On the Impact of Outdated Channel Information on the Capacity of Secondary User in Spectrum Sharing Environments

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***Abstract*—Conventional investigations on the capacity of a secondary link in spectrum sharing environments have assumed that a secondary user knows perfect channel information between the secondary transmitter and primary receiver. However, this channel information may be outdated at the secondary user because of the time-varying properties or feedback latency from the primary user. If the secondary user allocates transmission power using this outdated channel information, the interference power to the primary receiver will not satisfy the predetermined interference constraint.**

In this paper, we investigate the impact of outdated channel information between secondary and primary users in spectrum sharing environments. We begin by deriving the ergodic capacity of secondary user along with the optimum power allocation under the average received-power constraint. We also provide a closed-form expression for the ergodic capacity without interference from the primary transmitter, and the capacity bounds with interference from the primary transmitter. Moreover, we provide the power margin required to satisfy the interference outage probability at the primary user under the peak received-power constraint. Lastly, we derive the secondary user's ergodic capacity with and without interference from the primary transmitter. Comparisons done using simulations show the effects of the uncertainty of channel information and interference from the primary transmitter under both constraints.

***Index Terms*—Cognitive radio, spectrum sharing, outdated channel state information, interference constraint.**

I. Introduction

THERE has been considerable interest in recent years in spectrum sharing environments in cognitive radio (CR) systems [1][2]. These systems allow a secondary user to share the spectrum of the primary user if the interference power from the secondary user to the primary user can be limited in spectrum sharing environments [1][3]. Theoretically, spectrum sharing can greatly improve spectral utilization without affecting the existing legacy systems [4].

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A number of active studies have been conducted on spectrum sharing environments, especially on the secondary link capacity, based on using power control to satisfy interference power constraints [5][6]. Ghasemi *et al.* investigated the capacity of the secondary link under the peak and average interference constraints in Rayleigh and Nakagami fading models [5], and Kang *et al.* analyzed the ergodic and outage capacities of the secondary link in the delay-limited environments [6]. In those works, the capacity of the secondary link was analyzed under the assumption that the secondary user knows the channel information perfectly between the secondary transmitter (STx) and the primary receiver (PRx).

However, the obtained channel state information (CSI) is frequently outdated in practical systems [7]. In a spectrum sharing system, the problems caused by this outdated CSI are severe. In particular, it becomes difficult for the secondary system to perfectly measure the channel of the STx-PRx link in CR networks [8][9]. In general, the secondary user can obtain the channel information from the band manager mediating between the primary and secondary users [10], or from the PRx directly [11]. If the STx or PRx has mobility, the STx-PRx link channel is time-varying by Doppler effect, and the STx obtains outdated CSI due to feedback latency. If the transmission power of the secondary user is allocated using this outdated CSI, the interference power to the PRx does not satisfy the interference power constraint in a spectrum sharing environment, making it impossible for the secondary system to share the spectrum of the primary system. Therefore, our focus in this paper is on the problems related to the channel's outdated characteristic for STx-PRx link.

Fundamental capacity in fading environments with imperfect channel information was investigated in [8]. Musavian *et al.* analyzed the capacity of the secondary link when the secondary user is provided with only partial CSI for STx-PRx link. They assumed that the real channel of the STx-PRx link consists of the estimated channel and the estimation error term, and that the estimated channel information and the real channel are uncorrelated. However, the outdated channel at the secondary user and the real channel value are actually correlated as a result of the characteristics of the time-varying channel and feedback latency. Since the correlation means a reliability of channel information in the STx, this correlation information needs to be considered if one is to achieve the capacity enhancement for the secondary user.

In addition, the ergodic capacity of the secondary user and the statistics of the interference at the PRx under the peak received-power constraint were analyzed in [9]. Suraweera *et al.* provided the mean capacity of the secondary user when interference from the primary transmitter (PTx) to the secondary receiver (SRx) exists or is ignored, and analyzed the outage probability at the PRx. However, they did not provide the appropriate transmission power needed to satisfy the specific interference outage probability at the PRx. As with the average received-power constraint, the transmission power needed to satisfy the peak received-power constraint should be designed for the secondary user. In addition, all the scenarios researched in [9] were studied under the peak received-power constraint. Since the average received-power constraint is meaningful in regard to enhancing secondary user and primary user performance [12], the secondary user is bound to consider both constraints to improve its performance.

In CR networks, the primary user experiences the interference from the secondary user. For spectrum sharing, the interference statistics at the primary user are analyzed in [13][14]. These works show that the primary user is protected by the interference power constraints, and the secondary user can share the spectrum of the primary user within these constraints. However, the fact that the CSI on the STx-PRx link is outdated means that the secondary user cannot share the spectrum of the primary user, because doing so would violate the constraints. Moreover, the interference from the PTx generated by the primary user activity may also limit the performance of the secondary user. Thus, resolving the issues surrounding outdated CSI on the STx-PRx link and the interference from the PTx is critical for evaluating the secondary user performance in spectrum sharing environments. However, these critical factors were not considered or were considered only marginally in earlier works [8][9].

Therefore, this paper investigates the performance of secondary user while considering all these factors, i.e., the interference power constraints, the outdated CSI, and the interference from the PTx. We consider the performance of the secondary link under a general outdated correlation channel model with interference from the PTx. In this model, we propose power allocation schemes to overcome the violation of interference constraint due to the outdated CSI. Based on these power allocation schemes, we derive the ergodic capacity of secondary user. Building on this, we investigate the impact of the outdated channel and the interference from the PTx on the capacity of the secondary user under both constraints. Our goal is to give some insight into spectrum sharing that leads to efficient utilization of resources in more practical environments.

II. System and Channel Model with Outdated CSI

Let us consider a spectrum sharing system in which a secondary user can share the primary user's spectrum, as long as the amount of interference inflicted on the PRx is within a predetermined constraint. Fig. 1 shows the general model of a spectrum sharing system. It includes a STx, a SRx, a PTx, and a PRx. In this system, since the secondary user shares the spectrum of the primary user only within the predetermined interference power constraint, the PRx receives interference from the STx within the predetermined interference power. Furthermore, the SRx also receives interference from the PTx. In the previous works, this interference is sometimes considered [9] and sometimes not [6]. In order to investigate the effect of the interference from the PTx, we will consider not only the environment where the interference from the PTx is ignored, but also the environment where interference from the PTx exists.

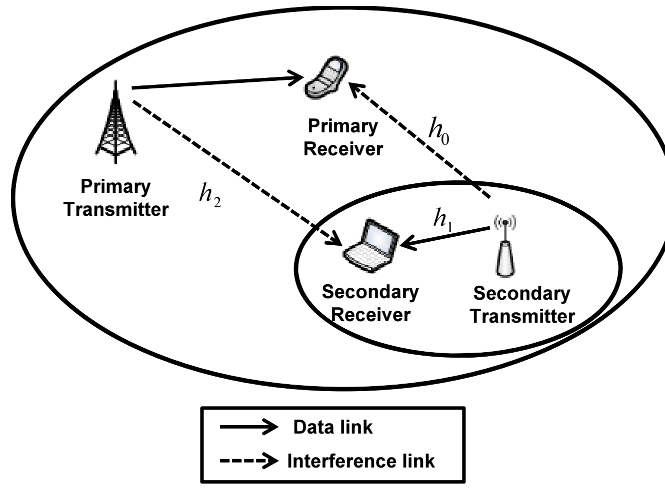

Fig. 1. General model of a spectrum sharing system. We assume that the interference from the primary transmitter to the secondary receiver exists or could be considered in the additive white Gaussian noise (AWGN) of the secondary receiver.

In spectrum sharing environments, there are two basic constraints on the secondary user transmission power: the average received-power constraint and the peak received-power constraint [5]. Under the average received-power constraint, the expected value of transmission power in the secondary system should be below the predetermined threshold. On the other hand, under the peak received-power constraint, the secondary system should always maintain the transmission power of the STx below the predetermined threshold. From the viewpoints of secondary user performance and primary user protection, we consider both constraints in outdated CSI.

Let us assume point-to-point flat Rayleigh fading channels. Let h_0, h_1 and h_2 denote instantaneous complex Gaussian channel values from the STx to the PRx and from the STx to the SRx, and from the PTx to the SRx, respectively, as shown in Fig. 1. The instantaneous channel gains are then denoted by $g_0 = |h_0|^2$, $g_1 = |h_1|^2$, and $g_2 = |h_2|^2$, respectively. In Rayleigh fading environments, the instantaneous channel gain would be exponentially distributed with a unit-mean. Additionally, since we are focusing only on the outdated channel effect, we will not be taking the path-loss and shadowing effect resulting from the geometry into account in this paper.

As mentioned above, the CSI on h_0 provided to the STx is outdated due to the time-varying nature of the wireless link. Basically, this imperfect CSI can be described using one of two models: the channel estimation error model and the correlation model. In the channel estimation error model, the outdated property of the channel is expressed by the variance of the error term [15][16][17]. This model is effective for analyzing practical channel estimation. Otherwise, in the correlation model, the outdated property is expressed by the correlation coefficient [18][19]. The effects of delay and time-varying of channel are analyzed based on this model. For the same purpose, we use the correlation model. In a general correlation model, h_0 can be expressed as follows [18]:

$$h_0 = \rho\hat{h}_0 + \sqrt{1-\rho^2}\tilde{h}_0, \tag{1}$$

where $\hat{h}_0$ is the outdated channel information which the secondary user knows, and $\tilde{h}_0$ is a complex Gaussian random variable with zero mean and unit variance, and is uncorrelated with $\hat{h}_0$. The correlation coefficient ρ is a constant which determines the average quality of the channel estimate over all channel states of h_0. In (1), h_0 and $\hat{h}_0$ are jointly Gaussian, and h_0 conditioned $\hat{h}_0$ follows a Gaussian distribution [19]:

$$h_0|\,\hat{h}_0 \;\sim\; \mathcal{CN}\left(\rho\hat{h}_0, 1-\rho^2\right), \tag{2}$$

where $\mathcal{CN}(a,b)$ denotes a complex Gaussian distribution with mean a and variance b. For the time-varying channels, the correlation coefficient takes the value $\rho = J_0\left(2\pi f_d \tau\right)$ in the Jakes' model, where $J_0\left(\cdot\right)$ is the zeroth order Bessel function of the first kind, f_d is the Doppler frequency, and τ is the delay parameter [20]. Additionally, we assume that the STx knows not only the outdated channel information, $\hat{h}_0$, but also the correlation coefficient, ρ.

III. Average Received-Power Constraint

In this section, we obtain the ergodic capacity of the secondary user under the average received-power constraint. We begin by showing how the constraint is violated when the secondary user obtains only the outdated CSI for the STx-PRx link. Then, we derive the optimal power allocation scheme to overcome the violation of interference constraint. Applying this power allocation scheme, we derive the ergodic capacity of the secondary user in two cases: 1) presence of interference from the PTx to the SRx 2) ignoring interference from the PTx when the PTx is located far away from the SRx.

A. Violation of Average Received-Power Constraint with Only Outdated CSI

Under the average received-power constraint, optimal power allocation is obtained via the water-filling-in-time solution corresponding to a constraint on the transmitted power [5]. When the secondary user has only outdated CSI for the STx-PRx link, the ergodic capacity represents the solution to the following optimization problem:

$$\begin{aligned}\max_{P(\hat{g}_0,g_1)\geq 0} \int_{\hat{g}_0}\int_{g_1} & B\log\left(1+\frac{g_1 P\left(\hat{g}_0,g_1\right)}{N_0 B}\right) \\ & \times f_{g_1}\left(g_1\right) f_{\hat{g}_0}(\hat{g}_0)dg_1 d\hat{g}_0,\end{aligned} \tag{3}$$

$$s.t. \quad E\left[\hat{g}_0 P\left(\hat{g}_0,g_1\right)\right] \leq I_{th}, \tag{4}$$

where I_{th} is the maximum interference power that the primary user can tolerate at its receiver, $P(\hat{g}_0,g_1)$ and B are the transmitted power at the STx and the total available bandwidth, respectively, N_0 is the power spectral density of the additive white Gaussian noise (AWGN) at the SRx, $\hat{g}_0 = \left|\hat{h}_0\right|^2$, and $E\left[\cdot\right]$ is the expectation operator. In this paper, $f_X(x)$ denotes the probability density function (PDF) of random variable X.

The solution to the optimization problem (3) can be found by using the Lagrangian optimization problem according to [5]

$$P(\hat{g}_0,g_1) = \left(\frac{1}{\lambda\hat{g}_0} - \frac{N_0 B}{g_1}\right)^+, \tag{5}$$

where $(\cdot)^+$ denotes $\max\left\{.,0\right\}$ and λ is determined such that the average received-power constraint is satisfied with equality in (4).

However, if the secondary user designs the transmission power in accordance with (5), the amount of interference to the primary user may not satisfy the average received-power constraint because of its outdated nature. We obtain the average interference power to the primary user using (5) as follows:

$$\begin{aligned} I_{th} &\geq E\left[g_0 P\left(\hat{g}_0,g_1\right)\right] \\ &= E\left[\rho^2\hat{g}_0 P\left(\hat{g}_0,g_1\right)\right] + E\left[\left(1-\rho^2\right)\tilde{g}_0 P\left(\hat{g}_0,g_1\right)\right] \\ &\quad + E\left[\rho\sqrt{1-\rho^2}\left(\hat{h}_0\tilde{h}_0^* + \hat{h}_0^*\tilde{h}_0\right)\right] \\ &= \rho^2 E[\hat{g}_0 P(\hat{g}_0,g_1)] + (1-\rho^2)E[P(\hat{g}_0,g_1)],\end{aligned} \tag{6}$$

where $(A)^*$ denotes the complex conjugate of A, and $\tilde{g}_0 = \left|\tilde{h}_0\right|^2$. In (6), the average transmission power at the secondary user $E\left[P\left(\hat{g}_0,g_1\right)\right]$ is as follows:

$$\begin{aligned} & E\left[P\left(\hat{g}_0,g_1\right)\right] \\ &= \int_0^\infty\int_{\hat{g}_0/\gamma}^\infty \left(\frac{1}{\lambda\hat{g}_0}+\frac{N_0 B}{g_1}\right) f_{g_1}(g_1) f_{\hat{g}_0}(\hat{g}_0)dg_1 d\hat{g}_0 \\ &= N_0 B\int_0^\infty \left(\frac{e^{-\frac{\hat{g}_0}{\gamma}}}{\hat{g}_0} + Ei\left(-\frac{\hat{g}_0}{\gamma}\right)\right) f_{\hat{g}_0}(\hat{g}_0)d\hat{g}_0 \\ &= -\frac{1}{\lambda}Ei\left(0\right) - N_0 B\log\left(1+\gamma\right),\end{aligned} \tag{7}$$

where $Ei(x)$ denotes the exponential-integral function given by $Ei\left(x\right) = -\int_{-x}^\infty e^{-t}t^{-1}dt$ [22, eq. (8.211.1)], and $\gamma = 1/(\lambda N_0 B)$.

In (7), since the value of $Ei(0)$ is infinite, the average transmission power at the STx is infinite. If the STx knows the STx-PRx link channel perfectly, i.e., ρ goes to 1, the average received-power constraint is not violated in (6), and the secondary user can share the spectrum of the primary user within the predetermined constraint. However, the value of the second term of (6) is infinite due to the outdated CSI from the STx-PRx link. If the STx is not able to obtain the perfect CSI for the STx-PRx link, the average received-power constraint is violated in (6). This means harmful interference to the primary user, with the result that the secondary user is not permitted to share the spectrum of the primary user. The above arguments confirm that the transmission power designs where only the outdated CSI is considered at the secondary user cannot satisfy the interference power constraint. Therefore, when allocating the transmission power, the secondary user needs to consider the correlation coefficient in the outdated CSI in order to satisfy the average received-power constraint.

B. Ergodic Capacity of the Secondary User via Optimal Power Allocation with Outdated CSI

In order to satisfy the average received-power constraint in an outdated channel environment, the correlation coefficient ρ should be considered if we are to find the optimal power allocation. Therefore, the ergodic capacity in (3) can be rewritten as follows:

$$\max_{P(\hat{g}_0,g_1)\geq 0} \int_{\hat{g}_0}\int_{g_1} B\log\left(1+\frac{g_1 P(\rho,\hat{g}_0,g_1)}{N_0 B}\right) \times f_{g_1}(g_1) f_{\hat{g}_0}(\hat{g}_0) dg_1 d\hat{g}_0, \tag{8}$$

$$s.t. \quad E\left[g_0 P(\rho,\hat{g}_0,g_1)\right] \leq I_{th}. \tag{9}$$

Using (6), the average received-power constraint (9) can be rewritten as

$$\int_{\hat{g}_0}\int_{g_1} \left(\hat{g}_0\rho^2 P(\rho,\hat{g}_0,g_1) + \left(1-\rho^2\right) P(\rho,\hat{g}_0,g_1)\right) \times f_{g_1}(g_1) f_{\hat{g}_0}(\hat{g}_0) dg_1 d\hat{g}_0 \leq I_{th}. \tag{10}$$

The solution of this optimization problem in (8) can be also found by using the Lagrangian optimization problem as shown here:

$$P(\rho,\hat{g}_0,g_1) = \left(\frac{1}{\lambda(\rho^2\hat{g}_0 + (1-\rho^2))} - \frac{N_0 B}{g_1}\right)^+. \tag{11}$$

In (11), λ is determined such that the average received-power constraint in (10) is equal to I_{th}, i.e.,

$$I_{th} = \int_{\hat{g}_0}\int_{g_1} \left(\frac{1}{\lambda} - \frac{(\rho^2\hat{g}_0 + (1-\rho^2))}{g_1} N_0 B\right)^+ \times f_{g_1}(g_1) f_{\hat{g}_0}(\hat{g}_0) dg_1 d\hat{g}_0. \tag{12}$$

By inserting $f_0(\hat{g}_0) = e^{-\hat{g}_0}$ and $f_1(g_1) = e^{-g_1}$ into (12), we can calculate

$$I_{th} = N_0 B\left[e^{\frac{-1+\rho^2}{\gamma}}\gamma - \Gamma\left(0, \frac{1-\rho^2}{\gamma}\right) + \rho^2 e^{-1+\frac{1}{\rho^2}}\Gamma\left(0, -1+\frac{1}{\rho^2}+\frac{1-\rho^2}{\gamma}\right)\right], \tag{13}$$

where $\Gamma(a,z)$ is an incomplete gamma function given by $\Gamma(a,z) = \int_z^\infty t^{a-1}e^{-t}dt$ [22 eq. (8.350.2)]. $\gamma = 1/(\lambda N_0 B)$ is determined from (13), for which the numerical integration is evaluated. When the transmission power at the STx is determined according to (11), we obtain the ergodic capacity of the secondary user.

We first obtain the ergodic capacity of the secondary user when the interference from the PTx is ignored due to the great distance between it and the SRx or the deep shadowing [9]. This is given by

$$C_{avg} = \int_{\hat{g}_0}\int_{g_1} \log\left(1+\frac{g_1 P(\rho,\hat{g}_0,g_1)}{N_0 B}\right) f_{g_1}(g_1) f_{\hat{g}_0}(\hat{g}_0) dg_1 d\hat{g}_0. \tag{14}$$

Substituting (11) into (14), it can be expressed as

$$\begin{aligned} C_{avg} &= \int_0^\infty e^{-\hat{g}_0}\int_{\frac{\rho^2\hat{g}_0+(1-\rho^2)}{\gamma}}^\infty \log\left(\frac{\gamma g_1}{\rho^2\hat{g}_0+(1-\rho^2)}\right) \times e^{-g_1} dg_1 d\hat{g}_0 \\ &= \int_0^\infty e^{-\hat{g}_0}\Gamma\left(0, \frac{\rho^2\hat{g}_0+(1-\rho^2)}{\gamma}\right) d\hat{g}_0 \\ &= e^{-1+\frac{1}{\rho^2}} Ei\left(\frac{\left(\rho^2-1\right)\left(\gamma+\rho^2\right)}{\gamma\rho^2}\right) - Ei\left(\frac{\rho^2-1}{\gamma}\right). \end{aligned} \tag{15}$$

The first integral can be evaluated with the help of [22 eq. (4.331.1)], and the second integral can be calculated by using the integration by parts. We can see that the ergodic capacity of the secondary user is affected by the correlation coefficient. By the property of the exponential integral function, the ergodic capacity of the secondary user decreases as the correlation coefficient decreases. Thus, in order to improve the ergodic capacity of the secondary user, it is important that the secondary user obtains accurate channel information on the STx-PRx link.

We next consider the ergodic capacity of the secondary user when interference from the PTx affects the SRx received signal. The signal-to-interference-and-noise ratio (SINR) η can then be written as

$$\eta = \frac{g_1 P(\rho,\hat{g}_0,g_1)}{N_0 B + g_2 P_{PTx}}, \tag{16}$$

where P_{PTx} is the transmission power at the PTx. The ergodic capacity of the secondary user can be calculated from

$$C_{avg} = \int_{g_2}\int_{\hat{g}_0}\int_{g_1} \log\left(1+\frac{\frac{\gamma g_1}{\rho^2\hat{g}_0+(1-\rho^2)}-1}{1+g_2\alpha}\right) \times f_{g_1}(g_1) f_{\hat{g}_0}(\hat{g}_0) f_{g_2}(g_2) dg_1 d\hat{g}_0 dg_2, \tag{17}$$

where α is the interference-to-noise ratio (INR) given by $\alpha = P_{PTx}/N_0 B$. Unfortunately, to the best of our knowledge, the integrals in (17) cannot be evaluated in closed form. Therefore, we first derive the upper bound of the ergodic capacity using Jensen's inequality [23]. The upper bound is derived as follows:

$$\begin{aligned} C_{avg} &\leq C_{avg}^{UB} \\ &= e^{\frac{1}{\alpha}} Ei\left(-\frac{1}{\alpha}\right) - e^{\frac{1+\beta}{\alpha}} Ei\left(-\frac{1+\beta}{\alpha}\right) + \log(1+\beta), \end{aligned} \tag{18}$$

$$\underbrace{\alpha = \frac{P_{PTx}}{N_0 B}}_{\text{Interfernce from PTx}}, \quad \underbrace{\beta = -\frac{e^{-1+\frac{1}{\rho^2}}\gamma Ei\left(\frac{\left(-1+\rho^2\right)\left(\gamma+\rho^2\right)}{\gamma\rho^2}\right)}{\rho^2}}_{\text{Outdated CSI of PRx}-\text{STx link}}. \tag{19}$$

Next, we derive the lower bound of the ergodic capacity using the log approximation. The lower bound is derived as follows:

$$C_{avg} \geq C_{avg}^{LB} = \frac{e^{\frac{\gamma-\alpha\left(1-\rho^2\right)}{\alpha\gamma}}\gamma Ei\left(-\frac{1}{\alpha}\right)}{\gamma+\rho^2} - Ei\left(\frac{-1+\rho^2}{\gamma}\right) - \frac{\rho^2\beta}{\gamma}. \tag{20}$$

The derivation processes for (18) and (20) are provided in Appendix I.

From the above results, the ergodic capacity of the secondary user decreases as the INR increases, similar to (15). Moreover, we can see that α is affected by the interference from the PTx, and that β is affected by the outdated channel of the STx-PRx link. Thus, through α and β, the interference from the PTx and the outdated property of the STx-PRx link both degrade the performance of the secondary user. In addition, since the transmission power is designed so as to consider the outdated CSI of the STx-PRx link, we can estimate that the performance degradation caused by the interference from the PTx is more serious than that caused by the outdated STx-PRx link CSI in (18) and (20). We will confirm the impacts of PTx interference and the outdated CSI of STx-PRx link using numerical results in Section V.

IV. Peak Received-Power Constraint

Under the peak received-power constraint, the secondary user can share the primary user's spectrum if the power of the received signal in the PRx is below a predetermined threshold, as follows [21]:

$$g_0 P(g_0) \leq I_{th}, \tag{21}$$

where $P(g_0)$ is the transmission power of the STx. In addition, there is also the maximum transmission power constraint, $P(g_0) \leq P_m$, where P_m is the maximum transmission power [9]. By those two constraints, the transmission power constraint of STx becomes as follows:

$$P\left(g_0\right) \leq \min\left\{\frac{I_{th}}{g_0}, P_m\right\}. \tag{22}$$

However, when the secondary user has only the outdated channel information on the STx-PRx link, $\hat{g}_0$, it becomes difficult to satisfy this constraint. Thus, in this section, we will derive the interference outage probability in the primary user, and will derive the power margin needed to satisfy a specific interference outage probability. Additionally, we will also derive the ergodic capacity of the secondary user.

A. Interference Outage Probability

Under the peak received-power constraint, when the secondary user has only the outdated channel information for the interference link, $\hat{g}_0$, it is impossible to satisfy this constraint. This is because it cannot be guaranteed that the interference power at the PRx will remain below the predetermined threshold at all times. After all, the secondary user must be silent at all times to satisfy such a constraint, which makes the capacity of the secondary link zero [8].

For this reason, it makes sense to consider a constraint based on a stochastic concept instead of the strict peak power constraint. The primary user should allow a certain percentage of outage, which is referred to as 'interference outage' [8]. The term 'interference outage' means that the interference power at the PRx exceeds the predetermined threshold for a certain percentage of the time. The interference outage constraint is defined as

$$\mathcal{P}_o \geq 1 - \Pr\left\{P(\hat{g}_0)g_0 \leq I_{th}\right\}. \tag{23}$$

When the secondary user knows only the outdated channel information, $\hat{g}_0$, the transmission power constraint (22) is rewritten as $P(\hat{g}_0) = \min\{I_{th}/\hat{g}_0, P_m\}$. Then, the interference outage probability is obtained as follows:

$$\begin{aligned}\mathcal{P}_o = {} & e^{-m} - e^{-m}Q_1\left(\sqrt{\frac{2\rho^2 m}{1-\rho^2}}, \sqrt{\frac{2m}{1-\rho^2}}\right) \\ & + \frac{1}{2}e^{-\frac{2m}{1-\rho^2}} I_0\left(\frac{2m}{1-\rho^2}\right),\end{aligned} \tag{24}$$

where $m = I_{th}/P_m$, and $I(\cdot)$ is the zeroth-order modified Bessel function of the first kind and

$$Q_1\left(a, b\right) = \int_b^\infty x e^{-\frac{x^2+a^2}{2}} I_0(ax)dx \tag{25}$$

is the first-order Marcum Q-function [24]. The derived process for (24) is provided in Appendix II.

When the maximum transmission power of STx is infinite, the interference outage probability is obtained as

$$\mathcal{P}_0 = e^0 - e^0 Q_1\left(0,0\right) + \frac{1}{2}e^0 I_0\left(0\right) = \frac{1}{2}, \tag{26}$$

where $Q_1(0,0) = 1$ and $I_0(0) = 1$ [24].

From (26), we can see that the interference outage probability is always 0.5 when the secondary user knows only the outdated CSI, $\hat{g}_0$. In other words, if the secondary user transmits its signal without considering the reliability of the PTx-SRx link, the probability that the interference power will exceed the predetermined threshold I_{th} is always half. That is because the probability that the magnitude of outdated channel gain is larger or smaller than that of the real channel gain is independent of the correlation coefficient. As a result, if the STx does not obtain perfect channel state information, the performance of the primary user is degraded, regardless of the feedback latency or time-varying channel characteristics. Therefore, under the peak received-power constraint, the power margin needed to consider the interference outage probability should be designed to prevent the performance degradation of the primary user.

B. Power Margin and Ergodic Capacity of Secondary User with Outdated CSI

Under the peak power constraint, the power margin that satisfies the predetermined interference outage probability can be expressed as follows:

$$P\left(\rho, \hat{g}_0\right) = \min\left\{k_m \frac{I_{th}}{\hat{g}_0}, P_M\right\}, \tag{27}$$

where k_m is the power margin factor. Our objective is to derive the value of k_m that satisfies the interference outage probability. When the proposed power margin is (27), the interference outage probability can be derived as

$$\begin{aligned}\mathcal{P}_o = e^{-m} - e^{-m}Q_1\left(\sqrt{\frac{2\rho^2 m}{1-\rho^2}}, \sqrt{\frac{2k_m m}{1-\rho^2}}\right)\\ - \frac{t}{r}Q_1\left(\sqrt{\frac{m(s-r)}{2}}, \sqrt{\frac{m(s+r)}{2}}\right)\\ + \frac{1}{2}\left(1+\frac{t}{r}\right)e^{-\frac{sm}{2}} I_0\left(\frac{2\rho m\sqrt{k_m}}{1-\rho^2}\right),\end{aligned} \tag{28}$$

$$\begin{aligned} s = \frac{2\left(1+k_m\right)}{1-\rho^2},\ t = \frac{2\left(1-k_m\right)}{1-\rho^2},\\ r = \frac{2\sqrt{\left(1+k_m\right)^2 - 4\rho^2 k_m}}{1-\rho^2}.\end{aligned} \tag{29}$$

Unfortunately, the power margin factor k_m needed to satisfy the predetermined interference outage probability $\mathcal{P}_o$ in (28) is not derived as a closed form. Hence, the power margin factor k_m can be numerically derived.

Otherwise, when the maximum transmission power at the STx doesn't exist, the power margin factor k can be derived. Then, the interference outage probability can be derived as

$$\mathcal{P}_0 = 1 - \Pr\left\{\frac{g_0}{\hat{g}_0} \le \frac{1}{k}\right\}. \tag{30}$$

In (30), the PDF of $g_0/\hat{g}_0$ can be expressed as

$$f_{\frac{g_0}{\hat{g}_0}}(x) = \frac{(1-\rho^2)(1+x)}{\{1+x(2-4\rho^2+x)\}^{3/2}}, \quad x \ge 0. \tag{31}$$

The derived process (31) is provided in Appendix III.

Therefore, the interference outage probability is obtained as a closed form:

$$\begin{aligned}\mathcal{P}_o &= 1 - \int_0^{1/k} \frac{(1-\rho^2)(1+x)}{\{1+x(2-4\rho^2+x)\}^{3/2}}dx\\ &= \frac{1}{2} + \frac{-1+k}{2\sqrt{\left(1+k\right)^2 - 4k\rho^2}}.\end{aligned} \tag{32}$$

The integral in (32) can be evaluated with the help of [22, eq. (2.264.6)].

In (32), we can see that the interference outage probability is a function of k and ρ. Equation (32) consists of $1/2$ and an adjustable term, depending on the power margin factor k and the correlation coefficient ρ. When the secondary user has only outdated channel information for the STx-PRx link, the interference outage probability is $1/2$ in (32), as compared to (27). Thus, our goal is to reduce the interference outage probability of the primary user by adjusting the power margin factor k.

From (32), the power margin factor needed to satisfy the predetermined $\mathcal{P}_o$ can be derived as

$$\begin{aligned}k &= \left(-1+2\rho^2\right)\\ &+ \frac{1-\rho^2-(1-2\mathcal{P}_0)\sqrt{(1-\rho^2)\left(1-(1-2\mathcal{P}_0)^2\rho^2\right)}}{2(1-\mathcal{P}_0)\mathcal{P}_0}.\end{aligned} \tag{33}$$

Using (33), the STx can adjust the transmission power to satisfy the interference outage constraint. The power margin factor is decided by the correlation coefficient and the predetermined interference outage probability. Thus, using the power margin factor k, the secondary user can share the spectrum of the primary user within the predetermined interference outage constraint. In order to analyze the ergodic capacity, hereinafter, we use the transmission power at the STx without P_m for analytical tractability.

Using this power margin factor, we derive the ergodic capacity of the secondary user when the interference from the PTx is weak or ignored. The ergodic capacity of the secondary user can then be derived as

$$C_{peak} = \int\limits_{g_1/\hat{g}_0} \log\left(1+\frac{g_1 P(\rho,\hat{g}_0)}{N_0 B}\right) f_{\frac{g_1}{\hat{g}_0}}(x)dx. \tag{34}$$

The PDF of $g_1/\hat{g}_0$ is known to have a log-logistic distribution of [5]

$$f_{\frac{g_1}{\hat{g}_0}}(x) = \frac{1}{\left(1+x\right)^2}, \quad x \ge 0. \tag{35}$$

Therefore, the ergodic capacity of the secondary user without the interference from the PTx can be expressed as

$$\begin{aligned}C_{peak} &= \int_0^\infty \log\left(1+\frac{kI_{th}}{N_0 B}x\right) \times \frac{1}{\left(1+x\right)^2}dx\\ &= \frac{k\mu\log\left(k\mu\right)}{-1+k\mu},\end{aligned} \tag{36}$$

where $\mu = I_{th}/N_0 B$.

As shown in (36), the ergodic capacity of the secondary user can be expressed by two terms: the given interference power threshold-to-noise ratio μ, and the power margin factor k. Since μ has fixed values in the given spectrum sharing environment, the ergodic capacity of the secondary link varies with the power margin factor k. From (36), since the power margin factor k consists of the correlation coefficient ρ and the predetermined interference outage probability $\mathcal{P}_o$, we confirm that the ergodic capacity of the secondary link is determined by the reliability of the STx-PRx link and the amount of interference to the PRx.

We next derive the ergodic capacity of the secondary user when interference exists from the PTx. The ergodic capacity of the secondary user can be expressed as

$$\begin{aligned}C_{peak} = \int\limits_{g_2}\int\limits_{\hat{g}_0}\int\limits_{g_1} \log\left(1+\frac{g_1 P\left(\rho,\hat{g}_0\right)}{N_0 B + g_2 P_{PTx}}\right)\\ \times f_{g_1}(g_1) f_{\hat{g}_0}(\hat{g}_0) f_{g_2}(g_2) dg_1 d\hat{g}_0 dg_2.\end{aligned} \tag{37}$$

As with (17), there is no way to find the closed form. Therefore, we derive the upper bound of (37) using Jensen's inequality [23]. The upper bound is derived as:

$$\begin{aligned}
C_{peak} &\leq C_{peak}^{UB} \\
&= \int_{g_1/\hat{g}_0} \log\left(1 + \int_{g_2} \frac{k\mu x}{1+g_2\alpha} f_{g_2}(g_2)dg_2\right) f_{\frac{g_1}{\hat{g}_0}}(x)dx \\
&= \int_0^{\infty} \log\left(1 + \frac{-k\mu e^{\frac{1}{\alpha}} x Ei\left(-\frac{1}{\alpha}\right)}{\alpha}\right) \times \frac{1}{(1+x)^2} dx \\
&= \frac{\upsilon \log(\upsilon)}{-1+\upsilon},
\end{aligned} \tag{38}$$

$$\upsilon = \underbrace{k}_{\text{outdated CSI of PRx}-\text{STx link}} \times \mu \times \underbrace{-\frac{e^{\frac{1}{\alpha}} Ei\left(-\frac{1}{\alpha}\right)}{\alpha}}_{\text{Interference from PTx}}. \tag{39}$$

Compared with (36), the term $-\frac{e^{\frac{1}{\alpha}}Ei\left(-\frac{1}{\alpha}\right)}{\alpha}$ is multiplied by $k\mu$ due to the interference from the PTx in (36). In interference-free environments, $-\frac{e^{\frac{1}{\alpha}}Ei\left(-\frac{1}{\alpha}\right)}{\alpha} \rightarrow 1$ as $\alpha \rightarrow 0$, and in infinite interference environments, $-\frac{e^{\frac{1}{\alpha}}Ei\left(-\frac{1}{\alpha}\right)}{\alpha} \rightarrow 0$ as $\alpha \rightarrow \infty$. Thus, as with the power margin factor k, the term $-e^{\frac{1}{\alpha}}Ei\left(-\frac{1}{\alpha}\right)/\alpha$ has a value between 0 and 1. Hence, the term $-\frac{e^{\frac{1}{\alpha}}Ei\left(-\frac{1}{\alpha}\right)}{\alpha}$ degrades the ergodic capacity of the secondary user. We can confirm that the ergodic capacity of the secondary user decreases because of the outdated CSI of the STx-PRx link and the interference from the PTx. Under the peak received-power constraint, the secondary transmitter should reduce its transmission power in order to prevent harmful interference to the primary user caused by the outdated CSI of the STx-PRx link. Especially, when the allowable interference outage probability at the primary user is very low, the secondary user should decrease its transmission power as the correlation coefficient decreases. Therefore, unlike the average received-power constraint, we can estimate that the ergodic capacity of the secondary user is sensitive to the uncertainty of the channel in the STx-PRx link. Using numerical results, we will check the effect of the correlation coefficient ρ and the INR α on the ergodic capacity of the secondary user.

V. Numerical Results

In this section, we confirm the analytical results derived in Sections III and IV via comparisons using Monte Carlo simulations. We begin by showing the ergodic capacity of the secondary user under the average received-power constraint via the interference threshold and the correlation coefficient in an outdated CSI environment. We then present the ergodic capacity of the secondary user under the peak received-power constraint in the same manner.

A. Average Received-Power Constraint

We begin by comparing the ergodic capacity of the secondary user under the average receive-power constraint for different correlation coefficients, ρ. Fig. 2 shows the ergodic capacity of the secondary user with respect to the interference

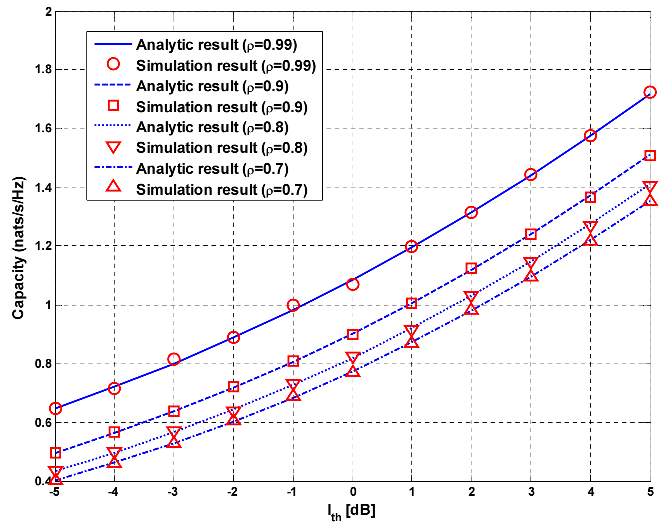

Fig. 2. Ergodic capacity of the secondary user under the average power constraint when the interference from the primary transmitter is ignored. Analytic results match simulation results well, and the ergodic capacity increases as the interference threshold increases because the permitted interference power increases.

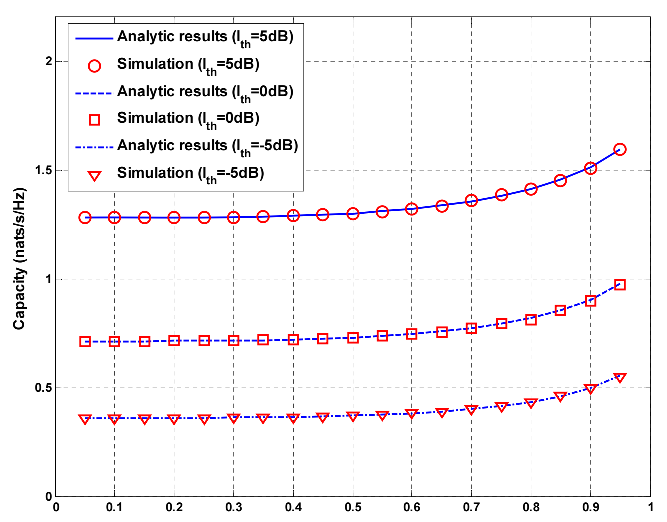

Fig. 3. Ergodic capacity of the secondary user for different interference power constraint values with respect to the correlation coefficient. Under the average received-power constraint, the ergodic capacity is insensitive to the correlation coefficient.

power constraint, I_{th}. As expected, the ergodic capacity degrades as the correlation coefficient decreases. In other words, the more accurate the obtained STx-PRx link CSI, the more enhanced the secondary user performance. Moreover, Fig. 2 shows that the loss of ergodic capacity caused by outdated channel information decreases as the correlation coefficient decreases. To observe the effect of correlation coefficient in more detail, we have plotted the ergodic capacity of the secondary user against the variation of the correlation coefficient in Fig. 3.

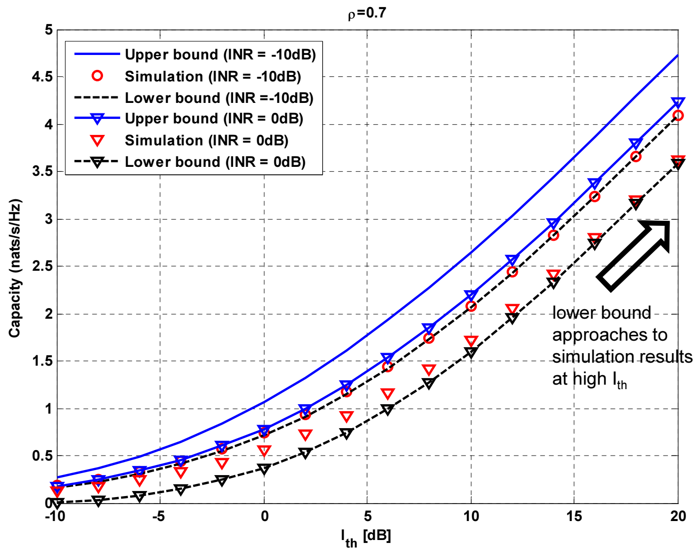

Fig. 4. Upper and lower bounds, and simulated results of ergodic capacity under the average received-power constraint when interference from the primary transmitter exists. The degradation of ergodic capacity due to interference is more serious than the uncertainty of the channel information.

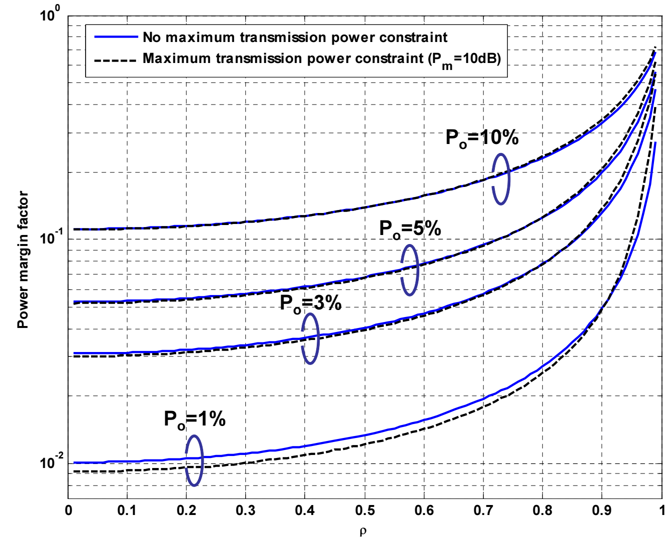

Fig. 5. Power margin factor (k, k_m) as a function of the correlation coefficient (ρ) and interference outage probability ($\mathcal{P}_o$). The secondary transmitter can increase transmission power as the correlation coefficient rises.

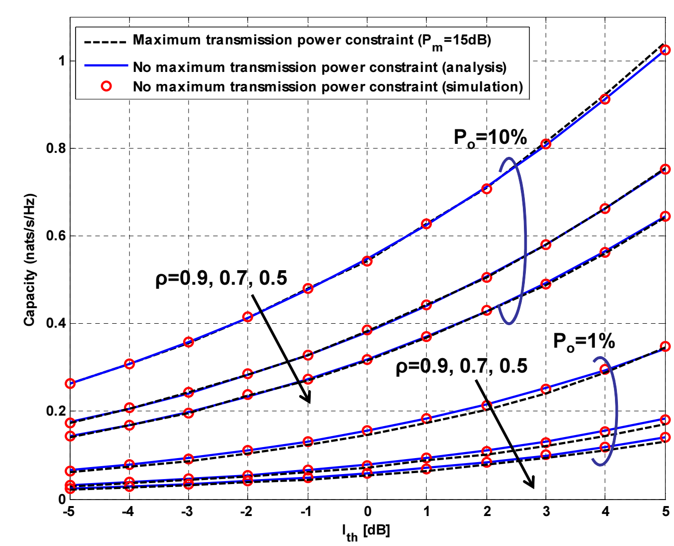

Fig. 6. Ergodic capacity of the secondary user versus an interference power threshold with various correlation coefficients. In order to improve the performance of the secondary link, more accurate CSI on the STx-PRx link should be obtained in the secondary system.

Fig. 3 depicts the ergodic capacity of the secondary user with respect to the correlation coefficient for the different interference threshold values. Fig. 3 shows that the loss of ergodic capacity due to the outdated channel is insensitive to the correlation coefficient, especially lower correlation coefficients. This is because the transmission power $P(\rho, \hat{g}_0, g_1)$ under the average received-power constraint is designed for both the correlation coefficient and the channel gain of the secondary link in terms of an average constraint. Thus, on average, since the transmission power $P(\rho, \hat{g}_0, g_1)$ can be increased without violating the average received-power constraint, the loss of ergodic capacity can be reduced regardless of the uncertainty of the channel information.

The resulting upper and lower bounds and simulated ergodic capacity when interference from the PTx is present are illustrated in Fig. 4. As shown in Fig. 4, the upper bound is close to the simulation results in low I_{th} regions. Otherwise, in high I_{th} regions, the lower bound approaches to the simulation results. Moreover, since the interference on the PTx-SRx link is not considered when designing of transmission power, degradation of ergodic capacity caused by the interference from the PTx is more serious than the effect of outdated CSI under the average received-power constraint as compared with the results of Fig. 3.

B. Peak Received-Power Constraint

Fig. 5 shows the power margin factor for different interference outage constraints versus the correlation coefficient. As one might expect, the power margin is small where the interference outage probability in the primary system is high. However, since a high interference outage probability causes performance degradation in the primary system, it is necessary to establish a proper constraint value that does not degrade the performance of the primary system. Furthermore, since the power margin is sensitive to the correlation coefficient, the more reliable CSI for the STx-PRx link ensures the growth of the secondary user transmission power, so that the secondary user achieves performance enhancement in the end. Additionally, if the maximum transmission power at the STx is sufficiently large (10dB in Fig. 5), k_m is close to k as shown in Fig. 5. That is because the probability to exceed the maximum transmission power at the STx is too small, especially high $\mathcal{P}_o$.

Fig. 6 shows the ergodic capacity of the secondary user when the interference from the PTx is ignored. As shown

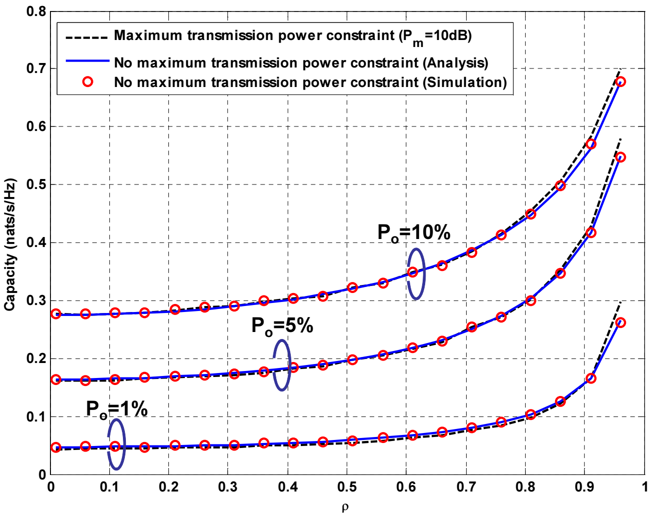

Fig. 7. Ergodic capacity of the secondary user versus a correlation coefficient with various interference outage probabilities. Under the peak received-power constraint, the ergodic capacity is sensitive to the correlation coefficient, especially to high correlation coefficients.

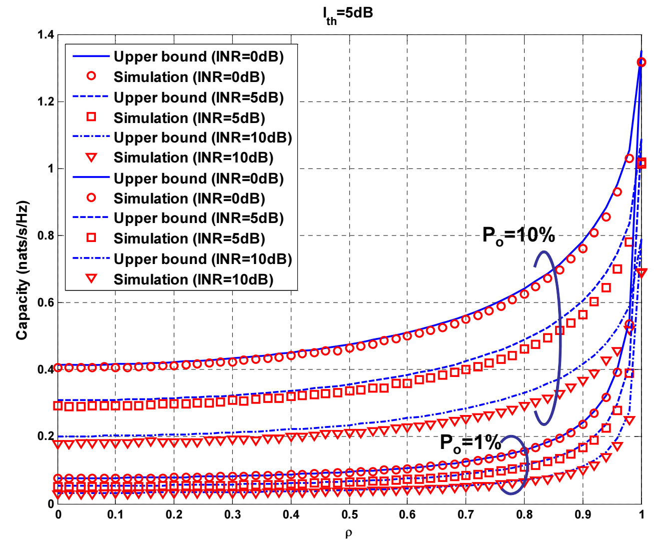

Fig. 8. Ergodic capacity of the secondary user versus correlation coefficient when the interference from the primary transmitter exists. The loss of ergodic capacity due to the uncertainty of the STx-PRx link is more significant than that due to interference from the primary transmitter, especially for high correlation coefficients.

in Fig. 6, since the peak received-power constraint is stricter than the average received-power constraint, we can find that the ergodic capacity to achieve the secondary user is smaller than that under the average received-power constraint. Hence, in order to improve the performance of the secondary link, more accurate CSI for the STx-PRx link should be obtained in the secondary system. In addition, as shown in Fig. 6, we confirm that the theoretical results are perfectly verified by Monte Carlo simulation. Furthermore, it is confirmed that the ergodic capacity with P_m is close to that without P_m. Thus, we can see that the analysis of ergodic capacity without P_m is meaningful.

Fig. 7 compares the ergodic capacity to the correlation coefficient for different interference outage constraints. Unlike the average received-power constraint, the ergodic capacity under the peak received-power constraint is sensitive to the correlation coefficient, especially to high correlation coefficients. Under the peak received-power constraint, since the transmission power is designed for the purpose of restricting any instantaneous constraint violation, the performance is sensitive to the reliability of the STx-PRx link. Particularly, in the case where the interference outage constraint is low, it is important that the channel uncertainty in the STx-PRx link be reduced.

Lastly, the ergodic capacity of the secondary user is illustrated in Fig. 8 when interference exits from the PTx. In Fig. 8, we assume that the interference power constraint is 5dB. As shown in Fig. 8, we find that the upper bound for the ergodic capacity is close to the simulated results. Fig. 8 shows clearly that the loss of ergodic capacity due do the uncertainty of the STx-PRx link is more significant than that due to the interference from the PTx, especially for a high correlation coefficient. When $\mathcal{P}_0 = 1\%$, we observe that the loss of ergodic capacity due to the uncertainty of the STx-PRx link for high correlation coefficients is more serious than that due to interference from the PTx. Therefore, under the peak received-power constraint, when enhancing the ergodic capacity of the secondary user, the outdated CSI of STx-PRx link should be considered to be of higher priority than the interference from the PTx.

VI. Conclusions and Future Works

In this paper, we investigated the effect of outdated CSI in spectrum sharing environments. We began by investigating the violation of average and peak received power constraints due to the outdated CSI. We therefore provided the power allocation schemes to overcome the constraint violation by the outdated CSI under each constraint. We then derived the ergodic capacity of the secondary user according to the interference from the PTx. Both analytical and simulated results demonstrated that the ergodic capacity of the secondary user under the average received-power constraint is more robust to the outdated channel environment than the peak received-power constraint. In addition, the interference generated by primary user activity has an important effect on the performance of the secondary user. Therefore, the secondary user should consider both the interference from the primary user and the outdated CSI if it is to be able to share the spectrum of the primary user. These observations give quantitative insight into the effect of the outdated CSI and the power allocation of secondary user on spectrum sharing system.

Possible topics for the future include extending our works to cover fully outdated CSI in spectrum sharing environments. Under the average received-power constraint, since the outdated CSI of the secondary link affects the power allocation at the STx, practical systems will require performance analysis in fully outdated CSI environments. A further topic for investigations is the effect of the geometry between the primary and secondary users in the outdated CSI environments. Since the geometry issue is important in spectrum sharing environments, the issue of analyzing the relationship between the outdated CSI and the geometry issue is an important one. These research topics will allow us to continue our identification and study of the practical issues in spectrum sharing environments.

APPENDIX I

We first derive the upper bound of the ergodic capacity of the secondary user when interference exists from the PTx. Using Jensen's inequality, we can derive the upper bound of (17). In order to derive a tighter bound, we apply Jensen's inequality to g_1 and $\hat{g}_0$. Therefore, the upper bound of the ergodic capacity is derived as follows:

$$\begin{aligned} C_{avg} &\leq C_{avg}^{UB} \\ &= \int\limits_{g_2} \log\left(1+\int\limits_{\hat{g}_0}\int\limits_{g_1}\frac{\frac{\gamma g_1}{\rho^2\hat{g}_0+(1-\rho^2)}-1}{1+g_2\alpha}f_1(g_1)f_0(\hat{g}_0)dg_1d\hat{g}_0\right) \\ &\quad\times f_2(g_2)dg_2 \\ &= \int_0^\infty \log\left(1+\int_0^\infty \frac{e^{-\frac{1-\rho^2(1-\hat{g}_0)}{\gamma}}\gamma}{(1-\rho^2(1-\hat{g}_0))(1+\alpha g_2)}e^{-\hat{g}_0}d\hat{g}_0\right) \\ &\quad\times e^{-g_2}dg_2. \end{aligned} \tag{40}$$

In (40), the upper bound of the ergodic capacity can be calculated as follows:

$$\begin{aligned} C_{avg}^{UB} &= \int_0^\infty \log\left(1-\frac{e^{-1+\frac{1}{\rho^2}}\gamma Ei\left(\frac{(-1+\rho^2)(\gamma+\rho^2)}{\gamma\rho^2}\right)}{\rho^2(1+\alpha g_2)}\right) \\ &\quad\times e^{-g_2}dg_2 \\ &= e^{\frac{1}{\alpha}}Ei\left(-\frac{1}{\alpha}\right)-e^{\frac{1+\beta}{\alpha}}Ei\left(-\frac{1+\beta}{\alpha}\right)+\log(1+\beta), \end{aligned} \tag{41}$$

$$\alpha=\frac{P_{PTx}}{N_0B},\quad \beta=-\frac{e^{-1+\frac{1}{\rho^2}}\gamma Ei\left(\frac{(-1+\rho^2)(\gamma+\rho^2)}{\gamma\rho^2}\right)}{\rho^2}. \tag{42}$$

In (41), the first integral can be evaluated with the help of [22, eq. (4.337.2)].

Next, we derive the lower bound of the ergodic capacity using the well-known log approximation as

$$\log(1+x)\approx\log(x)\quad\text{for large } x. \tag{43}$$

Thus, the log term in (17) has a lower bound as follows:

$$\begin{aligned} &\log\left(1+\frac{\frac{\gamma g_1}{\rho^2\hat{g}_0+(1-\rho^2)}-1}{1+g_2\alpha}\right) \\ &= \log\left(\frac{\frac{\gamma g_1}{\rho^2\hat{g}_0+(1-\rho^2)}}{1+g_2\alpha}+\underbrace{1-\frac{1}{1+g_2\alpha}}_{\delta}\right) \\ &\geq \log\left(\frac{\frac{\gamma g_1}{\rho^2\hat{g}_0+(1-\rho^2)}}{1+g_2\alpha}\right) \end{aligned} \tag{44}$$

In (17), since $\delta < 1$, this lower bound is a very tight bound.

The lower bound of the ergodic capacity is derived as

$$\begin{aligned} C_{avg}^{LB} &= \int\limits_{g_2}\int\limits_{\hat{g}_0}\int\limits_{g_1}\log\left(1+\frac{g_1P(\rho,\hat{g}_0,g_1)}{N_0B+g_2P_{PT}}\right) \\ &\quad\times f_1(g_1)f_0(\hat{g}_0)f_2(g_2)dg_1d\hat{g}_0dg_2 \\ &\underset{\mathbf{P1}}{=}\int_0^\infty \Gamma\left(0,\frac{\rho^2\hat{g}_0+(1-\rho^2)}{\gamma}\right)f_0(\hat{g}_0)d\hat{g}_0 \\ &\quad-\int_0^\infty\int_0^\infty e^{-\frac{\rho^2\hat{g}_0+(1-\rho^2)}{\gamma}}\log(1+g_2\alpha) \\ &\quad\times f_0(\hat{g}_0)f_2(g_2)d\hat{g}_0dg_2 \\ &\underset{\mathbf{P2}}{=}e^{-1+\frac{1}{\rho^2}}Ei\left(\frac{(-1+\rho^2)(\gamma+\rho^2)}{\gamma\rho^2}\right)-Ei\left(\frac{-1+\rho^2}{\gamma}\right) \\ &\quad-\int_0^\infty e^{\frac{-1+\rho^2}{\gamma}}\frac{\gamma\log(1+g_2\alpha)}{\gamma+\rho^2}f_2(g_2)dg_2 \\ &\underset{\mathbf{P3}}{=}\frac{e^{\frac{\gamma-\alpha(1-\rho^2)}{\alpha\gamma}}\gamma Ei\left(-\frac{1}{\alpha}\right)}{\gamma+\rho^2}-Ei\left(\frac{-1+\rho^2}{\gamma}\right)-\frac{\rho^2\beta}{\gamma}. \end{aligned} \tag{45}$$

In (45), the first integral (**P1**) is derived with the help of [22, eq. (4.331.2)], and the second integral (**P2**) is derived by using integration by parts, and the third integral (**P3**) is derived with the help of [22, eq. (4.331.2)].

APPENDIX II

We derive the interference outage probability when the maximum transmission power at the STx is P_m. The interference outage is then written as

$$\mathcal{P}_o=\int_m^\infty\int_0^{g_0} f_{g_0,\hat{g}_0}(x,y)\,dydx, \tag{46}$$

where $m = I_{th}/P_m$. The joint PDF of g_0 and $\hat{g}_0$ is presented as [9]

$$f_{g_0,\hat{g}_0}(x,y)=\frac{1}{(1-\rho^2)}e^{-\frac{x+y}{(1-\rho^2)}}I_0\left(\frac{2\rho\sqrt{xy}}{(1-\rho^2)}\right),\quad x,y\geq 0, \tag{47}$$

where $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind. Then, the interference outage probability can be written as

$$\mathcal{P}_0=\frac{1}{1-\rho^2}\int_m^\infty e^{-\frac{x}{1-\rho^2}}\underbrace{\int_0^x e^{-\frac{y}{1-\rho^2}}I_0\left(\frac{2\rho\sqrt{xy}}{1-\rho^2}\right)dy}_{B(x)}dx. \tag{48}$$

In (48), $B(x)$ can be derived with the help of [24. eq. (10)] as

$$B(x) = \frac{1-\rho^2}{2} e^{-\frac{\rho^2 x}{1-\rho^2}} \left(1 - Q_1\left(\sqrt{\frac{2\rho^2 x}{1-\rho^2}}, \sqrt{\frac{2x}{1-\rho^2}}\right)\right), \tag{49}$$

where $Q_1(a,b)$ is the first-order Marcum Q-function defined as (25). Thus, the outage interference can be rewritten as

$$\mathcal{P}_0 = \int_m^\infty e^{-x} dx - \underbrace{\int_m^\infty e^{-x} Q_1\left(\sqrt{\frac{2\rho^2 x}{1-\rho^2}}, \sqrt{\frac{2x}{1-\rho^2}}\right) dx}_{C}. \tag{50}$$

In (50), C can be expressed with help of [24, eq. (55)] as

$$C = e^{-m} Q_1\left(\sqrt{\frac{2\rho^2 m}{1-\rho^2}}, \sqrt{\frac{2m}{1-\rho^2}}\right) - \frac{1}{2} e^{-\frac{2m}{1-\rho^2}} I_0\left(\frac{2\rho m}{1-\rho^2}\right). \tag{51}$$

Therefore, the interference outage probability is derived as

$$\begin{aligned}\mathcal{P}_o = e^{-m} - e^{-m} Q_1\left(\sqrt{\frac{2\rho^2 m}{1-\rho^2}}, \sqrt{\frac{2m}{1-\rho^2}}\right) \\ + \frac{1}{2} e^{-\frac{2m}{1-\rho^2}} I_0\left(\frac{2\rho m}{1-\rho^2}\right).\end{aligned} \tag{52}$$

APPENDIX III

We derive the distribution of $g_0/\hat{g}_0$. Using (47), the distribution of $g_0/\hat{g}_0$ is obtained by the following integration:

$$\begin{aligned} f_{\frac{g_0}{\hat{g}_0}}(x) &= \int_0^\infty y \times f_{g_0,\hat{g}_0}(xy, y) dy \\ &= \int_0^\infty y \frac{1}{(1-\rho^2)} e^{-\frac{xy+y}{(1-\rho^2)}} I_0\left(\frac{2\rho\sqrt{x}y}{(1-\rho^2)}\right) dy \\ &= \int_0^\infty y \frac{1}{(1-\rho^2)} e^{-\frac{(x+1)}{(1-\rho^2)} y} J_0\left(\frac{2\rho\sqrt{x}i}{(1-\rho^2)} y\right) dy, \end{aligned} \tag{53}$$

where $J_0(\cdot)$ is a zeroth-order Bessel function of first kind. In (53), we use a relationship of $I_0(x) = J_0(ix)$. With the help of [22, eq. (6.623.1)], we can be evaluated the last integral, so that the pdf of $g_0/\hat{g}_0$ is derived as follows:

$$f_{\frac{g_0}{\hat{g}_0}}(x) = \frac{(1-\rho^2)(1+x)}{\{1+x(2-4\rho^2+x)\}^{3/2}}, \quad x \geq 0. \tag{54}$$

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