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phys 425 &01
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```
$Assumptions = {x, A, a, k, A1, a1, a2, m, ℏ, t, w} ∈ Reals
```

$(x \mid A \mid a \mid k \mid A_1 \mid a_1 \mid a_2 \mid m \mid \hbar \mid t \mid w) \in \text{Reals}$

Problem 1) 2.21 part a) && is used to indicate and conditions.

```
ψ[x_] := A * (e^(-a*Abs[x]))
Assuming[(A > 0 && a  > 0), Integrate[Conjugate[ψ[x]] * ψ[x], {x, -∞, ∞}]]
```

$$\frac{A^2}{a}$$

Problem 1) 2.21 part b)

```
Assuming[(k > 0 && a  > 0), Integrate[(Sqrt[a/(2*π)]) * (e^(-a*Abs[x])) * (e^(-i*k*x)), {x, -∞, ∞}]]
```

$$\frac{a^{3/2}\sqrt{\frac{2}{\pi}}}{(a - i\, k\, \text{Log}[e])\ (a + i\, k\, \text{Log}[e])}$$

Problem 2) 2.22 part a)

```
ψ1[x_] := A1 * (e^(-a1*x^2))
Assuming[(A1 > 0 && a1  > 0), Integrate[Conjugate[ψ1[x]] * ψ1[x], {x, -∞, ∞}]]
```

$$\frac{\sqrt{\frac{\pi}{2}}\, A_1^2}{\sqrt{a_1}}$$

Problem 2) 2.22) part b) sketch $|\Psi(x, 0)|^2$ and $|\Psi(x, t)|^2$ for a very large value of t. Let a_2=$\hbar$=m=1 for the plots and these are coded in as a_2= c , $\hbar$= d, m= f.

$$g[t\_] := \sqrt{1 + \left(\frac{2 * i * \hbar * a_2 * t}{m}\right)}$$

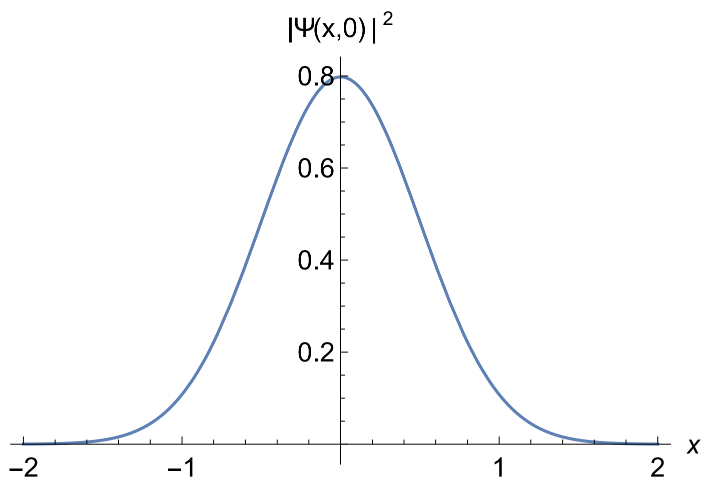
$$K1[x\_, t\_] := \left(\frac{2 * a_2}{\pi}\right)^{\frac{1}{4}} * \left(\frac{1}{g[t]}\right) * \left(e^{\frac{-(a_2 * x^2)}{(g[t])^2}}\right)$$

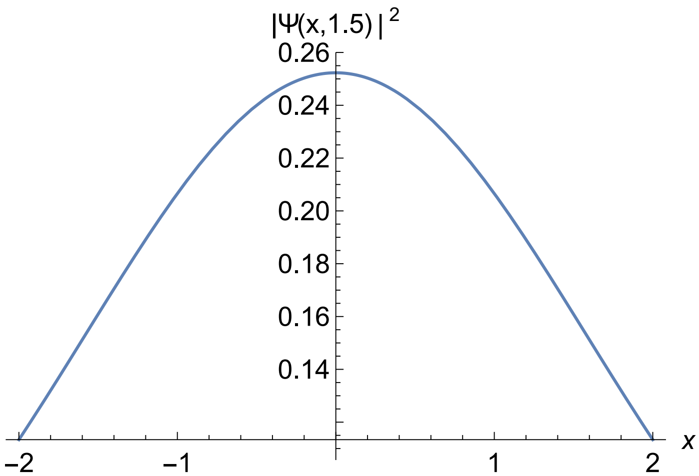
```
c := 1
d := 1
f := 1
```

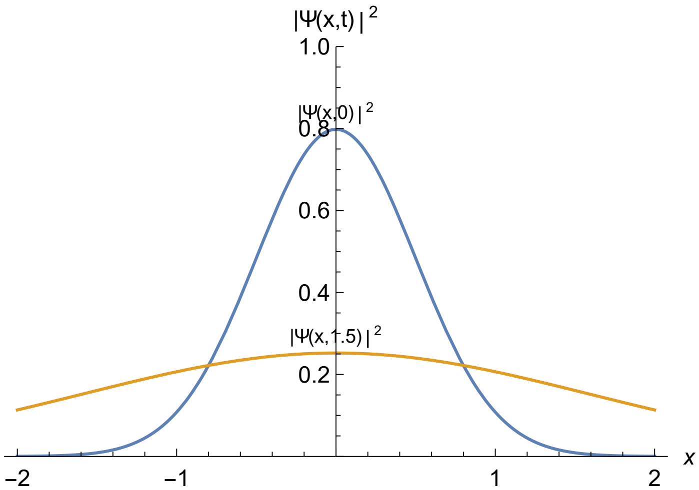
$$g2[t\_] := \sqrt{1 + \left(\frac{2 * i * d * c * t}{f}\right)}$$

$$K2[x\_, t\_] := \left(\frac{2 * c}{\pi}\right)^{\frac{1}{4}} * \left(\frac{1}{g2[t]}\right) * \left(e^{\frac{-(c * x^2)}{(g2[t])^2}}\right)$$

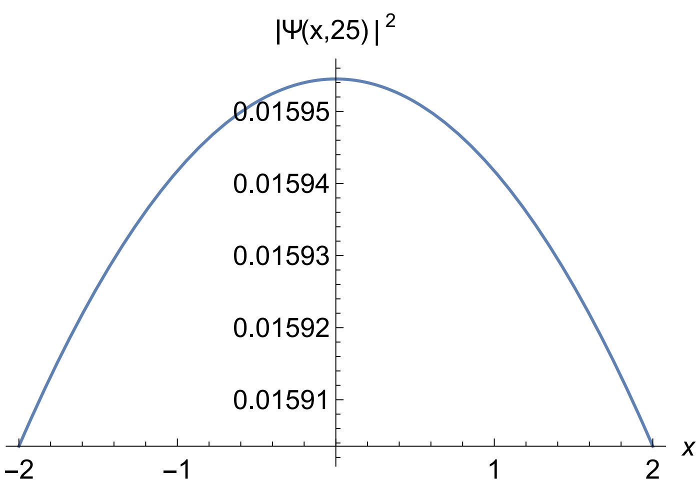
```
Plot[ (Conjugate[K2[x, 0]]) * (K2[x, 0]), {x, -2, 2},
 AxesLabel → {x, "|Ψ(x,0)|^2"}, AxesStyle → Directive[Black, 14]]
Plot[ (K2[x, 1.5]) * (Conjugate[K2[x, 1.5]]), {x, -2, 2},
 AxesLabel → {x, "|Ψ(x,1.5)|^2"}, AxesStyle → Directive[Black, 14]]
Plot[{(Conjugate[K2[x, 0]]) * (K2[x, 0]), (Conjugate[K2[x, 1.5]]) * (K2[x, 1.5])},
 {x, -2, 2}, PlotLabels → Placed[{"|Ψ(x,0)|^2", "|Ψ(x,1.5)|^2"}, Above],
 LabelStyle → Directive[Black, 10], AxesLabel → {x, "|Ψ(x,t)|^2"},
 AxesStyle → Directive[Black, 12], PlotRange → {0, 1}]
```








```
Plot[(K2[x, 25]) * (Conjugate[K2[x, 25]]), {x, -2, 2},
 AxesLabel → {x, "|Ψ(x,25)|²"}, AxesStyle → Directive[Black, 14]]
```




```
Integrate[Conjugate[K1[x, t]] * x * K1[x, t], {x, -∞, ∞}] (*it matches.*)
```

ConditionalExpression[0, $a_2 > 0$ && $m \neq 0$]

```
Integrate[Conjugate[K1[x, t]] * (ℏ/i) D[K1[x, t], {x, 1}], {x, -∞, ∞}]
```

ConditionalExpression[0, $a_2 > 0$ && $m \neq 0$]

$$w := \sqrt{\frac{a_2}{1 + \left(\frac{2*\hbar*a_2*t}{m}\right)^2}}$$

$$\text{probabilitydensity}[x\_, t\_] := \sqrt{\frac{2}{\pi}} * (w) * \left(e^{-2*w^2*x^2}\right)$$

```
Assuming[w > 0 && a₂ > 0, Integrate[x^2 * (probabilitydensity[x, t]), {x, -∞, ∞}]]
(* multiply the first term by m^2 and factor m^2 on the numerator and
 the second term by 4*a₂ to find the common denominator. Then we can
 cancel m^2 adn this matches with answer found using integral sheet.*)
```

$$\frac{1}{4\, a_2} + \frac{t^2\, \hbar^2\, a_2}{m^2}$$

```
Assuming[a₂ > 0 && m > 0,
 Integrate[(ℏ/i)^2 Conjugate[K1[x, t]] * D[K1[x, t], {x, 2}], {x, -∞, ∞}] ]
 (* This does simplify to a*ℏ^2*)
```

$$\hbar^2\, \text{Conjugate}\left[\frac{1}{\sqrt{m + 2\, i\, t\, \hbar\, a_2}}\right] a_2 \sqrt{m - 2\, i\, t\, \hbar\, a_2}$$

Problem 3) 2.23) part a) There is a function command for the the delta function called DiracDelta so let's confirm our calculations.

In[1]:=
```
Integrate[(x^3 - 3 * x^2 + 2 * x - 1) * DiracDelta[x + 2], {x, -3, 1}]
```

Out[1]= -25

In[2]:=
```
Integrate[(Cos[3 * x] + 2) * DiracDelta[x - π], {x, 0, ∞}]
```

Out[2]= 1

In[3]:=
```
Integrate[(e^(Abs[x]+3)) * DiracDelta[x - 2], {x, -1, 1}]
```

Out[3]= 0

In[4]:=
```
Integrate[(1/(2 π))^(1/2) * e^(-i*k*x) * DiracDelta[x], {x, -∞, ∞}]
```

Out[4]= $\frac{1}{\sqrt{2\,\pi}}$