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Q1 (a) Reflexive : for $\langle a,b\rangle \leq \langle a,b\rangle \iff a \leq a$ and $b \leq b$.

Thus Reflexive.

Anti Symmetric : For $\langle a,b\rangle \leq \langle c,d\rangle \iff a \leq c$ and $b \leq d$. — (A)

Then, $\langle c,d\rangle \leq \langle a,b\rangle \iff c \leq a$ and $d \leq b$. — (B)

From (A) and (B),

$a = c$ and $b = d$.

Thus, $\langle a,b\rangle = \langle c,d\rangle$

Thus, Antisymmetric.

Transitive : For $\langle a,b\rangle \leq \langle c,d\rangle \iff a \leq c$ and $b \leq d$ — (A)

For $\langle c,d\rangle \leq \langle e,f\rangle \iff c \leq e$ and $d \leq f$ — (B)

From (A) and (B), ~~$a \leq e$ and $b \leq f$~~

$a \leq c \leq e$ and $b \leq d \leq f$

$\Rightarrow a \leq e$ and $b \leq f$

$\Rightarrow \langle a,b\rangle \leq \langle e,f\rangle$

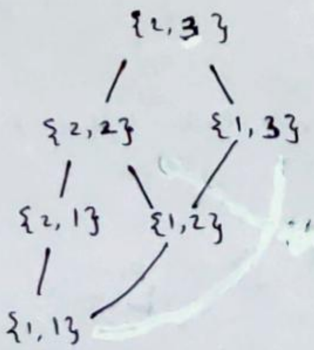
Thus, Transitive.

Since the set is Reflexive, Antisymmetric and transitive, it is a Partial ordering on $A_m \times A_n$

(b) $A_2 = \{1, 2\}$ $\Rightarrow A_2 \times A_3 = \{1,2\} \times \{1,2,3\}$

$A_3 = \{1, 2, 3\}$

$= \{\{1,1\}, \{1,2\}, \{1,3\}, \{2,1\}, \{2,2\}, \{2,3\}\}$



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(c) glb $(\langle a,b\rangle, \langle c,d\rangle)$ for any $a, c \in A_m$ and $b, d \in A_n$,

is $\langle \max(a,c), \max(b,d)\rangle$

lub $(\langle a,b\rangle, \langle c,d\rangle)$ for any $a, c \in A_m$ and $b, d \in A_n$

is $\langle \min(a,c), \min(b,d)\rangle$

Here, the $\max(a,c)$ returns the maximum value among a & c
the $\min(a,c)$ returns the minimum value among a & c.

~~Q2 For $\langle x,y\rangle$ and $\langle a,b\rangle$, since $\langle L, \cdot, +\rangle$ and $\langle M, \odot, \oplus\rangle$ are lattices, x~~

~~Q2 Since $\langle L, \cdot, +\rangle$ is a lattice, for any $\langle x, a\rangle$ and $\langle y, b\rangle \in L$,~~

~~such that~~

~~$\langle x, a\rangle \cdot \langle y, b\rangle$ exists and $\langle x, y\rangle + \langle a, b\rangle$ exists.~~

~~$\Rightarrow \langle x \cdot a, y \cdot b\rangle$ exists and $\langle x + a, y + b\rangle$ exists~~

~~Similarly, since $\langle M, \odot, \oplus\rangle$ is a lattice, for any~~

Q2 Since $\langle L, \cdot, +\rangle$ is a lattice, for any $\langle x, a\rangle \in L$,

$x \cdot a$ and $x + a$ exists.

Similarly, since $\langle M, \odot, \oplus\rangle$ is a lattice, for $\langle y, b\rangle \in M$,

$y \odot b$ and $y \oplus b$ exist.

Thus, for $\langle L \times M, \Delta, \nabla\rangle \Rightarrow \langle x,y\rangle \,\Delta\, \langle a,b\rangle = \langle x \cdot a, y \odot b\rangle$

Here, $x \cdot a \in L$ (meet) and $y \odot b$ exist $\in M$ (meet)

$\Rightarrow \langle x,y\rangle \,\Delta\, \langle a,b\rangle$ exists

$\langle x,y\rangle \,\nabla\, \langle a,b\rangle = \langle x + a, y \oplus b\rangle$

Here $x + a \in L$ (join) and $y \oplus b$ exist and $\in M$ (join)

$\Rightarrow \langle x,y\rangle \,\nabla\, \langle a,b\rangle$ exists.

Thus, $\langle L \times M, \Delta, \nabla\rangle$ is a lattice.