

Midterm

Instructions:

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- You have an hour and twenty minutes to solve this exam.
- There are a total of three questions on 7, one-sided pages, including this page. You can write on both sides of the page, and ask for extra sheets if needed. The last page has formulas and equations that you may find useful.
- Each question has multiple parts that will help you get to the final answer, as well as give you partial credit. Answer all questions, and with as many details as possible. Simplify all your answer to the maximum possible extent.

Points:

Problem 1	8
Problem 2	8
Problem 3	32

Problem 1 [30pts]: Let a system be given as

$$y(t) = x(r(t))$$

where $r(\cdot)$ is the ramp function, i.e. $r(t) = tu(t)$.

- Determine whether the system is linear.
- Determine whether the system is time-invariant.
- Determine whether the system is memoryless.
- Determine whether the system is causal.
- Determine whether the system is stable.
- Show that the system is not invertible.

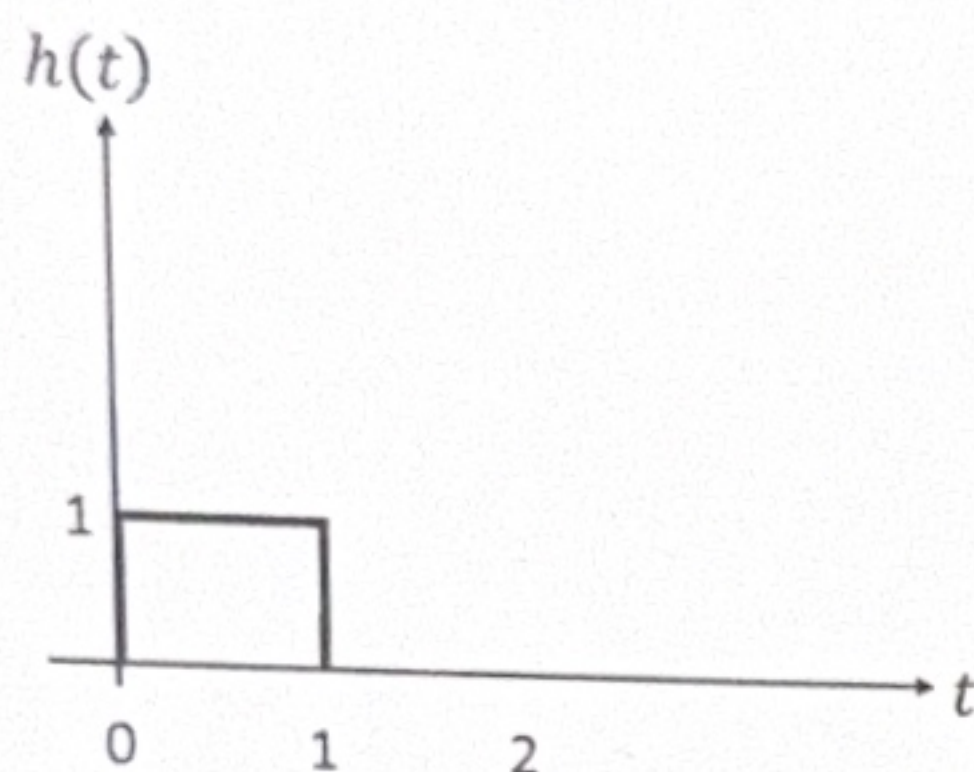
Hint: It will be helpful to draw the output for some arbitrary input.

- a) The system is NOT Linear ~~X~~ since

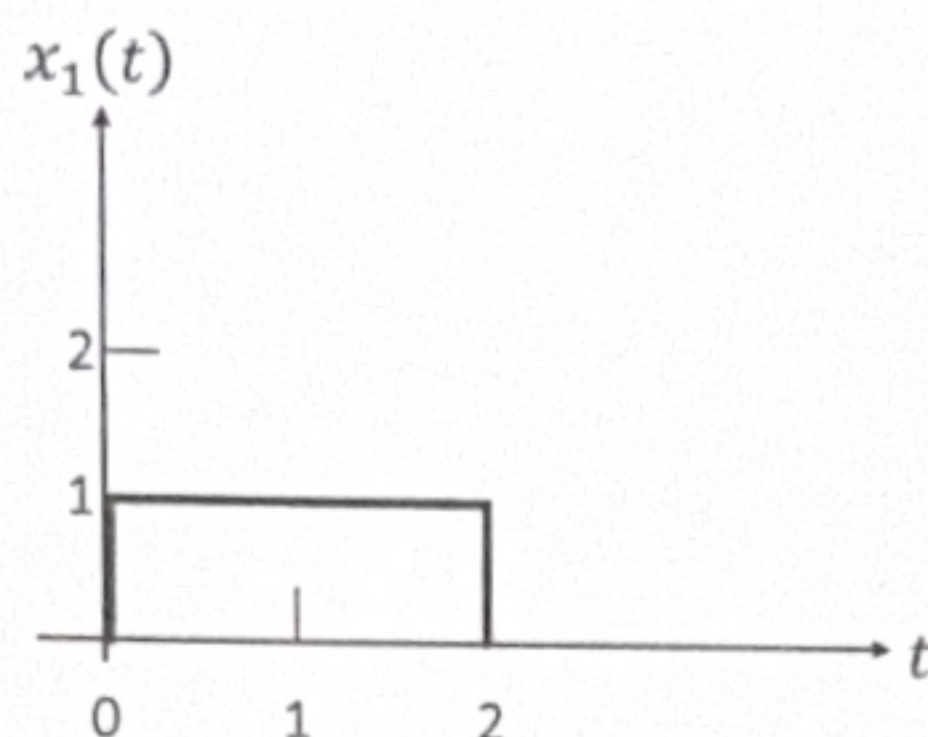
$$\begin{aligned} x_1(t) &\rightarrow x_1(t_1 u(t)) \rightarrow a x_1(t_1 u(t)) + b x_2(t_2 u(t)) \\ x_2(t) &\rightarrow x_2(t_2 u(t)) \end{aligned}$$
and the output is being scaled by t given its a ramp function
- b) The system is NOT ~~X~~ time-invariant since,

$$x(t) = x(t u(t)) \Rightarrow x(t - t_0) = x(t - t_0 u(t - t_0))$$
and the system is impacted by the time scaling of $t - t_0$ within $x(t)$ and also the unit step response of $u(t - t_0)$
- c) The system IS ~~X~~ memoryless since $y(t)$ at time t depends only on $x(t)$ at that instant. It doesn't refer to any past or future inputs for the output.
- d) The system IS ~~X~~ causal since it is memory-less & that it doesn't refer to any past inputs as well.
- e) The system is not ~~X~~ stable given that its a ramp function, it would be increasing linearly.
- f) its not invertible given that if $t < 0$, and that $r(t) = tu(t)$, any two distinct values of inputs would always result in 0.

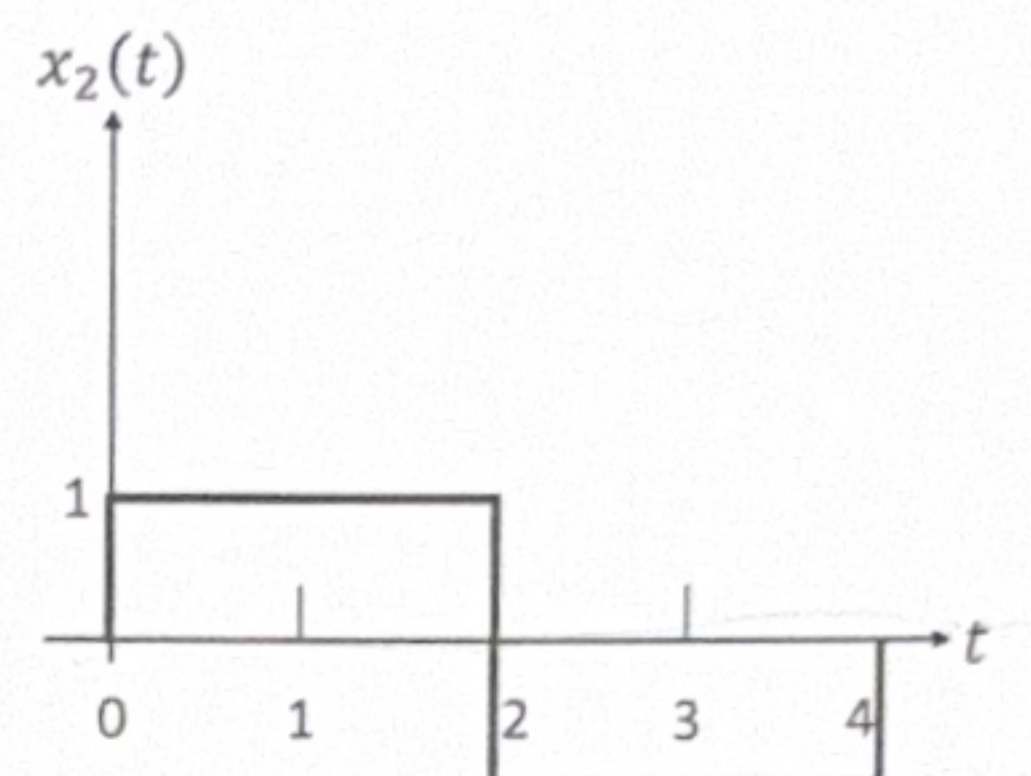
Problem 2 [30pts]: Consider impulse response shown in Fig. 1 (a) below. Answer the following questions.



(a)



(b)

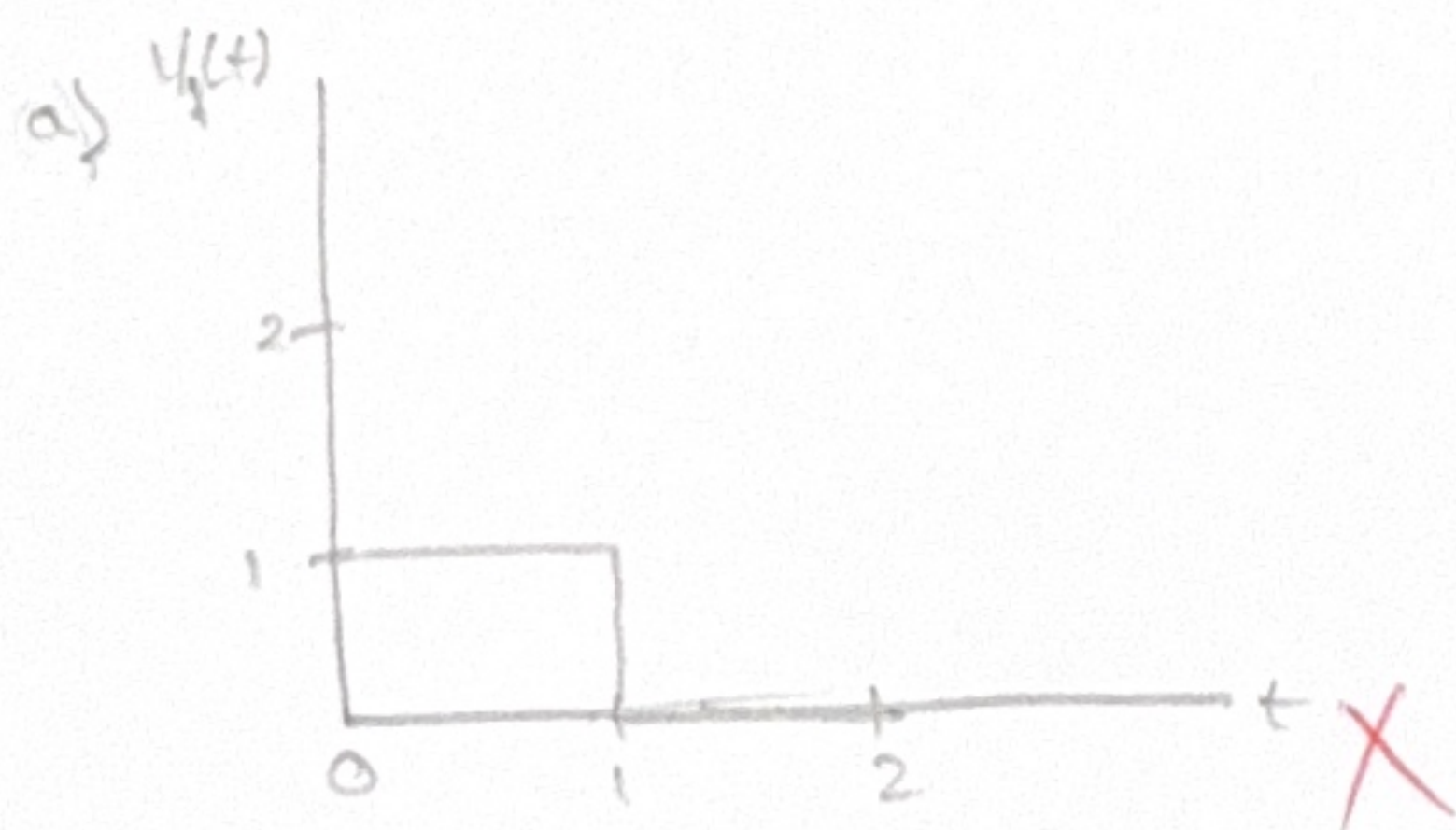


(c)

Figure 1: Plots for problem 3.

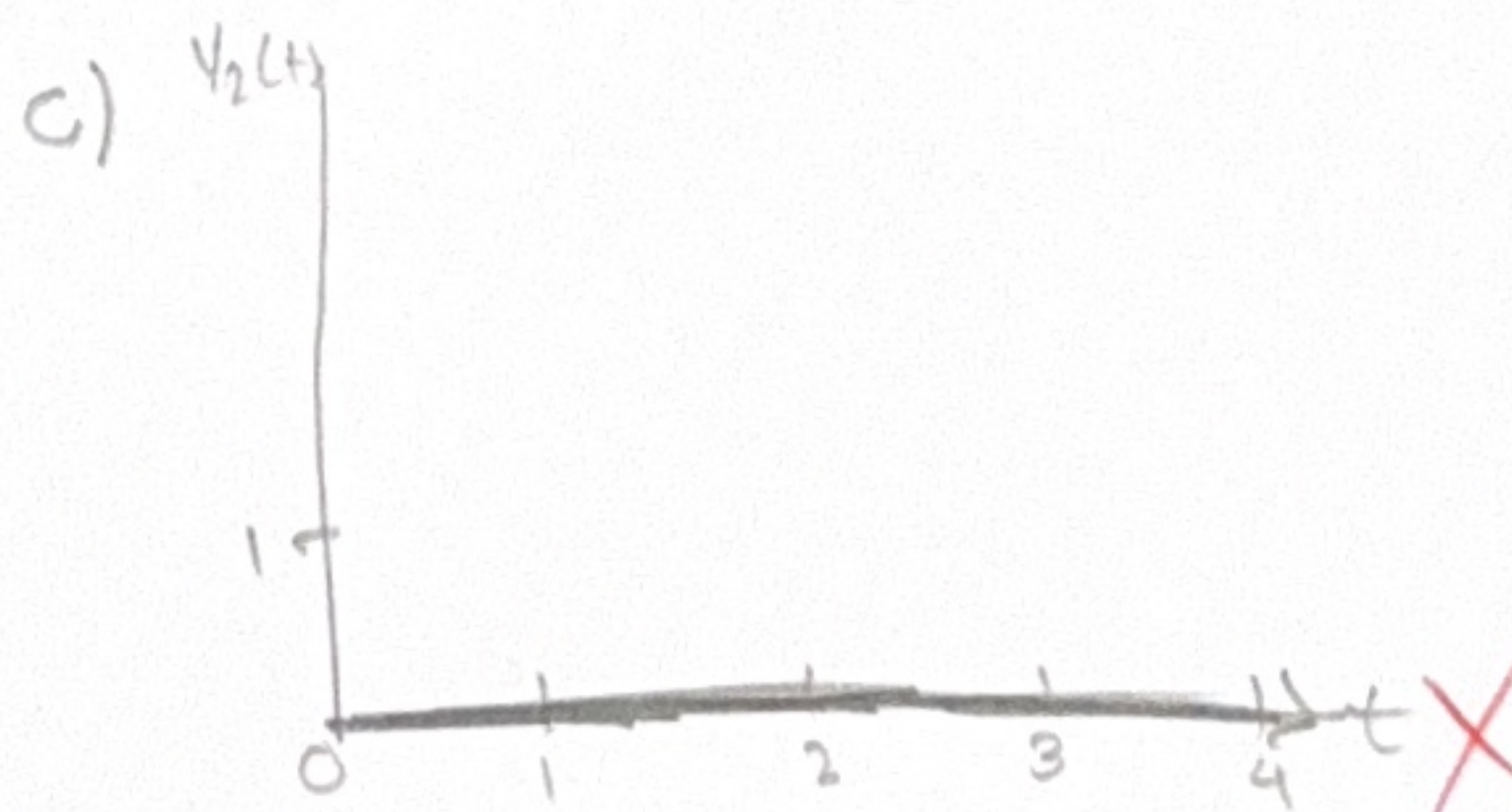
- Evaluate and plot the convolution output $y_1(t) = x_1(t) * h(t)$, where $x_1(t)$ is shown in Fig. 1 (b).
- Write down $x_2(t)$ in Fig. 1(c) in terms of $x_1(t)$.
- Now evaluate and plot the convolution output $y_2(t) = x_2(t) * h(t)$, where $x_2(t)$ is shown in Fig. 1 (c).

Hint: Use the linearity and time invariance of convolution to simplify your solution to this problem.



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$$b) \quad x_2(t) = \frac{(x_1(t) - x_1(t-2))h(t)}{y_2(t)} \quad \text{X} \quad -10$$



(it would be zero)

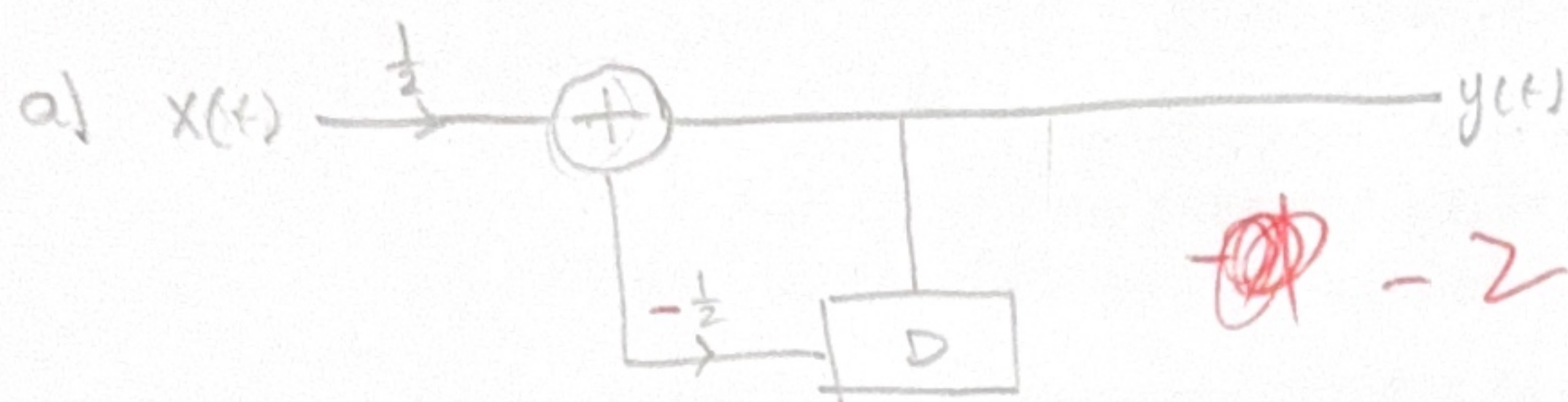
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Problem 3 [40pts]: Consider a causal LTI system given by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t).$$

Answer the following questions.

- [15 pts] Draw the block diagram for this LTI system. Mark all parts clearly.
- [15 pts] Evaluate the impulse response $h(t)$ of this system.
- [10 pts] Draw the impulse response $h(t)$.



$$y(t) = \frac{1}{2} \left(x(t) - \frac{dy(t)}{dt} \right)$$

$$y(t) = \frac{1}{2} x(t) - \frac{dy(t)}{2dt}$$

b)

$$\frac{dh(t)}{dt} + 2h(t) = \delta(t)$$

for $t > 0$ $h(t) = ce^{-t}u(t)$

$$\int_{-\infty}^{0^+} \frac{dh(t)}{dt} + 2h(t) = \delta(t)$$

$$\int_{-\infty}^{0^+} h(t) + 2H(t) = u(t)$$

for $t > 0$

$$h(t) = 0 \text{ for } t < 0$$

$$h(0^+) = 1$$

