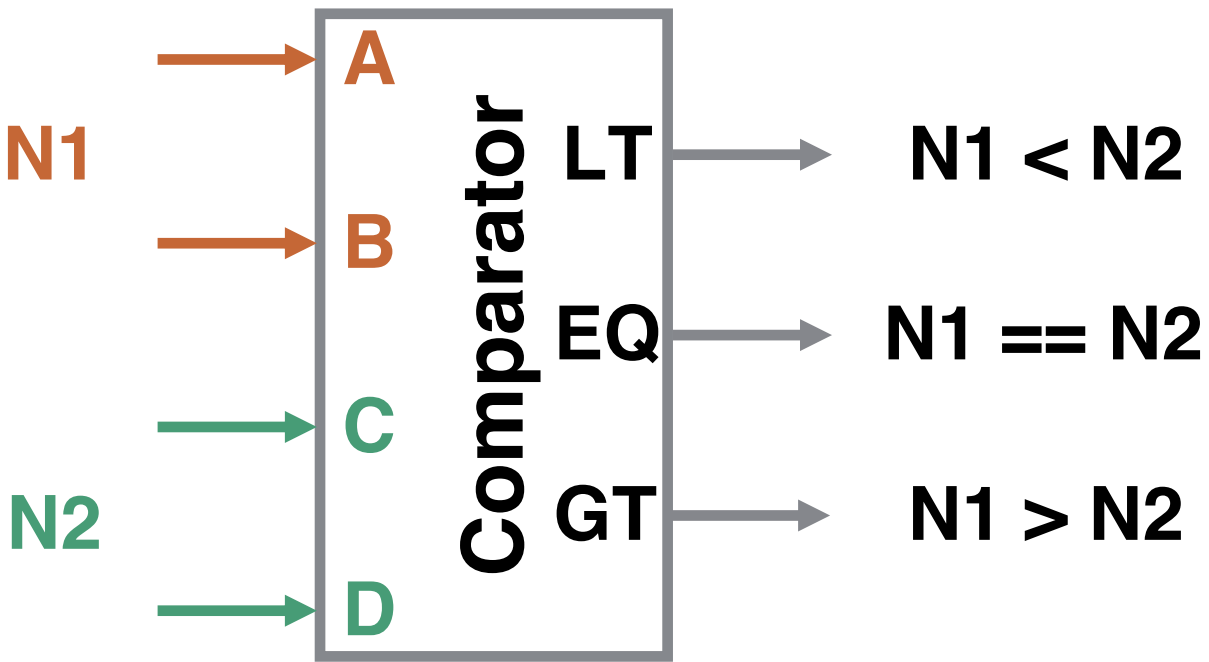


# Midterm review

Jia Chen  
jiac@ucr.edu

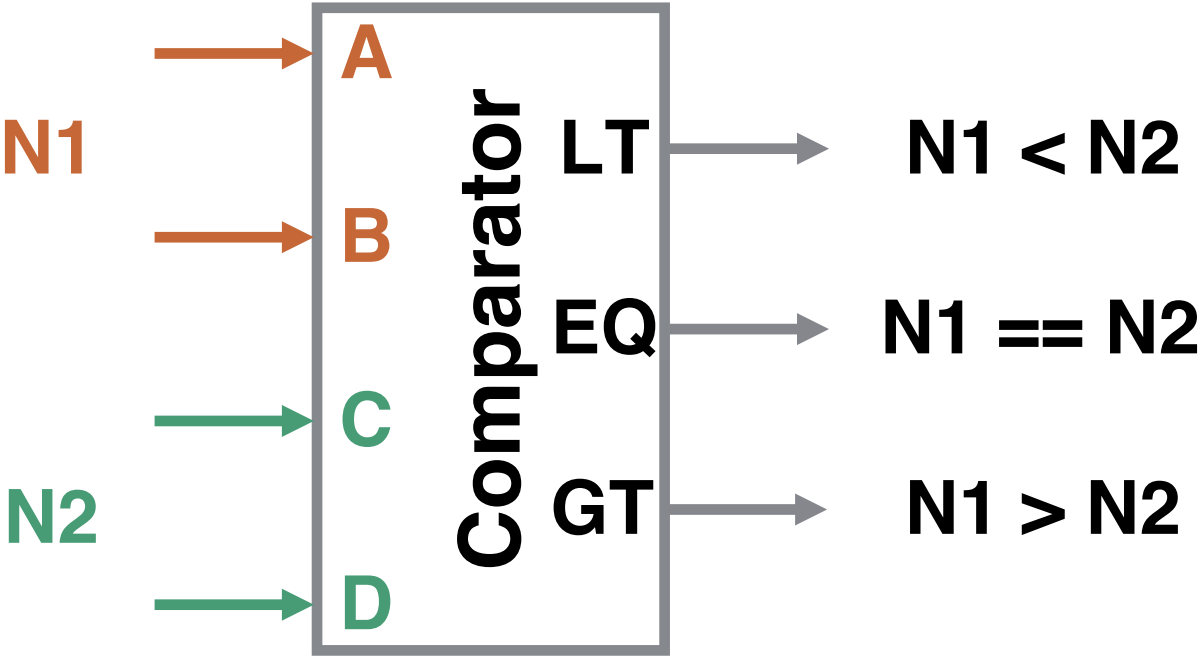
Q1: For the two-bit comparator, what's the minimum/simplest SOP presentation of LT?

Input				Output		
A	B	C	D	LT	EQ	GT
0	0	0	0	0	1	0
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	0	0
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	0	1	0
1	0	1	1	1	0	0
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		A'B'	A'B	AB	AB'	
		00	01	11	10	
C'D'	00	0	0	0	0	A'B'D
C'D	01	1	0	0	0	
CD	11	1	1	0	1	B'CD
CD'	10	1	1	0	0	
		A'C				

LT = A'B'D + A'C + B'CD

Q2: Find the minimum number of product term(s) to cover the following function and explain why.

Input			Output
A	B	C	
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
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0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

Answer: 1

		A'B'	A'B	AB	AB'
		0,0	0,1	1,1	1,0
C'	0	1	0	0	1
C	1	1	0	0	1

Output = B'

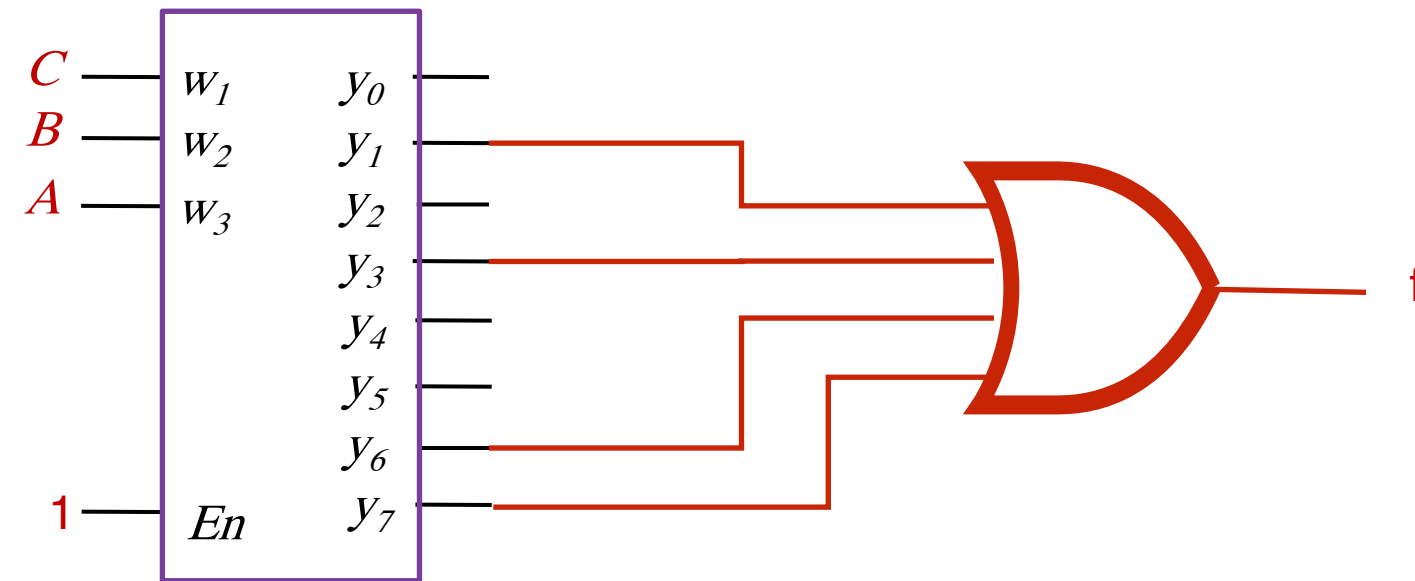
Literal is the simplest product term and the simplest sum term.

Q3: For the function  $f(A, B, C) = AB + A'C$ , show how the function  $f$  can be implemented using a 3-to-8 binary decoder and other necessary gate(s).



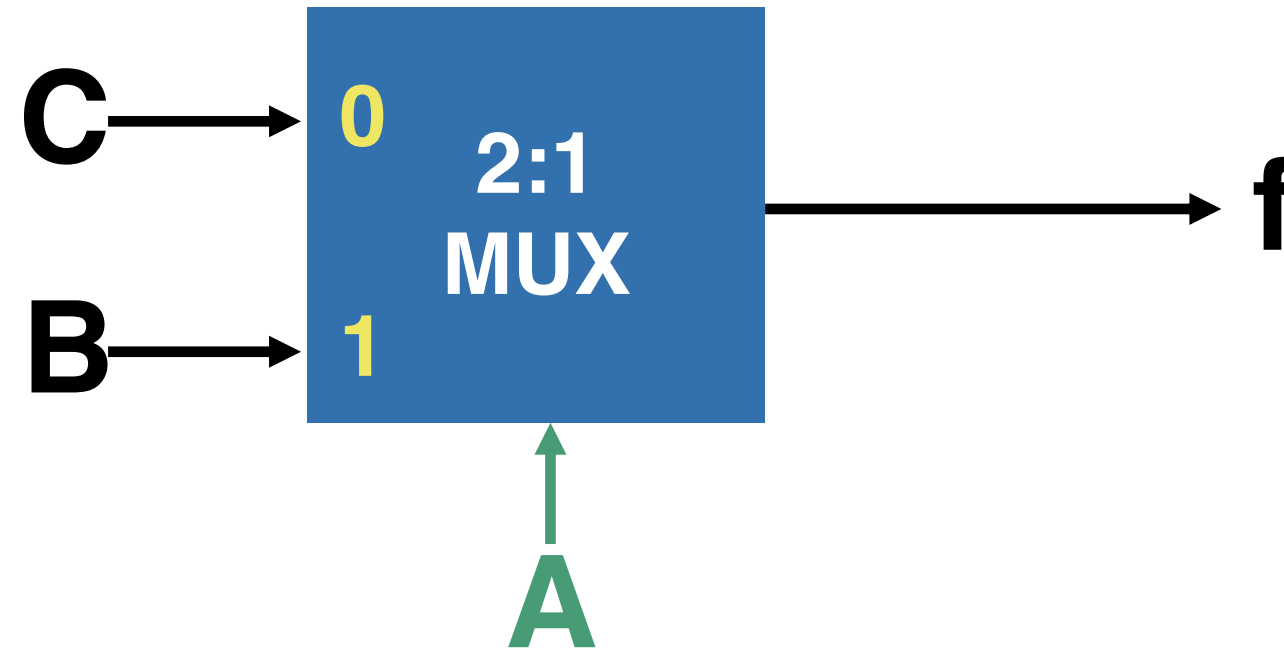
Q3: For the function  $f(A, B, C) = AB + A'C$ , show how the function  $f$  can be implemented using a 3-to-8 binary decoder and other necessary gate(s).

$$\begin{aligned} f(A, B, C) &= AB + A'C \\ &= ABC + ABC' + A'BC + A'B'C = m_7 + m_6 + m_3 + m_1 \end{aligned}$$



Q4: For the function  $f(A, B, C) = AB + A'C$ , show how the function  $f$  can be implemented using a MUX and gates if necessary.

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Q5: What's the value of  $F(a, b, c) = c' a + (b + a) c$  when  $a = 1$ ,  $b = 0$ , and  $c = 1$ ?

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$$F = 0^*1 + (0+1)1 = 0 + 1^*1 = 1$$

Q6: The binary number representation of decimal number 70 is ?

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Q7: What's the simplest SoP form for the function  $f(a, b, c, d) = m_1 + m_2 + m_3 + m_4$



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Please show all your work.

Q7: What's the simplest SoP form for the function  $f(a, b, c, d) = m_1 + m_2 + m_3 + m_4$ . Please show all your work.

**Method 1: using Boolean algebra**

$$\begin{aligned}
 f(a, b, c, d) &= m_1 + m_2 + m_3 + m_4 \\
 &= a'b'c'd + a'b'cd' + a'b'cd + a'bc'd' \\
 &= a'b'd + a'b'cd' + a'bc'd' \\
 &= (a'b'd + a'b'cd') + a'bc'd' \\
 &= a'b'(d + cd') + a'bc'd' \\
 &= a'b'(d + c) + a'bc'd' \\
 &= a'b'd + a'b'c + a'bc'd'
 \end{aligned}$$

**Method 2: using K-maps**

		a'b'	a'b	ab	ab'	
		00	01	11	10	
c'd'	00	0	1	0	0	a'bc'd'
c'd	01	1	0	0	0	
cd	11	1	0	0	0	
cd'	10	1	0	0	0	
		a'b'c		a'b'd		

Q8: What is a 2-input XOR gate's output value given input values  $a = 1$ ,  $b = 0$ ?

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1

Q9: Find the dual expression of  $(X' Y + Z) W'$

The dual form is  $(X' + Y)Z + W'$

Q10: What are the canonical SOP and POS (both) for each logic function. Please express each minterm and maxterm involved explicitly using the inputs, e.g., XYZ and A+B+C.

$$1) F = \sum_{Y, Z, X} (3, 2, 6) \quad \text{and} \quad 2) G(A, B, C) = \prod M(0, 3, 5, 6, 7)$$

Q11: simplify  $Y = \overline{A} \overline{B} + \overline{A} B \overline{C} + \overline{\overline{A} + \overline{B}}$   
as much as possible.



Q11:

$$Y = \overline{A} \overline{B} + \overline{A} B \overline{C} + \overline{\overline{A} + \overline{B}}$$

$$Y = A'B' + A'BC' + A'B$$

$$= (A'B' + A'B) + A'BC'$$

$$= A' + A'BC'$$

$$= A' (1 + BC')$$

$$= A' \cdot 1$$

$$= A'$$

Q12: Find the simplest SoP of

$$f(x_1, x_2, x_3) = \prod M(2, 3, 6)$$

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$$f(x_1, x_2, x_3) = \prod M(2, 3, 6)$$

		$x_2x_3$				
		00	01	11	10	
$x_1$	0	1	1	0	0	$x_2'$
	1	1	1	1	0	$x_1x_3$

$$f = x_2' + x_1x_3$$

Q13: Perform the following operations involving eight-bit 2's complement numbers and indicate whether arithmetic overflow occurs. What are the decimal values of the minuend and subtrahend?

$$\begin{array}{r} 0110101 \\ - 10110110 \\ \hline \end{array}$$

Q13: Perform the following operations involving eight-bit 2's complement numbers and indicate whether arithmetic overflow occurs. What are the decimal values of the minuend and subtrahend?

**Solution:**

$$\begin{array}{r} 01110101 \\ - 10110110 \\ \hline \end{array}$$
$$\begin{array}{r} 01000000 \\ + 01110101 \\ + 01001010 \\ \hline 10111111 \end{array}$$

The last two carry-outs  
are different. Thus, there  
is overflow.

The value of the minuend is  $1+4+16+32+64 = 117$ .

The value of the subtrahend is  $-(01001010) = -(2+8+64) = -74$

Q14: Use Boolean algebra laws/theorems to find the simplest SoP of y.

$$y(x_1, x_2, x_3) = \sum m(0, 3, 4, 7)$$

Q15: Use K-maps to find the simplest SoP of y. Show all your work.

$$y(x_1, x_2, x_3) = \prod M(1, 3, 7, 5, 6)$$

Q16: Use a 4-to-1 MUX and logic gates to design a circuit satisfying the following input ( $x_1, x_2, x_3$ ) and output (y) relationship  $y(x_1, x_2, x_3) = \prod M(1, 3, 7, 5, 6)$



Q17: Prove or disapprove the following equations.

(1)  $ab + bc = ab'(c + c')$

(2)  $abc + ab'c + a'c' = ab'c + ab'c' + abc$