In [1]:
```
# Kaushal Banthia
# 19CS10039
# Question 9
```

In [2]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

In [3]:
```
data = np.random.normal(size = (500, 2))

# data is a matrix with 2 features (x1 and x2) and 500 examples (500 random vectors)
```

In [4]:
```
y = []
for i in range(500):
  if data[i][0] * data[i][1] >= 0:
    y.append(1)
  else:
    y.append(-1)
```

In [5]:
```
df = pd.DataFrame(data)
```

In [6]:
```
df[2] = [1]*500        # column of 1s
df[3] = df[0]*df[1]    # column of x1*x2
df[4] = df[0]*df[0]    # column of x1^2
df[5] = df[1]*df[1]    # column of x2^2
df['y'] = y
```

In [7]:
```
df
```

Out[7]:

	0	1	2	3	4	5	y
0	0.662670	0.355158	1	0.235352	0.439131	0.126137	1
1	-0.982248	-0.254780	1	0.250257	0.964811	0.064913	1
2	1.292112	-0.617441	1	-0.797802	1.669553	0.381233	-1
3	-0.847413	0.749872	1	-0.635452	0.718109	0.562308	-1
4	0.074795	-1.066876	1	-0.079797	0.005594	1.138224	-1
...	...	...	...	...	...	...	...
495	0.169401	-0.677500	1	-0.114769	0.028697	0.459006	-1
496	0.042101	0.143737	1	0.006052	0.001773	0.020660	1
497	-0.295761	0.645985	1	-0.191057	0.087475	0.417297	-1
498	0.050363	0.579753	1	0.029198	0.002536	0.336114	1
499	0.218110	-0.147267	1	-0.032120	0.047572	0.021687	-1

500 rows × 7 columns

In [8]:

```
X = np.array(df.drop('y', axis = 1))
```

In [9]:
```
coeff = np.linalg.inv(X.T@X)@X.T@y
```

In [10]:
```
coeff
```

Out[10]:
```
array([ 6.94230086e-02, -2.41368450e-02, -2.37861495e-02,  6.69312577e-01,
        4.18716816e-02, -3.27387759e-04])
```

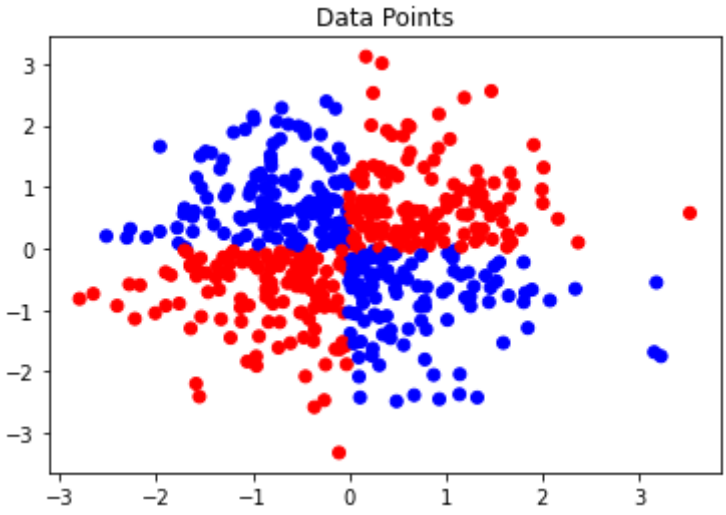
In [11]:
```
def predict(coeff, x):
  ans = np.dot(x, coeff)
  for i in range(len(ans)):
    if ans[i] >= 0:
      ans[i] = 1
    else:
      ans[i] = -1
  return ans
```

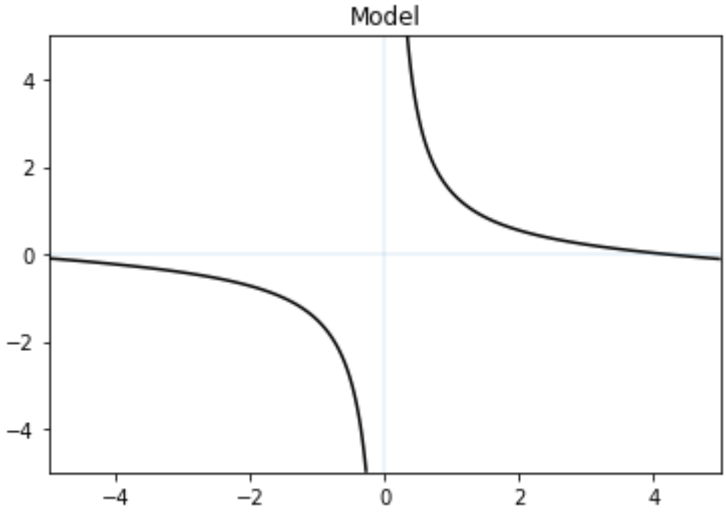
In [12]:
```
final_ans = predict(coeff, X)
```

In [13]:
```
# (a)

from sklearn.metrics import confusion_matrix
cf = confusion_matrix(y, final_ans)
print(cf)
print("Error Rate = " + str(100 * (cf[0][1] + cf[1][0])/500) + "%")
```

```
[[243   3]
 [ 21 233]]
Error Rate = 4.8%
```

In [14]:
```
# (b)
fig, ax = plt.subplots()
colors = {1:'red', -1:'blue'}
ax.scatter(df[0], df[1], c=df['y'].map(colors))
plt.title("Data Points")  # Red for 1 and Blue for -1
plt.show()

def axes():
    plt.axhline(0, alpha=.1)
    plt.axvline(0, alpha=.1)
x_ = np.linspace(-5, 5, 400)
y_ = np.linspace(-5, 5, 400)
x_, y_ = np.meshgrid(x_, y_)
a = coeff[0]
b = coeff[1]
c = coeff[2]
d = coeff[3]
e = coeff[4]
f = coeff[5]
axes()
plt.contour(x_, y_,(a*x_ + b*y_ + c + d*x_*y_ + e*x_*x_ + f*y_*y_), [1], colors='k')
plt.title("Model")
plt.show()
```





In [15]:

```
# (c)
# if the polynomial is f = x1x2, then
# Then the output is sign(x1x2)
# This is exactly how we defined our y to be
# Thus, there would be zero error
# This is demonstrated below
```

In [16]:

```
new_X = np.array(df[3])
new_ans = []

for i in new_X:
    if i >= 0:
        new_ans.append(1)
    else:
        new_ans.append(-1)
new_ans = np.array(new_ans)

new_cf = confusion_matrix(y, new_ans)
print(new_cf)
print("Error Rate = " + str(100 * (new_cf[0][1] + new_cf[1][0])/500) + "%")
```

```
[[246   0]
 [  0 254]]
Error Rate = 0.0%
```