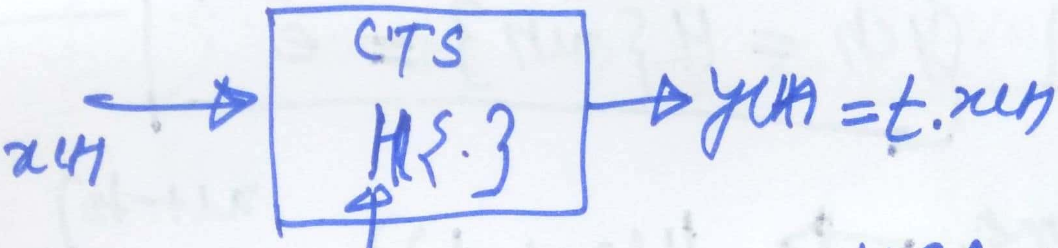
Problems on Time-Invariance / Time-Variance

Lecture 13

* Check whether following systems are TI / TV ?

⇒ (1) $y(t) = H\{x(t)\} = t \cdot x(t)$



Investigate whether $H\{\cdot\}$ is TI / TV ?

Step I) Input / output relationship ?

$$y(t) = t \cdot x(t)$$

Step II) Output due to i/p $x(t)$

$$H\{x(t)\} = t \cdot x(t).$$

Step III) Find the o/p due to delayed i/p.

$$\therefore \boxed{y_1(t) = H\{x(t-t_0)\} = t \cdot x(t-t_0)}$$

Step IV) Find delayed output by amount t_0

$$\therefore y(t)\big|_{t=t-t_0} = y(t-t_0) = (t-t_0) \cdot x(t-t_0)$$

(1)

$$y(t-t_0) \neq H\{x(t-t_0)\}.$$

$\therefore$ The given system $H\{\cdot\}$ is NOT time-invariant or It is <u>Time-Variant</u>.

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[2] $\boxed{y(t) = H\{x(t)\} = e^{x(t)}}$

Ans. $\Rightarrow$ $H\{x(t-t_0)\} = e^{x(t-t_0)}$

$$y(t-t_0) = e^{x(t-t_0)}$$

$\therefore$ $H\{x(t)\}$ is time-invariant.

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[3] $y(n) = \Phi\{x(n)\} = \cos[x(n)]$.

TI or TV ?

4) 5-point MA system

$$y[n] = \frac{1}{5}\sum_{k=-2}^{+2} x[n-k] \rightarrow S_1 = \text{Non-causal}$$

$$y[n] = \frac{1}{5}\sum_{k=0}^{4} x[n-k] \rightarrow S_2 = \underline{\text{causal}}$$

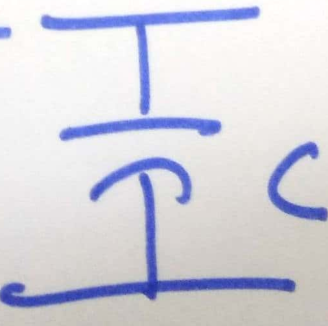
5) Problem

$v(t) = i(t) . R$

$i(t) = \frac{1}{R} v(t)$

causal

6) Problem



$y(t) = H\{x(t)\}$

$$y(t) = \frac{1}{C}\int_{\tau=-\infty}^{+t} x(\tau)\,d\tau$$

causal

*

7) $y[n] = D\{x[n]\} = \sum_{k=k_0}^{n} x(k)$

$n < k_0 \rightarrow$

let $n = k_0 - 1$

$y(k_0-1) = ??$

$\therefore D\{x[n]\}$ is Non-causal.

Problem:
8) $y[n] = D\{x[n]\} = \sum_{k=n-1}^{n+1} x(k)$

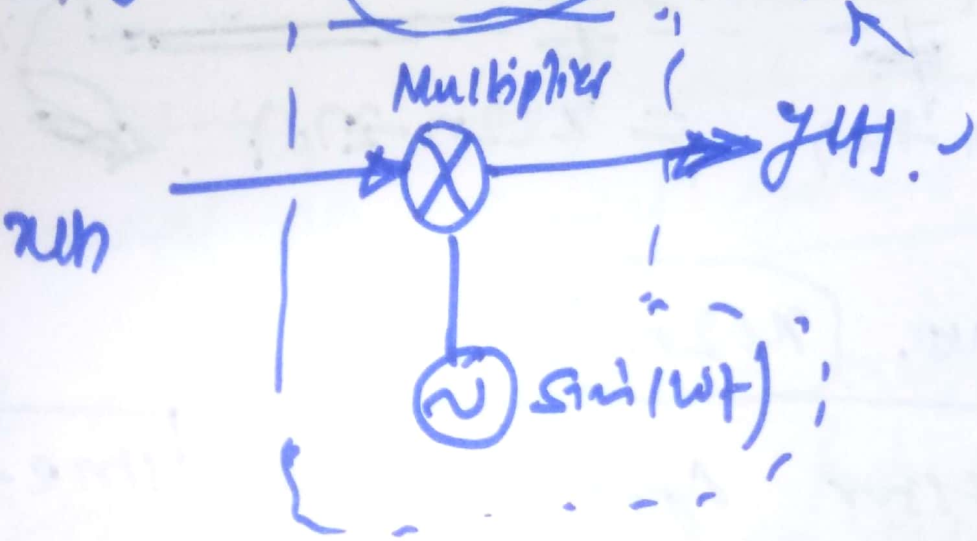
$D\{.\}$ Non-causal.

(9) $y[n] = D\{x[n]\} = a.x[n] + b.$

$D\{\cdot\}$ is TI or TV?

$D\{\cdot\}$ is TI.

(10) $y(t) = H\{x(t)\} = x(t) \times \sin(\omega t)$



$H\{\cdot\}$ is time-variant.

(11) $y[n] = D\{x[n]\} = x(k_0 n)$,
where k_0 is a positive integer, $k_0 = 2$

$$\boxed{y[n] = D\{x[n]\} = x(2n)}$$

Time compression system

Time-variant

$D\{x[n-n_0]\} = x(2n - n_0)$??

$\neq$

$y[n-n_0] = x(2n - 2n_0)$.

$x(2t)$ vs. $x(2n)$

Time compression systems are time-variant.

(12) $y[n] = x(3n)$

$y[n] = x(4n)$ → Time-invariant systems.

⋮

(7)

3) Causality: →

Measurement of temperature: →

$$y(t) = H\{x(t)\} = f\{x(t), x(t-t_1), x(t-t_2), \ldots\}.$$

The systems whose output is function of input at present time instant and/or previous time instant ($x(t-t_1)$, $x(t-t_2)$, ...), such systems are called as **causal systems** → Real-time systems.

On the other hand, if output of a system is function of future time instant values,

$x(t)$, $x(t-t_1)$

$$y(t) = f(x(t+t_1), x(t+t_2), \ldots)$$

then given system $H\{\cdot\}$ is said to be non-causal system. / Non-real-time

→ offline processing

Example: → Tutorial

Examine whether following systems are causal / non-causal ?

1) Moving Average (MA) system: →

$$x(n) \longrightarrow \boxed{\text{MA system } D\{\cdot\}} \longrightarrow y(n)$$

3-point MA system

$$y(n) = \frac{1}{3} \sum_{k=0}^{2} x(n-k) \qquad ; \qquad y(n) = \frac{1}{3} \sum_{k=-1}^{1} x(n-k)$$

$$y(n) = \frac{1}{3}\left[\underline{x(n)} + \underline{x(n-1)} + \underline{x(n-2)}\right] \qquad y(n) = \frac{1}{3}\left[x(n+1) + x(n) + x(n-1)\right]$$

⇓ ✓ → Real-time processing.

Causal or non-causal y(n)

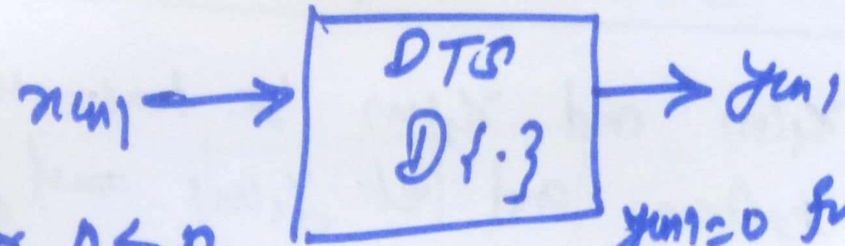
⇓ ⇒ offline process

Non-causal → seismic earthquake analysis x(n),

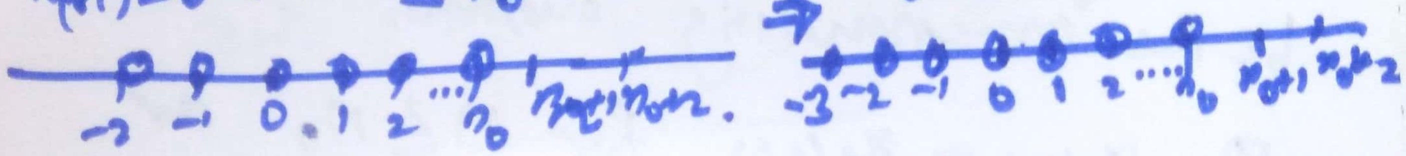
Average is moving over the data n, and hence, the name Moving Average.

Extension of Proposition 1.1 ~~and 1.2~~ :→

$\boxed{\text{Proposition 1.3}}$ Prove that for a discrete time linear and causal system, if $x[n] = 0$ for $n \le n_0$ then $y[n] = 0$ for $n \le n_0$



$x[n] = 0$ for $n \le n_0$ → $y[n] = 0$ for $n \le n_0$.



Proof:→

1) Necessary condition:→

The given system $D\{\cdot\}$ is linear.

From Proposition 1.1, we know that for a linear system if $x[n] = 0$ ∀ n then $y[n] = 0$ ∀ n.

Also the given system $D\{\cdot\}$ is also causal, which means $D\{\cdot\}$ is function of only present and/or past values of input, i.e., $x[n]$.

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$\therefore$ if $x(n) = 0$ for $n \leq n_0$

$\Rightarrow y(n) = 0$ for $n \leq n_0$.

Proof using sufficient condition :→

Let $x_1(n)$ and $x_2(n)$ be two inputs of the system and let $y_1(n)$ and $y_2(n)$ be the corresponding outputs.

If $x_1(n) = x_2(n)$ for $n \leq n_0$ or equivalently, $x(n) = x_1(n) - x_2(n) = 0$ for $n \leq n_0$

then the corresponding output will be

$y_1(n) = y_2(n)$ for $n \leq n_0$ or

$D\{0\} = 0$

equivalently,

$y(n) = y_1(n) - y_2(n) = 0$ for $n \leq n_0$

$y(n) = D\{x(n)\} = D\{x_1(n) - x_2(n)\}$

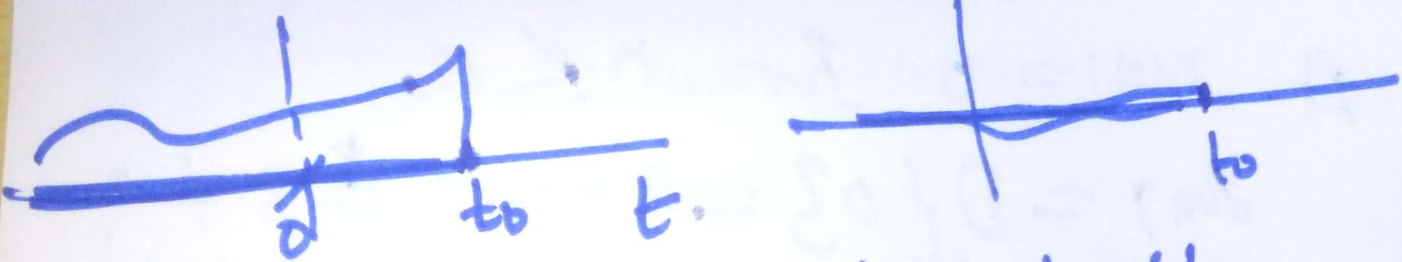
$y(n) = D\{x_1(n)\} - D\{x_2(n)\}$

$y(n) = y_1(n) - y_2(n) = 0$

Extension of Proposition 1.2 → CTS

$\boxed{\text{Proposition 1.4}}$ → For continuous-time linear and causal system, $H\{\cdot\}$, if $x(t) = 0$ for $t \le t_0$ then $y(t) = 0$ for $t \le t_0$

$$x(t) \rightarrow \boxed{\begin{matrix} CTS \\ H\{\cdot\} \end{matrix}} \rightarrow y(t)$$



Proof: → Homework (similar to Proposition 1.3)

Problem

2. Whether properties 1.3 & 1.4 are true for nonlinear systems?

Problem: Consider a system,

$$\boxed{y(n) = D\{x(n)\} = 2 \cdot x(n) + 3}$$

If $x(n) = 0$, $y(n) = 2 \times 0 + 3 = 3 \neq 0$

$y(n) \neq 0$

$\therefore$ The system is nonlinear

If $x(n) = 0$ for $n \leq n_0$

$y(n) = D\{0\} = 2 \times 0 + 3 = 3 \neq 0$ for $n \leq n_0$.

Property 1.3 requires system to be linear