# Multiple populations with correlated bad noise

Only one type of correlations ultimately limit the information content of large neural populations, and these are noise correlations that look exactly like the signal [1]. We call this 'bad noise'. The presence of bad noise in a large neural population generates a redundant code from which the information can be extracted in many different ways with negligible loss. In particular, distinct populations can share bad noise, in which case the brain could read out any or all of them with almost no change in information. One consequence is that every neuron would then have a correlation with behavior (choice correlation $C_k$) equal to the fraction of total information it carries [2], according to $C_k = d'_k/d'$ where $d'_k$ is the discriminability for neuron $k$ and $d'$ is the discriminability for the whole population. Here we analyze the information content and behavioral correlations of multiple populations that have overlapping sources of information. We derive new results that place constraints on the relevant correlations between areas, and we explain the following paradoxical effect: a population that is not decoded for behavior can have higher choice correlations than another population that is decoded optimally. These results provide a new framework for understanding the relationships between multiple populations in information processing.

**Supporting material**: We proceed by analyzing the covariance matrix of the bad noise. In previous work [1], we examined bad noise that was a rank-one perturbation to another ('good') covariance matrix $\Sigma_0$ that does not limit information. This produced a total neural noise covariance of $\Sigma = \Sigma_0 + \epsilon \boldsymbol{f}'\boldsymbol{f}'^\top$ where $\boldsymbol{f}(s)$ is the population tuning curve for a scalar stimulus $s$, $\boldsymbol{f}' = d\boldsymbol{f}/ds$ is the signal direction for fine discrimination tasks, and $\epsilon$ is the bad noise variance along this same direction. Here we extend this analysis to two populations, with mean responses $\boldsymbol{x}$ and $\boldsymbol{y}$ and corresponding discrimination signals $\boldsymbol{x}'$ and $\boldsymbol{y}'$. In each population, the bad noise has variances $\epsilon_{xx}$ and $\epsilon_{yy}$, forming *two* rank-one noise covariance matrices $\epsilon_{xx}\boldsymbol{x}'\boldsymbol{x}'^\top$ and $\epsilon_{yy}\boldsymbol{y}'\boldsymbol{y}'^\top$. Together, these noise modes can form a rank-two covariance, described by $UEU^\top$, where $U = \begin{pmatrix} \boldsymbol{x}' & 0 \\ 0 & \boldsymbol{y}' \end{pmatrix}$ is a $2 \times N$ block matrix reflecting the structure of the bad noise modes, and $E = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{xy} & \epsilon_{yy} \end{pmatrix}$ is a $2 \times 2$ matrix representing their covariance (Figure A). The total noise covariance matrix is then

$$\Sigma = \Sigma_0 + UEU^\top = \Sigma_0 + \begin{pmatrix} \epsilon_{xx}\, \boldsymbol{x}'\boldsymbol{x}'^\top & \epsilon_{xy}\, \boldsymbol{x}'\boldsymbol{y}'^\top \\ \epsilon_{xy}\, \boldsymbol{y}'\boldsymbol{x}'^\top & \epsilon_{yy}\, \boldsymbol{y}'\boldsymbol{y}'^\top \end{pmatrix} \tag{1}$$

We show that the information limits in one or both populations is determined entirely by $E$.

Once again, the only noise that limits information in both populations together is noise that looks like signal, and so the total information is $J = (1,1)E^{-1}(1,1)^\top$. With different covariances $E$, the populations can range from fully redundant to enormously synergistic, with information that can be arbitrarily larger than the information in the populations separately. For example, with perfect correlation between the bad noises, if they have different amplitudes then they can be perfectly distinguished from the signal, which must have the identical amplitude changes along $\boldsymbol{x}'$ and $\boldsymbol{y}'$. This generates boundless information. On the other hand, uncorrelated bad noises in each population separately span the bad noise for both populations together, and therefore generates limited information. Naturally, this information is the sum of the informations in the independent populations. And of course there is the original case, where the bad noises are perfectly correlated with the same amplitude, and thereby become entirely redundant.

Next we consider choice correlations, under conditions where only one population is decoded optimally, and the other is completely ignored. Under optimal decoding, we showed previously [2] that neurons have a choice correlation $C_k = d'_k/d'$. However, with correlated bad noise in different populations, an undecoded population has a choice correlation given by

$$C_k = \frac{\epsilon_{xy}}{\epsilon_{xx}} \frac{d'_k}{d'} \tag{2}$$

which can, surprisingly, be *larger* than the choice correlations for the population that actually drives behavior. These results are verified by simulation (Figure B). Note that the ratio $\epsilon_{xy}/\epsilon_{xx}$ can take any value, but places a constraint on the bad noise in the $\boldsymbol{y}$ population, according to $\epsilon_{yy} > \epsilon_{xy}^2 \epsilon_{xx}$. This pattern of choice correlations can coexist with reasonably efficient — but not perfect — decoding (Figure C).

Overall, this framework, and the noise covariance matrix $E$, provides simple way to think about multiple populations and their interactions, but with a massively reduced dimensionality that nonetheless preserves all relevant information.

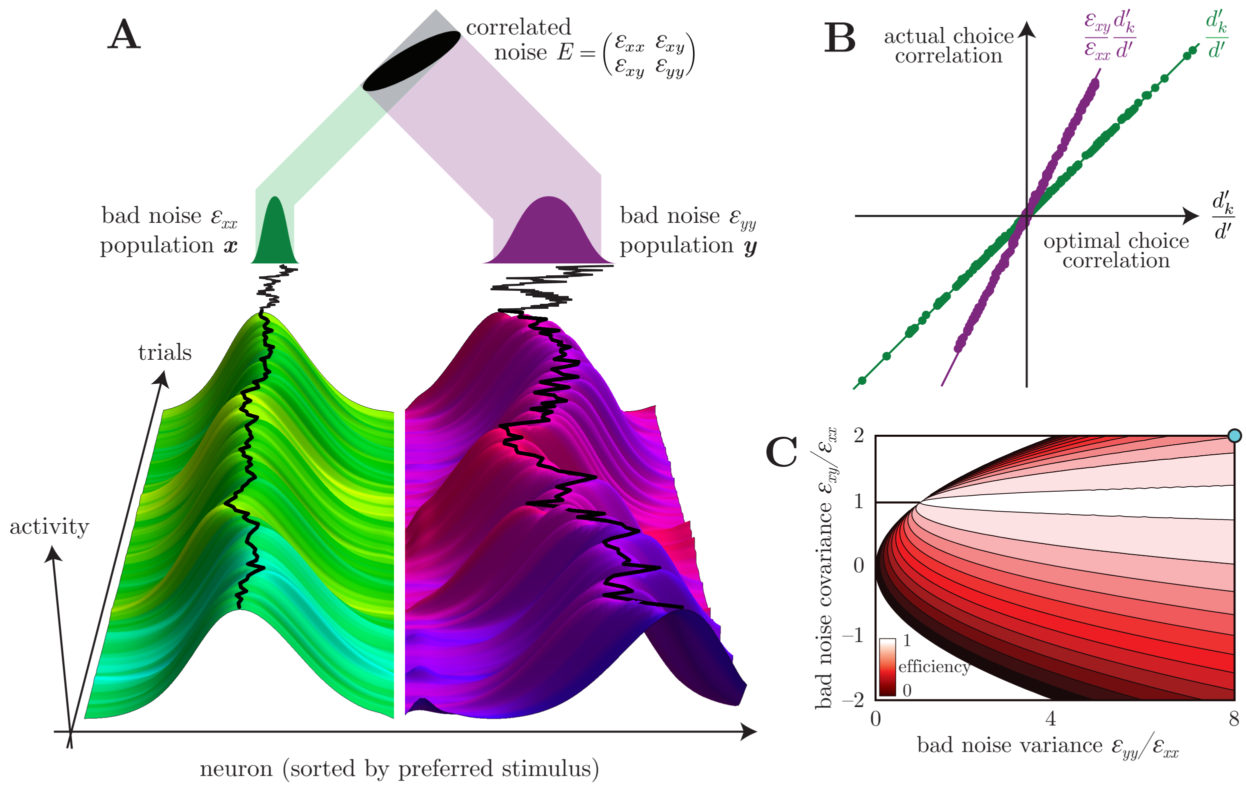


Figure 1: Effects of correlated bad noise modes between two populations. (**A**) shows two homogeneous populations driven by a constant stimulus, and bad noise (black lines) shift the population activities as if the stimulus had changed, and thus limit the reliability of the estimates to the variances $\epsilon_{xx}$ and $\epsilon_{yy}$. These noise sources can be correlated (black ellipse), in which case the two populations together will have more information than either population, and potentially much more. The matrix $E$ determines the correlation between the two populations and behavior if only one is decoded. Substantial deviations from optimal choice correlations seen in population $\boldsymbol{y}$ (**B**) occur only with an efficiency less than one (**C**, cyan dot).


[1] Moreno-Bote, Beck, Kanitscheider, Pitkow, Latham, Pouget. Information-limiting correlations (in review).

[2] Pitkow, Liu, Angelaki, DeAngelis, Pouget. How can single neurons predict behavior? Cosyne 2013.