CS 70 lecture #1: Introduction, Propositional logic.

Wason's experiment:

- 4 cards on table: 1 for Alice, Bob, Charlie, Donna.
- card contains person's destination on one side, mode of travel on other.
- Consider the theory: "if a person travels to Chicago, he/she flies"
- you see:

Alice	Bob	Charlie	Donna
Baltimore	drove	Chicago	flew

- To test theory, what do you need to flip?
- Answer: later.

Propositions

- Statements that are true or false.
  - ↳ Proposition vs opinion.
    - ↳ opinions can't have true/false values.
- Example:
  - $\sqrt{2}$ is irrational. → Proposition, true.
  - $4+5$ → Not a proposition.
  - Alice travelled to Chicago → Proposition False.

Propositional Forms.

- Put propositions together to make another.
- Conjunction: $P \land Q$ (P and Q)
  - → True if and only if P & Q are True.
- Disjunction: $P \lor Q$ (P or Q)
  - → False if and only if P & Q are False.
- Negation: $\sim P$
  - → False if P is True.
- You can put any propositions in these spots!
  - "$2+2=3$" $\land$ "$2+2=4$" → False Proposition.

Truth Tables for Propositional Forms:

P	Q	$P \wedge Q$	$P \vee Q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

($\wedge$, $\vee$ are commutitive).

$\sim(P \wedge Q) = \sim P \vee \sim Q$

- Both propositional forms have the same Truth Table.

*Demorgan's law for Negation: Distribute and flip.

$\sim(P \wedge Q) \equiv \sim P \vee \sim Q$

$\sim(P \vee Q) \equiv \sim P \wedge \sim Q$

Example:

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$

Cases:

P is True:

$T \wedge (Q \vee R) \equiv (Q \vee R)$

$(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$

P is False:

$F \wedge (Q \vee R) \equiv F$

$(F \wedge Q) \vee (F \wedge R) \equiv F \vee F \equiv F$

Similarly, $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

→ Foil! : $(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$

Implications:

$P \rightarrow Q$, if P then Q.

if P is true and $P \rightarrow Q$, Q is true.

if you $\underset{P}{\underset{\uparrow}{\text{stand}}}$ in the rain, then you'll $\underset{Q}{\underset{\uparrow}{\text{get}}}$ wet.

$P \to Q$ is false only if P is true & Q is false.

False implies nothing.

P being false means Q can be true or false.

P can be anything when Q is false.

if chemical plant pollutes rivers, fish die.

If fish die, did chemical plant pollute river?

Not necessarily.

Implication and English:

- if P, then Q.
- Q if P
- P only if Q
- P is sufficient for Q.

logically equivalent

P	Q	$P \to Q$	$\sim P \vee Q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Contrapositive, Converse.

- Contrapositive of $P \to Q$ is $\sim Q \to \sim P$
  - if plant pollutes, fish die.
  - If the fish don't die, plant does not pollute.
- Converse of $P \to Q$ is $Q \to P$.
  - Not logically equivalent.
- Equivalent: if $P \to Q$ and $Q \to P$ is P if and only if Q or $P \leftrightarrow Q$.

Variables

- Propositions?
  - $x > 2$
  - n is even & sum of 2 primes

  ] → Not, there are free variables.
- Predicates:
  - if you put a value in, they become a proposition
    - $Q(x)$: x is even. ← depends on x.
  - Wason's experiment
    - $F(x)$ = Person x flew
    - $C(x)$ = Person x went to Chicago.
    - $C(x) \to F(x)$

Quantifiers:

- There exists quantifier:

$(\exists x \in S)(P(x))$: There exists an x in S such that $P(x)$ is true.

- Converts predicates into statements.

Example:

$(\exists x \in \mathbb{R})(x = x^2)$

- For all quantifier:

$(\forall x \in S)(P(x))$: For all x in S, $P(x)$ is true.

Example:

Adding 1 makes a bigger number: $(\forall x \in \mathbb{R})(x+1>x)$

*Propositions have universes: $\mathbb{R}$ = all real #s. $\mathbb{N}$ → all natural #s.

Back to Wason's:

$P(x)$ = Person x went to Chicago. $Q(x)$ = Person x flew

Statement/Theory: $\forall x \in \{A, B, C, D\}\ P(x) \rightarrow Q(x)$

$P(A)$ = false. We don't care about $Q(A)$. ← can be anything.

$Q(B)$ = false. $P(B) \rightarrow Q(B) \equiv \sim Q(B) \rightarrow \sim P(B)$. So, $P(B)$ should be false.

$P(C)$ = true. $P(C) \rightarrow Q(C)$, so $Q(C)$ must be true.

$Q(D)$ = True. We don't care about $P(D)$

→ only have to flip over Bob, Charlie.

More Quantifiers:

$(\forall x \in N)(2x > x)$ → False. $x = 0$.

↳ $(\forall x \in N)(2x \geq x)$ → True.

$(\forall x \in N)(x > 5 \rightarrow x^2 > 25)$ ← implication restricts universe.

- There is a natural # that is the square of every natural number.

$(\exists y \in N)(\forall x \in N)(y = x^2)$ False.

- The square of every Natural # is a natural #.

$(\forall x \in N)(\exists y \in N)(y = x^2)$ → True.

Demorgan's for Quantifiers →

There is an x in S where $P(x)$ does not hold.

$\sim(\forall x \in S)(P(x)) \equiv \exists(x \in S)(\sim P(x))$.

Consider:

$\sim (\exists x \in S)(P(x))$

means that for all X in S, P(x) does not hold.

$\rightarrow (\forall x \in S)(\sim P(x))$.