"Moment Generating function -- Tranform a MGF of
a random variable which is non-central chi 2 distributed
to a find a MFG of a function of that random variable"

```
Integrate[ Exp[t x] Log[1 + x] 1 / 2 Exp[- (x + l) / 2 ]
  (x / l) ^ (K / 4 - l / 2) BesselI[K / 2 - 1, Sqrt[l x]], {x, 0, Infinity }]
```

$$\int_0^\infty \frac{1}{2} e^{\frac{1}{2}(-l-x)+t x} \left(\frac{x}{l}\right)^{\frac{K}{4}-\frac{l}{2}} \text{BesselI}\left[-1+\frac{K}{2}, \sqrt{l x}\right] \text{Log}[1+x]\, dx$$

"Case1: Integral to evalaute the density of the function
snr_s/(1 + snr_p) using Gamma approximation, which would help
us determine a closed form expression density for the capacity
C_1 = Log2(1 + snr_s/(1 + snr_p)): where the channel is
estimated by employing matched filtering for both the channels"

```
Clear[as, bs, ap2, bp2]
Simplify[Integrate[ 1 / bs^as 1 / Gamma[as] × 1 / bp2^ap2 1 / Gamma[ap2]
   y (y z) ^ (as - 1) Exp[- z y / bs] (y - 1) ^ (ap2 - 1) Exp[- (y - 1) / bp2],
  {y, 1, Infinity}, Assumptions → as > 0, Assumptions → bs > 0,
  Assumptions → ap2 > 0, Assumptions → bp2 > 0]]
```

$$\text{ConditionalExpression}\Big[\Big(\text{bp2}^{-\text{ap2}}\, \text{bs}^{-\text{as}}\, e^{\frac{1}{\text{bp2}}}\, z^{-1+\text{as}} \Big(\Big(\frac{1}{\text{bp2}}+\frac{z}{\text{bs}}\Big)^{-\text{ap2}-\text{as}} \text{Gamma}[\text{ap2}+\text{as}]\, \text{Hypergeometric1F1}\Big[1-\text{ap2}, 1-\text{ap2}-\text{as}, -\frac{\text{bs}+\text{bp2}\, z}{\text{bp2}\, \text{bs}}\Big] + 1/\text{Gamma}[-\text{as}]\, \text{Gamma}[\text{ap2}]\, \text{Gamma}[-\text{ap2}-\text{as}]\, \text{Hypergeometric1F1}\Big[1+\text{as}, 1+\text{ap2}+\text{as}, -\frac{\text{bs}+\text{bp2}\, z}{\text{bp2}\, \text{bs}}\Big]\Big)\Big) \Big/ (\text{Gamma}[\text{ap2}]\, \text{Gamma}[\text{as}]), \text{Re}\Big[\frac{1}{\text{bp2}}+\frac{z}{\text{bs}}\Big] > 0 \,\&\&\, \text{Re}[\text{ap2}] > 0\Big]$$

"Density Function for the capacity"

```
f [z_] :=
```

$$\Big(2\text{^}z\, \text{Log}[2]\, \text{bp2}^{-\text{ap2}}\, \text{bs}^{-\text{as}}\, e^{\frac{1}{\text{bp2}}}\, (2\text{^}z-1)^{-1+\text{as}} \Big(\Big(\frac{1}{\text{bp2}}+\frac{(2\text{^}z-1)}{\text{bs}}\Big)^{-\text{ap2}-\text{as}} \text{Gamma}[\text{ap2}+\text{as}]\, \text{Hypergeometric1F1}\Big[1-\text{ap2}, 1-\text{ap2}-\text{as}, -\frac{\text{bs}+\text{bp2}\,(2\text{^}z-1)}{\text{bp2}\, \text{bs}}\Big] + 1/\text{Gamma}[-\text{as}]\, \text{Gamma}[\text{ap2}]\, \text{Gamma}[-\text{ap2}-\text{as}]\, \text{Hypergeometric1F1}\Big[1+\text{as}, 1+\text{ap2}+\text{as}, -\frac{\text{bs}+\text{bp2}\,(2\text{^}z-1)}{\text{bp2}\, \text{bs}}\Big]\Big)\Big) \Big/ (\text{Gamma}[\text{ap2}]\, \text{Gamma}[\text{as}])$$

```
as = 4.8229
bs = .1402
ap2 = .8469
bp2 = 1.1808
Clear[as, bs, ap2, bp2]
```

Integrate[

$$\left(x\, 2^{\wedge}x\, \mathrm{Log}[2]\ bp2^{-ap2}\, bs^{-as}\, e^{\frac{1}{bp2}}\, (2^{\wedge}x - 1)^{-1+as} \left(\left(\frac{1}{bp2} + \frac{(2^{\wedge}x - 1)}{bs}\right)^{-ap2-as} \mathrm{Gamma}[ap2 + as]\right.\right.$$

$$\mathrm{Hypergeometric1F1}\left[1 - ap2,\ 1 - ap2 - as,\ -\frac{bs + bp2\,(2^{\wedge}x - 1)}{bp2\, bs}\right] +$$

$$1 / \mathrm{Gamma}[-as]\ \mathrm{Gamma}[ap2]\ \mathrm{Gamma}[-ap2 - as]$$

$$\left.\left.\mathrm{Hypergeometric1F1}\left[1 + as,\ 1 + ap2 + as,\ -\frac{bs + bp2\,(2^{\wedge}x - 1)}{bp2\, bs}\right]\right)\right) \Big/$$

(Gamma[ap2] Gamma[as]), {x, 0, Infinity}]

```
4.8229
0.1402
0.8469
1.1808
```

$$\int_0^{\infty} 889.811 \times 2^x\, (-1 + 2^x)^{3.8229}\, x$$

$$\Big((69.0078\ \mathrm{Hypergeometric1F1}[0.1531,\ -4.6698,\ -6.04054\ (0.1402 + 1.1808\ (-1 + 2^x))]) \Big/ (0.846883 + 7.13267\ (-1 + 2^x))^{5.6698} - 0.155554$$

$$\mathrm{Hypergeometric1F1}[5.8229,\ 6.6698,\ -6.04054\ (0.1402 + 1.1808\ (-1 + 2^x))]\Big)\, dx$$

"Case2: Integral to evalaute the density of the function snr_s/(1 + snr_p) using Gamma approximation, which would help us determine a closed form expression density for the capacity C_1 = Log2(1 + snr_s/(1 + snr_p)): where the channel is estimated by employing matched filtering for one channel and energy detection for the other channel"

```
Simplify[Integrate[1 / bs^as 1 / Gamma[as] × 1 / bp2^ap2
   1 / Gamma[ap2] y (y z) ^ (as - 1) Exp[- z y / bs] y^(ap2 - 1) Exp[- y / bp2],
  {y, 0, Infinity}, Assumptions → as > 0, Assumptions → bs > 0,
  Assumptions → ap2 > 0, Assumptions → bp2 > 0]]
```

$$\mathrm{ConditionalExpression}\left[\frac{bp2^{-ap2}\, bs^{-as}\, z^{-1+as}\left(\frac{1}{bp2} + \frac{z}{bs}\right)^{-ap2-as} \mathrm{Gamma}[ap2 + as]}{\mathrm{Gamma}[ap2]\ \mathrm{Gamma}[as]},\ \mathrm{Re}\left[\frac{1}{bp2} + \frac{z}{bs}\right] > 0\ \&\&\ as + \mathrm{Re}[ap2] > 0\right]$$

" Expected Value of the capacity C1 = C_1 = Log2(1 + snr_s/(1 + snr_p))"

Simplify[Integrate[

$$\frac{bp2^{-ap2}\, bs^{-as}\, z^{-1+as} \left(\frac{1}{bp2}+\frac{z}{bs}\right)^{-ap2-as} \text{Gamma}[ap2+as]}{\text{Gamma}[ap2]\,\text{Gamma}[as]} \text{Log2}[1+z],\ \{z, 0, \text{Infinity}\}]]$$

$$\text{ConditionalExpression}\Big[\Big(\left(\frac{1}{bp2}\right)^{-ap2-as} bp2^{-1-ap2} \left(\frac{bp2}{bs}\right)^{-ap2-as} bs^{-as} \Big(bp2\,\pi\,\text{Csc}[ap2\,\pi]\,\text{Gamma}[ap2+as]\,\text{Hypergeometric2F1}\left[ap2, ap2+as, 1+ap2, \frac{bs}{bp2}\right] + ap2 \left(\frac{bp2}{bs}\right)^{ap2} bs\,\text{Gamma}[-1+ap2]\,\text{Gamma}[1+as]\,\text{HypergeometricPFQ}\left[\{1, 1, 1+as\}, \{2, 2-ap2\}, \frac{bs}{bp2}\right]\Big)\Big) \Big/ (ap2\,\text{Gamma}[ap2]\,\text{Gamma}[as]\,\text{Log}[2]),\ \left(\text{Re}\left[\frac{bs}{bp2}\right] \geq 0 \,||\, \frac{bs}{bp2} \notin \text{Reals}\right) \&\& \text{Re}[ap2] > 0 \&\& \text{Re}[as] > -1 \&\& \text{Re}\left[\frac{1}{bp2}\right] > 0\Big]$$

"Integral to evalaute the density of the function snr_s using Gamma approximation, which would help us determine a closed form expression density for the capacity C_0 = Log2(1 + snr_s)"

```
Clear[as, bs]
Integrate[
 Log[1 + y] 1 / bs^as 1 / Gamma[as] y^(as - 1) Exp[- y / bs], {y, 0, Infinity}]
```

$$\text{ConditionalExpression}\Big[\frac{1}{\text{Gamma}[as]} \left(-\frac{1}{bs^2}\right)^{-as} bs^{-as} \Big(-\left(\frac{1}{bs}\right)^{as} \pi\,\text{Csc}[as\,\pi]\,\text{Gamma}\left[as, -\frac{1}{bs}\right] + 1/bs \left(-\frac{1}{bs}\right)^{as} \text{Gamma}[-1+as]\,\text{HypergeometricPFQ}\left[\{1, 1\}, \{2, 2-as\}, \frac{1}{bs}\right] + \text{Gamma}[as] \left(\left(\frac{1}{bs}\right)^{as} \pi\,\text{Csc}[as\,\pi] - \left(-\frac{1}{bs}\right)^{as} \text{Log}\left[\frac{1}{bs}\right] + \left(-\frac{1}{bs}\right)^{as} \text{PolyGamma}[0, as]\right)\Big),\ \text{Re}\left[\frac{1}{bs}\right] > 0 \&\& \text{Re}[as] > -1\Big]$$