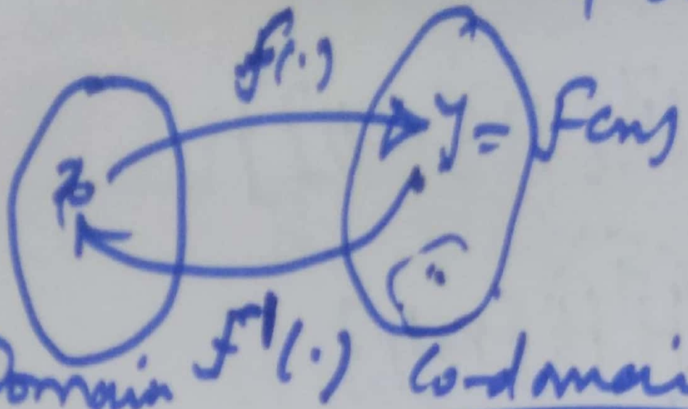
[1] Invertibility :→

Inverse function

One-one and onto.

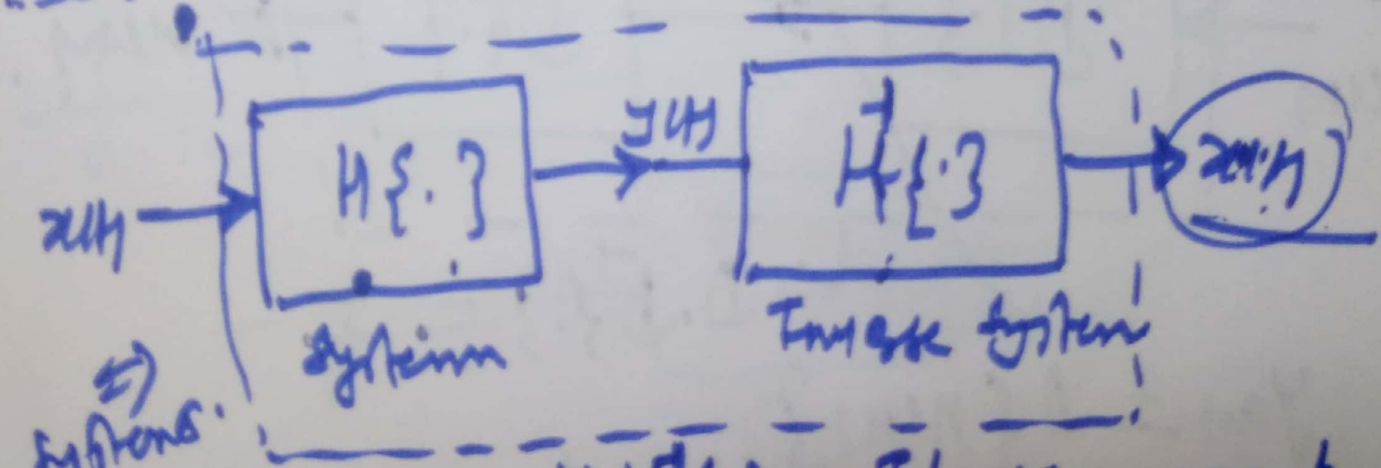


Function: $y = f(x)$

$\therefore\ x = f^{-1}(y) = f^{-1}(f(x))$.

$x = f^{-1}f(x)$

I



⇒ systems.

$\underline{H \cdot H^{-1}\{\cdot\}}$ = Identity operator.

Matrix Theory → $A \cdot I = A \Rightarrow A A^{-1} = I$

$$y(n) = H\{x(n)\}$$

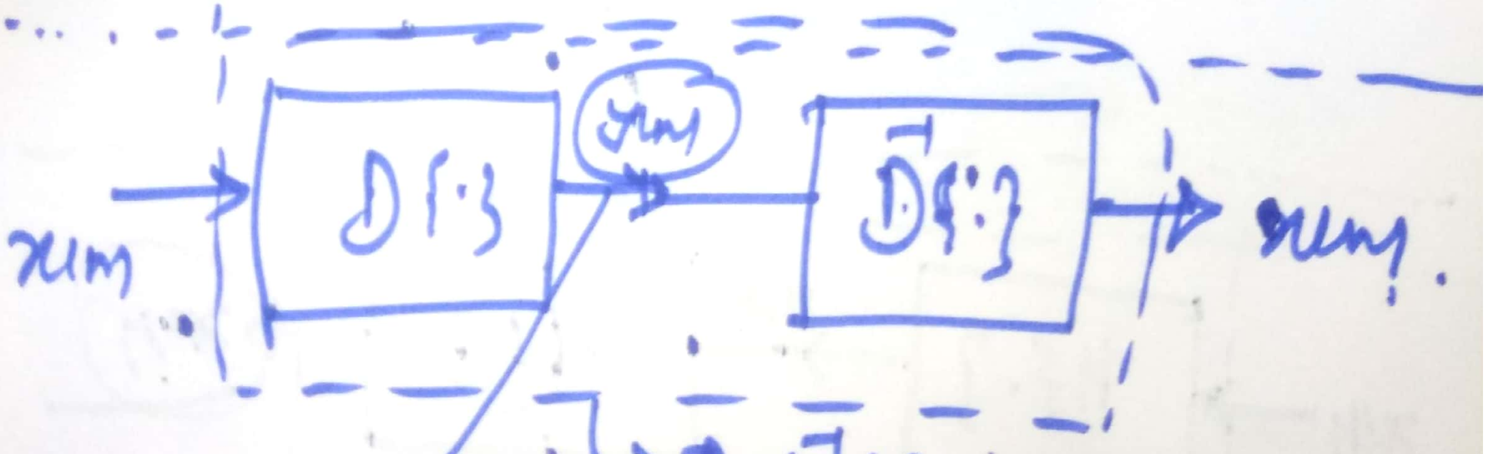
$$x(n) = H^{-1}\{y(n)\}$$

$$= H^{-1}\{H\{x(n)\}\}$$

$$x(n) = H^{-1}H\{x(n)\}$$

$$I$$

$$\boxed{x(n) = I \cdot x(n)}$$, where $H^{-1} \cdot H = I$.



$$D \cdot D^{-1}\{\cdot\} = I$$

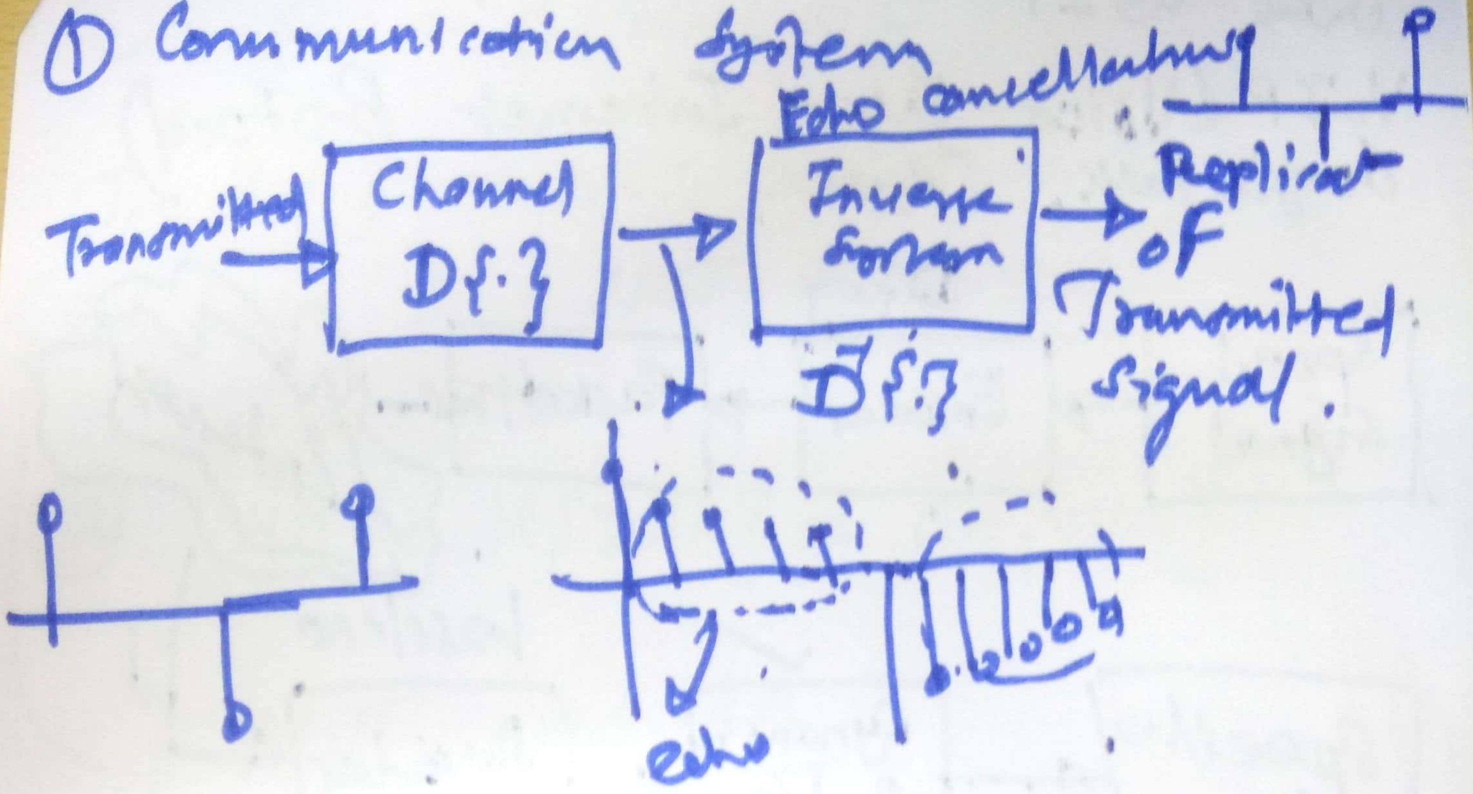
$$y(n) = D\{x(n)\}.$$

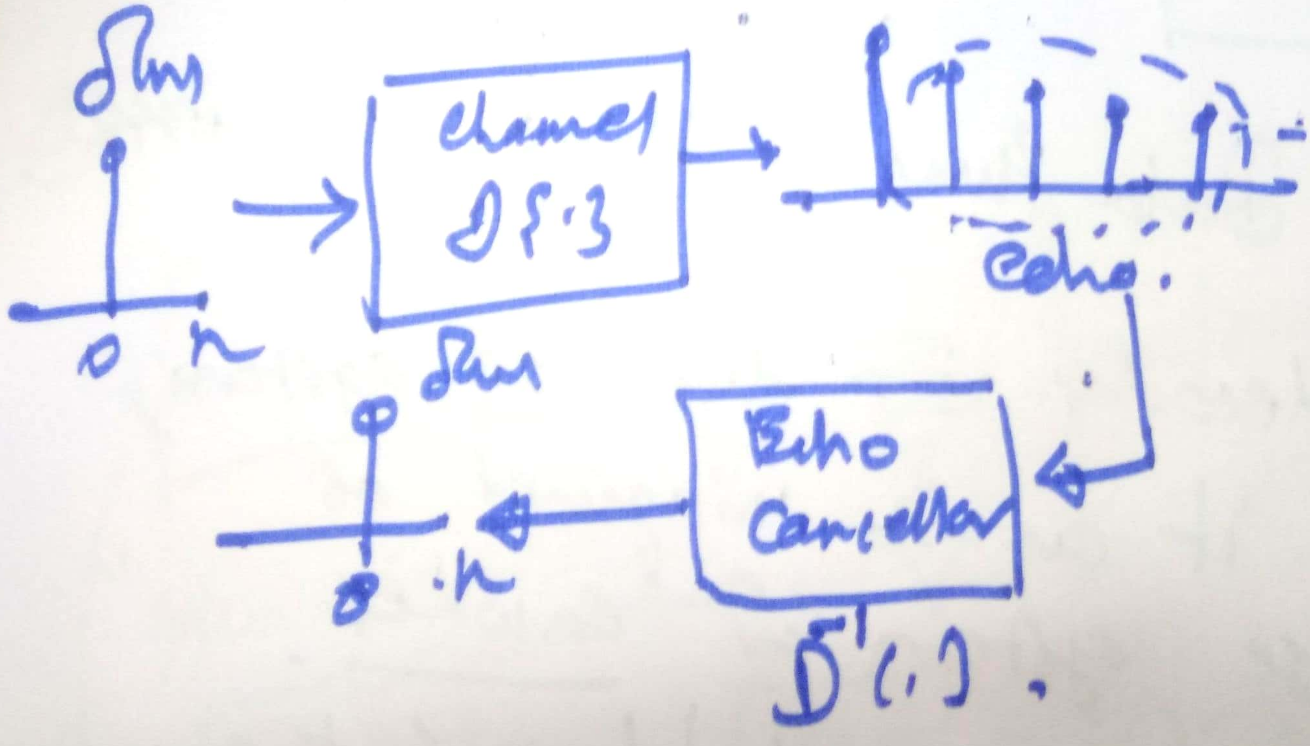
$$x(n) = D^{-1}\{y(n)\} = D^{-1}\{D\{x(n)\}\}$$

$$x(n) = D^{-1}D\{x(n)\} = I\{x(n)\}$$

Examples :-

① Communication system





② Transmission of Speech/Audio over VoIP

VoIP (Voice Over Internet Protocol)
Google Talk, Skype.

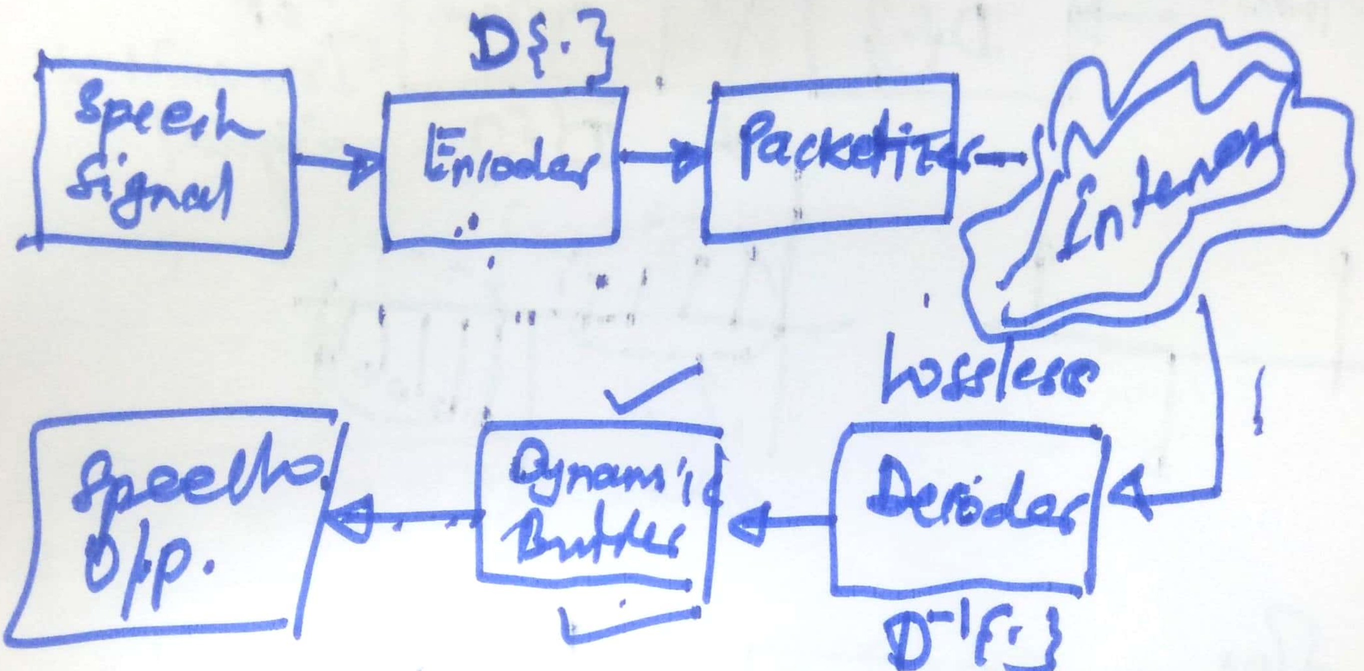


Fig. Data flow in VoIP Network.

If decoder is a lossless system, then it can be imagined as inverse system of encoder.
else if decoder is 'lossy' then it can't be inverse system of encoder.

Example

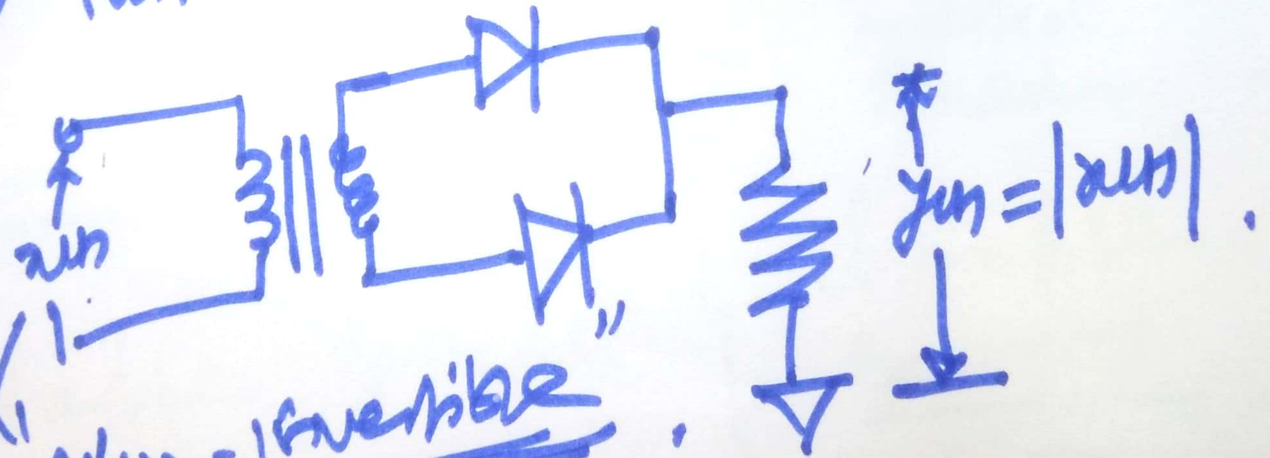
(1) $y(t) = H\{x(t)\} = 0$

The $H\{.\}$ is NOT one-to-one and hence it is NOT invertible.

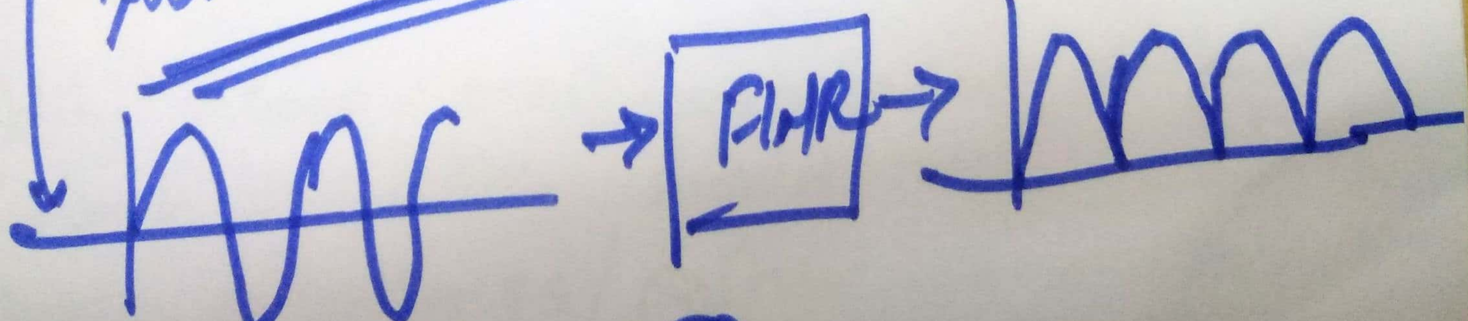
(2) $y(t) = H\{x(t)\} = \sin\{x(t)\}$

; NOT invertible.

(3) Full-wave rectifier



non-invertible.



4) $y(n) = D\{x(n)\} = x^2(n)$.

Non Invertible

5) $y(n) = D\{x(n)\} = n \cdot x(n)$.

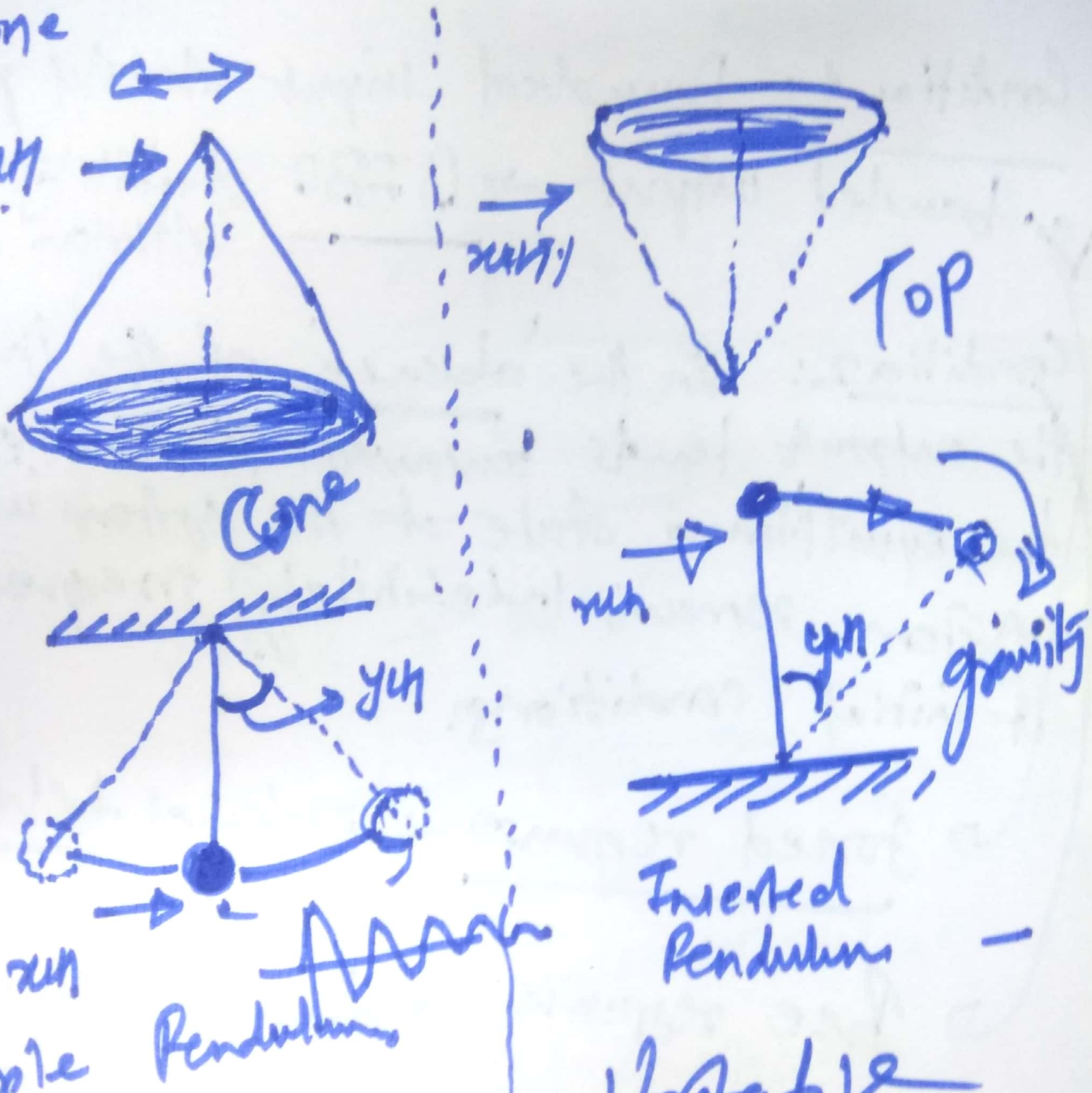
$x(n) = \left(\frac{1}{n}\right) \cdot y(n)$.

For $n=0$,

$x(n) = \infty$.

Non-invertible.

] Stability :→



Stable Systems | Unstable systems.

Stablity means small perturbation/distrubances in input excitation should result into small perturbances/disturbance in output response

Test for system's stability :→

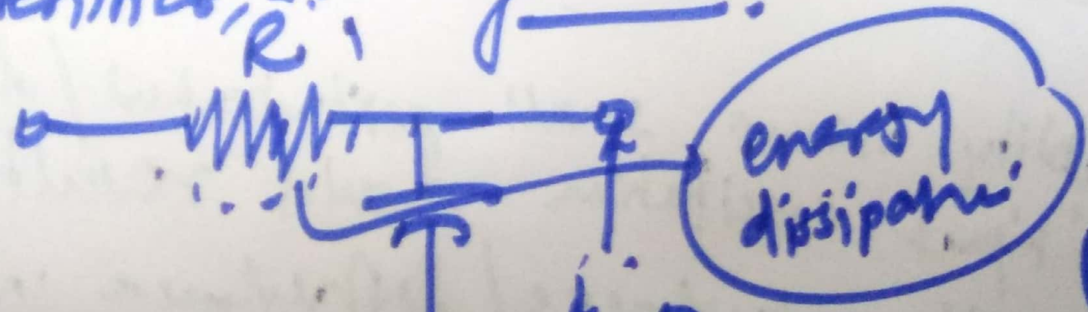
Condition 1: Bounded input should produce a bounded output → BIBO stability criterion.

Condition 2: In the 'absence' of the input, the output tends towards zero (i.e. the equilibrium state of the system where system remains indefinitely) irrespective of initial conditions.

→ Forced response → external behaviour

→ Free response. → internal behaviour

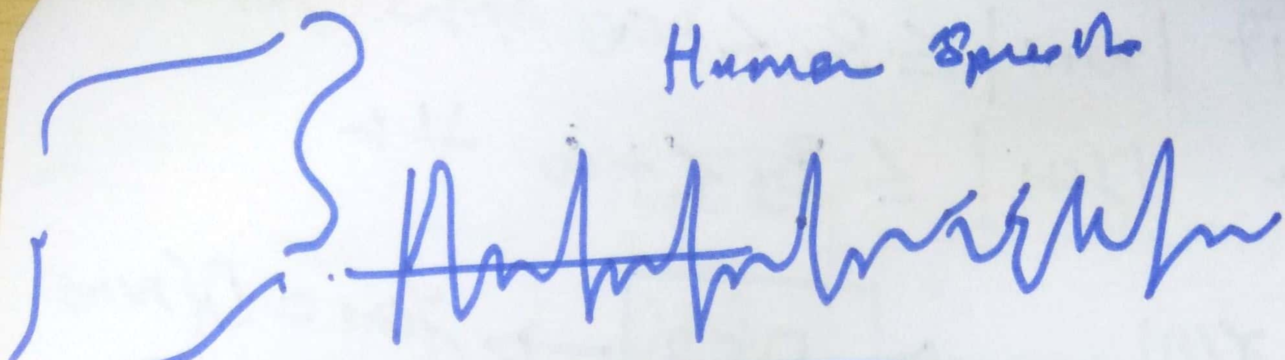
→ Stability indicates behavioural characteristics of systems.



mechanism of energy dissipation in real physical systems

Speech Production Mechanism

Human Speech



Vocal Tract System

↓

Stable → why?

↓

energy dissipation (How??)

↳ Lip radiation effect

↳ Vocal tract wall vibration.

↳ Friction of air particles in vocal tract walls →> Heat dissipation

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Applications of stability:

1) Control systems ✓
2) Power systems ✓
3) Electrical Network ✓
4) Design of Analog and Digital filters
5) Stability of Nonlinear systems

A system is said to be BIBO stable if $|x(n)| \le B_x < +\infty \ \forall n$ results in $|y(n)| \le B_y < +\infty \ \forall n$

$$x(n) \longrightarrow \boxed{D\{\cdot\}} \longrightarrow y(n) = D\{x(n)\}$$

Bounded Input

$$|x(n)| \le B_x < +\infty \ \forall n \Rightarrow |y(n)| \le B_y < +\infty \ \forall n$$

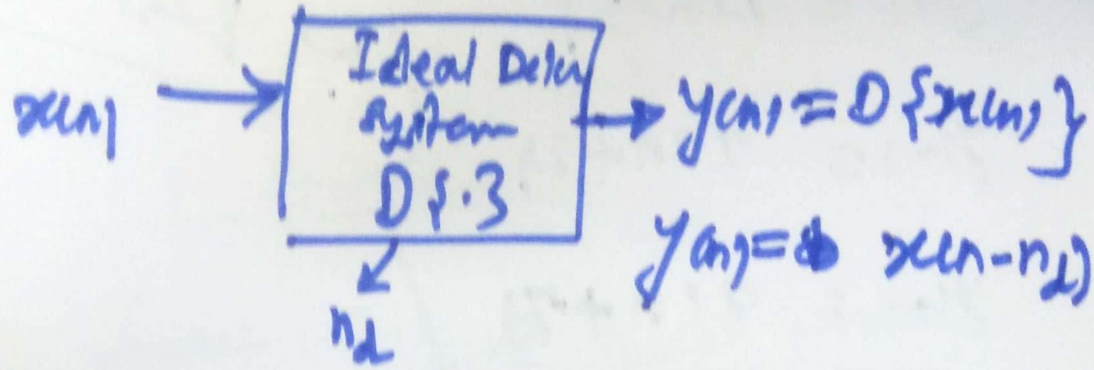
$\Rightarrow$ All the sample values of $x(n)$ are bounded/finite.

All the sample values of $y(n)$ are also bounded.

Then system $D\{\cdot\}$ is said to BIBO stable.

Problem: Check whether ideal delay system is BIBO stable?

Sol^n



$y(n) = \cancel{D}\; x(n-n_d)$

Step I) Input-output relations

$$y(n) = D\{x(n)\} = x(n-n_d).$$

Step II For BIBO stability, let input be bounded $\forall n$ i.e.

Let $|x(n)| \le B_x < +\infty \,;\ \forall n$

$$y(n) = x(n-n_d)$$

$$|y(n)| = |x(n-n_d)| \,;\ \forall n$$

$$\le B_x < +\infty \,;\ \forall n$$

$\therefore\ |y(n)| < +\infty \,;\ \forall n.$

$\therefore$ Output is also bounded $\forall n$.

$\therefore$ $D\{\cdot\}$ system is BIBO stable.

2) $y(n) = x(n-1)$

$y(n) = x(n-5)$

$y(n) = x(n+2)$

$y(n) = x(n+7)$

All these systems are BIBO stable.

What is the role of condition 2 ?!

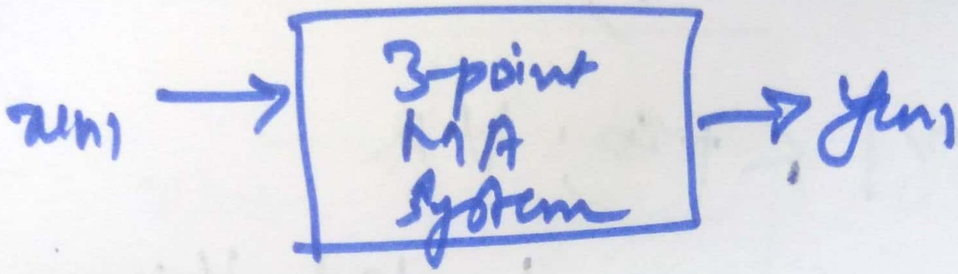
→ Using Proposition 1.1, for linear system if $x(n) = 0 \; \forall n$ then $y(n) = 0 \; \forall n$

⇒ For linear system origin is naturally the equilibrium point of system

⇒ Condition 2 for stability is inherently satisfied

⇒ Condition 2 needs to be verified for nonlinear systems in order to investigate their stability.

Problem Whether 3-point MA system is stable??



Step I Input-Output relationship for given system

$$y(n) = D\{x(n)\}$$

$$y(n) = \frac{1}{3}\sum_{k=-1}^{+1} x(n+k) = \frac{1}{3}\cancel{[x(n)}$$

$$\boxed{y(n) = \frac{1}{3}\left[x(n+1) + x(n) + x(n-1)\right]}$$

Step II For BIBO,

let $|x(n)| \le B_x < +\infty, \forall n$

$$|y(n)| = \left|\frac{1}{3}\left(x(n+1) + x(n) + x(n-1)\right)\right|$$

$$|y(n)| \le \frac{1}{3}\,\underbrace{|x(n+1)|}_{\to B_x} + \underbrace{|x(n)|}_{\to B_x} + \underbrace{|x(n-1)|}_{\to B_x}$$

$$|y(n)| \le B_{x_i} < +\infty \quad \forall n$$

$$|y(n)| < +\infty ; \forall n$$

$\Rightarrow$ Output $y(n)$ bounded $\forall n$

$\therefore$ BIBO stability criterion is ~~stab~~ satisfied

$\Rightarrow$ 3-point MA system is <u>stable</u>

Problem $y(n) = \frac{1}{3} \sum_{k=0}^{2} x(n-k)$

$\downarrow$ stable

5-point MA system

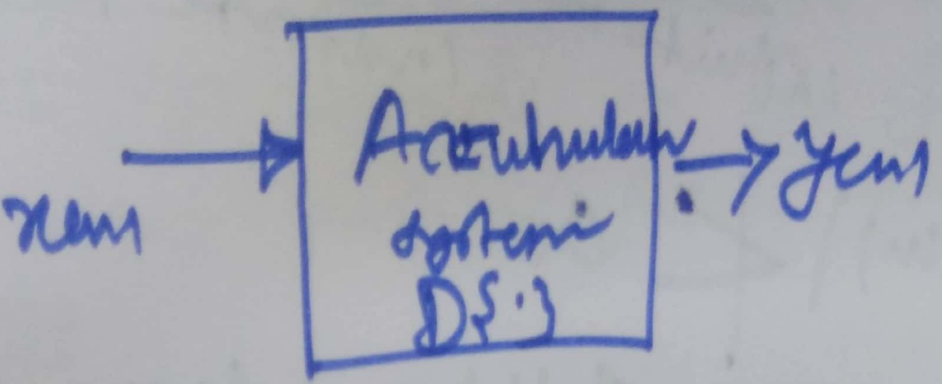
Problem

$y(n) = \frac{1}{5} \sum_{k=-2}^{+2} x(n-k)$ $\rightarrow$ stable

$y(n) = \frac{1}{5} \sum_{k=0}^{4} x(n-k)$ $\rightarrow$ <u>stable</u>

Problem Examine the stability of an accumulator system.

Soln



$$y(n) = D\{x(n)\}$$

Step I)

$$y(n) = \sum_{k=-\infty}^{n} x(k) \quad (I/O)$$

Step II Bounded I/p

$$\Rightarrow |x(n)| \le B_x < +\infty, \ \forall n.$$

$$|y(n)| = \left|\sum_{k=-\infty}^{n} x(k)\right|$$

$$|y(n)| \le \sum_{k=-\infty}^{n} |x(k)|$$

$$|y(n)| \le (\dots + |x(-2)| + |x(-1)| + |x(0)| + |x(1)| + \dots + |x(n)|)$$

15

$|y(n)| \le (\cdots + B_x + B_x + \cdots + B_x + B_x)$

infinite term — indefinite number

$|y(n)| \not< \infty$

$\therefore$ BIBO stability criterion is NOT satisfied

$\therefore$ Accumulator System is unstable

Problem: $y(n) = D\{x(n)\} = n \cdot x(n)$;

unstable.

$\lim_{n \to \infty} |y(n)| = \infty$..

[6] System with or without memory

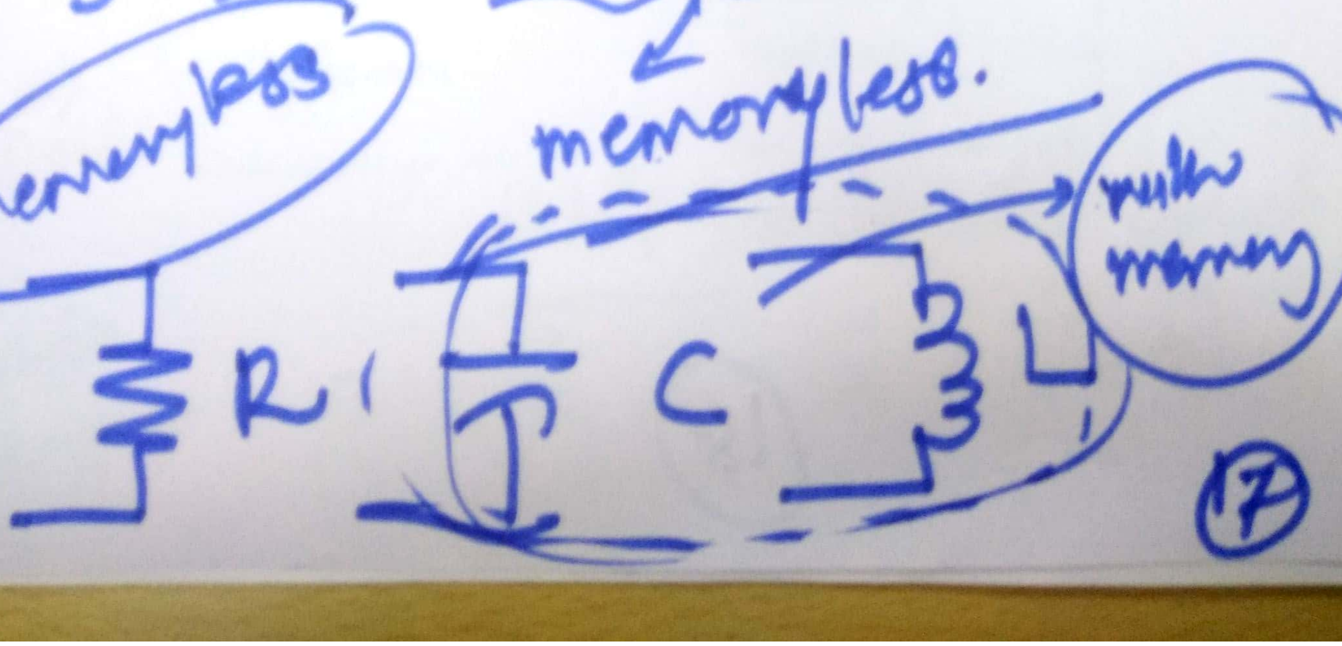
A system is said to possess memory if its o/p depends on (or function of) either past or future values of i/p.

$$y(n) = f(\ldots x(n-1),\ x(n+1) \ldots) = D\{x(n)\}$$

$D\{.\}$ is with memory.

$$y(n) = D\{x(n)\} = f(x(n))$$
$$= k\,x(n)$$

$D\{.\}$ is without memory or memoryless.



→ All memoryless systems are causal

→ ~~Whether~~ ~~all~~ The systems with memory may or may NOT be causal

→ Memory
- → future values
  - $x(n+1)$ → Non causal
- → Past values
  - $x(n-1)$ → Causal