PHY 303: Classical Electrodynamics
MONSOON SEMESTER 2022
TUTORIAL 05

1. A sphere of radius R carries a polarization

$$\mathbf{P}(\mathbf{r}) = kr\,\mathbf{r},$$

where k is a constant, $\mathbf{r}$ is the vector from the center, and $r = |\mathbf{r}|$. Calculate the bound charges σ_b and ρ_b.

2. Calculate the electric potential produced by a uniformly polarized sphere of radius a and center at the origin with electric polarization (dipole moment per unit volume) given by $P_0\,\hat{\mathbf{z}}$, using the result

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\big(\rho(\mathbf{r}') - \boldsymbol{\nabla}'\cdot\mathbf{P}(\mathbf{r}')\big)}{|\mathbf{r}-\mathbf{r}'|}d^3r' + \frac{1}{4\pi\epsilon_0}\oint_{\mathcal{S}} \frac{\mathbf{P}(\mathbf{r}')\cdot d\mathbf{a}'}{|\mathbf{r}-\mathbf{r}'|}.$$

Note that for this problem there is no (free) charge involved and outside the given sphere the polarization is zero. Obtain results for both interior ($|\mathbf{r}| < a$) and exterior ($|\mathbf{r}| > a$) regions using the following three approaches:

   (a) Consider the volume ($\mathcal{V}$) and the enclosing surface ($\mathcal{S}$) used in the integrals above as the ones approaching the surface and volume of the polarized sphere from inside, i.e., surface and volume same as the given sphere in a limiting sense, approaching from inside.

   (b) Consider $\mathcal{V}$ and $\mathcal{S}$ as the ones approaching the surface and volume of the polarized sphere from outside.

   (c) Consider $\mathcal{V}$ to be infinite.