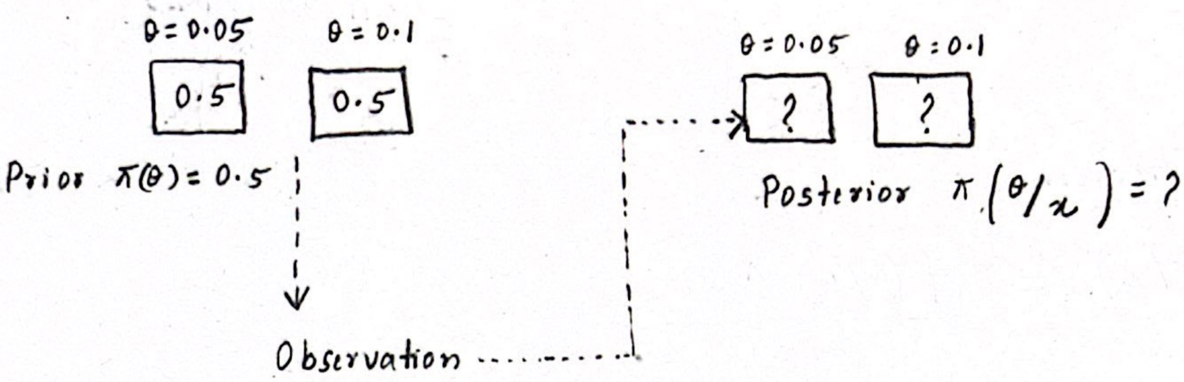
# Example 1

- A manufacturer claims that the shipment contains only 5% of defective items, but the inspector feels that in fact it is 10%. We have to decide whether to accept or to reject the shipment based on $\theta$, the proportion of defective parts. Before we see the real data, let's assign a 50-50 chance to both suggested values of $\theta$ i.e.

$$\pi(0.05) = \pi(0.10) = 0.5.$$

A random sample of 20 parts has 3 defective ones. Calculate the posterior distribution of $\theta$

## Solution




Let, $\theta$ represent the proportion of defective items.

We know, $\pi(\theta = 0.05) = \pi(\theta = 0.10) = 0.5$

A random sample of 20 parts $(n)$ has 3 defective items $(x)$. This process follows binomial distribution

i.e. $X \sim B(n, \theta)$.

Now,

for $\theta = 0.05$,

Likelihood for data, $f(x = 3/\theta = 0.05) = {}^{n}C_{x}\, \theta^{x} (1-\theta)^{n-x}$

$$= {}^{20}C_{3} (0.05)^3 (0.95)^{17}$$

$$= 0.0595$$

for $\theta = 0.10$,

Likelihood for data, $f(x = 3/\theta = 0.10) = {}^{n}C_{x}\, \theta^{x} (1-\theta)^{n-x}$

$$= {}^{20}C_{3} (0.1)^3 (0.9)^{17}$$

$$= 0.1901$$

Now,

Marginal probabibility of the data, $m(x=3)$

$$= \sum_{\theta} f(x/\theta)\,\pi(\theta)$$

$$= f(x=3/\theta=0.05)\,\pi(\theta=0.05)$$

$$+ f(x=3/\theta=0.1)\,\pi(\theta=0.1)$$

$$= 0.0595 * 0.5 + 0.1901 * 0.5$$

$$= 0.1248$$

Thus, Posterior distribution of $\theta$ is given as follows:

for $\theta = 0.05$,

$$\pi(\theta=0.05/x=3) = \frac{f(x=3/\theta=0.05)\,\pi(\theta=0.05)}{m(x=3)}$$

$$= \frac{0.0595 * 0.5}{0.1248}$$

$$= 0.2383$$

for $\theta = 0.10$,

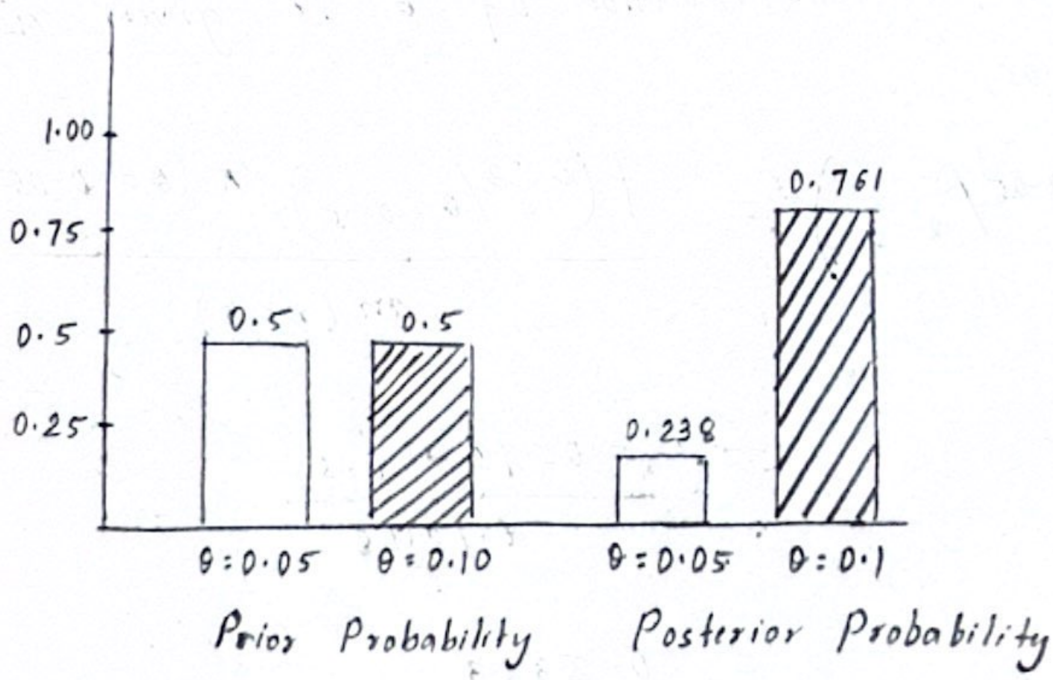
$$\pi\left(\theta = 0.10 / x = 3\right) = \frac{f\left(x = 3 / \theta = 0.10\right) \pi(\theta = 0.10)}{m(x = 3)}$$

$$= \frac{0.1901 * 0.5}{0.1248}$$

$$= 0.7616$$

Posterior distribution of $\theta$

| $\theta$ | 0.05 | 0.10 |
|---|---|---|
| $\pi(\theta/x)$ | 0.2383 | 0.7616 |




Interpretation: The manufacturer's claim that the defective item is 5% (0.238) is ~~more than~~ almost three times less likely than inspector's claim of 10%. Thus, it is better for the customer to reject the shipment. (0.761)

- Ultrasound tests done near the end of the first trimester of a pregnancy are often used to predict the sex of the baby. However, the errors made by radiologists in reading ultrasound results are not symmetric, in the following sense: girls are virtually always correctly identified as girls, while boys are sometimes misidentified as girls (in cases where the gender organ is not clearly visible, perhaps due to the child's position in the womb). More specifically, a leading radiologist states that

$$P(test + |G) = 1 \text{ and } P(test + |B) = 0.25$$

where "test +" denotes that the ultrasound test predicts the child is a girl. Thus, we have a 25% false positive rate for girl, but no false negatives.

# Example 2



## Introduction: An Example

- Suppose a particular woman's test comes back positive for girl, and we wish to know the probability she is actually carrying a girl. Assuming 48% of babies are girls, we can use Bayes Rules where "boy" and "girl" provide the J = 2 mutually exclusive and exhaustive cases
- Here we need to find $P(G|+)$

Example 2

Solution

$\pi(G) = 0.48$ $\pi(B) = 0.52$ (Prior)

Also given are the followings,

$P(\text{test}+ \mid G) = 1$

and $P(\text{test}+ \mid B) = 0.25$

Now, $P(G \mid \text{test}+) = ?$

- Marginal Probability for data $m(\text{test}+ = 1) = P(\text{test}+ \mid G)\pi(G) + P(\text{test}+ \mid B)\pi(B)$

$= 1 * 0.48 + 0.25 * 0.52$

$= 0.61$

Now, $P(G \mid \text{test}+) = \dfrac{P(\text{test}+ \mid G)\ \pi(G)}{m(\text{test}+)}$

$= \dfrac{1 * 0.48}{0.61}$

$= 0.7868$

Interpretation : For a positively tested result, the probability that the embryo is actually a girl is $\boxed{0.7868}$.