1. With $x_{-2} = 0, x_{-1} = 1$, and $x_n = x_{n-1} + x_{n-2}, \forall n \geq 0$ represents the Fibonacci series. In order to compute x_n, you need to know x_{n-1} and x_{n-2}. We can re-write this sequence using matrices as follows:

$$\begin{bmatrix} x_n \\ x_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_{n-1} \\ x_{n-2} \end{bmatrix}, \forall n \geq 0$$

   Using such a representation, can you derive a closed form expression for x_n that does not depend on any previous elements of the sequence?

2. Show that any $n \times n$ matrix A with real-valued entries, but considered as an operator on $\mathbb{C}^n$ has eigenvalues that are real or come in complex conjugate pairs.

3. Which of the following matrices are diaganolizable, assuming the field to be $\mathbb{R}$?

   (a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \end{bmatrix}$

   (c) $\begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

   (d) $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

   (e) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$. Try this example for both fields: $\mathbb{R}, \mathbb{C}$.

   (f) $A \in \mathbb{R}^{n \times n}, A^2 = A$.

4. Let $\frac{d}{dt} \in \mathcal{L}(U)$, where $U = span(\{1, \cos t, \sin t\})$, over $\mathbb{R}$, be the usual first derivative operator. If possible diagonalize $\frac{d}{dt}$, else, block-diagonalize.

5. Let $A \in \mathbb{R}^{m \times n}$ be any arbitrary matrix. Show that A maps all eigenvectors of $A^T A$ to eigenvectors of AA^T.

6. Let $T \in \mathcal{L}(\mathbb{R}^2)$. The matrix A that represents T in the standard $\mathbb{R}^2$ basis (x and y axis) has eigenvectors $(1, 3)$ and $(-2, 4)$ (represented in standard basis) with eigenvalues 2 and 3 respectively. Find the matrix A, and the matrix B that represents T with the given eigenvectors as the basis of $\mathbb{R}^2$.