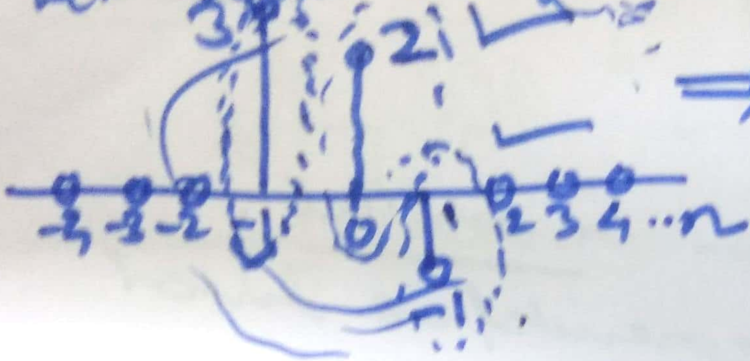
Sampling Property of Impulse signal:→

→ Chapter 7 → Sampling Theory (SAS Lecture 09)

→ Chapter 2 → " Convolution " → ??

Case I) Discrete-time signal $x(n)$

Let $x(n)$ be given by

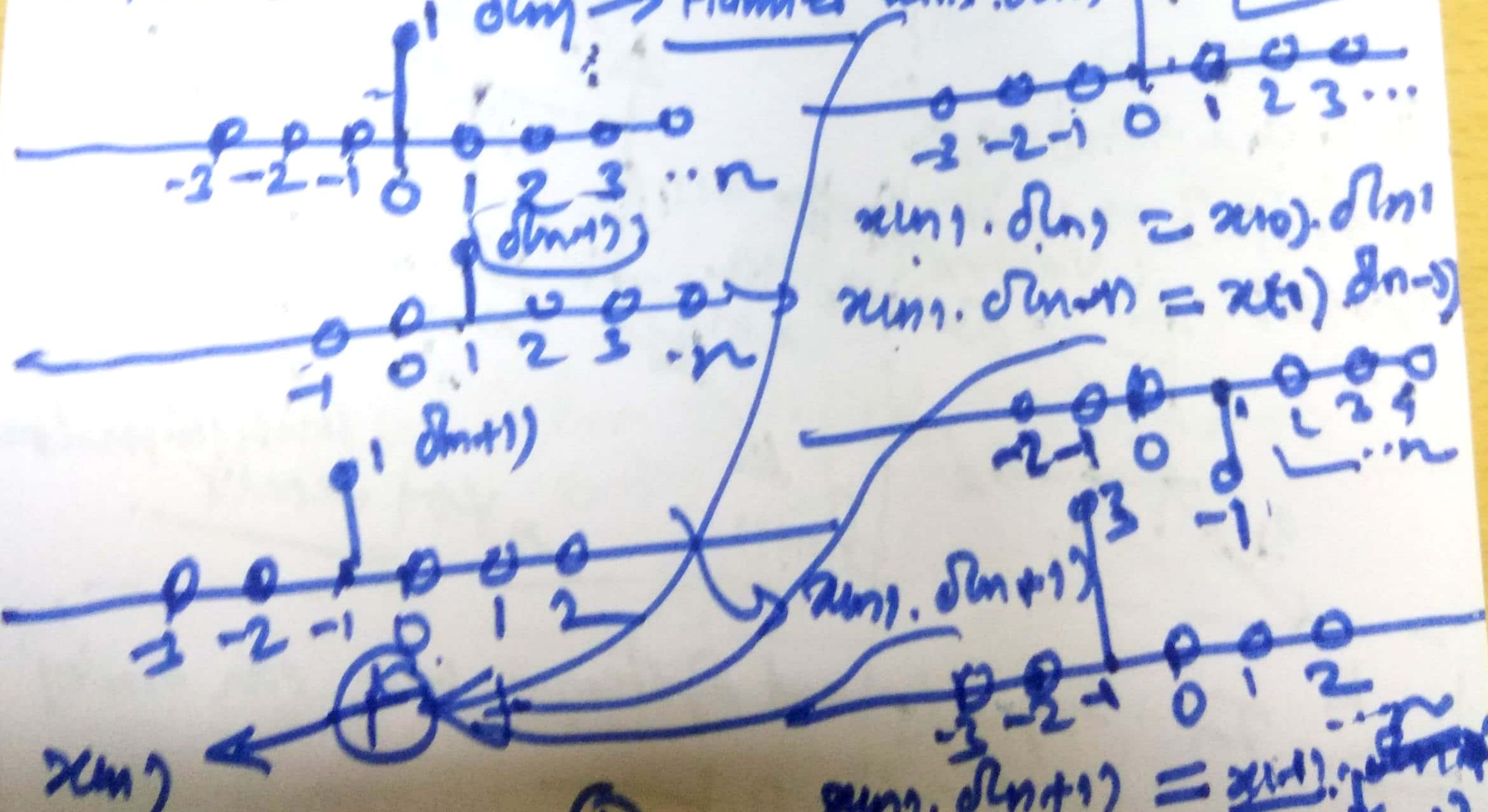


$\Rightarrow x(n) = \{..0, 3, \underset{\uparrow}{2}, -1, 0, ..\}$

$n = 0$

Question:- How $x(n)$ can be represented in terms of unit impulse signal $\delta(n)$?

$\delta(n) \rightarrow$ Hammer $x(n).\delta(n)$



$x(n).\delta(n) = x(0).\delta(n)$

$x(n).\delta(n-1) = x(1)\,\delta(n-1)$

$x(n).\delta(n+1)$

$x(n).\delta(n+1) = \underline{x(-1)}.\,\delta(n+1)$

$\hookrightarrow \times\ \delta(n+1)$

$x(n)$

①

$$\boxed{x(n) = \sum_{k=-\infty}^{+\infty} x(k)\cdot\delta(n-k)}$$

+ . . .

Coefficient of linear combination, shifted impulses

Sifting property of impulse signal.

Breaking

Concept of linear combination.

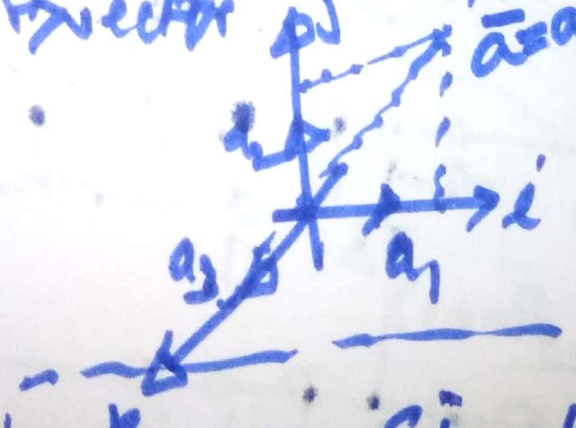
Here x(n) is represented as linear combination of shifted impulses

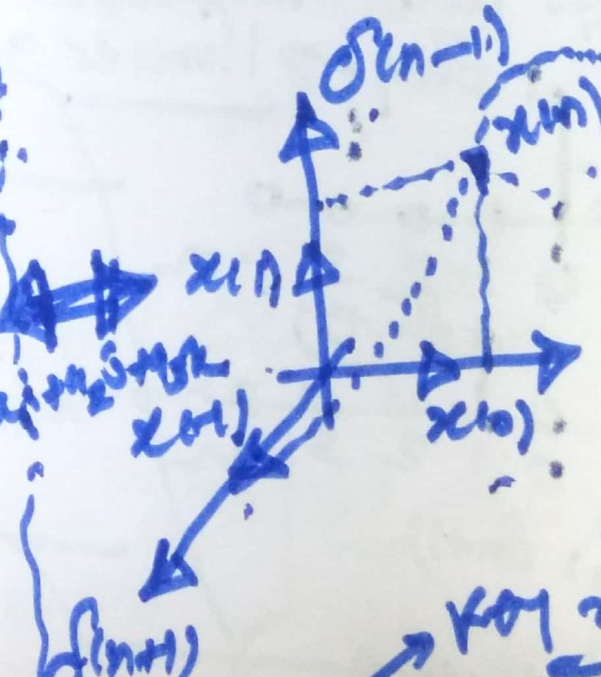
$\sum_{k=-\infty} x(k)\,\delta(n-k)$

x(n) ⇒

$\bar{a} = a_1 i + a_2 j + a_3 k$

$\{i, j, k\}$ → unit vector

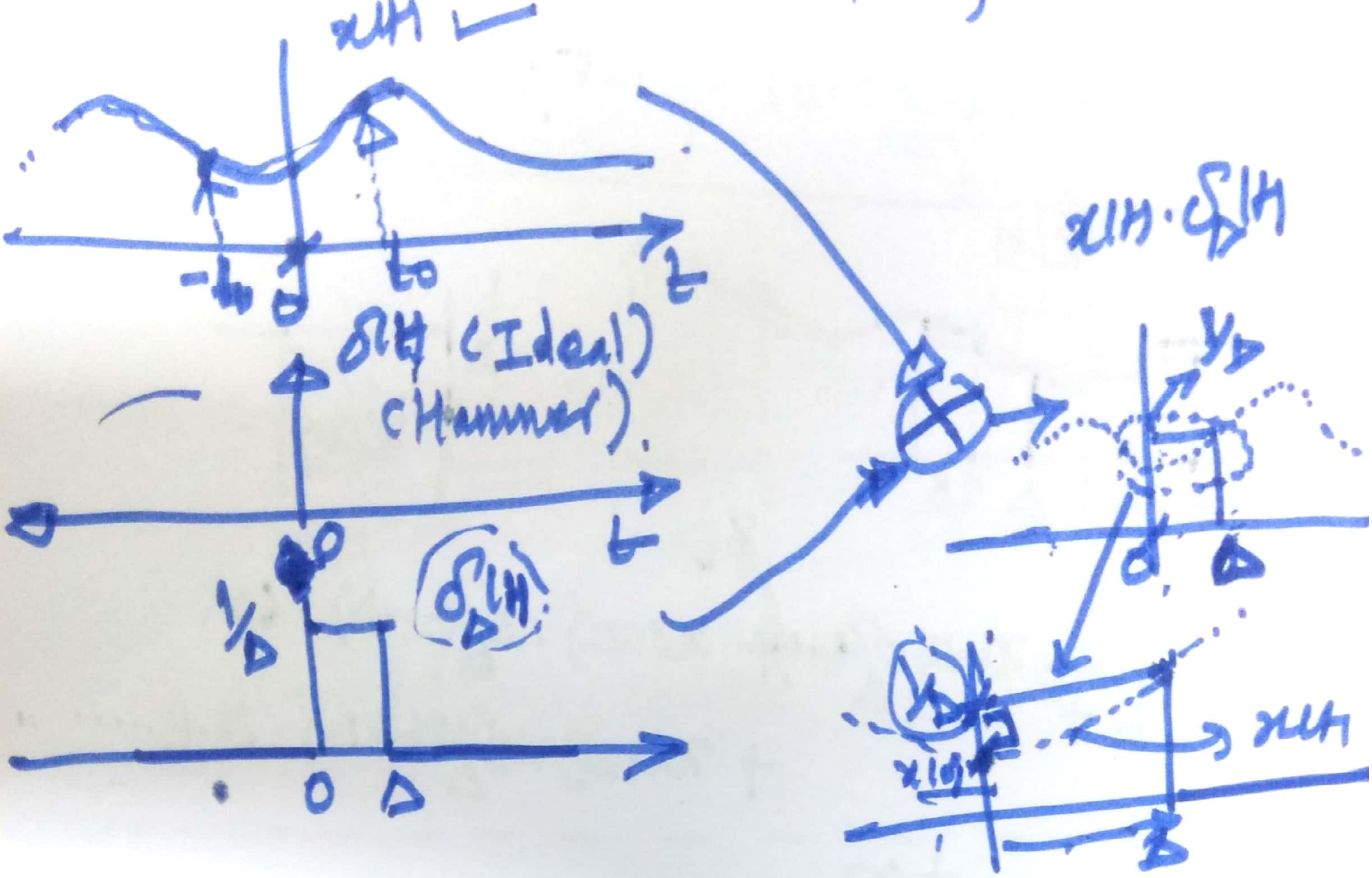




unit impulse vector result

In Signals and Systems, we can model signals as vectors.

Case II Continuous-time signal, $x(t)$



$$x(t)\cdot\delta_\Delta(t) \approx x(0)\cdot\delta_\Delta(t)$$

Take $\lim\limits_{\Delta\to 0}$ on both the sides, we get

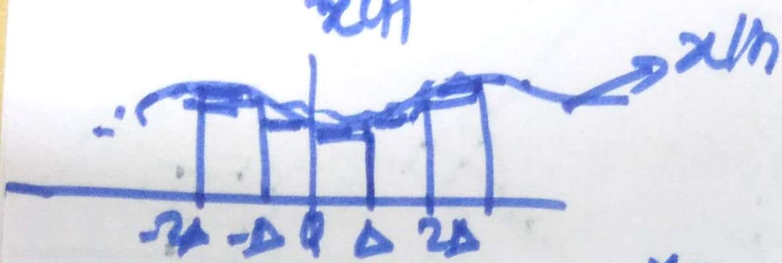
$$\lim_{\Delta\to 0}\left[x(t)\cdot\delta_\Delta(t)\right] \approx \lim_{\Delta\to 0}\left[x(0)\cdot\delta_\Delta(t)\right]$$

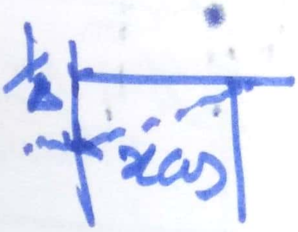
$$x(t)\left[\lim_{\Delta\to 0}\delta_\Delta(t)\right] \approx x(0)\left[\lim_{\Delta\to 0}\delta_\Delta(t)\right]$$

Apply limit

$$\boxed{x(t)\cdot\delta(t) = x(0)\cdot\delta(t)}$$

$$\boxed{\begin{aligned} x(t)\cdot\delta(t-t_0) &= x(t_0)\cdot\delta(t-t_0) \\ x(t)\cdot\delta(t+t_0) &= x(-t_0)\,\delta(t+t_0) \end{aligned}}$$





$\times\Delta$

$$\hat{x}(t) = \dots + x(-\Delta)\,\delta_\Delta(t+\Delta) + x(0\Delta)\cdot\delta_\Delta(t-\Delta)\cdot\Delta + x(1\Delta)\cdot\delta_\Delta(t+\Delta)\cdot\Delta + \dots$$

$$\hat{x}(t) = \sum_{k=-\infty}^{+\infty} x(k\Delta)\cdot\delta_\Delta(t-k\Delta)\cdot\Delta$$

Take $\lim_{\Delta\to 0}$ on both sides, we get

$$\lim_{\Delta\to 0}\hat{x}(t) = \lim_{\Delta\to 0}\sum_{k=-\infty}^{+\infty} x(k\Delta)\cdot\delta_\Delta(t-k\Delta)\,\Delta$$

Apply limit

$$\boxed{x(t) = \int_{\tau=-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau}$$

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Sifting Property

Discrete-time signal

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k]$$

$$\{\delta[n-k]\}_{k\in Z}$$

Continuous-time signal

$$x(t) = \int_{\tau=-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

$$\{\delta(t-\tau)\}_{\tau\in R}$$

Signals and Systems (CT 203)

Tutorial Sheet-02

DA-IICT, Gandhinagar.

1. For the following signals verify if they are of energy/power type. Accordingly compute the energy and power of the signal over a duration of T seconds
   (a) $g(t) = Ae^{j(2\pi f_0 t+\theta)}$
   (b) $g(t) = Ae^{-bt}(t > 0)$
   (c) $g(t) = t(t > 0)$
   (d) $g(t) = Kt^{1/4}(t > 0)$

2. Let $x_1(t)$ and $x_2(t)$ are periodic signals with fundamental periods T_1 and T_2 respectively. Under what condition is the sum $x(t) = x_1(t) + x_2(t)$ is periodic, and what is the fundamental period of $x(t)$ if it is periodic? Is $x(t) = \cos(60\pi t) + \sin(50\pi t)$ periodic? If yes find fundamental time period.

3. Determine the values of E_∞ and P_∞ for each of the following signals
   (a) $x_1(t) = e^{-5t}u(t)$
   (b) $x_2(t) = e^{j(4t+\pi/4)}$
   (c) $x_1(n) = \left(\frac{1}{3}\right)^n u(n)$

4. A continuous-time signal $x(t)$ is shown in Fig. 1. Sketch and label carefully each of the following signals:
   (a) $x(t-2)$
   (b) $x(3-t)$
   (c) $x(2t+1)$
   (d) $\left[x(t)+x(-t)\right]u(t)$

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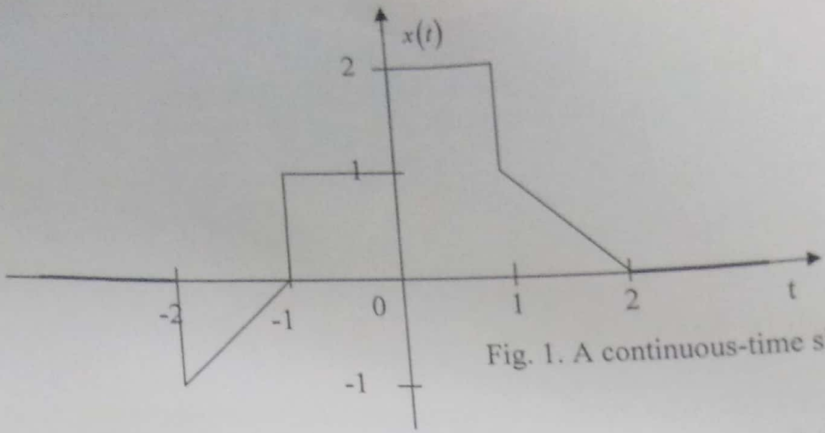

Fig. 1. A continuous-time signal

5. A discrete-time signal $x(n)$ is shown in Fig. 2. Sketch and label carefully each of the following signals:

(a) $x(n-3)$ (b) $x(3-n)$.

(c) $x(3n)$ (d) $x(n-2)\delta(n-2)$

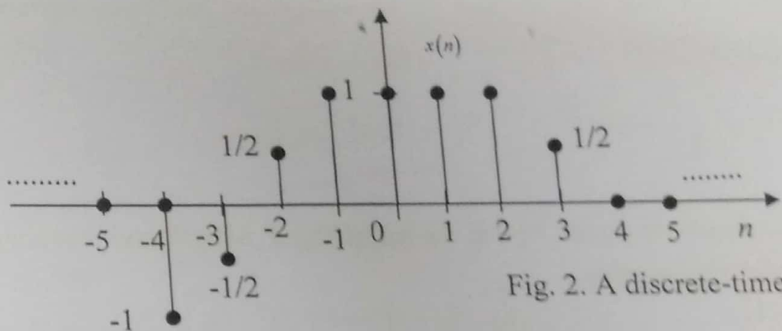

Fig. 2. A discrete-time signal

6. Determine whether each of the following signals is periodic

(a) $x_1(n) = u(n) + u(-n)$ (b) $x_2(t) = 2e^{j(t+\pi/4)}u(t)$

(c) $x_2(n) = u(n) + u(-n) - \delta(n)$ (d) $x_3(n) = \sum_{k=-\infty}^{+\infty} \{\delta(n-4k) - \delta(n-1-4k)\}$

7. Check whether the following results holds

(a) $e^{j(\omega_o+2\pi)t} = e^{j\omega_o t}, \quad t \in \mathbb{R}$ (b) $e^{j(\omega_o+2\pi)n} = e^{j\omega_o n}, \quad n \in \mathbb{Z}$

Is there any conclusion from the solution of above problem in the context of *periodicity* of continuous time vs. discrete time complex exponential signal?

8. Prove that complex exponential signal, i.e., $x(t) = e^{j\omega_o t}$ has infinite total energy but finite average power.

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Tutorial class 2

Problem 1)

① $g(t) = A \cdot e^{j(2\pi f_0 t + \theta)}$

$f(t) = e^{j\omega_0 t}$

② $g(t) = A \cdot e^{-bt} \quad (t > 0)$

$$E_\infty = \int_{-\infty}^{+\infty} |g(t)|^2 dt = \int_0^{+\infty} \left| A e^{-bt} \right|^2 dt$$

$$E_\infty = \frac{A^2}{2b} \rightarrow \text{Energy } \underline{\text{power}}.$$

$\Rightarrow$ Power $= 0$

$\hookrightarrow P_\infty = 0$

③ $g(t) = t; \quad (t > 0)$

$$E_\infty = \int_{-\infty}^{+\infty} |g(t)|^2 dt = \int_0^{+\infty} |t|^2 dt$$

$$= +\infty$$

④

$$P_{\infty} = \lim_{T \to \infty} \left(\frac{1}{2T}\right) \int_{-T}^{+T} |g(t)|^2 dt.$$

$$= \lim_{T \to \infty} \left(\frac{1}{2T}\right) \left[\int_{-T}^{+T} |t|^2 dt \right]$$

$$= \lim_{T \to +\infty} \left(\frac{1}{2T}\right) \left(\frac{t^3}{3}\right)_{-T}^{T}$$

$P_{\infty} = +\infty$

$\Rightarrow g(t) = t \ (t > 0)$ neither power nor energy signal

5) $g(t) = k\, t^{-1/4}, \ (t > 0)$.

$E_{\infty} = +\infty$

$P_{\infty} = ??$ $\Rightarrow$

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$x_1(t) = x(t+mT_1)$; $m \in \mathbb{Z}$

$x_2(t) = x(t+kT_2)$; $k \in \mathbb{Z}$.

$\because$ $x_1(t)$ & $x_2(t)$ are periodic signals.

$\therefore$ Their sum,

$$x(t) = x_1(t) + x_2(t)$$

$$x(t) = x_1(t+mT_1) + x_2(t+kT_2)$$

For $x(t)$ to be periodic with fundamental period 'T',

$$x(t) = x(t+T)$$

$$\cancel{= x_1 t}$$

$$\therefore x(t+T) = x_1(t+mT_1) + x(t+kT_2).$$

$$\Rightarrow \cancel{mT_1 = \ldots}$$

$$\boxed{mT_1 = kT_2 = T}$$

$$\frac{T_1}{T_2} = \left(\frac{k}{m}\right) = \underline{\text{rational}}$$

$T_1 = \frac{1}{30}$

$T_2 = \frac{1}{25}$.

$\frac{T_1}{T_2} = 5/6$

$\therefore T \Rightarrow 6T = 5T_2 = (1/5.) = 0.2$ seconds.

Problem (3) Find E_∞ and P_∞

i) $x_1(t) = e^{-5t} \cdot u(t)$

$E_\infty = \frac{1}{10}$
$P_\infty = 0$
$\Big\} \Rightarrow$ Energy signal

ii) $x_2(t) = e^{j \cdot (4t + \pi/4)}$.

$E_\infty = \infty$
$P_\infty = 1$
$\Big\} \Rightarrow$ Power signal.

iii) $x_1(n) = \left(\frac{1}{3}\right)^n \cdot u(n)$

$x(n) = \left(\frac{1}{2}\right)^n u(n)$

$E_\infty = 4/3$

$$E_\infty = \sum_{n=-\infty}^{+\infty} |x(n)|^2$$

$$E_\infty = \cancel{\sum_{n=0}^{+\infty} \left|\frac{1}{3}\right|^2}$$

$$= \sum_{n=-\infty}^{+\infty} \left| \left(\frac{1}{3}\right)^n \cdot \underline{u(n)} \right|^2$$

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$E_\infty = \sum_{n=0}^{+\infty} \left(\frac{1}{3}\right)^{2n} \Longrightarrow$$ Sum of infinite terms of G.P.

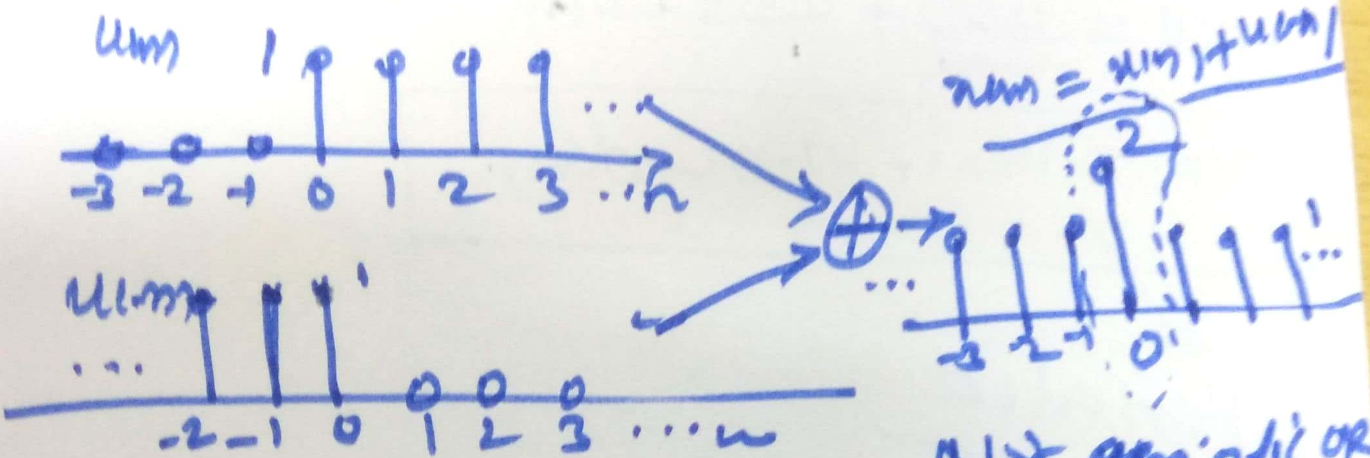
$$= \frac{a}{1-r}$$ $a = 1,\ r = 1/9$

$$E_\infty = \frac{1}{1 - 1/9} = 9/8$$

$P_\infty = ??$

Problem 6. Determine periodicity.

$x_1(n) = u(n) + u(-n)$

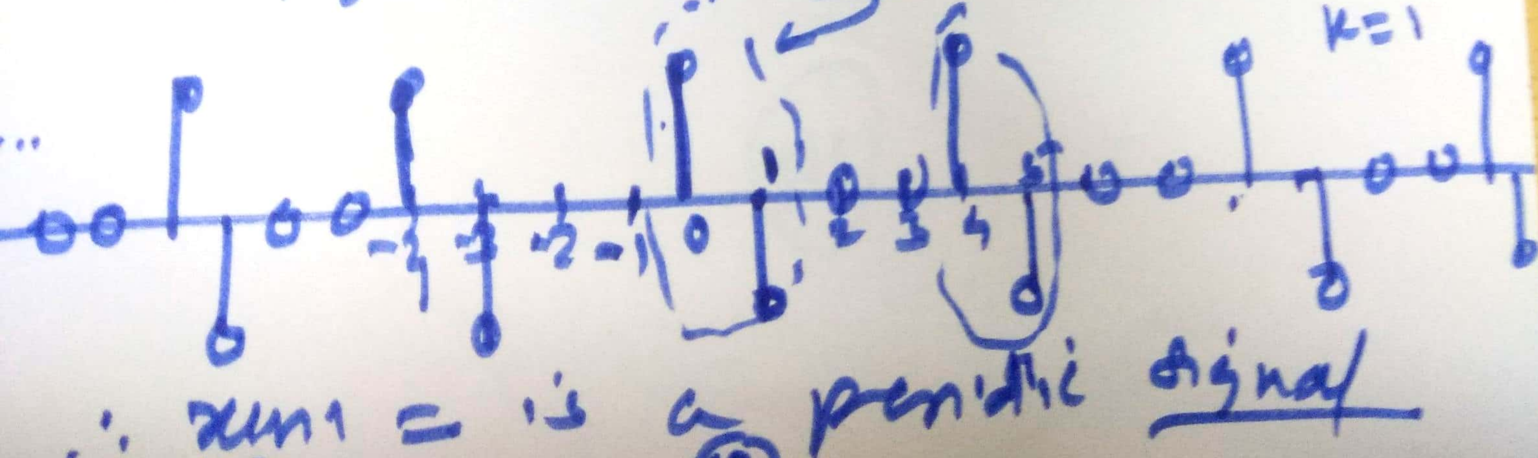


Not periodic OR aperiodic

$$x_3(n) = \sum_{k=-\infty}^{+\infty} \delta(n-4k) - \delta(n-1-4k)$$

$$= \dots + [\delta(n+4) - \delta(n+3)] + [\delta(n) - \delta(n-1)] + [\delta(n-4) - \delta(n-5)]$$

$k=1$



$\therefore x_3(n)$ = is a periodic signal

$$x_3(n) = \sum_{k=-\infty}^{+\infty} \{\delta(n-4k) - \delta(n-1-5k)\}$$

$\Rightarrow$ Not periodic

~~$x(n) = u(n) + u(-n) - \delta(n)$~~

$$\boxed{x(n) = u(n) + u(-n) - \delta(n)}$$

$\Rightarrow$ periodic

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{elsewhere} \end{cases}$$