Computational Numerical Methods

CS 374

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Table for Newton's forward interpolation.

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$
40	31				
		42			
			9		
50	73			-25	
		51			37
			-16		
60	124				
		35		12	
			-4		
70	159				
		31			
80	190				

Forward interpolation

$\Delta y_0 = y_1 - y_0$

$\Delta y_1 = y_2 - y_1$

$\nabla y_n = y_n - y_{n-1}$

To obtain a mark b/w 40 & 45.

Use the data in y_{45}.

$x_0 = 40$. $x = 45$.

$$p = \frac{x - x_0}{h} = \frac{45 - 40}{10} = 0.5$$

$$y_{45} = y_{40} + p\,\Delta y_{40} + \frac{p(p-1)}{2!}\Delta^2 y_{40} + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_{40} + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_{40}$$

$$= 31 + 0.5 \times 42 - \frac{0.5^2}{2!} \times 9 + \frac{0.5 \times 0.5 \times 1.5}{6}(-25) - \frac{0.5 \times 0.5 \times 1.5 \times 2.5}{24} \, 37$$

$$= \underline{47.867} \approx 48.$$

∴ The number of students get marks b/w 40 & 48 is

$48 - 31 = 17$

Repeat the same with backward interpolation.

Using Newton's backward interpolation.

$$y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n.$$

$$p = \frac{45-80}{10} = \frac{-35}{10} = -3.5$$

$$y_{45} = 190 + (-3.5)\,31 + \frac{(-3.5)(-2.5)}{2}(-4)$$

$$+ \frac{-3.5\,(-2.5)(-1.5)}{6}\,12 + \frac{-3.5\times(-2.5)\times(-1.5)\times(-0.5)}{24} \times 37.$$

$$= 47.87 \approx 48.$$

Ex Using NB interpolation construct an interpolating polynomial of degree 3 for the data.

$$f(-0.75) = -0.0718125$$

$$f(-0.5) = -0.02475$$

$$f(-0.25) = 0.3349375$$

$$f(0) = 1.10100$$

(Hence find $f(-1/3)$) ($= 0.1745$ 4 decimal places)

Eg. Given the data set

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

$x = x_0 + ph$

$x = x_n - ph$

$p = -\frac{(x - x_n)}{h}$

Evaluate $f(9)$ using Newton's divided difference formula.

x	y	$f[x_{i-1}, x_i]$	$f[x_{i-2}, x_{i-1}, x_i]$	3rd order $f(x_{i-3}, x_{i-2}, x_{i-1}, x_i)$	
5	150	$\frac{392-150}{7-5} = 121$	$\frac{265-121}{11-5} = 24$	1	0
7	392	$\frac{1452-392}{11-7} = 265$	~~27~~ 32	1	
11	1452	457	42		
13	2366	709			
17	5202				

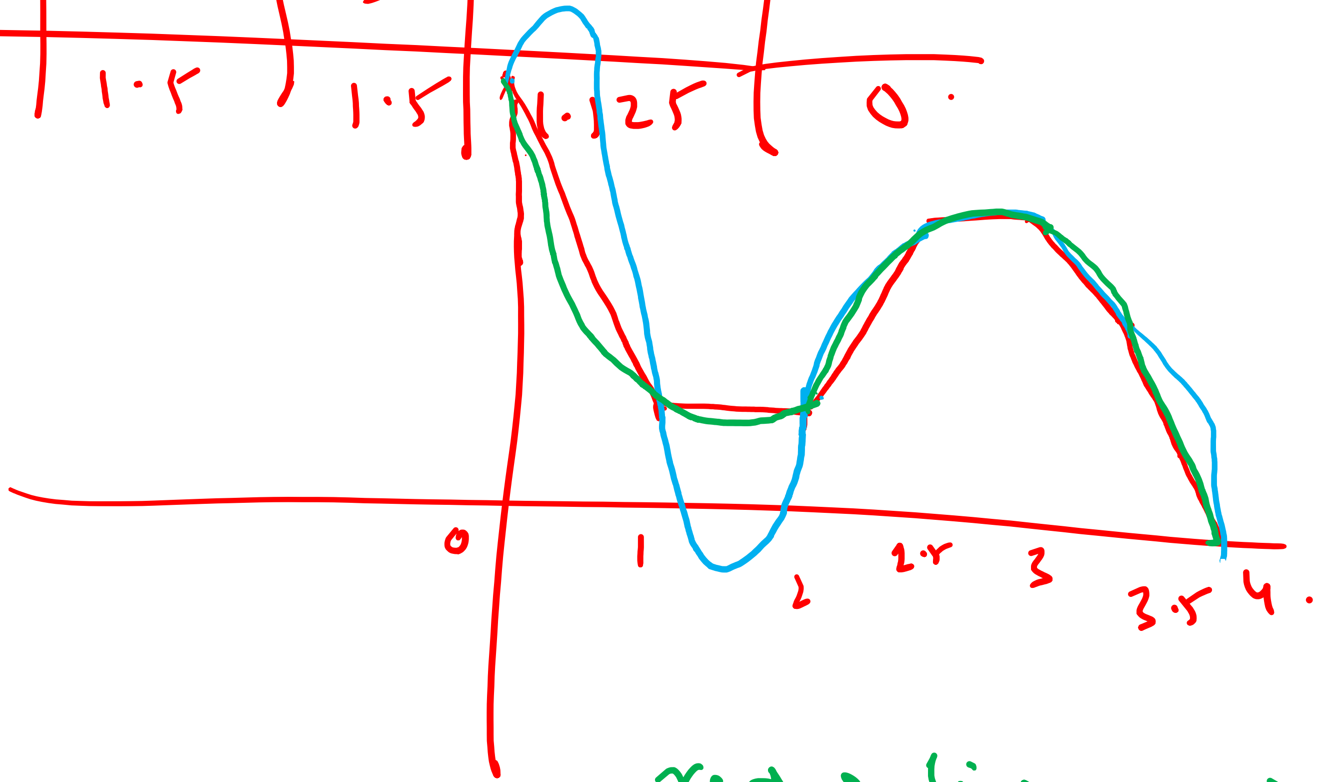
$$P_n(9) = f(5) + (9-5)\, f[5,7]$$
$$+ (9-5)(9-7)\, f[5,7,11]$$
$$+ (9-5)(9-7)(9-11)\, f[5,7,11,13]$$
$$+ (9-5)(9-7)(9-11)(9-13)\, f[5,7,11,13,17]$$

$$= 150 + 4 \times 121 + 8 \times 24 - 16 \times 1$$

$$= 810.$$

Spline Interpolation

x	0	1	2	2.5	3	3.5	4
y	2.5	0.5	0.5	1.5	1.5	1.125	0.



red → linear piecewise

blue → P_6.

green → quadratic piecewise.

For n data points $(x_1, y_1), (x_2, y_2) \dots (x_n, y_n)$.

$x_1 < x_2 < \dots < x_n$.

We need to seek a function defined on the the initial interval $[a, b]$ $[a = x_1, b = x_n]$ such that

$$S(x_i) = y_i \qquad \text{for } i = 1 \dots n.$$

① For smooth interpolation. $S'(x)$ & $S''(x)$ are continuous.

② The linear interpolation is to be followed closely.

(3) Hence $S'(x)$ must not change rapidly &

$S''(x)$ must be very small (For linear interpolation

$$\left.\begin{aligned} S'(x) &= c \\ S''(x) &= 0 \end{aligned}\right]$$