Computational Numerical Methods

CS 374

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Let $x_n \approx \alpha$.

$$\alpha - x_{n+1} = (\alpha - x_n)^2 \cdot \frac{-f''(c_n)}{2f'(x_n)}.$$

Then $$-\frac{f''(c_n)}{2f'(c_n)} \approx -\frac{f''(\alpha)}{2f'(\alpha)} = M$$

$$\alpha - x_{n+1} = (\alpha - x_n)^2 M.$$

Multiplying M in both sides.

$$M(\alpha - x_{n+1}) = [M(\alpha - x_n)]^2.$$

$$\therefore \quad M(\alpha - x_n) = \left[M(\alpha - x_{n-1})\right]^2$$

$$= \left[M(\alpha - x_{n-2})\right]^{2^2}$$

$$= \left[M(\alpha - x_0)\right]^{2^n}.$$

as per assumption $\alpha \approx x_n \Rightarrow \alpha - x_n \approx 0$

is possible when

$$|M(\alpha - x_0)| < 1$$

$$|\alpha - x_0| < \frac{1}{|M|} = \frac{2f'(\alpha)}{f''(\alpha)}.$$

For earlier considered problem

$$f(x) = x^6 - x - 1$$

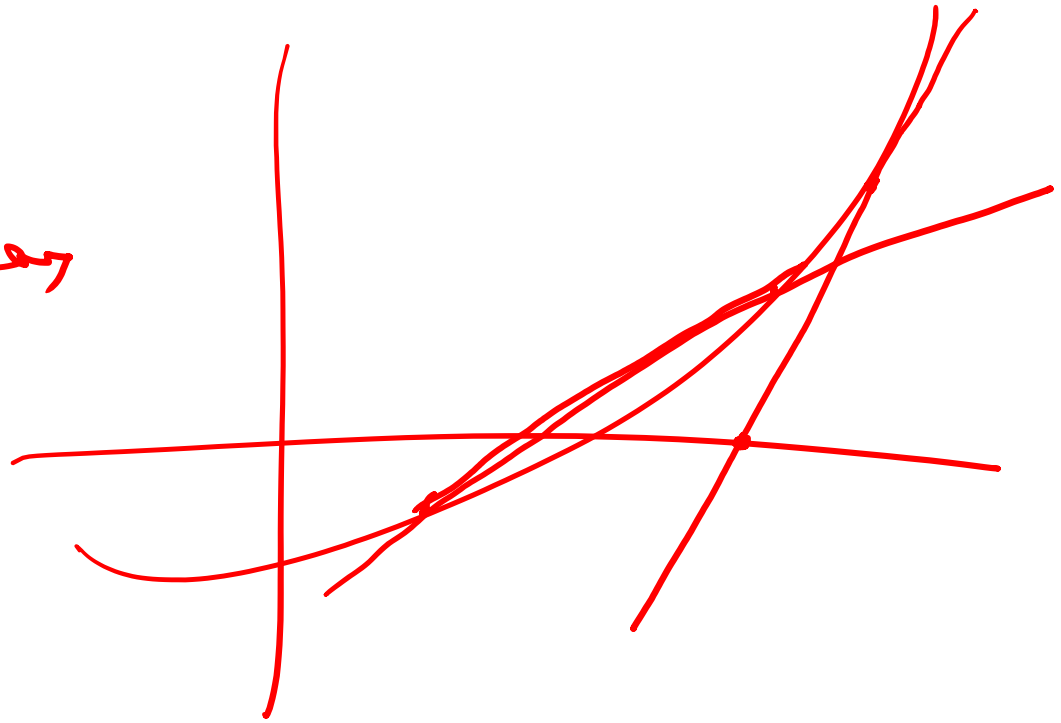
$$f'(x) = 6x^5 - 1 = 10.2870$$

$$f''(x) = 30x^4 = 49.737$$

$$\therefore \quad M = \frac{1}{M} = \frac{2f'(\alpha)}{f''(a)} = 0.4136$$

$$\therefore |\alpha - x_1| < 0.4136 \qquad \alpha = 1.134724$$

Secant Method



We have two initial guesses. say

~~Let~~ x_0 & x_1

With $(x_0, f(x_0))$, $(x_1, f(x_1))$

On one can draw a line.

which approximates $f(x)$

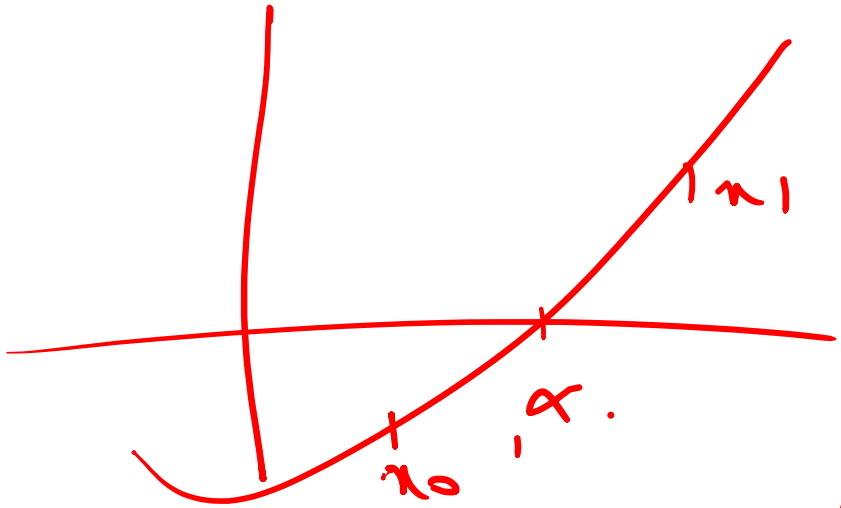
Then find the root of the line.

which will approximate the root of $f(x) = 0$

The eqn of the line joining.

$(x_0, f(x_0))$ $(x_1, f(x_1))$

$$p(x) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0) + f(x_0)$$

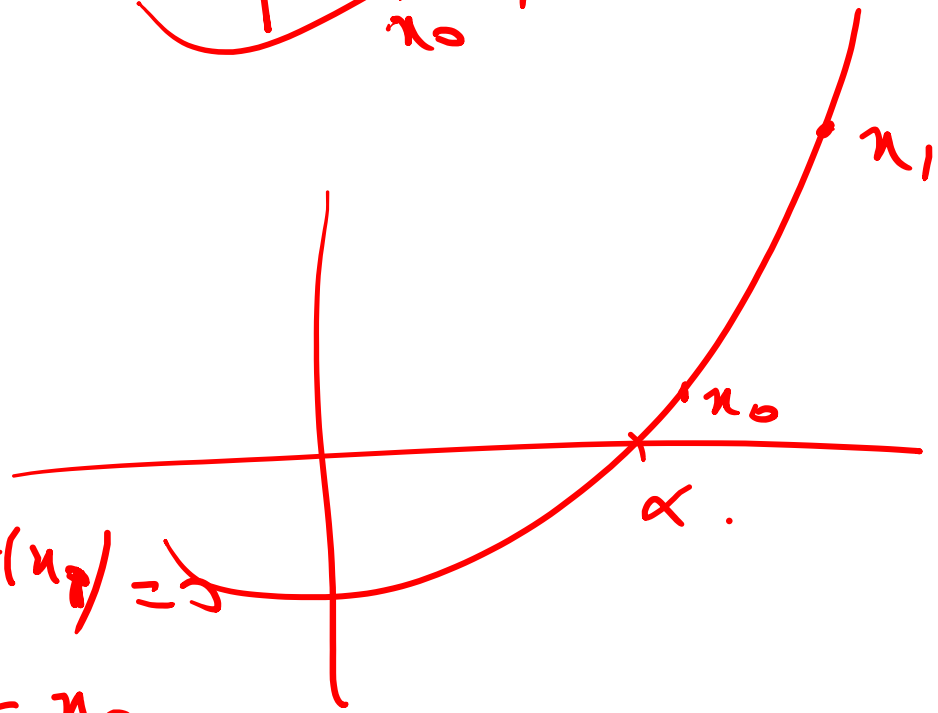


Get for root of this line.

$p(x_2) = 0$

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0}(x_2 - x_1) + f(x_1) = 0$$

$$x_2 = x_1 - f(x_1) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$



The general iterative formula.

$$x_{n+1} = x_n - f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \quad n \geq 1$$

For $f(x) = x^6 - x - 1$ find the root of $f(x) = 0$.

n	x_n	$f(x_n)$	$x_n - x_{n-1}$	$\alpha - x_{n-1}$	$f(x_n)\cdot\frac{x_n - x_{n-1}}{f(x_n)\cdot f(x_{n-1})}$
0	2	61			
1	1	-1	-1		
2.	1.01612903	-9.15×10^{-1}	1.61×10^{-2}.	1.35×10^{-1}	
3	1.19057777	6.57×10^{-1}	1.74×10^{-2}	1.19×10^{-1}	
4.	1.11765583	-1.68×10^{-1}	-7.29×10^{-2}.	-5.59×10^{-2}	
5.	1.13253155	-2.24×10^{-2}	1.49×10^{-2}.	1.71×10^{-2}	
6	1.13481681	9.54×10^{-4}	2.29×10^{-3}	2.19×10^{-3}	
7	1.13472365	-5.07×10^{-6}	-9.32×10^{-5}	-9.27×10^{-5}	
8	1.13472414	-1.13×10^{-9}	4.92×10^{-7}	4.92×10^{-7}.	

Task show that the error in secant method.

follows.

$$\alpha - x_{n+1} = (\alpha - x_n)(\alpha - x_{n-1}) \left[\frac{-f''(\xi_n)}{2f'(\xi_n)} \right]$$

where ξ_n is number b/w ~~the~~ the largest & smallest of x_n, x_{n-1}, α.

& ξ_n is a number b/w x_n & x_{n-1}