

Heaps

Priority Queue

Min priority queue

- like a "sack" but only lets you interact w/ the smallest item.

Solution: Tracking the M best things.

get Next Message(), add it to the priority queue

if size of priority queue $> M$, remove the smallest.

- How to make these?

Can't use: Arrays, Bushy BSTs, or HashTables, so use Heaps.

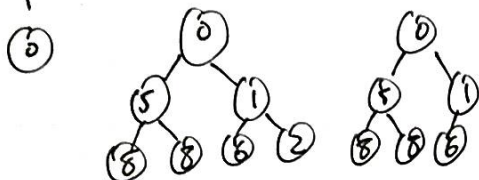
Heaps

Binary min heap = Binary tree that is complete & obeys the min-heap property.

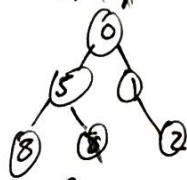
- Complete: Missing items only at bottom level (if any).
All nodes are as far left as possible.

- Min-heap: Every node is less than or equal to the children.

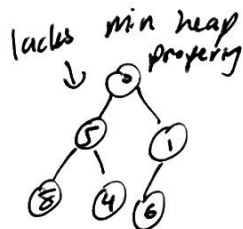
Examples:



Not Examples:



2 not all the way to left.
(Incomplete)

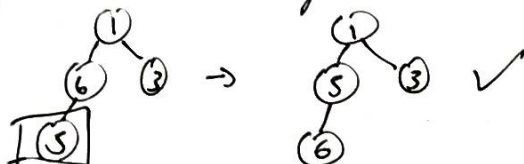


*Note: Every node should have 2 children unless at bottom.

Adding:

- Insert into an arbitrary leaf position. Have the node climb up and swap as you move along. ensures the right position.

Example:



Deletion of min value

- 1) Swap last item in heap into the root.
- 2) Sink the root into its correct spot.



Given a heap:

Priority queue \rightarrow

getSmallest() \rightarrow return root node

add(x) \rightarrow place in the ~~to~~ last position, then promote accordingly.

removeSmallest() \rightarrow remove root, promote rightmost leaf as root, change positions accordingly.

Heap implementation of a priority queue

	ordered array	Bushy BST	HashTable	Heap
add	$\Theta(n)$	$\Theta(\log n)$	$\Theta(1)$	$\Theta(\log n)$
getSmallest	$\Theta(1)$	$\Theta(\log n)$	$\Theta(n)$	$\Theta(1)$
removeSmallest	$\Theta(n)$	$\Theta(\log n)$	$\Theta(n)$	$\Theta(\log n)$

hard to handle items w/ same priority.

Notes:

- Priority is useful.
- Heap is $\log n$ time Amortized (resizes)
- BST can have a constant get smallest if you keep a pointer to the smallest value.
- Heaps handle duplicate priorities better than BSTs
- Array based heaps save memory.

keys: $[a, b, c, d, e, f, g, h, i]$ parent = $(k-1)/2$

parents: $[0, 0, 0, 1, 1, 2, 2, 3, 3]$

can show as:

$[a, b, c, d, e, f, g, h, i]$