Computational Numerical Methods

CS 374

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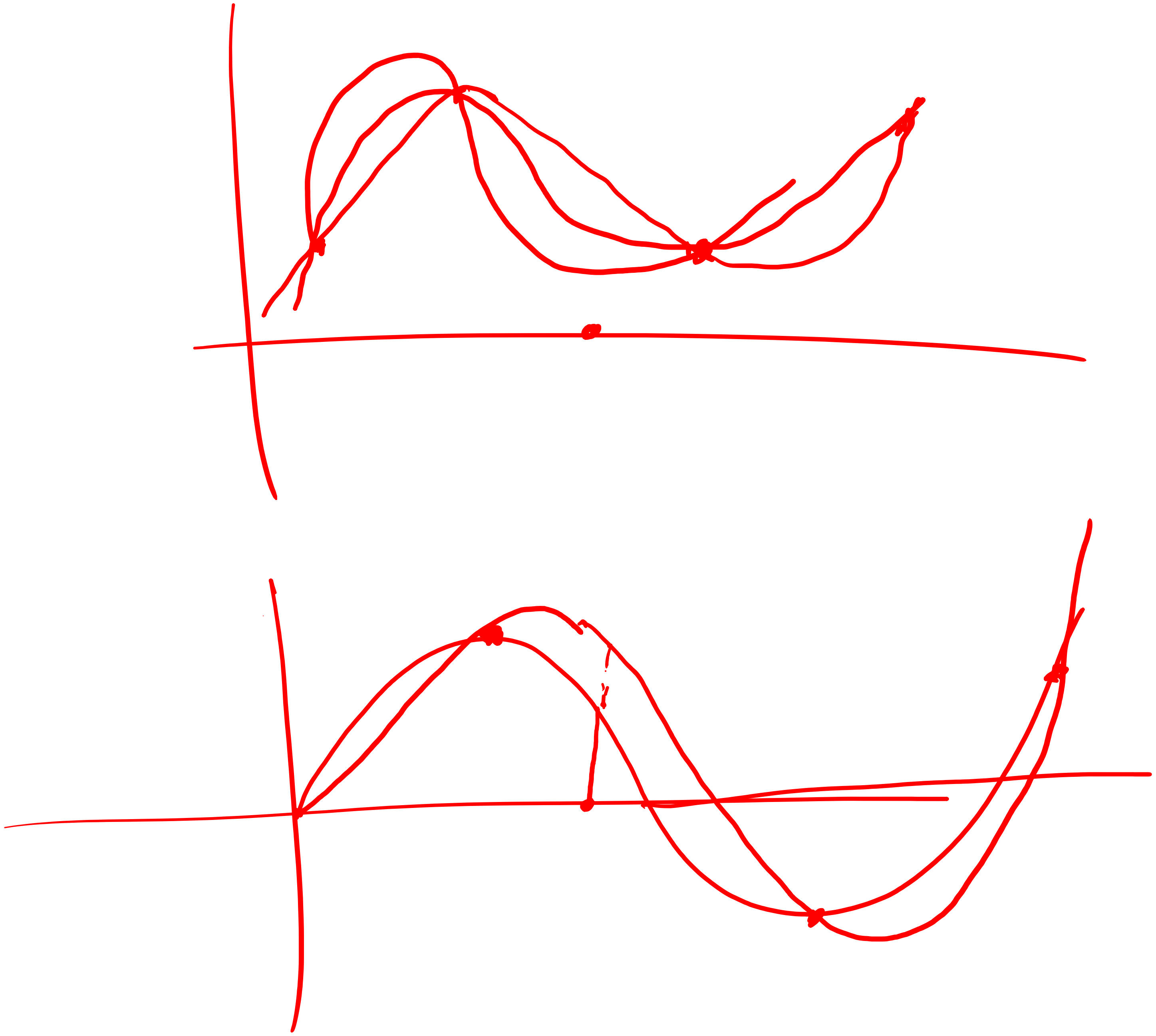
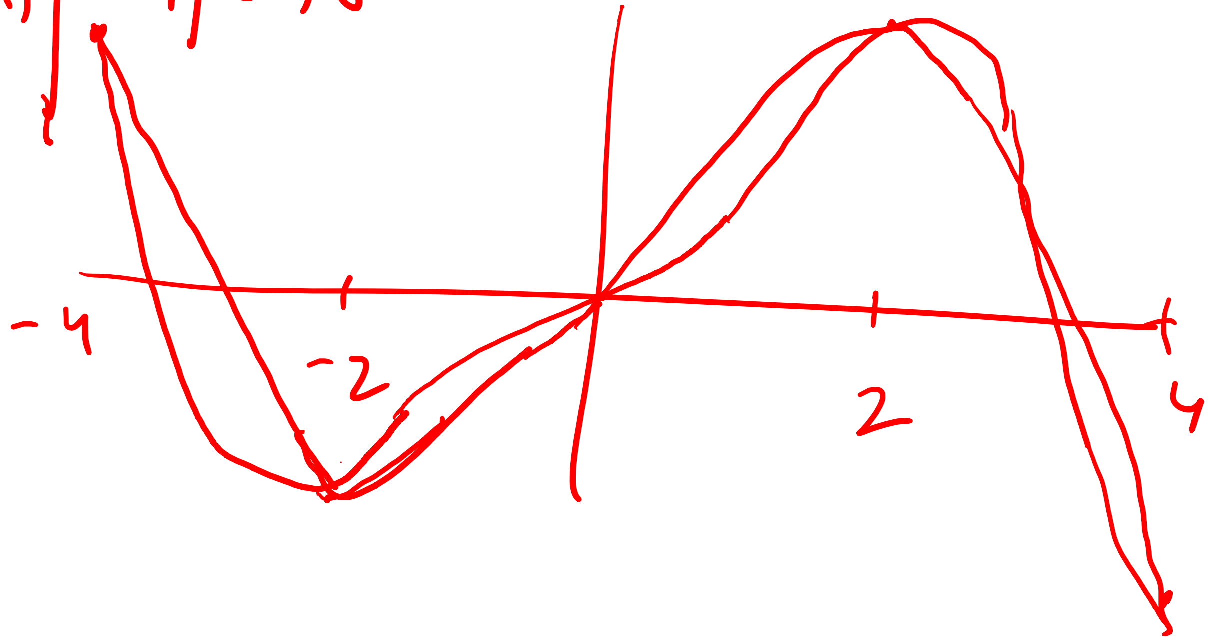
Interpolation

Polynomial interpolation



x	-4	-2	0	2	4
y	0.76	-0.91	0	0.91	-0.76

x	-4	-3	-2	-1	0	1	2	3	4
y	0.76	-0.14	-0.91	-0.84	0	0.84	0.91	0.14	0.76



Finding out the interpolating polynomials

x	x_0	x_1	x_2	$\cdots$	x_n
y	y_0	y_1	y_2		y_n

Goal → find out a polynomial $P(x)$

Let $P(x) = a_0 + a_1 x + \cdots + a_n x^n$.

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$$a_0 + a_1 x_0 + \cdots + a_n x_0^n = y_0$$

$$a_0 + a_1 x_1 + \cdots + a_n x_1^n = y_1$$

$$\vdots \qquad \vdots$$

$$a_0 + a_1 x_n + \cdots + a_n x_n^n = y_n$$

$$A a = \cancel{b}\, y$$

Vandermonde matrix.

$$A = \begin{array}{c} \begin{bmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^n \\ 1 & x_1 & x_1^2 & \cdots & x_1^n \\ \vdots & & & & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^n \end{bmatrix} \\ \begin{array}{ccccc} v_0 & v_1 & \cdots & & v_n \end{array} \end{array}$$

$$a = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} \qquad y = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_n \end{bmatrix}$$

$Aa = y$ is solvable if A^{-1} exist.

Let $v_0\ v_1 \ldots v_n$ are are the columns of A.

$$c_0 v_0 + c_1 v_1 + \cdots + c_n v_n = 0.$$

~~To show that~~

If $v_0, \dots v_n$ are L.I

then A^{-1} exist.

To show $v_0, v_1 \dots v_n$ are linearly independent one need to show

$c_0 v_0 + c_1 v_1 + \dots + c_n v_n = 0$ — (1)

$\Rightarrow C_0 = C_1 = \dots = C_n = 0$

$$\begin{bmatrix} & A & \end{bmatrix}_{3\times 3} = b \begin{bmatrix} \\ \\ \end{bmatrix}_{3\times 1}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \phi & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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let us consider the k-th eqn. of (1)

$C_0 + C_1 x_k + C_2 x_k^2 + \dots C_n x_k^n = 0$ — (2)

② will have ∞ number of soln.

which is possible only when the polynomial is a zero polynomial.

$\Rightarrow \quad c_0 = c_1 \cdots c_n = 0.$

$\therefore \quad A^{-1}$ exist.

$Aa = y$ will have uniqu soln.

Ex Let the following data points represent a function f.

x	0	0.5	1
$f(x)$	1	0.5242	-0.9037

- What is the exact expression of the function

(NOT Possible to find)

- Find $f(0.75)$

Interpolating polynomial

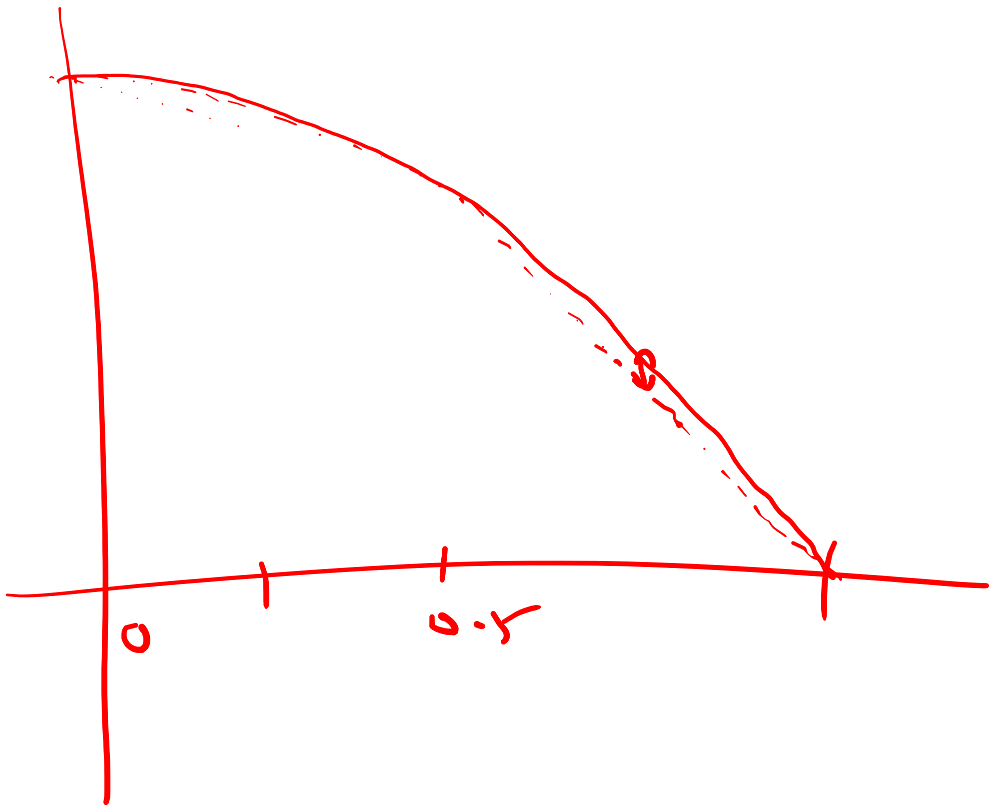
$$P(x) = -1.9042x^2 + 0.00051x + 1$$

$$P(0.75) = -0.0707.$$

Original function $\quad f(x) = \sin\left(\frac{\pi}{2} e^x\right)$

$$f(0.75) = -0.1827504$$

Relative error: $\underline{\underline{0.61}}$


0
0.5