# Algorithms Tutorial 1

1) (a) $f_1(n) \in \Theta(n!)$

(b) $f_2(n) \in \Theta(n^{3/2})$

(c) $f_3(n) \in \Theta(1)$

(d) $f_4(n) = n^{3/2} + n^{1/2}\log n \in \Theta(n^{3/2})$

(e) $f_5(n) \in \Theta(3^n)$

(f) $f_6(n) = 2^{(n+2)} = 2^n \cdot 2^2 = 4 \cdot 2^n \in \Theta(2^n)$

(g) $f_7(n) \in \Theta(n)$

Arranging them lowest to highest

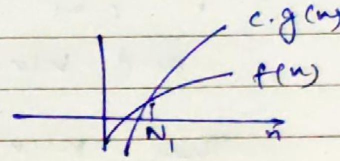
(c) < (g) < (b) = (d) < (f) < (e) < (a)

~~$\Rightarrow f_3(n) < f_7(n) < f_2(n) = f_4(n) < f_6(n) < f_1(n)$~~

$\Rightarrow f_3(n) < f_7(n) < f_2(n) = f_4(n) < f_6(n) < f_5(n) < f_1(n)$

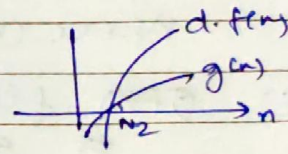
2) (a) for $f(n) = O(g(n))$,

$f(n) \leq c \cdot g(n)$ for ~~some~~ all $n \geq N_1$



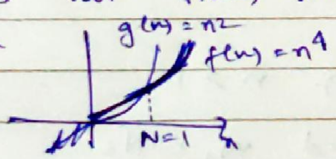

for $g(n) = O(f(n))$,

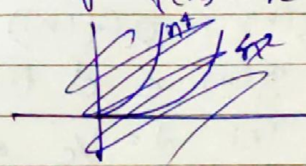
$g(n) \leq d \cdot f(n)$ for all $n > N_2$




This is not true, since if $f(n) = n^2$ and $g(n) = n^4$,

then



For $n = N_1$, $f(n) \leq 1 \cdot g(n)$ $(c = 1)$

Hence, $f(n) = O(g(n)) \Rightarrow n^2 = O(n^4)$

But the opposite is not true, since for any $d$ and any $N$, $g(n) = n^4$ will ~~never~~ sometime exceed $f(n) = n^2$

If we prove by contradiction, then, we say,

$g(n) = O(f(n)) \Rightarrow n^4 = O(n^2)$

$\Rightarrow n^4 < d \cdot n^2$ for ~~some~~ all $n \geq N_0$

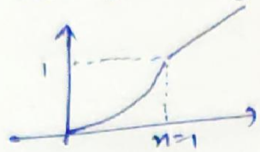
$\Rightarrow \underline{n^2 < d}$.

which is not true, since $\underline{n \geq N_0}$

Hence (a) is false

(b) $\min(f(n), g(n)) = O(f(n) + g(n))$

If $f(n) = n$, $g(n) = n^2$,
Then $\min(f(n), g(n))$ is shown as



But $O(f(n) + g(n)) = O(n + n^2)$ ~~$= O(n^2)$~~

$\min(f(n), g(n)) \le c \cdot (f(n) + g(n))$ for all $n \ge N$

Now, ~~If~~ since $f(n), g(n) \ge 0$ for all $n$

$\therefore f(n) + g(n) \ge f(n)$
$f(n) + g(n) \ge g(n)$

$\therefore c = 1$
and hence $c \cdot (f(n) + g(n)) \ge \min(f(n), g(n))$ for all $n$.

Thus, $\min(f(n), g(n)) = O(f(n) + g(n))$
Hence (b) is true

(c) ~~$O(n)$~~ $f(n) = \Omega(g(n))$
$\Rightarrow$ ~~$g(n) \Rightarrow$~~ $f(n) \ge c \cdot g(n) \quad n \ge N_1$

~~$\$$~~ $g(n) = O(f(n))$
$\Rightarrow g(n) \le d\, f(n) \quad n \ge N_2$
let $d = 1/c$ and $N_1 = N_2$
Then Both equations are true,

Hence (c) is True