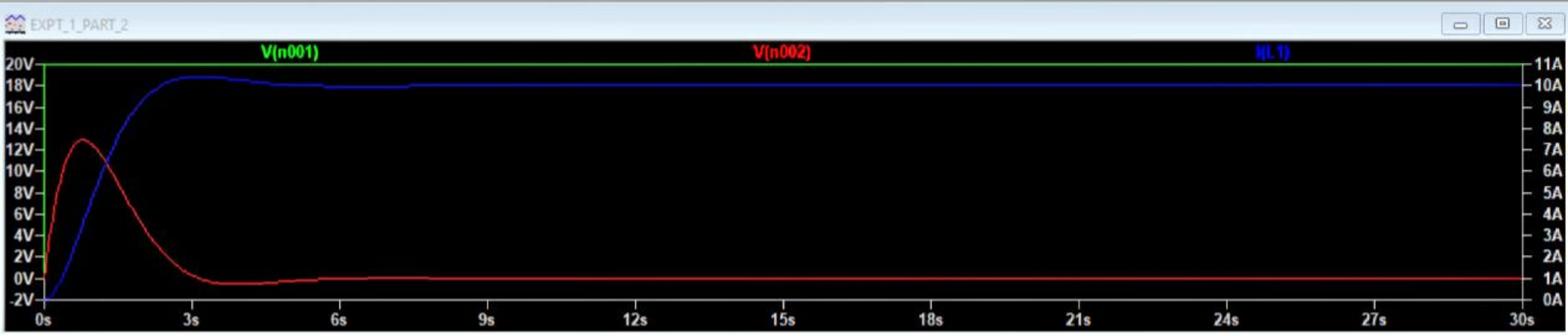
V(n001)
V(n002)
I(L1)
20V
18V
16V
14V
12V
10V
8V
6V
4V
2V
0V
10A
9A
8A
7A
6A
5A
4A
3A
2A
1A
0A
0s
3s
6s
9s
12s
15s
18s
21s
24s
27s
30s


EXPT_1_PART_2.asc
R1
2
V2
L1
2
C1
0.11111111111111
.tran 30
PULSE(0 20 0 1n 1n 100 1n 10)


EXPT_1_PART_2
V(n001)
V(n002)
I(L1)
20V
18V
16V
14V
12V
10V
8V
6V
4V
2V
0V
-2V
11A
10A
9A
8A
7A
6A
5A
4A
3A
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1A
0A
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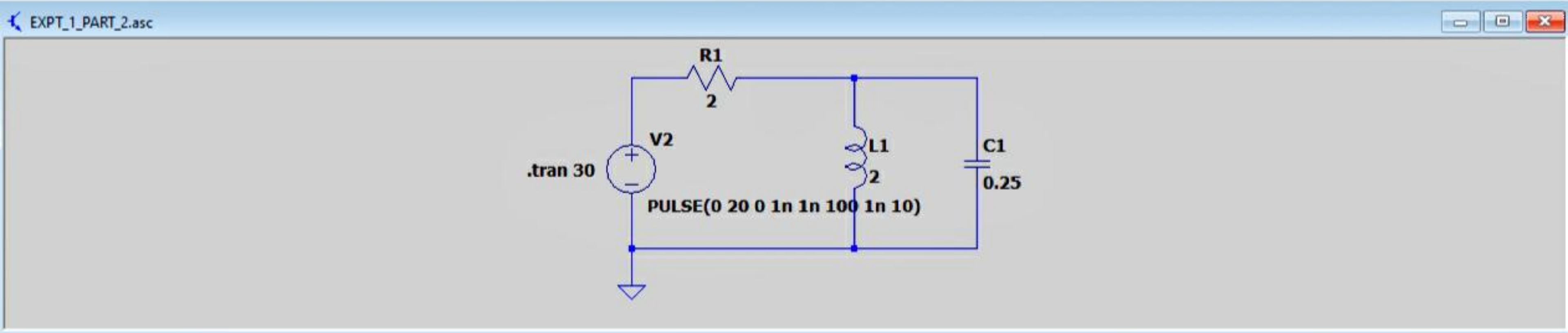
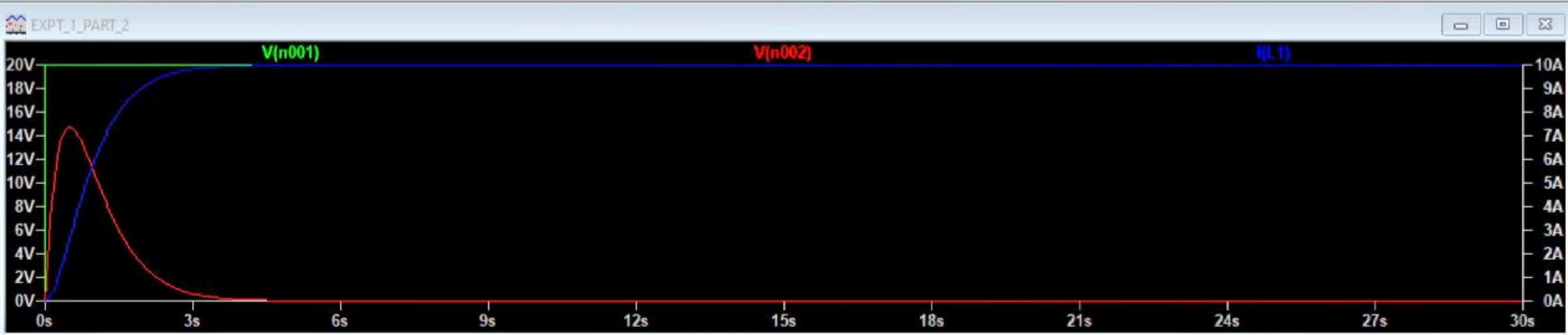
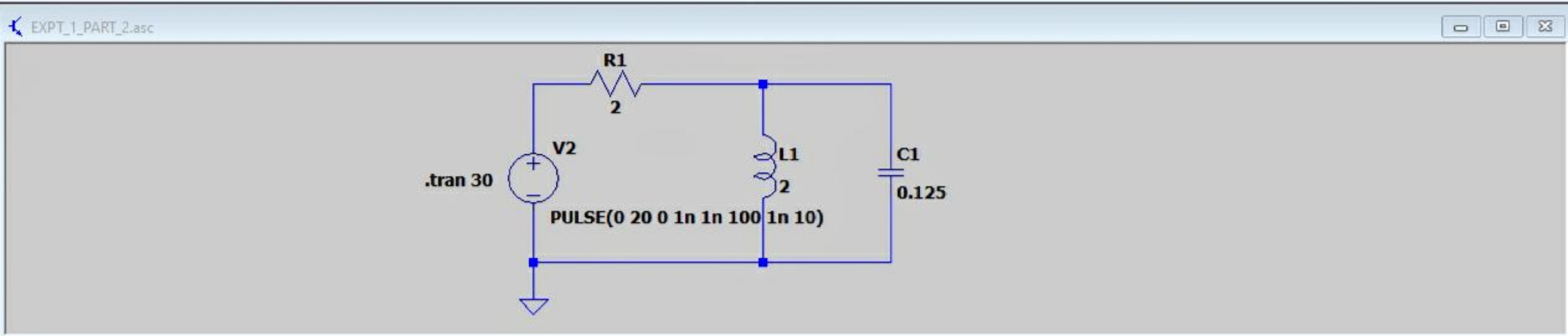
21s
24s
27s
30s


EXPT_1_PART_2.asc
R1
2
V2
.tran 30
L1
2
C1
0.25
PULSE(0 20 0 1n 1n 100 1n 10)



EXPT_1_PART_2
V(n001)
V(n002)
I(L1)
20V
18V
16V
14V
12V
10V
8V
6V
4V
2V
0V
10A
9A
8A
7A
6A
5A
4A
3A
2A
1A
0A
0s
3s
6s
9s
12s
15s
18s
21s
24s
27s
30s



EXPT_1_PART_2.asc
R1
2
V2
.tran 30
L1
2
C1
0.125
PULSE(0 20 0 1n 1n 100 1n 10)

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(B) Here since $V(0^-) = 5V$, thus we attach a 5V DC source to the capacitor in series and consider it uncharged, ie, $V_c(0^-) = 0$
Here $i(0^-) = 0$.

The equations would be similar to the previous with the exception of equation number (2)

$\Rightarrow$ From Equation (1) $\Rightarrow$ $20 - Ri_R(t) - \frac{L\,di(t)}{dt} = 0$ —(1)'

Also, we have $\Rightarrow$ $\frac{L\,di(t)}{dt} = 5 + V_c(t)$ $\Rightarrow$ $\frac{L\,d^2 i(t)}{dt^2} = \frac{dV_c(t)}{dt}$
(This gives $\frac{di}{dt}(0^-) = \frac{5}{2}$) —(2)'

From Equation (3) $\Rightarrow$ $i_R(t) = i(t) + i_c(t)$ —(3)'

From Equation (4) $\Rightarrow$ $i_c(t) = \frac{C\,dV_c(t)}{dt}$ —(4)'

Since equations (1)', (2)', (3)' and (4)' are same as the previous equation (1), (2), (3), (4) from part (A), we know that the corresponding differential equations will also be the same.

$\therefore$ we get $\frac{d^2 i(t)}{dt^2} + \frac{1}{2C}\frac{di(t)}{dt} + \frac{1}{2C} i(t) = \frac{5}{C}$ —(β)

Since the differential equation is the same, we can also say that the solutions are going to be the same too, for parts (i), (ii), (iii), ie, for the various values of the capacitors.

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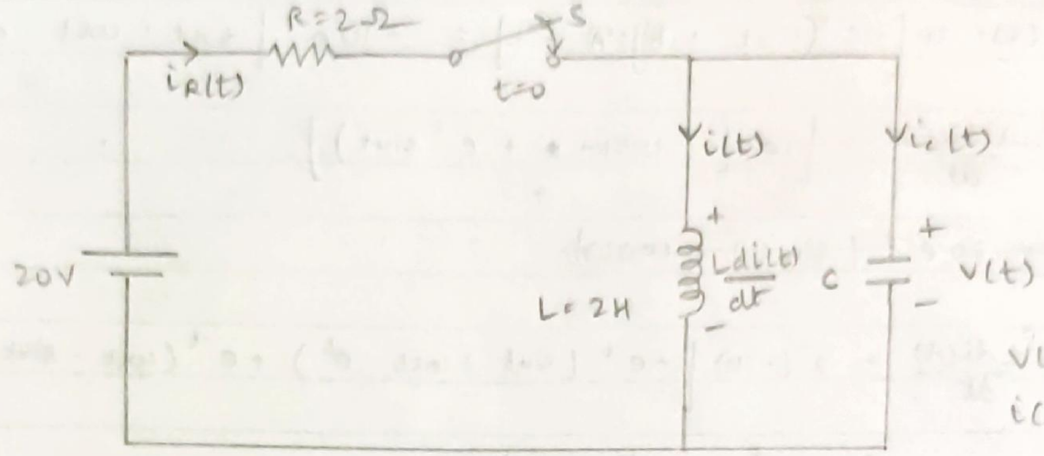
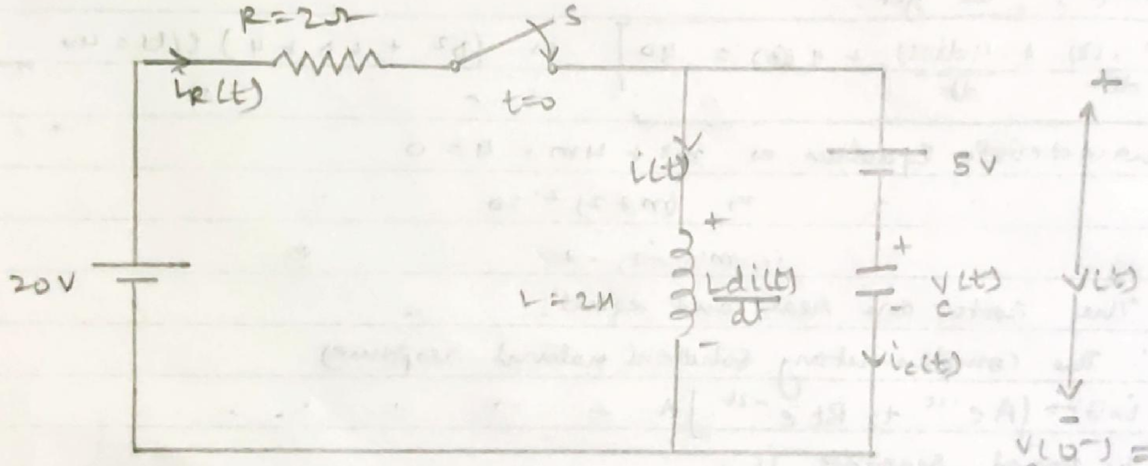


$v(0^-) = 5\,V$

$i(0^-) = 0$

$\equiv$



$V_c(0^-) = 0\,V$

$i(0^-) = 0$

→ For part (i), $C = \frac{1}{9} F$

From (β), we get

$\frac{d^2 i(t)}{dt^2} + \frac{9}{2}\frac{di(t)}{dt} + \frac{9}{2} i(t) = 45 \Rightarrow \left(D^2 + \frac{9}{2}D + \frac{9}{2}\right) i(t) = 45$

The characteristic equation is the same as before in part A (i)

$\therefore m = -3, -\frac{3}{2}$

$\therefore$ Natural response $i_n(t) = [Ae^{-3t} + Be^{-\frac{3t}{2}}]A$

Forced Response $= \frac{45}{(D^2 + \frac{9}{2}D + \frac{9}{2})}\Big|_{D=0}$ (Since $45 = 45e^{0t}$)

$i_f(t)$

$i_f(t) = 10 A$

$\therefore i(t) = i_n(t) + i_f(t) = [Ae^{-3t} + Be^{-\frac{3t}{2}} + 10]A$

Now $i(0^-) = A + B + 10 = 0 \Rightarrow 6A + 6B = -60$ — (i)

$\frac{di(0^-)}{dt} = -3A - \frac{3B}{2} = \frac{5}{2} \Rightarrow 6A + 3B = -5$ — (ii)

$\therefore 3B = -55 \Rightarrow B = \frac{-55}{3} \Rightarrow A = \frac{25}{3}$

$\therefore i(t) = \left[\frac{25}{3}e^{-3t} - \frac{55}{3}e^{-\frac{3t}{2}} + 10\right]A$

Now $\frac{L\,di(t)}{dt} = 2\left[-25e^{-3t} + \frac{55}{2}e^{-\frac{3t}{2}}\right] = 5 + V_C(t) = V(t)$

$\therefore \cancel{V(t) = \left[-50e^{-3t} + \frac{55}{2}e^{-\frac{3t}{2}}\right]V}$

$V(t) = [-50e^{-3t} + 55e^{-\frac{3t}{2}}]V$

→ For part (ii), $C = \frac{1}{4} F$

From (β), we get $\frac{d^2 i(t)}{dt^2} + 2\frac{di(t)}{dt} + 2i(t) = 20 \Rightarrow (D^2 + 2D + 2)i(t) = 20$

Characteristic equation is the same as before in part A (ii)

$\therefore m = -1 \pm j$

$\therefore$ Natural response $i_n(t) = [Ae^{(-1+j)t} + Be^{(-1-j)t}]A$

Forced Response $= \frac{20}{(D^2+2D+2)}\Big|_{D=0}$ (Since $20 = 20e^{0t}$)

$i_f(t)$

$i_f(t) = 10 A$

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$i(t) = i_n(t) + i_f(t) = [A e^{(-1+j)t} + B e^{(-1-j)t} + 10]$ A

Now $i(0^-) = A + B + 10 = 0 \Rightarrow A = -B - 10$ —(i)

$\frac{di(0^-)}{dt} = (-1+j)A - (1+j)B = \frac{5}{2}$ —(ii)

from (i) and (ii),

$(-1+j)(-B-10) - (1+j)B = \frac{5}{2}$

$\Rightarrow B(1-j) + 10(1-j) - (1+j)B = \frac{5}{2}$

$\Rightarrow -2jB + 10 - 10j = \frac{5}{2}$

$\Rightarrow -4jB + 20 - 20j = 5$

$\Rightarrow 4jB = 15 - 20j$

$\Rightarrow B = \frac{15 - 20j}{4j} = \frac{(-15j - 20)}{4}$

$\therefore A = -B - 10 = \frac{15j + 20 - 40}{4} = \frac{15j - 20}{4}$

$\therefore i(t) = \left[\frac{(15j - 20)}{4} e^{(-1+j)t} - \left(\frac{15j + 20}{4}\right) e^{(-1-j)t} + 10\right]$ A

$= e^{-t}\left[\frac{15j}{4} e^{jt} - 5e^{jt} - \frac{15j}{4} e^{-jt} - 5e^{-jt} + 10e^{t}\right]$ A

$= e^{-t}\left[-\frac{15}{2}\left(\frac{e^{jt} - e^{-jt}}{2j}\right) - 5\left(\frac{e^{jt} + e^{-jt}}{2}\right)\times 2 + 10e^{t}\right]$ A

$= e^{-t}\left[-\frac{15}{2}\sin t - 10\cos t + 10e^{t}\right]$ A

$i(t) = -\frac{5}{2} e^{-t}\left[3\sin t + 4\cos t - 4e^{t}\right]$ A

Now, $L\frac{di(t)}{dt} = (2)\frac{5}{2}\left\{e^{-t}\left[3\sin t + 4\cos t - 4e^{t}\right] - e^{-t}\left[3\cos t - 4\sin t - 4e^{t}\right]\right\}$ A

$= 5e^{-t}\left[7\sin t + \cos t\right] = v_c(t) + 5 = v(t)$

$\therefore v(t) = 5e^{-t}\left[7\sin t + \cos t\right]$ V

For (iii), $c = \frac{1}{8} F$

From β, we get

$$\frac{d^2 i(t)}{dt^2} + 4\frac{di(t)}{dt} + 4i(t) = 40 \Rightarrow (D^2 + 4D + 4)\, i(t) = 40$$

The characteristic equation is the same as before in part A (iii)

$\therefore m = -2, -2$

$\therefore$ Natural Response $i_n(t) = [Ae^{-2t} + Bte^{-2t}]$ A

Forced Response $i_f(t) = \left.\frac{40}{(D^2+4D+4)}\right|_{D=0}$ (since $40 = 40e^{0t}$)

$$i_f(t) = 10 \text{ A}$$

$$\therefore i(t) = i_n(t) + i_f(t) = [Ae^{-2t} + Bte^{-2t} + 10] \text{ A}$$

Now,

$i(0^-) = A + 10 = 0 \Rightarrow A = -10$

$\frac{di(0^-)}{dt} = -2A + B = \frac{5}{2} \Rightarrow -2(-10) + B = \frac{5}{2}$

$\Rightarrow 20 + B = \frac{5}{2}$

$\Rightarrow B = -\frac{35}{2}$

$$\therefore i(t) = [-10e^{-2t} - \frac{35}{2}te^{-2t} + 10] \text{ A}$$

$\therefore$ ~~v(t)~~ Now, $L\frac{di(t)}{dt} = 2\left[20e^{-2t} - \frac{35}{2}e^{-2t} + 35te^{-2t}\right]$

$= 2\left[\frac{5}{2}e^{-2t} + 35te^{-2t}\right]$

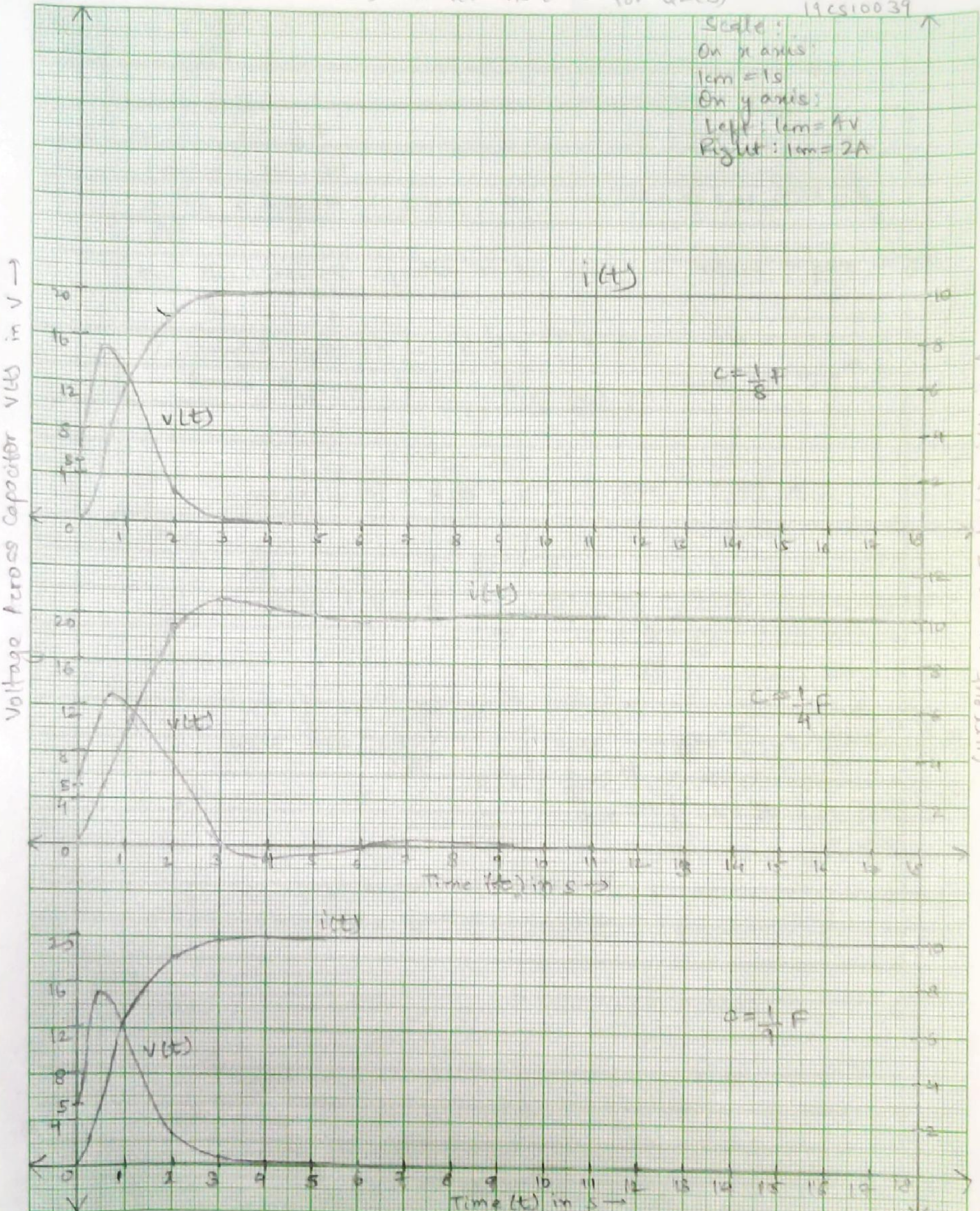
$= [e^{-2t}][5 + 70t] = V_c(t) + 5$

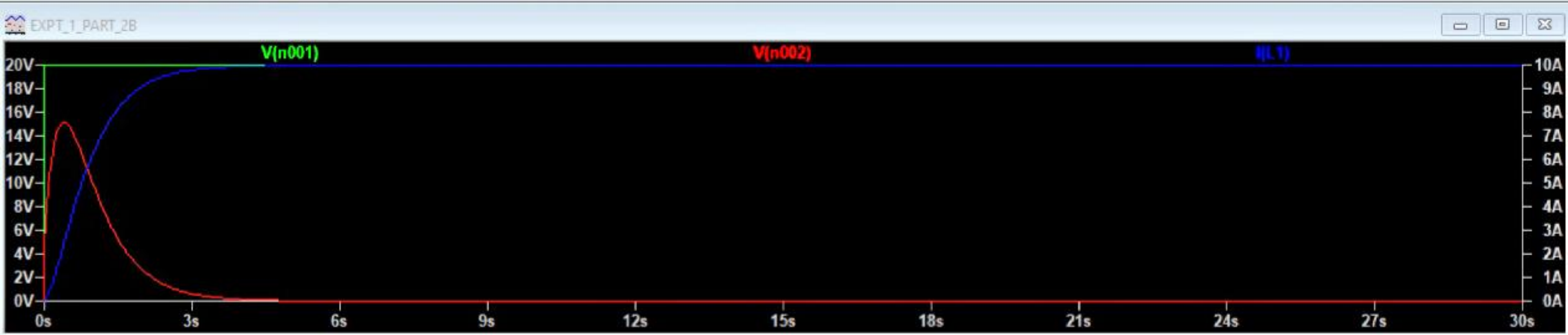
$= v(t)$

~~$\therefore v(t) = e^{-2t}(5 + 70t)$ V~~

$$v(t) = e^{-2t}(5 + 70t) \text{ V.}$$

V(t) and i(t) v/s time for Q2(B) Kaushal Banthia 19CS10039

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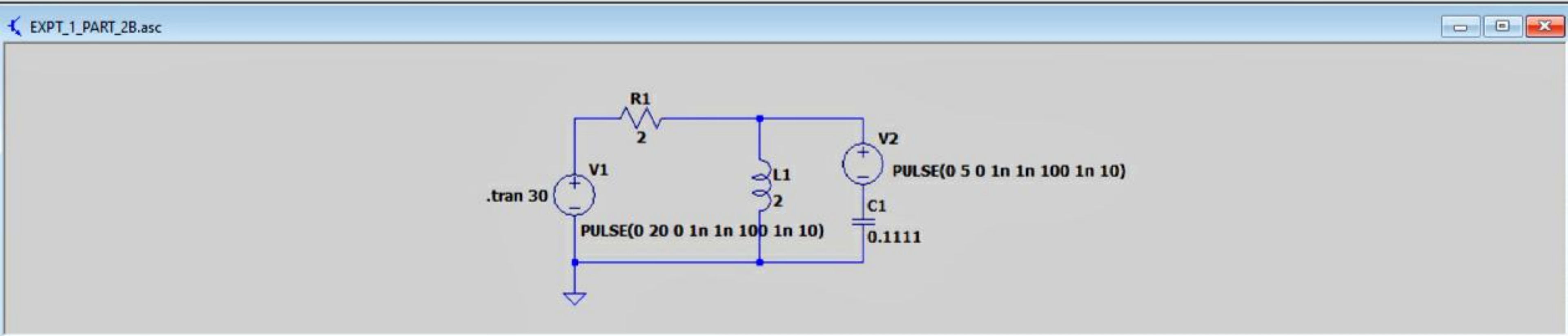
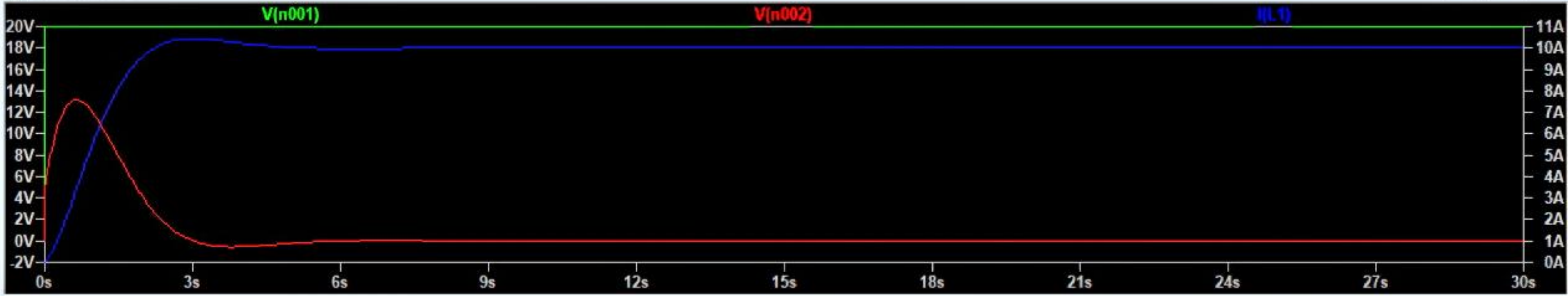
V(n001)
V(n002)
I(L1)
20V
18V
16V
14V
12V
10V
8V
6V
4V
2V
0V
10A
9A
8A
7A
6A
5A
4A
3A
2A
1A
0A
0s
3s
6s
9s
12s
15s
18s
21s
24s
27s
30s


EXPT_1_PART_2B.asc
R1
2
V1
.tran 30
PULSE(0 20 0 1n 1n 100 1n 10)
L1
2
V2
PULSE(0 5 0 1n 1n 100 1n 10)
C1
0.1111

EXPT_1_PART_2B



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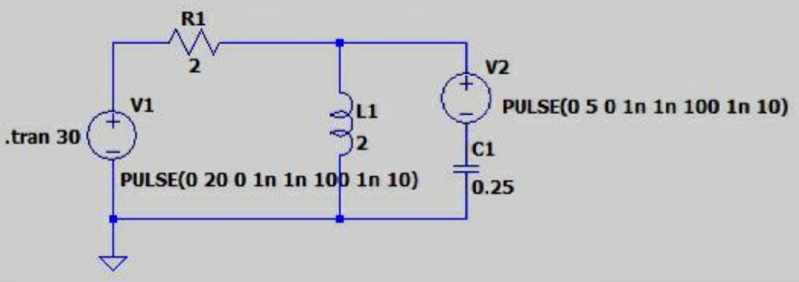
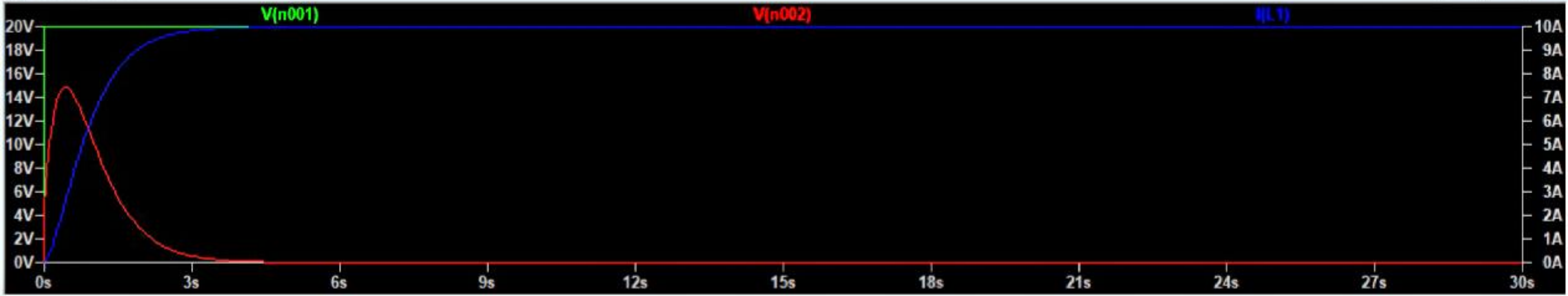


EXPT_1_PART_2B



EXPT_1_PART_2B.asc