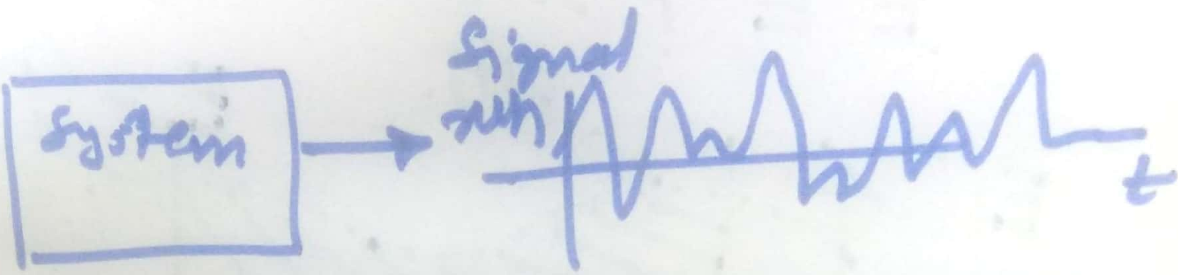
Alan V. Oppenheim, Nawab and Willsky
Signals and Systems.

Lecture 04

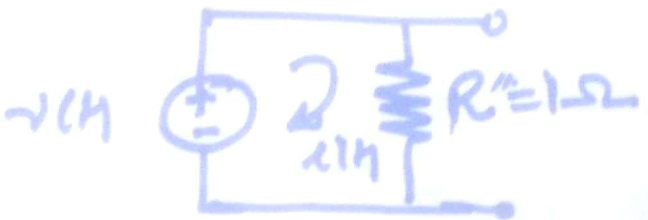
Concept of Energy and Power :→



Energy is required to generate a signal
= energy of a signal.

Energymeter :→ Monthly electricity bill

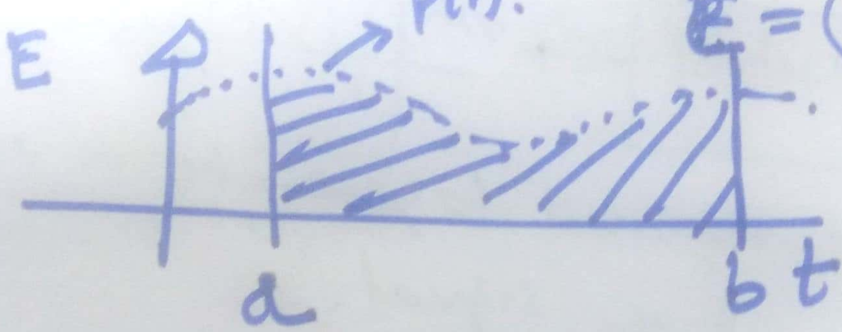
A simple electric circuit,



$\Rightarrow p(t) = v(t).i(t)$

$E = P \times t \Rightarrow y = f(x)$

$E = p(t) \times t$



$$E = \int_{t=a}^{t=b} p(t).\,dt$$

$$= \lim_{h\to 0} \sum_{\langle n\rangle} p(t_n).h$$

Ohm's law

$$E = \int_a^b v(t).\,i(t)\,dt \;; \quad v(t) = i(t).R$$

$$E = R\left[\int_a^b i^2(t)\,dt\right] \text{ OR } \left(\frac{1}{R}\right)\left[\int_a^b v^2(t)\,dt\right]$$

Excitation source → Abstract system → Output signal $\{x(t)\}$, $t \in [a,b]$

$x(t)$

Model voltage and current as signals

$$E = \int_{t=a}^{t=b} x^2(t)\, dt$$ = Energy in abstract sense.

$$E = (K) \cdot \left[\int_a^b x^2(t)\, dt \right]$$ = Energy in physical system.

$\{x(t)\}_{t \in [a,b]} \Longrightarrow \{x(t)\}_{t \in \mathbb{R}}$

Finite duration

Infinite duration

→ $x(t)$ can be real function or it can be complex function.

$$E = \int_a^b |x(t)|^2\, dt \quad \text{or} \quad E_{\infty} = \int_{-\infty}^{+\infty} |x(t)|^2\, dt$$

③ Energy for infinite duration signals.

\# $x(t) \longrightarrow \{x(n)\}$ $\quad n \in [N_1, N_2]$ $\quad \{x(n)\}$ $\quad n \in$

$$e_i = \sum_{n=N_1}^{N_2} |x(n)|^2 \longrightarrow \lim_{N\to\infty} e_i = \lim_{N\to+\infty} \sum_{n=-N}^{+N} |x(n)|^2$$

$$E = \int_a^b |x(t)|^2 dt$$

$$e_\infty = \sum_{n=-\infty}^{+\infty} |x(n)|^2$$

$$E_\infty = \lim_{T\to+\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$\boxed{E_\infty = \int_{-\infty}^{+\infty} |x(t)|^2 dt}$$

In communication system, we often deal with periodic signals, we define average power.

$$E = P \times t \quad [a, b]$$

$$\therefore P_{av} = \frac{E}{Time}$$

$$P_{av} = \frac{1}{b-a}\left[\int_{t=a}^{t=b} |x(t)|^2 dt\right]$$

$$P_\infty = \lim_{T\to+\infty} \frac{1}{2T}\left[\int_{-T}^{+T} |x(t)|^2\,dt\right]$$

Average power for continuous-time infinite duration signal is derived in limiting sense.

$$P_\infty = \lim_{T\to+\infty} \frac{E_\infty}{2T}$$

$$P_\infty = \lim_{T\to\infty} \left(\frac{1}{T}\right)\left[\int_{-T/2}^{+T/2} |x(t)|^2\,dt\right]$$

$$\boxed{P_\infty = \lim_{T\to\infty} \frac{E_\infty}{T}}$$

For discrete-time signal, $x(n)$.

$$P_{av} = \frac{1}{2N+1}\sum_{n=-N}^{+N} |x(n)|^2$$

$-N \; \cdots \; -2 \; -1 \; 0 \; 1 \; 2 \; \cdots \; N$

If $[x(n)]$, $n \in \mathbb{Z}$

$$P_\infty = \lim_{N\to+\infty} \left(\frac{1}{2N+1}\right)\left[\sum_{n=-N}^{+N} |x(n)|^2\right]$$

(5)

* Classification of signals Based on Energy / Power :-

Case I Signals having finite ~~energy~~ E_∞

i.e., $\boxed{E_\infty = K < +\infty} \Rightarrow$ Energy signals

$$\boxed{P_\infty = \lim_{T \to +\infty} \left(\frac{E_\infty}{2T}\right)}$$

$$= \lim_{T \to +\infty} \frac{(K)}{2T}$$

$$\boxed{P_\infty = 0.}$$

Infinite-duration signals

Case II) Signals with $(P_\infty > 0) \Rightarrow$ Power signals

$$P_\infty = \lim_{T \to +\infty} \frac{E_\infty}{2T} > 0$$

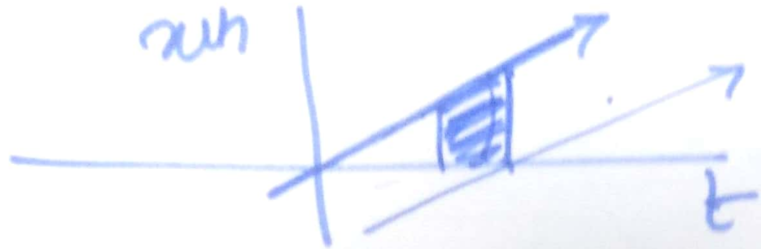
Communication systems

$$E_\infty = \lim_{T \to \infty} (P_\infty) \times (2T) = +\infty$$

Scanned by CamScanner

Case III Neither power signals nor energy signals.

$x(t) = t$



$$E_\infty = \int |x(t)|^2 dt = +\infty$$

$\rightarrow P_\infty = +\infty$

Example: Energy of vowels is relatively higher than consonants

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In Indian language, Indian languages are syllable-timed → English → phonemes

Marathi, Gujarati, Punjabi → syllable ⇒ cp|cp|

(7) Speech synthesis → Segment speech

Transformations of Independent Variable

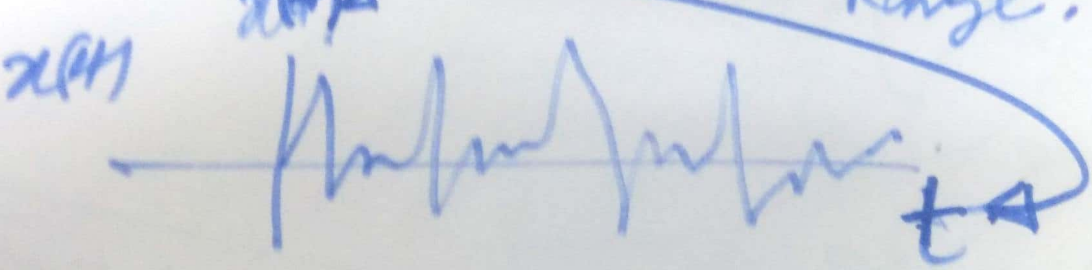
$y = f(x)$

$f(\cdot)$

Independent variable $\to x \to y = f(x) \to$ Dependent variable

Domain

Co-domain / Range



$t \to -t$ → Time Reversal

$t \to t - t_0$ or $t + t_0$

Time delay ($t - t_0$), Time advance ($t + t_0$)

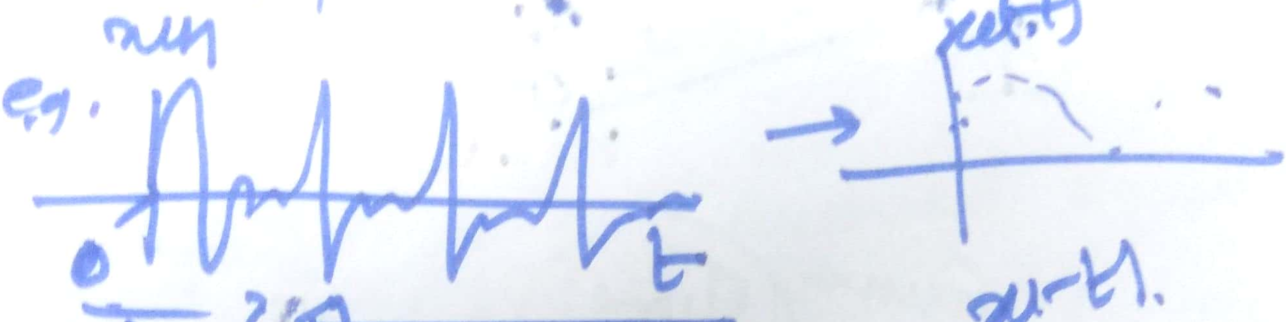
$t \to 2t$ or $t/2$ → Time Scaling

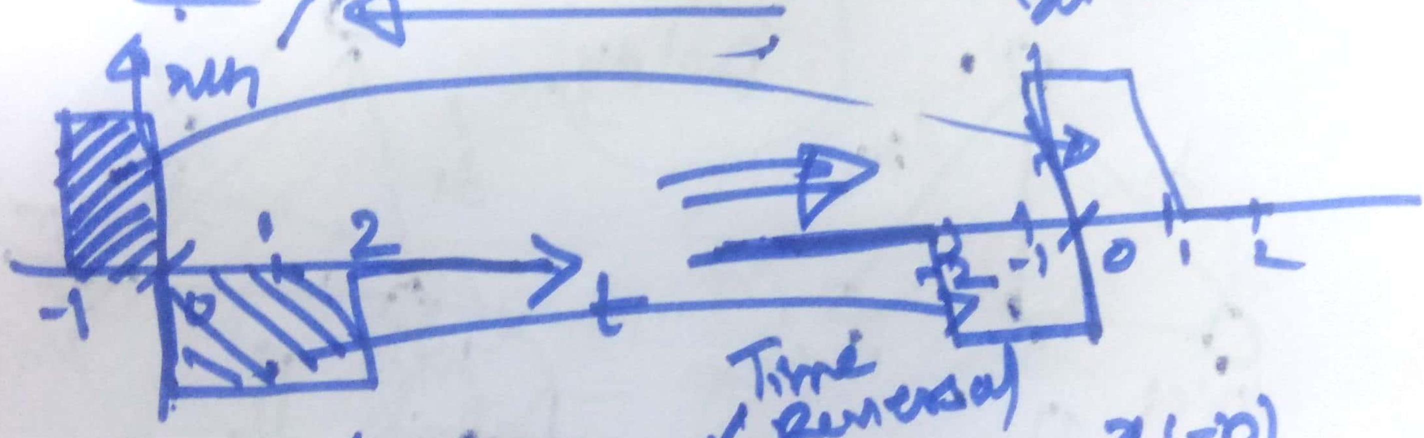
Independent Variable

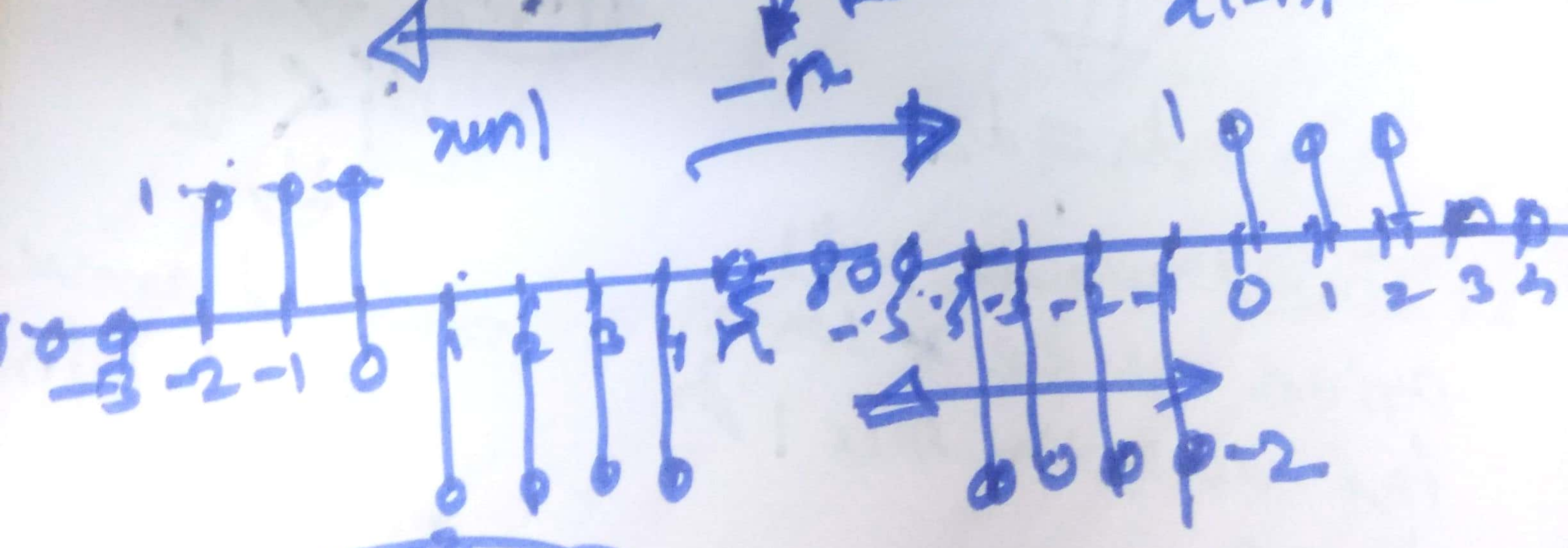
Time Reversal:—

Let x(t) be a signal then x(−t) is called time-reversed version of x(t).

e.g.

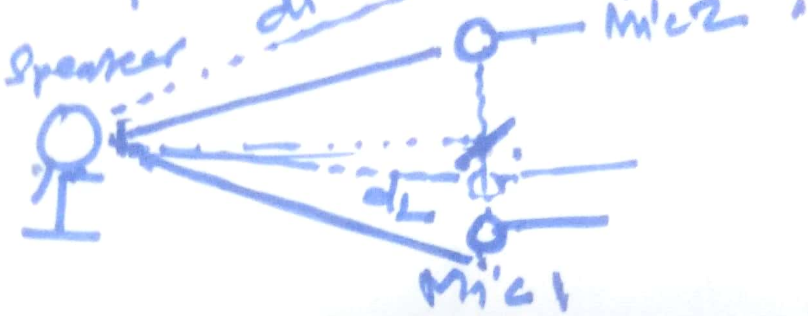




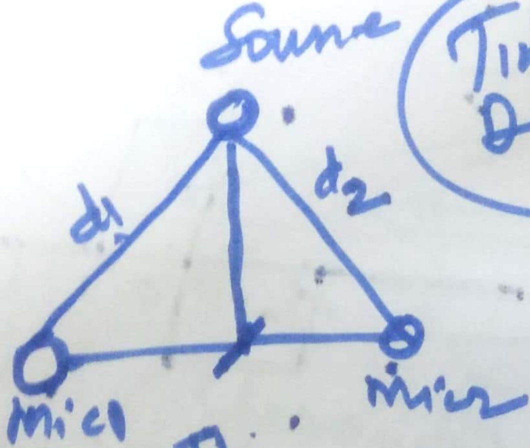


Reflection

(2) Time Delay or Time Advance.



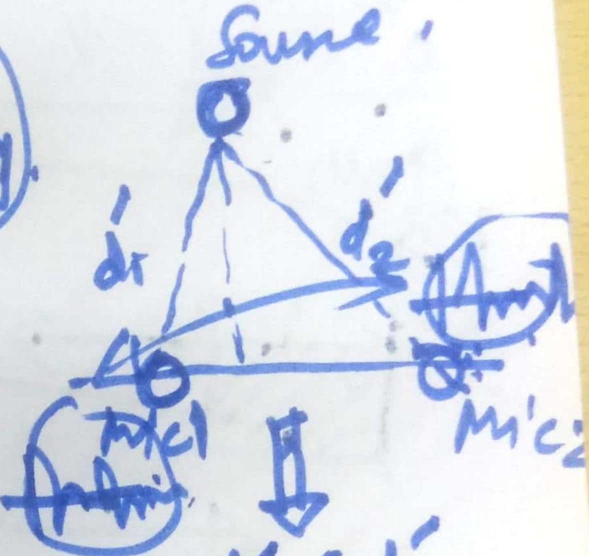
Time Delay



$\Downarrow$

$d_1 = d_2$

⇒ Sound wave will arrive at the same time at both mic 1 & mic 2



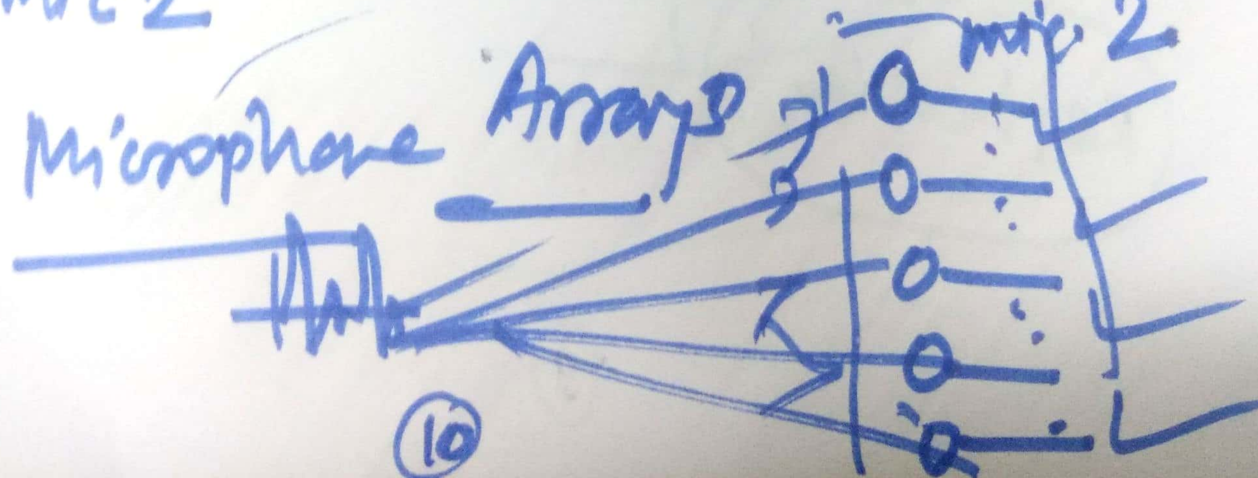
$\Downarrow$

$d_1' < d_2'$

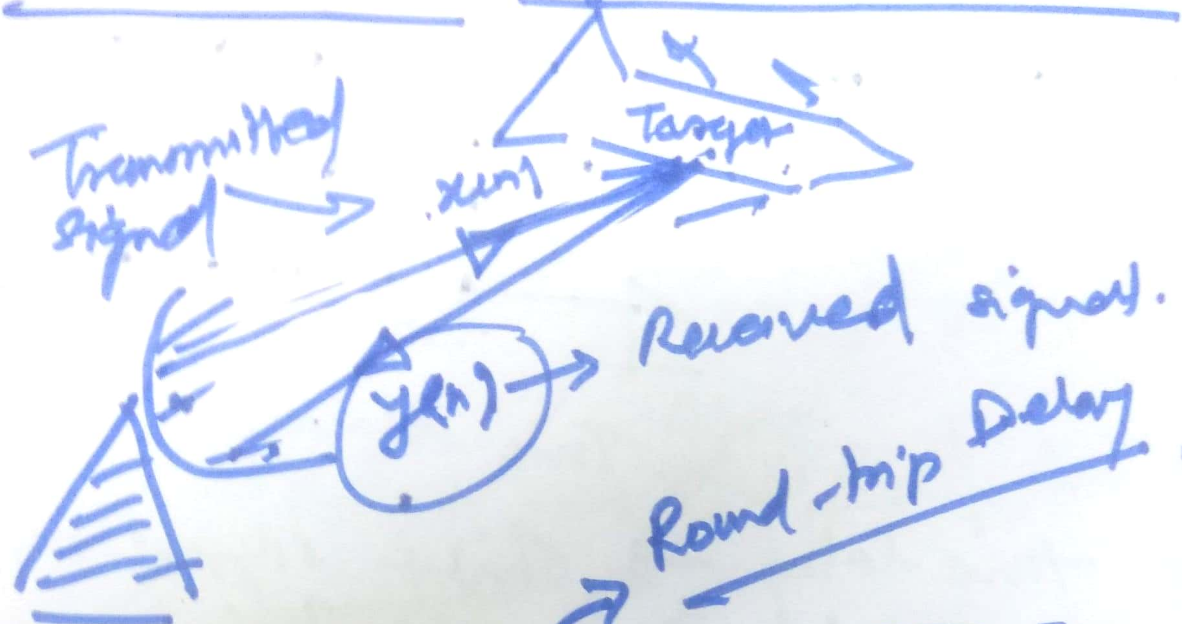
$\Downarrow$

⇒ Sound arrives at mic 1 first and then at mic 2

→ Microphone Arrays ⇒



Radar System : Target Detection System



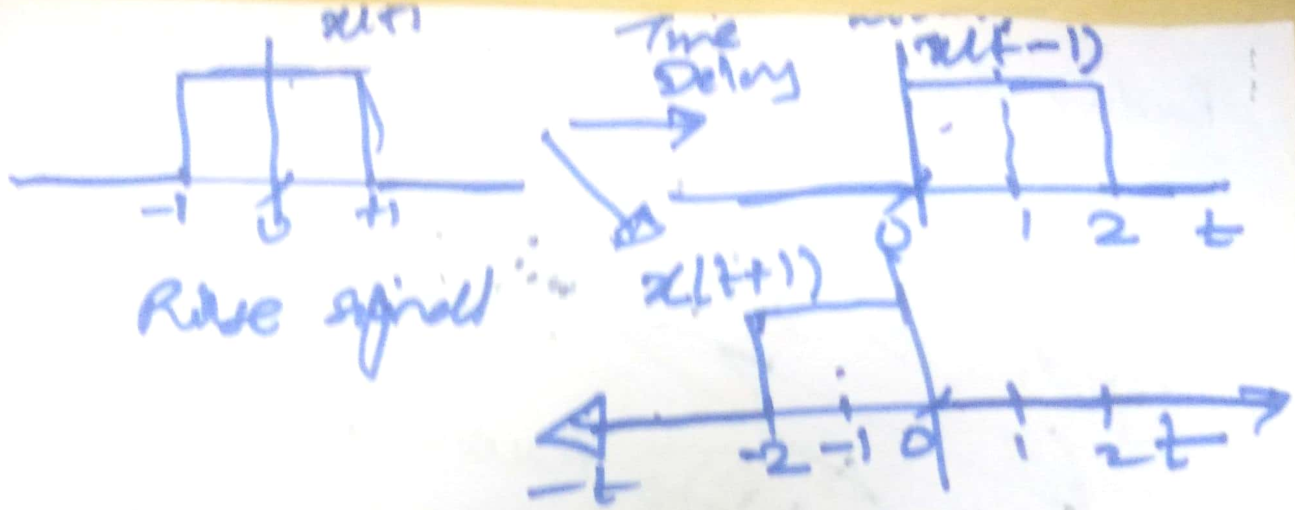
Round-trip Delay

$$\boxed{y(n) = \alpha \cdot x(n-D) + w(n)}$$

Attenuation factor

Communication noise.

$x(n) \longrightarrow x(n-D) \rightarrow$ Time delay version of $x(n)$



Time advance

For time delay → shift signal x(t) towards right by amount 't_0'

→ Time advance → shift x(t) towards left → by amount t_0.