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The Effects of Limited Channel Knowledge on Cognitive Radio System Capacity

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***Abstract*—We examine the impact of limited channel knowledge on the secondary user (SU) in a cognitive radio system. Under a minimum signal-to-interference noise ratio (SINR) constraint for the primary user (PU) receiver, we determine the SU capacity under five channel knowledge scenarios. We derive analytical expressions for the capacity cumulative distribution functions and the probability of SU blocking as a function of allowable interference. We show that imperfect knowledge of the PU–PU link gain by the SU-Tx often prohibits SU transmission or necessitates a high interference level at the PU. We also show that errored knowledge of the PU–PU channel is more beneficial than statistical channel knowledge and that imperfect knowledge of the SU-Tx to PU-Rx link has limited impact on SU capacity.**

***Index Terms*—Channel capacity, cognitive radio (CR), partial channel-state information (CSI).**

I. Introduction

A large body of work is now available on various aspects of cognitive radio (CR) systems, including fundamental information-theoretic capacity limits and performance analysis, which often assumes perfect secondary user (SU)-Tx to primary user (PU)-Rx channel state information (CSI) [1]–[7]. In practice, there is expected to be limited (or no) collaboration between PU and SU systems. Hence, an important question is the impact of the nature of channel knowledge on CR capacity. Several recent contributions have considered imperfect CSI [8]–[14]. In [8], mean and outage capacities along with optimum power allocation policies have been investigated for a CR system in a fading environment with imperfect CSI. Here, probabilistic constraints were employed to maintain an acceptably low probability that interference exceeded some target. In our work, we also use probabilistic constraints but apply them to a signal-to-interference noise ratio (SINR) target.

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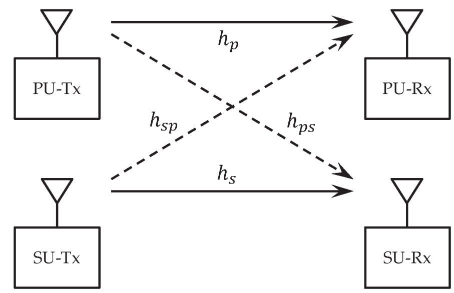

Fig. 1. System diagram.

This paper differs from the existing literature in several ways. There are four link gains in a two-user PU/SU channel to consider, and each of them may or may not be perfectly known at the SU transmitter. Previous studies [8]–[10], [12], [14] have only assumed imperfect knowledge of the SU-Tx to PU-Rx link. Additionally, in previous work, the effect of the interference from the PU-Tx on SU capacity is ignored. Moreover, we employ the SINR at the PU-Rx to impose probabilistic constraints to protect the PU-Rx, whereas prior works, with the exception of [10], have considered an interference outage constraint. Finally, we consider several cases where the imperfect CSI manifests itself in the form of statistical channel knowledge (i.e., knowledge of the mean link gains). Such a form of imperfect CSI is attractive from a practical standpoint since obtaining accurate knowledge is almost impossible for some links, such as the PU-Tx to PU-Rx link. Moreover, the mean value does not impose a large system burden as it only requires infrequent updates. Note that the inclusion of PU-Tx to SU-Rx interference and probabilistic constraints enables a rigorous evaluation of the benefits of various types of CSI. In this paper, we establish the following key observations and results:

1) In four of the five scenarios considered, we derive analytical expressions for the cumulative distribution function (cdf) of the SU SINR and use it to evaluate the SU capacity cdf.
2) For all scenarios, we derive the probability of SU blocking as a function of the permissible interference at the PU-Rx.
3) By evaluating our results for a range of system parameters, we demonstrate the importance of accurate knowledge of the PU-Tx to PU-Rx link at the SU-Tx.
4) We demonstrate the very high sensitivity of SU performance to the error in the estimation of the PU-Tx to PU-Rx and SU-Tx to PU-Rx links.
5) We show that errored knowledge of the PU-Tx to PU-Rx link and SU-Tx to PU-Rx link (if available) is better for SU capacity than knowledge of the mean link gains.
6) By considering a single probabilistic SINR constraint, a unified framework is presented that enables fair comparisons between different types of channel knowledge.

II. System Model

Consider a CR system (see Fig. 1) with the SU-Tx and PU-Tx transmitting simultaneously to their respective receivers. Independent point-to-point flat Rayleigh fading channels are assumed for all links. Let $g_{\rm p} = |h_{\rm p}|^2$, $g_{\rm s} = |h_{\rm s}|^2$, $g_{\rm ps} = |h_{\rm ps}|^2$, and $g_{\rm sp} = |h_{\rm sp}|^2$ denote the exponentially distributed instantaneous link gains of the PU-Tx to PU-Rx, SU-Tx to SU-Rx, PU-Tx to SU-Rx, and SU-Tx to PU-Rx links, respectively, with $\Omega_{\rm p} = \mathbb{E}(g_{\rm p})$, $\Omega_{\rm s} = \mathbb{E}(g_{\rm s})$, $\Omega_{\rm ps} = \mathbb{E}(g_{\rm ps})$, and $\Omega_{\rm sp} = \mathbb{E}(g_{\rm sp})$, where $\mathbb{E}(\cdot)$ denotes the expectation operator.

As described further in this section, the SU transmission under the SINR constraint is governed solely by the state of the $g_{\rm p}$ and $g_{\rm sp}$ links.[1] Thus, in this paper, we consider the following five scenarios for the knowledge of $g_{\rm p}$ and $g_{\rm sp}$ by the SU-Tx.

Scenario 1: The link gains $g_{\rm p}$ and $g_{\rm sp}$ are perfectly known. This clearly unrealistic scenario serves as a benchmark for comparison.

Scenario 2: The link gain $g_{\rm p}$ is perfectly known, whereas only the mean $\Omega_{\rm sp}$ of $g_{\rm sp}$ is known.

Scenario 3: The mean $\Omega_{\rm p}$ and the exact link gain $g_{\rm sp}$ are known. In contrast to Scenario 2, this case is considered mainly for completeness.

Scenario 4: Only the means $\Omega_{\rm p}$ and $\Omega_{\rm sp}$ are known. This scenario arises when only statistical information about the channels is available to the SU-Tx as a result of limited feedback resources.

Scenario 5: Only estimates of $g_{\rm p}$ and $g_{\rm sp}$ are available. This may arise due to channel estimation, feedback quantization, and delay.

Where possible, we impose a constraint $\gamma_{\rm T}$ on the PU-Rx SINR, denoted by $\gamma_{\rm p}$. Hence

$$\gamma_{\rm p} = \frac{P_{\rm p} g_{\rm p}}{P_{\rm s} g_{\rm sp} + \sigma_{\rm p}^2}, \quad \text{and} \quad \gamma_{\rm p} \geq \gamma_{\rm T} \tag{1}$$

where $\gamma_{\rm T}$ is an SINR threshold, $P_{\rm p}$ is the PU transmit power (assumed to be fixed and known to the SU-Tx in all scenarios), and $\sigma_{\rm p}^2$ is the additive white Gaussian noise (AWGN) variance at the PU-Rx. $P_{\rm s}$ is a provisional maximum value for the SU transmit power chosen to satisfy the relevant criteria, (1) in this case. The actual SU transmit power $P_{\rm t}$ is a function of $P_{\rm s}$. For example, if the PU-Rx SNR lies in the region $P_{\rm p} g_{\rm p}/\sigma_p^2 < \gamma_{\rm T}$, then (1) cannot be satisfied unless $P_{\rm s} < 0$, and as a result, $P_{\rm t} = 0$. If the PU SNR is above the SINR threshold $\gamma_{\rm T}$, the SU-Tx will adapt $P_{\rm s}$ to a maximum level satisfying (1) as determined under the five scenarios, regardless of the gain $g_{\rm s}$. We also impose a maximum SU transmit power constraint $P_{\rm m}$. Thus, in Scenario 1, where the SU-Tx knows $g_{\rm p}$, $P_{\rm t}$ is given by

$$P_{\rm t} = \begin{cases} 0, & \frac{P_{\rm p} g_{\rm p}}{\gamma_{\rm T}} < \sigma_{\rm p}^2 \\ \min(P_{\rm s}, P_{\rm m}), & \text{otherwise} \end{cases} \tag{2}$$

where $P_{\rm s}$ is obtained from (1) by solving $\gamma_{\rm T} = \gamma_{\rm p}$. Furthermore, the constraints described above can only be guaranteed if the SU-Tx has perfect knowledge of the gains $g_{\rm p}$ and $g_{\rm sp}$, i.e., under Scenario 1. In analysing Scenarios 2–5, we use probabilistic constraints. Hence, we require the SINR constraint to hold with an acceptably high probability $1 - \alpha$, where α is small.

In analysing the SU capacity, we first consider the SINR at the SU-Rx, which is denoted by $\gamma_{\rm I}$, i.e.,

$$\gamma_{\rm I} = \frac{P_{\rm t} g_{\rm s}}{P_{\rm p} g_{\rm ps} + \sigma_{\rm s}^2} \tag{3}$$

where $\sigma_{\rm s}^2$ is the AWGN variance at the SU-Rx, and $P_{\rm p} g_{\rm ps}$ is the interference from PU-Tx, treated as noise in the capacity calculations. We denote the probability density function (pdf) and cdf of $\gamma_{\rm I}$ by $f_{\gamma_{\rm I}}(x)$ and $F_{\gamma_{\rm I}}(x)$, respectively. The instantaneous SU capacity is given by $C = \log_2(1 + \gamma_{\rm I})$, where the mean $\bar{C}$ can be derived using $f_{\gamma_{\rm I}}(x)$ as

$$\bar{C} = \mathbb{E}(C) = \int_0^\infty \log_2(1 + x) f_{\gamma_{\rm I}}(x)\, dx. \tag{4}$$

The cdf of C can be obtained from $F_{\gamma_{\rm I}}(x)$ by noting that

$$F_C(y) = \Pr(\gamma_{\rm I} \leq 2^y - 1) = F_{\gamma_{\rm I}}(\tilde{y}) \tag{5}$$

where $\Pr(\cdot)$ denotes probability, and $\tilde{y} = 2^y - 1$. Using (3), we can express (5) as

$$\begin{aligned} F_{\gamma_{\rm I}}(\tilde{y}) &= \mathbb{E}_{g_{\rm ps}} \left\{ \Pr\left(P_{\rm t} g_{\rm s} < \tilde{y}\left(\sigma_{\rm s}^2 + P_{\rm p} g_{\rm ps}\right)\right) | g_{\rm ps} \right\} \\ &= \int_0^\infty F_\gamma\left(\tilde{y}\left(\sigma_{\rm s}^2 + P_{\rm p} v\right)\right) \frac{e^{-v/\Omega_{\rm ps}}}{\Omega_{\rm ps}}\, dv \end{aligned} \tag{6}$$

where we have defined $\gamma = P_{\rm t} g_{\rm s}$ with a cdf $F_\gamma(x)$. In what follows, we derive expressions for $F_\gamma(x)$, which, using (5) and (6), allows us to compute the capacity cdf.

We parameterize the main system variables by two key parameters. The first $c_1 = \Omega_{\rm sp}/\Omega_{\rm s}$ represents the ratio of the mean interference at the PU-Rx to the mean of the desired channel strength for the SU. The second $c_2 = \gamma_{\rm T}\sigma_{\rm p}^2/P_{\rm p}\Omega_{\rm p}$ is the ratio of the minimum target SINR to the mean SNR at the PU-Rx. Hence, increasing c_2 corresponds to reducing the allowable interference (with $c_2 = 1$ corresponding to zero average allowable interference).

III. Secondary User Capacity

The capacity mean in (4) and the cdf in (6) require knowledge of the distributions of $\gamma = P_{\rm t} g_{\rm s}$ and $\gamma_{\rm I}$. Hence, in this section, we derive the cdfs for γ and $\gamma_{\rm I}$ for Scenarios 1–4. For Scenario 5, an alternative approach is required (see Section III-E).

A. Scenario 1

In this scenario, $P_{\rm s}$ can be obtained directly from (1), giving

$$P_{\rm s} = \frac{\frac{P_{\rm p} g_{\rm p}}{\gamma_{\rm T}} - \sigma_{\rm p}^2}{g_{\rm sp}}. \tag{7}$$

We note that while we ignore the $P_{\rm t} = 0$ case in (2), the following derivation is valid since $\Pr(\gamma > 0) = 0$ for $P_{\rm t} \leq 0$. In finding $F_\gamma(x)$, we solve for the complementary cdf given by

$$\begin{aligned} \Pr(\gamma > x) &= \Pr\left(g_{\rm s} \min(P_{\rm m}, P_{\rm s}) > x\right) \\ &= \Pr\left(g_{\rm s} > \frac{x}{P_{\rm m}}, \left(\frac{P_{\rm p} g_{\rm p}}{\gamma_{\rm T}} - \sigma_{\rm p}^2\right) g_{\rm s} > x g_{\rm sp}\right). \end{aligned} \tag{8}$$

Noting that $g_{\rm p}$ is an exponentially distributed random variable, we can rewrite (8) by taking the conditional probability over $g_{\rm p}$ and then averaging over $g_{\rm s}$ and $g_{\rm sp}$. This gives

$$\Pr(\gamma > x) = \frac{e^{-\frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}\Omega_{\rm p}}}}{\Omega_{\rm sp}\Omega_{\rm s}} \int_{\frac{x}{P_{\rm m}}}^\infty \frac{e^{-\frac{u}{\Omega_{\rm s}}}}{\frac{\gamma_{\rm T} x}{P_{\rm p}\Omega_{\rm p} u} + \frac{1}{\Omega_{\rm sp}}}\, du. \tag{9}$$

[1] The link gains $g_{\rm s}$ and $g_{\rm ps}$ have an impact on the achievable SU capacity; however, the level of their knowledge by the SU-Tx does not impact the transmit power.

After simplifying (9), the cdf $F_\gamma(x) = 1 - \Pr(\gamma > x)$ can be shown to be [15, Eq. (3.351.2)]

$$F_\gamma(x) = 1 - e^{-\frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}\Omega_{\rm p}}}\left[e^{-\frac{x}{P_{\rm m}\Omega_{\rm s}}} - \frac{\Omega_{\rm sp}\gamma_{\rm T}x}{P_{\rm p}\Omega_{\rm p}\Omega_{\rm s}}e^{\frac{\Omega_{\rm sp}\gamma_{\rm T}x}{P_{\rm p}\Omega_{\rm p}\Omega_{\rm s}}} \times \Gamma\left(0, \frac{\Omega_{\rm sp}\gamma_{\rm T}x}{P_{\rm p}\Omega_{\rm p}\Omega_{\rm s}} + \frac{x}{P_{\rm m}\Omega_{\rm s}}\right)\right] \quad (10)$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function. Substituting (10) into (6) results in

$$\begin{aligned} F_{\gamma_I}(\tilde{y}) = 1 &- \frac{P_{\rm m}\Omega_{\rm s}e^{-\left(\frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}\Omega_{\rm p}} + \frac{\tilde{y}\sigma_{\rm s}^2}{P_{\rm m}\Omega_{\rm s}}\right)}}{P_{\rm m}\Omega_{\rm s} + \tilde{y}P_{\rm p}\Omega_{\rm ps}} \\ &+ \frac{\Omega_{\rm sp}\gamma_{\rm T}\tilde{y}}{\Omega_{\rm ps}\Omega_{\rm p}\Omega_{\rm s}P_{\rm p}}\exp\left\{\frac{\Omega_{\rm sp}\gamma_{\rm T}\sigma_{\rm p}^2}{\Omega_{\rm p}\Omega_{\rm s}P_{\rm p}}\left(\tilde{y} - \frac{\Omega_{\rm s}}{\Omega_{\rm sp}}\right)\right\} \\ &\times \int_0^\infty \left(\sigma_{\rm p}^2 + P_{\rm p}v\right)\exp\left\{\left(\frac{\Omega_{\rm sp}\gamma_{\rm T}}{\Omega_{\rm p}\Omega_{\rm s}}\tilde{y} - \frac{1}{\Omega_{\rm ps}}\right)v\right\} \\ &\times \Gamma\left(0, \frac{\Omega_{\rm sp}\gamma_{\rm T}P_{\rm m} + P_{\rm p}\Omega_{\rm p}}{P_{\rm p}P_{\rm m}\Omega_{\rm p}\Omega_{\rm s}}\left(\sigma_{\rm p}^2 + P_{\rm p}v\right)\tilde{y}\right)dv. \end{aligned} \quad (11)$$

To the best of the authors' knowledge, the integral in (11) does not have a closed-form solution. In Section IV, the capacity cdf results are obtained by numerical integration.

To obtain the expression for mean capacity, we can derive the pdf $f_{\gamma_{\rm I}}(x)$ by differentiating (11) with respect to $\tilde{y}$. Alternatively, using (6), we have

$$f_{\gamma_{\rm I}}(\tilde{y}) = \int_0^\infty \left(\sigma_{\rm p}^2 + P_{\rm p}v\right)f_\gamma\left(\tilde{y}\left(\sigma_{\rm p}^2 + P_{\rm p}v\right)\right)\frac{e^{-v/\Omega_{\rm ps}}}{\Omega_{\rm ps}}dv \quad (12)$$

where $f_\gamma(x)$ was computed in [16] as

$$\begin{aligned} f_\gamma(x) = e^{-\frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}\Omega_{\rm p}}}\Bigg[&\left(\frac{1}{P_{\rm m}\Omega_{\rm s}} - \frac{\Omega_{\rm sp}\gamma_{\rm T}}{P_{\rm p}\Omega_{\rm p}\Omega_{\rm s}}\right)e^{-\frac{x}{P_{\rm m}\Omega_{\rm s}}} \\ &+ e^{\frac{\Omega_{\rm sp}\gamma_{\rm T}x}{P_{\rm p}\Omega_{\rm p}\Omega_{\rm s}}}\left(\frac{(\Omega_{\rm sp}\gamma_{\rm T})^2x}{(P_{\rm p}\Omega_{\rm p}\Omega_{\rm s})^2} + \frac{\Omega_{\rm sp}\gamma_{\rm T}}{P_{\rm p}\Omega_{\rm p}\Omega_{\rm s}}\right) \\ &\times \Gamma\left(0, \frac{\Omega_{\rm sp}\gamma_{\rm T}x}{P_{\rm p}\Omega_{\rm p}\Omega_{\rm s}} + \frac{x}{P_{\rm m}\Omega_{\rm s}}\right)\Bigg]. \end{aligned} \quad (13)$$

The expression resulting from substituting (13) into (12) cannot be written in closed form. Thus, the mean capacity $\bar{C}$ must be calculated numerically by substituting (13) into (12) and (4).

B. Scenario 2

In Scenarios 2–5, with imperfect channel knowledge, the SU cannot guarantee that (1) is satisfied. Thus, we constrain the SU to satisfy (1) with an acceptably high probability $1 - \alpha$.

Hence, for Scenario 2, where the SU knows only the mean $\Omega_{\rm sp}$ of $g_{\rm sp}$, we consider the probability of satisfying the SINR constraint with a probability of $1 - \alpha$. This gives

$$\Pr\left(\frac{P_{\rm p}g_{\rm p}}{P_{\rm s}g_{\rm sp} + \sigma_{\rm p}^2} \geq \gamma_{\rm T}\middle| g_{\rm p}, \Omega_{\rm sp}\right) = 1 - \alpha. \quad (14)$$

Rewriting in terms of the cdf of $g_{\rm sp}$, we derive $P_{\rm s}$ as

$$P_{\rm s} = -\frac{P_{\rm p}g_{\rm p} - \gamma_{\rm T}\sigma_{\rm p}^2}{\ln(\alpha)\gamma_{\rm T}\Omega_{\rm sp}}. \quad (15)$$

Using (15), the complementary cdf of γ can be shown to be

$$\Pr(\gamma > x) = \mathbb{E}\left[\Pr(P_{\rm m}g_{\rm s} > x, P_{\rm s}g_{\rm s} > x | g_{\rm p})\right] \quad (16)$$

which can be expressed as

$$\Pr(\gamma > x) = \int_{\psi_0}^{\psi}\Pr\left(g_{\rm s} > \frac{x}{P_{\rm s}}\right)f_{g_{\rm p}}(y)\,dy + \int_{\psi}^{\infty}\Pr\left(g_{\rm s} > \frac{x}{P_{\rm m}}\right)f_{g_{\rm p}}(y)\,dy \quad (17)$$

where $\psi_0 = \gamma_{\rm T}\sigma_{\rm p}^2/P_{\rm p}$, and $\psi = \gamma_{\rm T}(\sigma_{\rm p}^2 - P_{\rm m}\ln(\alpha)\Omega_{\rm sp})/P_{\rm p}$. The lower integration limit in the first term of (17) takes into account the $P_{\rm t} = 0$ condition in (2). After some manipulation, we can simplify (17) to obtain $F_\gamma(x) = 1 - \Pr(\gamma > x)$ as

$$F_\gamma(x) = 1 - \exp\left\{-\frac{x}{P_{\rm m}\Omega_{\rm s}} - \frac{\psi}{\Omega_{\rm p}}\right\} - \frac{1}{\Omega_{\rm p}}\int_{\psi_0}^{\psi} e^{-\frac{\ln(\alpha)\gamma_{\rm T}\Omega_{\rm sp}x}{\left(\gamma_{\rm T}\sigma_{\rm p}^2 - P_{\rm p}y\right)\Omega_{\rm s}}}e^{-\frac{y}{\Omega_{\rm p}}}dy. \quad (18)$$

Once again, there exists no closed-form solution to the integral in (18). Following the same approach as in Scenario 1, we use (18) and (6) to find $F_{\gamma_{\rm I}}(\tilde{y})$ as

$$\begin{aligned} F_{\gamma_{\rm I}}(\tilde{y}) = 1 &- \frac{P_{\rm m}\Omega_{\rm s}e^{-\left(\frac{\tilde{y}\sigma_{\rm p}^2}{P_{\rm m}\Omega_{\rm s}} + \frac{\psi}{\Omega_{\rm p}}\right)}}{\Omega_{\rm ps}P_{\rm p}\tilde{y} + P_{\rm m}\Omega_{\rm s}} + \frac{1}{\Omega_{\rm s}}\int_{\psi_0}^{\psi} e^{-\left(\frac{\Omega_{\rm sp}\gamma_{\rm T}\sigma_{\rm p}^2\ln\alpha\tilde{y}}{\gamma_{\rm T}\sigma_{\rm p}^2\Omega_{\rm s} - P_{\rm p}\Omega_{\rm s}z} + \frac{z}{\Omega_{\rm s}}\right)} \\ &\times \frac{\gamma_{\rm T}\sigma_{\rm p}^2\Omega_{\rm s} - P_{\rm p}\Omega_{\rm s}z}{\gamma_{\rm T}\sigma_{\rm p}^2\Omega_{\rm s} + \Omega_{\rm sp}\gamma_{\rm T}P_{\rm p}\Omega_{\rm ps}\ln(\alpha)\tilde{y} - P_{\rm p}\Omega_{\rm s}z}dz. \end{aligned} \quad (19)$$

Here, again, the capacity cdf is obtained using (5) and numerically integrating (19).

To compute the SU mean capacity, we differentiate (19) with respect to $\tilde{y}$ to find the pdf

$$\begin{aligned} f_{\gamma_{\rm I}}(x) = &-\sigma_{\rm p}^2e^{-\left(\frac{x\sigma_{\rm p}^2}{P_{\rm m}\Omega_{\rm s}} + \frac{\psi}{\Omega_{\rm p}}\right)} + \frac{\Omega_{\rm sp}\gamma_{\rm T}\sigma_{\rm p}^2\ln(\alpha)}{\Omega_{\rm s}}\int_{\psi_0}^{\psi} e^{-\left(\frac{\Omega_{\rm sp}\gamma_{\rm T}\sigma_{\rm p}^2\ln(\alpha)x}{\gamma_{\rm T}\sigma_{\rm p}^2\Omega_{\rm s} - P_{\rm p}\Omega_{\rm s}z} + \frac{z}{\Omega_{\rm s}}\right)} \\ &\times \left(\gamma_{\rm T}\sigma_{\rm p}^2\Omega_{\rm s} + \Omega_{\rm sp}\gamma_{\rm T}P_{\rm p}\Omega_{\rm ps}\ln(\alpha)x - P_{\rm p}\Omega_{\rm s}z\right)^{-2} \\ &\times \left(\left(\gamma_{\rm T}\sigma_{\rm p}^2\Omega_{\rm s} - P_{\rm p}\Omega_{\rm s}z\right)\left(\Omega_{\rm sp}\gamma_{\rm T}P_{\rm p}\Omega_{\rm ps}\ln\alpha - 1\right) + \Omega_{\rm sp}\gamma_{\rm T}P_{\rm p}\Omega_{\rm ps}\ln(\alpha)x\right)dz. \end{aligned} \quad (20)$$

The mean capacity is then computed by substituting (20) into (4) and numerically integrating.

C. Scenario 3

In Scenario 3, where the SU has exact knowledge of g_{sp} and knows only the mean Ω_{p}, we once again satisfy the SINR constraint with a probability of $1-\alpha$. Hence

$$\Pr\left(\frac{P_{\mathrm{p}}g_{\mathrm{p}}}{P_{\mathrm{s}}g_{\mathrm{sp}}+\sigma_{\mathrm{p}}^2}\geq\gamma_{\mathrm{T}}\middle|\Omega_{\mathrm{p}},g_{\mathrm{sp}}\right)=1-\alpha. \tag{21}$$

Following the same approach as for Scenario 2, one can show that

$$P_{\mathrm{s}}=-\left(\frac{\ln(1-\alpha)P_{\mathrm{p}}\Omega_{\mathrm{p}}}{\gamma_{\mathrm{T}}}+\sigma_{\mathrm{p}}^2\right)\frac{1}{g_{\mathrm{sp}}}. \tag{22}$$

From (22), the SU SINR cdf $F_{\gamma_{\mathrm{I}}}(\tilde{y})$ can be derived as (see [17, App. A] for details)

$$F_{\gamma_I}(\tilde{y})=1-s(\tilde{y})-h(\tilde{y})\mathrm{E}_1\left(r(\tilde{y})\right) \tag{23}$$

where $s(\tilde{y})$, $h(\tilde{y})$, and $r(\tilde{y})$ are given by

$$s(\tilde{y})=\frac{K_1e^{-b\tilde{y}}}{1+a\tilde{y}},\quad h(\tilde{y})=\frac{K_2e^{-b\tilde{y}+r(\tilde{y})}}{\tilde{y}}$$
$$r(\tilde{y})=\frac{(P_{\mathrm{p}}\Omega_{\mathrm{ps}}\tilde{y}+P_{\mathrm{m}}\Omega_{\mathrm{s}})\left(\sigma_{\mathrm{s}}^2\Omega_{\mathrm{sp}}\tilde{y}+Q\Omega_{\mathrm{s}}\right)}{P_{\mathrm{m}}P_{\mathrm{p}}\Omega_{\mathrm{s}}\Omega_{\mathrm{ps}}\Omega_{\mathrm{sp}}\tilde{y}} \tag{24}$$

with constants $K_1=1-e^{Q/P_{\mathrm{m}}\Omega_{\mathrm{sp}}}$, $K_2=Q\Omega_{\mathrm{s}}e^{Q/P_{\mathrm{m}}\Omega_{\mathrm{sp}}}/P_{\mathrm{p}}\Omega_{\mathrm{ps}}\Omega_{\mathrm{sp}}$, $a=Pp\Omega_{\mathrm{ps}}/P_{\mathrm{m}}\Omega_{\mathrm{s}}$, and $b=\sigma_{\mathrm{s}}^2/P_{\mathrm{m}}\Omega_{\mathrm{s}}$. Hence

$$f_{\gamma_I}(\tilde{y})=-s'(\tilde{y})h'(\tilde{y})\mathrm{E}_1\left(r(\tilde{y})\right)+h(\tilde{y})r'(\tilde{y})\frac{e^{-r(\tilde{y})}}{r(\tilde{y})} \tag{25}$$

where $s'(\tilde{y})$, $h'(\tilde{y})$ and $r'(\tilde{y})$ are the derivatives of (24).

D. Scenario 4

When the SU-Tx has knowledge of only Ω_{p} and Ω_{sp}, then we have

$$\Pr\left(\frac{P_{\mathrm{p}}g_{\mathrm{p}}}{P_{\mathrm{s}}g_{\mathrm{sp}}+\sigma_{\mathrm{p}}^2}\geq\gamma_{\mathrm{T}}\middle|\Omega_{\mathrm{p}},\Omega_{\mathrm{sp}}\right)=1-\alpha. \tag{26}$$

Using conditioning and after some manipulation, P_{s} can be derived as

$$P_{\mathrm{s}}=\frac{P_{\mathrm{p}}\Omega_{\mathrm{p}}}{\gamma_{\mathrm{T}}\Omega_{\mathrm{sp}}}\left(\frac{e^{-\frac{\gamma_{\mathrm{T}}\sigma_{\mathrm{p}}^2}{P_{\mathrm{p}}\Omega_{\mathrm{p}}}}}{1-\alpha}-1\right). \tag{27}$$

Here, P_{s} and P_{t} are deterministic, depending on the system parameters. The latter is given by

$$P_{\mathrm{t}}=\begin{cases}0, & P_{\mathrm{s}}<0\\ P_{\mathrm{s}}, & 0<P_{\mathrm{s}}<P_{\mathrm{m}}\\ P_{\mathrm{m}}, & P_{\mathrm{s}}>P_{\mathrm{m}}.\end{cases} \tag{28}$$

Using (28), we obtain the cdf of $\gamma=P_{\mathrm{t}}g_{\mathrm{s}}$, which, when substituted into (6) and (5), results in

$$F_C(y)=1-\frac{P_{\mathrm{t}}\Omega_{\mathrm{s}}}{\tilde{y}P_{\mathrm{p}}\Omega_{\mathrm{ps}}+P_{\mathrm{t}}\Omega_{\mathrm{s}}}e^{-\frac{\tilde{y}\sigma_{\mathrm{s}}^2}{P_{\mathrm{t}}\Omega_{\mathrm{s}}}}. \tag{29}$$

To compute the mean capacity $\bar{C}$, we note that $F_{\gamma_I}(x)$ can be trivially derived from (29) and (5). Differentiating to obtain $f_{\gamma_I}(x)$ and substituting into (4), one obtains

$$\bar{C}=\frac{1}{\ln(2)}\int_1^\infty\left(\frac{\sigma_{\mathrm{s}}^2}{P_{\mathrm{p}}\Omega_{\mathrm{ps}}t+P_{\mathrm{t}}\Omega_{\mathrm{s}}}+\frac{P_{\mathrm{t}}P_{\mathrm{p}}\Omega_{\mathrm{s}}\Omega_{\mathrm{ps}}}{(P_{\mathrm{p}}\Omega_{\mathrm{ps}}t+P_{\mathrm{t}}\Omega_{\mathrm{s}})^2}\right)$$
$$\times\ln(t)e^{\frac{-t\sigma_{\mathrm{s}}^2}{P_{\mathrm{t}}\Omega_{\mathrm{s}}}}\,dt \tag{30}$$

where we have used the change of variable $t=1+x$.

E. Scenario 5

Here, the SU-Tx operates on estimates of g_{p} and g_{sp}. In such a case, we aim to satisfy

$$\Pr\left(P_{\mathrm{p}}g_{\mathrm{p}}\geq\gamma_{\mathrm{T}}P_{\mathrm{s}}g_{\mathrm{sp}}+\gamma_{\mathrm{T}}\sigma_{\mathrm{s}}^2|\hat{g}_{\mathrm{p}},\hat{g}_{\mathrm{sp}}\right)=1-\alpha \tag{31}$$

which must be solved for P_{s}. We use the classic model for imperfect CSI [9] given by $\hat{h}=\rho h+\sqrt{1-\rho^2}\epsilon$, where h is a generic channel coefficient, ρ controls the level of CSI, ϵ is statistically identical to h, and $\hat{g}=|\hat{h}|^2$. The complexity of (31) makes further capacity analysis infeasible. Instead, (31) is derived in [17, App. B] and is shown to be equivalent to

$$\sum_{j=0}^{\infty}\frac{(\lambda_1/2)^j}{j!}e^{-\lambda_1/2}\left(1-e^{-\frac{\lambda_2+\beta}{2}}e^{\frac{\lambda_2}{4(\delta+1)}}\sqrt{\frac{8}{\lambda_2(\delta+1)}}\right.$$
$$\times\sum_{r=0}^{j}\sum_{s=0}^{r}\left(\frac{\beta}{2}\right)^r\frac{\left(\frac{2\delta}{\beta(\delta+1)}\right)^s}{(r-s)!}$$
$$\left.\times M_{-s-1/2,0}\left(\frac{\lambda_2}{2(\delta+1)}\right)\right)=\alpha \tag{32}$$

where $\lambda_1=2\rho^2\hat{g}_{\mathrm{p}}/(\Omega_{\mathrm{p}}(1-\rho^2))$, $\lambda_2=2\rho^2\hat{g}_{\mathrm{sp}}/(\Omega_{\mathrm{sp}}(1-\rho^2))$, $\beta=2\sigma_{\mathrm{s}}^2\gamma_{\mathrm{T}}/(\Omega_{\mathrm{p}}(1-\rho^2)P_{\mathrm{p}})$, $\delta=\gamma_{\mathrm{T}}P_{\mathrm{s}}\Omega_{\mathrm{sp}}/(\Omega_{\mathrm{p}}P_{\mathrm{p}})$, and $M_{\mu,\nu}(\cdot)$ is a Whittaker function. P_{s} is then computed using a numerical root finder to solve (32), and the resulting value is used in capacity simulations.

F. SU Blocking

Using the results in Sections III-A–III-D, we derive the SU blocking conditions, that is, the probability or condition under which the SU cannot transmit due to the constraint (1).

In the case of Scenarios 1 and 2, where P_{s} is dependent on the instantaneous value of g_{p}, via (7) and (15), respectively, we can compute the probability of SU blocking by solving for $\Pr(P_{\mathrm{t}}\leq 0)$ or equivalently $\Pr(P_{\mathrm{s}}\leq 0)$. It is easily shown that for Scenarios 1 and 2

$$\Pr\left(P_{\mathrm{t}}\leq 0\right)=1-e^{-\frac{\gamma_{\mathrm{T}}\sigma_{\mathrm{p}}^2}{\Omega_{\mathrm{p}}P_{\mathrm{p}}}}=1-e^{-c_2}. \tag{33}$$

For Scenarios 3 and 4, the SU blocking condition is determined purely from the system parameters and can be obtained by setting $P_{\mathrm{s}}\leq 0$ in (22) and (27), respectively. Here, the SU blocking condition is related to α and c_2 by

$$P_{\mathrm{t}}=0\quad\text{if}\quad\alpha\leq 1-e^{-\frac{\gamma_{\mathrm{T}}\Omega_{\mathrm{p}}}{\sigma_{\mathrm{p}}^2P_{\mathrm{p}}}}=1-e^{-c_2}. \tag{34}$$

Using (34), we note that for small values of α, that is, where we guarantee the PU SINR constraint with high probability, the SU blocking condition is approximated by $\alpha\leq c_2$.

For Scenario 5, blocking occurs when (31) cannot be satisfied, even for $P_{\mathrm{s}}=0$. Hence, the boundary of the blocking region is equivalent to

$$\Pr\left\{\Pr\left(g_{\mathrm{p}}\geq\frac{\gamma_{\mathrm{T}}\sigma_{\mathrm{p}}^2}{P_{\mathrm{p}}}\middle|\hat{g}_{\mathrm{p}}\right)=1-\alpha\right\}. \tag{35}$$

In [17], the probability in (35) is rewritten as

$$\Pr\left(X\geq\frac{2c_2}{1-\rho^2}\middle|\hat{g}_{\mathrm{p}}\right)=1-\alpha \tag{36}$$

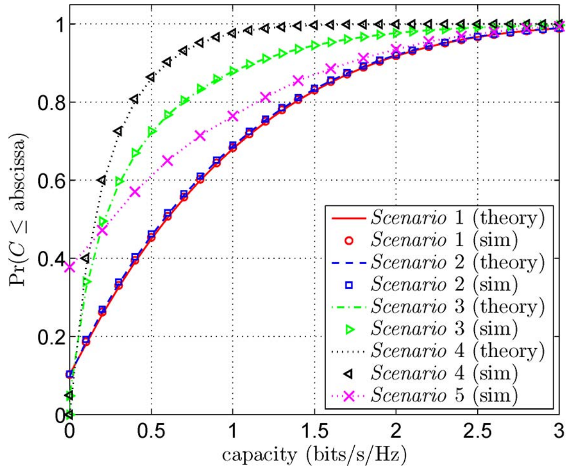

Fig. 2. SU capacity cdf for Scenarios 1–5 ($c_1 = c_2 = 0.1$).

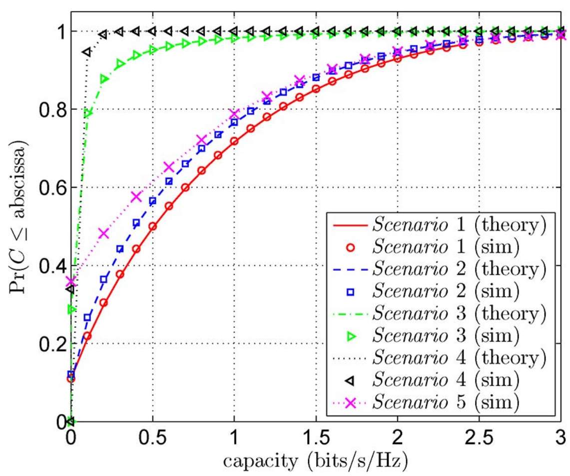

Fig. 3. SU capacity cdf for Scenarios 1–5 ($c_1 = 0.9$ and $c_2 = 0.1$).

where X is a noncentral Chi-squared variable with 2 degrees of freedom and noncentrality parameter $2\rho^2\hat{g}_\mathrm{p}/(\Omega_\mathrm{p}(1-\rho^2))$. We solve (36) by a simple root-finder to find the threshold value $\hat{g}_\mathrm{p} = g^*$, which satisfies (36). Then, the blocking probability is simply

$$\Pr(\hat{g}_\mathrm{p} < g^*) = 1 - e^{-g^*/\Omega_\mathrm{p}}. \tag{37}$$

IV. Simulation Results and Discussion

In all simulations, we have set $P_\mathrm{p}/\sigma_\mathrm{p}^2 = P_\mathrm{m}/\sigma_\mathrm{s}^2 = 0$ dB and $\Omega_\mathrm{p}/\sigma_\mathrm{p}^2 = \Omega_\mathrm{s}/\sigma_\mathrm{s}^2 = 5$ dB, where we assume $\sigma_\mathrm{p}^2 = \sigma_\mathrm{s}^2$. In Scenarios 2–5, we set $\alpha = 0.1$, and $\rho = 0.9$ is used in Scenario 5, unless otherwise indicated in the figures. Figs. 2, 3, and 5 show the SU capacity cdfs for various scenarios and a range of c_1, c_2 values. Fig. 5, with $c_1 = 0.01$, represents very favorable SU operating conditions. Fig. 3 $(c_1 = 0.1,\ c_1 = 0.9)$ represents increasingly difficult conditions for the SU.

From these results, we observe that Scenarios 1 and 2 result in similar performance, even in the case of $c_1 = 0.9$ (see Fig. 3), that is, where the SU interference is very prominent.

Furthermore, lack of knowledge of the PU–PU link (knowing only the mean Ω_p) greatly reduces the achievable capacity of the SU. This is shown in Figs. 2 and 3, where Scenarios 3 and 4 suffer a considerable loss in comparison to Scenarios 1 and 2. Hence, knowledge of g_p is more important than g_sp.

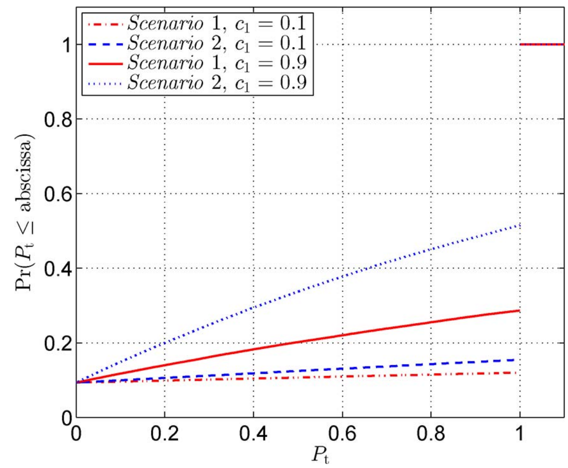

Fig. 4. SU transmit power cdf for Scenarios 1 and 2 ($c_1 = 0.1$ and $c_1 = 0.9$).

The dependence on c_1 can be observed by comparing Figs. 2 and 3. Under very favorable conditions, i.e., $c_1 = 0.01$, Scenarios 3 and 4 slightly outperform Scenarios 1 and 2. This seemingly counterintuitive result is due to the flexibility afforded by the probabilistic SINR constraint. This is confirmed by the additional cdfs for Scenarios 3 and 4 in Fig. 3, with $\alpha = 0.096$, where the protection of PU SINR with higher degree of certainty causes degradation of performance for Scenarios 3 and 4 below that for Scenarios 1 and 2.

From Fig. 3, we observe that placing the SU in a demanding environment $c_1 = 0.9$ results in very poor performance under Scenarios 3 and 4. Furthermore, the performance of Scenario 2 is noticeably degraded from that of Scenario 1. Further insight into this is provided by Fig. 4, which shows the cdf of the SU transmit power P_t for $c_1 = 0.1$ and $c_1 = 0.9$. We observe that in the latter case, the SU-Tx under Scenario 1 operates at maximum power $P_\mathrm{t} = 1$, with a likelihood of 70%, compared to approximately 50% for Scenario 2. This difference is much less pronounced for the less challenging case of $c_1 = 0.1$. Finally, based on Figs. 2 and 3, we observe that the performance under Scenario 5 is not highly dependent on the value of c_1.

Comparing the curves for Scenarios 3 and 4 with those for Scenario 5 in Fig. 3, we note that, for the most part, imperfect knowledge of the link gains is more beneficial than knowledge of their means. Only in the low capacity regime we observe that Scenarios 3 and 4 outperform Scenario 5, which has a relatively high blocking probability for the parameters considered. It should be noted, however, that blocking in Scenarios 3 and 4 is dictated by the parameter c_2, and thus, unless (34) is satisfied, the capacity cdfs for these scenarios originate at zero. Consequently, at higher capacity values, there exists a crossover point with Scenario 5.

Figs. 2 and 3 compare the scenarios using $c_2 = 0.1$, which is very generous to the SU. From (34), we see that SU transmission in Scenarios 3 and 4 occurs only for large values of α or for small values of c_2. That is, without knowledge of g_p, the SU can only operate if the PU is willing to accept large amounts of interference. Fig. 5 presents the capacity results for Scenarios 1 and 2 with the more realistic values of $c_2 = 0.5$ and $c_2 = 0.9$, where (34) prevents SU transmission under Scenarios 3 and 4. While SU transmission is possible under Scenario 5, we observe a high blocking probability of 0.73 and 0.88 for $c_2 = 0.5$ and $c_2 = 0.9$, respectively.

Fig. 6 shows the probability $\Pr(C \leq 0.5)$ as a function of c_1. As expected, for a constant c_2, the performance under Scenario 2 diverges from the baseline Scenario 1 with increasing c_1, that is, as the amount of interference to the PU increases.

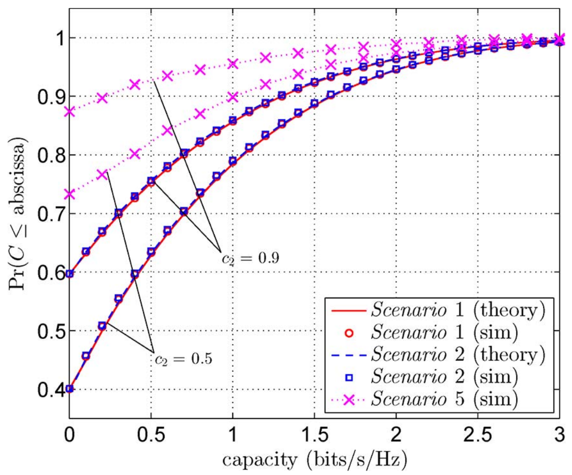

Fig. 5. SU capacity cdf for Scenarios 1, 2, and 5 ($c_1 = 0.01$, $c_2 = 0.5$, and $c_2 = 0.9$).

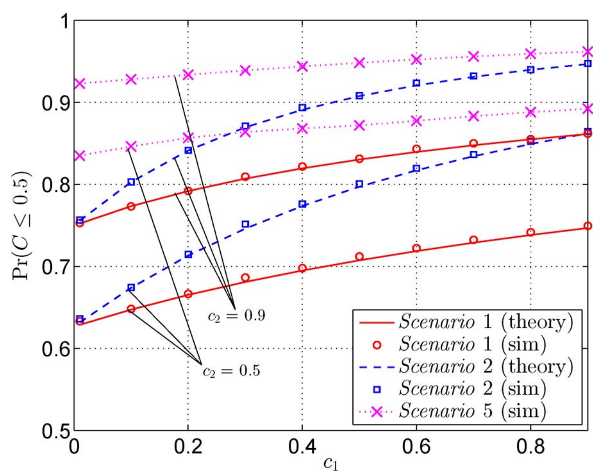

Fig. 6. SU capacity outage values for Scenarios 1, 2, and 5 versus c_1 ($c_2 = 0.5$ and $c_2 = 0.9$).

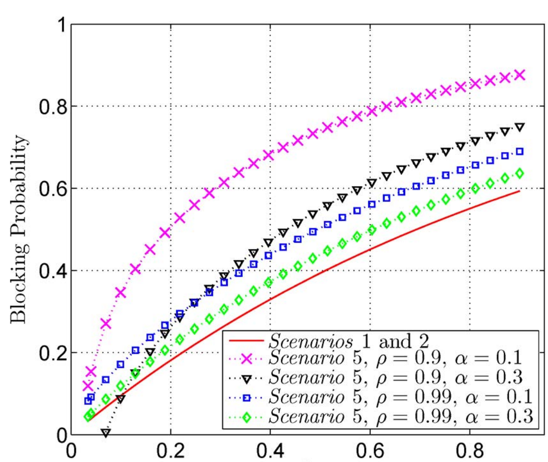

Fig. 7. Blocking probability for Scenarios 1, 2, and 5 versus c_2.

Finally, Fig. 7 shows the blocking probability for Scenarios 1, 2, and 5. We recall that the SU ability to transmit in Scenarios 3 and 4 is deterministic and governed by the blocking condition of (34). The results for Scenario 5 were obtained numerically via (36). We observe that as the channel knowledge error decreases ($\rho \to 1$), the blocking probability approaches that of Scenarios 1 and 2. Specifically, referring back to Fig. 5, we note from Fig. 7 where $\rho = 0.9$ that improving the channel estimate to $\rho = 0.99$ will reduce the blocking probability at $c_2 = 0.5$ and $c_2 = 0.9$ to 0.5 and 0.7, respectively, thus bringing the performance of Scenario 5 closer to that of Scenario 1. Similarly, relaxing the probabilistic SINR constraint by increasing α to 0.3 results in a significant reduction in blocking probability, as fully expected.

V. Conclusion

We have examined the effects of limited channel knowledge on the SU capacity. Considering five scenarios, we derived (in four cases) analytical expressions for the SU capacity cdf under a PU-Rx SINR constraint. We determined the SU blocking probability and blocking conditions as a function of the allowable interference at the PU-Rx. The results demonstrate the importance of the PU–PU CSI, which was shown to be much greater than that of the SU-Tx to PU-Rx link. Furthermore, we have shown that in challenging situations or in the presence of CSI error, there can be extremely large blocking probabilities for the SUs.

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Unified Analysis of Transmit Antenna Selection in MIMO Multirelay Networks

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***Abstract*—We present a unified asymptotic framework for transmit antenna selection in multiple-input multiple-output (MIMO) multirelay networks with Rician, Nakagami-m, Weibull, and generalized-$\mathcal{K}$ fading channels. We apply this framework to derive new closed-form expressions for the outage probability and symbol error rate (SER) of amplify-and-forward (AF) relaying in MIMO multirelay networks with two distinct protocols: 1) transmit antenna selection with receiver maximal-ratio combining (TAS/MRC) and 2) transmit antenna selection with receiver selection combining (TAS/SC). Based on these expressions, the diversity order and the array gain with M-ary phase-shift keying and M-ary quadrature-amplitude modulation are derived. We corroborate that the diversity order only depends on the fading distribution and the number of diversity branches, whereas the array gain depends on the fading distribution, the modulation format, the number of diversity branches, and the average per-hop signal-to-noise ratios (SNRs). We highlight that the diversity order of TAS/MRC is the same as TAS/SC, regardless of the underlying fading distribution. As such, we explicitly characterize the SNR gap between TAS/MRC and TAS/SC as the ratio of their respective array gains. An interesting observation is reached that for equal per-hop SNRs, the SNR gap between the two protocols is independent of the number of relays.**

***Index Terms*—Multiple-input multiple-output (MIMO), relays, transmit antenna selection with receive maximal-ratio combining (TAS/MRC), transmit antenna selection with receiver selection combining (TAS/SC).**

I. Introduction

Wireless relaying is currently under consideration in 3GPP LTE and IEEE 802.16m as a cost-effective application to support high data rates at the cell edge [1]. Indeed, the deployment of energy-efficient relays is acknowledged as a key strategy for coverage extension and capacity enhancement in future heterogeneous wireless networks [2]. In pursuit of peak data rates between 100 Mb/s and 1 Gb/s, recent standardization efforts have considered multiple-input–multiple-output (MIMO) processing in relay networks with up to four transmit/receive antennas at the base station, relay node, and user equipment [3].

MIMO relaying has attracted widespread interest in recent literature. One approach to MIMO relaying is transmit beamforming, which activates all the antennas at the source and the relays [4], [5]. In [4], the MIMO relay broadcast channel is analyzed, and a low-complexity beamforming optimization algorithm is proposed. Practical beamforming protocols are developed in [5] that are robust to limited and imperfect channel state information. An economical alternative to beamforming is antenna selection, which activates a single transmit and/or receive antenna. Antenna selection offers reduced feedback overhead, low implementation complexity, and low signal processing cost while preserving the diversity benefits of MIMO.

In this paper, we present a unified analytical framework for MIMO multirelay networks with two distinct antenna selection protocols, namely: 1) transmit antenna selection with receive maximal-ratio combining (TAS/MRC) and 2) transmit antenna selection with receive selection combining (TAS/SC). TAS/MRC and TAS/SC relaying have been previously considered in single-relay networks [6]–[9]. In TAS/MRC relaying, a single transmit antenna at the source and the relay offering the highest received signal-to-noise ratio (SNR) are selected, and all the receive antennas at the relay and destination are combined with MRC [6], [7]. In TAS/SC relaying, a single antenna pair that achieves the highest received SNR is selected in the source-to-relay link and the relay-to-destination link [8], [9]. The asymptotic analysis of single-antenna multirelay networks over Rician and Nakagami-m fading channels was previously presented in [10] and [11]. Recently, the performance of relaying in other fading distributions such as Rician, Weibull, and generalized-$\mathcal{K}$ fading has also come into consideration [12]. Indeed, these distributions have been proven to offer a close fit to experimental measurements in indoor and outdoor fading environments [13], [14]. In this paper, we consider TAS/MRC and TAS/SC relaying in a wide range of important fading channels, including Rician, Nakagami-m, Weibull, and generalized-$\mathcal{K}$.

Our aim is to address the following fundamental questions: *1) How much improvement does TAS/MRC offer relative to TAS/SC in MIMO multirelay networks?* and *2) What are the explicit network parameters that determine this improvement with higher order modulations* and *generalized fading environments?* Our response is in the form of new easy-to-calculate asymptotic expressions for the outage probability and symbol error rate (SER) of MIMO multirelay networks with M-ary phase-shift keying (M-PSK) and M-ary quadrature-amplitude modulation (M-QAM). Our results are valid for arbitrary L amplify-and-forward (AF) relays and N_{S}, N_{R}, and N_{D} antennas at the source, relays, and destination, respectively. Using these expressions, we explicitly characterize the diversity orders of TAS/MRC and TAS/SC for Rician, Nakagami-m, Weibull, and generalized-$\mathcal{K}$ fading channels. We confirm that the two protocols exhibit the same diversity order for each of the aforementioned fading channels. Due to this, one can conclude that the SNR gap between TAS/MRC and TAS/SC relaying is strictly characterized by their array gains. We highlight the interesting observation that the SNR gap is independent of L for equal per-hop SNRs. We concisely express this SNR gap in terms of the fading parameters and the number of antennas N_{S}, N_{R}, and N_{D}.

II. Protocol Description

Here, we detail two transmit-antenna-selection protocols for use in MIMO multirelay networks where the source transmits to the

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