University of California Riverside, Department of ECE
*EE110A: Signals and Systems*
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# Sample midterm and solutions

**Problem 1:** Let a system be given as

$$y(t) = x(t)u(t+1)$$

where $u(\cdot)$ is the usual step function.
**a)** Determine whether the system is linear.
**b)** Determine whether the system is time-invariant.
**c)** Determine whether the system is memoryless.
**d)** Determine whether the system is causal.
**e)** Determine whether the system is stable.
**f)** Determine whether the system is invertible.

**Solution:**

**a)** The system is **linear** because if the output to $x_1(t)$ and $x_2(t)$ are $y_1(t)$ and $y_2(t)$, respectively, the output to $[ax_1(t) + bx_2(t)]$ would be given by

$$y(t) = [ax_1(t) + bx_2(t)]u(t+1) = ax_1(t)u(t+1) + bx_2(t)u(t+1) = ay_1(t) + by_2(t) \ .$$

**b)** The system is **time-varying**, because if $y(t) = x(t)u(t+1)$, then the output to $x(t-t_0)$ would be

$$x(t-t_0)u(t+1) \neq y(t-t_0) \ .$$

**c)** The system is **memoryless**, because the output at time $t$ depends only on the input at time $t$.

**d)** The system is **causal**, because it is memoryless.

**e)** The system is **stable**, because if $|x(t)| \leq B$, then

$$|y(t)| = |x(t)u(t+1)| = |x(t)| \cdot |u(t+1)| \leq |x(t)| \leq B \ .$$

**f)** The system is **not invertible**, because $x_1(t) = 0$ and $x_2(t) = u(-t-2)$ give the same output, which is $y(t) = 0$.

**Problem 2:** A causal LTI system has the input-output relation given by

$$-y(t) + \frac{dy^2(t)}{dt^2} = x(t) - 2\frac{dx(t)}{dt} \ .$$

Find the impulse response of the system.

**Solution:**

Let us call the impulse response $h(t)$ as usual. For $t > 0$, the equation reduces to the homogeneous one

$$-h(t) + \frac{dh^2(t)}{dt^2} = 0 \ .$$

Trying the solution $h(t) = e^{\alpha t}$, we obtain

$$-e^{\alpha t} + \alpha^2 e^{\alpha t} = 0$$

or in other words,

$$\alpha^2 - 1 = 0 \ .$$

The roots of this quadratic equation are $\alpha = 1$ and $\alpha = -1$. Therefore the complete solution would be given by

$$h(t) = [c_1 e^t + c_2 e^{-t}]u(t) \ . \tag{1}$$

As always, we need $h(0^+)$ and $\frac{dh}{dt}(0^+)$ to find out $c_1$ and $c_2$. Writing the original differential equation as

$$-h(t) + \frac{dh^2(t)}{dt^2} = \delta(t) - 2\delta'(t) \ , \tag{2}$$

where $\delta'(t)$ is the derivative of the impulse, we can observe that $h(t)$ is allowed to have a discontinuity at $t = 0$ because the consequence of $\frac{dh}{dt}$ having an impulse in it, and $\frac{d^2h}{dt^2}$ having the derivative of an impulse, are both accounted for on the right-hand side.

Integrating the differential equation (2), we obtain

$$-H(t) + \frac{dh(t)}{dt} = u(t) - 2\delta(t) \tag{3}$$

where $H(t)$ is the anti-derivative (i.e., integral) of $h(t)$. Evaluating at $t = 0^+$ we get

$$-H(0^+) + \frac{dh}{dt}(0^+) = 1; .$$

Since $h(t)$ itself cannot have an impulse, $H(t)$ is continuous, and therefore $H(0^+) = H(0^-) = 0$ because of the initial rest assumption. Therefore,

$$\frac{dh}{dt}(0^+) = 1 \ . \tag{4}$$

Integrating (3) once more, we obtain

$$-HH(t) + h(t) = r(t) - 2u(t) \tag{5}$$

where $HH(t)$ is the integral of $H(t)$, and is itself also continuous, i.e., $HH(0^+) = HH(0^-) = 0$. Evaluating (5) at $t = 0^+$ we get

$$h(0^+) = -2 \; . \tag{6}$$

Equations (4) and (6) together will help us solve for $c_1$ and $c_2$ in (1). In particular,

$$h(0^+) = c_1 + c_2 = -2$$

and

$$\frac{dh}{dt}(0^+) = c_1 - c_2 = 1$$

and hence $c_1 = -0.5$ and $c_2 = -1.5$, and finally

$$h(t) = [-0.5e^t - 1.5e^{-t}]u(t) \; .$$