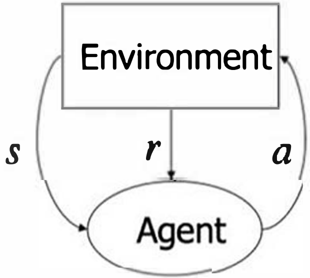
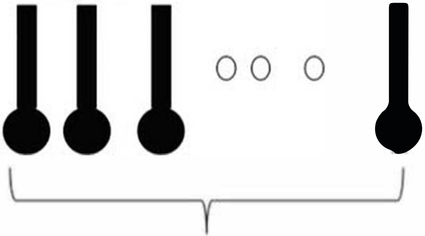
Learning with a critic

- Who is a critic?
  - Not a teacher
    - does not tell us what to do
  - only evaluates past performances
    - feedback scarce and delayed



- Learner (Agent)
  - interacts with an environment (at some state)
    - May change the state
  - gets reward / penalty sometimes (by the critic)
  - tries to reach a goal (a state)
  - Learns a series of actions to reach the goal.
    - Maximizes total rewards from any state.

A simple learner: K-Arm Bandit



K -levers

A simple machine with one state

$Q(a)$: Value of action a

Initially $Q(a)=0$ for all a

Store r_a after each a, $Q(a)=r_a$

Action (a):To pull a lever to win a reward (r_a).

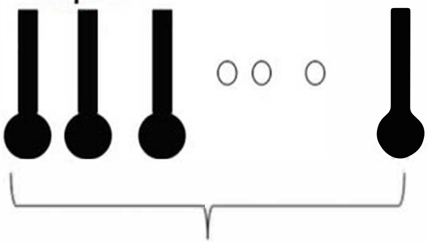
The task: to decide which lever to pull to maximize the reward

Supervised Learning: the correct class, namely, the lever leading to maximum earning labeled by a teacher.

Reinforcement learning: to try different levers and keep track of the best.

Choose $a*$ if $Q(a*) = \max_a Q(a)$

Nondeterministic: K-Arm Bandit



K -levers

Action (a):To pull a lever to win a reward r with $p(r|a)$.

The task: to decide which lever to pull to maximize the reward

Reinforcement learning: to try different levers and keep track of the best.

$Q_t(a)$: Estimate of the Value of action a at time t

→ an average of all past rewards when a was chosen

Delta rule: $Q_{t+1}(a) \leftarrow Q_t(a) + \eta[r_{t+1}(a) - Q_t(a)]$

η: learning factor decreasing with time

Choose $a*$ if $Q(a*) = \max_a Q(a)$

More complex environment

- Multiple states of the environment
- Action also affects the next state.
- The agent senses state (s_{t+1}) after an action (a_t).
  - may get reward (r_{t+1})
- Nondeterministic reward: p($r|s_i, a_j$)
- To learn Q(s_i, a_j): Value of taking action a_j in state s_i
- Rewards may be delayed
  - Need for immediate estimation of prospective reward

Learning a Markov Decision Process (MDP)

- An agent takes an action $a_t \in \mathbf{A}$ at state $s_t \in S$ at discrete time t.
  - $p(r_{t+1}|s_t, a_t)$
  - $p(a_{t+1}|s_t, a_t)$
- Reaching a terminal state and remains there for any action with prob. 1 without any reward.
- Episode or trial: The sequence of actions from the start to the terminal state.
- Policy: mapping from the states of the environment to actions: $\boldsymbol{\pi} : S \rightarrow \mathbf{A}$
- Value of a policy $\boldsymbol{\pi}$: $V^{\pi}(s_t) = E[\, r_{t+1} + r_{t+2} + \ldots + r_{t+T}]$

Finite horizon

May be infinite also.

Learning a policy

- Given p(r_{t+1}| s_t, a_t) and p(a_{t+1}|s_t, a_t)   (Model)
- Policy: mapping from the states to actions: $\pi : S \rightarrow A$
- Value of a policy π :
  - $V^{\pi}(s_t) = E[\, r_{t+1} + r_{t+2} + \dots + r_{t+T}]$
- Discounted value with infinite horizon:
  - $V^{\pi}(s_t) = E[\, r_{t+1} + \gamma\, r_{t+2} + \gamma^2\, r_{t+2} + \dots.]$
- Optimal policy π^* : $V^*(s_t) = \max_{\pi} [V^{\pi}(s_t)]$
- As an alternative: Learn Q(s_t, a_t): Value at state with action
- $V^*(s_t) = \max_a [Q(s_t, a_t)] = \max_a E[\, r_{t+1} + \gamma\, r_{t+2} + \gamma^2\, r_{t+2} + \dots.]$
  - $= \max_a E[\, r_{t+1} + \gamma\, V^{\pi}(s_{t+1})]$

Learning optimal policy

$$V^*(s_t) = \max_{a_t}\left(E[r_{t+1}] + \gamma \sum_{s_{t+1}} P(s_{t+1}|s_t, a_t)V^*(s_{t+1})\right)$$

$$Q^*(s_t, a_t) = \max_{a_t}\left(E[r_{t+1}] + \gamma \sum_{s_{t+1}} P(s_{t+1}|s_t, a_t)\max_{a_{t+1}}(Q^*(s_{t+1}, a_{t+1}))\right)$$

- $\pi^*(s_t)$: Choose a^* providing $V^*(s_t)$

Or

- $\pi^*(s_t)$: Choose a^* if $Q^*(s_t, a^*) = \max_a Q^*(s_t, a)$

PowerPoint Slide Show - [ReinforcementLearning]

Model based learning

- Model: p(r_{t+1}| s_t, a_t) and p(a_{t+1}|s_t, a_t) known.
- Directly solve for the optimal value function and policy
  - using dynamic programming

$$V^*(s) = \max_a \left(E[r|s,a] + \gamma \sum_{s'} P(s'|s,a)V(s') \right)$$

$$\pi^*(s) = \arg\max_a \left(E[r|s,a] + \gamma \sum_{s'} P(s'|s,a)V(s') \right)$$

- Two approaches
  - Value iteration algorithm
  - Policy iteration algorithm

Value iteration algorithm

Initialize $V(s)$ to arbitrary values

Repeat

For all $s \in S$

For all $a \in A$

$$Q(s,a) = \max_a \left(E[r|s,a] + \gamma \sum_{s'} P(s'|s,a) V(s') \right)$$

$$V(s) \leftarrow \max_a Q(s,a)$$

$$\pi^*(s) = \operatorname{argmax}_a \left(E[r|s,a] + \gamma \sum_{s'} P(s'|s,a) V(s') \right)$$

Until $V(s)$ converge

Policy iteration algorithm

Update policy directly from intermediate values

Initialize π to arbitrary values

Repeat

$\pi \leftarrow \pi'$

Compute values using π

$$V^{\pi}(s) = \max_a \left(E[r|s, \pi(s)] + \gamma \sum_{s'} P(s'|s, \pi(s)) V^{\pi}(s') \right)$$

Improve the policy at each stage.

$$\pi'(s) = \operatorname*{argmax}_a \left(E[r|s, a] + \gamma \sum_{s'} P(s'|s, a) V(s') \right)$$

Until $\pi = \pi'$

Temporal difference algorithm

- No prior knowledge of model
  - No $p(r_{t+1} | s_t, a_t)$ and $p(a_{t+1} | s_t, a_t)$
- requires exploration of the environment to query the model
  - to see the value of the next state and reward
- use this information to update the value of the current state
- called temporal difference algorithms
  - examines the difference between current estimate of the value and the discounted value of the next state and the reward received

Deterministic Environment

- For any state-action (s_t, a_t) power a single reward (r_{t+1}) and state transition (s_{t+1}) possible.

$$Q(s_t, a_t) = \left(r_{t+1} + \gamma \max_{a_{t+1}} \left(Q(s_{t+1}, a_{t+1})\right)\right)$$

- Update at every exploration
  - Add immediate reward with discounted estimate of the next state-action pair.
- Later updates more reliable
- Converge when all pairs are stable (little changes with iteration).

Nondeterministic environment

- varying reward or next state for a state-action pair
- keep a running average of values
  - Q-Learning

Delta rule: $Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \eta\,[r_{t+1} + \gamma \max\_a \{Q(s_{t+1}, a)\} - Q(s_t, a_t)]$

- Choose next action randomly (ε-Greedy sampling)
  - Sample an action uniformly with a prob. ε initially.
  - Actions providing higher values would have higher probability
    - Softmax over Q-values
      - using a Temperature Variable (T)
      - At every iteration T increases favoring higher Q values

$$p(a|s) = \frac{e^{\frac{Q(s,a)}{T}}}{\sum_b e^{\frac{Q(s,b)}{T}}}$$

Large number of states and actions

- Not feasible through tabular search.
- Use regression to predict Q values given current value, reward and next state.
  - Requires supervisory information or labels.

Summary

- RL: Learning with a critic
  - Different from supervised learning
  - labels or end results directly not available.
  - An agent observes state (s) changes on actions (a) , and may get reward (r) from a critic.
  - May be nondeterministic: $p(r|s,a)$, $p(s'|s,a)$
- To determine a policy: $\pi : \mathbf{S} \rightarrow \mathbf{A}$
  - For reaching goal from a state
- Model based learning:
  - Value iteration and Policy iteration
- Temporal difference algorithms

Deterministic

Nondeterministic

Q-Learning