Lecture 27

Understanding Fourier Series

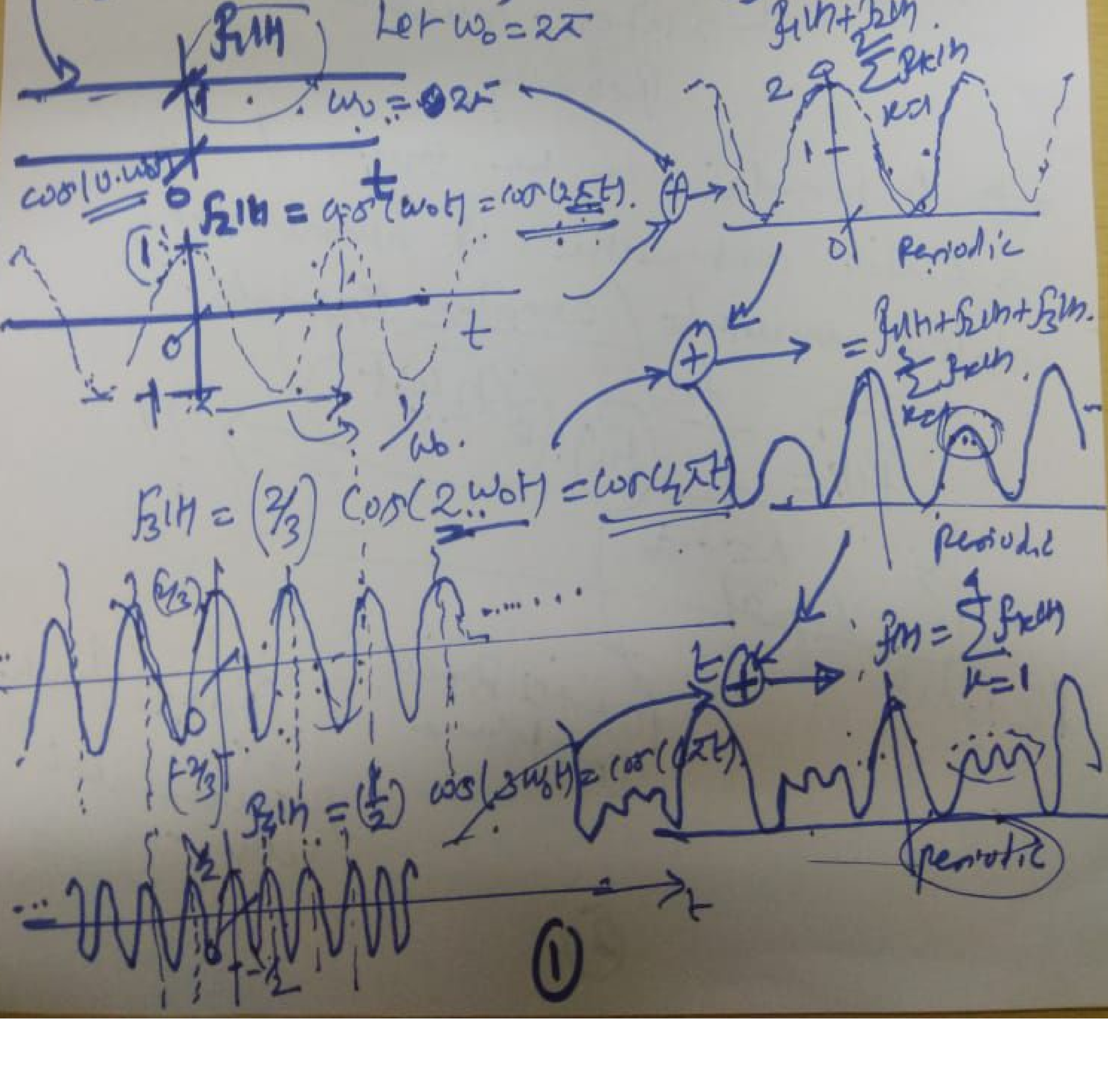
(Contd. Problem 5) → Last class.

$$f(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t)$$

$f_1(t) = 1 \;\forall\; t$, $f_3(t) = \left(\frac{2}{3}\right)\cos(2\omega_0 t)$

$f_2(t) = 1 \cdot \cos(\omega_0 t)$, $f_4(t) = \left(\frac{1}{2}\right) \cdot \cos(3\omega_0 t)$

Let $\omega_0 = 2\pi$



→ Each of the signals, i.e., $f_1(t)$, $f_2(t)$, $f_3(t)$, & $f_4(t)$ are periodic.

$\Rightarrow \{f_k(t)\}_{k\in[1,4]} \rightarrow$ harmonically related sinusoidal signals.

$\underbrace{\{0\cdot\omega_0, 1\omega_0, 2\omega_0, 3\omega_0\}}$

↳ integer multiple of fundamental frequency.

⇒ A periodic can be expressed as a linear combination of harmonically related sinusoids / complex exponents.

$$\boxed{f(t) = \sum_{n=-\infty}^{+\infty} F_n \cdot e^{jn\omega_0 t}} \rightarrow ??$$

Periodic signal

Fourier series coefficients

Basis function or Representation view

Find the exponential Fourier series representation for impulse-train signal.

Sol^n

Impulse-train ⇒ train of impulses

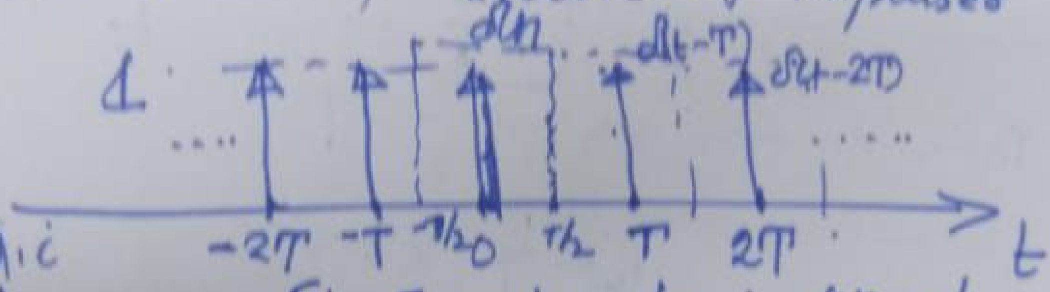


Fig. Impulse train signal.

$p(t)$

$$p(t) = \dots + \delta(t+T) + \delta(t) + \delta(t-T) + \delta(t-2T) + \dots$$

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT) \; ; \quad n \in \mathbb{Z}$$

$$\therefore p(t) = \sum_{n=-\infty}^{+\infty} P_n \cdot e^{jn\omega_0 t}$$

↳ Fourier series coefficient

$$P_n = \frac{1}{T}\int_0^{T} p(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} p(t) \cdot e^{-jn\omega_0 t}\, dt = \frac{1}{T}\left[\int_{-T/2}^{T/2} \delta(t)\, e^{-jn\omega_0 t}\, dt\right]$$

↳ ??

↳ 1

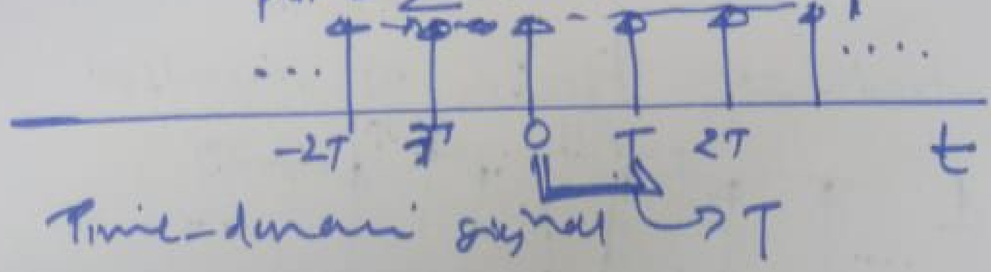
$$\therefore \boxed{P_n = \frac{1}{T} \;\; \forall n}$$

→ Fourier's

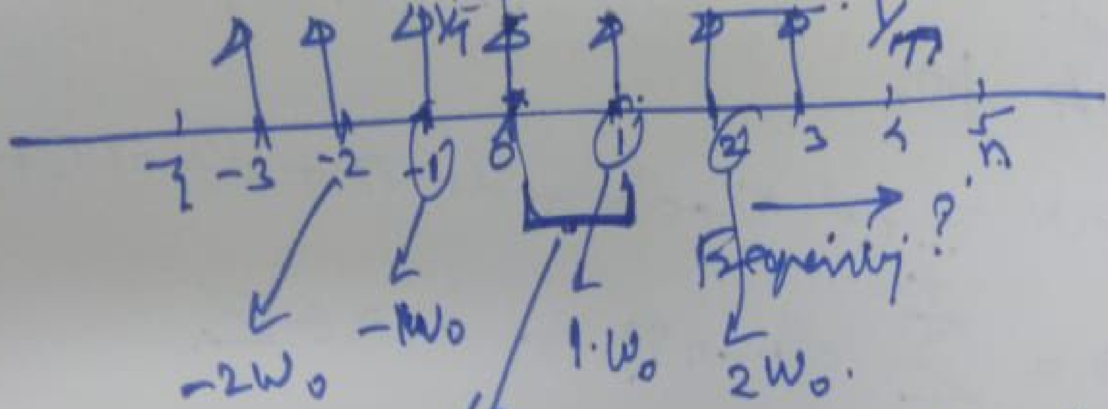
(3) Fourier coeff for $p(t)$ = impulse train signal.

$P_n = \frac{1}{nT}\ \forall n$

$p(t) = \sum_{n=-\infty} \delta(t-nT)$

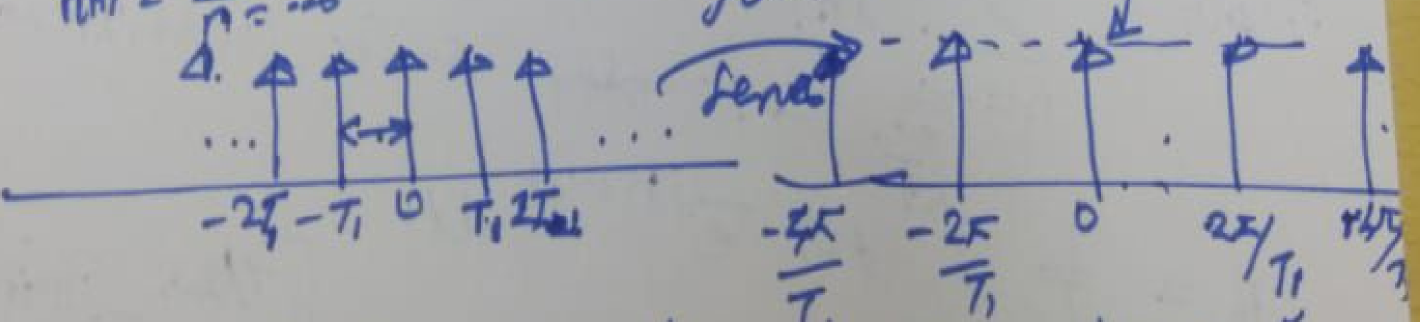


$P_n = \frac{1}{T},\ \forall n \in \mathbb{Z}$



$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT_1)$ FT. Fourier

$P_n = \frac{1}{T_1},\ n \in \mathbb{Z}$



⇒ If impulses are located closer to each other in time-domain, then in frequency domain impulses will be farther away from each other.

⇒ We can't achieve good time resolution and good frequency resolution at the same time.

④

⇒ Fourier series representation of impulse train signal is also an impulse train signal.

$\boxed{\text{Problem 7}}$ Find the exponential Fourier series representation for Pulse-train signal.

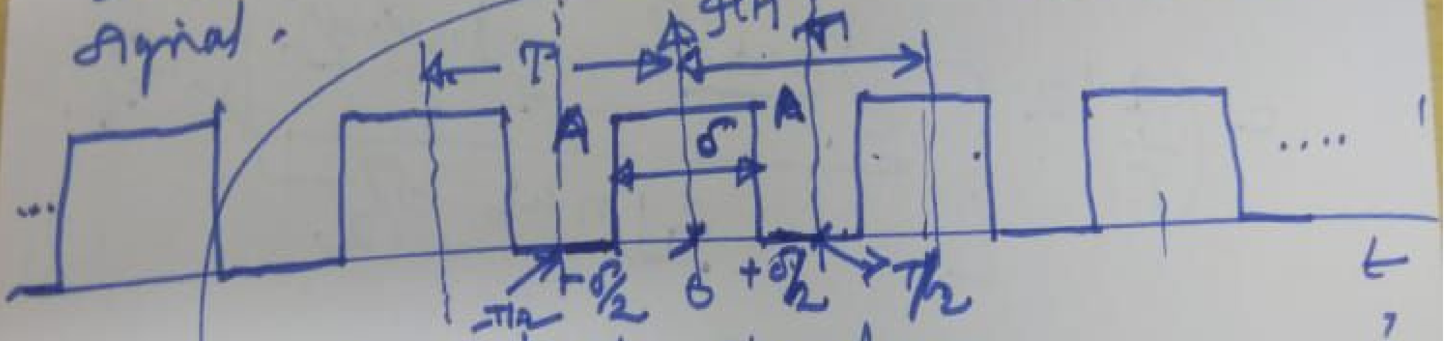


Fig. Pulse-train signal.

$$f(t) = \sum_{n=-\infty}^{+\infty} F_n \, e^{jn\omega_0 t}$$

Periodic signal.

Fourier series coefficient.

$$\therefore F_n = \frac{1}{T}\int_0^T f(t) \cdot e^{-jn\omega_0 t}\,dt.$$

$$= \frac{1}{T}\int_{-T/2}^{+T/2} f(t)\, e^{-jn\omega_0 t}\,dt$$

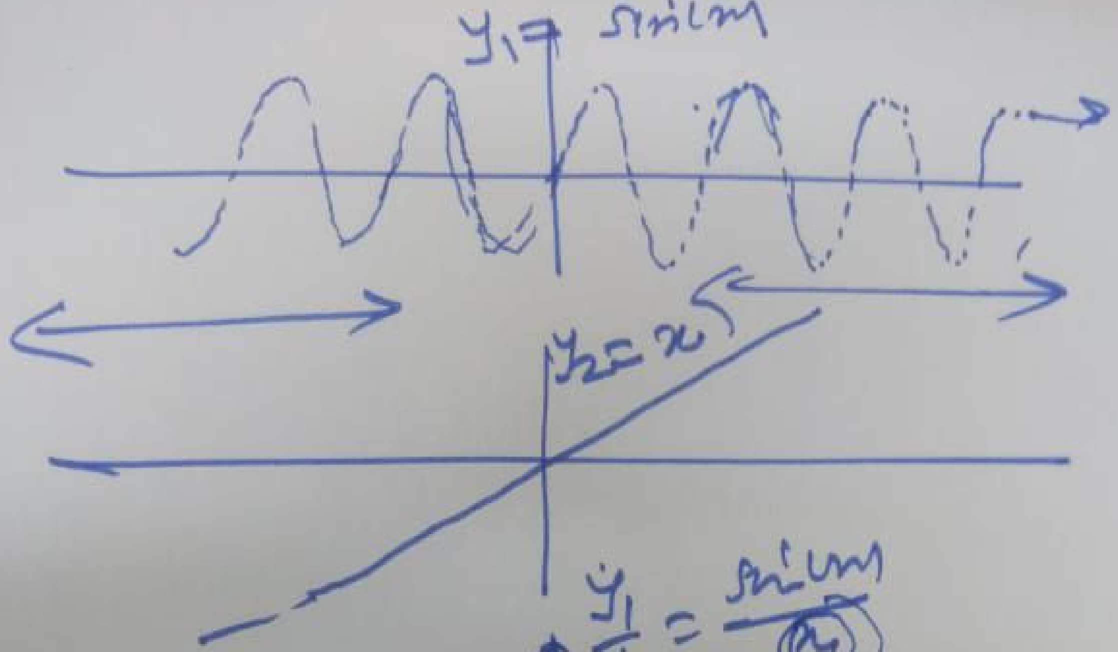
$$F_n = \frac{1}{T}\int_{-\delta/2}^{+\delta/2} A \cdot e^{-jn\omega_0 t}\, dt.$$

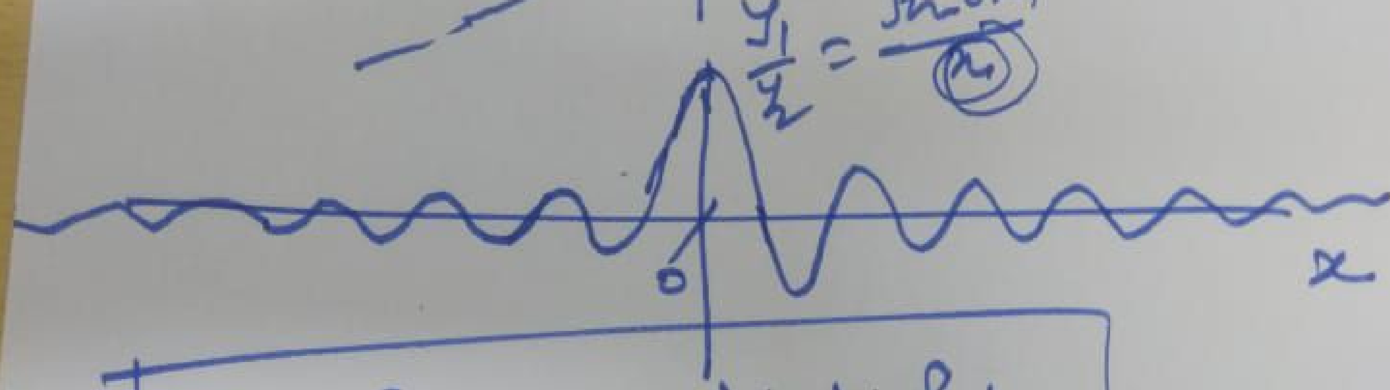
$$= \left(\frac{A}{T}\right) \cdot \left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{-\delta/2}^{\delta/2}$$

$$= \left(\frac{A}{T}\right) \cdot 2 \cdot \left[\frac{e^{jn\omega_0 \delta/2} - e^{-jn\omega_0 \delta/2}}{2jn\omega_0 \delta/2}\right] \times \delta/2$$

$$= \left(\frac{A\delta}{T}\right) \frac{\sin(n\omega_0 \cdot \delta/2)}{(n\omega_0 \delta/2)}$$

$$= \left(\frac{A \cdot \delta}{T}\right) \cdot \frac{\sin(x)}{(x)}$$

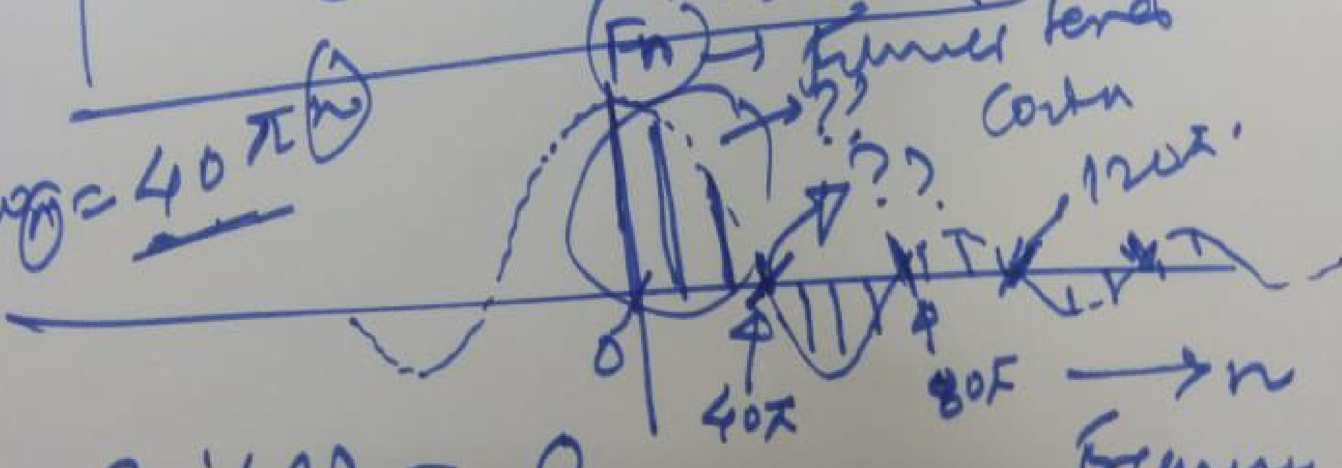




$$\boxed{F_n = \left(\frac{A\,\delta}{T}\right) \cdot \frac{\sin(n w_0 \delta/2)}{n w_0 \delta/2}}$$

$F_n \rightarrow$ Fourier terms coeff

$w_n = 40\pi (n)$



$\sin(\theta) = 0$

$\theta = k.\pi \;;\; k \in \mathbb{Z}$

7 $n.w_0(\delta/2) = k.\pi$ let $\boxed{\delta = 1/20.}$

$w_n = 40\pi k' \;,\; k \in \mathbb{Z}$