AMS 553.430 - Introduction to Statistics

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Introduction

Math 553.430 is one of the most important courses that is required/recommended for the engineering-based majors at Johns Hopkins University.

These notes are being live-TeXed, through I edot for Typos and add diagrams requiring the TikZ package separately. I am using Texpad on Mac OS X.

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Please email any corrections or suggestions to ksriniv4@jhu.edu.

Lecture 0 (2018-08-30)

Introduction to Probability (553.420) Review

Part 1 - Counting

1. Multiplication rule (Basic Counting Principle)
2. Combinations/Permutations
   - Sampling with or without replacement. $\Rightarrow$ Inclusion-Exclusion Principle
3. Birthday Problem
4. Matching Problem (inclusion-exclusion principle)
5. n balls going into m boxes (all are distinguishable)
6. Multinomial Coefficients e.g. assign A, B, C, D, to different students $\rightarrow$ anagram problem
7. Pairing Problem

$$2n \text{ people, paired up} \begin{cases} \text{ordered: } \binom{2n}{2,2,\cdots,2} & \text{e.g. different courts for players} \\ \\ \text{unordered: } \frac{\binom{2n}{2,2,\cdots,2}}{n!} \end{cases}$$

8. Partition of integers $\rightarrow \binom{n}{n+k-1}$ where n is the sum of integer and k is the number of partitions

Basics of Probability

<u>Axioms</u>

1. $0 \leq P(A) \leq 1, \forall A$
2. $P(\Omega) = 1 \rightarrow$ where Ω is the sample space
3. Countable additivity
   - if $A_1, \cdots, A_n$ are mutually exclusive, then

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = P(A_1) + P(A_2) + \cdots = \sum_{i=1}^{\infty} P(A_i)$$

$\Rightarrow P(A) = 1 - P(A^c)$
$P(A) = \dfrac{|A|}{|\Omega|}$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Bayes Rule

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{\sum_j P(B|C_j)P(C_j)} \qquad \underbrace{\bigcup_j C_j = \Omega}_{\text{partition of } \Omega}$$

Law of Total Probability

$$P(A) = \sum_j P(A|B_j)P(B_j) = \sum P(A \cap B_j) \qquad \underbrace{\bigcup_j B_j = \Omega}_{\text{partition of } \Omega}$$

Part 2 - Discrete and Continuous Random Variables

Function	**Discrete**	**Continuous**
Probability Function	PMF: $P(X = x)$	PDF: $f_x(x)$
Probability Distribution	$\sum_x P(X = x) = 1$	$\int_x f_x(x)dx = 1$
Expectation	$E[X] = \sum_x xP(X = x)$	E[X] = $\int_x xf(x)dx$
Variance	$Var[X] = E[X^2] - (E[X])^2$	$Var[X] = E[X^2] - (E[X])^2$

Law of the Unconscious Statistician (LOTUS)

1-dim $\quad E[g(x)] = \sum_x g(x)P(X = x) \Big/ E[g(x)] = \int_x g(x)f(x)dx$

2-dim $\quad E[g(X,Y)] = \sum_y \sum_x g(x,y)P(X = x, Y = y) \Big/ E[g(X,Y)] = \int_y \int_x g(x,y)f(x,y)dxdy$

Special notes on Discrete Distributions

Binomial Distribution

A sum of i.i.d. (identical, independent distribution) Bernoulli(p) R.V.

- Approximation method $\Rightarrow$ if n is large, p very small and $np < 10$.
  - use Poisson (np), otherwise preferably $p \approx \frac{1}{2}$
  - use Normal $(np, np(1-p))$
- Mode:
  - if $(n+1)p$ integer, mode = (n+1)p or (n+1)p - 1.
  - if $(n+1)p \notin \mathbb{Z}$ mode is $\lfloor (n+1)p \rfloor$
  - **Proof:** consider $\dfrac{P(X = x)}{P(X = x-1)}$ going below 1.

Negative Binomial

A sum of i.i.d Geometric(p) R.V.
■ a^{th} head before b^{th} tail

Example 1. A coin has probability p to land on a head, $q = 1 - p$ to land on a tail.
$P[5^{th}$tail occurs before the 10^{th} head]?