1 (a) $f_1(w,x,y,z) = \Sigma(0,2,4,5,6,7,8,10,13,15)$

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$	1	0	0	1
$\bar{w}x$	1	1	1	1
wx	0	1	1	0
$w\bar{x}$	1	0	0	1

layout

0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

Groups

$(0,2,4,6) \rightarrow \bar{w}\bar{z}$

$(0,2,8,10) \rightarrow \bar{x}\bar{z}$

$(5,7,13,15) \rightarrow xz$

$f_1(w,x,y,z) = \bar{w}\bar{z} + \bar{x}\bar{z} + xz$

(b) $f_2(w,x,y,z) = \Sigma(0,1,5,7,8,10,14,15)$

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$	1	1	0	0
$\bar{w}x$	0	1	1	0
wx	0	0	1	1
$w\bar{x}$	1	0	0	1

layout

0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

Groups

$(0,8) \rightarrow \bar{x}\bar{y}\bar{z}$

$(1,5) \rightarrow \bar{w}\bar{y}z$

$(7,15) \rightarrow xyz$

$(10,14) \rightarrow wy\bar{z}$

$f_2(w,x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{w}\bar{y}z + xyz + wy\bar{z}$

(c) $f_3(w,x,y,z) = \Sigma(2,3,12,13,14,15)$

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$	0	0	1	1
$\bar{w}x$	0	0	0	0
wx	1	1	1	1
$w\bar{x}$	0	0	0	0

layout

0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

Groups
(12, 13, 14, 15) → wx
(2, 3) → $\bar{w}\bar{x}y$

$f_3(w,x,y,z) = wx + \bar{w}\bar{x}y$

2)

a) $g_1(w,x,y,z) = \Pi(1,3,5,7,13,15)$

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$	0	1	1	0
$\bar{w}x$	0	1	1	0
wx	0	1	1	0
$w\bar{x}$	0	0	0	0

layout

0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

Groups

$(0,2,4,6,12,14,8,10) \rightarrow \bar{z} \rightarrow 0$

$(8,9,10,11) \rightarrow w\bar{x} \rightarrow \bar{w}+x$

$g_1(w,x,y,z) = z(\bar{w}+x)$

b) $g_2(w,x,y,z) = \Pi(1,3,6,9,11,12,14)$

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$	0	1	1	0
$\bar{w}x$	0	0	0	1
wx	1	0	0	1
$w\bar{x}$	0	1	1	0

layout

0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

Groups

$(0,2,8,10) \rightarrow \bar{x}\bar{z} \rightarrow x+z$

$(5,7,13,15) \rightarrow xz \rightarrow \bar{x}+\bar{z}$

$(0,4) \rightarrow \bar{w}\bar{y}\bar{z} \rightarrow w+y+z$

$g_2(w,x,y,z) = (x+z)(\bar{x}+\bar{z})(w+y+z)$

(c) $g_3(w,x,y,z) = \Pi(1,4,5,6,11,12,13,14,15)$

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$	0	1	0	0
$\bar{w}x$	1	1	0	1
wx	1	1	1	1
$w\bar{x}$	0	0	1	0

layout

0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

Groups

- $(0,2,8,10) \rightarrow \bar{x}\bar{z} \rightarrow x+z$
- $(3,7) \rightarrow \bar{w}yz \rightarrow w+\bar{y}+\bar{z}$
- $(8,9) \rightarrow w\bar{x}\bar{y} \rightarrow \bar{w}+x+y$

$g_3(w,x,y,z) = (x+z)(w+\bar{y}+\bar{z})(\bar{w}+x+y)$

(3) $h(w, x, y, z) = \Sigma(1, 2, 3, 5, 13) + \Sigma_\phi(6, 7, 8, 9, 11, 15)$

(a)

w, x \ y, z	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$h = z + \bar{w}y$

Sum of products

(b)

w, x \ y, z	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$h(w, x, y, z) = \Pi(0, 4, 10, 12, 14) \cdot \Pi_\phi(6, 7, 8, 9, 11, 15)$

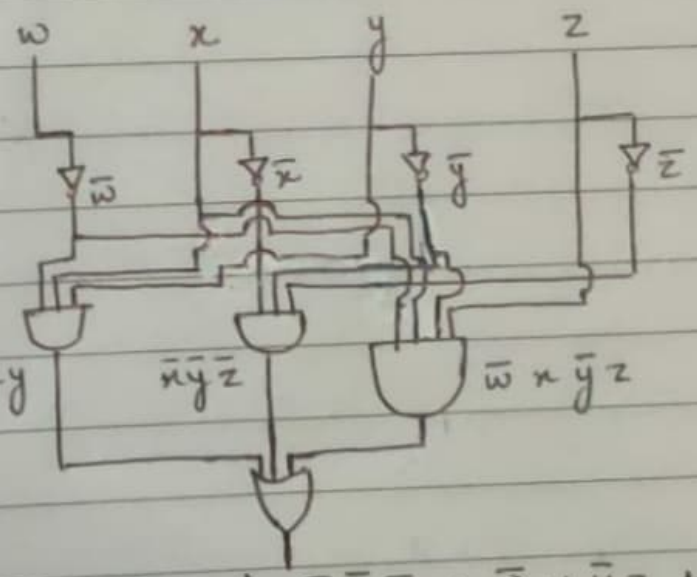
$h = (y + z)(\bar{w} + \bar{y})$

Product of sums

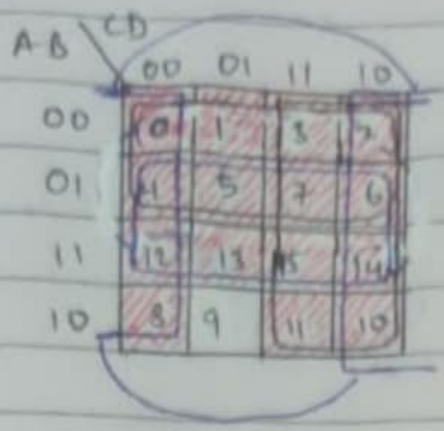
(4) $h(w, x, y, z) = \Sigma(0, 2, 3, 5, 8)$

w, x \ y, z	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

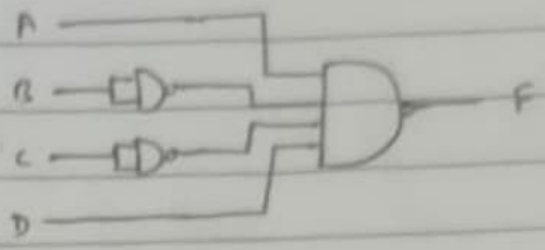
$h = \bar{x}\bar{y}\bar{z} + \bar{w}x\bar{y}z + \bar{w}\bar{x}y$



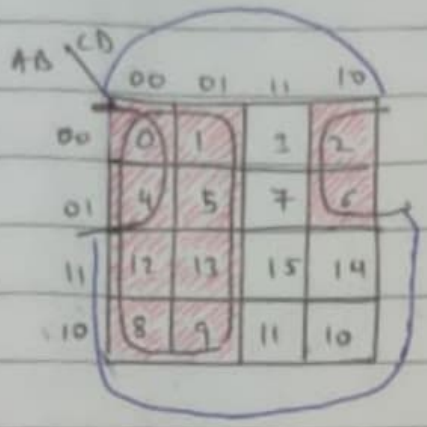
5 (a) $F_1(A, B, C, D) = A' + B + D' + B'C$



$F = A' + C + B + D'$

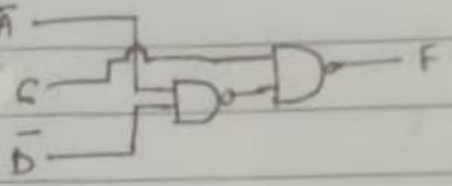


(b)



$F(A, B, C, D) = (A' + C' + D')(A' + C')(C' + D')$

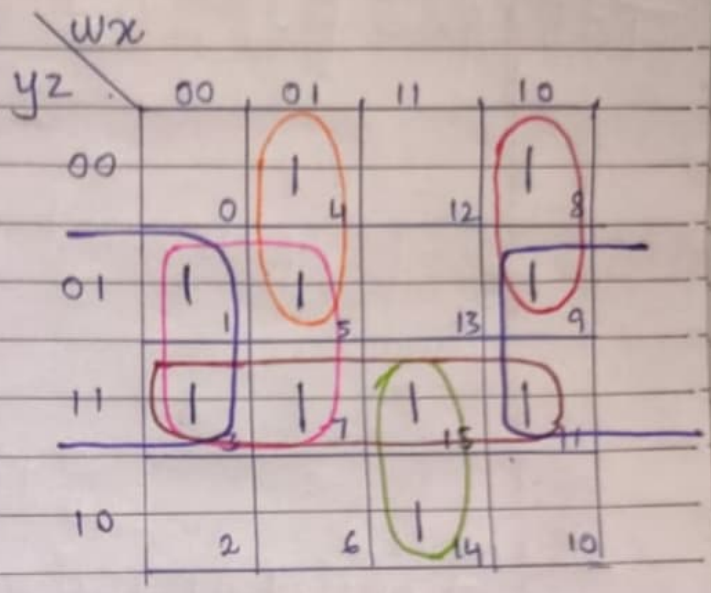
$F = C' + A'D'$



6 $f(v, w, x, y, z) = \Sigma(3, 6, 7, 8, 10, 12, 14, 17, 19, 20, 21, 24, 25, 27, 28)$

$f = v'wz' + vx'z + v'w'yz + v'w'xy + vw'xy' + wy'z'$

Q7. $T(w,x,y,z) = \Sigma(1,3,4,5,7,8,9,11,14,15)$



(a) PRIME IMPLICANTS:

$w'xy'$

$wx'y'$

wxy

$x'z$

$w'z$

yz

ESSENTIAL PRIME IMPLICANTS:

$w'xy'$, $wx'y'$, wxy

(b) MINIMAL EXPRESSIONS FOR T

1. first include all essential prime implicants
2. choose different subsets of the remaining prime implications which coner all the elements (the subset should be of minimum size)

SOLUTION - I

$T = w'xy' + wx'y' + wxy + w'z + yz$

SOLUTION - II

$T = w'xy' + wx'y' + wxy + x'z + w'z$

SOLUTION - III

$T = w'xy' + wx'y' + wxy + yz + x'z$

Q8)

PRIME IMPLICANTS

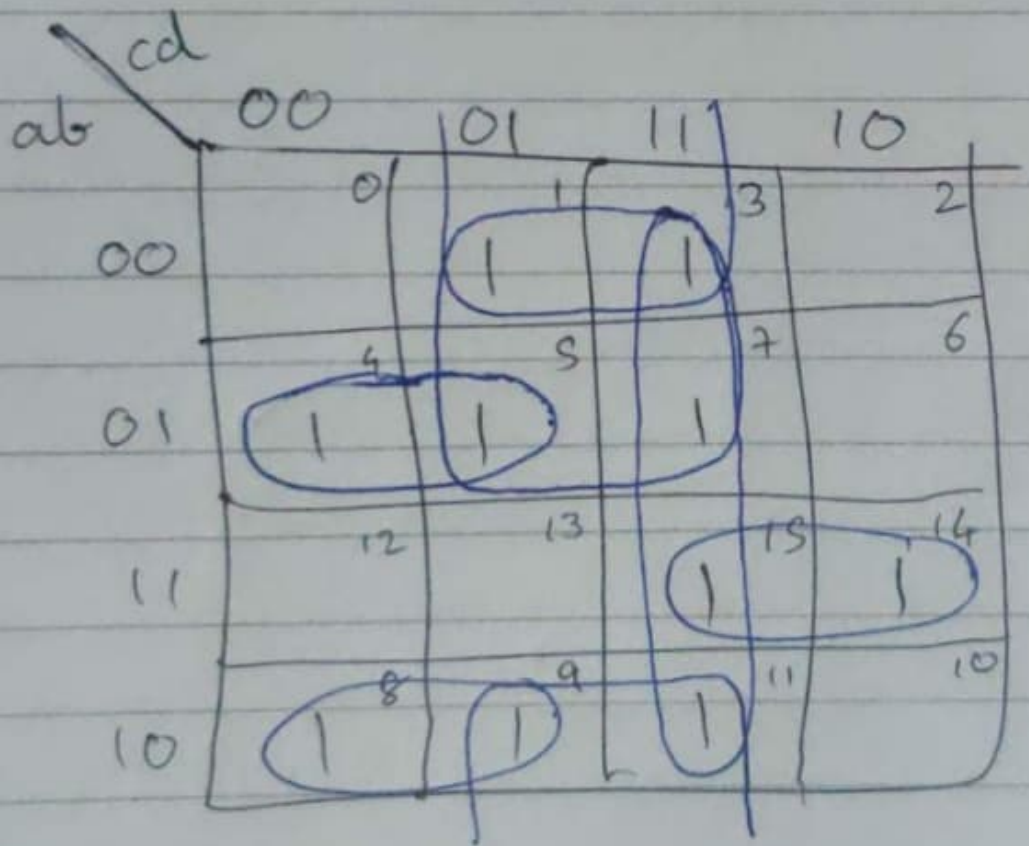
a'bc', abc, ab'c', a'd, b'd, cd (TOTAL 6)

ESSENTIAL

~~ESSENTIAL~~

ESSENTIAL

① a'bc'
(only cover for 4)



② abc
(only cover for 14)

③ ab'c'
(only cover for 8)

(TOTAL 3)

Q 9) Divide into groups

Group with k bits turned on → $\binom{n}{k}$ members
Group with k+1 " " " → $\binom{n}{k+1}$
→ .
→ .
→ $\binom{n}{n}$

when we collapse k and k+1 we would be collapsing a group with n-k 0's and k 1's with n-k-1 0's and k+1 1's ∴ in the output we would have n-k-1 0s, k 1's and 1 '-' symbol

the number of members of this group would be the permutation of n-k-1, k, 1 i.e.

$$\frac{n!}{(n-k-1)!\,k!}$$

we can continue collapsing such states and note that the number of 0s would decrease by 1 and the number of '-' would increase by 1

we can continue collapsing states till one of them has atleast a single 0.

Therefore in the final state where we get implicants all implicants would have k 1's and n-k '-'s.

∴ number of implicants = ∞
Permutation of k 1's and n-k '-'s

$$= \frac{(n-k+k)!}{k!\,(n-k)!}$$

$$= \frac{n!}{k!\,(n-k)!}$$

$$= \binom{n}{k}$$

Q10)

a) $h_1(w,x,y,z) = \Sigma(1,5,6,12,13,14) + \Sigma_D(2,4)$

(1) 0001 ✓
(2) 0010 ✓
(4) 0100 ✓
(5) 0101 ✓
(6) 0110 ✓
(12) 1100 ✓
(13) 1101 ✓
(14) 1110 ✓

(1,5) 0_01
(2,6) 0_10
(4,5) 010_ ✓
(4,6) 01_0 ✓
(4,12) _100 ✓
(5,13) _101 ✓
(6,14) _110 ✓
(12,13) 110_ ✓
(12,14) 11_0 ✓

(4,5,12,13) _10_
(4,6,12,14) _1_0

ignored

	0	1	(2)	3	(4)	5	6	7	8	9	10	11	12	13	14	15
$x\bar{y}$ (4,5,12,13)						X							X	X		
$x\bar{z}$ (4,6,12,14)							X						X		X	
$\bar{w}\bar{y}z$ (1,5)		X				X										
(2,6)							X									

redundant

$h_1(w,x,y,z) = x\bar{y} + x\bar{z} + \bar{w}\bar{y}z$

10 (b) $h_2(w,x,y,z) = \sum(0,1,5,7,8,10,14,15)$

(0) 0000 ✓
(1) 0001 ✓
(8) 1000 ✓
(5) 0101 ✓
(10) 1010 ✓
(7) 0111 ✓
(14) 1110 ✓
(15) 1111 ✓

(0,1) 000-
(0,8) -000
(1,5) 0-01
(8,10) 10-0
(5,7) 01-1
(10,14) 1-10
(7,15) -111
(14,15) 111-

	0	1	5	7	8	10	14	15
(0,1)	•	•						
(0,8)	•				•			
(1,5)	•		•					
(8,10)					•	•		
(5,7)			•	•				
(10,14)						•	•	
(7,15)				•				•
(14,15)							•	•

(0,8) → $\bar{x}\bar{y}\bar{z}$
(1,5) → $\bar{w}yz$
(7,15) → xyz
(10,14) → $wy\bar{z}$

$h_2(w,x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{w}\bar{y}z + xyz + wyz$

10(c) $h_3(w,x,y,z) = \Sigma(1,5,6,7,9,13,14,15,17,18,19,19,21,22,23,25,29,30)$

Ans :-

(1,5) 00-01 ✓
(1,9) 0-001 ✓
1,17 -0001 ✓
00001 ✓

(1,5,9,13) 0--01 ✓
(1,5,17,21) -0-01 ✓
(1,9,17,25) --001 ✓

(5) 00101 ✓
(6) 00110 ✓
(9) 01001 ✓
(17) 10001 ✓
(18) 10010 ✓

(5,7) 001-1 ✓
(5,13) 0-101 ✓
(5,21) -0101 ✓
(6,7) 001-0 ✓
6,14 0-110 ✓
6,22 -0110 ✓
(9,13) 01-01 ✓
(9,25) -1001 ✓
(17,19) 1001-1 ✓
(17,21) 10-01 ✓
(17,25) 1-001 ✓
(18,19) 1001- ✓
(18,22) 10-10 ✓

(5,7,13,15) 0--1-1
(5,7,21,23) -01-1
(5,13,21,29) --101 ✓
(6,7,14,15) 0-11-
(6,7,22,23) -011-
(6,14,22,30) --110
(9,13,25,29) -1-01 ✓
(17,19,21,23) 10--1
(17,21,25,29) 1--01 ✓
(18,19,22,23) 10-1-

7,15) 0-111 ✓
7,23) -0111 ✓
(13,15) 011-1 ✓
(13,29) -1101 ✓
(14,15) 0111- ✓
(14,30) -1110 ✓
(19,23) 10-11 ✓
(21,23) 101-1 ✓
(21,29) 1-101 ✓
(22,23) 1011- ✓
(22,30) 1-110 ✓

(1,5,9,13,17,21,23,29) ---01

(25,29) 11-01 ✓

01111 ✓
(23) 10111 ✓
(29) 11101 ✓
(30) 11110 ✓

→ Prime

	1	5	6	7	9	13	14	15	17	18	19	21	22	23	25	29	30
---01 → $\bar{y}z$ → (1,5,9,13,17,21,25,29) →	X	X			X	X			X			X			X	X	
0-1-1 → $\bar{v}xz$ → (5,7,13,15) →		X		X		X		X									X
--110 → $xy\bar{z}$ → (6,14,22,30) →			X				X						X				
10-1- → $v\bar{w}y$ → (18,19,22,23) →										X	X		X	X			
(5,7,21,23) →		X		X								X		X			
(6,7,14,15) →			X	X			X	X									
(6,7,22,23) →			X	X									X	X			
(17,19,21,23) →								X			X		X	X			

$$h_3(v,w,x,y,z) = \bar{y}z + \bar{v}xz + xy\bar{z} + v\bar{w}y$$