EE 115 Homework 3

1) (**30 points**) Sketch the spectra (i.e., Fourier transforms) of the following functions. If the spectrum is complex, sketch its real and imaginary parts (unless zero) separately.
   a) $m(t) = \mathtt{sinc}(2t) = \frac{\sin(\pi 2t)}{\pi 2t}$. Hint: $F\{\mathtt{sinc}(t)\} = \mathtt{rect}(f)$ and $F\{\mathtt{sinc}(at)\} = \frac{1}{a}\mathtt{rect}(f/a)$ with $a > 0$, $F\{\cdot\}$ denoting Fourier transform, and $\mathtt{rect}(f) = \begin{cases} 1, & |f| < 1/2 \\ 0, & |f| > 1/2 \end{cases}$.
   b) $m(t) 2\cos(2\pi 5t)$.
   c) $m(t) 2\sin(2\pi 5t)$.
2) (**30 points**) Assume $m(t) = 2\mathtt{sinc}(2t)$.
   a) Determine its Hilbert transform $\hat{m}(t)$. Hint: first determine the Fourier transform $\hat{M}(f)$ of $\hat{m}(t)$, and then compute the inverse Fourier transform of $\hat{M}(f)$.
   b) Sketch $M(f)$ and $\hat{M}(f)$ for $-\infty < f < \infty$.
   c) Sketch $m(t)$ and $\hat{m}(t)$ for $-\infty < t < \infty$.
3) (**40 points**) Let $u(t) = a(t)\cos(2\pi f_c t) - b(t)\sin(2\pi f_c t)$ where both $a(t)$ and $b(t)$ are baseband (or equivalently lowpass) signals with bandwidth $B \ll f_c$. Draw a diagram with mixers and lowpass filters (LPF) to retrieve $a(t)$ and $b(t)$ from $u(t)$.