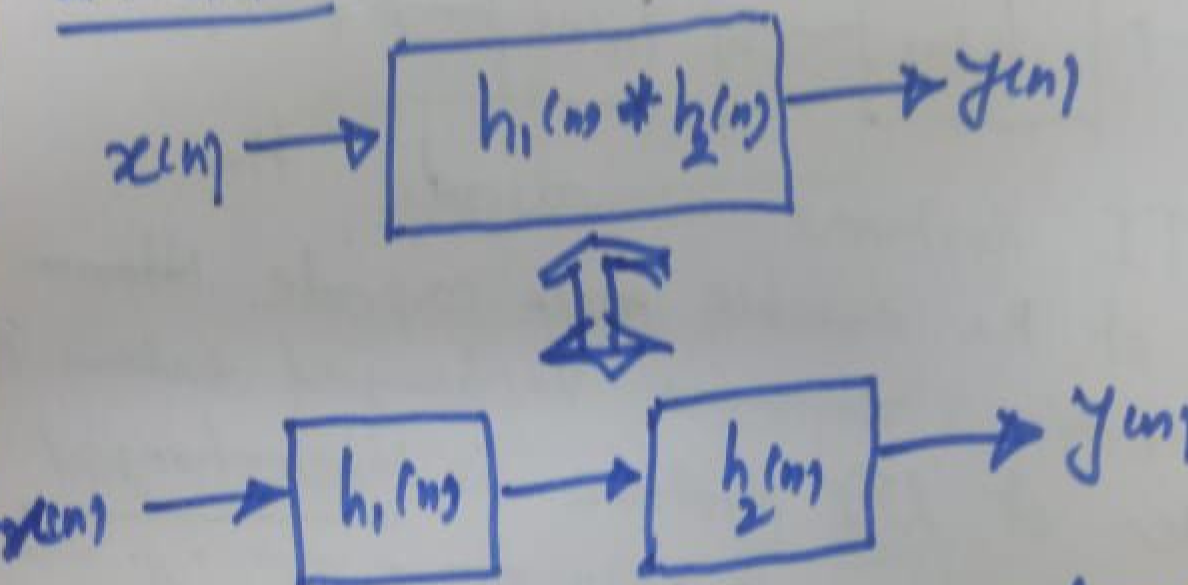
③ Associative Property:

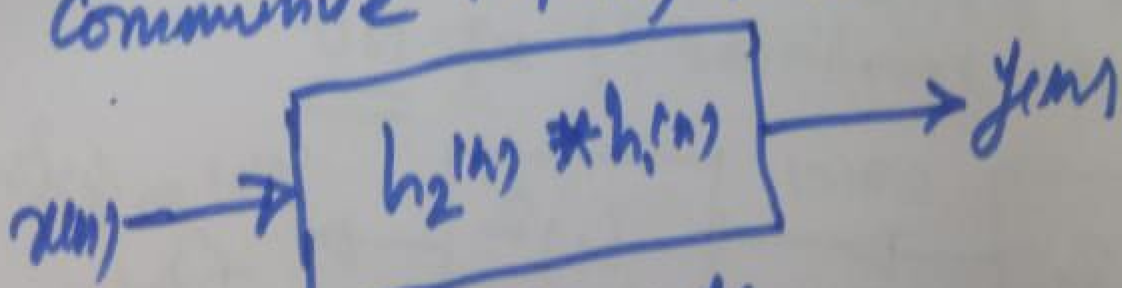
$$x(n) * h_1(n) * h_2(n) = [x(n) * h_1(n)] * h_2(n)$$
$$= x(n) * [h_1(n) * h_2(n)]$$

Proof ⇒ Homework ✓

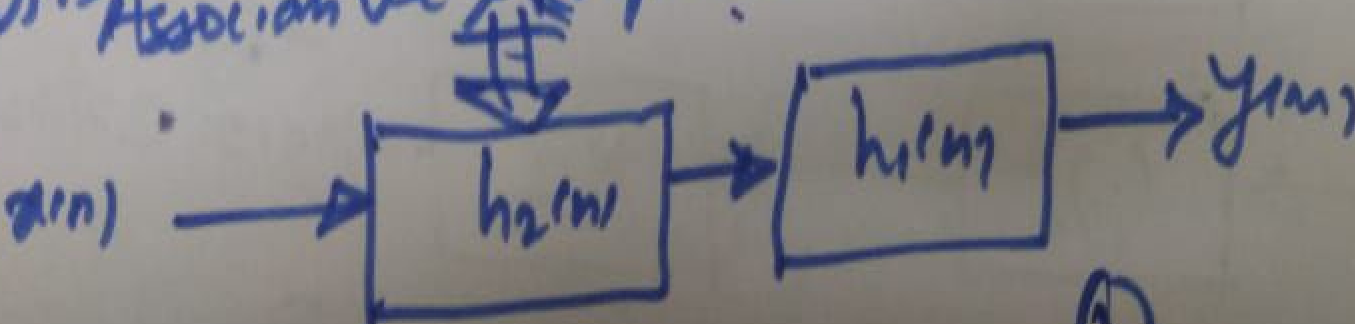
Inference:



Commutative Property of LTI system.



Associative Property



Using equivalence $A \Leftrightarrow B \Leftrightarrow C$

$\therefore A \Leftrightarrow C.$

$$x(n) \longrightarrow \underset{LTI}{\boxed{h_1(n)}} \longrightarrow \underset{LTI}{\boxed{h_2(n)}} \longrightarrow y(n)$$

$$\Updownarrow$$

$$x(n) \longrightarrow \boxed{h_2(n)} \longrightarrow \boxed{h_1(n)} \longrightarrow y(n).$$

For LTI systems in cascade, the output of the overall cascade interconnection remains unchanged even if the order of LTI systems is <u>interchanged</u>.

Q; whether above result holds true if systems are nonlinear?? $\Rightarrow$ <u>NO</u>

$$x(n) \longrightarrow \boxed{\times 2} \xrightarrow{2x(n)} \boxed{(.)^2} \longrightarrow y(n) = 4x^2(n)$$

$$\not\Updownarrow$$

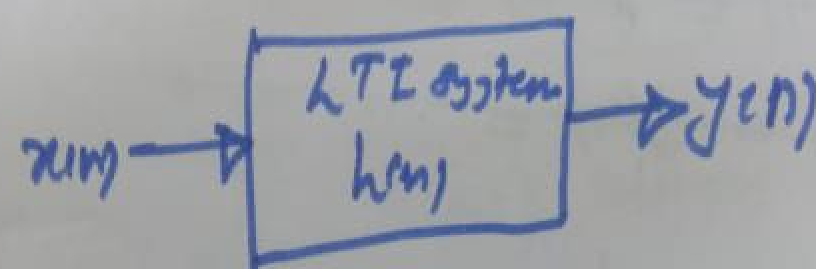
$$x(n) \longrightarrow \boxed{(.)^2} \longrightarrow \boxed{\times 2} \longrightarrow y(n) = 2x^2(n)$$

Properties of LTI systems

1) LTI system with or without memory

Problem: Find the condition on impulse response such that i) LTI system is without memory
ii) LTI system with memory.

Sol^n: →



$$y(n) = x(n) * h(n)$$

Using commutative property,

$$y(n) = h(n) * x(n)$$

$$y(n) = \sum_{k=-\infty}^{+\infty} h(k).\, x(n-k)$$

memoryless $y(n) = h(0)\, x(n)$

$$\boxed{y(n) = \cdots + h(-1)\, x(n+1) + h(0)\, x(n) + h(1)\, x(n-1) + \cdots}$$

Input-output relationship for an LTI system

Since the output $y(n)$ depends on the past and future values of input,

∴ the given LTI system is with memory,

$h(n) \neq 0$ for $n \neq 0$

For memoryless LTI system, we must choose,

~~$h(-1) = h(-2) = \cdots = 0$~~

~~$h(1) = h(2) = \cdots = 0$~~

$h(-1) = h(-2) = \cdots = 0$

$h(1) = h(2) = \cdots = 0$

$\cdots\cdots\cdots$

$$\therefore h(n) = \begin{cases} K, & n = 0 \\ 0 & n \neq 0 \end{cases}$$

$$\boxed{h(n) = K.\delta(n)}$$ → condition on impulse response for memoryless LTI

$y(n) = x(n) * h(n) = x(n) * K.\delta(n)$

$= K[x(n) * \delta(n)]$

$$\boxed{y(n) = K x(n)}$$

(4)

(1) Find condition on h[n] for Causal LTI system.

Soln. Method I

For causal LTI system, we must choose

$$h(-1) = h(-2) = \cdots = 0$$

$$\Rightarrow \boxed{h[n] = 0 \text{ for } n < 0}$$

$$y[n] = \sum_{k=-\infty}^{+\infty} h[k] \cdot x[n-k]$$

$$y[n] = \sum_{k=0}^{+\infty} h[k] \cdot x[n-k] \Rightarrow \text{O/P for causal LTI system}$$

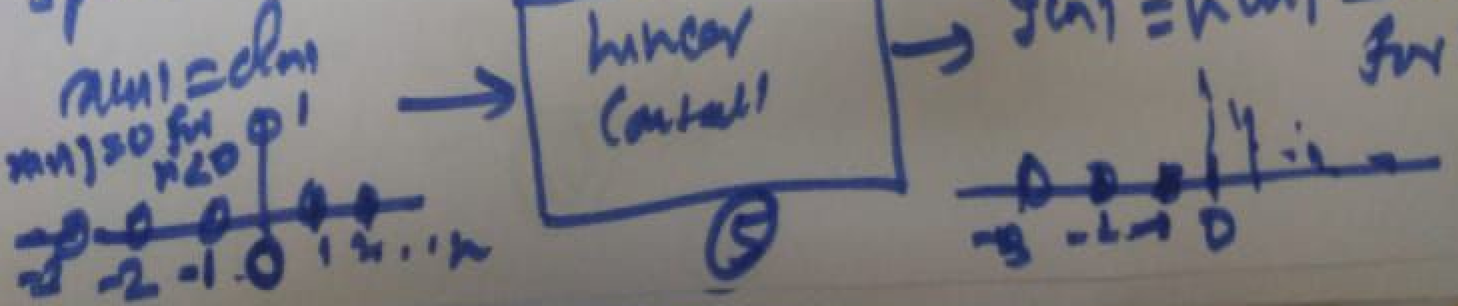
Soln Method II Using Proposition 1.3

Proposition 1.3 for linear and causal system

if $x[n] = 0$ for $n \le n_0$ then $y[n] = 0$ for $n \le n_0$

Special case, $x[n] = \delta[n] \Rightarrow y[n] = h[n]$

$y[n] = h[n] = 0$ for $n < 0$



Problem For causal signal, $x(n) = 0$ for $n <$

Proof: $\rightarrow$ $\boxed{x(n) * \delta(n) = x(n)}$

$$x(n) \longrightarrow \underset{}{\overset{LTI}{\boxed{\delta(n)}}} \longrightarrow x(n)$$

Commutative property,

$$\delta(n) * x(n) = x(n)$$

$$\delta(n) \longrightarrow \overset{LTI}{\boxed{h(n) = x(n)}} \longrightarrow x(n)$$

Using the result that for causal LTI system, $h(n) = 0$ for $n < 0$

$$\Rightarrow x(n) = 0 \text{ for } n < 0$$

Problem ③ Find condition in h(n) for stable LTI system.

Sol^n: → We have $y(n) = \sum_{k=-\infty}^{+\infty} h(k) \cdot x(n-k)$ —①

For BIBO stability criterion let us assume that the input $x(n)$ is bounded

i.e. $|x(n)| \leq B_x < +\infty, \forall n$

$\therefore |y(n)| = \left| \sum_{k=-\infty}^{+\infty} h(k) \cdot x(n-k) \right|$

$|y(n)| =$? ?

$$\boxed{\sum_{k=-\infty}^{+\infty} |h(k)| < +\infty}$$

For stable LTI system the impulse response of LTI system is absolutely summable.

Problem For LTI system,

$h(n) = a^n \cdot u(n)$ ✓

Case I) $|a| < 1 \rightarrow$ stable LTI, Case II $|a| > 1$ $\rightarrow$ Unstable LTI system.

$$\sum_{n=-\infty}^{+\infty} |h(n)| < +\infty$$

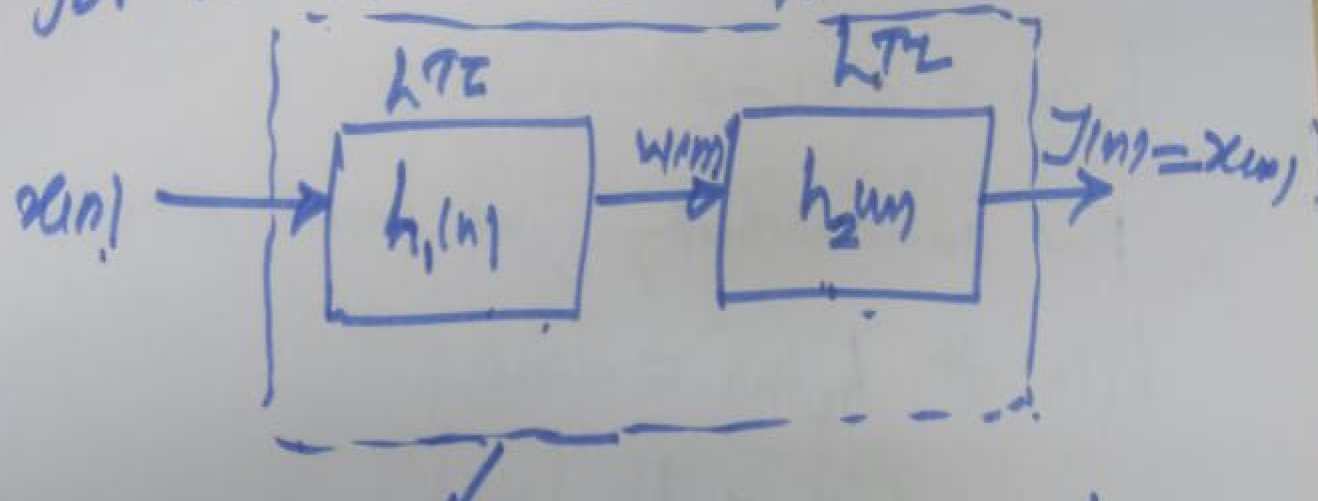
$$h(n) = \begin{cases} \frac{1}{3} & -1 \le n \le +1 \\ 0 & \text{elsewhere} \end{cases}$$

stable LTI system

$\boxed{\text{FIR systems are always stable LTI systems.}}$

Important property.

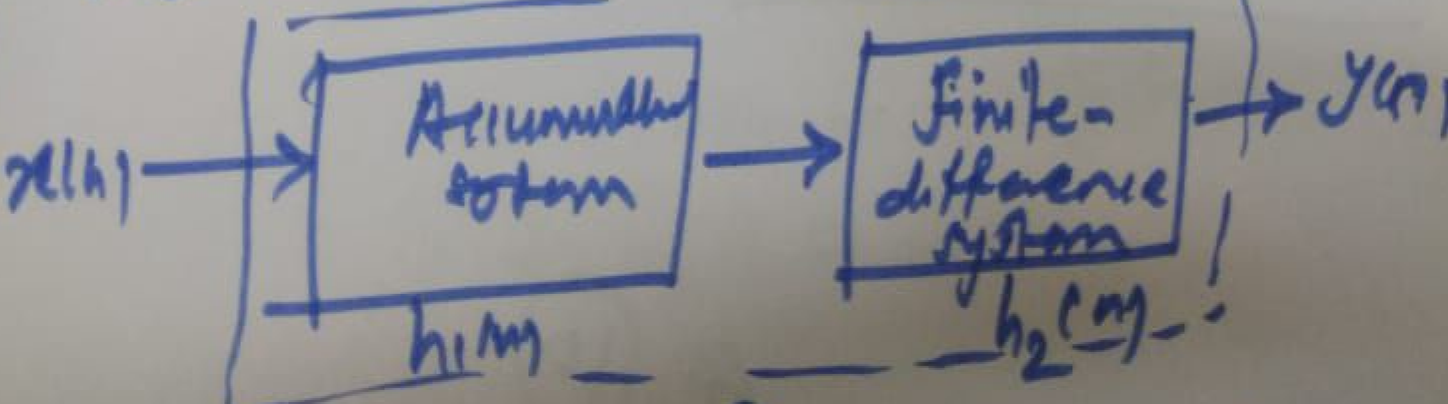
Problem (4) Find the condition on $h(n)$ for invertible LTI system.



$$h(n) = \delta(n) = h_1(n) * h_2(n)$$

$$\boxed{h_1(n) * h_2(n) = \delta(n)}$$

Problem: Prove that an accumulator system and finite-difference system are inverse of each other



$h_1(n) = u(n)$

$h_2(n) = \delta(n) - \delta(n-1)$

$\therefore$ For invertibility, we must prove

$h_1(n) * h_2(n) = \delta(n)$

$\therefore$ LHS $= h_1(n) * h_2(n)$

$= u(n) * [\delta(n) - \delta(n-1)]$

Distributive property

$= \underbrace{u(n) * \delta(n)} - u(n) * \delta(n-1)$

$= u(n) - u(n)$

$\boxed{h_1(n) * h_2(n) = \delta(n)}$ Hence proved.

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For continuous-time LTI systems

1) For causal LTI system,

$h(t) = 0$ for $t < 0$. [Homework]

2) For stable LTI system

$$\int_{t=-\infty}^{+\infty} |h(t)|\, dt < +\infty$$

$\Rightarrow$ Impulse response is absolutely integrable.

3) For memoryless LTI system

$$h(t) = k \cdot \delta(t)$$

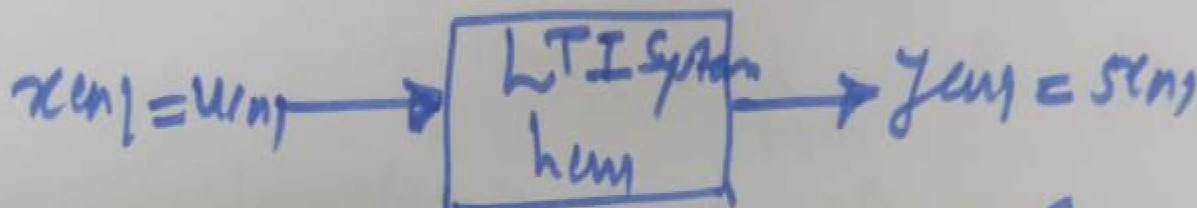
$$y(t) = k \cdot x(t).$$

4) For invertible LTI system

$$h_1(t) * h_2(t) = \delta(t)$$

Unit Step Response of an LTI system

Application: Used in electronic circuit theory,
control system
power systems.



Output of a system for unit step excitation source is called unit step response, s(n).

$$y(n) = x(n) * h(n)$$

$$s(n) = u(n) * h(n)$$

Commutative property.

$$s(n) = h(n) * u(n)$$

$$s(n) = \sum_{k=-\infty}^{+\infty} h(k)\, u(n-k)$$

$$\boxed{s(n) = \sum_{k=-\infty}^{n} h(k)}$$ → step response of an LTI system

Step response of an LTI system is dependent impulse response with accumulation operation.

$$\Rightarrow h(n) = s(n) - s(n-1)$$

For continuous-time system,

$$\boxed{\begin{aligned} s(t) &= \int_{\tau=-\infty}^{t} h(\tau)\, d\tau \\ \therefore h(t) &= \frac{d\,s(t)}{dt} \end{aligned}}$$

Since step response of an LTI system depends upon system's impulse response, $h(t)$. Thus, $h(t)$ is the key characterization for an LTI system.