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Department of Electrical Engineering

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EE 110B SIGNALS AND SYSTEMS
MIDTERM SOLUTIONS

Question 1)
a) We have

$$\begin{aligned} y[n]-y[n-1] &= \sum_{k=0}^{n}\alpha^k - \sum_{k=0}^{n-1}\alpha^k \\ &= (1+\alpha+\alpha^2+\ldots+\alpha^{n-1}+\alpha^n)-(1+\alpha+\alpha^2+\ldots+\alpha^{n-2}+\alpha^{n-1}) \\ &= \alpha^n\ . \end{aligned}$$

b) For the difference equation $y[n]-y[n-1]=\alpha^n$, we will try the particular solution $y_p[n]=K\alpha^n$ and homogeneous solution $y_h[n]=cr^n$. Substituting the particular solution, we obtain

$$K\alpha^n - K\alpha^{n-1} = \alpha^n$$

which can be simplified to

$$K(\alpha-1)=\alpha$$

and thus $K=\frac{\alpha}{\alpha-1}$. As for the homogeneous solution, we have

$$cr^n - cr^{n-1} = 0$$

which is equivalent to

$$c(r-1)=0$$

or $r=1$. Bringing all of this together,

$$\begin{aligned} y[n] &= y_p[n]+y_h[n] \\ &= K\alpha^n + cr^n \\ &= \frac{\alpha}{\alpha-1}\alpha^n + c1^n \\ &= \frac{\alpha^{n+1}}{\alpha-1}+c\ . \end{aligned}$$

Finally, we need to choose c so that $y[0]=1$:

$$\frac{\alpha}{\alpha-1}+c=1\ .$$

In other words,

$$c = 1-\frac{\alpha}{\alpha-1} = -\frac{1}{\alpha-1}\ .$$

In other words,

$$y[n]=\sum_{k=0}^{n}\alpha^k = \frac{\alpha^{n+1}}{\alpha-1}-\frac{1}{\alpha-1}=\frac{\alpha^{n+1}-1}{\alpha-1}\ .$$

c) Similar to part **a**, we can write

$$\begin{aligned}
y[n] - \alpha y[n-1] &= \sum_{k=0}^{n} \alpha^k - \alpha \sum_{k=0}^{n-1} \alpha^k \\
&= (1 + \alpha + \alpha^2 + \ldots + \alpha^{n-1} + \alpha^n) - \alpha(1 + \alpha + \alpha^2 + \ldots + \alpha^{n-2} + \alpha^{n-1}) \\
&= (1 + \alpha + \alpha^2 + \ldots + \alpha^{n-1} + \alpha^n) - (\alpha + \alpha^2 + \ldots + \alpha^{n-1} + \alpha^n) \\
&= 1\ .
\end{aligned}$$

d) This time we will try the particular solution $y_p[n] = K$ and homogeneous solution $y_h[n] = cr^n$ for this new difference equation. Substituting the particular solution, we obtain

$$K - \alpha K = 1$$

or $K = \frac{1}{1-\alpha}$. For the homogenous part,

$$cr^n - \alpha c r^{n-1} = 0$$

which is equivalent to

$$c(r - \alpha) = 0$$

or $r = \alpha$. Bringing all of this together,

$$\begin{aligned}
y[n] &= y_p[n] + y_h[n] \\
&= K + cr^n \\
&= \frac{1}{1-\alpha} + c\alpha^n\ .
\end{aligned}$$

Setting $y[0] = 1$:

$$1 = \frac{1}{1-\alpha} + c$$

or

$$c = 1 - \frac{1}{1-\alpha} = \frac{-\alpha}{1-\alpha}\ .$$

Therefore,

$$y[n] = \sum_{k=0}^{n} \alpha^k = \frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}\alpha^n = \frac{1-\alpha^{n+1}}{1-\alpha} = \frac{\alpha^{n+1}-1}{\alpha-1}\ .$$

Question 2) We have

$$x_1[n] - x_2[n+1] = \delta[n]\ .$$

Therefore, from the linearity and the time invariance of the system, we must also have

$$y_1[n] - y_2[n+1] = h[n]\ .$$

Since $y_1[n] = 0$, we have

$$h[n] = -y_2[n+1]\ .$$

We can either plot $h[n]$, or write

$$h[n] = \begin{cases} 1 & \text{if } n \bmod 3 = 0 \\ -1 & \text{if } n \bmod 3 = 1 \\ 0 & \text{if } n \bmod 3 = 2 \end{cases}$$