3) Time Scaling :→ $\boxed{\text{Lecture 06}}$

$$x(t) \xrightarrow{t = a\cdot t} \underbrace{x(a\cdot t)}$$

↑ Original signal

↓ Time-scaled version of x(t).

Case I) $\boxed{a > 1}$, $a = 2$

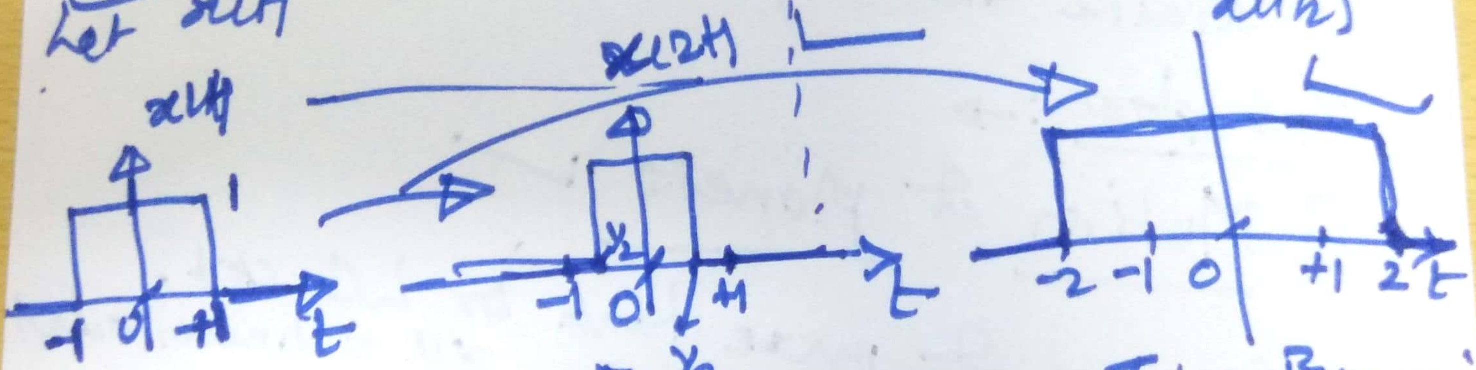
$\therefore x(at) = x(2t)$

$\boxed{\text{Example 1}}$

Let x(t)

Case II $a < 1$, $\boxed{a = 1/2}$

$\therefore x(at) = x(t/2)$



Time compression

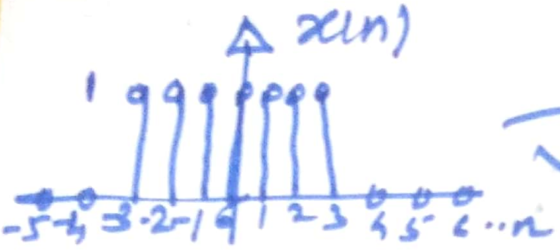
Time Expansion

$\boxed{\text{Example 2}}$ Playback of music recording at different speed than normal

→ x(2t) can be considered as playback of a song "twice" the speed than the original song

→ x(t/2) — " — " — "half" — " — " —

①



$x(2n)$? → $x(2t)$
$n \in \mathbb{Z}$ .. , $t \in \mathbb{R}$

$x(n/2)$? $x(t/2)$

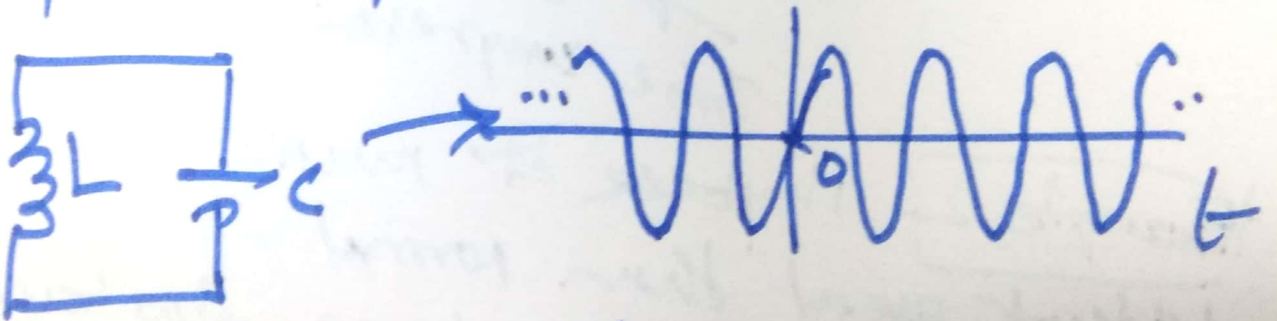
Chapter 7 → Sampling (?)

Periodic Signals: → Periodic signals occur naturally in various situations / scenarios in science and engineering..

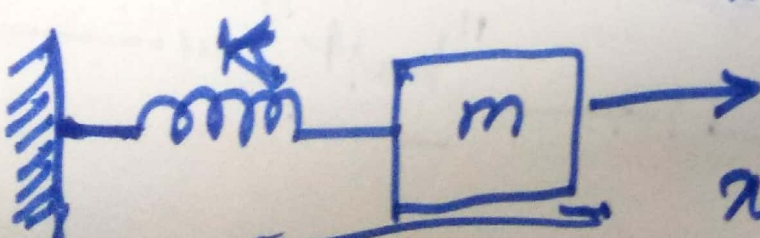
Examples: →

1. Motion of planets
2. Response of pure tank or LC ckt.

$x(t)$ = sinusoidal



3. Simple Harmonic Motion.

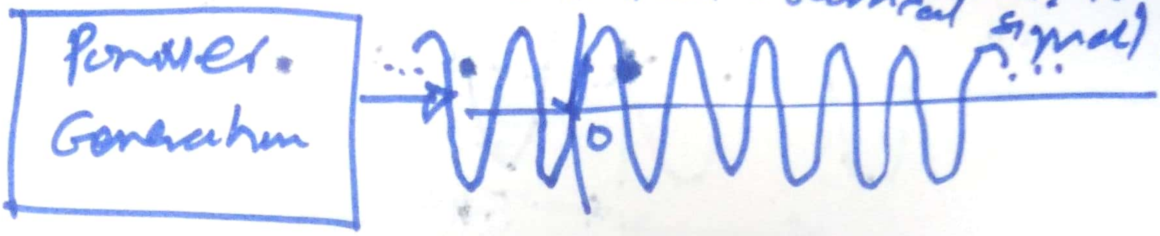


$x(t)$

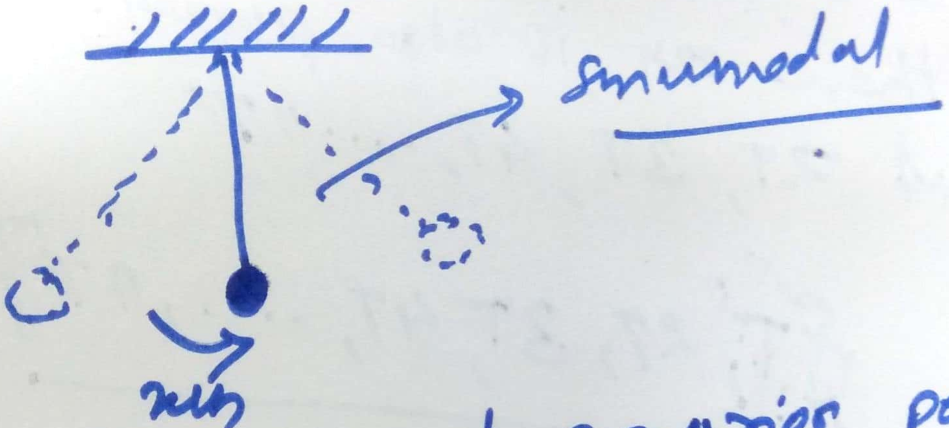
$$\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

$$x(t) = A\sin(\omega t + \phi).$$

(1) Electrical signal generated at power station.



(4) Simple Pendulum

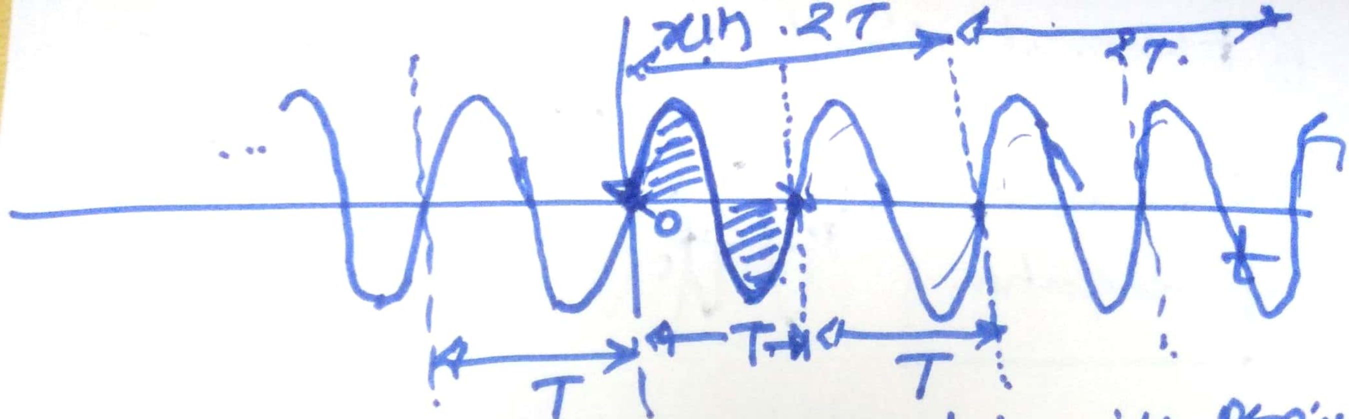


In nature, in several scenarios, periodic signals occur naturally.

⟹ Key motivation for studying periodic signals and their properties in our course on signals and systems.

Periodic signal [Defn]: — A signal x(t) is said to be periodic if it repeats after a certain interval time, say, T.

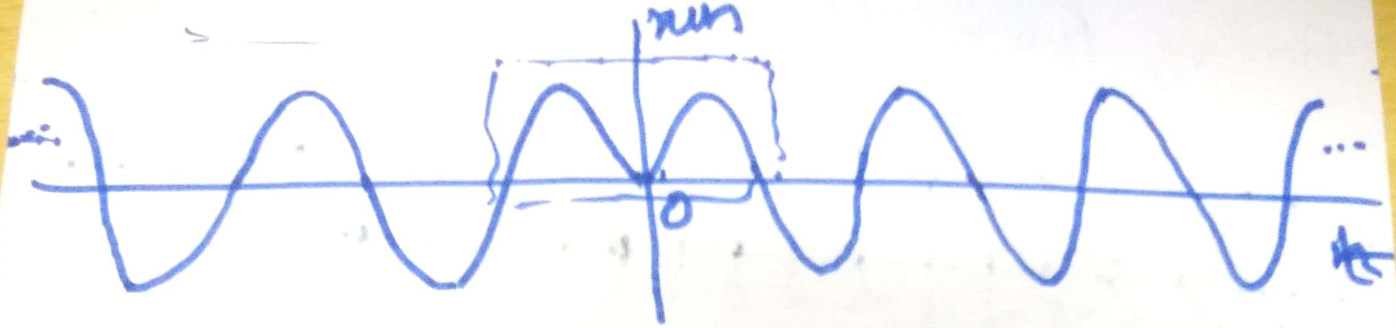
$$\therefore \boxed{x(t) = x(t+T)} \Rightarrow x(t) = \text{periodic} \Rightarrow \forall\, t \in \mathbb{R}$$



If a signal x(t) is periodic with period 'T', then x(t) is also periodic with period 2T, 3T, 4T, ... nT

$\{T, 2T, 3T, 4T, \ldots, nT\}$.

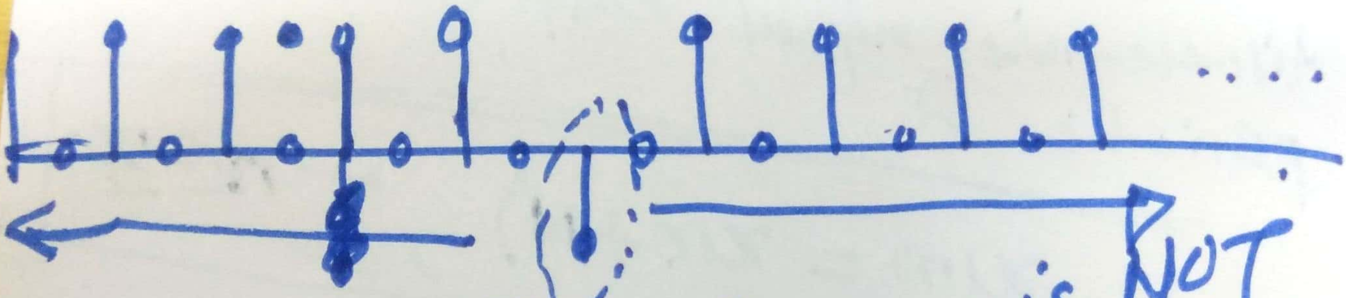
The least amongst this set of number is 'T' ⇒ Hence, 'T' is called as fundamental period of signal x(t).



Q: Whether x(t) is periodic ??

Ans: Not periodic $\because x(t) \neq x(t+T)\ \forall t$
or aperiodic

at $t = 0$



This feature at x(n) is NOT repeating at any other location

$\Rightarrow x(n) =$ aperiodic

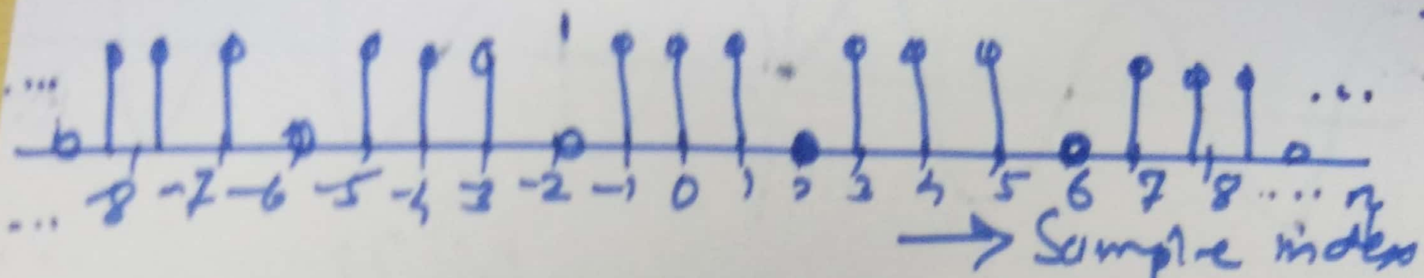
$$\boxed{\because x(n) \neq x(n+N)\ \forall n}$$

$$x(t) = x(t + n(T))\ ;\ n \in Z$$

$$x(n) = x(n + k(N))\ ;\ k \in Z$$

Similar, for a discrete-time signal $x(n)$

$x(n)$



Q. Whether $x(n)$ (shown above) is periodic?

Ans: Yes.

Fundamental Period = 04 = $\underline{\underline{N}}$

A discrete-time signal $x(n)$, is said to be periodic if

$$\boxed{x(n) = x(n+N), \ \forall n \in \mathbb{Z}}$$

Like in continuous-time signal ~~x(t)~~ x(t).

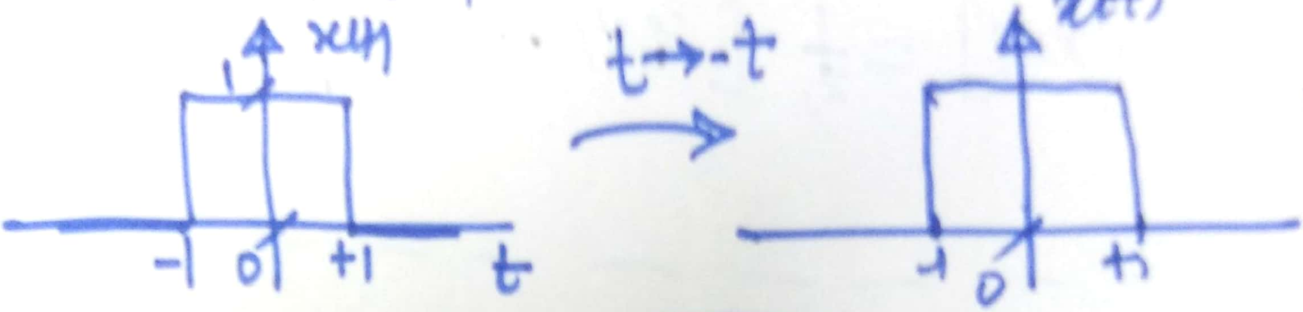
If a signal $x(n)$ is periodic for period N then it is also periodic for period 2N, 3N, ... nN

$$\{N, 2N, 3N, \ldots, nN\}.$$

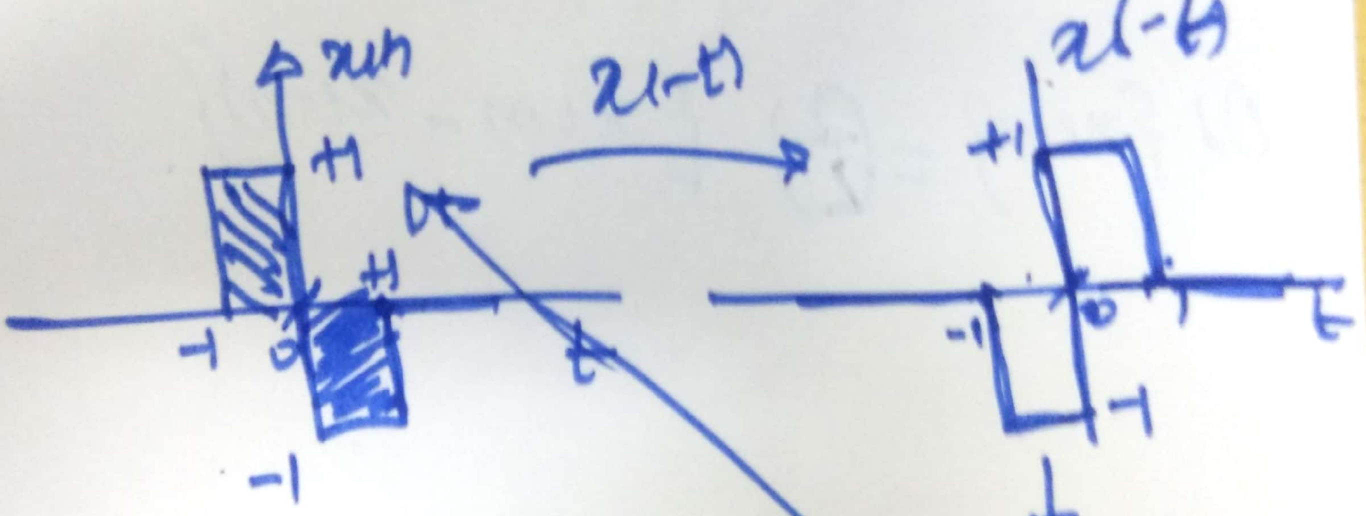
fundamental period of discrete-time periodic signal, $x(n)$.

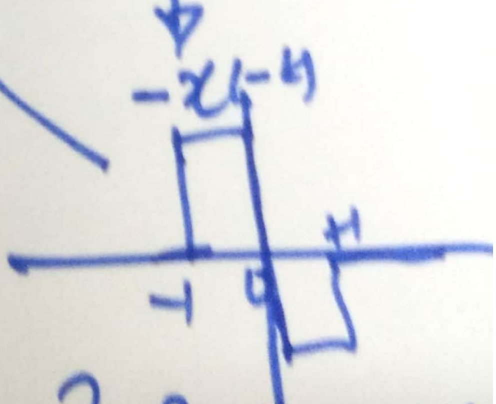
Even and Odd Signals:—

Even Signal: A signal is said to be 'even' signal if it is identical to its time-reversed version.



If $\therefore$ $\boxed{x(t) = x(-t)}$ $\Rightarrow$ $x(t)$ = even





$\Rightarrow$ $\boxed{x(t) = -x(-t)}$

$\Rightarrow$ $x(t)$ = Odd signal

$$\left.\begin{aligned} &\rightarrow x[n] = x[-n] \rightarrow \text{even} \\ & x[n] = -x[-n] \rightarrow \text{odd} \end{aligned}\right\} \text{Discrete-time signal}$$

Even Part of a Signal

$$Ev\{x(t)\} = \frac{1}{2}[x(t) + x(-t)]$$

$$Od\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$$

$$Ev\{x(n)\} = \frac{1}{2}\{x(n) + x(-n)\}$$

$$Od\{x(n)\} = \left(\frac{1}{2}\right)\cdot[x(n) - x(-n)].$$

* Exponential and Sinusoidal signals: →

→ These signals serves as basic building blocks to construct other signals and they occur frequently in several contexts in our course.

Case I

Continuous-time complex Exponential and sinusoidal signals: →

The continuous-time complex exponential signal is of the form.

$$\boxed{x(t) = C \cdot e^{a \cdot t}}$$

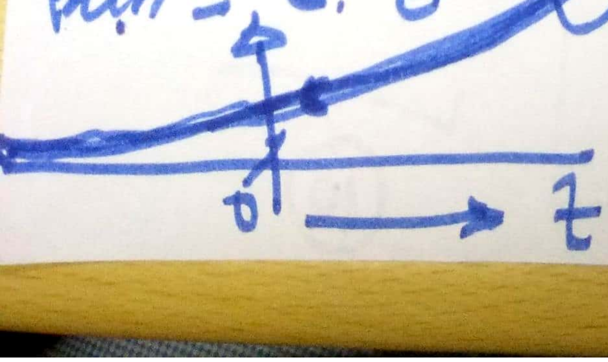
where 'C' and 'a' are in general complex nos.

Real Exponential signals ⇒ 'C' & 'a' are real.

$C > 0$

$a > 0$ $a = 2$ — population growth — chain reaction — exponential

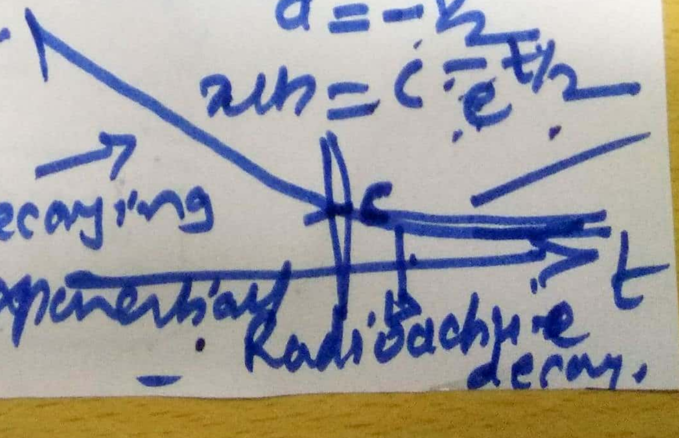
$x(t) = c \cdot e^{2t}$ → growing signal.



$a < 0$

$a = -1/2$

$x(t) = c \cdot e^{-t/2}$



Radioactive decay.

(9)

Periodic Complex Exponential and Sinusoidal Signal

$$x(t) = C \cdot e^{at}$$

Let $C = 1$

$$x(t) = e^{at}$$

Let 'a' be the purely imaginary,

i.e. $a = j\omega_0$

$$x(t) = e^{at} = e^{j\omega_0 t}$$

Goal: Investigate periodicity property of $x(t) = e^{j\omega_0 t}$.

If $x(t)$ is periodic,

$$x(t) = x(t + nT)$$

$$e^{j\omega_0 t} = e^{j\omega_0 (t+nT)}$$

$$e^{j\omega_0 t} = e^{j\omega_0 t} \cdot e^{j\omega_0 nT}$$

$$\therefore \boxed{e^{j\omega_0 nT} = 1}$$

$\boxed{\cos(\omega_0 n T) + j \sin(\omega_0 n T) = 1}$.

[~~Because of Euler's formula~~].

$\Rightarrow \omega_0 n T = 2\pi k; \; k \in \mathbb{Z}$.

$T = \left(\frac{2\pi}{\omega_0}\right) \cdot \left(k/n\right)$.

$\boxed{T = \frac{2\pi}{\omega_0}}$ or $\boxed{T = \frac{2\pi}{-\omega_0}}$

$\Rightarrow \boxed{\omega_0 = \frac{2\pi}{T}}$ or $\boxed{\omega_0 = \frac{-2\pi}{T}}$

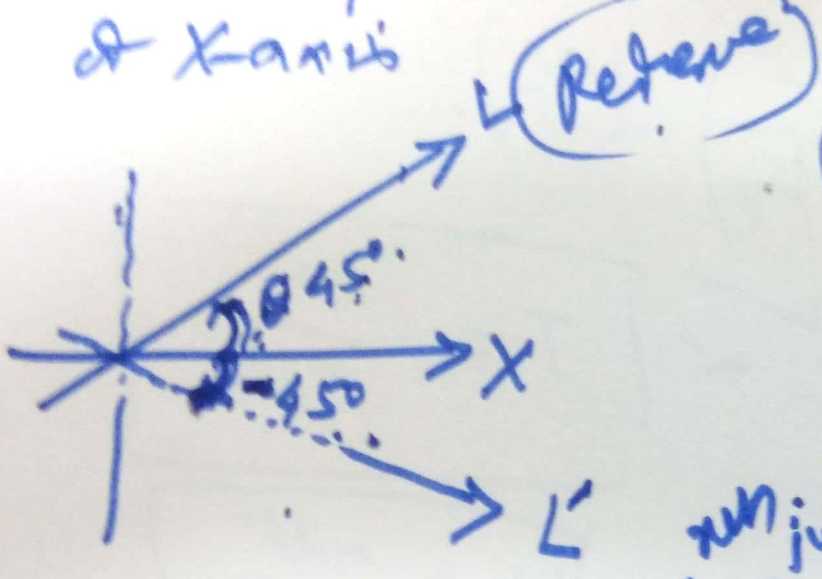
$\boxed{x(t) = e^{j\omega_0 t}}$ → has fundamental period $T = \frac{2\pi}{\omega_0}$ or $\frac{-2\pi}{\omega_0}$ as well and hence, it has fundamental frequency, $\omega_0 = \frac{2\pi}{T}$ or $\omega_0 = \frac{-2\pi}{T}$.

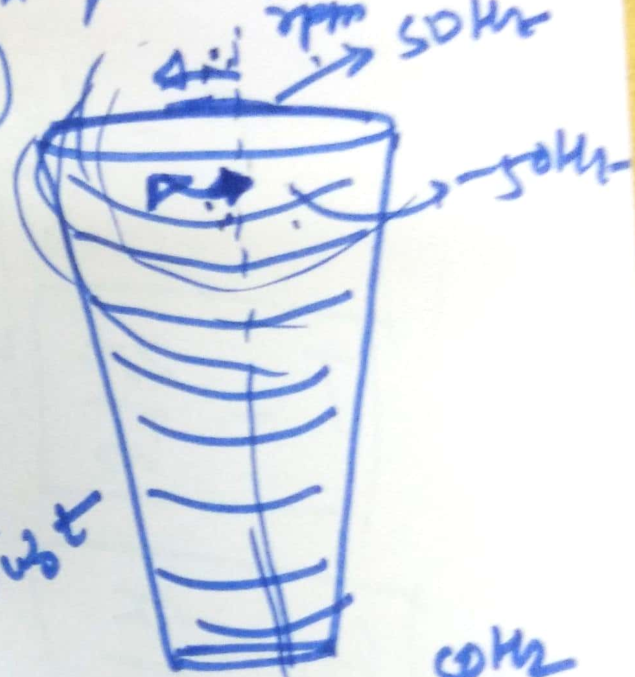
50Hz the ~~it has~~ -50 Hz frequency

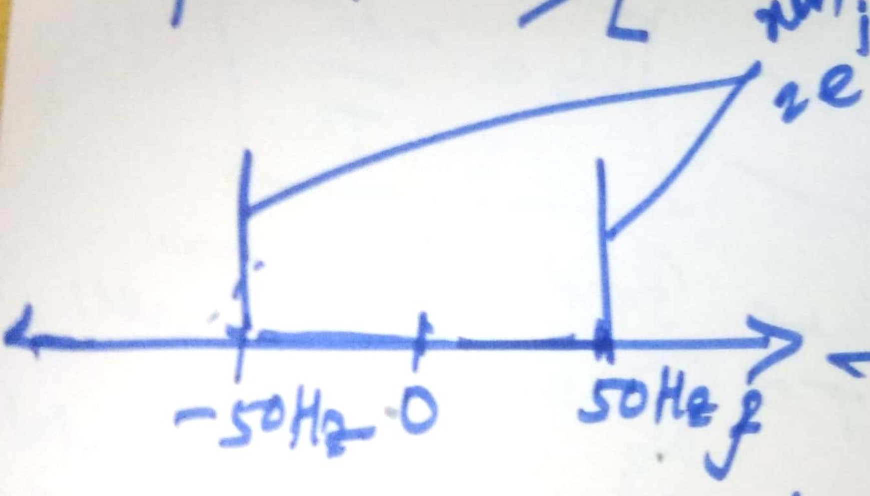
Concept of Negative Frequency :→

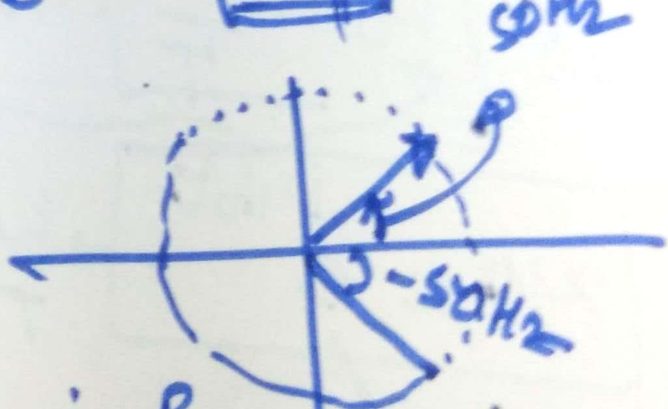
50Hz vs. -50Hz ??

Inclination of a line θ is nothing angle made by line L with positive direction of X-axis









Negative frequency

Reference
[Clockwise vs. anticlockwise

$e^{j\omega_0 t}$

Euler's relation

$$e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t).$$