Ergodic Capacity of Cognitive Radio Under Imperfect Channel-State Information

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***Abstract*—A spectrum-sharing communication system where the secondary user is aware of the instantaneous channel-state information (CSI) of the secondary link but knows only the statistics and an estimated version of the secondary transmitter–primary receiver link is investigated. The optimum power profile and the ergodic capacity of the secondary link are derived for general fading channels [with a continuous probability density function (pdf)] under the average and peak transmit power constraints and with respect to the following two different interference constraints: 1) an interference outage constraint and 2) a signal-to-interference outage constraint. When applied to Rayleigh fading channels, our results show, for example, that the interference constraint is harmful at the high-power regime, because the capacity does not increase with the power, whereas at the low-power regime, it has a marginal impact and no-interference performance, which corresponds to the ergodic capacity under average or peak transmit power constraint in the absence of the primary user, may be achieved.**

***Index Terms*—Cognitive radio (CR), ergodic capacity and interference outage constraint, optimal power allocation, spectrum sharing.**

I. Introduction

COGNITIVE RADIO (CR) techniques have been proposed to efficiently use the spectrum through an adaptive, dynamic, and intelligent process [1]. Spectrum utilization can be improved by permitting a secondary user (SU; who is not being serviced) to access a spectrum hole that is unoccupied by the primary user or to share the spectrum with the primary user under certain interference constraints [2]. CR refers to different approaches to this problem that seek to overlay, underlay, or interweave the SU's signals with the primary users' [3]. In the underlay settings, cognitive users can communicate, as long as the interference that was caused to noncognitive users is below a certain threshold. Overlay systems, on the contrary, adopt a less conservative policy by permitting cognitive and noncognitive users to simultaneously communicate by exploiting side information and using sophisticated coding techniques [4]. Perhaps the most conservative of the three systems is the interweave system, which permits cognitive users to communicate, provided that the actual spectrum is unoccupied by noncognitive users. More details on these three systems can be found, for example, in [3] and [4]. From an information-theoretical point-of-view, establishing the performance limits of these systems strongly relies on the available side information that a cognitive user has about the network nodes, e.g., channel-state information (CSI), coding techniques, and codebooks [5]–[8].

In this paper, we focus on a spectrum-sharing CR model under general fading channels, with continuous probability density functions (pdf's), where the primary user and the SU share the same spectrum under certain interference constraints. More specifically, we aim at analyzing the optimal power allocation and the ergodic capacity of the secondary link under limited channel knowledge at the secondary transmitter [9], [10].

Previous work has studied the impact of fading on the secondary-link capacity under the average or peak transmit power but assuming that the secondary transmitter is aware of the instantaneous CSI of the secondary transmitter–primary receiver (ST-PR) link [11]–[14]. Although the latter assumption generally guarantees an instantaneous limitation of the interference at the primary receiver (PR), it is quite strong to obtain such valuable CSI in the absence of an established cooperation protocol between the primary and the secondary links. Recall that protecting the primary user against interference may not be accurate if the CSI that was needed to estimate interference levels is coarsely precise, as shown in [15]. A step forward to address the problem in a more practical setting, considering imperfect CSI, has been realized in [16] and [17], where the capacity or a lower bound on it has been derived under the average received power or the average interference outage constraint, respectively, but neither an average nor a peak transmit power has been considered. The effect of ST-PR channel estimation at the SU on the ergodic capacity has also been analyzed under the peak transmit power and the peak interference constraint at the PR [18]. However, unless some assumptions on the interference (strong or weak) caused by the primary user at the secondary receiver are adopted in the more general interference channel model therein, the results obtained seem to be an achievable rate using the Gaussian codebook, because the capacity of the interference channel is still generally not known. Along similar lines, [19] considers the effect of statistical CSI rather than instantaneous channel estimation errors. Likewise, a rate-maximization problem of a secondary link where the cognitive users are equipped with multiple antennas and under an average transmit power along with an average interference constraints at the PR has been considered in [20]. The secondary transmitter (ST) has been assumed to know the mean or the covariance of

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the ST-PR CSI through feedback, and in both cases, algorithms have been proposed to find the instantaneous optimum rate. In [21], a sum-rate maximization of the secondary rates over a Gaussian multiple-access channel has been considered, under the assumption of an opportunistic interference cancellation at the secondary receiver. In [22], system-level capacity of a spectrum-sharing communication network, under received average interference power constraints, has been studied. Therein, the capacity of the following two scenarios has been analyzed: 1) a CR-based central access network and 2) a CR-assisted virtual multiple-input–multiple-output network.

To generalize the existing results and to provide a uniform framework of the performance limits of a spectrum-sharing protocol under the interference outage constraint, in this paper, we analyze the ergodic capacity under the following two different transmit power constraints: 1) a peak power constraint and 2) an average power constraint. In each case, the following two different interference constraints at the PR are considered: 1) an interference outage constraint and 2) a signal-to-interference (SI) outage constraint. The former outage events occur when the interference power at the PR is above a certain threshold, e.g., Q_{peak}, whereas the latter outage events happen when the ratio between the signal power and the interference power at the PR is below a certain threshold, e.g., I_{peak}. Note that these constraints are necessary to ensure low-error-probability decoding at the PR at the power- and interference-limited regimes, respectively. Furthermore, in our framework, we assume that the ST is provided only with imperfect ST-PR CSI. More specifically, our main contributions in this paper are listed as follows.

- Assuming that the ST is provided a noisy version of the ST-PR CSI, we introduce the instantaneous interference outage and SI outage constraints that aim at protecting the primary user, who operates in a stringent delay-sensitive mode.
- Subject to both an average and a peak power constraint and either an instantaneous interference outage or SI outage constraints, we derive the optimal power and the ergodic capacity of the SU, who operates in a spectrum-sharing mode with the primary user, and highlight the effect of CSI error on the performance.
- We show that, by letting the error variance of ST-PR CSI estimation tend toward one or zero, our framework naturally extends to the no- and perfect-ST-PR CSI cases, and hence, several previously reported results in the literature are retrieved as special cases.
- Specialized to Rayleigh fading channels, we provide asymptotic analysis of the derived results when the average or the peak power constraint tends to infinity.

This paper is organized as follows. Section II describes the system model. The optimal power profile and the ergodic capacity are derived according to an average and a peak transmit power constraint and under different outage constraints in Section III. Section IV addresses the perfect- and no-ST-PR CSI cases. In Section V, the derived results are applied to Rayleigh fading channels. Numerical results are briefly discussed in Section VI. Finally, Section VII concludes this paper.

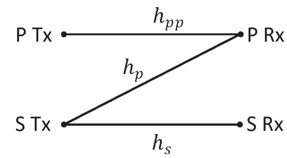

Fig. 1. Spectrum-sharing channel model.

Notation: The expectation operation is denoted by $\mathbb{E}\{\cdot\}$. The symbol $|x|$ is the modulus of the scalar x, whereas $[x]^+$ denotes $\max(0, x)$. The logarithm $\log(x)$ is the natural logarithm of x. A random variable is denoted by a boldface letter, e.g., $\mathbf{x}$, whereas the realization of $\mathbf{x}$ is denoted by x.

II. System Model

We consider a spectrum-sharing communication scenario, as depicted in Fig. 1, where an ST communicates with an SU receiver, under certain constraints that will be defined later, through a licensed bandwidth that is occupied by a primary user. The signal that was received at the SU is given by

$$\mathbf{r}_s(l) = \mathbf{h}_s(l)\mathbf{s}(l) + \mathbf{w}_s(l) \tag{1}$$

where l is the discrete-time index, $\mathbf{s}(l)$ is the channel input, $\mathbf{h}_s(l)$ is the complex channel gain, and $\mathbf{w}_s(l)$ is a zero-mean circularly symmetric complex white Gaussian noise with spectral density N_0 and is independent of $\mathbf{h}_s(l)$. The channel gains $\mathbf{h}_s(l)$, $\mathbf{h}_p(l)$, and $\mathbf{h}_{pp}(l)$ are assumed to be ergodic and stationary with a continuous pdf $f_{\mathbf{h}_s}(h_s)$, $f_{\mathbf{h}_p}(h_p)$, and $f_{\mathbf{h}_{pp}}(h_{pp})$, respectively. The ST is provided with the instantaneous CSI of the SU channel gain $\mathbf{h}_s(l)$. However, it is only provided with the statistics of $\mathbf{h}_p(l)$–$f_{\mathbf{h}_p}(h_p)$ and a noisy version of $\mathbf{h}_p(l)$, e.g., $\breve{\mathbf{h}}_p(l)$, which was obtained through a band manager who coordinates the primary user and the SU or through a feedback link from the primary user's receiver [6], [11], [16], [23] such that $f_{\mathbf{h}_p|\breve{\mathbf{h}}_p}(h_p|\breve{h}_p)$ is also known. To improve its instantaneous estimate of $\mathbf{h}_p(l)$, the ST further performs minimum-mean-square-error (MMSE) estimation to obtain $\hat{\mathbf{h}}_p(l) = \mathbb{E}[\mathbf{h}_p(l)|\breve{\mathbf{h}}_p(l) = \breve{h}_p(l)]$. Note that, to compute the MMSE estimate, the ST needs to know the conditional pdf of $\mathbf{h}_p(l)$, given $\breve{\mathbf{h}}_p(l)$, which it does. Therefore, the ST-PR channel estimation model can be written as

$$\mathbf{h}_p(l) = \sqrt{1-\sigma_p^2}\,\hat{\mathbf{h}}_p(l) + \sqrt{\sigma_p^2}\,\tilde{\mathbf{h}}_p(l) \tag{2}$$

where $\tilde{\mathbf{h}}_p(l)$ is the zero-mean unit-variance MMSE channel estimation error, and σ_p^2 is the MMSE error variance. Through well-known properties of the conditional mean, $\hat{\mathbf{h}}_p(l)$ and $\tilde{\mathbf{h}}_p(l)$ are uncorrelated. The channel estimation model (2) has widely been used in the channel estimation literature, e.g., in [24], and, recently, in a CR context, e.g., in [16] and [18]. Furthermore, because the channel and the estimation models defined in (1) and (2), respectively, are stationary and memoryless, the capacity-achieving statistics of the input $s(l)$ are memoryless independent and identically distributed (i.i.d.) as well. Therefore, for simplicity, we may drop the time index l in (1) and (2). A sufficient statistic (including a noise variance normalization) for detecting $\mathbf{s}$ from $\mathbf{r}_s$ in (1) is $\mathbf{y}_s = 1/\sqrt{N_0}((\mathbf{h}_s^*/|\mathbf{h}_s|)\mathbf{r}_s)$. The sufficient statistic $\mathbf{y}_s$ can be expressed by

$$\mathbf{y}_s = |\mathbf{h}_s|\mathbf{x} + \mathbf{z}_s \tag{3}$$

where $\mathbf{x} = 1/\sqrt{N_0}\mathbf{s}$, and $\mathbf{z}_s = 1/\sqrt{N_0}((\mathbf{h}_s^*/|\mathbf{h}_s|)\mathbf{w}_s)$ is a zero-mean unit-variance white Gaussian noise. Let $P = \mathbb{E}[\mathbf{x}^2] = 1/N_0\mathbb{E}[|\mathbf{s}|^2]$ be the normalized power at the ST. Because the sufficient statistic preserves the channel mutual information [25, Ch. 2], (3) does not entail any performance loss from a capacity point of view.

III. Ergodic Capacity

For the channel given by (3), the ergodic capacity [in nats per channel use (npcu)], with transmitter- and receiver-side information and under either an average or a peak transmit power constraint, can be expressed by [26]–[28]

$$C = \max_{P} \mathop{\mathbb{E}}_{\mathbf{h}_s}\left[\ln\left(1 + P\cdot|\mathbf{h}_s|^2\right)\right] \tag{4}$$

which is achievable using a variable-rate variable-power Gaussian codebook, as described in [26] and [28], or a simpler single Gaussian codebook with dynamic power allocation, as argued in [27]. If no power adaptation is used in (4), then because the channel is i.i.d., the ergodic capacity with perfect CSI at the transmitter is equal to where only perfect CSI at the receiver is available. Therefore, to get the benefit of channel knowledge at the transmitter, it is necessary to allow the power P to vary with the fading realizations h_s. If additional constraints will be considered, the ergodic capacity takes the form of the maximization (4) subject to these constraints. In particular, the derivation of the secondary-link ergodic capacity, in a CR setting, is subject to certain constraints related to the PR that depend on the ST-PR link h_p. Because the ST is provided an estimate of this channel gain $\hat{h}_p$, it is natural to let the power also vary with $\hat{h}_p$, i.e., $P = P(h_s, \hat{h}_p)$.

A. Average Transmit Power and Interference Outage Constraints

In this section, the transmit power P is subject to an average constraint, i.e., $\mathbb{E}[P(|\mathbf{h}_s|, |\hat{\mathbf{h}}_p|)] \le P_{avg}$, where the expectation is over both $\mathbf{h}_s$ and $\hat{\mathbf{h}}_p$. Moreover, because the instantaneous CSI to the PR is not available at the ST, the probability that the interference power at the PR is below a given positive threshold is always not null. However, we may resolve to tolerate a certain interference outage level and consequently compute the capacity link. Clearly, characterizing the capacity in terms of the interference outage constraint may be shown as a capacity-outage tradeoff: If no interference outage at the primary user will be tolerated, the capacity of the SU link is equal to zero. On the other hand, if a high interference outage is acceptable, the capacity of the SU link is equal to the capacity, because there is no interference constraint. The ergodic capacity can be derived by solving the optimization problem as

$$C = \max_{P(\mathbf{h}_s, \hat{\mathbf{h}}_p)} \mathop{\mathbb{E}}_{\mathbf{h}_s, \hat{\mathbf{h}}_p}\left[\ln\left(1 + P(\mathbf{h}_s, \hat{\mathbf{h}}_p)\cdot|\mathbf{h}_s|^2\right)\right] \tag{5}$$

$$\text{s.t.} \mathop{\mathbb{E}}_{\mathbf{h}_s, \hat{\mathbf{h}}_p}\left[P(\mathbf{h}_s, \hat{\mathbf{h}}_p)\right] \le P_{avg} \tag{6}$$

$$\text{Prob}\left\{P(\mathbf{h}_s, \hat{\mathbf{h}}_p)\cdot|\mathbf{h}_p|^2 \ge \mathcal{Q}_{peak} \mid \mathbf{h}_s = h_s, \hat{\mathbf{h}}_p = \hat{h}_p\right\} \le \epsilon. \tag{7}$$

Different from [16] and [17], (7) aims at reducing the instantaneous (not the average) interference power at the PR for all instantaneous values h_s and $\hat{h}_p$. For simplicity, we will assume that $\mathbf{h}_p$ and $\mathbf{h}_s$ are independent so that (7) is equivalent to

$$P(h_s, \hat{h}_p) \le \frac{\mathcal{Q}_{peak}}{F^{-1}_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(1-\epsilon)} \tag{8}$$

where $F^{-1}_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\cdot)$ is the inverse cumulative distribution function (cdf) of $|\mathbf{h}_p|^2$ conditioned on $\hat{\mathbf{h}}_p$. For $F^{-1}_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\cdot)$ to exist, it is sufficient that $f_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}$ be continuous and not null on an interval of its domain. Equation (8) can be interpreted as a variable peak transmit power constraint dictated by the interference constraint of the PR. Therefore, the problem at hand is now equivalent to the derivation of the ergodic capacity under both variable peak and average transmit power constraints. Recall that a somehow similar problem has been studied in [28], where the optimum power profile and the ergodic capacity have partially been found under both constant peak and average transmit power constraints but only in terms of Lagrangian multipliers. To provide a better understanding of the problem, an explicit solution in terms of system parameters is required. Furthermore, the peak constraint (8) now depends on the ST-PR channel estimate $\hat{\mathbf{h}}_p$; thus, it is of interest to analyze the impact of such an estimation on CR performances. Indeed, using an approach similar to the approach in [13] and [29] along with the Lagrangian method, it can be shown that the solution to the aforementioned convex optimization problem has the following water-filling power profile:

$$P(h_s, \hat{h}_p) = \min\left\{P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon), \left[\frac{1}{\lambda} - \frac{1}{|h_s|^2}\right]^+\right\} \tag{9}$$

where λ is the positive Lagrange multiplier that is associated with (6), and $P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon) = Q_{peak}/(F^{-1}_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(1-\epsilon))$ is the peak power constraint (8). Note that, although the optimal power profile (9) has a similar form as the corresponding power profile of a dynamic time-division multiple access in a cognitive broadcast channel (C-BC), as derived in [29, Th. 4.1, Case II],[1] and [13, Th. 2], our problem formulation is different from the formulation in [13] and [29] in the following ways.

[1]Although [29, Th. 4.1, Case II] deals with a C-BC with M primary users and K SUs, it is easy to see that, by setting $M = K = 1$ therein, the power profile in [29, Th. 4.1, Case II] coincides with (9).

- Our framework deals with a spectrum-sharing scenario where the SU has noisy CSI of the cross link and, thus, can capture the effect of such an uncertainty on the performance, whereas the formulation in [13] and [29], although more general, cannot encompass our setting.
- In our formulation, because the ST is aware only of a noisy version of the cross-link CSI, unlike [13] and [29], it cannot guarantee to limit the received interference power at the PR in every cross-link channel gain. Instead, the ST tries to opportunistically (using the cross-link channel gain estimation $\hat{h}_p$) protect the PR statistically by limiting the instantaneous outage probability at the PR according to (7).
- In both [13] and [29], the Lagrange multiplier that is associated with the average power constraint λ (g_{s1} in this paper) is numerically found without a sufficient analytical insight. Note that analytically finding λ (when possible) provides a better understanding of how the power profile and, hence, the capacity depend on system parameters. Therefore, we give an explicit solution of the optimal power (9) in terms of system parameters, and as such, our formulation turns out to be more eloquent and insightful.

First, let us define the function $G(x)$ by

$$G(x) = \frac{1 - F_{|\mathbf{h}_s|^2}(x)}{x} - \int_x^\infty \frac{f_{|\mathbf{h}_s|^2}(t)}{t}\, dt \tag{10}$$

for $x > 0$, where $F_{|\mathbf{h}_s|^2}$ is the cdf of $|\mathbf{h}_s|^2$. Because

$$\frac{f_{|\mathbf{h}_s|^2}(t)}{t} \leq \frac{f_{|\mathbf{h}_s|^2}(t)}{x}$$

for $t \geq x$ and $\int_x^\infty (f_{|\mathbf{h}_s|^2}(t)/x)dt$ exists, so does $\int_x^\infty (f_{|\mathbf{h}_s|^2}(t)/t)dt$, and hence, the function $G(\cdot)$ in (10) is well defined. Because $G(x) < 1/x$ and $G(\cdot)$ is a decreasing continuous positive-definite function and, thus, invertible on $(0, \infty)$, $G^{-1}(\cdot)$ is also a decreasing function on $(0, \infty)$. The purpose of defining such a function $G(\cdot)$ is to facilitate the presentation of the results in terms of system parameters. Clearly, for a system without a cognition constraint, the optimal power is the well-known water-filling given by $P(h_s) = [1/\lambda - 1/|h_s|^2]^+$, where λ is obtained by solving $\mathbb{E}_{|\mathbf{h}_s|^2}[[1/\lambda - 1/|h_s|^2]^+] = P_{avg}$. The left-hand side of the last equality is exactly $G(\lambda)$, and the definition in (10) follows from a simple integration by part. Note, for example, that the definition in (10) and the properties of the function $G(\cdot)$ (continuous, monotonically decreasing, positive-definite on $(0, \infty)$) hold true for all class of fading channels considered in this paper. Therefore, the optimum power profile can be derived through the first-order optimality conditions as follows (see the Appendix for the proof).

- If $P_{avg} \geq \mathbb{E}_{\hat{\mathbf{h}}_p}[P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon)]$, then we have

$$P(h_s, \hat{h}_p) = P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon). \tag{11}$$

- Otherwise, we have
  - $P_{avg} > G(P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon)^{-1})$

$$P(h_s, \hat{h}_p) = \begin{cases} 0 & |h_s|^2 < g_{s1} \\ \frac{1}{g_{s1}} - \frac{1}{|h_s|^2} & g_{s1} \leq |h_s|^2 < g_{s2} \\ P_{|\mathbf{h}_p|^2|\hat{h}_p}(\epsilon) & |h_s|^2 \geq g_{s2}. \end{cases} \tag{12}$$

  - $P_{avg} \leq G(P_{|\mathbf{h}_p|^2|\hat{h}_p}(\epsilon)^{-1})$

$$P(h_s, \hat{h}_p) = \begin{cases} 0 & |h_s|^2 < g_{s1} \\ \frac{1}{g_{s1}} - \frac{1}{|h_s|^2} & |h_s|^2 \geq g_{s1}. \end{cases} \tag{13}$$

Here, g_{s1} is obtained by satisfying the average power constraint (6) with equality, and $g_{s2} = (1/g_{s1} - P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon))^{-1}$. To express g_{s1} in terms of system parameters, let us define S_x as a parameterized set that characterizes the values of $\hat{\mathbf{h}}_p$ that satisfy the inequality $F_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(x\, Q_{peak}) < 1 - \epsilon$. Because the last inequality is equivalent to $x < 1/P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon)$, S_x also characterizes the values of $\hat{\mathbf{h}}_p$ for which $1/x - P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon) > 0$, i.e., $\mathcal{S}_x = \{\hat{h}_p \; | x < 1/(P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon))\}$. Note that substituting x by g_{s1}, for example, $S_{g_{s1}}$ would be the set of all $\hat{h}_p$ such that $g_{s2} > 0$, and hence, the power profile is given by (12). Therefore, g_{s1} can be expressed by (see the Appendix for the derivation)

$$g_{s1} = K^{-1}(P_{avg}) \tag{14}$$

where $K(x)$ is defined on $(0, P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon)]$ by (15), shown at the bottom of the page. Note that g_{s1} in (14) will be understood as the result of applying the inverse function of $K(x)$ to P_{avg}. It can easily be verified that the function $K(x)$ is continuous, monotonically decreasing, and invertible, which guarantees the existence of g_{s1}. Note that solving (14) to derive the optimal power profile is much more convenient than running a numerical optimization for each value of P_{avg}. Hence, combining (4) and (11)–(13), the secondary-link ergodic capacity under the average transmit power and interference outage constraints is given by (16), shown at the bottom of the next page. Note that (11) and the corresponding capacity expression in (16) clearly suggest that, at the high-power regime, the power profile and, hence, the capacity are impacted by the cross-link CSI only, irrespective of the secondary

$$K(x) = \begin{cases} G(x) - \underset{|\hat{\mathbf{h}}_p|^2 \in \mathcal{S}_x}{\mathbb{E}} \left[G\left(\left(1/x - P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon) \right)^{-1} \right) \right], & \text{if } x < 1/P_{|\mathbf{h}_p|^2|\hat{h}_p}(\epsilon) \\ G\left(1/P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon) \right), & \text{if } x = 1/P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon) \end{cases} \tag{15}$$

CSI. Such an insight provides, for example, useful design guidelines that cannot be gained straightforward from (9), which is another advantage of our explicit solution over previous works.

B. Average Transmit Power and SI Outage Constraints

In the problem formulation in Section III-A, the ST does its best to ensure as low interference as possible to the PR. However, (7) does not guarantee a low-outage performance of the primary link. Recall that the outage at the PR is defined as [30, Ch. 10]

$$P_{out} = \text{Prob}\left\{ \frac{P_{pp}|\mathbf{h}_{pp}|^2}{P(\mathbf{h}_s, \hat{\mathbf{h}}_p)|\mathbf{h}_p|^2} \leq \lambda_{th} \text{ or } P_{pp}|\mathbf{h}_{pp}|^2 \leq P_{th} \right\}$$

where λ_{th} and P_{th} are the SI power and signal power thresholds, respectively, and P_{pp} is the primary-link transmit power. For simplification, P_{pp} is assumed to be independent of h_{pp}. Should the primary link be in deep fade, there is no chance of reliably conveying any information on the primary link. A more engaged way of preventing interference outage at the PR, although requiring more CSI at the ST, would be to set a constraint on the SI power ratio as follows:

$$\text{Prob}\left\{ \frac{P_{pp}|\mathbf{h}_{pp}|^2}{P(\mathbf{h}_s, \hat{\mathbf{h}}_p)|\mathbf{h}_p|^2} \leq \lambda_{th} \mid \mathbf{h}_s = h_s, \hat{\mathbf{h}}_p = \hat{h}_p \right\} \leq \epsilon. \tag{17}$$

Letting $\boldsymbol{\beta} = |\mathbf{h}_p|^2/|\mathbf{h}_{pp}|^2$ and substituting $|\mathbf{h}_p|^2$ and Q_{peak} by $\boldsymbol{\beta}$ and P_{pp}/λ_{th}, respectively, it is easy to see that (17) is equivalent to (8). Therefore, the optimum power profile and the ergodic capacity are given by (11)–(13) and (16), respectively, using the previous substitution.

C. Peak Transmit Power and Interference or SI Outage Constraints

If, instead of the average transmit power constraint (6), a peak power constraint will be respected, i.e.,

$$P(h_s, \hat{h}_p) \leq P_{peak} \tag{18}$$

then either with the interference outage constraint (7) or the SI outage constraint (17), the optimum power profile consists of transmitting with the maximum power subject to two peak power constraints. That is, $P(h_s, \hat{h}_p)$ is given by

$$P(h_s, \hat{h}_p) = \frac{1}{h_{th}^2} \tag{19}$$

where $h_{th}^2 = (\min(P_{peak}, P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)))^{-1}$, and $\mathbf{X}$ is equal to either $|\mathbf{h}_p|^2$ in case of the interference outage constraint or $\boldsymbol{\beta}$ in case of the SI outage constraint. Furthermore, the ergodic capacity is equal to

$$C = \int\limits_{\hat{h}_p} \left[\int\limits_0^\infty \ln\left(1 + \frac{t}{h_{th}^2}\right) f_{|\mathbf{h}_s|^2}(t)\, dt \right] f_{\hat{\mathbf{h}}_p}(\hat{h}_p)\, d\hat{h}_p. \tag{20}$$

IV. Perfect and No Secondary Transmitter-Primary Receiver Channel-State Information Cases

In this section, the optimum power profile and the ergodic capacity, in case of perfect and no ST-PR CSI at the ST, are obtained as special cases by letting σ_p^2 in (2) tend toward 0 and 1, respectively.

A. Perfect ST-PR CSI

1) Average Transmit Power Constraint: Recall that this special case has been studied in [13], where the power profile has been derived, but in terms of a Lagrange multiplier. We show that our framework also captures this special case, and a more explicit solution is presented. Indeed, when the ST is provided the perfect instantaneous ST-PR channel gain h_p ($\mathbf{h}_p = \hat{\mathbf{h}}_p$), the interference outage in (7) is equal to zero ($\epsilon = 0$), and $P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon)$ in (8) reduces to

$$P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon) = \frac{\mathcal{Q}_{peak}}{|\hat{h}_p|^2}. \tag{21}$$

If the SI outage constraint will be fulfilled, then (17) may equivalently be expressed by

$$P_{\boldsymbol{\beta}|\hat{\mathbf{h}}_p}(\epsilon) = \frac{\mathcal{Q}_{peak}}{|\hat{h}_p|^2} \cdot \frac{1}{F^{-1}_{\frac{1}{|\mathbf{h}_{pp}|^2}}(1-\epsilon)} \tag{22}$$

where $Q_{peak} = P_{pp}/\lambda_{th}$. Using the aforementioned substitutions, the optimum power profile can consequently be obtained from (11)–(13). Noting that $P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)$ in (21) and (22) depends on $\hat{h}_p$ only through its norm, substituting $f_{\hat{\mathbf{h}}_p}$ by $f_{|\hat{\mathbf{h}}_p|^2}$, the ergodic capacity is obtained from (16), with $\mathcal{S}_{g_{s1}} = [\hat{h}_p^0, \infty[$ and $\hat{h}_p^0 = g_{s1} \times Q_{peak}$ in case of the interference outage constraint or $\hat{h}_p^0 = (g_{s1} \times Q_{peak})/(F^{-1}_{1/|\mathbf{h}_{pp}|^2}(1-\epsilon))$ in case of SI the outage constraint.

$$C = \begin{cases} \mathop{\mathbb{E}}\limits_{|\mathbf{h}_s|^2, \hat{\mathbf{h}}_p}\left[\ln\left(1 + P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon)|\mathbf{h}_s|^2\right)\right] & P_{avg} \geq \mathop{\mathbb{E}}\limits_{\hat{\mathbf{h}}_p}\left[P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon)\right] \\ \int\limits_{t \geq g_{s2}} \ln\left(\frac{t}{g_{s1}}\right) f_{|\mathbf{h}_s|^2}(t)\, dt - \int\limits_{\hat{h}_p \in \mathcal{S}_{g_{s1}}} \int\limits_{t \geq g_{s2}} \left[\ln\left(\frac{t}{g_{s1}}\right) - \ln\left(1 + P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon) t\right)\right] & \\ \times f_{|\mathbf{h}_s|^2}(t) f_{\hat{\mathbf{h}}_p}(\hat{h}_p)\, d\hat{h}_p dt & P_{avg} < \mathbb{E}_{\hat{\mathbf{h}}_p}\left[P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon)\right] \end{cases} \tag{16}$$

2) Peak Transmit Power Constraint: Similar to the previous case, the optimum power is given by (19), with h_{th}^2 computed using (21) or (22) for the signal or the SI outage constraint, respectively. The ergodic capacity can be simplified from (20) and is given by

$$C=\int_0^\infty \left[\int_0^\infty \ln\left(1+\frac{t}{h_{th}^2}\right) f_{|\mathbf{h}_s|^2}(t)\,dt\right] f_{|\hat{\mathbf{h}}_p|^2}(|\hat{h}_p|^2)d|\hat{h}_p|^2. \quad (23)$$

B. No ST-PR CSI

With no instantaneous ST-PR CSI provided ($\mathbf{h}_p = \tilde{\mathbf{h}}_p$), the ST can still rely on the statistics of $\mathbf{h}_p$ (through the pdf $f_{\mathbf{h}_p}(\cdot)$) to respect the interference constraints. Note that, now, the transmit power depends only on h_s, i.e., $P = P(h_s)$. In this case, the interference outage (8) and SI outage (17) become

$$P(h_s) \le P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon) \quad (24)$$

where $P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon) = Q_{peak}/F_{\mathbf{X}}^{-1}(1-\epsilon)$, with $\mathbf{X}$ again being equal to either $|\mathbf{h}_p|^2$ in case of the interference outage constraint or $\boldsymbol{\beta}$ in case of the SI outage constraint. Note that, in this case, $P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)$ in (24) takes a fixed value and does not depend on $\hat{\mathbf{h}}_p$ because no ST-PR CSI is assumed. That is, $P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon) = P_{\mathbf{X}}(\epsilon)$.

1) Average Transmit Power Constraint: The optimum power profile can be deduced from (11)–(13) by replacing $P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon)$ by $P_{\mathbf{X}}(\epsilon)$. Note that, because now the peak constraint $P_X(\epsilon)$ is constant, the optimal power is given by either (11) or (12), in which case, $g_{s1} = (G(x) - G((1/x - P_{\mathbf{X}}(\epsilon))^{-1}))^{-1}(P_{avg})$, or is given by (13), for which $g_{s1} = G^{-1}(P_{avg})$. Furthermore, the ergodic capacity is obtained by averaging (5) over $\mathbf{h}_s$ and is given by (25), shown at the bottom of the page.

2) Peak Transmit Power Constraint: The optimal power profile is given by (19), with $P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)$ defined as in (24). Furthermore, the ergodic capacity is equal to

$$C = \int_0^\infty \ln\left(1+\frac{t}{h_{th}^2}\right) f_{|\mathbf{h}_s|^2}(t)\,dt. \quad (26)$$

V. Application to Rayleigh Fading Channels

In this section, we assume that the channel gains $\mathbf{h}_s$, $\mathbf{h}_p$, and $\mathbf{h}_{pp}$ are i.i.d. zero-mean unit-variance circularly symmetric complex Gaussian random variables. Therefore, their square magnitude is exponentially distributed, the pdf's of $\boldsymbol{\beta}$, $1/|\mathbf{h}_p|^2$, and $|\mathbf{h}_p|^2$ and $\boldsymbol{\beta}$ conditioned on $\hat{\mathbf{h}}_p$ are defined for $t \ge 0$, respectively, by

$$f_{\boldsymbol{\beta}}(t) = \frac{1}{(1+t)^2} \quad (27)$$

$$f_{\frac{1}{|\mathbf{h}_{pp}|^2}}(t) = \begin{cases} \frac{1}{t^2}e^{-\frac{1}{t}}, & t>0 \\ 0, & \text{otherwise} \end{cases} \quad (28)$$

$$f_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(t) = \frac{1}{\sigma_p^2} e^{-\frac{t+\left(1-\sigma_p^2\right)|\hat{h}_p|^2}{\sigma_p^2}} I_0\left(2\sqrt{\frac{(1-\sigma_p^2)|\hat{h}_p|^2 t}{\sigma_p^4}}\right) \quad (29)$$

$$f_{\boldsymbol{\beta}|\hat{\mathbf{h}}_p}(t) = \frac{\sigma_p^4 + \left(\sigma_p^2 - \sigma_p^2|\hat{h}_p|^2 + |\hat{h}_p|^2\right)t}{\left(\sigma_p^2+t\right)^3} e^{-\frac{\left(1-\sigma_p^2\right)|\hat{h}_p|^2}{\sigma_p^2+t}} \quad (30)$$

where $I_0(\cdot)$ in (29) is the modified Bessel function of the first kind. Their cdf's can be obtained in a closed form through [30, Ch. 4 and 10] (for the last two pdf's) and are, respectively, given by

$$F_{\boldsymbol{\beta}}(t) = \frac{t}{1+t} \quad (31)$$

$$F_{\frac{1}{|\mathbf{h}_{pp}|^2}}(t) = \begin{cases} e^{-\frac{1}{t}}, & t>0 \\ 0, & \text{otherwise} \end{cases} \quad (32)$$

$$F_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(t) = 1 - \mathcal{Q}_1\left(\sqrt{\frac{2\left(1-\sigma_p^2\right)|\hat{h}_p|^2}{\sigma_p^2}}, \sqrt{\frac{2t}{\sigma_p^2}}\right) \quad (33)$$

$$F_{\boldsymbol{\beta}|\hat{\mathbf{h}}_p}(t) = \frac{t}{\sigma_p^2+t} e^{-\frac{\left(1-\sigma_p^2\right)|\hat{h}_p|^2}{\sigma_p^2+t}} \quad (34)$$

where $\mathcal{Q}_1(\alpha,\beta)$ in (33) is the first-order Marcum Q-Function defined by [30, Ch. 4]

$$\mathcal{Q}_1(\alpha,\beta) = \int_\beta^\infty x e^{-\frac{x^2+\alpha^2}{2}} I_0(\alpha x)\,dx. \quad (35)$$

The inverse cdf's of (31) and (32) are straightforward, whereas the inverse cdf's of (34) can easily be derived (after a few

$$C = \begin{cases} \underset{|\hat{\mathbf{h}}_s|^2}{\mathbb{E}}\left[\ln\left(1+P_{\mathbf{X}}(\epsilon)|h_s|^2\right)\right] & P_{avg} \ge P_{\mathbf{X}}(\epsilon) \\ \int_{g_{s1}}^{g_{s2}} \ln\left(\frac{t}{g_{s1}}\right) f_{|\mathbf{h}_s|^2}(t)\,dt + \int_{g_{s2}}^{\infty} \ln\left(1+P_{\mathbf{X}}(\epsilon)t\right) f_{|\mathbf{h}_s|^2}(t)\,dt & P_{\mathbf{X}}(\epsilon) > P_{avg} \ge G(P_{\mathbf{X}}(\epsilon)^{-1}) \\ \int_{g_{s1}}^{\infty} \ln\left(\frac{t}{g_{s1}}\right) f_{|\mathbf{h}_s|^2}(t)\,dt & P_{avg} < G(P_{\mathbf{X}}(\epsilon)^{-1}) \end{cases} \quad (25)$$

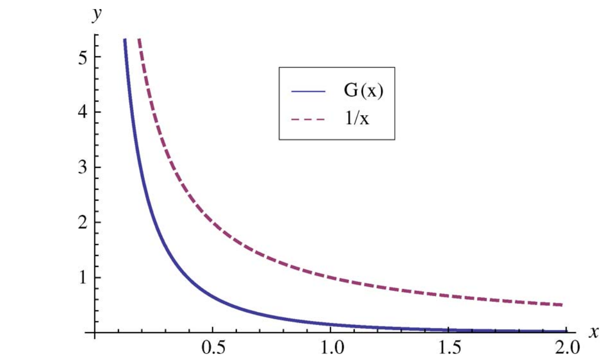

Fig. 2. Plot of $G(x)$ as defined in (10) for a Rayleigh fading SU channel, $G(x) = e^{-x}/x - \mathrm{E}_1(x)$, where $\mathrm{E}_1(x) = \int_x^\infty (e^{-t}/t)dt$ is the exponential integral function [31, Ch. 5].

manipulations) in terms of the principal branch of the Lambert W function $W(.)$ as follows:

$$F^{-1}_{\boldsymbol{\beta}|\hat{\mathbf{h}}_p}(u) = \frac{\sigma_p^2 W\left(\frac{(1-\sigma_p^2)|\hat{h}_p|^2}{\sigma_p^2} e^{-\frac{(1-\sigma_p^2)|\hat{h}_p|^2}{\sigma_p^2}} u\right)}{\frac{(1-\sigma_p^2)|\hat{h}_p|^2}{\sigma_p^2} - W\left(\frac{(1-\sigma_p^2)|\hat{h}_p|^2}{\sigma_p^2} e^{-\frac{(1-\sigma_p^2)|\hat{h}_p|^2}{\sigma_p^2}} u\right)} \tag{36}$$

for $u \in [0, 1[$. Because W is nonnegative definite over $[0, \infty[$, using the fact that $W(x \cdot e^x) = x$, it can easily be verified that $F^{-1}_{\boldsymbol{\beta}|\hat{\mathbf{h}}_p}(u)$ is also nonnegative definite over $u \in [0, 1[$. The inverse cdf of (33) is, unfortunately, tedious to derive in a closed form but can numerically be computed. Likewise, g_{s1} can numerically be computed using (14), where the function $G(\cdot)$, which is defined in (10), is now equal to

$$G(x) = \frac{e^{-x}}{x} - \mathrm{E}_1(x)$$

where $\mathrm{E}_1(x) = \int_x^\infty e^{-t}/t\, dt$ is the exponential integral function [31, Ch. 5]. A plot of $G(\cdot)$ is presented in Fig. 2.

A. Results for $0 < \sigma_p^2 < 1$

The results with regard to the optimum power profile and the ergodic capacity derived in Section III under the average and the peak transmit power are separately presented.

1) Average Transmit Power: The optimum power profile is directly obtained from (11)–(13) using (33) (to numerically compute g_{s2}) in case of the interference outage constraint. Instead, substituting $|\mathbf{h}_p|^2$ and Q_{peak} by $\boldsymbol{\beta}$ and P_{pp}/λ_{th}, respectively, in (11)–(13) and using (36), the optimum power profile is obtained in case of the SI outage constraint. Because $F_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}$ and $F_{\boldsymbol{\beta}|\hat{\mathbf{h}}_p}$ in (33) and (34) depend on $\hat{\mathbf{h}}_p$ only through its norm and because they are both strictly decreasing functions in $|\hat{h}_p|^2$ for $0 < \sigma_p^2 < 1$ and for a given t, again substituting $f_{\hat{\mathbf{h}}_p}$ by $f_{|\hat{\mathbf{h}}_p|^2}$, the ergodic capacity is obtained from (16), with $\mathcal{S}_{g_{s1}} = [[\hat{h}_p^0]^+, \infty[$, where $\hat{h}_p^0$ is the unique solution of $F_{\mathbf{X}|\hat{\mathbf{h}}_p}(g_{s1} \times Q_{peak}) = 1 - \epsilon$, and $\mathbf{X} = |\mathbf{h}_p|^2$ in case of the interference outage constraint or $\mathbf{X} = \boldsymbol{\beta}$ in case of the SI outage constraint. Nevertheless, in both cases, there is no closed form of the second integral in the capacity expression (16), although a numerical evaluation is easy to obtain.

Peak Transmit Power: The power profile for the interference and SI outage constraints is given by (19). Here, again, the outer integral in the capacity expression (20), which can also further be simplified as in (23), is not analytically easy to find and will numerically be computed.

B. Results for the No ST-PR CSI Case

When no ST-PR CSI is available at the ST ($\sigma_p^2 \to 1$), the capacity expression in (25) and (26) can interestingly be computed in a closed form. For convenience, the results with regard to the no ST-PR CSI case are presented in Table I. In this case, it is also of interest to point out the following facts.

1) Average Transmit Power Constraint: Based on (11)–(13), it is shown that, when $P_{avg} \to \infty$, $P(h_s) \to P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)$, where $\mathbf{X} = \mathbf{h}_p$ in case of the interference outage constraint and $\mathbf{X} = \boldsymbol{\beta}$ in case of the SI outage constraint. Consequently, the ergodic capacity is equal to

$$\lim_{P_{avg}\to\infty} C = e^{\frac{1}{P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)}} \mathrm{E}_1\left(\frac{1}{P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)}\right) \tag{37}$$

in agreement with [32, eq. (34)]. On the other hand, when no interference constraint is considered, i.e., $P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon) \to \infty$ (this condition can be obtained by letting $\epsilon \to 1$), the optimum power is given by (13), and the capacity is equal to

$$\lim_{P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)\to\infty} C = \mathrm{E}_1(g_{s1}) = \mathrm{E}_1\left(G^{-1}(P_{avg})\right) \tag{38}$$

in agreement with [26].

2) Peak Transmit Power Constraint: First, note that the optimal power profile is constant, regardless of the instantaneous CSI h_s. Hence, even without secondary CSI, we would achieve the same ergodic capacity in this case. Furthermore, if $P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon) > P_{peak}$, then the optimum power profile and the ergodic capacity are equal to a fading channel with a peak transmit power and without an interference constraint. Moreover, increasing the power above $P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)$ does not provide any capacity gain. Furthermore, the infinite peak transmit power and infinite interference outage constraints limits can be computed from (26) and are, respectively, given by

$$\lim_{P_{peak}\to\infty} C = e^{1/P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)} \mathrm{E}_1\left(1/P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)\right) \tag{39}$$

$$\lim_{P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)\to\infty} C = e^{1/P_{peak}} \mathrm{E}_1(1/P_{peak}). \tag{40}$$

C. Results for the Perfect ST-PR CSI Case

1) Average Transmit Power: When the ST is aware of the instantaneous ST-PR CSI ($\sigma_p^2 \to 0$), the power profile is similarly given by (11)–(13), along with (21) or (22) for the interference or SI outage constraints, respectively. The threshold h_p^0 in the capacity expression (16) can explicitly be computed in both cases and is equal to $\hat{h}_p^0 = g_{s1} \times Q_{peak}$ or $\hat{h}_p^0 = g_{s1} \times Q_{peak} \ln(1/1 - \epsilon)$, respectively. However, in both cases, there

TABLE I
SUMMARY RESULTS OF THE OPTIMUM POWER PROFILE AND THE ERGODIC CAPACITY OF A SPECTRUM-SHARING CR NETWORK WHERE ALL THE CHANNELS ARE I.I.D. GAUSSIAN AND WITH NO ST-PR CSI ($\sigma_p^2 \to 1$)

Average Transmit-Power And Interference Outage Constraints	Optimum Power Profile	given by (11), (12) and (13), with $P_{\|\mathbf{h}_p\|^2\|\hat{h}_p}(\epsilon) = -\frac{Q_{peak}}{\ln(\epsilon)}$
	Ergodic Capacity	$C = \begin{cases} \epsilon^{-1/Q_{peak}} \mathrm{E}_1\left(-\frac{\ln(\epsilon)}{Q_{peak}}\right) & P_{avg} \geq -\frac{Q_{peak}}{\ln(\epsilon)} \\ \mathrm{E}_1(g_{s1}) - \mathrm{E}_1(g_{s2}) + \epsilon^{-1/Q_{peak}} \mathrm{E}_1\left(\frac{g_{s2}^2}{g_{s2}-g_{s1}}\right) & -\frac{Q_{peak}}{\ln(\epsilon)} > P_{avg} \geq G\left(-\frac{\ln(\epsilon)}{Q_{peak}}\right) \\ \mathrm{E}_1(g_{s1}) & P_{avg} < G\left(-\frac{\ln(\epsilon)}{Q_{peak}}\right). \end{cases}$
Average Transmit-Power And SI Outage Constraints	Optimum Power Profile	given by (11), (12) and (13), with $P_{\|\mathbf{h}_p\|^2\|\hat{h}_p}(\epsilon) = \frac{\epsilon}{1-\epsilon} Q_{peak}$
	Ergodic Capacity	$C = \begin{cases} e^{\frac{(1-\epsilon)/\epsilon}{Q_{peak}}} \mathrm{E}_1\left(\frac{(1-\epsilon)/\epsilon}{Q_{peak}}\right) & P_{avg} \geq \frac{\epsilon}{1-\epsilon} Q_{peak} \\ \mathrm{E}_1(g_{s1}) - \mathrm{E}_1(g_{s2}) + e^{\frac{(1-\epsilon)/\epsilon}{Q_{peak}}} \mathrm{E}_1\left(\frac{g_{s2}^2}{g_{s2}-g_{s1}}\right) & \frac{\epsilon}{1-\epsilon} Q_{peak} > P_{avg} \geq G\left(\frac{(1-\epsilon)/\epsilon}{Q_{peak}}\right) \\ \mathrm{E}_1(g_{s1}) & P_{avg} < G\left(\frac{(1-\epsilon)/\epsilon}{Q_{peak}}\right). \end{cases}$
Peak Transmit-Power And Interference Outage Constraint	Optimum Power Profile	given by (19), with $h_{th}^2 = \left(\min\left(P_{peak}, -\frac{Q_{peak}}{\ln(\epsilon)}\right)\right)^{-1}$
	Ergodic Capacity	$C = e^{h_{th}^2} \mathrm{E}_1\left(h_{th}^2\right)$
Peak Transmit-Power And SI Outage Constraint	Optimum Power Profile	given by (19), with $h_{th}^2 = \left(\min\left(P_{peak}, \frac{\epsilon}{1-\epsilon} Q_{peak}\right)\right)^{-1}$
	Ergodic Capacity	$C = e^{h_{th}^2} \mathrm{E}_1\left(h_{th}^2\right)$

is no closed form of the second integral in the capacity expression (16), which may numerically be evaluated. Nevertheless, asymptotic analysis when P_{avg} (respectively, P_{peak}) is sufficiently large (no budget constraint) or the outage constraint is not effective (Q_{peak} is sufficiently high, for example) are provided as follows:

$$\lim_{P_{avg} \to \infty} C = \frac{Q_{eq} \ln(Q_{eq})}{Q_{eq} - 1} \tag{41}$$

where $Q_{eq} = Q_{peak}$ in the case of the interference constraint, and $Q_{eq} = Q_{peak} \ln(1/(1-\epsilon))$ in case of the SI outage constraint, whereas the capacity at an infinite outage constraint is given by (38) and is independent of the channel estimation quality.

2) Peak Transmit Power: In this case, the power profile (19) and the ergodic capacity (20) can be obtained using (21) or (22) for the interference or SI outage constraints, respectively. At a high-peak-power constraint, it can be shown that

$$\lim_{P_{peak} \to \infty} C = \frac{Q_{eq} \ln(Q_{eq})}{Q_{eq} - 1} \tag{42}$$

whereas the high-interference-outage constraint ergodic capacity is, again, independent of the channel estimation quality and is equal to where no ST-PR CSI is available (cf., Table 1).

VI. NUMERICAL RESULTS

In this section, numerical results are provided for Rayleigh fading channels, as derived in Section V. First, we show our results for an average transmit power constraint. Fig. 3 depicts the optimum power profile as a function of the secondary channel $|h_s|^2$ for a given $\hat{h}_p$ value. As shown in Fig. 3, the power profile has typically two or three regions, depending on the interference outage constraint value $P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)$. For a relatively high $P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)$, the power profile is similar to water-filling, as discussed in [26], whereas a low $P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)$ value limits the power profile, even when the secondary link $|h_s|^2$

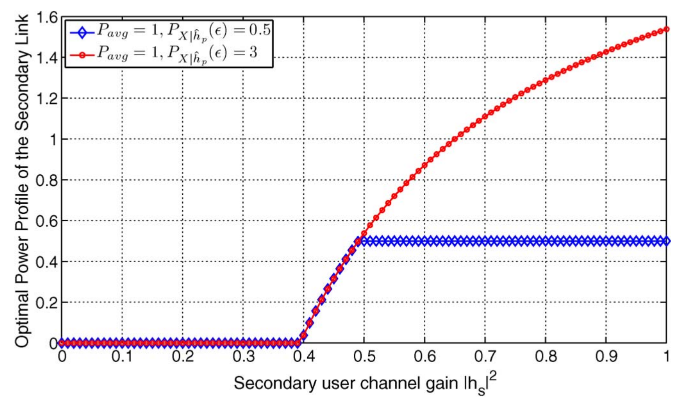

Fig. 3. Optimum power profile of the secondary link for a given $\hat{h}_p$ and under the average transmit-power constraint when $P_{avg} \leq \mathbb{E}_{\hat{\mathbf{h}}_p}[P_{|\mathbf{h}_p|^2|\hat{h}_p}(\epsilon)]$.

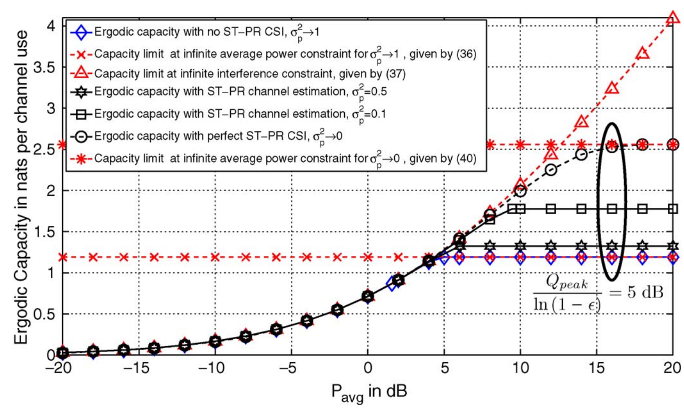

Fig. 4. Ergodic capacity of the secondary link under the average transmit power and interference outage constraints for different channel estimation error variance values σ_p^2.

is "good." This limitation behavior also affects the ergodic capacity, regardless of how the channel estimation quality is, as shown in Fig. 4. However, at the low-power regime ($P_{avg} \to 0$), the ergodic capacity is insensitive to the interference outage constraint, and equal capacity is achieved, regardless of $P_{\mathbf{X}|\hat{\mathbf{h}}_p}(\epsilon)$ and σ_p^2. The capacity at the infinite interference and infinite average power constraints are also shown in Fig. 4 as

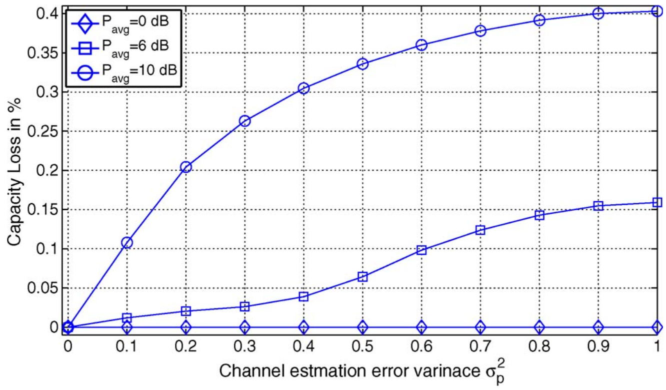

Fig. 5. Capacity-loss percentage of the secondary link under the average transmit power and interference outage constraints versus the channel estimation error variance σ_p^2 for different P_{avg} values.

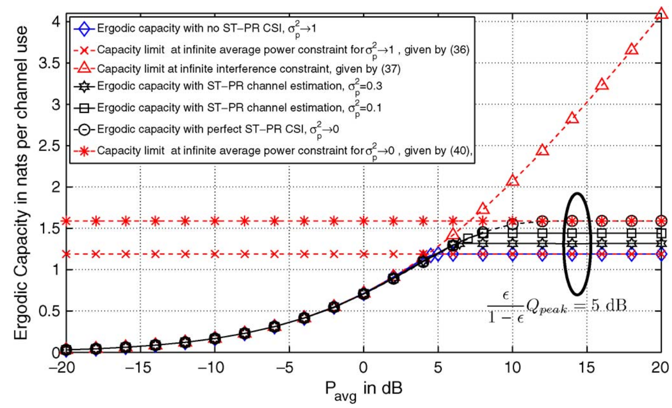

Fig. 6. Ergodic capacity of the secondary link under the average transmit power and SI outage constraints for different channel estimation error variance values σ_p^2.

performance limits. Interestingly, the first limit is achieved at the low-power regime. For example, when $Q_{peak} = 10$ and $\epsilon = \epsilon_1 = 4.2\%$ so that $Q_{peak}/\ln(1/\epsilon_1) = 5$ dB, the ergodic capacity with and without the interference constraint is the same for P_{avg} below 2 dB, irrespective of σ_p^2. The second limit is achieved at relatively high P_{avg} values and increasingly with respect to the channel estimation quality. In Fig. 5, the capacity loss percentage defined as the ratio of the difference between the ergodic capacity under perfect cross link CSI ($\sigma_p^2 \to 0$) and the same capacity where only a noisy cross link CSI with error variance σ_p^2 is available at the ST, over the capacity under perfect cross-link CSI, is plotted for different P_{avg} values. Fig. 5 confirms our previous observations: At low $P_{avg} = 0$ dB, the capacity loss is equal to zero, and the interference constraint has no effect on the secondary performance; at $P_{avg} = 6$ dB, the cross-link CSI uncertainty impacts the secondary capacity, and the loss may be more than 15% for a poor CSI quality ($\sigma_p^2 \geq 0.85$). However, note that an average CSI quality ($\sigma_p^2 \leq 0.5$) is enough to contain the capacity loss (less than 6%). On the contrary, at $P_{avg} = 10$ dB, the cross-link CSI quality is detrimental, because the capacity loss may reach up to 40%, and we require a "very good" CSI quality to reduce the capacity loss. For example, even with $\sigma_p^2 = 0.1$, the capacity loss is more than 10%. This case is, in fact, expected, because at the high-power regime, the capacity is dictated only by the interference constraint, as stipulated by (16).

To display results for the SI outage constraint, we set $P_{pp}/\lambda_{th} = Q_{peak} = 10$ and $\epsilon = \epsilon_2 = 24\%$ in Fig. 6 so that the interference and SI outage constraints are equal in the no ST-PR CSI case. That is, $1/(\ln(1/\epsilon_1))Q_{peak} = \epsilon_2/(1-\epsilon_2)Q_{peak} = 5$ dB, and hence, the corresponding ergodic capacities of the SU under the interference and SI outage constraints are equal. Note that, to achieve equal ergodic capacity in this case, ϵ_2 must be bigger than ϵ_1 (a higher SI outage should be tolerated), which implies that, at the no ST-PR CSI case, the SI outage constraint is more restrictive than the interference outage constraint from a capacity perspective when $P_{pp}/\lambda_{th} = Q_{peak}$. The evidence of this condition can be shown by first noting that the $\epsilon_1 - \epsilon_2$ region $\mathcal{R}_1$ defined by

$$\mathcal{R}_1 = \left\{ (\epsilon_1, \epsilon_2) \mid \ln \frac{1}{\epsilon_1} \geq \frac{1}{\epsilon_2} - 1 \right\} \tag{43}$$

corresponds to the region where the SU capacity under the SI outage constraint ϵ_2 is bigger than under the interference outage constraint ϵ_1 in the non–ST-PR CSI scenario. Then, using the inequality $x \geq \exp(1 - 1/x)$ for all $x > 0$, (43) directly implies that $\epsilon_1 \leq \epsilon_2$. The fact that a higher SI outage must be tolerated to achieve equal ergodic capacity than under the interference outage constraint is also shown for the perfect ST-PR CSI case, where $\epsilon_1 = 0$ by equalizing (21) and (22), which results in the $\epsilon_1 - \epsilon_2$ region $\mathcal{R}_2$ defined by

$$\mathcal{R}_2 = \{(\epsilon_1, \epsilon_2) \mid \epsilon_1 = 0, \epsilon_2 \geq 0.63.\} \tag{44}$$

Thus, all SI outage levels below this value provide an ergodic capacity that is smaller than the capacity that corresponds to an interference outage constraint. Only if the primary user is ready to accept an SI outage $\epsilon_2 \geq 63\%$, hence sacrificing its error probability performance, would the secondary link achieve a higher ergodic capacity than under the interference outage constraint. Furthermore, as shown in Fig. 6, even for such a high SI outage level ($\epsilon_2 = 24\%$), the secondary-link ergodic capacity is smaller than the capacity displayed in Fig. 4 for equal channel estimation quality. When $0 < \sigma_p^2 < 1$, this fact can similarly be explained by noting that the SU capacity under the SI outage is bigger than under the interference outage if and only if (iff) $(\epsilon_1, \epsilon_2) \in \mathcal{R}_3$, where $\mathcal{R}_3$ is defined by

$$\mathcal{R}_3 = \left\{ (\epsilon_1, \epsilon_2) \mid P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon_1) \leq P_{\boldsymbol{\beta}|\hat{\mathbf{h}}_p}(\epsilon_2) \right\} \tag{45}$$

for all $\hat{h}_p$. By letting $\hat{h}_p$ tend toward 0, it can be verified that $F^{-1}_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(x) \to \sigma_p^2 \ln 1/x$ and $F^{-1}_{\boldsymbol{\beta}|\hat{\mathbf{h}}_p}(x) \to \sigma_p^2 \ln x/1 - x$. Hence, as $\hat{h}_p$ tends toward 0, (45) implies (43), which itself implies that $\epsilon_1 \leq \epsilon_2$, as previously proved. Fig. 7 illustrates the $\epsilon_1 - \epsilon_2$ regions $\mathcal{R}_1$, $\mathcal{R}_2$, and $\mathcal{R}_3$ defined by (43)–(45), respectively.

Then, we show our results for the peak transmit power constraint. Although the interference outage constraint limits the capacity at the high-power regime, it has, again, no effect at the low-power regime, and no-interference performance is achieved, as shown in Fig. 8. Finally, note that, at the low-power regime, the peak constraint is more stringent than the average constraint from a capacity point of view, which is

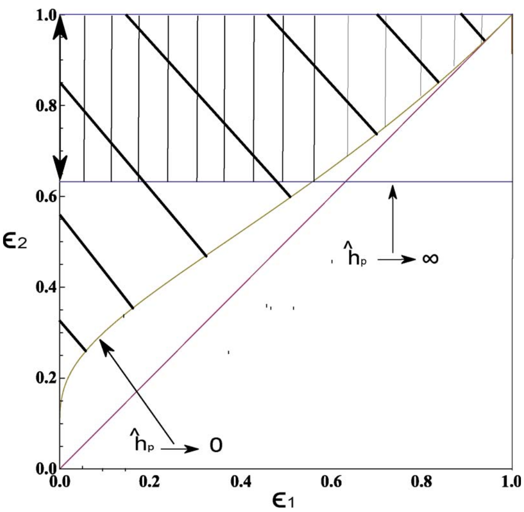

Fig. 7. $\epsilon_1 - \epsilon_2$ regions where the secondary-link ergodic capacity under the SI outage constraint is bigger than under the interference outage for $\sigma_p^2 = 1$ in oblique lines, for $\sigma_p^2 = 0$ coinciding with the y-axis such that $\epsilon_2 \geq 0.63$, and for $0 < \sigma_p^2 < 1$ in vertical lines.

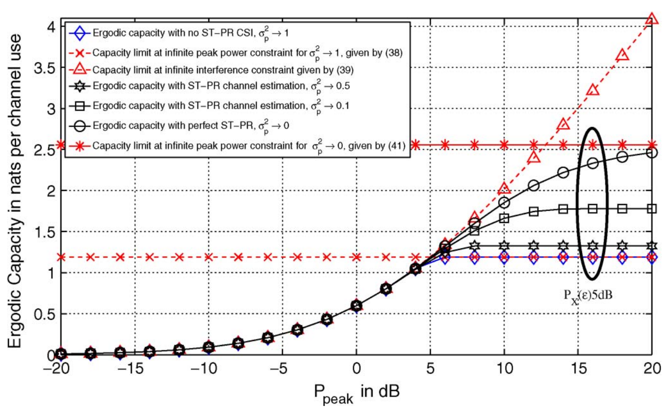

Fig. 8. Ergodic capacity of the secondary link under the peak transmit power constraint and for different channel estimation error variance values σ_p^2.

in agreement with [33]. For example, at a given interference outage constraint, e.g., $P_{\mathbf{X}}(\epsilon) = 5$ dB, the ergodic capacity provided by the peak transmit-power constraint $P_{peak} = 0$ dB is equal to 0.6 npcu, whereas for an average transmit-power $P_{avg} = 0$ dB, the ergodic capacity is equal to 0.71 npcu. This gap remarkably diminishes at the high-power regime, where the outage constraint dictates the power profile.

VII. Conclusion

A spectrum-sharing communication with ST-PR channel estimation at the SU has been addressed. The optimum power profile and the ergodic capacity have been derived for a class of fading channels with respect to an average or a peak transmit power, along with more realistic interference outage constraints. The impact of channel estimation quality on the ergodic capacity has been highlighted. In all cases, asymptotic analysis has been discussed to provide a better understanding of the performance limits of a spectrum-sharing protocol. Our framework generalizes and encompasses several existing results.

Appendix
Derivation of the Optimum Power Profile Given by (9) and, Equivalently, by (11)–(13)

To solve the optimization problem formulated by (5), (6), and (8), we first consider the set, e.g., $\mathcal{A}$, of all h_s and $\hat{h}_p$ values such that (8) is satisfied. That is, $\mathcal{A} = \{h_s, \hat{h}_p : P(h_s, \hat{h}_p) \leq P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon)\}$. Our optimization can thus be formulated by (5) and (6) over the set $\mathcal{A}$. Because (5) is concave, (6) is linear, and $\mathcal{A}$ is convex, the new equivalent optimization problem is concave over $\mathcal{A}$. The solution of (5) and (6) over all channel realizations h_s and $\hat{h}_p$ is known to be water-filling, as given by $P(h_s, \hat{h}_p) = [1/\lambda - 1/(|h_s|^2)]^+$. However, because we now optimize only over $\mathcal{A}$, the optimal power is capped by $P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon)$, which leads to (9).

Departing from (9), we show that the optimum power profile is given by (11)–(13). First, note that, because the optimization problem given by (5)–(7), is convex with a feasible point, λ in (9) can be found by solving the (concave) Lagrange dual problem defined by [34]

$$\min_{\lambda \geq 0} g(\lambda) \tag{46}$$

where $g(\lambda)$ is given by

$$g(\lambda) = \mathop{\mathbb{E}}_{\mathbf{h}_s, \hat{\mathbf{h}}_p} \left[\ln\left(1 + P^*(\mathbf{h}_s, \hat{\mathbf{h}}_p) \cdot |\mathbf{h}_s|^2\right) \right] - \lambda \left(\mathop{\mathbb{E}}_{\mathbf{h}_s, \hat{\mathbf{h}}_p} \left[P^*(\mathbf{h}_s, \hat{\mathbf{h}}_p) \right] - P_{avg} \right)$$

and $P^*(\mathbf{h}_s, \hat{\mathbf{h}}_p)$ is the optimal solution given by (9). Because $g(\lambda)$ is concave with a feasible minimum, we have the following two conditions.

C1. If $(\partial g / \partial \lambda)|_{\lambda=0} \geq 0$, then $\min_{\lambda \geq 0}\ g(\lambda) = g(0)$.

C2. Otherwise, $\min_{\lambda \geq 0}\ g(\lambda) = g(g_{s1})$ for some $g_{s1} > 0$.

Now, it can be easily verified that

$$\frac{\partial g}{\partial \lambda} = \left(P_{avg} - \mathop{\mathbb{E}}_{\mathbf{h}_s, \hat{\mathbf{h}}_p} \left[P^*(\mathbf{h}_s, \hat{\mathbf{h}}_p) \right] \right)$$

and thus

$$\left.\frac{\partial g}{\partial \lambda}\right|_{\lambda=0} = \left(P_{avg} - \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p} \left[P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon) \right] \right). \tag{47}$$

Combining (47) and conditions **C1** and **C2** together with (9) yields (11)–(13). It remains to prove (14). If **C2** holds, then by the complimentary slackness condition [34], the average power constraint is satisfied with equality, and we have

$$\begin{aligned} P_{avg} &= \mathop{\mathbb{E}}_{\mathbf{h}_s, \hat{\mathbf{h}}_p} \left[P^*(\mathbf{h}_s, \hat{\mathbf{h}}_p) \right] \\ &= \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p} \left[\mathop{\mathbb{E}}_{\mathbf{h}_s|\hat{\mathbf{h}}_p} \left[P^*(\mathbf{h}_s, \hat{\mathbf{h}}_p)\, |\hat{\mathbf{h}}_p \right] \right] \\ &= \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p \in \mathcal{S}_{g_{s1}}} \left[\mathop{\mathbb{E}}_{\mathbf{h}_s|\hat{\mathbf{h}}_p} \left[P^*(\mathbf{h}_s, \hat{\mathbf{h}}_p)\, |\hat{\mathbf{h}}_p \in \mathcal{S}_{g_{s1}} \right] \right] \\ &\quad + \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p \in \bar{\mathcal{S}}_{g_{s1}}} \left[\mathop{\mathbb{E}}_{\mathbf{h}_s|\hat{\mathbf{h}}_p} \left[P^*(\mathbf{h}_s, \hat{\mathbf{h}}_p)\, |\hat{\mathbf{h}}_p \in \bar{\mathcal{S}}_{g_{s1}} \right] \right]. \end{aligned} \tag{48}$$

Now, the outer expectation in the second term on the right-hand side of (48) is over all $\hat{\mathbf{h}}_p \in \bar{\mathcal{S}}_{g_{s1}}$, and hence, the power profile is given by (13). Therefore, we can compute the second term in (48) as follows:

$$\begin{aligned}
&\mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p \in \bar{\mathcal{S}}_{g_{s1}}}\left[\mathop{\mathbb{E}}_{\mathbf{h}_s|\hat{\mathbf{h}}_p}\left[P^*(\mathbf{h}_s, \hat{\mathbf{h}}_p)\,|\hat{\mathbf{h}}_p \in \bar{\mathcal{S}}_{g_{s1}}\right]\right] \\
&= \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p \in \bar{\mathcal{S}}_{g_{s1}}}\left[\mathop{\mathbb{E}}_{\mathbf{h}_s|\hat{\mathbf{h}}_p}\left[\frac{1}{g_{s1}} - \frac{1}{|h_s|^2}\right]^+\right] \\
&= \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p \in \bar{\mathcal{S}}_{g_{s1}}}\left[\mathop{\mathbb{E}}_{\mathbf{h}_s}\left[\frac{1}{g_{s1}} - \frac{1}{|h_s|^2}\right]^+\right] \qquad (49)\\
&= \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p \in \bar{\mathcal{S}}_{g_{s1}}}\left[G(g_{s1})\right] \qquad (50)
\end{aligned}$$

where (49) follows from the independence of $\mathbf{h}_s$ and $\hat{\mathbf{h}}_p$. On the contrary, the outer expectation in the first term on the right-hand side of (48) is over all $\hat{\mathbf{h}}_p \in \mathcal{S}_{g_{s1}}$, and hence, the power profile is given by (12). Therefore, we can compute the first term in (48) as follows

$$\begin{aligned}
&\mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p \in \mathcal{S}_{g_{s1}}}\left[\mathop{\mathbb{E}}_{\mathbf{h}_s|\hat{\mathbf{h}}_p}\left[P^*(\mathbf{h}_s, \hat{\mathbf{h}}_p)\,|\hat{\mathbf{h}}_p \in \mathcal{S}_{g_{s1}}\right]\right] \\
&= \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p \in \mathcal{S}_{g_{s1}}}\left[\mathop{\mathbb{E}}_{|\mathbf{h}_s|^2 \in [g_{s1}, g_{s2})}\left[\frac{1}{g_{s1}} - \frac{1}{|h_s|^2}\right]^+ \right.\\
&\qquad \left. + \mathop{\mathbb{E}}_{|\mathbf{h}_s|^2 \geq g_{s2}}\left[P_{|\mathbf{h}_p|^2|\hat{h}_p}(\epsilon)\right]\right] \\
&= \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p \in \mathcal{S}_{g_{s1}}}\left[G(g_{s1}) \right.\\
&\qquad \left. - \mathop{\mathbb{E}}_{|\mathbf{h}_s|^2 > g_{s2}}\left[\frac{1}{g_{s1}} - \frac{1}{|h_s|^2} - P_{|\mathbf{h}_p|^2|\hat{h}_p}(\epsilon)\right]^+\right] \\
&= \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p \in \mathcal{S}_{g_{s1}}}\left[G(g_{s1}) - G(g_{s2})\right] \\
&= \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p \in \mathcal{S}_{g_{s1}}}\left[G(g_{s1})\right] - \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p \in \mathcal{S}_{g_{s1}}}\left[G(g_{s2})\right]. \qquad (51)
\end{aligned}$$

Now, gathering (48), (50), and (51), we obtain

$$\begin{aligned}
P_{avg} &= \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p}\left[G(g_{s1})\right] - \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p \in \mathcal{S}_{g_{s1}}}\left[G(g_{s2})\right] \\
&= G(g_{s1}) - \mathop{\mathbb{E}}_{\hat{\mathbf{h}}_p \in \mathcal{S}_{g_{s1}}}\left[G(g_{s2})\right] \qquad (52)
\end{aligned}$$

where (52) follows, because g_{s1} depends only on P_{avg}. Using the fact that the function $K(x)$ in (15) is invertible on $(0, P_{|\mathbf{h}_p|^2|\hat{h}_p}(\epsilon)]$, (14) immediately follows. Finally, because $K(x)$ is monotonically decreasing, $g_{s1} < 1/(P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon))$ is equivalent to $P_{avg} > G(1/P_{|\mathbf{h}_p|^2|\hat{\mathbf{h}}_p}(\epsilon))$.

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REFERENCES

[1] J. Mitola, III and G. Q. Maguire, Jr., "Cognitive radio: Making software radios more personal," *IEEE Pers. Commun.*, vol. 6, no. 4, pp. 13–18, Aug. 1999.

[2] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[3] S. Srinivasa and S. Jafar, "Cognitive radios for dynamic spectrum access-the throughput potential of cognitive radio: A theoretical perspective," *IEEE Commun. Mag.*, vol. 45, no. 5, pp. 73–79, May 2007.

[4] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information-theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.

[5] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Trans. Inf. Theory*, vol. 52, no. 5, pp. 1813–1827, May 2006.

[6] A. Jovicic and P. Viswanath, "Cognitive radio: An information-theoretic perspective," *IEEE Trans. Inf. Theory*, vol. 55, no. 9, pp. 3945–3958, Sep. 2009.

[7] P.-H. Lin, S.-C. Lin, C.-P. Lee, and H.-J. Su, "Cognitive radio with partial channel-state information at the transmitter," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3402–3413, Nov. 2010.

[8] S. Rini, D. Tuninetti, and N. Devroye, "Inner and outer bounds for the Gaussian cognitive interference channel and new capacity results," *IEEE Trans. Inf. Theory*, vol. 58, no. 2, pp. 820–848, Feb. 2012.

[9] Z. Rezki and M.-S. Alouini, "On the capacity of cognitive radio under limited channel-state information," in *Proc. 7th ISWCS*, York, U.K., Sep. 2010, pp. 1066–1070.

[10] Z. Rezki and M.-S. Alouini, "On the capacity of cognitive radio under limited channel-state information over fading channels," in *Proc. IEEE ICC*, Jun. 2011, pp. 1–5.

[11] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649–658, Feb. 2007.

[12] L. Musavian and S. Aissa, "Capacity and power allocation for spectrum-sharing communications in fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 148–156, Jan. 2009.

[13] X. Kang, Y.-C. Liang, A. Nallanathan, H. Garg, and R. Zhang, "Optimal power allocation for fading channels in cognitive radio networks: Ergodic capacity and outage capacity," *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 940–950, Feb. 2009.

[14] D. Li, "Performance analysis of MRC diversity for cognitive radio systems," *IEEE Trans. Veh. Technol.*, vol. 61, no. 2, pp. 849–853, Feb. 2012.

[15] M. Hanif, P. Smith, and M. Shafi, "Performance of cognitive radio systems with imperfect radio environment map information," in *Proc. AusCTW*, Feb. 2009, pp. 61–66.

[16] L. Musavian and S. Aïssa, "Fundamental capacity limits of cognitive radio in fading environments with imperfect channel information," *IEEE Trans. Commun.*, vol. 57, no. 11, pp. 3472–3480, Nov. 2009.

[17] L. Musavian and S. Aïssa, "Outage-constrained capacity of spectrum-sharing channels in fading environments," *IET Commun.*, vol. 2, no. 6, pp. 724–732, Jul. 2008.

[18] H. Suraweera, P. Smith, and M. Shafi, "Capacity limits and performance analysis of cognitive radio with imperfect channel knowledge," *IEEE Trans. Veh. Commun.*, vol. 59, no. 4, pp. 1811–1822, May 2010.

[19] P. Dmochowski, H. Suraweera, P. Smith, and M. Shafi, "Impact of channel knowledge on cognitive radio system capacity," in *Proc. VTC*, Sep. 2010, pp. 1–5.

[20] L. Zhang, Y.-C. Liang, and Y. Xin, "Optimal transmission strategy for cognitive radio networks with partial channel-state information," in *Proc. Int. Conf. CrownCom*, May 2008, pp. 1–5.

[21] B. Maham, P. Popovski, X. Zhou, and A. Hjorungnes, "Cognitive multiple access network with outage margin in the primary system," *IEEE Trans. Wireless Commun.*, vol. 10, no. 10, pp. 3343–3353, Oct. 2011.

[22] C.-X. Wang, X. Hong, H.-H. Chen, and J. Thompson, "On capacity of cognitive radio networks with average interference power constraints," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 1620–1625, Apr. 2009.

[23] J. Peha, "Approaches to spectrum sharing," *IEEE Commun. Mag.*, vol. 43, no. 2, pp. 10–12, Feb. 2005.

[24] A. Pascual-Iserte, D. Palomar, A. Perez-Neira, and M. Lagunas, "A robust maximin approach for MIMO communications with imperfect channel-state information based on convex optimization," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 346–360, Jan. 2006.

[25] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: Wiley, 2006, ser. Wiley Series in Telecommunications and Signal Processing, 2nd ed.

[26] A. Goldsmith and P. Varaiya, "Capacity of fading channels with channel-side information," *IEEE Trans. Inf. Theory*, vol. 43, no. 6, pp. 1986–1992, Nov. 1997.

[27] G. Caire and S. Shamai, "On the capacity of some channels with channel-state information," *IEEE Trans. Inf. Theory*, vol. 45, no. 6, pp. 2007–2019, Sep. 1999.

[28] M. Khojastepour and B. Aazhang, "The capacity of average and peak power constrained fading channels with channel-side information," in *Proc. WCNC*, Atlanta, GA, Mar. 2004, pp. 77–82.

[29] R. Zhang, S. Cui, and Y.-C. Liang, "On ergodic sum capacity of fading cognitive multiple-access and broadcast channels," *IEEE Trans. Inf. Theory*, vol. 55, no. 11, pp. 5161–5178, Nov. 2009.

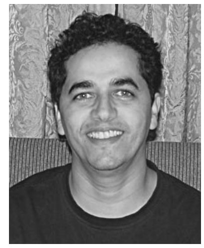
[30] M. K. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels (Wiley Series in Telecommunications and Signal Processing)*, 2nd ed. New York: Wiley, 2004.

[31] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*, 9th ed. New York: Dover, 1964.

[32] M.-S. Alouini and A. Goldsmith, "Capacity of Rayleigh fading channels under different adaptive transmission and diversity-combining techniques," *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1165–1181, Jul. 1999.

[33] R. Zhang, "On peak versus average interference power constraints for protecting primary users in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 2112–2120, Apr. 2009.

[34] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York: Cambridge Univ. Press, 2004.



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