SC224: Tutorial Sheet 4

Problems based on Special Continuous Random Variables.

Pb 1) The moment generating function of a random variable X is a function $M_X(t)$ of a free parameter t, defined by $M_X(t) = E[e^{tX}]$ (if it exists).

(i) Compute the moment generating functions for the following distributions.

a) Normal distribution with parameters μ and σ^2.

b) Exponential distribution with parameters λ.

c) Gamma distribution with parameter (α, λ).

d) Chi-Square distribution with n degrees of freedom.

e) t-distribution with n degrees of freedom.

(ii) Using the moment generating function, find the mean and variance of above mentioned distributions.

Pb 2) If U is uniformly distributed on $(0, 1)$, show that $a + (b - a)U$ is uniform on (a, b).

Pb 3) The annual rainfall in Cleveland, Ohio is approximately a normal random variable with mean 40.2 inches and standard deviation 8.4 inches. What is the probability that

(a) next year's rainfall will exceed 44 inches?

(b) the yearly rainfalls in exactly 3 of the next 7 years will exceed 44 inches?

Assume that if A_i is the event that the rainfall exceeds 44 inches in year i (from now), then the events A_i, $i \geq 1$, are independent.

Pb 4) There are two types of batteries in a bin. When in use, type i batteries last (in hours) an exponentially distributed time with rate λ_i, $i = 1, 2$. A battery that is randomly chosen from the bin will be a type i battery with probability p_i, $p_1 + p_2 = 1$. If a randomly chosen battery is still operating after t hours of use, what is the probability that it will still be operating after an additional s hours?

Pb 5) Evidence concerning the guilt or innocence of a defendant in a criminal investigation can be summarized by the value of an exponential random variable X whose mean μ depends on whether the defendant is guilty. If innocent, $\mu = 1$; if guilty, $\mu = 2$. The deciding judge will rule the defendant guilty if $X > c$ for some suitably chosen value of c.

(a) If the judge wants to be 95 percent certain that an innocent man will not be convicted, what should be the value of c?

(b) Using the value of c found in part (a), what is the probability that a guilty defendant will be convicted?

Pb 6) If X is a chi-square random variable with 6 degrees of freedom, find (a) $P[X < 6]$; (b) $P[3 < X < 9]$.

Pb 7) If X and Y are independent chi-square random variables with 3 and 6 degrees of freedom, respectively, determine the probability that $X + Y$ will exceed 10.

Pb 8) If T has a t-distribution with 8 degrees of freedom, find (a) $P[T = 1]$, (b) $P[T = 2]$, and (c) $P[-1 < T < 1]$.

Pb 9) If T_n has $t-$distribution with n degrees of freedom, find the distribution of T_n^2.

Problems based on the Central Limit Theorem, Sample Mean and Sample Variance.

Pb 1) The lifetime (in hours) of a type of electric bulb has expected value 500 and standard deviation 80. Approximate the probability that the sample mean of n such bulbs is greater than 525 when (a) $n = 4$; (b) $n = 16$; (c) $n = 36$; (d) $n = 64$.

Pb 2) If X is binomial with parameters $n = 150$, $p = .6$, compute the exact value of $P[X \leq 80]$ and compare with its normal approximation both (a) making use of and (b) not making use of the continuity correction.

Pb 3) Approximate the probability that the sum of 16 independent uniform (0, 1) random variables exceeds 10.

Pb 4) If 10 fair dice are rolled, approximate the probability that the sum of the values obtained (which ranges from 20 to 120) is between 30 and 40 inclusive.

Pb 5) Let $X_1, X_2, ..., X_n$ be a sample of values from some population. Assume that the probability mass function of each X_i, is

$$P[X = 0] = .2,\ P[X = 1] = .3, P[X = 3] = .5.$$

For $n = 2, 3, 4, 5$, and 6, determine $E[\overline{X}]$, $\text{Var}(\overline{X})$ and the sample variance S^2.

Pb 6) A six-sided die, in which each side is equally likely to appear, is repeatedly rolled until the total of all rolls exceeds 400. Approximate the probability that this will require more than 140 rolls.

Pb 7) A tobacco company claims that the amount of nicotine in its cigarettes is a random variable with mean 2.2 mg and standard deviation .3 mg. However, the sample mean nicotine content of 100 randomly chosen cigarettes was 3.1 mg. What is the approximate probability that the sample mean would have been as high or higher than 3.1 if the company's claims were true?