ESO207: Data Structures and Algorithms

Theoretical Assignment 1

Due Date: 12th February, 2022

Total Number of Pages: 2 Total Points 150

Instructions-

1. Start each problem from a new page. Write down your name, Roll number and problem number clearly for each problem.
2. Give a clear description of the algorithm either in your own words or pseudo-code or both. Unclear description will receive less marks.

Question 1. Guess the Number (30 points)

You are playing a game on a machine. First of all the machine chooses a number N. You do not know this number and you need to make guesses to find it. Suppose you choose the number k, the machine tells whether k is less than, equal to or greater than N. The game ends if k is equal to N. If you guess a number which is greater than N, then it is considered a bad guess. You cannot make more than 1 bad guess.

(a) (20 points) Design an algorithm to find the number N in $\mathbf{O}(\sqrt{\mathbf{N}})$ guesses.

(b) (10 points) Analyze the complexity of your algorithm.

Question 2. Wasp pollination of figsy (40 points)

A female wasp enters the syconium of an unripe fig through an opening known as the ostiole. The wasp lays eggs with some of the flowers in the syconium. Due to this interrelation, the wasp became a rare insect, because of global warming and industrialization, the number of fig trees decreasing. But we can find some of the wasps in different parts of the world in the present time. So scientists find an artificial technique to reproduce the wasps, for this; scientists take three wasps, mix their eggs and fertilize them into a lab. But they face only one problem: the three wasps should be the same family.
Here same family means, if you take any two wasps, lets assume ω_1, ω_2 any two wasps and S is a set of common ancestors of both wasps. Then, the third wasp ω_3 should belong to the set $S(\omega_3 \in S)$. Luckily scientists have the family structure of all the wasps in the form of a hierarchical tree (here, you can assume it as a binary search tree because each family member has a certain number as an ID).

(a) (20 points) Design an algorithm(pseudocode) to find out if the three wasps belong to the same family or not.

(b) (10 points) By mistake if the wasps are not from the same family and scientists mix their eggs then it can create a serious problem, for this your algorithm should always give correct results. Write down the proof of correctness.

(c) (10 points) As the family structure of the wasps is very big, your algorithm should handle all kinds of computation. So analyze the time complexity of your algorithm.(**Hint:** Instead of three if scientists want to mix n wasps , then will your code able to handle the computation)

Question 3. **Count it!** *(Divide and Conquer)* (50 points)

Consider a square grid of side length N, i.e. there are N rows and N columns in the grid. Each cell of the grid is either coloured white or black. You are also given a non-negative integer k. A rectangle in the grid is considered good if both the properties hold

1. Sides of the rectangle are parallel to the sides of the grid and are along the borders of the cells.
2. Number of black cells in the rectangle is at most k.

(a) (30 points) Design an algorithm to compute the number of good rectangles in a given square grid.

(b) (10 points) Give the proof of correctness of your algorithm.

(c) (10 points) Give a formal proof of the complexity of your algorithm.

The total marks for this problem will depend on the complexity of the algorithm as follows-

1. Maximum marks will be 50 for overall complexity $\mathbf{O(N^2 log(N)k)}$
2. Maximum marks will be 40 for overall complexity $\mathbf{O(N^2 log^2(N)k)}$
3. Maximum marks will be 15 for overall complexity $\mathbf{O(N^3)}$

Question 4. **Sending message** (30 points)

Bob wants to send a message to his friend Alice. Bob is afraid that someone can attack the server and read his message, but Bob knows cryptography very well. So he decrypts his message into an integer(in decimal form) **dec_msg**, which is non-negative and represented as a string. But he faces only one problem: he cannot send this message through the available server because it has its capacity. So, he decides to reduce the size of **dec_msg** by deleting **red_msg** digits from the **dec_msg**. By deleting **red_msg** digits, he wants that the resultant number should be smallest possible, so that he can send it easily.

(a) (15 points) Design an efficient algorithm(pseudocode) to find the smallest possible number. Here you can assume that **dec_msg** don't have any leading zeros and number of digits in **dec_msg** is greater than **red_msg**. Also you can assume **red_msg** as an integer. You can delete the digits in any order.

(b) (15 points) Analyze the time complexity of your algorithm and give the proof of correctness.