Computational Numerical Methods

CS 374

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Newton's interpolating polynomial

$$P_n(x) = A_0 + A_1(x-x_0) + A_2(x-x_0)(x-x_1)$$
$$\cdots + A_n(x-x_0)\ldots(x-x_{n-1})$$

for given data. $(x_0, y_0), (x_1, y_1) \ldots (x_n, y_n)$

we assume $x_0, x_1 \ldots x_n$ are equispaced.

$$x_n = x_0 + nh \qquad x_1 - x_0 = x_2 - x_1 = \ldots x_n - x_{n-1}$$

$\therefore\ y_0 = P_n(x_0)$

$y_0 = A_0$

$P_n(x_1) = y_1 = A_0 + A_1(x_1 - x_0)$

$y_1 = y_0 + A_1(h).$

$\Rightarrow \quad y_1 - y_0 = A_1(h).$

$\Delta y_0 = A_1 h.$

$A_1 = \dfrac{\Delta y_0}{h}.$

$x_1 - x_0 = h.$

$$y_2 = A_0 + A_1(x_2 - x_0) + A_2(x_2 - x_1)(x_2 - x_0)$$

$$y_2 - y_0 = \frac{\Delta y_0}{h} \cdot 2h + A_2(2h) \cdot h.$$

$$y_2 = A_0 + A_1(x_2 - x_0) + A_2(x_2 - x_0)(x_2 - x_1)$$

$$y_1 = A_0 + A_1(x_1 - x_0)$$

$$\Delta y_1 = y_2 - y_1 = A_1(x_2 - x_1) + A_2(x_2 - x_0)(x_2 - x_1)$$

$$= A_1 h + A_2 \cdot 2h \cdot h.$$

$$A_2 = \frac{1}{2 \cdot h^2}(\Delta y_1 - \Delta y_0)$$

$$= \frac{1}{2!\,h^2}(\underline{\Delta^2 y_0}).$$

Similarly

$$A_3 = \frac{1}{3!\,h^3}\Delta^3 y_0.$$

Substituting in newton's interpolating polynomial

$$y(x) = y_0 + \frac{\Delta y_0}{h}(x-x_0) + \frac{\Delta^2 y_0}{2!\,h^2}(x-x_0)(x-x_1)$$

$$+ \cdots + \frac{\Delta^n y_0}{n!\,h^n}(x-x_0)\cdots(x-x_{n-1})$$

Now we need to evaluate the value of y at some x say (x_0+ph) then.

$$x - x_0 = ph. \qquad x - x_1 = (p-1)h.$$

$$\cdots \quad (x - x_2) = (p-2)h \cdots$$

Substituting in $y(x) = y(x_0+ph). = y_p$

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \cdots + \frac{p(p-1)\cdots(p-n-1)}{n!}\Delta^n y_0$$

This is Newton's forward interpolation formula.

For backward interpolation one might consider:

$$x = x_n - ph$$

In that case the interpolation formula will lead to.

$$y_p = y_n + p \nabla y_n + \frac{p(p+1)}{2!} \nabla^2 y_n + \cdots \frac{p(p+1)(p+2)}{3!} \nabla^3 y_n + \cdots$$

Ex From the table estimate no of students who obtained marks b/w 40 & 45

marks	30-40	40-50	50-60	60-70	70-80
No. Students	31	42	51	35	31

Let us ~~cond~~ consider the data with cumulative frequency are given as follows

marks less than	40	50	60	70	80
No. of Students	31	73	124	159	190

Table for Newton's forward interpolation

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$
40	31				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

forward interpolation

To obtain a marks b/w 40 & 45.

We ~~the~~ data in y_{45}. $\qquad x_0 = 40$. $\quad x = 45$.

$$p = \frac{x - x_0}{h} = \frac{45 - 40}{10} = 0.5$$

$$y_{45} = y_{40} + p\,\Delta y_{40} + \frac{p(p-1)}{2!}\Delta^2 y_{40} + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_{40} + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_{40}$$

$$= 31 + 0.5 \times 42 + \frac{0.5^2}{2!} \times 9 + \frac{0.5 \times 0.5 \times 1.5}{6}(-25) + \frac{0.5 \times 0.5 \times 1.5 \times 2.5}{24} 37.$$

$$= 47.867 \approx 48.$$

$\therefore$ The number of students getting marks b/w 40 & 45 is

$48 - 31 = 17$

Repeat the same with backward interpolation.

Ex Using NB interpolation construct an interpolating polynomial of degree 3 for the data.

$$f(-0.75) = -0.0718125$$

$$f(-0.5) = -0.02475$$

$$f(-0.25) = 0.3349375$$

$$f(0) = 1.10100$$

Hence find $f(-1/3)$ $\left(= 0.1745 \text{ [illegible]}\right)$

Eg Given the data set

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

Evaluate $f(9)$ using Newton's divided difference formula.

x	y	$f[x_{i-1}, x_i]$	$f[x_{i-2}, x_{i-1}, x_i]$
5	150	$\frac{392-150}{7-5} = 121$	$\frac{265-121}{11-5}$
7	392	$\frac{1452-392}{11-7} = 265$	
11	1452	⋮	
13	2366		
17	5202		