

Computational Numerical Methods

CS 374

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Gauss elimination Method.

① Elimination

② Modification of right hand side.

③ Back substitution.

$$\left\{ \begin{array}{l} Ax = b. \\ \downarrow \\ Ux = \bar{b} \end{array} \right.$$

Steps	Addition/Subtraction	Multiplication	Division
1	$(n-1)^2$	$(n-1)^2$	$n-1$
2	$(n-2)^2$	$(n-2)^2$	$(n-2)$
⋮			
$n-1$	1	1	1
Total number of Arithmetic operations	$\frac{1}{6} n(n-1)(2n-1) +$	$\frac{1}{6} n(n-1)(2n-1) +$	$\frac{n(n-1)}{2}$

Jacobi's method.

$$\underline{Dx = b.}$$

$$A = D - C.$$

D = diagonal matrix with
the same diagonal entries
as A .

$$Ax = b.$$

$$(D - C)x = b.$$

$$\Rightarrow \underline{Dx = Cx + b.}$$

Let say initial solution is x^0 .

$x^{(1)}$ \rightarrow first iteration is given by

$$Dx^{(1)} = Cx^{(0)} + b.$$

$$Dx^{(2)} = Cx^{(1)} + b.$$

$$x^{(k)} = D^{-1}Cx^{(k-1)} + D^{-1}b.$$

~~Ex 2~~ Consider 3×3 2×3 .

$$\underline{a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1,}$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3.$$

$$x_1 = \frac{1}{a_{11}} (b_1 - a_{12}x_2 - a_{13}x_3)$$

$$x_2 = \frac{1}{a_{22}} (b_2 - a_{21}x_1 - a_{23}x_3)$$

$$x_3 = \frac{1}{a_{33}} (b_3 - a_{31}x_1 - a_{32}x_2)$$

$$x_1^{(k)} = \frac{1}{a_{11}} (b_1 - a_{12} x_2^{(k-1)} - a_{13} x_3^{(k-1)})$$

$$x_2^{(k)} = \frac{1}{a_{22}} (b_2 - a_{21} x_1^{(k-1)} - a_{23} x_3^{(k-1)})$$

$$x_3^{(k)} = \frac{1}{a_{33}} (b_3 - a_{31} x_1^{(k-1)} - a_{32} x_2^{(k-1)})$$

6x

$$6x_1 + x_2 + 2x_3 = -2$$

$$x_1 + 4x_2 + 0.5x_3 = 1$$

$$-x_1 + 0.5x_2 - 4x_3 = 0$$

Next

$$x_1 + 4x_2 + 0.5x_3 = 1$$

$$6x_1 + x_2 + 2x_3 = -2$$

$$-x_1 + 0.5x_2 - 4x_3 = 0$$

Exact solutions

$$x_1 = -0.441176, x_2 = 0.341176, x_3 = 0.152941$$

$$x^{(1)} = (-0.333333, 0.25, 0)$$

$$x^{(2)} = (-0.375, 0.333333, 0.114583)$$

$$x^{(3)} = (-0.427083, 0.329427, 0.135417)$$

$$x^{(4)} = (-\cancel{0.433377}, 0.339844, 0.147949.)$$

$$x^{(5)} = (-0.439290, 0.339851, \underline{0.150825})$$

For $x_1 + 4x_2 + 0.5x_3 = 1$

$$6x_1 + x_2 + 2x_3 = -2$$

$$-x_1 + 0.5x_2 - 4x_3 = 0$$

$$x^{(1)} = (1, -2, 0)$$

$$x^{(2)} = (9, -8, -0.5)$$

$$x^{(3)} = (33.25, -55, -3.25)$$