# 1 Estimate the Heading

## 1.1 Magnetometer Calibration

"Hard-iron" effects happen due to permanent magnetic interference from objects or components near the magnetometer, causing a constant offset in all three axes. The objective is to shift the center of the circular (or elliptical) data back to the origin.
**Offset Calculation**: Calculate the mean values for each axis x, y, z across all samples to estimate the offset:

$$\bar{x} = \frac{1}{N}\sum_{i}^{N} x_i \qquad \bar{y} = \frac{1}{N}\sum_{i}^{N} y_i \tag{1}$$

**Offset Removal**: Subtract these offsets from each data point:

$$x_{corrected} = x - \bar{x} \qquad y_{corrected} = y - \bar{y} \tag{2}$$

From the data collected, the x-offset and y-offset are observed as follows:
Hard Iron x_offset= -0.2365
Hard Iron y_offset= -0.08165

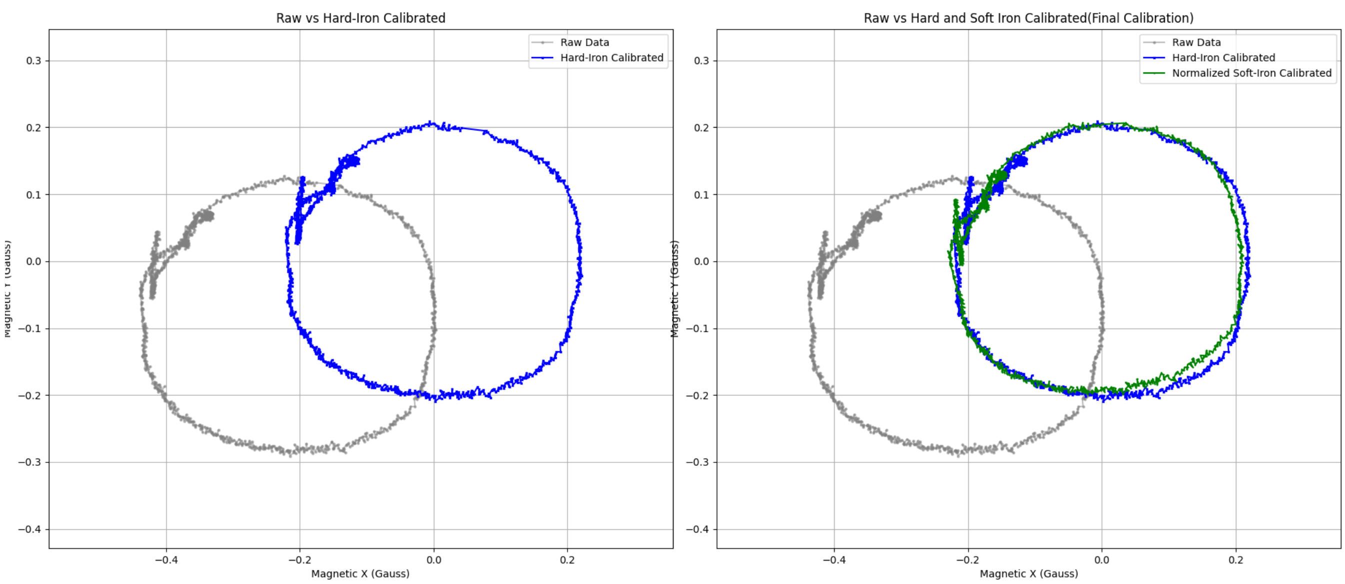


Figure 1: Magnetic Calibration

"Soft-iron" effects happen from distortions in the local magnetic field due to nearby ferromagnetic materials, causing the circular field strength around the magnetometer to appear elliptical and sometimes skewed. The objective is to normalize this ellipse to a circle, ensuring consistent scale on each axis.
**Mathematical Calculations:** Calculate the standard deviation $\sigma_x, \sigma_y, \sigma_z$ of each axis for the offset-corrected data. To correct the elliptical distortion, scale each axis by its standard deviation ratio to a common value (often the mean of the three standard deviations):

$$x_{final} = x_{corrected} * \frac{\sigma_{mean}}{\sigma_x} \qquad y_{final} = y_{corrected} * \frac{\sigma_{mean}}{\sigma_y} \qquad z_{final} = z_{corrected} * \frac{\sigma_{mean}}{\sigma_z} \tag{3}$$

where $\sigma_{mean} = \frac{\sigma_x+\sigma_y+\sigma_z}{3}$
By applying both hard-iron (offset) and soft-iron (scaling) corrections, the magnetometer data is calibrated, allowing for more accurate directional readings. Figure 1 compares the Hard Iron and Soft Iron corrections. **Q.** How did you calibrate the magnetometer from the data you collected? What were the sources of distortion present, and how do you know?
**Ans:** We calibrated the data as explained above. Sources for distortion could be because of metal components in

the car having magnetization. Other devices nearby with permanent magnets, can introduce hard-iron distortion. These were identified by the shift of the data from the origin. Soft Iron distortion was identified by observing an ellipse rather than circular mag_x vs mag_y.

## 1.2 Yaw Estimation

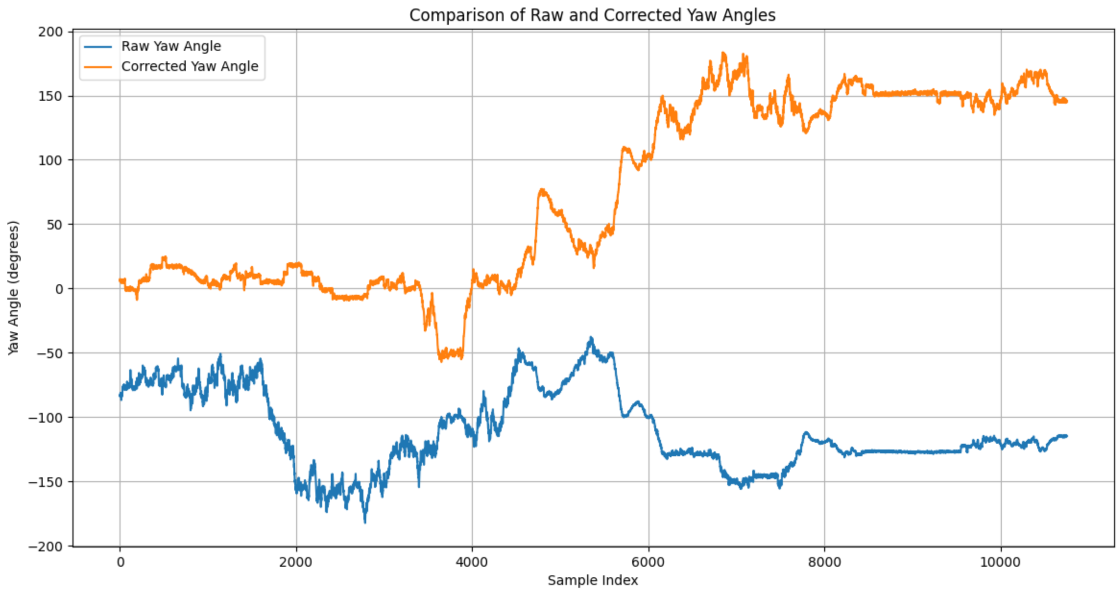


Figure 2: Raw Magnetic Yaw VS Corrected Magnetic Yaw

Yaw angle is found after the calibration of hard and soft iron of the magnetometer. It can also be found from integrating gyroscope data. Calculated yaw closely matches the form of the yaw angle calculated in the IMU. The yaw calculated from the gyroscope data is relatively smoother compared to the IMU yaw, while the yaw calculated from the magnetometer data is relatively rough.

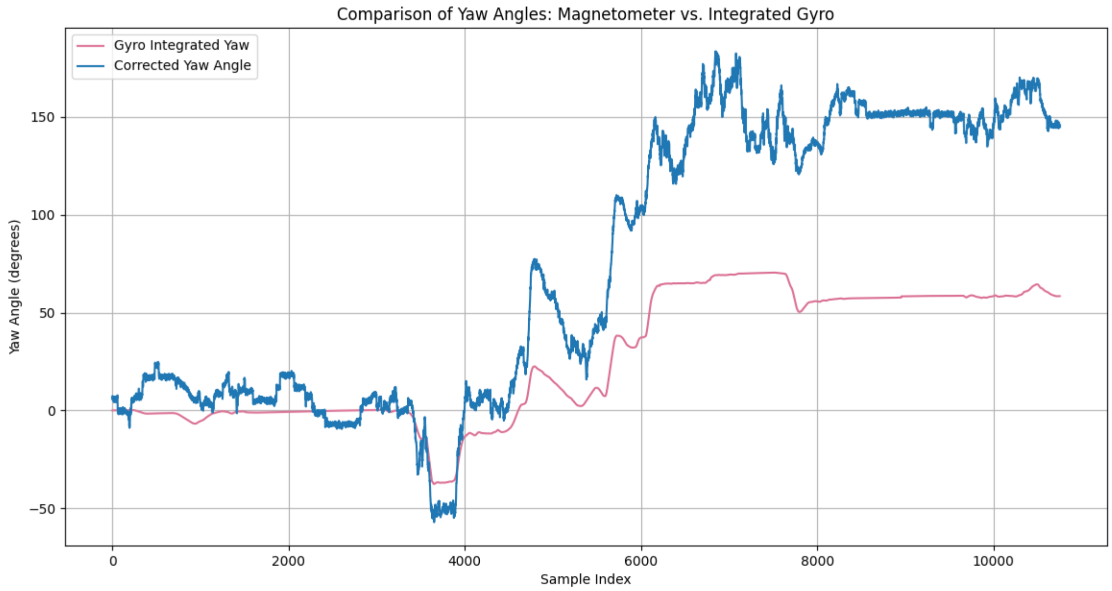


Figure 3: Magnetometer Yaw(Corrected) VS Gyro Integrated Yaw

**Question:** How did you use a complementary filter to develop a combined estimate of yaw? What components of the filter were present, and what cutoff frequency(ies) did you use?
I used a complementary filter to combine yaw estimates from the magnetometer and gyroscope, providing a reliable yaw angle. This approach takes in the strengths of both sensors: the magnetometer provides an absolute yaw reference but can be affected by noise, while the gyroscope offers smooth short-term yaw estimates but drifts over time.

**High Pass Filter (HPF):** Applied to the yaw angle derived from the gyroscope. The HPF removes low-frequency components, allowing only high-frequency, short-term changes from the gyroscope to pass through, effectively removing drift but keeping quick changes in yaw. LPF cut-off frequency - 0.1 Hz, used to filter out high-frequency noise in the magnetometer data.

**Low-Pass Filter (LPF):** Applied to the magnetometer-derived yaw. This filter removes high-frequency noise, keeping the slower, stable heading information provided by the magnetometer. HPF cut-off frequency - 0.0001 Hz, used to remove low-frequency drift in the gyroscope data.

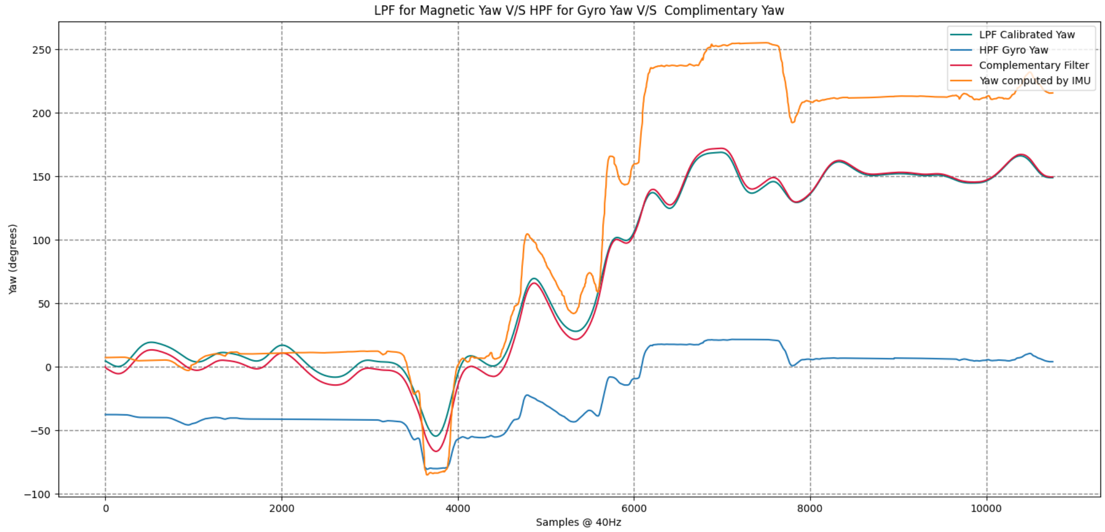


Figure 4: LPF, HPF, Complimentary Filter

**Question:** Which estimate or estimates for yaw would you trust for navigation? Why? (Your answer must not be the Yaw computed by the IMU)
**Ans:** For navigation, the complementary filter yaw estimate would be the most reliable choice. This estimate takes in the strengths of both the magnetometer and gyroscope, blending them to produce a stable, drift-free heading that maintains accuracy over time. Here's why:

**Drift Correction:** The high-pass filter applied to the gyroscope data removes low-frequency drift, a common issue with gyroscopes over time. This ensures that the yaw estimate doesn't deviate gradually, which is important for maintaining a correct heading in navigation.

**Noise Reduction:** The low-pass filter applied to the magnetometer data reduces high-frequency noise and short-term fluctuations. This provides a smooth baseline heading, which is especially useful when navigating in noisy magnetic environments.

## 2 Estimate the Forward Velocity

### 2.1 Velocity estimate from the GPS with Velocity estimate from accelerometer before adjustment

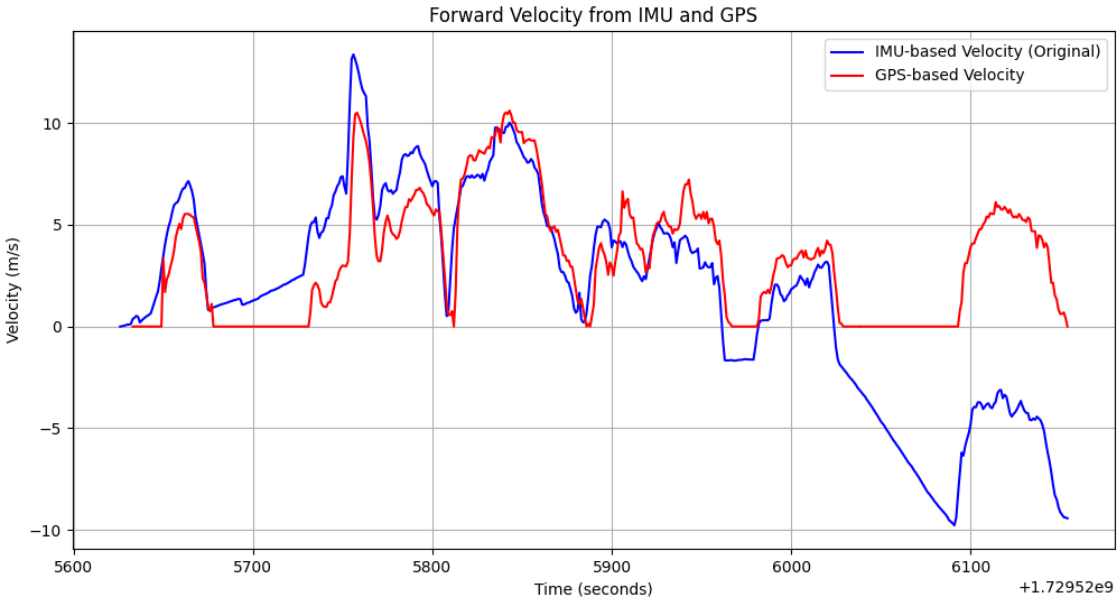


Figure 5: Forward Velocity before adjustment

The integrated velocity from the IMU shows a generally smooth, gradual increase in speed with time, which shows that the integration is capturing acceleration changes. However, there are instances where it increases even when the GPS velocity is decreasing, and it seems to keep speed longer than expected. This behavior shows drift in the IMU velocity estimate.

**Question:** Does the Integrated Velocity Make Sense?

**Ans:** The IMU-derived velocity gives a baseline trend but diverges over time due to accumulated drift errors. This drift is because any small biases or inaccuracies in the acceleration data become magnified through the integration process, causing the IMU velocity to continue increasing or decreasing even when the actual velocity (shown by GPS) is stable or changing direction.

**Question:** What adjustments did you make to the forward velocity estimate, and why?

**Low-Pass Filtering of IMU Acceleration Data:** IMU acceleration data often contains high-frequency noise due to sensor vibrations, environmental factors, or sensor limitations. By applying a low-pass filter, we isolate the lower-frequency components, representing more stable acceleration changes due to motion rather than sensor noise.

**Zeroing Negative Velocity Values:** Negative velocities in the context of forward motion are typically non-physical, as they imply that the vehicle is moving in reverse, which is not expected. These negative values arise from sensor noise or minor calculation errors. By setting negative values to zero, we ensure that the resulting velocity profile accurately reflects forward-only motion, aligning with the forward velocity measurement.

### 2.2 Velocity estimate from the GPS with Velocity estimate from accelerometer after adjustment

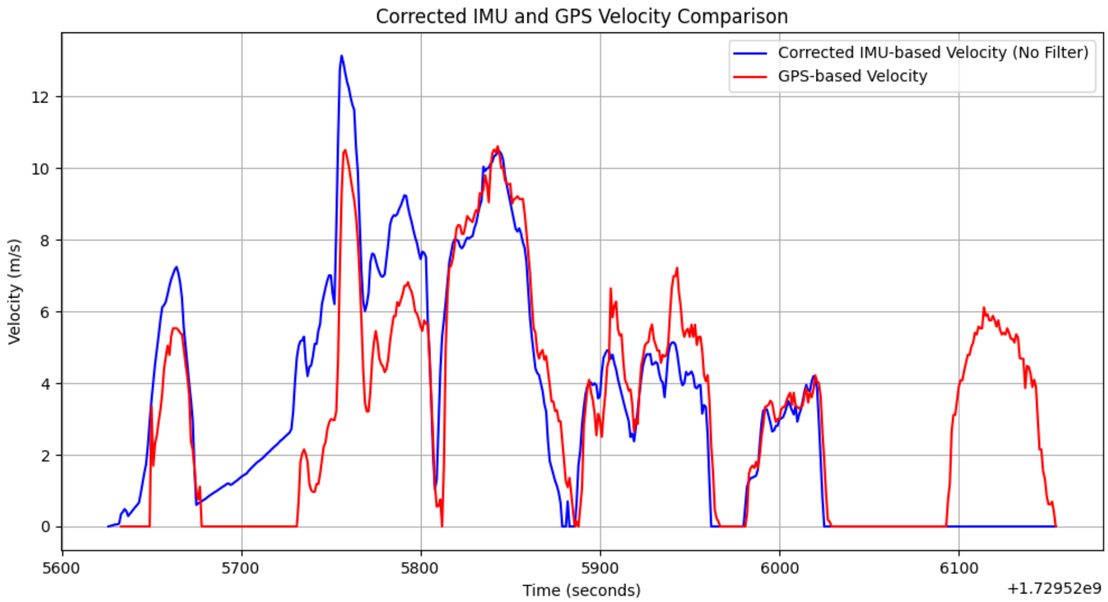


Figure 6: Forward Velocity after adjustment

**Question:** What discrepancies are present in the velocity estimate between accel and GPS. Why?

**Sensor Noise and Drift in IMU Data:** IMUs, are prone to noise and drift, which can accumulate over time. This drift can cause gradual inaccuracies in the integrated velocity estimate, as small errors in acceleration add up, especially in extended measurements. To get velocity from acceleration, we integrate the IMU's acceleration data over time. Any slight error in acceleration values will compound, as integration accumulates these errors.

**GPS Positional Update Rate and Delay:** GPS updates occur at a relatively low frequency (typically 1-10 Hz in standard setups), resulting in measurements that can lag behind real-time acceleration changes. GPS data also experiences slight delays due to satellite communication, which might not capture sudden changes accurately. GPS velocity estimates are based on changes in position between time steps.

Despite combining IMU and GPS timestamps, there can still be minor mismatches in the exact timing of each measurement due to slight variations in clock synchronization or measurement intervals.

## 3 Dead Reckoning with IMU

Dead reckoning is a navigation technique used to estimate the current position of an object by using a previously known position and using data on velocity, direction, and time elapsed since the last known position. It relies on integrating motion data (such as speed, heading, and time) to track movement over time, essentially calculating a new position based on movement from a starting point.

### 3.1 $\omega \dot{X}$ and $\ddot{y_{obs}}$ Plotted together

1. Integrate the forward velocity to obtain displacement and compare with GPS displacement.

2. We simplify the description of the motion by assuming that the vehicle is moving in a two-dimensional plane. Denote the position of the center-of-mass (CM) of the vehicle by (X,Y,0) and its rotation rate about the CM by $(0, 0, \omega)$. We denote the position of the inertial sensor in space by (x,y,0) and its position in the vehicle frame by $(x_c, 0, 0)$.

3. Then the acceleration measured by the inertial sensor (i.e. its acceleration as sensed in the vehicle frame) is:

$$\ddot{x_{obs}} = \ddot{X} - \omega\dot{Y} - \omega^2 x_c \qquad \ddot{y_{obs}} = \ddot{Y} - \omega\dot{X} - \dot{\omega}x_c \tag{4}$$

where all the quantities in these equations are evaluated in the vehicle frame. Assume that $\dot{Y} = 0$ (that is, the vehicle is not skidding sideways) and ignore the offset by setting $x_c = 0$ (meaning that the IMU is on the center of mass of the vehicle, i.e. the point about which the car rotates). Then the first equation above reduces to $\ddot{X} = \ddot{x_{obs}}$

4. Integrating this to obtain $\dot{X}$. Computing $\omega\dot{X}$ and comparing it to $\ddot{y_{obs}}$.

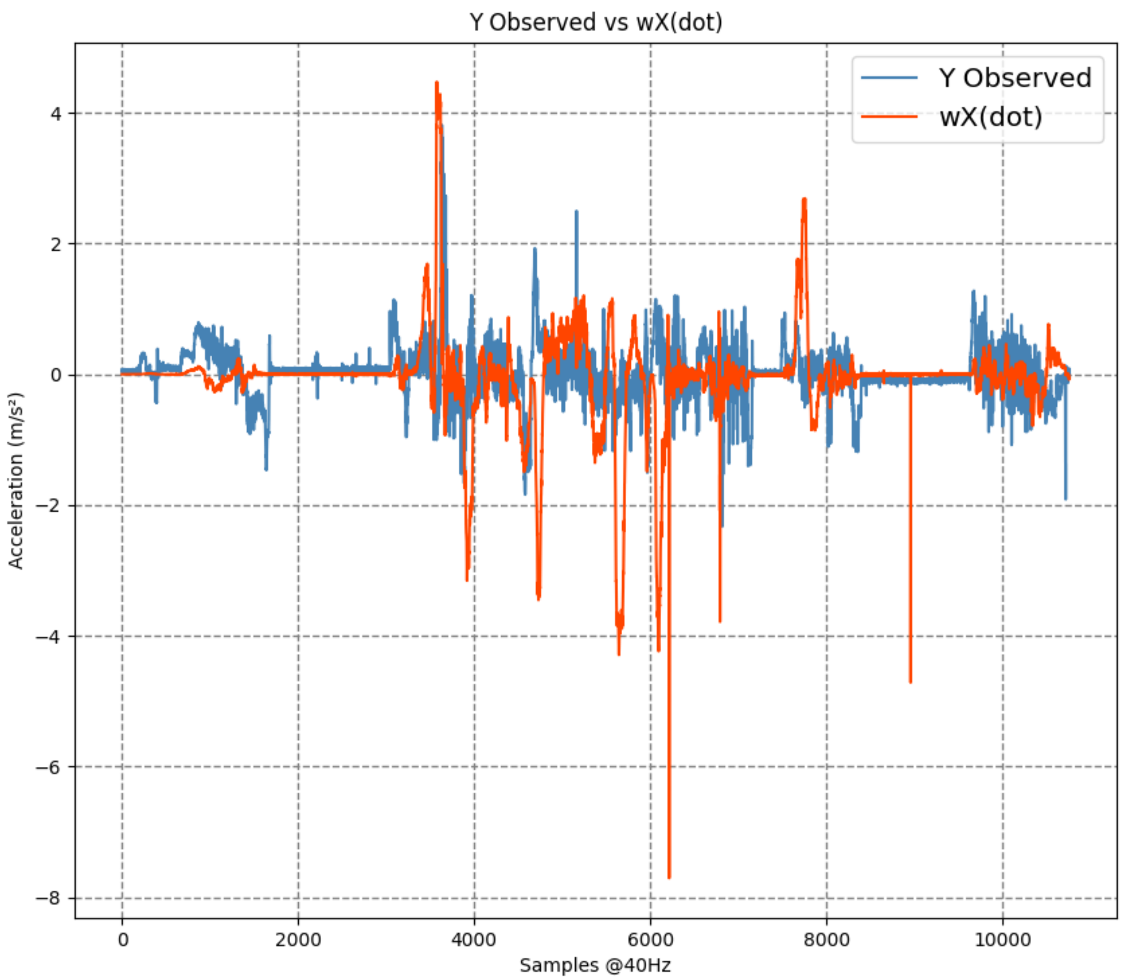


Figure 7: Comparing $\omega\dot{X}$ and $\ddot{y_{obs}}$

**Question:** How well do they agree? If there is a difference, what is it due to?
**Ans:**

**Measurement Noise:** Both the accelerometer and gyroscope have inherent noise, leading to discrepancies.

**Sensor Alignment:** Misalignment between IMU axes and the vehicle's movement introduce errors.

**External Influences:** Environmental factors, such as vehicle vibrations or uneven ground, affect the accelerometer's Y-axis readings independently of $\omega\dot{X}$.

### 3.2 Estimated Trajectory

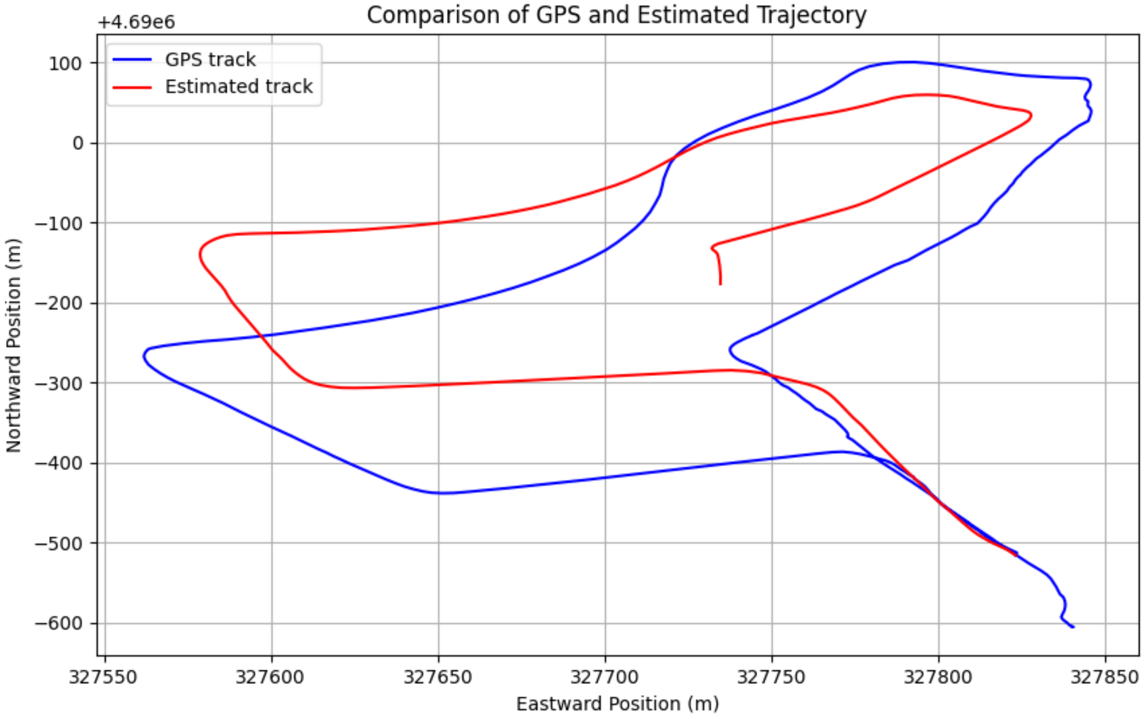


Figure 8: Estimated Trajectory vs GPS Trajectory

Scaling factor of 0.7 was used to fit both the graphs on the same plot. As seen, the starting point of the trajectories match with each other but as the plot progresses, the paths deviate due to noise and measurement errors.
**Question:** Given the specifications of the VectorNav, how long would you expect that it is able to navigate without a position fix? For what period of time did your GPS and IMU estimates of position match closely? (within 2m) Did the stated performance for dead reckoning match actual measurements? Why or why not?
**Ans:** The above plot shows that the GPS and IMU estimates of position match closely upto approximately the first 50 meters, however the effect of unaccounted slope and drift from the gyro sensor leads to higher displacement of the trajectories as we move ahead. The performance of dead reckoning did not match the actual measurements as ther could have been uncompensated biases and drift which caused faster deviation from true position. Also rapid changes in dynamics (e.g., sharp turns, rapid accelerations) increase the likelihood of error accumulation.

### 3.3 Finding $x_c$

To estimate $x_c$, we can analyze the acceleration equation provided and use the measured data from the IMU and GPS, which gives us the position R, velocity V, and acceleration at the IMU's location. The inertial sensor's displacement from the CM is $r = (x_c, 0, 0)$ so it lies only along the x-axis in the vehicle frame.
Rotation rate of the vehicle about the CM is $\omega = (0, 0, \omega)$, meaning only rotation around the z-axis.
Velocity of the IMU sensor (inertial sensor) is: $v = V + \omega r$
The acceleration of the IMU sensor is:
$\ddot{x} = \dot{v} + \omega \times v = \ddot{X} + \dot{\omega} \times r + \omega \times \dot{X} + \omega \times (\omega \times r)$
Then the range of possible $x_c$ values were found using the equation:
$x_c = \frac{(V - v)}{\omega}$
Using the above equation we come to know that $x_c$ and $\omega$ are inversely proportional, which means, If we take small values of $\omega$ it will give large value of $x_c$ and vice versa. This method may isolate the radius of rotation of the car making the turn. And, when the car was driving in a straight line at a constant speed V was set as the minimum velocity, which would return a range of more reasonable estimates.