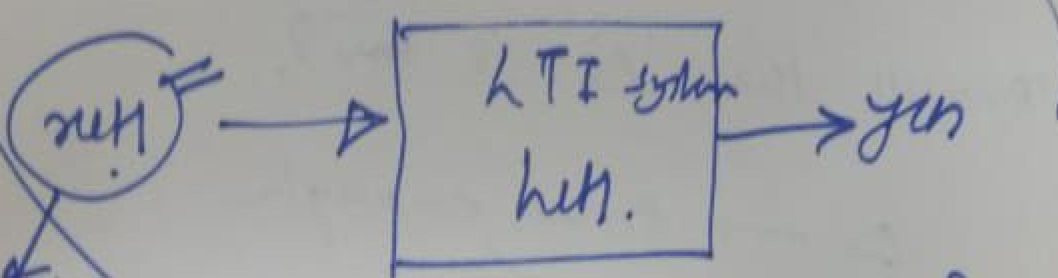
Singularity Functions [Contd.]

$\boxed{\text{Lecture 22}}$

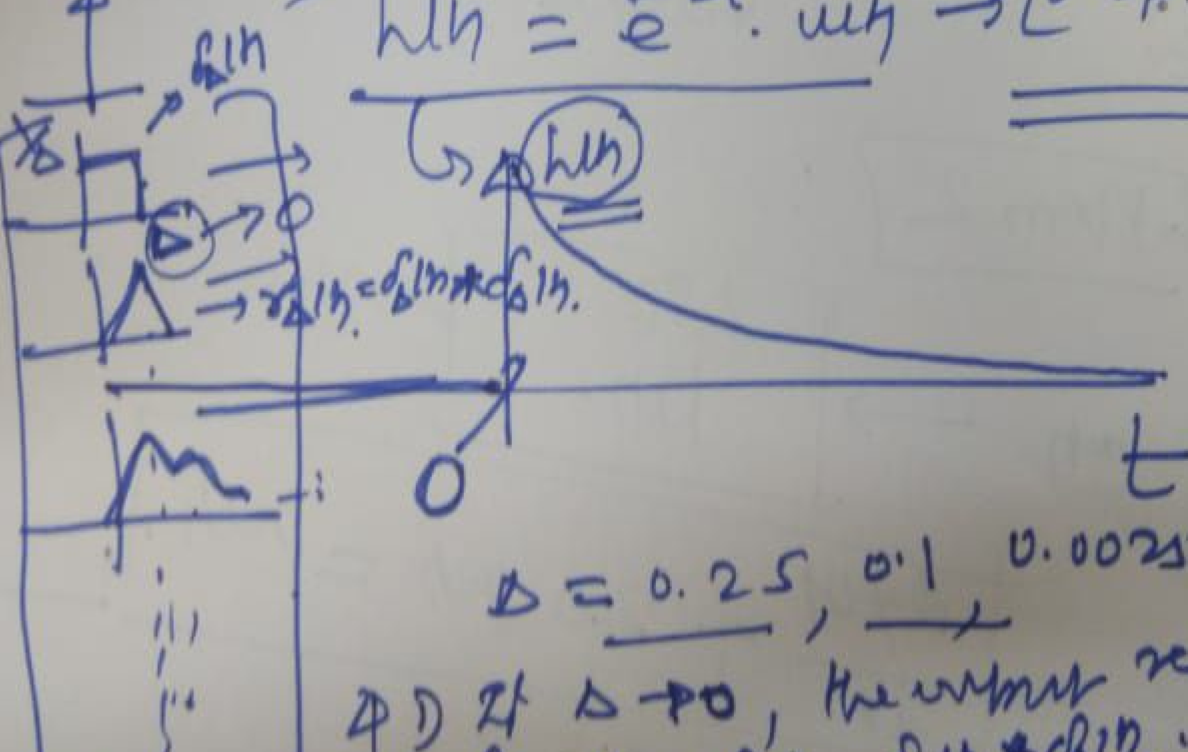
Problem $\boxed{2.16}$ consider the LTI system described by the first-order differential equation

$$\boxed{\frac{dy(t)}{dt} + 2y(t) = x(t)} \rightarrow \text{I/O relation (implicit).}$$

together with the condition of initial rest.



$$h(t) = e^{-2t}\cdot u(t) \rightarrow [\text{chapter 4}]$$



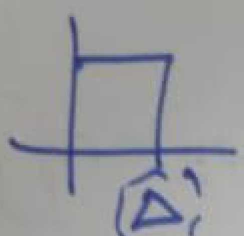
$\Delta = 0.25,\ 0.1,\ 0.0025$

4) If $\Delta \rightarrow 0$, the output response for $x(t) = \delta_\Delta(t),\ \delta_\Delta(t) * \delta_\Delta(t), \ldots$ tends ideal impulse response, $h(t)$

② All dissimilar looking signals such as $\delta_\Delta(t)$, $\delta_\Delta(t) * \delta_\Delta(t)$, $r_\Delta(t) * \delta_\Delta(t)$, etc. . . will behave like impulse

$\therefore$ In Signals and Systems, we refer impulse excitation as "impulse-like excitation".

How small the 'Δ' is ?



$\Delta \rightarrow$ small enough

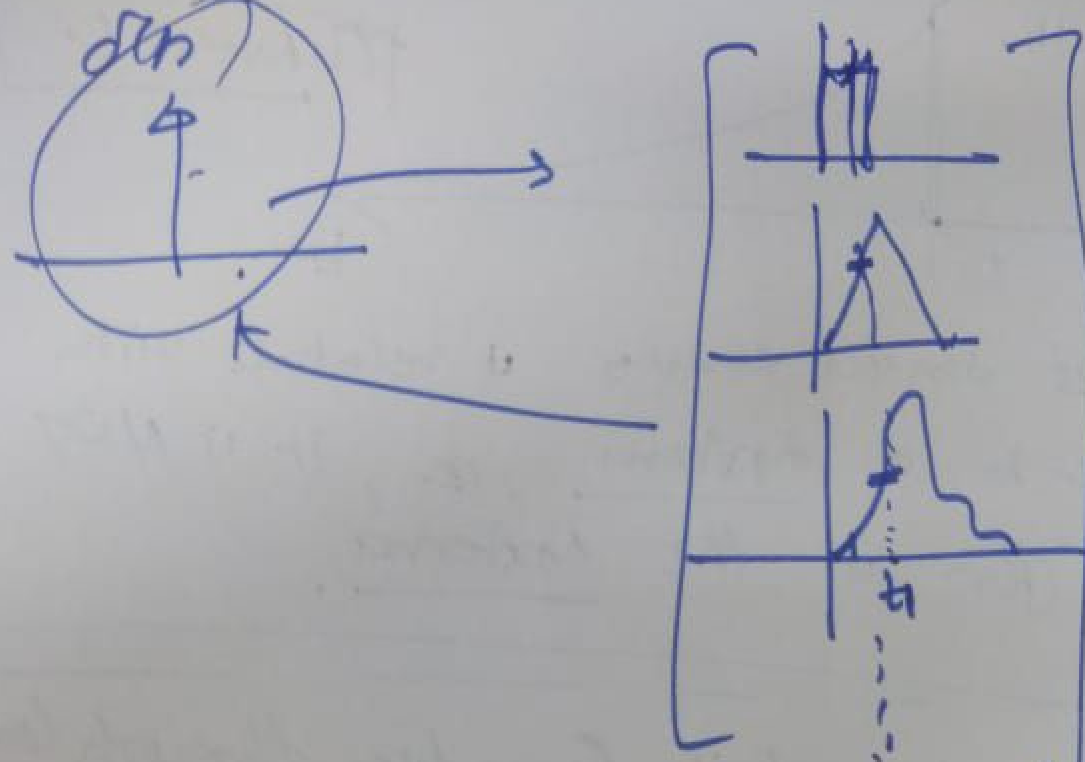
relative

[E] $\boxed{\text{Problem 2}}$:

$x(t) \longrightarrow \boxed{\begin{matrix}\text{LTI} \\ h(t)\end{matrix}} \longrightarrow y(t)$

$$\boxed{\frac{dy(t)}{dt} + 20\,y(t) = x(t)}$$

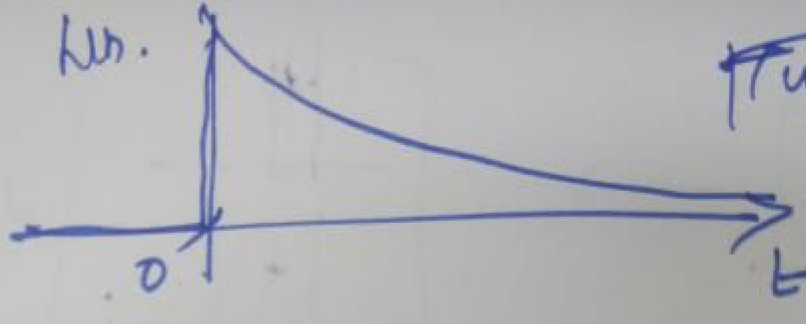
$$h(t) = e^{-20t} \cdot u(t).$$



⇒ We can't define impulse function the way we define a function in calculus i.e. what a function is at different va[lues] of independent variable rather we can de[fine] impulse as what it does under conv[olution]

⇒ Behaviour of impulse function under "convolution".

Tutorial 06

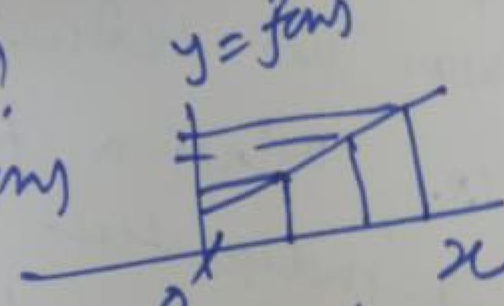


'Δ' is small enough is relative with respect to a system, i.e., it is NOT fixed for all the systems.

Defining the Unit Impulse through Convolution

To Define Impulse ??

In Calculus → $y = f(x)$



We define function w.r.t. each and every value of independent variable, function.

[1] Let $\boxed{x(t) = 1 \;\forall\; t}$

Using shifting property of impulse function

$$x(t) = x(t) * \delta(t)$$

Commutative property

$$\Rightarrow x(t) = \underbrace{\delta(t) * x(t)}$$

$$x(t) = \int_{-\infty}^{+\infty} \delta(\tau) \cdot x(t-\tau)\, d\tau$$

$$\boxed{\therefore 1 = \int_{-\infty}^{+\infty} \delta(\tau)\, d\tau}$$ = Area under impulse function unity.

22 classes.

Chapter I

Homework → { Above Sampling property of impulse
P.P. 131 – P32

Other singularity functions

1) Differentiator system:→

$$x(t) \longrightarrow \boxed{\text{Differentiator } \frac{d}{dt}(\cdot)} \longrightarrow y(t).$$

$$y(t) = \frac{dx(t)}{dt} = O/P.$$

∴ Impulse response, $u_1(t)$ → symbolic

$$u_1(t) = \frac{d}{dt}[\delta(t)]$$

2nd order differentiator system

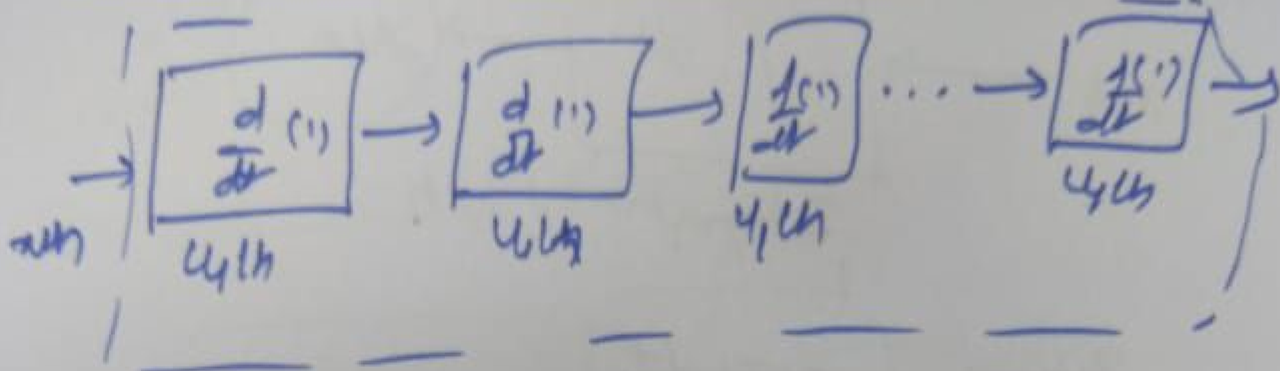
$$x(t) \longrightarrow \underset{u_1(t)}{\boxed{\frac{d}{dt}(\cdot)}} \longrightarrow \underset{u_1(t)}{\boxed{\frac{d}{dt}(\cdot)}} \longrightarrow y(t) = \frac{d^2}{dt^2}[x(t)]$$

what are [illegible]

The impulse response $u_1(t)$, $u_1(t)$

∴ $\boxed{u_2(t) = ?? \; u_1(t) * u_1(t)}$

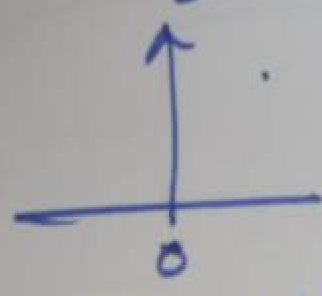
Cascade of 'K' differentiators



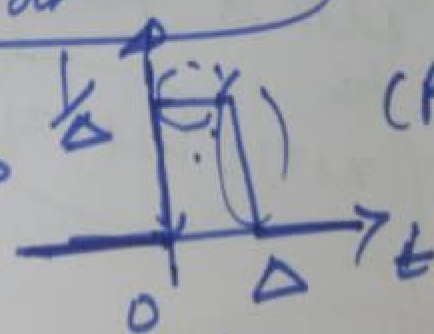
$u_k(t) = u_1(t) * u_1(t) * \cdots$ K times.

→ Overall impulse response → singularity function.

Q. What $u_1(t) = \frac{d}{dt}[\delta(t)]$ means?



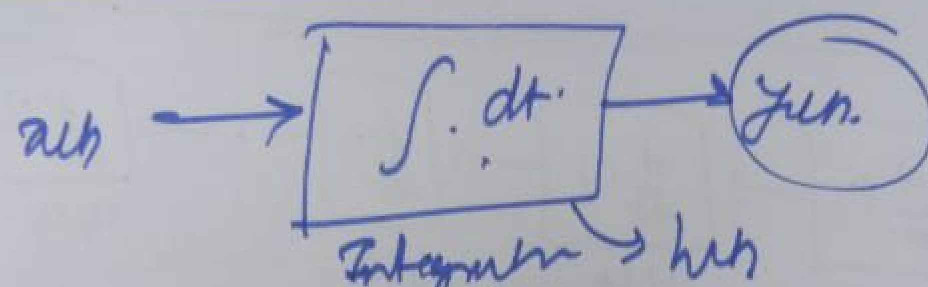
(Theoretical Impulse)



(Practical Impulse)

$\frac{d}{dt}(\cdot)$ → Unit doublet

* Integrator System



$$y(t) = \int_{\tau=-\infty}^{t} x(\tau)\, d\tau$$

Impulse response of integrator h(t) is

$h(t) = u(t)$ $\Rightarrow$ $h(t-\tau) = u(t-\tau)$.

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{\tau=-\infty}^{\tau=+\infty} x(\tau)\, u(t-\tau)\, d\tau$$

$$u(t-\tau) = \begin{cases} 1 & -\infty \le \tau < t \\ 0 & \text{else} \end{cases}$$

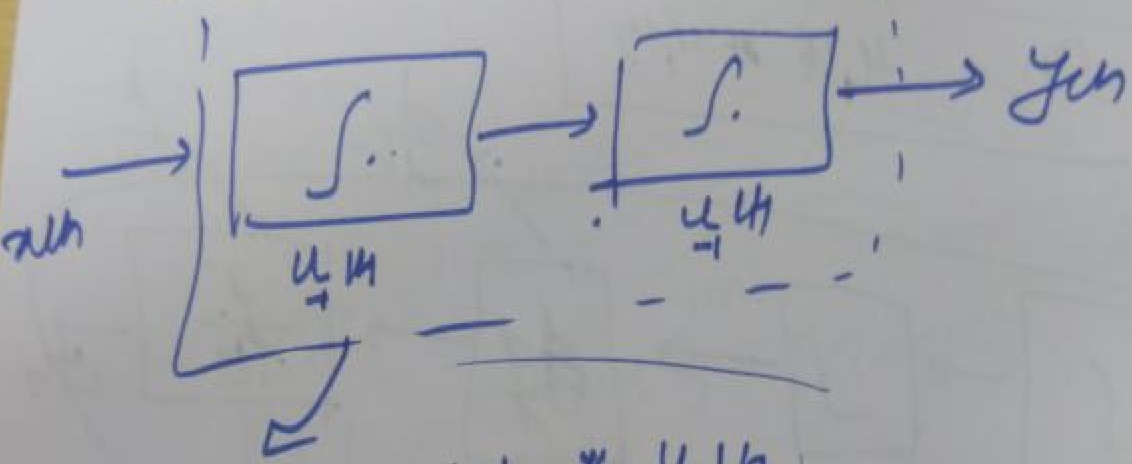
$$\boxed{y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau} \rightarrow \text{I/O relationship for integrator}$$

∴ For impulse response,

$$u_{-1}(t) = H\{\delta(t)\} = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

symbolic

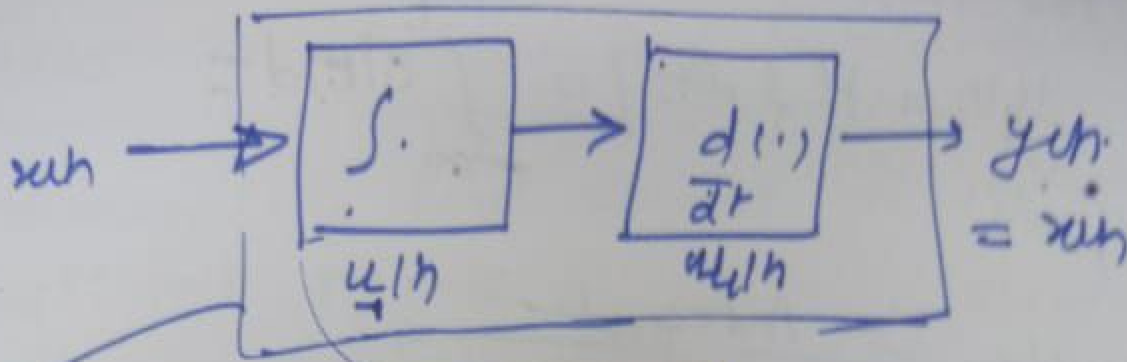
Cascade of Integrators



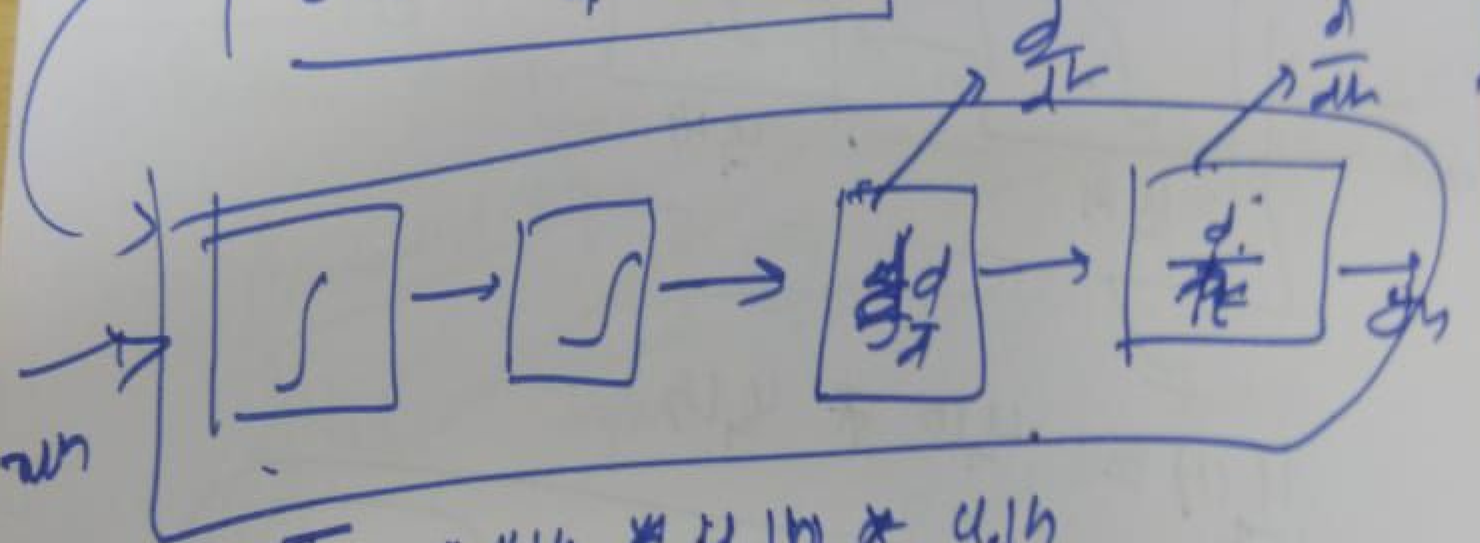
$$u_{-2}(t) = u_{-1}(t) * u_{-1}(t)$$

$$u_{-3}(t) = u_{-1}(t) * u_{-1}(t) * u_{-1}(t).$$

$\ldots u_{-k}(t) = u_{-1}(t) * u_{-1}(t) \ldots$ k times



$$\boxed{\delta(t) = u_{-1}(t) * u_1(t)}$$



$$u_{-1}(t) * u_{-1}(t) * u_1(t) * u_1(t)$$

$$= [u_{-1}(t) * u_1(t)] * [u_{-1}(t) * u_1(t)]$$

$$= \delta(t) * \delta(t)$$

$$= \delta(t)$$

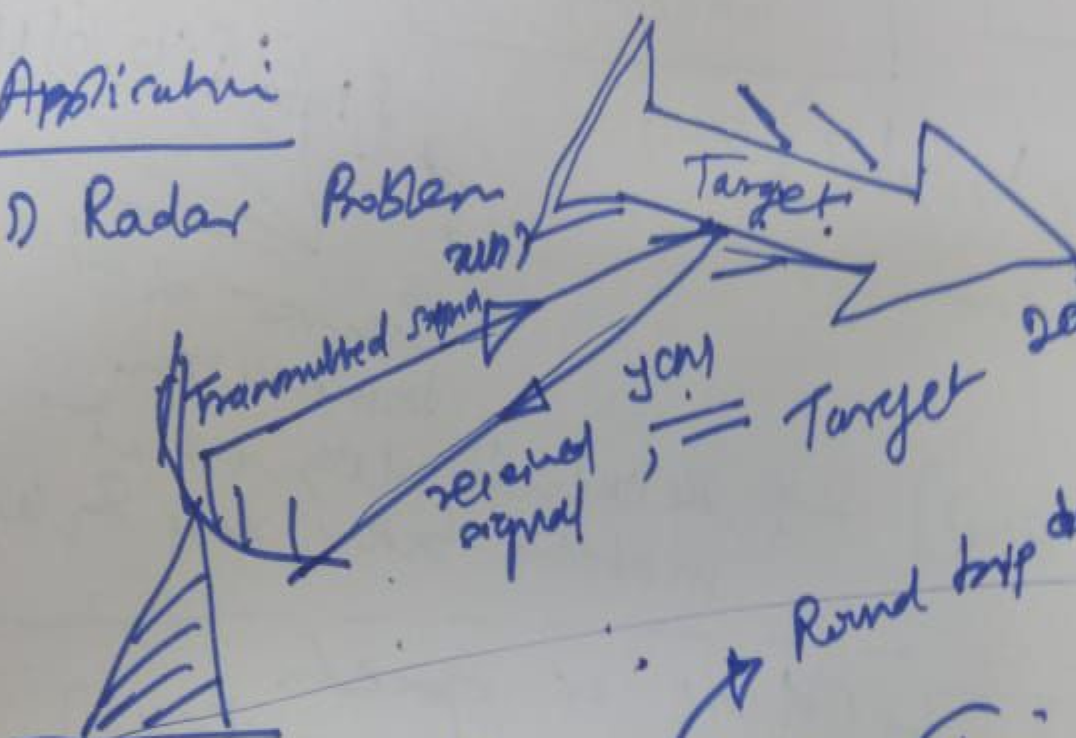
Cross-correlation and Autocorrelation:→

Motivation: To find degree of "match" between two signals

↙ closeness

Application

1) Radar Problem



y(n) = Target Detection Problem

Round trip delay

$$y(n) = \alpha\, x(n-D) + w(n)$$

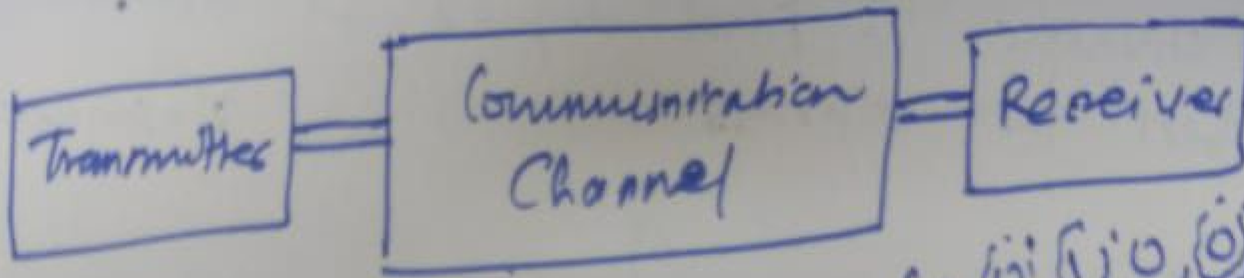
Received signal · Attenuation factor · Noise

Problem:→ To detect x(n) in y(n)?

→ If x(n) is present in y(n), then estimate round trip delay D,

$\{x(n),\ y(n)\}$

Application 2 | Digital communication



$\{1, 1, 0, 1, 0, 1, 1\} = x(n)$

bit-stream

$y(n) = \{1, 0, 1, 0, 0, 1, 0\}$

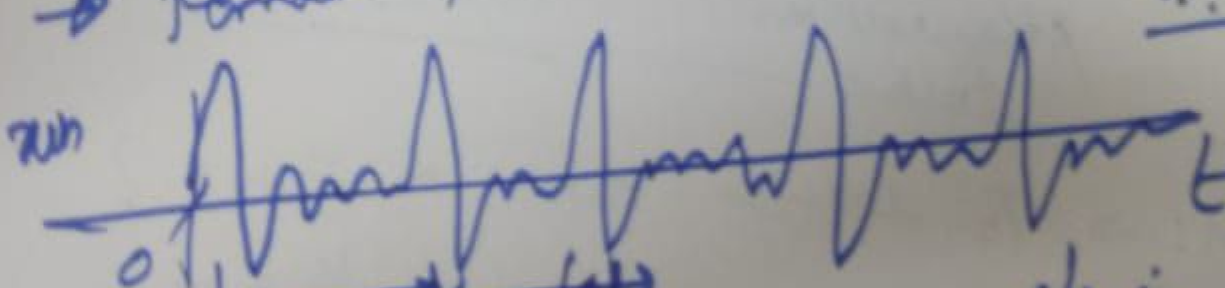
Errors in bit transmission

$\{x(n), y(n)\}$ → Relative degree of match betw. $x(n)$ & $y(n)$

Application 3 → Music or Speech [Human Voice].

→ Pitch frequency:

→ Female speaker have high pitch frequency



T_0 → Pitch Period ⟹ Using Autocorrelation function.