Answer of Question no (1)

Let $A = \begin{pmatrix} -12 & 7 \\ -7 & 2 \end{pmatrix}$

$\lambda = -5$

To find the eigen vector; we have

$(A - \lambda I)x = 0$

$\Rightarrow \cancel{\begin{pmatrix} -12-(-5) \end{pmatrix}}$

$\Rightarrow \begin{pmatrix} -12+5 & 7 \\ -7 & 2-5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} -7 & 7 \\ -7 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

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$$\Rightarrow \begin{cases} -7x_1 + 7x_2 = 0 \\ -7x_1 + 3x_2 = 0 \end{cases}$$

from above

$$x_1 = x_2$$

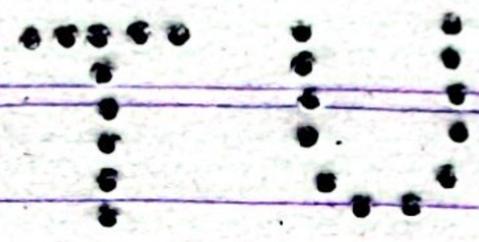
$$x_2 = -\frac{7}{3} x_1$$

$$v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$ is the unit eigen vector

$$v_2 = \begin{pmatrix} -\frac{7}{3} \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{7}{\sqrt{58}} \\ \frac{3}{\sqrt{58}} \end{pmatrix}$$ is the unit eigen vector.

$\frac{7}{3}\times 3 \quad 7^2$



U_1

$U_3\left(\frac{7}{1}\right)$

$U_2=\left(\frac{7}{3}\right)$

U_3

Answer of Question no (2)

The quadratic form $5x_1^2+5x_2^2-4x_1x_2$ can be expressed in matrix form as

$$x^TAx = (x_1\ x_2)\begin{pmatrix}5 & -2\\ -2 & 5\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}$$

where $A = \begin{pmatrix}5 & -2\\ -2 & 5\end{pmatrix}$ is a symmetric matrix

To find the eigen value of A; we have

$$|A-\lambda I| = 0$$

$$\Rightarrow \begin{vmatrix}5-\lambda & -2\\ -2 & 5-\lambda\end{vmatrix} = 0$$

$$\Rightarrow (5-\lambda)(5-\lambda)-4=0$$

$$\therefore \lambda_1 = 7 \text{ and } \lambda_2 = 3$$

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for $\lambda_1 = 7$

to find the eigenvector; we have;

$$(A - \lambda I)x = 0$$

$$\Rightarrow \begin{pmatrix} 5-7 & -2 \\ -2 & 5-7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} -2x_1 - 2x_2 = 0 \\ -2x_1 - 2x_2 = 0 \end{cases}$$

From the above; we get

$$x_1 = -x_2$$

for example

$$v_{\lambda_1 = 7} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$ is the eigen vector

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$$U_{\lambda=7} = \begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$ is the unit eigen vector crossponding to eigen value $\lambda_1 = 7$

$$\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$ is the maximum value of $5x_1^2 + 5x_2^2 - 4x_1x_2$ subject to the constraints $x^T x = 1$. Use use ~~m~~ greater value of λ which is 7

3

Answer of Question no (3)

Let $A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$

$A^T = \begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}$

Now,

$A^T A = \underset{3\times 2}{\begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}} \underset{2\times 3}{\begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}}$

$= \begin{pmatrix} 9+4 & 6+6 & 6-4 \\ 6+6 & 4+9 & 4-6 \\ 6-4 & 4-6 & 4-4 \end{pmatrix}$

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$$= \begin{pmatrix} 13 & 12 & 2 \\ 12 & 13 & -2 \\ 2 & -2 & 0 \end{pmatrix}$$

to find the eigen ~~vector~~ values of A^TA, we have

$$\begin{vmatrix} 13-\lambda & 12 & 2 \\ 12 & 13-\lambda & -2 \\ 2 & -2 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow 13-\lambda \begin{vmatrix} 13-\lambda & -2 \\ -2 & -\lambda \end{vmatrix} - 12 \begin{vmatrix} 12 & -2 \\ 2 & -\lambda \end{vmatrix}$$

$$\cancel{+2 \begin{vmatrix} 12 & 2 \\ 2 & -\lambda \end{vmatrix}}$$

$$+2 \begin{vmatrix} 12 & 13-\lambda \\ 2 & -2 \end{vmatrix} = 0$$

$$\Rightarrow (13-\lambda)\left(-\lambda(13-\lambda)-4\right)-12(-12\lambda+4)$$
$$+2\left(-24-2(13-\lambda)\right)=0$$

$$\Rightarrow (13-\lambda)(13\lambda-\lambda^2-4)+144\lambda-48$$
$$+\cancel{48}\ 2(-24-26+2\lambda)=0$$

$$\cancel{\Rightarrow (13-\lambda)(}$$

$$\Rightarrow 169\lambda-13\lambda^2-52-13\lambda+\lambda^3-\cancel{4\lambda}$$
$$+144\lambda-48-48-52+\cancel{4\lambda}=0$$

$$\Rightarrow \lambda^3 \cancel{-13\lambda^2}$$

$$\lambda_1=25,\ \cancel{\lambda_2=}$$

for $\lambda_1 = 25$

to find the eigen vector; we have

$$\begin{pmatrix} 13-25 & 12 & 2 \\ 12 & 13-25 & -2 \\ 2 & -2 & -25 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -12 & 12 & 2 \\ 12 & -12 & -2 \\ -2 & -2 & -25 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{cases} -12x_1 + 12x_2 + 2x_3 = 0 \\ 12x_1 - 12x_2 - 2x_3 = 0 \\ -2x_1 - 2x_2 - 25x_3 = 0 \end{cases}$$

1.5

Answer of Question no (9)

Given

$$Q(x) = x_1^2 - 6x_1x_2 - 9x_2^2$$

The quadratic form can be expressed as matrix form;

$$x^TAx = (x_1\ x_2)\begin{pmatrix} 1 & -3 \\ -3 & -9 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

where $A = \begin{pmatrix} 1 & -3 \\ -3 & -9 \end{pmatrix}$

is symmetric matrix

to find the eigenvalue of A we have

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & -3 \\ -3 & -9-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)(-9-\lambda) - 9 = 0$$

$\lambda_1 = 1.83$ and $\lambda_2 = -9.83$

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from above λ_1 is positive and λ_2 is negative. By the definition, ~~when~~ It is indefinite.

when $x^T A > 0$ ~~and~~ for some x and $x^T A < 0$ for other some x for $\forall x \neq 0$ then: this is indefinite

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Answer of Question no (6) (or)

(a)

Let U_1, U_2 be the eigenvectors associated with the eigenvalues λ_1, λ_2 of a 2×2 symmetric matrix respectively.

then

$$AV = (AU_1, AU_2)$$

$$= (\lambda U_1, \lambda U_2)$$

$$= \begin{pmatrix} \lambda U_{11} & \lambda U_{12} \\ \lambda U_{21} & \lambda U_{22} \end{pmatrix}$$

$$= \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

$$= V\Lambda$$

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$$VV^T = I_2$$

$$A = U\Lambda V^T$$

<u>Proved</u>

b) Given $v_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$$v_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$\therefore V = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}$$

$$V^T = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}$$

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UT

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

we have

$$A = V \Lambda V^T$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} \lambda_1 & \lambda_2 \\ \lambda_1 & -\lambda_2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} \lambda_1 + \lambda_2 & \lambda_1 - \lambda_2 \\ \lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 \end{pmatrix}$$

is the required 2×2 matrices A.

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$V^{-1}AV$

Answer of Question no (7)

(a) let A be an $n \times n$ matrix

$$V = \{v_1, \ldots, v_n\}$$

$$\Lambda = \{\lambda_1, \ldots, \lambda_n\}$$

we have

$$AV = \{Av_1, \ldots Av_n\}$$

$$= \{\lambda_1 v_1 \ldots \lambda_n v_n\}$$

$$= \Lambda V$$

where $\Lambda = \text{diag}\{\lambda_1, \ldots \lambda_n\}$ ← eigenvalue

$V = \{v_1, \ldots v_n\}$ ← eigenvector.

$$\therefore V^{-1}V = I$$

so $A = V^{-1}\Lambda V$

which proves that A has n linearly independent eigenvector & A is diagonalizable.

Answer of Question no (7) (b)

Given

$$A = \begin{pmatrix} -2 & 4 \\ -1 & -6 \end{pmatrix}$$

$$A^T = \begin{pmatrix} -2 & -1 \\ 4 & -6 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} \\ \\ \end{pmatrix}$$

To find the eigen value of $A^T A$ we have

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -2-\lambda & 4 \\ -1 & -6-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-2-\lambda)(-6-\lambda) + 4 = 0$$

$$\lambda_1 = -4$$

$U^T U$

to find the eigen vector.

$$\begin{pmatrix} -2+9 & 9 \\ -1 & -6+9 \end{pmatrix} \begin{pmatrix} n_2 \\ n_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2 & 9 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$= \begin{cases} 2n_1 + 9n_2 = 0 \\ -n_1 - 2n_2 = 0 \end{cases}$$

$$\therefore \quad n_1 = -2n_2$$

for exam

$$U_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$U_1 = \begin{pmatrix} -\frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}$$ → the unit eigen vector corresponding to $\lambda = -9$

Here λ has only one value & eigen vector also so

matrix $A = \begin{pmatrix} -2 & 9 \\ -1 & -6 \end{pmatrix}$ is not diagonalizable

6

Answer of Question no (8) (a)

If a is Real number

$$B_A(av, w) = (av)^T A w$$
$$= a v^T A w$$
$$= a\, B_A(v, w)$$

and

$$B_A(u+v, w) = (u+v)^T A w$$
$$= (u^T + v^T) A w$$
$$= u^T A w + v^T A w$$
$$= B_A(u, w) + B_A(v, w)$$

~~Thus~~ from above, $B_A(v, w) = v^T A w$, then $B_A(v, w)$ is linearly in the first variable v.

Answer of Question no (8) (b)

The given quadratic form $10x_1^2 - 8x_1x_2 + 4x_2^2$ can be expressed as matrix form;

$$x^T A x = (x_1 \; x_2) \begin{pmatrix} 10 & -4 \\ -4 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

where $A = \begin{pmatrix} 10 & -4 \\ -4 & 4 \end{pmatrix}$

is symmetric matrix

To find the eigenvalue of A; we have

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 10-\lambda & -4 \\ -4 & 4-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (10-\lambda)(4-\lambda) - 16 = 0$$

$$\lambda_1 = 12 \text{ and } \lambda_2 = 2$$

for $\lambda_1 = 12$

to find the eigenvector, we have

$$(A - \lambda I)x = 0$$

$$\Rightarrow \begin{pmatrix} 10-12 & -4 \\ -4 & 4-12 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -2 & -4 \\ -4 & -8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} -2x_1 - 4x_2 = 0 \\ -4x_1 - 8x_2 = 0 \end{cases}$$

from above, we get

$$x_1 = -2x_2$$

for example

$$v_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$ is the eigen vector

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$$v_1 = \begin{pmatrix} -\frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}$$ is the unit eigenvector corresponding to eigen value $\lambda = 12$

for $\lambda_2 = 2$

to find the eigenvector; we have

$$(A - \lambda I)x = 0$$

$$\Rightarrow \begin{pmatrix} 10-2 & -4 \\ -4 & 4-2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 8 & -4 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} 8x_1 - 4x_2 = 0 \\ -4x_1 + 2x_2 = 0 \end{cases}$$

From above; we get

$$2x_1 = x_2$$

for example

$$v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$ is the eigen vector

$$v_2 = \begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$$ is the unit eigen vector corresponding the eigen value $\lambda = 2$.

$\therefore$ v_1 & v_2 are orthogonal because their dot product is zero.

now

$$P = \begin{pmatrix} \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix}$$

$$P^T = \begin{pmatrix} \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix}$$

$$A = \begin{pmatrix} 10 & -4 \\ -4 & 4 \end{pmatrix}$$

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$$\hat{\Lambda} = P^T A P$$

$$= \begin{pmatrix} \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} 10 & -4 \\ -4 & 4 \end{pmatrix} \begin{pmatrix} \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix}$$

$$= \frac{1}{5} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 10 & -4 \\ -4 & 4 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$= \frac{1}{5} \begin{pmatrix} -20-4 & -8+4 \\ 10-8 & -4+8 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$= \frac{1}{5} \begin{pmatrix} -24 & -12 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$= \frac{1}{5} \begin{pmatrix} 48+12 & -24+24 \\ -4+4 & 2+8 \end{pmatrix}$$

$$= \frac{1}{5} \begin{pmatrix} 60 & 0 \\ 0 & 16 \end{pmatrix}$$

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$$= \begin{pmatrix} 12 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \Lambda$$

we have

$$x = Py$$

$$x^T A x = (Py)^T A Py$$

$$= P^T y^T A P y$$

$$= y^T P^T A P y$$

$$= y^T \Lambda y \quad (\because P^T A P = \Lambda)$$

So $Q' = y^T \Lambda y$

$$y^T \Lambda y = (y_1 \; y_2) \begin{pmatrix} 12 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$= 12y_1^2 + 2y_2^2$$

This is a quadratic form without the cross product term using eigenvalues and eigenvectors

Answer of Question no (9)

let $A = \begin{pmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{pmatrix}$

$$A^T = \begin{pmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{pmatrix}$$

now

$$A^TA = \begin{pmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{pmatrix}_{2\times3} \begin{pmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{pmatrix}_{3\times2}$$

$$= \begin{pmatrix} 9+36+36 & -3-12-12 \\ -3-12-12 & 1+4+4 \end{pmatrix}$$

$$= \begin{pmatrix} 81 & -27 \\ -27 & 9 \end{pmatrix}$$

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to find the eigenvalue of A^TA, we have

$$\begin{vmatrix} 81-\lambda & -27 \\ -27 & 9-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (81-\lambda)(9-\lambda) - 27^2 = 0$$

$$\cancel{\lambda_1 = 90}$$

$$\Rightarrow 729 - 81\lambda - 9\lambda + \lambda^2 - 729 = 0$$

$$\cancel{\Rightarrow \lambda^2 - 81\lambda}$$

$$\cancel{\text{For } \lambda_1 = 90}$$

$$\Rightarrow \lambda^2 - 90\lambda = 0$$

$$\lambda(\lambda - 90) = 0$$

$$\lambda = 90$$

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to find the eigenvector of A; we have

$$\begin{pmatrix} 81-90 & -27 \\ -27 & 9-90 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -9 & -27 \\ -27 & -81 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} -9x_1 - 27x_2 = 0 \\ -27x_1 - 81x_2 = 0 \end{cases}$$

from above we get

$$x_1 = -3x_2$$

for example

$$v_1 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

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$v_3 = \begin{pmatrix} -\frac{3}{\sqrt{90}} \\ \frac{1}{\sqrt{90}} \end{pmatrix}$ is the unit eigen vector corresponding to eigenvalue $\lambda = 90$

This has only one eigen value that is $\lambda_3 = 90$

Answer of Question no 9(b)

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}_{3\times 2} \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}_{2\times 3}$$

$$= \begin{pmatrix} 9+4 & 6+6 & 6-4 \\ 6+6 & 4+9 & 4-6 \\ 6-4 & 4-6 & 4+4 \end{pmatrix}$$

$$= \begin{pmatrix} 13 & 12 & 2 \\ 12 & 13 & -2 \\ 2 & -2 & 8 \end{pmatrix}$$

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to find eigen value, we have

$$\begin{vmatrix} 13-\lambda & 12 & 2 \\ 12 & 13-\lambda & -2 \\ 2 & -2 & 8-\lambda \end{vmatrix} = 0$$

$$13-\lambda \begin{vmatrix} 13-\lambda & -2 \\ -2 & 8-\lambda \end{vmatrix} - 12 \begin{vmatrix} 12 & -2 \\ 2 & 8-\lambda \end{vmatrix}$$

$$+2 \begin{vmatrix} 12 & 13-\lambda \\ 2 & -2 \end{vmatrix}$$

$$\Rightarrow (13-\lambda)((13-\lambda)(8-\lambda)-4)$$
$$-(12(12(8-\lambda)+4))+(2(-24+-2(13-\lambda)))$$

$$\lambda_1 = 25 \;,\; \lambda_2 = 9$$

3

Answer of Question no (10)

The reduced rowechelon form is when

i) If ~~non-z~~ zero elements are found they must be bottom of the matrix

ii) If non-zero elements founds top of the element be 1 which is leading element.

iii) the leading element must be adjust from top left to right below.

iv) The leading element 1 must be in top. then only this is called leading element.

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Example

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

is the example of reduced row echelon form

The augumented matrix of the given matrix is

$$\left(\begin{array}{ccc|c} 3 & -1 & 7 & 9 \\ 5 & 3 & 2 & 10 \\ 9 & 2 & -5 & 6 \end{array}\right)$$

$R_2 \to 3R_2 - 5R_1$

$R_3 \to R_3 - 3R_2$

$$\sim \left(\begin{array}{ccc|c} 3 & -1 & 7 & 9 \\ 0 & 14 & -29 & -15 \\ 0 & 5 & -26 & -21 \end{array}\right)$$

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$$\sim \left(\begin{array}{ccc|c} 3 & -1 & 7 & 9 \\ 0 & 14 & -29 & -15 \\ 0 & 5 & -26 & -21 \end{array}\right)$$

$$R_3 \to 14R_2 - 14R_3 \quad 5R_2$$

$$\sim \left(\begin{array}{ccc|c} 3 & -1 & 7 & 9 \\ 0 & 14 & -29 & -15 \\ 0 & 0 & & \end{array}\right)$$

$$R_3 \to 14R_3 - 5R_2$$

$$\sim \left(\begin{array}{ccc|c} 3 & -1 & 7 & 9 \\ 0 & 14 & -29 & 15 \\ 0 & 0 & -219 & -219 \end{array}\right)$$

$$R_3 \to \frac{R_3}{-219}$$

$$\sim \left(\begin{array}{ccc|c} 3 & -1 & 7 & 9 \\ 0 & 14 & -29 & 15 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

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$x = 1$
$y = 1$
$z = 1$

$R_1 \to R_1 - 7R_3$

$R_2 \to R_2 + 29R_3$

$$\sim \left(\begin{array}{ccc|c} 3 & -1 & 0 & 2 \\ 0 & 14 & 0 & 44 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$R_2 \to \frac{R_2}{14}$

$$\sim \left(\begin{array}{ccc|c} 3 & -1 & 0 & 2 \\ 0 & 1 & 0 & \frac{44}{14} \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$R_3 \to \frac{R_3}{14}$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

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$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$\therefore$ $x = 1$, $y = 1$ & $z = 1$ } Ans

6

Answer of Question no 9(a)

If A is an $m \times n$ matrix then all the eigenvalues of A^TA are non-negative.

$$\|AU_i\|^2 = (AU_i)^T AU_i$$
$$= U_i^T A^T A U_i$$
$$= U_i^T \lambda_i U_i$$
$$= \lambda_i U_i^T U_i$$
$$= \|AU_i\|^2$$

$\because$ square never be negative