Performance Analysis of Underlay Cognitive Radio Systems with Imperfect Channel Knowledge

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Abstract

In this paper, we study the performance of cognitive Underlay Systems (USs) that employ power control mechanism at the Secondary Transmitter (ST). Existing baseline models considered for the performance analysis either assume the knowledge of involved channels at the ST or retrieve this information by means of a feedback channel, however, such situations rarely exist in practice. Motivated by this fact, we propose a novel approach that incorporates the estimation of the involved channels at the ST, in order to characterize the performance of USs~~,~~ in terms of interference power received at primary receiver (or *primary interference*) and throughput at the secondary receiver (or *secondary throughput*), under realistic scenarios. Moreover, we apply an outage constraint that captures the impact of imperfect channel knowledge, particularly on the primary interference. Besides this, we employ a transmit power constraint at the ST to determine an operating regime for the US. We extend the performance analysis of the proposed approach to investigate the influence of channel fading. In this regard, we characterize the expressions of the primary interference and the secondary throughput, for the case, where the involved

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channels encounter Nakagami-m fading. Finally, we investigate a fundamental tradeoff between the estimation time and the secondary throughput allowing an optimized performance of the US.

Index Terms

Cognitive radio, Underlay system, Channel estimation, Estimation-throughput tradeoff, Operating regime

I. Introduction

Cognitive Radio (CR) communication is considered as one of the viable solutions that addresses the problem of spectrum scarcity of future wireless networks. Secondary access to the licensed spectrum can be broadly categorized into different CR paradigms, namely, interweave, underlay and overlay systems [2]. Among these, underlay and interweave systems are largely associated with techniques that applies to the physical layer, hence, can be considered feasible for hardware deployment. The interweave systems implement spectrum sensing to detect the presence of primary user signals to avoid causing interference to the primary system. In contrast to this, ~~the secondary system as~~ an Underlay System (US) exploits the interference tolerance capability of the primary systems that allows the secondary users to transmit also when the primary users are present. To accomplish this, the USs employ techniques such as power control to maintain the interference (power) received at the Primary Receiver (PR) below a specified level defined as Interference Threshold (IT) [3]. In this paper, we focus on performance characterization of US that employs power control at the Secondary Transmitter (ST).

A. *Motivation and Related Work*

The performance of the USs in context to primary system can be characterized in terms of interference (power) received at the PR. The power control mechanism can be implemented only if the knowledge of the *primary interference* channel between the ST and the PR is available at the ST. The preliminary investigations [3]–[5], considered for the performance analysis, assumed this knowledge to be perfectly known at the ST. Such situations, however, do not exist in practical implementations. To this end, performance analysis based on imperfect channel knowledge has recently received significant attention [6]–[9]. More particularly, [6]–[8]

consider that channel knowledge at the ST can be acquired from a band manager[1] [10] or directly over a feedback link from the PR [11]. From a deployment perspective, to establish such an inter-system communication is rather complicated. In addition, due to latency issues, the channel knowledge procured by implementing such solutions may be outdated [7], [8]. Besides that, for the feedback link, the primary systems are required to demodulate the secondary user signals. These issues render the hardware realizability of the approach proposed in [6]–[8] challenging. Motivated by this, in this paper, we propose an estimation of the involved channels directly at the secondary system.

Besides the primary interference, the secondary system's performance can be characterized in terms of throughput at the SR (*secondary throughput*). Moreover, the knowledge of the secondary throughput at the ST can be utilized to implement a band allocation policy pertaining to a desired quality of service. However, to characterize the secondary throughput, the ST (along with the primary interference channel, which is associated with power control mechanism) requires the knowledge of *access* channel between the ST and the SR, and *secondary interference* channel between the PT and the SR. The performance analysis, depicted in [6]–[8], has been limited to the estimation of the primary interference channel. Apart from this, particularly for the primary and secondary interference channels, the channel estimation involves different (primary and secondary) systems. This is possible, when the ST and the SR have the knowledge of the primary user signal. From the deployment perspective, it is necessary to select the estimation techniques such that complexity and versatility (unknown primary signals) requirements are fulfilled. To this end, similar to [12], we employ received power based estimation at the ST and SR for the interference channels.

From the discussion above, it is clear that the performance of the USs can be depicted in terms of the primary interference (at the PR) and the secondary throughput (at the SR), however, a certain time needs to be allocated by the Secondary User (SU) for the channel estimation. Since the aspect concerning the time allocation has not been taken into account in any of the previous investigations [6]–[8], the performance of the USs in terms of secondary throughput is overestimated. Moreover, the power control is dependent on the knowledge of the primary interference channel, its imperfect knowledge (or the induced variations) affect the primary

[1] An entity that mediates between the primary and the secondary systems.

interference, which in certain cases may exceed the IT. Under such conditions, the conventional constraint imposed in [3]–[5] is strictly violated. As a result, this *excessive interference* originated from imperfect channel knowledge (of the primary interference channel) may seriously degrade the performance of the primary systems.

It is worth noticing that the aforementioned variations can be effectively controlled by increasing the estimation time. However, from the SU's perspective, this increase in estimation time decreases the secondary throughput. This is due to the fact that more time is allocated for channel estimation and less for data transmission, which consequently leads to a *performance degradation*. Clearly, the excessive interference and secondary throughput are dependent on the estimation time, therefore, it is interesting to investigate a fundamental relationship between these two parameters that jointly characterize the performance of the USs. In this paper, we explore this relationship between the estimation time and secondary throughput such that excessive interference at the PR is constrained. It is important to note that previous investigations [6]–[8] have considered the channel estimation, ~~however,~~ the influence of channel estimation, in terms of excessive interference and performance degradation is still lacking.

B. Contributions

In this paper, we provide the following contributions:

1) Analytical Framework: The main contribution of this paper is to derive an analytical framework for underlay systems that employ a power control mechanism and implement estimation of the interacting channels, namely (i) primary interference channel between the ST and the PR, (ii) secondary interference channel between the PT and the SR, and (iii) access channel between the ST and the SR. To analyze the performance of proposed framework, we derive expressions of the performance parameters such as secondary throughput at the SR and primary interference at the PR pertaining to short-term and long analyses.

The imperfect channel knowledge translates to the variations in the performance parameters. We characterize these variations in the performance parameters in terms of their cumulative distribution functions. Particularly, these variations in the primary interference that exceed the IT may seriously disrupt the operation of the primary system. To control this excessive interference such that it remains below a tolerable limit, we propose to employ an outage constraint over the primary interference. In addition to this, we further capture the variations in the secondary throughput in terms of its expected value.

2) Operating Regime: The power control at the ST depends inversely on the received signal to noise power ratio, which depicts the quality of the primary interference channel. In practice, the power control is limited by the maximum transmit power. In this context, we characterize this performance limit of the secondary systems as an operating regime, which is illustrated as a lower bound on the received signal to noise ratio.

3) Estimation-throughput Tradeoff: Besides the imperfect channel knowledge, we propose a time allocation for the channel estimation in the secondary system frame structure. This allocation, however, corresponds to a performance degradation in terms of the secondary throughput. Along with secondary throughput, the channel estimation time is also associated with the excessive interference at the PR. We analyze this relationship between the primary interference and secondary throughput in terms of the estimation-throughput tradeoff. This tradeoff depicts a suitable estimation time that achieves a maximum secondary throughput for the USs.

C. Organization

The subsequent sections of the paper are organized as follows: Section II describes the system model that includes the deployment scenario, medium access and the signal model. It further presents the problem description and the proposed approach. Section III characterizes the distribution functions of the performance parameters and establishes the estimation-throughput tradeoff subject to outage constraint on the primary interference received at the PR. Section IV analyzes the numerical results based on the obtained expressions. Finally, Section V concludes the paper.

II. System Model

A. Underlay scenario and Medium access

Cognitive Small Cell (CSC), a CR application, characterizes a small cell deployment that fulfills the spectral requirements for Mobile Stations (MSs) operating indoor, cf. Fig. 1. For the disposition of the CSC in the network, the following key elements are essential: a CSC-Base Station (CSC-BS), a Macro Cell-Base Station (MC-BS) and MS [12]. Considering the fact that the power control is employed at the CSC-BS, the CSC-BS and the MS represent ST and SR,

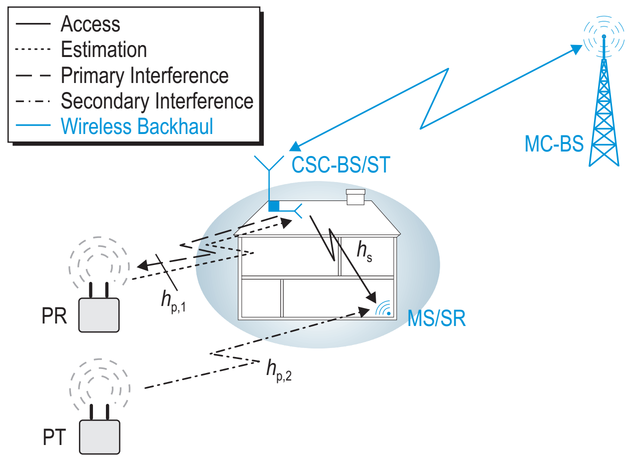

Fig. 1. A cognitive small cell scenario demonstrating: (i) the underlay paradigm, (ii) the associated network elements, which constitute Cognitive Small Cell-Base Station/Secondary Transmitter (CSC-BS/ST), Mobile Station/Secondary Receiver (MS/SR), Macro Cell-Base Station (MC-BS) and Primary Transmitter (PT), (iii) the interacting channels: primary interference channel, secondary interference channel and access channel.

respectively. In order to acquire channel knowledge concerning the primary interference channel, the ST listens to the transmissions from the PR. In this work, we consider those primary systems where the PR perform transmissions interchangeably over time with the PT. These transmisions can occur over the same band (time division duplexing TDD) or over separate bands (half-duplex frequency division duplexing FDD). In cellular networks, these duplexing modes are effectively deployed in the Long Term Evolution standard [13]. The ST follows these deuplexing modes to exploit channel reciprocity principle and determine the primary interference (power) received at the PR, thus, controls its power for transmitting signals over the access channel such that it satisfies the interference constraint by operaing at the IT. Particularly for half-duplex FDD, it is assumed that the coherence bandwidth is large as compared to frequency separation between the estimation channel and the band of interest.

We propose to employ a slotted medium access for the US, where the time axis is segmented into frames. As depicted in Fig. 2, the frame duration T is chosen in such a way that the frames are aligned to the primary users' transmissions. In order to incorporate channel estimation, we further propose to employ a periodic channel estimation[2], according to which, US uses a time

[2]This frame structure is similar to the periodic sensing followed by the interweave systems [14].

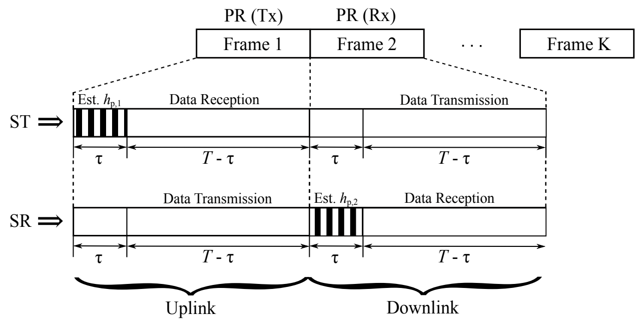

Fig. 2. Frame structure of the USs illustrating the time allocation for channel estimation and data transmission from the perspective of a ST and a SR. Here, we consider power control estimation of the interacting channels and the data transmission from the perspective of the ST. In this regard, corresponding to the uplink and the downlink, the primary interference and secondary interference channel estimation occur at the ST and the SR, respectively. PR (Tx)/PR (Rx) presents the transmission/reception of the primary signal from the PR/PT to the PT/PR.

interval $\tau(< T)$ to perform channel estimation followed by data transmission $T - \tau$, cf. Fig. 2. In order to consider variations due to channel fading, we assume that the interacting channels remain constant over two frame durations ($2T$). Hence characterized by the fading process, every alternating frame observes a different received power. Since the channel knowledge is essential to employ power control such that the primary users are sufficiently protected from the excessive interference induced due to channel estimation, it is reasonable to exercise estimation followed by data transmission with power control in the remaining time $T - \tau$ for each frame.

In accordance with the half duplexing modes, the ST and the SR perform received power estimation to acquire the knowledge of primary and secondary interference channel over consecutive frames, as illustrated in Fig. 2. For primary systems that implement full FDD, the proposed frame structure can be adapted such that the primary and secondary interference channel estimation occurs in a single frame. Besides that, since the access channel estimation is performed by listening to the pilot symbols from the SR, no time resources are allocated for access channel estimation in the frame structure, hence τ is utilized for the estimation of the interference channels only. At first, we consider the proposed frame structure for a short-term analysis, i.e., the performance is analyzed for a certain channel gain, without taking into account the effect of channel fading. Next, by including channel fading, we carry out a long-term analysis of the proposed framework. In accordance with the nature of regulatory policies, the aforementioned analyses can be considered for the system design.

B. Signal model

During the estimation phase, the discrete and complex signal received from the PR at the ST is given by [1]

$$y_{\text{rcvd}}[n] = h_{\text{p},1} \cdot x_{\text{tran}}[n] + w_{\text{s}}[n], \tag{1}$$

where $x_{\text{tran}}[n]$ corresponds to a discrete and complex sample transmitted by the PR with transmit power P_{tran} known at the ST, $|h_{\text{p},1}|^2$ represents the power gain for the interference channel and $w_{\text{s}}[n]$ is circularly symmetric complex Additive White Gaussian Noise (AWGN) at the ST with $\mathcal{CN}(0, \sigma^2)$.

During data transmission phase, the interference signal received at the PR is given by

$$y_{\text{p}}[n] = h_{\text{p},1} \cdot x_{\text{cont}}[n] + w_{\text{p}}[n], \tag{2}$$

and on the other side, the received signal at the SR follows

$$y_{\text{s}}[n] = h_{\text{s}} \cdot x_{\text{cont}}[n] + h_{\text{p},2} \cdot x_{\text{tran}}[n] + w_{\text{s}}[n], \tag{3}$$

where $x_{\text{cont}}[n]$ corresponds to a discrete and complex sample transmitted by the ST with controlled power P_{cont}, and $x_{\text{tran}}[n]$ is the transmit signal from the PT with transmit power P_{tran}[3]. Further, $|h_{\text{s}}|^2$ and $|h_{\text{p},2}|^2$ represent the power gain for the access channel and secondary interference channel, respectively and $w_{\text{p}}[n]$ is AWGN at the PR with $\mathcal{CN}(0, \sigma^2)$.

C. Problem description

According to the existing investigations (also referred as ideal model), a ST as an US is required to control its transmit power in such a way that the primary interference (power) received P_{p} at the PR is below IT θ_{I} [3]

$$P_{\text{p}} = |h_{\text{p},1}|^2 P_{\text{cont}} \leq \theta_{\text{I}}. \tag{4}$$

With controlled power at the ST determined using (4), the capacity at the SR is defined as

$$\text{C}_{\text{s}} = \log_2 \left(1 + \frac{|h_{\text{s}}|^2 P_{\text{cont}}}{|h_{\text{p},2}|^2 P_{\text{tran}} + \sigma^2} \right). \tag{5}$$

[3]To avoid the use of an extra notation, we choose the transmit power to be the same for the PT and the PR, however, in practice, it may have a different value. In such situations also, for the proposed framework, its ignorance at the SR doesn't affect the analysis of the secondary interference.

From the deployment perspective, the ideal model illustrated in (4) and (5) has following issues:

- Without the knowledge of the primary interference channel $h_{\text{p},1}$, it is impossible to employ the power control based on (4).
- Furthermore, along with P_{cont}, the knowledge of the access channel h_{s} and secondary interference channel $h_{\text{p},2}$ is required to determine C_{s} according to (5).

The ideal model considers the perfect knowledge of these channels, namely, $h_{\text{p},1}$, $h_{\text{p},2}$ and h_{s} at the ST, which is not available in practice. In this regard, we incorporate channel estimation in the system model. The imperfect channel knowledge, however, translates to variations in the performance parameters, P_{p} and C_{s}. Particularly, a variation in P_{p} that exceeds the θ_{I} causes excessive interference at the PR, thereby violating the interference constraint illustrated (4). Unless captured, this excessive interference may seriously degrade the performance of the US. Since perfect knowledge of the involved channels is depicted by the ideal model, no performance degradation in times the time resources allocated for the channel estimation is considered.

D. *Proposed approach*

In order to facilitate channel estimation for the USs, it is essential to take the aforementioned issues in account. To accomplish this, the following strategy is proposed in the paper.

- At first, we consider the estimation of involved channels. In this regard, we propose to employ received power estimation for the interference channels and pilot based estimation for the access channel.
- To capture the effect of imperfect channel knowledge, we characterize the variations in the estimated parameters (namely, received power for the interference channels and power gain for the access channels) in terms of their cumulative distribution function.
- We translate the aforementioned variations to the performance parameters, such as primary interference and secondary throughput in terms of their cumulative distribution functions. More specifically, using the characterization of the primary interference, we propose a novel power control mechanism that regulates the excessive interference at the PR.
- Finally, using the derived expressions, we analyze a relationship between the estimation time and expected secondary throughput for the USs. We extend the proposed framework (also referred as estimation model) to analyze the impact of channel fading on the performance of the system.

Since the channel estimation in context to cognitive radio systems involves two different (primary and secondary) systems, the channel estimation techniques should be selected in such a way that (i) low complexity and (ii) versatility towards unknown primary user signals requirements are respected. Similar situation, however, in context to interweave systems has been deeply investigated in [12], according to which, the authors in [12] propose to employ received power estimation for the channels between the primary and secondary users and pilot based estimation for the access channel. In this paper, we propose to employ the channel estimation techniques in context to the underlay systems. It is also worth noticing that, since the signal model in [12] (Orthogonal Frequency Division Multiplexing transmission) differ from the one (constant power transmission) depicted in this paper, we reconsider the mathematical expressions for the derivation of the performance parameters. In the following paragraphs, we consider the estimation of the involved channels.

1) Estimation of primary interference channel: Given

$$P_{\text{rcvd,ST}} = |h_{\text{p},1}|^2 P_{\text{tran}} + \sigma^2 \tag{6}$$

and the knowledge of PR's transmit power P_{tran}, the ST listens to the transmissions from the PR and acquires the knowledge of $|\hat{h}_{\text{p},1}|^2$ indirectly by estimating the received power $\hat{P}_{\text{rcvd,ST}} = \frac{1}{\tau f_{\text{s}}} \sum_{n}^{\tau f_{\text{s}}} |y_{\text{rcvd}}[n]|^2$ in the uplink, cf. Fig. 2. Besides the knowledge of $h_{\text{p},2}$, the knowledge of P_{tran} at the ST is critical for the employing the power control. However, this knowledge can be retrieved by considering the specifications of different wireless standards such GSM, EDGE and LTE, etc. [15]. $\hat{P}_{\text{rcvd,ST}}$ estimated using τf_{s} samples follows a non-central chi-squared distribution $F_{\hat{P}_{\text{rcvd,ST}}} \sim \mathcal{X}_1^2(\lambda_{\text{p},1}, \tau f_{\text{s}})$ with non-centrality parameter $\lambda_{\text{p},1} = \tau f_{\text{s}} |h_{\text{p},1}|^2 P_{\text{tran}}/\sigma^2 = \tau f_{\text{s}} \gamma$ [16], where γ is defined as the ratio of the received power (from the PR) to noise at the ST and τf_{s} corresponds to the degrees of freedom. For analytical tractability, we consider the following approximation.

Approximation 1: For all degrees of freedom, the $\mathcal{X}_1^2$ distribution can be approximated by a Gamma distribution [17]. The parameters of the Gamma distribution are obtained by matching the first two central moments to those of $\mathcal{X}_1^2$.

Lemma 1: The cumulative distribution function of $\hat{P}_{\text{rcvd,ST}}$ is characterized as

$$F_{\hat{P}_{\text{rcvd,ST}}}(x) \approx 1 - \Gamma\left(a_{\text{p},1}, \frac{x}{b_{\text{p},1}}\right), \tag{7}$$

$$\text{where } a_{\text{p},1} = \frac{\tau f_{\text{s}} (1+\gamma)^2}{2+4\gamma} \text{ and } b_{\text{p},1} = \frac{\sigma^2 (2+4\gamma)}{\tau f_{\text{s}} (1+\gamma)}, \tag{8}$$

and $\Gamma(\cdot,\cdot)$ represents the regularized upper-incomplete Gamma function [17].

Proof: Applying Approximation 1 to $\mathcal{X}_1^2(\lambda_{\text{p},1}, \tau f_{\text{s}})$ yields (7). ■

2) Estimation of access channel: In the uplink, the pilot signal received from the SR undergoes matched filtering and demodulation at the ST, hence, we employ a pilot-based estimation at the ST to acquire the knowledge of the access channel. According to [18], the maximum-likelihood estimate with N_{s} pilot symbols is given by

$$h_{\text{s}} = \hat{h}_{\text{s}} + \frac{\sum_n^{N_{\text{s}}} p[n]}{2N_{\text{s}}}, \tag{9}$$

where $p[n]$ denotes the discrete pilot symbol and $\frac{\sum_n^{N_{\text{s}}} p[n]}{2N_{\text{s}}}$ represents the estimation error. As a result, the estimate $\hat{h}_{\text{s}}$ is unbiased, efficient, i.e., achieves the Cramér-Rao bound with equality, with asymptotic variance $\mathbb{E}\left[|h_{\text{s}} - \hat{h}_{\text{s}}|^2\right] = \frac{\sigma^2}{2N_{\text{s}}}$ [18]. Hence, $\hat{h}_{\text{s}}$ conditioned on h_{s} follows a Gaussian distribution

$$\hat{h}_{\text{s}}|h_{\text{s}} \sim \mathcal{N}\left(h_{\text{s}}, \frac{\sigma^2}{2N_{\text{s}}}\right). \tag{10}$$

Consequently, the estimated power gain $|\hat{h}_{\text{s}}|^2$ follows a non-central chi-squared $\mathcal{X}_1^2(\lambda_{\text{s}}, 1)$ distribution with 1 degree of freedom and non-centrality parameter $\lambda_{\text{s}} = \frac{2N_{\text{s}}|h_{\text{s}}|^2}{\sigma^2}$.

Lemma 2: The cumulative distribution function of $|\hat{h}_{\text{s}}|^2$ is characterized as

$$F_{|\hat{h}_{\text{s}}|^2}(x) \approx 1 - \Gamma\left(a_{\text{s}}, \frac{x}{b_{\text{s}}}\right), \tag{11}$$

$$\text{where } a_{\text{s}} = \frac{(1+\lambda_{\text{s}})^2}{2+4\lambda_{\text{s}}} \text{ and } b_{\text{s}} = \frac{\sigma^2(2+4\lambda_{\text{s}})}{(1+\lambda_{\text{s}})}. \tag{12}$$

Proof: Applying Approximation 1 to $\mathcal{X}_1^2(\lambda_{\text{s}}, 1)$ yields (11). ■

3) Estimation of secondary interference channel: In the downlink, the SR cancels the ST pilot signal over the access channel in order to estimate the secondary interference (power) received from the PT, refer to (3). The power estimated over the signal $h_{\text{p},1}x_{\text{tran}} + w_{\text{s}}$ corresponds to the interference plus noise power ($\hat{P}_{\text{rcvd,SR}} = |h_{\text{p},2}|^2 P_{\text{tran}} + \sigma^2$, where $P_{\text{rcvd,SR}}$ represents the true value). To characterize the secondary throughput, $\hat{P}_{\text{rcvd,SR}}$ is made available to the ST over a low rate feedback channel. Similar to $\hat{P}_{\text{rcvd,ST}}$, $\hat{P}_{\text{rcvd,SR}}$ follows a non-central chi-squared distribution $\mathcal{X}_1^2(\lambda_{\text{s}}, \tau f_{\text{s}})$, wit non-centrality parameter $\lambda_{\text{p},2} = \tau f_{\text{s}}|h_{\text{p},2}|^2 P_{\text{tran}}/\sigma^2$.

Lemma 3: The cumulative distribution function of $\hat{P}_{\text{rcvd,ST}}$ is characterized as

$$F_{\hat{P}_{\text{rcvd,SR}}}(x) \approx 1 - \Gamma\left(a_{\text{p},2}, \frac{x}{b_{\text{p},2}}\right), \tag{13}$$

$$\text{where } a_{\mathrm{p},2} = \frac{(\tau f_\mathrm{s} + \lambda_{\mathrm{p},2})^2}{2\tau f_\mathrm{s} + 4\lambda_{\mathrm{p},2}} \text{ and } b_{\mathrm{p},2} = \frac{\sigma^2(2\tau f_\mathrm{s} + 4\lambda_{\mathrm{p},2})}{(\tau f_\mathrm{s} + \lambda_{\mathrm{p},2})}. \tag{14}$$

Proof: Applying Approximation 1 to $\mathcal{X}_1^2(\lambda_{\mathrm{p},2}, \tau f_\mathrm{s})$ yields (13). ■

III. Theoretical Analysis

A. Short-term performance analysis

In this section, we investigate the performance of the USs for a certain frame, in this sense, the involved channels $h_{\mathrm{p},1}$, $h_{\mathrm{p},2}$ and h_s are deterministic. At first, we employ an outage probability constraint[4] ρ_out at the ST to capture the variations in the P_p incurred due to channel estimation, defined as

$$\mathbb{P}\left(P_\mathrm{p}(= |\hat{h}_{\mathrm{p},1}|^2 P_\mathrm{cont}) \geq \theta_\mathrm{I}\right) \leq \rho_\mathrm{out},$$

$$\mathbb{P}\left(\underbrace{\left(\frac{\hat{P}_{\mathrm{rcvd,ST}} - \sigma^2}{P_\mathrm{tran}}\right)}_{=|\hat{h}_{\mathrm{p},1}|^2,\text{ cf. (6)}} P_\mathrm{cont} \geq \theta_\mathrm{I}\right) \leq \rho_\mathrm{out}. \tag{15}$$

Besides the outage constraint, P_cont is limited by a predefined transmit power P_full. To capture this aspect, the transmit power constraint at the ST is defined as

$$P_\mathrm{cont} \leq P_\mathrm{full}. \tag{16}$$

Based on the aforementioned constraints, we determine the expression of controlled power for the proposed framework.

Lemma 4: Subject to the outage constraint and transmit power constraint, the controlled power at the ST is given by

$$P_\mathrm{cont} = \begin{cases} \frac{\theta_\mathrm{I} P_\mathrm{tran}}{\left(b_{\mathrm{p},1}\Gamma^{-1}(1-\rho_\mathrm{out}, a_{\mathrm{p},1}) - \sigma^2\right)} & \text{if } P_\mathrm{cont} < P_\mathrm{full} \\ P_\mathrm{full} & \text{if } P_\mathrm{cont} \geq P_\mathrm{full} \end{cases}, \tag{17}$$

where $a_{\mathrm{p},1}$ and $b_{\mathrm{p},1}$ are defined in (8) and $\Gamma^{-1}(\cdot,\cdot)$ is the inverse function of regularized upper-incomplete Gamma function [17].

Proof: Substituting the distribution function for $\hat{P}_{\mathrm{rcvd,ST}}$, defined in (7), in (15) and combining with (16) yields (17). ■

[4]The outage constraint is common tool for designing communication system that ensures the outage occurs no more than a certain percentage of time.

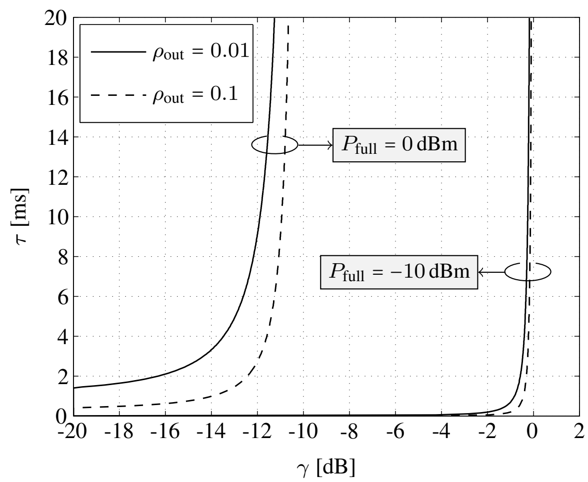

Fig. 3. An illustration of operating regime (γ^*) for the US depicted in terms of estimation time (τ) and the ratio of the received signal power (from the PR) to noise (γ) at the ST.

Clearly, P_{cont} increases with increase in $|h_{\text{s}}|^2$, which depicts low γ, consequently a better performance in terms of secondary throughput is achieved by the US for low γ, however with the presence to P_{full} an upper limit is imposed on the achievable performance. We define this performance limit in terms of γ as an operating regime γ^* for the US.

Corollary 1: Subject to a transmit power constraint, an operating regime at the ST is defined as[5]

$$\rho_{\text{out}} \leq 1 - \Gamma\left(a_{\text{p},1}, \frac{1}{b_{\text{p},1}}\left(\frac{\theta_{\text{I}} P_{\text{tran}}}{P_{\text{full}}} + \sigma^2\right)\right). \tag{18}$$

Proof: Substituting P_{cont}, cf. (17), in (16) results in (18). Replacing (18) with equality yields γ^*. ■

In other words, below a certain $\gamma(\leq \gamma^*)$ performance gain is witnessed by the CR system, cf. Fig. 7. As a result, by replacing γ^* in the following expression of secondary throughput, we determine the performance limits of operation for the US.

Next, we capture the variations in the secondary throughput in terms of its expected value. To accomplish this, we determine the cumulative distribution function (cdf) of the capacity C_{s} over the access channel.

[5]Please note that τf_{s} and γ are included in the parameters $a_{\text{p},1}$ and $b_{\text{p},1}$, cf. (7).

Lemma 5: The cdf of capacity $\mathrm{C_s} = \log_2\left(1 + \frac{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{tran}}}{\hat{P}_{\mathrm{rcvd,SR}}}\right)$ is given by

$$F_{\mathrm{C_s}}(x) = \int_0^x f_{\mathrm{C_s}}(t)dt, \tag{19}$$

where the $f_{\mathrm{C_s}}(x)$ probability density function (pdf) is given by

$$f_{\mathrm{C_s}}(x) = 2^x \ln 2 \frac{(2^x-1)^{a_{\mathrm{s}}-1}\Gamma(a_{\mathrm{s}}+a_{\mathrm{p},2})}{\Gamma(a_{\mathrm{s}})\Gamma(a_{\mathrm{p},2})(b_{\mathrm{s}}P_{\mathrm{tran}})^{a_{\mathrm{s}}} b_{\mathrm{p},2}^{a_{\mathrm{p},2}}} \left(\frac{1}{b_{\mathrm{p},2}} + \frac{2^x-1}{b_{\mathrm{s}}P_{\mathrm{tran}}}\right). \tag{20}$$

Proof: See Appendix A. ■

Besides the outage constraint on the primary interference, the expected secondary throughput over the access channel at the SR is defined as

$$R_{\mathrm{s}}(\tau) = \mathbb{E}_{\mathrm{C_s}}\left[\frac{T-\tau}{T}\log_2\left(1 + \frac{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{cont}}}{\hat{P}_{\mathrm{rcvd,SR}}}\right)\right], \tag{21}$$

where $\mathbb{E}_{\mathrm{C_s}}[\cdot]$ corresponds to an expectation over $\mathrm{C_s}$, whose pdf is characterized in Lemma 5.

At this stage, it is worthy to note that P_{cont} and R_{s} depend on τ, cf. (17) and (30), respectively. Hence, the proposed framework exhibits a fundamental tradeoff between the estimation time and the achievable secondary throughput.

Theorem 1: The expected achievable secondary throughput subject to the outage constraint on the received power at the PR and transmit power constraint at the ST is defined as

$$R_{\mathrm{s}}(\tilde{\tau}) = \max_{\tau} \; R_{\mathrm{s}}(\tau), \tag{22}$$
$$\text{s.t. } (15),\ (16),$$

where $R_{\mathrm{s}}(\tilde{\tau})$ corresponds to optimum throughput at $\tilde{\tau}$.

Proof: The constrained optimization problem is solved by substituting P_{cont} from Lemma 3, determined by applying outage and transmit power constraints defined in (15) and (16), in (30).

Using the probability density function $f_{\mathrm{C_s}}$ in (20) to determine an expression of expected throughput as a function of τ. Solving numerically this expression yields $\tilde{\tau}$ and $R_{\mathrm{s}}(\tilde{\tau})$. ■

B. Long-term performance analysis

Here, our objective is to investigate the long-term performance of the proposed approach. In this regard, we consider the channel gains $h_{\mathrm{p},1}$, $h_{\mathrm{p},2}$ and h_{s} are subject to Nakagami-m fading model, therefore, the power gains $|h_{\rm p,1}|^2$, $|h_{\rm p,2}|^2$ and $|h_{\rm s}|^2$ follow a Gamma distribution [19] with cumulative distribution functions

$$F_{|h_{\rm p,1}|^2}(x) = \gamma\left(m_{\rm p,1}, \frac{m_{\rm p,1}x}{|h_{\rm p,1}|^2}\right), \tag{23}$$

$$F_{|h_{\rm p,2}|^2}(x) = \gamma\left(m_{\rm p,2}, \frac{m_{\rm p,2}x}{|h_{\rm p,2}|^2}\right), \tag{24}$$

$$F_{|h_{\rm s}|^2}(x) = \gamma\left(m_{\rm s}, \frac{m_{\rm s}x}{|h_{\rm s}|^2}\right), \tag{25}$$

where $m_{\rm p,1}$, $m_{\rm p,2}$ and $m_{\rm s}$ represent the m parameter, respectively, and $\gamma(\cdot,\cdot)$ is a regularized lower-incomplete Gamma function [17]. The performance analysis subject to channel fading has been considered by Ghasemi *et al.* [4], [20]. However, the authors in [4], [20] evaluated average capacity under an average interference constraint. In order to make a fair comparison (benchmark) of our proposed approach, we slightly modify the existing investigations, according to which, the variations in context to channel fading are quantified based on an outage constraint on the primary interference, given by

$$\mathbb{E}_{|h_{\rm p,2}|^2,|h_{\rm s}|^2}\left[\mathrm{C_s}(|h_{\rm p,2}|^2, |h_{\rm s}|^2)\right], \tag{26}$$

$$\text{s.t. } \mathbb{P}(P_{\rm p}(|h_{\rm p,1}|^2) \geq \theta_{\rm I}) \leq \rho_{\rm out}, \tag{27}$$

where $\mathbb{E}_{|h_{\rm p,2}|^2,|h_{\rm s}|^2}[\cdot]$ corresponds to exception with respect to $|h_{\rm p,2}|^2, |h_{\rm s}|^2$. Despite the knowledge of the fading model, similar to short-term analysis, the characterization in (26) and (27) assumes the perfect knowledge of the power gains $(|h_{\rm p,1}|^2, |h_{\rm p,2}|^2, |h_{\rm s}|^2)$ of the corresponding channels. In this regard, we extend our performance analysis to study the effect of channel fading on the performance of the proposed framework.

At first, we determine the expression of the outage constraint on the primary interference

$$\mathbb{E}_{|h_{\rm p,1}|^2}\left[\mathbb{P}\left(\left(\frac{\hat{P}_{\rm rcvd,ST}(|h_{\rm p,1}|^2) - \sigma^2}{P_{\rm tran}}\right)P_{\rm cont} \geq \theta_{\rm I}\right)\right] \leq \rho_{\rm out}. \tag{28}$$

In contrast to the constraint in (27), (28) consider the expectation $\mathbb{E}_{|h_{\rm p,1}|^2}[\cdot]$ over the estimated value of the channel $|\hat{h}_{\rm p,1}|^2$, determined in terms of $\hat{P}_{\rm rcvd,ST}$. Based on (28) and transmit power constraint defined in (16), we obtain the expression of controlled power subject to long-term analysis.

Lemma 6: Subject to the outage constraint and transmit power constraint, the controlled power at the ST in context to Nakagami-m fading is given by

$$P_{\text{cont}} = \begin{cases} \text{Solving for } P_{\text{cont}},\ \int_0^\infty \Gamma\left(a_{\text{p},2}\left(|h_{\text{p},1}|^2, \frac{x}{b_{\text{p},1}(|h_{\text{p},1}|^2)}\right)\right) dF_{|h_{\text{p},1}|^2} = \rho_{\text{out}} & \text{if } P_{\text{cont}} < P_{\text{full}} \\ P_{\text{full}} & \text{if } P_{\text{cont}} \geq P_{\text{full}} \end{cases}, \tag{29}$$

where $a_{\text{p},1}$ and $b_{\text{p},1}$ are defined in (8) and $F_{|h_{\text{p},1}|^2}(x)$ is defined in (23).

Proof: Since it is complicated to obtain a closed form expression of the integral in (29), ~~in this regard,~~ we evaluate the controlled power numerically. ■

Upon determining the controlled power in Lemma 6 that constrains the primary interference, we determine the expression of the secondary throughput.

$$R_{\text{s}}(\tau) = \mathbb{E}_{\text{C}_{\text{s}},|h_{\text{p},2}|^2,|h_{\text{s}}|^2}\left[\frac{T-\tau}{T}\log_2\left(1 + \frac{|\hat{h}_{\text{s}}|^2 P_{\text{cont}}}{\hat{P}_{\text{rcvd,ST}}(|h_{\text{p},2}|^2)}\right)\right], \tag{30}$$

where $\mathbb{E}_{\text{C}_{\text{s}},|h_{\text{p},2}|^2,|h_{\text{s}}|^2}[\cdot]$ corresponds to an expectation over C_{s}[6], $|h_{\text{p},2}|^2$, $|h_{\text{s}}|^2$, whose cdfs are characterized in Lemma 5, (24) and (25), respectively. Next, we characterize the estimation-throughput tradeoff for the long-term analysis of the proposed framework.

Theorem 2: The expected achievable secondary throughput subject to the outage constraint on the received power at the PR and transmit power constraint at the ST in context to Nakagami-m fading is defined as

$$R_{\text{s}}(\tilde{\tau}) = \max_{\tau} R_{\text{s}}(\tau), \tag{31}$$
$$\text{s.t. } (28),\ (16),$$

where $R_{\text{s}}(\tilde{\tau})$ corresponds to optimum throughput at $\tilde{\tau}$.

IV. Numerical Analysis

Here, we investigate the performance of the US based on the proposed model. To accomplish this: (i) we perform simulations to validate the expressions obtained, (ii) we analyze the performance loss incurred due to the estimation. In this regard, we consider the ideal model for benchmarking and evaluating the performance loss. Unless stated explicitly, the following choice of the parameters given is table I is considered for the analysis.

TABLE I

PARAMETERS FOR NUMERICAL ANALYSIS

Parameter	**Value**
f_{s}	1 MHz
$\lvert h_{\mathrm{p},1}\rvert^2$	−100 dB
$\lvert h_{\mathrm{p},2}\rvert^2$	−100 dB
$\lvert h_{\mathrm{s}}\rvert^2$	−80 dB
θ_{I}	−110 dB
T	100 ms
ρ_{out}	$\{0.01,\ 0.10\}$
P_{full}	$\{10,\ 0\}$ dBm
σ^2	−100 dBm
γ	0 dB
P_{tran}	0 dBm
N_{s}	10

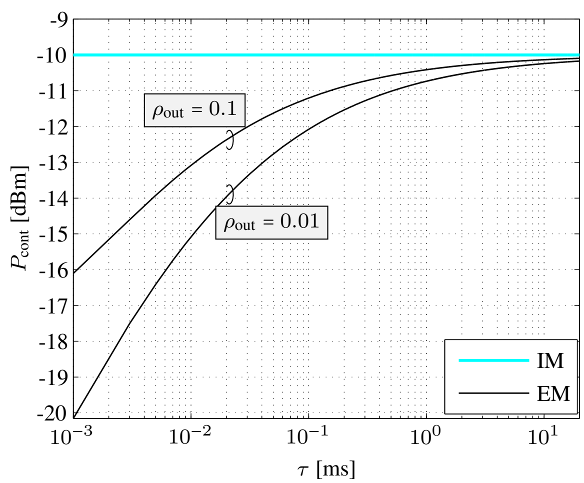

Fig. 4. Control power versus estimation time.

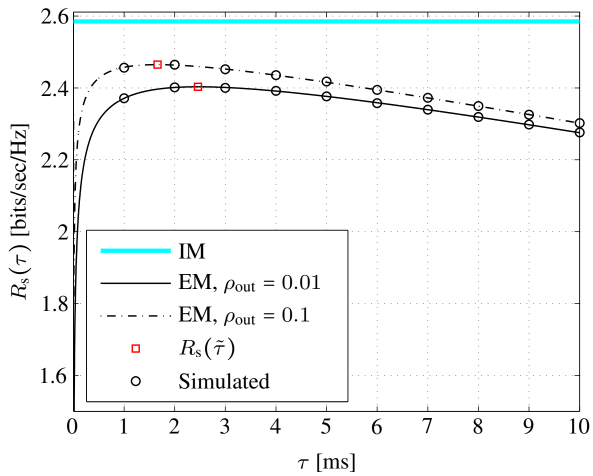

Fig. 5. Estimation-throughput tradeoff with $\gamma = 0\,\text{dB}$, $\rho_{\text{out}} \in \{0.01, 0.1\}$ and $P_{\text{full}} = 0\,\text{dBm}$.

A. *Short-term analysis*

Fig. 5 analyzes performance of US in terms of estimation-throughput tradeoff, cf. Theorem 1, corresponding to the Ideal Model (IM) and the Estimation Model (EM). It is indicated that the estimation-throughput tradeoff yields a suitable estimation time $\tilde{\tau}$ that results in an optimum throughput $R_s(\tilde{\tau})$. Hereafter, for the analysis, we consider the theoretical expressions and choose to operate at suitable estimation time. To procure further insights, the variation of $R_s(\tilde{\tau})$ with γ for different choices of P_{full} and ρ_{out} are considered in Fig. 10. It is observed that $R_s(\tilde{\tau})$ gets saturated below a certain γ, thereby limiting the performance of the US. Particularly for $P_{\text{full}} = -10\,\text{dBm}$, a severe performance loss indicated by the margin between the IM and the EM is witnessed by the US for $\gamma \le -2\,\text{dB}$.

B. *Long-term analysis*

V. Conclusion

In this paper, we studied the performance of the USs from a deployment perspective. In this view, a novel model that incorporates channel estimation has been proposed. To capture the impact of imperfect channel knowledge, an outage constraint that forbids performance degradation in terms of interference power received at the primary receiver has been employed. With

[6] It is worth noticing that C_s entails the variations due to channel estimation $|\hat{h}_s|^2$ and $\hat{P}_{\text{rcvd,ST}}$.

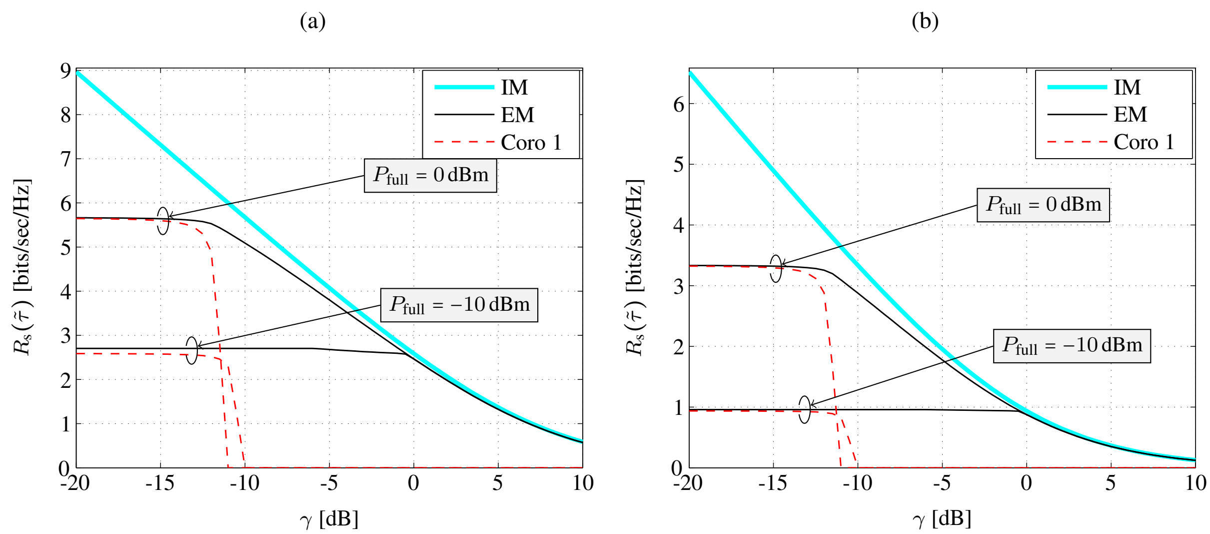

Fig. 6. Optimum throughput $(R_s(\tilde{\tau}))$ versus the ratio of received power to noise (γ) with $\rho_{\text{out}} \in \{0.01, 0.1\}$ and $P_{\text{full}} \in \{-10, 0\}$ dBm.

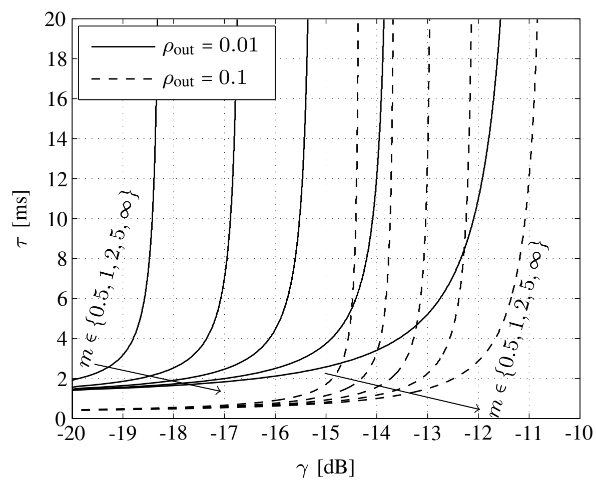

Fig. 7. An illustration of operating regime (γ^*) for the US depicted in terms of estimation time (τ) and the ratio of the received power (from the PR) to noise (γ) at the ST.

the inclusion of a transmit power constraint, an operating regime that determines performance limits for the US has been established. Further, a power control mechanism subject to the outage and transmit power constraints has been proposed. Finally, the estimation-throughput tradeoff has been investigated to determine the achievable secondary throughput for the US. ~~In future work, we plan to extend the proposed analysis to include the effect of channel fading in order to characterize the long-term performance of the USs.~~

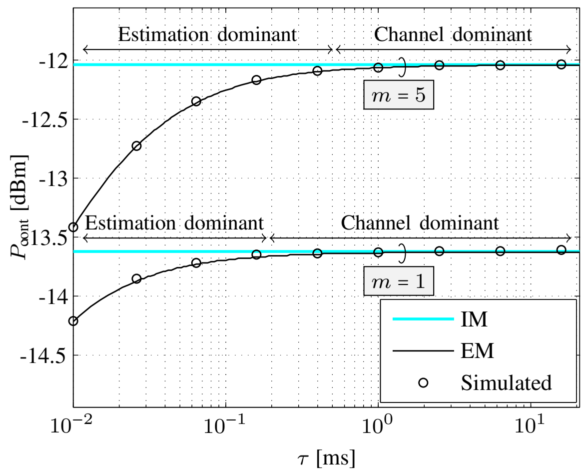

Fig. 8. Control power versus estimation time.

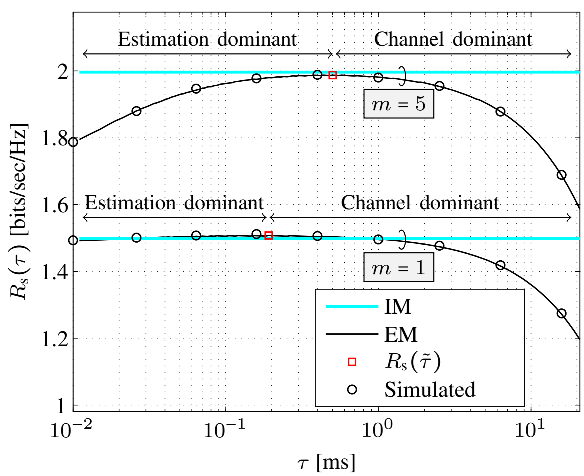

Fig. 9. Estimation-throughput tradeoff.

Appendix
Proof of Lemma 5

Proof: For simplification, we break the expression $\frac{|\hat{h}_s|^2 P_{\text{tran}}}{\hat{P}_{\text{rcvd,SR}}}$ and determine the pdf for $f_{|\hat{h}_s|^2 P_{\text{tran}}}$ and $f_{\hat{P}_{\text{rcvd,ST}}}$ separately. Using (11) in Lemma 2, the pdf of $f_{|\hat{h}_s|^2 P_{\text{tran}}}$ is determined as

$$f_{|\hat{h}_s|^2 P_{\text{tran}}}(x) = \frac{1}{\Gamma(a_s)(b_s P_{\text{tran}})^{a_s}} x^{a_s - 1} \exp\left(-\frac{x}{b_s P_{\text{tran}}}\right), \tag{32}$$

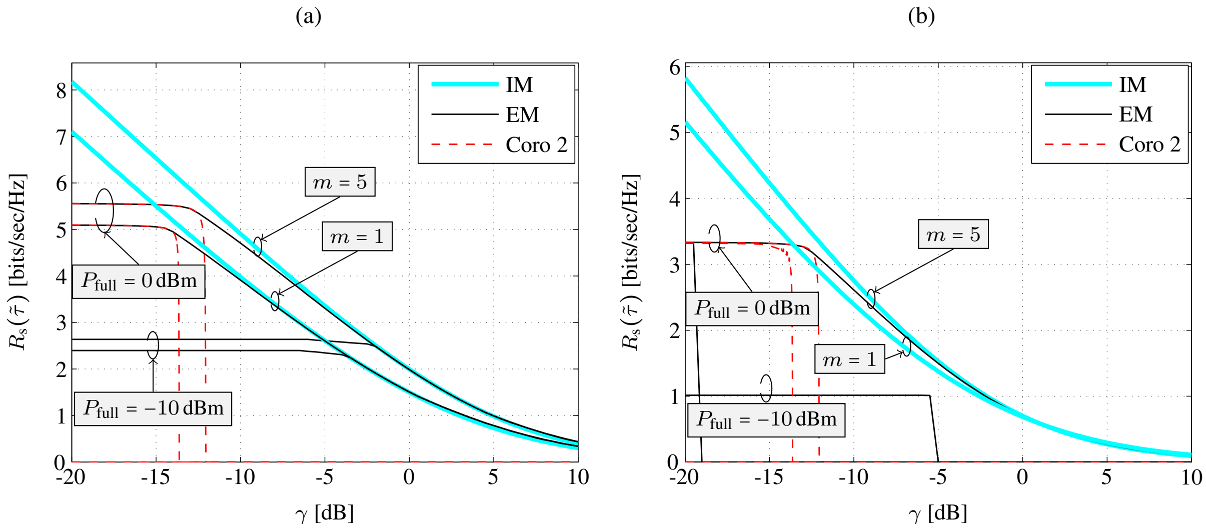

Fig. 10. Optimum throughput $(R_{\mathrm{s}}(\tilde{\tau}))$ versus the ratio of received power to noise (γ) for Nakagmi-m fading channels with $\rho_{\mathrm{out}} \in \{0.01, 0.1\}$ and $P_{\mathrm{full}} \in \{-10, 0\}$ dBm.

where a_{s} and b_{s} are defined in (12). Similarly, using Lemma 3, the pdf of $f_{\hat{P}_{\mathrm{rcvd,SR}}}$ is characterized as

$$f_{\hat{P}_{\mathrm{rcvd,ST}}}(x) = \frac{1}{\Gamma(a_{\mathrm{p,2}})(b_{\mathrm{p,2}})^{a_{\mathrm{p,2}}}} x^{a_{\mathrm{p,2}}-1} \exp\left(-\frac{x}{b_{\mathrm{p,2}}}\right), \tag{33}$$

where $a_{\mathrm{p,2}}$ and $b_{\mathrm{p,2}}$ are defined in (14).

Using (32) and (33), we apply Mellin transform [21] to determine the pdf of $\frac{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{tran}}}{\hat{P}_{\mathrm{rcvd,SR}}}$ as

$$f_{\frac{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{tran}}}{\hat{P}_{\mathrm{rcvd,SR}}}}(x) = \frac{(x)^{a_{\mathrm{s}}-1}\Gamma(a_{\mathrm{s}} + a_{\mathrm{p,2}})}{\Gamma(a_{\mathrm{s}})\Gamma(a_{\mathrm{p,2}})(b_{\mathrm{s}} P_{\mathrm{tran}})^{a_{\mathrm{s}}} b_{\mathrm{p,2}}^{a_{\mathrm{p,2}}}} \left(\frac{1}{b_{\mathrm{p,2}}} + \frac{x-1}{b_{\mathrm{s}} P_{\mathrm{tran}}}\right). \tag{34}$$

Finally, substituting the expression $\frac{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{tran}}}{\hat{P}_{\mathrm{rcvd,SR}}}$ in $\mathrm{C_s}$ yields (20). ■

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