1. Which of the following sets with the corresponding binary operation (denoted by $*$) forms a Group? If it is not a group, which Group axiom does it violate? If it is a group, is the group abelian?

   (a) Let $G = \{z \in \mathbb{C} \mid |z| = 1\}$, with the binary operation $*$ denoting the usual complex number multiplication.

   (b) Let $G = \{A \in \mathbb{R}^{n \times n} \mid A^T = A, A \text{ is invertible}\}$ with the binary operation $*$ denoting usual matrix multiplication.

   (c) $G = \mathbb{R}$, with $\forall a, b \in G, a * b := \exp(a) \cdot \exp(b)$, where $\cdot$ denotes usual real number multiplication.

   (d) $G = \{0, 1, 2, 3\}$ with the binary operation $*$ denoting multiplication modulo-4.

   (e) $G = \{1, 2, 3, 4\}$ with the binary operation $*$ denoting multiplication modulo-5.

2. For the given sets V, fields $\mathbb{F}$, and the binary operations $+, \cdot$, find out whether $(V, +, \cdot)$ forms a vector space over $\mathbb{F}$ or not.

   (a) Let $V = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$, and $\mathbb{F} = \mathbb{R}$. The binary operations are: $+$ is the same as the addition of vectors in $\mathbb{R}^3$, and scalar multiplication $\cdot$ is the same as that of $\mathbb{R}$ with $\mathbb{R}^3$.

   (b) Let $V = \mathcal{C}[a, b]$ be the set of real-valued continuous functions defined on the interval $[a, b] \subset \mathbb{R}$, $\mathbb{F} = \mathbb{R}$, with the following binary operations: $\forall f, g \in V, (f + g)(x) := f(x) + g(x), \forall x \in [a, b]$, and $\forall a \in \mathbb{R}, \forall f \in V, (a \cdot f)(x) := a \times f(x), \forall x \in [a, b]$.

   (c) Let $V = \mathbb{R}, \mathbb{F} = (\mathbb{Z}_2, \oplus, \times)$, where $\oplus$ denotes addition modulo-2, and $\times$ is the usual multiplication. The binary operations are: Addition is the usual real number addition, scalar multiplication is also the usual real number multiplication.

   (d) Let $V = (0, \infty) \subset \mathbb{R}, \mathbb{F} = \mathbb{R}$. The binary operations are as follows: $\forall p, q \in V, p + q := p \times q$, where $\times$ is the usual real number multiplication, and $\forall a \in \mathbb{R}, \forall p \in V, a \cdot p = p^a$.

   (e) $V = \mathbb{C}, \mathbb{F} = \mathbb{R}$ with usual complex number addition and multiplication between real and complex numbers defining the two binary operations.