EE115: Lab 1

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Question 1: AM Power and Efficiency

(a)

Generated a Gaussian random sequence $m[k]$ of length $N = 200$ using MATLAB's `randn` function. This produces a zero-mean, unit-variance random signal representing the message waveform.

(b)

The minimum of the sequence was found as $m_{\min} = -2.417465$. Defined $M_0 = -m_{\min} = 2.417465$ so that $m_{\min} = -M_0$.

(c)

Normalized the sequence so its minimum becomes -1:

$$m_n[k] = \frac{1}{M_0} m[k].$$

After normalization, $\min(m_n[k]) = -1.000000$, confirming correct scaling.

(d)

Computed the average power of $m_n[k]$:

$$P_m = \frac{1}{N} \sum_{k=1}^{N} m_n^2[k] = 0.190670.$$

(e)

Plotted the AM efficiency:

$$\eta_{AM} = \frac{a_{mod} P_m}{1 + a_{mod} P_m}$$

versus P_m for $a_{mod} = 1.0, 0.75, 0.5$ using MATLAB. The plot (`eta_vs_Pm.png`) shows that efficiency increases with a_{mod} and P_m but never exceeds 50%.

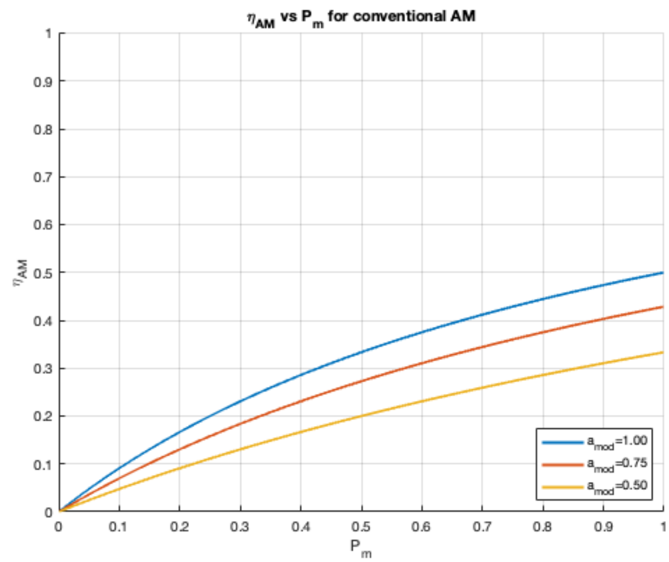


Figure 1: Task 1 MATLAB plot of η_{AM} against P_m for the simulated data.

(f)

Evaluated the AM efficiency at $P_m = 0.190670$:

a_{mod}	η_{AM}
1.00	0.1601
0.75	0.1251
0.50	0.0870

Table 1: Measured AM efficiencies at $P_m = 0.190670$.

Theoretical Upper Bound (as $P_m \to 1$):

a_{mod}	$\eta_{AM,max}$	**Percent**
1.00	0.5000	50.0%
0.75	0.4286	42.9%
0.50	0.3333	33.3%

Table 2: Theoretical AM efficiency limits as $P_m \to 1$.

Takeaway. Conventional AM wastes carrier power. Even under 100% modulation ($a_{mod} = 1$), efficiency is capped at 50%. Lower modulation depth or smaller P_m further reduce η_{AM}.

Question 1 MATLAB Output

Listing 1: Task 1 MATLAB Command Window Output

```
-------------------- Task 1: Random Signal & AM Efficiency --------------------
(a) Generated Gaussian sequence m[k] of length N=200
(b) Minimum of m[k] is m_min = -2.417465, so M0 = -m_min = 2.417465
(c) After normalization, min(m_n) = -1.000000 (should be -1)
(d) Average power Pm of m_n[k] = 0.190670
(e) Plotted eta_AM vs P_m for a_mod = 1, 0.75, 0.5
(f) eta_AM evaluated at your Pm from (d): Pm = 0.190670
    a_mod = 1.00 -> eta_AM = 0.160136
    a_mod = 0.75 -> eta_AM = 0.125111
    a_mod = 0.50 -> eta_AM = 0.087037

Theoretical upper bound as Pm -> 1:
    a_mod = 1.00 -> max eta_AM = 0.5000 (i.e., 50.0%)
    a_mod = 0.75 -> max eta_AM = 0.4286 (i.e., 42.9%)
    a_mod = 0.50 -> max eta_AM = 0.3333 (i.e., 33.3%)

Takeaway: Conventional AM uses a large carrier term (the "+1"). Even with 100% modulation
     (a_mod=1) and the most energetic message (P_m->1), eta_AM <= 0.5 (50%). Lower a_mod
    or lower P_m pushes efficiency below that.
```

Question 2: DC Blocker (RC High-Pass) Response

We analyze the simple RC high-pass (used as a DC blocker) with frequency response

$$H(f) \;=\; \frac{j2\pi f}{j2\pi f + \frac{1}{RC}}.$$

(a) Plot $|H(f)|$ over $|f| < B$ for $RC \in \{0.01, 0.1, 1, 10\}$

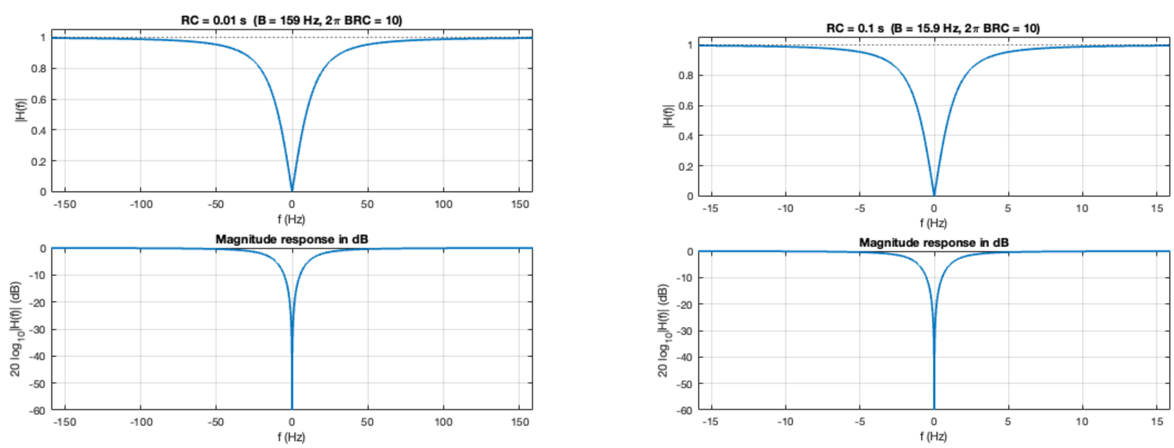
For each RC, choose B so that $2\pi BRC = 10 \Rightarrow B = \dfrac{10}{2\pi RC}$. If you saved `Hmag_linear.png` from MATLAB in the same folder, it will be included below.

(b) Plot $20\log_{10}|H(f)|$ (dB) versus f

To avoid the singularity at $f = 0$, plot over $f \in [10^{-3}, B]$. Limit the vertical axis to $[-60, 0]$ dB. If you saved `Hmag_dB.png`, it will be included below.

Observed MATLAB Plots (from simulation results)

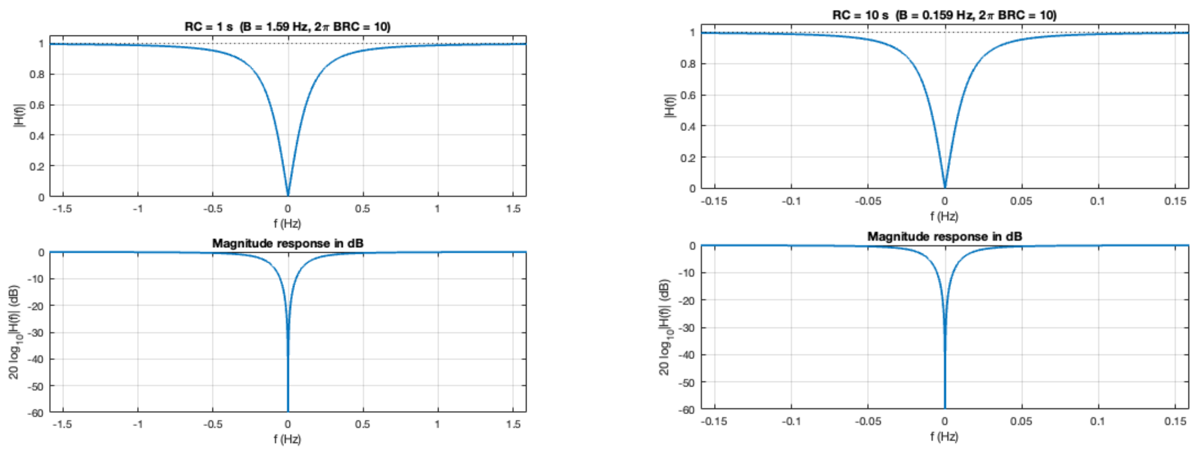
The following plots illustrate the magnitude and dB responses for various RC values. Each figure pair shows the linear and logarithmic magnitude responses for a chosen RC.



(a) RC = 0.01 s
(B = 159 Hz, $2\pi BRC = 10$)

(b) RC = 0.1 s
(B = 15.9 Hz, $2\pi BRC = 10$)

Figure 2: Magnitude and dB responses for smaller time constants.



(a) RC = 1.0 s
(B = 1.59 Hz, $2\pi BRC = 10$)

(b) RC = 10 s
(B = 0.159 Hz, $2\pi BRC = 10$)

Figure 3: Magnitude and dB responses for larger time constants.

(c) Acceptable RC for passband flatness over $|f| \geq 20$ Hz

We require $|H(f)| \geq 0.95$ at $f_0 = 20$ Hz for the real-valued baseband $m_n(t)$ that occupies $\pm[20\text{ Hz}, 5\text{ kHz}]$. Using

$$|H(f)| = \frac{2\pi|f|}{\sqrt{(2\pi f)^2 + \left(\frac{1}{RC}\right)^2}},$$

the constraint $|H(f_0)| \geq 0.95$ implies

$$\frac{2\pi f_0}{\sqrt{(2\pi f_0)^2 + (1/RC)^2}} \geq 0.95 \Rightarrow \frac{1}{RC} \leq 0.328684 \cdot 2\pi f_0 \Rightarrow RC \geq \frac{1}{0.328684 \cdot 2\pi \cdot 20} \approx \boxed{0.0242\text{ s}}.$$

Thus any $\boxed{RC \geq 0.024\text{ s}}$ keeps the attenuation at 20 Hz within ≈ 0.45 dB, and is flatter at higher frequencies. In practice, choose R and C to satisfy this while meeting input impedance and size constraints.

Question 2 MATLAB Output

Listing 2: Task 2 MATLAB Command Window Output

```
---- Task 2: RC high-pass (DC blocker) ----

(a) RC = 0.01 s | fc = 15.92 Hz | B = 159.2 Hz | |H(B)| = 0.9950
(a) RC = 0.1 s | fc = 1.592 Hz | B = 15.92 Hz | |H(B)| = 0.9950
(a) RC = 1 s | fc = 0.1592 Hz | B = 1.592 Hz | |H(B)| = 0.9950
(a) RC = 10 s | fc = 0.01592 Hz | B = 0.1592 Hz | |H(B)| = 0.9950

(c) To keep |H(f)| >= 0.95 for |f| >= 20 Hz:
   Need 2*pi*f_min*RC >= 3.0424 -> RC >= 0.024211 s (~ 24.211 ms)
   Acceptable RC range: RC >= 0.024211 s.
```

Appendix: MATLAB code for Task 1

Listing 3: Task 1 MATLAB script.

```
%% AM power/efficiency experiment
% Task 1: (a)-(f)
clear; clc; close all;

%% -------- Parameters you can tweak --------
N = 200; % sequence length (>=200 per problem)
seed = 12345; % RNG seed for reproducibility
amods = [1.0, 0.75, 0.5]; % a_mod values for part (e)
Pm_grid = linspace(1e-3, 0.999, 2000); % 0 < Pm < 1 for plotting
%% ------------------------------------------

fprintf('------------------- Task 1: Random Signal & AM Efficiency -------------------\n'
    );

%% (a) Generate Gaussian random sequence m[1..N]
rng(seed);
m = randn(1, N); % zero-mean Gaussian
fprintf('(a) Generated Gaussian sequence m[k] of length N=%d\n', N);

%% (b) Minimum value and denote it by -M0
m_min = min(m);
M0 = -m_min; % so that m_min = -M0
fprintf('(b) Minimum of m[k] is m_min = %.6f, so M0 = -m_min = %.6f\n', m_min, M0);

%% (c) Normalize so the minimum becomes -1: m_n[k] = (1/M0) * m[k]
mn = (1 / M0) * m;
mn_min = min(mn);
fprintf('(c) After normalization, min(m_n) = %.6f (should be -1)\n', mn_min);

%% (d) Average power of m_n[k]: Pm = (1/N) * sum( m_n[k]^2 )
Pm = mean(mn.^2);
fprintf('(d) Average power Pm of m_n[k] = %.6f\n', Pm);

%% (e) Conventional AM: u_AM(t) = A_c * (a_mod*m_n(t) + 1) * cos(2*pi*f_c*t)
% Power efficiency: eta_AM = (a_mod * Pm) / (1 + a_mod * Pm)
% Plot eta_AM vs Pm for each a_mod (0 < Pm < 1)
figure('Color','w'); hold on; grid on;
for a = amods
    eta = (a .* Pm_grid) ./ (1 + a .* Pm_grid);
    plot(Pm_grid, eta, 'LineWidth', 1.8);
end
xlabel('P_m','Interpreter','tex');
ylabel('\eta_{AM}');
title('\eta_{AM} vs P_m for conventional AM');
legend(arrayfun(@(x) sprintf('a_{mod}=%.2f', x), amods, 'UniformOutput', false), ...
       'Location', 'southeast');
xlim([0 1]); ylim([0 1]);

fprintf('(e) Plotted eta_AM vs P_m for a_mod = 1, 0.75, 0.5\n');
```

```
%% (f) Evaluate eta_AM at the Pm computed in (d), for each a_mod
fprintf('(f) eta_AM evaluated at your Pm from (d): Pm = %.6f\n', Pm);
for a = amods
    eta_at_Pm = (a * Pm) / (1 + a * Pm);
    fprintf(' a_mod = %.2f -> eta_AM = %.6f\n', a, eta_at_Pm);
end

%% Bonus: theoretical upper bounds (when Pm -> 1)
fprintf('\nTheoretical upper bound as Pm -> 1:\n');
for a = amods
    eta_max = (a * 1.0) / (1 + a * 1.0);
    fprintf(' a_mod = %.2f -> max eta_AM = %.4f (i.e., %.1f%%)\n', a, eta_max, 100*
        eta_max);
end
```

Appendix: MATLAB code for Task 2

Listing 4: Task 2 MATLAB script (DC Blocker magnitude and dB plots).

```
%% Task 2: DC blocker (RC high-pass) quality
% H(f) = (j*2*pi*f) / (j*2*pi*f + 1/(RC))
% Plots for RC = [0.01, 0.1, 1, 10] with B chosen s.t. 2*pi*B*RC = 10
% Then compute the RC range that keeps |H(f)| >= 0.95 for |f| >= 20 Hz.

clear; clc; close all;

RC_list = [0.01, 0.1, 1, 10]; % seconds
db_floor = -60; % for part (b) y-axis
Npts = 20001; % dense frequency grid

fprintf("---- Task 2: RC high-pass (DC blocker) ----\n\n");

for k = 1:numel(RC_list)
    RC = RC_list(k);

    % ----- (a) pick B so that 2*pi*B*RC = 10 -> a decade above corner -----
    B = 10/(2*pi*RC);
    f = linspace(-B, B, Npts); % |f| < B
    H = (1j*2*pi*f) ./ (1j*2*pi*f + 1/RC);
    mag = abs(H);
    mag_dB = 20*log10(max(mag, 10^(db_floor/20))); % avoid -Inf at f=0

    % quick printouts
    fc = 1/(2*pi*RC);
    mag_at_B = (2*pi*B*RC)/sqrt(1+(2*pi*B*RC)^2); % should be ~0.995
    fprintf("(a) RC = %-6.3g s | fc = %-9.4g Hz | B = %-10.4g Hz | |H(B)| = %.4f\n", ...
           RC, fc, B, mag_at_B);

    % ---- plots for this RC (a) linear magnitude; (b) 20*log10|H| with y in [-60, 0] dB
    figure('Color','w');
    tl = tiledlayout(2,1,'TileSpacing','compact');
```

```
    % (a) |H(f)|
    nexttile;
    plot(f, mag, 'LineWidth', 1.8); grid on;
    xlabel('f (Hz)'); ylabel('|H(f)|');
    title(sprintf('RC = %.3g s (B = %.3g Hz, 2\\pi BRC = 10)', RC, B));
    xlim([-B, B]); ylim([0 1.05]);
    hold on; yline(1,'k:');

    % (b) 20log10|H(f)|
    nexttile;
    plot(f, mag_dB, 'LineWidth', 1.8); grid on;
    xlabel('f (Hz)'); ylabel('20 log_{10}|H(f)| (dB)');
    title('Magnitude response in dB');
    xlim([-B, B]); ylim([db_floor 0]);
end

%% (c) Pick RC so that |H(f)| >= 0.95 for |f| >= 20 Hz
% |H| = x/sqrt(1+x^2) with x = 2*pi*f*RC >= alpha = 0.95
% Solve: x >= alpha / sqrt(1 - alpha^2)

alpha = 0.95; % tolerance
f_min = 20; % Hz, band starts at 20 Hz
x_min = alpha/sqrt(1 - alpha^2); % ~3.042
RC_min = x_min / (2*pi*f_min); % seconds

fprintf("\n(c) To keep |H(f)| >= %.2f for |f| >= %d Hz:\n", alpha, f_min);
fprintf(" Need 2*pi*f_min*RC >= %.4f -> RC >= %.6f s (~ %.3f ms)\n", x_min, RC_min, 1e3*
    RC_min);
fprintf(" Acceptable RC range: RC >= %.6f s.\n\n", RC_min);
```