Find the domain and range of the function

Q19. $f(x, y) = x^2 + y^2$

The given function is defined for any value of x and y. Thus, the domain of the given function is entire plane of $\mathbb{R}^2$.

$\therefore$ Domain$_f$ = $\{(x, y) \in \mathbb{R}^2\}$

For Range of f

As we know $x^2 + y^2 \geq 0$. Thus,

$0 \leq z < \infty$

$\therefore$ Range$_f$ = $[0, \infty)$

Q20. $f(x, y) = e^{xy}$

The given function f is defined for any value of x and y. Thus, the domain of f is entire plane of $\mathbb{R}^2$.

$\therefore$ Domain$_f$ = $\{(x, y) \in \mathbb{R}^2\}$

Range of f

As we know $e^{xy} > 0$. Thus,

$0 < z < \infty$

$\therefore$ Range$_f$ = $(0, \infty)$

Q21. $g(x, y) = x\sqrt{y}$

The given function g is defined for any value of x and for $y \geq 0$. Thus, domain of g is a subset plane of $\mathbb{R}^2$ such that $y \geq 0$.

$$\therefore \text{Domain}_{fg} = \{(x, y) \in \mathbb{R}^2 \mid y \geq 0\}$$

<u>For Range of g</u>

As we know; $-\infty \leq x\sqrt{y} < \infty$. Thus,

$$-\infty \leq z < \infty$$

$$\therefore \text{Range}_g = \cancel{\{(\infty, \infty)\}}\ (\infty, \infty)$$

Q22. $f(x, y) = y/\sqrt{x}$

The given function f is defined for all values of y and any value of $x > 0$. Thus, domain of f is a subset plane of $\mathbb{R}^2$ where $x > 0$.

$$\therefore \text{Domain}_f = \{(x, y) \in \mathbb{R}^2 \mid x > 0\}$$

<u>For range of f</u>

We know $-\infty < y/\sqrt{x} < \infty$. Thus,

$$-\infty < z < \infty$$

$$\therefore \text{Range}_f = (-\infty, \infty) \quad \text{or } \mathbb{R}$$

Q23. $z = \dfrac{x+y}{xy}$

The given function is defined for all values of x and y except $xy = 0$. Thus, domain of given function is the subset of plane $\mathbb{R}^2$ such that $xy \neq 0$.

$$\therefore \text{Domain}_f = \{(x, y) \in \mathbb{R}^2 \mid xy \neq 0\}$$

(ii) Range of f

As we know $-\infty < \frac{x+y}{xy} < \infty$. Thus,

$$-\infty < z < \infty$$

$\therefore$ $\text{Range}_f = (-\infty, \infty)$ or $\mathbb{R}$

Q24 $z = \frac{xy}{x-y}$

The given function is defined for all values of x and y such that $x \neq y$. Thus, domain of given function is the subset of plane $\mathbb{R}^2$ such that $x \neq y$.

$\therefore$ $\text{Domain}_f = \{(x, y) \in \mathbb{R}^2 \mid x \neq y\}$

For range of f

As we know, $-\infty < \frac{xy}{x-y} < \infty$, thus,

$$-\infty < z < \infty$$

$\therefore$ $\text{Range}_f = (-\infty, \infty)$ or $\mathbb{R}$

Q25 $f(x,y) = \sqrt{4 - x^2 - y^2}$

The domain D of f is the set of points (x, y) such that

$$4 - x^2 - y^2 \geq 0$$

Thus, D is the set of all points lying on or inside the circle $x^2 + y^2 \leq 4$

$\therefore$ $\text{Domain}_f = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 4\}$

Range of f

We have, $z = \sqrt{4 - x^2 - y^2} = \sqrt{4 - (x^2 + y^2)}$

Since, $x^2 + y^2 \geq 0$ we obtain,

$0 \le z \le \sqrt{4} = 2$

$\therefore \text{Range}_f = [0, 2]$

Q26. $f(x,y) = \sqrt{4 - x^2 - 4y^2}$
$= \sqrt{4 - (x^2 + 4y^2)}$

The domain D of f is the set of all points (x, y) such that

$$4 - (x^2 + 4y^2) \ge 0$$

Thus, D is the set of all points lying on or inside the ellipse $x^2 + 4y^2 \le 4$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{1} \le 1$$

$\therefore \text{Domain}_f = \{(x,y) \in \mathbb{R}^2 \mid x^2 + 4y^2 \le 4\}$

Range of f

We have, $f(x,y) = \sqrt{4 - (x^2 + 4y^2)}$

Since, $x^2 + 4y^2 \ge 0$, we obtain,

$0 \le z \le \sqrt{4} = 2$

$\therefore \text{Range}_f = [0, 2]$

Q27. $f(x,y) = \arccos(x+y)$

The domain D of f is the set of all points (x, y) such that

$$-1 \le x+y \le 1$$

This is because the value of $\cos\theta$ lies between -1 and $+1$.

$\therefore \text{Domain}_f = \{(x,y) \in \mathbb{R}^2 \mid -1 \le x+y \le 1\}$

Q27. $f(x,y) = \arccos(x+y)$

The domain D of f is set of all points (x,y) such that

$$-1 \le x+y \le +1$$

$$\therefore \text{Domain}_f = \{(x,y) \in \mathbb{R}^2 \mid -1 \le x+y \le 1\}$$

For Range f

As we know the value of $\cos^{-1}(x+y)$ lies between -1 and $+1$.

For -1,

$z = \cos^{-1}(-1) = \pi$

For $+1$,

$z = \cos^{-1}(1) = 0$

$$\therefore \text{Range}_f = [0, \pi]$$

Q28. $f(x,y) = \arcsin(y/x)$

The domain D of f is set of points (x,y) such that

$-1 \le y/x \le 1$ where $x \ne 0$

$\Rightarrow -x \le y \le x$

$$\therefore \text{Domain}_f = \{(x,y) \in \mathbb{R}^2 \mid -x \le y \le x,\ x \ne 0\}$$

For range of f

As we know the value of $\sin^{-1}(y/x)$ lies between -1 and $+1$.

For -1,

$z = \sin^{-1}(-1) = -\frac{\pi}{2}$

For $+1$,

$z = \sin^{-1}(1) = \pi/2$

$$\therefore \text{Range}_f = [-\pi/2, \pi/2]$$

Q29. $f(x,y) = \ln(4-x-y)$

The domain D of f is a set of points (x,y) such that

$$4-x-y > 0$$

$$\Rightarrow x+y < 4$$

Thus domain is a plane defined by the condition $x+y<4$

$\therefore \text{Domain}_f = \{(x,y) \in \mathbb{R}^2 \mid x+y<4\}$

For range of f

As we know $x+y<4$, we obtain,

$$0 \le z < \infty$$

$$\therefore \text{Range}_f = [0, \infty)$$

Q30. $f(x,y) = \ln(xy-6)$

The domain D of f is a set of points (x,y) such that

$$xy-6 > 0$$

$$\therefore xy > 6$$

$\therefore \text{Domain}_f = \{(x,y) \in \mathbb{R}^2 \mid xy>6\}$

For range f

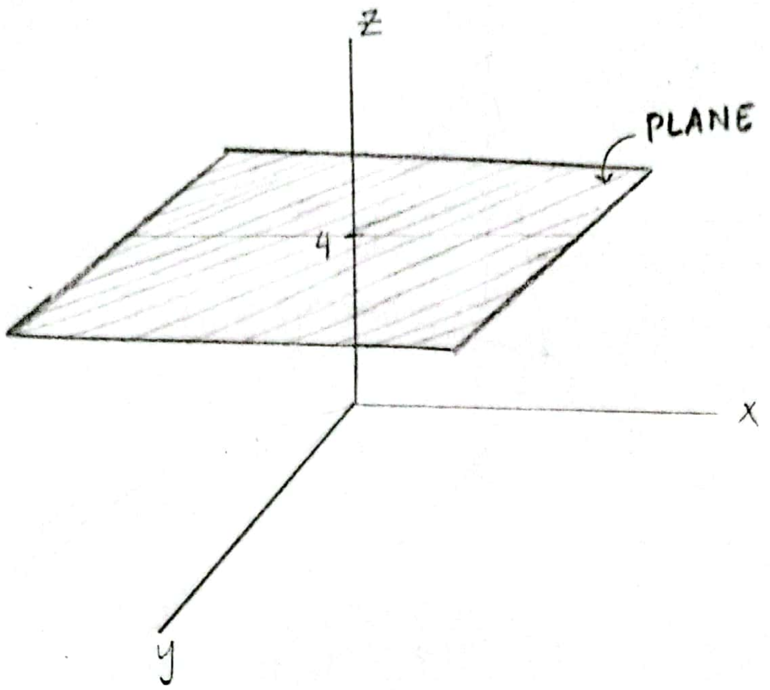
We have, $z = \ln(xy-6)$

Since $xy>6$, we obtain,

$$0 \le z < \infty$$

$$\therefore \text{Range}_f = [0, \infty)$$

33. $f(x, y) = 4$



The surface represented by $f(x,y) = 4$ (i.e. $z = 4$) is a plane parallel to xy plane that includes the point $(0, 0, 4)$ in it.

34. $f(x, y) = 6 - 2x - 3y$

As we know,

$$z = f(x, y) = 6 - 2x - 3y.$$

$\Rightarrow$ $2x + 3y + z = 6$ — (i) is a plane in Euclidean space.

Put $x = y = 0$ in (i), we get:

$z = 6$

Put $x = z = 0$ in (i), we get:

$3y = 6$

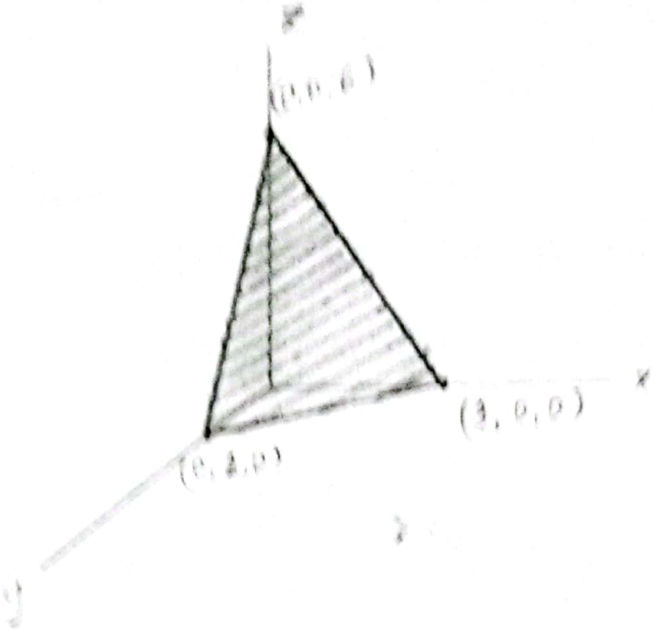
$\therefore y = 2$

Put $y = z = 0$ in (i), we get:

$2x = 6$

$\therefore x = 3$

Thus, points $(3, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 6)$ lie in the plane (i).

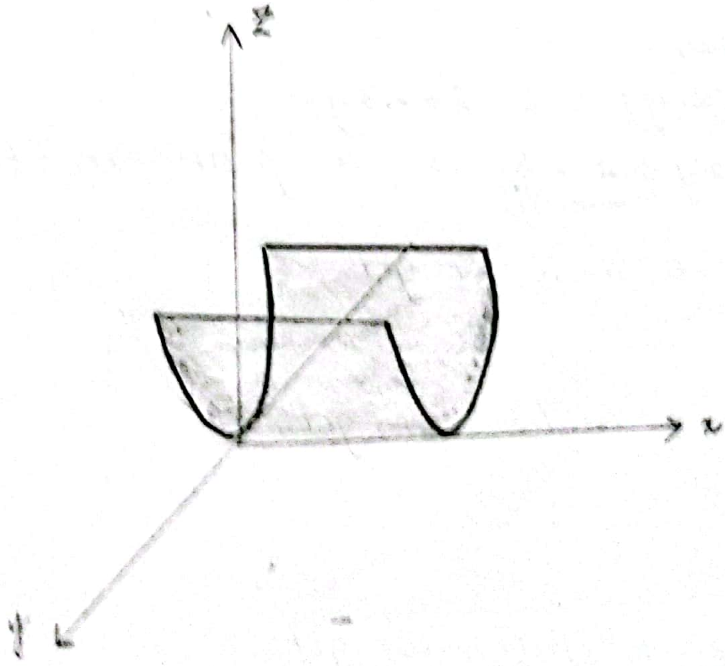


35. $f(x,y) = y^2$

As we know, $z = f(x,y)$. Thus,

$z = y^2$ —— ①

Equation ① represents a parabola. In Euclidian space, it represents a parabolic surface as shown in figure below.



Q36. $g(x,y) = \frac{1}{2}y$

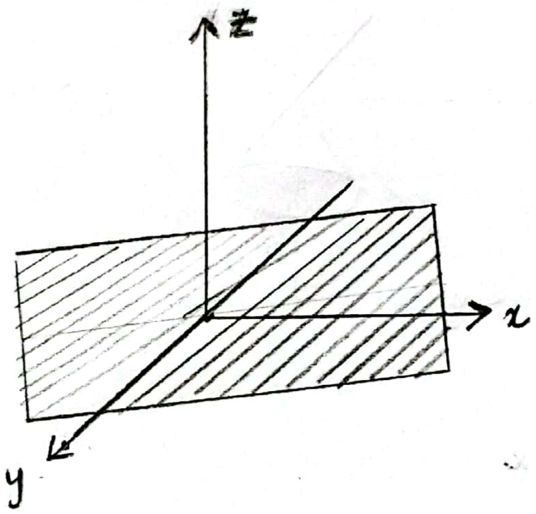
We know,

$z = g(x,y)$. Thus,

$z = \frac{y}{2}$

$\Rightarrow 2z - y = 0$ —— (i)

Equation (i) represents a plane that contains origin in Euclidean space. It is as shown in figure below:



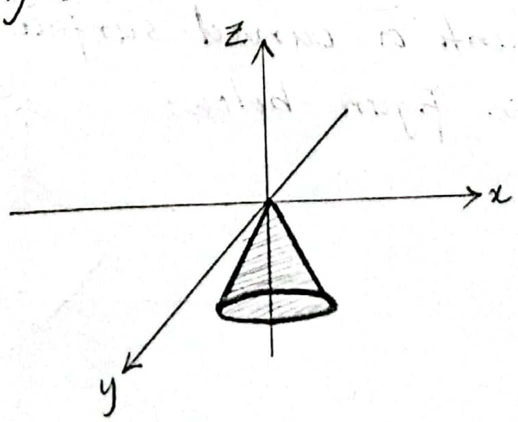
Q37. $z = -x^2 - y^2$

The above equation can be written as:

$z = -(x^2 + y^2)$

$\Rightarrow x^2 + y^2 = -z$ —— (i)

The above equation represents a downward facing cone as shown in diagram below:



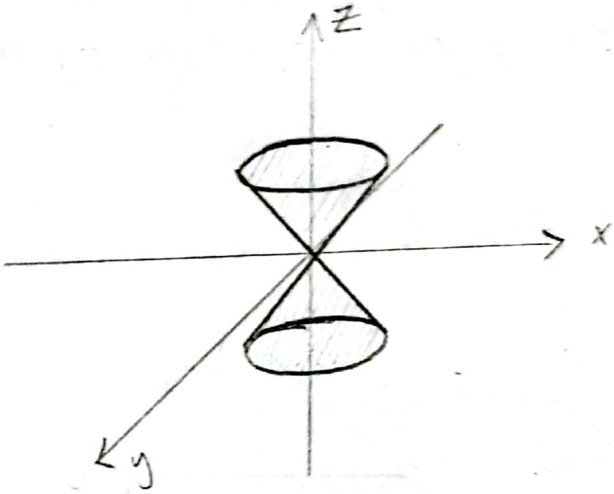
Q38. $z = \frac{1}{2}\sqrt{x^2+y^2}$

Squaring both sides, we get:

$4z^2 = x^2+y^2$

$\therefore x^2+y^2 = 4z^2$ ——— (i)

Above equation represents the equation of a double napped cone. The sketch is as shown below:



Q39. $f(x,y) = e^{-x}$

We know,

$z = f(x,y)$. Thus,

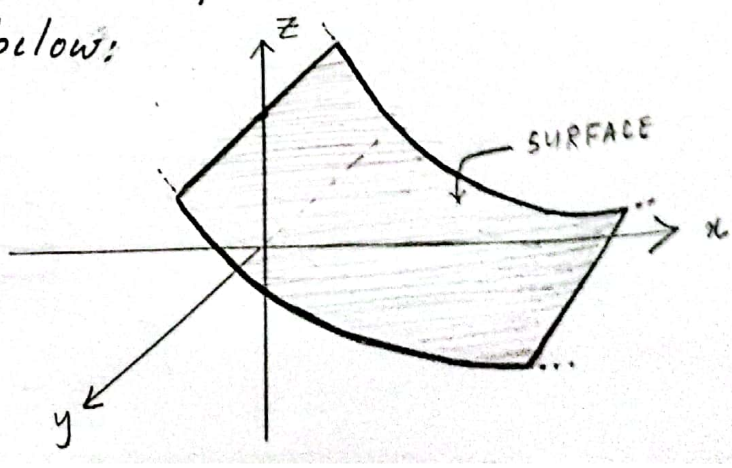
$z = e^{-x}$ ——— (i)

Put $x = 0$,

$z = e^{-0} = e^{0} = 1$

~~Thus~~ From (i), we can see that $z \longrightarrow 0$ as $x \longrightarrow \infty$.

Thus, eq^n (i) represents a curved surface in Euclidean space as shown in figure below:



Describe the level curves of the function. Sketch a contour map of the surface using level curves for the given c-values.

The level curves of a function of two variables are the curves with equations $f(x,y) = c$, where c is a constant (in the range of f). A graph of the various level curves of a function is called a <u>contour map</u>.

Q49. $z = x+y$

Take c-values from $c_0 = 0$ to $c_4 = 4$.

Then for $c_0 = 0$,

$f(x,y) = 0$

$\Rightarrow x+y = 0$

$\Rightarrow y = -x$

x	0	1	-1
y	0	-1	1

for $c_1 = 1$,

$f(x,y) = c_1$

$\Rightarrow x+y = 1$

$\Rightarrow y = 1-x$

x	1	0
y	0	1

for $c_2 = 2$,

$f(x,y) = c_2$

$\Rightarrow x+y = 2$

$\Rightarrow y = 2-x$

x	2	0
y	0	2

for $c_3 = 3$,

$f(x,y) = c_3$

$\Rightarrow x+y = 3$

$\Rightarrow y = 3-x$

x	0	3
y	3	0

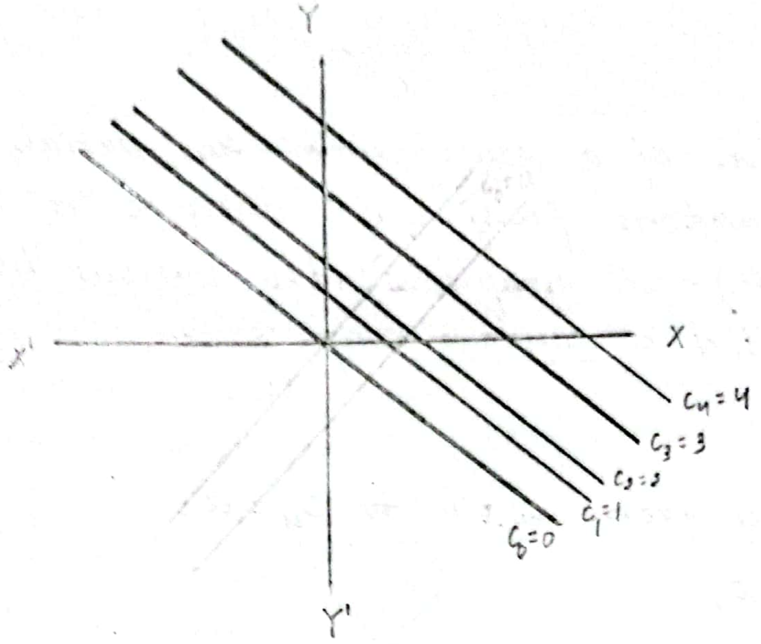
for $c_4 = 4$,

$f(x,y) = c_4$

$\Rightarrow x+y = 4$

x	4	0
y	0	4

CONTOUR MAP:



Q50. $f(x,y) = 6-2x-3y$

Put $c_0=0, \ldots, c_2=2$ for above function. Then,

for $c_0=0$, $f(x,y)=0$

$\Rightarrow 6-2x-3y=0$

$\Rightarrow 2x+3y-6=0$

x	3	0
y	0	2

Put $c_1=1$, $f(x,y)=1$

$\Rightarrow 6-2x-3y=1$

$\Rightarrow 2x+3y-5=0$

x	0	$5/2$
y	$5/3$	0

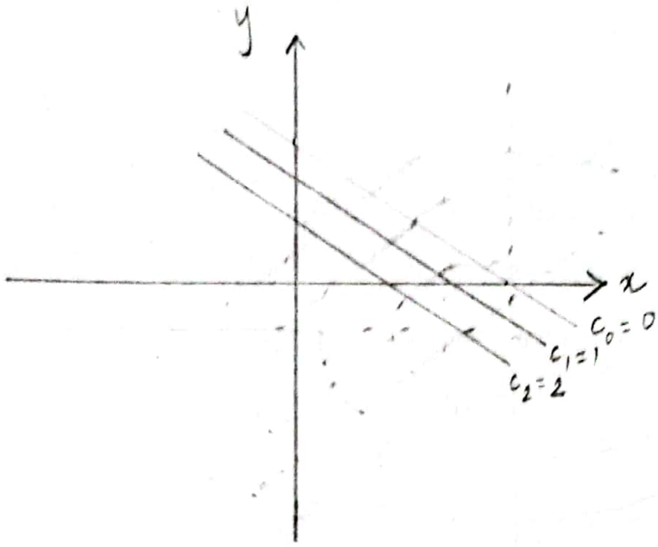
Put $c_2=2$, $f(x,y)=2$

$\Rightarrow 6-2x-3y=2$

$\Rightarrow 2x+3y-4=0$

x	0	2
y	$4/3$	0

CONTOUR MAP:



Q51. $z = x^2 + 4y^2$

Put $c_0 = 0, \dots, c_3 = 3$ for above function. Then,

for $c_0 = 0$, $f(x, y) = 0$

$\Rightarrow x^2 + 4y^2 = 0$

x	0
y	0

for $c_1 = 1$, $f(x, y) = 1$

$\Rightarrow x^2 + 4y^2 = 1$

x	± 1	0
y	0	$\pm 1/2$

for $c_2 = 2$, $f(x, y) = 2$

$\Rightarrow x^2 + 4y^2 = 2$

x	$\pm\sqrt{2}$	0
y	0	$\pm 1/\sqrt{2}$

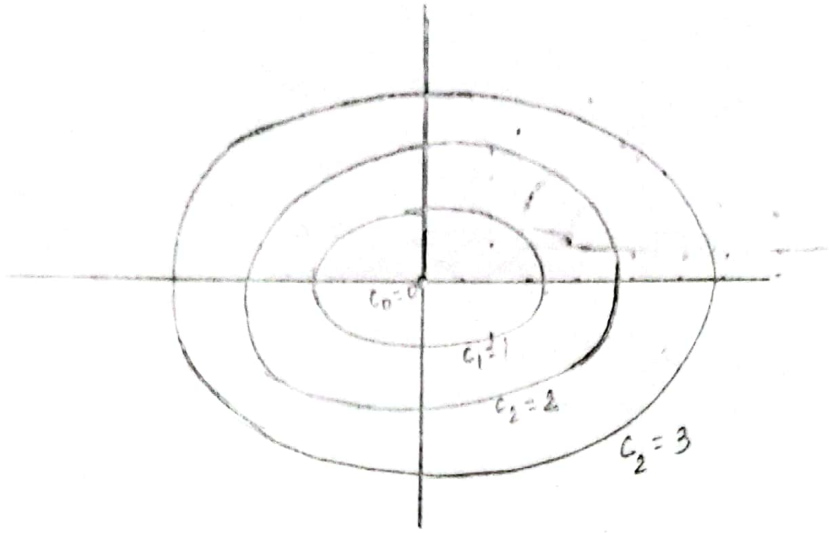
for $c_3 = 3$, $f(x, y) = 3$

$\Rightarrow x^2 + 4y^2 = 3$

x	0	$\pm\sqrt{3}$
y	$\pm 3/\sqrt{4}$	0

Here, the level curves form ellipses.

CONTOUR MAP:



Q 52. $f(x,y) = \sqrt{9-x^2-y^2}$

Put $c_0=0, \dots, c_2=2$ in above function. Thus,

for $c_0=0$, $f(x,y)=c_0$

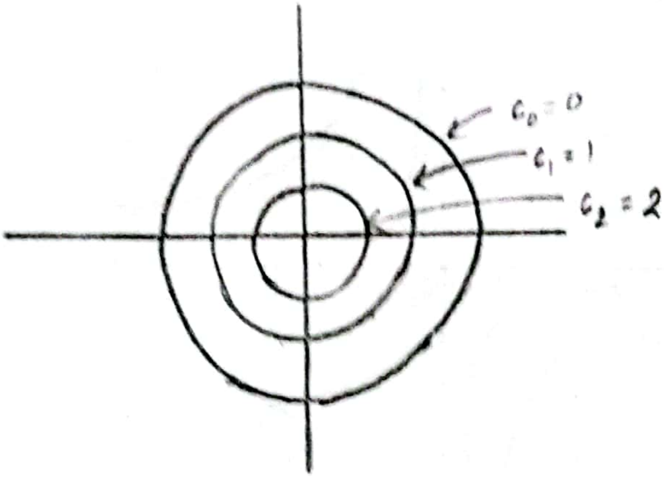
$$\Rightarrow \sqrt{9-x^2-y^2} = 0$$
$$\Rightarrow 9-x^2-y^2=0$$
$$\therefore x^2+y^2=9$$

for $c_1=1$, $f(x,y)=c_1$

$$\Rightarrow \sqrt{9-x^2-y^2} = 1$$
$$\Rightarrow 9-x^2-y^2=1$$
$$\therefore x^2+y^2 = 8$$

for $c_2=2$, $f(x,y)=c_2$

$$\Rightarrow \sqrt{9-x^2-y^2} = 2$$
$$\Rightarrow 9-x^2-y^2=4$$
$$\therefore x^2+y^2=5$$



Q53.

$f(x, y) = xy$

Put $c_0 = 0, \ldots, c_2 = 2$ in above function. Then,

for $c_0 = 0$,

$f(x, y) = c_0$

$\Rightarrow xy = 0$

x	0
y	0

for $c_1 = 1$,

$f(x, y) = c_1$

$\Rightarrow xy = 1$

x	1	5	1/5
y	1	1/5	5

for $c_2 = 2$,

$f(x, y) = 2$

$\Rightarrow xy = 2$

x	2	1	$\sqrt{2}$
y	1	2	$\sqrt{2}$

Here, the level curves produce a rectangular hyperbola.

CONTOUR MAP: