Pseudo BER based SNR Estimation for Energy Detection Scheme in Cognitive Radio

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Abstract—**In this paper, an energy detection scheme based on the pseudo bit error rate (BER) estimation is investigated. In the proposed scheme, the basic strategy is to use the pseudo BER to estimate the signal-noise-ratio (SNR) of primary signal for energy detection at the cognitive users. Threshold and sensing length are updated with the estimated SNR afterward. We have shown that this modified energy detector outperforms traditional energy detector with the noise uncertainty and reliably alleviate the SNR wall effect.**

Keywords-Cognitive radio; energy detection; pseudo BER; SNR estimation

I. INTRODUCTION

The traditional static spectrum allocation has been suffering from inefficiency to meet the growing demands of the wireless services and applications. In recent years, the Federal Communications Commission (FCC) is developing policies and the licensed TV bands are about to be opened up wherein the idle spectrum segments can be accessed by the unlicensed wireless devices beneficially [1]. Cognitive Radio (CR), introduced in [2], has been regarded as a talented technology to revolutionize the existing spectrum utilization. Strict requirements that the harmful interference to the incumbent wireless services should be refrained from have been specified while reutilizing the spectrum opportunities. Accordingly, the CR devices are obligated to be equipped with essential functionalities that continuously monitor the contemporary status of the spectrum utilization, dynamically access the unused spectral holes across time and space, or immediately vacate the occupied spectrum resources while the Primary Users (PU) transmission are detected [3]. Among those functionalities discussed in the literatures, the spectrum sensing is evidently the critical one that enables the CR devices effectively sense the spectrum opportunities or reliably detect the weak primary signals of possibly different types. Numerous sensing methods proposed thus far can be primarily classified into three broad categories: the energy detection (ED) [4], the matched filtering detection [6] and the cyclostationary detection [5]. The ED is recognized as the optimum detector with no prior knowledge of the primary signals at the CR receiver. If the deterministic pattern (e.g., pilot, operating frequency, or modulation) is known, the optimum detector should be the matched filtering. However, the matched filtering detection requires perfect knowledge of the primary signals which may be impractical as there would be limited cooperation between the PU and the CR users. Moreover, the cyclostationary detection exploits the inherent periodicity of the primary signals, leading to more accurate detection whereas the evaluation is mathematically intractable and the empirical simulations are required to identify the thresholds. A more detailed survey of the spectrum sensing techniques for CR users can be found in [5] and the references therein.

In virtue of the hardware simplicities, the ED is considered as a common way for the spectrum sensing in CR. The performance of the energy-based detection in various static scenarios has been discussed while the background noise is assumed stable. However, the noise level actually varies with the time due to the temperature variations, ambient interference, etc., which yields the noise uncertainty [7]. Therefore, the practical performance would degrade significantly from the theoretic analysis. This limitation of the sensitiveness to the noise uncertainty is a so-called *signal-to-noise* (SNR) *wall* which is defined in [8] as the minimum SNR below which the desired performance could not be attained even with the arbitrarily long sensing period. The cross-correlation based ED has been suggested in [9] to alleviate the SNR wall effect. Moreover, Mariani [10] has even revealed that the intrinsic cause of the SNR wall is the insufficient estimation of the noise power and applied the maximum likelihood (ML) estimation for the improvements. However, previous studies mainly focused on the performance enhancement whereas the system complexities and the detection duration were traded for that, which motivates our work in this paper. Since the idea shows that the noise power is changing all the time and the SNR wall is inevitable, we propose an SNR estimation method based on a pseudo bit error rate (BER) scheme for the ED and study the detection performance in a dynamic way while the system simplicity and the sensing duration are both considered.

The paper is organized as follows. In section II, we describe the ED for the CR devices and introduce the SNR wall problem. In section III, we develop the pseudo BER based SNR estimation for the ED, which seeks to minimize the average detection error rate during the whole time. Simulation results and discussion are provided in section IV. Finally, the paper is concluded in section V.

II. Problem Formulation

A. Signal Model

We consider the detection of the primary signal with a zero-mean additive white Gaussian noise (AWGN). The ED is performed to identify the presence or the absence of the primary signal, viz., the active PU. A binary hypothesis testing at the n-th time instant is given as

$$\mathcal{H}_0 \ : \ y(n) = w(n) \tag{1}$$

$$\mathcal{H}_1 \ : \ y(n) = s(n) + w(n) \tag{2}$$

where $\mathcal{H}_0$ stands for the absence of the primary signal $s(n)$, i.e. the received signal $y(n)$ contains only the noise, $w(n) \sim \mathcal{CN}(0,\sigma^2)$, and $\mathcal{H}_1$ represents the presence of the primary signal $s(n)$ coexisting with the noise $w(n)$.

The statistic for the ED is written as

$$Y = \sum_{n=1}^{N} |y(n)|^2 \tag{3}$$

where N is the sensing length in accordance with the number of the observation measurements per detection. Consequently, the decision statistic Y follows a chi-square distribution with the degree of freedom N in the case of $\mathcal{H}_0$ and a non central chi-square distribution with the degree of freedom N and a non centralized parameter γ in the case of $\mathcal{H}_1$:

$$Y \sim \begin{cases} \chi_N{}^2 \ , & \mathcal{H}_0 \\ \chi_N{}^2(\gamma), & \mathcal{H}1 \end{cases} \tag{4}$$

where γ is defined as the average SNR:

$$\gamma = \frac{P}{\sigma^2}, \text{ where } P = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} |s(n)|^2 \tag{5}$$

The performance of the detection is generally evaluated in terms of two probabilities:

$$P_{\mathrm{d}} = \Pr(Y > \lambda \mid \mathcal{H}_1) \tag{6}$$

$$P_{\mathrm{f}} = \Pr(Y > \lambda \mid \mathcal{H}_0) \tag{7}$$

P_{d} indicates the probability of detecting a primary signal under $\mathcal{H}_1$ and P_{f} shows the probability of false alarm under $\mathcal{H}_0$ while λ is the corresponding threshold. For the detection performance in the CR, P_{f} should be kept as small as possible to avoid the underutilization of the transmission opportunities; whereas a larger P_{d} is imperative and implies a better protection of the PU communication. Observe from the Eqs. (6) and (7) that the decision threshold λ and the sensing length N can be chosen by finding an optimal balance between the desired P_{d} and P_{f}. With the central limit theorem (CLT), it gives the minimum value of the sensing length $N_{\min} = 2[\mathcal{Q}^{-1}(P_{\mathrm{f}}) - \mathcal{Q}^{-1}(P_{\mathrm{d}})]^2 \gamma^{-2}$ in [8] by eliminating the λ. We note that if the perfect knowledge of the noise is known in advance, despite the arbitrarily low SNR, the desired performance can be obtained by appropriately choosing the sensing length and the threshold.

B. SNR Wall Problem

On the contrary, the background noise is variable at all time as an uncertain aggregation of the thermal noise, the leakage of signals from other bands due to the receiver nonlinearity, etc. Let consider the noise power σ^2 uniformly distributed in the interval $[\sigma_n^2/\rho, \rho\sigma_n^2]$, where σ_n^2 is the nominal noise power and $\rho > 1$ is the parameter of the uncertainty. Thus, the required performance, expressed as the probabilities pair of (P_{d}^*, P_{f}^*), should be satisfied for all the σ^2

$$P_{\mathrm{d}}^* \le \min_{\sigma^2 \in [1/\rho\sigma_n{}^2, \rho\sigma_n{}^2]} P_{\mathrm{d}} \tag{8}$$

$$P_{\mathrm{f}}^* \ge \max_{\sigma^2 \in [1/\rho\sigma_n{}^2, \rho\sigma_n{}^2]} P_{\mathrm{f}} \tag{9}$$

As a result, the uncertainty interval for the SNR can be expressed accordingly as $[\gamma_{\mathrm{n}}/\rho, \rho\gamma_{\mathrm{n}}]$ and $\gamma_{\mathrm{n}} = P/\sigma_n{}^2$ is the nominal SNR.

By the approximation and the simplification, the minimum sensing length is given as [8]

$$N_{\min} = 2[\mathcal{Q}^{-1}(P_{\mathrm{f}}^*) - \mathcal{Q}^{-1}(P_{\mathrm{d}}^*)]^2 (\gamma_{\mathrm{n}} - (\rho - 1/\rho))^{-2} \tag{10}$$

Clearly, as the $\gamma_{\mathrm{n}} \to \rho - 1/\rho$ the $N_{\min} \to \infty$, which has been called the "SNR Wall" phenomenon in [8]. To be exact, the required performance could not be achieved even with the arbitrarily long sensing length under certain noise uncertainty. In other words, the detection performance of the ED is sensitive to the noise uncertainty. In order to alleviate the SNR wall effect, the noise uncertainty should be mitigated; alternatively, the SNR uncertainty should be mitigated. Our objective in this paper is to propose an SNR estimation method at a run-time based on the pseudo BER for the ED scheme, and investigate the adaption of the threshold and sensing length dynamically, which could enhance the detection performance with certain constraints on the probability pair $(P_{\mathrm{d}}^*, P_{\mathrm{f}}^*)$.

III. Pseudo BER based SNR Estimation for Energy Detection

It has been established in the previous section that the detection probability is the function of the SNR level, the sensing length N and the threshold λ. Inspired by the exclusive relationship between the P_{d} and the γ, in this section, we introduce a pseudo BER based SNR estimation scheme (Fig. 1) for the ED. Within the scheme, we can estimate the SNR γ

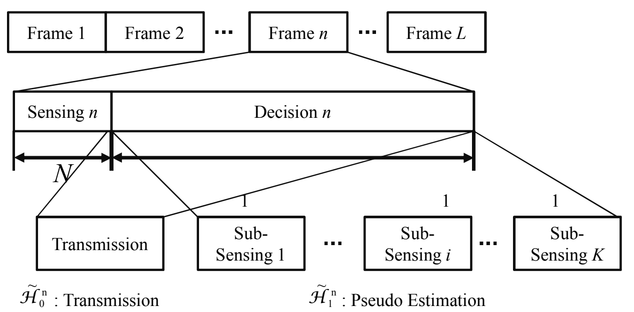

Figure 1 Frame structure of pseudo BER based SNR estimation for ED

frame by frame and then adapt the threshold λ according to the estimated SNR, which might diminish the SNR uncertainty, alleviate the SNR wall effect and improve the sensing reliability.

A. Frame Structure

Figure 1 shows the frame structure designed for the pseudo BER based SNR estimation scheme where each frame consists of one sensing slot and one decision slot. Follow the n-th sensing slot at the start of the n-th frame, the n-th decision is made for the rest of the frame. Differences from the existing frame structure are:

- Under the n-th decision $\tilde{\mathcal{H}}_0^n$: CR starts transmission during the n-th decision slot.
- Under the n-th decision $\tilde{\mathcal{H}}_1^n$: CR divides the n-th decision slot into K sub-slots and repeats the sub-sensing every sub-slot.
- The i-th sub-sensing: CR repeats the ED with the offset thresholds. Output sequence is 1 for sub-decision $\tilde{\mathcal{H}}_1^n(i)$ and 0 for decision $\tilde{\mathcal{H}}_0^n(i)$ at i-th sub-slot.

where $\tilde{\mathcal{H}}_0$ and $\tilde{\mathcal{H}}_1$ denotes the decision assuming the absence and the presence of the active PU, respectively.

B. Pseudo BER based SNR Estimation

As the exclusive relationship between the SNR and the error rate is inherent, it is intuitive to infer the instantaneous SNR from the error rate and eventually ease the SNR uncertainty. However, as a matter of fact, the true copy of the signal is unavailable at the receiver under the operating condition, not to mention the accurate error rate. The pseudo error rates described in [11] have been extrapolated for the error rate monitoring. Several pseudo error rates were firstly obtained with modified thresholds and then extrapolated to determine the actual error rate. Further, the performance of the on-line pseudo error monitoring is detailed in [12] which show that the intentionally degraded error rates overcome the shortcomings of the long evaluation time and the interruption to the data traffic while monitoring the actual error rate. In this paper, we propose an SNR estimation method based on the pseudo BER by exploiting the error monitoring method as described in [13].

In accordance with the frame structure in Fig.1, the proposed scheme for each sub-slot is illustrated in Fig.2. The sub-slot ED is performed if and only if the primary signal has been detected at the beginning sensing slot of the current frame. In the scheme, there includes two branches with two offset

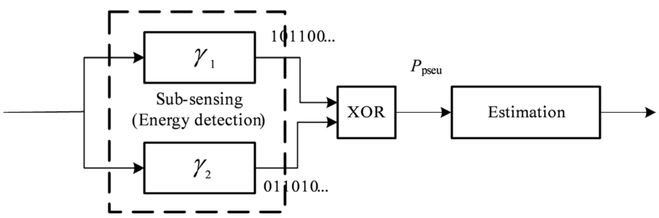

Figure 2 Pseudo BER based estimation scheme

thresholds, $\lambda_1 = (1-\alpha)\lambda$ and $\lambda_2 = (1+\alpha)\lambda$, respectively. λ is the threshold for the previous sensing slot and α represents the parameter for some offset.

Under the decision $\tilde{\mathcal{H}}_1^n$ at the n-th sensing slot, each branch outputs a 1 for the sub-decision $\tilde{\mathcal{H}}_1^n(i)$ or a 0 for the sub-decisio $\tilde{\mathcal{H}}_0^n(i)$ at the i-th sub-slot. After that, the exclusive OR (XOR) operation is executed on the decision sequences to count the disagreements. Hence, the pseudo BER P_{pseu} is obtained. It can easily be shown that the P_{pseu} can be expressed mathematically by the probabilities of detection with different offset thresholds as

$$\begin{aligned} P_{\text{pseu}} &= \Pr(Y > \lambda_1 \mid \tilde{\mathcal{H}}_1^n) - \Pr(Y > \lambda_2 \mid \tilde{\mathcal{H}}_1^n) \\ &= \Pr[(1-\alpha)\lambda < Y < (1+\alpha)\lambda \mid \tilde{\mathcal{H}}_1^n] \end{aligned} \tag{11}$$

It has been demonstrated in the references that the pseudo BER plotted against SNR yields a curve similar in shape to a BER vs. SNR curve but with larger value. The desired relationship between the P_{pseu} and the P_{d} can thus be defined as

$$\log P_{\text{pseu}}(\gamma) = M + [1 - \log \hat{P}_{\text{d}}(\gamma)] \tag{12}$$

where the parameter $M > 0$ is the “gain” of the pseudo BER over the actual BER relate to the SNR value. The caret over the quantity indicates an estimation of the quantity.

The general steps carried out for the pseudo BER based SNR estimation can be generalized as follows: while the decision $\tilde{\mathcal{H}}_1^n$ is assumed at the sensing slot, start the sub-sensing at the decision slot; then take the XOR operation on the outputs of the decision sequences and the pseudo BER P_{pseu} is thus obtained; estimate the probability of detection $\hat{P}_{\text{d}}$ from (12); find out the estimated $\hat{\gamma}$ eventually from the inverse function of the P_{d} in virtue of the statistic characteristics or just simply search within a lookup table. For simplicity, the inverse function of P_{d} can be approximated to be the inverse Q function if N is sufficiently large due to the central limit theory.

C. System Adaption for Energy Detection

As mentioned before, both the probabilities P_{d} and P_{f} are essential to the spectrum reutilization of the CR. In order to evaluate the detection performance for the CR, we firstly define the detection error rate (DER) as

$$P_{\text{e}}(\gamma) = [1 - P(\mathcal{H}_1)] * P_{\text{f}} + P(\mathcal{H}_1) * (1 - P_{\text{d}}) \tag{13}$$

where $P(\mathcal{H}_1)$ is the prior probability of an active PU relative to the spectrum utilization status. The optimization criterion for the performance can be expressed as

$$\begin{aligned} &\min_{N,\lambda} P_{\text{e}}(\hat{\gamma}) \\ &s.t. P_{\text{f}} \le P_{\text{f}}^* \text{and} \quad P_{\text{d}} \ge P_{\text{d}}^* \end{aligned} \tag{14}$$

where the P_{d}^* and the P_{f}^* are the target probabilities. Consider the actual dynamic scenarios with noise uncertainty, in spite of the fluctuation, the SNR level would maintain during several frames. Therefore, with the proposed scheme, sub-sensing is carried out after the assertion of the active PU within the current frame and the pseudo BER based SNR estimation followed to update the estimation. With the estimation $\hat{\gamma}$, given desired pair of probabilities (P_d^*, P_f^*), we could always find the optimal sensing length N and threshold λ for the subsequent frame according to (14), which undoubtedly minimize the DER. Thus, the DER might be minimized with the approximated SNR level based on the proposed scheme and meanwhile the SNR wall effect could be alleviated for the reduction of the noise uncertainty.

IV. SIMULATION AND RESULTS

In this section, we evaluate the proposed pseudo BER based SNR estimation scheme for the ED. Given the target probabilities P_d^* =0.9 and P_f^* =0.1, the sensing length and the threshold are assumed to be the optimal selection for the nominal γ_n at the very beginning.

A. SNR Wall

We firstly confirm the SNR wall effect owing to the SNR uncertainty, viz. the noise uncertainty on the traditional ED. Figure 3 indicates the SNR wall effect with three degrees of the uncertainty. Firstly, we note that, to achieve the required performance, the ED should increase the sensing length as the nominal SNR level decreases. Secondly, for the given uncertainty $X = 10\log_{10}\rho$ with different values of 1dB, 0.1dB and 0.001dB, the SNR walls locate around -3.3 dB, -13.4dB and -33.4dB, respectively. Especially, with the noise uncertainty X=1dB, the necessary sensing length should approach to infinity as the nominal SNR is about -3.3dB, which is called as the SNR wall. We can also find that, with the less noise uncertainty, the SNR wall would be smaller and the ED can be satisfying even within the lower SNR regime. Without loss of generality, in the following simulations, the noise uncertainty parameter we choose to be $X = 1$dB and the SNR wall is around -3.3dB accordingly.

B. Pseudo BER based SNR Estimation

Figure 4 shows the performance of the pseudo BER based SNR estimation. For the given offset parameter α =0.2, the detection probability $\hat{P}_d$ could be well-determined by the P_{pseu} with (12). The estimated $\hat{\gamma}$ can thus be obtained by calculating the inverse function of the P_d. Figure 4 shows the relationship between the estimated $\hat{\gamma}$ and the actual γ with the lengths of pseudo sequence increasing from 1000 to 100000. It is clear that the longer sequence realizes the better estimation performance. However, the pseudo sequence could not be very long as the estimation is executed during one frame and certainly time limited. We employ here the sequence length to be 1000, which is both applicable and accurate enough.

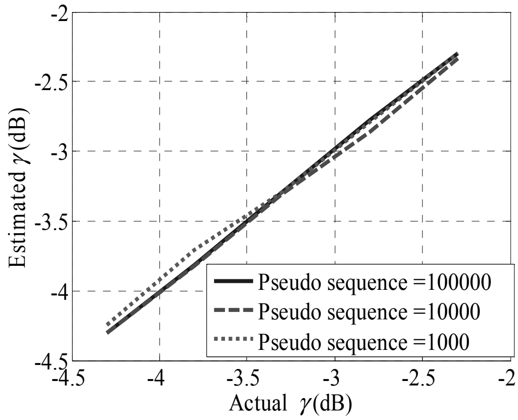

Figure 4 SNR estimation performances

C. Detection Error Rate Optimization

Eventually, we will give some simulation results for the scheme evaluation. The simulation parameters are: the prior probability of the active PU $P(\mathcal{H}_1) = 0.7$, and the offset parameter $\alpha = 0.2$. Figure 5 shows the comparison of the proposed scheme and the traditional scheme in term of the receiver operating characteristic (ROC) curve. The detection performance has been intuitively improved with the proposed scheme over the traditional one.

Moreover, Fig. 6 compares the DERs between the proposed scheme and the traditional ED scheme. With the uncertainty parameter $X = 1$dB, the sensing length N is adaptive for the proposed scheme ranging from 60 to 140 whereas 500 for the traditional ED. Most of the DERs for the proposed scheme have been found below 0.1, which is the preset target DER that

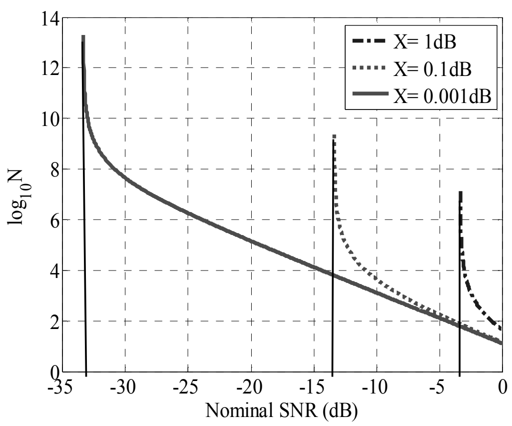

Figure 3 SNR wall with different uncertainty parameters *X*

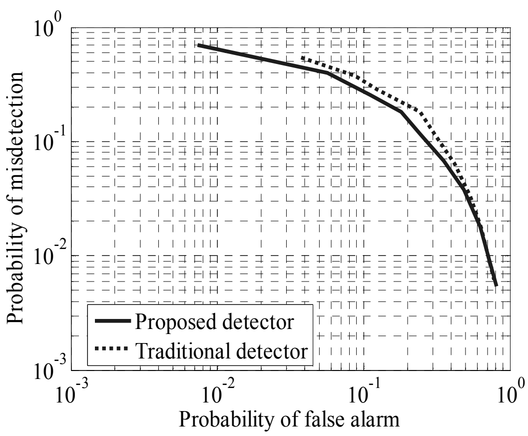

Figure 5 ROC of the detection performance

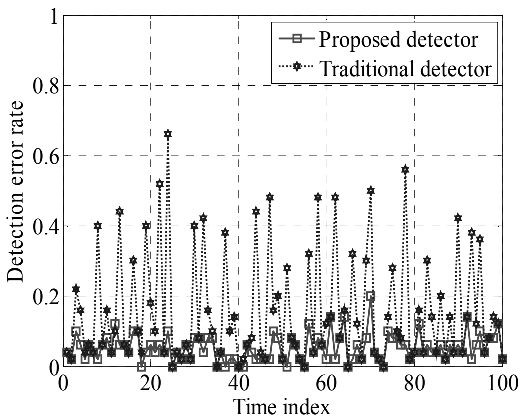

Figure 6 Comparison of the detection performance

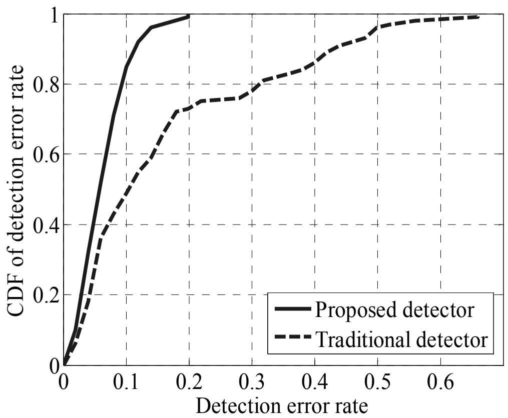

Figure 7 Comparison of the empirical CDFs for detection error rate

can be calculated in advance. However, the traditional detector acts much worse with the same noise uncertainty as most of the DERs far exceed the target DER within the SNR uncertainty interval [-4.3dB, -2.3dB].

The comparison of the empirical CDFs is given in Fig.7. It shows that more than 80% simulation results of the proposed scheme are satisfied to be lower than target DER= 0.1 under the given noise uncertainty. Whereas almost half the simulation results of the traditional detector suffers from the poor performance (The DER is large than 0.1 due to the same noise uncertainty). This means that the pseudo BER based SNR estimation has alleviated the SNR wall effect on the ED greatly and desirable performance can be achieved.

V. CONCLUSIONS

In this paper we proposed the pseudo BER based SNR estimation for the ED scheme with certain constraints on the probability of detection and the probability of false alarm. An optimization method for the threshold adaptation has been applied to combat with the SNR wall effect. The proposed scheme is evaluated and its effectiveness has been confirmed within the given noise uncertainty interval. In particular, as a straightforward transformation of the traditional ED scheme, the proposed scheme manages to achieve the required performance even within short sensing duration.

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