Computational Numerical Methods

CS 374

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Solving System of Linear Equation

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$\vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n.$$

$Ax = b$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & \dots & \dots & a_{2n} \\ \vdots & & & \\ a_{n1} & \dots & \dots & a_{nn} \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\begin{bmatrix} 1 & * & * & \cdots & * \\ 0 & 1 & * & & * \\ \vdots & \vdots & 1 & \cdots & * \\ 0 & 0 & 0 & & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & 0 \\ 0 & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

R. r_2 -

Q1 Gauss Elimination method.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \quad E1 \times \frac{1}{a_{11}} \times a_{21}$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \quad E2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \quad E3$$

$$\Downarrow$$

$E1$

$E2^{(2)} \quad E_2 - \frac{a_{21}}{a_{11}} E_1$

$E_3^{(2)} - \quad E_3 - \frac{a_{31}}{a_{11}} E_1$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$0 + \underline{a_{22}^{(2)}}x_2 + a_{23}^{(2)}x_3 = b_2^{(2)}$$

$$0 + a_{32}^{(2)}x_2 + a_{33}^{(2)}x_3 = b_3^{(2)}$$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$0 \quad a_{22}^{(2)}x_2 + a_{23}^{(2)}x_3 = b_2^{(2)}$$

$$a_{33}^{(3)}x_3 = b_3^{(3)} \qquad E_3^{(2)} - \frac{a_{32}^{(2)}}{a_{22}^{(2)}} E_2^{(2)}$$

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & b_2^{(2)} \\ 0 & 0 & a_{33}^{(3)} & b_3^{(2)} \end{array}\right]$$

$$\underline{x_3 = \frac{b_3^{(2)}}{a_{33}^{(3)}}} \qquad x_2 = \frac{1}{a_{22}^{(2)}}\left[b_2^{(2)} - a_{23}^{(2)} \cdot \frac{b_3^{(2)}}{a_{33}^{(2)}}\right]$$

Example

$$6x_1 + 2x_2 + 2x_3 = -2$$

$$2x_1 + \frac{2}{3}x_2 + \frac{1}{3}x_3 = 1$$

$$x_1 + 2x_2 - x_3 = 0.$$

Actual solution.

$$\left.\begin{aligned} x_1 &= 2.6 \\ x_2 &= -3.8 \\ x_3 &= -5. \end{aligned}\right)$$

Compute using 4 digit rounding.

$$6x_1 + 2x_2 + 2x_3 = -2.$$

$$0 + \underline{0.0001}\,x_2$$

$$6x_1 + 2x_2 + 2\cdot x_3 = -2$$

$$\underline{2}x_1 + 0.6667x_2 + 0.333x_3 = 1$$

$$\underline{x_1} + 2x_2 - x_3 = 0$$

$$\downarrow$$

$$\begin{cases} 6x_1 + 2x_2 + 2x_3 = -2 \\ 0 + 0.0001x_2 - 0.3333x_3 = 1.6667 \\ 0 + \underline{1.6667}x_2 - 1.3333x_3 = 0.3334 \end{cases}$$

$$6x_1 + 2x_2 + 2x_3 = -2.$$

$$0 + 0.0001x_2 - 0.3333x_3 = 1.6667$$

$$0 \quad 0 \quad 5555x_3 = -27790$$

Back tracing we get.

$$\begin{cases} x_3 = -5.003 \\ x_2 = 0 \\ x_1 = 1.335. \end{cases}$$

Modified Gauss elimination. with partial pivot

~~Use~~ let $S_1 = \max \{ |a_{11}|, |a_{21}|, |a_{31}| \}$.

Using the modified method solve the previous problem

$$6 \cdot x_1 + 2x_2 + 2x_3 = -2$$

$$0 = \cancel{2} x_1 + 0.6667x_2 + 0.3333x_3 = 1$$

$$0 = x_1 + 2x_2 - x_3 = 0$$

Step 1

$$\max(6, 2, 1) = 6$$

$$6x_1 + 2x_2 + 2x_3 = \cancel{2}\ -2$$

$$0 + 0.0001x_2 - 0.3333x_3 = 1.6667$$

$$0 + 1.6667x_2 - 1.3333x_3 = 0.3334.$$

$$\max(1.6667, 0.0001) = 1.6667$$

$$6x_1 + 2x_2 + 2x_3 = -2$$

$$1.6667x_2 - 1.3333x_3 = 0.3334$$

$$0.0001x_2 - 0.3333x_3 = 1.6667.$$

$$5x_1 + 6x_2 + 2x_3 = -2.$$

$$0 \quad 1.6667x_2 - 1.3333x_3 = 0.3334$$

$$0 \quad 0 \quad -0.3332x_3 = 1.6667.$$

$x_3 = -5.003$ $\quad x_2 = -3.801$, $\quad x_1 = 2.602.$

$x_3 = -5$ $\quad x_2 = -3.8$, $\quad x_1 = 2.6$