Sensing-Throughput Tradeoff for Cognitive Radio Systems: A Deployment-Centric Viewpoint

A. Kaushik*, S. K. Sharma†, S. Chatzinotas†, B. Ottersten†, F. K. Jondral*
*Communications Engineering Lab, Karlsruhe Institute of Technology (KIT), Germany
†SnT - securityandtrust.lu, University of Luxembourg, Luxembourg
Contact Information: Ankit.Kaushik@kit.edu

Abstract

Understanding the true performance of a cognitive radio system is a challenging task. In order to enable secondary access to the licensed spectrum, several paradigms has been conceptualized. Of these, interweave systems that employ spectrum sensing has been widely investigated in the literature. According to which, the interference is avoided by sustaining probability of detection above a desired level. Upon satisfying the constraint, the secondary transmitter intends to optimize its throughput. In literature, this optimization has been characterized in terms of sensing-throughput tradeoff. However, this characterization depicts an ideal scenario, in practice, the system requires the knowledge of the interacting channels at the secondary transmitter. This knowledge is essential, if we consider a hardware deployment. Motivated by this fact, we propose a novel system model that incorporate channel estimation. However, with the inclusion of channel estimation, distortion is induced in the system. More specifically, the distortion in the probability of detection may increase the interference at the primary users. In order to capture these distortions, we employ average and outage constraint on the probability of detection. Based on these constraints, we investigate the true performance of the interweave system. Following the analysis, we observe that ideal scenario overestimates the performance of the system.

I. Introduction

We are currently in the phase of conceptualizing the requirements of the fifth-generation (5G) of wireless standards. One of the major requirement includes the improvement in the areal capacity (bits/s/m^2) by a factor of 1000 [1]. A large contribution of this demand is procured by means of an extension to the existing spectrum. Due to the static allocation of the spectrum specially below 5 GHz which is appropriate for mobile communications, it is on the verge of scarcity. Given the spectrum is utilized efficiently, it is possible to overcome this scarcity in the spectrum. In this perspective, cognitive radio is envisaged as one of the viable solutions that addresses the problem of spectrum scarcity ~~below 5 GHz.~~

An access to the Primary User (PU) spectrum is an outcome to the strategy employed by the Secondary User (SU). Based on the strategies described in the literature, all cognitive radio systems dealing with an dynamic access to the spectrum fall mainly under three categories namely, interweave, underlay and overlay [2]. In Interweave Systems (IS), the SUs render an interference-free access to the PU by exploiting spectral holes in different domains such as time, frequency, space and polarization existing in the licensed spectrum, whereas underlay systems enable an interference-tolerant access under which the SUs are allowed to use the primary spectrum as long as they respect the interference constraints of the Primary Receivers (PRs). Besides that, overlay systems consider participation of higher layers for enabling spectral coexistence between two or more wireless networks. Due to its ease in deployment, IS is mostly preferred for performing analysis among these paradigms. In this context, this paper focuses on the performance analysis of the ISs from a deployment perspective.

Motivation and Related Work

Sensing is an integral part of the IS. At the Secondary Transmitter (ST), sensing is necessary for detecting the presence and absence of a primary signal, thereby protecting the PRs against harmful interference. Sensing at the ST is accomplished by listening to the power received from the PT. For detecting a primary signal, several techniques such as energy detection, matched filtering, cyclostationary and feature-based detection exist [3], [4]. Because of its versatility towards unknown primary signals, energy detection has been extensively investigated in the literature [5]–[9]. According to energy detection, the decision is accomplished by comparing the power received at the ST to a threshold. In reality, the ST encounters variation in the received power due to existence of thermal noise at the receiver and fading in the channel. This leads to sensing errors described as misdetection or false alarm. These sensing errors limit the performance of the IS. Therefore, it is essential to characterize the expressions of probability of detection and probability of false alarm.

In particular, probability of detection is critical for the primary system because it precludes the PR from the interference induced by the ST. As a result, sustaining a desired value of probability of detection is of paramount importance to the secondary system [10]. Hence, it is necessary to characterize the probability of detection. Urkowitz [5] introduced a probabilistic model that establishes a fundamental framework for characterizing the sensing errors, however, the characterization consisted of only noise in the system. Besides noise, including the effect of fading in the system model is a challenging task. In this regard, different approaches has been investigated in the literature. Most prevalent approach involves an employment of a fading model. With this, it is possible to average out the variation in received power arising because of fading. Some of the most common models used for modelling fading include – Rice, Rayleigh, Nakagami-m and Log-normal [11]. Following this approach, several studies [6]–[8] came forward to determine analytical expression of expected probability of detection or distribution function of the received power. From the perspective of a cognitive radio system, this approach has some major drawbacks. Fading models depict a long-term characterization of the system, however ignores severe interference for short durations that may lead to an outage of the primary systems. Moreover, fading models are specific to underlying scenario, hence, knowledge of the fading model is necessary at the ST. Apart from that, fading model incurs certain model parameters. As a result, estimation of these parameters at the ST [12], particularly during the initial phase of the deployment, depicts an additional overhead. Therefore, imposes a certain delay in the system. In practice, due to mobility, most systems are likely to disobey the stationary condition, thus over time, it becomes challenging to track these model parameters. Consequently, these drawbacks constrict the applicability of this approach to cognitive radio systems.

To overcome these drawbacks, a different approach that exploits the coherence property of the channel in time domain is applied. Whereby, a frame structure is introduced such that the channel is considered to remain constant for the frame duration. Clearly, this approach precludes itself from estimating or tracking the model parameters. Hence, following this short-term approach, it is feasible to characterize the performance of the system for each frame. In this paper, we apply this approach for the performance characterization. Irrespective of the approach followed, the knowledge of the received power is an absolute paramount for characterizing the probability of detection. Most previous works assume this knowledge is available at the ST. In practice, this knowledge is not available, thus, needs to be estimated. Hence, characterization of the probability of detection with received power estimation is an open problem.

Apart from the PU constraint defined using probability of detection, probability of false alarm accounts for the throughput attained by the secondary system at the Secondary Receiver (SR). The characterization of the probability of false alarm requires the knowledge of noise power. Subject to a given uncertainty [13], this knowledge can be acquired through hardware calibration. Upon sustaining the probability of detection at a desired level, the ST aims to optimize its throughput. This phenomenon is characterized as a sensing-throughput tradeoff by Liang *et. al.* [14]. At the ST, the sensing-through tradeoff renders a suitable sensing time that achieves an optimum throughput for a given received power. Several contributions [14]–[17]

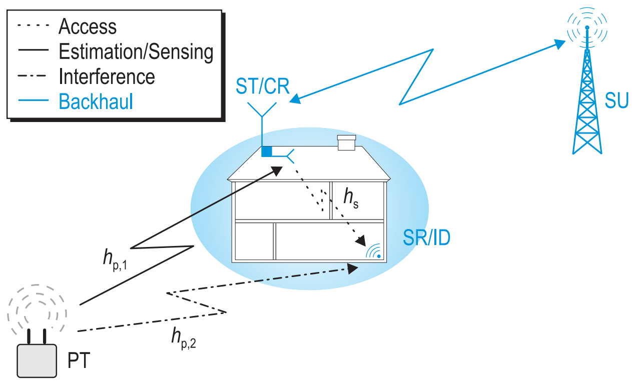

Fig. 1. A scenario demonstrating the interweave paradigm.

have considered this tradeoff to characterize the performance of the IS. These contributions investigated in the literature assume the knowledge of the interacting channels at the ST, which include an access link from the ST and an interference link from PT to the SR, hence, are considered idealistic and implausible for deployment. In practice, the knowledge of these channels is not available, thus, needs to be estimated by the SR and made available to the ST through a feedback channel. Moreover, due to the presence of noise in the system, this knowledge is never perfect.

In order to breakthrough these bottlenecks, following solutions are proposed in the paper. Firstly, we consider received power estimation at the ST that allows us to constrain the probability of detection at a desired level. In accordance with the short-term approach, to realize received power estimation at the ST, we allocate a certain time interval for the estimation within the frame duration. With the inclusion of estimation in the system, a certain performance loss in terms of (i) temporal resources and (ii) estimation error is anticipated. A preliminary analysis of this performance loss was done in [18], where, it was shown that at low signal to noise ratio, imperfect estimation of received power results in large distortion in probability of detection, hence, causing severe degradation in the performance of IS. However, the performance degradation was determined by means of lower and upper bounds. Herein, we consider a more exact analysis, whereby we capture the variations in probability of detection by characterizing its distribution function. Using this, we apply new probabilistic constraints on the probability of detection that allow IS to operate at low signal to noise ratio. These constraints are categorized as average and outage constraints corresponds to an aggressive or conservative approach followed by the SU towards the primary system.

Besides that, we include channel estimation at the SR to acquire the knowledge of the interacting channels. Those systems that entails transmitter information (such as matched filter, pilot symbols, modulation type and time-frequency synchronization) at the receiver retreive this knowledge directly by listening to the pilot data sent by the transmitter [19]–[22]. Other systems with either no access to this information or requiring minimum overhead at the receiver procure channel knowledge indirectly, thereby estimating a different parameter that entails the channel gain. These parameters correspond to received signal strength or received power. Recently, pilot based estimation [23], [24] and received power estimation [25] have been applied to procure channel information and depict the performance of cognitive systems. However, the analysis was restricted to underlay systems, where the emphasis was laid on modelling the interference at the PR. In this paper, we extend this notion to IS and employ pilot based estimation for the access link and received power based estimation for the interference link, cf. Fig. 1. Upon acquiring the knowledge of received power, access channel and the interference channel at the ST, we depict the performance loss due to the estimation for the IS in terms of sensing-throughput tradeoff.

Contributions

The major contributions of the paper can be summarized as follows:

- The main goal of the analysis is to establish a system model that includes received power estimation and channel estimation. With the inclusion of estimation in the system, a certain amount to performance loss is incurred in the system. By characterizing the estimation of these parameters according to proposed model, we investigate the true performance of the IS.
- To capture the effect of received power estimation on the probability of detection on the system, we characterize the distribution function of the probability of detection. Based on this, we employ either an average constraint or an outage constraint as a primary user constraint on the probability of detection. Furthermore, to include the effect of channel estimation on the SR's throughput, we characterize the density function for the capacity.
- We consider sensing-throughput tradeoff to optimize the throughput corresponding to the sensing time while sustaining the underlying constraints.
- Additionally, we depict an inherent tradeoff between estimation time, sensing time and throughput. We exploit this tradeoff to determine a suitable estimation time that characterizes an optimal performance.

Organization

The subsequent sections of the paper are organized as follows:

II. System Model

Interweave scenario

Cognitive Relay (CR) [26] characterizes a small cell deployment that fulfills the spectral requirements for Indoor Devices (IDs). Fig. 1 illustrates a snapshot of a CR scenario to depict the interaction between the CR with PT and ID, where CR and ID represents the ST and SR respectively. In [26], the challenges involved while deploying the CR as an IS were presented. For simplification, a constant false alarm rate was considered in the system model. Now, we extend the analysis to employ a constant detection rate. We consider a slotted medium access for the IS, where the time axis is segmented into frames of length T. The frame structure is analog to the one illustrated in [14]. According to which, the ST employs periodic sensing, hence, each consists of a sensing slot τ_{sen} and the remaining duration $T - \tau_{\text{sen}}$ is utilized for data transmission. With a relatively small choice of T, allignment of the IS to primary system's medium access is not mandotory.

Signal model

The discrete signal received illustrating the presence $(\mathcal{H}_1)$ or absence $(\mathcal{H}_0)$ of a primary signal is given by

$$y_{\text{ST}}[n] = \begin{cases} h_{\text{p,1}} \cdot x_{\text{PT}}[n] + w[n] & : \mathcal{H}_1 \\ w[n] & : \mathcal{H}_0 \end{cases}, \tag{1}$$

where $x_{\text{PT}}[n]$ corresponds to a discrete sample transmitted by the PT, $|h_{\text{p,1}}|^2$ represents the power gain for the channel for a specific frame and $w[n]$ is circularly symmetric complex Additive White Gaussian Noise (AWGN) at the ST. x_{PT} is an i.i.d. (independent and identically distributed) random process. As the channel $h_{\text{p,1}}$ is independent to $x_{\text{PT}}[n]$ and $w[n]$ is an i.i.d. Gaussian random process with zero mean and variance $\mathbb{E}\left[|w[n]|^2\right] = \sigma^2$, the y_{ST} is also an i.i.d. random process.

Similar to (1), during data transmission, the discrete received signal at the SR conditioned over the probability of detection (P_{d}) and probability of false alarm (P_{fa}) is given by

$$y_{\text{ST}}[n] = \begin{cases} h_{\text{s}} \cdot x_{\text{ST}}[n] + h_{\text{p,2}} \cdot x_{\text{PT}}[n] + w[n] & : 1 - \text{P}_{\text{d}} \\ h_{\text{s}} \cdot x_{\text{ST}}[n] + w[n] & : 1 - \text{P}_{\text{fa}} \end{cases}, \tag{2}$$

where $x_{\text{ST}}[n]$ is an i.i.d. random process and corresponds to discrete signal transmitted by the ST. Further, $|h_{\text{s}}|^2$ and $|h_{\text{p,2}}|^2$ represent the power gains for channel, cf. Fig. 1.

Sensing

Following the frame structure, ST performs sensing for a duration of τ_{sen}, cf. Fig. 2. Test statistics $\mathrm{T}(\mathbf{y})$ at the ST is evaluated as

$$\mathrm{T}(\mathbf{y}) = \frac{1}{\tau_{\text{sen}} f_{\text{s}}} \sum_{n}^{\tau_{\text{sen}} f_{\text{s}}} |y_{\text{ST}}[n]|^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \mu, \tag{3}$$

where μ is the threshold and $\mathbf{y}$ is a vector with $\tau_{\text{sen}} f_{\text{s}}$ samples. $\mathrm{T}(\mathbf{y})$ represents a random variable, whereby the characterization of the distribution function depends on the underlying hypothesis. Corresponding to $\mathcal{H}_0$ and $\mathcal{H}_1$, $\mathrm{T}(\mathbf{y})$ follows a central chi-squared $\mathcal{X}^2$ and a non-central chi-squared $\mathcal{X}_1^2$ distribution [27].

Considering large number of samples $f_{\text{s}}\tau_{\text{sen}}$ are used to evaluate $\mathrm{T}(\mathbf{y})$. It is possible to apply Central Limit Theorem to approximate the distribution functions of the mentioned hypotheses as Gaussian distribution. In this sense, the probability of detection P_{d} and the probability of false alarm P_{fa} corresponding to (3) is determined as [13]

$$\mathrm{P}_{\text{d}}(\mu, \tau_{\text{sen}}, P_{\text{Rx,ST}}) = \mathcal{Q}\left(\frac{\mu - P_{\text{Rx,ST}}}{\sqrt{\frac{2}{\tau_{\text{sen}} f_{\text{s}}}} P_{\text{Rx,ST}}} \right), \tag{4}$$

$$\mathrm{P}_{\text{fa}}(\mu, \tau_{\text{sen}}) = \mathcal{Q}\left(\frac{\mu - \sigma^2}{\sqrt{\frac{2}{\tau_{\text{sen}} f_{\text{s}}}} \sigma^2} \right), \tag{5}$$

where $\mathcal{Q}(\cdot)$ represents Q-function [28].

Sensing-Throughput tradeoff

Following the characterization of P_{fa} and P_{d}, Liang *et. al.* [14] established a fundamental tradeoff between the sensing time and secondary throughput attained subject to a desired probability of detection $\bar{\mathrm{P}}_{\text{d}}$. This tradeoff is represented as

$$\tilde{R}_{\text{s}}(\tilde{\tau}_{\text{sen}}) = \max_{\tau_{\text{sen}}} R_{\text{s}}(\tau_{\text{sen}}) = \frac{T - \tau_{\text{sen}}}{T} \Big[C_0 (1 - \mathrm{P}_{\text{fa}}) \mathrm{P}(\mathcal{H}_0) + C_1 (1 - \mathrm{P}_{\text{d}}) \mathrm{P}(\mathcal{H}_1) \Big], \tag{6}$$

$$\text{s.t. } \mathrm{P}_{\text{d}} \geq \bar{\mathrm{P}}_{\text{d}}, \tag{7}$$

$$\text{where } C_0 = \log_2\left(1 + |h_{\text{s}}|^2 \frac{P_{\text{Tx,ST}}}{\sigma^2}\right) \tag{8}$$

$$\text{and } C_1 = \log_2\left(1 + \frac{|h_{\text{s}}|^2 P_{\text{Tx,ST}}}{|h_{\text{p,2}}|^2 P_{\text{Tx,PT}} + \sigma^2}\right) = \log_2\left(1 + \frac{|h_{\text{s}}|^2 P_{\text{Tx,ST}}}{P_{\text{Rx,SR}}}\right). \tag{9}$$

$\mathrm{P}(\mathcal{H}_0)$ and $\mathrm{P}(\mathcal{H}_1)$ are the probabilities of occurrence for the respective hypothesis. In other words, using (6), the ST determines a sensing time $\tau_{\text{sen}} = \tilde{\tau}_{\text{sen}}$, such that the throughput is optimized $\tilde{R}_{\text{s}}$ subject to

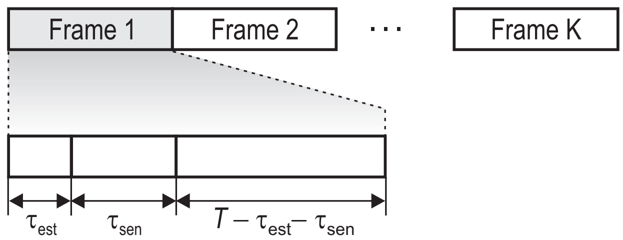

Fig. 2. Frame structure of interweave system with received power estimation.

a desired probability of detection, given by (7). From the deployment perspective, the tradeoff depicted above has following issues:

- Without the knowledge of the received power, it is not feasible to characterize $\mathrm{P_d}$. This further leaves the characterization of the throughput (6) impossible and the constraint defined in (7) invalid.
- For characterizing the throughput in terms of C_0 and C_1 at the SR, the knowledge of the interacting channels is required at the ST, cf. (8) and (9).

With these issues, it is unreasonable to depict the performance of IS based on such characterization. In the subsequent section, we propose an estimation model that complements these issues, thereby including the estimation of received power and interacting channels at the ST. Based on the proposed model, we investigate the performance of the IS.

III. Estimation Model

Estimation of Received Power

Unlike [14], the proposed frame structure uses τ_{est} to estimate and τ_{sen} to sense the received power, where $\tau_{\mathrm{est}}, \tau_{\mathrm{sen}}$ correspond to time intervals and $\tau_{\mathrm{est}} + \tau_{\mathrm{sen}} < T$, cf. Fig. 2. Characterized by the fading process, each frame witnesses a different received power. Therefore, to sustain a desired probability of detection, it is important to perform estimation τ_{est} followed by sensing τ_{sen} for each frame. The remaining time $T - (\tau_{\mathrm{est}} + \tau_{\mathrm{sen}})$ is utilized for data transmission.

In the estimation phase, the received power estimated at the ST is given as [5]

$$\hat{P}_{\mathrm{Rx,ST}} = \frac{1}{\tau_{\mathrm{est}} f_{\mathrm{s}}} \sum_{n}^{\tau_{\mathrm{est}} f_{\mathrm{s}}} |y_{\mathrm{ST}}[n]|^2. \tag{10}$$

$P_{\mathrm{Rx,ST}}$ determined in (10) using $\tau_{\mathrm{est}} f_{\mathrm{s}}$ samples follows a non central chi-squared distribution $\mathcal{X}_1^2$ [27]. Applying the central limit theorem, this distribution can be approximated as the Gaussian distribution

$$\hat{P}_{\mathrm{Rx,ST}} \sim \mathcal{N}\left(P_{\mathrm{Rx,ST}}, \frac{2}{\tau_{\mathrm{est}} f_{\mathrm{s}}} P_{\mathrm{Rx,ST}}^2\right). \tag{11}$$

Estimation of Interacting Channels

Besides estimation of the received power at the ST, we consider the channel estimation of $h_{\mathrm{p},2}$ and h_{s} at the SR, cf. Fig. 1. First, we consider the estimation of h_{s}. As the signal received from the ST undergoes matched filtering and demodulation at the SR, hence, it is reasonable to consider pilot based estimation for h_{s}. Now, to accomplish pilot based channel estimation, the SR aligns itself to pilot symbols transmitted by the ST. Without loss of generality, the pilot symbols are considered to be +1. The simplified signal model of the discrete pilot symbols at the output of the demodulator is given by [21]

$$p[n] = \sqrt{E_{\mathrm{s}}} h_{\mathrm{s}} + w[n], \tag{12}$$

where the $w[n]$ represents a $\mathcal{CN}(0, \sigma^2)$ and E_{s} denotes the pilot energy. The maximum likelihood estimate, representing a sample average of N_{s} pilot symbols, is given by [20]

$$h_{\mathrm{s}} = \hat{h}_{\mathrm{s}} + \underbrace{\frac{\sum_i^{N_{\mathrm{s}}} n[i]}{\sqrt{E_{\mathrm{s}}} N_{\mathrm{s}}}}_{\epsilon}, \tag{13}$$

where ϵ denotes the estimation error. $\hat{h}_{\mathrm{s}}$ is unbiased efficient and achieves a Cramer-Rao bound with equality, with variance $\sigma_\epsilon^2 = \mathbb{E}\left[|h_{\mathrm{s}} - \hat{h}_{\mathrm{s}}|^2\right] = \sigma^2/(2E_{\mathrm{s}}N_{\mathrm{s}})$ [21]. Consequently, $\hat{h}_{\mathrm{s}}$ conditioned on h_{s} follows a Gaussian distribution

$$\hat{h}_{\mathrm{s}} | h_{\mathrm{s}} \sim \mathcal{N}(h_{\mathrm{s}}, \sigma_\epsilon^2) \tag{14}$$

As a result, the power gain $|\hat{h}_{\mathrm{s}}|^2$ follows a $\mathcal{X}_1^2(\lambda_{\mathrm{s}}, 1)$ distribution, where $\lambda_{\mathrm{s}} = \frac{|h_{\mathrm{s}}|^2}{\sigma_\epsilon^2}$.

In addition, under $\mathcal{H}_0$, the SR performs received power estimation by listening to the transmission from the PT. This is done to characterize interference from the PT. The discrete signal model at the SR is given as

$$y_{\mathrm{ST}} = h_{\mathrm{p},2} \cdot x_{\mathrm{PT}}[n] + w[n]. \tag{15}$$

Analog to $\hat{P}_{\mathrm{Rx,ST}}$, the received power at the SR from the PT follows

$$\begin{aligned} \hat{P}_{\mathrm{Rx,SR}} &= \frac{1}{N_{\mathrm{p}}} \sum_n^{N_{\mathrm{p}}} |h_{\mathrm{s}} x_{\mathrm{PT}}[n] + w[n]|^2 \\ &\sim \mathcal{X}_1^2(\lambda_{\mathrm{p}}, N_{\mathrm{p}}), \end{aligned} \tag{16}$$

where $\lambda_{\mathrm{p}} = N_{\mathrm{p}} \frac{P_{\mathrm{Tx,PT}}}{\sigma^2}$ and N_{p} corresponds to degree of freedom or number of samples used for estimation.

Assumptions

To simplify the analysis and sustain analytical tractability for the proposed model, several assumptions considered in the paper are summarized as follows:

- We consider that all transmitted signals are subjected to distance dependent path loss and the small scale fading gains. The coherence time for the channel gain is greater than the frame duration. With no loss of generality, we consider that the channel gains include distance dependent path loss and small scale gain.
- We consider disjoint sets of samples for estimation and sensing for a certain frame. However, in practice, it is possible to utilize the samples used in the estimation phase for sensing purpose as well, which leads to an improvement in the performance in terms of the number of samples utilized for sensing.
- We apply Central limit theorem to approximate the non central chi-squared distribution as Gaussian distribution [13].
- We approximate non-central chi-squared distribution as Gamma distribution [29].

The estimation of received power at the ST and interacting channels translate to the distortion in the performance parameters $\mathrm{P_d}$ and $\tilde{R}_\mathrm{s}$. This distortion is captured by characterizing distribution function function for $f_{\mathrm{P_d}}$ and density functions for f_{C_0} and f_{C_1}.

Lemma 1: The distribution function of $\mathrm{P_d}$ is characterized as

$$F_{\mathrm{P_d}}(x) = 1 - \mathcal{Q}\left(\frac{\mu}{\left(\sqrt{\frac{2}{\tau_\mathrm{sen} f_\mathrm{s}}}\mathcal{Q}^{-1}(x) + 1\right)} \Bigg/ \sqrt{\frac{2}{\tau_\mathrm{est} f_\mathrm{s}}} P_\mathrm{Rx,ST}\right). \tag{17}$$

Proof: The distribution function of $\mathrm{P_d}$ is defined as

$$F_{\mathrm{P_d}}(x) = \mathbb{P}(\mathrm{P_d} \le x) \tag{18}$$

Using (4)

$$= \mathbb{P}\left(\mathcal{Q}\left(\frac{\mu - \hat{P}_\mathrm{Rx,ST}}{\sqrt{\frac{2}{\tau_\mathrm{sen} f_\mathrm{s}}}}\right) \le x\right) \tag{19}$$

$$= 1 - \mathbb{P}\left(\hat{P}_\mathrm{Rx,ST} \le \frac{\mu}{\mathcal{Q}^{-1}(x)\sqrt{\frac{2}{\tau_\mathrm{sen} f_\mathrm{s}}}}\right) \tag{20}$$

Replacing the distribution function of $\hat{P}_\mathrm{Rx,ST}$ in (20), we obtain an expression of $F_{\mathrm{P_d}}$. ■

From (17), it is clearly observed that $F_{\mathrm{P_d}}$ depends on $P_\mathrm{Rx,ST}$, τ_sen and τ_est. Next, we characterize the probability density functions (pdfs) for C_0 and C_1.

Lemma 2: The pdf of C_0 is defined as

$$f_{C_0}(x) = 2^x \ln 2 \frac{(2^x - 1)^{a_\mathrm{s}-1}}{\Gamma(a_\mathrm{s}) b_\mathrm{s}^{a_\mathrm{s}}} \exp\left(-\frac{2^x - 1}{b_\mathrm{s}}\right), \tag{21}$$

where

$$a_\mathrm{s} = \frac{\left(\sigma_\epsilon^2 \frac{\sigma^2}{P_{\mathrm{Tx,ST}}} + |h_\mathrm{s}|^2\right)^2}{\sigma_\epsilon^2 \frac{\sigma^2}{P_{\mathrm{Tx,ST}}} \left(2\sigma_\epsilon^2 \frac{\sigma^2}{P_{\mathrm{Tx,ST}}} + 4|h_\mathrm{s}|^2\right)} \text{ and } b_\mathrm{s} = \frac{\sigma_\epsilon^2 \frac{\sigma^2}{P_{\mathrm{Tx,ST}}} \left(2\sigma_\epsilon^2 \frac{\sigma^2}{P_{\mathrm{Tx,ST}}} + 4|h_\mathrm{s}|^2\right)}{\left(\sigma_\epsilon^2 \frac{\sigma^2}{P_{\mathrm{Tx,ST}}} + |h_\mathrm{s}|^2\right)}. \tag{22}$$

Proof: Following the pdf of $|\hat{h}_\mathrm{s}|^2$ in (14), the pdf $|\hat{h}_\mathrm{s}|^2 \frac{P_{\mathrm{Tx,ST}}}{\sigma^2}$ is given by

$$f_{|\hat{h}_\mathrm{s}|^2 \frac{P_{\mathrm{Tx,ST}}}{\sigma^2}}(x) = \frac{P_{\mathrm{Tx,ST}}}{\sigma^2} \frac{1}{\sigma_\epsilon^2} \frac{1}{2} \exp\left[-\frac{1}{2}\left(x\sigma_\epsilon^2 \frac{\sigma^2}{P_{\mathrm{Tx,ST}}} + \lambda_\mathrm{s}\right)\right] \left(\frac{x\sigma_\epsilon^2}{\lambda_\mathrm{s}} \frac{\sigma^2}{P_{\mathrm{Tx,ST}}}\right)^{\frac{N_\mathrm{s}}{4} - \frac{1}{2}} I_{\frac{N_\mathrm{s}}{2}-1}\left(\sqrt{\lambda_\mathrm{s} x \sigma_\epsilon^2 \frac{\sigma^2}{P_{\mathrm{Tx,ST}}}}\right)$$

Approximating $\mathcal{X}_1^2(\cdot,\cdot)$ with Gamma distribution $\Gamma(a_\mathrm{s}, b_\mathrm{s})$ gives [29]

$$\approx \frac{1}{\Gamma(a_\mathrm{s})} \frac{x^{a_\mathrm{s}-1}}{b_\mathrm{s}^{a_\mathrm{s}}} \exp\left(-\frac{x}{b_\mathrm{s}}\right), \tag{23}$$

where the parameters $a_\mathrm{s}, b_\mathrm{s}$ are determined by comparing the first two central moments of the two distribution. Finally, by substituting the expression of C_0 in (8) yields 21. ■

Lemma 3: The pdf of C_1 is given by

$$f_{C_1}(x) = 2^x \ln 2 \frac{(2^x - 1)^{a_\mathrm{s}-1} \Gamma(a_\mathrm{s} + a_\mathrm{p})}{\Gamma(a_\mathrm{s})\Gamma(a_\mathrm{p}) b_\mathrm{s}^{a_\mathrm{s}} b_\mathrm{p}^{a_\mathrm{p}}} \left(\frac{1}{b_\mathrm{p}} + \frac{2^x - 1}{b_\mathrm{s}}\right), \tag{24}$$

where

$$a_\mathrm{p} = \frac{N(1 + \frac{P_{\mathrm{Tx,PT}}}{\sigma^2})^2}{(2 + 4\frac{P_{\mathrm{Tx,PT}}}{\sigma^2})} \text{ and } b_\mathrm{p} = \frac{\sigma^2(2 + 4\frac{P_{\mathrm{Tx,PT}}}{\sigma^2})}{N(1 + \frac{P_{\mathrm{Tx,PT}}}{\sigma^2})}, \tag{25}$$

where a_s and b_s are defined in (21).

Proof: Giving a close look at the expression of C_1 in (9), $1 + \frac{P_{\mathrm{Tx,PT}}}{\sigma^2} = \frac{\hat{P}_{\mathrm{Rx,SR}}}{\sigma^2}$. Thereby following the characterization $\hat{P}_{\mathrm{Rx,SR}}$ in (16), the pdf of $1 + \frac{P_{\mathrm{Tx,PT}}}{\sigma^2}$ is determined as

$$f_{\frac{\hat{P}_{\mathrm{Rx,SR}}}{\sigma^2}} = \frac{N_\mathrm{p}}{\sigma^2} \frac{1}{2} \exp\left[-\frac{1}{2}\left(\frac{xN_\mathrm{p}}{\sigma^2} + \lambda_\mathrm{p}\right)\right] \left(\frac{xN_\mathrm{p}}{\sigma^2 \lambda_\mathrm{s}}\right)^{\frac{N_\mathrm{s}}{4} - \frac{1}{2}} I_{\frac{N_\mathrm{p}}{2}-1}\left(\sqrt{\frac{\lambda_\mathrm{p} x \sigma^2}{N_\mathrm{p}}}\right)$$

Following the approach similar to Lemma 2, we approximate the $\mathcal{X}_1^2(\cdot,\cdot)$ with Gamma distribution $\Gamma(a_\mathrm{p}, b_\mathrm{p})$ gives

$$\approx \frac{1}{\Gamma(a_\mathrm{p})} \frac{x^{a_\mathrm{p}-1}}{b_\mathrm{p}^{a_\mathrm{p}}} \exp\left(-\frac{x}{b_\mathrm{p}}\right). \tag{26}$$

Analogous to Lemma 2, the parameters $a_{\rm p}, b_{\rm p}$ are determined by comparing the first two central moments. Based on pdfs $f_{|\hat{h}_{\rm s}|^2 \frac{P_{\rm Tx,ST}}{\sigma^2}}$ and $f_{\frac{\hat{P}_{\rm Rx,SR}}{\sigma^2}}$ in (22), we apply Mellin transform to determine the pdf of $|\hat{h}_{\rm s}|^2 \frac{P_{\rm Tx,ST}}{\sigma^2} / \frac{\hat{P}_{\rm Rx,SR}}{\sigma^2}$ as [30]

$$f_{\frac{P_{\rm Tx,ST}}{\sigma^2} / \frac{\hat{P}_{\rm Rx,SR}}{\sigma^2}}(x) = \frac{x^{a_{\rm s}-1}\Gamma(a_{\rm s}+a_{\rm p})}{\Gamma(a_{\rm s})\Gamma(a_{\rm p}) b_{\rm s}^{a_{\rm s}} b_{\rm p}^{a_{\rm p}}} \left(\frac{1}{b_{\rm p}} + \frac{x}{b_{\rm s}}\right).$$

Finally, by substituting the expression of C_1 in (9) yields 24. ■

Sensing-throughput tradeoff

Here, we establish the sensing-throughput tradeoff for the estimation model. Considering the distortions in the $\rm P_d$, we establish two new primary user constraints at the PR, namely, an average constraint and an outage constraint on the probability of detection. Based on these constraints, we characterize the performance of the IS.

Theorem 1: Subject to an average constraint on $\rm P_d$ at the PR, the sensing-throughput tradeoff is given by

$$\tilde{R}_{\rm s}^{\rm AC}(\tilde{\tau}_{\rm sen}^{\rm AC}) = \max_{\tau_{\rm sen}} \mathbb{E}_{{\rm P_d}, C_0, C_1}\left[R_{\rm s}(\tau_{\rm sen})\right] = \frac{T - \tau_{\rm sen} - \tau_{\rm est}}{T} \Big[\mathbb{E}_{C_0}\left[C_0\right](1 - {\rm P_{fa}}){\rm P}(\mathcal{H}_0) + \mathbb{E}_{C_1}\left[C_1\right](1 - \mathbb{E}_{\rm P_d}\left[{\rm P_d}\right]){\rm P}(\mathcal{H}_1) \Big], \tag{27}$$

$$\text{s.t. } \mathbb{E}_{\rm P_d}\left[{\rm P_d}\right] \le \bar{\rm P}_{\rm d}. \tag{28}$$

$\mathbb{E}_{\rm P_d}[\cdot]$ represents the expectation with respect to $\rm P_d$, $\mathbb{E}_{{\rm P_d}, C_0, C_1}[\cdot]$ denotes the expectation with respect to $\rm P_d$, C_0 and C_1, and $\bar{\rm P}_{\rm d}$ represents the threshold for the probability of detection.

Theorem 2: Subject to an outage constraint on $\rm P_d$ at the PR, the sensing-throughput tradeoff is given by

$$\tilde{R}_{\rm s}^{\rm OC}(\tilde{\tau}_{\rm sen}^{\rm OC}) = \max_{\tau_{\rm sen}} \mathbb{E}_{{\rm P_d}, C_0, C_1}\left[R_{\rm s}(\tau_{\rm sen})\right] = \frac{T - \tau_{\rm sen} - \tau_{\rm est}}{T} \Big[\mathbb{E}_{C_0}\left[C_0\right](1 - {\rm P_{fa}}){\rm P}(\mathcal{H}_0) + \mathbb{E}_{C_1}\left[C_1\right](1 - \mathbb{E}_{\rm P_d}\left[{\rm P_d}\right]){\rm P}(\mathcal{H}_1) \Big], \tag{29}$$

$$\text{s.t. } \mathbb{P}({\rm P_d} \le \bar{\rm P}_{\rm d}) \le \kappa, \tag{30}$$

where κ represents the outage constraint.

In order to solve the constrained optimization problems illustrated in Theorem 1 and Theorem 2, following approach is considered. As a first step, an underlying constraint is employed to determine μ as a function of the $\tau_{\rm sen}$ and $\tau_{\rm est}$. Its value is substituted in (4) and (5) to determine $\rm P_{fa}$ and $\rm P_d$. Finally, replacing $\mathrm{P_d}$ and $\mathrm{P_{fa}}$ inside (27) and (29), we obtain an expression of the throughput in terms of the τ_{sen}, the τ_{est} and the system parameters. Using this expression, we consider the variation of expected throughput along τ_{est} and τ_{sen}, thereby determining an optimum performance.

Corollary 1: For the average constraint, the analytical expression $\mathbb{E}_{\mathrm{P_d}}[\mathrm{P_d}]$ did not lead to a closed form expression. In this context, we procure the threshold μ_{AC} for the average constraint numerically.

Corollary 2: For this case, we determine the threshold μ_{OC} based on the outage constraint. This is accomplished by combining the expression of $F_{\mathrm{P_d}}$ in (17) with the outage constraint (30)

$$P(\mathrm{P_d} \leq \bar{\mathrm{P}}_{\mathrm{d}}) = F_{\mathrm{P_d}}(\bar{\mathrm{P}}_{\mathrm{d}}) \leq \kappa. \tag{31}$$

Rearranging (31) gives

$$\mu_{\text{OC}} \geq P_{\text{Rx,ST}} \left(1 + \mathcal{Q}^{-1}(1-\kappa)\sqrt{\frac{2}{\tau_{\text{est}} f_{\text{s}}}}\right) \cdot \left(1 + \mathcal{Q}^{-1}(\bar{\mathrm{P}}_{\mathrm{d}})\sqrt{\frac{2}{\tau_{\text{sen}} f_{\text{s}}}}\right). \tag{32}$$

The expected throughput based on the average and outage constraint can be determined by inserting the respective thresholds μ_{AC} and μ_{OC} in (27) and (29) and evaluating the expectation over $\mathrm{P_d}$, C_0 and C_1 using Lemma 1, Lemma 2 and Lemma 3. Hence, according to estimation model the sensing-throughput tradeoff based on new constraints is depicted by considering the variation of expected throughput against the variation of sensing time. Consequently, an estimation-throughput tradeoff is depicted by following a variation of optimum expected throughput (optimized over the sensing time) against the estimation time.

IV. Numerical Analysis

Here, we analyze the performance of the IS in terms of sensing-throughput tradoff subject to new interference constraints.

V. Conclusion

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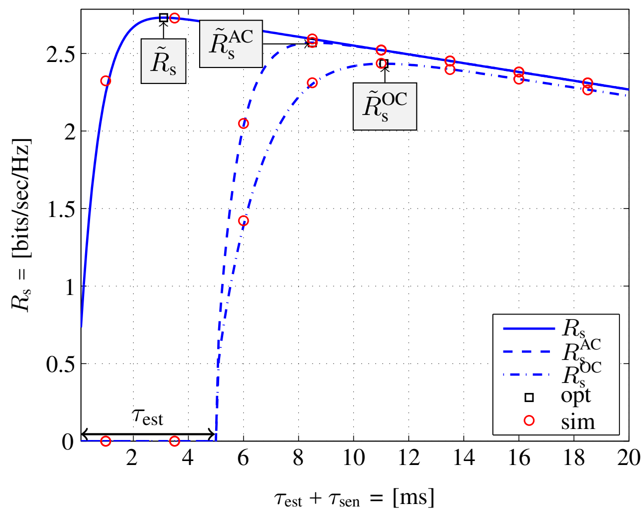

Fig. 3. Sensing-throughput tradeoff for the ideal and estimation models with $\gamma_{rcvd} = -10\,\text{dB}$ and $\tau_{est} = 5\,\text{ms}$.

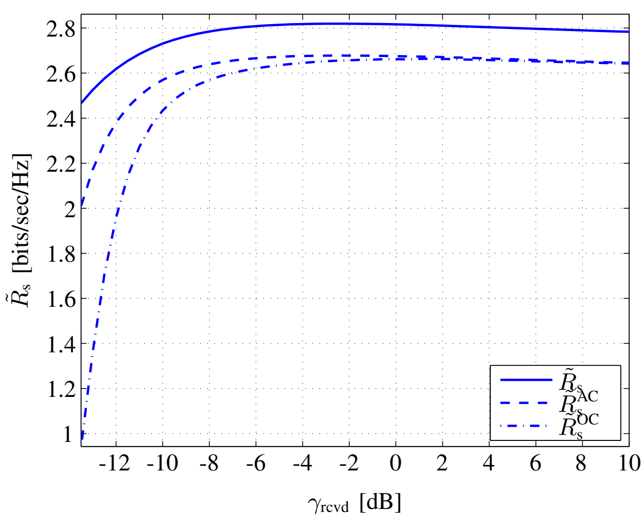

Fig. 4. Distortion in optimum throughput versus the γ_{rcvd} with $\tau_{est} = 5\,\text{ms}$.

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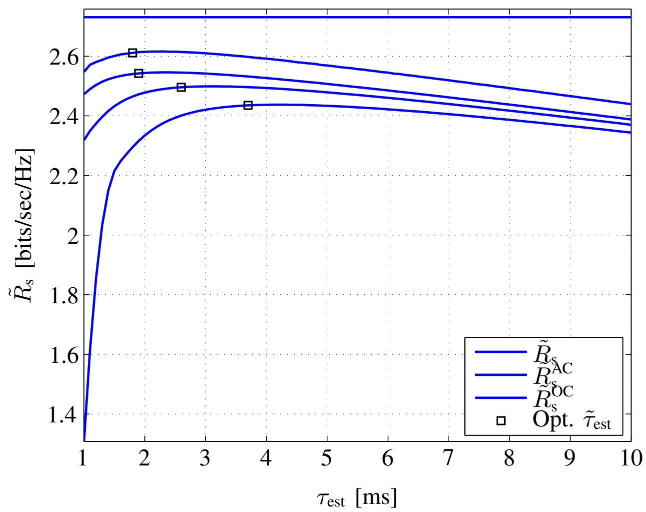

Fig. 5. Optimum throughput-estimation tradeoff for the adjacent and outage constraint with $\gamma_{rcvd} = -10\,$dB.

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