Periodicity

SAS Lecture 08

$x[n] = x[n+N]$ ← [Periodicity]

Let $x[n] = e^{jw_0 n}$

N = fundamental period.

$$e^{jw_0 n} = e^{jw_0(n+N)}$$

$$e^{jw_0 n} = e^{jw_0 n} \cdot e^{jw_0 N}$$

$$\therefore e^{jw_0 N} = 1$$

By Euler's relation,

$$\boxed{\cos(w_0 N) + j\sin(w_0 N) = 1}$$

$\therefore w_0 N = 2\pi K$; $K \notin R$

$K \in Z$.

$$w_0 = \left(\frac{2\pi}{N}\right) \cdot K$$

$w_0 = \frac{2\pi}{N}$ = fundamental frequency

$$\boxed{w_K = K w_0} = \text{harmonics}.$$

1

$$\phi_k(n) = e^{jk\omega_0 n}$$

$$\phi_k(n) = e^{j\omega_k \cdot n}$$

$$\phi_k(n) = \left\{ e^{j\omega_k n} \right\} = \left\{ e^{j\omega_1 n}, e^{j\omega_2 n}, e^{j\omega_0 n}, e^{j\omega_1 n}, e^{j\omega_2 n}, \dots \right\}$$

$k, n \in Z$,

set of harmonically related discrete time periodic complex exponential signals.

$$e^{j\omega t} \quad vs. \quad e^{j\omega n}$$

$$\phi_k(t) = \left\{ e^{j\omega_k t} \right\} \quad vs. \quad \phi_k(n) = \left\{ e^{j\omega_k n} \right\}$$

$k \rightarrow k+N$

$$\phi_k(n) \Rightarrow e^{j\omega_k n} \Rightarrow e^{jk \cdot \omega_0 n} = e^{jk\left(\frac{2\pi}{N}\right) \cdot n}$$

$k = k+N$

$$\therefore \phi_{k+N} = e^{j(k+N)\cdot\left(\frac{2\pi}{N}\right)\cdot n} = e^{jk\omega_0 n} = \{\phi_k(n)\}$$

$$\phi_{k+N}(n) = \phi_k(n)$$

Periodicity (with a period N) for $\phi_k(n)$

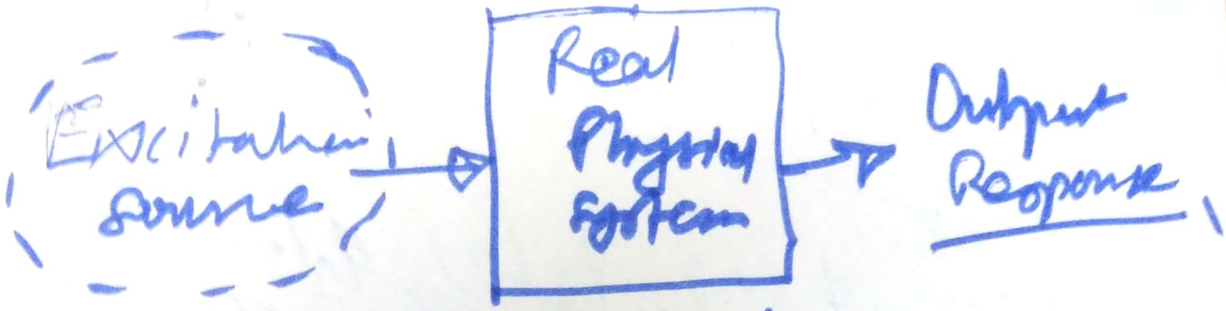
$$\{\underbrace{\phi_0(n), \phi_1(n), \phi_2(n), \phi_3(n), \ldots \phi_{N-1}(n)}_{\text{"unique signals"}}, \phi_N(n)\}$$

"unique signals"

$\boxed{\text{Study Table 1.1, pp. 26}}$

Oppenheim's book

* Standard Excitation Signals: →



[Musical Pillars, RC circuit]

Impulse

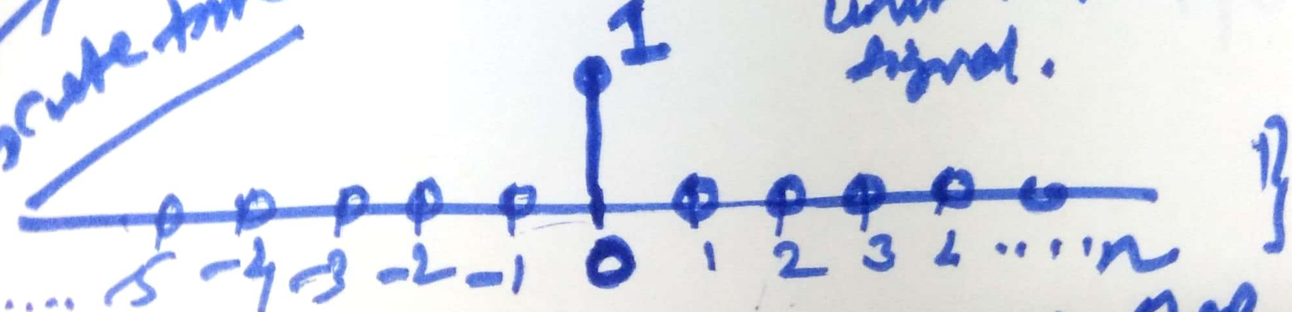
Unit step

Sinusoid.

1) Discrete-time unit impulse and unit step signals:

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0 & \text{elsewhere} \end{cases}$$

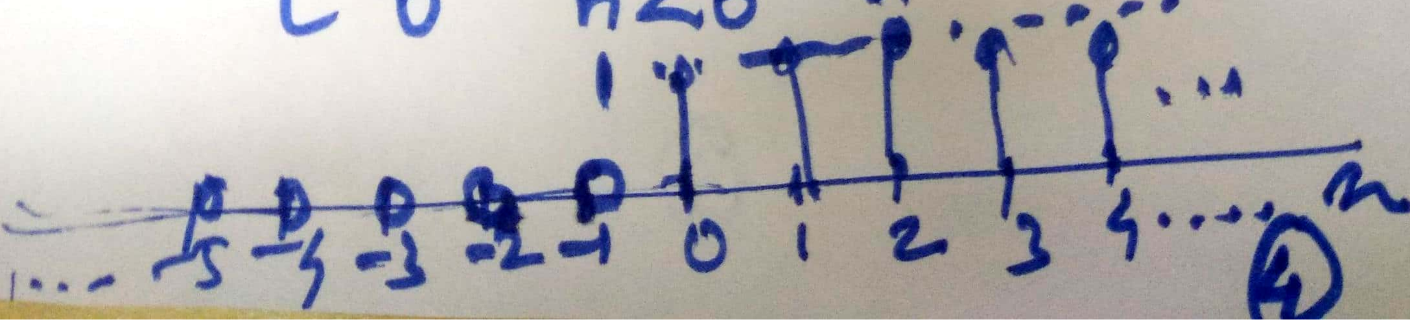
Discrete time

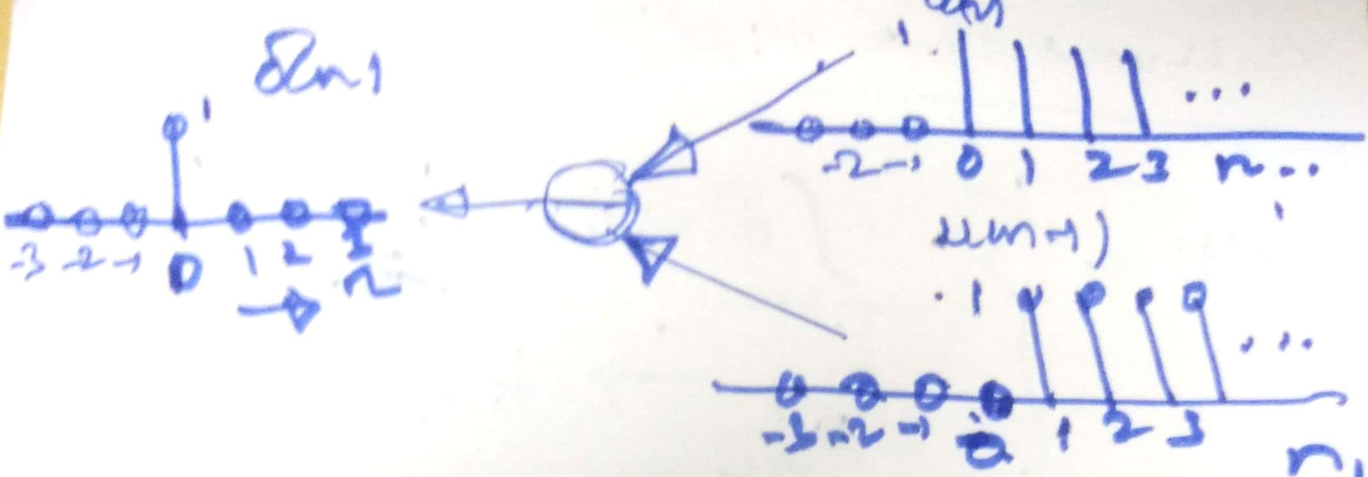


unit impulse signal.

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Unit step signal

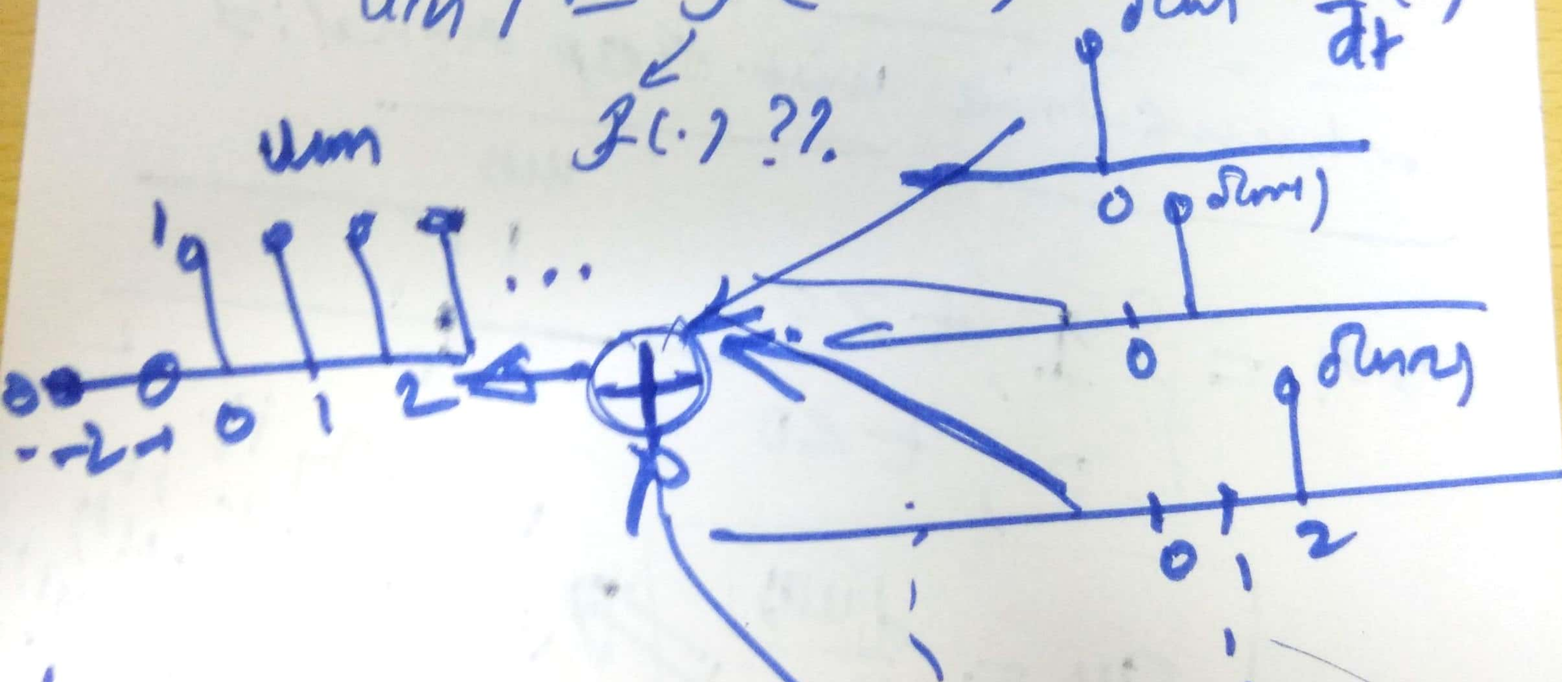




$$\therefore\ \boxed{\delta(n) = u(n) - u(n-1)} \rightarrow \text{Backward difference} \rightarrow \frac{d}{dt}(\cdot)$$

$$u(n) = \mathcal{F}(\delta(n))$$

$\mathcal{F}(\cdot)$??

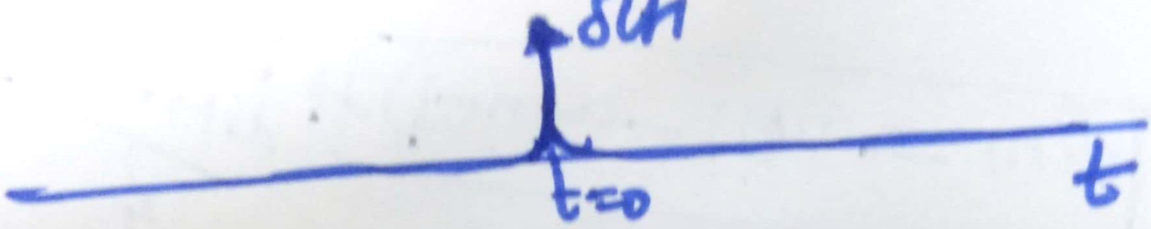


$$\therefore\ u(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \cdots + \delta(n-\infty)$$

$$\boxed{u(n) = \sum_{k=0}^{+\infty} \delta(n-k)}$$

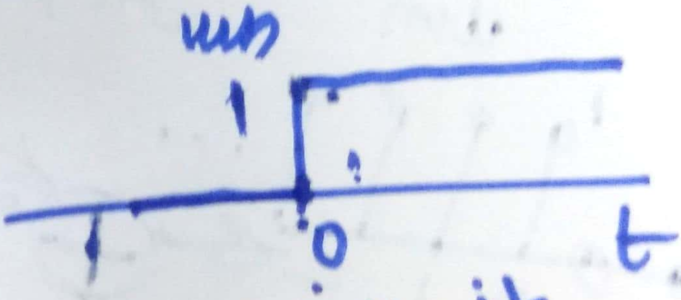
Continuous-time impulse signal:→

$$\delta(t) = \begin{cases} \int\limits_{\tau \in R} \delta(\tau)\,d\tau = 1, & t=0 \\ 0, & t \neq 0 \end{cases}$$



Continuous-time unit step signal:→

$$u(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}$$



$$\boxed{\delta(t) = \frac{d[u(t)]}{dt}} \Rightarrow$$

formally it is derivative to differentiate u(t) at t=0 since u(t) is discontinuous fun at t=0

$$\therefore\ u(t) = \int_{\tau=-\infty}^{t} \delta(\tau)\,d\tau$$

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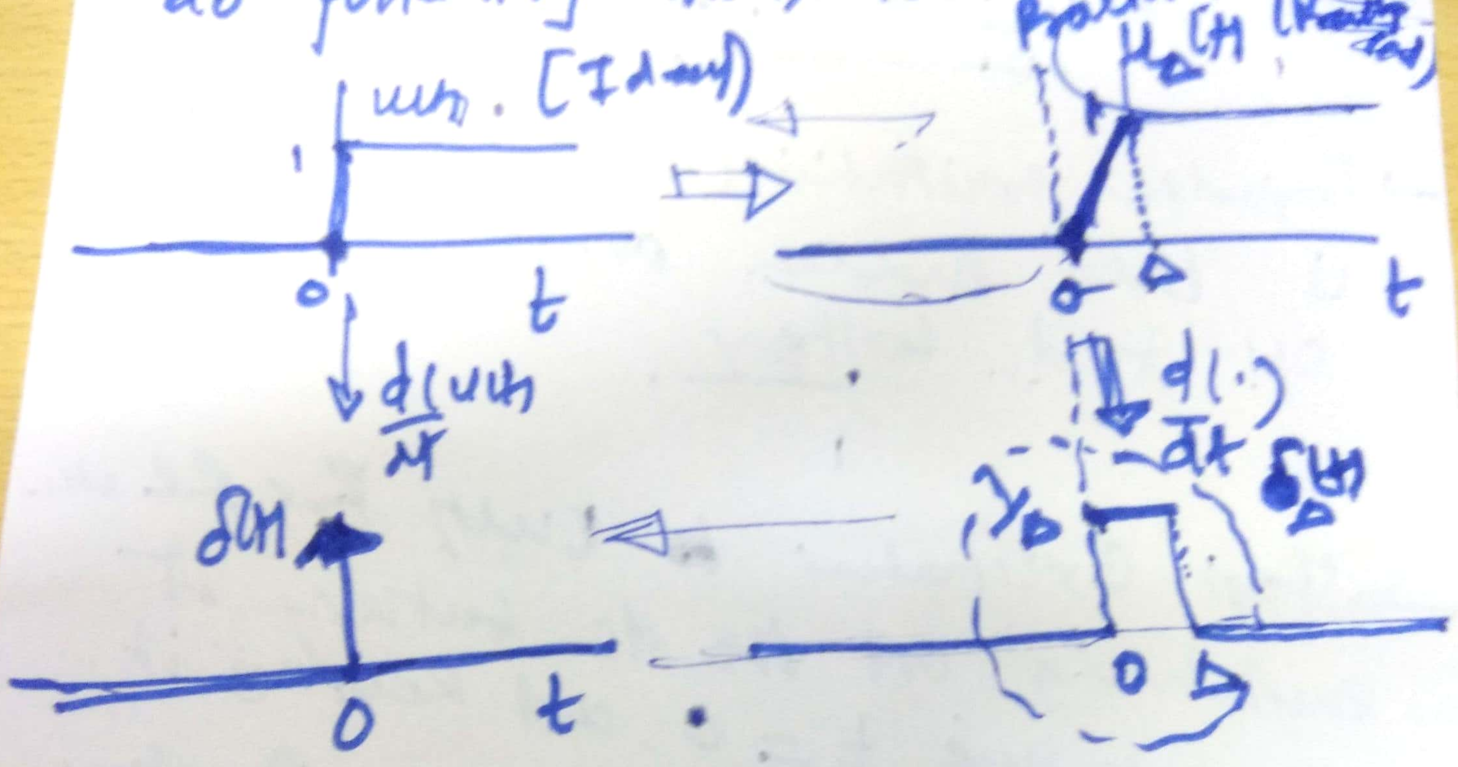
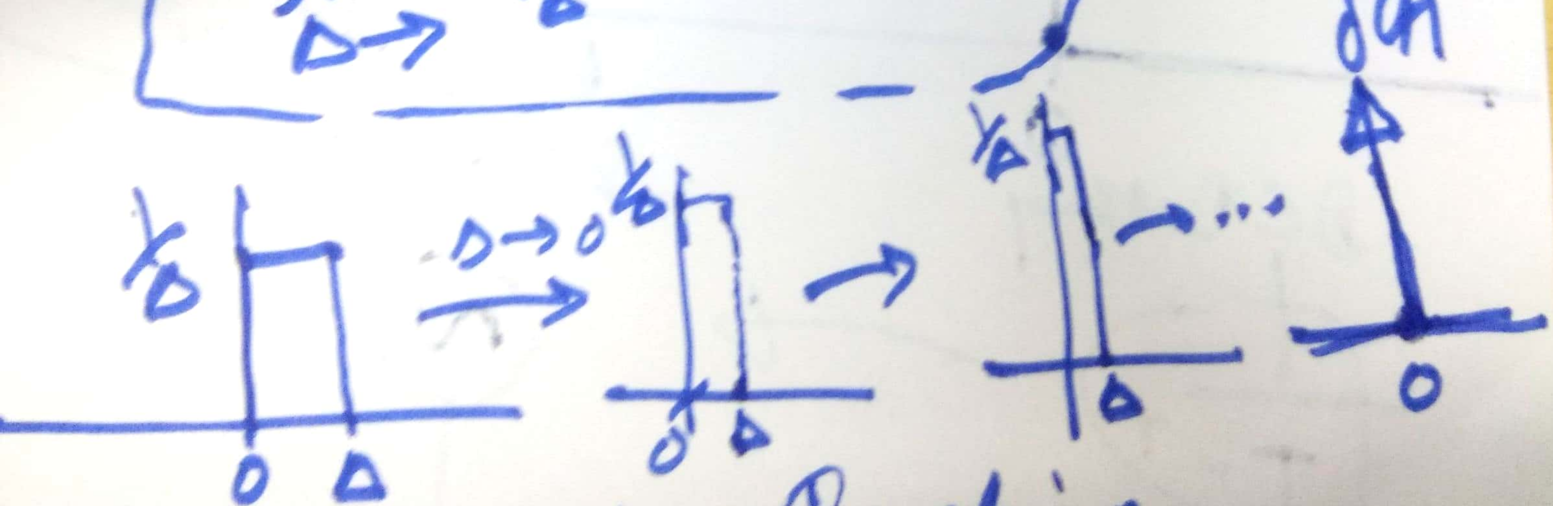
In order to alleviate this issue we do following modification

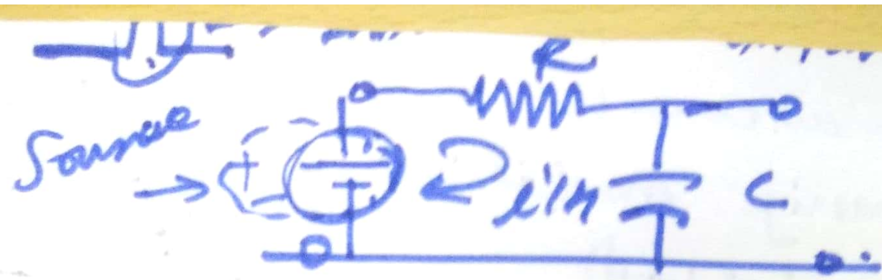


$$\lim_{\Delta \to 0} u_\Delta(t) \to u(t)$$

$$\lim_{\Delta \to 0} \delta_\Delta(t) \to \delta(t).$$



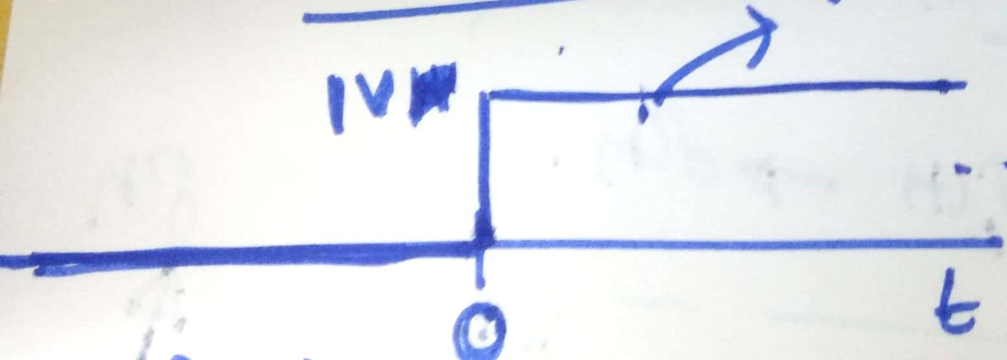
Dirac Delta Function



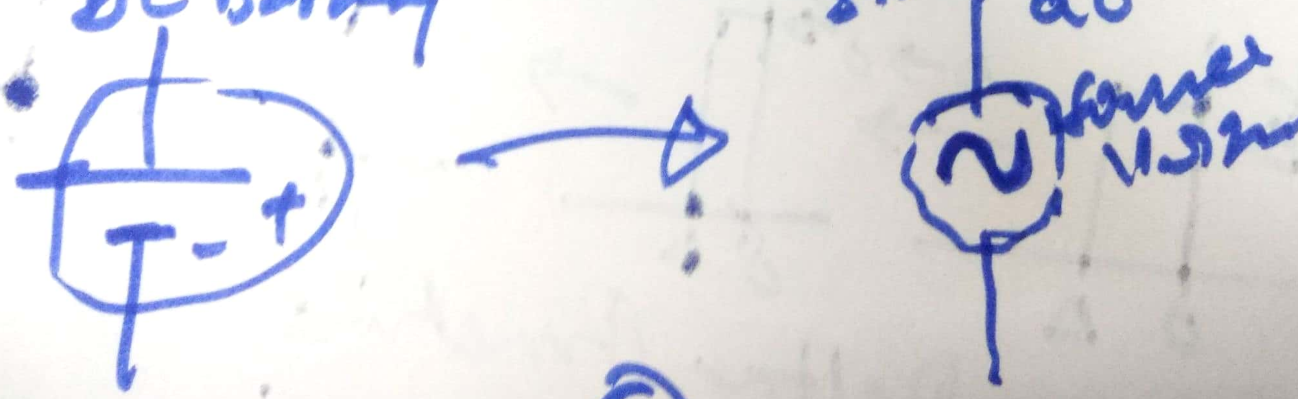
→ Impulse excitation ($\delta(t)$) for RC ckt is like sudden switching ON & OFF of dc battery.

→ Step Excitation to RC ckt for switching ON the dc battery at time $t = 0$ and keeping it ON forever

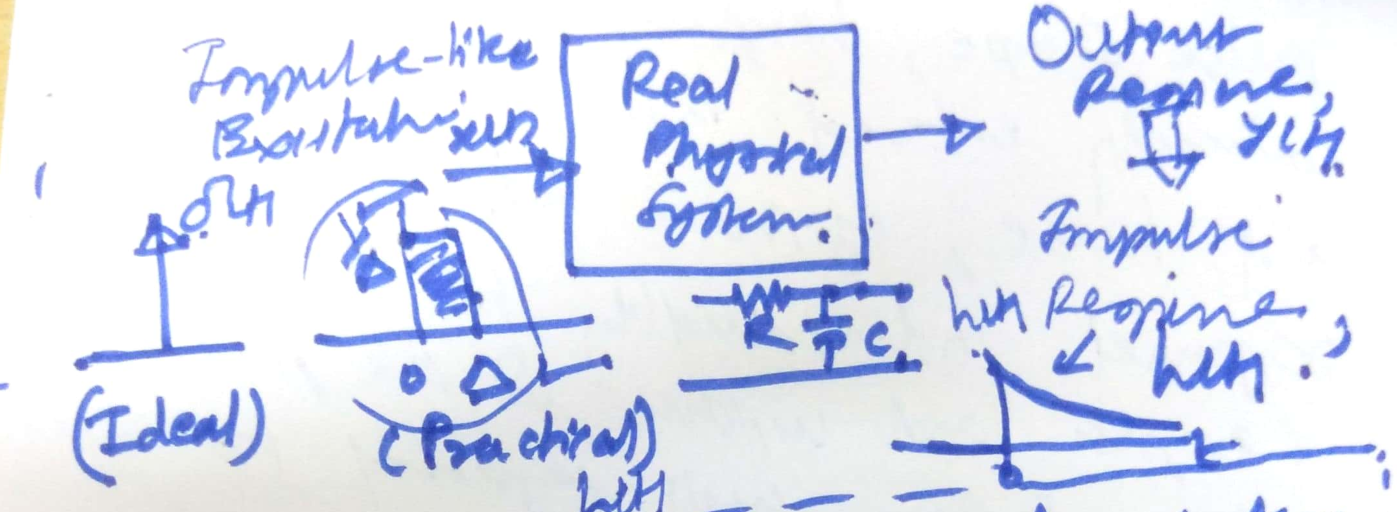
unit step Excitation.



Sinusoidal excitation



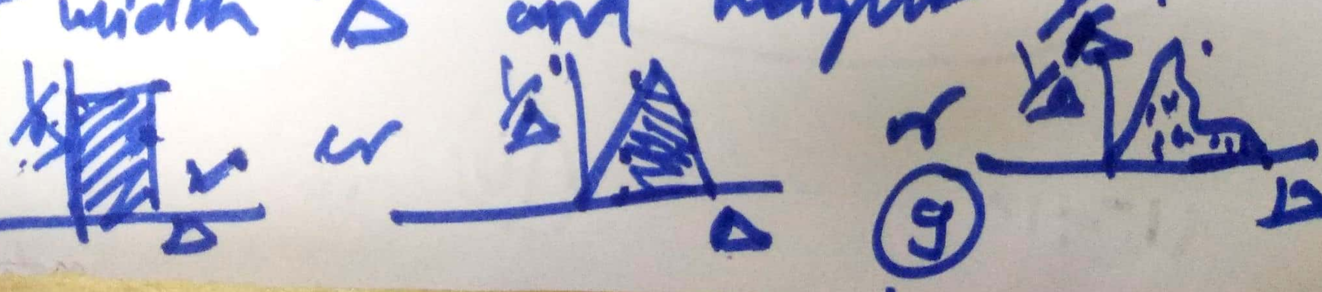
Impulse Excitation to a Real Physical System



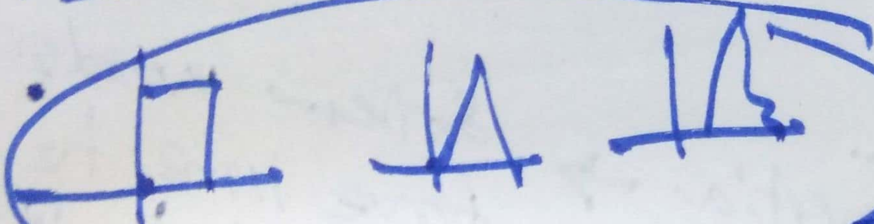
Impulse Response: Output of a system to impulse-like excitation

Real Physical System → Inertia → System needs some time to respond to input excitation.

Let system is excited by a pulse of arbitrary shape (rectangular, triangular, of width 'Δ' and height '1/Δ'.



⇒ Due to this elapsed time, the individual effects pertaining to pulse shape, height, and width have already entered into the systems and hence, system will NOT respond individually to shape, height and width of the pulse rather system will respond to Net Integrating Effect of pulse ⇒ its • its area.



$$\int_{\tau \in R} \delta(\tau)\, d\tau = 1$$

capture Net Integrating effect of excitation

Example of System's Inertia:→

Weight Control System: