Lecture 12

Q. Whether the converse of proposition 1.1 is true?

Q. Means if a system gives zero o/p for zero i/p then whether the system is linear? [Converse of Proposition 1.1]

$\Rightarrow$ $\boxed{y(t) = x^2(t)}$

if $x(t) = 0 \;\forall t$, $y(t) = (0)^2 = 0$

However, the system $y(t) = x^2(t)$ is NOT linear or Nonlinear.

$y(t) = 2x(t) + 5$ $\Rightarrow$ Nonlinear system

$x(t) = 0$, $y(t) = 5 \neq 0$

$y(t) = 2x(t) \rightarrow$ linear system

$x(t) = 0$, $\Rightarrow y(t) = 0$

Conclusion: — Converse of Proposition 1.1 and Proposition 1.2 is NOT always true

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② Check whether $y(n) = x^2(n)$. is linear

Soln:→ $y_1(n) = H\{x_1(n)\} = x_1^2(n)$

$y_2(n) = H\{x_2(n)\} = x_2^2(n)$.

$\therefore H\{x_1(n) + x_2(n)\} = [x_1(n) + x_2(n)]^2$

$= x_1^2(n) + x_2^2(n) + 2x_1(n)x_2(n)$ —①

$H\{x_1(n)\} + H\{x_2(n)\} = x_1^2(n) + x_2^2(n)$ —②

$H\{x_1(n)+x_2(n)\} \neq H\{x_1(n)\} + H\{x_2(n)\}$

Additivity is NOT satisfied.

Homogenity: ~~NOT~~ satisfied ✓

Conclusion: $y(n) = x^2(n)$. does not obey Principle of superposition

$\therefore y(n) = x^2(n)$ is Nonlinear. ②

② $y(n) = D\{x(n)\} = e^{x(n)}$

Sol^n: →

$$x(n) \longrightarrow \boxed{D\{\cdot\}} \longrightarrow y(n) = D\{x(n)\} = e^{x(n)}$$

$$x_1(n) \longrightarrow y_1(n) = e^{x_1(n)}$$

$$x_2(n) \longrightarrow y_2(n) = e^{x_2(n)}$$

$$\therefore D\{x_1(n) + x_2(n)\} = e^{x_1(n) + x_2(n)}.$$

$$D\{x_1(n) + x_2(n)\} = e^{x_1(n)} \cdot e^{x_2(n)} \quad \text{— ①}$$

$$\therefore D\{x_1(n)\} + D\{x_2(n)\} = y_1(n) + y_2(n)$$

$$= e^{x_1(n)} + e^{x_2(n)} \quad \text{— ②}$$

$$D\{x_1(n) + x_2(n)\} \overset{?}{\neq} D\{x_1(n)\} + D\{x_2(n)\}$$

$\therefore$ Additivity is NOT satisfied

$\therefore D\{x(n)\} = e^{x(n)}$ ③ is <u>Nonlinear</u>.

5) Consider the system $y(n) = H\{x(n)\}$
$y(n) = Im\{x(n)\}$
where $x(n)$ is a complex signal.
Comment on linearity of system $H\{\cdot\}$?

Soln → ∵ $x(n)$ is given to be a complex signal,

let $x(n) = x_R(n) + j\, x_I(n)$

$\therefore y(n) = H\{x(n)\} = H\{x_R(n) + j\, x_I(n)\}$
$= Im\{x(n)\}$
$y(n) = x_I(n) = H\{x(n)\}$

$\therefore H\{x_1(n)\} = x_{1I}(n)$
$H\{x_2(n)\} = x_{2I}(n)$

$\therefore H\{x_1(n) + x_2(n)\} = H\{x_{1R}(n) + j x_{1I}(n) + x_{2R}(n) + j x_{2I}(n)\}$
$= H\{x_{1R} + x_{2R} + j[x_{1I} + x_{2I}]\}$
$= x_{1I}(n) + x_{2I}(n)$

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Homogenity $\Rightarrow$ $y(n) = e^{x(n)}$

↳ Not satisfied

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4) $y(n) = H\{x(n)\} = t \cdot x(n)$

$x_1(n) \rightarrow$ $y_1(n) = H\{x_1(n)\} = t\, x_1(n)$

$x(n) \rightarrow \boxed{H\{\cdot\}} \rightarrow y(n) = H\{x(n)\}$

$y(n) = t \cdot x(n)$

$x_2(n) \rightarrow$ $y_2(n) = t \cdot x_2(n)$.

$\therefore$ Additivity $\Rightarrow$ satisfied ✓

Homogenity $\Rightarrow$ satisfied ✓

$\therefore$ $y(n) = H\{x(n)\} = t \cdot x(n)$ obey Principle of Superposition

$\therefore$ $y(n) = H\{x(n)\} = t \cdot x(n)$ is a linear system

$\therefore H\{x_1(n) + x_2(n)\} = H\{x_1(n)\} + H\{x_2(n)\}$

$\Rightarrow$ $H\{\cdot\}$ satisfy additivity property.

$H\{a \cdot x(n)\} = a\, H\{x(n)\}$.

case II
if $a = 2$

$H\{2x(n)\} = 2\, H\{x(n)\}$.

$\downarrow$

$H\{2(x_R + j x_I)\} = H\{2x_R + j\, 2x_I\}$

$= 2x_I$

$\because H\{x(n)\} = x_I$

$\therefore 2 \cdot H\{x(n)\} = 2x_I$

$H\{2 \cdot x(n)\} = 2 \cdot H\{x(n)\}$

$\Rightarrow$ Homogenity is satisfied

$\therefore$ the system $H\{\cdot\}$ is linear.

(b)

Case II if $(a) = 2j$

$$H\{a \cdot x(t)\} = 2x_I(t)$$

$$a \cdot H\{x(t)\} = 2j \, x_I(t)$$

$\therefore H\{a \cdot x(t)\} \neq a \, H\{x(t)\}$ if a is complex.

$\Rightarrow$ Homogeneity property is NOT satisfied.

$\Rightarrow$ The system $y(t) = Im\{x(t)\}$ is nonlinear.

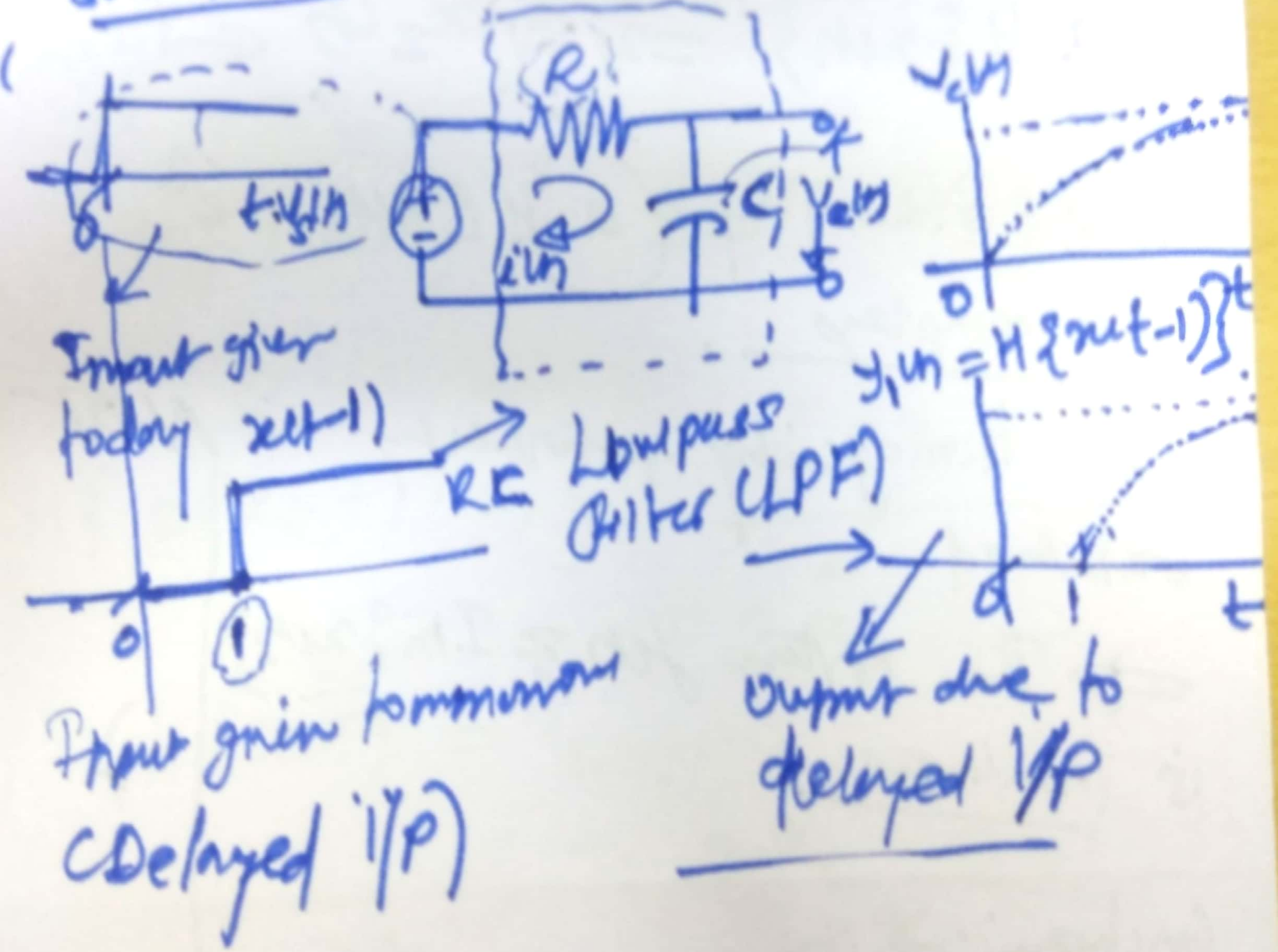
Conclusion: $\rightarrow$

(1) For a system to be linear it should satisfy both additivity & homogeneity.

(2) Signals as well as scaling constants (such as, 'a') are allowed to be complex.

[2] Time Invariance :-

Goal: In Engineering Design, we want or desire consistency in design and system behaviour.



Q.1) What is time-invariance?

↳ Output of the system ~~gives~~ i/p consistent output irrespective of when i/p is applied

10-15 ~~course~~ course

LTI ✓

Q.2) Why "time-invariance" is desirable in Engineering Design

⇒ If we invest efforts, cost, time to design an electronic ckt, say RC LPF, then if it gives output response that varying w.r.t. time of application of input source voltage or input excitation source then as designer, it becomes difficult to predict system's behaviour in future and thus, its generalization is also in question.?

Condition for Time Invariance:

Case I) CTS

$x(t) \rightarrow \boxed{H\{\cdot\}} \rightarrow y(t) = H\{x(t)\}$

Input given today

o/p obtained today

↳ $x(t-t_0) \rightarrow y_1(t) = H\{x(t-t_0)\}$

o/p due to delayed i/p

∴ The system ~~is~~ $H\{\cdot\}$ said to be time-invariant if:

$$y(t) = y_1(t)$$

$$\Rightarrow H\{x(t)\} = H\{x(t-t_0)\}$$

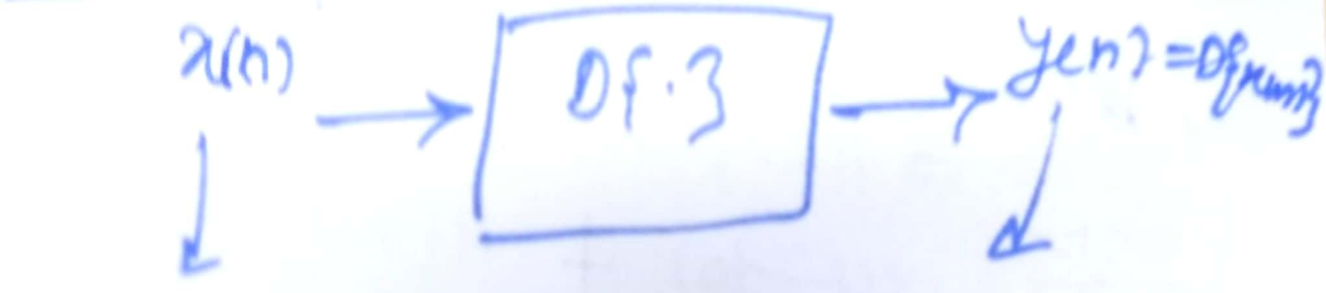
o/p due to i/p given today

o/p due to delayed input.

else if $H\{x(t)\} \neq H\{x(t-t_0)\}$

~~then then~~ then the system $H\{\cdot\}$ is NOT time-invariant or it is time-varying

Conditions for Time Invariance (TI) DTS



$x(n-n_0)$ $\quad$ ~~$y(n-n_0)$~~ $\quad y_1(n) = D\{x(n-n_0)\}$

$\therefore$ $D\{\cdot\}$ is TI if $y(n) = y_1(n)$

$$\Rightarrow \underbrace{D\{x(n)\}}_{\text{O/P for i/p given today}} = \underbrace{D\{x(n-n_0)\}}_{\text{O/P for input given by a delay of}}$$

$\therefore$ The system $D\{\cdot\}$ is Time-Variant (TV)

$$D\{x(n)\} \neq D\{x(n-n_0)\}.$$

Procedure to check TI vs. TV ?

Step ~~I~~ IV) $y(t-t_0) = H\{x(t-t_0)\} \Rightarrow$

$\Rightarrow H\{\cdot\}$ is TI

~~other~~ $y(t-t_0) \neq H\{x(t-t_0)\}$

$\Rightarrow H\{\cdot\}$ is TV

Step I Find the input-output relation for the given system

Step II Find the output of the system for the i/p applied today, i.e.

$y(t) = H\{x(t)\}$

Step III) Find the output of the system for the i/p applied tomorrow.

$y_1(t) = H\{x(t-t_0)\}$ delay i/p w.r.t. $x(t)$.

Step IV) Find the system O/P due to shift/delay in time i.e. $t \to t-t_0$ in $y(t)$

$y(t) \Rightarrow y(t-t_0) = y(t)\Big|_{t=t_0}$

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