Signals and Systems (CT 203)

Tutorial Sheet-12 DA-IICT, Gandhinagar.

1. Find the frequency response, $H(\omega)$, of the causal and stable LTI systems which are modeled by second-order differential equations
   - $\frac{d^2y(t)}{dt^2}+4\frac{dy(t)}{dt}+4y(t)=x(t)$
   - $5\frac{d^2y(t)}{dt^2}+4\frac{dy(t)}{dt}+5y(t)=7x(t)$
   - $\frac{d^2y(t)}{dt^2}+20\frac{dy(t)}{dt}+y(t)=x(t)$
   - $5\frac{d^2y(t)}{dt^2}+4\frac{dy(t)}{dt}+5y(t)=7x(t)+\frac{1}{3}\frac{dx(t)}{dt}$
2. Consider a continuous-time LTI system with frequency response, $H(\omega)=|H(\omega)|e^{j\measuredangle H(\omega)}$ and real impulse response $h(t)$. Suppose that we apply an input $x(t)=\cos(\omega_o t+\phi_o)$ to this system. The resulting output can be shown to be of the form

$$y(t)=Ax(t-t_o)$$

where A is a non-negative real number representing an amplitude- scaling factor and t_o is a time delay.

   - Express A in terms of $|H(\omega)|$
   - Express t_o in terms of $<\measuredangle H(\omega)$ (i.e. Phase of $H(\omega)$)
3. Consider a discrete-time LTI system with frequency response, $H(e^{j\omega})=|H(e^{j\omega})|e^{j\measuredangle H(e^{j\omega})}$ and real impulse response $h(n)$. Suppose that we apply an input $x(n)=\cos(\omega_o n+\phi_o)$ to this system. The resulting output can be shown to be of the form

$$y(n)=|H(e^{j\omega_o})|x(n-n_o)$$

   provided that $\measuredangle H(e^{j\omega_o})$ and ω_o are related in a particular way. Determine this relationship.

4. Consider the following frequency response for a causal and stable LTI system:
   $H(\omega)=\frac{1-j\omega}{1+j\omega}.$
   - Show that $|H(\omega)|=A,$ and determine the value of A
   - Determine the *group delay*, $\tau(\omega)$ of the system.

5. Consider a continuous-time ideal bandpass filter whose frequency response is

$$H(\omega)=\begin{cases}1, & \omega_c \le |\omega| \le 3\omega_c \\ 0, & otherwise\end{cases}.$$

- If $h(t)$ is the impulse response of this filter, determine a function $g(t)$ such that
$$h(t)=\left(\frac{\sin(\omega_c t)}{\pi t}\right)g(t).$$
- As ω_c is increased, does the impulse response of the filter get more concentrated or less concentrated about origin?

6. Consider a discrete-time ideal highpass filter whose frequency response is

$$H\left(e^{j\omega}\right)=\begin{cases}1, & \pi-\omega_c \le |\omega| \le \pi \\ 0, & |\omega| \le \pi-\omega_c\end{cases}$$

- If $h(n)$ is the impulse response of this filter, determine a function $g(n)$ such that
$$h(n)=\left(\frac{\sin(\omega_c n)}{\pi n}\right)g(n).$$
- As ω_c is increased, does the impulse response of the filter get more concentrated or less concentrated about origin?

7. A causal LTI filter has the frequency response $H(\omega)$ shown in Fig.1. For each of the input signals below, determine the filtered output signal $y(t)$.

- $x(t)=e^{jt}$
- $x(t)=\sin(\omega_o t)u(t)$
- $X(\omega)=\dfrac{1}{(j\omega)(6+j\omega)}$
- $X(\omega)=\dfrac{1}{2+j\omega}$

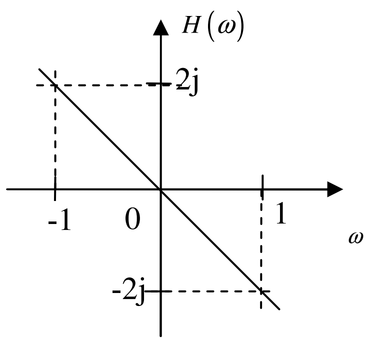

Fig. 1. Frequency response of LTI system