Binary logic using spatially patterned deaths in chemical oscillators

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Abstract

Fascinating spatiotemporal patterns have been obtained using the microfluidic system of chemical oscillators. The possibility of the existence of logic gates in such systems in such assemblies was explored by transforming the Spatially Patterned Oscillator Death (SPOD) state. We showed that such deterministic gates indeed exist. A 'parity checker,' a NOT gate, an OR gate and a NOR gate were obtained easily. The presence of the NOR gate endorses the possibility of achieving universal computation using the said arrays but the NAND gate is still to be found.

Motivation

Synchronisation amongst chemical reaction-diffusion systems or 'chemical-oscillators' has been studied in detail both experimentally and theoretically [1, 2]. The temporal evolution of certain reaction solutions in micro-fluidic assemblies [3] gives interesting spatial patterns (e.g. Belousov-Zhabotinsky reaction). We use the FitzHugh-Nagumo (FHN) model to mimic these spatiotemporal patterns and examine the possibility of constructing 'logic gates' [4] using such oscillators.

Main Objectives

1. Mimic the spatiotemporal patterns in chemical oscillators.
2. Identify the most suitable candidate amongst various patterns for constructing 'logic gates'.
3. Determinie appropriate parameter domains for obtaining the said candidate pattern.
4. Identify the threshold for discern the two binary states, 0 and 1.

FHN Model

The FHN model can be used to mimic the spatiotemporal patterns originating in arrays of chemical oscillators. It consists of two variables viz. fast activation variable, u and slow inactivation variable v, also known as the excitatory and inhibitory variables respectively. The model is described as follows:

$$\dot{u} = f(u,v) = u(1-u)(u-\alpha) - v \tag{1a}$$
$$\dot{v} = g(u,v) = \epsilon(ku - v - b) \tag{1b}$$

where the parameters α and k describe the kinetics, ϵ characterises the recovery rate, and b is a measure of the asymmetry of an entity in the system.

Constructing the array of oscillators

We considered a system of N relaxation oscillators, interacting in some particular topological configuration. The FHN equations dictated the dynamics of each oscillator. The values of the parameters used are: $\alpha = 0.139$, $k = 0.6$ and $\epsilon = 0.001$. The neighbouring oscillators are coupled diffusively via the inactivation variable v. Dynamics of the resulting system can be given as:

$$\dot{u}_i = f(u_i, v_i) \tag{2a}$$
$$\dot{v}_i = g(u_i, v_i) + D_v(v_{i-1} + v_{i+1} - 2v_i) \tag{2b}$$
$$\dot{v}_0 = D_v(v_1 - v_0) \tag{2c}$$
$$\dot{v}_{N+1} = D_v(v_N - v_{N+1}). \tag{2d}$$

Here $i = 1, 2, ..., N$ and D_v represents the strength of the coupling between neighbouring relaxation oscillators through the inhibitory variables. For various regions in the $b - D_v$ domain, interesting spatiotemporal patterns are obtained.

Spatially patterned death in oscillators - the suitable candidate

The spatially patterned oscillator deaths (SPOD) state can be obtained for only certain regions in the $b - D_v$ space; we considered values of b and D_v to be in the vicinity 0.16 and 0.002, however, these choices are not unique.

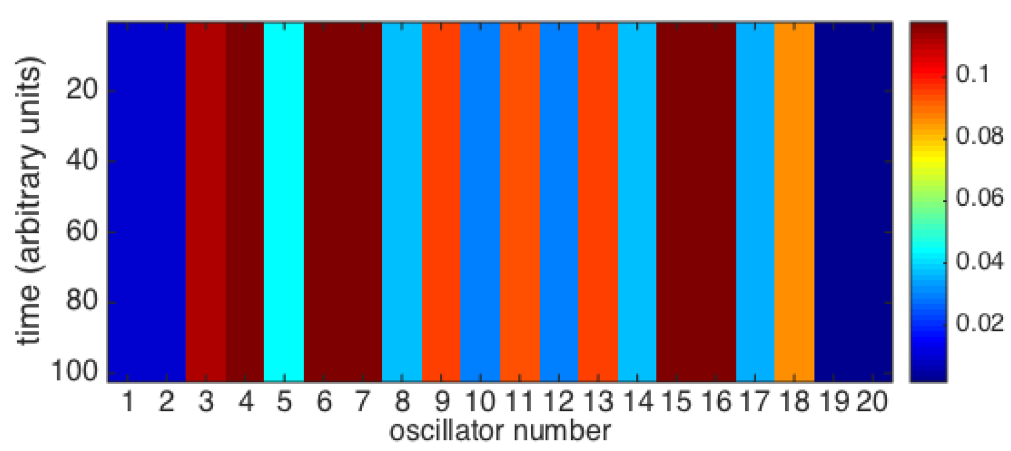

Figure 1: Spatially patterned deaths in array of chemical oscillators.

The SPOD state is most suited for developing logic gates because:

1. Information can easily be encoded into the SPOD states in the form of binaries. The high modes can be assigned the value 1 and the low ones 0.
2. The SPOD state remains steady with the evolution of time unless a change is triggered externally.

Discerning 0 and 1

To select a discriminating threshold value for discriminating between 0 and 1, we found out what steady values of modes do the oscillators obtain for different N when the initial conditions were chosen randomly. A clear inclination towards two values or rather two ranges of values were observed and the mean of these 2 ranges of values can be considered the threshold between 0 and 1. This threshold didn't vary much with N.

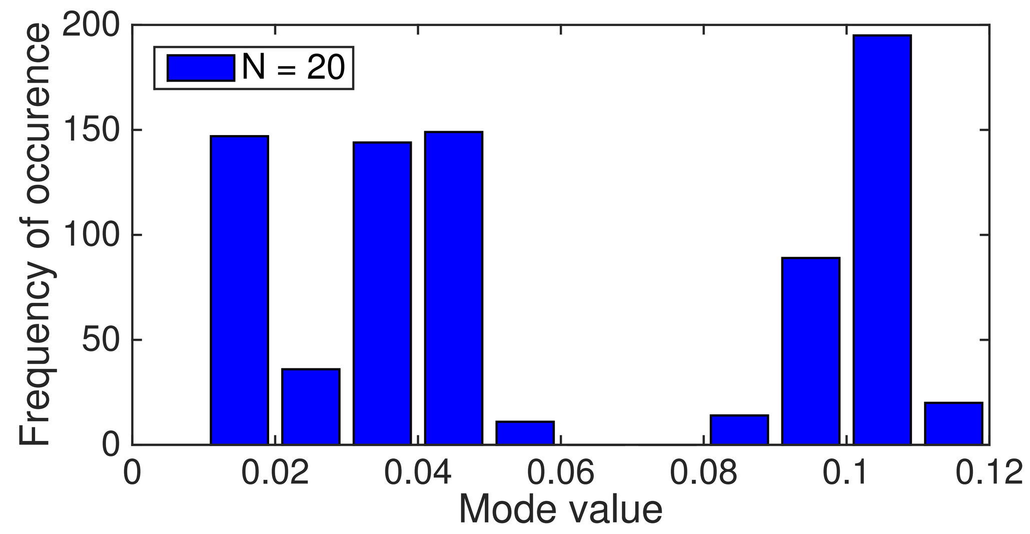

Figure 2: Frequency of occurrence of various modes.

After selecting the appropriate threshold the SPOD state looked as:

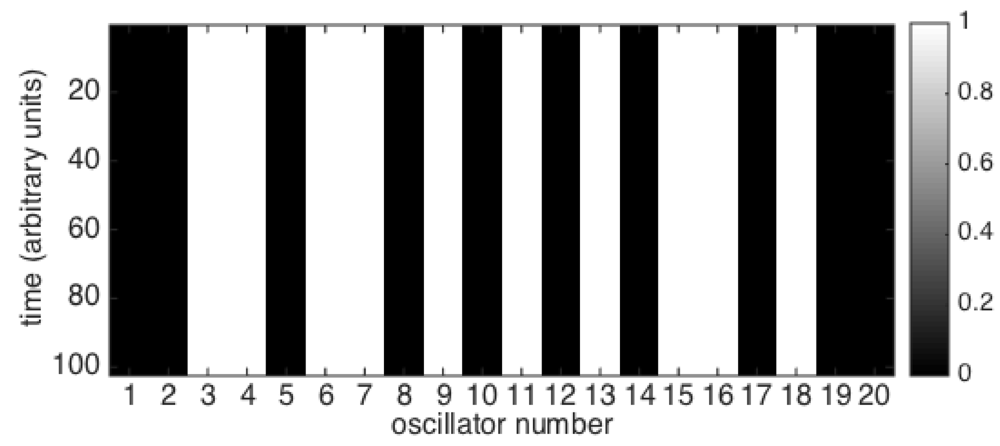

Figure 3: Binary values in SPOD state

Results

We considered a system of 20 oscillators, in which the value attained by the 10^{th} and the 11^{th} oscillators were taken as inputs. A perturbation of a finite amplitude was applied for a certain time duration to different configurations of oscillators. This induced changes in the values attained by the oscillators. The value attained by the 11^{th} oscillator was considered the output.

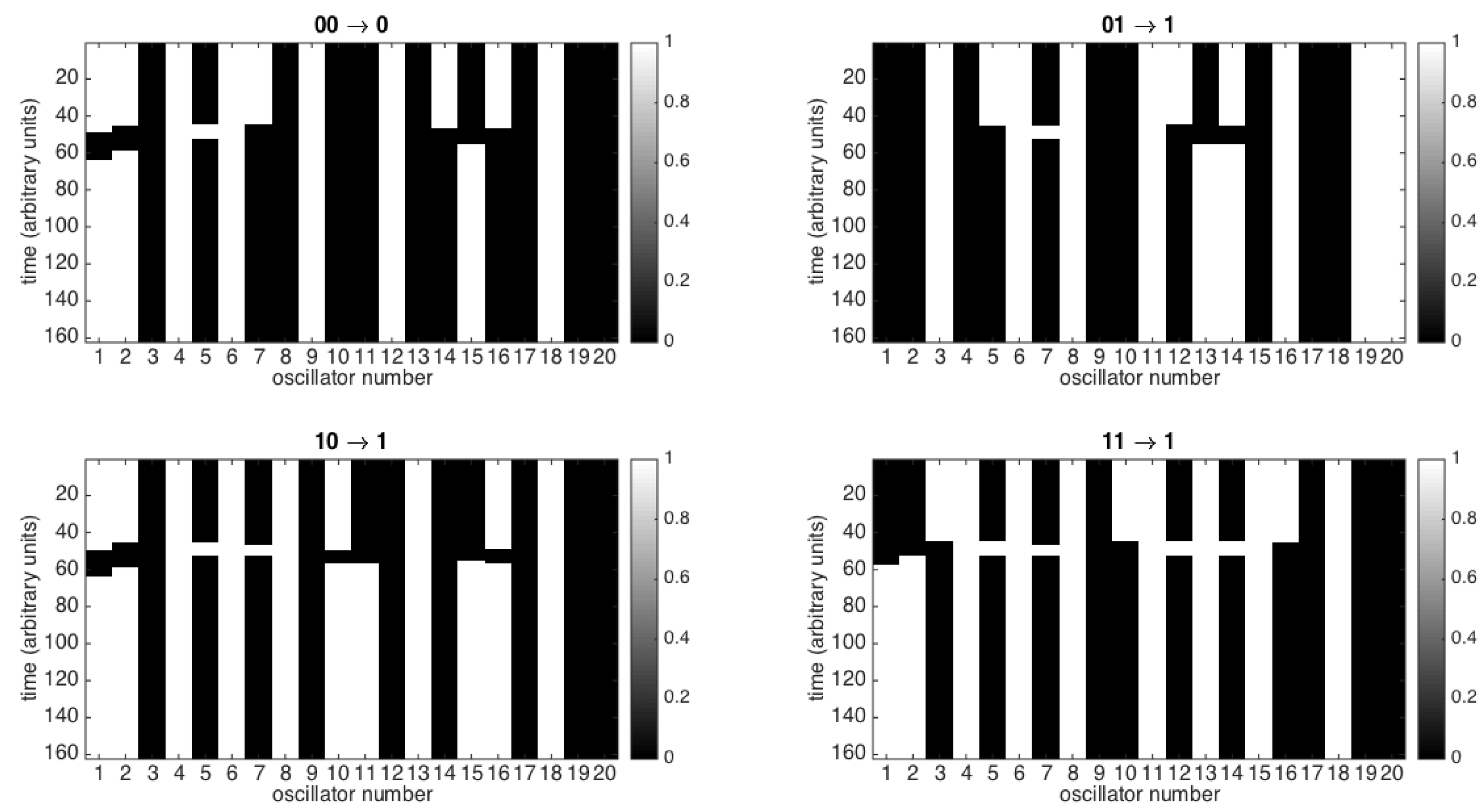

Figure 4: The OR gate constructed using SPOD logic.

Conclusions

- We verified that using arrays of chemical oscillators for computation using binary logic is indeed possible.
- We readily constructed a parity checker, an OR gate, a NOT gate and a NOR gate.

Forthcoming Research

- Appropriate configuration for perturbation is to be found to generate a NAND gate.
- Staggering of gates in the aforementioned array of chemical oscillators is to be investigated.

References

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