PHY305 Tutorial 2

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Q1

Consider a particle with a wave-function,

$$\psi(x, y, z) = N(x + y + z)e^{-(x^2+y^2+z^2)/\alpha^2} \tag{1}$$

where N is a normalization constant and α is a parameter. We measure the values of L^2 and L. Find the probabilities that the measurements yield:
a) $L^2 = 2\hbar^2$, $L_z = 0$ b) $L^2 = 2\hbar^2$, $L_z = \hbar$ c) $L^2 = 2\hbar^2$, $L_z = -\hbar$
Use know relations,

$$Y_1^1(\theta, \phi) = -\sqrt{\frac{3}{8\pi}} Sin\theta e^{i\phi}$$

$$Y_1^0(\theta, \phi) = -\sqrt{\frac{3}{4\pi}} Cos\theta$$

$$Y_1^{-1}(\theta, \phi) = -\sqrt{\frac{3}{8\pi}} Sin\theta e^{-i\phi}$$

Q2

At $t = 0$ the hydrogen atom is in the state

$$\psi(t = 0) = \frac{1}{\sqrt{2}}(\psi_{100} - \psi_{200}) \tag{2}$$

Calculate the expected value of r as a function of time.

Q3

Consider a particle of mass m and energy $E > 0$ moving in infinite spherical well of radius a,
a) Show that the Schrodinger equation can be reduced to the following

$$\frac{d^2 R_{nl}}{dz^2} + \frac{2}{z}\frac{dR_{nl}}{dz} + \left(1 - \frac{l(l+1)}{z^2}\right) R_{nl} = 0, \tag{3}$$

where $Z = kr$ and $k^2 = 2mE/\hbar^2$
b)Show that the energy is then given by

$$E_{nl} = \frac{\hbar^2}{2ma^2} z_{nl}^2 \tag{4}$$

where z_{nl} is the nth zero of the Bessel function solution of the radial Schrodinger equation.