Computational Numerical Methods

CS 374

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Lagranges form of interpolation

x_i	x_0	x_1	$\dots$	x_n
y_i	y_0	y_1		y_n

$$P_n(x) = \sum_{k=0}^{n} y_k l_k(x)$$

where

$$l_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \left(\frac{x - x_i}{x_k - x_i} \right)$$

$$l_k(x) = \frac{(x - x_0)(x - x_1)\dots(x - x_{k-1})(x - x_{k+1})\dots(x - x_n)}{(x_k - x_0)(x_k - x_1)\dots(x_k - x_{k-1})(x_k - x_{k+1})\dots(x_k - x_n)}$$

① : degree of $\underline{P_n(x) \leq n}$.

② Interpolating conditions

$$p_n(x_i) = y_i \quad i = 1 \dots n.$$

③ $$l_k(x) = \frac{(x-x_0)(x-x_1)\dots(x-x_{k-1})(x-x_{k+1})\dots(x-x_n)}{(x_k-x_0)(x_k-x_1)\dots(x_k-x_{k-1})(x_k-x_{k+1})\dots(x_k-x_n)}$$

$l_k(x_0) = 0$

$l_k(x_1) = 0$

$\vdots$

$l_k(x_n) = 0$

$l_k(x_k) = 1$

$$l_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{x-x_i}{x_k-x_i}.$$

$$l_k(x_i) = \begin{cases} 0 & \text{if } i \neq k. \\ 1 & \text{if } i = k \end{cases}$$

$$P_n(x) = \sum_{k=0}^{n} y_k \cdot l_k(x)$$

$$P_n(x_0) = y_0 .$$

$$P_n(x_1) = y_1$$

$$\vdots$$

$$P_n(x_n) = y_n .$$

$$P_n(x) = y_0 l_0(x) + y_1 l_1(x) + \cdots + y_n l_n(x)$$

Ex Consider $f(x) = e^x$.

x	0.82	0.83
y	$e^{0.82} \approx 2.2705$	≈ 2.293319

Using the data find out $e^{0.826}$

$$P_1(x) = 2.2705\left(\frac{x-0.83}{-0.01}\right) + 2.29319\left(\frac{x-0.82}{0.01}\right)$$

$$= 2.2819x + 0.399342$$

$$P_1(0.826) = 2.2841914$$

$$e^{0.826} = 2.2841638$$

Relative error.

1.205×10^{-5}

Ex

$$f(x) = e^x.$$

x	0.82	0.83	0.84
y	2.2705	2.293319	2.316367

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{x^2 - 1.67x + 0.6972}{0.0002}.$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{x^2 - 1.66x + 0.6888}{-0.0001}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{x^2 - 1.65x + 0.6806}{0.0002}$$

$$P_2(x) = 2.2705 \times l_0(x) + 2.293319\, l_1(x) + 2.316367\, l_2(x)$$

$$P_2(x) = 1.155x^2 + 0.37605x + 1.185521$$

$$P_2(0.826) = 2.28416708$$

$$\underline{\text{Relative error}} = \underline{\underline{1.44 \times 10^{-5}}}$$

Newton's interpolating polynomial

$P_{n+1} = P_n + \text{something.}$

$P_{n+1}(x_{n+1}) = y_{n+1}$

$P_{n+1}(x_n) = y_n.$

$$P_{n+1} = P_n + c(x-x_0)(x-x_1)\dots(x-x_n)$$

$$c = \frac{y_{n+1} - y_n}{(x_{n+1}-x_0)(x_{n+1}-x_1)\dots(x_{n+1}-x_n)}$$

General form of Newton's interpolating polynomial.

$$P_n(x) = A_0 + A_1(x - x_0) + A_2(x-x_0)(x-x_1) + \cdots + A_n \prod_{i=0}^{n-1} (x - x_i)$$

$$A_0 = y_0 = P_n(x_0)$$

$$P_n(x_1) = y_1 = A_0 + A_1(x_1 - x_0)$$