Markov Decision Processes

REINFORCEMENT LEARNING WITH GYMNASIUM IN PYTHON

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datacamp

MDP

- Models RL environments mathematically

Complex environment

- States
- Actions
- Rewards
- Transition probabilities

MDP

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- Models RL environments mathematically

Complex environment

- States
- Actions
- Rewards
- Transition probabilities

MDP

Solve with model-based RL techniques

Markov property

- Future state depends only on *current state* and *action*

Frozen Lake as MDP

- Agent must reach goal without falling into holes

Frozen Lake as MDP - states

- Positions agent can occupy

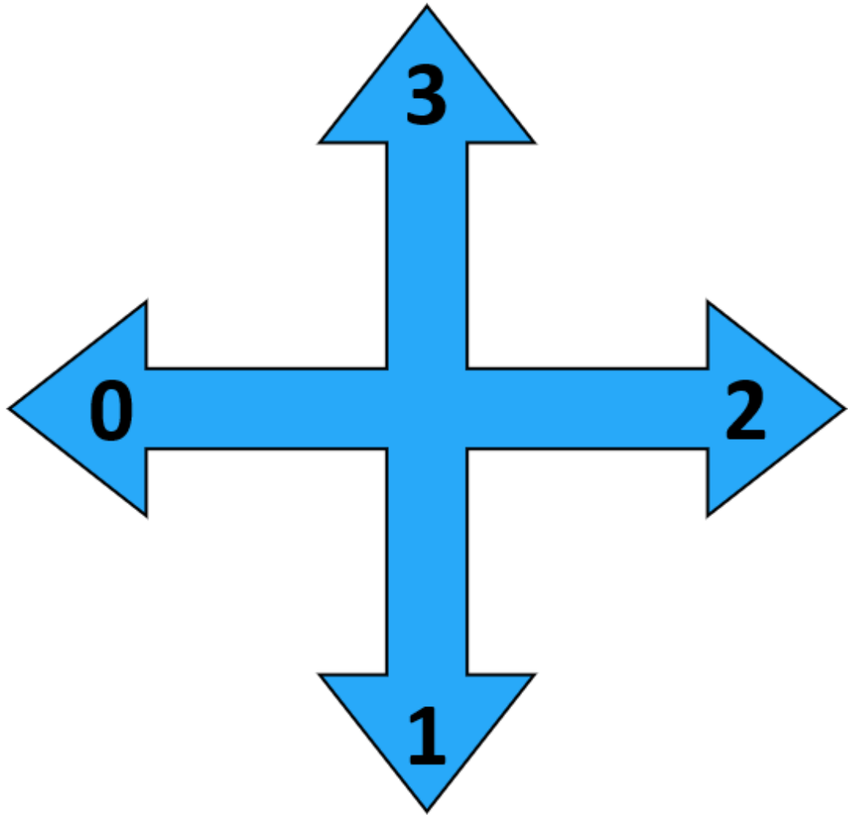


Frozen Lake as MDP - terminal states

- Lead to episode termination

Frozen Lake as MDP - actions

- Up, down, left, right



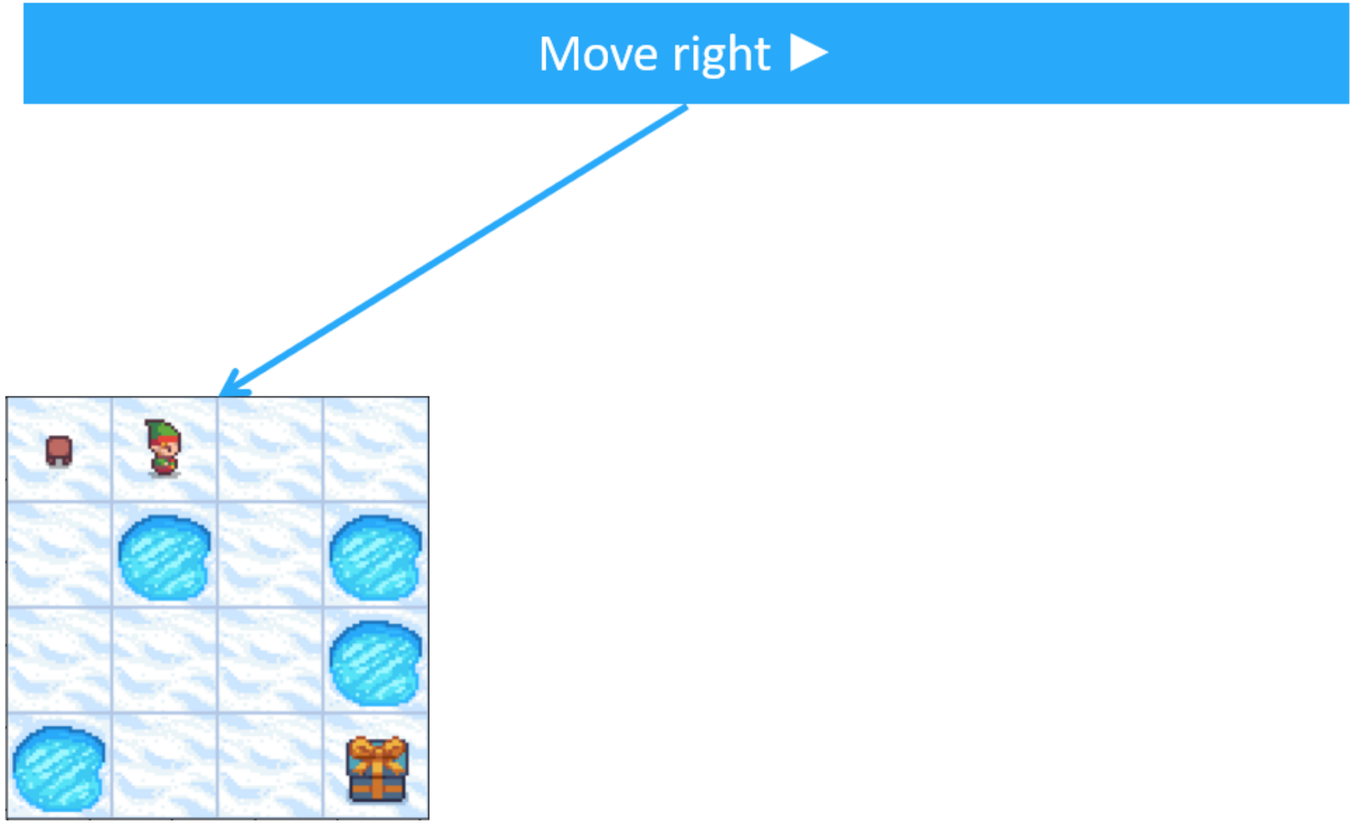
Frozen Lake as MDP - transitions

- Actions don't necessarily lead to expected outcomes

Move right ▶

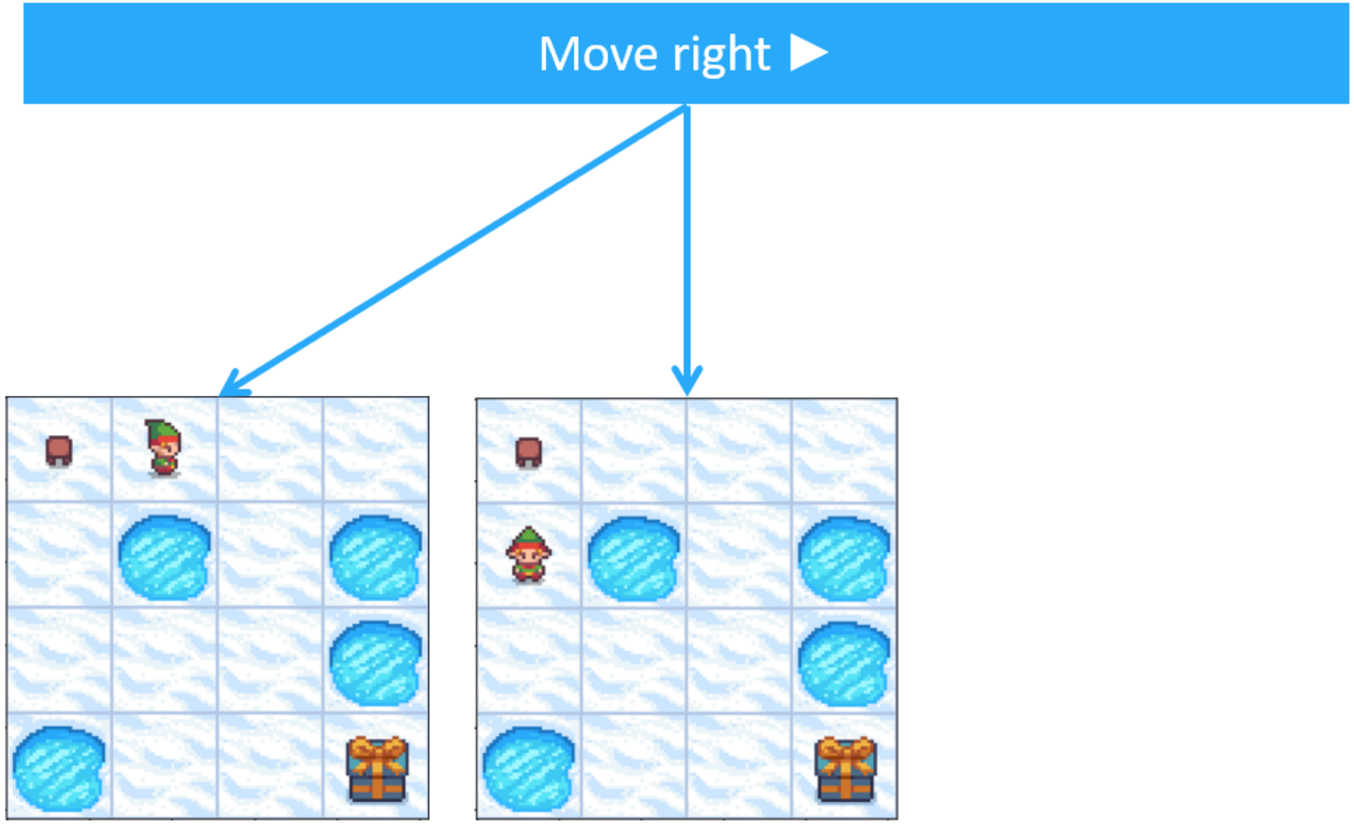
Frozen Lake as MDP - transitions

- Actions don't necessarily lead to expected outcomes



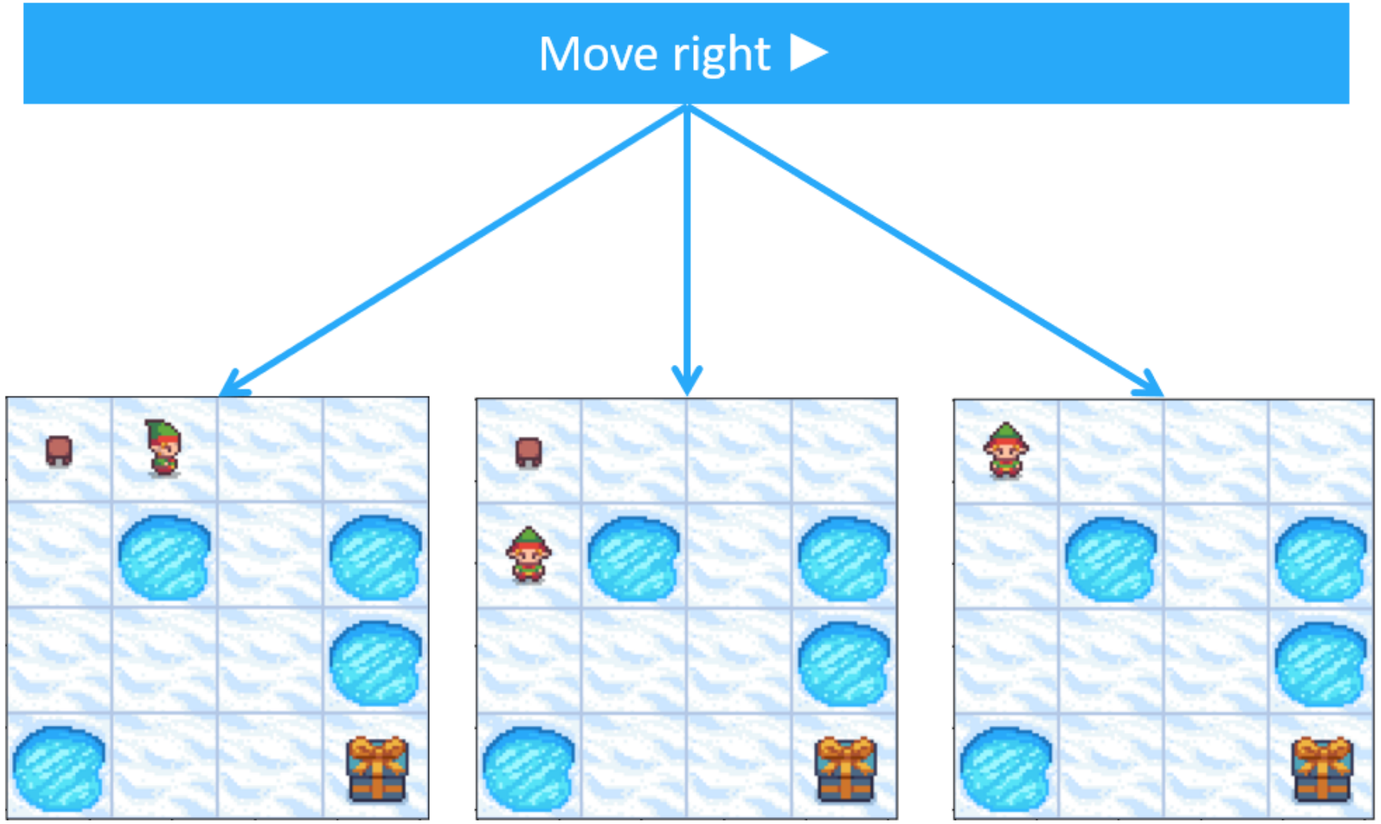
Frozen Lake as MDP - transitions

- Actions don't necessarily lead to expected outcomes



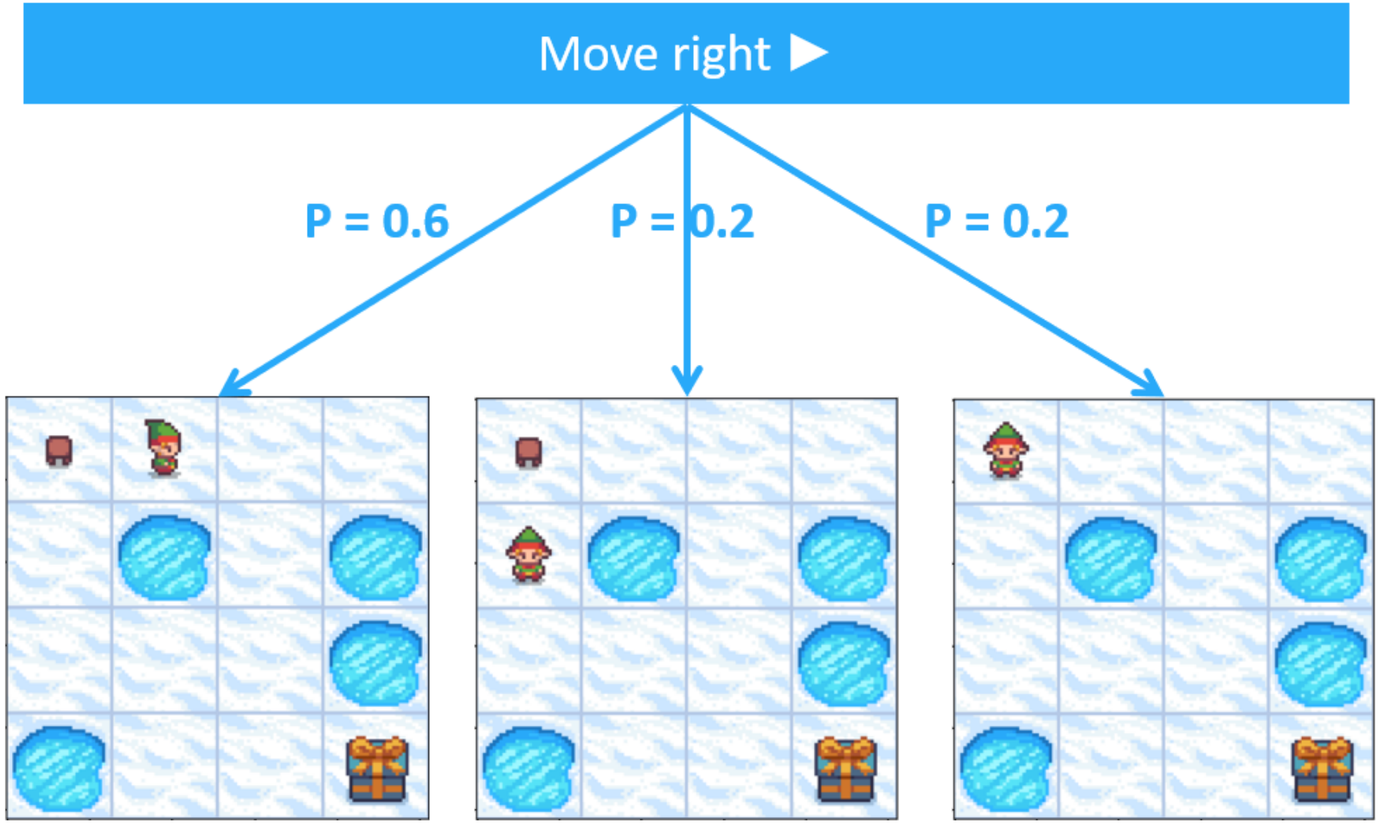
Frozen Lake as MDP - transitions

- Actions don't necessarily lead to expected outcomes



Frozen Lake as MDP - transitions

- Actions don't necessarily lead to expected outcomes



- **Transition probabilities:** likelihood of reaching a state given a state and action

Frozen Lake as MDP - rewards

- Reward only given in goal state

Positive reward

Gymnasium states and actions

```
import gymnasium as gym


env = gym.make('FrozenLake', is_slippery=True)
print(env.action_space)
print(env.observation_space)
print("Number of actions: ", env.action_space.n)
print("Number of states: ", env.observation_space.n)
```

```
Discrete(4)
Discrete(16)
Number of actions: 4
Number of states: 16
```

Gymnasium rewards and transitions

`env.unwrapped.P` : dictionary where keys are state-action pairs

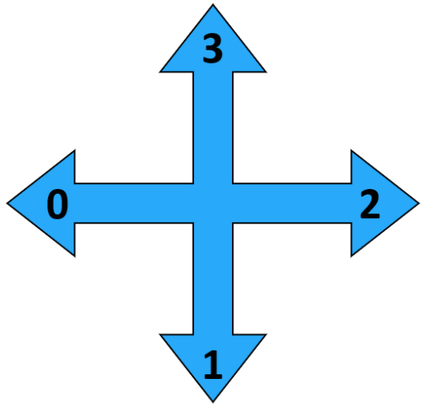
```
print(env.unwrapped.P[state][action])
```

```
[
  (probability_1, next_state_1, reward_1, is_terminal_1),
  (probability_2, next_state_2, reward_2, is_terminal_2),
  etc.
]
```

Gymnasium rewards and transitions - example

```
state = 6
action = 0
print(env.unwrapped.P[state][action])
```

```
[(0.3333333333333333, 2, 0.0, False),
(0.3333333333333333, 5, 0.0, True),
(0.3333333333333333, 10, 0.0, False)]
```



Let's practice!

REINFORCEMENT LEARNING WITH GYMNASIUM IN PYTHON

Policies and state-value functions

REINFORCEMENT LEARNING WITH GYMNASIUM IN PYTHON

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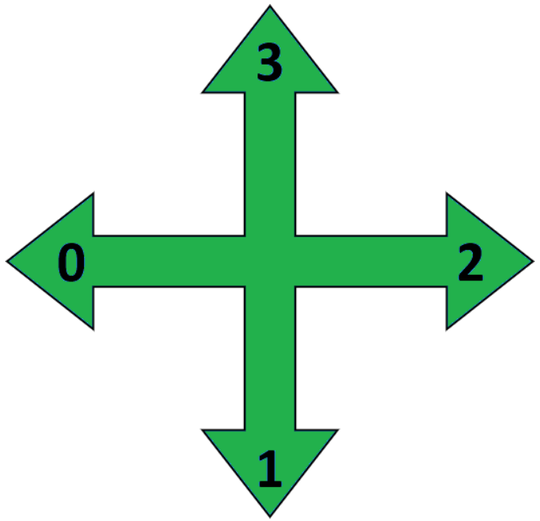
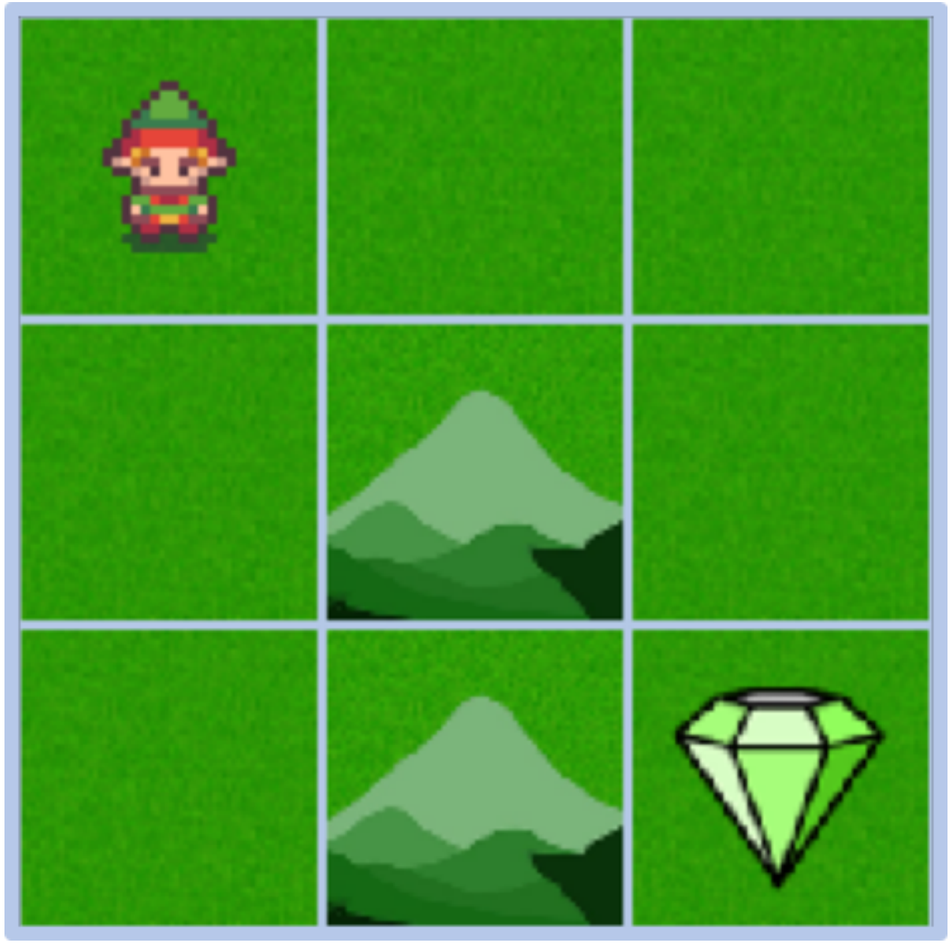
Policies

- RL objective → formulate effective policies
- Specify which action to take in each state to maximize return

Grid world example

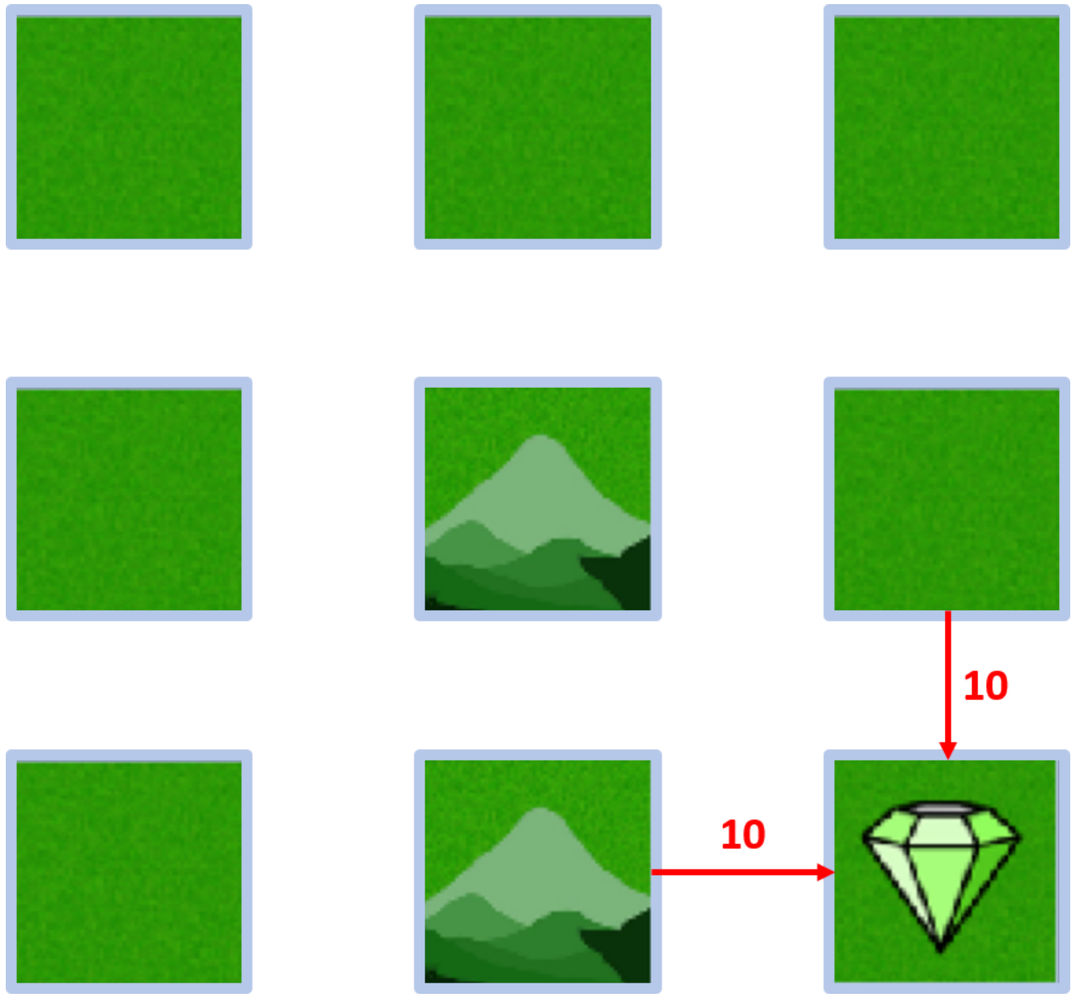
- Agent aims to reach diamond while avoiding mountains
- Nine states
- Deterministic movements





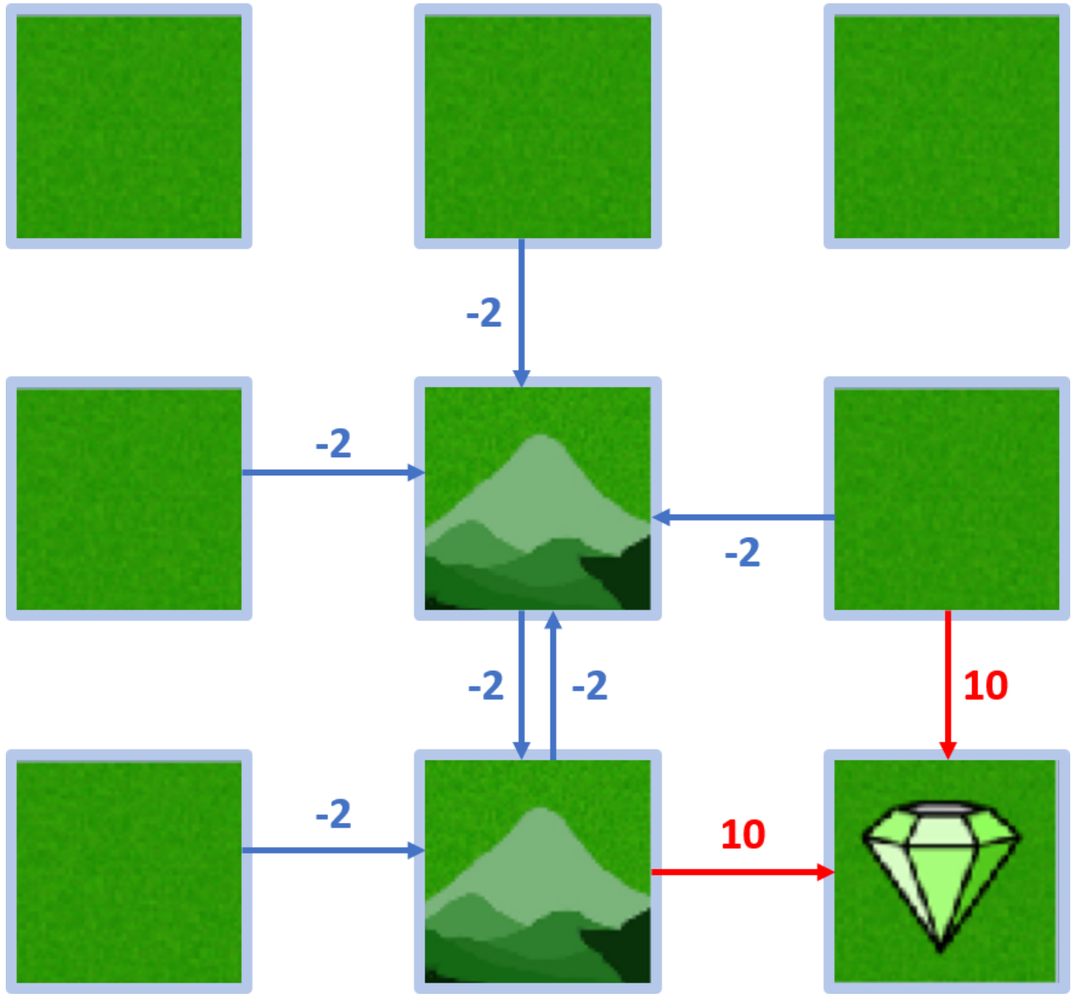
Grid world example - rewards

- Given based on states:
  - Diamond: +10



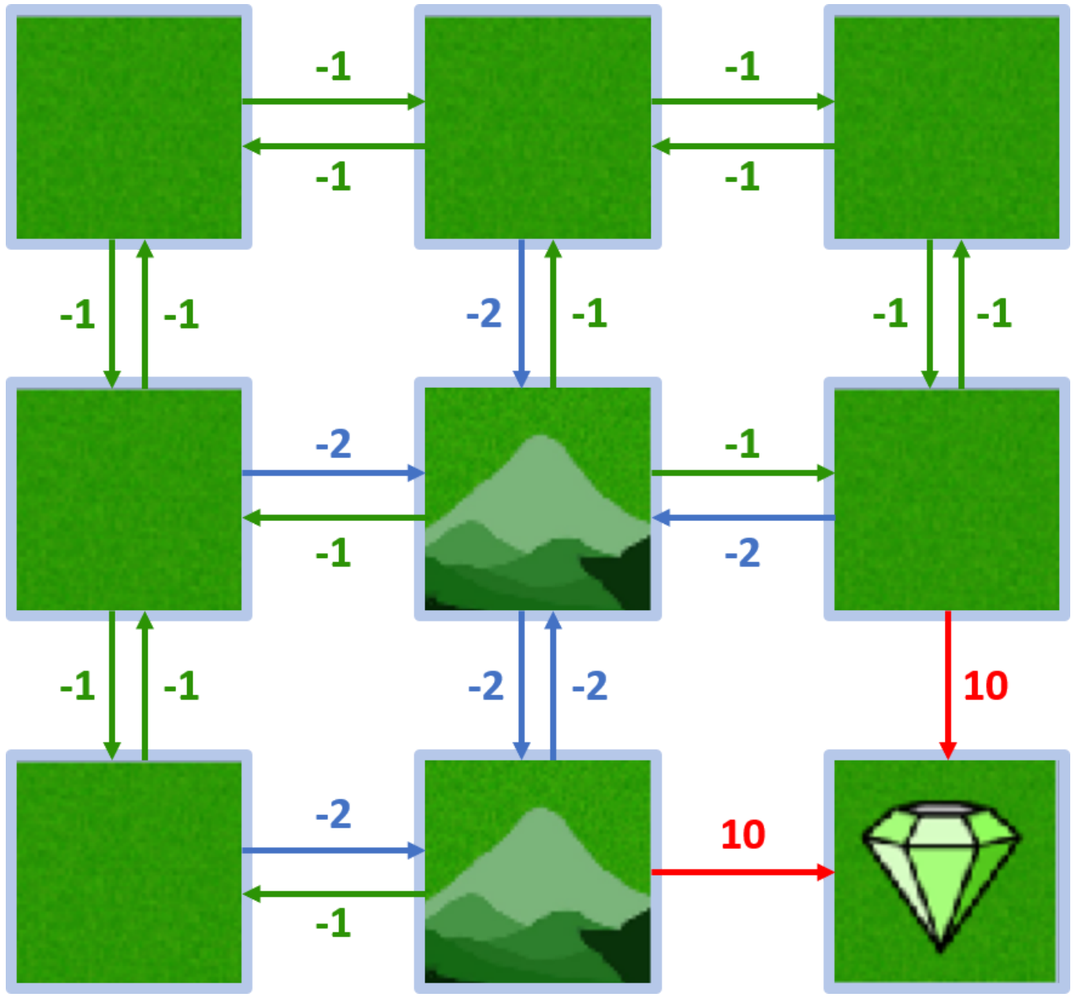
Grid world example - rewards

- Given based on states:
  - Diamond: +10
  - Mountain: -2



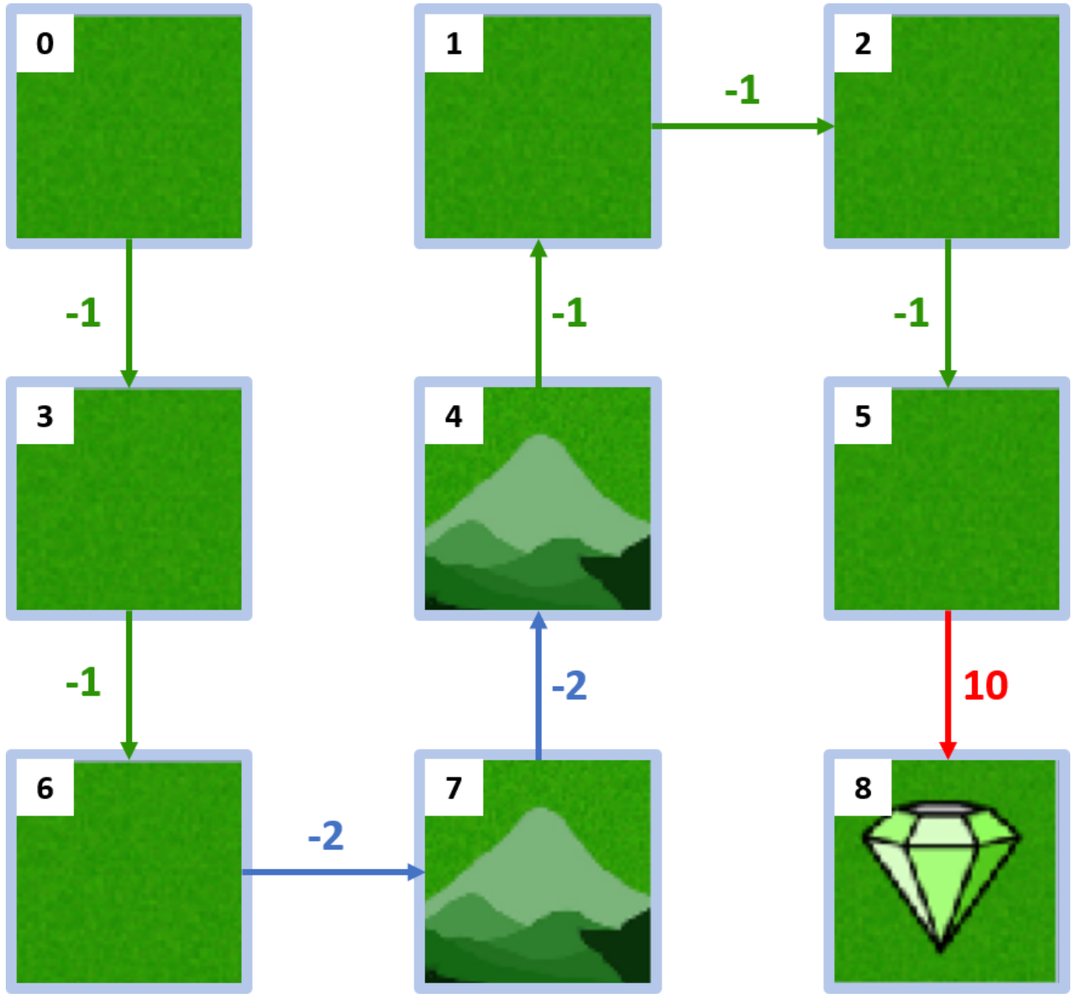
Grid world example - rewards

- Given based on states:
  - Diamond: +10
  - Mountain: -2
  - Other states: -1



Grid world example: policy

```
# 0: left, 1: down, 2: right, 3: up
policy = {
    0:1, 1:2, 2:1,
    3:1, 4:3, 5:1,
    6:2, 7:3
}

state, info = env.reset()
terminated = False
while not terminated:
    action = policy[state]
    state, reward, terminated, _, _ = env.step(action)
```



State-value functions

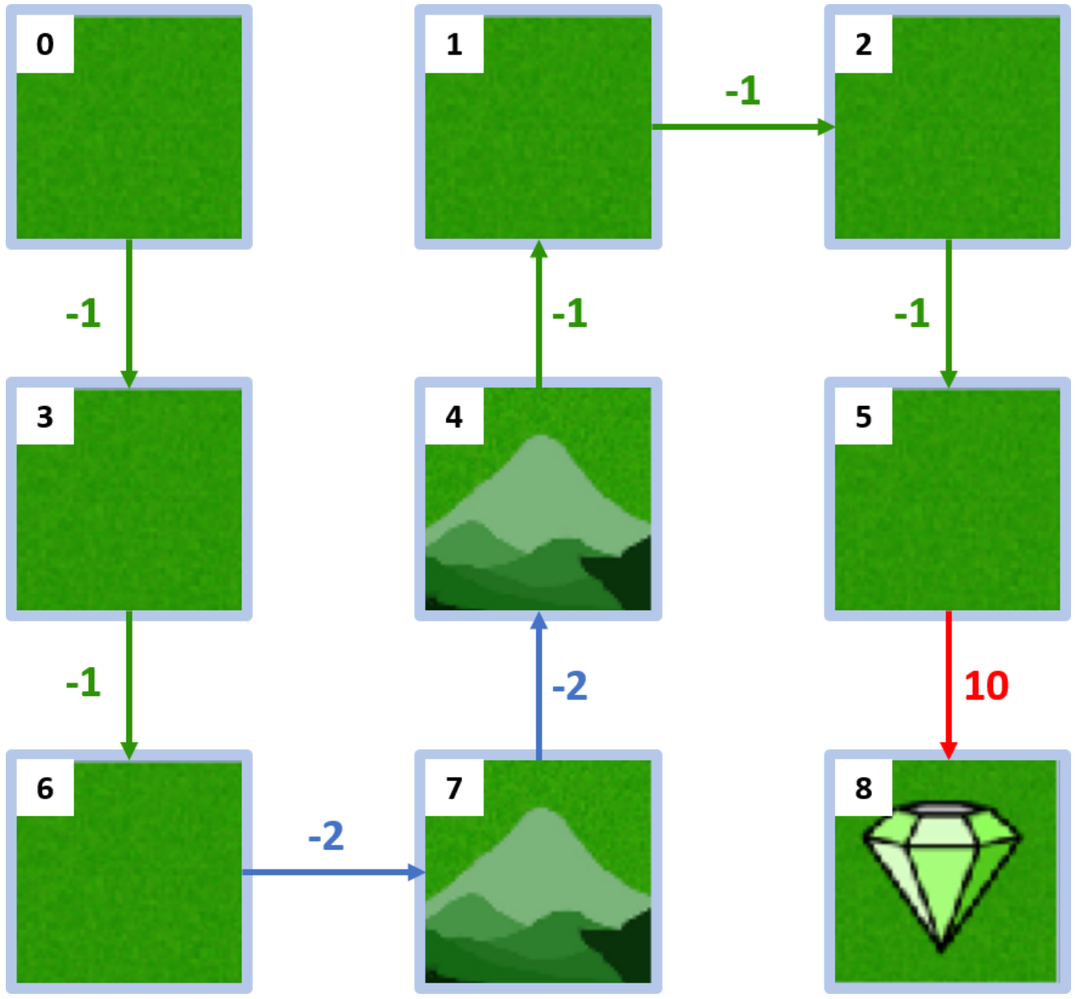
- Estimate the state's worth
- Expected return starting from state, following policy

$$V(s) = r_{s+1} + \gamma r_{s+2} + \gamma^2 r_{s+3} + \cdots + \gamma^{n-1} r_{s+n}$$

$State - value\ of\ s$

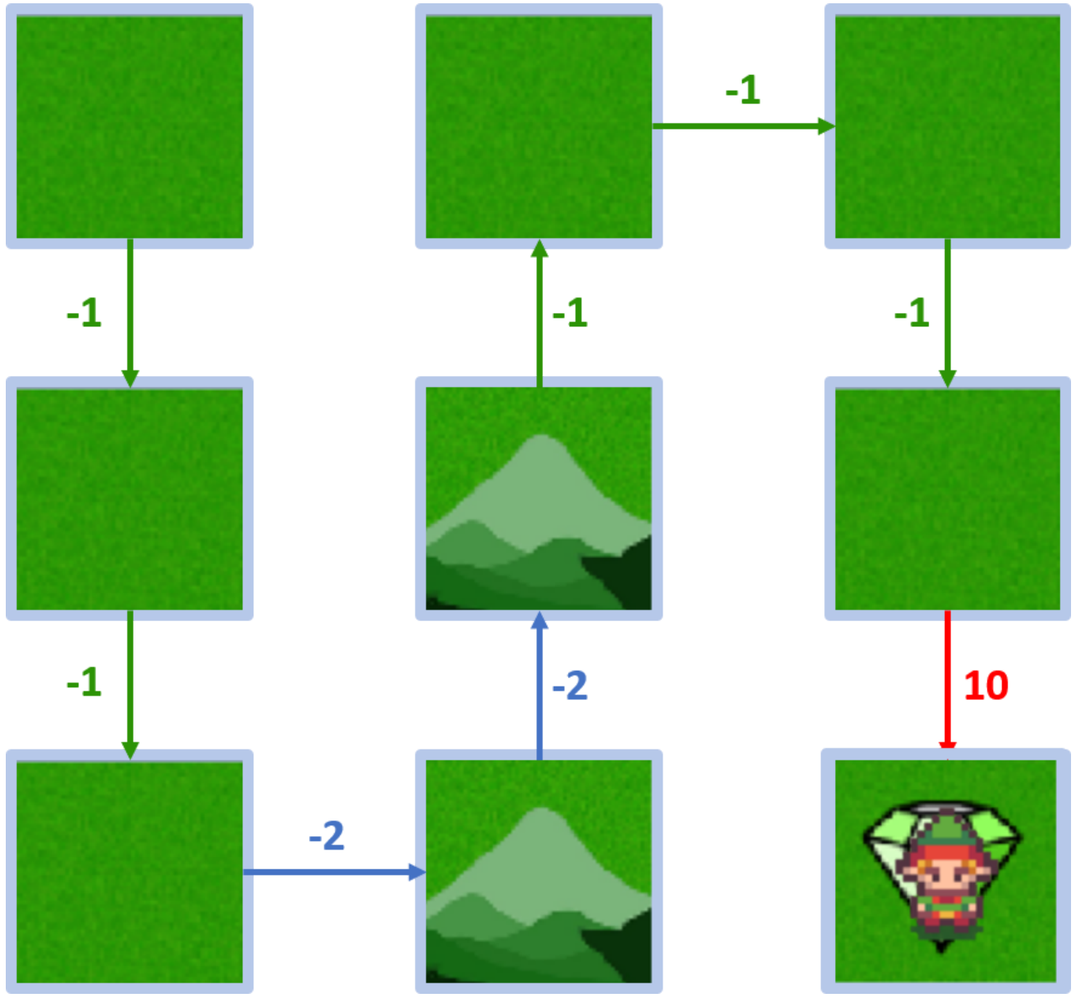
Sum of discounted rewards collected by
- starting in state s
- and following the policy

Grid world example: State-values

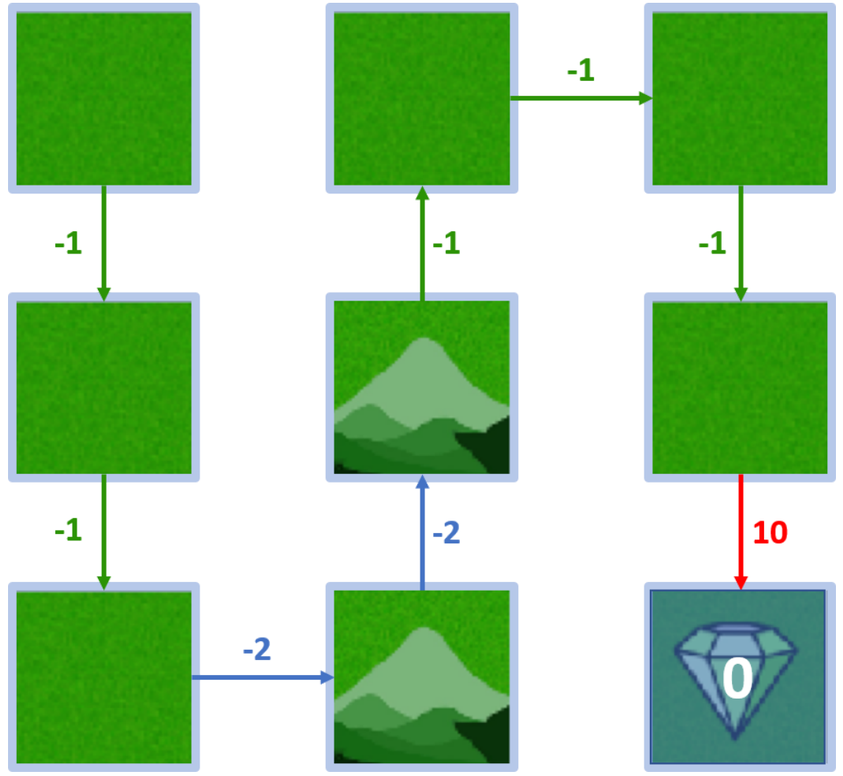


- Nine states → nine state-values
- Discount factor: $\gamma = 1$

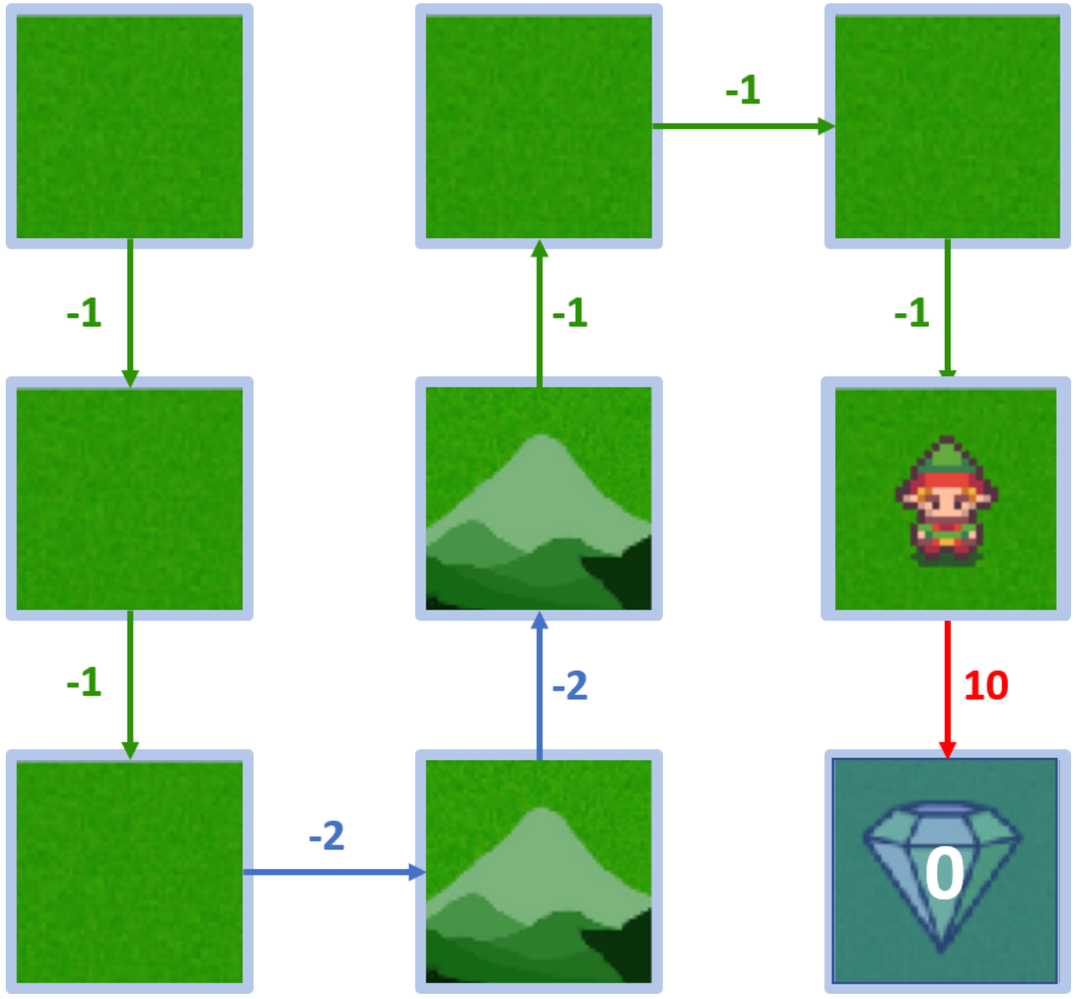
Value of goal state



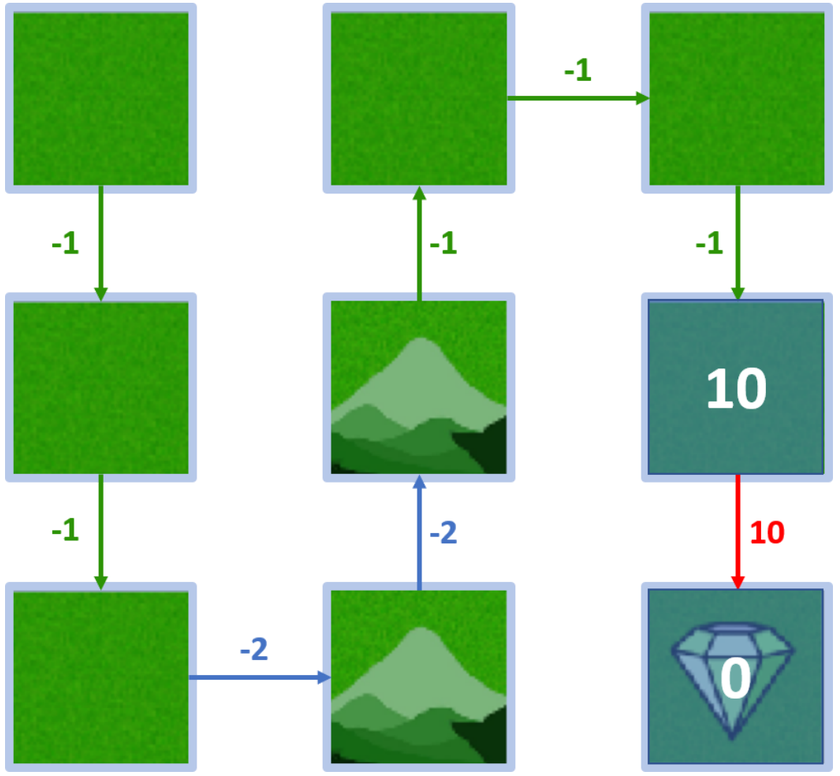
- Starting in goal state, agent doesn't move
- $V(goal\ state) = 0$



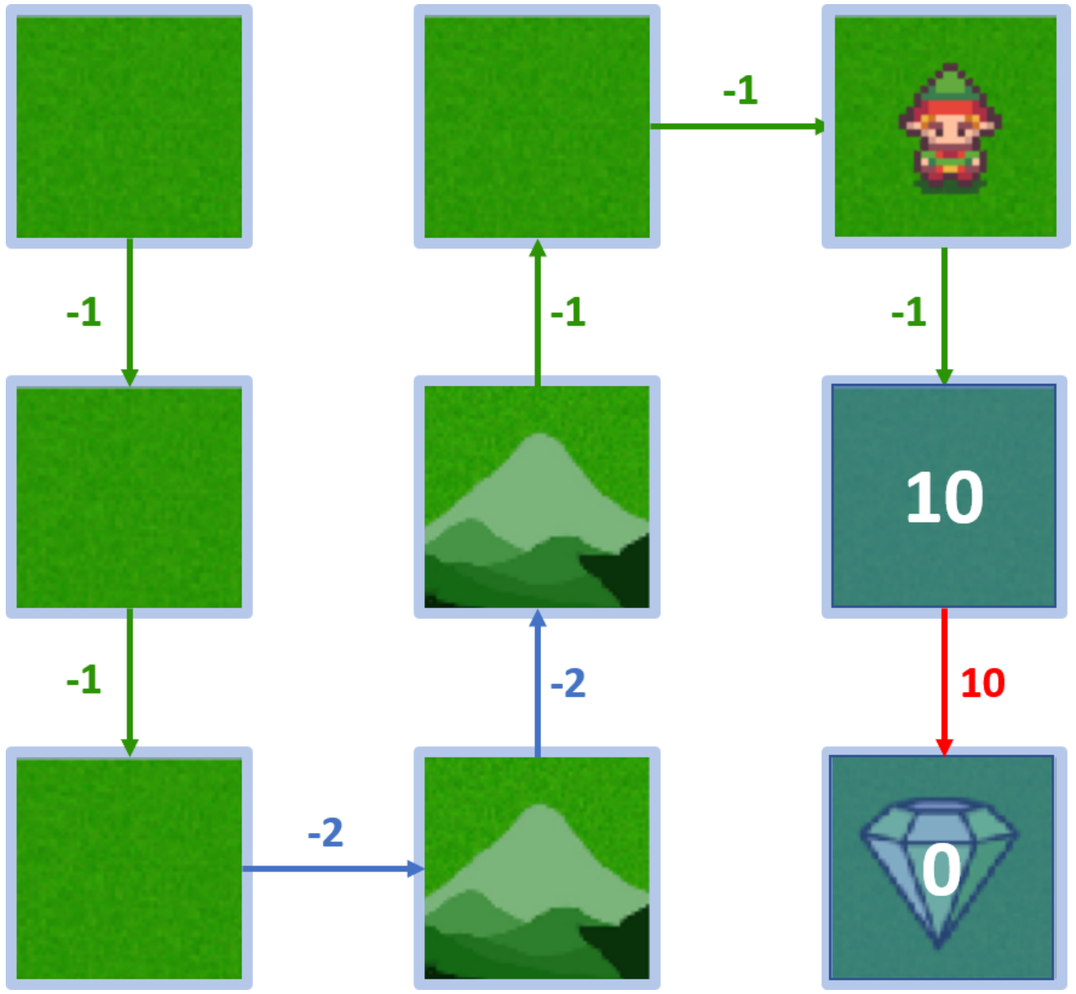
Value of state 5



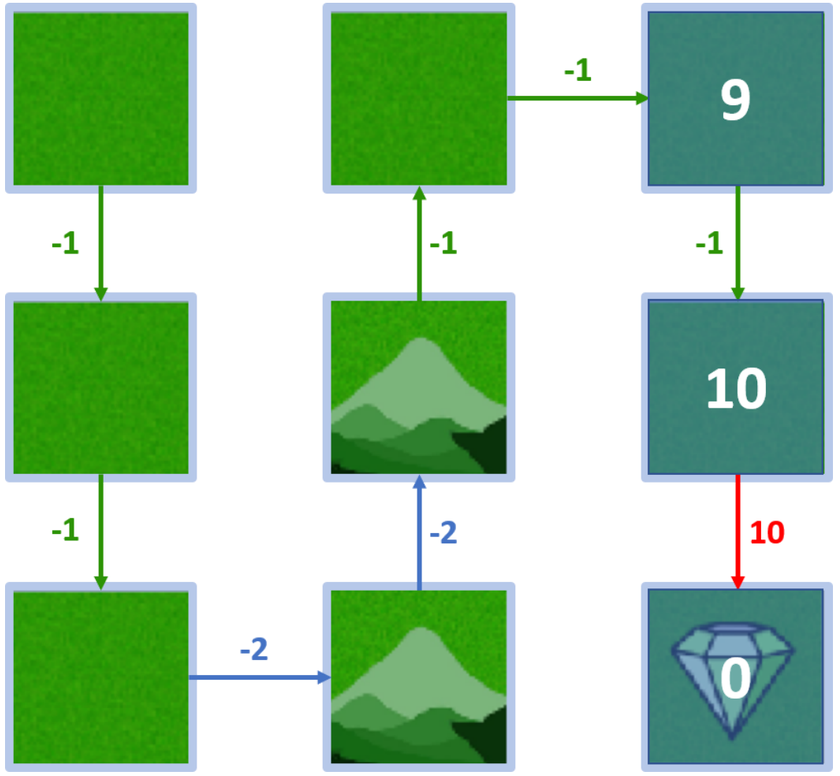
- Starting in 5, agent moves to goal
- $V(5) = 10$



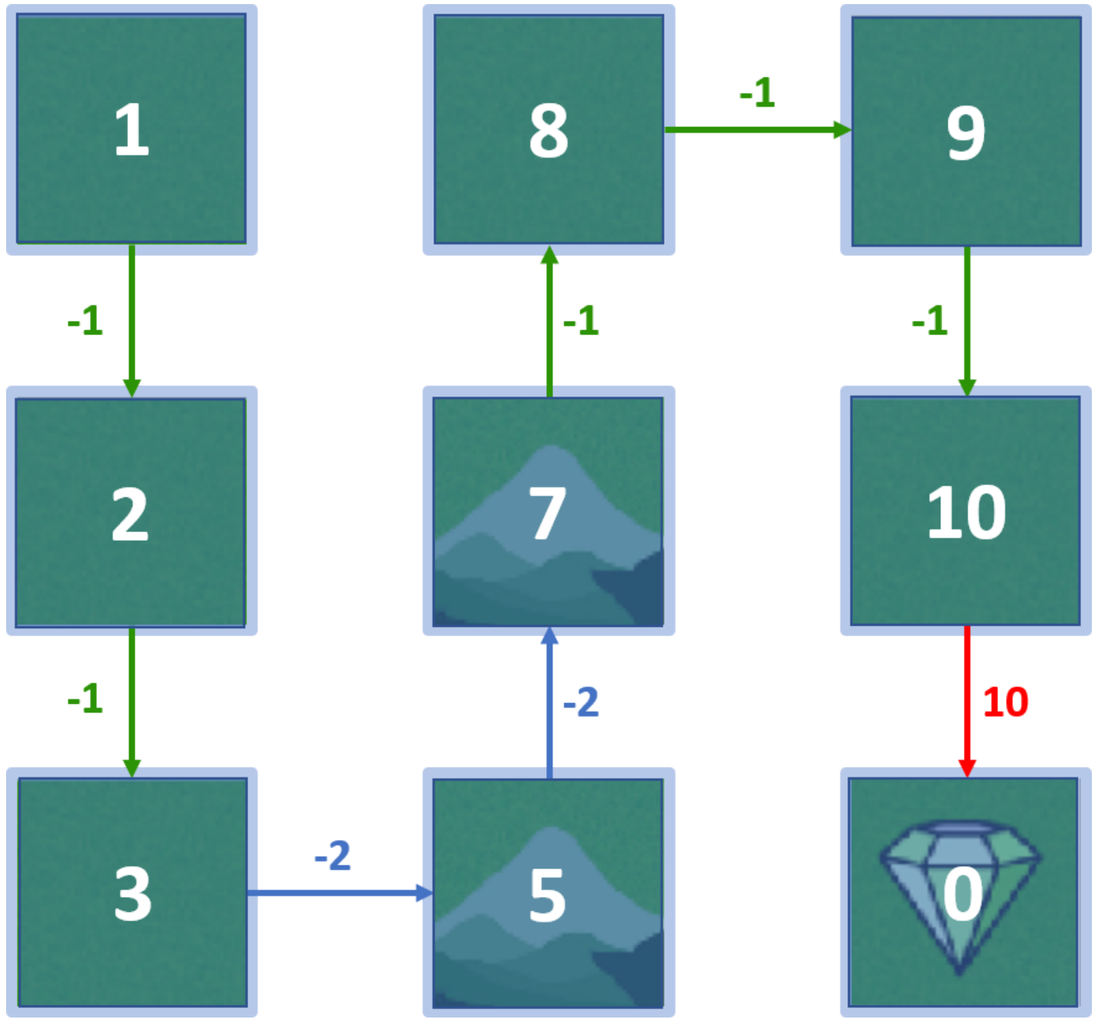
Value of state 2



- Starting in 2, rewards: $-1, 10$
- $V(2) = (1 \times -1) + (1 \times 10) = 9$



All state values



Bellman equation

- Recursive formula
- Computes state-values

$$V(s) = r_{s+1} + \gamma V(s+1)$$

$V(s)$: $State - value\ of\ s$

$r_{s+1} + \gamma V(s+1)$: Sum of:
- immediate reward
- discounted value of the next state

Computing state-values

```
def compute_state_value(state):
    if state == terminal_state:
        return 0

    action = policy[state]
    _, next_state, reward, _ = env.unwrapped.P[state][action][0]
    return reward + gamma * compute_state_value(next_state)
```

$$V(s) = r_{s+1} + \gamma V(s+1)$$

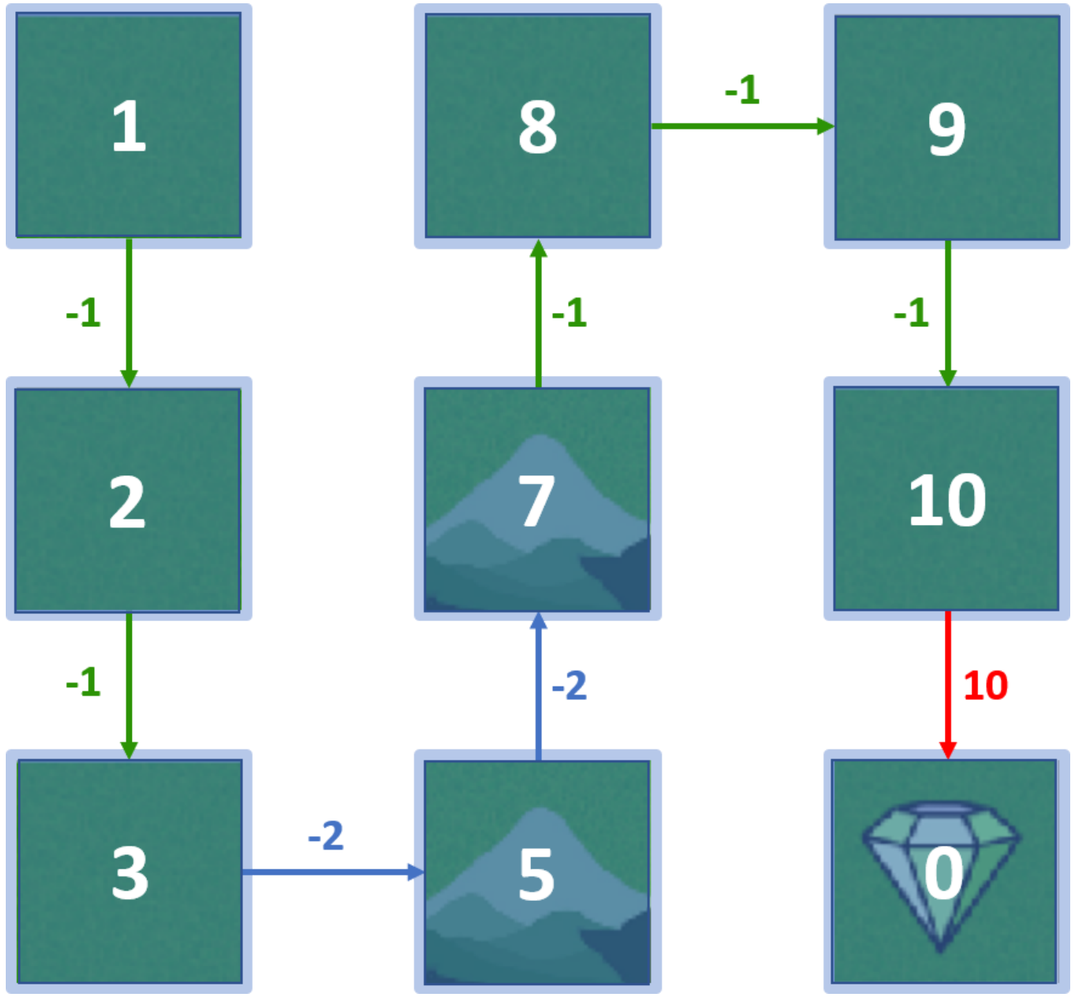
$State - value\ of\ s$

Sum of:
- immediate reward
- discounted value of the next state

Computing state-values

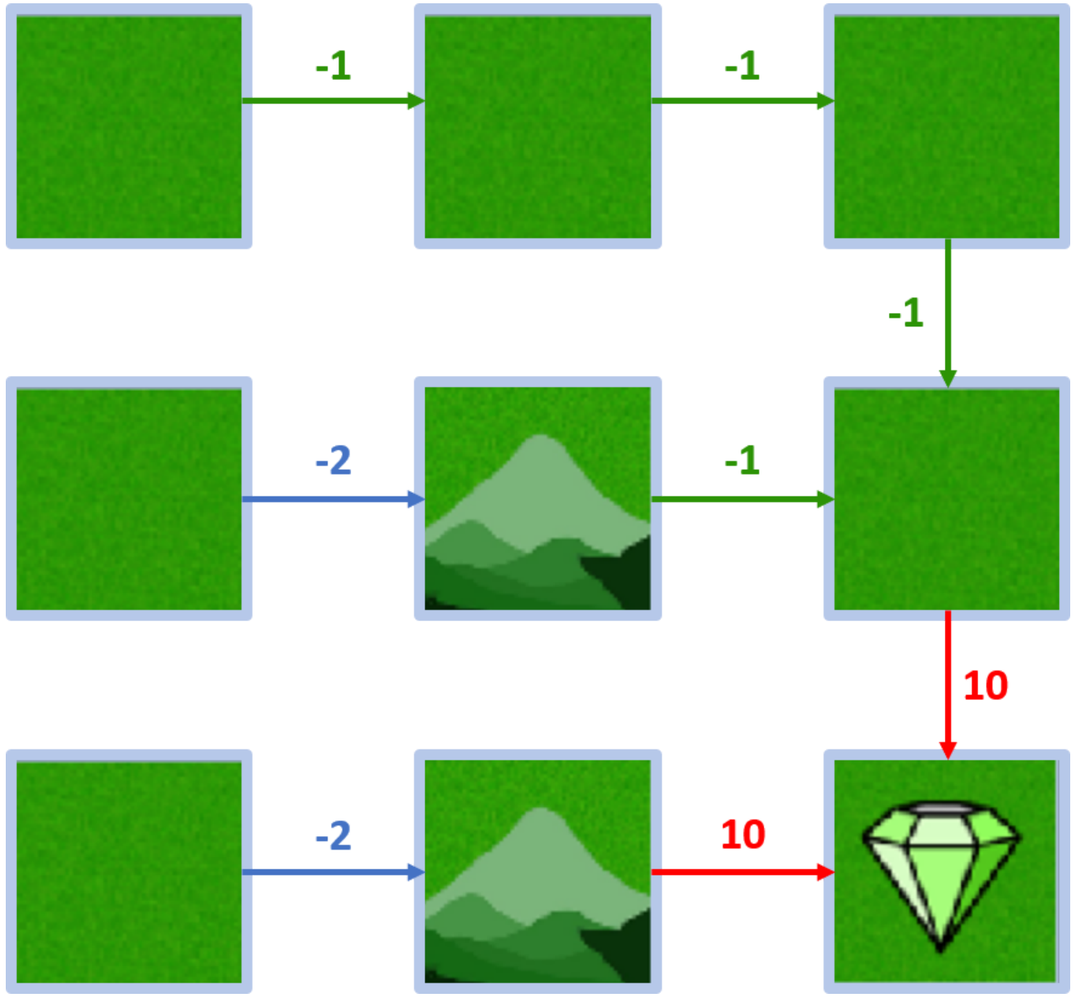
```
terminal_state = 8
gamma = 1

V = {state: compute_state_value(state)
     for state in range(num_states)}

print(V)
```

```
{0: 1, 1: 8, 2: 9,
 3: 2, 4: 7, 5: 10,
 6: 3, 7: 5, 8: 0}
```



Changing policies

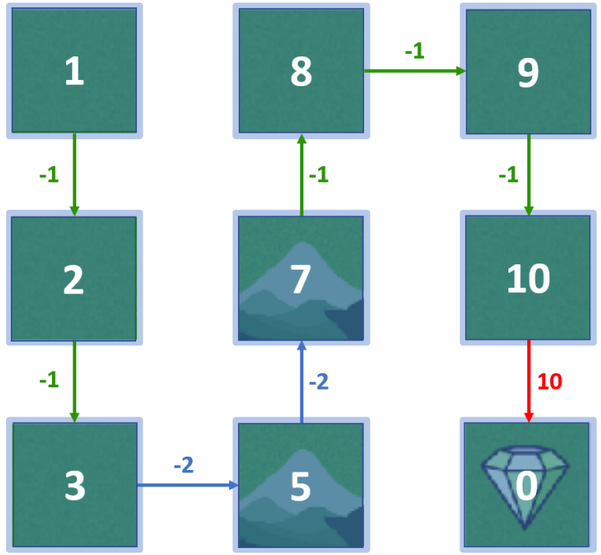
```
# 0: left, 1: down, 2: right, 3: up
policy_two = {
    0:2, 1:2, 2:1,
    3:2, 4:2, 5:1,
    6:2, 7:2
}
V_2 = {state: compute_state_value(state)
     for state in range(num_states)}
print(V_2)
```



Comparing policies

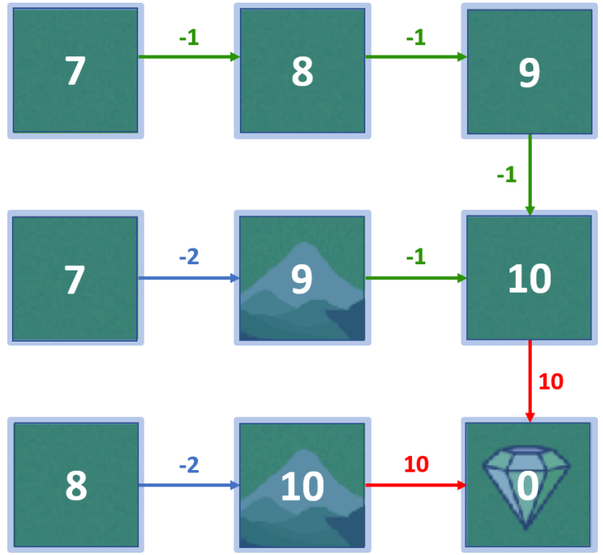
State-values for policy 1

```
{0: 1, 1: 8, 2: 9,
 3: 2, 4: 7, 5: 10,
 6: 3, 7: 5, 8: 0}
```



State-values for policy 2

```
{0: 7, 1: 8, 2: 9,
 3: 7, 4: 9, 5: 10,
 6: 8, 7: 10, 8: 0}
```



Let's practice!

REINFORCEMENT LEARNING WITH GYMNASIUM IN PYTHON

Action-value functions

REINFORCEMENT LEARNING WITH GYMNASIUM IN PYTHON

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Action-value functions (Q-values)

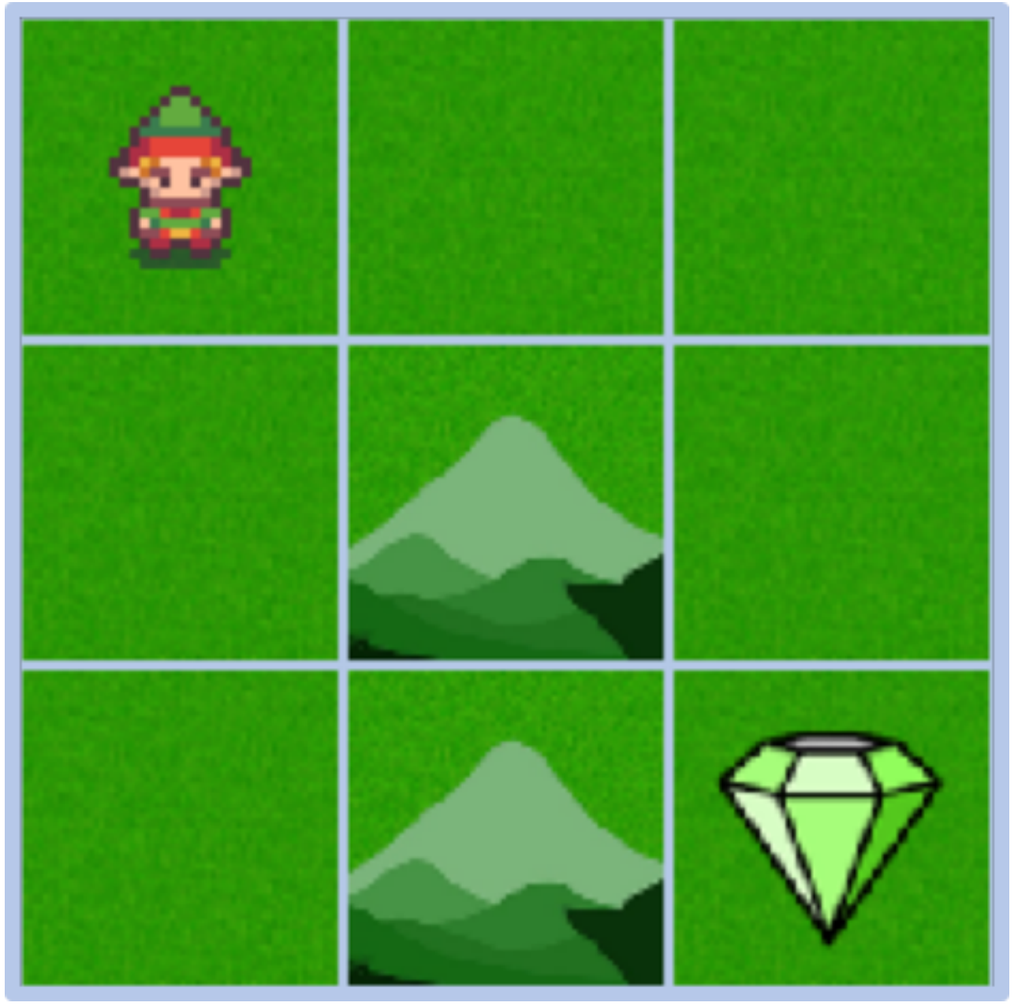
- Expected return of:
  - Starting at a state s
  - Taking action a
  - Following the policy
- Estimates desirability of actions within states

$$Q(s,a) = r_a + \gamma V(s+1)$$

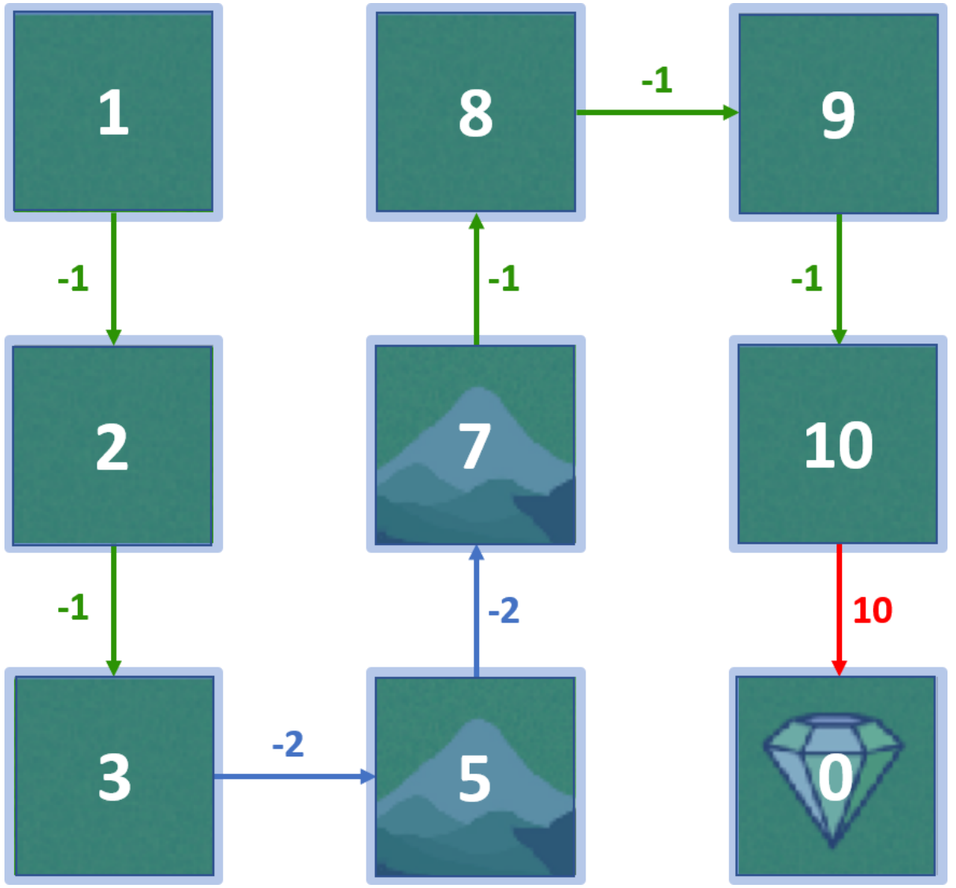
$Action - value\ of\ state\ s, action\ a$

Sum of:
– reward received after performing action a in state s
– discounted value of the next state resulting from action a

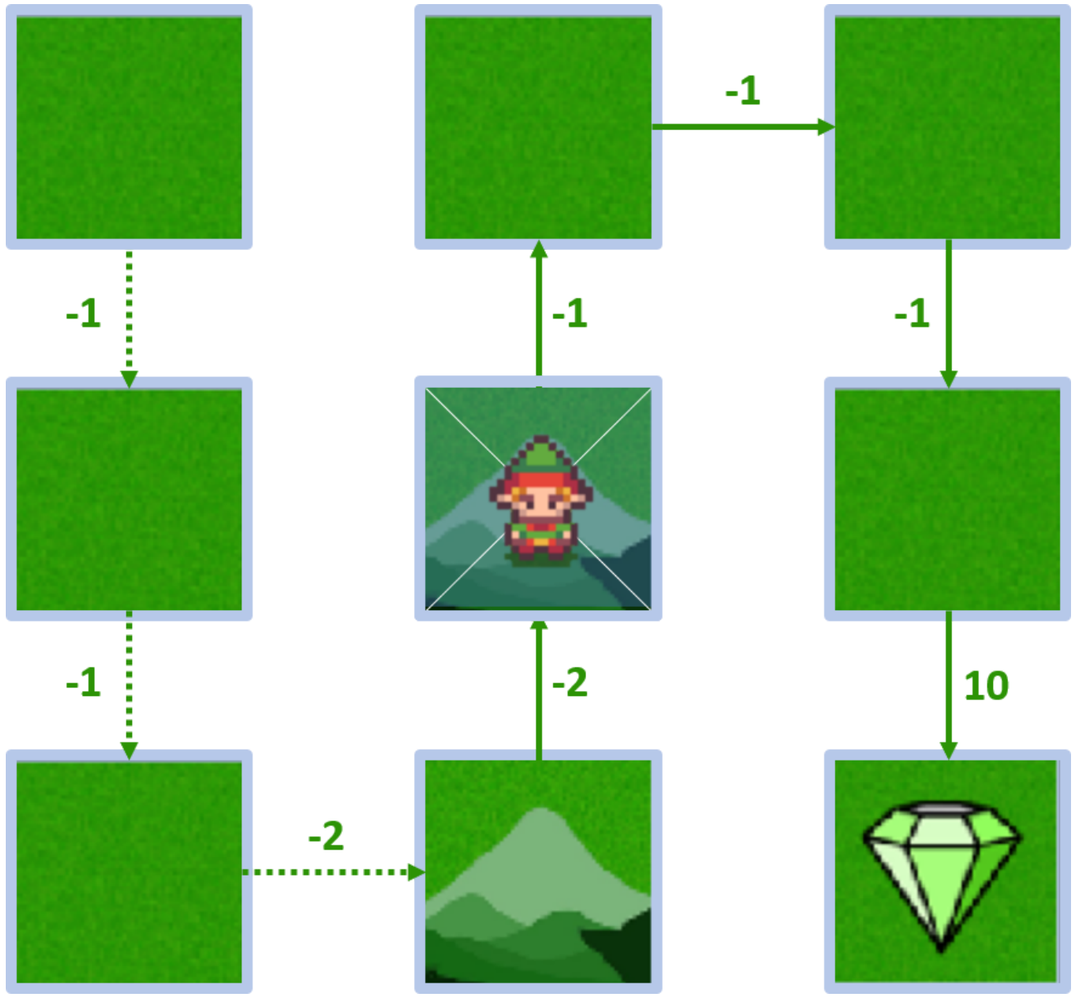
Grid world



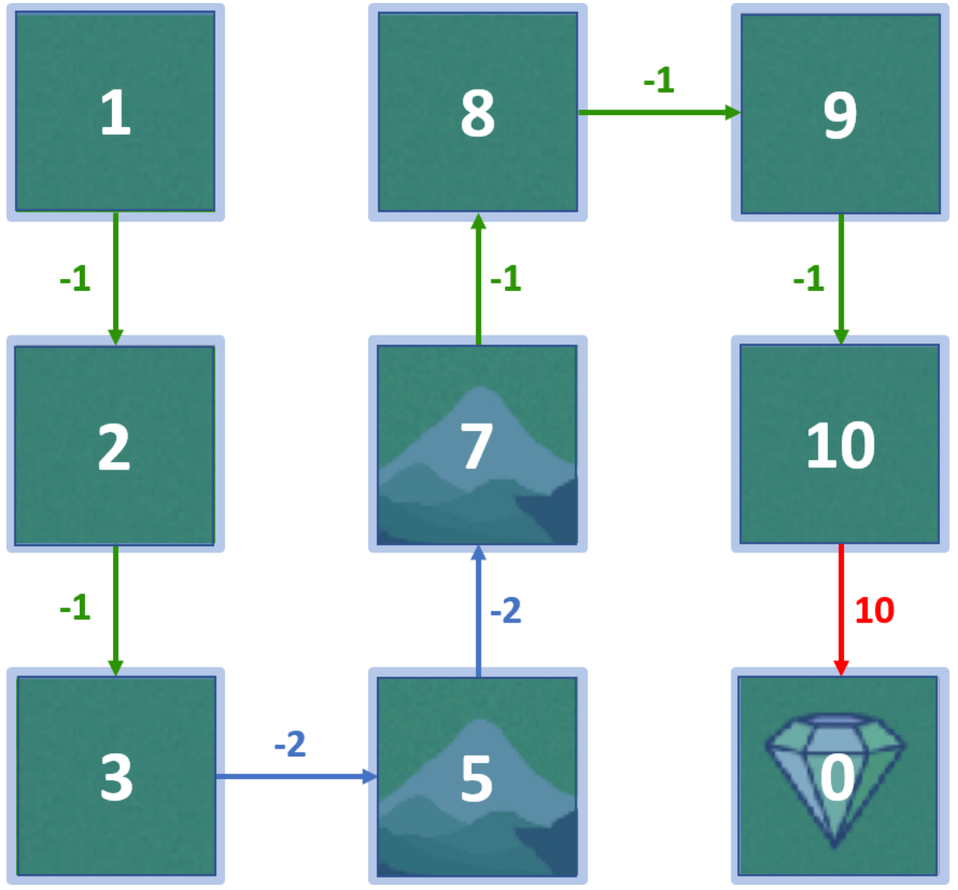
- State-values



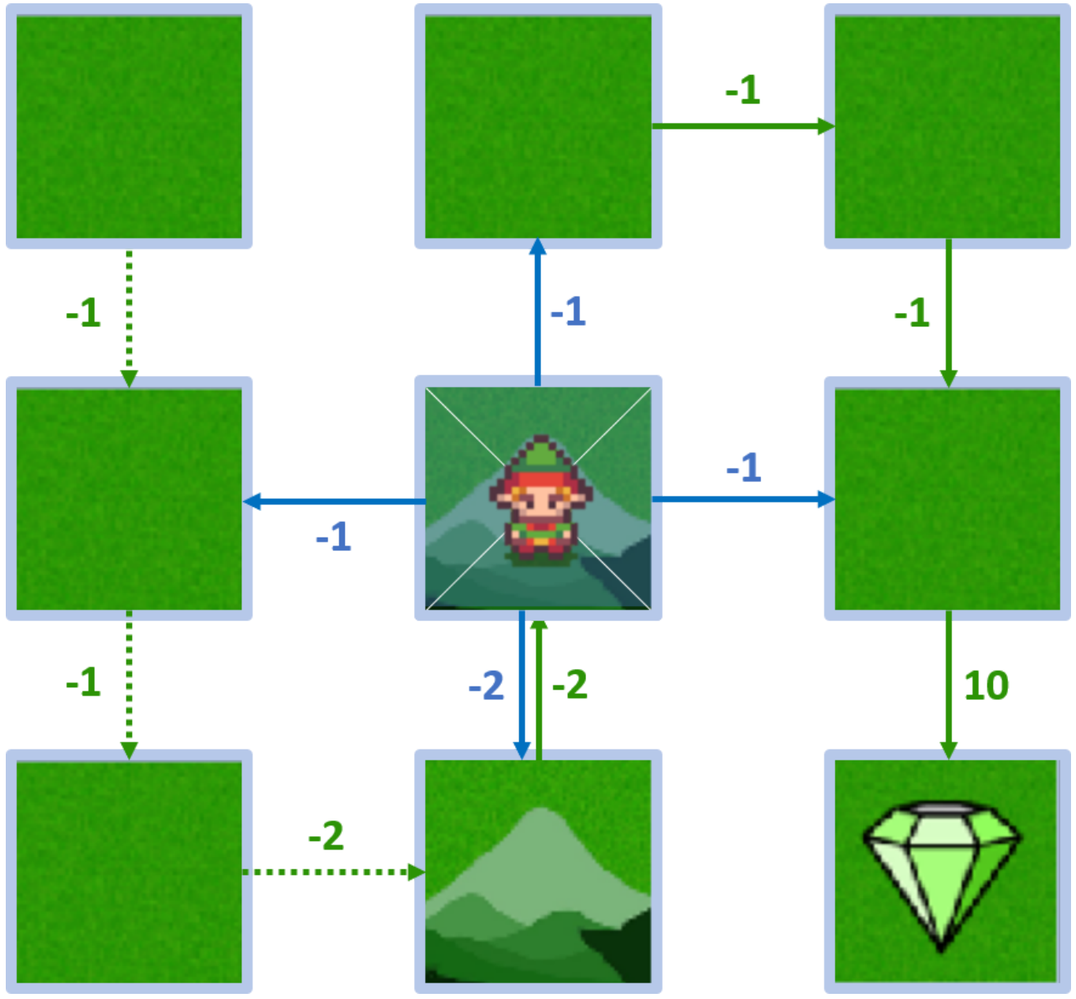
Q-values - state 4



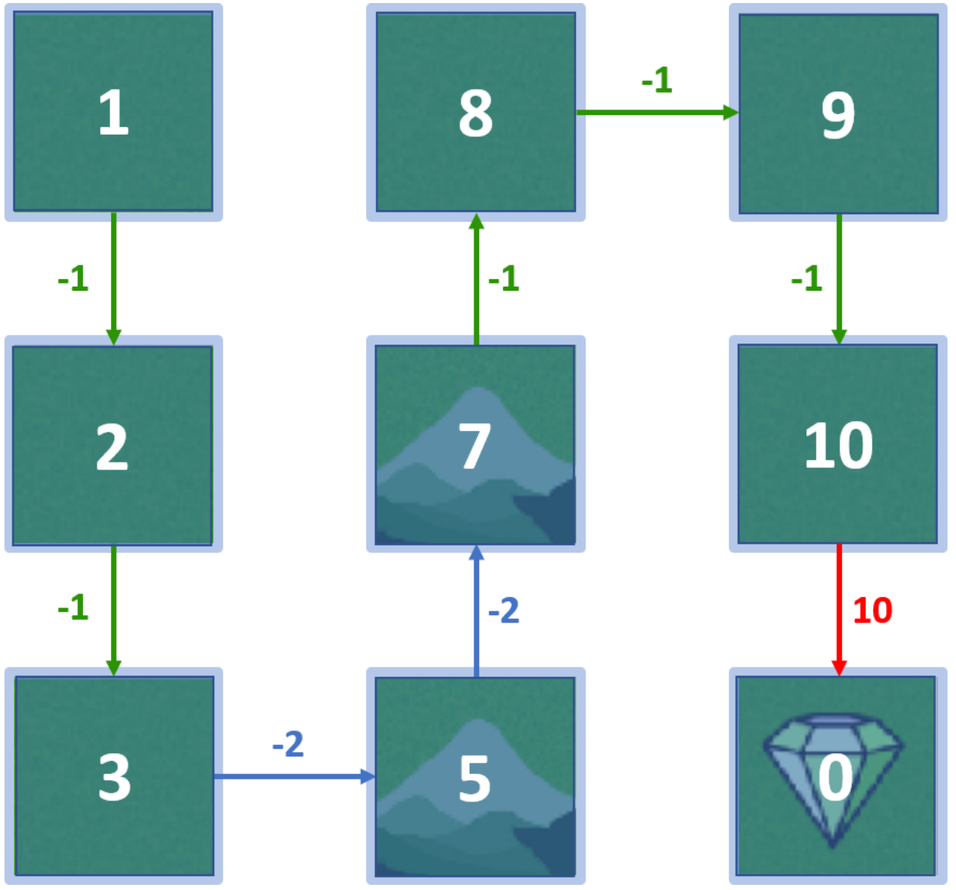
- Agent born in state 4



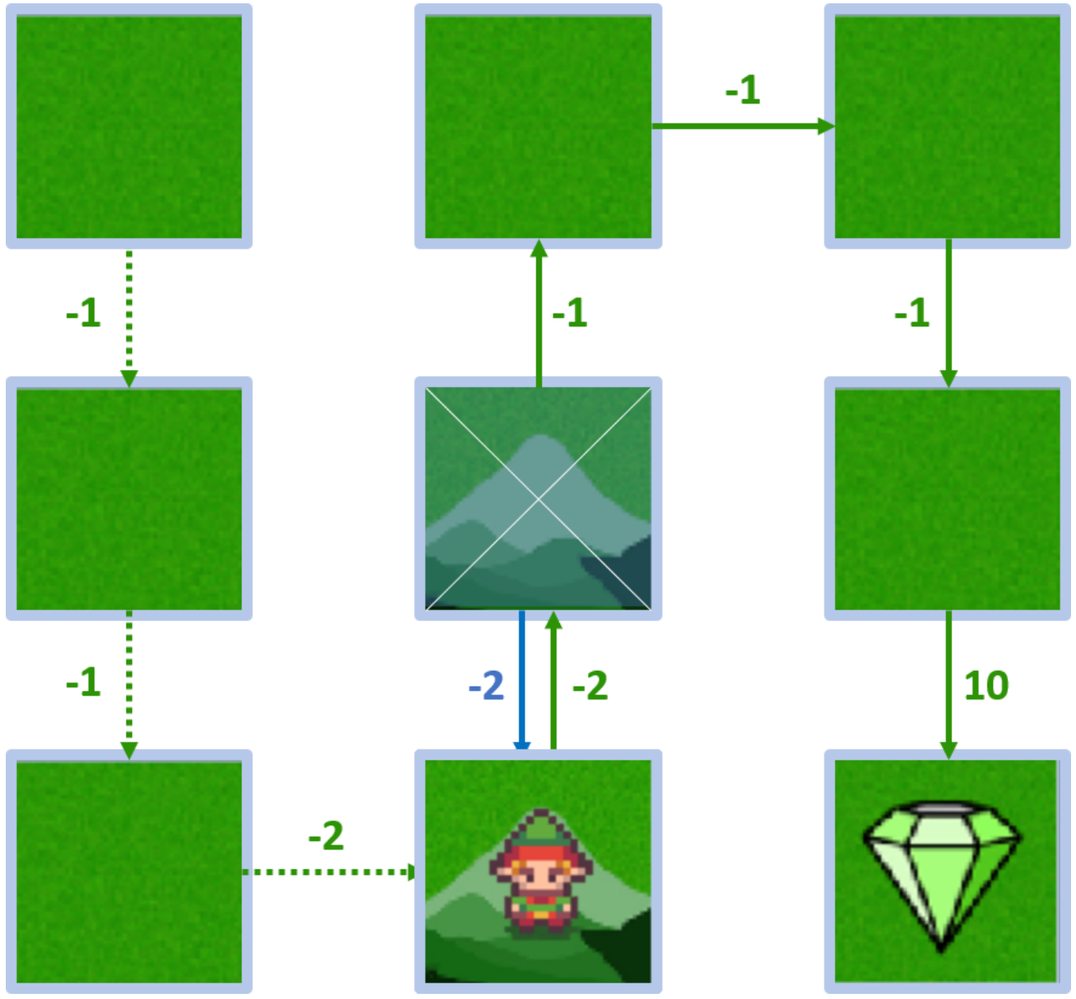
Q-values - state 4



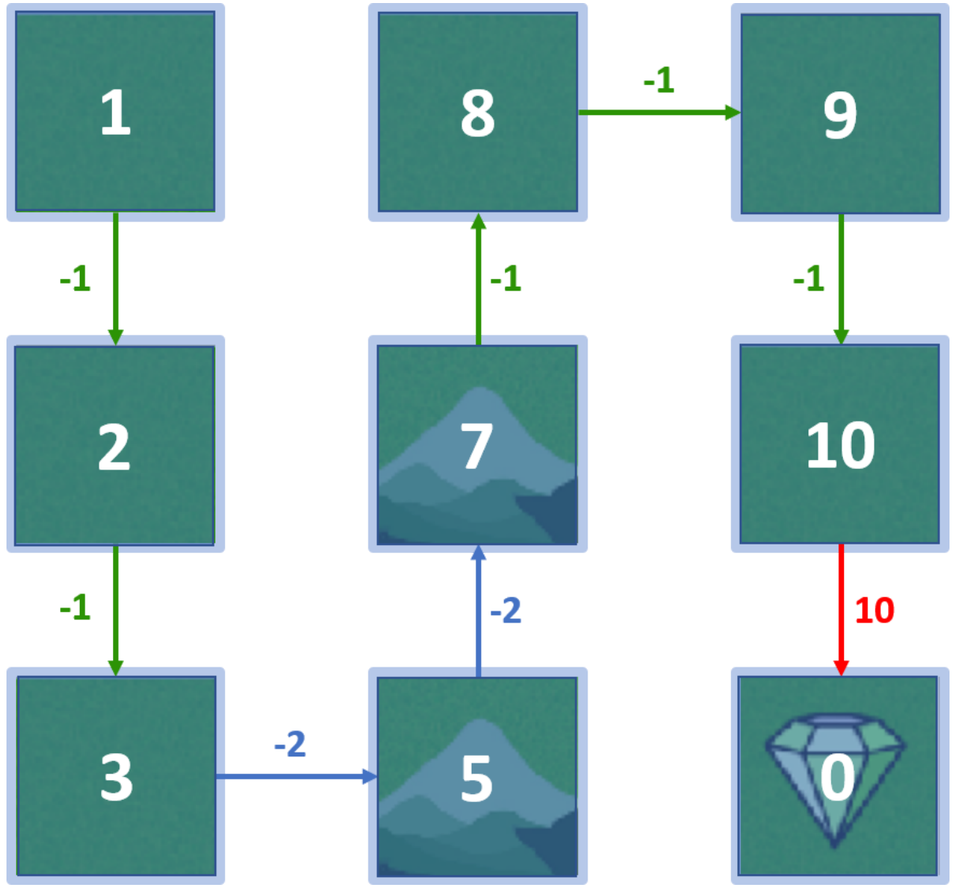
- Agent can move up, down, left, right



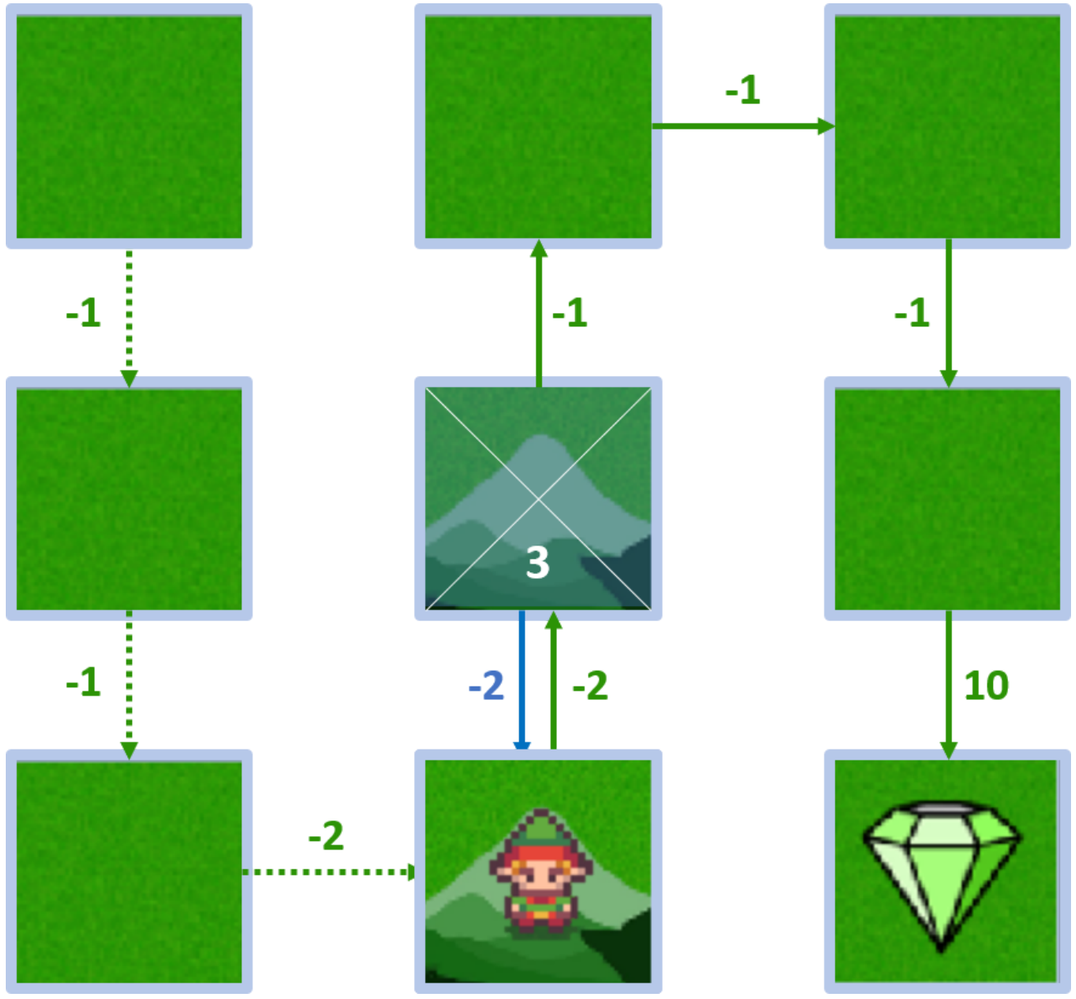
State 4 - action down



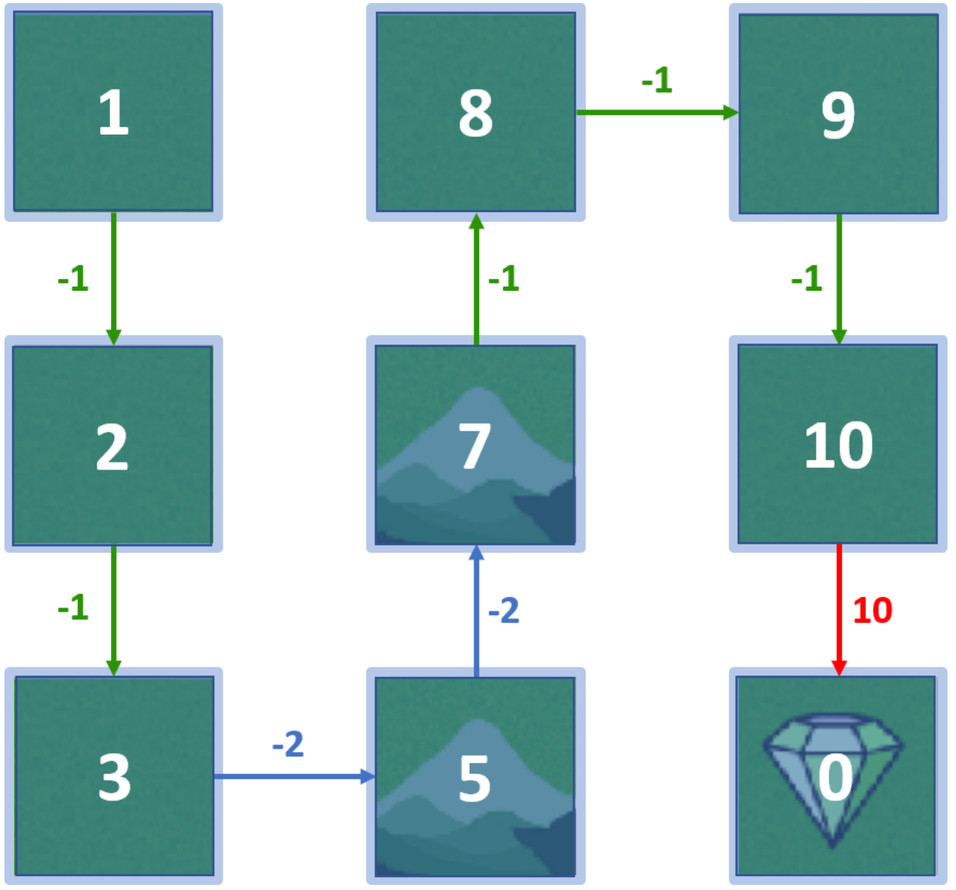
- Reward: -2, state-value: 5



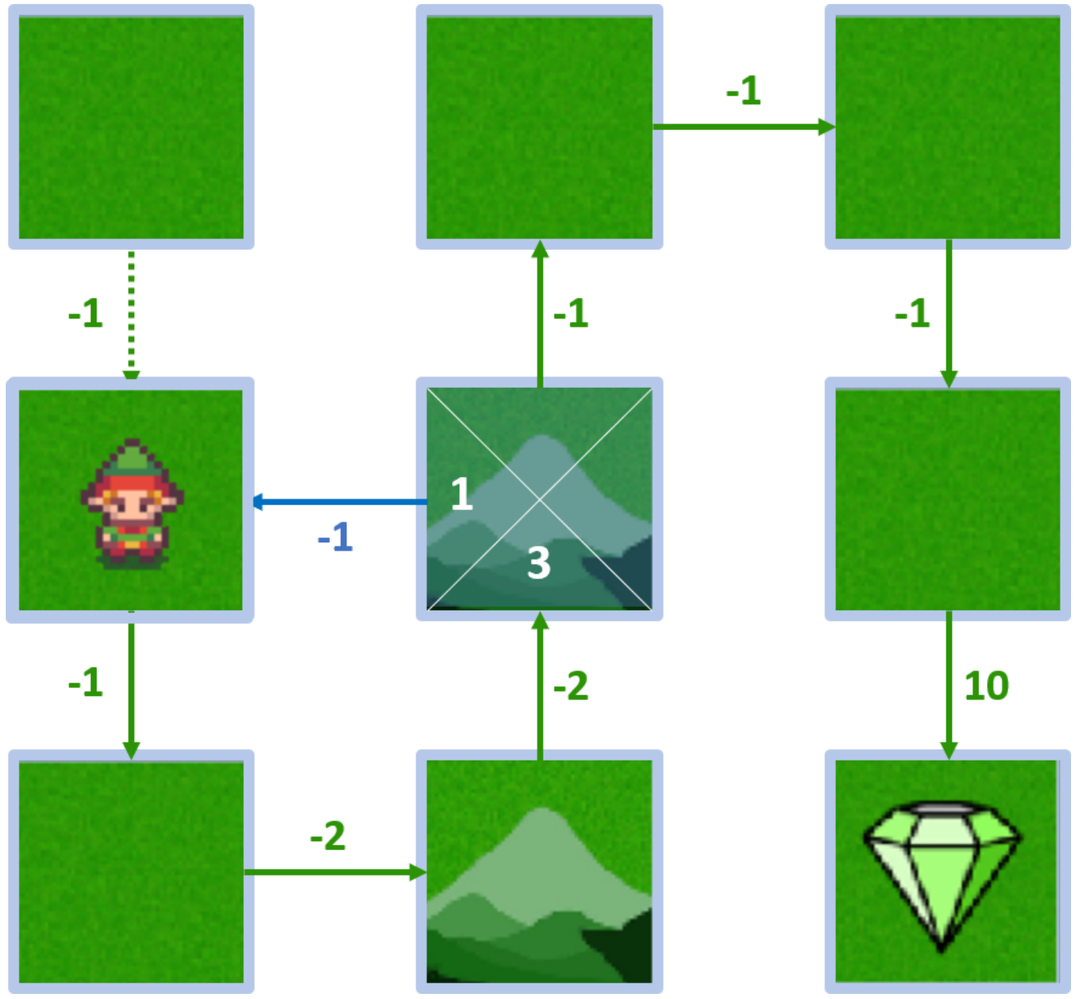
State 4 - action down



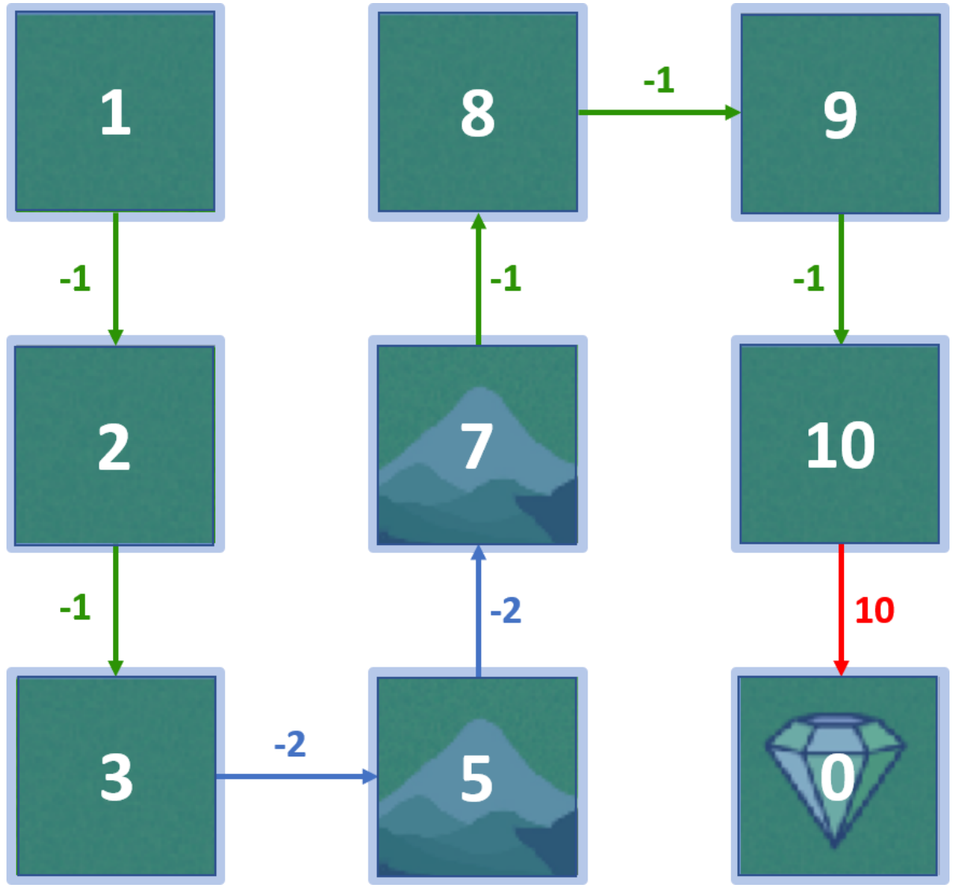
- $Q(4, \text{down}) = -2 + 1 \times 5 = 3$



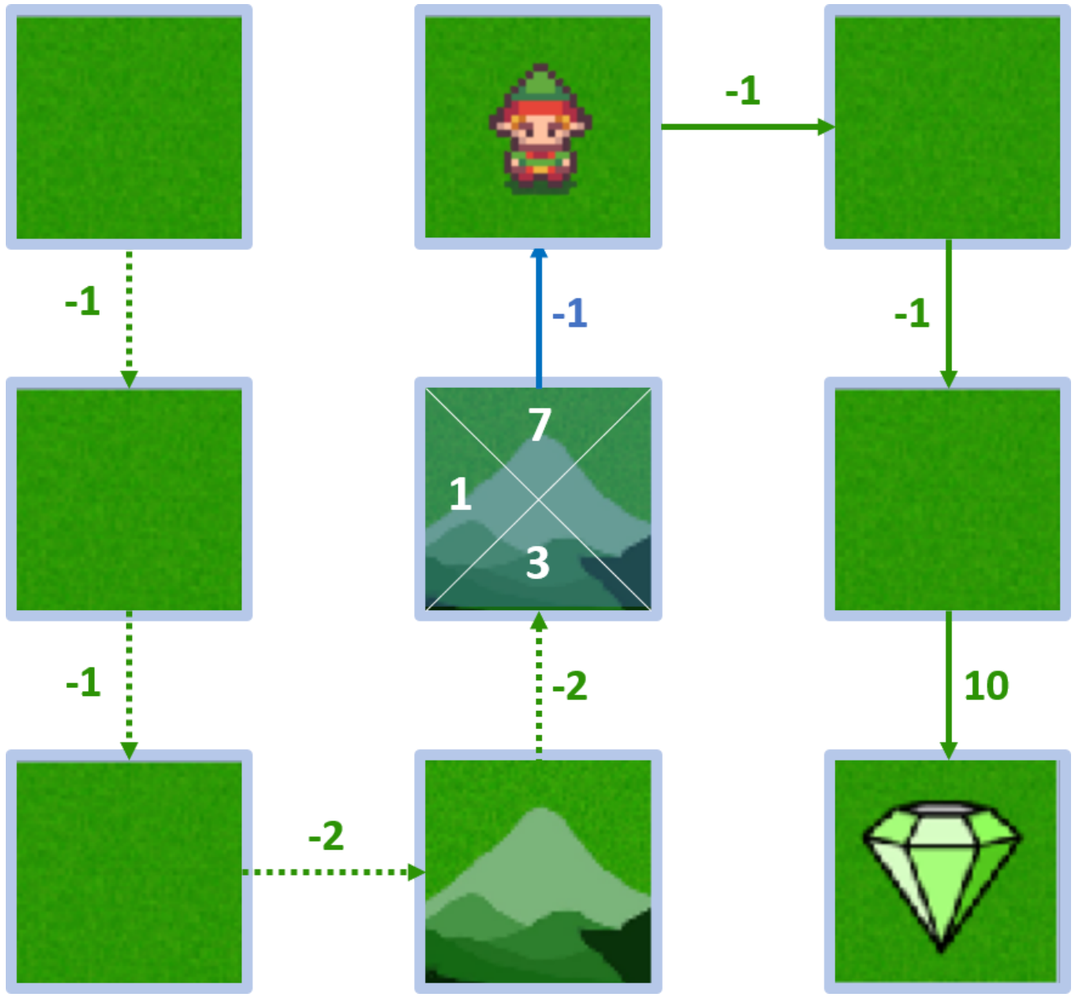
State 4 - action left



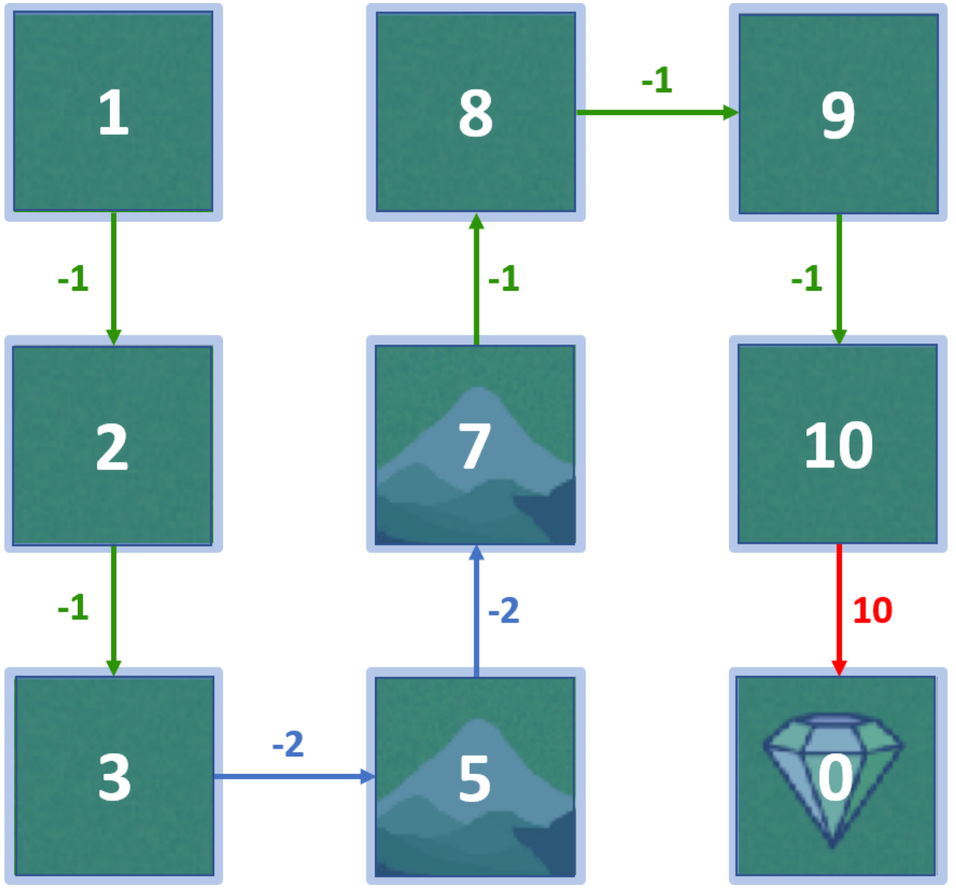
- $Q(4, \text{left}) = -1 + 1 \times 2 = 1$



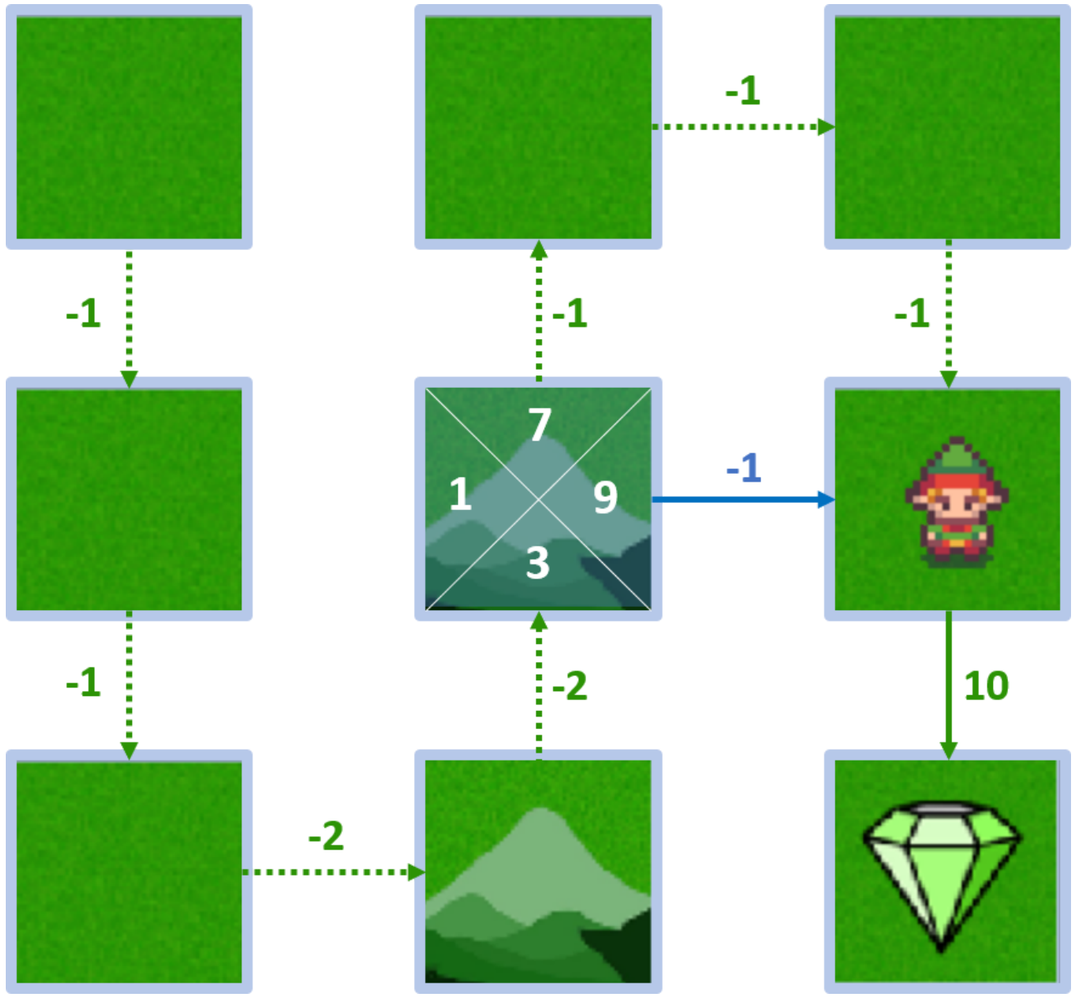
State 4 - action up



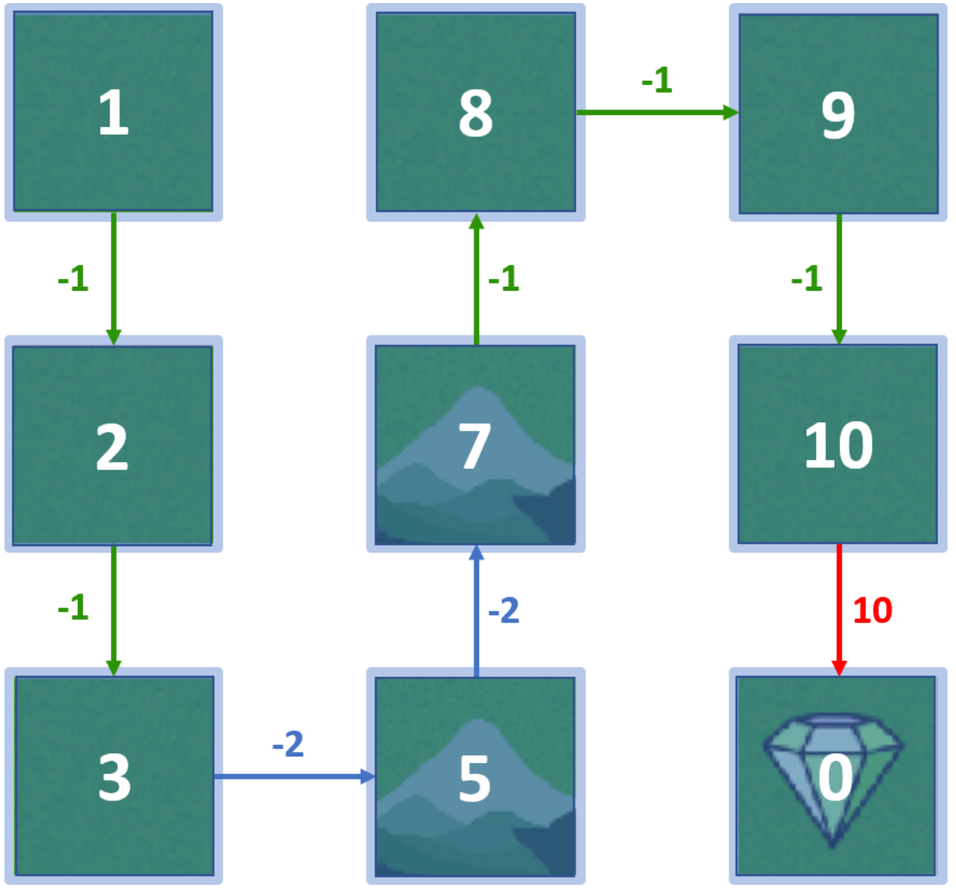
- $Q(4, \text{up}) = -1 + 1 \times 8 = 7$



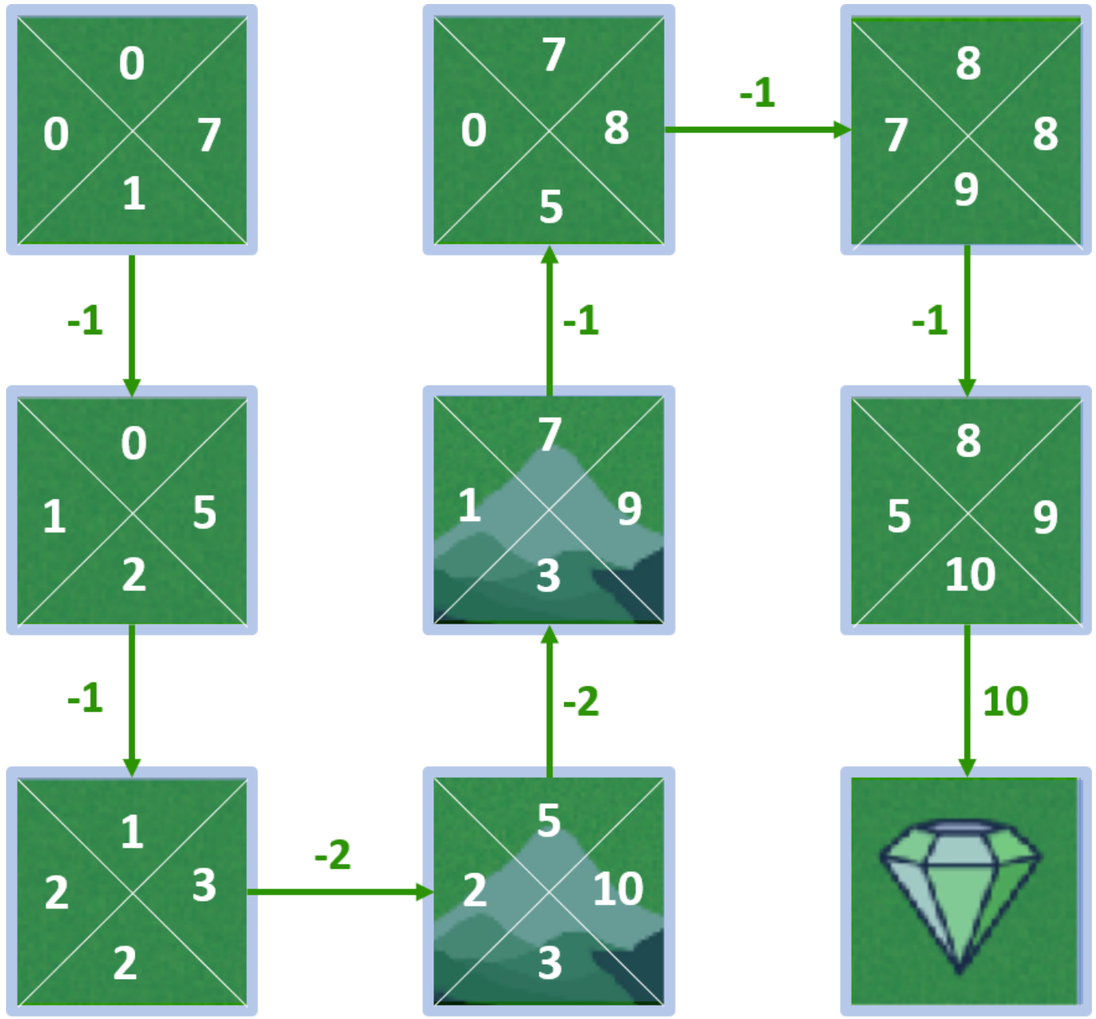
State 4 - action right



- $Q(4, \text{right}) = -1 + 1 \times 10 = 9$



All Q-values



Computing Q-values

```
def compute_q_value(state, action):
    if state == terminal_state:
        return None
    _, next_state, reward, _ = env.unwrapped.P[state][action][0]
    return reward + gamma * compute_state_value(next_state)
```

$$Q(s,a) = r_a + \gamma V(s+1)$$

$Action - value\ of\ state\ s, action\ a$

Sum of:

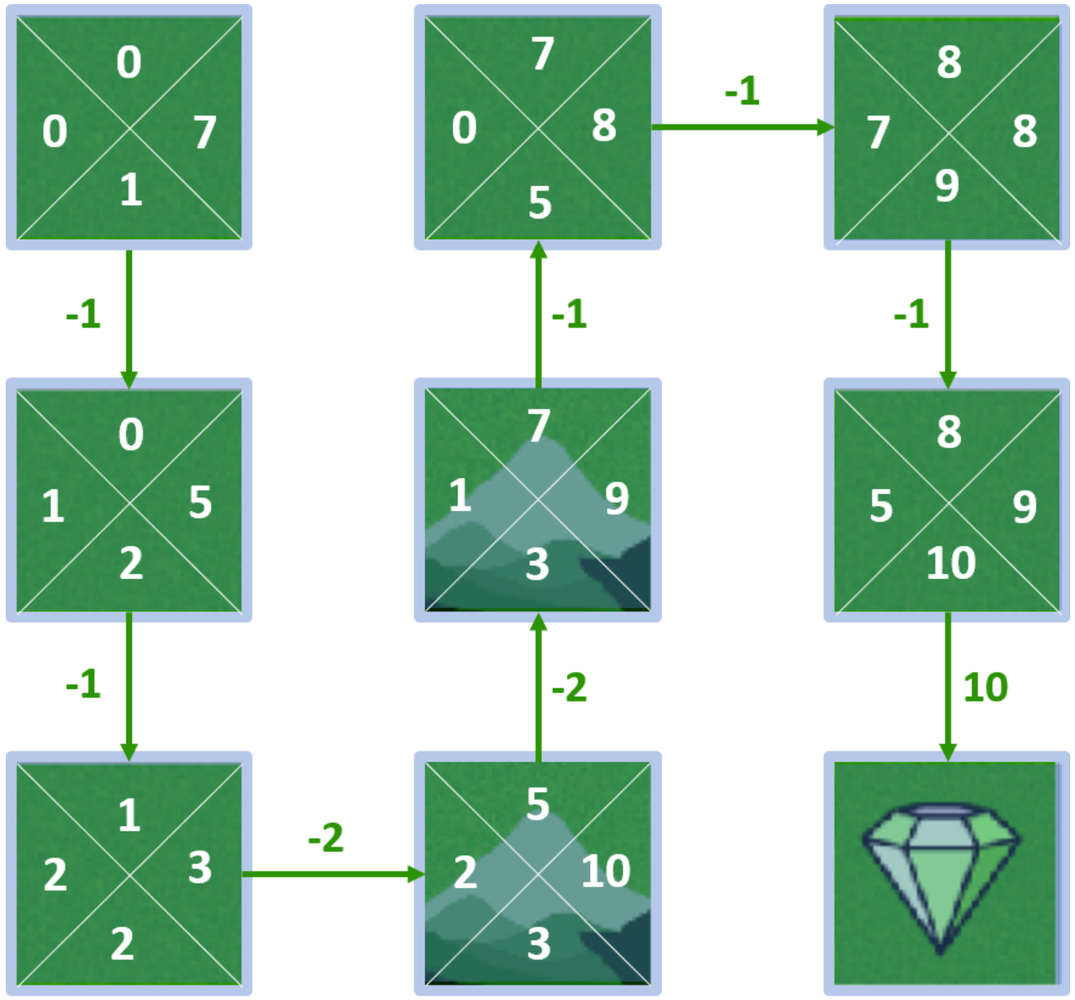
- reward received after performing action a in state s
- discounted value of the next state resulting from action a

Computing Q-values

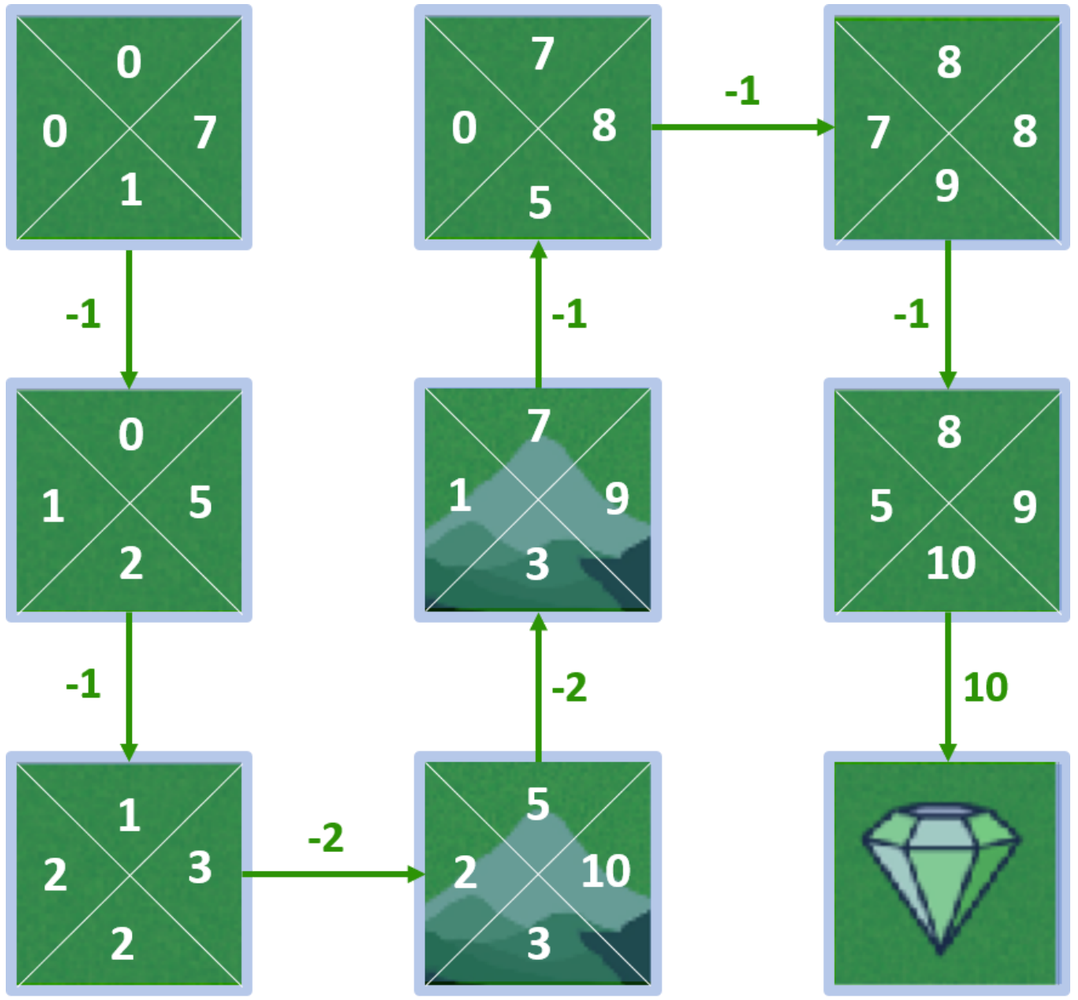
```
Q = {(state, action): compute_q_value(state, action)
     for state in range(num_states)
     for action in range(num_actions)}

print(Q)
```

Computing Q-values

```
{(0, 0): 0, (0, 1): 1, (0, 2): 7, (0, 3): 0,
 (1, 0): 0, (1, 1): 5, (1, 2): 8, (1, 3): 7,
 (2, 0): 7, (2, 1): 9, (2, 2): 8, (2, 3): 8,
 (3, 0): 1, (3, 1): 2, (3, 2): 5, (3, 3): 0,
 (4, 0): 1, (4, 1): 3, (4, 2): 9, (4, 3): 7,
 (5, 0): 5, (5, 1): 10, (5, 2): 9, (5, 3): 8,
 (6, 0): 2, (6, 1): 2, (6, 2): 3, (6, 3): 1,
 (7, 0): 2, (7, 1): 3, (7, 2): 10, (7, 3): 5,
 (8, 0): None, (8, 1): None, (8, 2): None, (8, 3): None}
```

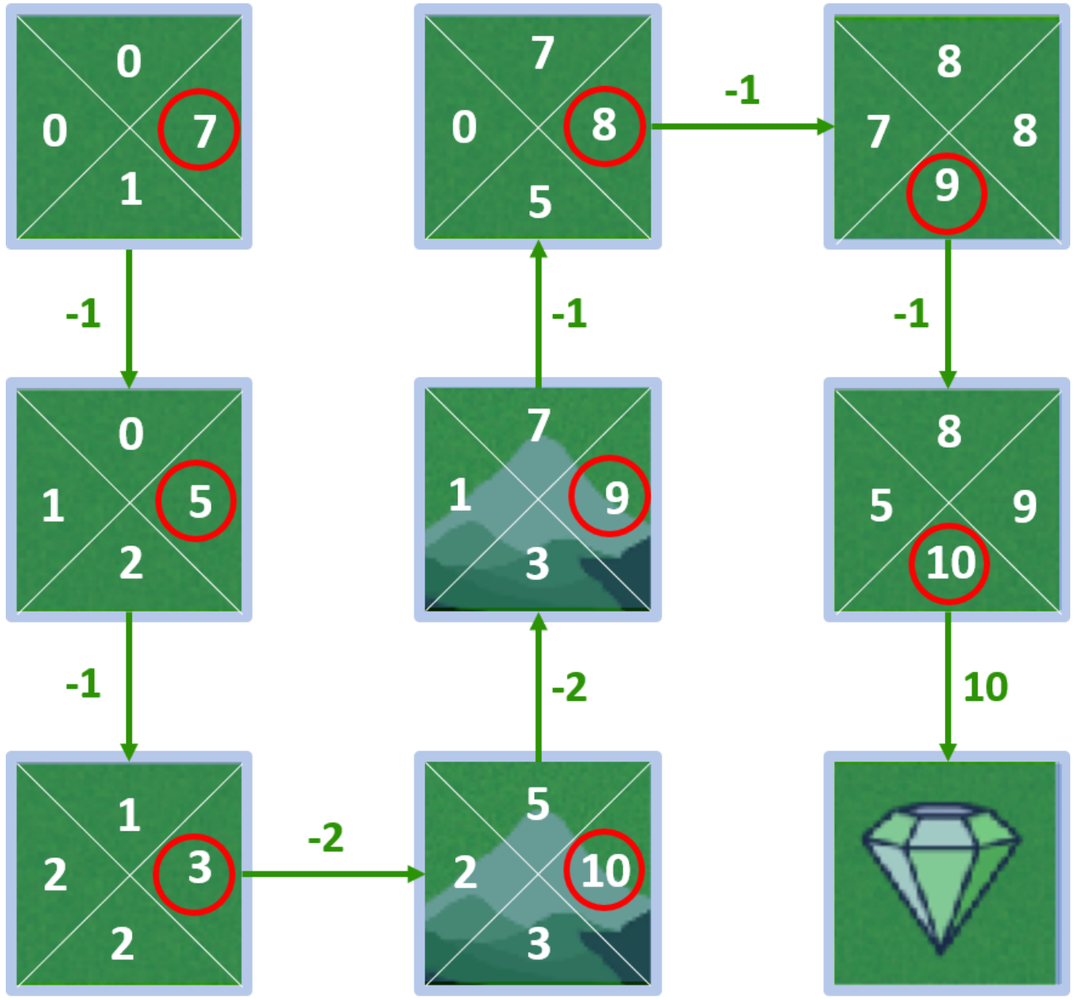


Improving the policy

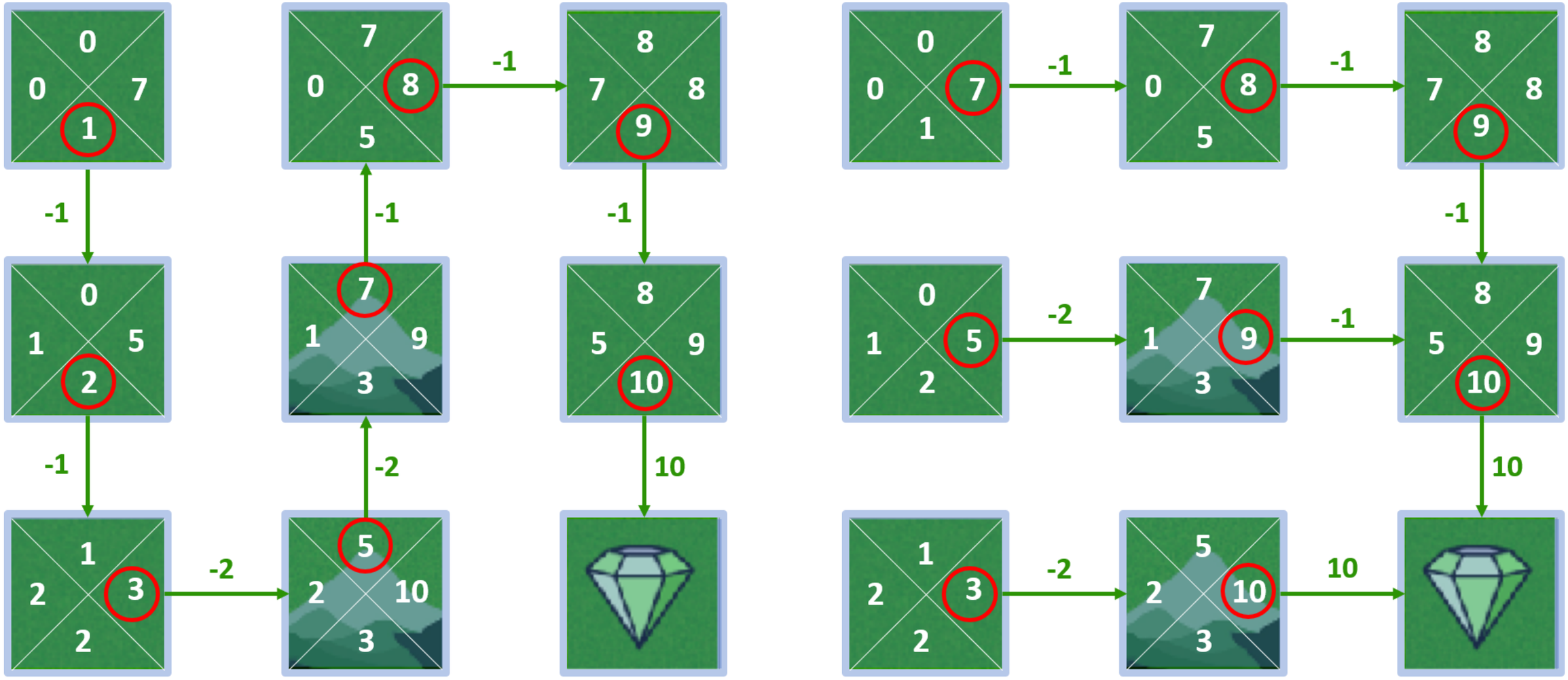


Improving the policy

- Selecting for each state the action with highest Q-value



Improving the policy



Old policy

Improving the policy

```
improved_policy = {}

for state in range(num_states-1):
    max_action = max(range(num_actions), key=lambda action: Q[(state, action)])
    improved_policy[state] = max_action

print(improved_policy)
```

```
{0: 2, 1: 2, 2: 1,
 3: 2, 4: 2, 5: 1,
 6: 2, 7: 2}
```

Let's practice!

REINFORCEMENT LEARNING WITH GYMNASIUM IN PYTHON

Policy iteration and value iteration

REINFORCEMENT LEARNING WITH GYMNASIUM IN PYTHON

Fouad Trad
Machine Learning Engineer

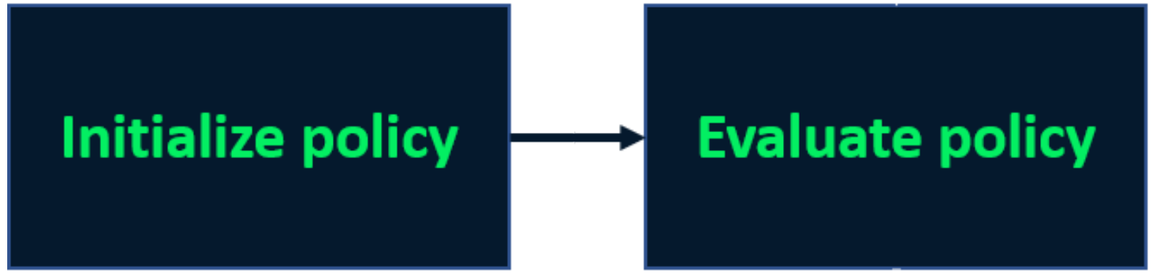
datacamp

Policy iteration

- Iterative process to find optimal policy

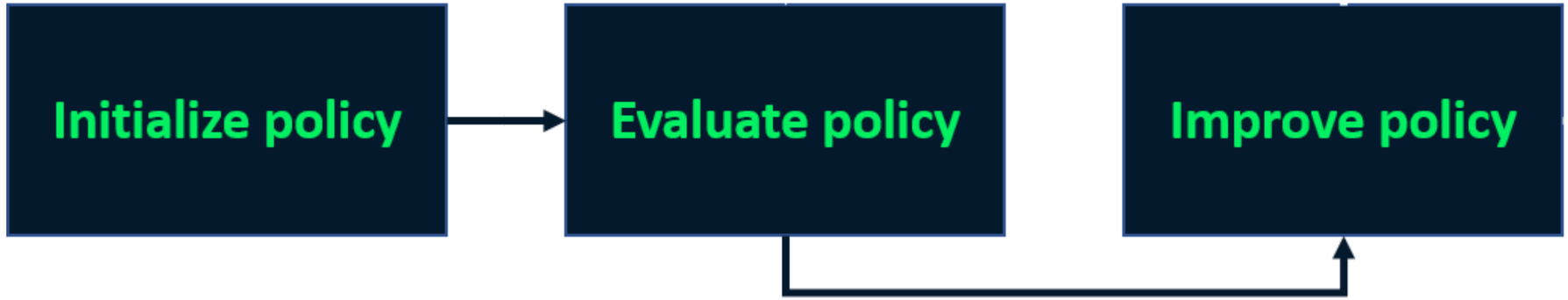
Policy iteration

- Iterative process to find optimal policy



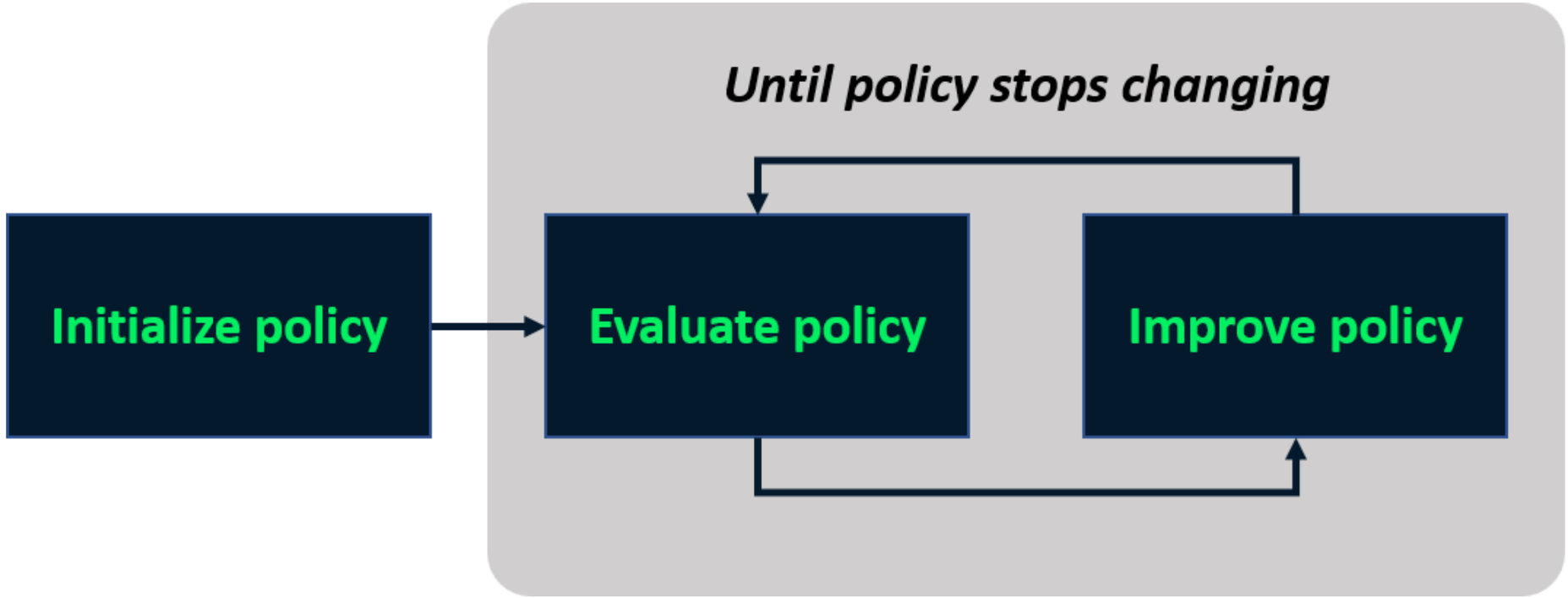
Policy iteration

- Iterative process to find optimal policy



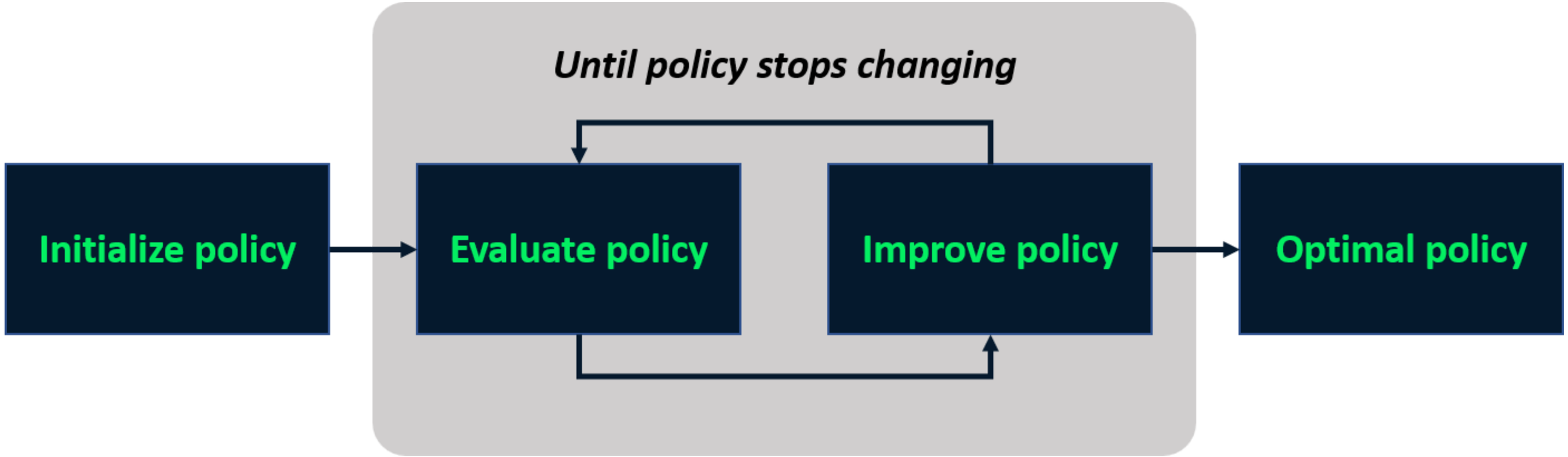
Policy iteration

- Iterative process to find optimal policy



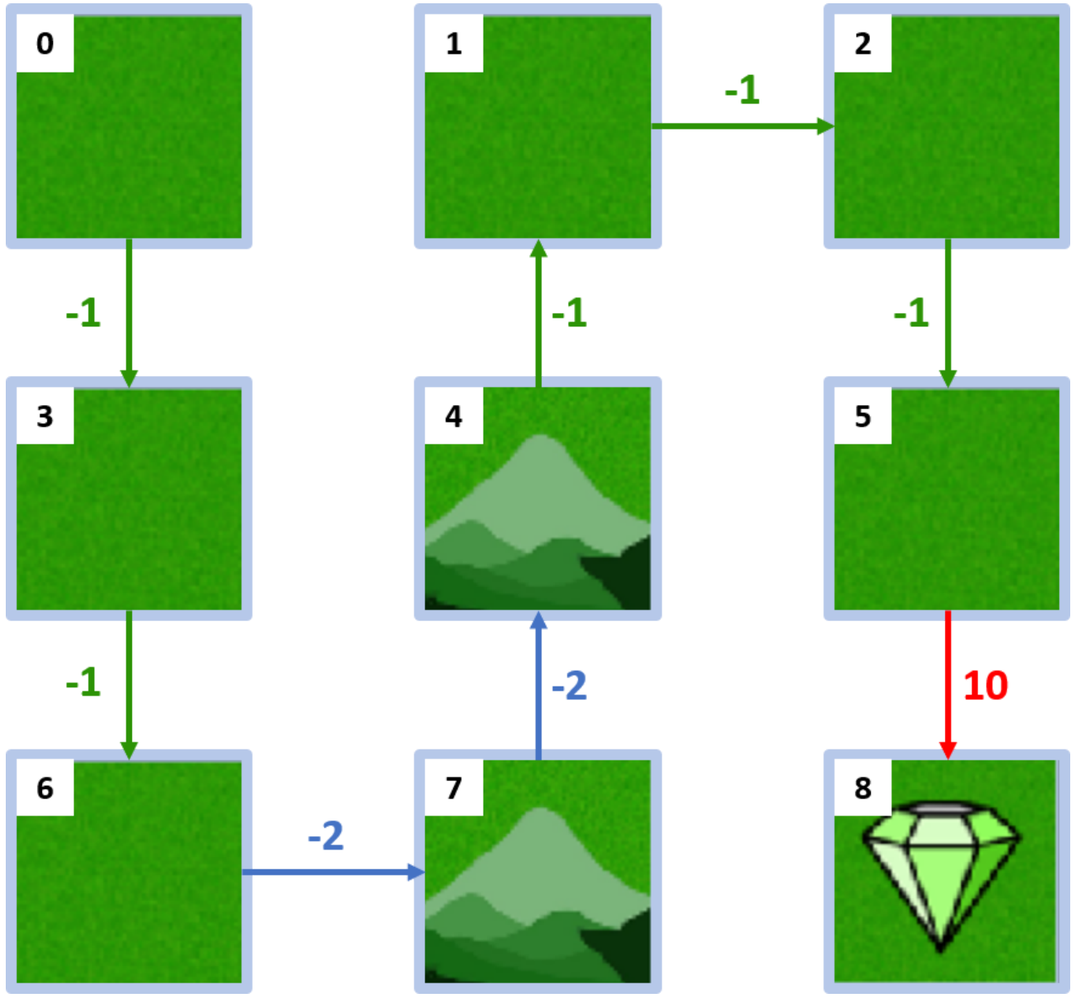
Policy iteration

- Iterative process to find optimal policy



Grid world

```
policy = {
    0:1, 1:2, 2:1,
    3:1, 4:3, 5:1,
    6:2, 7:3
}
```



Policy evaluation

```
def policy_evaluation(policy):
    V = {state: compute_state_value(state, policy) for state in range(num_states)}
    return V
```

Policy improvement

```
def policy_improvement(policy):
    improved_policy = {s: 0 for s in range(num_states-1)}
    Q = {(state, action): compute_q_value(state, action, policy)
      for state in range(num_states) for action in range(num_actions)}

    for state in range(num_states-1):
        max_action = max(range(num_actions), key=lambda action: Q[(state, action)])
        improved_policy[state] = max_action

    return improved_policy
```

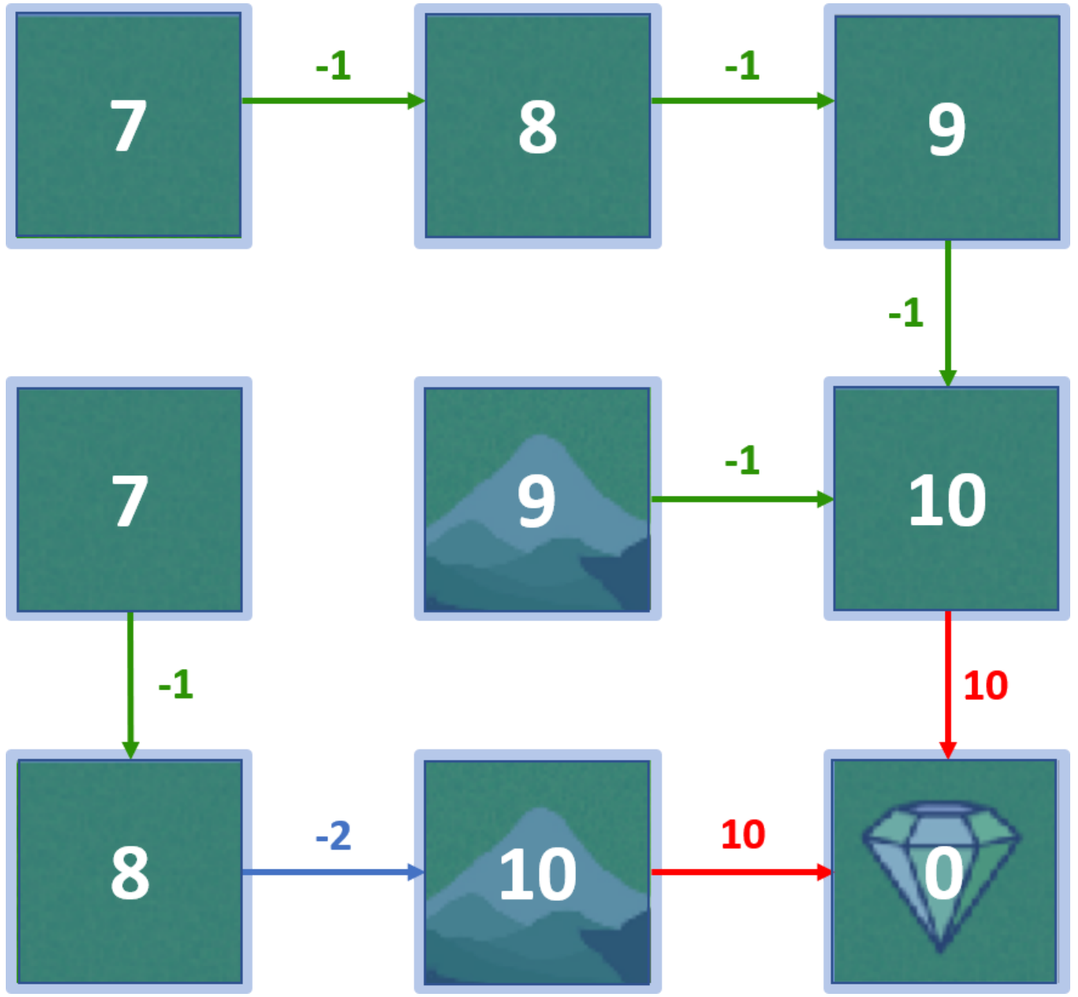
Policy iteration

```
def policy_iteration():
    policy = {0:1, 1:2, 2:1, 3:1, 4:3, 5:1, 6:2, 7:3}
    while True:
        V = policy_evaluation(policy)
        improved_policy = policy_improvement(policy)

        if improved_policy == policy:
            break
        policy = improved_policy

    return policy, V
```

Optimal policy

```
policy, V = policy_iteration()
print(policy, V)
```

```
{0: 2, 1: 2, 2: 1,
 3: 1, 4: 2, 5: 1,
 6: 2, 7: 2}

{0: 7, 1: 8, 2: 9,
 3: 7, 4: 9, 5: 10,
 6: 8, 7: 10, 8: 0}
```



Value iteration

- Combines policy evaluation and improvement in one step
  - Computes optimal state-value function
  - Derives policy from it

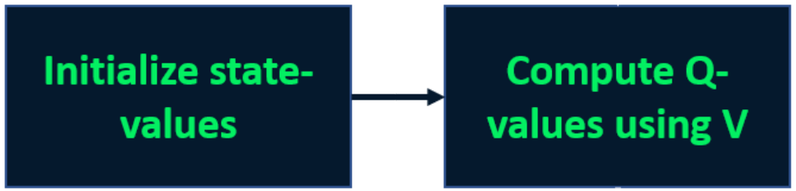
Initialize state-values

V

0	0	0
0	0	0
0	0	0

Value iteration

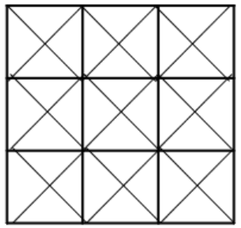
- Combines policy evaluation and improvement in one step.
  - Computes optimal state-value function
  - Derives policy from it



V

0	0	0
0	0	0
0	0	0

Q



Value iteration

- Combines policy evaluation and improvement in one step.
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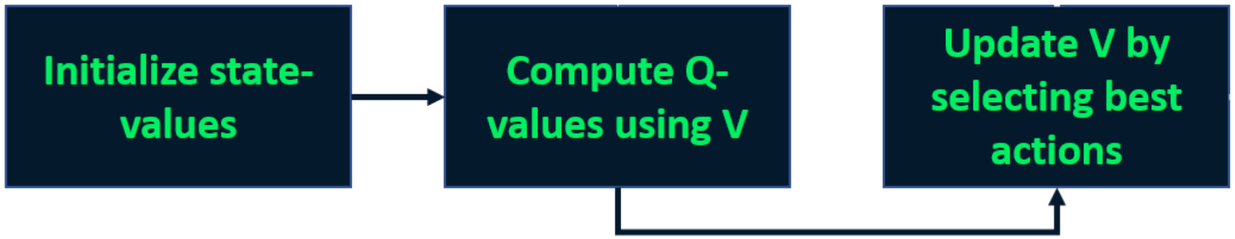


V

0	0	0
0	0	0
0	0	0

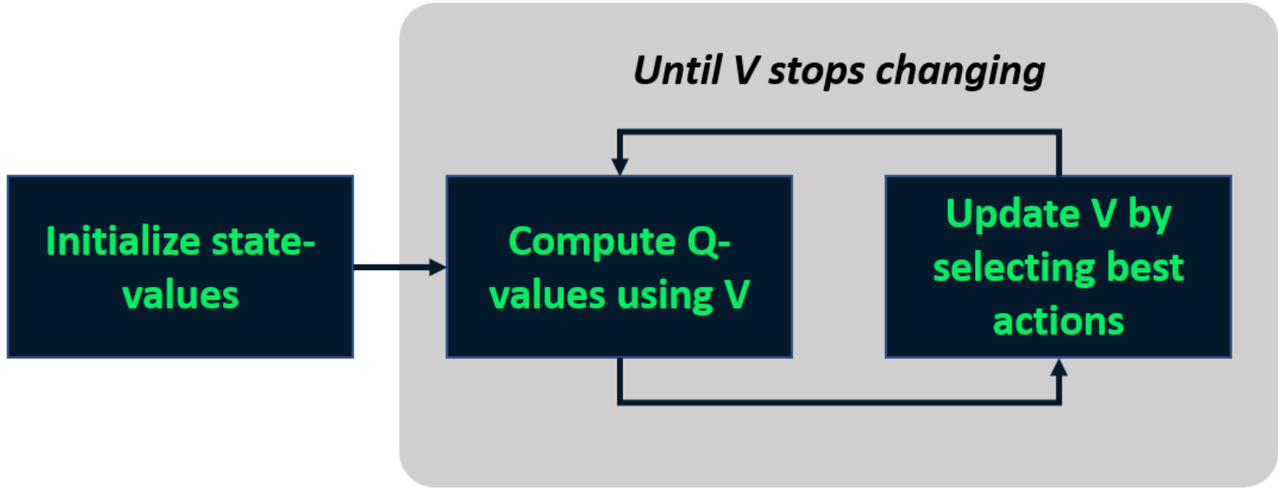
Q

V

0	0	0
0	0	6
0	4	0

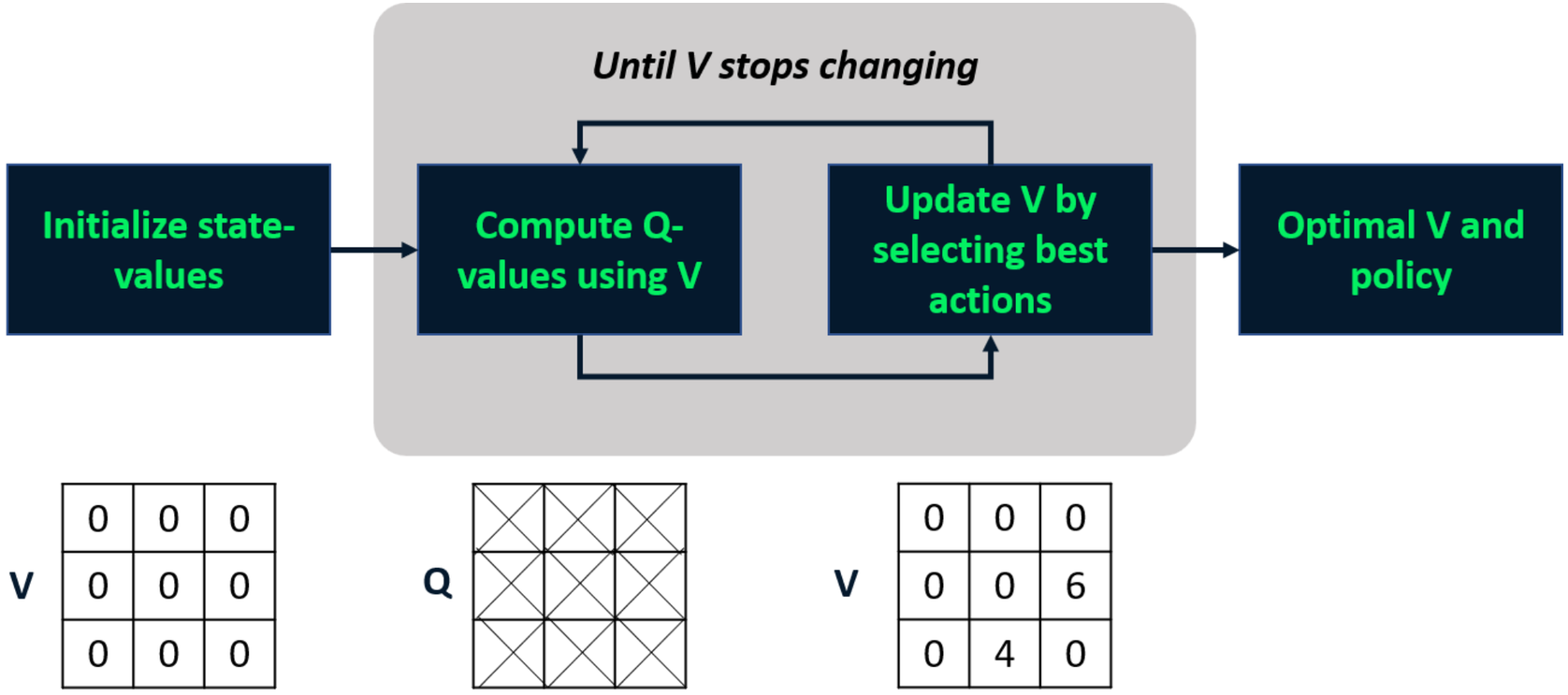
Value iteration

- Combines policy evaluation and improvement in one step.
  - Computes optimal state-value function
  - Derives policy from it



Value iteration

- Combines policy evaluation and improvement in one step.
  - Computes optimal state-value function
  - Derives policy from it



Implementing value-iteration

```
V = {state: 0 for state in range(num_states)}
policy = {state:0 for state in range(num_states-1)}
threshold = 0.001

while True:
    new_V = {state: 0 for state in range(num_states)}
    for state in range(num_states-1):
        max_action, max_q_value = get_max_action_and_value(state, V)
        new_V[state] = max_q_value
        policy[state] = max_action

    if all(abs(new_V[state] - V[state]) < thresh for state in V):
        break
    V = new_V
```

Getting optimal actions and values

```
def get_max_action_and_value(state, V):
    Q_values = [compute_q_value(state, action, V) for action in range(num_actions)]
    max_action = max(range(num_actions), key=lambda a: Q_values[a])
    max_q_value = Q_values[max_action]
    return max_action, max_q_value
```

Computing Q-values

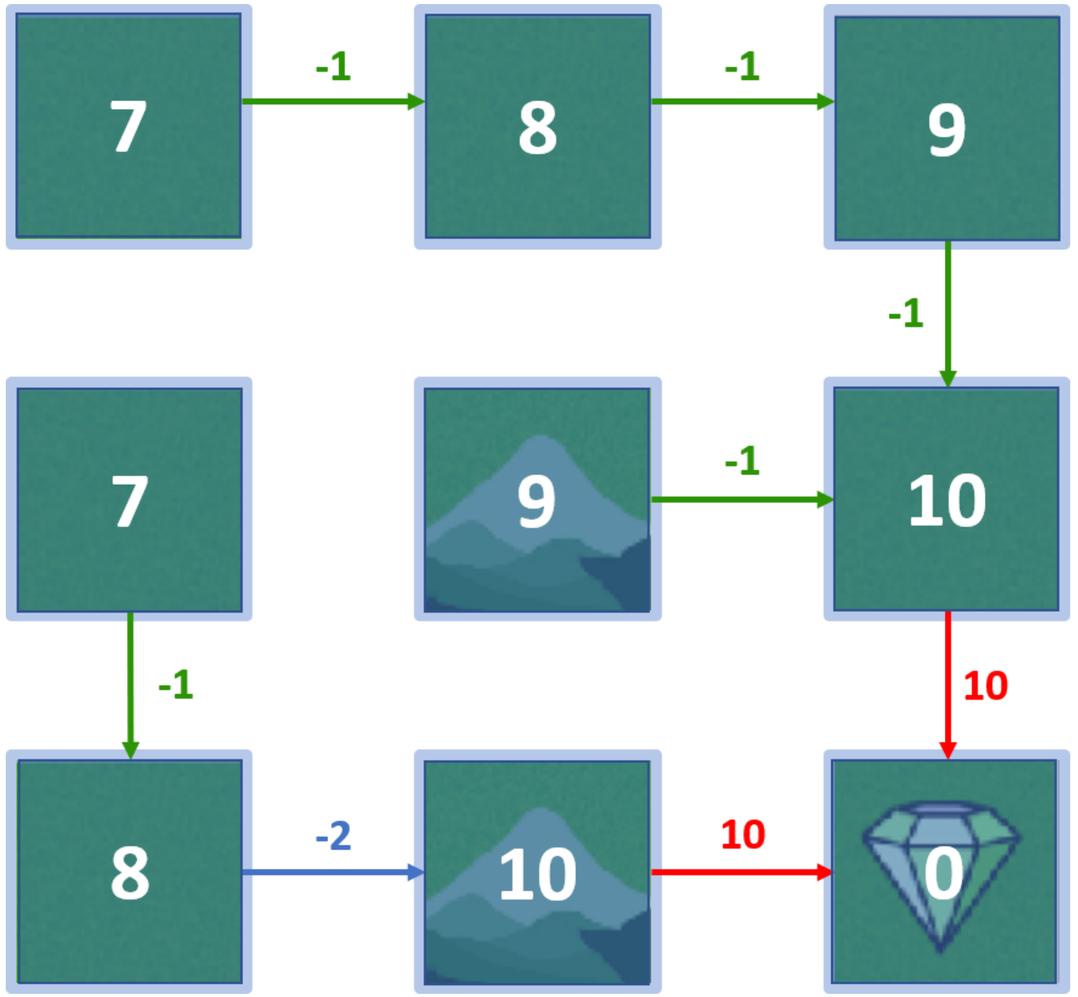
```
def compute_q_value(state, action, V):
    if state == terminal_state:
        return None
    _, next_state, reward, _ = env.P[state][action][0]
    return reward + gamma * V[next_state]
```

Optimal policy

```
print(policy, V)
```

```
{0: 2, 1: 2, 2: 1,
 3: 1, 4: 2, 5: 1,
 6: 2, 7: 2}

{0: 7, 1: 8, 2: 9,
 3: 7, 4: 9, 5: 10,
 6: 8, 7: 10, 8: 0}
```



Let's practice!

REINFORCEMENT LEARNING WITH GYMNASIUM IN PYTHON