19CS10039
Kaushal Banthia

8 Given: $\hat{z}_{t+1} = \theta_1 z_t + \theta_2 z_{t-1} + \dots + \theta_M z_{t-M+1}$

$t = M, M+1, \dots 100$

(a) We need to find parameters $\theta_1, \theta_2, \dots, \theta_M$ to minimize the sum of squared errors. ~~~~

$(\hat{z}_{t+1} - z_{t+1})^2$. for $t = M, M+1, \dots 99$ (observed data points only upto z_{100}) $\hookrightarrow z_{t+1}$

This can be written as $\|A\theta - b\|^2$.

We need to minimize this value, where A ~~has~~ is a matrix with dimension $(100-M) \times M$.

(b) $A = \begin{bmatrix} z_M & z_{M-1} & z_{M-2} & \cdots & z_1 \\ z_{M+1} & z_M & & & z_2 \\ \vdots & \vdots & & & \vdots \\ z_{99} & z_{98} & & & z_{99-M+1} \end{bmatrix}_{(100-M)\times M}$

$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_M \end{bmatrix}_{M\times 1}$

$b = \begin{bmatrix} z_{M+1} \\ z_{M+2} \\ \vdots \\ \cancel{z_{99}} \\ z_{100} \end{bmatrix}_{(100-M)\times 1}$

(c) The values along the any diagonal are the same. Thus, for a constant value of $(i-j)$, a_{ij} remains constant.

This makes the matrix A special (Toeplitz matrix)

(d) Dimension $(A) = (100-M) \times M$

$\therefore$ Rank $(A) \leq \min(100-M, M)$

~~If Rank (A) is always~~

$\therefore$ Rank (A) can never exceed 50.