University of California Riverside, Department of ECE
EE110A: Signals and Systems
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Homework 1 solutions

Problem 1 [4pts]: Consider the complex-valued exponential signal

$$x(t) = Ae^{\alpha t + j\omega t}$$

a) Evaluate the real and imaginary components of $x(t)$.

b) Evaluate the odd and even components of $x(t)$. (Odd and even parts for complex functions are obtained in exactly the same way that they are obtained for real functions.)

Solution:

Here we note that j is an imaginary number and that A and α are real-valued variables.

a)

$$Ae^{\alpha t + jwt} = Ae^{\alpha t}e^{jwt}$$

Using Euler's decomposition formula:

$$\begin{aligned} e^{jwt} &= \cos(wt) + j\sin(wt) \\ Ae^{\alpha t + jwt} &= Ae^{\alpha t}(\cos(wt) + j\sin(wt)) \\ Ae^{\alpha t + jwt} &= (Ae^{\alpha t}\cos(wt)) + j(Ae^{\alpha t}\sin(wt)) \end{aligned}$$

Real part is $(Ae^{\alpha t}\cos(wt))$
Imaginary part is $(Ae^{\alpha t}\sin(wt))$.

b) In order to decompose into even and odd parts we use

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

Here, $x(t) = Ae^{\alpha t + jwt}$
Using the derivation in part (a):

$$x_e(t) = \frac{Ae^{\alpha t + jwt} + Ae^{-\alpha t - jwt}}{2}$$

$$x_e(t) = \frac{(Ae^{\alpha t}\cos(wt)) + j(Ae^{\alpha t}\sin(wt)) + (Ae^{-\alpha t}\cos(wt)) - j(Ae^{-\alpha t}\sin(wt))}{2}$$

$$x_e(t) = \frac{((Ae^{\alpha t} + Ae^{-\alpha t})\cos(wt)) + ((jAe^{\alpha t} - jAe^{-\alpha t})sin(wt))}{2}$$

$$x_o(t) = \frac{Ae^{\alpha t + jwt} - Ae^{-\alpha t - jwt}}{2}$$

$$x_o(t) = \frac{(Ae^{\alpha t}\cos(wt)) + j(Ae^{\alpha t}\sin(wt)) - (Ae^{-\alpha t}\cos(wt)) - j(Ae^{-\alpha t}\sin(wt))}{2}$$

$$x_o(t) = \frac{((Ae^{\alpha t} - Ae^{-\alpha t})\cos(wt)) + ((jAe^{\alpha t} + jAe^{-\alpha t})sin(wt))}{2}$$

Problem 2 [8pts]: Let $x(t) = e^{-2t}u(t)$. Carefully sketch the following $y(t)$:
a) $y(t) = x(t)u(3-2t)$
b) $y(t) = x(t)^2$
c) $y(t) = x(t-3) + x(3-t)$
d) $y(t) = x(2-t)u(t-3)$

Solution:

a) We have

$$y(t) = e^{-2t}u(t)u(3-2t)\ .$$

Now since

$$u(3-2t) = \begin{cases} 1 & t \leq 1.5 \\ 0 & t > 1.5 \end{cases}$$

$y(t)$ can alternatively be written as

$$y(t) = \begin{cases} 0 & t < 0 \\ e^{-2t} & 0 \leq t \leq 1.5 \\ 0 & t > 1.5 \end{cases}\ .$$

Using this, $y(t)$ can be more easily sketched as below:

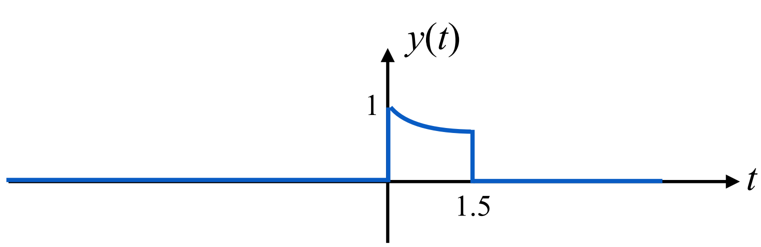

Figure 1: Plot for part (a)

b) We can write

$$\begin{aligned} y(t) &= x(t)^2 \\ &= \left[e^{-2t}\right]^2 u(t)^2 \\ &= e^{-4t} u(t)\ . \end{aligned}$$

The sketch is as below:

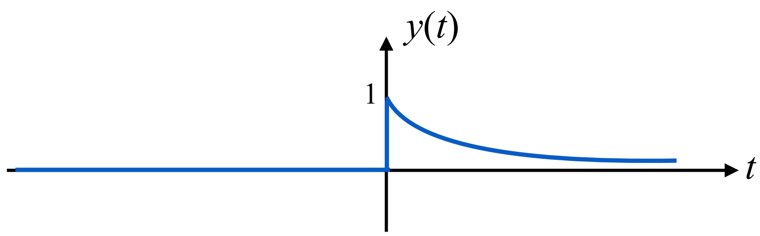

Figure 2: Plot for part (b)

c) The best way to sketch $y(t)$ is to understand that it is the sum of $x(t-3)$, the original signal $x(t)$ shifted to the right by 3, and $x(3-t)$, the original signal time reversed and also shifted to the right by 3. The signal $y(t)$ can therefore be sketched as below:

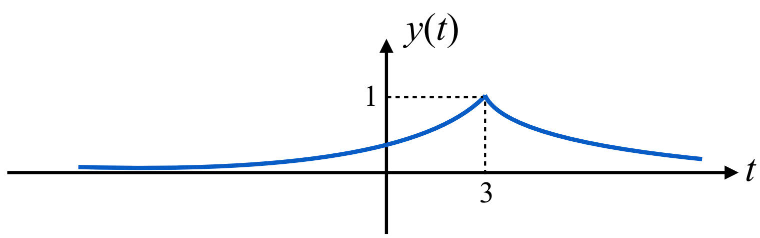

Figure 3: Plot for part (c)

d) Observe that

$$y(t) = e^{-2(2-t)} u(2-t) u(t-3)\ .$$

Now, for all $t < 3$, $u(t-3) = 0$. Similarly, for all $t > 2$, $u(2-t) = 0$. Therefore, for all values of t,

$$u(t-3)u(2-t) = 0\ ,$$

which implies that $y(t) = 0$. So there is really nothing to sketch. The graph is the same as the x-axis.

Problem 3 [8pts]: Consider the system given by

$$y(t) = x(t) + (t+1)^2\, x(t-3)\ \ .$$

Determine whether or not this system is linear, time-invariant, memoryless, and causal.

Solution:

Linearity: If $y_1(t)$ and $y_2(t)$ are the outputs to inputs $x_1(t)$ and $x_2(t)$, respectively, i.e.,

$$\begin{aligned} y_1(t) &= x_1(t) - (t+1)^2\, x_1(t-3) \\ y_2(t) &= x_2(t) - (t+1)^2\, x_2(t-3)\ , \end{aligned}$$

then we need to check whether $ax_1(t) + bx_2(t)$ yields $ay_1(t) + by_2(t)$ for any a and b. Indeed, the output to $ax_1(t) + bx_2(t)$ is

$$\begin{aligned} y'(t) &= ax_1(t) + bx_2(t) - (t+1)^2\big[ax_1(t-3) + bx_2(t-3)\big] \\ &= ax_1(t) - (t+1)^2\, ax_1(t-3) + bx_2(t) - (t+1)^2\, bx_2(t-3) \\ &= a\big[x_1(t) - (t+1)^2\, x_1(t-3)\big] + b\big[x_2(t) - (t+1)^2\, x_2(t-3)\big] \\ &= ay_1(t) + by_2(t)\ . \end{aligned}$$

So the system is linear.

Time invariance: For time invariance, if $x(t)$ yields $y(t)$, then $x(t-t_0)$ should yield $y(t-t_0)$. However, the output to $x(t-t_0)$ is given by

$$\begin{aligned} y'(t) &= x(t-t_0) - (t+1)^2\, x(t-t_0-3) \\ &= x(t-t_0) - (t-t_0+1+t_0)^2\, x(t-t_0-3) \\ &= x(t-t_0) - \big[(t-t_0+1)^2 + t_0^2 + 2(t-t_0+1)t_0\big]\, x(t-t_0-3) \\ &= x(t-t_0) - (t-t_0+1)^2\, x(t-t_0-3) - \big[t_0^2 + 2(t-t_0+1)t_0\big]\, x(t-t_0-3) \\ &= y(t-t_0) - \big[t_0^2 + 2(t-t_0+1)t_0\big]\, x(t-t_0-3) \\ &\neq y(t-t_0)\ . \end{aligned}$$

So, the system is time-varying.

Memory: Does $y(t)$ depend only on present value of $x(t)$ but not on its past or future values? No, it depends not only on $x(t)$ but also on $x(t-3)$. So the system has memory.

Causality: Does $y(t)$ depend only on present and past values of $x(t)$ but not on its future values? Yes, and more specifically, it only depends on $x(t)$ and $x(t-3)$. So the system is causal.