EE 115 Homework 2

1) (**40 points**) Let $x(t) = 3(\sin(100\pi t) + \sin(200\pi t))\cos(1000\pi t)$. Also assume that $h(t)$ has the Fourier transform $H(f) = \texttt{rect}(f/220) = \begin{cases} 1, & |f| < 110; \\ 0, & |f| > 110. \end{cases}$ Recall the *modulation theory* learned in classes and simplify the following expressions of signals:

   a) $y_1(t) = h(t) * (x(t)\cos(1000\pi t))$
   b) $y_2(t) = h(t) * (x(t)\cos(1000\pi t + \pi/4)$
   c) $y_3(t) = h(t) * (x(t)\sin(1000\pi t))$
   d) $y_4(t) = h(t) * (x(t)\cos(1010\pi t))$

   where $*$ denotes the continuous-time convolution. **Hint:** $x(t)$ here can be expressed as $m(t)\cos(1000\pi t)$ where $m(t)$ is a message signal and $\cos(1000\pi t)$ is the carrier signal with the carrier frequency equal to 500. You can apply the frequency-domain method as follows: determine and sketch the Fourier transform $M(f)$ of $m(t)$; determine and sketch the Fourier transform $X(f) = \frac{1}{2}M(f-500) + \frac{1}{2}M(f+500)$ of $x(t)$; then, *for example*, $x(t)\cos(1000\pi t + \phi)$ has the Fourier transform $V(f) \doteq \frac{e^{j\phi}}{2}X(f-500) + \frac{e^{-j\phi}}{2}X(f+500)$ which can be sketched as well; then determine $Y(f) = H(f)V(f)$ which simply selects the frequency components within $|f| < 110$; finally take the inverse Fourier transform of $Y(f)$ to get $y(t)$.

2) (**40 points**) Let $m(t) = \sum_{k=-\infty}^{\infty}(\texttt{rect}(t-2k) - \texttt{rect}(t-2k-1))$ and $x(t) = (m(t) + \alpha)\cos(2\pi f_c t)$. Let $y(t)$ be the output of an ideal *envelope detector* driven by $x(t)$. Determine and sketch $y(t)$ for each of the following cases:

   a) $\alpha = 0$
   b) $\alpha = 0.5$
   c) $\alpha = 1$
   d) $\alpha = 1.5$

   Hint: If $e(t)\cos(2\pi f_c t)$ is the input to an ideal envelope detector, then the output is $|e(t)|$.

3) (**30 points**) Let a message waveform be $m(t) = \sin(20\pi t) + 2\sin(30\pi t)$ and its AM modulated signal be $x_{AM}(t) = 10(m(t) + 3)\cos(100\pi t)$.

   a) Determine and sketch the *amplitude spectrum* of $x_{AM}(t)$.
   b) Determine the AM *modulation index* assuming $\min_t m(t) = -3$.
   c) Determine the *power efficiency* of this AM signal.