Operating Characteristics of Underlay Cognitive Relay Networks

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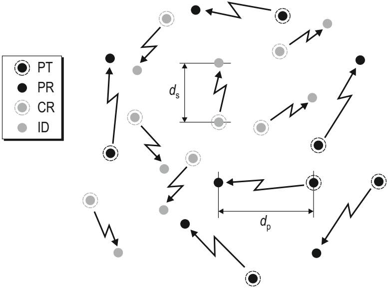
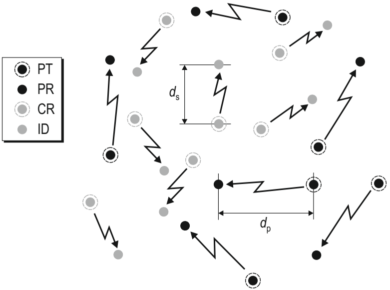
Problem Statement



Motivation:

- Goldsmith *et. al.* described different paradigms for shared access: overlay, underlay and interweave.
- For the underlay system it is important to characterize the interference caused by other transmitters in the system namely, PT and ST.
- At network level, stochastic geometry (SG) offers an analytical tractable model to characterize interference at PRs and SRs and perform analysis for CRN.

Problem Statement



Issues:

- Systems with sensing are prone to imperfections.
- Also, sensing introduces dependency in the model, ignoring this dependency may distort the true performance of the system.
- In most works, the performance of the CRN is restricted to the outage probability at the PRs only.

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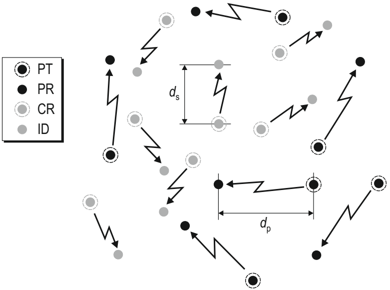
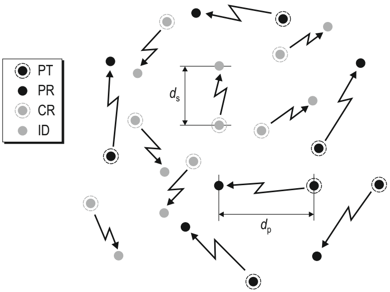


Contributions:

- We do not perform sensing, we are able to obtain exact closed-form expressions.
- The expressions obtained from our model can serve as a lower performance bound (LPB).
- We consider outage probability constraints at the PR and SR jointly and derive operating characteristics (OC) for the CRN.
- We perform the quantitative analysis for the CRN operating in indoor and outdoor scenarios.

System Model

- We assume the same transmit power for all CRs P_s and preclude any form of cooperation or coordination among them.
- We do not involve sensing at CRs, P_s can be regulated to sustain the constraint at the PR.
- Network Layer:
  PTs and STs/CRs are modelled by a stationary 2-D PPP Φ_{PT}, Φ_{CR} with densities λ_p, λ_s.
- Medium Access layer:
  All active PTs and CRs follow a time synchronous slotted medium access.
- Physical layer:
  All transmitted signals undergo distance dependent path loss $||\cdot||^{-\alpha}$, where $\alpha > 2$ and frequency-flat Rayleigh fading.



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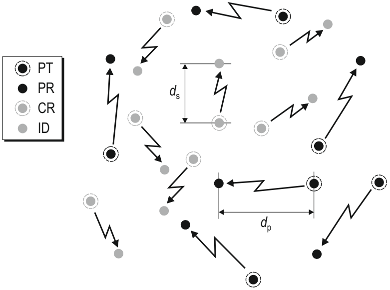

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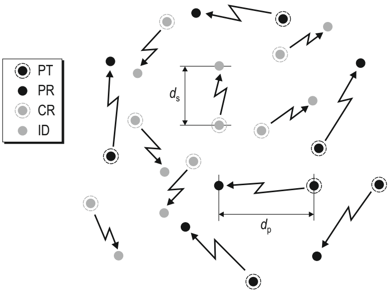

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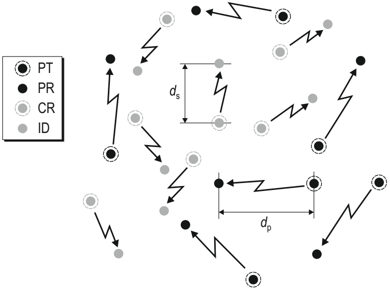

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Interference Analysis at PR

- SIR_{PR} at a hypothetical PR

$$\text{SIR}_{\text{PR}} = \frac{P_{\text{p}} g_{o,\text{p}} d_{\text{p}}^{-\alpha}}{\sum_{i \in \Phi_{\text{PT}}} P_{\text{p}} g_i \|X_i\|^{-\alpha} + \sum_{j \in \Phi_{\text{CR}}} P_{\text{s}} g_j \|Y_j\|^{-\alpha}}$$

- Outage probability at PR

$$\mathbb{P}(\text{SIR}_{\text{PR}} < N_{\text{p}}) = \text{p}_{\text{out,p}} \leq \epsilon_{\text{p}} \qquad (1)$$

N_{p} is SIR_{PR} threshold and ϵ_{p} is outage probability constraint at PR

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- Success probability at a PR in absence of CRs

$$\kappa = \mathbb{P}\left(\frac{P_{\text{p}} g_{o,\text{p}} d_{\text{p}}^{-\alpha}}{\sum_{i \in \Phi_{\text{PT}}} P_{\text{p}} g_i \|X_i\|^{-\alpha}} > N_{\text{p}}\right)$$

$$= \exp\left(-\frac{2\pi^2 \lambda_{\text{p}} c_1^{\frac{2}{\alpha}}}{\alpha \sin\left(\frac{2\pi}{\alpha}\right)}\right), \text{where } c_1 = N_{\text{p}} d_{\text{p}}^{\alpha}$$

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- Relative degradation of success probability at PR

$$\theta = \frac{1-\epsilon_{\text{p}}}{\kappa}$$

- For sustaining (1), the maximum transmit power P_{s} at CRs is

$$P_{\text{s}}^{*} \leq \frac{P_{\text{p}}}{N_{\text{p}}} \left(\frac{\alpha \sin\left(\frac{2\pi}{\alpha}\right)}{2\pi^2 \lambda_{\text{s}} d_{\text{p}}^2} \ln\left(\frac{\kappa}{1-\epsilon_{\text{p}}}\right)\right)^{\frac{\alpha}{2}} .$$

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Interference Analysis at PR

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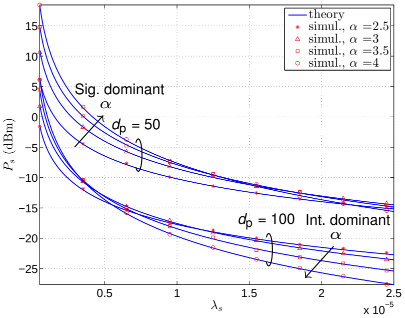

θ	0.95
N_{p}	10
P_{p}	10 dBm
λ_{p}	10^{-6}nodes/m^2

Interference Analysis at ID

- $\mathrm{SIR_{ID}}$ at a hypothetical ID

$$\mathrm{SIR_{ID}} = \frac{P_{\mathrm{s}}^{*} g_{o,\mathrm{s}} d_{\mathrm{s}}^{-\alpha}}{\sum_{i \in \Phi_{\mathrm{PT}}} P_{\mathrm{p}} g_i \|X_i\|^{-\alpha} + \sum_{j \in \Phi_{\mathrm{CR}}} P_{\mathrm{s}} g_j \|Y_j\|^{-\alpha}}.$$

- The outage probability of $\mathrm{SIR_{ID}}$

$$\mathbb{P}(\mathrm{SIR_{ID}} < N_{\mathrm{s}}) = \mathrm{p_{out,s}} \equiv \mathbb{P}(\mathrm{C_{ID}} < R_{\mathrm{s}})$$

where, $\mathrm{C_{ID}}$ is capacity at ID, N_{s} is $\mathrm{SIR_{ID}}$ threshold and R_{s} is $\mathrm{C_{ID}}$ threshold at ID.

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Interference Analysis at ID

- $\mathrm{SIR}_{\mathrm{ID}}$ at a hypothetical ID

$$\mathrm{SIR}_{\mathrm{ID}} = \frac{P_{\mathrm{s}}^{*} g_{o,\mathrm{s}} d_{\mathrm{s}}^{-\alpha}}{\sum_{i \in \Phi_{\mathrm{PT}}} P_{\mathrm{p}} g_i \|X_i\|^{-\alpha} + \sum_{j \in \Phi_{\mathrm{CR}}} P_{\mathrm{s}} g_j \|Y_j\|^{-\alpha}}.$$

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where, C_{ID} is capacity at ID, N_{s} is $\mathrm{SIR}_{\mathrm{ID}}$ threshold and R_{s} is C_{ID} threshold at ID.

- Substituting P_{s}^{*} from interference analysis at PR

$$p_{\mathrm{out,s}} = 1 - (1 - \epsilon_{\mathrm{p}})^{N_{\mathrm{s}}^{\frac{2}{\alpha}} m},$$

$$\text{where } m = \frac{2\pi^2 \lambda_{\mathrm{s}} d_{\mathrm{s}}^2}{\alpha \sin\left(\frac{2\pi}{\alpha}\right) \ln\left(\frac{\kappa}{1-\epsilon_{\mathrm{p}}}\right)}$$

Interference Analysis at ID

- $\mathrm{SIR}_{\mathrm{ID}}$ at a hypothetical ID

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where, C_{ID} is capacity at ID, N_{s} is $\mathrm{SIR}_{\mathrm{ID}}$ threshold and R_{s} is C_{ID} threshold at ID.

- Operating characteristics for the CRN.
  For given $\psi_{\mathrm{s}}, \epsilon_{\mathrm{p}}$ the following must hold

$$\psi_{\mathrm{s}} \geq \mathbb{P}(\mathrm{C}_{\mathrm{ID}} < R_{\mathrm{s}}) = 1 - (1 - \epsilon_{\mathrm{p}})^{\left(2^{R_{\mathrm{s}}} - 1\right)^{\frac{2}{\alpha}} m},$$

Interference Analysis at ID

- $\mathrm{SIR_{ID}}$ at a hypothetical ID

$$\mathrm{SIR_{ID}} = \frac{P_s^* g_{o,s} d_s^{-\alpha}}{\sum_{i \in \Phi_{PT}} P_p g_i \|X_i\|^{-\alpha} + \sum_{j \in \Phi_{CR}} P_s g_j \|Y_j\|^{-\alpha}}.$$

- The outage probability of $\mathrm{SIR_{ID}}$

$$\mathbb{P}(\mathrm{SIR_{ID}} < N_s) = p_{\mathrm{out,s}} \equiv \mathbb{P}(\mathrm{C_{ID}} < R_s)$$

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  For given ψ_s, ϵ_p the following must hold

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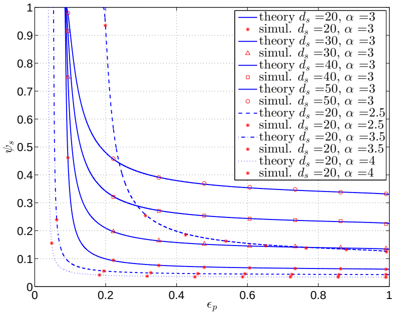

N_p	10
d_p	50 m
P_p	10 dBm
R_s	2 bits/sec/Hz
λ_s	10^{-5}nodes/m^2
λ_p	10^{-6}nodes/m^2

Performance Analysis at ID

- Consider constraint at PR is fulfilled.

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Performance Analysis at ID

- Consider constraint at PR is fulfilled.
- The expected capacity at the ID

$$\mathbb{E}[\mathrm{C}_{\mathrm{ID}}] = \frac{1}{\ln 2} \int_{0}^{\infty} \frac{1}{1+x} e^{-\mu x^{\frac{2}{\alpha}}} \mathrm{d}x,$$

where $\mu = -m \ln(1-\epsilon_{\mathrm{p}})$ and $\mu \geq 0$.
For $\alpha = 4$,

$$\mathbb{E}[\mathrm{C}_{\mathrm{ID}}] = \frac{1}{\ln 2} \left[\sin(\mu) \left(\frac{\pi}{2} - \mathrm{si}(\mu) \right) - \cos(\mu)\, \mathrm{ci}(\mu) \right],$$

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- The variance of capacity at the ID

$$\mathrm{Var}[C_{\mathrm{ID}}] = \frac{2}{(\ln 2)^2} \int\limits_{0}^{\infty} \frac{\ln(1+x)}{1+x} e^{-\mu x^{\frac{2}{\alpha}}} \mathrm{d}x - \mathbb{E}[C_{\mathrm{ID}}]^2.$$

Performance Analysis at ID

- Consider constraint at PR is fulfilled.
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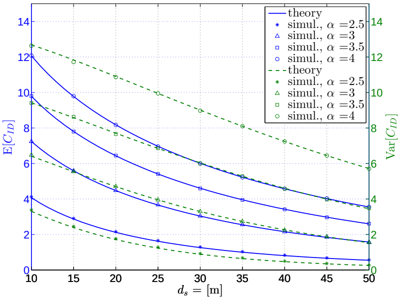

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Conclusion

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- The paper provides an extension to the concept of cognitive relay to cognitive relay network.
- Stochastic geometry is used to model the locations of the primary and secondary systems.
- We establish a lower performance bound to benchmark the performance of systems that include model inaccuracies and sensing.
- Furthermore, we obtain OC to jointly analyze the performance of primary and secondary systems.
- Based on the expressions obtained and the system parameters defined for an indoor scenario, it is indicated that the CRN operating indoor are propitious for the system.

Thank you for attention!

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