Sensing-based Spectrum Sharing in Cognitive Radio Networks

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***Abstract*— In this paper, a new spectrum sharing model called sensing-based spectrum sharing is proposed for cognitive radio networks. This model consists of two phases: in the first phase, the secondary user (SU) listens to the spectrum allocated to primary user (PU) to detect the state of PU; in the second phase, SU adapts its transit power based on the sensing results. If the PU is inactive, the SU allocates the transmission power based on its own benefit. However, if the PU is active, interference power constraint is imposed in order to protect the PU. By studying the ergodic capacity of SU, we show that this spectrum sharing model can achieve a higher capacity of SU link and improve the spectrum utilization compared to conventional opportunistic spectrum access or simple spectrum sharing. Using the dual decomposition method, we find the optimal power allocation policies and the optimal sensing time for fading channels to achieve the ergodic capacity of the SU link considering both transmit and interference power constraints. Finally, the numerical results are presented to validate the analytical results.**

I. Introduction

Due to the convenience brought to people's life by wireless products, 21st century has witnessed a great increase of wireless devices and applications. However, with most of the radio spectrum being already allocated based on the traditional fixed spectrum allocation policy, the compelling need of radio spectrum to accommodate more upcoming wireless applications poses a serious problem for the future development of the wireless communications. On the other hand, a recent report published by the Federal Communication Commission (FCC) reveals that most of the licensed spectrum is rarely utilized continuously across time and space [1]. This motivates the proposition and development of cognitive radio (CR) and cognitive radio network (CRN) [2].

CR is a kind of highly intelligent wireless devices, which are able to adjust their transmission parameters such as transmit power and transmission frequency band based on the environment [2], [3]. CRN is a network which includes CRs. In a CRN, the ordinary wireless devices are referred as PUs and the CRs are referred as SUs. A CRN can be formed by either allowing the SUs to opportunistically operate in the frequency bands originally allocated to the PUs or by allowing the SUs to coexist with the PUs as long as the quality of service (QoS) of the PUs is not degraded by the interference caused by the SUs. The former transmission model is known as *opportunistic spectrum access*, and the latter transmission model is known as *spectrum sharing*.

In this paper, we propose a new transmission model referred as *sensing-based spectrum sharing*. In our proposed model, the SU first does spectrum sensing to detect whether the PU is on or off, then adapts its transmission power according to the detection result. In this new model, the SU can access the frequency band allocated to the PU at any time no matter whether the PU is active or not. If the PU is active, the SU transmits with a low power to avoid causing harmful interference to the PU. If the PU is not active, the SU transmits with a higher power to achieve a higher transmission rate. This is different from either *opportunistic spectrum access* or *spectrum sharing*. Since in the *opportunistic spectrum access* transmission model, the SU can transmit only when it detects *spectrum holes* [3], which is the time duration that PU is not transmitting over the band. In the *spectrum sharing* transmission model, the SU can transmit at any time without detecting whether the PU is active or not, however, it has to restrict its transmission power in order not to cause harmful interference to PU for the whole transmission process. By studying the ergodic capacity of the SU link based on this new transmission model, we have shown that this model would achieve a higher ergodic capacity of the SU link compared to *opportunistic spectrum access* or *spectrum sharing*.

In this paper, we are interested in the ergodic capacity of the SU link for our sensing-based spectrum sharing model, and consider the optimization problem over the transmit power and the sensing time under joint the constraint on the transmit and interference powers. Similar to [4], the ergodic capacity here is defined as the maximum achievable rate averaged over all the fading states of the SU's channel. Traditionally, ergodic capacity is studied under transmit power constraint [5]. Motivated by the interference temperature concept in [3], people realize that it is more reasonable to study ergodic capacity under interference power constraint for CRNs [6]. In [7], the author points out that it is more practical to study ergodic capacity under joint the transmit power constraint and interference power constraint. Thus, in this paper, we study the ergodic capacity under joint the transmit power constraint and interference power constraint.

II. System Model and Spectrum-sensing Model

A. System model

In this paper, we consider a simple CRN with one primary link and one secondary link. The primary link is consisted

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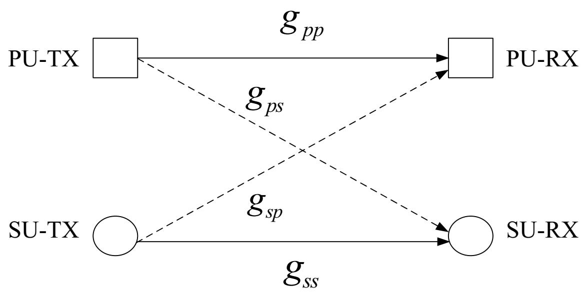

Fig. 1. System model for sensing-based spectrum sharing

of a PU transmitter (PU-TX) and a PU receiver (PU-RX). The secondary link is consisted of a SU transmitter (SU-TX) and a SU receiver (SU-RX). We assume the two links use the same frequency band. Thus, if the two links coexists, there will be interference between them. The primary link, secondary link and the interference links are all assumed to be block fading channels [8]. The additive white gaussian noises (AWGN) at PU-RX and SU-RX are denoted by n_0 and n_1, respectively. Besides, n_0 and n_1 are assumed to be independent and zero mean with the distribution $\mathcal{CN}(0, N_0)$ (circularly symmetric complex gaussian). Moreover, as shown in Fig. 1, the instantaneous channel power gains for primary link, secondary link, the link between PU-TX and SU-RX, and the link between SU-TX and PU-RX are denoted by g_{pp}, g_{ss}, g_{ps} and g_{sp}, respectively. All the channel power gains are assumed to be ergodic , stationary and available to the SU-TX and SU-RX.

B. Spectrum-sensing Model

Spectrum-sensing is the technique to decide the active/idle state of the PU between the following two hypotheses:

$$\begin{array}{lll} \mathcal{H}_0: & y(i) = n(i) & i = 1, 2, \cdots, N \\ \mathcal{H}_1: & y(i) = h(i) * x(i) + n(i) & i = 1, 2, \cdots, N \end{array} \tag{1}$$

where $y(i)$ is the signal received by SU, $x(i)$ is the signal sent by PU, $h(i)$ is the channel between PU-TX and SU-TX, $n(i)$ is the AWGN with zero mean and variance σ_n^2, and N is the number of samples, which equals to $N = f_s\tau$, τ is the sensing time, f_s is the sampling frequency.

When the energy detector is used, based on the PDF of the test static, the probability of detection is given by [9]

$$P_d = \mathcal{Q}\left(\left(\frac{\varepsilon}{\sigma_n^2} - \gamma - 1\right)\sqrt{\frac{\tau f_s}{2\gamma + 1}}\right), \tag{2}$$

where ε is the detection threshold, γ is the received SNR at the SU-TX. $\mathcal{Q}(\cdot)$ is the complementary cumulative distribution function of the standard Gaussian, i.e., $\mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{t^2}{2}\right) dt$.

Besides, for a target probability of detection $\bar{P}_d$, the probability of false alarm is given by [9], expressed as

$$P_f = \mathcal{Q}\left(\sqrt{2\gamma + 1}\mathcal{Q}^{-1}\left(\bar{P}_d\right) + \sqrt{\tau f_s}\gamma\right). \tag{3}$$

III. Sensing-based Spectrum Sharing

A. Problem Formulation

For the sensing-based spectrum sharing transmission protocol, we assume that each frame consists of one sensing slot, whose time duration is τ, and one data transmission slot, whose time duration is $T - \tau$. During the data transmission slot, SU adapts its transmit power based on the sensing result obtained in the sensing slot. If $\mathcal{H}_0$ holds, SU transmits with a high power $P_s^{(0)}$; if $\mathcal{H}_1$ holds, SU transmits with a relatively low power $P_s^{(1)}$ to reduce the interference caused to PU. Therefore, when $\mathcal{H}_0$ holds, the instantaneous transmission rate, if we denote it by r_0, is

$$r_0 = \log_2\left(1 + \frac{g_{ss}P_s^{(0)}}{N_0}\right); \tag{4}$$

when $\mathcal{H}_1$ holds, the instantaneous transmission rate, if we denote it by r_1, is

$$r_1 = \log_2\left(1 + \frac{g_{ss}P_s^{(1)}}{g_{ps}P_p + N_0}\right), \tag{5}$$

where P_p is the transmission power of PU when PU is active. Thus, according to the real state of PU and the sensing results, there are four possible scenarios for our sensing-based spectrum sharing model, which are listed in the following table.

PU's State	Sensing Results	Related Probability	Power	Rate
Active	$\mathcal{H}_1$	P_d	$P_s^{(1)}$	r_1
Active	$\mathcal{H}_0$	$P_m = 1 - P_d$	$P_s^{(0)}$	r_0
Idle	$\mathcal{H}_1$	P_f	$P_s^{(1)}$	r_1
Idle	$\mathcal{H}_0$	$1 - P_f$	$P_s^{(0)}$	r_0

As can be seen from table, when miss detection happens, SU transmits with the high power $P_s^{(0)}$, and this will cause harmful interference to PU, which is the case that should be avoided. Similarly, when false alarm happens, SU transmits with low power $P_s^{(1)}$, and this will decrease the spectrum utilization, which is also a case that should be avoided. In practical, P_d is set to be larger than a threshold P_{th}, i.e., $P_d \geq P_{th}$, such that the miss detection case can be ignored. P_f is also set to a sufficiently small value such that the false alarm case can also be ignored. Thus, if we denote the probability for which PU is idle as $\mathscr{P}(\mathcal{H}_0)$, and denote the probability for which PU is active as $\mathscr{P}(\mathcal{H}_1)$, the probability for which SU transmits with r_0 can be approximated by $\mathscr{P}(\mathcal{H}_0)(1 - P_f)$; the probability for which SU transmits with r_1 can be approximated by $\mathscr{P}(\mathcal{H}_1)P_d$.

Thus, under the above assumption, the ergodic capacity for the sensing-based spectrum sharing model is formulated as

$$\begin{aligned} &\underset{\{P_s^{(0)}, P_s^{(1)}, \tau\}}{\text{maximize}}\ C = \mathbb{E}\left\{\frac{T-\tau}{T}\left[\mathscr{P}(\mathcal{H}_0)(1-P_f)r_0 + \mathscr{P}(\mathcal{H}_1)P_d r_1\right]\right\}, \\ &\text{subject to } P_d \geq P_{th},\ P_s^{(0)} \geq 0,\ P_s^{(1)} \geq 0,\ T \geq \tau \geq 0, \\ &\qquad \mathscr{P}(\mathcal{H}_0)(1-P_f)\mathbb{E}\{P_s^{(0)}\} + \mathscr{P}(\mathcal{H}_1)P_d\mathbb{E}\{P_s^{(1)}\} \leq P_{av}, \\ &\qquad \mathbb{E}\{g_{sp}P_s^{(1)}\} \leq Q_{av}, \end{aligned} \tag{6}$$

where P_{av} stands for the average transmit power limit, and Q_{av} stands for the average interference power limit. Besides, P_d and P_f are related to τ by (2) and (3), respectively.

This problem is complicated and difficult to solve directly. Therefore, before solving this problem, we consider a simplified version of this problem referred as perfect sensing.

B. Perfect Sensing

In practical, when SU does spectrum sensing, its objective is to achieve a high detection probability and a small false alarm probability within a limited sensing time. An ideal scenario is achieving $P_d = 1$ and $P_f = 0$ with the sensing time $\tau = 0$, and this scenario is referred as perfect sensing in this paper. In this scenario, the problem defined by (6) is simplified to

$$\begin{aligned}\underset{\{P_s^{(0)}\geq 0, P_s^{(1)}\geq 0\}}{\text{maximize}} \quad & C = \mathscr{P}(\mathcal{H}_0)C_0 + \mathscr{P}(\mathcal{H}_1)C_1, \\ \text{subject to} \quad & \mathscr{P}(\mathcal{H}_0)\mathbb{E}\{P_s^{(0)}\} + \mathscr{P}(\mathcal{H}_1)\mathbb{E}\{P_s^{(1)}\} \leq P_{av}, \\ & \mathbb{E}\{g_{sp}P_s^{(1)}\} \leq Q_{av}, \end{aligned} \tag{7}$$

where $C_0 = \mathbb{E}\{r_0\}$ and $C_1 = \mathbb{E}\{r_1\}$.

It is evident that the problem defined by (7) is a convex optimization problem over $P_s^{(0)}$, $P_s^{(1)}$. However, there is a coupling constraint, which makes the problem difficult to solve. Therefore, we have to decompose the above problem into a two-level optimization problem by employing the dual decomposition method illustrated in [10].

First, we form the partial lagrangian by introducing Lagrange multiplier λ only for the coupling constraint in (7),

$$\begin{aligned} L(P_s^{(0)}, P_s^{(1)}, \lambda) &= \mathscr{P}(\mathcal{H}_0)C_0 + \mathscr{P}(\mathcal{H}_1)C_1 \\ &- \lambda\left\{\mathscr{P}(\mathcal{H}_0)\mathbb{E}\{P_s^{(0)}\} + \mathscr{P}(\mathcal{H}_1)\mathbb{E}\{P_s^{(1)}\} - P_{av}\right\}, \end{aligned} \tag{8}$$

Then, the dual function can be written as

$$\begin{aligned} q(\lambda) = \mathscr{P}(\mathcal{H}_1) & \sup_{P_s^{(1)}\geq 0} \left\{C_1 - \lambda\mathbb{E}\{P_s^{(1)}\}\middle| \mathbb{E}\{g_{sp}P_s^{(1)}\} \leq Q_{av}\right\} \\ + \mathscr{P}(\mathcal{H}_0) & \sup_{P_s^{(0)}\geq 0} \left\{C_0 - \lambda\mathbb{E}\{P_s^{(0)}\}\right\} + \lambda P_{av}. \end{aligned} \tag{9}$$

The dual optimization problem is

$$\underset{\lambda \geq 0}{\text{minimize}} \quad q(\lambda). \tag{10}$$

This is also known as the master problem (MP).

For a given λ, the dual function can be decomposed into the following two subproblems:

Subproblem 1 (SP1):

$$\underset{P_s^{(0)}\geq 0}{\text{maximize}} \quad C_0 - \lambda\mathbb{E}\{P_s^{(0)}\}. \tag{11}$$

Subproblem 2 (SP2):

$$\underset{P_s^{(1)}\geq 0}{\text{maximize}} \quad C_1 - \lambda\mathbb{E}\{P_s^{(1)}\}, \tag{12}$$

$$\text{subject to} \quad \mathbb{E}\{g_{sp}P_s^{(1)}\} \leq Q_{av}. \tag{13}$$

It is easy to observe that both SP1 and SP2 are convex optimization problems, by writing their lagrangian functions and applying the Karush-Kuhn-Tucker (KKT) conditions, the optimal power are obtained as

$$P_s^{(0)} = \left(\frac{1}{\lambda} - \frac{N_0}{g_{ss}}\right)^+, \tag{14}$$

$$P_s^{(1)} = \left(\frac{1}{\lambda + \mu g_{sp}} - \frac{g_{ps}P_p + N_0}{g_{ss}}\right)^+, \tag{15}$$

where $(\cdot)^+$ denotes $\max\{\cdot, 0\}$, and μ is obtained by substituting (15) into $\mathbb{E}\{g_{sp}P_s^{(1)}\} = Q_{av}$. Numerically, μ can be obtained by the bisection search.

Once the optimal solution for SP1 and SP2 are obtained, the following subgradient algorithm can be applied to solve the MP, which requires the calculation of the subgradient of $q(\lambda)$ at each iteration. The subgradient of $q(\lambda)$ is given by proposition 1.

The subgradient algorithm

1) Initialization: λ_1, $k = 1$,
2) repeat
   a) calculate $P_{s,k}^{(0)}$ and $P_{s,k}^{(1)}$ by (14) and (15)
   b) calculate the subgradient at λ_k by
      $P_{av} - \mathscr{P}(\mathcal{H}_0)\mathbb{E}\{P_{s,k}^{(0)}\} - \mathscr{P}(\mathcal{H}_1)\mathbb{E}\{P_{s,k}^{(1)}\}$
   c) update λ_{k+1} by
      $\lambda_{k+1} = \lambda_k + \alpha\left\{\mathscr{P}(\mathcal{H}_0)\mathbb{E}\{P_{s,k}^{(0)}\} + \mathscr{P}(\mathcal{H}_1)\mathbb{E}\{P_{s,k}^{(1)}\} - P_{av}\right\}$
3) stop, when $|\lambda_{k+1} - \lambda_k| \leq \epsilon$

where α is the step size, and ϵ is a given small constant.

Proposition 1: Assume $P_{s,i}^{(0)}$ and $P_{s,i}^{(1)}$ are the optimal power allocation for ith iteration, the subgradient for $q(\lambda)$ is $P_{av} - \mathscr{P}(\mathcal{H}_0)\mathbb{E}\{P_{s,i}^{(0)}\} - \mathscr{P}(\mathcal{H}_1)\mathbb{E}\{P_{s,i}^{(1)}\}$.

Proof: Let λ' be a feasible value for $q(\lambda)$, then if we can prove $q(\lambda') \geq q(\tilde{\lambda}) + (\lambda' - \tilde{\lambda})S$ holds for any λ', then S must be a subgradient of $q(\tilde{\lambda})$ at $\tilde{\lambda}$.

Denote

$$S\left(P_s^{(0)}, P_s^{(1)}\right) = \mathscr{P}(\mathcal{H}_0)\mathbb{E}\{P_s^{(0)}\} + \mathscr{P}(\mathcal{H}_1)\mathbb{E}\{P_s^{(1)}\} - P_{av},$$

we have

$$\begin{aligned} q(\lambda) &= \sup_{\{P_s^{(0)}, P_s^{(1)}\geq 0\}} \left\{L\left(P_s^{(0)}, P_s^{(1)}, \lambda\right)\middle| \mathbb{E}\{g_{sp}P_s^{(1)}\} \leq Q_{av}\right\} \\ &= \mathscr{P}(\mathcal{H}_0)C_0' + \mathscr{P}(\mathcal{H}_1)C_1' - \lambda' S\left(P_s^{(0)\prime}, P_s^{(1)\prime}\right) \\ &\overset{a}{\geq} \mathscr{P}(\mathcal{H}_0)\tilde{C}_0 + \mathscr{P}(\mathcal{H}_1)\tilde{C}_1 - \lambda' S\left(\tilde{P_s^{(0)}}, \tilde{P_s^{(1)}}\right) \\ &= \mathscr{P}(\mathcal{H}_0)\tilde{C}_0 + \mathscr{P}(\mathcal{H}_1)\tilde{C}_1 - \lambda' S\left(\tilde{P_s^{(0)}}, \tilde{P_s^{(1)}}\right) \\ &+ \tilde{\lambda} S\left(\tilde{P_s^{(0)}}, \tilde{P_s^{(1)}}\right) - \tilde{\lambda} S\left(\tilde{P_s^{(0)}}, \tilde{P_s^{(1)}}\right) \\ &= q(\tilde{\lambda}) + (\lambda' - \tilde{\lambda})\left\{-S\left(P_s^{(0)\prime}, P_s^{(1)\prime}\right)\right\}, \end{aligned}$$

where $P_s^{(0)\prime}$ and $P_s^{(1)\prime}$ is the optimal solution of (9) when $\lambda = \lambda'$, and $\tilde{P_s^{(0)}}$ and $\tilde{P_s^{(1)}}$ is the optimal solution of (9) when $\lambda = \tilde{\lambda}$. The inequality a results from the fact that $P_s^{(0)\prime}$ and $P_s^{(1)\prime}$ is the optimal solution under λ'. ■

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C. Imperfect Sensing

In this part, we study the problem defined by (6), which is referred as the imperfect sensing problem in this paper. First, we show that the objective function of (6) is maximized when $P_d = P_{th}$, if $P_{th} > 0.5$ and $\mathscr{P}(\mathcal{H}_0) > \mathscr{P}(\mathcal{H}_1)$. To prove this, we need the following Lemma.

Lemma 1: The function $f(\gamma) = \left(\frac{\varepsilon}{\sigma_n^2} - \gamma - 1\right)^2 \frac{\tau f_s}{2\gamma+1}$ is an increasing function of γ for the range of $P_d > 0.5$.

Proof:

$$\frac{df(\gamma)}{d\gamma} = -2\tau f_s \frac{\left(\frac{\varepsilon}{\sigma_n^2} - \gamma - 1\right)\left(\frac{\varepsilon}{\sigma_n^2} + \gamma\right)}{(2\gamma+1)^2} \overset{a}{\geq} 0, \tag{16}$$

where a results from the fact that $\frac{\varepsilon}{\sigma_n^2} - \gamma - 1 \leq 0$ for the range of $P_d > 0.5$. Thus, the function is obviously an increasing function of γ for the range of $P_d > 0.5$. ■

Proposition 2: The objective function of (6) is maximized when $P_d = P_{th}$, if $P_{th} > 0.5$ and $\mathscr{P}(\mathcal{H}_0) > \mathscr{P}(\mathcal{H}_1)$.

Proof: Denote

$$J_1(P_d) = \mathscr{P}(\mathcal{H}_0)\left[1 - \mathcal{Q}\left(\sqrt{2\gamma+1}\mathcal{Q}^{-1}(P_d) + \sqrt{\tau f_s}\gamma\right)\right] r_0,$$
$$J(P_d) = \mathscr{P}(\mathcal{H}_0)(1-P_f)r_0 + \mathscr{P}(\mathcal{H}_1)P_d r_1.$$

Thus, from (3), we have

$$J(P_d) = J_1(P_d) + \mathscr{P}(\mathcal{H}_1)P_d r_1, \tag{17}$$

Then, if we denote $t = \mathcal{Q}^{-1}(P_d)$, we have

$$\begin{aligned} \frac{dJ_1(P_d)}{dt} &= \frac{\mathscr{P}(\mathcal{H}_0)r_0\sqrt{2\gamma+1}}{\sqrt{2\pi}} \exp\left(-\frac{\left(\sqrt{2\gamma+1}t + \sqrt{\tau f_s}\gamma\right)^2}{2}\right) \\ &\overset{a}{=} \frac{\mathscr{P}(\mathcal{H}_0)r_0\sqrt{2\gamma+1}}{\sqrt{2\pi}} \exp\left(-\frac{\left(\left(\frac{\varepsilon}{\sigma_n^2} - 1\right)\sqrt{\tau f_s}\right)^2}{2}\right), \end{aligned}$$

where a results from the fact that $t = \left(\frac{\varepsilon}{\sigma_n^2} - \gamma - 1\right)\sqrt{\frac{\tau f_s}{2\gamma+1}}$ which can be obtained from (2).

$$\begin{aligned} \frac{d(P_d)}{dt} &= -\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) \\ &= -\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\left(\left(\frac{\varepsilon}{\sigma_n^2} - \gamma - 1\right)\sqrt{\frac{\tau f_s}{2\gamma+1}}\right)^2}{2}\right). \end{aligned}$$

Therefore, it follows

$$\begin{aligned} \frac{dJ_1(P_d)}{dP_d} &= \frac{dJ_1(P_d)}{dt} \frac{1}{\frac{dP_d}{dt}} = -\mathscr{P}(\mathcal{H}_0)r_0\sqrt{2\gamma+1} \\ &\times \exp\left(\frac{\left(\left(\frac{\varepsilon}{\sigma_n^2} - \gamma - 1\right)\sqrt{\frac{\tau f_s}{2\gamma+1}}\right)^2}{2} - \frac{\left(\left(\frac{\varepsilon}{\sigma_n^2} - 1\right)\sqrt{\tau f_s}\right)^2}{2}\right) \\ &\leq -\mathscr{P}(\mathcal{H}_0)r_0, \end{aligned} \tag{18}$$

where the inequality results from Lemma 1, and the equality holds when $\gamma = 0$.

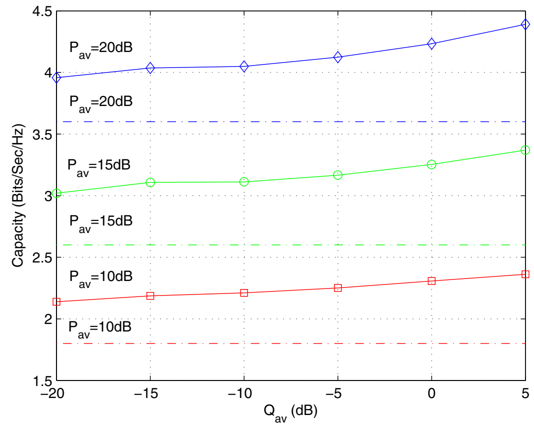

Fig. 2. Capacities for different P_{av} under $\mathscr{P}(\mathcal{H}_0) = 0.6$ for perfect sensing scenario vs. Q_{av}

Thus, from (17), we have

$$\frac{dJ(P_d)}{dP_d} \leq -\mathscr{P}(\mathcal{H}_0)r_0 + \mathscr{P}(\mathcal{H}_1)r_1 \leq 0. \tag{19}$$

This inequality results from the fact that $r_0 \geq r_1$, and $\mathscr{P}(\mathcal{H}_0) > \mathscr{P}(\mathcal{H}_1)$.

Then, (19) indicates that the objective function is a decreasing function of P_d, thus the maximum value is obtained when $P_d = P_{th}$. ■

Now we can remove the constraint $P_d \geq P_{th}$ in (6) by setting P_d equal to the constant P_{th}. Then, it is easy to observe that the objective function of (6) is a concave function with respect to $P_s^{(0)}$ and $P_s^{(1)}$. Besides, it also can be verified that the objective function of (6) is concave function with respect to τ when $P_f < 0.5$. Therefore, the problem defined by (6) now becomes a convex optimization problem.

Then, from (6), it is observed that for a given sensing time τ, the optimal power allocation $P_s^{(0)}$ and $P_s^{(1)}$ can be obtained by the same method used to solve the perfect sensing problem. Therefore, for each τ in the feasible region, the corresponding optimal power allocation $P_s^{(0)}$ and $P_s^{(1)}$ and the maximum C can be obtained. Then, the τ with the largest C must be the optimal sensing time and the corresponding $P_s^{(0)}$ and $P_s^{(1)}$ must be the optimal power allocation for (6). This is due to the convexity of the problem.

IV. Numerical Results

In this section, we present the numerical results for the proposed studies under the Rayleigh fading channels. All the channel power gains are assumed to be exponentially distributed random variables with unit-mean. N_0 is assumed to be 1. The frame duration is chosen to be $T = 100ms$, the number of frame simulated is 10000, and the target detection probability P_{th} is set to 0.9 with $\gamma = -15dB$. The transmit power of PU is assumed to be $10dB$.

Fig. 2 shows the ergodic capacities under joint the transmit and interference power constraints for $\mathscr{P}(\mathcal{H}_0) = 0.6$ for the perfect sensing scenario. The dash dotted line shows the

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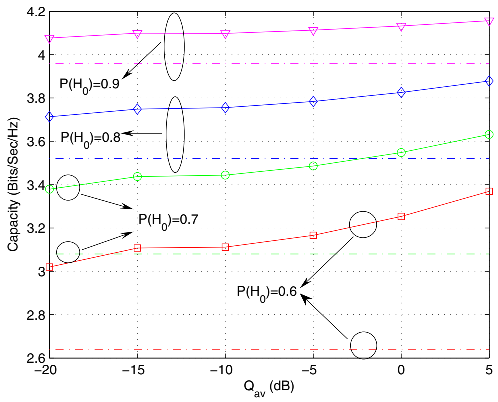

Fig. 3. Capacities for different $\mathscr{P}(\mathcal{H}_0)$ under $P_{av} = 15dB$ for perfect sensing scenario vs. Q_{av}

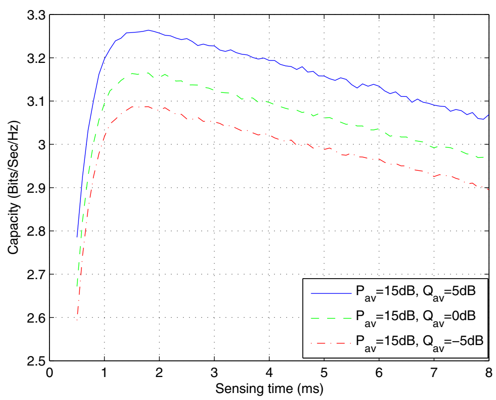

Fig. 4. Capacities for different Q_{av} under $\mathscr{P}(\mathcal{H}_0) = 0.6$ for imperfect sensing vs. τ

ergodic capacities under the same setup for the opportunistic access model. It is clear from the figure that, the capacities for our transmission model increase with the increase of P_{av} and Q_{av}. However, the capacities for the opportunistic access model only increase with the increase of P_{av}. This is due to the reason that opportunistic access model only allows transmission when PU is absent, thus the capacities are limited only by the transmit power. Besides, for the same transmit power constraint, it is seen that the capacities for our sensing-based spectrum sharing model are always larger then those of the opportunistic access model. This shows superiority of the sensing-based spectrum sharing model.

Fig. 3 shows the capacities for different $\mathscr{P}(\mathcal{H}_0)$ under the same transmit power constraint $P_{av} = 15dB$. It is clear that the capacities increase with the increase of the $\mathscr{P}(\mathcal{H}_0)$. This is reasonable due to the fact that a large $\mathscr{P}(\mathcal{H}_0)$ indicates a high probability that the PU is idle and more chances that SU can transmit with a high power. Besides, the dash dotted line shows the capacities under the same setup for the opportunistic spectrum access model. It is evident from the figure that the capacities for our sensing-based spectrum sharing model are always larger than those of the opportunistic spectrum access model. However, the capacity gains decrease with the increase of $\mathscr{P}(\mathcal{H}_0)$. This is because with the increase of $\mathscr{P}(\mathcal{H}_0)$, the capacities gains obtained from sharing spectrum with PU decreases with the decrease of the coexistence probability.

Fig. 4 shows capacities for imperfect sensing case vs. the sensing time τ for $\mathscr{P}(\mathcal{H}_0) = 0.6$. It is clear from the figure that the capacity is a concave function with respect to τ. Besides, it is also noticed that the optimal sensing time for all the three set of parameters labeled in the figure is almost the same. Besides, compare Fig. 4 with Fig. 2, it is easier to see that under the same transmit and interference power constraints, the capacity for imperfect sensing case is always lower than that of the perfect sensing case, which indicates that the capacity obtained under perfect sensing can be served as a upper bound of imperfect sensing.

V. Conclusions

CR is a promising technology to deal with the spectrum scarcity problem by exploring the currently under-utilized spectrum usage pattern. In this paper, we propose a new transmission model for CR, which is referred as sensing-based spectrum sharing. Under this new model, we study the ergodic capacity of SU link under joint the transmit and interference power constraint. We then obtain the optimal power control policy and the sensing time by formulating the problem as an optimization problem. Due to the complexity of this problem, we first study the perfect sensing scenario, then extend the results to the imperfect sensing scenario. Finally, numerical results are given to validate the theoretical analysis and show the superiority of our new transmission model.

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