1. Show that the similarity relation on all $n \times n$ matrices is an equivalence relation.
2. Show that eigenvalues of an upper-triangular matrix are its diagonal elements.
3. For any matrix $A \in \mathbb{C}^{n \times n}$, show that the determinant of A is equal to the product of its eigenvalues.
4. Let C_1, C_2, C_3 be three electives, for which 20%, 50% and 30% of your seniors registered, respectively. After the add-drop period, C_1 had 50% of its originally registered students, while it got 20% and 30% of the students earlier registered in C_2 and C_3 respectively. Similarly, C_2 had 70% of its originally registered students, while it got 20% and 10% of the students earlier registered in C_1 and C_3 respectively.

   Given that all students participate in the registration, find the distribution of students in the electives, if they are allowed to change their registration infinitely often, but the migration pattern of the registration remains as described above.
5. For matrices with the following characteristic polynomials, conclude whether they are diagonalizable, not diagonalizable, or may be diagonalizable: (a) $c(x) = (x-1)(x-3)$, (b) $c(x) = (x-1)^2(x-3)$, (c) $c(x) = x^2(x-1)$, with $Rank(A) = 1$.
6. Solve the following coupled linear constant coefficient differential equation:

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

   Find the values of $x(10), y(10)$, given that $x(0) = y(0) = 1$.