EE110B - Signals and Systems
Winter 2025

Lab 6

In this lab, we will explore what is known as first- and second-order infinite impulse response (IIR) filters.

Task 1: Consider the LTI system

$$y[n] - ay[n-1] = (1-a)x[n]$$

with some real $a < 1$.

a) Find the transfer function $H(e^{j\omega})$ of this system.
b) Plot $20\log_{10}|H(e^{j\omega})|$ in the interval $-\pi \leq \omega \leq \pi$ for $a = 0.1$, $a = 0.3$, $a = 0.7$, $a = 0.9$, and $a = 0.99$ on the same plot. Do you always observe a low-pass filter? What characteristic of the filter seem to change?
c) This log-plot is said to be in dB (decibel) scale. On the same plot, also draw a line corresponding to -3dB. The interval of ω for which $20\log_{10}|H(e^{j\omega})|$ is above this line is commonly accepted as the **passband** of the filter, and the length of the interval is referred to as the **bandwidth**. For each value of a above, determine (by estimating from the plot) the bandwidth.

Task 2: Now consider the system

$$y[n] - 2r\cos(\theta)y[n-1] + r^2y[n-2] = x[n]$$

with some $0 < r < 1$, and $0 \leq \theta \leq \pi$.

a) Find the transfer function $H(e^{j\omega})$ of this system.
b) Plot $20\log_{10}|H(e^{j\omega})|$ in the interval $-\pi \leq \omega \leq \pi$ for $r = 0.5$ and $\theta = 0$, $\theta = 0.25\pi$, $\theta = 0.5\pi$, $\theta = 0.8\pi$, $\theta = \pi$. How does the filter change this time?
c) The passband and bandwidth is determined with respect to the maximum of $20\log_{10}|H(e^{j\omega})|$ (which can always be brought to 0dB by multiplying $x[n]$ in the difference equation with an appropriate constant). For each value of θ above, determine the passband and the bandwidth.