

# DSAA Mid 1

- ✓ Basics
- ✓ Operations on Signals
- ✓ Characteristics of Signals
- ✓ Properties of a System
- ✓ Convolution
- ✓ Vector Algebra

## Mid 1

- **Signal** : Function that conveys information about behaviour and attributes of some phenomenon.
- **Sampling** - While storing an analogue signal in digital media, it's sampled at fixed intervals are stored, the process is called **sampling**.  
*Discretisation of time-space — Sampling.*
- Interval at which samples are recorded is called **sampling rate**.
- **Nyquist's Theorem** : A signal with maximum frequency **F** can be recovered accurately if it's **sampling frequency  $\geq 2F$** .
- **Quantisation** : The amplitude space is also discretised for storage, process called quantisation.  
*Quantisation is a lossy process.*
- Analogue to digital because-
  - Cheaper.
  - Allows for more sophisticated algorithms.
  - Easy storage, reproduction.
  - Modular and flexible.
  - Easy offline and remote processing.
- *Digital signals are slower than analogue to transmit though.*
- **Interpolation** : Digital to analogue conversion.
- **Aliasing** : A false signal, close enough to the real one.
- Reconstruction from quantised sample (Interpolation):
  - Zero Order Hold** : Keeping the original signal's value until a new value is reached.
  - Nearest Neighbour** : Value equal to the nearest neighbour, not

for realtime data. Delay with realtime.

**Linear** : Linear between points.

**Sync** : Some weird function.

– **Operations on Signals :**

*Shifting* :  $x(n-2)$

*Flipping* :  $x(-n)$

*Scaling* :  $x(b*n)$

To be done precisely in this order Shift — Flip — Scale

- Scaling causes loss of info, to avoid this loss, while sampling, take a greater sampling frequency than  $2F$ .

– **Characteristics of Signals :**

**Odd Even** —  $x(-n) = -x(n)$  ,  $x(-n) = x(n)$

**Periodic** —  $x(N + n) = x(n)$

**Energy** —  $E = \sum_{(-\infty, \infty)} |x(t)|^2$

**Power** —  $\lim_{N \rightarrow \infty} \sum_{(-N, N)} 1/(2N + 1) * \text{Energy}$

**Unit** —  $x(n) = 1$  for all  $n \neq 0$

**$\delta$  (Delta)** — opposite of Unit

**$r(n)$**  —  $n$  for all  $n$ , 0 for 0

**$e(n)$**  —  $|a|^n * u(n)$

– **Properties of a Systems:** (Averaging Filter given as example)

Causal vs Non-Causal

Static vs Dynamic

Linear (Must satisfy Additivity and Homogeneity)

Time Invariance

– **Linear Time Invariant Systems** - Linear and Time invariant.

- **Impulse Response** - Characterises the system, can predict output for any given input to the system.

– **Convolution**

- Derivation of convolution equation, and the equation itself. —

**BLUE DENOTES NOTEBOOK**

- That one convolution example in the notebook, for understanding how to convolve graphically.
- Flipping and sliding for convolution — **zeroth value at overlap of centres.**
- Matrix form for convolution.
- **Algorithm for convolution in 2D.**

- **Properties of Convolution**
  - Commutative.
  - Distributive.
  - Associative
- Representing functions as flow diagram. ( $Z^x$ ) for delay.
- Averaging Filter, equations and convolution.
- Gaussian averaging filter FORMULA for 1D and 2D.
- Moving average filter.
- Recursive moving average, equations and flow diagram.
- Recursive mean.
- Recursive Variance.
- **Linear Regression**, derivation + Polynomial regression.
- Proof that the line passes through means of x and y.
- **Sinusoids and Exponentials**
- Finding whether an expression is periodic, using  $(\omega_{not})N = 2\pi k$   
N must be integral for integral K for periodicity.
- Complex Exponential generating machine.
- **Wagon Wheel Effect.**
- **Vector Spaces** : two rules for qualification. (*Additive Closure, and constant multiplicative Closure*)
- **Spanning Set of a Vector Space** : Any element defined as linear combination of other elements.
- **Linear Independence** : Statement expression.
- **Basis** : A linearly independent set which spans V is called the basis of V.  
*Revisit Crammer's Rules*
- **Inner Product**: Formula.  $\sum_{i=1}^{N-1} u(i)v(i)$  Vectors multiplied position by position then added.
- **Orthogonal** : If inner product of 2 vectors is 0, they are orthogonal.
- **Norm** : Inner product with itself.
- **Orthonormal** : If orthogonal, and vectors are unit vectors.
- **Theorem** : If a subset S of V, of non-zero vectors is orthogonal, then S is linearly independent. - **PRACTICE PROOF AGAIN**
- **Theorem of Orthogonal Decomposition** - See notes for expression and proof.

