19CS10039
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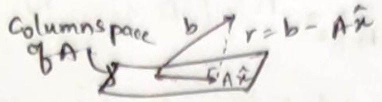
4) $A = \begin{bmatrix} | & | & | & & | \\ a_1 & a_2 & a_3 & \cdots & a_n \\ | & | & | & & | \end{bmatrix}$



Any vector in the plane, that represents the subspace which is column space of A, can be represented as linear combination of columns of A.

$\hat{x}$ is coefficient vector of the linear combination which gives closest vector to b in colspace (A),

Now, $b - A\hat{x}$ is perpendicular to the plane. Thus, all vectors in the plane are perpendicular to $b - A\hat{x}$.

Since $a_1, a_2, \ldots a_n$ all lie in this plane, all are perpendicular to $(b - A\hat{x})$. Thus, their dot products are zero.

$\Rightarrow a_1^T \cdot (b - A\hat{x}) = 0$, $a_2^T \cdot (b - A\hat{x}) = 0$ .... $a_n^T (b - A\hat{x}) = 0$.

This gives rise to the name of "normal equations".
This justifies the name.

Grouped as one, they are written as $A^T(b - A\hat{x}) = 0$.

$\therefore A^T b = A^T A \hat{x}$.

In case the columns of A are not linearly independent, then there can be infinite solutions, because the columns are dependent, we have $Ay = 0$ for some $y \neq 0$.

If the LS solution is $\hat{x}$, then $\|A\hat{x} - b\|_2^2 = \min_{x \in \mathbb{R}^n} \|Ax - b\|_2^2$,

then any solution of the type $\hat{x} + \tau y$ is another solution. $(\tau \in \mathbb{R})$

This is due to the fact, $A(\hat{x} + \tau y) - b = A\hat{x} + \tau Ay - b$
$= A\hat{x} - b$

$(\because Ay = 0)$