Hybrid Underlay/Overlay Cognitive Radio System with Hierarchical Modulation in the Presence of Channel Estimation Error

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***Abstract*—In this paper, we study the performance of hybrid cognitive radio (CR) system in the presence of channel estimation error (CER) in terms of bit-error-probability (BEP) and outage probability. We assume simultaneous switching between the underlay and overlay modes of CR where in, the secondary user (SU) probabilistically accesses the primary user's (PU) channel in both underlay and overlay modes to utilize the benefits of hybrid CR system. The PU uses two-layer $2/4-ASK$ constellation with unequal level of bit protection against the interference imposed by the SU's transmitter. The SU wishes to opportunistically use the PU's channel to transmit its own data while maintaining the performance of the PU's layer with the worst level of protection at a desired value. We derive the SU's maximum allowable transmit power in the overlay mode to guarantee the PU reliable communication then, we discuss about the optimum value of the hybrid switching rate that minimizes the outage probability of the SU along with, deriving the exact BEP and outage probability expressions for the secondary system as well as that of the first and the second layers of the PU system.**

I. Introduction

With the rapid growing uses of wireless communication and its applications such as sensor networks, mobile internet, high-speed wireless access technologies, and also vast growing number of subscribers with different levels of priorities, it's indispensable to exploit the inefficient usages of the limited communication resources specially radio spectrum. Cognitive radio (CR) has been accepted as a technology that would help utilizing the available spectrum more efficiently and effectively by allowing non-legitimate users to access the licensed bands [1], [2]. Typically in a CR system, transmission happens in one of the underlay or overlay sharing methods or a combination of which that is called the hybrid underlay/overlay spectrum sharing [3]–[5]. In the underlay method, the secondary user (SU) utilizes the licensed spectrum with a proper level of power, in order to keep the level of interference imposed to the primary user (PU) under a given threshold that is tolerable by it. In an overlay CR system, the cognitive users opportunistically scess the licensed spectrum holes without causing any interference (if the spectrum holes are detected correctly) to the PU. Subsequently, the SU can transmit with its maximum power. In the hybrid underlay/overlay spectrum sharing, the SU adaptively switches between the two mentioned methods based on spectrum occupancy, i.e., the SU transmits with the overlay method in the case that the PU is idle and transmits with the underlay method when the CR detects PU activity.

Recently investigating the impact of imperfect knowledge of channel state information (CSI) on wireless communication systems has gained growing attentions. In the practical applications, CSI is available through channel estimation algorithms such as pilot symbol aided modulation (PSAM) [6] or minimum mean square error (MMSE) [7]. Since estimation algorithms in the reality are exposed to have error, it is of great importance to study the effect of CER on the performance of communication systems. Therefore, it would seem necessary to investigate the effect of CER on the hybrid CR system that in, the SU switches between the underlay and overlay modes and experiences incorrect detection of the PU along with having CER [8]. In the literature, the CSI estimation error is modeled as a complex Gaussian random variable (RV) which is added to the actual channel as shown in [7].

In this paper, we investigate the effect of imperfect CSI on hybrid underlay/overlay CR system where in, the SU detects the channel utilization of a PU and decides to transmit opportunistically with adaptive transmission power based on the interference link condition. We assume that the PU has two classes of information bits with unequal error protection importance which are transmitted using hierarchical $2/4-ASK$ constellation. We discuss the effects of imperfect CSI on both layers of the PU's constellation as well as the SU in terms of average bit-error-rate (BER) and outage probability. At the end, we discuss about the optimal value of hybrid switching rate that minimizes the average outage probability of the SU while keeping the average outage probability of the second layer of the PU at a required value.

The remainder of this paper is organized as follows: Section II presents the basic assumptions and transmission model. The performance analysis of the considered model is investigated in Section III. The optimal value of the hybrid switching rate is discussed in Section IV. The simulation results are provided in Section V. Finally, Section VI concludes the paper.

II. Basic Assumptions and Transmission Model

A hybrid underlay/overlay CR system is considered in this paper where in, the SU by monitoring the licensed spectrum, wishes to opportunistically access the PU's channel in both

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underlay and overlay modes of CR system. The SU transmits in the overlay mode if it detects a spectrum hole and if not, it adapts its power based on the PU's interference constraint and transmits in the underlay mode. We assume that there is an interference link caused by the SU's transmitter at the primary receiver which is modeled as $f \sim CN(0, \sigma_f^2)$. We further assume that the primary receiver puts its best effort to have f with nearly no error . The primary transmitter, primary receiver, secondary transmitter and the secondary receiver are denoted by PT, PR, ST, and SR respectively as depicted in Fig. 1. The PU utilizes the two-layer $2/4-ASK$ constellation with modulation parameter α which, enables mapping of the most significant information bits into the first layer and information bits with the least significance to the second layer of constellation as shown in Fig 2. In our model, the primary and the secondary users estimate their own channel thus, suffer from CER. We model the imperfectness of CSI of the primary and the secondary links as follows

$$h = \hat{h} + e_1 \quad , \quad g = \hat{g} + e_2, \tag{1}$$

where $\hat{h} \sim CN(0, \sigma_{\hat{h}}^2)$ and $\hat{g} \sim CN(0, \sigma_{\hat{g}}^2)$ are channel estimates and $e_1 \sim CN(0, \sigma_{e_1}^2)$, $e_2 \sim CN(0, \sigma_{e_2}^2)$ are CERs. Also, $h \sim CN(0, \sigma_h^2)$ and $g \sim CN(0, \sigma_g^2)$ are exact channel coefficients that are not available at receivers due to the estimation error. The channel coefficients are assumed to be flat fading and constant during one symbol transmission. We assume MMSE estimation in which, the estimate and the error are orthogonal [9]. In this case, $\hat{h}$ and h, $\hat{g}$ and g would be jointly complex normal Gaussian distributed RV with correlation factor ρ_1 and ρ_2 given by [10]

$$\rho_1 = \frac{\sigma_{\hat{h}}}{\sigma_h} = \frac{1}{\sigma_h}\sqrt{\sigma_h^2 - \sigma_{e_1}^2} \tag{2}$$

$$\rho_2 = \frac{\sigma_{\hat{g}}}{\sigma_g} = \frac{1}{\sigma_g}\sqrt{\sigma_g^2 - \sigma_{e_2}^2} \tag{3}$$

Also the additive noises at PR and SR nodes are modeled as $CN(0, \sigma_n^2)$. Based on the PU's activity, the SU transmits in three different modes. If the SU detects no PU activity, it transmits in the overlay mode with transmitted power $E_{s,O}$. When the SU detects PU activity, it adapts its power to transmit in the underlay mode in the way that, the SU is allowed to transmit with its maximum power, E_s^{max}, if $|f|^2 \leq \frac{T}{E_s^{max}}$ that in, T is the interference temperature that indicates the maximum allowable interference level at the primary's receiver [11]–[13]. Lastly in the third mode, the SU transmits with power $T/|f|^2$ if $|f|^2 > \frac{T}{E_s^{max}}$. The performance analysis of the SU and PU greatly depend on the probability of detection P_d, probability of false alarm P_f and the PU activity. P_d refers to the probability that the SU detects the PU's spectrum occupation correctly, and P_f is the probability that the SU detects the spectrum occupation while it is not occupied actually. Note that P_d and P_f do not necessarily add to one because they are two different definitions. π_0 and π_1 denote the steady state probability that the PU's channel is empty or occupied respectively. ε denotes the hybrid rate between the underlay and overlay cognitive modes where, $\varepsilon = 0$ means fully overlay mode and $\varepsilon = 1$ means fully underlay mode and also $0 < \varepsilon < 1$ represents combination of underlay and overlay modes. Throughout this paper, indices: max, out, e, P, S, l_1, l_2, U, O and I mean, maximum value, outage probability, error probability, PU, SU, PU's first layer, PU's second layer, underlay mode, overlay mode and interference, respectively.

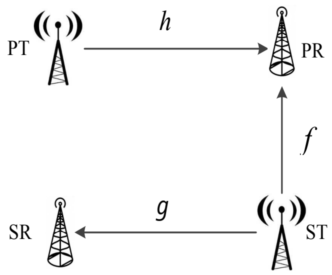

Fig. 1. System Model.

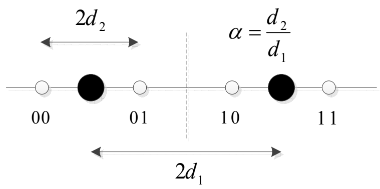

Fig. 2. Generalized hierarchical $2/4 - ASK$ constellation.

III. Performance Analysis

The received signals at the PU's and SU's receivers are respectively

$$r_p = \sqrt{E_p} h x_p + \sqrt{E_s} f x_s + n_p = \sqrt{E_p}\hat{h} x_p + n'_p \tag{4}$$

$$r_s = \sqrt{E_s} g x_s + n_s = \sqrt{E_s}\hat{g} x_s + n'_s \tag{5}$$

where in, E_p and E_s are the primary and the secondary users transmitted energy; x_p is a $2/4 - ASK$ symbol of the PU [14] and x_s is a BPSK symbol of the secondary user; n_p and n_s are additive noises at the receiver's terminals; $n'_p = \sqrt{E_p} e_1 x_p + \sqrt{E_s} f x_s + n_p$ and $n'_s = \sqrt{E_s} e_2 x_s + n_s$.

A. Outage Analysis

In this section we examine the hybrid CR system of this paper's interest in sense of outage probability. Note that in all over the paper by the term outage probability, we mean the probability of the SNR falling below a threshold value at a desired receiver side. The average outage probability of the PU in hybrid CR system is given by

$$P_{out,P} = P_d \left[\varepsilon P_{out,P,U} + (1-\varepsilon) P_{out,P,O}\right] + (1 - P_d) P_{out,P,O,I} \tag{6}$$

where in, $P_{out,P,U}$ is the outage probability of the PU when the SU transmits in the underlay mode due to the correct detection of PU's activity. For the first layer of hierarchical constellation

$$P_{out,P,l_1,U} = 1 - \exp\left(-\frac{\gamma_1 D}{\sigma_{\hat{h}}^2}\right) \times \frac{1 - \exp\left(-\frac{T}{E_s^{max}}\left(\frac{\gamma_1 C_1^U}{\sigma_{\hat{h}}^2(1-\gamma_1 A)} + \frac{1}{\sigma_I^2}\right)\right)}{\sigma_{e_1}^2 \sigma_I^2 \left(\frac{\gamma_1 B}{\sigma_{\hat{h}}^2(1-\gamma_1 A)} + \frac{1}{\sigma_{e_1}^2}\right)\left(\frac{\gamma_1 C_1^U}{\sigma_{\hat{h}}^2(1-\gamma_1 A)} + \frac{1}{\sigma_I^2}\right)} + \left(1 - \frac{\exp\left(-\frac{\gamma_1(C_2^U + D)}{\sigma_{\hat{h}}^2(1-\gamma_1 A)}\right)}{\sigma_{e_1}^2\left(\frac{\gamma_1 B}{\sigma_{\hat{h}}^2(1-\gamma_1 A)} + \frac{1}{\sigma_{e_1}^2}\right)}\right)\exp\left(-\frac{T}{\sigma_I^2 E_s^{max}}\right) \quad (7)$$

that in the above, $A = \alpha^2$, $B = 1 + \alpha^2$, $C_1^U = \frac{E_s^{max}}{Ep}B$, $C_2^U = \frac{T}{Ep}B$, $D = \frac{\sigma_n^2}{E_p}B$ and γ_1 is the SNR threshold for the first layer. It is remarkable to know that the above was obtained by calculating the outage probability for the PU receiver with setting condition on the SU transmitting energy. Since in the underlay mode of SU the transmitting power is chosen according to the level of interfering link, the outage probability of the PU in this case would be the sum of PU outage probability once when the SU energy is equal to $E_s = E_s^{max}$ and once for the case that in the SU energy is $E_s = T/|f|^2$. For the second layer by using the same technique in obtaining (7) we have

$$P_{out,P,l_2,U} = 1 - \exp\left(-\frac{\gamma_2 D'}{\sigma_{\hat{h}}^2}\right) \times \frac{1 - \exp\left(-\frac{T}{E_s^{max}}\left(\frac{\gamma_2 C_1'^U}{\sigma_{\hat{h}}^2} + \frac{1}{\sigma_I^2}\right)\right)}{\sigma_{e_1}^2 \sigma_I^2 \left(\frac{\gamma_2 B'}{\sigma_{\hat{h}}^2} + \frac{1}{\sigma_{e_1}^2}\right)\left(\frac{\gamma_2 C_1'^U}{\sigma_{\hat{h}}^2} + \frac{1}{\sigma_I^2}\right)} + \left(1 - \frac{\exp\left(-\frac{\gamma_2(C_2'^U + D')}{\sigma_{\hat{h}}^2}\right)}{\sigma_{e_1}^2\left(\frac{\gamma_2 B'}{\sigma_{\hat{h}}^2} + \frac{1}{\sigma_{e_1}^2}\right)}\right)\exp\left(-\frac{T}{\sigma_I^2 E_s^{max}}\right) \quad (8)$$

where in, $B' = \frac{1+\alpha^2}{\alpha^2}$, $C_1'^U = \frac{E_s^{max}}{E_p}B'$, $C_2'^U = \frac{T}{E_p}B'$, $D' = \frac{\sigma_n^2}{E_p}B'$ and γ_2 is the SNR threshold for the second layer. Please note that the definition of α is given in Fig. 2. $P_{out,P,O}$ is the outage probability of the PU when the SU correctly detects PU's activity. In this case the PU does not experience interference from the SU. For the first layer we have

$$P_{out,P,l_1,O} = 1 - \frac{\exp\left(-\frac{\gamma_1 D}{\sigma_{\hat{h}}^2(1-\gamma_1 A)}\right)}{\sigma_{e_1}^2\left(\frac{\gamma_1 B}{\sigma_{\hat{h}}^2(1-\gamma_1 A)} + \frac{1}{\sigma_{e_1}^2}\right)} \quad (9)$$

and also for the second layer we get

$$P_{out,P,l_2,O} = 1 - \frac{\exp\left(-\frac{\gamma_2 D'}{\sigma_{\hat{h}}^2}\right)}{\sigma_{e_1}^2\left(\frac{\gamma_2 B'}{\sigma_{\hat{h}}^2} + \frac{1}{\sigma_{e_1}^2}\right)} \quad (10)$$

$P_{out,P,l_1,O,I}$ indicates the outage probability of the PU when the SU's decision shows the absence of PU activity while the PU's channel is occupied actually, for the first layer it's obtained by

$$P_{out,P,l_1,O,I} = 1 - \frac{\exp\left(-\frac{\gamma_1 D}{\sigma_{\hat{h}}^2}\right)}{\sigma_{e_1}^2 \sigma_I^2 \left(\frac{\gamma_1 B}{\sigma_{\hat{h}}^2(1-\gamma_1 A)} + \frac{1}{\sigma_{e_1}^2}\right)\left(\frac{\gamma_1 C^O}{\sigma_{\hat{h}}^2(1-\gamma_1 A)} + \frac{1}{\sigma_I^2}\right)} \quad (11)$$

and for the second layer we have

$$P_{out,P,l_2,O,I} = 1 - \frac{\exp\left(-\frac{\gamma_2 D'}{\sigma_{\hat{h}}^2}\right)}{\sigma_{e_1}^2 \sigma_I^2 \left(\frac{\gamma_2 B'}{\sigma_{\hat{h}}^2} + \frac{1}{\sigma_{e_1}^2}\right)\left(\frac{\gamma_2 C'^O}{\sigma_{\hat{h}}^2} + \frac{1}{\sigma_I^2}\right)} \quad (12)$$

where in, $C^O = \frac{E_{s,O}}{E_p}B$, $C'^O = \frac{E_{s,O}}{E_p}B'$. The outage probability of the SU is given by [4]

$$P_{out,S} = 1 - \left[P_{out,S,O} + \varepsilon(1-\mu)P_{out,S,U}\right] \quad (13)$$

where in, μ is a cost caused by monitoring the interference link f in an underlay mode. $P_{out,S,O}$ and $P_{out,S,U}$ determine the outage probabilities of the SU when the secondary system is working in the overlay and underlay mode respectively. In the overlay mode the SU transmits with $E_s = E_s^O$ thus we have

$$P_{out,S,O} = \frac{\exp\left(-\frac{\gamma_1 \sigma_n^2}{E_s^O \sigma_{\hat{g}}^2}\right)}{\sigma_{e_2}^2\left(\frac{\gamma_1}{\sigma_{\hat{g}}^2} + \frac{1}{\sigma_{e_2}^2}\right)}\left[\pi_0(1-P_f) + \pi_1(1-P_d)\right] \quad (14)$$

In the underlay mode since the SU transmitter's energy must be adapted regarding the PU's outage probability criteria similar to the procedure of calculating (7) we have

$$P_{out,S,U} = \left[\pi_0 P_f + \pi_1 P_d\right]\left(1 - \left(1 - \frac{\exp\left(-\frac{\gamma_1 \sigma_n^2}{E_s^{max}\sigma_{\hat{g}}^2}\right)}{\sigma_{e_2}^2\left(\frac{\gamma_1}{\sigma_{\hat{g}}^2} + \frac{1}{\sigma_{e_2}^2}\right)}\right) \times \left(1 - \exp\left(-\frac{T}{\sigma_I^2 E_s^{max}}\right)\right) + \frac{\exp\left(-\frac{T}{E_s^{max}}\left(\frac{\gamma 1 \sigma_n^2}{T\sigma_{\hat{g}}^2} + \frac{1}{\sigma_I^2}\right)\right)}{\sigma_{e_2}^2 \sigma_I^2\left(\frac{\gamma_1}{\sigma_{\hat{g}}^2} + \frac{1}{\sigma_{e_2}^2}\right)\left(\frac{\gamma_1 \sigma_n^2}{\sigma_{T\hat{g}}^2} + \frac{1}{\sigma_I^2}\right)} - \exp\left(-\frac{T}{\sigma_I^2 E_s^{max}}\right)\right) \quad (15)$$

B. BEP Analysis

The average BEP of the PU in hybrid CR system is given by

$$P_{e,P} = P_d\left[\varepsilon P_{e,P,U} + (1-\varepsilon)P_{e,P,O}\right] + (1-P_d)P_{e,P,O,I} \quad (16)$$

where in, $P_{e,P,U}$ is the average BEP of the PU when the SU works in the underlay mode, $P_{e,P,O}$ is the average BEP of the PU when the SU works in the overlay mode without causing interference to the PU because of correct sensing of the spectrum, and $P_{e,P,O,I}$ is the average BEP of the PU under the condition that the SU wrongly detects the absence of the PU and hence, working in the overlay mode with causing interference to the PU.

According to the error probability of the first and the second layer of the $2-4$ ASK hierarchical modulation in AWGN channel [14] we have

$$P_{e,P,l_1,U} = Pr\left\{|f|^2 < \frac{T}{E_s^{max}}\right\} \tag{17}$$
$$\times\left[\frac{1}{2}\left(1-\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta_{1,U}\bar{\gamma}_p}{1+0.5\beta_{1,U}\bar{\gamma}_p}}-\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta_{2,U}\bar{\gamma}_p}{1+0.5\beta_{2,U}\bar{\gamma}_p}}\right)\right]$$
$$+Pr\left\{|f|^2 > \frac{T}{E_s^{max}}\right\}$$
$$\times\left[\frac{1}{2}\left(1-\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta_{3,U}\bar{\gamma}_p}{1+0.5\beta_{3,U}\bar{\gamma}_p}}-\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta_{4,U}\bar{\gamma}_p}{1+0.5\beta_{4,U}\bar{\gamma}_p}}\right)\right]$$

that in, $\gamma_p = \frac{E_p|h|^2}{\sigma_n^2}$ and the coefficients $\beta_{i,U},\ i=1,2,3,4$ that depend on the hierarchical parameter α, the PU's transmitted power and the SU's maximum allowable power are obtained by the below expressions

$$\beta_{1,U} = \frac{\frac{(1+\alpha)^2}{1+\alpha^2}}{1+\frac{E_s^{max}\sigma_I^2}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}, \beta_{2,U} = \frac{\frac{(1+\alpha)^2}{1-\alpha^2}}{1+\frac{E_s^{max}\sigma_I^2}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}$$
$$\beta_{3,U} = \frac{\frac{(1+\alpha)^2}{1+\alpha^2}}{1+\frac{T}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}, \beta_{4,U} = \frac{\frac{(1+\alpha)^2}{1-\alpha^2}}{1+\frac{T}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}$$

Note that (17) is obtained by averaging the error probability expressions of $2/4-ASK$ hierarchical modulation in AWGN channel over the PU's source to destination channel estimate. The average error probability of the first layer of the PU in the case that the SU transmits in the overlay mode when there is no PU activity is given by

$$P_{e,P,l_1,O} = \tag{18}$$
$$\frac{1}{2}\left(1-\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta_{1,O}\bar{\gamma}_p}{1+0.5\beta_{1,O}\bar{\gamma}_p}}-\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta_{2,O}\bar{\gamma}_p}{1+0.5\beta_{2,O}\bar{\gamma}_p}}\right)$$

where the coefficients $\beta_{i,O},\ i=1,2$ are defined as follows

$$\beta_{1,O} = \frac{\frac{(1+\alpha)^2}{1+\alpha^2}}{1+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}, \beta_{2,O} = \frac{\frac{(1+\alpha)^2}{1-\alpha^2}}{1+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}$$

The average BEP for the first layer of the PU when the SU imposes interference to the PU's receiver is given by

$$P_{e,P,l_1,O,I} = \tag{19}$$
$$\frac{1}{2}\left(1-\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta_{1,O,I}\bar{\gamma}_p}{1+0.5\beta_{1,O,I}\bar{\gamma}_p}}-\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta_{2,O,I}\bar{\gamma}_p}{1+0.5\beta_{2,O,I}\bar{\gamma}_p}}\right)$$

where

$$\beta_{1,O,I} = \frac{\frac{(1+\alpha)^2}{1+\alpha^2}}{1+\frac{E_{s,O}\sigma_I^2}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}, \beta_{2,O,I} = \frac{\frac{(1+\alpha)^2}{1-\alpha^2}}{1+\frac{E_{s,O}\sigma_I^2}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}$$

Similarly, for the second layer we have

$$P_{e,P,l_2,U} = Pr\left\{|f|^2 < \frac{T}{E_s^{max}}\right\}\times \tag{20}$$
$$\left[\frac{1}{2}\left(1-\rho_1\sqrt{\frac{0.5\beta'_{1,U}\bar{\gamma}_p}{1+0.5\beta'_{1,U}\bar{\gamma}_p}}-\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta'_{2,U}\bar{\gamma}_p}{1+0.5\beta'_{2,U}\bar{\gamma}_p}}\right.\right.$$
$$\left.\left.+\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta'_{3,U}\bar{\gamma}_p}{1+0.5\beta'_{3,U}\bar{\gamma}_p}}\right)\right]+Pr\left\{|f|^2 > \frac{T}{E_s^{max}}\right\}\times$$
$$\left[\frac{1}{2}\left(1-\rho_1\sqrt{\frac{0.5\beta'_{4,U}\bar{\gamma}_p}{1+0.5\beta'_{4,U}\bar{\gamma}_p}}-\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta'_{5,U}\bar{\gamma}_p}{1+0.5\beta'_{5,U}\bar{\gamma}_p}}\right.\right.$$
$$\left.\left.+\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta'_{6,U}\bar{\gamma}_p}{1+0.5\beta'_{6,U}\bar{\gamma}_p}}\right)\right]$$

where the coefficients $\beta'_{i,U},\ i=1,\ldots,6$ are obtained by

$$\beta'_{1,U} = \frac{\frac{(2-\alpha)^2}{1+\alpha^2}}{1+\frac{E_s^{max}\sigma_I^2}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}, \beta'_{2,U} = \frac{\frac{(2+\alpha)^2}{1+\alpha^2}}{1+\frac{E_s^{max}\sigma_I^2}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}},$$
$$\beta'_{3,U} = \frac{\frac{\alpha^2}{1+\alpha^2}}{1+\frac{E_s^{max}\sigma_I^2}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}, \beta'_{4,U} = \frac{\frac{(2-\alpha)^2}{1+\alpha^2}}{1+\frac{T}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}$$
$$\beta'_{5,U} = \frac{\frac{(2+\alpha)^2}{1+\alpha^2}}{1+\frac{T}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}, \beta'_{6,U} = \frac{\frac{\alpha^2}{1+\alpha^2}}{1+\frac{T}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}$$

The average BEP of the second layer of the PU when the SU detects an spectrum hole is given by

$$P_{e,P,l_2,O} = \frac{1}{2}\left(1-\rho_1\sqrt{\frac{0.5\beta'_{1,O}\bar{\gamma}_p}{1+0.5\beta'_{1,O}\bar{\gamma}_p}}\right. \tag{21}$$
$$\left.-\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta'_{2,O}\bar{\gamma}_p}{1+0.5\beta'_{2,O}\bar{\gamma}_p}}+\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta'_{3,O}\bar{\gamma}_p}{1+0.5\beta'_{3,O}\bar{\gamma}_p}}\right)$$

that in the above

$$\beta'_{1,O} = \frac{\frac{(2-\alpha)^2}{1+\alpha^2}}{1+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}, \beta'_{2,O} = \frac{\frac{(2+\alpha)^2}{1+\alpha^2}}{1+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}, \beta'_{3,O} = \frac{\frac{\alpha^2}{1+\alpha^2}}{1+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}$$

when the SU misdetects the PU's activity, the average BEP for the second layer becomes

$$P_{e,P,l_2,O,I} = \frac{1}{2}\left(1-\rho_1\sqrt{\frac{0.5\beta'_{1,O,I}\bar{\gamma}_p}{1+0.5\beta'_{1,O,I}\bar{\gamma}_p}}\right. \tag{22}$$
$$\left.-\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta'_{2,O,I}\bar{\gamma}_p}{1+0.5\beta'_{2,O,I}\bar{\gamma}_p}}+\frac{1}{2}\rho_1\sqrt{\frac{0.5\beta'_{3,O,I}\bar{\gamma}_p}{1+0.5\beta'_{3,O,I}\bar{\gamma}_p}}\right)$$

that in

$$\beta'_{1,O,I} = \frac{\frac{(2-\alpha)^2}{1+\alpha^2}}{1+\frac{E_{s,O}\sigma_I^2}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}, \beta'_{2,O,I} = \frac{\frac{(2+\alpha)^2}{1+\alpha^2}}{1+\frac{E_{s,O}\sigma_I^2}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}},$$
$$\beta'_{3,O,I} = \frac{\frac{\alpha^2}{1+\alpha^2}}{1+\frac{E_{s,O}\sigma_I^2}{\sigma_n^2}+\frac{E_p\sigma_{e_1}^2}{\sigma_n^2}}$$

The average BEP of the SU in hybrid CR system is given by

$$P_{e,S} = P_d\left[\varepsilon P_{e,S,U} + (1-\varepsilon)P_{e,S,O}\right] + (1-P_d)P_{e,S,O} \quad (23)$$

where

$$P_{e,S,U} = \frac{1}{2}Pr\left\{|f|^2 < \frac{T}{E_s^{max}}\right\}\left(1-\rho_2\sqrt{\frac{E_s^{max}\bar{\gamma}_s}{1+E_s^{max}\bar{\gamma}_s}}\right)$$
$$+\frac{1}{2}Pr\left\{|f|^2 < \frac{T}{E_s^{max}}\right\}\left(1-\rho_2\sqrt{\frac{\frac{T}{\sigma_I^2}\bar{\gamma}_s}{1+\frac{T}{\sigma_I^2}\bar{\gamma}_s}}\right)$$

that in, $\gamma_s = \frac{E_s|g|^2}{\sigma_n^2}$ and also, $P_{e,S,O}$ would be

$$P_{e,S,O} = \frac{1}{2}\left(1-\rho_2\sqrt{\frac{E_{s,O}\bar{\gamma}_s}{1+E_{s,O}\bar{\gamma}_s}}\right)$$

IV. Switching Rate Optimization

In this section, the optimal value of the hybrid rate and the transmit power of the SU in the overlay mode are obtained aiming at minimize the outage probability of the SU when the outage probability of the second layer of the PU meets a given requirement. Therefor, we solve the optimization problem

$$\min_{\varepsilon, E_{s,O}} \left[1-\left[P_{out,S,O} + \varepsilon(1-\mu)P_{out,S,U}\right]\right] \quad (24)$$

$$s.\ t. \begin{cases} P_{out,P,l_2} \le P_{out}^{th} & (34.a) \\ E_{s,O} \le E_s^{max} & (34.b) \end{cases}$$

From the constraint $P_{out,P,l_2} \le P_{out}^{th}$ and using (6), the maximum allowable secondary transmitter energy in the overlay mode that maintains (24.a) is obtained as below

$$E_{s,O}(\varepsilon) = \frac{E_p\sigma_{\hat{h}}^2 \exp\left(-\frac{\gamma_2 D\prime}{\sigma_{\hat{h}}^2}\right)}{B\sigma_{e_1}^2\sigma_I^2\gamma_2\left(1-\frac{P_{out}^{th}-P_dK(\varepsilon)}{1-P_d}\right)\left(\frac{\gamma_2 B\prime}{\sigma_{\hat{h}}^2}+\frac{1}{\sigma_{e_1}^2}\right)} \quad (25)$$
$$-\frac{E_p\sigma_{\hat{h}}^2}{\sigma_I^2\gamma_2 B}$$

where

$$K(\varepsilon) = P_{out,P,l_2,O} + \varepsilon(P_{out,P,l_2,U} - P_{out,P,l_2,O}) \quad (26)$$

E_s^{max} can be obtained by (25) for $\varepsilon = 0$. By obtaining $E_{s,O}$ in terms of ε, the optimization problem in (24) reduces to

$$\min_{\varepsilon}\left[1-\left[P_{out,S,O}+\varepsilon(1-\mu)P_{out,S,U}\right]\right] \quad (27)$$

The optimum value of ε in (27) can only be found numerically.

V. Simulation Results

In this section, we present Monte-Carlo simulation results to evaluate the analysis of the average error and outage probabilities obtained in the previous sections. In all simulations we assume that, $E_p = 1$, $\sigma_n^2 = 1$, $T = 1$, $P_d = 0.8$, $P_f = 0.2$, $\sigma_I^2 = 0.1$, $\sigma_{\hat{h}}^2 = 1$ and $\sigma_g^2 = 1$ unless otherwise noted. It's mentionable that the parameters' setup can take any value within their acceptable range. A comparison between the BER of the first and the second layers of the PU for two different values of α is depicted in Fig. 3. In this simulation we assume that, $\sigma_{e_1}^2 = 0.05$ and $\varepsilon = 0.2$. As expected, the BER of the first layer is far better than the second layer since the hierarchical constellation suggests and also, because of CER, the BERs tend to a constant value in high SNRs. The illustration of the effect of CER on the BERs of the primary and the secondary users along with the comparison between the obtained analytical expressions and the simulation results are provided in Fig. 4. In this simulation we assume that $\alpha = 0.3$. Fig. 5 compares the simulation results and the analytical expressions of the outage probability of two users along with the effect of CER on their performance. It's assumed that, $\alpha = 0.2$, $\gamma_1 = 10[dB]$ and $\gamma_2 = 0[dB]$. As expected, the outage probability of the PU's second layer is worse than that of the first layer. The optimal values of hybrid

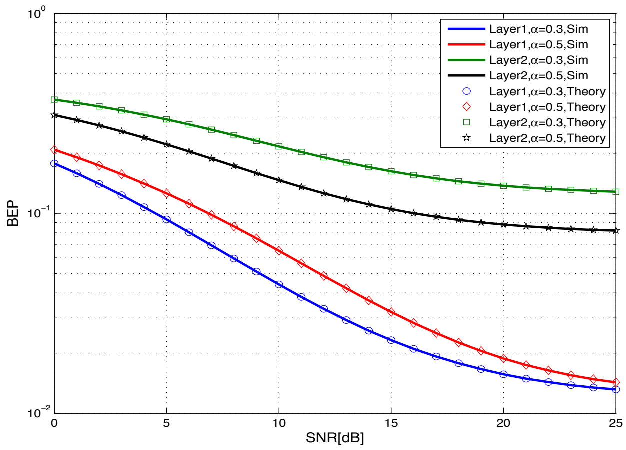

Fig. 3. Comparison between the BERs of the first and the second layer of the PU for two different values of α with $\sigma_{e_1}^2 = 0.05$ and $\sigma_I^2 = 0.1$.

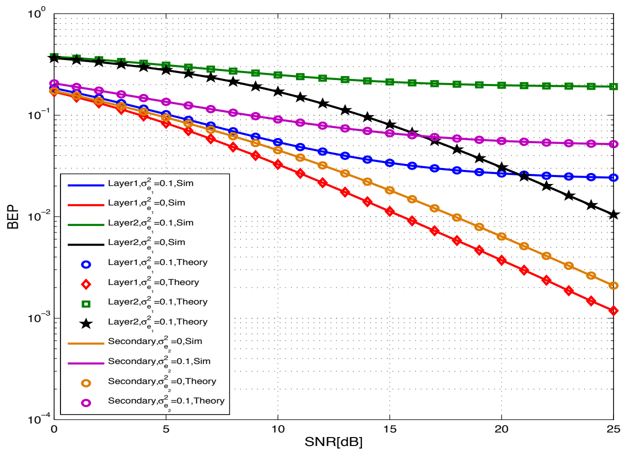

Fig. 4. The effect of CER on the BER of the primary and the secondary users in hybrid underlay/overlay mode with $\alpha = 0.3$.

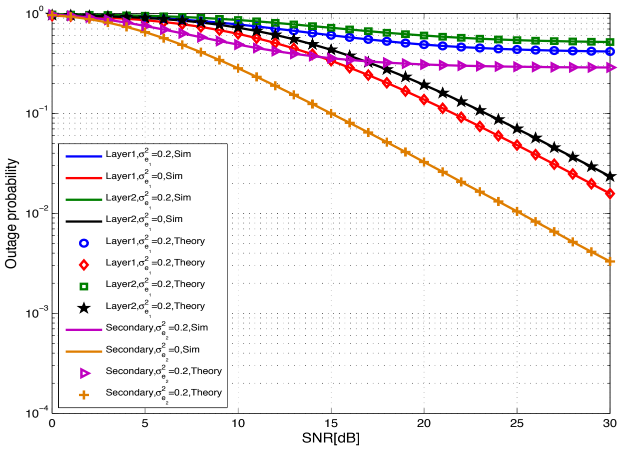

Fig. 5. The effect of CER on primary's and secondarys' outage probability in hybrid underlay/overlay mode with $\alpha = 0.2$, $\gamma_1 = 10[dB]$ and $\gamma_2 = 0[dB]$.

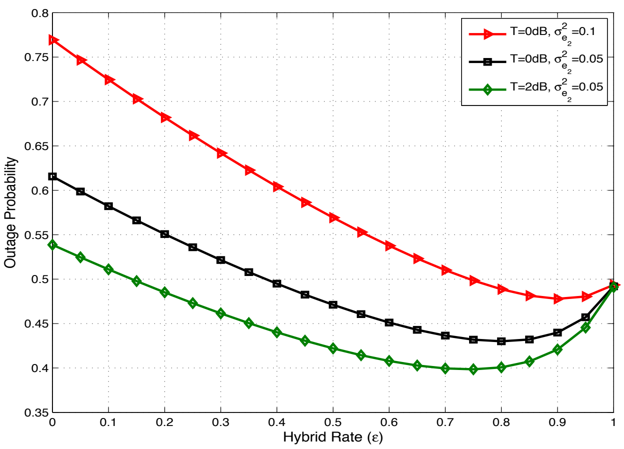

Fig. 6. Outage probability of the SU $P_{out,S}$ versus hybrid rate ε for different values of CER $\sigma^2_{e_2}$ and T. $\pi_0 = \pi_1 = 0.5$, $P_f = 0.25$, $P_d = 0.8$, $E_p = 1$, $E_s^{max} = 1.5$, $\gamma_1 = 10[dB]$, $P_{out}^{th} = -10[dB]$ and $\mu = 0.3\varepsilon$.

rate for different values of interference temperature T and the SU's CER $\sigma^2_{e_2}$ are depicted in Fig. 6. In this simulation we presume that, $\pi_0 = \pi_1 = 0.5$, $P_f = 0.25$, $P_d = 0.8$, $E_p = 1$, $E_s^{max} = 1.5$, $\gamma_1 = 10[dB]$, $P_{out}^{th} = -10[dB]$ and $\mu = 0.3\varepsilon$. As shown, with fixed CER, an increase in the value of the interference temperature results in an decrease in the value of hybrid rate. In other words, the SU must switch to the underlay mode more frequent rather than the overlay mode to maintain the interference constraint imposed by the PU. On the other hand, while the interference temperature is kept at a fixed value, an increase in the value of CER causes the optimal value of the hybrid rate to be increased thus, the SU transmits in the overlay mode more than the underlay mode.

VI. Conclusion

We investigated the performance analysis of hybrid Underlay/Overlay CR system in terms of average error and outage probabilities while the primary and the secondary users' channels suffer from CER. We derived exact expressions for the average error and outage probabilities demonstrating the effect of CER. We discussed the optimum value of the hybrid rate that minimizes the outage probability of the SU while keeping the outage probability of the PU's worst layer of hierarchical modulation below a desired level. As expected, in the presence of CER, the performance of the system degrades. In contrast, based on the obtained results in the hybrid CR system, we conclude that CER causes the SU to stay in the overlay mode more frequent rather than in the underlay mode to maintain an acceptable performance.

Acknowledgment

This work was supported by Ooredoo under the project QUEX-Qtel-09/10-10.

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