Surface Integral

$$\iint_S \vec{F}\cdot d\vec{s} = \iint_S \vec{F}\cdot\vec{n}\, ds$$

i.e. $d\vec{s} = \vec{n}\, ds$

where, $\vec{n} = \dfrac{\nabla\phi}{|\nabla\phi|}$; ϕ is given.

& $ds = \dfrac{dx\,dy}{|\vec{n}\cdot\vec{k}|}$ or

$\dfrac{dy\,dz}{|\vec{n}\cdot\vec{i}|}$ or

$\dfrac{dx\,dz}{|\vec{n}\cdot\vec{j}|}$

Green's Theorem

Relationship betⁿ line integral and surface integral.

$$\int_C (f_1 dx + f_2 dy) = \iint_S \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right) dx\, dy$$

Application of Green Theorem

+ Helps us compute the area enclosed by closed curve C.

$$\text{Area } (A) = \frac{1}{2}\int_C [x\,dy - y\,dx]$$

Stoke's Theorem

General case of Green's theorem. Also relates line integral with surface integral.

$$\int_C \vec{F}\cdot d\vec{r} = \iint_S \text{Curl } \vec{F}\cdot\vec{n}\, ds$$

Gauss Divergence Theorem

If $\vec{F}$ is a vector differentiable function, then the normal surface integral of $\vec{F}$ over the closed surface S enclosed the volume V is equal to the volume integral of divergence of $\vec{F}$ over V.

i.e.

$$\iint_S \vec{F}\cdot\vec{n}\, ds = \iiint_V (\nabla\cdot\vec{F})\, dV$$