# PHY 305 4

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## Q1

Consider a two dimensional physical system. The kets $|\psi_1\rangle$ and $|\psi_2\rangle$ form an orthonormal basis of state space. We define a new basis $|\phi_1\rangle$ and $|\phi_2\rangle$ by

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle) \tag{1}$$

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle - |\psi_2\rangle) \tag{2}$$

An operator $\mathbf{A}$ is represented in the $|\psi_i\rangle$ basis by the matrix

$$a_{ij} = \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix} \tag{3}$$

Find the representation of $\mathbf{A}$ in the basis $|\phi_i\rangle$ [Hint:Find $\tilde{a}_{ij} = \langle\phi_i|\,\mathbf{A}\,|\phi_j\rangle$ ]

## Q2

Consider an operator $\mathbf{p}$ such that

$$[\mathbf{p}, \mathbf{H}] = 0 \tag{4}$$

show that the expectation value $<\mathbf{p}>$ is a constant for any wave function $\psi(x,t)$. Note that the operator does not have an explicit dependence on time, and therefore the time dependence is entirely due to the fact that the wave function evolves according to Schrodinger equation.

## Q3

a) Let $\mathbf{x(t)}$ and $\mathbf{p(t)}$ be the position and momentum operator for a free particle in 1D in the Heisenberg picture, Evaluate

$$[\mathbf{x(t)}, \mathbf{x(0)}], \qquad [\mathbf{p(t)}, \mathbf{p(0)}] \tag{5}$$

b) Consider the time-independent Schrodinger Hamiltonian for a spinning particle in a uniform and constant magnetic field of magnitude B along the 3rd-direction:

$$H = \lambda B S_3 \tag{6}$$

Here $\lambda$ is a (real) constant that relates the dipole moment to the spin. Find the explicit time evolution for the Heisenberg operators $\hat{S}_1(t)$, $\hat{S}_2(t)$, and $\hat{S}_3(t)$ associated with the Schrodinger operators $S_1$, $S_2$, and $S_3$.