Tribhuvan University

Institute of Sciences and Technology

SCHOOL OF MATHEMATICAL SCIENCES

First Assessment 2080

MATH

Subject: **Mathematics for Data Science**
Course No: **MDS 504**
Level: **MDS /I Year /I Semester**

Full Marks: 45
Pass Marks:22.5
Time: 2 hrs

Candidates are required to give their answer in their own words as far as practicable.
Attempt All Questions.

Group A [5×3 = 15]

1. What is the parallel coordinates method? Explain with an example. What is the use of this method in data science?
2. Define L_1, L_2 and L_∞ norms on $\mathbf{R}^n$. Calculate L_1, L_2 and L_∞ norms of the vector $x = (1, -1, 0, \ldots, 0, 2)$ on $\mathbf{R}^n$.
3. What is the angle between the diagonal of theunit cube in the positive orthant and the vector e_1 in $\mathbf{R}^3$?
4. Show that $u=\begin{pmatrix}-1\\1\end{pmatrix}$ and $v=\begin{pmatrix}-1\\--1\end{pmatrix}$ are orthogonal in $\mathbf{R}^2$ and find corresponding orthonormal basis for $\mathbf{R}^2$.
5. Prove that eigen vectors v_1 and v_2 that correspond to distinct eigen values λ_1 and λ_2 of a 2×2 matrixare linearly independent.

Group B [5×6 = 30]

6. Let S and T be matrix transformations defined by $S(y) = Ay$ and $T(x) = Bx$, where
$$A=\begin{pmatrix}1 & 2 & 0\\0 & 1 & 1\end{pmatrix} \text{ and } B=\begin{pmatrix}3 & 0\\5 & -2\\0 & 1\end{pmatrix}.$$
   a) What are the domains and codomains of S and T? Why is the compositetransformation $S\circ T$ defined? What are thedomain and the codomain of $S\circ T$?
   b) Let $x=\begin{pmatrix}x_1\\x_2\end{pmatrix}$. Determine $T(x)$.
   c) Find $(S\circ T)(x)$.
   d) Find a matrix C so that $(S\circ T)(x) = Cx$.
   e) Show that $S\circ T$ is linear.

7. Prove that if θ is the angle between two non-zero vectors x, y in $\mathbf{R}^n$, then
$$x \cdot y = \|x\|\|y\|\cos\theta.$$
8. Let
$$V=\left\{\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} : x_1 + x_2 + x_3 = 0\right\},\ B=\left\{\begin{pmatrix}1\\-1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\--1\end{pmatrix}\right\}$$
Show that V is a subspace of $\mathbf{R}^3$, and B is a basis for V.

OR

Show that the following set of vectors is a basis for $\mathbf{R}^3$, and then express the standard basis vectors: e_1, e_2, e_3 in terms of these:
$$u_1=\begin{pmatrix}4\\2\\1\end{pmatrix},\ u_2=\begin{pmatrix}-5\\2\\3\end{pmatrix},\ u_3=\begin{pmatrix}1\\3\\0\end{pmatrix}.$$
9. Consider the following matrix: $A=\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$.
   a) What can we say about the action of A on an arbitrary vector?
   b) What are examples of eigen values and eigen vectors of this matrix ?
   c) What does the discussion for this example illustrate?

OR

Find the eigenvalues and eigenvectors of $A=\begin{pmatrix} 5 & -2 \\ 7 & -4 \end{pmatrix}$. State and sketch the effect of multiplying the eige vector by the matrix A.

10. Let A be a square matrix with eigen vector u belonging to Eigen value λ. Prove that
   a) If m is a natural number then λ^m is an eigen value of the matrix A^m with the *same* eigen vector u.
   b) If the matrix A is invertible then the eigen value of the inverse matrix A^{-1} is $1/\lambda = \lambda^{-1}$ with the *same* eigen vector u.

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Tribhuvan University

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SCHOOL OF MATHEMATICAL SCIENCES

First Reassessment 2081

Subject: **Mathematics for Data Science** *Full Marks*: 45

Course No: **MDS 504** *Pass Marks*:22.5

Level: **MDS /I Year /I Semester** *Time*: 2 hrs

Candidates are required to give their answer in their own words as far as practicable.

Attempt All Questions.

Group A [5×3 = 15]

1. How are grayscale images stored in a computer? Explain it in brief.
2. Let $u=\begin{pmatrix}1\\2\end{pmatrix}$ and $v=\begin{pmatrix}-1\\1\end{pmatrix}$.
   a) Write the vector $\begin{pmatrix}3\\2\end{pmatrix}$ in terms of the vectors u and v.
   b) Show that the vectors u and v span $\mathbf{R}^2$.
3. Define linearly independent vectors. Prove that an orthogonal set of nonzero vectors in a vector space is linearly independent.
4. Show that $u=\begin{pmatrix}1\\1\end{pmatrix}$ and $\mathbf{v}=\begin{pmatrix}1\\-1\end{pmatrix}$ are orthogonal in $\mathbf{R}^2$ and find corresponding orthonormal basis for $\mathbf{R}^2$.
5. Prove that eigen vectors v_1 and v_2 that correspond todistinct eigenvalues λ_1 and λ_2 of a 2×2 matrix are linearly independent.

Group B [5×6 = 30]

6. Explain four major ways to view a matrix. Prove that if $S:\mathbf{R}^l \to \mathbf{R}^m$ and $T: \mathbf{R}^n \to \mathbf{R}^l$ are linear transformations, given by matrices A and B, respectively, then, the composition $S\circ T: \mathbf{R}^n \to \mathbf{R}^m$ is a linear transformation and is given by AB.
7. By showing that the L_1-norm satisfies each of the conditions inthe definition of a norm prove thisis a norm. First do this for $\mathbf{R}^2$, and then do this for $\mathbf{R}^n$.

   OR

   Define the span of a set. Prove that span $\{v_1,v_2,\ldots,v_k\}\subseteq V$ is a subspace of a vector space V. Also, show that if $x\in\mathbf{R}^2$, such that $x\neq 0$, then $x^\perp$ is a subspace of $\mathbf{R}^2$.
8. Show that the following set of vectors is a basis for $\mathbf{R}^3$, and then express the standard basis vectors: e_1, e_2, e_3 in terms of these:

   $$u_1=\begin{pmatrix}4\\2\\1\end{pmatrix},\ u_2=\begin{pmatrix}-5\\2\\3\end{pmatrix},\ u_3=\begin{pmatrix}1\\3\\0\end{pmatrix}.$$

   OR

   Let V be a subspace of $\mathbf{R}^n$ and w a vector in $\mathbf{R}^n$.
   a) If $\{v_1,v_2,\ldots,v_k\}$ is an orthogonal basis for V, derive the expression for the projection of w onto V.
   b) If $\{v_1,v_2,\ldots,v_k\}$ is an orthonormal basis for V, derive the expression for the projection of w onto V.
9. Consider the following matrix: $A=\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$.
   a) What can we say about the action of A onan arbitrary vector?
   b) What are examples of eigenvalues andeigenvectors of this matrix ?
   c) What does the discussion for this exampleillustrate?
10. Find the eigenvalues and eigenvectors of $A=\begin{pmatrix}2 & 0\\1 & 3\end{pmatrix}$. State and sketch the effect of multiplying the eigenvector by the matrix A.

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Tribhuvan University
Institute of Sciences and Technology

SCHOOL OF MATHEMATICAL SCIENCES

Second Assessment 2081

Subject: **Mathematics for Data Science**
Course No: **MDS 504**
Level: **MDS /I Year /I Semester**

Full Marks: 45
Pass Marks: 22.5
Time: 2 hrs

Candidates are required to give their answers in their own words as far as practicable.
Attempt All Questions.

Group A [5×3 = 15]

1. If $\lambda = 1, 5$ eigenvalues of the matrix $\begin{pmatrix} 7 & 4 \\ -3 & -1 \end{pmatrix}$, find a basis for the eigenspace corresponding to each eigenvalue.

2. Find the maximum value of $9x_1^2 + 4x_2^2 + 4x_3^2$ subject to the constraints $x^Tx = 1$ and $x^Tu_1 = 0$, where $u_1 = (1, 0, 0)$. Find x where it is attained. Here, u_1 is a unit eigen vector corresponding to the greatest eigenvalue $\lambda = 9$ of the matrix of the quadratic form.

3. Find the singular values of the matrix $\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{pmatrix}$.

4. Consider the quadratic form $Q(x) = 2x_1^2 + 4x_1x_2 - 4x_3x_1 - x_2^2 + 8x_3x_2 - x_3^2$. Decide whether this quadratic form is positive, negative or indefinite.

5. Determine if the following homogeneous system has a nontrivial solution. Then describe the solution set.
$3x_1 + 5x_2 - 4x_3 = 0,\ -3x_1 - 2x_2 + 4x_3 = 0,\quad 6x_1 + x_2 - 8x_3 = 0.$

Group B [5×6 = 30]

6. Consider the matrix: $A = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$.
   a) What can we say about the action of A on an arbitrary vector ?
   b) What are examples of eigenvalues and eigenvectors of this matrix?
   c) What does the discussion for this example illustrate?

   OR

   a) Let v_1, v_2 be the eigenvectors associated with the eigenvalues λ_1, λ_2 of a 2×2 symmetric matrix A respectively. Prove that if $\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$ and $V = (v_1 v_2)$, then $A = V\Lambda V^T$.
   b) Find all 2×2 matrices A which admit the normalized eigenvectors $v_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $v_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ with the corresponding eigenvalues λ_1 and λ_2.

7. a) Let A be an $n \times n$ matrix. Prove that if A has n linearly independent eigenvectors, then A is diagonalizable.
   b) Show that the matrix $A = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix}$ is not diagonalizable.

8. a) Prove that if A is a symmetric $n\times n$ matrix and $B_A(v,w) = v^T Aw$, then $B_A(v,w)$ is linear in the first vari

b) Write the quadratic form $10x_1^2 - 8x_1 x_2 + 4x_2^2$ as $x^T Ax$. Transform it into a quadratic form without the cross

product term using eigenvalues and eigenvectors.

9. Find an SVD of the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{pmatrix}$.

OR

a) Prove that if A is an $m\times n$ matrix, then all the eigenvalues of $A^T A$ are non-negative.

b) Find the eigenvalues and eigenvectors of $A^T A$ where $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ -2 & 1 \end{pmatrix}$.

10. What is reduced row echelon form? Illustrate with an example of an augmented matrix of order 4×5. Solve the following linear system by placing the augmented matrix in reduced row echelon form.

$2x + y - z = 2, 4x + 3y + 2z = -3,\ \ 6x - 5y + 3z = -14.$

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Tribhuvan University
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SCHOOL OF MATHEMATICAL SCIENCES
Second Reassessment 2081

Subject: **Mathematics for Data Science**
Course No: **MDS 504**
Level: **MDS /I Year /I Semester**

Full Marks: 45
Pass Marks: 22.5
Time: 2 hrs

Candidates are required to give their answers in their own words as far as practicable.
Attempt All Questions.

Group A [5×3 = 15]

1. The matrix $\begin{pmatrix} -12 & 7 \\ -7 & 2 \end{pmatrix}$ has only one eigenvalue −5. Determine a set of basis vectors for E_λ and plot the eigen space E_λ.
2. Find the maximum value of $5x_1^2 + 5x_2^2 - 4x_1x_2$ subject to the constraints $x^Tx = 1$. Find x where it is attained.
3. Find the singular values of the matrix $\begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$.
4. Consider the quadratic form $Q(x) = x_1^2 - 6x_1x_2 - 9x_2^2$. Decide whether thisquadratic form is positive, negative, indefinite, positive semi-definite or negative semi-definite.
5. What is a linear homogeneous system?Prove that if**u** and **v** be solutions of a linear homogeneous system **Ax** = **0**, for anyscalars k and c the vector k**u**+ c**v**is also a solution of **Ax** = **0**. What does this statement mean?

Group B [5×6 = 30]

6. Consider the matrix:$A = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$.
   a) What can we say about the action of Aonan arbitrary vector ?
   b) What are examples of eigenvalues andeigenvectors of this matrix?
   c) What does the discussion for this exampleillustrate?

OR

   a) Let v_1, $v_2$be the eigenvectors associated with theeigenvalues λ_1, $\lambda_2$of a 2 × 2 symmetric matrix Arespectively. Prove that if$\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}andV = (v_1 v_2)$,then
   $A = V \Lambda V^T$.
   b) Find all 2 × 2 matrices A which admit the normalized eigenvectors $v_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $v_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$with the corresponding eigenvalues $\lambda_1$and λ_2.
7. a) Let Abe an $n\times n$matrix. Prove that if Ahas $n$linearlyindependent eigenvectors, then Aisdiagonalizable.
   b) Show that the matrix $A = \begin{pmatrix} -2 & 4 \\ -1 & -6 \end{pmatrix}$isnotdiagonalizable.
8. a) Prove that if A is a symmetric $n\times n$ matrix and $B_A(v,w) = v^TAw$, then $B_A(v,w)$ is linear in thefirst variable v.
   b) Write the quadratic form $10x_1^2 - 8x_1x_2 + 4x_2^2$as x^TAx. Transform it into a quadratic form without the cross product term using eigenvalues and eigenvectors.
9. Find an SVD of the matrix$\begin{pmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{pmatrix}$.

OR

   a) Prove that if A is an $m\times n$ matrix, then all the eigenvalues of $A^TA$are non-negative.
   b) Find the eigenvalues and eigenvectors of $A^TA$where $A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$.
10. What isareduced rowechelon form? Explain with an example. Solve the following linear system by placing the augmented matrix in reduced rowechelon form.
    $3x - y + 7z = 9$, $5x + 3y + 2z = 10$, $9x + 2y - 5z = 6$.

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Tribhuvan University
Institute of Science and Technology
2080
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Subject: **Mathematics for Data Science**
Course No: **MDS 504**
Level: **MDS /I Year /I Semester**

Full Marks:45
Pass Marks:22.5
Time:2 hrs

Candidates are required to give their answer in their own words as far as practicable.
Attempt All questions.

Group A [5 × 3 = 15]

1. Are the following sets form the subspace of R^2? Justify.

 a) The set $S = \{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : y + z = 2$ and $x, y, z \in R \}$ in the vector space R^3.

 b) The closed L_2 ball ,$B(0,3)=\{X=\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in R^2 : \|X\| \leq 9\}$. [1.5+1.5]

2. What is rank of a matrix? Reduce the matrix $A = \begin{pmatrix} 3 & 1 & 3 & 8 \\ 2 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 8 & 4 & 11 & 17 \end{pmatrix}$ to Echelon form and hence find the rank. [1+2]

3. Let $T : R^2 \to R^2$ be defined by $T(1, 0) = (1, 3)$, $T(1, 2) = (0, -2)$. If T is linear, find the formula for $T(x, y)$. What is the matrix represented by T relative to the standard basis? [1.5+1.5]

4. What is meant by linear independence of a set of vectors. Let V and W be two vector spaces over the field F and $T: V \to W$ be a linear transformation. Then prove that the kernel of T is a subspace of V and the image of T is a subspace of W. [1+1+1]

5. What is quadratic form? Let S be a 3×3 square matrix with a quadratic form in 3 variables. Then there exists a 3×3 symmetric matrix T such that $X^T S X = X^T T X, \forall X \in R^3$. [1+2]

Group B

[5 ×6 = 30]

6. What is L_∞ norm on a vector n-space R^n? Write any two properties. Let $\|.\|$ be the Euclidean norm , X and Y be two vectors in R^n. State and prove the Triangle Inequality and Parallelogram Law. Verify Cauchy -Schwarz inequality for $X = (1, 3)$ and $Y = (2, 1)$.
[1+2+2+1]

7. Define Kernel and Image of a linear transformation $T: V \rightarrow W$. If $v_1, v_2, ..., v_n$ are linearly independent vectors in V and $KerT = \{0_V\}$.Is the set { $T(v_1), T(v_2), ..., T(v_n)$} forms linearly independent vectors in W? Justify. Also, show that the set {(0, 1), (1, 1)} of vectors span R^2.
[1+2.5+2.5]

OR

What is quadratic form. Let A be a 3 ×3 square matrix with a quadratic form in 3 variables. Then there exists a 3 ×3 symmetric matrix B such that $X^TAX = X^TBX, \forall X \in R^3$. Further, express the quadratic form $x_1^2 + x_1x_2 - 4x_3x_1 + 2x_2x_3 - 4x_3^2$ as the difference of squares.
[1+2.5+2.5]

8. Let $A = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$ be a symmetric matrix. Find the orthogonal matrix D such that $D^{-1}AD$ is a diagonal matrix.Let $u_1, u_2, ..., u_n$ be the eigenvectors associated with the eigenvalues $\lambda_1, \lambda_2, ..., \lambda_n$ of a $n \times n$ symmetric matrix B respectively, then prove that $B = \lambda_1 u_1 u_1^T + \lambda_2 u_2 u_2^T + ... + \lambda_n u_n u_n^T$..
[3+3]

9. Explain the role of basic linear algebra techniques that are useful in the study of data science. Describe how the various concepts of Vector Spaces are applied in Machine Learning.
[2+4]

OR

Define four fundamental subspaces of a matrix A. Determine the values of the constants a and b for which the system $3x - 2y + z = b$, $5x - 8y + 9z = 3$, $2x + y + az = -2$ has a unique solution, no solution and infinitely many solutions.
[2+4]

10. Discuss in brief about singular value decomposition? Find the singular value decomposition of the matrix $\begin{pmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{pmatrix}$.
[1+5]

Tribhuvan University.
Institute of Science and Technology
2078

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Master Level / I Year /First Semester/ Science
Data Science (MDS 504)
(Mathematics for Data Science)

Full Marks: 45
Pass Marks: 22.5
Time: 2 hours

Attempt All Questions. Write your answer in detail as far as possible.

Group A

(3×5=15)

1. Show that
   a. The line $x_2 = ax_1$ is a subspace $\mathbb{R}^2$.
   b. The set of points that is the union of two lines through the origin is not a subspace.

2. Let

$$v_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix},\quad v_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}.$$

Show that $B = \{v_1,\ v_2\}$ is an orthonormal basis for $\mathbb{R}^2$. Find a vector $x \in \mathbb{R}^2$ with respect to the basis B.

3. Without calculation, find one eigenvalue and two linearly independent eigenvectors of

$$A = \begin{pmatrix}4 & 4 & -4\\4 & 4 & -4\\4 & 4 & -4\end{pmatrix}.$$

Justify your answer.

4. Let $Q(x) = 3x_1^2 + 9x_2^2 + 8x_1x_2$. Find (a) the maximum value of $Q(x)$ subject to the constraint $x^Tx = 1$, (b) a unit vector u where this maximum is attained, and (c) the maximum of $Q(x)$ subject to the constraints $x^Tx = 1$ and $x^Tu = 0$.

5. Describe and compare the solution sets of $x_1 + 9x_2 - 4x_3 = 0$ and $x_1 + 9x_2 - 4x_3 = 2$

Group B

(6×5=30)

6. Give a geometric description of span (v) and span (u, v). Consider the vectors $u = \begin{pmatrix}1\\2\end{pmatrix}$ and

$$v = \begin{pmatrix}-1\\1\end{pmatrix}.$$

   a. Write the vector $\begin{pmatrix}3\\2\end{pmatrix}$ in terms of the vectors u and v.
   b. Show that the vectors u and v span $\mathbb{R}^2$.

MDS504-2078

7. Let $u_1 = (2\ 0\ 0)^T, u_2 = (0\ 1\ 1)^T$ and $u_3 = (0\ 1\ -1)^T$. Find the orthonormal set associated with the set $S = \{u_1, u_2, u_3\}$.

   Prove that an orthogonal set of nonzero vectors in a vector space is linearly independent.

8. consider the matrix : $A = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$. What can you say about the action of A on an arbitary vector?

   What are examples of eigenvalues/eigenvectors of this matrix? What does this discussion for this example illustrate?

   OR

   Let v_1, v_2 be the eigenvectors associated with the eigenvalues λ_1, λ_2 of a 2×2 symmetric matrix A respectively. Prove that $A = \lambda_1 v_1 v_1^T + \lambda_2 v_2 v_2^T$ (1)

9. Prove that if B is a symmetric bilinear function on $\mathbb{R}^n$, then it is of the form $B = B_A(v, w) = v^T A w$, for some unique symmetric matrix A.

   OR

   Express the quadratic form $Q(x) = x_1x_2 - x_1x_3 + x_2x_3$ as a sum of squares.

10. Find the SVD of $A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$. If A an invertible $n\times n$ matrix, what is the relationship between the singular values of A and A^{-1}?

TRIBHUVAN UNIVERSITY

INSTITUTE OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICAL SCIENCES

First Assessment 2078

Subject: **Mathematics for Data Science**
Course No.: MDS 504
Level: **Master in Data Science/I Semester**

Full Marks: 45
Pass Marks: 22.5
Time: 2:00 hr

Attempt ***ALL questions****. Write your answer in detail as far as possible.*

Group A [3 × 5 = 15]

1. Show that

 (a) The line $x_2 = ax_1$ (in usual notations, $y = ax$) is a subspace $\mathbb{R}^2$.

 (b) The line $x_2 = ax_1 + b$ (perhaps more familiar as $y = ax + b$) is not a subspace $\mathbb{R}^2$ for $b \neq 0$.

2. Show that any vector in $\mathbb{R}^3$ can be expressed as a linear combination of the three unit basis vectors in $\mathbb{R}^3$. Also, show that a linear combination of the three unit basis vectors in $\mathbb{R}^3$ equals to 0 if and only if all coefficients in the linear combination are zeros.

3. What is the parallel coordinates method? Explain with explain with an example. What is the use of this method in data science?

4. Find a basis for the solution space of the equation $x + y - z = 0$.

5. Let $u_1 = (1, 2, 2, -1)$, $u_2 = (1, 1, -1, 1)$, $u_3 = (-1, 1, -1, -1)$ and $B = \{u_1, u_2, u_3\}$ an orthogonal basis for $V = \text{span}\,(u_1, u_2, u_3)$. Find the projection of $w = (0, 1, 2, 3)$ onto V.

Group B [6 × 5 = 30]

6. (a) By showing that the L_∞-norm satisfies each of the conditions in the definition of a norm prove this is a vector norm for $\mathbb{R}^n$.

 (b) Let $x = (x_1, ..., x_n) \in \mathbb{R}^n$ be a vector with $x_i = i^{-1}$. Compute the 1-norm, the 2-norm, and the ∞-norm of x.

OR

Prove that if x and y are vectors in $\mathbb{R}^n$, then

(a) $|x \cdot y| \leq \|x\|_2 \|y\|_2$.

7. Let $u_1 = (2\ 0\ 0)'$, $u_2 = (0\ 1\ 1)'$ and $u_3 = (0\ 1\ -1)'$. Find the orthonormal set associated with the set $S = \{u_1, u_2, u_3\}$.
Prove that an orthogonal set of nonzero vectors in a vector space is linearly independent.

8. consider the matrix : $A = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$. What can you say about the action of A on an arbitary vector?

What are examples of eigenvalues/eigenvectors of this matrix? What does this discussion for this example illustrate?

OR

Let v_1, v_2 be the eigenvectors associated with the eigenvalues λ_1, λ_2 of a 2×2 symmetric matrix A respectively. Prove that $A = \lambda_1 v_1 v_1^T + \lambda_2 v_2 v_2^T$. (1)

9. Prove that if B is a symmetric bilinear function on $\mathbb{R}^n$, then it is of the form $B = B_A(v, w) = v^T A w$, for some unique symmetric matrix A.

OR

Express the quadratic form $Q(x) = x_1x_2 - x_1x_3 + x_2x_3$ as a sum of squares.

10. Find the SVD of $A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$. If A an invertible $n \times n$ matrix, what is the relationship between the singular values of A and A^{-1}?