An Estimation of Primary User's SNR for Spectrum Sensing in Cognitive Radios

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Abstract- The accuracy for any spectrum sensing technique in cognitive radio (CR) is affected by the transmission channel parameters such as the signal to noise ratio (SNR). In this paper, a special approach is used to estimate the SNR in the received primary user (PU) signal. This approach is based on using filter bank transform which is called hybrid slantlet transform (HST). This transform decomposes the input PU signal into some sub-bands where some of these sub-bands give the approximate signal power with low noise affects and the others give the approximate noise power. The results for this approach are compared with iterated discrete wavelet transform (DWT) approach. Moreover, these results are shown the superior for HST approach in comparison with iterated DWT approach in reduction of the system complexity computational that it is reduced by 9.75%.

Keywords- Spectrum Sensing, Primary User, Hybrid Slantlet Transform, Signal to Noise Ratio Estimation, Computational Complexity.

I. INTRODUCTION

Wireless communication systems and networks are currently working based on static spectrum allocation to the service providers and their costumers for exclusive use on a long-term basis and over large geographical areas. The exclusive radio frequency (RF) spectrum allocation was an efficient method for interference alleviation among adjacent bands [1]. However, the fixed spectrum allocation causes inefficient use of spectrum, since most of the channels actively transmit the information only for short period while a certain part of the spectrum is unused when and where the licensed users are off. Therefore, the lack of RF spectrum is not a result of scarcity of spectrum but a result of wasteful fixed spectrum assignments [2]. In order to mitigate the problem of the lack of RF spectrum, cognitive radio (CR) has been proposed to ease the spectrum sharing to increase the spectrum efficiency [3]. Therefore, the larger density of wireless employers can be accommodated without needing for a new RF spectrum

In cognitive radio networks, it can be noted that there are two types of users, the primary users (PUs) who are the licensed frequency band users and the secondary users (SUs) who are the users that want to use the frequency band of the PUs [4]. Secondary users perform the sensing

of the spectrum and the sensed information is analyzed to make correct decision in timely and accurate manner [5].

Many techniques have been proposed to detect whether the PU is on or off, such as energy detection (ED), matched filtering detection (MFD) and cyclostationary feature detection (CFD). Energy detection can detect any type of PU signal including known and unknown when the signal to noise ratio (SNR) is high enough [6], [7]. Matched filtering is known as the optimum method in the additive white Gaussian noise (AWGN) channel when the transmitted PU signal is known by the PU [8]. Cyclostationary feature detector can detect any PU signal which has cyclostationary period feature [9].

The most important features that should be available in any spectrum sensing technique are accuracy, flexibility, low complexity, high speed, low power consumption, and good detection performance at very low SNR. Therefore, the energy detection is the most popular method for spectrum sensing because of its low computational and implementation complexity. However, the energy detection is highly affected by the fluctuation of the noise power would lead to degrading of the detection performance. Some approaches have been proposed to alleviate the effect for the noise uncertainty on the performance of detection [10]. One of these techniques is based on the estimation of the noise power of the PU signal. Moreover, discrete wavelet packet transform (DWPT) has been used to estimate the noise power in the AWGN channel [11].

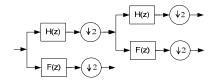
In this paper, a special approach is used to estimate the SNR that affects on the sensing performance with low computational complexity in comparison with Discrete Wavelet Transform (DWT). This approach likes DWT where it is based on using the filter bank transform to decompose the PU signal into some sub-bands and it is called hybrid slantlet transform (HST). Furthermore, this approach reduces the sensing time and power consumption of SU in comparison with DWT. Those reductions produce from the reduction for the number of operations that carry out between the PU's signal and HST filters.

The rest of this paper is organized as follows. A brief description and comparison for slantlet transform (ST) with DWT will be given in section II. In section III, HST based energy detection algorithm is presented. Section IV will include comparison between HST and DWT computational

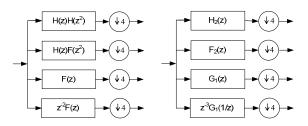
complexities. Simulation results and analysis are presented in section V. Conclusions are given in section VI.

II. SLANTLET TRANSFORM

Slantlet transform (ST) is a set of digital filters which includes low pass filter (LPF), band pass filter (BPF) and high pass filter (HPF). Moreover, it is an enhanced form of orthogonal iterated DWT using Daubechies 2 (D₂) coefficients [12]. ST can give improved time localization with shorter lengths of filters. Consequently, it is suitable to analyze and process the non-stationary signal with low computational complexity in comparison with iterated DWT [12], [13]. In spite of ST using filters with lengths shorter than DWT filters, it maintains on the characteristics of DWT filters design such as orthogonality, with a two zero moments. Fig. 1 shows two-levels of DWT, its equivalent structure (iterated), and ST filter banks.



(a) Two-level DWT filter bank.



- (b) Equivalent structure for (a).
- (c) Two-level ST filter bank.

Figure 1. Two-level filter bank for (a) DWT, (b) an equivalent structure (iterated) for (a), and (c) ST respectively [12].

In Fig. 1. (c), the filters coffecients for ST can be calculated by using [13, eqs. 1-6]. A comparison between ST and iterated DWT using D_2 in point of filters' coffecients values at each sample and filters' response with respect to radian freaquency is explained in Fig. 2.

From Fig. 2, it is clear that the filters $H_2(z)$, $F_2(z)$, and $G_1(z)+z^{-3}G_1(1/z)$ are LPF, BPF, and HPF respectively. The LPF and BPF separate the low and middle components of the input signal frequency respectively, whereas the HPF separates the high frequency components for the same signal. Moreover, the coefficients that are given by LPF are called the scaling (approximation) coefficients, but the coefficients that are given by BPF and HPF are called wavelet (details) coefficients. The scaling coefficients

represent the approximated signal information with low noise effects. On the other hand, the wavelet coefficients represent the approximated noise information that is inculed in the signal [14]. In this paper, the wavelet coefficients will be called the hybrid slantlet coefficients because hybrid slantlet trnasform (HST) is used instead of DWT.

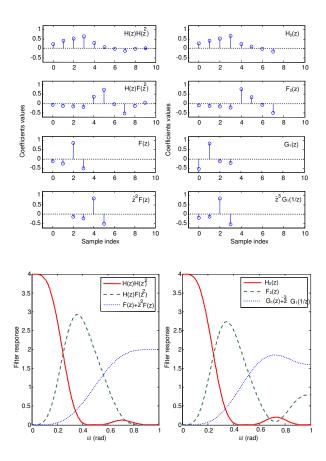


Figure 2. Comparison of two-level iterated DWT filter bank using Daubechies 2 (D_2) coefficients (left-hand side) and two-level ST filter bank (right-hand side) [12].

III. SYSTEM MODEL

In the energy detector, the input BPF limits the band of the input PU signal. Consequently, the received PU signal at the output of the sampler is given by:

$$y(n) = as(n) + w(n)$$
 $n = 0,1,...,N-1$ (1)

where a is 1 when the PU is on and 0 when the PU is off, s(n) is the PU signal to be sensed with zero mean and variance of σ_s^2 , w(n) is the AWGN with zero mean and variance σ_w^2 , and N is the number of samples.

Then, the received signal is squaring and averaging, and the energy is compared with a specific threshold. When the energy of the signal is greater than the threshold, the PU signal is present. Otherwise the PU signal is considered to be absent. In the sensing model that is used in this paper, the signal and noise samples are modelled as Gaussian random process.

At very large N and by using the central limit theorem (CLT), the decision statistic has Normal distribution. When there is no PU signal, the mean and variance for decision statistic are $N\sigma_w^2$ and $2N\sigma_w^2$ respectively. But, if there is PU signal, the mean and variance will be $N(\sigma_w^2 + \sigma_s^2)$ and $2N(\sigma_w^2 + \sigma_s^2)^2$ respectively. Thus, the detection probability (P_d) and false alarm probability (P_f) , that give the spectrum sensing performance, can be evaluated by [15]:

$$P_f = \mathbb{Q}\left(\frac{\lambda - N}{\sqrt{2N}}\right) \tag{2}$$

$$P_{d} = \mathbb{Q}\left(\frac{\lambda - N(1+\gamma)}{\sqrt{2N(1+\gamma)}}\right) \tag{3}$$

where $\mathbb{Q}(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt$, λ is the decision threshold and γ is the signal to noise ratio.

From (2), the decision threshold can be calculated in terms of P_f and N:

$$\lambda = N + \sqrt{2N} \, \mathbb{Q}^{-1}(P_f) \tag{4}$$

By using the decision threshold value that calculated by (4), the P_d under estimated signal to noise ratio, will be:

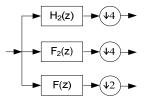
$$P_d = \mathbb{Q}\left(\frac{\lambda - N(1 + \gamma^*)}{\sqrt{2N(1 + \gamma^*)}}\right) \tag{5}$$

where γ^* is the estimated SNR using HST ($\gamma^* = \frac{\sigma_S^{2^*}}{\sigma_{co}^{2^*}}$).

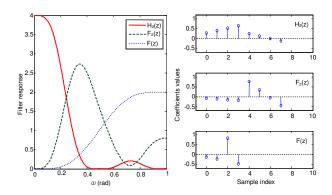
In this work, hybrid slantlet transform (HST) has been used by the SU to estimate the PU's SNR. HST is a mixed transform between DWT and ST where the LPF and BPF are the same for that are used in ST. Moreover, the HPF is the same one that is used in DWT. Fig. 3 shows the filters response and filters samples for HST.

If the received PU signal is a periodic signal over period T and by using its HST representation, the received PU signal's power can be calculated in time domain as in (6) or by using it is HST representation as in (7) after further simplifications.

$$P = \frac{1}{T} \int_{0}^{T} y^2(t)dt \tag{6}$$



(a) Two-level HST filter bank.



- (b) Filters response versus frequency.
- (c) Coefficients values versus sample index.

Figure 3. Two-level HST: (a) filter bank, (b) filters reponses versus angular frequency, and coefficients values versus order of the samples.

$$P = \frac{1}{T} \left[\sum_{k} c_{j_0,k}^2 + \sum_{j \ge j_0} \sum_{k} s_{j,k}^2 \right]$$
 (7)

where j, j_0 , and k are number of decomposition levels, first decomposition level and order of the sub-band respectively. Furthermore, $\mathbb{C}^2_{j_0,k}$, and $\mathbb{S}^2_{j,k}$ scaling coefficients and hybrid slantlet coefficients respectively [12].

IV. COMPLEXITY COMPARISON BETWEEN HST AND ITERATED DWT

As mentioned in section II, the ST has filters with shorter lengths in comparison with iterated DWT filters. As a result, the total number of real multiplications and additions between the PU signal and HST filters is less than that between iterated DWT filters and PU signal. This leads to reduce the SU's system complexity, sensing time and handset power consumption.

The computational complexity depends on the number of the operations that carries out between the input signal and analyzed filters. In another words, the number of the multiplications and additions for the convolution process between the signal and the filters.

The computational complexity for iterated DWT can be calculated by (8) and (9):

$$C_{DWT} = M_{DWT} + A_{DWT} \tag{8}$$

$$C_{DWT} = N(6(2^{i}) - 5) - 3(2^{i} - 1) + \sum_{k=1}^{i} (N((6)2^{k} - 5) - 3(2^{k} - 1))$$
 (9)

where C_{DWT} , M_{DWT} , and A_{DWT} are the total computational complexity, total number of multiplications, and total number of additions respectively for the iterated DWT, and i is the number of analysis levels for the input signal.

On the other hand, the computational complexity for HST can be calculated by (10) and (11):

$$C_{HST} = M_{HST} + A_{HST} \tag{10}$$

$$C_{HST} = 2^{i+1}(2N-1) - N + 1 + \sum_{k=1}^{i} (2^{k+1}(2N-1) - N + 1)$$
 (11)

where C_{HST} , M_{HST} , and A_{HST} are total computational complexity, total number of multiplications, total number of additions respectively for the HST.

In order to make the two transforms with the same computational complexity, the number of input signal samples or the number of decomposition levels in HST should be increased. This will be explained in (12) and (13):

$$C_{HST} = C_{DWT} \tag{12}$$

For i = 2 and after some mathematical analysis, the number of signal samples in HST that gives approximately the same computational complexity for DWT is:

$$N_{HST} = \left[\frac{45N_{DWT} - 4}{37} \right] \tag{13}$$

where N_{HST} and N_{DWT} are the number of samples for HST and iterated DWT respectively. The function $\lfloor x \rfloor$ gives the smallest integer number that is less than or equal to x.

V. SIMULATION RESULTS AND ANALYSIS

In this section, the simulation results will be given and analyzed for the proposed spectrum sensing technique. In addition, the proposed approach is compared with another approach has been designed by using DWT.

The input PU signal is chosen as QPSK (quadrature phase shift keying) signal with a different number of samples (1024, 2048 and 4096) and carrier frequency 3 kHz. The sampling frequency (f_s) is equal to 62.5 kHz which is a typical FFT (fast Fourier transform) bin resolution of an experimental energy detection implementation [16]. Moreover, two-level of

decomposition for the PU signal has been used for both HST and iterated DWT.

Fig. 4 shows the comparison of the detection probability for the proposed approach which is based on HST with another approach based on iterated DWT under the same number of input signal samples. As shown in Fig. 4, the probabilities of detection are approximately equal for both approaches and for the different cases of signal samples' number. However, the computational complexity for HST is less than the computational complexity for the iterated DWT. Moreover, when N=1024, the computational complexities for the iterated DWT and HST are 46059 and 37871 operations respectively. In this paper, we have used a large number of samples to make the difference in the complexity is clear.

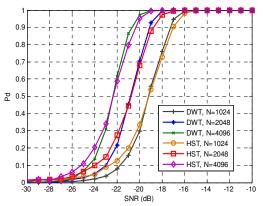


Figure 4. Comparison of the detection probability for the HST and iterated DWT under the same number of samples and for different N.

Fig. 5 explains the comparison of the detection probability for the proposed HST approach and iterated DWT approach under the same computational complexity. From this figure, it is clear the superiority of the HST over the iterated DWT for the spectrum sensing by depending on the estimation of the PU's SNR.

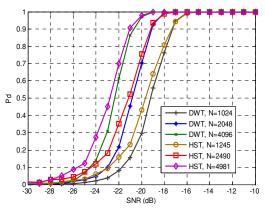


Figure 5. Comparison of the detection probability for the HST and iterated DWT under the same complexity and for different *N*.

The constant false alarm rate (CFAR) principle has been used in Fig. 4 and Fig. 5 to find the decision threshold value where P_t has been set to 0.01.

Fig. 6 and Fig. 7 illustrate the comparison of the detection probability for the proposed HST approach with the iterated DWT approach under the same number of samples and the same complexity respectively. In Fig. 6, the number of samples is set to 1024 for both approaches, but in Fig. 7 the number of samples are 1024 and 1245 for the HST and iterated DWT respectively. Moreover, different values of the P_f have been chosen to show the detection performance.

In Fig. 6, the detection performance for both approaches is nearly equal, but the computational complexity for the HST is less than the computational complexity for the iterated DWT. However, the HST detection performance is better than the iterated DWT detection performance when the comparison is carried out under the same complexity as shown in Fig. 7.

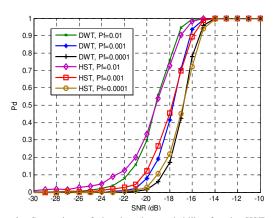


Figure 6. Comparison of the detection probability for the HST and iterated DWT under the same number of samples and for different P_f .

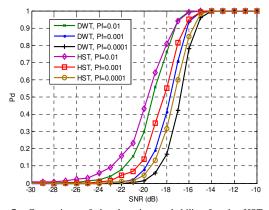


Figure 7. Comparison of the detection probability for the HST and iterated DWT under the same complexity and for different P_f .

Fig. 8 shows the computational complexities for iterated DWT and HST filter banks versus the number of signal samples. From this figure, it's clear the difference in the number of operations between the DWT and HST where for i=2 and N=4096, $C_{SHT}=151535$ and $C_{DWT}=184299$ respectively. Therefore, 32764 operations is the number of the operations which is reduced by HST.

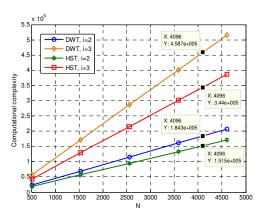


Figure 8. Comparison of the computational complexity for the HST and iterated DWT filter bank versus the number of input signal samples.

VI. CONCLUSIONS

In this paper, hybrid slantlet transform is proposed to estimate the primary user's SNR to enhance the detection performance of the cognitive radio user. A simulation results have been carried out to find the detection probability for different cases of N and P_f . Moreover, the results showed the superiority of the HST over the iterated DWT in detection performance if the comparison is done under the same computational complexity. However, the performance of detection is approximately same if the comparison is done under the same number of input signal samples. But, in this case, the computational complexity for the HST is less than the computational complexity of the iterated DWT. Further improvement for the SU detection performance can be done by reducing the noise impacts which is estimated by our proposed approach.

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