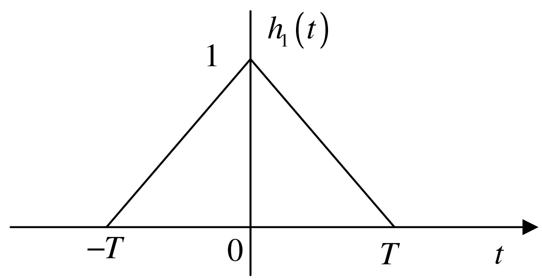
Signals and Systems (CT 203)

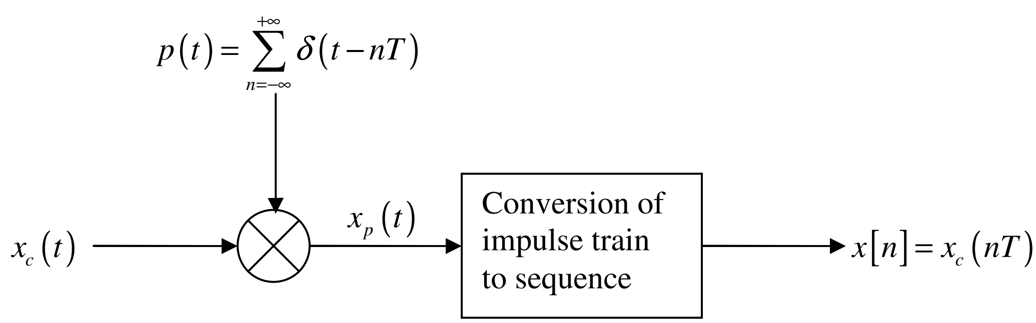
1. A signal $x(t)$ undergoes a zero-order hold operation with an effective sampling period T to produce a signal $x_o(t)$. Let $x_1(t)$ denote the result of a first-order hold operation on the samples of $x(t)$, i.e.,

$$x_1(t) = \sum_{n=-\infty}^{+\infty} x(nT)\, h_1(t-nT),$$

where $h_1(t)$ is the function shown below. Specify the frequency response of the filter that produces $x_1(t)$ as its output when $x_o(t)$ is the input.



2. Consider a band-limited signal $x_c(t)$ that is sampled at a rate higher than the Nyquist rate. The samples, spaced T seconds apart, are then converted to a sequence $x[n]$, as shown below.



Determine the relation between the energy E_d of the sequence $x[n]$, the energy E_c of the original continuous-time signal sequence $x_c(t)$ and the sampling interval T.

3. Consider the signal, $x(t) = e^{-5t}u(t-1)$
   (a) Evaluate its Laplace transform, $X(s)$ and specify ROC.
   (b) Determine the values of the finite numbers A and t_o such that the Laplace transform, $G(s)$ of

   $$g(t) = Ae^{-5t}u(-t-t_o)$$

   has the same algebraic form as $X(s)$. What is the ROC corresponding to $G(s)$?

4. Consider the signal, $x(t) = e^{-5t}u(t) + e^{-\beta t}u(t)$,

   And denote its Laplace transform by $X(s)$. What are the constraints placed on the real and imaginary parts of β if the region of convergence of $X(s)$ is $\mathrm{Re}\{s\} > -3$?

5. Find the system function or transfer function, $H(s)$, of the causal and stable LTI systems which are modeled by second-order differential equations

   - $\dfrac{d^2y(t)}{dt^2} + 3\dfrac{dy(t)}{dt} + 2y(t) = x(t)$
   - $5\dfrac{d^2y(t)}{dt^2} + 4\dfrac{dy(t)}{dt} + 5y(t) = 7x(t)$
   - $\dfrac{d^2y(t)}{dt^2} + 20\dfrac{dy(t)}{dt} + y(t) = x(t)$
   - $5\dfrac{d^2y(t)}{dt^2} + 4\dfrac{dy(t)}{dt} + 5y(t) = 7x(t) + \dfrac{1}{3}\dfrac{dx(t)}{dt}$

6. From each of the system function derived in above problem, find the corresponding *impulse response*, $h(t)$, by using inverse Laplace transform method.