Computational Numerical Methods

CS 374

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Task show that the error in secant method.

follows.

$$\alpha - x_{n+1} = (\alpha - x_n)(\alpha - x_{n-1}) \left[\frac{-f''(\xi_n)}{2f'(\xi_n)}\right]$$

where ξ_n is number b/w ~~the~~ the largest & smallest of x_n, x_{n-1}, α.

& ξ_n is a number b/w x_n & x_{n-1}

Also establish the relation

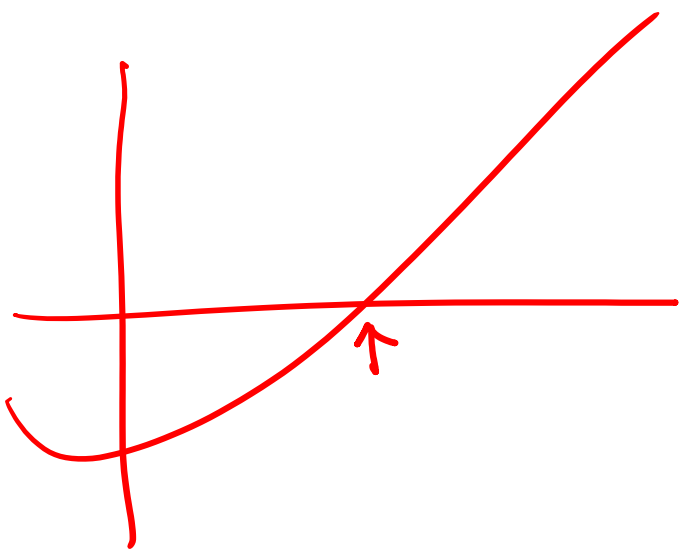
$$|\alpha - x_{n+1}| \approx c\,|\alpha - x_n|^{1.62}.$$

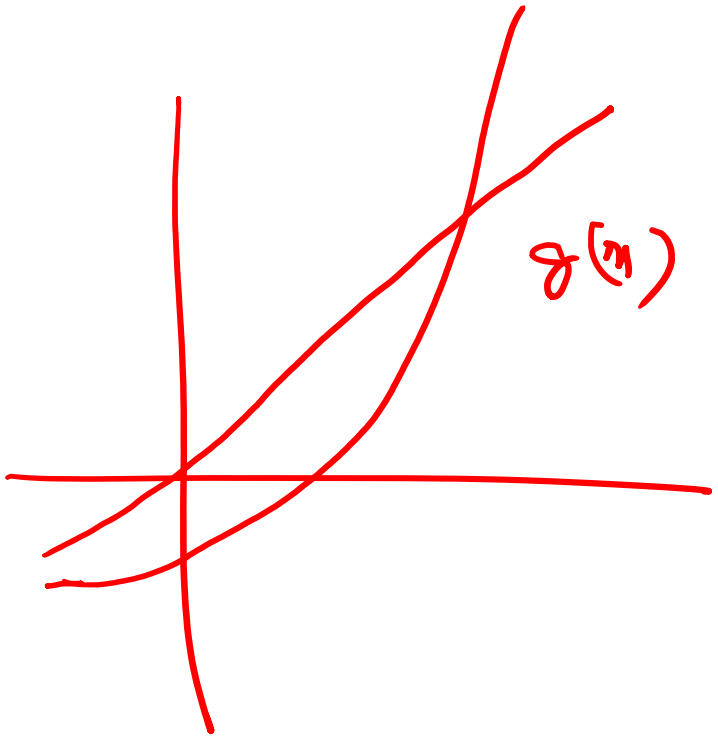
Fixed point Iteration.

$f(x) = 0$.

$x = g(x)$.

Soln of $f(x)$





Example

$x^2 - x - 2 = 0$

$x = x^2 - 2$

$x = \pm\sqrt{x+2}$

$x = 1 + \frac{2}{x}$.

In fixed point iteration.

$$x = g(x)$$

$$x_{n+1} = g(x_n) \quad \text{for } n = 0, 1, 2, \ldots$$

Steps

Step 1 : Choose an initial guess x_0.

Step 2 : Define the iteration method as

$$x_{n+1} = g(x_n) \quad n = 0, 1, 2, \ldots$$

Let $x = \pm\sqrt{x+2}$.

$x_0 = 2$.

$x_1 = -2$.

Since we expect $x = g(n)$

we have to define $g(n)$ in such a way that this value should belong in the domain of g.

This kind of functions are called <u>self map</u>.

$a \le x \le b \Rightarrow a \le g(x) \le b$

(1) $x_{n+1} = g(x_n)$ $\quad n = 0, 1, \ldots$

Choose $g(x)$ in such a way that the sequence of iterations are well defined & can be calculated.

(2) The sequence $x_1, \ldots x_n \ldots$ converges to some point ξ.

(3) The limit ξ is a fixed point of $g(x)$

i.e. $\xi = g(\xi)$

To achieve $f \circ g = g \circ f$ we need $g(\xi)$ to be

& continuous $\longrightarrow$ [why ??]

$x_1 \dots x_n \dots$

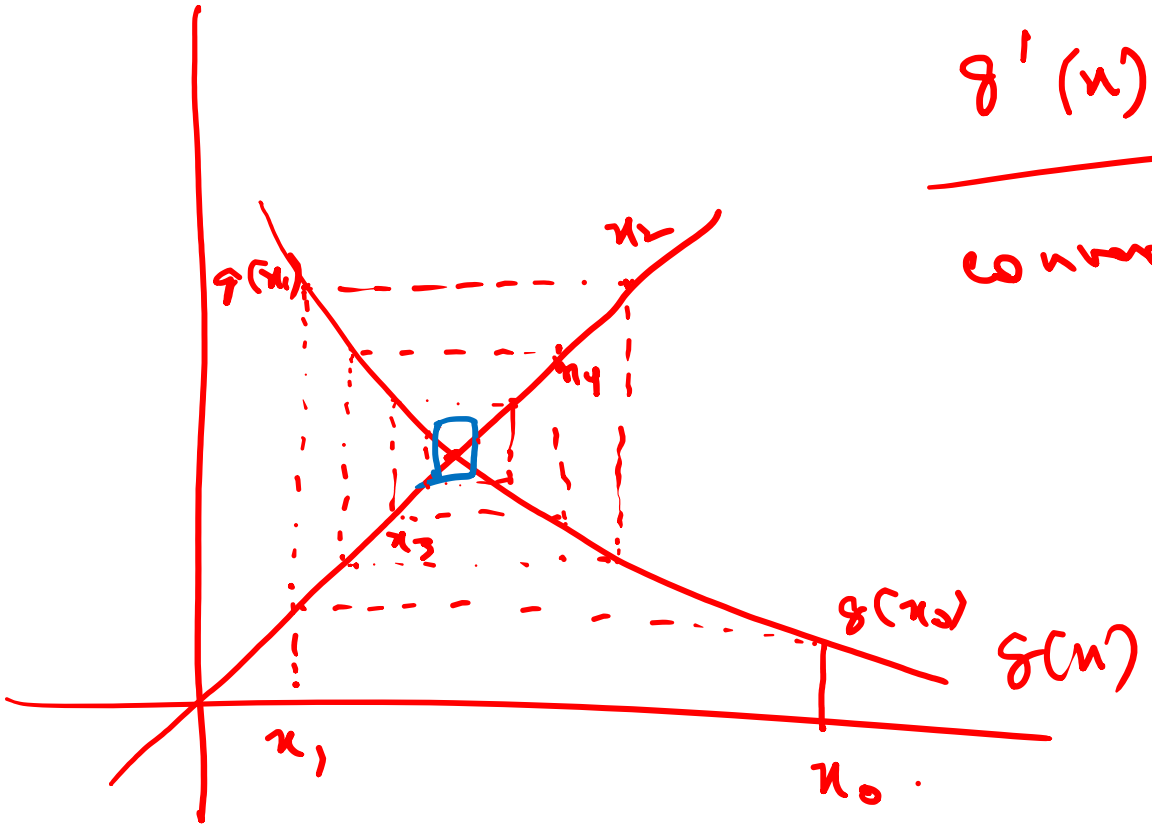
$\lim_{n \to \infty} x_n = \xi$

$\Downarrow$

$\lim_{n \to \infty} g(x_n) = g(\xi)$

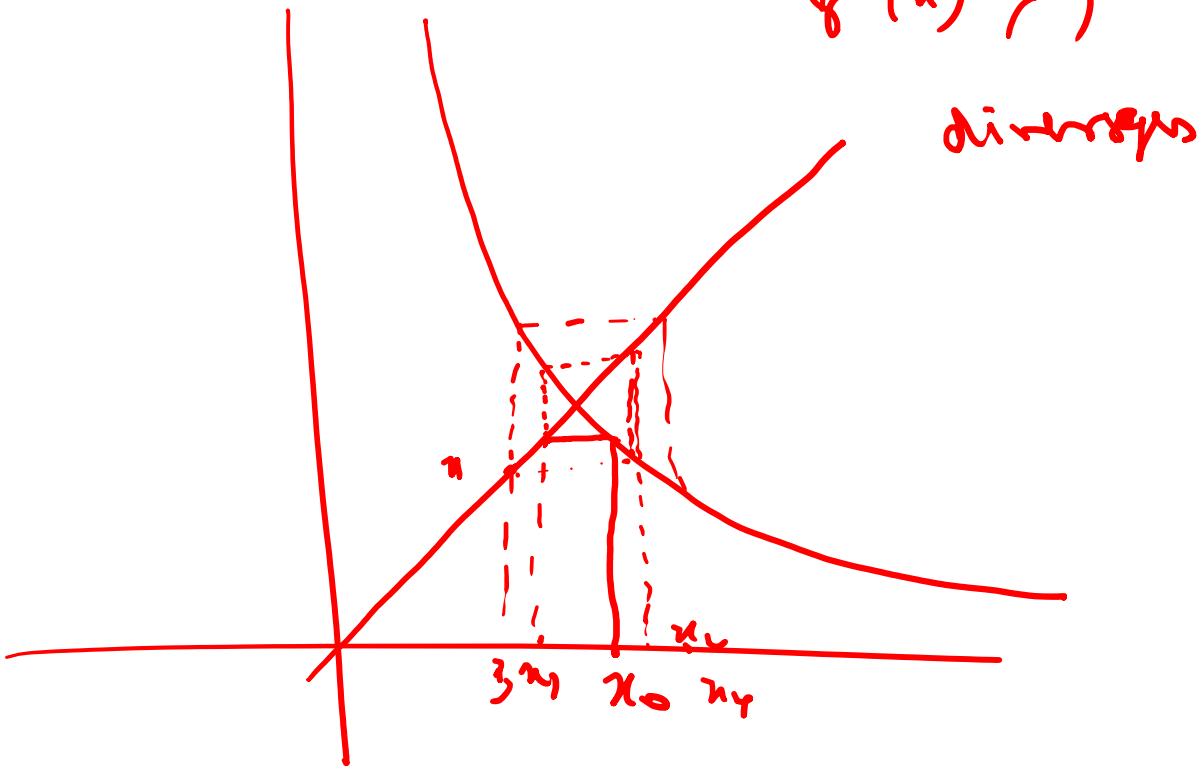
$g'(x) < 1$

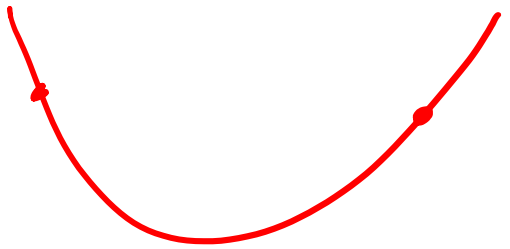
converges.



$g'(x) > 1$

diverges





Q. Find the root of $x = 0.1 + 0.3 \cos x$.

$$f(x) = -x + 0.3 \cos x + 0.1$$

$\varepsilon = 0.0001$