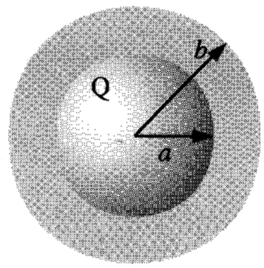
PHY 303: Classical Electrodynamics
MONSOON SEMESTER 2022
TUTORIAL 06

1. A metal sphere of radius a carries a charge Q (free). It is surrounded, out to radius b, by a linear dielectric material of permittivity ϵ; see the figure.

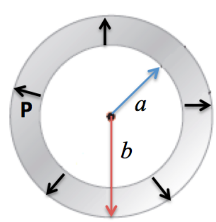


Calculate the following:

(a) The electric field in the regions $r < a, a < r < b$ and $r > b$.
(b) The polarization induced in the dielectric.
(c) The polarization volume and surface charge densities (bound charge densities).

2. A long straight wire, carrying uniform line charge λ, is surrounded by rubber insulation out to a radius a. Find the electric displacement vector. What is the electric field in the region outside the rubber insulation? Can we calculate the electric field inside the insulation using the given information?

3. A thick spherical shell (inner radius a, outer radius b) is made of dielectric material with a "frozen-in" polarization $\mathbf{P}(\mathbf{r}) = \frac{k}{r}\hat{\mathbf{r}}$, where k is a constant and r is the radial distance from the center; see the figure below. There is no *free* charge in the problem, $Q_f = 0$. Find the electric field in all three regions by two different methods:



(a) Locate all the bound (polarized) charges, and use Gauss's law written in terms of $\mathbf{E}$, $\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = Q_{\text{total,enc}} = Q_{f,\text{enc}} + Q_{b,\text{enc}}$, to calculate the field it produces. Here $Q_{f,\text{enc}}$ and $Q_{b,\text{enc}}$ refer to the enclosed free charges and enclosed bound charges, respectively.

(b) Use Gauss's law written in terms of $\mathbf{D}$, $\oint \mathbf{D} \cdot d\mathbf{S} = Q_{f,\text{enc}}$, to find $\mathbf{D}$, and then obtain $\mathbf{E}$ using the relation $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$.

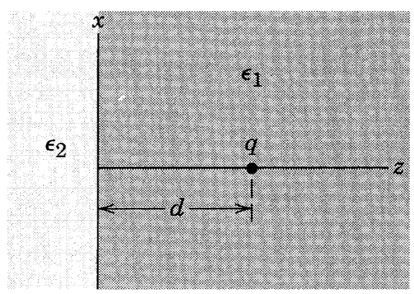
4. In the classroom lectures, we derived the electric field inside a sphere of linear dielectric material in an otherwise uniform electric field $\mathbf{E}_0 = E_0\hat{\mathbf{z}}$. Recalculate it using the following method of successive approximations: First pretend the field inside is just $\mathbf{E}_0$, and use relation $\mathbf{P} = \epsilon_0\chi_e\mathbf{E}$ to write down the resulting polarization $\mathbf{P}_0$ (*assuming that the final equilibrium stage has not been reached yet*). This polarization generates a field of its own*, $\mathbf{E}_1$, which in turn modifies the polarization by an amount $\mathbf{P}_1$, which further changes the field by an amount $\mathbf{E}_2$, and so on. The resulting field is $\mathbf{E}_0 + \mathbf{E}_1 + \mathbf{E}_2 + \cdots$. Sum this infinite series, and compare with the answer (*final situation in the equilibrium*) derived in the class.

   [*The inside-electric-field produced by a sphere with fixed polarization $P_0\hat{\mathbf{z}}$ is $-\frac{P_0}{3\epsilon_0}\hat{\mathbf{z}}$. This can be proved by taking the negative-gradient of the inside-potential expression derived in Problem 2 of Tutorial 5.]

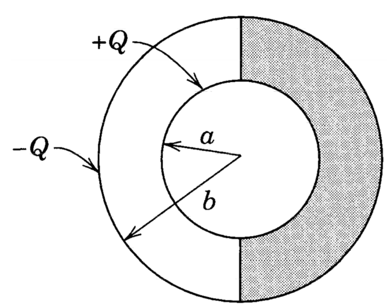
5. Consider a point charge q embedded in a semi-infinite dielectric medium with permittivity ϵ_1 a distance d away from a plane interface (say $z = 0$ plane) that separates the first medium from another semi-infinite dielectric medium with permittivity ϵ_2.

   (a) Calculate the electric potential (for both $z < 0$ and $z > 0$) for this set up using the method of images. (Refer to J. D. Jackson.)

   (b) Recalculate the electric potential by calculating the bound charges and then calculating the overall potential by superimposing potentials due to these and the (free) charge q. (Refer to D. J. Griffiths.)



6. Two concentric conducting spheres of inner and outer radii a and b, respectively, carry charges $\pm Q$. The empty space between the spheres is half-filled by a hemispherical shell of dielectric (of dielectric constant ϵ/ϵ_0), as shown in the figure.



   (a) Find the electric field everywhere between the spheres.

   (b) Calculate the surface-charge distribution on the inner sphere.

   (c) Calculate the polarization-charge density (bound surface charge density) induced on the surface of the dielectric at $r = a$.