Polynomial time reduction

$A \leq_p B$

Thm $A \leq_p B$ and B is in P then A is in P.

Contrapositive :- $A \leq_p B$, if A is hard then B is hard.

Transitivity: If $A \leq_p B$, $B \leq_p C$ then $A \leq_p C$

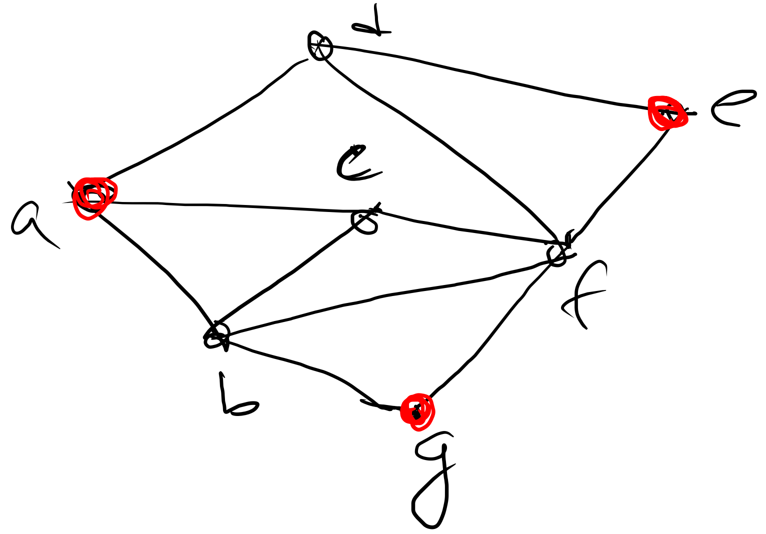
H.W.

Exm

Indebendent set problem;

Input: A graph $G(V,E)$, find a set $V' \subseteq V$ of maximum size such that no two vertices in V' are adjacent.

Exm



$\{b, d\}$

$\{g, a, e\}$

Does $|V'| \geq K$ or not?

Vertex cover problem

Given a graph $G(V,E)$ find ~~a~~ a set $V' \subseteq V$ of minimum size such that at least one end-point of each edge must belongs to V'.

$\{b, c, d, f\}$

$\boxed{\text{Does } |V'| \leq K \text{ or not?}}$

show that $IS_D \leq_p VC_D$

Two thing we need to show:

(i) $x \xrightarrow{f} f(x)$

x is a yes-instance of A iff $f(x)$ is a yes instance of B

(ii) $f(x)$ is computed in polynomial time.

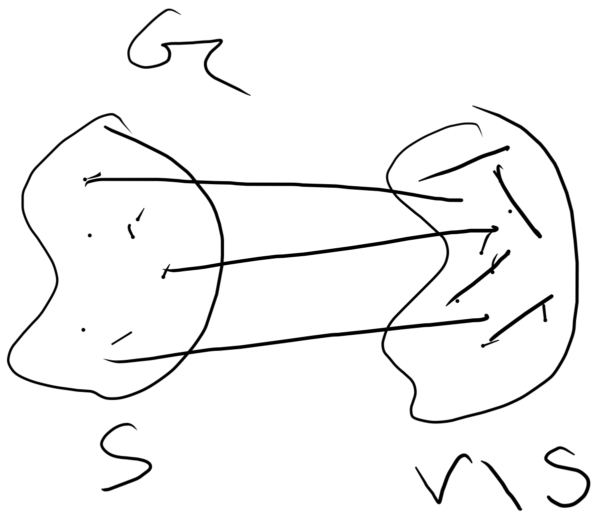
(i) $x \rightarrow f(x)$

- we are given the IS_D instance $G(V, E)$
- we take the same graph $G(V, E)$ as an instance of VC_D

we now prove the following:

Thm Let G be a graph. Let S be an independent set of size at least K iff $\{V \setminus S\}$ is a vertex cover of G of size at most $(n-K)$.

Proof ($\Rightarrow$) Assume S is an independent set.
we need to prove that $\{V \setminus S\}$ is a vertex cover.



($\Leftarrow$)

(ii) Trivial as we take the same graph.

Ex^m **Set cover problem**

Given a universe $U = \{a_1, a_2, \ldots, a_n\}$ and a collection of subsets of U i.e. $C = \{S_1, S_2, \ldots, S_m\}$

Find a subcollection $C' \subseteq C$ of minimum size such that each element in U belongs to at least one set in C'.

Ex^m

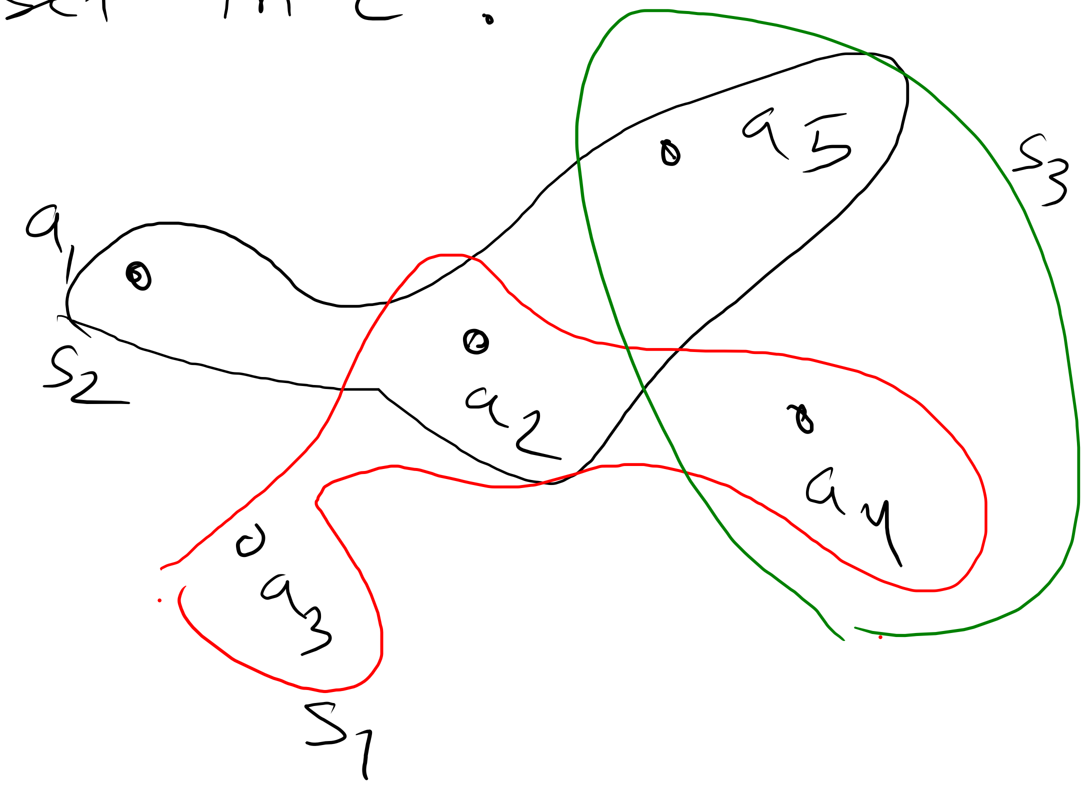
$U = \{a_1, a_2, a_3, a_4, a_5\}$

$C = \{S_1, S_2, S_3\}$

$S_1 = \{a_2, a_3, a_5\}$

$S_2 = \{a_1, a_2, a_4\}$

$S_3 = \{a_4, a_5\}$



solution $= \{S_1, S_2\}$

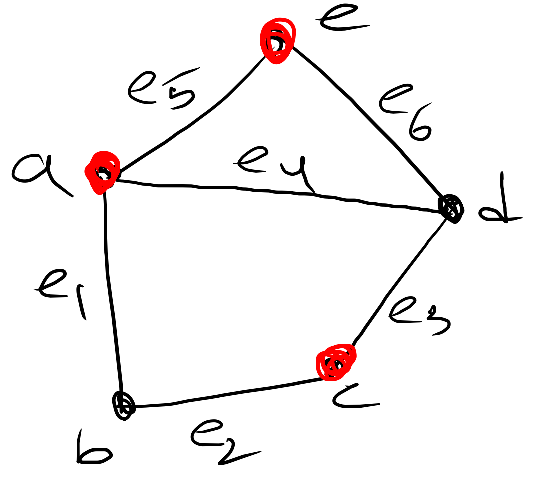
Show that $VC_D \le_p SC_D$

We are given a VC_D problem instance.

A graph is given $G(V, E)$

$$G \xrightarrow{f} I_G(U, C)$$

instance of SC_D depends on G.



$E \Longleftrightarrow U$

$V \quad U = \{e_1, e_2, e_3, e_4, e_5, e_6\}$

For each vertex $v \in V$ take a set S_v in C.

$C = \{S_a, S_b, S_c, S_d, S_e\}$

where $S_a = \{e_1, e_4, e_5\}$

$S_b = \{e_1, e_2\}$

$S_c = \{e_2, e_3\}$

$S_d = \{e_3, e_4, e_6\}$

$S_e = \{e_5, e_6\}$

$x \xrightarrow{f} f(x)$

For each edge e_i take an element in U

for each vertex $v \in G$ take a set S_v in C.

where S_v contains all elements whose corresponding edges are incident on v,

Thm: G has a vertex cover of size at most K iff $I_G(U,C)$ has a set cover of size at most K.

Further the reduction takes polynomial time. Why?