Computational Numerical Methods

CS 374

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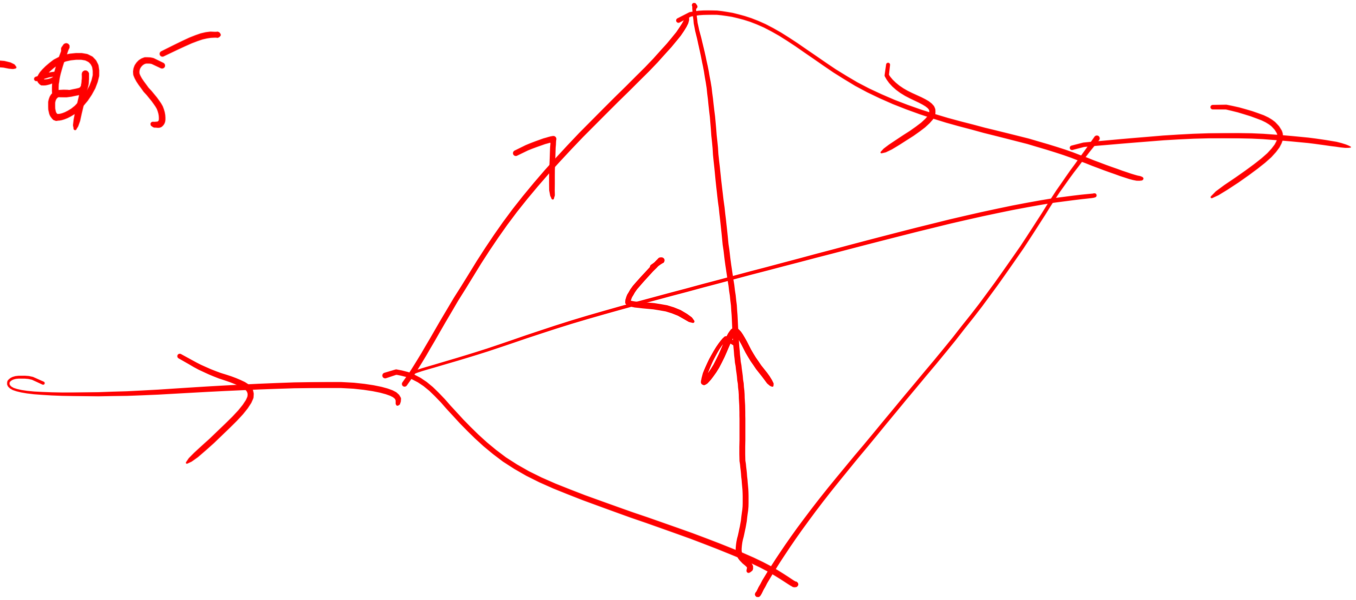
$$\left.\begin{aligned} a_1 x + b_1 y + c_1 z &= d_1 \\ a_2 x + b_2 y + c_2 z &= d_2 . \end{aligned}\right\}$$

$$\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \in \mathrm{Col}\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}\right.$$

$$x + y + z = 2$$

$$2x + 2y + 2z = \cancel{4}\ \cancel{4}\ 5$$



$$f(x) = p_n(x)$$

$x = a$.

$$f(x) = f(a) + (x-a) f'(a)$$

a	b	c	d	e
f(a)	f(b)	f(c)	f(d)	f(e)

$$\frac{dy}{dx} = f'(x) f(x)$$

$$\frac{dy}{dt} = 3x + 5y$$

$$\frac{dx}{dt} = 5x + 9yx \, .$$

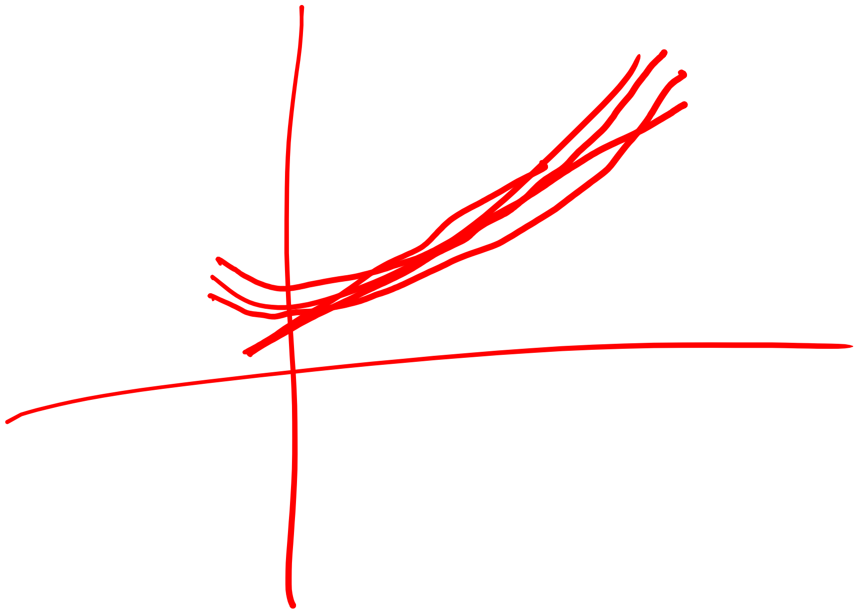
Taylor's polynomials

$f(x)$ $\underline{p_1(x)}$ $\underline{x = a}$

$$\begin{cases} f(a) = p_1(a) \\ f'(a) = p_1'(a) \end{cases}$$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2$$

Let $f(x) = e^x$. around $x=0$



$$P_1(x) = 1 + x.$$

$$P_2(x) = 1 + x + \frac{x^2}{2}.$$

$$P_3(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$$

Taylor's approximation to e^x

x	$P_1(x)$	$P_2(x)$	$P_3(x)$	e^x.
-1.0	0	0.5	0.333	0.36788
-0.5	0.5	0.625	0.60417	0.60653
0	1	1	1	1
0.5	1.5	1.625	1.64583	1.64872
1.0	2	2.5	2.6667	2.71828

Error in Taylor's polynomial approximation

Taylor's remainder

Assume that $f(x)$ has $n+1$ continuous derivatives on an interval $\alpha \leq x \leq \beta$ & let the point a belongs to that interval. For the taylor's polynomial $P_n(x)$ let $R_n(x) = f(x) - P_n(x)$ denote the remainder in approximating $f(x)$ by $P_n(x)$. Then

$$R_n(x) = \frac{(x-a)^{n+1}}{(n+1)!} f^{(n+1)}(c_x)$$

with c_x is a unknown point b/w a & x.

Consider $f(x) = \log x$. Find the Taylor's approximation around $x = 1$

$$P_1(x) = (x-1)$$

$$P_2(x) = (x-1) - \frac{(x-1)^2}{2}$$

$$P_3(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}.$$

$$P_n(x) = \cancel{(x-1)} \sum_{j=1}^{n} \frac{(-1)^{j-1}}{j} (x-1)^j$$

for $y = \ln(x)$

#1 Try ~~a~~ considering the function with error.

$\epsilon < 0.01$ at $x = 2$

#2. Plot n vs error. at $x = 2, 3, 4$.