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# Tutorial 2

**Pb 6** $P_{ij}$ : probability of receiving signal $s_j$, given that signal $s_i$ has been transmitted.

| | | Reception | | |
|---|---|---|---|---|
| | | S1 | S2 | S3 |
| Transmission | S1 | 0.8 | 0.1 | 0.1 |
| | S2 | 0.05 | 0.9 | 0.05 |
| | S3 | 0.02 | 0.08 | 0.9 |

(a) $P_i$ : probability that signal $s_i$ has been transmitted

$$P_1 = P_2 = P_3 = \frac{1}{3} \quad \because \text{equally likely.}$$

→ Let A be the event that signal $s_1$ has been transmitted.

→ Let B be the event that signal $s_2$ has been received.

$$P\left(\frac{A}{B}\right) = \frac{\cancel{P(B)}\; P(A)\,P(B/A)}{P(A)P(B/A) + P(A')P(B/A')} \quad \text{Bayes Theorem}$$

$$P\left(\frac{A}{B}\right) = \frac{P_1 \times P_{12}}{P_1 P_{12} + P_2 P_{22} + P_3 P_{32}}$$

$$\therefore P\left(\frac{A}{B}\right) = \frac{(1/3)(0.1)}{(1/3)(0.1) + (1/3)(0.9) + (1/3)(0.08)}$$

$$\therefore P\left(\frac{A}{B}\right) = 0.0926$$

(b) Let $C$ be the event that signal $S_3$ was received

$\therefore$ By ~~to~~ Theorem of Total Probability

$$P(C) = P_1 P\left(\frac{C}{1}\right) + P_2 P\left(\frac{C}{2}\right) + P_3 P\left(\frac{C}{3}\right)$$

$$\therefore P(C) = \left(\frac{1}{3}\right)(0.1) + \left(\frac{1}{3}\right)(0.05) + \left(\frac{1}{3}\right)(0.9)$$

$$\therefore P(C) = 0.35$$

Probability of receiving two consecutive $S_3$ signals is given by $= P(C) \times P(C)$

$$\therefore P(X) = (0.35)(0.35)$$

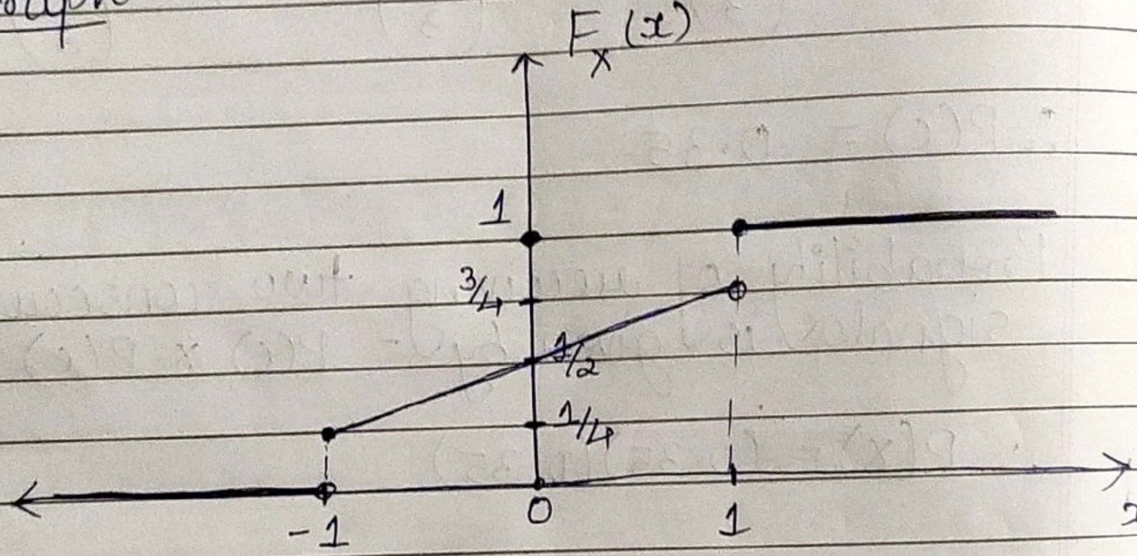
$$\therefore \underline{P(X) = 0.1225}$$

Problems based on Random Variables:

**Pb 4.** X be a r.v.

$$F_X(x) = \begin{cases} 0 & x < -1 \\ \frac{x+2}{4} & -1 \le x < 1 \\ 1 & x \geq 1 \end{cases}$$

**Graph:**




(i) $P(\{-\frac{1}{2} < X \le \frac{1}{2}\}) = F_X\left(\frac{1}{2}\right) - F_X\left(\frac{-1}{2}\right)$

$\therefore P(\{-\frac{1}{2} < X \le \frac{1}{2}\}) = \left(\frac{2+\frac{1}{2}}{4}\right) - \left(\frac{2-\frac{1}{2}}{4}\right)$

$\therefore P(\{-\frac{1}{2} < X \le \frac{1}{2}\}) = \frac{1}{4}$

(ii) $P(\{X=0\}) = F_X(0) - F_X(0^-)$

$\therefore P(\{X=0\}) = \frac{2}{4} - \frac{2}{4} = 0$

(iii) $P(\{-1 \leq X < 1\}) = F_X(1^-) - F_X(-1^-)$

$\therefore P(\{-1 \leq X < 1\}) = \frac{3}{4} - 0 = \frac{3}{4}$

(iv) $P(\{-1 < X < 1\}) = F_X(1^-) - F_X(-1)$

$\therefore P(\{-1 < X < 1\}) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

Pb 7 (b)

$$p_X(x) = \begin{cases} \frac{c\lambda^x}{x!} & x \in \{1, 2, ---\} \\ 0 & \text{otherwise} \end{cases}$$

Using property, $\sum_{x=-\infty}^{\infty} p_X(x) = 1$

$\therefore \sum_{x=1}^{\infty} \frac{c\lambda^x}{x!} = 1$

$\therefore c\left[\lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + ----\right] = 1$

$\therefore c[e^\lambda - 1] = 1 \quad \left(\because e^\lambda = 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + --\right)$

$\therefore \boxed{c = \frac{1}{e^\lambda - 1}}$

(i) $P(\{X > 3\}) = \sum_{x=4}^{\infty} c \dfrac{\lambda^x}{x!}$

$$\therefore P(\{X > 3\}) = \left(\frac{1}{e^{\lambda}-1}\right)\left[\frac{\lambda^4}{4!} + \frac{\lambda^5}{5!} + \cdots\right]$$

$$\therefore P(\{X > 3\}) = \frac{1}{e^{\lambda}-1}\left[e^{\lambda} - 1 - \lambda - \frac{\lambda^2}{2!} - \frac{\lambda^3}{3!}\right]$$

$$\therefore P(\{X > 3\}) = \frac{e^{\lambda}-1}{e^{\lambda}-1} - \left(\frac{1}{e^{\lambda}-1}\right)\left(\lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!}\right)$$

$$\therefore P(\{X > 3\}) = 1 - \frac{1}{6}\left(\frac{\lambda^3 + 3\lambda^2 + 6\lambda}{e^{\lambda}-1}\right)$$

(ii) $P(\{X \leq 3\}) = \sum_{x=1}^{3} c \dfrac{\lambda^x}{x!}$

$$\therefore P(\{X \leq 3\}) = \frac{1}{e^{\lambda}-1}\left[\lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!}\right]$$

$$\therefore P(\{X \leq 3\}) = \frac{1}{6}\left[\frac{\lambda^3 + 3\lambda^2 + 6\lambda}{e^{\lambda}-1}\right]$$

(iii) $P(\{3 < X < 4\}) = 0$

(iv) $P(\{1 < X < 2\}) = 0$

Pb 7) (c)

$$f_X(x) = \begin{cases} cxe^{-x^2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Using property, $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

$\therefore \int_0^{\infty} cxe^{-x^2}\,dx = 1$

let $x^2 = u$

$2x\,dx = du$

$\therefore \int_0^{\infty} \frac{c}{2} e^{-u}\,du = 1$

$\therefore \frac{c}{2}(-1)\left[e^{-u}\right]_0^{\infty} = 1$

$\therefore c = 2$

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(i) $P(\{X>3\}) = 2\int_3^{\infty} x e^{-x^2}\, dx$

let $x^2 = u$

$2x\,dx = du$

$\therefore P(\{X>3\}) = \frac{2}{2}\int_9^{\infty} e^{-u}\, du$

$\therefore P(\{X>3\}) = (-1)[0-e^{-9}]$

$\therefore \underline{P(\{X>3\}) = e^{-9}}$

(ii) $P(\{X\le 3\}) = 1 - P(\{X>3\})$

$\therefore \underline{P(\{X\le 3\}) = 1-e^{-9}}$

(iii) $P(\{3<X<4\}) = 2\int_3^{4} x e^{-x^2}\, dx$

let $x^2 = u$

$2x\,dx = du$

$\therefore P(\{3<X<4\}) = \frac{2}{2}\int_9^{16} e^{-u}\, du$

$\therefore \underline{P(\{3<X<4\}) = e^{-9} - e^{-16}}$

(iv) $P(\{1 < X < 2\}) = 2\int_1^2 x e^{-x^2} dx$

let $x^2 = u$

$2x\,dx = du$

$\therefore P(\{1 < X < 2\}) = \int_1^4 e^{-u} du$

$\therefore P(\{1 < X < 2\}) = e^{-1} - e^{-4}$