Equations Used

$$-EI\frac{d^2\theta^*}{ds^{*2}} = \frac{1}{2}\rho u^2 C_D \int_{s^*}^{l} \cos^2(\theta)\cos(\theta-\theta^*)ds + \frac{1}{2}\rho u^2 C_L \int_{s^*}^{l} \cos^2(\theta)\sin(\theta-\theta^*)ds \tag{1}$$

$$-\frac{d^2\theta^*}{ds^{*2}} = \frac{\rho u^2 l^3}{2EI}\left[C_D \int_{s^*}^{1} \cos^2(\theta)\cos(\theta-\theta^*)ds + C_L \int_{s^*}^{1} \cos^2(\theta)\sin(\theta-\theta^*)ds\right] \tag{2}$$

$$-\frac{d^2\theta^*}{ds^{*2}} = \frac{\rho u^2 l^3}{2EI}\left[\int_{s^*}^{1} \cos^2(\theta)\left(C_D\cos(\theta-\theta^*) + C_L\sin(\theta-\theta^*)\right)ds\right] \tag{3}$$

$$\begin{aligned}-\frac{d^2\theta^*}{ds^{*2}} = \frac{\rho u^2 l^3}{2EI}\sqrt{C_D^2+C_L^2}\Bigg[\int_{s^*}^{1} \cos^2(\theta)\Bigg(&\frac{C_D}{\sqrt{C_D^2+C_L^2}}\cos(\theta-\theta^*) \\ &+ \frac{C_L}{\sqrt{C_D^2+C_L^2}}\sin(\theta-\theta^*)\Bigg)ds\Bigg]\end{aligned} \tag{4}$$

$$-\frac{d^2\theta^*}{ds^{*2}} = \frac{\rho u^2 l^3}{2EI}\sqrt{C_D^2+C_L^2}\left[\int_{s^*}^{1} \cos^2(\theta)\sin\left(\theta-\theta^* + \sin^{-1}\left(\frac{C_D}{\sqrt{C_D^2+C_L^2}}\right)\right)ds\right] \tag{5}$$

$$-\frac{d^2\theta^*}{ds^{*2}} = \mathrm{Ca}\sqrt{C_D^2+C_L^2}\left[\int_{s^*}^{1} \cos^2(\theta)\sin\left(\theta-\theta^*+\phi\right)ds\right] \tag{6}$$

where,

$$\begin{aligned}\mathrm{Ca} &= \text{Cauchy Number} \\ &= \frac{\rho u^2 l^3}{2EI} \\ C_D &= 1.95 \\ C_L &= 0.178 \\ \phi &= \sin^{-1}\left(\frac{C_D}{\sqrt{C_D^2+C_L^2}}\right)\end{aligned}$$

And the boundary conditions being,

$$\begin{aligned}\theta^*(0) &= 0 \\ \left.\frac{d\theta^*}{ds^*}\right|_{s^*=1} &= 0 \\ \left.\frac{d^2\theta^*}{ds^{*2}}\right|_{s^*=1} &= 0\end{aligned}$$