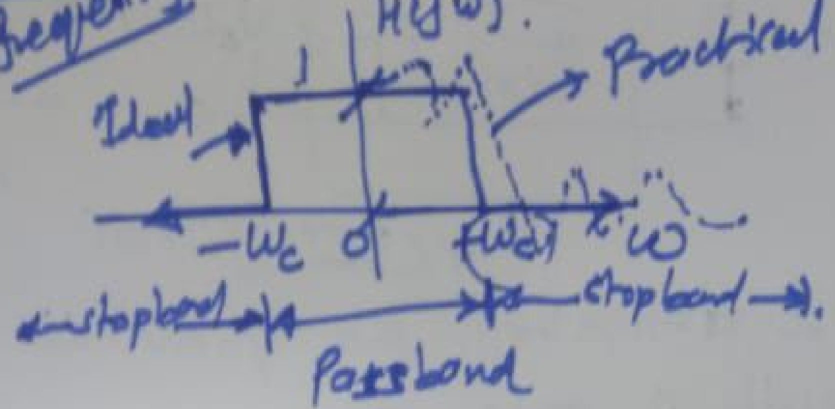
Lecture 30

* Frequency Selective Filters: →

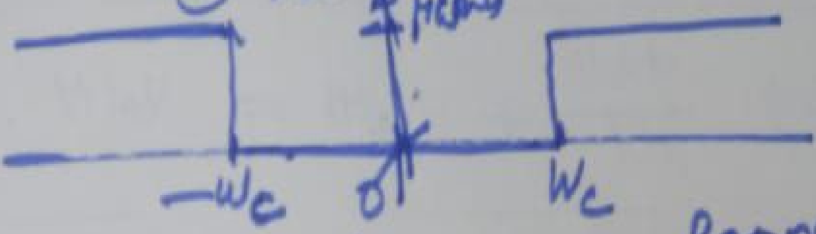
Ideal Characteristics: →

① Lowpass Filter

W_c = cut-off frequency

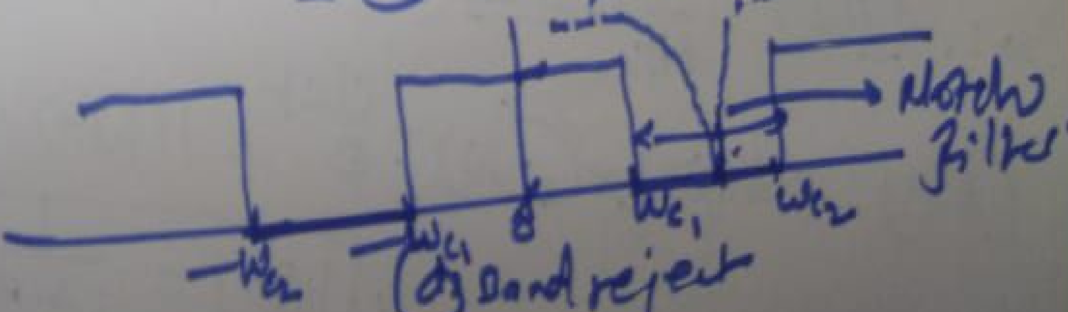


(a) Lowpass.



(b) Highpass

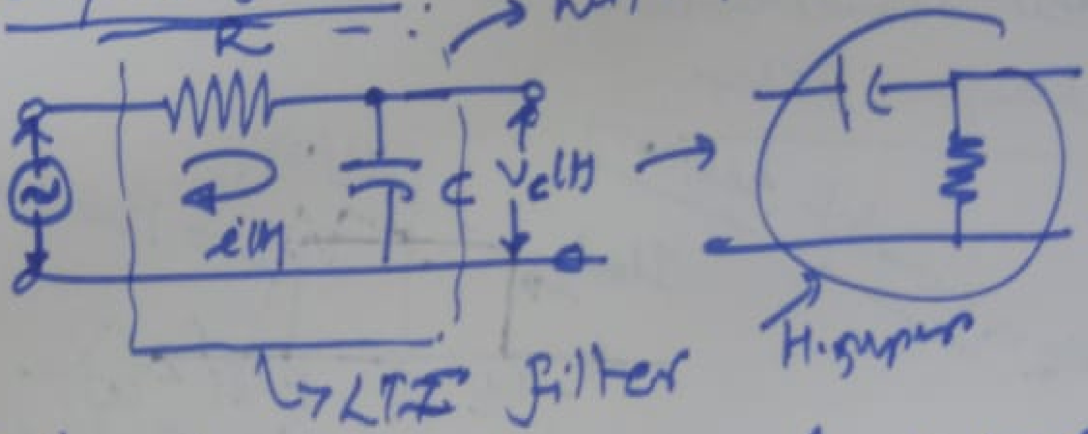
(c) Bandpass



(d) Band reject

Example of Continuous-time Filter Described by Differential Equation:

→ Lowpass filter : $h(t) \rightarrow H(j\omega)$



DE describing the dynamics of series RC circuit is given by

$$(RC)\frac{dv_c(t)}{dt} + v_c(t) = v_s(t).$$

Find the frequency response, $H(j\omega)$ of above RC circuit (LTI filter), we assume $\boxed{v_s(t) = e^{j\omega t}}$

then $v_c(t) = H(j\omega) \cdot e^{j\omega t}$

$$\therefore (RC)\frac{d}{dt}\left[H(j\omega) \cdot e^{j\omega t}\right] + H(j\omega) \cdot e^{j\omega t} = e^{j\omega t}$$

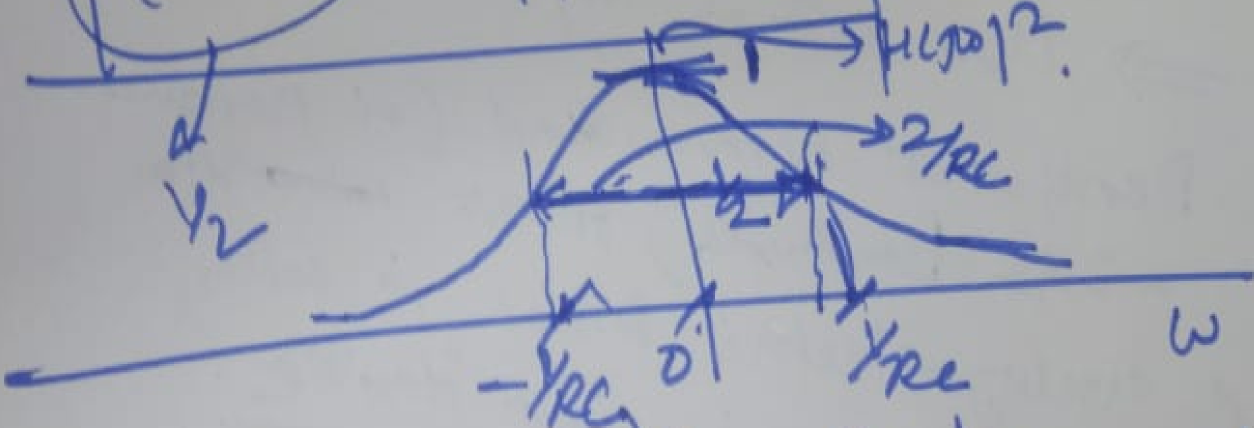
$$(RC) \times H(j\omega) \times (j\omega) \cdot e^{j\omega t} + H(j\omega) \cdot e^{j\omega t} = e^{j\omega t}$$

$$\therefore \boxed{H(j\omega) = \frac{1}{1 + RC\,j\omega}}$$

$$\therefore \quad |H(j\omega)| = \frac{|1|}{|1+j\omega RC|}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+R^2C^2\omega^2}}$$

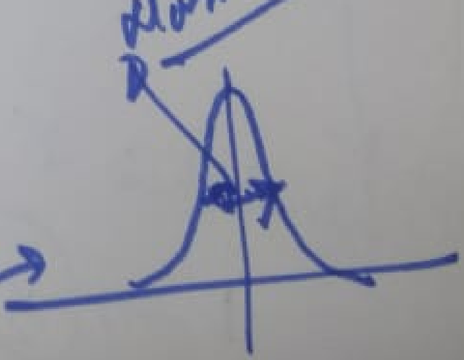
$$\boxed{|H(j\omega)|^2 = \frac{1}{1+R^2C^2\omega^2}}$$ = Power in frequency domain



$$\frac{1}{1+R^2C^2\omega^2} = \frac{1}{2}$$

$Q \uparrow \propto \frac{1}{B \downarrow}$

Narrow band



$$\therefore \quad \boxed{\omega = \pm\frac{1}{RC}}$$

$$\therefore \quad \boxed{\omega_{\text{Half power}} = 2/RC}$$

For High quality factor filter, we desire narrow $-3dB$ bandwidth or half-power bandwidth

⇒ RC ↑

⇒ Large value of R and C

⇒ Heat dissipation / temperature effect

⇒ energy loss into the circuit.

⇒ More cost of implementation.

∴ Design of narrowband (or narrow −3dB Bandwidth) filters ~~using~~ via analog electric filter is costly, time-consuming, less flexible.

Limitations of analog filters. —

$1 \rightarrow \frac{1}{2} \rightarrow 10 \log_{10}(\frac{1}{2}) = -3$

decibel

Chapter 6

4

The impulse response of series RC circuit is

$$h(t) = e^{-t/RC} . u(t).$$

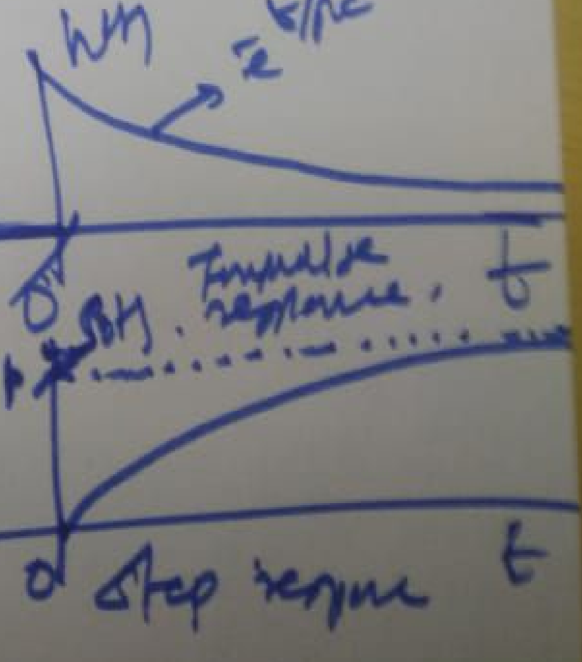
∴ The step response of series RC circuit is given,

$$s(t) = h(t) * u(t)$$

$$= \int_{-\infty}^{\infty} h(\tau)\, \underline{\underline{u(t-\tau)}}\, d\tau$$

$$= \int_{-\infty}^{t} e^{-\tau/RC} . u(\tau)\, u(t-\tau)\, d\tau$$

$$\boxed{s(t) = \left[1 - e^{-t/RC}\right] . u(t)}$$



Problem: Discrete-time or Digital filter

Filter described by difference equation

$$\boxed{y[n] - a \cdot y[n-1] = x[n]}$$

$H(e^{j\omega}) = ??$

For frequency response, substitute, $x[n] = e^{j\omega n}$

Using eigenfunction theory,

If $x[n] = e^{j\omega n}$, $\longrightarrow y[n] = H(e^{j\omega}) \cdot e^{j\omega n}$ — (1)

$$\boxed{H(e^{j\omega}) = \frac{1}{1 - a e^{-j\omega}}}$$

$$\boxed{h[n] = a^n \cdot u[n]}$$

$$\boxed{s[n] = u[n] * h[n] = \frac{1 - a^{n+1}}{1 - a} \cdot u[n]}$$

step response

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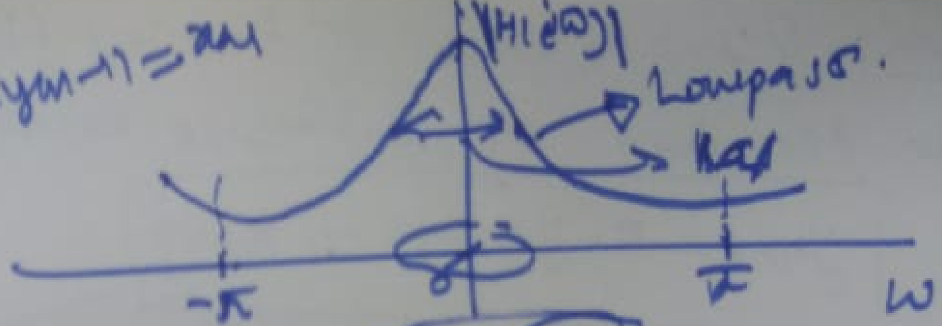
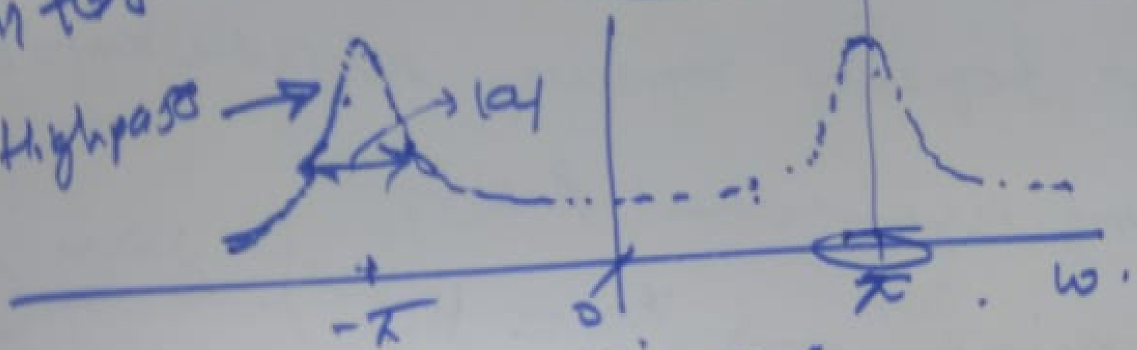
$y(n) - a\,y(n-1) = x(n)$



$a = 0.6.$

$y(n) + (a)\,y(n-1) = x(n)$



$a = -0.6$

Inference: ① The sign of filter coefficient 'a' dictates the characteristic of frequency response from ~~...~~ lowpass to highpass and vice-a-versa.

② The magnitude of filter coefficient $|a|$ will dictate −3dB bandwidth of the filter or size of passband of the filter

⇒ ∴ Digital filters offers more flexibility in implementation

On the Developement of Fourier Transform

Chapter 4

[Continuous-time Fourier Transform (CTFT)]

<u>Motivation</u> :→ Extension of study done in chapter 3 (for periodic signals) to Frequency-domain analysis of aperiodic signal.

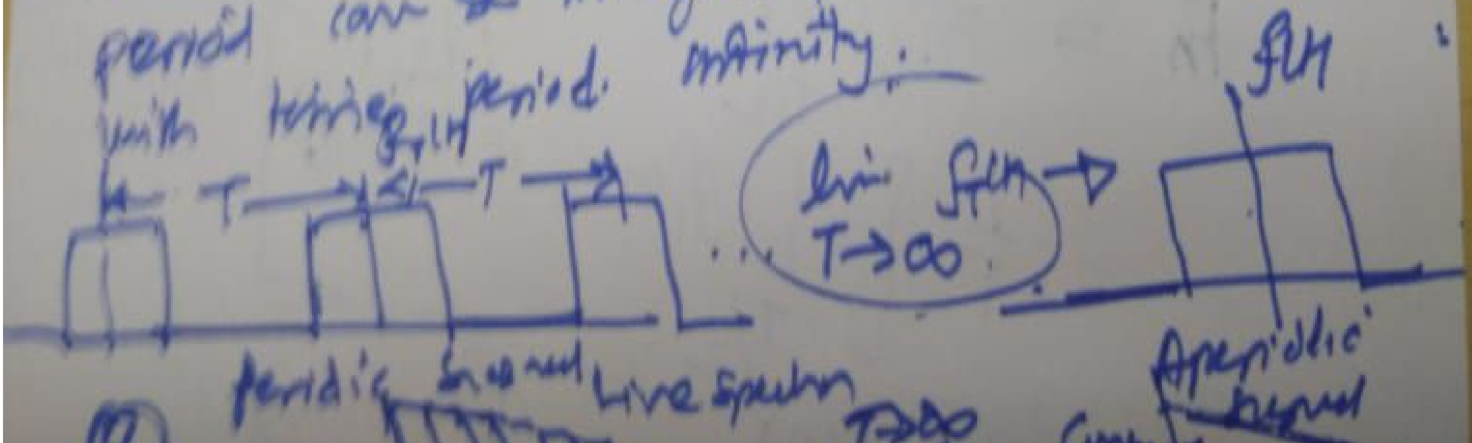
Chapter 3
Fourier Series
(line spectrum)
(Periodic signal)

⟹

Chapter 4
Fourier Transform
Aperiodic Signal
Continuous-Spectrum

Fourier Original Thought [Contribution]

Fourier reasoned the fact that an aperiodic periodic signal can be imagined as periodic signal with time period. period infinity.



Let $f_T(t)$ = be the periodic signal with
∴ fundamental period, T, and
fundamental frequency, $\omega_0 = \frac{2\pi}{T}$.

$f(t)$ = aperiodic signal.

$$\lim_{T \to \infty} \underbrace{f_T(t)}_{\text{Periodic}} = \underbrace{f(t)}_{\text{Aperiodic}} \quad \text{—— (1)}$$

∵ $f_T(t)$ is periodic signal, we can represent $f_T(t)$ via Exponential Fourier series.

$$\therefore f_T(t) = \sum_{n=-\infty}^{+\infty} F_n \cdot e^{jn\omega_0 t} \quad \text{—— (2)} \; ; \; \omega_0 = \frac{2\pi}{T}.$$

$$F_n = \frac{1}{T} \int_0^{T} f_T(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T} \left[\int_{-T/2}^{T/2} f_T(t)\, e^{-j\omega_n t}\, dt \right], \quad n\omega_0 = \omega_n$$

$$F_n = \frac{1}{T} F(\omega_n). \quad \text{—— (3)}$$

where $F(\omega_n) = \int_{-T/2}^{T/2} f_T(t) \cdot e^{-j\omega_n t} \, dt$ ... (3)

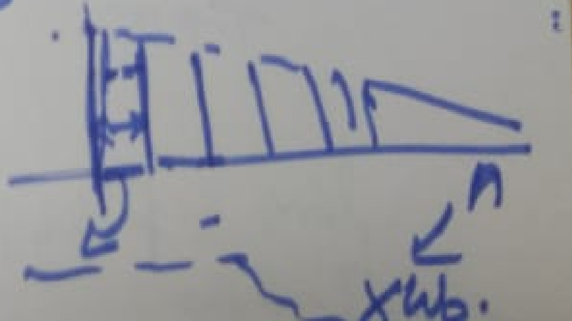
Use eqn (3) in eqn (2), we get

$$f_T(t) = \sum_{n=-\infty}^{+\infty} \underbrace{\left[\frac{1}{T} F(\omega_n)\right]}_{F_n} e^{jn\omega_0 t}$$

Take $\lim_{T\to\infty}$ on both the sides

$$\lim_{T\to\infty} f_T(t) = \lim_{T\to\infty} \sum_{n=-\infty}^{+\infty} \left[\left(\frac{1}{T}\right) F(\omega_n)\right] e^{jn\omega_0 t} \quad (\omega_0/2\pi)$$

$\omega_0 = \frac{2\pi}{T}$, $\omega_n = n\omega_0$



$T\to\infty$, $\omega_0 \downarrow$, $\omega_n \approx \omega$

$$\lim_{T\to\infty} f_T(t) = \frac{1}{2\pi}\left\{\lim_{\omega_0\to 0} \sum_{n=-\infty}^{+\infty} \left(F(\omega_n)\cdot e^{j\omega_n t}\right)\times\omega_0\right\}$$

Applying limits,

$$\boxed{f(t) = \frac{1}{2\pi} \int_{\omega=-\infty}^{\omega=+\infty} F(\omega) \cdot e^{j\omega t} \, d\omega}$$

$F(\omega_n) \cdot e^{j\omega_n t}$

$$\omega_0 = \frac{2\pi}{T}$$

$$F(\omega_n) = \int_{-T/2}^{+T/2} f_T(t) \cdot e^{-j\omega_n t} \, dt$$

Take $\lim_{T\to\infty}$ on both the sides

$$\lim_{T\to\infty} F(\omega_n) = \lim_{T\to\infty} \int_{-T/2}^{+T/2} f_T(t) \cdot e^{-j\omega_n t} \, dt$$

Assume $\lim \int = \int \lim$

$\Downarrow$

$$\lim_{\omega_0\to 0} F(\omega_n) = \lim_{T\to\infty} \int_{-T/2}^{T/2} \left[\lim_{T\to\infty} f_T(t)\right] \left[\lim_{T\to\infty} e^{-j\omega_n t}\right] dt$$

$e^{j\omega t}$

$\omega_n \to \omega$, $\omega_0 \to 0$

$$F(\omega) = \int_{t=-\infty}^{t=+\infty} f(t) \cdot e^{-j\omega t}\, dt \quad \text{— [A]}$$

[Analysis Eqn]

$$f(t) = \frac{1}{2\pi} \int_{\omega=-\infty}^{\omega=+\infty} F(\omega) \cdot e^{j\omega t}\, d\omega \quad \text{— [B]}$$

[Synthesis Eqn]

CTFT pair for continuous-time aperiodic signal. $f(t)$

$F(\omega) = \mathcal{F}\{f(t)\}$

$\mathcal{F}^{-1}\{F(\omega)\} = f(t)$

Fourier transform operator