Simulation of Random Variables

Firstly, using inbuilt MATLAB routines, we find the given PDFs in the question, i.e., normal, exponential and uniform. Then using acceptance-rejection method, the PDF is generated and compared with the theoretical values.

Theoretical Means:
Norm (2,2) = 2
Exp (2) = 1/λ = 0.5
Uniform [2,4] = (a+b)/2 = 3

Theoretical Variance:
Norm (2,2) = 2
Exp (2) = $1/\lambda^2$ = 0.25
Uniform [2,4] = $(b-a)^2/12$ = 0.3333

As noticed from the various PDF plots and the MATLAB outputs we can conclude that for all cases the theoretical means and variances are extremely close to the rejection method mean and variances. We can also notice that as the sample size increases. The reject means and variance values tend to get closer and closer to the theoretical values. Further we can also notice the difference between MATLAB routine values and rejection method values decreasing as the sample size increases.

The difference in the values is due to the rejection sampling. The following is the formula used while performing Acceptance-Rejection Method:

$$f(x) \leq cg(x)$$

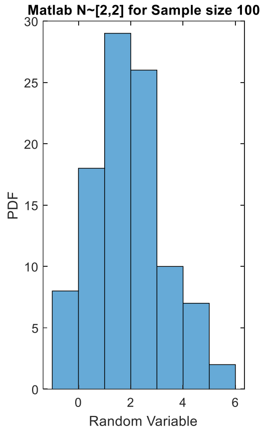
For this procedure we first select a density g(x). Further we have to find a constant c which satisfies the above criterion. For each of the cases i.e., Normal, Exponential and Uniform, the value of c will be altered in the code as per the parameters given in the PDF. Next, we create uniform random variable 'u' and another random variable 'v' from 'g'. For these parameters if the below condition is satisfied then we accept the value. Else we reject it.

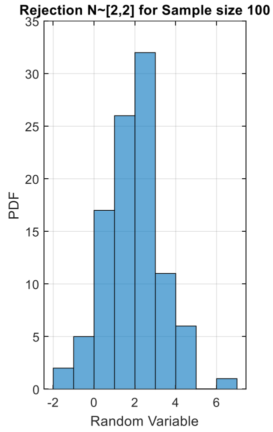
$$c * u \leq f(x)/g(x)$$

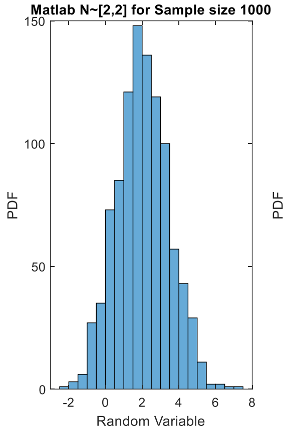
Since the acceptance of the sample is dependent on a random number generation, there is a possibility that the wrong sample could be exited. Hence as the number of samples increases T = 100, 1000, 10000 the possibility of wrongly accepted samples decreases and hence the difference between theoretical and sampled rejection decreases. Therefore, we see an improvement.

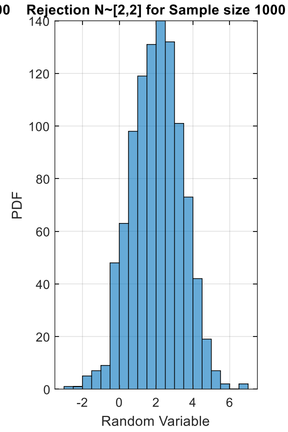
Normal Distribution~ (2,2)

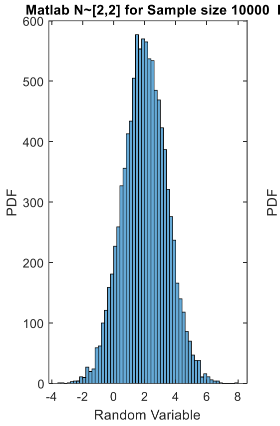
Plots:

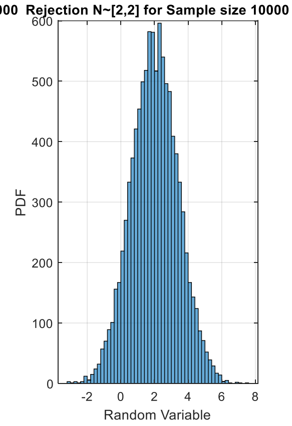












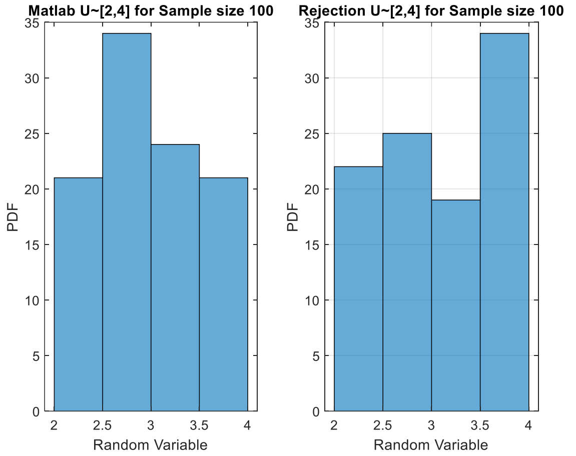
Command Window Output:

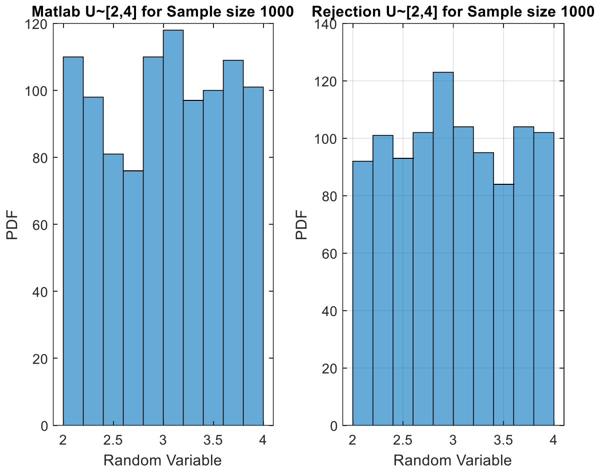
Sample size N = 100
Matlab Mean for M = 1.891865
Matlab Variance for M = 1.934688
Rejection Mean for M = 2.000617
Rejection Variance for M = 1.765717

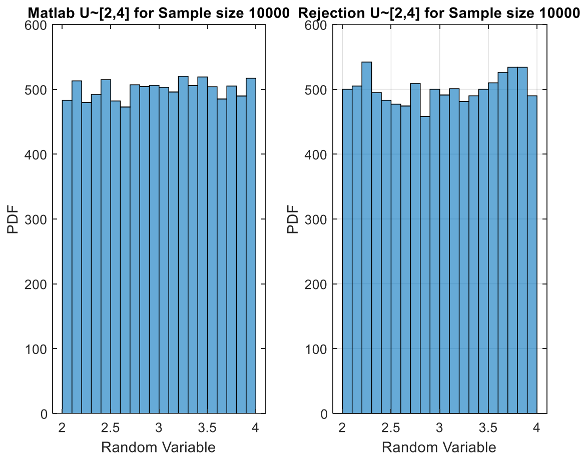
Sample size N = 1000
Matlab Mean for M = 2.032410
Matlab Variance for M = 1.996155
Rejection Mean for M = 2.035714
Rejection Variance for M = 1.875607

Sample size N = 10000
Matlab Mean for M = 2.002758
Matlab Variance for M = 1.993446
Rejection Mean for M = 2.009800
Rejection Variance for M = 1.940575

Uniform ~ [2,4]





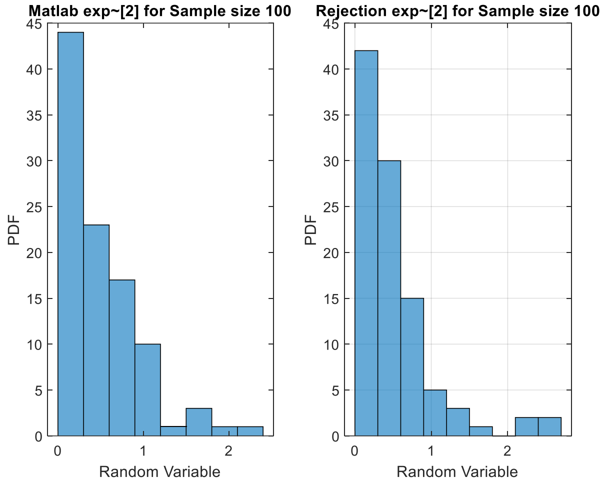


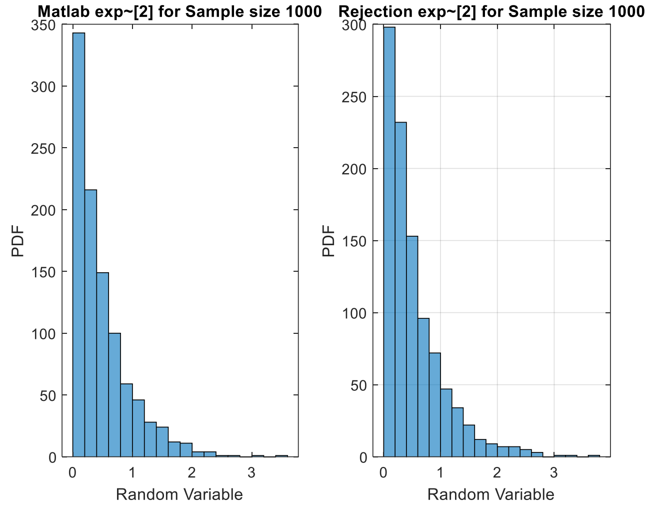
Command Window Output:
Sample size N = 100
Matlab Mean for M = 2.976303
Matlab Variance for M = 0.296751
Rejection Mean for M = 3.047399
Rejection Variance for M = 0.321389

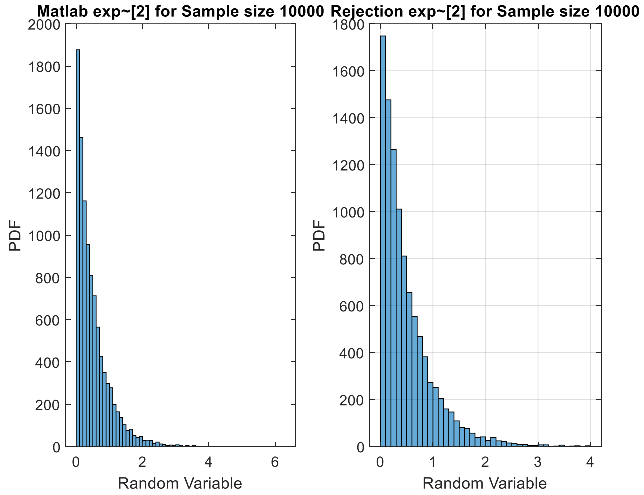
Sample size N = 1000
Matlab Mean for M = 3.018530
Matlab Variance for M = 0.339206
Rejection Mean for M = 3.002614
Rejection Variance for M = 0.325403

Sample size N = 10000
Matlab Mean for M = 3.005260
Matlab Variance for M = 0.332455
Rejection Mean for M = 3.005976
Rejection Variance for M = 0.338321

Exponential ~ (2)







Command Window Output:

Sample size N = 100
Matlab Mean for M = 0.456147
Matlab Variance for M = 0.248850
Rejection Mean for M = 0.497990
Rejection Variance for M = 0.186008

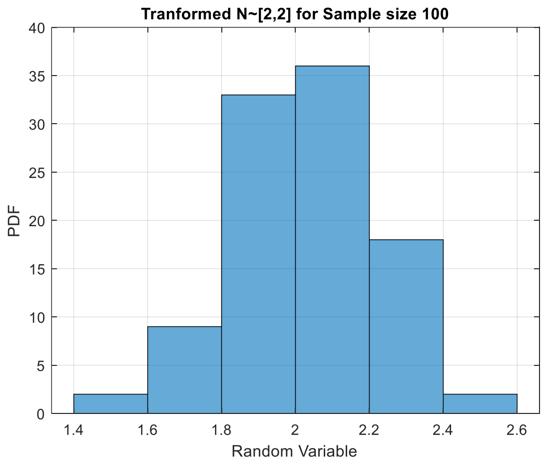
Sample size N = 1000
Matlab Mean for M = 0.483969
Matlab Variance for M = 0.233352
Rejection Mean for M = 0.513335
Rejection Variance for M = 0.272034

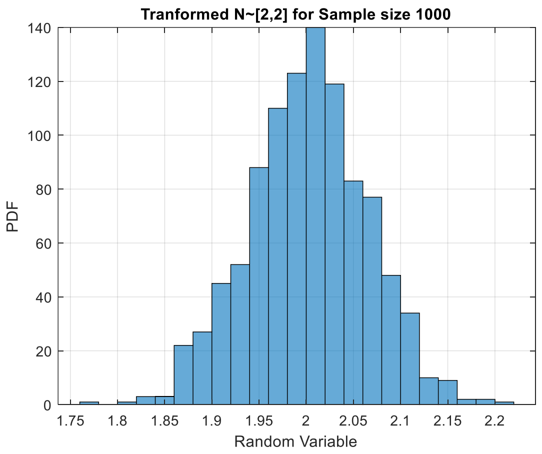
Sample size N = 10000
Matlab Mean for M = 0.504667
Matlab Variance for M = 0.262069
Rejection Mean for M = 0.502516
Rejection Variance for M = 0.245258

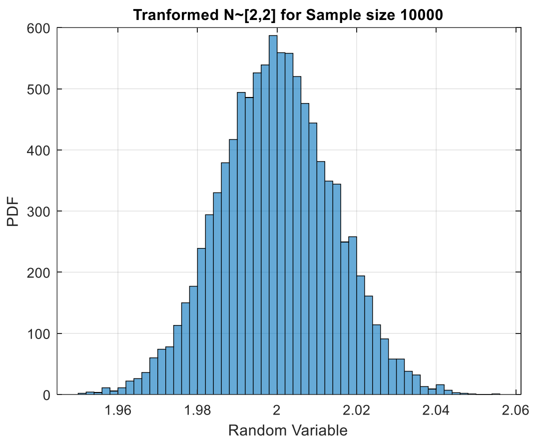
Transforming Random Variables

As per the question we are required to define a random variable $Y_i = (1/T)\Sigma_i X_l$. We have to generate PDFs for Normal distribution, Exponential and Uniform for T = 100, 1000, 10000. The task in hand is to basically perform central limit theorem. Which states that as the sample size increases, the sampling distribution of the mean of Xi which is Y can be approximated to the normal distribution. As we can notice from the below figures. For each of the following distributions, after the transformation is applied, we notice a normal distribution PDF. This can be proved by noticing that the middle of each histogram or the bin with the highest number of random variables aligns itself at the mean. For example, for Norm ~ (2,2) we notice that as the sample size increases (T) the distribution has aligned itself around 2 which is the mean for the given PDF. Similar trends can be observed for exp ~ (2) and Uniform ~ [2,4].

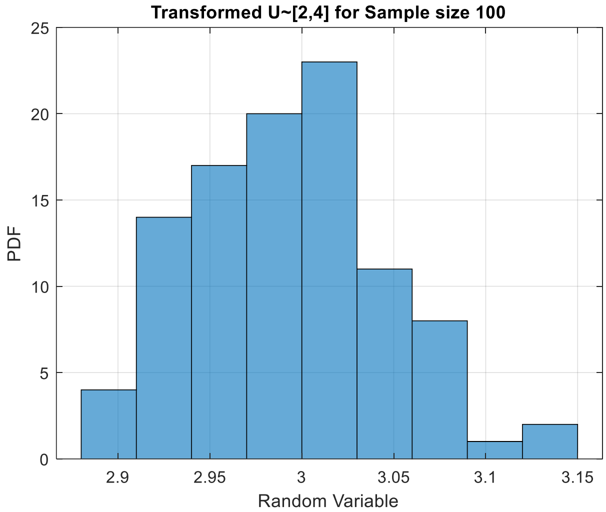
Normal Distribution~ (2,2)

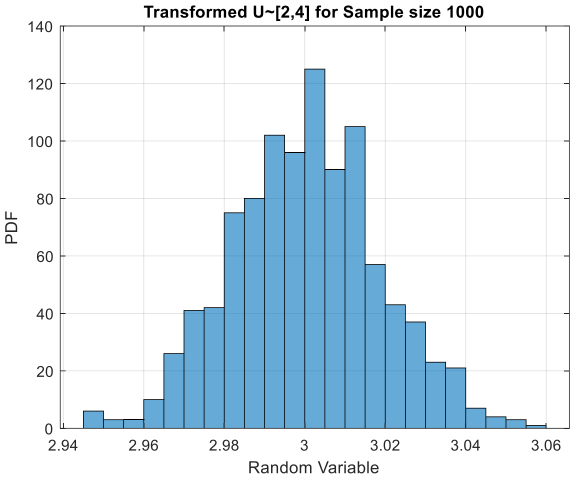


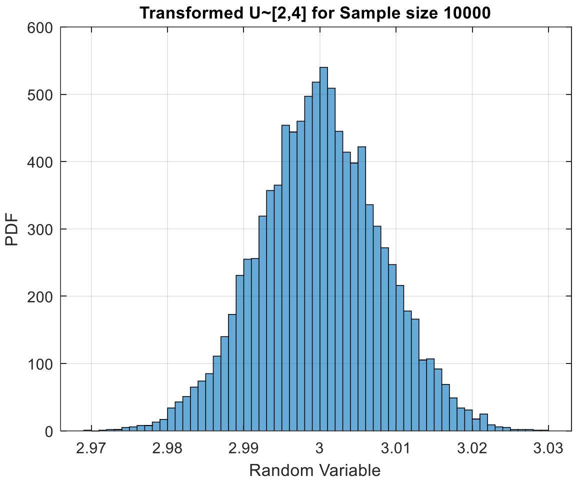




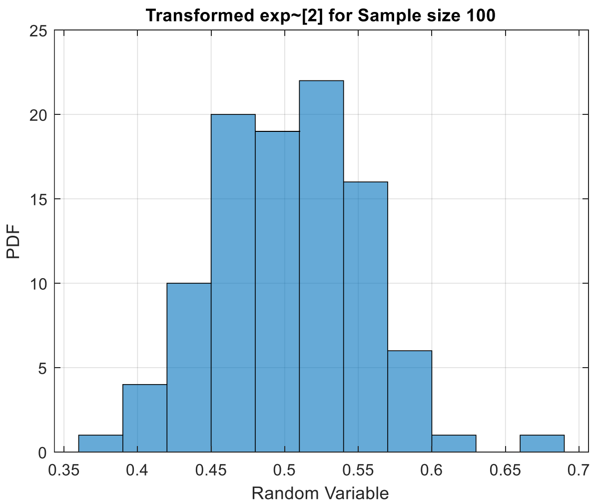
Uniform ~ [2,4]

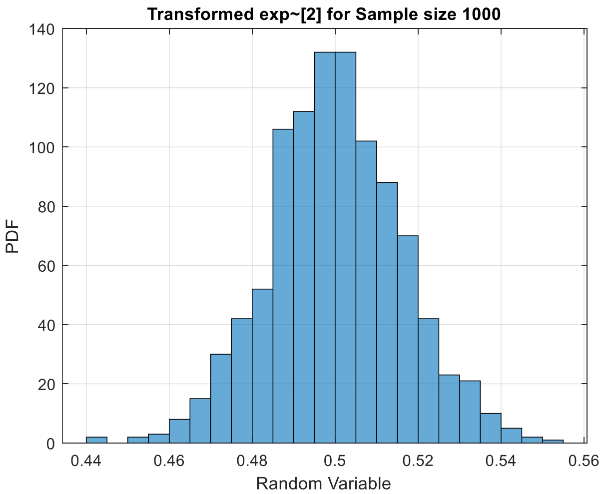


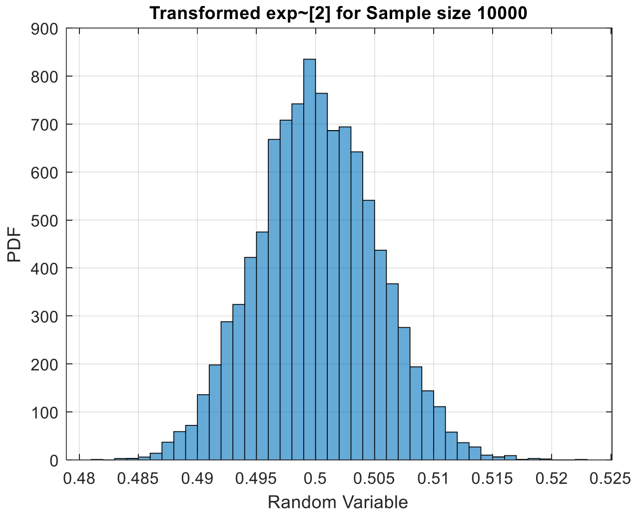




Exponential ~ (2)






Transformed exp~[2] for Sample size 10000
PDF
Random Variable
900
800
700
600
500
400
300
200
100
0
0.48
0.485
0.49
0.495
0.5
0.505
0.51
0.515
0.52
0.525

Convergence of Random Variables

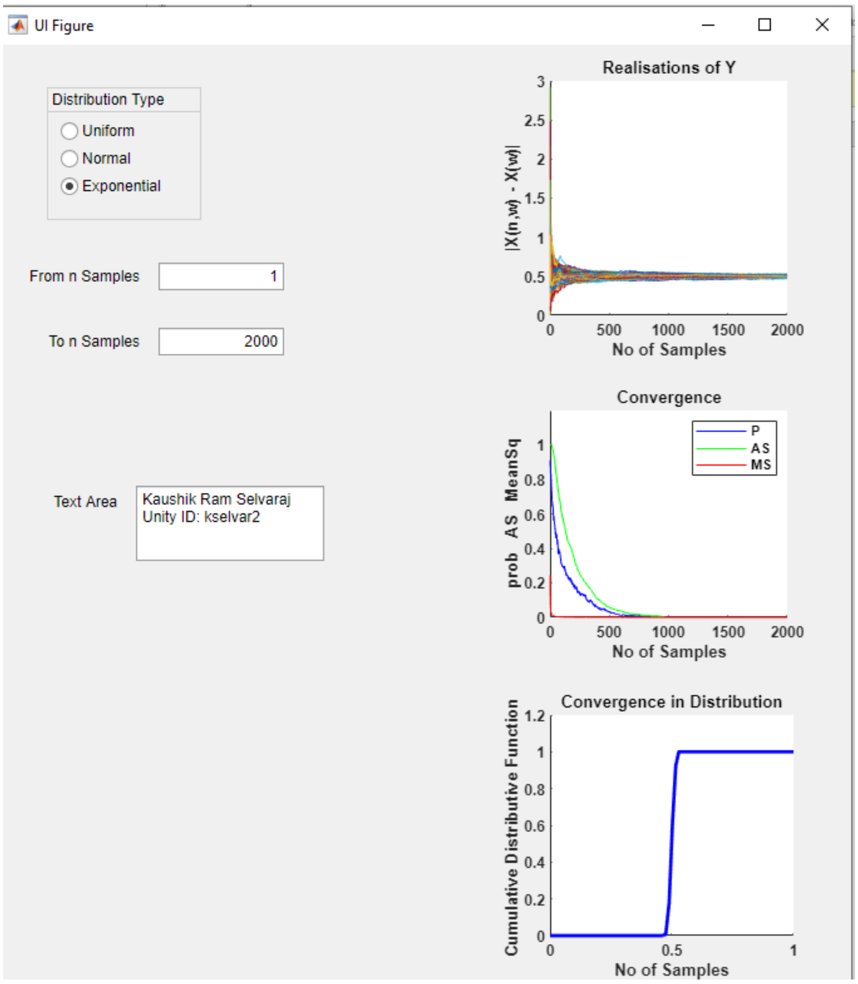
In this question we have been asked to make a GUI for plotting the various convergence. That is Convergence in Mean square, Convergence in Probability, Convergence in distribution and Almost Sure convergence. From the paper given for reference, it has been concluded that we consider sample size T = 2000 and the number of realisations as M = 500. Similar procedure as Question 2 has been carried out to Obtain Y. For the first graph to be plotted, a conclusion has been drawn from the law of large numbers that the convergence of Y for a significantly large number of random variables, the normalised mean will take the values close to the mean of the distribution used for the sample. In out case it is Normalisation, Exponential and Uniform distributions. In the below figure it can be noticed in the Realisations of Y graph that for all three distributions the normalised mean is close to the mean of the distribution.

For the GUI creation, inbuilt MATLAB GUI was utilised. With the help of Radio buttons this GUI allows the user to choose between the distribution type and also change the sampling range as per their needs. A similar method was followed in the reference paper provided in the question

GUI Screenshots

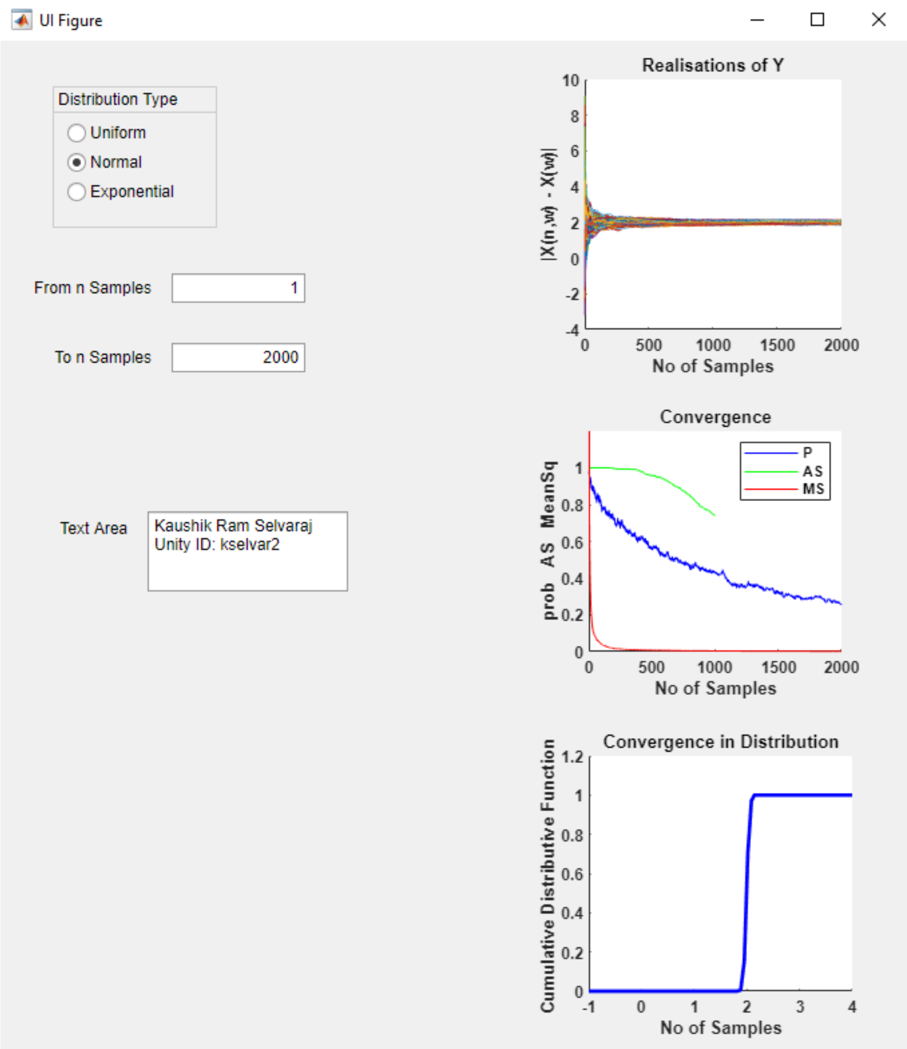
Exponential ~ (2)

First considering the Exponential Distribution. As mentioned above we can notice that as the number of samples increase the Realisations of Y converge and take close to the value of the mean of the distribution used for the samples. In this case the Mean is 0.5. We can also notice that the convergence in probability, mean square and almost sure convergence are converging towards zero and in the bottom, most graph the convergence in distribution of Y to the Exponential distribution has a mean of 0.5 which can be clearly seen and verified from the graph below.



Normal Distribution ~ (2,2)

Now considering the Normal Distribution. As mentioned above we can notice that as the number of samples increase the Realisations of Y converge and take close to the value of the mean of the distribution used for the samples. In this case the Mean is 2. We can also notice that the convergence in probability, mean square and almost sure convergence are converging towards zero and in the bottom, most graph the convergence in distribution of Y to the Normal distribution has a mean of 2 which can be clearly seen and verified from the graph below.



Uniform Distribution ~ [2,4]

Now considering the Uniform Distribution. As mentioned above we can notice that as the number of samples increase the Realisations of Y converge and take close to the value of the mean of the distribution used for the samples. In this case the Mean is 2. We can also notice that the convergence in probability, mean square and almost sure convergence are converging towards zero and in the bottom, most graph the convergence in distribution of Y to the Uniform distribution has a mean of 3 which can be clearly seen and verified from the graph below.