Analyzing IBM's Growth Through Logistic Modeling

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CS-302, Modeling and Simulation

In this report, we used the logistic equation to model the growth of IBM's revenue and workforce size (Human Resource). The parameters and constants were adjusted to minimize the mean and standard deviation of the relative error. We have also plotted the correlation between IBM Revenue and Workforce.

I. INTRODUCTION AND MODELING

The Logistic Equation, given by

$$\ddot{x} \equiv \frac{dx}{dt} = \mathcal{F}(x) = ax - bx^2 \tag{1}$$

describes a fundamental non-linear system where a and b are fixed parameters. Solving this equation with the initial condition $x(0) = x_0$, $k = a/b$, we obtain the following solution:

$$x(t) = \frac{kx_0e^{at}}{k + x_0(e^{at} - 1)} \tag{2}$$

The time scale for the nonlinear growth is expressed as:

$$t_{nl} = \frac{1}{a} \cdot \ln\left(\frac{k}{x_0} - 1\right) \tag{3}$$

In the context of modeling the Annual Revenue of IBM, the logistic equation can be applied as:

$$\dot{R} \equiv \frac{dR}{dt} = \mathcal{R}(R) = \rho_1 R - \rho_2 R^2 \tag{4}$$

where, $\mathcal{R}(R)$ represents IBM's revenue in US dollars and t is time in years. Here, ρ_1 and ρ_2 replace parameters a and b , and the maximum revenue k_R is given by $k_R = \rho_1/\rho_2$.
Similarly, to model the growth of IBM's Human Resources, we use a logistic equation:

$$\dot{H} \equiv \frac{dH}{dt} = \mathcal{H}(H) = \eta_1 H - \eta_2 H^2 \tag{5}$$

where, $\mathcal{H}(H)$ represents IBM's human resources and η_1 and η_2 are analogous to the parameters a and b, with $k_H = \eta_1/\eta_2$.
To explore the interdependent growth of IBM revenue (R) and human resources (H), we define a coupled autonomous dynamical system described by $\dot{R} = \mathcal{R}(H, R)$ and $\dot{H} = \mathcal{H}(H, R)$. By introducing new variables $V = R^{-1} - {k_R}^{-1}$ and $U = H^{-1} - {k_H}^{-1}$, and defining the parameter $\beta = \rho_1/\eta_1$, the relationship between human resources and revenue simplifies into the following power-law expression:

$$V \sim U^{\beta} \tag{6}$$

II. RESULTS

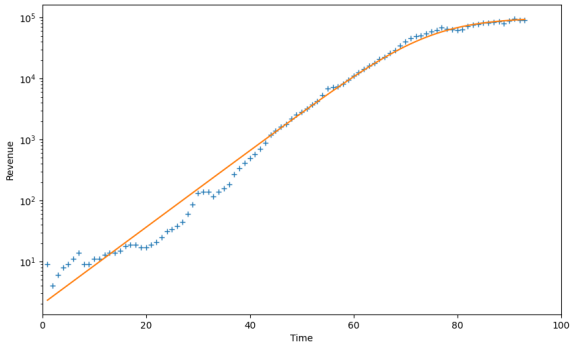

FIG. 1: The dotted graph shows the Annual Revenue of IBM and The smooth curve is the logistic function, as given by Eq. 4. The parameter values to fit the revenue growth are $\rho_1 = 0.145$ year^{-1} and $k_R = \$100$ billion (the predicted maximum revenue that IBM can earn).

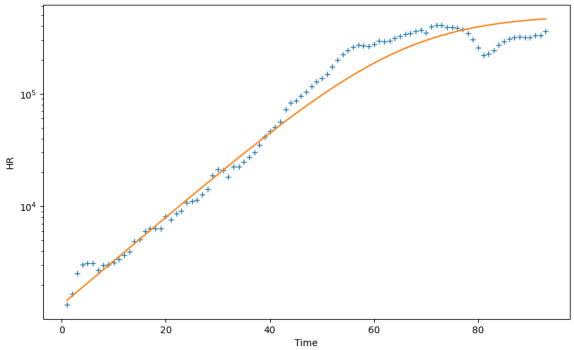

FIG. 2: The dotted graph shows the Human Resource of IBM and The smooth curve is the logistic function, as given by Eq. 5. The parameter values to fit the revenue growth are $\eta_1 = 0.09$ year^{-1} and $k_H = 500000$ (the predicted maximum revenue that IBM can earn).

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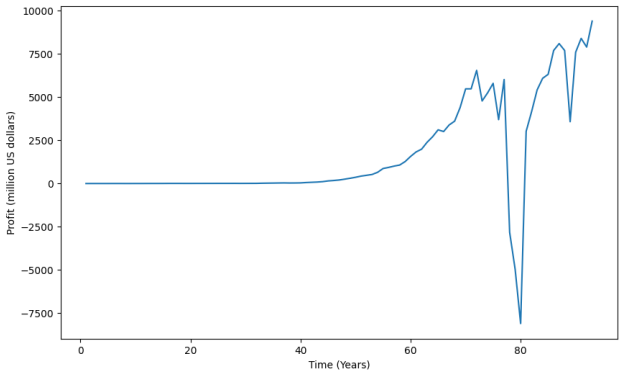

FIG. 3: The net annual earnings (the profit P) of IBM grow steadily till about 75-80 years (the early years of the 1990s). Around this time IBM suffered major losses in its net earnings ($8 billion in 1993), and this time scale corresponds closely to the time scale for the onset of nonlinear saturation in revenue growth, which is also 75-80 years.

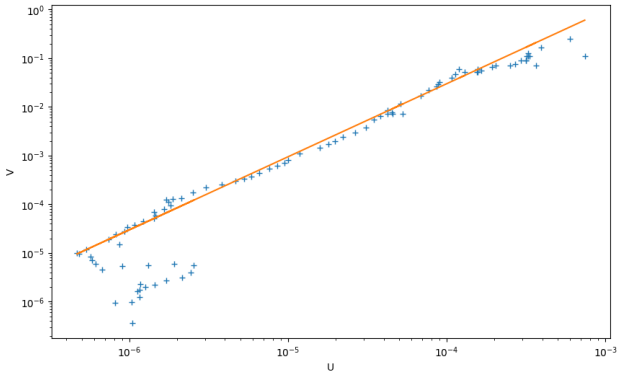

FIG. 4: Fitting Eq. 6 to the correlated growth of H and R, with $\beta = 1.5$ (close to $\beta = \rho 1/\eta_1 \simeq 1.6$). The graph illustrates the relationship between IBM's revenue and workforce on a log-log scale. The cusp in the data points at the bottom left is due to human resource loss around 75-80 years. We have taken proportionality constant for $V \sim U^{\beta}$ as $3 * 10^4$.

III. STATISTICAL ANALYSIS

1. Annual Revenue
   (a) `Relative Mean:  0.024998`
   (b) `Relative Standard Deviation:  0.487041`

2. Human Resources
   (a) `Relative Mean:  0.095333`
   (b) `Relative Standard Deviation:  0.299811`

IV. CONCLUSIONS

1. The revenue growth of IBM follows a logistic model, where the trend becomes non-linear around the 70-80 year mark. This aligns with the calculated t_{nl} value, confirming that IBM's revenue growth hits saturation around this period. The sharp change in trend suggests that external factors or internal limitations start to influence revenue generation beyond this point.

2. The data indicates that IBM's profit starts declining around the 80th year, which could suggest financial difficulties or a shift in market conditions. This period also coincides with the onset of non-linear saturation in revenue growth, suggesting a potential link between revenue stagnation and profit decline. The loss of $8 billion in 1993 further supports the idea that IBM faced significant financial difficulties during this period.

3. There is a strong correlation between IBM's revenue and its human resources, as indicated by coupled logistic growth models. When a company faces financial difficulties, it reduces its workforce, likely as a cost-cutting measure. This is evident from the drop in human resources around the time of declining profits, reinforcing the power-law relationship given by Eq. 6 between revenue and human resources.

[1] Arnab K. Ray, *Logistic modeling of economic dynamics* (September 6, 2023).