Sensing-Throughput Tradeoff for Cognitive Radio as Interweave System~~s~~: A Deployment-Centric Viewpoint

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Abstract

Cognitive Radio is envisaged as one of potential candidates that addresses the issue of spectrum scarcity. Secondary access to the licensed spectrum is viable only if the interference is avoided at the primary system. In this regard, different paradigms have been conceptualized in the exisiting literature. Of these, Interweave Systems (ISs) that employ spectrum sensing have been widely investigated. This makes performance characterization of a cognitive radio system critical, hence, an interesting research problem. Baseline models investigated in the literature characterize the performance of IS in terms of sensing-throughput tradeoff, however, this characterization channels' knowledge at secondary transmitter, in practice, this knowledge is unavailable. As a result, these models depicts an ideal scenario, thereby making the performance characterization based on these models disputable and impractical for hardware deployment. Motivated by this fact, we establish a novel model that incorporates channel estimation in the system model. Based on the proposed model, this work investigates the impact of estimation on the performance of IS. More particularly, the variation induced in the probability of detection affects the performance of the detector, that may result in severe interference at the primary users. To capture this effect, we propose to employ two constraints namely, average and outage constraints, in order to capture the performance of the IS while considering the variation induced due to the imperfect channel

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estimation. Our analysis reveals that the ideal scenario considerably overestimates the performance of the IS.

I. Introduction

We are currently in the phase of conceptualizing the requirements of the fifth-generation (5G) of wireless standards. One of the major requirements includes the improvement in the areal capacity (bits/s/m^2) by a factor of 1000 [2]. A large contribution of this demand is procured by means of an extension to the existing spectrum. Due to the static allocation of the spectrum specially below 6 GHz, which is appropriate for mobile communications, it is on the verge of scarcity. However, it is possible to overcome this scarcity, provided it is utilized efficiently. In this perspective, Cognitive Radio (CR) is foreseen as one of the viable solution that addresses the problem of spectrum scarcity. Over the past one and a half decade, the notion of CR has evolved at a tremendous pace right from its origin by Mitola *et al.* in 1999 [3], consequently, has acquired certain maturity. However, from a deployment perspective, this technology is still in its preliminary phase. In this view, it is imperative to make substantial efforts that enable disposition of this concept over a hardware platform.

An access to the Primary User (PU) spectrum is an outcome to the paradigm employed by the Secondary User (SU). Based on the paradigms described in the literature, all cognitive radio systems that deal dynamic access to the spectrum fall mainly under three categories namely, interweave, underlay and overlay systems [4]. As Interweave Systems (IS), the SUs render an interference-free access to the PU by exploiting spectral holes in different domains such as time, frequency, space and polarization existing in the licensed spectrum, whereas underlay systems enable an interference-tolerant access under which the SUs are allowed to use the licensed spectrum as long as they respect the interference constraints of the Primary Receivers (PRs). Besides that, overlay systems consider participation of higher layers for enabling spectral coexistence between two or more wireless networks. Due to its ease in deployment, IS is mostly preferred for performing analysis among these paradigms. Motivated by these facts, this paper focuses on the performance analysis of the ISs from a deployment perspective.

Motivation and Related Work

Spectrum sensing is an integral part of the IS. At the Secondary Transmitter (ST), sensing is necessary for detecting the presence and absence of a primary signal, thereby protecting the PRs against harmful interference. Sensing at the ST is accomplished by listening to the signal transmitted by the PT. For detecting a primary signal, several techniques such as energy detection, matched filtering, cyclostationary and feature-based detection exist [5], [6]. Because of its versatility towards unknown primary signals and its low computational as well as deployment complexity requirement, energy detection has been extensively investigated in the literature [7]–[11]. According to energy detection, the decision is accomplished by comparing the power received at the ST to a threshold. In reality, the ST encounters variation in the received power due to existence of thermal noise at the receiver and fading in the channel. This leads to sensing errors described as misdetection or false alarm, thus, limit the performance of the IS. In order to determine the performance of the detector, it is essential to characterize the expressions of probability of detection and probability of false alarm.

In particular, probability of detection is critical for the primary system because it precludes the PR from the interference induced by the ST. As a result, sustaining a target of probability of detection is of paramount importance to the secondary system [12]. Therefore, the characterization of the probability of detection is a prime requirement. In this context, Urkowitz [7] introduced a probabilistic model that establishes a fundamental framework for characterizing the sensing errors, however, the characterization accounted only noise in the system. Besides noise, fading in the channel causes further variation in the received power. To encounter this variation, mainly two approaches have been investigated [8]–[10], [13]–[15].

The most prevalent approach involves the application of a fading model, whereby the system averages out the variation in the received power. Subject to the deployment scenario[1], different fading models – Rice, Rayleigh, Nakagami-m and Log-normal can be employed [16]. The analytical expressions for the expected probability of detection for these fading models are characterized in [8]–[10]. Considering a cognitive radio system, this approach has some major drawbacks. Fading models depict a long-term characterization of the system, however, short-term interference that may lead to outage is ignored, thereby deteriorating the performance of the primary system. Moreover, fading models are specific to the deployment scenario, hence, the knowledge of the fading model is necessary while designing a CR system. Apart from that, every fading model incurs certain model parameters. As a result, estimation of these parameters at the ST [17], particularly during the initial phase of the deployment, depicts an additional overhead. With this, a certain delay is imposed in the secondary system. In practice, due to mobility, most systems are likely to violate stationarity over a large duration. Thus, it becomes challenging to track these model parameters. As a consequence, these drawbacks constrict the applicability of this approach to practical cognitive radio systems.

[1]These scenarios include urban, suburban and rural or indoor and outdoor from a different perspective.

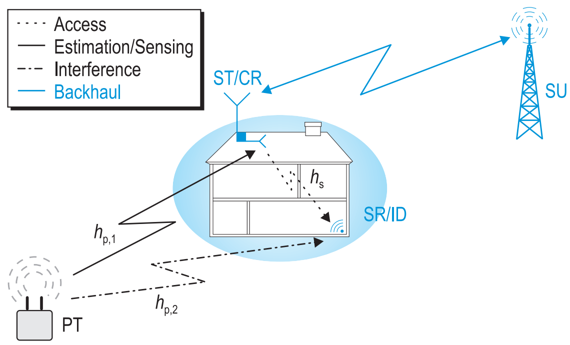


Fig. 1. A scenario demonstrating the interweave paradigm and the interacting channels.

To overcome these aforementioned drawbacks, an alternative approach that exploits the channel coherence in time domain is established in [13]–[15]. In this regard, a frame structure is introduced such that the channel is considered to remain constant over the frame duration, however, upon exceeding the frame duration, the system may witness a different realization of the channel, thus, the channel is regarded as unknown. In this way, we preclude the variations due to the channel, thereby eluding the deployment of the fading model and model complexities thereafter. As a consequence, this approach carries out a short-term performance characterization of the IS in which the parameters are optimized for a given frame. Motivated by these facts, we focus on the latter approach for performing analysis.

Problem Formulation

Recently, the performance characterization of cognitive radio systems in terms of sensing-throughput tradeoff has received significant attention [13], [15], [18], [19]. According to Liang *et al.* [13], the ST assures a reliable detection of primary signal, thereby sustaining the probability of detection above a desired level with an objective of optimizing the throughput at the Secondary Receiver (SR). In this way, the sensing-throughput tradeoff renders a suitable sensing time that achieves an optimum throughput for a given received power. However, to characterize the probability of detection and throughput, the system requires the knowledge of interacting channels, namely, a sensing channel, an access channel and an interference channel, cf. Fig. 1. Several contributions investigated in the literature assume this knowledge to be present at the ST. As a result, the existing solutions are considered idealistic and implausible for deployment. The knowledge of these channels is not available in reality, thus, needs to be estimated at the ST. Unlike the sensing channel, the access and interference channels have to be estimated at the SR and made available at the ST through a feedback channel.

Following the above discussion, it is apparent that received power, that entails the sensing channel, is crucial for characterizing the probability of detection, hence, is essential for evaluating the detector's performance. With unknown channel, it is reasonable to include received power estimation for each frame [1]. Now, inherent to the estimation process, a variation is induced in the probability of detection. In this sense, characterizing the performance of detector with received power estimation remains an open problem. Besides probability of detection, probability of false alarm largely accounts for the throughput attained by the secondary system at the SR. The characterization of the probability of false alarm requires the knowledge of noise power. Subject to a given uncertainty [20], this knowledge can be acquired through hardware calibration.

In order to breakthrough these bottlenecks, following stratergy is employed in the paper. Firstly, we consider received power estimation at the ST that allows us to constrain the probability of detection at a desired level. However, with the inclusion of estimation, the system anticipates: (i) a performance loss in terms of temporal resources used and (ii) a variation in the performance parameters due to imperfect estimation. A preliminary analysis of this performance loss was carried out in [1], where, it was revealed that at low signal to noise ratio regime, imperfect estimation of received power corresponds to large variation in probability of detection, hence, cause a severe degradation in the performance of the IS. However, the performance degradation was determined by means of lower and upper bounds. In this work, we consider a more exact analysis, whereby we capture the variation in probability of detection by characterizing its distribution function. Using this, we apply new probabilistic constraints on the probability of detection that allow IS to operate at low signal to noise ratio.

Besides that, we include channel estimation at the SR to acquire the knowledge of the access and interference channels. It is well-known that systems with transmitter information (such as matched filter, pilot symbols, modulation type and time-frequency synchronization) at the receiver acquire channel knowledge by listening to the pilot data sent by the ST [21]–[24]. Other systems, where the receiver possess either no access to this information or limited by hardware complexity, procure channel knowledge indirectly by estimating a different parameter, for instance, received signal strength or received power, that entails the channel knowledge. Recently, pilot based estimation [25], [26] and received power estimation [27] have been applied to obtain channel knowledge and depict the performance of cognitive radio systems. However, the analysis was restricted to underlay systems, where the emphasis was laid on modelling the interference at the PR. In this paper, we extend this notion to the IS, hence, employ pilot based estimation for the access channel and received power based estimation for the sensing and interference channels, cf. Fig. 1. Upon acquiring the knowledge of these channels at the ST, we depict the impact of imperfect channel estimation on the performance of the IS in terms of sensing-throughput tradeoff.

Contributions

The major contributions of the paper can be summarized as follows:

- The main goal of the paper is to establish a system model that constitutes channel estimation. With the inclusion of estimation, the system witnesses variation in the performance parameters and a certain performance loss. Based on the proposed model, this work investigates these two aspects and characterize the true performance of the IS.
- To capture the variation induced in the system, we characterize the distribution functions of performance parameters, such as, probability of detection and capacities at the SR. More importantly, we utilize the distribution function for the probability of detection to establish an average constraint or an outage constraint as new PU constraints on the probability of detection.
- Subject to the new constraints, we establish the expressions of sensing-throughput tradeoff to capture the variation and evaluate the performance loss in the IS.
- Finally, we depict a fundamental tradeoff between estimation time, sensing time and throughput. We exploit this tradeoff to determine a suitable estimation and sensing time that depicts an optimal performance of the IS.

Organization

The subsequent sections of the paper are organized as follows: Section II describes the system model that includes the interweave scenario, the signal model and performance characterization for the ideal scenario. Section III presents the proposed model that incorporates channel estimation and characterizes the distribution functions of the performance parameters. Section IV establishes the sensing-throughput tradeoff subject to average and outage constraints. Section V analyzes the numerical results based on the obtained expressions. Finally, Section VI concludes the paper.

II. System Model

Interweave scenario

Cognitive relay [28] characterizes a small cell deployment that fulfills the spectral requirements for indoor devices. Fig. 1 illustrates a snapshot of a cognitive relay scenario to depict the interaction between the cognitive relay with PT and indoor devices, where cognitive relay and indoor devices represents the ST and SR, respectively. In [28], the challenges involved while deploying the cognitive relay as an IS were presented. For simplification, a PU constraint based on probability of false alarm rate was considered in

TABLE I

DEFINITIONS OF ACRONYMS AND NOTATIONS USED

Acronyms and Notations	**Definitions**
AC, OC	Average Constraint, Outage Constraint
CR	Cognitive Radio
IM, EM	Ideal Model, Estimation Model
IS	Interweave System
PU - PT, PR	Primary User - Primary Transmitter, Primary Receiver
SU - ST, SR	Secondary User - Secondary Transmitter, Secondary Receiver
$\mathcal{H}_1, \mathcal{H}_0$	Signal plus noise hypothesis, noise only hypothesis
f_{s}	Sampling frequency
$\tau_{\mathrm{est}}, \tau_{\mathrm{sen}}$	Estimation time, sensing time interval
T	Frame duration
$\mathrm{P_d}, \mathrm{P_{fa}}$	Probability of detection, probability of false alarm
$\bar{\mathrm{P}}_{\mathrm{d}}$	Target probability of detection
κ	Outage Constraint over probability of detection
$h_{\mathrm{p,1}}, h_{\mathrm{p,2}}, h_{\mathrm{s}}$	Channel coefficient for the link PT-ST, PT-SR, ST-SR
$\gamma_{\mathrm{p,1}}, \gamma_{\mathrm{p,2}}, \gamma_{\mathrm{s}}$	Signal to noise ratio for the link PT-ST, PT-SR, ST-SR
R_{s}	Throughput at SR
$\mathrm{C}_0, \mathrm{C}_1$	Shanon capacity at SR without and with interference from PT
μ	Threshold for the energy detector
$F_{(\cdot)}$	Cumulative distribution function of random variable $(\cdot)$
$f_{(\cdot)}$	Probability density function of random variable $(\cdot)$
$\hat{(\cdot)}$	Estimated value of $(\cdot)$
$\tilde{(\cdot)}$	Optimum value of $(\cdot)$
$\mathbb{E}_{(\cdot)}$	Expectation with respect $(\cdot)$
$\mathbb{P}$	Probability measure
$\mathbf{T}(\cdot)$	Test statistics
$x_{(\cdot)}[n], y_{(\cdot)}[n]$	n^{th} sample of the transmitted discrete signal, received discrete signal at $(\cdot)$
$P_{\mathrm{Tx,}\,(\cdot)}, P_{\mathrm{Rx,}\,(\cdot)}$	Power transmitted, power received at $(\cdot)$
σ_x^2, σ_w^2	Signal variance at PT, noise variance at ST and SR
$\Gamma(\cdot)$	Gamma function
$\Gamma(\cdot,\cdot)$	Regularized incomplete upper Gamma function
$\Gamma^{-1}(\cdot,\cdot)$	Inverse of regularized incomplete upper Gamma function
$\mathcal{N}, \mathcal{X}^2, \mathcal{X}_1^2$	Normal, central chi-squared, non-central chi-squared distribution
N_{s}	Number of pilot symbols used for pilot based estimation at the SR for h_{s}
$N_{\mathrm{p,2}}$	Number of samples used for received power estimation at the SR for $h_{\mathrm{p,2}}$
a_1, b_1, a_2, b_2	Model parameters of Gamma distribution
λ	Non-centrality parameter of $\mathcal{X}_1^2$ distribution

the system model. With the purpose of improving system's reliability, we extend the analysis to employ a PU constraint on the probability of detection.

As a follow-on from the ideal model depicted in [13], we consider a slotted medium access for the IS, where the time axis is segmented into frames of length T. According to which, the ST employs periodic sensing, hence, each frame consists of a sensing slot τ_{sen} and the remaining duration $T - \tau_{\mathrm{sen}}$ is utilized for data transmission. For small T relative to the PU's expected ON/OFF period, the requirement of the ST to be in alignment to PUs' medium access can be relaxed [29]–[31].

Signal model

Subject to the underlying hypothesis that illustrates the presence $(\mathcal{H}_1)$ or absence $(\mathcal{H}_0)$ of primary signal, the discrete signal received at the ST is given by

$$y_{\mathrm{ST}}[n] = \begin{cases} h_{\mathrm{p},1} \cdot x_{\mathrm{PT}}[n] + w[n] & : \mathcal{H}_1 \\ w[n] & : \mathcal{H}_0 \end{cases}, \tag{1}$$

$x_{\mathrm{PT}}[n]$ corresponds to a discrete sample transmitted by the PT, $|h_{\mathrm{p},1}|^2$ represents the power gain of the sensing channel for a given frame and $w[n]$ is additive white Gaussian Noise (AWGN) at the ST. The primary signal $x_{\mathrm{PT}}[n]$ can be modelled as: (i) PSK modulated signal, or (ii) Gaussian signal. The signals that are prone to high inter symbol interference or entail precoding can be modelled as Gaussian signal [13]. For this paper, we focus our analysis on the latter case. As a result, the mean and variance for the signal and noise are determined as $\mathbb{E}\left[x_{\mathrm{PT}}[n]\right] = 0$, $\mathbb{E}\left[w[n]\right] = 0$, $\mathbb{E}\left[|x_{\mathrm{PT}}[n]|^2\right] = \sigma_x^2$ and $\mathbb{E}\left[|w[n]|^2\right] = \sigma_w^2$. The channel $h_{\mathrm{p},1}$ is considered to be independent to $x_{\mathrm{PT}}[n]$ and $w[n]$, thus, y_{ST} is also an i.i.d. random process.

Similar to (1), during data transmission, the discrete received signal at the SR conditioned over the probability of detection ($\mathrm{P_d}$) and probability of false alarm ($\mathrm{P_{fa}}$) is given by

$$y_{\mathrm{SR}}[n] = \begin{cases} h_{\mathrm{s}} \cdot x_{\mathrm{ST}}[n] + h_{\mathrm{p},2} \cdot x_{\mathrm{PT}}[n] + w[n] & : 1 - \mathrm{P_d} \\ h_{\mathrm{s}} \cdot x_{\mathrm{ST}}[n] + w[n] & : 1 - \mathrm{P_{fa}} \end{cases}, \tag{2}$$

where $x_{\mathrm{ST}}[n]$ corresponds to discrete signal transmitted by the ST. Further, $|h_{\mathrm{s}}|^2$ and $|h_{\mathrm{p},2}|^2$ represent the power gains for access and interference channels, cf. Fig. 1.

Sensing

Following the frame structure, ST performs sensing for a duration of τ_{sen}. The test statistics $\mathrm{T}(\mathbf{y})$ at the ST is evaluated as

$$\mathrm{T}(\mathbf{y}) = \frac{1}{\tau_{\text{sen}} f_{\text{s}}} \sum_{n}^{\tau_{\text{sen}} f_{\text{s}}} |y_{\text{ST}}[n]|^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \mu, \tag{3}$$

where μ is the threshold and $\mathbf{y}$ is a vector with $\tau_{\text{sen}} f_{\text{s}}$ samples. $\mathrm{T}(\mathbf{y})$ represents a random variable, whereby the characterization of the distribution function depends on the underlying hypothesis. Corresponding to $\mathcal{H}_0$ and $\mathcal{H}_1$, $\mathrm{T}(\mathbf{y})$ follows a central chi-squared ($\mathcal{X}^2$) distribution [32]. As a result, the probability of detection ($\mathrm{P_d}$) and the probability of false alarm ($\mathrm{P_{fa}}$) corresponding to (3) is determined as [20]

$$\mathrm{P_d}(\mu, \tau_{\text{sen}}, P_{\text{Rx,ST}}) = \Gamma\left(\frac{\tau_{\text{sen}} f_{\text{s}}}{2}, \frac{\tau_{\text{sen}} f_{\text{s}} \mu}{2 P_{\text{Rx,ST}}}\right), \tag{4}$$

$$\mathrm{P_{fa}}(\mu, \tau_{\text{sen}}) = \Gamma\left(\frac{\tau_{\text{sen}} f_{\text{s}}}{2}, \frac{\tau_{\text{sen}} f_{\text{s}} \mu}{2 \sigma_w^2}\right), \tag{5}$$

where $\Gamma(\cdot,\cdot)$ represents a regularized upper Gamma function [33].

Sensing-Throughput tradeoff

Following the characterization of $\mathrm{P_{fa}}$ and $\mathrm{P_d}$, Liang *et. al.* [13] established a tradeoff between the sensing time and secondary throughput (R_{s}) attained subject to a target probability of detection ($\bar{\mathrm{P}}_{\mathrm{d}}$). This tradeoff is represented as

$$\tilde{R}_{\text{s}}(\tilde{\tau}_{\text{sen}}) = \max_{\tau_{\text{sen}}} R_{\text{s}}(\tau_{\text{sen}}) = \frac{T - \tau_{\text{sen}}}{T}\Big[\mathrm{C}_0(1 - \mathrm{P_{fa}})\mathbb{P}(\mathcal{H}_0) + \mathrm{C}_1(1 - \mathrm{P_d})\mathbb{P}(\mathcal{H}_1)\Big], \tag{6}$$

$$\text{s.t. } \mathrm{P_d} \geq \bar{\mathrm{P}}_{\mathrm{d}}, \tag{7}$$

$$\text{where } \mathrm{C}_0 = \log_2\left(1 + |h_{\text{s}}|^2 \frac{P_{\text{Tx,ST}}}{\sigma_w^2}\right) = \log_2(1 + \gamma_{\text{s}}) \tag{8}$$

$$\text{and } \mathrm{C}_1 = \log_2\left(1 + \frac{|h_{\text{s}}|^2 P_{\text{Tx,ST}}}{|h_{\text{p,2}}|^2 P_{\text{Tx,PT}} + \sigma_w^2}\right) = \log_2\left(1 + \frac{|h_{\text{s}}|^2 P_{\text{Tx,ST}}}{P_{\text{Rx,SR}}}\right) = \log_2\left(1 + \frac{\gamma_{\text{s}}}{\gamma_{\text{p,2}} + 1}\right). \tag{9}$$

$\mathbb{P}(\mathcal{H}_0)$ and $\mathbb{P}(\mathcal{H}_1)$ are the probabilities of occurrence for the respective hypothesis, whereas $\gamma_{\text{p,2}}$ and γ_{s} correspond to signal to noise ratio for the links PT-SR and SR-SR, respectively. In other words, using (6), the ST determines a suitable sensing time $\tau_{\text{sen}} = \tilde{\tau}_{\text{sen}}$, such that the throughput is optimized ($\tilde{R}_{\text{s}}$) subject to a target probability of detection, cf. (7). From the deployment perspective, the tradeoff depicted above has fundamental issues:

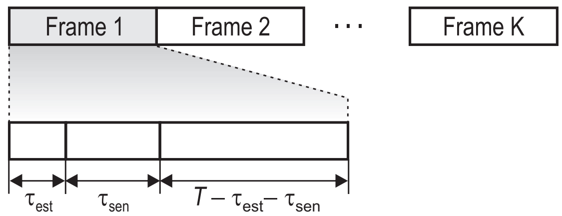


Fig. 2. Frame structure of interweave system with received power estimation.

- Without the knowledge of the received power (sensing channel), it is not feasible to characterize $\mathrm{P_d}$. This leaves the characterization of the throughput (6) impossible and the constraint defined in (7) inappropriate.
- Moreover, the knowledge of the interacting channels is required at the ST, cf. (8) and (9) for characterizing the throughput in terms of $\mathrm{C_0}$ and $\mathrm{C_1}$ at the SR.

With these issues, it is unreasonable to depict the performance of IS based on the ideal model, mentioned above. In the subsequent section, we propose an estimation model that complements these issues, thereby including the estimation of the sensing channel at the ST and interference and access channel at the SR. Based on the proposed model, we then investigate the performance of the IS.

III. Proposed Model

The inclusion of estimation of the interacting channels causes variation in the parameters $\mathrm{P_d}$, $\mathrm{C_0}$ and $\mathrm{C_1}$. Unless characterized, this variation may be hazardous for a hardware deployment. In this view, we include the estimation of the interacting channels in the system model, thereby characterizing the variation in $\mathrm{P_d}$, $\mathrm{C_0}$ and $\mathrm{C_1}$ by means of distribution functions. Equipped with these expressions, we finally obtain a characterization of sensing-throughput tradeoff. To include channel estimation, we propose a frame structure includes estimation τ_{est}, sensing τ_{sen} and data transmission $T - (\tau_{\mathrm{est}} + \tau_{\mathrm{sen}})$, where $\tau_{\mathrm{est}}, \tau_{\mathrm{sen}}$ correspond to time intervals and $\tau_{\mathrm{est}} + \tau_{\mathrm{sen}} < T$, cf. Fig. 2. In this regard, the following subsections consider the estimation of the interacting channels.

Estimation of sensing channel ($h_{p,1}$)

Following the previous discussions, the ST acquires the knowledge of $h_{\mathrm{p},1}$ by estimating its received power. The estimated received power is required for the characterization of $\mathrm{P_d}$, thereby evaluating the detector performance.

Under $\mathcal{H}_1$, the received power estimated during the estimation phase at the ST is given as [7]

$$\hat{P}_{\text{Rx,ST}} = \frac{1}{\tau_{\text{est}} f_{\text{s}}} \sum_{n}^{\tau_{\text{est}} f_{\text{s}}} |y_{\text{ST}}[n]|^2. \tag{10}$$

$\hat{P}_{\text{Rx,ST}}$ determined in (10) using $\tau_{\text{est}} f_{\text{s}}$ samples follows a central chi-squared distribution $\mathcal{X}^2$ [32]. The cumulative distribution function of $\hat{P}_{\text{Rx,ST}}$ is given by

$$F_{\hat{P}_{\text{Rx,ST}}}(x) = \Gamma\left(\frac{\tau_{\text{est}} f_{\text{s}}}{2}, \frac{\tau_{\text{est}} f_{\text{s}} x}{2 P_{\text{Rx,ST}}}\right). \tag{11}$$

Estimation of access channel (h_s)

The signal received from the ST undergoes matched filtering and demodulation at the SR, hence, it is reasonable to employ pilot based estimation for h_{s}. Unlike received power estimation, pilot based estimation renders a direct estimation of the channel. Now, to accomplish pilot based estimation, the SR aligns itself to pilot symbols transmitted by the ST. Without loss of generality, the pilot symbols are considered to be +1. Under $\mathcal{H}_1$, the discrete pilot symbols at the output of the demodulator is given by [23]

$$p[n] = \sqrt{E_{\text{s}}} h_{\text{s}} + w[n], \tag{12}$$

where E_{s} denotes the pilot energy. The maximum likelihood estimate, representing a sample average of N_{s} pilot symbols, is given by [22]

$$h_{\text{s}} = \hat{h}_{\text{s}} + \underbrace{\frac{\sum_i^{N_{\text{s}}} n[i]}{2\sqrt{E_{\text{s}}} N_{\text{s}}}}_{\epsilon}, \tag{13}$$

where ϵ denotes the estimation error. The estimate $\hat{h}_{\text{s}}$ is unbiased, efficient and achieves a Cramer-Rao bound with equality, with variance $\mathbb{E}\left[|h_{\text{s}} - \hat{h}_{\text{s}}|^2\right] = \sigma_w^2/(2 E_{\text{s}} N_{\text{s}})$ [23]. Consequently, $\hat{h}_{\text{s}}$ conditioned on h_{s} follows a Gaussian distribution

$$\hat{h}_{\text{s}} | h_{\text{s}} \sim \mathcal{N}\left(h_{\text{s}}, \frac{\sigma_w^2}{2 E_{\text{s}} N_{\text{s}}}\right) \tag{14}$$

As a result, the power gain $|\hat{h}_{\text{s}}|^2$ follows a non-central chi-squared ($\mathcal{X}_1^2$) distribution with 1 degree of freedom and non-centrality parameter $\lambda = \frac{|h_{\text{s}}|^2}{\frac{\sigma_w^2}{2 E_{\text{s}} N_{\text{s}}}}$.

Estimation of interference channel ($h_{p,2}$)

In addition, analog to sensing channel, the SR performs received power estimation by listening to the transmission from the PT. The knowledge of $h_{\text{p},2}$ is required to characterize interference from the PT. Under $\mathcal{H}_0$, the discrete signal model at the SR is given as

$$y_{\text{SR}}[n] = h_{\text{p},2} \cdot x_{\text{PT}}[n] + w[n]. \tag{15}$$

The received power at the SR from the PT given by

$$\hat{P}_{\text{Rx,SR}} = \frac{1}{N_{\text{p},2}} \sum_{n}^{N_{\text{p},2}} |y_{\text{SR}}[n]|^2, \tag{16}$$

follows a $\mathcal{X}^2$ distribution, where $N_{\text{p},2}$ corresponds to the number of samples used for estimation.

Assumptions and Approximations

To simplify the analysis and sustain analytical tractability for the proposed model, several assumptions ~~are~~ considered in the paper are summarized as follows:

- We consider that all transmitted signals are subjected to distance dependent path loss and small scale fading gain. With no loss of generality, we consider that the channel gains include distance dependent path loss and small scale gain. Moreover, the coherence time for the channel gain is considered to be greater than the frame duration[2].
- We consider disjoint sets of samples for estimation and sensing for a certain frame. However, in practice, it is possible to utilize the samples used in the estimation phase for sensing purpose as well, which leads to an improvement in the performance in terms of the number of samples utilized for sensing.
- We assume perfect knowledge of the noise power in ~~the~~ system, however, the uncertainty in noise power can be captured as a bounded interval [20]. Inserting this interval in the derived expressions, cf. Section IV, the performance of the IS can be expressed in terms of the best and the worst bounds.
- For all degrees of freedom, $\mathcal{X}_1^2$ distribution can be approximated as Gamma distribution [34]. The parameters of the Gamma distribution are computed by comparing its first two central moments with those of $\mathcal{X}_1^2$.
- In addition, the system model stipulates the knowledge of the underlying hypothesis at the ST and the SR. Upon the application of PU traffic models proposed in [29]–[31], it is possible to acquire

[2]In the scenarios where the coherence time exceeds the frame duration, in such cases our characterization depicts a lower performance bound.

this knowledge with high probability. Moreover, it is reasonable that the ST and the SR perform estimation independently. Considering a realistic situation, it is possible that SR might not accomplish estimation in each frame, under such circumstances, the ST utilizes the previous estimation value for the analysis.

- Finally, it is assumed that the estimation time for the channels h_{s} and $h_{\text{p,2}}$ is smaller than $h_{\text{p,1}}$, that is $N_{\text{p,2}}, N_{\text{s}} < \tau_{\text{sen}} f_{\text{s}}$. Hence, it is sufficient to incorporate $\max(\tau_{\text{est}} f_{\text{s}}, N_{\text{s}}, N_{\text{p,2}}) = \tau_{\text{est}} f_{\text{s}}$ in the expression of the throughput.

IV. Theoretical Analysis

At this stage, it is evident that the variation due to imperfect channel estimation translates to the variation of the performance parameters $\text{P}_{\text{d}}, \text{C}_0$ and C_1, which are fundamental to sensing-throughput tradeoff. Below, we capture this variation by characterizing their cumulative distribution functions $F_{\text{P}_{\text{d}}}$, F_{C_0} and F_{C_1}, respectively.

Lemma 1: The cumulative distribution function of P_{d} is characterized as

$$F_{\text{P}_{\text{d}}}(x) = 1 - \Gamma\left(\frac{\tau_{\text{est}} f_{\text{s}}}{2}, \frac{\tau_{\text{est}} f_{\text{s}} \tau_{\text{sen}} f_{\text{s}} \mu}{4 P_{\text{Rx,ST}} \Gamma^{-1}\left(\frac{\tau_{\text{sen}}}{2}, x\right)}\right), \tag{17}$$

where $\Gamma^{-1}(\cdot,\cdot)$ is inverse function of regularized upper Gamma function [33].

Proof: The cumulative distribution function of P_{d} is defined as

$$F_{\text{P}_{\text{d}}}(x) = \mathbb{P}(\text{P}_{\text{d}} \leq x) \tag{18}$$

Using (4)

$$= \mathbb{P}\left(\Gamma\left(\frac{\tau_{\text{sen}} f_{\text{s}}}{2}, \frac{\tau_{\text{est}} f_{\text{s}} \mu}{2 \hat{P}_{\text{Rx,ST}}}\right) \leq x\right) \tag{19}$$

$$= 1 - \mathbb{P}\left(\hat{P}_{\text{Rx,ST}} \leq \frac{\mu \tau_{\text{sen}} f_{\text{s}}}{2\Gamma^{-1}\left(\frac{\tau_{\text{sen}} f_{\text{s}}}{2}, x\right)}\right) \tag{20}$$

Replacing the cumulative distribution function of $\hat{P}_{\text{Rx,ST}}$ in (20), we obtain an expression of $F_{\text{P}_{\text{d}}}$. ■

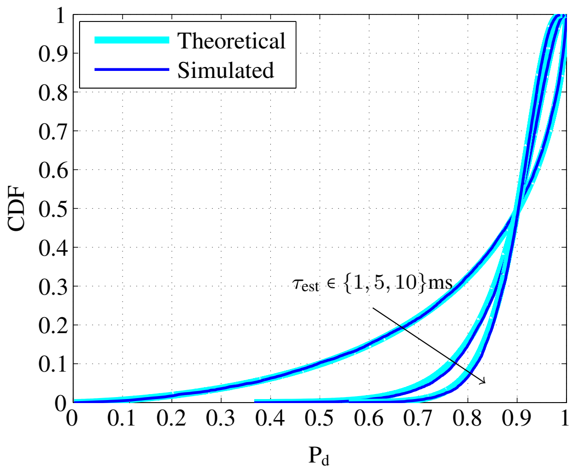
From (17), it is clearly observed that $F_{\text{P}_{\text{d}}}$ depends on $P_{\text{Rx,ST}}$, τ_{sen} and τ_{est}.

Lemma 2: The cumulative distribution function of C_0 is defined as

$$F_{\text{C}_0}(x) = \int_0^{\infty} f_{\text{C}_0}(x) dx, \tag{21}$$

where

$$f_{\text{C}_0}(x) = 2^x \ln 2 \frac{(2^x - 1)^{a_1 - 1}}{\Gamma(a_1) b_1^{a_1}} \exp\left(-\frac{2^x - 1}{b_1}\right), \tag{22}$$



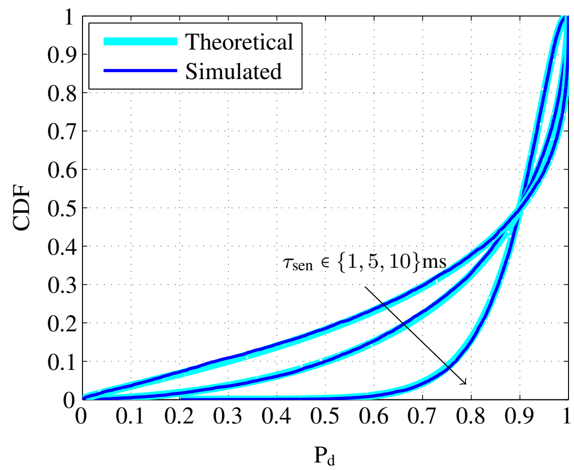


Fig. 3. CDF of P_d for different τ_{est} and τ_{sen}.

and

$$a_1 = \frac{\left(\frac{\sigma_w^2}{2E_\text{s}N_\text{s}}\frac{\sigma_w^2}{P_{\text{Tx,ST}}} + |h_\text{s}|^2\right)^2}{\frac{\sigma_w^2}{2E_\text{s}N_\text{s}}\frac{\sigma_w^2}{P_{\text{Tx,ST}}}\left(2\frac{\sigma_w^2}{2E_\text{s}N_\text{s}}\frac{\sigma_w^2}{P_{\text{Tx,ST}}} + 4|h_\text{s}|^2\right)} \text{ and } b_1 = \frac{\frac{\sigma_w^2}{2E_\text{s}N_\text{s}}\frac{\sigma_w^2}{P_{\text{Tx,ST}}}\left(2\frac{\sigma_w^2}{2E_\text{s}N_\text{s}}\frac{\sigma_w^2}{P_{\text{Tx,ST}}} + 4|h_\text{s}|^2\right)}{\left(\frac{\sigma_w^2}{2E_\text{s}N_\text{s}}\frac{\sigma_w^2}{P_{\text{Tx,ST}}} + |h_\text{s}|^2\right)}. \tag{23}$$

Proof: Following the pdf of $|\hat{h}_\text{s}|^2$ in (14), the pdf $|\hat{h}_\text{s}|^2\frac{P_{\text{Tx,ST}}}{\sigma_w^2}$ is given by

$$f_{\frac{|\hat{h}_\text{s}|^2 P_{\text{Tx,ST}}}{\sigma_w^2}}(x) = \frac{P_{\text{Tx,ST}}}{\sigma_w^2}\frac{1}{\frac{\sigma_w^2}{2E_\text{s}N_\text{s}}}\frac{1}{2}\exp\left[-\frac{1}{2}\left(x\frac{\sigma_w^2}{2E_\text{s}N_\text{s}}\frac{\sigma_w^2}{P_{\text{Tx,ST}}} + \lambda\right)\right]\left(\frac{x\frac{\sigma_w^2}{2E_\text{s}N_\text{s}}}{\lambda}\frac{\sigma_w^2}{P_{\text{Tx,ST}}}\right)^{\frac{N_\text{s}}{4}-\frac{1}{2}}$$
$$I_{\frac{N_\text{s}}{2}-1}\left(\sqrt{\lambda x\frac{\sigma_w^2}{2E_\text{s}N_\text{s}}\frac{\sigma_w^2}{P_{\text{Tx,ST}}}}\right).$$

Approximating $\mathcal{X}_1^2(\cdot,\cdot)$ with Gamma distribution $\Gamma(a_1, b_1)$ gives [34]

$$f_{\frac{|\hat{h}_\text{s}|^2 P_{\text{Tx,ST}}}{\sigma_w^2}} \approx \frac{1}{\Gamma(a_1)}\frac{x^{a_1-1}}{b_1^{a_1}}\exp\left(-\frac{x}{b_1}\right), \tag{24}$$

where the parameters a_1 and b_1 in (24) are determined by comparing the first two central moments of the two distributio[illegible]nally, by substituting the expression of C_0 in (8) yields (22). ■

Lemma 3: The cumulative distribution function of C_1 is given by

$$F_{\text{C}_1}(x) = \int_0^\infty f_{\text{C}_1}(x)dx, \tag{25}$$

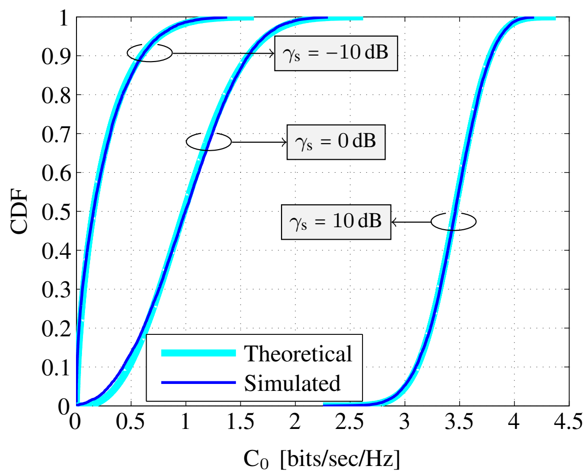


Fig. 4. CDF of C_0 for different values of $\gamma_s \in \{-10, 0, 10\}$dB.

where

$$f_{C_1}(x) = 2^x \ln 2 \frac{(2^x - 1)^{a_1 - 1}\Gamma(a_1 + a_2)}{\Gamma(a_1)\Gamma(a_2) b_1^{a_1} b_2^{a_2}} \left(\frac{1}{b_2} + \frac{2^x - 1}{b_1} \right), \tag{26}$$

and

$$a_2 = \frac{N_{p,2}}{2} \text{ and } b_2 = \frac{2P_{Rx,ST}}{\sigma_w^2 N_{p,2}}, \tag{27}$$

where a_1 and b_1 are defined in (23).

Proof: For simplification, we disintegrate the expression $\left(\frac{|\hat{h}_s|^2 P_{Tx,ST}}{\hat{P}_{Rx,SR}}\right)$ in (9), as $E_1 = \left(\frac{|\hat{h}_s|^2 P_{Tx,ST}}{\sigma_w^2}\right)$ and $E_2 = \left(\frac{\hat{P}_{Rx,SR}}{\sigma_w^2}\right)$, where $C_1 = \log_2\left(1 + \frac{E_1}{E_2}\right)$. The pdf of the expression E_1 is determined in (24).

Following the characterization $\hat{P}_{Rx,SR}$ in (16), the pdf of E_2 is determined as

$$f_{\frac{\hat{P}_{Rx,SR}}{\sigma_w^2}} = \frac{N_{p,2}\sigma_w^2}{P_{Rx,ST}} \frac{1}{2^{\frac{N_{p,2}}{2}}\Gamma\left(\frac{N_{p,2}}{2}\right)} \left(x \frac{N_{p,2}\sigma_w^2}{P_{Rx,ST}} \right)^{\frac{N_{p,2}}{2} - 1} \exp\left(-x \frac{N_{p,2}\sigma_w^2}{2P_{Rx,ST}} \right). \tag{28}$$

Using the characterizations of pdfs $f_{\frac{|\hat{h}_s|^2 P_{Tx,ST}}{\sigma_w^2}}$ and $f_{\frac{\hat{P}_{Rx,SR}}{\sigma_w^2}}$, we apply Mellin transform [35] to determine the pdf of $\frac{E_1}{E_2}$ as

$$f_{\frac{|\hat{h}_s|^2 P_{Tx,ST}}{\sigma_w^2} \Big/ \frac{\hat{P}_{Rx,SR}}{\sigma_w^2}}(x) = \frac{x^{a_1 - 1}\Gamma(a_1 + a_2)}{\Gamma(a_1)\Gamma(a_2) b_1^{a_1} b_2^{a_2}} \left(\frac{1}{b_2} + \frac{x}{b_1} \right). \tag{29}$$

Finally, substituting the expression $\frac{E_1}{E_2}$ in C_1 yields (26). ■

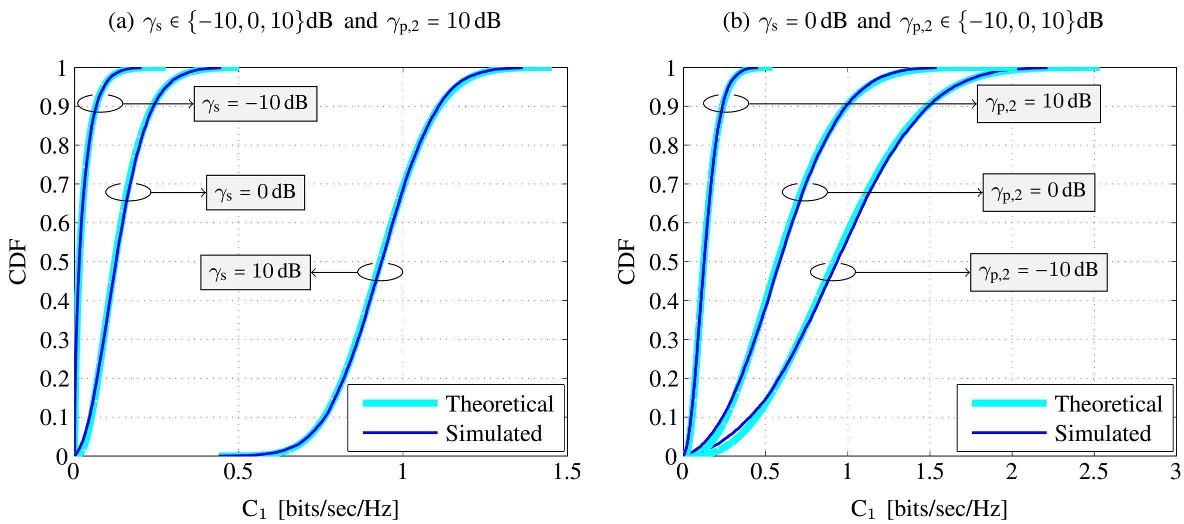


Fig. 5. CDF of C_1 for different γ_s and $\gamma_{p,2}$.

Sensing-throughput tradeoff

Here, we establish sensing-throughput tradeoff for the estimation model that includes estimation time and incorporates variation in the performance parameter. Most importantly, by capturing this variation, we establish two new primary user constraints at the PR, namely, an average constraint and an outage constraint on the probability of detection. These constraints constrict the harmful interference at the PR. Based on these constraints and a certain choice of estimation time τ_{est}, we characterize the sensing-throughput tradeoff for the IS.

Theorem 1: Subject to an average constraint on P_d at the PR, the sensing-throughput tradeoff is given by

$$\tilde{R}_s(\tilde{\tau}_{sen}) = \max_{\tau_{sen}} \mathbb{E}_{P_d, C_0, C_1}\left[R_s(\tau_{sen})\right] = \frac{T - \tau_{sen} - \tau_{est}}{T}\Big[\mathbb{E}_{C_0}\left[C_0\right](1 - P_{fa})\mathbb{P}(\mathcal{H}_0) + \mathbb{E}_{C_1}\left[C_1\right](1 - \mathbb{E}_{P_d}\left[P_d\right])\mathbb{P}(\mathcal{H}_1)\Big], \quad (30)$$

$$\text{s.t. } \mathbb{E}_{P_d}\left[P_d\right] \le \bar{P}_d. \quad (31)$$

$\mathbb{E}_{P_d}[\cdot]$ represents the expectation with respect to P_d, $\mathbb{E}_{P_d, C_0, C_1}[\cdot]$ denotes the expectation with respect to P_d, C_0 and C_1. Unlike (7), $\bar{P}_d$ in (30) represents the constraint on expected probability of detection.

Theorem 2: Subject to an outage constraint on P_d at the PR, the sensing-throughput tradeoff is given by

$$\tilde{R}_{\mathrm{s}}(\tilde{\tau}_{\mathrm{sen}}) = \max_{\tau_{\mathrm{sen}}} \mathbb{E}_{\mathrm{P_d},\mathrm{C}_0,\mathrm{C}_1}\left[R_{\mathrm{s}}(\tau_{\mathrm{sen}})\right] = \frac{T-\tau_{\mathrm{sen}}-\tau_{\mathrm{est}}}{T}\Bigg[\mathbb{E}_{\mathrm{C}_0}\left[\mathrm{C}_0\right](1-\mathrm{P_{fa}})\mathbb{P}(\mathcal{H}_0)+ \mathbb{E}_{\mathrm{C}_1}\left[\mathrm{C}_1\right](1-\mathbb{E}_{\mathrm{P_d}}\left[\mathrm{P_d}\right])\mathbb{P}(\mathcal{H}_1)\Bigg], \tag{32}$$

$$\text{s.t. } \mathbb{P}(\mathrm{P_d} \le \bar{\mathrm{P}}_{\mathrm{d}}) \le \kappa, \tag{33}$$

where κ represents the outage constraint.

In order to solve the constrained optimization problems illustrated in Theorem 1 and Theorem 2, the following approach is considered. As a first step, an underlying constraint is employed to determine μ as a function of the τ_{sen} and τ_{est}. Its value is substituted in (4) and (5) to determine $\mathrm{P_{fa}}$ and $\mathrm{P_d}$. Finally, replacing $\mathrm{P_d}$ and $\mathrm{P_{fa}}$ inside (30) and (32), we obtain an expression of the throughput in terms of τ_{sen}, τ_{est} and the system parameters. Using this expression, we consider the variation of expected throughput along τ_{est} and τ_{sen}, thereby determining an optimum throughput.

Corollary 1: For the average constraint, the analytical expression $\mathbb{E}_{\mathrm{P_d}}\left[\mathrm{P_d}\right]$ did not lead to a closed form expression. In this context, we procure the μ for the average constraint numerically.

Corollary 2: For this case, we determine the μ based on the outage constraint. This is accomplished by combining the expression of $F_{\mathrm{P_d}}$ in (17) with the outage constraint (33)

$$P(\mathrm{P_d} \le \bar{\mathrm{P}}_{\mathrm{d}}) = F_{\mathrm{P_d}}(\bar{\mathrm{P}}_{\mathrm{d}}) \le \kappa. \tag{34}$$

Rearranging (34) gives

$$\mu \ge \frac{4P_{\mathrm{Rx,ST}}\Gamma^{-1}\left(1-\kappa, \frac{\tau_{\mathrm{est}}f_{\mathrm{s}}}{2}\right)\Gamma^{-1}\left(1-\bar{\mathrm{P}}_{\mathrm{d}}, \frac{\tau_{\mathrm{sen}}f_{\mathrm{s}}}{2}\right)}{\tau_{\mathrm{est}}f_{\mathrm{s}}\tau_{\mathrm{sen}}f_{\mathrm{s}}} \tag{35}$$

Upon replacing the respective thresholds in $\mathrm{P_d}$ and $\mathrm{P_{fa}}$ and evaluating the expectation over $\mathrm{P_d}$, C_0 and C_1 using the distribution functions characterized in Lemma 1, Lemma 2 and Lemma 3, we determine the expected throughput as a function of sensing and estimation time. As a consequence, for a certain estimation time, sensing-throughput tradeoff that depicts the variation of expected throughput against the sensing time is established based on the average and outage constraint. In contrast to the ideal model, the sensing-throughput tradeoff substantiated by the estimation model incorporates the channel knowledge is qualified for characterizing the performance of IS.

Corollary 3: Subsequently, following the variation of optimum expected throughput $\tilde{R}_{\mathrm{s}}(\tau_{\mathrm{est}}, \tilde{\tau}_{\mathrm{sen}})$ (optimized over the sensing time) against the estimation time, we depict an estimation-sensing-throughput tradeoff. Based on this new tradeoff established, we determine the suitable estimation $\tau_{\mathrm{est}} = \tilde{\tau}_{\mathrm{est}}$ and sensing time $\tau_{\mathrm{sen}} = \tilde{\tau}_{\mathrm{sen}}$ that achieves an optimum throughput $\tilde{R}_{\mathrm{s}}(\tilde{\tau}_{\mathrm{est}}, \tilde{\tau}_{\mathrm{sen}})$ for the IS.

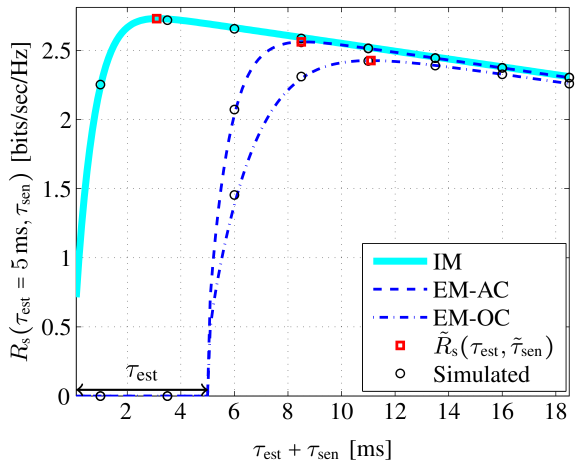


Fig. 6. Sensing-throughput tradeoff for the ideal and estimation models with $\gamma_{p,1} = -10\,\text{dB}$, $\tau_{est} = 5\,\text{ms}$ and $\kappa = 0.05$.

V. Numerical Results

Here, we investigate the performance of the IS based on the proposed model. To accomplish this: (i) we perform simulations to validate the expressions obtained, (ii) we analyze the performance loss incurred due to the estimation. In this regard, we consider the ideal model for benchmarking and evaluating the performance loss, (iii) we establish mathematical justification to the considered approximations. Although, the expressions derived using our sensing-throughput analysis are general and applicable to all cognitive radio systems, ~~however,~~ the parameters are selected in such a way that they closely relate to the deployment scenario described in Fig. 1. Unless stated explicitly, the following choice of the parameters is considered for the analysis, $f_s = 1\,\text{MHz}$, $h_{p,1} = h_{p,2} = -100\,\text{dB}, h_s = -80\,\text{dB}$, $T = 100\,\text{ms}$, $\bar{\text{P}}_d = 0.9$, $\kappa = 0.05$ $\sigma_w^2 = -100\,\text{dBm}$, $\gamma_{p,1} = -10\,\text{dB}$, $\gamma_{p,2} = -10\,\text{dB}$, $\gamma_s = 10\,\text{dB}$, $\sigma_x^2 = P_{Tx,PT} = -10\,\text{dBm}$ $P_{Tx,ST} = -10\,\text{dBm}$, $\mathbb{P}(\mathcal{H}_1) = 1 - \mathbb{P}(\mathcal{H}_0) = 0.2$, $\tau_{est} = 5\,\text{ms}$, $N_s = 10$ and $N_{p,2} = 1000$.

Firstly, we analyze the performance of the IS in terms of sensing-throughput tradeoff corresponding to the ideal model (IM) and estimation model (EM) by fixing $\tau_{est} = 5\,\text{ms}$, cf. Fig. 6. In contrast to constraint on P_d for the ideal model, we employ average constraint (EM-AC) and outage constraint (EM-OC) for the proposed estimation model. With the inclusion of received power estimation in the frame structure, the ST procures no throughput at the SR for the interval τ_{est}. For the given cases, namely, IM, EM-AC and EM-OC, a suitable sensing time that results in an optimum throughput $\tilde{R}_s(\tau_{est} = 5\,\text{ms}, \tilde{\tau}_{sen})$ is determined. Hence, a performance degradation is depicted in terms of the optimum throughput, cf. Fig. 6. For $\kappa = 0.05$, it is observed that the outage constraint is more sensitive to the performance loss in comparison to average constraint. It is clear that the analysis illustrated Fig. 6 is obtained for a certain

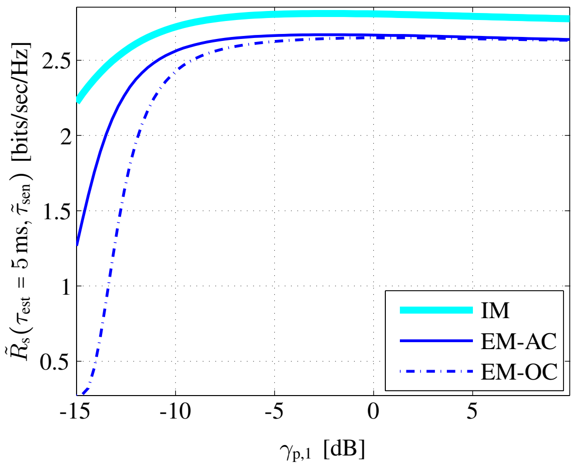


Fig. 7. Optimum throughput versus the $\gamma_{p,1}$ with $\tau_{est} = 5\,\text{ms}$.

choice of system parameters, particularly $\gamma_{p,1} = -10\,\text{dB}$, $\tau_{est} = 5\,\text{ms}$ and $\kappa = 0.05$. To acquire more insights, we consider the effect of variation of these parameters on the performance of IS, subsequently.

Hereafter, for the analysis, we consider the theoretical expressions and choose to operate at suitable sensing time. Next, we determine the variations of the optimum throughput against the received signal to noise ratio $\gamma_{p,1}$ at the ST with $\tau_{est} = 5\,\text{ms}$, cf. Fig. 7. For $\gamma_{p,1} < -5\,\text{dB}$, the estimation model incurs a significant performance loss. This clearly reveals that the ideal model overestimates the performance of IS. Hence, it is perceived that despite loss in performance, the estimation model is capable of precluding interference at the PR, hence, assuring reliability to the system.

Upon optimizing the secondary throughput, it is interesting to analyze the variation of optimum throughput against the estimation time. Corresponding to the estimation model, Fig. 8 illustrates a fundamental tradeoff between the estimation time, the sensing time and the throughput. This can be explained from the fact that low values of estimation time result in large variation in P_d. To counteract and satisfy the average and outage constraints, the corresponding thresholds shift to a lower value. This causes an increase in P_{fa}, thereby increasing the sensing-throughput curvature. As a result, the optimum sensing time is obtained at a higher value. However, beyond a certain value ($\tilde{\tau}_{est}$), a further increase in estimation time slightly contributes to performance improvement and largely consumes the time resources. As a consequence to the estimation-sensing-throughput tradeoff, we determine the suitable estimation time that yields an optimum throughput $\tilde{R}_s(\tilde{\tau}_{est}, \tilde{\tau}_{sen})$. Besides that, we consider the variation of optimum throughput for different values of the outage constraint, cf. Fig. 8. It is observed that for the selected the choice of κ, the outage constraint is stringent as compare to the average constraint, hence, results in a

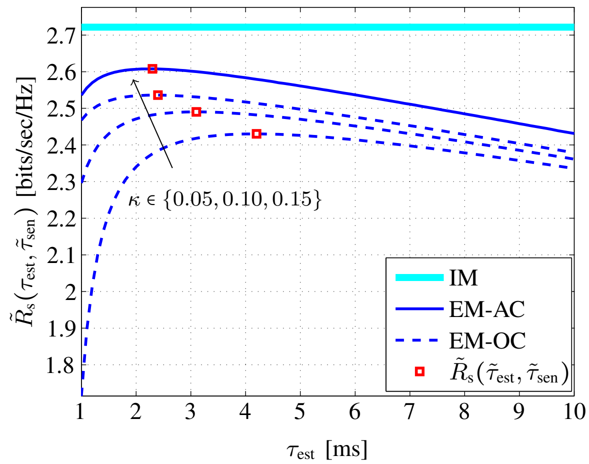


Fig. 8. Estimation-throughput tradeoff for the adjacent and outage constraint with $\gamma_{p,1} = -10$ dB, where the throughput is maximized over the sensing time, $\tilde{R}_s(\tau_{est}, \tilde{\tau}_{sen})$. Estimation-throughput tradeoff is utilized to determine a suitable estimation time $\tilde{\tau}_{est}$ that optimizes the throughput, $\tilde{R}_s(\tilde{\tau}_{est}, \tilde{\tau}_{sen})$.

(a) $\mathrm{P_d}$ verses τ_{est} (b) $\mathrm{P_{fa}}$ verses τ_{est}

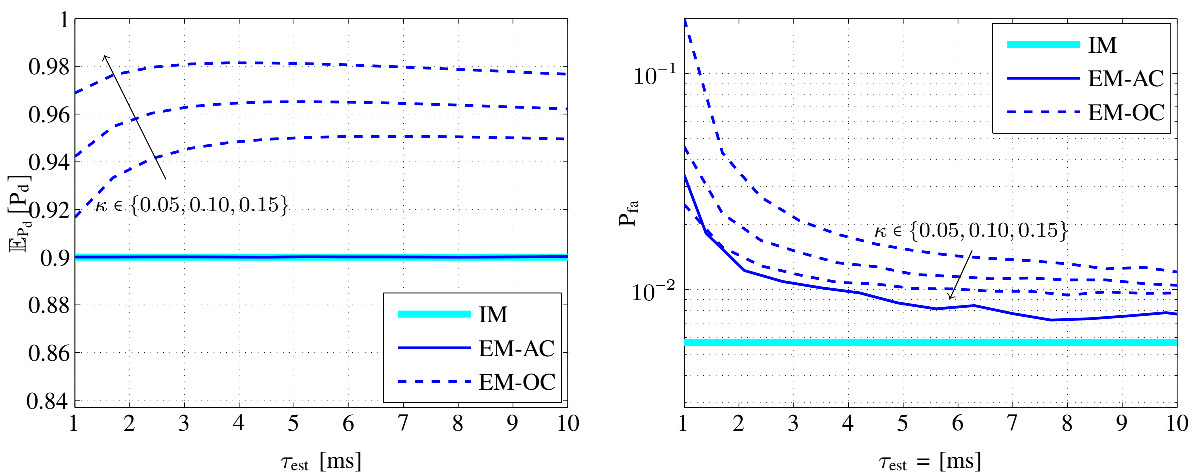


Fig. 9. Variation of $\mathbb{E}_{\mathrm{P_d}}[\mathrm{P_d}]$ and $\mathrm{P_{fa}}$ verses the τ_{est}, where the throughput is maximized over the sensing time, $\tilde{R}_s(\tau_{est}, \tilde{\tau}_{sen})$.

lower optimum throughput. Thus, depending on the nature of policy (aggressive or conservative) followed by the regulatory towards interference at the primary system, it is possible to define κ accordingly at the system design.

To procure further insights, we investigate the variations of expected $\mathrm{P_d}$ and $\mathrm{P_{fa}}$ against the estimation time. From Fig. 9a, it is noticed that the expected $\mathrm{P_d}$ corresponding to the outage constraint is strictly above the desired level $\bar{\mathrm{P}}_\mathrm{d}$ for all values of estimation time, however, for lower value of estimation time, this margin reduces. This is based on the fact that lower estimation time shifts the probability mass of $\mathrm{P_d}$, to lower value, cf. Fig. 3a. Besides that, based on the previous discussion, it was analyzed that $\mathrm{P_{fa}}$ accounts for a large contribution to the throughput. According to Fig. 9b, $\mathrm{P_{fa}}$ witnesses a large improvement in performance in the regime $\tau_{\text{est}} \leq 3\,\text{ms}$, however, saturates in the regime $\tau_{\text{est}} \geq 3\,\text{ms}$, thus, provides further justification to the variation of $\tilde{R}_{\text{s}}(\tau_{\text{est}}, \tilde{\tau}_{\text{sen}})$ against τ_{est} characterized as estimation-sensing-throughput tradeoff depicted in Fig. 8.

VI. Conclusion

In this paper, we have investigated the performance of cognitive radio as interweave system from a deployment perspective. It has been argued that the knowledge of the interacting channels is a key aspect that enables the performance characterization of the IS in terms of sensing-throughput tradeoff. In this regard, a novel model that facilitates channel estimation and captures the effect of estimation in the system has been proposed. As a major outcome of the analysis, it has been justified that the existing model, illustrating an ideal scenario, overestimates the performance of the interweave system, hence, less suitable for deployment. Moreover, it has been indicated that the variation induced in the system, specially in the probability of detection may severely degrade the performance of primary system. To overcome this situation, average and outage constraints as primary user constraints have been employed. As a consequence, for the estimation model, novel expressions for sensing-throughput tradeoff based on the mentioned constraints have been established. Lastly, by analyzing the estimation-sensing-throughput tradeoff, suitable estimation time and sensing time that optimizes the throughput at the secondary receiver has been determined. In our future work, we plan to extend the proposed analysis for the hybrid cognitive radio which combines the advantages of interweave and underlay techniques.

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