UNIVERSITY OF CALIFORNIA, RIVERSIDE
Department of Electrical Engineering
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EE110B-SIGNALS AND SYSTEMS
HOMEWORK 1 SOLUTIONS

Problem 1: Determine whether or not each of the following signals is periodic. If the signal is periodic, specify its fundamental period.
a) $x[n] = \cos(0.9\pi n)$
b) $x[n] = \cos(\pi^2 n)$
c) $x[n] = e^{j\pi 0.16 n}$
d) $x[n] = e^{j\pi\sqrt{2}n}$

Solution: We know that $x[n]$ is periodic if and only if there exists an N such that

$$x[n+N] = x[n] \ .$$

But we also know that it suffices to check if $\frac{\omega}{\pi}$ is a rational number for any sine wave. That means only the signals in (a) and (c) are periodic. Still, to double check and gain experience, let us go through each case:
a) $x[n+N] = x[n]$ translates to

$$\cos(0.9\pi(n+N)) = \cos(0.9\pi n)$$

which is the same as

$$\cos(0.9\pi n + 0.9\pi N) = \cos(0.9\pi n) \ .$$

But this is satisfied if and only if $0.9\pi N = k2\pi$ for some integer k, or equivalently, if and only if

$$\frac{k}{N} = \frac{9}{20} \ .$$

Choosing $k = 9$ and $N = 20$ satisfies the last equality. Thus, $N = 20$ is a period. In fact, it is the *fundamental* period, because there cannot be a smaller N accompanied by an integer k such that $\frac{k}{N} = \frac{9}{20}$.

b) We need to find an N such that

$$\cos(\pi^2(n+N)) = \cos(\pi^2 n)$$

which is the same as $\pi^2 N = k2\pi$ for some integer k. But that means

$$\frac{k}{N} = \frac{\pi}{2} \ .$$

Since there can be no such (k, N) pair, $x[n]$ is not periodic.

c) For $x[n]$ to be periodic, we need

$$e^{j\pi 0.16[n+N]} = e^{j\pi 0.16 n}$$

which simplifies to

$$e^{j\pi 0.16 N} = 1 \ .$$

In other words, there must exist k such that $\pi 0.16 N = k2\pi$, or

$$\frac{k}{N} = \frac{8}{100} \ .$$

That means $N = 100$ is a period. But it is not the fundamental period, because $\frac{8}{100} = \frac{2}{25}$ and $N = 25$ is the smallest period.

d) For this $x[n]$,

$$e^{j\pi\sqrt{2}[n+N]} = e^{j\pi\sqrt{2}n}$$

or equivalently

$$e^{j\pi\sqrt{2}N} = 1 \ .$$

That, in turn, is possible if and only if $\pi\sqrt{2}N = k2\pi$ which is the same as

$$\frac{k}{N} = \frac{\sqrt{2}}{2} \ .$$

Since that is impossible, $x[n]$ is not periodic.

Problem 2: Consider the system given by

$$y[n] = x[n] - 0.005n\, x[n-2] \ .$$

Determine whether this system is linear, time-invariant, causal, and stable.

Solution:
Linearity: If $y_1[n]$ and $y_2[n]$ are the outputs to inputs $x_1[n]$ and $x_2[n]$, respectively, i.e.,

$$\begin{aligned} y_1[n] &= x_1[n] - 0.005n\, x_1[n-2] \\ y_2[n] &= x_2[n] - 0.005n\, x_2[n-2] \ , \end{aligned}$$

then we need to check whether $ax_1[n] + bx_2[n]$ yields $ay_1[n] + by_2[n]$ for any a and b. Indeed, the output to $ax_1[n] + bx_2[n]$ is

$$\begin{aligned} y'[n] &= ax_1[n] + bx_2[n] - 0.005n(ax_1[n-2] + bx_2[n-2]) \\ &= ax_1[n] - 0.005n\, ax_1[n-2] + bx_2[n] - 0.005n\, bx_2[n-2] \\ &= ay_1[n] + by_2[n] \ . \end{aligned}$$

So the system is linear.

Time invariance: For time invariance, if $x[n]$ yields $y[n]$, then $x[n-n_0]$ should yield $y[n-n_0]$. However, the output to $x[n-n_0]$ is given by

$$\begin{aligned} y'[n] &= x[n-n_0] - 0.005n\, x[n-n_0-2] \\ &= x[n-n_0] - 0.005(n-n_0+n_0)\, x[n-n_0-2] \\ &= x[n-n_0] - 0.005(n-n_0)\, x[n-n_0-2] - 0.005n_0 x[n-n_0-2] \\ &= y[n-n_0] - 0.005n_0 x[n-n_0-2] \ . \end{aligned}$$

So, the system is time-varying.

Causality: Does $y[n]$ depend only on $x[n], x[n-1], x[n-2], \ldots$ but not on $x[n+1], x[n+2], \ldots$? Yes, and more specifically, it only depends on $x[n]$ and $x[n-2]$. So the system is causal.

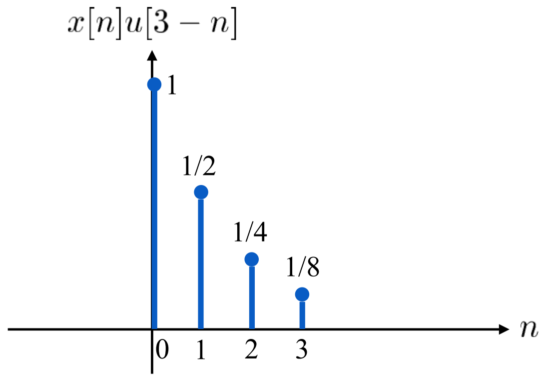
Stability: The system is not stable, because a bounded input as simple as $x[n] = 1$ creates an output which is given by

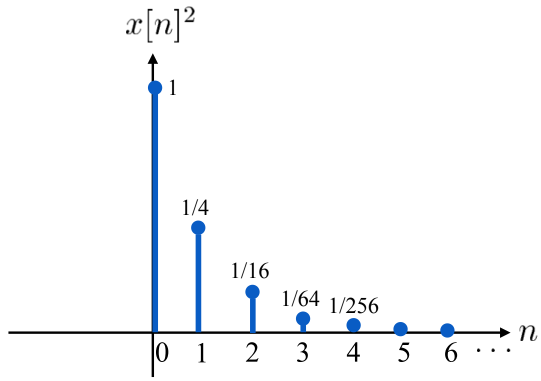
$$y[n] = 1 - 0.005n$$

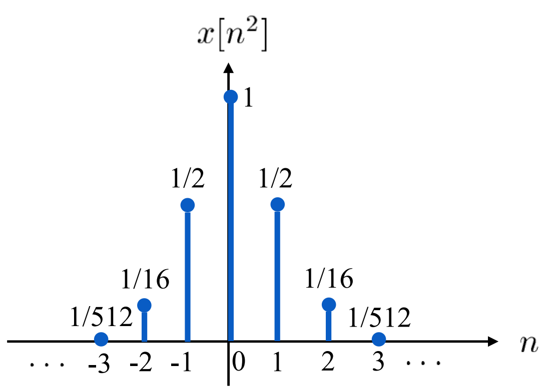
which goes to $-\infty$ when n goes to ∞.

Problem 3: Let $x[n] = 2^{-n}u[n]$. Carefully sketch the following $y[n]$:
a) $y[n] = x[n]u[3-n]$
b) $y[n] = x[n]^2$
c) $y[n] = x[n^2]$

Solution:







Problem 4: Determine whether the following transformations are invertible. If it is, express $x[n]$ in terms of $y[n]$.
a) $y[n] = nx[n]$
b) $y[n] = \begin{cases} x[n-1] & n \geq 1 \\ 0 & n = 0 \\ x[n] & n \leq -1 \end{cases}$
c) $y[n] = \begin{cases} x[n/2] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$

d) $y[n] = x[n]x[n-1]$.

Solution:
a) Not invertible, because $x[n] = \delta[n]$ and $x[n] = 2\delta[n]$ yield the same output, which is $y[n] = 0$.
b) Invertible because

$$x[n] = \begin{cases} y[n+1] & n \geq 0 \\ y[n] & n \leq -1 \end{cases}$$

c) Invertible, because $x[n] = y[2n]$.
d) Not invertible, because $x[n] = \delta[n]$ and $x[n] = 2\delta[n]$ yield the same output, which is $y[n] = 0$.