1. Convert the following Augmented matrices into REF form and then find *all* solutions to the system of equations (if any).

   (a) $\left[\begin{array}{ccccc|c} 1 & 2 & -1 & 0 & 2 & 2 \\ 2 & 4 & 0 & 2 & 1 & -1 \\ 1 & -2 & 1 & 2 & 0 & 0 \\ -3 & -4 & 1 & 1 & 2 & 3 \end{array}\right]$

   (b) $\left[\begin{array}{cc|c} 2 & 3 & 1 \\ 1 & 3/2 & 1/2 \\ -3 & -9/2 & -3/2 \\ 6 & 9 & 3 \end{array}\right]$

   (c) $\left[\begin{array}{ccc|c} 2 & 4 & 1 & 3 \\ 5 & 10 & 5/2 & 15/2 \\ -1 & -2 & -1/2 & -3/2 \end{array}\right]$

   (d) $\left[\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 1 \\ 2 & 4 & 2 & -2 & 2 \\ 1 & 2 & 1 & -1 & 1 \\ -1 & -2 & -1 & 1 & 1 \end{array}\right]$

2. Each equation in the system given in Question (1b) can be thought of as representing a straight line in $\mathbb{R}^2$. Plot all the 4 lines from the system, and interpret the number of solutions from your plot. Now, can you add one extra equation to this system of linear equations so as to obtain (1) Unique solution to the new system of equations, (2) Multiple solutions to the new system of equations, and (3) No solutions for the new system of Equations? Repeat this for Question (1a), (without the visualization).

3. Consider the following Augmented Matrix: $\left[\begin{array}{cccc|c} 2 & 3 & -1 & 1 & 5 \\ 2 & 1 & 0 & 1 & 4 \\ 1 & 0 & 2 & -1 & 2 \\ 4 & 1 & 3 & k & 2 \end{array}\right]$, where $k \in \mathbb{R}$. Can you find values of k for which the system has (1) a unique solution, (2) multiple solutions, and (3) no solution.

4. Given a linear system of equations $Ax = b$, with $A \in \mathbb{R}^{n \times n}, b \in \mathbb{R}^n$, find the number of real number multiplications (incl. divisions) and additions (incl. subtractions) in order to obtain the REF form, in the worst case.