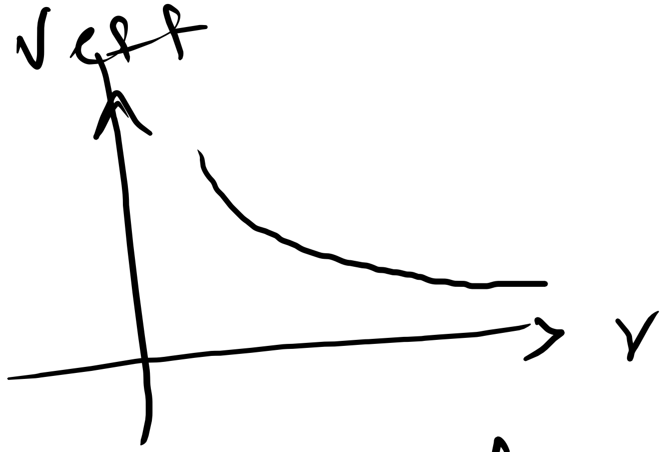
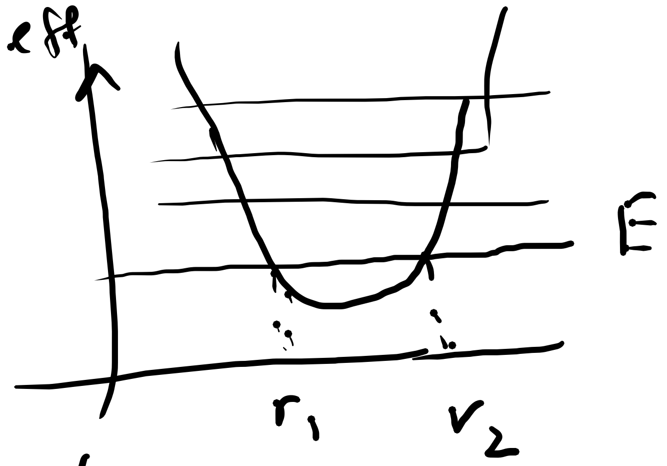
# LECTURE 12

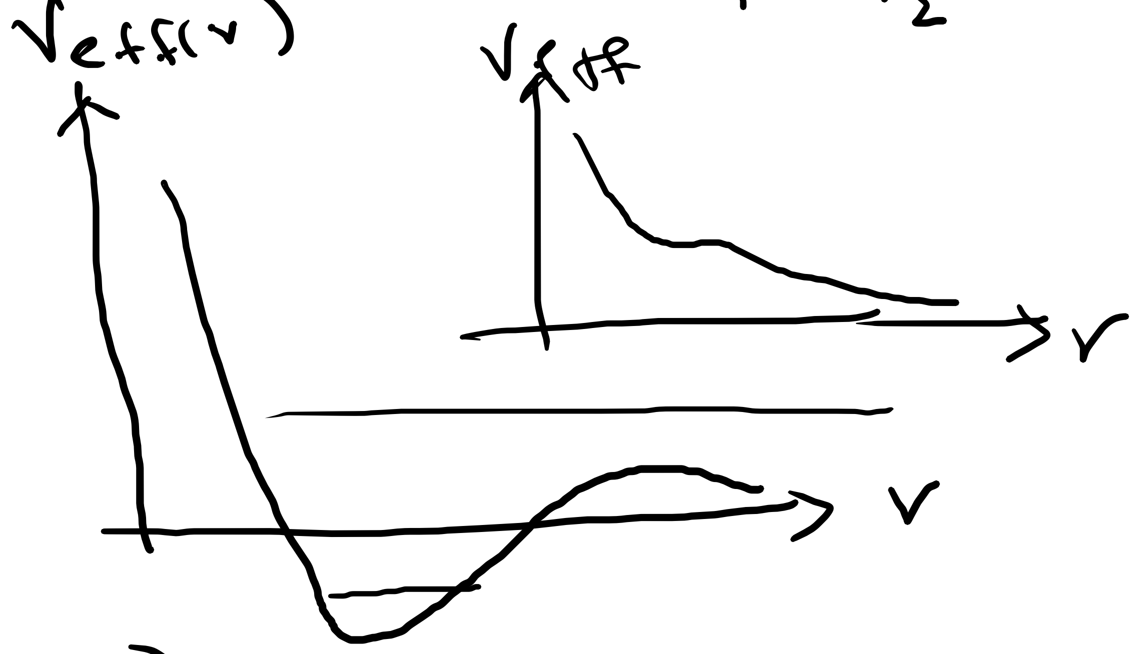
Typo :- $\frac{dv}{dt} = g + F_{drag}$ .



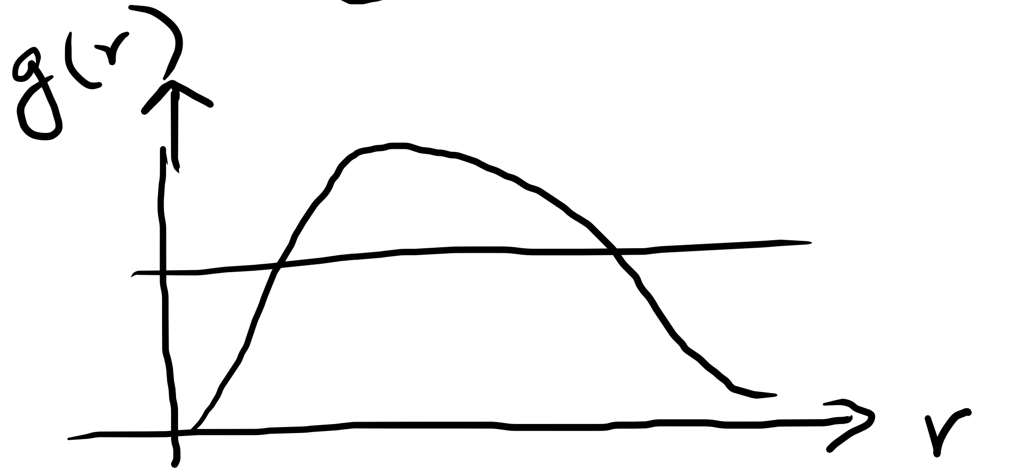

Recap : $V(r) = -V_0 e^{-\lambda^2 r^2}$

$$V_{eff}(r) = \frac{L^2}{2mr^2} - V_0 e^{-\lambda^2 r^2}$$




$$\frac{dV_{eff}(r)}{dr} = 0$$

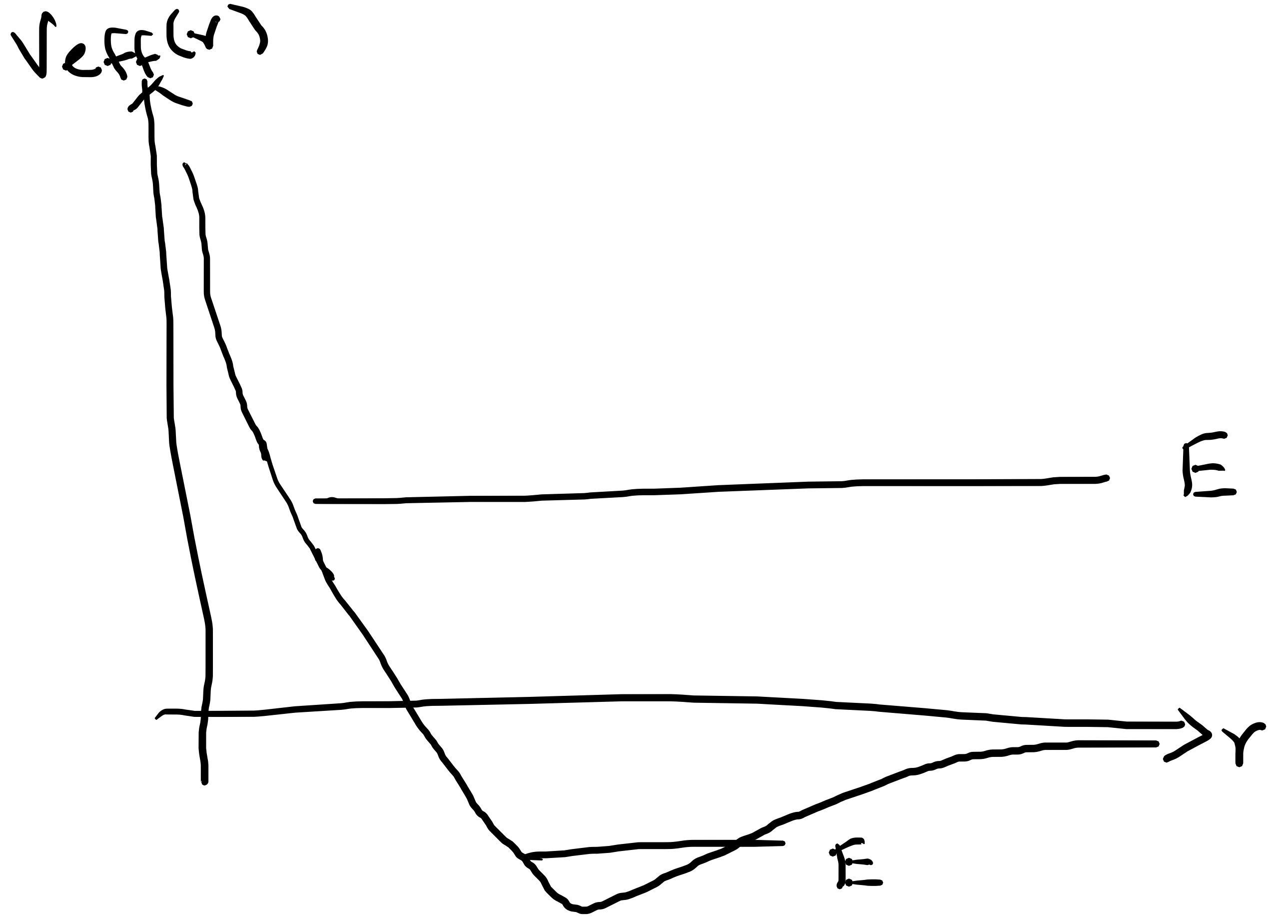
$$\Rightarrow L^2 = (2mV_0\lambda^2)\underbrace{r^4 e^{-\lambda^2 r^2}}_{g(r)}$$



Example:- $V(r) = -\alpha/r$

$$V_{eff}(r) = \frac{L^2}{2mr^2} + V(r)$$

$$= \frac{L^2}{2mr^2} - \alpha/r$$

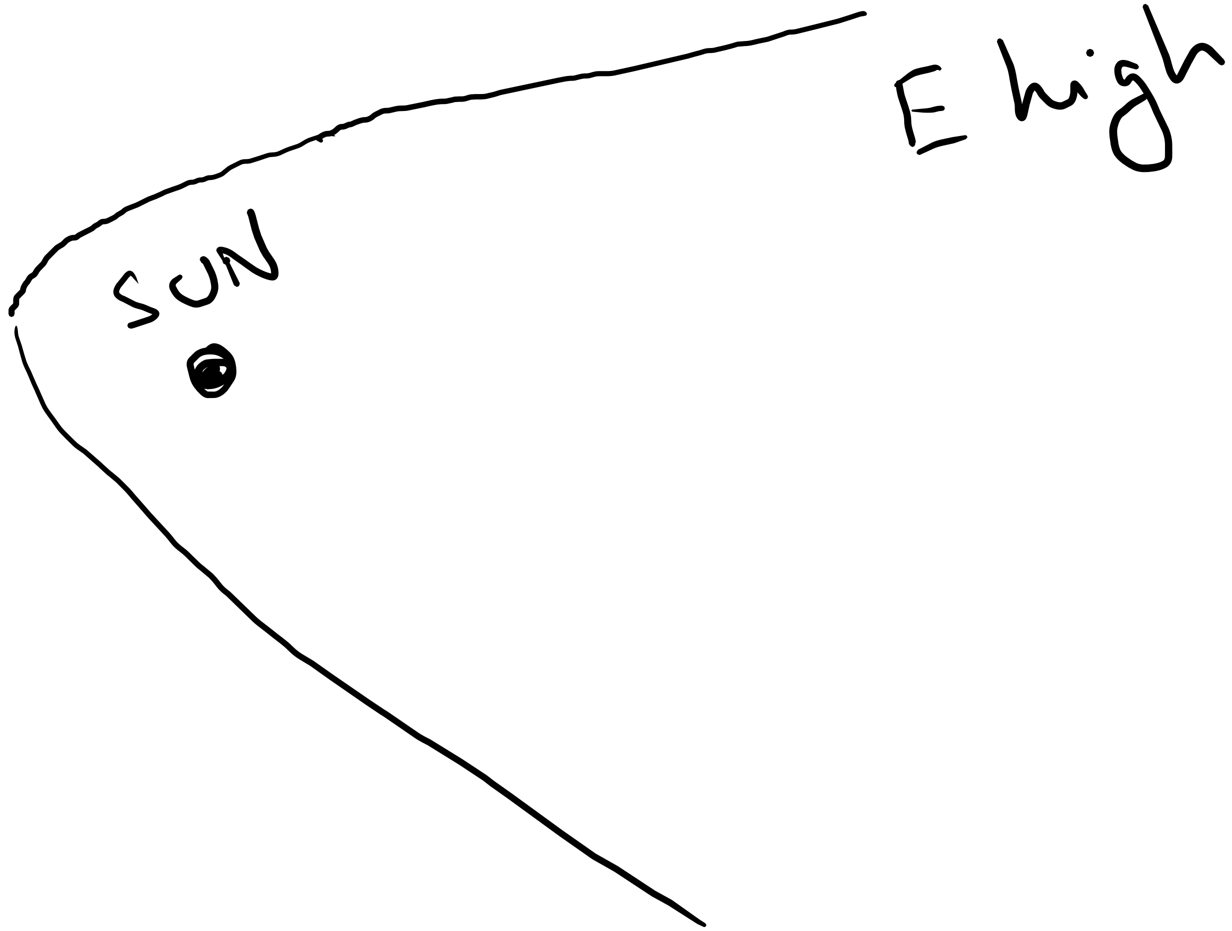



$$\frac{dV_{eff}}{dr} = -\frac{L^2}{mr^3} + \frac{\alpha}{r^2} = 0$$

$$\Rightarrow \quad r = \frac{L^2}{m\alpha} = r_*$$

$$V_{eff}(r_*) = \frac{L^2}{2mr_*^2} - \frac{\alpha}{r_*} = -\frac{m\alpha^2}{2L^2} < 0$$

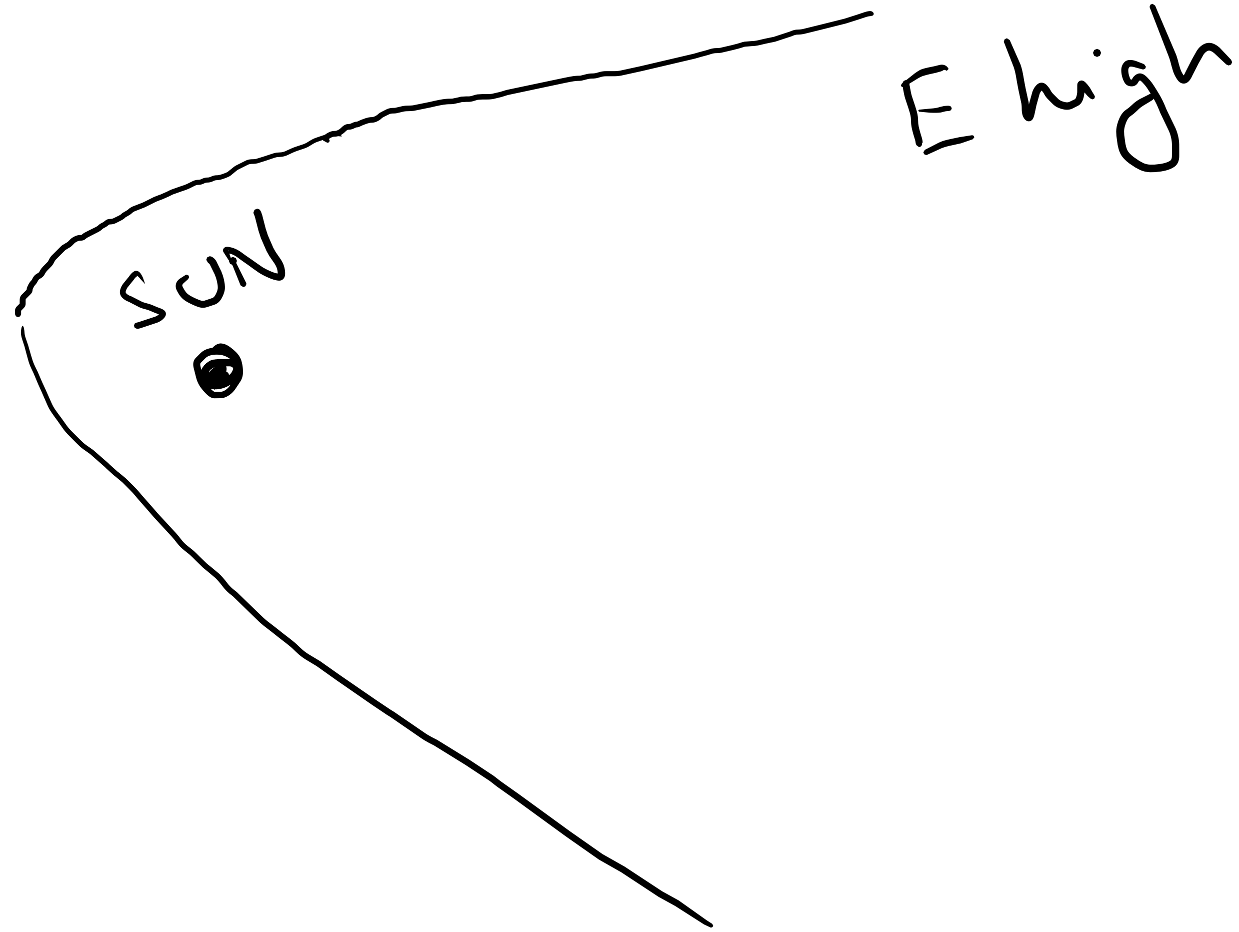
<u>Physical situation (Roughly)</u>

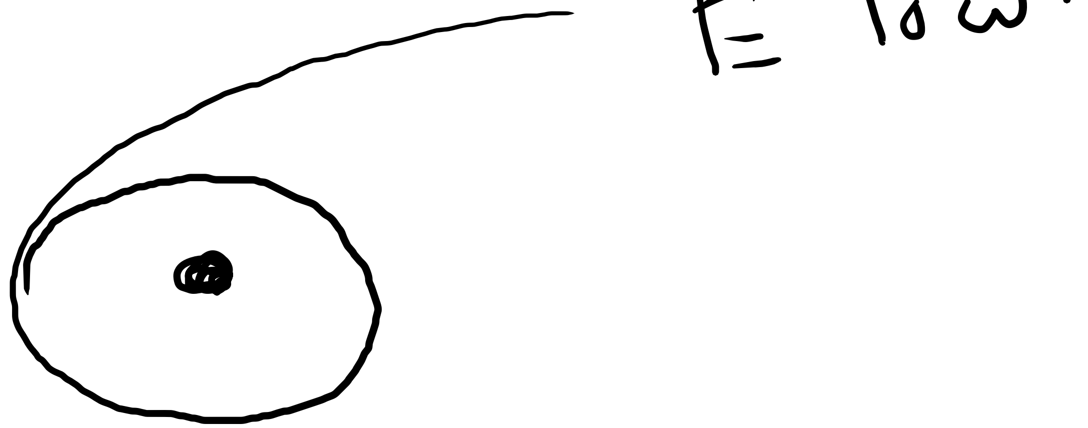




<u>Ref</u> :- Gregory
Classical mechanics.

Physical situation (Roughly)



Get exact eqn. for the orbit for $V(r) = -\alpha/r$.

EOM can be written in the form,

$$\frac{L^2}{m^2 r^4}\left[\left(\frac{dr}{d\theta}\right)^2 + r^2\right] = \frac{2(E-V)}{m}$$

Convenient to change variables, $u = \frac{1}{r}$

$$\frac{dr}{d\theta} = \frac{dr}{du}\frac{du}{d\theta} = -\frac{1}{u^2}\left(\frac{du}{d\theta}\right)$$

$$\frac{L^2}{m^2}u^4\left[\frac{1}{u^4}\left(\frac{du}{d\theta}\right)^2 + \frac{1}{u^2}\right] = \frac{2(E-V)}{m}$$

$$\Rightarrow \quad \left(\frac{du}{d\theta}\right)^2 + u^2 = \frac{2m(E-V)}{L^2}$$

$$\left(\frac{du}{d\theta}\right)^2 + u^2 = \frac{2m(E-V)}{L^2}$$

$$\Rightarrow \left(\frac{du}{d\theta}\right)^2 = -u^2 + \frac{2mE}{L^2} + \frac{2m\alpha}{L^2}u$$

$$\Rightarrow \left(\frac{du}{d\theta}\right)^2 = -\left(u^2 - 2\frac{m\alpha}{L^2}u + \frac{m^2\alpha^2}{L^4} - \frac{m^2\alpha^2}{L^4}\right) + \frac{2mE}{L^2}$$

$$= -\underbrace{\left(u - \frac{m\alpha}{L^2}\right)}_{z}{}^2 + B^2$$

$$\frac{dz}{d\theta} = \sqrt{B^2 - z^2}$$

$$\Rightarrow \int \frac{dz}{\sqrt{B^2 - z^2}} = \int d\theta = \theta - \theta_0 .$$

$$\Rightarrow \quad \cos^{-1}\left(z/B\right) = \theta - \theta_0$$

Can set $\theta_0 = 0$.

$$z = B\cos(\theta)$$

Substituting,

$$\frac{1}{r} = \frac{m\alpha}{L^2} + \frac{m\alpha}{L^2}\underbrace{\left(1 + \frac{2EL^2}{m\alpha^2}\right)^{1/2}}_{\epsilon}\cos\theta.$$

$$\Rightarrow \quad \frac{1}{r} = \frac{m\alpha}{L^2}\left(1 + \epsilon\cos\theta\right).$$

$$\cos\theta = -1/\epsilon$$

$$\epsilon = \underbrace{\left(1 + \frac{2EL^2}{m\alpha^2}\right)^{1/2}}$$

$$\epsilon = 0$$

$$\Rightarrow 1 + \frac{2EL^2}{m\alpha^2} = 0$$

$$\Rightarrow E = -\frac{m\alpha^2}{2L^2}$$

$$\Rightarrow \quad \cos^{-1}\left(\frac{z}{B}\right) = \theta - \theta_0$$

Can set $\theta_0 = 0$.

$$z = B\cos(\theta)$$

Substituting,

$$\frac{1}{r} = \frac{m\alpha}{L^2} + \frac{m\alpha}{L^2}\underbrace{\left(1 + \frac{2EL^2}{m\alpha^2}\right)^{1/2}}_{\epsilon}\cos\theta.$$

$$\Rightarrow \quad \frac{1}{r} = \frac{m\alpha}{L^2}\left(1 + \epsilon\cos\theta\right).$$

$$\cos\theta = -1/\epsilon$$

$$\epsilon = \underbrace{\left(1 + \frac{2EL^2}{m\alpha^2}\right)^{1/2}}$$

$$\epsilon = 0$$

$$\Rightarrow 1 + \frac{2EL^2}{m\alpha^2} = 0$$

$$\Rightarrow E = -\frac{m\alpha^2}{2L^2}$$

Case I :- $\epsilon > 1$

$$r_{min} = \frac{L^2}{m\alpha(1+\epsilon)}$$

Since, there will be a value of $\theta$ such that $\cos\theta = -1/\epsilon$

$$r_{max} = \infty .$$

$$\frac{1}{2} m \frac{d}{dt}\left(\dot{r}^2 + r^2 \dot{\theta}^2\right) = \frac{d}{dt} \int dr\, f(r) .$$

$$\Rightarrow \frac{1}{2} m \left(\dot{r}^2 - r^2 \dot{\theta}^2\right) - \int dr\, f(r) = \underline{E} .$$

Had used :- $\dot{r} f(r) = \frac{d}{dt} \int dr\, f(r)$

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$$F(x) = \int_a^x dt\, f(t)$$

$$F'(x) = f(x) .$$

$$\frac{d}{dt}\int dr\, f(r)$$

$$f(r)\frac{dr}{dt} = \dot{r} f(r)$$

$$(r + \Delta r) - r$$