# On Combined Beamforming and OSTBC over the Cognitive Radio Z-Channel with Partial CSI

Efthymios Stathakis, Mikael Skoglund, and Lars K. Rasmussen
School of Electrical Engineering and ACCESS Linnaeus Center
KTH - Royal Institute of Technology, Stockholm, Sweden
Email:{eftsta,skoglund,lkra}@kth.se

***Abstract*—We consider a pair of secondary nodes (SU) coupled, in Z-topology, with multiple pairs of primary nodes (PU). The secondary (cognitive) transmitter is combining beamforming with orthogonal space-time block coding (BOSTBC) and operates under Quality-of-Service (QoS) constraints that must be guaranteed for the primary receivers $(\mathrm{PU_{Rx}})$. The cognitive link is designed assuming imperfect channel state information (CSI) for all links, available at the SU transmitter $(\mathrm{SU_{Tx}})$. Under this premise we characterize the optimal design in terms of CSI quality and interference and evaluate their impact on the performance of BOSTBC transmission in underlay cognitive networks.**

## I. Introduction

The increasing demand for wireless services has rendered spectrum scarcity the bottleneck of modern communications. One popular approach to meeting this challenge relies on Cognitive Radios (CR); smart devices able to sense their local environment [1]. The CR paradigm targets efficient spectrum allocation, by encouraging vacant band acquisition (*Interweave CR*) by secondary users or by allowing simultaneous transmission (*Underlay*, *Overlay CR*) of both primary and secondary users [2]. All approaches, however, are driven by the requirement to maintain a minimum QoS level for PUs.

Spectrum utilization is favored in overlay and underlay networks, with the latter being a subset (of assumptions) of the former and thus being easier for us to analyse. Underlay CRs operate in occupied PU bands, under primary QoS constraints and *no* cooperation with PUs. Due to concurrent transmission, underlay networks carry the challenges of traditional interference networks. Thus, to simplify analysis we study a relaxation of the CR channel, the cognitive Z-channel, where interference is present only on the $\mathrm{SU_{Tx}}-\mathrm{PU_{Rx}}$ link. In this context single-user techniques can be adapted and re-implemented to test their feasibility under the underlay assumptions.

The current work investigates, in the aforementioned direction, the performance of combined beamforming and orthogonal space-time block coding (OSTBC). This approach was pioneered in [3],[4] in order to combine the advantages of OSTBCs [5], a class of codes with full diversity and linear complexity maximum likelihood (ML) decoding; and beamforming, a technique which adapts signal transmission and power allocation to channel conditions. Various extensions have been considered, see [6], [7] and references therein, however they have been directed towards single-user networks.

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Analysis of beamforming for various types of CR networks, under primary QoS constraints and perfect CSI has been extensively studied, see [8] for a comprehensive overview. Design of robust beamforming strategies, for the Z-channel, with partial CSI has been been considered in [9], where the authors assumed *structured* uncertainty channel models. Analysis of BOSTBC transmission, in the same context, under the assumption of perfect CSI was done in [10], however suboptimal solutions were proposed.

In this paper, we employ BOSTBC on the secondary link of the Z-channel and analyze its performance under partial CSI and primary receive QoS constraints. We adopt a *statistical* uncertainty model in order to characterize the quality of side-information, available at the cognitive transmitter. The optimal transmit design, minimizing the error probability, turns out to be a convex optimization problem [11], therefore it admits efficient, though not analytical, solutions. In the absence of closed-form solutions we assess the impact of channel mismatch errors on system performance by characterizing the structure of the optimal design in terms of CSI quality. We further propose a way to model interference constraints such that primary QoS satisfaction is design dependent.

The paper is organized as follows. The system model is described in Section II and problem formulation and analysis are provided in Section III. Numerical results are presented in Section IV and finally Section V concludes the paper.

## II. System Model

### A. Network Model

The basic cognitive Z-channel, shown in Fig. 1a, is a special case of the cognitive interference channel, obtained from the latter by removing the $\mathrm{PU_{Tx}}-\mathrm{SU_{Rx}}$ link. This occurs when for various reasons, e.g. distance, the $\mathrm{PU_{Tx}}-\mathrm{SU_{Rx}}$ link is practically inactive, i.e. the received power at the $\mathrm{SU_{Rx}}$ node from the $\mathrm{PU_{Tx}}$ node is negligible. Moreover, we assume that the $\mathrm{SU_{Tx}}$ node has partial CSI for the $\mathrm{SU_{Tx}}-\mathrm{SU_{Rx}}$ link and all $\mathrm{SU_{Tx}}-\mathrm{PU_{Rx}}$ links and the $\mathrm{SU_{Rx}}$ node has full CSI of the $\mathrm{SU_{Tx}}-\mathrm{SU_{Rx}}$ link, in order to perform ML-detection.

### B. Channel Model

Our network, depicted in Fig. 1b, comprises $K$ primary $\mathrm{PU_{Tx}}-\mathrm{PU_{Rx}}$ links and a single cognitive pair with all nodes carrying multiple antennas (MIMO). We use $M_k, N_k$ to denote the number of transmit and receive antennas for the

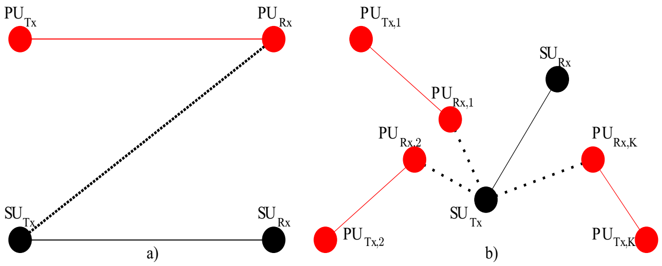


Fig. 1. a) The basic cognitive Z-channel and b) our system with multiple Z-channels. Red lines refer to primary and black to cognitive users, respectively.

$k$-th primary link. Further, we define $M, N$ as the number of $\mathrm{SU_{Tx}}$ and $\mathrm{SU_{Rx}}$ antennas, respectively. The $\mathrm{SU_{Tx}} - \mathrm{SU_{Rx}}$ channel is denoted by $\mathbf{H} \in \mathbb{C}^{M\times N}$, where element $h_{ij}^H$ describes the channel between transmit antenna $i$ and receive antenna $j$, with $(\cdot)^H$ denoting conjugate transpose. Channel statistics are captured by the vector $\mathbf{h} = \mathrm{vec}(\mathbf{H})$, which is modeled as complex Gaussian, i.e. $\mathbf{h} \sim \mathbb{CN}(\mathbf{m}, \mathbf{K})$. Similarly, each $\mathrm{SU_{Tx}} - \mathrm{PU_{Rx}}$ cross-link channel $\mathbf{G}_j \in \mathbb{C}^{M\times N_j}$ is fully characterized by its corresponding vector $\mathbf{g}_j$, distributed according to $\mathbb{CN}(\mathbf{m}_j, \mathbf{K}_{jj})$. We restrict our attention to non-singular covariance matrices that have Kronecker structure [12], i.e. $\mathbf{K} = \mathbf{K}_r \otimes \mathbf{K}_t$ (each $\mathbf{K}_{jj}$ is similarly defined), with $\mathbf{K}_t, \mathbf{K}_r$ denoting transmit and receive covariance matrices.

System design concerns secondary communication, which is assumed to be quasi-static block fading for all links. During each block, $\mathrm{SU_{Tx}}$ sends a codeword of the form $\mathbf{C} = \mathbf{W}\bar{\mathbf{C}}$ where $\mathbf{W} \in \mathbb{C}^{M\times M}$ is a beamforming matrix and $\bar{\mathbf{C}} \in \mathbb{C}^{M\times L}$ is a mapping of the information vector $\mathbf{c}$, with BPSK elements $c_i$, on rate-one $(L = M)$ OSTBC designs [5]. The received matrix, in each block, is written as

$$\mathbf{Y} = \mathbf{H}^H\mathbf{C} + \mathbf{N}, \tag{1}$$

where $\mathbf{N} \sim \mathbb{CN}\left(\mathbf{0}, \sigma_n^2\mathbf{I}_N\right)$ is the additive white complex Gaussian noise term, with variance $\sigma_n^2$.

### C. CSI Model

To capture the impact of partial CSI we assume imperfect channel estimates $\hat{\mathbf{h}}, \{\hat{\mathbf{g}}_j\}_{j=1}^K$, available at $\mathrm{SU_{Tx}}$. Our statistical uncertainty model is determined by the distribution of $\tilde{\mathbf{h}} = \mathbf{h}|\hat{\mathbf{h}}$, which relies on the distribution of $\mathbf{h}^o = [\mathbf{h}^T\hat{\mathbf{h}}^T]^T$. If $\mathbf{h}^o \sim \mathbb{CN}(\mathbf{m}^o, \mathbf{K}^o)$ then $\tilde{\mathbf{h}} \sim \mathbb{CN}(\tilde{\mathbf{m}}, \tilde{\mathbf{K}})$ with components determined from $\mathbf{m}^o, \mathbf{K}^o$[13]. Since the conditional mean $\tilde{\mathbf{m}}$ carries information regarding the actual channel realization and the conditional covariance $\tilde{\mathbf{K}}$ measures the mismatch, we characterize uncertainty using a single parameter $\delta$, and model asymptotic cases as: a) no CSI $\delta \to 0, \tilde{\mathbf{m}} \to \mathbf{0}$ and $\tilde{\mathbf{K}} \to \mathbf{K}$, b) full CSI $\delta \to 1, \tilde{\mathbf{m}} \to \mathbf{h}$ and $\|\tilde{\mathbf{K}}\| \to \mathbf{0}$, where $\|\cdot\|$ denotes spectral norm for matrices and Euclidean norm for vectors. Similar, to the above, definitions hold for each $\mathrm{SU_{Tx}} - \mathrm{PU_{Rx,j}}$ link where the random vector $\tilde{\mathbf{g}}_j = \mathbf{g}_j|\hat{\mathbf{g}}_j$ and the respective asymptotic cases are characterized using $\tilde{\mathbf{m}}_j, \tilde{\mathbf{K}}_{jj}$ and $\delta_j$.

### D. Interference Power Model

A critical aspect of CR networks is interference management since the QoS of the PUs must be maintained, above a predefined threshold, during cognitive operation. A common approach is to impose a constraint $\Gamma_j$ on the actual received inteference power $P_j^{int} = \|\mathbf{G}_j^H\mathbf{W}\bar{\mathbf{C}}\|_F^2$, of the $j$-th link $\mathrm{SU_{Tx}} - \mathrm{PU_{Rx,j}}$, expressed as $P_j^{int} \le \Gamma_j$, with $\|\cdot\|_F$ denoting the Frobenius norm. In our model the estimated inflicted interference $\tilde{P}_j^{int}$ on $\mathrm{PU_{Rx,j}}$, measured at $\mathrm{SU_{Tx}}$, conditioned on its side-information is

$$\begin{aligned}\tilde{P}_j^{int} &= \mathbf{E}\left\{\left\|\mathbf{G}_j^H\mathbf{W}\bar{\mathbf{C}}\right\|_F^2|\hat{\mathbf{G}}_j\right\} \overset{(a)}{=} M\mathbf{E}\left\{\mathrm{tr}(\mathbf{G}_j^H\mathbf{Z}\mathbf{G}_j)|\hat{\mathbf{G}}_j\right\}\\ &\overset{(b)}{=} M\mathbf{E}\left\{(\mathbf{g}_j^H(\mathbf{I}_N\otimes\mathbf{Z})\mathbf{g}_j)|\hat{\mathbf{g}}_j\right\} \overset{(c)}{=} M\mathrm{tr}\left(\mathbf{Z}\boldsymbol{\Theta}_j\right),\end{aligned}$$

$(a)$ follows by $\bar{\mathbf{C}}\bar{\mathbf{C}}^H = M\mathbf{I}_M$ for codeword bits $c_i = \pm 1$ and $\mathbf{Z} = \mathbf{W}\mathbf{W}^H$, $(b)$ follows by the identity $\mathrm{tr}(\mathbf{A}^H\mathbf{B}\mathbf{C}) = \mathrm{vec}(\mathbf{A})^H(\mathbf{I}\otimes\mathbf{B})\mathrm{vec}(\mathbf{C})$ for matrices $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ and $(c)$ follows by reformulating the quadratic as a trace, bringing expectation inside and setting $\tilde{\mathbf{R}}_{jj} = \mathbf{E}\left\{\mathbf{g}_j\mathbf{g}_j^H|\hat{\mathbf{g}}_j\right\}$. Note that $\tilde{\mathbf{R}}_{jj} = \tilde{\mathbf{K}}_{jj} + \tilde{\mathbf{m}}_j\tilde{\mathbf{m}}_j^H$, hence $\boldsymbol{\Theta}_j$ is, by construction, the sum of the $M\times M$ blocks on the diagonals of $\tilde{\mathbf{K}}_{jj}, \tilde{\mathbf{m}}_j\tilde{\mathbf{m}}_j^H$.

In general $\tilde{P}_j^{int} \neq P_j^{int}$ hence a beamformer designed to satisfy $\tilde{P}_j^{int} \le \Gamma_j$ does not necessarily satisfy $P_j^{int} \le \Gamma_j$. Failure to meet the actual constraint can be viewed as *system outage* and one way to resolve this is to make the right-hand-side (RHS) of the interference constraint an increasing function $I_j = f(\delta_j, \Gamma_j) \in [0, \Gamma_j]$ of the uncertainty measure $\delta_j$; hence the system design will now be subject to the constraint $\tilde{P}_j^{int} \le I_j$ which becomes tighter with diminishing CSI quality. For example, $I_j = \delta_j\Gamma_j, \delta_j^2\Gamma_j$ penalize differently the RHS depending on how our system rates QoS violations.

## III. Problem Formulation and Analysis

In this section we embed the problem discussed in [3] to our system model in order to assess CSI impact on the secondary link performance. The initial design considers the ML rule in order to determine $P(\mathbf{C}_l \to \mathbf{C}_m|\mathbf{C}_l, \hat{\mathbf{h}})$, the probability of erroneously choosing codeword $\mathbf{C}_m$, given the CSI and the actual sent codeword $\mathbf{C}_l$. After some manipulations, detailed in [3], we get to minimize an upper bound on $P(\cdot|\cdot)$ which is equivalent to maximizing the following concave function

$$\begin{aligned}l(\mathbf{Z}) = &-\tilde{\mathbf{m}}^H\tilde{\mathbf{K}}^{-1}((\mathbf{I}_N\otimes\mathbf{Z}\alpha) + \tilde{\mathbf{K}}^{-1})^{-1}\tilde{\mathbf{K}}^{-1}\tilde{\mathbf{m}}\\ &+ \log\det((\mathbf{I}_N\otimes\mathbf{Z}\alpha) + \tilde{\mathbf{K}}^{-1}),\end{aligned} \tag{2}$$

where $\alpha = \frac{\mu_{\min}^2}{4\sigma_n^2}, \mu_{\min} = \min\left\{\|\mathbf{c}_l - \mathbf{c}_m\| \,\forall\, l \neq m\right\}$ and log base is $e$. In the context of the Z-channel we need to maximize $l(\mathbf{Z})$ under a power constraint $(\mathrm{tr}(\mathbf{Z}) \le 1)$ and interference constraints (Section II-D); hence the generic design can be posed as the following convex optimization problem

$$\begin{aligned}&\text{maximize } l(\mathbf{Z})\\ &\text{subject to } \mathbf{Z}\in\mathcal{S},\end{aligned} \tag{3}$$

where $\mathcal{S} = \{\mathrm{tr}(\mathbf{Z}) \le 1, \mathrm{tr}(\mathbf{Z}\boldsymbol{\Theta}_j) \le {}^{I_j}/_{M}, j = 1,\ldots,K, \mathbf{Z}\succeq\mathbf{0}\}$. The presence of multiple constraints hinders the development of analytical solutions. However, inspired by [8], we can obtain the form of the optimal point $\mathbf{Z}^{\mathrm{opt}}$, in special instances of $l(\mathbf{Z})$ presented below. By means of the Lagrangian $L(\mathbf{Z}, \boldsymbol{\eta})$, where $\boldsymbol{\eta} = [\eta_i]_{i=0}^K$ is the dual vector associated with the linear constraints, the optimal value $p^\star$ can be determined as

$$p^\star = \min_{\boldsymbol{\eta}\succeq\mathbf{0}}\max_{\mathbf{Z}\succeq\mathbf{0}} L(\mathbf{Z}, \boldsymbol{\eta}). \tag{4}$$

If the Lagrangian relaxation is easily solved with respect to (w.r.t) the primal variable $\mathbf{Z}$, then we can *efficiently* find

$\mathbf{Z}^{\text{opt}}$ using the projected subgradient method (PSM) [14], by alternating between the inner (primal) and the outer (dual) optimization problems, with guaranteed convergence. When $\mathbf{Z}^{\text{opt}}$ is obtained we can decompose it as $\mathbf{Z}^{\text{opt}} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ and choose the optimal beamformer as $\mathbf{W}^{\text{opt}} = \mathbf{U}\mathbf{\Lambda}^{1/2}$.

### A. No CSI for $SU_{Tx} - SU_{Rx}$ link

When the $\text{SU}_{\text{Tx}}$ node has no CSI for the cognitive link, i.e. $\delta \to 0, \tilde{\mathbf{m}} \to \mathbf{0}$ and $\tilde{\mathbf{K}} \to \mathbf{K}$, then the objective function $l(\mathbf{Z})$ tends to $l_1(\mathbf{Z}) = \log\det\left[(\mathbf{I}_N \otimes \mathbf{Z})\alpha + \mathbf{K}^{-1}\right]$. Adding power and interference constraints yields the following problem.

$$\begin{aligned} &\text{maximize} \;\; \log\det\left[(\mathbf{I}_N \otimes \mathbf{Z})\alpha + \mathbf{K}^{-1}\right] \\ &\text{subject to} \;\; \mathbf{Z} \in \mathcal{S}. \end{aligned} \tag{5}$$

For ease of exposition we define the matrix $\mathbf{T} = \eta_0 \mathbf{I}_M + \sum_{j=1}^{K} \eta_j \mathbf{\Theta}_j$, with decomposition $\mathbf{T} = \mathbf{U}_T \mathbf{\Lambda}_T \mathbf{U}_T^H$ and let $\mathbf{S} \equiv \mathbf{T}^{1/2} = \mathbf{U}_T \mathbf{\Lambda}_T^{1/2} \mathbf{U}_T^H$. Recalling that $\mathbf{K} = \mathbf{K}_r \otimes \mathbf{K}_t$, define matrices $\mathbf{R} = \mathbf{K}_t^{1/2}\mathbf{S}^{-1}, \hat{\mathbf{Z}} = \mathbf{S}\mathbf{Z}\mathbf{S}$ and the decompositions $\mathbf{R} = \mathbf{U}_R \mathbf{\Sigma}_R \mathbf{V}_R^H$, $\hat{\mathbf{Z}} = \mathbf{U}_{\hat{Z}} \mathbf{\Lambda}_{\hat{Z}} \mathbf{U}_{\hat{Z}}^H$ and $\mathbf{K}_r = \mathbf{U}_{K_r} \mathbf{\Lambda}_{K_r} \mathbf{U}_{K_r}^H$. The optimal point is given by the following proposition.

**Proposition 1.** *The form of the optimal point, solving* (5)*, is* $\mathbf{Z}^{\text{opt}} = \mathbf{S}^{-1}\mathbf{V}_R(\mathbf{\Lambda}_{\hat{Z}})^+ \mathbf{V}_R^H \mathbf{S}^{-1}$*, where* $(\cdot)^+$ *denotes element-wise maximum of the entries of* $\mathbf{\Lambda}_{\hat{Z}}$ *and 0.*

*Proof*: We only sketch the proof here, due to page constraints. Let $L_1(\mathbf{Z}, \boldsymbol{\eta})$ be the Lagrangian of (5), written in terms of the variables $\hat{\mathbf{Z}}, \mathbf{R}, \mathbf{K}_r$, defined above. Invoking the invariance of trace and determinant operators to unitary transformations and applying Hadamard's inequality yield $\mathbf{U}_{\hat{Z}} = \mathbf{V}_R$ and render (4) separable in the diagonal elements $\lambda_{\hat{Z}}(i)$ of $\mathbf{\Lambda}_{\hat{Z}}$. Each $\lambda_{\hat{Z}}(i)$ can then be easily determined, by solving the equation $\partial L_1/\partial \lambda_{\hat{Z}}(i) = 0$ with bisection and projecting the solution on the non-negative orthant. In the special case of uncorrelated antennas the design further simplifies as follows.

**Corollary 1.** *For uncorrelated antennas,* $\mathbf{K} = \sigma_h^2 \mathbf{I}_{MN}$*, we have* $\mathbf{Z}^{\text{opt}} = \mathbf{U}_T \mathbf{\Lambda}_T^{-1/2}(N\mathbf{I}_M - \frac{1}{\tilde{\alpha}}\mathbf{\Lambda}_T)^+ \mathbf{\Lambda}_T^{-1/2}\mathbf{U}_T^H$*,* $\tilde{\alpha} = \sigma_h^2 \alpha$.

*Proof*: If $\mathbf{K} = \sigma_h^2 \mathbf{I}_{MN}$ then $\mathbf{R} \equiv \mathbf{S}$ and the result follows by extrapolating the previous proof concept and noting that, in this special case, $\partial L_1/\partial \lambda_{\hat{Z}}(i) = 0$ has a closed-form solution which is $\lambda_{\hat{Z}}(i) = (N - \frac{1}{\tilde{\alpha}}\lambda_T(i))^+$, where $\lambda_T(i)$ denotes the $i$-th diagonal element of matrix $\mathbf{\Lambda}_T$.

### B. Full CSI for $SU_{Tx} - SU_{Rx}$ link

When the $\text{SU}_{\text{Tx}}$ node has full CSI for the cognitive link, i.e. $\delta \to 1, \tilde{\mathbf{m}} \to \mathbf{h}$ and $\|\tilde{\mathbf{K}}\| \to \mathbf{0}$, then from [3], the objective function $l(\mathbf{Z})$ tends to $l_2(\mathbf{Z}) = \mathbf{h}^H (\mathbf{I}_N \otimes \mathbf{Z})\, \mathbf{h} = \text{tr}(\mathbf{Z}\mathbf{H}\mathbf{H}^H)$. In this context the beamformer design problem becomes

$$\begin{aligned} &\text{maximize} \;\; \text{tr}(\mathbf{Z}\mathbf{H}\mathbf{H}^H) \\ &\text{subject to} \;\; \mathbf{Z} \in \mathcal{S}. \end{aligned} \tag{6}$$

The form of $\mathbf{Z}^{\text{opt}}$ is given by the following proposition.

**Proposition 2.** *The form of the optimal point, solving* (6)*, is* $\mathbf{Z}^{\text{opt}} = \mathbf{v}\mathbf{v}^H$*, where* $\mathbf{v}$ *is the principal generalized eigenvector associated with the maximum generalized eigenvalue of the pair of matrices* $(\mathbf{H}\mathbf{H}^H, (1/B)\mathbf{T})$*, where matrix* $\mathbf{T}$ *is defined as in section III-A and* $B = \eta_0 + \frac{1}{M}\sum_{j=1}^{K} \eta_j I_j$.

*Proof*: The basic idea of the proof is that the Lagrangian $L_2(\mathbf{Z}, \boldsymbol{\eta})$ of (6), when written using the definitions of $(\mathbf{T}, B)$, looks like a scaled, by $1/B$, version of the Lagrangian of problem (6) with just one constraint in the feasible set, i.e. $\mathcal{S} = \{\text{tr}(\mathbf{Z}\mathbf{T}) \le 1\}$. For the latter problem, though, the solution can be easily shown to be the principal generalized eigenvector of the pair of matrices $(\mathbf{H}\mathbf{H}^H, \mathbf{T})$.

### C. A Simplified Channel Model

As in [3], a tractable instance of (2) can be realized by considering Rayleigh channels $\mathbf{h}, \{\mathbf{g}_j\}_{j=1}^K$ and estimates $\hat{\mathbf{h}}, \{\hat{\mathbf{g}}_j\}_{j=1}^K$ with i.i.d components, i.e. $\mathbf{h}, \hat{\mathbf{h}} \sim \mathbb{C}\mathcal{N}(\mathbf{0}, \sigma_h^2 \mathbf{I}_{MN})$ and cross-covariance matrix $\bar{\mathbf{K}} = \mathbf{K}_{h\hat{h}} = \delta \sigma_h^2 \mathbf{I}_{MN}$. (Identical definitions hold for the cross-link channels each with individual variance $\sigma_{g_j}^2$.) We can then easily derive $\tilde{\mathbf{m}} = \delta \hat{\mathbf{h}}$ and $\tilde{\mathbf{K}} = \sigma_h^2(1 - |\delta|^2)\mathbf{I}_{MN}$. Similarly we get $\tilde{\mathbf{m}}_j = \delta_j \hat{\mathbf{g}}_j$ and $\tilde{\mathbf{K}}_{jj} = \sigma_{g_j}^2(1 - |\delta_j|^2)\mathbf{I}_{MN_j}$, from which we calculate $\mathbf{\Theta}_j = N_j \sigma_{g_j}^2 (1 - |\delta_j|^2)\mathbf{I}_M + |\delta_j|^2 \hat{\mathbf{G}}_j \hat{\mathbf{G}}_j^H$. If we plug these statistics in the generic objective (2), we obtain a special instance $l_3(\mathbf{Z})$ of $l(\mathbf{Z})$ and problem (3) is now equivalent to

$$\begin{aligned} &\text{maximize} \;\; N\log\det(\mathbf{Z}\bar{\alpha} + \mathbf{I}_M) - \text{tr}[(\mathbf{Z}\bar{\alpha} + \mathbf{I}_M)^{-1}\, \bar{\mathbf{Y}}] \\ &\text{subject to} \;\; \mathbf{Z} \in \mathcal{S}, \end{aligned} \tag{7}$$

where $\bar{\alpha} = \alpha \sigma_h^2 \left(1 - |\delta|^2\right)$, $\bar{\mathbf{Y}} = \frac{|\delta|^2}{\sigma_h^2(1-|\delta|^2)} \hat{\mathbf{H}}\hat{\mathbf{H}}^H$. Defining $\mathbf{T}, \mathbf{S}$ as in Section III-A, we further let $\bar{\mathbf{T}} = \mathbf{T} - \mathbf{\Phi}$ and $\bar{\mathbf{S}} = \bar{\mathbf{T}}^{1/2}$, where $\mathbf{\Phi}$ is the dual variable associated with $\mathbf{Z}$. If we form the Lagrangian $L_3'(\mathbf{Z}, \mathbf{\Phi}, \boldsymbol{\eta})$, then the characterization of the solution stems from the following proposition.

**Proposition 3.** *The form of the optimal point, solving* (7)*, is* $\mathbf{Z}^{\text{opt}} = \frac{1}{2}\bar{\mathbf{S}}^{-1}\big[N\mathbf{I}_M + (N^2\mathbf{I}_M + \frac{4}{\bar{\alpha}}\bar{\mathbf{S}}\bar{\mathbf{Y}}\bar{\mathbf{S}})^{\frac{1}{2}}\big]\bar{\mathbf{S}}^{-1} - \frac{1}{\bar{\alpha}}\mathbf{I}_M$.

*Proof*: Set the gradient of $L_3'(\mathbf{Z}, \mathbf{\Phi}, \boldsymbol{\eta})$ w.r.t $\mathbf{Z}$ equal to zero to get the unconstrained minimizer and solve the resulting equation as in [6]. In this result $\mathbf{\Phi}$ does not have an immediate physical interpretation, except that it controls the eigenvalues of $\mathbf{Z}^{\text{opt}}$. We can derive a suboptimal solution, without invoking $\mathbf{\Phi}$, if we keep the constraint $\mathbf{Z} \succeq \mathbf{0}$ implicit and write the corresponding Lagrangian $L_3(\mathbf{Z}, \boldsymbol{\eta})$. In this case, the solution will have the form described in the following result.

**Corollary 2.** *A suboptimal solution to* (7) *is given by* $\mathbf{Z}^{\text{sub}} = \left\{\frac{1}{2}\mathbf{S}^{-1}\big[N\mathbf{I}_M + (N^2\mathbf{I}_M + \frac{4}{\bar{\alpha}}\mathbf{S}\bar{\mathbf{Y}}\mathbf{S})^{\frac{1}{2}}\big]\mathbf{S}^{-1} - \frac{1}{\bar{\alpha}}\mathbf{I}_M\right\}^+$*, where* $\{\cdot\}^+$ *denotes projection on the set of positive semidefinite (PSD) matrices.*

*Proof*: The proof is similar to that of Proposition 3 except that the unconstrained minimizer of $L_3$ is projected on the set of PSD matrices. The motivation for the presentation of this result lies in the fact that $\mathbf{Z}^{\text{sub}}$ depends only on quantities that have physical interpretations. We highlight three aspects here:
**Remark 1**: In the case of no CSI ($\delta \to 0, \bar{\mathbf{Y}} \to \mathbf{0}$), for the direct-link $\text{SU}_{\text{Tx}} - \text{SU}_{\text{Rx}}$, we recover from $\mathbf{Z}^{\text{sub}}$ the solution of Corollary 1; thus this relaxation can always attain the upper bound on $P(\cdot|\cdot)$, obtained by the optimal solution of the no CSI case, and make it tighter, in general. This is an indication that this solution will normally have acceptable performance, especially when the Bit-Error-Rate (BER) gap between no and full CSI scenarios, for the direct-link, is not large.

**Remark 2**: The parameter $\bar{\alpha}$ can be interpreted as the signal to noise ratio (SNR). Observing that the $\log\det(\cdot)$ term in $l_3(\mathbf{Z})$ enforces $\mathbf{Z} \succ -\frac{1}{\bar{\alpha}}\mathbf{I}_M$ then with increasing $\bar{\alpha}$ the solution is "pushed", in the limit of $\bar{\alpha} \to \infty$, towards the interior of the set of PSD matrices, thus the PSD constraint on $\mathbf{Z}$ becomes inactive. In that case $\mathbf{\Phi}^{\text{opt}} = \mathbf{0}$ implying that $\mathbf{Z}^{\text{opt}} \equiv \mathbf{Z}^{\text{sub}}$.

**Remark 3**: It is important to note that the same strategy was adopted to prove the results in sections III-A, III-B, i.e. the constraint $\mathbf{Z} \succeq \mathbf{0}$ was kept implicit in the Lagrangian functions $L_1(\mathbf{Z}, \boldsymbol{\eta}), L_2(\mathbf{Z}, \boldsymbol{\eta})$ and the unconstrained minimizer w.r.t $\mathbf{Z}$ was projected on the set of PSD matrices. However, in those two cases, *optimality of the solutions can be established* with mathematical and graphical arguments because of the special structure of the resulting problems.

### D. Implications of the Solutions

The results stated in the previous subsections can give us insight into the behaviour of the candidate beamformers in order to assess how CSI quality and interference affect the design. All solutions have in common two things:

i. Diminishing CSI quality for cross-links, i.e. decreasing $\delta_j$, shrinks the feasible set $\mathcal{S}$ and in the limit, when no CSI is available for at least one pair, e.g. $\delta_j \to 0$ for cross-link $\text{SU}_{\text{Tx}} - \text{PU}_{\text{Rx},j}$, then $\mathcal{S} = \emptyset$ and no transmission is attempted, to avoid violation of primary QoS.

ii. The mathematical form of the solutions, as expressed in the aforementioned results, indicates that the optimal design attempts to balance cross-link interference with direct-link channel statistics. The beamformer properly combines the interference subspace, associated with the eigenvectors of $\mathbf{T}$, with the $\text{SU}_{\text{Tx}} - \text{SU}_{\text{Rx}}$ channel subspace, associated with the eigenvectors of $\tilde{\mathbf{K}}, \mathbf{H}$ or $\hat{\mathbf{H}}$, to minimize the bound on $P(\cdot|\cdot)$.

It has also been seen that solving the Lagrangian functions, w.r.t $\mathbf{Z}$, has the complexity of a matrix eigen-decomposition; hence the solution can be efficiently implemented. In this case, as stated in the beginning of this section, we can construct an iterative algorithm based on the PSM, which is much simpler to implement than interior-point based solvers [15]. Since the crucial step of the algorithm corresponds to the solution of the inner (primal) optimization problem in (3), which can be easily found, a method iterating between the primal and the dual problem can be a tractable solution. Based on this observation we present below the skeleton of this algorithm.

**Algorithm 1** PSM for BOSTBC

Choose $\epsilon, \{\eta_j^{(1)}\}_{j=1}^{K}$ and stepsize rule $\alpha_k$;
Set $k = 1, l_i(\mathbf{Z}^{(0)}) = -\infty$ *% i=1,2,3*
**repeat**
  $\mathbf{T}^{(k)} := \sum_{j=1}^{K} \eta_j^{(k)} \mathbf{\Theta}_j + \eta_0^{(k)} \mathbf{I}_M$;
  Calculate $\mathbf{Z}^{(k)}$; *% The expressions for* $\mathbf{Z}^{(k)}, l_i(\mathbf{Z}^{(k)})$
  Calculate $l_i(\mathbf{Z}^{(k)})$; *% depend on the examined case*
  $s_j^{(k)} := I_j - \text{tr}(\mathbf{Z}^{(k)} \mathbf{\Theta}_j)$, $j = 1, \ldots, K$;
  $\eta_j^{(k+1)} := (\eta_j^{(k)} - \alpha_k s_j^{(k)})^+$, $j = 1, \ldots, K$;
  $\Delta l := |l_i(\mathbf{Z}^{(k)}) - l_i(\mathbf{Z}^{(k-1)})|$; $k := k + 1$; *% i=1,2,3*
**until** $\Delta l \leq \epsilon$
$\mathbf{Z}^{(k)} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H; \mathbf{W} = \mathbf{U}\mathbf{\Lambda}^{1/2}$

## IV. Numerical Results

In this section we provide numerical results in order to evaluate CSI impact. We consider a network with one primary link and we adopt the simplified channel model, described in Section III-C, with parameters $M = N = 2, N_1 = 1$ and variances $\sigma_h^2 = \sigma_{g_1}^2 = 1$ and a fixed $\Gamma_1 = M$. The results are plotted using the *nominal* SNR, defined as $\overline{\text{SNR}} = P/\sigma_n^2$ where $P = \mathbf{E}\{\|\mathbf{H}^H\mathbf{C}\|_F^2\}$ and $\sigma_n^2 = \mathbf{E}\{\|\mathbf{N}\|_F^2\}$ denote the total average received signal power and noise power, respectively. We use $\overline{\text{SNR}}$ since an optimum solution might not satisfy $\|\mathbf{Z}\|_F^2 = 1$, hence the actual SNR will be lower than $P/\sigma_n^2$. However our system is evaluated for fixed noise variance, irrespectively of how transmit power is utilized. Albeit the results should then be plotted against $\sigma_n^2$, we use instead $\overline{\text{SNR}}$ as a familiar measure of representation.

Fig. 2 shows BER performance for different choices of system parameters. Comparison of the results for $(\delta,\delta_1) = (1, 0.7)$ and $(\delta,\delta_1) = (0.7, 1)$ reveals that direct-link CSI is more critical, in terms of BER, than cross-link CSI. The former pair, with $\delta = 1$, has an observable performance gap, of the order of 1.5dB, w.r.t the BER of the latter with $\delta = 0.7$, even though $\delta + \delta_1 = 1.7$ for both setups. For fairness of comparison, it should be noted here that the latter pair induces no primary QoS violation since the cross-link channel $\mathbf{G}_1$ is perfectly known. This observation raises a trade-off for the BER w.r.t the CSI quality, which occurs when $\delta + \delta_1 = \text{constant}$. Moreover we observe that for fixed $\delta$, degradation with diminishing $\delta_1$, is negligible. It is also seen that the suboptimal solution proposed by corollary 2 performs well, in terms of BER, for this setup.

Fig. 3 illustrates the impact of cross-link CSI quality $\delta_1$ for two differents functions, a linear $f_1(\delta_1, M) = \delta_1 M$ and a quadratic $f_2(\delta_1, M) = \delta_1^2 M$, of the RHS of the interference power constraint. Note that, full CSI ($\delta = 1$) is assumed for the $\text{SU}_{\text{Tx}} - \text{SU}_{\text{Rx}}$ link, hence the solution is a single-mode beamformer, given by Proposition 2. In general, we see that the system exhibits acceptable performance even for moderate values of CSI, up to $\delta_1 = 0.5$, and only the quadratic function incurs observable degradation, when $\delta_1 = 0.5$, w.r.t the perfect cross-link CSI case. The linear function, though, renders the system quite robust to cross-link mismatch errors. For example, at BER$= 10^{-3}$ there is less than 2dB loss between $\delta_1 = 1$ and $\delta_1 = 0.5$ cases. This is an indication that the single-mode beamformer is quite insensitive to directional rotations, incurred by balancing signal with interference subspace, and it is mainly the power back-off, due to the constraints, which causes performance degradation.

Fig. 4 shows the outage probability (ratio) $P_{out}$ for different cases. We observe that, for fixed $\delta = 0.7$, $P_{out}$ curves shift to the right, with growing SNR, and $P_{out}$ drops. This occurs because from Proposition 3 we see that, with increasing $\bar{\alpha}$, the beamformer is putting more weight on interference suppression, described by the terms $\bar{\mathbf{S}}$ and $\mathbf{S}$. Another notable result is the gap between $P_{out}$ curves w.r.t the RHS function $f_1, f_2$, for $\delta = 1$. Since the quadratic function $f_2$ lies below the linear $f_1$, in the range $\delta_1 \in [0, 1]$, imperfect CSI induces tighter constraints, when $f_2$ is employed, hence the beamformer is

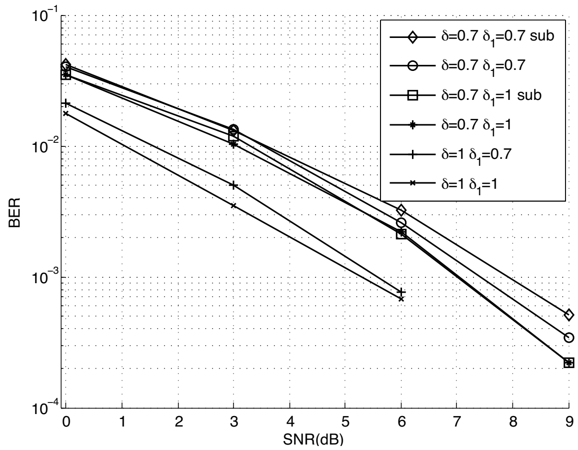


Fig. 2. BER performance for various system parameters. Results generated with the suboptimal solution of corollary 2 are indicated with "sub". The RHS function for the interference power constraint was $f_1 = \delta_1 \Gamma_1$. The SNR label corresponds to the nominal SNR as defined in the beginning of Section IV.

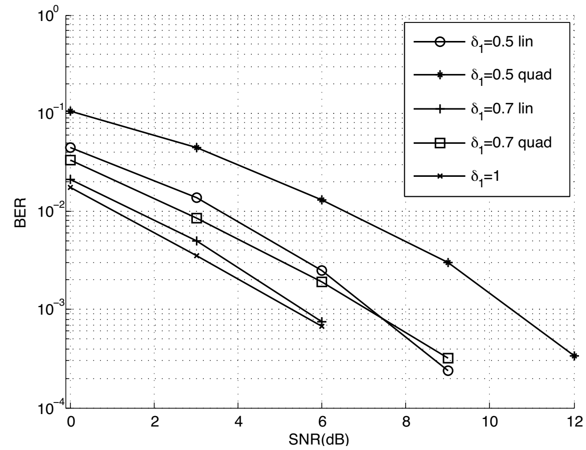


Fig. 3. BER comparison for two RHS functions of the interference power constraints. The terms "lin","quad" correspond to $f_1 = \delta_1 \Gamma_1, f_2 = \delta_1^2 \Gamma_1$. For $\delta_1 = 1$ it holds that $f_1 = f_2$ and the curve represents both cases.

less likely to violate primary QoS. We can also mention here that results for $\delta = 1$ do not vary with $\overline{\text{SNR}}$, because $\overline{\text{SNR}}$ is not involved in the asymptotic solution for $\delta = 1$. We can further combine the results of Figures 3 and 4 to construct an interesting trade-off, between secondary user performance and primary QoS violation, depending on the cross-link CSI level $\delta_1$ and the choice of the function $f(\delta_1)$.

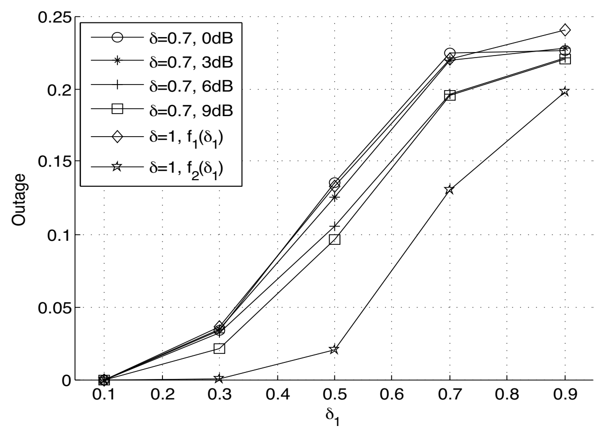


Fig. 4. The primary QoS violation ratio for various levels of cross-link CSI quality $\delta_1$ and RHS function $f_1(\delta_1), f_2(\delta_1)$. For $\delta = 0.7$ only the optimal beamformer of Proposition 3 was considered.

## V. Conclusions

In this paper we analyzed the performance of BOSTBC transmission in the context of underlay cognitive networks. Under the assumption of imperfect CSI we characterized the form of the optimal design in terms of the involved components (channels and CSI) and proposed a model for the QoS constraints that takes into account the quality of CSI. We further provided theoretical insights into the impact of various system parameters and further presented an iterative algorithm to solve the beamforming design problem.

From a practical viewpoint it would be useful to investigate proper parameter selection in order to speed up the PSM-based algorithms. Moreover, numerical evaluation can be extended to other scenarios and cover more aspects. One such direction would be the introduction of multiple PU links in order to analyse the aggregate impact, on system performance and outage, of having different CSI qualities. The current work indicates that supplementing the cognitive transmitter with side-information of the wireless environment can lead to feasible underlay cognitive system realizations with a compromise between secondary performance and primary QoS satisfaction.

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