DA-IICT Gandhinagar
SC 224, Winter 2022-2023
End Semester Exam: BTech Semester IV (ICT & ICT-CS)
Date: 09/05/2023, Duration: 120 minutes
Maximum Marks: 50

General Instructions

- Answer all questions.
- Kindly ensure that no relevant books, notes are open during the exam duration.

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1. We want to model traffic flow on the highway shown in Fig. 1:

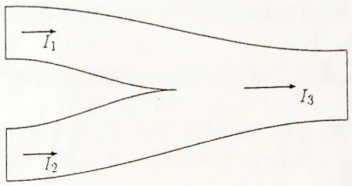

Figure 1: Highway representation.

Let X_1 denotes the number of cars passing on I_1 during one minute, X_2 for I_2 and X_3 for I_3. We assume that X_1 and X_2 are independent and follow a Poisson law with parameters λ_1 and λ_2 respectively. What can you say about X_3? **[4 marks]**

2. Consider a source $\mathcal{S}$ which emits certain symbols (messages) with certain probabilities given as:

Symbols	s_1	s_2	s_3	s_4	s_5	s_6	s_7
Probability	0.50	0.26	0.11	0.04	0.04	0.03	0.02

(a) Using the binary Huffman coding scheme, obtain the codeword for each of the symbols of source $\mathcal{S}$ and calculate the efficiency of this Huffman coding scheme **[8 marks]** = 1.98, 99%

(b) Suppose the source $\mathcal{S}$ emits infinite messages with probability given as $P(s_i) = 2^{-i}$; $i = \{1, 2, 3 \cdots\}$. Calculate the entropy of the source. **[4 marks]** =2

3. An exam of 100 marks is conducted for a batch of 178 students where the marks obtained by the students are normally distributed. A random sample with marks of 25 students is collected with the average marks being 42 marks.

(a) Considering the variance of the marks of the batch to be 9 marks, estimate the average marks of the entire batch with a confidence level of (i) 95% and (ii) 98% **[3 marks]**

(b) Considering the variance of the marks of the chosen sample to be 7 marks, estimate the average marks of the entire batch with a confidence level of (i) 95% and (ii) 98% [3 marks]

(c) The course instructor conducting the exam claims that the average marks of the batch is 46 with a standard deviation of 2.5 marks. Use 95% confidence level and test for hypothesis that true average marks of the batch is 46 marks. [4 marks]

Note: Relevant z and t multiplier values: $Z_{0.025} = 1.96$, $Z_{0.01} = 2.33$, $t_{0.025,24} = 2.064$, $t_{0.01,24} = 2.492$.

4. Consider a linear regression model illustrating the scenario where output is zero corresponding to zero input, i.e,

$$y_i = \beta x_i + \epsilon_i; \quad i = 1, 2, \cdots, n \tag{1}$$

Here, x_i, y_i represent the set of input and output values for the ith set of data and β is the fixed value unknown parameter. The random errors ϵ_i's are considered to be independent and identically distributed (IID) following Normal distribution where $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$

(a) Derive the Least Squares estimator $\hat{\beta}$ of the parameter β. [3 marks]

(b) Obtain the expected value $E\{\hat{\beta}\}$ and variance $Var(\hat{\beta})$ of the estimator $\hat{\beta}$. For the fitted model represented as $\hat{y}_i = \hat{\beta} x_i$, compute the variance of the residual represented as $r_i = y_i - \hat{y}_i$. [3+3 marks]

(c) The metric α is defined as

$$\alpha = \frac{Var(r_i)}{Var(\hat{\beta})} - x_i^2 \tag{2}$$

Given that $n = 10$ and $x_i = \left(\frac{1}{2}\right)^i$, obtain the value of the metric α. [4 marks]

5. A machine produces parts whose diameter X (in centimeters) has approximately a Gaussian distribution $N(\mu, \sigma^2 = 0.0001)$. What should the value of μ be so that no more than 1% of the parts have a diameter greater than 3 cm? Assuming $\mu = 3$, calculate the PMF of $Y := |X - 3|$. [7 marks] $\mu < 2.97$

6. (St. Petersburg paradox:) You toss independently a fair coin and you count the number of tosses until the first tail appears. If this number is n, you receive 2^n dollars. What is the expected amount that you will receive? How much would you be willing to pay to play this game? [4 marks] ∞ I'll play