Cross-correlation and Autocorrelation.

Lecture 23

Definition: → Let $x(n)$ and $y(n)$ be the given two signals.

Problem: To find their degree of closeness.

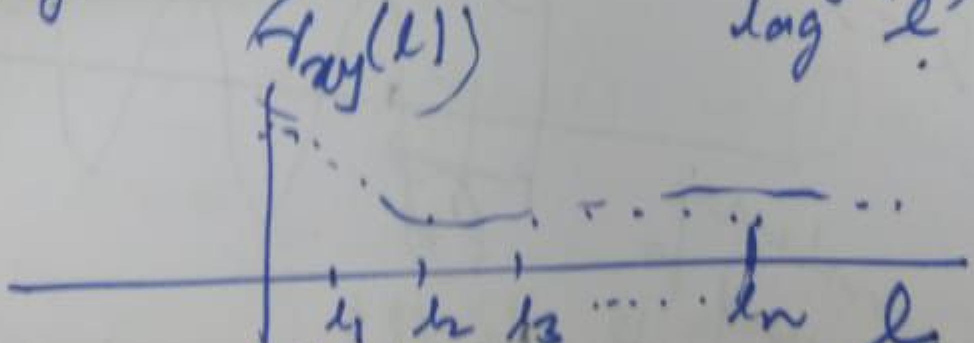
Cross-correlation function [$x(n)$, $y(n)$]

$$r_{xy}(l) = \sum_{n=-\infty}^{+\infty} x(n) \cdot y(n-l)$$

function

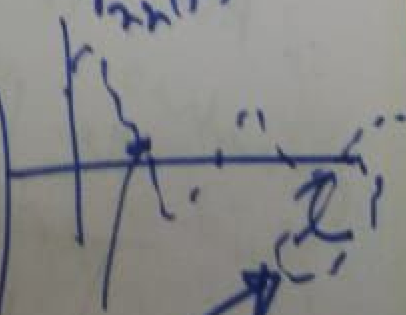
lag element, fixed

Delay $y(n)$ with lag 'l'



Autocorrelation function →

$$\boxed{r_{xx}(l) = \sum_{n=-\infty}^{+\infty} x(n) \cdot x(n-l)}$$

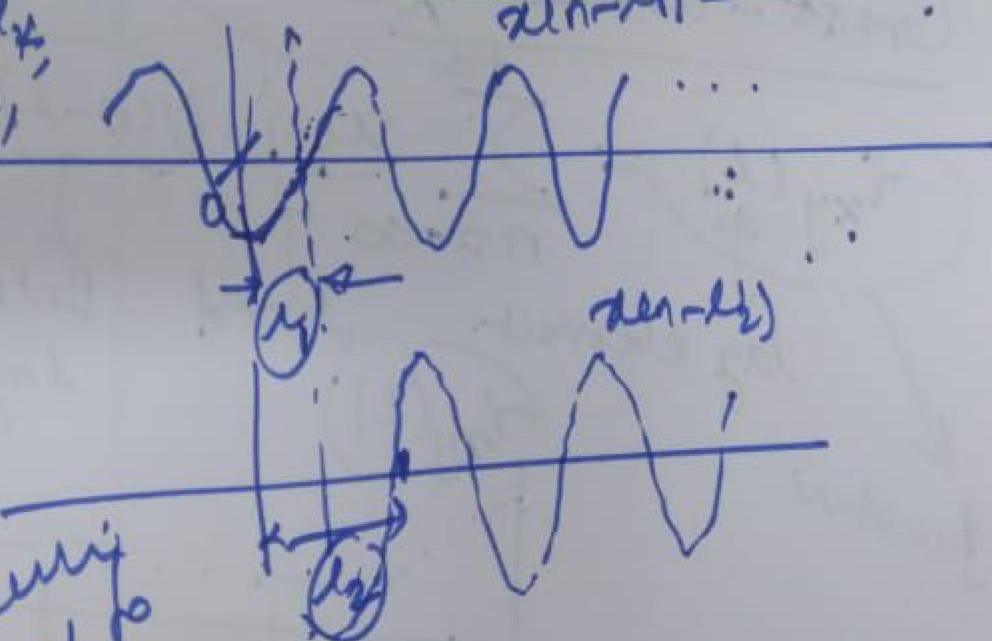


lag element

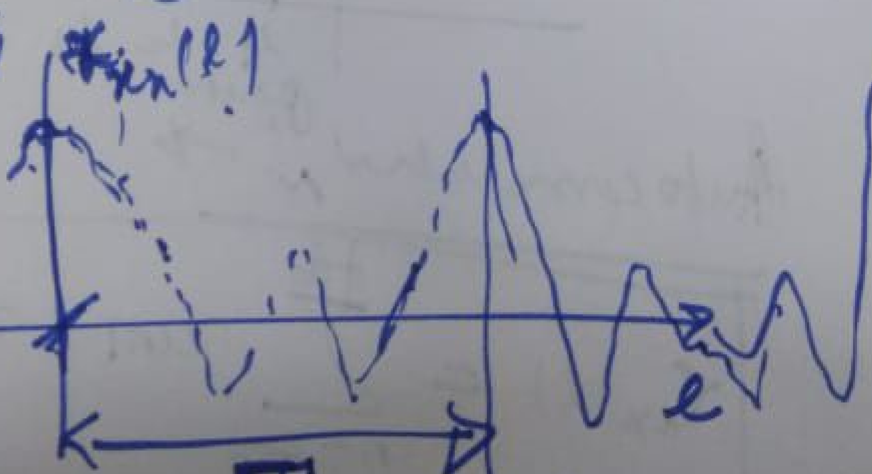
$T \rightarrow$ period of 'sine' func

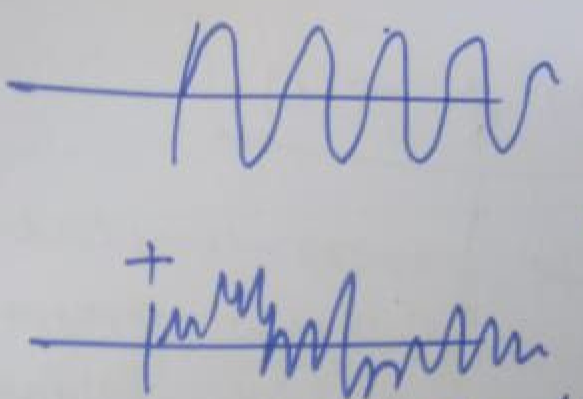
$$r_{xx}(l) = \sum_{n=-\infty}^{+\infty} x(n) \cdot x(n-l)$$

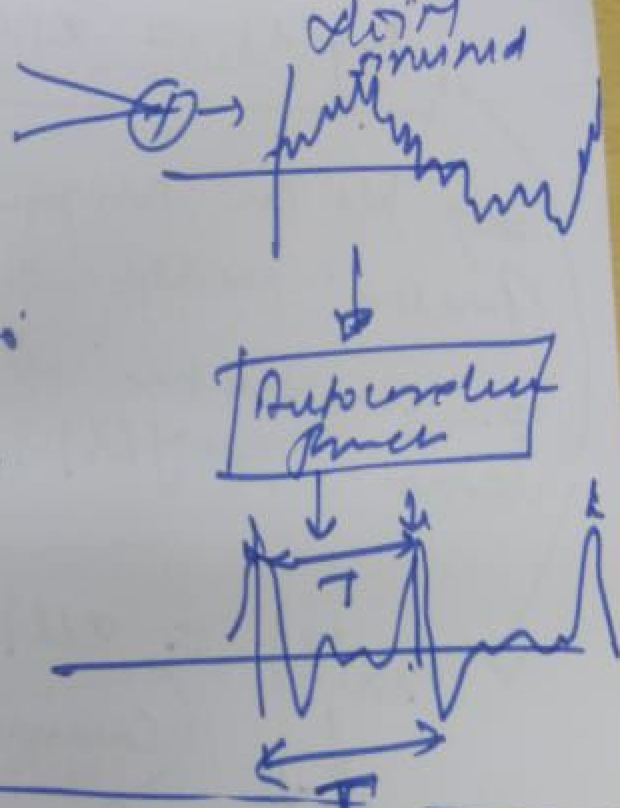
$l = l_1, l_2, l_3, \ldots, l = T$



∴ Autocorrelation function is used to find the periodicity of a periodic signal.







Autocorrelation function is "robust" under signal degradation condition.

Cross-correlation vs. Convolution (why?)

$$r_{xy}(l) = \sum_{n=-\infty}^{+\infty} x(n) \cdot y(n-l) \quad \text{--- (1)}$$

$$x(n) * h(n) = \sum_{k=-\infty}^{+\infty} x(k) \cdot h(n-k) \quad \text{--- (2)}$$

$$x(n) * y(n) = \sum_{k=-\infty}^{+\infty} x(k)\, y(n-k) \quad \text{--- (3)}$$

$$\boxed{r_{xy}(l) = x(l) * y(-l)} = \sum_{n=-\infty}^{+\infty} x(n) \cdot y(l-n)$$

$$x(l) * y(l) = \sum_{(k)} x(k) \cdot y(l-k) \quad (3)$$

$$\boxed{r_{xy}(l) = x(l) * y(-l)}$$

⇒ We can compute cross-correlation function using a convolution program i.e. convolution of $x(l)$ with flipped version of $y(l)$.

$$r_{xx}(l) = x(l) * x(-l)$$

$l = -l$

(Commutative property of convolution)

$$= x(-l) * x(l)$$

$$= x(+l) * x(l).$$

$\Rightarrow \therefore r_{xx}(l)\Big|_{l=-l}$

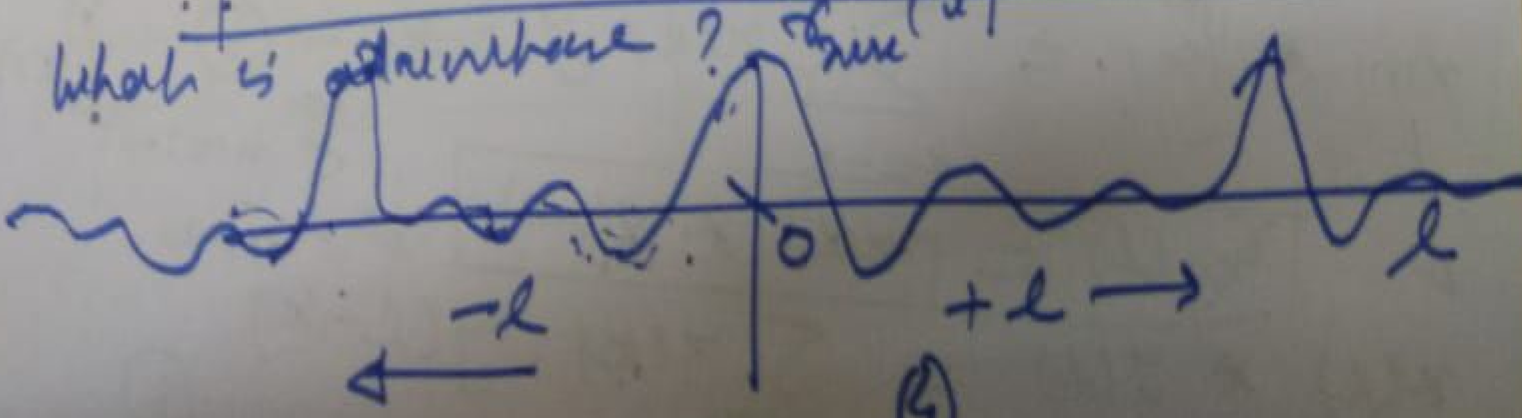
$r_{xx}(-l)$

⇒ We don't need to compute $r_{xx}(l)$ for negative value of l.

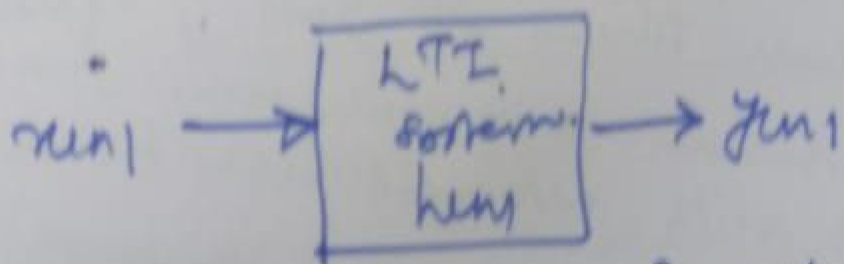
$\therefore$ * $\boxed{r_{xx}(l) = r_{xx}(-l)}$

Autocorrelation function is an even function

What is advantage? $r_{xx}(l)$



Response of LTI System in terms of Cross-correlation & Autocorrelation



Find cross-correlation of output with i/p.

$$\boxed{y(n) = x(n) * h(n)}$$

$$\therefore r_{xy}(l) = x(l) * y(-l)$$

$$r_{yx}(l) = y(l) * x(-l)$$

$$y(l) = x(l) * h(l) = y(n)\big|_{n=l}$$

$$\therefore r_{yx}(l) = [x(l) * h(l)] * x(-l)$$

Using Associative Property

$$r_{yx}(l) = [x(l) * x(-l)] * h(l)$$

(5) $$\boxed{r_{yx}(l) = r_{xx}(l) * h(l).}$$

Cross-correlation of the output y(n) and i/p x(n) with LTI system is convolution of autocorrelation function with system's impulse response

$$r_{yy}(l) = r_{xx}(l) * r_{hh}(l)$$

Autocorrelation function of $y(n)$ is equal to convolution of autocorrelation function of i/p $x(n)$ with autocorrelation function of LTI system's impulse response.

Topic: Prove that autocorrelation function at lag zero (i.e., $l=0$) is energy of signal.

Proof: $x(n)$ = Given signal

$$r_{xx}(l) = \sum_{n=-\infty}^{+\infty} x(n) \cdot x(n-l)$$

energy

For lag zero, $l = 0$.

$$\therefore r_{xx}(0) = \sum_{n=-\infty}^{+\infty} x(n)\, x(n) = \sum_{n=-\infty}^{+\infty} x^2(n)$$

Hence proved.

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Fourier Kingdom
or
Fourier Analysis

Fourier transform has ruled over signal processing and communication for 150 years. ⟹ and hence, it continues to do so.

Signals and Systems

$\boxed{\text{Signals and Systems}}$

Study ~~of~~ Linear Combination.

Basis Function / Basis Vectors / Representative Vectors

Shifted