Achievable Transmission Rate of the Secondary User in Cognitive Radio Networks with Hybrid Spectrum Access Strategy

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***Abstract*—A hybrid spectrum access strategy which is different from traditional underlay or overlay strategy is proposed for cognitive radio networks. In proposed strategy, the secondary users (SU) cooperatively sense the state of the primary user (PU) in a given spectrum band. Based on the sensing results and an additional requirement on the bit error rate, the SU adapts its transmit power and modulation level. Specifically, if the PU is inactive, the SU allocates its power subject to a peak transmit power constraint. Else if the PU is active, the average interference power constraint would be further imposed to protect the primary link. We assume that the maximal ratio combing (MRC) technique is available at the secondary receiver. Imperfect spectrum sensing and imperfect estimations of the channel power gain from the secondary transmitter to primary receiver are also considered. Achievable transmission rate of the SU over Rayleigh fading channel is formulated as an optimization problem and solved by using Lagrange dual decomposition method. Simulation results demonstrate that the transmission rate of the SU with hybrid strategy is significantly improved when compared with the traditional ones.**

***Index Terms*—Cognitive radio, achievable transmission rate, spectrum access strategy, dual decomposition, cooperative spectrum sensing.**

I. Introduction

IN cognitive radio (CR) applications, two classical spectrum access strategies, overlay and underlay strategies are often employed by the secondary users (SU) to better utilize the spectrum bands which are licensed to the primary users (PU). It is obvious that there are some fundamental limitations to the SU's transmission activities in both of the strategies. In overlay strategy, although it can reduce the potential interference to the PU, simultaneously possible spectrum sharing opportunities are also missed because the feature of interference tolerance of the PU can't be utilized. The SU's transmission capability is limited for underlay access because the interference power constraint at the primary receiver is always active regardless of the PU's states. Achievable transmission rates of the SU with underlay or overlay in different conditions have been investigated in some literature [1]–[5]. However, mixed or hybrid strategy which combines the two classical strategies is preferred to improve spectrum efficiency. Currently, some work about achievable transmission rate of hybrid spectrum access strategy has been reported, e.g. [6]–[8], but there are still some unsolved issues. First, most existing work is based on the assumption that the channel power gain from the secondary transmitter to the primary receiver is perfectly estimated. Secondly, the impact of imperfect spectrum sensing is not considered. Thirdly, the quality of service (QoS) of the secondary link is ignored. These assumptions would not be so realistic in practice, because (i) it is usually difficult for the SU to gather full information of channel power gain from the secondary transmitter to the primary receiver since the non-collaboration between the PU and SU, and (ii) assuming the spectrum sensing to be performed perfectly is practically unrealistic and (iii) QoS guarantee of the secondary link is important for practical wireless applications.

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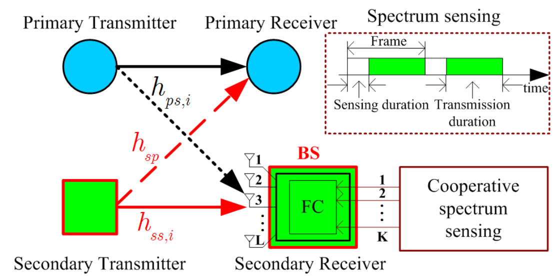

Fig. 1. Wireless scenario of this work.

In this letter, we present a hybrid strategy which enables the SU to switch between the two classical strategies according to the spectrum sensing results. If the PU is detected as inactive, the SU allocates the transmit power based on its own benefit (overlay). If the PU is active, the SU may transmit with a lower power to avoid generating harmful interference to the PU (underlay). The problem of SU's achievable transmission rate with this strategy is formulated as an optimization problem and derived by using Lagrange dual decomposition method. The impact of imperfect estimation of the interference channel's power gain is also discussed. To demonstrate the superiority of the hybrid strategy, for a given bit error rate (BER) target, achievable transmission rates of different strategies over Rayleigh fading channel are discussed and compared.

II. System Model and Assumptions

As shown in Fig. 1, we consider a wireless scenario which consists of one primary link and one micro cognitive radio network (CRN). The primary link includes one primary transmitter and one primary receiver. Considering a secondary transmitter in the CRN is trying to access the spectrum band which has been assigned to the primary link, and transmit data to its own base station (BS: the secondary receiver). The CR BS has L antennas, and we suppose that the maximal ratio combing (MRC) technique is applied. And h_{sp}, $h_{ss,i}$, $h_{ps,i}$ denote the instantaneous channel power gains from the secondary transmitter to the primary receiver, from the secondary transmitter to the i^{th} branch of the secondary receiver, from the primary transmitter to the i^{th} branch of the secondary receiver, respectively. We assume that all links are flat fading channels, and the power gains of all channels are independent and identically distributed (i.i.d), following Rayleigh fading distribution with unit mean. As mentioned before, the secondary transmitter gathers partial information of h_{sp}, which is denoted as $\bar{h}_{sp}$. Thus the estimation error can be computed by $\Delta_{h_{sp}} = h_{sp} - \bar{h}_{sp}$, $\Delta_{h_{sp}}$ and $\bar{h}_{sp}$ follow Rayleigh distribution with means e and $(1-e)$, respectively. The noise of all channels is assumed to be the AWGN with zero mean and variance σ_n^2.

Cooperative spectrum sensing among K secondary users is employed. Spectrum sensing is performed periodically every T milliseconds, as shown in Fig. 1, t milliseconds for spectrum sensing and the remaining $(T-t)$ milliseconds for data transmission. If the received signal is sampled with f_s, the total samples for spectrum sensing in each frame is $N = f_s t$. If energy detection is applied, thus when N is large enough, the probability of detection $P_{d,j}$ and the probability of false alarm $P_{f,j}$ for the j^{th} SU in the CRN are given as follows [9]:

$$P_{d,j} = Q\left(\left(\frac{\kappa}{\sigma_n^2} - \gamma_j - 1\right)\sqrt{\frac{N}{2\gamma_j+1}}\right) \tag{1}$$

$$P_{f,j} = Q\left(\left(\frac{\kappa}{\sigma_n^2} - 1\right)\sqrt{N}\right) \tag{2}$$

where γ_j is the received signal-to-noise ratio (SNR) of the PU signal, κ is the detection threshold, and $Q(\cdot)$ is the complementary distribution function of the standard Gaussian which is defined as $Q(x) = (1/2\pi)\int_x^{+\infty} e^{(-(t^2/2))}dt$. For a target $P_{d,j}$, the corresponding $P_{f,j}$ can be obtained by $P_{f,j} = Q(\sqrt{2\gamma_j+1}Q^{-1}(P_{d,j}) + \sqrt{N}\gamma_j)$. On the other hand, for a target $p_{f,j}$, the corresponding $P_{d,j}$ can be denoted as $P_{d,j} = Q(\frac{1}{\sqrt{2\gamma_j+1}}(Q^{-1}(P_{f,j}) - \sqrt{N}\gamma_j))$. Once the detection decisions have been made by each SU, they would be sent to the BS (FC: Fusion Center) to make a final decision. The probability of detection (P_d) and probability of false alarm (P_f) of the final decision with *logical-OR fusion rule* are given by $P_d = 1 - \prod_{j=1}^{K}(1-P_{d,j})$ and $P_f = 1 - \prod_{j=1}^{K}(1-P_{f,j})$. For the *logical-AND fusion rule*, these two probabilities can be expressed as $P_d = \prod_{j=1}^{K} P_{d,j}$ and $P_f = \prod_{j=1}^{K} P_{f,j}$. It is easy to know that the probabilities of miss detection and no false alarm are $(1-P_d)$ and $(1-P_f)$, respectively.

III. Problem Formulation

Based on the system model and assumptions, we assume perfect synchronization and symbol timing recovery are achieved, thus the SNR of the secondary receiver with L MRC diversity branches is given by

$$\gamma_s = \begin{cases} \sum_{i=1}^{L} \frac{P_{T,(H_0,H_1)} h_{ss,i}}{n_{ss,i} + P_p h_{ps,i}} & \text{if PU is present} \\ \sum_{i=1}^{L} \frac{P_{T,(H_0,H_1)} h_{ss,i}}{n_{ss,i}} & \text{if PU is absent} \end{cases} \tag{3}$$

where $n_{ss,i}$ is the noise power of the i_{th} branch of the secondary link, P_p is the transmit power of the primary transmitter, thus the interference power of the i_{th} branch can be denoted as $P_p h_{ps,i}$($h_{ps,i}$ is not needed to know in practical applications), and $P_{T,(H_0,H_1)}$ is the transmit power of the secondary transmitter. If the PU is detected to be active, the secondary transmitter transmits with power P_{T,H_1}; otherwise the secondary transmitter transmits with power P_{T,H_0}. We suppose that M-QAM signal constellation is employed in the CRN, thus the BER bound of M-QAM is given as follow [1]:

$$P_{err} \le 0.2e^{-1.5\gamma_s/(M-1)} \tag{4}$$

where P_{err} is the instantaneous BER of the secondary link and M is the modulation level, respectively. Thus the transmission rate of the SU R_b can be determined by $R_b = R_s \log_2(M)$, where R_s denotes the symbol rate. From equation (4), for a given BER target P_{err}, the optimal modulation level of the secondary transmitter is

$$M = 1 + G\gamma_s \tag{5}$$

where $G = -\frac{1.5}{\ln(5P_{err})}$. Therefore the achievable transmission rate of the SU can be written as:

$$R_b = (1 - \frac{t}{T})\{\mathbb{E}[\log_2(1 + G\gamma_s)]\} \tag{6}$$

where $\mathbb{E}(\cdot)$ is the expectation operator. From equation (3) we can see that γ_s has four different values under different conditions. We assume that the real active probability of the PU is P_{H1}, and P_{H0} denotes the probability when PU is inactive $(P_{H0} + P_{H1} = 1)$. Based on the aforementioned discussions, (6) can be rewritten as:

$$\begin{aligned} R_b = (1 - \frac{t}{T})\{\mathbb{E}[&P_{H1}P_dR_d + P_{H1}(1-P_d)R_{md} + \\ &P_{H0}P_fR_f + P_{H0}(1-P_f)R_{nf}]\} \end{aligned} \tag{7}$$

where $R_d = \log_2(1 + G\sum_{i=1}^{L}\frac{P_{T,H_1}h_{ss,i}}{n_{ss,i}+P_ph_{ps,i}})$, $R_{md} = \log_2(1 + G\sum_{i=1}^{L}\frac{P_{T,H_0}h_{ss,i}}{n_{ss,i}+P_ph_{ps,i}})$, $R_f = \log_2(1 + G\sum_{i=1}^{L}\frac{P_{T,H_1}h_{ss,i}}{n_{ss,i}})$ and $R_{nf} = \log_2(1 + G\sum_{i=1}^{L}\frac{P_{T,H_0}h_{ss,i}}{n_{ss,i}})$. In practice, P_d is usually closer to 1. Meanwhile, P_f is controlled to be low, i.e., usually less than 0.1. Therefore the items in (7) including $(1-P_d)$ and P_f are relatively small. Moreover, we assume that P_{H1} is small, e.g., less than 0.4. Thus the achievable transmission rate in (7) can be approximated to

$$R_b = (1 - \frac{t}{T})\{\mathbb{E}[P_{H1}P_dR_d + P_{H_0}(1-P_f)R_{nf}]\} \tag{8}$$

In practical implementation, transmit power of the secondary transmitter would be limited according to the operation range of the power amplifiers. Thus we impose a peak transmit power limit at the secondary transmitter which is given by

$$P_{T,(H_0,H_1)} \le P_{\max} \tag{9}$$

where $P_{\max}$ denotes the maximum available transmit power of the secondary transmitter. To protect the PU's normal transmission, we also impose a constraint on the average interference power limited at the primary receiver. The constraint is denoted as $\mathbb{E}[h_{sp}P_{T,H_0}(1-P_d)+h_{sp}P_{T,H_1}P_d] \leq I_{\max}$, and it can be further reduced as follow since P_d is close to 1:

$$\mathbb{E}\left[h_{sp}P_{T,H_1}P_d\right] \leq I_{\max} \tag{10}$$

where $I_{\max}$ is the maximum average interference power that the primary receiver can tolerate. As the perfect information of h_{sp} is unavailable, we have $\mathbb{E}\left[h_{sp}P^*_{T,H_1}P_d\right] = \mathbb{E}\left[(\bar{h}_{sp}+\Delta_{h_{sp}})P^*_{T,H_1}P_d\right] \leq I_{\max}$, in this case, the constraint in (10) can be rewritten as

$$\mathbb{E}\left[\bar{h}_{sp}P^*_{T,H_1}P_d\right] \leq I_{\max} - \mathbb{E}\left[\Delta_{h_{sp}}P^*_{T,H_1}P_d\right] \tag{11}$$

where P^*_{T,H_1} denotes the transmit power in this condition. Therefore the achievable transmission rate of the SU can be obtained by solving the problem $\mathcal{O}_{R_b}$:

$$\mathcal{O}_{R_b}: \max_{\{P_{T,(H_0,H_1)}\geq 0,\ 0<t<T\}} R_b \tag{12}$$

$$\textit{s.t.: } (9) \text{ and } (10) \tag{13}$$

when perfect estimation of the h_{sp} is not available, (10) in (13) should be replaced with (11).

IV. Achievable Transmission Rate

Directly solving the problem $\mathcal{O}_{R_b}$ is difficult since it is not a convex optimization problem. P_d and P_f are related to t by a nonlinear function, i.e., Q function. But when t is given at first, P_d and P_f can be determined thus the objective function in (12) is a concave function with respect to $P_{T,H0}$ and $P_{T,H1}$. Thus it is easy to show that this problem can be evaluated by solving the two following optimization problems:

$$\mathcal{O}_1: \max_{\{P_{T,H_1}\geq 0\}} \mathbb{E}[R_d], \quad \textit{s.t.: } (9) \text{ and } (10) \tag{14}$$

$$\mathcal{O}_2: \max_{\{P_{T,H_0}\geq 0\}} \mathbb{E}[R_{nf}], \quad \textit{s.t.: } (9) \tag{15}$$

For a particular fading state $(h_{sp}, h_{ss,i}, h_{ps,i})$, the two problems can be easily shown equivalent to $\{\mathcal{O}_1{}' : \max\ R_d\}$ and $\{\mathcal{O}_2{}' : \max\ R_{nf}\}$. We note that $g(x) = \log_2(1+\frac{a}{b+x})$ is a convex function for $a \geq 0$, $b \geq 0$ and $x \geq 0$. Thus, the two problems $\mathcal{O}_1{}'$ and $\mathcal{O}_2{}'$ can be solved by applying convex optimization methods. In $\mathcal{O}_1{}'$, both transmit power and average interference power constraints are present, and P_{T,H_1} are coupled by them, we can relax the coupled constraints by using dual decomposition method [10]. By introducing the dual variables associated with these constraints, the dual function of problem $\mathcal{O}_1{}'$ with perfect h_{sp} can be written as:

$$\begin{aligned} L(P_{T,H_1},\lambda_1,\mu_1) = R_d - \lambda_1\{[h_{sp}P_{T,H_1}P_d] - I_{\max}\} \\ -\mu_1(P_{T,H_1}-P_{\max}) \end{aligned} \tag{16}$$

where λ_1 and μ_1 are the nonnegative dual variables. According to the Karush-Kuhn-Tuker (KKT) conditions, the optimal transmit power can be found by solving the equation $\partial L(P_{T,H_1},\lambda_1,\mu_1)/\partial P_{T,H_1} = 0$. Let $\Re_1 \triangleq \sum_{i=1}^{L} \frac{Gh_{ss,i}}{n_{ss,i}+P_p h_{ps,i}}$, thus we have

$$P_{T,H1} = \left(\frac{1}{\mu_1+\lambda_1 h_{sp}P_d} - \frac{1}{\Re_1}\right)^+ \tag{17}$$

where$(\cdot)^+$ stands for $\max(0,\cdot)$. If the estimated h_{sp} is imperfect, it can be easily obtained that the optimal transmit power P^*_{T,H_1} in this condition has the same structure as (17), except for the interference power constraint and the h_{sp} should be replaced with $\bar{h}_{sp}$. Following the similar way, let $\Re_2 \triangleq \sum_{i=1}^{L} \frac{Gh_{ss,i}}{n_{ss,i}}$, P_{T,H_0} also can be solved and given by

$$P_{T,H0} = \left(\frac{1}{\mu_0} - \frac{1}{\Re_2}\right)^+ \tag{18}$$

Once the optimal transmit power is computed, accordingly, the optimal modulation level also can be obtained by substituting $P_{T,(H_0,H_1)}$ into (5). Based on the constraints with which they are associated, μ_0, μ_1 and λ_1 can be obtained by using the bisection search method as [11]. Finally, the corresponding achievable transmission rate is computed according to (8).

V. Simulation Results

To sufficiently protect the normal transmission activities of the primary link, when $\gamma_j = -15$ dB, in the first case, we set the target detection probability of each SU in the CRN to be $P_{d,j} = 0.9$ (**TPD** case: target probability of detection case); while in the second case, the target probability of false alarm of each SU is set to be $P_{f,j} = 0.05$ (**TPF** case: target probability of false alarm case). Other simulation parameters are set as follows: $T = 50$ ms, $n_{ss,i} = 1$ dBm, $f_s = 12$ MHz, $R_s = 6$ Mbps, $P_p = 10$ dBm and $P_{H1} = 0.4$. All the curves in the figures are averaged from 20000 independent trials.

Fig. 2 shows achievable transmission rates of the SU with different strategies under constraints on average interference power at the primary receiver and peak transmit power at the secondary transmitter in TPD and TPF cases. It is clear that the transmission rate increases with the rise of peak transmit power of the secondary transmitter and interference power tolerance of the primary receiver when hybrid strategy is employed. For overlay access strategy, the transmission rate only varies with peak transmit power constraint, because it is allowed to transmit only at the moment when the PU is inactive. In underlay access strategy, the interference power constraint is always active regardless of the PU's presence or absence. Thus the interference power at the primary receiver is the dominant constraint in this condition. From the figure we can see that the SU's transmission rate is also increased when the $I_{\max}$ becomes larger but it doesn't vary with the peak transmit power constraint. Compared with the two classical strategies, the hybrid strategy achieves the highest transmission rate both in TPD and TPF cases, which proves its superiority to the traditional ones. We also consider the case when the SU is with perfect spectrum sensing. In this case, $P_d = 1$, $P_f = 0$ and $t \approx 0$. Obviously, the rate is improved, and it gives the maximum rate of the secondary link can achieve in perfect condition, though this assumption is practically unrealistic.

From Fig. 3, we can see that there is an optimal sensing duration for each scenario. And the transmission rate of the SU is improved with the increasing of the diversity branches. This is reasonable, because diversity combing technique combines signals received over several antenna branches would improve the SNR of the signal received. Compared with logical-OR fusion rule, lower transmission rate is achieved by logical-AND rule when the sensing duration is large enough. But

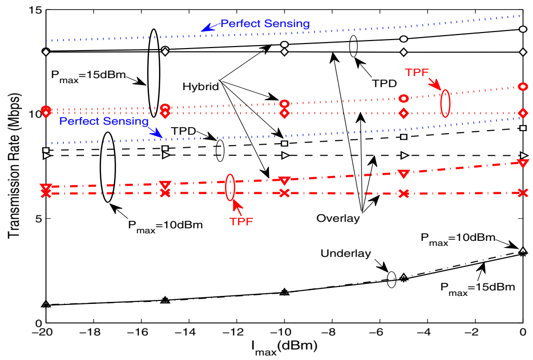

Fig. 2. Transmission rates of different strategies with $L = 2$, $K = 5$, $t = 2$ ms, $P_{err} = 10^{-3}$, and logical-OR fusion rule is employed.

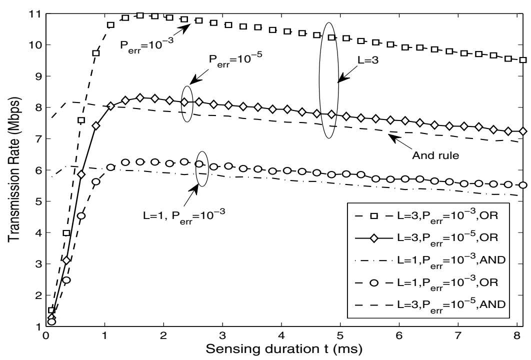

Fig. 3. Transmission rates of the hybrid strategy versus t under different conditions with $K = 5$, $P_{\max} = 10$ dBm and $I_{\max} = -5$ dBm, TPD case.

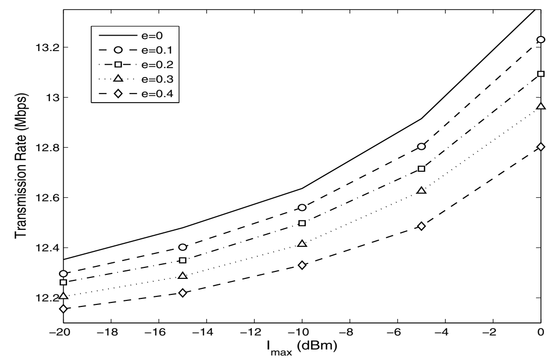

Fig. 4. Transmission rates of the hybrid strategy with imperfect channel power gain estimations, where $K = 5$, $t = 2$ ms, $L = 3$, $P_{\max} = 15$ dBm and $P_{err} = 10^{-5}$, logical-OR rule is applied, TPD case.

when the sensing duration is small, better transmission performance is obtained by the logical-AND rule. This is because the sensing duration affects both the P_d and P_f of the whole system. Therefore, according to (8), transmission rate of the SU varies with the sensing duration when different fusion rules are employed. In addition, the transmission performance is also affected by the BER target. If a lower BER is required, lower transmission rate would be achieved. Under the same constraint, to guarantee better BER performance of the secondary link, lower modulation level would be selected. Thus the transmission rate is decreased.

Fig. 4 demonstrates the impact of interference channel power gain estimation error to achievable transmission rate versus different $I_{\max}$. From the figure, it is obvious that the transmission rate decreases as e increases. From equation (11), larger e always makes the interference power constraint be more stricter than no estimation error.

VI. Conclusion

A spectrum access strategy which is different from traditional underlay and overlay strategies and referred to as hybrid strategy over Rayleigh fading channel is presented. Achievable transmission rate of the SU with this strategy is studied when consider the constraints not only on transmit power at the secondary transmitter and interference power at the primary receiver, but also the BER requirement of the secondary link. When diversity combing technique is available at the secondary receiver, benefiting much from cooperative spectrum sensing, the rate of the SU with hybrid strategy is significantly improved when compared with traditional ones. Simulations also show that an optimum sensing duration exists in such a strategy. In addition, the estimation errors of interference channel power gains would decrease the transmission performance of the secondary link.

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