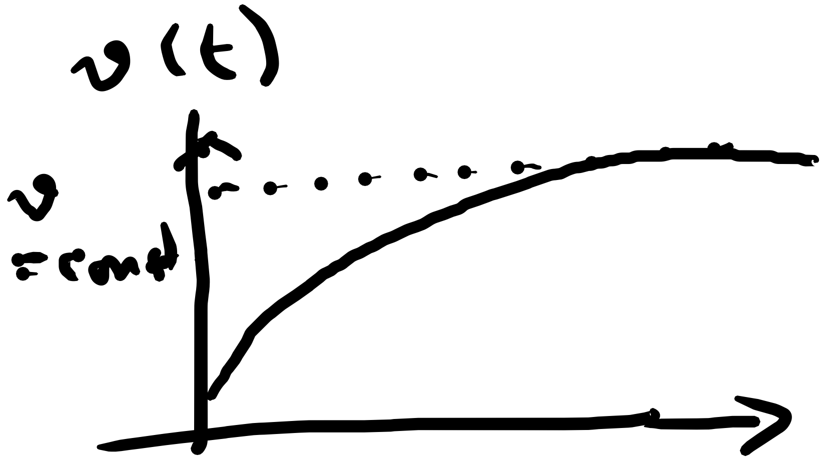
LECTURE 7

▨ Recap.

— Particle moving through a viscous medium.

- EOM: $m \dfrac{dv}{dt} = mg - \underbrace{\beta v}_{F_{drag}}$.



PROB:- (MOTION OF A BICYCLE THROUGH RESISTIVE MEDIUM).

- Observation:- Earlier, a point mass had been considered, now we consider an object with non-zero, finite dimensions.
  - Bicycle + rider experience drag force due to wind-resistance. F_{drag} increases with velocity.
  - There is no external force on the bicycle.

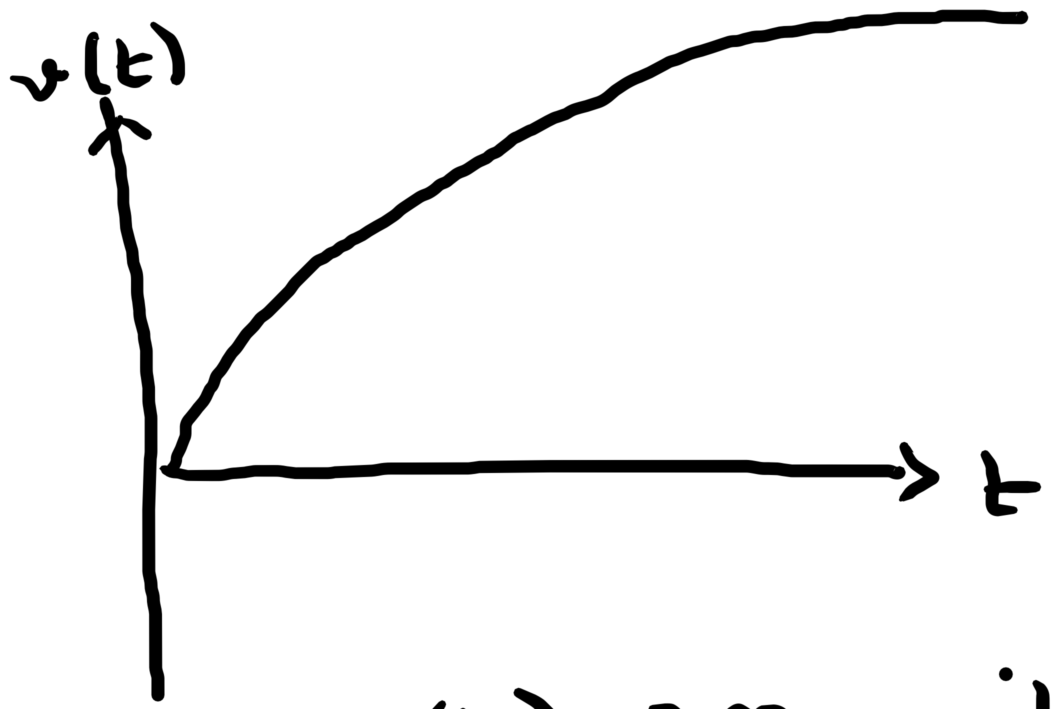
Assumption:- Let $\underset{\sim}{P}$ = power produced by the rider = const.

$$E = \frac{1}{2} m v^2$$

$$P = \frac{dE}{dt} = \frac{1}{2} m \frac{d}{dt}(v^2)$$

$$\Rightarrow P = m v \frac{dv}{dt}$$

$$\Rightarrow \boxed{\frac{dv}{dt} = \frac{P}{mv}} \Rightarrow \text{Eqn. of motion.}$$



$v(t) \to \infty$ as $t \to \infty$

Integrating,

$$\int v \, dv = \int \frac{P}{m} dt$$

$$\Rightarrow v^2 - v_0^2 = \frac{2P}{m} t$$

$$\Rightarrow v^2 = v_0^2 + (2P/m) t.$$

Discretise using Euler scheme,

$$\frac{v_{n+1} - v_n}{\Delta t} = \frac{P}{m v_n}$$

$$\Rightarrow \quad v_{n+1} = v_n + \frac{P}{m v_n} \Delta t .$$

Consider $n = 0$.

$$v_1 = v_0 + \frac{P}{m v_0} \Delta t .$$

Initial velocity $v(t=0) = v_0$ cannot be set equal to zero, other the 2nd term in the above eqn. blows up.

This is unphysical.

— Incorporate some drag force into the force.

$$F_{drag} \equiv f(v)$$

To construct an approximation for $f(v)$ for low values of v, expand in a Taylor series.

$$F_{drag} = f(v) = f(0) + f'(0)\,v + \tfrac{1}{2}f''(0)\,v^2 + \cdots$$

$F_{drag} = 0$ when $v = 0$. So, from physical consideration

$$f(0) = 0.$$

$f'(0),\ f''(0) < 0$, since F_{drag} opposes motion.

$$|f'(0)| = B_1 \qquad \tfrac{1}{2}|f''(0)| = B_2$$

$- F_{drag} = -B_1 v - B_2 v^2$, where, $B_1, B_2 > 0$.

Resultant EOM is,

$$m\frac{dv}{dt} = \frac{P}{v} + F_{drag}$$

$$\Rightarrow \frac{dv}{dt} = \frac{P}{mv} + (-B_1 v - B_2 v^2)$$

☐ Qualitative behaviours of solution :-

- Consider a situation in which $B_2 v^2$ is neglected.

  Effective model becomes,

$$\frac{dv}{dt} = \frac{P}{mv} - B_1 v$$

At low v, P/mv dominates in the R.H.S.

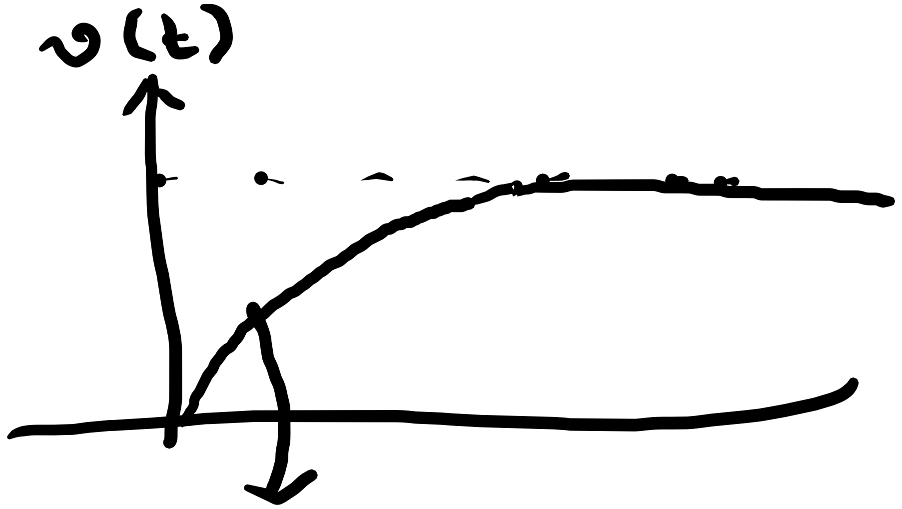
$$\frac{dv}{dt} \approx \frac{P}{mv} \Rightarrow \quad v(t) = \sqrt{v_0^2 + 2Pt/m}$$

For higher v, $-B_1 v$ dominates in the R.H.S

$$\frac{dv}{dt} \approx -B_1 v$$

$$\Rightarrow \quad v(t) \sim e^{-B_1 t} + \text{const.}$$

$$v_1(t \to \infty) \Rightarrow \text{const.}$$



can be approximated by a parabola.

In other words,

$$v(t) = \sqrt{v_0^2 + 2Pt/m}.$$

— Now, recall, $F_{drag} = -B_1 v - B_2 v^2$

— Take $\underline{-B_2 v^2}$. Can the form of B_2 be guessed?

<u>Option 1</u>: Dimensional analysis: $B_2 \equiv B_2(\rho, A)$,

$\rho \equiv$ density of the medium

A : Surface area of the bicycle + rider.

$$[F_{drag}] = \rho^{\alpha} A^{\beta} (v^2)^{\gamma}$$

$$\Rightarrow \quad MLT^{-2} = (ML^{-3})^{\alpha} (L^2)^{\beta} (L^2 T^{-2})^{\gamma}$$

$$= M^{\alpha} L^{2\beta - 3\alpha + 2\gamma} T^{-2\gamma}$$

- Comparing both sides,

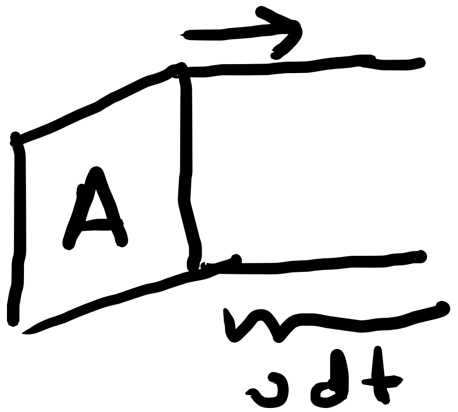
$$y = A x^{\alpha}$$

$$\left.\begin{aligned} \alpha &= 1 \\ 2\beta + 2\gamma - 3\alpha &= 1 \\ 2\gamma &= 2 \end{aligned}\right\} \quad \alpha = \beta = \gamma = 1$$

$$F_{drag}^{(?)} \sim \underline{C \rho A v^2}$$

Option 2:- "Exact" physical considerations.

As cyclist moves, air is pushed out of the way.
Work is being done against this drag force.



Consider infinitesimal interval dt.
Cyclist covers distance $v\,dt$.

- Assume elastic collision. Mass of medium contained in volume will then also be pushed back with velocity v.

$E_{air} \sim \frac{1}{2} m_{air} v^2 \rightarrow$ as a result of work done against drag force.

$$F_{drag}\, v\, dt = E_{air} = \frac{1}{2} \rho A v\, dt\, v^2$$

$$\Rightarrow F_{drag} = -\frac{1}{2} \rho A v^2$$

Mass of air displaced in time dt is ,

$$m_{air} \sim \rho A v \, dt \,.$$