Throughput Maximization of the Cognitive Radio Using Hybrid (Overlay-Underlay) Approach with Energy Harvesting

Muhammad Usman and Insoo Koo
School of Electrical and Computer Engineering
University of Ulsan,
South Korea
email: usman@nwfpuet.edu.pk, iskoo@ulsan.ac.kr

Abstract*—Energy management is an acute challenge for battery powered devices. In this work, we consider a cognitive radio (CR) equipped with energy harvested finite capacity battery. The cognitive radio harvests energy from radio frequency (RF) source (primary channel) as well as from non-RF sources (ambient sources such as wind, solar, and vibration). Energy harvesting from the primary channel occurs only if the channel is found in busy state and energy of the signal is above a certain threshold. The harvested energy is consumed in channel sensing, processing and subsequent transmission. The spectrum sensing decision is controlled optimally by partially observable Markov decision process (POMDP) according to the available energy of the CR and status of the primary channel. Such decision results in an optimized utilization of energy. The cognitive radio enhances throughput by transmitting in the hybrid mode adapting the overlay or underlay mode depending on the sensing and transmission energy. Throughput improvement of the cognitive radio is shown through simulations where throughput in the hybrid mode is compared with the overlay-only mode. Simulations also show the effect of channel idle probability and harvested energy on the throughput.

***Keywords*—energy efficiency, energy harvested cognitive radio, hybrid overlay-underlay cognitive radio, throughput maximization, POMDP.**

I. INTRODUCTION

The recent shift in trend towards wireless applications stresses allocation of more frequency bands. However, the limited wireless spectrum is a shared natural resource which becomes insufficient to accommodate the growing demands of wireless services. It has also been reported that most of the time and at majority of the locations, the statically allocated spectrum is not efficiently utilized. According to Federal Communications Commission (FCC), utilization of the assigned spectrum varies from 15% to 85% depending upon spatiotemporal variations [1]. Scarcity and underutilization of the spectrum has ensued the advent of cognitive radio (CR). CR utilizes the spectrum which is primarily allocated to the primary network intelligently, flexibly, and efficiently [2]-[3]. In cognitive radio network, the unlicensed or cognitive user (CU) opportunistically uses the primary network spectrum without causing harmful interference to the licensed or primary user (PU). The CU operates in one of two modes; overlay or underlay. In overlay mode, the CU uses the primary network spectrum when it is not used by the PU and upon the arrival of the PU, CU instantly vacates the frequency band. This requires CU to detect status of the PU promptly and accurately. In underlay mode, the CU can coexist with the PU. However, the CU should keep the interference level below a threshold called interference temperature. We are considering both modes of the CU, i.e. overlay and underlay, to enhance throughput. The CU transmits in overlay mode when it finds the channel idle, on the other hand, if the channel is found busy then it may either transmit in underlay mode or harvest energy from the channel depending on the interference generated by the CU. Transmission in the presence of PU increases throughput of the CU which is the main objective of this paper.

Energy management of a finite capacity battery operated device is a challenging task. In this paper, we assume that the CU is equipped with a rechargeable and energy harvested battery. CU harvests energy from natural ambient sources such as wind, solar, and vibrations as well as from the wireless primary channel [4]-[5]. Energy of the CU is consumed optimally by using an appropriate action through POMDP. In the active mode, CU consumes the harvested energy in spectrum sensing, subsequent processing, and transmission whereas, in the idle mode, energy is consumed only to keep the CU in sleep mode. Energy harvesting from the wireless channel occurs when CU finds the primary channel in busy mode. Status of the primary channel can either be busy or idle representing presence or absence of the primary user, respectively.

Recently many researchers considered the problem of energy management and throughput maximization. In [6] sensor node with an energy harvester is considered for throughput maximization. In [7], variable sensing time and stationary Markov process is used to switch between idle and active states. In [8], POMDP is used to select the optimal choice between sensing and idle actions. However, all of the above work consider only overlay transmission mode and fixed harvested energy. In [9], statistical analysis of the PU with no consideration of energy is used for the selection of transmission mode.

In this paper, we assume that the CU operates in a time slotted manner. In each time slot, of duration T, the CU optimally decides an action; to sense the channel or to stay idle based on the available energy and belief about the inactivity of the PU. The Partially Observable Markov Decision Process (POMDP) framework is used for action

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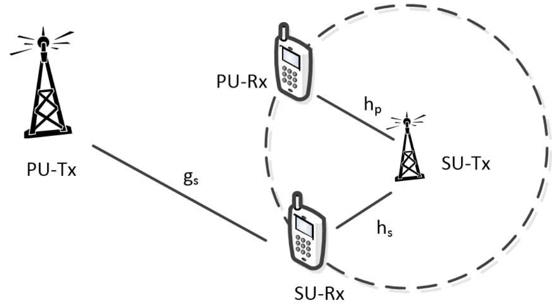

Fig. 1. System model.

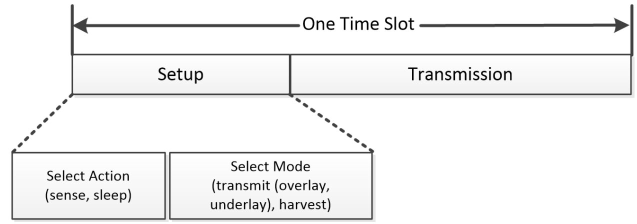

Fig. 2. Frame Structure.

selection. After spectrum sensing, energy thresholds are considered for the mode selection of transmission. The CU transmits in hybrid mode (overlay-underlay) to achieve the maximum throughput and maintain the quality of service (QoS) of the PU at the same time.

The remaining part of this paper is organized as; in Section II we present the system model. In Section III, we elaborate action and transmission mode selection. Simulation results are presented in Section IV. Finally, the paper is concluded in Section V.

II. System Model

We consider a cognitive radio network (CRN) operating as a secondary network in the range of primary network as shown in Fig. 1. The CRN consists of *N*-CU transmitter-receiver pairs. We assume that the secondary transmitter has the information of channel gains i.e. the channel gain (h_s) between the secondary transmitter and receiver, the channel gain (h_p) between the secondary transmitter and the primary receiver, and the channel gain (g_s) between the primary transmitter and the secondary receiver [10]. All the channels suffer from Rayleigh fading gains, i.e. channel gains are assumed to be *iid* (independent and identically distributed) exponential random variables with unit mean. The secondary receiver provides the channel state information (CSI) including the channel gain h_s as feedback to the transmitter [11]. The channel gain h_p can be obtained by using a sensor node detector approach [12].

Since the CU operates in a time slotted fashion, therefore, at the start of each time slot it decides, by using POMDP, whether to sense the channel or stay idle as shown in Fig. 2. The decision is based on energy of the CU and idle probability of the primary channel. The channel may be found either in busy (*B*) or in free (*F*) state which can be modelled by a two state Markov chain. The states represents presence or absence of the PU, respectively. The transition probability from state a to state b is denoted by p_{ab}. We assume that the PU traffic is highly correlated in time [9], i.e. $p_{BB} > p_{BF}$.

A. Throughput and Energy Model

Transmission between the secondary transmitter and receiver occurs successful when outage does not occur. Outage happens when the throughput falls below a certain threshold. The throughput of the CU can be described in terms of the outage as

$$C = \hat{p}_{out} = 1 - p_{out} \tag{1}$$

where C is one of C_O, the throughput in overlay mode, or C_U, the throughput in underlay mode and p_{Out} is the outage probability and $\hat{p}_{Out}$ is the probability when outage does not occur. The throughput in overlay mode, C_O , is given by

$$C_O = p_t(1-p_f)\,\hat{p}^{O}_{out} + (1-p_t)\,p_m\,\hat{p}^{OI}_{out} \tag{2}$$

where p_t is the probability that PU is absent or the primary channel is idle, p_f is the probability of false alarm, $\hat{p}^{O}_{out}$ is the probability, when outage does not occur, of the secondary link for no interference from PU, p_m is the probability of misdetection, and $\hat{p}^{OI}_{out}$ is the probability, when outage does not occur, of the secondary link for interference from PU due to misdetection. $\hat{p}^{O}_{out}$ and $\hat{p}^{OI}_{out}$ can be calculated as follows

$$\begin{aligned} \hat{p}^{O}_{out} &= \Pr[\log_2(1+h_s P^{O}_s) > R_{th}] \\ &= \exp(-\frac{2^{R_s}-1}{P^{O}_s}) \end{aligned} \tag{3}$$

where R_{th} is the minimum data rate of secondary link when no outage occurs and P^{O}_s is the transmission power of CU in overlay mode.

$$\begin{aligned} \hat{p}^{OI}_{out} &= \Pr[\log_2(1+\frac{h_s P^{O}_s}{1+g_s P_p}) > R_{th}] \\ &= \frac{\exp(-\frac{2^{R_{th}}-1}{P^{O}_s})}{1+\frac{2^{R_{th}}-1}{P^{O}_s}P_p} \end{aligned} \tag{4}$$

where P_p is the peak transmit power of primary link. The throughput in underlay mode, C_U , is given as

$$C_U = p_t p_f \hat{p}^{U}_{out} + (1-p_t)(1-p_m)\hat{p}^{UI}_{out} \tag{5}$$

where $\hat{p}^{U}_{out}$ is the probability, outage does not occur, of the secondary link when the CU switches to underlay mode by false alarm and $\hat{p}^{UI}_{out}$ is the probability, outage does not occur, in underlay when the PU is actually present. Similarly, $\hat{p}^{U}_{out}$ and $\hat{p}^{UI}_{out}$ can be calculated as follows.

$$\hat{p}_{out}^{U} = \Pr\left[\log_2(1+h_s P_s^U) > R_{th}\right] \tag{6}$$

$$= \left(\exp\left(-\frac{I_T}{P_{s,\max}}\right)\right)\left(\exp\left(-\frac{2^{R_{th}}-1}{P_{s,\max}}\right)\right)$$

$$+\exp\left(-\frac{I_T}{P_{s,\max}}\right) - \frac{\exp\left(-\frac{I_T}{P_{s,\max}}\left(1+\frac{2^{R_{th}}-1}{I_T}\right)\right)}{1+\frac{2^{R_{th}}-1}{I_T}}$$

where P_s^U is the transmission power of CU in underlay mode, I_T is the interference temperature, and $P_{s,max}$ is the maximum transmission power of CU.

$$\hat{p}_{out}^{UI} = \Pr\left[\log_2\left(1+\frac{h_s P_s^U}{1+g_s P_p}\right) > R_{th}\right]$$

$$= \left(\exp\left(-\frac{I_T}{P_{s,\max}}\right)\right)\left(\frac{\exp\left(-\frac{2^{R_{th}}-1}{P_{s,\max}}\right)}{1+\frac{P_p}{P_{s,\max}}(2^{R_{th}}-1)}\right) \tag{7}$$

$$+\exp\left(-\frac{I_T}{P_{s,\max}}\right) - \frac{\exp\left(\frac{1}{P_p}-\frac{I_T}{P_p(2^{R_{th}}-1)}\right)I_T}{P_p(2^{R_{th}}-1)}$$

$$\times E_1\left(\frac{(I_T+2^{R_{th}}-1)(P_{s,\max}+P_p(2^{R_{th}}-1))}{P_{s,\max}P_p(2^{R_{th}}-1)}\right)$$

where E_1 is the exponential integral function.

The rechargeable finite capacity battery of each CU stores the harvested energy from ambient sources and primary channel. The CU harvests e_h and e_p packets of energy from ambient sources and primary channel, respectively. Energy from ambient sources is harvested by the CU in parallel with the normal operation, on the other hand, energy from the channel can only be harvested if it is found in busy state. We assume that packets of energy harvested from ambient sources and primary channel both follows a Poisson process with mean $e_{h\mu}$ and $e_{p\mu}$, respectively. The probability distribution of e_h and e_p are given below.

$$\Pr(e_h = j) = e^{-e_{h\mu}} \frac{(e_{h\mu})^j}{j!} \qquad j = 1,2,3,...,m. \tag{8}$$

$$\Pr(e_p = i) = e^{-e_{p\mu}} \frac{(e_{p\mu})^i}{i!} \qquad i = 1,2,...,n. \tag{9}$$

We assume that the CU receiver reports the channel state information (CSI), i.e. channel gain (h_s), to the CU transmitter over a causal feedback channel. The CU transmitter adapts its parameters, especially the transmission power, accordingly. The communication channel gain is mainly effected by position and distance of the receiver from transmitter. Since random location of the CU receiver follows a Poisson process [13], therefore we assume that the channel gain (h_s) also follows a Poisson process with mean $h_{s\mu}$. As the transmission power (energy) is adjusted in accordance with the channel gain, so the probability distribution of transmission power can be described in terms of the channel gain between CU transmitter and receiver as follows

$$\Pr(h_s = k) = e^{-h_{s\mu}} \frac{(h_{s\mu})^k}{k!} \qquad k = 1,2,...,K. \tag{10}$$

III. ACTION AND TRANSMISSION MODE SELECTION

At the start of each time slot, based on the previous actions and observations, CU has a belief, p_t, about the channel i.e. idle probability of the primary channel. Depending on the belief and available energy (e_a), CU takes an optimal action using POMDP and further updates its belief, energy, and other parameters for the next time slot. The action taken by CU may either be to stay idle (SI) or sense (S). The choice of action is driven by a reward (throughput). Selection of the action and transmission mode is shown graphically in the flow chart given in Fig. 3. For every action $a_i(e_a, p_t, \mathrm{s}) \in A = \{SI, S\}$ in the state space $s = \{B, F\}$, CU gets a reward $R(e_a, p_t, a_i)$. If CU decides to stay idle, the obtained reward is

$$R(e_a, p_t, SI) = 0. \tag{11}$$

Staying idle is beneficial for energy conservation but it wastes the opportunity for transmission, which has a negative impact on the throughput. On the other hand, CU will get the following reward when channel sensing is chosen.

$$R(e_a, p_t, S) = \frac{T-\tau}{T} C \tag{12}$$

where C is the throughput whose value is dependent on the observation and τ is the sensing time. Spectrum sensing and transmission results in the consumption of energy but it also increases the opportunity for the throughput. POMDP is used to take an action i.e. to stay idle or to sense the channel, based on the available energy and primary network dynamics which maximizes the throughput. Our objective is to maximize the discounted reward function in a particular time slot. In POMDP, the expected value of a policy or the maximum achievable reward function starting from the current slot is represented by the value function $V(e_a, p_t)$, and is given by

$$V(e_a, p_t) = \max_{a_k, a_{k+1} \ldots \varepsilon A} \mathrm{E}\left\{ \begin{array}{c} \sum_{n=k}^{\infty} \left(\gamma^{n-k} R(e_{an}, p_{tn}, a_n) \mid e_k = e_a, p_k = p_t\right) \\ 0 \le \gamma < 1 \end{array} \right\} \tag{13}$$

where γ is the discount factor and is used to value the current reward more than future rewards. The value function satisfies the following Bellman equation and is solved by the value iteration method [14].

$$V(e_a, p_t) =$$
$$R(e_a, p_t, a_i) + \left\{ \sum_{e'}^{\infty} \gamma^{t-k} \sum_{i=1}^{7} \Pr(Z_i) \Pr(e_a \to e' \mid Z_i) V(e', p_{t+1}) \mid e_a, p_t \right\} \tag{14}$$

where $\Pr(Z_i)$ is the probability of observation Z_i and $\Pr(e_a \to e'|Z_i)$ is the transition probability from e_a to e' (updated energy state for the next slot). When the CU decides an action, one of the following observations may occur.

Observation 1 (Stay Idle): CU does not carry out spectrum sensing due to insufficient energy or due to the CU's belief about the channel. An updated belief, p_{t+1}, that channel would be free in the next time slot is given as

$$p_{t+1} = p_t p_{FF} + (1-p_t) p_{BF}. \tag{15}$$

CU does not get any reward in this case and we have

$$R(e_a, p_t, SI|Z_1) = 0 \tag{16}$$

The energy and transition probability, $\Pr(e_a \to e'|Z_1)$, will be updated, respectively, as follows

$$\begin{aligned} &e' = e_a + e_h - e_i \\ &\Pr(e_a \to e'|Z_1) = \Pr\left(e_h = e_j^h\right) \quad j = 1,2,\ldots,m. \end{aligned} \tag{17}$$

Observation 2 (Overlay-mode): CU decides to sense the channel consuming energy e_s in time τ, and finds the channel in idle state. The belief for next slot is updated as follows

$$p_{t+1} = p_{FF}. \tag{18}$$

In this case, CU gets a reward in the form of transmission as given below

$$R(e_a, p_t, S|Z_2) = \frac{T-\tau}{T} C_O. \tag{19}$$

The probability of observation is computed as

$$\Pr(Z_2) = p_t(1-p_f)\Pr(e_t \le e_{to}). \tag{20}$$

The updated energy and transition probability is given by

$$\begin{aligned} &e' = e_a + e_h - e_s - e_t \\ &\Pr(e_a \to e'|Z_2) = \Pr(e_h = e_j^h)\Pr(e_t = e_k^t) \end{aligned} \tag{21}$$

where e_s is the sensing energy, $j = 1,2,\ldots,m$ and $k = 1,2,\ldots,K$.

Observation 3 (Underlay-mode): CU finds the channel in the busy state but CU can coexist with PU i.e. $\lambda_o < E_{se} < \lambda_u$. CU transmits in underlay mode and updates belief for the next slot as

$$p_{t+1} = p_{BF}. \tag{22}$$

The reward in the form of data transmitted is given by

$$R(e_a, p_t, S|Z_5) = \frac{T-\tau}{T} C_U. \tag{23}$$

The probability of observation is computed as

$$\Pr(Z_5) = (1-p_t)(p_d)\Pr(e_t \le e_{tu}). \tag{24}$$

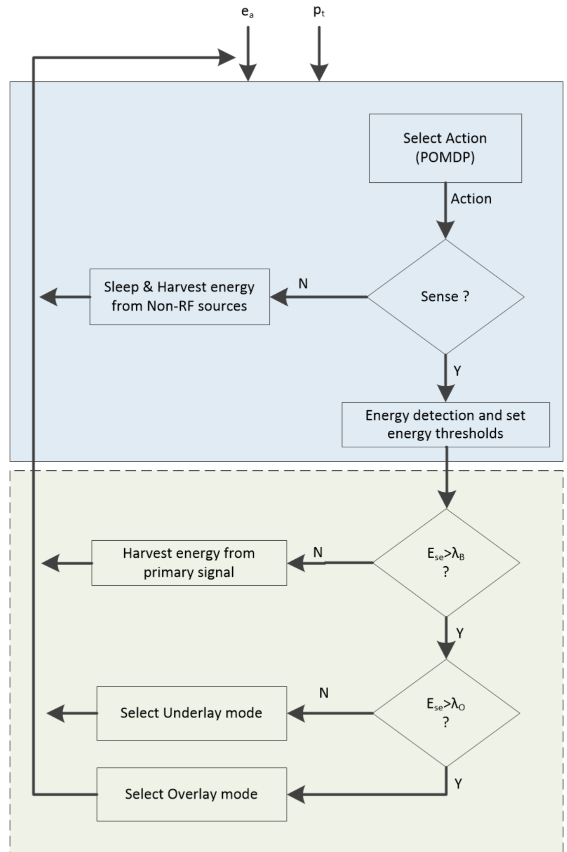

Fig. 3. Flow chart of the proposed system

The updated energy and transition probability are given, respectively, by

$$\begin{aligned} &e' = e_a + e_h - e_s - e_t \\ &\Pr(e_a \to e'|Z_5) = \Pr(e_h = e_j^h)\Pr(e_t = e_k^t) \end{aligned} \tag{25}$$

where $j = 1,2,\ldots,m$ and $k = 1,2,\ldots,K$.

Observation 4 (Energy harvesting): CU senses the channel and finds it in state B i.e. $E_{se} > \lambda_B$. In this case CU harvests energy from the primary signal and refrains from transmission for the remaining time slot. The updated belief is given as

$$p_{t+1} = p_t^u p_{FF} + (1-p_t^u) p_{BF} \tag{26}$$

where $p_t^u = \dfrac{p_t p_f}{p_t p_f + (1-p_t) p_d}$.

As no data transmission occurs so the reward is

$$R(e_a, p_t, S|Z_7) = 0. \tag{27}$$

The probability of observation is given as

$$\Pr(Z_7) = p_t p_f(e_s) + (1-p_t) p_d. \tag{28}$$

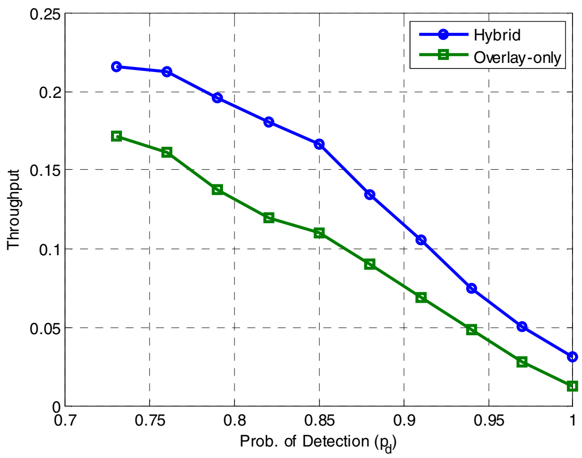

Fig. 4. Comparison of the throughput of hybrid CU with overlay-only CU

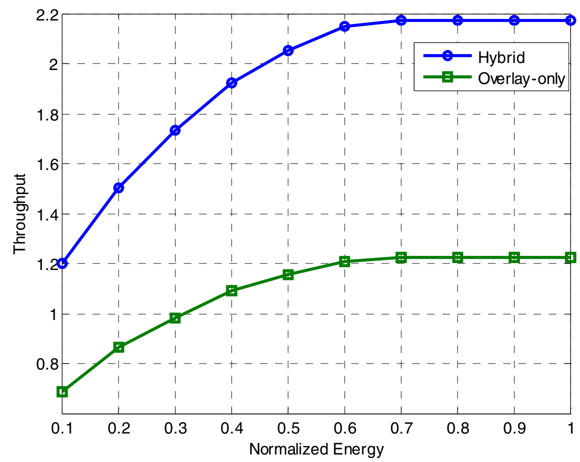

Fig. 5. Throughput of the CU in the hybrid and overlay-only modes versus the normalized harvested energy.

The updated energy and transition probability are computed, respectively, as

$$\begin{aligned} &e' = e_a + e_h + e_p - e_s \\ &\Pr(e_a \rightarrow e' | Z_7) = \Pr(e_h = e_j^h)\Pr(e_p = e_i^p) \end{aligned} \tag{29}$$

where $j = 1,2, \dots, m$ and $i = 1,2, \dots, n$.

The transmission mode (overlay or underlay) is determined on the basis of predefined energy thresholds. Energy of the signal is computed through energy detection method and is compared against the predefined thresholds. Let E_{se} be the signal energy and λ_O and λ_U be the predefined overlay and underlay thresholds, respectively, and λ_B be the underlay maximum threshold. If the signal energy is below the overlay threshold, i.e. $E_{se} < \lambda_O$, then the CU will transmit in overlay mode, on the other hand, if energy of the signal is above the overlay threshold but lower than the underlay threshold, i.e. $\lambda_O < E_{se} < \lambda_U$, then CU will adopt the underlay transmission mode. Notice that to keep the interference caused by CU in the underlay mode below the interference temperature, the transmission energy in underlay mode is kept below the overlay mode. If the signal energy is greater than the underlay maximum threshold, i.e. $\lambda_B < E_{se}$, then the channel is determined to be in busy state, meaning that CU is not allowed to transmit, rather the CU will harvest energy from the channel.

IV. Simulation Results

In this section, we show the performance effectiveness of our proposed system in terms of throughput. For simulation purposes, we consider that the value of SNR is -10dB, primary channel idle probability is 0.7, transition probability from idle state to itself and from busy to idle state is 0.7 and 0.3, respectively. The minimum required data rate when outage does not occur is 0.4bps/Hz. The interference temperature is 2dB, peak transmit power is 10dB, and duration of each slot is 20ms. Capacity of the battery is 20units, mean value of harvested energy from channel and from other sources are 1unit and 3units, respectively.

Fig. 4 illustrates throughput of the hybrid overlay-underlay energy harvested and overlay-only cognitive radio systems. Our proposed system harvests energy from the wireless primary signal as well as from the ambient sources and uses POMDP for action selection. Overlay-only cognitive radio harvests energy from the ambient sources only and do not use any access strategy i.e. POMDP for selecting an action to sense the channel. It is evident from the figure that our proposed scheme performs better than the overlay-only cognitive radio systems. The reason is that our proposed scheme uses POMDP to decide the action of channel sensing and subsequent transmission by considering energy of the CU and hence maximizes the throughput. For example, in the proposed scheme, if the available energy is not enough for transmission, CU would refrain from channel sensing and would harvest energy from the channel along with the ambient sources which will be utilized for transmission in the next slot. Whereas, in the overlay-only system, the user spends energy in channel sensing that further reduces the available insufficient energy for transmission. In next time slot, CU would not be able again to transmit due to insufficient energy even if the channel is free.

Fig. 5 exhibits the relationship of throughput and harvested energy. The harvested energy comes from non-RF (ambient sources) and RF (primary channel) sources. In the figure throughput of the CU operating in hybrid and overlay-only mode are shown. The CU employing hybrid mode shows enhanced throughput compared to the overlay-only CU. It is clear from the figure that throughput of the CU increases with the normalized harvested energy. Since the sensing action is taken on the basis of available energy of the CU and idle probability of the channel, therefore more harvested energy increases the chances of transmission which subsequently enhances the throughput.

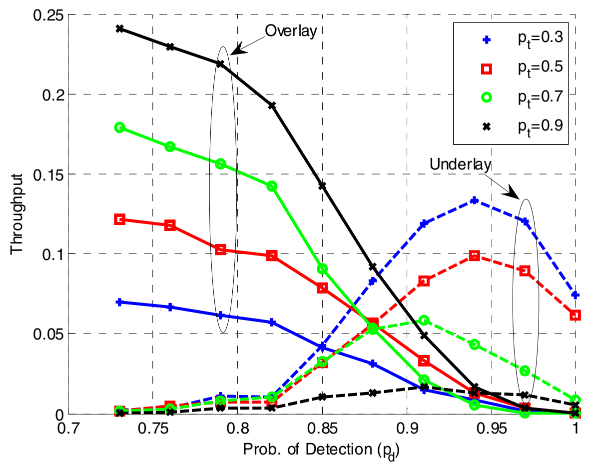

Fig. 6. Throughput of the CU for different values of the idle probability of the primary channel.

Fig. 6 demonstrates the throughput of hybrid overlay-underlay energy harvesting cognitive radio for different values of channel idle probability p_t. It is observed from the figure that as the channel idle probability increases, throughput of the CU in overlay mode increases. On the other hand, throughput of the CU in underlay mode increases as the channel idle probability decreases. The obvious reason for the above observed phenomenon is that the CU can transmit in the overlay mode only if the primary channel is free. When idle probability of a channel decreases, transmission in the overlay mode decreases which consequently decreases the throughput. On the other side, CU in the underlay mode can transmit even in the presence of the PU. When the channel idle probability decreases, transmission in the underlay mode increases and vice versa.

V. CONCLUSION

Increasing throughput is a desirable feature of every wireless equipment. However, the increased throughput is usually achieved at the cost of more consumed energy. In this paper, we have shown throughput improvement of the cognitive radio with efficient usage of the energy. Cognitive radio efficiently consumes energy by selecting an optimal action between sensing and staying idle through POMDP which considers the available energy of the CU and idle probability of the primary channel. After deciding to sense, the cognitive radio either i) transmits in overlay or underlay mode to enhance throughput and also harvest energy simultaneously from non-RF sources to increase energy or ii) harvests from non-RF and RF sources simultaneously to increase energy. It is shown through simulation that throughput increases with harvested energy and channel idle probability.

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