Embedded Control Laboratory

Ball on Inclined Plane

Report of Lab Exercise 1

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Group 04

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1. Introduction

System consists of a ball rolling on an inclined plane, whose angle is controlled by a servo motor. Angle of the servo motor arm can be regulated with the help of a micro controller mounted to a PCB. LED optical sensors are used to get the position of the ball along the plane.

2. Objectives

- Understand the mathematical and physical principals involved in the control system.
- Determining relation between the angle of plane in terms of the angle of servo motor using free body diagram.
- Writing a SCILAB script to plot the plane angle for various values of servo motor arm.
- Comparing the plots with linear approximation and determining the range of the motor arm angles for which the linear approximation stays valid.

3. Model Representation

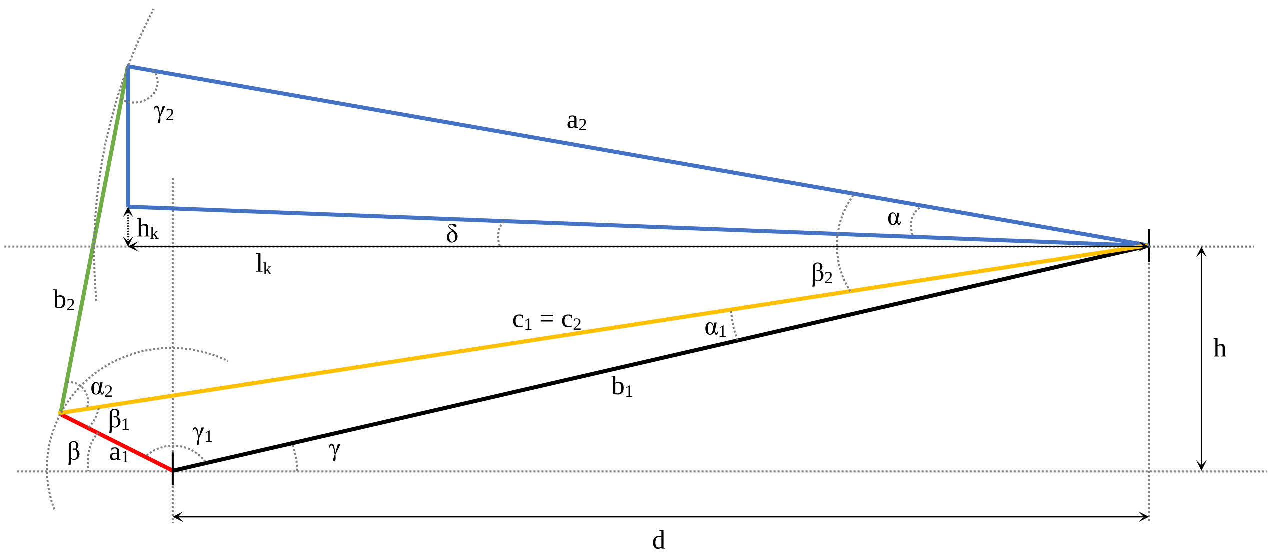

Fig. 1: Model of the inclined plane (case 1: β >= - γ)

4. Equations Used

Name	Formula and value	SCILAB Value
D	105mm	$105e^{-3}$
H	19mm	$19e^{-3}$
$l=a_2$	118.72mm	$118.72e^{-3}$
$r=a_1$	18mm	$18e^{-3}$
$c=b_2$	36.22mm	$36.22e^{-3}$
h_k	17mm	$17e^{-3}$
l_k	117.5mm	$117.5e^{-3}$

α = Plane angle

β = Angle made by the motor arm

The values mentioned below are constants and used throughout the experiments.

$$b_1=sqrt(d^2+h^2)=sqrt(105^2+19^2)=106.70mm$$

$$\gamma= \tan^{-1}(h/d)=\tan^{-1}(19/105)=10.26^0$$

$$\delta=\tan^{-1}(h_k/l_k)=\tan^{-1}(17/117.5)=8.23^0$$

A first linear approximation for the function is:

$$\alpha = (a1/a2)*\beta$$

Case 1: $\beta >= -\gamma$

Sum of the angles at the pivot point of the plane:

$$\alpha+ \delta+ \gamma= \alpha_1+\beta_2 \text{ ---------------------- } (1)$$

Sum of angles at the servo motor arm:

$$\beta + \gamma_1 + \gamma = \pi \text{ ----------------------- } (2)$$

From the above equation:

$$\gamma_1 = \pi - \beta - \gamma \text{ ----------------------- } (3)$$

From Cosine laws:

$$c_1\char`^2 = a_1\char`^2 + b_1\char`^2 - 2*a_1*b_1*\cos\gamma_1 \text{ ------------------------ } (4)$$

$$\alpha_1=\mathrm{Cos}^{-1}((c_1\char`^2+b_1\char`^2-a_1\char`^2)/2*c_1*b_1) \text{ ----------------------- } (5)$$

$$\beta_2=\mathrm{Cos}^{-1}((a_2\char`^2+c_1\char`^2-b_2\char`^2)/2*a_2*c_1) \text{ ----------------------- } (6)$$

Substitute the values obtained from the above equations in the equations below to obtain α:

$$\alpha =\alpha_1+\beta_2-\delta-\gamma \text{ ----------------------- } (7)$$

Case 2: $\beta < -\gamma$

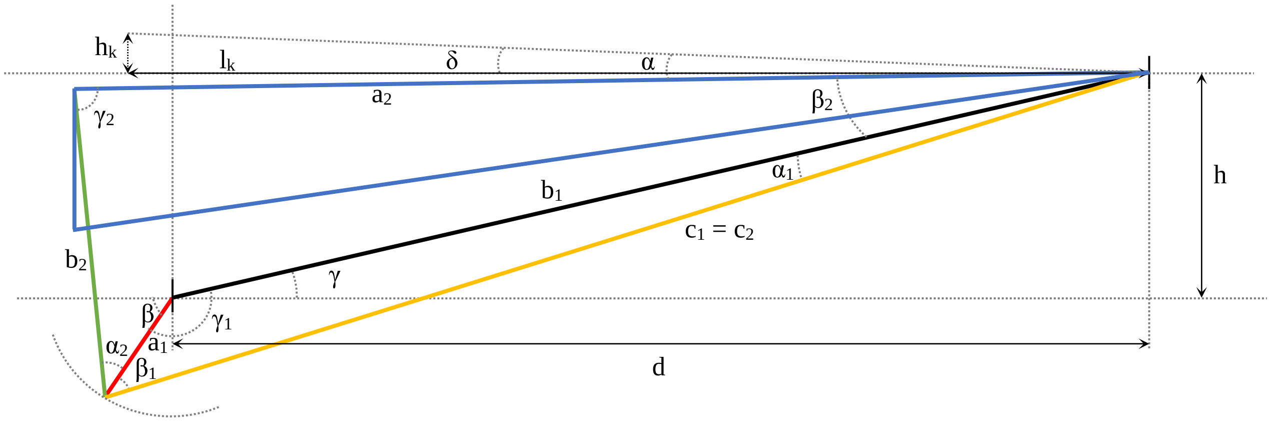

Fig. 2: Model of the inclined plane (case 1: $\beta < -\gamma$)

Sum of the angles at the pivot point of the plane:

$$\alpha + \delta + \gamma = -\alpha_1 + \beta_2 \text{-----------------------} (8)$$

Sum of angles at the servo motor arm:

$$\beta + \gamma_1 + \gamma = \pi \text{-----------------------} (9)$$

From the above equation:

$$\gamma_1 = \pi - \beta - \gamma \text{-----------------------} (10)$$

From Cosine laws:

$$c_1\verb|^|2 = a_1\verb|^|2 + b_1\verb|^|2 - 2*a_1*b_1*\cos\gamma_1 \text{-----------------------} (11)$$

$$\alpha_1 = \mathrm{Cos}^{-1}((c_1\verb|^|2+b_1\verb|^|2-a_1\verb|^|2)/2*c_1*b_1) \text{ -----------------------} (12)$$

$$\beta_2 = \mathrm{Cos}^{-1}((a_2\verb|^|2+c_1\verb|^|2-b_2\verb|^|2)/2*a_2*c_1) \text{ -----------------------} (13)$$

Substitute the values obtained from the above equations in the equations below to obtain α:

$$\alpha = -\alpha_1 + \beta_2 - \delta - \gamma \text{-----------------------} (14)$$

5. SCILAB Code:

```
//Defining the INPUT values
d=105;
h=19;
a2=118.72;
a1=18;
b2=36.22;
hk=17;
lk=117.5;

//Calculation Formulae
b1=sqrt(d^2+h^2);
gam=atand(h/d);
del=atand(hk/lk);

//Defining the Beta values range
bet=linspace(-90,90);

//Linear Approximation
alp=(a1/a2)*bet;
plot(bet,alp)

//Cases: 1. Angle β >= Angle -γ  2. Angle β < Angle -γ
function alp=f(bet)
   gam1=180-gam-bet;
   c1=sqrt(a1^2+b1^2-2*a1*b1*cosd(gam1));
   alp1=acosd((c1^2+b1^2-a1^2)/(2*c1*b1));
   bet2=acosd((a2^2+c1^2-b2^2)/(2*a2*c1));
   if(bet>=-gam) then
      alp=alp1+bet2-gam-del;
   else
      alp=-alp1+bet2-gam-del;
   end
endfunction
plot(bet,f,'r')

legend('Linear Approximation: α=(a1/a2)β','α =f(β) from (-pi/2 to pi/2)', 'in_lower_right')
xtitle('Graph α =f(β)','β','α')
set(gca(),"grid",[1,1])
```

6. Result Plot:

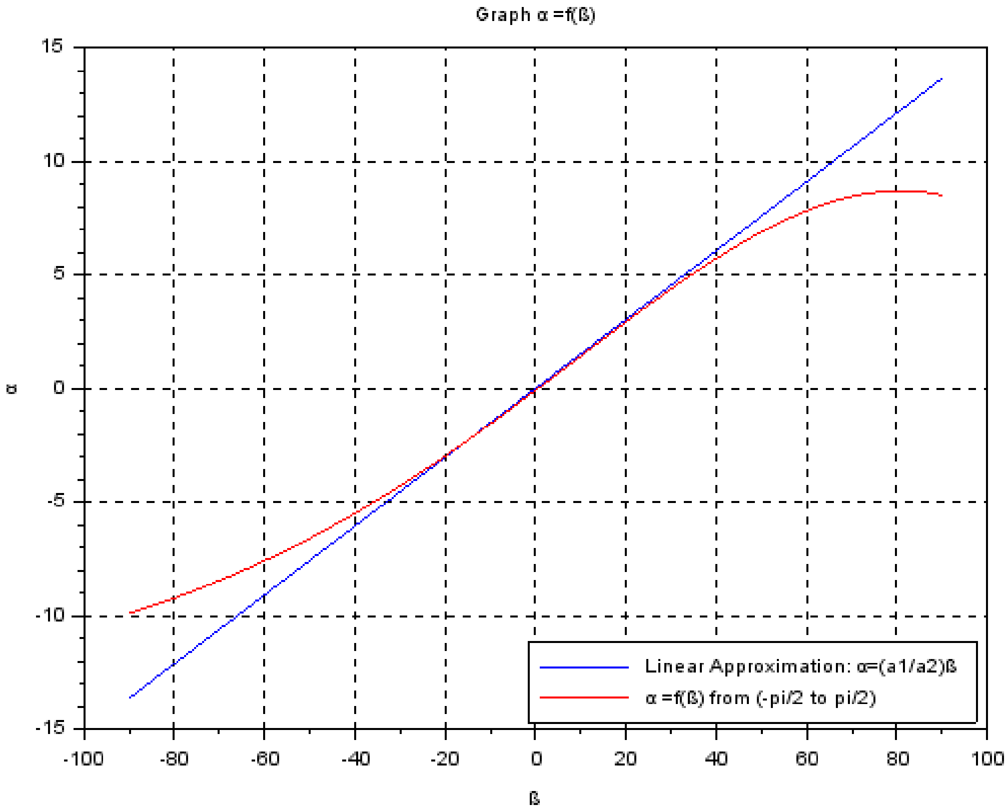

Fig. 3: α vs β for β ranging from -90° to +90°

7. Q and A:

1. *Explain with reasons why the approximation is not so good if angle β is greater than 40 degrees or smaller than -40 degrees?*
    - From the above graph, we can see that the function α =f(ß) is almost linear in the range of angle β from + 40 to – 40. But as the β value moves out of this range the curve deviates from linear plot.

2. *In your SCILAB simulation, maximize the range of the angle β to ±300, plot and explain your output graph.*
    - From the Graphs plotted, we can conclude the relation between α and β as follows:
        a. As the angle β varies from -300 to +300, the angle α varies periodically.
        b. When the angle β reaches near 90 degree, the angle α reaches its maximum and starts descending until it reaches the minimum and this cycle repeats for every period of angle β.
        c. Thus the plot is symmetric and varies from that of linear approximation.

Graph:

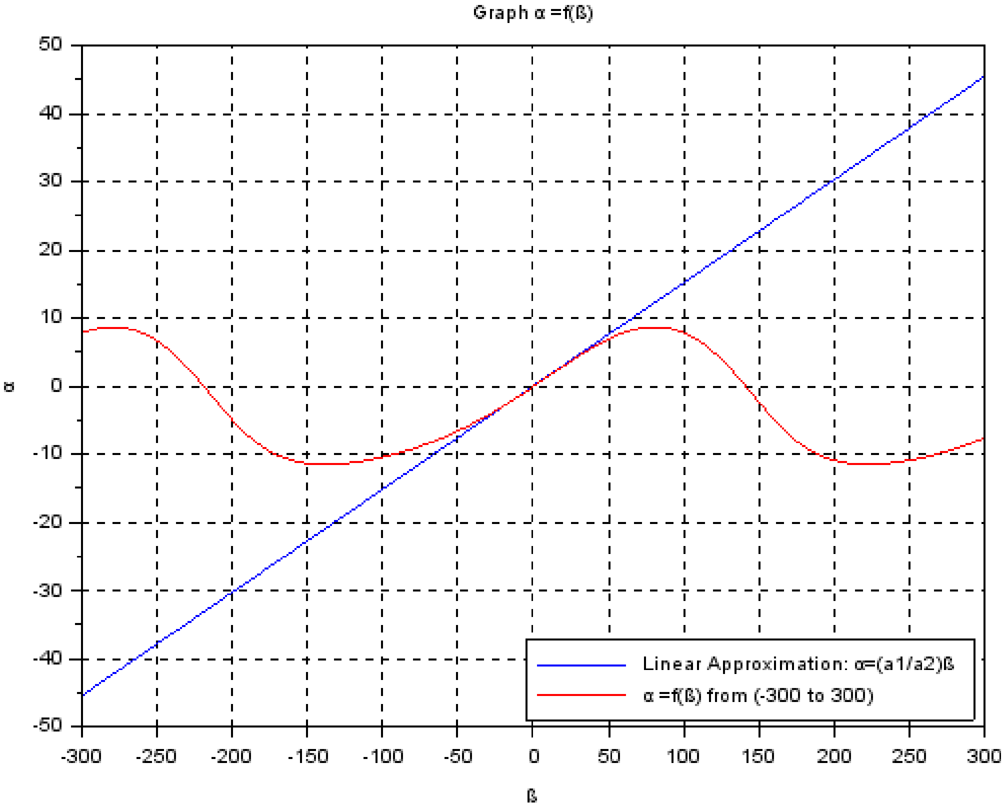

Fig. 4: α vs β for β ranging from -300° to +300°

SCILAB Code:

```
//Defining the INPUT values
d=105;
h=19;
a2=118.72;
a1=18;
b2=36.22;
hk=17;
lk=117.5;

//Calculation Formulae
b1=sqrt(d^2+h^2);
gam=atand(h/d);
del=atand(hk/lk);

//Defining the Beta values range
bet=linspace(-300,300);

//Linear Approximation
alp=(a1/a2)*bet;
plot(bet,alp)
```

```
//Cases based on angle α1
function alp=f(bet)
   gam1=180-gam-bet;
   c1=sqrt(a1^2+b1^2-2*a1*b1*cosd(gam1));
   alp1=acosd((c1^2+b1^2-a1^2)/(2*c1*b1));
   bet2=acosd((a2^2+c1^2-b2^2)/(2*a2*c1));
   if((bet>=-gam && bet<=(180-gam))||(bet>=-300 && bet<=(-180-gam))) then
      alp=alp1+bet2-gam-del;
   else
      alp=-alp1+bet2-gam-del;
   end
endfunction
plot(bet,f,'r')

legend('Linear Approximation: α=(a1/a2)β','α =f(β) from (-300 to 300)',
'in_lower_right')
xtitle('Graph α =f(β)','β','α')
set(gca(),"grid",[1,1])
```

3. *What are the mechanical problems when the black rod is too long or too short? (with plots and arguments)*
   - The mechanism involved in this Ball on inclined Plane (BOIP) is a four bar mechanism. According to Grashof's theorem for a 4-bar mechanism,
     a. there exists at least one link which can fully revolve with respect to other three links if sum of the lengths of the shortest and the longest links is lesser than the sum of the lengths of the other two, i.e…, $l+s < p+q$
     b. and none of the four links can make a full revolution if sum of the lengths of the shortest and the longest links is greater than the sum of the lengths of the other two, i.e…, $l+s > p+q$

     where l = longest link,
     s = smallest link &
     p & q are intermediary links

Case 1: Black rod is too long

- When the black rod is too long, then the sum of the lengths of the longest and the smallest links is less than that of intermediary links.

  i.e. $a1+a2 < b1+b2$
- In this case shortest rod a1 (motor arm) will act as a crank and the a2 as the rocker which follows the linear motion.
- The behaviour of the angle α with respect to β is as shown in the below graph.

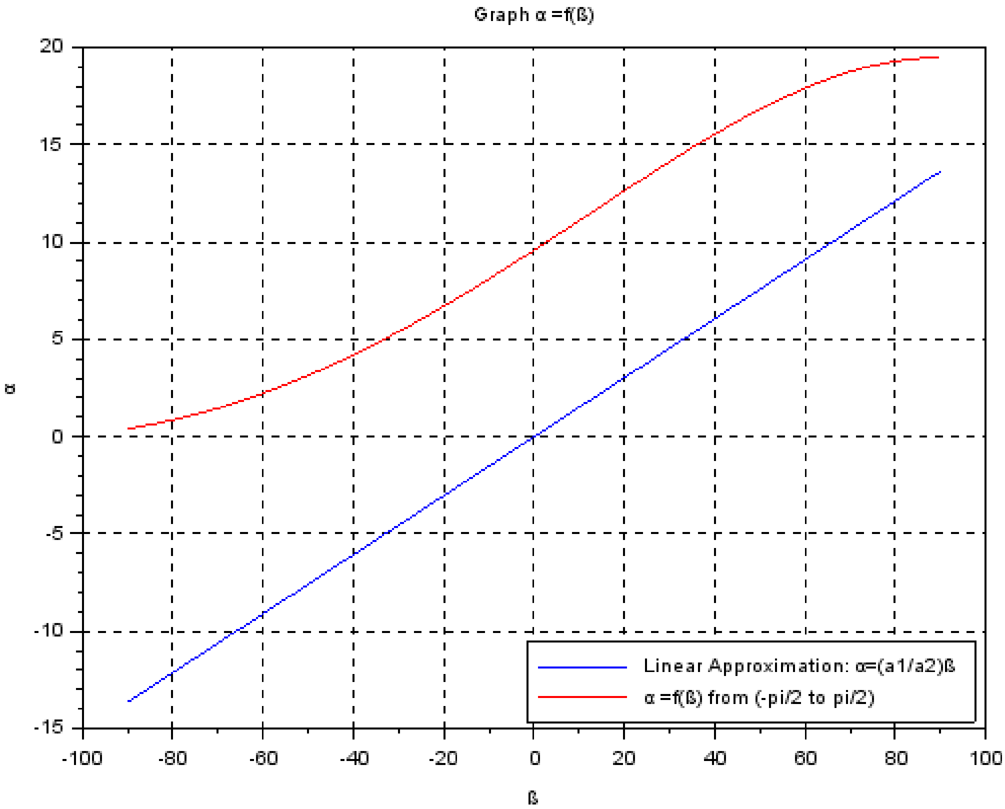

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Fig. 5: α vs β when black rod is too long

Case 2: Black rod is too short

- When the black rod is too short, then the sum of the lengths of the longest and the smallest links is greater than that of intermediary links.

  i.e. $a1+a2 > b1+b2$
- In this case, none of the links will rotate but the a1 (motor arm) will just oscillate.
- Similarly, the behaviour of angle α with respect to β when the length of the black rod is decreased is as shown in the below graph.

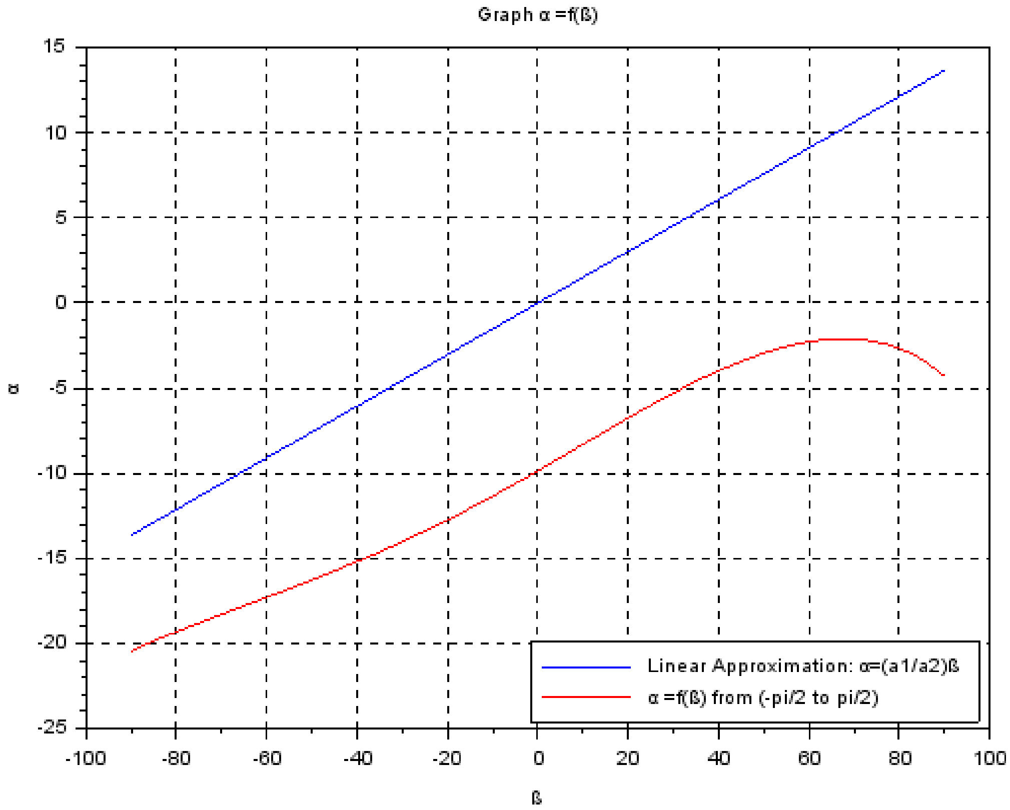

Fig. 6: α vs β when black rod is too short

For both the cases, it can be observed that the angle α cannot be controlled properly by the angle β and so the system becomes unstable in these cases.