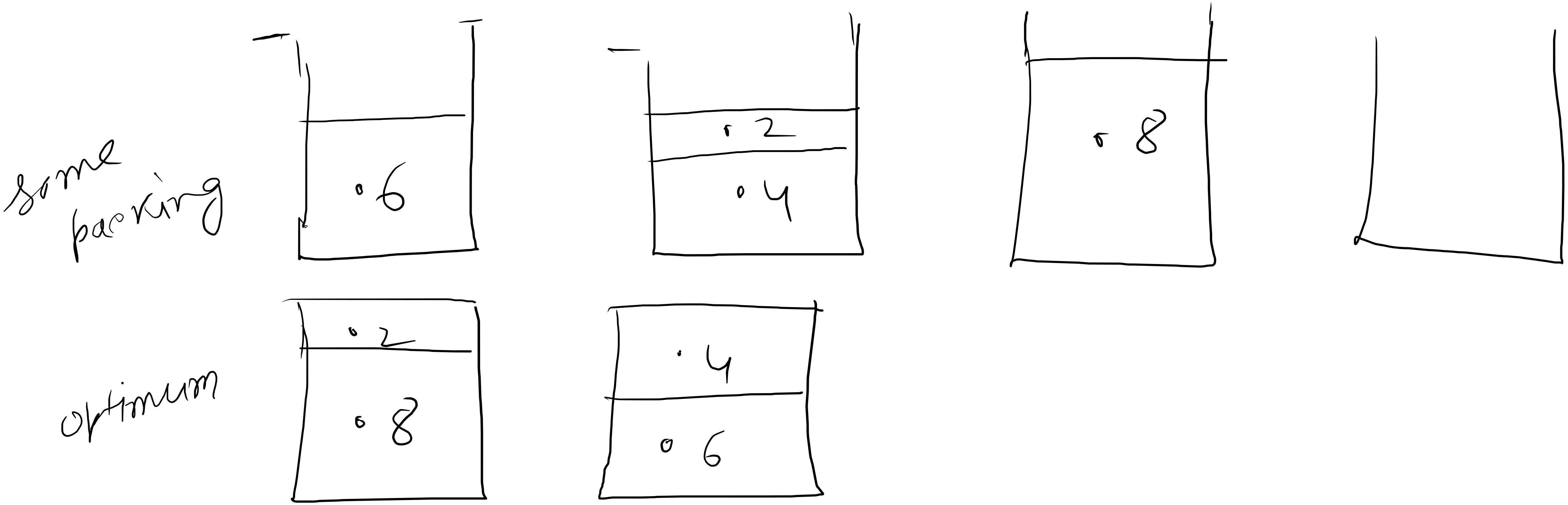
Approximation

Bin packing problem

Assume there are n items of sizes < 1
They need to fit in minimum number of unit size boxes.

$.4, .6, .2, .8$



Best-fit

⇒ first-fit

Next-fit

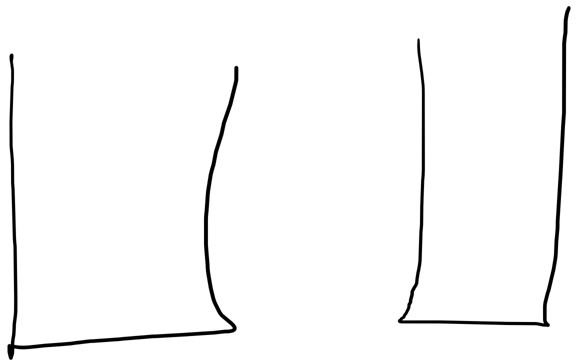
First-fit

- traverse from 1st to last bin.
- if it fits in a bin - place it there
- otherwise open a new bin

Correctness: trivially.

running time: $O(n^2)$

Approximation factor:



can these two bins are both are less than half full.

Not the case here.

If C bins are used by the algorithm.

Let C^* be the optimum number of bins can be used.

$$c^* \geq \sum_{i=1}^{n} a_i > \frac{c-1}{2}$$

$$\Rightarrow 2c^* > c-1$$

$$\Rightarrow c < 2c^* - 1$$

$\Rightarrow c \leq 2c^*$ as c & c^* are integers.

<u>minimize makespan</u>

Scheduling jobs on identical parallel machines.

<u>Input</u>: n jobs $j_1, j_2, \dots, j_n$ each job j_i has processing time t_i. There are m identical machines.

<u>output</u>: Assign jobs into those machines such that the maximum load to any machine is minimum.

5, 7, 3, 2, 4

m_1	5	4
m_2	7	
m_3	3	2

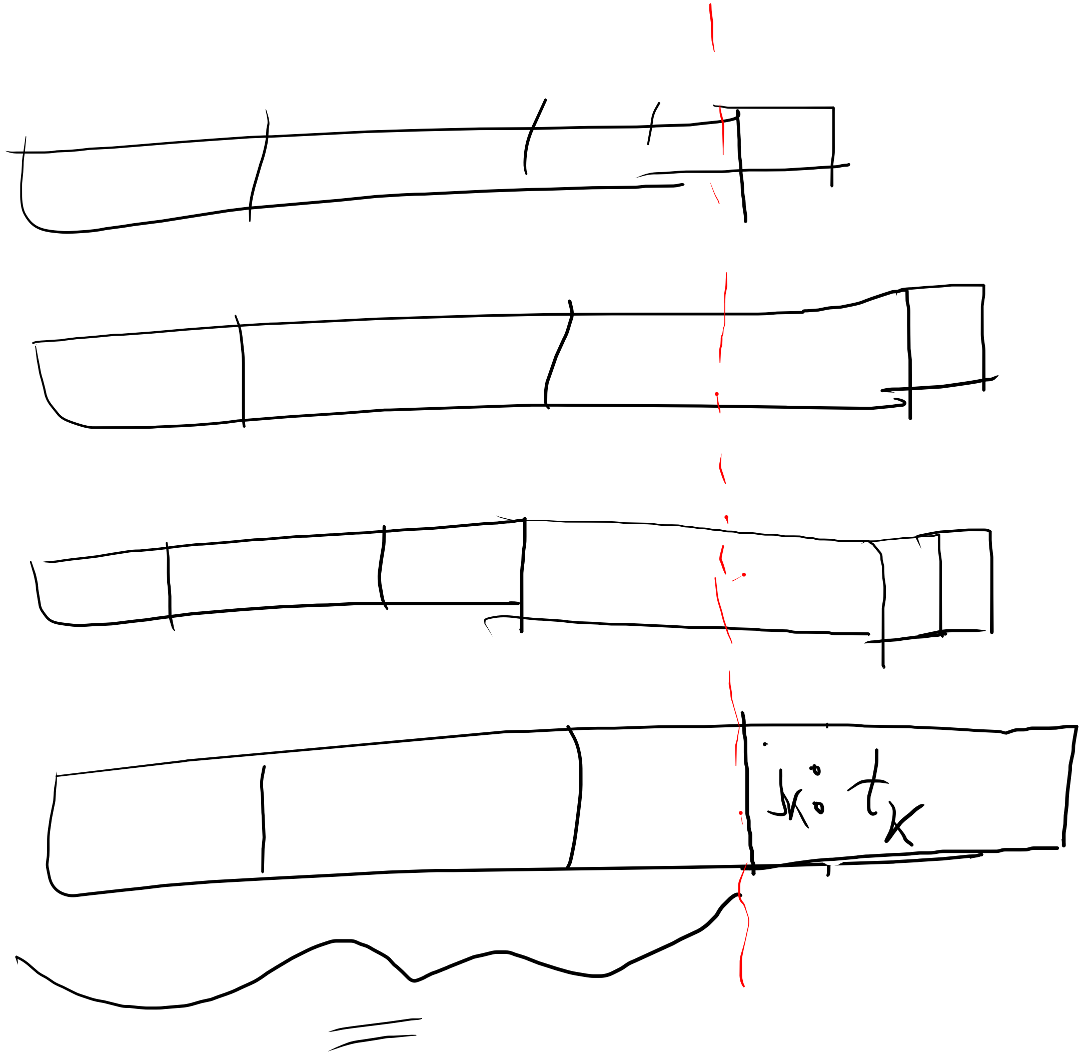
m_1	7	
m_2	5	2
m_3	4	3

Algo: - Take jobs one-by-one.
- Assign to a machine whose load is minimum so far.

Approx factor:

opt is the optimum load

$$\text{opt} \geq \frac{\sum t_i}{m} \qquad\qquad \text{opt} \geq t_{max}$$



d' be the makespan just before adding j_k.

$$d' \le \sum_{i=1}^{n} \frac{t_i}{m}$$

Total makespan $= \displaystyle\sum_{i=1}^{n} \frac{t_i}{m} + \underbrace{t_k}_{t_{max}}$

$\le opt + opt$

$= 2\,opt$.