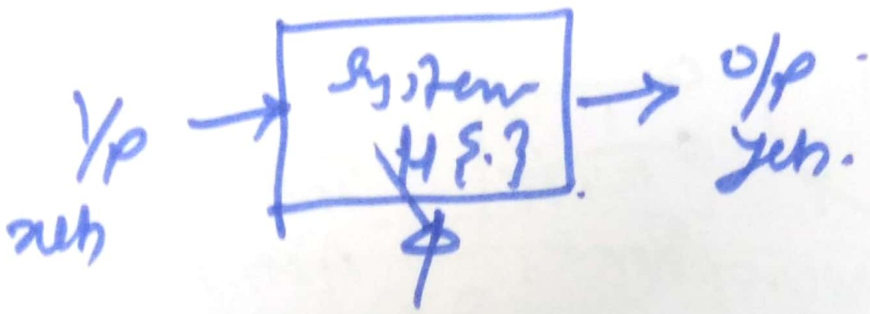
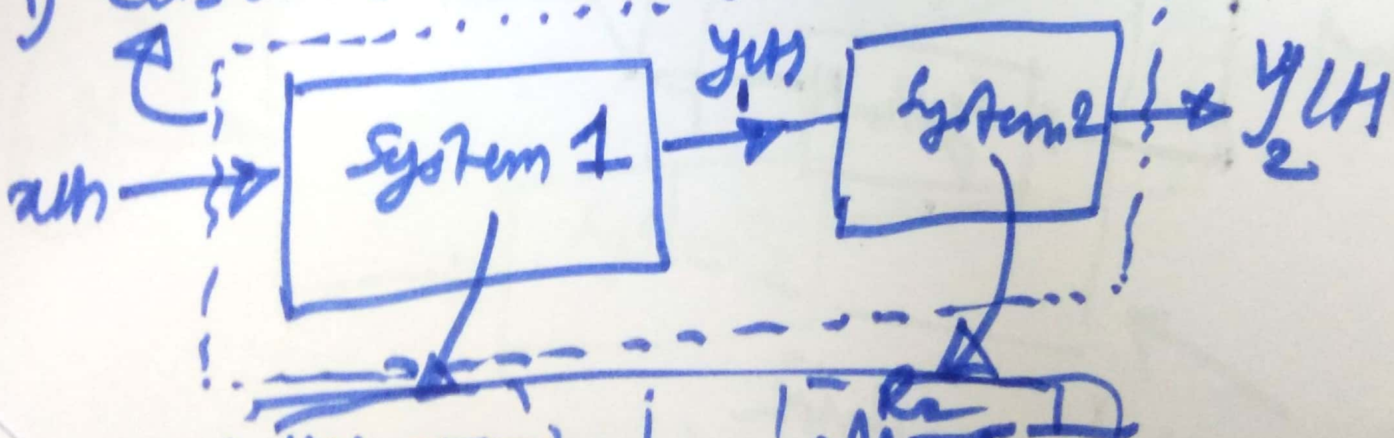
System Interconnections: → $\boxed{\text{Lecture 11}}$



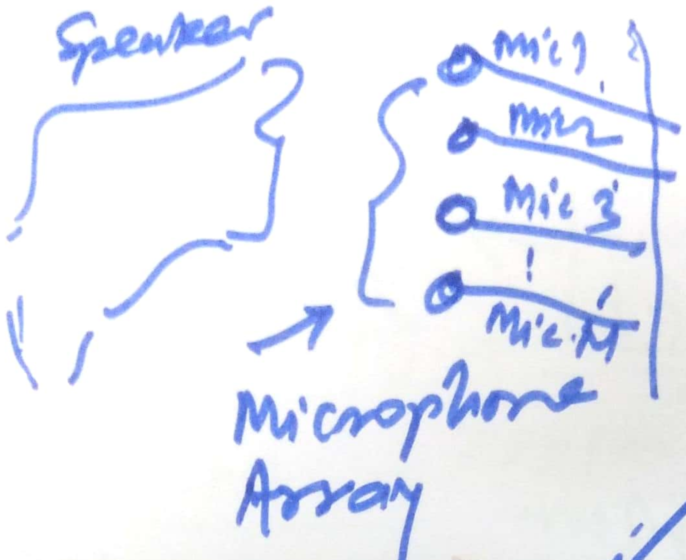
1 system

However, in realistic scenarios practical applications of signals & systems, we have many systems interconnected.

1) Cascade Interconnection

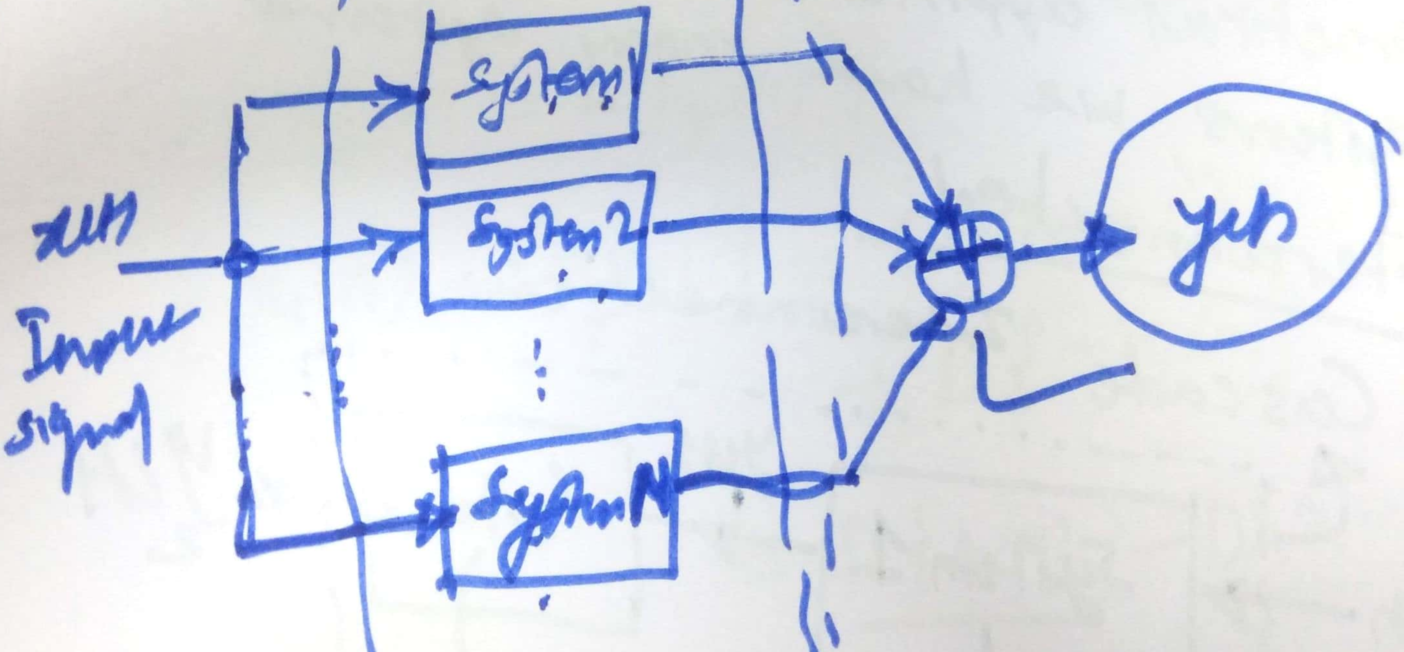


2) Parallel Interconnection



Commercial success of Voice Assistants OR Intelligent Personal Assistants (IPAs)

Speech Technology



Parallel Interconnection

4) Feedback Interconnection

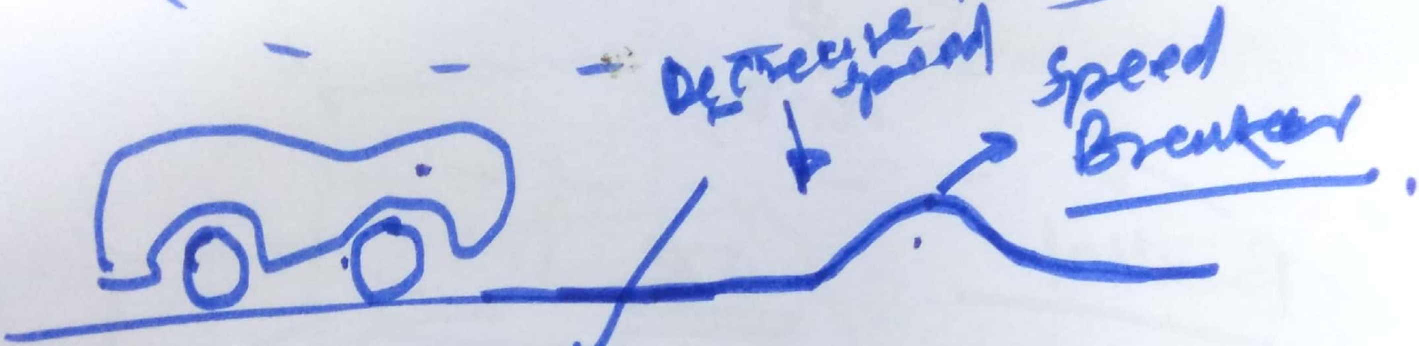
→ Level Control System

→ Weight control system.

→ Temperature control system

→ Pressure control system

→ Flow control system



control action or Process of regulation

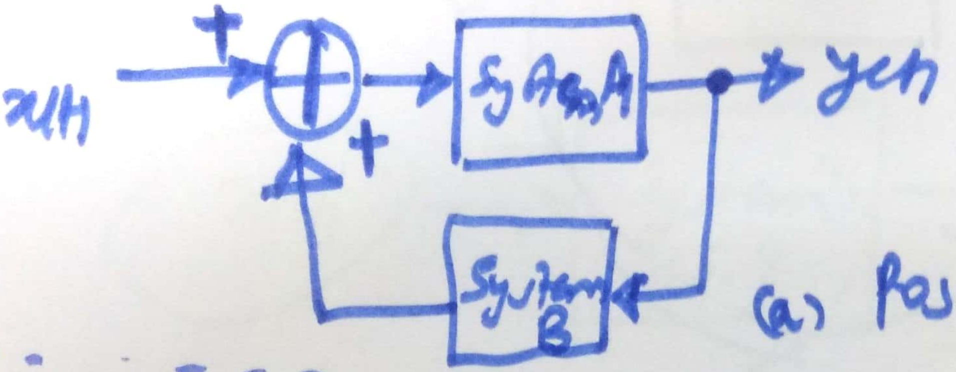
→ 40°C

iron press ⇒ Temperature setting

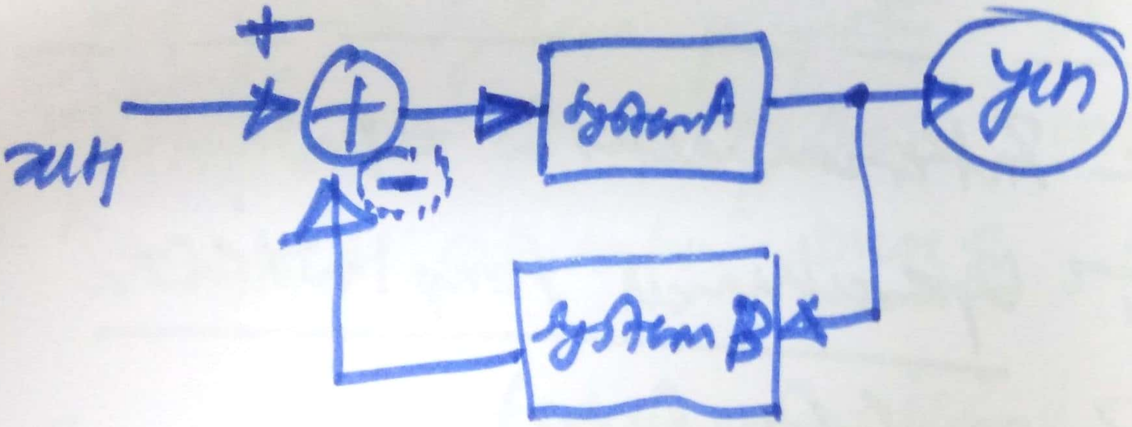
Feedback Interconnection

Positive Feedback

Negative Feedback.



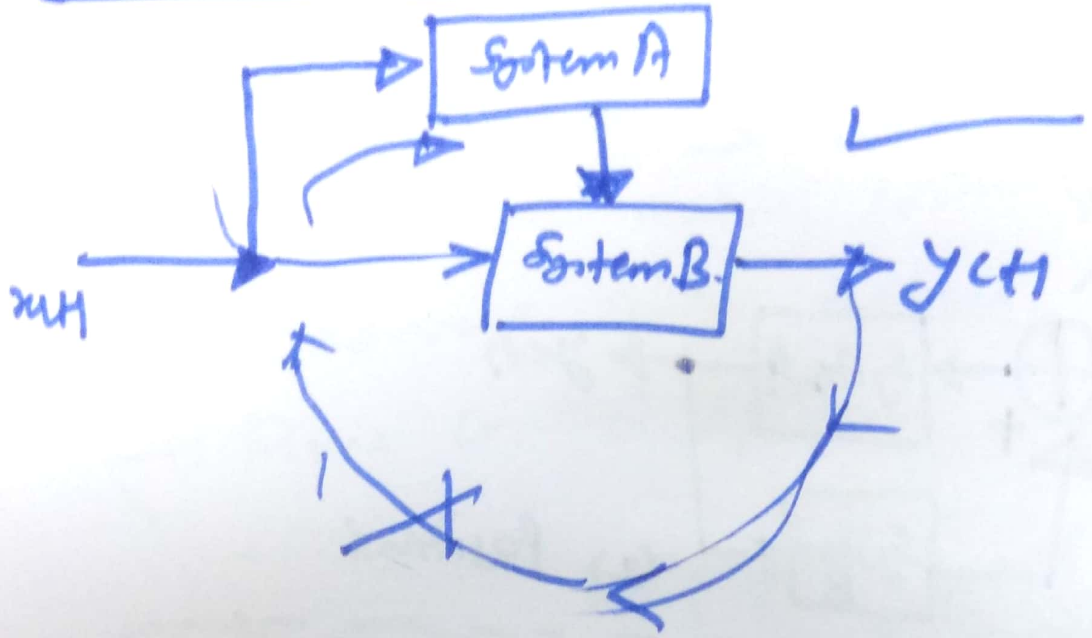
(a) Positive



(b) Negative.

Process of regulation.

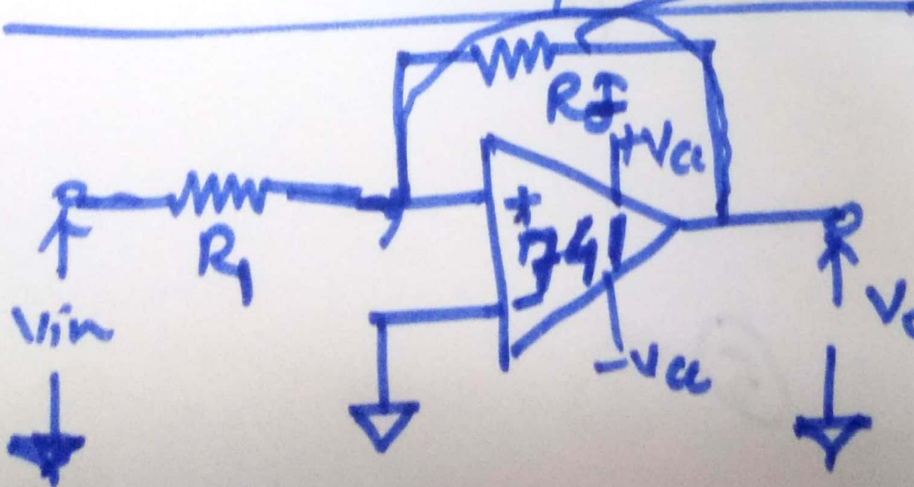
Feed Forward Interconnection



Feedback Interconnection ⇒ Design of Operational Amplifiers

(Op-Amp)

Feedback Amplifiers



→ feedback register.

$$\frac{V_{out}}{V_{in}} = \left(1 + \frac{R_f}{R_1}\right)$$

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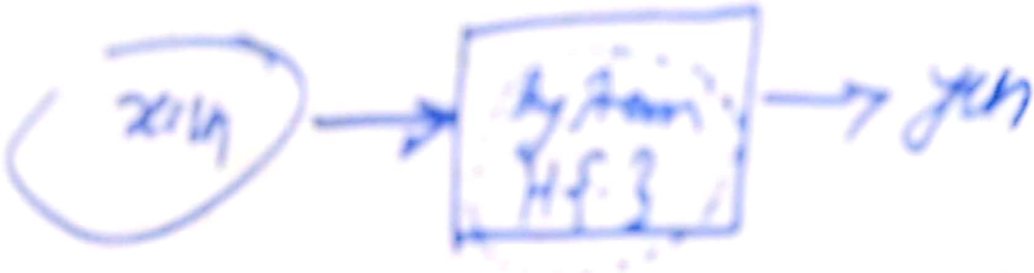
* Properties of systems: →

Control is the concern of signals & systems

1) Linearity: →

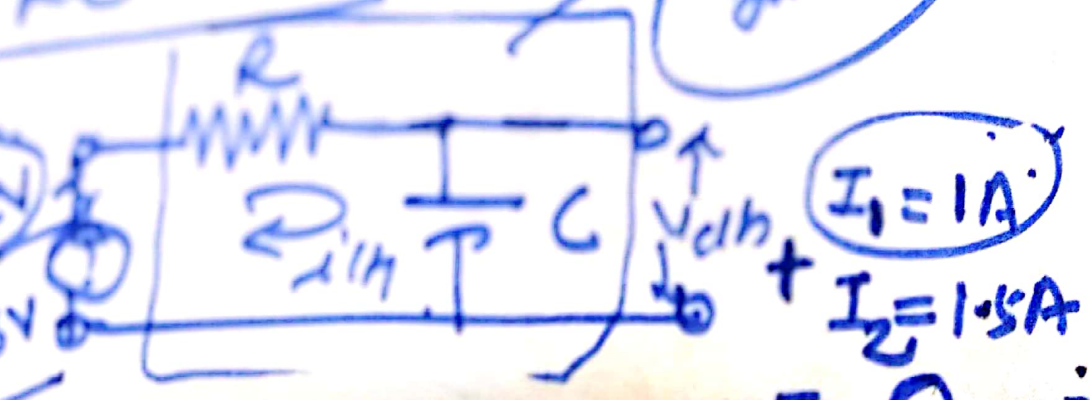


H{.} → system property → is an inherent attribute of a system and is independent of input excitation source.

Principle of superposition: →

A series RC circuit: → element system

Additivity



$K V_1 = 2 \times 2V = 4V$ → $K I_1 = 2 \times 1A = 2A$.

Homogenity

$x_1(n)$ $\qquad$ $y_1(n) = H\{x_1(n)\}$.

$x(n) \longrightarrow \boxed{\begin{matrix}\text{System} \\ H\{\cdot\}\end{matrix}} \longrightarrow y(n) = H\{x(n)\}$.

$x_2(n)$ $\qquad$ $y_2(n) = H\{x_2(n)\}$.

$$x_1(n) + x_2(n) \longrightarrow \left\{ \begin{matrix} y_1(n) + y_2(n) \\ = H\{x_1(n)\} \\ + H\{x_2(n)\} \end{matrix} \right.$$

Additivity

$$\left.\begin{matrix} a_1\, x_1(n) \longrightarrow a_1\, y_1(n) \\ a_2 \cdot x_2(n) \longrightarrow a_2 \cdot y_2(n) \end{matrix}\right] \text{Homogeneity}$$

Scalars

In sum, if a system satisfy both additivity and homogeneity properties then the system is said to obey <u>Principle of Superposition</u>.

Applications of Linearity :→

→ Electrical Network Theory.

→ Control systems

Linearity for DTS

$x_1(n)$ $\qquad$ $y_1(n)$

$x(n) \rightarrow \boxed{D\{\cdot\}} \rightarrow y(n) = D\{x(n)\}$

$x_2(n)$ $\qquad$ $y_2(n)$

Additivity

$x_1(n) + x_2(n) \xrightarrow{\quad} ?\ y_1(n) + y_2(n)$

individual excitation signals

$a_1 x_1(n) \longrightarrow a_1 y_1(n)$

$a_2 x_2(n) \longrightarrow a_2 y_2(n)$ } Homogeneity

DTS

$x(n) = \sum_{\langle k \rangle} a_k x_k(n) \rightarrow$ Linear system $\rightarrow$ ~~y(n)~~ $y(n)$

scalar

$y(n) = \sum_{\langle k \rangle} a_k y_k(n)$

$x(n) \longrightarrow \boxed{CTS} \longrightarrow y(n).$

$x(n) = \sum_{k=1} a_k x_k(n) \longrightarrow \boxed{\text{Linear System}} \longrightarrow y(n) = \sum_{k=1} a_k y_k(n)$

Principle of superposition

$a_1 x_1(n) + a_2 x_2(n) \longrightarrow a_1 y_1(n) + a_2 y_2(n)$

X X X X

Problems on Linearity Property:→

1) For the system $y(n) = H\{x(n)\} = 2x(n) + 5$, Comment on linearity?

Soln:

$$x(n) \rightarrow \boxed{H\{\cdot\}} \rightarrow y(n) = 2x(n) + 5$$

Step I) Input-output relationship.
↳ I/O.

$$y(n) = H\{x(n)\} = 2 \cdot x(n) + 5$$

I/O

Step II) Find o/p due to $x_1(n)$ & $x_2(n)$.

$\therefore$ Let $y_1(n) = H\{x_1(n)\} = 2x_1(n) + 5$

$y_2(n) = H\{x_2(n)\} = 2x_2(n) + 5$

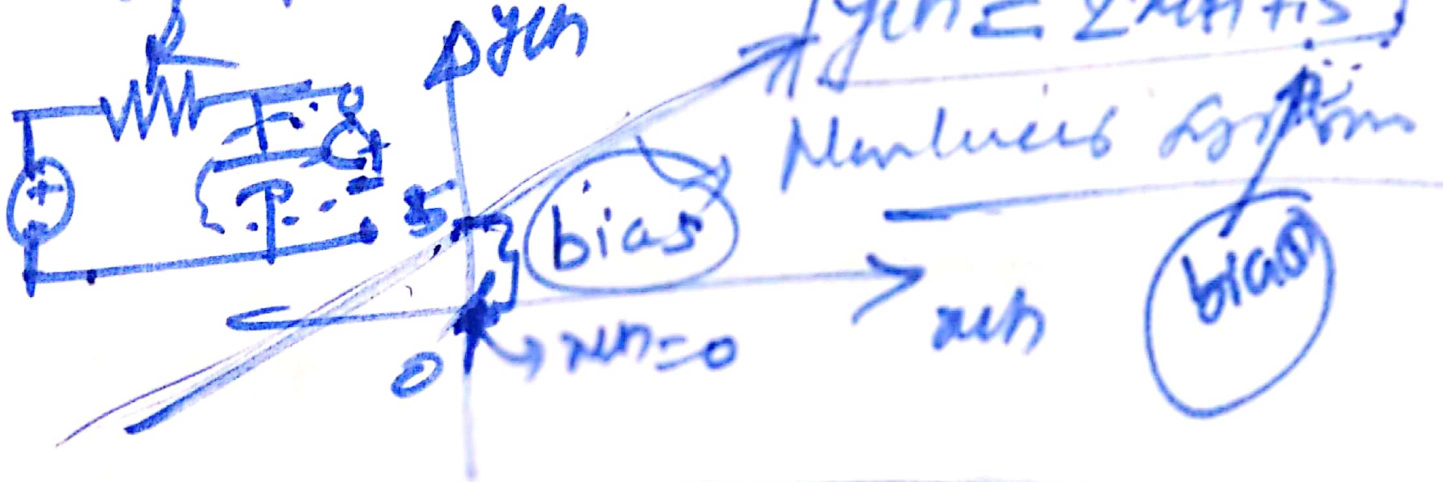
$$y_1(n) + y_2(n) = 2x_1(n) + 2x_2(n) + 10 \quad \text{[A]}$$

Step III Find $H\{x_1(n) + x_2(n)\}$

$$\therefore y_3(n) = H\{x_1(n) + x_2(n)\} = 2[x_1(n) + x_2(n)] + 5 \quad \text{[B]}$$

$\therefore$ The $H\{\cdot\}$ does not obey Principle of superposition.

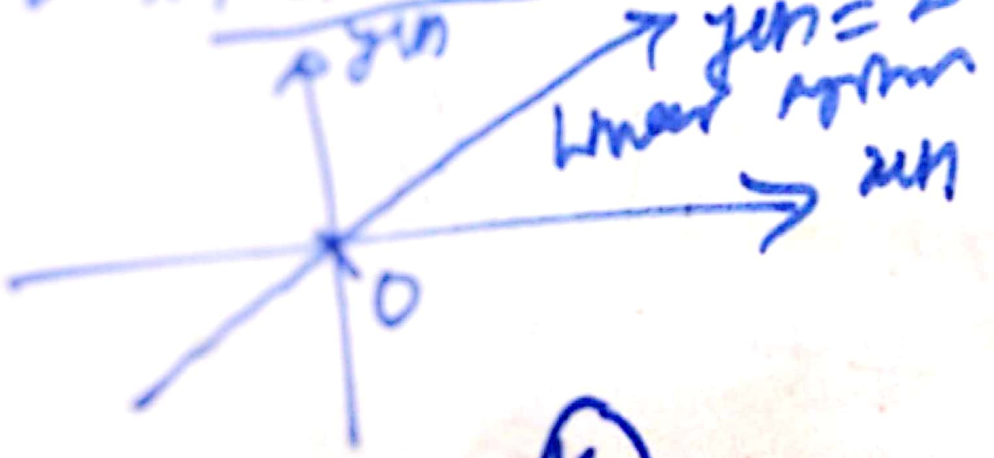
$\Rightarrow$ The system $H\{\cdot\}$ is NOT linear or nonlinear system



(2) $y(n) = H\{x(n)\} = 2x(n).$

Whether system $H\{x(n)\}$ is linear ??

Sol^n:-> $H\{\cdot\}$ is linear :->



(4)

From eqn (A) & eqn (B)

$H\{x_1(n) + x_2(n)\} \neq y_1(n) + y_2(n)$.

$\therefore$ Additivity property is NOT satisfied.

Step IV) To check for Homogeneity

$\therefore H\{a \cdot x(n)\} = \cancel{2 \cdot a x}$

$2ax(n) + 5$ (A1)

$y(n) = H\{x(n)\} = 2x(n) + 5$

$a \cdot y(n) = a H\{x(n)\} = a\left[2x(n) + 5\right]$

$a \cdot y(n) = 2ax(n) + 5a$ (B1)

From eqn (A1) & eqn (B1)

$\Rightarrow H\{a \cdot x(n)\} \neq a \cdot y(n)$

$\therefore$ The system $H\{\cdot\}$ is disobeying homogeneity property.

(13)

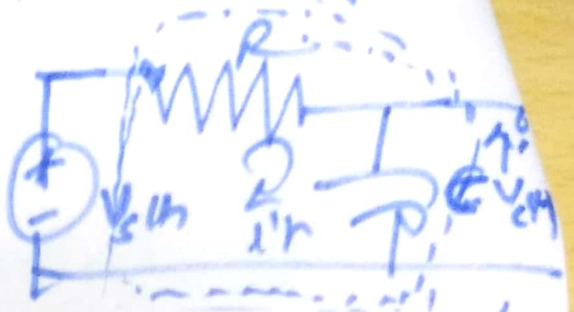
Linear system

If capacitor 'C' is discharged before application of source voltage, i.e. $v_s(t) = 0$ V & $v_c(t) = 0$

zero i/p

zero i/p

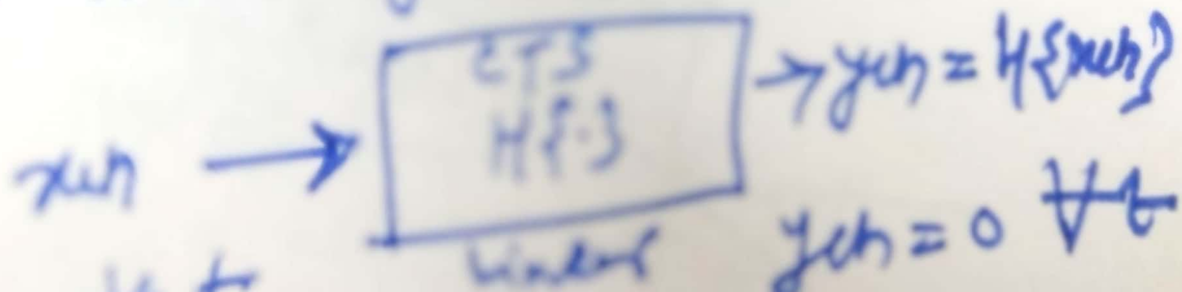
Nonlinear



If capacitor 'C' is NOT discharged before application of source voltage i.e. $v_s(t) = 0$ and $v_c(t) \neq 0$

zero i/p ⇒ Non-zero o/p.

Proposition 1.2: → Prove that for a continuous time linear system zero input must give zero output



$y(t) = 0 \ \forall t$

$x(t) = 0 \ \forall t$

Question 11 : → Prove that for a discrete time linear system zero input must give zero output.

$$x(n) \longrightarrow \boxed{\begin{matrix}\text{DTS} \\ D\{\cdot\} \\ \text{Linear}\end{matrix}} \longrightarrow y(n) = D\{x(n)\}$$

$x(n) = 0 \ \forall n$ $\qquad$ $y(n) = 0 \ \forall n$

0	0	0	0	0	0	0	
-3	-2	-1	0	1	2	3	... n

$\Longrightarrow$

0	0	0	0	0	0	0	
-3	-2	-1	0	1	2	3	... n

Proof : → Consider a discrete-time $D\{\cdot\}$ with I/O relationship

$$y(n) = D\{x(n)\}.$$

Given that $D\{\cdot\}$ = linear system

$\Longrightarrow$ $D\{\cdot\}$ satisfy both additivity & homogeneity

a) Proof using homogeneity : → $D\{0\} = 0$

$$y(n) = D\{x(n)\}$$

$$D\{a . x(n)\} = a . D\{x(n)\} = a . y(n)$$

Let $a = 0$ $\therefore$ $D\{\underline{0 . x(n)}\} = \underline{0} . y(n)$

$D\{0\} = 0$

$x(n) = 0 \; \forall n \quad y(n) = 0 \; \forall n.$

b) Proof using additivity

Let $x(n) = x(n) - x(n)$

$D\{x_1(n) + x_2(n)\} = D\{x_1(n)\} + D\{x_2(n)\}$

$= y_1(n) + y_2(n)$

Let $x_2(n) = -x_1(n)$

$D\{x_1(n) - x_1(n)\} = D\{x_1(n)\} + D\{-x_1(n)\}$

$= D\{x_1(n)\} - D\{x_1(n)\}$

$D\{x_1(n) - x_1(n)\} = y_1(n) - y_1(n)$

$D\{0\} = 0$

$\Rightarrow$ if $x(n) = 0 \; \forall n \Rightarrow y(n) = 0 \; \forall n$

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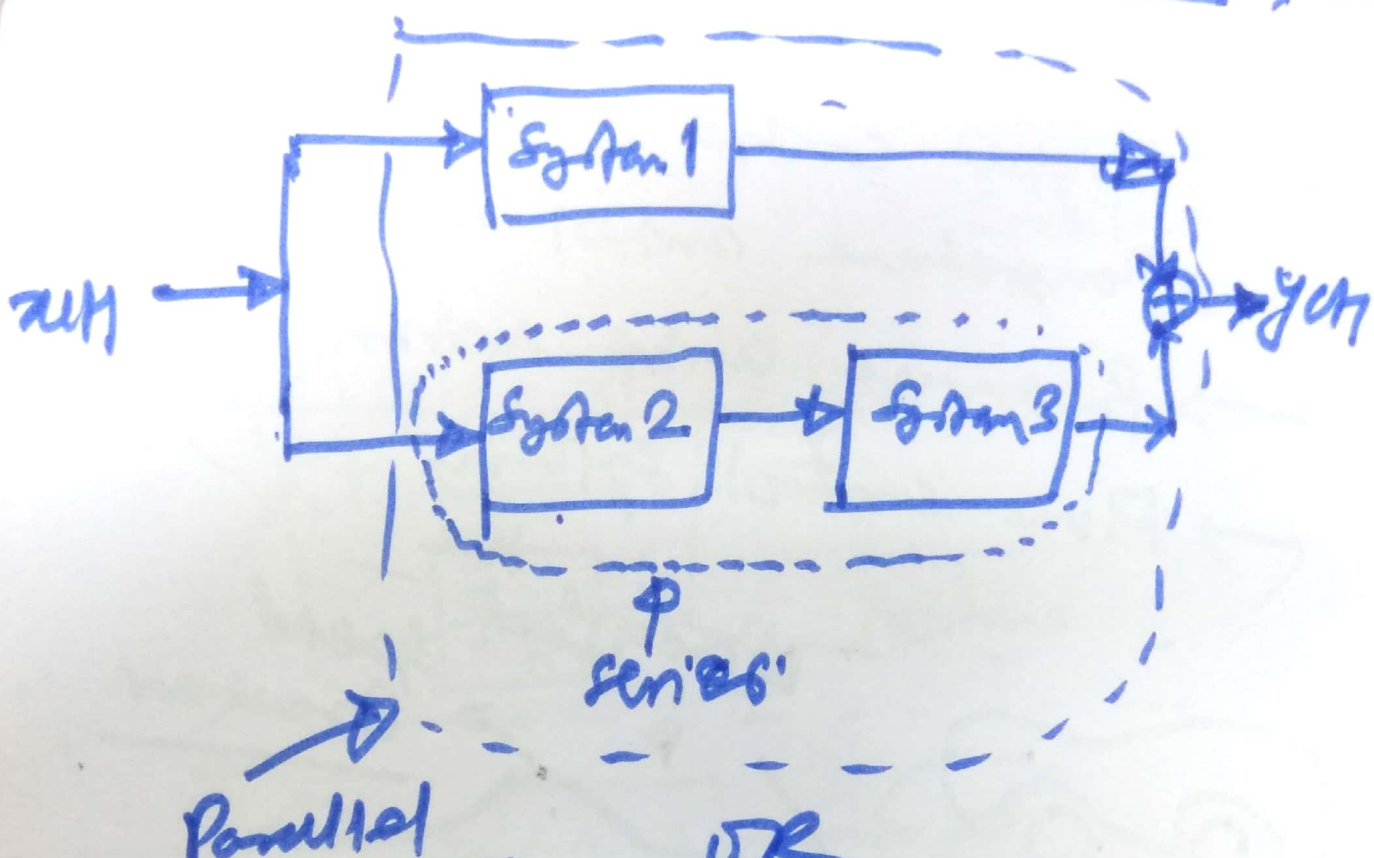
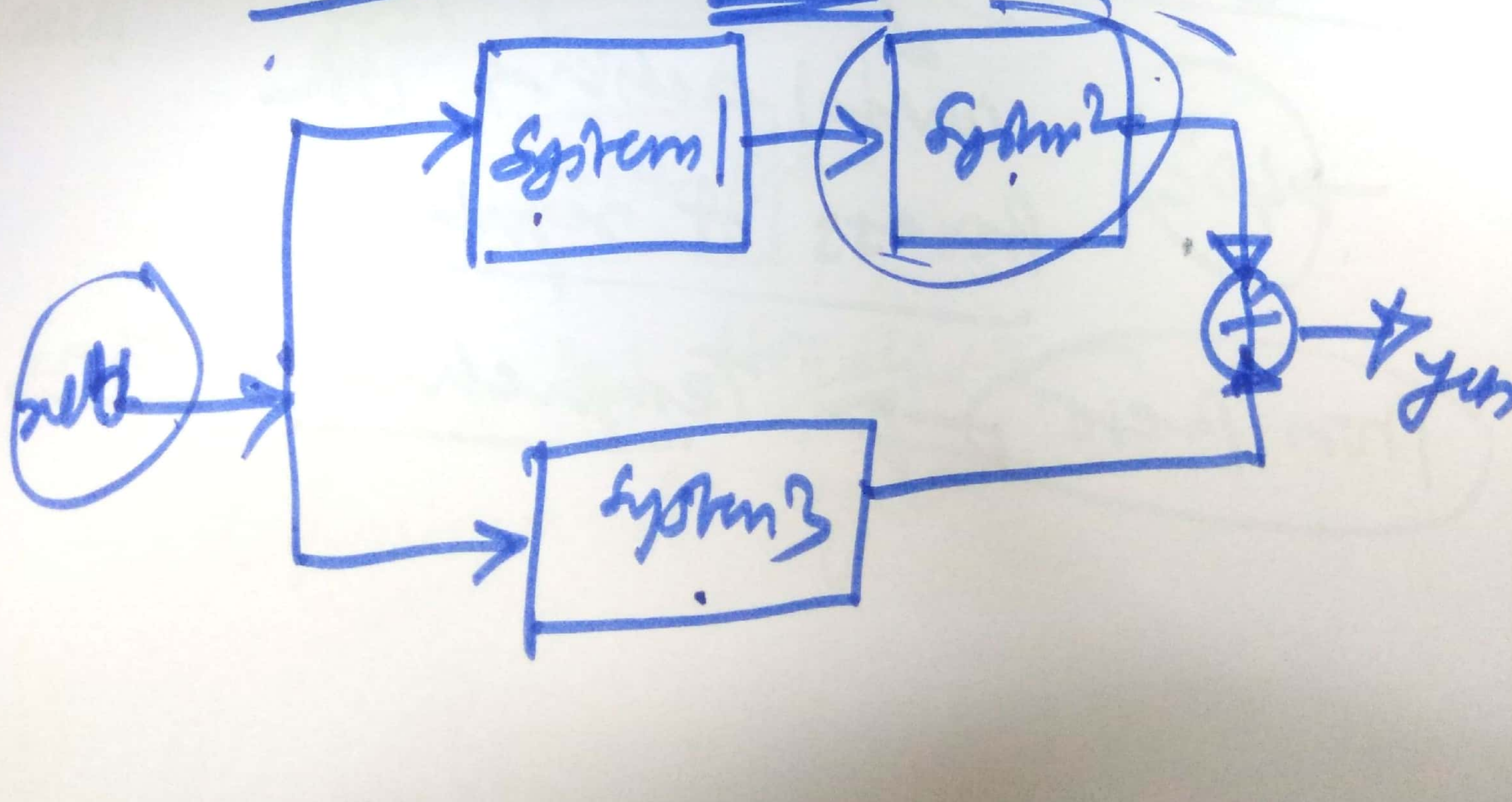
3) Series – Parallel Interconnection



OR



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