Introduction to Statistics (MAT 283)

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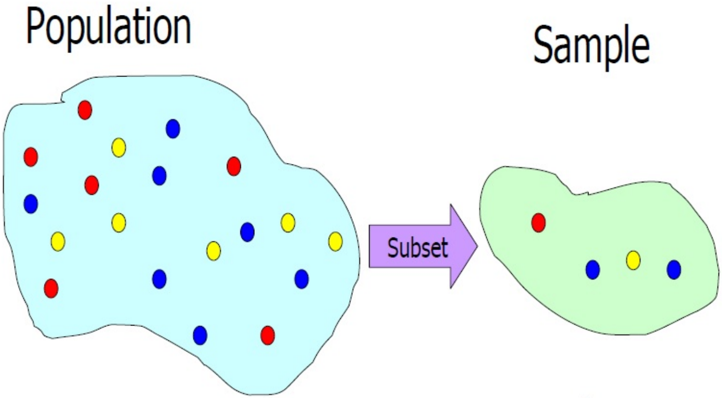
Statistics is the discipline that concerns the collection, organization, displaying, analysis, interpretation and presentation of data.

Population:
In statistics, a population is a set of similar items or events which is of interest for some question or experiment.

Sample:
A sample is a subset of the population.


Population
Sample
Subset

Two main branches of statistics:

Descriptive statistics: Summarize data from a sample using indexes such as the mean or standard deviation.

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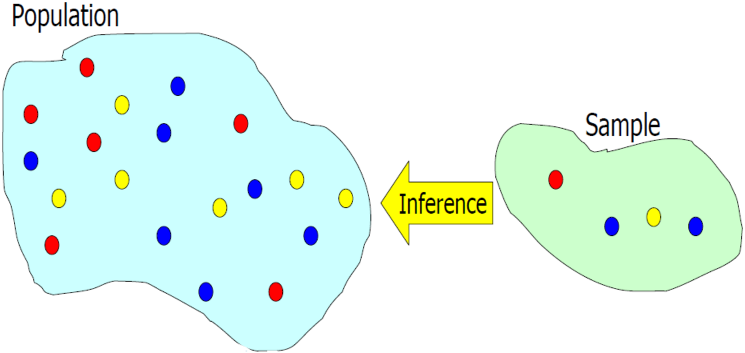
Inferential statistics:
Inferential statistics is used to make predictions or comparisons about a larger group (a population) using information gathered about a small part of that population.


Population
Inference
Sample

Sir Ronald A Fisher
(17 February 1890 – 29 July 1962)

Prasanta C. Mahalanobis
(29 June 1893– 28 June 1972)

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Probability Theory:

- Probability of Events, Conditional Probability and Bayes Theorem
- Random Variables and Distribution Functions
- Moments of Random Variables and Chebychev Inequality
- Some Special Discrete Distributions, Some Special Continuous Distributions
- Bivariate Random Variables and Product Moments of Bivariate Random Variables
- Some Special Discrete Bivariate Distributions, Some Special Continuous Bivariate Distributions
- Functions of Random Variables and Their Distribution, Laws of Large Numbers, The Central Limit Theorem

Statistics:

- Sampling Distributions
- Estimators of Parameters
- Test of Statistical Hypotheses
- Linear Regression

Recommended Books:

- Prasanna Sahoo, Probability and Mathematical Statistics
- Miller & Miller, John E. Freund's Mathematical Statistics with Applications
- John A. Rice, Mathematical Statistics and Data Analysis
- Larry Wasserman, All of Statistics: A Concise Course in Statistical Inference

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RANDOM EXPERIMENT: An experiment whose outcomes can not be predicted with certainty.

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EXAMPLES:

1. The sample space for the possible outcomes of one toss of a coin is

$$S_1 = \{H, T\}$$

where H and T stand for head and tail.

2. If we toss it twice then

$$S_2 = \{HH, HT, TH, TT\}$$

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3. If a coin is tossed until a head appears for the first time, this

could happen on the first toss, the second toss, the third toss, the fourth toss, . . ., and there are infinitely many possibilities. For this experiment we obtain the sample space

$$S_3 = \{H, TH, TTH, TTTH, TTTTH, \ldots\}$$

with an unending sequence of elements.

Discrete Sample Space:

If a sample space contains a finite number of elements or an infinite though countable number of elements, it is said to be discrete.

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Continuous Sample Space: If a sample space consists of a continuum, such as all the points of a line segment or all the points in a plane, it is said to be continuous.

• Continuous sample spaces arise in practice whenever the outcomes of experiments are measurements of physical properties, such as temperature, speed, pressure, length that are measured on continuous scales.

EXAMPLE: Choosing a point from the interval $(0, 1)$. The sample space

$$S = (0, 1)$$

is continuous.

EVENT: An event is a subset of a sample space.

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EXAMPLE: Tossing a die. The sample space is
$S = \{1, 2, 3, 4, 5, 6\}$.$E = \{2, 4, 6\}$ is an event, which can be described in words as "the number is even".

EVENT SPACE: A subset E of the sample space S is said to be an event if it belongs to a collection $\mathcal{F}$ of subsets of S satisfying the following three rules:

(a) $S \in \mathcal{F}$

(b) If $E \in \mathcal{F}$ then $E^c \in \mathcal{F}$

(c) If $E_j \in \mathcal{F}$ then $\bigcup_{j=1}^{\infty} E_j \in \mathcal{F}$.

The collection $\mathcal{F}$ is called an event space or a σ-field.

PROBABILITY MEASURE: Let S be the sample space of a random experiment. A probability measure $P : \mathcal{F} \to [0, 1]$ is a set function which assigns real numbers to the various events of S satisfying

(a) $P(E) \geq 0$ for all event $E \in \mathcal{F}$

(b) $P(S) = 1$

(c) If $E_1,\ E_2, \ldots, E_k, \ldots$ are mutually disjoint events of S, then

$$P(\bigcup_{j=1}^{\infty} E_j) = \sum_{j=1}^{\infty} P(E_j).$$

PROPERTIES OF PROBABILITY MEASURE:

- $P(\phi) = 0$
- $P(E^c) = 1 - P(E)$
- $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

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If A is an event in a discrete sample space S, then $P(A)$ equals the sum of the probabilities of the individual outcomes comprising A.

EXAMPLE: A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find $P(G)$, where G is the event that a number greater than 3 occurs on a single roll of the die.

If an experiment can result in any one of n different equally likely outcomes, and if of these m outcomes together constitute event A, then the probability of event A is

$$P(A) = \frac{m}{n}.$$

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EXAMPLE: If we twice flip a balanced coin, what is the probability of getting at least one head?