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1)Problem 2.7)a)

In[10]:= $\$Assumptions = \{x, t, m, a, A, g, \omega, \hbar, A_1, A_2, A_3, E2\} \in Reals$

Out[10]= $(x \mid t \mid m \mid a \mid A \mid g \mid \omega \mid \hbar \mid A_1 \mid A_2 \mid A_3 \mid E2) \in Reals$

$$F1[x\_] = Piecewise\left[\left\{\left\{(A*x), 0 \le x \le \frac{a}{2}\right\}, \left\{A*(a-x), \frac{a}{2} \le x \le a\right\}\right\}\right]$$

$$\begin{cases} A\,x & 0 \le x \le \frac{a}{2} \\ A\,(a-x) & \frac{a}{2} \le x \le a \\ 0 & True \end{cases}$$

Now if we set this expression equal to 1 and solve for A and take the positive root we get our normalization constant.

$$Integrate\left[(F1[x])^2, \{x, 0, a\}\right]$$

$$\begin{cases} \frac{a^3 A^2}{12} & a > 0 \\ 0 & True \end{cases}$$

Let us redefine our piecewise with the normalization constant such that we can graph it to see if it matches.Let us call this new one F2[x_].

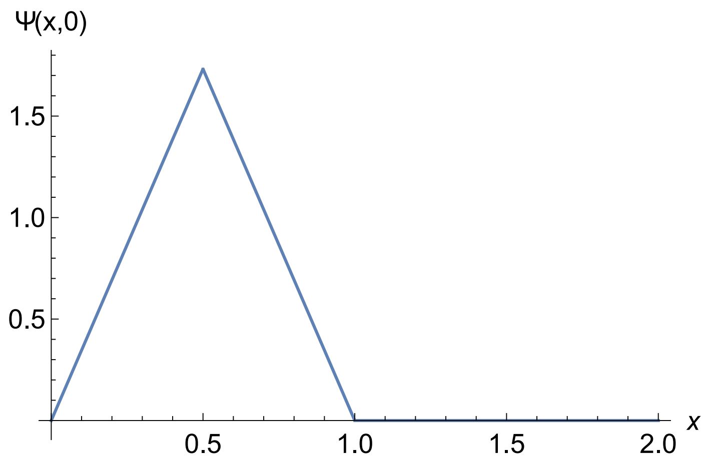
$$F2[x\_] = Piecewise\left[\left\{\left\{\left(\left(\sqrt{\frac{12}{a^3}}\right)*x\right), 0 \le x \le \frac{a}{2}\right\}, \left\{\left(\sqrt{\frac{12}{a^3}}\right)*(a-x), \frac{a}{2} \le x \le a\right\}\right\}\right]$$

$$\begin{cases} 2\sqrt{3}\sqrt{\frac{1}{a^3}}\,x & 0 \le x \le \frac{a}{2} \\ 2\sqrt{3}\sqrt{\frac{1}{a^3}}\,(a-x) & \frac{a}{2} \le x \le a \\ 0 & True \end{cases}$$

We must set a to be something in order to graph it. Let a = 1.

```
a := 1
```

```
Plot[F2[x], {x, 0, 2}, AxesLabel → {x, "Ψ(x,0)"}, AxesStyle → Directive[Black, 14]]
```




```
(*2.7part b) checking integration*)
$Assumptions = {n} ∈ Integers
```

$$\text{Integrate}\left[(x)*\left(\text{Sin}\left[\frac{n*\pi*x}{g}\right]\right), \left\{x, 0, \frac{g}{2}\right\}\right]$$

n ∈ Integers

$$-\frac{g^2\left(n\pi\,\text{Cos}\left[\frac{n\pi}{2}\right]-2\,\text{Sin}\left[\frac{n\pi}{2}\right]\right)}{2\,n^2\,\pi^2}$$

$$\text{Integrate}\left[-1*(x)*\left(\text{Sin}\left[\frac{n*\pi*x}{g}\right]\right), \left\{x, \frac{g}{2}, g\right\}\right]$$

$$\frac{g^2\left(-n\pi\,\text{Cos}\left[\frac{n\pi}{2}\right]+2\left((-1)^n\,n\pi+\text{Sin}\left[\frac{n\pi}{2}\right]\right)\right)}{2\,n^2\,\pi^2}$$

$$\text{Integrate}\left[(g)*\left(\text{Sin}\left[\frac{n*\pi*x}{g}\right]\right), \left\{x, \frac{g}{2}, g\right\}\right]$$

$$\frac{g^2\left((-1)^{1+n}+\text{Cos}\left[\frac{n\pi}{2}\right]\right)}{n\pi}$$

$$\text{Simplify}\left[-\frac{g^2\left(n\pi\,\text{Cos}\left[\frac{n\pi}{2}\right]-2\,\text{Sin}\left[\frac{n\pi}{2}\right]\right)}{2\,n^2\,\pi^2}+\frac{g^2\left(-n\pi\,\text{Cos}\left[\frac{n\pi}{2}\right]+2\left((-1)^n\,n\pi+\text{Sin}\left[\frac{n\pi}{2}\right]\right)\right)}{2\,n^2\,\pi^2}+\frac{g^2\left((-1)^{1+n}+\text{Cos}\left[\frac{n\pi}{2}\right]\right)}{n\pi}\right]$$

$$\frac{2\,g^2\,\text{Sin}\left[\frac{n\pi}{2}\right]}{n^2\,\pi^2}$$

$$\text{Sum}\left[\frac{1}{((2*j)+1)^2}, \{j, 0, \text{Infinity}\}\right]$$

$$\frac{\pi^2}{8}$$

2)Problem 2.10)a) Just checking the integral for to obtain A_1 normalization constant.

$$\text{F3[x\_] := } \left(\frac{m * \omega}{\pi * \hbar}\right)^{\frac{1}{4}} * e^{\frac{-m*\omega*x^2}{2*\hbar}}$$

$$\text{F4[x\_] := } \left(\frac{(A_1)}{\sqrt{2*\hbar*m*\omega}}\right) * (2*m*\omega*x) * (\text{F3[x]})$$

$$\text{Integrate}\left[x^2 * e^{\frac{-m*\omega*x^2}{\hbar}}, \{x, -\infty, \infty\}\right]$$

$$\text{ConditionalExpression}\left[\frac{\sqrt{\pi}}{2\left(\frac{m\,\omega}{\hbar}\right)^{3/2}}, m\,\omega\,\hbar > 0\right]$$

(*It works. The command below works too because we know ω $\hbar$ and m positive non zero real numbers.*)

$$\text{Integrate}\left[\text{Abs}[\text{F4}[x]]^2, \{x, -\infty, \infty\}\right]$$

$$\begin{cases} A_1^2 & m\,\omega\,\hbar > 0 \\ \text{Integrate}\left[\frac{2\, e^{-\text{Re}\left[\frac{m x^2 \omega}{\hbar}\right]} \text{Abs}\left[\frac{m\, x\, \omega \left(\frac{m\,\omega}{\hbar}\right)^{1/4} A_1}{\sqrt{m\,\omega\,\hbar}}\right]^2}{\sqrt{\pi}}, \{x, -\infty, \infty\}, \right. & \text{True} \\ \quad \left. \text{Assumptions} \to (x \mid t \mid m \mid a \mid A \mid g \mid \omega \mid \hbar \mid A_1 \mid A_2) \in \text{Reals} \,\&\&\, m\,\omega\,\hbar \le 0\right] & \end{cases}$$

(*Checking the integral to get normalization constant A_2. We should get $\frac{1}{\sqrt{2!}}$. First define Piecewise for ψ_2(x).Again we know m,ω,$\hbar$ are positive non-zero real numbers*)

$$\text{F5[x\_] := } (A_2) * \left(\frac{2*m*\omega*x^2}{\hbar} - 1\right) * (\text{F3[x]})$$

$$\text{Integrate}\left[\text{Abs}[\text{F5}[x]]^2, \{x, -\infty, \infty\}\right]$$

$$\begin{cases} 2\,A_2^2 & m\,\omega\,\hbar > 0 \\ \text{Integrate}\left[\frac{e^{-\text{Re}\left[\frac{m x^2 \omega}{\hbar}\right]} \text{Abs}\left[\left(-1+\frac{2 m x^2 \omega}{\hbar}\right)\left(\frac{m\,\omega}{\hbar}\right)^{1/4} A_2\right]^2}{\sqrt{\pi}}, \{x, -\infty, \infty\}, \right. & \text{True} \\ \quad \left. \text{Assumptions} \to (x \mid t \mid m \mid a \mid A \mid g \mid \omega \mid \hbar \mid A_1 \mid A_2) \in \text{Reals} \,\&\&\, m\,\omega\,\hbar \le 0\right] & \end{cases}$$

Now we must declare the constants that we ha have in our functions to be able to graph those functions. I want to keep those equations in the same form so I shall use the following to declare constants. Let m be represented by b (m=b=1), (ω=c= 1),(and $\hbar$ = d = 1). Now let us redefine those functions and I shall call these H[x].

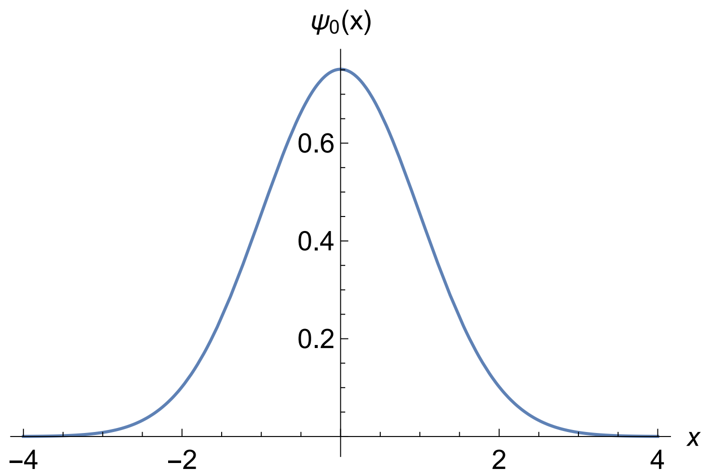
$$\text{H3[x\_] := } \left(\frac{b*c}{\pi*d}\right)^{\frac{1}{4}} * e^{\frac{-b*c*x^2}{2*d}}$$

$$\text{H4[x\_] := } \left(\frac{1}{\sqrt{1!}}\right) * \left(\sqrt{\frac{2*b*c}{d}}\right) * (x) * (\text{H3[x]})$$

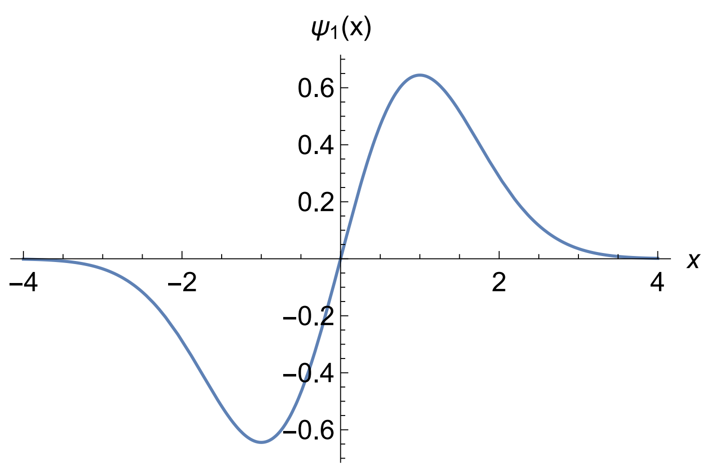
$$\text{H5[x\_] := } \left(\sqrt{\frac{1}{2!}}\right) * \left(\frac{2*b*c*x^2}{d} - 1\right) * (\text{H3[x]})$$

```
(* Now let us declare the constants as planned and go for the plots.*)
b := 1
c := 1
d := 1
```

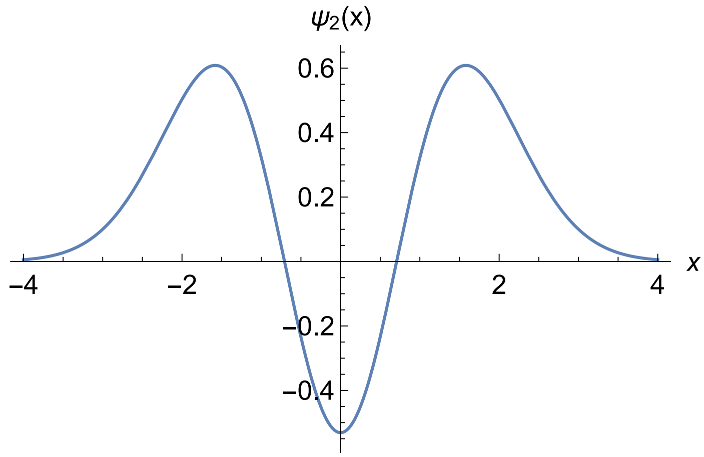
```
P3 = Plot[H3[x], {x, -4, 4}, AxesLabel → {x, "ψ0(x)"}, AxesStyle → Directive[Black, 14]]
```



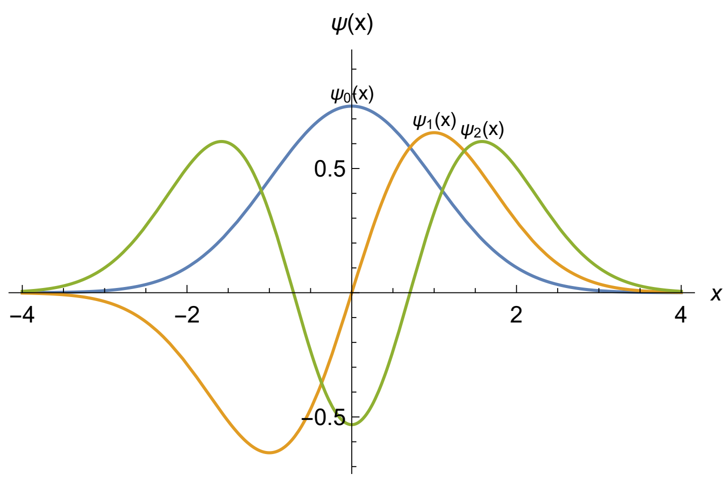

```
P4 = Plot[H4[x], {x, -4, 4}, AxesLabel → {x, "ψ1(x)"}, AxesStyle → Directive[Black, 14]]
```



```
P5 = Plot[H5[x], {x, -4, 4}, AxesLabel → {x, "ψ₂(x)"}, AxesStyle → Directive[Black, 14]]
```




```
Plot[{H3[x], H4[x], H5[x]}, {x, -4, 4},
 PlotLabels → Placed[{"ψ₀(x)", "ψ₁(x)", "ψ₂(x)"}, Above],
 LabelStyle → Directive[Black, 10],
 AxesLabel → {x, "ψ(x)"}, AxesStyle → Directive[Black, 12]]
```



2)Problem 2.10)part)c) Just checking the integrals such that we can show orthonormality. i will call these set of functions J[x].Again we know m,ω,$\hbar$ are positive non-zero real numbers

$$J1[x\_] := (x) * \left(e^{\left(\frac{-m*\omega*x^2}{\hbar}\right)}\right)$$

```
Integrate[J1[x], {x, -∞, ∞}]
```

ConditionalExpression[0, m ω ℏ > 0]

$$J2[x\_] := (x)^3 * \left(e^{\left(\frac{-m*\omega*x^2}{\hbar}\right)}\right)$$

```
Integrate[J2[x], {x, -∞, ∞}]
```

ConditionalExpression[0, m ω ℏ > 0]

3)Problem 2.11)a) Here we have to check a Whole Bunch of integral values.Let us define the same functions but now in terms of α and ξ (shortcut is esc x esc). We can call these functions K[x]. first let us declare the constant α and ξ.

```
α := (m * ω / (π * ℏ))^(1/4)

ξ := (Sqrt[m * ω / ℏ]) * (x)

K[ξ_] := α * ⅇ^(-(ξ)^2/2)

(*Just to check*)
Assuming[m * ω * ℏ > 0, Integrate[Abs[K[ξ]]^2, {x, -∞, ∞}]]
```

1

```
Integrate[(K[ξ]) * ξ * (K[ξ]), {x, -∞, ∞}]
```

ConditionalExpression[0, Re[$\frac{m\,\omega}{\hbar}$] ≥ 0]

```
(*I get that Mathematica is assuming different things for these codes,
which is why I am getting  two different results,
but how can I rectify this? Because the same function definition
 gave me the correct answers for the momentum expectation values.*)

Integrate[(K[ξ]) * (ξ)^2 * (K[ξ]), {x, -∞, ∞}]
```

ConditionalExpression[$\frac{1}{2}$, Re[$\frac{m\,\omega}{\hbar}$] > 0]

```
Integrate[(Abs[K[ξ]]^2) * (ξ)^2, {x, -∞, ∞}]
```

ConditionalExpression[$\frac{m\,\omega\sqrt{\text{Abs}\left[\frac{m\,\omega}{\hbar}\right]}}{2\,\hbar\,\text{Re}\left[\frac{m\,\omega}{\hbar}\right]^{3/2}}$, Re[$\frac{m\,\omega}{\hbar}$] > 0]

```
Integrate[(ℏ/ⅈ) * (K[ξ]) * (D[K[ξ], x]), {x, -∞, ∞}]
```

ConditionalExpression[0, Re[$\frac{m\,\omega}{\hbar}$] ≥ 0]

```
Integrate[(ℏ/ⅈ)^2 * (K[ξ]) * (D[K[ξ], {x, 2}]), {x, -∞, ∞}]
```

ConditionalExpression[$\frac{m\,\omega\,\hbar}{2}$, Re[$\frac{m\,\omega}{\hbar}$] > 0]

```
(*Here weare defining ψ1 interms of ξ and ψ0(ξ)and checking normalization after that.*)

K1[ξ] := Sqrt[2] * ξ * (K[ξ])
(*It works.*)Assuming[m * ω * ℏ > 0, Integrate[Abs[K1[ξ]]^2, {x, -∞, ∞}]]
```

1

```
(*checking expectation value of <x>*)Integrate[(K1[ξ] * ξ * K1[ξ]), {x, -∞, ∞}]
```

ConditionalExpression$\left[0, \mathrm{Re}\left[\frac{m\,\omega}{\hbar}\right] \geq 0\right]$

```
Integrate[(K1[ξ] * (ξ)^2 * K1[ξ]), {x, -∞, ∞}]
```

ConditionalExpression$\left[\frac{3}{2}, \mathrm{Re}\left[\frac{m\,\omega}{\hbar}\right] > 0\right]$

Integrate$\left[\left(\frac{\hbar}{i}\right) * \left(K1[\xi]\right) * \left(D[K1[\xi], x]\right), \{x, -\infty, \infty\}\right]$

ConditionalExpression$\left[0, \mathrm{Re}\left[\frac{m\,\omega}{\hbar}\right] \geq 0\right]$

Integrate$\left[\left(\frac{\hbar}{i}\right)^2 * \left(K1[\xi]\right) * \left(D[K1[\xi], \{x, 2\}]\right), \{x, -\infty, \infty\}\right]$

ConditionalExpression$\left[\frac{3\,m\,\omega\,\hbar}{2}, \mathrm{Re}\left[\frac{m\,\omega}{\hbar}\right] > 0\right]$

Problem 4)2.13) part a)

In[1]:= E1[n_] := $(\hbar * \omega) * \left(n + \frac{1}{2}\right)$

ψ_0[x_] := $(1) * \left(\frac{m * \omega}{\pi * \hbar}\right)^{\frac{1}{4}} * e^{\frac{-m*\omega*x^2}{2*\hbar}}$

ψ_1[x_] := $\left(\sqrt{\frac{2 * m * \omega}{\hbar}}\right) * x * (\psi_0[x])$

ψ_2[x_] := $\left(\sqrt{\frac{1}{2!}}\right) * \left(\frac{2 * m * \omega * x^2}{\hbar} - 1\right) * (\psi_0[x])$

Ψ[x_, t_] := $(A_3) * \left(\left((3) * (\psi_0[x]) * \left(e^{-i*\left(\frac{(E1[0]*t)}{\hbar}\right)}\right)\right) + \left((4) * (\psi_1[x]) * \left(e^{-i*\left(\frac{(E1[1]*t)}{\hbar}\right)}\right)\right)\right)$

(* This is not the right form because we need to find c_n's*)

In[6]:= Integrate[(Ψ[x, 0]) * (Ψ[x, 0]), {x, -∞, ∞}]
(*Note Capital E is a pre-defined function for exponent Do Not use E[]*)

Out[6]= ConditionalExpression$\left[25\,A_3^2, \mathrm{Re}\left[\frac{m\,\omega}{\hbar}\right] > 0\right]$

In[7]:= $\Psi_1[x\_, t\_] := \left(\frac{1}{5}\right) * \left(\left((3) * (\psi_0[x]) * \left(e^{-i * \left(\frac{(E1[0]*t)}{\hbar}\right)}\right)\right) + \left((4) * (\psi_1[x]) * \left(e^{-i * \left(\frac{(E1[1]*t)}{\hbar}\right)}\right)\right)\right)$

$\Psi_1{}^*[x\_, t\_] := \left(\frac{1}{5}\right) * \left(\left((3) * (\psi_0[x]) * \left(e^{+i * \left(\frac{(E1[0]*t)}{\hbar}\right)}\right)\right) + \left((4) * (\psi_1[x]) * \left(e^{+i * \left(\frac{(E1[1]*t)}{\hbar}\right)}\right)\right)\right)$

$\text{Integrate}\left[\left(\frac{\hbar}{i}\right) * \left(\Psi_1{}^*[x, t]\right) * \left(D[\Psi_1[x, t], \{x, 1\}]\right), \{x, -\infty, \infty\}\right]$

Out[9]= $\text{ConditionalExpression}\left[-\frac{12}{25}\sqrt{2}\sqrt{\frac{m\,\omega}{\hbar}}\,\hbar\,\text{Sin}[t\,\omega], \text{Re}\left[\frac{m\,\omega}{\hbar}\right] > 0\right]$

Let m=ω=$\hbar$=1 and E_0= $\frac{\hbar\omega}{2}$.

In[24]:= $\text{Integrate}\left[(\psi_0[x]) * (\psi_0[x]), \left\{x, -\sqrt{\left(\frac{2*1}{2}\right)}, +\sqrt{\left(\frac{2*1}{2}\right)}\right\}\right]$

$\text{Erf}\left[\sqrt{\frac{m\,\omega}{\hbar}}\right]$ (*Reminder m=ω=$\hbar$=1*)

In[27]:= $N\left[\text{Erf}\left[\sqrt{1}\right], 6\right]$ (*where N numerical value function.*)

Out[27]= 0.842701

In[28]:= $P_c = \left(1 - N\left[\text{Erf}\left[\sqrt{1}\right], 6\right]\right)$ (* where P_c is the complement. *)

Out[28]= 0.157299