Midterm Exam 2

Date	:	Wednesday 29th March, 2023, from 4:30 pm to 6:30 pm
Total Marks	:	50
Notes:	:	All questions are scored (no optionals).
		Do not forget to write your name and student ID below.
		Fully circle your selected option in the MCQ section.

Name: ______________________ **Student ID**: ______________________

Reference:

1 Fill In the Blanks:

Instructions

- ▷ There are total 16 questions in this section that carry total of 28 marks.
- ▷ There is no negative marking in this section.

Questions

1. (2 points) Name two reasons why a sinusoidal electromagnetic waveform is used as a carrier waveform by the modulator (and frequency upconvertor) to transmit the informative messages.
   - EM waves - after emission from an antenna - can travel large distances.
   - The choice of carrier frequency allows frequency division multiplexing of multiple simultaneous transmissions.
   - The carrier frequency can be varied to optimize the hardware design (e.g., the larger the carrier frequency, the smaller is the size of the transmit antenna needed).

2. (2 points) What are two advantages of a digital communication system compared to an analog communication system.
   - Digital communication system provide better resilience against the deletrious effect of the channel, including the noise.
   - Many performance enhancing signal processing techniques, such as source coding, channel coding, encryption, etc., require digital processing.
   - Digital technology, for which the primary currency is bits (Digital Integrated Circuits (ICs), and in general, the computers and the smartphones and their networks), has become very powerful and are inexpensive to manufacture.
   - Digital Communications allows integration of voice, video and data on a single packet networking system.

- Easier to preserve privacy by encryption of digital messages.
- Compared to analog communication systems, digital communications provides a better tradeoff of bandwidth efficiency against energy efficiency (i.e., exchange the power with the bandwidth).

3. (2 points) Why is a digital communication system called digital when the transmitted signal (typically a modulated EM wave) is entirely analog in nature?
   - The digital communication system always transmits one of a finite number of messages or symbols. It does so by mapping these discrete informative-bearing symbols onto amplitude and/or phase and/or frequency of the transmitted modulated EM wave.
   - In contrast, the analog communication system transmits a continuously-valued analog message signal, such as speech or video camera signal.

4. (2 points) How does a digital communication system with a family of modulation and coding schemes provide the scheme of throughput (data rate) adaptation, and why is this beneficial?
   - The throughput adaptation increases the coding rate and the modulation complexity when the signal to noise ratio (SNR) becomes high (since the high code rate and higher order modulation schemes require a high SNR). The benefit achieved is that they allow a greater throughput (data rate) to be delivered to the receiver. When the SNR drops (e.g., because the receiver – the cellphone – goes behind/inside a building, etc.), the throughput adaptation switches to a low coding rate and lower complexity modulation schemes that deliver smaller throughput but are resilient to degraded SNR conditions.
   - The net effect is that – as the SNR received by the receiver varies from low to high and back – the delivered throughput is adjusted. It does not happen that the throughput does not increase even when a high SNR is available or that the communication system fails when the SNR becomes low.

5. (2 points) Briefly describe Incremental Redundancy (IR) Type II Hybrid ARQ (H-ARQ):
   - In the H-ARQ with IR, the transmitter first sends only a few parity bits along with the message (informative) bits. Although these few parity bits do not provide a strong protection against the noise, the transmitter still takes a chance with the hope that the noise introduced by the channel is small and the transmitted message is successfully decoded at the receiver. If that happens, a high code rate is achieved and the valuable transmission resource is not wasted in transmitting the overhead parity bits.
   - If the first transmission fails, the transmitter - instead of resending the entire encoded packet again, as it happens in the conventional ARQ - just sends a few more additional parity bits. The receiver combines these newly received parity bits with the information + parity bits received in the first packet to perform decoding. These additional parity bits may allow the decoder to recover the transmitted message without wasting the transmission resource in duplicate transmission of the entire packet.

   .

6. (2 points) Prove that the dot product of $\cos(2\pi f_c t)$ with itself over $t \in [0, T_{sym}]$ equals $T_{sym}/2$ (you may assume $T_{sym} \propto f_c^{-1}$).

$$\int_0^{T_{sym}} \cos(2\pi f_c t)\, \cos(2\pi f_c t)\, dt = \frac{1}{2} \int_0^{T_{sym}} (1 - \cos(4\pi f_c t))\, dt = T_{sym}/2,$$

where $\int_0^{T_{sym}} \cos(4\pi f_c t)\, dt = 0$ since $T_{sym} \propto f_c^{-1}$. Even when T_{sym} is not proportional to f_c^{-1}, but as long as $T_{sym} \gg f_c^{-1}$, almost all of the positive half-cycles of the cosine waveform get canceled out by the negative cycles of this waveform in this integral, with the result that is nearly (for all practical purposes) zero.

7. (2 points) State the complex-envelope (CE) notation of the transmitted modulated signal $s(t)$ as a function of the inphase and the quadrature modulating information components s_i and s_q and the carrier frequency f_c. From this CE notation, derive (show the steps of your derivation) the cartesian coodinate representation (i.e., the inphase/quadrature notation) and the polar coodinate representation (i.e., the magnitude and representation) of $s(t)$.

   - CE notation: $s(t) = \Re\{(s_i + js_q)\exp(j2\pi f_c t)\}$
   - Cartesian Coordinate notation:

$$
\begin{aligned}
s(t) &= \Re\{(s_i + js_q)\exp(j2\pi f_c t)\} \\
&= \Re\{s_i\cos(2\pi f_c t) - s_q\sin(2\pi f_c t) + j(s_q\cos(2\pi f_c t) + s_i\sin(2\pi f_c t))\} \\
&= s_i\cos(2\pi f_c t) - s_q\sin(2\pi f_c t)
\end{aligned}
$$

   - Polar Coordinate notation: using the given definitions of a and θ,

$$
\begin{aligned}
s(t) &= \Re\{(s_i + js_q)\exp(j2\pi f_c t)\} \\
&= \Re\{a\exp(j\theta)\exp(j2\pi f_c t)\} \\
&= \Re\{a\exp(j(2\pi f_c t + \theta))\} \\
&= a\cos(2\pi f_c t + \theta).
\end{aligned}
$$

8. (1 point) State the two requirements for the inphase and the quadrature signal waveforms $g_i(t)$ and $g_q(t)$ so that they can be used at a modulator.

   - Each signal should be normalized to have unit energy, i.e., the dot product of the signal with itself should be unity. Specifically $\int_0^{T_{sym}} g_i^2(t)\,dt = \int_0^{T_{sym}} g_q^2(t)\,dt = 1$.
   - The two signals should be orthogonal to each other, i.e., their dot product should be zero. Specifically, $\int_0^{T_{sym}} g_i(t)g_q(t)\,dt = 0$.

9. (2 points) Suppose a digital communication system sends one bit at a time. The modulator transmits $s_0(t)$ to send bit $X = 0$ and it uses $s_1(t)$ to send bit $X = 1$, where $s_0(t)$ and $s_1(t)$ are the modulated waveforms shown in Fig. 3. Draw appropriate $g_i(t)$ and $g_q(t)$ given these two modulated waveforms. Represent $s_0(t)$ and $s_1(t)$ as vectors in a two-dimensional space whose $\hat{i}$ and $\hat{j}$ vectors are $g_i(t)$ and $g_q(t)$, respectively.

   - The waveforms $g_i(t)$ and $g_q(t)$ are shown in Fig. 1.
   - The vectorial representation of $s_0(t)$ and $s_1(t)$ is shown in Fig. 2.

10. (2 points) Given that the transmitter uses two orthogonal waveforms $g_i(t)$ and $g_q(t)$ for transmitting the modulated signal $s(t)$, the noise component at the receiver - regardless of what its exact shape is - can *always* be written as $n(t) = n_i g_i(t) + n_q g_q(t) + u(t)$, where $u(t)$ is the component of the noise $n(t)$ that lies in a vector subspace that is orthogonal to the two-dimensional space formed by $g_i(t)$ and $g_q(t)$, i.e., the dot product of $u(t)$ with either $g_i(t)$ or $g_q(t)$ is zero. Show that $u(t)$ – no matter how powerful or strong it is – has *no* effect on the receiver (demodulator) operation.

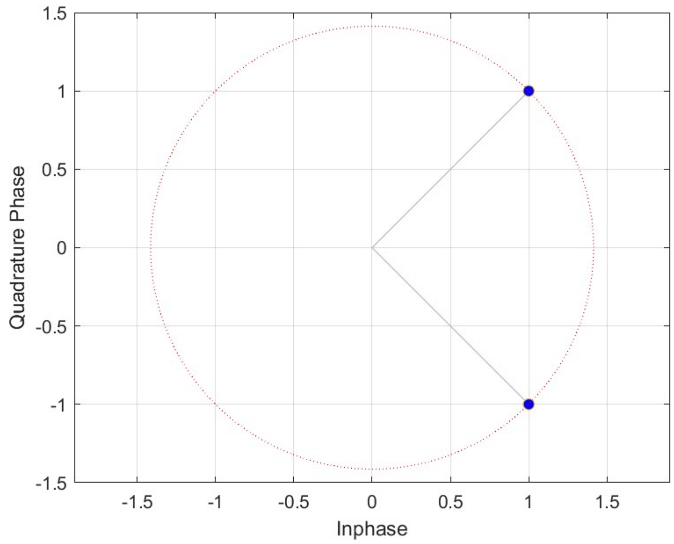

Figure 1: Solution for Problem 9.

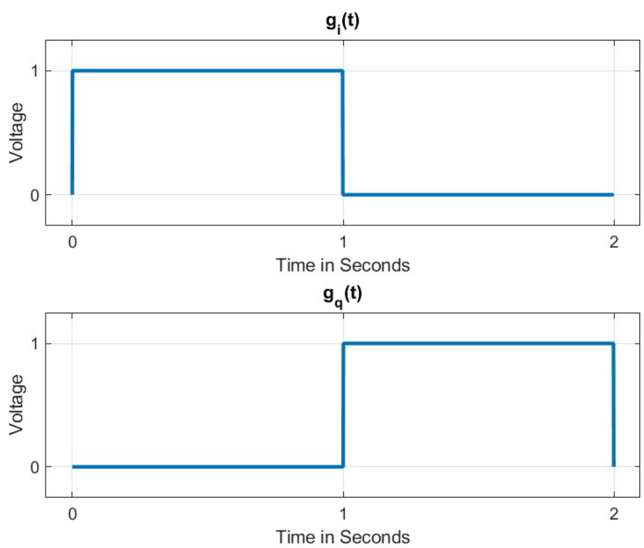

Figure 2: Solution for Problem 9.

- The receiver performs an analysis operation on the received signal $r(t)$. In this operation, $r(t)$ is projected on $g_i(t)$ and $g_q(t)$. As this projection is a linear operation (vector dot product), its output corresponding to $u(t)$ part of $r(t)$ is the same as the output obtained when $u(t)$ itself is projected on to $g_i(t)$ and $g_q(t)$. However, the projection of $u(t)$ – given the definition of $u(t)$ – on $g_i(t)$ and $g_q(t)$ is zero. This shows that the output of the receiver's analysis operation is not affected (it remains unchanged) by $u(t)$.

11. (1 points) Let $\bar{m_n^2}$ be the power in a normalized version $m_n(t)$ of the message signal $m(t)$ – where the normalization is performed such that the minimum value of $m_n(t) = -1$. Let a_{mod} be the modulation

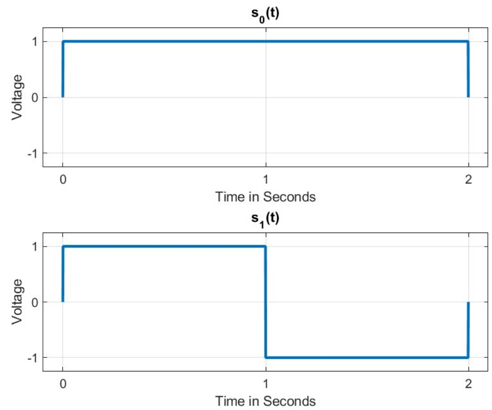

Figure 3: Two signals for Problem 9.

index of the DSB-FC (Conventional) AM. Express the power efficiency η_{AM} of the conventional AM as a function of $\bar{m_n^2}$ and a_{mod}.

$\eta_{\text{AM}} = \dfrac{a_{\text{mod}}^2 \, \bar{m_n^2}}{1 + a_{\text{mod}}^2 \, \bar{m_n^2}}$.

12. (2 points) Name four blocks that a digital communication system has which an analog communication system does not have.

    1. A/D converter
    2. Source coding
    3. Encryption
    4. Channel coding

13. (2 points) Show that the F.T. of a signal that exhibits time-domain symmetry (i.e., it is an even signal $s_e(t) = s_e(-t)$) is real-valued.

    - Any signal $s(t)$ can be written as a summation of an even signal $s_e(t)$ and an odd signal $s_o(t)$, i.e., $s(t) = s_e(t) + s_o(t)$.
    - An odd signal is the one whose integration from $-\infty$ to $+\infty$ in the time variable t variable is zero.
    - An odd signal times an even signal is an odd signal. An even signal times an even signal is an even signal. Similarly, an odd signal times an odd signal is an even signal.
    - The complex exponential $\exp(j2\pi f_c t) = \cos(2\pi f_c t) + j\sin(2\pi f_c t)$ has the real part which is an even signal and imaginary part which is an odd signal.
    - Therefore,

$$\begin{aligned} S(f) &= \int_{-\infty}^{\infty} s(t) \exp(-j2\pi f_c t)\, dt \\ &= \int_{-\infty}^{\infty} s_e(t)\cos(2\pi f_c t) + s_o(t)\cos(2\pi f_c t) - j\left(s_e(t)\sin(j2\pi f_c t) + s_o(t)\sin(2\pi f_c t)\right) dt \\ &= \int_{-\infty}^{\infty} s_e(t)\cos(2\pi f_c t)\, dt - j\left(\int_{-\infty}^{\infty} s_o(t)\sin(2\pi f_c t)\, dt\right) \end{aligned}$$

    - If the signal is an even signal, $s_o(t) = 0$ and therefore, the above integral becomes real-valued.

14. (2 points) Prove that if modulo-two sum of two binary error patterns $\mathbf{e}_1$ and $\mathbf{e}_2$ is a valid binary codeword $\mathbf{c}$, each error pattern has the same syndrome.

   - Since $\mathbf{e}_1 + \mathbf{e}_2 = \mathbf{c}$, the syndrome $\mathbf{H}\,(\mathbf{e}_1 + \mathbf{e}_2) = \mathbf{H}\,\mathbf{c} = \bar{0}$, where $\bar{0}$ is an all-zero vector of length $U = N - K$ bits.
   - Therefore, $\mathbf{H}\,\mathbf{e}_1 = \bar{0} - \mathbf{H}\,\mathbf{e}_2 = \mathbf{H}\,\mathbf{e}_2$.
   - Since $\mathbf{H}\,\mathbf{e}$ is the syndrome vector associated with the binary error vector $\mathbf{e}$, we conclude that the syndrome vectors associated with $\mathbf{e}_1$ and $\mathbf{e}_2$ are identical.

15. (1 points) Why is DSB-SC AM preferred over DSB-FC AM?

   - The DSB-SC AM does not waste extra power in transmitting a carrier signal that conveys no information to the receiver.

16. (1 points) Why is DSB-FC (Conventional) AM preferred over DSB-SC AM?

   - The DSB-FC AM allows making the receiver design simpler. A simpler envelope detector is sufficient instead of a phase coherent receiver. This makes the AM radio receiver less expensive.

2 Multiple Choice Questions:

Instructions

- ▷ This is MCQ Answer Key of IV.
- ▷ There are total 15 questions in this section that carry a maximum of 22 marks.
  - → If the question is answered incorrectly, negative one-fourth of the total marks shown next to each question will be given.
- ▷ Circle the correct option. No tick marks or partial circling, etc.
  - → If more than one option is circled, zero marks will be given for the question.
- ▷ Note: *Volume* $V(N; t_c)$ of a Hamming Sphere of radius t_c is defined as the number of (binary) codewords of length N bits whose Hamming weight is less than or equal to t_c bits.

Questions

Version	I	II	III	IV
Question 1				
Answer (a):	-0.25 pts	1 pt	-0.25 pts	-0.25 pts
Answer (b):	1 pt	-0.25 pts	-0.25 pts	-0.25 pts
Answer (c):	-0.25 pts	-0.25 pts	1 pt	-0.25 pts
Answer (d):	-0.25 pts	-0.25 pts	-0.25 pts	1 pt
Question 2				
Answer (a):	-0.25 pts	-0.25 pts	-0.25 pts	-0.25 pts
Answer (b):	-0.25 pts	-0.25 pts	-0.25 pts	1 pt
Answer (c):	1 pt	-0.25 pts	-0.25 pts	-0.25 pts
Answer (d):	-0.25 pts	1 pt	1 pt	-0.25 pts
Question 3				
Answer (a):	-0.25 pts	-0.75 pts	-0.5 pts	2 pts

Version	I	II	III	IV
Answer (b):	1 pt	-0.75 pts	-0.5 pts	-0.5 pts
Answer (c):	—	3 pts	2 pts	-0.5 pts
Answer (d):	—	-0.75 pts	-0.5 pts	-0.5 pts
Question 4				
Answer (a):	-0.25 pts	-0.25 pts	1 pt	-0.25 pts
Answer (b):	1 pt	1 pt	-0.25 pts	-0.25 pts
Answer (c):	—	-0.25 pts	-0.25 pts	-0.25 pts
Answer (d):	—	-0.25 pts	-0.25 pts	1 pt
Question 5				
Answer (a):	-0.25 pts	1 pt	-0.25 pts	-0.25 pts
Answer (b):	1 pt	-0.25 pts	-0.25 pts	1 pt
Answer (c):	-0.25 pts	—	-0.25 pts	—
Answer (d):	-0.25 pts	—	1 pt	—
Question 6				
Answer (a):	-0.25 pts	1 pt	3 pts	-0.25 pts
Answer (b):	-0.25 pts	-0.25 pts	-0.75 pts	1 pt
Answer (c):	1 pt	—	-0.75 pts	—
Answer (d):	-0.25 pts	—	-0.75 pts	—
Question 7				
Answer (a):	-0.25 pts	-0.25 pts	1 pt	-0.25 pts
Answer (b):	-0.25 pts	-0.25 pts	-0.25 pts	-0.25 pts
Answer (c):	-0.25 pts	1 pt	—	-0.25 pts
Answer (d):	1 pt	-0.25 pts	—	1 pt
Question 8				
Answer (a):	-0.25 pts	-0.25 pts	1 pt	-0.25 pts
Answer (b):	1 pt	-0.25 pts	-0.25 pts	1 pt
Answer (c):	-0.25 pts	-0.25 pts	—	-0.25 pts
Answer (d):	-0.25 pts	1 pt	—	-0.25 pts
Question 9				
Answer (a):	-0.5 pts	1 pt	-0.25 pts	1 pt
Answer (b):	2 pts	-0.25 pts	1 pt	-0.25 pts
Answer (c):	-0.5 pts	-0.25 pts	-0.25 pts	-0.25 pts
Answer (d):	-0.5 pts	-0.25 pts	-0.25 pts	-0.25 pts
Question 10				
Answer (a):	1 pt	-0.25 pts	1 pt	-0.25 pts
Answer (b):	-0.25 pts	-0.25 pts	-0.25 pts	-0.25 pts
Answer (c):	-0.25 pts	1 pt	-0.25 pts	1 pt
Answer (d):	-0.25 pts	-0.25 pts	-0.25 pts	-0.25 pts
Question 11				
Answer (a):	-0.25 pts	-0.25 pts	-0.25 pts	-0.25 pts
Answer (b):	1 pt	-0.25 pts	1 pt	1 pt
Answer (c):	-0.25 pts	1 pt	-0.25 pts	-0.25 pts
Answer (d):	-0.25 pts	-0.25 pts	-0.25 pts	-0.25 pts
Question 12				
Answer (a):	-0.75 pts	1 pt	-0.5 pts	-0.25 pts
Answer (b):	3 pts	-0.25 pts	-0.5 pts	-0.25 pts
Answer (c):	-0.75 pts	-0.25 pts	2 pts	1 pt
Answer (d):	-0.75 pts	-0.25 pts	-0.5 pts	-0.25 pts
Question 13				
Answer (a):	-0.25 pts	-0.25 pts	-0.25 pts	-0.5 pts
Answer (b):	-0.25 pts	1 pt	1 pt	2 pts

Version	I	II	III	IV
Answer (c):	-0.25 pts	-0.25 pts	-0.25 pts	-0.5 pts
Answer (d):	1 pt	-0.25 pts	-0.25 pts	-0.5 pts
Question 14				
Answer (a):	-0.25 pts	2 pts	1 pt	1 pt
Answer (b):	-0.25 pts	-0.5 pts	-0.25 pts	-0.25 pts
Answer (c):	-0.25 pts	-0.5 pts	-0.25 pts	-0.25 pts
Answer (d):	1 pt	-0.5 pts	-0.25 pts	-0.25 pts
Question 15				
Answer (a):	2 pts	-0.5 pts	-0.25 pts	-0.75 pts
Answer (b):	-0.5 pts	-0.5 pts	-0.25 pts	3 pts
Answer (c):	-0.5 pts	2 pts	-0.25 pts	-0.75 pts
Answer (d):	-0.5 pts	-0.5 pts	1 pt	-0.75 pts