

EQUATIONS USED

$$EI \frac{d^2 \theta^*}{ds^{*2}} = \frac{1}{2} \rho u^2 C_D \int_{s^*}^l \cos^2 \theta \cos(\theta - \theta^*) ds + \frac{1}{2} \rho u^2 C_L \int_{s^*}^l \cos^2 \theta \sin(\theta - \theta^*) ds \quad (1)$$

$$\frac{d^2 \theta^*}{ds^{*2}} = \frac{\rho u^2 l}{2EI} \left[C_D \int_{s^*}^1 \cos^2 \theta \cos(\theta - \theta^*) ds + C_L \int_{s^*}^1 \cos^2 \theta \sin(\theta - \theta^*) ds \right] \quad (2)$$

$$\frac{d^2 \theta^*}{ds^{*2}} = \frac{\rho u^2 l}{2EI} \left[\int_{s^*}^1 \cos^2 \theta (C_D \cos(\theta - \theta^*) + C_L \sin(\theta - \theta^*)) ds \right] \quad (3)$$

$$\frac{d^2 \theta^*}{ds^{*2}} = \frac{\rho u^2 l}{2EI} \sqrt{C_D^2 + C_L^2} \left[\int_{s^*}^1 \cos^2 \theta \left(\frac{C_D}{\sqrt{C_D^2 + C_L^2}} \cos(\theta - \theta^*) + \frac{C_L}{\sqrt{C_D^2 + C_L^2}} \sin(\theta - \theta^*) \right) ds \right] \quad (4)$$

$$\frac{d^2 \theta^*}{ds^{*2}} = \frac{\rho u^2 l}{2EI} \sqrt{C_D^2 + C_L^2} \left[\int_{s^*}^1 \cos^2 \theta \sin \left(\theta - \theta^* + \sin^{-1} \left(\frac{C_D}{\sqrt{C_D^2 + C_L^2}} \right) \right) ds \right] \quad (5)$$

$$\frac{d^2 \theta^*}{ds^{*2}} = a \left[\int_{s^*}^1 \cos^2 \theta \sin \left(\theta - \theta^* + \sin^{-1} \left(\frac{C_D}{\sqrt{C_D^2 + C_L^2}} \right) \right) ds \right] \quad (6)$$

$$\frac{d^3 \theta^*}{ds^{*3}} = a \left[-\cos^2 \theta^* \frac{C_D}{\sqrt{C_D^2 + C_L^2}} - \int_{s^*}^1 \cos^2 \theta \cos \left(\theta - \theta^* + \sin^{-1} \left(\frac{C_D}{\sqrt{C_D^2 + C_L^2}} \right) \right) ds \right] \quad (7)$$

$$\frac{d^4 \theta^*}{ds^{*4}} = a \left[2 \cos \theta^* \sin \theta^* \frac{C_D}{\sqrt{C_D^2 + C_L^2}} + \cos^2 \theta^* \frac{C_L}{\sqrt{C_D^2 + C_L^2}} - \int_{s^*}^1 \cos^2 \theta \sin \left(\theta - \theta^* + \sin^{-1} \left(\frac{C_D}{\sqrt{C_D^2 + C_L^2}} \right) \right) ds \right] \quad (8)$$

$$\frac{d^4 \theta^*}{ds^{*4}} = a \left[\sin 2\theta^* \frac{C_D}{\sqrt{C_D^2 + C_L^2}} + \cos^2 \theta^* \frac{C_L}{\sqrt{C_D^2 + C_L^2}} \right] - \frac{d^2 \theta^*}{ds^{*2}} \quad (9)$$

$$\frac{d^4 \theta^*}{ds^{*4}} + \frac{d^2 \theta^*}{ds^{*2}} = \frac{a}{\sqrt{C_D^2 + C_L^2}} [C_D \sin 2\theta^* + C_L \cos^2 \theta^*] \quad (10)$$

$$\frac{d^4 \theta^*}{ds^{*4}} + \frac{d^2 \theta^*}{ds^{*2}} = \frac{a}{\sqrt{C_D^2 + C_L^2}} \left[\frac{C_L}{2} + C_D \sin 2\theta^* + \frac{C_L}{2} \cos 2\theta^* \right] \quad (11)$$

$$\frac{d^4 \theta^*}{ds^{*4}} + \frac{d^2 \theta^*}{ds^{*2}} = \frac{a}{\sqrt{C_D^2 + C_L^2}} \left[\frac{C_L}{2} + \sqrt{C_D^2 + \frac{C_L^2}{4}} \left(\frac{C_D}{\sqrt{C_D^2 + \frac{C_L^2}{4}}} \sin 2\theta^* + \frac{C_L}{2\sqrt{C_D^2 + \frac{C_L^2}{4}}} \cos 2\theta^* \right) \right] \quad (12)$$

$$\frac{d^4 \theta^*}{ds^{*4}} + \frac{d^2 \theta^*}{ds^{*2}} = a(\lambda + b \sin(2\theta^* + \mu)) \quad (13)$$

where,

$$\begin{aligned} C_D &= 1.95 \\ C_L &= 0.178 \\ \lambda &= \frac{C_L}{2\sqrt{C_D^2 + C_L^2}} \\ &= 0.00454521 \\ b &= \frac{\sqrt{C_D^2 + \frac{C_L^2}{4}}}{\sqrt{C_D^2 + C_L^2}} \\ &= 0.99689635 \\ \mu &= \sin^{-1} \left(\frac{C_L}{2\sqrt{C_D^2 + \frac{C_L^2}{4}}} \right) \\ &= 0.04561 \end{aligned}$$

And the boundary conditions being,

$$\theta^*(0) = 0 \quad (14)$$

$$\frac{d\theta^*}{ds^*}(1) = 0 \quad (15)$$

$$\frac{d^2 \theta^*}{ds^{*2}}(1) = 0 \quad (16)$$

$$\frac{d^3 \theta^*}{ds^{*3}}(1) = -a \frac{C_D}{\sqrt{C_D^2 + C_L^2}} \cos^2 \theta^*(1) \quad (17)$$

$$= -0.99585965a \cos^2 \theta^*(1) \quad (18)$$