LECTURE 26

$$M\ddot{x} = -Kx.$$

Transformation :- $x = S\xi$.

$$\ddot{x} = -M^{-1}Kx.$$

$$\Rightarrow S\ddot{\xi} = -(M^{-1}K)S\xi$$

$$\Rightarrow S^{-1}S\ddot{\xi} = -S^{-1}(M^{-1}K)S\xi.$$

$$\Rightarrow \ddot{\xi} = -S^{-1}(M^{-1}K)S\xi$$

$$\Rightarrow \ddot{\xi} = -(S^{-1}K'S)\xi \quad \Rightarrow \underline{\ddot{\xi} = -K'_D\xi.}$$

$$\xi = S^{-1}x.$$

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

$$\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = S^{-1}x = \begin{pmatrix} & \\ & \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\ddot{\xi} = -K'_D\,\xi. \quad = -\begin{pmatrix} k_{11} & 0 \\ 0 & k_{22} \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}.$$

$$2\ddot{x}_1 + \omega^2(5x_1 - 3x_2) = 0$$

$$2\ddot{x}_2 + \omega^2(-3x_1 + 5x_2) = 0$$

$$M = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$K = \begin{pmatrix} 5 & -3 \\ -3 & 5 \end{pmatrix}\omega^2 .$$

$$M^{-1}K = \omega^2 \begin{pmatrix} \frac{5}{2} & -\frac{3}{2} \\ -\frac{3}{2} & \frac{5}{2} \end{pmatrix} = K'$$

Normal mode freqs. will be eigenvalues of the above matrix K'.

$$|K' - \lambda I| = 0 \; . \Rightarrow \quad \begin{vmatrix} \frac{5}{2} - \lambda & -\frac{3}{2} \\ -\frac{3}{2} & \frac{5}{2} - \lambda \end{vmatrix} = 0 .$$

$$\Rightarrow \quad \lambda_1 = 1 \times \omega^2$$

$$\lambda_2 = 4 \times \omega^2$$

Case :- One eigenvalue is zero.

$$\ddot{\xi}_1 = 0.$$

$$\Rightarrow \xi_1 = At + B .$$

$$x_1 + x_2 = At + B.$$

$$\ddot{\xi}_1 = -\lambda_1 \xi_1$$

$$\ddot{\xi}_2 = -\lambda_2 \xi_2$$

$$\ddot{\xi}_1 = -\text{eig}_1 \xi_1$$

$$\ddot{\xi} = -K_D' \xi.$$

$$= -\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \xi.$$

Eigenvalues of K' are ω^2, $4\omega^2$.

Eigenvector is :- $e = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$.

$K'e = \lambda e$.

$\underbrace{K' \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \lambda_1 \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}}$

$e_1 - e_2 = 0$

$\Rightarrow e_1 = e_2$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\underbrace{K' \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \lambda_2 \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}}$.

$e_1 + e_2 = 0$

$\Rightarrow e_1 = -e_2$ $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$$K'\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \lambda_1 \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$$

$$\Rightarrow \quad \omega^2 \begin{pmatrix} \frac{5}{2} & -\frac{3}{2} \\ -\frac{3}{2} & \frac{5}{2} \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \omega^2 \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$$

$$\left.\begin{aligned} \frac{5}{2} e_1 - \frac{3}{2} e_2 &= e_1 \\ -\frac{3}{2} e_1 + \frac{5}{2} e_2 &= e_2 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} \frac{3}{2} e_1 - \frac{3}{2} e_2 &= 0 \\ -\frac{3}{2} e_1 + \frac{3}{2} e_2 &= 0 \end{aligned}\right\} \Rightarrow e_1 = e_2$$

Similarly for λ_2.

$$e = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad e = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

$$S = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad S^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}.$$

Verify

$$S^{-1} K' S = \begin{pmatrix} \omega^2 & 0 \\ 0 & 4\omega^2 \end{pmatrix}.$$

S has been obtained.

$$\xi = S^{-1} x. = S^{-1} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Rightarrow \begin{aligned} \xi_1 &= \tfrac{1}{2}(x_1 + x_2) \\ \xi_2 &= \tfrac{1}{2}(x_1 - x_2). \end{aligned}$$

Normal coord eqns

$$\left.\begin{aligned} \ddot{\xi}_1 &= -\omega^2 \xi_1 \\ \ddot{\xi}_2 &= -4\omega^2 \xi_2 \end{aligned}\right\} \Rightarrow \begin{aligned} (\ddot{x}_1 + \ddot{x}_2) &= -\omega^2 (x_1 + x_2) \\ (\ddot{x}_1 - \ddot{x}_2) &= -4\omega^2 (x_1 - x_2) . \end{aligned}$$

Inhomogenous ODEs :- Superposition of terms in R.H.S

$$y'' + y' - 2y = e^x + 4\sin 2x + (x^2 - x),$$

$$y_c'' + y_c' - 2y_c = 0$$

$$\Rightarrow (D+2)(D-1)y_c = 0$$

$$y_c = Ae^x + Be^{-2x}$$

$$y_p'' + y_p' - 2y_p = e^x + 4\sin 2x + (x^2 - x).$$

$$\Rightarrow (D+2)(D-1)y_p = \underline{e^x} + 4\,\underline{\sin 2x} + \underline{(x^2 - x)}.$$

y_{p_1}, y_{p_2}, y_{p_3}.

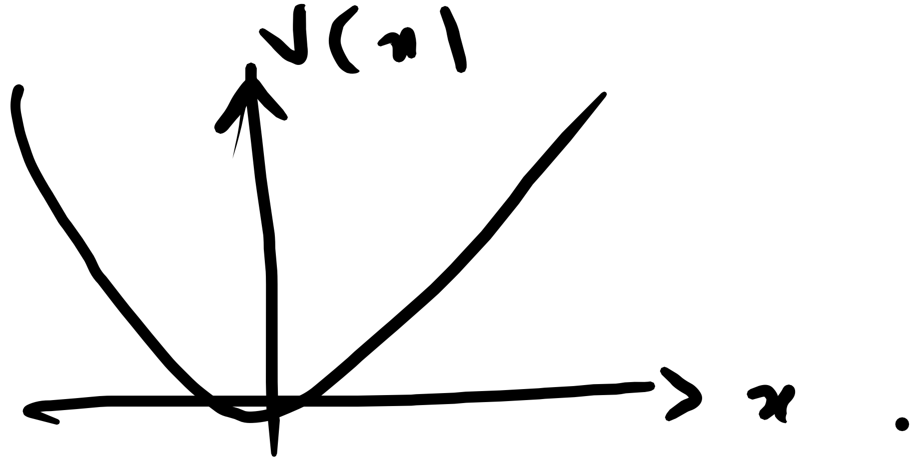
$$y_p = y_{p_1} + y_{p_2} + y_{p_3}$$

$$(D+2)(D-1)(y_{p_1} + y_{p_2} + y_{p_3}) = e^x + 4\sin 2x + (x^2 - x).$$

VII Taylor expansion of $V(x)$.

$$V(x) = V(x_0) + V'(x_0)(x-x_0) + \frac{1}{2}V''(x_0)(x-x_0)^2 + \cdots$$

If x_0 is extremum,



$$V(x) = V(x_0) + \frac{1}{2}V''(x_0)(x-x_0)^2$$

$$F = -\frac{dV}{dx} = -V''(x)(x-x_0).$$