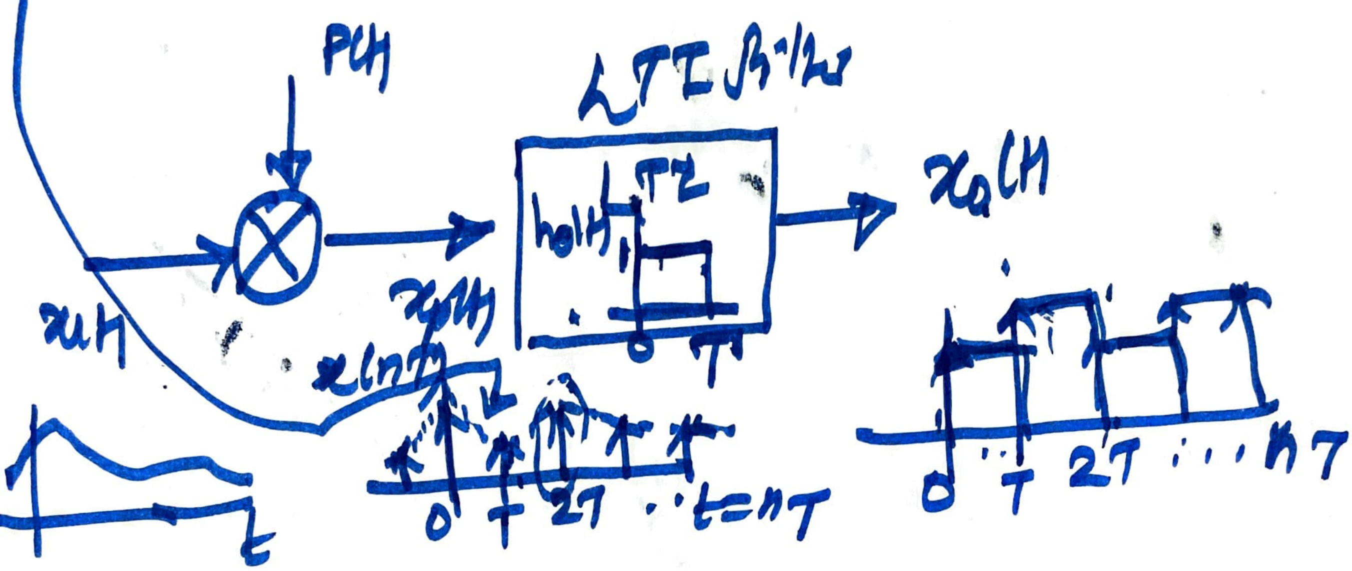
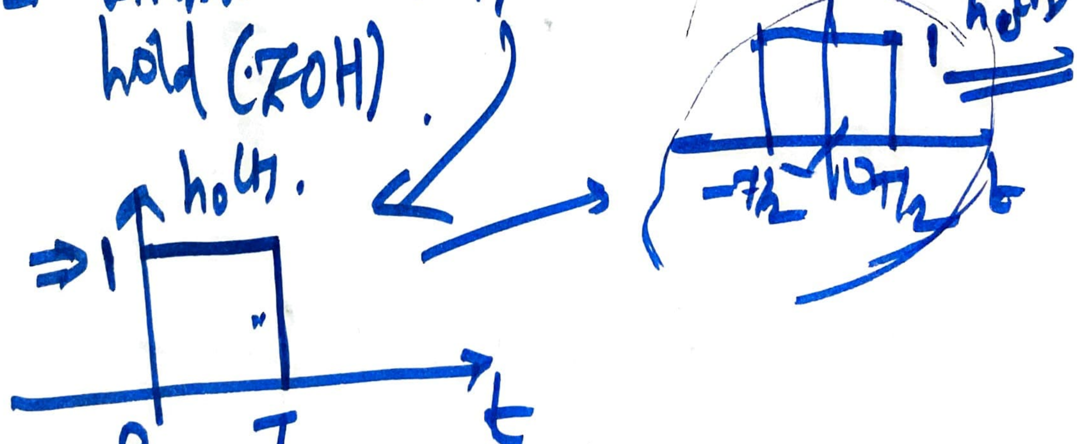
Lecture 41

Sampling With a Zero-Order Hold (ZOH):→

In practice, it is difficult to transmit narrow and large amplitude impulses (for $x_p(t)$) due to hardware limitations.

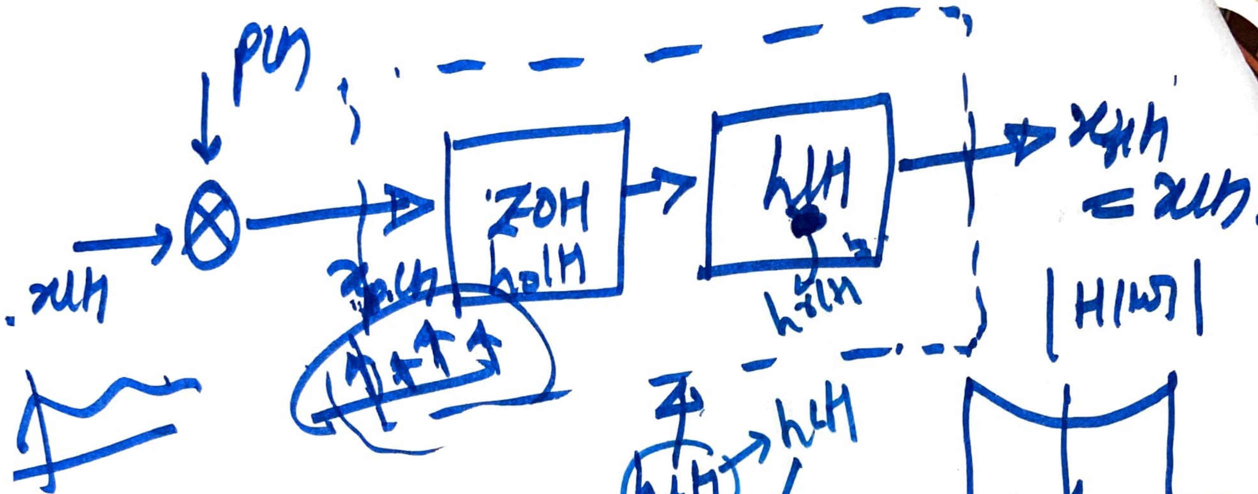


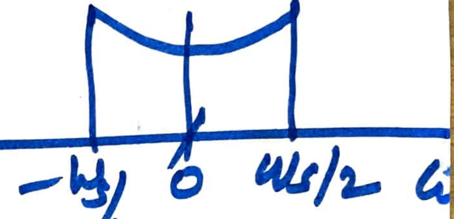
$h_0(t)$ = Impulse response of zero-order hold (ZOH)



∴ Frequency response of ZOH

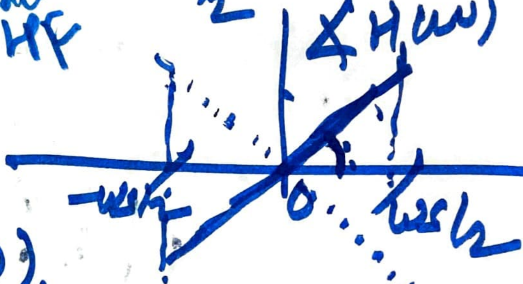
$$\therefore \mathcal{F}\{h_0(t)\} = e^{-j\omega T/2}\left[\frac{2\sin(\omega T/2)}{\omega}\right] = H_0(\omega).$$





$h(t) = h_0(t) * h_r(t)$

Reconstruction filter.



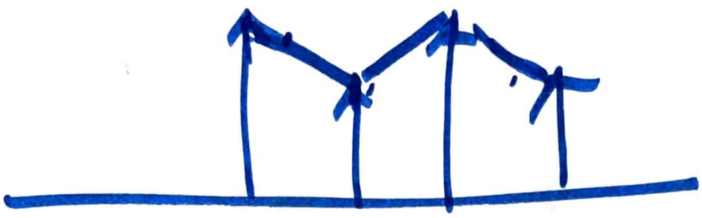
$\therefore\ H_0(w) = H_0(w) \cdot H_r(w)$. $\to H(w)$

$$H_r(w) = \frac{H_0(w)}{H_0(w)} = \frac{H_0(w)}{e^{-jwT/2}\,\frac{\sin(wT/2)}{w}}$$

$$\boxed{H_r(w) = e^{jwT/2} \cdot \left[\frac{H_0(w)}{\frac{\sin(wT/2)}{w}}\right]}$$

H(w)

1.2 Reconstruction of a signal from its Samples Using Interpolation →



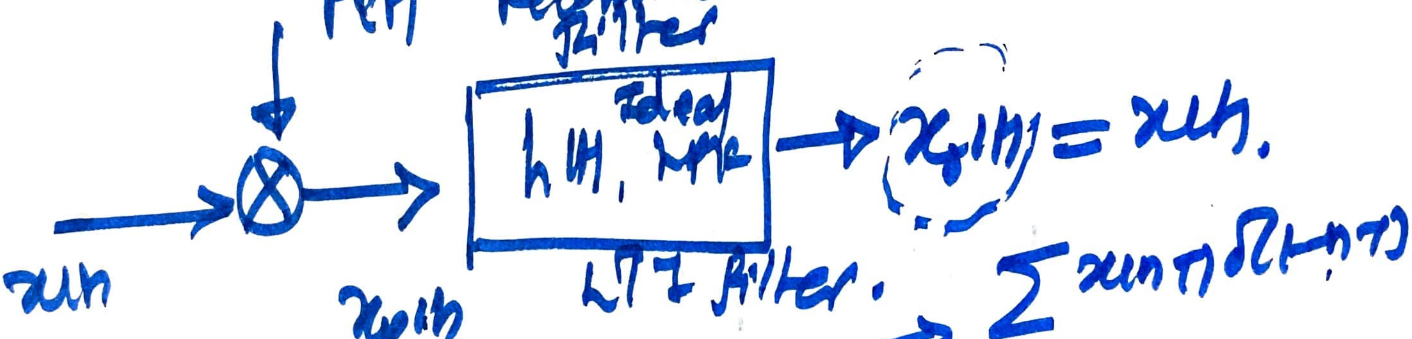
Linear interpolation

Polynomial fit → for interpolation

spline functions. → Michael Unser

To

PCM Reconstruction

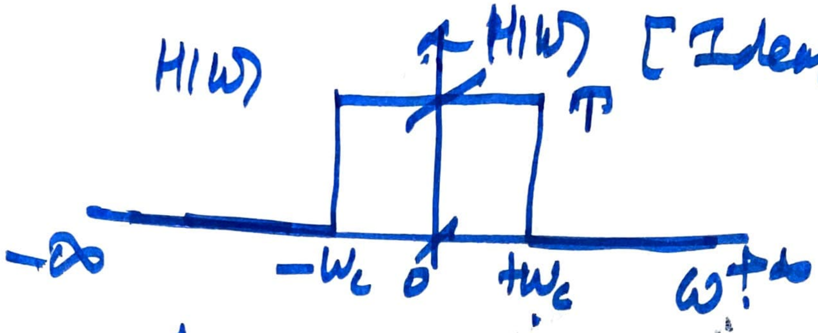


$$\therefore\ x_r(t) = x_p(t) * h(t).$$

$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT) \cdot h(t-nT)$$

$\Rightarrow$

[Ideal LPF]



$$h(t) = \mathcal{F}^{-1}\{H(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(\omega)\, e^{+j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{+\omega_c} T \cdot e^{j\omega t}\, d\omega$$

⋮

$$h(t) = T \cdot \frac{\sin(\omega_c t)}{\pi t}$$

$$\boxed{h(t) = \left(\frac{\omega_c T}{\pi}\right) \times \frac{\sin(\omega_c t)}{(\omega_c t)}}$$

∴ Proved.

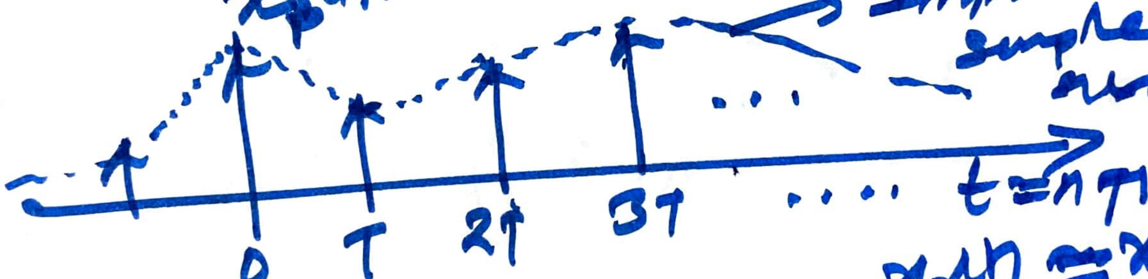
4B.

$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT) \, \underbrace{\frac{\omega_c T}{\pi} \, \frac{\sin(\omega_c (t-nT))}{\omega_c (t-nT)}}_{h(t-nT)}$$

$x_r(t)$ → Reconstructed Analog signal

$x(nT)$ → Impulse-train sampled signal.

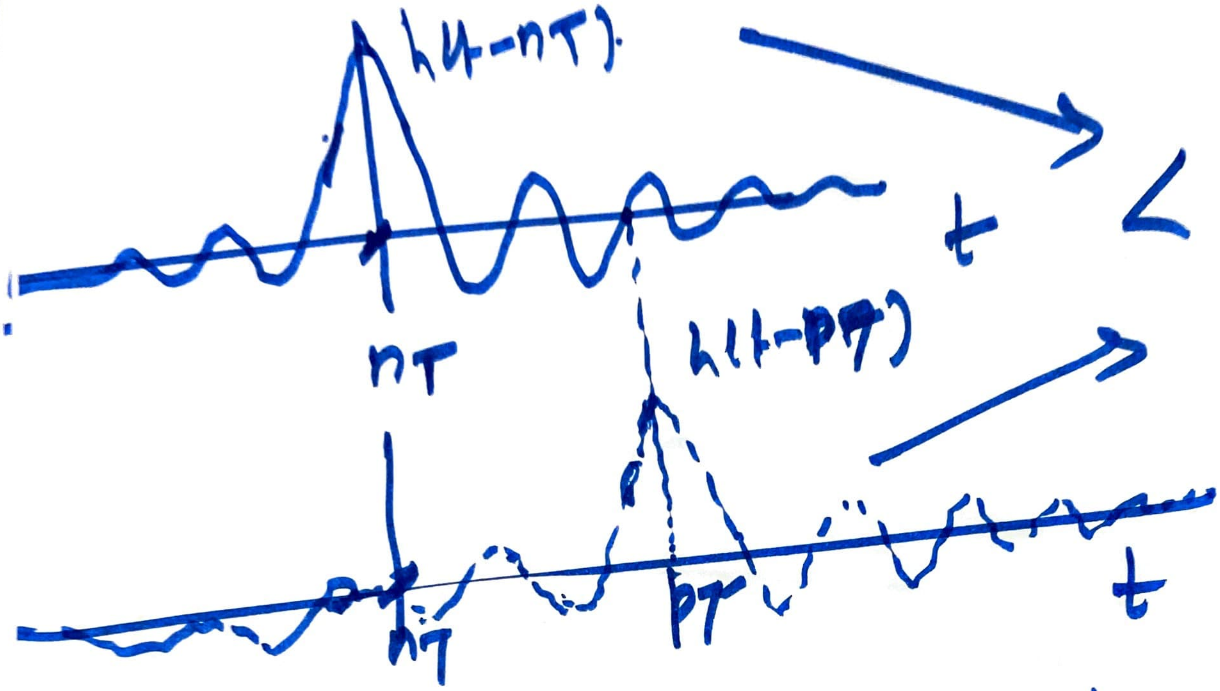
$x(t)$ → Bandlimited analog signal.



Reconstructed analog signal.

$\{h(t-nT)\}_{n \in \mathbb{Z}}$ → infinite shifted "sinc" impulse responses

$$\langle h(t-nT),\ h(t-pT)\rangle = 0 \text{ for } n \neq p$$



$\langle\ ,\ \rangle = 0$

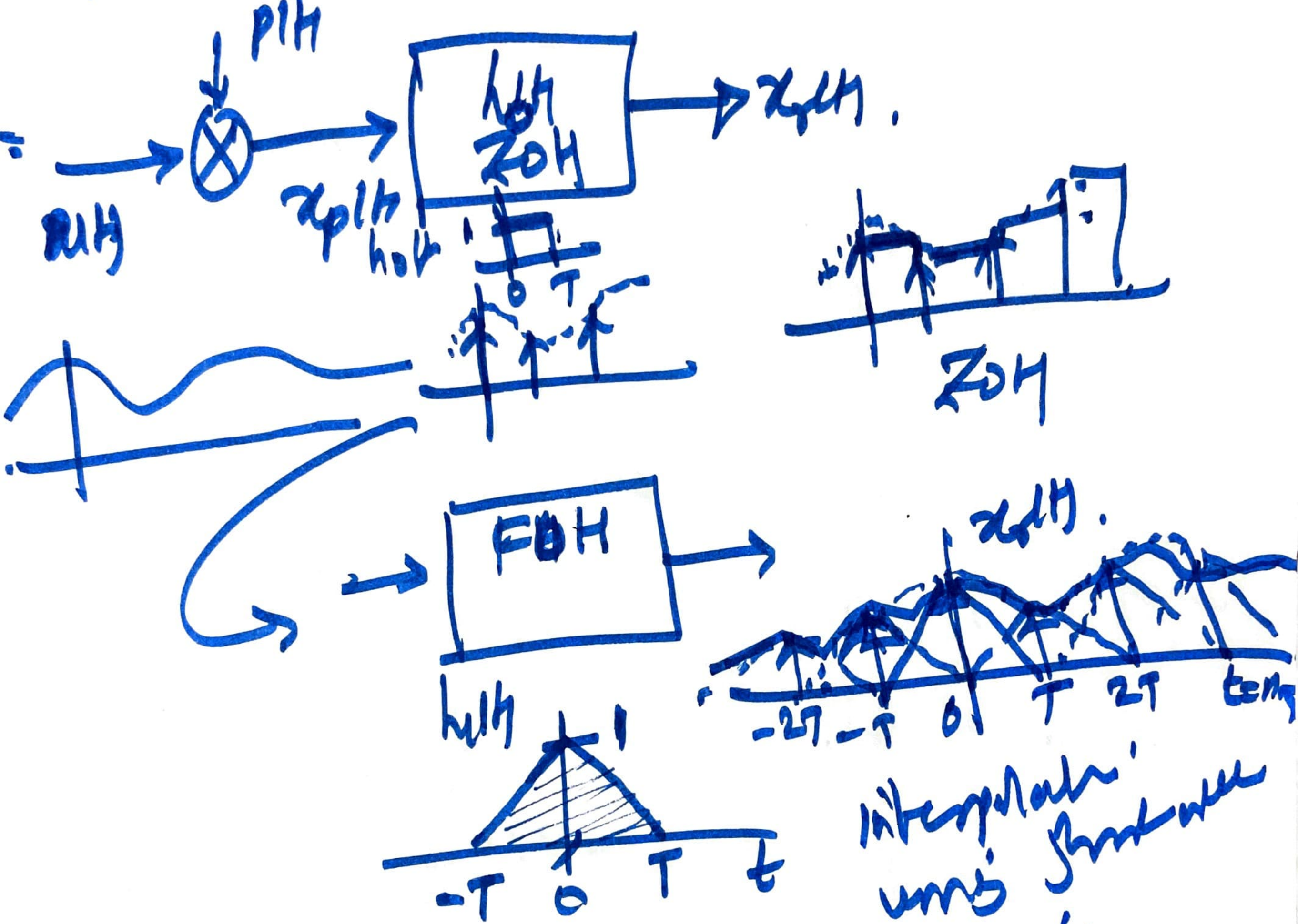
$\Rightarrow \{h(t-nT)\}_{n\in\mathbb{Z}}$ for orthogonal basis representation for signal reconstruction.

'sinc' interpolation

→ Band limited

Higher order Holds :→

First-order Hold :→



interpolation using straight line

FOH is smooth interpolation than ZOH.

→ SOH [Second Order Hold]

⇒ The output of second-order hold provides an interpolation of the samples values that is continuous and has a continuous first derivative and discontinuous second derivative.

The n^{th} order hold will have continuous $(n-1)^{th}$ derivative and discontinuous n^{th} derivative.

→ Quantification of smoothness of interpolation.

$\boxed{7.3}$ The Effect of Undersampling [Aliasing in time-domain]

$x(t) = \cos(\omega_0 t)$

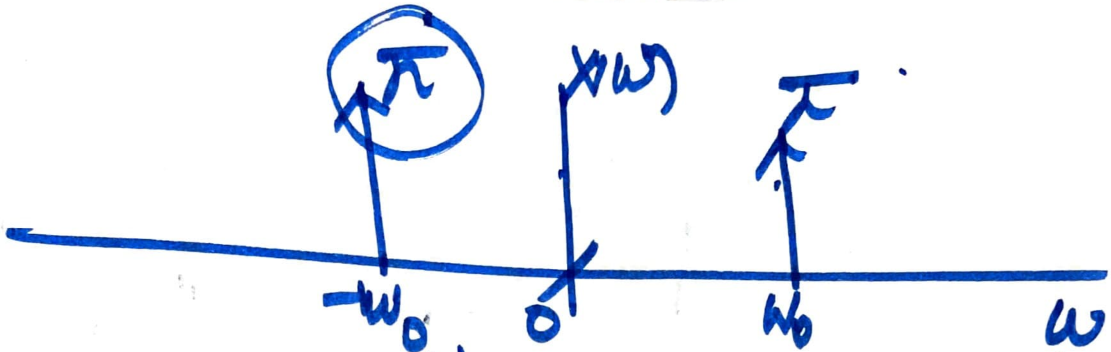
$\downarrow$

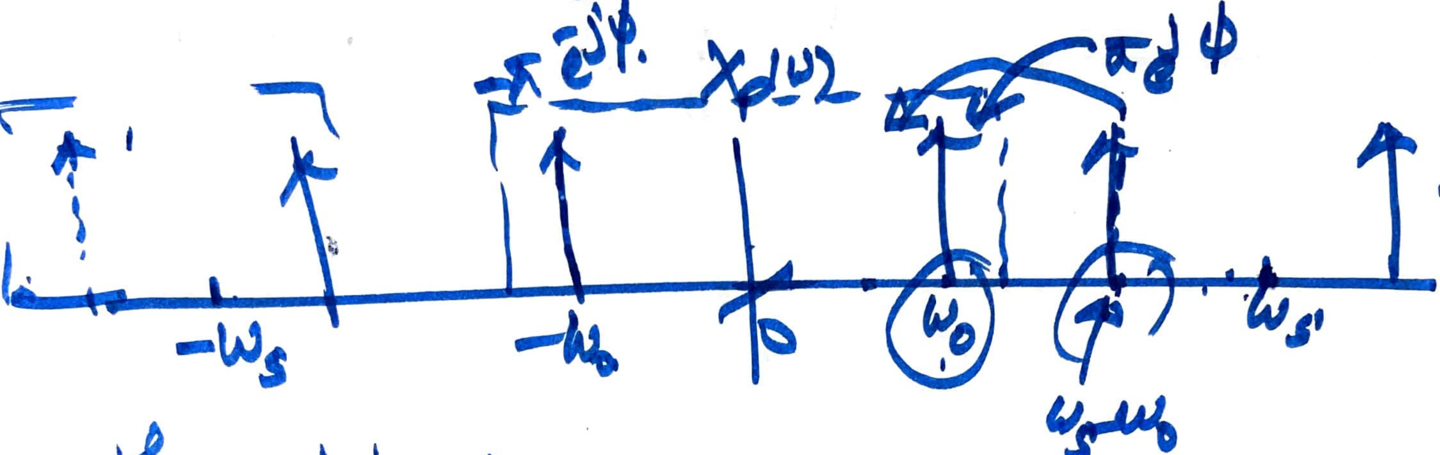
$x_p(t) = x(t) \cdot p(t) = \cos(\omega_0 t) \times \sum_{n=-\infty}^{\infty} \delta(t-nT)$

$X_p(\omega) = \underline{F\{\cos(\omega_0 t)\}} * F\{\sum_{n=-\infty}^{\infty} \delta(t-nT)\}$

$\pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)] * \frac{2\pi}{T} \sum \delta(\omega - k\omega_s)$

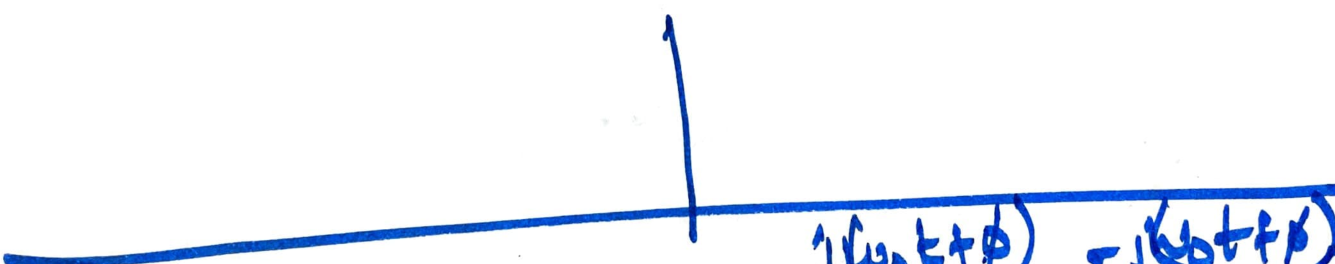
$\Rightarrow$





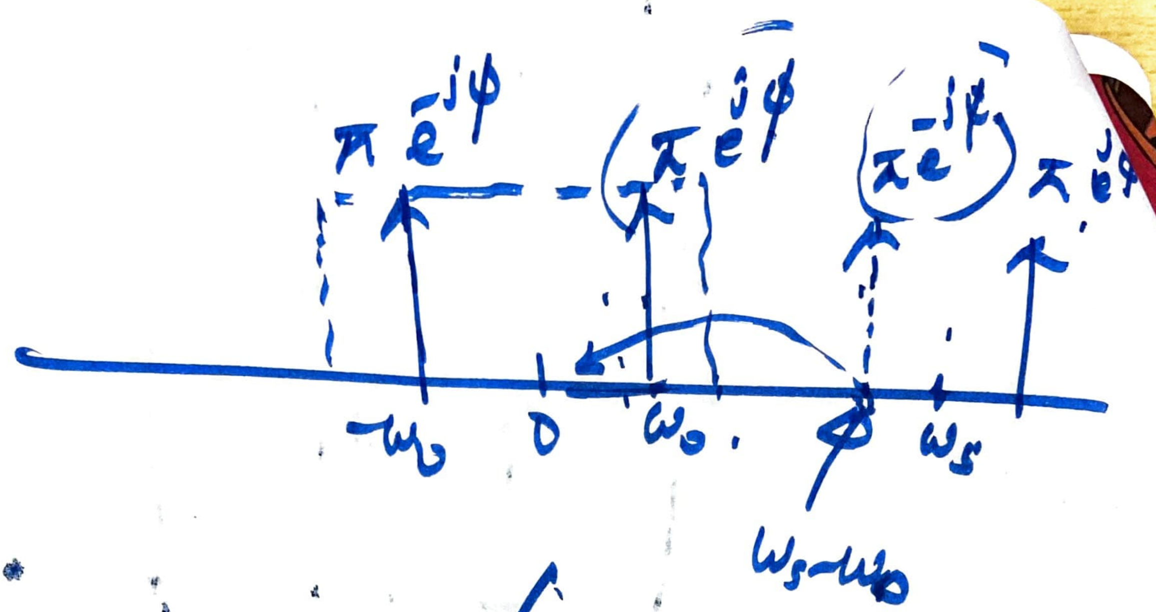
For aliasing,

$$\omega_s - \omega_0 < \omega_0 \Rightarrow \omega_s < 2\omega_0$$



$$F\{\cos(\omega_0 t + \phi)\} \rightarrow F\left\{\frac{e^{j(\omega_0 t+\phi)} + e^{-j(\omega_0 t+\phi)}}{2}\right\}$$

$$= e^{j\phi}[\ \cdot\ ] + e^{-j\phi}[\ ?\ ]$$



$x(t) = \cos(\omega_0 t + \phi)$

$x_r(t) = \cos(\omega_0 t - \phi)$

phase reversal.

Stroboscopic effect

[Design of transducer]

Western Movies.

7.4 Discrete-time Processing of Continuous-Time Signals :→

Computer System → sound card ↙ A/D + D/A.

→ record speech, audio → A/D.

& play speech, audio, music → D/A.

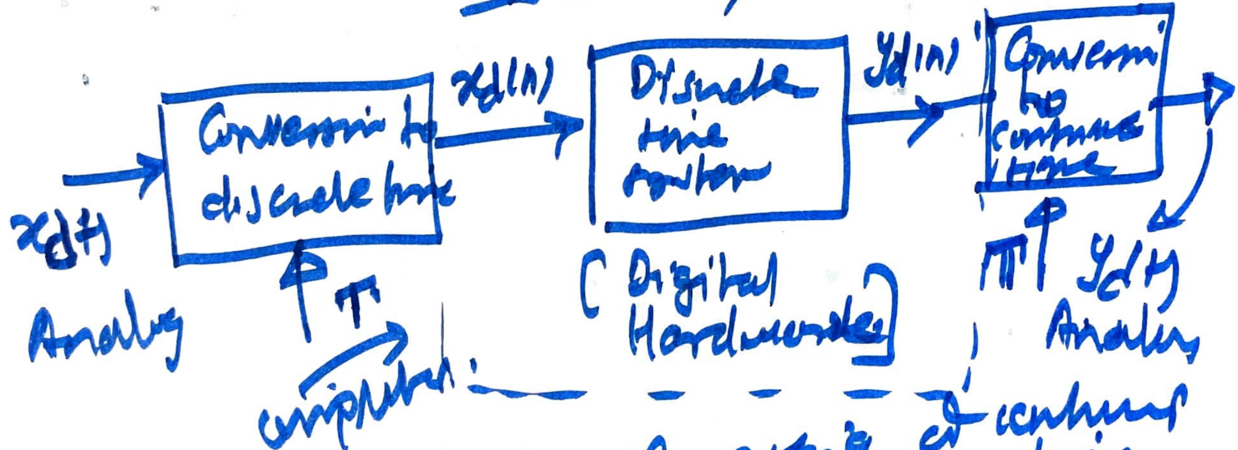


Fig. Discrete-time Processing of continuous-time signals.

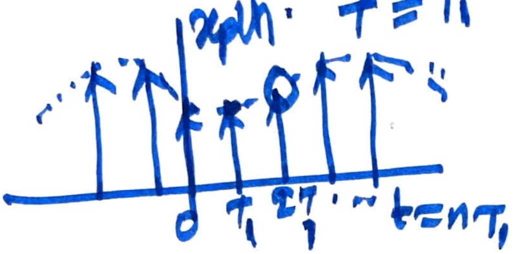
$x_d(n) \rightarrow x_c(t)$ } How ??

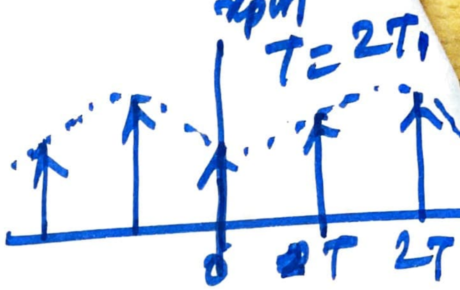
$y_c(t) \longrightarrow y_d(n)$

$x_d(n) = x_c(nT)$, $y_d(n) = y_c(nT)$

→ Sampling interval plays a crucial role,

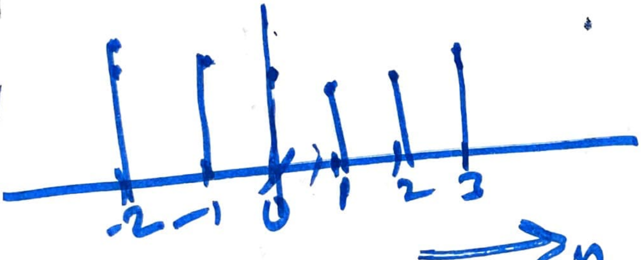
$x(t) \rightarrow x_p(t)$

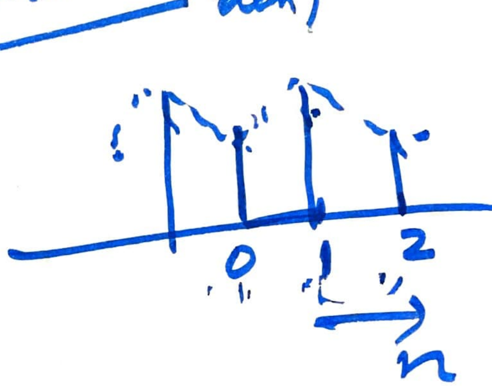




$x(n)$

Time Normalization





$$x_p(t) = \sum_{n=-\infty}^{+\infty} x_c(nT)\,\delta(t-nT)$$

$$X_p(\omega) = CTFT\{x_p(t)\} = F\{x_p(t)\}$$

$$= F\left\{\sum_{n=-\infty}^{+\infty} x_c(nT)\cdot\delta(t-nT)\right\}$$

Linearity property.

$$X_p(\omega) = \sum_{n=-\infty}^{+\infty} \mathcal{F}\{x_c(nT) \cdot \delta(t-nT)\}$$

$$= \sum_{n=-\infty}^{+\infty} x_c(nT)\, \mathcal{F}\{\delta(t-nT)\}$$

analog frequency

$$X_p(\omega) = \sum_{n=-\infty}^{+\infty} x_c(nT)\, e^{-j\omega nT} \quad \text{—(A)}$$

DTFT of $x_d(n)$

$$\therefore X_d(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} x_d(n)\, e^{-j\Omega n} \quad \text{—(B)}$$

[DTFT]

(A) & (B)

Design of filters

$$\boxed{\Omega = \omega T}$$

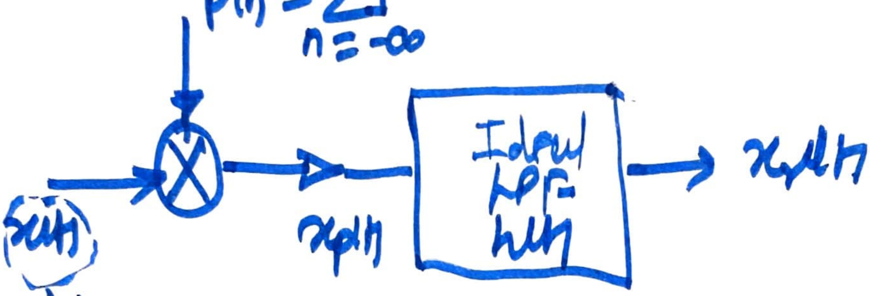
Ω: Analog frequency (rad/sec)

ω: Digital frequency (rad)

T: Sampling interval (sec).

* Limitations of Shannon's Sampling Paradigm: →

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$



Shannon's sampling paradigm requires x(t) to be band-limited

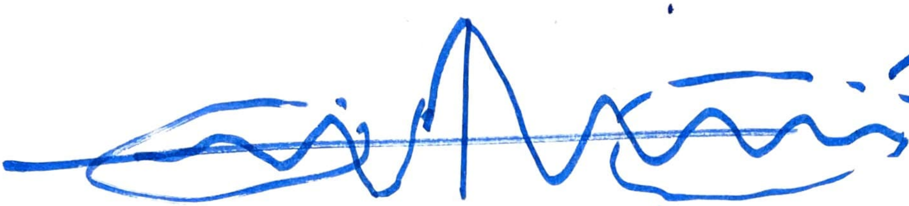
↳ most of the signals in nature are not bandlimited and hence we need to do → Ideal LPF before impulse train sampling

Anti-aliasing filter → pre-filter   p(t)   Reconstruction filter → post filter.

→ Since pre-filter & post filter are ideal LPF → difficult to realize in practical hardware circuits.

h(t) = IIR & non-causal (sinc).

(2) Ideal LPF → "sinc" as impulse response → interpolation → mosaic effect



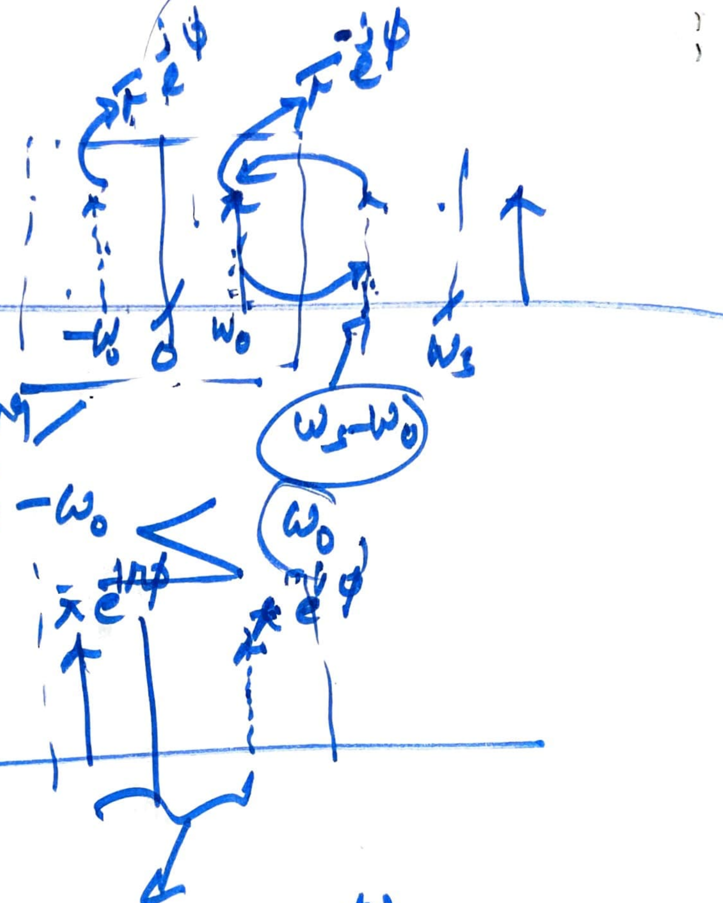
slow decay of sinc

Image processing application.

(3) Signals in nature are NOT strictly band limited ⇒ prefilter

$\cos(\omega_0 t + \phi)$

$\cos(\omega_s t + \phi)$



Phase reversed due to undersampling

$\cos(\omega_0 t - \phi)$