Strassen's Matrix Multiplication

Complete Course on Algorithm - Part II

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Matrix - Addition

$n \times n \Rightarrow n^2$ | ~~DAC~~

$2 \times 2 \Rightarrow 2^2$

$$A = \begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & \multicolumn{3}{c}{\left[\begin{matrix} - & - & - \\ - & - & - \end{matrix}\right]} \\ 2 & & & \end{array} \quad n \times n$$

$\cancel{m \times n}$

$\cancel{2 \times 3}$

$$B = \begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & \multicolumn{3}{c}{\left[\begin{matrix} - & - & - \\ - & - & - \end{matrix}\right]} \\ 2 & & & \end{array} \quad n \times n$$

$\cancel{m \times n}$

$\Rightarrow \left(\frac{n}{2}\right)$ $\cancel{2 \times 3}$

$\frac{n}{2} \times \frac{n}{2}$

$C = A + B$

$\Downarrow$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & \multicolumn{3}{c}{\left[\begin{matrix} - & - & - \\ - & - & - \end{matrix}\right]} \\ 2 & & & \end{array} \quad n \times n$$

$\cancel{m \times n}$

$\cancel{2 \times 3}$

c-cont - n

```
for(i = 1 to n)
  for(j = 1 to n)
    c[i][j] = A[i][j] + B[i][j]
```

2×3 ele

$\parallel$

$2 \times 3 \times 1$-add

$2 \times 3 \times C$

$n \times n \times C$

$n \times n \times C$

$\rightarrow O(n^2)$

Matrix – multiplication

~~DAC~~

365

$$A = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix} \end{array} \quad m \times n$$

2×3

$$B = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} \end{array}$$

4

3×3 $\quad n \times p$

$C = A * B$

$2 \times \boxed{3 \quad 3} \times 3$

$$= \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix} \end{array}$$

$C_{11} = A_{11} * B_{11} + A_{12} * B_{21} + A_{13} * B_{31}$

C – contain – $2 * 3$ – ele

– $m \times p$ – ele

$m \times f$ – 2×3 $\quad O(mnp)$ $\quad O(n^3)$ $\Longleftarrow$ – $m \times p * n$ – mu for (i = 1 to n) → n

for (j = 1 to n) → n

for (k = 1 to n) → n   (1, 2, 3)

$c[i][j] += a[i][k] * b[k][j]$

$O(n^3)$

$c[1][1] = 0 + a[1][1] * b[1][1] + a[1][2] * b[2][1] + a[1][3] * b[3][1]$

With – DAC $\Longrightarrow$ Matrix multiplication

2×2 (or) 1×1 small problem

$$A = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 3 & 4 & 4 & 5 \\ 3 & 4 & 7 & 8 \\ 5 & 6 & 9 & 10 \end{bmatrix} \qquad B = \begin{bmatrix} 11 & 21 & 51 & 61 \\ 30 & 41 & 71 & 81 \\ 31 & 88 & 1 & 2 \\ 42 & 36 & 5 & 7 \end{bmatrix}$$

A_{11} A_{12} A_{21} A_{22}

B_{11} B_{12} B_{21} B_{22}

$2 \times 2 \Leftarrow 4 \times 4 / 2$

$4 \times 4 / 2$

$C = A \otimes B$

$\Downarrow$ 4×4 4×4

4×4

$$C = \begin{array}{cc} C_{11} & C_{12} \\ \left[\begin{array}{cc|cc} 5 & 6 & 9 & 10 \\ 7 & 8 & 11 & 12 \\ \hline 13 & 14 & 17 & 18 \\ 15 & 16 & 19 & 20 \end{array}\right] & \\ C_{21} & C_{22} \end{array}$$

4×4

2×2

2

$$C_{11} = A_{11} \otimes B_{11} + A_{12} \otimes B_{21}$$

$$C_{12} = A_{11} \otimes B_{12} + A_{12} \otimes B_{22}$$

$$C_{21} = A_{21} \otimes B_{11} + A_{22} \otimes B_{21}$$

$$C_{22} = A_{21} \otimes B_{12} + A_{22} \otimes B_{22}$$

$1 - \frac{mm}{} - 4 \times 4 \Longrightarrow 8 - \frac{mm}{2 \times 2} + 4 - \frac{mA}{2 \times 2}$

$1 - mm - 16 \times 16 \Longrightarrow 8 - \underset{8 \times 8}{mm} + 4 - \underset{8 \times 8}{mA}$

$1 - mm - n \times n \Longrightarrow 8 - \underset{n/2 \times n/2}{mm} + 4 - \underset{n/2 \times n/2}{mA}$

$T(n)$ = Time complexity of multiply 2- matrices of size $n \times n$ w/ DAC

RR

add

$n \times n \Rightarrow n^2$

$2 \times 2 \Rightarrow 2^2$

$n/2 \times n/2 \Rightarrow (n/2)^2$

$$T(n) = \begin{cases} O(1) & \text{if } n \le 2 \\ 8T(n/2) + 4\,O\left(\frac{n}{2}\right)^2 & \text{if } n > 2 \end{cases}$$

$$T(n) = \boxed{8T(n/2) + n^2}$$

$\{ \underline{\underline{8n}} \Longrightarrow$ Static space

$= n^{\log_2 8} \Longrightarrow \theta(n^3)$ ✓

$\Downarrow$

According to Strassen's

$\Downarrow$

$$T(n) = \begin{cases} O(1) & \text{if } \le 2 \\ 7T(n/2) + 18\,\theta\left(\frac{n}{2}\right)^2 & \text{if } n > 2 \end{cases}$$

$$= 2T(n/2) + 18 \times \frac{n^2}{4}$$

$$T(n) = 2T(n/2) + 4.5\, n^2$$

$$\boxed{T(n) = 2T(n/2) + n^2}$$

$5n$

$$= n^{\log_2 7} \Longrightarrow \Theta\left(n^{2.81}\right)$$

2×3

$$\begin{bmatrix} \_ & \_ & \_ & 0 \\ \_ & \_ & \_ & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3×4

$$\begin{bmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3×3

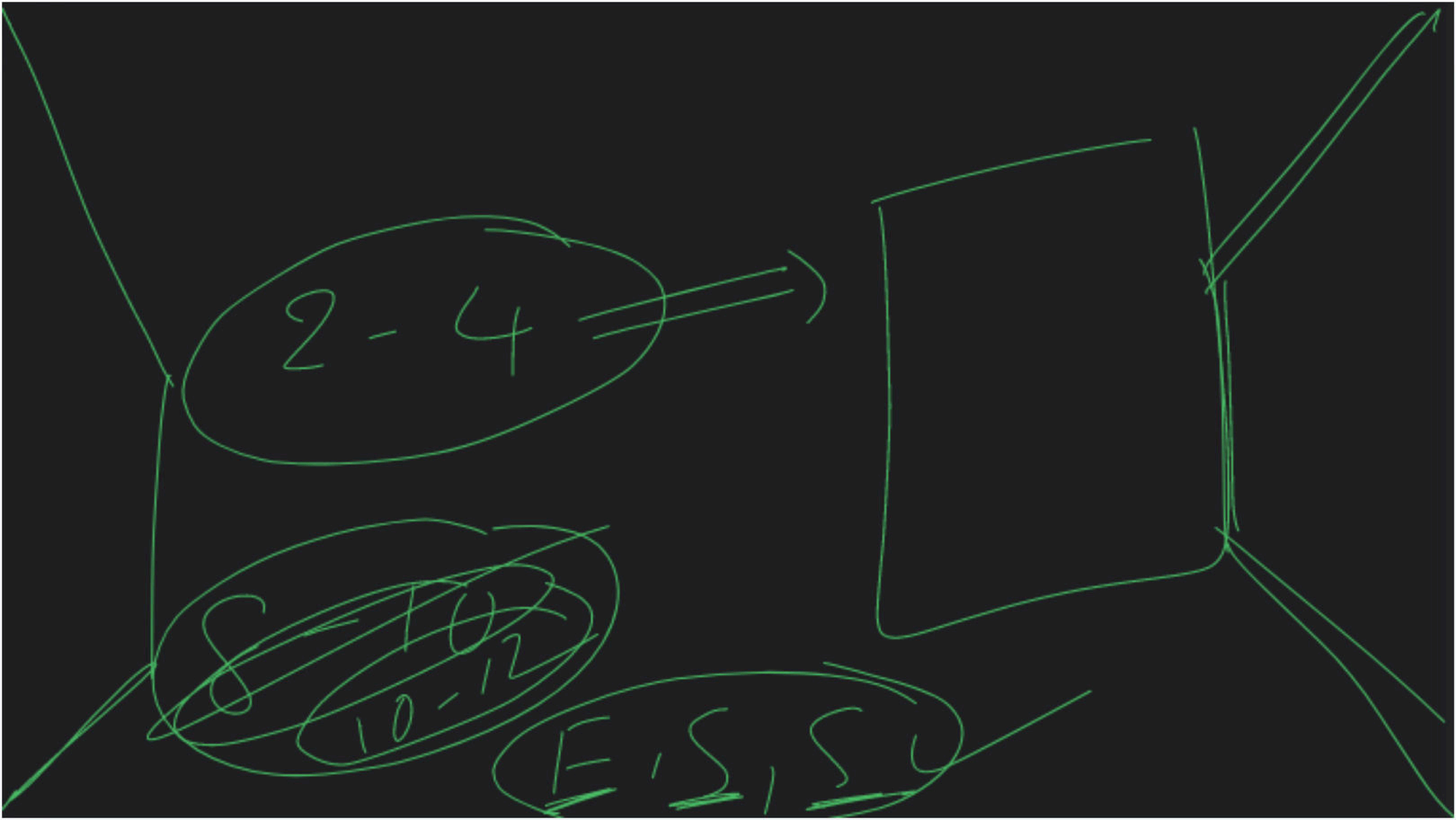
$\downarrow$

4×4

3×3

$\downarrow$

4×4


2-4
8
10
10-12
E, S, S, U