Computational Numerical Methods

CS 374

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Root finding

$ax + b = c$.

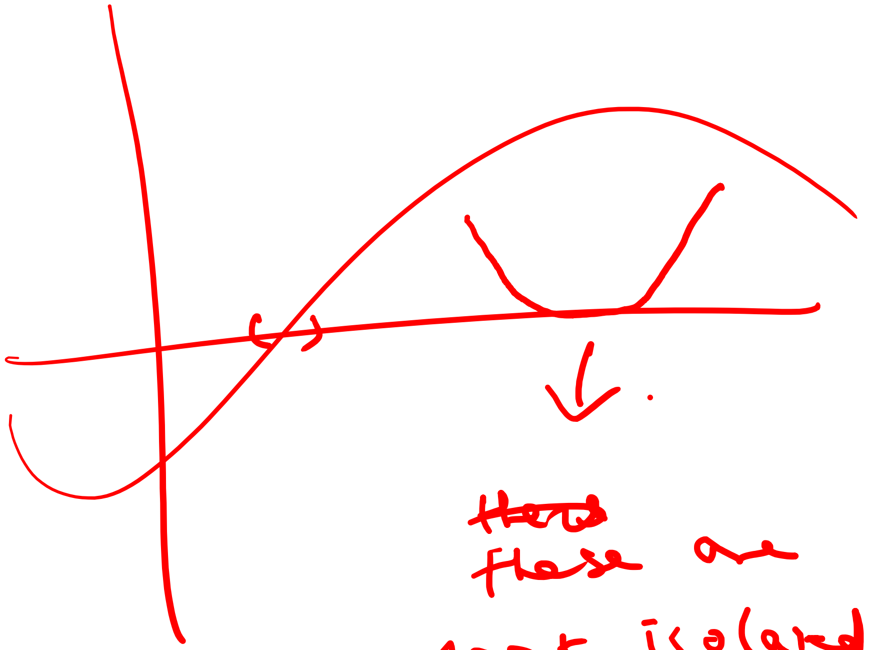
Nonlinear equations

$$f(x) = x^6 - x - 1 = 0.$$

$f(x) = 0$ find the roots

where:

- $f : [a, b] \to \mathbb{R}$.
- $f \in C^1[a, b]$.
- roots are isolated.



~~Here~~ these are not isolated roots

Root of a eqn

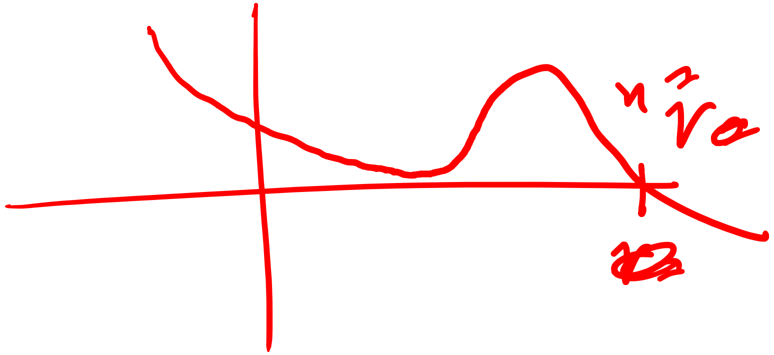
$r \in \mathbb{R}$, $f(r) = 0 \Rightarrow r$ is a root.

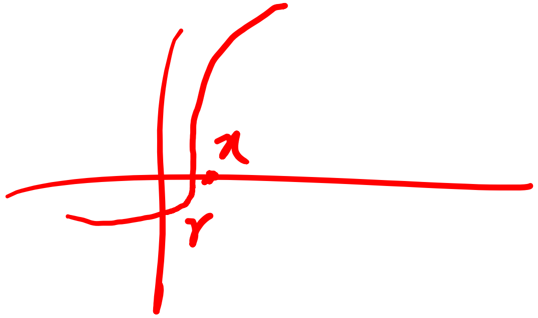
Approximate root

Let r is root.

- $|x - r|$ is very small
- $f(x) \approx 0$

$\Rightarrow x$ is approximate root.





only $|x - r|$ is not sufficient.

Iterative method.

$$x_{n+1} = f(x_{n}, x_{n-1}, \dots x_1)$$

Starting step

initial guess / values.

$$x_n = x_{n-1} + x_{n-2}.$$

1, 1, 2, 3, 5, 8 . . . . .

Improvement step.

~~Rate~~ Iteration process to reduce the error

Two kinds of iterative method.

① Closed domain method / Bracketing methods.

- Bisection method. →
- Regular Falsi method.

Adv → it converges.

dis → need some idea about original solⁿ.

② Open domain / Non bracketing method.

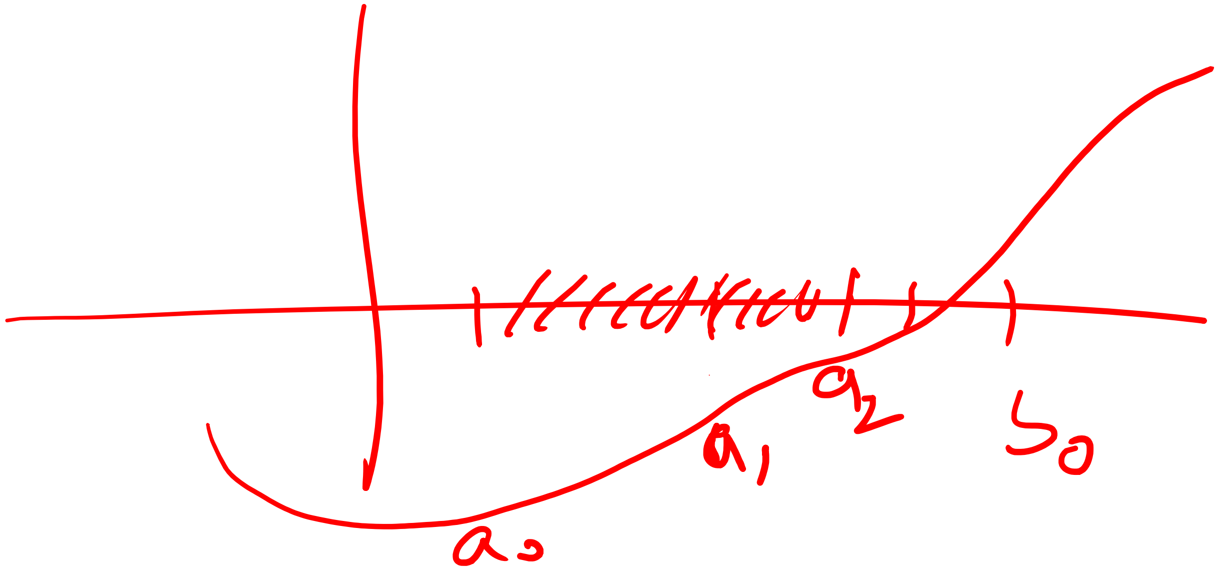
- Secant method.
- Newton raphson methods.
- Fixed point iteration.

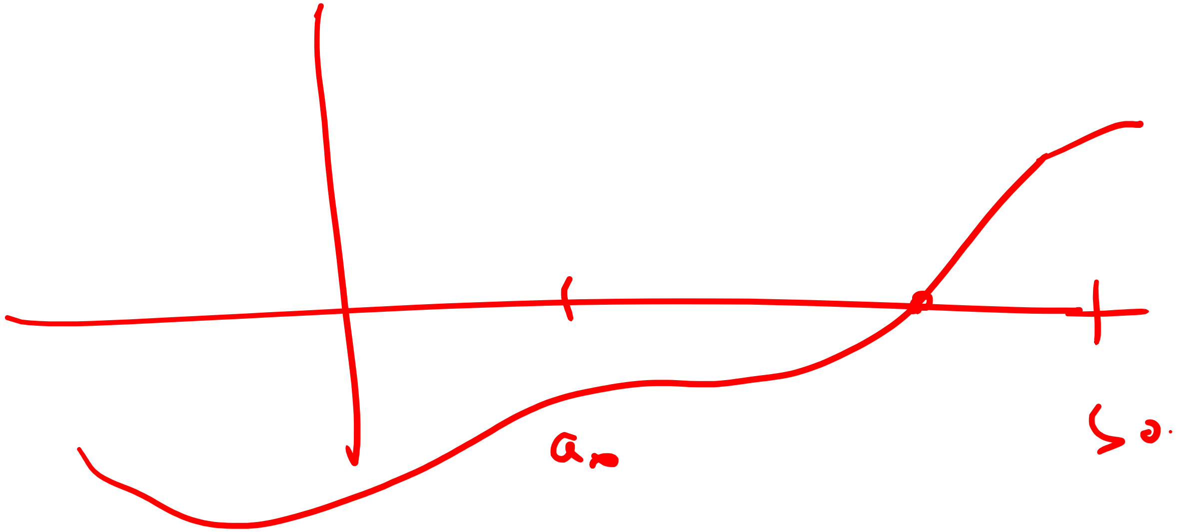
Adv: no need for idea of original solⁿ

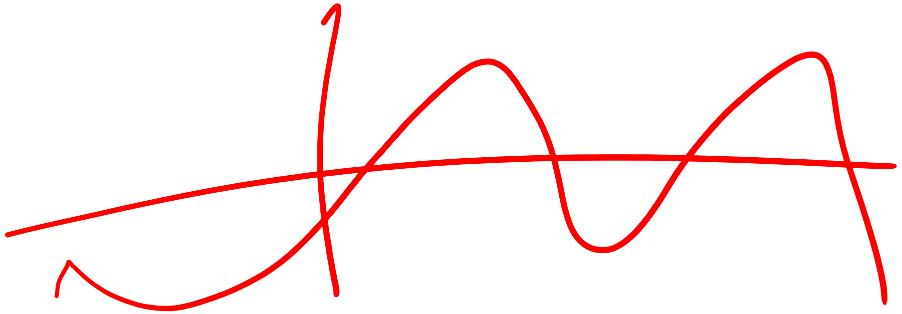
dis.adv: may non converge.

Method of Bisection

- $f(x)$ exist in $[a_0, b_0]$
- $f(x)$ is ~~cont~~ $f : [a_0, b_0] \to \mathbb{R}$. is continuous
- $f(a_0)\, f(b_0) < 0$







Algorithm

Step 1

for $n = 0, 1, 2 \ldots$

$$x_{n+1} = \frac{a_n + b_n}{2}.$$

which is the mid point of the interval $[a_n, b_n]$

Step 2

One of the following of the two cases hold.

- x_{n+1} solves the non linear eqn $\Rightarrow f(x_{n+1}) = 0$
- Either $f(a_n) f(x_{n+1}) < 0$ or $f(b_n) \cdot f(x_{n+1}) < 0$

define

$$[a_{n+1}, b_{n+1}] = \begin{cases} [a_n, x_{n+1}] & \text{if } f(a_n)\cdot f(x_{n+1}) \leq 0 \\ [x_{n+1}, b_n] & \text{if } f(b_n) f(x_{n+1}) < 0 \end{cases}$$

Step 3 Stop the iteration if one of the following happens.

- Case 1 in step 2 to hold $=0$ ~~Cas~~ Consider x_{n+1} as the root
- $(b_{n+1} - a_{n+1})$ is sufficiently small then x_{n+2} ~~as~~ can be considered as the root.

$f(x) = x^6 - x - 1$ Find a root accurate to within $\varepsilon = 0.001$

Step:

$f(1) = -1$

$f(2) = 61$

$\therefore f(1) \cdot f(2) < 0$

$\Rightarrow \exists$ a root b/w $[1, 2]$.

$c = \frac{1+2}{2} = 1.5$

approximate root

$x = \underline{\underline{1.1338}}$

iteration	a_n	b_n	c	b−c	f(c)
1	1	2	1.5	0.5	8.8906
2	1	1.5	1.25	0.25	1.5647
3	1	1.25	1.125	0.125	−0.0977
4	1.125	1.25	1.1875	0.0625	0.6167
5	1.125	1.1875	1.1562	0.0312	0.2337
9	1.1328	1.1367	1.1348	0.002	0.0004
10	1.1328	1.1348	$\underline{1.1338}$	0.00098	−0. 0.096