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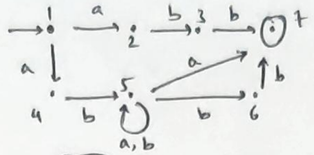
Q1) (a) Regular expression is $abb + ab(a^*b^*)^*(a+bb)$

(b) Its NFA is




$\delta(1, a) = \{2, 4\}$ New

$\delta(1, b) = \phi$

$\delta(2, a) = \phi$

$\delta(2, b) = 3$

$\delta(3, a) = \phi$

$\delta(3, b) = 7$

$\delta(4, a) = \phi$

$\delta(4, b) = 5$

$\delta(5, a) = \{5, 7\}$ New

$\delta(5, b) = \{5, 6\}$ New

$\delta(6, a) = \phi$

$\delta(6, b) = 7.$

$\delta(7, a) = \phi$

$\delta(7, b) = \phi$

$\delta(\{2, 4\}, a) = \delta(2, a) \cup \delta(4, a)$
$= \phi$

$\delta(\{2, 4\}, b) = \delta(2, b) \cup \delta(4, b)$
$= \{3, 5\}$ New

$\delta(\{5, 7\}, a) = \delta(5, a) \cup \delta(7, a)$
$= \{5, 7\}$

$\delta(\{5, 7\}, b) = \delta(5, b) \cup \delta(7, b)$
$= \{5, 6\}$

$\delta(\{5, 6\}, a) = \delta(5, a) \cup \delta(6, a)$
$= \{5, 7\}$

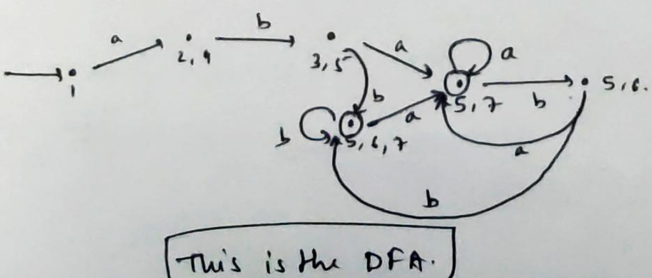
$\delta(\{5, 6\}, b) = \delta(5, b) \cup \delta(6, b)$
$= \{5, 6, 7\}$ New

$\delta(\{3, 5\}, a) = \delta(3, a) \cup \delta(5, a)$
$= \{5, 7\}$

$\delta(\{3, 5\}, b) = \delta(3, b) \cup \delta(5, b)$
$= \{5, 6, 7\}$

$\delta(\{5, 6, 7\}, a) = \delta(5, a) \cup \delta(6, a) \cup \delta(7, a)$
$= \{5, 7\}$

$\delta(\{5, 6, 7\}, b) = \delta(5, b) \cup \delta(6, b) \cup \delta(7, b)$
$= \{5, 6, 7\}$




This is the DFA.

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Q2)

(a) Regular expression for $L_2$ over $\{a, b, c\}$ is:

~~$(a^* b^* c^*)^{4i} + b$~~

$(a^* c^*)^* (a^* b c^*)^{4i} (a^* b c^*) (a^* c^*)^*$

~~$(a^* b c^*)^{4i} (a^*bc^*)(a^* c^*)^*$~~

This ensures any number of a's and c's, but only $4i+1$ b's.

(b) Here cycleshift $(A) = \{yx \mid xy \in A$ for some $x, y \in \Sigma^*\}$

Since there is closure under reversal,

thus if $xy \in A$ is regular,

then $yx \in$ cycleshift $(A)$ is also regular.

Hence proved (By using closure under reversal).

Q4) a) There are 2 equivalent classes, so we define

excess $(x) = \#a(x) - \#b(x) = e(x)$

$[u] = \{x \in \{a, b\}^* \mid e(x) = e(u)\}$

There are infinitely many such equivalent classes (since it maps to $\mathbb{Z}$) $\Rightarrow$ This implies

$\Rightarrow$ Not Regular

For the accepted class, $e(x) = 2021$. (given)

$\Rightarrow e(xz) = e(x) + e(z)$ ; $e(yz) = e(y) + e(z)$

$\therefore x \equiv_{L_4(a)} y$ iff $\forall z \in \{a, b\}^*$ (for $xz \in L_4(a)$ iff $yz \in L_4(a)$)

~~iff $yz \in$~~

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Q4) b) The equivalent classes are.

$e(x) \mod 2021$.

where $e(x)$ is excess $(x) = \# a(x) - \# b(x) = e(x)$

Here there are only finite number of such classes.

$\Rightarrow$ Only 2021 such classes. They are

$[\{0, 1, 2, 3, 4, 5, \ldots, 2019, 2020\}]$

Since it is finite

$\Rightarrow$ Regular language.

For the accepted class, $e(x) \mod 2021 = 0$

$e(xz) = e(x) + e(z)$ ; $e(yz) = e(y) + e(z)$

$x \equiv_{L_4(b)} y$ iff $\forall z \{a, b\}^* \left( \begin{array}{l} \text{for } xz \in L_4(b) \\ \Leftrightarrow yz \in L_4(b) \end{array} \right)$

Q3 a) $L_3(a) = \{\gamma \beta \gamma \mid \gamma \in \{0,1\}^* \text{ \& } \beta \in \{0,1\}^*\}$ be regular.

Let $p$ be pumping lemma constant.

$\Rightarrow w = 0^p 1 . 1 . 0^p 1 \in L_3(a)$

By Pumping lemma,

Pump from ~~from~~ first p. characters. ($0^l$ pumped out)

$\Rightarrow w' = 0^q 1 . 1 . 0^p 1$ where $q$ not equal to $p$.
$= 0^{(p-l)} 1 . 1 . 0^p 1$ since $p - l \neq l$ because $l \geq 1$.

Hence $w' \in L$.

Thus, there is contradiction

Thus $L_3(a)$ is not regular.