1. Let $A \in \mathbb{R}^{m\times n}$ be a given matrix. If $B = EA$, where E is a matrix representing a product of elementary row transformations (incl. row exchanges), then which of the following statements are true? Justify with a proof.

   (a) $C(A) = C(B)$
   (b) $C(A^T) = C(B^T)$
   (c) $N(A) = N(B)$
   (d) $N(A^T) = N(B^T)$

2. Find the linearly independent rows and columns (and hence also the row and column rank) of the following matrices by first obtaining the REF form:

   (a)
$$A = \begin{bmatrix} 2 & -3 & 1 & 4 \\ -2 & 3 & -1 & -4 \\ 4 & -6 & 2 & 8 \\ 6 & -9 & 3 & 12 \end{bmatrix}$$

   (b)
$$A = \begin{bmatrix} 1 & 2 & 1 & -2 \\ 2 & 4 & 0 & 1 \\ -2 & -3 & 0 & 5 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

   (c)
$$A = \begin{bmatrix} 2 & -1 \\ 5 & 2 \\ 1 & 3 \\ 4 & 1 \end{bmatrix}$$

   (d)
$$A = \begin{bmatrix} 1 & 2 & 3 & \dots & 10 \end{bmatrix}$$

3. Let $A \in \mathbb{R}^{4\times 6}$ be a matrix such that any solution of the system $Ax = \mathbf{0}$ has the general form:
$$\begin{bmatrix} 2p+q \\ p \\ 3q-r \\ q \\ r \\ 4p+2r \end{bmatrix}, \forall p, q, r \in \mathbb{R}.$$
   Determine the column rank of A.

4. Let $A \in \mathbb{R}^{2\times n}, n \geq 2$. Show that row rank$(A) =$ col rank(A).