Set D

CT216: Introduction to Communication Systems

Winter 2023

Final Exam

Date	:	Monday 8th May, 2023, from 8 am to 11 am
Total Marks	:	50
Notes:	:	All questions are scored (no optionals).
		Do not forget to write your name and student ID below.
		Fully fill the OMR bubble for your selected option in the MCQ section.
		Answers written or OMR bubble filled using pencil will not be graded.

Name: ______________________ **Student ID:** ______________________

1 Multiple Choice Questions:

Instructions

- ▷ This is MCQ Version IV of IV.
- ▷ There are total 30 questions in this section that carry a maximum of 55 marks. If the question is answered incorrectly, negative one-fourth of the total marks shown next to each question will be given.
- ▷ An OMR sheet is provided on the last page. Fill the bubble corresponding to the correct option. No tick marks or partial filling of the bubble, etc. If more than one bubble is filled. zero marks will be given for the question. Do not use a pencil to fill the bubble – the answer bubbled in with pencil, even if it is correct, will not be graded.

Questions

1. (2 points) Which of the following is a proper choice for the modulation matrix $\mathbf{G}_{mod}$ at the modulator and the demodulator of a communication system?

   (a) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \\ -1 & -1 \end{bmatrix}$

   (b) $\begin{bmatrix} 0.57 & -0.57 \\ -0.57 & 0.57 \\ 0.57 & 0.57 \end{bmatrix}$

   (c) $\begin{bmatrix} 0.35 & 0.53 \\ 0.5 & -0.8 \\ -0.8 & -0.26 \end{bmatrix}$

   (d) $\begin{bmatrix} 0.707 & 1 & 0.707 \\ -0.707 & 0 & 0.707 \end{bmatrix}$

2. (2 points) You have the choice of using either (a) 16-QAM modulation scheme with channel coding rate $r = 4/5$ or (b) BPSK modulation with coding rate of $r = 1/10$. There are two different communication system/channel scenarios: (c) the transmit power is limited and the receiver is expected to operate with a low value of signal to noise ratio (SNR) or average E_S/N_0, and (d) the transmit power can be increased to a high value and the receiver operates at a high E_S/N_0. Which of the following is the correct design choice?

   (a) Design choice (a) for systems (c) and (d) and design choice (b) for system (d)
   (b) Design choice (a) for system (c) and design choice (b) for systems (c) and (d)
   (c) Design choice (a) for system (c) and design choice (b) for system (d)
   (d) Design choice (a) for system (d) and design choice (b) for system (c)

3. (1 points) The decision boundary for the BFSK modulation with equiprobable symbols

   (a) is the same as the Y-axis
   (b) depends on the specific modulator
   (c) Passes through the origin and it is inclined
   (d) is located at the origin

4. (2 points) What is the variance of the average of 10 realizations of the Bernouilli($q = 0.1$) random variable?[6]

   (a) 0.9
   (b) 1
   (c) 0.1
   (d) 0.009

5. (1 points) For a given length N of transmitted codewords, if the minimum Hamming Distance $d^H_{\min}$ is desired to be increased, the code rate r should be

   (a) reduced
   (b) not possible to determine without specifying the exact channel coding scheme
   (c) kept constant
   (d) increased

6. (2 points) For a rate $r = 2/3$ SPC code, the probabilities that the two of three received bits are unity are given as p_1 and p_2. The odds in favor of the third bit equal to unity (the posterior probability ratio) is given as

   (a) $\dfrac{p_1 p_2}{p_1 p_2 + (1-p_1)(1-p_2)}$
   (b) $\dfrac{p_1(1-p_2) + (1-p_1)p_2}{p_1 p_2}$
   (c) $\dfrac{p_1(1-p_2) + (1-p_1)p_2}{p_1 p_2 + (1-p_1)(1-p_2)}$
   (d) $\dfrac{p_1 p_2}{(1-p_1)(1-p_2)}$

[6] To answer this question, you will need to know (i) what is the (as a function of q) of the RV that is obtained by summing 10 Bernouilli(q) RVs. (ii) how does this variance change when this sum is divided by 10 to obtain the average.

7. (2 points) Let an OOK transmitter send bit $X = 1$ at $+5$ volts with probability q. The transmitted signal is corrupted with additive Gaussian channel with zero mean and unit variance. The posterior on $X = 0$ given a measured voltage r at the channel output is

(a) $\dfrac{\exp\left(-r^2/2\right)(1-q)}{\exp\left(-(r-5)^2/2\right)q}$

(b) $\dfrac{\exp\left(-(r-5)^2/2\right)q}{\exp\left(-r^2/2\right)(1-q)+\exp\left(-(r-5)^2/2\right)q}$

(c) $\dfrac{\exp\left(-(r-5)^2/2\right)q}{\exp\left(-r^2/2\right)(1-q)}$

(d) $\dfrac{\exp\left(-r^2/2\right)(1-q)}{\exp\left(-r^2/2\right)(1-q)+\exp\left(-(r-5)^2/2\right)q}$

8. (2 points) Suppose a communication transmitter uses two orthogonal waveforms

$$g_i(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & \text{otherwise} \end{cases} \quad ; \quad g_q(t) = \begin{cases} 1, & 1 \le t < 2 \\ 0, & \text{otherwise} \end{cases}.$$

Which one of the four options shown in Fig. 3 is the correct waveform for a modulated transmitted waveform which has magnitude of 2 and phase of 60^o.

(a) (b)
(b) (d)
(c) (a)
(d) (c)

9. (2 points) The error probability for the BFSK whose bits are transmitted with energy of E_b and whose equiprobable symbols are transmitted with symbol energy E_s is given as

(a) $Q\left(\sqrt{2E_s/N_0}\right)$
(b) $Q\left(\sqrt{E_s/N_0}\right)$
(c) $Q\left(\sqrt{2E_b/N_0}\right)$
(d) $Q\left(\sqrt{E_b/\sigma_n^2}\right)$

10. (2 points) Suppose a transmitter is sending information bits at $R_b = 1000$ bits/second over a BSC with cross-over probability $p = 0.5$. The maximum possible data rate at which the receiver can receive the transmitted information reliably equals

(a) 500 bps
(b) $1000 \times H_b(p)$ bps
(c) 0 bps
(d) 250 bps

11. (2 points) The average energy per symbol for a 16-QAM constellation is given as 10 units-squared. The minimum Euclidean distance equals

(a) 2.5 units

(b) 2 units
(c) $\sqrt{2.5}$ units
(d) 5 units

12. (2 points) Compare 16-PSK with 16-QAM. Which has greater spectral efficiency? Which has better energy efficiency?

    (a) 16-PSK has the same energy efficiency as 16-QAM. 16-QAM has a higher spectral efficiency than 16-PSK.
    (b) 16-PSK has the same spectral efficiency as 16-QAM. 16-QAM has a higher energy efficiency than 16-PSK.
    (c) 16-QAM has a higher spectral efficiency, 16-PSK has a higher energy efficiency.
    (d) 16-PSK has a higher spectral efficiency, 16-QAM has a higher energy efficiency.

13. (2 points) When the receiver is nearly certain that the transmitter has sent bit $X = 1$, the posterior likelihood ratio or the odds in favor of $X = 0$ becomes

    (a) 0
    (b) 0.5
    (c) 1
    (d) ∞

14. (1 points) The inner or the dot product between two functions $p(x)$ and $q(x)$ of a continuous variable x can be defined as

    (a) $p(x)\,q(x)$
    (b) $p(x)\,q(x)\,\sin(\theta)$
    (c) $p(x)q(x)\cos(\theta)$
    (d) $\int_{-\infty}^{\infty} p(x)\,q(x)\,dx$

15. (2 points) What is the modulation index of the DSB-FC (Conventional) AM signal whose TDV is shown in Fig. 2?

    (a) $a_{mod} = 1.75$
    (b) $a_{mod} = 0.35$
    (c) $a_{mod} = 0.7$
    (d) $a_{mod} = 0.5$

16. (1 point) Compared to the union bound, the true error probability of a demodulator

    (a) can be either greater or less depending on the channel state.
    (b) is less.
    (c) is greater.
    (d) is equal.

17. (2 points) Which of the two constellation diagrams shown in Fig. 4 correspond to a modulation scheme with lower probability of symbol errors for a given average E_s/N_0?

(a) Choice A
(b) Both Choice A and Choice B give the same probability of symbol errors for a given average E_s/N_0
(c) Choice B
(d) Either Choice A or Choice B depending on the value of E_s/N_0

18. (2 points) One of the four sets of signals shown in Fig. 1 is not a valid choice of the modulation function set $g_I(t)$ and $g_Q(t)$. Which one?

(a) a
(b) b
(c) c
(d) d

19. (2 points) The informative source produces a bit rate $R_b = 20000$ bits/sec. What is the symbol rate in kilo-symbols per second after a rate $r = 1/4$ channel encoder and 16-QAM modulator?

(a) 5
(b) 20
(c) 40
(d) 10

20. (3 points) Suppose the BEC erasure probability is p_1. What is the probability of an erasure in a message from VN to CN if the probability of erasure of the CN to VN message is p_2 and the degree of VN is d_v?

(a) $(p_1 \times p_2)^{d_v - 1}$
(b) $(p_1 \times p_2)^{d_v}$
(c) $p_1 \left(p_2^{d_v - 1}\right)$
(d) $p_2 \left(p_1^{d_v - 1}\right)$

21. (2 points) The decision boundary of an M-ary PSK receiver is a function of the noise variance:

(a) Never
(b) Always
(c) When M symbols are equally likely to be transmitted
(d) When M symbols are not equally likely to be transmitted

22. (1 point) Modern coding techniques such as LDPC codes and Polar Codes (used in 5G) and Turbo Codes in 4G are considered to be powerful since they have the potential to turn a communication channel into

(a) A binary symmetric channel

(b) A waterfall channel
(c) A binary erasure channel
(d) An on-off channel

23. (2 points) Suppose a symbol with duration $T_{sym} = 2$ seconds is located on the constellation diagram at the coordinates $[s_i, s_q] = [2\sqrt{3}, 2]$. The transmitted RF signal at radio frequency $f_c = 100$ Hz is written as

(a) $2\cos(200\pi t + \pi/6)$
(b) $4\cos(200\pi t + \pi/3)$
(c) $4\cos(200\pi t + \pi/6)$
(d) $2\cos(200\pi t + \pi/3)$

24. (3 points) Suppose the M symbols at an M-ary PSK transmitter are located on a circle of squared-radius E_s units and the additive Gaussian noise channel has the power spectral density of N_0 units. The symbol detection error probability of an M-ary PSK receiver using the Improved Union Bound, which considers only the two nearest neighbor symbols, equals

(a) $2Q\left(\sqrt{2E_s}\sin(\pi/M)/N_0\right)$
(b) $Q\left(\sqrt{2E_s}\sin(2\pi/M)/N_0\right) + Q\left(\sqrt{E_s}\sin(2\pi/M)/N_0\right)$
(c) $Q\left(\sqrt{2E_s}\sin(\pi/M)/N_0\right) + Q\left(\sqrt{E_s}\sin(\pi/M)/N_0\right)$
(d) $2Q\left(\sqrt{E_s}\sin(2\pi/M)/N_0\right)$

25. (2 points) The parity check matrix for a rate $r = 1/3$ repetition code is given as

(a) $\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$
(b) $\mathbf{H} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$
(c) $\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$
(d) $\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

26. (1 point) Which of the following is an example of sliding-window ARQ technique?

(a) Hybrid ARQ
(b) Selective Repeat ARQ
(c) Round-robin ARQ
(d) Stop and Wait ARQ

27. (2 points) For a rate $r = 1/3$ repetition code, the *log* likelihood ratios (LLRs) in favor of the two of three received bits are unity are given as ξ_1 and ξ_2. The LLR in favor of the third bit equal to unity

is given as

(a) $\xi_1 \times (1-\xi_2) + (1-\xi_1) \times \xi_2$
(b) $\xi_1 \times \xi_2$
(c) $\xi_1 + \xi_2$
(d) $\xi_1 \times \xi_2 + (1-\xi_1) \times (1-\xi_2)$

28. (1 points) The EM waves $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ satisfy the two requirements for the quadrature modulation signals $g_I(t)$ and $g_Q(t)$. True or False?

(a) True.
(b) False.

29. (2 points) Suppose the BPSK transmitter sends bits $X = 1$ and $X = 0$ with equal probability. When the BPSK receiver does not register any measurable voltage at the output of the additive Gaussian noise channel, the posterior likelihood ratio or the odds in favor of $X = 1$ becomes

(a) ∞
(b) 1
(c) 0
(d) $-\infty$

30. (2 points) The decision region for an M-ary QAM symbol may sometimes not include that symbol itself. When?

(a) Never
(b) The M symbols are not equally likely to be transmitted
(c) The M symbols are equally likely to be transmitted
(d) the additive noise introduced by the channel is very large

2 Fill In the Blanks

Instructions

- ▷ There are total 16 questions in this section that carry total of 45 marks.
- ▷ There is no negative marking in this section.

Part 1

1. (1 point) In the presence of additive Gaussian noise, what is the name of the receiver which is equivalent to the Bayesian ML receiver?

2. (2 points) What are the full forms of the EVM and BLER?

3. (1 point) Energy of a symbol located on a constellation diagram is defined as

4. (1 point) Energy of a time domain signal $s(t)$ over a duration 0 to T seconds is formulated as

5. (1 point) What is functional form of the integral $\frac{1}{\sqrt{2\pi\sigma_n^2}} \int_0^\infty \exp\left(-\frac{(x+a)^2}{2\sigma_n^2}\right) dx$

6. (1 point) The OOK transmitter uses +5 volts to transmit bit $X = 1$ and 0 volts to send $X = 0$. When the output of the channel is r volts, the ML OOK demodulator rule is

7. (2 points) The power of the additive Gaussian distributed noise signal $n(t)$ at time $t = 5$ is given as $P_n(t = 5) = 10$ Watts. Can this statement be correct? Briefly describe your answer.

8. (2 points) The maximum rate of information transfer over the BSC(p) (i.e., the mutual information between RVs corresponding to the input and the output of the channel) is given as

Part 2

9. (3 points) Write expression for the Union Bound on the error probability for the Constellation A in Fig. 5:

10. (3 points) Write expression for the Union Bound on the error probability for the Constellation B in Fig. 5:

11. (4 points) Are the following three cases possible? For each case, if the answer is 'no', explain why. If it is 'yes', provide an example of modulation and coding design that achieves the stated relationship.

1. Energy per bit E_b is smaller than the energy per symbol E_S, i.e., $E_b < E_S$.
2. $E_b = E_S$.
3. $E_b > E_S$.

12. (4 points) Determine the symbol rate, bandwidth efficiency in bps/Hz and required received information bit energy to noise ratio E_b/N_0 to support a data rate $R_b = 10$ Mbps for the following cases:

    1. QPSK modulation, code rate $r = 0.5$, required $E_S/N_0 = 1.2$ dB
    2. 32-APSK modulation, code rate $r = 0.8$, required $E_S/N_0 = 14$ dB

13. (5 points) When the transmitted bit X is modeled as the Bernouilli($q = 0.5$) RV, and the channel is the BEC(p), what are (i) the conditional entropy $H(X\,|\,Y)$ and (ii) the maximum rate of information transfer (i.e., mutual information)? Provide the derivation steps.

14. (5 points) When X is the Bernouilli(p) RV, provide expressions for (i) for the expected number k of ones in the sequences of length N bits generated by N realizations of X; (ii) the number of length N binary sequences that have these number of ones.

15. (5 points) Consider a beamformer with two antenna elements. A user terminal located on the ground at an angle of $\breve{\theta}$ (relative to line connecting the antenna elements) transmits a signal $s(t)$. Resultant signals at the output of the two antenna elements are given by $r_1(t) = \alpha s(t)$ and $r_2(t) = \alpha \exp(-j\theta)\, s(t)$ (α is the path loss attenuation and the phase θ is a one-to-one function of the Direction of Arrival (DoA) angle $\breve{\theta}$). Beamformer output is given as $r(t) = r_1(t) + \exp(j\phi)\, r_2(t)$, where $\exp(j\phi)$ is a complex-valued scalar weighting that the beamformer applies to antenna 2 relative to antenna 1. Define $\varphi = \phi - \theta$. Determine the closed-form expression of the magnitude response of beamformer, defined as $\left|\frac{r(t)}{\alpha s(t)}\right|$. For which value of θ, is the response of the beamformer the greatest? For which θ does it become zero?

16. (5 points) There are two twenty-faced marbles A and B. On each face of each marble, one of five symbols $(C, T, 5, 1, 6)$ is written. Number of times these symbols appear on the faces of each marble is listed in Table 1. Your friend chooses a marble at random, rolls the chosen marble 5 times and tells you that the outcomes turned out to be C, T, 5, 1, 6. Given this *evidence*, what is the ratio of probabilities that your friend chose marble A versus the probability that he/she chose marble B? From this ratio, determine the probability that the chosen marble is marble A.

Table 1: Marble Characteristics for Problem 16

Symbol	C	T	5	1	6
Number of Faces of Marble A	8	3	4	2	3
Number of Faces of Marble B	4	6	6	3	1

Part 3

- Please mention three concepts of CT216 that you liked and that will stay with you.
- Wish you all the best. Have a great summer break.

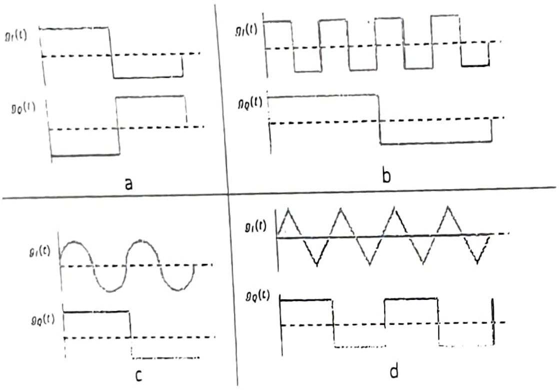

Figure 1: Two signals for Problem 18.

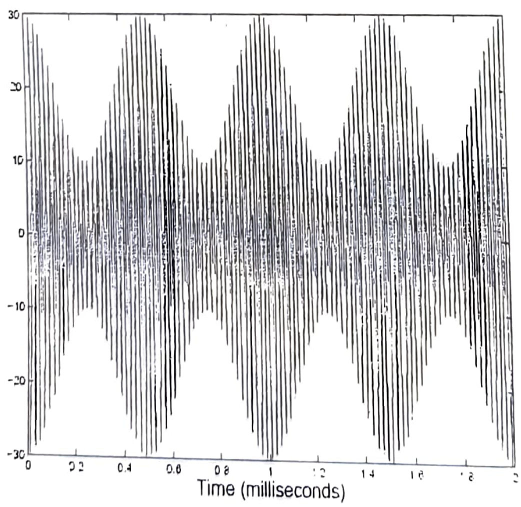

Figure 2: DSB-FC Waveform for Problem 15.

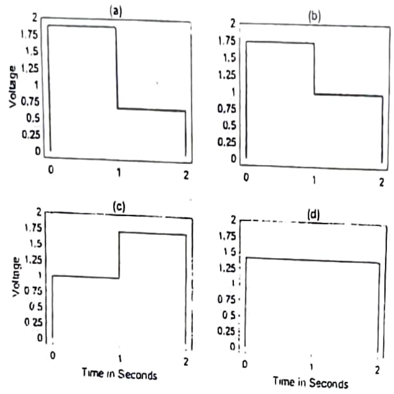

Figure 3: Modulated signal waveforms for Problem 8.

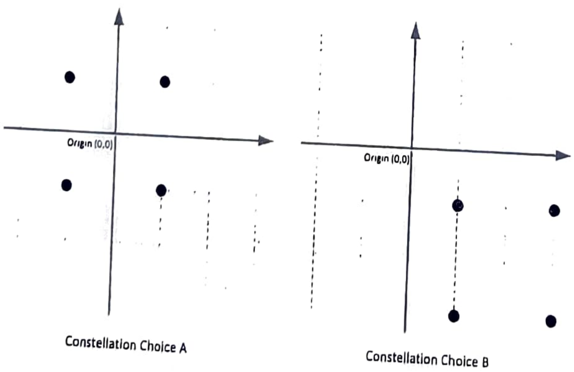

Figure 4: Signal Constellation for Problem 17.

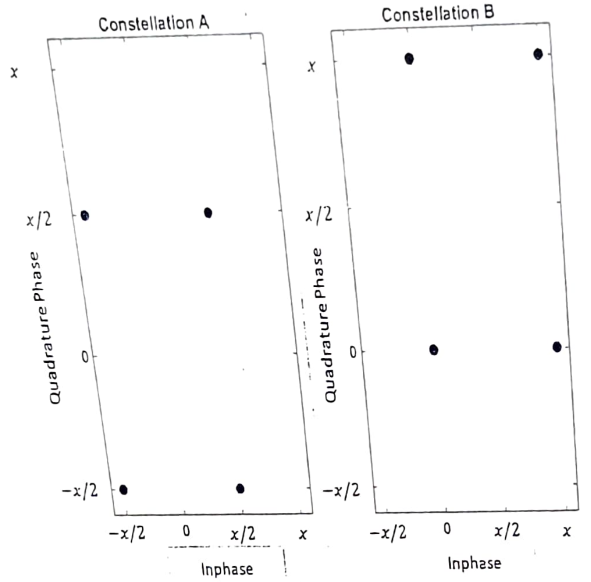

Figure 5: Constellations A and B for Problems 9 and 10, respectively.

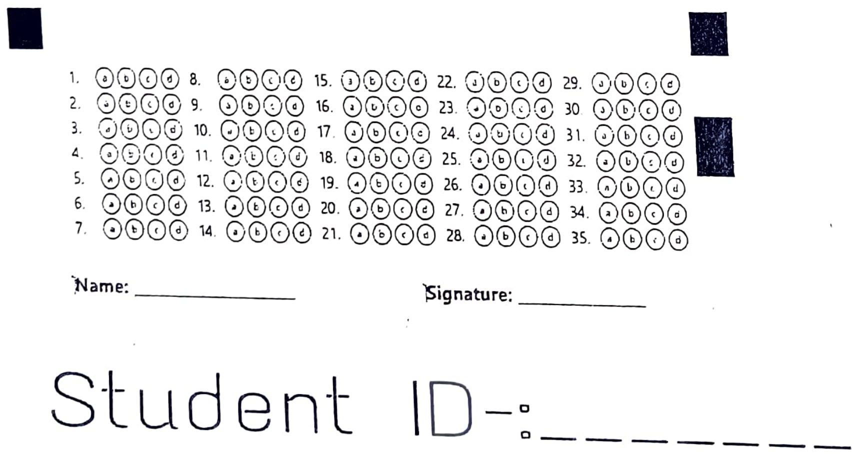

Figure 6: Indicate your MCQ answers by filling up this OMR sheet using a ball-point pen. Place your name and student ID.