1) a) Ideal delay system $\boxed{\text{Tutorial 04}}$

$$y(n) = x(n-n_d) = D\{x(n)\} \rightarrow I/O$$

$$\therefore D\{x_1(n)\} = y_1(n) = x_1(n-n_d).$$

$$\therefore D\{x_2(n)\} = y_2(n) = x_2(n-n_d).$$

$$\therefore D\{x_1(n)+x_2(n)\} = x_1(n-n_d)+x_2(n-n_d) \quad \text{— (A)}$$

$$\therefore y_1(n)+y_2(n) = x_1(n-n_d)+x_2(n-n_d) \quad \text{— (B)}$$

$$D\{x_1(n)+x_2(n)\} = y_1(n)+y_2(n)$$

$\therefore$ Additivity property is satisfied.

$$\therefore D\{a.x(n)\} = a\,x(n-n_d).$$

$$\therefore a.y(n) = a.x(n-n_d)$$

$$\therefore D\{a.x(n)\} = a.y(n).$$

$\therefore$ Homogenity is satisfied.

$\therefore$ Ideal delay system is <u>linear system</u>

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1. (b) To prove Moving Average (MA) system is linear.

$$y[n] = D\{x[n]\} = \frac{1}{(N_1+N_2+1)} \sum_{k=-N_1}^{+N_2} x[n-k]$$

I/O

1) Additivity ? → satisfied.

2) Homogenity → ?? satisfied.

∴ Moving Average system is linear.

Q-2) a) $\boxed{y(n) = D\{x(n)\} = g(n).\ x(n)} \rightarrow I/O$

$I/O \rightarrow$

i) Stability. BIBO, let $|x(n)| \leq B_x < +\infty$

$$|y(n)| = |g(n).\ x(n)|.$$

$$|y(n)| \leq |g(n)|\ |x(n)|.$$

If $|g(n)| < +\infty\ \forall n$ then $= k$

$$|y(n)| \leq k.\ B_x$$

$\therefore |y(n)| < +\infty\ \forall n$

$\therefore$ If the number $|g(n)|$ is finite the given system $D\{\cdot\}$ is <u>stable</u>

else system $D\{\cdot\}$ is <u>unstable</u>.

ii) <u>Causality:</u>

Since the o/p of system depends upon only on the present value of input, i.e. $x(n)$, the given system $D\{\cdot\}$ is causal. - ②

iii) Linearity.

Additivity. ✓

Homogenity. ✓

$\therefore D\{\cdot\}$ is linear

iv) Time-Invariance

$$\therefore D\{x(n-n_0)\} = y(n-n_0).$$

$$g(n) . x(n-n_0) \neq g(n-n_0) . x(n-n_0)$$

$\therefore$ Given system is time-variant OR NOT time-invariant.

v) Memoryless

$$\boxed{y(n) = g(n) . x(n)}$$

Since $y(n)$ depends only on present i/p, the given system is memoryless.

b) $y(n) = D\{x(n)\} = \sum_{k=n_0}^{n} x(k)$

Sol^n i) Stability.

Unstable

ii) linearity. → linear

$n = n_0 + 1$
$n > n_0$.

iii) Causality ⇒ for $n < n_0$

$y(n) = \sum_{k=n_0}^{n} x(k)$.

$n = n_0 - 1$

$n = n_0 - 1$

$\therefore y(n_0 - 1) = \sum_{k=n_0}^{n_0 - 1} x(k) = x(n_0 - 1) + x(n_0)$

∴ Given system is Non-causal.

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iii) <u>Linearity</u>.

linear.

iv) <u>Time-invariance</u>

$$D\{x(n-n_0)\} \neq y(n-n_0).$$

$$\sum_{k=n_0}^{n} x(k-n_0) \overset{??}{\Longleftrightarrow} \sum_{k=n_0}^{n-n_0} x(k)$$

$$\Downarrow$$

$$\sum_{k=0}^{n-n_0} x(k) \overset{?}{\not\Longleftrightarrow}$$

$\therefore$ Given system is time-<u>variant</u>.

v) <u>Memoryless</u> ??

The system $D\{\cdot\}$ is <u>with memory</u>.

c) $y(n) = D\{x(n)\} = e^{x(n)}$ (??).

⇒ i) Stable :→ ✓ BIBO ✓

ii) Causal ✓ $D\{x(n-n_0)\} = e^{x(n-n_0)}$

$y(n-n_0) = e^{x(n-n_0)}$

iii) linear → Nonlinear

iv) TI/TV → TI ✓

v) Memoryless → ✓

d) $y(n) = D\{x(n)\} = \underline{a \cdot x(n) + b}$.

i) Stability :→
Stable if 'a' and 'b' are finite.

ii) Causal :→ causal ✓

iii) linearity : Nonlinear

iv) TI/TV → TI

v) Memoryless ✓

(P)

(3) Answer

Let $x(n) = \delta(n) + 2 \cdot \delta(n-1) - \delta(n-3)$

and $h(n) = 2 \cdot \delta(n+1) + 2 \cdot \delta(n-1)$

a) $y_1(n) = x(n) * h(n) = \sum_{k=-\infty}^{+\infty} x(k) h(n-k)$

We use properties of convolution sum

$\therefore y_1(n) = [\delta(n) + 2\delta(n-1) - \delta(n-3)]$

$* [2\delta(n+1) + 2\delta(n-1)]$

Using distributive property

$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$

$\therefore y_1(n) = \{\delta(n) * 2\delta(n+1) + 2\delta(n-1) * 2\delta(n-1)$
$- \delta(n-3) * 2\delta(n+1)\} + \{\delta(n) * 2\delta(n-1)$
$+ 2\delta(n+1) * 2\delta(n-1) - \delta(n-3) * 2\delta(n-1)\}$

(B)

Using identity property of discrete-time impulse function,

$$x[n] * \delta[n] = x[n]$$

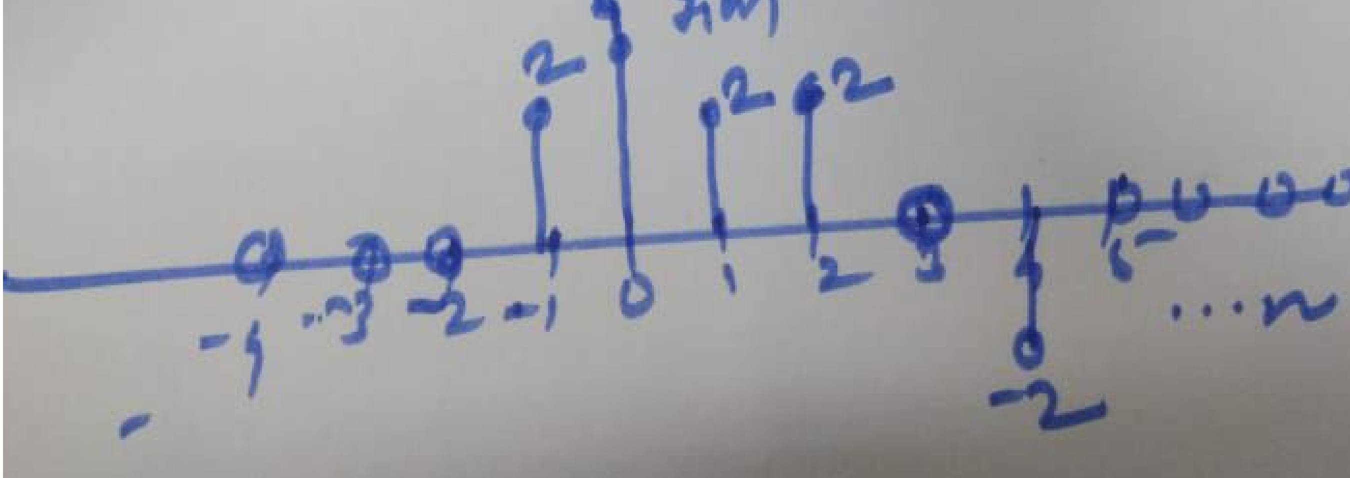
$$\therefore \quad x[n-n_0] * \delta[n] = x[n-n_0]$$

$$x[n] * \delta[n-n_0] = x[n-n_0]$$

$$x[n] * \delta[n+n_0] = x[n+n_0]$$

$$\therefore \quad y_1[n] = \underline{2\delta[n+1]} + 4\delta[n] \underline{- 2\delta[n-2]} + 2\delta[n-1] + \underline{4\delta[n-2]} - 2\delta[n-4]$$

$$\therefore \quad y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-2] + 2\delta[n-1] - 2\delta[n-4]$$



(b) $y_2(n) = x(n+2) * h(n)$

plot $y_2(n)$

Homework

$\Rightarrow$

$$y_2(n) = -2\delta(n-2) + 2\delta(n) + 2\delta(n+1) + 4\delta(n+2) + 2\delta(n+3)$$

(c) $y_3(n) = x(n) * h(n+2)$

$$y_3(n) = y_2(n)$$

Problem 4) $I = \int_{0}^{\infty} \delta(t+3/4)\, e^{-t}\, dt$

Property of impulse function??

$\delta(t+3/4) \Rightarrow 0$

$t = -3/4 \notin [0,\infty]$

$\therefore I = 0$

Problem 5 a) $y(t) = t^2 \cdot x(t-1)$

Linear

TI or TV ?? $\Rightarrow$ TV.

b) $y(n) = x^2(n-2)$

linear? $\rightarrow$ Nonlinear

TI/TV $\rightarrow$ Time Invariant

Problem 6. Prove that the system given by the following input - output relationship is nonlinear

$$y(n) = D\{x(n)\} = \overset{*}{x}(n)$$ $\rightarrow$ complex conjugate

Proof: $\rightarrow$

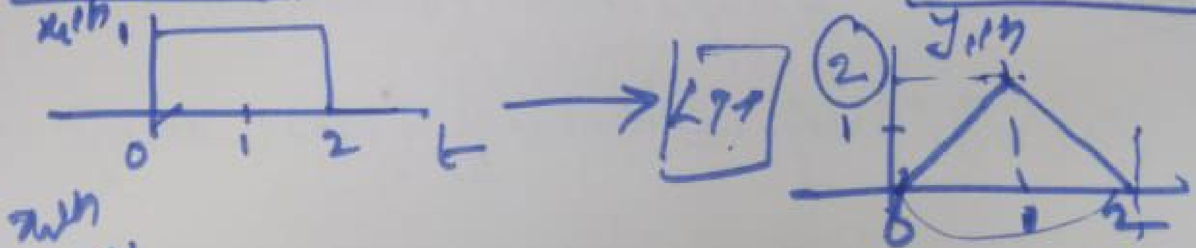
Additivity : $\rightarrow$ satisfied.

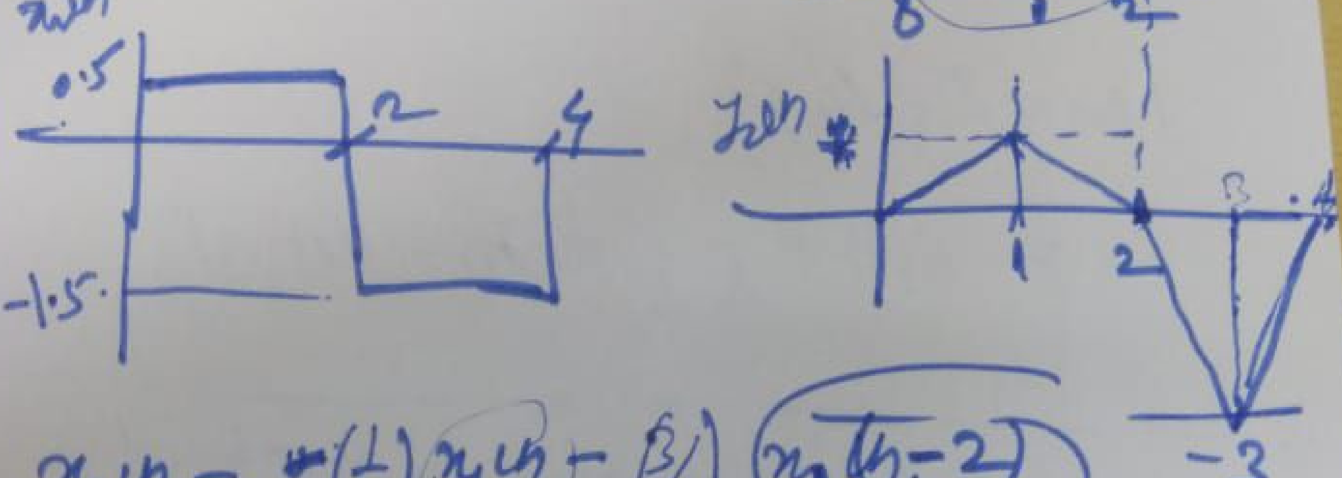
Homogenity $\longrightarrow$ Not satisfied if scaling constant 'a' is complex

$\therefore$ Given system $D\{x(n)\} = \overset{*}{x}(n) =$ Nonlinear

Problem 1

Tutorial 05





$$x_2(t) = \left(\frac{1}{2}\right) x_1(t) - \left(\frac{3}{2}\right) x_1(t-2)$$

$$y_2(t) = \left(\frac{1}{2}\right) y_1(t) - \left(\frac{3}{2}\right) y_1(t-2)$$

Problem 2

a) Ideal delay system

I/O $y[n] = D\{x[n]\} = x[n-n_d]$

$\therefore h[n] = \delta[n-n_d]$

c) Forward diff

$h[n] = \delta[n+1] - \delta[n]$

(1) Linear Interpolation system

$$y(n) = D\{x(n)\} = x(n) + \frac{1}{2}\{x(n+1) - x(n+1)\}$$

$$\therefore h(n) = D\{\delta(n)\}$$

$$= \delta(n) + \frac{1}{2}\{\delta(n+1) - \delta(n+1)\}$$

Problem 3. Consider an input signal $x(n)$ and an unit impulse response sequence $h(n)$ given by

$$x(k) = \left(\frac{1}{2}\right)^{k-2} \cdot u(k-2)$$

$$x(n) = \left(\frac{1}{2}\right)^{n-2} \cdot u(n-2)$$

$$h(n) = u(n+2) \rightarrow h(n-k) = u(n-k+2)$$

Determine and plot $y(n) = x(n) * h(n)$.

Soln: $y(n) = x(n) * h(n)$

$$= \sum_{k=-\infty}^{+\infty} x(k)\, h(n-k)$$

(K4)

$$\therefore y(n) = \sum_{k=-\infty}^{+\infty} \Big[\underbrace{\left(\tfrac{1}{2}\right)^{k-2} u(k-2)}_{x(k)}\Big] \cdot u(n-k+2)$$

$$u(k-2) = \begin{cases} 1 & k-2 \geq 0 \\ 0 & k-2 < 0 \end{cases}$$

$$u(n-k+2) = \begin{cases} 1 & n-k+2 \geq 0 \rightarrow k \leq n+2 \\ 0 & n-k+2 < 0 \end{cases}$$

$$n > k \qquad n+2 \rightarrow n-2 ??$$

$$\therefore y(n) = \underbrace{\cancel{\sum_{k=-\infty}^{2} (\cdot)}}_{0} + \sum_{k=2}^{n+2} (\cdot) + \underbrace{\cancel{\sum_{k=n+2}^{\infty} (\cdot)}}_{0}$$

$$y(n) = \sum_{k=2}^{n+2} \left(\tfrac{1}{2}\right)^{k-2} \cdot \underbrace{\cancel{u(k-2)}}_{1} \cdot \underbrace{u(n-k+2)}_{1}$$

Let $m = k-2$

$$15 = \sum_{m=0}^{n} \left(\tfrac{1}{2}\right)^{m} \xrightarrow{GP} = 1 \left[\frac{1-\left(\tfrac{1}{2}\right)^{n+1}}{1-\tfrac{1}{2}}\right]$$

$$y_{n+1} = 2\left[1-\left(\frac{1}{2}\right)^{n+1}\right].$$

u_n

??

$\therefore u_{n+1}$