Lecture 10

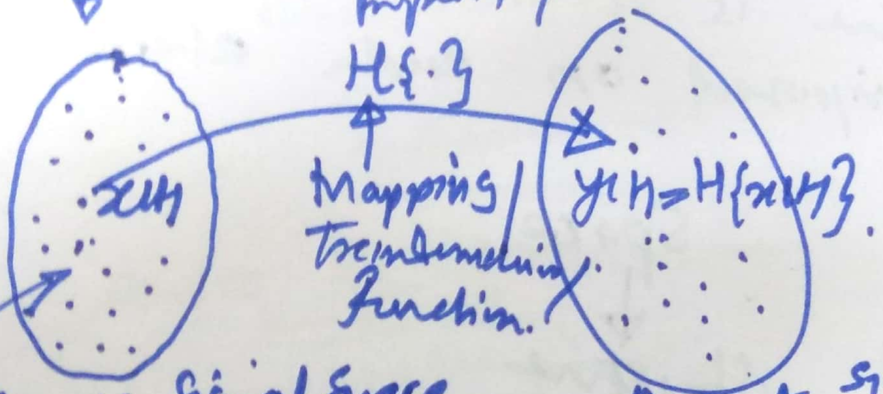
* Continuous-time and Discrete-Time Systems :→

System : { software, hardware }

1) CTS :→

Input Excitation signal $x(t)$ → System $H\{\cdot\}$ → Output Response $y(t) = H\{x(t)\}$.

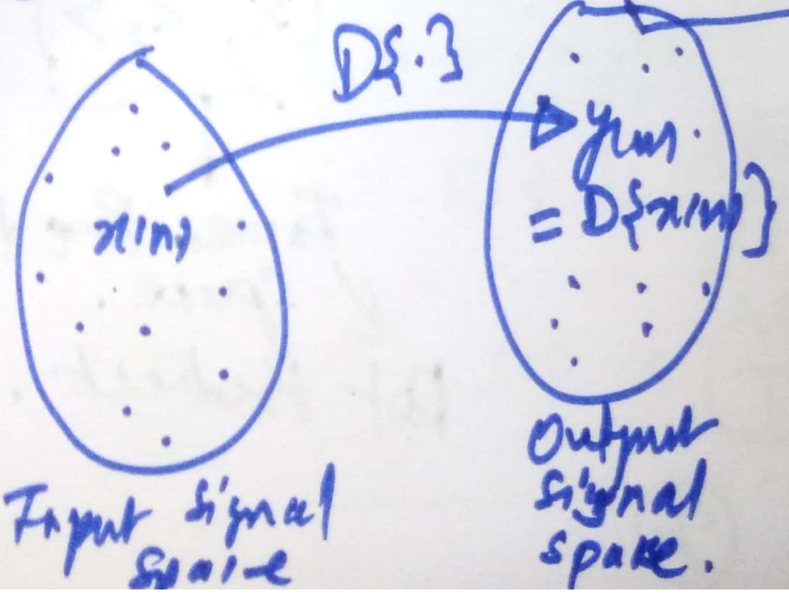
Property / Characteristics



Each dot corresponds to a signal $x(t)$

2) DTS

$x[n]$ → $D\{\cdot\}$ → $y[n] = D\{x[n]\}$.



①

Set → collection of objects {Tables, chairs, ~~[illegible]~~}

Space

Set + Structure ⇒ Space.

Structure ↓ Constraint / Condition

→ Structure is a constraint / condition that is imposed on each element of a set

Space

↓

Structure

↓ Algebraic

↓ $(X, +, \cdot)$

↓ Vector Space

↓ Geometric

↓ $(X, \langle , \rangle)$

↓ Inner Product Space.

↙ Dot Product.

CTS: $y(t) = H\{x(t)\}$.

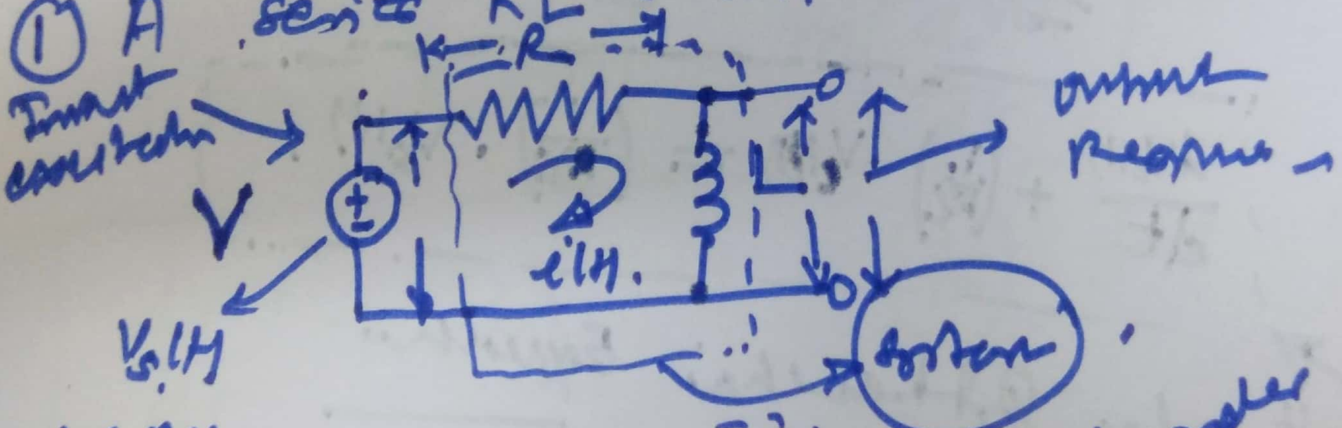
DTS: $y[n] = D\{x[n]\}$.

~~{H} or {D~~

$H\{\cdot\}$ or $D\{\cdot\}$ are called as system transformation or system properties or system characteristics.

Examples of systems: →

① A series RL circuit: →



KVL

$$V_s(t) = V_R(t) + V_L(t)$$

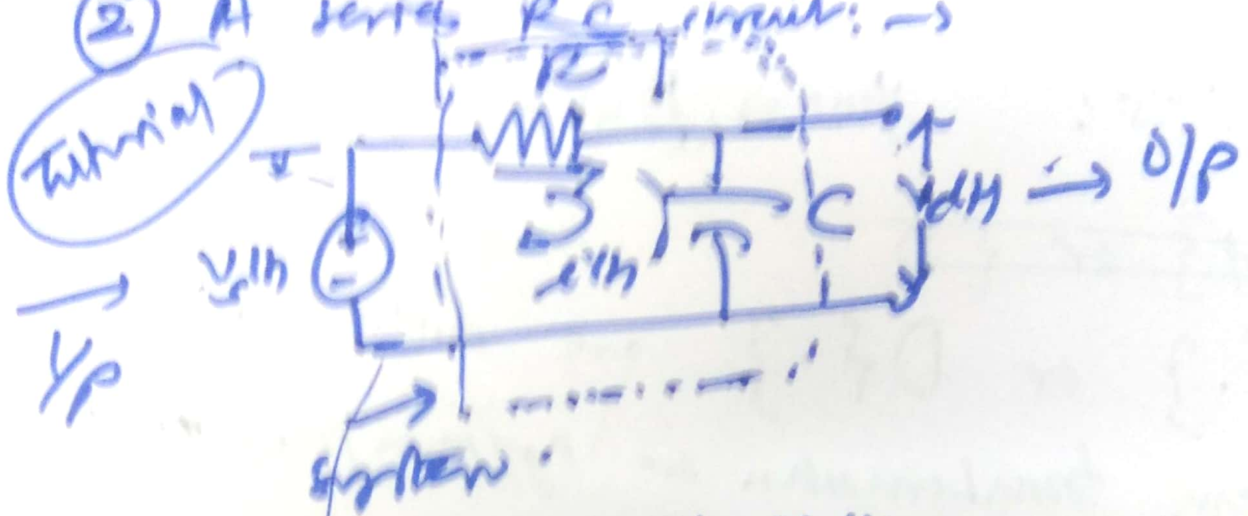
$$V u(t) = i(t)\cdot R + L\frac{di(t)}{dt}$$

1st order linear constant coefficient Differential equation

$$\frac{di(t)}{dt} + \left(\frac{R}{L}\right)\cdot i(t) = \left(\frac{V}{L}\right)\cdot u(t)$$

② A series RC circuit: →

Tutorial



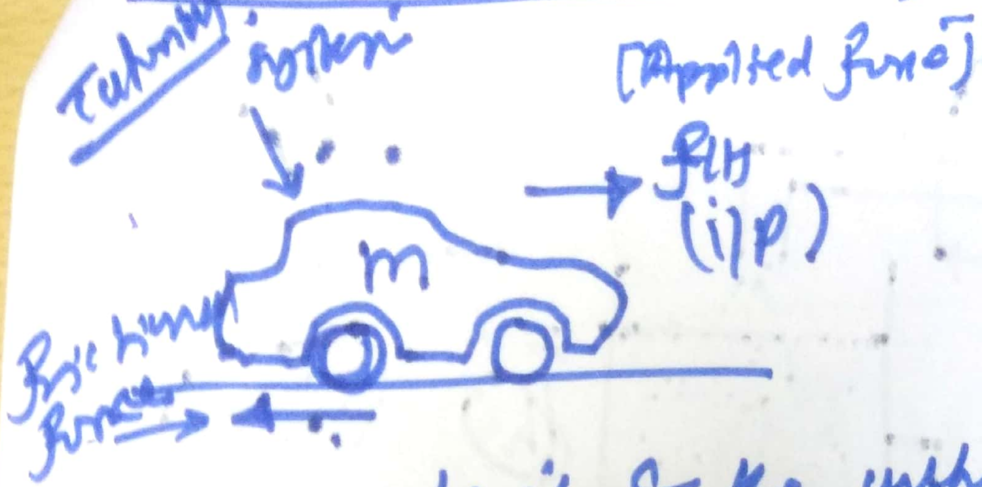
$$\underline{\underline{i(t)}} = \frac{v_s(t) - v_c(t)}{R} \quad \text{--- (1)}$$

$$i(t) = C\,\frac{dv_c(t)}{dt} \quad \text{--- (2)}$$

$$\frac{dv_c(t)}{dt} + \left(\frac{1}{RC}\right) v_c(t) = \left(\frac{1}{RC}\right) \cdot v_s(t).$$

1st order differential Equation

① An automobile system: →



v(t) → velocity of the automobile as the o/p.

$$F = m(a)$$

$$a = \frac{dv(t)}{dt}$$

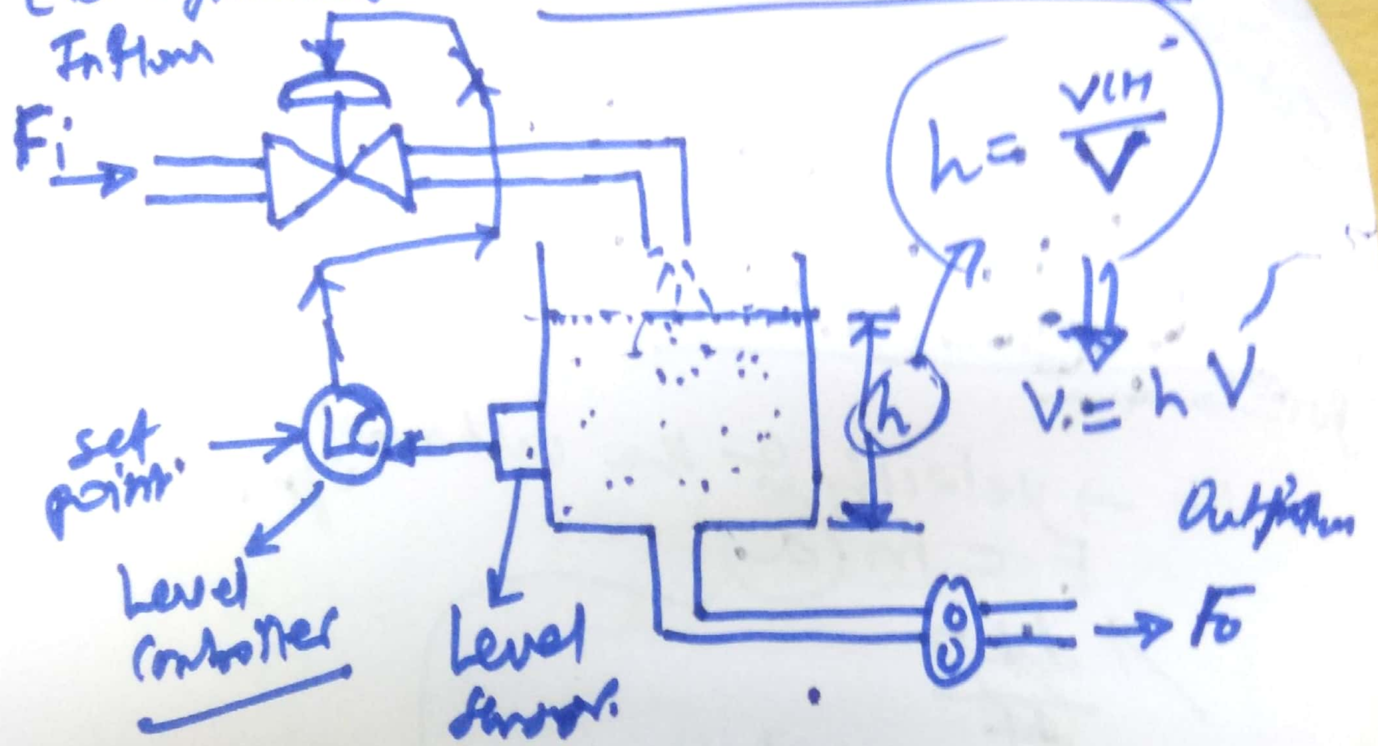
Net force acting on body.

$$f(t) - \beta . v(t) = m\left(\frac{dv(t)}{dt}\right)$$

$$\boxed{\frac{dv(t)}{dt} + \left(\frac{\beta}{m}\right) . v(t) = \left(\frac{1}{m}\right) . f(t)}$$

1st order differential equation.

[4] Dynamics of Level Control System:



Level Control Loop

→ The rate of change of tank contents equals the difference b/w inflow & outflow

$$\frac{d(V(h))}{dt} = F_i - F_o$$

F_i = input flow to the tank

F_o = output — " —

$V(h)$ = tank content at a particular time

$$V \frac{dh}{dt} = F_i - F_o$$

⑥

Assuming a linear model, i.e., flow of fluid out of the tank is proportional to the head of the liquid / fluid

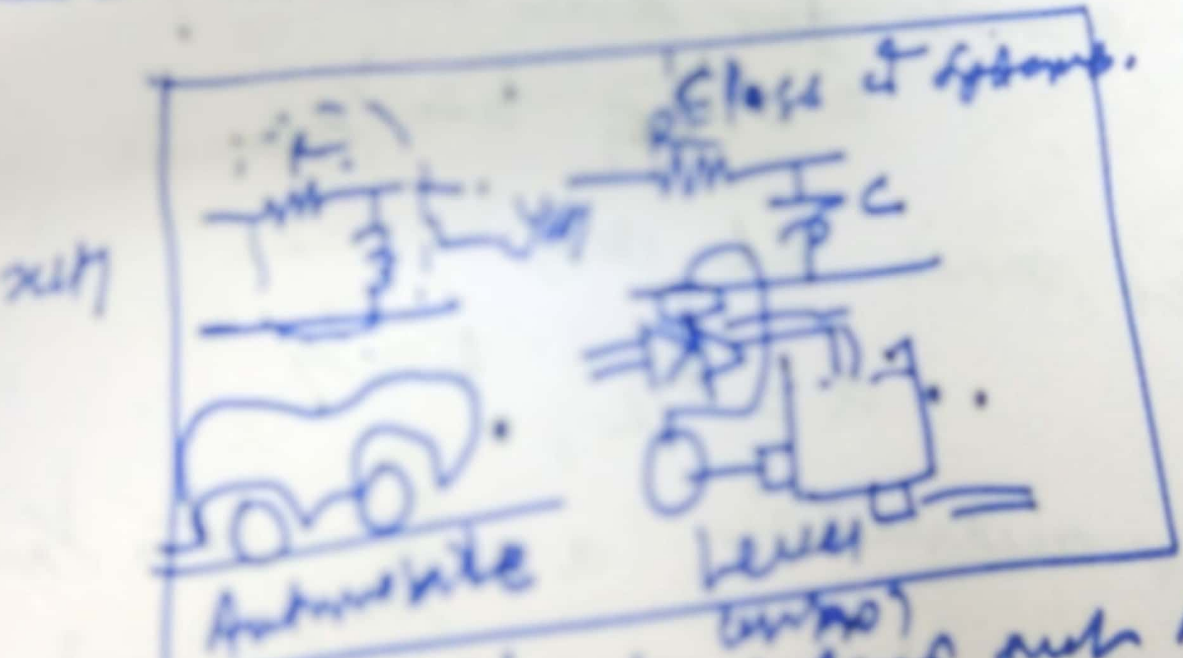
$F_o = K h$ o/p

$$\frac{dh}{dt} + \left(\frac{K}{V}\right) h = \left(\frac{1}{V}\right) F_i$$

i/p.

V

1st order differential equation



All together different systems such as RC, RL automobile, level control have dynamics described by a common 1^{n}- order differential equation.

7

$$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$$

$$\boxed{\frac{dy(t)}{dt} + a \cdot y(t) = b\, x(t)}$$

Mathematical model of a class of systems described by 1^{st} order DE. where constants 'a' and 'b' are dependent on actual physical system

For example, for series RC ckt,

i) $a = 1/RC$, $b = 1/RC$

ii) For series RL ckt

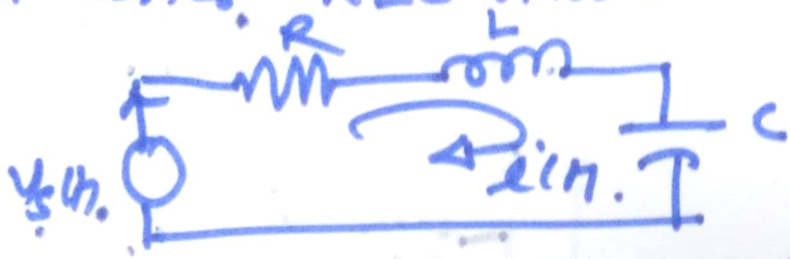
$\therefore a = R/L$, $b = 1/L$

iii) For automobile, $a = B/m$, $b = 1/m$,

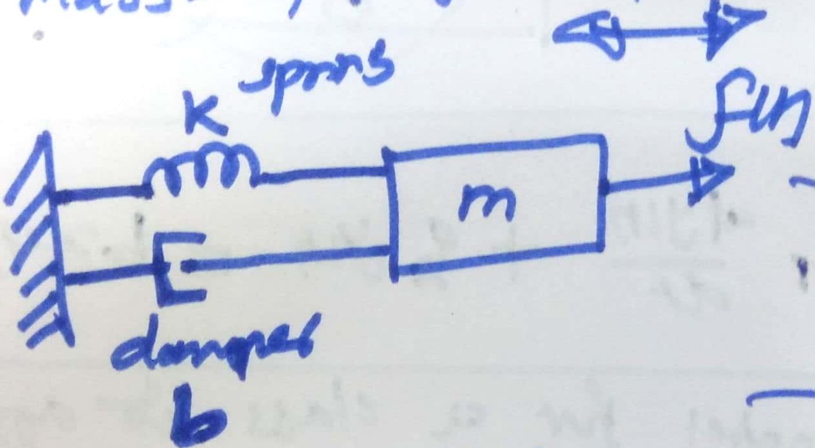
iv) For level control, $a = \frac{K}{V}$, $b = \frac{1}{V}$.

Tutorial

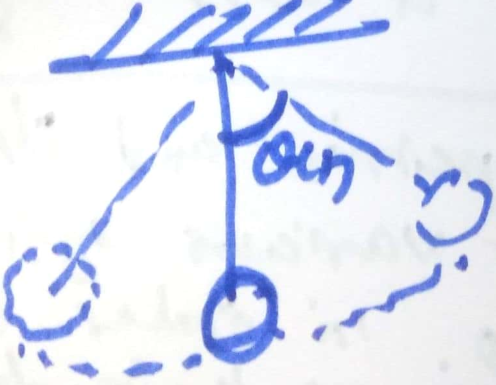
1) A series RLC circuit



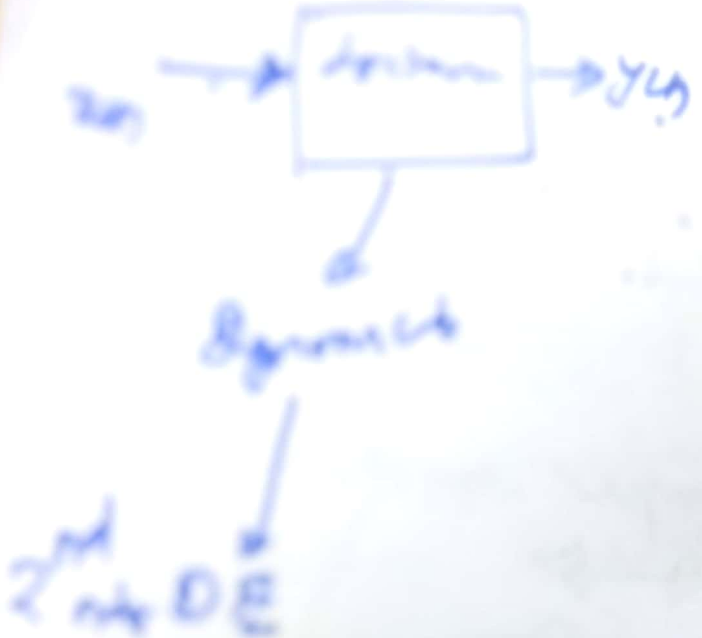
2) Mass-spring-damper system

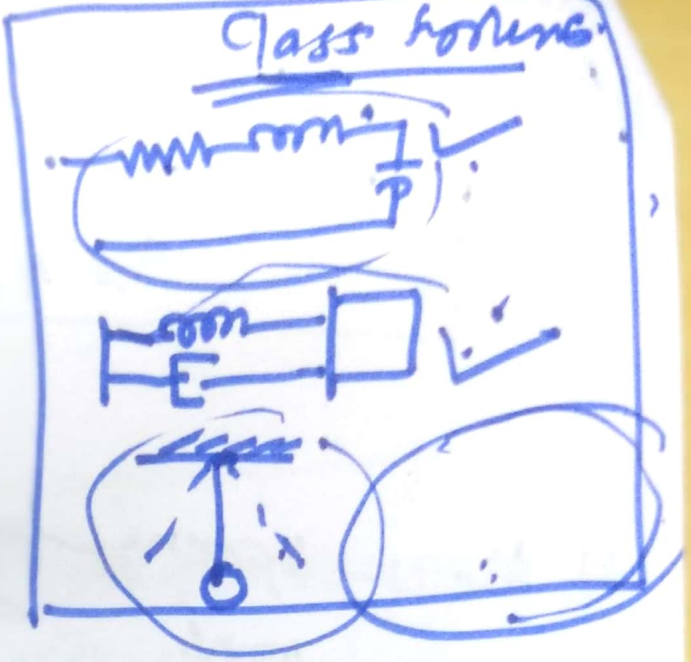


3) Simple pendulum



second order differential equation





$$a_0 \frac{d^2y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_2\, y(t) = b\, x(t).$$

Mathematical model for a class of systems

Q. What is key goal of course on Signals & Systems

→ ① To analyze properties and structures or characteristics of various systems in a common class. in order to get deeper understanding of their dynamics and behaviour. ⇒ To develop a common mathematical model such as differential equation

② We can use common mathematical (DE) to model many other physical system in the same common class.

Example : We can model speech production mechanism, seismic signal recording, o/p of communication channel, etc.