EE 115 Homework 1

1) (**30 points**) Consider a periodic message signal $m(t) = m(t+T)$ which satisfies

$$m(t) = \begin{cases} 3 & 0 \le t < \frac{T}{3} \\ -3 & -\frac{T}{3} \le t < 0 \\ 0 & \frac{T}{3} \le t < \frac{T}{2} \\ 0 & -\frac{T}{2} \le t < -\frac{T}{3} \end{cases} \tag{1}$$

For the above $m(t)$, determine the following:

a) The mean of $m(t)$, i.e., $\lim_{\tau\to\infty} \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} m(t)dt$; the energy of $m(t)$ within $|t| < T/2$; and the average power of $m(t)$ for $-\infty < t < \infty$.

b) Simplify the expression of the coefficients $c_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} m(t)e^{-j2\pi\frac{k}{T}t}dt$ in the Fourier series expansion $m(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi\frac{k}{T}t}$. What is the value of c_0, and Why? Is c_k for any k real, imaginary or complex, and why? Is c_k an even or odd function of k, and why?

c) Sketch the real and imaginary parts of the Fourier transform $M(f)$ of $m(t)$ for $|f| < \frac{6}{T}$.

2) (**30 points**) Determine the complex envelope of each of the following signals with respect to $\cos(2\pi f_c t)$:

a) $a(t)\cos(2\pi f_c t + \theta(t)) + b(t)\cos(2\pi f_c t + \phi(t))$.

b) $a(t)\cos(2\pi f_c t) - b(t)\sin(2\pi f_c t) + c(t)\sin(2\pi f_c t + \phi(t))$.

c) $a(t)\sin(2\pi(f_c + \Delta)t + \theta(t))$.

3) (**40 points**) Consider the DSB-SC signal $u(t) = m(t)\cos(500\pi t)$ where $m(t) = \sin(10\pi t) + 2\cos(20\pi t)$. Determine the following:

a) The spectrum $M(f)$ of $m(t)$, and the spectrum $U(f)$ of $u(t)$.

b) Sketch each of $|U(f)|$, $Re(U(f))$ and $Im(U(f))$ versus f.

c) The demodulator of the DSB-SC signal starts with a mixer which produces $y(t) = u(t)\cos(500\pi t)$. Then you can apply a lowpass filter to $y(t)$ to obtain $m(t)$. Determine the frequency response of the required lowpass filter.