Computational Numerical Methods

CS 374

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Approximate root

$x = \underline{\underline{1.1338}}$

iteration	a_n	b_n	c	$b-c$	$f(c)$
1	1	2	1.5	0.5	8.8906
2	1	1.5	1.25	0.25	1.5647
3	1	1.25	1.125	0.125	-0.0977
4	1.125	1.25	1.1875	0.0625	0.6167
5	1.125	1.1875	1.1562	0.0312	0.2337
9	1.1328	1.1367	1.1348	0.002	0.0004
10	1.1328	1.1348	$\underline{1.1338}$	0.00098	-0.0096

Convergence & Error ~~or~~ estimation of ~~iteration~~ Bisection method

<u>Hypothesis:</u> let $f:[a_0, b_0] \to R$. be continuous function. such that the number $f(a_0)$ & $f(b_0)$ is of opposite sign..

<u>conclusion</u> There exist an $r \in [a_0, b_0]$ s.t $f(r) = 0$. & the iterative sequence of approximate solⁿ $\{x_n\}$ converges. to r.

\# For each $n = 0, 1, 2, \ldots$ we have the error.

$$|x_{n+1} - r| = \left(\frac{1}{2}\right)^{n+1} (b_0 - a_0)$$

$$b_n - a_n = \frac{1}{2}\left[b_{n-1} - a_{n-1}\right] = \frac{1}{2^2}\left[b_{n-2} - a_{n-2}\right]$$

$$= \frac{1}{2^n}\left(b_0 - a_0\right)$$

$$\lim_{n \to \infty} b_n - a_n = 0$$

$a_n \longleftarrow | \longrightarrow b_n$

c_n.

$$\Rightarrow \lim_{n \to \infty} b_n = \lim_{n \to \infty} a_n .$$

x_{n+1} is the mid point of the interval $[a_n, b_n]$

$$\Rightarrow a_n \le x_{n+1} \le b_n .$$

Using sandwich theorem

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = \lim_{n\to\infty} x_{n+1} = \gamma .$$

Error

Let a_n, b_n & c_n are the nth computed values ~~of the~~ a_0, b_0 & c.

$$b_{n+1} - a_{n+1} = \frac{1}{2}(b_n - a_n) \qquad n \geq 1$$

$$\cancel{b_{n+1}} \quad b_n - a_n = \boxed{\frac{1}{2^n}(b_0 - a_0)} \qquad \cancel{\frac{1}{2^{n-1}}(b_0 - a_0)}$$

~~ob~~ where ~~b_0, a_0 denote~~ $b_0 - a_0$ denote the length of the original interval.

Since the root r is either in the interval $[a_n, c_n]$ or, $[c_n, b_n]$ we have

$$|r - c_n| \le c_n - a_n = b_n - c_n.$$

$$= \frac{1}{2}(b_n - a_n)$$

$$\boxed{|r - c_n| \le \frac{1}{2^{n+1}}(b_0 - a_0)}$$