**PHY 303: Classical Electrodynamics**

MONSOON SEMESTER 2022

TUTORIAL 02

1. Consider a uniformly charged ring having a total charge $Q$ and of radius $a$ lying in the $xy$-plane, with its center coinciding with the origin. Find out the expression of the corresponding volume charge density $\rho(\mathbf{r})$ in cylindrical coordinates with the help of Dirac delta function(s).

2. The electric field due to a certain charge density $\rho(\mathbf{r})$ is given by

$$\mathbf{E}(\mathbf{r}) = \begin{cases} \mathbf{0} & \text{for } r < R, \\ \dfrac{Q}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}} & \text{for } r > R. \end{cases}$$

   With the aid of Gauss's law in differential form, find out the charge density $\rho(\mathbf{r})$ and interpret it physically. Moreover, based on your answer for the charge density, verify that the discontinuity in the electric field is as expected.

3. Derive Green's theorem by using the vector function $\mathbf{A}(\mathbf{r}) = \phi(\mathbf{r})\nabla\psi(\mathbf{r}) - \psi(\mathbf{r})\nabla\phi(\mathbf{r})$ in the divergence theorem. Here, $\phi(\mathbf{r})$ and $\psi(\mathbf{r})$ are arbitrary scalar functions.

Challenge Problem:

Solve the differential equation $\nabla^2_{\mathbf{R}}\,\mathcal{G}(\mathbf{R}) = -4\pi\delta(\mathbf{R})$ using the Fourier transform method to show that $\mathcal{G}(\mathbf{R}) = 1/|\mathbf{R}|$. Here $\mathbf{R} = X\hat{\mathbf{x}} + Y\hat{\mathbf{y}} + Z\hat{\mathbf{z}}$ and $\nabla^2_{\mathbf{R}}$ represents the Laplacian involving the $X, Y, Z$ variables.