EE 115 Lab 5

Consider the FM signal

$$u_{FM}(t) = \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau) \tag{1}$$

where f_c is a (large) radio carrier frequency and $m(t)$ is a message signal. The bandwidth of the FM signal is approximately equal to

$$W = 2B_m(\beta + 1) \tag{2}$$

where B_m is the bandwidth of $m(t)$ and $\beta = \frac{\Delta f_{max}}{B_m} = \frac{k_f \max_t |m(t)|}{B_m}$ (FM modulation index).

If $m(t) = \cos(2\pi f_m t)$, we can let $B_m = f_m$ and $\beta = \frac{k_f}{f_m}$. Furthermore, it can be shown that the complex envelope of $u_{FM}(t)$ is

$$u(t) = e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi f_m n t} \tag{3}$$

and its bandwidth is approximately equal to $B_u = K f_m = K B_m$ if $J_n(\beta)$ for $|n| > K$ can be all neglected. Here $J_n(\beta)$ is the Bessel function of the first find of order n.

Let

$$u_K(t) = \sum_{n=-K}^{K} J_n(\beta) e^{j2\pi f_m n T}. \tag{4}$$

Then the power of $u_K(t)$ is

$$P_{u_K} = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} |u_K(t)|^2 dt = J_0^2(\beta) + 2\sum_{n=1}^{K} J_n^2(\beta). \tag{5}$$

Since the power of $u(t) = e^{j\beta \sin(2\pi f_m t)}$ is

$$P_u = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} |u(t)|^2 dt = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} 1 \cdot dt = 1, \tag{6}$$

then $\lim_{K\to\infty} P_{u_K} = J_0^2(\beta) + 2\sum_{n=1}^{\infty} J_n^2(\beta) = 1$.

1) Compute and plot "$1 - P_{u_K}$ versus K" for various β. To show details of $1 - P_{u_K}$ when it is small, also plot "$10\log_{10}(1 - P_{u_K})$ versus K".
2) Explain why $1 - P_{u_K}$ is the power of the error function $u(t) - u_K(t)$.
3) Explain why the bandwidth of $u(t)$ is approximately equal to $B_u = f_m(\beta + 1)$.