PHY 305 3

September 2022

Q1.
Consider the following relations,

$$J_{+} = J_1 + iJ_2, \tag{1}$$

$$J_{-} = J_1 - iJ_2, \tag{2}$$

$$J_{+}|lm> = \hbar\sqrt{l(l+1-m(m+1))}|l, m+1> \tag{3}$$

$$J_{-}|lm> = \hbar\sqrt{l(l+1-m(m-1))}|l, m-1> \tag{4}$$

$$J_3|lm> = m\hbar|lm> \tag{5}$$

$$J^2|lm> = l(l+1)\hbar^2|lm> \tag{6}$$

For a system of spin l=1, find the matrix representations of J_1, J_2, J_3 and J^2 in the basis of eigen vectors of J_3 and J^2.

Q2
Given the form of the the Pauli matrices,

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{7}$$

Prove the following statements,

$$\{\sigma_i, \sigma_j\} = 2\delta_{ij} \tag{8}$$

$$det(\sigma_i) = -1 \tag{9}$$

$$Tr(\sigma_i) = 0 \tag{10}$$

$$[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k \tag{11}$$

Q3.

If $\hat{L^2}$ can be written as $\hat{L^2} = \hat{L}_1^2 + \hat{L}_2^2 + \hat{L}_3^2$, construct $\hat{L}_1, \hat{L}_2, \hat{L}_3$ and $\hat{L^2}$ in Spherical polar coordinate system from Cartesian coordinate system and show that,

$$[\hat{L^2}, \hat{L_3}] = 0. \tag{12}$$

Comment on the physical meaning of the result.

Recognise the deferential form of angular momentum operator $\hat{L}$ form which we can directly read of the form of $\hat{\vec{L}}$ and subsequently $\hat{L^2}$ in spherical polar coordinates.

Apply $\hat{L^2}$ and $\hat{L_3}$ to the solution $\psi(r, \theta, \phi)$ of the Schrodinger equation of the hydrogen atom and state their eigen values.