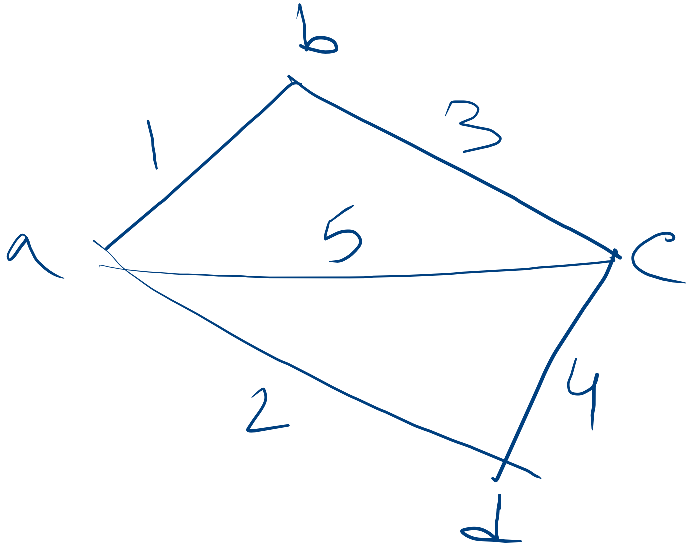
Shortest path problem

- single source shortest path problem.

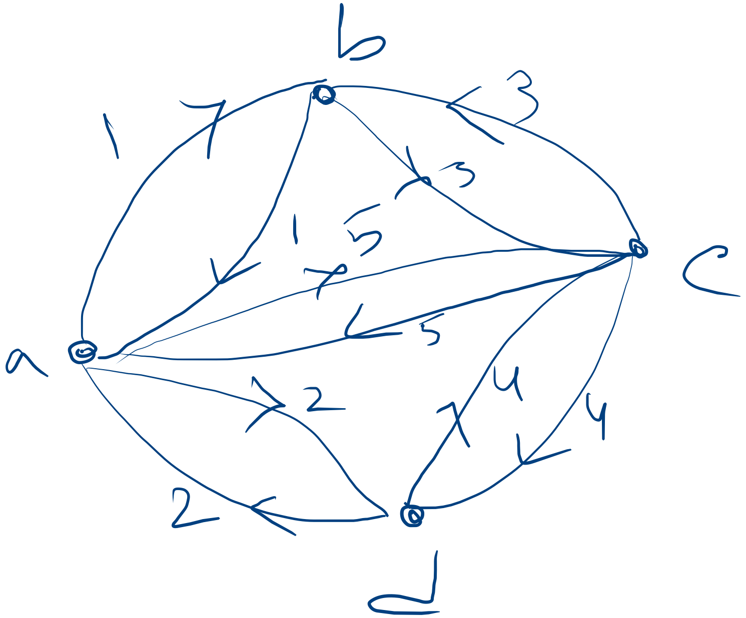
  Given s find shortest path from s to all other vertices

  Given s, t ,, ,, ,, from s to t.

⇒ we will concentrate on directed graphs only.
why?.



⇒

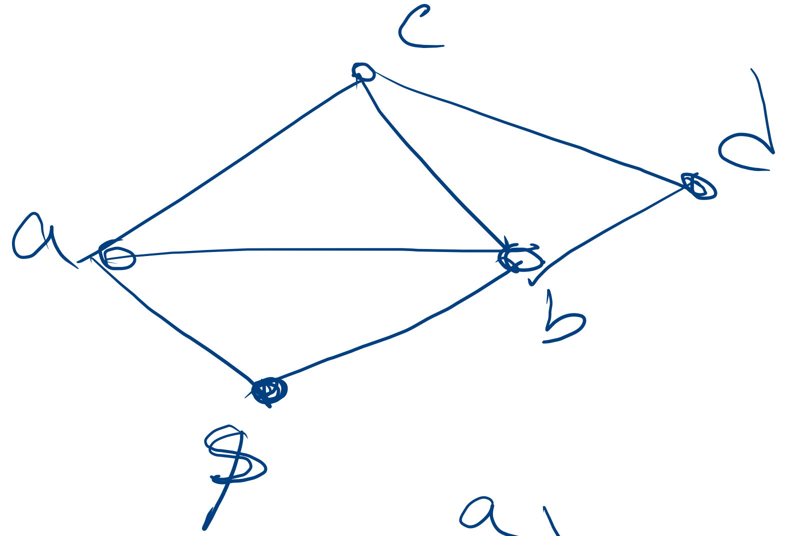


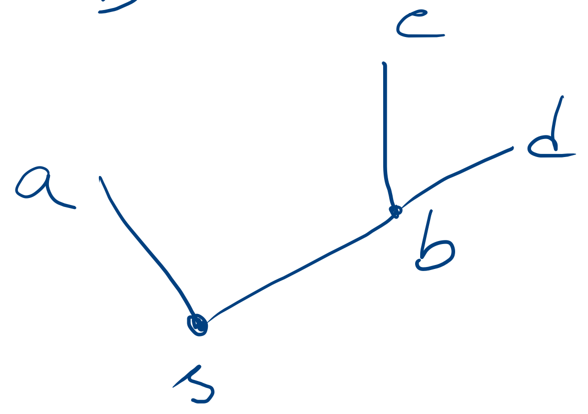
H.W.

Special cases 1

— Assume the edge weights are all 1.

— use BFS algorithm.





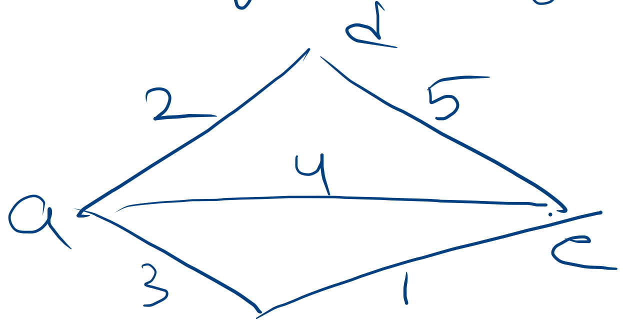
Time: $O(|V|+|E|)$

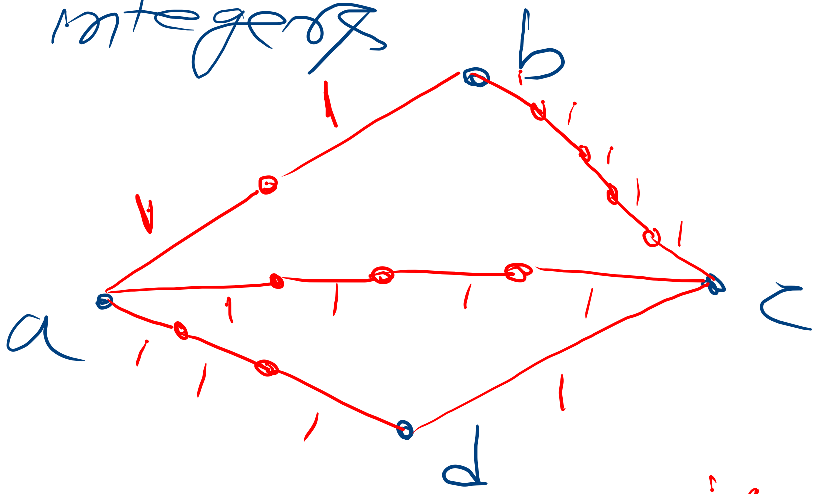
$|V| = n$

$|E| = m$

Special case 2

— All edge weights are integers





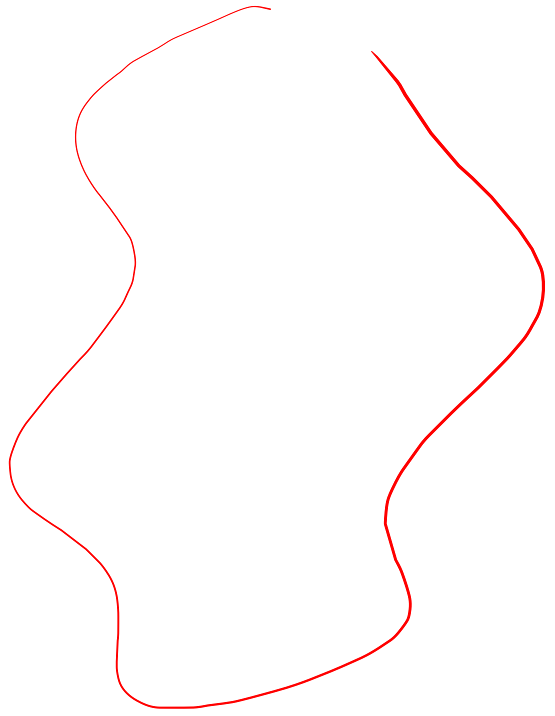
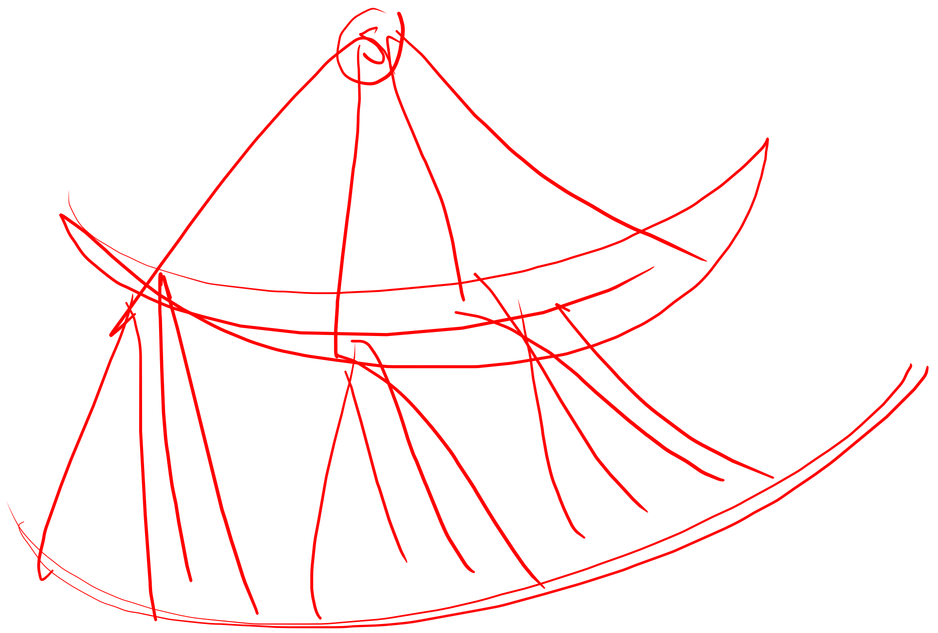
L = maximum number of edges added in an old edge

vertices = $mL + n$

edges = mL

running time: $O(mL+n)$

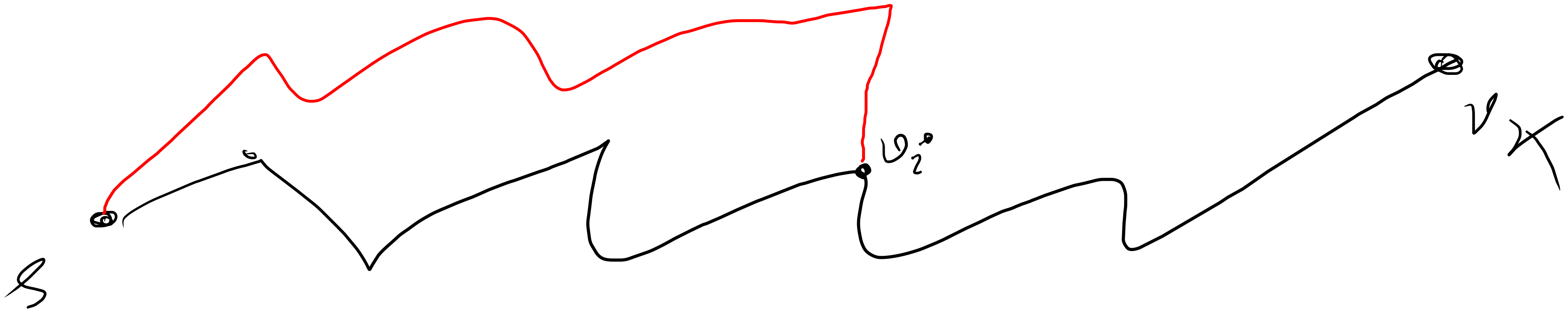
For large L this algorithm is not efficient.

Let G ~~be~~ a directed graph with non-negative edge weights: let $dist(s, v)$ is the shortest path weight from s to v.

If $s = v_0 \rightarrow v_1 \rightarrow v_2 \cdots \rightarrow v_k$, ~~be~~ the shortest path from s to v_k then for $1 \le i \le k$

- $s = v_0 \rightarrow v_1 \rightarrow \cdots \rightarrow v_i$ is a shortest path from s to v_i
- $dist(s, v_i) \le dist(s, v_k)$



General case

$d[v] \leftarrow$ shortest path distance from s to v
$\pi[v] \leftarrow$ parent of v.

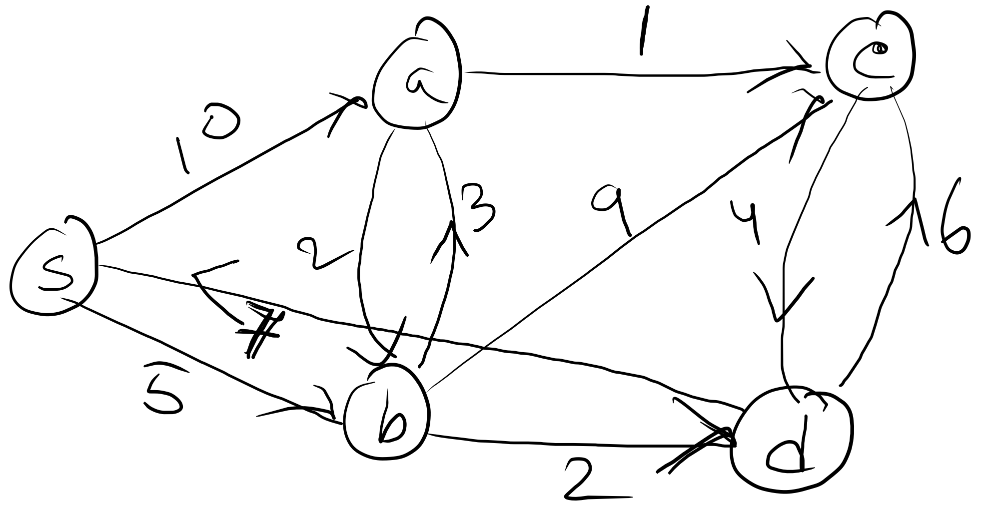
Dijkstra's algorithm

Dijkstra (G, w, s)
$d[s] = 0$
for each vertex $v \in V \setminus \{s\}$
$d[v] \leftarrow \infty$, $\pi[v] \leftarrow NIL$

$S \leftarrow \phi$ (empty set)
$Q \leftarrow V$ // $Q \leftarrow$ a priority queue
while $Q \neq \phi$
$u = \text{Extract-min}(Q)$
$S = S \cup \{u\}$
for each vertex $v \in Adj[u]$
if $d[v] > d[u] + w(u,v)$
$d[v] = d[u] + w(u,v)$
$\pi[v] \leftarrow u$

Running time:

$O(|V|) \cdot T_{\text{extract-min}} + O(|E|)\, T_{\text{decrease-key}}$



Q	d	π
s	0	NIL
a	∞	NIL
b	∞	NIL
c	∞	NIL
d	∞	NIL

$S = \{\ \}$

Q	d	π
s	0	NIL
a	10	s
b	5	s
c	∞	NIL
d	∞	NIL

$s \leftarrow$ Extract-min (Q)

$S = \{s\}$

$b \leftarrow$ Extract-min (Q) $\quad S = \{s, b\}$

	Q	d	π
X	s	0	NIL
	a	~~10~~ 8	~~s~~ b
X	b	5	s
	e	~~∞~~ 14	b
	d	~~∞~~ 7	b

Final table

Q	d	π
s	0	NIL
a	8	b
b	5	s
c	9	a
d	7	b

Shortest paths:

s to a

$s \to b \to a$ weight is 8