Quality-of-Service Based Power Allocation in Spectrum-Sharing Channels

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***Abstract*—In this paper, we propose a quality-of-service (QoS) constrained power and rate allocation scheme for spectrum sharing systems. In particular, we assume existence of secondary users, who are allowed to access the spectrum occupied by a primary user subject to satisfying interference-power limitations. Specifically, we assume that the successful operation of the primary user requires a minimum-rate to be supported by its channel for a certain percentage of time, and obtain an average interference-power constraint that is required to be fulfilled by the secondary user. Applying this limitation, we obtain the maximum arrival-rate supported by a Rayleigh block-fading channel subject to satisfying a given statistical delay QoS constraint. In this respect, we derive an optimal adaptation policy that maximizes the effective capacity of the channel, and provide closed-form expressions for the power allocation and the effective capacity. In addition, we obtain closed-form expressions for the expenditure-power that is required at the secondary transmitter to achieve the above-mentioned capacity metric.**

I. Introduction

Recent advances in spectrum-sharing techniques have enabled different wireless communications technologies to coexist and cooperate towards achieving a better gain from the limited spectrum resources. A major trend in this regard started when spectrum utilization measurements showed that most of the allocated spectrum experiences low utilization [1]. In fact, the spectrum regulatory bodies have traditionally granted license for spectrum utilization with compulsory and detailed transmission guidelines, and allocated proper guard bands between neighboring frequency bands to avoid mutual interference. This conservative approach has left non-negligible parts of the spectrum to be wasted and inefficiently utilized. On the other hand, this century's growth of wireless applications and demands has caused the frequency allocation table for wireless services to become oversaturated. These key observations motivated the concept of opportunistic spectrum access, or the so-called cognitive radio (CR), which offers a tremendous potential to improve the utilization of the radio spectrum.

While offering attractive promises, CR faces various challenges, such as proposing protocols that exploit their capabilities and defining the fundamental performance limits of this radio technology, in order to achieve the capability of using the spectrum in an opportunistic manner. In addition, CR is required to determine the spectrum band allocation that meets the QoS requirements for different users with various applications. Indeed, QoS guarantees play a critical role in next-generation wireless networks. In particular, in systems that carry real-time or delay-sensitive applications, we need to ensure that the delay adheres to the service requirements.

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In wireless networks, as the capacity varies as a function of channel gain, supporting delay QoS constraints is very challenging. In fact, it is impossible to satisfy a deterministic delay QoS constraint in Rayleigh fading channels [2]. In this paper, we consider statistical delay QoS constraint which means that the delay is required to be lower than a specific threshold only for a certain percentage of time [3], and focus on maximizing the throughput of spectrum-sharing systems while satisfying the delay QoS constraint. Recently, the concept of effective capacity has been introduced by Wu and Negi [2] as a link-layer channel model for supporting QoS requirements. The effective capacity is the dual of the effective bandwidth [3], and can be interpreted as the maximum constant arrival-rate that can be supported by the channel given that the delay QoS requirement of the system is satisfied.

Considering average input transmit power constraint, an optimum rate and power allocation strategy that maximizes the effective capacity of the channel in fading environment has been proposed in [4]. On the other hand, considering spectrum-sharing channels, where different users share the same spectrum, a constraint on the received-power at a third party user's receiver, e.g., primary's receiver, may be a more relevant constraint than a maximum on the transmit power [5]. In this respect, different capacity metrics, e.g., ergodic and outage, of a system with constraints on the received-power at the primary's receiver in fading environments have been derived in [6]. Here, we investigate the maximum throughput of the secondary user's channel with delay QoS constraint by obtaining the effective capacity of the channel and proposing an optimum rate and power allocation scheme under interference-power constraint. In particular, we assume that the successful operation of the primary user requires a minimum-rate to be supported by its channel for a certain percentage of time, and determine a lower bound on the effective capacity of the secondary user's link in Rayleigh block-fading channel, when

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the transmission power of the secondary user adheres with the above-mentioned interference limitation. We further obtain closed-form expressions for the effective capacity bound and its power allocation.

II. SYSTEM MODEL

We consider a spectrum-sharing system in which a secondary user is allowed to use the spectrum occupied by a primary user, as long as the latter is provided with a minimum-rate, $R_{\min}$, for a certain percentage of time. We further assume that in the secondary user communication system, the upper layer packets are organized into frames, stored in the transmit buffer, with the same time duration, T_f, at the data-link layer. These frames are then divided into bit-streams to be transmitted through the channel.

We consider a discrete-time block-fading channel for the secondary and primary user links. Hence, the received signal at the secondary receiver, $y_{\mathrm{s}}[n]$, depends on the transmitted signal $x_{\mathrm{s}}[n]$ according to

$$y_{\mathrm{s}}[n] = \sqrt{h_{\mathrm{s}}[n]}x_{\mathrm{s}}[n] + z_{\mathrm{s}}[n], \tag{1}$$

whereby n indicates the time index, $h_{\mathrm{s}}[n]$ denotes the channel gain between the transmitter and the receiver of the secondary user, and $z_{\mathrm{s}}[n]$ represents the additive white Gaussian noise (AWGN). In this model, we treat the interference from the transmission of the primary user on the secondary's receiver as additive noise. We consider that the primary user transmits with constant power P_{p}, and define the channel gain between the secondary's transmitter and the primary's receiver by $h_{\mathrm{sp}}[n]$, and the one between the transmitter and the receiver of the primary user with $h_{\mathrm{p}}[n]$. We further assume that $h_{\mathrm{s}}[n]$, $h_{\mathrm{p}}[n]$, and $h_{\mathrm{sp}}[n]$ are independent and identically distributed (i.i.d.) and are independent from the noise and assume that the channel envelopes are distributed according to Rayleigh fading distribution with unit variance. Channel gains are stationary and ergodic random processes, and the noise power spectral density and the received signal bandwidth are denoted by N_0 and B, respectively

We also assume that the transmission technique of the secondary user must satisfy a statistical delay QoS constraint. We emphasize on the fact that the probability for the queue length of the transmit buffer exceeding a certain threshold, x, decays exponentially as a function of x, [3], [4], and define θ as a delay QoS exponent such that

$$\theta = -\lim_{x\to\infty} \frac{\ln\left(\Pr\left\{q(\infty) > x\right\}\right)}{x}, \tag{2}$$

where $q(n)$ indicates the transmit buffer length at time n and $\Pr\{a \le b\}$ is the probability that the inequality $a \le b$ holds true. It is worth noting that $\theta \to 0$ corresponds to a system with no delay constraint while $\theta \to \infty$ implies strict delay constraint. Considering θ as the QoS constraint in our system, we obtain the secondary user's maximum supported arrival-rate given that the QoS constraint is satisfied.

Hereafter, the fading is assumed to remain constant over the duration of each frame. Furthermore, we assume that perfect knowledge of h_{sp}[1] is available to the receiver and transmitter of the secondary user. The information about h_{sp} can be carried out by a band manager that intervenes between the primary and secondary users [7], or can be directly fed back from the primary's receiver to the secondary user as introduced in [8], [9], which propose algorithms that allow the primary and secondary users to be able to collaborate and exchange the channel state information.

III. INTERFERENCE-POWER CONSTRAINT

In this section, we study the constraint on the service-outage level of the primary user considering that the information about h_{sp} is available to the secondary user. We recall that the transmission power of the secondary user is limited such that the primary receiver is provided with a minimum-rate for a certain percentage of time [10]. Hence, we have

$$\Pr\left\{ \ln\left(1 + \frac{P_{\mathrm{p}} h_{\mathrm{p}}}{P(\theta, h_{\mathrm{s}}, h_{\mathrm{sp}})h_{\mathrm{sp}} + N_0 B}\right) \le R_{\min} \right\} \le P_{\mathrm{p}}^{\mathrm{out}}, \tag{3}$$

where $P(\theta, h_{\mathrm{s}}, h_{\mathrm{sp}})$ denotes the transmit power of the secondary user as a function of θ, h_{s} and h_{sp}, $P_{\mathrm{p}}^{\mathrm{out}}$ indicates the percentage of time during which the rate of the primary user is lower than $R_{\min}$, and P_{p} is its corresponding input transmit power. We now simplify (3) as follows:

$$\begin{aligned} P_{\mathrm{p}}^{\mathrm{out}} &\ge \Pr\left\{h_{\mathrm{p}} \le k\left(P(\theta, h_{\mathrm{s}}, h_{\mathrm{sp}})h_{\mathrm{sp}} + N_0 B\right)\right\} \\ &= \int_0^\infty \int_0^\infty \int_0^{k(\theta, P(\theta, h_{\mathrm{s}}, h_{\mathrm{sp}})h_{\mathrm{sp}} + N_0 B)} f_{h_{\mathrm{p}}}(h_{\mathrm{p}}) \\ &\qquad \times f_{h_{\mathrm{sp}}}(h_{\mathrm{sp}}) f_{h_{\mathrm{s}}}(h_{\mathrm{s}}) dh_{\mathrm{p}} dh_{\mathrm{sp}} dh_{\mathrm{s}} \\ &= \mathcal{E}_{h_{\mathrm{s}}, h_{\mathrm{sp}}} \left\{1 - e^{-k(P(\theta, h_{\mathrm{s}}, h_{\mathrm{sp}})h_{\mathrm{sp}} + N_0 B)}\right\}, \end{aligned} \tag{4}$$

where $k = \dfrac{e^{R_{\min}} - 1}{P_{\mathrm{p}}}$, $f_x(x)$ indicates the probability density function (PDF) of the random variable x, and $\mathcal{E}_{h_{\mathrm{s}}, h_{\mathrm{sp}}}$ defines the expectation over the joint PDF of h_{s} and h_{sp}. Satisfying the constraint (4) guarantees that the achievable-rate of the primary user is bigger than $R_{\min}$ for at least $(1 - P_{\mathrm{p}}^{\mathrm{out}})$ percentage of time.

We now further simplify (4) as follows. Define $z = P(\theta, h_{\mathrm{s}}, h_{\mathrm{sp}})h_{\mathrm{sp}}$ and $f(z) = 1 - e^{-k(z + N_0 B)}$, which can be shown to be a strictly concave function of z. Then, by applying the Jensen's inequality on the right-hand-side of (4), we get

$$1 - e^{-k\left(\mathcal{E}_{h_{\mathrm{s}}, h_{\mathrm{sp}}}\{P(\theta, h_{\mathrm{s}}, h_{\mathrm{sp}})h_{\mathrm{sp}}\} + N_0 B\right)} \le P_{\mathrm{p}}^{\mathrm{out}}, \tag{5}$$

which can be simplified to

$$\mathcal{E}_{h_{\mathrm{s}}, h_{\mathrm{sp}}} \left\{P(\theta, h_{\mathrm{s}}, h_{\mathrm{sp}})h_{\mathrm{sp}}\right\} \le Q, \tag{6}$$

with $Q = \dfrac{-\ln\left(1 - P_{\mathrm{p}}^{\mathrm{out}}\right)}{k} - N_0 B$, which we refer to as *interference-limit*. It is worth noting that when $Q \le 0$, no feasible power allocation satisfying (6) exists and, as such, the capacity lower bound is zero. In the following, we assume

[1] Hereafter, we omit the time index n wherever it is clear from the context.

$Q > 0$ which yields

$$P_{\mathrm{p}}^{\mathrm{out}} > 1 - e^{-kN_0B}. \tag{7}$$

In the next section, we derive the effective capacity of the secondary user's fading channel taking into account the above-presented constraint on the received-power at the primary user.

IV. Effective Capacity

As stated earlier, effective capacity was initially defined in [2] as the dual concept of effective bandwidth. In this section, we first introduce this concept and then find the maximum effective capacity of a Rayleigh fading channel under spectrum-sharing constraint (6).

We start by defining $\{R[i], i = 1, 2, \dots\}$ as the stochastic service process which is assumed to be stationary and ergodic. Assuming that the function

$$\Lambda(-\theta) = \lim_{n\to\infty} \frac{1}{n} \ln\left(\mathcal{E}\left\{e^{-\theta\sum_{i=1}^{n} R[i]}\right\}\right), \tag{8}$$

exists, the effective capacity is defined as [2]

$$\begin{aligned} E_c(\theta) &= \frac{-\Lambda(-\theta)}{\theta} \\ &= -\lim_{n\to\infty} \frac{1}{n\theta} \ln\left(\mathcal{E}\left\{e^{-\theta\sum_{i=1}^{n} R[i]}\right\}\right). \end{aligned} \tag{9}$$

Note that the effective capacity (9) shows the maximum arrival-rate that can be supported by the channel under the constraint of QoS exponent θ, which can be interpreted as the delay constraint. Moreover, in block-fading channels, where the sequence $R[i]$, $i = 1, 2, \dots$, is uncorrelated, the effective capacity can be simplified to

$$E_c(\theta) = -\frac{1}{\theta} \ln\left(\mathcal{E}\left\{e^{-\theta R[i]}\right\}\right). \tag{10}$$

We now obtain the optimum power and rate allocation that maximizes the effective capacity of the channel with the QoS constraint exponent θ, given in (2). We proceed by deriving the instantaneous channel capacity $R[i]$ as

$$R[i] = T_f B \ln\left(1 + \frac{P\left(\theta, h_{\mathrm{s}}[i], h_{\mathrm{sp}}[i]\right) h_{\mathrm{s}}[i]}{N_0 B}\right).$$

Therefore, the effective capacity can be formulated according to

$$\begin{aligned} E_c^{\mathrm{opt}}(\theta) = \max\Bigg\{ &-\frac{1}{\theta} \\ &\times \ln\left(\mathcal{E}_{h_{\mathrm{s}},h_{\mathrm{sp}}}\left\{e^{-\theta T_f B \ln\left(1+\frac{P(\theta,h_{\mathrm{s}},h_{\mathrm{sp}})h_{\mathrm{s}}}{N_0B}\right)}\right\}\right)\Bigg\} \\ \text{s.t.}\quad & \mathcal{E}_{h_{\mathrm{s}},h_{\mathrm{sp}}}\left\{P\left(\theta, h_{\mathrm{s}}, h_{\mathrm{sp}}\right) h_{\mathrm{sp}}\right\} \le Q, \\ & P(\theta,h_{\mathrm{s}},h_{\mathrm{sp}}) \ge 0, \quad \forall\ h_{\mathrm{s}},h_{\mathrm{sp}}, \end{aligned} \tag{11}$$

with $E_c^{\mathrm{opt}}(\theta)$ indicating the maximum on the effective capacity. Now, by using the fact that the function $\ln(x)$ is a monotonically increasing function of x, one can show that the solution for the maximization problem in (11) is the same as the one for the following minimization problem:

$$\begin{aligned} &\min\left\{\mathcal{E}_{h_{\mathrm{s}},h_{\mathrm{sp}}}\left\{\left(1 + \frac{P\left(\theta, h_{\mathrm{s}}, h_{\mathrm{sp}}\right) h_{\mathrm{s}}}{N_0 B}\right)^{-\theta T_f B}\right\}\right\} \\ &\text{s.t.} \int_0^\infty \int_0^\infty P\left(\theta, h_{\mathrm{s}}, h_{\mathrm{sp}}\right) h_{\mathrm{sp}} f_{h_{\mathrm{s}}}(h_{\mathrm{s}}) f_{h_{\mathrm{sp}}}(h_{\mathrm{sp}}) dh_{\mathrm{s}} dh_{\mathrm{sp}} \le Q, \\ &\quad P(\theta,h_{\mathrm{s}},h_{\mathrm{sp}}) \ge 0, \quad \forall\ h_{\mathrm{s}},h_{\mathrm{sp}}. \end{aligned} \tag{12}$$

The solution for the minimization problem in (12) can be obtained by using Lagrangian optimization approach. Define

$$\begin{aligned} &L\left(P(\theta,h_{\mathrm{s}},h_{\mathrm{sp}}), \lambda_0\right) \\ &\quad = \mathcal{E}_{h_{\mathrm{s}},h_{\mathrm{sp}}}\left\{\left(1 + \frac{P\left(\theta, h_{\mathrm{s}}, h_{\mathrm{sp}}\right) h_{\mathrm{s}}}{N_0 B}\right)^{-\theta T_f B}\right\} \\ &\qquad + \lambda_0\left(\mathcal{E}_{h_{\mathrm{s}},h_{\mathrm{sp}}}\left\{P(\theta,h_{\mathrm{s}},h_{\mathrm{sp}}) h_{\mathrm{sp}}\right\} - Q\right). \end{aligned}$$

Using the fact that the solution for the optimum power allocation should satisfy $\frac{\partial L\left(P(\theta,h_{\mathrm{s}},h_{\mathrm{sp}}), \lambda_0\right)}{\partial P(\theta,h_{\mathrm{s}},h_{\mathrm{sp}})} = 0$, we get

$$P(\theta,h_{\mathrm{s}},h_{\mathrm{sp}}) = N_0 B \left[\frac{\beta^{\frac{1}{1+\alpha}}}{h_{\mathrm{sp}}^{\frac{1}{1+\alpha}} h_{\mathrm{s}}^{\frac{\alpha}{1+\alpha}}} - \frac{1}{h_{\mathrm{s}}}\right]^+, \tag{13}$$

where $\alpha = \theta T_f B$, $\beta = \frac{\gamma_0 \alpha}{N_0 B}$, $[x]^+$ denotes $\max\{0, x\}$, and $\gamma_0 = \frac{1}{\lambda_0}$ must be determined such that the interference-power constraint (6) is satisfied with equality. Therefore, the power allocation policy can be expanded as

$$\begin{aligned} &P(\theta,h_{\mathrm{s}},h_{\mathrm{sp}}) \\ &\quad = \begin{cases} N_0 B \left(\dfrac{\beta^{\frac{1}{1+\alpha}}}{h_{\mathrm{sp}}^{\frac{1}{1+\alpha}} h_{\mathrm{s}}^{\frac{\alpha}{1+\alpha}}} - \dfrac{1}{h_{\mathrm{s}}}\right) & \text{if} \quad h_{\mathrm{sp}} \le \beta h_{\mathrm{s}}, \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \tag{14}$$

Now, define the random variable v as $v = \frac{h_{\mathrm{sp}}}{h_{\mathrm{s}}}$. By using the fact that the distribution of the ratio between two Gamma distributed random variables with parameters α_1 and α_2 is a beta prime distribution with parameters α_1 and α_2 [11], we can find the distribution of the random variable v as $f_v(v) = \frac{1}{(v+1)^2}$. The optimal cutoff threshold γ_0 can now be found by setting the inequality (6) to equality and evaluating the integration as follows:

$$\begin{aligned} \frac{Q}{N_0 B} &= \int_0^\beta \left(\beta^{\frac{1}{1+\alpha}} v^{\frac{\alpha}{1+\alpha}} - v\right) \frac{1}{(v+1)^2}\, dv \\ &= \underbrace{\frac{\beta^2}{(1+\beta)^2} \int_0^1 (1-x)^{\frac{\alpha}{1+\alpha}} \left[1 - \frac{\beta}{1+\beta} x\right]^{-2} dx}_{\mathcal{I}} \\ &\quad - \left[\ln(1+v) + \frac{1}{1+v}\right]_0^\beta, \end{aligned} \tag{15}$$

where $x = 1 - \frac{v}{\beta}$. To obtain a closed-form expression for the first part of (15), $\mathcal{I}$, we start by recalling the following identity [12]

$$
{}_2F_1(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \times \int_0^1 t^{b-1}(1-t)^{-b+c-1}(1-tz)^{-a}dt,
$$
$$
\text{for } \mathrm{Re}(c) > \mathrm{Re}(b) > 0 \text{ and } |z| < 1, \tag{16}
$$

where $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$ indicates the Gamma function and ${}_2F_1(a,b;c;z)$ denotes the Gauss's hypergeometric function. Then, by setting $b = 1$, $c = \frac{\alpha}{1+\alpha} + 2$, $a = 2$, and $z = \frac{\beta}{1+\beta}$, into (16), $\mathcal{I}$ can be expanded as

$$
\mathcal{I} = \frac{\beta^2\Gamma\left(\frac{\alpha}{1+\alpha}+2\right)}{(1+\beta)^2\Gamma\left(\frac{\alpha}{1+\alpha}+1\right)} \; {}_2F_1\left(2, 1; \frac{\alpha}{1+\alpha}+2; \frac{\beta}{1+\beta}\right). \tag{17}
$$

Now, by inserting (17) into (15) and using the equality $\Gamma(1+z) = z\Gamma(z)$, we can get a closed-form expression for (15) according to

$$
\frac{Q}{N_0B} = \frac{\beta^2(1+\alpha)}{(1+\beta)^2(1+2\alpha)} \; {}_2F_1\left(2, 1; \frac{\alpha}{1+\alpha}+2; \frac{\beta}{1+\beta}\right) - \ln(1+\beta) - \frac{1}{1+\beta} + 1, \tag{18}
$$

from which a solution for the optimal cutoff threshold γ_0 can be obtained. We now evaluate the integration in the effective capacity expression (11) as follows:

$$
\begin{aligned}
E_c^{\text{opt}}(\theta) &= -\frac{1}{\theta}\ln\left(\mathcal{E}_v\left\{\left(1+\left[\beta^{\frac{1}{1+\alpha}}v^{\frac{-1}{1+\alpha}}-1\right]^+\right)^{-\alpha}\right\}\right) \\
&= -\frac{1}{\theta}\ln\left(\beta^{\frac{-\alpha}{1+\alpha}}\int_0^\beta \frac{v^{\frac{\alpha}{1+\alpha}}}{(1+v)^2}\,dv + \int_\beta^\infty \frac{1}{(1+v)^2}\,dv\right) \\
&= -\frac{1}{\theta}\ln\left(\frac{\beta(1+\alpha)}{(1+\beta)^2(1+2\alpha)} \times {}_2F_1\left(2, 1; \frac{\alpha}{1+\alpha}+2; \frac{\beta}{1+\beta}\right) + \frac{1}{1+\beta}\right),
\end{aligned} \tag{19}
$$

thus leading to a closed-form expression for the effective capacity of the secondary user's link. Finally, we find the average expenditure-power required to achieve $E_c^{\text{opt}}(\theta)$ as follows:

$$
\frac{P_{\text{avg}}}{N_0B} = \mathcal{E}_{h_\text{s},h_\text{sp}}\left\{\left[\frac{\beta^{\frac{1}{1+\alpha}}}{h_\text{sp}^{\frac{1}{1+\alpha}}h_\text{s}^{\frac{\alpha}{1+\alpha}}} - \frac{1}{h_\text{s}}\right]^+\right\} \tag{20}
$$
$$
= \frac{\beta(1+\alpha)}{\alpha(1+\beta)} \; {}_2F_1\left(1, 1; \frac{1+2\alpha}{1+\alpha}; \frac{\beta}{1+\beta}\right) - \Big[\mathrm{E1}((1+\beta)h_\text{s}) - \mathrm{E1}(h_\text{s})\Big]_{h_\text{s}=0}, \tag{21}
$$

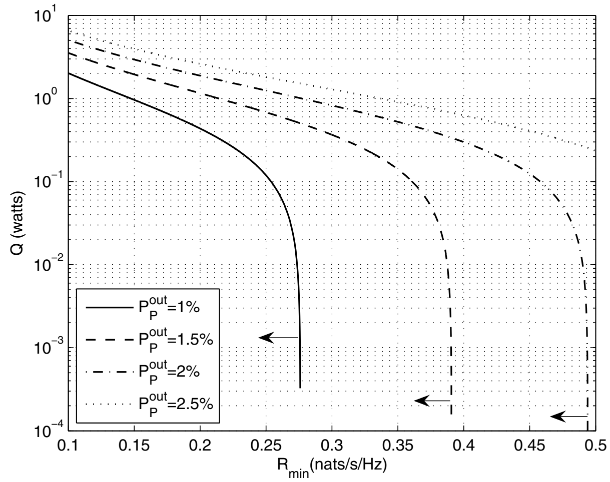

Fig. 1: *Interference-limit* versus $R_{\min}$ for various outage probabilities.

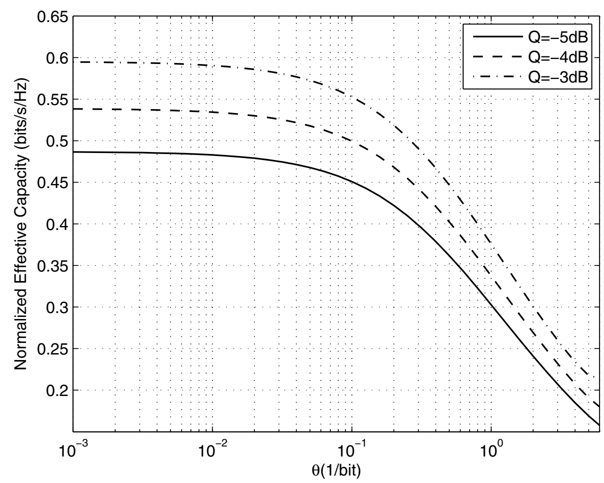

Fig. 2: Normalized effective capacity versus *interference-limit* for various delay QoS exponents, θ.

where $\mathrm{E1}(x)$ denotes the exponential-integral function given by $\mathrm{E1}(x) = \int_x^\infty \frac{e^{-t}}{t}dt$ [12]. Due to space limitations, the proof on how (21) can be obtained is provided in the journal version of this paper. Now, by using the Puiseux series of the exponential integral function given by $\mathrm{E1}(z) = -\gamma - \ln(z) - \sum_{n=1}^{\infty}\frac{(-1)^n z^n}{n.n!}$ [12], where $\gamma = 0.577216$ is the Euler-Mascheroni constant [13], one can show that $[\mathrm{E1}((1+\beta)h_\text{s}) - \mathrm{E1}(h_\text{s})]_{h_\text{s}=0} = -\ln(1+\beta)$.

V. Numerical Results

In this section, we numerically illustrate the *interference-limit*, the average expenditure-power, and the effective capacity of Rayleigh block-fading channels under interference-power constraint (6). Hereafter, we assume $N_0B = 1$ and $T_fB = 1$.

Fig. 1 provides the plots for the *interference-limit* versus the minimum-rate required by the primary user with $P_\text{p} = 15$dB. The arrows indicate the regions at which (7) is satisfied.

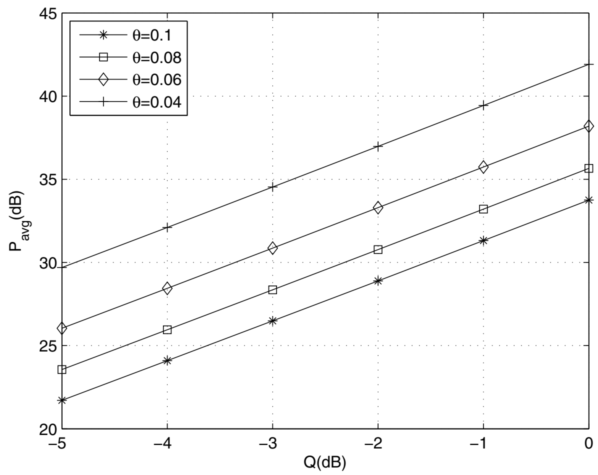

Fig. 3: Average expenditure-power required to achieve effective capacity versus *interference-limit* for various QoS exponents, θ.

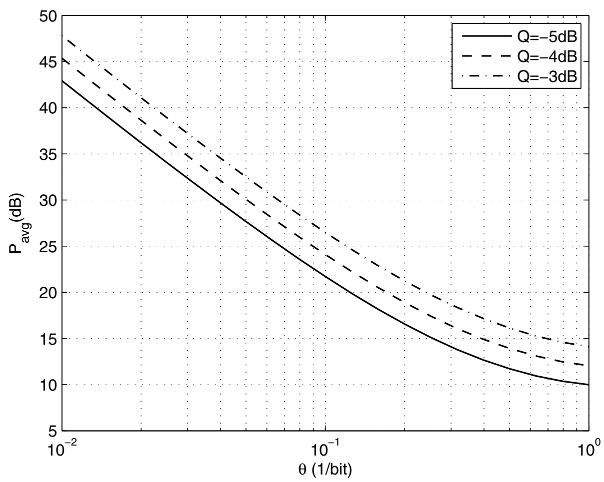

Fig. 4: Average expenditure-power required to achieve effective capacity versus QoS exponent, θ, for various *interference-limit values*.

The figure shows that beyond certain thresholds for $R_{\min}$, the *interference-limit* decreases rapidly as the minimum rate, $R_{\min}$, increases or the outage probability decreases.

In Fig. 2, the normalized effective capacity for a Rayleigh block-fading channel under average interference-power constraint (6) is plotted versus the delay QoS exponent for various *interference-limit* values. We observe that the capacity increases as θ decreases, however, the gain in the effective capacity decreases for lower values of θ.

The plots for the average transmit power required to achieve the maximum on the effective capacity, P_{avg}, under interference-power constraint (6) versus *interference-limit* and delay QoS exponent, θ, are provided in Fig. 3 and Fig. 4, respectively. In particular, Fig. 3 shows that the transmit power required to achieve E_c^{opt} increases as the *interference-limit* increases or θ decreases. Fig. 4, on the other hand, shows that the effect of decreasing the QoS exponent on P_{avg} is more significant for smaller values of θ. Specifically, when θ tends to zero, or equivalently when no delay QoS constraint exists, in which case the effective capacity is equal to the ergodic capacity, Fig. 4 reveals that P_{avg} tends to infinity.

VI. Conclusion

We investigated the QoS-constrained capacity gains that can be achieved in time-varying fading channels under spectrum-sharing constraints. In particular, we assumed that the spectrum band occupied by a primary user, whom should be supported with a minimum-rate for a certain percentage of time, may be accessed and utilized by a secondary user, as long as the latter adheres to the rate requirement of the primary user. In this respect, we obtained an average interference-constraint. We then determined the effective capacity of the channel under consideration given that the delay QoS is satisfied. We also obtained the optimal power allocation strategy to achieve the maximum effective capacity, and further derived closed-form expression for the capacity and its corresponding power allocation policy under Rayleigh fading. Finally, we obtained closed-form expressions for the average expenditure-power required to achieve the effective capacity. Numerical results corroborating our theoretical analysis were also provided. In particular, it was shown that the effective capacity increases as the delay QoS exponent decreases, and at the same time the expenditure-power required to achieve the effective capacity increases significantly.

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