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Indian Institute of Technology Kanpur
CS637 Embedded and Cyber-Physical Systems
Homework Assignment 3
Deadline: September 16, 2022

Total: **40 marks**

1. Write the answers **neatly** in the given boxes.
2. You may discuss the solutions with the other students, but you have to write them in your own words.

Problem 1. (10 points) Provide the state-space representation of the dynamics of a DC Motor. Assume that there is no additional load on the motor. Next, Design a Simulink model to capture the dynamics and simulate the model for an input PWM voltage signal with magnitude 1V, frequency 1 kHz and duty cycle 0.1. Assume that the kinetic friction of the motor is negligible. Take the values of the other parameters from Example 7.13 in [LS15].

[LS15] Edward A. Lee and Sanjit A. Seshia, Introduction to Embedded Systems, A Cyber-Physical Systems Approach, Second Edition, http://LeeSeshia.org, ISBN 978-1-312-42740-2, 2015.

To describe the **electrical behaviour**:
Let $\omega : \mathbb{R} \to \mathbb{R}$ represent the angular velocity of the motor
The voltage and current through the motor to satisfy the following equation

$$v(t) = Ri(t) + L\frac{di(t)}{dt}$$

back electromagnetic force - torque resisting the rotation

$$v(t) = Ri(t) + L\frac{di(t)}{dt} + k_b\omega(t)$$

where k_b is back electromagnetic force constant

$$\implies \frac{di(t)}{dt} = v(t) - Ri(t) - k_b\omega(t)$$

To describe the **mechanical behaviour**:
The torque T on the motor is proportional to the current flowing through the motor, adjusted by friction and any torque that might be applied by the mechanical load

$$T(t) = k_T i(t) - \eta\omega(t) - \tau(t)$$

k_T is an empirically determined motor torque constant
η is the kinetic friction
τ is the torque applied by the
By Newton's second law, this needs to be equal to the moment of inertia I times the angular acceleration, so

$$I\frac{d\omega(t)}{dt} = k_T i(t) - \eta\omega(t) - \tau(t)$$

$$\implies \frac{d\omega(t)}{dt} = \frac{1}{I}(k_T i(t) - \eta\omega(t) - \tau(t))$$

Given kinetic friction of motor is negligible, therefore $\eta = 0$
Also, assume that there is no additional load on the motor, so the equation becomes

$$\frac{d\omega(t)}{dt} = \frac{k_T}{I}i(t)$$

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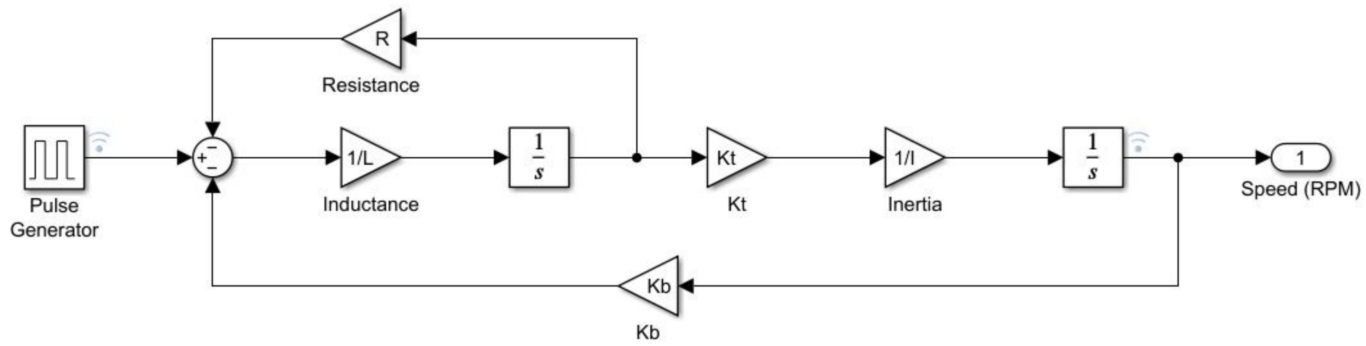
Motor's overall behaviour is governed by the following equations :

$$\frac{di(t)}{dt} = v(t) - Ri(t) - k_b\omega(t)$$

$$\frac{d\omega(t)}{dt} = \frac{k_T}{I}i(t)$$

Given, $I = 3.88 \times 10^{-7}$ kg-meters^2
$k_b = 2.75 \times 10^{-4}$ volts/RPM
$k_T = 5.9 \times 10^{-3}$ newton meters/ampere
$R = 1.71$ ohms
$L = 1.1 \times 10^{-4}$ henrys

The design of the motor's model in simulink :



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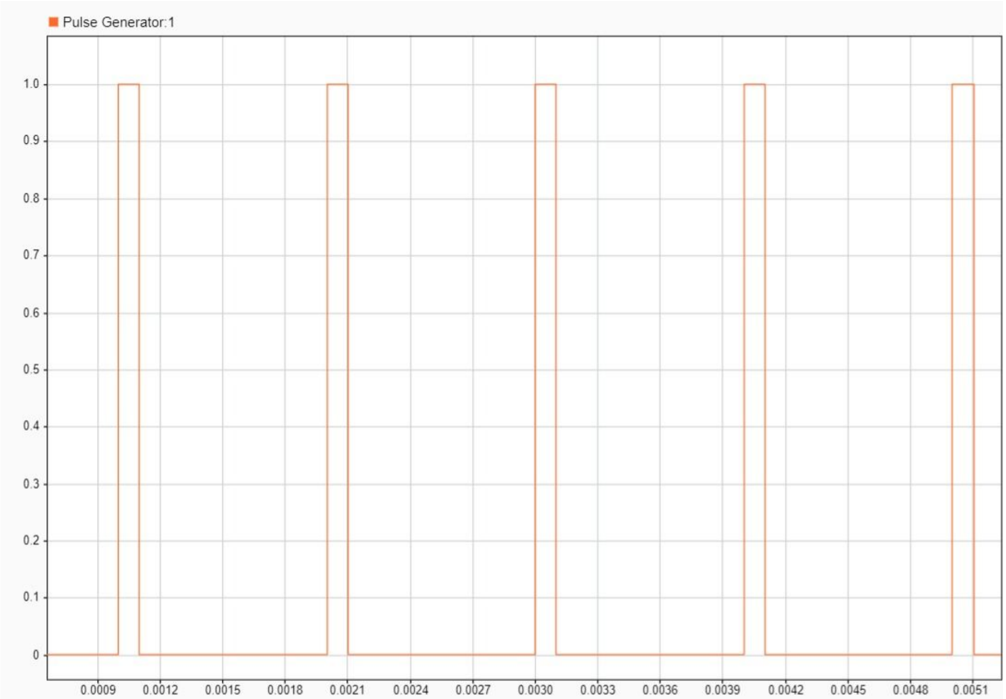
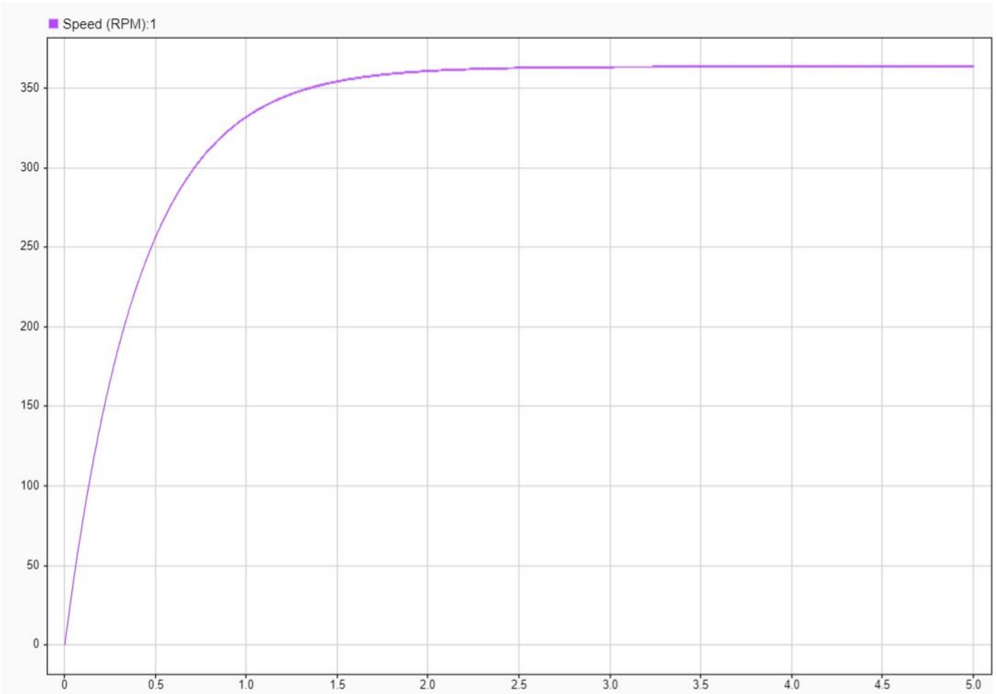
The plot of input signal of Amplitude 1V, Frequencey 1kHz and Duty Cycle 10% :



The plot of angular velocity of the motor vs time :



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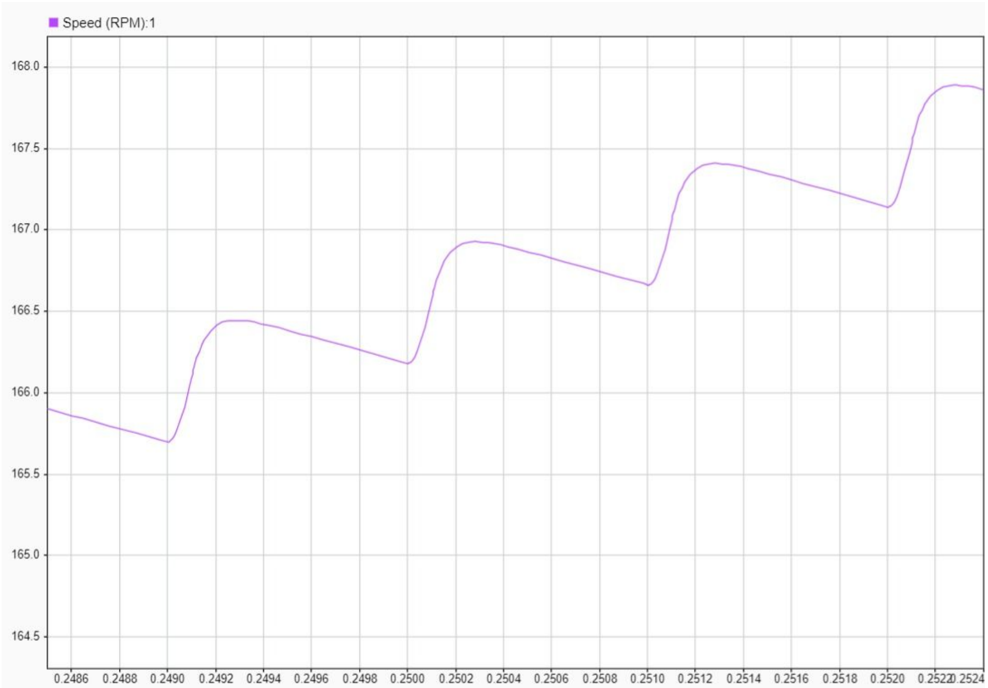

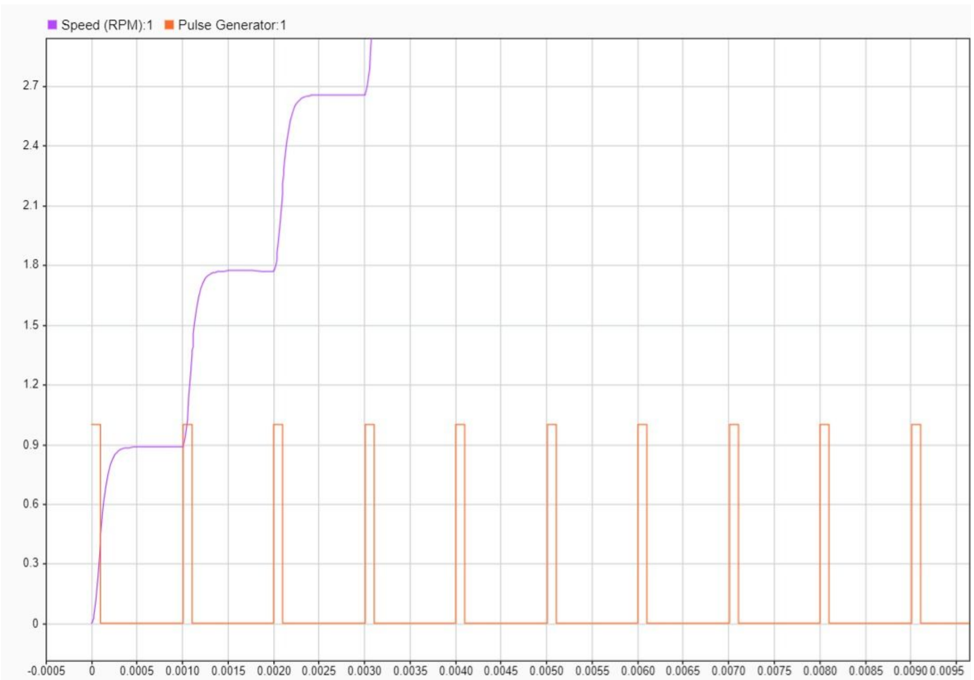

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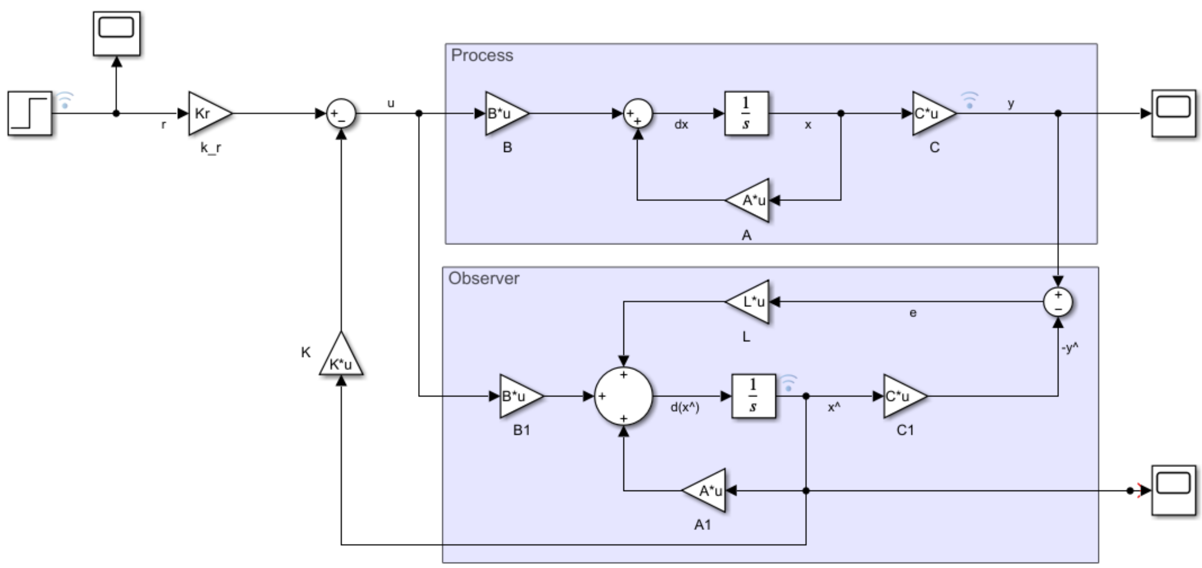
Problem 2. (20 points) Consider the vehicle steering control problem in Example 6.4 in [AM09]. Assume that $k_1 = 1$, $k_2 = 1.6$, and $k_r = 1$. Model the control system in Simulink using double precision floating point arithmetic. Now replace the model of the controller with the ones that use 16 bit and 8-bit fixed-point arithmetic. In each case, determine the fixed-point data types precisely. Plot the difference between the first state for the floating-point controller and that for the fixed-point controllers. Generate code for both the floating point controller and the fixed-point controllers using different optimization options. Describe your experience with code generation.

[AM09] K. J. Astrom and R. M. Murray. Feedback Systems: An Introduction for Scientists and Engineers. Princeton University Press, 2009.
http://www.cds.caltech.edu/~murray/books/AM05/pdf/am08-complete_22Feb09.pdf.

Simulink model of the vehicle steering control system :



The following values were taken for the above model :

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0.4 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = 0$$

Given $K = \begin{bmatrix} 1 & 1.6 \end{bmatrix}$, we find the closed-loop poles of the system using MATLAB:

```
Acl = A - B * K;
syscl = ss(Acl, B, C, D);
p = pole(syscl)
```

Therefore, we get $p = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

Now, using $L = place(A', C', p).'$, we get the estimator gain L of Observer, where $L = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

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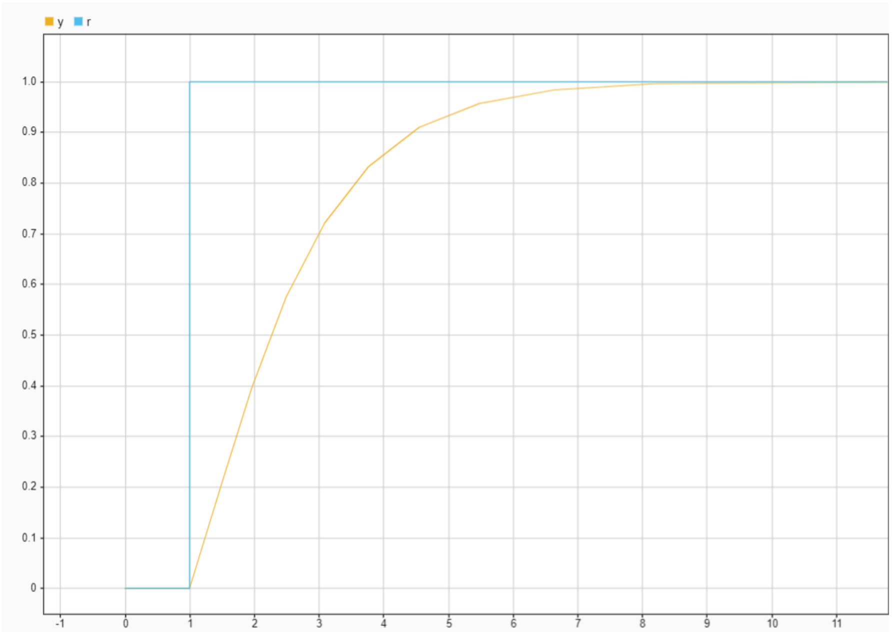
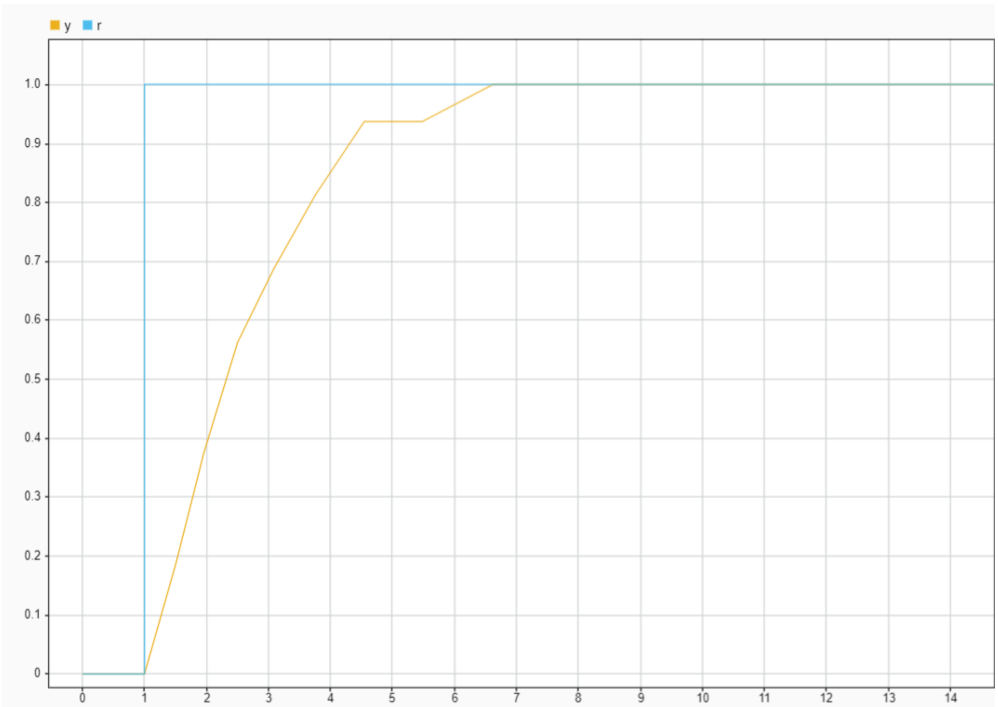
Following are the **Lateral Displacement** vs **Time** plots of the control system
y : Actual Displacement (yellow)
r : Reference (blue)

Plot for **double precision floating point** controller (data type used: double) :



Plot for **8-bit Fixed point** controller (data type used: sfix8_En4) :
Word Length - 8 bits and Fractional Length - 4 bits



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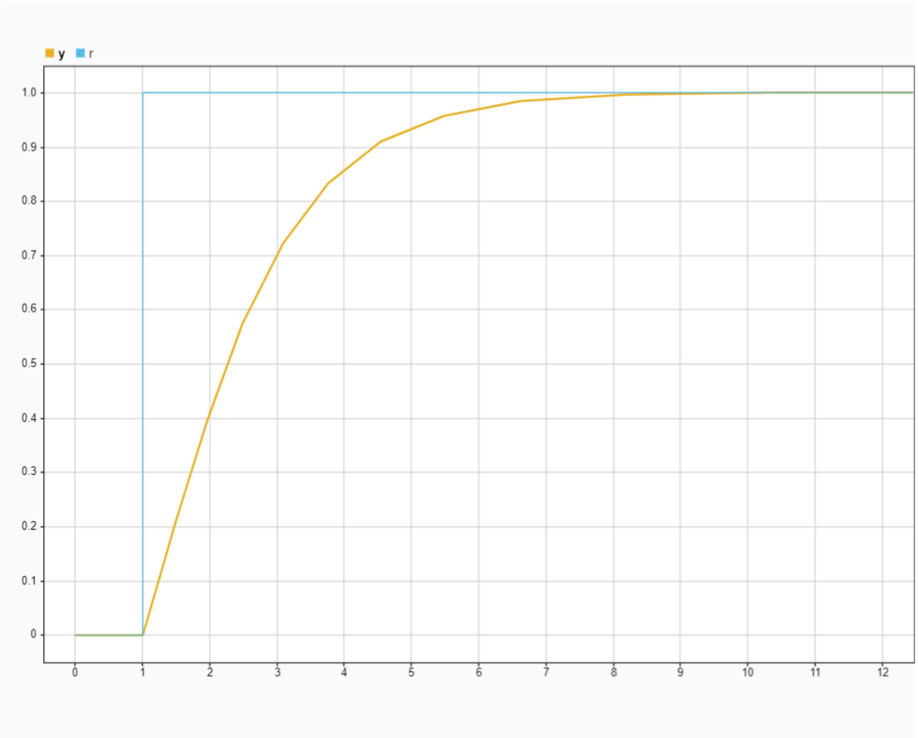
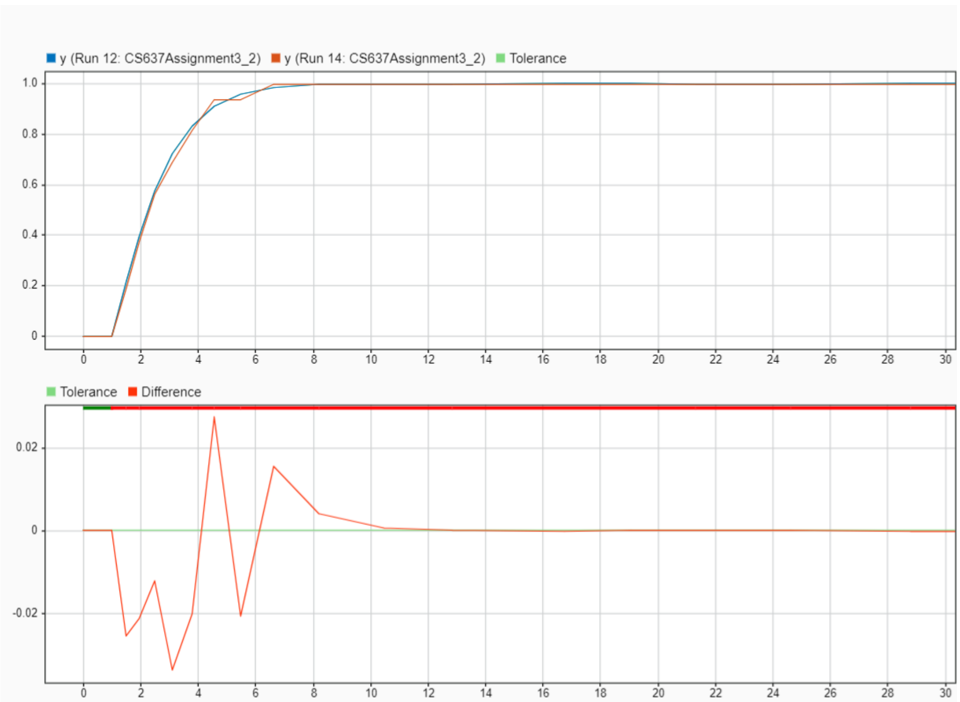
Plot for **16-bit Fixed point** controller (data type used: sfix16_En8) :
Word Length - 16 bits and Fractional Length - 8 bits



Comparing the plots of different precision configurations

Floating Double (blue) V/S 8-bit Fixed (orange) precision :



The difference is high in this case, as the precision in the 8-bit case is significantly lower than double.
Code generation is faster in case of 8-bit fixed point prescision than double.

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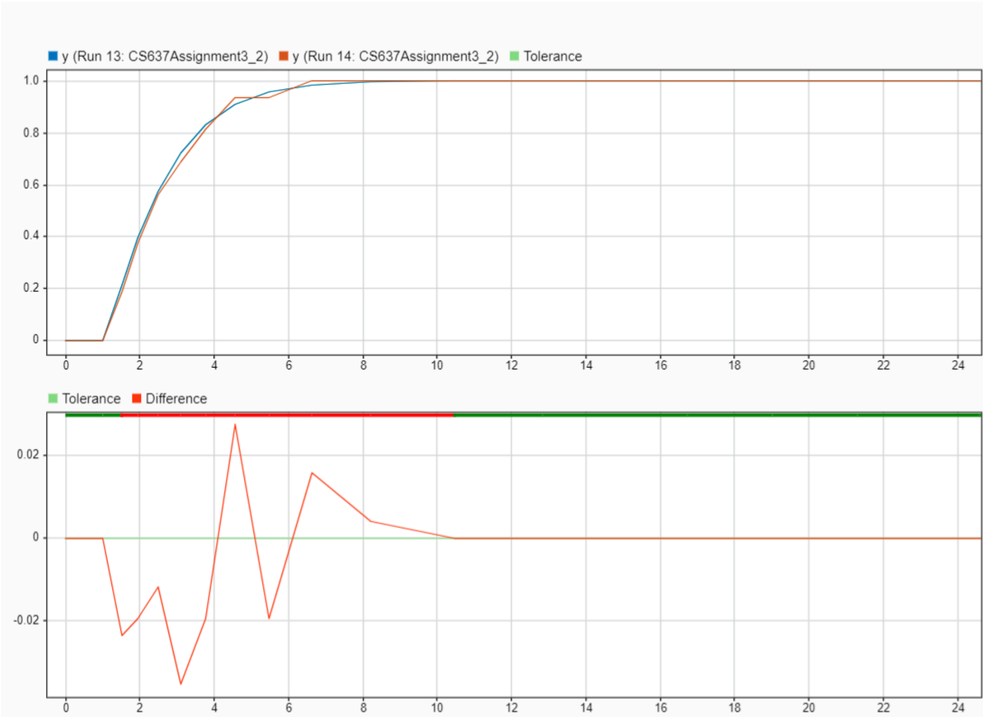
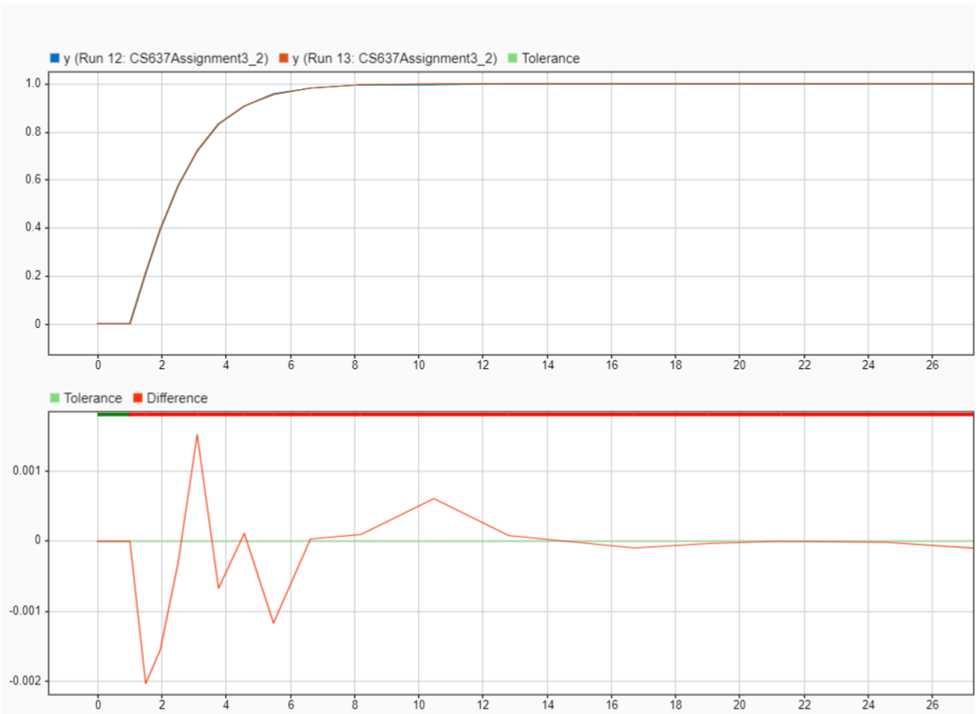
16-bit Fixed (blue) V/S 8-bit Fixed (orange) precision :



The difference is significant in this case, although not as high as the previous one.
Code generation speed is almost similar in both cases.

Floating Double (blue) V/S 16-bit Fixed (orange) precision :



The difference is almost negligible in this case.
Code generation of 16-bit is slightly faster than double.

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Problem 3. (10 points) Work out Problem 1 in the Exercises of Chapter 9 in [LS15].
[LS15] Edward A. Lee and Sanjit A. Seshia, Introduction to Embedded Systems, A Cyber-Physical Systems Approach, Second Edition, http://LeeSeshia.org, ISBN 978-1-312-42740-2, 2015.

(a) Consider the case where N is 16. How many cache misses will there be?

Solution: Keeping in mind that cache blocks are just 8 bytes in size. Given the size of an *int* (in this case, 4 bytes), we can see that the cache blocks of $data[0]$ and $data[1]$ will be identical, as will those of $data[2]$ and $data[3]$, and so on.

A cache miss will occur in the first for loop reading $data[i]$ for every even i between 0 and 15 if N is 16. The full array's worth of data will be cached by the time the first for loop finishes. As a result, we won't ever have a cache miss on any read while the second for loop is running.

This results in a grand total of 8 cache misses.

(b) Now suppose that N is 32. Recompute the number of cache misses.

Solution: When N is 32, the cache block mapped to by $data[i]$ and $data[i + 16]$ will be the same. Therefore, the block holding $data[i]$ will be evicted on every read to $data[i + 16]$ in the first for loop. Therefore, exactly like the first for loop, the second for loop will cause a cache miss on each even element in the array.

As a result, the total number of cache misses in this scenario will amount to 16 times 2, which is equal to 32.

(c) Now consider executing for $N = 16$ on a 2-way set-associative cache with parameters $(m, S, E, B) = (32, 8, 2, 4)$. In other words, the block size is halved, while there are two cache lines per set. How many cache misses would the code suffer?

Solution: In the first for loop, the code would incur 16 cache misses, one for each array element read. The decrease in block size accounts for the doubling of misses. Reading $data[2 * i]$ also pushes $data[2 * i + 1]$ into the cache in section (a), but this is not the case here.

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