Outage Performance Study of Cognitive Relay Networks with Imperfect Channel Knowledge

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***Abstract*—This letter presents the outage performance analysis for a cognitive relay network (CRN) with imperfect channel knowledge estimations. Under the condition of imperfect channel state information (CSI) estimations of interference links between primary and secondary systems, especially the links from both secondary transmitter and secondary relays to primary users, the exact outage probability is derived over Rayleigh fading channels and is verified through simulations. The simulation results show that: 1) increasing the number of relays is an effective approach to mitigate the outage performance degradation caused by the imperfect CSI; 2) the imperfect CSI of the secondary transmitter-primary user (PU) link has a greater impact on the outage performance than that of the secondary relay-PU link.**

***Index Terms*—Cognitive relay networks, imperfect channel knowledge, outage performance.**

I. Introduction

COGNITIVE relay network (CRN) [1] combines cognitive radio technique and cooperative relay technology which has received a considerable research attention [2][3]. In [5] the effect of imperfect channel state information (CSI) in cooperative selective amplify-and-forward (SAF) networks is studied, which shows that the diversity order tends to zero due to imperfect channel state information. In [6] the authors study the asymptotic outage performance considering the imperfect CSI of links inside the secondary system. However [6] does not analyze the channel estimations between primary and secondary system. In [7] and [8], imperfect CSI estimations of the links between primary users and secondary system are introduced to study the average capacity and bit-error-rate (BER) under the average and peak transmission power. But both studies do not consider the cognitive relay system, especially for the practical multiple relay systems. In [4], spectrum sharing cognitive multiple relay networks with imperfect CSI is considered. However, [4] only chooses partial AF relay selection, while DF relay selection protocol has not been considered. Meanwhile in the system model of [4], there is no direct link for the secondary system and the different degree of imperfectness on different interference links being addressed, which is an important issue in practical scenarios.

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In this paper, the imperfect CSI of interference links from both secondary transmitter and secondary relays to primary users is analyzed in case of the cognitive multi-relay networks with selective decode-and-forward (SDF) protocol based on our previous study [3]. As shown in Fig. 1, SS-PU re-sends the interference link between the source (SS) and the primary user (PU), and SR-PU re-sends the interference link between the relay (SR) and the primary user (PU). In this scenario, we have derived a closed form expression for the probability of outage that occurs due to imperfect CSI estimation. The expression is a function of the number of relays. The major contributions of this paper are summarized as follows:

- The CSI imperfection for the links SS-PU and SR-PU does not affect the diversity order for cognitive relay networks.
- Increasing the number of relays is an effective way to improve the outage performance. This implies that, in practice, if CSI knowledge is difficult to obtain, then an increased number of relays can mitigate the performance loss due to imperfect CSI.
- The imperfection of SS-PU link has a stronger impact on the outage performance than that of the SR-PU link. In practice, more effort should be put on the channel knowledge for the SS-PU link than the relay-PU link.

The rest of this paper is organized as follows: Section II describes the system model for the analysis of cognitive relay networks with imperfect channel knowledge. The exact outage probability is derived over Rayleigh fading channels to study the impact of imperfect CSI estimations in Section III. Simulation results are obtained and analyzed in Section IV. Finally, conclusions are drawn in Section V.

II. System Model

A cognitive relay network (CRN) with imperfect channel state information knowledge in a spectrum sharing scenario is considered in this letter, as illustrated in Fig.1. It involves a primary user (PU) and a secondary system. The secondary system is a cooperative relay communication system which consists of a source (SS), a destination (SD) and N relays (SR). In this letter we consider the imperfections of the interference links, i.e., the link between SS and PU, and the links between SR and PU. Time division multiple access (TDMA) based selection decode-and-forward (SDF) cooperative protocol is used for transmissions. The whole transmission process of secondary system consists of two phases. In the first phase, the SS broadcasts messages, the SD and relays receive signals. In the second phase, the best relay with maximum signal-to-noise ratio (SNR) in the second hop link is selected from the successfully decoding relay set, then the best relay

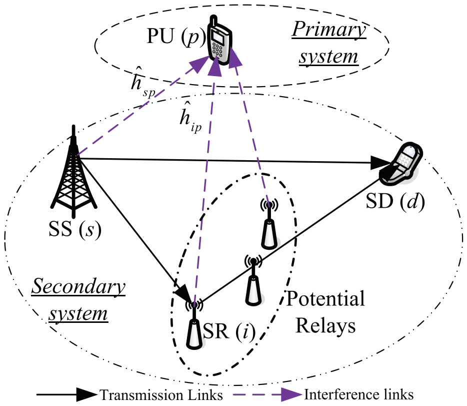

Fig. 1. System model.

repeats source messages and forwards to the SD. Finally, the SD combines the signals from the SS and the best relay with maximum ratio combine (MRC) to decode source messages.

It is assumed in this letter that all the links are independent and identically distributed (*i.i.d.*) zero-mean Rayleigh flat fading channels with unit variance. The channel gains are exponentially distributed. It is denoted that the channel gain of the link between transmitter $x = \{s, i\}$ and receiver $y = \{p, d, i\}$ as $g_{xy} = |h_{xy}|^2$ which is exponentially distributed with mean λ_{xy}. To simplify the analysis, it is assumed that $\lambda_{si} = \lambda_{sr}$, $\lambda_{ip} = \lambda_{rp}$, $\lambda_{id} = \lambda_{rd}$. The thermal noise of the receivers are modeled as complex additive white Gaussian noises (AWGN) with variance σ^2.

In practical networks, due to channel estimation errors, mobility, feedback delay, limited feedback or feedback quantization, obtaining the full CSI is difficult, and often only partial CSI information can be acquired. Thus the analysis for imperfect channel knowledge will be very beneficial for the design of cognitive relay system. In this letter, the commonly-used equation is used to reflect the impact of imperfections of the channel state information [7],

$$\hat{h}_{xy} = \rho h_{xy} + \sqrt{1-\rho^2}\varepsilon. \tag{1}$$

Where h_{xy} denotes the ideal channel coefficients of the $x-y$ link, and $\hat{h}_{xy}$ is the channel estimation available at the secondary system which is uncorrelated with h_{xy}. ε is a Gaussian random variable (RV) with zero mean and variance $\frac{1}{\lambda_{xy}}$. The correlation coefficient ρ $(0 \leq \rho \leq 1)$ is a constant that determines the average quality of the channel estimation over all channel states which reflects the degree of imperfections of CSI estimations. For example, when $\rho = 1$ the channel knowledge is perfect and the full CSI is available, while $\rho = 0$ means that the CSI estimation is completely random. It is easily derived that $\hat{g}_{xy} = |\hat{h}_{xy}|^2$ is also exponentially distributed with mean λ_{xy}.

In this letter the channel knowledge between the primary system and the secondary system is assumed to be imperfect, and the channel knowledge among the secondary system is perfect. In this way, the channel knowledge of the interference links are imperfect. To simplify the analysis we consider a more general case, that is, the SS-PU link and SR-PU link have different correlation coefficients for the imperfect CSI knowledge as follows,

$$\hat{h}_{sp} = \rho_1 h_{sp} + \sqrt{1-\rho_1^2}\varepsilon. \tag{2}$$

$$\hat{h}_{ip} = \rho_2 h_{ip} + \sqrt{1-\rho_2^2}\varepsilon \quad (i = 1, 2, ..., N). \tag{3}$$

Where, ρ_1 and ρ_2 represent the correlation coefficients for the CSI estimation of the SS-PU link and the SR-PU link, respectively.

III. Outage Performance Analysis

First we consider the symmetrical case, that is, $\rho_1 = \rho_2 = \rho$. In order to guarantee that the interference power from the secondary system to PU is not exceeding Q, the maximum transmission power of SS and i-th Relay should be $\frac{Q}{\hat{g}_{sp}}$ and $\frac{Q}{\hat{g}_{ip}}$, respectively, as follows,

$$P_s = \frac{Q}{\hat{g}_{sp}} \quad and \quad P_i = \frac{Q}{\hat{g}_{ip}}. \tag{4}$$

However, since that the channel knowledge estimation is imperfect for the interference channel, the actual interference power from the SS to primary user (PU) will probably exceed the maximum tolerated interference power Q. It is denoted that the actual interference power from SS to the primary user as,

$$I_s = P_s \cdot g_{sp} = \frac{Q \cdot g_{sp}}{\hat{g}_{sp}}. \tag{5}$$

Using the joint probability density function (PDF) of g_{sp} and $\hat{g}_{sp}$, we can get the cumulative density function (CDF) of I_s as,

$$F_{I_s} = Pr\{I_s < z\} = \frac{1}{2}\{1 + \frac{1-\frac{Q}{z}}{\sqrt{(1+\frac{Q}{z})^2 - \frac{4\rho^2 Q}{z}}}\}. \tag{6}$$

From equation it is indicated that, the probability of the actual interference power from SS to PU is less than Q is $Pr\{I_s < Q\} F_{I_s}(Q) = 0.5$, thus under the condition of imperfect CSI knowledge, the probability of guaranteeing the interference constraint will only be half, which is far more from our expectation. Therefore it is crucial to set a new interference power constraint $\hat{Q}$ $(\hat{Q} < Q)$, which can be used for limiting the transmission power of the SS. Meanwhile, let p be the probability that the actual interference power from SS to PU is less than Q, which can be regarded as a quality for interference. We define an equivalent factor (EF) $\mu_1 = \frac{\hat{Q}_1}{Q}$, which represents the ratio of new maximum interference power constraint $\hat{Q}_1$ and the old one Q during the first transmission phase. From (III) we can get,

$$p = \frac{1}{2}\{1 + \frac{1-\mu_1}{\sqrt{(1+\mu_1)^2 - 4\rho_1^2\mu_1}}\}. \tag{7}$$

Since $0 < \mu_1 < 1$ we can get the new maximum tolerated interference power $\hat{Q}_1$ in the first transmission phase as $\hat{Q}_1 = \mu_1 Q = \frac{1-(2p-1)^2(2\rho_1^2-1)-2(2p-1)\sqrt{(1-\rho_1^2)[1-\rho_1^2(2p-1)^2]}}{4p(1-p)} Q$. The new maximum tolerated interference power $\hat{Q}_2$ in the second transmission phase is similar to Q_1,

For asymmetrical cases, the SS-PU link and SR-PU link have different correlation coefficients for the imperfect CSI

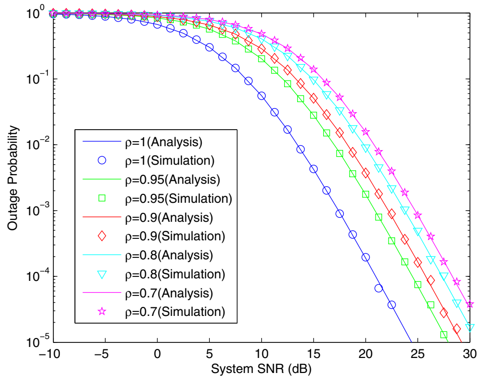

Fig. 2. Outage probability under different imperfections of estimation errors ρ, N=2.

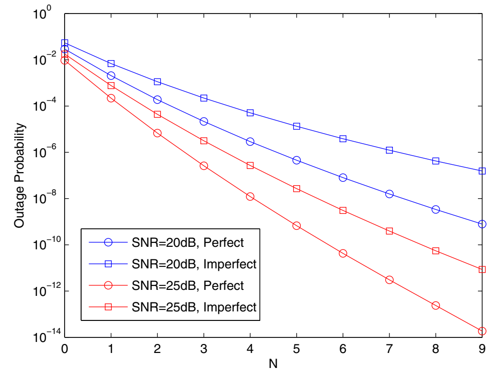

Fig. 3. Outage probability under different number of relays N.

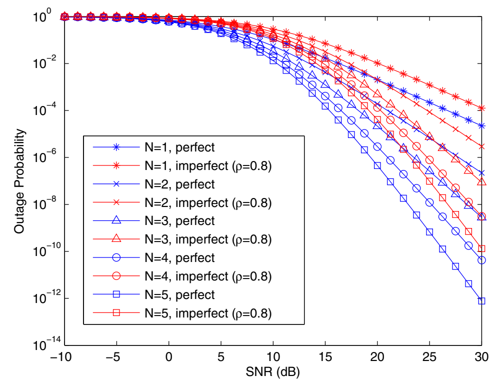

Fig. 4. Outage probability under different SNR with various relay number N.

knowledge. Under this condition, the new tolerated interference power constraints for the SS-PU and SR-PU links are $\hat{Q}_1$, $\hat{Q}_2$. The transmit power for source node SS and relay node SR_i are $P_s = \frac{\hat{Q}_1}{\hat{g}_{sp}}$, $P_{R_i} = \frac{\hat{Q}_2}{\hat{g}_{ip}}$. The received SNR at SD and i-th relay node SR_i can be written as, $\gamma_{sd} = \frac{\hat{Q}_1 g_{sd}}{\sigma^2 \hat{g}_{sp}}$ and $\gamma_{si} = \frac{\hat{Q}_1 g_{si}}{\sigma^2 \hat{g}_{sp}}$.

It is assumed that the transmission rate of secondary system is R, it is defined $k = 2^{2R} - 1$, and if the i-th relay SR_i can successfully decode the signal from SS its SNR should be $\gamma_{si} \geq k$. Let $\Gamma = \{1, 2, ..., N\}$ be the set for all the relays, and $R(s)$ denotes the successful decoding relay set in the first phase. If $R(s)$ is empty, then the received SNR at SD in the second phase γ_{rd} is zero, if $R(s)$ is not empty, the best relay among the set $R(s)$ will be selected, then the SNR at SD is

$$\gamma_{rd} = max_{i \in R(s)}(\gamma_{id}) = max_{i \in R(s)}(\frac{\hat{Q}_2 g_{id}}{\sigma^2 \hat{g}_{ip}}). \qquad (8)$$

Since that g_{id} and $\hat{g}_{ip}$ are independent exponentially distributed with parameters λ_{rd} and λ_{rp}, the cumulative density function (CDF) can be obtained as,

$$\begin{aligned} F_{\gamma rd}(x) &= Pr(\gamma_{rd} \leq x) \\ &= \big(1 - \frac{\lambda_{rp}\hat{Q}_2}{x\lambda_{rd}\sigma^2 + \lambda_{rp}\hat{Q}_2}\big)^n \\ &= \sum_{j=0}^{n} C_n^j (-1)^j (\frac{\lambda_{rp}\hat{Q}_2}{\lambda_{rd}\sigma^2})^j (x + \frac{\lambda_{rp}\hat{Q}_2}{\lambda_{rd}\sigma^2})^{-j}. \quad (9) \end{aligned}$$

Hence, the equivalent received SNR at the SD is $\gamma_{sd} + \gamma_{rd}$ in a whole transmission process, and the mutual information of secondary system is $C = 1/2 \log(1 + \gamma_{sd} + \gamma_{rd})$, the outage occurs when $\gamma_{sd} + \gamma_{rd} < k$, in this way, the outage probability of secondary system is calculated in (10), where $\gamma_Q = \frac{Q}{\sigma^2}$, $U_1 = 1 - (2p-1)^2(2\rho_1^2 - 1) - 2(2p - 1)\sqrt{(1-\rho_1^2)[1-\rho_1^2(2p-1)62]}$, $U_2 = 1 - (2p-1)^2(2\rho_2^2 - 1) - 2(2p-1)\sqrt{(1-\rho_2^2)[1-\rho_2^2(2p-1)62]}$, $V = 4p(1-p)$, $A = \frac{\lambda_{sr}k(n+i)V + \lambda_{sp}U_1\lambda_Q}{\lambda_{sd}}$, $B = \frac{\lambda_{rp}U_2\lambda_Q}{\lambda_{rd}}$, $C = kV + \frac{\lambda_{rp}U_2\lambda_Q}{\lambda_{rd}}$.

IV. Analytical and Simulation Results

The parameters used for numerical analysis and simulations are: $\lambda_{sp} = \lambda_{sd} = \lambda_{sr} = \lambda_{rp} = \lambda_{rd} = 1$, R=1bps/Hz.

Fig.2 shows the outage probability under different correlation coefficient for channel estimations ρ with N=2, it can be seen that the theoretical analytical results are very close to the simulation results. From Fig.2 it can also be observed that the slopes of these curves are the same for different CSI estimation errors which indicates that the diversity order is not be influenced by the channel estimation errors. However the channel estimation errors greatly impact on the outage performance. For instance, the smaller of ρ (large channel estimation errors), the worse of outage performance.

Fig.3 shows that when the number of relay N increases, the outage performance gap between ideal and imperfect CSI estimations increases too. For example, when system SNR=20dB, the outage probability for imperfect CSI estimation ($\rho = 0.8$) is 7.7dB and 12.4dB more than that of ideal CSI when N=2 and N=4, respectively. However, increasing the relay number N can mitigate the outage performance loss of secondary system due to imperfect estimations. From Fig.4 presents the outage probability of different SNRs with various number of relays. When system SNR is large than 25.6dB the outage probability for imperfect CSI (with $\rho = 0.8$) at N=3 will be superior to that of ideal CSI at N=2. When system SNR is large than 18.8dB, the outage probability for imperfect CSI ($\rho = 0.8$) at N=4 is superior to that of ideal CSI at N=2. This is due to that although channel estimation errors decrease the outage performance, the larger number of relay N can increase the diversity order, which overcomes the impact of channel estimation errors. Thus increasing the number of relays is an

$$
\begin{aligned}
P_{out} =& \Pr(C<R, R(s)=\emptyset)+\Pr(C<R, R(s)\neq\emptyset)\\
=& \Pr\left[\bigcap_{i\in\Gamma}\left(\gamma_{si}<k\right), \gamma_{sd}<k\right]+\Pr\left[\bigcap_{i\in R(s)}\left(\gamma_{si}\geq k\right), \bigcap_{i\notin R(s)}\left(\gamma_{si}<k\right), \gamma_{sd}+\gamma_{rd}<k\right]\\
=& \int_0^\infty\left(1-e^{-\frac{\sigma^2\lambda_{sr}kx}{\tilde{Q}_1}}\right)^N\left(1-e^{-\frac{\sigma^2\lambda_{sd}kx}{\tilde{Q}_1}}\right)\lambda_{sp}e^{-\lambda_{sp}x}dx\\
&+\sum_{n=1}^{N}\left\{C_N^n\int_0^k\left[\int_0^\infty\left(e^{-\frac{\sigma^2\lambda_{sr}kx}{\tilde{Q}_1}}\right)^n\left(1-e^{-\frac{\sigma^2\lambda_{sr}kx}{\tilde{Q}_1}}\right)^{N-n}\left(1-e^{-\frac{\sigma^2\lambda_{sd}(k-y)x}{\tilde{Q}_1}}\right)\lambda_{sp}e^{-\lambda_{sp}x}dx\right]\mathrm{f}_{\gamma_{rd}}(y)dy\right\}\\
=& \sum_{i=0}^{N}\left\{\frac{\mathrm{C}_N^i(-1)^i\lambda_{sd}\lambda_{sp}U_1V\gamma_Q k}{\left(\lambda_{sr}kiV+\lambda_{sp}U_1\gamma_Q\right)\left[k\left(\lambda_{sr}i+\lambda_{sd}\right)V+\lambda_{sp}U_1\gamma_Q\right]}\right\}\\
&+\sum_{n=1}^{N}\sum_{i=0}^{N-n}\sum_{j=1}^{n}\left\{C_N^nC_{N-n}^iC_n^j(-1)^{i+j+1}j\frac{\lambda_{sp}}{\lambda_{sd}}\left(\frac{\lambda_{rp}}{\lambda_{rd}}\right)^jU_1{U_2}^j{\gamma_Q}^{j+1}\left[\frac{1}{jA}\left(\frac{1}{B^j}-\frac{1}{C^j}\right)\right.\right.\\
&\left.\left.-\sum_{m=1}^{j}\frac{1}{(j+1-m)\left(A+C\right)^m}\left(\frac{1}{B^{j+1-m}}-\frac{1}{C^{j+1-m}}\right)-\frac{1}{(A+C)^{j+1}}\ln\frac{C\left(A+C-B\right)}{AB}\right]\right\}
\end{aligned}
\tag{10}
$$

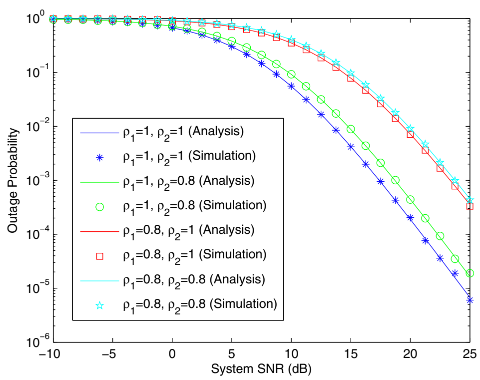

Fig. 5. Outage probability for the imperfections of both SS-PU (ρ_1) and SR-PU (ρ_2) links.

effective approach to mitigate the influence of imperfect CSI.

The outage probability for the imperfections of both SS-PU and SR-PU links is demonstrated in Fig.5. It can be seen that the outage performance in the case of the SR-PU link being imperfect is very similar to that when both SS-PU and SR-PU links are ideal estimations. However the outage performance in case of only the SS-PU link being imperfect is far more worse than that when both the links are in ideal estimations. This phenomenon shows that the imperfection of SS-PU link has a greater impact on the outage performance than that of the relay-PU links. Thus more effort should be put on the channel knowledge for the SS-PU link than the relay-PU links in practise.

V. Conclusion

This letter presented a further study about the outage performance for a cognitive relay networks with imperfect channel knowledge estimation based on our previous study[2][3]. The exact outage probability has been derived over Rayleigh fading channels under the condition of imperfect channel state information (CSI) estimations of the interference links between the primary and secondary systems. The analytical study was validated through simulations. It has been shown that increasing the number of relays is an effective way to obtain the similar outage performance in order to complement the imperfect channel knowledge. The imperfection of secondary transmitter-primary user link had a greater impact on the outage performance than that of the secondary relay-PU link. Therefore, more effort should be concentrated on the channel knowledge estimations of the links between the secondary transmitters and primary users.

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