1^{st} Insem
Marks=80

Signals and Systems (CT 203)

Course Instructor: Dr. Hemant A. Patil

Duration=2 hrs **01/09/2016** **DA-IICT Gandhinagar.**

Instructions: 1. Assume suitable data or certain assumptions whenever necessary
2. All questions are compulsory and no clarifications during examination..
3. Do not write answer to a question in split manner (i.e., at different locations).

1. (a) For a linear system, if input signal $x(n)=0,\forall n$ then what will be the output? (Clearly justify your answer.
(4M)

(b) Develop mathematical model (i.e., differential equation) for series RC circuit. From you results, what is the order of differential equation?
(4M)

(c) Find the E_∞ and P_∞ for the signal $x(t)=5t, \quad t>0$. Whether $x(t)$ is power or energy signal? (4M)

(d) What is key difference between discrete-time harmonically related complex exponential signal, $\phi_k(n)=\{\exp(jk(2\pi/N)n)\}_{k\in Z}$ w.r.t its continuous-time counterpart , $\phi_k(t)=\{\exp(jk\omega_o t)\}_{k\in Z}$? What is the meaning of harmonic here?
(4M)

2. (a) Illustrate with suitable example as to *dead time* of a system contributes to inertia (of real physical system). From this, justify as to how a real physical system responds to pulse input of a width Δ and height $\frac{1}{\Delta}$?
(5M)

(b) If $x(n)=\{4,\underset{\uparrow}{3},2,-2\}$ and $h(n)=\{\underset{\uparrow}{3},2,1\}$, find $y(n)=x(n)*h(n)$ using graphical Method using LTI concept (not by any other method).
(5M)

(c) Consider a linear time-invariant (LTI) system whose response to the signal $x_1(t)$ in Fig. 1a is the signal $y_1(t)$ in Fig. 1b. Determine and sketch carefully response of the system to the input $x_2(t)$ shown in Fig. 1c.
(4M)

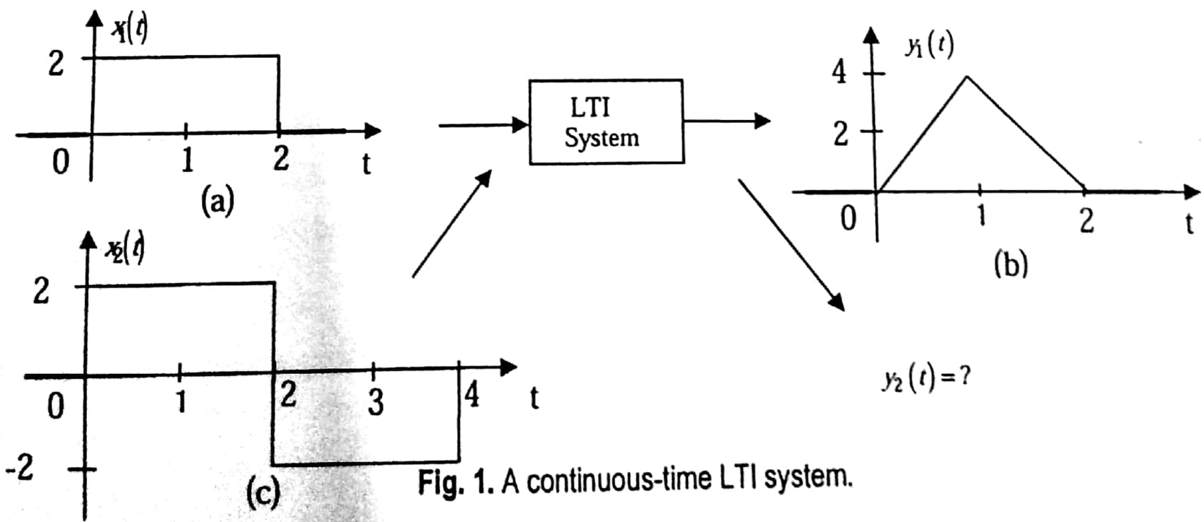

Fig. 1. A continuous-time LTI system.

(d) Using computer simulation of derivative operation , develop the discrete-time counterpart of N^{th} order linear constant coefficient differential equation to model many physical systems.
(4M)

12

3 2 1
4 12 8 4
3 9 6 3

3. (a) For the system shown in Fig. 2, justify whether the system is

- stable
- causal
- linear
- time-invariant
- memoryless

$$x(n) \longrightarrow \boxed{T\{.\}} \longrightarrow y(n) = T\{x(n)\} = e^{x(n)}$$

Fig. 2. Discrete-time system.

(6M)

(b) Whether a half-wave rectifier circuit can have an *inverse* system? Justify (3M)

(c) Evaluate the integral $\int_{-8}^{\infty} \delta(t+5)\, e^{-3t}\, dt.$ (3M)

(d) Using sifting property of discrete-time impulse signal and representation of a vector using linear combination of unit vectors, justify that we can indeed model *signals as vectors*. (3M)

4. (a) Using discrete-time *sifting* property of impulse signal and *convolution* sum, give a logical reasoning to justify that impulse response of an LTI system characterizes an LTI system *completely*. (3M)

(b) Whether the *finite-difference* system and *accumulator* system are *inverse* of each other? Clearly justify your answer. $x(n, n_2)$ $h(n) = u(n)$ (4M)
$u_n =$

(c) Justify clearly that unit impulse response of a *cascade* of two LTI systems does not depend on the order in which they are cascaded. Whether this is true nonlinear system? (4M)

(d) Whether system described by input-output relationship $y(t) = H\{x(t)\} = \mathrm{Im}\{x(t)\}$ is linear? Clearly justify your answer. (4M)

5. (a) For an LTI system if input signal, $x[n] = \begin{cases} 2 & 0 \le n \le 8 \\ 0 & otherwise \end{cases}$ and impulse response,

$h[n] = b^n u[n]$, find the output response, $y[n]$ of the system. (3M)

(b) Consider an auditorium with an echo problem. We can model the acoustics of the auditorium as an LTI system with an impulse response $h(t)$. Let $h_k(t)$ representing the gain factor on the k^{th} echo resulting from an initial acoustic excitation impulse $\delta(t)$. Find the impulse response of the auditorium. (5M)

(c) Prove that complex periodic exponential signal, $x(t) = e^{j5\omega_o t}$, has infinite signal energy but finite average power. (4M)

(d) If $x[n] = e^{j\omega n}$ is given as input to an LTI system having impulse response, $h[n]$ find the output $y[n]$, of this LTI system? (4M)

***** All the best *****

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2nd In-Sem Signals and Systems (CT 203)

Marks=85 Course Instructor: Prof. Hemant A. Patil

Duration=2 hrs Date: 18/10/2016 DA-IICT Gandhinagar

Instructions: 1. Assume suitable data or certain assumptions whenever necessary
2. All questions are compulsory and no clarifications during examination.
3. Do not write answer to a question in split manner (i.e., at different locations).

1. (a) Consider an LTI system for which the inputs and output are related by a time shift of 7, i.e., $y(t)=x(t-7)$. Find the impulse response of this system. If the input to this system is $x(t)=e^{j4t}$, find the output and associated eigenvalue of this LTI system. (3M)

(b) Consider a periodic signal, $x(t)=\sum_{n=-3}^{+3} X_n e^{jn\omega_o t}$ (with $\omega_o=2\pi$) which is given as input to an LTI system having impulse response $h(t)=e^{-t}u(t)$. Using connection between Fourier series and LTI filtering, find the expression for output $y(t)$. Whether the output, $y(t)$ is periodic? Justify your answer. (5M)

(c) State *Weierstrass-Stone Approximation Theorem* and demonstrate its use in approximating a frequency response of an LTI system in terms of polynomials. What are various advantage of working with polynomials in engineering design ? (5M)

(d) Justify clearly that sine and consine functions in frequency-domain are crude version of highpass and lowpass filters, respectively. (**Hint:** Perform eigenfunction analysis on finite difference system and two-point moving average system. Finally, compare the corresponding frequency response, i.e., $H(e^{j\omega})$). (6M)

2. (a) If a signal $x(t)$ having $X(\omega)$ (as shown in Fig. 1(b)) is given to input to system shown in Fig. 1(a), derive the expression for $Y(\omega)$, $W(\omega)$ and $F(\omega)$ and finally plot them. (6M)

(**Hint:** Modulation Theorem)

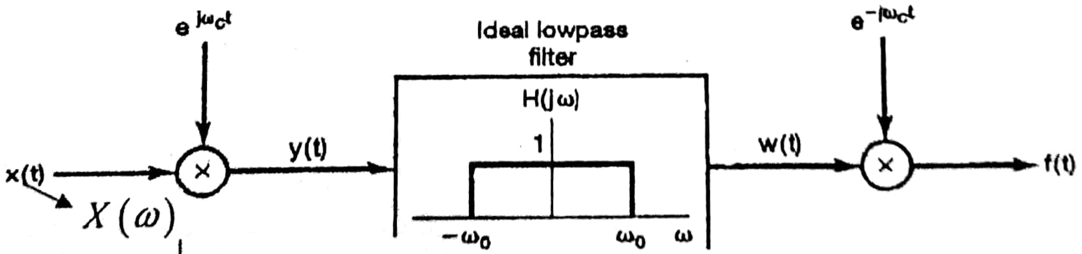


Fig. 1(a)

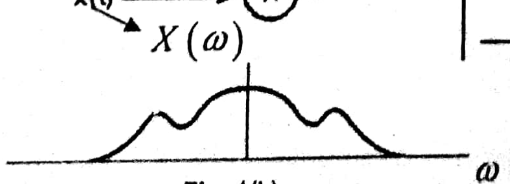


Fig. 1(b)

(b) In the context of filter design, one of the distinct features of digital signal processing as opposed to analog signal processing is to be able to achieve smaller -3 dB (half power) bandwidth. With suitable example (of analog filter, say RC lowpass) and counterexample (of digital filter, say, $y(n)-ay(n-1)=x(n)$), justify your answer. (6M)

(c) Consider an auditorium with an echo problem. We can model the acoustics of the auditorium as an LTI system with an impulse response consisting of an impulse train with the k^{th} impulse in the train corresponding to the k^{th} echo. Suppose that in this particular case the impulse response is

$$h(t) = \sum_{k=-\infty}^{+\infty} e^{-kT}\delta(t-kT)$$

where the factor e^{-kT} represents the attenuation of the k^{th} echo. In order to make a high-quality recording from the stage, the effect of the echoes must be removed by performing some processing of sounds sensed by the recording equipment. Let $G(j\omega)$ denote the frequency response of the LTI system to be used to process the sensed acoustic signal. Choose $G(j\omega)$ so that the echoes are completely removed and the resulting signal is a faithful reproduction of the original stage sounds. (6M)

(d) Find the Hilbert transform of the signal $\sin(4t)$. (4M)

3. (a) Fig. 2 shows a linear system that consists of an integrator. To this system, pair of rectangular pulses, i.e., $x(t)$ is given as input. Find the Fourier transform of output signal $y(t)$. Finally, plot the magnitude spectrum of $y(t)$. (4M)

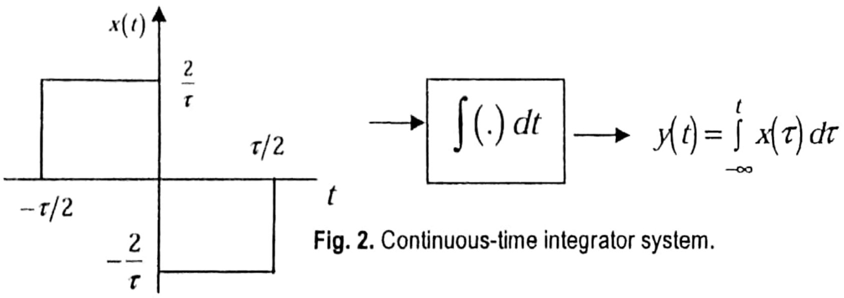


Fig. 2. Continuous-time integrator system.

(b) Find $\delta(t)*\delta(t)*\delta(t)*......*\delta(t)=?$ From this justify clearly that impulse signal can defined in terms of its behavior under convolution rather than its value at each independent variable. (4M)

(c) Verify Parseval's energy equivalence for the signal $x(t)=e^{-t}u(t)$. Determine the frequency ω_1 (rad/sec) so that the energy contributed by the spectral components of all the frequencies below ω_1 is 95 % of the total signal energy. (5M)

(d) Prove that Fourier transform of *autocorrelation* function of discrete-time real signal is its energy spectral density (ESD) (Note: This result is often called as **Weiner-Khinchin theorem**). (2M)

(e) If $f(t)$ is a real signal, then develop an algorithm or expression to detect instantaneous frequency (IF) of signal. (**Hint:** Hilbert transform). (3M)

4. (a) If $x(t)$ and $y(t)$ are two functions, then their *cross-correlation* (i.e., $R_{xy}(\tau)$) and *autocorrelation*

function (i.e., $R_{xx}(\tau)$) are defined as $R_{xy}(\tau) = \int_{t=-\infty}^{+\infty} x(t+\tau)y(t)\,dt$ and (3M)

$$R_{xx}(\tau) = \int_{t=-\infty}^{+\infty} x(t+\tau)x(t)\,dt$$

i. What is the relationship between $R_{xy}(\tau)$ and $R_{yx}(\tau)$?

ii. Suppose that $y(t) = x(t+5)$. Express $R_{xy}(\tau)$ and $R_{yy}(\tau)$ in terms of $R_{xx}(\tau)$.

(b) Let $x(t)$ and $y(t)$ be two real signals. Then the cross-correlation function of $x(t)$ and $y(t)$ is defined by $\phi_{xy}(t) = \int_{-\infty}^{+\infty} x(t+\tau)y(\tau)\,d\tau$ and the autocorrelation of $x(t)$ is defined as $\phi_{xx}(t) = \int_{-\infty}^{+\infty} x(t+\tau)x(\tau)\,d\tau$. Suppose that $x(t)$ is a pulse signal centered around origin with height 1 and width 1; which is given as input to an LTI system with a real-valued impulse response as $h(t) = e^{-at}u(t),\ a > 0$. Compute $\Phi_{xx}(\omega)$, $\Phi_{xy}(\omega)$ and $\Phi_{yy}(\omega)$. (6M)

(c) One technique for building a d.c. power supply is to take an ac signal and full wave rectify it. That is, we put the a.c. signal $x(t)$ through a system that produces $y(t) = |x(t)|$ as its output.

i. Sketch the input and output waveforms if $x(t) = \cos(t)$. What are the fundamental periods of the input and output?

ii. If $x(t) = \cos(t)$, determine the coefficients of the Fourier series for the input.

iii. What is the amplitude of the dc component of the input signal? (6M)

(d) Develop the **Poisson formula,** $\sum_{n=-\infty}^{+\infty} e^{-jnT\omega} = \frac{2\pi}{T} \sum_{n=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi}{T} n\right)$. (6M)

(e) In DTFT framework, a more general form of Parseval's theorem is given by

$$\sum_{n=-\infty}^{+\infty} x[n]\,y^*[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\,Y^*(e^{j\omega})\,d\omega.$$

Using this result, determine the numerical value of the sum

$$\sum_{n=-\infty}^{+\infty} \frac{\sin(\pi n/4)}{2\pi n} \cdot \frac{\sin(\pi n/6)}{5\pi n}$$ (5M)

******* All the best *******

Signals and Systems (CT 203)

Endsem

Course Instructor: Prof. Hemant A. Patil

Marks=135 Duration=3 hrs Date: November 29, 2016 DA-IICT, Gandhinagar.

Instructions: 1. Assume suitable data or certain assumptions whenever necessary.

2. All questions are compulsory and no clarifications during examination.

3. Do not write answer to a question in split manner (i.e., at different locations).

1. (a) Find the frequency response, $H(e^{j\omega})$, (using DTFT approach) of the linear time-invariant (LTI) system whose input and output satisfy the difference equation (2M)

$$y(n)-\frac{1}{2}y(n-1)+\frac{1}{5}y(n-2)=x(n)+2x(n-1)+x(n-2)$$

(b) Consider an LTI system with input $x(n)=(0.2)^n u(n)$ and impulse response $h(n)=(0.6)^n u(n)$. Determine the output using DTFT approach. (5M)

(c) Find the impulse response (using DTFT method) of a ~~causal LTI~~ system that is characterized by the difference equation $y(n)-\frac{3}{4}y(n-1)+\frac{1}{8}y(n-2)=2x(n)$. (5M)

(d) Find the impulse response of ideal highpass filter whose frequency-domain characteristics is shown in Fig. 1. (3M)

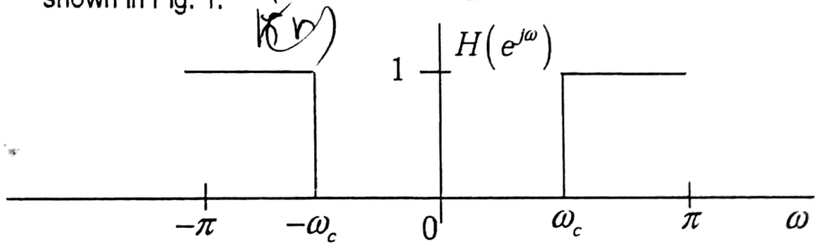


Fig.1. Frequency response of a discrete-time ideal highpass filter.

2. (a) Consider an LTI system for which the input $x(n)$ and output $y(n)$ satisfy the linear constant coefficient difference equation

$$y[n]-\frac{1}{2}y[n-1]=x[n]+\frac{1}{3}x[n-1].$$ (4M)

Find the system function $H(z)$ and corresponding impulse response of this system if ROC is $|z|>1$.

(b) Consider an discrete-time LTI system having *even* impulse response $h[n]$ (i.e., $h[n]=h[-n]$) with rational z-transform $H(z)$. (6M)

i) From the definition of the z-transform, show that $H(z)=H\left(\frac{1}{z}\right)$.

ii) From your result obtained in part i) above, show that if a pole of $H(z)$ occurs at $z=z_o$ then a pole must also occur at $z=\frac{1}{z_o}$.

iii) Similarly, show that if a zero of $H(z)$ occurs at $z=z_o$ then a zero must also occur at $z=\frac{1}{z_o}$.

(c) Consider a system function for discrete-time 2nd order resonator. (12M)

$$H(z)=\frac{1}{(1-p_1z^{-1})(1-p_2z^{-1})}=\frac{1}{1-0.9z^{-1}+0.81z^{-2}}$$

i) Sketch the pole-zero plot for this digital resonator
ii) Whether this resonator represents a stable system?
iii) At which frequency (approximate) resonance would occur?
iv) What is the -3dB bandwidth (approximate) of this resonator if the sampling frequency is 8 kHz?
v) Find and plot the impulse response of this 2nd order resonator. Whether it is IIR or FIR? Justify your answer.
vi) If the pole radius would have been greater than one, then whether still resonator represents a stable system? Justify

(d) Prove that the impulse response of 0-Hz digital resonator (resonance at 0-Hz frequency) is a cascade of two integrators. (4M)

3. (a) Find the Laplace-transform of the signal $x(t) = te^{-at}u(t)$. (4M)

(b) In the network shown in Fig. 2, capacitor is initially uncharged and inductor is unfluxed (i.e., initial current is zero) and switch *K* is closed at *t=0*. Solve for the current *i(t)*, using the Laplace transformation method. (5M)

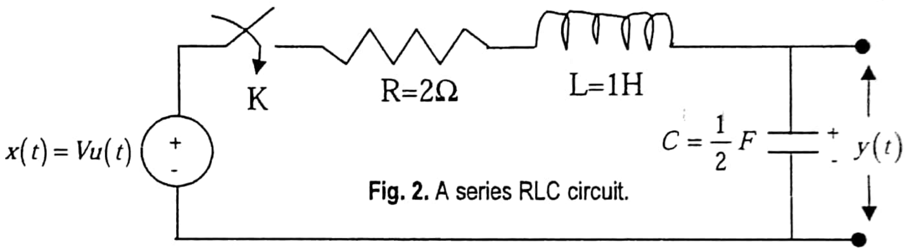


Fig. 2. A series RLC circuit.

(c) A pressure gauge that can be modeled as an LTI system has a unit step response $s(t)$ given by $(1-e^{-t}-te^{-t})u(t)$. Find the input pressure $x(t)$ to the pressure gauge which produces that output response as $(2-3e^{-t}+e^{-3t})u(t)$. (**Hint:** First find $h(t)$ from $s(t)$ and then use convolution theorem in s-domain). (5M)

(d) Consider a class of continuous-time second order system whose system function is given by

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad (10M)$$

i) Plot the pole-zero of this system for a1) $\xi > 1$, b1) $\xi \gg 1$, c1) $0 < \xi < 1$.
ii) If $\xi < 0$ whether given $H(s)$ represents *stable* second order system? Justify
iii) Find the expression for impulse response of this system with $0 < \zeta < 1$.
iv) Using expression for Bode plot, prove that $|H(j\omega)|$ of this system has a maximum value at $\omega_{max} = \omega_n\sqrt{1-2\zeta^2}$ and the value at this point is $|H(j\omega_{max})| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$.
v) Which parameter in the given system function controls the sharpness and width (i.e., quality of resonance) of the spectral peak in associated frequency response $H(j\omega)$?

4. (a) Find $|H(e^{j\omega})|$, $\measuredangle H(e^{j\omega})$ and corresponding group delay function for following LTI filter. (7M)

$$H\left(e^{j\omega}\right)=e^{-j\omega}\frac{1-\frac{1}{4}e^{j\omega}}{1-\frac{1}{4}e^{-j\omega}}.$$

(b) Fig. 3 shows the plot of group delay function for an allpass filter and its impulse response. Relate the occurrence of different frequency components in impulse response. Clearly justify your answer. (3M)

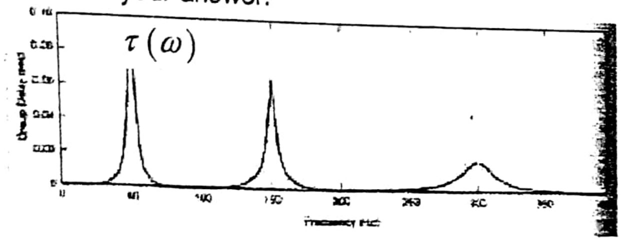


Fig. 3. (a) Group delay function for allpass filter.

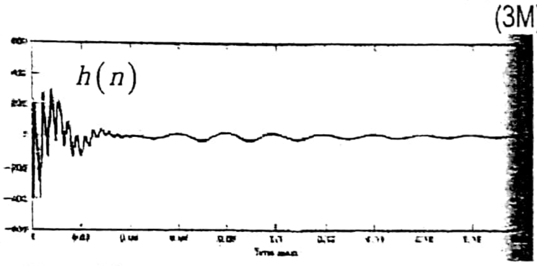


Fig. 3. (b) corresponding impulse response.

(c) Find the step response and then time constant of first order RC filter. (5M)

(d) Differential describing dynamics of automobile suspension system is given by

$$M\frac{d^2y(t)}{dt^2}+b\frac{dy(t)}{dt}+ky(t)=kx(t)+b\frac{dx(t)}{dt}.$$ (5M)

where M is mass of the chassis and k and b are the spring and *shock absorber* constants. Find the Bode plot (magnitude) of this system. From this Bode plot, justify the location of cutoff frequency of this filter to serve the purpose of shock absorber in order to have smooth ride?

5. (a) Draw standard three-step sampling setup and hence, discuss practical limitations of Shannon's sampling theory? (3M)

(b) Find the frequency response of first order hold (FOH) used in interpolation? (3M)

(c) Let $x(t)=\cos(\omega_o t)$ be sampling with sampling frequency ω_s. With suitable plots in frequency-domain, demonstrate the effect of oversampling ($\omega_s > 2\omega_o$) and undersampling ($\omega_s < 2\omega_o$). (4M)

(d) Justify stroboscopic effect using phase reversal phenomenon (due to undersampling) (3M)

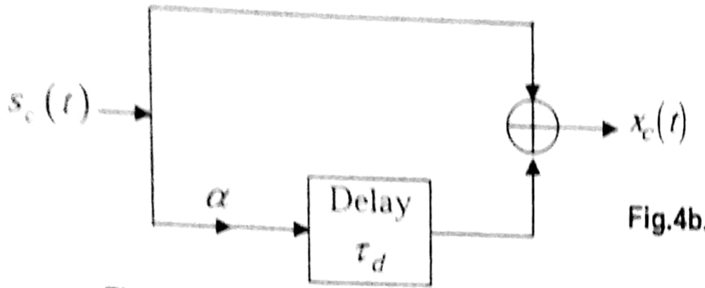
(e) In the bandlimited sinc interpolation, establish the fact that integer translation of sinc functions, i.e., $\{h(t-nT)\}_{n\in Z}$ is mutually orthogonal to each other. (3M)

(f) PCM framework, find the bit rate to transmit 5 seconds of speech over landline telephone. (3M)

6. (a) A simple model of multipath communication channel is indicated in Fig. 4. Assume that $s_c(t)$ is bandlimited such that $S_c(j\omega)=0$ for $|\omega|\geq \pi/T$ and that $x_c(t)$ is sampled with a sampling period T to obtain the sequence $x(n)=x_c(nT)$ (9M)

- Determine the Fourier transform of $x_c(t)$ and the Fourier transform of $x(n)$ in terms of $S_c(j\omega)$

- We want to simulate the multipath system with a discrete-time system by choosing $H(e^{j\omega})$ in Fig. 4b so that the output $r(n) = x_c(nT)$ when the input is $s(n) = s_c(nT)$ in Fig. 4b. Determine $H(e^{j\omega})$ in terms of T and τ_d.
- Determine the impulse response $h(n)$ in Fig.4b when (i) $\tau_d = T$ and (ii) $\tau_d = T/2$



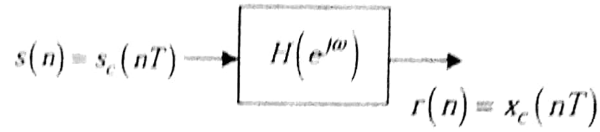


Fig.4b. Discrete-time simulation for multipath system.

Fig.4a. Model for multipath communication channel system.

(b) In long-distance telephone communication, an echo is sometimes encountered due to the transmitted signal being reflected at the receiver, sent back down the line, reflected again at the transmitter, and returned to the receiver. The impulse response for a system which models this effect is shown in Fig. 5, where we have assumed that only one echo is received. The parameter T corresponds to the one-way travel time along the communication channel, and the parameter α represents the attenuation in amplitude between transmitter and receiver. (6M)

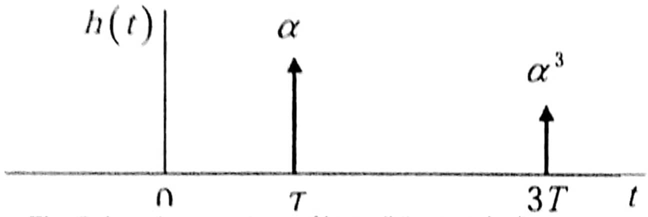


Fig. 5. Impulse response of long-distance telephone communication channel.

- Determine the system function $H(s)$ and associated ROC for the system.
- Whether $H(s)$ is ratio of polynomials?
- Whether $H(s)$ have both zeros and poles? If yes, find them.

(c) Justify with suitable example that our ability to represent data in digital memory (and hence, ability to delay data) helps in achieving the frequency selectivity in design of digital filter. (5M)

(d) In many practical situations we are faced with the problem of recovering a signal that has been blurred by a *convolution* process as shown in Fig.6 (a) with corresponding blurring impulse response in Fig.6 (b). Suggest some signal processing of blurred signal $y(n)$ to recover $x(n)$. Find the frequency response of your proposed deblurring system. (6M)

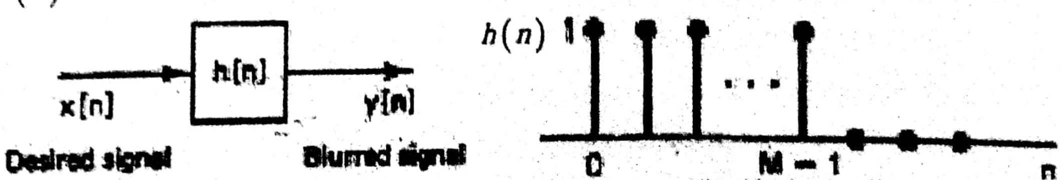


Fig.6. (a) LTI model for blurring mechanism (b) corresponding blurring impulse response.

(e) Find the frequency response of downsampling by 2 system? Justify your answer by time-scaling property of DTFT and CTFT. (5M)