UNIVERSITY OF CALIFORNIA, RIVERSIDE
Department of Electrical Engineering

EE 110B SIGNALS AND SYSTEMS
WINTER 2024 MIDTERM SOLUTIONS

Question 1)

a) The homogenous solution must satisfy

$$y_h[n] - y_h[n-1] = 0 \ .$$

Guessing the typical solution kr^n, we obtain

$$kr^n - kr^{n-1} = 0 \ ,$$

which simplifies to $r = 1$, and therefore

$$y_h[n] = k \ .$$

b) Following the hint, let

$$y_p[n] = an^3 + bn^2 + cn$$

and substitute into the difference equation:

$$\begin{aligned}
n^2 &= an^3 + bn^2 + cn - a(n-1)^3 - b(n-1)^2 - c(n-1) \\
&= an^3 + bn^2 + cn - a(n^3 - 3n^2 + 3n - 1) - b(n^2 - 2n + 1) - c(n-1) \\
&= a(3n^2 - 3n + 1) + b(2n - 1) + c \\
&= 3an^2 + (2b - 3a)n + c - b + a
\end{aligned}$$

indicating that $a = \frac{1}{3}$, $b = \frac{1}{2}$, and $c = \frac{1}{6}$. Therefore,

$$y_p[n] = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n \ .$$

c) We can bring the two together and write

$$\begin{aligned}
y[n] &= y_h[n] + y_p[n] \\
&= \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n + k \ .
\end{aligned}$$

Noting that $y[0] = 0$, we obtain $k = 0$, and therefore,

$$\begin{aligned}
y[n] &= \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n \\
&= \frac{2n^3 + 3n^2 + n}{6} \\
&= \frac{n(2n^2 + 3n + 1)}{6} \\
&= \frac{n(n+1)(2n+1)}{6} \ .
\end{aligned}$$

Question 2)
Since this is a linear and time-invariant system, for any a and b, it must output

$$ay_1[n] + by_2[n]$$

when the input is

$$ax_1[n] + bx_2[n] \ .$$

Taking $a = b = 0.5$, one can observe that

$$0.5x_1[n] + 0.5x_2[n] = \delta[n] \ .$$

Therefore, by definition of the *impulse response*, the output to $0.5x_1[n] + 0.5x_2[n]$ will be nothing but $h[n]$. In other words,

$$h[n] = 0.5y_1[n] + 0.5y_2[n] = \begin{cases} 1 & n = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

Question 3)
The key is to observe that

$$y[n] = 2x[n] + x[n-1] + x[n+1] \ .$$

Using this relation, one can write

$$\begin{aligned} b_k &= 2a_k + a_k e^{-jk\omega_0} + a_k e^{jk\omega_0} \\ &= 2a_k \left(1 + \frac{e^{-jk\omega_0} + e^{jk\omega_0}}{2}\right) \\ &= 2a_k \left(1 + \cos(k\omega_0)\right) \\ &= \frac{1}{5}\left(1 + \cos\left(\frac{k\pi}{5}\right)\right) \ . \end{aligned}$$