Indian Institute of Technology Kanpur

Department of Mathematics and Statistics

Time Series Analysis (MTH442)

Assignment 1, Due date: September 9, 2024, Monday

Answers should be provided neatly. In case the handwriting is unreadable, the instructor and the teaching assistants hold the right to give a zero score. In case of cheating, all students involved will get zero, irrespective of who copied from whom. Write on a paper using some dark ink and scan using a good scanner to obtain a PDF file. For the coding related question, use R markdown and generate the PDF output with echo = TRUE mode (so that the codes are also visible along with the outputs). Finally, join the two PDFs and submit the single PDF file only. The final file should be named as RollNo_Lastname_Firstname.pdf. If the file is not submitted in this nomenclature format, marks will be zero. No request should be made in that case.

1. Consider the time series $X_t = \beta_1 + \beta_2 t + W_t$, where β_1 and β_2 are known constants and W_t is a white noise process with variance σ_W^2. Show that the mean of the moving average $V_t = \frac{1}{2q+1}\sum_{j=-q}^{q} X_{t-j}$ is $\beta_1 + \beta_2 t$, and give a simplified expression for the autocovariance function. (0.5+1=1.5 points)

2. A time series with a periodic component can be constructed from $X_t = U_1 \sin(2\pi\omega_0 t) + U_2 \cos(2\pi\omega_0 t)$, where U_1 and U_2 are independent random variables with zero means and $E(U_1^2) = E(U_2^2) = \sigma^2$. The constant ω_0 determines the period or time it takes the process to make one complete cycle. Show that this series is weakly stationary with autocovariance function $\gamma(h) = \sigma^2 \cos(2\pi\omega_0 h)$. (1 point)

3. Consider the two series $X_t = W_t$ and $Y_t = W_t - \theta W_{t-1} + U_t$, where W_t and U_t are independent white noise series with variances σ_W^2 and σ_U^2, respectively, and θ is an unspecified constant. (a) Express the ACF, $\rho_Y(h)$, for $h = 0, \pm1, \pm2, \ldots$ of the series Y_t as a function of σ_W^2, σ_U^2, and θ. (b) Determine the CCF, $\rho_{X,Y}(h)$ relating X_t and Y_t. (0.75+0.75=1.5 points)

4. Consider the series $X_t = \sin(2\pi U t), t = 1, 2, \ldots$, where U has a uniform distribution on the interval $(0, 1)$. (a) Prove X_t is weakly stationary. (b) Prove X_t is not strictly stationary. (1+1=2 points)

5. Prove that the autocovariance function of a stationary process, is a nonnegative definite function. Also, verify that the sample autocovariance is a nonnegative definite function. (0.5+1=1.5 points)

6. Given an observed data vector $\boldsymbol{y}$ of length T, we define the information for discriminating between two densities in the same family, indexed by a parameter $\boldsymbol{\theta}$, say $f(\boldsymbol{y};\boldsymbol{\theta}_1)$ and $f(\boldsymbol{y};\boldsymbol{\theta}_2)$, as
$$I(\boldsymbol{\theta}_1;\boldsymbol{\theta}_2) = T^{-1} E_1 \log \frac{f(\boldsymbol{y};\boldsymbol{\theta}_1)}{f(\boldsymbol{y};\boldsymbol{\theta}_2)}$$
where E_1 denotes expectation with respect to the density determined by $\boldsymbol{\theta}_1$. For the Gaussian regression model, the parameters are $\boldsymbol{\theta} = (\boldsymbol{\beta}', \sigma^2)'$. Show that (1 point)
$$I(\boldsymbol{\theta}_1;\boldsymbol{\theta}_2) = \frac{1}{2}\left(\frac{\sigma_1^2}{\sigma_2^2} - \log\left(\frac{\sigma_1^2}{\sigma_2^2}\right) - 1\right) + \frac{1}{2}\frac{(\boldsymbol{\beta}_1 - \boldsymbol{\beta}_2)'\boldsymbol{Z}'\boldsymbol{Z}(\boldsymbol{\beta}_1 - \boldsymbol{\beta}_2)}{T\sigma_2^2}.$$

7. Johnson & Johnson data market share price data is given as `jj` within the `R` package `astsa`. We consider the transformation $X_t = \log(Y_t)$. In our case, time t is in quarters $(1960.00, 1960.25, \ldots)$

so one unit of time is a year. Fit the regression model

$$X_t = \beta t + \alpha_1 Q_1(t) + \alpha_2 Q_2(t) + \alpha_3 Q_3(t) + \alpha_4 Q_4(t) + W_t,$$

where $Q_i(t) = 1$ if time t corresponds to quarter $i = 1, 2, 3, 4$, and zero otherwise. The $Q_i(t)$'s are called indicator variables. We will assume that W_t is a Gaussian white noise sequence. (a) If the model is correct, what is the estimated average annual increase in the logged earnings per share? (b) If the model is correct, does the average logged earnings rate increase or decrease from the third quarter to the fourth quarter? And, by what percentage does it increase or decrease? (c) Graph the data, X_t, and superimpose the fitted values, say $\widehat{X}_t$, on the graph. Examine the residuals, $X_t - \widehat{X}_t$, and state your conclusions. Does it appear that the model fits the data well (do the residuals look white)? (0.5+0.5+0.5=1.5 points)