האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Is the Speed of Ocean Surface Currents Weibull Distributed?

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אוניברסיטת בן-גוריון בנגב
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INTRODUCTION

- Ocean surface currents are crucial for:
  - regulating Earth's climate by **transporting heat.**
  - **movement of marine life** across the world and regulating temperature of marine ecosystems.
  - determining the **course of navigation** while traversing the ocean.
- Underlying causes: wind stress, buoyancy fluctuations, tides, wave dynamics etc.
- **Nontrivial** to understand piece-wise contribution of each.
- Very little known about the shape and behaviour of their statistical distribution in time. But WHY would we want to know anything about it?
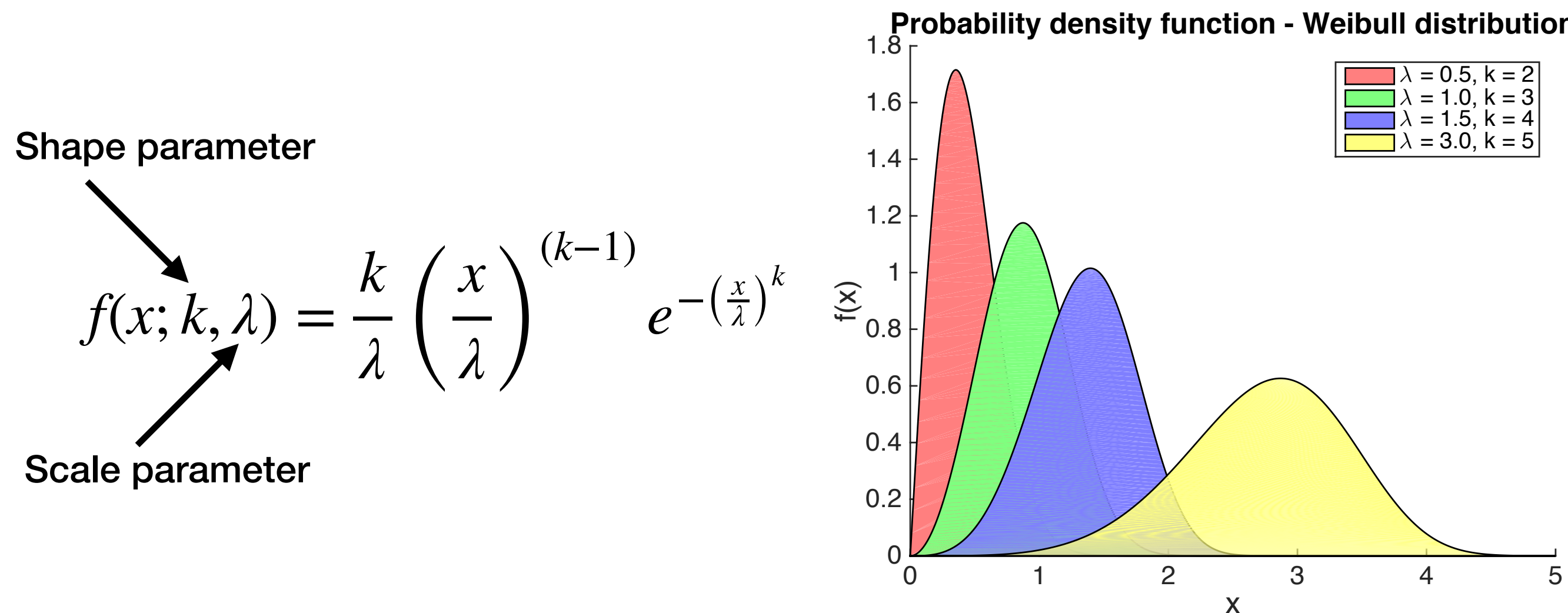
MOTIVATION

- Ocean surface currents are also a potential source of electric energy, possibly **better than winds.**
- Accurate information about these distributions can be utilised as a bench mark for improved **ocean and climate modelling**
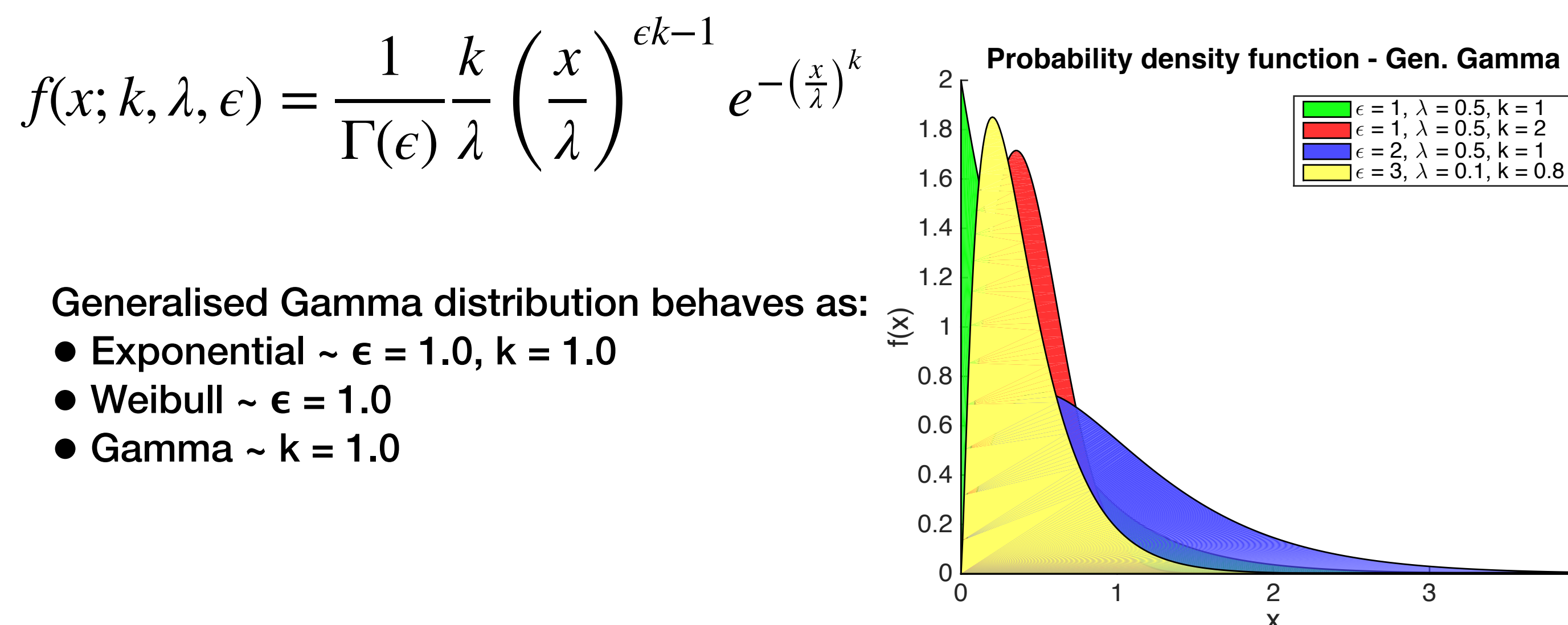
AGENDA

- Test the long standing hypothesis that the pdfs under consideration are **Weibull** distributed.
- If the hypothesis is rejected, test whether the alternate, the **Generalised Gamma** distribution is better suited for the pdfs.
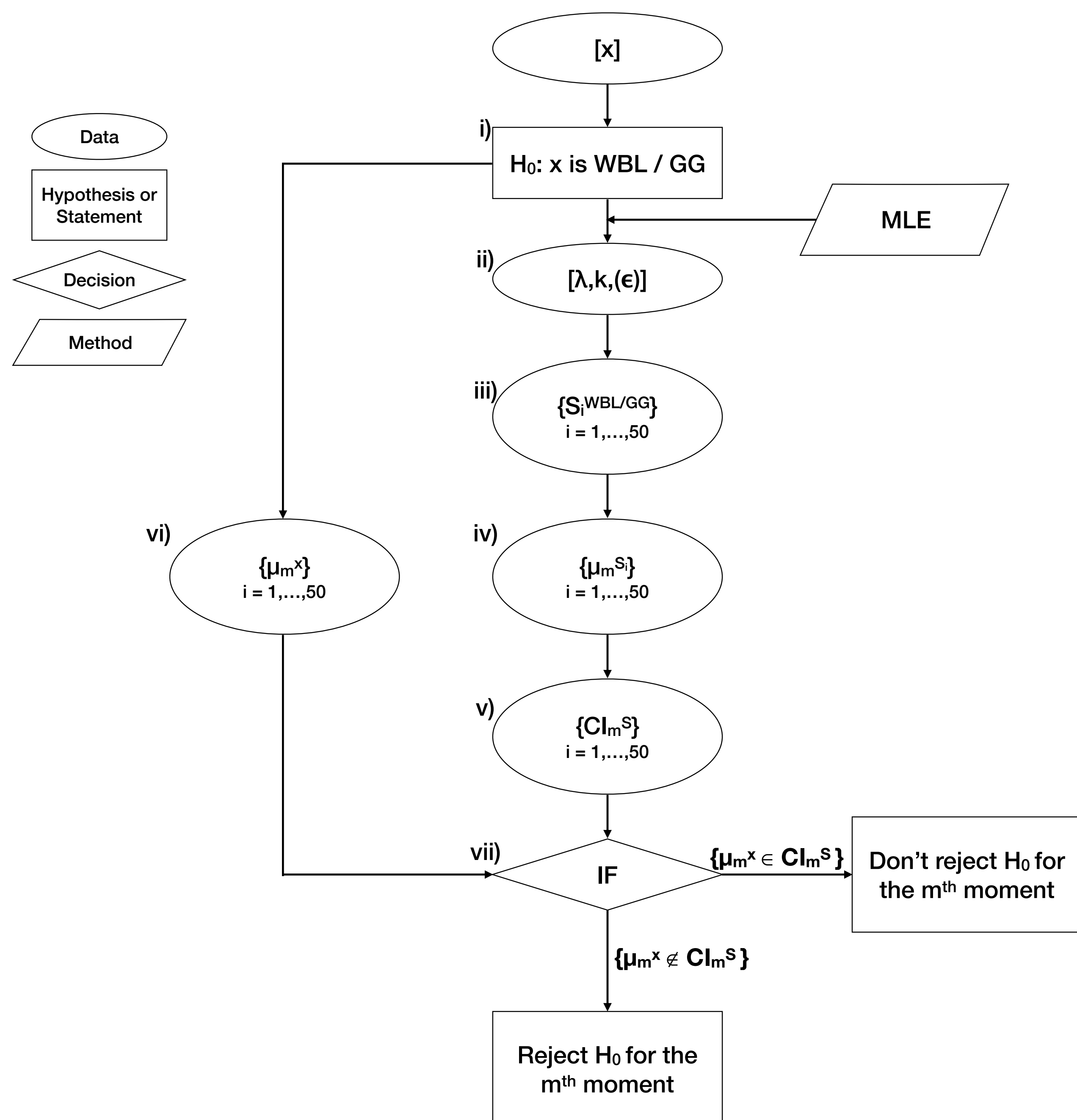
WEIBULL DISTRIBUTION

Shape parameter

$$f(x; k, \lambda) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{(k-1)} e^{-\left(\frac{x}{\lambda}\right)^k}$$

Scale parameter

Probability density function - Weibull distribution

- $\lambda = 0.5, k = 2$
- $\lambda = 1.0, k = 3$
- $\lambda = 1.5, k = 4$
- $\lambda = 3.0, k = 5$

GENREALISED GAMMA DISTRIBUTION

$$f(x; k, \lambda, \epsilon) = \frac{1}{\Gamma(\epsilon)} \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{\epsilon k - 1} e^{-\left(\frac{x}{\lambda}\right)^k}$$

Generalised Gamma distribution behaves as:
- Exponential ~ $\epsilon = 1.0$, $k = 1.0$
- Weibull ~ $\epsilon = 1.0$
- Gamma ~ $k = 1.0$

Probability density function - Gen. Gamma

- $\epsilon = 1, \lambda = 0.5, k = 1$
- $\epsilon = 1, \lambda = 0.5, k = 2$
- $\epsilon = 2, \lambda = 0.5, k = 1$
- $\epsilon = 3, \lambda = 0.1, k = 0.8$

METHOD

- Data
- Hypothesis or Statement
- Decision
- Method

[x]

i) H_0: x is WBL / GG — MLE

ii) $[\lambda, k, (\epsilon)]$

iii) $\{S_i^{WBL/GG}\}$ i = 1,...,50

iv) $\{\mu_m^S\}$ i = 1,...,50

vi) $\{\mu_m^x\}$ i = 1,...,50

v) $\{CI_m^S\}$ i = 1,...,50

vii) IF — $\{\mu_m^x \in CI_m^S\}$ Don't reject H_0 for the mth moment
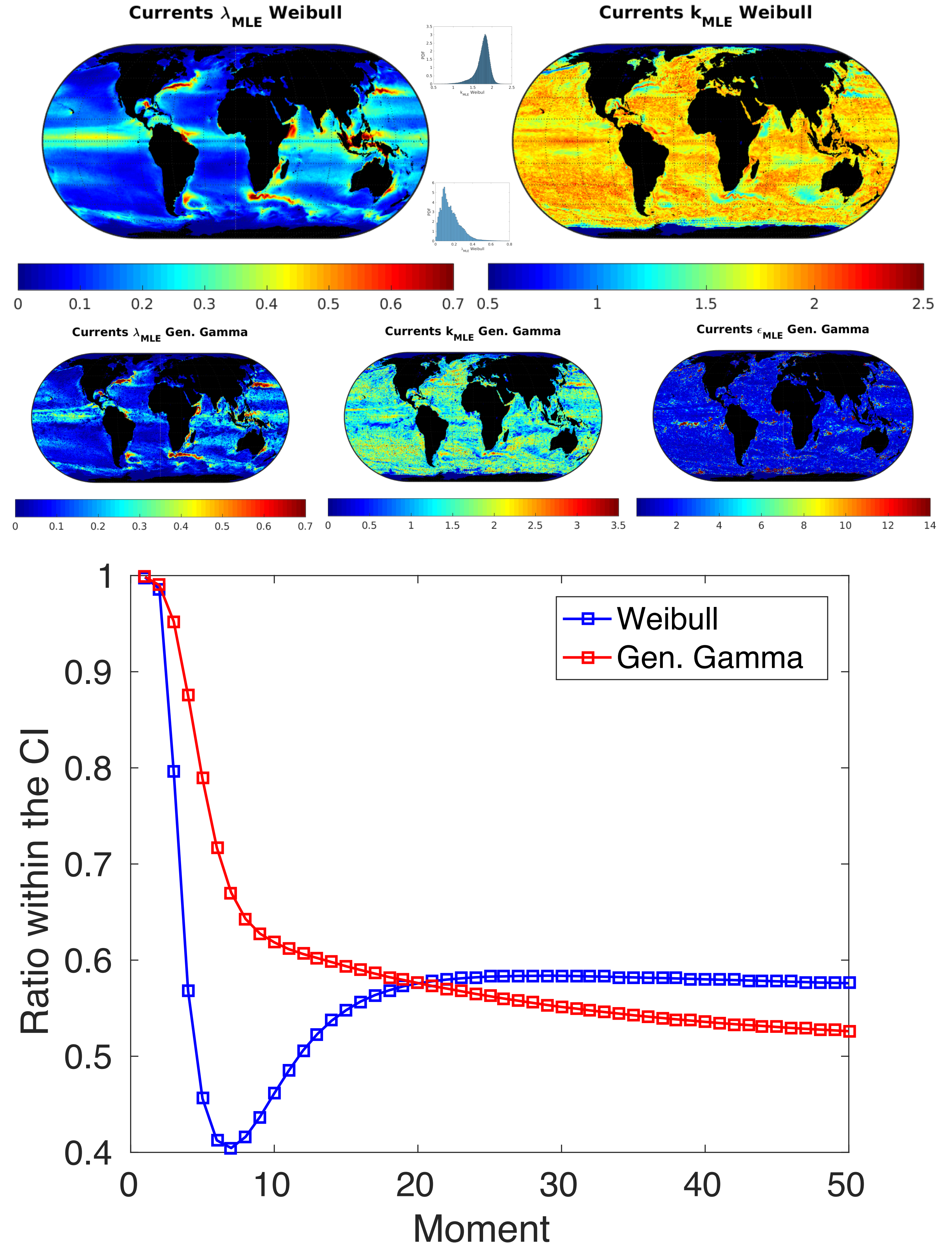
$\{\mu_m^x \notin CI_m^S\}$
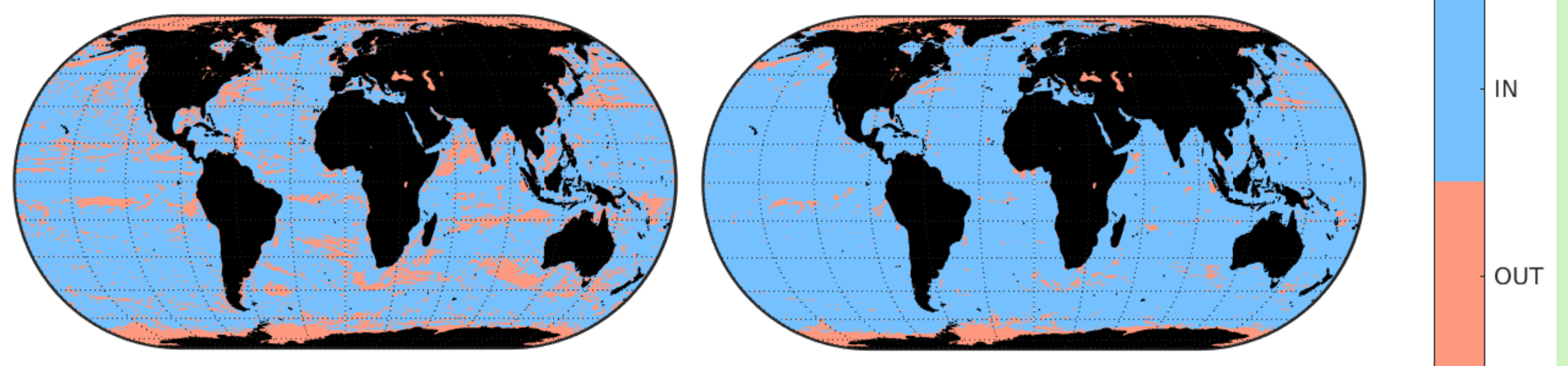
Reject H_0 for the mth moment

The analysis starts by assuming
i) the sample at hand (x) are Weibull (WBL) /Gen. Gamma (GG) distributed; therefore,
ii) calculate the MLEs λ, k, (ε) of x.
iii) using the estimated parameters, generate a set of random WBL/GG surrogate series $\{S^{WBL/GG}\}$
iv) calculate up to the 50th moment for each surrogate distribution; v) estimate the confidence interval for each moment of the surrogates.
vi) calculate up to the 50th moment of the original sample x; finally;
vii) benchmark the values of the moments of x against the boundary levels of the corresponding confidence interval if the moment is within its confidence interval, then conclude that x is WBL/GG for that moment else it is not.

RESULTS

Currents λ_{MLE} Weibull

Currents k_{MLE} Weibull

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7   0.5 1 1.5 2 2.5

Currents λ_{MLE} Gen. Gamma

Currents k_{MLE} Gen. Gamma

Currents ϵ_{MLE} Gen. Gamma

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7   0 0.5 1 1.5 2 2.5 3 3.5   2 4 6 8 10 12 14

- Weibull
- Gen. Gamma

Ratio within the CI — Moment

Currents μ_3 within Weibull C.I. ~ 78%

Currents μ_3 within Gen. Gamma C.I. ~ 94%

IN

OUT

CONCLUSION

- Successful statistical analysis of the available time-series.
- **Surface Current speeds are not ALWAYS Weibull distributed** over the global marine surface.
- **Generalised Gamma distribution is a better approximation** for a larger number of the global marine surface time series in comparison to the Weibull.
- The **Generalised Gamma distribution gives a better estimate of μ_3** (the third moment) for time-series of the Currents.
- This is relevant because for many applications, for example, the electric power available from these currents ~ $A\rho \langle U^3 \rangle / 2$.

DISCUSSION

- A similar analysis was carried out by Dr. Salvatore Campisi-Pinto for time-series of winds over the global surface and the results were consistent.
- Other methods were also used to further validate the hypothesis for both the winds and the currents and the results were robust.

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