# Signals and Systems (CT 203)

Tutorial Sheet-10 DA-IICT, Gandhinagar.

1. Find the Discrete-time Fourier transform (DTFT) of

   (a) $x(n) = (0.2)^n u(n)$

2. Prove that the DTFT of discrete-time signal $x(n)$, is periodic, i.e.,

   $X\left(e^{j(\omega+2\pi)}\right) = X\left(e^{j\omega}\right)$ where, $X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{+\infty} x(n) e^{-j\omega n}$

3. Prove following *time* shifting and *frequency* shifting property of DTFT

$$x(n-n_o) \overset{F}{\leftrightarrow} e^{-j\omega n_o} X\left(e^{j\omega}\right) \quad \text{(Time shifting)}$$

$$e^{j\omega_o n} x(n) \overset{F}{\leftrightarrow} X\left(e^{j(\omega-\omega_o)}\right) \quad \text{(Frequency Shifting)}$$

4. Prove *Parseval's* relation for DTFT

$$\sum_{n=-\infty}^{+\infty} \left|x(n)\right|^2 = \frac{1}{2\pi}\int_{2\pi} \left|X\left(e^{j\omega}\right)\right|^2 d\omega$$

   Explain the fundamental concept conveyed by this theorem.

5. Prove the *convolution* property for DTFT, if $y(n) = x(n) * h(n)$, then

$$Y\left(e^{j\omega}\right) = X\left(e^{j\omega}\right) H\left(e^{j\omega}\right)$$

6. Prove the *multiplication* property for DTFT, if $y(n) = x(n)h(n)$, then

$$Y\left(e^{j\omega}\right) = \frac{1}{2\pi}\int_{2\pi} X\left(e^{j\theta}\right) H\left(e^{j(\omega-\theta)}\right) d\theta = \frac{1}{2\pi}\left[X\left(e^{j\omega}\right) * H\left(e^{j\omega}\right)\right]$$

7. A causal LTI system is characterized by the difference equation

$$y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = 2x(n)$$

   Determine the *impulse response,* $h(n)$, of the LTI system.