UNIVERSITY OF CALIFORNIA, RIVERSIDE
Department of Electrical Engineering
WINTER 2025
EE110B-SIGNALS AND SYSTEMS
HOMEWORK 7

Please turn in before Friday, March 14th, 2025, 11:59pm

Problem 1: Consider the signal

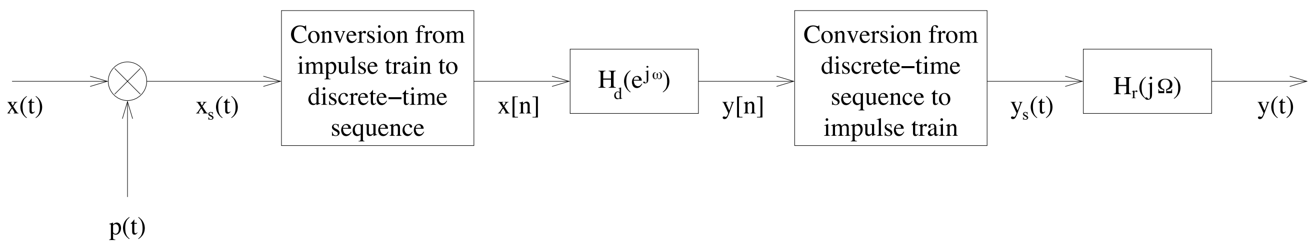
$$x_c(t) = \left(\frac{\sin(\pi t)}{t}\right)^2$$

which is discretized by sampling in time via

$$x_d[n] = x_c(nT) .$$

a) What is the bandwidth Ω_M of this signal?
Hint: $x_c(t)$ is nothing but multiplication of a sinc function with itself.
b) What is the maximum value of T so that $x_c(t)$ can be recovered from $x_d[n]$?
c) If we use $T = 1$, and try to reconstruct $x_c(t)$ from $x_d[n]$ using the standard ideal filter of bandwidth $\frac{\pi}{T}$, what do we get as a result?

Problem 2: Consider the standard block diagram for discrete-time processing of a band-limited continuous-time signal $x(t)$:



Here, $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$, and $X_c(j\Omega) = 0$ for $|\Omega| > \frac{\pi}{T}$. So, if $H_d(e^{j\omega}) = 1$, and if $H_r(j\Omega)$ is an ideal reconstruction filter, we have $y(t) = x(t)$. However, instead of the ideal reconstruction filter, suppose that we have at our disposal

$$H_r(j\Omega) = \frac{T}{1 + j\frac{3T}{\pi}\Omega}$$

which can be implemented easily using first-order linear circuitry. The potential problems caused by using this instead of an ideal filter are, of course, (i) the filter may not efficiently eliminate the replicas of $X_c(j\Omega)$ in $X_s(j\Omega)$, and (ii) the shape of $X_c(j\Omega)$ is distorted by the reconstruction filter. Therefore, in general we will have $y(t) \neq x(t)$.

(a) Sketch both the ideal reconstruction filter and $|H_r(j\Omega)|$, and discuss why the first problem mentioned above is not very severe.
(b) To eliminate the second problem, design a discrete-time filter $H_d(e^{j\omega})$. Sketch its magnitude.
(c) Find the corresponding discrete-time impulse response $h[n]$ by inverting $H_d(e^{j\omega})$ using

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

Caution: Treat $n = 0$ and $n \neq 0$ separately in the above integral.