

MTH442 Assignment 4

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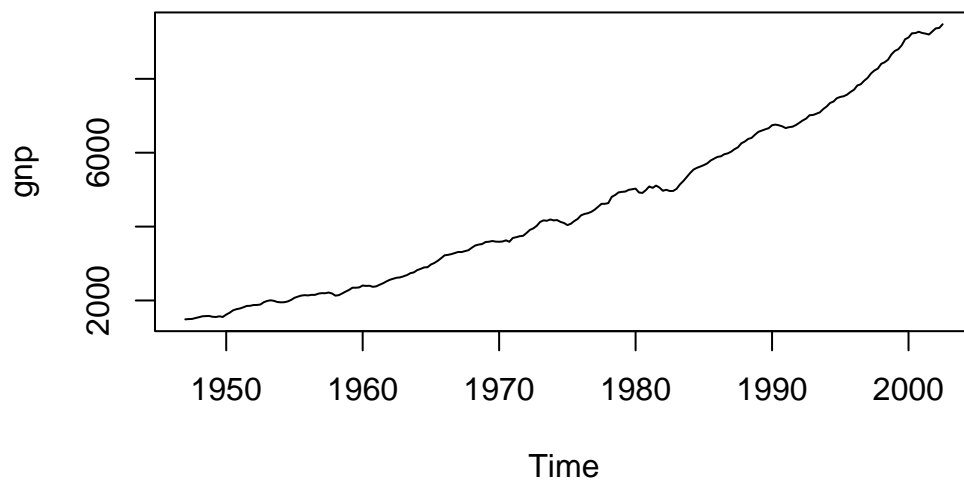
2024-10-31

Problem 3

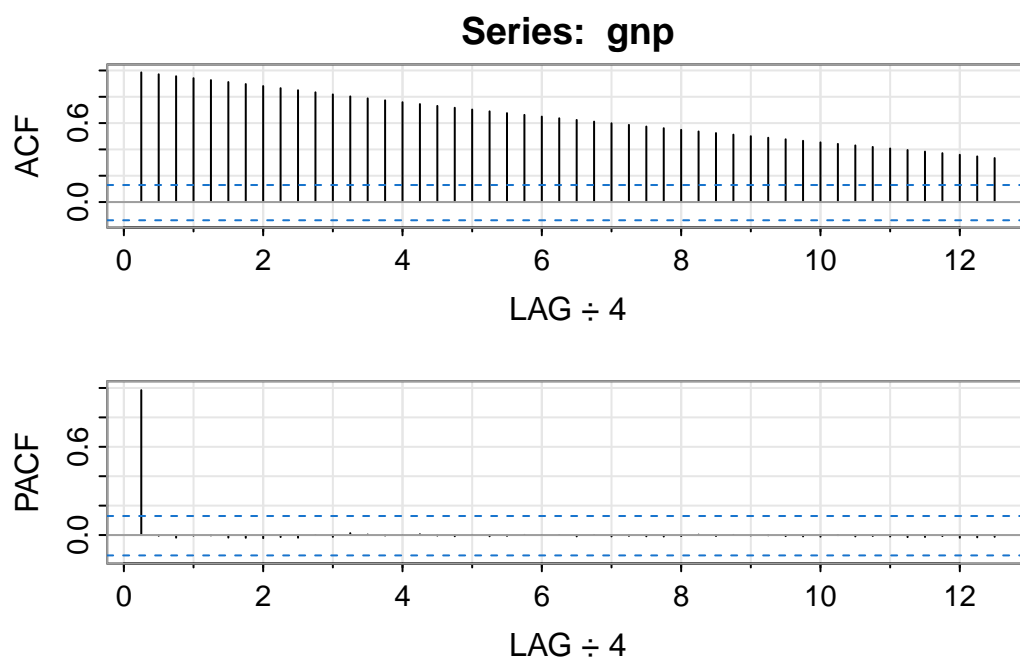
We discussed quarterly United States Gross National Product (GNP) data analysis in the lab. We said that AR(1) and MA(2) are two possible models for the differenced log GNP data. We discussed the model diagnostics for MA(2) model but also pointed out that AR(1) is preferable. Show the model diagnostics for AR(1) model. Repeat the diagnostics for ARMA(1, 2) model and compare the results.

Plot the data

```
library(astsa)
plot(gnp)
```



```
acf2(gnp, 50)
```



[,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]

```

ACF 0.99 0.97 0.96 0.94 0.93 0.91 0.90 0.88 0.87 0.85 0.83 0.82 0.80
PACF 0.99 0.00 -0.02 0.00 0.00 -0.02 -0.02 -0.02 -0.01 -0.02 0.00 -0.01 0.01
      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
ACF 0.79 0.77 0.76 0.74 0.73 0.72 0.7 0.69 0.68 0.66 0.65 0.64
PACF 0.00 0.00 0.00 0.01 0.00 -0.01 0.0 -0.01 -0.01 0.00 0.00 0.00
      [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37]
ACF 0.62 0.61 0.60 0.59 0.57 0.56 0.55 0.54 0.52 0.51 0.5 0.49
PACF -0.01 0.00 -0.01 -0.01 -0.01 -0.01 -0.01 0.00 -0.01 0.00 0.0 0.00
      [,38] [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46] [,47] [,48] [,49]
ACF 0.48 0.47 0.45 0.44 0.43 0.42 0.41 0.40 0.38 0.37 0.36 0.35
PACF -0.01 -0.01 -0.01 0.00 -0.01 -0.01 -0.01 -0.01 -0.01 -0.01 -0.02 -0.02
      [,50]
ACF 0.33
PACF -0.01

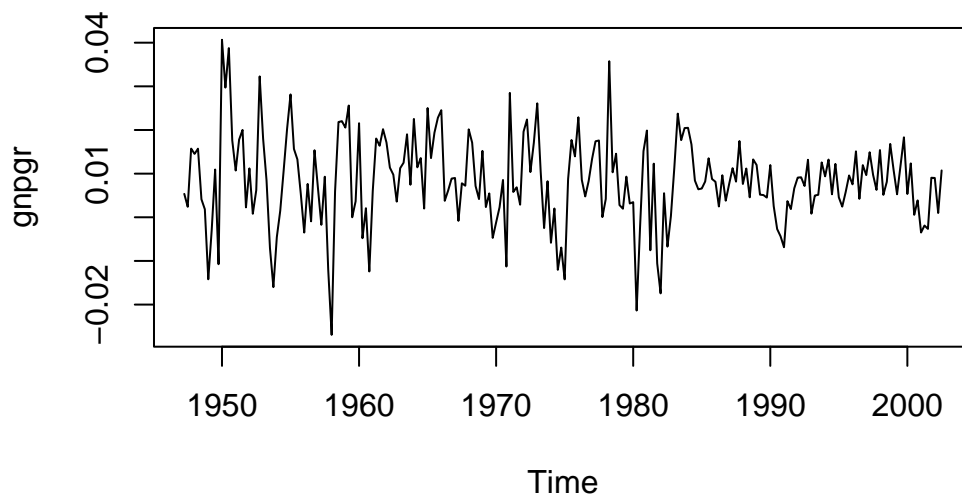
```

Plot Differenced Log GNP

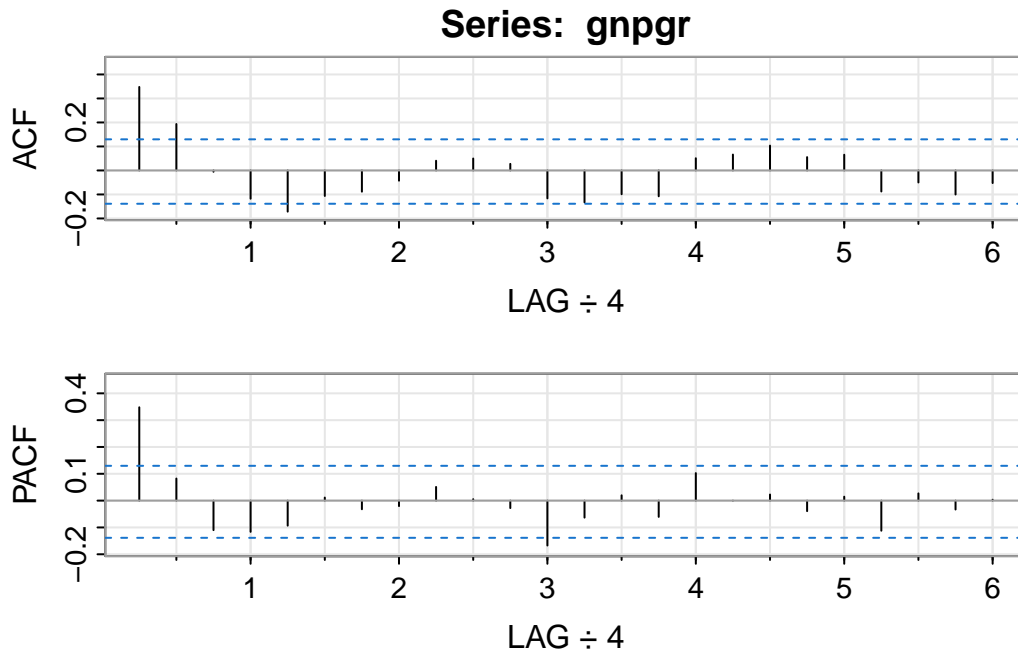
```

gnpgr = diff(log(gnp)) # growth rate
plot(gnpgr)

```



```
acf2(gnpgr, 24)
```



	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]	[,13]
ACF	0.35	0.19	-0.01	-0.12	-0.17	-0.11	-0.09	-0.04	0.04	0.05	0.03	-0.12	-0.13
PACF	0.35	0.08	-0.11	-0.12	-0.09	0.01	-0.03	-0.02	0.05	0.01	-0.03	-0.17	-0.06
	[,14]	[,15]	[,16]	[,17]	[,18]	[,19]	[,20]	[,21]	[,22]	[,23]	[,24]		
ACF	-0.10	-0.11	0.05	0.07	0.10	0.06	0.07	-0.09	-0.05	-0.10	-0.05		
PACF	0.02	-0.06	0.10	0.00	0.02	-0.04	0.01	-0.11	0.03	-0.03	0.00		

Model Diagnostics for AR(1)

```
# Fit AR(1) model
cat("\nAR(1) Model Diagnostics:\n")
```

AR(1) Model Diagnostics:

```
ar1_fit = sarima(gnpgr, 1, 0, 0)
```

```
initial value -4.655919
iter 2 value -4.655921
iter 3 value -4.655922
iter 4 value -4.655922
iter 5 value -4.655922
iter 5 value -4.655922
iter 5 value -4.655922
final value -4.655922
converged
```

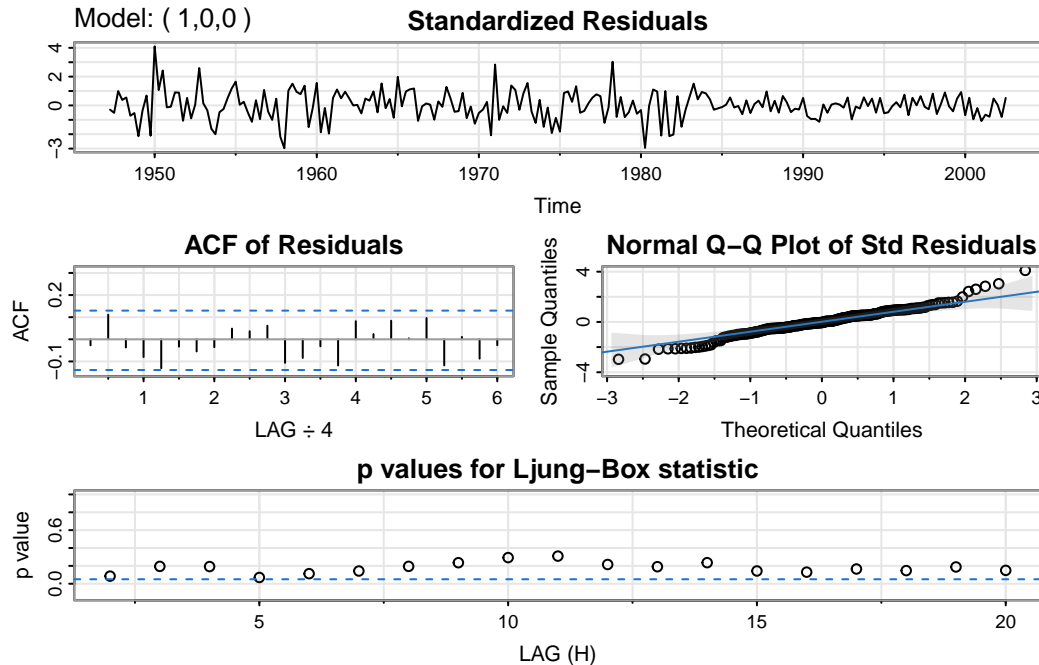
[illegible]

Coefficients:

	Estimate	SE	t.value	p.value
ar1	0.3467	0.0627	5.5255	0
xmean	0.0083	0.0010	8.5398	0

sigma^2 estimated as 9.029569e-05 on 220 degrees of freedom

AIC = -6.44694 AICc = -6.446693 BIC = -6.400958



```
# Display AR(1) coefficients and standard errors
cat("\nAR(1) Model Summary:\n")
```

AR(1) Model Summary:

```
ar1_fit$tttable
```

	Estimate	SE	t.value	p.value
ar1	0.3467	0.0627	5.5255	0
xmean	0.0083	0.0010	8.5398	0

The above figure displays a plot of the standardized residuals, the ACF of the residuals, a boxplot of the standardized residuals, and the p-values associated with the Q-statistic at lags $H=2$ through $H=20$.

Like the MA(2) model, inspecting the time plot of standardized residuals in Figure 3.16 reveals no clear patterns. However, some outliers are apparent, with a few values exceeding 3 standard deviations in magnitude. The autocorrelation function (ACF) of these residuals does not indicate any deviation from the model assumptions, and the Q-statistics at the shown lags are not statistically significant. Additionally, the normal Q-Q plot of residuals suggests the assumption of normality is largely valid, aside from the presence of potential outliers.

Model Diagnostics for ARMA(1,2)

```
# Fit ARMA(1,2) model
cat("\nARMA(1,2) Model Diagnostics:\n")
```

ARMA(1,2) Model Diagnostics:

```
arma12_fit = sarima(gnpgr, 1, 0, 2)
```

[illegible]

Coefficients:

	Estimate	SE	t.value	p.value
ar1	0.2407	0.2066	1.1651	0.2453

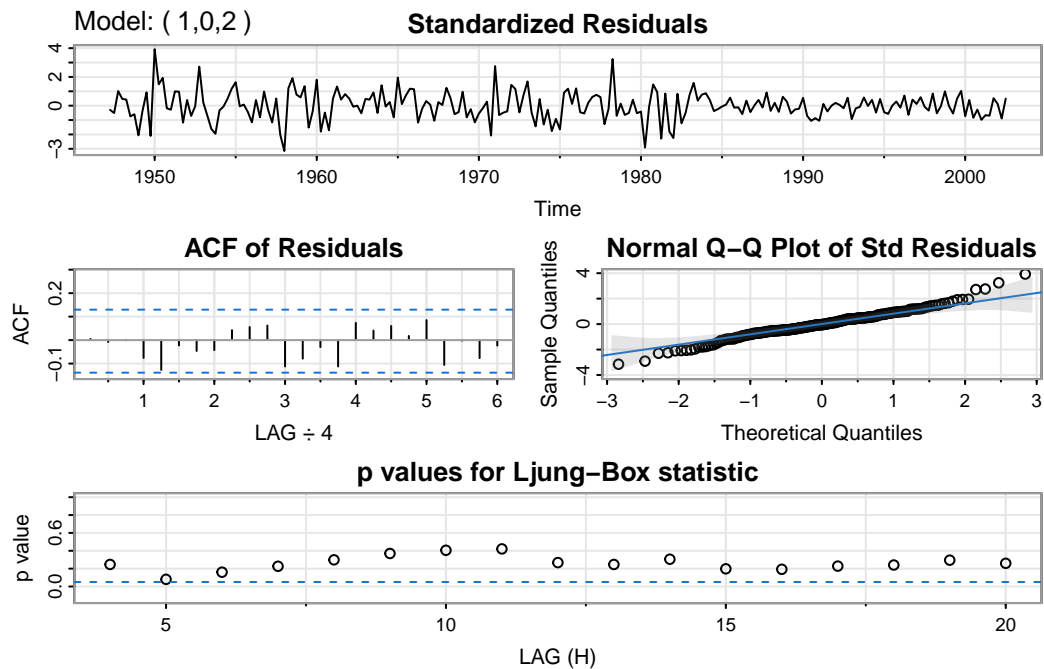
```

ma1      0.0761 0.2026  0.3754  0.7077
ma2      0.1623 0.0851  1.9084  0.0577
xmean    0.0083 0.0010  8.0774  0.0000

```

σ^2 estimated as 8.877466e-05 on 218 degrees of freedom

AIC = -6.445712 AICc = -6.444882 BIC = -6.369075



```

# Display ARMA(1,2) coefficients and standard errors
cat("\nARMA(1,2) Model Summary:\n")

```

ARMA(1,2) Model Summary:

```
arma12_fit$tttable
```

	Estimate	SE	t.value	p.value
ar1	0.2407	0.2066	1.1651	0.2453
ma1	0.0761	0.2026	0.3754	0.7077
ma2	0.1623	0.0851	1.9084	0.0577
xmean	0.0083	0.0010	8.0774	0.0000

The results are very similar to the AR(1) and MA(2). Q-Q plot here is a bit better than the previous cases with most of the residuals following the normality assumption.

Comparing AR(1) and ARMA(1,2) using Information Criteria

```
# Calculate AIC and BIC for AR(1)
ar1_aic = ar1_fit$ICs[1]
ar1_aicc = ar1_fit$ICs[2]
ar1_bic = ar1_fit$ICs[3]

# Calculate AIC and BIC for ARMA(1,2)
arma12_aic = arma12_fit$ICs[1]
arma12_aicc = arma12_fit$ICs[2]
arma12_bic = arma12_fit$ICs[3]

# Compare AIC and BIC
cat("AR(1) AIC:", ar1_aic, "AICc:", ar1_aicc, "BIC:", ar1_bic, "\n")
```

AR(1) AIC: -6.44694 AICc: -6.44693 BIC: -6.400958

```
cat("ARMA(1,2) AIC:", arma12_aic, "AICc:", arma12_aicc, "BIC:", arma12_bic, "\n")
```

ARMA(1,2) AIC: -6.445712 AICc: -6.444882 BIC: -6.369075

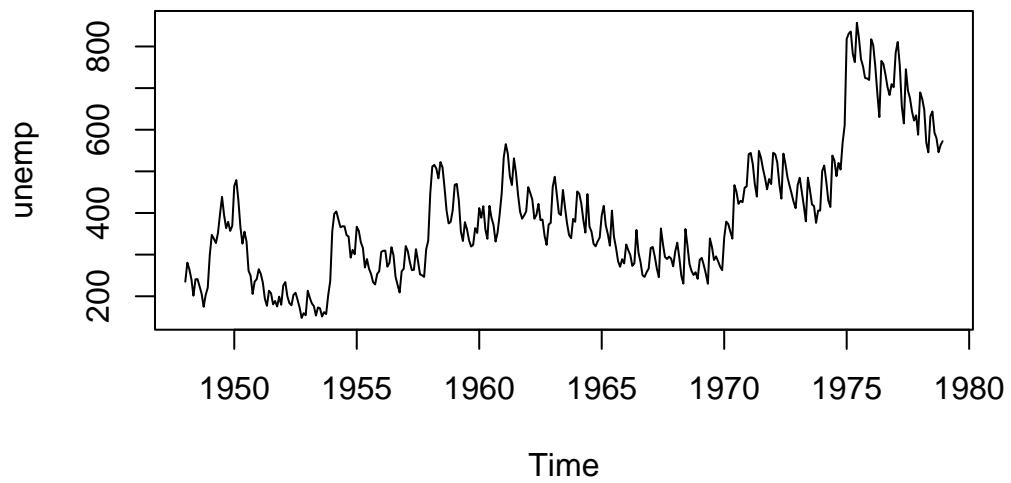
We can see that the AIC, AICc, as well as BIC all prefer the AR(1) model over ARMA(1,2). The AR(1) can be considered as the preferred model in this case. Moreover, pure auto-regressive models are easier to work with.

Problem 4

Fit a seasonal ARIMA model of your choice to the unemployment data in **unemp** from the R package *astsa*. Use the estimated model to forecast the next 12 months.

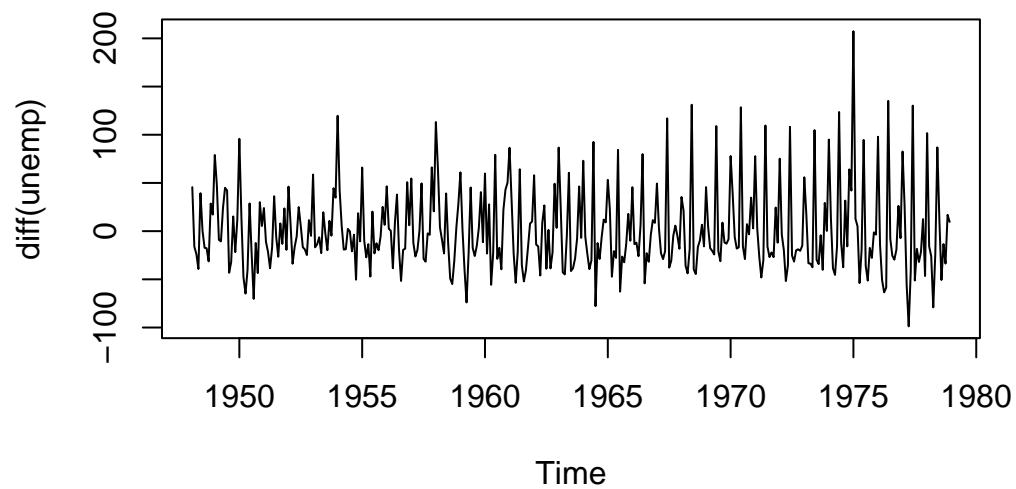
Plot the unemp data

```
plot(unemp)
```



Plot the Differenced unemp

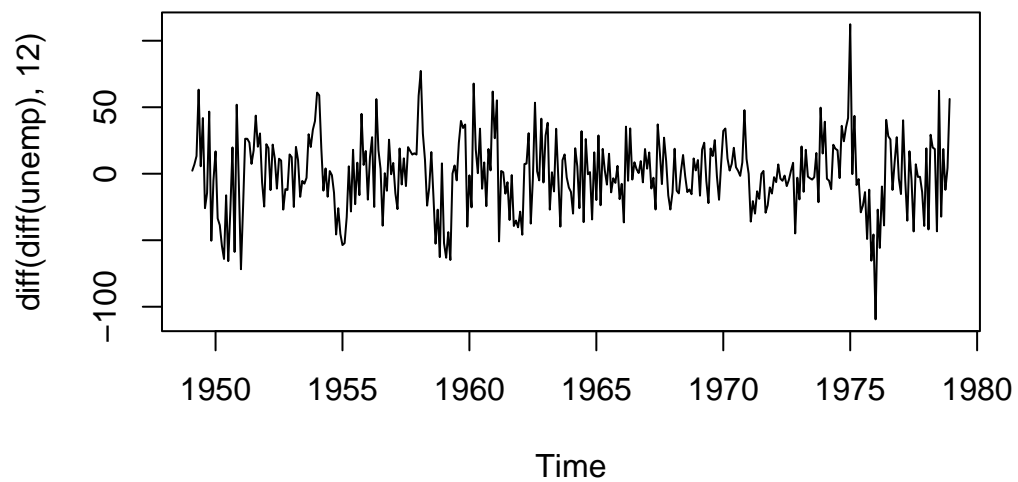
```
plot(diff(unemp))
```



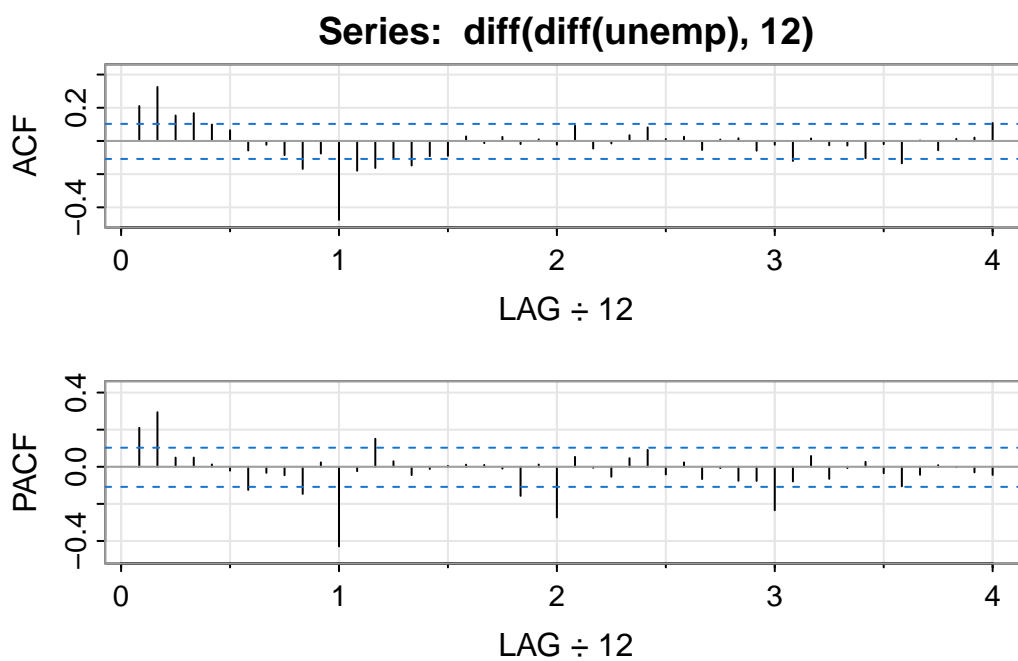
Plot Difference of Differenced unemp at lag 12

Since the given data varies over months, thus there may be a seasonal pattern over months of a year. Therefore, it is reasonable to assume that Differencing at lag 12 may give us a stationary time with which can continue our analysis.

```
plot(diff(diff(unemp),12))
```



```
acf2(diff(diff(unemp), 12))
```



[,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]

```

ACF  0.21 0.33 0.15 0.17 0.10  0.06 -0.06 -0.02 -0.09 -0.17 -0.08 -0.48 -0.18
PACF 0.21 0.29 0.05 0.05 0.01 -0.02 -0.12 -0.03 -0.05 -0.15  0.02 -0.43 -0.02
      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
ACF  -0.16 -0.11 -0.15 -0.09 -0.09  0.03 -0.01  0.02 -0.02  0.01 -0.02  0.09
PACF  0.15  0.03 -0.04 -0.01  0.00  0.01  0.01 -0.01 -0.16  0.01 -0.27  0.05
      [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37]
ACF  -0.05 -0.01  0.03  0.08  0.01  0.03 -0.05  0.01  0.02 -0.06 -0.02 -0.12
PACF -0.01 -0.05  0.05  0.09 -0.04  0.02 -0.07 -0.01 -0.08 -0.08 -0.23 -0.08
      [,38] [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46] [,47] [,48]
ACF   0.01 -0.03 -0.03 -0.10 -0.02 -0.13  0.00 -0.06  0.01  0.02  0.11
PACF  0.06 -0.07 -0.01  0.03 -0.03 -0.11 -0.04  0.01  0.00 -0.03 -0.04

```

The ACF and PACF confirm our assumption of seasonal pattern. Here, we can observe the seasonal lags in the ACF cut off after lag 12, whereas the seasonal lags in the PACF tail off at lags 12, 24, 36, and so on. This indicates a clear SMA(1) pattern.

Fit a SMA(1) model

Now, we fit an SARIMA(0,1,0) \times (0,1,1)₁₂ to x_t and look at the ACF and PACF of the residuals

```
sma_fit <- sarima(unemp, 0,1,0, 0,1,1,12)
```

```

initial  value 3.338111
iter    2 value 3.154910
iter    3 value 3.119632
iter    4 value 3.113045
iter    5 value 3.112067
iter    6 value 3.108906
iter    7 value 3.108728
iter    8 value 3.108723
iter    9 value 3.108720
iter    9 value 3.108720
iter    9 value 3.108720
final    value 3.108720
converged
initial  value 3.111599
iter    2 value 3.111596
iter    3 value 3.111594
iter    3 value 3.111594
iter    3 value 3.111594
final    value 3.111594

```

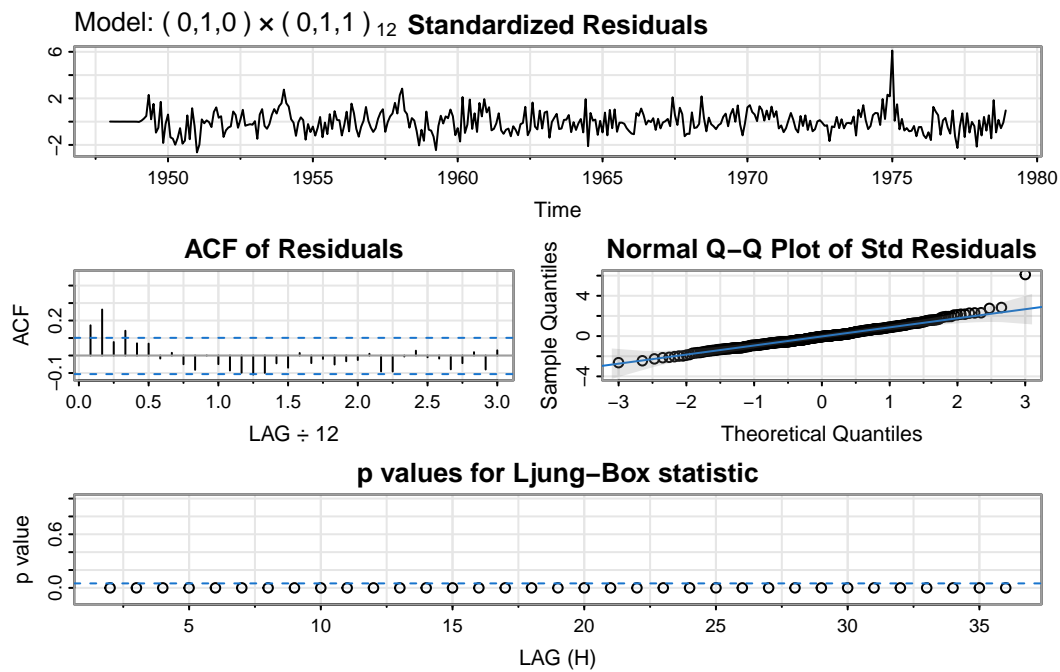
[illegible]

Coefficients:

	Estimate	SE	t.value	p.value
sma1	-0.7334	0.0366	-20.025	0

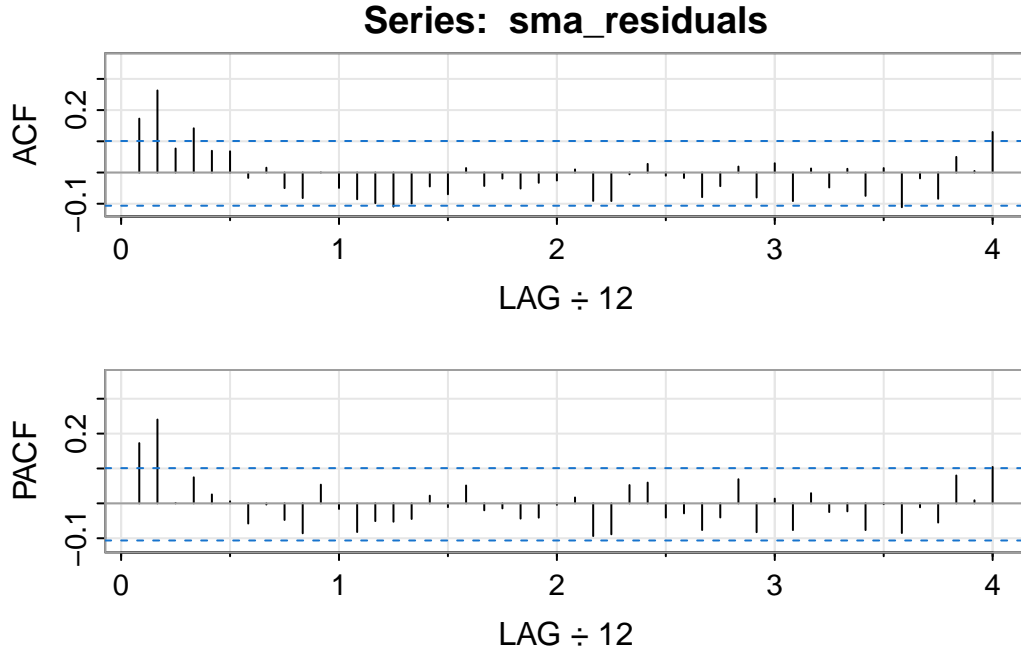
sigma^2 estimated as 491.4653 on 358 degrees of freedom

AIC = 9.072207 AICc = 9.072239 BIC = 9.093842



```
# Extract residuals
sma_residuals <- sma_fit$fit$residuals

acf2(sma_residuals)
```

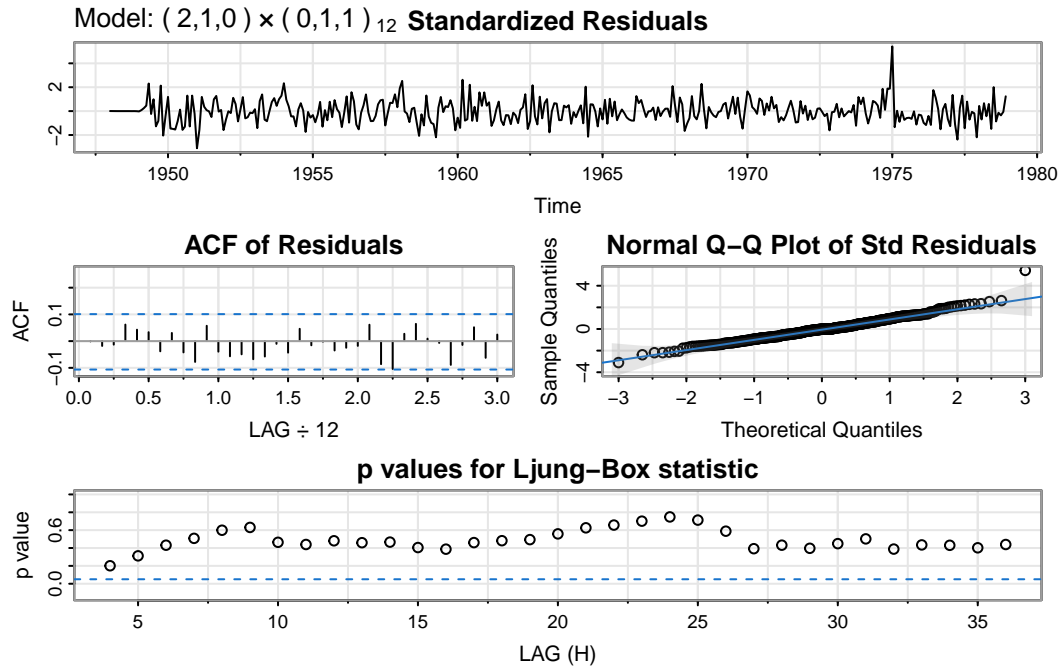


	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]	[,13]
ACF	0.17	0.26	0.08	0.14	0.07	0.07	-0.02	0.02	-0.05	-0.08	0.00	-0.05	-0.09
PACF	0.17	0.24	0.00	0.07	0.03	0.01	-0.06	0.00	-0.05	-0.09	0.05	-0.02	-0.08
	[,14]	[,15]	[,16]	[,17]	[,18]	[,19]	[,20]	[,21]	[,22]	[,23]	[,24]	[,25]	
ACF	-0.10	-0.11	-0.10	-0.04	-0.07	0.01	-0.04	-0.02	-0.05	-0.03	-0.03	0.01	
PACF	-0.05	-0.05	-0.04	0.02	-0.01	0.05	-0.02	-0.01	-0.04	-0.04	0.00	0.02	
	[,26]	[,27]	[,28]	[,29]	[,30]	[,31]	[,32]	[,33]	[,34]	[,35]	[,36]	[,37]	
ACF	-0.09	-0.09	-0.01	0.03	-0.01	-0.02	-0.08	-0.04	0.02	-0.08	0.03	-0.09	
PACF	-0.09	-0.09	0.05	0.06	-0.04	-0.03	-0.08	-0.04	0.07	-0.08	0.01	-0.08	
	[,38]	[,39]	[,40]	[,41]	[,42]	[,43]	[,44]	[,45]	[,46]	[,47]	[,48]		
ACF	0.01	-0.05	0.01	-0.07	0.01	-0.11	-0.02	-0.08	0.05	0.00	0.13		
PACF	0.03	-0.02	-0.02	-0.08	0.00	-0.09	-0.01	-0.05	0.08	0.01	0.10		

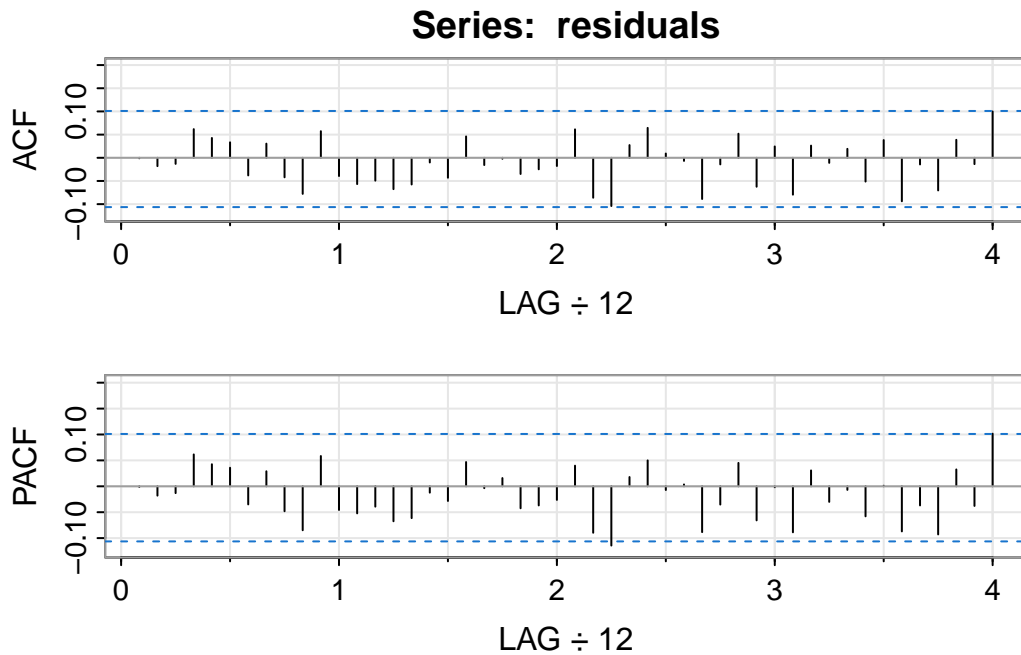
The within-season portion of the ACF gradually tapers off, while the PACF either cuts off at lag 2 or also tapers. These observations indicate that an AR(2) or ARMA(1,1) model could be suitable for the within-season component of the model.

Fit SARIMA with AR(2) component

SARIMA(2, 1, 0) × (0, 1, 1)₁₂



```
# Extract residuals
residuals <- sarima_model$fit$residuals
acf2(residuals)
```



	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]	[,13]
ACF	0	-0.02	-0.01	0.06	0.04	0.03	-0.04	0.03	-0.04	-0.08	0.06	-0.04	-0.06
PACF	0	-0.02	-0.01	0.06	0.04	0.04	-0.03	0.03	-0.05	-0.08	0.06	-0.05	-0.05
	[,14]	[,15]	[,16]	[,17]	[,18]	[,19]	[,20]	[,21]	[,22]	[,23]	[,24]	[,25]	
ACF	-0.05	-0.07	-0.06	-0.01	-0.04	0.05	-0.02	0.00	-0.03	-0.02	-0.02	0.06	
PACF	-0.04	-0.07	-0.06	-0.01	-0.03	0.05	0.00	0.02	-0.04	-0.04	-0.03	0.04	
	[,26]	[,27]	[,28]	[,29]	[,30]	[,31]	[,32]	[,33]	[,34]	[,35]	[,36]	[,37]	
ACF	-0.09	-0.10	0.03	0.06	0.01	-0.01	-0.09	-0.01	0.05	-0.06	0.02	-0.08	
PACF	-0.09	-0.11	0.02	0.05	-0.01	0.00	-0.09	-0.04	0.05	-0.07	0.00	-0.09	
	[,38]	[,39]	[,40]	[,41]	[,42]	[,43]	[,44]	[,45]	[,46]	[,47]	[,48]		
ACF	0.03	-0.01	0.02	-0.05	0.04	-0.09	-0.01	-0.07	0.04	-0.01	0.1		
PACF	0.03	-0.03	-0.01	-0.06	0.00	-0.09	-0.04	-0.09	0.03	-0.04	0.1		

```
# Print MSE
mse <- mean(residuals^2)
mse
```

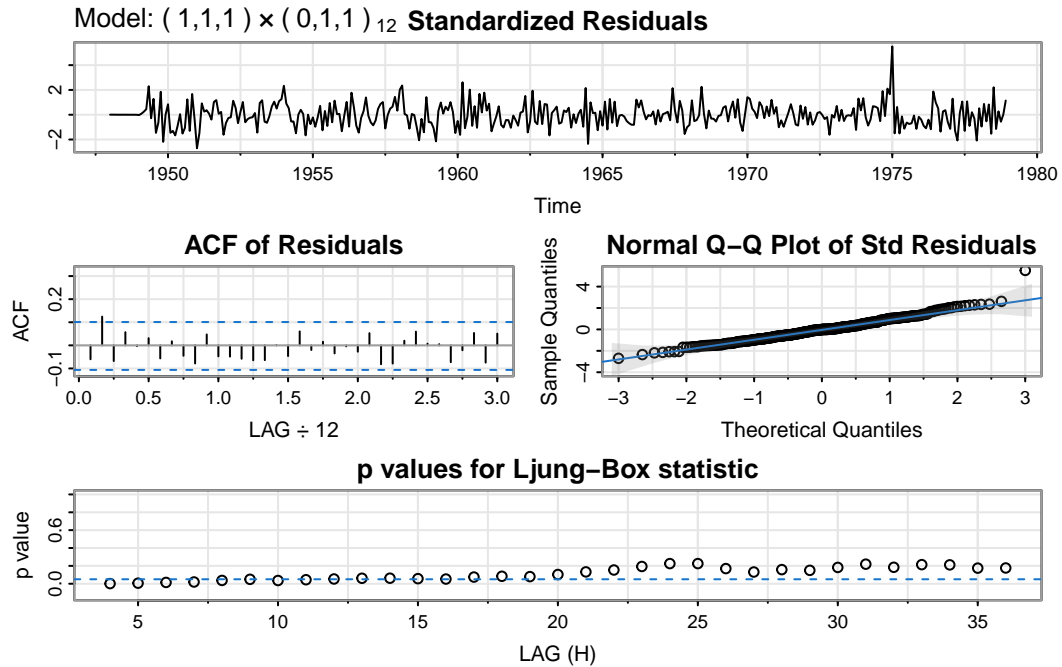
```
[1] 433.9248
```

Fit SARIMA with ARMA(1,1) component

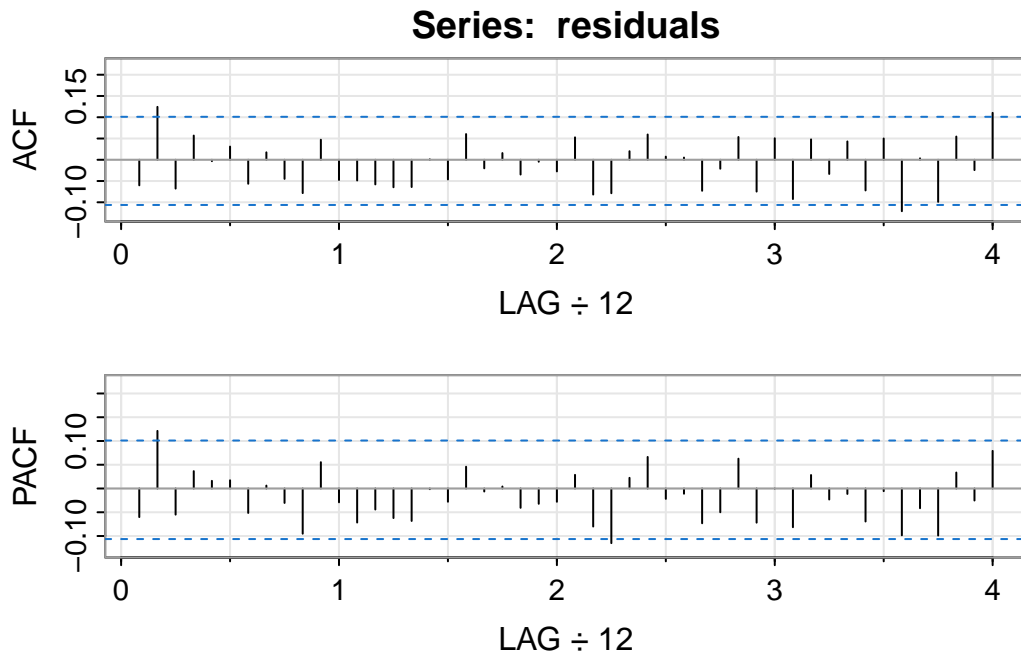
SARIMA(1,1,1) \times (0,1,1)₁₂

```
sarima_model <- sarima(unemp, 1, 1, 1, 0, 1, 1, 12)
```

```
initial value 3.339497
iter 2 value 3.185867
iter 3 value 3.137572
iter 4 value 3.103005
iter 5 value 3.102720
iter 6 value 3.096553
iter 7 value 3.095284
iter 8 value 3.093866
iter 9 value 3.093270
iter 10 value 3.091924
iter 11 value 3.084816
iter 12 value 3.078887
iter 13 value 3.076774
iter 14 value 3.075302
iter 15 value 3.075003
iter 16 value 3.074831
```

```
# Extract residuals
residuals <- sarima_model$fit$residuals
acf2(residuals)
```



	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]	[,13]
ACF	-0.06	0.12	-0.07	0.06	0.00	0.03	-0.06	0.02	-0.05	-0.08	0.05	-0.05	-0.05
PACF	-0.06	0.12	-0.05	0.04	0.02	0.02	-0.05	0.01	-0.03	-0.10	0.06	-0.03	-0.07
	[,14]	[,15]	[,16]	[,17]	[,18]	[,19]	[,20]	[,21]	[,22]	[,23]	[,24]	[,25]	
ACF	-0.06	-0.06	-0.06	0	-0.05	0.06	-0.02	0.02	-0.03	0.00	-0.03	0.05	
PACF	-0.04	-0.06	-0.07	0	-0.03	0.05	-0.01	0.00	-0.04	-0.03	-0.03	0.03	
	[,26]	[,27]	[,28]	[,29]	[,30]	[,31]	[,32]	[,33]	[,34]	[,35]	[,36]	[,37]	
ACF	-0.08	-0.08	0.02	0.06	0.01	0.01	-0.07	-0.02	0.05	-0.07	0.05	-0.09	
PACF	-0.08	-0.12	0.02	0.07	-0.02	-0.01	-0.07	-0.05	0.06	-0.07	0.00	-0.08	
	[,38]	[,39]	[,40]	[,41]	[,42]	[,43]	[,44]	[,45]	[,46]	[,47]	[,48]		
ACF	0.05	-0.03	0.04	-0.07	0.05	-0.12	0.00	-0.1	0.05	-0.02	0.11		
PACF	0.03	-0.02	-0.01	-0.07	-0.01	-0.10	-0.04	-0.1	0.03	-0.03	0.08		

```
# Print MSE
mse <- mean(residuals^2)
mse
```

```
[1] 441.8145
```

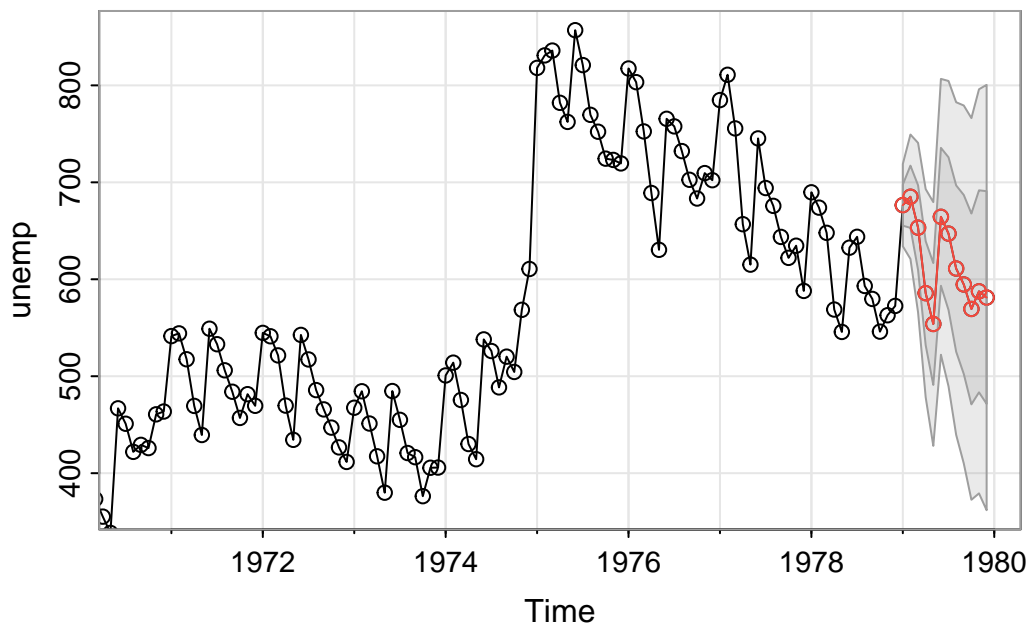
From the above two plots we can see that the first model is way better than the second one as the first one has MSE lower than the second model. Also the Q-statistic in the first model is not significant compared to the second which implies that the residuals are more likely to be white noise in the first case.

Therefore, we prefer the first SARIMA model.

Forecasting

Forecasting using both models for the next 12 months.

```
# SARIMA(2,1,0) ×(0,1,1)12
sarima.for(unemp, 12, 2, 1, 0, 0, 1, 1, 12)
```



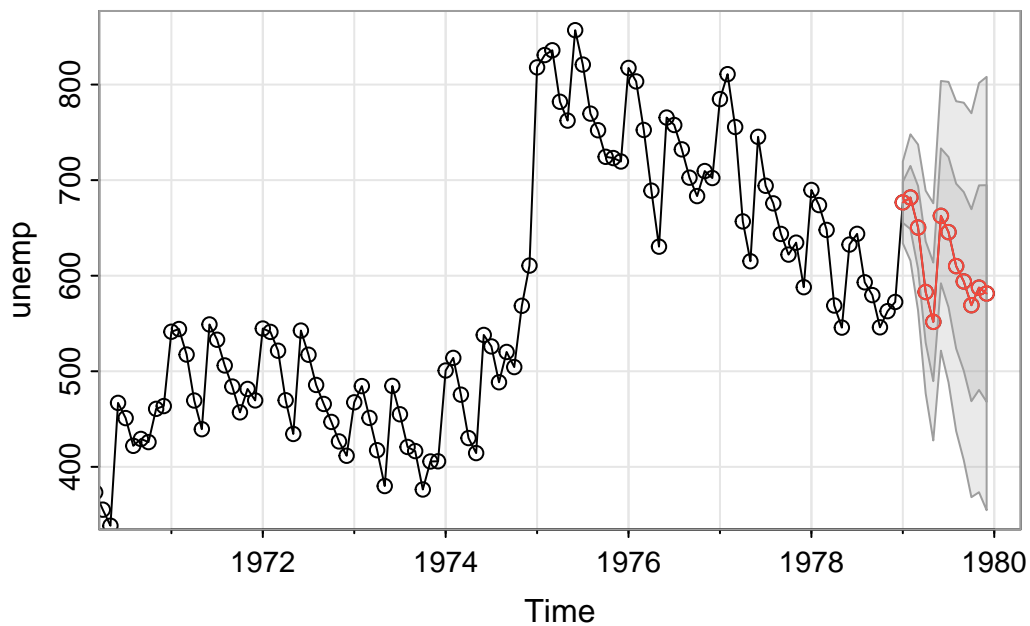
\$pred

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
1979	676.4664	685.1172	653.2388	585.6939	553.8813	664.4072	647.0657	611.0828
	Sep	Oct	Nov	Dec				
1979	594.6414	569.3997	587.5801	581.1833				

\$se

	Jan	Feb	Mar	Apr	May	Jun	Jul
1979	21.20465	32.07710	43.70167	53.66329	62.85364	71.12881	78.73590
	Aug	Sep	Oct	Nov	Dec		
1979	85.75096	92.28663	98.41329	104.19488	109.67935		

```
# SARIMA(1,1,1) ×(0,1,1)12
sarima.for(unemp, 12, 1, 1, 1, 0, 1, 1, 12)
```



\$pred

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
1979	676.7140	681.8208	650.4030	582.9454	551.6973	662.6317	645.5813	610.0253
	Sep	Oct	Nov	Dec				
1979	593.9988	569.1150	587.5246	581.3489				

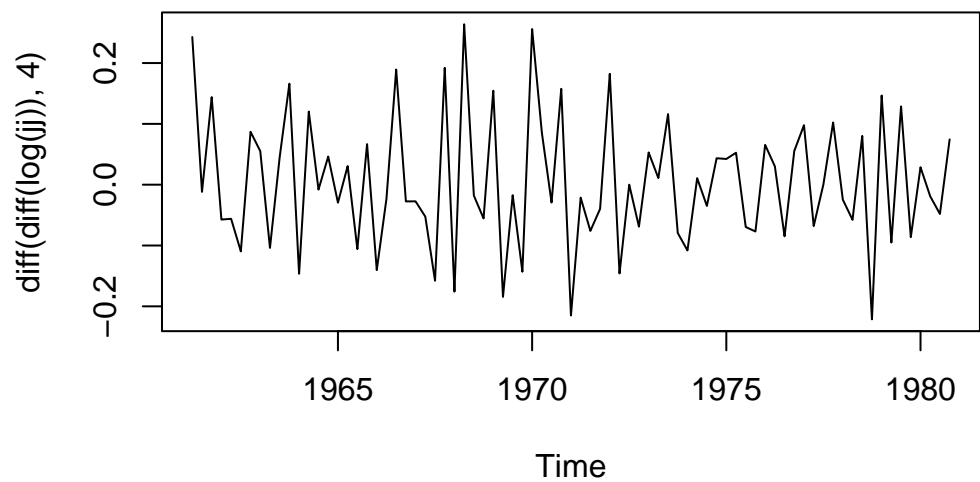
\$se

	Jan	Feb	Mar	Apr	May	Jun	Jul
1979	21.39655	33.05952	43.42253	53.03046	62.04995	70.56057	78.61490
	Aug	Sep	Oct	Nov	Dec		
1979	86.25504	93.51784	100.43660	107.04161	113.36045		

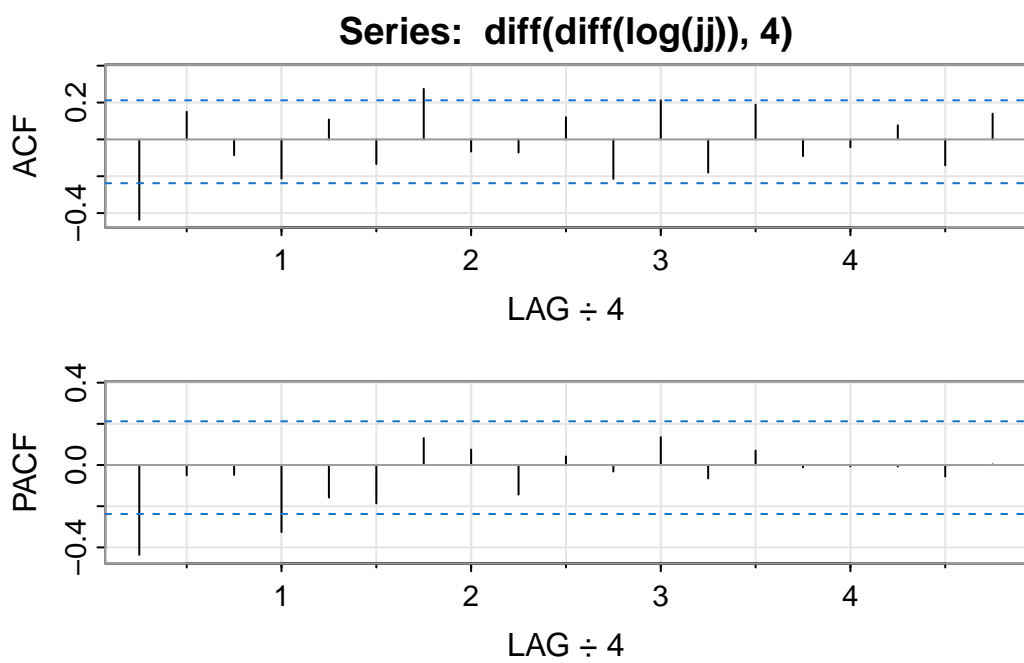
Problem 5

Fit an appropriate seasonal ARIMA model to the log-transformed Johnson and Johnson earnings series (`jj` from the R package *astsa*) discussed in Lecture 2. Use the estimated model to forecast the next 4 quarters.

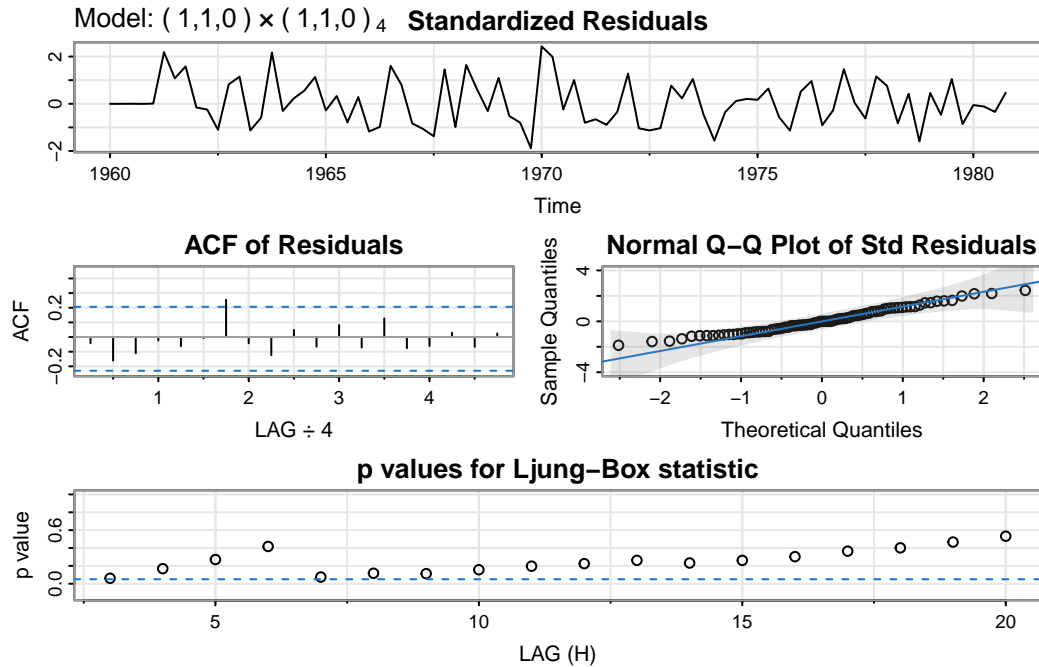
```
plot(diff(diff(log(jj)),4))
```



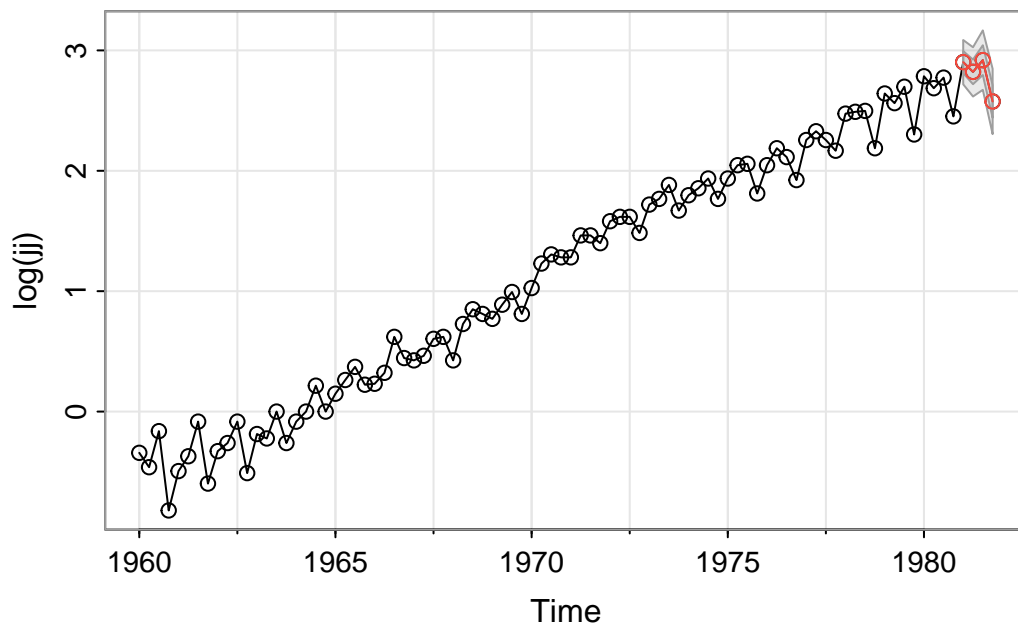
```
acf2(diff(diff(log(jj)),4))
```



[,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12]



```
sarima.for(log(jj),4,1,1,0,1,1,0,4)
```



\$pred

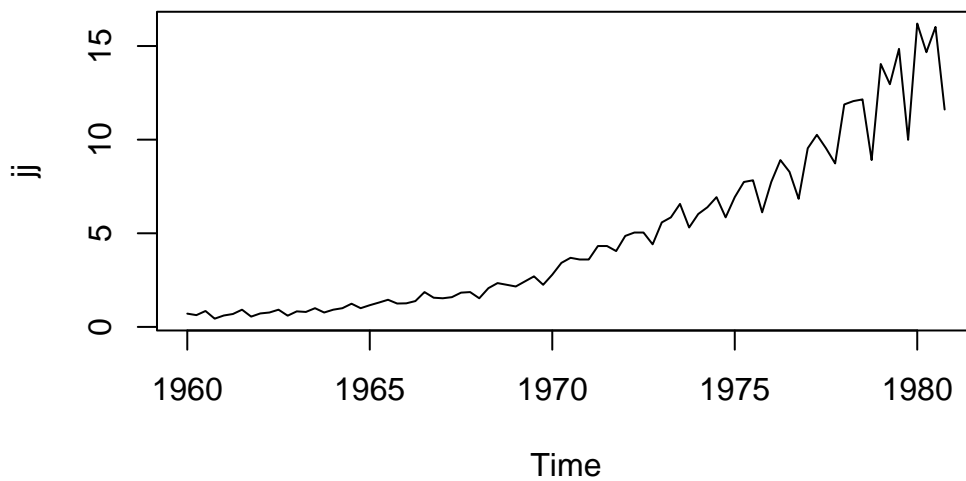
	Qtr1	Qtr2	Qtr3	Qtr4
1981	2.902126	2.821452	2.919034	2.575784

\$se

	Qtr1	Qtr2	Qtr3	Qtr4
1981	0.09202127	0.10226343	0.12338542	0.13568573

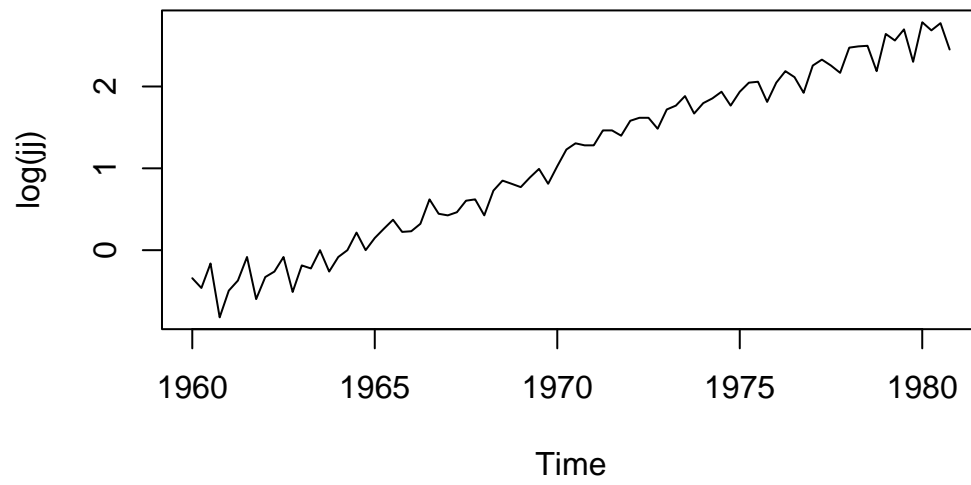
Plot the jj data

```
plot(jj)
```



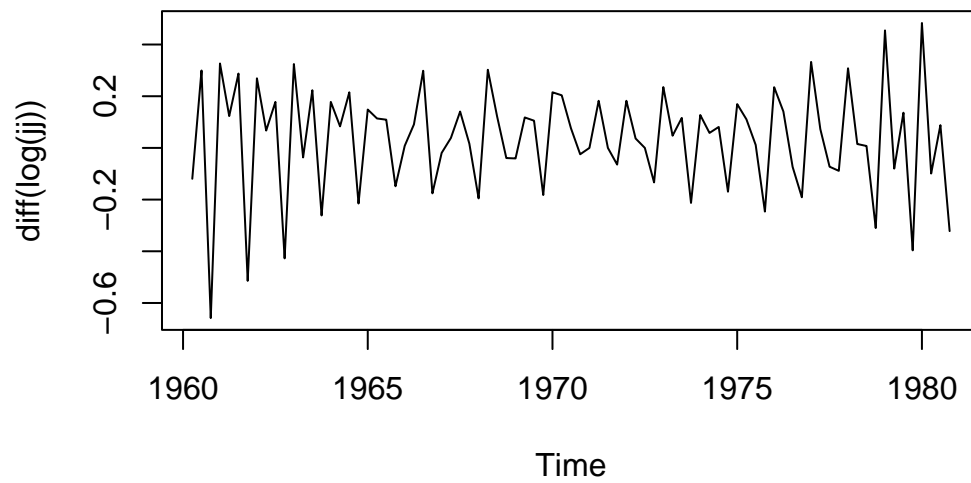
Apply Log transformation

```
plot(log(jj))
```



Plot the Differenced log jj data to remove trend

```
plot(diff(log(jj)))
```

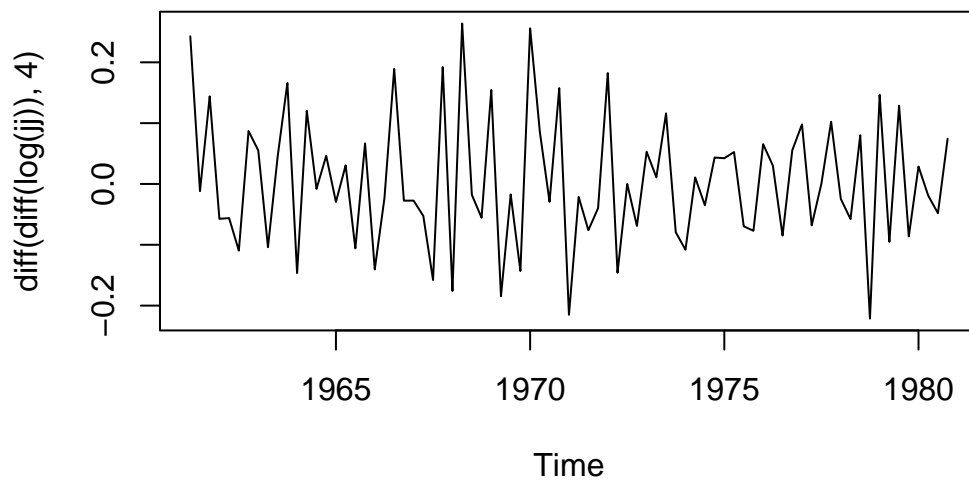


Difference at lag 4 over the Differenced log data

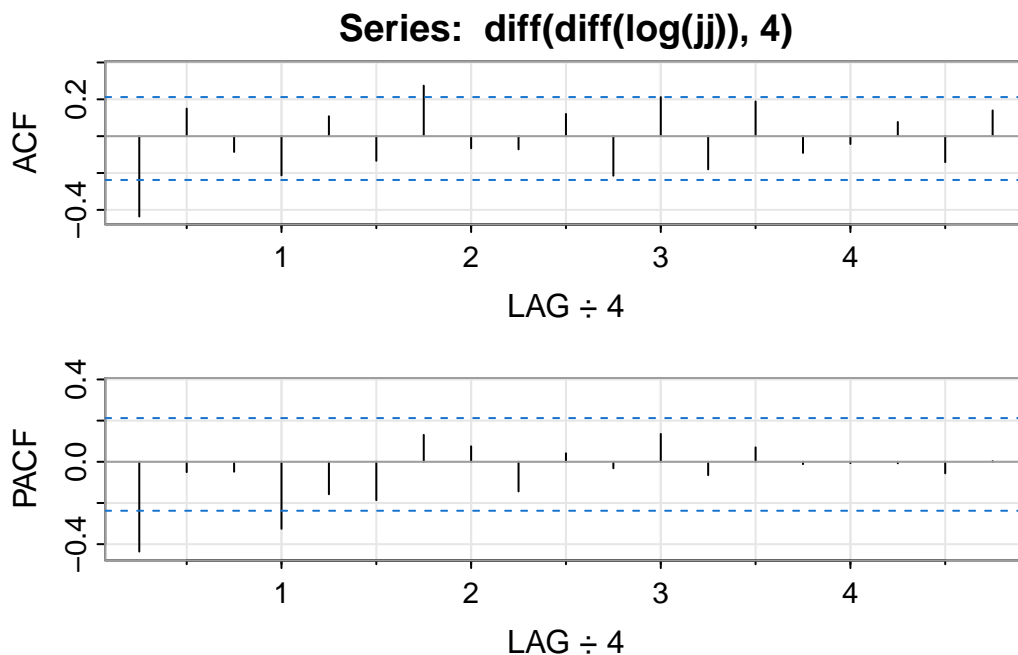
Since the data varies over quarters, there may be seasonal patterns over the quarters of each year.

Thus we take difference over lag 4.

```
plot(diff(diff(log(jj)), 4))
```



```
acf2(diff(diff(log(jj)), 4))
```



	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]
ACF	-0.44	0.15	-0.09	-0.21	0.11	-0.13	0.27	-0.07	-0.07	0.12	-0.21	0.21
PACF	-0.44	-0.05	-0.05	-0.33	-0.16	-0.19	0.13	0.08	-0.14	0.04	-0.03	0.14

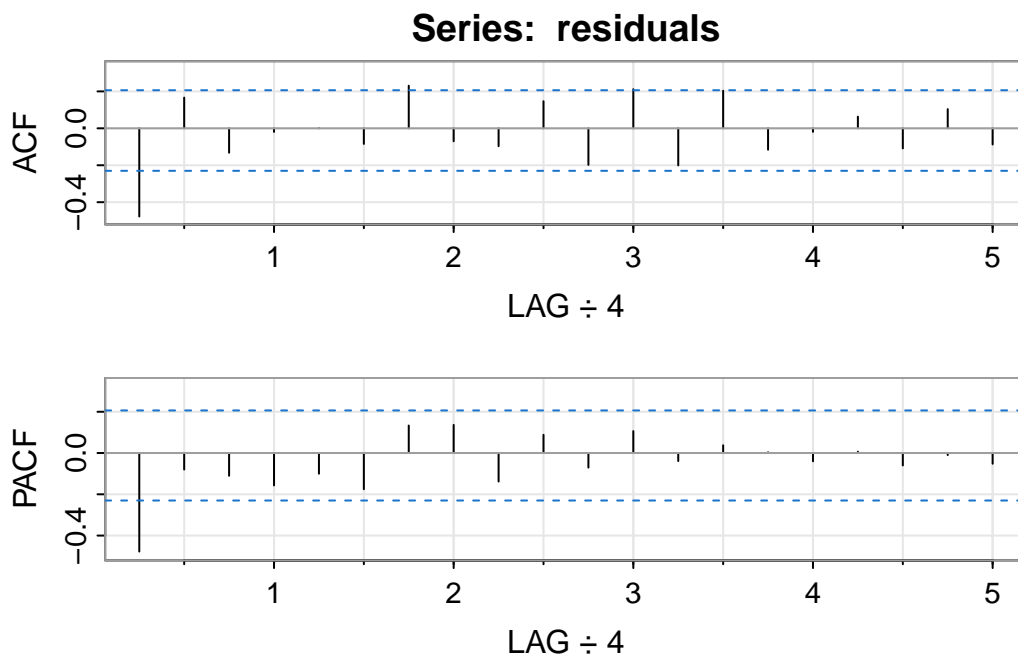
	[,13]	[,14]	[,15]	[,16]	[,17]	[,18]	[,19]
ACF	-0.18	0.19	-0.09	-0.04	0.08	-0.14	0.14
PACF	-0.06	0.07	-0.01	-0.01	-0.01	-0.06	0.00

The PACF reveals a large correlation at the seasonal lag 4, so an SAR(1) seems appropriate.

Fit SAR(1)

```
sar_model <- sarima(log(jj), 0,1,0, 1,1,0, 4)
```

```
initial value -2.237259
iter 2 value -2.262782
iter 3 value -2.262979
iter 4 value -2.262980
iter 4 value -2.262980
final value -2.262980
converged
initial value -2.241179
```

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]
ACF	-0.48	0.17	-0.13	-0.02	0.0	-0.08	0.23	-0.07	-0.10	0.15	-0.20	0.21
PACF	-0.48	-0.08	-0.11	-0.16	-0.1	-0.18	0.13	0.14	-0.14	0.09	-0.07	0.11

	[,13]	[,14]	[,15]	[,16]	[,17]	[,18]	[,19]	[,20]
ACF	-0.20	0.20	-0.12	-0.02	0.06	-0.11	0.10	-0.09
PACF	-0.04	0.04	0.00	-0.04	0.01	-0.06	-0.01	-0.05

As the ACF and PACF of the residuals tails off at seasonal lag 4, this reveals an ARMA(1,1) correlation structure for the within the seasons.

Fit SARIMA(1,1,0)x(1,1,0) and SARIMA(1,1,1)x(1,1,0) and compare them

```
# SARIMA(1,1,0)x(1,1,0)4
sarima_model1 <- sarima(log(jj), 1,1,0, 1,1,0, 4)
```

```
initial value -2.232392
iter 2 value -2.403794
iter 3 value -2.409520
iter 4 value -2.410263
iter 5 value -2.410266
iter 6 value -2.410266
```


BIC
-1.758525

```

initial    value -2.232392
iter      2 value -2.397976
iter      3 value -2.442269
iter      4 value -2.447299
iter      5 value -2.452117
iter      6 value -2.454541
iter      7 value -2.455805
iter      8 value -2.455938
iter      9 value -2.455938
iter     10 value -2.455941
iter     11 value -2.455942
iter     12 value -2.455943
iter     13 value -2.455943
iter     13 value -2.455943
final     value -2.455943
converged

initial    value -2.411490
iter      2 value -2.411708
iter      3 value -2.412085
iter      4 value -2.412085
iter      5 value -2.412087
iter      5 value -2.412087
iter      5 value -2.412087
final     value -2.412087
converged

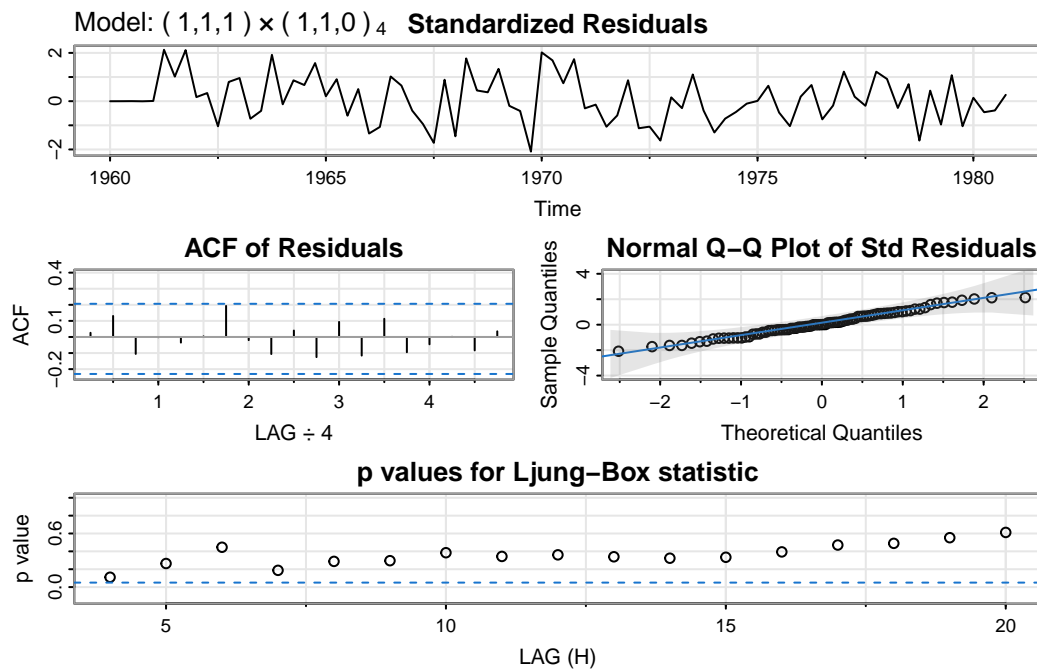
<><><><><><><><><><><><><><>

Coefficients:
            Estimate      SE t.value p.value
ar1      -0.0141  0.2221  -0.0635  0.9495
ma1      -0.6700  0.1814  -3.6940  0.0004
sar1     -0.3265  0.1320  -2.4728  0.0156

```

σ^2 estimated as 0.007912779 on 76 degrees of freedom

AIC = -1.885031 AICc = -1.880981 BIC = -1.765059



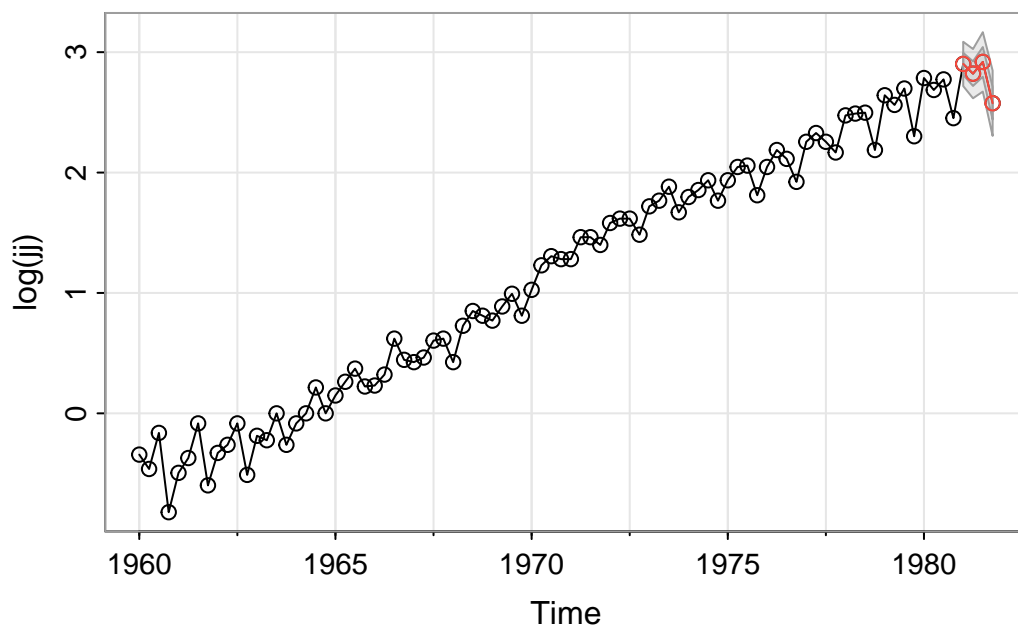
```
sarima_model2$ICs[3]
```

BIC
-1.765059

Both the models perform well. Moreover, the second model has lower BIC, thus we prefer the $\text{SARIMA}(1, 1, 1) \times (1, 1, 0)_4$ model

Forecasting

```
sarima.for(log(jj), 4, 1, 1, 0, 1, 1, 0, 4)
```



```
$pred
      Qtr1      Qtr2      Qtr3      Qtr4
1981 2.902126 2.821452 2.919034 2.575784
```

```
$se
      Qtr1      Qtr2      Qtr3      Qtr4
1981 0.09202127 0.10226343 0.12338542 0.13568573
```