SC223 Linear Algebra
Autumn 2023-24
Practice Questions

Note: The following questions have been taken from various online sources, and are meant for your practice. Based on these questions, do not make any assumptions about the kind of questions that you will face in the In-Sem examination. Do not expect me to share solutions of these questions.

1. Let $U = \{u_1, \ldots, u_n\}$ be a set of linearly independent vectors in a vector space V. If $w \in V$, but $w \notin span(U)$, show that $U \cup \{w\}$ is linearly independent.
2. If V is an n-dimensional vector space and if S is a set in V with exactly n vectors, then S is a basis for V if either S spans V or S is linearly independent.
3. Let V be an n-dimensional vector space. Show that a subset W of V containing n elements is linearly independent if and only if W spans V.
4. Find the basis of the subspace $U = \{(x, y, z) \in \mathbb{R}^3 \mid x + 2y - 5z = 0\}$ of $\mathbb{R}^3$. Also find a subspace W, and a basis of W such that $U \oplus W = \mathbb{R}^3$. If W unique?
5. Let $A \in \mathbb{R}^{m \times n}$ be a given matrix. If the rank of A is less than m, can you conclude whether A is one-one (injective) or not? How about A being onto?
6. Let U and W be subspaces of a vector space V. Show that if $U \cup W$ is a subspace then either $U \subseteq W$ or $W \subseteq U$.
7. Let U and W be subspaces of V such that $V = U + W$. If for some $v \in V, v \neq \theta$ it is given that $v = u + w, u \in U, w \in W$ is a unique decomposition, then is it true that $V = U \oplus W$?
8. Let U be a subset of a vector space V. Show that $span(U)$ is the smallest subspace containing U.
9. Let U be subspace of a FDVS V. Show that there exists a subspace W of V such that $V = U \oplus W$.
10. Can you have two 2-dimensional subspaces of $\mathbb{R}^3$ whose direct sum yields $\mathbb{R}^3$?
11. Let V be an n-dimensional vector space. Let $T_1, T_2 \in \mathcal{L}(U, V)$. Show that the composition of these two, i.e., $T_1 \cdot T_2$ is a linear operator on V. With basis β of U, if the matrix representation of T_1 and T_2 are $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times n}$ respectively, then find the matrix representation of the linear operator $T_1 \cdot T_2$.
12. Consider the vector space $V = \mathcal{C}[0, 1]$, the set of all real-valued continuous functions defined on the interval $[0, 1] \subset \mathbb{R}$. Let $U = span(\{t, t^4\}), t \in [0, 1]$ and $W = \{h \in V \mid \int_0^1 t\, h(t)\, dt = 0, \int_0^1 t^4\, h(t)\, dt = 0\}$ be two of its subspaces. Show that $V = U \oplus W$.
13. Given that vector spaces U and V are finite-dimensional, find the dimension of the vector space $\mathcal{L}(U, V)$.