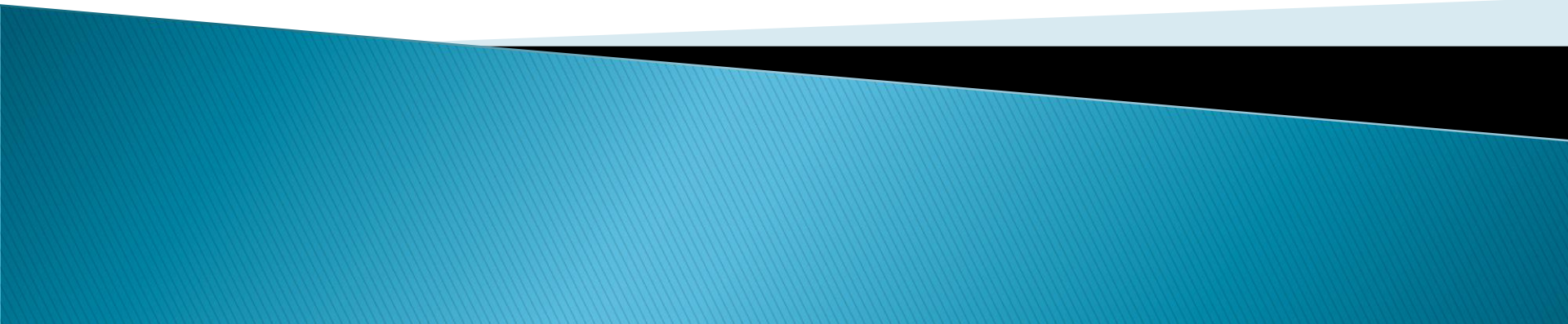


Game theory

Impartial Games



Combinatorial games

- ▶ Two-person games with perfect information
- ▶ No chance moves
- ▶ A win-or-lose outcome
- ▶ **Impartial games**
 - Set of moves available from any given position is the same for both players
- ▶ **partizan games**
 - Each player has a different set of possible moves from a given position
 - Eg: Chess

Take away games – an example

- ▶ We start with 'n' coins, in each turn the player can take away 1, 3 or 4 coins
- ▶ Question:
 - What is the value of 'n' for which the first player can win the game if both the players play optimally ??

Solution for example

- ▶ L – Losing position
- ▶ W – Winning position

L	W	L	W	W	W	W	L	W	L	W	W	W	W	L	W
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

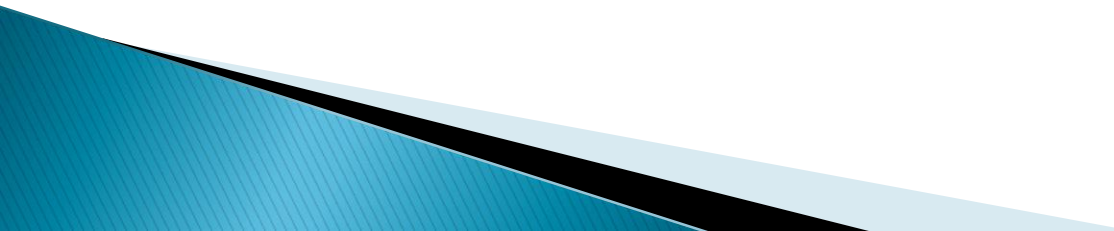
- ▶ 0,2,7,9,14,16,... are losing positions
- ▶ $7N, 7N+2$ are losing positions for $N \geq 0$

Properties of positions

- ▶ All terminal positions are losing.
- ▶ If a player is able to move to a losing position then it is a winning position.
- ▶ If a player is able to move only to the winning positions then it is a losing position.

WL – Algorithm

```
boolean isWinning(position pos) {  
    moves[] = possible positions to which we  
               can move from the position pos;  
    for (all x in moves)  
        if (!isWinning(x))  
            /* can move to Losing pos */  
            return true;  
    return false;  
}
```



Try this problem

- ▶ <http://www.spoj.pl/problems/NGM/>
- ▶ <http://www.spoj.pl/problems/MCOINS/>

Game of Nim

- ▶ There are 'n' piles of coins.
- ▶ In each turn a player chooses one pile and takes at least one coin from it. If a player is unable to move he loses.

Solution of Nim Game

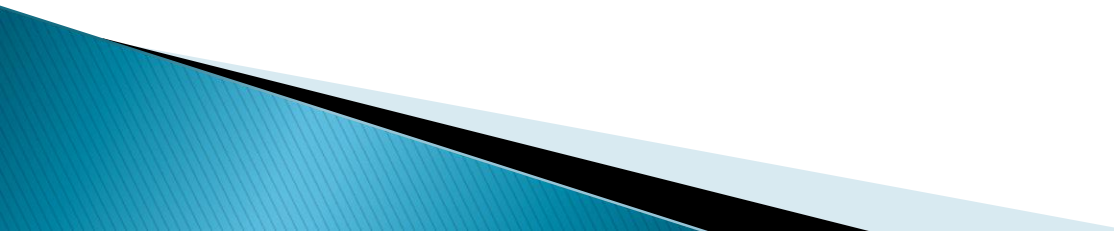
- ▶ Let n_1, n_2, \dots, n_k , be the sizes of the piles. It is a losing position for the player whose turn it is if and only if $n_1 \text{ xor } n_2 \text{ xor } \dots \text{ xor } n_k = 0$.
- ▶ Why does it work ??

Why does it work??

- ▶ From the losing positions we can move only to the winning ones:
 - if xor of the sizes of the piles is 0 then it will be changed after our move
- ▶ From the winning positions it is possible to move to at least one losing:
 - if xor of the sizes of the piles is not 0 we can change it to 0 by finding the left most column where the number of 1s is odd, changing one of them to 0 and then by changing 0s or 1s on the right side of it to gain even number of 1s in every column.

Composite Games – Grundy numbers

```
int grundyNumber(position pos) {  
    moves[] = possible positions to which I can  
               move from pos set s;  
  
    for (all x in moves)  
        insert into s grundyNumber(x);  
    //return the smallest non-negative integer  
    //not in the set s;  
  
    int ret=0;  
    while (s.contains(ret)) ret++; return ret;  
  
}
```



Practice problems

- ▶ <http://www.spoj.pl/problems/QCJ3/>
- ▶ <http://www.spoj.pl/problems/RESN04/>
- ▶ <http://www.spoj.pl/problems/MMMGAME>
- ▶ <http://pclub.in/index.php/wpc-archives/16-kodefest-solutions/87-problem-e>
- ▶ <http://www.spoj.pl/problems/PEBBMOV/>
- ▶ <http://www.codechef.com/problems/CHEFBRO>
- ▶ <http://www.spoj.pl/problems/HUBULLU/>
- ▶ SRM 330 DIV I Hard

- ▶ <http://www.codechef.com/problems/BIGPIZA>
- ▶ <http://projecteuler.net/problem=301>
- ▶ <http://www.codeforces.com/contest/87/problem/C>
- ▶ **EASY DP:**
<http://www.spoj.pl/problems/CRSCNTRY>

References

- ▶ http://www.math.ucla.edu/~tom/Game_Theory/comb.pdf
- ▶ <http://community.topcoder.com/tc?module=Static&d1=tutorials&d2=algorithmGames>