# LECTURE 32

- de Broglie hypothesis :- wave particle duality.

$$p = \frac{h}{\lambda} \qquad E = \frac{hc}{\lambda} = hf. \qquad h = \text{Planck's const}.$$

$$\lambda = \frac{c}{f}.$$

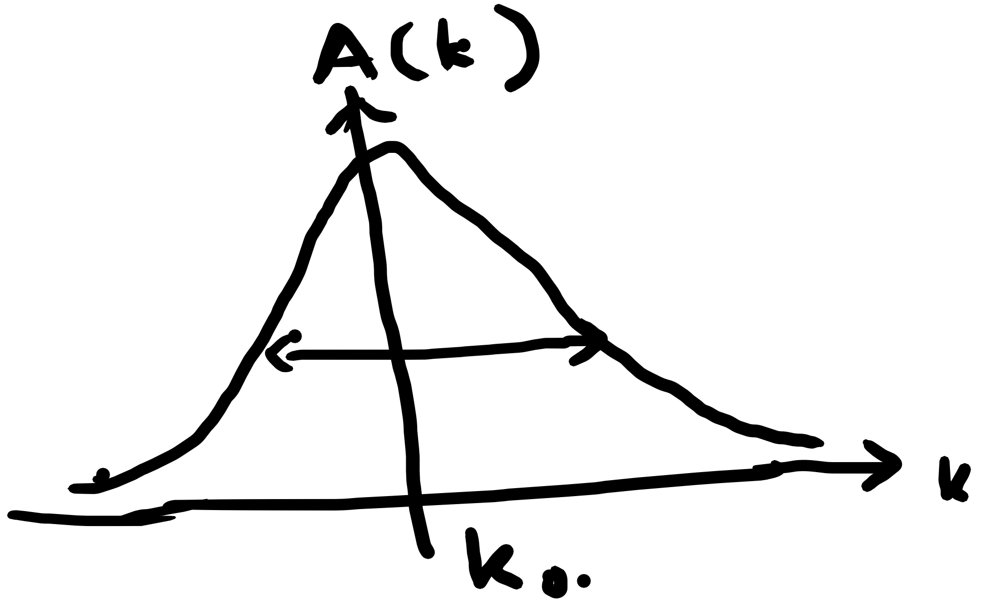
$$\Rightarrow p = \hbar k \qquad E = \hbar\omega.$$

— Wave delocalized. $\quad e^{i(kx - \omega t)}$.

— Way to localize :-

$$\psi(x) = \int dk\, A(k) \cos kx.$$
$$= \int dk\, A(k)\, e^{ikx}.$$

— $A(k) = A_0 \, e^{-(k-k_0)^2/2(\Delta k)^2}$

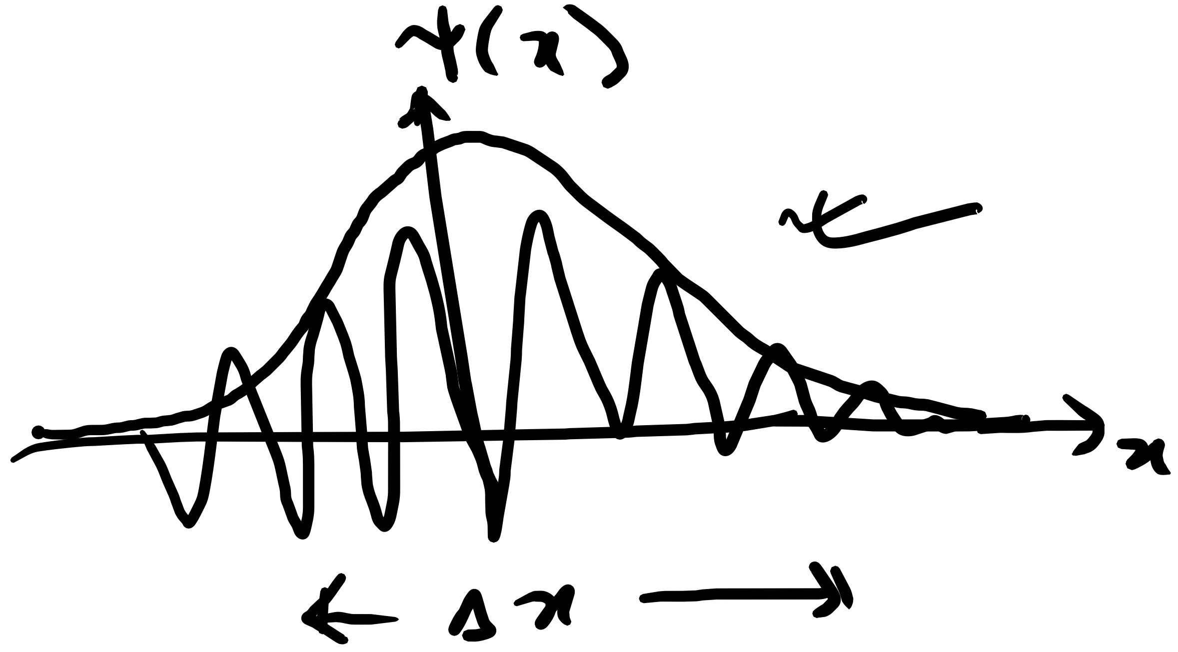


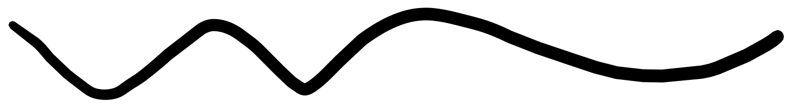

$$\psi(x) = A_0 \int_{-\infty}^{\infty} dk \; e^{-(k-k_0)^2/2(\Delta k)^2} \cos kx$$

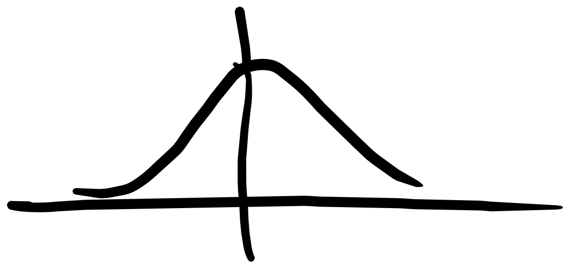
$$= A_0 \, \Delta k \; e^{-(\Delta k x)^2/2} \cos(k_0 x) .$$

$$e^{-(\Delta k x)^2/2} .$$

$$I = \int_{-\infty}^{\infty} dx \, e^{-\alpha x^2} \propto \sqrt{\frac{1}{\alpha}}$$





$\psi \rightarrow$ wave function.

- Intuitively, $\Delta x$ = "uncertainty" in position
  $\Delta k$ = " " " in $k$.

$\psi(x)$

$p = \hbar k$.

$$\Delta x \propto \frac{1}{\Delta k}. \quad \text{OR} \quad \Delta x \propto \frac{1}{\Delta p}.$$

## ☑ HEISENBERG'S UNCERTAINTY RELATIONS.

$$\Delta x \, \Delta p \geq \frac{\hbar}{2} \qquad \Delta E \, \Delta t \geq \frac{\hbar}{2}$$

- If $x$ is completely specified, $(\Delta x = 0)$, $\Delta p \to \infty$, i.e., momentum is completely unknown and vice versa.

## II GENERALIZE TO WAVE-PULSES / PACKETS IN MOTION.

$$\psi(x) = A\left[\cos k_1 x + \cos k_2 x\right]$$

$$\psi(x,t) = A\left[\cos(k_1 x - \omega_1 t) + \cos(k_2 x - \omega_2 t)\right]$$

$$= 2A \cos \frac{k_1 x - \omega_1 t + k_2 x - \omega_2 t}{2} \cos \frac{k_1 x - \omega_1 t - k_2 x + \omega_2 t}{2}$$

$$= 2A \cos\left(\frac{k_1 + k_2}{2} x - \frac{\omega_1 + \omega_2}{2} t\right) \cos\left(\frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t\right).$$

$$c = \lambda f = \left(\frac{2\pi}{k}\right)\left(\frac{\omega}{2\pi}\right) = \frac{\omega}{k} \Rightarrow \boxed{c = \frac{\omega}{k}} \Rightarrow \text{phase velocity.}$$

$$\cos(kx - \omega t).$$

- $v_{group} := \dfrac{\Delta\omega}{\Delta k}$.
- Generalizing to infinitesimal limit:- $v_{group} = \dfrac{d\omega}{dk}$.
- Group vel: of de Broglie waves:

$p = \dfrac{h}{\lambda}$ $\quad E = hf = \hbar\omega$. $\quad$ For free particle $E = p^2/(2m)$

$= \hbar k$.

$$v_g = \frac{d(E/\hbar)}{d(p/\hbar)} = \frac{dE}{dp} = \frac{p}{m} = v_{particle}.$$

Ex: $v_{phase} = \sqrt{\frac{g\lambda}{2\pi}}$ , $v_g = ?$

$v_{phase} = \frac{\omega}{k}$ (by defn).

$k = \frac{2\pi}{\lambda}$.

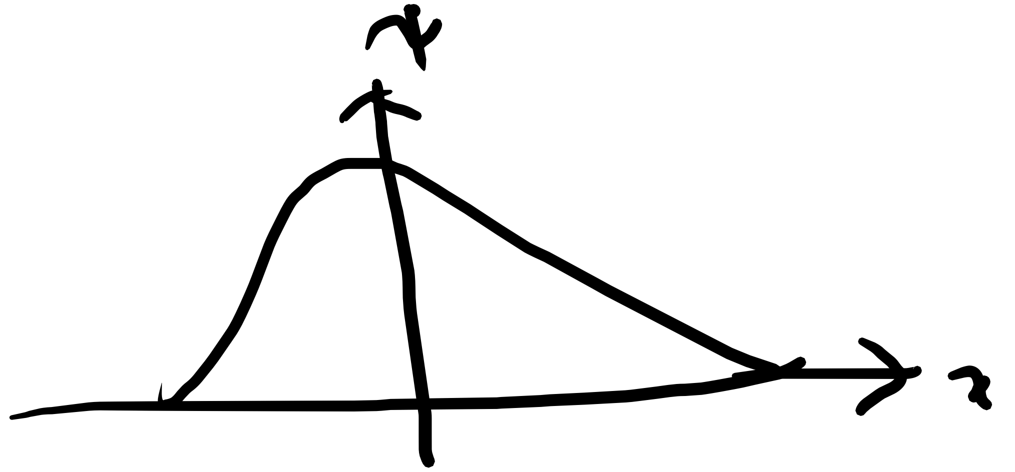
$\Rightarrow \frac{\omega}{k} = \sqrt{\frac{g\lambda}{2\pi}} = \sqrt{\frac{g}{k}}$

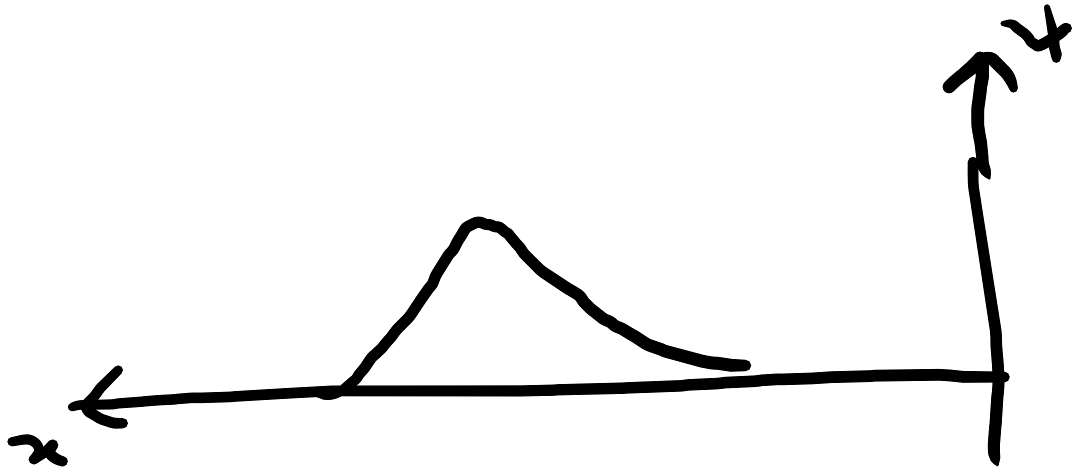
$\Rightarrow \omega = \sqrt{gk}$.

$v_g = \frac{d\omega}{dk} =$

VI Summary: — Possible to localize a wave to a region, but not to a specific point.

— Uncertainty related to a wave packet.



# BORN INTERPRETATION

Probability of finding a particle associated with $\psi(x,t)$ in the region between $x$ and $x+dx$ is,

$$P(x,t)\,dx = |\psi(x,t)|^2\,dx.$$

- $\psi(x,t)$ is not physically significant on its own.

$$\psi(x,t) = \int_{-\infty}^{+\infty} dk \, A(k) \, e^{i(kx-\omega t)} .$$

$$E = \hbar\omega , \qquad p = \hbar k .$$

$$\psi(x,t) = \int_{-\infty}^{+\infty} dp \, A(p) \, e^{i(px-Et)/\hbar} .$$

$$= \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} dp \, \phi(p) \, e^{i(px-Et)/\hbar}$$

(change notation for convenience).

$$E = \frac{p^2}{2m} + V(x) .$$

$$\psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} dp \, \phi(p) \, e^{i(px - \frac{p^2}{2m}t)/\hbar} \, e^{-iV(x)}$$