Lyapunov Based Design for Two Wheeled Robot

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A. Abstract

A two-wheeled bot is an inherently unstable system that needs the use of a controller to keep it stable. To determine the stability of this nonlinear dynamical system, a Lyapunov-based control system technique is applied. The robot's dynamics can be determined by system identification, which entails providing some data to the system and obtaining the system's input output relationship. In this situation, however, we obtain the dynamics by examining the free body diagram and applying Newton's second law. The foundations underlying nonlinear control system design are Lyapunov's stability theorems. We propose a Lyapunov function that takes into account the dynamics in order to ensure that we achieve Global Asymptotic Stability (GAS). Simulations in Matlab have been used to evaluate performance.

Keywords: Two-wheeled robot, self-balancing robot, nonlinear control, Lyapunov-based control.

B. Introduction

Because non-linear control design has more complications than linear control design, it is not as simple as designing a linear system. The non-linear systems are approximated as linear systems through linearization around an operating point and is commonly used to simplify the control design of a non-linear system. Local asymptotic stability refers to the fact that the intended control system is only valid in a limited region surrounding the operational point as the linearization basis. It means the stability is not valid for the entire operating region. Non-linear control design techniques can help to accomplish the stabilization of the Two wheeled bot for the entire operating region.

Over the last decade, two-wheeled robots have been the focus of research. The robot's fundamental structure consists of a body supported by two wheels. The concept of a two-wheeled bot was inspired by the inverted pendulum. We must build a controller/torque input that allows the two-wheeled bot to remain stable and avoid falling. The controller must be able to create the appropriate torque for the bot to be stable, hence the angle of inclination must be established.

C. Control System Design

Base on the free body diagram, the working torques on the robot body are evaluated at the wheel axis. Applying the Newton's second law results in the following dynamic equations:

$$\sum M = I\ddot{\theta} \tag{1}$$

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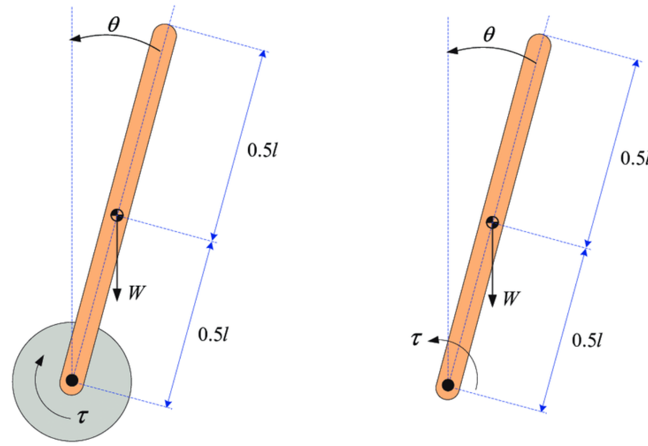

$$\frac{1}{2}mgl\sin\theta - \tau = I\ddot{\theta} \tag{2}$$

where,
M is moment
I is inertia of the robot body,
θ is the pitching angle which is the robot body deflection with respect to vertical axis,
m is the mass of robot body,
g is the gravity acceleration,
l is the length of the robot body,
$\ddot{\theta}$ is the pitching acceleration of the robot body, and
τ is the input torque to the robot body. In this case, τ is generated by a motor and is used as the counter torque to stabilize the robot.

The system state can be defined as:

$$x_1 = \theta \tag{3}$$

$$x_2 = \dot{\theta} \tag{4}$$

Now, we get robot dynamics in terms of states as,

$$\dot{x_1} = x_2 \tag{5}$$

$$\dot{x_2} = \frac{mgl}{2I}\sin x_1 - \frac{1}{I}\tau \tag{6}$$

For ease of design, let us replace

$$\frac{mgl}{2I} = k_1$$

$$\frac{1}{I} = k_2$$

.

we get,

$$\dot{x_2} = k_1 \sin x_1 - k_2\tau \tag{7}$$

Let the lyapunov function be,

$$V = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 \tag{8}$$

Taking the time derivative of lyapunov function, we get,

$$\dot{V} = x_1\dot{x_1} + x_2\dot{x_2} \tag{9}$$

substituting the dynamics now, we get

$$\dot{V} = x_1(x_2) + x_2(k_1 \sin x_1 - k_2\tau) \tag{10}$$

Now we have bad cross terms, so we need to design τ such that we cancel out these terms.

Therefore, designing

$$\tau = \frac{k_1}{k_2}\sin x_1 + \frac{1}{k_2}x_2 + \lambda x_2$$

where, λ is the feedback term.

Now substituting τ in $\dot{V}$, we get

$$\dot{V} = x_1x_2 + x_2(k_1 \sin x_1 - k_2(\frac{k_1}{k_2}\sin x_1 + \frac{1}{k_2}x_2 + \lambda x_2)) \tag{11}$$

Now, simplifying and cancelling some terms, we get

$$\dot{V} = -\lambda x_2^2 \tag{12}$$

Also, if we substitute τ in equation (7), we get

$$\dot{x_2} = -x_1 - \lambda x_2$$

Therefore, the time derivative of lyapunov function is Negative semi definite. Since, V is radially unbounded, we can say that the equilibrium points are globally stable.

Now, let's try Lasalle,

$\dot{V} = 0$ implies $x_2 = 0$ and $x_2 = 0$ implies $x_1 = 0$

Therefore, by Lasalle's invariance principle, we can say that the equilibrium points are globally asymptotically stable and the closed loop system is stable and we can conclude through Lasalle, that the states will return to origin when the robot is disturbed.

Now, for further analysis, let's consider a constant reference. Therefore, from equation (2), we have

$$\frac{1}{2}mgl\sin\theta - \tau = I\ddot{\theta} \tag{13}$$

$$\frac{1}{2I}mgl\sin\theta - \frac{\tau}{I} = \ddot{\theta} \tag{14}$$

we know,

$$\frac{mgl}{2I} = k_1$$

$$\frac{1}{I} = k_2$$

.

$$k_1 \sin\theta - k_2\tau = \ddot{\theta} \tag{15}$$

Therefore,

$$e = \theta_d - \theta$$

$$\dot{e} = -\dot{\theta}$$

$$\ddot{e} = -\ddot{\theta}$$

$$r = \dot{e} + \alpha e$$

$$\dot{r} = \ddot{e} + \alpha\dot{e}$$

substituting the value of $\ddot{e}$ and multiplying by m, we get

$$m\dot{r} = -m\ddot{\theta} + m\alpha\dot{e}$$

Now let us consider lyapunov candidate as,

$$V = \frac{1}{2}e^2 + \frac{1}{2}mr^2 \tag{16}$$

Taking the time derivative of lyapunov function, we get,

$$\dot{V} = e\dot{e} + rm\dot{r} \tag{17}$$

substituting $\dot{e}$ and $m\dot{r}$,

$$\dot{V} = e(r - \alpha e) + r(-m\ddot{\theta} + m\alpha\dot{e}) \tag{18}$$

Now substituting $\ddot{\theta}$, we get

$$\dot{V} = e(r - \alpha e) + r(-mk_1 \sin\theta + mk_2\tau + m\alpha\dot{e}) \tag{19}$$

Now, designing,

$$\tau = \frac{k_1}{k_2}\sin\theta - \frac{\alpha\dot{e}}{k_2} - e - \beta r$$

Now, substituting τ and simplifying we get,

$$\dot{V} = -\alpha e^2 - \beta r^2 \tag{20}$$

Now, for third analysis let us consider we have time varying reference.

$$e = \theta_d - \theta$$

$$\dot{e} = \dot{\theta}_d - \dot{\theta}$$

$$\ddot{e} = \ddot{\theta}_d - \ddot{\theta}$$

we know,

$$r = \dot{e} + \alpha e$$

$$\dot{r} = \ddot{e} + \alpha\dot{e}$$

substituting the value of $\ddot{e}$ and multiplying by m, we get

$$m\dot{r} = m\ddot{\theta}_d - m\ddot{\theta} + m\alpha\dot{e}$$

Now let us consider lyapunov candidate as,

$$V = \frac{1}{2}e^2 + \frac{1}{2}mr^2 \tag{21}$$

Taking the time derivative of lyapunov function, we get,

$$\dot{V} = e\dot{e} + rm\dot{r} \tag{22}$$

substituting $\dot{e}$ and $m\dot{r}$,

$$\dot{V} = e(r - \alpha e) + r(m\ddot{\theta}_d - m\ddot{\theta} + m\alpha\dot{e})$$

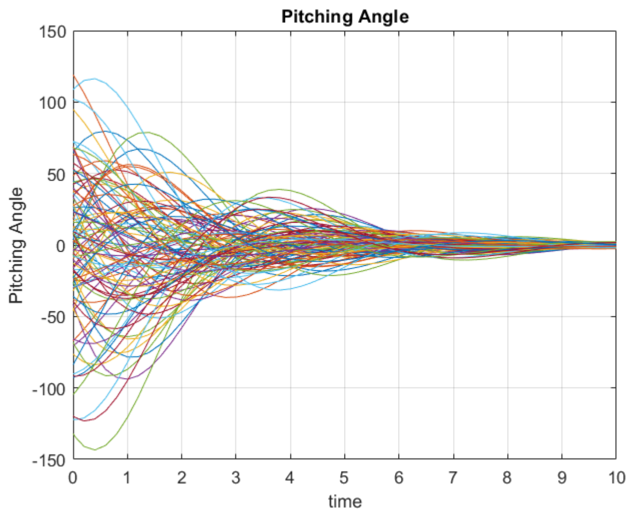

Fig. 1. Pitching Angle.

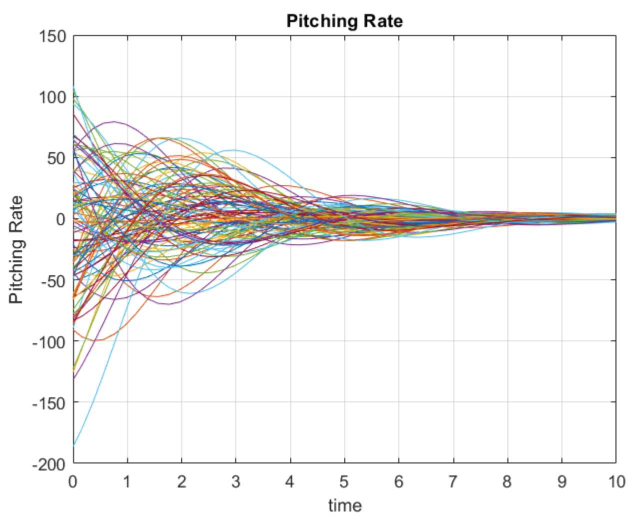

Fig. 2. Pitching Rate.

Now, substituting the value of $m\ddot{\theta}$, we get

$$\dot{V} = er - \alpha e^2 + r(m\ddot{\theta}_d - (-\ mk_1 \sin\theta + mk_2\tau) + m\alpha\dot{e})$$

$$\dot{V} = er - \alpha e^2 + r(m\ddot{\theta}_d + \ mk_1 \sin\theta - mk_2\tau + m\alpha\dot{e})$$

Now, designing,

$$\tau = \frac{1}{k_2}\ddot{\theta}_d + \frac{k_1}{k_2}\sin\theta - \frac{\alpha\dot{e}}{k_2} - e - \beta r$$

Now, substituting τ and simplifying we get,

$$\dot{V} = -\alpha e^2 - \beta r^2 \tag{23}$$

$$\dot{V} = -2[\alpha, \beta]V$$
$$\dot{V} = -2KV$$

Therefore, we can say that as the time derivative of lyapunov candidate is Negative Definite and the lyapunov function is radially unbounded, we have globally exponential tracking.

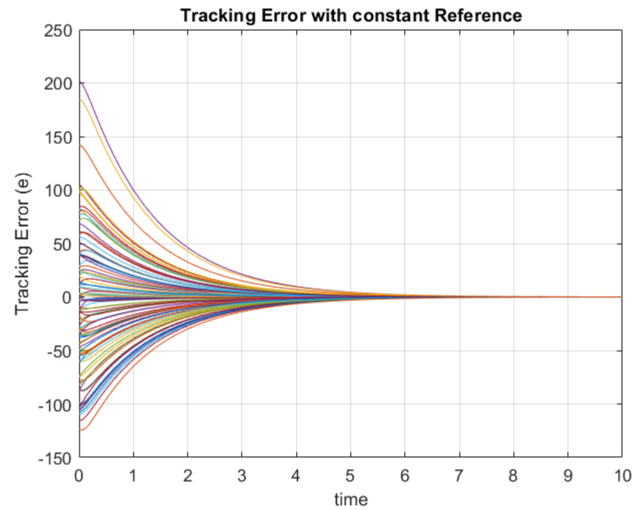

Fig. 3. Tracking error with constant reference.

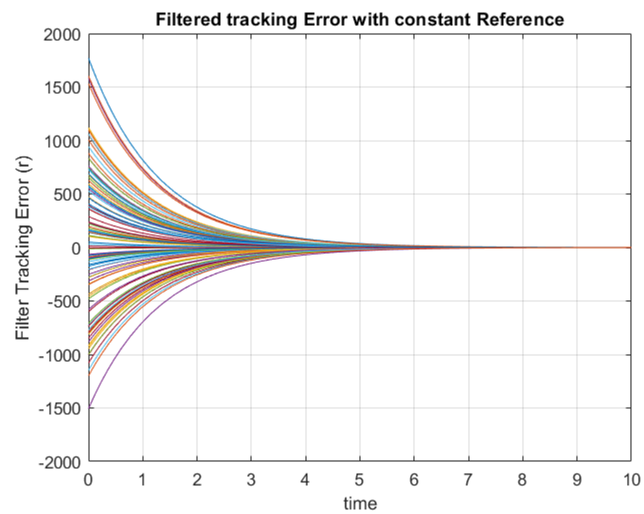

Fig. 4. Filtered Tracking Error with constant reference.

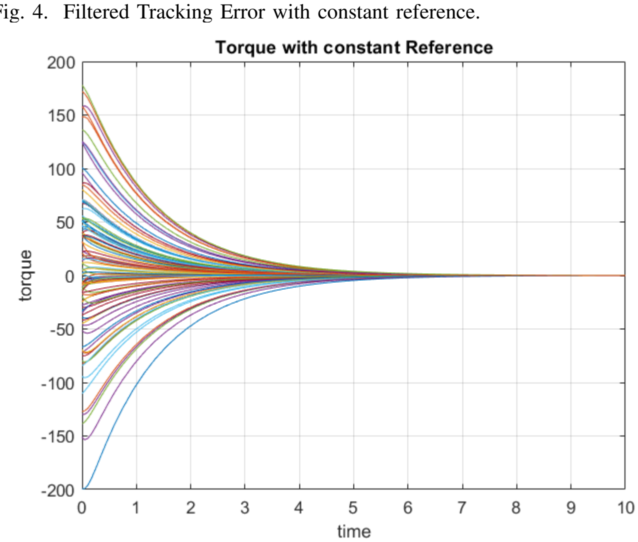

Fig. 5. Torque with constant reference.

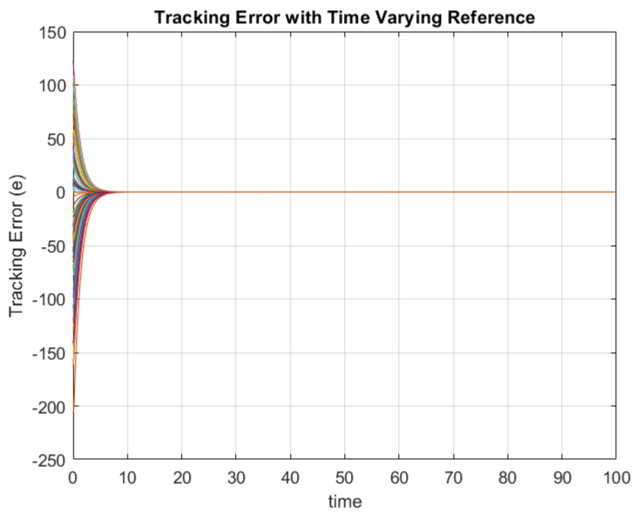

Fig. 6. Tracking error with Time varying reference.

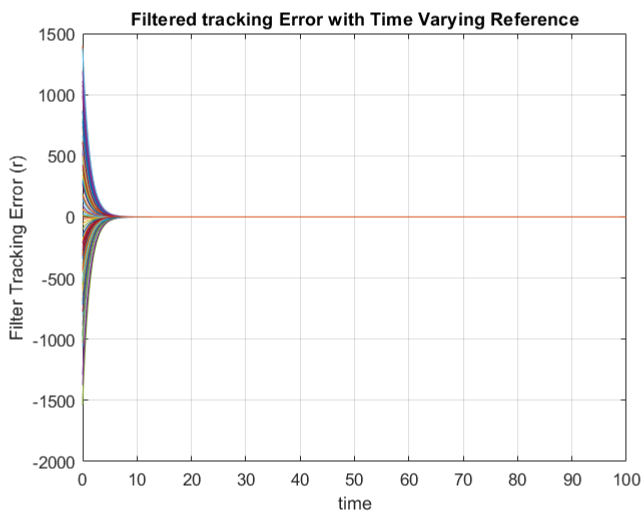

Fig. 7. Filtered Tracking Error with Time Varying reference.

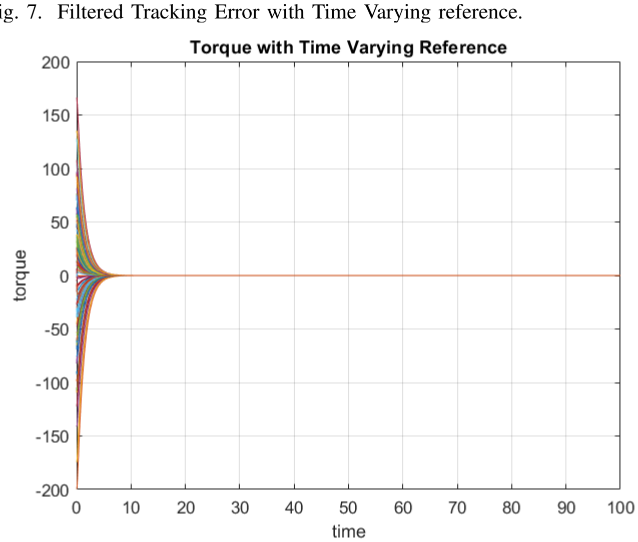

Fig. 8. Torque with Time varying reference.

D. Simulation

Simulation Parameters			
Parameter	Symbol	Value	Unit
Mass of the rod	m	0.1	kg
Length of the rod	l	20	cm
Inertia of the rod	I	13.408	$kg.cm^2$
Control Gain	λ	2	

The plotting is done to check whether our design matches our simulation results. We plotted the Monte Carlo simulation for random initial values of x_1 and x_2. As we increase the value of λ, it converges faster. For the third analysis, where we have time varying reference, the reference considered was,

$$\theta_d \ (t) = \bar{\theta_d} \ \sin(2\pi f_{\theta d} \ t) \tag{24}$$

$$\dot{\theta_d} \ (t) = (2\pi f_{\theta d}) \ \bar{\theta_d} \ \cos(2\pi f_{\theta d} \ t) \tag{25}$$

$$\ddot{\theta_d} \ (t) = -(2\pi f_{\theta d})^2 \ \bar{\theta_d} \ \sin(2\pi f_{\theta d} \ t) \tag{26}$$

E. Conclusion:

In conclusion, we can say with lyapunov based design technique we can achieve stability for non linear systems. We can conclude from the plots that are simulation results matches our stability results. The controller tracks the reference perfectly, as we can see the tracking error and filtered tracking error converges for constant reference and time varying reference.

F. References:

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