Embedded Control Laboratory

Ball on inclined Plane (BoiP)

Figure 1 below shows the real system consisting of a ball, an inclined plane, a servo motor as the actuator to manipulate the angle of the inclined plane, optical sensors to measure the ball's position, and the printed circuit board (PCB) with a micro controller.

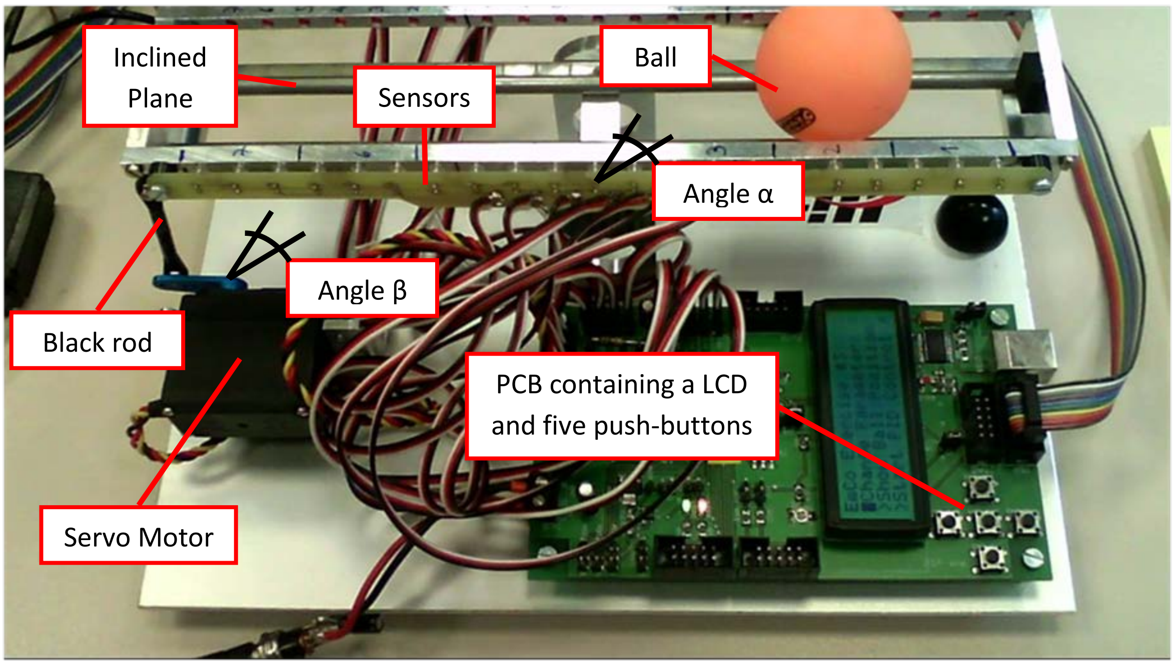

Figure 1: Ball-on-inclined-plane with sensors, an actuator (servo motor) and a PCB

The goals of the three exercises are sequentially the modeling, the simulation, and the programming for controlling the position of the ball on the inclined plane. The exercises are divided into the following tasks:

1. Calculation of the inclined angle of the plane α as a function of the arm angle of the servo motor β
   - Study and understand the given physical relationships which describe the ball acceleration and position on the plane with respect to the plane inclined angle α.
   - Understand the presented simplified mathematical model and its geometrical relationships.
   - Program the formulas and obtain the plot with the help of SCILAB.
2. Modeling and simulation of the BoiP system with closed-loop control in Xcos
   - Construct the mechanical model (BoiP) in XCos with angle β as input and the ball position as output.
   - Understand the concept and the working principle of the PID-controller.
   - Create a closed-loop configuration with a PID-controller, BoiP, and a reference value.
   - Tune PID parameters to achieve good system performance.

3. Calibration and Control of the real BioP system with a PID-controller in embedded C language
    - Self-study: ATmega128, AVR studio, and FreeRTOS
    - Calibrate the system with parameters.
    - Implement a discrete PID-controller in the code.
    - Understand the working principle of the tasks and the queues provided in the code.

Exercise 1 – Mathematical construction of the controlled system

Figure 2 shows the forces acting on the ball:

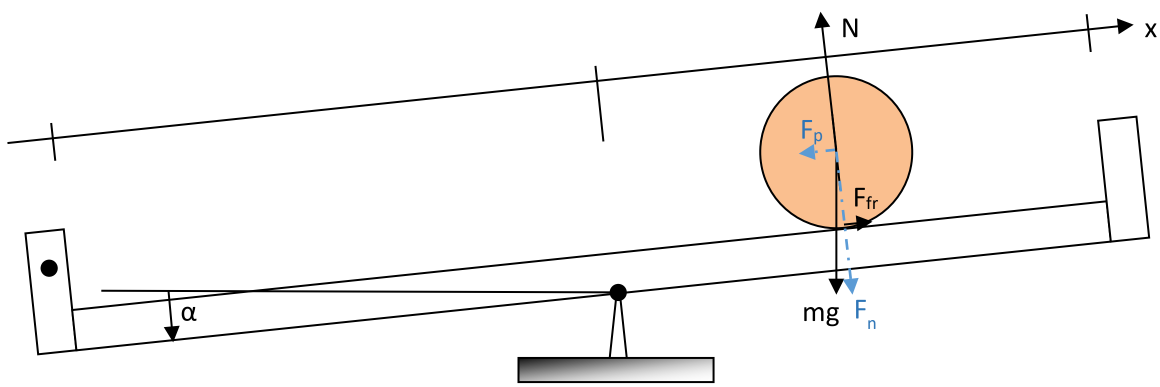

Figure 2: Acting forces on the ball

The gravitational force parallel to the inclined plane is given by

$$F_p = m \cdot g \cdot \sin(\alpha)$$

Friction force produces the rotational movement of the ball with radius R

$$F_{fr} = J_K \cdot \frac{a}{R^2} = \frac{2}{5} \cdot m \cdot a$$

with the ball's moment of inertia

$$J_K = \frac{2}{5} \cdot m \cdot R^2$$

With Newton's 2nd law of motion the sum of all forces equals the mass of the ball multiplied by its acceleration.

$$\sum F = F_p - F_{fr} = m \cdot a$$

$$\sum F = m \cdot g \cdot \sin(\alpha) - \frac{2}{5} \cdot m \cdot a = m \cdot a$$

$$m \cdot g \cdot \sin(\alpha) = \frac{7}{5} \cdot m \cdot a$$

Aerodynamic drag is neglected in this analysis. Moreover, neither slip nor bounces occur during the movement of the ball. After rearrangement of the last equation the acceleration of the ball is now a function of the angle α

$$a = \frac{5}{7} \cdot g \cdot \sin(\alpha)$$

The integration of the acceleration yields the speed, and with another integration we obtain the position of the ball. The result is the function: $x = f(\alpha, t)$. For a constant angle α the formula for the position becomes

$$x = \int v\,dt = \iint a\,dt^2 = \frac{5}{7} \cdot g \cdot \sin(\alpha) \cdot \iint dt^2 = \frac{5}{14} g \sin(\alpha)\, t^2 + C_1 t + C_2$$

We further assume the angle α is due to mechanical limitations bounded by 10 and -10 degrees. The half-length l of the inclined plane is 0.11872 m. The maximum roll time of the ball is given by the next formula with the angle α at maximum value. The speed is initially zero, so the integration constant C_1 will be zero. C_2 denotes the initial position.

$$x - C_2 = 2 \cdot l = \frac{5}{14} \cdot g \cdot \sin(\alpha) \cdot {t_{max}}^2$$

After formula re-organization

$$t_{max} = \sqrt{\frac{28 \cdot l}{5 \cdot g \cdot \sin(\alpha)}} = \sqrt{\frac{28 \cdot 0.11872m}{5 \cdot 9.81 \frac{m}{s^2} \cdot \sin(10^{\circ})}}$$

The speed is the first derivative of the position

$$v(t) = \frac{dx}{dt} = \frac{5}{7} \cdot g \cdot \sin(\alpha) \cdot t$$

Inserting t_{max} into the speed formula we acquire the maximum speed of the ball on the inclined plane

$$v_{max} = \frac{5}{7} \cdot g \cdot \sin(\alpha) \cdot t_{max} = \sqrt{\frac{5 \cdot 5 \cdot g^2 \cdot \sin^2(\alpha)}{7 \cdot 7} \cdot \frac{28 \cdot l}{5 \cdot g \cdot \sin(\alpha)}}$$

$$v_{max} = \sqrt{\frac{20}{7} \cdot g \cdot \sin(\alpha) \cdot l}$$

Figure 3 schematically shows the inclined plane (in blue) and its connection to the servo motor. Line b_2 depicts an imaginary connection between the plane edge and one end of the motor arm. Line a_1 represents the servo motor arm (Compare with Figure 1). Line b_1 depicts an imaginary line between the axis of the servo motor and the pivot point of the plane. Line a_2 corresponds to half a plane. h_k is the length between the bottom edge of the plane and the joint of the plane and the black rod. By means of this construction, the rotational motion of the servo motor controls the inclined angle of the plane on which the ball rolls. In Figure 3, the angle α describes the inclination of the plane and the angle β corresponds to the angle of the servo motor arm. The angle β has a value range of $\pm\frac{\pi}{2}$. Arising from the joint position, a small offset angle δ has to be taken into consideration. With the help of laws of cosine, the angle α can be expressed as a function of the angle β, i.e. $\alpha = f(\beta)$.

As our controlled system has the input β and the output x, it is necessary to derive the mathematical relationships between the two. Combining all the above analyses, we obtain the system relationship from $\alpha = f(\beta)$ and $x = f(\alpha)$. In the first exercise, we concentrate on deriving the former relationship. The latter will be utilized in the second exercise.

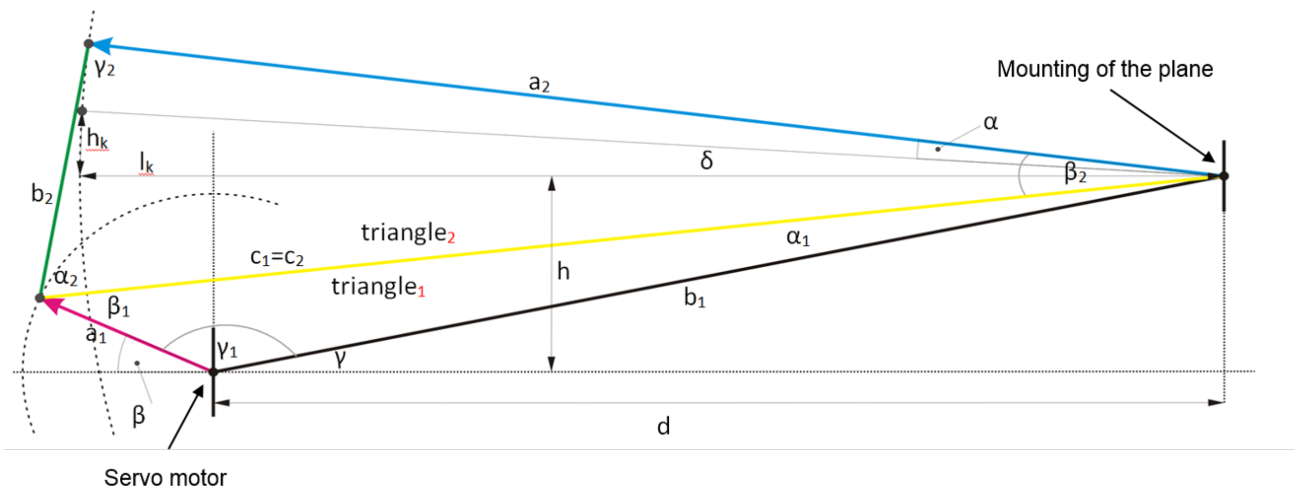

Figure 3: Model of the inclined plane

The following table shows characteristic values of the physical model.

Name	Formula and value	SCILAB value
d	105 mm	105e-3
h	19 mm	19e-3
l = a_2	118.72 mm	118.72e-3
r = a_1	18 mm	18e-3
c = b_2	36.22 mm	36.22e-3
h_k	17 mm	17e-3
l_k	117.5 mm	117.5e-3

Table 1: Characteristic values of the physical model

The following equations are helpful for building the SCILAB script for the model

- Constants:

$$b_1 = \sqrt{d^2 + h^2}$$

$$\gamma = \tan^{-1}\left(\frac{h}{d}\right); \ \delta = \tan^{-1}\left({}^{h_k}\!/_{l_k}\right);$$

where $\tan^{-1}$ denotes the inverse function of tangent, and often also named as arctan (scilab: atan())

- Sum of angles at the pivot point of the plane:

$$\alpha + \delta + \gamma = \alpha_1 + \beta_2$$

- Law of cosines for two triangles:

$$c_1{}^2 = a_1{}^2 + b_1{}^2 - 2a_1 b_1 \cdot \cos\gamma_1$$
$$a_1{}^2 = c_1{}^2 + b_1{}^2 - 2c_1 b_1 \cdot \cos\alpha_1$$
$$b_2{}^2 = a_2{}^2 + c_1{}^2 - 2a_2 c_1 \cdot \cos\beta_2$$

- Sum of angles at the servo motor arm:

$$\beta + \gamma_1 + \gamma = \pi$$

Combining the above equations leads to the desired function

$$\alpha = f(\beta)$$

A first linear approximation for the function is

$$\alpha = \frac{a_1}{a_2}\beta$$

All equations should be implemented with SCILAB and the result should be compared with the first linear approximation.

To-Do list for Exercise 1:

1.) Understand the physical principles and the mathematical analyses

2.) Build a SCILAB script for acquiring the relationship between α and β (with β limited to $\pm\frac{\pi}{2}$) and present the relationship in a SCILAB plot.
   - Two vector constructs for angle β
     - First case: $\beta \geq (-\gamma)$
     - Second case: $\beta < (-\gamma)$
   - Find the right equations for both vectors
   - Combine the two cases in one plot $\alpha = f(\beta)$
   - Obtain the limits for angle β from the plot when the angle α is limited to ±5 degrees
   - Plot the first linear approximation in the same plot

3.) Answer the following questions:
   - Explain with reasons why the approximation is not so good if angle β is greater than 40 degrees or smaller than -40 degrees.
   - In your SCILAB simulation, maximize the range of the angle β to ±300 degrees, plot, and explain your output graph.
   - What are the mechanical problems when the black rod is too long or too short? (with plots and arguments)

Submission files:

- The report should contain the following with detailed explanations:
  - Mathematical analyses of the equations used
  - Implemented code
  - Result plots
  - Answers to the questions

- The zipped file for submission should include the below mentioned files:
  - Report in pdf
  - Working scilab file for β limited to $\pm\frac{\pi}{2}$ degrees
  - Working scilab file for β limited to ± 300 degrees

- The zipped file should be named as follows:
  group<your group ID>_lab<number>.zip/rar
  (e.g. group1_lab1.zip).

  and be sent by E-mail before the deadline to: **ec_lab@eti.uni-siegen.de**

- Please use the file "Lab report cover sheet example" as cover sheet for your report

- The file "Guideline for the lap report" summarizes some important information that may help you prepare a good lab report