MTH442 Assignment 2

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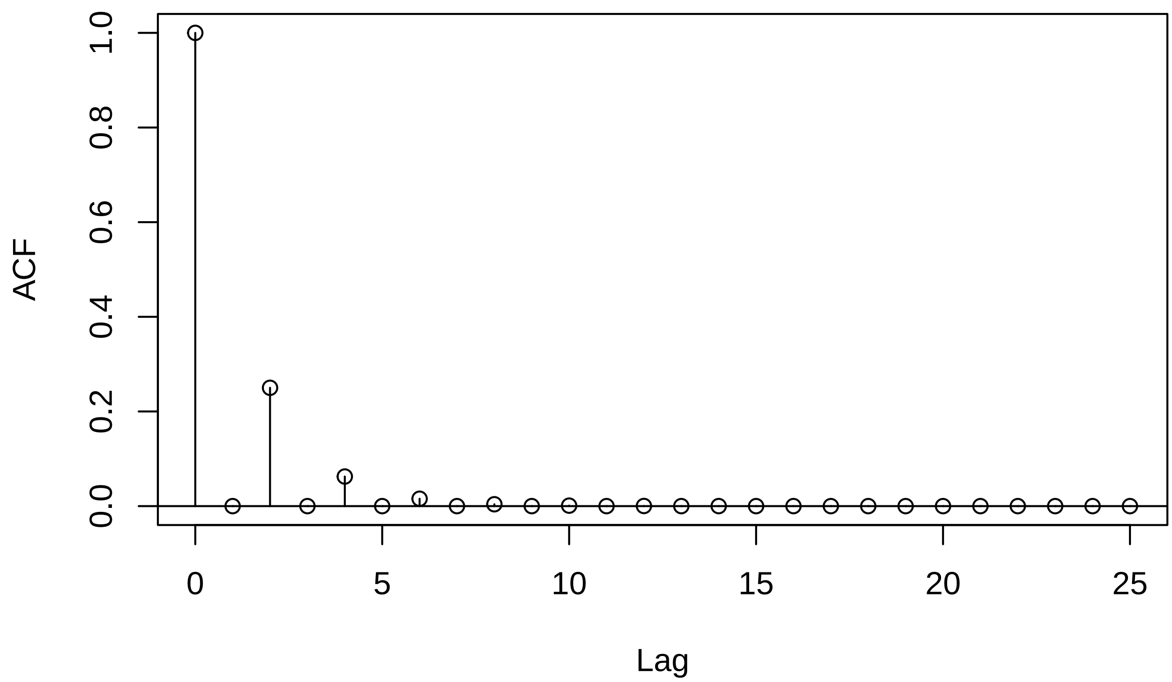
```
knitr::opts_chunk$set(echo = TRUE)

# Load the required packages
library(ggplot2)
library(astsa)
library(dplyr)
library(tidyr)
```

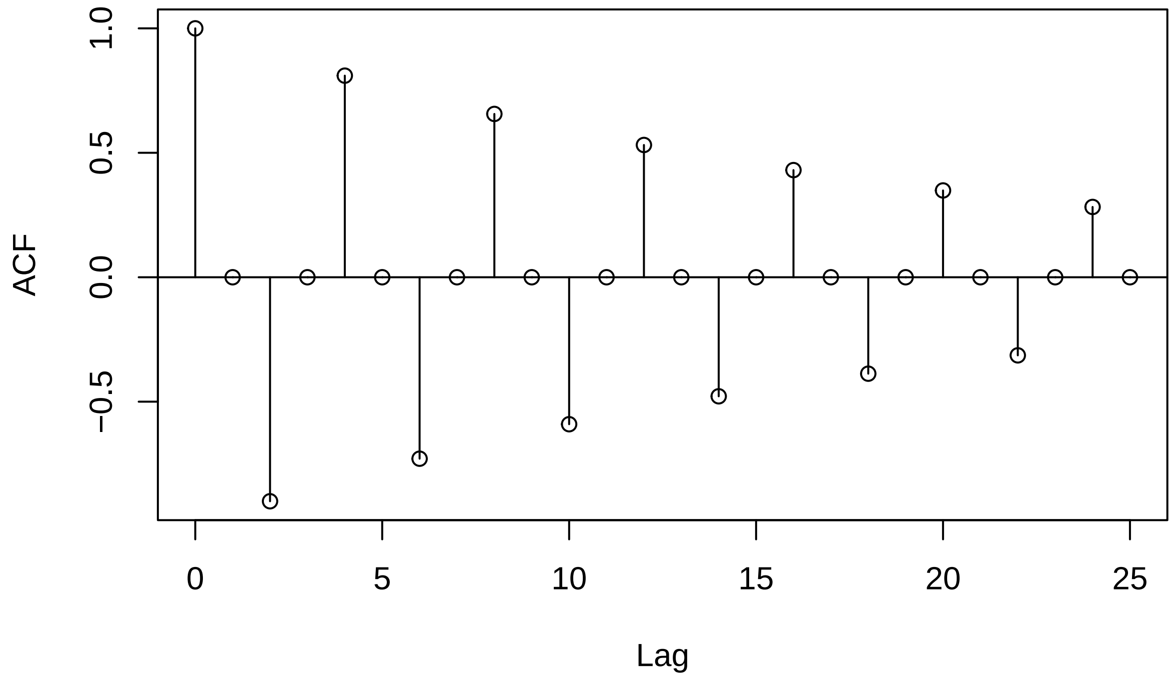
Problem 4

4. For the AR(2) models given by $X_t = 0.25X_{t-2} + W_t$ and $X_t = -0.9X_{t-2} + W_t$, find the roots of the autoregressive polynomials, and then plot their ACFs, $\rho(h)$. ($0.5 \times 2 = 1$ point)

```
u = ARMAacf(ar=c(0,0.25), lag.max=25)
plot(0:25, u, type="h", xlab="Lag", ylab="ACF")
points(0:25, u)
abline(h=0)
```



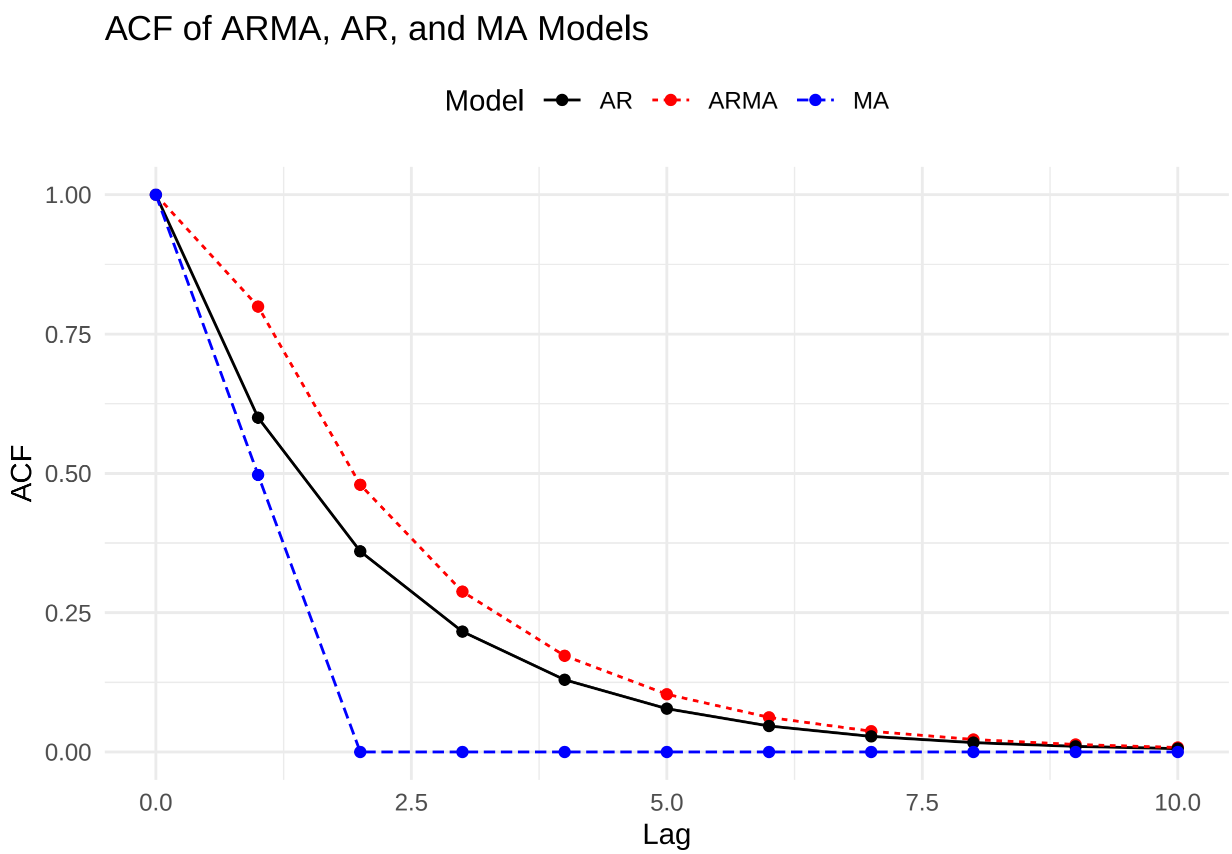

```r
u = ARMAacf(ar=c(0,-.9), lag.max=25)
plot(0:25, u, type="h", xlab="Lag", ylab="ACF")
points(0:25, u)
abline(h=0)
```


ACF
1.0
0.5
0.0
−0.5
0
5
10
15
20
25
Lag

Problem 6

6. Plot the theoretical ACFs (all derived in the class) of the three series AR(1)=ARMA(1,0) $X_t = \phi X_{t-1} + W_t$, MA(1)=ARMA(0,1) $X_t = W_t + \theta W_{t-1}$, and ARMA(1,1) $X_t = \phi X_{t-1} + W_t + \theta W_{t-1}$ on the same graph for $\phi = 0.6$, $\theta = 0.9$, and comment on the diagnostic capabilities of the ACF in this case. (1 point)

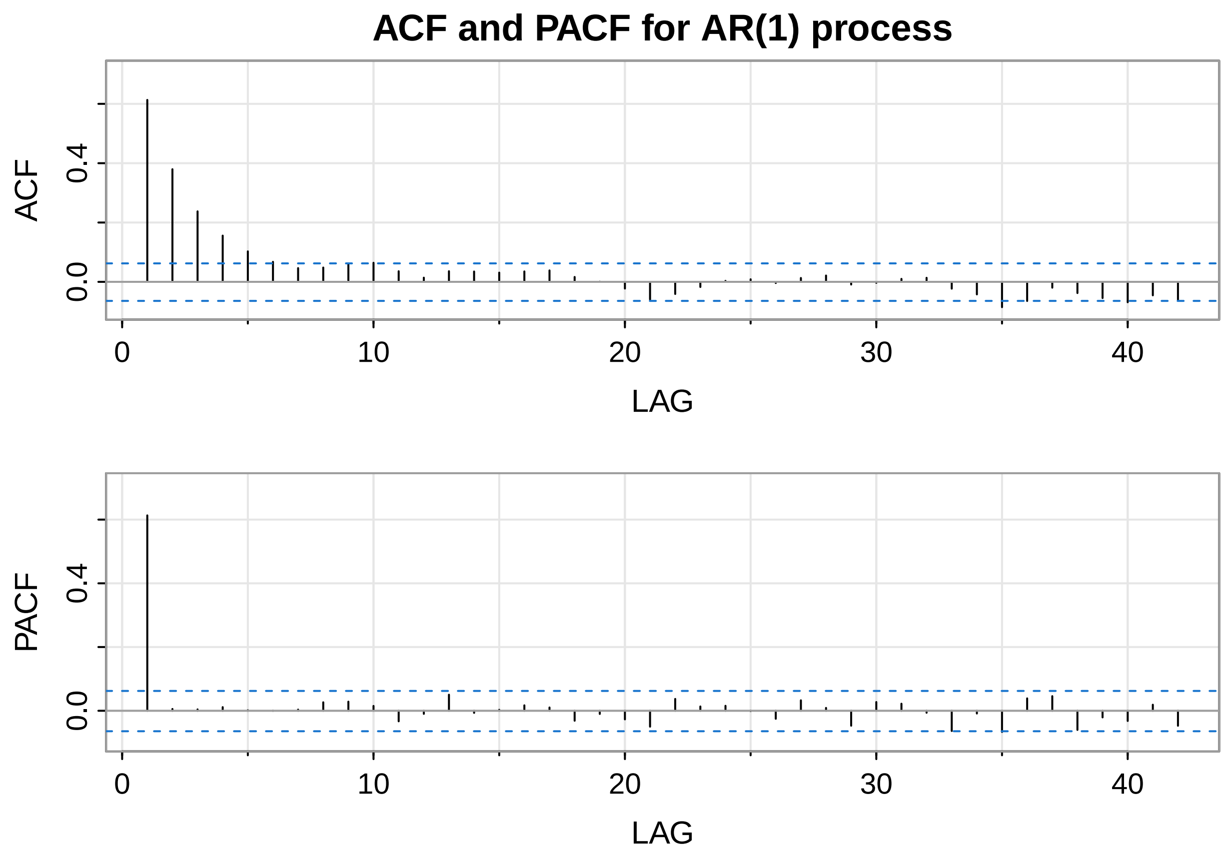
```r
# Generate ACF values for the models
u1 <- ARMAacf(0.6, 0.9, lag.max=10)
u2 <- ARMAacf(0.6, 0, lag.max=10)
u3 <- ARMAacf(0, 0.9, lag.max=10)

# Create a data frame for ggplot
df <- data.frame(
  Lag = 0:10,
  ARMA = u1,
  AR = u2,
  MA = u3
) %>%
  pivot_longer(cols = c("ARMA", "AR", "MA"), names_to = "Model", values_to = "ACF")

# Plot using ggplot
ggplot(df, aes(x = Lag, y = ACF, color = Model)) +
  geom_line(aes(linetype = Model)) +
  geom_point() +
  labs(title = "ACF of ARMA, AR, and MA Models", x = "Lag", y = "ACF") +
  scale_color_manual(values = c("black", "red", "blue")) +
  theme_minimal() +
  theme(legend.position = "top")
```


ACF of ARMA, AR, and MA Models
Model
AR
ARMA
MA
1.00
0.75
0.50
0.25
0.00
ACF
0.0
2.5
5.0
7.5
10.0
Lag

Problem 7

7. Generate $n = 1000$ observations from each of the models in the previous question and comment on the important patterns in the ACF and PACF plots. (1 point)

```
ar = arima.sim(list(order=c(1,0,0), ar=.6), n=1000)
acf2(ar, main="ACF and PACF for AR(1) process")
```

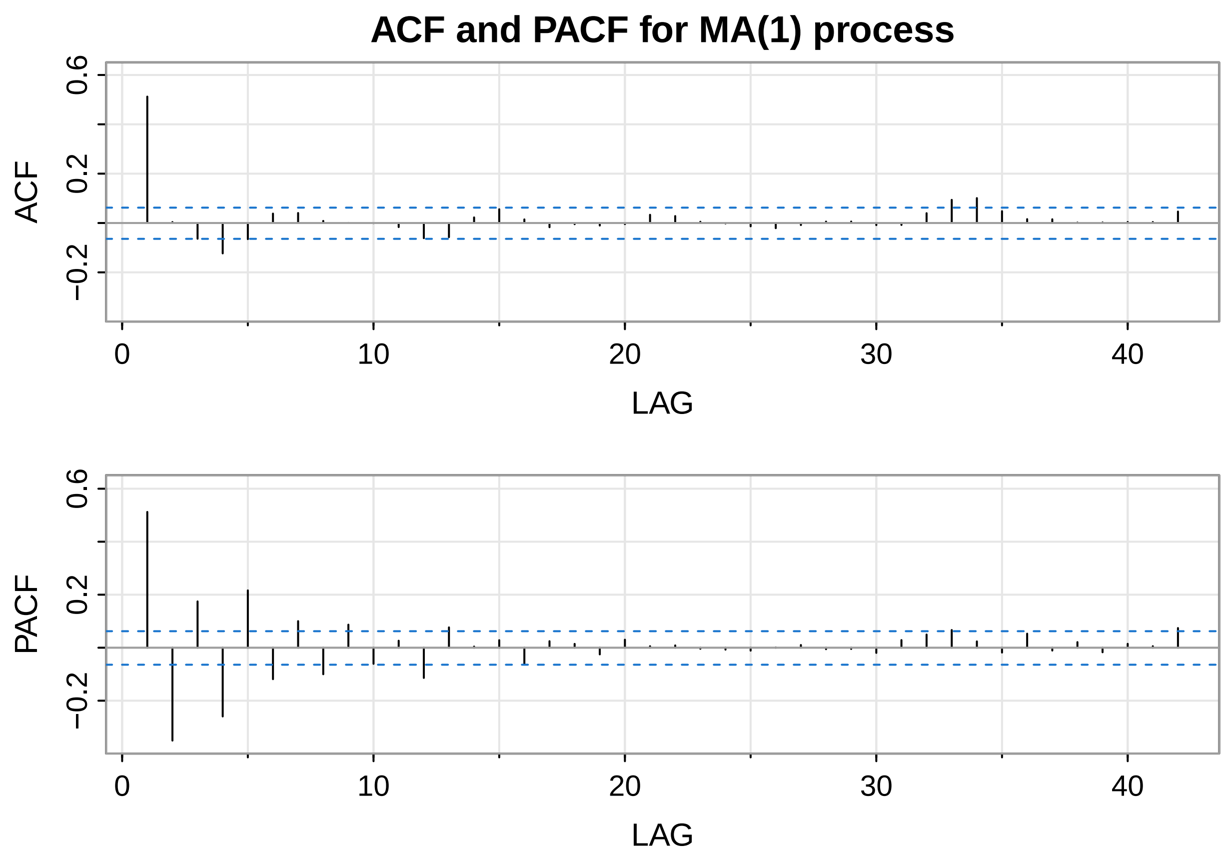



```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13] [,14]
## ACF  0.61 0.38 0.24 0.16  0.1 0.07 0.05 0.05 0.06  0.06  0.04  0.01  0.04  0.03
## PACF 0.61 0.01 0.00 0.01  0.0 0.00 0.00 0.03 0.03  0.02 -0.03 -0.01  0.05 -0.01
##      [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25] [,26]
## ACF   0.03  0.04  0.04  0.02  0.00 -0.02 -0.06 -0.04 -0.02  0.00  0.01  0.00
## PACF  0.00  0.02  0.01 -0.03 -0.01 -0.03 -0.05  0.04  0.01  0.02  0.00 -0.03
##      [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37] [,38]
## ACF   0.01  0.02 -0.01  0.00  0.01  0.01 -0.02 -0.04 -0.09 -0.06 -0.02 -0.04
## PACF  0.03  0.01 -0.05  0.03  0.02 -0.01 -0.06 -0.01 -0.07  0.04  0.05 -0.06
##      [,39] [,40] [,41] [,42]
## ACF  -0.05 -0.07 -0.05 -0.06
## PACF -0.02 -0.03  0.02 -0.05
```

Comments on AR(1):

- **ACF**: The ACF plot shows a gradual decrease or tapering off of the autocorrelation coefficients as the lag increases. This pattern is characteristic of an autoregressive (AR) process, where the current value of the series is dependent on its past values.
- **PACF**: The PACF plot exhibits a sharp cutoff after the first lag, indicating that the partial autocorrelations beyond the first lag are effectively zero. This is a typical signature of an AR(1) or first-order autoregressive process, where the current value of the series is linearly dependent only on the immediately preceding value.

```
ma = arima.sim(list(order=c(0,0,1), ma=.9), n=1000)
acf2(ma, main="ACF and PACF for MA(1) process")
```

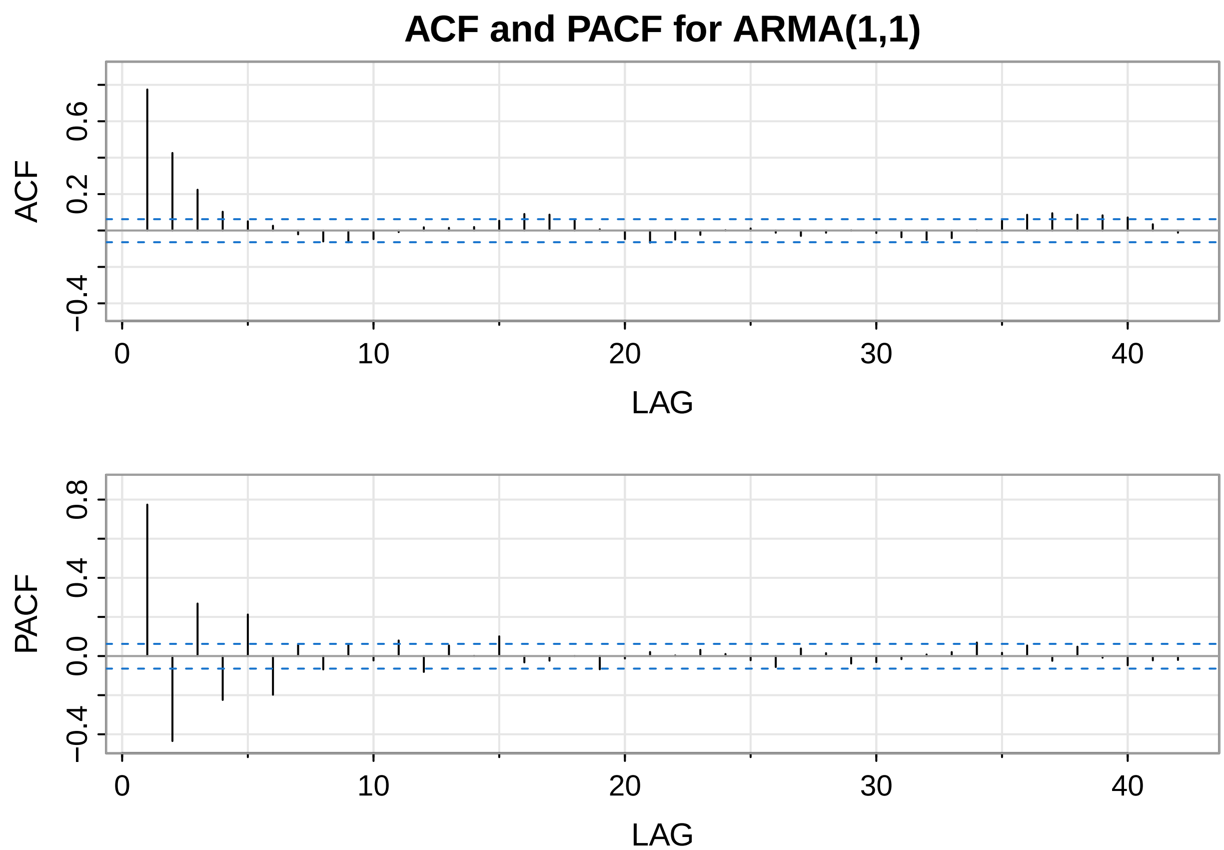



```
##      [,1]  [,2]  [,3]  [,4]  [,5]  [,6] [,7]  [,8] [,9] [,10] [,11] [,12] [,13]
## ACF  0.51  0.00 -0.06 -0.12 -0.07  0.04 0.04  0.01 0.00  0.00 -0.02 -0.06 -0.06
## PACF 0.51 -0.35  0.17 -0.26  0.22 -0.12 0.10 -0.10 0.09 -0.06  0.03 -0.11  0.08
##      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
## ACF   0.02  0.06  0.01 -0.02  0.00 -0.01  0.00  0.03  0.03  0.01  0.00 -0.01
## PACF  0.00  0.03 -0.06  0.02  0.01 -0.03  0.03  0.01  0.01  0.00 -0.01 -0.01
##      [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37]
## ACF  -0.02 -0.01  0.01  0.01 -0.01 -0.01  0.04  0.09  0.10  0.05  0.02  0.02
## PACF  0.00  0.01 -0.01  0.00 -0.02  0.03  0.05  0.07  0.02 -0.02  0.05 -0.01
##      [,38] [,39] [,40] [,41] [,42]
## ACF   0.00  0.00  0.00  0.00  0.05
## PACF  0.02 -0.02  0.01  0.01  0.07
```

Comments on MA(1):

- **ACF**: The ACF plot exhibits a sharp cutoff after the first lag, with the autocorrelation coefficients becoming effectively zero beyond this point. This pattern is characteristic of a moving average (MA) process, where the current value of the series is dependent on the immediate past random shocks or errors.
- **PACF**: The PACF plot shows a gradual tapering off of the partial autocorrelation coefficients as the lag increases. This behavior is consistent with an MA process, where the current value depends linearly on the immediately preceding random shock or error term.

```
arma = arima.sim(list(order=c(1,0,1), ar=.6, ma=.9), n=1000)
acf2(arma, main="ACF and PACF for ARMA(1,1)")
```




```
##      [,1]  [,2] [,3]  [,4] [,5]  [,6]  [,7]  [,8]  [,9] [,10] [,11] [,12] [,13]
## ACF  0.77  0.43 0.22  0.10 0.05  0.03 -0.02 -0.06 -0.06 -0.05 -0.01  0.02  0.01
## PACF 0.77 -0.43 0.27 -0.22 0.21 -0.20  0.06 -0.07  0.06 -0.02  0.08 -0.08  0.05
##      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
## ACF   0.02  0.05  0.09  0.09  0.05  0.01 -0.05 -0.07 -0.05 -0.02  0.00  0.01
## PACF  0.00  0.10 -0.03 -0.02  0.00 -0.07 -0.01  0.02  0.00  0.03  0.01 -0.02
##      [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37]
## ACF  -0.01 -0.03 -0.01  0.00 -0.01 -0.04 -0.05 -0.04  0.00  0.05  0.09  0.09
## PACF -0.06  0.04  0.02 -0.04 -0.03 -0.02  0.01  0.02  0.07  0.02  0.05 -0.02
##      [,38] [,39] [,40] [,41] [,42]
## ACF   0.09  0.08  0.07  0.03 -0.01
## PACF  0.05 -0.01 -0.05 -0.02 -0.02
```

Comments on ARMA(1,1):

- **ACF**: The ACF plot shows a gradual tapering off of the autocorrelation coefficients as the lag increases. This pattern is characteristic of a combined autoregressive and moving average (ARMA) process, where the current value of the series is dependent on both its past values and past random shocks or errors.
- **PACF**: The PACF plot also exhibits a gradual decline in the partial autocorrelation coefficients as the lag increases. This behavior is consistent with an ARMA process, where the current value depends linearly on both its own past values and the immediately preceding random shocks or error terms.