3SAT is NP-complete.

we have seen $IS_D \leq_p VC_D$

Also we have seen VC_D is in NP.

we know that IS_D is NP-complete (somehow we know)

The we conclude VC_D is also NP-complete.

An alternative proof of VC_D is NP-complete.

by giving a <u>reduction</u> from 3-SAT problem.

poly-time.

$3SAT \leq_p VCD$

$$\phi \xrightarrow{f} G_\phi(V, E)$$

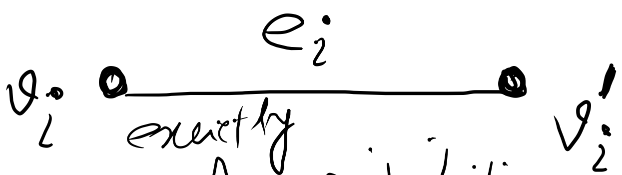
We show that ϕ is satisfiable iff G_ϕ has a vertex cover of size $\leq k$.

<u>Gadget reduction</u>

The graph G_ϕ contains <u>gadgets</u> that mimics the variables and clauses of ϕ.

variable gadget

For each variable x_i, we take a graph as,



There are two exactly possibilities of minimum vertex cover.

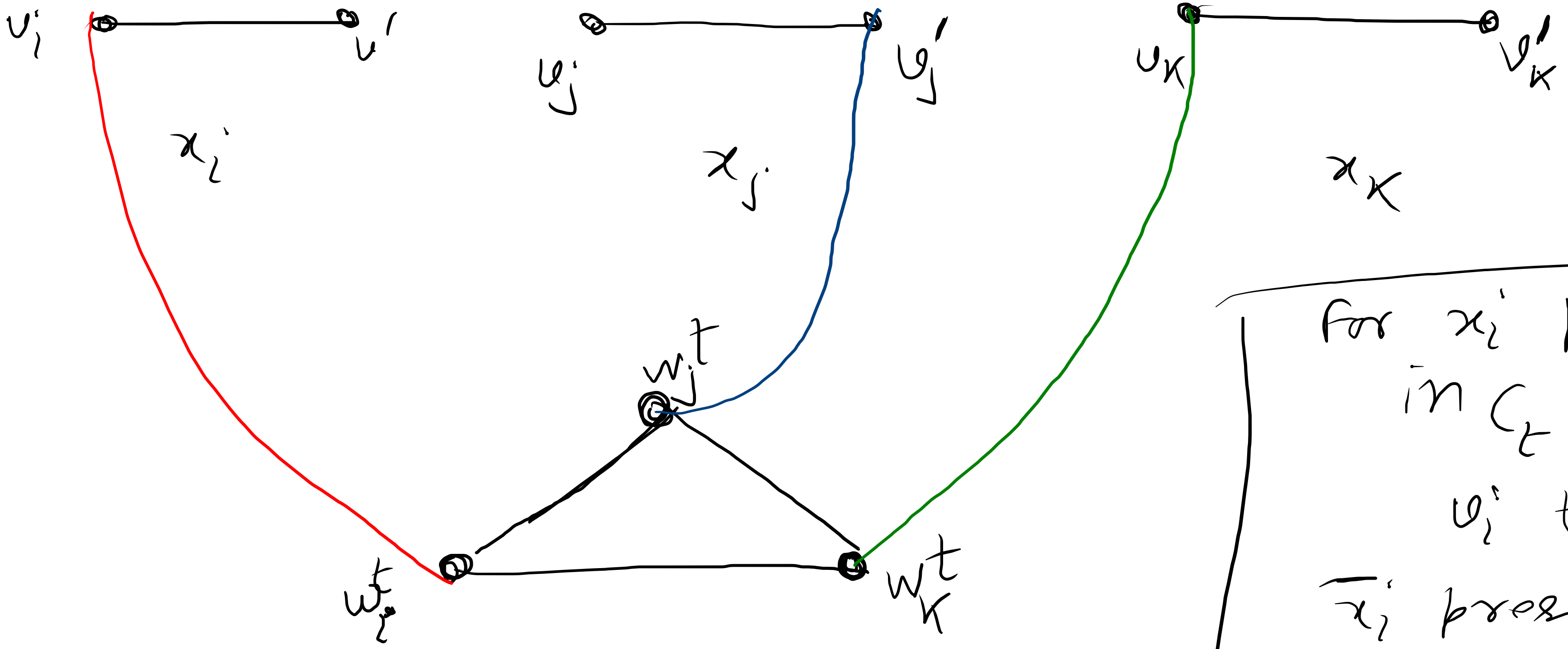
we pick in the vertex cover $v_i \rightarrow$ corresponding to literal x_i

" " " " " " $v_i' \rightarrow$ " " " $\overline{x_i}$

clause gadget

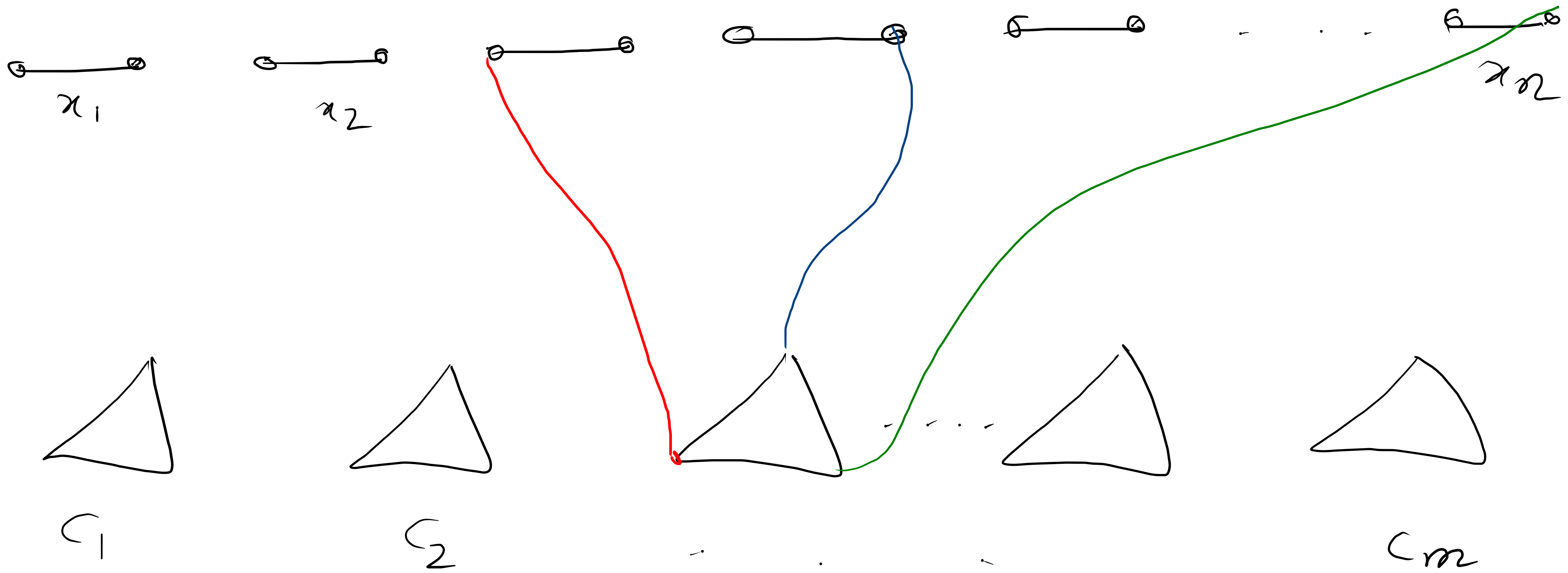
$$C_t = (x_i \vee \bar{x}_j \vee x_k)$$

For a clause take 3 vertices (w_t^i, w_t^j, w_k^t) and 3 edges that makes a triangle.



For x_i present in C_t connect v_i to w_i^t

$\bar{x}_i$ present in C_t connect v_i' to w_j^t



We now prove ϕ is satisfiable iff G_ϕ has a vertex cover of size $n+2m$

Assume ϕ is satisfiable

It has a satisfying assignment that assign to variables that evaluates ϕ to be 1.

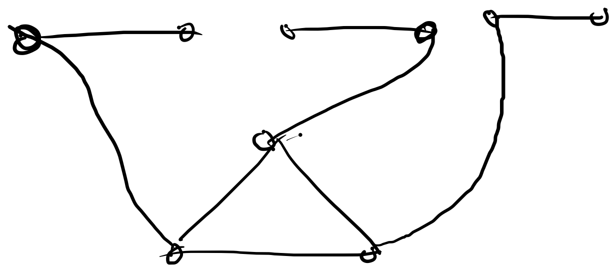
$$T : \{x_1 \cdots x_n\} \longrightarrow \{0, 1\}$$

$T(x_i) = 1$ then take v_i in the solution

$T(x_i) = 0$ " " v_i' " " "

So we select a total of n vertices.

So we can pick at most $2m$ vertices covering the remaining edges.



For each clause $C_t = (x_i \vee \bar{x}_j \vee x_k)$

we consider a true literal and then select the other two vertices from clause gadget in the vertex cover.

Total we ~~select~~ $n + 2m$ vertices.

<u>otherside</u>

suppose there is a ~~vertex~~ cover of size at most $n+2m$.

For each variable gadget at least one ~~vertex~~ vertex is required to cover the edge

For each clause gadget at least 2 vertices are required.

$\Rightarrow$ at least $n+2m$ vertices are required.

$\Rightarrow$ we need exactly $n+2m$ vertices.

$\Rightarrow$ exactly n vertices from variable gadget
  " $2m$ " " clause "

each variable and clause gadgets are independent

=> Each variable gadget exactly 1 vertex is required

" clause " " 2 vertices are "

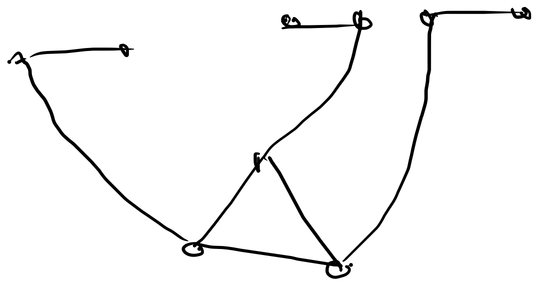
We now consider the assignment as follows.

we take x_i to be 1 if v_i is picked

x_i to be 0 if v_i' " "

Now we prove that each clause is satisfiable.

$$C_t = (x_i \vee \bar{x}_j \vee x_k)$$



since exactly 2 vertices are picked.
one of the three cross edges must be covered
by variable vertex.

That vertex is corresponding to a true literal.

makes the clause satisfiable.

	vertices	edges
variable gadget	$2n$	n
clause gadget	$3m$	$3m$
	$3m+2n$	$3m+n$.