EXAMPLE PLROBLEM

Solving a System of Linear Equations using Gauss Elimination Matrix Method

Let us consider a system of linear equation as follows –

$$\begin{aligned} 2x - y + 3z &= 4 \\ x + y - 2z &= 3 \\ 3x - 2y + 2z &= 2 \end{aligned}$$

Writing the above equation in matrix form:

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & -2 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

We will solve this system using the Gauss elimination matrix method.

Step 1: Augmented Matrix

We form the augmented matrix:

$$\begin{bmatrix} 2 & -1 & 3 & 4 \\ 1 & 1 & -2 & 3 \\ 3 & -2 & 2 & 2 \end{bmatrix}$$

Step 2: Row Operations

We perform row operations to obtain an upper triangular matrix. Performing $R_1 \to \frac{1}{2}R_1$, we get:

$$\begin{bmatrix} 1 & -1/2 & 3/2 & 2 \\ 1 & 1 & -2 & 3 \\ 3 & -2 & 2 & 2 \end{bmatrix}$$

Performing $R_2 \to R_2 - R_1$, we get:

$$\begin{bmatrix} 1 & -1/2 & 3/2 & 2 \\ 0 & 3/2 & -7/2 & 1 \\ 3 & -2 & 2 & 2 \end{bmatrix}$$

Performing $R_3 \to R_3 - 3R_1$, we get:

$$\begin{bmatrix} 1 & -1/2 & 3/2 & 2 \\ 0 & 3/2 & -7/2 & 1 \\ 0 & -1/2 & -5/2 & -4 \end{bmatrix}$$

Performing $R_3 \to R_3 + \frac{1}{3}R_2$, we get:

$$\begin{bmatrix} 1 & -1/2 & 3/2 & 2 \\ 0 & 3/2 & -7/2 & 1 \\ 0 & 0 & -22/6 & -11/3 \end{bmatrix}$$

Step 3: Back Substitution

Now, we solve the equations backward starting from the last row:

$$-\frac{22}{6}z = -\frac{11}{3} \implies z = 1$$

Substitute $z = 1$ into the second row:

$$\frac{3}{2}y - \frac{7}{2}(1) = 1 \implies y = 3$$

Finally, substitute $z = 1$ and $y = 3$ into the first row:

$$x - \frac{1}{2}(3) + \frac{3}{2}(1) = 2 \implies x = 2$$

Step 4: Solution

Therefore, the solution to the system of equations is:

$$\boxed{x = 2, \quad y = 3, \quad z = 1}$$