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(3)(a) If $\frac{a}{b} = 2^i$ then $\frac{b}{a} = 2^{-i} = 2^j$

∴ Symmetric

⇒ $\frac{a}{a} = 1 = 2^0$ ⇒ ~~Transitive~~ Reflexive

⇒ If $\frac{a}{b} = 2^i$ and $\frac{b}{c} = 2^j$

Then $\frac{a}{b} \cdot \frac{b}{c} = 2^i \cdot 2^j$

⇒ $\frac{a}{c} = 2^{(i+j)} = 2^k$ ∴ Transitive

∴ R is equivalence relation.

(b) ~~[2n]~~ { $[1] = \{1, 1\times 2, 1\times 2^2 \dots\}$

$[3] = \{3, 3\times 2, 3\times 2^2 \dots\}$

$[5] = \{5, 5\times 2, 5\times 2^2 \dots\}$

⋮

$[2n+1] = \{2n+1, (2n+1)\times 2, (2n+2)\times 2^2 \dots\}$

∴ The equivalences class is the set of odd natural numbers

(c) R is reflexive and transitive

For antisymmetry, if (a,b) and $(b,a) \in R$ then $a = b$

Here ~~it~~ not necessary as even when $a \neq b$,

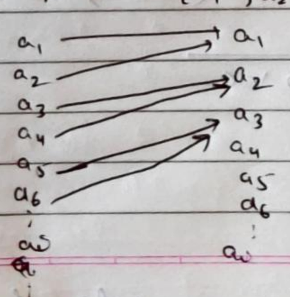
$(a,b) \in R$ and $(b,a) \in R$

as $\frac{a}{b} = 2^i$ as $\frac{b}{a} = 2^{-i} = 2^j$

∴ Not a partial Order.

(4) Let the infinite set A be $\{a_1, a_2, a_3 \dots a_i \dots\}$

Now we map(h)



The mapping can be done as

$$
\left.
\begin{aligned}
\therefore\ h(a_1) &= a_1 \\
h(a_2) &= a_1 \\
h(a_3) &= a_2 \\
h(a_4) &= a_2 \\
h(a_5) &= a_3 \\
h(a_6) &= a_3 \\
&\ \vdots
\end{aligned}
\right\}
\qquad
\begin{aligned}
&\cancel{h(a_{2n})} = a \\
&h(a_{2n}) = a_n \\
&h(a_{2n-1}) = a_n
\end{aligned}
$$

$\downarrow$

This gives us a mapping that is onto but not surjective.

(1) This program gives a n whose sum is equal to the sum of the initial n.

Now, since a is not divisible by 3 $\therefore n := a^{2020}$ is not divisible by 3. Thus sum of digits of n is not a multiple of 3.

↳ Loop Invariance.

Now after termination of loop, Assume all 10 digits are different. Then we have {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} arranged in a certain order.

$\Rightarrow$ sum of digits $= 0 + 1 + 2 + 3 + 4 \ldots + 9$

$= 45$

$\Rightarrow$ Divisible by 3.

$\Rightarrow$ This is contradiction.

$\therefore$ Not all 10 digits are different

$\Rightarrow$ At least 1 digit must be repeated.

~~This can also be done by Pigeon hole~~ as, Since ~~sum of n cannot be divided by~~ 3 $\Rightarrow$ sum of n ~~cannot be~~ ~~3~~

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(2) Any rational number can be approximated to a nearby real number (Dirichlet's Approximation)

$\Rightarrow c_0 \pi \approx d_0$

$\Rightarrow \sin(d_0) \approx 0 \therefore 0 < \sin(d_0) < 2^{-2020}$