University of California at Riverside

Department of Electrical Engineering

EE 110A: Signals and Systems

Final Exam (Dec 8th, 2012)

Question	MAX	GRADE
1	X	
2	X	
3	X	
4	X	
5	X	
6	X	
TOTAL	100	

Surname **First name**

Student ID Number

Time: 150 Minutes

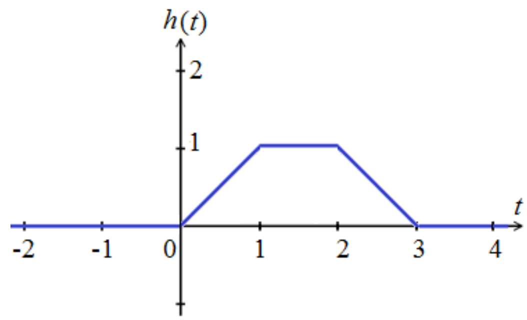
Instructions:

- **Attempt all questions.**
- **One page of one-sided hand-written letter-size information sheet is allowed.**
- **No other reference material is allowed.**
- **No calculator of any kind is allowed.**
- **All answers should be written in booklet.**
- **In addition to final answers, you need to include necessary steps to show your derivations.**
- **Enjoy your exam, and *Good Luck!***

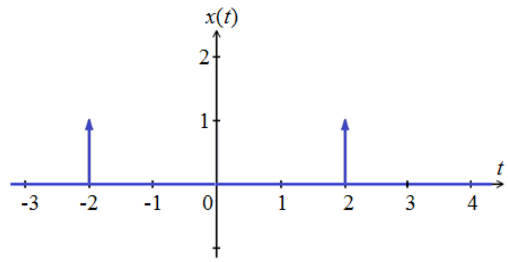
Instructor's Name: Dr. Hamed Mohsenian-Rad

<u>Question One</u>:

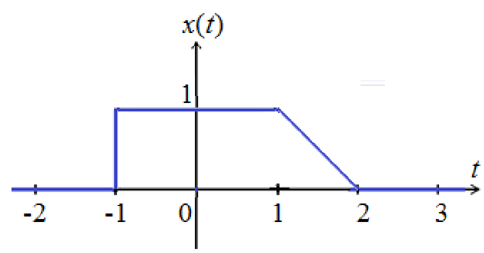
Part 1) Consider an LTI system with unit impulse response $h(t)$:



Plot the output of this system to the following input?



Part 2) Given the following signal, sketch signal $y(t) = x(1-t)u(t)$.

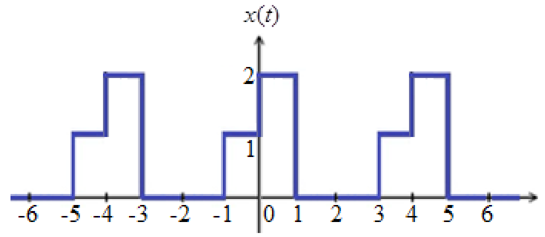


<u>Question Two</u>:

Part 1) A continuous-time periodic signal $x(t)$ is real-valued and has a fundamental period $T = 4$. The nonzero Fourier series coefficients for $x(t)$ are $a_0 = 2$, $a_2 = a_{-2} = 4$, and $a_4 = a_{-4} = 5$.

Express $x(t)$ in form of $x(t) = \sum_{k=-\infty}^{\infty} A_k \cos(\omega_k t + \phi_k)$.

Part 2) Determine the Fourier series representation, and its coefficients a_k (i.e., for $k = 0$ and $k \neq 0$) of the following periodic signal:

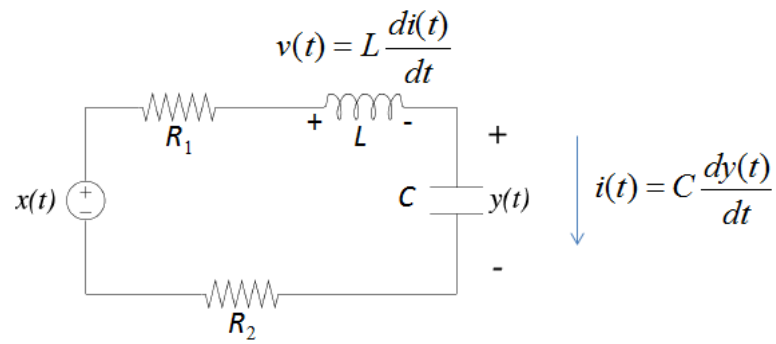


Question Three:

Part 1) Let $X(j\omega)$ be the Fourier Transform of signal $x(t)$. Determine the Fourier Transform of signal $y(t) = x(1-2t) - x(-2-t) + 2x(t+1)$ in terms of $x(j\omega)$.

Part 2) Find the signal $x(t)$ that has Fourier Transform $X(j\omega) = 2\delta(\omega - 2) - 5\delta(\omega + 3) + \delta(\omega)$.

<u>Question Five</u>: Consider the following RLC circuit with two resistors, one capacitor, and one inductor. The input signal is $x(t)$ and the output signal is $y(t)$. The parameters are $R_1 = R_2 = 2$, $L = 1$, $C = \frac{1}{3}$.



Part 1) Obtain the transfer function of the system.

Part 2) Identify the zeros and poles of the transfer function.

Part 3) Is the system stable? Justify your answer.

Part 4) Assume that $h(t)$ denotes unit impulse response and $s(t)$ denote the unit step response for this system. Calculate the following signal values:

$h(0^+) =$

$s(+\infty) =$

<u>Question Four</u>: Consider an LTI system with the following Transfer Function: $H(s) = \dfrac{1}{s+1}$.

Part 1) What is the output of this system to an **aperiodic** signal $x(t)$ with Fourier Transform:

$$X(j\omega) = \frac{1}{2+j\omega}?$$

Part 2) What is the output a **periodic** signal $x(t)$ with Fourier series representation:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\pi t}, \qquad a_k = \begin{cases} 1 & k = 0 \\ 0.5 & k = 1,-1 \\ 0 & otherwise \end{cases}.$$

Question Four: Assume that signal $y(t) = x(t) * h(t)$, where $x(t) = e^{-2t}u(t)$ and $h(t) = e^{-(t-3)}u(t-3)$.
Part 1) Find the Fourier Transform $Y(j\omega)$ for signal $y(t)$.

Part 2) Find an expression for signal $y(t)$ itself.
[**Hint**: You can either use the results in Part 1 or directly calculate the convolution integral].

Appendix: The following Laplace Transformation table may assist you:

- $\mathcal{L}[a_1x_1(t) + a_2x_2(t)] = a_1\mathcal{L}[x_1(t)] + a_2\mathcal{L}[x_2(t)]$
- $\mathcal{L}[x(t-t_0)] = e^{-st_0}\mathcal{L}[x(t)]$
- $\mathcal{L}[e^{s_0t}x(t)] = X_L(s-s_0)$
- $\mathcal{L}[x(at)] = \frac{1}{|a|}X_L(\frac{s}{a})$
- $\mathcal{L}[x^*(t)] = X_L^*(s^*)$
- $\mathcal{L}[x(t) * y(t)] = X_L(s)Y_L(s)$
- $\mathcal{L}[\frac{d}{dt}x(t)] = sX_L(s)$
- $\mathcal{L}[tx(t)] = -\frac{dX_L(s)}{ds}$
- $\mathcal{L}[\int_{-\infty}^{t} x(\tau)d\tau] = \frac{1}{s}X_L(s)$