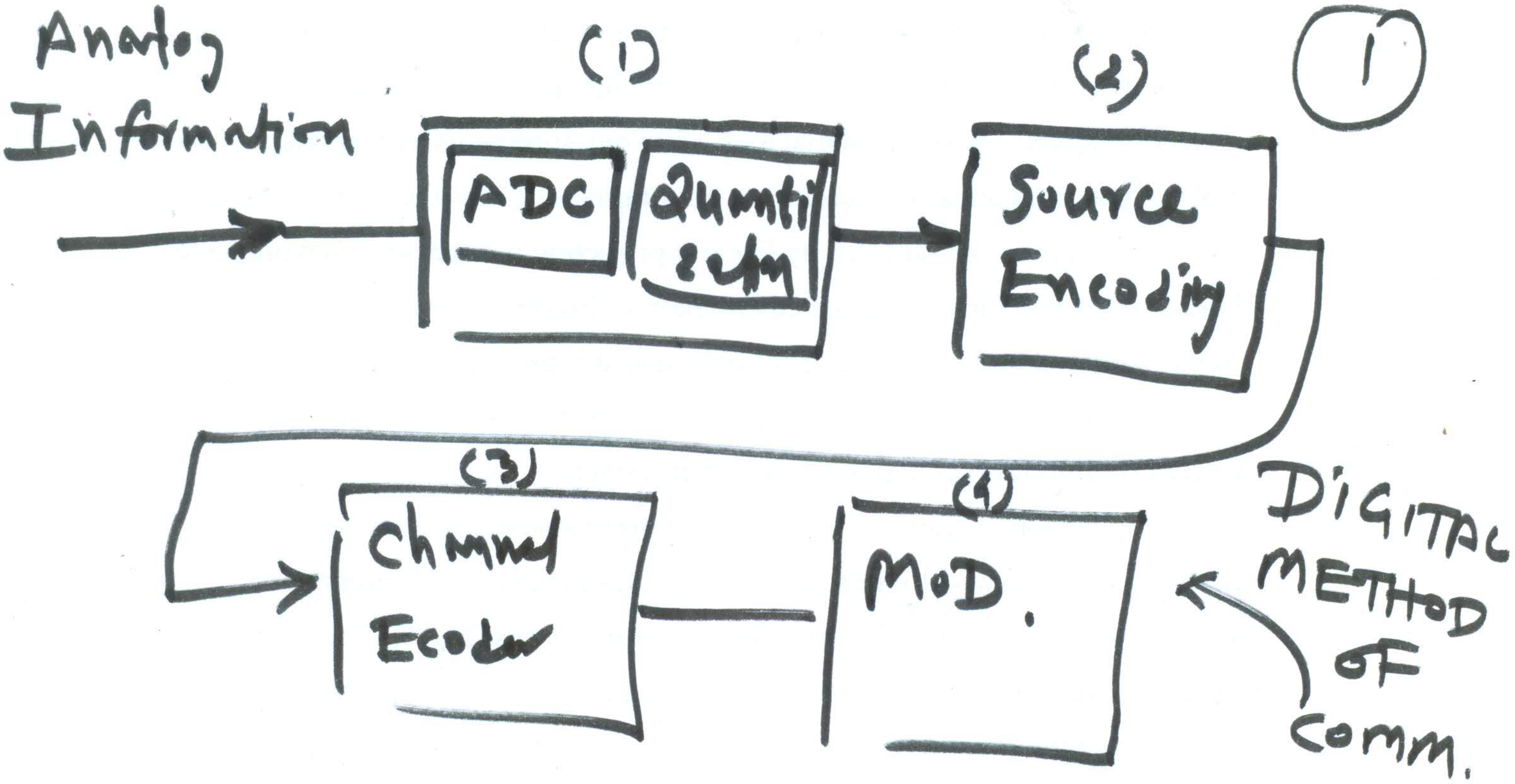
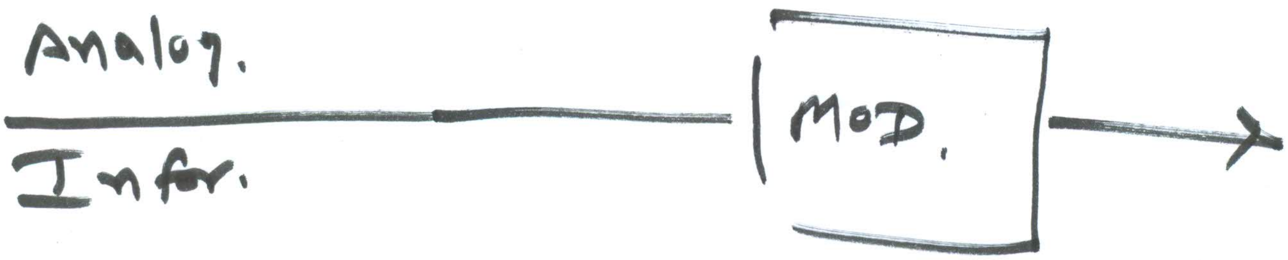
CT216

Analog Modulation: Handwritten Notes

3rd April 2023

Both AM & FM are called Analog methods of Communication.





$$s(t) \longleftrightarrow S(f)$$

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$s_1(t)$ $\quad$ $s_1(f)$

$$A\, e^{j2\pi f_c t} \longleftrightarrow A\,\delta(f-f_c)$$

$s_2(t)$ $\quad$ $S_2(f)$

$$\sum_{k=1}^{N} A_k\, e^{j2\pi f_{c,k} t} \longleftrightarrow \sum_{k=1}^{N} A_k\,\delta(f-f_{c,k})$$

$$\boxed{s_1(t)\cdot e^{j2\pi F t} \longleftrightarrow S(f) * \delta(f-F)}$$

$$= A\, e^{j2\pi (f_c+F)t} \longleftrightarrow A\,\delta(f-(f_c+F))$$

$$s_2(t)\cdot e^{j2\pi F t}$$

$$\longleftrightarrow \sum A_k\,\delta(f-(f_{c,k}+F))$$

MODULATION THEOREM OF F.T.

Time-Domain Multiplication with $e^{j2\pi Ft}$ $\longleftrightarrow$ Shift of Spectrum by F Hz

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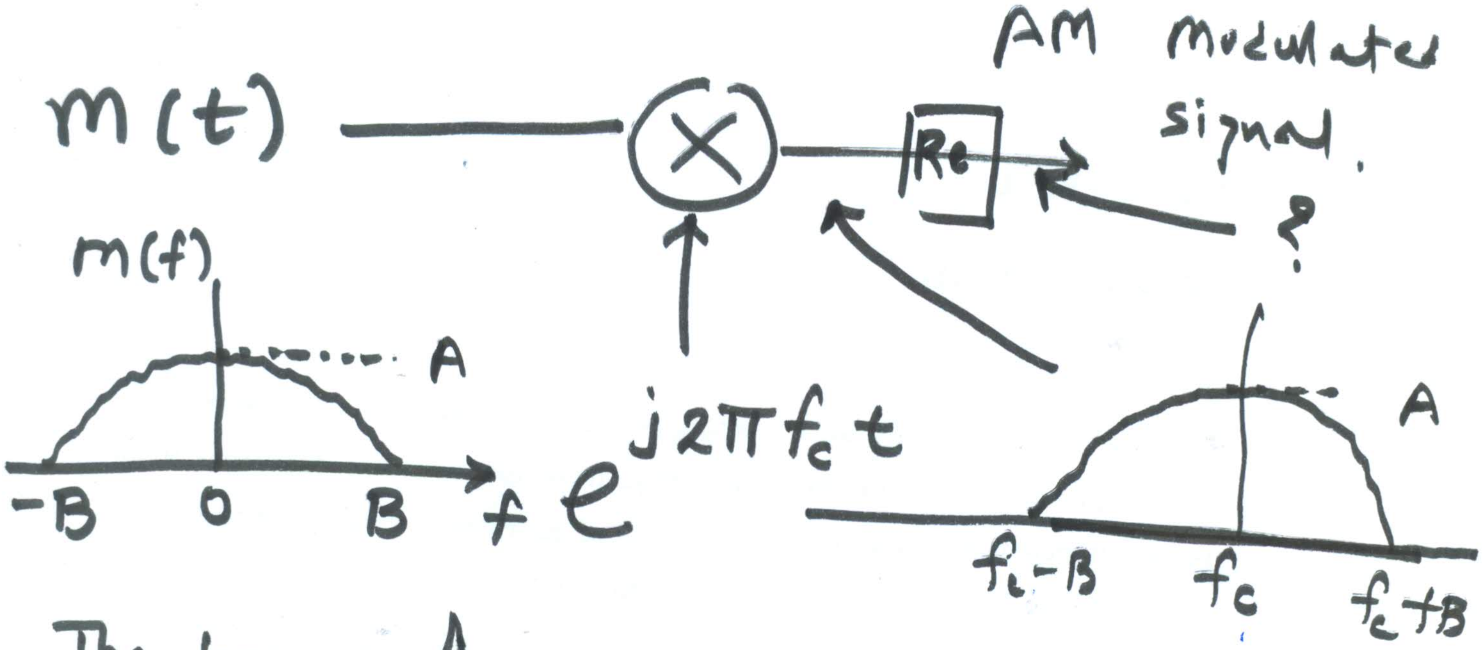

The larger f_c, carrier freq.

The required size of antenna is smaller.

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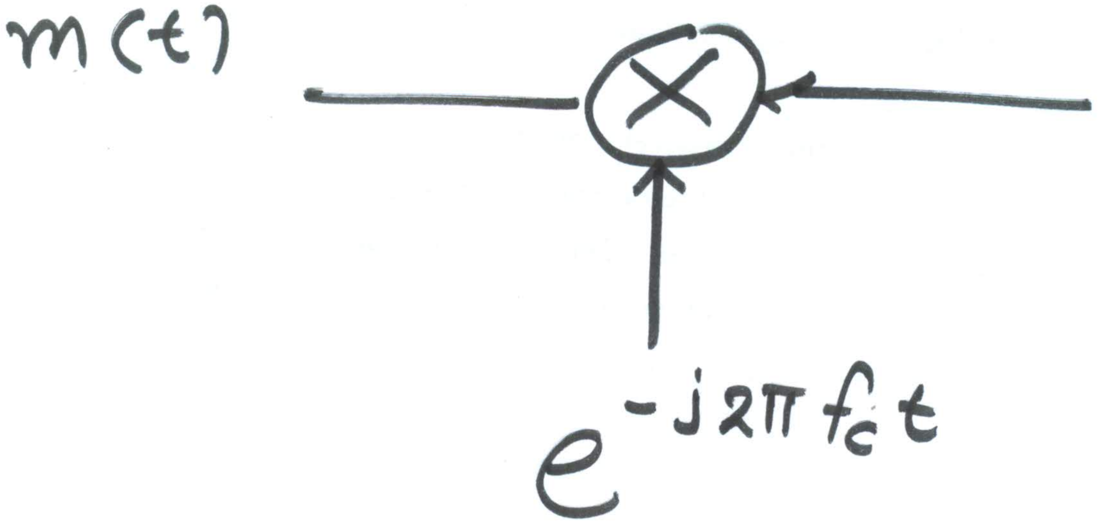
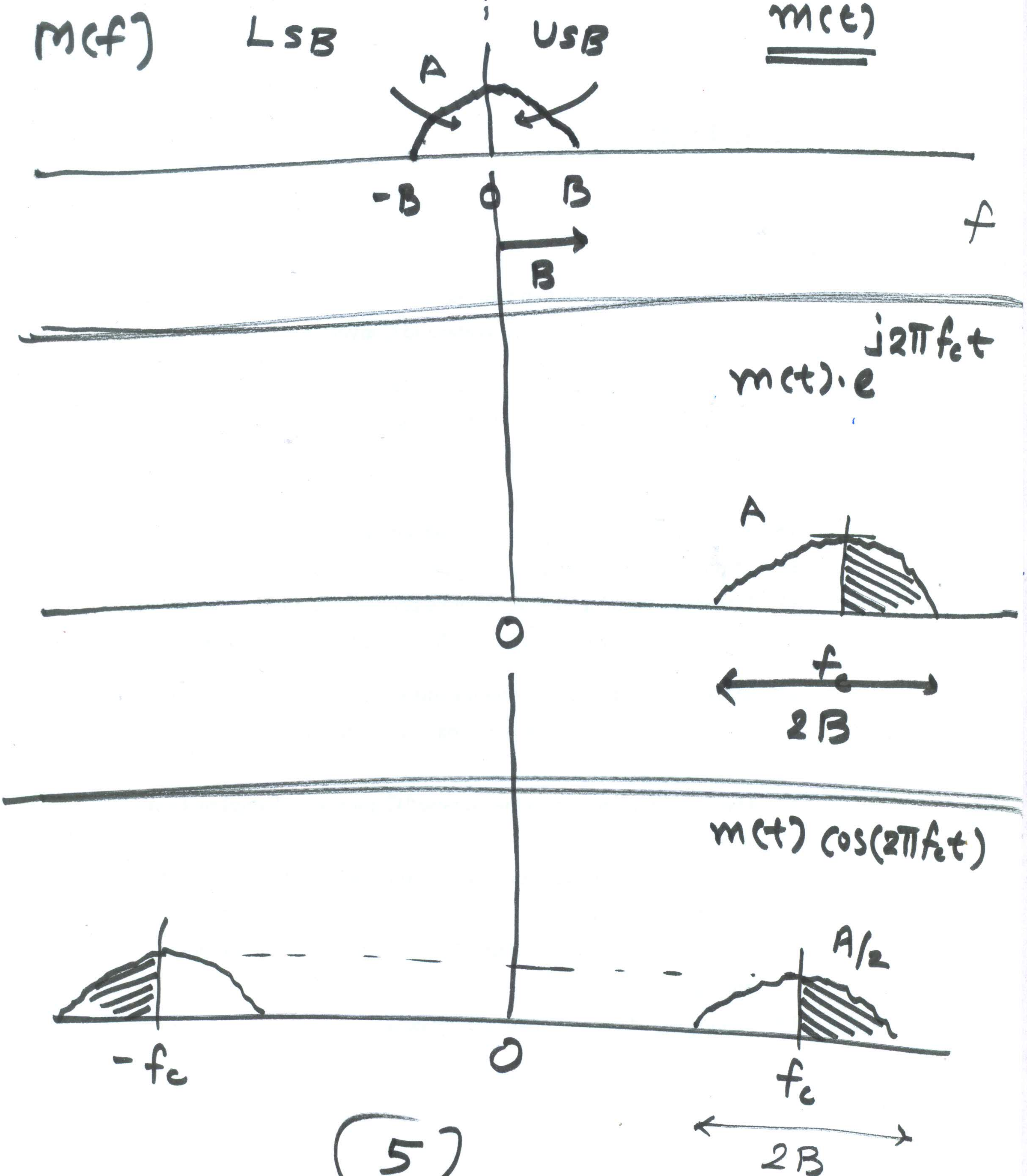
Demodulation



Spectrum of $m(t)$



An Effect of Modulation:

→ Negative (or mathematically defined but practically not realized] frequencies become real in some sense, positive-valued.

→ As a consequence, The message signal of bandwidth B Hz becomes a modulated signal of 2B Hz.

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→ An improved scheme of single sideband (SSB) does not have This disadv. of doubling The bandwidth.

$$s(t) = m(t)\, e^{j 2\pi f_c t}$$

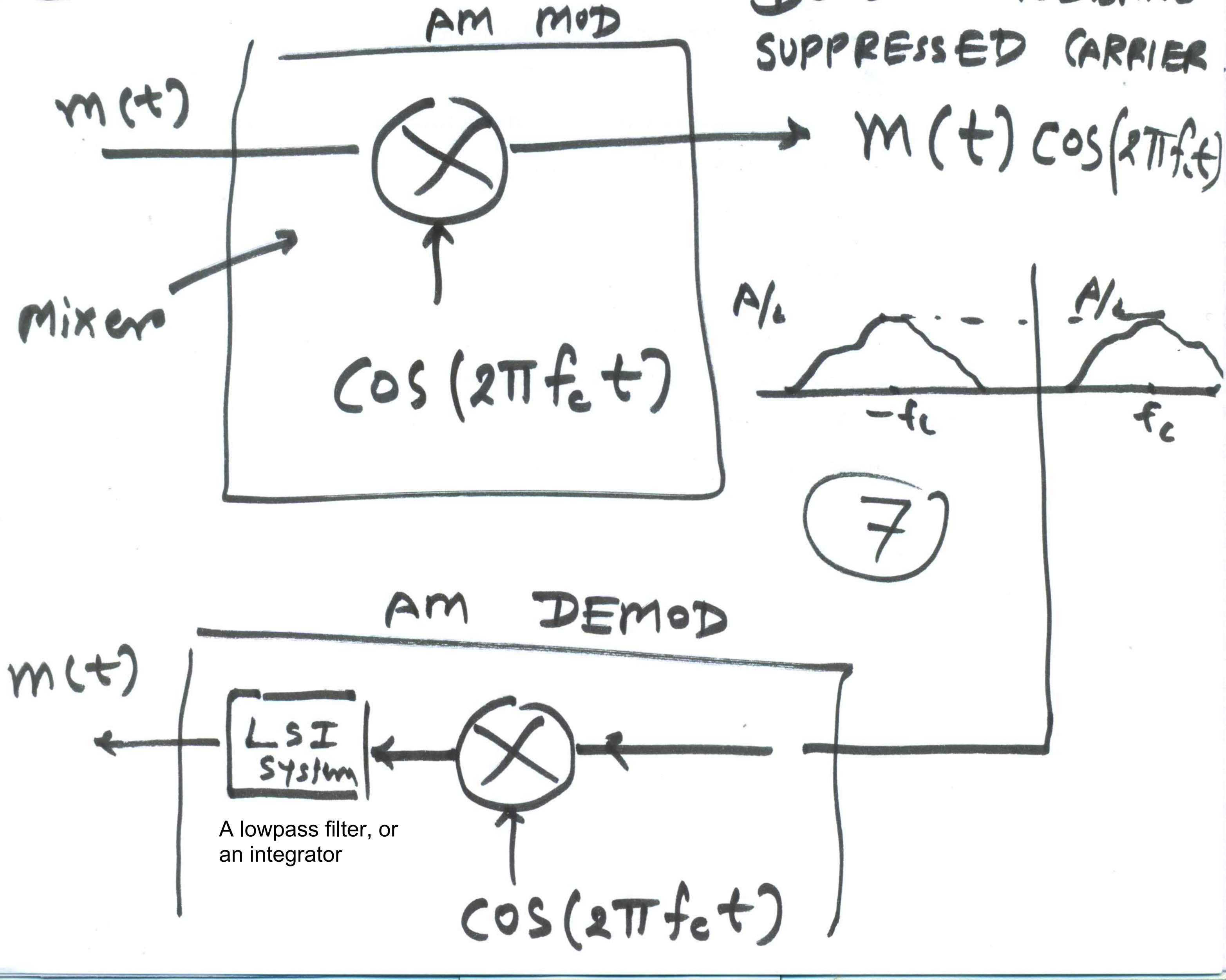
$$= m(t) \cos(2\pi f_c t) + j\, m(t) \sin(2\pi f_c t)$$

$$\Rightarrow \operatorname{Re}\{s(t)\} = m(t) \cdot \cos(2\pi f_c t)$$

A practical implementation of

AM Radio: (DSB-SC)

DOUBLE SIDEBAND SUPPRESSED CARRIER



A mixer is required for practical realization of ~~an~~ a DSB-SC AM Radio.

→ Topic of electronic circuit design

→ Typically makes use of diodes

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→ At an abstract level, the operation of a diode can be modelled as a nonlinear operation.

$$x(t) \longrightarrow \boxed{\text{DIODE}} \longrightarrow y(t) = x^2(t) + \ldots$$

If diode input is $x(t) + y(t)$

$\Rightarrow$ Output will contain one term which is the required time-domain product term $x(t) \cdot y(t)$

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1) A signal $f(t)$ is called an even signal if (They have mirror symmetry around $t=0$)

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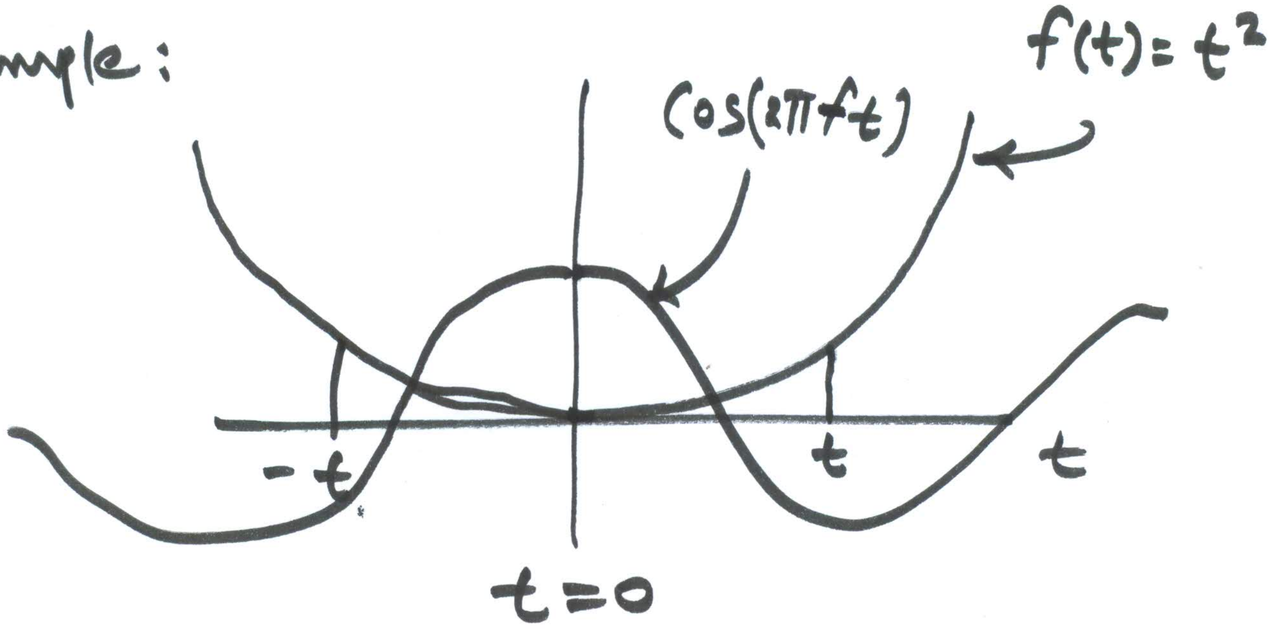
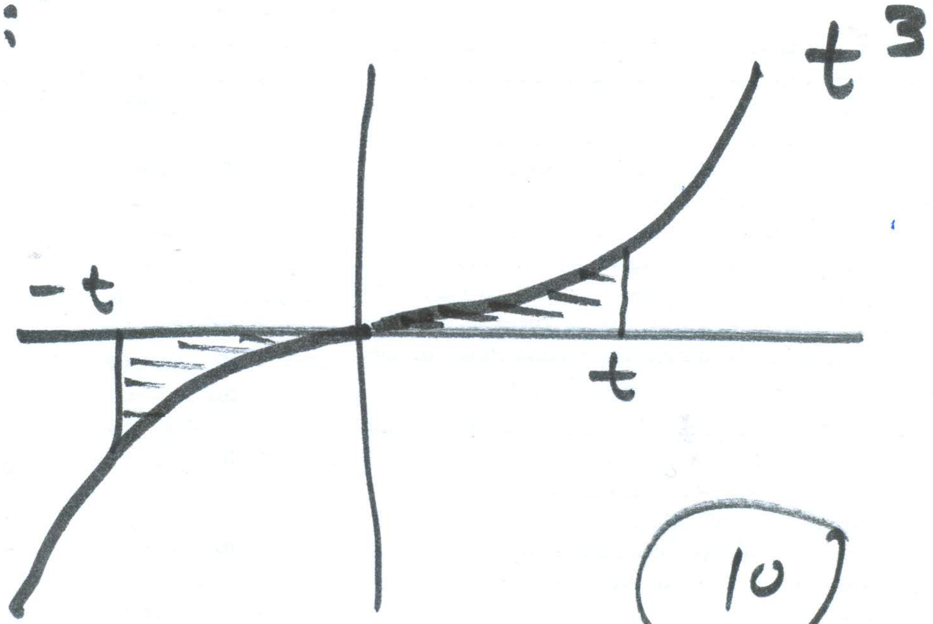
$$f(t) = f(-t)$$

Example:



A signal is called an odd signal if $f(t) = -f(-t)$

Example:



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2) $$\int_{-T}^{T} \text{ODD signal } dt = 0$$

3) ODD × EVEN = ODD

EVEN × ODD = ODD

ODD × ODD = EVEN

EVEN × EVEN = EVEN

4) $$\int_{-T}^{T} \text{ODD} \cdot \text{EVEN} \, dt = 0$$

WHY ?

$\Rightarrow$ ODD & EVEN signals are orthogonal :

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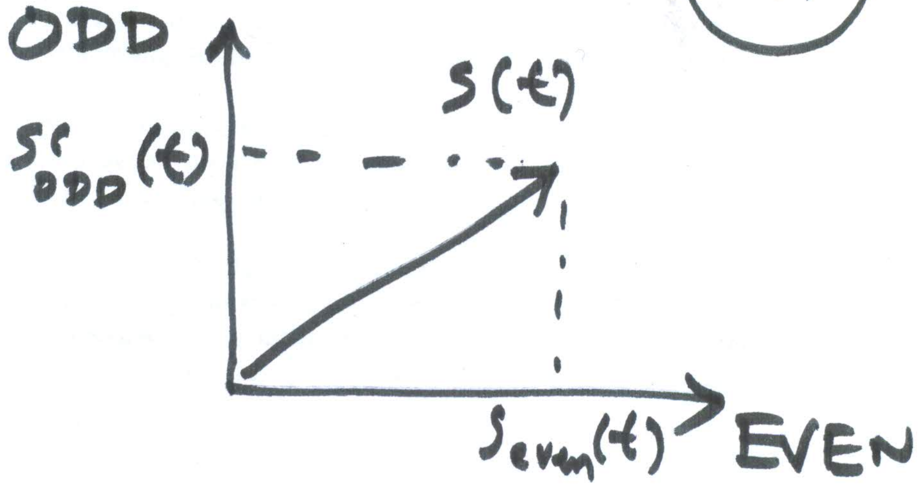

5) Any Signal $s(t)$ can be represented as

$$s(t) = S_{even}(t) + S_{ODD}(t)$$

$$S(f) = \int_{-\infty}^{\infty} s(t)\, e^{-j2\pi f t}\, dt$$

PROVE THIS.

$$S(f) = \int S_{even}(t) \cos(2\pi f_{\bullet} t)\, dt$$

$$+\ j \int S_{ODD}(t) \sin(2\pi f t)\, dt$$

Therefore, if $s(t)$ is an even/ODD signal, i.e., $S_{ODD}(t) = 0$ for all values of time t, the F.T. is not complex-valued, it is real-valued/imaginary.

In general, when $s(t) = s_{even}(t) + s_{ODD}(t)$, the F. T. has both real and imaginary components.

However, provided $s(t)$ is real-valued, the F. T. will exhibit (complex-conjugate) mirror symmetry, i.e.,

$$\boxed{S(-f) = S^*(f)}$$

This gives rise to USB and LSB which are mirrors of each other.

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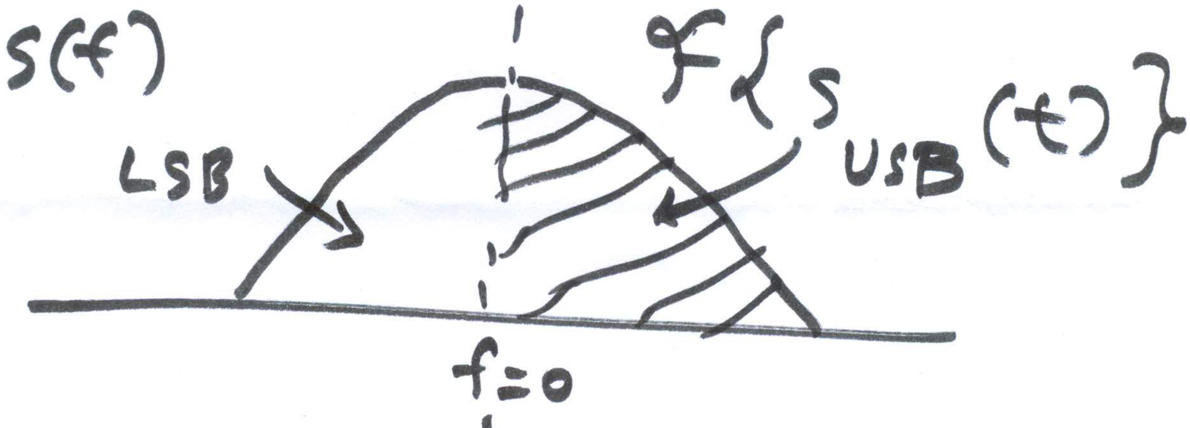
What happens to The real-valued information signal

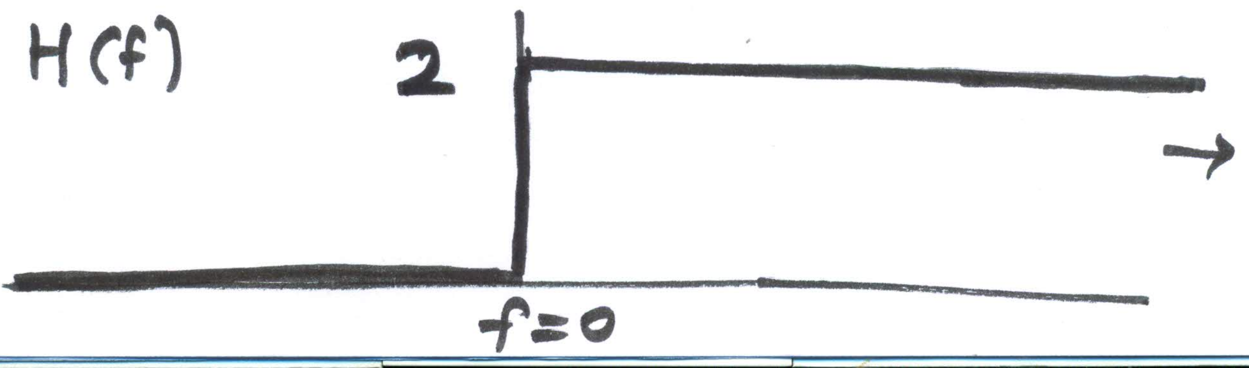
$s(t)$ if we remove one of The

two sidebands?

- Derivation is for a case where LSB is removed.
- Extend This to a case where USB is removed



To remove LSB, multiply $S(f)$ by $H(f)$



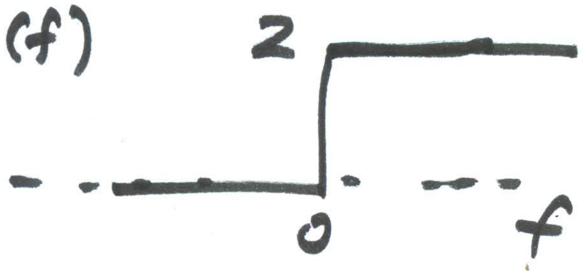
$$S_{USB}(f) = S(f) \cdot H(f)$$

$$\Rightarrow \quad s_{USB}(t) = s(t) * h(t)$$

where $h(t) = \mathcal{F}^{-1}\left[H(f)\right]$

$$= \delta(t) + j\,\frac{1}{\pi t} \,,$$

when $H(f)$



$$\Rightarrow \quad s_{USB}(t) = s(t) * \left(\delta(t) + j\,\frac{1}{\pi t}\right)$$

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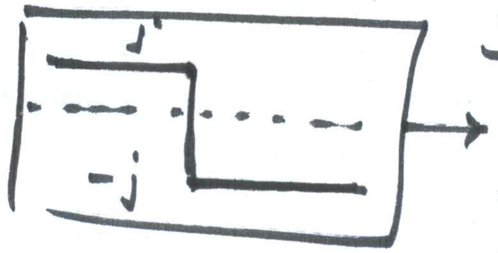
$$= s(t) * \delta(t) + j\left[s(t) * \frac{1}{\pi (t)}\right]$$

$$= s(t) + j\,\hat{s}(t)$$

A brief look at Hilbert Transform :

$$s(t) \longrightarrow \boxed{\overset{\text{H.T.}}{1/\pi t}} \longrightarrow \hat{s}(t)$$

$S(f) \longrightarrow$



$$\hat{S}(f) = \begin{cases} -j\,S(f), & f \geq 0 \\ j\,S(f), & f < 0 \end{cases}$$

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The USB signal in time domain

$$S_{USB}(t) = s(t) + j\,\hat{s}(t)$$

→ This signal is still around 0 Hz, also known as at "baseband"

The AM signal at carrier frequency f_c is given by

$$S^{AM}_{SSB}(t) = \mathrm{Re}\Big\{ \underbrace{S_{SSB}(t)\, e^{j2\pi f_c t}}_{\text{Complex-valued}} \Big\}$$

Real-valued

$S'_{SSB}(t)$

$$= \frac{1}{2}\Big(S'_{SSB}(t) + \big(S'_{SSB}\big)^{*}(t) \Big)$$

$$= \frac{1}{2}\Big(S_{SSB}(t)\, e^{j2\pi f_c t} + S^{*}_{SSB}(t)\, e^{-j2\pi f_c t} \Big)$$

$$\Downarrow$$

$$S^{AM}_{SSB}(f) = \frac{1}{2}\Big(S_{SSB}(f - f_c) + S_{SSB}\big(-(f + f_c)\big) \Big)$$

$$S_{SSB}^{AM}(t) = \mathrm{Re}\left\{\left[s(t) + j\,\hat{s}(t)\right] e^{j2\pi f_c t}\right\}$$

Complex-number representation

$$= s(t)\cos(2\pi f_c t) - \hat{s}(t)\sin(2\pi f_c t)$$

Cartesian coordinate representation

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$$= \underbrace{R(t)}_{\sqrt{s^2(t)+\hat{s}^2(t)}}\cos\left(2\pi f_c t + \theta(t)\right)$$

Polar

$$\theta(t) = \tan^{-1}\left(\frac{\hat{s}(t)}{s(t)}\right)$$

AM

Phase Modulation