Statistical Tests (Non Parametric)

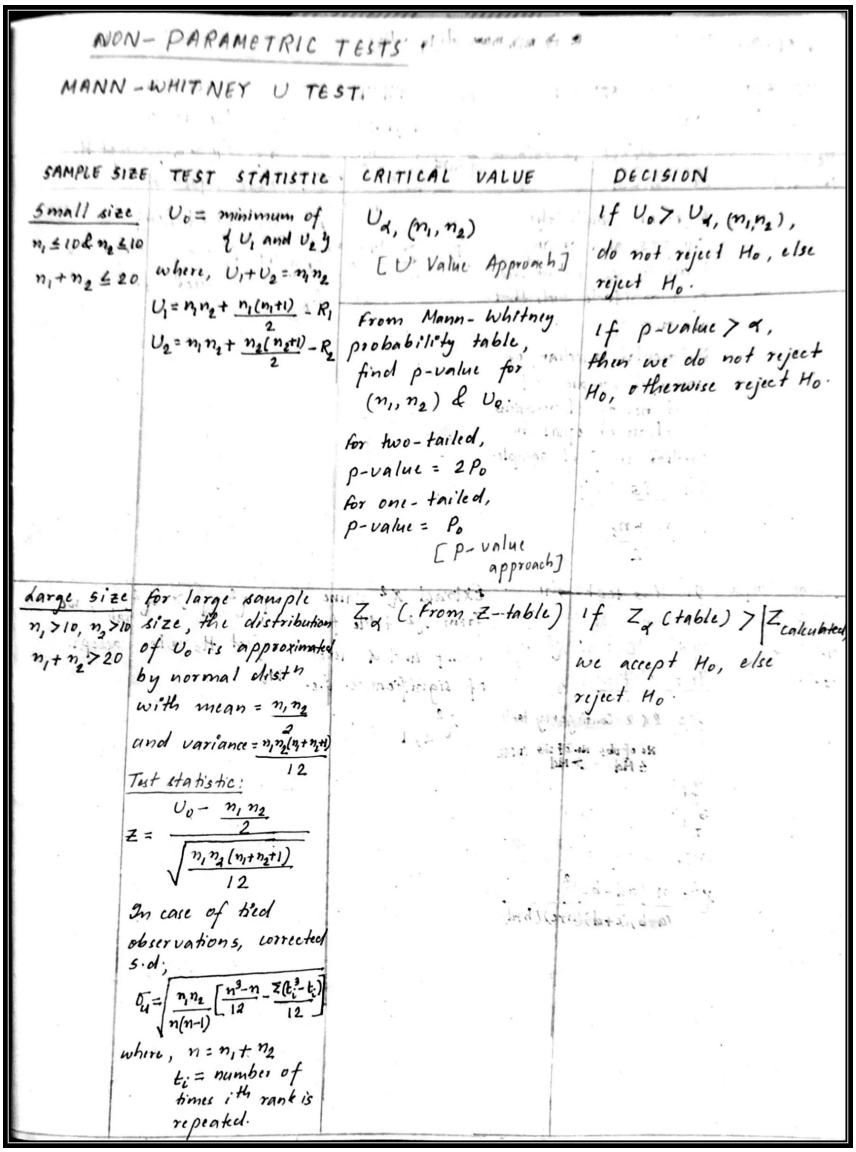
Master in Data Science

September 2024

Introduction

Non- parametric tests are those tests that do not involve any parameter of the population. Some common non-parametric tests are tabulated below:

Mann-Whitney U Test



NON- PARAMETRIC TESTS

MANN - WHITNEY U TEST.

SAMPLE SIZE	TEST STATISTIC	CRITICAL VALUE	DECISION
Small size $n_1 \leq 10$ & $n_2 \leq 10$ $n_1 + n_2 \leq 20$	U_0 = minimum of $\{U_1$ and $U_2\}$ where, $U_1 + U_2 = n_1 n_2$ $U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$ $U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$	$U_\alpha, (n_1, n_2)$ [U Value Approach]	If $U_0 > U_\alpha, (n_1, n_2)$, do not reject H_0, else reject H_0.
		From Mann- Whitney probability table, find p-value for (n_1, n_2) & U_0: for two-tailed, p-value = $2P_0$ for one-tailed, p-value = P_0 [p-value approach]	If p-value $> \alpha$, then we do not reject H_0, otherwise reject H_0.
large size $n_1 > 10$, $n_2 > 10$ $n_1 + n_2 > 20$	for large sample size, the distribution of U_0 is approximated by normal dist^n with mean = $\frac{n_1 n_2}{2}$ and variance = $\frac{n_1 n_2 (n_1+n_2+1)}{12}$ Test statistic: $z = \frac{U_0 - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 (n_1+n_2+1)}{12}}}$ In case of tied observations, corrected s.d; $\sigma_U = \sqrt{\frac{n_1 n_2}{n(n-1)}\left[\frac{n^3-n}{12} - \frac{\sum(t_i^3 - t_i)}{12}\right]}$ where, $n = n_1 + n_2$ t_i = number of times i^{th} rank is repeated.	Z_α (from z-table)	If Z_α (table) $> \lvert Z_{calculated} \rvert$ we accept H_0, else reject H_0.

Median Test

MEDIAN TEST

a ⇒ minimum of (n_1 and K)

SAMPLE SIZE	TEST STATISTIC	CRITICAL VALUE	DECISION
$n_1 \leq 10$ & $n_2 \leq 10$ [Small]	'a' Test-statistic is obtained as follows: * Combine both the samples and arrange them in ascending order of magnitude such that $n = n_1 + n_2$ * Calculate median of combined sample and count no. of observations less than or equal to median in first sample. This is 'a'. $K = \frac{n_1 + n_2}{2}$	$P_0 = P(A \geq a)$; $P(A=a) = \frac{{}^{n_1}C_a \times {}^{n_2}C_{K-a}}{{}^{n_1+n_2}C_K}$ * $P_0 = \sum_{a}^{k} \frac{{}^{n_1}C_a \, {}^{n_2}C_{K-a}}{{}^{n_1+n_2}C_K}$	for two-tailed: If $2P_0 > \alpha$, accept H_0 else reject H_0 for one-tailed: If $P_0 > \alpha$, accept H_0 else reject H_0
[Large Sample Size] $n_1 > 10$ & $n_2 > 10$	In this test, the median test is equivalent to χ^2-test Test-statistic: χ^2 Use 2 X 2 contingency table	Extract χ^2 value from χ^2 table for 1 dof and α level of significance. i.e. $\chi^2_{\alpha,1}$	If $\chi^2_{cal} > \chi^2_{tab}$, we reject H_0 else accept H_0.

	No of obs ≤ Md	No of obs > Md	TOTAL
Sample I	a	b	a+b
Sample II	c	d	c+d
Total	a+c	b+d	a+b+c+d

$$\chi^2 = \frac{n(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

Kolmogorov Smirnov Test

KOLMOGOROV– SMIRNOV TEST

SAMPLE	TEST STATISTIC	CRITICAL VALUE	DECISION
One Sample To check if there is significant difference between observed and expected frequency	D_0 = Maximum $\lvert F_e(x) - F_o(x) \rvert$ where, $F_e(x) = \frac{cf_e}{n}$ cf_e = expected cumulative frequency f_e = expected frequency $= np_i$ $= \frac{\Sigma f}{\text{no. of categories}}$ $F_o(x) = \frac{cf_o}{n}$; cf_o = observed cumulative frequency	$D_{tabulated} = D_{n,\alpha}$	If $D_0 \geq D_{n,\alpha}$, then we reject H_0 otherwise do not reject H_0.
TWO SAMPLES **Small sample size:** $n_1 = n_2 \leq 40$ or $n_1 \neq n_2 \leq 20$	D_0 = Maximum $\lvert F(x) - F(y) \rvert$ where, $F(x) = \frac{cf_x}{n_1}$, $F(y) = \frac{cf_y}{n_2}$	$D_{tabulated} = D_{(n_1,n_2),\alpha}$	If $D_{(n_1,n_2),\alpha} > D_0$, we don't reject H_0, else we do.
large sample size: $n_1, n_2 > 40$ (for $n_1 = n_2$) $n_1, n_2 > 20$ (for $n_1 \neq n_2$)	D_0 = Maximum $\lvert F(x) - F(y) \rvert$ (for Two Tailed Test)	$D_{tabulated} = D_{(n_1,n_2),\alpha}$	If $D_{(n_1,n_2),\alpha} > D_0$, we do not reject H_0, else we do.
	$\chi^2 = 4D_0^2 \left(\frac{n_1 n_2}{n_1 + n_2}\right)$ (for One Tailed Test)	$\chi^2_{calculated} = \chi^2_{(\alpha,2)}$ ↑ degree of freedom	If $\chi^2_{cal} \geq \chi^2_{(\alpha,2)}$, we reject H_0, else we accept H_0

Wilcoxon and Kruskal Wallis H Test

WILCOXON MATCHED PAIR SIGN RANKED TEST

(used for small, usually $n \leq 20$ data)

SAMPLE	TEST STATISTIC	CRITICAL VALUE	DECISION
Use for small sample size ($n \leq 20$).	T = Minimum of $\{S(+), S(-)\}$ where, $S(+)$ = Sum of ranks of difference with '+' sign $S(-)$ = Sum of ranks of difference with '−' sign.	$T_{tabulated} = T_{\alpha, n_e}$ n_e = effective sample size $= n - t$ t = no. of differences with zero.	If $T_{\alpha, n_e} \geq T$, we reject H_0, else accept H_0.

KRUSKAL WALLIS H-TEST

SAMPLE	TEST STATISTIC	CRITICAL VALUE	DECISION
Small sample: $n_i \leq 5$ and $K = 3$	$H = \frac{12}{n(n+1)} \left(\sum_{i=1}^{K} \frac{R_i^2}{n_i} \right) - 3(n+1)$ where, $n = n_1 + n_2 + \ldots + n_K$ If there is tie in observations, then corrected H, $H_{corr} = \frac{H}{C.F.}$ C.F = Correction factor $= 1 - \frac{\sum (t_i^3 - t_i)}{n^3 - n}$	Obtain the p-value.	If p-value $> \alpha$, we accept H_0, else we reject H_0.
Large sample: $n_i > 5$ and $K > 3$	Same test statistic as above	$\chi^2_{tabulated} = \chi^2_{\alpha, (K-1)}$ ↓ degree of freedom	If $H < \chi^2_{tab}$, accept H_0 else reject H_0.

Friedman F Test

FRIEDMAN F-TEST

k = no. of samples, n = size of each sample

Sample	Test Statistic	Critical Value	Decision
Small Sample $2 \le n \le 9$ and $k=3$ and $2 \le n \le 4$ and $k=4$	$F_r = \frac{12}{nk(k+1)} \left(\sum_{i=1}^{k} R_i^2 \right) - 3n(k+1)$ If tied case then, then corrected value of F_r is $F_{r\,\text{corrected}} = \frac{F_r}{C.F.}$; $C.F. = 1 - \frac{\sum (t_j^3 - t_i)}{n(k^3-k)}$ t_i = no. of times i^{th} rank is repeated	P-value is obtained from Friedman probability table.	If P-value > α, we accept H_0, else we reject H_0.
Large sample $n > 5$ & $k > 3$	Same test statistic as above	$\chi^2_{\text{tabulated}} = \chi^2_{\alpha,(k-1)}$ ↓ degree of freedom	If $F_r < \chi^2_{tab}$, we accept H_0, else reject H_0.