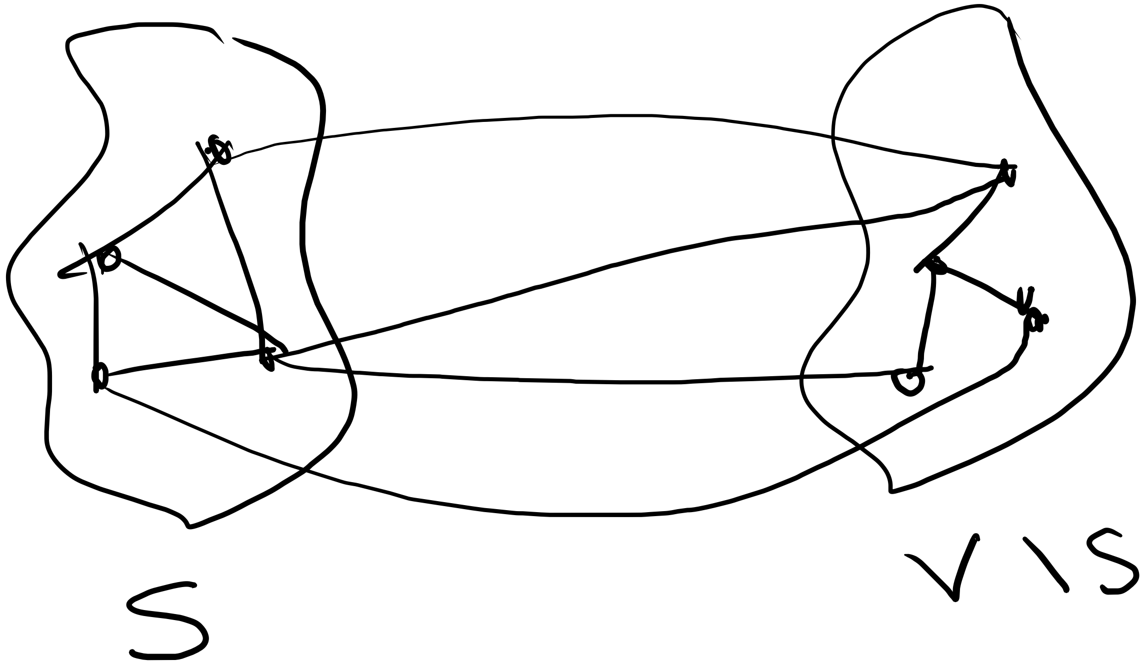
Correctness of MST algorithms

- There are many MST finding algorithms
- All of them rely on some basic properties of MST
    - cut property
    - cycle ,,

- Assume that edge costs are distinct.

cut: partition of vertex set of a graph in S and $V \setminus S$

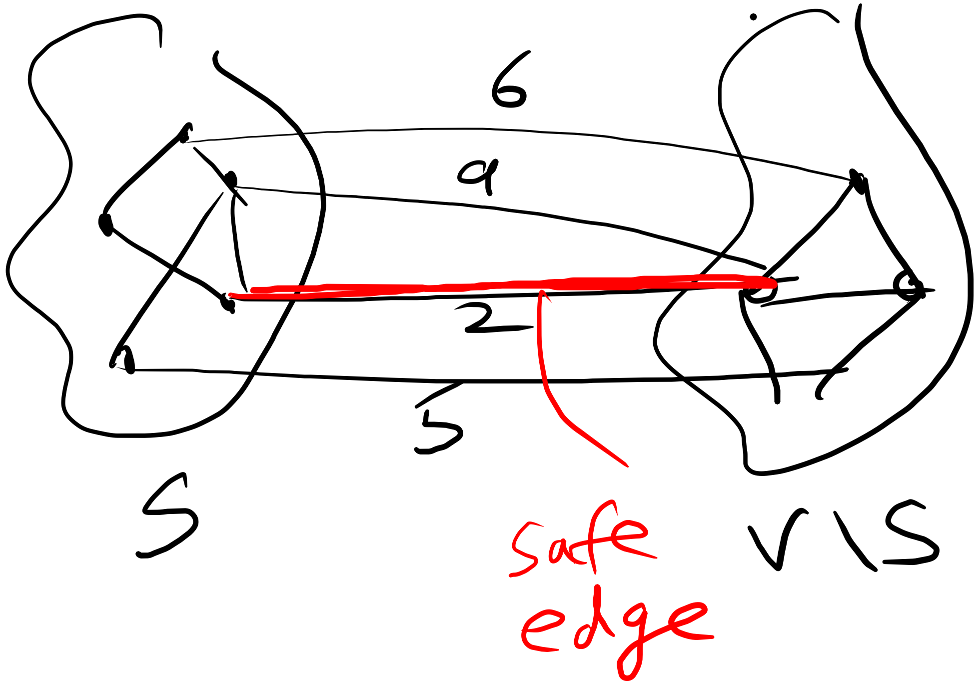


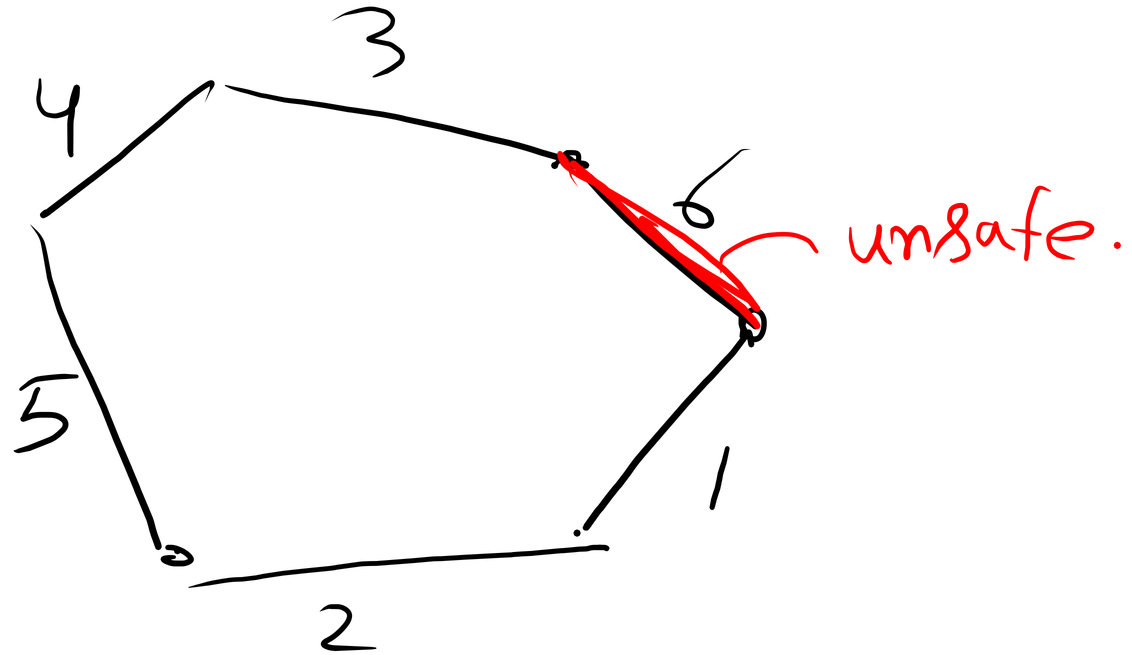
edges in the cut: edges crossing the cut.
one end in S and another in $V \setminus S$

Safe and unsafe edges

safe edge: unique minimum weight edge crossing a cut.

unsafe edge: unique maximum weight edge of a cycle.





cut property and its correctness

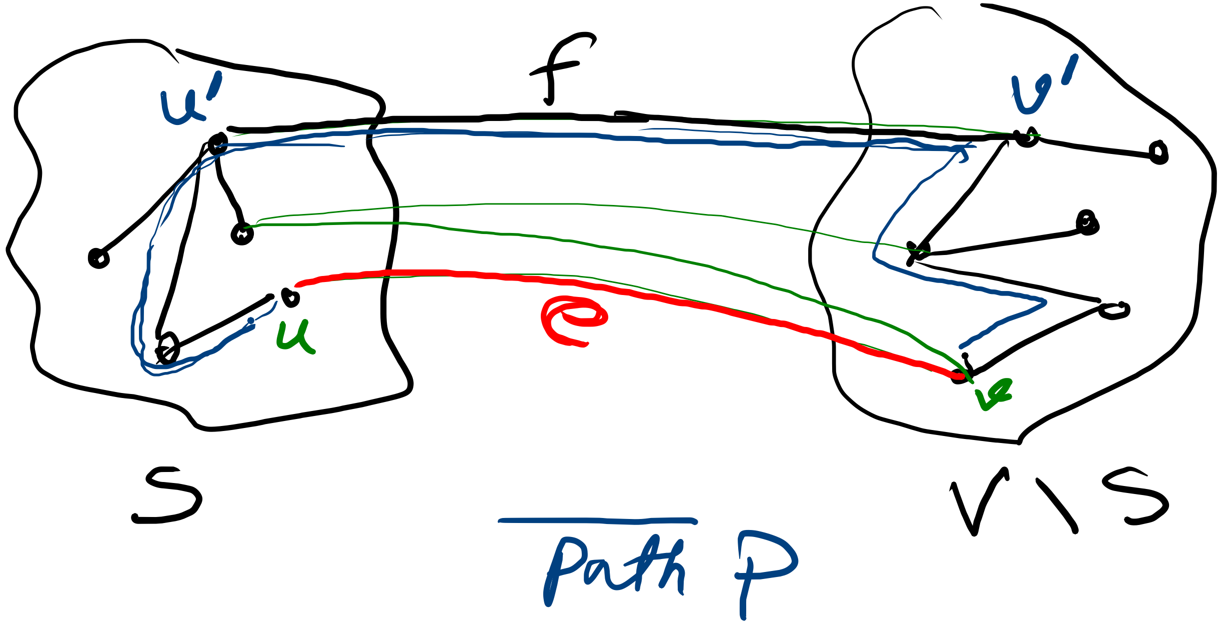
cut property: Let S be any subset of vertices of V and $e=(u,v)$ be the minimum cost edge with one end point in S and the other in $V \setminus S$. Then every MST contains e.

Proof By contradiction.

- G_T be a MST that does not contain e.

- Assume that $u \in S$ then $v \notin S$

- Since G_T is a MST there is a unique path P from u to v.



- $f = (u', v')$
- v' is the first vertex on P (starting from u) in $V \setminus S$
- u' is just before v'
- consider the graph $G_{T'} = (G_T \setminus \{f\}) \cup \{e\}$

If $G_{T'}$ is a spanning tree → we need to prove.

we have a contradiction due to G_T is a MST

<u>claim</u> $G_{T'}$ is a spanning tree

i) $G_{T'}$ is connected. ✓

ii) $G_{T'}$ is a tree ✓

iii) $G_{T'}$ has lower cost than G_T ✓

cycle property :-

H. W.

- e = (u, v) be a edge of smallest weight not creating any cycle.

- e does not belong to a single component.

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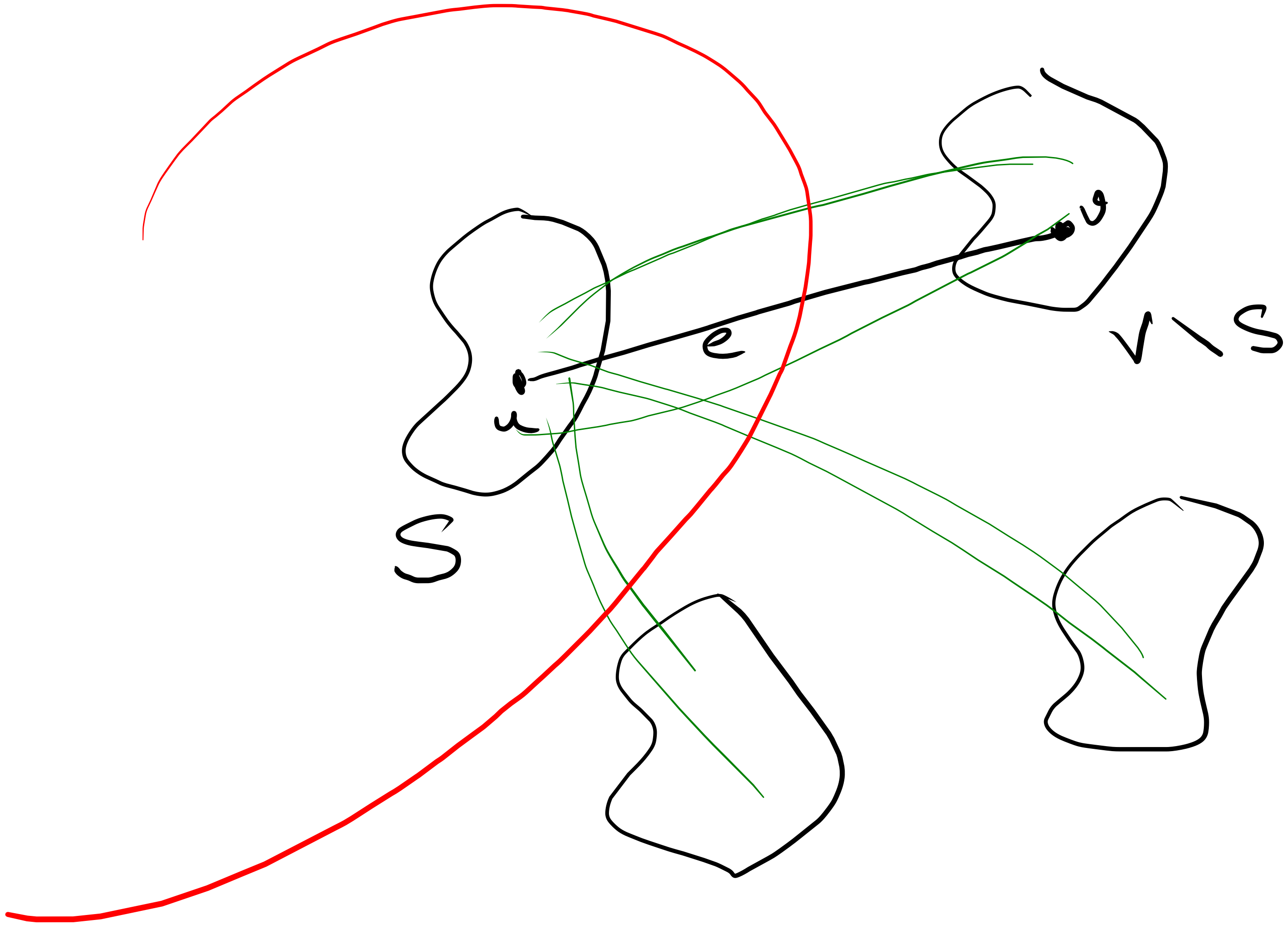

Boruvka's algo
Reverse delete

H.W.

Running time

G has n vertices
m edges

Kruskal $\rightarrow O(m \lg m + m \cdot n)$

$\rightarrow O(mn)$

Prims $\rightarrow O(mn)$

Bouruvka's $\rightarrow T(n, m) = T(\frac{n}{2}, m) + O(m)$

Reverse delete $\rightarrow O(mn)$