

# **EE-110A: Signals and Systems**

## **Final Review**

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**Course materials courtesy of Prof. Ertem Tuncel**

# Final exam

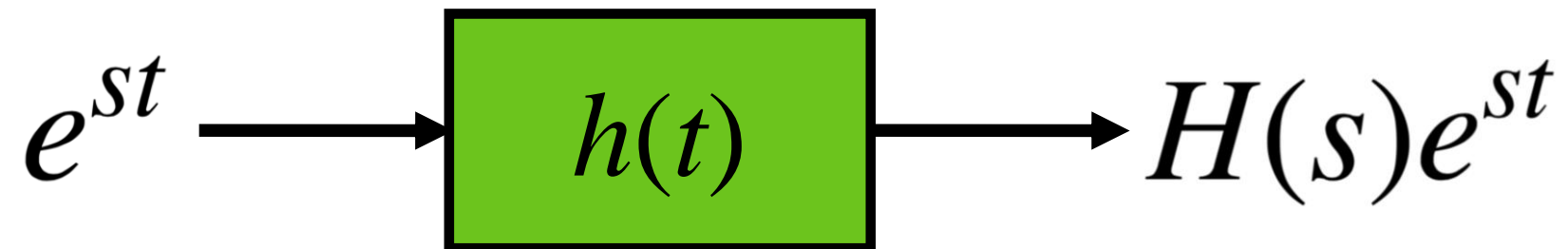
- *Emphasis* on
  - Laplace transform
- Content prior to the midterm exam is also useful!
  - e.g., convolution, Fourier transform definition

# The Laplace transform

- The Laplace transform is then defined as

$$H(s) = \int_{-\infty}^{\infty} h(t)e^{-st} dt$$

- For any  $s$ , we have



- If we specialize this to  $s = j\Omega$ , we get back CTFT:

$$H(j\Omega) = \int_{-\infty}^{\infty} h(t)e^{-j\Omega t} dt$$

# Partial fraction expansion

- Example: Find  $x(t)$  if

$$X(s) = \frac{s^2}{s^2 + 1} \quad \text{with ROC: } \operatorname{Re}\{s\} > 0$$

$$X(s) = 1 - \left( \frac{A}{s - j} + \frac{B}{s + j} \right) \quad A = -0.5j \quad B = 0.5j$$

- Therefore, the solution must be given by

$$\begin{aligned} x(t) &= \delta(t) + 0.5je^{jt}u(t) - 0.5je^{-jt}u(t) \\ &= \delta(t) - \frac{e^{jt} - e^{-jt}}{2j} u(t) = \delta(t) - \sin(t)u(t) \end{aligned}$$

# Partial fraction expansion

- Example: Find  $x(t)$  if

$$X(s) = \frac{s + 2}{s^2 + 7s + 12} \quad \text{ROC: } \text{Re}(s) > 3$$

# System properties re-revisited

- We can tell from the Laplace transform and its ROC whether the LTI system is causal, stable, and even invertible.
  - **Causality:** We already saw that the ROC has to be of the form  $\operatorname{Re}\{s\} > \alpha$  and must include  $s = \infty$ .
  - **Stability:** Recall that if the system is stable,

$$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$

# System properties re-revisited

- **Stability:** Recall that if the system is stable,

$$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$

But we also have

$$\left| \int_{-\infty}^{\infty} h(t) e^{-j\Omega t} dt \right| \leq \int_{-\infty}^{\infty} |h(t) e^{-j\Omega t}| dt = \int_{-\infty}^{\infty} |h(t)| dt < \infty$$

Hence, stability implies the existence of CTFT

Stability  $\implies$  ROC includes the imaginary axis

It can be shown that  $\Longleftarrow$  is also true.

# System properties re-revisited

- So, what does it take to have both causality and stability?
  - All poles must be on the left side of the imaginary axis (including the hidden ones)

- Example: The following is causal and stable.

$$H(s) = \frac{s - 1}{(s + 2)(s + 3)} \quad \text{with ROC: } \operatorname{Re}\{s\} > -2$$

- Example: The following is stable but not causal.

$$H(s) = \frac{(s - 1)s^2}{(s + 1)(s + 5)} \quad \text{with ROC: } \operatorname{Re}\{s\} > -1$$



# System properties re-revisited

- **Invertibility:** At first, it looks like every LTI system with non-empty ROC is invertible.

- The inverse is simply  $G(s) = \frac{1}{H(s)}$

- However, for practicality, we need both the system and its inverse to be causal and stable.
- This implies that not only all poles, but also all zeros must be on the left side of the imaginary axis (including the hidden ones).

# System properties re-revisited

$$H(s) = \frac{s + 3}{(s + 1)(s + 5)} \quad \text{with ROC: } \operatorname{Re}\{s\} > -1$$

- There is still a hidden zero at  $s = \infty$  so the inverse won't be causal.

- Example:

$$H(s) = \frac{(s + 3)(s + 2)}{(s + 1)(s + 5)} \quad \text{with ROC: } \operatorname{Re}\{s\} > -1$$

- This one has a causal and stable inverse.