Running time of Ford-Fulkerson

initialization — $O(|E|)$

Find residual path $O(|V|+|E|)$

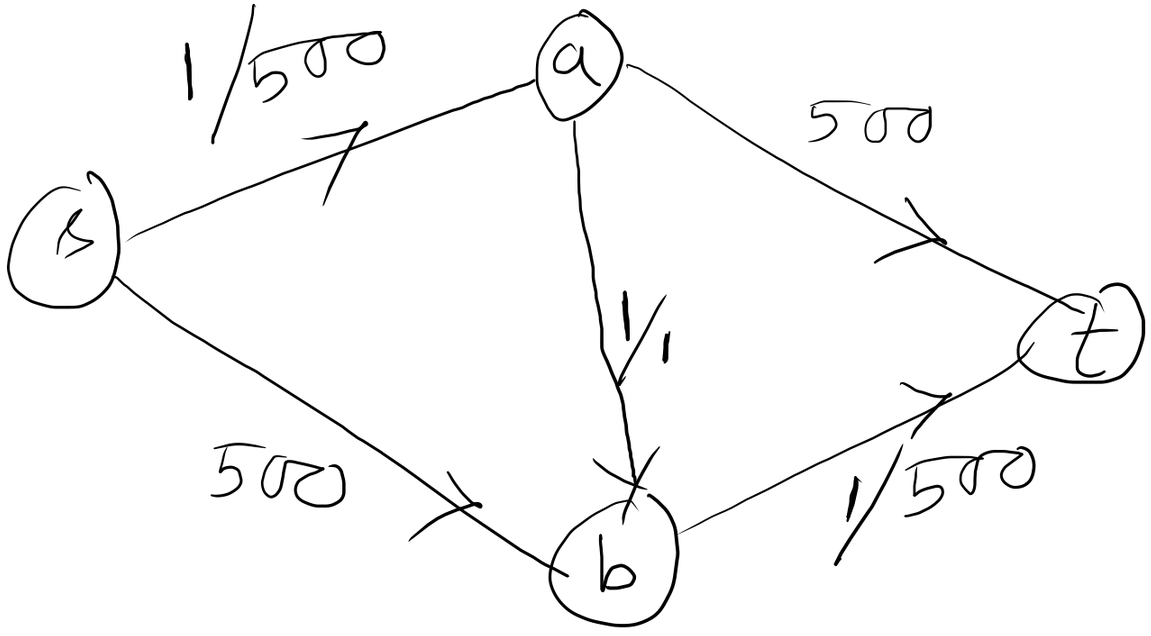
Bottleneck - capacity : $O(|E|)$

update the flow : $O(|E|)$

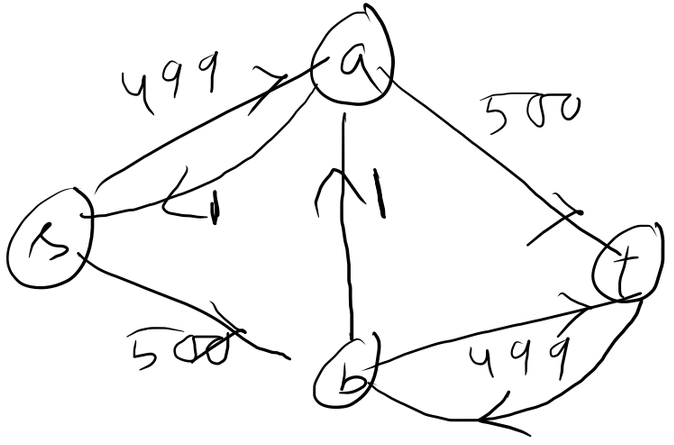
update residual graph : $O(|E|)$

Let # Augment operations is C

Total time : $O(C(|V|+|E|))$



$s \to a \to b \to t$



$s \to b \to a \to t$

Here c is $|f^*|$

- integer array $A[1 \ldots n]$
- Find indices i and j, $i < j$
  $A[i] - A[j]$ minimum.

Brute force

- consider each pair i, j such that $i < j$.
- Track the minimum of $A[i] - A[j]$.

$O(n^2)$

$A[1 \ldots n]$

A 5 2 9 6 3 1 4

create a new array B such that

$B[n] = A[n]$

for $i = n-1$ down to 1

$B[i] = \max(A[i], B[i+1])$

B

9	9	9	6	4	4	4

$min = -7$

$i = 2$

$K = 2$

$l = 3$

$min = \infty$

for $i = 1$ to $n-1$

$t = A[i] - B[i+1]$

if $t < min$ then

$min = t$

$K = i$

for $l = K+1$ to n

If $A[K] - A[l] == min$

return the pair (K, l)

A is a m×n matrix

1> element sorted rowwise

2> " " columnwise

Given x,

Find x in A.

5	9	16	20	25
6	12	18	25	32
9	13	25	29	35
13	29	36	41	45

Trivial: $O(mn)$

- Search all element one by one

Better Binary search each row: $O(m \log n)$

OR

" " column: $O(n \log m)$

$K == A[1][n]$

$K > A[1][n]$

Search $(A[2 \ldots m][1 \ldots n], K)$

$K < A[1][n]$

Search $(A[1 \ldots m][1 \ldots n-1], K)$

Running time : $O(m+n)$