Analyzing GDP Growth of Major Countries Through Logistic Modeling

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CS-302, Modeling and Simulation

In this report , we have applied the logistic growth model to analyze the relationship between GDP and trade in six major economies: the USA, China, Japan, Germany, the UK, and India. By fitting World Bank data, we explore economic growth patterns and the correlation between GDP and trade. Our findings highlight distinct growth behaviors across countries, providing insights into their long-term economic trajectories.

I. INTRODUCTION AND MODELING

The Logistic Equation, given by

$$\dot{x} \equiv \frac{dx}{dt} = \mathcal{F}(x) = ax - bx^2 \tag{1}$$

describes a fundamental non-linear system where a and b are fixed parameters. Solving this equation with the initial condition $x(0) = x_0$, $k = a/b$, we obtain the following solution:

$$x(t) = \frac{kx_0e^{at}}{k + x_0(e^{at} - 1)} \tag{2}$$

In the context of modeling he relationship between GDP(G) and Trade(T), we introduce a logistic equation for GDP growth:

$$\dot{G} \equiv \frac{dG}{dt} = \mathcal{G}(G) = \gamma_1 G - \gamma_2 G^2 \tag{3}$$

where, $\mathcal{G}(G)$ represents country's GDP in US dollars and t is time in years. Here, γ_1 and γ_2 replace parameters a and b, governing GDP expansion and its eventual saturation and the maximum value of GDP k_G is given by $k_G = \gamma_1/\gamma_2$.

Similarly, the Trade Value follows the following equation:

$$\dot{T} \equiv \frac{dT}{dt} = \mathcal{T}(T) = \tau_1 T - \tau_2 T^2 \tag{4}$$

where $\mathcal{T}(T)$ represents Trade Values and τ_1 and τ_2 are analogous to the parameters a and b, with $k_T = \tau_1/\tau_2$.

The high correlation between GDP and trade across all the six countries suggests a coupled dynamic system governed by the equations: $\dot{G} = \mathcal{G}(G, T)$ and $\dot{T} = \mathcal{T}(G, T)$.
Assuming a linear relationship, the GDP-trade dependence follows a power-law scaling:

$$G \sim T^{\alpha} \tag{5}$$

where, $\alpha = \gamma_1/\tau_1$.

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II. RESULTS

A. GDP and Trade Plots:

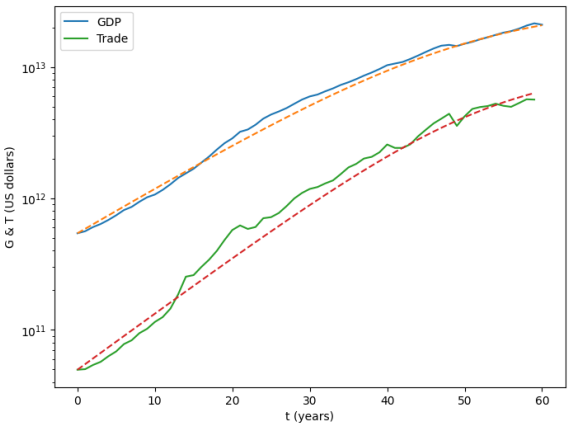

FIG. 1: Analyzing the growth patterns of GDP (top plot) and trade (bottom plot).The dotted curves follow the logistic equation, with parameter values given in Table I. Both plots have 1960 as the reference year.

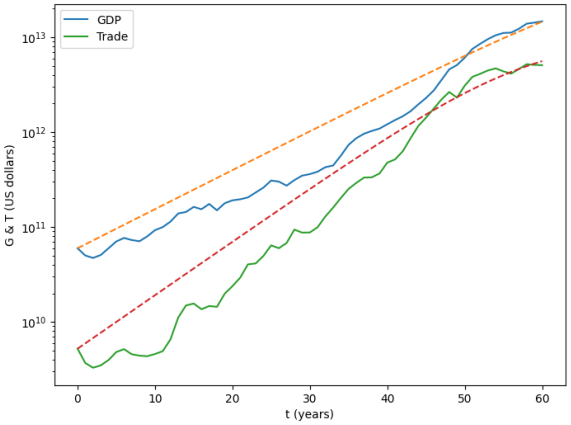

FIG. 2: Analyzing the growth patterns of GDP (top plot) and trade (bottom plot) for China. The dotted curves follow the logistic equation, with parameter values given in Table I. The reference year is 1960.

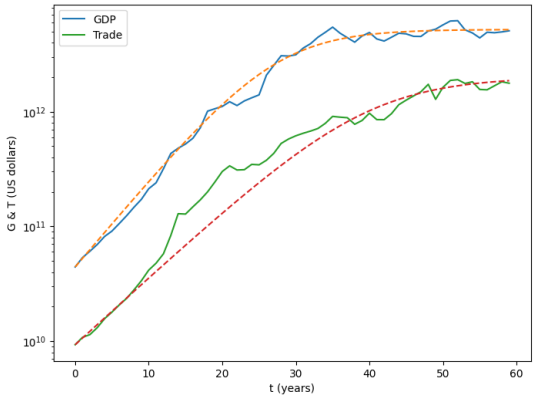

FIG. 3: Analyzing the growth patterns of GDP (top plot) and trade (bottom plot) for Japan. The dotted curves follow the logistic equation, with parameter values given in Table I. The reference year is 1960.

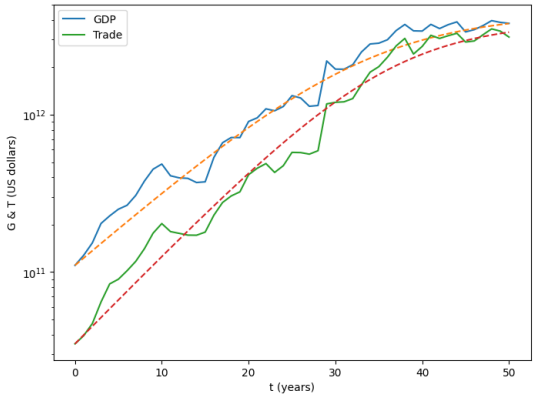

FIG. 4: Analyzing the growth patterns of GDP (top plot) and trade (bottom plot) for Germany. The dotted curves follow the logistic equation, with parameter values given in Table I. The reference year is 1970.

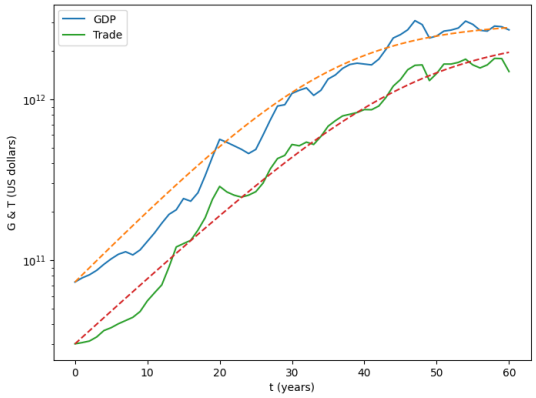

FIG. 5: Analyzing the growth patterns of GDP (top plot) and trade (bottom plot) for the UK. The dotted curves follow the logistic equation, with parameter values given in Table I. The reference year is 1960.

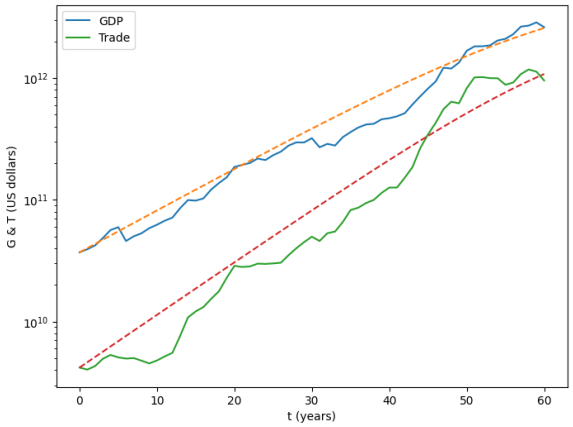

FIG. 6: Analyzing the growth patterns of GDP (top plot) and trade (bottom plot) for India. The dotted curves follow the logistic equation, with parameter values given in Table I**??**. The reference year is 1960.

Country Name	γ_1 (per annum)	k_G (Billion USD)	G_0 (Trillion USD)
USA	0.080	30.0	543
China	0.095	80.0	59.7
Japan	0.175	5.2	44.3
Germany	0.110	4.4	110
UK	0.105	3.0	73.2
India	0.080	6.0	37

TABLE I: GDP growth parameters for selected countries.

Country Name	τ_1 (per annum)	k_T (Trillion USD)	T_0 (Billion USD)
USA	0.099	10.0	49.8
China	0.130	10.0	5.22
Japan	0.135	2.0	9.31
Germany	0.130	3.9	35
UK	0.095	2.5	30.2
India	0.100	3.0	4.18

TABLE II: Trade growth parameters for selected countries.

B. GDP vs Trade Plots:

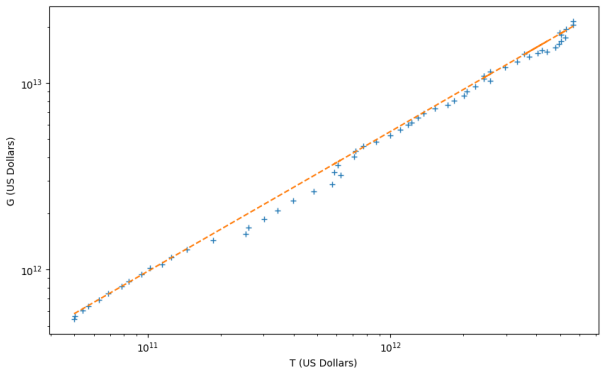

FIG. 7: Plot of GDP versus trade for the USA from 1960 to 2019. The dotted line follows Eq. 5 with $\alpha = 0.75$ and Scaling Factor $= 5.5 \times 10^3$, showing the relationship between economic growth and trade.

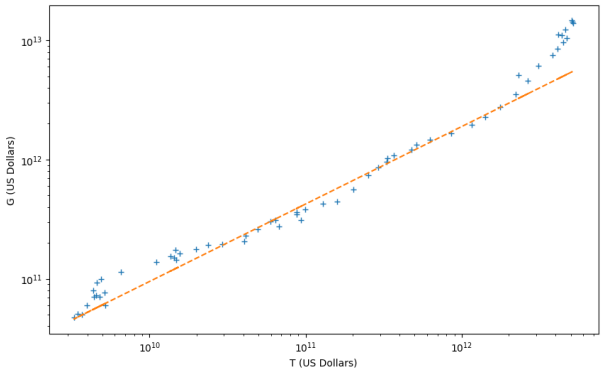

FIG. 8: Plot of GDP versus trade for China from 1960 to 2020. The dotted line follows Eq. 5 with $\alpha = 0.65$ and Scaling Factor $= 3 \times 10^4$, illustrating China's economic development alongside trade.

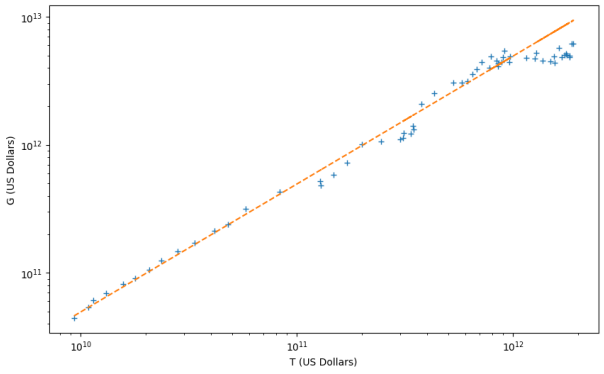

FIG. 9: Plot of GDP versus trade for Japan from 1960 to 2020. The dotted line follows Eq. 5 with $\alpha = 1.00$ and Scaling Factor $= 4.95$, highlighting Japan's robust trade and economic growth.

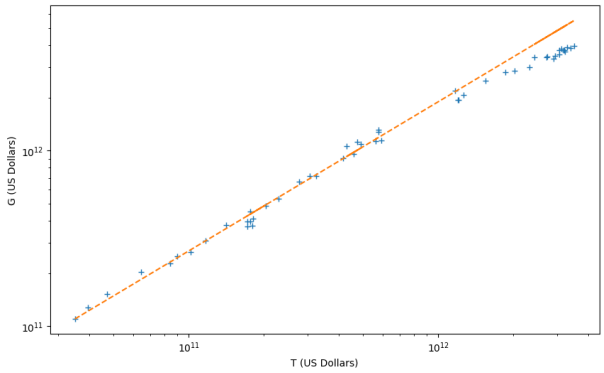

FIG. 10: Plot of GDP versus trade for Germany from 1970 to 2020. The dotted line follows Eq. 5 with $\alpha = 0.85$ and Scaling Factor $= 1.2 \times 10^2$, reflecting Germany's economic expansion and its trade patterns.

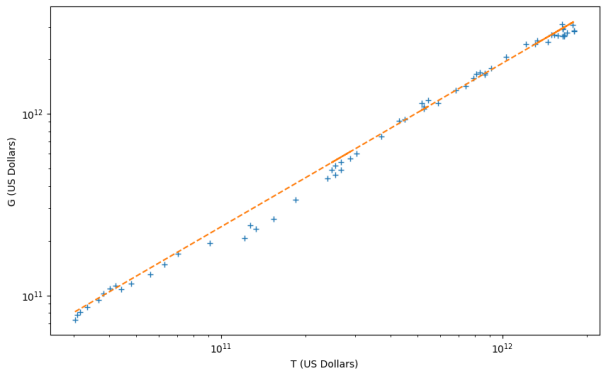

FIG. 11: Plot of GDP versus trade for the UK from 1960 to 2020. The dotted line follows Eq. 5 with $\alpha = 0.90$ and Scaling Factor $= 30$, showcasing the UK's economic and trade dynamics.

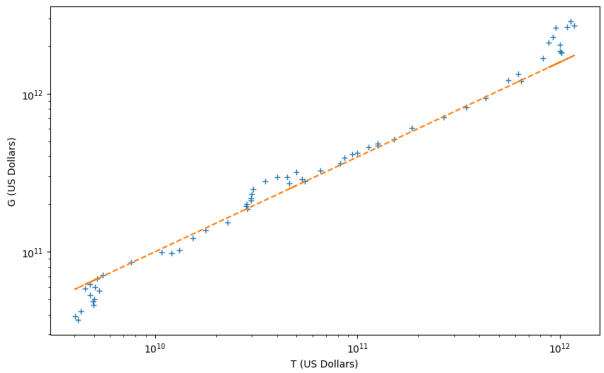

FIG. 12: Plot of GDP versus trade for India from 1960 to 2020. The dotted line follows Eq. 5 with $\alpha = 0.60$ and Scaling Factor $= 10^5$, demonstrating India's trade and economic growth trajectory.

C. Comparison Plots:

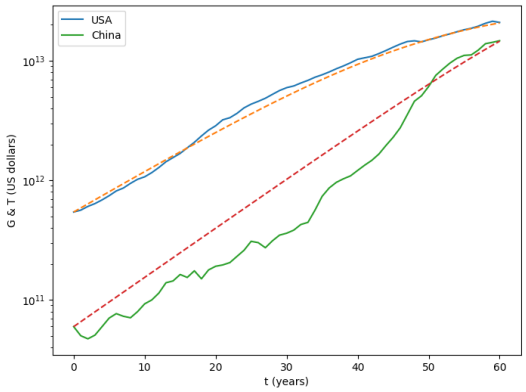

FIG. 13: Comparison of GDP growth between the USA and China from 1960 to 2020. The logistic model (dotted curve) fits the USA's GDP growth more closely than China's, with both countries' data showing significant differences in growth patterns. The model parameters are given in Table 1.

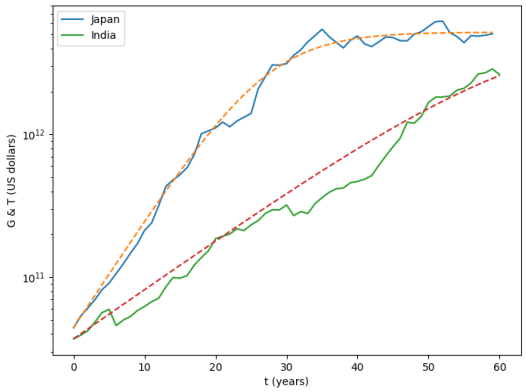

FIG. 14: GDP growth comparison of Japan and India from 1960 to 2020. Japan's growth shows early steep acceleration, later stagnating, while India's steady growth surpasses Japan's by 2020. Extrapolated logistic curves suggest India will continue to outpace Japan in the future.

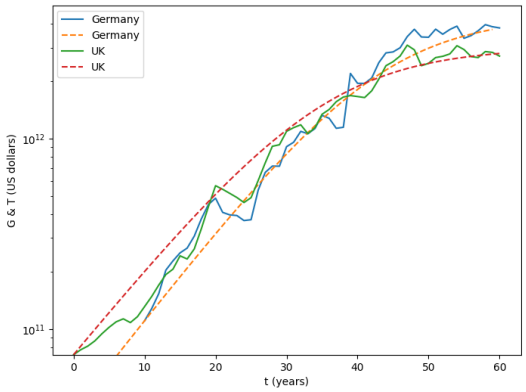

FIG. 15: GDP growth of Germany and the UK, starting in 1960 for the UK and 1970 for Germany. The logistic models show a similar trend until 2000, after which Germany's GDP surpasses the UK's, a shift captured by the intersection of the curves.

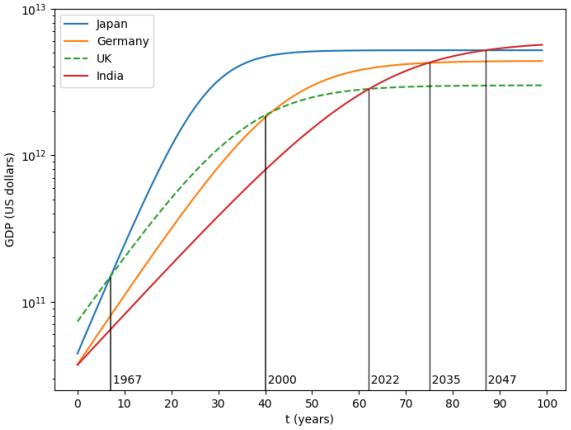

FIG. 16: Long-term GDP competitiveness among Japan, Germany, the UK, and India. Historical GDP intersections occurred in 1966 and 2000 when Japan and Germany overtook the UK. Future predictions suggest India will surpass Germany in 2035 and Japan in 2047, becoming the leader after 2022.

D. Predictions for India's GDP:

1. India's GDP is projected to reach $4 Trillion US Dollars by the year 2033.

2. India's GDP is expected to grow to $5 Trillion US Dollars by the year 2044.

3. Assuming India's GDP grows exponentially as $x = x_0 \times e^{at}$ with the same parameters, it will reach approximately 39.02 Trillion US Dollars by 2047.

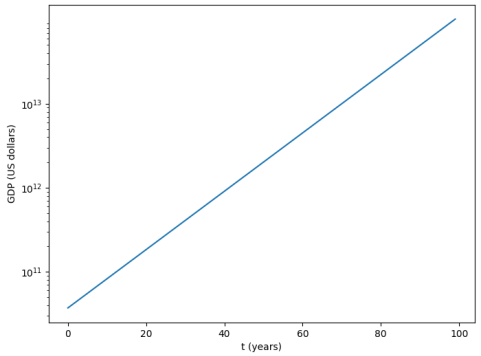

FIG. 17: Log-Normal Plot if GDP Grows Exponentially.

III. STATISTICAL ANALYSIS

Country Name	Mean	Standard Deviation
USA	0.04922	0.086612
China	-0.356831	0.248342
Japan	-0.033641	0.125207
Germany	0.107963	0.18553
UK	-0.097231	0.16097
India	-0.136483	0.172744

TABLE III: Relative Mean and Standard Deviation of GDP

Country Name	Mean	Standard Deviation
USA	0.12001	0.203252
China	-0.379053	0.328575
Japan	0.252728	0.401554
Germany	0.056289	0.233909
UK	-0.000398	0.165624
India	-0.178285	0.345566

TABLE IV: Relative Mean and Standard Deviation of Trade

IV. CONCLUSIONS

1. The results show that GDP and trade grow quickly at first but slow down over time, following a pattern called logistic growth. This means that while economies expand rapidly in the beginning, their growth eventually stabilizes due to limits like resource availability and market saturation. In simple terms, no economy can grow forever - at some point, it reaches a peak where further growth slows down.

2. The results show that GDP and trade are closely linked, following a pattern called a power-law relationship. This means that when a country's GDP increases, its trade grows even faster. In simple terms, richer countries tend to trade much more than poorer ones, and their trade expands quickly as their economy grows. This shows that trade plays an important role in helping economies grow.

3. The plotted data shows that Japan, Germany, and the UK have either already reached or are close to reaching their peak GDP levels, meaning their growth is slowing down. On the other hand, India is still in a phase of rapid economic expansion and has significant room for further growth before stabilizing. This highlights a key difference between well-developed economies, which experience slower growth, and emerging economies like India, which are still growing at a fast pace.

[1] Arnab K. Ray, *Logistic modeling of economic dynamics* (September 6, 2023).