Logest common subsequence (LCS)

Subsequence:

A B B C B A B

then B C B B is a subsequence

- A sequence whose order is same as in the given sequence.
- They may not be consecutive.

Common subsequence:

X = B A A B C B A B C

Y = A B B C B A C

A B B B C

A B C B A C

longest.

A sequence that is subsequence to both x and y.

ABC is a common subsequence.

Logest: A common subsequence whose length is largest possible.

A simple algorithm

X, Y $\quad |Y| < |X| \quad |Y| = n$, $|X| = m$

consider all possible subsequences of Y

compare with X whether any of the subsequence is common to X

return the largest one.

running time : $O(m \cdot 2^n)$

$X = x_1 x_2 \cdots x_m$

$Y = y_1 y_2 \cdots y_n$

$X_i = x_1 x_2 \cdots x_i$

$Y_j = y_1 y_1 \cdots y_j$

Z be a LCS of X and Y

$Z_K = z_1 z_2 \cdots z_K$

$x_m = y_n$

z_K is a member of LCS of X and Y

Z_{K-1} is a LCS of X_{m-1} and Y_{n-1}

$x_m \neq y_n$

$z_K \neq x_m$

Z_K is a LCS of X_{m-1} and Y_n

$z_K \neq y_n$

Z_K is a LCS of X_m and Y_{n-1}

Recursive definition

$c[i\ j] \leftarrow$ optimum length of an LCS for X_i and Y_j

$$c[i,j] = \begin{cases} 0 & \text{when } i = 0 \text{ or } j = 0 \\ c[i-1, j-1] + 1 & \text{if } i, j > 0 \text{ and } x_i = y_j \\ \max\{c[i-1, j], c[i, j-1]\} & \text{if } i, j > 0 \text{ and } x_i \neq y_j \end{cases}$$

Compute the value of the optimum solution

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LCS-length (X, Y)
m ← length(X) , n ← length(Y)
let C[0..m, 0...n] be a new table
let B[0..m, 0..n] be a new table.
for i = 1 to m
        C[i,0] = 0
for j = 1 to n
        C[0,j] = 0
for i = 1 to m
    for j = 1 to n
        if x_i = y_j
            C[i,j] = C[i-1,j-1] + 1
            b[i,j] = "↖"
        else if C[i-1, j] ≥ C[i, j-1]
            C[i,j] = C[i-1, j]
            b[i,j] = "↑"
        else
            C[i, j] = C[i, j-1]
            b[i, j] = "←"
return C, b
```

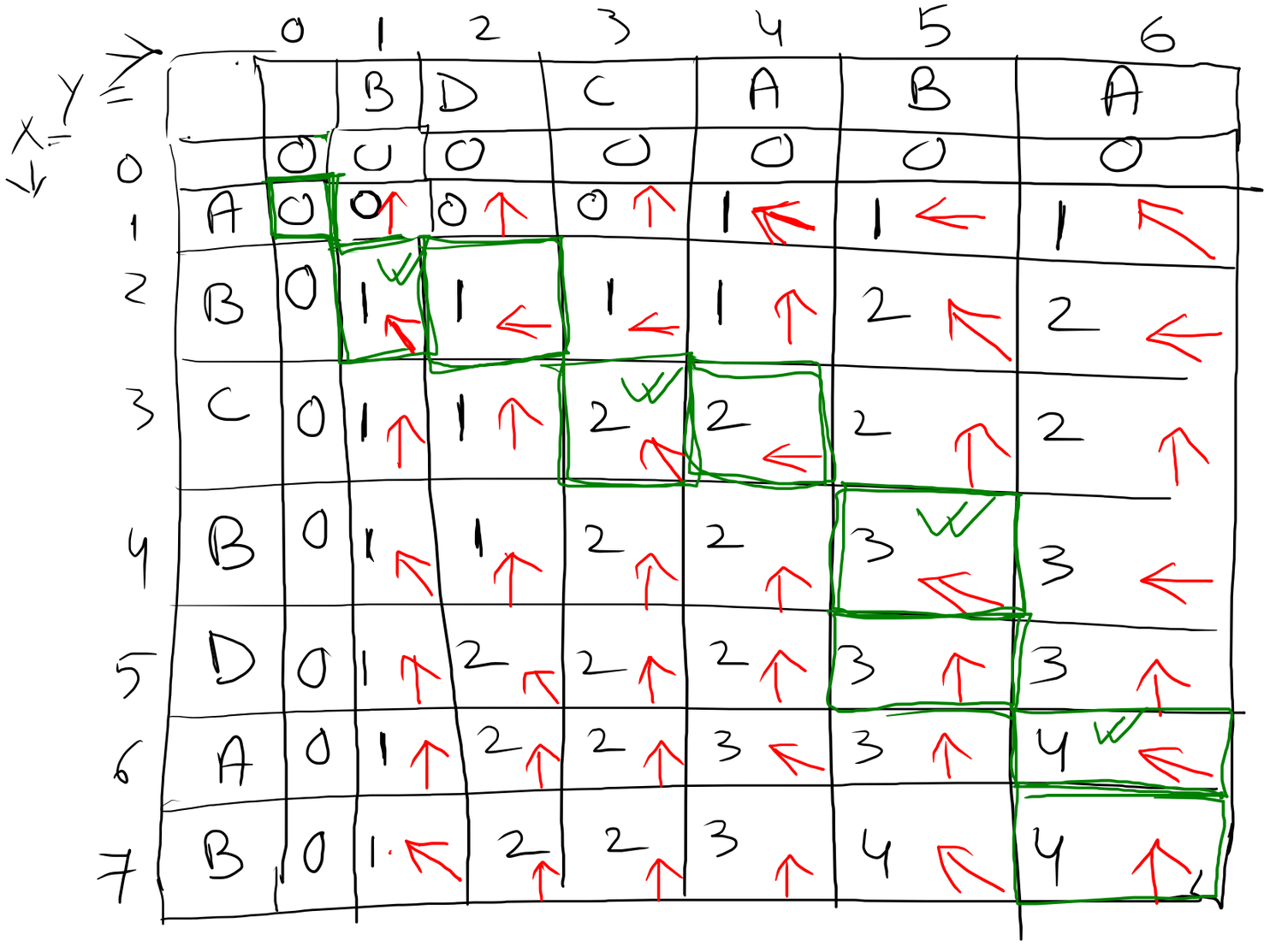
Running time:

$O(mn)$

Ex^m

X = A B C B D A B

Y = B D C A B A



BCBA