Statistically Coordinated Precoding for the MISO Cognitive Radio Channel

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***Abstract*—In this work[1], we study a cognitive radio setting consisting of a primary multiple-antenna transmitter (TX) serving a single-antenna primary user (PU) and a secondary multiple-antenna TX serving a secondary user (SU). The main specificity of this work is that we let the primary TX coordinate its transmit strategy with the secondary TX, while considering a realistic channel state information (CSI) scenario where each TX has solely access to the instantaneous knowledge of its direct channel and the statistics of the multi-user channel. This setting gives rise to a *Team Decision* problem where the TXs aim at cooperating on the basis of individual information. We develop a novel coordination scheme where the TXs coordinate *without any exchange of information or any iteration* to guarantee the fulfillment of the primary constraint while maximizing the rate of the SU. The coordination is done on the basis of statistical information such that the coordination can be optimized off-line. The proposed scheme outperforms conventional schemes from the literature and has low complexity. It can thus be used in settings with low signal processing capabilities and a weak backhaul infrastructure.**

I. Introduction

Cognitive radio has been lately suggested as a promising technology in view of increasing spectral efficiency in wireless communications [1]. In the *underlay* cognitive radio approach, a primary operator allows the simultaneous use of its spectral resources by an unlicensed secondary system, on condition that harmful interference emitted by the secondary transmitter will not exceed a prescribed maximum tolerated level [2]. Under such a setup, efficient schemes have been developed in multiple-input multiple-output (MIMO) settings to maximize the rate of the SUs subject to given constraints over the interference suffered by the PUs [3]–[5]. In practice, it is expected to be difficult for the secondary TX to obtain accurately the multi-user CSI. In particular, the CSI exchange between TXs (if possible) is likely to introduce delay which significantly impacts the performance of the precoding.

Therefore, an extensive literature has been focused on designing iterative distributed approaches being robust to imperfect CSI or requiring only local CSI [See [6] and references therein]. Game theoretic iterative approaches have been also suggested as a way to avoid the need for exchanging the global multi-user CSI for spectrum sharing approaches [7] as well as for underlay cognitive radio [8].

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Conventional cognitive radio approaches, however, assume that the primary TX should not adapt to the presence of the secondary TX. As the goal of the PU is solely to achieve its primary rate constraint in any case, it leads to the PU transmitting with higher power than required, thus pointlessly limiting the performance of the SU. This is likely to be too pessimistic as the exchange of long-term information between TXs does not represent a major hurdle and allows for the primary TX to adapt to the transmission of the secondary TX. How to efficiently exploit this multi-user statistical information is exactly the focus of this work.

Specifically, we propose a new coordination paradigm where the primary TX exploits its locally available CSI and the statistical information of the multi-user channel to coordinate with the secondary TX so as to guarantee its desired quality-of-service. Each TX knows only the realization of the direct channel and cooperates with the other TX which hence does not share the same CSI: this falls in the category of *Team Decision* problems [9] discussed in other wireless settings in [10], [11]. A similar scenario was already investigated in [12]. However, the scheme developed in [12] has high complexity (it requires multiple iterations of Monte-Carlo averaging) and it has to be run for each channel realization. It also requires the exchange of information between the TXs, which is not possible in our setting.

In particular, our main contributions are as follows.

- Considering a cognitive setting with a rate constraint for the PU, we develop a *statistically coordinated precoding* scheme which outperforms conventional approaches from the literature without requiring any exchange of the instantaneous channel state between the TXs. The coordination can be optimized off-line on the basis of statistical information only and is characterized by low complexity.
- The proposed statistical coordination approach for Team Decision problems is very general and can be applied in other optimization problems in wireless communications.

II. System and Channel Model

The cognitive setting studied is composed of a primary TX (TX p) communicating with a PU while a secondary TX (TX s) transmits to a SU. Assuming an underlay approach, both TX p and TX s share the same frequency band. TX j with $j \in \{s,p\}$ is equipped with M_j antennas while each of the users is equipped with a single antenna.

The signal received at user i is written as

$$y_i = \boldsymbol{h}_{i,i}^{\mathrm{H}} \mathbf{w}_i s_i + \boldsymbol{h}_{i,\bar{i}}^{\mathrm{H}} \mathbf{w}_{\bar{i}} s_{\bar{i}} + n_i, \qquad i \in \{p, s\}, \tag{1}$$

where $\boldsymbol{h}_{i,j} \in \mathbb{C}^{M_j}, i, j \in \{p, s\}$ are distributed as $\mathcal{N}_{\mathbb{C}}(\mathbf{0}_{M_j}, \mathbf{R}_{i,j})$ to model a Rayleigh fading scenario. We restrict in this work to $\mathbf{R}_{j,j} = \mathbf{I}_{M_j}, \forall j$, while the extension to correlated direct channels will be tackled in future works. The vector $\mathbf{w}_j \in \mathbb{C}^{M_j}$ denotes the beamformer at TX j and it is assumed that $\mathbf{w}_j = \sqrt{P_j}\boldsymbol{u}_i$ with $\|\mathbf{u}_j\|^2 = 1$. Finally, we consider Gaussian noise $n_i \sim \mathcal{CN}(0, N_0), i = \{p, s\}$ and we assume that the users data symbols to be transmitted are also Gaussian distributed.

The instantaneous rate of user i is then given by [13]

$$R_i(\mathbf{w}_p, \mathbf{w}_s) = \log_2\left(1 + \frac{P_i |\boldsymbol{h}_{i,i}^{\mathrm{H}} \boldsymbol{u}_i|^2}{N_0 + P_{\bar{i}} |\boldsymbol{h}_{i,\bar{i}}^{\mathrm{H}} \boldsymbol{u}_{\bar{i}}|^2}\right). \tag{2}$$

III. Problem Formulation

We consider in this work the realistic CSI configuration where TX j has instantaneous knowledge of its direct channel $\boldsymbol{h}_{j,j}$ and of the multi-user covariance matrices $\mathbf{R}_{i,j}$, $i, j \in \{s, p\}$. The multi-user covariance matrices are long-term informations and can be exchanged through low capacity/high-delay links. The Team Decision problem can be formulated as a functional optimization problem with the appropriate functional dependencies:

$$\begin{aligned} &(\mathbf{w}_p^\star, \mathbf{w}_s^\star) = \operatorname{argmax} \mathbb{E}\left[R_s(\mathbf{w}_p(\mathbf{h}_{p,p}), \mathbf{w}_s(\mathbf{h}_{s,s}))\right] \\ &\text{subject to} \quad \mathbb{E}\left[R_p(\mathbf{w}_p(\mathbf{h}_{p,p}), \mathbf{w}_s(\mathbf{h}_{s,s}))\right] \geq \tau > 0, \\ &0 \leq \|\mathbf{w}_p(\mathbf{h}_{p,p})\|^2 \leq P_p^{\max}, 0 \leq \|\mathbf{w}_s(\mathbf{h}_{s,s})\|^2 \leq P_s^{\max} \end{aligned} \tag{P1}$$

with, for $j \in \{s, p\}$,

$$\begin{array}{rccc} \boldsymbol{w}_j : & \mathbb{C}^{M_j} & \rightarrow & \mathbb{C}^{M_j} \\ & \boldsymbol{h}_{j,j} & \mapsto & \boldsymbol{w}_j(\boldsymbol{h}_{j,j}) \end{array} \tag{3}$$

Remark 1. The distributed aspect of the team decision problem is reflected in the particular functional dependencies of the two precoders. Note further that we will omit in the following to mention explicitly the dependencies of the beamformers. □

Optimization problem (P1) is supposed to be feasible which means that the PU rate objective satisfies:

$$\mathbb{E}\left[\log_2\left(1 + \frac{P_p^{\max}\|\boldsymbol{h}_{p,p}\|^2}{N_0}\right)\right] \geq \tau. \tag{4}$$

The functional optimization is especially difficult to handle due to the expectation of the logarithm. Hence, we will consider an approximated problem to develop practical solutions while keeping the important features of the optimization problem (P1). Our first step consists in applying Jensen's inequality for the expectations over the cross-channels (known at none of the TXs) by exploiting the convexity of the function $\log(1+\frac{1}{x})$. This gives

$$\begin{aligned} \mathbb{E}[R_i(\mathbf{w}_p, \mathbf{w}_s)] \geq& \mathbb{E}\left[\log_2\left(1 + \frac{|\mathbf{h}_{i,i}^H \mathbf{w}_i|^2}{N_0 + \mathbb{E}_{\mathbf{h}_{i,\bar{i}}}[|\mathbf{h}_{i,\bar{i}}^H \mathbf{w}_{\bar{i}}|^2]}\right)\right] \\ &= \mathbb{E}\left[\log_2\left(1 + \frac{P_i |\mathbf{h}_{i,i}^H \mathbf{u}_i|^2}{N_0 + P_{\bar{i}} \mathbf{u}_{\bar{i}}^{\mathrm{H}} \mathbf{R}_{i,\bar{i}} \mathbf{u}_{\bar{i}}}\right)\right] \\ &\triangleq \mathbb{E}\left[\tilde{R}_i(\mathbf{w}_p, \mathbf{w}_s)\right]. \end{aligned} \tag{5}$$

Remark 2. It is possible to apply Jensen's inequality solely for the cross-channel thanks to the independence of the precoding vectors from the cross-channels. □

Importantly, since we have obtained a lower bound for the ergodic rates, the approximation comes at the cost of a possible lower rate for the SU, but preserves the feasibility of the rate constraint at the PU.

For tractability, we focus on slow power control, which we denote by $\bar{P}_j$, for $j \in \{s, p\}$. This yields the following optimization problem:

$$\begin{aligned} &(\bar{P}_p^\star \boldsymbol{u}_p^\star, \bar{P}_s^\star \boldsymbol{u}_s^\star) = \operatorname{argmax} \mathbb{E}\left[\tilde{R}_s(\bar{P}_p \boldsymbol{u}_p(\mathbf{h}_{p,p}), \bar{P}_s \boldsymbol{u}_s(\mathbf{h}_{s,s}))\right] \\ &\text{subject to} \quad \mathbb{E}\left[\tilde{R}_p(\bar{P}_p \boldsymbol{u}_p(\mathbf{h}_{p,p}), \bar{P}_s \boldsymbol{u}_s(\mathbf{h}_{s,s}))\right] \geq \tau, \\ &0 \leq \bar{P}_p \leq P_p^{\max}, 0 \leq \bar{P}_s \leq P_s^{\max}. \end{aligned} \tag{P1'}$$

IV. Preliminary Results

We start by providing some preliminary results which will be used in the design of the novel transmission scheme.

Proposition 1. *The rate constraint at the PU is fulfilled with equality by any optimal solution of* (P1')*, i.e.,*

$$\mathbb{E}\left[\tilde{R}_p(\bar{P}_p^\star \boldsymbol{u}_p^\star, \bar{P}_s^\star \boldsymbol{u}_s^\star)\right] = \tau. \tag{6}$$

Proof. The objective $\mathbb{E}[\tilde{R}_s(\bar{P}_p \boldsymbol{u}_p, \bar{P}_s \boldsymbol{u}_s)]$ is monotonically decreasing with respect to $\bar{P}_p$, while $\mathbb{E}[\tilde{R}_p(\bar{P}_p \boldsymbol{u}_p, \bar{P}_s \boldsymbol{u}_s)]$ is monotonically increasing and continuous in $\bar{P}_p$. Hence, if the primary constraint is not fulfilled with equality, one can increase the objective by reducing power $\bar{P}_p$. This is always feasible because $\tau > 0$ implies that $\bar{P}_p^\star > 0$. □

Proposition 2. *The optimal solution of* (P1') *is obtained when one of the two TXs emits with full power, i.e., when $\bar{P}_p^\star = P_p^{\max}$ or $\bar{P}_s^\star = P_s^{\max}$.*

Proof. Let us consider an optimal solution and write $\bar{P}_i^\star = \alpha_i^\star \bar{P}^\star$ for some $\alpha_i^\star \geq 0$. It is trivial that both the rate of the PU and the rate of the SU are increasing in $\bar{P}^\star$. If none of the two TXs transmits with full power, this means that it is possible to transmit with $\bar{P}^{'} > \bar{P}^\star$. The transmission using $(\alpha_p^\star \bar{P}^{'} \boldsymbol{u}_p^\star, \alpha_s^\star \bar{P}^{'} \boldsymbol{u}_s^\star)$ is feasible and achieves a larger objective, which contradicts the optimality of $(\bar{P}_p^\star \boldsymbol{u}_p^\star, \bar{P}_s^\star \boldsymbol{u}_s^\star)$. □

V. Statistically Coordinated Precoding

A. General Approach

To tackle the difficult optimization problem (P1'), we restrict the space of possible precoding decisions to a codebook of precoding strategies $\{\mathscr{S}_1, \ldots, \mathscr{S}_K\}$. Strategy $\mathscr{S}_j$ refers in fact to a couple $(\bar{P}_p^{\mathscr{S}_j}\boldsymbol{u}_p^{\mathscr{S}_j}, \bar{P}_s^{\mathscr{S}_j}\boldsymbol{u}_s^{\mathscr{S}_j})$. The restriction to a codebook of strategies renders the optimization problem tractable as it allows to exhaustively evaluate the performance of each transmit strategy so as to choose the most efficient one. This means finding a strategy $\mathscr{S}^\star$ which fulfills $\forall j$,

$$\mathbb{E}\Big[\tilde{R}_s(\bar{P}_p^{\mathscr{S}^\star}\boldsymbol{u}_p^{\mathscr{S}^\star}, \bar{P}_s^{\mathscr{S}^\star}\boldsymbol{u}_s^{\mathscr{S}^\star})\Big] \geq \mathbb{E}\Big[\tilde{R}_s(\bar{P}_p^{\mathscr{S}_j}\boldsymbol{u}_p^{\mathscr{S}_j}, \bar{P}_s^{\mathscr{S}_j}\boldsymbol{u}_s^{\mathscr{S}_j})\Big] \quad (7)$$

while satisfying at the same time the primary rate constraint. In principle, any codebook can be used, and adding elements to the codebooks leads to a performance improvement. This requires however the computation of the expectations in (7), which in general cannot be done in closed form.

In order to be able to evaluate analytically (7), we restrict in the following to a particular codebook made of two strategies, denoted by $\mathscr{S}$ and $\mathscr{P}$. These two precoding strategies are designed based on the results obtained in Section IV and such that the expectations can be computed in closed-form. Specifically, we use the fact that the primary constraint is fulfilled with equality (Proposition 1) and that at least one of the TXs emits with full power (Proposition 2).

B. Precoding Strategy $\mathscr{P}$

In this precoding strategy –*primary oriented*–, TX p transmits with its full power $\bar{P}_p = P_p^{\max}$ and using the matched precoder $\boldsymbol{u}_p^{\mathrm{MF}}$ defined as

$$\boldsymbol{u}_p^{\mathrm{MF}} \triangleq \frac{\boldsymbol{h}_{p,p}}{\|\boldsymbol{h}_{p,p}\|}. \quad (8)$$

The secondary TX then transmits using the statistical ZF precoder $\boldsymbol{u}_s^{\mathrm{sZF}}$ equal to

$$\boldsymbol{u}_s^{\mathrm{sZF}} \triangleq \operatorname*{argmin}_{\boldsymbol{u}} \boldsymbol{u}^{\mathrm{H}}\mathbf{R}_{p,s}\boldsymbol{u}. \quad (9)$$

It remains to choose $\bar{P}_s$ so as to fulfill the rate constraint at the PU. Replacing the precoder expressions of (8) and (9) and the power of the primary TX in the rate constraint gives

$$\mathbb{E}\left[\log_2\left(1+\frac{\bar{P}_p^{\max}\|\mathbf{h}_{p,p}\|^2}{N_0+\bar{P}_s\lambda_{min}(\mathbf{R}_{p,s})}\right)\right] \geq \tau \quad (10)$$

where $\lambda_{min}(\mathbf{R}_{p,s})$ denotes the minimum eigenvalue of matrix $\mathbf{R}_{p,s}$. For clarity, we also introduce

$$\bar{\gamma}_p \triangleq \frac{\bar{P}_p^{\max}}{N_0+\bar{P}_s\lambda_{min}(\mathbf{R}_{p,s})}. \quad (11)$$

The only random variable inside (10) is $\mathbf{h}_{p,p}$ which is distributed as $\mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{R}_{p,p}=\mathbf{I}_{M_p})$. Hence, the expectation in (10) can be computed using [14, eq. (41)] as

$$\mathbb{E}[\log_2\left(1+\bar{\gamma}_p\|\mathbf{h}_{p,p}\|^2\right)] = \log_2(e)e^{\frac{1}{\bar{\gamma}_p}}\sum_{k=0}^{M-1}\frac{\Gamma(-k,\frac{1}{\bar{\gamma}_p})}{\bar{\gamma}_p^k} \quad (12)$$

where $\Gamma(\cdot)$ is the incomplete Gamma function [15, eq. (8.350.2), p. 949] defined as

$$\Gamma\left(\alpha,\frac{1}{\bar{\gamma}}\right) \triangleq \int_x^\infty t^{\alpha-1}e^{-t}\mathrm{d}t. \quad (13)$$

It remains then to determine $\bar{P}_s$ to fulfill the ergodic rate constraint with equality. This can be easily done by bisection using (12). Indeed, the ergodic rate of the PU is non-increasing in the power $\bar{P}_s$ and the rate constraint is fulfilled by assumption for $\bar{P}_s = 0$. If the constraint is also fulfilled for $\bar{P}_s = P_s^{\max}$, the secondary TX transmits with its full power. Otherwise, a simple bisection algorithm over $\bar{P}_s$ converges to a power level $\bar{P}_s$ fulfilling (10) with equality.

C. Precoding Strategy $\mathscr{S}$

In this precoding strategy –*secondary oriented*–, TX s transmits with its full power $\bar{P}_s = P_s^{\max}$ and using the matched precoder $\boldsymbol{u}_s^{\mathrm{MF}}$ defined as

$$\mathbf{u}_s^{\mathrm{MF}} \triangleq \frac{\mathbf{h}_{s,s}}{\|\mathbf{h}_{s,s}\|}. \quad (14)$$

The primary TX then transmits with the statistical ZF precoder

$$\boldsymbol{u}_p^{\mathrm{sZF}} \triangleq \operatorname*{argmin}_{\boldsymbol{u}} \boldsymbol{u}^{\mathrm{H}}\mathbf{R}_{s,p}\boldsymbol{u}. \quad (15)$$

It remains to choose $\bar{P}_p$ so as to fulfill the ergodic primary rate constraint. Replacing the precoder expressions of (14) and (15) and the power of the secondary TX in the primary rate constraint gives

$$\mathbb{E}\left[\log_2\left(1+\frac{\bar{P}_p|\boldsymbol{h}_{p,p}^{\mathrm{H}}\mathbf{u}_p^{\mathrm{sZF}}|^2}{N_0+\bar{P}_s^{\max}(\mathbf{u}_s^{\mathrm{MF}})^{\mathrm{H}}\mathbf{R}_{p,s}\mathbf{u}_s^{\mathrm{MF}}}\right)\right] \geq \tau. \quad (16)$$

We can once more use Jensen's inequality for convex functions for the expectation over $\boldsymbol{h}_{s,s}$ to obtain that

$$\begin{aligned}
&\mathbb{E}\left[\log_2\left(1+\frac{\bar{P}_p|\mathbf{h}_{p,p}^{\mathrm{H}}\mathbf{u}_p^{\mathrm{sZF}}|^2}{N_0+\bar{P}_s^{\max}(\mathbf{u}_s^{\mathrm{MF}})^{\mathrm{H}}\mathbf{R}_{p,s}\mathbf{u}_s^{\mathrm{MF}}}\right)\right] \\
&\geq \mathbb{E}\left[\log_2\left(1+\frac{\bar{P}_p|\mathbf{h}_{p,p}^{\mathrm{H}}\mathbf{u}_p^{\mathrm{sZF}}|^2}{N_0+\mathbb{E}_{\boldsymbol{h}_{s,s}}\left[\bar{P}_s^{\max}(\mathbf{u}_s^{\mathrm{MF}})^{\mathrm{H}}\mathbf{R}_{p,s}\mathbf{u}_s^{\mathrm{MF}}\right]}\right)\right] \quad (17) \\
&\geq \mathbb{E}\left[\log_2\left(1+\frac{\bar{P}_p|\boldsymbol{h}_{p,p}^{\mathrm{H}}\boldsymbol{u}_p^{\mathrm{sZF}}|^2}{N_0+\bar{P}_s^{\max}\frac{1}{M}\mathrm{tr}(\mathbf{R}_{p,s})}\right)\right]. \quad (18)
\end{aligned}$$

The beamforming vector $\boldsymbol{u}_p^{\mathrm{sZF}}$ depends only on the long-term CSI such that the random variable $|\mathbf{h}_{p,p}^{\mathrm{H}}\mathbf{u}_p^{\mathrm{sZF}}|^2$ can be seen to be exponentially distributed. Similarly to precoding strategy $\mathscr{P}$, we start by introducing for clarity $\bar{\gamma}_s$ as

$$\bar{\gamma}_s \triangleq \frac{\bar{P}_p}{N_0+\bar{P}_s^{\max}\frac{1}{M}\mathrm{tr}(\mathbf{R}_{p,s})}. \quad (19)$$

Using this notation, the lower-bound in (18) is easily shown to be equal to [14, eq. (34)]

$$\mathbb{E}\left[\log_2\left(1+\bar{\gamma}_s|\boldsymbol{h}_{p,p}^{\mathrm{H}}\boldsymbol{u}_p^{\mathrm{sZF}}|^2\right)\right] = \log_2(e)e^{\frac{1}{\bar{\gamma}_s}}\mathrm{E}_1\left(\frac{1}{\bar{\gamma}_s}\right). \quad (20)$$

In the same way as for strategy $\mathscr{P}$, it is possible to use the closed-form expression (20) to control the power used at TX p and find by bisection the minimal power $\bar{P}_p$ such that

$$\mathbb{E}\left[\log_2\left(1+\bar{\gamma}_s|\boldsymbol{h}_{p,p}^{\mathrm{H}}\boldsymbol{u}_p^{\mathrm{sZF}}|^2\right)\right] \geq \tau. \quad (21)$$

Remark 3. In contrast to strategy $\mathscr{P}$, strategy $\mathscr{S}$ reduces the feasibility region since the secondary TX transmits with full power which reduces the rate of the PU. Hence, a preliminary condition before using strategy $\mathscr{S}$ is to verify that it allows to fulfill the PU rate constraint, i.e., that $\bar{P}_p^{\max}$ satisfies (21). □

VI. Statistically Coordinated Precoding

We have described two precoding strategies which will form the building blocks for our algorithm. It remains now to show how to combine them to obtain an efficient solution to the optimization problem (P1').

Given the statistical parameters of a channel, we study whether it is possible to fulfill the primary constraint using precoding strategy $\mathscr{S}$. If equation (21) is satisfied when using $\bar{P}_p = P_p^{\max}$, this means that it is possible to fulfill the primary constraint using precoding strategy $\mathscr{S}$. This equation requires only the knowledge of statistical information and not of the instantaneous channel realizations. Hence, it can be verified at the two TXs whether equation (21) can be satisfied.

If this criterion is satisfied, both TXs transmit according to precoding strategy $\mathscr{S}$. Otherwise, both TXs transmit according to precoding strategy $\mathscr{P}$. The choice between the two precoding schemes is done by computing (21) and we call therefore this step the statistical *handshake* as it allows for the TXs to coordinate on the basis of statistical information. This coordination step needs only to be run again when the CSI statistics have changed. For the sake of clarity, the different steps of the algorithm are put together in the table below.

Algorithm 1 Statistically Coordinated Precoding Scheme

- Handshake: Verify whether coordination criterion (21) holds with $\bar{P}_p = P_p^{\max}$. If this is the case use precoding strategy $\mathscr{S}$, otherwise use precoding strategy $\mathscr{P}$.
- Precoding strategy $\mathscr{P}$:
  - TX p transmits with $\boldsymbol{u}_p^{\mathrm{MF}}$ in (8) and $\bar{P}_p = P_p^{\max}$
  - TX s transmits with $\boldsymbol{u}_s^{\mathrm{sZF}}$ in (9) and a power level $\bar{P}_s$ fulfilling (10) with equality (obtained by bisection)
- Precoding strategy $\mathscr{S}$:
  - TX s transmits with $\boldsymbol{u}_s^{\mathrm{MF}}$ in (14) and $\bar{P}_s = P_s^{\max}$
  - TX p transmits with $\boldsymbol{u}_p^{\mathrm{sZF}}$ in (15) and a power level $\bar{P}_p$ fulfilling (21) with equality (obtained by bisection)

VII. Simulations

We will now evaluate the performance of the novel coordination algorithm via Monte-Carlo simulations. In particular, we compare our scheme to the conventional *interference temperature* approach. Hence, we will first adapt this approach in the case of an ergodic rate constraint for the PU. We also provide an upperbound to evaluate the potential losses.

A. Interference Temperature Approach

We consider in this reference scheme that the primary TX transmits using $\boldsymbol{u}_p^{\mathrm{MF}}$ and $\bar{P}_p^{\max}$. Because of the absence of coordination between the TXs, this is necessary to be sure to fulfill the ergodic rate constraint at the PU. We consider then an interference temperature constraint where the secondary TX maximizes its signal strength subject to a constraint on the average interference level, which we denote by $\mathcal{I}$. We then fix the beamformer of the secondary TX as

$$\boldsymbol{u}_s^{\mathrm{TP}} \triangleq \underset{\boldsymbol{u}}{\operatorname{argmax}} \frac{\boldsymbol{u}^{\mathrm{H}} \boldsymbol{h}_{s,s} \boldsymbol{h}_{s,s}^{\mathrm{H}} \boldsymbol{u}}{\boldsymbol{u}^{\mathrm{H}} \mathbf{R}_{p,s} \boldsymbol{u}} \tag{22}$$

and the power is then given by

$$\bar{P}_s = \frac{\mathcal{I}}{\mathbb{E}\left[(\boldsymbol{u}_s^{\mathrm{TP}})^{\mathrm{H}} \mathbf{R}_{p,s} \boldsymbol{u}_s^{\mathrm{TP}}\right]}. \tag{23}$$

It remains then to determine the maximal interference temperature $\mathcal{I}$, i.e., such that

$$\mathbb{E}\left[\log_2\left(1 + \frac{P_p^{\max} \|\boldsymbol{h}_{p,p}^{\mathrm{H}}\|^2}{N_0 + \mathcal{I}}\right)\right] = \tau. \tag{24}$$

Such an interference level $\mathcal{I}$ can be easily found by bisection as described in Section V.

B. Upperbound

Optimal precoders are not easily derived as they strike a trade-off between maximizing the signal strength and minimizing the interference. However, an upperbound can be obtained by considering that it is possible to achieve the optimal value in both cases. It then remains to find the slow power control solving:

$$\begin{aligned} &\max_{\bar{P}_s, \bar{P}_p} \mathbb{E}\left[\log_2\left(1 + \frac{\bar{P}_s \|\boldsymbol{h}_{s,s}^{\mathrm{H}}\|^2}{N_0 + \bar{P}_p \lambda_{\min}(\mathbf{R}_{s,p})}\right)\right] \\ &\text{subject to } \mathbb{E}\left[\log_2\left(1 + \frac{\bar{P}_p \|\boldsymbol{h}_{p,p}^{\mathrm{H}}\|^2}{N_0 + \bar{P}_s \lambda_{\min}(\mathbf{R}_{p,s})}\right)\right] \geq \tau, \\ &\qquad 0 \leq \bar{P}_p \leq P_p^{\max}, 0 \leq \bar{P}_s \leq P_s^{\max}. \end{aligned} \tag{25}$$

Similarly to Proposition 2, it is straightforward that one of the TXs will transmit with its full power. Hence, we try whether the ergodic constraint is fulfilled using $\bar{P}_s = P_s^{\max}$ and $\bar{P}_p = P_p^{\max}$. If this is the case, the secondary TX transmits with full power and the primary TX finds by bisection a power level to fulfill the ergodic constraint with equality. Otherwise, the primary TX transmits with full power and it is the secondary TX which finds its power level by bisection to fulfill the ergodic constraint with equality.

Remark 4. We consider the same partial CSI assumptions when computing this upper-bound. Considering full CSI knowledge would lead to a very loose upperbound. □

C. Simulation Results

We now compare the performance of our algorithm to the interference temperature approach and to the upperbound presented above. We use Monte-Carlo simulations with 10000 channel realizations for a network with $M_p = M_s = 3$ antennas per TX and with the correlation matrices being written as follows for a given $\rho \in [0, 1]$:

$$\mathbf{R}_{p,p} = \mathbf{R}_{s,s} = \mathbf{I}_3, \quad \mathbf{R}_{p,s} = \mathbf{R}_{s,p} = \begin{bmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{bmatrix}. \tag{26}$$

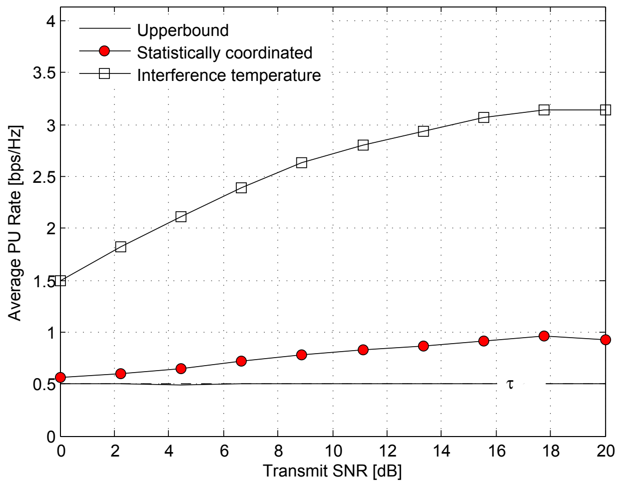

Fig. 1: Ergodic rate of the primary user

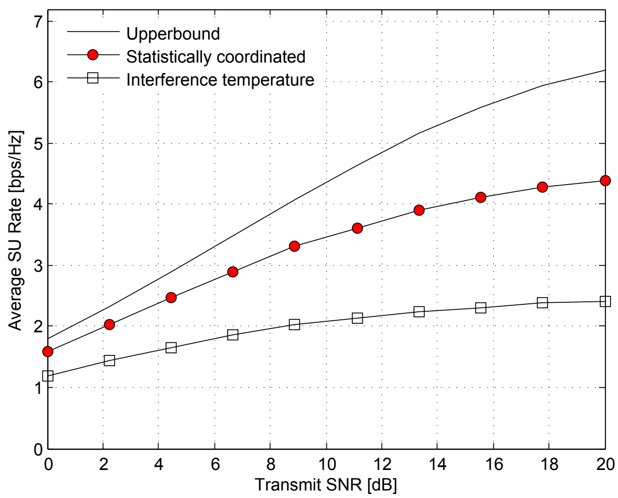

Fig. 2: Ergodic rate of the secondary user

Considering $\rho = 0.5$ and $\tau = 0.5$ bps/Hz, we show in Fig. 1 the rate of the PU while the rate of the SU is shown in Fig. 2. As expected all three approaches satisfy the rate constraint at the PU. The outerbound achieves exactly the PU rate constraint following the centralized power control in (25). The proposed algorithm allows to control the rate of the PU and translates this improved coordination in a strong performance improvement for the SU. Note that the upperbound achieves a lower PU rate as the goal of the optimization is not to maximize the rate of the PU. However, it outperforms as expected both alternative approaches in terms of the SU rate.

In fact, using small values of τ, our approach becomes then very close to the upper-bound. For large values of τ, there is then not much room for the secondary TX and our approach becomes less efficient. In particular, it is then even slightly-outperformed by the interference temperature approach which uses a more efficient definition of the statistical ZF. This is not a fundamental limitation and will be investigated in the future.

VIII. Conclusions

Considering a realistic CSI configuration where each TX has only access to its own channel, we propose a scheme where the secondary TX and the primary TX coordinate on the basis of the statistical information of the channel. The proposed scheme outperforms conventional approaches from the literature at the price of only low CSI and computation requirements as it is not necessary for the TXs to share any instantaneous CSI, and the coordination can be optimized off-line. The proposed approach is a novel method to deal with Team Decision problems and has the potential to be generalized to many other network configurations. Although the proposed method is practically interesting, getting closer to the optimal solution is an interesting and challenging topic for future research.

References

[1] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, 2005.

[2] E. Biglieri, A. J. Goldsmith, L. J. Greenstein, N. Mandayam, and H. V. Poor, *Principles of cognitive radio*. Cambridge University Press, 2012.

[3] R. Zhang and Y.-C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE Journal of Sel. Topics in Sign. Process.*, vol. 2, no. 1, pp. 88–102, Feb. 2008.

[4] R. Zhang, "Optimal power control over fading cognitive radio channel by exploiting primary user CSI," in *Proc. IEEE Global Communications Conference (GLOBECOM)*, 2008.

[5] Y. Huang and D. P. Palomar, "Rank-constrained separable semidefinite programming with applications to optimal beamforming," *IEEE Trans. Signal Process.*, vol. 58, no. 2, pp. 664–678, Feb. 2010.

[6] K. Wang, N. Jacklin, Z. Ding, and C. Chi, "Robust MISO transmit optimization under outage-based QoS constraints in two-tier heterogeneous networks," *IEEE Trans. on Wireless Commun.*, vol. 12, no. 4, pp. 1536–1276, Apr. 2013.

[7] B. Wang, Y. Wu, and K. J. R. Liu, "Game theory for cognitive radio networks: An overview," *Elsevier Computer Networks*, pp. 2537–2561, 2010.

[8] G. Scutari and D. P. Palomar, "MIMO cognitive radio: A game theoretical approach," *IEEE Trans. Signal Process.*, vol. 58, no. 2, pp. 761–780, Feb. 2010.

[9] R. Radner, "Team decision problems," *The Annals of Mathematical Statistics*, 1962.

[10] R. Zakhour and D. Gesbert, "Team decision for the cooperative MIMO channel with imperfect CSIT sharing," in *Proc. Information Theory and Applications Workshop (ITA)*, 2010.

[11] P. de Kerret and D. Gesbert, "CSI sharing strategies for transmitter cooperation in wireless networks," *IEEE Wireless Commun. Mag.*, vol. 20, no. 1, pp. 43–49, Feb. 2013.

[12] M. C. Filippou, G. A. Ropokis, and D. Gesbert, "A team decisional beamforming approach for underlay cognitive radio networks," in *Proc. IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, 2013.

[13] T. Cover and A. Thomas, *Elements of information theory*. Wiley-Interscience, Jul. 2006.

[14] M. S. Alouini and A. J. Goldsmith, "Capacity of Rayleigh fading channels under different adaptive transmission and diversity-combining techniques," *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1165–1181, Jul. 1999.

[15] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products*, 7th ed. Elsevier/Academic Press, Amsterdam, 2007.