Fundamental Capacity Limits of Cognitive Radio in Fading Environments with Imperfect Channel Information

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***Abstract*—In this paper, we analyze the capacity gains of opportunistic spectrum-sharing channels in fading environments with imperfect channel information. In particular, we consider that a secondary user may access the spectrum allocated to a primary user as long as the interference power, inflicted at the primary's receiver as an effect of the transmission of the secondary user, remains below predefined power limits, average or peak, and investigate the capacity gains offered by this spectrum-sharing approach when only partial channel information of the link between the secondary's transmitter and primary's receiver is available to the secondary user. Considering average received-power constraint, we derive the ergodic and outage capacities along with their optimum power allocation policies for Rayleigh flat-fading channels, and provide closed-form expressions for these capacity metrics. We further assume that the interference power inflicted on the primary's receiver should remain below a peak threshold. Introducing the concept of *interference-outage*, we derive lower bounds on the ergodic and outage capacities of the channel. In addition, we obtain closed-form expressions for the expenditure-power required at the secondary transmitter to achieve the above-mentioned capacity metrics. Numerical simulations are conducted to corroborate our theoretical results.**

***Index Terms*—Cognitive radio, spectrum sharing, received-power constraint, ergodic capacity, outage capacity, fading channels.**

I. INTRODUCTION

COGNITIVE radio (CR), formally defined as *a radio that can change its transmitter parameters based on interaction with the environment in which it operates* [1], is emerging as the solution for the spectrum scarcity in next-generation wireless networks. This technology, first introduced by J. Mitola in [2], has a tremendous potential to improve the utilization of the radio spectrum by implementing an efficient sharing of the licensed spectrum. It consists of an intelligent wireless communications technique which enables secondary users to use spectral bands that are licensed to primary users, as long as the former do not cause harmful interference to the latter [3], [4].

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CR technology needs to enable essential functionalities so as to achieve the capability of using and sharing the spectrum in an opportunistic manner. As the first step, CR is required to sense the spectrum holes and to adapt to the changes in its surrounding environment in order to satisfy interference limitations to privileged users of the band [3]. Hence, CR is required to detect the neighboring receivers of the primary users. However, as the receivers are passive equipments, it is difficult to measure the channel between the secondary's transmitter and the primary's receiver. Therefore, most of the recent works focus on detecting active primary transmitters in the band [5]–[7]. In particular, [7] shows that cooperation between secondary users in detecting the primary transmitter can significantly increase the accuracy of the spectrum sensing techniques. However, none of the studies in [5]–[7] provide information about the location of the primary receivers.

Noting that the interference actually takes place at the receiver, interference-based detection methods are required. In this respect, [8] introduces measurement scenarios for assessing harmful interference on the licensed device and quantifies the effect of factors such as power control and directional antennas on the interference level. In [9], a direct receiver detection method, where the leakage power of the local oscillator emitted by the radio frequency front-end of the primary receiver is used to detect a primary receiver, is presented.

On the other hand, the spectrum holes, detected through spectrum sensing, show different characteristics according to the time-varying radio environment, interference level, link layer delay, etc [3]. Hence, the cognitive radio network should be able to decide on the best spectrum band allocation that meets the quality of service (QoS) requirements of different users. This decision can be made by assessing the channel capacity, which is known as the most important factor for spectrum characterization.

In this respect, the capacity of CR is studied in [10]–[12]. In particular, [12] characterizes the maximum achievable rate of a cognitive user under interference from the primary user. On the other hand, the capacity subject to spectrum-sharing constraints, i.e., constraints on the received-power at a third party's receiver, has been investigated in [13] and [14] for additive white Gaussian noise (AWGN) and fading channels, respectively. In addition, different capacity metrics, e.g., ergodic, outage, and minimum-rate, of Rayleigh fading channels with joint peak and average interference power constraints on the primary receiver are obtained in [15]. The results of [14] and [15], however, rely on the availability of perfect knowledge of the link between the secondary's transmitter and primary's receiver to the secondary user; an assumption which is hard to obtain in practice. On the other hand, [16] assumes that the secondary user is allowed to access the spectrum band occupied by a primary user as long as the latter is provided with a minimum rate for a certain percentage of time, and obtains lower bounds on the outage capacity of the secondary user's link when only imperfect channel information is available to the secondary user.

In this paper, assuming that the secondary user is provided with only partial information of the link between its transmitter and primary's receiver, we consider average or peak received-power constraints at the primary receiver, and investigate the capacity limits of the secondary user channel in Rayleigh fading environment. Different capacity notions, e.g., ergodic capacity, delay-limited capacity, and outage capacity, are of importance when assessing the capacity limits of fading channels [17], [18]. In particular, ergodic capacity is defined as the maximum of the long-term average rate with arbitrary small probability of error that can be achieved when the transmission time is long enough to reveal the long-term ergodic properties of the channel. Hence, in systems with no constraints on the delay, the ergodic capacity represents the relevant performance limit indicator [17], [18]. The capacity of a fading point-to-point channel subject to average transmit-power constraint, under the assumption of perfect channel side information (CSI) at both the receiver and transmitter, has been derived in [19]. Closed-form expressions for the channel capacity of the above system have also been derived in [20].

On the other hand, delay-limited capacity which defines the rate that can be maintained in all fading states is a more appropriate capacity notion in wireless systems that carry out real-time delay-sensitive applications, such as voice and video [21]. In this case, the transmitter uses the CSI to maintain a constant received-power or, equivalently, inverts the channel fading. Using channel inversion, the capacity of fading channels with transmit-power constraint has been derived in [19]. This metric corresponds to the capacity that can be achieved in all fading states while meeting the power constraint. However, in extreme fading cases, e.g., Rayleigh fading, this capacity is zero as the transmitter has to spend a huge amount of power for channel states in deep fade to achieve a constant-rate. To alleviate this problem, an adaptive transmission technique, referred to as truncated channel inversion with fixed rate (*tifr*), which can achieve non-zero constant rates was introduced in [19]. This technique maintains a constant received-power for channel fades above a given cutoff depth. Outage capacity refers to the rate that can be achieved with an outage probability according to a certain threshold [17].

Here, we consider partial channel information for the secondary user and obtain the ergodic capacity, capacity with *tifr* transmission policy and outage capacity along with their power allocation policies under average received-power constraint at a third party's receiver. Average received-power constraint is imposed when the performance of the primary's receiver depends on the average signal-to-interference-and-noise ratio (SINR). On the other hand, we consider that the interference power inflicted at the primary's receiver as an effect of the secondary transmission should remain below a predefined peak threshold. Then, introducing the concept of *interference-outage*, we obtain lower bounds on the outage and ergodic capacities of the channel. Moreover, we find the expenditure-power required at the secondary transmitter to achieve these capacity levels.

In the following, the system and channel models are introduced in Section II. The ergodic and outage capacities of a Rayleigh fading channel, under average received-power constraint at the primary receiver and imperfect channel knowledge, are derived in closed-form in Section III. Section IV considers the peak received-power constraint case and presents the channel capacity lower bounds. Section V provides numerical results and discussions followed by summarizing conclusions given in Section VI.

II. System and Channel Models

We consider a spectrum-sharing system in which a secondary user is allowed to use the spectrum occupied by a primary user, as long as the amount of interference power inflicted at the receiver of the primary user is within predefined constraint on the average or peak powers. We consider a discrete-time flat-fading channel, where the received signal $y_{\mathrm{s}}[n]$ at the receiver of the secondary user depends on the transmitted signal $x_{\mathrm{s}}[n]$ according to

$$y_{\mathrm{s}}[n] = c_{\mathrm{s}}[n]x_{\mathrm{s}}[n] + z_{\mathrm{s}}[n], \tag{1}$$

where n indicates the time index, $c_{\mathrm{s}}[n]$ is the complex channel gain between the transmitter and the receiver of the secondary user, and $z_{\mathrm{s}}[n]$ represents the AWGN. We define the channel gain between the transmitter of the secondary user and the receiver of the primary user by $c_{\mathrm{p}}[n]$, and assume that $c_{\mathrm{s}}[n]$ and $c_{\mathrm{p}}[n]$ are independent and identically distributed (i.i.d.) zero mean circularly symmetric complex Gaussian (ZMCSCG) random variables with real and imaginary parts having variances of 0.5. Furthermore, the channel gains are assumed to be independent from the noise. The noise power spectral density and the received signal bandwidth are denoted by N_0 and B, respectively.

We further assume that perfect knowledge of $c_{\mathrm{s}}[n]$ is available at the receiver and transmitter of the secondary user. However, the secondary user is only provided with some partial information of $c_{\mathrm{p}}[n]$, namely, $\check{c}_{\mathrm{p}}[n]$, where $c_{\mathrm{p}}[n]$ and $\check{c}_{\mathrm{p}}[n]$ are jointly ergodic and stationary Gaussian distributed processes. The information about $c_{\mathrm{p}}[n]$ can be carried out by a band manager that mediates between the primary and secondary users [14], [22], or can be directly fed back from the primary's receiver to the secondary's transmitter [23]. In particular, an algorithm for exchanging the CSI between the primary and secondary users is provided in [23].

The secondary user performs minimum mean square error (MMSE) estimation of $c_{\mathrm{p}}[n]$ given $\check{c}_{\mathrm{p}}[n]$, such that $\hat{c}_{\mathrm{p}}[n] = \mathcal{E}\left[c_{\mathrm{p}}[n]\middle|\check{c}_{\mathrm{p}}[n], \check{c}_{\mathrm{p}}[n-1], \cdots\right]$. The MMSE estimation error is denoted by $\tilde{c}_{\mathrm{p}}[n] = c_{\mathrm{p}}[n] - \hat{c}_{\mathrm{p}}[n]$, where, by the property of MMSE estimation, $c_{\mathrm{p}}[n]$ and $\hat{c}_{\mathrm{p}}[n]$ are uncorrelated, and $\hat{c}_{\mathrm{p}}[n]$ and $\tilde{c}_{\mathrm{p}}[n]$ are ZMCSCG distributed with variances $\frac{1-\sigma_{\mathrm{e}}^2}{2}$ and $\frac{\sigma_{\mathrm{e}}^2}{2}$, respectively. Hereafter, we omit the time index n as it is clear from the context. Also, for the ease of notations, we refer to channel power gains by h_{p}, h_{s}, $\hat{h}_{\mathrm{p}}$, and the channel power gain estimation error by $\tilde{h}_{\mathrm{p}}$, where $h = |c|^2$.

In the following, we derive the ergodic capacity and, for non-ergodic channels, the outage capacity, of the secondary user's fading link taking into account average or peak constraints on the received-power at the primary receiver when only imperfect knowledge of $h_{\rm p}$ is available at the secondary's transmitter.

III. Average Received-Power Constraint

In this section, we consider a fading channel with average received-power constraint at a third party's receiver which can be defined as follows:

$$Q_{\rm avg} \geq \mathcal{E}_{h_{\rm s},\hat{h}_{\rm p},\tilde{h}_{\rm p}}\left\{P(h_{\rm s},\hat{h}_{\rm p})h_{\rm p}\right\} \quad (2)$$

$$= \mathcal{E}_{h_{\rm s},\hat{h}_{\rm p},\tilde{h}_{\rm p}}\left\{P(h_{\rm s},\hat{h}_{\rm p})\left(\hat{h}_{\rm p}+\tilde{h}_{\rm p}\right)\right\} \quad (3)$$

$$= \mathcal{E}_{h_{\rm s},\hat{h}_{\rm p}}\left\{P(h_{\rm s},\hat{h}_{\rm p})\hat{h}_{\rm p}\right\} + \sigma_{\rm e}^2 P_{\rm avg}, \quad (4)$$

where $P(h_{\rm s},\hat{h}_{\rm p})$ denotes the transmit power as a function of $h_{\rm s}$ and $\hat{h}_{\rm p}$, $P_{\rm avg} = \mathcal{E}_{h_{\rm s},\hat{h}_{\rm p}}\left\{P(h_{\rm s},\hat{h}_{\rm p})\right\}$ indicates the average transmit power, $\mathcal{E}_{h_{\rm s},\hat{h}_{\rm p},\tilde{h}_{\rm p}}$ defines the expectation over the joint probability density function (PDF) of $h_{\rm s}$, $\hat{h}_{\rm p}$ and $\tilde{h}_{\rm p}$, and $\mathcal{E}_{h_{\rm s},\hat{h}_{\rm p}}$ indicates the expectation over the joint PDF of $h_{\rm s}$ and $\hat{h}_{\rm p}$. To derive (4) from (3), we use the fact that $\hat{h}_{\rm p}$ and $\tilde{h}_{\rm p}$ are independent.

A. *Ergodic Capacity*

Here we obtain the ergodic capacity of a Rayleigh fading channel under the average received-power constraint (4). By adapting the approach in [19] to our system model, the ergodic capacity can be shown to be achieved through optimal utilization of the input power over time such that the received-power constraint is met. Therefore, the ergodic capacity in this case represents the solution to the following problem:

$$\frac{C_{\rm er}}{B} = \max_{P(h_{\rm s},\hat{h}_{\rm p})>0} \; \mathcal{E}_{h_{\rm s},\hat{h}_{\rm p}}\left\{\ln\left(1+\frac{P(h_{\rm s},\hat{h}_{\rm p})h_{\rm s}}{N_0 B}\right)\right\} \quad (5)$$

$$\text{s.t.} \quad \mathcal{E}_{h_{\rm s},\hat{h}_{\rm p}}\left\{P(h_{\rm s},\hat{h}_{\rm p})\hat{h}_{\rm p}\right\} + \sigma_{\rm e}^2 P_{\rm avg} \leq Q_{\rm avg}. \quad (6)$$

The solution of the optimization problem in (5) can be found by using Lagrangian optimization approach according to

$$P(h_{\rm s},\hat{h}_{\rm p}) = \max\left\{0, \left(\frac{\gamma_0}{\hat{h}_{\rm p}+\sigma_{\rm e}^2} - \frac{N_0 B}{h_{\rm s}}\right)\right\}, \quad (7)$$

where γ_0 must be determined such that the average received-power constraint is satisfied with equality. The power allocation policy (7) implies that the transmission is suspended when the link between the transmitter and receiver of the secondary user is weak compared to $\hat{h}_{\rm p}+\sigma_{\rm e}^2$. Considering the conditions $P(h_{\rm s},\hat{h}_{\rm p}) \geq 0$ and $\hat{h}_{\rm p} \geq 0$, the power allocation policy can be simplified to

$$P(h_{\rm s},\hat{h}_{\rm p}) = \begin{cases} \dfrac{\gamma_0}{\hat{h}_{\rm p}+\sigma_{\rm e}^2} - \dfrac{N_0 B}{h_{\rm s}}, & \hat{h}_{\rm p} \leq \dfrac{\gamma_0 h_{\rm s}}{N_0 B} - \sigma_{\rm e}^2 \text{ and} \\ & h_{\rm s} \geq \dfrac{N_0 B \sigma_{\rm e}^2}{\gamma_0}, \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

We now simplify the average received-power constraint by inserting (8) into (2) at equality, thus yielding

$$Q_{\rm avg} = \int_{\frac{N_0 B \sigma_{\rm e}^2}{\gamma_0}}^{\infty} \int_0^{\frac{\gamma_0 h_{\rm s}}{N_0 B}-\sigma_{\rm e}^2} \left(\gamma_0 - N_0 B \frac{\hat{h}_{\rm p}+\sigma_{\rm e}^2}{h_{\rm s}}\right) \times f_{\hat{h}_{\rm p}}(\hat{h}_{\rm p}) f_{h_{\rm s}}(h_{\rm s}) d\hat{h}_{\rm p} dh_{\rm s}, \quad (9)$$

where $f_x(x)$ represents the PDF of the random variable x. We now evaluate the integration in (9) by inserting $f_{h_{\rm s}}(h_{\rm s}) = e^{-h_{\rm s}}$ and $f_{\hat{h}_{\rm p}}(\hat{h}_{\rm p}) = \frac{1}{1-\sigma_{\rm e}^2}\, e^{-\frac{\hat{h}_{\rm p}}{1-\sigma_{\rm e}^2}}$ into (9) as follows:

$$Q_{\rm avg} = \int_{\frac{N_0 B \sigma_{\rm e}^2}{\gamma_0}}^{\infty} e^{-h_{\rm s}} \int_0^{\frac{\gamma_0 h_{\rm s}}{N_0 B}-\sigma_{\rm e}^2} \left(\gamma_0 - N_0 B \frac{\hat{h}_{\rm p}+\sigma_{\rm e}^2}{h_{\rm s}}\right) \times \frac{e^{-\frac{\hat{h}_{\rm p}}{1-\sigma_{\rm e}^2}}}{1-\sigma_{\rm e}^2}\, d\hat{h}_{\rm p} dh_{\rm s} \quad (10)$$

$$= -N_0 B\left(1-\sigma_{\rm e}^2\right) e^{\frac{\sigma_{\rm e}^2}{1-\sigma_{\rm e}^2}} \mathrm{Ei}\left(-\frac{\sigma_{\rm e}^2}{1-\sigma_{\rm e}^2} - \frac{N_0 B \sigma_{\rm e}^2}{\gamma_0}\right) + \gamma_0 e^{-\frac{N_0 B \sigma_{\rm e}^2}{\gamma_0}} + N_0 B \mathrm{Ei}\left(-\frac{N_0 B \sigma_{\rm e}^2}{\gamma_0}\right), \quad (11)$$

where $\mathrm{Ei}(x)$ denotes the exponential-integral function given by $\mathrm{Ei}(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt$ [24]. A solution for γ_0 can now be obtained from (11), for which the integrations are evaluated.

We further obtain a closed-form expression for the ergodic capacity of the system under consideration as follows:

$$\frac{C_{\rm er}}{B} = \int_0^{\infty} \frac{e^{-\frac{\hat{h}_{\rm p}}{1-\sigma_{\rm e}^2}}}{1-\sigma_{\rm e}^2} \int_{N_0 B \frac{\hat{h}_{\rm p}+\sigma_{\rm e}^2}{\gamma_0}}^{\infty} \ln\left(\frac{\gamma_0 h_{\rm s}}{N_0 B\left(\hat{h}_{\rm p}+\sigma_{\rm e}^2\right)}\right) \times e^{-h_{\rm s}} dh_{\rm s} d\hat{h}_{\rm p} \quad (12)$$

$$= e^{\frac{\sigma_{\rm e}^2}{1-\sigma_{\rm e}^2}} \mathrm{Ei}\left(-\frac{\sigma_{\rm e}^2}{1-\sigma_{\rm e}^2} - \frac{N_0 B \sigma_{\rm e}^2}{\gamma_0}\right) - \mathrm{Ei}\left(-\frac{N_0 B \sigma_{\rm e}^2}{\gamma_0}\right). \quad (13)$$

Note that applying the above result (13) to the perfect channel knowledge case, where $\sigma_{\rm e}^2 = 0$, yields the result in [14].

In addition, we obtain the average expenditure-power required to achieve the ergodic capacity as

$$P_{\rm avg} = \int_0^{\infty} \frac{e^{-\frac{\hat{h}_{\rm p}}{1-\sigma_{\rm e}^2}}}{1-\sigma_{\rm e}^2} \int_{N_0 B \frac{\hat{h}_{\rm p}+\sigma_{\rm e}^2}{\gamma_0}}^{\infty} \left(\frac{\gamma_0}{\hat{h}_{\rm p}+\sigma_{\rm e}^2} - \frac{N_0 B}{h_{\rm s}}\right) \times e^{-h_{\rm s}} dh_{\rm s} d\hat{h}_{\rm p} \quad (14)$$

$$= -\left(\frac{\gamma_0}{1-\sigma_{\rm e}^2} + N_0 B\right) e^{\frac{\sigma_{\rm e}^2}{1-\sigma_{\rm e}^2}} \mathrm{Ei}\left(-\frac{\sigma_{\rm e}^2}{1-\sigma_{\rm e}^2} - \frac{N_0 B \sigma_{\rm e}^2}{\gamma_0}\right) + N_0 B \mathrm{Ei}\left(-\frac{N_0 B \sigma_{\rm e}^2}{\gamma_0}\right). \quad (15)$$

Note that by using (15), one can show that the average transmit power to achieve the ergodic capacity of a Rayleigh fading channel under average received-power constraint with perfect channel knowledge at the secondary transmitter, $\sigma_{\rm e}^2 = 0$, is infinite.

B. *Capacity with tifr Transmission Policy*

In this section, we determine the capacity of a Rayleigh fading channel with *tifr* transmission policy under average received-power constraint at a third party's receiver, when only partial information about $h_{\rm p}$ is available to the secondary user. We recall that in *tifr* technique, the transmitter inverts the channel fading so as to maintain a constant-rate at the receiver. Defining γ_0 as the cutoff value for $\frac{\hat{h}_{\rm p}+\sigma_{\rm e}^2}{h_{\rm s}}$, over which the transmission is suspended, we can then express the power allocation policy according to

$$P(h_{\rm s},\hat{h}_{\rm p}) = \begin{cases} \frac{\alpha}{h_{\rm s}}, & \hat{h}_{\rm p} \le (\gamma_0 h_{\rm s} - \sigma_{\rm e}^2) \text{ and } h_{\rm s} \ge \frac{\sigma_{\rm e}^2}{\gamma_0}, \\ 0, & \text{otherwise,} \end{cases} \quad (16)$$

where γ_0 and α must be found such that the average received-power constraint is satisfied. Accordingly, the capacity of the fading channel with *tifr* transmission policy under average received-power constraint at a third party's receiver can be found as

$$\frac{C_{\rm tifr}}{B} = \max_{\gamma_0 \ge 0} \ln\left(1 + \frac{\alpha}{N_0 B}\right) \times \Pr\left\{\frac{\hat{h}_{\rm p}+\sigma_{\rm e}^2}{h_{\rm s}} \le \gamma_0\right\}, \quad (17)$$

and the constraint can be simplified to

$$\int_{\frac{\sigma_{\rm e}^2}{\gamma_0}}^{\infty} \alpha \int_0^{\gamma_0 h_{\rm s} - \sigma_{\rm e}^2} \frac{\hat{h}_{\rm p}+\sigma_{\rm e}^2}{h_{\rm s}} \, f_{\hat{h}_{\rm p}}(\hat{h}_{\rm p}) f_{h_{\rm s}}(h_{\rm s}) d\hat{h}_{\rm p} dh_{\rm s} \le Q_{\rm avg}.$$

We now derive a closed-form expression for α as follows:

$$\begin{aligned} \frac{Q_{\rm avg}}{\alpha} &= \int_{\frac{\sigma_{\rm e}^2}{\gamma_0}}^{\infty} e^{-h_{\rm s}} \int_0^{\gamma_0 h_{\rm s} - \sigma_{\rm e}^2} \left(\frac{\hat{h}_{\rm p}+\sigma_{\rm e}^2}{h_{\rm s}}\right) \frac{e^{-\frac{\hat{h}_{\rm p}}{1-\sigma_{\rm e}^2}}}{1-\sigma_{\rm e}^2} \, d\hat{h}_{\rm p} dh_{\rm s} \\ &= -\frac{\gamma_0(1-\sigma_{\rm e}^2)}{\gamma_0+1-\sigma_{\rm e}^2} e^{-\frac{\sigma_{\rm e}^2}{\gamma_0}} - \mathrm{Ei}\left(-\frac{\sigma_{\rm e}^2}{\gamma_0}\right) \\ &\quad + (1-\sigma_{\rm e}^2) e^{\frac{\sigma_{\rm e}^2}{1-\sigma_{\rm e}^2}} \mathrm{Ei}\left(-\frac{\sigma_{\rm e}^2}{1-\sigma_{\rm e}^2} - \frac{\sigma_{\rm e}^2}{\gamma_0}\right). \end{aligned} \quad (18)$$

Recalling that the transmission is cut off when the ratio $\frac{\hat{h}_{\rm p}+\sigma_{\rm e}^2}{h_{\rm s}}$ is higher than γ_0, we can express the outage probability according to

$$P_{\rm out} = 1 - \Pr\left\{\hat{h}_{\rm p} + \sigma_{\rm e}^2 \le \gamma_0 h_{\rm s}\right\} \quad (19)$$

$$= 1 - \int_0^{\infty} \frac{e^{-\frac{\hat{h}_{\rm p}}{1-\sigma_{\rm e}^2}}}{1-\sigma_{\rm e}^2} \int_{\frac{\hat{h}_{\rm p}+\sigma_{\rm e}^2}{\gamma_0}}^{\infty} e^{-h_{\rm s}} dh_{\rm s} d\hat{h}_{\rm p} \quad (20)$$

$$= 1 - \frac{\gamma_0 e^{-\frac{\sigma_{\rm e}^2}{\gamma_0}}}{\gamma_0 + 1 - \sigma_{\rm e}^2}, \quad (21)$$

which leads to the following closed-form expression for $C_{\rm tifr}$

$$\frac{C_{\rm tifr}}{B} = \max_{\gamma_0 \ge 0} \ln\left(1 + \frac{\alpha}{N_0 B}\right) \times \frac{\gamma_0 e^{-\frac{\sigma_{\rm e}^2}{\gamma_0}}}{\gamma_0 + 1 - \sigma_{\rm e}^2}. \quad (22)$$

The maximum on the right-hand-side of (22) can be found by searching numerically for the optimal value of γ_0. On the other hand, to find the outage capacity, the cutoff value γ_0 must be selected so as to achieve a specific permitted outage level using (21).

In the case when perfect knowledge of $h_{\rm p}$ is available to the transmitter of the secondary user, i.e., ($\sigma_{\rm e}^2 = 0$), the received-power constraint simplifies to $\frac{Q_{\rm avg}}{\alpha} = \ln(1+\gamma_0) - \frac{\gamma_0}{1+\gamma_0}$, and hence, the closed-form expression for the capacity can be simplified to

$$\left.\frac{C_{\rm tifr}}{B}\right|_{\sigma_{\rm e}^2=0} = \max_{\gamma_0 \ge 0} \ln\left(1 + \frac{Q_{\rm avg}}{N_0 B\left(\ln(1+\gamma_0) - \frac{\gamma_0}{1+\gamma_0}\right)}\right) \times \frac{\gamma_0}{1+\gamma_0}, \quad (23)$$

where the solution for the optimal cutoff threshold, γ_0, when $Q_{\rm avg} \ge 0.28 N_0 B$, can be obtained from

$$U(\gamma_0)\left(U(\gamma_0) + \frac{Q_{\rm avg}}{N_0 B}\right) \ln\left(1 + \frac{Q_{\rm avg}}{N_0 B U(\gamma_0)}\right) - \frac{\gamma_0 Q_{\rm avg}}{N_0 B} = 0, \quad (24)$$

where $U(\gamma_0) = \ln(1+\gamma_0) - \frac{\gamma_0}{1+\gamma_0}$. The proof for (24) is provided in Appendix I. In this case of operation, $\sigma_{\rm e}^2 = 0$, the cutoff threshold required to achieve a certain outage probability can be obtained as $\gamma_0 = \frac{1}{P_{\rm out}} - 1$, and hence, a closed-form expression for the channel capacity while satisfying a certain outage probability, $P_{\rm out}$, can be found as

$$\left.\frac{C_{\rm out}}{B}\right|_{\sigma_{\rm e}^2=0} = \ln\left(1 + \frac{Q_{\rm avg}}{N_0 B\left(-\ln(P_{\rm out}) - 1 + P_{\rm out}\right)}\right) \times (1 - P_{\rm out}). \quad (25)$$

Note that the outage capacity can be obtained as $C_{\rm outage} = \frac{C_{\rm out}}{1-P_{\rm out}}$.

On the other hand, one can obtain the average expenditure-power to achieve the capacity with *tifr* transmission policy as

$$P_{\rm avg} = \alpha \int_{\frac{\sigma_{\rm e}^2}{\gamma_0}}^{\infty} \frac{e^{-h_{\rm s}}}{h_{\rm s}} \int_0^{\gamma_0 h_{\rm s} - \sigma_{\rm e}^2} \frac{e^{-\frac{\hat{h}_{\rm p}}{1-\sigma_{\rm e}^2}}}{1-\sigma_{\rm e}^2} d\hat{h}_{\rm p} dh_{\rm s} \quad (26)$$

$$= \alpha e^{\frac{\sigma_{\rm e}^2}{1-\sigma_{\rm e}^2}} \mathrm{Ei}\left(-\frac{\sigma_{\rm e}^2}{1-\sigma_{\rm e}^2} - \frac{\sigma_{\rm e}^2}{\gamma_0}\right) - \alpha \mathrm{Ei}\left(-\frac{\sigma_{\rm e}^2}{\gamma_0}\right). \quad (27)$$

Finally, the average transmit power required to achieve $C_{\rm tifr}$ in a system with perfect channel knowledge can be found as

$$\left.P_{\rm avg}\right|_{\sigma_{\rm e}^2=0} = \alpha \int_0^{\infty} \frac{e^{-h_{\rm s}}}{h_{\rm s}} \int_0^{\gamma_0 h_{\rm s}} e^{-h_{\rm p}} dh_{\rm p} dh_{\rm s} \quad (28)$$

$$= \alpha\left(\mathrm{Ei}\left(-(1+\gamma_0)h_{\rm s}\right) - \mathrm{Ei}\left(-h_{\rm s}\right)\right)\Big|_{h_{\rm s}=0} \quad (29)$$

$$= \alpha \ln(1+\gamma_0), \quad (30)$$

where to derive (30) from (29), we use the Puiseux series of the exponential integral function given as $\mathrm{Ei}(z) = \gamma + \ln(z) + z + \frac{1}{4}z^2 + \cdots$, where the denominators of the coefficients are given by $n.n!$ [24] and $\gamma = 0.577216$ is the Euler-Mascheroni constant [25].

IV. Peak Received-Power Constraint

We now assume that the performance of the primary's receiver depends on the instantaneous SINR. In this case, the secondary user's transmission has to adhere to limitations on the ensuing peak received-power at the primary's receiver, where the constraint can be formulated according to

$$P(h_{\rm s},\hat{h}_{\rm p}) h_{\rm p} \le Q_{\rm peak}. \quad (31)$$

As such, the average interference inflicted on the primary receiver is also limited to $Q_{\rm peak}$. Satisfying (31), however, is impossible, since the secondary user is only provided with an imperfect estimation of the channel between its transmitter and the primary receiver, $\hat{h}_{\rm p}$, and as such it can not guarantee that the interference level at the primary's receiver, $P(h_{\rm s},\hat{h}_{\rm p})\hat{h}_{\rm p} + P(h_{\rm s},\hat{h}_{\rm p})\tilde{h}_{\rm p}$, remains below the peak threshold at all times. Therefore, under strict peak received-power constraint, the secondary user has to be silent at all times to satisfy such constraint and hence, the capacity is zero.

Here, we assume that while the average interference should still remain below $Q_{\rm peak}$, the primary user allows a certain percentage of outage in a sense that the received-power at the primary receiver as an effect of the transmission of the secondary user can exceed the peak threshold only for a certain percentage of time, $P_{\rm o}$, which hereafter is referred to by *interference-outage*. For instance, assume that the primary user requires a minimum-rate to be supported by its transmission for a certain percentage of time, during which the secondary user must satisfy the peak interference constraint set by the primary receiver. The secondary user can benefit from exceeding this constraint for the rest of the time. The primary user may wish to charge the secondary users for the allowed *interference-outage* level.

In this case, the received-power constraint can be defined as

$$\Pr\left\{P(h_{\rm s},\hat{h}_{\rm p})h_{\rm p} \geq Q_{\rm peak}\right\} \leq P_{\rm o}, \tag{32}$$

where $Q_{\rm peak}$ indicates the peak power limit. We now simplify (32) as follows

$$P_{\rm o} \geq \Pr\left\{\tilde{h}_{\rm p} \geq \frac{Q_{\rm peak}}{P(h_{\rm s},\hat{h}_{\rm p})} - \hat{h}_{\rm p}\right\} \tag{33}$$

$$= \int_0^\infty e^{-h_{\rm s}} \int_0^\infty e^{-\frac{Q_{\rm peak}}{\sigma_{\rm e}^2 P(h_{\rm s},\hat{h}_{\rm p})} + \frac{\hat{h}_{\rm p}}{\sigma_{\rm e}^2}} \frac{e^{-\frac{\hat{h}_{\rm p}}{1-\sigma_{\rm e}^2}}}{1-\sigma_{\rm e}^2}\, d\hat{h}_{\rm p} dh_{\rm s}. \tag{34}$$

Therefore, satisfying constraint (34) guarantees that the received-power at the primary user remains below the peak constraint $Q_{\rm peak}$ for $(1-P_{\rm o})$ percentage of time. Note that (34) is not a convex function of $P(h_{\rm s},\hat{h}_{\rm p})$, and hence, obtaining the optimum power allocation policy that achieves the channel capacity is complicated. However, a lower bound on the capacity can be found by assuming that the inequality (34) is satisfied at all times which yields

$$P(h_{\rm s},\hat{h}_{\rm p}) \leq \frac{Q_{\rm peak}}{\hat{h}_{\rm p} - \sigma_{\rm e}^2 \ln P_{\rm o}}. \tag{35}$$

One can show that in the case when $P_{\rm o} \leq \frac{1}{e} = 36\%$, satisfying (35) implies that the average interference power remains below $Q_{\rm peak}$. In the following, we obtain the capacity of the secondary user link under interference power constraint (35).

A. *Ergodic Capacity*

In this section, we obtain a lower bound on the ergodic capacity that can be achieved by the secondary user's Rayleigh fading channel subject to the imposed power constraint by the primary user (35) when only imperfect knowledge of $h_{\rm p}$ is available to the secondary user. Since no other limitation has been imposed on the transmit power of the secondary user, a lower bound on the ergodic capacity can be achieved by transmitting at the instantaneous maximum allowed power $P(h_{\rm s},\hat{h}_{\rm p}) = \frac{Q_{\rm peak}}{\hat{h}_{\rm p} - \sigma_{\rm e}^2 \ln P_{\rm o}}$. In such case, the ergodic capacity can be obtained as

$$\frac{C_{\rm er}}{B} = \int_0^\infty \frac{e^{-\frac{\hat{h}_{\rm p}}{1-\sigma_{\rm e}^2}}}{1-\sigma_{\rm e}^2} \int_0^\infty \ln\left(1 + \frac{Q_{\rm peak}}{N_0 B\left(\hat{h}_{\rm p} - \sigma_{\rm e}^2 \ln P_{\rm o}\right)} h_{\rm s}\right) \times e^{-h_{\rm s}} dh_{\rm s} d\hat{h}_{\rm p} \tag{36}$$

$$= \frac{Q_{\rm peak}}{Q_{\rm peak} - N_0 B(1-\sigma_{\rm e}^2)} \left(e^{-\frac{\sigma_{\rm e}^2 \ln P_{\rm o}}{1-\sigma_{\rm e}^2}} \mathrm{Ei}\left(\frac{\sigma_{\rm e}^2 \ln P_{\rm o}}{1-\sigma_{\rm e}^2}\right) - e^{-\frac{N_0 B \sigma_{\rm e}^2 \ln P_{\rm o}}{Q_{\rm peak}}} \mathrm{Ei}\left(\frac{N_0 B \sigma_{\rm e}^2 \ln P_{\rm o}}{Q_{\rm peak}}\right)\right). \tag{37}$$

We now find the average expenditure-power at the secondary transmitter that is required to achieve the lower bound on the ergodic capacity, (37), as follows:

$$P_{\rm avg} = \int_0^\infty \frac{e^{-\frac{\hat{h}_{\rm p}}{1-\sigma_{\rm e}^2}}}{1-\sigma_{\rm e}^2} \int_0^\infty \frac{Q_{\rm peak}}{\hat{h}_{\rm p} - \sigma_{\rm e}^2 \ln P_{\rm o}} e^{-h_{\rm s}} dh_{\rm s} d\hat{h}_{\rm p} \tag{38}$$

$$= -\frac{Q_{\rm peak}}{1-\sigma_{\rm e}^2} e^{-\frac{\sigma_{\rm e}^2 \ln P_{\rm o}}{1-\sigma_{\rm e}^2}} \mathrm{Ei}\left(\frac{\sigma_{\rm e}^2 \ln P_{\rm o}}{1-\sigma_{\rm e}^2}\right). \tag{39}$$

B. *Capacity with tifr Transmission Policy*

In this section, we determine the capacity of a Rayleigh fading channel with *tifr* transmission policy under peak power constraint as given in (35). Here, the secondary's transmitter inverts the channel fading so as to obtain a constant-rate at its receiver in non-outage states. However, the transmit power is limited to $\frac{Q_{\rm peak}}{\hat{h}_{\rm p} - \sigma_{\rm e}^2 \ln P_{\rm o}}$, hence, under peak power constraint, outage is unavoidable for most fading channels. Now, let us define γ_0 as the cutoff threshold for the ratio $\frac{\hat{h}_{\rm p} - \sigma_{\rm e}^2 \ln P_{\rm o}}{h_{\rm s}}$ over which the transmission is suspended. Therefore, the power allocation can be found as

$$P(h_{\rm s},\hat{h}_{\rm p}) = \begin{cases} \dfrac{\alpha}{h_{\rm s}} & \dfrac{\hat{h}_{\rm p} - \sigma_{\rm e}^2 \ln P_{\rm o}}{h_{\rm s}} \leq \gamma_0, \\ 0, & \text{otherwise} \end{cases} \tag{40}$$

and the peak power constraint (35) simplifies to $\alpha \leq \frac{Q_{\rm peak} h_{\rm s}}{\hat{h}_{\rm p} - \sigma_{\rm e}^2 \ln P_{\rm o}}$, and hence, α can be found as $\alpha = \frac{Q_{\rm peak}}{\gamma_0}$. Furthermore, we derive closed-form expression for the outage probability according to

$$P_{\rm out} = 1 - \int_0^\infty \frac{e^{-\frac{\hat{h}_{\rm p}}{1-\sigma_{\rm e}^2}}}{1-\sigma_{\rm e}^2} \int_{\frac{\hat{h}_{\rm p} - \sigma_{\rm e}^2 \ln P_{\rm o}}{\gamma_0}}^\infty e^{-h_{\rm s}} dh_{\rm s} d\hat{h}_{\rm p} \tag{41}$$

$$= 1 - \frac{\gamma_0 e^{\frac{\sigma_{\rm e}^2 \ln P_{\rm o}}{\gamma_0}}}{\gamma_0 + 1 - \sigma_{\rm e}^2}, \tag{42}$$

which leads to the following closed-form expression for $C_{\rm tifr}$:

$$\frac{C_{\rm tifr}}{B} = \max_{\gamma_0 \geq 0} \ln\left(1 + \frac{Q_{\rm peak}}{N_0 B \gamma_0}\right) \times \frac{\gamma_0 e^{\frac{\sigma_{\rm e}^2 \ln P_{\rm o}}{\gamma_0}}}{\gamma_0 + 1 - \sigma_{\rm e}^2}. \tag{43}$$

In addition, by following similar steps as in (40) to (43), the capacity with *tifr* transmission policy of a Rayleigh fading channel under peak received-power constraint at a third party's

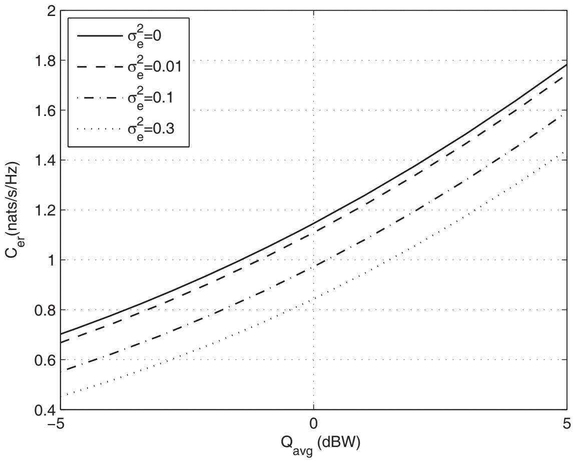

Fig. 1: Ergodic capacity under average received-power constraint and imperfect channel knowledge.

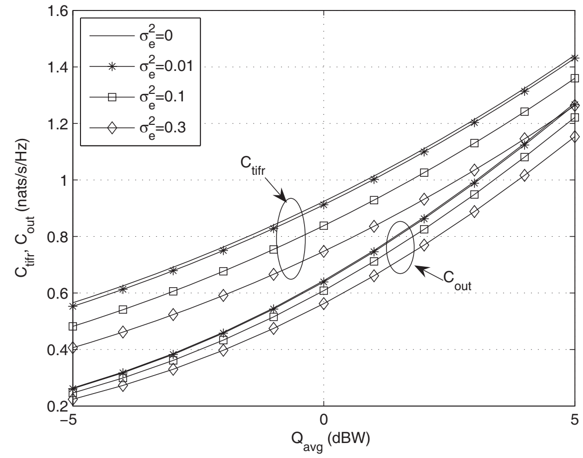

Fig. 2: Capacity with *tifr* transmission policy and C_{out} with $P_{\text{out}} = 0.2$ under average received-power constraint for various σ_{e}^2.

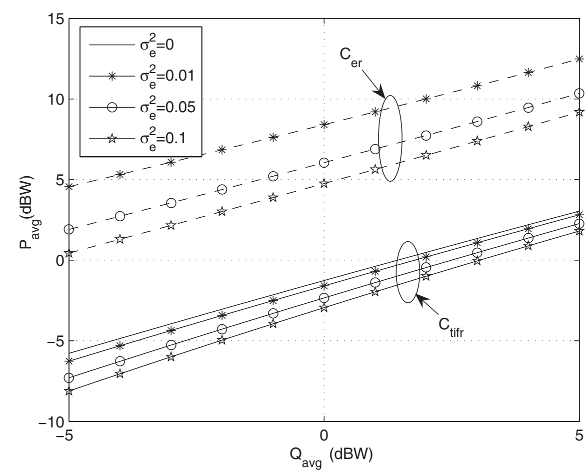

Fig. 3: Expenditure-power required to achieve C_{er} and C_{tifr} under average received-power constraint for various σ_{e}^2.

receiver with perfect channel knowledge, $\sigma_{\text{e}}^2 = 0$, and zero *interference-outage*, $P_{\text{o}} = 0$, can be obtained according to

$$\frac{C_{\text{tifr}}}{B}\bigg|_{\substack{\sigma_{\text{e}}^2=0\\P_{\text{o}}=0}} = \ln\left(1+\frac{Q_{\text{peak}}}{N_0 B\gamma_0}\right) \times \frac{\gamma_0}{\gamma_0+1}, \tag{44}$$

where a solution for the optimal cutoff threshold, γ_0, can be obtained from

$$\ln\left(1+\frac{Q_{\text{peak}}}{N_0 B\gamma_0}\right) - \frac{Q_{\text{peak}}\,(1+\gamma_0)}{N_0 B\gamma_0 + Q_{\text{peak}}} = 0. \tag{45}$$

The proof for (45) is provided in Appendix II. We further obtain the average expenditure power required to achieve C_{tifr} as follows:

$$\begin{aligned} P_{\text{avg}} &= \alpha\int_{\frac{-\sigma_{\text{e}}^2\ln P_{\text{o}}}{\gamma_0}}^{\infty} \frac{e^{-h_{\text{s}}}}{h_{\text{s}}} \int_0^{\gamma_0 h_{\text{s}}+\sigma_{\text{e}}^2\ln P_{\text{o}}} \frac{e^{-\frac{\hat{h}_{\text{p}}}{1-\sigma_{\text{e}}^2}}}{1-\sigma_{\text{e}}^2}\, d\hat{h}_{\text{p}} dh_{\text{s}} \\ &= \alpha e^{\frac{-\sigma_{\text{e}}^2\ln P_{\text{o}}}{1-\sigma_{\text{e}}^2}} \text{Ei}\left(\frac{\sigma_{\text{e}}^2\ln P_{\text{o}}}{1-\sigma_{\text{e}}^2} + \frac{\sigma_{\text{e}}^2\ln P_{\text{o}}}{\gamma_0}\right) - \alpha\text{Ei}\left(\frac{\sigma_{\text{e}}^2\ln P_{\text{o}}}{\gamma_0}\right). \end{aligned}$$

On the other hand, by following similar steps as in (29) to (30), one can find the average expenditure power required to achieve the capacity under *tifr* transmission policy of a system with perfect channel knowledge and zero *interference-outage* according to

$$P_{\text{avg}}\Big|_{\substack{\sigma_{\text{e}}^2=0\\P_{\text{o}}=0}} = \frac{Q_{\text{peak}}}{\gamma_0}\ln(1+\gamma_0). \tag{46}$$

Note that by using (46) and the inequality $\ln(1+x) \le x$ [24], one can show that $P_{\text{avg}}\Big|_{\substack{\sigma_{\text{e}}^2=0\\P_{\text{o}}=0}} \le Q_{\text{peak}}$.

V. Numerical Results

In this section, we numerically illustrate the capacity of a Rayleigh fading channel under received-power constraints at a third party's receiver, when only partial information of the link between the secondary's transmitter and primary's receiver is available to the secondary user. In the plotted results, we assume $N_0 B = 1$.

A. *Average Received-Power Constraint*

We start by comparing the ergodic capacity of a fading channel under average received-power constraint for different values of channel estimation error variance, σ_{e}^2. Fig. 1 shows that the ergodic capacity degrades as the channel estimation error variance increases. The loss in capacity, however, is not significant, e.g., for $\sigma_{\text{e}}^2 = 0.1$, the capacity degrades less than 0.2nats/s/Hz when compared to a system with perfect channel knowledge.

The capacity with *tifr* transmission policy and the capacity with respect to the outage probability $P_{\text{out}} = 0.2$ are examined in Fig. 2, which shows that capacity decreases as the channel estimation error variance increases. The capacity with *tifr* transmission policy degrades less than 0.1nats/s/Hz, for a channel estimation error variance $\sigma_{\text{e}}^2 = 0.1$, when compared to a system with perfect channel knowledge. In addition, comparing Fig. 1 and Fig. 2, we observe that the capacity gain of C_{er} compared to C_{tifr} is less than 0.2nats/s/Hz for $-5\text{dBW} \le Q_{\text{avg}} \le 5\text{dBW}$. Furthermore, a comparison of the expenditure-power required for the ergodic capacity and capacity with *tifr* transmission policy under average received-power constraint is provided in Fig. 3, wherein plots for P_{avg} in dBW are provided for various values of σ_{e}^2. It can be seen

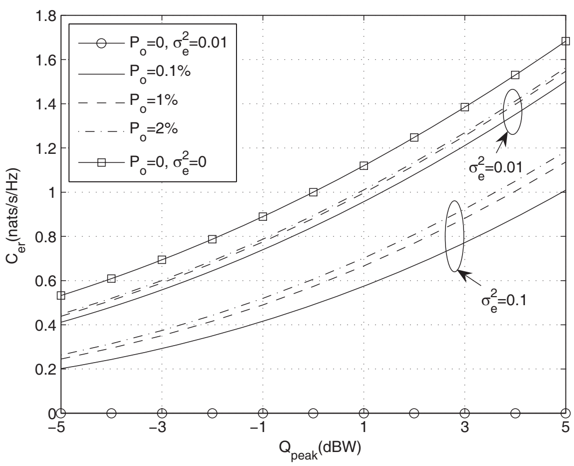

Fig. 4: Ergodic capacity lower bound under peak received-power constraint for various values of interference-outage P_o.

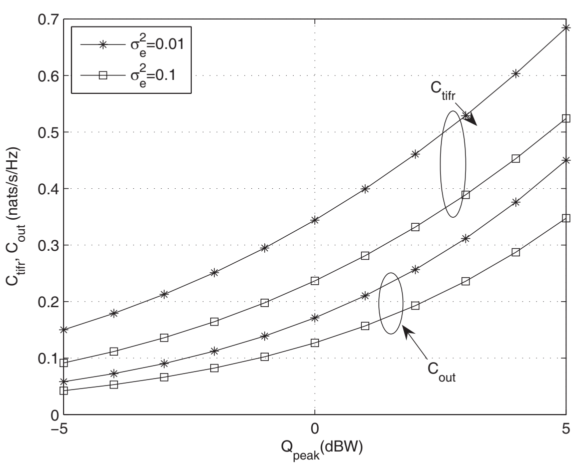

Fig. 5: Lower bounds on C_tifr and C_out with $P_\mathrm{out} = 0.2$, under peak received-power constraint with $P_\mathrm{o} = 1\%$ for various σ_e^2.

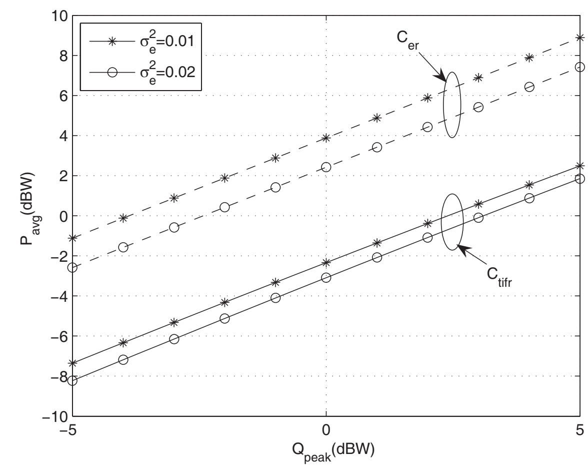

Fig. 6: Expenditure-power required to achieve the C_er and C_tifr lower bounds under peak received-power constraint with $P_\mathrm{o} = 1\%$.

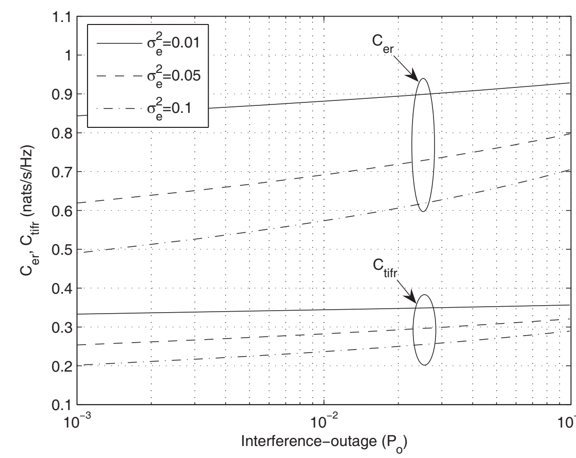

Fig. 7: Ergodic capacity and C_tifr lower bounds versus the interference-outage with $Q_\mathrm{peak} = 0\mathrm{dBW}$ for various σ_e^2.

that P_avg decreases as the channel estimation deteriorates. This can be justified by referring to (4) which relates the average transmit power to the channel estimation error variance through $P_\mathrm{avg} = \dfrac{Q_\mathrm{avg} - \mathcal{E}_{h_\mathrm{s},\hat{h}_\mathrm{p}}\left\{P(h_\mathrm{s},\hat{h}_\mathrm{p})\hat{h}_\mathrm{p}\right\}}{\sigma_\mathrm{e}^2}$. Furthermore, the figure reveals that the transmit-power required to achieve the ergodic capacity is much higher than the required P_avg to achieve C_tifr.

B. *Peak Received-Power Constraint*

The ergodic capacity of a Rayleigh fading channel with power constraint (35) for systems with perfect and imperfect channel estimation at the secondary transmitter are provided in Fig. 4. It is shown that for imperfect channel estimation, even a small amount of *interference-outage* permitted by the primary receiver, e.g., $P_\mathrm{o} = 0.1\%$, results in a significant gain in the ergodic capacity compared to a system with strict peak received-power constraint, $P_\mathrm{o} = 0$. However, the increase in the ergodic capacity gain of a system with $P_\mathrm{o} = 2\%$ *interference-outage* is small when compared to a system with $P_\mathrm{o} = 0.1\%$. Furthermore, the comparison of Fig. 1 and Fig. 4 shows that the capacity loss as an effect of imposing a peak constraint on the received-power at the primary receiver is not significant as long as some level of *interference-outage*, even as low as $P_\mathrm{o} = 0.1\%$, is allowed by the primary receiver.

The capacity with *tifr* transmission policy of a Rayleigh fading channel under peak received-power constraint (35) with $P_\mathrm{o} = 1\%$ is plotted in Fig. 5. The figure also includes the capacity results under channel inversion technique satisfying certain outage probability $P_\mathrm{out} = 0.2$. The figure reveals that C_tifr of a system with peak received-power constraint degrades significantly when compared to C_tifr of a system with average received-power constraint.

In addition, Fig. 6 provides the plots for the average expenditure-power required to achieve the ergodic capacity and capacity with *tifr* transmission policy of a Rayleigh fading channel under peak power constraint (35) with $P_\mathrm{o} = 1\%$. As observed, the transmit power required to achieve the ergodic capacity is higher than the required P_avg to achieve C_tifr.

Finally, the ergodic capacity and the capacity with *tifr* transmission policy of a Rayleigh fading channel with power constraint (35) and $Q_\mathrm{peak} = 0\mathrm{dBW}$ are plotted versus the *interference-outage* in Fig. 7, which shows that the capacity increases monotonically as a function of P_o, however, the slope of the capacity gain is small. For instance, in a system with $\sigma_\mathrm{e}^2 = 0.01$, allowing only 0.1% of *interference-outage* is almost as good as allowing $P_\mathrm{o} = 1\%$.

VI. Conclusions

We considered a spectrum-sharing system and quantified the capacity gains of Rayleigh fading channel under received-power constraints at a third party's receiver with and without channel estimation error. In particular, we assumed that the primary user imposes a constraint either on the average or on the peak received-power at its receiver as an effect of the transmission of the secondary user. We further assumed that the peak received-power constraint should be satisfied only for a certain percentage of time and introduced the concept of *interference-outage*.

In this respect, we determined the ergodic capacity, the capacity with *tifr* transmission policy and the outage capacity under average received-power constraint at the primary receiver when only partial channel information of the link between the secondary's transmitter and primary's receiver is available to the secondary user. For systems with peak received-power constraint, we provided lower bounds on the above-mentioned channel capacity metrics. We also obtained the optimal power allocation strategies to achieve the channel capacity, and further derived closed-form expressions for the capacity metrics under Rayleigh fading. In addition, we determined the average transmit power required to achieve the ergodic capacity and the capacity with *tifr* transmission policy. Numerical results corroborating our theoretical analysis were also provided. In particular, it was shown that the ergodic capacity achieves higher data rate than the capacity with *tifr* transmission policy, however, it requires much higher transmit power compared to $C_{\rm tifr}$; a result that motivates further investigations for systems with joint transmit and received-power constraints.

Appendix I

We define parameters $c \triangleq \frac{Q_{\rm peak}}{N_0 B}$, $x \triangleq \dfrac{\gamma_0}{\gamma_0+1}$, $v(x) \triangleq \ln(1-x)+x$, $f(x) \triangleq 1-\dfrac{c}{v(x)}$, and $g(x) \triangleq \ln(f(x))\,x$, which lead to a simplified expression for $C_{\rm tifr}$ under average received-power constraint according to

$$\left.\frac{C_{\rm tifr}}{B}\right|_{\sigma_e^2=0} = \max_{0\le x<1} g(x), \tag{47}$$

Furthermore, by using the equality $\log(1-x) = -x-\dfrac{x^2}{2}-\dfrac{x^3}{3}-\cdots$, for $-1<x<1$, [24], one can simplify $v(x)$ as $v(x)=-\dfrac{x^2}{2}-\dfrac{x^3}{3}-\cdots\le 0$. We further obtain the first and second derivatives of $v(x)$ with respect to x as follows:

$$v'(x) = -\frac{x}{1-x}\le 0 \quad\text{and}\quad v''(x) = -\frac{1}{(1-x)^2}\le 0. \tag{48}$$

In order to prove (24), we need to show that for $c\ge 0.28$ the maximum on $g(x)$ occurs when x satisfies $g'(x)=0$. We start by proving that $g'(x)|_{x=0}\ge 0$ as follows:

$$\begin{aligned} g'(x) &= \frac{f'(x)}{f(x)}x + \ln(f(x)) \qquad (49)\\ &= \frac{1}{f(x)}\left(\frac{cv'(x)}{v^2(x)}x + (f(x)-1)\left(1+\frac{1}{2}\left(\frac{f(x)-1}{f(x)}\right)\right.\right.\\ &\qquad \left.\left. + \frac{1}{3}\left(\frac{f(x)-1}{f(x)}\right)^2+\cdots\right)\right) \qquad (50)\\ &= -\frac{c}{f(x)v(x)}\left(\frac{-1}{(1-x)w(x)}+1\right.\\ &\qquad \left. + \frac{f(x)-1}{2f(x)} + \frac{1}{3}\left(\frac{f(x)-1}{f(x)}\right)^2+\cdots\right), \qquad (51)\end{aligned}$$

where $w(x) = -\dfrac{v(x)}{x^2} = \dfrac{1}{2}+\dfrac{x}{3}+\cdots$, and to derive (50) from (49) we use the fact that $f(x)\ge 1$ and the equality $\ln z = \left(\dfrac{z-1}{z}\right)+\dfrac{1}{2}\left(\dfrac{z-1}{z}\right)^2+\dfrac{1}{3}\left(\dfrac{z-1}{z}\right)^3+\cdots$, for $z\ge 0.5$ [24]. Now by using $\lim_{x\to 0}\dfrac{f(x)-1}{f(x)}=1$ and $f(1)=1$, one can show that $g'(x)|_{x=0}\ge 0$ and $g'(x)|_{x=1}\le 0$.

We now find the condition under which the second derivative of $g(x)$ with respect to x, $g''(x)$, is non-positive. We proceed by simplifying $g''(x)$ as follows:

$$\begin{aligned} g''(x) &= \frac{f''(x)}{f(x)}x - \frac{(f'(x))^2}{f^2(x)}x + 2\frac{f'(x)}{f(x)} \qquad (52)\\ &= \frac{cx}{f(x)v^2(x)(1-x)^2}\left(\frac{2}{w(x)}\right.\\ &\qquad \left. + \frac{-c}{w(x)(c-v(x))} - 3 + 2x\right). \qquad (53)\end{aligned}$$

Therefore, $g''(x)\le 0$ implies

$$c \ge -v(x)\left(\frac{1}{w(x)(3-2x)-1}-1\right). \tag{54}$$

Furthermore, the maximum on the right-hand-side of (54) can be found numerically to be lower than 0.28. Hence, we conclude that when $c\ge 0.28$, the second partial derivative of $g(x)$ with respect to x is non-positive and as such, by using $g'(x)|_{x=0}\ge 0$ and $g'(x)|_{x=1}\le 0$, one can show that the maximum on $g(x)$ occurs when x satisfies $g'(x)=0$. Note that when $N_0B=1$, the condition $c\ge 0.28$ implies $Q_{\rm avg}\ge -12.73$dBW. Finally, by inserting $c=\dfrac{Q_{\rm peak}}{N_0B}$ and $x\triangleq\dfrac{\gamma_0}{\gamma_0+1}$ into (49) and using $g'(x)=0$, one can prove (24).

Appendix II

We define $\lambda_0 \triangleq \dfrac{1}{\gamma_0}$ and $k(\lambda_0) \triangleq \dfrac{\ln(1+c\lambda_0)}{\lambda_0+1}$ which lead to a simplified expression for the capacity with *tifr* transmission policy under peak received-power constraint with perfect channel knowledge and zero *interference-outage* according to

$$\left.\frac{C_{\rm tifr}}{B}\right|_{\substack{\sigma_e^2=0\\ P_o=0}} = \max_{\lambda_0\ge 0} k(\lambda_0). \tag{55}$$

We need to prove that $k(\lambda_0)$ is at its maximum when λ_0 satisfies the equality $k'(\lambda_0)=0$. We start by taking the first partial derivative of $f(\lambda_0)$ with respect to λ_0 such that

$$k'(\lambda_0) = \frac{1}{(\lambda_0+1)^2}\left(\frac{c(\lambda_0+1)}{1+c\lambda_0} - \ln(1+c\lambda_0)\right). \tag{56}$$

Now define the function $l(\lambda_0) = \dfrac{c(\lambda_0+1)}{1+c\lambda_0} - \ln(1+c\lambda_0)$. We further prove that $l(\lambda_0)$ is a strictly decreasing function in λ_0 by taking the first derivative of $l(\lambda_0)$ with respect to λ_0 which yields

$$l'(\lambda_0) = \frac{-c^2(\lambda_0+1)}{(1+c\lambda_0)^2} \leq 0. \tag{57}$$

In addition, one can show that $l(\lambda_0)|_{\lambda_0=0} = c \geq 0$ and $l(\lambda_0)|_{\lambda_0=\infty} = -\infty$. Now choose λ_{zero} such that $l(\lambda_0)\big|_{\lambda_0=\lambda_{\text{zero}}} = 0$. Given that the denominator of $k'(\lambda_0)$ (56) is always positive, then, $k'(\lambda_0)|_{\lambda_0\leq\lambda_{\text{zero}}} \geq 0$ and $k'(\lambda_0)|_{\lambda_0>\lambda_{\text{zero}}} < 0$, which imply that the maximum of $k(\lambda_0)$ occurs at λ_{zero}. Therefore, the solution for the optimum cutoff threshold can be expressed according to (45).

References

[1] Federal Communications Commission. (2003, Dec.) Notice of proposed rule making and order, (ET docket no. 03-108). [Online]. Available: http://hraunfoss.fcc.gov/edocs_public/attachmatch/FCC-03-322A1.pdf

[2] J. Mitola III, “Cognitive radio for flexible mobile multimedia communications," in *Proc. IEEE Int. Workshop Mobile Multimedia Commun. (MoMuC)*, San Diego, CA, USA, Nov. 1999, pp. 3-10.

[3] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, “NeXt generation/dynamic spectrum access/cognitive radio wireless networks: a survey," *Computer Networks: Int. J. Computer Telecommun. Networking*, vol. 50, no. 13, pp. 2127-2159, Sept. 2006.

[4] S. Haykin, “Cognitive radio: brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201-220, Feb. 2005.

[5] F. F. Digham, M.-S. Alouini, and M. K. Simon, “On the energy detection of unknown signals over fading channels," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Seoul, South Korea, May 2003, pp. 3575-3579.

[6] G. Ganesan and Y. Li, “Cooperative spectrum sensing in cognitive radio networks," in *Proc. IEEE DySPAN*, Baltimore, USA, Nov. 2005, pp. 137-143.

[7] A. Ghasemi and E. Sousa, “Collaborative spectrum sensing for opportunistic access in fading environments," in *Proc. IEEE DySPAN*, Baltimore, USA, Nov. 2005, pp. 131-136.

[8] T. X. Brown, “An analysis of unlicensed device operation in licensed broadcast service bands," in *Proc. IEEE Dynamic Spectrum Access Networks (DySPAN)*, Baltimore, USA, Nov. 2005, pp. 11-29.

[9] B. Wild and K. Ramchandran, “Detecting primary receivers for cognitive radio applications," in *Proc. IEEE Dynamic Spectrum Access Networks (DySPAN)*, Baltimore, USA, Nov. 2005, pp. 124-130.

[10] S. A. Jafar and S. Srinivasa, “Capacity limits of cognitive radio with distributed and dynamic spectral activity," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Istanbul, Turkey, June 2006, pp. 5742-5747.

[11] N. Devroye, P. Mitran, and V. Tarokh, “Achievable rates in cognitive radio channels," *IEEE Trans. Inf. Theory*, vol. 52, no. 5, pp. 1813-1827, May 2006.

[12] A. Jovičić and P. Viswanath, “Cognitive radio: an information-theoretic perspective," in *Proc. IEEE Int. Sym. Inform. Theory (ISIT)*, Seattle, Washington, USA, July 2006, pp. 2413-2417.

[13] M. Gastpar, “On capacity under received-signal constraints," in *Proc. 42nd Annual Allerton Conf. Commun. Control Comp.*, Monticello, USA, Sept. 2004.

[14] A. Ghasemi and E. S. Sousa, “Capacity of fading channels under spectrum-sharing constraints," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Istanbul, Turkey, June 2006, pp. 4373-4378.

[15] L. Musavian and S. Aïssa, “Capacity and power allocation for spectrum-sharing communications in fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 148-156, Jan. 2009.

[16] ——, “Outage-constrained capacity of spectrum-sharing channels in fading environments," *IET Commun.*, vol. 2, no. 6, pp. 724-732, July 2008.

[17] E. Biglieri, J. Proakis, and S. Shamai (Shitz), “Fading channels: Information-theoretic and communications aspects," *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2619-2692, Oct. 1998.

[18] G. Caire and S. Shamai (Shitz), “On the capacity of some channels with channel state information," *IEEE Trans. Inf. Theory*, vol. 45, no. 6, pp. 2007-2019, Sept. 1999.

[19] A. J. Goldsmith and P. P. Varaiya, “Capacity of fading channels with channel side information," *IEEE Trans. Inf. Theory*, vol. 43, no. 6, pp. 1986-1992, Nov. 1997.

[20] M.-S. Alouini and A. J. Goldsmith, “Capacity of Rayleigh fading channels under different adaptive transmission and diversity-combining techniques," *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1165-1181, July 1999.

[21] L. Lifang, N. Jindal, and A. J. Goldsmith, “Outage capacities and optimal power allocation for fading multiple-access channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 4, pp. 1326-1347, Apr. 2005.

[22] J. M. Peha, “Approaches to spectrum sharing," *IEEE Commun. Mag.*, vol. 43, no. 2, pp. 10-12, Feb. 2005.

[23] A. Jovičić and P. Viswanath, “Cognitive radio: an information-theoretic perspective," submitted to *IEEE Trans. Inf. Theory*, May 2006. [Online], Available: http://arxiv.org/PS_cache/cs/pdf/0604/0604107v2.pdf.

[24] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*. New York: Dover, 1965.

[25] R. M. Young, *Euler's Constant*, *Math. Gaz.*, vol. 75, 1991.

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