Kidney exchange problem

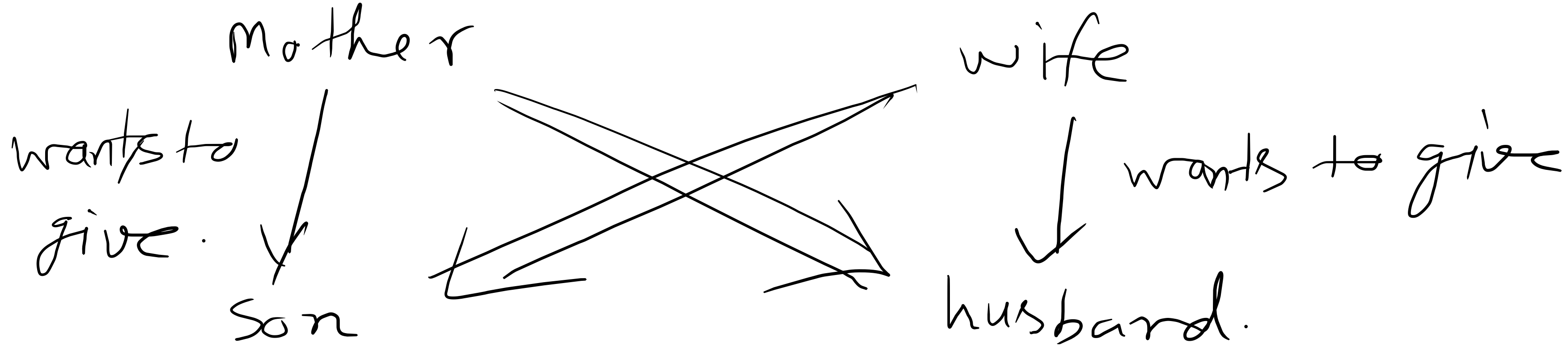
Basic facts:

~~1.~~ People can survive with one kidney only.

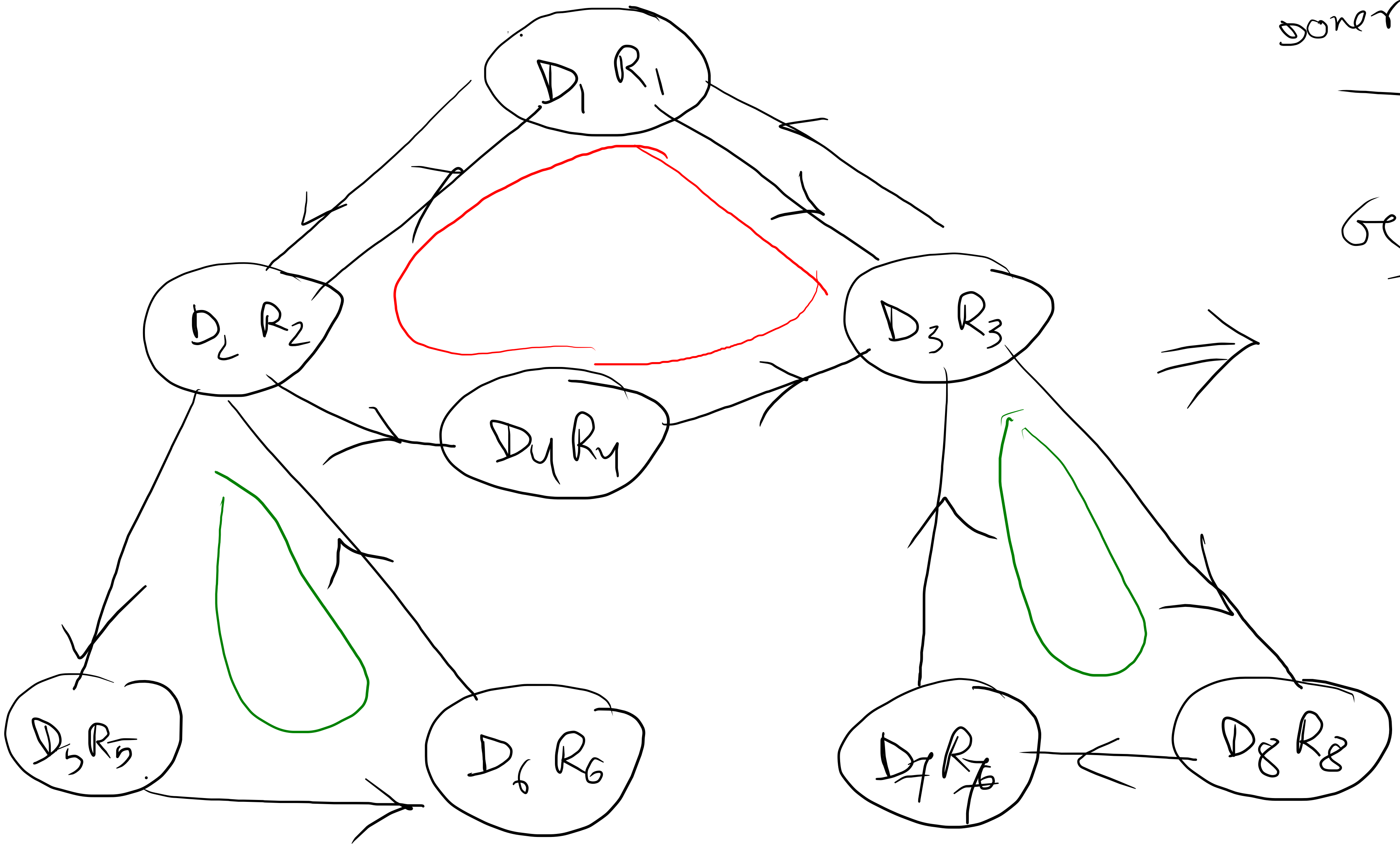
~~2.~~ Thousands of patients need only one kidney.

~~3~~ Thousands of them die because of unavailability of kidney

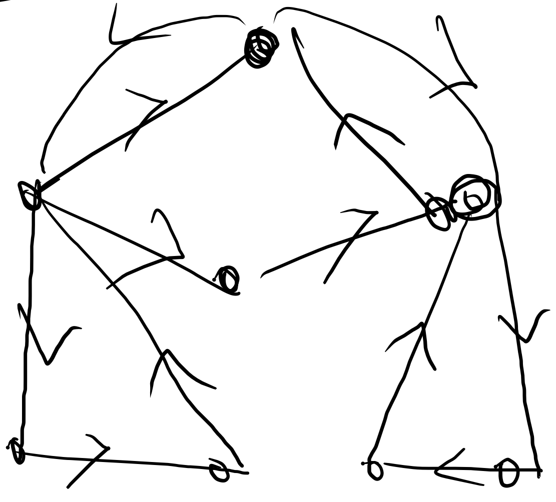
— compatibility issue.



donor$_i$ → receiver$_i$



Graph problem



- Given a directed graph
- Find a set of disjoint cycles that cover maximum vertices.

Disjoint cycle cover problem:

It is NP-complete.

cope with NP-complete problems

Optimization problems

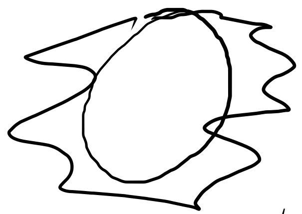
- They are everywhere.
- They are incredibly hard to solve.
- But we need to solve them. They are important.
- Many of them are NP-complete.

coping methods

- Special cases — (general problem is NP-complete a special case may be solved in poly time).

  Exm: vertex cover problem in bipartite graph can be solved in poly time.

- Approximation algorithms →



  In polynomial time find near optimum solution.

- Heuristics → difficult to measure the solution.

- Exponential time algorithm →

- Parameterised algorithms. → running time is polynomial on the input size and the given parameter.

Approximation algorithms

- It is a polynomial time algorithms.
- It gives a solution as close to the optimum.

- A be a minimization problem.
  - opt $\leftarrow$ optimum solution.
  - algo $\leftarrow$ algorithm "

$$\alpha = \frac{algo}{opt.}$$ is called the approximation for the minimization problem.

$$\boxed{\alpha > 1}$$

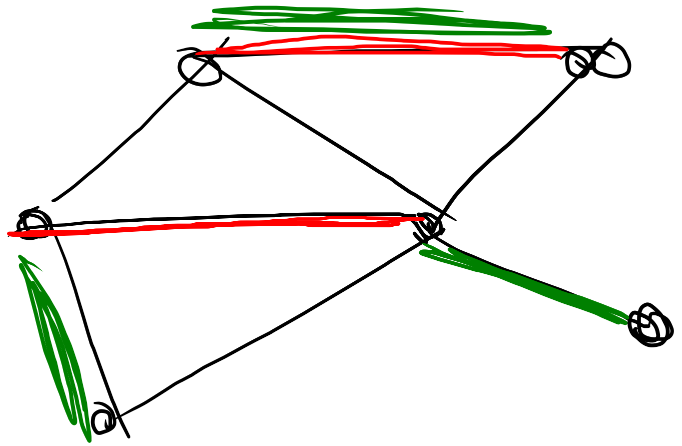
vertex cover problem

Input: $G(V,E)$

output: $V' \subseteq V$ of minimum cardinality that cover all edges.

Approximation algorithm

maximal matching : $F \subseteq E$ such that no two edges are adjacent.



maximum : $|F|$ is maximum

maximal : can not add one extra edge in F.

Approx Algo:

1. Find a ~~maximal~~ maximum matching $M \subseteq E$

2. Return the set C of end-point of each edge in M as a vertex cover.

We need to prove:

i) C is a vertex cover.

ii) Algo takes poly time

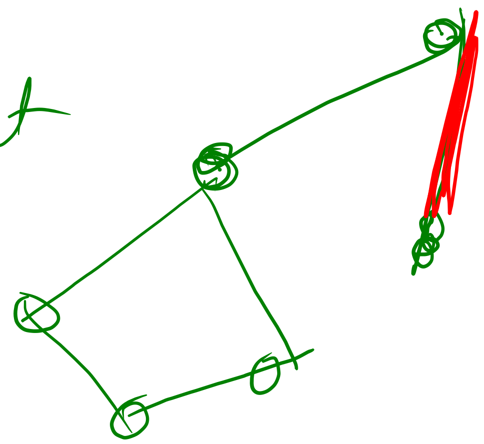
iii) Approximation factor : 2.

i) C is a vertex cover.

Let C is not a vertex cover

- ∃ an edge e whose both end point not in C.
- add e to M.

makes increment in size of M.



$\Rightarrow\!\!\Leftarrow$

ii) ~~A~~lgo takes poly time.

straightforward.

iii) Approximation factor:

opt ← optimum vertex cover. size

$$opt \geq |M| \quad \text{How they related.}$$

$$|C| = 2|M| \leq 2 \cdot opt$$

$$\frac{|C|}{opt} = 2$$