

Boolean expressions and Karnaugh map (K-map)

Forms of Boolean Expressions:

- Sum-of-products form (SOP) – first the product (AND) terms are formed then these are summed (OR) – eg: $ABC + DEF + GHI$
- Product-of-sum form (POS) – first the sum (OR) terms are formed then the products are taken (AND) – eg: $(A+B+C)(D+E+F)(G+H+I)$
- It is possible to convert between these two forms using Boolean algebra (DeMorgan's)
- Canonical form is not efficient but sometimes useful in analysis and design
- In an expression in canonical form, every variable appears in every term

$$f(A,B,C, D) = ABC'D + AB'CD + AB'CD'$$

Karnaugh map:

- *A grid of squares – each square represents one minterm
- *Only one variable changes between adjacent squares
- *Squares on edges are considered adjacent to squares on opposite edges
- *Karnaugh maps become clumsier to use with more than 4 variables
- *For each product term, write a 1 in all the squares which are included in the term, 0 elsewhere
- *Minimization is done by spotting patterns of 1's and 0's. Simple theorems are then used to simplify the Boolean description of the patterns. Pairs of adjacent 1's – remember that adjacent squares differ by only one variable – hence the combination of 2 adjacent squares has the form $P(A+A') = P$.
- *Adjacent Pairs – The same idea extends to pairs of pairs

Tutorial-5

Boolean expression and K-map

1. **Boolean algebra:** Find a simplified Boolean expression for the truth table (shown below) using K-map,

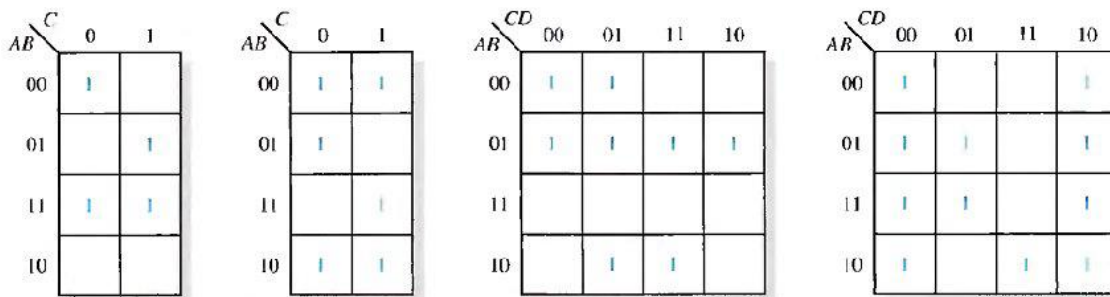
x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

2. **K-maps from a given Boolean expression:** Design a K-map for the following Boolean expressions,

(i). $A'.B+B.C'.D'$; (ii). $AB'C + A'B' + ABC'D$

3. **Simplification of Boolean expression using K-map:** Simplify the following Boolean expression using K-map, $f(A,B,C,D)=A'.B.D'+B.C.D+A'.B.C'.D+C.D$

4. **K-map:** Group the 1s in each of the Karnaugh maps (shown in figure),



5. **Minimization of 5-variable expressions:** Use a Karnaugh map to minimize the following standard SOP 5-variable expression:

$$F = A'B'C'D'E' + A'B'CD'E' + A'BCD'E' + A'BC'D'E' + A'B'C'D'E + A'BCD'E + A'BCDE + AB'C'D'E' + AB'C'D'E + ABCD'E + ABCDE + AB'CDE$$