PHY305 Tutorial 1

Noel Jonathan Jobu

2nd September, 2022

Q1

Determine the spherical harmonic function which are joint eigenfunction of L^2 and L_z. Consider the state $|l, l>$ and $Y_l^l(\theta, \phi) =< \theta, \phi|l, l >$. We know that $L_+|l, l >= 0$.
Use the spherical coordinate form of L_+ to determine the wave-function $Y_l^l(\theta, \phi)$ (in position space). Use the explicitly known dependence on $m_l = l$.
HINT: Assume

$$< \theta, \phi|l, l >= c_l f(\theta) e^{il\phi} \tag{1}$$

and determine $f(\theta)$ and c_l.
Then verify that $L_- Y_1^1(\theta, \phi) = h\sqrt{2} Y_0^1(\theta, \phi)$

Q2

The wavefunction of an electron in a hydrogen like atom is $\psi(r) = Ce^{-r/a}$, where $a = a_0/Z; a_0 \sim 0.5\mathring{A}$ is the Bohr radius,
(a) Compute the normalization constant.
(b) If the nucleus number is A=173 and Z=70, what is the probability that the electron is in the nucleus? assume the radius of the nucleus is $1.2 \times A^{1/3} fm$.
(c) What is the probability that the electron is in the region $x, y, z > 0$?

Q3

Write the Schrodinger equation for two-dimensional hydrogen atom. Suppose that the potential energy is $-e^2/r$, where $r = \sqrt{x^2 + y^2}$. Using separation of variables, find the radial and the angular equations. Solve the angular equation. Describe the quantum numbers that characterise the bound states and the degeneracies of the system.