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Q1 (a) $a_n = \sum_{i=n}^{\infty} \frac{2^i}{i!} = \frac{2^n}{n!} + \sum_{n+1}^{\infty} \frac{2^i}{i!}$

$= \frac{2^n}{n!} + a_{n+1}$

$\therefore a_n = a_{n-1} - \frac{2^{n-1}}{(n-1)!}$

Now, generating function $A(x) = a_0 x + (a_0 - 1) x + (a_1 - 2) x^2 + \left(a_2 - \frac{2^2}{2!}\right) x^3 \ldots$

$A(x) = a_0 + x A(x) - x\left(1 + 2x + \frac{2^2 x^2}{2!} \ldots\right)$

$= a_0 + x A(x) - x e^{2x}$

(e^{2x})

$\therefore$ we get $A(x) = \frac{a_0 - x e^{2x}}{1-x}$

$$\boxed{A(x) = \frac{e^2 - x e^{2x}}{1-x}}$$

(b) $\sum_{n=0}^{\infty} a_n = A(1) =$

$= \lim_{x \to 1} \frac{e^2 - x e^{2x}}{1-x}$

$\Rightarrow A(1) = \lim_{x \to 1} \frac{-[e^{2x} + 2x e^{2x}]}{-1}$

$= e^2 + 2e^2 = 3e^2.$

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Q3 $T(n) = \sqrt{n}\, T(\sqrt{n}) + n(\log_2 n)^d$

$\Rightarrow \frac{T(n)}{n} = \frac{T(\sqrt{n})}{\sqrt{n}} + (\log_2 n)^d$ (Dividing by n)

$\Rightarrow n = 2^{2^k}$

$\Rightarrow \frac{T(2^{2^k})}{2^{2^k}} = \frac{T(2^{2^{k-1}})}{2^{2^{k-1}}} + (2^k)^d.$

Let $\frac{T(2^{2^k})}{2^{2^k}} = S(k)$

$\Rightarrow S(k) = S(k-1) + 2^{kd}.$

$\Rightarrow S(k) = S(k-2) + 2^{(k-1)d} + 2^{kd}$

$\Rightarrow S(k) = S(k-3) + 2^{(k-2)d} + 2^{(k-1)d} + 2^{kd}$

$\vdots$

$S(k) = S(0) + 2^{(k-(k-1))d} + \cdots + 2^{(k-2)d} + 2^{(k-1)d} + 2^{kd}.$

$S(k) = S(0) + \frac{2^{kd}}{2^{(k-1)d}} + \cdots + \frac{2^{kd}}{2^{2d}} + \frac{2^{kd}}{2^{d}} + \frac{2^{kd}}{2^{(0)d}}.$

$S(k) = S(0) + 2^{kd}\underbrace{\left[\frac{1}{2^{(k-1)d}} + \cdots + \frac{1}{2^{2d}} + \frac{1}{2^d} + \frac{1}{2^{(0)d}}\right]}_{\text{GP of } k \text{ terms with Common factor } \frac{1}{2^d}.}$

$\Rightarrow S(k) = S(0) + 2^{kd}\left[\frac{\left(\frac{1}{2^d}\right)^k - 1}{\frac{1}{2^d} - 1}\right]$

$\Rightarrow T(2^{2^k}) = T(2) + 2^{kd}\left[\frac{\frac{1}{2^{kd}} - 1}{\frac{1}{2^d} - 1}\right]$

$\Rightarrow T(2^{2^k}) = 2 + \left[\frac{1 - 2^{kd}}{1 - 2^d}\right] \cdot 2^d.$

$\Rightarrow \boxed{T(n) = 2 + 2^d\left[\frac{1 - 2^{kd}}{1 - 2^d}\right]}$

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Q4 $a_n = a_{n-1} + 8a_{n-2} - 12a_{n-3} + 2^n$

Homogenous Part

$\Rightarrow x^3 - x^2 - 8x + 12 = 0$

$\Rightarrow x = +2, +2, -3,$

$\therefore a_n^{(h)} = (A_1 + A_2 n)(2^n) + A_3(-3)^n$

For particular solution,

$a_n^{(p)} = A_4 2^n \cdot n^2$

$\Rightarrow A_4(2^n) \cdot n^2 = A_4(2^{n-1})(n-1)^2 + 8A_4(2^{n-2})(n-2)^2 - 12A_4(2^{n-3})(n-3)^2 + 2^n$

$\Rightarrow$ The n^2 terms and the n terms cancel out

~~$\Rightarrow A_4(2^n)(n^2) = A_4(2^{n-1})(n^2-2n+1) + 8A_4(2^{n-2})$~~

$\Rightarrow A_4 n^2 = A_4(2^{-1})(n^2-2n+1) + 8A_4(2^{-2})(n^2-4n+4) - 12A_4(2^{-3})(n^2-6n+9) + 1$

$\Rightarrow \cancel{8A_4 n^2} = 4A_4(\cancel{n^2} - 6n + 9) + 16A_4(\cancel{n^2} - 4n + 4) - 12A_4(\cancel{n^2} - 6n + 9) + 8$

$\therefore \boxed{A_4 = \frac{1}{5}}$ On comparing constant terms

$\therefore a_n = A_1 2^n + A_2 \cdot n \cdot 2^n + A_3(-3)^n + \frac{1}{5} n^2 (2^n)$

$a_0 = A_1 + A_3 = 1$ — (1)

$a_1 = \frac{2}{5} + 2A_1 + 2A_2 - 3A_3 = 1$ — (2)

$a_2 = \frac{16}{5} + 4A_1 + 8A_2 + 9A_3 = \frac{83}{5}$ — (3)

$\therefore$ From (1), (2), (3) $\Rightarrow A_1 = \frac{2}{5}$, $A_2 = \frac{4}{5}$, $A_3 = \frac{3}{5}$

$$\therefore \boxed{a_n = \frac{(2 + 4n + n^2) \cdot (2)^n + 3(-3)^n}{5}}$$