

Example 4.5.f. A construction firm has recently sent in bids for 3 jobs worth (in profits) 10, 20, and 40 (thousand) dollars. If its probabilities of winning the jobs are respectively .2, .8, and .3, what is the firm's expected total profit?

Solution. Letting $X_i, i = 1, 2, 3$ denote the firm's profit from job i , then

$$\text{total profit} = X_1 + X_2 + X_3$$

and so

$$E[\text{total profit}] = E[X_1] + E[X_2] + E[X_3]$$

Now

$$E[X_1] = 10(.2) + 0(.8) = 2$$

$$E[X_2] = 20(.8) + 0(.2) = 16$$

$$E[X_3] = 40(.3) + 0(.7) = 12$$

and thus the firm's expected total profit is 30 thousand dollars. ■

Example 4.5.g. A secretary has typed N letters along with their respective envelopes. The envelopes get mixed up when they fall on the floor. If the letters are placed in the mixed-up envelopes in a completely random manner (that is, each letter is equally likely to end up in any of the envelopes), what is the expected number of letters that are placed in the correct envelopes?

Solution. Letting X denote the number of letters that are placed in the correct envelope, we can most easily compute $E[X]$ by noting that

$$X = X_1 + X_2 + \cdots + X_N$$

where

$$X_i = \begin{cases} 1 & \text{if the } i\text{th letter is placed in its proper envelope} \\ 0 & \text{otherwise} \end{cases}$$

Now, since the i th letter is equally likely to be put in any of the N envelopes, it follows that

$$P\{X_i = 1\} = P\{i\text{th letter is in its proper envelope}\} = 1/N$$

and so

$$E[X_i] = 1P\{X_i = 1\} + 0P\{X_i = 0\} = 1/N$$

Hence, from Equation (4.5.2) we obtain that

$$E[X] = E[X_1] + \cdots + E[X_N] = \left(\frac{1}{N}\right) N = 1$$