Final Exam

Date	:	Monday 8th May, 2023, from 8 am to 11 am
Total Marks	:	50
Notes:	:	All questions are scored (no optionals).
		Do not forget to write your name and student ID below.
		Fully fill the OMR bubble for your selected option in the MCQ section.
		Answers written or OMR bubble filled using pencil will not be graded.

Name: _____ **Student ID**: _____

1 Multiple Choice Questions:

Instructions

- ▷ This is MCQ Answer Key of IV.
- ▷ There are total 30 questions in this section that carry a maximum of 55 marks. If the question is answered incorrectly, negative one-fourth of the total marks shown next to each question will be given.
- ▷ An OMR sheet is provided on the last page. Fill the bubble corresponding to the correct option. No tick marks or partial filling of the bubble, etc. If more than one bubble is filled, zero marks will be given for the question. Do not use a pencil to fill the bubble – the answer bubbled in with pencil, even if it is correct, will not be graded.

Questions

Version	I	II	III	IV
Question 1				
Answer (a):	-0.5 pts	-0.5 pts	2 pts	-0.5 pts
Answer (b):	2 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (c):	-0.5 pts	-0.5 pts	-0.5 pts	2 pts
Answer (d):	-0.5 pts	2 pts	-0.5 pts	-0.5 pts
Question 2				
Answer (a):	-0.5 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (b):	-0.5 pts	2 pts	2 pts	-0.5 pts
Answer (c):	2 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (d):	-0.5 pts	-0.5 pts	-0.5 pts	2 pts
Question 3				
Answer (a):	2 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (b):	-0.5 pts	-0.5 pts	2 pts	-0.5 pts
Answer (c):	-0.5 pts	-0.5 pts	-0.5 pts	2 pts
Answer (d):	-0.5 pts	2 pts	-0.5 pts	-0.5 pts

Version	I	II	III	IV
Question 4				
Answer (a):	-0.5 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (b):	-0.5 pts	2 pts	-0.5 pts	-0.5 pts
Answer (c):	2 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (d):	-0.5 pts	-0.5 pts	2 pts	2 pts
Question 5				
Answer (a):	-0.5 pts	-0.75 pts	-0.5 pts	1 pt
Answer (b):	-0.5 pts	3 pts	2 pts	-0.25 pts
Answer (c):	-0.5 pts	-0.75 pts	-0.5 pts	-0.25 pts
Answer (d):	2 pts	-0.75 pts	-0.5 pts	-0.25 pts
Question 6				
Answer (a):	-0.5 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (b):	-0.5 pts	-0.5 pts	-0.5 pts	2 pts
Answer (c):	2 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (d):	-0.5 pts	2 pts	2 pts	-0.5 pts
Question 7				
Answer (a):	-0.5 pts	-0.25 pts	-0.5 pts	-0.5 pts
Answer (b):	2 pts	1 pt	-0.5 pts	-0.5 pts
Answer (c):	-0.5 pts	-0.25 pts	2 pts	-0.5 pts
Answer (d):	-0.5 pts	-0.25 pts	-0.5 pts	2 pts
Question 8				
Answer (a):	-0.5 pts	-0.25 pts	-0.25 pts	-0.5 pts
Answer (b):	-0.5 pts	-0.25 pts	-0.25 pts	-0.5 pts
Answer (c):	-0.5 pts	2 pts	-0.25 pts	-0.5 pts
Answer (d):	2 pts	-0.25 pts	1 pt	2 pts
Question 9				
Answer (a):	-0.75 pts	-0.25 pts	-0.5 pts	-0.5 pts
Answer (b):	-0.75 pts	1 pt	-0.5 pts	2 pts
Answer (c):	-0.75 pts	—	-0.5 pts	-0.5 pts
Answer (d):	3 pts	—	2 pts	-0.5 pts
Question 10				
Answer (a):	-0.25 pts	-0.5 pts	1 pt	-0.5 pts
Answer (b):	1 pt	-0.5 pts	-0.25 pts	-0.5 pts
Answer (c):	-0.25 pts	2 pts	—	2 pts
Answer (d):	-0.25 pts	-0.5 pts	—	-0.5 pts
Question 11				
Answer (a):	-0.75 pts	2 pts	-0.5 pts	-0.5 pts
Answer (b):	-0.75 pts	-0.5 pts	-0.5 pts	2 pts
Answer (c):	3 pts	-0.5 pts	2 pts	-0.5 pts
Answer (d):	-0.75 pts	-0.5 pts	-0.5 pts	-0.5 pts
Question 12				
Answer (a):	-0.5 pts	-0.25 pts	-0.5 pts	-0.25 pts
Answer (b):	-0.5 pts	-0.25 pts	-0.5 pts	2 pts
Answer (c):	-0.5 pts	1 pt	-0.5 pts	-0.25 pts
Answer (d):	2 pts	-0.25 pts	2 pts	-0.25 pts
Question 13				
Answer (a):	-0.25 pts	2 pts	-0.5 pts	2 pts
Answer (b):	-0.25 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (c):	2 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (d):	-0.25 pts	-0.5 pts	2 pts	-0.5 pts
Question 14				

Version	I	II	III	IV
Answer (a):	-0.5 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (b):	-0.5 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (c):	2 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (d):	-0.5 pts	2 pts	2 pts	2 pts
Question 15				
Answer (a):	-0.5 pts	-0.5 pts	3 pts	-0.5 pts
Answer (b):	2 pts	2 pts	-0.75 pts	-0.5 pts
Answer (c):	-0.5 pts	-0.5 pts	-0.75 pts	-0.5 pts
Answer (d):	-0.5 pts	-0.5 pts	-0.75 pts	2 pts
Question 16				
Answer (a):	-0.5 pts	-0.25 pts	-0.5 pts	-0.25 pts
Answer (b):	-0.5 pts	1 pt	-0.5 pts	1 pt
Answer (c):	-0.5 pts	-0.25 pts	-0.5 pts	-0.25 pts
Answer (d):	2 pts	-0.25 pts	2 pts	-0.25 pts
Question 17				
Answer (a):	-0.5 pts	3 pts	-0.5 pts	2 pts
Answer (b):	-0.5 pts	-0.75 pts	2 pts	-0.5 pts
Answer (c):	-0.5 pts	-0.75 pts	-0.5 pts	-0.5 pts
Answer (d):	2 pts	-0.75 pts	-0.5 pts	-0.5 pts
Question 18				
Answer (a):	-0.5 pts	-0.5 pts	-0.75 pts	2 pts
Answer (b):	-0.5 pts	-0.5 pts	-0.75 pts	-0.5 pts
Answer (c):	2 pts	2 pts	3 pts	-0.5 pts
Answer (d):	-0.5 pts	-0.5 pts	-0.75 pts	-0.5 pts
Question 19				
Answer (a):	-0.25 pts	2 pts	-0.5 pts	-0.5 pts
Answer (b):	1 pt	-0.5 pts	-0.5 pts	2 pts
Answer (c):	-0.25 pts	-0.5 pts	2 pts	-0.5 pts
Answer (d):	-0.25 pts	-0.5 pts	-0.5 pts	-0.5 pts
Question 20				
Answer (a):	-0.5 pts	-0.25 pts	-0.5 pts	-0.75 pts
Answer (b):	-0.5 pts	-0.25 pts	2 pts	-0.75 pts
Answer (c):	-0.5 pts	1 pt	-0.5 pts	3 pts
Answer (d):	2 pts	-0.25 pts	-0.5 pts	-0.75 pts
Question 21				
Answer (a):	-0.5 pts	2 pts	-0.25 pts	-0.5 pts
Answer (b):	-0.5 pts	-0.5 pts	-0.25 pts	-0.5 pts
Answer (c):	2 pts	-0.5 pts	1 pt	-0.5 pts
Answer (d):	-0.5 pts	-0.5 pts	-0.25 pts	2 pts
Question 22				
Answer (a):	-0.5 pts	-0.5 pts	-0.25 pts	-0.25 pts
Answer (b):	2 pts	2 pts	-0.25 pts	-0.25 pts
Answer (c):	-0.5 pts	-0.5 pts	1 pt	-0.25 pts
Answer (d):	-0.5 pts	-0.5 pts	-0.25 pts	1 pt
Question 23				
Answer (a):	2 pts	2 pts	2 pts	-0.5 pts
Answer (b):	-0.5 pts	-0.5 pts	-0.25 pts	-0.5 pts
Answer (c):	-0.5 pts	-0.5 pts	-0.25 pts	2 pts
Answer (d):	-0.5 pts	-0.5 pts	-0.25 pts	-0.5 pts
Question 24				
Answer (a):	1 pt	-0.5 pts	2 pts	3 pts

Version	I	II	III	IV
Answer (b):	-0.25 pts	-0.5 pts	-0.5 pts	-0.75 pts
Answer (c):	-0.25 pts	2 pts	-0.5 pts	-0.75 pts
Answer (d):	-0.25 pts	-0.5 pts	-0.5 pts	-0.75 pts
Question 25				
Answer (a):	-0.5 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (b):	-0.5 pts	2 pts	2 pts	-0.5 pts
Answer (c):	-0.5 pts	-0.5 pts	-0.5 pts	2 pts
Answer (d):	2 pts	-0.5 pts	-0.5 pts	-0.5 pts
Question 26				
Answer (a):	2 pts	-0.5 pts	2 pts	-0.25 pts
Answer (b):	-0.5 pts	-0.5 pts	-0.5 pts	1 pt
Answer (c):	-0.5 pts	2 pts	-0.5 pts	-0.25 pts
Answer (d):	-0.5 pts	-0.5 pts	-0.5 pts	-0.25 pts
Question 27				
Answer (a):	-0.25 pts	2 pts	-0.5 pts	-0.5 pts
Answer (b):	1 pt	-0.5 pts	-0.5 pts	-0.5 pts
Answer (c):	—	-0.5 pts	-0.5 pts	2 pts
Answer (d):	—	-0.5 pts	2 pts	-0.5 pts
Question 28				
Answer (a):	-0.25 pts	2 pts	2 pts	-0.25 pts
Answer (b):	1 pt	-0.5 pts	-0.5 pts	1 pt
Answer (c):	-0.25 pts	-0.5 pts	-0.5 pts	—
Answer (d):	-0.25 pts	-0.5 pts	-0.5 pts	—
Question 29				
Answer (a):	-0.5 pts	-0.5 pts	-0.5 pts	-0.5 pts
Answer (b):	-0.5 pts	-0.5 pts	-0.5 pts	2 pts
Answer (c):	2 pts	2 pts	2 pts	-0.5 pts
Answer (d):	-0.5 pts	-0.5 pts	-0.5 pts	-0.5 pts
Question 30				
Answer (a):	-0.5 pts	-0.5 pts	1 pt	-0.5 pts
Answer (b):	-0.5 pts	-0.5 pts	-0.25 pts	2 pts
Answer (c):	2 pts	2 pts	-0.25 pts	-0.5 pts
Answer (d):	-0.5 pts	-0.5 pts	-0.25 pts	-0.5 pts

2 Fill In the Blanks

Instructions

- ▷ There are total 16 questions in this section that carry total of 45 marks.
- ▷ There is no negative marking in this section.

Part 1

1. (1 point) In the presence of additive Gaussian noise, what is the name of the receiver which is equivalent to the Bayesian ML receiver?
   - Minimum Euclidean Distance Receiver

2. (2 points) What are the full forms of the EVM and BLER?

   - Error Vector Magnitude
   - Block Error Rate

3. (1 point) Energy of a symbol located on a constellation diagram is defined as

   - squared distance of the symbol from the origin

4. (1 point) Energy of a time domain signal $s(t)$ over a duration 0 to T seconds is formulated as

   - $\int_0^T |s(t)|^2 \, dt$

5. (1 point) What is functional form of the integral $\frac{1}{\sqrt{2\pi\sigma_n^2}} \int_0^\infty \exp\left(-\frac{(x+a)^2}{2\sigma_n^2}\right) dx$

   - $Q\left(\frac{a}{\sigma_n}\right)$

6. (1 point) The OOK transmitter uses +5 volts to transmit bit $X = 1$ and 0 volts to send $X = 0$. When the output of the channel is r volts, the ML OOK demodulator rule is

   - Set the demodulated bit $\hat{X} = 1$ if $r \geq 2.5$ volts; else set $\hat{X} = 0$.

7. (2 points) The power of the additive Gaussian distributed noise signal $n(t)$ at time $t = 5$ is given as $P_n(t = 5) = 10$ Watts. Can this statement be correct? Briefly describe your answer.

   - False, the instantaneous power of a random signal, such as the noise signal, is not defined. Power of a random signal can, e.g., be obtained by taking a statistical average over time domain, which is not possible to do at a given time instant.

8. (2 points) The maximum rate of information transfer over the BSC(p) (i.e., the mutual information between RVs corresponding to the input and the output of the channel) is given as

   - $1 - H_b(p) = 1 + p\log_2(p) + (1-p)\log_2(1-p)$.

Part 2

9. (3 points) Write expression for the Union Bound on the error probability for the Constellation A in Fig. 5:

   - The Euclidean distance between the nearest two neighbors of a given constellation symbol equals $d = \sqrt{2E_S}$. The distance to the third symbol along the diagonal is $d' = 2\sqrt{E_S}$.
   - Plugging the values of d and d' in the formula for the UB, where the pairwise error probability is given as $Q\left(()\, d/sqrt2N_0\right)$, we obtain the following union bound on the symbol error probability p_e:
   - $p_e < 2Q\left(\sqrt{E_S/N_0}\right) + Q\left(\sqrt{2E_S/N_0}\right)$
   - A note to the TA evaluator: if someone has used just the nearest neighbors and given the answer $p_e < 2Q\left(\sqrt{E_S/N_0}\right)$, please check their answer to the next question and give full marks if the answer there also is correct assuming just the two nearest neighbors. Else give zero marks.

10. (3 points) Write expression for the Union Bound on the error probability for the Constellation B in Fig. 5:

- For the constellation B, the Euclidean distance between the nearest two neighbors of a given constellation symbol equals $d = \sqrt{E_S}$. The distance to the third symbol along the diagonal is $d' = \sqrt{2E_S}$.
- Therefore the union bound becomes
- $p_e < 2Q\left(\sqrt{E_S/2N_0}\right) + Q\left(\sqrt{E_S/N_0}\right)$

11. (4 points) Are the following three cases possible? For each case, if the answer is 'no', explain why. If it is 'yes', provide an example of modulation and coding design that achieves the stated relationship.

1. Energy per bit E_b is smaller than the energy per symbol E_S, i.e., $E_b < E_S$.
2. $E_b = E_S$.
3. $E_b > E_S$.

- Note: $E_b = \dfrac{E_S}{k \times r}$, where r is the code rate, $k = \log_2 M$ and M is the size of the modulation constellation. All three cases are possible, for example, for:
- Case (i): set $r = 1$ and $k = \log_2 M > 1$ (QPSK, 8-PSK, etc.).
- Case (ii): set $r = 0.5$ and $k = \log_2 M = 2$ (QPSK).
- Case (iii): set $r = 0.1$ and $k = \log_2 M = 2$.

12. (4 points) Determine the symbol rate, bandwidth efficiency in bps/Hz and required received information bit energy to noise ratio E_b/N_0 to support a data rate $R_b = 10$ Mbps for the following cases:

1. QPSK modulation, code rate $r = 0.5$, required $E_S/N_0 = 1.2$ dB
2. 32-APSK modulation, code rate $r = 0.8$, required $E_S/N_0 = 14$ dB

- Note: $E_b = \dfrac{E_S}{k \times r}$ and $R_b = R_{sym} \times k \times r$.
- For QPSK modulation, $k = 2$. The coding rate r is given as 0.5. Therefore, $k \times r = 1$. Therefore,
  - ▷ $E_b/N_0 = E_S/N_0 = 1.2$ dB and
  - ▷ $R_b = R_{sym}$, i.e., the spectral efficiency, which can be approximated as $R_b/R_{sym} = k \times r$ bits/symbol equals 1.
- For 32-APSK modulation, $k = 5$. The coding rate r is given as 0.8. Therefore, $k \times r = 4$. Therefore,
  - ▷ $E_b/N_0 = E_S/N_0 - 10\log_{10}(4) = 14 - 6 = 8$ dB and
  - ▷ $R_b = 4R_{sym}$, i.e., the spectral efficiency, which can be approximated as $R_b/R_{sym} = k \times r$ bits/symbol equals 4.

13. (5 points) When the transmitted bit X is modeled as the Bernouilli($q = 0.5$) RV, and the channel is the BEC(p), what are (i) the conditional entropy $H(X\,|\,Y)$ and (ii) the maximum rate of information transfer (i.e., mutual information)? Provide the derivation steps.

- Since this is a BEC, $H(X\,|\,Y = 0) = 0$ bits. Similarly, $H(X\,|\,Y = 1) = 0$ bits. In both the cases, the receiver has no uncertainty regarding the state of X.
- $H(X\,|\,Y = e) = 1$ bit, since the receiver has maximal uncertainty about the state of X.
- The conditional entropy $H(X\,|\,Y)$ is the expectation of the above three values. To calculate this expectation, we need to calculate the probabilities $P(Y = 0), P(Y = 1)$ and $P(Y = e)$. However, the first two probabilities get multiplied with $H(X\,|\,Y$ values which are zero and need not be calculated. The probability $P(Y = e)$ is clearly p.

- Therefore, $H(X\,|\,Y) = 1$ and $I(X;Y) = H(X) - H(X\,|\,Y) = 1 - p$.

14. (5 points) When X is the Bernouilli(p) RV, provide expressions for (i) for the expected number k of ones in the sequences of length N bits generated by N realizations of X; (ii) the number of length N binary sequences that have these number of ones.

    - $p \times N$ and $\binom{N}{N\times p}$, respectively

15. (5 points) Consider a beamformer with two antenna elements. A user terminal located on the ground at an angle of $\breve{\theta}$ (relative to line connecting the antenna elements) transmits a signal $s(t)$. Resultant signals at the output of the two antenna elements are given by $r_1(t) = \alpha s(t)$ and $r_2(t) = \alpha \exp(-j\theta)\, s(t)$ (α is the path loss attenuation and the phase θ is a one-to-one function of the Direction of Arrival (DoA) angle $\breve{\theta}$). Beamformer output is given as $r(t) = r_1(t) + \exp(j\phi)\, r_2(t)$, where $\exp(j\phi)$ is a complex-valued scalar weighting that the beamformer applies to antenna 2 relative to antenna 1. Define $\varphi = \phi - \theta$. Determine the closed-form expression of the magnitude response of beamformer, defined as $\left|\dfrac{r(t)}{\alpha s(t)}\right|$. For which value of θ, is the response of the beamformer the greatest? For which θ does it become zero?

    - The following is the solution to this problem.

$$\begin{aligned}
\left|\frac{r(t)}{\alpha s(t)}\right| &= |1 + \exp(j(\phi - \theta)| = |1 + \exp(j\varphi)| \\
&= |(1 + \cos(\varphi)) + j\sin(\varphi)| \\
&= \sqrt{(1 + \cos(\varphi))^2 + \sin^2(\varphi)} \\
&= \sqrt{2 + 2\cos(\varphi)} \\
&= \sqrt{2(1 + \cos(\varphi))} \\
&= \sqrt{2 \times 2\cos^2(\varphi/2)} \\
&= 2\cos(\varphi/2)
\end{aligned}$$

    - The response of the beamformer is the greatest when $\varphi = \phi - \theta = 0^o$ since the $\cos(\varphi/2)$ term becomes maximum. Thus, the beamformer provides the maximum gain when the signal arrives in the direction $\phi = \theta$.
    - The beamformer response becomes zero when $\varphi/2 = 90^o$, i.e., $\phi - \theta = 180^o$, i.e., when the beamformer coefficient phase $\phi = \theta + 180^o$.

16. (5 points) There are two twenty-faced marbles A and B. On each face of each marble, one of five symbols $(C, T, 5, 1, 6)$ is written. Number of times these symbols appear on the faces of each marble is listed in Table 2. Your friend chooses a marble at random, rolls the chosen marble 5 times and tells you that the outcomes turned out to be C, T, 5, 1, 6. Given this *evidence*, what is the ratio of probabilities that your friend chose marble A versus the probability that he/she chose marble B? From this ratio, determine the probability that the chosen marble is marble A.

- The Bayesian likelihood functions, i.e., probability of evidence given the cause, are given as follows:

$$\begin{aligned} P(CT516 \,|\, A) &= \frac{8}{12} \times \frac{3}{9} \times \frac{4}{10} \times \frac{2}{5} \times \frac{3}{4} \\ &= \frac{2}{3} \times \frac{1}{3} \times \frac{2}{5} \times \frac{2}{5} \times \frac{3}{4} \\ &= \frac{2}{75} \\ P(CT516 \,|\, B) &= \frac{4}{12} \times \frac{6}{9} \times \frac{6}{10} \times \frac{3}{5} \times \frac{1}{4} \\ &= \frac{1}{3} \times \frac{2}{3} \times \frac{3}{5} \times \frac{3}{5} \times \frac{1}{4} \\ &= \frac{1}{50} \end{aligned}$$

- Since the priors are equal, the ratio of the posteriors is the same as the ratio of the likelihood functions. Therefore, the required ratio is given as

$$\frac{P(A \,|\, CT516)}{P(B \,|\, CT516)} = \frac{P(CT516 \,|\, A)}{P(CT516 \,|\, B)} = \frac{2}{75} \times \frac{50}{1} = \frac{4}{3}$$

- The probability that Marble A was chosen is

$$P(A \,|\, CT516) = \frac{\frac{4}{3}}{\frac{4}{3} + 1} = \frac{4}{7}$$

Table 2: Marble Characteristics for Problem 16

Symbol	C	T	5	1	6
Number of Faces of Marble A	8	3	4	2	3
Number of Faces of Marble B	4	6	6	3	1

Part 3

- Please mention three concepts of CT216 that you liked and that will stay with you.
- Wish you all the best. Have a great summer break.

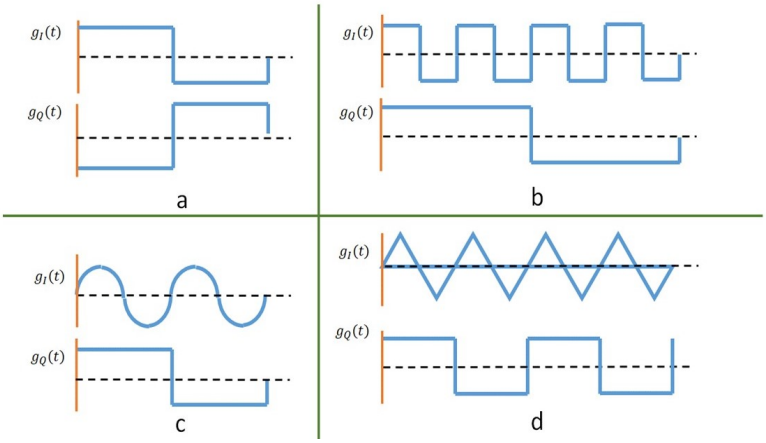

Figure 1: Two signals for Problem **??**.

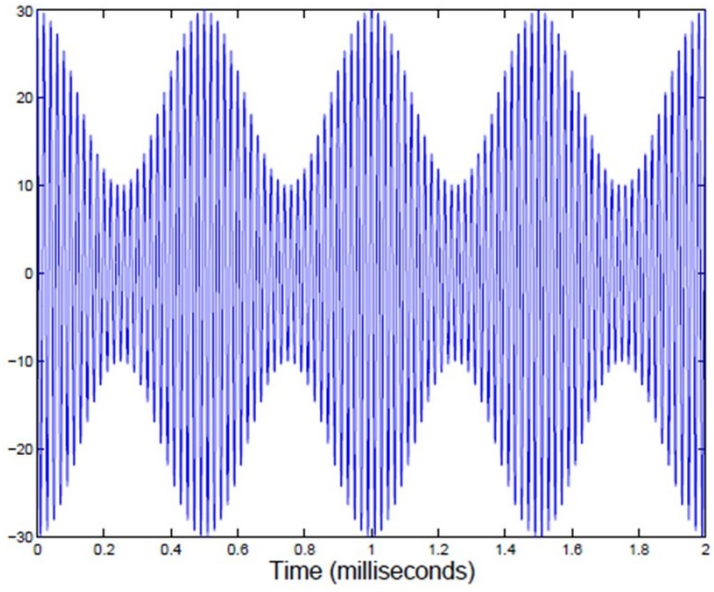

Figure 2: DSB-FC Waveform for Problem **??**.

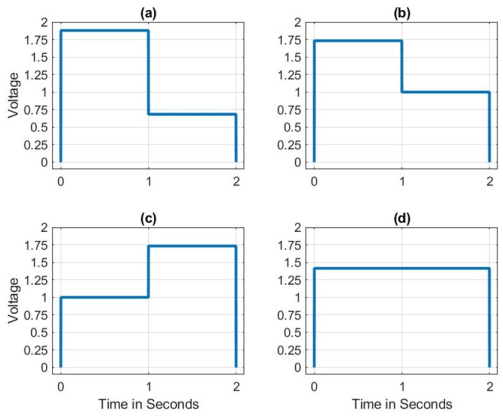

Figure 3: Modulated signal waveforms for Problem **??**.

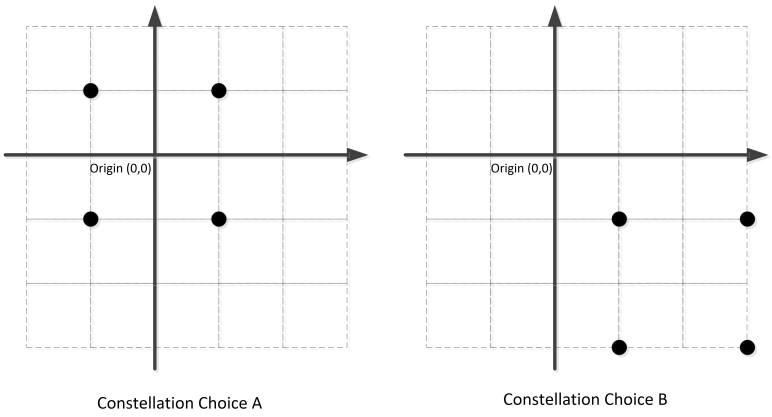

Figure 4: Signal Constellation for Problem **??**.

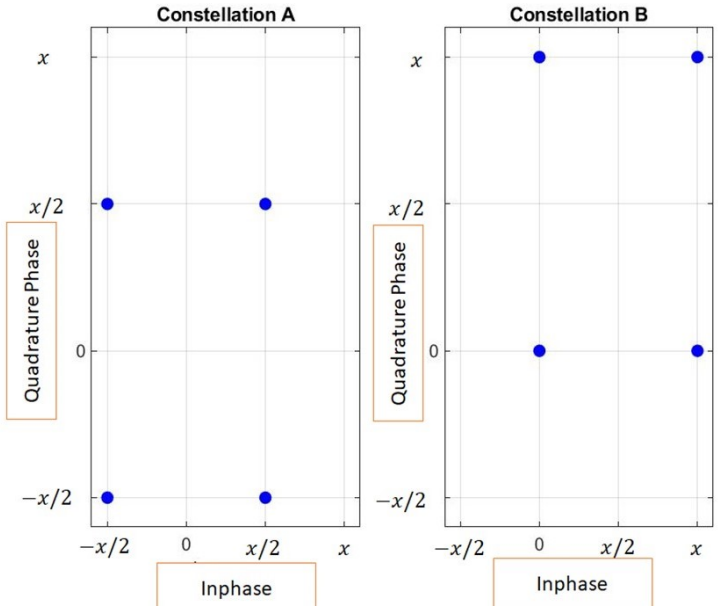

Figure 5: Constellations A and B for Problems 9 and 10, respectively.

Figure 6: Indicate your MCQ answers by filling up this OMR sheet using a ball-point pen. Place your name and student ID.