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(2) (a) LHS $= \text{avg}(\alpha x + \beta 1_n) = 1_n^T \cdot \dfrac{(\alpha x + \beta \cdot 1_n)}{n}$ $\left(\because \text{avg}(x) = \dfrac{1_n^T \cdot x}{n}\right)$

$$= \frac{1_n^T \alpha x + 1_n^T \cdot \beta \cdot 1_n}{n}$$

$$= \alpha\left(\frac{1_n^T \cdot x}{n}\right) + \beta\left(\frac{1_n^T \cdot 1_n}{n}\right)$$

$$= \alpha\, \text{avg}(x) + \beta \frac{n}{n}$$

$$= \alpha\, \text{avg}(x) + \beta = \text{RHS}.$$

$\therefore \text{avg}(\alpha x + \beta 1_n) = \alpha\, \text{avg}(x) + \beta$.

~~LHS =~~

~~(b) $\text{std}(\alpha x + \beta 1_n) = \dfrac{\left\| (\alpha x + \beta 1_n) - 1_n^T \dfrac{(\alpha x + \beta 1_n)}{n} \cdot 1_n \right\|_2}{\sqrt{n}}$~~ $\left(\because \text{std}(x) = \cancel{\dfrac{\| x - 1_n^T \cdot \frac{x}{n} 1_n \|_2}{\sqrt{n}}}\right)$

~~$= \dfrac{\left[(\alpha x + \beta 1_n) - \dfrac{1_n^T(\alpha x + \beta 1_n)}{n} \cdot 1_n\right]^T \left[(\alpha x + \beta 1_n) - \dfrac{1_n^T(\alpha x + \beta 1_n) \cdot 1_n}{n}\right]}{\sqrt{n}}$~~

~~$(\because \|x\|_2 = \sqrt{x^T x})$~~

(b) We have $[\text{rms}(x)]^2 = [\text{avg}(x)]^2 + [\text{std}(x)]^2$

$\Rightarrow \text{std}(x) = \sqrt{[\text{rms}(x)]^2 - [\text{avg}(x)]^2}$

$\Rightarrow \text{std}(x) = \sqrt{\dfrac{x^T x}{n} - \left(\dfrac{1_n^T \cdot x}{n}\right)^2}$

LHS =

$\therefore \text{std}(\alpha x + \beta \cdot 1_n) = \sqrt{\dfrac{(\alpha x + \beta 1_n)^T (\alpha x + \beta 1_n)}{n} - \left(\dfrac{1_n^T (\alpha x + \beta 1_n)}{n}\right)^2}$

$$= \sqrt{\frac{(\alpha x^T + \beta 1_n^T)(\alpha x + \beta 1_n)}{n} - \frac{(1_n^T \alpha x + 1_n^T \beta 1_n)^2}{n^2}}$$

$$= \sqrt{\frac{\alpha^2 x^T x + 2\alpha\beta x^T 1_n + \beta^2 n}{n} - \frac{(\alpha^2 (1_n^T x)^2 + \beta^2 n^2 + 2\alpha\beta n 1_n^T x)}{n^2}}$$

$$= \sqrt{\frac{\alpha^2 x^T x}{n} + \cancel{\frac{2\alpha\beta x^T 1_n}{n}} + \cancel{\beta^2} - \alpha^2\left(\frac{1_n^T x}{n}\right)^2 - \cancel{\beta^2} - \cancel{\frac{2\alpha\beta 1_n^T x}{n}}}$$

$$= \sqrt{\alpha^2 \frac{x^T x}{n} - \alpha^2 \left(\frac{1_n^T x}{n}\right)^2}$$

$$= \sqrt{(\alpha^2)\left[\frac{x^T x}{n} - \left(\frac{1_n^T x}{n}\right)^2\right]} = |\alpha| \sqrt{\frac{x^T x}{n} - \left(\frac{1_n^T x}{n}\right)^2}$$

$= |\alpha|\, \text{std}(x) = \text{RHS}$ $\therefore \text{std}(\alpha x + \beta \cdot 1_n) = |\alpha|\, \text{std}(x)$