On the Performance Analysis of Underlay Cognitive Radio Systems: A Deployment Perspective

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Abstract

In this paper, we study the performance of cognitive Underlay Systems (USs) that employ power control mechanism at the Secondary Transmitter (ST) from a deployment perspective. Existing baseline models considered for performance analysis either assume the knowledge of involved channels at the ST or retrieve this information by means of a feedback channel, however, such situations rarely exist in practice. Motivated by this fact, we propose a novel approach that incorporates estimation of the involved channels at the ST, in order to characterize the performance of the USs in terms of interference power received at the primary receiver (or *primary interference*) and throughput at the secondary receiver (or *secondary throughput*). Moreover, we apply an outage constraint that captures the impact of imperfect channel knowledge, particularly on the primary interference. Besides this, we employ a transmit power constraint at the ST to classify the operation of the USs in terms of an interference-limited regime and a power-limited regime. In addition, we extend the performance analysis of the proposed approach to investigate the influence of channel fading. In this regard, we characterize the expressions of the uncertain interference and the secondary throughput for the case where the involved channels encounter Nakagami-m fading. Finally, we investigate a fundamental tradeoff between the estimation time and the secondary throughput depicting an optimized performance of the US.

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The preliminary analysis of this paper has been presented at ICC 2015 in London, UK [1].

This work was partially supported by the National Research Fund, Luxembourg under the CORE projects "SeMIGod" and "SATSENT".

Index Terms

Cognitive radio, Underlay system, Channel estimation, Estimation-throughput tradeoff

I. Introduction

Cognitive Radio (CR) communications is considered as one of the viable solutions that addresses the problem of spectrum scarcity of future wireless networks. Secondary access to the licensed spectrum can be broadly categorized into different CR paradigms, namely, interweave, underlay and overlay systems [2]. Among these, underlay and interweave systems are largely associated with the techniques applicable at the physical layer and therefore can be considered feasible for hardware deployment. The interweave systems employ spectrum sensing to detect the presence of primary user signals while avoiding harmful interference to the primary system. On the other hand, an Underlay System (US) exploits the interference tolerance capability of the primary systems that allows the secondary users to transmit even in the presence of the primary users. To accomplish this, the USs employ techniques such as power control to maintain the interference (power) received at the Primary Receiver (PR) below a specified level defined as Interference Threshold (IT) [3]. In this paper, we focus on performance characterization of the US that employs power control at the Secondary Transmitter (ST).

A. Motivation and Related Work

In order to consider shared access to the licensed spectrum, it is essential to characterize the performance of a CR system jointly in terms of the primary and the secondary systems. With regard to the primary system, the performance of a US is characterized in terms of interference (power) received at the PR, which arises due to concurrent data transmission over the same channel by the secondary system. Recently, power control at the ST has emerged as an effective way of regulating of the interference induced by the ST. However, the power control primarily requires the knowledge of the *primary interference* channel[1] between the ST and the PR at the ST. The preliminary investigations [3]–[7], considered for the performance evaluation, assume this knowledge to be perfectly known at the ST. Such situations rarely exist in practical

[1]Here, the knowledge refers to the channel state information.

implementations. In order to address this, the performance analysis based on imperfect channel knowledge has been dealt extensively in [8]–[18].

It is worth noticing that the majority of these works [8], [10], [11] in reference to imperfect channel knowledge consider that the channel's knowledge at the ST is obtained from a band manager[2], an approach proposed in [19]. Whereas [9], [13] rely on the presence of a feedback link from the PR to the ST [20]. The fact is, the feasibility of the band manager or feedback link across two completely different systems is unrealistic from a practical standpoint. In addition, due to latency, the channel knowledge obtained while implementing these approaches may be outdated, as considered in [9]–[11], [13]. Besides, even if we assume the existence of the feedback link, the demodulation of the secondary user signals at the PR and a resource (time) allocation explicitly for communicating the channel state impose an additional overhead for the primary system. With these issues in hand, the hardware implementation of the US in reference to the aforementioned approaches becomes challenging. In contrast to these approaches, we propose a novel strategy according to which the channel estimation is employed directly at the secondary system. Thus, by avoiding the realization of the band manager or feedback link and the issues related to it, in this paper, we focus on the key aspects that leverage the hardware deployment of the US.

Along with the performance of the primary system, the data rate received at the SR for the link between the ST and the SR also contributes significantly to the overall performance of the USs [7]–[9], [11], [13]–[15]. As a matter of fact, the knowledge of the data rate at the ST can be utilized for guaranteeing a certain quality of service, thereby determining potential applications or prominent use cases for the CR system. For instance, using this knowledge, the CR system is allowed to execute a band allocation policy, based on which, the ST can relinquish those channels that are largely responsible for causing interference at the PR. In order to characterize the data rate, the ST (along with the primary interference channel, which is associated with power control mechanism) requires the knowledge of *access* channel between the ST and the SR, and *secondary interference* channel between the PT and the SR. Despite these facts, the performance characterization of the data rate in reference to the estimation of the access and the secondary interference channels are not considered [8]–[11], [13], [14] or only marginally [12],

[2]An entity that mediates between the primary and the secondary systems.

[15] ~~in the previous works~~.

From a deployment perspective, it is worthy to understand that the primary and the secondary interference channels, which are representative of the channels that exist between different (primary and secondary) systems. This signifies that in order to carry channel estimation based on the conventional ~~estimation~~ techniques such as pilot-based channel estimation, which is majority employed in previous works, a preliminary processing in the form of synchronization and demodulation of the baseband signal of the primary user is necessary. With the existence of multiple wireless standards and their complexity prelude us from deploying a dedicated circuitry corresponding to each primary user signals [21]. In this regard, it is necessary to select the estimation techniques such that complexity and versatility (to the unknown primary user signals) requirements are satisfied. In this paper, similar to [22], we address this critical problem by employing a received power-based estimation at the ST and the SR for the interference channels. In contrast to the interweave scenario considered in [22], we investigate an underlay scenario herein. The employment of received power-based channel estimation allow us to examine the performance of the USs under conditions that are closer to the real environment. To support the previous statement, a successful deployment of the received power-based channel estimation at the ST (however, limited to only primary interference channel) in context to the USs has recently emerged in [23].

From the discussion above, it is clear that the performance of the USs can be depicted in terms of the achievable *secondary throughput* for a certain interference threshold (IT). However, a certain time needs to be allocated by the secondary user for channel estimation. Since the aspect concerning the time allocation for the channel estimation has not been taken into account in any of the previous investigations related to the cognitive USs [8]–[15], [17], the performance of the USs in terms of the secondary throughput is overestimated. Moreover, the power control depends on the knowledge of the primary interference channel. Its imperfect knowledge (or the induced variations) leads to uncertainty in the interference, which in certain cases may exceed the IT. Under such conditions, the conventional constraint imposed in [3], [4], [6], [7] is strictly violated. As a result, this *uncertain interference* originated from imperfect channel knowledge (of the primary interference channel) may seriously degrade the performance of the primary systems. In order to tackle this issue, we propose an outage constraint (also referred as interference constraint) that regulates the uncertain interference caused at the PR.

Besides, it is worth noticing that the uncertain interference is associated with the estimation time and the control power. In order to quantify this, we represent a power control mechanism as a function of the estimation time such that the outage constraint is fulfilled. An increase in the estimation time reduces the uncertain interference, which increases the control power, thereby increasing the secondary throughput. From the secondary system's perspective, this increase in estimation time decreases the secondary throughput because more time is allocated for channel estimation and less for data transmission. In this paper, we examine this relationship between the estimation time and the secondary throughput while constraining the uncertain interference below a desired level. It is important to note that although the previous studies have considered channel estimation, the influence of imperfect channel knowledge depicted in terms of the time allocation and the uncertain interference in the considered underlay scenario is still lacking.

B. Contributions

In this paper, we provide the following contributions:

1) Analytical Framework: The main contribution of this paper is to derive an analytical framework for underlay CR systems that employ a power control mechanism and incorporate the estimation of the following interacting channels: (i) primary interference channel between the ST and the PR, (ii) secondary interference channel between the PT and the SR, and (iii) access channel between the ST and the SR. In contrast to existing works that demand the presence of a band manager or a feedback link in order to retrieve channel estimates, we propose to employ channel estimation at the secondary system. In order to facilitate the deployment of the USs, we propose to employ received power-based channel estimation, specially for the interference channels so that low complexity and versatility to estimate primary user signals is accomplished. Clearly, the channel estimation is detrimental to the performance of the US, leading to performance degradation. By comparing its performance with the ideal scenario (with perfect channel knowledge), we study the performance degradation caused due to imperfect channel knowledge.

Besides, we characterize these variations due to imperfect channel knowledge in the performance parameters, which include interference at the PR and secondary throughput at the SR in terms of their cumulative distribution functions pertaining to the deterministic (not random) and the random behaviour (channel fading) of the interacting channels. Particularly, these variations leads to uncertainty in the interference that may seriously disrupt the operation of the primary system. To regulate this uncertain interference below a tolerable limit, we propose to employ an outage constraint over the uncertain interference. In addition to this, we further capture the variations in the secondary throughput in terms of its expected value.

2) Interference-limited and Power-limited Regimes: The power control at the ST depends on the received signal to noise power ratio of the link between the PR and the ST, which characterizes the quality of the primary interference channel. In this paper, we characterize the power control in terms of the estimation time and the signal to noise ratio such that the outage constraint is satisfied. In practice, the power control is limited by the maximum transmit power. Due to this limitation, good channel conditions (symbolize low signal to noise power, do not translate into performance gains for the USs. We study this behaviour of the USs in terms of performance bound which is illustrated as a relation between the received signal to noise ratio and the estimation time. As depicted later in Fig. 3, based on this performance bound, we classify the operation of the US as the interference-limited and the power-limited regimes.

3) Estimation-throughput Tradeoff: Besides, we propose a successful incorporation of the time allocated for the channel estimation in the secondary system's frame structure. The time allocation dedicated to channel estimation causes a linear decrease in the secondary throughput. Therefore, a low estimation time increases the secondary throughput, since less time is allocated for channel estimation. On the other side, its low value increases the uncertain interference, thus, requires a severe power control that ultimately reduces the secondary throughput. We study the association of the estimation time in reference to the time allocation and the power control to derive a fundamental tradeoff between the estimation time and the secondary throughput such that the uncertain interference is kept below a desired level. The theoretical as-well-as numerical analysis of this tradeoff in realistic underlay scenarios that takes imperfect channel estimation into account is a significant aspect investigated in this paper. Finally, we employ this tradeoff to derive a suitable estimation time that achieves a maximum secondary throughput for the USs. In other words, the considered tradeoff signifies the fact that the performance degradation in terms of secondary throughput can be effectively controlled only if the estimation time is selected appropriately.

4) Estimation-dominant and Channel-dominant Regimes: For the random channel, we classify the variations in the interference arising due to channel estimation and channel fading as an

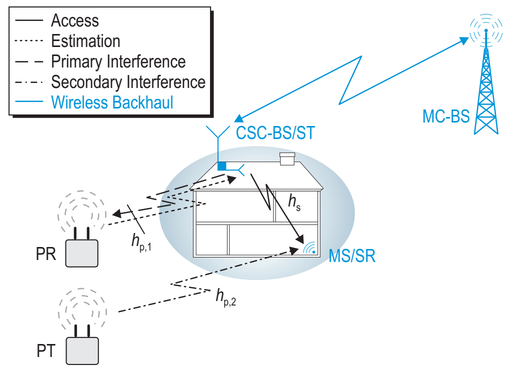

Fig. 1. A cognitive small cell scenario demonstrating: (i) the underlay paradigm, (ii) the associated network elements, which constitute Cognitive Small Cell-Base Station/Secondary Transmitter (CSC-BS/ST), Mobile Station/Secondary Receiver (MS/SR), Macro Cell-Base Station (MC-BS) and Primary Transmitter (PT), (iii) the interacting channels: primary interference channel, secondary interference channel and access channel.

estimation-dominant regime and a channel-dominant regime, respectively. Based on this analysis, it is revealed that a suitable selection of estimation time can achieve a performance gain (in terms of the secondary throughput) closer to the one predicted by the existing models that consider perfect channel knowledge of the interacting channels.

C. Organization

The subsequent sections of the paper are organized as follows: Section II presents the system model that describes the deployment scenario, the medium access and the signal model. It further presents the problem description and the proposed approach. Section III characterizes the cumulative distribution functions of the performance parameters and establishes the estimation-throughput tradeoff subject to the outage constraint on the uncertain interference and transmit power constraint at the ST. Section IV analyzes the numerical results based on the obtained expressions. Finally, Section V concludes the paper. Table I lists the definitions of acronyms and important mathematical notations used throughout the paper.

II. System Model

A. Underlay Scenario and Medium Access

The Cognitive Small Cell (CSC), a CR application, characterizes a small cell deployment that fulfills the spectral requirements of the Mobile Stations (MSs) operating indoor, Fig. 1. For the disposition of the CSC in the network, the following key elements are essential: a CSC-Base Station (CSC-BS), a Macro Cell-Base Station (MC-BS) and an MS [22]. Considering the fact that the power control is employed at the CSC-BS, the CSC-BS and the MS represent ST and SR,

TABLE I

DEFINITIONS OF ACRONYMS AND NOTATIONS USED

Acronyms and Notations	**Definitions**
CR	Cognitive Radio
CSC, CSC-BS, MC-BS, MS	Cognitive Small Cell, Cognitive Small Cell-Base Station, Macro Cell-Base Station, Mobile Station
IM, EM	Ideal Model, Estimation Model
US	Underlay System
PT, PR, ST, SR	Primary Transmitter, Primary Receiver, Secondary Transmitter, Secondary Receiver
f_{s}	Sampling frequency
τ, T	Estimation time interval, Frame duration
ρ_{out}	Outage probability constraint
P_{full}	Maximum transmit power or transmit power constraint at ST
P_{cont}	Control power at ST
$h_{\mathrm{p},1}, h_{\mathrm{p},2}, h_{\mathrm{s}}$	Channel coefficient for the link PR-ST (primary interference channel), PT-SR (secondary interference channel), ST-SR (access channel)
γ	Signal to noise ratio for the link PR-ST at ST
$R_{\mathrm{s}}, \mathrm{C}_{\mathrm{s}}$	Throughput at SR, Data rate at SR
$F_{(\cdot)}$	Cumulative distribution function of random variable $(\cdot)$
$f_{(\cdot)}$	Probability density function of random variable $(\cdot)$
$\hat{(\cdot)}$	Estimated value of $(\cdot)$
$\tilde{(\cdot)}$	Suitable value of the parameter $(\cdot)$ that achieves maximum performance
$\mathbb{E}_{(\cdot)}$	Expectation with respect to $(\cdot)$
$\mathbb{P}$	Probability measure
P_{tran}	Transmit power for the primary system
$P_{\mathrm{rcvd,ST}}, P_{\mathrm{rcvd,SR}}$	Power received at ST from PR, interference power received at SR from PT
σ^2	noise variance for primary and secondary systems
N_{s}	Number of pilot symbols used for pilot-based estimation at the ST for h_{s}

respectively. In order to acquire channel knowledge concerning the primary interference channel, the ST listens to the transmissions from the PR. In this work, we consider those primary systems where the PR performs transmissions interchangeably over time (time division duplexing TDD and half-duplex frequency division duplexing FDD) or frequency (full-duplex FDD) with the PT. These transmissions can occur over the same band (TDD) or over separate bands (half-duplex and full-duplex FDD). In cellular networks, these duplexing modes are effectively deployed in the Long Term Evolution (LTE) standard [24]. The ST follows these duplexing modes to exploit channel reciprocity principle and determine the interference (power) received at the PR, thus, controls its power for transmitting signals over the access channel such that it satisfies the interference constraint by operating at the IT. Particularly for half-duplex and full duplex FDD, it is assumed that the coherence bandwidth is large as compared to frequency separation between the estimation channel and the band of interest.

We propose to employ a slotted medium access for the US, where the time axis is segmented into frames. As depicted in Fig. 2, the frame duration T is chosen in such a way that the frames

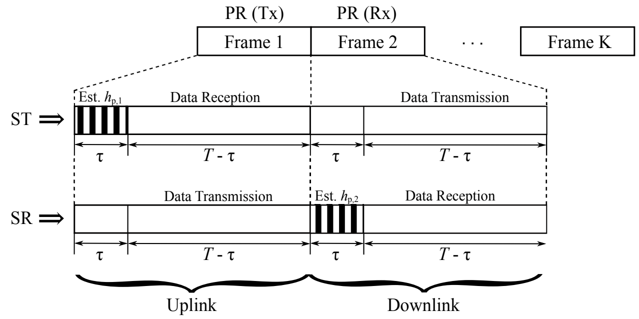

Fig. 2. Frame structure of the USs illustrating the time allocation for channel estimation and data transmission from the perspective of a ST and a SR. In this regard, corresponding to the uplink and the downlink, the primary interference and secondary interference channel estimation occur at the ST and the SR, respectively. PR (Tx)/PR (Rx) presents the transmission/reception of the primary signal from the PR/PT to the PT/PR.

are aligned to the primary users' transmissions. In order to incorporate channel estimation, we further propose to employ a periodic channel estimation[3], according to which, the US uses a time interval $\tau(< T)$ to perform channel estimation followed by data transmission $T - \tau$, see Fig. 2. In order to consider variations due to channel fading, we assume that the interacting channels remain constant over at least two frame durations ($2T$). Based on this assumption, every alternating transmission frame observes a different received power, consider Fig. 2. Since the channel knowledge is essential to employ power control such that the primary users are sufficiently protected from the uncertain interference induced due to imperfect channel knowledge, it is reasonable to carry out estimation for τ time interval followed by data transmission with power control in the remaining time $T - \tau$ for each frame.

In accordance with the half duplexing modes, the ST and the SR perform received power-based estimation to acquire the knowledge of the primary and the secondary interference channel over consecutive frames, as illustrated in Fig. 2. For primary systems and secondary systems that implement full-duplex FDD, the proposed frame structure can be adapted such that the primary and secondary interference channel estimation occurs in a single frame. Besides this, since the access channel estimation is performed by listening to the pilot symbols transmitted by the SR, classified as pilot-based channel estimation, which are relatively small in comparison to the samples used for performing received power-based channel estimation. Considering the fact that the time allocation of the pilot symbols do not affect the overall performance of the USs, no time resources are allocated for access channel estimation in the frame structure. Hence, τ is utilized for the estimation of the interference channels only. At first, we consider the proposed frame

[3]This frame structure is similar to the periodic sensing followed by the interweave systems [25].

structure for a deterministic channel, i.e., the performance is analyzed for a certain channel gain (path-loss channel), without taking into account the effect of channel fading. Next, we carry out the performance analysis of the proposed framework by taking channel fading into account.

B. Signal Model

In the uplink, during the estimation phase, the discrete and complex signal received from the PR at the ST is given by

$$y_{\text{rcvd}}[n] = h_{\text{p},1} \cdot \sqrt{P_{\text{tran}}} \cdot x_{\text{tran}}[n] + w_{\text{s}}[n], \tag{1}$$

where $x_{\text{tran}}[n]$ corresponds to a discrete and complex sample transmitted by the PR with unit power, P_{tran} is the transmit power known at the ST and $|h_{\text{p},1}|^2$ represents the power gain for the primary interference channel and $w_{\text{s}}[n]$ is circularly symmetric complex Additive White Gaussian Noise (AWGN) at the ST with $\mathcal{CN}(0, \sigma^2)$.

In the downlink, during data transmission phase, the signal received at the PR is given by

$$y_{\text{p}}[n] = h_{\text{p},1} \cdot \sqrt{P_{\text{cont}}} \cdot x_{\text{cont}}[n] + w_{\text{p}}[n], \tag{2}$$

and on the other side, the received signal at the SR follows

$$y_{\text{s}}[n] = h_{\text{s}} \cdot \cdot \sqrt{P_{\text{cont}}} \cdot x_{\text{cont}}[n] + h_{\text{p},2} \cdot \sqrt{P_{\text{tran}}} \cdot x_{\text{tran}}[n] + w'_{\text{s}}[n], \tag{3}$$

where $x_{\text{cont}}[n]$ corresponds to a discrete and complex sample transmitted by the ST with unit power, P_{cont} is the controlled transmit power and $x_{\text{tran}}[n]$ is the transmit signal from the PT with transmit power P_{tran}[4] Further, $|h_{\text{s}}|^2$ and $|h_{\text{p},2}|^2$ represent the power gain for the access channel and the secondary interference channel, respectively. $w_{\text{p}}[n]$ and $w'_{\text{s}}[n]$ is AWGN at the PR and at the SR, respectively, with $\mathcal{CN}(0, \sigma^2)$[5].

[4]Please note that the transmit power at the PT and the PR might have different values. In contrast to the knowledge of the transmit power at the PR required to carry out the power control at the ST, in reference to the proposed framework, the knowledge of the transmit power at the PT is not necessary at the secondary system. Hence, its ignorance at the SR doesn't affect the analysis concerning the secondary interference. For clarity of the exposition, we choose P_{tran} for denoting the transmit power for the PT and for the PR. With no loss of generality, for the analysis, the PT and the PR are alloted same transmit power.

[5]In practice, the noise power at the ST, the SR and the PR have different values. The fact is, only the signal to noise ratio received at the ST, and SR and the PR, respectively, are effected due to these noise powers. Since these signal to noise ratios are already included in the performance analysis, the assignment of different notations to these noise powers are excluded in the expressions.

C. Problem Description

According to the existing investigations (also referred as ideal model), an ST of an US is required to control its transmit power in such a way that the interference received P_{p} at the PR is below IT θ_{I} [3]

$$P_{\mathrm{p}} = |h_{\mathrm{p},1}|^2 P_{\mathrm{cont}} \leq \theta_{\mathrm{I}}. \tag{4}$$

With the controlled power at the ST determined using (4), the data rate at the SR is defined as

$$\mathrm{C_s} = \log_2\left(1 + \frac{|h_{\mathrm{s}}|^2 P_{\mathrm{cont}}}{|h_{\mathrm{p},2}|^2 P_{\mathrm{tran}} + \sigma^2}\right). \tag{5}$$

where $\mathrm{C_s}$ represents the data rate over the access channel. From the deployment perspective, the ideal model depicted in (4) and (5) has following issues:

- Without the knowledge of the primary interference channel $h_{\mathrm{p},1}$, it is impossible to employ the power control based on (4).
- Furthermore, along with P_{cont}, the knowledge of the access channel h_{s} and the secondary interference channel $h_{\mathrm{p},2}$ is required to determine $\mathrm{C_s}$ according to (5).

The ideal model considers the perfect knowledge of the aforementioned channels at the ST, which is not available in practice. In this regard, we incorporate channel estimation in the system model. The imperfect channel knowledge, however, translates to the variations in the performance parameters, P_{p} and $\mathrm{C_s}$. Particularly, a variation in P_{p} that exceeds θ_{I} causes uncertain interference at the PR, thereby violating the interference constraint illustrated in (4). Unless captured, this uncertain interference may seriously degrade the performance of the US. Since the ideal model assumes the perfect knowledge of the involved channels, it fails to depict the degradation in the performance due to the time resources allocated for the channel estimation.

D. Proposed Approach

In order to facilitate channel estimation for the USs, it is essential to take the aforementioned issues into account. To accomplish this, the following strategy is proposed in the paper.

- At first, we consider the estimation of involved channels. In this regard, we propose to employ a received power-based estimation for the interference channels and a pilot-based estimation for the access channel.

- To capture the effect of the imperfect channel knowledge, we characterize the variations in the estimated parameters (namely, received power for the interference channels and power gain for the access channels) in terms of their cumulative distribution functions.
- Further, we translate the aforementioned variations to the performance parameters, such as uncertain interference and secondary throughput in terms of their cumulative distribution functions. More specifically, using the characterization of the uncertain interference, we propose a novel power control mechanism that regulates the uncertain interference at the PR.
- Finally, using the derived expressions, we analyze a relationship between the estimation time and the expected secondary throughput for the USs. We extend the proposed framework (also referred as estimation model) to analyze the impact of channel fading on the performance of the system.

Since the channel estimation in the context of CR systems involves two different (primary and secondary) systems, suitable channel estimation techniques should be selected such that the following requirements: (i) low complexity and (ii) versatility towards unknown primary user signals, essential from the deployment perspective, are respected. Similar problem for the interweave CR systems has been deeply investigated in [22], where the authors propose to employ a received power-based estimation for the channels between the primary and secondary users and a pilot-based estimation for the access channel. In contrast to this, we propose to employ received power-based and pilot-based estimation techniques in the underlay CR systems. It is also worth stating that, since the signal model in [22] (orthogonal frequency division multiplexing transmission) differs from the one (constant power transmission) studied in this paper, we derive new mathematical expressions for the performance parameters. In the following paragraphs, we consider the estimation of the power gains of the primary interference channel $|\hat{h}_{\mathrm{p,1}}|^2$, the access channel $|\hat{h}_{\mathrm{s}}|^2$ and the secondary interference channel $|\hat{h}_{\mathrm{p,2}}|^2$.

1) Estimation of primary interference channel: Considering

$$P_{\mathrm{rcvd,ST}} = |h_{\mathrm{p,1}}|^2 P_{\mathrm{tran}} + \sigma^2, \tag{6}$$

and the knowledge of PR's transmit power P_{tran}, the ST employs received power-based estimation to obtain the knowledge of $|h_{\mathrm{p,1}}|^2$. To accomplish this, in reference to (1), the ST listens to the transmissions from the PR and acquires the knowledge of $|\hat{h}_{\mathrm{p,1}}|^2$ indirectly by estimating the power received in the uplink as $\hat{P}_{\text{rcvd,ST}} = \frac{1}{\tau f_s} \sum_n^{\tau f_s} |y_{\text{rcvd}}[n]|^2$, where f_{s} being the sampling frequency and τ represents the estimation time interval. f_{s} and τ are such that the number of samples τf_{s} is an integer. The estimated received power $\hat{P}_{\text{rcvd,ST}}$ is utilized to determine the control power P_{cont} at which the data transmission over the downlink is carried out, Fig. 2. In accordance to the received power-based estimation for the channel $|h_{\text{p,1}}|^2$, it is noticed that the knowledge of P_{tran} at the ST is essential for the characterization of the power control mechanism (considered later in Lemma 4). However, this knowledge can be retrieved by considering the specifications of different wireless standards such GSM, EDGE and LTE, etc. [26]. It is well-known that certain standards (considered as primary systems) follow adaptive modulation and coding, which can consequently change P_{tran}. Under these situations, the ST can employ more complex techniques such as pilot assisted techniques in order to determine P_{tran} for the given frame.

For a certain value of $|h_{\text{p,1}}|^2$, the received power at the ST estimated using τf_{s} samples follows a non-central chi-squared distribution $F_{\hat{P}_{\text{rcvd,ST}}} \sim \mathcal{X}_1^2(\lambda_{\text{p,1}}, \tau f_{\text{s}})$ with non-centrality parameter $\lambda_{\text{p,1}} = \tau f_{\text{s}} |h_{\text{p,1}}|^2 P_{\text{tran}} / \sigma^2 = \tau f_{\text{s}} \gamma$ [27], where γ is defined as the ratio of the received power (from the PR) to noise at the ST and τf_{s} corresponds to the degrees of freedom. For analytical tractability, we consider the following approximation.

Approximation 1: For all degrees of freedom, the $\mathcal{X}_1^2$ distribution can be approximated by a Gamma distribution [28]. The parameters of the Gamma distribution are obtained by matching the first two central moments to those of $\mathcal{X}_1^2$.

Lemma 1: The cumulative distribution function of $\hat{P}_{\text{rcvd,ST}}$ is characterized as

$$F_{\hat{P}_{\text{rcvd,ST}}}(x) \approx 1 - \Gamma\left(a_{\text{p,1}}, \frac{x}{b_{\text{p,1}}}\right), \tag{7}$$

$$\text{where } a_{\text{p,1}} = \frac{\tau f_{\text{s}}(1+\gamma)^2}{2+4\gamma} \text{ and } b_{\text{p,1}} = \frac{\sigma^2(2+4\gamma)}{\tau f_{\text{s}}(1+\gamma)}, \tag{8}$$

and $\Gamma(\cdot,\cdot)$ represents the regularized upper-incomplete Gamma function [28].

Proof: Applying Approximation 1 to $\mathcal{X}_1^2(\lambda_{\text{p,1}}, \tau f_{\text{s}})$ yields (7). ■

2) Estimation of access channel: In the uplink, the pilot signal transmitted by the SR undergoes matched filtering and demodulation at the ST, hence, we employ a pilot-based estimation at the ST to acquire the knowledge of the access channel. According to [29], the maximumlikelihood estimate with N_s pilot symbols is given by

$$h_\mathrm{s} = \hat{h}_\mathrm{s} + \frac{\sum_n^{N_\mathrm{s}} p[n]}{2N_\mathrm{s}}, \tag{9}$$

where $p[n]$ denotes the discrete pilot symbol and $\frac{\sum_n^{N_\mathrm{s}} p[n]}{2N_\mathrm{s}}$ represents the estimation error. With no loss of generality, the pilot symbols are considered to be +1. As a result, the estimate $\hat{h}_\mathrm{s}$ is unbiased, efficient, i.e., achieves the Cramér-Rao bound with equality, with asymptotic variance $\mathbb{E}\left[|h_\mathrm{s} - \hat{h}_\mathrm{s}|^2\right] = \frac{\sigma^2}{2N_\mathrm{s}}$ [29]. Hence, $\hat{h}_\mathrm{s}$ conditioned on h_s follows a Gaussian distribution

$$\hat{h}_\mathrm{s}|h_\mathrm{s} \sim \mathcal{N}\left(h_\mathrm{s}, \frac{\sigma^2}{2N_\mathrm{s}}\right). \tag{10}$$

Consequently, for a certain value of $|h_\mathrm{s}|^2$, the estimated power gain $|\hat{h}_\mathrm{s}|^2$ follows a non-central chi-squared $\mathcal{X}_1^2(\lambda_\mathrm{s}, 1)$ distribution with 1 degree of freedom and non-centrality parameter $\lambda_\mathrm{s} = \frac{2N_\mathrm{s}|h_\mathrm{s}|^2}{\sigma^2}$.

Lemma 2: The cumulative distribution function of $|\hat{h}_\mathrm{s}|^2$ is characterized as

$$F_{|\hat{h}_\mathrm{s}|^2}(x) \approx 1 - \Gamma\left(a_\mathrm{s}, \frac{x}{b_\mathrm{s}}\right), \tag{11}$$

$$\text{where } a_\mathrm{s} = \frac{(1+\lambda_\mathrm{s})^2}{2+4\lambda_\mathrm{s}} \text{ and } b_\mathrm{s} = \frac{\sigma^2(2+4\lambda_\mathrm{s})}{(1+\lambda_\mathrm{s})}. \tag{12}$$

Proof: Applying Approximation 1 to $\mathcal{X}_1^2(\lambda_\mathrm{s}, 1)$ yields (11). ■

3) Estimation of secondary interference channel: In the downlink, the SR cancels the ST pilot signal over the access channel in order to estimate the secondary interference (power) received from the PT, (3). The power estimated over the signal $h_{\mathrm{p},1} \cdot \sqrt{P_\mathrm{tran}} \cdot x_\mathrm{tran}[n] + w'_\mathrm{s}[n]$ corresponds to the interference plus noise power ($P_\mathrm{rcvd,SR} = |h_{\mathrm{p},2}|^2 P_\mathrm{tran} + \sigma^2$, where $P_\mathrm{rcvd,SR}$ represents the true value, consider (5)). The estimated received power at the SR is determined as $\hat{P}_\mathrm{rcvd,SR} = \frac{1}{\tau f_\mathrm{s}} \sum_n^{\tau f_\mathrm{s}} |h_{\mathrm{p},1} \cdot \sqrt{P_\mathrm{tran}} \cdot x_\mathrm{tran}[n] + w'_\mathrm{s}[n]|^2$. To characterize the secondary throughput, $\hat{P}_\mathrm{rcvd,SR}$ is made available to the ST over a low rate feedback channel. Similar to $\hat{P}_\mathrm{rcvd,ST}$, for a certain value of $|h_{\mathrm{p},2}|^2$, $\hat{P}_\mathrm{rcvd,SR}$ follows a non-central chi-squared distribution $\mathcal{X}_1^2(\lambda_{\mathrm{p},2}, \tau f_\mathrm{s})$, with non-centrality parameter $\lambda_{\mathrm{p},2} = \tau f_\mathrm{s} |h_{\mathrm{p},2}|^2 P_\mathrm{tran}/\sigma^2$.

Lemma 3: The cumulative distribution function of $\hat{P}_\mathrm{rcvd,ST}$ is characterized as

$$F_{\hat{P}_\mathrm{rcvd,SR}}(x) \approx 1 - \Gamma\left(a_{\mathrm{p},2}, \frac{x}{b_{\mathrm{p},2}}\right), \tag{13}$$

$$\text{where } a_{\mathrm{p},2} = \frac{(\tau f_\mathrm{s} + \lambda_{\mathrm{p},2})^2}{2\tau f_\mathrm{s} + 4\lambda_{\mathrm{p},2}} \text{ and } b_{\mathrm{p},2} = \frac{\sigma^2(2\tau f_\mathrm{s} + 4\lambda_{\mathrm{p},2})}{(\tau f_\mathrm{s} + \lambda_{\mathrm{p},2})}. \tag{14}$$

Proof: Applying Approximation 1 to $\mathcal{X}_1^2(\lambda_{\text{p},2}, \tau f_\text{s})$ yields (13). ■

It is important to note that, in this paper, we are dealing with a single PT and a single PR. However, in practice, it is possible that the ST and the SR accumulate significant interference (defined as aggregate interference) from other PRs and PTs (co-channel interference due to frequency reuse) in the network [30], [31] over the primary interference channel and the secondary interference channel, respectively. For the secondary interference channel, the only difference is that the SR now estimates the aggregate interference. Due to this, the expression of $\hat{P}_{\text{rcvd,SR}}$ in the throughput remains unchanged. On the other side, by estimating the aggregate interference on the primary interference channel, the ST overestimates $\hat{P}_{\text{rcvd,ST}}$ and exercises a greater power control. Even for such a case, the outage constraint on the primary interference channel to the desired PR is satisfied, which consequently reduces the secondary throughput.

III. Theoretical Analysis

A. Deterministic Channel

In this section, we investigate the performance of the USs on a single-frame basis. In this sense, the involved channels $h_{\text{p},1}$, $h_{\text{p},2}$ and h_s are deterministic (not random). At first, we employ an outage probability constraint[6] ρ_{out} on the interference to capture the variations in the P_p incurred due to channel estimation, defined as

$$\mathbb{P}\left(P_\text{p} = |\hat{h}_{\text{p},1}|^2 P_{\text{cont}} \geq \theta_\text{I}\right) \leq \rho_{\text{out}}. \tag{15}$$

Substituting $|\hat{h}_{\text{p},1}|^2$ from (6) yields

$$\mathbb{P}\left(\left(\frac{\hat{P}_{\text{rcvd,ST}} - \sigma^2}{P_{\text{tran}}}\right) P_{\text{cont}} \geq \theta_\text{I}\right) \leq \rho_{\text{out}}. \tag{16}$$

Besides the outage constraint, P_{cont} is limited by a predefined transmit power P_{full}. To capture this aspect, the transmit power constraint at the ST is defined as

$$P_{\text{cont}} \leq P_{\text{full}}. \tag{17}$$

[6]The outage constraint is commonly used parameter for designing communication system that ensures the outage occurs no more than a certain percentage of time.

We consider that the same power is allocated to all the symbols transmitted within a frame by the ST. In this regard, the transmit power constraint on symbol basis and frame basis is equivalent. As a consequence, the constraint depicted in (17) is applicable the both the aforementioned scenarios. Based on the constraints in (16) and (17), we determine the expression of the controlled power for the proposed framework.

Lemma 4: Subject to the outage constraint on the uncertain interference and the transmit power constraint at the ST, the controlled power at the ST is given by

$$P_{\mathrm{cont}} = \begin{cases} \frac{\theta_{\mathrm{I}} P_{\mathrm{tran}}}{\left(b_{\mathrm{p},1} \Gamma^{-1}(\rho_{\mathrm{out}}, a_{\mathrm{p},1}) - \sigma^2\right)} & \text{if } P_{\mathrm{cont}} < P_{\mathrm{full}} \\ P_{\mathrm{full}} & \text{if } P_{\mathrm{cont}} \geq P_{\mathrm{full}} \end{cases}, \tag{18}$$

where $a_{\mathrm{p},1}$ and $b_{\mathrm{p},1}$ are defined in (8) and $\Gamma^{-1}(\cdot,\cdot)$ is the inverse function of regularized upper-incomplete Gamma function [28].

Proof: Substituting the distribution function for $\hat{P}_{\mathrm{rcvd,ST}}$, defined in (7), in (16) and combining with (17) yields (18). ■

Clearly, the performance of the US improves over the access channel (in terms of the secondary throughput) with P_{cont}, but P_{cont} increases when $|h_{\mathrm{p},1}|^2$ is such that (high) a low γ is achieved. The real world wireless systems are limited by the transmit power P_{full}, which bounds the performance of the USs. In order to understand the effect of the power limitation on the USs, we analyze this performance bound in terms of the signal to ratio at the ST (γ)[7]. Therefore, by substituting P_{full} against P_{cont} determined in Lemma 4, we determine a performance bound for the US characterized as γ^*, in terms of γ for a certain value of the estimation time.

Corollary 1: Subject to the outage constraint on the uncertain interference and the transmit power constraint at the ST, the performance bound (γ^*) of the USs that employ power control at the secondary is defined as

$$\Gamma\left(\frac{\tau f_{\mathrm{s}}(1+\gamma^*)^2}{2+4\gamma^*}, \frac{\tau f_{\mathrm{s}}(1+\gamma^*)}{\sigma^2(2+4\gamma^*)}\left(\frac{\theta_{\mathrm{I}} P_{\mathrm{tran}}}{P_{\mathrm{full}}} + \sigma^2\right)\right) = \rho_{\mathrm{out}}. \tag{19}$$

Proof: Substituting P_{cont} with P_{full} in (16) and reformulating gives

$$\mathbb{P}\left(\hat{P}_{\mathrm{rcvd,ST}} \geq \frac{\theta_{\mathrm{I}} P_{\mathrm{tran}}}{P_{\mathrm{full}}} + \sigma^2\right) \leq \rho_{\mathrm{out}}. \tag{20}$$

[7] Signal to noise ratio is mostly used as a design parameter for characterizing the performance of a wireless system.

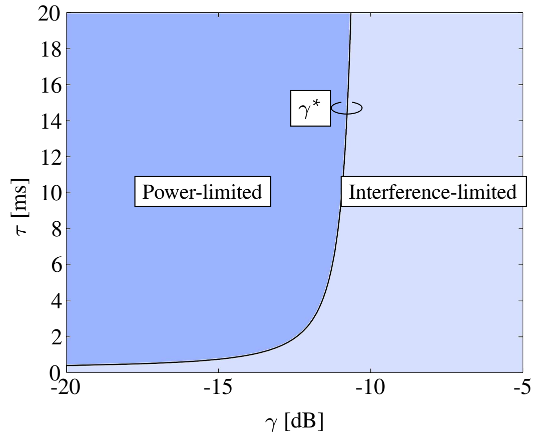

Fig. 3. An illustration of the interference-limited and the power-limited regimes (γ^*) for the US depicted in terms of estimation time (τ) and the ratio of the received power (from the PR) to noise (γ) at the ST.

Using (7) in Lemma 1 gives

$$\Gamma\left(\frac{\tau f_{\mathrm{s}}(1+\gamma)^2}{2+4\gamma}, \frac{\tau f_{\mathrm{s}}(1+\gamma)}{\sigma^2(2+4\gamma)}\left(\frac{\theta_{\mathrm{I}} P_{\mathrm{tran}}}{P_{\mathrm{full}}}+\sigma^2\right)\right) \leq \rho_{\mathrm{out}}. \tag{21}$$

Substituting γ with γ^* and replacing the expression in (21) with equality yields (19). ■

Remark 1: Fig. 3 analyzes the variations of γ^* with τ. Using the expression γ^* obtained in Corollary 1, we classify the operation of the USs into the following regimes: (i) interference-limited regime and (ii) power-limited regime. Inside the interference-limited regime $\gamma > \gamma^*$, due to good quality of the channel (unfavourable to USs), the system is limited due to the exceeding level of the uncertain interference, which can be regulated effectively by employing power control at the US to satisfy the given outage constraint. At $\gamma = \gamma^*$, the ST operates at the maximum control power $P_{\mathrm{cont}} = P_{\mathrm{full}}$ while respecting the tolerance limits defined for the uncertain interference. From a different perspective, the situation $\gamma = \gamma^*$ also represents those USs that are unable to carry out power control. With regard to the outage constraint and the lack of the power control, for a given choice of γ^*, such systems can operate only at a specific value of τ.

On the other side, the region $\gamma \leq \gamma^*$, which depicts a weak link quality between the ST and the PR, hence, is beneficial to the secondary user. However, due to the transmit power constraint, the USs are confined to operate at P_{full}. As a result, these favorable conditions does not translate to any performance gain. Therefore, this regime is characterized as a power-limited regime. Besides, Fig. 3 the variation of γ^* along the estimation time illustrates an interesting observation described as follows: for low values of the estimation time, $\gamma^* \to -\infty$, which signifies that low τ increases the uncertainty in the interference. In order to regulate the level of this uncertainty, US has to be proactive in terms of power control mechanism to be able to satisfy the interference constraint. It is also observed that as $\tau \to \infty$, γ^* converges asymptotically to a certain value. This signifies the fact that additional time resources allocated for the channel estimation time doesn't account to any significant improvement in the terms of the uncertain interference. As a result, the performs in the form of power control gets saturated.

Next, we capture the variations in the secondary throughput in terms of its expected value. To accomplish this, the cumulative distribution function (cdf) of the data rate[8]

$$\hat{\mathrm{C}}_{\mathrm{s}} = \log_2\left(1 + \frac{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{cont}}}{\hat{P}_{\mathrm{rcvd,SR}}}\right) \tag{22}$$

over the access channel evaluated at the ST. It is worth noticing the fact that unlike C_{s} defined in (5), $\hat{\mathrm{C}}_{\mathrm{s}}$ entails the random behaviour due to the estimation of $|\hat{h}_{\mathrm{s}}|^2$ and $\hat{P}_{\mathrm{rcvd,SR}}$.

Lemma 5: The cdf of data rate $\hat{\mathrm{C}}_{\mathrm{s}}$ is given by

$$F_{\hat{\mathrm{C}}_{\mathrm{s}}}(x) = \int_0^x f_{\hat{\mathrm{C}}_{\mathrm{s}}}(t)dt, \tag{23}$$

where the $f_{\hat{\mathrm{C}}_{\mathrm{s}}}(x)$ probability density function (pdf) is given by

$$f_{\hat{\mathrm{C}}_{\mathrm{s}}}(x) = 2^x \ln 2 \frac{(2^x-1)^{a_{\mathrm{s}}-1}\Gamma(a_{\mathrm{s}}+a_{\mathrm{p},2})}{\Gamma(a_{\mathrm{s}})\Gamma(a_{\mathrm{p},2})(b_{\mathrm{s}}P_{\mathrm{tran}})^{a_{\mathrm{s}}} b_{\mathrm{p},2}^{a_{\mathrm{p},2}}}\left(\frac{1}{b_{\mathrm{p},2}} + \frac{2^x-1}{b_{\mathrm{s}}P_{\mathrm{tran}}}\right). \tag{24}$$

Proof: See Appendix A. ■

In consideration to the Approximation 1, which is applied to obtain the cdfs' of $\hat{P}_{\mathrm{rcvd,ST}}$, $|\hat{h}_{\mathrm{s}}|^2$ and $\hat{P}_{\mathrm{rcvd,SR}}$ in Lemma 5, the theoretical expression of the cdf depicted in (23) is validated by means of simulations in Fig. 4 with different choices of system parameters, which include $\gamma = 10\,\mathrm{dB}$, $P_{\mathrm{cont}} = 0\,\mathrm{dBm}$, $\frac{|h_{\mathrm{p},2}|^2 P_{\mathrm{tran}}}{\sigma^2} \in \{-10, 0, 10\}\mathrm{dB}$ and $\tau = \{0.1, 1, 10\}\mathrm{ms}$.

Besides the outage constraint on the primary interference, the expected secondary throughput over the access channel at the SR is defined as

$$R_{\mathrm{s}}(\tau) = \frac{T-\tau}{T}\mathbb{E}_{\hat{\mathrm{C}}_{\mathrm{s}}}\left[\hat{\mathrm{C}}_{\mathrm{s}}\right], \tag{25}$$

where $\mathbb{E}_{\hat{\mathrm{C}}_{\mathrm{s}}}[\cdot]$ corresponds to an expectation over $\hat{\mathrm{C}}_{\mathrm{s}}$, whose pdf is characterized in Lemma 5.

Remark 2: At this point, it is well-known that the performance degradation due to channel estimation in the form of the secondary throughput is inherent to the USs. Specifically, the time

[8]Please note, we have introduced the following terms the data rate C_{s} and the throughput R_{s} to make a clear distinction between the instantaneous data rate and its average value over the frame duration.

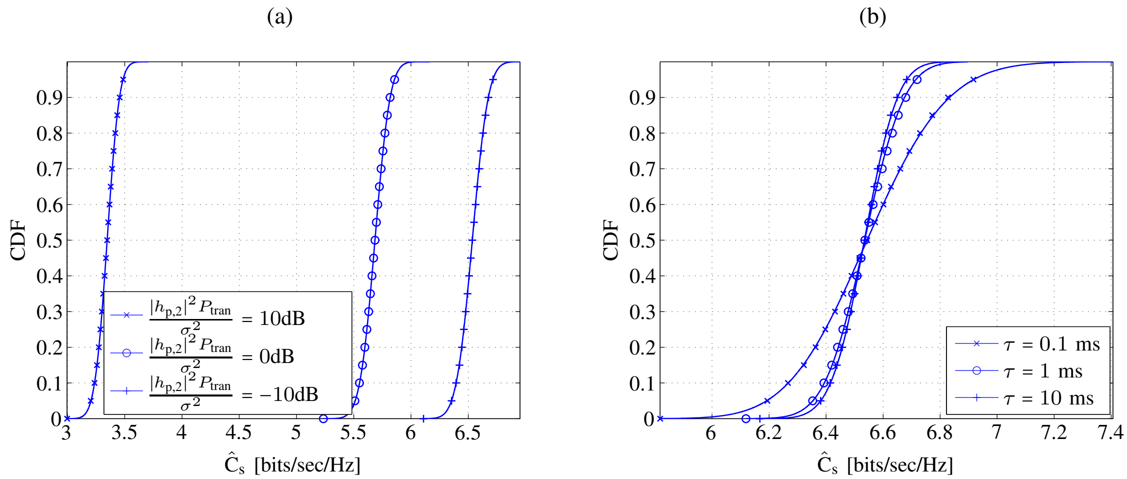

Fig. 4. CDF of $\hat{C}_s$ for different received signal to noise ratio $\frac{|h_{p,2}|^2 P_{tran}}{\sigma^2}$ over the secondary interference channel and estimation time τ. (a) $\frac{|h_{p,2}|^2 P_{tran}}{\sigma^2} \in \{-10, 0, 10\}$dB and $\tau = 1$ ms, (b) $\tau = \{0.1, 1, 10\}$ms and $\frac{|h_{p,2}|^2 P_{tran}}{\sigma^2} = 0$dB, for the following values of the system parameters, $\gamma = 10$ dB, $P_{cont} = 0$ dBm. The solid line represents the values computed using the analytical expressions while the markers represent the values obtained after performing the simulations.

allocation and the uncertain interference are responsible of this degradation. The power control, determined in Lemma 4, represented as a function of estimation time is able to regulate the uncertain interference. As discussed previously in Remark 1, the low estimation time enables a severe control in power, thereby reducing the throughput. On the other hand, the time resources allocated for the channel estimation also decrease the throughput. This phenomenon can be captured by observing the variation of the secondary throughput along the estimation time such that the constraints depicted in (16) and (17) are fulfilled. Below, Problem 1 captures this relationship between the estimation time and the secondary throughput defined as estimation-throughput tradeoff. More importantly, we utilize this tradeoff to determine a suitable estimation time at which maximum throughput at the SR is achieved.

Problem 1: The expected achievable secondary throughput subject to the outage constraint on the uncertain interference and the transmit power constraint at the ST is defined as

$$R_s(\tilde{\tau}) = \max_{\tau} \; R_s(\tau), \qquad (26)$$
$$\text{s.t. } (16),\ (17),$$

where $R_s(\tilde{\tau})$ corresponds to optimum throughput at $\tilde{\tau}$.

Proof: The constrained optimization problem is solved by substituting P_{cont} from Lemma 3, determined by applying the outage and the transmit power constraints defined in (16) and (17), in (25). The pdf of $\hat{C}_s$, determined in (24), is used to evaluate the expectation. Following this, we obtain an expression of the expected secondary throughput as a function of τ[9]

$$R_{\mathrm{s}}(\tau) = \frac{T-\tau}{T} \int_0^{\infty} x f_{\hat{\mathrm{C}}_{\mathrm{s}}}(x) dx. \tag{27}$$

Solving numerically the expression in (27) yields $\tilde{\tau}$ and $R_{\mathrm{s}}(\tilde{\tau})$. ■

Corollary 2: Problem 1 considers the optimization of the expected secondary throughput for the proposed framework that employ power control and considers the effect of the imperfect channel knowledge. In accordance to Corollary 1, these USs correspond to those USs that operate in the interference-limited regime $\gamma^* \geq \gamma$. Besides, it is interesting to compare its performance with those CR systems that employ channel estimation (as proposed in the paper) and satisfy the outage constraint on the primary interference, however, employ no power control, i.e., operate at P_{full}. With regard to Corollary 1, these systems correspond to the one operating on the curve $\gamma^* = \gamma$. For the latter approach, the secondary throughput is obtained by substituting P_{cont} with P_{full} in (25), where τ in (25) is determined using Corollary 1. Using such a comparison, we are able to emphasize on the benefits procured the USs by employing power control.

B. Random Channel

Here, our objective is to investigate the performance of the proposed approach, where the interacting channels encounter quasi-static block fading. Following the frame structure in Fig. 2, the alternating transmissions observe a different channel. In this regard, we characterize the channel gains $h_{\mathrm{p},1}$, $h_{\mathrm{p},2}$ and h_{s} according to Nakagami-m fading model. As a consequence, the power gains $|h_{\mathrm{p},1}|^2$, $|h_{\mathrm{p},2}|^2$ and $|h_{\mathrm{s}}|^2$ follow a Gamma distribution [32], whose corresponding cumulative distribution functions are defined as

$$F_{|h_{\mathrm{p},1}|^2}(x) = 1 - \Gamma\left(m_{\mathrm{p},1}, \frac{m_{\mathrm{p},1} x}{|\bar{h}_{\mathrm{p},1}|^2}\right), \tag{28}$$

$$F_{|h_{\mathrm{p},2}|^2}(x) = 1 - \Gamma\left(m_{\mathrm{p},2}, \frac{m_{\mathrm{p},2} x}{|\bar{h}_{\mathrm{p},2}|^2}\right), \tag{29}$$

$$F_{|h_{\mathrm{s}}|^2}(x) = 1 - \Gamma\left(m_{\mathrm{s}}, \frac{m_{\mathrm{s}} x}{|\bar{h}_{\mathrm{s}}|^2}\right), \tag{30}$$

where $m_{\mathrm{p},1}$, $m_{\mathrm{p},2}$ and m_{s} represent the m parameter, whereas $|\bar{h}_{\mathrm{p},1}|^2$, $|\bar{h}_{\mathrm{p},2}|^2$ and $|\bar{h}_{\mathrm{s}}|^2$ are the expected values for channels $|h_{\mathrm{p},1}|^2$, $|h_{\mathrm{p},2}|^2$ and $|h_{\mathrm{s}}|^2$, respectively. $\Gamma(\cdot,\cdot)$ is a regularized upper-incomplete Gamma function [28]. The performance analysis subject to channel fading has been

[9]Please note that $a_{\mathrm{p},2}$ and $b_{\mathrm{p},2}$ are also functions of τ, see (14).

considered by Ghasemi *et al.* [4], [33]. However, the authors in [4], [33] evaluated average data rate under an average interference constraint. The influence of channel fading (however, without channel estimation) has been quantified in terms of the outage constraint on the primary interference, is given by

$$\max_{P_{\text{cont}}} \mathbb{E}_{|h_{\text{p},2}|^2,|h_{\text{s}}|^2}\left[\frac{|h_{\text{s}}|^2 P_{\text{cont}}}{|h_{\text{p},2}|^2 P_{\text{tran}} + \sigma^2}\right], \tag{31}$$

$$\text{s.t. } \mathbb{P}(P_{\text{p}} = |h_{\text{p},1}|^2 P_{\text{cont}} \geq \theta_{\text{I}}) \leq \rho_{\text{out}}, \tag{32}$$

where $\mathbb{E}_{|h_{\text{p},2}|^2,|h_{\text{s}}|^2}[\cdot]$ corresponds to expectation with respect to $|h_{\text{p},2}|^2, |h_{\text{s}}|^2$. Despite the knowledge of the fading model, similar to the ideal model depicted for the deterministic channel (Section II-C), the characterization in (31) and (32) assumes the perfect knowledge[10] of the power gains $(|h_{\text{p},1}|^2, |h_{\text{p},2}|^2, |h_{\text{s}}|^2)$ of the corresponding channels. In view of this, we extend our proposed framework to investigate the effect of the random channel (channel fading) on the performance of the USs.

In this regard, we first determine the expression of the outage constraint on the uncertain interference

$$\overbrace{\mathbb{E}_{|h_{\text{p},1}|^2}\Bigg[\underbrace{\mathbb{P}\left(\left(\frac{\hat{P}_{\text{rcvd,ST}} - \sigma^2}{P_{\text{tran}}}\right) P_{\text{cont}} \geq \theta_{\text{I}}\right)}_{\text{Channel Estimation}}\Bigg]}^{\text{Channel Fading}} \leq \rho_{\text{out}}, \tag{33}$$

$$\tag{34}$$

where $\hat{P}_{\text{rcvd,ST}}$ depends on the underlying value of $|h_{\text{p},1}|^2$. In contrast to the constraint in (32), (33) captures the variations due to the channel estimation ($\mathbb{P}(\cdot)$ determined in terms of $\hat{P}_{\text{rcvd,ST}}$) and the channel fading ($\mathbb{E}_{|h_{\text{p},1}|^2}[\cdot]$).

Based on (33) and transmit power constraint defined in (17), we further obtain the expression of the controlled power (P_{cont}) for the case with random channel.

Lemma 6: Subject to the outage constraint on the uncertain interference and the transmit power constraint at the ST, the controlled power at the ST under Nakagami-m fading is given

[10]For the channel fading case, this knowledge signifies the expected values.

by

$$P_{\text{cont}} = \begin{cases} \text{Solving for } P_{\text{cont}}, \\ \int\limits_0^\infty \Gamma\left(\frac{\tau f_{\text{s}}(1+xP_{\text{tran}}/\sigma^2)^2}{2+4xP_{\text{tran}}/\sigma^2}, \frac{\tau f_{\text{s}}(1+xP_{\text{tran}}/\sigma^2)}{\sigma^2(2+4xP_{\text{tran}}/\sigma^2)}\left(\frac{\theta_{\text{I}} P_{\text{tran}}}{P_{\text{cont}}} + \sigma^2\right)\right) dF_{|h_{\text{p},1}|^2}(x) = \rho_{\text{out}} & \text{if } P_{\text{cont}} < P_{\text{full}} \\ P_{\text{full}} & \text{if } P_{\text{cont}} \geq P_{\text{full}} \end{cases} \quad (35)$$

, where $F_{|h_{\text{p},1}|^2}(\cdot)$ is defined in (28).

Proof: Since it is complicated to obtain a closed form expression of the integral in (35), we evaluate the controlled power numerically. ■

Similar to deterministic channel (in Corollary 1), below, we determine the performance bound (γ^*) in terms of γ and a certain value of estimation time for the random channel. To this end, we substitute P_{cont} with P_{full} in the expression (35)

$$\int\limits_0^\infty \Gamma\left(\frac{\tau f_{\text{s}}(1+xP_{\text{tran}}/\sigma^2)^2}{2+4xP_{\text{tran}}/\sigma^2}, \frac{\tau f_{\text{s}}(1+xP_{\text{tran}}/\sigma^2)}{\sigma^2(2+4xP_{\text{tran}}/\sigma^2)}\left(\frac{\theta_{\text{I}} P_{\text{tran}}}{P_{\text{full}}} + \sigma^2\right)\right) dF_{|h_{\text{p},1}|^2}(x) \leq \rho_{\text{out}}, \quad (36)$$

In order to obtain γ^*, we evaluate the integral on the left side to obtain function of $|h_{\text{p},1}|^2$ and τ. Since no closed form expression of this function is obtained, we represent this function as

$$g(|h_{\text{p},1}|^2, \tau f_{\text{s}}) \leq \rho_{\text{out}}.$$

Substituting $|h_{\text{p},1}|^2 = \frac{\gamma^* \sigma^2}{P_{\text{tran}}}$ and replacing with equality, we determine γ^* for the random channel as

$$g\left(\frac{\gamma^* \sigma^2}{P_{\text{tran}}}, \tau f_{\text{s}}\right) = \rho_{\text{out}}.$$

Remark 3: Fig. 5 analyzes the variation of γ^* with τ for different $m \in \{0.5, 1, 2, 5, \infty\}$, where $m = \infty$ depicts a path-loss channel (or converges to a deterministic channel). It is observed that γ^* attains a lower value as fading becomes more severe, hence enables the USs to operate at low γ by extending the interference-limited regime. Following the analysis from Remark 1, this also reflects that the power control becomes more proactive as the severity in fading increases. In addition, it is noticed that the path-loss channel is more sensitive to the estimation time as compared to the random channels.

Upon determining the controlled power in Lemma 6 that constrains the primary interference, we determine the expression of the secondary throughput.

$$R_{\text{s}}(\tau) = \mathbb{E}_{\hat{\text{C}}_{\text{s}}, |h_{\text{p},2}|^2, |h_{\text{s}}|^2}\left[\frac{T-\tau}{T}\hat{\text{C}}_{\text{s}}\right], \quad (37)$$

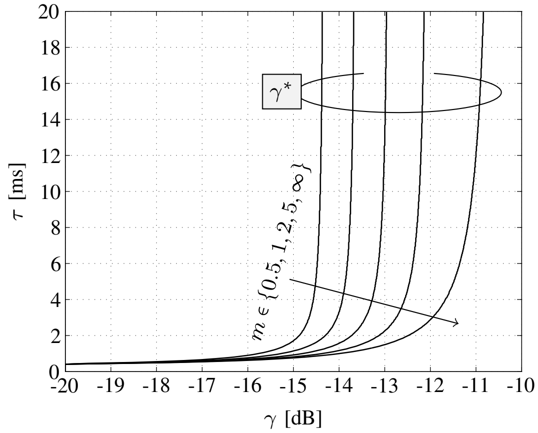

Fig. 5. An extension of the interference-limited and the power-limited regimes for the US to the random channels, where the channels are subject to Nakagami-m fading. The performance bound (γ^*) is depicted in terms of estimation time (τ). The different curves demonstrates the severity ($m \in \{0, 5, 1, 2, 5, \infty\}$) in fading observed by the channels.

where $\mathbb{E}_{\hat{\mathrm{C}}_{\mathrm{s}}, |h_{\mathrm{p,2}}|^2, |h_{\mathrm{s}}|^2}[\cdot]$ corresponds to an expectation over $\hat{\mathrm{C}}_{\mathrm{s}}$, $|h_{\mathrm{p,2}}|^2$ and $|h_{\mathrm{s}}|^2$, whose cdfs are characterized in Lemma 5, (29) and (30), respectively. It is worth noticing that $\hat{\mathrm{C}}_{\mathrm{s}}$ captures the variations due to channel estimation $|\hat{h}_{\mathrm{s}}|^2$ and $\hat{P}_{\mathrm{rcvd,ST}}$, (22), however due to channel fading, the underlying values of the channels $|h_{\mathrm{p,2}}|^2$ and $|h_{\mathrm{s}}|^2$ are random. In this context, we perform an expectation with respect to $|h_{\mathrm{p,2}}|^2$ and $|h_{\mathrm{s}}|^2$ as depicted in (37). Next, we characterize the estimation-throughput tradeoff for the random channel.

Problem 2: The expected achievable secondary throughput subject to the outage constraint on the uncertain interference and the transmit power constraint at the ST under Nakagami-m fading is defined as

$$R_{\mathrm{s}}(\tilde{\tau}) = \max_{\tau} \; R_{\mathrm{s}}(\tau), \qquad (38)$$
$$\text{s.t. } (33), \; (17),$$

where $R_{\mathrm{s}}(\tilde{\tau})$ corresponds to optimum throughput at $\tilde{\tau}$.

Corollary 3: Here, we extend the approach depicted Corollary 2 to compare the performance with those CR systems that employ channel estimation, operate at constant power, satisfy the outage constraint and are subjected to Nakagami-m fading. The expected secondary throughput for this particular approach is obtained by replacing P_{cont} in the expression in (37) with P_{full}, where τ is determined using (36).

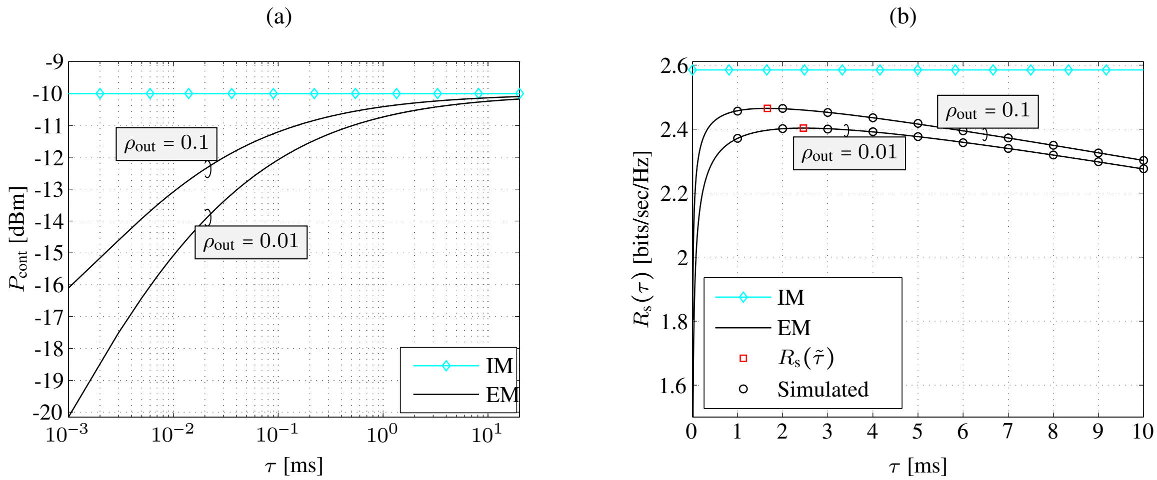

Fig. 6. (a) Control power versus estimation time with $\gamma = 0\,$dB, $\rho_{out} \in \{0.01, 0.1\}$ and $P_{full} = 0\,$dBm. (b) Estimation-throughput tradeoff with $\gamma = 0\,$dB, $\rho_{out} \in \{0.01, 0.1\}$ and $P_{full} = 0\,$dBm.

IV. Numerical Analysis

Here, we evaluate the performance of the US based on the proposed model. To accomplish this: (i) we perform simulations to validate the expressions obtained, (ii) we analyze the performance loss incurred due to the estimation. In addition, we consider the ideal model to benchmark and evaluate the performance loss. Unless stated explicitly, the parameters are considered for the analysis, $f_s = 1\,$MHz, $|h_{p,1}|^2 = -100\,$dB, $|h_{p,2}|^2 = -100\,$dB, $|h_s|^2 = -80\,$dB, $\theta_I = -110\,$dBm, $T = 100\,$ms, $\rho_{out} = 0.10$, $P_{full} = 0\,$dBm, $\sigma^2 = -100\,$dBm, $\gamma = 0\,$dB, $P_{tran} = 0\,$dBm, $N_s = 10$ and $m \in \{1, 5\}$.

A. Deterministic Channel

At first, we evaluate the performance of the proposed framework in context to the deterministic channel. Fig. 6a considers the variation of P_{cont} versus τ, Corollary 1. It is noticed that the ST controls its transmit power (P_{cont}) more severely for low values of τ, consequently affecting the link budget for the access channel. Besides, Fig. 6b analyzes the performance of the US in terms of the estimation-throughput tradeoff, Problem 1, corresponding to the Ideal Model (IM) and the proposed Estimation Model (EM). It can be depicted that the estimation-throughput tradeoff yields a suitable estimation time $\tilde{\tau}$ that results in an optimum throughput $R_s(\tilde{\tau})$. Hereafter, for the performance analysis with respect to the deterministic channel, we consider the theoretical expressions and choose to operate at the suitable estimation time.

To procure further insights, it is necessary to consider the variation of $R_s(\tilde{\tau})$ with γ with dif-

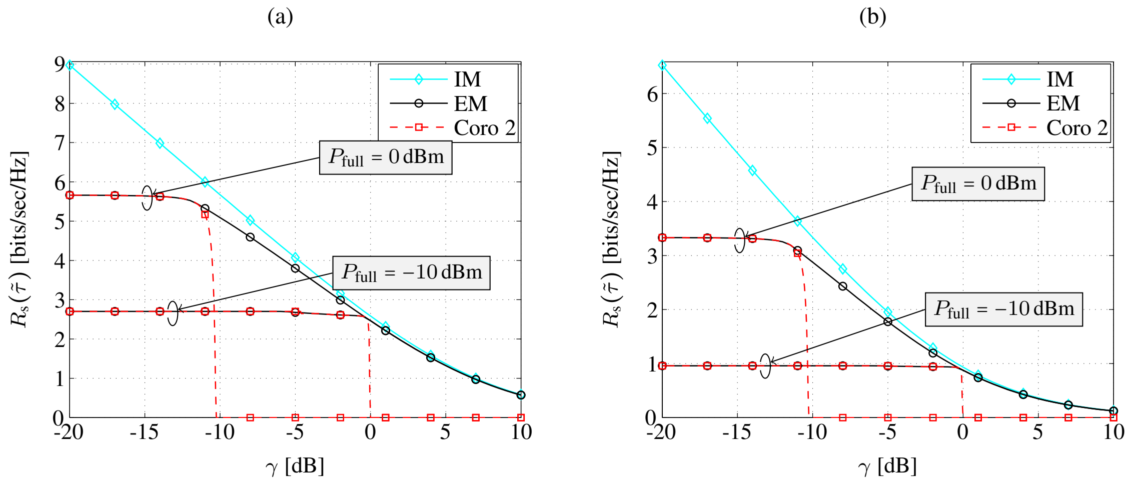

Fig. 7. Optimum throughput $(R_s(\tilde{\tau}))$ versus the ratio of the received power to noise (γ) with $\rho_{\text{out}} = 0.1$ and $P_{\text{full}} \in \{-10, 0\}$dBm for (a) $|h_{\text{p},2}|^2 = -100$ dBm and (b) $|h_{\text{p},2}|^2 = -90$ dBm, which translate to an interference power (from the PT) to noise ratio of (a) 0 dB and (b) 10 dB, respectively, at the SR.

ferent choices of the secondary interference at the SR (regulated using $|h_{\text{p},2}|^2 \in \{-90, -100\}$dBm), Fig. 7. It is observed that due to the employed transmit power constraint, $R_s(\tilde{\tau})$ gets saturated below a certain γ, thereby limiting the performance of the US, depicted in Corollary 2. Upon increasing P_{full} from -10 dB to 0 dB, the point where saturation is achieved shifts to a lower γ. This is due to the fact that higher P_{full} extends the interference-limited regime to a lower γ. In this context, it can be stated that the secondary system exploits the benefits of operating at a high controlled power because of a low γ. Particularly for $P_{\text{full}} = -10$ dBm, a severe performance loss indicated by the margin between the IM and the EM is witnessed by the US for $\gamma \leq -2$ dB. This signifies that the consideration of the maximum transmit power of the ST is essential while designing the system. Besides this, Fig. 7 depicts the performance of the USs with no power control, proposed in Corollary 2. As indicated in Fig. 3, beyond a certain $\gamma = \gamma^*$, the USs with no power control delivers no throughput. In order to avoid such situations, the US can exercise power control in order to deliver non-zero throughput.

B. Random Channel

Here, we evaluate the performance of the proposed framework, where the interacting channels are under the influence of Nakagami-m fading. For simplification of the analysis, we assume the m parameter to be same for all the involved channels. In addition, we investigate the performance under following fading scenarios: (i) severe fading $m = 1$, which corresponds to

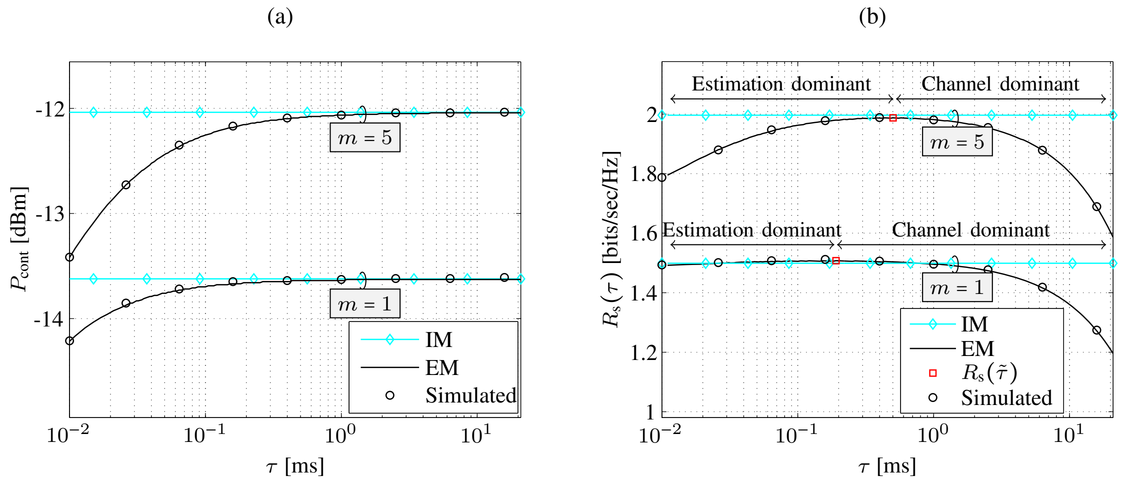

Fig. 8. (a) Control power versus estimation time with $\gamma = 0\,$dB, $\rho_{\text{out}} = 0.1$ and $P_{\text{full}} = 0\,$dBm, where Nakagami-m fading. (b) Estimation-throughput tradeoff with $\gamma = 0\,$dB, $\rho_{\text{out}} = 0.1$ and $P_{\text{full}} = 0\,$dBm with Nakagami-m fading. The plot classifies the estimation time into the estimation-dominant and the channel-dominant regime

Rayleigh fading[11], and (ii) mild fading $m = 5$. First, we analyze the variation of P_{cont} along the estimation time. It is observed that the mild fading scenario ($m = 5$) is more sensitive to the estimation time, see Fig. 8a. Furthermore, comparing analysis for the deterministic channel in Fig. 6a, the power control according to the EM saturates with IM at a smaller τ.

Besides, we capture the influence of the random channels on the performance in terms of the estimation-throughput, cf. Problem 2. In this regard, the estimation-throughput tradeoff for corresponding to the fading scenarios is illustrated in Fig. 8b. Similar to the case with the deterministic channel, it is depicted that for a suitable choice of the estimation time, the performance of the proposed framework that captures the imperfect channel knowledge is comparable to the ideal conditions in terms of the achievable secondary throughput. Since the USs are subjected to the variations from the estimation and fading, we classify the estimation time into an estimation-dominant regime and a channel-dominant regime. These regimes signify that the estimation time can only reduce the imperfections (incurred in the USs) due to the channel estimation, however, beyond a certain estimation time ($\tilde{\tau}$), the time resources allocated for channel estimation slightly contributes to the performance improvement (in terms of the power control, which finally affect the secondary throughput) and largely to the performance degradation (due to the factor $\frac{T-\tau}{T}$ in

[11] Please note that the intention here is to consider the impact of severity in fading on the performance of the US with regard to the channel estimation. The value $m = 1$, which corresponds to a Rayleigh fading, is an obvious representative of a severe fading scenario.

(a) (b)

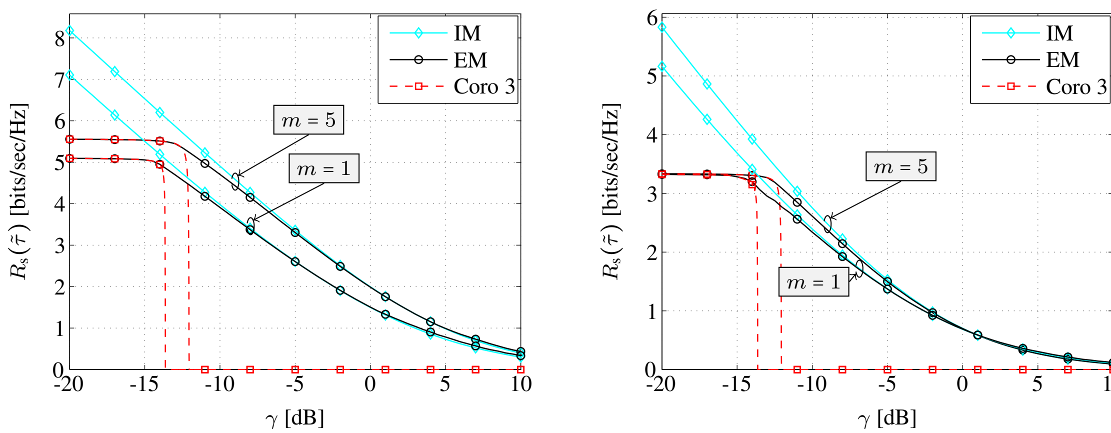

Fig. 9. Optimum throughput ($R_s(\tilde{\tau})$) versus the ratio of the received power to noise (γ) for Nakagami-m fading with $\rho_{\text{out}} = 0.1$ and $P_{\text{full}} = 0$ dBm for (a) $|h_{\text{p,2}}|^2 = -100$ dBm and (b) $|h_{\text{p,2}}|^2 = -90$ dBm, which correspond to an interference power (from the PT) to noise ratio of (a) 0 dB and (b) 10 dB, respectively, at the SR.

(37)) in the secondary throughput.

Upon determining the optimum secondary throughput ($R_s(\tilde{\tau})$) using the estimation-throughput tradeoff, we consider the variation of the $R_s(\tilde{\tau})$ along the received signal to noise ratio for different choices of the secondary interference in Fig. 9. It is observed that for a large range ($\gamma \geq -10$ dB), the optimum secondary throughput determined by the EM closely follows the throughput depicted by the IM. In addition, Fig. 9 considers the performance of the USs with no power control, Corollary 3. Following the discussion in Remark 3, it was indicated the performance bound (γ^*) shifts to a lower γ when fading becomes severe, Fig. 5. This effect is finally translated to the secondary throughput, where $m = 1$ approaches the region with no throughput at a lower γ as compared to $m = 5$, consider Fig. 9.

V. Conclusion

In this paper, we studied the performance of USs from a deployment perspective by laying emphasis on the fact that the knowledge of the interacting channels is pivotal to the CR systems. In this view, a novel approach that incorporates estimation of the involved channels at the secondary system has been proposed. Considering the time resources utilized for the channel estimation and the uncertainty due its imperfect knowledge in the analysis, it been shown that the channel estimation has a detrimental effect on the performance, leading to its degradation. To tackle the uncertain interference, an outage constraint that precisely regulates the uncertain interference at the PR has been employed. Besides, it has been observed that the imposition of power control at the ST is limited by the maximum transmit power. This limitation, complementing with the channel estimation has been studied in terms of the interference-limited and the power-limited regimes to determine the performance bounds of the USs. As a result, a power control mechanism subject to the outage and the transmit power constraints has been proposed. Finally, from the perspective of a system designer, an estimation-throughput tradeoff has been established that allow us to determine the achievable secondary throughput for the US. In consideration to the channel fading, it has been observed that the performance degradation is highly prone to scenarios that are subject to mild fading.

In our future work, we plan to extend the proposed analysis to capture the influence of multiple primary and secondary users present in the network on the performance of the underlay systems. In addition, the analysis presented in the paper considers symmetric fading, i.e., the channel gains are subjected to the same value of m (which means $m_{\mathrm{p},1} = m_{\mathrm{p},2} = m_{\mathrm{s}}$). However, depending on the deployment scenario, the derived expressions can be utilized to realize asymmetric fading ~~(proposed in [5])~~ by substituting different values of m corresponding to ~~the~~ different channels. In this regard, we extend the proposed framework to study the influence of asymmetric fading on the performance in our future work.

Appendix
Proof of Lemma 5

Proof: For simplification, we consider $\frac{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{tran}}}{\hat{P}_{\mathrm{rcvd,SR}}}$ as individual expressions $|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{tran}}$ and $\hat{P}_{\mathrm{rcvd,SR}}$ and determine the pdf for $f_{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{tran}}}$ and $f_{\hat{P}_{\mathrm{rcvd,ST}}}$ separately. Using (11) in Lemma 2, the pdf of $f_{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{tran}}}$ is determined as

$$f_{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{tran}}}(x) = \frac{1}{\Gamma(a_{\mathrm{s}})(b_{\mathrm{s}} P_{\mathrm{tran}})^{a_{\mathrm{s}}}} x^{a_{\mathrm{s}}-1} \exp\left(-\frac{x}{b_{\mathrm{s}} P_{\mathrm{tran}}}\right), \tag{39}$$

where a_{s} and b_{s} are defined in (12). Similarly, using Lemma 3, the pdf of $f_{\hat{P}_{\mathrm{rcvd,SR}}}$ is characterized as

$$f_{\hat{P}_{\mathrm{rcvd,ST}}}(x) = \frac{1}{\Gamma(a_{\mathrm{p},2})(b_{\mathrm{p},2})^{a_{\mathrm{p},2}}} x^{a_{\mathrm{p},2}-1} \exp\left(-\frac{x}{b_{\mathrm{p},2}}\right), \tag{40}$$

where $a_{\mathrm{p},2}$ and $b_{\mathrm{p},2}$ are defined in (14).

Using (39) and (40), we apply Mellin transform [34] to determine the pdf of $\frac{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{tran}}}{\hat{P}_{\mathrm{rcvd,SR}}}$ as

$$f_{\frac{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{tran}}}{\hat{P}_{\mathrm{rcvd,SR}}}}(x) = \frac{(x)^{a_{\mathrm{s}}-1}\Gamma(a_{\mathrm{s}} + a_{\mathrm{p},2})}{\Gamma(a_{\mathrm{s}})\Gamma(a_{\mathrm{p},2})(b_{\mathrm{s}} P_{\mathrm{tran}})^{a_{\mathrm{s}}} b_{\mathrm{p},2}^{a_{\mathrm{p},2}}} \left(\frac{1}{b_{\mathrm{p},2}} + \frac{x-1}{b_{\mathrm{s}} P_{\mathrm{tran}}}\right). \tag{41}$$

Finally, substituting the expression $\frac{|\hat{h}_{\mathrm{s}}|^2 P_{\mathrm{tran}}}{\hat{P}_{\mathrm{rcvd,SR}}}$ in C_{s} yields (24). ■

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