SC224: Tutorial Sheet 1

Problems based on Basic Counting Principle, and Permutation and Combination

Pb 1) Eighteen workers are to be assigned to 18 different jobs, one to each job. How many different assignments are possible?

Pb 2) How many 3-letter code words are possible using the first 8 letters of the alphabet if:

(a) No letter can be repeated? (b) Letters can be repeated?

(c) Adjacent letters can not be alike.

Pb 3) From a committee of 8 people, in how many ways can we choose a chair and a vice-chair, assuming one person can not hold more than one position?

Pb 4) Serial numbers for a product are to be made using 2 letters followed by 3 numbers. If the letters are to be taken from the first 8 letters of the alphabet with no repeats and the numbers from the 10 digits 0 through 9 with no repeats, how many serial numbers are possible?

Pb 5) In how many ways can r objects be selected from a set of n objects if the order of selection is also to be considered?

Pb 6) In a cricket 11 is to be selected out of 14 players of whom 5 are bowlers. Find the number of ways in which this can be done so as to include at least 3 bowlers.

Pb 7) How many triangles can be formed by joining the vertices of a hexagon?

Pb 8) There are 10 points in a plane. No three points are in the same straight line excepting 4 points which are collinear. Find the number of straight lines obtained by joining the points?

Pb 9) How many ways to form a 4-digit numbers from the set {0,1,2,3,4,5} (repetition is not allowed)?

Problems based on Axioms of Probability, Conditional Probability and Baye's Theorem

Pb 1) Let P be a **probability function** defined on the sample space Ω. Then

(i) $P(\phi) = 0$. (ii) $\forall\ E \subseteq \Omega$, $0 \leq P(E) \leq 1$ and $P(\overline{E}) = 1 - P(E)$.

(iii) If $E_i \subseteq \Omega,\ i = 1, 2, .., n$, and $E_i \cap E_j = \phi,\ i \neq j$, then $P\left(\bigcup\limits_{i=1}^{n} E_i\right) = \sum\limits_{i=1}^{n} P(E_i)$.

(iv) Let $E_1, E_2 \subseteq \Omega$ such that $E_1 \subseteq E_2$, then

$$P(E_2 - E_1) = P(E_2) - P(E_1) \text{ and } P(E_1) \leq P(E_2).$$

(v) For $E_1, E_2 \subseteq \Omega$, we have $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$.

(vi) (**Bonferroni's Inequality**) For $E_1, E_2 \subseteq \Omega$, $P(E_1 \cap E_2) \geq P(E_1) + P(E_2) - 1$.

Pb 2) (i) Let $B_1, B_2, ..., B_n$ be a partition of the Sample space Ω. Show that, for any $A \subset \Omega$, we have: $P(A) = \sum_{k=1}^{n} P(A \cap B_k)$. (By an partition of the sample space Ω, we mean a countable collection $\{B_i\}_{i\in\Lambda}$ of mutually exclusive events B_i in Ω such that $\bigcup_{i\in\Lambda} B_i = \Omega$).

(ii) Use (i) to show that for events A, B and C:

$$P(A) = P(A \cap B) + P(A \cap C) + P(A \cap B^c \cap C^c) - P(A \cap B \cap C).$$

Pb 3) For events $A_1, A_2, ..., A_n$ in the Sample space Ω, establish the following result:

$$P(A_1 \cap A_2 \cap .... \cap A_n) \geq P(A_1) + P(A_2) + ... + P(A_n) - (n-1).$$

Pb 4) Two dice are thrown simultaneously. If a sum equal to five or seven is obtained, then a coin is tossed. Write down the sample space for this experiment. Given that the coin has been tossed, what is the probability that a "2" has been rolled with the second die?

Pb 5) Consider four coding machines M_1, M_2, M_3 and M_4 producing binary codes 0 and 1. The machine M_1 produces codes 0 and 1 with respective probabilities $\frac{1}{4}$ and $\frac{3}{4}$. The code produced by machine M_k is fed into machine M_{k+1} (k=1,2,3) which may either leave the received code unchanged or may change it. Suppose that each of the machines M_2, M_3 and M_4 change the received code with probability $\frac{3}{4}$. Given that the machine M_4 has produced code 1, find the conditional probability that the machine M_1 produced code 0.

Pb 6) A communication system transmits three signals: s_1, s_2 and s_3, with equal probability. The reception is sometimes erroneous, because of the "Noise". It was found, experimentally, that the probability p_{ij} of receiving the signal s_j, given that the signal s_i has been transmitted, is given by the following table:

		Reception		
		s_1	s_2	s_3
Transmission	s_1	0.8	0.1	0.1
	s_2	0.05	0.9	0.05
	s_3	0.02	0.08	0.90

a) What is the probability that the signal s_1 has been transmitted, given that the signal s_2 has been received?

b) If we assume that the transmissions are independent, what is the probability of receiving two consecutive s_3 signals?

Pb 7) A locality has n houses numbered $1, 2, ..., n$ and a terrorist is hiding in one of these houses. Let H_j denote the event that the terrorist is hiding in house number j, $j = 1, 2, ..., n$ and let $P(H_j) = p_j \in (0, 1)$, $j = 1, 2, ..., n$. During a search operation, let F_j denote the event that search of the house number j will fail to nab the terrorist there and let $P(F_j|H_j) = r_j \in (0, 1)$, $j = 1, 2, ..., n$. For each $i, j \in \{1, 2, ..., n\}$, $i \neq j$, show that H_j and F_j are negatively associated but H_i and F_j are positively associated. Interpret these findings.

Pb 8) Give an example, which illustrates that, in general, pairwise independence of a collection of events may not imply their independence.