1. Let $V = \mathcal{C}[0,1]$ denote the vector space of all real valued continuous functions defined on the interval $[0,1] \subset \mathbb{R}$. Define $N : V \to \mathbb{R}$ as
$$\forall f \in V, N(f) = \sup_{x \in [0,1]} \{|f(x)|\}.$$
Is N a valid norm on V?

2. Let $(V, ||\cdot||)$ be a NVS. Show that $\forall x, y \in V, ||x - y|| \geq ||x|| - ||y||$.

3. Show that in an IPS $(V, \langle \cdot, \cdot \rangle)$, the parallelogram identity holds: $\forall u, v \in V, 2(||u||^2 + ||v||^2) = ||u+v||^2 + ||u-v||^2$. Why is it named so?

4. Let $(V, \langle \cdot, \cdot \rangle_{L_2})$ be the inner product space of all polynomials of degree at most 2 over one variable and real coefficients, seen as real valued functions on the interval $[-1,1] \subset \mathbb{R}$, where
$$\forall f, g \in V, \ \ \langle f, g \rangle_{L_2} = \int_{-1}^{1} f(x)g(x)\, dx.$$
Find an orthonormal basis of V.

5. Let $V = \mathbb{R}^{n \times n}$. Define $f : V \times V \to \mathbb{R}$ as $\forall A, B \in V, f(A, B) = trace(A^T B)$, where for $Q \in V, trace(Q) := \sum_{i=1}^{n} Q_{i,i}$. Is f an inner-product on V?