Lecture 39: Reductions, Algorithmic Bounds, NP Completeness.

Best self-extracting bitstream is the program that you used to make the image.

Given a bitstream B, what's the shortest bitstream C_B that outputs B.

Kolmogorov Complexity:

The Java-Kolmogorov complexity $K_j(B)$ is the length of the shortest Java program that generates B. length in Bytes.

- There is an answer! It might be hard to find.

Fact 1: Kolmogorov Complexity is independent of language.

Fact 2: It's impossible to write a program that calculates Kolmogorov Complexity of any bitstream.

- Corollary: It's impossible to write the "perfect" compression algorithm

How hard is the independent set?

Does there exist a independent set of size k? (Such that vertices don't touch)

- For each of the 2^N possible colorings:
  - Check if # of colored vertices is equal to k: $O(N)$
  - For each red vertex, check that neighbors are white $O(kN)$
  - If both checks succeed, return true.
  - If either check fails, go on to next coloring.
- RT: $O(k*N*2^N) = O(N^2 * 2^N)$ $k \leq N$

Reductions:

- Useful tool. Using one algorithm to solve another prob.
- X reduces to Y if instance of X can be transformed into an instance of Y that solves X.

3SAT: Given a boolean formula, does there exist a truth value for boolean vars that obey a set of 3 var disjunctive constraints?

Example: (x1 || x2 || !x3) && (x1 || !x1 || x1) && (x2 || x3 || x4)

Solution: x1 = T, x2 = T, x3 = T, x4 = F

Independent-set cracks 3SAT.

Complexity Classes:

decision problems: yes or no problems.

a problem is in complexity class P if:
- It is a decision problem
- An answer can be found in $O(N^k)$ for some k.

a problem is in class NP if:
- it is a decision problem
- A "yes" answer can be verified in $O(N^k)$ time for some k. We can verify a specific answer "yes" in $O(N^k)$.

a problem is NP-Complete if:
- it is a member of NP
- it cracks every other problem in NP.

3SAT is NP complete. → it cracks any problem for which a "yes" answer can be efficiently verified.

As NP-complete problems grow it gets easier to find more NP-complete problems.