

SC223 - Linear Algebra

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Lecture 29



October 17, 2023

Change of Basis (Similarity Transformation)

- Let $T \in \mathcal{L}(U, V)$.
- We have seen how to compute $[T]_{\beta_U}^{\beta_V}$, the matrix representation of T w.r.t the basis β_U and β_V .
- What happens if we choose a different basis, say α_U and α_V . Are $[T]_{\beta_U}^{\beta_V}$ and $[T]_{\alpha_U}^{\alpha_V}$ different?
- How are they related?

Change of Basis (Similarity Transformation)

- Let $T \in \mathcal{L}(U, V)$.
- Let $\beta_U = \{u_1, \dots, u_n\}$ and $\beta_V = \{v_1, \dots, v_m\}$ be basis of U and V , and let $[T]_{\beta_U}^{\beta_V}$ denote the matrix representation of T w.r.t β_U and β_V .

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- Now, $U \cong \mathbb{F}^n$, and $V \cong \mathbb{F}^m$.

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- Now, $U \cong \mathbb{F}^n$, and $V \cong \mathbb{F}^m$.
- Let $N_{\beta_U} \in \mathcal{L}(U, \mathbb{F}^n)$ be defined as $N_{\beta_U}(u_1) = e_1^n, \dots, N_{\beta_U}(u_n) = e_n^n$,

$$\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{n \times 1} \quad \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{n \times 1}$$

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$$\forall x \in U, N_{\beta_U}(x) = [x]_{\beta_U} \quad e_1^m = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{m \times 1}$$
$$\forall y \in V, M_{\beta_V}(y) = [y]_{\beta_V}$$

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- $x \in U, N_{\beta_U}(x) = [x]_{\beta_U}$, and $y \in V, M_{\beta_V}(y) = [y]_{\beta_V}$.

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- $x \in U, N_{\beta_U}(x) = [x]_{\beta_U}$, and $y \in V, M_{\beta_V}(y) = [y]_{\beta_V}$.
- Let $\alpha_U = \{p_1, \dots, p_n\}, \alpha_V = \{q_1, \dots, q_m\}$, $U \cong \mathbb{F}^n$, be different set of basis vector for U and V resp.

$$U \rightarrow \mathbb{F}^n$$

$$V \rightarrow \mathbb{F}^m$$

$$N_{\alpha_U}(p_i) = e_i^n, \quad i=1, \dots, n$$

$$M_{\alpha_V}(q_i) = e_i^m, \quad i=1, \dots, m$$

$$N_{\alpha_U}(x) = [x]_{\alpha_U}$$

$$M_{\alpha_V}(y) = [y]_{\alpha_V}$$

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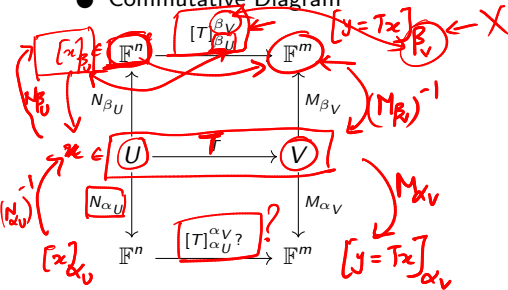
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- $x \in U, N_{\alpha_U}(x) = [x]_{\alpha_U}$, and $y \in V, M_{\alpha_V}(y) = [y]_{\alpha_V}$.
- Given $[T]_{\beta_U}^{\beta_V}$, how to compute $[T]_{\alpha_U}^{\alpha_V}$?

- Commutative Diagram

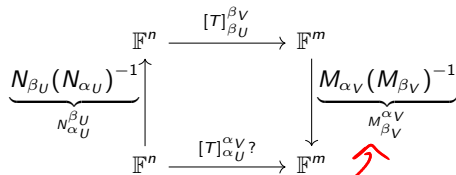
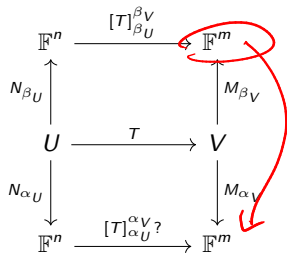


$$[y]_{\alpha_v} = \underbrace{M_{\alpha_v}^{-1} M_{P_v}^{-1}}_{P_v} [T]_{P_v}^T N_{P_v} N_{\alpha_v}^{-1} [x]_{\alpha_v}$$

$$[y]_{\alpha_V} = [T]_{\alpha_V}^{\alpha_V} [x]_{\alpha_V}$$

$$\left(\begin{matrix} N & B & N^{-1} \\ P & & \end{matrix} \right) ?$$

● Commutative Diagram



$M_{\alpha_V} M_{\beta_V}^{-1}$

$M_{\beta_V}^{\alpha_V}$

● Commutative Diagram

$$\begin{array}{ccc}
 \mathbb{F}^n & \xrightarrow{[T]_{\beta_U}^{\beta_V}} & \mathbb{F}^m \\
 N_{\beta_U} \uparrow & & \uparrow M_{\beta_V} \\
 U & \xrightarrow{T} & V \\
 N_{\alpha_U} \downarrow & & \downarrow M_{\alpha_V} \\
 \mathbb{F}^n & \xrightarrow{[T]_{\alpha_U}^{\alpha_V} ?} & \mathbb{F}^m
 \end{array}$$

$$\begin{array}{ccc}
 \mathbb{F}^n & \xrightarrow{[T]_{\beta_U}^{\beta_V}} & \mathbb{F}^m \\
 \underbrace{N_{\beta_U} (N_{\alpha_U})^{-1}}_{N_{\alpha_U}^{\beta_U}} \uparrow & & \downarrow \underbrace{M_{\alpha_V} (M_{\beta_V})^{-1}}_{M_{\beta_V}^{\alpha_V}} \\
 \mathbb{F}^n & \xrightarrow{[T]_{\alpha_U}^{\alpha_V} ?} & \mathbb{F}^m
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$$\begin{array}{ccc}
 [x]_{\beta_U} = N_{\alpha_U}^{\beta_U} [x]_{\alpha_U} & \xrightarrow{[T]_{\beta_U}^{\beta_V}} & [y]_{\beta_V} = [T]_{\beta_U}^{\beta_V} [x]_{\beta_U} \\
 N_{\alpha_U}^{\beta_U} \uparrow & & \downarrow M_{\beta_V}^{\alpha_V} \\
 [x]_{\alpha_U} & \xrightarrow{[T]_{\alpha_U}^{\alpha_V} ?} & [y]_{\alpha_V} = M_{\beta_V}^{\alpha_V} [y]_{\beta_V}
 \end{array}$$

Commutative Diagram

$$\begin{array}{ccc}
 \mathbb{F}^n & \xrightarrow{[T]_{\beta_U}^{\beta_V}} & \mathbb{F}^m \\
 N_{\beta_U} \uparrow & & \uparrow M_{\beta_V} \\
 U & \xrightarrow{T} & V \\
 N_{\alpha_U} \downarrow & & \downarrow M_{\alpha_V} \\
 \mathbb{F}^n & \xrightarrow{[T]_{\alpha_U}^{\alpha_V?}} & \mathbb{F}^m
 \end{array}$$

$$\begin{array}{ccc}
 \mathbb{F}^n & \xrightarrow{[T]_{\beta_U}^{\beta_V}} & \mathbb{F}^m \\
 \underbrace{N_{\beta_U}(N_{\alpha_U})^{-1}}_{N_{\alpha_U}^{\beta_U}} \uparrow & & \downarrow \underbrace{M_{\alpha_V}(M_{\beta_V})^{-1}}_{M_{\beta_V}^{\alpha_V}} \\
 \mathbb{F}^n & \xrightarrow{[T]_{\alpha_U}^{\alpha_V?}} & \mathbb{F}^m
 \end{array}$$

$$\begin{array}{ccc}
 [x]_{\beta_U} = N_{\alpha_U}^{\beta_U}[x]_{\alpha_U} & \xrightarrow{[T]_{\beta_U}^{\beta_V}} & [y]_{\beta_V} = [T]_{\beta_U}^{\beta_V}[x]_{\beta_U} \\
 N_{\alpha_U}^{\beta_U} \uparrow & & \downarrow M_{\beta_V}^{\alpha_V} \\
 [x]_{\alpha_U} & \xrightarrow{[T]_{\alpha_U}^{\alpha_V?}} & [y]_{\alpha_V} = M_{\beta_V}^{\alpha_V}[y]_{\beta_V}
 \end{array}$$

Thus,

$$[y]_{\alpha_V} = M_{\beta_V}^{\alpha_V} [T]_{\beta_U}^{\beta_V} N_{\alpha_U}^{\beta_U} [x]_{\alpha_U}, \forall x \in U$$

Commutative Diagram

$$\begin{array}{ccc}
 \mathbb{F}^n & \xrightarrow{[T]_{\beta_U}^{\beta_V}} & \mathbb{F}^m \\
 N_{\beta_U} \uparrow & & \uparrow M_{\beta_V} \\
 U & \xrightarrow{T} & V \\
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$$\begin{array}{ccc}
 \mathbb{F}^n & \xrightarrow{[T]_{\beta_U}^{\beta_V}} & \mathbb{F}^m \\
 \underbrace{N_{\beta_U} (N_{\alpha_U})^{-1}}_{N_{\alpha_U}^{\beta_U}} \uparrow & & \downarrow \underbrace{M_{\alpha_V} (M_{\beta_V})^{-1}}_{M_{\beta_V}^{\alpha_V}} \\
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$$\begin{array}{ccc}
 [x]_{\beta_U} = N_{\alpha_U}^{\beta_U} [x]_{\alpha_U} & \xrightarrow{[T]_{\beta_U}^{\beta_V}} & [y]_{\beta_V} = [T]_{\beta_U}^{\beta_V} [x]_{\beta_U} \\
 N_{\alpha_U}^{\beta_U} \uparrow & & \downarrow M_{\beta_V}^{\alpha_V} \\
 [x]_{\alpha_U} & \xrightarrow{[T]_{\alpha_U}^{\alpha_V} ?} & [y]_{\alpha_V} = M_{\beta_V}^{\alpha_V} [y]_{\beta_V}
 \end{array}$$

Thus,

$$[y]_{\alpha_V} = M_{\beta_V}^{\alpha_V} [T]_{\beta_U}^{\beta_V} N_{\alpha_U}^{\beta_U} [x]_{\alpha_U}, \forall x \in U$$

$$[T]_{\alpha_U}^{\alpha_V} = M_{\beta_V}^{\alpha_V} [T]_{\beta_U}^{\beta_V} N_{\alpha_U}^{\beta_U}$$

- $[T]_{\alpha_U}^{\alpha_V} = M_{\beta_V}^{\alpha_V} [T]_{\beta_U}^{\beta_V} N_{\alpha_U}^{\beta_U}$

- $[T]_{\alpha_U}^{\alpha_V} = M_{\beta_V}^{\alpha_V} [T]_{\beta_U}^{\beta_V} N_{\alpha_U}^{\beta_U}$
- For a linear operator $T : U \rightarrow U$, assume $\beta_U = \beta_V = \beta$ and $\alpha_U = \alpha_V = \alpha$.

$$\underline{[T]_{\alpha}^{\alpha}} = \underbrace{M_{\beta}^{\alpha}}_{\beta} \underbrace{[T]_{\beta}^{\beta}}_{\beta} \underbrace{N_{\alpha}^{\beta}}_{\alpha}$$

$N_{\beta}^{\alpha} = (N_{\alpha}^{\beta})^{-1}$?

- $[T]_{\alpha_U}^{\alpha_V} = M_{\beta_V}^{\alpha_V} [T]_{\beta_U}^{\beta_V} N_{\alpha_U}^{\beta_U}$
- For a linear operator $T : U \rightarrow U$, assume $\beta_U = \beta_V = \beta$ and $\alpha_U = \alpha_V = \alpha$.
- In this case, $[T]_{\alpha}^{\alpha} = M_{\beta}^{\alpha} [T]_{\beta}^{\beta} N_{\alpha}^{\beta}$.

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- Note that $M_{\beta}^{\alpha} = (N_{\alpha}^{\beta})^{-1}$.

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- In this case, $[T]_{\alpha}^{\alpha} = M_{\beta}^{\alpha} [T]_{\beta}^{\beta} N_{\alpha}^{\beta}$.
- Note that $M_{\beta}^{\alpha} = (N_{\alpha}^{\beta})^{-1}$. Denote M_{β}^{α} by S , which gives us $[T]_{\alpha}^{\alpha} = S [T]_{\beta}^{\beta} S^{-1}$.

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- In this case, $[T]_{\alpha}^{\alpha} = M_{\beta}^{\alpha} [T]_{\beta}^{\beta} N_{\alpha}^{\beta}$.
- Note that $M_{\beta}^{\alpha} = (N_{\alpha}^{\beta})^{-1}$. Denote M_{β}^{α} by S , which gives us $[T]_{\alpha}^{\alpha} = S [T]_{\beta}^{\beta} S^{-1}$.
- **Similar matrices and Similarity transformation:** We say two matrices A and B are similar if there exists an invertible matrix, say S such that $B = SAS^{-1}$. The transformation $A \mapsto SAS^{-1}$ is said to be a similarity transformation of A by S .

$$[T]_{\text{time}}^{\text{time}} = (N_{\text{time}}^{\text{freq}})^{-1} [T]_{\text{freq}}^{\text{freq}} N_{\text{freq}}^{\text{time}}$$

$T : U \rightarrow U$
 β is a basis of U .

$$[T]_{\beta}^{\beta} \rightarrow [T]_{\alpha}^{\alpha}$$

$$[1]_{\beta}^{\beta} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

↓

$$\alpha = \{\alpha_1, \alpha_2, \alpha_3\}$$

$$\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \lambda_1 x_1 \\ \lambda_2 x_2 \\ \lambda_3 x_3 \end{bmatrix}$$