Signals and Systems (CT 203)

Tutorial Sheet-13 DA-IICT, Gandhinagar.

1. By computing the group delay at three selected frequencies, verify that the frequency response $H(\omega)=\frac{1}{j\omega+1}$ has nonlinear phase characteristics.
2. Consider a discrete-time lowpass filter whose impulse response $h[n]$ is known to be real and whose frequency response magnitude is given by

$$\left|H\left(e^{j\omega}\right)\right|=\begin{cases}1, & |\omega|\le\frac{\pi}{4}\\ 0, & otherwise\end{cases}$$

   Determine and sketch real valued impulse response $h[n]$ for this filter when the corresponding group delay function is given as $\tau(\omega)=5$.

3. Consider a continuous-time causal and stable LTI system whose input $x(t)$ and output $y(t)$ are related by the differential equation

$$\frac{dy(t)}{dt}+5y(t)=2x(t)$$

   What is the final value of the step response $s(t)$ of this filter? Also determine the value of t_o for which

$$s(t)\big|_{t=t_o}=s(t_o)=s(\infty)\left[1-\frac{1}{e^2}\right].$$

4. A real-valued signal $x(t)$ is known to be uniquely determined by its sampling frequency when the sampling frequency is $\omega_s=10,000\pi$. For what values of ω is $X(\omega)$ guaranteed to be zero?
5. A continuous-time signal $x(t)$ is obtained at the output of an ideal *lowpass* filter with cutoff frequency $\omega_c=1,000\pi$. If impulse-train sampling is performed on $x(t)$, which of the following sampling periods would guarantee that $x(t)$ can be recovered from its sampled version using an appropriate lowpass filter?
   (a) $T=0.5\times10^{-3}$, (b) $T=2\times10^{-3}$, (c) $T=10^{-4}$

6. The frequency which, under the sampling theorem, must be exceeded by the sampling frequency is called as *Nyquist* rate. Determine the *Nyquist* rate corresponding to the signal

$$x(t)=\frac{\sin(4,000\pi t)}{\pi t}$$