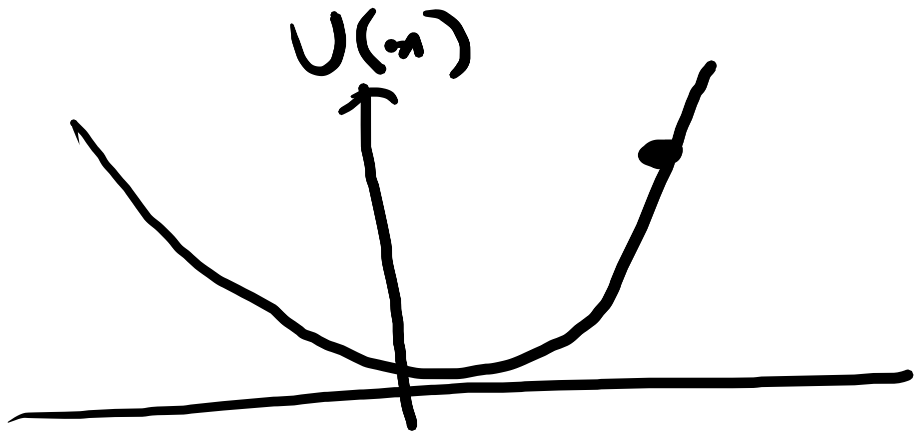
LECTURE 6

PROB:- (RADIOACTIVITY).

PHEN:- - Atomic nuclei = protons + neutrons held together by nuclear forces.



- Stability of nuclei based on balance of forces, instability is caused by "excess" protons or neutrons

- Has been observed that unstable nuclei tend to decay to stable nuclei emitting α, β and γ radiation.

– Emissions constitute 'radioactivity.'

<u>Observations</u> :- 'Radioactivity' decreases with time exponentially.

<u>Aim</u> :- Set up a mathematical model accurately describing this behaviour.

<u>Assumptions</u> :- Assume that radioactivity is going to be prop. to total # of unstable nuclei

Let λ be prob. of decay of unstable nuclei / time.

$$\text{Prob.} = \frac{\#\text{ of favourable outcomes}}{\text{Total }\#\text{ of possible outcomes.}}$$

$$\text{Prob. of decay} = \frac{\#\text{ of particles that decay}}{\text{Total }\#\text{ of particles.}}$$

Consider infinitesimal time interval dt.

$dN = \#$ of nuclei that decay

$N =$ Total $\#$ of nuclei

$$\frac{dN}{N} = -\lambda\, dt \quad \Rightarrow \quad \boxed{\frac{dN}{dt} = -\lambda N}$$

Initial condition:- $N(t=0) = N_0$.

— Analytical sol^{n}: constructed by direct integration,

$$\int \frac{dN}{N} = -\lambda \int dt$$

$$\Rightarrow \quad \ln N = -\lambda t + C_1$$

Using initial condition :- $\quad C_1 = \ln N_0$

$$\ln \frac{N}{N_0} = -\lambda t$$

$$\Rightarrow \quad N = N_0 e^{-\lambda t}$$

$\lambda_2 > \lambda_1$

$$t = \frac{1}{\lambda} \ln \frac{N_0}{N}$$

- Half-life → Time taken for half the nuclei to decay.

PROB: (PARTICLE MOVING UNDER GRAVITY)

Particle is thrown upwards with velocity v_0.
Find velocity and position. given

$x(t=0) = x_0$

$v(t=0) = v_0$

$$\frac{dv}{dt} = -g$$

$$\Rightarrow v = v_0 - gt.$$

$$\frac{dx}{dt} = v_0 - gt \quad \Rightarrow x(t) = x_0 + v_0 t - \frac{1}{2} g t^2$$

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Suppose, we want to solve,

$$\frac{d^2x}{dt^2} = -g \ .$$

$$\frac{d^2x}{dt^2} \approx \frac{x(t+h) - 2x(t) + x(t-h)}{h^2} = -g$$

$$x(t+h) = 2x(t) - x(t-h) - h^2 g \ .$$

Consider the first step. $t = t_0$

$$x(t_0+h) = 2x(t_0) - x(t_0-h) - h^2 g \ .$$

– Use the info. $v(t=0) = v_0$

$$v(t) = \frac{dx(t)}{dt}$$

$$\Rightarrow \quad v(t_0) = \frac{x(t_0) - x(t_0 - h)}{h}$$

(Backward diff. formula for derivative)

$$\Rightarrow \quad v_0 = \frac{x(t_0) - x(t_0 - h)}{h}$$

$$\Rightarrow \quad h v_0 = x(t_0) - x(t_0 - h)$$

$$\Rightarrow \quad x(t_0 - h) = x(t_0) - h v_0$$

Substitute $x(t_0 - h)$ and solve for $x(t)$

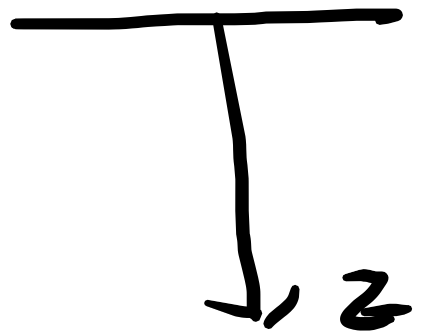
Note that forward diff. would not be useful in this case

PROB:- (PARTICLE FALLING THROUGH A VISCOUS MEDIUM).

Medium offers resistance to motion of the particle.

Observation:- Faster the particle moves, more is the resistance.

Assumption:- Assume that this viscous force $\propto v$



EOM:

$$m\frac{d\vec{v}}{dt} = (m\vec{g} - \beta\vec{v})$$

$$\Rightarrow \quad m\frac{dv}{dt} = mg - \beta v$$

$$- \quad m\frac{dv}{dt} = mg - \beta v.$$

$$\Rightarrow \frac{-\beta\, dv}{mg-\beta v} = -\beta\frac{dt}{m}$$

Integrating,

$$\ln(mg-\beta v) = -\frac{\beta t}{m} + c_1$$

Using initial condition, $v(0) = v_0$.

$$\ln(mg-\beta v_0) = c_1$$

$$\Rightarrow \ln(mg-\beta v) - \ln(mg-\beta v_0) = -\beta t/m.$$

$$\Rightarrow \ln\left(\frac{mg-\beta v}{mg-\beta v_0}\right) = -\beta/m\, t.$$

$$\boxed{t \to \infty \Rightarrow \text{asymptotic}}$$

$$\frac{mg-\beta v}{mg-\beta v_0} = e^{-\frac{\beta}{m}t}$$

Solve, for v.

$$mg-\beta v = (mg-\beta v_0)\, e^{-\frac{\beta}{m}t}$$

$$\Rightarrow \beta v = mg + (\beta v_0 - mg)\, e^{-\frac{\beta}{m}t}.$$

$$\Rightarrow v = \frac{mg}{\beta} + \left(v_0 - \frac{mg}{\beta}\right) e^{-\frac{\beta}{m}t}.$$

- Any physical reason to expect that at $t \to \infty$, $v \to$ constant?

- Yes: Recall eom.

$$m \frac{dv}{dt} = \underbrace{mg - \beta v}_{\text{Opposing forces.}}$$

$$0 = mg - \beta v \Rightarrow v = \frac{mg}{\beta}$$

TERMINAL VELOCITY