

Lab Assignment 11

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Problem 1

Consider the Aircraft Damage Data in which the response variable is the number of locations where damage was inflicted on the aircraft. The regressor x_1 is an indicator variable (type of aircraft), and the other regressors x_2 and x_3 are bomb load (in tons) and total months of aircrew experience. Fit Poisson regression model and test the hypothesis for the significance of regressors.

```
# Create the dataset
aircraft_data <- data.frame(
  y = c(0,1,0,0,0,0,1,0,0,2,1,1,1,1,2,3,1,1,1,2,0,1,1,2,
        5,1,1,5,5,7),
  x1 = c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,1,
        1,1,1,1,1,1),
  x2 = c(4,4,4,5,5,5,6,6,6,7,7,7,8,8,8,7,7,7,10,10,10,12,
        12,12,8,8,8,14,14,14),
  x3 = c(91.5,84.0,76.5,69.0,61.5,80.0,72.5,65.0,57.5,
        50.0,103.0,95.5,88.0,80.5,73.0,
        116.1,100.6,85.0,69.4,53.9,112.3,96.7,81.1,
        65.6,50.0,120.0,104.4,88.9,73.7,57.8)
)

# Fit Poisson regression model
poisson_model <- glm(y ~ x1 + x2 + x3,
  family = poisson(link = "log"),
  data = aircraft_data)

# Display model summary
summary(poisson_model)
```

```
Call:
glm(formula = y ~ x1 + x2 + x3, family = poisson(link = "log"),
    data = aircraft_data)
```

Coefficients:

| | Estimate | Std. Error | z value | Pr(> z) |
|-------------|-----------|------------|---------|----------|
| (Intercept) | -0.406023 | 0.877489 | -0.463 | 0.6436 |
| x1 | 0.568772 | 0.504372 | 1.128 | 0.2595 |
| x2 | 0.165425 | 0.067541 | 2.449 | 0.0143 * |
| x3 | -0.013522 | 0.008281 | -1.633 | 0.1025 |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

Null deviance: 53.883 on 29 degrees of freedom
 Residual deviance: 25.953 on 26 degrees of freedom
 AIC: 87.649

Number of Fisher Scoring iterations: 5

```
# Calculate pseudo R-squared (1 - residual deviance/null deviance)
pseudo_r2 <- 1 - poisson_model$deviance/poisson_model$null.deviance

# Perform analysis of deviance
anova_result <- anova(poisson_model, test = "Chisq")

# Print additional model statistics
cat("\nModel Statistics:\n")
```

Model Statistics:

```
cat("Null Deviance:", poisson_model$null.deviance,
    "on", poisson_model$df.null, "degrees of freedom\n")
```

Null Deviance: 53.88306 on 29 degrees of freedom

```
cat("Residual Deviance:", poisson_model$deviance,
    "on", poisson_model$df.residual, "degrees of freedom\n")
```

Residual Deviance: 25.95316 on 26 degrees of freedom

```
cat("AIC:", poisson_model$aic, "\n")
```

AIC: 87.64922

```
cat("Pseudo R-squared:", pseudo_r2, "\n")
```

Pseudo R-squared: 0.5183429

```
# Display ANOVA results
cat("\nAnalysis of Deviance Table:\n")
```

Analysis of Deviance Table:

```
print(anova_result)
```

Analysis of Deviance Table

Model: poisson, link: log

Response: y

Terms added sequentially (first to last)

| | Df | Deviance | Resid. Df | Resid. Dev | Pr(>Chi) |
|------|----|----------|-----------|------------|---------------|
| NULL | | | 29 | 53.883 | |
| x1 | 1 | 15.5996 | 28 | 38.283 | 7.827e-05 *** |
| x2 | 1 | 9.6492 | 27 | 28.634 | 0.001894 ** |
| x3 | 1 | 2.6811 | 26 | 25.953 | 0.101543 |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
# Calculate and print odds ratios and confidence intervals
coef_exp <- exp(coef(poisson_model))
conf_int <- exp(confint(poisson_model))
```

Waiting for profiling to be done...

```
coef_table <- cbind(Estimate = coef_exp,
                    CI_lower = conf_int[,1],
                    CI_upper = conf_int[,2])

cat("\nExponentiated Coefficients (Rate Ratios) with 95% CI:\n")
```

Exponentiated Coefficients (Rate Ratios) with 95% CI:

```
print(coef_table)
```

| | Estimate | CI_lower | CI_upper |
|-------------|-----------|-----------|----------|
| (Intercept) | 0.6662950 | 0.1109979 | 3.504006 |
| x1 | 1.7660977 | 0.6529760 | 4.792330 |
| x2 | 1.1798949 | 1.0358915 | 1.351870 |
| x3 | 0.9865687 | 0.9704577 | 1.002674 |

```
# Perform Walt's test for analysis of deviance
library(lmtest)
```

Loading required package: zoo

Attaching package: 'zoo'

The following objects are masked from 'package:base':

as.Date, as.Date.numeric

```
walt_result <- waldtest(poisson_model)
cat("\nWald's Test Results:\n")
```

Wald's Test Results:

```
print(walt_result)
```

Wald test

Model 1: $y \sim x1 + x2 + x3$

Model 2: $y \sim 1$

| | Res.Df | Df | F | Pr(>F) |
|---|--------|----|--------|---------------|
| 1 | 26 | | | |
| 2 | 29 | -3 | 8.7541 | 0.0003508 *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Problem 2

Following Table contains data from an experiment conducted to investigate the three factors $x1$ = length, $x2$ = amplitude, and $x3$ = load on the cycles to failure y of worsted yarn. The regressor variables are coded, and readers who have familiarity with designed experiments will recognize that the experimenters here used a 33 factorial design. Fit a GLM using the gamma distribution and the log link.

```
# Create the dataset
yarn_data <- data.frame(
  y = c(674, 1414, 3636, 338, 1022, 1568, 170, 442, 1140, 370, 1198,
        3184, 266, 620, 1070, 118, 332, 884, 292, 634, 2000, 210,
        438, 566, 90, 220, 360),
  x1 = c(-1, 0, 1, -1, 0, 1, -1, 0, 1, -1, 0, 1, -1, 0, 1, -1, 0, 1,
        -1, 0, 1, -1, 0, 1, -1, 0, 1),
  x2 = c(-1, -1, -1, 0, 0, 0, 1, 1, 1, -1, -1, -1, 0, 0, 0, 1, 1, 1,
        -1, -1, -1, 0, 0, 0, 1, 1, 1),
  x3 = c(-1, -1, -1, -1, -1, -1, -1, -1, -1, -1, 0, 0, 0, 0, 0, 0, 0, 0,
        0, 1, 1, 1, 1, 1, 1, 1, 1, 1)
)

# Fit Gamma regression model with log link
gamma_model <- glm(y ~ x1 + x2 + x3, family = Gamma(link = "log"), data = yarn_data)

# Display model summary
summary(gamma_model)
```

```
Call:
glm(formula = y ~ x1 + x2 + x3, family = Gamma(link = "log"),
    data = yarn_data)
```

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|----------|------------|---------|--------------|
| (Intercept) | 6.34891 | 0.03422 | 185.511 | < 2e-16 *** |
| x1 | 0.84251 | 0.04192 | 20.100 | 4.34e-16 *** |
| x2 | -0.63132 | 0.04192 | -15.062 | 2.10e-13 *** |
| x3 | -0.38513 | 0.04192 | -9.188 | 3.68e-09 *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for Gamma family taken to be 0.0316243)

Null deviance: 22.88613 on 26 degrees of freedom
Residual deviance: 0.76939 on 23 degrees of freedom
AIC: 332.76

Number of Fisher Scoring iterations: 5

```
# Calculate pseudo R-squared (1 - residual deviance/null deviance)
pseudo_r2 <- 1 - gamma_model$deviance/gamma_model$null.deviance

# Perform analysis of deviance
anova_result <- anova(gamma_model, test = "Chisq")

# Print additional model statistics
cat("\nModel Statistics:\n")
```

Model Statistics:

```
cat("Null Deviance:", gamma_model$null.deviance,
    "on", gamma_model$df.null, "degrees of freedom\n")
```

Null Deviance: 22.88613 on 26 degrees of freedom

```
cat("Residual Deviance:", gamma_model$deviance,  
    "on", gamma_model$df.residual, "degrees of freedom\n")
```

Residual Deviance: 0.7693924 on 23 degrees of freedom

```
cat("AIC:", gamma_model$aic, "\n")
```

AIC: 332.757

```
cat("Pseudo R-squared:", pseudo_r2, "\n")
```

Pseudo R-squared: 0.9663817

```
# Display ANOVA results  
cat("\nAnalysis of Deviance Table:\n")
```

Analysis of Deviance Table:

```
print(anova_result)
```

Analysis of Deviance Table

Model: Gamma, link: log

Response: y

Terms added sequentially (first to last)

| | Df | Deviance | Resid. Df | Resid. Dev | Pr(>Chi) |
|------|----|----------|-----------|------------|---------------|
| NULL | | | 26 | 22.8861 | |
| x1 | 1 | 12.6758 | 25 | 10.2104 | < 2.2e-16 *** |
| x2 | 1 | 6.8645 | 24 | 3.3459 | < 2.2e-16 *** |
| x3 | 1 | 2.5765 | 23 | 0.7694 | < 2.2e-16 *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
# Calculate and print odds ratios and confidence intervals
coef_exp <- exp(coef(gamma_model))
conf_int <- exp(confint(gamma_model))
```

Waiting for profiling to be done...

```
coef_table <- cbind(Estimate = coef_exp,
                    CI_lower = conf_int[,1],
                    CI_upper = conf_int[,2])

cat("\nExponentiated Coefficients (Rate Ratios) with 95% CI:\n")
```

Exponentiated Coefficients (Rate Ratios) with 95% CI:

```
print(coef_table)
```

| | Estimate | CI_lower | CI_upper |
|-------------|-------------|-------------|-------------|
| (Intercept) | 571.8703665 | 535.1664797 | 612.0105689 |
| x1 | 2.3221878 | 2.1365750 | 2.5239201 |
| x2 | 0.5318889 | 0.4899942 | 0.5773708 |
| x3 | 0.6803594 | 0.6260482 | 0.7393754 |

```
wald_result <- waldtest(poisson_model)
cat("\nWald's test results:\n")
```

Wald's test results:

```
print(wald_result)
```

Wald test

Model 1: y ~ x1 + x2 + x3

Model 2: y ~ 1

| | Res.Df | Df | F | Pr(>F) |
|---|--------|----|--------|---------------|
| 1 | 26 | | | |
| 2 | 29 | -3 | 8.7541 | 0.0003508 *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1