T.U

1)

Given,

$$\text{Matrix}(A) = \begin{pmatrix} -12 & 7 \\ -\cancel{1}7 & 2 \end{pmatrix}$$

eigen value(λ) = -5

For eigen vector, when $\lambda = -5$,

We have,

$$(A - \lambda I)x = 0$$

$$\Rightarrow \left[\begin{pmatrix} -12 & 7 \\ -\cancel{1}7 & 2 \end{pmatrix} - \begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix} \right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\Rightarrow \begin{pmatrix} -12+5 & 7 \\ -\cancel{1}7 & 2+5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$\Rightarrow \begin{pmatrix} -\cancel{1}7 & 7 \\ -\cancel{1}7 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

This gives equation

$$-\cancel{1}7x_1 + 7x_2 = 0$$

$$\Rightarrow 7x_2 = \cancel{1}7x_1$$

$$\Rightarrow x_1 = x_2$$

There fore, the eigen vector $= \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and

~~unit~~ normalized eigen vector is $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

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$$\Rightarrow \begin{pmatrix} -12+5 & 7 \\ -7 & 2+5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$\Rightarrow \begin{pmatrix} -7 & 7 \\ -7 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

This gives equation

$$-7x_1 + 7x_2 = 0$$

$$\Rightarrow 7x_2 = 7x_1$$

$$\Rightarrow x_1 = x_2$$

There fore, the eigen vector = $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and

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The eigen space E_λ when $\lambda = -5$ is the eigen vector.

So,

The eigen space $E_{-5} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

2.5

2) Given, $5x_1^2 + 5x_2^2 - 4x_1x_2$

For maximum value,

$$\begin{aligned}
&5x_1^2 + 5x_2^2 - 4x_1x_2 \\
&\le 5x_1^2 + 5x_2^2 - 5x_1x_2 \\
&\le 5(x_1^2 + x_2^2 - x_1x_2) \\
&\le 5 \qquad (\because x^Tx = 1)
\end{aligned}$$

So the maximumvalue is 5.

The quadratic form can be written as,

$$(x_1\ x_2)\begin{pmatrix}5 & -2\\ -2 & 5\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}$$

$\Rightarrow x^TAx$, So, $A = \begin{pmatrix}5 & -2\\ -2 & 5\end{pmatrix}$

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3) ~~Re~~ Given

$$\text{Matrix } (A) = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$$

We have,

$$A^TA = \begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$$

$$= \cancel{3\times3+2\times2} \begin{pmatrix} 13 & 12 & 2 \\ 12 & 13 & -2 \\ 2 & -2 & 8 \end{pmatrix}$$

Now, For eigen value of A^TA ~~we hav~~ we know,

$$|A^TA - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 13-\lambda & 12 & 2 \\ 12 & 13-\lambda & -2 \\ 2 & -2 & 8 \end{vmatrix} = 0$$

$$\Rightarrow (13-\lambda)(13-\lambda)8 + 12\times(-2)\times2 + 2\times12\times(-2$$
$$- 12\times12\times8 - \{(13-\lambda)\times(-2)\times(-2) - \{2\times(13-\lambda)\times2$$

$$\Rightarrow (169-13\lambda-13\lambda+\lambda^2)8 - 48 - 48 - 1152 - 52 + 4\lambda$$
$$-52 + 4\lambda = 0$$

$$\Rightarrow 1352 - 208\lambda + 8\lambda^2 - 1352 + 8\lambda = 0$$

$$\Rightarrow 8\lambda^2 - 200\lambda = 0 \Rightarrow \lambda^2 - 25\lambda = 0$$

Solving this equation we get eigen value 25,

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so the singular value is $\sigma_1 = \sqrt{\lambda_1} = 5$.

1.5

4 Given,

$$Q(x) = x_1^2 - 6x_1x_2 - 9x_2^2$$

This Quadratic form can be written as,

$$\begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ -3 & -9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Which is in the form of $x^T A x$

so, $A = \begin{pmatrix} 1 & -3 \\ -3 & -9 \end{pmatrix}$

For eigen value of A

We know,

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & -3 \\ -3 & -9-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)(-9-\lambda) - 9 = 0$$

$$\Rightarrow \lambda^2 - \lambda + 9\lambda - 9 - 9 = 0$$

$$\Rightarrow \lambda^2 + 8\lambda - 18$$

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6)

a) Given,

eigen vectors, v_1, v_2

eigen values λ_1, λ_2

of 2×2 symmetric matrix

Then, we know,

$$AV = Av_1 \quad Av_2$$
$$= \lambda_1 v_1 \quad \lambda_2 v_2 \qquad (\because Av = \lambda v)$$
$$= \begin{matrix} \lambda_1 v_{11} & \lambda_2 v_{21} \\ \lambda_1 v_{12} & \lambda_2 v_{22} \end{matrix}$$
$$= \begin{pmatrix} v_{11} & v_{21} \\ v_{12} & v_{22} \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$
$$= V \Lambda$$

Then, Multiplying both sides by V^T

$$AVV^T = V\Lambda V^T$$

$$(\because VV^T = 1)$$

$$A = V\Lambda V^T$$

Hence, If $\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$ and $V = (v_1 \; v_2)$ then,

$$A = V\Lambda V^T$$

3

b) Given, normalized eigen vectors,

$$v_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \quad v_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

eigen values λ_1 and λ_2

Then we know,
$$V = \begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{pmatrix}$$

$$A = V \Lambda V^T$$

$$= \begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{pmatrix}\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}\begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{pmatrix}$$

$$= \begin{pmatrix}1/\sqrt{2}\,\lambda_1 & 1/\sqrt{2}\,\lambda_2\\ 1/\sqrt{2}\,\lambda_1 & -1/\sqrt{2}\,\lambda_2\end{pmatrix}\begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{pmatrix}$$

$$= \begin{pmatrix}\lambda_1+\lambda_2 & \lambda_1-\lambda_2\\ \lambda_1-\lambda_2 & \lambda_1+\lambda_2\end{pmatrix}$$

Therefore, $A = \begin{pmatrix}\lambda_1+\lambda_2 & \lambda_1-\lambda_2\\ \lambda_1-\lambda_2 & \lambda_1+\lambda_2\end{pmatrix}$

3

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(a) Given,

eigen vectors $(v) = v_1, \dots, v_n$

eigen values $(\lambda) = \lambda_1, \dots, \lambda_n$

$A = n \times n$ matrix

Then we know,

$\cancel{AV = Av_1, \dots,}$

$AV = (Av_1 \dots Av_n)$ (∵

$= (\lambda_1 v_1, \dots \lambda_n v_n)$

$= V \Lambda$

Here, Λ is diagonal matrix.

Then,

$AVV^T = V \Lambda V^T$

$\Rightarrow A = \Lambda V V^T$ (∵ $VV^T =$

$\Rightarrow A = \Lambda$

So, A is digonalizable.

(b) Given, Matrix $A = \begin{pmatrix} -2 & 4 \\ -1 & -6 \end{pmatrix}$

For eigen value of A

we know

$|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -2-\lambda & 4 \\ -1 & -6-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-2-\lambda)(-6-\lambda) + 4 = 0$$

$$\Rightarrow \lambda^2 + 6\lambda + 2\lambda + 12 + 4 = 0$$

$$\Rightarrow \lambda^2 + 8\lambda + 16 = 0$$

$$\Rightarrow (\lambda+4)(\lambda+4) = 0$$

There for the eigen value is $\lambda = -4$.

Since, it has only one eigen value, so it cannot be diagonalizable.

6

8 ⓐ

Given,

$B_A(v,w) = v^T A w$

A is symmetric $n \times n$ matrix.

let α be a scalar Then,

$$B_A(\alpha v, w) = (\alpha v)^T B_A w$$

$$= \alpha (v^T) B_A w$$

$$= \alpha (v^T B_A w)$$

$$= \alpha \; B_A(v,w)$$

Also,

$$B_A(u+v, w) = (u+v)^T B_A w$$
$$= (u^T+v^T) B_A w$$
$$= u^T B_A w + v^T B_A w$$
$$= B(u,w) + B(v,w)$$

Therefore, $B_A(v,w)$ is linear in the first variable

(b) Given, quadratic form,
$$10x_1^2 - 8x_1x_2 + 4x_2^2$$
Then ~~this~~ can be written as,

$$(x_1\ x_2)\begin{pmatrix} 10 & -4 \\ -4 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

which is in the form of $x^T A x$

so,
$$A = \begin{pmatrix} 10 & -4 \\ -4 & 4 \end{pmatrix}$$

Then, $A^T A = \begin{pmatrix} 10 & -4 \\ -4 & 4 \end{pmatrix}\begin{pmatrix} 10 & -4 \\ -4 & 4 \end{pmatrix}$
$$= \begin{pmatrix} 116 & -56 \\ -56 & 32 \end{pmatrix}$$

For eigen value of A^TA
we know,

$$|A^TA - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 116-\lambda & -56 \\ -56 & 32-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (116-\lambda)(32-\lambda) - 3136 = 0$$

$$\Rightarrow 3712 - 116\lambda - 32\lambda + \lambda^2 - 3136 = 0$$

$$\Rightarrow \lambda^2 - 148\lambda + 576 = 0$$

Solving this equation we get,
eigen values ($\lambda_1 = 144$, $\lambda_2 = 4$)

singular value $\sigma_1 = 12$, $\sigma_2 = 2$.

(b) Given quadratic form,

$$10x_1^2 - 8x_1x_2 + 4x_2^2$$

can be written as,

$$(x_1 \ x_2)\begin{pmatrix} 10 & -4 \\ -4 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

which is in the form of x^TAx

so, $A = \begin{pmatrix} 10 & -4 \\ -4 & 4 \end{pmatrix}$.

TU

For eigen value of A

We know, $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} 10-\lambda & -4 \\ -4 & 4-\lambda \end{vmatrix}=0$$

$$\Rightarrow (10-\lambda)(4-\lambda)-16=0$$

$$\Rightarrow 40-4\lambda-10\lambda+\lambda^2-16=0$$

$$\Rightarrow \lambda^2-14\lambda+24=0$$

$$\Rightarrow (\lambda-12)(\lambda-2)=0$$

The eigen values are $(\lambda_1=12,\ \lambda_2=2)$.

Then,

~~This~~ for non cross product term,

~~we know~~; we have

$$\Lambda=\begin{pmatrix} 12 & 0 \\ 0 & 2 \end{pmatrix}$$

Then,

$$y^T\Lambda y = \begin{pmatrix} y_1 & y_2 \end{pmatrix}\begin{pmatrix} 12 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$= \begin{pmatrix} 12y_1 & 0 \\ 0 & 2y_2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$= 12y_1^2+2y_2^2$$

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This is the without cross product term

UT

(9). Given $A = \begin{pmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{pmatrix}$

Then $A^T A = \begin{pmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{pmatrix} \begin{pmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{pmatrix}$

$= \begin{pmatrix} 81 & -27 \\ -27 & 9 \end{pmatrix}$

For eigen value of $A^T A$
we know,

$|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 81-\lambda & -27 \\ -27 & 9-\lambda \end{vmatrix} = 0$

$\Rightarrow (81-\lambda)(9-\lambda) - 27^2 = 0$

$\Rightarrow 729 - 9\lambda - 81\lambda + \lambda^2 - 27^2 = 0$

$\Rightarrow \lambda^2 - 90\lambda = 0$

~~$\Rightarrow \lambda(\lambda - 90) = 0$~~

The eigen value is $\lambda_1 = 90$.

Then, singular value is $\sigma_1 = \sqrt{90}$

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Then, $\Sigma = \begin{pmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

When, $\lambda_1 = \sqrt{90}$

For, Eigen vector

$$(A - \lambda I)x = 0$$

$$\Rightarrow \begin{pmatrix} 81-\sqrt{90} & -27 \\ -27 & 9-\sqrt{90} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$\Rightarrow (81-\sqrt{90})x_1 - 27x_2 = 0$$

$$-27x_1 + (9-\sqrt{90})x_2 = 0.$$

3

(10).

$$\begin{cases} 3x - y + 7z = 9 \\ 5x + 3y + 2z = 10 \\ 9x + 2y - 5z = 6 \end{cases}$$

The augmented matrix is:-

$$\left[\begin{array}{ccc|c} 3 & -1 & 7 & 9 \\ 5 & 3 & 2 & 10 \\ 9 & 2 & -5 & 6 \end{array}\right]$$

$R_1 \leftarrow R_1/3$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/3 & 7/3 & 3 \\ 5 & 3 & 2 & 10 \\ 9 & 2 & -5 & 6 \end{array}\right]$$

$R_2 \leftarrow R_2 - 5R_1$

$R_3 \leftarrow R_3 - 9R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/3 & 7/3 & 3 \\ 0 & 14/3 & -29/3 & -5 \\ 0 & -1 & -26 & -12 \end{array}\right]$$

$R \Leftarrow R_3 \Leftrightarrow R_2$

$3 - 5/3$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/3 & 7/3 & 3 \\ 0 & -1 & -26 & -12 \\ 0 & 4/3 & -29/3 & -5 \end{array}\right]$$

$R_2 \leftarrow -R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/3 & 7/3 & 3 \\ 0 & 1 & 26 & 12 \\ 0 & 14/3 & -29/3 & -5 \end{array}\right]$$

$R_3 \leftarrow R_3 - 14/3 R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/3 & 7/3 & 3 \\ 0 & 1 & 26 & 12 \\ 0 & 0 & -131 & -5 \end{array}\right]$$

$R_3 \leftarrow R - \frac{?}{?} R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/3 & 7/3 & 3 \\ 0 & 1 & 26 & 12 \\ 0 & 0 & 1 & -61 \end{array}\right]$$

The valu is ~~xpe~~ $x=1, y=1, z=1$.

(9)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Therefor the value of $x=1$, $y=1$, $z=1$ is solved using reduced rowechelon form.

Reduced row echolen form is a type of form where ~~p~~ leading pivot element in ~~ar~~ row is 1 and all other ~~non-diagonal~~ are 0.

4

(4) $Q(x) = x_1^2 - 6x_1x_2 - 9x_2^2$.

The quadratic form can be written as,

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which is in form $x^T A x$

so, $A = \begin{pmatrix} 1 & -3 \\ -3 & -9 \end{pmatrix}$

We know for eigen value,

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & -3 \\ -3 & -9-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)(-9-\lambda) - 9 = 0$$

$$\Rightarrow \lambda^2 + 8\lambda - 18 = 0$$

Solving this equation we get eigen values. ~~$-4+34$,~~ 0.

$-4+\sqrt{34}, 0$.

So, this quadratic form is negative semi definite as its eigen value is negative and 0.

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