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Experiment 2 Part 1

Q1

Aim : To solve a series RLC and get the voltage across the capacitor and the current across the circuit. as a function of time

Since the capacitor and the inductor are initially uncharged, ie, $i(0^-) = 0$ and $v(0^-) = 0$, we dont need to add any extra voltage or current sources. Also, since $i(t) = c\frac{dv(t)}{dt}$ $\therefore \frac{dv(0^-)}{dt} = i(0^-) = 0$

→ For (i) $C = \frac{25}{9}$ F

Using Kirchhoff's Voltage Law in the loop,

$$1 - R\,i(t) - L\frac{di(t)}{dt} - v(t) = 0. \quad (1)$$

Now, $i(t) = c\frac{dv(t)}{dt} = \frac{25}{9}\frac{dv(t)}{dt}$ (2)

$\therefore$ ~~$1 - 2i(t) - c\frac{dv(t)}{dt} - v(t)$~~

$\Rightarrow$ From (1) and (2),

$$1 - 2\left(\frac{25}{9}\frac{dv(t)}{dt}\right) - \frac{25}{9}\frac{d^2v(t)}{dt^2} - v(t) = 0$$

$$\Rightarrow \frac{25}{9}\frac{d^2v(t)}{dt^2} + 2\left(\frac{25}{9}\frac{dv(t)}{dt}\right) + v(t) = 1$$

(a) $\Rightarrow$ $\boxed{\frac{d^2v(t)}{dt^2} + 2\frac{dv(t)}{dt} + \frac{9}{25}v(t) = \frac{9}{25}}$ $\Rightarrow \left(D^2 + 2D + \frac{9}{25}\right)v(t) = \frac{9}{25}$

(b) Characteristic Equation $\Rightarrow m^2 + 2m + \frac{9}{25} = 0$

$$\Rightarrow \left(m + \frac{1}{5}\right)\left(m + \frac{9}{5}\right) = 0$$

$$\Rightarrow \boxed{m = -\frac{1}{5}, -\frac{9}{5}}$$

The roots are real and distinct.

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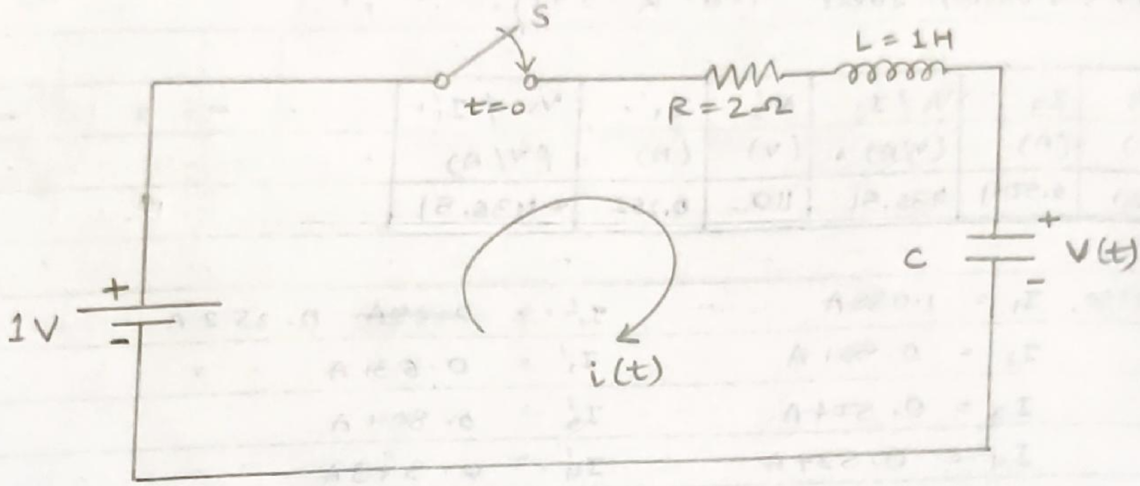

$i(0^-) = 0$

$V(0^-) = 0$

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(c) Thus, the complementary solution ~~is~~ (natural response) is

$$V_n(t) = \left(Ae^{-\frac{1}{5}t} + Be^{-\frac{9}{5}t}\right) V$$

The forced response is

$$V_f(t) = \left.\frac{9/25}{\left(D^2 + 2D + \frac{9}{25}\right)}\right|_{D=0} \quad \left(\text{since } \frac{9}{25} = \frac{9e^{0t}}{25}\right)$$

$$V_f(t) = 1 V.$$

$$\therefore V(t) = V_n(t) + V_f(t) = \left(Ae^{-\frac{1}{5}t} + Be^{-\frac{9}{5}t} + 1\right) V$$

Using initial conditions,

$$V(0^-) = A + B + 1 = 0$$

$$\therefore A + B = -1 \quad \text{--- (i)}$$

Also, $\frac{dv(0^-)}{dt} = 0 \Rightarrow \frac{-A}{5} - \frac{9}{5}B = 0$

$$\Rightarrow A = -9B \quad \text{--- (ii)}$$

from (i) and (ii), $-9B + B = -1 \Rightarrow 8B = 1$

$$\Rightarrow B = \frac{1}{8}$$

$$\therefore A = -9B = -\frac{9}{8}$$

$$\therefore v(t) = \left[-\frac{9}{8}e^{-\frac{1}{5}t} + \frac{1}{8}e^{-\frac{9}{5}t} + 1\right] V.$$

$$i(t) = C\frac{dv(t)}{dt} = \left(\frac{25}{9}\right)\frac{9}{40}\left[e^{-\frac{1}{5}t} - e^{-\frac{9}{5}t}\right] A$$

$$= \left[e^{-\frac{1}{5}t} - e^{-\frac{9}{5}t}\right] A$$

$$= \frac{5}{8}\left[e^{-\frac{1}{5}t} - e^{-\frac{9}{5}t}\right] A$$

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→ For(ii) $C = \frac{1}{2} F$

We have previously derived,

$1 - R\,i(t) - L\frac{di(t)}{dt} - v(t) = 0$ and $i(t) = C\frac{dv(t)}{dt}$

Thus, $1 - \frac{2dv(t)}{2\,dt} - \frac{1}{2}\frac{d^2v(t)}{dt^2} - v(t) = 0$

$\Rightarrow \frac{1}{2}\frac{d^2v(t)}{dt^2} + \frac{dv(t)}{dt} + v(t) = 1$

(a) $\Rightarrow \boxed{\frac{d^2v(t)}{dt^2} + 2\frac{dv(t)}{dt} + 2v(t) = 2}$ $\Rightarrow (D^2 + 2D + 2)\,v(t) = 2$

(b) Characteristic equation $\Rightarrow m^2 + 2m + 2 = 0.$

$\Rightarrow (m+1)^2 + 1 = 0$

$\Rightarrow m+1 = \pm\sqrt{-1}$

$\Rightarrow m+1 = \pm j$

$\Rightarrow m = -1 \pm j$

The roots are complex and distinct.

(c) The complementary solution (natural responses) is

$V_n(t) = [A e^{(-1+j)t} + B e^{(-1-j)t}]\,V.$

The forced response is

$V_f(t) = \left.\frac{2}{(D^2+2D+2)}\right|_{D=0}$ (since $2 = 2e^{0t}$)

$V_f(t) = 1\ V$

$\therefore v(t) = V_n(t) + V_f(t) = [A e^{(-1+j)t} + B e^{(-1-j)t} + 1]\,V.$

Using initial conditions,

$v(0^-) = A + B + 1 = 0$ $\therefore A + B = -1$ $\Rightarrow A = -B - 1$ —(i)

$\frac{dv}{dt}(0^-) = A(-1+j) + B(-1-j) = 0$ —(ii)

From (i) & (ii) $\Rightarrow (-B-1)(-1+j) + B(-1-j) = 0$

$\Rightarrow -B(-1+j) - (-1+j) - B(1+j) = 0$

$\Rightarrow -B[-1+j+1+j] + 1 - j = 0$

$\Rightarrow -B[2j] = j - 1$

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$$\Rightarrow B = \frac{1-j}{2j} = \frac{-j-1}{2}$$

$$\therefore A = -B-1$$

$$A = \frac{1+j}{2} - 1 = \frac{-1+j}{2}$$

$$\therefore v(t) = \left[\frac{(-1+j)}{2} e^{(-1+j)t} + \frac{(-j-1)}{2} e^{(-1-j)t} + 1\right] V$$

$$v(t) = e^{-t}\left[\left(\frac{-1+j}{2}\right) e^{jt} + \left(\frac{-j-1}{2}\right) e^{-jt} + e^{t}\right] V.$$

$$v(t) = e^{-t}\left[\frac{-e^{jt} + je^{jt} - je^{-jt} - e^{-jt}}{2} + e^{t}\right] V$$

$$v(t) = e^{-t}\left[-\left(\frac{e^{jt} + e^{-jt}}{2}\right) + \left(\frac{e^{jt} - e^{-jt}}{-2j}\right) + e^{t}\right] V.$$

$$v(t) = e^{-t}\left[-\cos t - \sin t + e^{t}\right] V.$$

$$\therefore i(t) = c\frac{dv(t)}{dt} = \frac{1}{2}(-e^{-t})(-\cos t - \sin t + e^{t}) + \frac{1}{2}e^{-t}(\sin t - \cos t + e^{t})$$

$$= \frac{1}{2}e^{-t}(\cos t + \sin t - e^{t}) + \frac{1}{2}e^{-t}(\sin t - \cos t + e^{t})$$

$$= \frac{1}{2}e^{-t}(2\sin t) \cancel{= e^{-t}}$$

$$\therefore i(t) = e^{-t}\sin t \ A$$

→ For (iii), $C = 1F$

From previous derivation,

$$1 - Ri(t) - L\frac{di(t)}{dt} - v(t) = 0 \quad \text{and} \quad i(t) = c\frac{dv(t)}{dt}$$

$$\Rightarrow 1 - 2\frac{dv(t)}{dt} - \frac{d^2v(t)}{dt^2} - v(t) = 0$$

(a) $\Rightarrow \boxed{\dfrac{d^2v(t)}{dt^2} + 2\dfrac{dv(t)}{dt} + v(t) = 1}$ $\Rightarrow (D^2 + 2D + 1)v(t) = 1$

(b) characteristic Equation $\Rightarrow m^2 + 2m + 1 = 0 \Rightarrow m = -1, -1$

The roots are real and equal

V(t) and i(t) v/s time (t) for Q1

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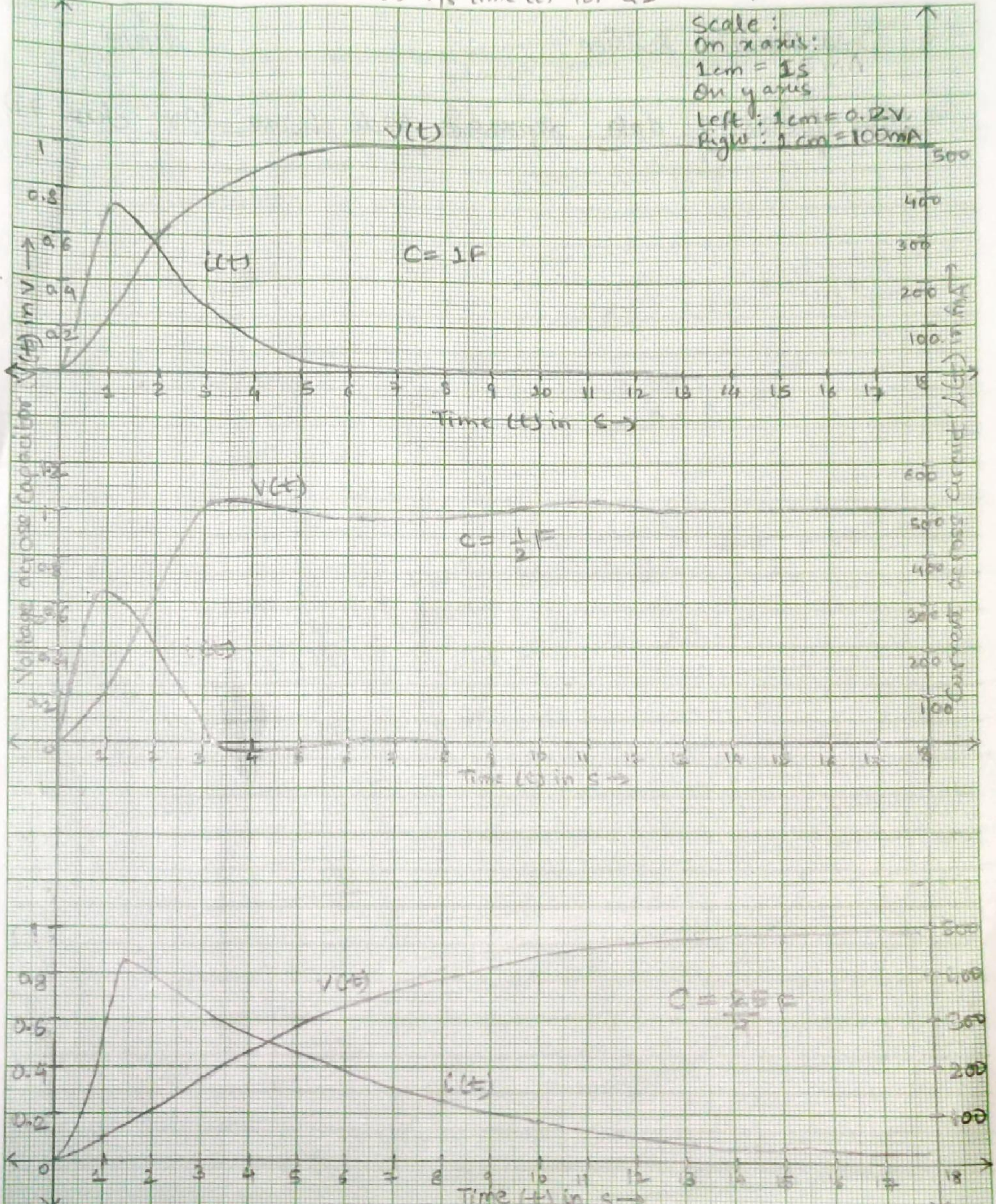

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(c) The complementary solution (natural response) is

$V_n(t) = [Ae^{-t} + Bte^{-t}]\,V$

The forced response is

$V_f(t) = \dfrac{1}{(D^2+2D+1)|_{D=0}}$ (since $1 = e^{0t}$)

$V_f(t) = 1\,V$

∴ $v(t) = V_n(t) + V_f(t) = [Ae^{-t} + Bte^{-t} + 1]\,V$

Using initial conditions,

$v(0^-) = A + \cancel{B+}\,1 = 0$ ∴ $A\,\cancel{+B} = -1$ —(i)

$\dfrac{dv}{dt}(0^-) = -A + B = 0$ ∴ $A = B$ —(ii)

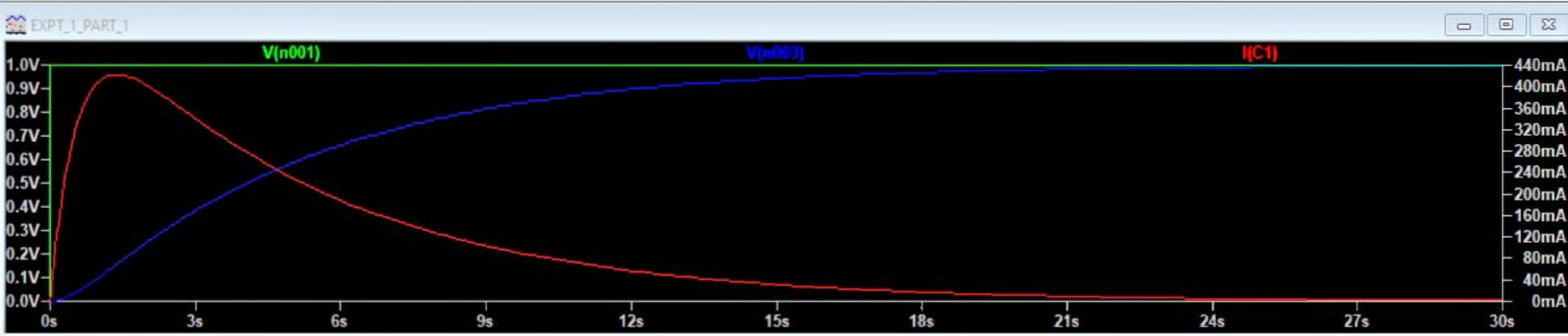
from (i) and (ii), $A = B = -1$

∴ $v(t) = [-1e^{-t} - 1te^{-t} + 1]\,V. = [-e^{-t} - te^{-t} + 1]\,V$

$i(t) = C\,\dfrac{dv(t)}{dt}$

∴ $i(t) = [\;e^{-t} - \;e^{-t} + \;te^{-t}]\,V$

$i(t) = \;te^{-t}\,V.$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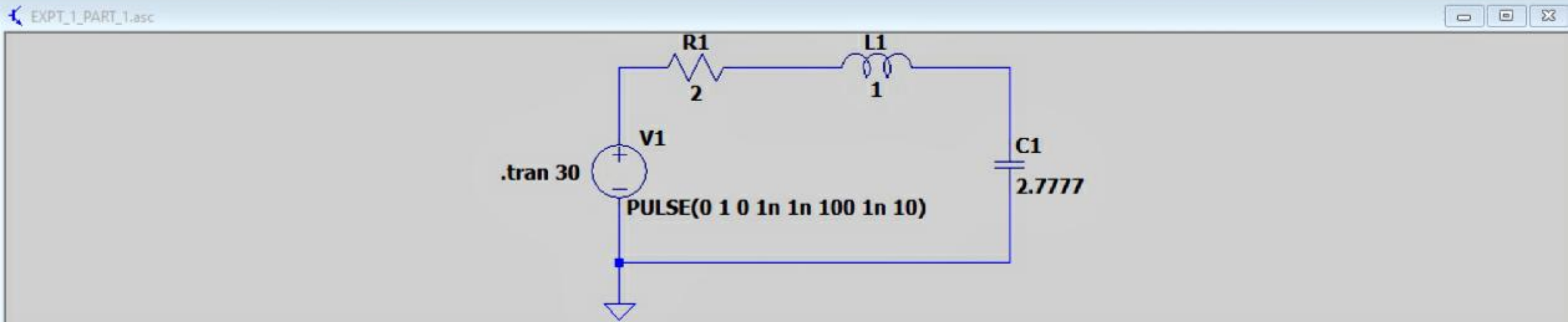
V(n001)
V(n003)
I(C1)
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
440mA
400mA
360mA
320mA
280mA
240mA
200mA
160mA
120mA
80mA
40mA
0mA
0s
3s
6s
9s
12s
15s
18s
21s
24s
27s
30s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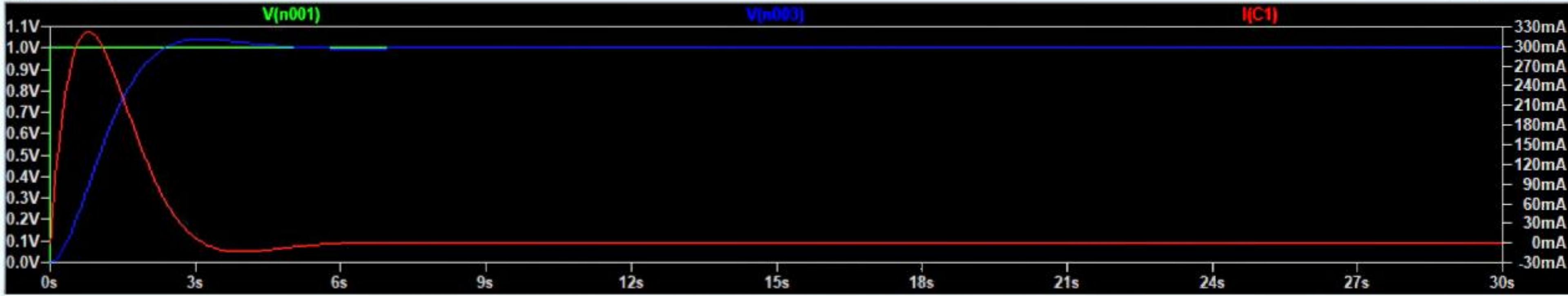
R1
2
L1
1
V1
.tran 30
PULSE(0 1 0 1n 1n 100 1n 10)
C1
2.7777

EXPT_1_PART_1



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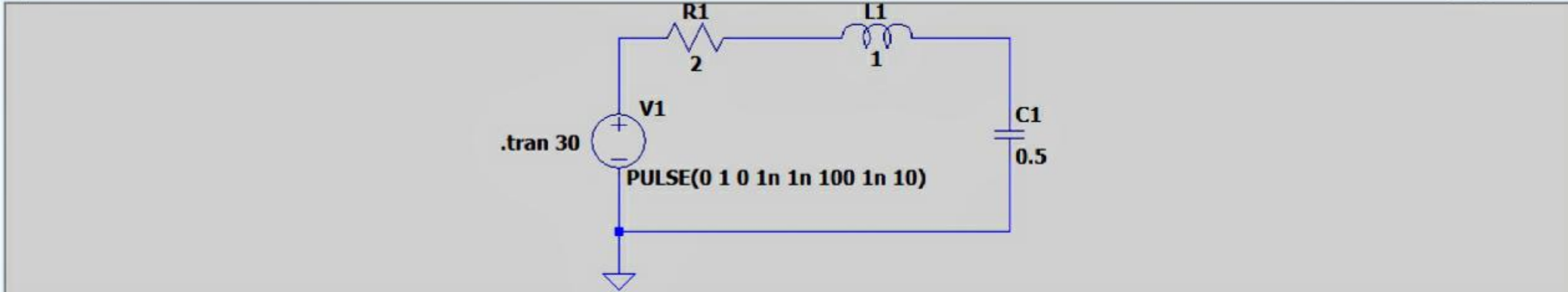

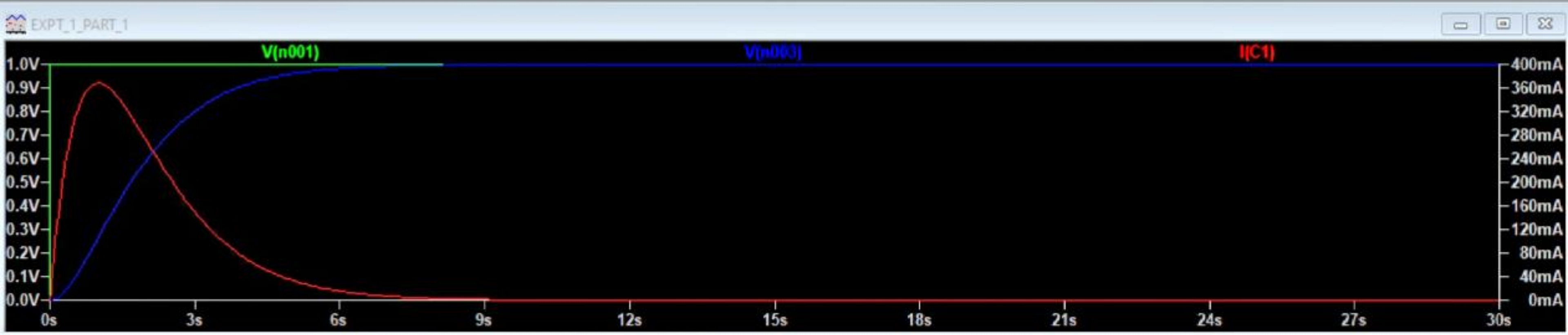
EXPT_1_PART_1
V(n001)
V(n003)
I(C1)
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
400mA
360mA
320mA
280mA
240mA
200mA
160mA
120mA
80mA
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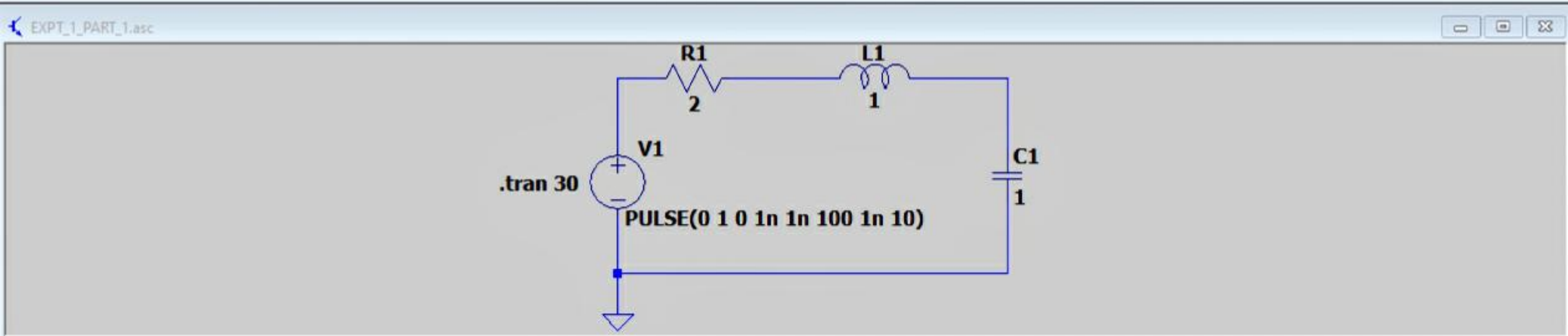
21s
24s
27s
30s


EXPT_1_PART_1.asc
R1
2
L1
1
V1
C1
1
.tran 30
PULSE(0 1 0 1n 1n 100 1n 10)