Sensing-Throughput Tradeoff for Interweave Cognitive Radio System: A Deployment-Centric Viewpoint

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Abstract

Secondary access to the licensed spectrum is viable only if interference is avoided at the primary system. In this regard, different paradigms have been conceptualized in the existing literature. Of these, interweave systems (ISs) that employ spectrum sensing have been widely investigated. Baseline models investigated in the literature characterize the performance of IS in terms of a sensing-throughput tradeoff, however, this characterization assumes the knowledge of the involved channels at the secondary transmitter, which is unavailable in practice. Motivated by this fact, we establish a novel approach that incorporates channel estimation in the system model, and consequently investigate the impact of imperfect channel knowledge on the performance of the IS. More particularly, the variation induced in the detection probability affects the detector's performance at the secondary transmitter, which may result in severe interference at the primary users. In this view, we propose to employ average and outage constraints on the detection probability, in order to capture the performance of the IS. Our analysis reveals that with an appropriate choice of the estimation time determined by the proposed

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approach, the degradation in performance of the IS can be effectively controlled, and subsequently the achievable secondary throughput can be significantly enhanced.

Index Terms

Cognitive radio, Interweave system, Sensing-throughput tradeoff, Spectrum Sensing, Channel estimation

I. Introduction

We are currently in the phase of conceptualizing the requirements of the fifth generation (5G) of mobile wireless systems. One of the major goals is to improve the areal capacity (bits/s/m^2) by a factor of 1000 [2]. A large contribution of this demand is procured by means of an extension to the existing spectrum. Recently, the spectrum beyond $6\,$GHz, which largely entails the millimeter wave is envisaged as a powerful source of spectrum for 5G systems. However, the millimeter wave technology is still in its nascent stage and along with complex regulatory requirements in this regime, it is surmounted by key challenges like propagation loss, low efficiency of radio frequency components such as power amplifiers, small size of the antenna and link acquisition that need to be addressed [3]. Therefore, to capture a deeper insight of its feasibility in 5G, it is essential to overcome the aforementioned challenges in the forthcoming future.

In contrast to the spectrum beyond $6\,$GHz, the spectrum below $6\,$GHz, which is appropriate especially for mobile communications, presents an alternative solution. Due to its static allocation, this spectrum is on the verge of depletion. However, it is possible to overcome this scarcity if we manage to utilize this radio spectrum efficiently. In this perspective, cognitive radio (CR) is foreseen as one of the potential contenders that addresses the problem of spectrum scarcity. Over the past one and a half decade, this notion has evolved at a tremendous pace right from its origin by Mitola *et al.* in 1999 [4] and consequently, it has acquired certain maturity. However, from a deployment perspective, this technology is still in its preliminary phase. In this view, it is imperative to make substantial efforts that enable the placement of this concept over a hardware platform.

An access to the licensed spectrum is an outcome to the paradigm employed by the secondary user (SU). Based on the paradigms described in the literature, all CR systems that provide dynamic access to the spectrum fall mainly under three categories, namely, interweave, underlay and overlay systems [5]. In interweave systems (ISs), the SUs render an interference-free access to the licensed spectrum by exploiting spectral holes in different domains such as time, frequency, space and polarization, whereas underlay systems enable an interference-tolerant access under which the SUs are allowed to use the licensed spectrum (e.g. Ultra Wide Band) as long as they respect the interference constraints of the primary receivers (PRs). Besides that, overlay systems consider participation of higher layers for enabling spectral coexistence between two or more wireless networks. Due to its ease of deployment, IS is mostly preferred not only for performing theoretical analysis but for practical implementation as well. Motivated by these facts, this paper focuses on the performance analysis of the ISs from a deployment perspective.

A. *Motivation and Related Work*

Spectrum sensing is an integral part of ISs. At the secondary transmitter (ST), sensing is necessary for detecting the presence or absence of a primary signal, thereby protecting the PRs against harmful interference. Sensing at the ST is accomplished by listening to the signal transmitted by the primary transmitter (PT). For detecting a primary signal, several techniques such as energy, matched filtering, cyclostationary and feature-based detection exist [6], [7]. Because of its versatility towards unknown primary signals and its low computational complexity, energy detection has been extensively investigated in the literature [8]–[12]. In energy detection, the decision is accomplished by comparing the power received at the ST to a threshold. In reality, the ST encounters variation in the received power due to existence of thermal noise at the receiver and channel fading. This leads to sensing errors described as misdetection or false alarm, that limits the performance of the IS. In order to determine the performance of the detector, it is essential to characterize the expressions of detection probability and false alarm probability.

In particular, detection probability is critical for ISs because it protects the PR from the interference induced by the ST. As a result, the ISs have to ensure that they operate above a target detection probability [13]. Therefore, the characterization of the detection probability becomes absolutely necessary for the performance analysis of the IS. In this context, Urkowitz [8] introduced a probabilistic model that establishes a fundamental framework for characterizing the sensing errors, however, the characterization accounts only for noise in the system. To encounter the variation caused by channel fading, a frame structure is introduced such that the channel is considered to remain constant over the frame duration, however, upon exceeding the frame duration, the system may witness a different realization of the channel. Based on this frame structure, the performance of the IS has been investigated in terms of deterministic channel [14]–[16] and random channel[1] [9]–[11]. Complementing the analysis in [14]–[16], in this paper, we consider the involved channels to be deterministic.

Besides detection probability, false alarm probability has a large influence on the throughput attained by the secondary system at the secondary receiver (SR). Recently, the performance characterization of CR systems in terms of sensing-throughput tradeoff has received significant attention [14], [16]–[18]. According to Liang *et al.* [14], the ST assures a reliable detection of a primary signal by retaining the detection probability above a desired level with an objective of maximizing the throughput at the SR. In this way, the sensing-throughput tradeoff delivers a suitable sensing time that achieves a maximum throughput. However, to characterize the detection probability and throughput, the system requires the knowledge of interacting channels, namely, a *sensing* channel, an *access* channel and an *interference* channel, refer Fig. 1[2]. To the best of the authors' knowledge, the baseline models investigated in the literature assume the knowledge of these channels to be available at the ST. However, in practice, this knowledge is not available, thus, needs to be estimated at the ST. As a result, from a deployment perspective, the existing solutions for the IS are considered inaccurate for performing analysis.

Following the previous discussion, it is apparent that the knowledge of the sensing, interference and access channels is crucial for characterizing the detection probability and the secondary throughput. In practice, this knowledge can be estimated either (i) directly by using the conventional channel estimation techniques such as training sequence based [19] and pilot based [20], [21] channel estimations or (ii) indirectly by estimating the received signal to noise ratio [22], [23]. Since the sensing and interference channels represent the channels between two different (priamry and secondary) systems, it becomes necessary to select the estimation methods in such a way that low complexity and versatility (towards different PU signal) requirements are satisfied, this issue render the existing estimation techniques [22], [23] unsuitable for the

[1]In literature, deterministic and random channel are interpreted as path-loss and fading channel, respectively.

[2]As the interference to the PR is controlled by a regulatory constraint over the detection probability, in this view, the interaction with the PR is excluded in the considered scenario [14].

hardware implementations. To this end, we propose a received power based estimation at the ST and the SR. Since the access channel corresponds to link between the ST and the SR, we propose to employ conventional channel estimation techniques such as pilot based channel estimation at the SR. Inherent to the estimation process, the variations due the channel estimation translate to the performance parameters, namely detection probability and secondary throughput. In particular, the variations induced in the detection probability may result in harmful interference at the PR, hence, severely degrade the performance of a CR system. In this sense, performance characterizing of an IS with imperfect channel knowledge remains an open problem. Hence, to characterize the performance of the IS in terms of sensing-throughput tradeoff, it is essential to capture these variations in an accurate manner.

B. Contributions

The major contributions of this paper can be summarized as follows:

- The main goal of the paper is to derive an analytical framework that constitutes the estimation of: (i) sensing channel at the ST, (ii) access and (iii) interference channels at the SR. With the inclusion of estimation, the system witnesses variations in the performance parameters and a certain performance loss. This work investigates these two aspects and characterizes the performance of the IS under realistic conditions. We propose a received power estimation for the sensing and interference channels, which offers a good tradeoff between reliability of the estimates, complexity and applicability of the proposed framework to larger range of primary systems, in order words, facilitates the hardware realizability of the proposed framework.
- To capture the variations induced in the system, we characterize the distribution functions of performance parameters such as detection probability and achievable secondary throughput. More importantly, by utilizing the distribution function of the detection probability to propose new primary user (PU) constraints on the detection probability. In this way, the proposed approach is able to control the level of interference at the PR due to the imperfect channel knowledge. Subject to the new constraints, we establish expressions of sensing-throughput trade-off that captures these variations and evaluates the performance loss in terms of the achievable throughput of the IS.

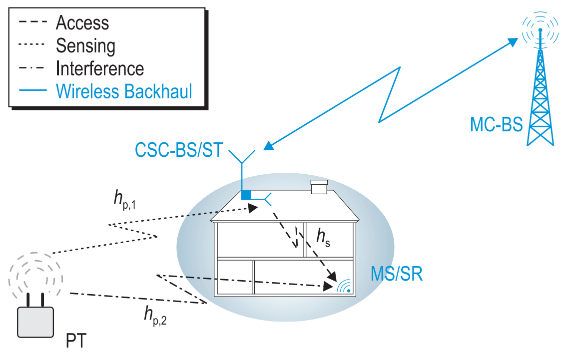

Fig. 1. A cognitive small cell scenario demonstrating: (i) the interweave paradigm, (ii) the associated network elements, which constitute cognitive small cell-base station/secondary transmitter (CSC-BS/ST), mobile station/secondary receiver (MS/SR), macro cell-base station (MC-BS) and primary transmitter (PT), (iii) the interacting channels: sensing, access and interference.

- Finally, we depict a fundamental tradeoff between estimation time, sensing time and achievable secondary throughput. We exploit this tradeoff to determine a suitable estimation and sensing time that depicts the maximum achievable performance of the IS.

C. *Organization*

The subsequent sections of the paper are organized as follows: Section II describes the system model that includes the deployment scenario and the signal model. Section III presents the problem description and the proposed approach that incorporates channel estimation. Section IV characterizes the distribution of the performance parameters and establishes the sensing-throughput tradeoff subject to average and outage constraints. Section V analyzes the numerical results based on the obtained expressions. Finally, Section VI concludes the paper. Table I lists the definitions of acronyms and important mathematical notations used throughput the paper.

II. System Model

A. *Deployment Scenario*

Cognitive small cell (CSC), a CR application, characterizes a small cell deployment that fulfills the spectral requirements for mobile stations (MSs) operating indoor, refer Fig. 1. For the disposition of the CSC in the network, the following key elements are essential: a CSC-base station (CSC-BS), a macro cell-base station (MC-BS) and MS, refer Fig. 1. MSs are the indoor devices served by the CSC-BS over an access channel (h_{s}). Furthermore, the MC-BS

TABLE I

DEFINITIONS OF ACRONYMS AND NOTATIONS USED

Acronyms and Notations	**Definitions**
AC, OC	average constraint, outage constraint
CR	cognitive radio
CSC, CSC-BS, MC-BS, MS	cognitive small cell, cognitive small cell-base station, macro cell-base station, mobile station
IM, EM	ideal model, estimation model
IS	interweave system
PU - PT, PR	primary user - primary transmitter, primary receiver
SU - ST, SR	secondary user - secondary transmitter, secondary receiver
$\mathcal{H}_1, \mathcal{H}_0$	Signal plus noise hypothesis, noise only hypothesis
f_{s}	Sampling frequency
$\tau_{\mathrm{est}}, \tau_{\mathrm{sen}}$	Estimation time, sensing time interval
T	Frame duration
$\mathrm{P_d}, \mathrm{P_{fa}}$	Probability of detection, false alarm probability
$\bar{\mathrm{P}}_{\mathrm{d}}$	Target detection probability
κ	Outage constraint over detection probability
$h_{\mathrm{p,1}}, h_{\mathrm{p,2}}, h_{\mathrm{s}}$	Channel coefficient for the link PT-ST, PT-SR, ST-SR
$\gamma_{\mathrm{p,1}}, \gamma_{\mathrm{p,2}}, \gamma_{\mathrm{s}}$	Signal to noise ratio for the link PT-ST, PT-SR, ST-SR
R_{s}	Throughput at SR
$\mathrm{C}_0, \mathrm{C}_1$	Shanon capacity at SR without and with interference from PT
μ	Threshold for the energy detector
$F_{(\cdot)}$	Cumulative distribution function of random variable $(\cdot)$
$f_{(\cdot)}$	Probability density function of random variable $(\cdot)$
$\hat{(\cdot)}$	Estimated value of $(\cdot)$
$\tilde{(\cdot)}$	Suitable value of the parameter $(\cdot)$ that achieves maximum performance
$\mathbb{E}_{(\cdot)}$	Expectation with respect to $(\cdot)$
$\mathbb{P}$	Probability measure
$\mathbf{T}(\cdot)$	Test statistics
σ_x^2, σ_w^2	Signal variance at PT, noise variance at ST and SR
N_{s}	Number of pilot symbols used for pilot based estimation at the SR for h_{s}
$N_{\mathrm{p,2}}$	Number of samples used for received power estimation at the SR for $h_{\mathrm{p,2}}$

is connected to several CSC-BSs over a wireless backhaul[3]. Moreover, the transmissions from the PT can be listened by the CSC-BS and the MS over sensing ($h_{\text{p},1}$) and interference channel ($h_{\text{p},2}$), respectively. Considering the fact that the IS is employed at the CSC-BS, the CSC-BS and the MS represent ST and SR, respectively. A hardware prototype of the CSC-BS operating as IS was presented in [24]. For simplification, a PU constraint based on false alarm probability was considered in [24]. With the purpose of improving system's reliability, we extend the analysis to employ a PU constraint on the detection probability.

As a follow-on from the ideal model depicted in [14], we consider a slotted medium access for the IS, where the time axis is segmented into frames of length T, according to which, the ST employs periodic sensing. Hence, each frame consists of a sensing slot τ_{sen} and the remaining duration $T - \tau_{\text{sen}}$ is utilized for data transmission. For small T relative to the PU's expected ON/OFF period, the requirement of the ST to be in alignment to PUs' medium access can be relaxed [25]–[27].

B. Signal model

Subject to the underlying hypothesis that illustrates the presence ($\mathcal{H}_1$) or absence ($\mathcal{H}_0$) of a primary signal, the discrete and real signal received at the ST is given by

$$y_{\text{ST}}[n] = \begin{cases} h_{\text{p},1} \cdot x_{\text{PT}}[n] + w[n] & : \mathcal{H}_1 \\ w[n] & : \mathcal{H}_0 \end{cases}, \tag{1}$$

where $x_{\text{PT}}[n]$ corresponds to a discrete and real sample transmitted by the PT, $|h_{\text{p},1}|^2$ represents the power gain of the sensing channel for a given frame and $w[n]$ is additive white Gaussian noise at the ST. According to [14], the signal $x_{\text{PT}}[n]$ transmitted by the PUs can be modelled as: (i) phase shift keying modulated signal, or (ii) Gaussian signal. The signals that are prone to high inter-symbol interference or entail precoding can be modelled as Gaussian signals. For this paper, we focus our analysis on the latter case. As a result, mean and variance for the signal and the noise are determined as $\mathbb{E}\left[x_{\text{PT}}[n]\right] = 0$, $\mathbb{E}\left[w[n]\right] = 0$, $\mathbb{E}\left[|x_{\text{PT}}[n]|^2\right] = \sigma_x^2$ and $\mathbb{E}\left[|w[n]|^2\right] = \sigma_w^2$. The channel $h_{\text{p},1}$ is considered to be independent of $x_{\text{PT}}[n]$ and $w[n]$, thus, y_{ST} is also an independent and identically distributed (i.i.d.) random process.

[3]A wireless backhaul is a point-to-point wireless link between the CSC-BS and MC-BS that relays the traffic generated from the CSC to the core network.

Similar to (1), during data transmission, the discrete and real received signal at the SR conditioned on the detection probability ($\mathrm{P_d}$) and false alarm probability ($\mathrm{P_{fa}}$) is given by

$$y_{\mathrm{SR}}[n] = \begin{cases} h_{\mathrm{s}} \cdot x_{\mathrm{ST}}[n] + h_{\mathrm{p,2}} \cdot x_{\mathrm{PT}}[n] + w[n] & : 1 - \mathrm{P_d} \\ h_{\mathrm{s}} \cdot x_{\mathrm{ST}}[n] + w[n] & : 1 - \mathrm{P_{fa}} \end{cases}, \tag{2}$$

where $x_{\mathrm{ST}}[n]$ corresponds to discrete and real sample transmitted by the ST. Further, $|h_{\mathrm{s}}|^2$ and $|h_{\mathrm{p,2}}|^2$ represent the power gains for access and interference channels, refer Fig. 1.

III. Problem Description and Proposed Approach

A. *Problem Description*

In accordance to conventional frame structure, ST performs sensing for a duration of τ_{sen}. The test statistics $\mathrm{T}(\mathbf{y})$ at the ST is evaluated as

$$\mathrm{T}(\mathbf{y}) = \frac{1}{\tau_{\mathrm{sen}} f_{\mathrm{s}}} \sum_{n=1}^{\tau_{\mathrm{sen}} f_{\mathrm{s}}} |y_{\mathrm{ST}}[n]|^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \mu, \tag{3}$$

where μ is the decision threshold and $\mathbf{y}$ is a vector with $\tau_{\mathrm{sen}} f_{\mathrm{s}}$ samples. $\mathrm{T}(\mathbf{y})$ represents a random variable, whereby the characterization of the distribution function depends on the underlying hypothesis. Corresponding to $\mathcal{H}_0$ and $\mathcal{H}_1$, $\mathrm{T}(\mathbf{y})$ follows a central chi-squared ($\mathcal{X}^2$) distribution [28]. As a result, the detection probability ($\mathrm{P_d}$) and the false alarm probability ($\mathrm{P_{fa}}$) corresponding to (3) are determined as [29]

$$\mathrm{P_d}(\mu, \tau_{\mathrm{sen}}, P_{\mathrm{Rx,ST}}) = \Gamma\left(\frac{\tau_{\mathrm{sen}} f_{\mathrm{s}}}{2}, \frac{\tau_{\mathrm{sen}} f_{\mathrm{s}} \mu}{2 P_{\mathrm{Rx,ST}}}\right), \tag{4}$$

$$\mathrm{P_{fa}}(\mu, \tau_{\mathrm{sen}}) = \Gamma\left(\frac{\tau_{\mathrm{sen}} f_{\mathrm{s}}}{2}, \frac{\tau_{\mathrm{sen}} f_{\mathrm{s}} \mu}{2 \sigma_w^2}\right), \tag{5}$$

where $P_{\mathrm{Rx,ST}}$ is the power received over the sensing channel and $\Gamma(\cdot,\cdot)$ represents a regularized incomplete upper Gamma function [30].

Following the characterization of $\mathrm{P_{fa}}$ and $\mathrm{P_d}$, Liang *et al.* [14] established a tradeoff between the sensing time and secondary throughput (R_{s}) subject to a target detection probability ($\bar{\mathrm{P}}_{\mathrm{d}}$).

This tradeoff is represented as

$$R_{\mathrm{s}}(\tilde{\tau}_{\mathrm{sen}}) = \max_{\tau_{\mathrm{sen}}} R_{\mathrm{s}}(\tau_{\mathrm{sen}}) = \frac{T - \tau_{\mathrm{sen}}}{T}\left[\mathrm{C}_0(1 - \mathrm{P}_{\mathrm{fa}})\mathbb{P}(\mathcal{H}_0) + \mathrm{C}_1(1 - \mathrm{P}_{\mathrm{d}})\mathbb{P}(\mathcal{H}_1)\right], \tag{6}$$

$$\text{s.t. } \mathrm{P}_{\mathrm{d}} \geq \bar{\mathrm{P}}_{\mathrm{d}}, \tag{7}$$

$$\text{where } \mathrm{C}_0 = \log_2\left(1 + |h_{\mathrm{s}}|^2 \frac{P_{\mathrm{Tx,ST}}}{\sigma_w^2}\right) = \log_2\left(1 + \gamma_{\mathrm{s}}\right) \tag{8}$$

$$\text{and } \mathrm{C}_1 = \log_2\left(1 + \frac{|h_{\mathrm{s}}|^2 P_{\mathrm{Tx,ST}}}{|h_{\mathrm{p,2}}|^2 P_{\mathrm{Tx,PT}} + \sigma_w^2}\right) = \log_2\left(1 + \frac{|h_{\mathrm{s}}|^2 P_{\mathrm{Tx,ST}}}{P_{\mathrm{Rx,SR}}}\right) = \log_2\left(1 + \frac{\gamma_{\mathrm{s}}}{\gamma_{\mathrm{p,2}} + 1}\right), \tag{9}$$

where $\mathbb{P}(\mathcal{H}_0)$ and $\mathbb{P}(\mathcal{H}_1)$ are the probabilities of occurrence for the respective hypothesis, whereas $\gamma_{\mathrm{p,2}}$ and γ_{s} correspond to signal to noise ratio for the links PT-SR and ST-SR, respectively. Moreover, $P_{\mathrm{Tx,ST}}$ and $P_{\mathrm{Tx,PT}}$ represent the transmit power at the PT and the ST, whereas $P_{\mathrm{Rx,SR}}$ corresponds to the received power at the SR. In other words, using (6), the ST determines a suitable sensing time $\tau_{\mathrm{sen}} = \tilde{\tau}_{\mathrm{sen}}$, such that the throughput is maximized subject to a target detection probability, refer (7). From the deployment perspective, the tradeoff depicted above has the following fundamental issues:

- Without the knowledge of the received power (sensing channel), it is not feasible to characterize P_{d}. This leaves the characterization of the throughput (6) impossible and the constraint defined in (7) inappropriate.
- Moreover, the knowledge of the interference and access channels is required at the ST, refer (8) and (9) for characterizing the throughput in terms of C_0 and C_1 at the SR.

Taking into account these issues, it is not sensible to employ the performance analysis depicted by the ideal model for hardware implementation. In the subsequent section, we propose an estimation model that addresses these issues, thereby including the estimation of the sensing channel at the ST, and interference and access channels at the SR. Based on the proposed approach, we then investigate the performance of the IS in terms of the sensing-throughput tradeoff.

B. *Proposed Approach*

In order to overcome these difficulties, the following strategy is pursued in this paper. As a first step, we consider the estimation of involved channels. In order to characterize the detection probability, we propose a received power estimation at the ST for the sensing channel, in this way, we ensure that detection probability operates above a desired level. We further propose an estimation of the access channel and the interference channel at the ST and the SR, respectively, to characterize the secondary throughput. Next, we characterize the distribution functions of estimated parameters to capture effect of variations due to channel estimation. In order to investigate the performance of the IS subject to channel estimation, we characterize these variations in the performance parameters, which include detection probability and secondary throughput, in terms of their distribution functions. As a last step, we employ these characterization to present a sensing-throughput tradeoff that includes imperfect channel knowledge, and subsequently determines the achievable secondary throughput at a suitable sensing time.

It is well-known that systems with transmitter information (which includes the filter parameters, pilot symbols, modulation type and time-frequency synchronization) at the receiver acquire channel knowledge by listening to the pilot data sent by the ST [20], [21], [31], [32]. Other systems, where the receiver possesses either no access to this information or limited by hardware complexity, procure channel knowledge indirectly by estimating a different parameter that entails the channel knowledge, for instance, received signal strength or received power. Recently, pilot based estimation [33], [34] and received power estimation [35] have been applied to obtain channel knowledge for CR systems. However, the analysis was restricted to underlay systems, where the emphasis was laid on modelling the interference at the PR. In this paper, we introduce this concept to the IS, hence, we propose to employ pilot based estimation for the access channel and received power based estimation for the sensing and interference channels. Unlike the sensing channel, the access and interference channels have to be estimated at the SR and made available at the ST over a low-rate feedback channel. As a result, by proposing received power estimation and pilot based estimation, we ensure the versatility and the low complexity requirements of the CR system, which are absolutely essential from the deployment perspective.

However, with the inclusion of this estimation, the system anticipates: (i) a performance loss in terms of temporal resources used and (ii) variations in the aforementioned performance parameters due to estimation. A preliminary analysis of this performance loss was carried out in [1], where it was revealed that in low signal to noise ratio regime, imperfect knowledge of received power corresponds to large variation in detection probability, hence, causing a severe degradation in the performance of the IS. However, this performance degradation was determined by means of lower and upper bounds. In this work, we consider a more exact analysis, whereby we capture the variations in detection probability by characterizing its distribution function, and

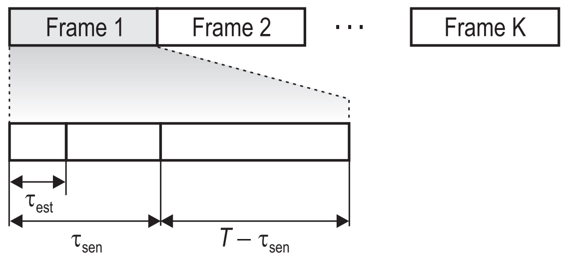

Fig. 2. Frame structure of interweave system with received power estimation.

subsequently apply new probabilistic constraints on the detection probability, which allows ISs to operate at low signal to noise ratio regime.

To include channel estimation, we propose a frame structure that constitutes a estimation τ_{est}, a sensing τ_{sen} and data transmission $T-\tau_{\text{sen}}$, where τ_{est} and τ_{sen} correspond to time intervals and $0 < \tau_{\text{est}} \leq \tau_{\text{sen}} < T$, refer Fig. 2. Since the estimated values of the interacting channels are required for determining the suitable sensing time, the sequence depicted in Fig. 2, whereby estimation followed by sensing is reasonable for the hardware deployment. To avail the estimates for the interference and access channels at the ST, a low-rate feedback channel from the SR to the ST is required for the proposed approach. Particularly for the sensing channel, it is worthy to note that the samples used for estimation can be combined with the samples acquired for sensing[4] such that the time resources within the frame duration are utilized efficiently. In this sense, as proposed in the frame structure in Fig. 2, the characterization of the detector's performance incurs the estimation samples. In the following subsections, we consider the estimation of the involved channels.

1) Estimation of sensing channel ($h_{p,1}$): Following the previous discussions, the ST acquires the knowledge of $h_{\text{p},1}$ by estimating its received power. The estimated received power is required for the characterization of P_{d}, thereby evaluating the detector performance.

Under $\mathcal{H}_1$, the received power estimated during the estimation phase at the ST is given as [8]

$$\hat{P}_{\text{Rx,ST}} = \frac{1}{\tau_{\text{est}} f_{\text{s}}} \sum_{n=1}^{\tau_{\text{est}} f_{\text{s}}} |y_{\text{ST}}[n]|^2. \tag{10}$$

$\hat{P}_{\text{Rx,ST}}$ determined in (10) using $\tau_{\text{est}} f_{\text{s}}$ samples follows a central chi-squared distribution $\mathcal{X}^2$ [28].

[4]That is why, the sensing phase incorporates the estimation phase, see Fig. 2.

The cumulative distribution function (CDF) of $\hat{P}_{\text{Rx,ST}}$ is given by

$$F_{\hat{P}_{\text{Rx,ST}}}(x) = 1 - \Gamma\left(\frac{\tau_{\text{est}} f_{\text{s}}}{2}, \frac{\tau_{\text{est}} f_{\text{s}} x}{2 P_{\text{Rx,ST}}}\right). \tag{11}$$

2) Estimation of access channel (h_s): The signal received from the ST undergoes matched filtering and demodulation at the SR, hence, it is reasonable to employ pilot based estimation for h_{s}. Unlike received power estimation, pilot based estimation renders a direct estimation of the channel. Now, to accomplish pilot based estimation, the SR aligns itself to pilot symbols transmitted by the ST. Under $\mathcal{H}_0$, the discrete and real pilot symbols at the output of the demodulator is given by [21]

$$p[n] = \sqrt{E_{\text{s}}} h_{\text{s}} + w[n], \tag{12}$$

where E_{s} denotes the pilot energy. Without loss of generality, the pilot symbols are considered to be +1. The maximum likelihood estimate, representing a sample average of N_{s} pilot symbols, is given by [20]

$$h_{\text{s}} = \hat{h}_{\text{s}} + \underbrace{\frac{\sum_{n}^{N_{\text{s}}} p[n]}{2 N_{\text{s}}}}_{\epsilon}, \tag{13}$$

where ϵ denotes the estimation error. The estimate $\hat{h}_{\text{s}}$ is unbiased, efficient and achieves a Cramér-Rao bound with equality, with variance $\mathbb{E}\left[|h_{\text{s}} - \hat{h}_{\text{s}}|^2\right] = \sigma_w^2/(2N_{\text{s}})$ [21]. Consequently, $\hat{h}_{\text{s}}$ conditioned on h_{s} follows a Gaussian distribution.

$$\hat{h}_{\text{s}} | h_{\text{s}} \sim \mathcal{N}\left(h_{\text{s}}, \frac{\sigma_w^2}{2N_{\text{s}}}\right). \tag{14}$$

As a result, the power gain $|\hat{h}_{\text{s}}|^2$ follows a non-central chi-squared ($\mathcal{X}_1^2$) distribution with 1 degree of freedom and non-centrality parameter $\lambda = \frac{2N_{\text{s}}|h_{\text{s}}|^2}{\sigma_w^2}$.

3) Estimation of interference channel ($h_{p,2}$): In addition, analog to sensing channel, the SR performs received power estimation by listening to the transmission from the PT. The knowledge of $h_{\text{p,2}}$ is required to characterize interference from the PT. Under $\mathcal{H}_1$, the discrete signal model at the SR is given as

$$y_{\text{SR}}[n] = h_{\text{p,2}} \cdot x_{\text{PT}}[n] + w[n]. \tag{15}$$

The received power at the SR from the PT given by

$$\hat{P}_{\text{Rx,SR}} = \frac{1}{N_{\text{p,2}}} \sum_{n=1}^{N_{\text{p,2}}} |y_{\text{SR}}[n]|^2, \tag{16}$$

follows a $\mathcal{X}^2$ distribution, where $N_{\text{p,2}}$ corresponds to the number of samples used for estimation.

C. Validity of the estimates

At this stage, it is known that the estimates $\hat{P}_{\text{Rx,ST}}$, $|\hat{h}_{\text{s}}|^2$ and $\hat{P}_{\text{Rx,SR}}$ exhibit the knowledge corresponding to the involved channels, however, in order to certify their validity, specially $\hat{P}_{\text{Rx,ST}}$ and $\hat{P}_{\text{Rx,SR}}$, it is necessary to ensure the presence of the PU signal for that particular frame. This issue has recently received significant attention [22], [36], whereby Chavali *et al.* [22] proposed a detection followed by estimation of the signal to noise ratio, whereas [36] implemented a blind technique for the estimating signal power of non-coherent PU signals. In this paper, we propose a different methodology, according to which, we apply a coarse detection on the estimates $\hat{P}_{\text{Rx,ST}}$, $\hat{P}_{\text{Rx,SR}}$ at the end of the estimation phase τ_{est}, in this way, we assure the validity of the estimates without trading against the complexity of the estimators employed by the secondary system. Moreover, by performing a joint estimation and (coarse) detection, we propose an efficient way of utilizing the time resources within the frame duration. During the detection phase τ_{sen}, we consider these estimates along with samples acquired during the estimation phase to conduct a fine detection of the PU signals thereby improving the performance of the detector. At system design, a suitable choice of the time interval τ_{est} can be performed to ensure reliability of the coarse detector. With the existence of separate control channel such as cognitive pilot channel, the reliability of the coarse detection can be further enhanced by exchanging the detection results among the ST and the SR.

D. Assumptions and Approximations

To simplify the analysis and sustain analytical tractability for the proposed approach, several assumptions considered in the paper are summarized as follows:

- We consider that all transmitted signals are subjected to distance dependent path loss and small scale fading gain. With no loss of generality, we consider that the channel gains include distance dependent path loss and small scale gain. Moreover, the coherence time for the channel gain is considered to be greater than the frame duration[5].
- We assume perfect knowledge of the noise power in the system, however, the uncertainty in noise power can be captured as a bounded interval [29]. Inserting this interval in the

[5]In the scenarios where the coherence time exceeds the frame duration, in such cases our characterization depicts a lower performance bound.

derived expressions, refer Section IV, the performance of the IS can be expressed in terms of the upper and the lower bounds.

- For all degrees of freedom, $\mathcal{X}_1^2$ distribution can be approximated by Gamma distribution [37]. The parameters of the Gamma distribution are obtained by matching the first two central moments to those of $\mathcal{X}_1^2$.

IV. Theoretical Analysis

At this stage, it is evident that the variation due to imperfect channel knowledge translates to the variations of the performance parameters $\mathrm{P_d}, \mathrm{C_0}$ and $\mathrm{C_1}$, which are fundamental to sensing-throughput tradeoff. Below, we capture these variations by characterizing their cumulative distribution functions $F_{\mathrm{P_d}}$, $F_{\mathrm{C_0}}$ and $F_{\mathrm{C_1}}$, respectively.

Lemma 1: The cumulative distribution function of $\mathrm{P_d}$ is characterized as

$$F_{\mathrm{P_d}}(x) = 1 - \Gamma\left(\frac{\tau_{\mathrm{est}} f_{\mathrm{s}}}{2}, \frac{\tau_{\mathrm{est}} f_{\mathrm{s}} \tau_{\mathrm{sen}} f_{\mathrm{s}} \mu}{4 P_{\mathrm{Rx,ST}} \Gamma^{-1}(\frac{\tau_{\mathrm{sen}}}{2}, x)}\right), \tag{17}$$

where $\Gamma^{-1}(\cdot,\cdot)$ is inverse function of regularized upper Gamma function [30].

Proof: The cumulative distribution function of $\mathrm{P_d}$ is defined as

$$F_{\mathrm{P_d}}(x) = \mathbb{P}(\mathrm{P_d}(\mu, \tau_{\mathrm{sen}}, \hat{P}_{\mathrm{Rx,ST}}) \leq x). \tag{18}$$

Using (4)

$$= \mathbb{P}\left(\Gamma\left(\frac{\tau_{\mathrm{sen}} f_{\mathrm{s}}}{2}, \frac{\tau_{\mathrm{est}} f_{\mathrm{s}} \mu}{2 \hat{P}_{\mathrm{Rx,ST}}}\right) \leq x\right), \tag{19}$$

$$= 1 - \mathbb{P}\left(\hat{P}_{\mathrm{Rx,ST}} \leq \frac{\mu \tau_{\mathrm{sen}} f_{\mathrm{s}}}{2\Gamma^{-1}\left(\frac{\tau_{\mathrm{sen}} f_{\mathrm{s}}}{2}, x\right)}\right). \tag{20}$$

Replacing the cumulative distribution function of $\hat{P}_{\mathrm{Rx,ST}}$ in (20), we obtain an expression of $F_{\mathrm{P_d}}$. ■

Lemma 2: The cumulative distribution function of $\mathrm{C_0}$ is defined as

$$F_{\mathrm{C_0}}(x) = \int_0^x f_{\mathrm{C_0}}(t) dt, \tag{21}$$

where

$$f_{\mathrm{C_0}}(x) = 2^x \ln 2 \frac{(2^x - 1)^{a_1 - 1}}{\Gamma(a_1) b_1^{a_1}} \exp\left(-\frac{2^x - 1}{b_1}\right), \tag{22}$$

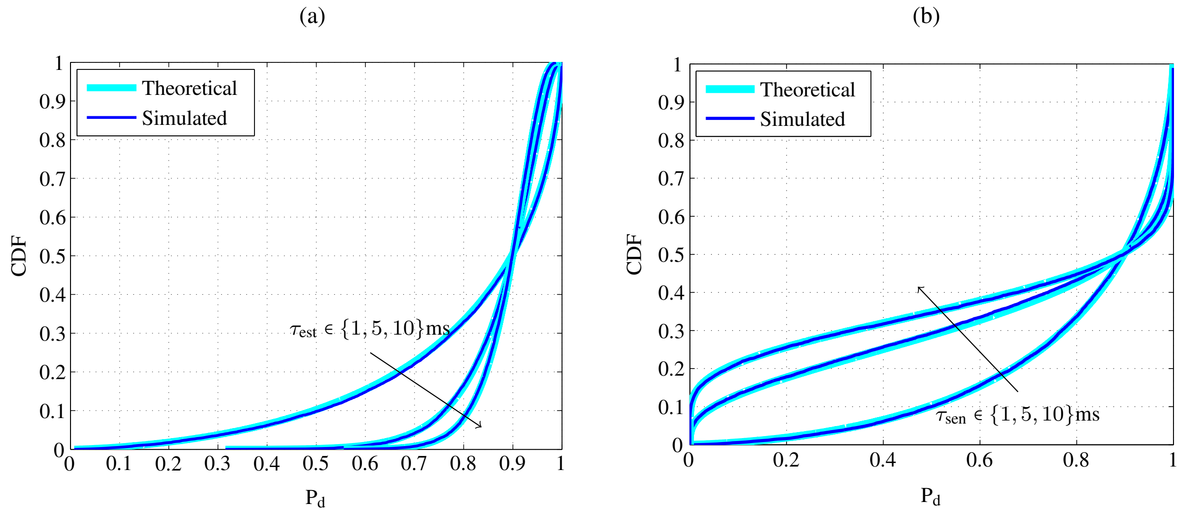

Fig. 3. CDF of $\mathrm{P_d}$ for different τ_{est} and τ_{sen}. (a) $\tau_{\mathrm{est}} \in \{1, 5, 10\}$ms and $\tau_{\mathrm{sen}} = 1$ ms, (b) $\tau_{\mathrm{est}} = 1$ ms and $\tau_{\mathrm{sen}} \in \{1, 5, 10\}$ms.

and

$$a_1 = \frac{\left(\frac{\sigma_w^4}{2N_\mathrm{s}P_{\mathrm{Tx,ST}}} + |h_\mathrm{s}|^2\right)^2}{\frac{\sigma_w^4}{2N_\mathrm{s}P_{\mathrm{Tx,ST}}}\left(2\frac{\sigma_w^4}{2N_\mathrm{s}P_{\mathrm{Tx,ST}}} + 4|h_\mathrm{s}|^2\right)} \text{ and } b_1 = \frac{\frac{\sigma_w^4}{2N_\mathrm{s}P_{\mathrm{Tx,ST}}}\left(2\frac{\sigma_w^4}{2N_\mathrm{s}P_{\mathrm{Tx,ST}}} + 4|h_\mathrm{s}|^2\right)}{\left(\frac{\sigma_w^4}{2N_\mathrm{s}P_{\mathrm{Tx,ST}}} + |h_\mathrm{s}|^2\right)}. \tag{23}$$

Proof: Following the probability density function (pdf) of $|\hat{h}_\mathrm{s}|^2$ in (14), the pdf $|\hat{h}_\mathrm{s}|^2\frac{P_{\mathrm{Tx,ST}}}{\sigma_w^2}$ is given by

$$f_{\frac{|\hat{h}_\mathrm{s}|^2 P_{\mathrm{Tx,ST}}}{\sigma_w^2}}(x) = \frac{2N_\mathrm{s}P_{\mathrm{Tx,ST}}}{\sigma_w^4}\frac{1}{2}\exp\left[-\frac{1}{2}\left(x\frac{\sigma_w^4}{2N_\mathrm{s}P_{\mathrm{Tx,ST}}} + \lambda\right)\right]\left(\frac{x}{\lambda}\frac{\sigma_w^4}{2N_\mathrm{s}P_{\mathrm{Tx,ST}}}\right)^{\frac{N_\mathrm{s}}{4}-\frac{1}{2}}$$
$$\times I_{\frac{N_\mathrm{s}}{2}-1}\left(\sqrt{\lambda x\frac{\sigma_w^4}{2N_\mathrm{s}P_{\mathrm{Tx,ST}}}}\right),$$

where $I_{(\cdot)}(\cdot)$ represents the modified Bessel function of first kind [30]. Approximating $\mathcal{X}_1^2(\cdot,\cdot)$ with Gamma distribution $\Gamma(a_1, b_1)$ [37] gives

$$f_{\frac{|\hat{h}_\mathrm{s}|^2 P_{\mathrm{Tx,ST}}}{\sigma_w^2}} \approx \frac{1}{\Gamma(a_1)}\frac{x^{a_1-1}}{b_1^{a_1}}\exp\left(-\frac{x}{b_1}\right), \tag{24}$$

where the parameters a_1 and b_1 in (24) are determined by comparing the first two central moments of the two distributions. Finally, by substituting the expression of C_0 in (8) yields (22). ■

Lemma 3: The cumulative distribution function of C_1 is given by

$$F_{\mathrm{C}_1}(x) = \int_0^x f_{\mathrm{C}_1}(t)dt, \tag{25}$$

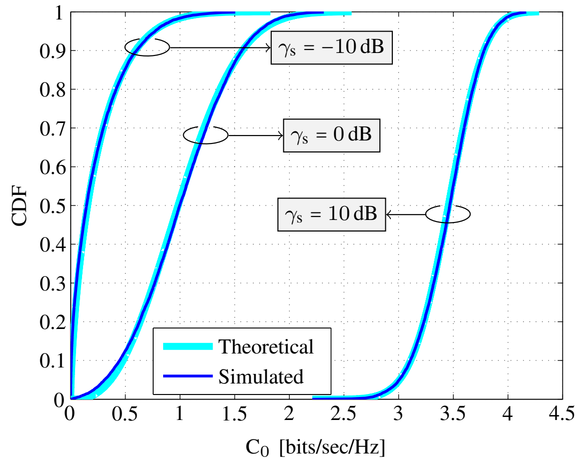

Fig. 4. CDF of C_0 for different values of $\gamma_s \in \{-10, 0, 10\}$dB.

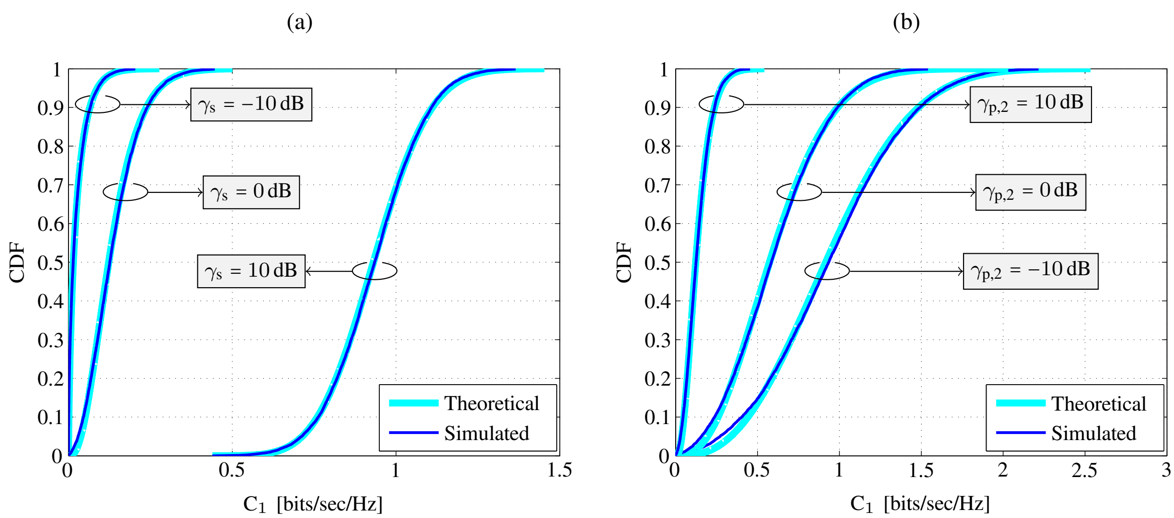

Fig. 5. CDF of C_1 for different γ_s and $\gamma_{p,2}$. (a) $\gamma_s \in \{-10, 0, 10\}$dB and $\gamma_{p,2} = 10\,$dB, (b) $\gamma_s = 0\,$dB and $\gamma_{p,2} \in \{-10, 0, 10\}$dB.

where

$$f_{C_1}(x) = 2^x \ln 2 \frac{(2^x - 1)^{a_1 - 1}\Gamma(a_1 + a_2)}{\Gamma(a_1)\Gamma(a_2) b_1^{a_1} b_2^{a_2}} \left(\frac{1}{b_2} + \frac{2^x - 1}{b_1} \right), \tag{26}$$

and

$$a_2 = \frac{N_{p,2}}{2} \text{ and } b_2 = \frac{2 P_{Rx,SR}}{\sigma_w^2 N_{p,2}}, \tag{27}$$

where a_1 and b_1 are defined in (23).

Proof: See Appendix A. ■

The theoretical expressions of the distribution functions depicted in Lemma 1, Lemma 2 and Lemma 3 are validated by means of simulations in Fig. 3, Fig. 4 and Fig. 5, respectively, with different choices of system parameters, these include $\tau_{\text{est}} \in \{1, 5, 10\}$ms, $\tau_{\text{sen}} = \{1, 5, 10\}$ms, $\gamma_{\text{s}} \in \{-10, 0, 10\}$dB and $\gamma_{\text{p,2}} \in \{-10, 0, 10\}$dB.

A. Sensing-throughput tradeoff

Here, we establish sensing-throughput tradeoff for the estimation model that includes estimation time and incorporates variations in the performance parameter. Most importantly, by capturing these variations, we establish two new PU constraints at the PR, namely, an average constraint and an outage constraint on the detection probability. These constraints restrain the harmful interference at the PR. Based on these constraints and a certain choice of estimation time τ_{est}, we characterize the sensing-throughput tradeoff for the IS.

Theorem 1: Subject to an average constraint on P_{d} at the PR, the sensing-throughput tradeoff is given by

$$\begin{aligned} R_{\text{s}}(\tilde{\tau}_{\text{est}}, \tilde{\tau}_{\text{sen}}) &= \max_{\tau_{\text{est}}, \tau_{\text{sen}}} \mathbb{E}_{\text{P}_{\text{d}}, \text{C}_0, \text{C}_1} \left[R_{\text{s}}(\tau_{\text{est}}, \tau_{\text{sen}}) \right], \\ &= \frac{T - \tau_{\text{sen}}}{T} \Big[\mathbb{E}_{\text{C}_0}\left[\text{C}_0\right](1 - \text{P}_{\text{fa}})\mathbb{P}(\mathcal{H}_0) + \mathbb{E}_{\text{C}_1}\left[\text{C}_1\right](1 - \mathbb{E}_{\text{P}_{\text{d}}}\left[\text{P}_{\text{d}}\right])\mathbb{P}(\mathcal{H}_1) \Big], \qquad (28) \\ \text{s.t. } & \mathbb{E}_{\text{P}_{\text{d}}}\left[\text{P}_{\text{d}}\right] \leq \bar{\text{P}}_{\text{d}}, \qquad (29) \\ \text{s.t. } & 0 < \tau_{\text{est}} \leq \tau_{\text{sen}} \leq T, \end{aligned}$$

where $\mathbb{E}_{\text{P}_{\text{d}}}[\cdot]$ represents the expectation with respect to P_{d}, $\mathbb{E}_{\text{P}_{\text{d}}, \text{C}_0, \text{C}_1}[\cdot]$ denotes the expectation with respect to P_{d}, C_0 and C_1. Unlike (7), $\bar{\text{P}}_{\text{d}}$ in (28) represents the constraint on expected detection probability.

Proof: See Appendix B. For simplification, the proof of Theorem 1 is included in the proof of Theorem 2. ■

Theorem 2: Subject to an outage constraint on P_{d} at the PR, the sensing-throughput tradeoff is given by

$$\begin{aligned} R_{\text{s}}(\tilde{\tau}_{\text{est}}, \tilde{\tau}_{\text{sen}}) &= \max_{\tau_{\text{est}}, \tau_{\text{sen}}} \mathbb{E}_{\text{P}_{\text{d}}, \text{C}_0, \text{C}_1} \left[R_{\text{s}}(\tau_{\text{est}}, \tau_{\text{sen}}) \right], \\ &= \frac{T - \tau_{\text{sen}}}{T} \Big[\mathbb{E}_{\text{C}_0}\left[\text{C}_0\right](1 - \text{P}_{\text{fa}})\mathbb{P}(\mathcal{H}_0) + \mathbb{E}_{\text{C}_1}\left[\text{C}_1\right](1 - \mathbb{E}_{\text{P}_{\text{d}}}\left[\text{P}_{\text{d}}\right])\mathbb{P}(\mathcal{H}_1) \Big], \qquad (30) \\ \text{s.t. } & \mathbb{P}(\text{P}_{\text{d}} \leq \bar{\text{P}}_{\text{d}}) \leq \kappa, \qquad (31) \\ \text{s.t. } & 0 < \tau_{\text{est}} \leq \tau_{\text{sen}} \leq T, \end{aligned}$$

where κ represents the outage constraint.

Proof: See Appendix B. ∎

As a consequence, for a certain estimation time, the sensing-throughput tradeoff that depicts the variation of expected throughput against the sensing time is established based on the average and outage constraints. In contrast to the ideal model, the sensing-throughput tradeoff substantiated by the estimation model, which incorporates the channel knowledge is qualified for characterizing the performance of IS.

Remark 1: Herein, based on the estimation model, we establish a fundamental relation between estimation time (regulates the variation in the detection probability according to the PU constraint), sensing time (represents the detector performance) and achievable throughput, this relationship is characterized as estimation-sensing-throughput tradeoff. Based on this tradeoff, we determine the suitable estimation $\tau_{\text{est}} = \tilde{\tau}_{\text{est}}$ and sensing time $\tau_{\text{sen}} = \tilde{\tau}_{\text{sen}}$ that attains a maximum achievable throughput $R_{\text{s}}(\tilde{\tau}_{\text{est}}, \tilde{\tau}_{\text{sen}})$ for the IS.

Corollary 1: Theorems 1 and 2 consider the optimization of the average throughput to incorporate the effect of variations due to channels estimation, and subsequently determine the suitable sensing and estimation time. Here, we investigate an alternative approach to the optimization problem described in (6) to capture these variations. According to which, for a certain estimation time τ_{est}, the suitable sensing time subject to the average constraint is determined as

$$\begin{aligned} \tilde{\tau}_{\text{sen}} &= \underset{\tau_{\text{sen}}}{\operatorname{argmax}}\, R_{\text{s}}(\tau_{\text{est}}, \tau_{\text{sen}}), \\ &= \frac{T - \tau_{\text{sen}}}{T}\left[\mathrm{C}_0(1 - \mathrm{P}_{\text{fa}})\mathbb{P}(\mathcal{H}_0) + \mathrm{C}_1(1 - \mathrm{P}_{\text{d}})\mathbb{P}(\mathcal{H}_1)\right], \\ &\text{s.t. } \mathbb{E}_{\mathrm{P}_{\text{d}}}[\mathrm{P}_{\text{d}}] \le \bar{\mathrm{P}}_{\text{d}}, \\ &\text{s.t. } 0 < \tau_{\text{est}} \le \tau_{\text{sen}} \le T. \end{aligned} \tag{32}$$

Similarly, the suitable sensing time subject to the outage constraint is determined as

$$\begin{aligned} \tilde{\tau}_{\text{sen}} &= \underset{\tau_{\text{sen}}}{\operatorname{argmax}}\, R_{\text{s}}(\tau_{\text{est}}, \tau_{\text{sen}}), \\ &= \frac{T - \tau_{\text{sen}}}{T}\left[\mathrm{C}_0(1 - \mathrm{P}_{\text{fa}})\mathbb{P}(\mathcal{H}_0) + \mathrm{C}_1(1 - \mathrm{P}_{\text{d}})\mathbb{P}(\mathcal{H}_1)\right], \\ &\text{s.t. } \mathbb{P}(\mathrm{P}_{\text{d}} \le \bar{\mathrm{P}}_{\text{d}}) \le \kappa, \\ &\text{s.t. } 0 < \tau_{\text{est}} \le \tau_{\text{sen}} \le T. \end{aligned} \tag{33}$$

Now, the suitable sensing time evaluated in (32) and (33) entails the variations due to channel estimation. Hence, the average secondary throughput subject to the average and outage constraint that captures the variations in suitable sensing time and performance parameters is determined as

$$\mathbb{E}_{\mathrm{P_d},\mathrm{C_0},\mathrm{C_1},\tilde{\tau}_{\mathrm{sen}}}\left[R_{\mathrm{s}}(\tau_{\mathrm{est}},\tilde{\tau}_{\mathrm{sen}})\right]. \tag{34}$$

Following Remark 1, we further optimize the average throughput defined in (34) over the estimation time

$$R_{\mathrm{s}}(\tilde{\tau}_{\mathrm{est}},\tilde{\tau}_{\mathrm{sen}}) = \max_{\tau_{\mathrm{est}}} \mathbb{E}_{\mathrm{P_d},\mathrm{C_0},\mathrm{C_1},\tilde{\tau}_{\mathrm{sen}}}\left[R_{\mathrm{s}}(\tau_{\mathrm{est}},\tilde{\tau}_{\mathrm{sen}})\right], \tag{35}$$

where $\mathbb{E}_{\mathrm{P_d},\mathrm{C_0},\mathrm{C_1},\tilde{\tau}_{\mathrm{sen}}}[\cdot]$ corresponds to an expection over $\mathrm{P_d}, \mathrm{C_0}, \mathrm{C_1}, \tilde{\tau}_{\mathrm{sen}}$. In this way, we establish an estimation-sensing-throughput tradeoff for the alternative approach to determine the suitable estimation time.

Proof: According to [14], there exists no closed-form expression of $\tilde{\tau}_{\mathrm{sen}}$, this renders the analytical tractability of its distribution function difficult. In this view, we capture the performance of the alternative approach by means of simulations. ■

V. Numerical Results

Here, we investigate the performance of the IS based on the proposed approach. To accomplish this: (i) we perform simulations to validate the expressions obtained, (ii) we analyze the performance loss incurred due to the estimation. In this regard, we consider the ideal model for benchmarking and evaluating the performance loss, (iii) we establish mathematical justification to the considered approximations. Although, the expressions derived using our sensing-throughput analysis are general and applicable to all CR systems, the parameters are selected in such a way that they closely relate to the deployment scenario described in Fig. 1. Unless stated explicitly, the choice of the parameters given in Table II is considered for the analysis.

Firstly, we analyze the performance of the IS in terms of sensing-throughput tradeoff corresponding to the ideal model (IM) and estimation model (EM) by fixing $\tau_{\mathrm{est}} = 5\,\mathrm{ms}$, refer Fig. 6. In contrast to constraint on $\mathrm{P_d}$ for the ideal model, we employ average constraint (EM-AC) and outage constraint (EM-OC) for the proposed estimation model. With the inclusion of received power estimation in the frame structure, the ST procures no throughput at the SR for the interval

TABLE II

PARAMETERS FOR NUMERICAL ANALYSIS

Parameter	**Value**
f_{s}	1 MHz
$h_{\mathrm{p,1}}, h_{\mathrm{p,2}}$	−100 dB
h_{s}	−80 dB
T	100 ms
$\bar{\mathrm{P}}_{\mathrm{d}}$	0.9,
κ	0.05
σ_w^2	−100 dBm
$\gamma_{\mathrm{p,1}}$	−10 dB
$\gamma_{\mathrm{p,2}}$	−10 dB
γ_{s}	10 dB
$\sigma_x^2 = P_{\mathrm{Tx,PT}}$	−10 dBm
$P_{\mathrm{Tx,ST}}$	−10 dBm
$\mathbb{P}(\mathcal{H}_1) = 1 - \mathbb{P}(\mathcal{H}_0)$	0.2
τ_{est}	5 ms
N_{s}	10
$N_{\mathrm{p,2}}$	1000

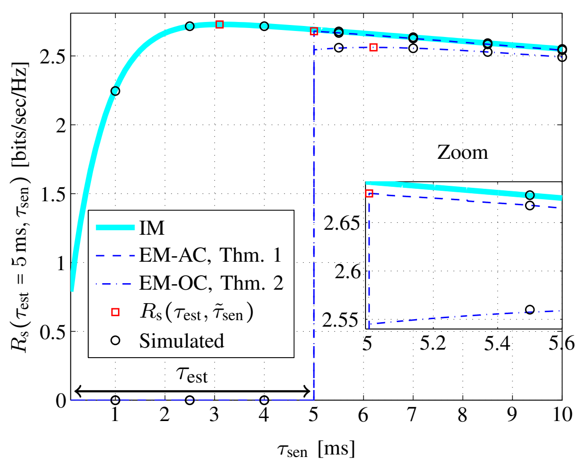

Fig. 6. Sensing-throughput tradeoff for the ideal and estimation models with $\gamma_{\mathrm{p,1}} = -10\,\mathrm{dB}$, $\tau_{\mathrm{est}} = 5\,\mathrm{ms}$ and $\kappa = 0.05$.

τ_{est}. For the given cases, namely, IM, EM-AC and EM-OC, a suitable sensing time that results in a maximum throughput $R_{\mathrm{s}}(\tau_{\mathrm{est}} = 5\,\mathrm{ms}, \tilde{\tau}_{\mathrm{sen}})$ is determined. Hence, a performance degradation is depicted in terms of the achievable throughput, refer Fig. 6. For $\kappa = 0.05$, it is observed that the

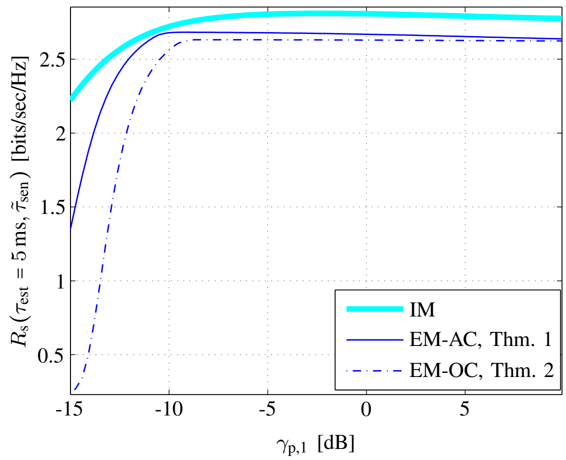

Fig. 7. Achievable throughput versus the $\gamma_{\mathrm{p},1}$ with $\tau_{\mathrm{est}} = 5\,\mathrm{ms}$.

outage constraint is more sensitive to the performance loss in comparison to average constraint. It is clear that the analysis illustrated Fig. 6 is obtained for a certain choice of system parameters, particularly $\gamma_{\mathrm{p},1} = -10\,\mathrm{dB}$, $\tau_{\mathrm{est}} = 5\,\mathrm{ms}$ and $\kappa = 0.05$. To acquire more insights, we consider the effect of variation of these parameters on the performance of IS, subsequently.

Hereafter, for the analysis, we consider the theoretical expressions and choose to operate at suitable sensing time. Next, we determine the variations of the achievable throughput against the received signal to noise ratio $\gamma_{\mathrm{p},1}$ at the ST with $\tau_{\mathrm{est}} = 5\,\mathrm{ms}$, refer Fig. 7. For $\gamma_{\mathrm{p},1} < -5\,\mathrm{dB}$, the estimation model incurs a significant performance loss. This clearly reveals that the ideal model overestimates the performance of IS. Hence, it is perceived that despite loss in performance, the estimation model is capable of precluding interference at the PR, hence, assuring reliability to the system.

Upon maximizing the secondary throughput, it is interesting to analyze the variation of achievable throughput with the estimation time. Corresponding to the estimation model, Fig. 8 illustrates a tradeoff among the estimation time, the sensing time and the throughput, refer Remark 1. This can be explained from the fact that low values of estimation time result in large variation in $\mathrm{P_d}$. To counteract and satisfy the average and outage constraints, the corresponding thresholds shift to a lower value. This causes an increase in $\mathrm{P_{fa}}$, thereby increasing the sensing-throughput curvature. As a result, the suitable sensing time is obtained at a higher value. However, beyond a certain value ($\tilde{\tau}_{\mathrm{est}}$), a further increase in estimation time slightly contributes to performance improvement and largely consumes the time resources. From Fig. 8, it is worthy

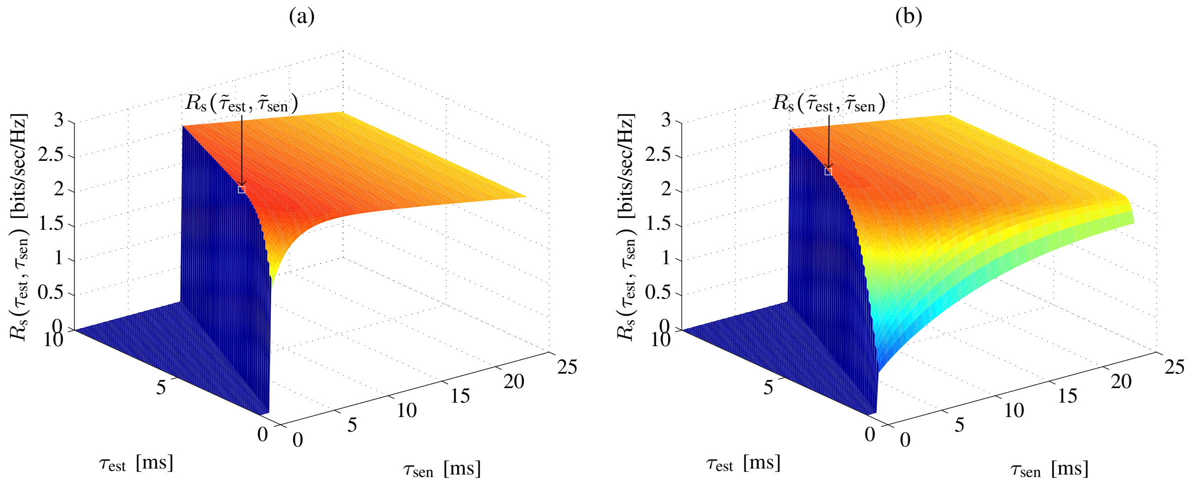

Fig. 8. Estimation-sensing-throughput tradeoff for the estimation model for (a) average constraint and (b) outage constraint with $\kappa = 0.05$.

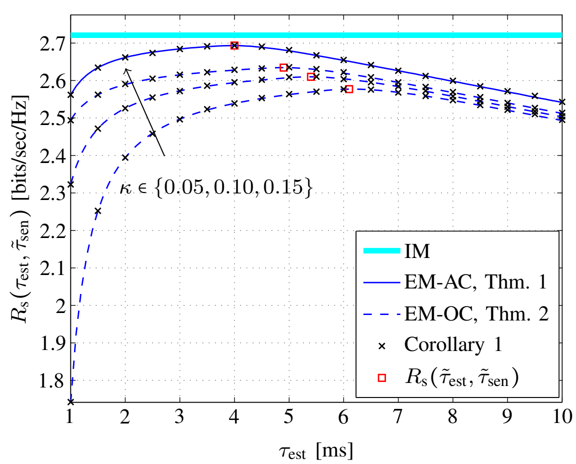

Fig. 9. Estimation-sensing-throughput tradeoff for the average and outage constraints with $\gamma_{p,1} = -10\,\text{dB}$, where the throughput is maximized over the sensing time, $R_s(\tau_{est}, \tilde{\tau}_{sen})$. Estimation-sensing-throughput tradeoff is utilized to determine a suitable estimation time $\tilde{\tau}_{est}$ that maximizes the throughput, $R_s(\tilde{\tau}_{est}, \tilde{\tau}_{sen})$.

to note that the function $R_s(\tau_{est}, \tau_{est})$ is well-behaved in the region $0 < \tau_{est} \leq \tau_{sen} \leq T$ and consists of an optimum value representing global maximum. As a consequence to the estimation-sensing-throughput tradeoff, we determine the suitable estimation time that yields an achievable throughput $R_s(\tilde{\tau}_{est}, \tilde{\tau}_{sen})$.

Besides that, we consider the variation of achievable throughput for different values of the outage constraint, refer Fig. 9. It is observed that for the selected choice of κ, the outage

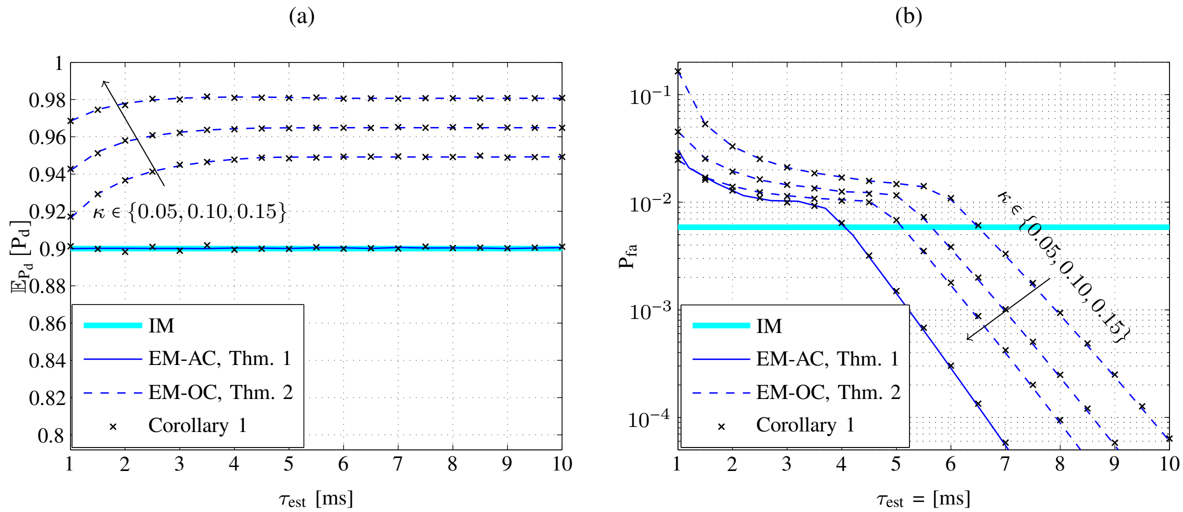

Fig. 10. Variation of $\mathbb{E}_{\mathrm{P_d}}[\mathrm{P_d}]$ and $\mathrm{P_{fa}}$ versus the τ_{est}, where the secondary throughput is maximized over the sensing time, $R_{\mathrm{s}}(\tau_{\mathrm{est}}, \tilde{\tau}_{\mathrm{sen}})$. (a) Expected $\mathrm{P_d}$ versus τ_{est}, (b) $\mathrm{P_{fa}}$ versus τ_{est}.

constraint is severe as compared to the average constraint, hence, results in a lower throughput. Thus, depending on the nature of policy (aggressive or conservative) followed by the regulatory bodies towards interference at the primary system, it is possible to define κ accordingly during the system design. Morover, it is witnessed that the alternative approach proposed in Corollary 1 does not present any noticeable performance difference depicted in terms of achievable throughput corresponding to one charaterized in Theorems 1 and 2.

To procure further insights, we investigate the variations of expected $\mathrm{P_d}$ and $\mathrm{P_{fa}}$ with the estimation time. From Fig. 10a, it is observed that the expected $\mathrm{P_d}$ corresponding to the outage constraint is strictly above the desired level $\bar{\mathrm{P}}_{\mathrm{d}}$ for all values of estimation time, however, for lower values of estimation time, this margin reduces. This is based on the fact that lower estimation time shifts the probability mass of $\mathrm{P_d}$, to a lower value, refer Fig. 3a. Besides that, based on the previous discussion, it was analyzed that $\mathrm{P_{fa}}$ accounts for a large contribution to the throughput. According to Fig. 10b, $\mathrm{P_{fa}}$ witnesses a large improvement in performance in the regime $\tau_{\mathrm{est}} \leq 3\,\mathrm{ms}$, however, saturates in the regime $\tau_{\mathrm{est}} \geq 3\,\mathrm{ms}$, thus, provides further justification to the variation of $R_{\mathrm{s}}(\tau_{\mathrm{est}}, \tilde{\tau}_{\mathrm{sen}})$ against τ_{est} characterized as estimation-sensing-throughput tradeoff depicted in Fig. 9.

VI. Conclusion

In this paper, we have investigated the performance of cognitive radio as an interweave system from a deployment perspective. It has been argued that the knowledge of the interacting channels is a key aspect that enables the performance characterization of the interweave system in terms of sensing-throughput tradeoff. In this regard, a novel model that facilitates channel estimation and captures the effect of estimation in the system model has been proposed. As a major outcome of the analysis, it has been justified that the existing model, illustrating an ideal scenario, overestimates the performance of the interweave system, hence, less suitable for deployment. Moreover, it has been indicated that the variation induced in the system, specially in the detection probability may severely degrade the performance of the primary system. To overcome this situation, average and outage constraints as primary user constraints have been employed. As a consequence, for the proposed estimation model, novel expressions for sensing-throughput tradeoff based on the mentioned constraints have been established. More importantly, by analyzing the estimation-sensing-throughput tradeoff, suitable estimation time and sensing time that maximizes the secondary throughput have been determined. In our future work, we plan to extend the proposed analysis for the hybrid cognitive radio system that combines the advantages of interweave and underlay techniques.

Appendix

A. Proof of Lemma 3

Proof: For simplification, we break down the expression $\left(\frac{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{Tx,ST}}}{\hat{P}_{\mathrm{Rx,SR}}}\right)$ in (9), as $E_1 = \left(\frac{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{Tx,ST}}}{\sigma_w^2}\right)$ and $E_2 = \left(\frac{\hat{P}_{\mathrm{Rx,SR}}}{\sigma_w^2}\right)$, where $\mathrm{C}_1 = \log_2\left(1 + \frac{E_1}{E_2}\right)$. The pdf of the expression E_1 is determined in (24). Following the characterization $\hat{P}_{\mathrm{Rx,SR}}$ in (16), the pdf of E_2 is determined as

$$f_{\frac{\hat{P}_{\mathrm{Rx,SR}}}{\sigma_w^2}} = \frac{N_{\mathrm{p},2}\sigma_w^2}{P_{\mathrm{Rx,SR}}} \frac{1}{2^{\frac{N_{\mathrm{p},2}}{2}}\Gamma\left(\frac{N_{\mathrm{p},2}}{2}\right)} \left(x\frac{N_{\mathrm{p},2}\sigma_w^2}{P_{\mathrm{Rx,SR}}}\right)^{\frac{N_{\mathrm{p},2}}{2}-1} \exp\left(-x\frac{N_{\mathrm{p},2}\sigma_w^2}{2P_{\mathrm{Rx,SR}}}\right). \tag{36}$$

Using the characterizations of pdfs $f_{\frac{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{Tx,ST}}}{\sigma_w^2}}$ and $f_{\frac{\hat{P}_{\mathrm{Rx,SR}}}{\sigma_w^2}}$, we apply Mellin transform [38] to determine the pdf of $\frac{E_1}{E_2}$ as

$$f_{\frac{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{Tx,ST}}}{\sigma_w^2} \Big/ \frac{\hat{P}_{\mathrm{Rx,SR}}}{\sigma_w^2}}(x) = \frac{x^{a_1-1}\Gamma(a_1+a_2)}{\Gamma(a_1)\Gamma(a_2)b_1^{a_1}b_2^{a_2}}\left(\frac{1}{b_2}+\frac{x}{b_1}\right). \tag{37}$$

Finally, substituting the expression $\frac{E_1}{E_2}$ in C_1 yields (26). ■

B. Proof of Theorems 1 and 2

Proof: In order to solve the constrained optimization problems illustrated in Theorem 1 and Theorem 2, the following approach is considered. As a first step, an underlying constraint is employed to determine μ as a function of the τ_{sen} and τ_{est}.

For the average constraint, the expression $\mathbb{E}_{\text{P}_\text{d}}\left[\text{P}_\text{d}\right]$ in (29) did not lead to a closed form expression, consequently, no analytical expression of μ is obtained. In this context, we procure μ for the average constraint numerically from (29).

Next, we determine μ based on the outage constraint. This is accomplished by combining the expression of F_{P_d} in (17) with the outage constraint (31)

$$P(\text{P}_\text{d} \leq \bar{\text{P}}_\text{d}) = F_{\text{P}_\text{d}}(\bar{\text{P}}_\text{d}) \leq \kappa. \tag{38}$$

Rearranging (38) gives

$$\mu \geq \frac{4 P_{\text{Rx,ST}} \Gamma^{-1}\left(1-\kappa, \frac{\tau_{\text{est}} f_\text{s}}{2}\right) \Gamma^{-1}\left(\bar{\text{P}}_\text{d}, \frac{\tau_{\text{sen}} f_\text{s}}{2}\right)}{\tau_{\text{est}} \tau_{\text{sen}} (f_\text{s})^2}. \tag{39}$$

Given that the random variables P_d, C_0 and C_1 are functions of $\hat{P}_{\text{Rx,ST}}$, and $|\hat{h}_\text{s}|^2$ and $\hat{P}_{\text{Rx,SR}}$, which are independent random variables. Applying the independence property to obtain

$$\mathbb{E}_{\text{P}_\text{d},\text{C}_0,\text{C}_1}\left[\text{C}_0(1-\text{P}_{\text{fa}}) + \text{C}_1(1-\text{P}_\text{d})\right] = \mathbb{E}_{\text{C}_0}\left[\text{C}_0\right](1-\text{P}_{\text{fa}}) + \mathbb{E}_{\text{C}_1}\left[\text{C}_1\right]\mathbb{E}_{\text{P}_\text{d}}\left[(1-\text{P}_\text{d})\right]$$

in (28) and (30). Upon replacing the respective thresholds in P_d and P_{fa} and evaluating the expectation over P_d, C_0 and C_1 using the distribution functions characterized in Lemma 1, Lemma 2 and Lemma 3, we determine the expected throughput as a function of sensing and estimation time. ■

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