"Determine the density function"

```
FullSimplify[
 D[1 / 2 Erf[(e / ((Sqrt[2] ts InverseErfc[2 x]  + 1)) - p) / (Sqrt[2] te p)], x]]
```

$$\frac{e\, e^{\text{InverseErfc}[2\,x]^2-\frac{\left(p-\frac{e}{1+\sqrt{2}\ ts\ \text{InverseErfc}[2\,x]}\right)^2}{2\,p^2\,te^2}}\ ts}{p\ te\left(1+\sqrt{2}\ ts\ \text{InverseErfc}[2\,x]\right)^2}$$

"Determine the expected value"

$$\text{Integrate}\left[x\ \frac{e\, e^{\text{InverseErfc}[2\,x]^2-\frac{\left(-p+\frac{e}{1+\sqrt{2}\ ts\ \text{InverseErfc}[2\,x]}\right)^2}{2\,p^2\,te^2}}\ ts}{p\ te\left(1+\sqrt{2}\ ts\ \text{InverseErfc}[2\,x]\right)^2},\ \{x,\ 0,\ 1\}\right]$$

$$\int_0^1 \frac{e\, e^{\text{InverseErfc}[2\,x]^2-\frac{\left(-p+\frac{e}{1+ts\ \text{InverseErfc}[2\,x]}\right)^2}{p^2\,te^2}}\ ts\ x}{p\ te\ (1+ts\ \text{InverseErfc}[2\,x])^2}\, dx$$

"Numerical Integration"

```
e = 0.1182 * 10^-9
ts = 0.0239
te = 0.0447
p = 1.3162 * 10^-10
```

$$\texttt{NIntegrate}\left[\frac{e\, e^{\texttt{InverseErfc}[2\,x]^2-\frac{\left(-p+\frac{e}{1+\sqrt{2}\;ts\;\texttt{InverseErfc}[2\,x]}\right)^2}{2\,p^2\,te^2}}\,ts}{p\;te\left(1+\sqrt{2}\;ts\;\texttt{InverseErfc}[2\,x]\right)^2},\ \{x, 0, 1\},\ \texttt{PrecisionGoal}\to 12,\ \texttt{MaxRecursion}\to 40\right]$$

$$\texttt{NIntegrate}\left[x\,\frac{e\, e^{\texttt{InverseErfc}[2\,x]^2-\frac{\left(-p+\frac{e}{1+\sqrt{2}\;ts\;\texttt{InverseErfc}[2\,x]}\right)^2}{2\,p^2\,te^2}}\,ts}{p\;te\left(1+\sqrt{2}\;ts\;\texttt{InverseErfc}[2\,x]\right)^2},\ \{x, 0, 1\},\ \texttt{PrecisionGoal}\to 12,\ \texttt{MaxRecursion}\to 40\right]$$

$$\texttt{NIntegrate}\left[x\,\frac{e\, e^{\texttt{InverseErfc}[2\,x]^2-\frac{\left(-p+\frac{e}{1+\sqrt{2}\;ts\;\texttt{InverseErfc}[2\,x]}\right)^2}{2\,p^2\,te^2}}\,ts}{p\;te\left(1+\sqrt{2}\;ts\;\texttt{InverseErfc}[2\,x]\right)^2},\ \{x, 0, 0.9999999999999999999\},\ \texttt{PrecisionGoal}\to 12,\ \texttt{MaxRecursion}\to 40\right] - f[0.9999999999999999999]$$

1.182×10^{-10}

0.0239

0.0447

1.3162×10^{-10}

1.

0.97926

$0.97926+\frac{72.4118}{\texttt{sqrt}[2]^2}$

0.0239`

0.0632

1.3162×10^{-10}

1.

0.976926

$0.976926+\frac{41.6059}{\texttt{sqrt}[2]^2}$

0.0158

0.0365

1.3162×10^{-10}

1.

0.997751

"Take 2"

```
Integrate[0.5 Erfc[(e - x) / (ts x)] x Exp[-(x - p)^2 / (2 te p)], {x, 0, Infinity}]
```

$$\int_0^\infty 0.5\, e^{-\frac{(-p+x)^2}{2\,p\,te}}\, x\, \mathrm{Erfc}\left[\frac{e-x}{ts\, x}\right] dx$$

"Take3"

```
Integrate[Exp[-( x - p)^2 / (2 p / ts )] Exp[-(p - pb)^2 / (2 pb / te)],
 {p, -Infinity, Infinity}]
```

$$\int_{-\infty}^\infty e^{-\frac{(p-pb)^2\, te}{2\,pb}-\frac{ts\,(-p+x)^2}{2\,p}}\, dp$$

"Moment Generating function"

"Determine the moments of the density for Pd --> Under the approximation, when the estimated power is Gaussian distributed "

```
Integrate[
 Exp[K x t / (1 - 2 t)] / (1 - 2 t)^(K / 2) Exp[-(x - p)^2 / (ts p)], {x, 0, Infinity}]
```

$$\mathrm{ConditionalExpression}\left[\frac{1}{2\sqrt{\frac{1}{p\,ts}}}\, e^{\frac{K\,p\,t\,(4+t\,(-8+K\,ts))}{4\,(1-2\,t)^2}}\sqrt{\pi}\,(1-2\,t)^{-K/2}\,\mathrm{Erfc}\left[\frac{p\sqrt{\frac{1}{p\,ts}}\,(2+t\,(-4+K\,ts))}{-2+4\,t}\right],\right.$$
$$\left.\left(\mathrm{Re}\left[\frac{K\,t}{1-2\,t}+\frac{2}{ts}\right]<0\ \&\&\ \mathrm{Re}\left[\frac{1}{p\,ts}\right]\geq 0\right)\ ||\ \mathrm{Re}\left[\frac{1}{p\,ts}\right]>0\right]$$

"First Central Moment"

Simplify[

$$D\left[Log\left[\frac{1}{2\sqrt{\frac{1}{p\,ts}}}\,e^{\frac{K\,p\,t\,(4+t\,(-8+K\,ts))}{4\,(1-2\,t)^2}}\,\sqrt{\pi}\,(1-2\,t)^{-K/2}\,Erfc\left[\frac{p\sqrt{\frac{1}{p\,ts}}\,(2+t\,(-4+K\,ts))}{-2+4\,t}\right]\right],\right.$$

$$\left.\{t,1\}\right]\ /.\ t\to 0\Big]$$

$$K\left(1+p+\frac{e^{-\frac{p}{ts}}}{\sqrt{\pi}\,\sqrt{\frac{1}{p\,ts}}\,Erfc\left[-p\,\sqrt{\frac{1}{p\,ts}}\right]}\right)$$

"Second Central Moment"

Simplify[

$$D\left[Log\left[\frac{1}{2\sqrt{\frac{1}{p\,ts}}}\,e^{\frac{K\,p\,t\,(4+t\,(-8+K\,ts))}{4\,(1-2\,t)^2}}\,\sqrt{\pi}\,(1-2\,t)^{-K/2}\,Erfc\left[\frac{p\sqrt{\frac{1}{p\,ts}}\,(2+t\,(-4+K\,ts))}{-2+4\,t}\right]\right],\right.$$

$$\left.\{t,2\}\right]\ /.\ t\to 0\Big]$$

$$\left(e^{-\frac{2p}{ts}}\,K\left(-2\,K\,p\,ts-\frac{1}{\sqrt{\frac{1}{p\,ts}}}2\,e^{p/ts}\,(-4+K\,p)\,\sqrt{\pi}\,Erfc\left[-p\,\sqrt{\frac{1}{p\,ts}}\right]+e^{\frac{2p}{ts}}\,\pi\,(4+p\,(8+K\,ts))\,Erfc\left[-p\,\sqrt{\frac{1}{p\,ts}}\right]^2\right)\right)\Big/\left(2\,\pi\,Erfc\left[-p\,\sqrt{\frac{1}{p\,ts}}\right]^2\right)$$

"Determine the moments of the density for Pd -->
When the estimated power is non-central Chi2 distributed "

```
Integrate[Exp[K1 x t / (1 - 2 t)] / (1 - 2 t) ^ (K1 / 2) 1 / 2 Exp[- (x + l) / 2]
  (x / l) ^ (K2 / 4 - 1 / 2) BesselI[K2 / 2 - 1, Sqrt[l x]], {x, 0, Infinity} ]
```

$$ConditionalExpression\left[e^{\frac{K1\,l\,t}{1-2\,(1+K1)\,t}}\left(\frac{1}{l}\right)^{\frac{1}{4}(-2+K2)}\,l^{\frac{1}{4}(-2+K2)}\,(1-2\,t)^{-K1/2}\left(1+K1+\frac{K1}{-1+2\,t}\right)^{-K2/2},\right.$$
$$\left.Re\left[\frac{1-2\,(1+K1)\,t}{-1+2\,t}\right]<0\,\&\&\,\left(Im\left[\sqrt{l}\right]>0\,||\,Re\left[\sqrt{l}\right]>0\right)\&\&\,Re[K2]>0\right]$$

"First Central Moment"

$$\mathbf{Simplify}\left[\mathbf{D}\left[\mathbf{Log}\left[e^{\frac{K1\,l\,t}{1-2\,(1+K1)\,t}}\left(\frac{1}{l}\right)^{\frac{1}{4}\,(-2+K2)}\,l^{\frac{1}{4}\,(-2+K2)}\,(1-2\,t)^{-K1/2}\left(1+K1+\frac{K1}{-1+2\,t}\right)^{-K2/2}\right],\ \{t,1\}\right]\ /.\ t\to 0\right]$$

$$K1\,(1+K2+l)$$

"Second Central Moment"

$$\mathbf{Simplify}\left[\mathbf{D}\left[\mathbf{Log}\left[e^{\frac{K1\,l\,t}{1-2\,(1+K1)\,t}}\left(\frac{1}{l}\right)^{\frac{1}{4}\,(-2+K2)}\,l^{\frac{1}{4}\,(-2+K2)}\,(1-2\,t)^{-K1/2}\left(1+K1+\frac{K1}{-1+2\,t}\right)^{-K2/2}\right],\ \{t,2\}\right]\ /.\ t\to 0\right]$$

$$2\,K1\,(1+(2+K1)\,K2+2\,(1+K1)\,l)$$

```
ClearAll[e, ts, te, p]
Integrate[
 (e Exp[InverseErfc[2 x]^2] ts) / (p te (sqrt[2] * ts InverseErfc[2 x])^2), x]
```

$$e \Big/ \left(p\,\sqrt{\pi}\ te\ ts\ \mathrm{InverseErfc}[2\,x]\ \mathrm{sqrt}[2]^2\right)$$

$$f[x\_] := \left(e\left(1/(2\,\pi)\,\mathrm{ExpIntegralEi}\left[-\mathrm{InverseErfc}[2\,x]^2\right]+\frac{x}{\sqrt{\pi}\ \mathrm{InverseErfc}[2\,x]}\right)\right)\Big/\left(p\ te\ ts\ \mathrm{sqrt}[2]^2\right)$$

```
Evaluate[f[0.9999999999999]]
```

$$-\frac{0.108574\,e}{p\ te\ ts\ \mathrm{sqrt}[2]^2}$$

"Density of probability of density -- sent signal is Gaussian distributed"

```
FullSimplify[D[ GammaRegularized[
   Kest Ksen mu / (2 prcvd InverseGammaRegularized[x, Ksen / 2]), Kest / 2 ], x]]
```

$$-\Bigg(e^{\text{InverseGammaRegularized}\left[x,\frac{\text{Ksen}}{2}\right]}\,\text{Kest Ksen mu InverseGammaRegularized}\left[x,\frac{\text{Ksen}}{2}\right]^{-1-x}\Bigg(1\Big/x^2\;\;2\,\text{HypergeometricPFQ}\Big[\{x,x\},\{1+x,1+x\},-\text{InverseGammaRegularized}\left[x,\frac{\text{Ksen}}{2}\right]\Big]\,\text{InverseGammaRegularized}\left[x,\frac{\text{Ksen}}{2}\right]^{x}+(-2+\text{Ksen})\,\text{Gamma}[x]\,\text{Log}\Big[\text{InverseGammaRegularized}\left[x,\frac{\text{Ksen}}{2}\right]\Big]+2\left(\text{Gamma}[x]-\text{Gamma}\Big[x,\text{InverseGammaRegularized}\left[x,\frac{\text{Ksen}}{2}\right]\Big]\right)\text{PolyGamma}[0,x]\Bigg)\Bigg(\text{MeijerG}\Big[\{\{\},\{1,1\}\},\Big\{\Big\{0,0,(\text{Kest Ksen mu})\Big/\Big(2\,\text{prcvd InverseGammaRegularized}\left[x,\frac{\text{Ksen}}{2}\right]\Big)\Big\},\{\}\Big\},\frac{\text{Kest}}{2}\Big]-\text{Gamma}\Big[(\text{Kest Ksen mu})\Big/\Big(2\,\text{prcvd InverseGammaRegularized}\left[x,\frac{\text{Ksen}}{2}\right]\Big),\frac{\text{Kest}}{2}\Big]\Big(\text{Log}[2]-\text{Log}[\text{Kest}]+\text{PolyGamma}\Big[0,(\text{Kest Ksen mu})\Big/\Big(2\,\text{prcvd InverseGammaRegularized}\left[x,\frac{\text{Ksen}}{2}\right]\Big)\Big]\Big)\Bigg)\Bigg)\Bigg/\Bigg(4\,\text{prcvd Gamma}\Big[(\text{Kest Ksen mu})\Big/\Big(2\,\text{prcvd InverseGammaRegularized}\left[x,\frac{\text{Ksen}}{2}\right]\Big)\Big]\Bigg)$$

"2 method, integrating over the function for probability of detection"

```
Integrate[ GammaRegularized[Ksen x / (2 y), Ksen / 2]
  1 / (2^(Kest / 2) Gamma[Kest / 2]) prcvd / Kest
  (Kest y / prcvd)^(Kest / 2 - 1) Exp[-Kest y / (2 prcvd)], {y, 0, Infinity}]
```

$$\int_0^{\infty}\left(2^{-\text{Kest}/2}\,e^{-\frac{\text{Kest y}}{2\,\text{prcvd}}}\,\text{prcvd}\left(\frac{\text{Kest y}}{\text{prcvd}}\right)^{-1+\frac{\text{Kest}}{2}}\text{GammaRegularized}\left[\frac{\text{Ksen x}}{2\,y},\frac{\text{Ksen}}{2}\right]\right)\Bigg/\left(\text{Kest Gamma}\left[\frac{\text{Kest}}{2}\right]\right)dy$$

```
GammaRegularized[0.4, 0.2]
```

"3 method, Approximation integrating over the function for probability of detection, where function is approximated using the CLT"

```
Integrate[
 1 / 2 Erfc[(mu - y) / (test y)] 1 / (2^(Kest / 2) Gamma[Kest / 2]) prcvd / Kest
  (Kest y / prcvd)^(Kest / 2 - 1) Exp[-Kest y / (2 prcvd)], {y, 0, Infinity}]
```

$$\int_0^{\infty}\left(2^{-1-\frac{\text{Kest}}{2}}\,e^{-\frac{\text{Kest y}}{2\,\text{prcvd}}}\,\text{prcvd}\left(\frac{\text{Kest y}}{\text{prcvd}}\right)^{-1+\frac{\text{Kest}}{2}}\text{Erfc}\left[\frac{\text{mu}-y}{\text{test y}}\right]\right)\Bigg/\left(\text{Kest Gamma}\left[\frac{\text{Kest}}{2}\right]\right)dy$$