Lecture 21

Tutorial 05

Part II

Problem 4:

Let $x(t) = u(t-3) - u(t-5)$ and

$h(t) = e^{-3t} \cdot u(t)$. Find the following

a) $y(t) = x(t) * h(t)$?

b) $g(t) = \frac{d}{dt}[x(t)] * h(t)$?

c) Relationship betw $g(t)$ and $y(t)$?

Sol^n: —

$\because x(t) = u(t-3) - u(t-5)$

$u(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}$ $\quad \therefore u(t-3) = \begin{cases} 1, & t-3>0 \\ 0 & t-3<0 \end{cases}$

$u(t-5) = \{ ??$

$x(t) = \begin{cases} 1, & 3 \leq t \leq 5 \\ 0 & \text{otherwise.} \end{cases}$

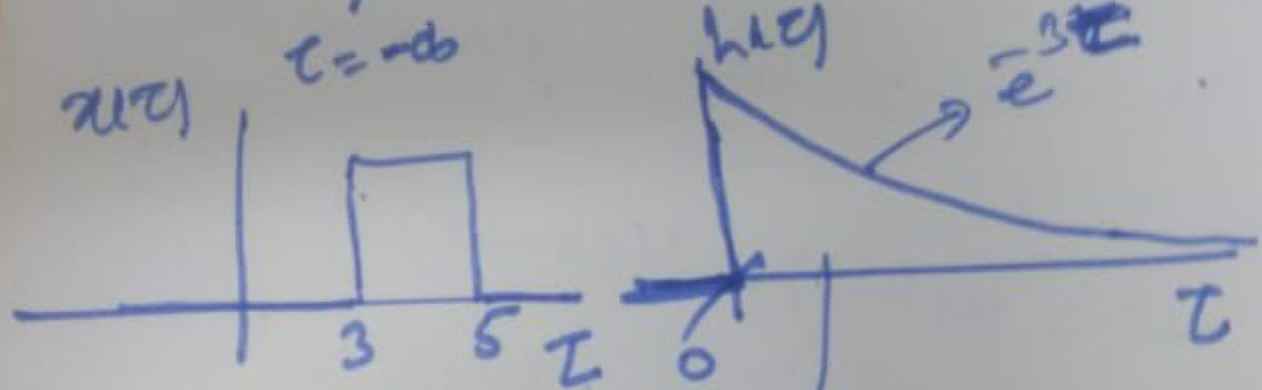
$h(t) = \begin{cases} e^{-3t}, & t \geq 0 \\ 0 & t<0 \end{cases}$

(a) $y(t) = x(t) * h(t)$

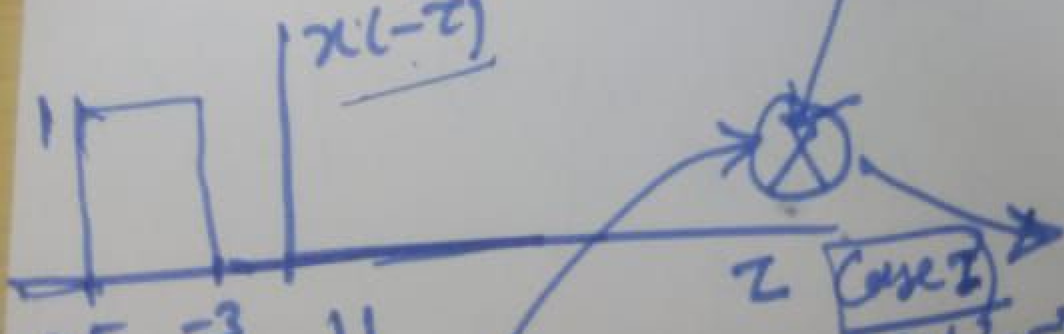
$$\therefore\ y(t) = \int_{\tau=-\infty}^{+\infty} x(\tau)\cdot h(t-\tau)\,d\tau$$

$$\Longrightarrow\ y(t)\big|_{t=0} = y(0) = \int_{\tau=-\infty}^{+\infty} x(-\tau)\,h(\tau)\,d\tau$$

$$y(t) = \int_{\tau=-\infty}^{+\infty} x(t-\tau)\,h(\tau)\,d\tau$$



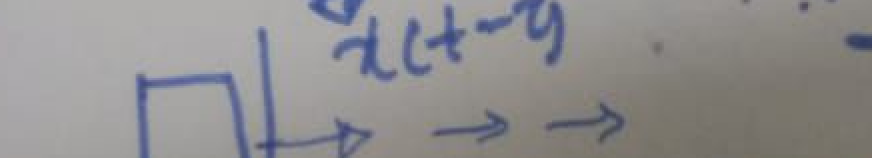
⇓ flip.



Case I

$\therefore\ x(t-\tau)\,h(\tau) = 0$

⇓ $x(t-\tau)$



For

$$y(t) = \int_{\tau=-\infty}^{+\infty} x(t-\tau)\,h(\tau)\,d\tau = 0 \quad \text{for } t < 3$$

Case II)

$$y(t) = \int_0^{t-3} \boxed{x(t-\tau)}\, h(\tau)\, d\tau .$$

$$= \int_0^{t-3} 1 \cdot e^{-3\tau}\, d\tau$$

$$y(t) = \frac{1}{3}\left[1 - e^{3(3-t)}\right] \quad \text{for } 3 \le t < 5$$

Case III)

$$y(t) = \int_{t-5 \,\to\, t-3}^{t-3 \,\to\, t-3} x(t-\tau) \cdot h(\tau)\, d\tau$$

$$= ? \qquad \text{for } t \geq 5$$

$$\therefore y(t) = \begin{cases} 0 & t < 3 \\ \frac{1}{3}\left[1 - e^{3(3-t)}\right], & 3 \le t < 5 \\ ?\, , & t > 5 \end{cases}$$

(b) $g(t) = \frac{d}{dt}(x(t)) * h(t)$.

$x(t) = u(t-3) - u(t-5)$

$\therefore \frac{dx(t)}{dt} = \frac{d}{dt}[u(t-3)] - \frac{d}{dt}u(t-5)$

$= \delta(t-3) - \delta(t-5)$

$\therefore g(t) = [\delta(t-3) - \delta(t-5)] * h(t)$.

$g(t) = h(t-3) - h(t-5)$

$g(t) = e^{-3(t-3)} u(t-3) - e^{-3(t-5)} \cdot u(t-5)$.

$$g(t) = \begin{cases} 0 & , t < 3 \\ e^{-3(t-3)} & 3 \leq t < 5 \\ e^{-3(t-3)} - e^{-3(t-5)}, & t > 5 \end{cases}$$

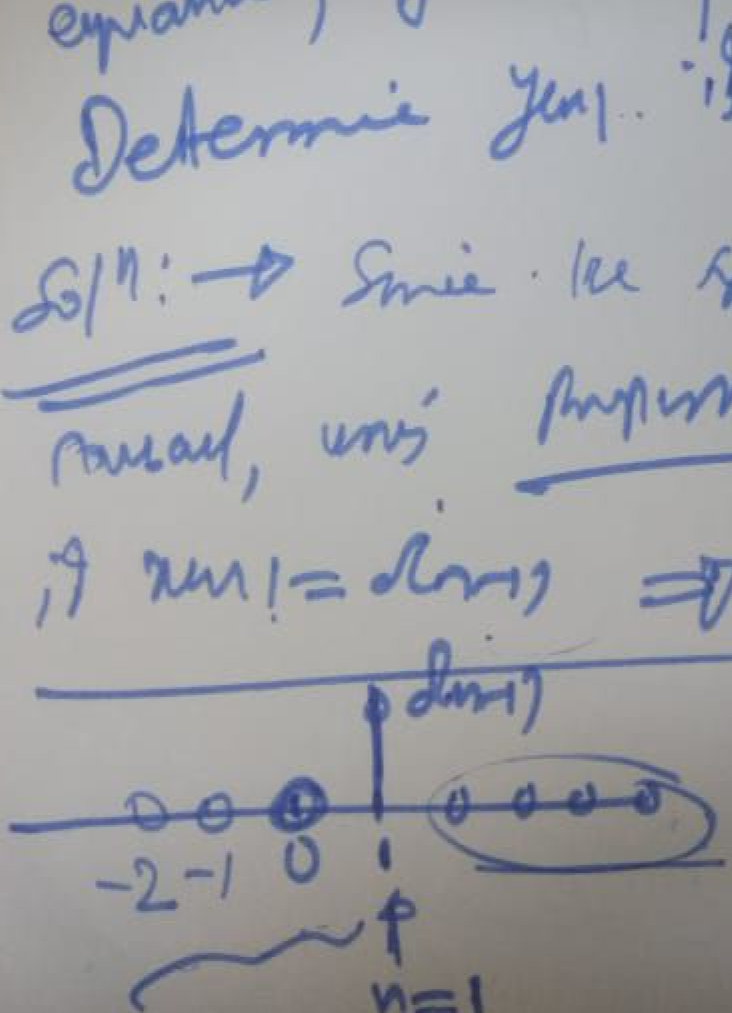
(c) $g(t)$ vs. $y(t)$??

$$\boxed{g(t) = \frac{dy(t)}{dt}}$$

Problem (5) Consider a causal LTI system whose input $x[n]$ and output $y[n]$ are related by the difference equation, $y[n] = \frac{1}{4}y[n-1] + x[n]$.
Determine $y[n]$ if $x[n] = \delta[n-1]$.

Sol^n: → Since the system is LTI and causal, using Proposition 1.3.

if $x[n] = \delta[n-1] \Rightarrow y[n] = 0$ for $n < 1$



(5)

Given difference eqn for LTI system is

$$y(n) = \frac{1}{4} y(n-1) + x(n)$$

Let $x(n) = \delta(n-1)$

$$y(n) = \frac{1}{4} y(n-1) + \delta(n-1)$$

$n=0$

$$\therefore y(0) = \frac{1}{4} y(-1) + \delta(-1)$$

$$= \frac{1}{4} \times 0 + 0$$

$\therefore y(0) = 0$

$y(1) =$

$y(2) =$

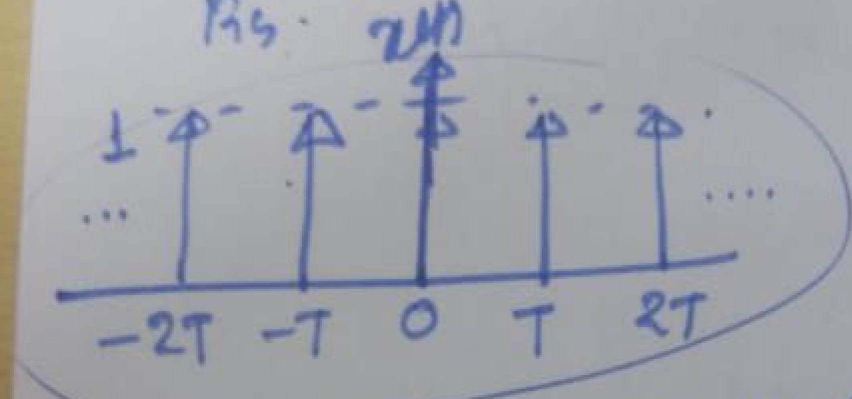
$y(3) =$

$\vdots$

$$\boxed{y(n) = \left(\frac{1}{4}\right)^{n-1} \cdot u(n-1)}$$

⑥

Problem 6 Let $h(t)$ be the triangular pulse shown in fig. and let $x(t)$ be the impulse train signal shown in fig.



Determine and sketch $y(t) = x(t) * h(t)$ for following values of 'T'?

a) $T=4$, b) $T=2$, c) $T=3/2$, d) $T=1$

Soln: $\rightarrow y(t) = x(t) * h(t)$.

$$x(t) = \sum_{k=-\infty}^{+\infty} \delta(t-kT)$$

$$\therefore y(t) = \left[\sum_{k=-\infty}^{+\infty} \delta(t-kT)\right] * h(t).$$

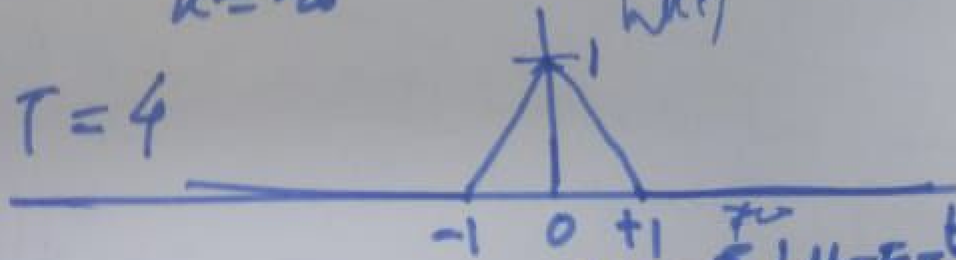
Using distributive property,

$$y(t) = \ldots\ldots + \delta(t+T) * h(t) + \delta(t) * h(t) + \delta(t-T) * h(t) + \ldots\ldots$$

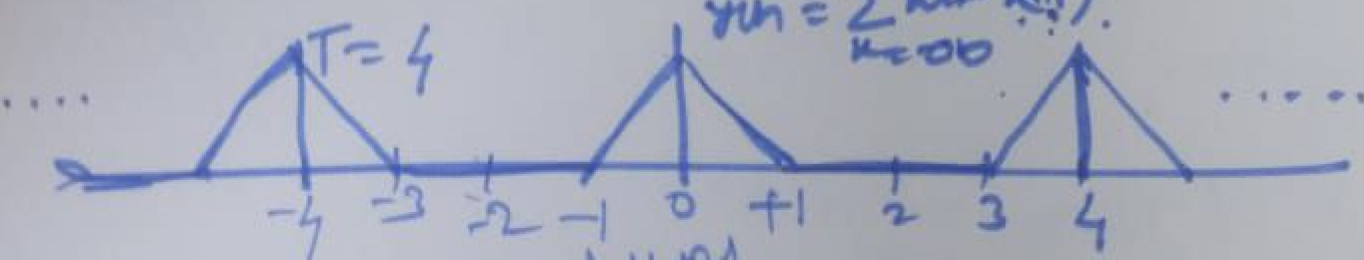
$$y(t) = \ldots\ldots + h(t+T) + h(t) + h(t-T) + \ldots\ldots$$

$$y(t) = \sum_{k=-\infty}^{+\infty} h(t-kT)$$

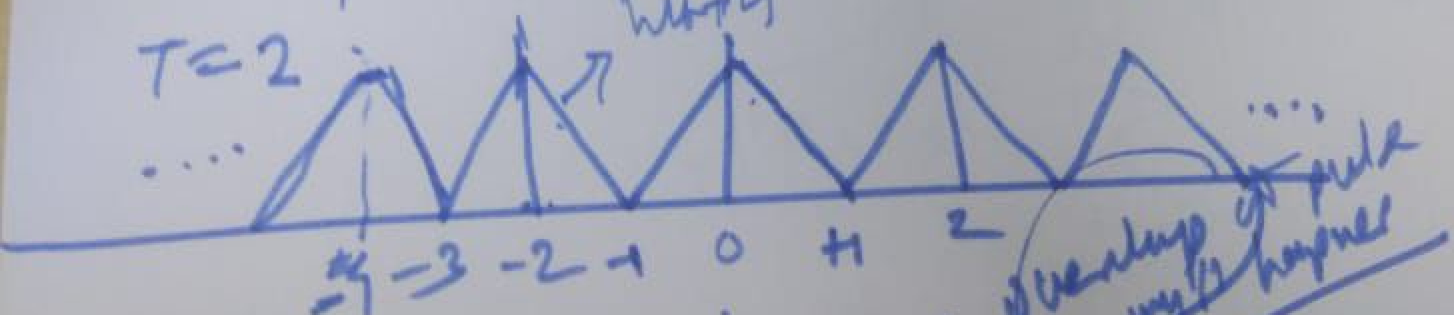
a) $T = 4$



$y(t) = \sum_{k=-\infty}^{+\infty} h(t-kT)$



$T = 2$

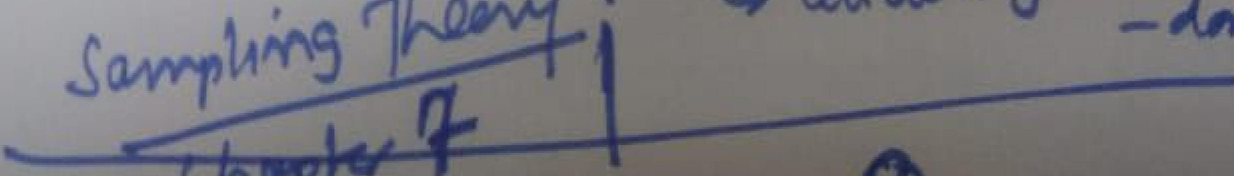


$T = 3/2$

aliasing in time-domain

$T = 1$ Sampling Theory

Chapter 7



Tutorial 06

Q.1)

Since systems are equivalent so the overall impulse response of cascade interconnection is

$$h(n) = h_1(n) * h_2(n)$$

$$h(n) = \sum_{k=-\infty}^{+\infty} h_1(k) \cdot h_2(n-k)$$

$$h(n) = \sum_{k=-\infty}^{+\infty} \left(\frac{1}{3}\right)^k \cdot \underline{u(k)} \cdot \left(\frac{1}{6}\right)^{n-k} \cdot \underline{u(n-k)}$$

⋮

$$h(n) = \left(\frac{1}{2}\right) \left[1 - \left(\frac{1}{3}\right)^{n+1}\right] u(n).$$

$$= \left(\frac{1}{6}\right)^n \left[2^{n+1} - 1\right]$$

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2.2) Investigate stability & causality.

i) Ideal (delay) system

$y[n] = D\{x[n]\} = x[n - n_d]$ I/O

$h[n] = D\{\delta[n]\} = \delta[n - n_d]$.

Causal ✓

For Stability $\sum_{\langle K \rangle} |h(k)| < +\infty$

∴ Stable. $\sum_{\langle K \rangle} |\delta(x - n_d)| < +\infty$ ✓

+ causal

For LTI system, $h[n]$

$h[n] = 0$ for $n < 0$

ii) $y[n] = D\{x[n]\} = \sum_{k=-\infty}^{n} x[k]$.

$\Rightarrow$ Accumulator system.

$h[n] = u[n]$.

$h[n] = 0$ for $n < 0$??

$\therefore$ causal ✓

$\sum_{<k>} |h[k]| = \sum_{<k>} |u[k]| \neq \infty$

$\therefore$ Unstable

iii) $y[n] = x[n+1] - x[n] \rightarrow$ forward difference

$h[n] = \delta[n+1] - \delta[n]$

$h[n] = 0$ for $n < 0 \Rightarrow$ ✗

Non causal $\Rightarrow$

Stable.

iv) $y[n] = D\{x[n]\} = x[n] - x[n-1]$

$\Rightarrow$ Backward difference

$h[n] = \delta[n] - \delta[n-1]$. LTI

$\boxed{h[n] = 0 \text{ for } n < 0 \;!?}$

∴ Causal system

BIBO stable

Section 2.5 Singularity Functions

Singular ??

Singular Matrix

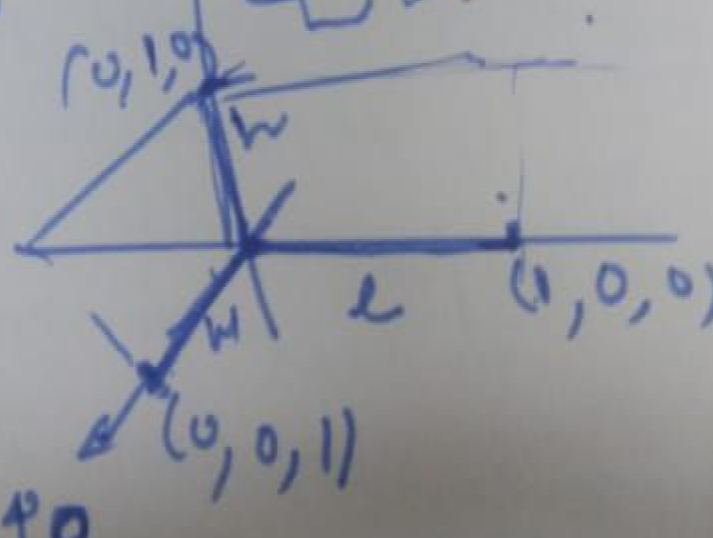
'A' singular → $|A| = 0$

The term singular were behaviour of functions / vectors

Geometrically determinant of a matrix indicates volume of parallelepiped in n-dimension for n×n square matrix.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$$

Volume = $l \times h \times w$



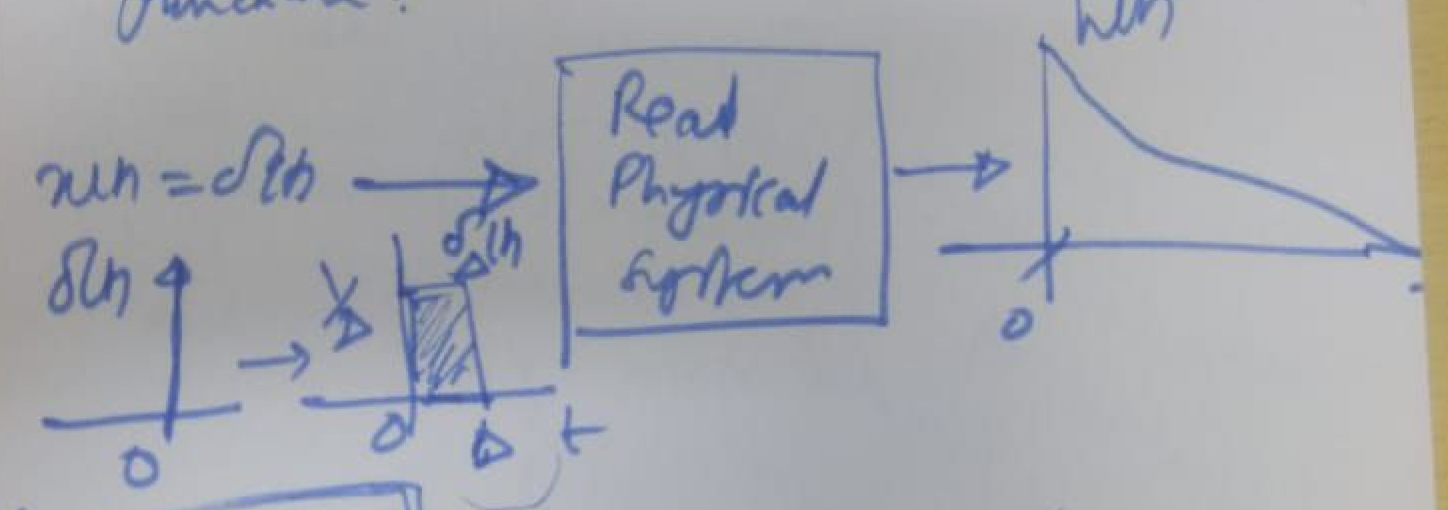
$A^{-1} = \dfrac{Adj(A)}{|A|}$ $|A| \neq 0$

'∞'

(13)

In Signals and Systems, continuous-time impulse signal, $\delta(t)$, is an example of singularity function.

Why to study $\delta(t)$ through singularity function?



$\boxed{\text{In chapter 1}}$ we studied that the output response of this real physical system will not be dependent on individual pulse shape, pulse width, pulse height, rather its net integrating effect i.e. its "area" $\Rightarrow$ primarily due to system's inertia.

⇒ We will quantify this analysis via singularity function

connected to LTI systems behaviour for "impulse-like" excitation.

Objective: To understand how impulse signal $\delta(t)$ "behaves" under convolution

LTI

$$y(t) = x(t) * h(t) = \int_{\tau=-\infty}^{+\infty} x(\tau) h(t-\tau) d\tau$$

sifting property of impulse

$$x(t) = \int_{\tau=-\infty}^{+\infty} x(\tau) \cdot \delta(t-\tau) d\tau$$

$$x(t) = x(t) * \delta(t) \quad \text{— (1)}$$

Let $x(t) = \delta(t)$ in eqn (1)

$$\therefore \delta(t) = \delta(t) * \delta(t) \quad \text{— (2)}$$

Use eqn (2) in eqn (1), we get

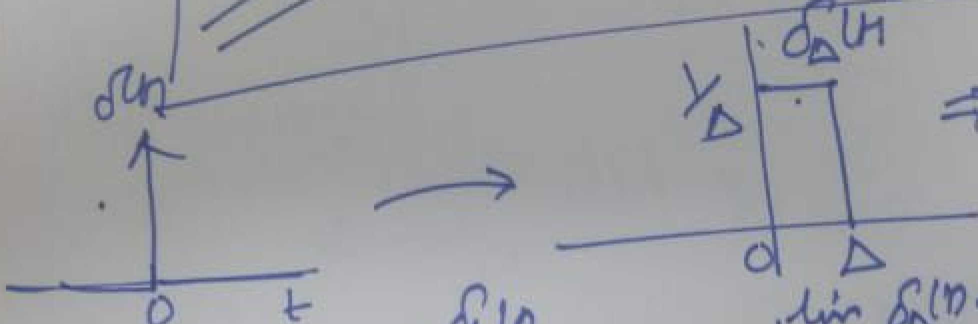
$$x(t) = x(t) * \delta(t) * \delta(t) \quad \text{— (3)}$$

$x(t) = \delta(t)$ in eqn (3)

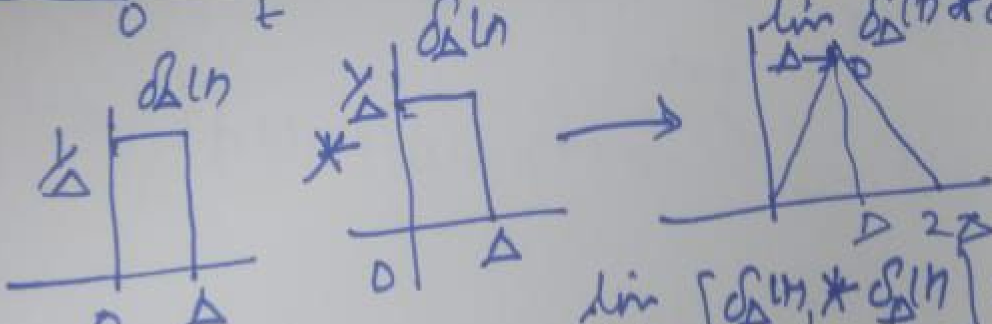
$$\therefore \delta(t) = \delta(t) * \delta(t) * \delta(t) \quad \text{— (4)}$$

(4) $\rightarrow$ (1)

$$\boxed{\delta(t) = \delta(t) * \delta(t) * \delta(t) * \cdots \text{ infinite times}}$$



$$\Rightarrow \lim_{\Delta \to 0} \delta_\Delta(t) \to \delta(t) =$$



$$\lim_{\Delta \to 0} \delta_\Delta(t) * \delta_\Delta(t) = \lim_{\Delta \to 0} r_\Delta(t) =$$

$$\lim_{\Delta \to 0} \left[\delta_\Delta(t) * \delta_\Delta(t)\right] = \lim_{\Delta \to 0} \delta_\Delta(t) * \lim_{\Delta \to 0} \delta_\Delta(t)$$

$$= \delta(t) * \delta(t)$$

$$= \delta(t)$$

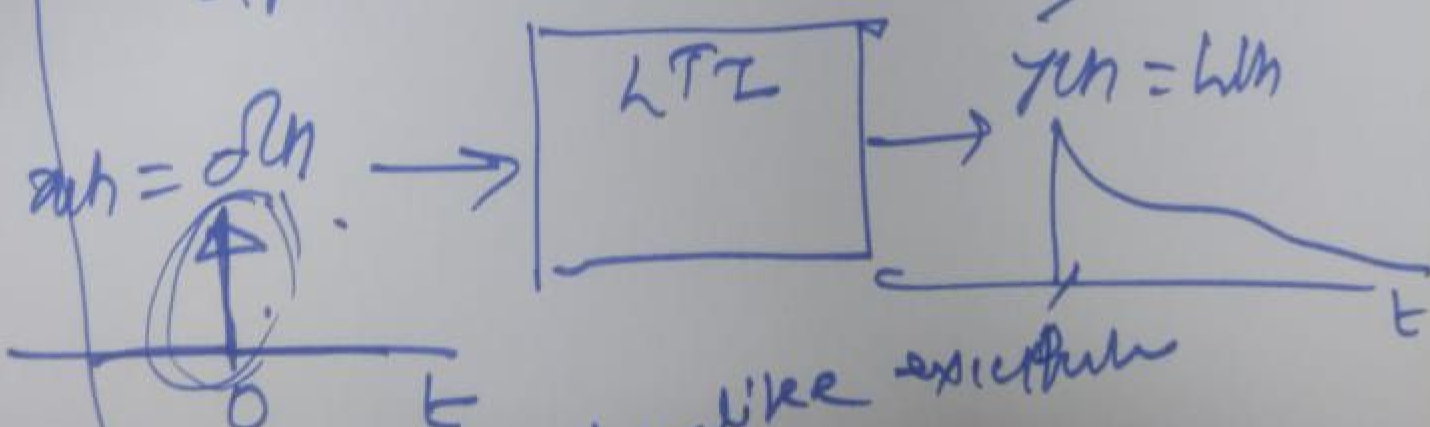
$$\lim_{\Delta \to 0} r_\Delta(t) * \delta_\Delta(t) \rightarrow \delta(t)$$

$\lim_{\Delta \to 0}(\ldots) \Rightarrow \delta(t)$

Impulse excitation signal for LTI system system

Different ones dissimilar looking signals $\Rightarrow$ will behave like impulse under convolution

$x(n) = \delta(n)$ $\longrightarrow$ LTI $\longrightarrow$ $y(n) = h(n)$ → Impulse Response



Impulse-like excitation

Impulse-like