Introduction to Statistics (MAT 283)

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Table of Contents

SOME SPECIAL CONTINUOUS DISTRIBUTIONS

1. Uniform Distribution

A random variable X is said to be uniform on the interval $[a, b]$ if its probability density function is of the form

$$f(x) = \frac{1}{b-a}, \; a \leq x \leq b$$

where a and b are constants.

We denote a random variable X with the uniform distribution on the interval $[a, b]$ as $X \sim \text{UNIF}(a, b)$.

APPLICATION: Random number generation.

THEOREM: If X is a uniform random variable on the interval $[a, b]$, then the mean, variance and moment generating functions are respectively given by

$$E(X) = \mu_X = \frac{b+a}{2}$$

$$\text{Var}(X) = \sigma_X^2 = \frac{(b-a)^2}{12}$$

$$M_X(t) = \begin{cases} 1 & \text{if} \quad x = 0 \\ \frac{e^{tb}-e^{ta}}{t(b-a)} & \text{otherwise.} \end{cases}$$

EXERCISE: Suppose $Y \sim \text{UNIF}(0, 1)$ and $Y = \frac{1}{4}X^2$. What is the probability density function of X?

2. **Exponential Distribution:** A continuous random variable is said to be an exponential random variable with parameter θ if it's probability density function is of the form

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & \text{if} \quad x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

where $\theta > 0$.

If a random variable X has an exponential density function with parameter θ, then we denote it by writing $X \sim \text{EXP}(\theta)$.

APPLICATION: To model lifetime of electronic components.

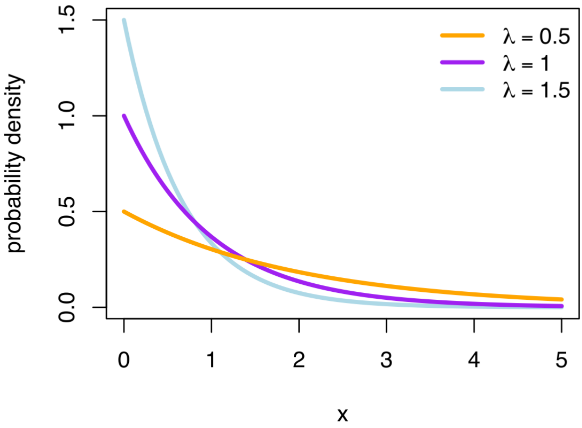
Theorem: If $X \sim EXP(\theta)$, then $E(X) = \frac{1}{\theta}$ and $Var(X) = \frac{1}{\theta^2}$.

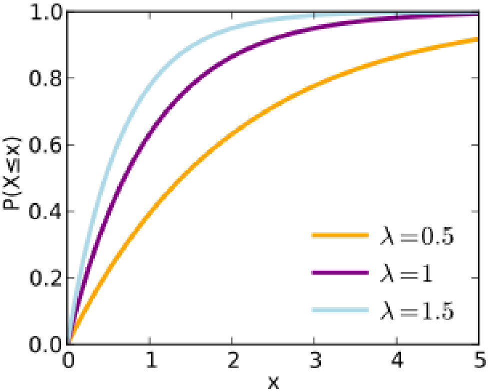
Theorem: If $X \sim EXP(\theta)$, then $E(X) = \frac{1}{\theta}$ and $Var(X) = \frac{1}{\theta^2}$.

Alternate Definition of Exponential Distribution: A continuous random variable is said to be an exponential random variable with parameter $\lambda = \frac{1}{\theta}$ if it's probability density function is of the form

$$f(x;\theta) = \begin{cases} \lambda e^{-\lambda x} & \text{if} \quad x > 0 \\ \\ 0 & \text{otherwise.} \end{cases}$$

where $\lambda > 0$.


λ = 0.5
λ = 1
λ = 1.5
probability density
0.0
0.5
1.0
1.5
0
1
2
3
4
5
x


1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
x
P(X≤x)
λ=0.5
λ=1
λ=1.5

EXERCISE: What is the cumulative distribution function of a random variable which has an exponential distribution with variance 25?