PES UNIVERSITY

Ph.D. / MTech by Research course work syllabus

Mathematics

List of courses

#	Course Code	Course Title	Credits
1.	UE22MA841A	Number Theory	
2.	UE22MA842A	Advanced Graph Theory	
3.	UE22MA843A	Applied Numerical Analysis	
4.	UE22MA844A	Advanced Fluid Mechanics	
5.	UE22MA845A	Graph Theory	
6.	UE23MA841A	Mathematical Modeling - I	
7.	UE23MA842A	Mathematical Modeling - II	
8.	UE23MA843A	Advanced Numerical Techniques	
9.	UE23MA844A	Complex Analysis	
10.	UE23MA845A	Functional Analysis	
11.	UE23MA846A	Topology	
12.	UE23MA847A	Advanced Differential Equations	

UE22MA841A: Number Theory

Unit1:-

Sum product identities: Introduction, Euler's identity for infinite products, Jacobi's triple product identity, Quintuple product identity and its applications. *(Refernce Book no.1 – Unit 0).*

Unit 2:-

Basic Hypergeometric Series: Introduction, hypergeometric and basic hypergeometric series , Hein's transformation formula for $_2\phi_1$ series, Hein's q-analogue of Guass summation formula, Application of Bailey's transformation to Basic series, Basic bilateral series, Ramanujan's sum for .
$_{r+1}\phi_r$. *(Refernce Book no.2 – Unit 1).*

Unit 3:-

Elementary Partition Theory: Introduction, Graphical Representation, Euler's Partition Theorem, Searching for Partition Identities . *(Refernce Book no.4 – Chapter -12).*

Unit 4:-

The Rogers-Ramanujan identities and Generating functions : A fundamental type of partition identity , Discovering the first Rogers-Ramanujan identity , Looking for a bijective proof of the first Rogers-Ramanujan identity , Euler's theorem , Euler's pentagonal number theorem , Congruences for p(n) , Rogers-Ramanujan revisited. *(Refernce Book no.3– Unit 4 and 5).*

Unit 5:-

Basic contour integrals: Introduction , Watson's contour integral representation for $_2\phi_1$(a,b;c;q,z) series . Analytic continuation of $_2\phi_1$(a,b;c;q,z) , q-analog of Barnes first and second lemmas, Analytic continuation of $_{r+1}\phi_r$ series. *(Refernce Book no.2– Unit 4).*

Reference Books:

1. Shaun Cooper - **Ramanujan's Theta Functions**, Springer International Publishing   AG2017.
2. G. Gasper and M. Rahman- **On Basic Hypergeometric Series**, Second Ed., Encycl.Math. Applics., Vol 35, Cambridge University Press, Cambridge, 2004.
3. George E. Andrews and Kimmo Eriksson- **Integer Partitions**, Published By the press Syndicate of The University of Cambridge 2004.
4. George E . Andrews - **Number Theory** ,1971 ,w. b. saunders company ,Print No.: 987654321.

UE22MA842A: Advanced Graph Theory

Unit I:

Spectral properties of graphs: The spectrum of an undirected graph, spectrum of some graphs: complete graph, cycle, regular graphs, line graphs, complete Bipartite graphs, spectra of product graphs: Cartesian, Kronecker, Bipartite double and strong products, Cayley graphs, (Reference Book No.2). Ramanujan graphs. (Reference book no. 4).

Unit II:

Graph Energy: The energy of a graph , energy of standard graphs. Other graph energies: Laplacian energy,Distance energy, LEL and Incidence energy (Reference book no.1). skew energy of a directed graph (Reference paper no.5).

Unit III:

Bounds for energy and Eigen values of graphs: Preliminary bounds, upper bounds for general graphs, Bipartite graphs and regular graphs, lower bounds. (Reference book No. 1)

Unit IV:

Equienergetic and Integral graph: Equienergetic graphs, Equienergetic trees, Hypoenergetic trees and Strongly hypoenergetic trees (Reference book nos.1). Integral graphs, construction of integral graphs by means of certain operations, Integral trees. (Reference Paper No. 6)

Unit V:

Applications of Spectral Graph Theory: A simple construction of expander graphs: squaring graphs, Relative spectral gap, Line graphs, The Spectrum of the line graph, The whole construction, Pseudo-random generators via random walks on graphs, Random walk generators, Formalizing the problem, matrix norm. (Reference book no.3)

References:
1. Graph Energy by Xueliang Li, Yongtang shi, Ivan Gutman, Springer, 2012.
2. Spectra of graph by Andries E. Brouwer,Willem H. Haemers. (Internet Version)
3. Spectral and Algebraic Graph Theory, Daniel A. Spielman, Yale University, 2019.
4. A text book of Graph theory, R. Balakrishnan and K. Ranganathan, Second Edition, Springer, 2012.
5. The skew energy of a digraph, Linear algebra and its applications, C.Adiga, R. Balakrishna, Wasin So, 432(2010),1825-1835.
6. A survey on integral graphs , K. Balińska, D. Cvetković, Z. Radosavljević, S. Simić, D.

Stevanović, Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. 13 (2002), 42–65.

UE22MA843A: Applied Numerical Analysis

Unit I:

Numerical solution of ordinary differential equations: Picard's method, Improvements of Euler method, Runge – Kutta methods for second, third, fourth and higher order. Multistep methods: Milne's method, Heun's method.

Unit II:

General methods for boundary value problems: Finite difference method, The Shooting method, The cubic spline method, The polynomial method. Methods of approximation: The Rayleigh – Ritz method and Galerkin method.

Unit III:

 Finite difference approximations to derivatives: The Laplace equation solution techniques – Jacobi's method, Liebmann method(Gauss – Seidel method), Successive over - relaxation method(SOR method), The Alternating direction implicit method(ADI method) with four internal mesh points.

Unit IV:

Numerical solution of partial differential equation: The parabolic equation – Bender – Schmidt recurrence relation, Crank – Nicolson formula, Jacobi's iteration formula, Gauss – Seidel iteration formula, Successive over relaxation(SOR).

Unit V:

 Numerical integration: Integration with unequal segments, open integration formulas, Romberg's method, Adaptive quadrature method, Gauss quadrature – two point Gauss Legendre formula.

Textbooks :

1) Numerical Methods for Engineers, sixth edition, Steven C. Chapra and Raymond P. Canale, McGraw Hill Education (India) Edition 2012.
2) Introductory Methods of Numerical analysis, fourth edition, S. S. Sastry, Asoke K. Ghosh, Prentice – Hall of India Private limited, M-97, Connaught Circus, New Delhi, 2007.

Reference Books :

1) Applied Numerical Analysis, fourth edition, Curtis F. Gerald and Patrick O. Wheatley, Addison – Wesley Publishing Company.
2) Numerical Methods and Applications, Ward Cheney and David Kincaid, Cengage Learning.

UE22MA844A: Advanced Fluid Mechanics

Unit 1:

Boundary layer: Boundary layer approximation, governing equations for Blasius boundary layer equation, stretching sheet problems (horizontal, vertical and inclined) and Falkner- Skan family of equations. Flow past a wedge and a flat plate (horizontal and vertical). Liquid thin film flows.

Unit 2:

Linear and nonlinear natural convective instabilities: Governing equation for Rayleigh-Benard and Benard-Maragoni convections in Newtonian and Oldroyd-B liquids for different velocities and thermal boundary conditions. Local and global nonlinear stability analysis-Lorenz and Ginzburg- Landau model for stress-free isothermal boundary combination in the case of Newtonian liquids. Critical points of the linear autonomous system in the case of linearized Lorenz model. Illustration of the energy method.

Unit 3:

Classical and modern method of solving linear and nonlinear differential equations: Variational method of Galerkin and Rayleigh Ritz, Homotopy method, differential transform method and Pade approximation, perturbation method, shooting method,Tanhmethod, variational formulation of boundary value problem.

Unit 4:

Thermal instability: The equation of motion, The exact equations, The Boussinesq equations. The stability problems, The linearized equations, the boundary conditions, normal modes. General stability characteristics, exchange of stabilities, avariational principle.

Unit 5:

Numerical computations using Scilab; Numerical solution of differential equations, numerical solutions of initial and boundary value problem, numerical integration, numerical differentiation, matrix manipulations, optimization techniques, two and three dimensional plots, parametric plots, typesetting capabilities for labels and text in plots, direct control of final graphics size resolutions.

Text and Reference Books:

Jain, Iyengar and Jain, "Numerical methods for Scientific Engineering Computations", Wiley Eastern, 1993.

T Cebeci and P Bradshaw, "Physical and Computational Aspect of Convective Heat Transfer", Springer-Verlag, 1988.

S Chandrashekar, "Hydrodynamic and Hydro magnetic Stability", Oxford University Press, 2006.

P G Drazin and W H Reid, "Hydrodynamic Instability", Cambridge University Press, 2006.

J S Jurner, "Buoyancy Effects in Fluids", Cambridge University Press, 1973.

D J Tritton, "Physical Fluid Dynamics", Van Nostrand Reinhold Company, England, 1979.

J K Platten and J C Legros, "Convection in Liquids", Springer-Verlag, 1984.

UE22MA845A: Graph Theory

Unit-1:-

Planarity: Definition of Plane and Planar Graph, Eulers formula , Kuratowski's theorem. Genus,Thickness , Coarseness, crossing number (Statement only). (*Unit 11 in Reference book 1*)

Unit-2:-

Colorability: The Chromatic number, The five colour theorem, The four colour theorem, Uniquely colorable graph, Critical graphs, The Chromatic polynomial. (*Unit 12 in Reference book 1*).

Unit-3:-

Matrix Representation of Graph: Incidence Matrix , Submatrices of A(G), Circuit Matrix, Fundamental Circuit Matrix and Rank of B An Application to a Switching Network, Cut-set Matrix, Path Matrix , Adjacency Matrix. (*Unit 7 in Reference book 2*).

Unit-4:-

Directed Graphs: Introduction to Directed Graph, Some Types of Digraphs, Digraphs and Binary Relations, Directed Paths and Connectedness, Euler Digraphs, Trees with Directed Edges, Fundamental Circuits in Digraphs ,Adjacency matrix of a Digraph. (*Unit 9 in Reference book 2*).

Unit-5:-

Trees and Distance: Introduction to Tree, Distance and Spanning tree in a Graph. Enumeration of Trees, Minimum spanning tree, Shortest paths.(*Unit 2.1,2.2 and 2.3 in Reference book 3*). Rooted trees, Weighted Trees and Prefix codes(Definition only)(*Unit 12.2, 12.4 in Reference book 4*),
Trees in Computer Science. (*Unit 2.3 in Reference book 3*)

Reference Books:

1. Frank Harary-**Graph theory**, Addition Wesley Reading Mass,1969.
2. Narsingh Deo-**Graph theory with Applications to Engineering and Computer Science,** Published by Asoke K. Ghosh, PHI learning private Limited Delhi-110092 ,2017.
3. Douglas .B. West- **Introduction to Graph Theory** –Pearson Education (Singapore) Pvt. Ltd, 2002.   (ISBN 81-7808-830-4)

UE23MA841A: Mathematical Modeling – I

Unit 1:
Introduction to Mathematical Modeling: History of Mathematical Modeling, Importance of Mathematical Modeling, Latest developments in Mathematical Modeling, Limitations of Mathematical Modeling, How to build Mathematical models, Mathematical models and functions – Logistic function and Logistic growth model, Gompertz function and Gompertz growth model. Functional responses in population dynamics -Holling Type I, Type II and Type III responses.

Unit 2:
Difference Equations with Applications: Formation of difference equations, Linear difference equations with constant coefficients, Solution of Homogeneous equations, Equilibria and Stability – Linear difference equations System of linear difference equations, Non linear difference equations, Linear Models – Population model involving growth, Linear Prey – Predator model, Non Linear model -Logistic model, Richer's model.

Unit 3:
Mathematical Modeling Through Ordinary Differential Equations of First Order : Steady state solutions, Stability, Linearization and local stability Analysis, Routh-Hurwitz Stability Criteria, Linear and Non-linear Population Growth and Decay models, Effects of Immigration and Emigration on population size, Mathematical modeling in dynamics, Carbon dating, Growth and decay of current in LR circuit.

Unit 4:
Mathematical Modeling Through System of Ordinary Differential Equations of First Order : Phase plane diagram of Linear systems, Mathematical modeling in Population Dynamics, Compartment models, Mathematical models in Medicine, Arms race models and Combat models – Conventional combat model, Guerrilla combat model, Mixed combat model.

Unit 5:
Mathematical Modeling Through Ordinary Differential Equations of Second Order : Mathematical modelling of Circular motion and rectilinear motion, Mathematical modelling of Mechanical Oscillations – Horizontal oscillations, Vertical oscillations, Damped and forced oscillations

Reference Books:
1. J. N. Kapur_Mathematical Modeling, Wiley Eastern Ltd., 1998
2. J. N. Kapur_Mathematical Models in Biology and Medicine, Affiliated East West Press Private Ltd, New Delhi, 2000
3. Sandip Banerjee_Mathematical Modeling Models, Analysis and Applications, CRC Press, 2014

UE23MA842A: Mathematical Modeling – II

Unit 1:

Mathematical modeling of epidemics: Simple epidemic model : Deterministic models without removal, General deterministic model with removal, General deterministic model with removal and immigration, formulation, solution, interpretation, and limitations, SI, SIS, SIR and SIRS model.

Unit 2:

Two Species Population Models : Types of interaction between two species, Simple Lotka-Volterra prey-predator model, formulation, solution, interpretation, Special case of basic competition model, Difference equations model for competition.

Unit 3:

Multi Species Population Models : Volterra model for n species : The basic system of equations, Existence of constant of motion, Stability of Equilibrium position, Long – time averages of Powers and products of Species population, particular case of two species. May's multispecies predator-prey model : The basic model, Generalization of the basic model, Comparison of May's model and Volterra's model having anti symmetry

Unit 4:

Optimal Exploitation Models : Growth of population with Harvesting : Logistic model with constant harvesting rate, Harvesting rate being proportional to Population size, Growth of Predator – prey populations with harvesting, Growth of population in competition under harvesting, Optimal Utilization of Renewable Resources : Optimization problem in harvesting of fish.

Unit 5:

Mathematical Models in Pharmacokinetics: Basic equations and their solutions, Solutions for special cases, Determination of transfer coefficients and compartment volumes, Mathematical techniques used in compartment analysis, Drug delivery problem using difference equations, Drug distribution in the body using Ordinary differential Equation.

Reference Books:

1. J. N. Kapur_Mathematical Models in Biology and Medicine, Affiliated East West Press Private Ltd, New Delhi, 2000 [Chapters 5,6,7 & 8]
2. J. N. Kapur_Mathematical Modeling, Wiley Eastern Ltd., 1998 [Chapter 3]

3. Elizabeth S. Allman John A. Rhodes, Mathematical Models In Biology An Introduction, Cambridge University Process, 2004
4. Sandip Banerjee,Mathematical Modeling Models, Analysis and Applications, CRC Press, 2014
5. Fred Brauer, Carlos Castillo-Chavez, Zhilan , Feng, Mathematical Models in Epidemiology, Springer, 2019
6. J. D. Murray,Mathematical Biology I. An Introduction, Springer, 2002

UE23MA843A: ADVANCED NUMERICAL TECHNIQUES

Unit 1:

Iterative methods for linear systems: Classical iterative methods (Jacobi, Gauss-Seidel and successive over relaxation (SOR) methods), Krylov subspace methods; GMRES, Conjugate gradient biconjugate-gradient (BiCG), BiCGStab methods, preconditioning techniques, parallel implementations.

Unit 2:

Finite difference method: Explicit and implicit schemes, Crank-Nicolson schemes, consistence, stability and convergence, Lax's equivalence theorem, numerical solutions to elliptic, parabolic and hyperbolic partial differential equations. Dirichlet, Neumann and mixed problems.

Unit 3:

Approximate method of solution: Galerkin method, properties of Galerkin approximations, Petrov-Galerkin method, generalized Galerkin method.

Unit 4:

Numerical integration: Newton's cotes formula, Simpson's rules, Weddle's rule, Gaussian Quadrature – Gauss Legendre and Gauss Chebyshev methods, double integration

Unit 5:

Eigen values and vectors: Eigen values and Eigenvectors, bounds on Eigen values, Jacobi's method for symmetric matrices, given method for symmetric matrices, householder's method for symmetric matrices, largest Eigen value by power method.

Text and Reference Books:

1. M K Jain, S R K Iyengar and R K Jain, "Numerical methods for scientific and engineering Computation, 6th edition, New Age, 2012.
2. S S Sastry, "Introductory methods of numerical analysis", 4th edition, PHI, 2011.
3. C T Kelley, "Iterative Methods for Linear and Nonlinear Equations", SIAM publications, Philadelphia, 1995.
4. G D Smith, "Numerical Solution of Partial Differential Equations", Oxford University Press, 1985.

UE23MA844A: COMPLEX ANALYSIS

UNIT 1:

Analytic functions. Harmonic conjugates. Elementary functions. Mobius Transformation. Con- formal mappings. Cauchy's Theorem and Integral formula. Morera's Theorem. Cauchy's Theorem for triangle and rectangle. Cauchy's Theorem in a disk. Zeros of Analytic function. The index of a closed curve. counting of zeros. Principle of analytic Continuation. Liouville's Theo- rem. Fundamental theorem of algebra.

UNIT 2:

Series. Uniform convergence. Power series. Radius of convergences. Power series representation of Analytic function. Relation between Power series and Analytic function. Taylor's series. Laurent's series.

UNIT 3:

Rational Functions. Singularities. Poles. Classification of Singularities. Characterization of removable Singularities. poles. Behavior of an Analytic function at an essential singular point. Entire and Meromorphic functions. The Residue Theorem.

UNIT 4:

Evaluation of Definite integrals, argument principle, Rouche's Theorem. Schwarz lemma. Open mapping theorem and Maximum modulus theorem and its applications. Convex functions. Hadamard's Three circle theorem.

UNIT 5:

Phragmen- Lindelof theorem. The Riemann mapping theorem. Weierstrass factorization theorem. Harmonic functions. Mean Value theorem. Poisson´s formula. Poisson's Integral formula. Jensen's formula. Poisson's-Jensen's formula.

TEXT BOOKS:

1) J. B. Conway. Functions of one complex variable. Narosa. 1987.

2) L.V. Ahlfors. Complex Analysis. McGraw Hill. 1986.
3) T. W. Gamelin. Complex Analysis. Springer-Verlag. 2006

REFERENCE BOOKS:

1) R. Nevanlinna. Analytic functions. Springer. 1970.
2) E. Hille. Analytic Theory. Vol-I. Ginn. 1959.

UE23MA845A: FUNCTIONAL ANALYSIS

UNIT 1:

Normed linear spaces: Norm on a linear space: Example of norm linear spaces. Semi-norms and quotient spaces. Measurable functions and L^p spaces. Inner Product Space.

UNIT 2:

Banach Spaces: Incomplete norm Linear Space. Completion of norm linear spaces. Properties of Banach Spaces. Schauder Basis and Separability. Heine Borel Theorem and Riesz Lemma. Best approximation theorem and projection theorem.

UNIT 3:

Operation on Norm Linear Spaces: Bounded Operators. Norm on $B(X, Y)$. Riesz- representation theorem. Completeness of $B(X, Y)$.

UNIT 4:

Hilbert Spaces: Orthonormal Set and Orthonormal Basis. Bessel's Inequality. Fourier Ex- pension and Parseval's Formula. Riesz-Fischer Theorem.

UNIT 5:

Hahn-Bannach Theorem and its consequences: The extension theorem and its consequences. Closed Graph Theorem and its consequences: Closed Graph Theorem. Bounded Inverse Theorem. Open Mapping Theorem.

TEXT BOOKS:

1) M. T. Nair, Functional Analysis a first course, PHI

2) B. V. Limaye, Functional Analysis (Wiley Eastern).

REFERENCE BOOKS:

1.   G. F. Simmons, Introduction to Topology and Modern Analysis (McGraw-Hill International Edition).
2.   G. Backman and L. Narici, Functional Analysis (Academic).
3.   P.R. Halmos, Finite dimensional vector spaces (Van Nostrand), 1958.
4.   E. Kreyszig, Introduction to Functional Analysis with Applications (John Wiley and Sons).

UE23MA846A: Topology

UNIT 1:

Introduction to Topology : Definition and examples of topological spaces. Basis for a topology. Product Topology. Subspace Topology. Neighborhoods and Limit points. Closed Sets. Limit points. Closure. Interior and Boundary of a set. Hausdorff Space. (Excluding the concept of finer and coarser, order topology, box topology)

UNIT 2:

Continuous Functions: Definition of continuous function. Equivalent definitions of continuity. and Homeomorphism. Pasting lemma. Maps into Product Spaces. Metric topology. Sequence Lemma. Quotient Topology.

UNIT 3:

Connected spaces: Definition and examples. Union of connected set having a point in com- mon is connected. Image of connected space is connected. Cartesian product of connected space is connected. path connected spaces. example of a topological space which is connected but not path connected (topologist's sine curve). Components and path components forms an equivalence relation.

UNIT 4:

Compactness: Definition and Examples of Compact Spaces. Closed subspace of compact space is compact. Compact subspace of a Hausdorff space is closed. Image of compact set is compact under a continuous map. The product of finitely many compact space is compact. Compactness and finite intersection property. Lebesgue number lemma. Uniform continuity and compact- ness.

UNIT 5:

Separation Axioms: First countable and Second Countable topological space. Hausdorff Space. Regular Space. Normal Space. Necessary and Sufficient condition for Regular and Nor- mal Spaces. Subspace of regular is regular. subspace of normal is normal. Urysohn's Lemma. Urysohn Metrization theorem. Tietze Extension Theorem. Tychnoff Theorem.

TEXT BOOKS:

1. G. F. Simmons. Introduction to Topology and Modern Analysis. Tata McGraw-Hill Education. 1963
2. J. Munkres. Topology. Pearson Education India. 2nd Edition. 2007

REFERENCE BOOKS:

1) J L. Kelley. General Topology. Van Nostrand. Princeton. 1955
2) J. B. Conway. A course in point set topology. UTM Series. Springer. 2013

3) K. D. Joshi. Topology. New Age International Private limited. 1983
4) M. A. Armstrong. Basic Topology. Springer India .1983.

UE23MA847A: Advanced Differential Equations

Unit 1:

Existence and uniqueness theory: Existence and uniqueness of solutions of initial value problems for system of first order differential equations. Existence and uniqueness theorem for a linear system.

Unit 2:

System of linear differential equations: Homogeneous and inhomogeneous linear systems, linear equations with constant coefficients, fundamental matrix

Unit 3:

Linear differential equations with periodic coefficients: Floquet theory, stability for linear systems. Principle oflinearized stability. Stability for autonomous systems. Liapunov functions. Plane autonomous systems. Periodic solutions of plane autonomous systems.

Unit 4:

Characteristics and classification of Partial differential equations: Method of characteristics for first order partial differential equations and classification of second order partial differential equations.

Unit 5:

Nonlinear partial differential equations: Nonlinear first order partial differential equations. Conservation laws. Lax- Oleinik formula. Riemann's problem. Long time behavior separation of variables. Similarity solutions. Transform methods. Converting nonlinear partial differential equations into linear partial differential equation.

Text and Reference Books:

R Grimshaw," Nonlinear ordinary differential equations", Blackwell Scientific publications, 1990.

Lawrence C Evans, "Partial Differential Equations", American Mathematical Society, 1991.

David Betounes, "Differential equations: Theory and applications", Springer, 2010.

L Perko, "Differential equations and dynamical systems", Springer, 2001.