Lecture 38

Q. Justification for the term Group Delay Function

Why $\tau(\omega) = -\frac{d}{d\omega}[\angle H(\omega)]$ is called so ??

Problem :→ Consider the impulse response of an all ~~pile~~ pass system with a group delay that varies with frequency. The frequency response $H(\omega)$ [or $H(j\omega)$] for this problem is ~~the~~ product of three factors.

$$H(\omega) = \prod_{i=1}^{3} H_i(\omega) = H_1(\omega) \cdot H_2(\omega) H_3(\omega)$$

where

$$H_i(\omega) = \text{all-pass characteristics} = \frac{a_i + jb_i}{a_i - jb_i}$$

$$H_i(\omega) = \frac{1 + \left(\frac{j\omega}{\omega_i}\right)^2 - 2j\,\xi_i\,(\omega/\omega_i)}{1 + \left(\frac{j\omega}{\omega_i}\right)^2 + 2j\,\xi_i\,(\omega/\omega_i)}$$

$\omega_i = 2\pi f_i$

$f_1 \cong 50\,Hz$, $f_2 \cong 150\,Hz$, $f_3 \cong 300\,Hz$

$|H_i(\omega)| = 1$

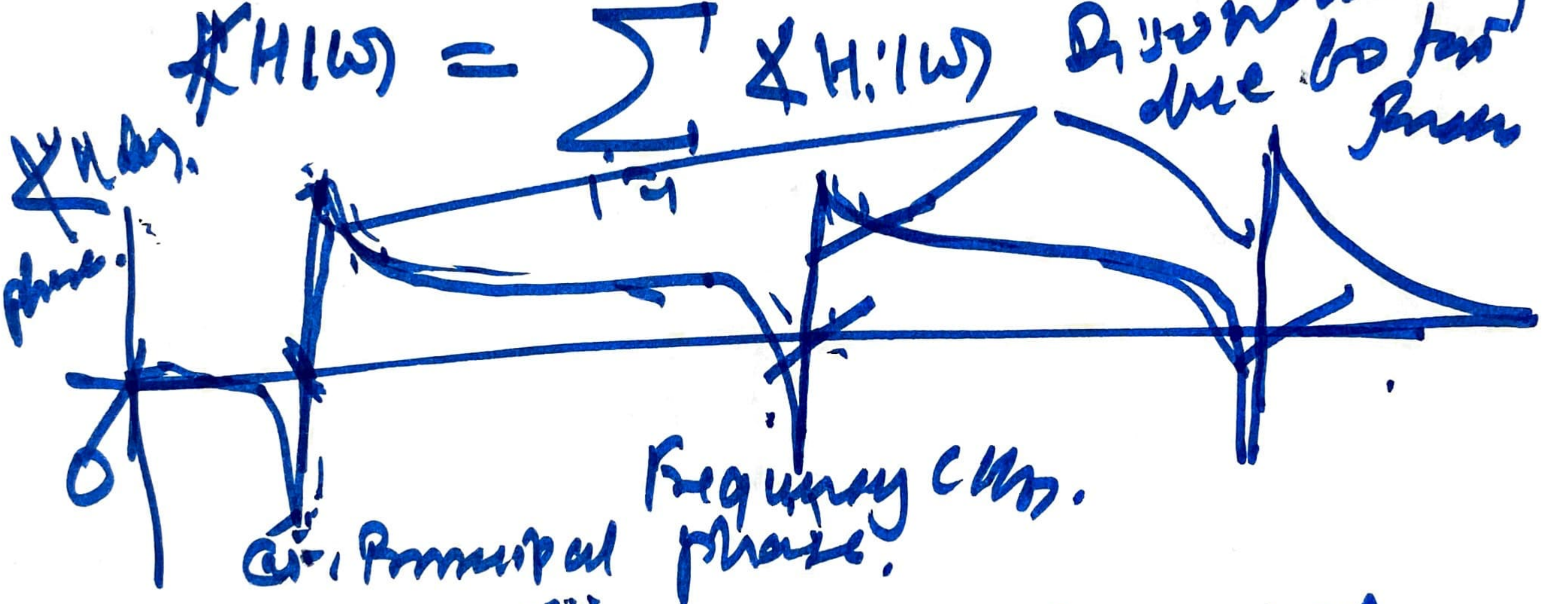
1

The phase for each $H_i(\omega)$ can be determined as

$$\measuredangle H_i(\omega) = -2\tan^{-1}\left(\frac{2\xi_i(\omega/\omega_i)}{1-(\omega/\omega_i)^2}\right)$$

and

$$\measuredangle H(\omega) = \sum_{i=1}^{3} \measuredangle H_i(\omega)$$



$$\tan^{-1}(2\pi + \theta) = \tan^{-1}(4\pi \pm \theta) = \tan^{-1}(2\pi k + \theta) = \tan^{-1}(\theta)$$

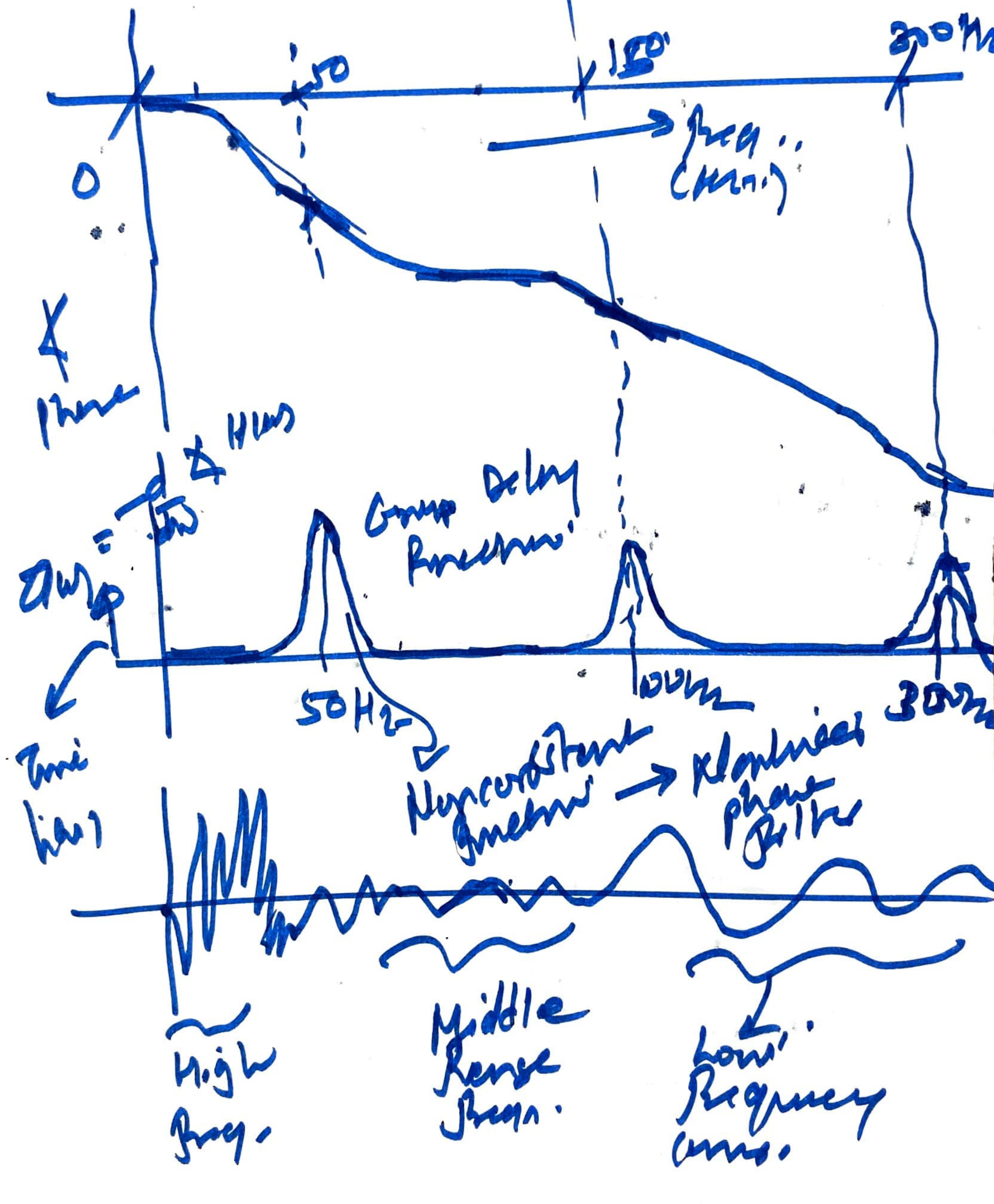
Phase wrapping.

↳ It creates discontinuity in phase

Hence

$$\therefore\ T(\omega) = \left(\frac{-d}{d\omega}\measuredangle H(\omega)\right)$$ is not possible to evaluate at discontinuity

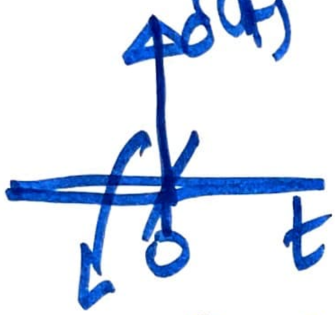
∴ We perform phase unwrapping.

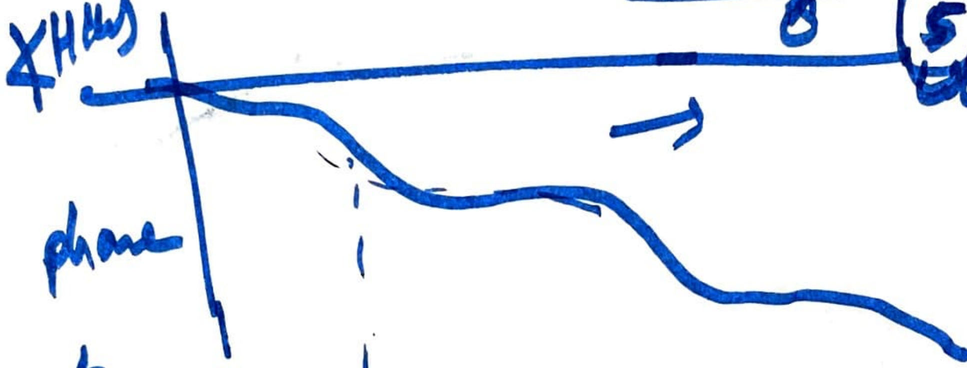


Meaning of the term Group Delay.

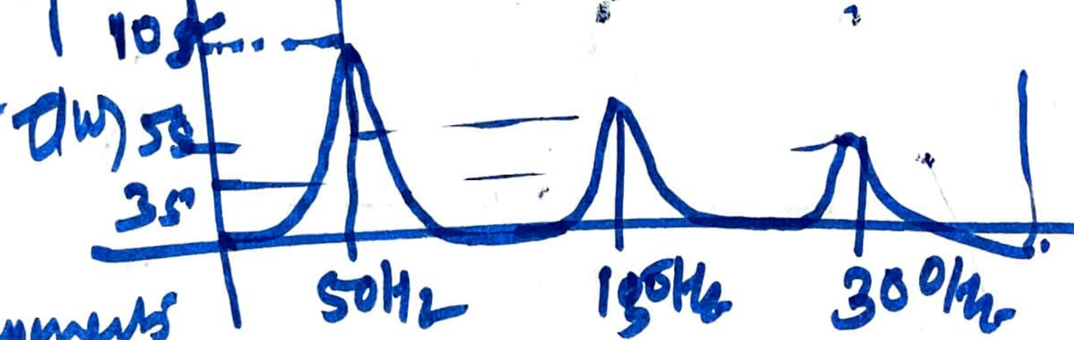
$x(t) \longrightarrow$ [Nonlinear phase, $h(t) \to H(\omega)$] $\longrightarrow y(t) = \delta(t-5)$

$x(t) = \delta(t)$..





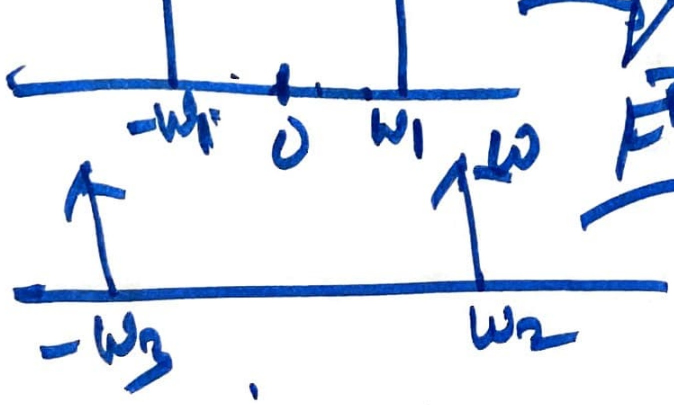
$\mathcal{F}\{\delta(t)\} = 1$



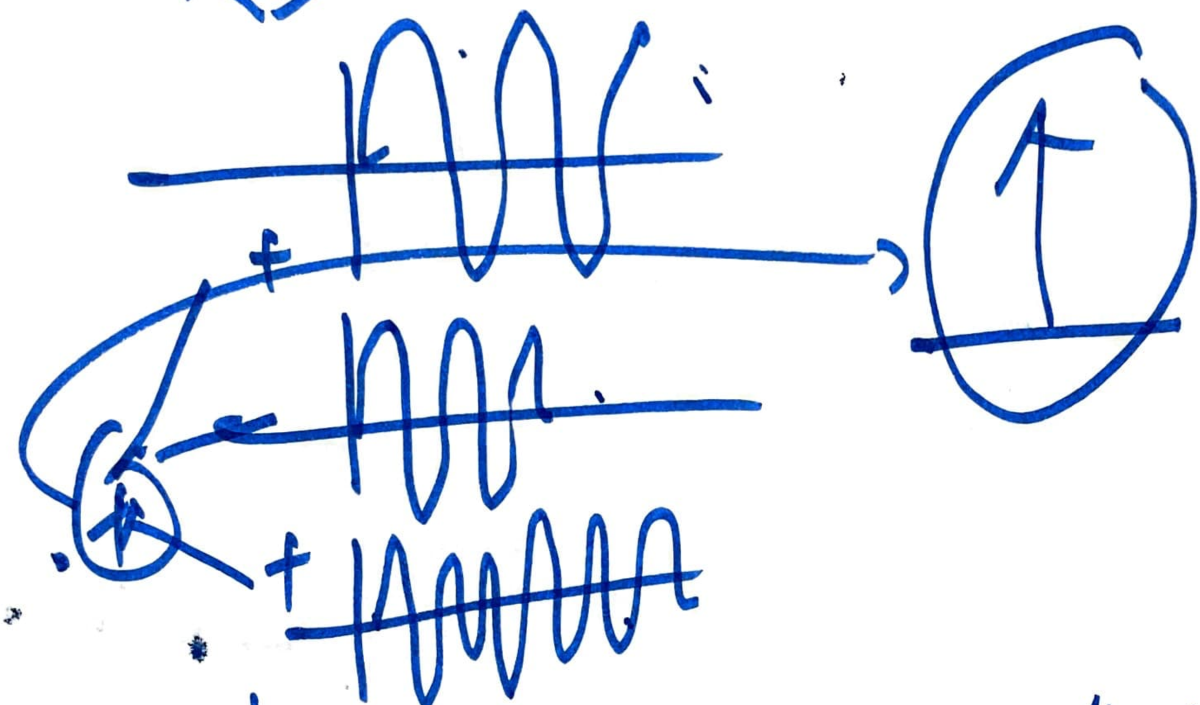
⇒ Propagate contains all the freq. components

Frequency (Hz).

$= \sum \delta(\omega)$



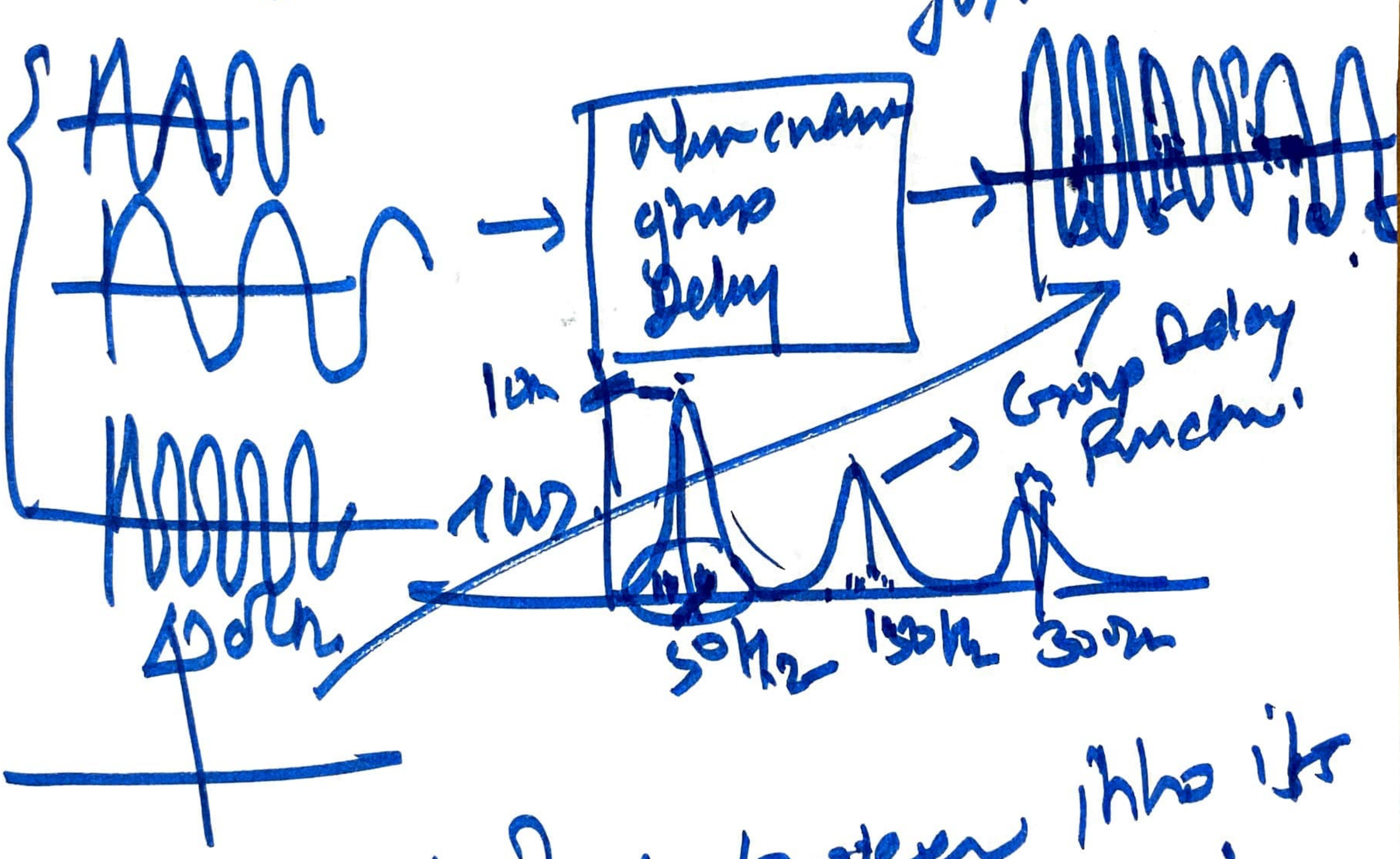
$$\delta(t) \cong \sum_{\langle n \rangle} \cos(\omega_n t)$$



Impulse ! is such a signal that is formed by adding infinite number sinusoids of infinitesimal small frequencies.

∴ When an impulse is passed through a linear phase filter then the output signal is $y(t) = \delta(t-5)$ i.e. all the infinite sinusoids experience the same amount of delay equal to the slope of phase function of the filter i.e. all the sinusoids in $\delta(t)$ align in such a way that they form the impulse at a delay of 5 seconds.

On the other hand, if we pass the same impulse $\delta(t)$ through a filter having nonlinear phase characteristics



∴ $\delta(t)$ [Impulse] is broken into its different components appearing at different locations (determined by characteristics of group delay function $\tau(\omega)$). This phenomenon is called as pulse dispersion

* Applications of Group Delay Function.
GDS.

1) Telecommunication Networks.
GDS is used to assess quality of telecommunication (transmission characteristics). → Bell Labs, USA.
$\boxed{PP.435}$

2) Speech Technology
(Design of Vocoder → Prof. Heidiki Kawahara, Japan.

3) GDS is used as feature set for speech and speaker recognition, anti-spoofing algorithms in voice biometrics,

4) Detection of Normal vs. Whispered speech
↳ Amazon has a patent
↳ Voice Assistants

(6) τ(ω) is to perform speech segmentation at syllable-level

कोकोर = कोर + कोर

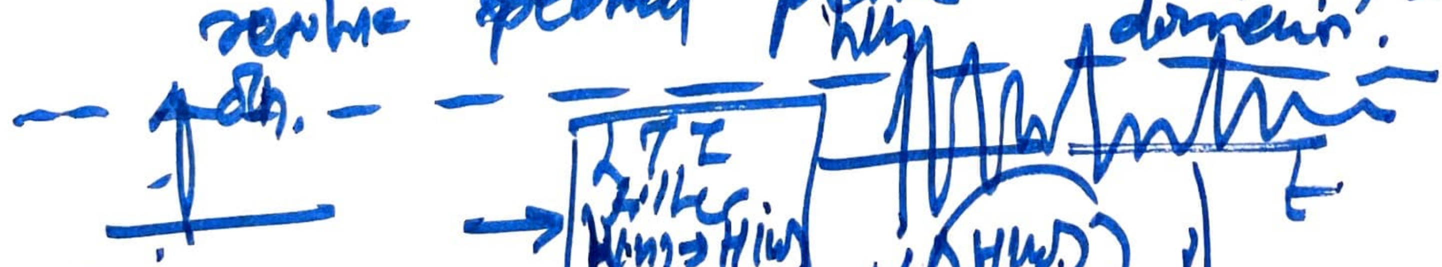
syllable

τ(ω)

(7) τ(ω) is ~~not~~ noise-robust

[Autocorrelation function $R_{xx}(\tau)$ is also noise-robust]

(8) τ(ω) has good spectral resolution ⇒ good capability to resolve spectral peaks in frequency-domain.



$x(n) =$

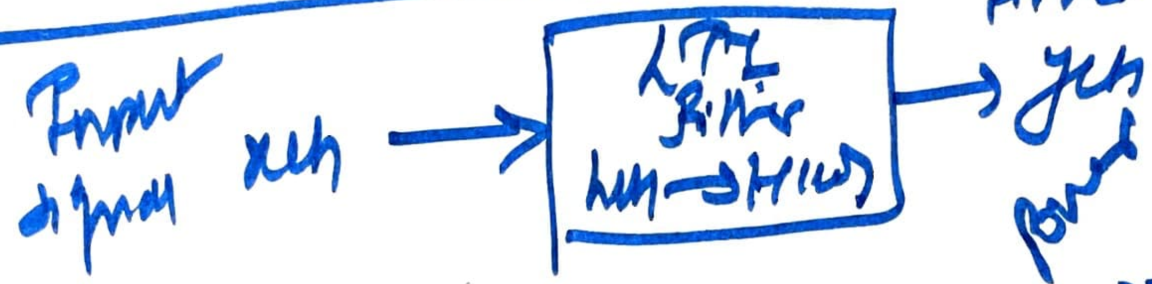
$H(\omega) = |H(\omega)| \cdot e^{j\theta(H(\omega))}$

$\tau(\omega) = -\frac{d}{d\omega}(\angle H(\omega))$. $y(n) = h(n)$

system characteristics (8)

6.2.3

Log-Magnitude and Bode Plots



$y(n) = x(n) * h(n)$

Convolution Theorem

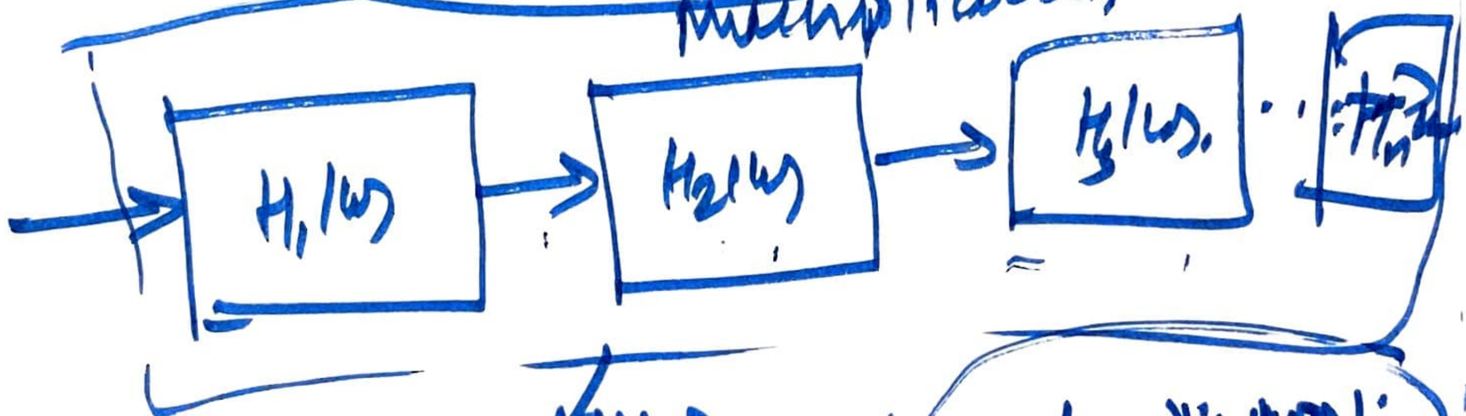
$\therefore Y(\omega) = X(\omega) \cdot H(\omega)$

$|Y(\omega)| = |X(\omega)| \cdot |H(\omega)|$

multiplication

$\Rightarrow |H(\omega)| = \frac{|Y(\omega)|}{|X(\omega)|}$

$\therefore |H(\omega)|^2 = \frac{|Y(\omega)|^2}{|X(\omega)|^2}$



$H(\omega)$

$\log_{10} |Y(\omega)| = \log_{10} |X(\omega)| + \log_{10} |H(\omega)|$

$\log_{10} |H(\omega)|$

$|H(\omega)|^2 = \frac{|Y(\omega)|^2}{|X(\omega)|^2} = \frac{\text{Output Power}}{\text{Input Power}}$

Power ratio.

(9)

decibel $= 10\log_{10}|H(\omega)|^2$ dB

Cascade of ten LTI systems.

In the honour of inventor of Telephone

Bell Labs ⇒ Alexander Graham Bell

$20\log_{10}|H(\omega)| = 0\ \text{dB}$

$1 \rightarrow$ ⇒ LTI filter has magnitude spectrum $=1$

$20\log_{10}|H(\omega)| = 20\ \text{dB}$

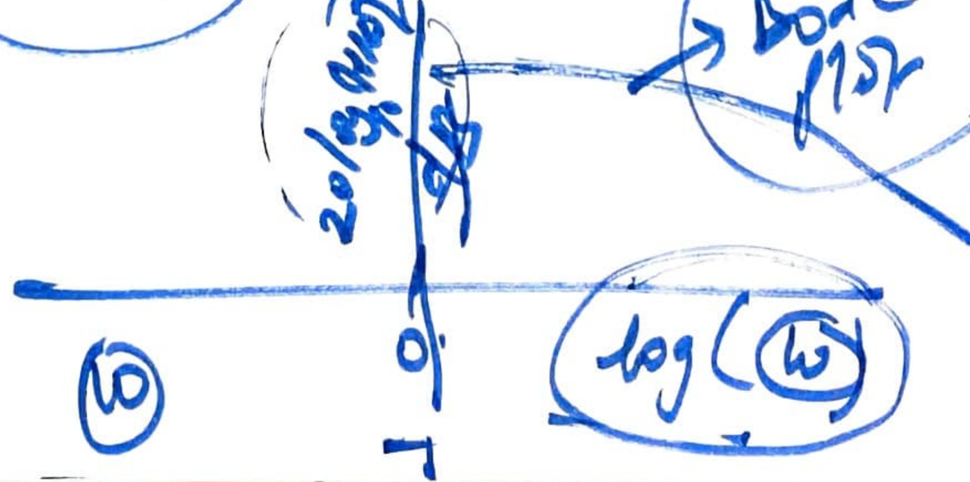
$\Rightarrow |H(\omega)| = 10$ = gain of 10.

$20\log_{10}|H(\omega)| = -20\ \text{dB} \Rightarrow |H(\omega)| = 0.1$

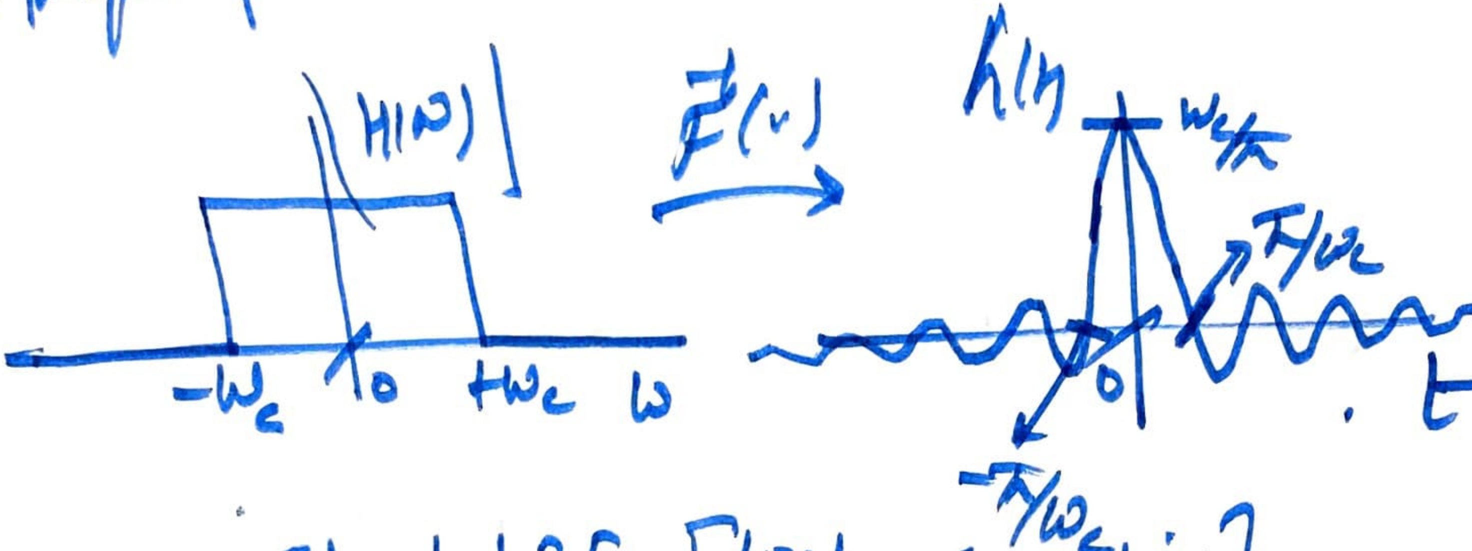
Bode plot

CTFT

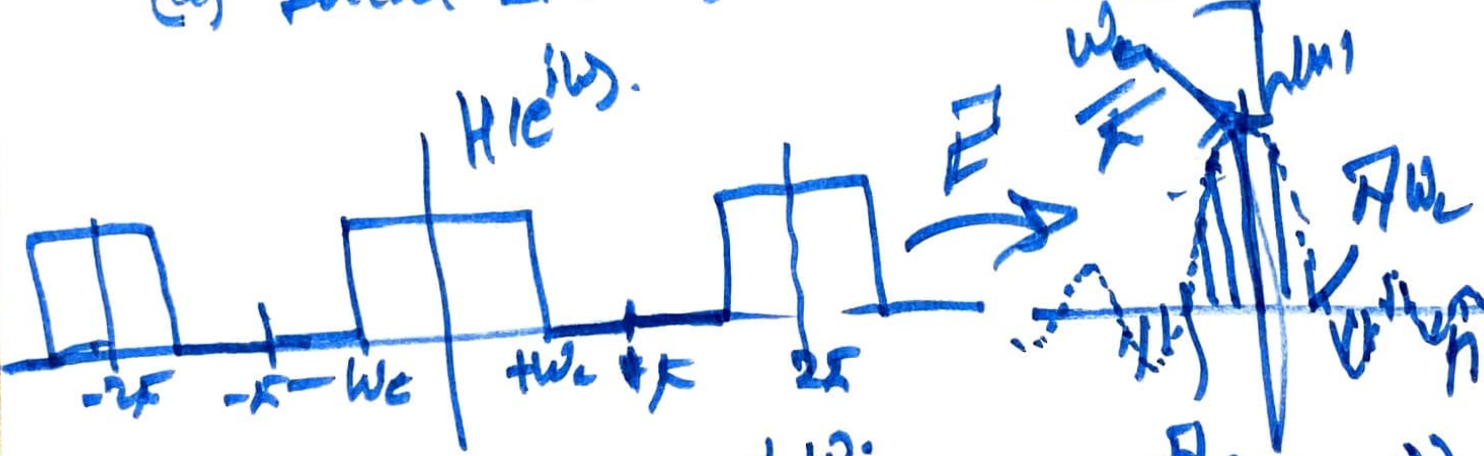
aperiodic in frequency

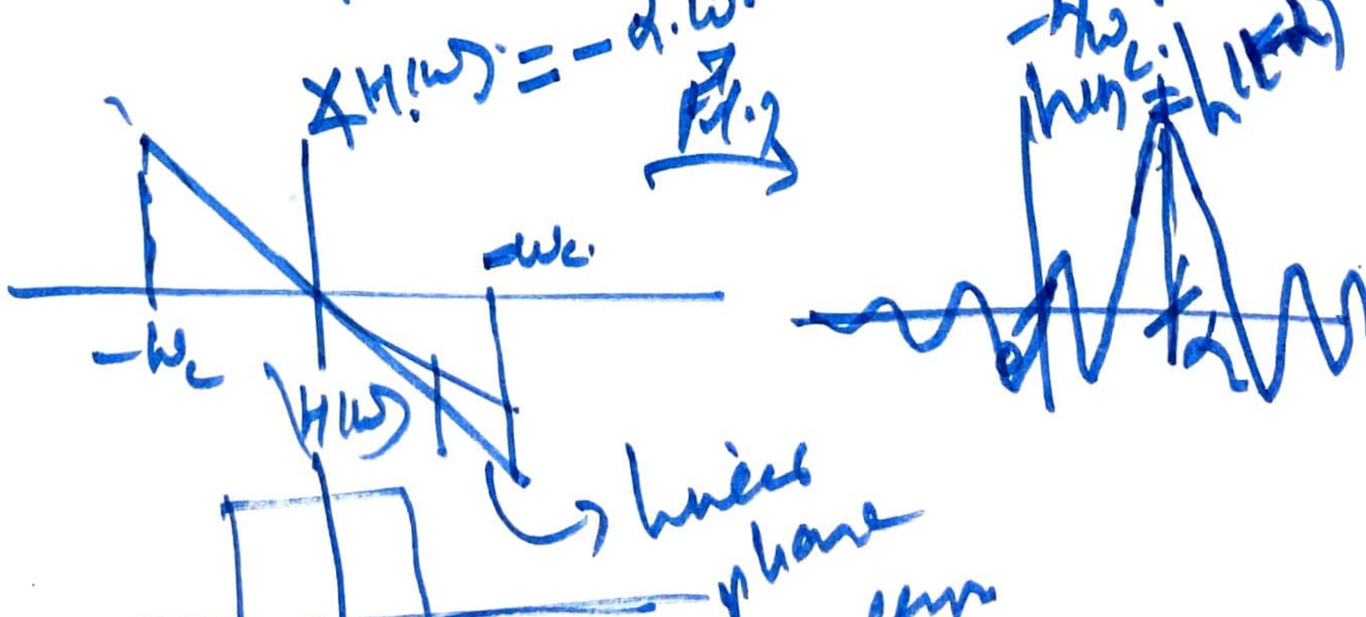


3 | Time domain properties of Ideal frequency selective filters.



(a) Ideal LPF [continuous time]



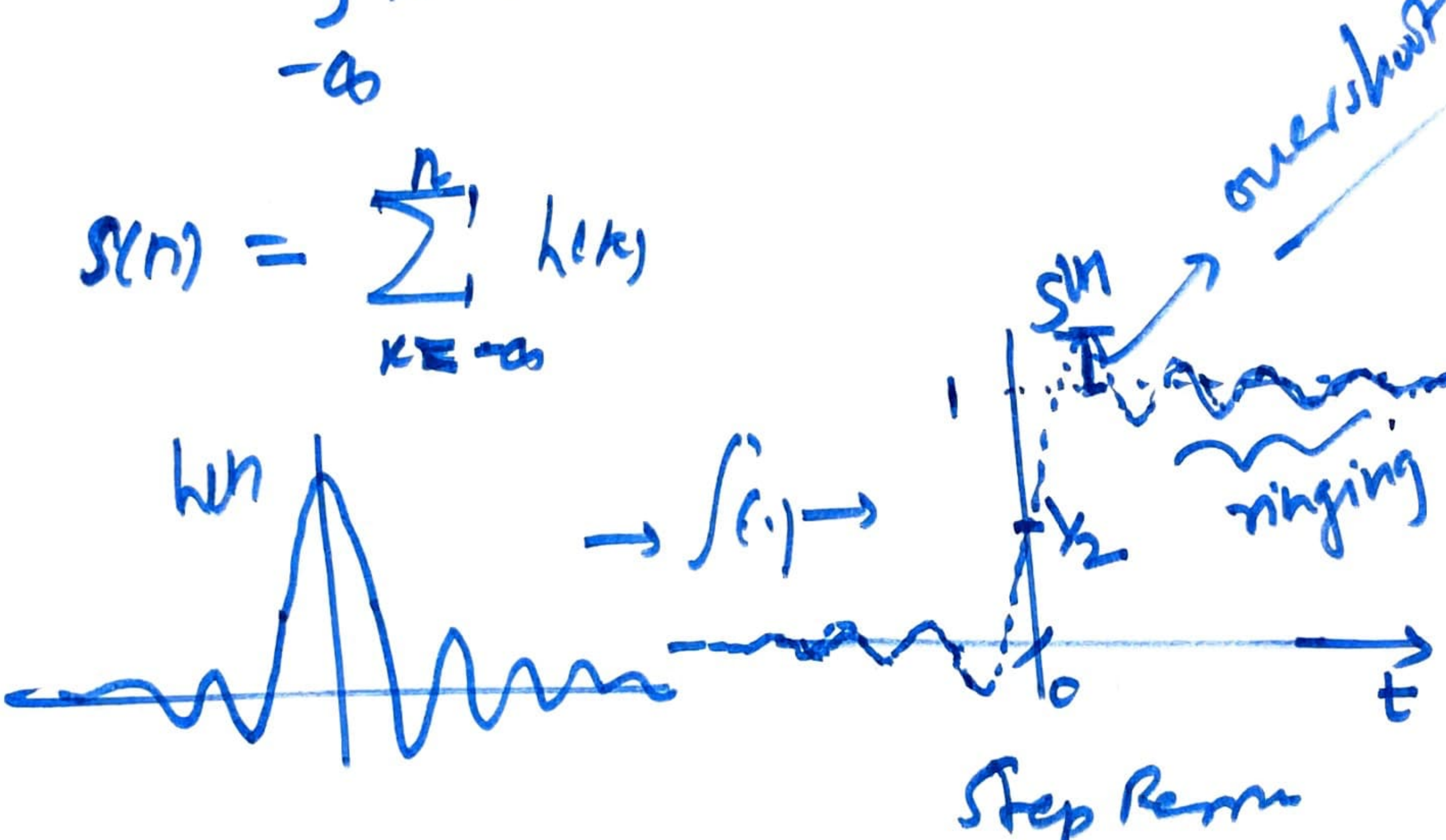


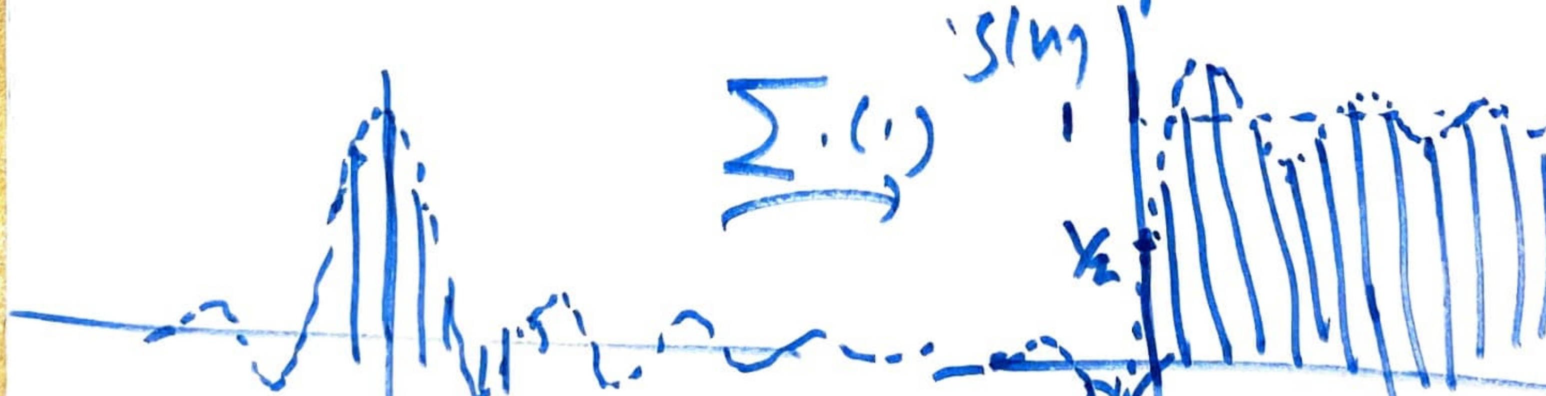
linear phase characteristics

Step Response of the LTI filter

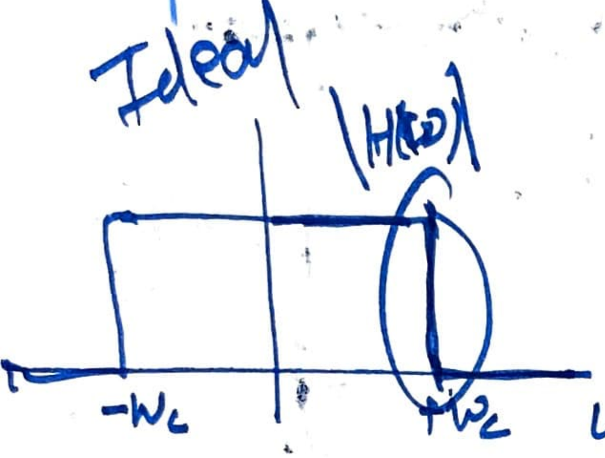
$$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$$

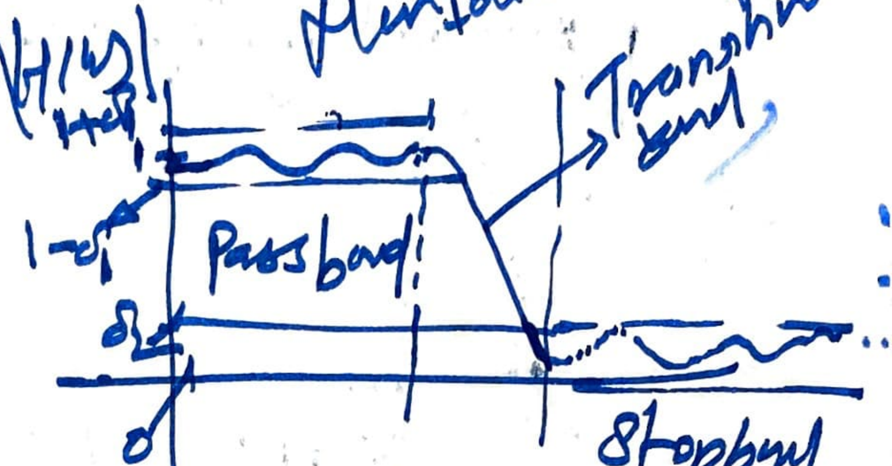
$$s(n) = \sum_{k=-\infty}^{n} h(k)$$





4] Time-domain and Frequency-Domain Aspects of Non-Ideal Filters.





Ideal

δ_1 = passband ripple

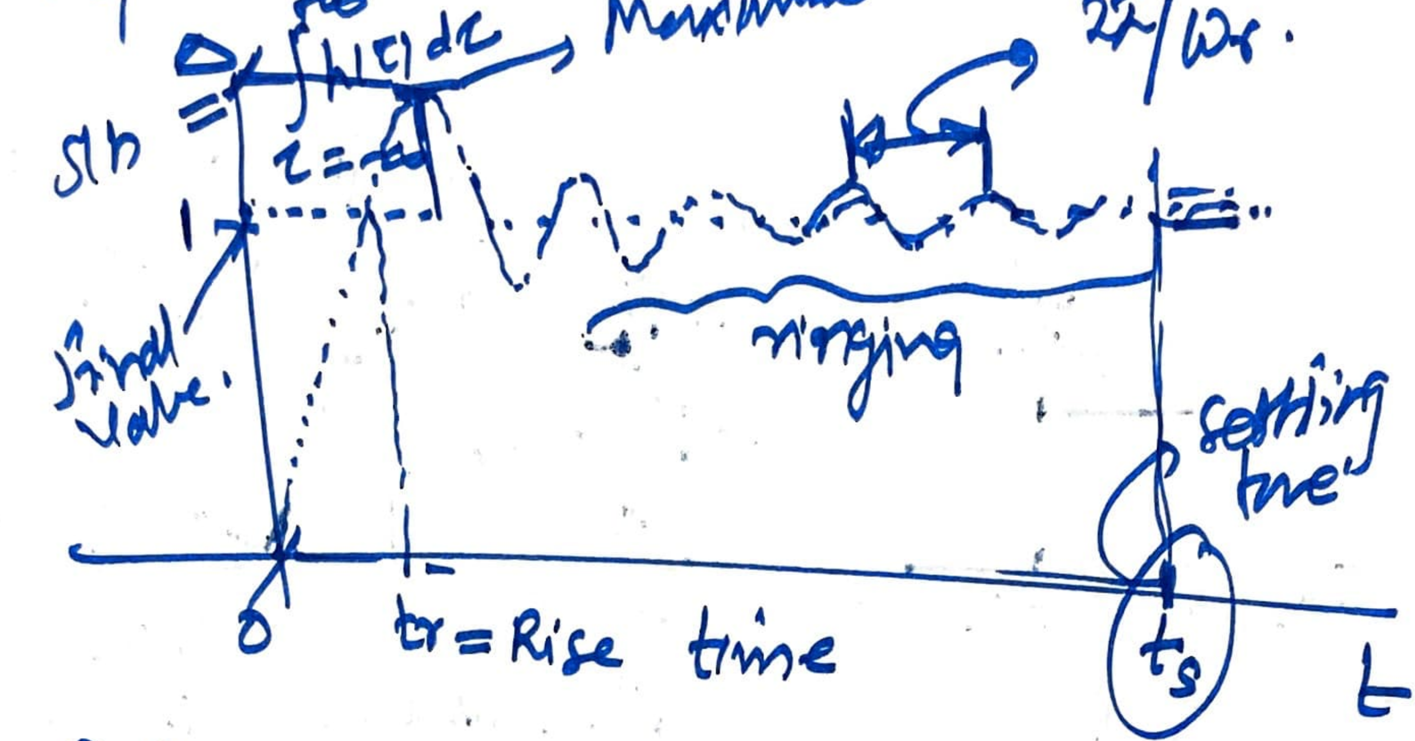
δ_2 = stopband ripple

$x(t) = x_1(t) + x_2(t)$

$X(\omega) = X_1(\omega) + X_2(\omega) \rightarrow X_1(\omega)$ & $X_2(\omega)$ are overlapping spectra

To achieve sharp transition band, we need large cascade / value of electronic components R, L, C or resistors, multiplier and adders. ⇒ very costly

Step Response



1) Rise time (t_r) ⇒ the interval over which the step response of the filter rises towards its final value

2) Ringing → Oscillatory behaviour of step response after rise time 't_r'. If ringing is present.

3) Overshoot (Δ) →

4) Ringing frequency (ω_r)

5) Settling time (t_s) ⇒ time required for the step response to settle down to within a specified tolerance of its final value.