EE/CS120A-Logic Design

Homework 2- Solution

Problem 1: Please simplify both sides of the following logic equation as much as you can and prove that the logic equation is true.

$$x_1 \cdot \bar{x}_3 + \bar{x}_2 \cdot \bar{x}_3 + x_1 \cdot x_3 + \bar{x}_2 \cdot x_3 = \bar{x}_1 \cdot \bar{x}_2 + x_1 \cdot x_2 + x_1 \cdot \bar{x}_2$$

$$x_1 \cdot \bar{x}_3 + \bar{x}_2 \cdot \bar{x}_3 + x_1 \cdot x_3 + \bar{x}_2 \cdot x_3 = \bar{x}_1 \cdot \bar{x}_2 + x_1 \cdot x_2 + x_1 \cdot \bar{x}_2$$

Solution:

The left-hand side can be manipulated as follows

$$\begin{aligned} \text{LHS} &= x_1 \cdot \bar{x}_3 + x_1 \cdot x_3 + \bar{x}_2 \cdot \bar{x}_3 + \bar{x}_2 \cdot x_3 \\ &= x_1 \cdot (\bar{x}_3 + x_3) + \bar{x}_2 \cdot (\bar{x}_3 + x_3) \\ &= x_1 \cdot 1 + \bar{x}_2 \cdot 1 \\ &= x_1 + \bar{x}_2 \end{aligned}$$

The right-hand side can be manipulated as

$$\begin{aligned} \text{RHS} &= \bar{x}_1 \cdot \bar{x}_2 + x_1 \cdot (x_2 + \bar{x}_2) \\ &= \bar{x}_1 \cdot \bar{x}_2 + x_1 \cdot 1 \\ &= \bar{x}_1 \cdot \bar{x}_2 + x_1 \\ &= x_1 + \bar{x}_1 \cdot \bar{x}_2 \\ &= x_1 + \bar{x}_2 \end{aligned}$$

Thus, LHS = RHS.

Problem 2: Determine whether or not the following expressions are valid, i.e., whether the left- and right-hand sides represent the same function.

(a) $\bar{x}_1 x_3 + x_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 + x_1 \bar{x}_2 = \bar{x}_2 x_3 + x_1 \bar{x}_3 + x_2 \bar{x}_3 + \bar{x}_1 x_2 x_3$

(b) $x_1 \bar{x}_3 + x_2 x_3 + \bar{x}_2 \bar{x}_3 = (x_1 + \bar{x}_2 + x_3)(x_1 + x_2 + \bar{x}_3)(\bar{x}_1 + x_2 + \bar{x}_3)$

(c) $(x_1 + x_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2) = (x_1 + x_2)(x_2 + x_3)(\bar{x}_1 + \bar{x}_3)$

Solution:

(a) Yes

(b) Yes

(c) No

Problem 3: Find the dual of a logic function F = (X'Y + Z)W'

Solution:

The dual of F is (X' + Y) Z + W'

Problem 4: Write canonical SOP and POS (both) for each logic function:

1) F = $\sum_{A,\,B,\,C}(1, 2, 4, 6)$ and 2) G = $\prod_{W,X,Y}(0, 2, 3, 6, 7)$

Solution:

1) Canonical SOP: F = $A'B'C + A'BC' + AB'C' + ABC'$

   Canonical POS: F = $(A + B + C)(A + B' + C')(A' + B + C')(A' + B' + C')$

2) Canonical SOP: F = $W'X'Y + WX'Y' + WX'Y$

   Canonical POS: F = $(W + X + Y)(W + X' + Y)(W + X' + Y')(W' + X' + Y)(W' + X' + Y')$

Problem 5: Use Boolean algebra laws/theorems to show that

$$x_1' x_2' x_3 + x_1' x_2 x_3' + x_1' x_2 x_3 + x_1 x_2' x_3' + x_1 x_2' x_3 + x_1 x_2 x_3' + x_1 x_2 x_3 = x_1 + x_2 + x_3$$

Solution:

$LHS = x_1' x_2' x_3 + x_1' x_2 x_3' + x_1' x_2 x_3 + x_1 x_2' x_3' + x_1 x_2' x_3 + x_1 x_2 x_3' + x_1 x_2 x_3$

$= x_1' x_2' x_3 + x_1' x_2 x_3 + x_1' x_2 x_3' + x_1' x_2 x_3 + x_1 x_2' x_3' + x_1 x_2' x_3 + x_1 x_2 x_3' + x_1 x_2 x_3$

$= (x_1' x_2' x_3 + x_1' x_2 x_3) + (x_1' x_2 x_3' + x_1' x_2 x_3) + (x_1 x_2' x_3' + x_1 x_2' x_3) + (x_1 x_2 x_3' + x_1 x_2 x_3)$

$= x_1' x_3 + x_1' x_2 + x_1 x_2' + x_1 x_2$

$= x_1' x_3 + (x_1' x_2 + x_1 x_2) + (x_1 x_2' + x_1 x_2)$

$= x_1' x_3 + x_2 + x_1$

$= (x_1' x_3 + x_1) + x_2$

$= x_3 + x_1 + x_2$ = *RHS*

Problem 6: Use Boolean algebra laws/theorems to show that

$$(x_1 + x_2 + x_3)(x_1 + x_2 + x_3')(x_1 + x_2' + x_3)(x_1 + x_2' + x_3')(x_1' + x_2 + x_3)(x_1' + x_2 + x_3')(x_1' + x_2' + x_3) = x_1 x_2 x_3$$

Solution:

$LHS = (x_1 + x_2 + x_3)(x_1 + x_2 + x_3')(x_1 + x_2' + x_3)(x_1 + x_2' + x_3')(x_1' + x_2 + x_3)(x_1' + x_2 + x_3')(x_1' + x_2' + x_3)$

$= [(x_1 + x_2 + x_3)(x_1 + x_2 + x_3')]\,[(x_1 + x_2' + x_3)(x_1 + x_2' + x_3')]\,[(x_1' + x_2 + x_3)(x_1' + x_2 + x_3')]\,[(x_1' + x_2 + x_3)(x_1' + x_2' + x_3)]$

$= [x_1 + x_2][x_1 + x_2'][x_1' + x_2][x_1' + x_3]$

$= [(x_1 + x_2)(x_1 + x_2')][(x_1 + x_2)(x_1' + x_2)][x_1' + x_3]$

$= x_1 x_2 (x_1' + x_3)$

$= x_1 (x_1' + x_3) x_2$

$= x_1 x_3 x_2 = RHS$