Introduction to Mathematica:

Mathematica is a great program that we can use to accomplish laborious math and allows us to focus on the concepts at large. Not only this we can also use this to advance out perspective on things, by this I mean we can use *Mathematica* for visualization of three-dimenisonal graphs that would otherwise be taxing on our brain. Visualization is an important aspect of Mathematics and Physics. The more we are able to do this the better we get at the concepts which is an important task when it comes to learning. Mathematica allows us to get closer to these concepts which I find to be very useful. The purpose of this book is to provide guidance on how to use *Mathematica* for the Mathematics behind our physics in a step-by-step tutorial fashion. Just like life we first want to familiarize ourselves with the atmosphere before we step into it like dipping our toes in the water before jumping in. This is the kind of riute that we are going to follow familiarize ourselves with the syntax of *Mathematica* and the basic input elements/components of *Mathematica* before going into the specific tasks like graphing and matrices. The topics that we see in terms of Mathematics include Graphing of functions, Calculus, Progressions, Series,Matrices et cetera. We are basically looking at the method for our Mathematics that we must know in order to do great things with *Mathematica*.

Let's Get Started with *Mathematica*: Why is Synatx important?

We find ourselves using a variety of interactive software apps/programs for various tasks but this is a form of communication that you have with said software apps/programs and in order to get what we need we should be able to effectively communicate with it. In order to get *Mathematica* to do what you want we should be able to tell it what exactly we want it do so that it can perform this without errors. This in computer language is effectively syntax we are telling *Mathematica* what we want it to do in the right order.This is the place where we are going to discuss some tips in order to get started with *Mathematica*. To begin with a computer program is just as smart as oneself because people tend to carry this misconception about technology and a magical self correcting abilty it has. This idea does not give a person the right to improve ones ability which is an essential tool. Then what is computer program good at? The computer program is good carrying out complex,challenging problems, and visual representaion in a much more effective and less time consuming manner. This is a good thing to understand about computer programs. It also takes one out of their comfort zone which is not a bad thing in fact it is good we would be able to limit complacency as much as we can. Let us press on with this type of understanding of a computer program. In *Mathematica* we type everything in a cell. This indication that we are typing in a particular cell visually appears on the far right of the *Mathematica* page (on the left hand side of the scroll page up/down toolbar) it looks like the right square bracket] (no pun intended) this square bracket elongates verticallly depending on the amount we enter into a cell. This idea of a cell is similar to when we use Microsoft Excel and also useful.

This way we can enter plain text and mathematical expressions together and maintain a continuous flow. It does not have to be one or the other and leads to a lot of possibilities. In order for us to higlight an entire cell we would not need to us e the cursor and highlight the cell instead we can take the pointer and click on the desired square bracket which means we have now higlighted the entire cell and this leads to changes that we want to make for the enitre cell for example type of font and/or size. This can save some time here and there.

Mathematica's primary function is to be able to perform mathematical computations and in turn visual representations if need be so everytime we star to enter text into a cell *Mathematica* assumes it is a mathematical expression of some sort and diplays it in a blue color. In order to convert this into just a text cell we can click on the square bracket (cell) and use the toolbar at the tap and follow these steps: Cell>Convert To>Text Display or we can click on the square bracket (cell) and press <ALT>+7 this converts a cell into specifically into a text cell. The other alternative is to keep on typing the string until *Mathematica* displays a dropdown menu at the point where we are typing and suggest to convert to text cell which can also be used. We find that every mathematical thing we type into Mathematica is an expression.It has an interface with the user, and it has a kernel (core of the program). The kernel is responsible for all the hard work when we evaluate an expression. We want to use *Mathematica* usefully and effectively so we need to be able to tell it what to do exactly in the right format. This is where commands come into play. *Mathematica* is filled with loads of mathematical commands and we need to enter it in the right format. Every *Mathematica* command need to be capitalized and there are some commands that would require more capitalization. These commands must be followed by a pair of square brackets this is the step we must take in order to be able to evaluate an expression. When we want to evaluate the expression we must press <Shift>+<Enter>. This in turn evaluates the expression and displays the output provided we type in the command without an error (syntax!). Let us look at a simple example.

```
Subtract[545, 5]
```

```
540
```

```
Subtract[5, 545]
```

```
-540
```

Notice that the order matters here because Mathematica subtracts the second entry from the first one so we need to keep that in mind.

When typing in commands like these to Matheamatica we might make a mistake and get an error message. This is natural but a great thing about *Mathematica* is that we can back to the spot where there is an error and fix the mistake which is a great thing. This is unlike MatLab if you don't write it in the script window and instead write in the command window and run the code you will get an error message if there is an error in the command but we cannot go back to that place to fix it we have to start over.This is a huge advantage when using *Mathematica* and it shows that *Mathematica* is user friendly. There are other things that we need to keep in mind when entering in a command or checking the input once an error mesage is diplayed.
i)Check for capitalization when we enter in a command.
ii) The importance of commas will be seen when we look at more commands especially with matrices because the position of the comma can change the entire mathematical meaning of the expression.
iii)Square Brackets! When typing a command it should be followed with a pair of [ ] because this let's *Mathematica* know what to evaluate. DO NOT interchange parenthesis with square brackets [ ] or squiggly bracket { } as each of these have a specific purpose in *Mathematica*.
iv) When in doubt HELP yourself out! The help menu is there on the toolbar at the top of the *Mathematica* page right next to Palletes and Window. Do the following : Help>Wolfram Documentation. This takes you to the Wolfram documentation directly where you can type in question on how to type a particular command in *Mathematica*.Other places to look for help with *Mathematica* are places like Google and other forums.
v) There other ways to execute certain commands as well this depends completely upon oneself and what they want to do. For example the Subtract command could have simply been performed like this 545 - 5 = 540 or 5 - 545= -540 where 545 - 5 and 5 - 545 is the input and 540 and -540 would have been the output that Mathemactica displayed.
vi) We can also type in simple commands rather than performing the formula on *Mathematica* like the command Mean[] typing this is much faster compared to actually taking the numbers multiplying it wtih their frequency and dividing by the sum of the numbers in the list. Other simple commands like Median[],Plus[],Times[],and Divide[]. This depends on whether we want to do it this way or not.
vii) Formatting and Typesetting are key when it comes to entering in more complicatd commands into *Mathematica.*Palletes are useful in this case just like how it is useful to a painter because we can grab various things from it. One of the most useful thing that we can grab from it is the Classroom Assistant.We can access it throught the Toolbar at the top where we have palletes : Palletes> Classroom Assistant. This saves a lot of time when typing in

sophisticated commands as we can grab numbers like pi and formatting for division,vectors,and matrices et cetera.
viii) The classroom assistant comes into play here especially when pi or exponential function to the x comes into play because rather than typing in 3.141592653.... just type in N[π,1000] and watch *Mathematica* do the rest we have just told Matheamtica to evaluate pi to 1000-digit precision.This can be very useful when dealing with chaotic systems like Lorenz Strange Attractor where a very minute (small) change in the initial condition could have a drastic effect on the motion later. The N[ ] command evaluates a digit to an n-digit precision.e^(variable) can be found in the Classroom Assistant>Basic Commands under Elementary Functions where we have e^(var.), Sine,Cosine,Tangent,Sinh,Cosh,Tanh,Pi et cetera. It is important to use e^(var.) from the Classroom Asisstant because it lowers the chances of error in the syntax and will in turn yield an output.
ix) Since we were lookin at how to access functions like pi exp^(var) et cetera. Here are a few shortcuts that we can take in order to access it.
a)infinity- use the following command- ESCinfESC.
b)exponential-use the following commandESCeeESC.
c)Degree-use the following commandESCdegESC.
d) Iota (*i)*-use the following command ESC iiESC.
e)Theta (θ)- use The following command ESCthESC.
x) One of the biggest things that we should also focus on is the explicti operations rather than implicit operations *Mathematica* does recognize impicit multiplication et cetera but typing it in an explicit way helps the chances of reducing your own errors which is a very important thing.
xi) Here are a few more useful things that we will need later so it is better to introduce it now than on the spot. First -> to execute this press the dash button and<SHIFT>+ greater than button then press the space bar button. This symbol transforms the thing on the left hand side to the thing on the right hand side for example if we have a->b and if we have expression a+3b=8 this rule will a with b and the expression becomes 4b=8 we will see this use later. Replacing is also a useful thing to know in *Mathematica*. Another useful command to know in *Mathematica* is the % or Out[n] where n is the n^{th} output. The ESC aliasESCis a very useful thing to know because we can type the alias of a symbol and use ESC aliasESC to produce it instead of through the color pallette for example: to get pi we type ESC pESC this is much faster and saves a lot of time.This command refers to the most recent output and uses it in the next evaluation if referred to.The ReplaceAll in *Mathematica* is represented by /. this command takes the given expression and applies the rules that are given to it once or the command Replace which is also applied once by default.

```
Replace[x^3, x^3 → a + c]
```

```
a + c
```

```
ReplaceAll[x^3, x^3 → a + c]
```

```
a + c
```

```
{x, x^4, y} /. {x → b, y → c^2}
```

$\{b, b^4, c^2\}$

This is an example in which we are replacing an expression in three different ways and we can see that the replace rule is not caaried further down in the command it is applied once by default. Now we will dive into specific Mathematical topics and how to do this in *Mathematica.*

Let's Get Started with Different Mathematical Topics in *Mathematica*:

Trignometric Functions:

Trigonometric functions are very useful and we will be using it later a lot more so we can start with them as it will be good pratice to check the syntax in our commands. Again we can find the trigonometric functions in in the Palletes>Classroom Assistant>Basic Commands>Elementary Functions and we can use them just like any other commands like we have seen. Let us use the Sine function now and see what happens. We must remember one thing when using trigonometric functions that the arugment inside the Sine function is assumed to be given in the measure of radians unless it is specified to be in degrees. We can also evaluate our Sine[ ] answer to a n-digit precision. Evaluations at special arguments of a trigonometric function produces an exact value.

Sin[π / 3]

$$\frac{\sqrt{3}}{2}$$

Sin[(3 * π) / 5]

$$\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}$$

N[Sin[(3 * π) / 5], 10]

0.9510565163

Sin[30 Degree]

$$\frac{1}{2}$$

We just saw different ways to enter a trigonometric function depending on the way we want our input/answer. We also saw the case where we can evaluate a trigonometric function in degrees as *Mathematica* by default assumes that we entered the argument in radians unless it is specified. Notice that the when we entered 30 in the argument there is also a Degree which is capitalized this is because *Mathematica* is using Degree as a command so we must capitalize it. We can see that the same procedure would work for a Cosine or Tangent function.

ArcCos[0]

$$\frac{\pi}{2}$$

Notice that the answer is produced in a measure of radians which can then be converted to degrees. using the conversion factor of $\frac{180}{\pi}$.

$$(\%) * \left(\frac{180}{\pi}\right)$$

90

This if you notice is reported in degrees and we just used the % command this takes the most recent output that was evaluated and uses this in the next evaluation that we type in.

We can also have *Mathematica* play with some trigonometric identities for using these commands:

```
TrigExpand[Cos[2 * x]]
```

$\text{Cos}[x]^2 - \text{Sin}[x]^2$

```
TrigFactor[Cos[x]^2 - Sin[x]^2]
```

$2\,\text{Sin}\left[\frac{\pi}{4} - x\right]\,\text{Sin}\left[\frac{\pi}{4} + x\right]$

```
TrigReduce[Cos[x]^2 - Sin[x]^2]
```

Cos[2 x]

We just saw three different commands to play with our trignometric identities. The TrigExpand command expands the trigonometric function using a trigonometric identity.Whereas the TrigFactor command factors out common expressions using trig identities/rules.Finally, The TrigReduce command Reduces a trigonometric expansion to a known trigonometric identity as much as possible.

Basic Math:

Mathematica allows us to do basic math just like as we would on a calculator This comes in very handy when we need it because we can just type out basic operations instead of typing in the command for it. Let's look at an example:

```
5 * 3
```

15

```
Times[5, 3]
```

15

The former in this case is much faster which is why doing basic calculations is much easy in *Mathematica*. We can also have *Mathematica* give us an answer in scientific notation. This is a simple command but it's use will be immense because we can save some time as we don't have to worry about the 10^n factor. *Mathematica* will take care of that for us.

```
ScientificForm[3 459 338.334]
```

3.45934×10^6

We can also evaluate an aswer in scientific notation and n-digit precision. A list of elements in *Mathematica* is given by { } so a list containing a,c,d,e is typed into *Mathematica* like this {a,b,c,d,e} this is useful because a command can now be applied to a list of things. This command ScientificForm[expr,n] displays the answer in scientific notation to a n-digit precsion.

```
ScientificForm[{67 844 500.0, 0.000067844500, 67 844.45}, 3]
```

$\{6.78 \times 10^7, 6.78 \times 10^{-5}, 6.78 \times 10^4\}$

Algebra:

Algebra can be done in Mathematica in a very useful manner. Algebraic simplification can be time consuming and *Mathematica* can do this efficiently.

```
(7 - x) * (5 + x) * (3 + x^2)
```

$(7 - x)\,(5 + x)\,(3 + x^2)$

```
Expand[%]
```

$105 + 6x + 32x^2 + 2x^3 - x^4$

```
Factor[105 + 6 x + 32 x^2 + 2 x^3 - x^4]
```

$-(-7 + x)(5 + x)(3 + x^2)$

Notice that there is a minus sign in front using the general associative property of multiplication and multiply the minus sign in then we get the same answer as Out[14]. We just used to Factor and Reduce algebraic expression and now We can use the Solve Command to solve simultaneous equations.

```
Solve[3 * x + 4 * y == 7 && 2 * x - 7 * y == 4, {x, y}]
```

$\left\{\left\{x \to \frac{65}{29}, y \to \frac{2}{29}\right\}\right\}$

The == command can be used when representing an equation for the Solve command.The && symbol is used to represent the AND function we use this because we want to solve for the values that solves one equation and the other not just one of them.We can also use the Solve command to solve for Polynomial Functions.

```
Solve[x^2 - 7 * x + 10 == 0, x]
```

$\{\{x \to 2\}, \{x \to 5\}\}$

This is a quick, simple, and a useful command. We can now move onto *Mathematica* commands for Calculus and see how to tkae Derivative and Multiple integrals et cetera.

A Little Bit of Calculus with Mathematica:

We shall see how to input calculus related computations onto *Mathematica* and this will be very useful based of What we have done because *Mathematica* can do these computations much faster and give us the right answer provided we communicate with *Mathematica* correctly. We can take derivatives in multiple ways in *Mathematica*.The _ when used in *Mathematica* represents any expression or we can use Blank[h]

```
f[x_] = 2 * x^3 + Sin[x]
```

```
2 x^3 + Sin[x]
```

```
f'[x_]
```

```
Cos[x_] + 6 x_^2
```

```
f[x_] = 2 * x^3 + Sin[x]
```

```
2 x^3 + Sin[x]
```

```
f''[x_]
```

```
12 x_ - Sin[x_]
```

Here we took the first and second derivative of the function f[x_] that we defined and sincewe just have to use the addition rule when differentiating we just take the derivative of the individual pieces and it turn out to be right. We can take this is other ways as well we can now take thie derivative as a partial derivative $\frac{\partial f(x)}{\partial x}$.The command that we can use for this is the following: D[expr, {var, order}].

```
D[f[x], x]
```

$6\,x^2 + \mathrm{Cos}[x]$

```
D[f[x], {x, 2}]
```

$12\,x - \mathrm{Sin}[x]$

We just evaluated this expression in two different ways but we can do it in another way as well. For this we would need the help of the Pallete.

$\partial_x f[x]$

$6\,x^2 + \mathrm{Cos}[x]$

$\partial_x \partial_x f[x]$

$12\,x - \mathrm{Sin}[x]$

We can take even higher order derivatives this way. In total we have three different ways to take a derivative as we have seen so far and these different methods yielded the same answer and we can see that it is the right if we actually take the derivatives. Now let's look at Integration and we will first look at an indefinite integral and just like taking derivative in *Mathematica* we have multiple commands that we can use to integrate.If we use the type setting feature then we do it like this:

$$\int x * \mathrm{Cos}[2 * x]\, dx$$

$$\frac{1}{4} \mathrm{Cos}[2\,x] + \frac{1}{2} x\, \mathrm{Sin}[2\,x]$$

```
Integrate[x * Cos[2 * x], x]
```

$$\frac{1}{4} \mathrm{Cos}[2\,x] + \frac{1}{2} x\, \mathrm{Sin}[2\,x]$$

$$\int_{\pi}^{\pi/2} x * \mathrm{Cos}[2 * x]\, dx$$

$$-\frac{1}{2}$$

```
Integrate[x * Cos[2 * x], {x, π, π/2}]
```

$$-\frac{1}{2}$$

We can also do multiple integrals. The mulitple integrals in *Mathematica* are defined in an iterative manner so it does the outermost integral that we have typed last and the innermost first.The command is given in the following way: Integrate[exp,r{var1,min1,max1},{var2,min2,max2}].

$$\int_0^{\pi/2} \int_0^2 x * \mathrm{Cos}[y]\, dx\, dy$$

2

Notice that if we had switched the order of our variables our answer would be different. This would be a detail that we want to pay attention to becuase sometimes a multiple integral can go from straightforward to being very complicated and we have kept this evaluation consistent even if we have used multiple commands to evaluate it.

```
Integrate[x * Cos[y], {x, 0, 2}, {y, 0, π/2}]
```

2

The best thing to do when doing multiple integrals is to use the typesetting format because that is the format that is similar to what we would have one paper visually.This forces us to think about the order *Mathematica* is performing the integral in which in turn reduces the chance of errors in our evaluation. Next we can look at the Wronskian as it is a very useful tool when it comes to tell whether a set of solutions are linearly dependent/independent and it is frequently used in the study of differential equations.

```
Wronskian[{Sin[x], Cos[x]}, x]
```

-1

This shows that set of solutions are linearly independent.

```
Wronskian[{Sin[x], 2 Sin[x]}, x]
```

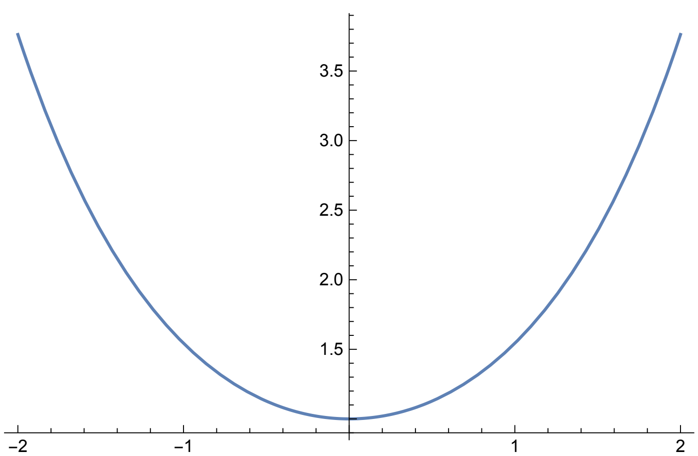
0

This shows that set of solutions are linearly dependent. Next we shall look at plotting various graphs on to *Mathematica*.

Plotting Graphs with Mathematica:

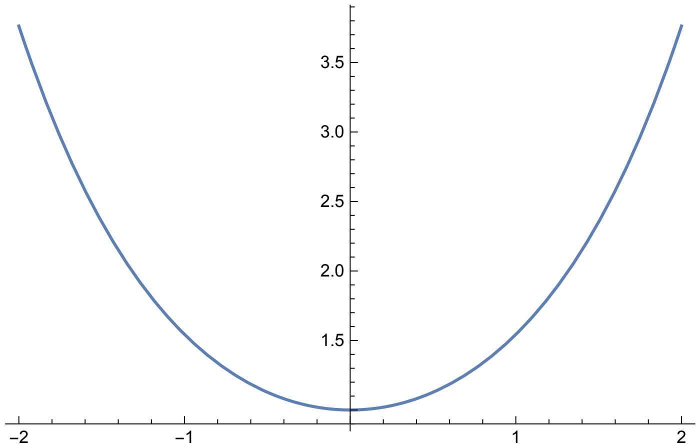
We want to be able to plot graphs once we know a certain function bacause we would be able to tell a lot more about what is going on with a fuction. Visualizing can be a huge challenge when given a hard problem and visualizing it could be the key to bringing one closer to the solution. It is very helpful and it is like adding another dimension to the problem (HaHa).This is what we will focus on this section plotting different kinds of graphs. Let’s try plotting a regular 2-dimensional plot.

```
Plot[Cosh[x], {x, -2, 2}]
```



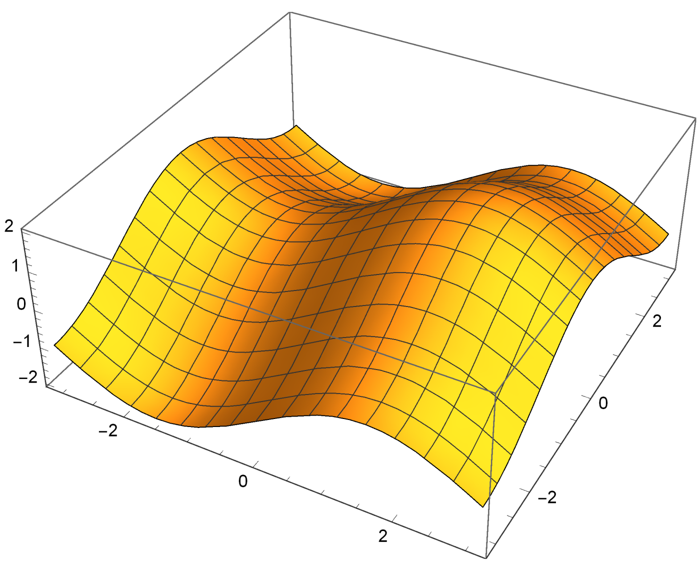
Let' s plot it again but using the formula for Cosh[x] to see if Mathematica can be trusted.We know that $\text{Cos}[x] = \frac{(e^x+e^{-x})}{2}$.

```
Plot[(e^x + e^-x)/2, {x, -2, 2}]
```

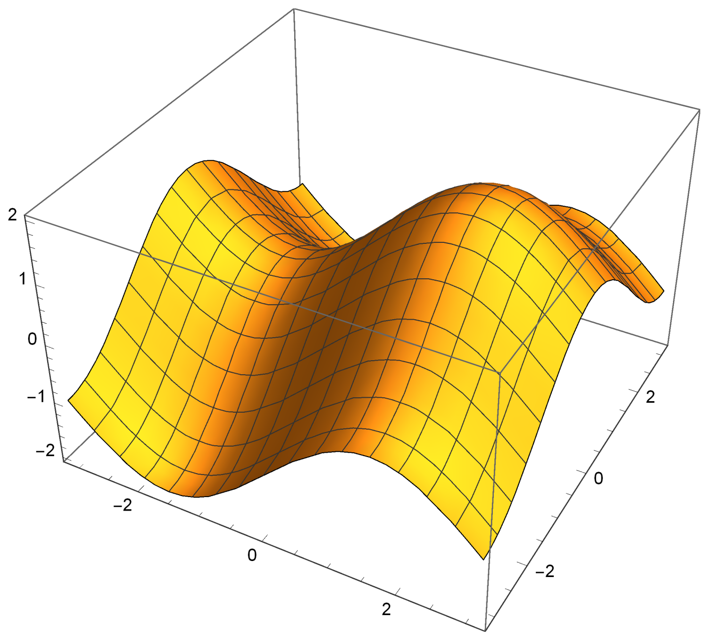


The graphs do match and we can trust *Mathematica* if we enter our commands correctly in order to see if something matches up correctly. As promised let’s add another dimension.The Plot3D[expr,{x,min1,max1},{y,min2,-max2},{z,min3,max3}]

```
Plot3D[Sin[x] + Cos[y] , {x, -π, π}, {y, -π, π}]
```

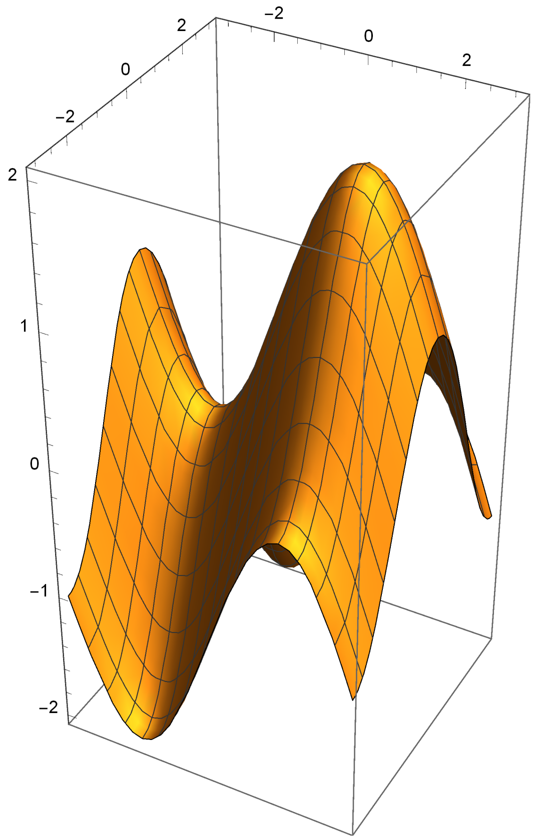


```
Plot3D[Sin[x] + Cos[y], {x, -π, π}, {y, -π, π}, BoxRatios → Automatic]
```



BoxRatios can be used to accentuate a particular feature about a graph such as a saddle point et cetera.When we type in Automatic it uses the scale provided from our PlotRange which is a command that tells *Mathematica* the Range of the plot or the spectrum of the output.We can also adjust the BoxRatios using a list.

```
Plot3D[Sin[x] + Cos[y], {x, -π, π}, {y, -π, π}, BoxRatios → {1, 1, 2}]
```



As we can see this really does enhance the the features of this grpah a lot more such as the saddle points.Let's play with graphs even more and try useful commands that we can use to tell what is going on if a graph looks complicated.

```
MyFunction[x_, y_] := 125. / ((x^2 - x0)^2 + (y^2 - y0)^2)
```

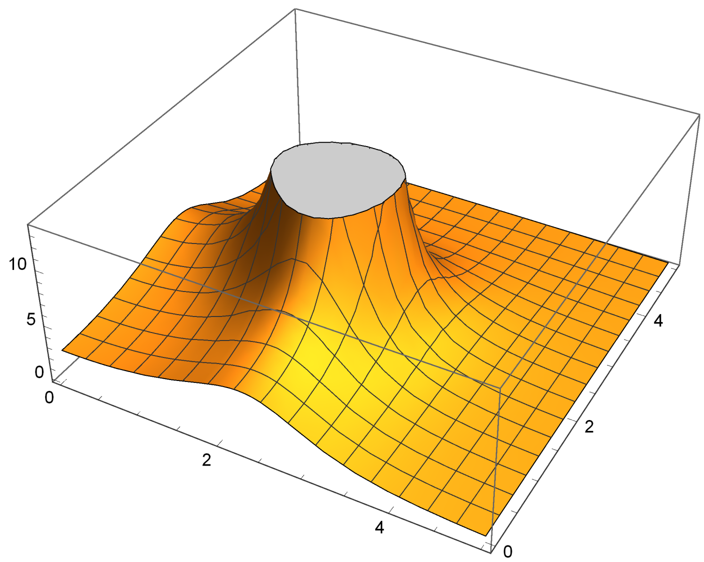
SInce *Mathematica* does not know about the variables x0 and y0 it is in blue,this is because Mathematica doesn't not know about these variables yet. We need a way to temporarily define the values for x0 and y0. Here is how we do it.

```
MyFunction[1, 1] /. {x0 → 5, y0 → 5}
```

```
3.90625
```

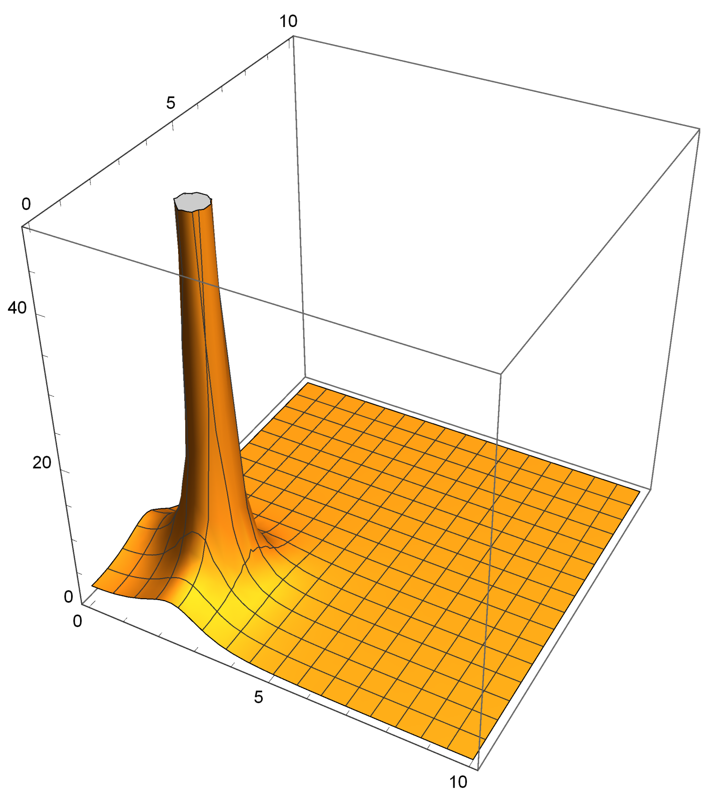
Mathematica would have returned a fraction. If we define our numbers as integers, without decimal points, then Mathematica does the math as a rational expression so it would have returned a fraction when we evaluate that.

```
Plot3D[MyFunction[x, y] /. {x0 → 5, y0 → 5}, {x, 0, 5}, {y, 0, 5}]
```

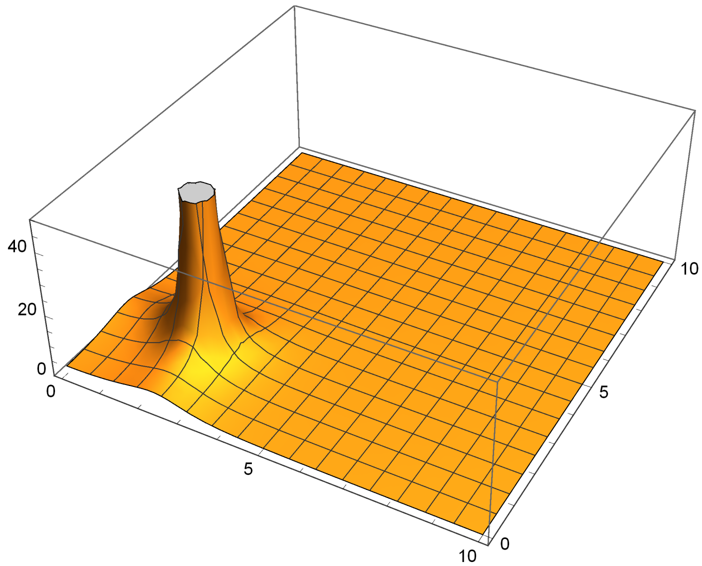


Now let us try some mor grapahical tools that can be added to this plot with the ones that we know already.

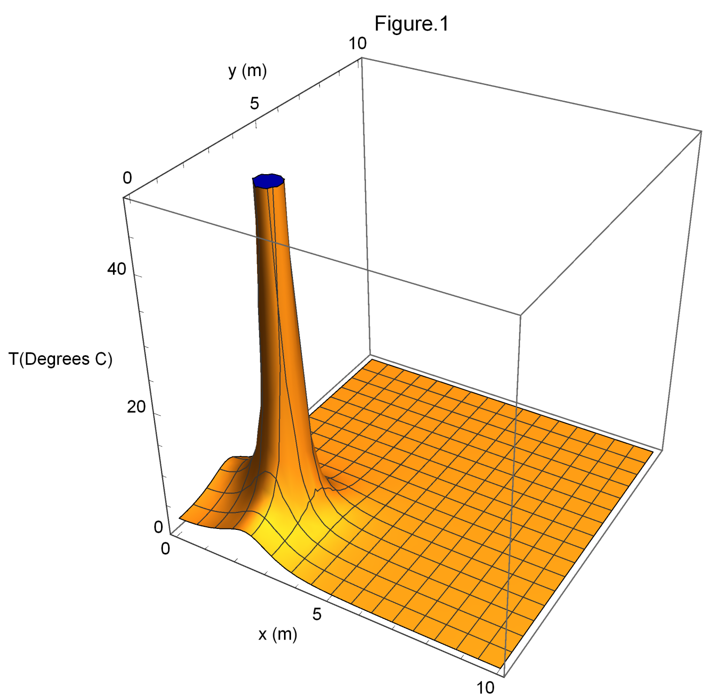
```
Plot3D[MyFunction[x, y] /. {x0 → 5, y0 → 5}, {x, 0, 10},
 {y, 0, 10}, PlotRange → {0, 50}, BoxRatios → {4, 4, 4}]
```



```
Plot3D[MyFunction[x, y] /. {x0 → 5, y0 → 5}, {x, 0, 10}, {y, 0, 10}, PlotRange → {0, 50}]
```

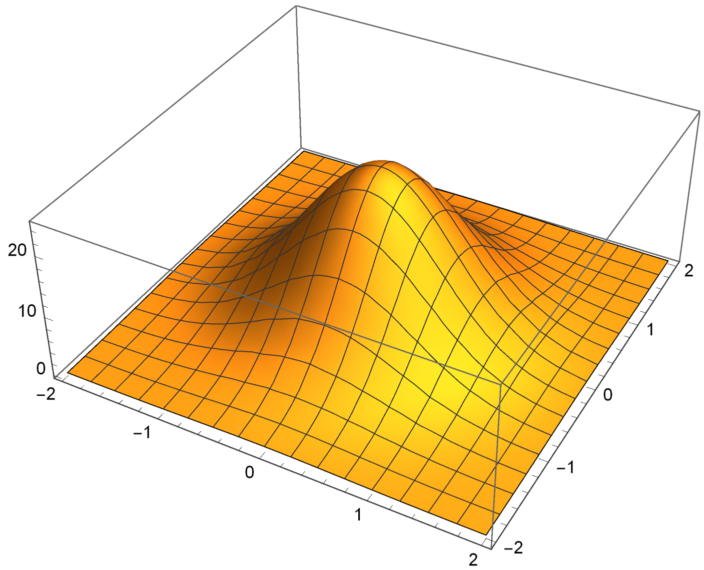



```
Plot3D[MyFunction[x, y] /. {x0 → 5, y0 → 5}, {x, 0, 10}, {y, 0, 10}, PlotRange → {0, 50},
 ClippingStyle → {Green, Blue}, AxesLabel → {"x (m)", "y (m)", "T(Degrees C)"},
 BoxRatios → {2, 2, 2}, PlotLabel → "Figure.1"]
```



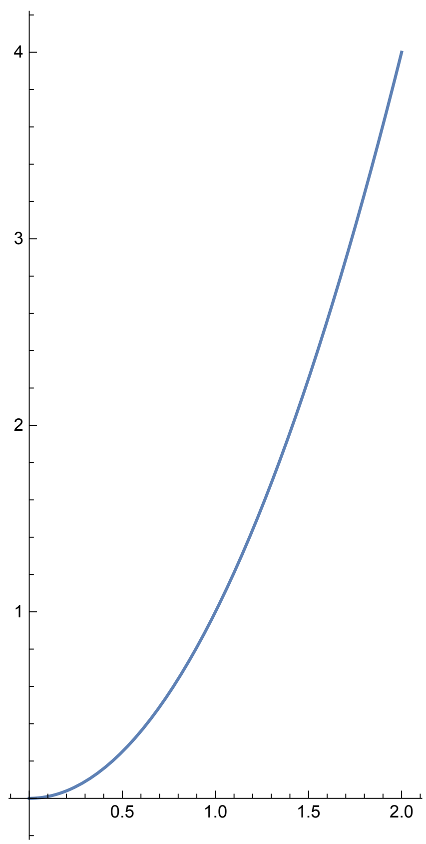
This evaluation had a lot of commands that makes the final graph look very good.If we look at the graph it is being clipped when this function exceeds the range and we can tell *Mathematica* what to do When this clipping occurs so we use the ClippingStyle command and color it blue as we can see. We can use AxesLabel to Label each of our axes and this is an example to show the labeling of the axes.PlotLabel is used to label the plot and PlotRange is used to specify the range for our plot.Let’s explore further and look at ParametricPlots.

```
Plot3D[25 * e^(-x^2 - y^2), {x, -2, 2}, {y, -2, 2}]
```



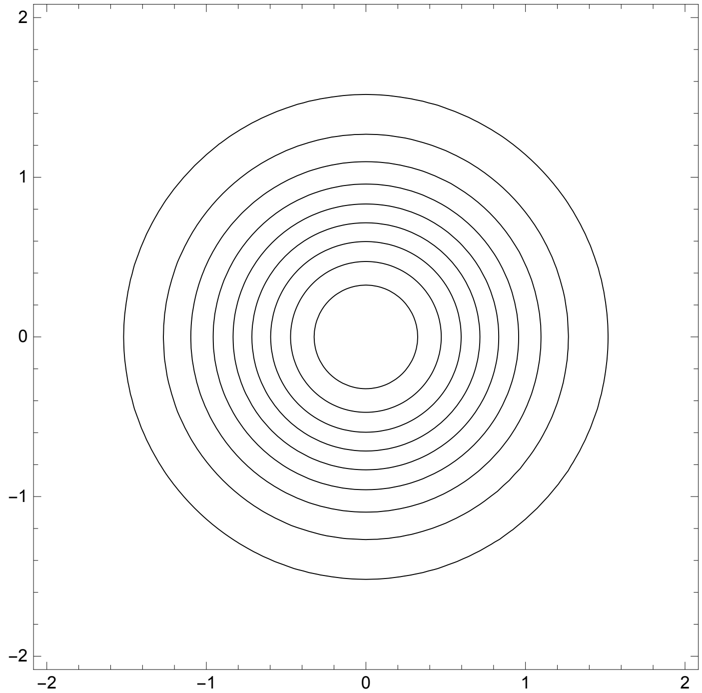
Now that we graphed our function right here we can go ahead with our parametric plot using our parametric equations and we just pick values for a and b as of now

```
P1 = ParametricPlot[{a * t, b * t^2} /. {a → 1, b → 1}, {t, 0, 2}]
```



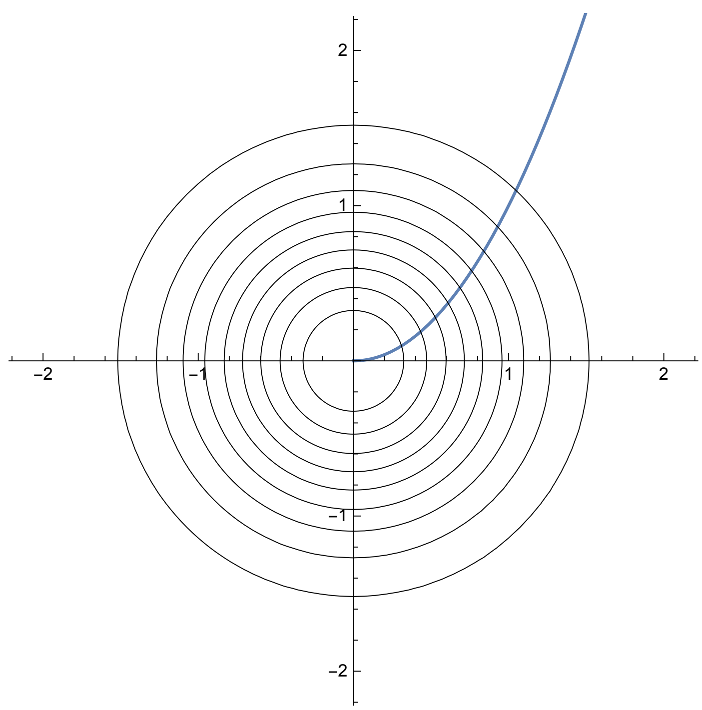
Here we use the contour plot for our originial function.

```
P2 = ContourPlot[25 * e^(-x^2-y^2), {x, -2, 2}, {y, -2, 2}, ContourShading → None]
```

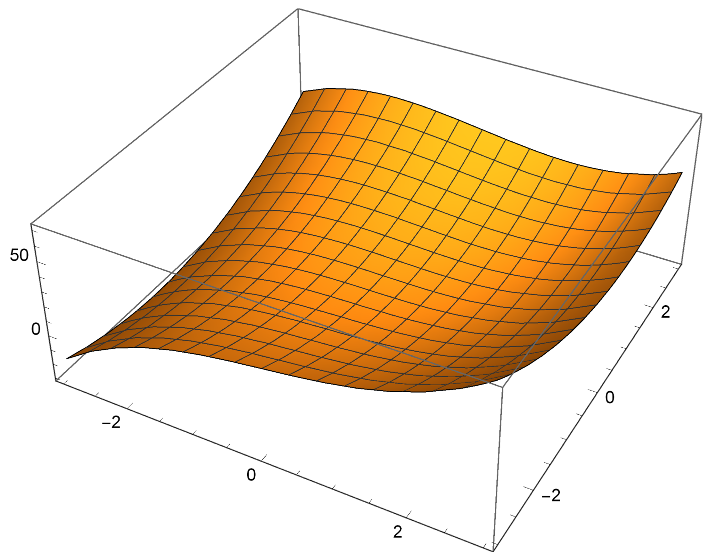


We use the Show command with our P1 &P2 to see where these graphs merge (at values of t) we can see the use of this we can back and forth between our function x & y and also with x(t) & y(t) to see the interdependency.

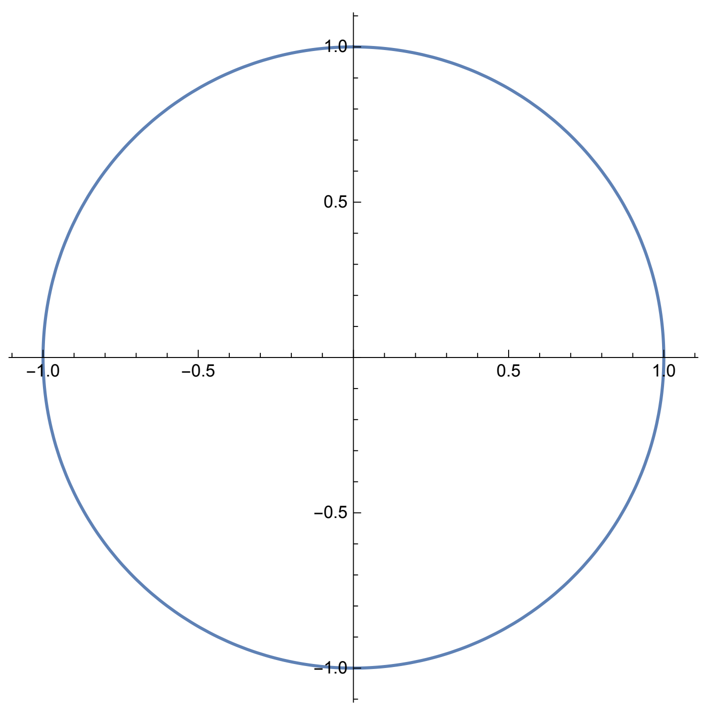
```
Show[P1, P2, PlotRange → {{-2, 2}, {-2, 2}}]
```



```
Plot3D[x^3 - 2 * x * y + 3 * y^2, {x, -3, 3}, {y, -3, 3}]
```




```
ParametricPlot[{Sin[t], Cos[t]}, {t, 0, 2 π}]
```



These are good uses that we have seen for our Graphing abilites with *Mathematica.*We are now going to shift gears and move onto series and see where that leads.

Series with Mathematica:

Series are great tools that we have for solving a lot of problems and it is one of the best ways to approximate something when we don't the value of a function at that exact point. Series improves ones ability of pattern recognition.We are often intrested to see the value it converges to for a converging series.

We see that an arithmetic Series has the general formula a_n=a_0+(n)d where {n,0,N-1} so if a_0=3 and d=5 (common difference) and we want the sum of this arithmetic series from {n,0,99} till first N=100 terms. The we just use the Sum command: Sum[expr,{var,min,max}

```
Sum[3 + (n * 5), {n, 0, 99}]
```

```
25 050
```

The sum of an arithmetic series is given by this general formula: S_n= $\frac{N}{2}$(a_0+a_n)= $\frac{N}{2}$(a_0 +a_0+(N-1)d)=$\frac{N}{2}$(2a_0+(N-1)d). In this case a_0=3, d=5, and N=100 terms then we get the following: S_n= $\frac{100}{2}$(2*(3)+(100-1)*5)=?

$$\left(\frac{100}{2}\right) (2 * (3) + (100 - 1) * 5)$$

```
25 050
```

As we can see the answers are the same using the tqo different ways so we can trust *Mathematica*'s evaluation provided we type it in correctly.Let us move onto Geometric series is defined by a_n= $a_0 r^n$ where r is the common ratio between $\frac{a_n}{a_{n-1}}$. If r=3 and a_0=2 then ths Sum from {n,0,9} n= 0 to N-1. S_n= $\frac{(a_0(1-r^n))}{(1-r)}$.

```
Sum[((2 * (1 - 3^n)) / (1 - 3)), {n, 0, 9}]
```

```
29 514
```

This is sum of this geometric series when |r|>1.When |r|<1 we get the following convergence for the sum of geometric series S_∞= $\frac{a_0}{1-r}$.If $a_0 = 2$ and $r = \frac{1}{3}$.

```
(2) / (1 - (1 / 3))
```

```
3
```

We can also find Series approximations for a series that we define for example functions like Cosh[x] and we can do this command for a power series.

```
MyCosh = Series[Cosh[x], {x, 0, 11}]
```

$$1 + \frac{x^2}{2} + \frac{x^4}{24} + \frac{x^6}{720} + \frac{x^8}{40\,320} + \frac{x^{10}}{3\,628\,800} + O[x]^{12}$$

The O[x] term means "Higher Order powers". *Mathematica* produces a series "Object" which means that it is not a simple polynomial. The O[x] is a sign that this is not a simple polynomial. If we want to treat it as a regular polynomial, then we have to use the Normal function.

```
MySeries2 = Normal[MyCosh]
```

$$1 + \frac{x^2}{2} + \frac{x^4}{24} + \frac{x^6}{720} + \frac{x^8}{40\,320} + \frac{x^{10}}{3\,628\,800}$$

Now the O[x] term is gone so we can treat this as a regular polynomial and perform algebra with this evaluation.

```
Expand[(1/x) * MySeries2]
```

$$\frac{1}{x} + \frac{x}{2} + \frac{x^3}{24} + \frac{x^5}{720} + \frac{x^7}{40\,320} + \frac{x^9}{3\,628\,800}$$

Let us look at a Taylor Series Approximation example:

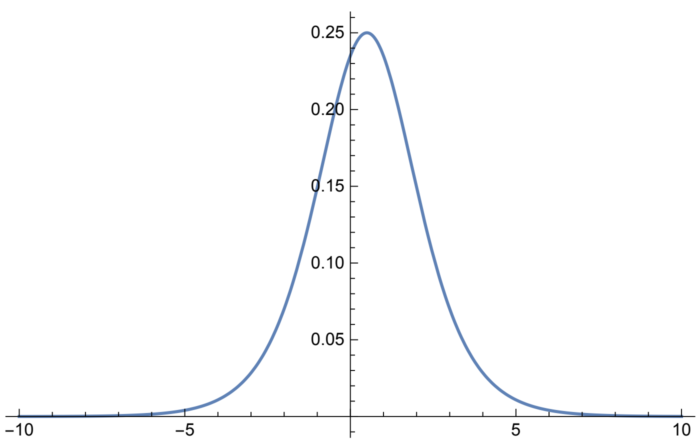
$$\texttt{MyF1[x\_] :=} \frac{e^{(0.5-x)}}{\left(1 + e^{(0.5-x)}\right)^2}$$

This function can also be defined with = sign but it matters when we try to do more sophisticated computations with this. When we want to find the gradient using FIndMaximum this may not show the actual Maximum value if defined with the equal sign.

$$\texttt{MyF2[x\_] =} \frac{e^{(0.5-x)}}{\left(1 + e^{(0.5-x)}\right)^2}$$

$$\frac{e^{0.5-x}}{\left(1 + e^{0.5-x}\right)^2}$$

```
Plot[MyF2[x], {x, -10, 10}]
```




```
FindMaximum[MyF2[x], x]
```

{0.25, {x → 0.5}}

Sometimes the when computing the gradient as we are with the FindMaximum here it might not be able to actually find the maximum due to the difference in the equal sign so it si better to use :=.

```
FindMaximum[MyF1[x], {x, 0, 1}]
```

{0.25, {x → 0.5}}

```
MySeries = Series[MyF1[x], {x, 1, 8}]
```

$$0.235004 - 0.0575568\,(x-1) - 0.0481784\,(x-1)^2 + 0.0174593\,(x-1)^3 + 0.00565084\,(x-1)^4 - 0.00325263\,(x-1)^5 - 0.000405096\,(x-1)^6 + 0.00047421\,(x-1)^7 - 1.27193\times10^{-6}\,(x-1)^8 + O[x-1]^9$$

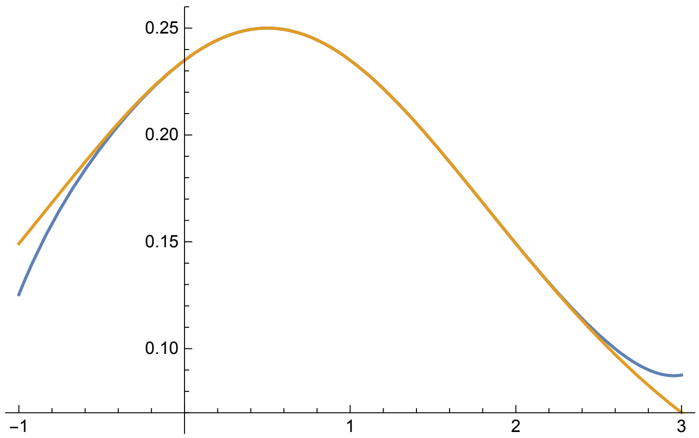
```
MySeriesN = Normal[MySeries]
```

$0.235004 - 0.0575568\,(-1+x) - 0.0481784\,(-1+x)^2 + 0.0174593\,(-1+x)^3 + 0.00565084\,(-1+x)^4 - 0.00325263\,(-1+x)^5 - 0.000405096\,(-1+x)^6 + 0.00047421\,(-1+x)^7 - 1.27193\times10^{-6}\,(-1+x)^8$

```
MySeriesN2 = Expand[MySeriesN]
```

$0.234946 + 0.0580716\,x - 0.0501954\,x^2 - 0.0128999\,x^3 - 0.000848814\,x^4 + 0.00920758\,x^5 - 0.00376018\,x^6 + 0.000484385\,x^7 - 1.27193\times10^{-6}\,x^8$

```
Plot[{MySeriesN2, MyF2[x]}, {x, -1, 3}]
```



Basically, what we tried to do here was approximate the function using the Taylor series sxpansion which we calculated upto eight terms and we tried to overlay teh graph with the function and the Taylor Series Expansion and we find that our approximation is pretty good and matches function well as we add more terms onto our approximation the fit will be much better and the deviation at the end from the true value will be much closer to the actual value.

Complex Variables with Mathematica:

Complex variables incorporate a Real part and an Imaginary Part in there numerical value which is typically denoted with a z= a±b*i* wher the Real Part RE[z]=a which is a real number and the other part could lead to a confusion but a the Imaginary Part IMg[z]=b which is also a real number the imaginary part is not b*i*.Let's see how to defina a complex number.

```
MyZ = 4 - 2 ⅈ
```

4 - 2 ⅈ

```
Arg[MyZ]
```

$-\text{ArcTan}\left[\frac{1}{2}\right]$

Now that we have the Argument w can use th N[ ] to get a numerical value.

```
N[%]
```

-0.463648

By now we know that angle is given in radians and the Modulus of Z is given by the following command Abs[z].

```
Abs[MyZ]
```

$2\sqrt{5}$

If we want to put z into polar form we do the following:

```
MyZ2 = Abs[MyZ] * (Cos[Arg[MyZ]] + ⅈ * Sin[Arg[MyZ]])
```

4 - 2 ⅈ

▶ convert to exponential MyZ2

$\left(2\sqrt{5}\right) e^{i\left(-\text{ArcTan}\left[\frac{1}{2}\right]\right)}$

After this command is run we can use the suggestion bar to convert it in terms of the exponential form.

```
MyZ3 = %
```

$\left(2\sqrt{5}\right) e^{i\left(-\text{ArcTan}\left[\frac{1}{2}\right]\right)}$

$$\text{MyZ4} = 2\sqrt{5} * \text{Exp}\left[i * \left(-\text{ArcTan}\left[\frac{1}{2}\right]\right)\right]$$

$2\sqrt{5}\, e^{-i\,\text{ArcTan}\left[\frac{1}{2}\right]}$

```
MyCFunction = Tanh[z] * Exp[z]
```

$e^{z}\,\text{Tanh}[z]$

```
ComplexExpand[MyCFunction /. z → x + ⅈ * y]
```

$$-\frac{e^{x}\,\text{Sin}[y]\,\text{Sin}[2\,y]}{\text{Cos}[2\,y]+\text{Cosh}[2\,x]}+\frac{e^{x}\,\text{Cos}[y]\,\text{Sinh}[2\,x]}{\text{Cos}[2\,y]+\text{Cosh}[2\,x]}+i\left(\frac{e^{x}\,\text{Cos}[y]\,\text{Sin}[2\,y]}{\text{Cos}[2\,y]+\text{Cosh}[2\,x]}+\frac{e^{x}\,\text{Sin}[y]\,\text{Sinh}[2\,x]}{\text{Cos}[2\,y]+\text{Cosh}[2\,x]}\right)$$

```
ComplexExpand[MyCFunction /. z → x + ⅈ * y] +
  ⅈ (ⅇ^x Cos[y] Cosh[x] Sin[y] + ⅇ^x Cos[y] Sin[y] Sinh[x])
```

$$-\frac{e^{x}\,\text{Sin}[y]\,\text{Sin}[2\,y]}{\text{Cos}[2\,y]+\text{Cosh}[2\,x]}+i\left(e^{x}\,\text{Cos}[y]\,\text{Cosh}[x]\,\text{Sin}[y]+e^{x}\,\text{Cos}[y]\,\text{Sin}[y]\,\text{Sinh}[x]\right)+\frac{e^{x}\,\text{Cos}[y]\,\text{Sinh}[2\,x]}{\text{Cos}[2\,y]+\text{Cosh}[2\,x]}+i\left(\frac{e^{x}\,\text{Cos}[y]\,\text{Sin}[2\,y]}{\text{Cos}[2\,y]+\text{Cosh}[2\,x]}+\frac{e^{x}\,\text{Sin}[y]\,\text{Sinh}[2\,x]}{\text{Cos}[2\,y]+\text{Cosh}[2\,x]}\right)$$

Now basically what we did was us the command and expand this complex expression and now by adding $i\,(e^{x}\,\text{Cos}[y]\,\text{Cosh}[x]\,\text{Sin}[y]+e^{x}\,\text{Cos}[y]\,\text{Sin}[y]\,\text{Sinh}[x]$ we have now broken the function to the Real part U(x,y) & V(x,y).

Linear Algebra with Mathematica:

Linear Algebra is very useful for us because it comes with loads of tricks that we can use to olve problems about a linear system it and In Physics we are always dealing with systems. If we are dealing with a linear system then we are going to use system of equations and if we have three unknowns we need three equation to solve for it and get three values these equations would have to linearly independent.Let us consider this Linear system of equations:

$$4x - 7y + 2z = 16$$
$$-3x + 1y - 5z = -21$$
$$2x - 6y + 5z = 7$$

```
A1 = {{a, -b, c}, {-d, e, f}, {g, -h, j}}
```

```
{{a, -b, c}, {-d, e, f}, {g, -h, j}}
```

In this case {a,-b,c} corresponds to the first row and the comma (,) inside the{ } sperate one column form the other and , in this spot },{ seperates one row from the next.

```
A2 = {{4, -7, 2}, {-3, 1, -5}, {2, -6, 5}}
```

```
{{4, -7, 2}, {-3, 1, -5}, {2, -6, 5}}
```

```
Out[2] // MatrixForm
```

$$\begin{pmatrix} 4 & -7 & 2 \\ -3 & 1 & -5 \\ 2 & -6 & 5 \end{pmatrix}$$

In order to see A2 in MatrixFOrm we use the following Command Out[n]//MatrixForm.We can use the fact that $A\vec{x} = \vec{b}$ and solve this system of equations.

```
b1 = {16, -21, 7}
```

```
{16, -21, 7}
```

```
Inverse[A2]
```

$$\left\{\left\{\frac{25}{103}, -\frac{23}{103}, -\frac{33}{103}\right\}, \left\{-\frac{5}{103}, -\frac{16}{103}, -\frac{14}{103}\right\}, \left\{-\frac{16}{103}, -\frac{10}{103}, \frac{17}{103}\right\}\right\}$$

We just used the Inverse command to find the inverse of our matrix A (A^{-1}).If we want to find the determinant then we can use the command: Det[expr] and we know it has to be nonzero in this case as we found an inverse.

```
Det[A2]
```

```
-103
```

If we want multiply two matrices we just do the following command to find the product of two matrices Dot (.)

```
B1 = {{1, 2, -3}, {-4, 3, 2}, {4, -8, -9}}
```

```
{{1, 2, -3}, {-4, 3, 2}, {4, -8, -9}}
```

```
(A2) . (B1)
```

```
{{40, -29, -44}, {-27, 37, 56}, {46, -54, -63}}
```

```
Out[10] // MatrixForm
```

$$\begin{pmatrix} 40 & -29 & -44 \\ -27 & 37 & 56 \\ 46 & -54 & -63 \end{pmatrix}$$

When doing Matrix Multiplication it is important to remember the order the multiplication is applied in is important i.e AB≠BA it is true in some cases but that is very rare.Another most important rule doing Matrix Multiplication is that the columns of the first matrix must be equal to the rows of the second matrix in order for matrix multiplication to work we then end up with R1 rows of the first matrix and C2 columns of the second matrix R1 × C2.Those are the check that we can put in place when multiplying Matrices. We will continue with more linear algebra.

Let us look at Row Reduced Echelon form we are just trying to simplify the system of equations and eventually solve for the unknowns (augment matrix).It is defined as a command in *Mathematica* so the command is RowReduce[ ].

```
RowReduce[{{12, -26, 34, 54}, {-30, 65, -85, -46}}]
```

$$\left\{\left\{1, -\frac{13}{6}, \frac{17}{6}, 0\right\}, \{0, 0, 0, 1\}\right\}$$

Since we have entered this augmented matrix after row reduction we 0=1 which cannot NOT BE TRUE so this matrix is inconsistent. Let’s RowReduce A2 and since we have a solution already and an inverse we know we cannot get an inconsistent matrix.

```
A3 = {{4, -7, 2, 16}, {-3, 1, -5, -21}, {2, -6, 5, 7}}
{{4, -7, 2, 16}, {-3, 1, -5, -21}, {2, -6, 5, 7}}
```

```
Out[21] // MatrixForm
```

$$\begin{pmatrix} 4 & -7 & 2 & 16 \\ -3 & 1 & -5 & -21 \\ 2 & -6 & 5 & 7 \end{pmatrix}$$

```
RowReduce[A3]
```

$$\left\{\left\{1, 0, 0, \frac{652}{103}\right\}, \left\{0, 1, 0, \frac{158}{103}\right\}, \left\{0, 0, 1, \frac{73}{103}\right\}\right\}$$

```
Out[23] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & \frac{652}{103} \\ 0 & 1 & 0 & \frac{158}{103} \\ 0 & 0 & 1 & \frac{73}{103} \end{pmatrix}$$

As we can see we do get a solution because we have Row reduced it to the simplest form and since it is an augmented matrix we are left with the unique solution.

```
LinearSolve[A2, b1]
```

$$\left\{\frac{652}{103}, \frac{158}{103}, \frac{73}{103}\right\}$$

Here we clearly see that solutions match each other This is a good way to confirm becuase we used three different wayst to solve this Linear system we can RowReduce[ ] the augmented matrix, use

LinearSolve[ ] ,and finding $A^{-1}(\vec{b})$. These Methods as we saw all worked and it up to us choose which method we want. we will now look at Eigenvalues and Eigenvectors [Eigen(space/system)]. We know that if λ is an eigenvalue of Matrix T (Linear transformation) and that $\vec{v} \neq \vec{0}$,then we know that $T^n \vec{v} = (\lambda)^n \vec{v}$.

```
A4 = {{2, 3}, {7, 9}}
```

{{2, 3}, {7, 9}}

```
Eigenvalues[A4]
```

$\left\{\frac{1}{2}\left(11+\sqrt{133}\right), \frac{1}{2}\left(11-\sqrt{133}\right)\right\}$

```
Eigenvectors[A4]
```

$\left\{\left\{\frac{1}{14}\left(-7+\sqrt{133}\right), 1\right\}, \left\{\frac{1}{14}\left(-7-\sqrt{133}\right), 1\right\}\right\}$

```
Eigensystem[A4]
```

$\left\{\left\{\frac{1}{2}\left(11+\sqrt{133}\right), \frac{1}{2}\left(11-\sqrt{133}\right)\right\}, \left\{\left\{\frac{1}{14}\left(-7+\sqrt{133}\right), 1\right\}, \left\{\frac{1}{14}\left(-7-\sqrt{133}\right), 1\right\}\right\}\right\}$

We jsut basically lookaed at ways to either get the eigenvalue or the eigenvector using the command EigenValues[ ] or EigenVectors[ ] respectively. The other option is we can get both of it usin the command Eigensystem[ ] where the first eigenvalue and the second eigenvalue this is the first list and then we are given eigenvectors coressponding to the eigenvalues in that order.

Orthogonality with Mathematica:

Dot product or the scalar product take two vectors and finally we are left with a scalar, hence scalar product. It can also be calculated using A•B= |A||B| Cos[ϑ]. This is the following command that we need to input to calculte a dot product.

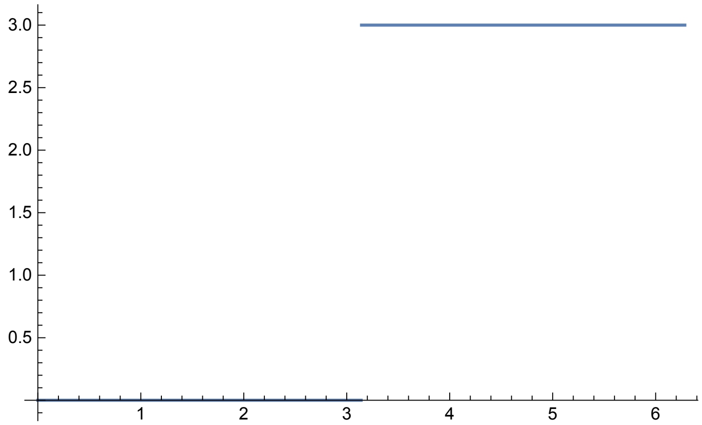
```
{1, 2, -3} . {i, j, k}
```

i + 2 j - 3 k

This in terms of functions is defined to be the inner product where this notates inner product <f(x)|g(x)> and in *Mathematica* we can use the following command Inner[f,{list1},{list2},h] .When we look at Fourier series knowing how to enter a Piecewise Function is important because a square wave or triangle wave for example has sharp edges that show discontiniuity so it cannot be defined as a continuous function. We need to use the following command for Piecewise functions : Piecewise[ ]

```
MyF[x_] := Piecewise[{{-3, x < 0}, {0, 0 < x < π}, {3, π < x < 2 * π}}]
```

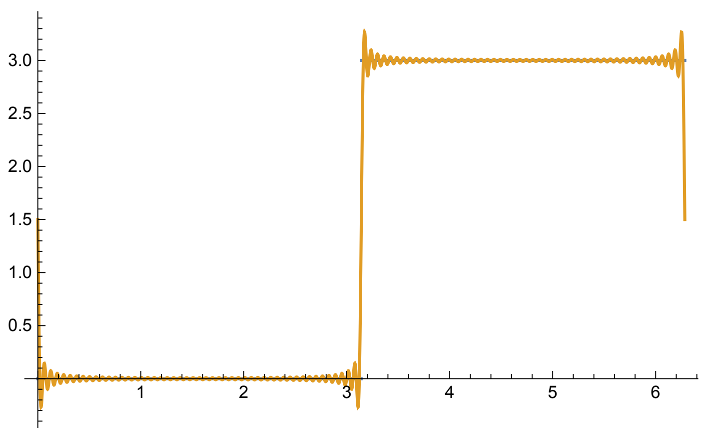
```
Plot[MyF[x], {x, 0, 2 * π}]
```




```
MySquareApprox[x_, m_] := (+1.5) + Sum[-6/(n * π) * Sin[n * x], {n, 1, m, 2}]
```

We wrote the two in there because we know that the even terms are zero in this case so we are skipping every other term and want only the odd ones.

```
Plot[{MyF[x], MySquareApprox[x, 100]}, {x, 0, 2 * π}]
```



As we can see that we have a good square wave fuction approxiamtion using the Fourier series and if we add more terms this aprroximation will get better.

References:

2) "Wolfram Language & System Documentation Center." Wolfram Language & System Docum

3) "Math Methods Sample Student Mathematica Notebook". 2012. PDF.