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A MULTIVARIATE APPROACH TO CONSTRUCTION CONTRACT BIDDING MARK-UP STRATEGIES

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Corrigendum

The paper contains the following errors:

1. Eqn (21) should read

$$\hat{\beta}_j = \frac{\sum_{i=1}^{r} \delta_{ij} (y_{ij} - \alpha_i) / \sigma_i^2}{\sum_i \delta_{ij} / \sigma_i^2}$$

In practice, however, convergence problems occur with this formulation and the original eqn (21) was used as an approximation.

2. Eqn (23) should read

$$\hat{\sigma}_i^2 = \frac{1}{n_i} \sum_i \sum_j \delta_{ij} (y_{ij} - \alpha_i - \beta_j)^2$$

3. Fig 1 legend should read:

$c_e(x)$
$f_{55}(x)$
$X_e = X_{ev}$

4. Figs 2 and 3 $f_{ss}(x)$ should read $f_{55}(x)$.

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A multivariate approach to contract bidding strategies in the construction industry is presented. This represents a radical departure from previous work in the field by using all available data on competing bidders. 'Optimal', 'no loss' and 'break even' mark up strategies are derived and methods of parameter estimation proposed. A case study shows how the three strategic mark up values are calculated against known competitors.

Key Words: Bidding Strategy; Multivariate Analysis; Construction Contracts

INTRODUCTION

Many contracts for goods and services are let on the basis of sealed bid auctions. The usual conditions for the auction are that interested suppliers may enter, by a stipulated date and time, a bid for the amount they wish to be paid should they become a party to the contract. All bids are delivered separately and simultaneously in sealed envelopes to the procurer who then opens the envelopes. The procurer inspects the bids and generally enters into a written contract with the lowest bidder based on the amount stated in the bid.

Construction contracts are typical in this respect. Each bidding contractor estimates his likely costs of carrying out the work detailed in the project plans and schedules and adds a percentage mark-up to form the bid value. The value of the mark-up crucially influences the chances of a bidder winning the contract and the subsequent profit should the contract be secured and the work be completed. Clearly, a low mark-up value should increase the chance of winning but decrease the profit, whilst a high mark-up should increase the profit but decrease the chances of winning.

Strategic mark-up bidding assumes that the bidder applies a mark-up which happens to produce a satisfactory balance between the probability of the winning the contract and the profit generated as a result of winning the contract. A special case of strategic mark-up bidding is optimal bidding, defined as applying a mark-up which happens to maximise expected profit, ie., the product of the probability of winning the contract and the profit generated as a result of winning the contract[1].

The literature on strategic mark-up bidding is quite extensive and several reviews have been published (eg [2]). All the work to date has been based on two bivariate models. The Friedman[1] model compares the strategic bidder with individual competitors while Hanssmann and Rivett[3] compare the strategic bidder with lowest bidders. However, the Friedman model has been frequently criticised as demanding unrealistic amounts of data to estimate the model parameters (eg [4]) especially for construction contract auctions (eg. [5], and [6]). The Hanssmann and Rivett model partially solves this by reducing the number of parameters in the model and thus the data demands, but with loss of predictive power.

Multivariate methods offer a means of better utilisation of all available data, depending on the adequacy of certain assumptions concerning the statistical properties of bids. In this case an individual bidder is not restricted to data for auctions in which he has been a participant, as is the case with bivariate approaches. Instead, he is able to incorporate data for **all** auctions in which his competitors, and potential competitors, have been participants, irrespective of the individual bidder's participation. This increases, by several orders of magnitude, the amount of data available for estimating the model parameters.

Recent empirical studies[7] indicate that, with suitably transformed data, the assumptions implicit in multivariate approaches may not be unduly violated in construction contract auctions.

This paper considers, via a case study, the use of one such multivariate approach for deriving 'optimal' and other strategic mark up values against known competitors. Firstly the multivariate approach is introduced and the probability of a bidder underbidding his competitors formulated. This is then extended, by the inclusion of a mark-up decision variable, into a formulation for 'optimal', 'no loss' and 'breakeven' strategies. Maximum likelihood estimators are proposed for obtaining values of the basic parameters in the model from which it is shown how the other parameters may also be estimated. The paper describes how these parameters were estimated from the 'live' case study data and the three strategic mark-up values obtained against known competitors.

THE MULTIVARIATE APPROACH

Profit depends on the value of the mark up multiplier, *v*. A low mark up increases the chance of acquiring a contract, but with little profit, while conversely a high mark up gives a larger profit, but with little chance of acquiring the contract. We propose a model for the probability of obtaining a contract as a function of bid, *x*, or equivalently, *v*. Since $v=x/c$ where *c* is the cost estimate, we can choose an additive formulation if we work on a log scale.

If for a particular contract, y_i, $i=1,2 \ldots ,n$ are the (log transformed) bids, treated as continuous random variables with joint probability density function $f(y_1, \ldots ,y_n)$ then

$$P\left(y_1 < y_i, \forall i \neq 1\right) = \int_{y_1=-\infty}^{\infty} \int_{y_2=y_1}^{\infty} \int_{y_3=y_1}^{\infty} f\left(y_1 y_2 y_3 \ldots y_n\right) dy_n \ldots dy_3 dy_2 dy_1 \qquad \textbf{(1)}$$

where $f(\ldots)$ is the joint probability density function of the *n* bids (*n* is assumed to be known). Now assuming the variables are **independent**, it follows from (1) that

$$P\left(y_1 < y_i, \forall i \neq 1\right) = \int_{-\infty}^{\infty} f_1\left(y_1\right) \prod_{i=2}^{n} \left\{ \int_{y_i=y_1}^{\infty} f_i\left(y_i\right) dy_i \right\} dy_1 \qquad \textbf{(2)}$$

Optimal bidding

Skitmore[7] has proposed the model

$$y_i \sim N\left(\mu_i, \sigma_i^2\right), \text{ independently} \qquad \textbf{(3)}$$

where $y_i = \ln\left(x_i - mw\right)$, x_i is the bid, *w* is a parameter estimated by the value of the lowest bid entered for the contract and *m* is a modifying constant. In an empirical analysis of two independent sets of construction contract auctions, Skitmore has also shown that any value of the modifying constant in the range 0≤m≤0.8 will provide a good normalising transformation[7].

Here we adopt the special case where m=0, and thus $y_i = \ln(x_i)$. If $\mu_1 = 0$, the probability of bidder 1 entering the lowest bid now becomes

$$\int_{-\infty}^{\infty} \frac{e^{-z_1^2/2}}{\sqrt{2\pi}} \prod_{i=2}^{n} K(z_i) dz_1 \tag{4}$$

where

$$K(x) = \int_x^{\infty} \frac{e^{-z^2/2}}{\sqrt{2\pi}} dz \tag{5}$$

and

$$z_i = \frac{\sigma_1 z_1 - \mu_i}{\sigma_i} \tag{6}$$

Applying a mark up multiplier of v to x_i affects the mean in model (3) by an amount of **ln**(v), with the probability, $P(v)$, of entering the lowest bid given by (4) but with z_i in (6) now replaced by

$$z_i = \frac{\sigma_1 z_1 + \ln v - \mu_i}{\sigma_i} \tag{7}$$

Replacing a bidder's bids with his/her cost estimates, v then represents a mark up multiplier and the objective is to find the mark up v_o to maximise expected profit. Since profit can be taken as zero for those contracts which we do not win, we need to compute

$$\text{Expected profit} = E[P] = E\left(\frac{X_e v - A}{A} \middle| win \right) P(v), \tag{8}$$

where X_e = cost estimate (a random variable); A = unknown, but assumed fixed, actual cost.

Assuming cost estimates are unconditionally unbiased[8], the actual cost can be estimated by the expected value of X_e, $E[X_e] = \mu_e$, say. Substituting in (8) gives the estimated expected profit as

$$E[P] = E\left(\frac{X_e v - \mu_e}{\mu_e} \middle| win \right) P(v) \tag{9}$$

which is explicitly

$$E[P] = \int \frac{yv - \mu_e}{\mu_e} C_e(y) dy \tag{10}$$

where $C_e(x) = f_e(x \mid win)P(v)$ and $f_e(x \mid win)$ is the probability density function (pdf) of X_e given that the contract is won. From Bayes' formula, we have the conditional distribution of

$$z_1 = \frac{\ln vx - \mu_1}{\sigma_1} \tag{11}$$

(a standardised, transformed bid), given that the contract is won, given by

$$p(z_1 | win) = \frac{p(win | z_1) p(z_1)}{p(win)} \tag{12}$$

and $p(win)$ is simply the normalisation constant, $P(v)$, given by (4), (5) and (7). In the case of model (3)

$$C_e(x) = \frac{e^{-z_1^2/2}}{\sqrt{2\pi}} \left(\prod_{j=2}^{n} K(z_j) \right) \frac{dz_1}{dx} \tag{13}$$

where

$$z_1 = \frac{\ln x - \ln v}{\sigma_1} \tag{14}$$

$$z_1 = \frac{\sigma_1 z_1 + \ln v - \mu_j}{\sigma_j}, \qquad j \geq 2 \tag{15}$$

and

$$\frac{dz_1}{dx} = \frac{1}{\sigma_1 x}. \tag{16}$$

Loss Oriented Strategies - 'No loss' and 'Breakeven'

As an alternative to maximising expected profit, a bidder may prefer to restrict the probability of making a loss in some way. One such strategy we term a 'no loss' strategy where the mark up, v_n, required is the one which results in a (conditional) probability of 0.05 of making a loss. This is given where

$$\int_{-\infty}^{\mu_e} \frac{C_e(x)dx}{P(v)} = p, \tag{17}$$

where $P(v)$ is given by (4) and $C_e(x)$ is given by (13) and we solve v for p=0.05.

By adjusting p, the probability of making a loss can be set to any desired value (the 'breakeven' mark-up, v_b, is obtained by solving (17) for p=0.50).

Estimation of μ_i and σ_i

From data on r bidders over c contracts, Skitmore's[7] empirical analysis of construction contract bids found that bids may be adequately modelled by

$$\ln x_{ij} = y_{ij} \; iid \; N\left(\alpha_i + \beta_j, \sigma_i^2\right) \tag{18}$$

where x_{ij} is the ith bidder's bid for the jth contract, α_i is a bidder location parameter, β_j is a contract datum parameter (so $\alpha_i + \beta_j \approx \mu_{ij}$). We estimate α_i, β_j, and σ_i by maximising the likelihood of (18). The log-likelihood is

$$\ln L = -\sum_{i=1}^{r} \frac{n_i \ln \sigma_i^2}{2} - \frac{1}{2} \sum_{i=1}^{r} \frac{1}{\sigma_i^2} \sum_{j=1}^{c} \delta_{ij} \left(y_{ij} - \alpha_i - \beta_j\right)^2 \tag{19}$$

where δ_{ij}=1 if bidder i bids for contract j, and 0 if bidder i does not bid for contract j,

$$n_i = \sum_{j=1}^{c} \delta_{ij} \tag{20}$$

The maximum likelihood estimates of the α_i, β_j and σ_i^2 are

$$\hat{\beta}_j = \sum_{i=1}^{l} \delta_{ij} \frac{y_{ij} - \alpha_i}{n_i} \tag{21}$$

$$\hat{\alpha}_i = \sum_{j=1}^{c} \delta_{ij} \frac{y_{ij} - \beta_j}{n_i} \tag{22}$$

$$\hat{\sigma}_i^2 = \sum_{j=1}^{c} \delta_{ij} \left(\frac{y_{ij} - \alpha_i - \beta_j}{n_i}\right)^2 \tag{23}$$

The numerical procedure for solving these equations involves initialising all α_i = 0 and iterating (21) and (22) to convergence. The estimates of σ_i^2 provided by (23) are adjusted for bias (approximately) by multiplying by the factor

$$\frac{n_i}{(n_i - 1)\left(1 - \frac{c-1}{N-r}\right)} \tag{24}$$

where

$$N = \sum_{j=1}^{c} n_i \tag{25}$$

and for once only bidders (n_i = 1) a weighted average of the unbiased variance estimates with weights

$$\frac{n_i - r - 1}{N - 1} \tag{26}$$

For computational purposes it is unnecessary to introduce once only bidders until after convergence of the iteration procedure.

Application to estimating v for a future contract

Since the α_i are bidder effects, we can assume that they will have the same value for a future contract and so we can use the estimates obtained above. The β value for a future contract is unknown to us but is tyhe same for all bidders in the contract and need not concern us. The only problem remaining in (10) is the unconditional mean cost estimate μ_e.

Estimation of μ_e

As μ_e is the expected value of the (unconditional) cost estimate, ie. setting ln(v)=0, we can find an explicit value. The pdf is that of a log normal distribution, so μ_e is (recalling that μ_I=0)

$$\mu_e = \exp\ \sigma^2/2 \tag{27}$$

A CASE STUDY

To illustrate the practical application of this approach, a set of 'live' bidding data has been analysed. These were donated by a construction company operating in the London area and covered all the company's building contract bidding activities during a 12 month period in the early 1980s for a total of 86 contracts. To preserve confidentiality, all the bidders were assigned a code at random, the bidder providing the data having code 304. Bidder 304's cost estimates were included in the data set instead of his bids. Some of the data were incomplete; that is the value of some bids or the identity of the bidders were not known by the company. In several cases it was possible to supplement these data from a bidding information agency in the London area. The 51 resulting contracts for which a full set of bids, together wit the identity of the bidder, were available for analysis are given in Table 1.
By first transforming the cost estimates to bids to

$$y_{ij} = \ln x_{ij} \tag{28}$$

and then applying the iterative eqns (21) and (22), the estimates α_i and σ_i were obtained for all participants, $\alpha_{304} = 0$.

For simplicity, bidder 55 was chosen for analysis, bidder 55 having entered bids for 20 of the contracts in the data. As a result of the iterative procedure, the values of $\hat{\alpha}$ and $\hat{\sigma}$ for bidder 304's cost estimates and bidder 55's bids were found to be 0, 0.00137, 0.06922, and 0.00257 respectively.

The probability function for bidder 304's profit was obtained from (4) for a series of mark up multipliers, v, the area under which was computed by a suitable quadrature method for each percentage point. Expected profit was calculated by substituting for μ_e in (27), and then finding the v which maximises $E[P]$ in (10). The estimated values of μ_e, $P(v)$, and $E[P]$ for a range of mark up multipliers are given in Table 2.

Figs 1 to 3 show the probability curves for bidders 304 and 55 together with bidder 304's profit with 0, 5 and 10% mark-up values.

It can be seen that the profit probability curve becomes progressively flatter with the mean value, $E[P]/P(v)$, progressively diminishing relative to the mark up multiplier - a phenomenon often termed 'the winner's curse' (see Thaler[9] for a review of the literature).

Visual inspection of Table 2 indicates the maximum expected profit to be around 2.7% at a mark up multiplier of 8%. By using a method of successive approximation, the exact optimal mark up multiplier was found to be 7.80737% (2.68163% expected profit). The mark-up multipliers for the 'no loss' (p=0.05) and 'breakeven' (p=0.5) strategies were found to be 7.50510 and 0.57660 respectively.

The results for bidders 55 and 221 are given in Table 3. The $\hat{\alpha}$ and $\hat{\sigma}$ values for bidder 221 were 0.07971 and 0.00385 respectively and the 'no loss', 'break even' and 'optimal' mark up multipliers were found to be 8.39080, 1.05870 and 6.61799 (1.45403 expected profit) respectively.

CONCLUSIONS

The application of multivariate methods to sealed bid mark-up strategies offers a potential improvement on previous bivariate methods in providing a means of better utilisation of all available data. Since it has been shown that, with suitably transformed data, the necessary assumptions concerning the statistical properties of bids may not be unduly violated in construction contract auctions[7], a multivariate model has been proposed for deriving 'optimal' and other strategic mark up multipliers. A means of estimating the model parameters was described and this has been applied in a case study to obtain the strategic mark-up multipliers required against known competitors.

The major untested assumptions in the method are that (1) the variables are independent (ie., bidders do not change their behaviour depending on who their competitors are), this could not be true if competitors used the method proposed here; (2) the variables are intertemporaly fixed (ie., bidders do not change their behaviour over time), (3) variability is stochastic (ie., the error term is truly random), (4) the estimated cost is unconditionally unbiased (ie., the conditions supporting 'winner's curse' apply), (5) the number and identity of competitors are known in advance of bidding. Assumptions (1)-(3) are also implicit in the Friedman model, and assumption (4) has been considered to be reasonable by Flanagan and Norman[8], who have extensive knowledge of the construction industry. Although not formally allowed in many sealed bid auctions, assumption (5) is generally accepted as being reasonable in construction contract auctions where such information may be purchased for the purpose. In the absence of certain knowledge of (5) however, probabilistic methods are available, and these have been examined to some extent by Friedman and others.

The multivariate approach described in this paper is still very much in its formative stages and it is clear that some work is yet needed before it can be applied with confidence in 'real-world' auctions. In addition to testing the assumptions in the model, the paucity of data in the field from which to estimate parameter values suggests that the resulting strategic mark-up multiplier estimates, though unbiased, may not be very accurate. The consequent opportunity loss may well be a significant factor. If this is the case, then it will be appropriate for future work in the field to consider the minimisation of such opportunity loss as a strategic option in itself.

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TABLE 1*: The 51 bids available for analysis*

Project	Bdr	Bid	Bdr	Bid	Bdr	Bid	Bdr	Bid	Bdr	Bid	Bdr	Bid	Bdr	Bid	Bdr	Bid	Bdr	Bid
1	150	1454515	55	1514865	304	1475398	134	1468775	154	1447867	73	1457977	1	1386652				
2	304	535608	291	502042	154	529744	157	516376	1	505291								
3	75	1333142	217	1331156	304	1366863	281	1266892	115	1276787	93	1277652	360	1865545	1	1271146		
4	304	696743	292	696972	237	701062	79	637815	361	697826	280	637815						
5	55	404110	304	422297	97	413224	117	389196	362	417489	157	389848	1	389214				
6	134	2116877	99	2169966	293	2187991	304	2161120	221	2198655	137	2296108	8	2165611	117	2153344	294	2133608
	1	2058210																
7	304	3065742	150	3119689	170	3141641	134	3153800	191	3249927	55	3269768	187	3335993	1	2919754		
8	221	7925257	304	7351929	247	7374650	20	6900000	1	7035339								
9	118	871520	137	899935	304	902378	291	914393	83	950737	221	996483						
10	304	1063337	251	1154023	173	1102272	201	1079657	1	1012702								
11	154	1759614	281	1792123	157	1838532	170	1918066	304	1947733	308	1784215	1	1811845				
12	304	1126816	201	1146398	154	1169795	24	1227296	280	1312527	221	1399472	1	1053099				
13	304	698005	268	625501	308	630288	55	666545	1	652341								
14	364	588810	365	584833	79	639229	145	646341	304	682802	154	691474						
15	303	1429218	291	1493849	304	1511033	12	1521628	366	1526377	55	1717715						
16	6	842319	304	870894	185	883617												
17	367	284947	356	292692	368	294694	152	303700	85	307282	134	313203	369	315727	118	333597	370	334353
	304	348969																
18	150	461444	304	483862	308	482241	55	447021	154	493417	311	455480						
19	280	2858191	371	2947007	134	2950723	304	2999999	60	3093587	6	3099528	266	3278229	170	3325198	55	3333793
	1	2884614																
20	276	7831865	304	7837276	256	7859122	55	7904172	152	8047230	293	8145323	117	8279564	134	8657685	1	7646123
21	55	3971051	304	3854074	372	4724785	154	3955009	373	3944772	79	3731543	237	4001188	1	3705840		
22	292	573485	291	596737	134	597730	201	613528	304	615015	170	621223	1	580203				
23	304	1610942	163	1623447	173	1646286	268	1663742	152	1700000	1	1558574						
24	64	1196036	374	1199328	187	1206837	304	1226589	291	1262082	152	1271000	170	1295954	254	1302161	1	1179413
25	137	2636397	150	2654728	187	2673906	55	2685127	304	2762123	152	2845567						
26	24	469663	268	476784	286	485870	55	486485	122	504026	263	529468	304	540814	1	515061		
27	201	1526553	152	1533719	148	1698797	304	1876612	1	1770389								
28	201	2106139	304	2175928	308	2210065	280	2223710	221	2255246	117	2296623	266	2331830	1	2062491		

29	102	499888	55	559596	217	592026	170	602042	304	608957	134	619065						
30	304	2639525	308	2842407	280	2874130	55	2861665	152	2736300	256	2770720	1	2538005				
31	304	732572	365	599429	145	623906	79	691759	154	744332	364	607065						
32	134	546641	268	539565	55	608242	24	538382	170	599934	304	559351	1	530190				
33	221	792966	99	811788	308	819971	55	847621	137	847892	304	853793						
34	152	2085151	107	2130217	280	2150583	115	2203956	137	2219653	154	2241687	304	2325900				
35	268	821617	115	844579	303	848459	304	871927	106	872215	375	935765	1	830407				
36	304	792474	24	747374	217	778559	252	743788	268	808345	170	835465	1	754737				
37	304	7279854	60	7650271	308	7029448	150	6631664	193	7089879	170	7230120	247	6986341	191	7143710	266	6794553
	1	7067819																
38	304	592096	150	573997	217	518613	121	508985	376	544480	1	550787						
39	348	538600	377	567031	378	621365	268	699839	72	825451	190	991468	304	1001254				
40	154	2087946	276	2104017	186	2183122	304	2205359	280	2212382	112	2267987	221	2332476	294	2400000		
41	247	1503739	24	1536654	304	1576905	154	1583595	294	1616432	157	1704995	1	1530976				
42	191	3624453	221	3694803	304	3732133	170	3751115	193	3773967	55	3866339	134	3922937	281	4122448	1	3641105
43	157	629164	173	695284	311	723315	266	729305	304	743578	379	768189						
44	304	2252833	24	2264310	112	2274380	191	2323385	55	2384494	1	2187217						
45	163	1202916	55	1268733	221	1291365	304	1294986										
46	217	2968891	280	2772626	186	2822857	134	2972189	276	2821600	304	2857275	221	2793000	1	2787585		
47	286	1398400	152	1401500	237	1427140	304	1436804	301	1453070	55	1511643	371	1591986	83	1665760	1	1381542
48	294	698161	24	709676	291	758565	304	789355	134	797926	55	842684	252	751677	1	751767		
49	31	248733	291	251007	252	251415	380	261286	304	264933								
50	293	358840	217	362370	304	386983	381	421797	154	456272	1	351803						
51	317	527692	311	570874	75	588854	173	609221	308	636451	304	694297	1	645858				

TABLE 2*: Results for a range of mark up multipliers (against bidder 55)*

v(%)	*v*	ln*(v)*	*P*(*v*)	*E*[*P*]	*E*[*P*]/*P*(*v*)
0.0	1.00	0.00000	0.86476	-0.48082	-0.55602
1.0	1.01	0.00995	0.82730	0.25725	0.31095
2.0	1.02	0.01980	0.78427	0.91031	1.16071
3.0	1.03	0.02956	0.73610	1.46750	1.99362
4.0	1.04	0.03922	0.68351	1.92084	2.81025
5.0	1.05	0.04879	0.62748	2.26607	3.61136
6.0	1.06	0.05827	0.56918	2.50317	4.39784
7.0	1.07	0.06766	0.50989	2.63646	5.17060
8.0	1.08	0.07696	0.45094	2.67433	5.93061
9.0	1.09	0.08618	0.39357	2.62858	6.67881
10.0	1.10	0.09531	0.33892	2.51345	7.41610

TABLE 3*: Results for a range of mark up multipliers (against bidders 55 and 221)*

v(%)	*v*	ln*(v)*	*P*(*v*)	*E*[*P*]	*E*[*P*]/*P*(*v*)
0.0	1.00	0.00000	0.99219	-0.70731	-0.92451
1.0	1.01	0.00995	0.99951	-0.08586	-0.12074
2.0	1.02	0.01980	1.00625	0.43161	0.66160
3.0	1.03	0.02956	1.01238	0.83968	1.42326
4.0	1.04	0.03922	1.01787	1.13790	2.16519
5.0	1.05	0.04879	1.02272	1.33097	2.88848
6.0	1.06	0.05827	1.02692	1.42837	3.59430
7.0	1.07	0.06766	1.03050	1.44345	4.28383
8.0	1.08	0.07696	1.03350	1.39210	4.95823
9.0	1.09	0.08618	1.03598	1.29136	5.61863
10.0	1.10	0.09531	1.03798	1.15792	6.26609

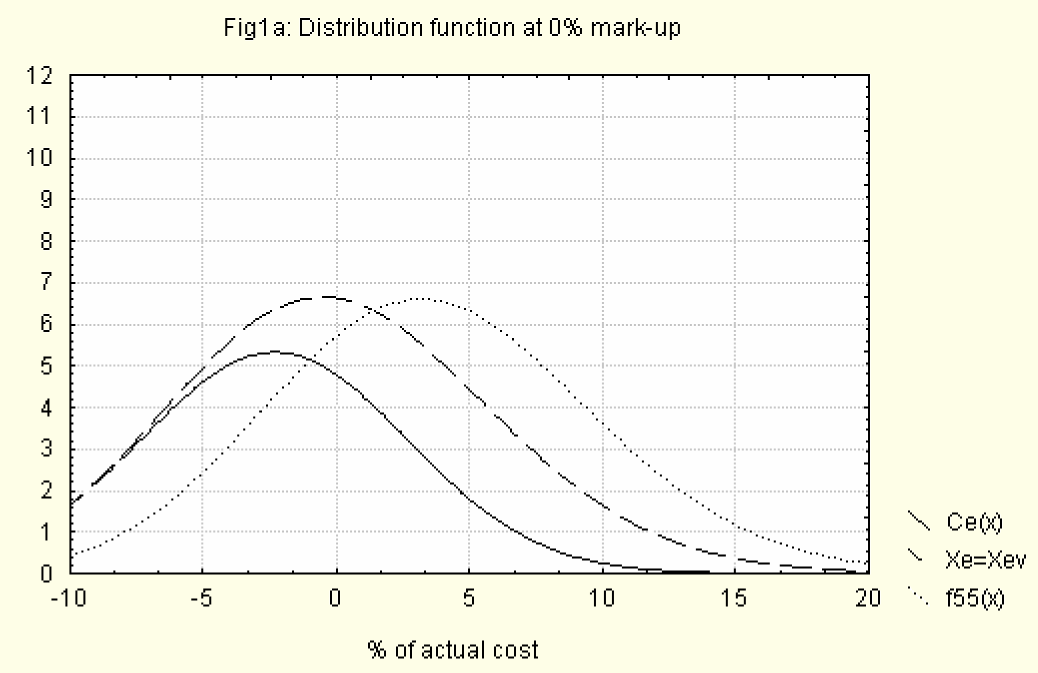

Fig1a: Distribution function at 0% mark-up
12
11
10
9
8
7
6
5
4
3
2
1
0
-10
-5
0
5
10
15
20
Ce(x)
Xe=Xev
f55(x)
% of actual cost

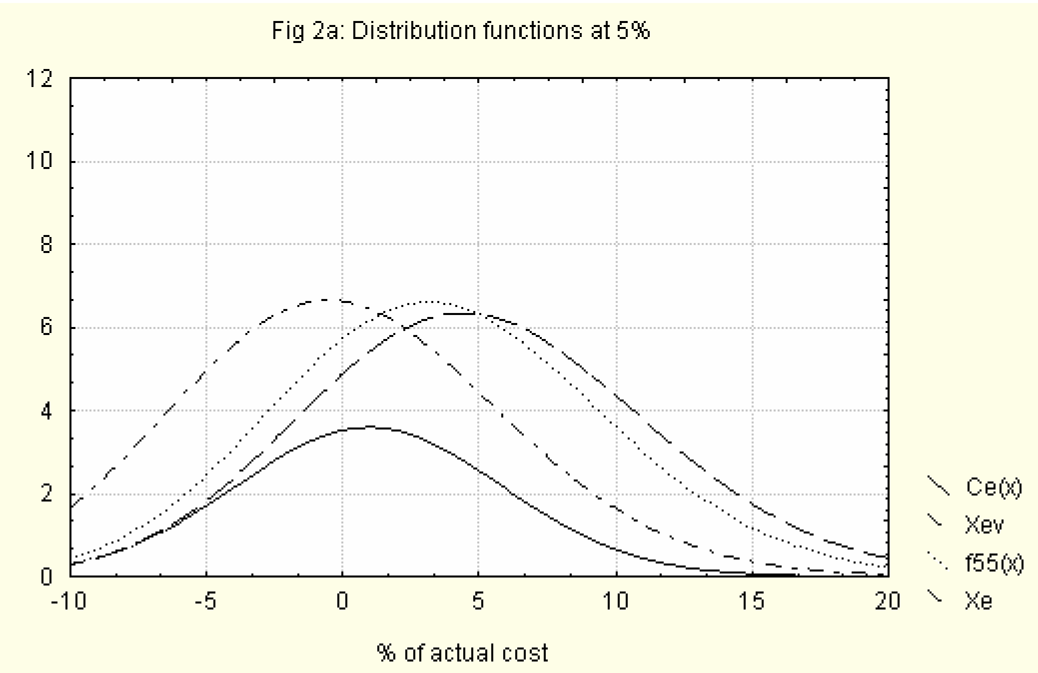

Fig 2a: Distribution functions at 5%
12
10
8
6
4
2
0
-10
-5
0
5
10
15
20
Ce(x)
Xev
f55(x)
Xe
% of actual cost

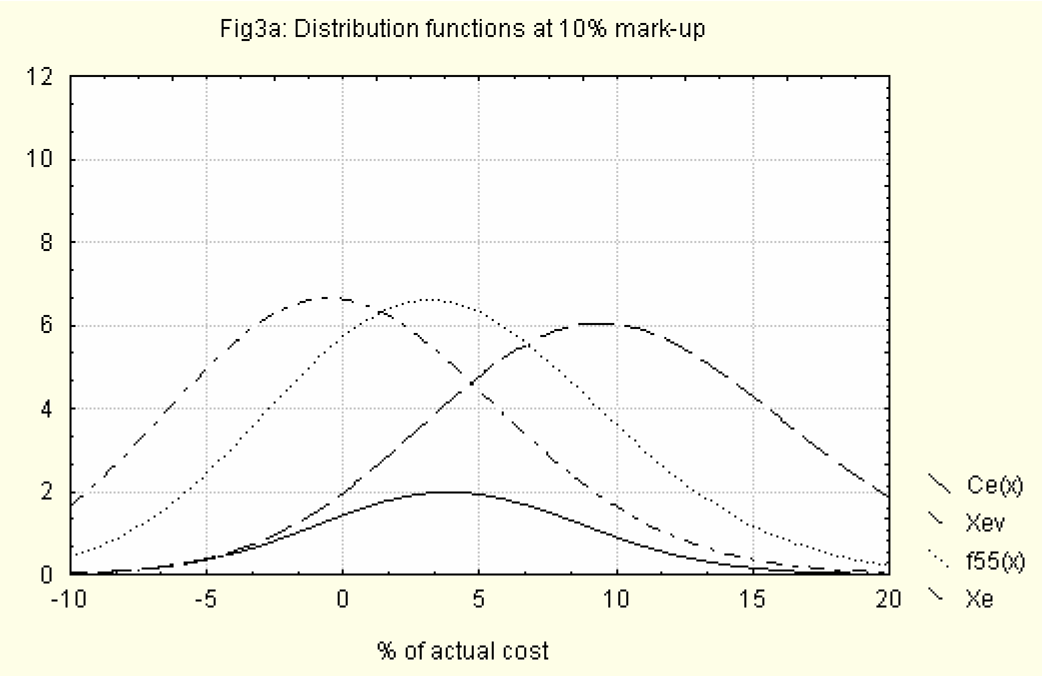
Fig3a: Distribution functions at 10% mark-up
12
10
8
6
4
2
0
-10
-5
0
5
10
15
20
% of actual cost
Ce(x)
Xev
f55(x)
Xe