MTH442 Assignment 5

Kaushik Raj Nadar (208160499)

2024-11-14

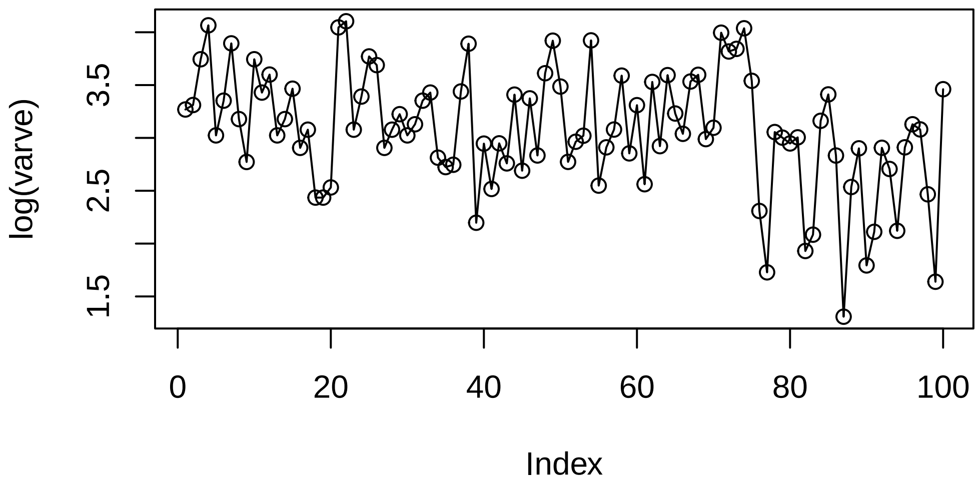
Problem 3

Consider the univariate state-space model given by state conditions $X_0 = W_0$, $X_t = X_{t-1} + W_t$ and observations $Y_t = X_t + V_t$, $t = 1, 2, \ldots$, where W_t and V_t are independent, Gaussian, white noise processes with $\mathrm{Var}(W_t) = \sigma_W^2$ and $\mathrm{Var}(V_t) = \sigma_V^2$. (a) Show that Y_t follows an IMA(1,1) model, that is, ∇Y_t follows an MA(1) model. (b) Fit the model specified in part (a) to the logarithm of the glacial varve series, available as `varve` from the `astsa` package, and summarize the findings. (1+2 = 3 points)

Visualise the Varve Data

```
library(astsa)

x = log(varve[1:100])
plot(x, type="o", ylab="log(varve)")
```



Fit the IMA(1,1) Model

```
model = sarima(x, 0,1,1 )
```

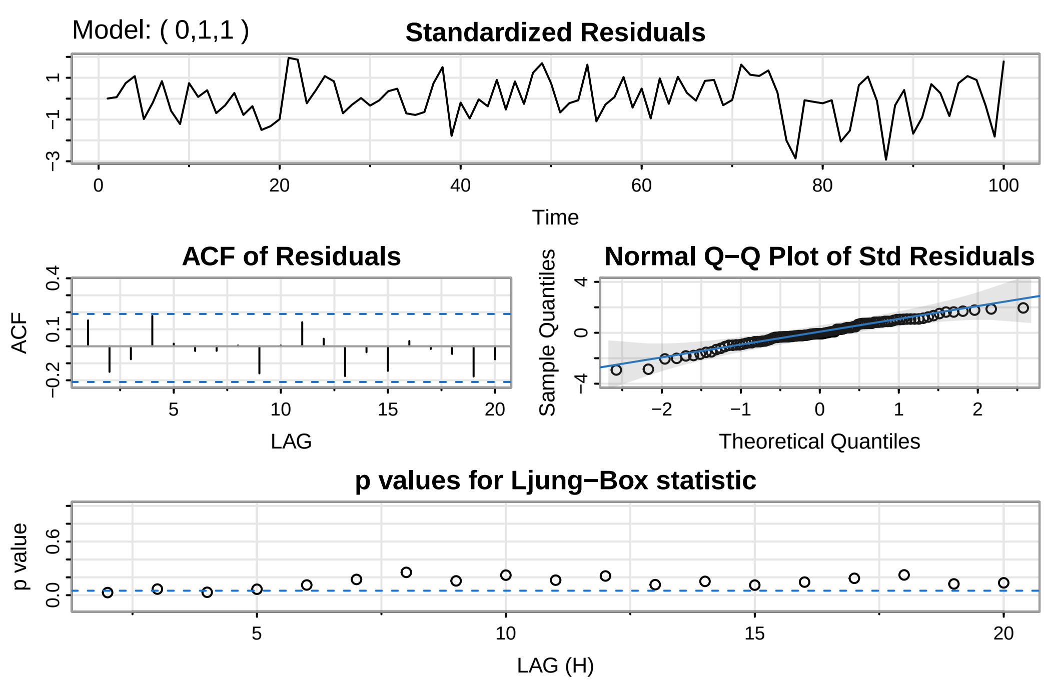
```
initial  value -0.417129
iter   2 value -0.492628
iter   3 value -0.595765
iter   4 value -0.614431
iter   5 value -0.614618
iter   6 value -0.616969
iter   7 value -0.617765
iter   8 value -0.618830
iter   9 value -0.619414
iter  10 value -0.619502
iter  11 value -0.619535
iter  12 value -0.619535
iter  13 value -0.619537
iter  14 value -0.619537
iter  14 value -0.619537
iter  14 value -0.619537
final  value -0.619537
```

```
converged
initial  value -0.612816
iter   2 value -0.612975
iter   3 value -0.613267
iter   4 value -0.613391
iter   5 value -0.613392
iter   6 value -0.613392
iter   6 value -0.613392
final  value -0.613392
converged
<><><><><><><><><><><><><><>

Coefficients:
         Estimate     SE  t.value p.value
ma1       -0.8952 0.0708 -12.6462  0.0000
constant  -0.0084 0.0062  -1.3498  0.1802

sigma^2 estimated as 0.2884843 on 97 degrees of freedom

AIC = 1.671699  AICc = 1.672961  BIC = 1.750339
```



We can see that the residuals behave like white noise and the IMA(1,1) model fit is consistent with theory.

Estimate of theta using IMA(1,1)

```
model$fit$coef[1]
```

```
       ma1
-0.8952357
```

Estimate of Rho(1) using IMA(1,1)

```
model$fit$coef[1]/(1+model$fit$coef[1]^2)
```

```
       ma1
-0.4969537
```

Fitting State Space Model

```
y = log(varve)
num = length(y)
mu0=y[1]
Sigma0=var(y[1:10])
# likelihood
Linn=function(para){
cQ = para[1]
cR= para[2]
kf = Kfilter(y, 1, mu0, Sigma0, 1, cQ, cR)
return(kf$like)
}
init.par=c(.1,.1)
(est = optim(init.par, Linn, NULL, method='BFGS', hessian=TRUE,
control=list(trace=1,REPORT=1)))
```

```
initial  value 2023.409987
iter   2 value 1742.756163
iter   3 value 1736.077915
iter   4 value 1735.079872
iter   5 value 1732.170187
iter   6 value 1728.060365
```

```
iter   7 value 1362.787765
iter   8 value 981.796241
iter   9 value 598.991902
iter  10 value 562.463276
iter  11 value 249.374615
iter  12 value 173.209337
iter  13 value -16.064648
iter  14 value -40.111355
iter  15 value -78.821607
iter  16 value -92.396181
iter  17 value -102.392870
iter  18 value -122.521452
iter  19 value -141.329624
iter  20 value -141.520531
iter  21 value -141.607111
iter  22 value -141.634741
iter  23 value -141.655686
iter  24 value -141.658078
iter  25 value -141.660543
iter  25 value -141.660545
iter  25 value -141.660545
final  value -141.660545
converged

$par
[1] 0.1109679 0.4259291

$value
[1] -141.6605

$counts
function gradient
      79       25

$convergence
[1] 0

$message
NULL

$hessian
         [,1]     [,2]
```

```
[1,] 4489.114 2284.473
[2,] 2284.473 5487.897
```

```
SE = sqrt(diag(solve(est$hessian)))

# Summary of estimation
estimate = c(sig.w=est$par[1], sig.v=est$par[2])
cbind(estimate, SE)
```

```
       estimate         SE
sig.w 0.1109679 0.01681172
sig.v 0.4259291 0.01520511
```

Estimate of Rho(1)

```
-1*estimate[2]*estimate[2]/(2*estimate[2]*estimate[2] + estimate[1]*estimate[1])
```

```
     sig.v
-0.4835879
```

We can see that the State Space model estimates match with that of IMA(1,1).