AMS 553.361 - Introduction to Optimization

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Introduction

Math 110.304 is one of the semi-important courses that is required/recommended for the engineering-based majors at Johns Hopkins University.

These notes are being live-TeXed, through I edot for Typos and add diagrams requiring the TikZ package separately. I am using Texpad on Mac OS X.

I would like to thank Zev Chonoles from The University of Chicago and Max Wang from Harvard University for providing me with the inspiration to start live-TeXing my notes. They also provided me the starting template for this, which can be found on their personal websites.

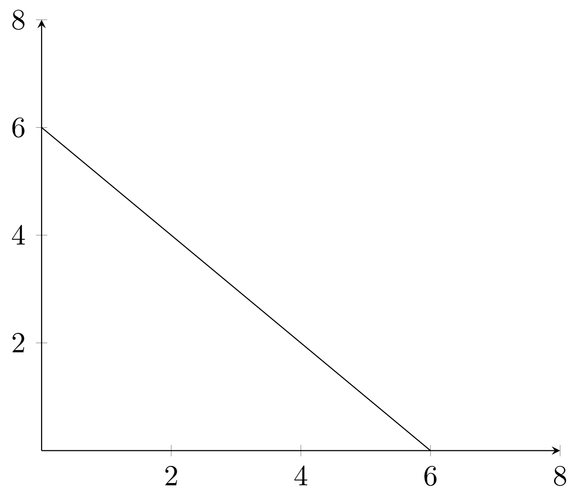
Please email any corrections or suggestions to `ksriniv4@jhu.edu`.

Lecture 1 (2018-08-30)

Introduction

Example 1. Up to 6 units of two nutrients can be added to solution and we require the number of units of nutrient 2 has to be atleast (natural) logarithm of # units of nutrient one. Goal: choose x_1 = number units of nutrient 1, choose x_2 = number units of nutrient 2. To maximize expected height of plant $1 + x_1^2(x_2-1)^3 e^{-x_1-x_2}$

$$\begin{aligned} \text{Maximise} \quad & 1 + x_1^2(x_2-1)^3 e^{-x_1-x_2} \\ & x_1 + x_2 \leq 6 \\ & x_2 > \log x_1 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{aligned}$$



generic optimization problem: say $\subseteq \mathbb{R}$

$$f : \underbrace{\mathbb{S}}_{\text{feasible region}} \rightarrow \underbrace{\mathbb{R}}_{\text{objective function}}$$

$$x^* \begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad \text{optimal}$$

however optimal objective function value is1.2677

Definition 1. We say x^* is an optimal solution if

- $x^* \in \mathbb{S}$
- For any $y \in \mathbb{S}$

Example 2. Find $\min \log x$ s.t. $-\infty \leq x \leq 7 \rightarrow$ unbounded, has no solution.

Example 3. Find $\min \log x$ s.t. $1 < x \leq 7 \rightarrow$ bounded, but is also no solution. (as we can go 1.000001)

Example 4. Find min $\log x$ s.t. $x > 1$, $x \leq 0.5 \rightarrow$ infeasible!!!

Example 5. minimize $3 + (x-1)^2$ s.t. $1 \leq x \leq 3 \rightarrow$ feasible as optimal solution $x^* = 2$ is an interior part of feasible region.

$$
\begin{aligned}
f(x) &= 3 + (x-1)^2 \\
f'(x) &= 0 \\
f''(x) &> 0
\end{aligned}
$$

Example 6. min $3 + (x-2)^2$ s.t. $x \geq 10 \rightarrow$ optimal solution is $x^* = 10$. but $f'(x) \neq 0$. But is not an interior point - it is a boundary point of feasible region.