

Computational Numerical Methods

CS 374

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Approximate root

$$x = \underline{\underline{1.1338}}$$

iteration	a_n	b_n	c	$b-c$	$f(c)$
1	1	2	1.5	0.5	8.8906
2	1	1.5	1.25	0.25	1.5647
3	1	1.25	1.125	0.125	-0.0977
4	1.125	1.25	1.1875	0.0625	0.6167
5	1.125	1.1875	1.1562	0.0312	0.2337
6	1.1562	1.1875	1.1719	0.0156	0.0457
7	1.1562	1.1719	1.1641	0.0078	0.0057
8	1.1641	1.1719	1.1680	0.0039	0.0004
9	1.1328	1.1367	1.1348	0.002	0.0004
10	1.1328	1.1368	<u>1.1338</u>	0.00398	0.0006

Convergence & Error estimation of ~~Newton~~ Bisection method

Hypothesis: let $f: [a_0, b_0] \rightarrow \mathbb{R}$ be continuous function.

such that the number $f(a_0)$ & $f(b_0)$ is of opposite sign.

Conclusion There exist an $r \in [a_0, b_0]$ s.t. $f(r) = 0$.
If the iterative sequence of approximate solⁿ $\{x_n\}$ converges to r .

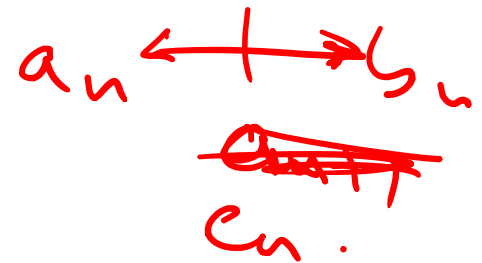
For each $n = 0, 1, 2, \dots$ we have the error.

$$|x_{n+1} - r| = \left(\frac{1}{2}\right)^{n+1} (b_0 - a_0)$$

$$b_n - a_n = \frac{1}{2} [b_{n-1} - a_{n-1}] = \frac{1}{2^2} [b_{n-2} - a_{n-2}]$$

$$= \frac{1}{2^n} (b_0 - a_0)$$

$$\lim_{n \rightarrow \infty} b_n - a_n = 0$$



$$\Rightarrow \lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} a_n$$

x_{n+1} is the mid point of the interval $[a_n, b_n]$

$$\Rightarrow a_n \leq x_{n+1} \leq b_n$$

Using sandwich theorem

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} x_{n+1} = r.$$

Error Let a_n, b_n & c_n are the n th computed values ~~of the~~ a_0, b_0 & c .

$$\underline{b_{n+1} - a_{n+1} = \frac{1}{2} (b_n - a_n)} \quad n \geq 1$$

$$\underline{b_n - a_n = \frac{1}{2^{n-1}} (b_0 - a_0)}$$

where ~~b_0, a_0~~ denote $b_0 - a_0$ denote the length of the original interval.

Since the root r is either in the interval $[a_n, c_n]$
or, $[c_n, b_n]$ we have

$$|r - c_n| \leq c_n - a_n = b_n - c_n.$$

$$= \frac{1}{2} (b_n - a_n)$$

$$\underline{|r - c_n| \leq \frac{1}{2^{n+1}} (b_0 - a_0)}$$