DA-IICT Gandhinagar
Tutorial Sheet: SC224

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1. A civil engineer is analyzing the compressive strength of concrete. Compressive strength is normally distributed with $\sigma^2 = 1000\ (psi)^2$. A random sample of 12 specimens has a mean compressive strength of $\bar{x} = 3250\ psi$

   (a) Construct a 95% two-sided confidence interval on mean compressive strength.

   (b) Construct a 99% two-sided confidence interval on mean compressive strength. Compare the width of this confidence interval with the width of the one found in part (a).

2. Let X_1 and X_2 be independent random variables with mean μ and variance σ^2. Suppose that we have two estimators of μ:

$$\hat{\theta}_1 = \frac{X_1 + X_2}{2} \tag{1}$$

   and

$$\hat{\theta}_2 = \frac{X_1 + 3X_2}{4} \tag{2}$$

   (a) Are both the estimators $\hat{\theta}_1$ and $\hat{\theta}_2$, unbiased estimators of μ.

   (b) Obtain the variance of both the estimators $\hat{\theta}_1$ and $\hat{\theta}_2$.

3. A survey of the various prices for a bicycle has been conducted in 16 stores which were selected at random in the city. The prices were noted as following:
   $95, 108, 97, 112, 99, 106, 105, 100, 99, 98, 104, 110, 107, 111, 103, 110.$

   Assuming that the prices of bicycle follow a normal distribution with variance of 25 and an unknown mean:

   (a) What is the distribution of the sample mean?

   (b) Determine the confidence interval at 95% for the population mean.

4. The average heights of a random sample of 400 people from New York is 1.75 m. It is known that the heights of the population are random variables that follow a normal distribution with a variance of 0.16.

   (a) Determine the interval (lower and upper confidence limits) of 95% confidence for the average heights of the population.

   (b) With a confidence level of 90%, what would be the minimum sample size needed for the true mean of the heights to be less than 2 cm from the sample mean?

5. (a) Let $Y_1, Y_2, \cdots Y_n$ be the samples from the distribution whose PDF is

$$f(y) = e^{-(y-\theta)} \qquad y \geq 0 \tag{3}$$

Determine the maximum likelihood estimator of θ.

(b) If the $Y_i's$ mentioned above be the samples from the Normal distribution having known mean μ. Determine the Maximum Likelihood estimator for σ^2 and find the expected value of the estimator.

6. Let $X_1, X_2, \cdots X_n$ be the samples (of size n) from a Normal population having mean μ_1 and variance σ_1^2. Meanwhile $Y_1, Y_2, \cdots Y_n$ be the samples (of size m) from a different Normal population having mean μ_2 and variance σ_2^2. It is considered that any two samples are independent of each other. The weighted Mean is a parameter of interest here which is defined as $\mu_w = 3\mu_1 + 2\mu_2$.

(a) Find the interval estimates of the weighted mean μ_w with 95% confidence.

(b) Find the interval estimates of the weighted mean μ_w with 99% confidence.