Lecture 24

Fourier Transform / Fourier Analysis.

→ Fourier transform has ruled over signal processing and communications.

→ Fourier Analysis in other courses core + elective

→ Analog Communication and transmission line

→ Digital Communication

→ Digital Signal Processing

→ Control Systems.

→ Digital Image Processing

→ Speech Signal Processing.

② $\mathbb{Z}, R, R^+, \mathbb{Z}^+, \mathbb{Z}^-, C$

Signals and Systems

Study of Linear Combinations.

[Set of basis Functions / basis (Linear Algebra) or Representative vectors.]

Shifted Impulses (Chapter 1)

$\{\delta[n-k]\}_{k\in\mathbb{Z}}$

$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\cdot\delta[n-k]$

[Sifting Property of discrete impulse]

$\{\delta(t-\tau)\}_{\tau\in R}$

$x(t) = \int_{\tau=-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau$

Sifting Proper of continuous-time impulse

Shifted Impulse Responses. [Chapter 2]

$\{h[n-k]\}_{k\in\mathbb{Z}}$

$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\cdot h[n-k]$

[Convolution Sum]

? $\{h(t-\tau)\}_{\tau\in R}$

$y(t) = \int_{\tau=-\infty}^{+\infty} x(\tau)\,h(t-\tau)\,d\tau$

[Convolution Integral]

Set of Complex Exponential Signals (?)

Set of complex Exponentials

$\{e^{j\omega t}\}$

- $\omega \in \mathbb{R}$
  - $t \in \mathbb{R}$ (chapter 4)
    - $\{e^{j\omega t}\}$
    - $\omega, t \in \mathbb{R}$
    - $X(\omega) = \int x(t) \cdot e^{-j\omega t} dt$.
    - $t \in \mathbb{R}$ → Continuous-time Fourier transform
  - ~~$t\in$~~ $n \in \mathbb{Z}$ (chapter 5)
    - $\{e^{j\omega n}\}$
    - $n \in \mathbb{Z}, \omega \in \mathbb{R}$
    - $X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(n) \cdot e^{-j\omega n}$
    - Discrete-time Fourier Transform
- $\omega_k = k \cdot \omega_0$ (chapter 3)
  - $t \in \mathbb{R}$
    - $\{e^{jk\omega_0 t}\}$
    - $x(t) = \sum_{k=-\infty}^{+\infty} F_k \cdot e^{jk\omega_0 t}$ (0 to N-1)
    - Continuous-time Fourier series Representation
  - $n \in \mathbb{Z}$
    - Discrete-time Fourier series Representation

→ Chapter 1 and Chapter 2 can be understood as study of linear combination with shifted impulses and shifted impulse responses as basis functions, respectively.

→ (Chapter 3, Chapter 4) and chapter 5 can also be understood as study of linear combination with set of complex exponential functions such as

$\{e^{jk\omega_0 t}\}_{t\in\mathbb{R},\, k\in\mathbb{Z}}$, & $\{e^{jk\omega_0 n}\}_{k,n\in\mathbb{Z}}$ → chapter 3,

$\{e^{j\omega t}\}_{\omega, t\in\mathbb{R}}$ → chapter 4

$\{e^{j\omega n}\}_{n\in\mathbb{Z},\, \omega\in\mathbb{R}}$ → chapter 5

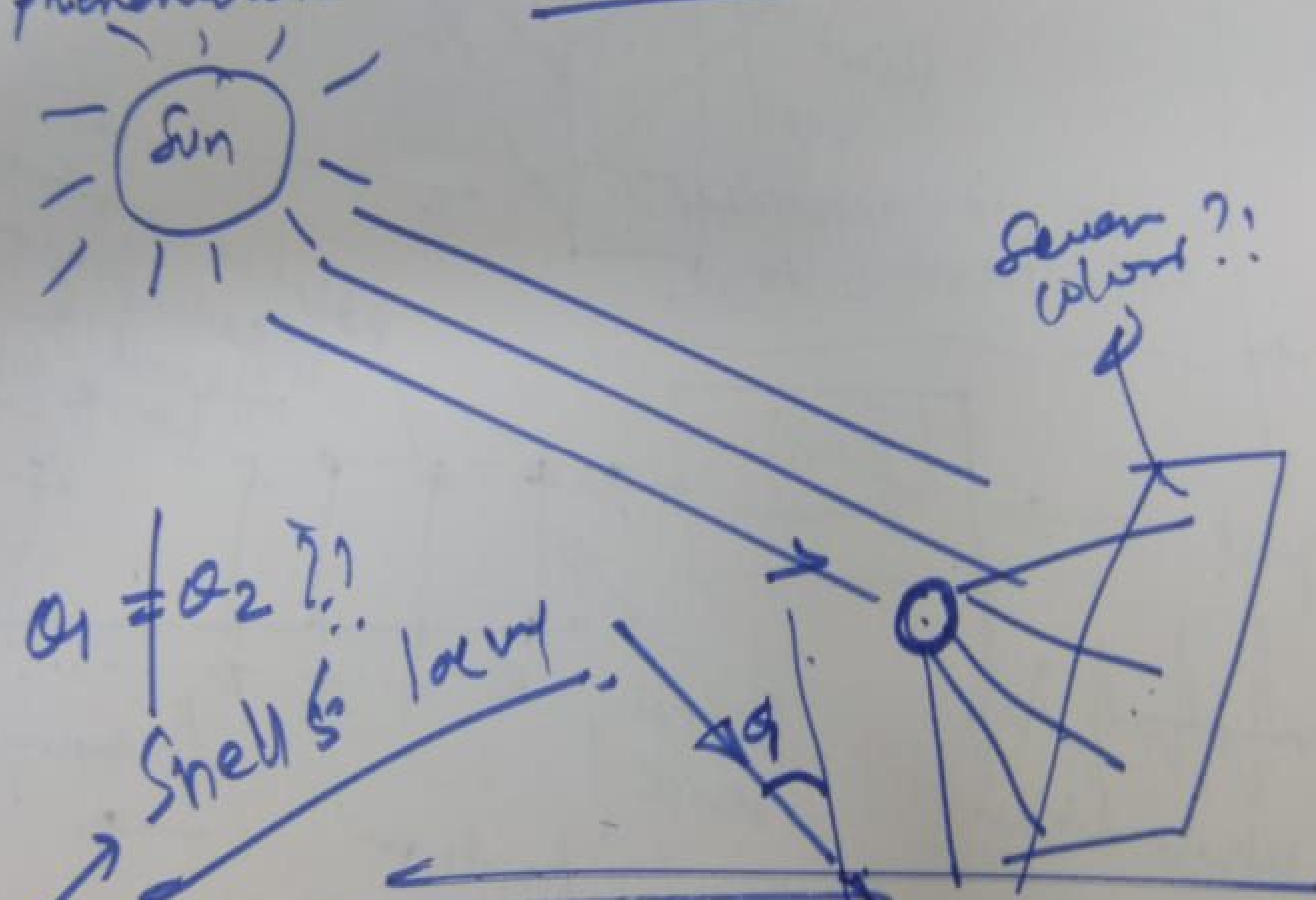
→ ∴ Course on Signals and Systems is nothing but study of linear combinations with different basis functions

↳ A kind of <u>linear algebra</u>

History of Fourier Analysis:-

Classic verse (historical) which began in 1000 years before.

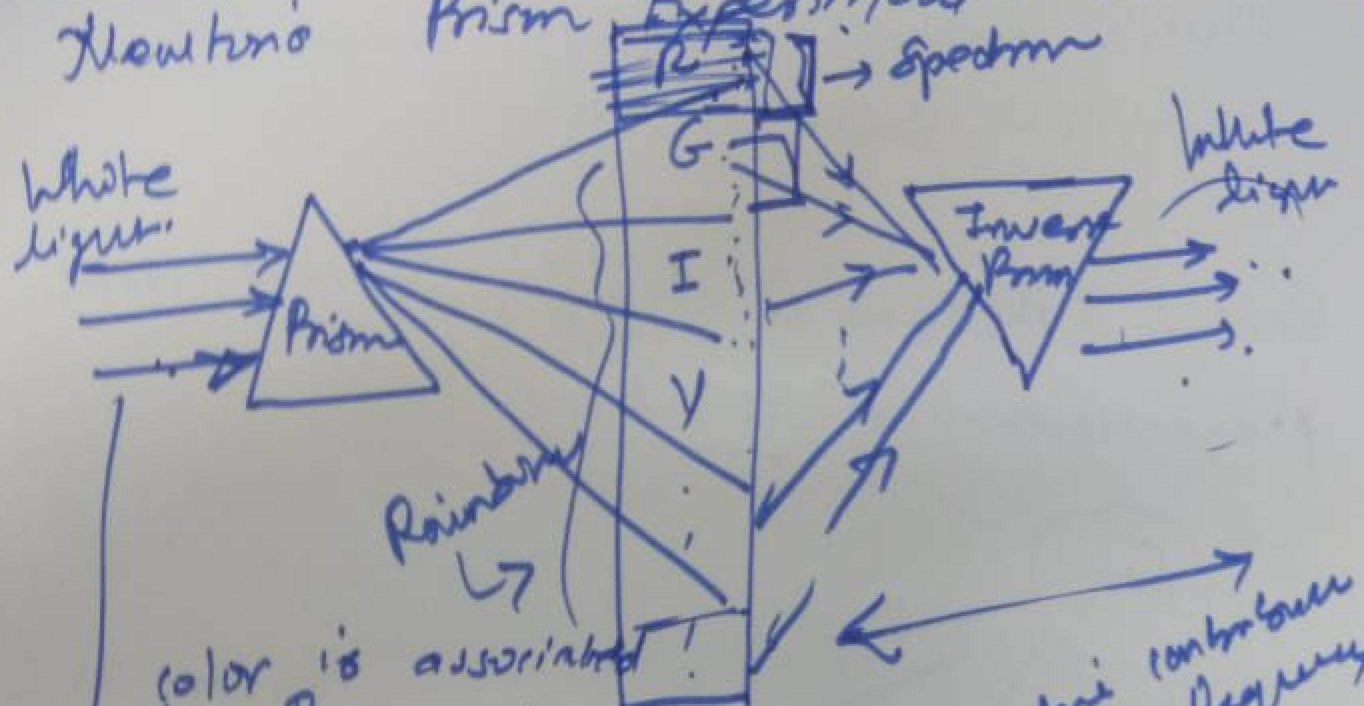
Question (Problem of study): To investigate phenomenon of rainbow.



$$\boxed{\frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{v_1}{v_2} = \frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2}}$$

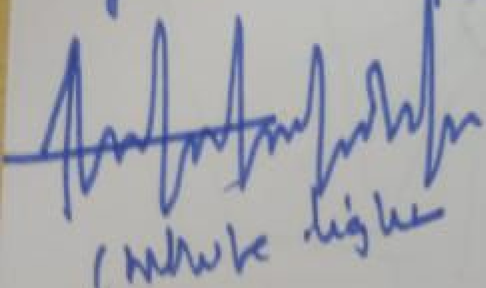
Sir Issac Newton :→

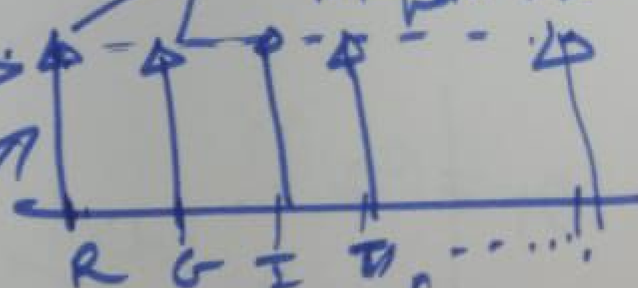
Newton's Prism Experiment :→



color is associated with frequency range.

Signal → Fourier Transform →





Relative contribution of each frequency & intensity hence color

Fourier series / Fourier transform continuous.

Frequency →

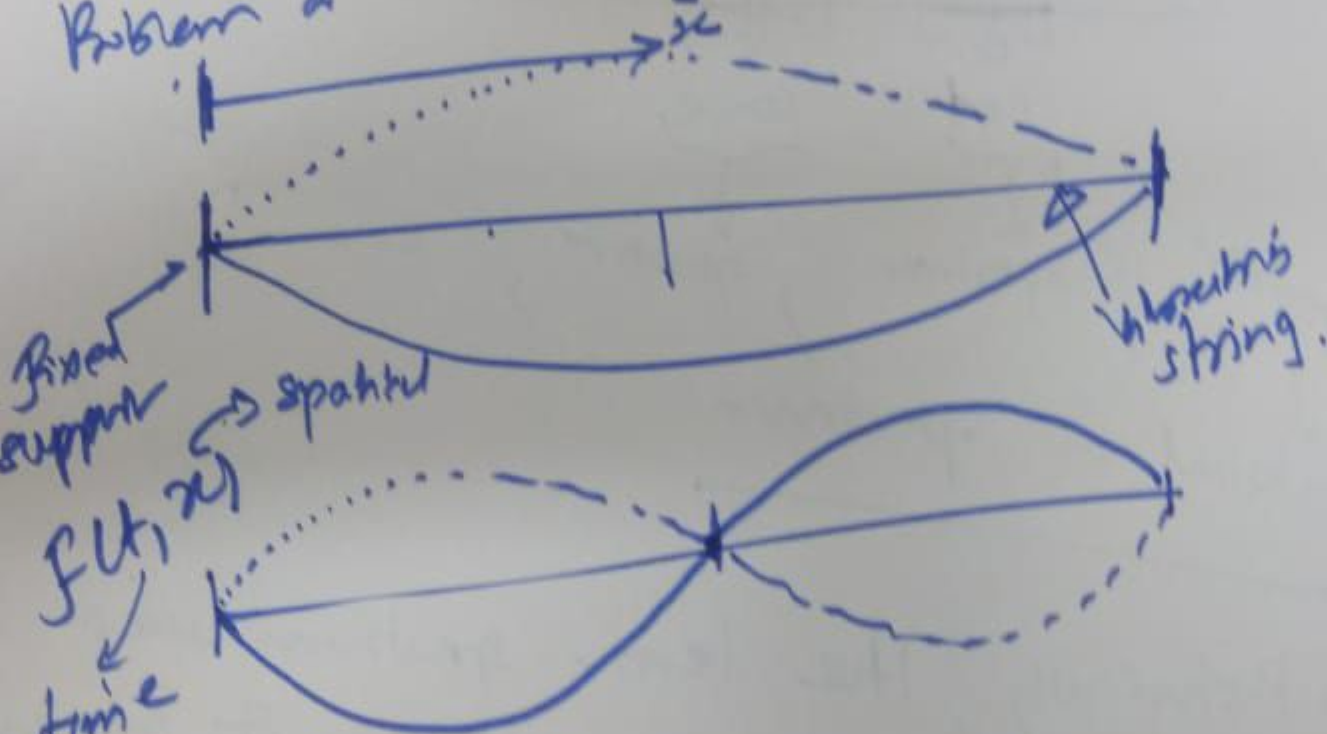
Signal Spectrum → Inverse Fourier Transform → Reconstructed Signal

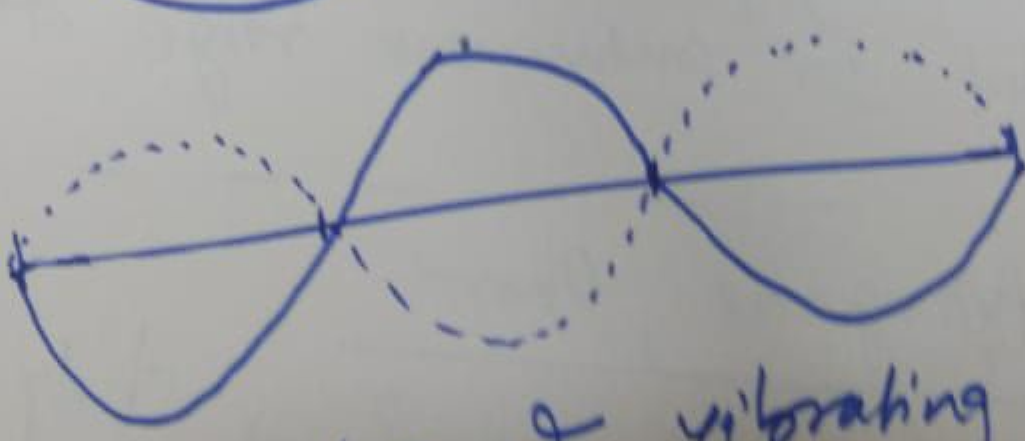
Modern History of Fourier Analysis.

17th century :→

1703 : {Bernoulli, Euler}

→ Problem of Vibrating string

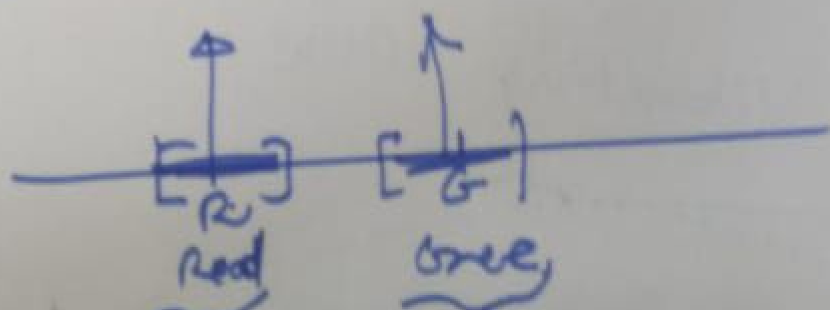




Normal modes of vibrating string.

The key idea thought was to represent modes of vibrating string as linear combination of sines and cosines

Due to Sir Newton, spectrum is defined as range of frequencies associated with a color



Spectra = { spectrum , spectrum , --- }

↓ plural of spectrum

→ Historically, the term spectrum was coined for continuous range of frequencies

→ Pythagoras, Ohm

↓ { Music and Acoustics } → study of sound waves

$$f(t,x) = \sum_{2\cdot 7} \left[\sin(\cdot) + \cos(\cdot)\right]$$

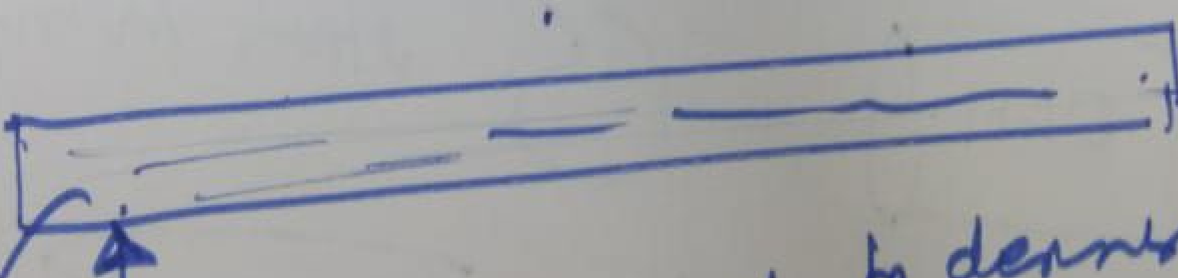
1759.
J. L. Lagrange → opposed the idea represented by above eqn.

Jean Baptiste Joseph Fourier

J. B. J. Fourier

Thermodynamics

Heat Diffusion Process.



Fourier series was used to describe the heat diffusion process through a body. ⟹ Linear combination of sines and cosines

Fourier's research paper

J. L. Lagrange

x, y, t.

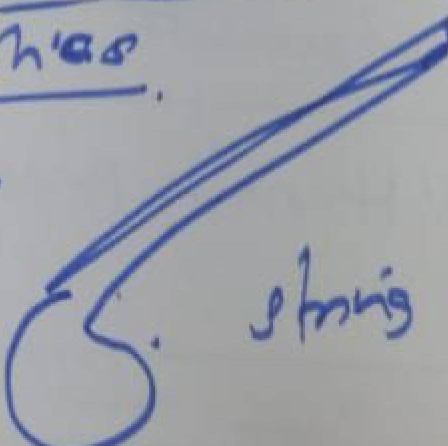
Rejected Fourier's Paper

Book : Analytical Theory of Heat.

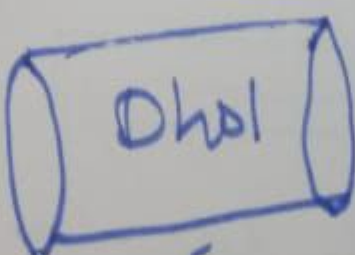
Institute of France.

Music and Acoustics.

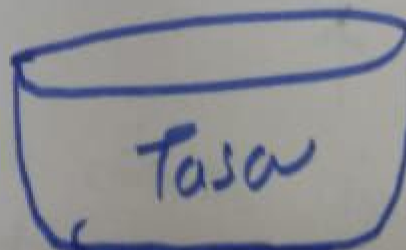
Musical Instrument



String Instrument

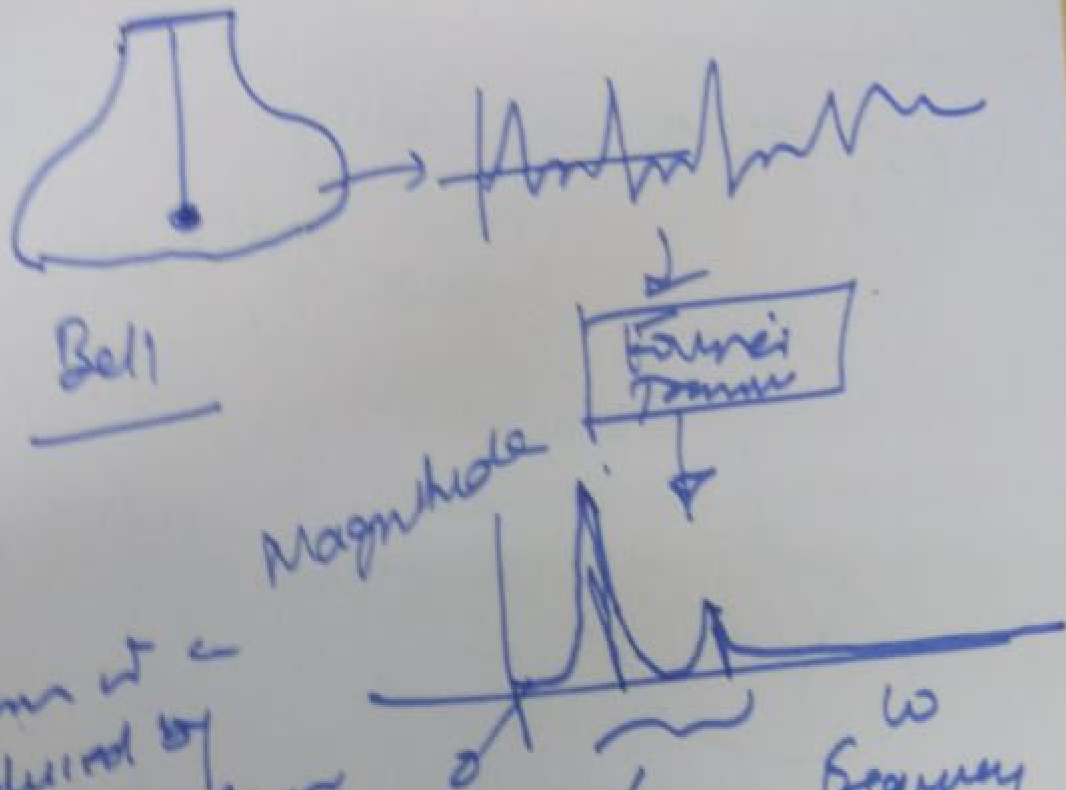


Dafli



goat skin

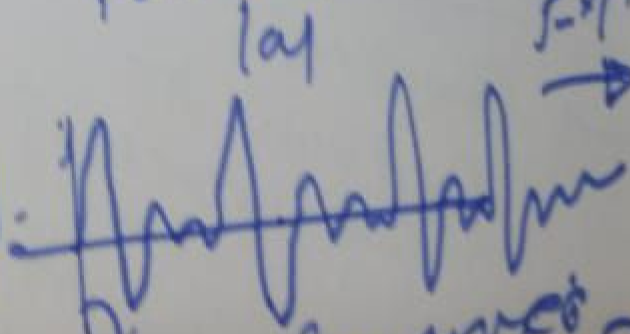
Eigenmodes of vibrations of Tabla → Sir C. V. Raman



The spectrum of a signal produced by real physical system, captures information about system characteristic, system architecture.

Spectrum of bell sound.

Human speech /a/



Resonant frequencies of vocal tract. $\longrightarrow \{\omega_1, \omega_2, \omega_3, \omega_4\}$

Speech recognition

Speaker identification

Technological perspective

1965, IBM → Cooley and Tukey.

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$ CTFT.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\omega n}$$ DTFT

To implement this in computationally effective way.

Fast Fourier Transform (FFT): →

FFT → Practical Application of FT became possible & commercially feasible

(12)

⇒ Special purpose microprocessor to compute FFT via standalone hardware

Response of an LTI system for Sinusoidal Excitation →

$$x(n) \longrightarrow \boxed{\begin{array}{c}\text{LTI} \\ \text{system} \\ h(n)\end{array}} \longrightarrow y(n)$$

$\therefore\ y(n) = x(n) * h(n)$

Using Commutative Property.

$$y(n) = h(n) * x(n)$$

$$y(n) = \sum_{k=-\infty}^{+\infty} h(k) . x(n-k)$$

Let input excitation signal be given by

$$x(n) = e^{jwn} = \text{complex exponential signal.}$$

$$\therefore\ y(n) = \sum_{k=-\infty}^{+\infty} h(k) .. e^{jw(n-k)}$$

constant

$$= \sum_{k=-\infty}^{+\infty} h(k) .. e^{jwn} \times e^{-jwk}$$

Numerical Analysis:

$$\int_0^1 \frac{dx}{1+x^2} = \left[\tan^{-1}(x)\right]_0^1$$

$$\int_0^1 \frac{dx}{1+x^5} = ??$$

eigen → proper

Fourier Analysis is useful.

$$y(n) = \underbrace{\left\{\sum_{k=-\infty}^{+\infty} h(k)\, e^{-jwk}\right\}}_{\text{eigenvalue}} \cdot e^{jwn}$$

$$\boxed{y(n) = \underbrace{H(e^{jw})}_{\text{Eigen value}} \cdot \underbrace{e^{jwn}}_{\text{Input Excitation}}}, \text{ where } H(e^{jw}) = \sum_{n=-\infty}^{+\infty} h(n)\, e^{-jwn}$$

eigenfunction / eigen vector

Impulse response of an LTI system

→ Impulse response of an LTI system characterises an LTI system completely.

⟹ So is its frequency ~~response~~, $H(e^{jw})$.

(14)

For an LTI system, if excited by ~~eigen~~ eigenfunction, $e^{j\omega n}$ then o/p response, $y[n]$, also contains $e^{j\omega n}$ multiplied by LTI system's eigenvalue, $H(e^{j\omega})$

$$\therefore \underbrace{y[n]}_{\text{O/P of LTI system}} = \underbrace{H(e^{j\omega})}_{\text{Eigenvalue / System's Frequency Response}} \cdot \underbrace{e^{j\omega n}}_{\text{Eigenfunction}}$$