SC222 : Tutorial Sheet 9

Problems based on parameter estimation.

1. . Determine the maximum likelihood estimates of a and λ when $X1, ..., Xn$ is a sample from the Pareto density function,

$$f(x) = \begin{cases} \lambda a^\lambda x^{-(\lambda+1)}, & if \ x \geq a \\ 0, & if \ x < a \end{cases} \tag{1}$$

2. An electric scale gives a reading equal to the true weight plus a random error that is normally distributed with mean 0 and standard deviation $\sigma = 0.1 \, mg$. Suppose that the results of five successive weighing of the same object are as follows:

$$3.142, 3.163, 3.155, 3.150, 3.141.$$

   (a) Determine a 95 percent confidence interval estimate of the true weight.
   (b) Determine a 99 percent confidence interval estimate of the true weight.

3. The PCB concentration of a fish caught in Lake Michigan was measured by a technique that is known to result in an error of measurement that is normally distributed with a standard deviation of $0.08 \, ppm$ (parts per million). Suppose the results of 10 independent measurements of this fish are,

$$11.2, 12.4, 10.8, 11.6, 12.5, 10.1, 11.0, 12.2, 12.4, 10.6$$

   (a) Give a 95 percent confidence interval for the PCB level of this fish.
   (b) Give a 95 percent lower confidence interval.
   (c) Give a 95 percent upper confidence interval.

4. The standard deviation of test scores on a certain achievement test is 11.3. If a random sample of 81 students had a sample mean score of 74.6, find a 90 percent confidence interval estimate for the average score of all students.

5. A sample of 20 cigarettes is tested to determine nicotine content and the average value observed was $1.2 \, mg$. Compute a 99 percent two-sided confidence interval for the mean nicotine content of a cigarette if it is known that the standard deviation of a cigarette's nicotine content is $\sigma = 0.2 \, mg$.

6. A random sample of $1,200$ engineers included 48 Hispanic Americans, 80 African Americans, and 204 females. Determine 90 percent confidence intervals for the proportion of all engineers who are,

   (a) female;
   (b) Hispanic Americans or African Americans.

7. To estimate p, the proportion of all newborn babies that are male, the gender of $10,000$ newborn babies was noted. If $5,106$ of them were male, determine (a) a 90 percent and (b) a 99 percent confidence interval estimate of p.

8. In a recent study, 79 of 140 meteorites were observed to enter the atmosphere with a velocity of less than 25 miles per second. If we take $\hat{p} = 79/140$ as an estimate of the probability that an arbitrary meteorite that enters the atmosphere will have a speed less than 25 miles per second, what can we say, with 99 percent confidence, about the maximum error of our estimate?