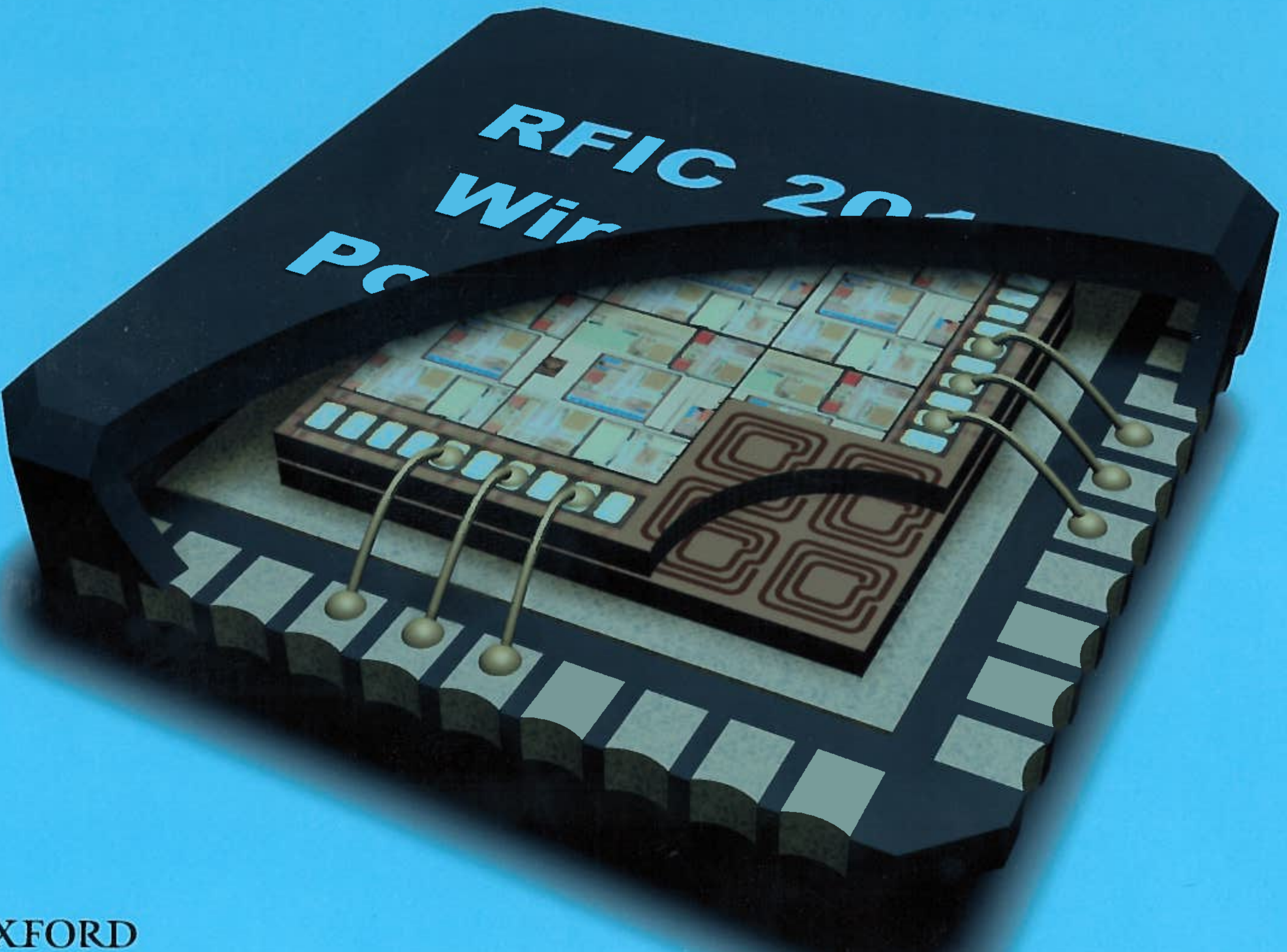
SEDRA/SMITH

Instructor's Solutions Manual for
Microelectronic Circuits

SEVENTH EDITION

By Adel S. Sedra



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Contents

Preface

This Instructor's Solution Manual (ISM) contains complete solutions for all 485 exercises and 1,545 end-of-chapter problems included in the book ***Microelectronic Circuits*, Seventh Edition** by Adel S. Sedra and Kenneth C. Smith.

Most of the solutions are new; however, I have used and/or adapted some of the solutions from the ISM of the sixth edition. Credit for these goes to the problem solvers listed therein.

This manual has greatly benefited from the careful work of the accuracy checkers listed below. These colleagues and friends worked diligently to ensure that the 2,030 solutions are free of error. Despite all of our combined efforts, however, there is little doubt that some errors remain, and for these I take full responsibility. I will be most grateful to instructors who discover errors and point them out to me. Please send all corrections and comments by email to: sedra@uwaterloo.ca.

A number of individuals made significant contributions to the production of the manual. Jennifer Rodrigues, who typed many of the previous editions of the book including the seventh, undertook the mammoth task of typing the ISM. Her work has been excellent, as usual, and she has maintained her good humour throughout the long journey.

At OUP, two individuals played key roles in shepherding through the ISM: Senior Production Editor Jane Lee and Associate Editor Christine Mahon. I am grateful to both of them for their excellent work and for their thoughtfulness. I have also benefited from the guidance of Engineering's Senior Acquisitions Editor, Nancy Blaine.

Adel Sedra
Waterloo, Ontario, Canada
January 2015

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- Professor Tony Chan Carusone, University of Toronto - Assisted by graduate students Jeffrey Wang and Luke Wang
- Professor Vincent Gaudet, University of Waterloo
- Professors Shahriar Mirabbasi and Mandana Amiri, University of British Columbia
- Professor Wai Tung Ng, University of Toronto
- Professor Olivier Trescases, University of Toronto
- Professor Amir Yazdani, Ryerson University

Ex: 1.1 When output terminals are open-circuited, as in Fig. 1.1a:

For circuit a. $v_{oc} = v_s(t)$

For circuit b. $v_{oc} = i_s(t) \times R_s$

When output terminals are short-circuited, as in Fig. 1.1b:

For circuit a. $i_{sc} = \dfrac{v_s(t)}{R_s}$

For circuit b. $i_{sc} = i_s(t)$

For equivalency

$R_s i_s(t) = v_s(t)$

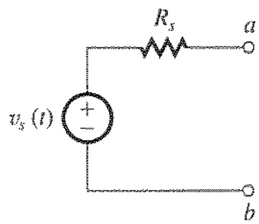


Figure 1.1a

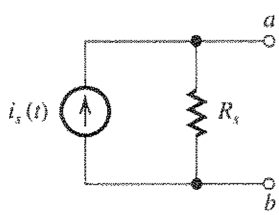
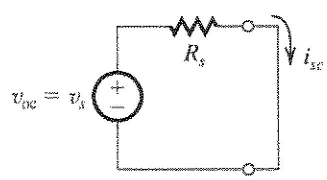


Figure 1.1b

Ex: 1.2



$v_{oc} = 10$ mV

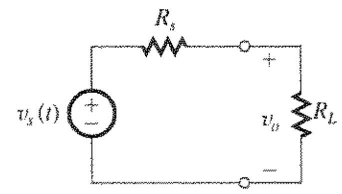
$i_{sc} = 10\ \mu$A

$$R_s = \frac{v_{oc}}{i_{sc}} = \frac{10\text{ mV}}{10\ \mu\text{A}} = 1\text{ k}\Omega$$

Ex: 1.3 Using voltage divider:

$$v_o(t) = v_s(t) \times \frac{R_L}{R_s + R_L}$$

Given $v_s(t) = 10$ mV and $R_s = 1$ kΩ.



If $R_L = 100$ kΩ:

$$v_o = 10\text{ mV} \times \frac{100}{100+1} = 9.9\text{ mV}$$

If $R_L = 10$ kΩ:

$$v_o = 10\text{ mV} \times \frac{10}{10+1} \simeq 9.1\text{ mV}$$

If $R_L = 1$ kΩ:

$$v_o = 10\text{ mV} \times \frac{1}{1+1} = 5\text{ mV}$$

If $R_L = 100\ \Omega$:

$$v_o = 10\text{ mV} \times \frac{100}{100 + 1\text{ K}} \simeq 0.91\text{ mV}$$

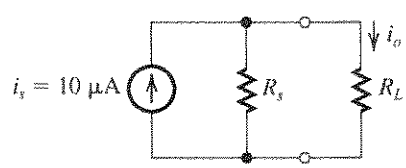
For $v_o = 0.8 v_s$,

$$\frac{R_L}{R_L + R_s} = 0.8$$

Since $R_s = 1$ kΩ,

$R_L = 4$ kΩ

Ex: 1.4 Using current divider:



$$i_o = i_s \times \frac{R_s}{R_s + R_L}$$

Given $i_s = 10\ \mu$A, $R_s = 100$ kΩ.

For $R_L = 1$ kΩ, $i_o = 10\ \mu\text{A} \times \dfrac{100}{100+1} = 9.9\ \mu$A

For $R_L = 10$ kΩ, $i_o = 10\ \mu\text{A} \times \dfrac{100}{100+10} \simeq 9.1\ \mu$A

For $R_L = 100$ kΩ, $i_o = 10\ \mu\text{A} \times \dfrac{100}{100+100} = 5\ \mu$A

For $R_L = 1$ MΩ, $i_o = 10\ \mu\text{A} \times \dfrac{100\text{ K}}{100\text{ K} + 1\text{ M}}$

$\simeq 0.9\ \mu$A

For $i_o = 0.8 i_s$, $\dfrac{100}{100 + R_L} = 0.8$

$\Rightarrow R_L = 25$ kΩ

Ex: 1.5 $f = \frac{1}{T} = \frac{1}{10^{-3}} = 1000\text{ Hz}$

$\omega = 2\pi f = 2\pi \times 10^3\text{ rad/s}$

Ex: 1.6 (a) $T = \frac{1}{f} = \frac{1}{60}\text{ s} = 16.7\text{ ms}$

(b) $T = \frac{1}{f} = \frac{1}{10^{-3}} = 1000\text{ s}$

(c) $T = \frac{1}{f} = \frac{1}{10^6}\text{ s} = 1\ \mu\text{s}$

Ex: 1.7 If 6 MHz is allocated for each channel, then 470 MHz to 806 MHz will accommodate

$\frac{806 - 470}{6} = 56$ channels

Since the broadcast band starts with channel 14, it will go from channel 14 to channel 69.

Ex: 1.8 $P = \frac{1}{T}\int_0^T \frac{v^2}{R}\,dt$

$= \frac{1}{T} \times \frac{V^2}{R} \times T = \frac{V^2}{R}$

Alternatively,

$P = P_1 + P_3 + P_5 + \cdots$

$= \left(\frac{4V}{\sqrt{2}\pi}\right)^2 \frac{1}{R} + \left(\frac{4V}{3\sqrt{2}\pi}\right)^2 \frac{1}{R}$

$+ \left(\frac{4V}{5\sqrt{2}\pi}\right)^2 \frac{1}{R} + \cdots$

$= \frac{V^2}{R} \times \frac{8}{\pi^2} \times \left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots\right)$

It can be shown by direct calculation that the infinite series in the parentheses has a sum that approaches $\pi^2/8$; thus P becomes V^2/R as found from direct calculation.

Fraction of energy in fundamental

$= 8/\pi^2 = 0.81$

Fraction of energy in first five harmonics

$= \frac{8}{\pi^2}\left(1 + \frac{1}{9} + \frac{1}{25}\right) = 0.93$

Fraction of energy in first seven harmonics

$= \frac{8}{\pi^2}\left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49}\right) = 0.95$

Fraction of energy in first nine harmonics

$= \frac{8}{\pi^2}\left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81}\right) = 0.96$

Note that 90% of the energy of the square wave is in the first three harmonics, that is, in the fundamental and the third harmonic.

Ex: 1.9 (a) D can represent 15 distinct values between 0 and +15 V. Thus,

$v_A = 0\text{ V} \Rightarrow D = 0000$

$v_A = 1\text{ V} \Rightarrow D = 0001$

$v_A = 2\text{ V} \Rightarrow D = 0010$

$v_A = 15\text{ V} \Rightarrow D = 1111$

(b) (i) +1 V (ii) +2 V (iii) +4 V (iv) +8 V

(c) The closest discrete value represented by

D is 5 V; thus $D = 0101$. The error is -0.2 V, or $-0.2/5.2 \times 100 = -4\%$.

Ex: 1.10 Voltage gain $= 20 \log 100 = 40$ dB

Current gain $= 20 \log 1000 = 60$ dB

Power gain $= 10 \log A_p = 10 \log (A_v A_i)$

$= 10 \log 10^5 = 50$ dB

Ex: 1.11 $P_{dc} = 15 \times 8 = 120$ mW

$P_L = \frac{(6/\sqrt{2})^2}{1} = 18$ mW

$P_{dissipated} = 120 - 18 = 102$ mW

$\eta = \frac{P_L}{P_{dc}} \times 100 = \frac{18}{120} \times 100 = 15\%$

Ex: 1.12 $v_o = 1 \times \frac{10}{10^6 + 10} \simeq 10^{-5}\text{ V} = 10\ \mu\text{V}$

$P_L = v_o^2/R_L = \frac{(10 \times 10^{-6})^2}{10} = 10^{-11}$ W

With the buffer amplifier:

$v_o = 1 \times \frac{R_i}{R_i + R_s} \times A_{vo} \times \frac{R_L}{R_L + R_o}$

$= 1 \times \frac{1}{1+1} \times 1 \times \frac{10}{10+10} = 0.25$ V

$P_L = \frac{v_o^2}{R_L} = \frac{0.25^2}{10} = 6.25$ mW

Voltage gain $= \frac{v_o}{v_s} = \frac{0.25\text{ V}}{1\text{ V}} = 0.25$ V/V

$= -12$ dB

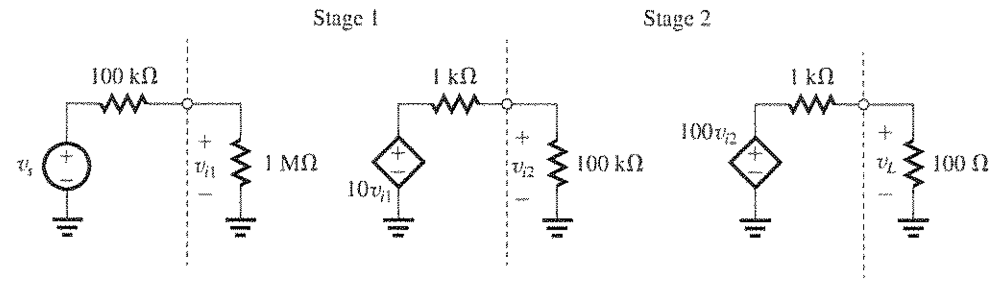
Power gain $(A_p) \equiv \frac{P_L}{P_i}$

where $P_L = 6.25$ mW and $P_i = v_i i_1$,

$v_i = 0.5$ V and

$i_i = \frac{1\text{ V}}{1\text{ M}\Omega + 1\text{ M}\Omega} = 0.5\ \mu\text{A}$

This figure belongs to Exercise 1.15.



Thus,

$P_i = 0.5 \times 0.5 = 0.25\ \mu\text{W}$

and

$$A_p = \frac{6.25 \times 10^{-3}}{0.25 \times 10^{-6}} = 25 \times 10^3$$

$10 \log A_p = 44$ dB

Ex: 1.13 Open-circuit (no load) output voltage = $A_{vo} v_i$

Output voltage with load connected

$$= A_{vo} v_i \frac{R_L}{R_L + R_o}$$

$$0.8 = \frac{1}{R_o + 1} \Rightarrow R_o = 0.25\ \text{k}\Omega = 250\ \Omega$$

Ex: 1.14 $A_{vo} = 40$ dB $= 100$ V/V

$$P_L = \frac{v_o^2}{R_L} = \left(A_{vo} v_i \frac{R_L}{R_L + R_o}\right)^2 \Big/ R_L$$

$$= v_i^2 \times \left(100 \times \frac{1}{1+1}\right)^2 \Big/ 1000 = 2.5\ v_i^2$$

$$P_i = \frac{v_i^2}{R_i} = \frac{v_i^2}{10{,}000}$$

$$A_p \equiv \frac{P_L}{P_i} = \frac{2.5 v_i^2}{10^{-4} v_i^2} = 2.5 \times 10^4\ \text{W/W}$$

$10 \log A_p = 44$ dB

Ex: 1.15 Without stage 3 (see figure above)

$$\frac{v_L}{v_s} = \left(\frac{1\ \text{M}\Omega}{100\ \text{k}\Omega + 1\ \text{M}\Omega}\right)(10)\left(\frac{100\ \text{k}\Omega}{100\ \text{k}\Omega + 1\ \text{k}\Omega}\right) \times (100)\left(\frac{100}{100 + 1\ \text{k}\Omega}\right)$$

$$\frac{v_L}{v_s} = (0.909)(10)(0.9901)(100)(0.0909)$$

$= 81.8$ V/V

Ex: 1.16 Refer the solution to Example 1.3 in the text.

$$\frac{v_{i1}}{v_s} = 0.909\ \text{V/V}$$

$v_{i1} = 0.909\ v_s = 0.909 \times 1 = 0.909$ mV

$$\frac{v_{i2}}{v_s} = \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s} = 9.9 \times 0.909 = 9\ \text{V/V}$$

$v_{i2} = 9 \times v_S = 9 \times 1 = 9$ mV

$$\frac{v_{i3}}{v_s} = \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s} = 90.9 \times 9.9 \times 0.909$$

$= 818$ V/V

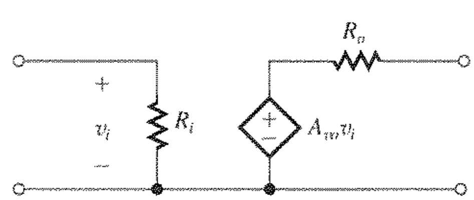
$v_{i3} = 818\ v_s = 818 \times 1 = 818$ mV

$$\frac{v_L}{v_s} = \frac{v_L}{v_{i3}} \times \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s}$$

$= 0.909 \times 90.9 \times 9.9 \times 0.909 \simeq 744$ V/V

$v_L = 744 \times 1$ mV $= 744$ mV

Ex: 1.17 Using voltage amplifier model, the three-stage amplifier can be represented as



$R_i = 1$ MΩ

$R_o = 10$ Ω

$A_{vo} = A_{v1} \times A_{v2} \times A_{v3} = 9.9 \times 90.9 \times 1 = 900$ V/V

The overall voltage gain

$$\frac{v_o}{v_s} = \frac{R_i}{R_i + R_s} \times A_{vo} \times \frac{R_L}{R_L + R_o}$$

For $R_L = 10\ \Omega$:

Overall voltage gain

$$= \frac{1\text{ M}}{1\text{ M} + 100\text{ K}} \times 900 \times \frac{10}{10 + 10} = 409\text{ V/V}$$

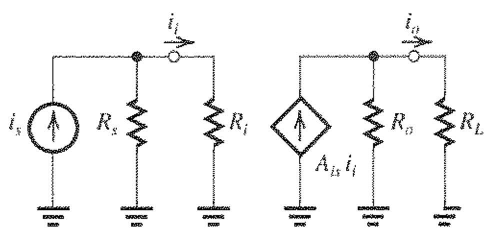
For $R_L = 1000\ \Omega$:

Overall voltage gain

$$= \frac{1\text{ M}}{1\text{ M} + 100\text{ K}} \times 900 \times \frac{1000}{1000 + 10} = 810\text{ V/V}$$

$\therefore$ Range of voltage gain is from 409 V/V to 810 V/V.

Ex: 1.18



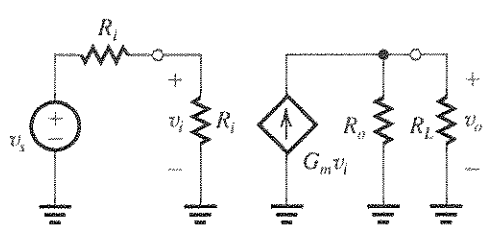
$$i_i = i_s \frac{R_s}{R_s + R_i}$$

$$i_o = A_{is} i_i \frac{R_o}{R_o + R_L} = A_{is} i_s \frac{R_s}{R_s + R_i} \frac{R_o}{R_o + R_L}$$

Thus,

$$\frac{i_o}{i_s} = A_{is} \frac{R_s}{R_s + R_i} \frac{R_o}{R_o + R_L}$$

Ex: 1.19



$$v_i = v_s \frac{R_i}{R_i + R_s}$$

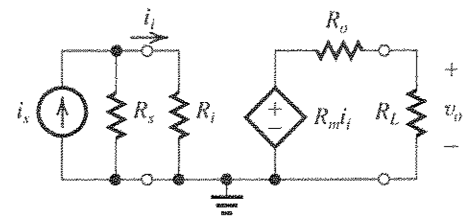
$$v_o = G_m v_i (R_o \parallel R_L)$$

$$= G_m v_s \frac{R_i}{R_i + R_s} (R_o \parallel R_L)$$

Thus,

$$\frac{v_o}{v_s} = G_m \frac{R_i}{R_i + R_s} (R_o \parallel R_L)$$

Ex: 1.20 Using the transresistance circuit model, the circuit will be



$$\frac{i_i}{i_s} = \frac{R_s}{R_i + R_s}$$

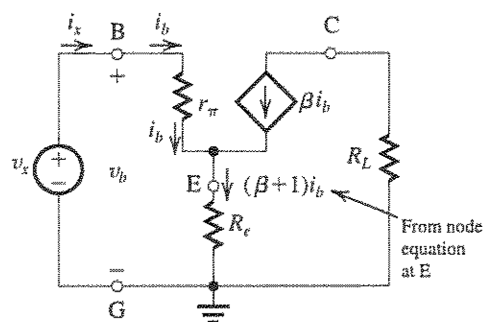
$$v_o = R_m i_i \times \frac{R_L}{R_L + R_o}$$

$$\frac{v_o}{i_i} = R_m \frac{R_L}{R_L + R_o}$$

$$\text{Now } \frac{v_o}{i_s} = \frac{v_o}{i_i} \times \frac{i_i}{i_s} = R_m \frac{R_L}{R_L + R_o} \times \frac{R_s}{R_i + R_s}$$

$$= R_m \frac{R_s}{R_s + R_i} \times \frac{R_L}{R_L + R_o}$$

Ex: 1.21



$$v_b = i_b r_\pi + (\beta + 1) i_b R_e$$

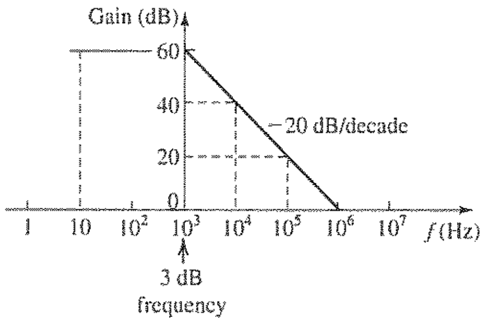
$$= i_b [r_\pi + (\beta + 1) R_e]$$

But $v_b = v_x$ and $i_b = i_x$, thus

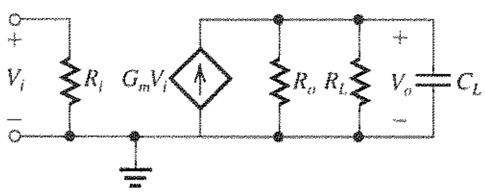
$$R_{in} \equiv \frac{v_x}{i_x} = \frac{v_b}{i_b} = r_\pi + (\beta + 1) R_e$$

Ex: 1.22

f	Gain
10 Hz	60 dB
10 kHz	40 dB
100 kHz	20 dB
1 MHz	0 dB



Ex: 1.23



$$V_o = G_m V_i [R_o \parallel R_L \parallel C_L]$$

$$= \frac{G_m V_i}{\frac{1}{R_o} + \frac{1}{R_L} + sC_L}$$

Thus, $\dfrac{V_o}{V_i} = \dfrac{G_m}{\frac{1}{R_o} + \frac{1}{R_L}} \times \dfrac{1}{1 + \dfrac{sC_L}{\frac{1}{R_o} + \frac{1}{R_L}}}$

$$\frac{V_o}{V_i} = \frac{G_m(R_L \parallel R_o)}{1 + sC_L(R_L \parallel R_o)}$$

which is of the STC LP type.

DC gain $= G_m(R_L \parallel R_o) = 10 \times (R_L \parallel 50)$

To obtain a dc gain of at least 40 dB (i.e., 100), $10(R_L \parallel 50) \geq 100$

$\Rightarrow R_L \geq 12.5\ \text{k}\Omega$

$$\omega_0 = \frac{1}{C_L(R_L \parallel R_o)}$$

$$= \frac{1}{C_L(12.5 \parallel 50) \times 10^3}$$

For ω_0 to be a least $2\pi \times 100 \times 10^3$, the highest value allowed for C_L is

$$C_L = \frac{1}{2\pi \times 10^5 \times 10 \times 10^3} = 159.2\ \text{pF}$$

Ex: 1.24 Refer to Fig. E1.24

$$\frac{V_2}{V_s} = \frac{R_i}{R_s + \frac{1}{sC} + R_i} = \frac{R_i}{R_s + R_i} \frac{s}{s + \frac{1}{C(R_s + R_i)}}$$

which is an HP STC function.

$$f_{3\text{dB}} = \frac{1}{2\pi C(R_s + R_i)} \leq 100\ \text{Hz}$$

$$C \geq \frac{1}{2\pi(1 + 9)10^3 \times 100} = 0.16\ \mu\text{F}$$

Ex: 2.1 The minimum number of terminals required by a single op amp is 5: two input terminals, one output terminal, one terminal for positive power supply, and one terminal for negative power supply.

The minimum number of terminals required by a quad op amp is 14: each op amp requires two input terminals and one output terminal (accounting for 12 terminals for the four op amps). In addition, the four op amps can all share one terminal for positive power supply and one terminal for negative power supply.

Ex: 2.2 Relevant equations are:

$v_3 = A(v_2 - v_1)$; $v_{Id} = v_2 - v_1$,

$v_{Icm} = \frac{1}{2}(v_1 + v_2)$

(a)

$$v_1 = v_2 - \frac{v_3}{A} = 0 - \frac{2}{10^3} = -0.002 \text{ V} = -2 \text{ mV}$$

$$v_{Id} = v_2 - v_1 = 0 - (-0.002) = +0.002 \text{ V}$$

$$= 2 \text{ mV}$$

$$v_{Icm} = \frac{1}{2}(-2 \text{ mV} + 0) = -1 \text{ mV}$$

(b) $-10 = 10^3(5 - v_1) \Rightarrow v_1 = 5.01$ V

$$v_{Id} = v_2 - v_1 = 5 - 5.01 = -0.01 \text{ V} = -10 \text{ mV}$$

$$v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(5.01 + 5) = 5.005 \text{ V}$$

$$\simeq 5 \text{ V}$$

(c)

$$v_3 = A(v_2 - v_1) = 10^3(0.998 - 1.002) = -4 \text{ V}$$

$$v_{Id} = v_2 - v_1 = 0.998 - 1.002 = -4 \text{ mV}$$

$$v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(1.002 + 0.998) = 1 \text{ V}$$

(d)

$$-3.6 = 10^3[v_2 - (-3.6)] = 10^3(v_2 + 3.6)$$

$$\Rightarrow v_2 = -3.6036 \text{ V}$$

$$v_{Id} = v_2 - v_1 = -3.6036 - (-3.6)$$

$$= -0.0036 \text{ V} = -3.6 \text{ mV}$$

$$v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}[-3.6 + (-3.6036)]$$

$$\simeq -3.6 \text{ V}$$

Ex: 2.3 From Fig. E2.3 we have: $v_3 = \mu v_d$ and

$$v_d = (G_m v_2 - G_m v_1)R = G_m R(v_2 - v_1)$$

Therefore:

$$v_3 = \mu G_m R(v_2 - v_1)$$

That is, the open-loop gain of the op amp is $A = \mu G_m R$. For $G_m = 10$ mA/V and

$\mu = 100$, we have:

$A = 100 \times 10 \times 10 = 10^4$ V/V, or equivalently, 80 dB.

Ex: 2.4 The gain and input resistance of the inverting amplifier circuit shown in Fig. 2.5 are $-\frac{R_2}{R_1}$ and R_1, respectively. Therefore, we have:

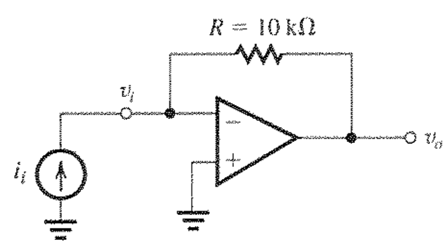
$R_1 = 100$ kΩ and

$$-\frac{R_2}{R_1} = -10 \Rightarrow R_2 = 10\,R_1$$

Thus:

$$R_2 = 10 \times 100 \text{ k}\Omega = 1 \text{ M}\Omega$$

Ex: 2.5



From Table 1.1 we have:

$R_m = \left.\frac{v_o}{i_i}\right|_{i_o=0}$; that is, output is open circuit

The negative input terminal of the op amp (i.e., v_i) is a virtual ground, thus $v_i = 0$:

$$v_o = v_i - Ri_i = 0 - Ri_i = -Ri_i$$

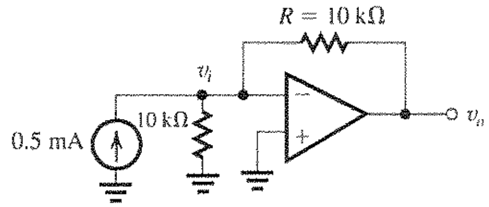
$$R_m = \left.\frac{v_o}{i_i}\right|_{i_o=0} = -\frac{Ri_i}{i_i} = -R \Rightarrow R_m = -R$$

$$= -10 \text{ k}\Omega$$

$R_i = \frac{v_i}{i_i}$ and v_i is a virtual ground ($v_i = 0$),

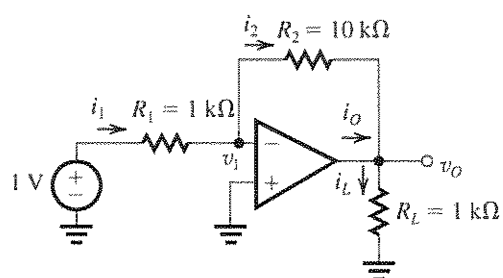
thus $R_i = \frac{0}{i_i} = 0 \Rightarrow R_i = 0\ \Omega$

Since we are assuming that the op amp in this transresistance amplifier is ideal, the op amp has zero output resistance and therefore the output resistance of this transresistance amplifier is also zero. That is $R_o = 0\ \Omega$.



Connecting the signal source shown in Fig. E2.5 to the input of this amplifier, we have:
v_i is a virtual ground that is $v_i = 0$, thus the current flowing through the 10-kΩ resistor connected between v_i and ground is zero. Therefore,
$v_o = v_i - R \times 0.5\ \text{mA} = 0 - 10\ \text{k}\Omega \times 0.5\ \text{mA}$
$= -5\ \text{V}$.

Ex: 2.6



v_1 is a virtual ground, thus $v_1 = 0$ V

$$i_i = \frac{1\ \text{V} - v_1}{R_1} = \frac{1 - 0}{1\ \text{k}\Omega} = 1\ \text{mA}$$

Assuming an ideal op amp, the current flowing into the negative input terminal of the op amp is zero. Therefore, $i_2 = i_1 \Rightarrow i_2 = 1$ mA

$$v_O = v_1 - i_2 R_2 = 0 - 1\ \text{mA} \times 10\ \text{k}\Omega = -10\ \text{V}$$

$$i_L = \frac{v_o}{R_L} = \frac{-10\ \text{V}}{1\ \text{k}\Omega} = -10\ \text{mA}$$

$$i_O = i_L - i_2 = -10\ \text{mA} - 1\ \text{mA} = -11\ \text{mA}$$

Voltage gain $= \dfrac{v_O}{1\ \text{V}} = \dfrac{-10\ \text{V}}{1\ \text{V}} = -10$ V/V or 20 dB

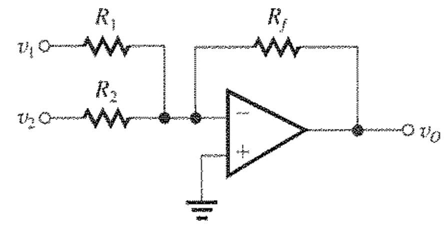
Current gain $= \dfrac{i_L}{i_1} = \dfrac{-10\ \text{mA}}{1\ \text{mA}} = -10$ A/A or 20 dB

Power gain

$$= \frac{P_L}{P_i} = \frac{-10(-10\ \text{mA})}{1\ \text{V} \times 1\ \text{mA}} = 100\ \text{W/W or 20 dB}$$

Note that power gain in dB is $10 \log_{10} \left|\dfrac{P_L}{P_i}\right|$.

Ex: 2.7



For the circuit shown above we have:

$$v_O = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2\right)$$

Since it is required that $v_O = -(v_1 + 5v_2)$,

we want to have:

$$\frac{R_f}{R_1} = 1 \quad \text{and} \quad \frac{R_f}{R_2} = 5$$

It is also desired that for a maximum output voltage of 10 V, the current in the feedback resistor not exceed 1 mA.

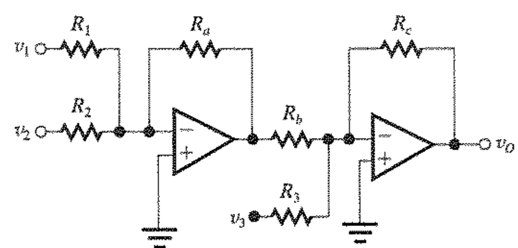
Therefore

$$\frac{10\ \text{V}}{R_f} \leq 1\ \text{mA} \Rightarrow R_f \geq \frac{10\ \text{V}}{1\ \text{mA}} \Rightarrow R_f \geq 10\ \text{k}\Omega$$

Let us choose R_f to be 10 kΩ, then

$$R_1 = R_f = 10\ \text{k}\Omega \text{ and } R_2 = \frac{R_f}{5} = 2\ \text{k}\Omega$$

Ex: 2.8



$$v_O = \left(\frac{R_a}{R_1}\right)\left(\frac{R_c}{R_b}\right)v_1 + \left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right)v_2 - \left(\frac{R_c}{R_3}\right)v_3$$

We want to design the circuit such that

$$v_O = 2v_1 + v_2 - 4v_3$$

Thus we need to have

$$\left(\frac{R_a}{R_1}\right)\left(\frac{R_c}{R_b}\right) = 2, \left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right) = 1, \text{ and } \frac{R_c}{R_3} = 4$$

From the above three equations, we have to find six unknown resistors; therefore, we can arbitrarily choose three of these resistors. Let us choose $R_a = R_b = R_c = 10\ \text{k}\Omega$.

Then we have

$$R_3 = \frac{R_c}{4} = \frac{10}{4} = 2.5\ \text{k}\Omega$$

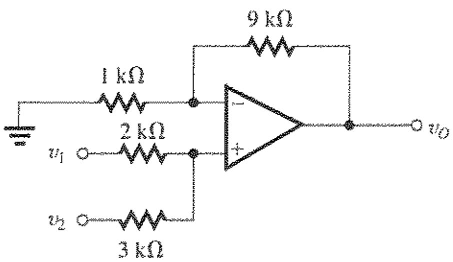
$$\left(\frac{R_a}{R_1}\right)\left(\frac{R_c}{R_b}\right) = 2, \Rightarrow \frac{10}{R_1} \times \frac{10}{10} = 2$$

$$\Rightarrow R_1 = 5\ \text{k}\Omega$$

$$\left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right) = 1 \Rightarrow \frac{10}{R_2} \times \frac{10}{10} = 1$$

$$\Rightarrow R_2 = 10\ \text{k}\Omega$$

Ex: 2.9 Using the superposition principle to find the contribution of v_1 to the output voltage v_O, we set $v_2 = 0$



v_+ (the voltage at the positive input of the op amp is: $v_+ = \frac{3}{2+3}v_1 = 0.6v_1$

Thus $v_O = \left(1 + \frac{9\ \text{k}\Omega}{1\ \text{k}\Omega}\right)v_+ = 10 \times 0.6v_1 = 6v_1$

To find the contribution of v_2 to the output voltage v_O we set $v_1 = 0$.

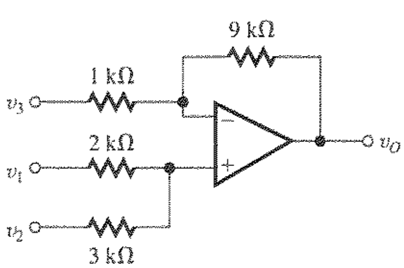
Then $v_+ = \frac{2}{2+3}v_2 = 0.4v_2$

Hence

$$v_O = \left(1 + \frac{9\ \text{k}\Omega}{1\ \text{k}\Omega}\right)v_+ = 10 \times 0.4v_2 = 4v_2$$

Combining the contributions of v_1 and v_2 to v_O, we have $v_O = 6v_1 + 4v_2$

Ex: 2.10



Using the superposition principle to find the contribution of v_1 to v_O, we set $v_2 = v_3 = 0$. Then we have (refer to the solution of Exercise 2.9): $v_O = 6v_1$

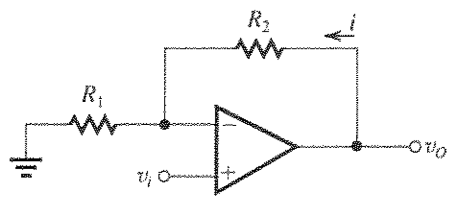
To find the contribution of v_2 to v_O, we set $v_1 = v_3 = 0$, then: $v_O = 4v_2$

To find the contribution of v_3 to v_O we set $v_1 = v_2 = 0$, then

$$v_O = -\frac{9\ \text{k}\Omega}{1\ \text{k}\Omega}v_3 = -9v_3$$

Combining the contributions of v_1, v_2, and v_3 to v_O we have: $v_O = 6v_1 + 4v_2 - 9v_3$.

Ex: 2.11



$$\frac{v_O}{v_I} = 1 + \frac{R_2}{R_1} = 2 \Rightarrow \frac{R_2}{R_1} = 1 \Rightarrow R_1 = R_2$$

If $v_O = 10$ V, then it is desired that $i = 10\ \mu\text{A}$.

Thus,

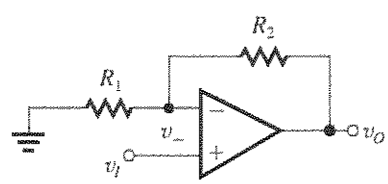
$$i = \frac{10\ \text{V}}{R_1 + R_2} = 10\ \mu\text{A} \Rightarrow R_1 + R_2 = \frac{10\ \text{V}}{10\ \mu\text{A}}$$

$R_1 + R_2 = 1\ \text{M}\Omega$ and

$R_1 = R_2 \Rightarrow R_1 = R_2 = 0.5\ \text{M}\Omega$

Ex: 2.12

(a)



$$v_I - v_- = v_O/A \Rightarrow v_- = v_I - v_O/A \quad (1)$$

But from the voltage divider across v_O,

$$v_- = v_O\frac{R_1}{R_1 + R_2} \quad (2)$$

Equating Eq. (1) and Eq. (2) gives

$$v_O\frac{R_1}{R_1 + R_2} = v_I - \frac{v_O}{A}$$

which can be manipulated to the form

$$\frac{v_O}{v_I} = \frac{1 + (R_2/R_1)}{1 + \frac{1 + (R_2/R_1)}{A}}$$

(b) For $R_1 = 1\ \text{k}\Omega$ and $R_2 = 9\ \text{k}\Omega$ the ideal value for the closed-loop gain is $1 + \frac{9}{1}$, that is, 10. The actual closed-loop gain is $G = \frac{10}{1 + 10/A}$.

If $A = 10^3$, then $G = 9.901$ and

$$\epsilon = \frac{G - 10}{10} \times 100 = -0.99\% \simeq -1\%$$

For $v_I = 1$ V, $v_O = G \times v_I = 9.901$ V and

$$v_O = A(v_+ - v_-) \Rightarrow v_+ - v_- = \frac{v_O}{A} = \frac{9.901}{1000}$$

$\simeq 9.9$ mV

If $A = 10^4$, then $G = 9.99$ and $\epsilon = -0.1\%$.

For $v_I = 1$ V, $v_O = G \times v_I = 9.99$ V,

therefore,

$$v_+ - v_- = \frac{v_O}{A} = \frac{9.99}{10^4} = 0.999\ \text{mV} \simeq 1\ \text{mV}$$

If $A = 10^5$, then $G = 9.999$ and $\epsilon = -0.01\%$

For $v_I = 1$ V, $v_O = G \times v_I = 9.999$ thus,

$$v_+ - v_- = \frac{v_O}{A} = \frac{9.999}{10^5} = 0.09999\ \text{mV}$$

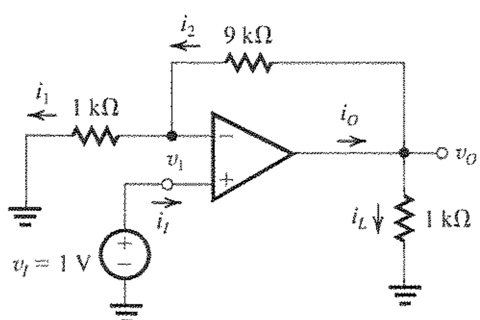
$\simeq 0.1$ mV

Ex: 2.13

$i_I = 0$ A, $v_1 = v_I = 1$ V

$$i_1 = \frac{v_1}{1\ \text{k}\Omega} = \frac{1\ \text{V}}{1\ \text{k}\Omega} = 1\ \text{mA}$$

$i_2 = i_1 = 1$ mA



$$v_O = v_1 + i_2 \times 9\ \text{k}\Omega = 1 + 1 \times 9 = 10\ \text{V}$$

$$i_L = \frac{v_O}{1\ \text{k}\Omega} = \frac{10\ \text{V}}{1\ \text{k}\Omega} = 10\ \text{mA}$$

$i_O = i_L + i_2 = 11$ mA

$$\frac{v_O}{v_I} = \frac{10\ \text{V}}{1\ \text{V}} = 10\ \text{V/V or 20 dB}$$

$$\frac{i_L}{i_I} = \frac{10\ \text{mA}}{0} = \infty$$

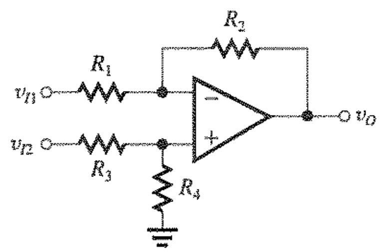
$$\frac{P_L}{P_I} = \frac{v_O \times i_L}{v_I \times i_I} = \frac{10 \times 10}{1 \times 0} = \infty$$

Ex: 2.14

(a) Load voltage $= \frac{1\ \text{k}\Omega}{1\ \text{k}\Omega + 1\ \text{M}\Omega} \times 1\ \text{V} \simeq 1\ \text{mV}$

(b) Load voltage = 1 V

Ex: 2.15



(a) $R_1 = R_3 = 2\ \text{k}\Omega$, $R_2 = R_4 = 200\ \text{k}\Omega$

Since $R_4/R_3 = R_2/R_1$ we have:

$$A_d = \frac{v_O}{v_{I2} - v_{I1}} = \frac{R_2}{R_1} = \frac{200}{2} = 100\ \text{V/V}$$

(b) $R_{id} = 2R_1 = 2 \times 2\ \text{k}\Omega = 4\ \text{k}\Omega$

Since we are assuming the op amp is ideal,

$R_o = 0\ \Omega$

(c)

$$A_{cm} \equiv \frac{v_O}{v_{Icm}} = \frac{R_4}{R_3 + R_4}\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

The worst-case common-mode gain (i.e., the largest A_{cm}) occurs when the resistor tolerances are such that the quantity in parentheses is maximum. This in turn occurs when R_2 and R_3 are at their highest possible values (each one percent above nominal) and R_1 and R_4 are at their lowest possible values (each one percent below nominal), resulting in

$$A_{cm} = \frac{R_4}{R_3 + R_4}\left(1 - \frac{1.01 \times 1.01}{0.99 \times 0.99}\right)$$

$$|A_{cm}| \simeq \frac{R_4}{R_3 + R_4} \times 0.04 \simeq \frac{200}{202} \times 0.04 \simeq 0.04\ \text{V/V}$$

The corresponding CMRR is

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{100}{0.04} = 2500$$

or 68 dB.

Ex: 2.16 We choose $R_3 = R_1$ and $R_4 = R_2$. Then for the circuit to behave as a difference amplifier with a gain of 10 and an input resistance of 20 kΩ, we require

$$A_d = \frac{R_2}{R_1} = 10 \text{ and}$$

$$R_{Id} = 2R_1 = 20\text{ k}\Omega \Rightarrow R_1 = 10\text{ k}\Omega \text{ and}$$

$$R_2 = A_d R_1 = 10 \times 10\text{ k}\Omega = 100\text{ k}\Omega$$

Therefore, $R_1 = R_3 = 10\text{ k}\Omega$ and

$R_2 = R_4 = 100\text{ k}\Omega$.

Ex: 2.17 Given $v_{Icm} = +5$ V

$$v_{Id} = 10 \sin \omega t \text{ mV}$$

$$2R_1 = 1\text{ k}\Omega,\ R_2 = 0.5\text{ M}\Omega$$

$$R_3 = R_4 = 10\text{ k}\Omega$$

$$v_{I1} = v_{Icm} - \frac{1}{2}v_{Id} = 5 - \frac{1}{2} \times 0.01 \sin \omega t$$

$$= 5 - 0.005 \sin \omega t \text{ V}$$

$$v_{I2} = v_{Icm} + \frac{1}{2}v_{Id}$$

$$= 5 + 0.005 \sin \omega t \text{ V}$$

$$v_-(\text{op amp } A_1) = v_{I1} = 5 - 0.005 \sin \omega t \text{ V}$$

$$v_-(\text{op amp } A_2) = v_{I2} = 5 + 0.005 \sin \omega t \text{ V}$$

$$v_{Id} = v_{I2} - v_{I1} = 0.01 \sin \omega t$$

$$v_{O1} = v_{I1} - R_2 \times \frac{v_{Id}}{2R_1}$$

$$= 5 - 0.005 \sin \omega t - 500\text{ k}\Omega \times \frac{0.01 \sin \omega t}{1\text{ k}\Omega}$$

$$= (5 - 5.005 \sin \omega t) \text{ V}$$

$$v_{O2} = v_{I2} + R_2 \times \frac{v_{Id}}{2R_1}$$

$$= (5 + 5.005 \sin \omega t) \text{ V}$$

$$v_+(\text{op amp } A_3) = v_{O2} \times \frac{R_4}{R_3 + R_4} = v_{O2}\frac{10}{10 + 10}$$

$$= \frac{1}{2}v_{O2} = \frac{1}{2}(5 + 5.005 \sin \omega t)$$

$$= (2.5 + 2.5025 \sin \omega t)\text{V}$$

$$v_-(\text{op amp } A_3) = v_+(\text{op amp } A_3)$$

$$= (2.5 + 2.5025 \sin \omega t) \text{ V}$$

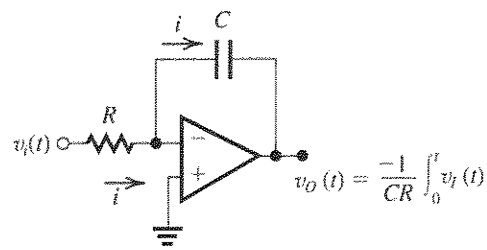
$$v_O = \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right)v_{Id}$$

$$\frac{10\text{ k}\Omega}{10\text{ k}\Omega}\left(1 + \frac{0.5\text{ M}\Omega}{0.5\text{ k}\Omega}\right) \times 0.01 \sin \omega t$$

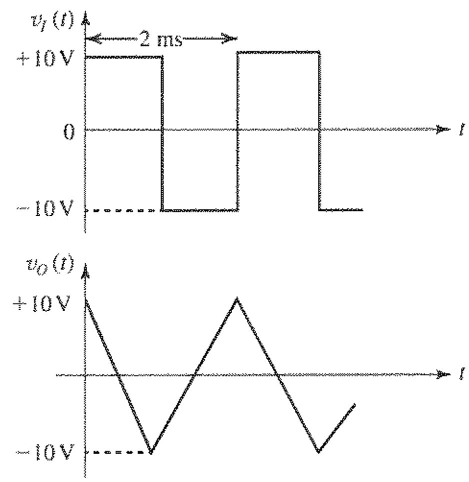
$$= 1(1 + 1000) \times 0.01 \sin \omega t$$

$$= 10.01 \sin \omega t \text{ V}$$

Ex: 2.18



The signal waveforms will be as shown.



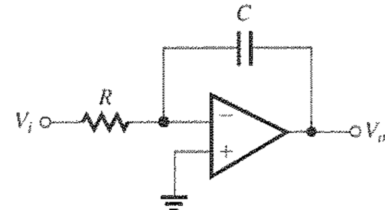
When $v_I = +10$ V, the current through the capacitor will be in the direction indicated, $i = 10\text{ V}/R$, and the output voltage will decrease linearly from +10 V to −10 V. Thus in $(T/2)$ seconds, the capacitor voltage changes by 20 V. The charge equilibrium equation can be expressed as

$$i(T/2) = C \times 20 \text{ V}$$

$$\frac{10}{R}\frac{T}{2} = 20C \Rightarrow CR = \frac{10T}{40} = \frac{1}{4} \times 2 \times 10^{-3}$$

$$= 0.5 \text{ ms}$$

Ex: 2.19



The input resistance of this inverting integrator is R; therefore, $R = 10\text{ k}\Omega$.

Since the desired integration time constant is 10^{-3} s, we have: $CR = 10^{-3}$ s $\Rightarrow$

$$C = \frac{10^{-3}\ \text{s}}{10\ \text{k}\Omega} = 0.1\ \mu\text{F}$$

From Eq. (2.27) the transfer function of this integrator is:

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{1}{j\omega CR}$$

For $\omega = 10$ rad/s, the integrator transfer function has magnitude

$$\left|\frac{V_o}{V_i}\right| = \frac{1}{1 \times 10^{-3}} = 100\ \text{V/V}$$

and phase $\phi = 90°$.

For $\omega = 1$ rad/s, the integrator transfer function has magnitude

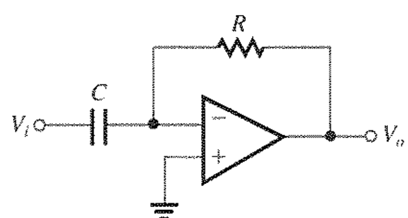
$$\left|\frac{V_o}{V_i}\right| = \frac{1}{1 \times 10^{-3}} = 1000\ \text{V/V}$$

and phase $\phi = 90°$.

The frequency at which the integrator gain magnitude is unity is

$$\omega_{\text{int}} = \frac{1}{CR} = \frac{1}{10^{-3}} = 1000\ \text{rad/s}$$

Ex: 2.20



$C = 0.01\ \mu$F is the input capacitance of this differentiator. We want $CR = 10^{-2}$ s (the time constant of the differentiator); thus,

$$R = \frac{10^{-2}}{0.01\ \mu\text{F}} = 1\ \text{M}\Omega$$

From Eq. (2.33), the transfer function of the differentiator is

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -j\omega CR$$

Thus, for $\omega = 10$ rad/s the differentiator transfer function has magnitude

$$\left|\frac{V_o}{V_i}\right| = 10 \times 10^{-2} = 0.1\ \text{V/V}$$

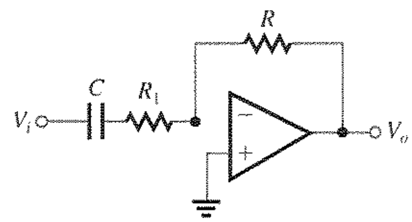
and phase $\phi = -90°$.

For $\omega = 10^3$ rad/s, the differentiator transfer function has magnitude

$$\left|\frac{V_o}{V_i}\right| = 10^3 \times 10^{-2} = 10\ \text{V/V}$$

and phase $\phi = -90°$.

If we add a resistor in series with the capacitor to limit the high-frequency gain of the differentiator to 100, the circuit would be:



At high frequencies the capacitor C acts like a short circuit. Therefore, the high-frequency gain of this circuit is: $\frac{R}{R_1}$. To limit the magnitude of this high-frequency gain to 100, we should have:

$$\frac{R}{R_1} = 100 \Rightarrow R_1 = \frac{R}{100} = \frac{1\ \text{M}\Omega}{100} = 10\ \text{k}\Omega$$

Ex: 2.21

Refer to the model in Fig. 2.26 and observe that

$$v_+ - v_- = V_{OS} + v_2 - v_1 = V_{OS} + v_{Id}$$

and since $v_O = v_3 = A(v_+ - v_-)$, then

$$v_O = A(v_{Id} + V_{OS}) \qquad (1)$$

where $A = 10^4$ V/V and $V_{OS} = 5$ mV. From Eq. (1) we see that $v_{Id} = 0$ results in $v_O = 50$ V, which is impossible; thus the op amp saturates and $v_O = +10$ V. This situation pertains for $v_{Id} \geq -4$ mV. If v_{Id} decreases below -4 mV, the op-amp output decreases correspondingly. For instance, $v_{Id} = -4.5$ mV results in $v_O = +5$ V; $v_{Id} = -5$ mV results in $v_O = 0$ V; $v_{Id} = -5.5$ mV results in $v_O = -5$ V; and $v_{Id} = -6$ mV results in $v_O = -10$ V, at which point the op amp saturates at the negative level of -10 V. Further decreases in v_{Id} have no effect on the output voltage. The result is the transfer characteristic sketched in Fig. E2.21. Observe that the linear range of the characteristic is now centered around $v_{Id} = -5$ mV rather than the ideal situation of $v_{Id} = 0$; this shift is obviously a result of the input offset voltage V_{OS}.

Ex: 2.22 (a) The inverting amplifier of -1000 V/V gain will exhibit an output dc offset voltage of $\pm V_{OS}(1 + R_2/R_1) =$ ± 3 mV $\times$ $(1 + 1000) = \pm 3.03$ V. Now, since the op-amp saturation levels are ± 10 V, the room left for output signal swing is approximately ± 7 V. Thus to avoid op-amp saturation and the attendant clipping of the peak of the output sinusoid, we must limit the peak amplitude of the input sine wave to approximately $7\ \text{V}/1000 = 7$ mV.

(b) If at room temperature (25°C), V_{OS} is trimmed to zero and (i) the circuit is operated at a constant temperature, the peak of the input sine wave can be increased to 10 mV. (ii) However, if the circuit is to operate in the temperature range of 0°C to 75°C (i.e., at a temperature that deviates from room temperature by a maximum of 50°C), the input offset voltage will drift from by a maximum of 10 μV/°C × 50°C = 500 μV or 0.5 mV. This will reduce the allowed peak amplitude of the input sinusoid to 9.5 mV.

Ex: 2.23

(a) If the amplifier is capacitively coupled in the manner of Fig. 2.31(a), then the input offset voltage V_{OS} will see a unity-gain amplifier [Fig. 2.31(b)] and the cc offset voltage at the output will be equal to V_{OS}, that is, 3 mV. Thus, almost the entire output range of ± 10 V will be available for signal swing, allowing a sine-wave input of approximately 10-mV peak without the risk of output clipping. Obviously, in this case there is no need for output trimming.

(b) We need to select a value of the coupling capacitor C that will place the 3-dB frequency of the resulting high-pass STC circuit at 100 Hz, thus

$$100 = \frac{1}{2\pi C R_1}$$

$$\Rightarrow C = \frac{1}{2\pi \times 100 \times 1 \times 10^3} = 1.6\ \mu\text{F}$$

Ex: 2.24 From Eq. (2.37) we have:

$$V_O = I_{B1} R_2 \simeq I_B R_2$$

$$= 100\ \text{nA} \times 1\ \text{M}\Omega = 0.1\ \text{V}$$

From Eq. (2.39) the value of resistor R_3 (placed in series with positive input to minimize the output offset voltage) is

$$R_3 = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{10\ \text{k}\Omega \times 1\ \text{M}\Omega}{10\ \text{k}\Omega + 1\ \text{M}\Omega}$$

$$= 9.9\ \text{k}\Omega$$

$$R_3 = 9.9\ \text{k}\Omega \simeq 10\ \text{k}\Omega$$

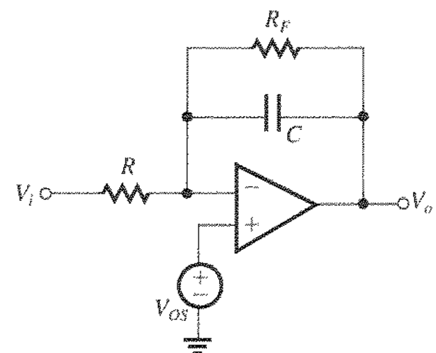
With this value of R_3, the new value of the output dc voltage [using Eq. (2.40)] is:

$$V_O = I_{OS} R_2 = 10\ \text{nA} \times 10\ \text{k}\Omega = 0.01\ \text{V}$$

Ex: 2.25 Using Eq. (2.41) we have:

$$v_O = V_{OS} + \frac{V_{OS}}{CR} t \Rightarrow 12 = 2\ \text{mV} + \frac{2\ \text{mV}}{1\ \text{ms}} t$$

$$\Rightarrow t = \frac{12\ \text{V} - 2\ \text{mV}}{2\ \text{mV}} \times 1\ \text{ms} \simeq 6\ \text{s}$$



With the feedback resistor R_F, to have at least ± 10 V of output signal swing available, we have to make sure that the output voltage due to V_{OS} has a magnitude of at most 2 V. From Eq. (2.36), we know that the output dc voltage due to V_{OS} is

$$V_O = V_{OS}\left(1 + \frac{R_F}{R}\right) \Rightarrow 2\ \text{V} = 2\ \text{mV}\left(1 + \frac{R_F}{10\ \text{k}\Omega}\right)$$

$$1 + \frac{R_F}{10\ \text{k}\Omega} = 1000 \Rightarrow R_F \simeq 10\ \text{M}\Omega$$

The corner frequency of the resulting STC network is $\omega_0 = \dfrac{1}{CR_F}$

We know $RC = 1$ ms and

$$R = 10\ \text{k}\Omega \Rightarrow C = 0.1\ \mu\text{F}$$

$$\text{Thus } \omega_0 = \frac{1}{0.1\ \mu\text{F} \times 10\ \text{M}\Omega} = 1\ \text{rad/s}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi} = 0.16\ \text{Hz}$$

Ex: 2.26

$$20 \log A_0 = 106\ \text{dB} \Rightarrow A_0 = 200{,}000\ \text{V/V}$$

$$f_t = 3\ \text{MHz}$$

$$f_b = f_t/A_0 = \frac{3\ \text{MHz}}{200{,}000} = 15\ \text{Hz}$$

At f_b, the open-loop gain drops by 3 dB below its value at dc; thus it becomes 103 dB.

For $f \gg f_b$, $|A| \simeq f_t/f$; thus

At $f = 300$ Hz, $|A| = \dfrac{3 \text{ MHz}}{300 \text{ Hz}} = 10^4$ V/V

or 80 dB

At $f = 3$ kHz, $|A| = \dfrac{3 \text{ MHz}}{3 \text{ kHz}} = 10^3$ V/V

or 60 dB

At $f = 12$ kHz, which is two octaves higher than 3 kHz, the gain will be $2 \times 6 = 12$ dB below its value at 3 kHz; that is, $60 - 12 = 48$ dB.

At $f = 60$ kHz, $|A| = \dfrac{3 \text{ MHz}}{60 \text{ kHz}} = 50$ V/V

or 34 dB

Ex: 2.27

$A_0 = 10^6$ V/V or 120 dB

The gain falls off at the rate of 20 dB/decade. Thus, it reaches 40 dB at a frequency four decades higher than f_b,

$10^4 f_b = 10 \text{ kHz} \Rightarrow f_b = 1 \text{ Hz}$

The unity-gain frequency f_t will be two decades higher than 10 kHz, that is,

$f_t = 100 \times 10 \text{ kHz} = 1 \text{ MHz}$

Alternatively, we could have found f_t from

$f_t = A_0 f_b = 10^6 \times 1 \text{ Hz} = 1 \text{ MHz}$

At a frequency $f \gg f_b$,

$|A| = f_t/f$

For $f = 1$ kHz, $|A| = \dfrac{1 \text{ MHz}}{1 \text{ kHz}} = 10^3$ V/V or 60 dB

Ex: 2.28

$20 \log A_0 = 106 \text{ dB} \Rightarrow A_0 = 200{,}000$ V/V

$f_t = 2$ MHz

For a noninverting amplifier with a nominal dc gain of 100,

$$1 + \frac{R_2}{R_1} = 100$$

Since the nominal dc gain is much lower than A_0,

$$f_{3\text{dB}} \simeq f_t \bigg/ \left(1 + \frac{R_2}{R_1}\right)$$

$$= \frac{2 \text{ MHz}}{100} = 20 \text{ kHz}$$

Ex: 2.29 For the input voltage step of magnitude V the output waveform will still be given by the exponential waveform of Eq. (2.58) if

$$\omega_t V \le SR$$

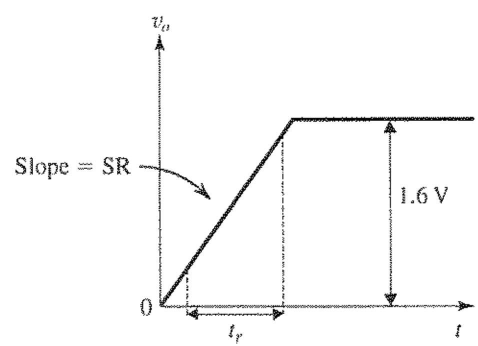
that is, $V \le \dfrac{SR}{\omega_t} \Rightarrow V \le \dfrac{SR}{2\pi f_t}$ resulting in

$$V \le 0.16 \text{ V}$$

From Appendix F we know that the 10% to 90% rise time of the output waveform of the form of Eq. (2.58) is $t_r \simeq 2.2 \times \text{time constant} = \dfrac{2.2}{\omega_t}$.

Thus, $t_r \simeq 0.35$ μs

If an input step of amplitude 1.6 V (10 times as large compared to the previous case) is applied, the output will be slew-rate limited and thus linearly rising with a slope equal to the slew rate, as shown in the following figure.



$$t_r = \frac{0.9 \times 1.6 - 0.1 \times 1.6}{1 \text{ V}/\mu\text{s}}$$

$\Rightarrow t_r = 1.28$ μs

Ex: 2.30 From Eq. (2.59) we have:

$$f_M = \frac{SR}{2\pi V_{O\ \max}} = 15.915 \text{ kHz} \simeq 15.9 \text{ kHz}$$

Using Eq. (2.60), for an input sinusoid with frequency $f = 5 f_M$, the maximum possible amplitude that can be accommodated at the output without incurring SR distortion is:

$$V_O = V_{O\ \max}\left(\frac{f_M}{5 f_M}\right) = 10 \times \frac{1}{5} = 2 \text{ V (peak)}$$

Ex: 3.1 $T = 50$ K

$n_i = BT^{3/2}e^{-Eg/(2kT)}$

$= 7.3 \times 10^{15}(50)^{3/2}\ e^{-1.12/(2\times 8.62\times 10^{-5}\times 50)}$

$\simeq 9.6 \times 10^{-39}/\text{cm}^3$

$T = 350$ K

$n_i = BT^{3/2}e^{-Eg/(2kT)}$

$= 7.3 \times 10^{15}(350)^{3/2}\ e^{-1.12/(2\times 8.62\times 10^{-5}\times 350)}$

$= 4.15 \times 10^{11}/\text{cm}^3$

Ex: 3.2 $N_D = 10^{17}/\text{cm}^3$

From Exercise 3.1, n_i at

$T = 350\text{ K} = 4.15 \times 10^{11}/\text{cm}^3$

$n_n = N_D = 10^{17}/\text{cm}^3$

$$p_n \cong \frac{ni^2}{N_D}$$

$$= \frac{(4.15 \times 10^{11})^2}{10^{17}}$$

$= 1.72 \times 10^6/\text{cm}^3$

Ex: 3.3 At 300 K, $n_i = 1.5 \times 10^{10}/\text{cm}^3$

$p_p = N_A$

Want electron concentration

$$= n_p = \frac{1.5 \times 10^{10}}{10^6} = 1.5 \times 10^4/\text{cm}^3$$

$$\therefore N_A = p_p = \frac{ni^2}{n_p}$$

$$= \frac{(1.5 \times 10^{10})^2}{1.5 \times 10^4}$$

$= 1.5 \times 10^{16}/\text{cm}^3$

Ex: 3.4 (a) $v_{n\text{-drift}} = -\mu_n E$

Here negative sign indicates that electrons move in a direction opposite to E.

We use

$$v_{n\text{-drift}} = 1350 \times \frac{1}{2 \times 10^{-4}}\ \because 1\ \mu\text{m} = 10^{-4}\text{ cm}$$

$= 6.75 \times 10^6\text{ cm/s} = 6.75 \times 10^4\text{ m/s}$

(b) Time taken to cross 2-μm

$$\text{length} = \frac{2 \times 10^{-6}}{6.75 \times 10^4} \simeq 30\text{ ps}$$

(c) In n-type silicon, drift current density J_n is

$J_n = qn\mu_n E$

$$= 1.6 \times 10^{-19} \times 10^{16} \times 1350 \times \frac{1\text{ V}}{2 \times 10^{-4}}$$

$= 1.08 \times 10^4\text{ A/cm}^2$

(d) Drift current $I_n = AJ_n$

$= 0.25 \times 10^{-8} \times 1.08 \times 10^4$

$= 27\ \mu\text{A}$

Note that $0.25\ \mu\text{m}^2 = 0.25 \times 10^{-8}\text{ cm}^2$.

Ex: 3.5 $J_n = qD_n \dfrac{dn(x)}{dx}$

From Fig. E3.5,

$n_0 = 10^{17}/\text{cm}^3 = 10^5/(\mu\text{m})^3$

$D_n = 35\text{ cm}^2/\text{s} = 35 \times (10^4)^2\ (\mu\text{m})^2/\text{s}$

$= 35 \times 10^8\ (\mu\text{m})^2/\text{s}$

$$\frac{dn}{dx} = \frac{10^5 - 0}{1} = 10^5\mu\text{m}^{-4}$$

$$J_n = qD_n\frac{dn(x)}{dx}$$

$= 1.6 \times 10^{-19} \times 35 \times 10^8 \times 10^5$

$= 56 \times 10^{-6}\text{ A}/\mu\text{m}^2$

$= 56\ \mu\text{A}/\mu\text{m}^2$

For $I_n = 1\text{ mA} = J_n \times A$

$$\Rightarrow A = \frac{1\text{ mA}}{J_n} = \frac{10^3\ \mu\text{A}}{56\ \mu\text{A}/(\mu\text{m})^2} \simeq 18\ \mu\text{m}^2$$

Ex: 3.6 Using Eq. (3.21),

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$$

$D_n = \mu_n V_T = 1350 \times 25.9 \times 10^{-3}$

$\cong 35\text{ cm}^2/\text{s}$

$D_p = \mu_p V_T = 480 \times 25.9 \times 10^{-3}$

$\cong 12.4\text{ cm}^2/\text{s}$

Ex: 3.7 Equation (3.26)

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0}$$

$$= \sqrt{\frac{2\epsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)V_0}$$

$$W^2 = \frac{2\epsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)V_0$$

$$V_0 = \frac{1}{2}\left(\frac{q}{\epsilon_s}\right)\left(\frac{N_A N_D}{N_A + N_D}\right)W^2$$

Ex: 3.8 In a p^+n diode $N_A \gg N_D$

Equation (3.26) $W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_0}$

We can neglect the term $\frac{1}{N_A}$ as compared to $\frac{1}{N_D}$, thus

$$W \simeq \sqrt{\frac{2\epsilon_s}{qN_D}\cdot V_0}$$

Equation (3.27) $x_n = W\frac{N_A}{N_A+N_D}$

$$\simeq W\,\frac{N_A}{N_A}$$

$$= W$$

Equation (3.28), $x_p = W\frac{N_D}{N_A+N_D}$

since $N_A \gg N_D$

$$\simeq W\,\frac{N_D}{N_A} = W\Big/\left(\frac{N_A}{N_D}\right)$$

Equation (3.29), $Q_J = Aq\left(\frac{N_AN_D}{N_A+N_D}\right)W$

$$\simeq Aq\frac{N_AN_D}{N_A}W$$

$$= AqN_DW$$

Equation (3.30), $Q_J = A\sqrt{2\epsilon_s q\left(\frac{N_AN_D}{N_A+N_D}\right)V_0}$

$\simeq A\sqrt{2\epsilon_s q\left(\frac{N_AN_D}{N_A}\right)V_0}$ since $N_A \gg N_D$

$$= A\sqrt{2\epsilon_s qN_DV_0}$$

Ex: 3.9 In Example 3.5, $N_A = 10^{18}/\text{cm}^3$ and $N_D = 10^{16}/\text{cm}^3$

In the n-region of this pn junction

$$n_n = N_D = 10^{16}/\text{cm}^3$$

$$p_n = \frac{n_i^2}{n_n} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^4/\text{cm}^3$$

As one can see from above equation, to increase minority-carrier concentration (p_n) by a factor of 2, one must lower N_D ($= n_n$) by a factor of 2.

Ex: 3.10

Equation (3.41) $I_S = Aqn_i^2\left(\frac{D_p}{L_pN_D}+\frac{D_n}{L_nN_A}\right)$

since $\frac{D_p}{L_p}$ and $\frac{D_n}{L_n}$ have approximately similar values, if $N_A \gg N_D$, then the term $\frac{D_n}{L_nN_A}$ can be neglected as compared to $\frac{D_p}{L_pN_D}$

$$\therefore I_S \cong Aqn_i^2\frac{D_p}{L_pN_D}$$

Ex: 3.11 $I_S = Aqn_i^2\left(\frac{D_p}{L_pN_D}+\frac{D_n}{L_nN_A}\right)$

$$= 10^{-4}\times1.6\times10^{-19}\times(1.5\times10^{10})^2$$

$$\times\left(\frac{10}{5\times10^{-4}\times\frac{10^{16}}{2}}+\frac{18}{10\times10^{-4}\times10^{18}}\right)$$

$$= 1.46\times10^{-14}\text{ A}$$

$$I = I_S(e^{V/V_T}-1)$$

$$\simeq I_Se^{V/V_T} = 1.45\times10^{-14}e^{0.605/(25.9\times10^{-3})}$$

$$= 0.2\text{ mA}$$

Ex: 3.12 $W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(V_0-V_F)}$

$$= \sqrt{\frac{2\times1.04\times10^{-12}}{1.6\times10^{-19}}\left(\frac{1}{10^{18}}+\frac{1}{10^{16}}\right)(0.814-0.605)}$$

$$= 1.66\times10^{-5}\text{ cm} = 0.166\ \mu\text{m}$$

Ex: 3.13 $W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(V_0+V_R)}$

$$= \sqrt{\frac{2\times1.04\times10^{-12}}{1.6\times10^{-19}}\left(\frac{1}{10^{18}}+\frac{1}{10^{16}}\right)(0.814+2)}$$

$$= 6.08\times10^{-5}\text{ cm} = 0.608\ \mu\text{m}$$

Using Eq. (3.29),

$$Q_J = Aq\left(\frac{N_AN_D}{N_A+N_D}\right)W$$

$$= 10^{-4}\times1.6\times10^{-19}\left(\frac{10^{18}\times10^{16}}{10^{18}+10^{16}}\right)\times6.08\times10^{-5}\text{ cm}$$

$$= 9.63\text{ pC}$$

Reverse current $I = I_S = Aqn_i^2\left(\frac{D_p}{L_pN_D}+\frac{D_n}{L_nN_A}\right)$

$$= 10^{-14}\times1.6\times10^{-19}\times(1.5\times10^{10})^2$$

$$\times\left(\frac{10}{5\times10^{-4}\times10^{16}}+\frac{18}{10\times10^{-4}\times10^{18}}\right)$$

$$= 7.3\times10^{-15}\text{ A}$$

Ex: 3.14 Equation (3.48),

$$C_{j0} = A\sqrt{\left(\frac{\epsilon_s q}{2}\right)\left(\frac{N_A N_D}{N_A + N_D}\right)\left(\frac{1}{V_0}\right)}$$

$$= 10^{-4}\sqrt{\left(\frac{1.04 \times 10^{-12} \times 1.6 \times 10^{-19}}{2}\right)}$$

$$\sqrt{\left(\frac{10^{18} \times 10^{16}}{10^{18} + 10^{16}}\right)\left(\frac{1}{0.814}\right)}$$

$= 3.2$ pF

Equation (3.47),

$$C_j = \frac{C_{j0}}{\sqrt{1 + \dfrac{V_R}{V_0}}}$$

$$= \frac{3.2 \times 10^{-12}}{\sqrt{1 + \dfrac{2}{0.814}}}$$

$= 1.72$ pF

Ex: 3.15 $C_d = \dfrac{dQ}{dV} = \dfrac{d}{dV}(\tau_T I)$

$$= \frac{d}{dV}[\tau_T \times I_S(e^{V/V_T} - 1)]$$

$$= \tau_T I_S \frac{d}{dV}(e^{V/V_T} - 1)$$

$$= \tau_T I_S \frac{1}{V_T} e^{V/V_T}$$

$$= \frac{\tau_T}{V_T} \times I_S e^{V/V_T}$$

$$\cong \left(\frac{\tau_T}{V_T}\right) I$$

Ex: 3.16 Equation (3.51),

$$\tau_p = \frac{L_p^2}{D_p}$$

$$= \frac{(5 \times 10^{-4})^2}{10}$$

$= 25$ ns

Equation (3.57),

$$C_d = \left(\frac{\tau_T}{V_T}\right) I$$

In Example 3.6, $N_A = 10^{18}/\text{cm}^3$,

$N_D = 10^{16}/\text{cm}^3$

Assuming $N_A \gg N_D$,

$\tau_T \simeq \tau_p = 25$ ns

$$\therefore C_d = \left(\frac{25 \times 10^{-9}}{25.9 \times 10^{-3}}\right) 0.1 \times 10^{-3}$$

$= 96.5$ pF

Ex: 4.1 Refer to Fig. 4.3(a). For $v_I \geq 0$, the diode conducts and presents a zero voltage drop. Thus $v_O = v_I$. For $v_I < 0$, the diode is cut off, zero current flows through R, and $v_O = 0$. The result is the transfer characteristic in Fig. E4.1.

Ex: 4.2 See Fig. 4.3a and 4.3b. During the positive half of the sinusoid, the diode is forward biased, so it conducts resulting in $v_D = 0$. During the negative half cycle of the input signal v_I, the diode is reverse biased. The diode does not conduct, resulting in no current flowing in the circuit. So $v_O = 0$ and $v_D = v_I - v_O = v_I$. This results in the waveform shown in Fig. E4.2.

Ex: 4.3 $\hat{i}_D = \dfrac{\hat{v}_I}{R} = \dfrac{10\text{ V}}{1\text{ k}\Omega} = 10\text{ mA}$

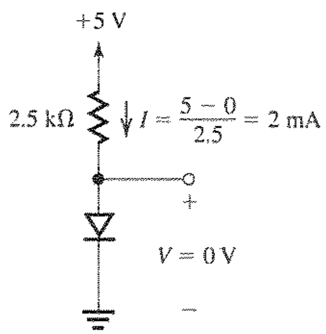
dc component of $v_O = \dfrac{1}{\pi}\hat{v}_O$

$= \dfrac{1}{\pi}\hat{v}_I = \dfrac{10}{\pi}$

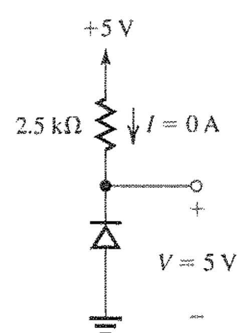
$= 3.18$ V

Ex: 4.4

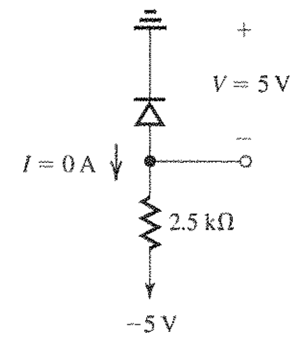
(a)



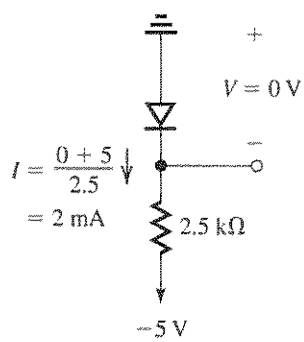
(b)



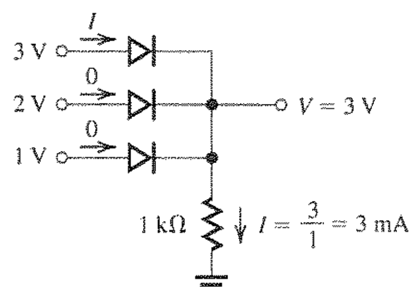
(c)



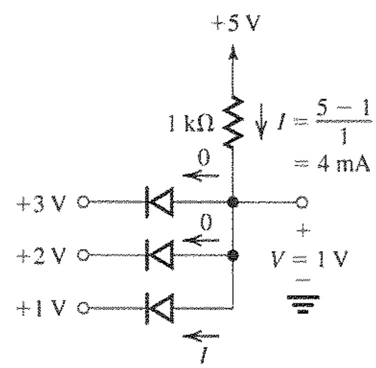
(d)



(e)



(f)



Ex: 4.5 $V_{\text{avg}} = \dfrac{10}{\pi}$

$$50 + R = \frac{\frac{10}{\pi}}{1\text{ mA}} = \frac{10}{\pi}\text{ k}\Omega$$

$\therefore\ R = 3.133$ kΩ

Ex: 4.6 Equation (4.5)

$$V_2 - V_1 = 2.3\ V_T \log\left(\frac{I_2}{I_1}\right)$$

At room temperature $V_T = 25$ mV

$$V_2 - V_1 = 2.3 \times 25 \times 10^{-3} \times \log\left(\frac{10}{0.1}\right)$$

$= 115$ mV

Ex: 4.7 $i = I_S e^{v/V_T}$ (1)

$1\ (\text{mA}) = I_S e^{0.7/V_T}$ (2)

Dividing (1) by (2), we obtain

$i\ (\text{mA}) = e^{(v-0.7)/V_T}$

$\Rightarrow v = 0.7 + 0.025 \ln(i)$

where i is in mA. Thus,

for $i = 0.1$ mA,

$v = 0.7 + 0.025 \ln(0.1) = 0.64$ V

and for $i = 10$ mA,

$v = 0.7 + 0.025 \ln(10) = 0.76$ V

Ex: 4.8 $\Delta T = 125 - 25 = 100°\text{C}$

$I_S = 10^{-14} \times 1.15^{\Delta T}$

$= 1.17 \times 10^{-8}$A

Ex: 4.9 At 20°C $I = \dfrac{1\ \text{V}}{1\ \text{M}\Omega} = 1\ \mu\text{A}$

Since the reverse leakage current doubles for every 10°C increase, at 40°C

$I = 4 \times 1\ \mu\text{A} = 4\ \mu\text{A}$

$\Rightarrow V = 4\ \mu\text{A} \times 1\ \text{M}\Omega = 4.0$ V

@ 0°C $\quad I = \dfrac{1}{4}\ \mu\text{A}$

$\Rightarrow V = \dfrac{1}{4} \times 1 = 0.25$ V

Ex: 4.10 a. Use iteration:

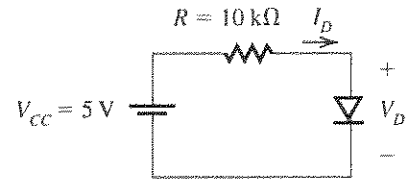
Diode has 0.7 V drop at 1 mA current.

Assume $V_D = 0.7$ V

$I_D = \dfrac{5 - 0.7}{10\ \text{k}\Omega} = 0.43$ mA

Use Eq. (4.5) and note that

$V_1 = 0.7\ \text{V}, \quad I_1 = 1$ mA



$$V_2 - V_1 = 2.3 \times V_T \log\left(\frac{I_2}{I_1}\right)$$

$$V_2 = V_1 + 2.3 \times V_T \log\left(\frac{I_2}{I_1}\right)$$

First iteration

$$V_2 = 0.7 + 2.3 \times 25 \times 10^{-3} \log\left(\frac{0.43}{1}\right)$$

$= 0.679$ V

Second iteration

$I_2 = \dfrac{5 - 0.679}{10\ \text{k}\Omega} = 0.432$ mA

$$V_2 = 0.7 + 2.3 \times 25.3 \times 10^{-3} \log\left(\frac{0.432}{1}\right)$$

$= 0.679\ \text{V} \simeq 0.68$ V

we get almost the same voltage.

$\therefore$ The iteration yields

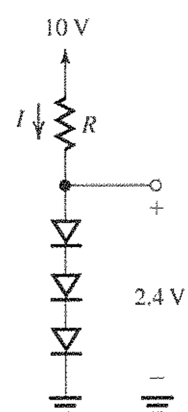
$I_D = 0.43\ \text{mA}, \ V_D = 0.68$ V

b. Use constant voltage drop model:

$V_D = 0.7$ V constant voltage drop

$I_D = \dfrac{5 - 0.7}{10\ \text{k}\Omega} = 0.43$ mA

Ex: 4.11



Diodes have 0.7 V drop at 1 mA

$\therefore 1\ \text{mA} = I_S e^{0.7/V_T}$ (1)

At a current I(mA),

$I = I_S e^{V_D/V_T}$ (2)

Using (1) and (2), we obtain

$I = e^{(V_D-0.7)/V_T}$

For an output voltage of 2.4 V, the voltage drop across each diode $= \frac{2.4}{3} = 0.8$ V

Now I, the current through each diode, is

$I = e^{(0.8-0.7)/0.025}$

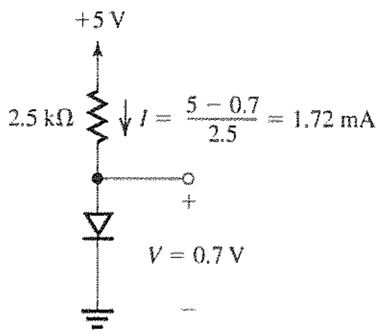
$= 54.6$ mA

$R = \frac{10 - 2.4}{54.6 \times 10^{-3}}$

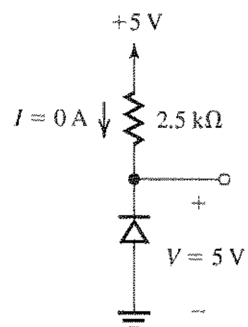
$= 139\ \Omega$

Ex: 4.12

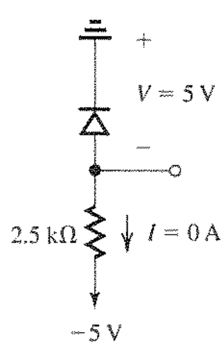
(a)



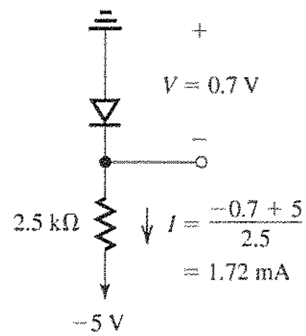
(b)



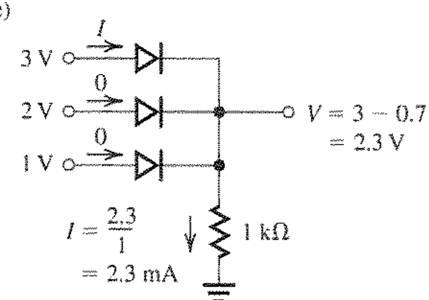
(c)



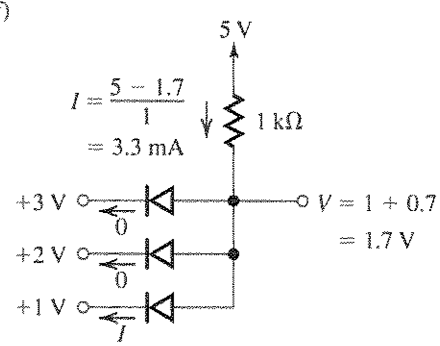
(d)



(e)



(f)



Ex: 4.13 $r_d = \frac{V_T}{I_D}$

$I_D = 0.1$ mA $\quad r_d = \frac{25 \times 10^{-3}}{0.1 \times 10^{-3}} = 250\ \Omega$

$I_D = 1$ mA $\quad r_d = \frac{25 \times 10^{-3}}{1 \times 10^{-3}} = 25\ \Omega$

$I_D = 10$ mA $\quad r_d = \frac{25 \times 10^{-3}}{10 \times 10^{-3}} = 2.5\ \Omega$

Ex: 4.14 For small signal model,

$$\Delta i_D = \Delta v_D / r_d \tag{1}$$

where $r_d = \frac{V_T}{I_D}$

For exponential model,

$i_D = I_S e^{V/V_T}$

$$\frac{i_{D2}}{i_{D1}} = e^{(V_2-V_1)/V_T} = e^{\Delta v_D/V_T}$$

$$\Delta i_D = i_{D2} - i_{D1} = i_{D1}e^{\Delta v_D/V_T} - i_{D1}$$

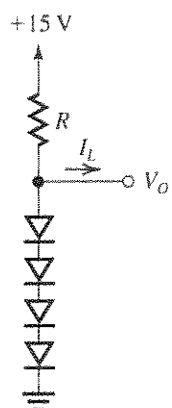
$$= i_{D1}\left(e^{\Delta v_D/V_T} - 1\right) \qquad (2)$$

In this problem, $i_{D1} = I_D = 1$ mA.

Using Eqs. (1) and (2) with $V_T = 25$ mV, we obtain

	Δv_D (mV)	Δi_D (mA) small signal	Δi_D (mA) exponential model
a	−10	−0.4	−0.33
b	−5	−0.2	−0.18
c	+5	+0.2	+0.22
d	+10	+0.4	+0.49

Ex: 4.15



a. In this problem, $\dfrac{\Delta V_O}{\Delta i_L} = \dfrac{20\text{ mV}}{1\text{ mA}} = 20\ \Omega$.

∴ Total small-signal resistance of the four diodes $= 20\ \Omega$

∴ For each diode, $r_d = \dfrac{20}{4} = 5\ \Omega$.

But $r_d = \dfrac{V_T}{I_D} \Rightarrow 5 = \dfrac{25\text{ mV}}{I_D}$.

∴ $I_D = 5$ mA

and $R = \dfrac{15-3}{5\text{ mA}} = 2.4\text{ k}\Omega$.

b. For $V_O = 3$ V, voltage drop across each diode $= \dfrac{3}{4} = 0.75$ V

$$i_D = I_S e^{V/V_T}$$

$$I_S = \frac{i_D}{e^{V/V_T}} = \frac{5\times10^{-3}}{e^{0.75/0.025}} = 4.7\times10^{-16}\text{ A}$$

c. If $i_D = 5 - i_L = 5 - 1 = 4$ mA.

Across each diode the voltage drop is

$$V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$$

$$= 25\times10^{-3}\times\ \ln\left(\frac{4\times10^{-3}}{4.7\times10^{-16}}\right)$$

$$= 0.7443\text{ V}$$

Voltage drop across 4 diodes

$= 4\times0.7443 = 2.977$ V

so change in $V_O = 3 - 2.977 = 23$ mV.

Ex: 4.16 For a zener diode

$$V_Z = V_{Z0} + I_Z r_Z$$

$$10 = V_{Z0} + 0.01\times50$$

$$V_{Z0} = 9.5\text{ V}$$

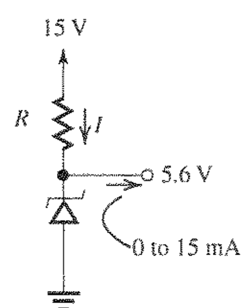
For $I_Z = 5$ mA,

$$V_Z = 9.5 + 0.005\times50 = 9.75\text{ V}$$

For $I_Z = 20$ mA,

$$V_Z = 9.5 + 0.02\times50 = 10.5\text{ V}$$

Ex: 4.17



The minimum zener current should be

$5\times I_{Zk} = 5\times1 = 5$ mA

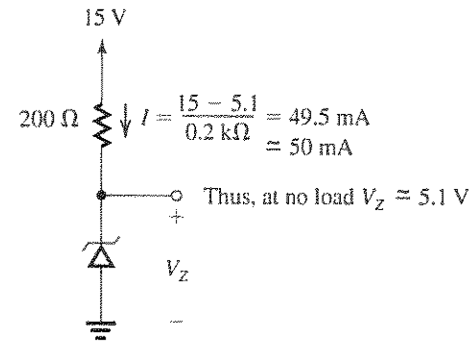
Since the load current can be as large as 15 mA, we should select R so that with $I_L = 15$ mA, a zener current of 5 mA is available. Thus the current should be 20 mA, leading to

$$R = \frac{15-5.6}{20\text{ mA}} = 470\ \Omega$$

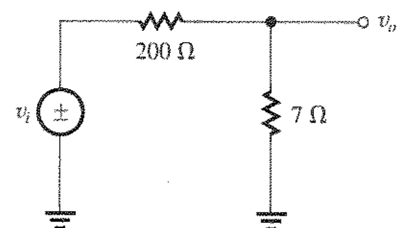
Maximum power dissipated in the diode occurs when $I_L = 0$ is

$$P_{\max} = 20\times10^{-3}\times5.6 = 112\text{ mV}$$

Ex: 4.18

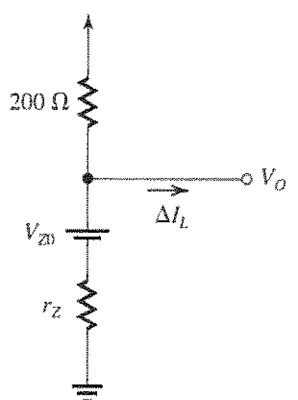


For line regulation:



$$\text{Line regulation} = \frac{v_o}{v_i} = \frac{7}{200 + 7} = 33.8\ \frac{\text{mV}}{\text{V}}$$

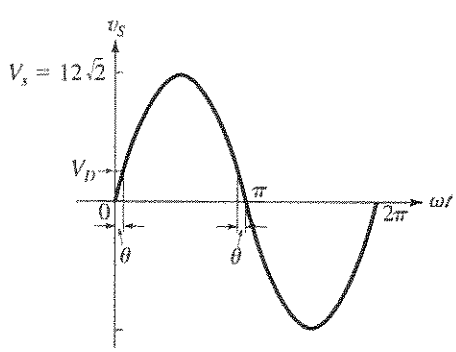
For load regulation:



$$\frac{\Delta V_O}{\Delta I_L} = \frac{-\Delta I_L(r_Z \parallel 200\ \Omega)}{\Delta I_L\ \text{mA}}$$

$$= -6.8\,\frac{\text{mV}}{\text{mA}}$$

Ex: 4.19



a. The diode starts conduction at

$v_S = V_D = 0.7$ V

$v_S = V_s \sin \omega t$, here $V_s = 12\sqrt{2}$

At $\omega t = \theta$,

$v_S = V_s \sin \theta = V_D = 0.7$ V

$12\sqrt{2} \sin \theta = 0.7$

$$\theta = \sin^{-1}\left(\frac{0.7}{12\sqrt{2}}\right) \simeq 2.4^\circ$$

Conduction starts at θ and stops at $180 - \theta$.

$\therefore$ Total conduction angle $= 180 - 2\theta = 175.2^\circ$

b. $$v_{O,\text{avg}} = \frac{1}{2\pi}\int_{\theta}^{(\pi-\theta)} (V_s \sin \phi - V_D)\, d\phi$$

$$= \frac{1}{2\pi}[-V_s \cos \phi - V_D\phi]_{\phi=\theta}^{\phi=\pi-\theta}$$

$$= \frac{1}{2\pi}[V_s \cos \theta - V_s \cos(\pi - \theta) - V_D(\pi - 2\theta)]$$

But $\cos \theta \simeq 1$, $\cos(\pi - \theta) \simeq -1$, and

$\pi - 2\theta \simeq \pi$

$$v_{O,\text{avg}} = \frac{2V_s}{2\pi} - \frac{V_D}{2}$$

$$= \frac{V_s}{\pi} - \frac{V_D}{2}$$

For $V_s = 12\sqrt{2}$ and $V_D = 0.7$ V

$$v_{O,\text{avg}} = \frac{12\sqrt{2}}{\pi} - \frac{0.7}{2} = 5.05\ \text{V}$$

c. The peak diode current occurs at the peak diode voltage.

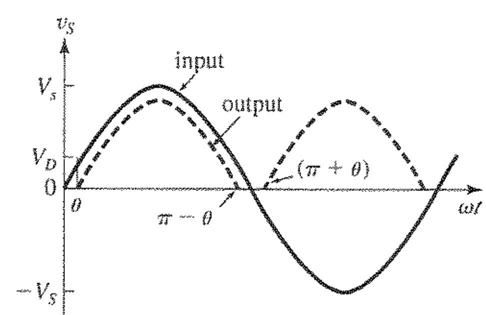
$$\therefore \hat{i}_D = \frac{V_s - V_D}{R} = \frac{12\sqrt{2} - 0.7}{100}$$

$= 163$ mA

PIV $= +V_S = 12\sqrt{2}$

$\simeq 17$ V

Ex: 4.20



a. As shown in the diagram, the output is zero between $(\pi - \theta)$ to $(\pi + \theta)$

$= 2\theta$

Here θ is the angle at which the input signal reaches V_D.

$\therefore\ V_s \sin\theta = V_D$

$$\theta = \sin^{-1}\left(\frac{V_D}{V_s}\right)$$

$$2\theta = 2\sin^{-1}\left(\frac{V_D}{V_s}\right)$$

b. Average value of the output signal is given by

$$V_O = \frac{1}{2\pi}\left[2 \times \int_{\theta}^{(\pi-\theta)} (V_s \sin\phi - V_D)\, d\phi\right]$$

$$= \frac{1}{\pi}[-V_s \cos\phi - V_D\phi]_{\phi=\theta}^{\pi-\theta}$$

$$\simeq 2\frac{V_s}{\pi} - V_D, \qquad \text{for } \theta \text{ small.}$$

c. Peak current occurs when $\phi = \frac{\pi}{2}$.

Peak current

$$= \frac{V_s \sin(\pi/2) - V_D}{R} = \frac{V_s - V_D}{R}$$

If v_S is 12 V(rms),

then $V_s = \sqrt{2} \times 12 = 12\sqrt{2}$

$$\text{Peak current} = \frac{12\sqrt{2} - 0.7}{100} \simeq 163 \text{ mA}$$

Nonzero output occurs for angle $= 2(\pi - 2\theta)$

The fraction of the cycle for which $v_O > 0$ is

$$= \frac{2(\pi - 2\theta)}{2\pi} \times 100$$

$$= \frac{2\left[\pi - 2\sin^{-1}\left(\frac{0.7}{12\sqrt{2}}\right)\right]}{2\pi} \times 100$$

$\simeq 97.4\ \%$

Average output voltage V_O is

$$V_O = 2\frac{V_s}{\pi} - V_D = \frac{2 \times 12\sqrt{2}}{\pi} - 0.7 = 10.1 \text{ V}$$

Peak diode current $\hat{i}_D$ is

$$\hat{i}_D = \frac{V_s - V_D}{R} = \frac{12\sqrt{2} - 0.7}{100}$$

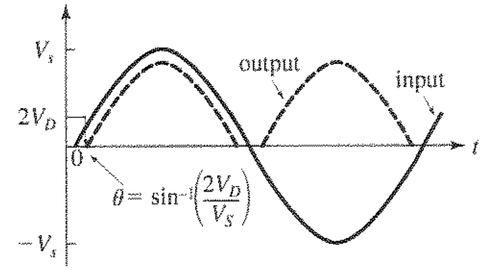
$= 163$ mA

$\text{PIV} = V_s - V_D + V_S$

$= 12\sqrt{2} - 0.7 + 12\sqrt{2}$

$= 33.2$ V

Ex: 4.21



$$\text{(a)}\ V_{O,\text{avg}} = \frac{1}{2\pi}\int (V_s \sin\phi - 2V_D)\, d\phi$$

$$= \frac{2}{2\pi}[-V_s \cos\phi - 2V_D\phi]_{\phi=\theta}^{\pi-\theta}$$

$$= \frac{1}{\pi}[V_s \cos\phi - V_s \cos(\pi - \theta) - 2V_D(\pi - 2\theta)]$$

But $\cos\theta \approx 1$,

$\cos(\pi - \theta) \approx -1$

$\pi - 2\theta \approx \pi$. Thus

$$\Rightarrow V_{O,\text{avg}} \simeq \frac{2V_s}{\pi} - 2V_D$$

$$= \frac{2 \times 12\sqrt{2}}{\pi} - 1.4 = 9.4 \text{ V}$$

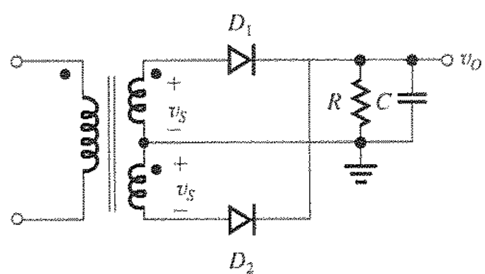
$$\text{(b) Peak diode current} = \frac{\text{Peak voltage}}{R}$$

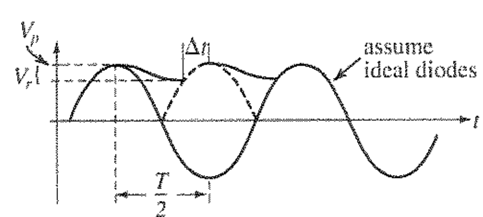
$$= \frac{V_s - 2V_D}{R} = \frac{12\sqrt{2} - 1.4}{100}$$

$= 156$ mA

$\text{PIV} = V_s - V_D = 12\sqrt{2} - 0.7 = 16.3$ V

Ex: 4.22 Full-wave peak rectifier:





The ripple voltage is the amount of voltage reduction during capacitor discharge that occurs when the diodes are not conducting. The output voltage is given by

$$v_O = V_p e^{-t/RC}$$

$V_p - V_r = V_p e^{-\frac{T/2}{RC}}$ $\leftarrow$ discharge is only half the period. We also assumed $\Delta t \ll \frac{T}{2}$.

$$V_r = V_p\left(1 - e^{-\frac{T/2}{RC}}\right)$$

$e^{-\frac{T/2}{RC}} \simeq 1 - \frac{T/2}{RC}$, for $CR \gg T/2$

Thus $V_r \simeq V_p\left(1 - 1 + \frac{T/2}{RC}\right)$

$V_r = \frac{V_p}{2fRC}$ (a) Q.E.D.

To find the average diode current, note that the charge supplied to C during conduction is equal to the charge lost during discharge.

$$Q_{\text{SUPPLIED}} = Q_{\text{LOST}}$$

$i_{C\text{av}}\Delta t = CV_r$ SUB (a)

$$(i_{D,\text{av}} - I_L)\,\Delta t = C\frac{V_p}{2fRC} = \frac{V_p}{2fR}$$

$$= \frac{V_p \pi}{\omega R}$$

$$i_{D,\text{av}} = \frac{V_p \pi}{\omega \Delta t R} + I_L$$

where $\omega\Delta t$ is the conduction angle.

Note that the conduction angle has the same expression as for the half-wave rectifier and is given by Eq. (4.30),

$\omega \Delta t \cong \sqrt{\frac{2V_r}{V_p}}$ (b)

Substituting for $\omega\Delta t$, we get

$$\Rightarrow i_{D\text{av}} = \frac{\pi V_p}{\sqrt{\frac{2V_r}{V_p}} \cdot R} + I_L$$

Since the output is approximately held at V_p, $\frac{V_p}{R} \approx I_L$. Thus

$$\Rightarrow i_{D\text{av}} \cong \pi I_L \sqrt{\frac{V_p}{2V_r}} + I_L$$

$= I_L\left[1 + \pi\sqrt{\frac{V_p}{2V_r}}\right]$ Q.E.D.

If $t = 0$ is at the peak, the maximum diode current occurs at the onset of conduction or at $t = -\omega\Delta t$.

During conduction, the diode current is given by

$$i_D = i_C + i_L$$

$$i_{D,\max} = C\left.\frac{dv_S}{dt}\right|_{t=-\omega\Delta t} + i_L$$

assuming i_L is const. $i_L \simeq \frac{V_p}{R} = I_L$

$$= C\frac{d}{dt}(V_p \cos \omega t) + I_L$$

$$= -C \sin \omega t \times \omega V_p + I_L$$

$$= -C \sin(-\omega\Delta t) \times \omega V_p + I_L$$

For a small conduction angle

$\sin(-\omega\Delta t) \approx -\omega\Delta t$. Thus

$$\Rightarrow i_{D,\max} = C\omega\Delta t \times \omega V_p + I_L$$

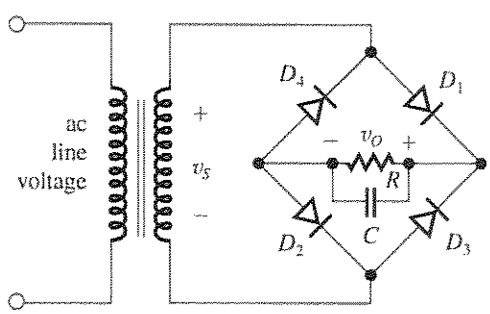
Sub (b) to get

$$i_{D,\max} = C\sqrt{\frac{2V_r}{V_p}}\omega V_p + I_L$$

Substituting $\omega = 2\pi f$ and using (a) together with $V_p/R \simeq I_L$ results in

$i_{D\max} = I_L\left[1 + 2\pi\sqrt{\frac{V_p}{2V_r}}\right]$ Q.E.D.

Ex: 4.23



The output voltage, v_O, can be expressed as

$$v_O = (V_p - 2V_D)\,e^{-t/RC}$$

At the end of the discharge interval

$$v_O = V_p - 2V_D - V_r$$

The discharge occurs almost over half of the time period $\simeq T/2$.

For time constant $RC \gg \frac{T}{2}$

$$e^{-t/RC} \simeq 1 - \frac{T}{2} \times \frac{1}{RC}$$

$$\therefore V_P - 2V_D - V_r = (V_p - 2V_D)\left(1 - \frac{T}{2} \times \frac{1}{RC}\right)$$

$$\Rightarrow V_r = (V_p - 2V_D) \times \frac{T}{2RC}$$

Here $V_p = 12\sqrt{2}$ and $V_r = 1$ V

$V_D = 0.8$ V

$$T = \frac{1}{f} = \frac{1}{60}\text{ s}$$

$$1 = (12\sqrt{2} - 2 \times 0.8) \times \frac{1}{2 \times 60 \times 100 \times C}$$

$$C = \frac{(12\sqrt{2} - 1.6)}{2 \times 60 \times 100} = 1281\ \mu\text{F}$$

Without considering the ripple voltage, the dc output voltage

$= 12\sqrt{2} - 2 \times 0.8 = 15.4$ V

If ripple voltage is included, the output voltage is

$$= 12\sqrt{2} - 2 \times 0.8 - \frac{V_r}{2} = 14.9\text{ V}$$

$$I_L = \frac{14.9}{100\ \Omega} \simeq 0.15\text{ A}$$

The conduction angle $\omega\Delta t$ can be obtained using Eq. (4.30) but substituting $V_p = 12\sqrt{2} - 2 \times 0.8$:

$$\omega\Delta t = \sqrt{\frac{2V_r}{V_p}} = \sqrt{\frac{2 \times 1}{12\sqrt{2} - 2 \times 0.8}}$$

$= 0.36$ rad $= 20.7°$

The average and peak diode currents can be calculated using Eqs. (4.34) and (4.35):

$$i_{Dav} = I_L\left(1 + \pi\sqrt{\frac{V_p}{2V_r}}\right), \text{ where } I_L = \frac{14.9\text{ V}}{100\ \Omega},$$

$V_p = 12\sqrt{2} - 2 \times 0.8$, and $V_r = 1$ V; thus

$i_{Dav} = 1.45$ A

$$i_{Dmax} = I\left(1 + 2\pi\sqrt{\frac{V_p}{2V_r}}\right)$$

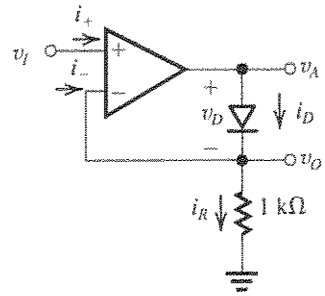
$= 2.76$ A

PIV of the diodes

$= V_S - V_{DO} = 12\sqrt{2} - 0.8 = 16.2$ V

To provide a safety margin, select a diode capable of a peak current of 3.5 to 4A and having a PIV rating of 20 V.

Ex: 4.24



The diode has 0.7 V drop at 1 mA current.

$i_D = I_S e^{v_D/V_T}$

$$\frac{i_D}{1\text{ mA}} = e^{(v_D - 0.7)/V_T}$$

$$\Rightarrow v_D = V_T \ln\left(\frac{i_D}{1\text{ mA}}\right) + 0.7\text{ V}$$

For $v_I = 10$ mV, $\quad v_O = v_I = 10$ mV

It is an ideal op amp, so $i_+ = i_- = 0$.

$$\therefore i_D = i_R = \frac{10\text{ mV}}{1\text{ k}\Omega} = 10\ \mu\text{A}$$

$$v_D = 25 \times 10^{-3} \ln\left(\frac{10\ \mu\text{A}}{1\text{ mA}}\right) + 0.7 = 0.58\text{ V}$$

$v_A = v_D + 10$ mV

$= 0.58 + 0.01$

$= 0.59$ V

For $v_I = 1$ V

$v_O = v_I = 1$ V

$$i_D = \frac{v_O}{1\text{ k}\Omega} = \frac{1}{1\text{ k}\Omega} = 1\text{ mA}$$

$v_D = 0.7$ V

$V_A = 0.7\text{ V} + 1\text{ k}\Omega \times 1\text{ mA}$

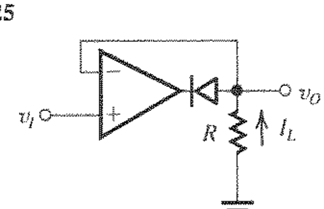
$= 1.7$ V

For $v_I = -1$ V, the diode is cut off.

$\therefore v_O = 0$ V

$v_A = -12$ V

Ex: 4.25



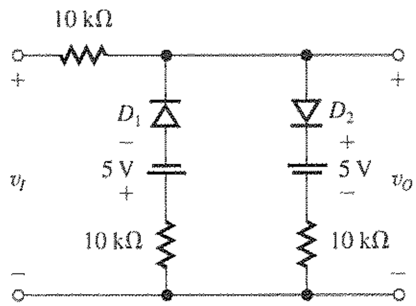
$v_I > 0 \sim$ diode is cut off, loop is open, and the opamp is saturated:

$v_O = 0$ V

$v_I < 0 \sim$ diode conducts and closes the negative feedback loop:

$v_O = v_I$

Ex: 4.26



Both diodes are cut off

for $-5\text{V} \le v_I \le +5\text{V}$

and $v_O = v_I$

For $v_I \le -5$ V, diode D_1 conducts and

$$v_O = -5 + \frac{1}{2}(+v_I + 5)$$

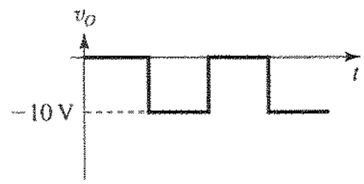
$$= \left(-2.5 + \frac{v_I}{2}\right) \text{ V}$$

For $v_I \ge 5$ V, diode D_2 conducts and

$$v_O = +5 + \frac{1}{2}(v_I - 5)$$

$$= \left(2.5 + \frac{v_I}{2}\right) \text{ V}$$

Ex: 4.27 Reversing the diode results in the peak output voltage being clamped at 0 V:



Here the dc component of $v_O = V_O = -5$ V

Ex: 5.1

$C_{ox} = \dfrac{\epsilon_{ox}}{t_{ox}} = \dfrac{34.5 \text{ pF/m}}{4 \text{ nm}} = 8.625 \text{ fF}/\mu\text{m}^2$

$\mu_n = 450 \text{ cm}^2/\text{V}\cdot\text{S}$

$k'_n = \mu_n C_{ox} = 388\ \mu\text{A/V}^2$

$V_{OV} = (v_{GS} - V_t) = 0.5 \text{ V}$

$g_{DS} = \dfrac{1}{1 \text{ k}\Omega} = k'_n \dfrac{W}{L} V_{OV} \Rightarrow \dfrac{W}{L} = 5.15$

$L = 0.18\ \mu\text{m}$, so $W = 0.93\ \mu\text{m}$

Ex: 5.2 $C_{ox} = \dfrac{\epsilon_{ox}}{t_{ox}} = \dfrac{34.5 \text{ pF/m}}{4 \text{ nm}} = 8.6 \text{ fF}/\mu\text{m}^2$

$\mu_n = 450 \text{ cm}^2/\text{V}\cdot\text{s}$

$k'_n = \mu_n C_{ox} = 387\ \mu\text{A/V}^2$

$I_D = \dfrac{1}{2} k'_n \dfrac{W}{L} V_{OV}^2 = 0.3 \text{ mA},\ \dfrac{W}{L} = 20$

$\therefore\ V_{OV} = 0.28 \text{ V}$

$V_{DS,\text{ min}} = V_{OV} = 0.28$ V, for saturation

Ex: 5.3 $I_D = \dfrac{1}{2} k'_n \dfrac{W}{L} V_{OV}^2$ in saturation

Change in I_D is:

(a) double L, 0.5

(b) double W, 2

(c) double V_{OV}, $2^2 = 4$

(d) double V_{DS}, no change (ignoring length modulation)

(e) changes (a)–(d), 4

Case (c) would cause leaving saturation if

$V_{DS} < 2V_{OV}$

Ex: 5.4 For saturation $v_{DS} \geq V_{OV}$, so V_{DS} must be changed to $2V_{OV}$

$I_D = \dfrac{1}{2} k'_n \dfrac{W}{L} V_{OV}^2$, so I_D increases by a factor of 4.

Ex: 5.5 $V_{OV} = 0.5$ V

$g_{DS} = k'_n \dfrac{W}{L} V_{OV} = \dfrac{1}{1 \text{ k}\Omega}$

$\therefore\ k_n = k'_n \dfrac{W}{L} = \dfrac{1}{1 \times 0.5} = 2 \text{ mA/V}^2$

For $v_{DS} = 0.5 \text{ V} = V_{OV}$, the transistor operates in saturation, and

$I_D = \dfrac{1}{2} k'_n \dfrac{W}{L} V_{OV}^2 = 0.25 \text{ mA}$

Similarly, $V_{DS} = 1$ V results in saturation-mode operation and $I_D = 0.25$ mA.

Ex: 5.6 $V_A = V'_A L = 50 \times 0.8 = 40 \text{ V}$

$\lambda = \dfrac{1}{V_A} = 0.025 \text{ V}^{-1}$

$V_{DS} = 1 \text{ V} > V_{OV} = 0.5 \text{ V}$

$\Rightarrow$ Saturation: $I_D = \dfrac{1}{2} k'_n \dfrac{W}{L} V_{OV}^2 (1 + \lambda V_{DS})$

$I_D = \dfrac{1}{2} \times 200 \times \dfrac{16}{0.8} \times 0.5^2 (1 + 0.025 \times 1)$

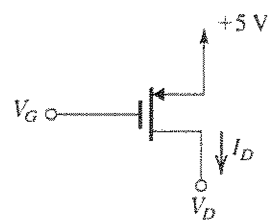
$= 0.51$ mA

$r_o = \dfrac{V_A}{I_D} = \dfrac{40}{0.5} = 80 \text{ k}\Omega$

where I_D is the value of I_D without channel-length modulation taken into account.

$r_o = \dfrac{\Delta V_{DS}}{\Delta I_O} \Rightarrow \Delta I_O = \dfrac{2 \text{ V}}{80 \text{ k}\Omega} = 0.025 \text{ mA}$

Ex: 5.7



$V_{tp} = -1$ V

$k'_p = 60\ \mu\text{A/V}^2$

$\dfrac{W}{L} = 10 \Rightarrow k_p = 600\ \mu\text{A/V}^2$

(a) Conduction occurs for $V_{SG} \geq |V_{tp}| = 1$ V

$\Rightarrow V_G \leq 5 - 1 = 4$ V

(b) Triode region occurs for $V_{DG} \geq |V_{tp}| = 1$ V

$\Rightarrow V_D \geq V_G + 1$

(c) Conversely, for saturation

$V_{DG} \leq |V_{tp}| = 1$ V

$\Rightarrow V_D \leq V_G + 1$

(d) Given $\lambda \cong 0$

$I_D = \dfrac{1}{2} k'_p \dfrac{W}{L} |V_{OV}|^2 = 75\ \mu\text{A}$

$\therefore\ |V_{OV}| = 0.5 \text{ V} = V_{SG} - |V_{tp}|$

$\Rightarrow V_{SG} = |V_{OV}| + |V_{tp}| = 1.5$ V

$V_G = 5 - |V_{SG}| = 3.5$ V

$V_D \leq V_G + 1 = 4.5$ V

(e) For $\lambda = -0.02\ \text{V}^{-1}$ and $|V_{OV}| = 0.5$ V,

$$I_D = 75\ \mu\text{A and } r_o = \frac{1}{|\lambda| I_D} = 667\ \text{k}\Omega$$

(f) At $V_D = 3$ V, $V_{SD} = 2$ V

$$I_D = \frac{1}{2} k_n' \frac{W}{L} |V_{OV}|^2 (1 + |\lambda||V_{SD}|)$$

$$= 75\ \mu\text{A}\ (1.04) = 78\ \mu\text{A}$$

At $V_D = 0$ V, $V_{SD} = 5$ V

$$I_D = 75\ \mu\text{A}\ (1.10) = 82.5\ \mu\text{A}$$

$$r_o = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{3\ \text{V}}{4.5\ \mu\text{A}} = 667\ \text{k}\Omega$$

which is the same value found in (c).

Ex: 5.8

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{OV}^2 \Rightarrow 0.3 = \frac{1}{2} \times \frac{60}{1000}$$

$$\times \frac{120}{3} V_{OV}^2 \Rightarrow$$

$$V_{OV} = 0.5\ \text{V} \Rightarrow V_{GS} = V_{OV} + V_t = 0.5 + 1$$

$$= 1.5\ \text{V}$$

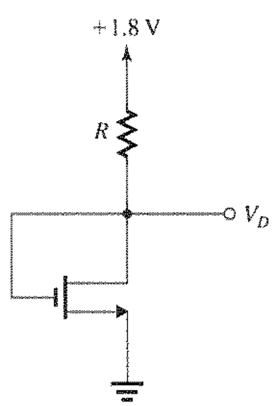
$$V_S = -1.5\ \text{V} \Rightarrow R_S = \frac{V_S - V_{SS}}{I_D}$$

$$= \frac{-1.5 - (-2.5)}{0.3}$$

$$R_S = 3.33\ \text{k}\Omega$$

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{2.5 - 0.4}{0.3} = 7\ \text{k}\Omega$$

Ex: 5.9



$$V_{tn} = 0.5\ \text{V}$$

$$\mu_n C_{ox} = 0.4\ \text{mA/V}^2$$

$$\frac{W}{L} = \frac{0.72\ \mu\text{m}}{0.18\ \mu\text{m}} = 4.0$$

$$\lambda = 0$$

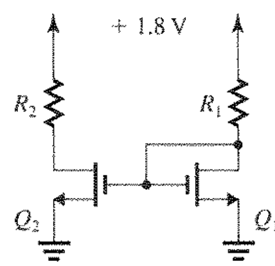
Saturation mode ($v_{GD} = 0 < V_{tn}$):

$$V_D = 0.7\ \text{V} = 1.8 - I_D R_D$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_D - V_{tn})^2 = 0.032\ \text{mA}$$

$$\therefore R = \frac{1.8 - 0.7}{0.032\ \text{mA}} = 34.4\ \text{k}\Omega$$

Ex: 5.10



Since Q_2 is identical to Q_1 and their V_{GS} values are the same,

$$I_{D2} = I_{D1} = 0.032\ \text{mA}$$

For Q_2 to operate at the triode–saturation boundary, we must have

$$V_{D2} = V_{OV} = 0.2\ \text{V}$$

$$\therefore R_2 = \frac{1.8\ \text{V} - 0.2\ \text{V}}{0.032\ \text{mA}} = 50\ \text{k}\Omega$$

Ex: 5.11 $R_D = 12.4 \times 2 = 24.8\ \text{k}\Omega$

$V_{GS} = 5$ V, assume triode region:

$$\left.\begin{aligned} I_D &= k_n' \frac{W}{L}\left[(V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right] \\ I_D &= \frac{V_{DD} - V_{DS}}{R} \end{aligned}\right\} \Rightarrow$$

$$\frac{5 - V_{DS}}{24.8} = 1 \times \left[(5 - 1)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$\Rightarrow V_{DS}^2 - 8.08 V_{DS} + 0.4 = 0$$

$$\Rightarrow V_{DS} = 0.05\ \text{V} < V_{OV} \Rightarrow \text{triode region}$$

$$I_D = \frac{5 - 0.05}{24.8} = 0.2\ \text{mA}$$

Ex: 5.12 As indicated in Example 5.6,

$V_D \geq V_G - V_t$ for the transistor to be in the saturation region.

$$V_{D\min} = V_G - V_t = 5 - 1 = 4\ \text{V}$$

$$I_D = 0.5\ \text{mA} \Rightarrow R_{D\max} = \frac{V_{DD} - V_{D\min}}{I_D}$$

$$= \frac{10 - 4}{0.5} = 12\ \text{k}\Omega$$

Ex: 5.13

$I_D = 0.32\text{ mA} = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2 = \frac{1}{2} \times 1 \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.8\text{ V}$

$V_{GS} = 0.8 + 1 = 1.8\text{ V}$

$V_G = V_S + V_{GS} = 1.6 + 1.8 = 3.4\text{ V}$

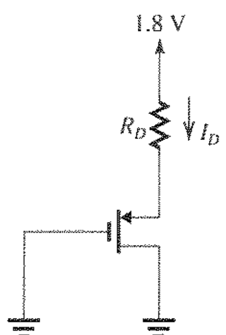
$R_{G2} = \frac{V_G}{I} = \frac{3.4}{1\ \mu\text{A}} = 3.4\text{ M}\Omega$

$R_{G1} = \frac{5 - 3.4}{1\ \mu\text{A}} = 1.6\text{ M}\Omega$

$R_S = \frac{V_S}{0.32} = 5\text{ k}\Omega$

$V_D = 3.4\text{ V}$, then $R_D = \frac{5 - 3.4}{0.32} = 5\text{ k}\Omega$

Ex: 5.14



$V_{tp} = -0.4\text{ V}$

$k_p' = 0.1\text{ mA/V}^2$

$\frac{W}{L} = \frac{10\ \mu\text{m}}{0.18\ \mu\text{m}} \Rightarrow k_p = 5.56\text{ mA/V}^2$

$V_{SG} = |V_{tp}| + |V_{OV}|$

$= 0.4 + 0.6 = 1\text{ V}$

$V_S = +1\text{ V}$

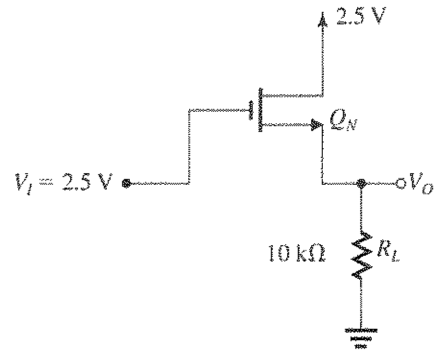
Since $V_{DG} = 0$, the transistor is operating in saturation, and

$I_D = \frac{1}{2}k_p'V_{OV}^2 = 1\text{ mA}$

$\therefore\ R = \frac{1.8 - 1}{1} = 0.8\text{ k}\Omega = 800\ \Omega$

Ex: 5.15 $v_I = 0$: since the circuit is perfectly symmetrical, $v_O = 0$ and therefore $V_{GS} = 0$, which implies that the transistors are turned off and $I_{DN} = I_{DP} = 0$.
$v_I = 2.5$ V: if we assume that the NMOS is turned on, then v_O would be less than 2.5 V, and this implies that PMOS is off ($V_{SGP} < 0$).

$I_{DN} = \frac{1}{2}k_n'\frac{W}{L}(V_{GS} - V_t)^2$



$I_{DN} = \frac{1}{2} \times 1(2.5 - V_O - 1)^2$

$I_{DN} = 0.5(1.5 - V_O)^2$

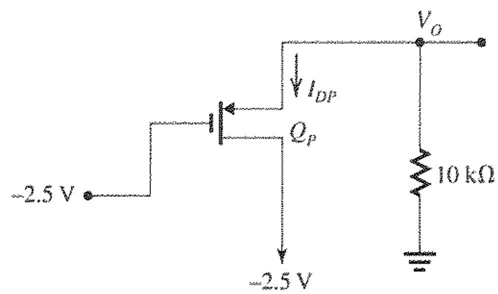
Also: $V_O = R_L I_{DN} = 10 I_{DN}$

$I_{DN} = 0.5(1.5 - 10I_{DN})^2$

$\Rightarrow 100I_{DN}^2 - 32I_{DN} + 2.25 = 0 \Rightarrow I_{DN}$

$= 0.104\text{ mA}$

$I_{DP} = 0$, $V_O = 10 \times 0.104 = 1.04\text{V}$



$V_I = -2.5$ V: Again if we assume that Q_p is turned on, then $V_O > -2.5$ V and $V_{GS1} < 0$, which implies that the NMOS Q_N is turned off.

$I_{DN} = 0$

Because of the symmetry,

$I_{DP} = 0.104$,

$V_O = -I_{DP} \times 10\text{ k}\Omega$

$= -1.04\text{ V}$

Ex: 5.16 $V_t = 0.8 + 0.4\left[\sqrt{0.7 + 3} - \sqrt{0.7}\right]$

$= 1.23\text{ V}$

Ex: 5.17 $v_{DS\min} = v_{GS} + |V_t|$

$= 1 + 2 = 3\text{ V}$

$I_D = \frac{1}{2} \times 2\,[1 - (-2)]^2$

$= 9\text{ mA}$

Ex: 6.1 $i_C = I_S\, e^{v_{BE}/V_T}$

$$v_{BE2} - v_{BE1} = V_T \ln\left[\frac{i_{C2}}{i_{C1}}\right]$$

$$v_{BE2} = 700 + 25 \ln\left[\frac{0.1}{1}\right]$$

$= 642$ mV

$$v_{BE3} = 700 + 25 \ln\left[\frac{10}{1}\right]$$

$= 758$ mV

Ex: 6.2 $\therefore \alpha = \dfrac{\beta}{\beta + 1}$

$$\frac{50}{50+1} < \alpha < \frac{150}{150+1}$$

$0.980 < \alpha < 0.993$

Ex: 6.3 $I_C = I_E - I_B$

$= 1.460$ mA $-$ 0.01446 mA

$= 1.446$ mA

$$\alpha = \frac{I_C}{I_E} = \frac{1.446}{1.460} = 0.99$$

$$\beta = \frac{I_C}{I_B} = \frac{1.446}{0.01446} = 100$$

$I_C = I_S e^{v_{BE}/V_T}$

$$I_S = \frac{I_C}{e^{v_{BE}/V_T}} = \frac{1.446}{e^{700/25}}$$

$$= \frac{1.446}{e^{28}} \text{ mA} = 10^{-15} \text{ A}$$

Ex: 6.4 $\beta = \dfrac{\alpha}{1-\alpha}$ and $I_C = 10$ mA

For $\alpha = 0.99$, $\beta = \dfrac{0.99}{1 - 0.99} = 99$

$$I_B = \frac{I_C}{\beta} = \frac{10}{99} = 0.1 \text{ mA}$$

For $\alpha = 0.98$, $\beta = \dfrac{0.98}{1 - 0.98} = 49$

$$I_B = \frac{I_C}{\beta} = \frac{10}{49} = 0.2 \text{ mA}$$

Ex: 6.5 Given:

$I_S = 10^{-16}$ A, $\beta = 100$, $I_C = 1$ mA

We write

$$I_{SE} = I_{SC}/\alpha = I_S = \left(1 + \frac{1}{\beta}\right)$$

$= 10^{-16} \times 1.01 = 1.01 \times 10^{-16}$ A

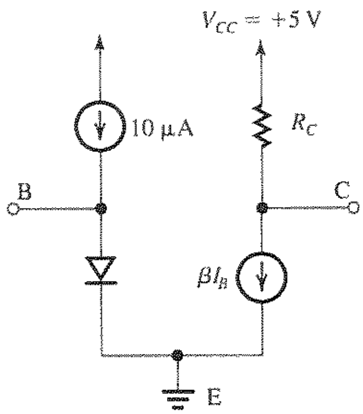
$$I_{SB} = \frac{I_S}{\beta} = \frac{10^{-16}}{100} = 10^{-18} \text{ A}$$

$$V_{BE} = V_T \ln\left[\frac{I_C}{I_S}\right] = 25 \ln\left[\frac{1 \text{ mA}}{10^{-16}}\right]$$

$= 25 \times 29.9336$

$= 748$ mV

Ex: 6.6



$v_{BE} = 690$ mV

$I_C = 1$ mA

For active range $V_C \geq V_B$,

$$R_{C\max} = \frac{V_{CC} - 0.690}{I_C}$$

$$= \frac{5 - 0.69}{1}$$

$= 4.31$ kΩ

Ex: 6.7 $I_S = 10^{-15}$ A

$\text{Area}_C = 100 \times \text{Area}_E$

$I_{SC} = 100 \times I_S = 10^{-13}$ A

Ex: 6.8 $i_C = I_S e^{v_{BE}/V_T} - I_{SC} e^{v_{BC}/V_T}$

for $i_C = 0$

$I_S e^{v_{BE}/V_T} = I_{SC} e^{v_{BC}/V_T}$

$$\frac{I_{SC}}{I_S} = \frac{e^{v_{BE}/V_T}}{e^{v_{BC}/V_T}}$$

$= e^{(v_{BE} - v_{BC})/V_T}$

$$\therefore V_{CE} = V_{BE} - V_{BC} = V_T \ln\left[\frac{I_{SC}}{I_S}\right]$$

For collector Area $= 100 \times$ Emitter area

$$V_{CE} = 25 \ln\left[\frac{100}{1}\right] = 115 \text{ mV}$$

Ex: 6.9 $I_C = I_S e^{v_{BE}/V_T} - I_{SC} e^{v_{BC}/V_T}$

$$I_B = \frac{I_S}{\beta} e^{v_{BE}/V_T} + I_{SC} e^{v_{BC}/V_T}$$

$$\beta_{\text{forced}} = \left.\frac{I_C}{I_B}\right|_{\text{sat}} < \beta$$

$$= \beta \frac{I_S e^{v_{BE}/V_T} - I_{SC} e^{v_{BC}/V_T}}{I_S e^{v_{BE}/V_T} + \beta I_{SC} e^{v_{BC}/V_T}}$$

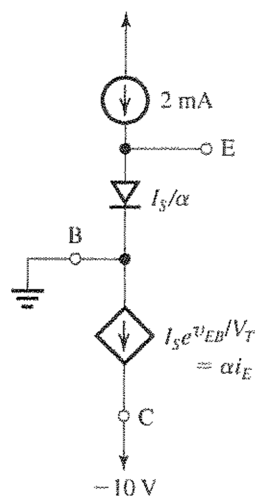
$$= \beta \frac{I_S e^{(v_{BE}-v_{BC})/V_T} - I_{SC}}{I_S e^{(v_{BE}-v_{BC})/V_T} + \beta I_{SC}}$$

$$= \beta \frac{e^{V_{CEsat}/V_T} - I_{SC}/I_S}{e^{V_{CEsat}/V_T} + \beta I_{SC}/I_S} \qquad \text{Q.E.D.}$$

$$\beta_{\text{forced}} = 100 \frac{e^{200/25} - 100}{e^{200/25} + 100 \times 100}$$

$$= 100 \times 0.2219 \approx 22.2$$

Ex: 6.10



$$I_E = \frac{I_S}{\alpha} e^{v_{BE}/V_T}$$

$$2 \text{ mA} = \frac{51}{50} 10^{-14} e^{v_{BE}/V_T}$$

$$V_{BE} = 25 \ln\left[\frac{2}{10^3} \times \frac{50}{51} \times 10^{14}\right]$$

$$= 650 \text{ mV}$$

$$I_C = \frac{\beta}{\beta+1} I_E = \frac{50}{51} \times 2$$

$$= 1.96 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{1.96}{50} \Rightarrow 39.2\ \mu\text{A}$$

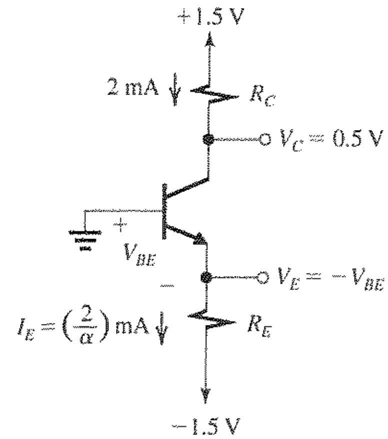
Ex: 6.11 $I_C = I_S e^{v_{BE}/V_T} = 1.5$ A

$$\therefore V_{BE} = V_T \ln\left[1.5/10^{-11}\right]$$

$$= 25 \times 25.734$$

$$= 643 \text{ mV}$$

Ex: 6.12



$$R_C = \frac{1.5 - V_C}{I_C} = \frac{1.5 - 0.5}{2}$$

$$= 0.5 \text{ k}\Omega = 500\ \Omega$$

Since at $I_C = 1$ mA, $V_{BE} = 0.8$ V, then at $I_C = 2$ mA,

$$V_{BE} = 0.8 + 0.025 \ln\left(\frac{2}{1}\right)$$

$$= 0.8 + 0.017$$

$$= 0.817 \text{ V}$$

$$V_E = -V_{BE} = -0.817 \text{ V}$$

$$I_E = \frac{2 \text{ mA}}{\alpha} = \frac{2}{0.99} = 2.02 \text{ mA}$$

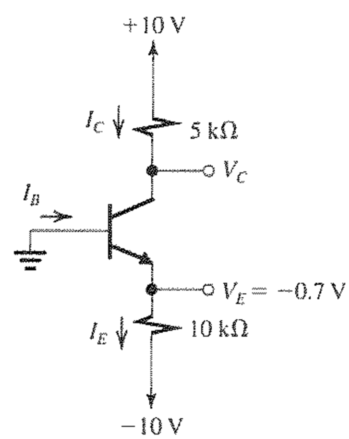
$$I_E = \frac{V_E - (-1.5)}{R_E}$$

Thus,

$$R_E = \frac{-0.817 + 1.5}{2.02} = 0.338 \text{ k}\Omega$$

$$= 338\ \Omega$$

Ex: 6.13



$$I_E = \frac{V_E - (-10)}{10} = \frac{-0.7 + 10}{10}$$

$= 0.93$ mA

Assuming active-mode operation,

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.93}{50 + 1} = 0.0182 \text{ mA}$$

$= 18.2\ \mu$A

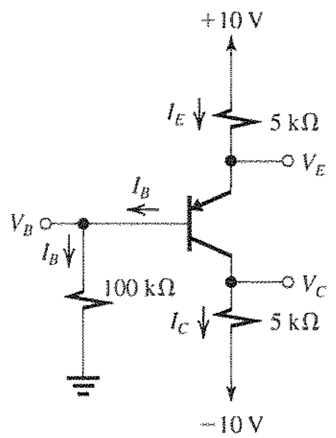
$I_C = I_E - I_B = 0.93 - 0.0182 = 0.91$ mA

$V_C = 10 - I_C \times 5$

$= 10 - 0.91 \times 5 = 5.45$ V

Since $V_C > V_B$, the transistor is operating in the active mode, as assumed.

Ex: 6.14



$V_B = 1.0$ V

Thus,

$$I_B = \frac{V_B}{100 \text{ k}\Omega} = 0.01 \text{ mA}$$

$V_E = +1.7$ V

Thus,

$$I_E = \frac{10 - V_E}{5 \text{ k}\Omega} = \frac{10 - 1.7}{5} = 1.66 \text{ mA}$$

and

$$\beta + 1 = \frac{I_E}{I_B} = \frac{1.66}{0.01} = 166$$

$\Rightarrow \beta = 165$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{165}{165 + 1} = 0.994$$

Assuming active-mode operation,

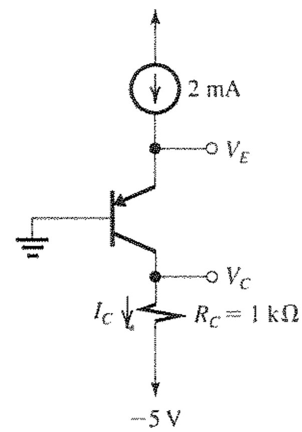
$I_C = \alpha I_E = 0.994 \times 1.66 = 1.65$ mA

and

$V_C = -10 + 1.65 \times 5 = -1.75$ V

Since $V_C < V_B$, the transistor is indeed operating in the active mode.

Ex: 6.15



The transistor is operating at a constant emitter current. Thus, a change in temperature of +30°C results in a change in V_{EB} by

$\Delta V_{EB} = -2 \text{ mV} \times 30 = -60$ mV

Thus,

$\Delta V_E = -60$ mV

Since the collector current remains unchanged at αI_E, the collector voltage does not change:

$\Delta V_C = 0$ V

Ex: 6.16 Refer to Fig. 6.19(a):

$$i_C = I_S e^{v_{BE}/V_T} + \frac{v_{CE}}{r_o} \tag{1}$$

Now using Eqs. (6.21) and (6.22), we can express r_o as

$$r_o = \frac{V_A}{I_S e^{v_{BE}/V_T}}$$

Substituting in Eq. (1), we have

$$i_C = I_S e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right)$$

which is Eq. (6.18). Q.E.D.

Ex: 6.17 $r_o = \dfrac{V_A}{I_C} = \dfrac{100}{I_C}$

At $I_C = 0.1$ mA, $r_o = 1$ MΩ

At $I_C = 1$ mA, $r_o = 100$ kΩ

At $I_C = 10$ mA, $r_o = 10$ kΩ

Ex: 6.18 $\Delta I_C = \dfrac{\Delta V_{CE}}{r_o}$

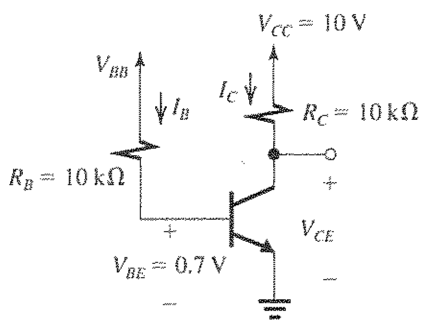
where

$r_o = \dfrac{V_A}{I_C} = \dfrac{100}{1} = 100\ \text{k}\Omega$

$\Delta I_C = \dfrac{11-1}{100} = 0.1\ \text{mA}$

Thus, I_C becomes 1.1 mA.

Ex: 6.19



(a) For operation in the active mode with $V_{CE} = 5$ V,

$I_C = \dfrac{V_{CC} - V_C}{R_C} = \dfrac{10-5}{10} = 0.5\ \text{mA}$

$I_B = \dfrac{I_C}{\beta} = \dfrac{0.5}{50} = 0.01\ \text{mA}$

$V_{BB} = V_{BE} + I_B R_B$

$= 0.7 + 0.01 \times 10 = 0.8\ \text{V}$

(b) For operation at the edge of saturation,

$V_{CE} = 0.3\ \text{V}$

$I_C = \dfrac{V_{CC} - V_{CE}}{R_C} = \dfrac{10-0.3}{10} = 0.97\ \text{mA}$

$I_B = \dfrac{I_C}{\beta} = \dfrac{0.97}{50} = 0.0194\ \text{mA}$

$V_{BB} = V_B + I_B R_B$

$= 0.7 + 0.0194 \times 10 = 0.894\ \text{V}$

(c) For operation deep in saturation with $\beta_{\text{forced}} = 10$, we have

$V_{CE} \simeq 0.2\ \text{V}$

$I_C = \dfrac{10-0.2}{10} = 0.98\ \text{mA}$

$I_B = \dfrac{I_C}{\beta_{\text{forced}}} = \dfrac{0.98}{10} = 0.098\ \text{mA}$

$V_{BB} = V_B + I_B R_B$

$= 0.7 + 0.098 \times 10 = 1.68\ \text{V}$

Ex: 6.20 For $V_{BB} = 0$ V, $I_B = 0$ and the transistor is cut off. Thus,

$I_C = 0$

and

$V_C = V_{CC} = +10\ \text{V}$

Ex: 6.21 Refer to the circuit in Fig. 6.22 and let $V_{BB} = 1.7$ V. The current I_B can be found from

$I_B = \dfrac{V_{BB} - V_B}{R_B} = \dfrac{1.7-0.7}{10} = 0.1\ \text{mA}$

Assuming operation in the active mode,

$I_C = \beta\, I_B = 50 \times 0.1 = 5\ \text{mA}$

Thus,

$V_C = V_{CC} - R_C I_C$

$= 10 - 1 \times 5 = 5\ \text{V}$

which is greater than V_B, verifying that the transistor is operating in the active mode, as assumed.

(a) To obtain operation at the edge of saturation, R_C must be increased to the value that results in $V_{CE} = 0.3$ V:

$R_C = \dfrac{V_{CC} - 0.3}{I_C}$

$= \dfrac{10-0.3}{5} = 1.94\ \text{k}\Omega$

(b) Further increasing R_C results in the transistor operating in saturation. To obtain saturation-mode operation with $V_{CE} = 0.2$ V and $\beta_{\text{forced}} = 10$, we use

$I_C = \beta_{\text{forced}} \times I_B$

$= 10 \times 0.1 = 1\ \text{mA}$

The value of R_C required can be found from

$R_C = \dfrac{V_{CC} - V_{CE}}{I_C}$

$= \dfrac{10-0.2}{1} = 9.8\ \text{k}\Omega$

Ex: 6.22 Refer to the circuit in Fig. 6.23(a) with the base voltage raised from 4 V to V_B. If at this value of V_B, the transistor is at the edge of saturation then,

$V_C = V_B - 0.4\ \text{V}$

Since $I_C \simeq I_E$, we can write

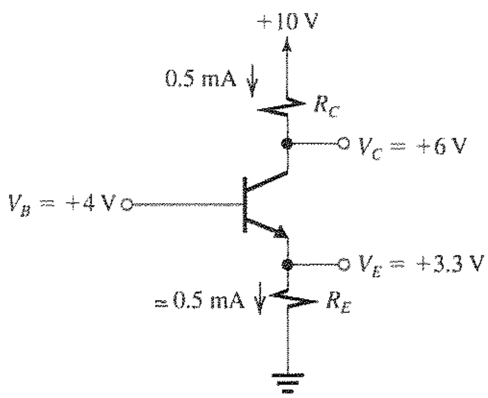
$\dfrac{10 - V_C}{R_C} = \dfrac{V_E}{R_E} = \dfrac{V_B - 0.7}{R_E}$

Thus,

$$\frac{10-(V_B-0.4)}{4.7}=\frac{V_B-0.7}{3.3}$$

$\Rightarrow V_B = +4.7$ V

Ex: 6.23



To establish a reverse-bias voltage of 2 V across the CBJ,

$V_C = +6$ V

From the figure we see that

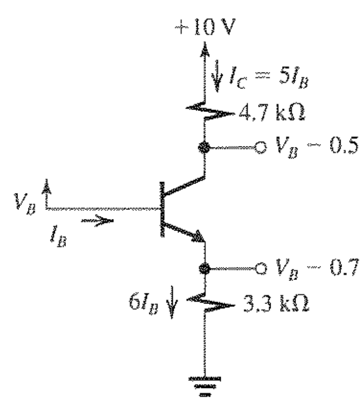
$$R_C = \frac{10-6}{0.5} = 8\text{ k}\Omega$$

and

$$R_E = \frac{3.3}{0.5} = 6.6\text{ k}\Omega$$

where we have assumed $\alpha \simeq 1$.

Ex: 6.24



The figure shows the circuit with the base voltage at V_B and the BJT operating in saturation with $V_{CE} = 0.2$ V and $\beta_{\text{forced}} = 5$.

$$I_C = 5I_B = \frac{10-(V_B-0.5)}{4.7} \quad (1)$$

$$I_E = 6I_B = \frac{V_B-0.7}{3.3} \quad (2)$$

Dividing Eq. (1) by Eq. (2), we have

$$\frac{5}{6} = \frac{10.5-V_B}{V_B-0.7} \times \frac{3.3}{4.7}$$

$\Rightarrow V_B = +5.18$ V

Ex: 6.25 Refer to the circuit in Fig. 6.26(a). The largest value for R_C while the BJT remains in the active mode corresponds to

$V_C = +0.4$ V

Since the emitter and collector currents remain unchanged, then from Fig. 6.26(b) we obtain

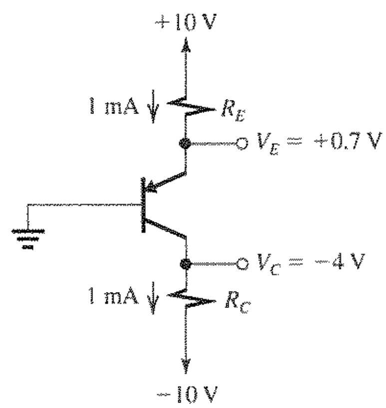
$I_C = 4.6$ mA

Thus,

$$R_C = \frac{V_C-(-10)}{I_C}$$

$$= \frac{+0.4+10}{4.6} = 2.26\text{ k}\Omega$$

Ex: 6.26



For a 4-V reverse-biased voltage across the CBJ,

$V_C = -4$ V

Refer to the figure.

$$I_C = 1\text{ mA} = \frac{V_C-(-10)}{R_C}$$

$$\Rightarrow R_C = \frac{-4+10}{1} = 6\text{ k}\Omega$$

$$R_E = \frac{10-V_E}{I_E}$$

Assuming $\alpha = 1$,

$$R_E = \frac{10-0.7}{1} = 9.3\text{ k}\Omega$$

Ex: 6.27 Refer to the circuit in Fig. 6.27:

$$I_B = \frac{5 - 0.7}{100} = 0.043 \text{ mA}$$

To ensure that the transistor remains in the active mode for β in the range 50 to 150, we need to select R_C so that for the highest collector current possible, the BJT reaches the edge of saturation, that is, $V_{CE} = 0.3$ V. Thus,

$$V_{CE} = 0.3 = 10 - R_C I_{C\max}$$

where

$$I_{C\max} = \beta_{\max} I_B$$

$$= 150 \times 0.043 = 6.45 \text{ mA}$$

Thus,

$$R_C = \frac{10 - 0.3}{6.45} = 1.5 \text{ k}\Omega$$

For the lowest β,

$$I_C = \beta_{\min} I_B$$

$$= 50 \times 0.043 = 2.15 \text{ mA}$$

and the corresponding V_{CE} is

$$V_{CE} = 10 - R_C I_C = 10 - 1.5 \times 2.15$$

$$= 6.775 \text{ V}$$

Thus, V_{CE} will range from 0.3 V to 6.8 V.

Ex: 6.28 Refer to the solution of Example 6.10.

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + [R_{BB}/(\beta + 1)]}$$

$$= \frac{5 - 0.7}{3 + (33.3/51)} = 1.177 \text{ mA}$$

$$I_C = \alpha I_E = 0.98 \times 1.177 = 1.15 \text{ mA}$$

Thus the current is reduced by

$$\Delta I_C = 1.28 - 1.15 = 0.13 \text{ mA}$$

which is a -10% change.

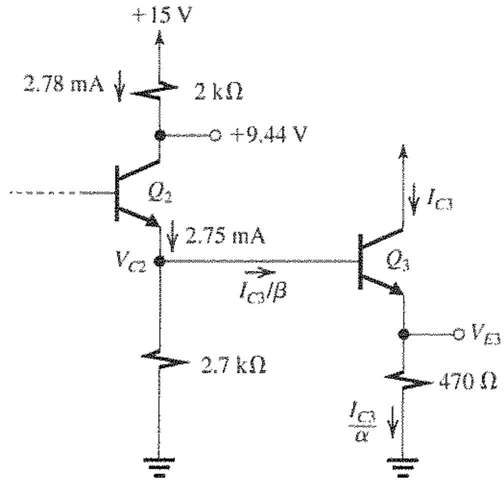
Ex: 6.29 Refer to the circuit in Fig. 6.30(b). The total current drawn from the power supply is

$$I = 0.103 + 1.252 + 2.78 = 4.135 \text{ mA}$$

Thus, the power dissipated in the circuit is

$$P = 15 \text{ V} \times 4.135 \text{ mA} = 62 \text{ mW}$$

Ex: 6.30



From the figure we see that

$$V_{E3} = \frac{I_{C3}}{\alpha} \times 0.47$$

$$V_{C2} = V_{E3} + 0.7 = \frac{I_{C3}}{\alpha} \times 0.47 + 0.7 \qquad (1)$$

A node equation at the collector of Q_2 yields

$$2.75 = \frac{V_{C2}}{2.7} + \frac{I_{C3}}{\beta}$$

Substituting for V_{C2} from Eq. (1), we obtain

$$2.75 = \frac{(0.47\, I_{C3}/\alpha) + 0.7}{2.7} + \frac{I_{C3}}{\beta}$$

Substituting $\alpha = 0.99$ and $\beta = 100$ and solving for I_{C3} results in

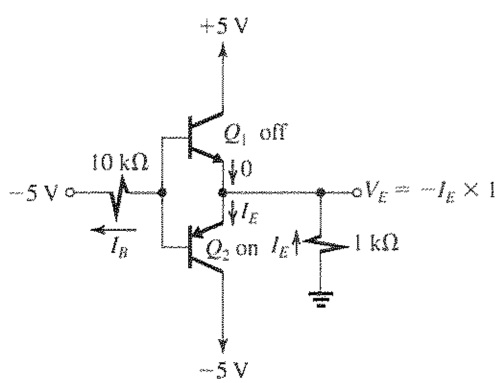
$$I_{C3} = 13.4 \text{ mA}$$

Now, V_{E3} and V_{C2} can be determined:

$$V_{E3} = \frac{I_{C3}}{\alpha} \times 0.47 = \frac{13.4}{0.99} \times 0.47 = +6.36 \text{ V}$$

$$V_{C2} = V_{E3} + 0.7 = +7.06 \text{ V}$$

Ex: 6.31



From the figure we see that Q_1 will be off and Q_2 will be on. Since the base of Q_2 will be at a voltage higher than -5 V, transistor Q_2 will be operating in the active mode. We can write a loop equation for the loop containing the 10-kΩ resistor, the EBJ of Q_2 and the 1-kΩ resistor:

$$-I_E \times 1 - 0.7 - I_B \times 10 = -5$$

Substituting $I_B = I_E/(\beta + 1) = I_E/101$ and rearranging gives

$$I_E = \frac{5 - 0.7}{\frac{10}{101} + 1} = 3.9 \text{ mA}$$

Thus,

$$V_E = -3.9 \text{ V}$$

$$V_{B2} = -4.6 \text{ V}$$

$$I_B = 0.039 \text{ mA}$$

Ex: 6.32 With the input at + 10 V, there is a strong possibility that the conducting transistor Q_1 will be saturated. Assuming this to be the case, the analysis steps will be as follows:

$$V_{CEsat}|_{Q_1} = 0.2 \text{ V}$$

$$V_E = 5 \text{ V} - V_{CEsat} = +4.8 \text{ V}$$

$$I_{E1} = \frac{4.8 \text{ V}}{1 \text{ k}\Omega} = 4.8 \text{ mA}$$

$$V_{B1} = V_E + V_{BE1} = 4.8 + 0.7 = +5.5 \text{ V}$$

$$I_{B1} = \frac{10 - 5.5}{10} = 0.45 \text{ mA}$$

$$I_{C1} = I_{E1} - I_{B1} = 4.8 - 0.45 = 4.35 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{4.35}{0.45} = 9.7$$

which is lower than $\beta_{\min}$, verifying that Q_1 is indeed saturated.

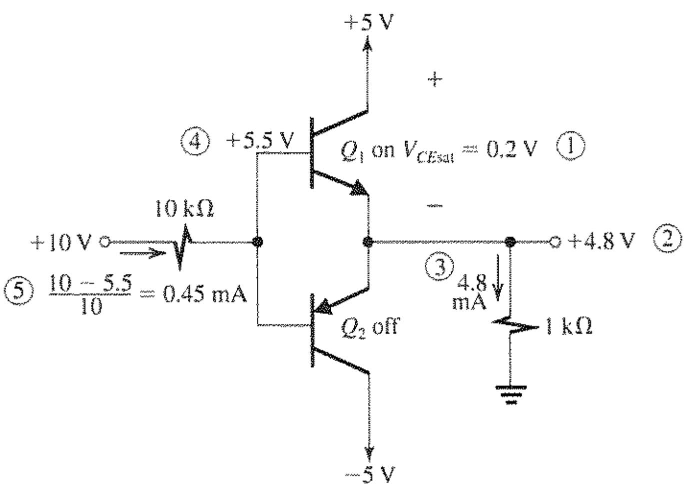
Finally, since Q_2 is off,

$$I_{C2} = 0$$

Ex: 6.33 $V_O = +10 - BV_{BCO} = 10 - 70$

$= -60$ V

This figure belongs to Exercise 6.32.



Ex: 7.1 Refer to Fig. 7.2(a) and 7.2(b).

Coordinates of point A: V_t and V_{DD}; thus 0.4 V and 1.8 V. To determine the coordinates of point B, we use Eqs. (7.7) and (7.8) as follows:

$$V_{OV}\big|_B = \frac{\sqrt{2k_nR_DV_{DD}+1}-1}{k_nR_D}$$

$$= \frac{\sqrt{2\times 4\times 17.5\times 1.8+1}-1}{4\times 17.5}$$

$= 0.213$ V

Thus,

$$V_{GS}\big|_B = V_t + V_{OV}\big|_B = 0.4+0.213 = 0.613 \text{ V}$$

and

$$V_{DS}\big|_B = V_{OV}\big|_B = 0.213 \text{ V}$$

Thus, coordinates of B are 0.613 V and 0.213 V. At point C, the MOSFET is operating in the triode region, thus

$$i_D = k_n\left[(v_{GS}\big|_C - V_t)v_{DS}\big|_C - \frac{1}{2}v_{DS}^2\big|_C\right]$$

If $v_{DS}\big|_C$ is very small,

$$i_D \simeq k_n(v_{GS}\big|_C - V_t)v_{DS}\big|_C$$

$$= 4(1.8-0.4)v_{DS}\big|_C$$

$$= 5.6v_{DS}\big|_C, \text{ mA}$$

But

$$i_D = \frac{V_{DD}-v_{DS}\big|_C}{R_D} \simeq \frac{V_{DD}}{R_D} = \frac{1.8}{17.5} = 0.1 \text{ mA}$$

Thus, $v_{DS}\big|_C = \frac{0.1}{5.6} = 0.018$ V $= 18$ mV, which is indeed very small, as assumed.

Ex: 7.2 Refer to Example 7.1 and Fig. 7.4(a).

Design 1:

$V_{OV} = 0.2$ V, $V_{GS} = 0.6$ V

$I_D = 0.8$ mA

Now,

$$A_v = -k_nV_{OV}R_D$$

Thus,

$$-10 = -0.4\times 10\times 0.2\times R_D$$

$$\Rightarrow R_D = 12.5 \text{ k}\Omega$$

$$V_{DS} = V_{DD} - R_DI_D$$

$$= 1.8 - 12.5\times 0.08 = 0.8 \text{ V}$$

Design 2:

$R_D = 17.5$ kΩ

$$A_v = -k_nV_{OV}R_D$$

$$-10 = -0.4\times 10\times V_{OV}\times 17.5$$

Thus,

$V_{OV} = 0.14$ V

$$V_{GS} = V_t + V_{OV} = 0.4+0.14 = 0.54 \text{ V}$$

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2$$

$$= \frac{1}{2}\times 0.4\times 10\times 0.14^2 = 0.04 \text{ mA}$$

$R_D = 17.5$ kΩ

$$V_{DS} = V_{DD} - R_DI_D$$

$$= 1.8 - 17.5\times 0.04 = 1.1 \text{ V}$$

Ex: 7.3

$$A_v = -\frac{I_CR_C}{V_T}$$

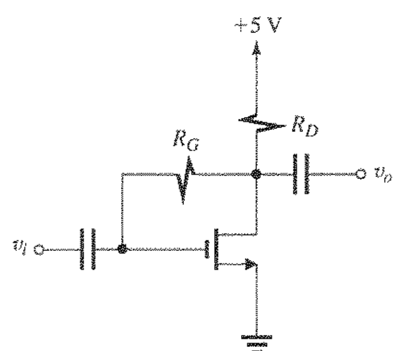
$$-320 = -\frac{1\times R_C}{0.025} \Rightarrow R_C = 8 \text{ k}\Omega$$

$$V_C = V_{CC} - I_CR_C$$

$$= 10 - 1\times 8 = 2 \text{ V}$$

Since the collector voltage is allowed to decrease to +0.3 V, the largest negative swing allowed at the output is $2 - 0.3 = 1.7$ V. The corresponding input signal amplitude can be found by dividing 1.7 V by the gain magnitude (320 V/V), resulting in 5.3 mV.

Ex: 7.4



Refer to the solution of Example 7.3. From Eq. (7.47), $A_v \equiv \frac{v_o}{v_i} = -g_mR_D$ (note that R_L is absent).

Thus,

$g_mR_D = 25$

Substituting for $g_m = k_nV_{OV}$, we have

$k_nV_{OV}R_D = 25$

where $k_n = 1\ \text{mA/V}^2$, thus

$$V_{OV} R_D = 25 \qquad (1)$$

Next, consider the bias equation

$$V_{GS} = V_{DS} = V_{DD} - R_D I_D$$

Thus,

$$V_t + V_{OV} = V_{DD} - R_D I_D$$

Substituting $V_t = 0.7$ V, $V_{DD} = 5$ V, and

$$I_D = \frac{1}{2} k_n V_{OV}^2 = \frac{1}{2} \times 1 \times V_{OV}^2 = \frac{1}{2} V_{OV}^2$$

we obtain

$$0.7 + V_{OV} = 5 - \frac{1}{2} V_{OV}^2 R_D \qquad (2)$$

Equations (1) and (2) can be solved to obtain

$$V_{OV} = 0.319\ \text{V}$$

and

$$R_D = 78.5\ \text{k}\Omega$$

The dc current I_D can be now found as

$$I_D = \frac{1}{2} k_n V_{OV}^2 = 50.9\ \mu\text{A}$$

To determine the required value of R_G we use Eq. (7.48), again noting that R_L is absent:

$$R_{\text{in}} = \frac{R_G}{1 + g_m R_D}$$

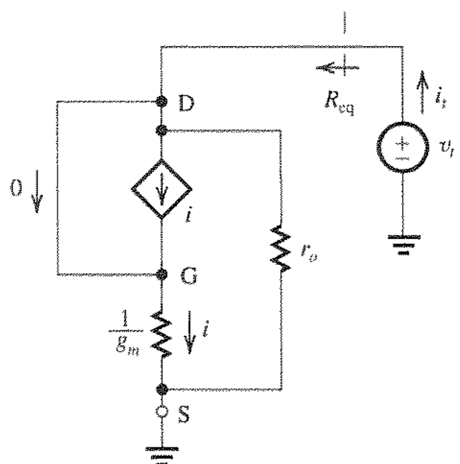
$$0.5\ \text{M}\Omega = \frac{R_G}{1 + 25}$$

$$\Rightarrow R_G = 13\ \text{M}\Omega$$

Finally, the maximum allowable input signal $\hat{v}_i$ can be found as follows:

$$\hat{v}_i = \frac{V_t}{|A_v| + 1} = \frac{0.7\ \text{V}}{25 + 1} = 27\ \text{mV}$$

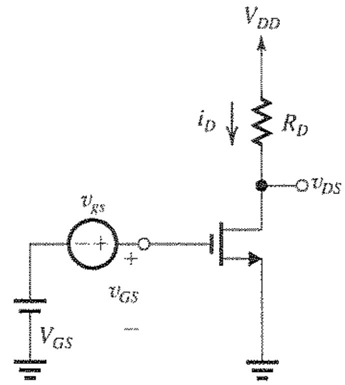
Ex: 7.5



$$i_t = \frac{v_t}{r_o} + i = \frac{v_t}{r_o} + g_m v_t$$

$$\therefore R_{\text{eq}} = \frac{v_t}{i_t} = r_o \| \frac{1}{g_m}$$

Ex: 7.6



$V_{DD} = 5$ V

$V_{GS} = 2$ V

$V_t = 1$ V

$\lambda = 0$

$k'_n = 20\ \mu\text{A/V}^2$

$R_D = 10\ \text{k}\Omega$

$\frac{W}{L} = 20$

(a) $V_{GS} = 2\ \text{V} \Rightarrow V_{OV} = 1\ \text{V}$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 = 200\ \mu\text{A}$$

$$V_{DS} = V_{DD} - I_D R_D = +3\ \text{V}$$

(b) $g_m = k'_n \frac{W}{L} V_{OV} = 400\ \mu\text{A/V} = 0.4\ \text{mA/V}$

(c) $A_v = \frac{v_{ds}}{v_{gs}} = -g_m R_D = -4\ \text{V/V}$

(d) $v_{gs} = 0.2 \sin\omega t$ V

$v_{ds} = -0.8 \sin\omega t$ V

$v_{DS} = V_{DS} + v_{ds} \Rightarrow 2.2\ \text{V} \le v_{DS} \le 3.8\ \text{V}$

(e) Using Eq. (7.28), we obtain

$$i_D = \frac{1}{2} k_n (V_{GS} - V_t)^2 + k_n (V_{GS} - V_t) v_{gs} + \frac{1}{2} k_n v_{gs}^2$$

$$i_D = 200 + 80 \sin\omega t + 8 \sin^2\omega t,\ \mu\text{A}$$

$= [200 + 80\sin\omega t + (4 - 4\cos 2\omega t)]$

$= 204 + 80\sin\omega t - 4\cos 2\omega t$, μA

I_D shifts by 4 μA.

Thus,

$$2\text{HD} = \frac{\hat{i}_{2\omega}}{\hat{i}_\omega} = \frac{4\ \mu\text{A}}{80\ \mu\text{A}} = 0.05\ (5\%)$$

Ex: 7.7

(a) $g_m = \dfrac{2I_D}{V_{OV}}$

$I_D = \dfrac{1}{2}k_n'\dfrac{W}{L}V_{OV}^2 = \dfrac{1}{2} \times 60 \times 40 \times (1.5 - 1)^2$

$I_D = 300\ \mu\text{A} = 0.3$ mA, $V_{OV} = 0.5$ V

$g_m = \dfrac{2 \times 0.3}{0.5} = 1.2$ mA/V

$r_o = \dfrac{V_A}{I_D} = \dfrac{15}{0.3} = 50\ \text{k}\Omega$

(b) $I_D = 0.5\ \text{mA} \Rightarrow g_m = \sqrt{2\ \mu_n C_{ox}\dfrac{W}{L}I_D}$

$= \sqrt{2 \times 60 \times 40 \times 0.5 \times 10^3}$

$g_m = 1.55$ mA/V

$r_o = \dfrac{V_A}{I_D} = \dfrac{15}{0.5} = 30\ \text{k}\Omega$

Ex: 7.8

$I_D = 0.1$ mA, $g_m = 1$ mA/V, $k_n' = 50\ \mu\text{A/V}^2$

$g_m = \dfrac{2I_D}{V_{OV}} \Rightarrow V_{OV} = \dfrac{2 \times 0.1}{1} = 0.2$ V

$I_D = \dfrac{1}{2}k_n'\dfrac{W}{L}V_{OV}^2 \Rightarrow \dfrac{W}{L} = \dfrac{2I_D}{k_n' V_{OV}^2}$

$= \dfrac{2 \times 0.1}{\dfrac{50}{1000} \times 0.2^2} = 100$

Ex: 7.9

$g_m = \mu_n C_{ox}\dfrac{W}{L}V_{OV}$

Same bias conditions, so same V_{OV} and also same L and g_m for both PMOS and NMOS.

$\mu_n C_{ox} W_n = \mu_p C_{ox} W_p \Rightarrow \dfrac{\mu_p}{\mu_n} = 0.4 = \dfrac{W_n}{W_p}$

$\Rightarrow \dfrac{W_p}{W_n} = 2.5$

Ex: 7.10

$I_D = \dfrac{1}{2}k_p'\dfrac{W}{L}(V_{SG} - |V_t|)^2$

$= \dfrac{1}{2} \times 60 \times \dfrac{16}{0.8} \times (1.6 - 1)^2$

$I_D = 216\ \mu$A

$g_m = \dfrac{2I_D}{|V_{OV}|} = \dfrac{2 \times 216}{1.6 - 1} = 720\ \mu$A/V

$= 0.72$ mA/V

$\lambda = 0.04 \Rightarrow V_A' = \dfrac{1}{\lambda} = \dfrac{1}{0.04} = 25\ \text{V}/\mu\text{m}$

$r_o = \dfrac{V_A' \times L}{I_D} = \dfrac{25 \times 0.8}{0.216} = 92.6\ \text{k}\Omega$

Ex: 7.11

$g_m r_o = \dfrac{2I_D}{V_{OV}} \times \dfrac{V_A}{I_D} = \dfrac{2V_A}{V_{OV}}$

$V_A' \times L = V_A$

$L = 0.8\ \mu\text{m} \Rightarrow g_m r_o = \dfrac{2 \times 12.5 \times 0.8}{0.2}$

$= 100$ V/V

Ex: 7.12

Given: $g_m = \left.\dfrac{\partial i_C}{\partial v_{BE}}\right|_{i_C = I_C}$

where $I_C = I_S e^{V_{BE}/V_T}$

$\dfrac{\partial i_C}{\partial v_{BE}} = \dfrac{I_S e^{V_{BE}/V_T}}{V_T} = \dfrac{I_C}{V_T}$

Thus,

$g_m = \dfrac{I_C}{V_T}$

Ex: 7.13

$g_m = \dfrac{I_C}{V_T} = \dfrac{0.5\ \text{mA}}{25\ \text{mV}} = 20$ mA/V

Ex: 7.14

$I_C = 0.5$ mA (constant)

$\beta = 50 \qquad \beta = 200$

$g_m = \dfrac{I_C}{V_T} = \dfrac{0.5\ \text{mA}}{25\ \text{mV}}$

$= 20\ \text{mA/V} \qquad = 20\ \text{mA/V}$

$I_B = \dfrac{I_C}{\beta} = \dfrac{0.5}{50} = \dfrac{0.5}{200}$

$= 10\ \mu\text{A} \qquad = 2.5\ \mu\text{A}$

$r_\pi = \dfrac{\beta}{g_m} = \dfrac{50}{20} = \dfrac{200}{20}$

$= 2.5\ \text{k}\Omega \qquad = 10\ \text{k}\Omega$

Ex: 7.15

$\beta = 100 \quad I_C = 1$ mA

$$g_m = \frac{1 \text{ mA}}{25 \text{ mV}} = 40 \text{ mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C} \simeq \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \ \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

Ex: 7.16

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{25 \text{ mV}} = 40 \text{ mA/V}$$

$$A_v = \frac{v_{ce}}{v_{be}} = -g_m R_C$$

$$= -40 \times 10$$

$$= -400 \text{ V/V}$$

$$V_C = V_{CC} - I_C R_C$$

$$= 15 - 1 \times 10 = 5 \text{ V}$$

$$v_C(t) = V_C + v_c(t)$$

$$= (V_{CC} - I_C R_C) + A_v v_{be}(t)$$

$$= (15 - 10) - 400 \times 0.005 \sin\omega t$$

$$= 5 - 2 \sin\omega t$$

$$i_B(t) = I_B + i_b(t)$$

where

$$I_B = \frac{I_C}{\beta} = \frac{1 \text{ mA}}{100} = 10 \ \mu\text{A}$$

and $i_b(t) = \dfrac{g_m v_{be}(t)}{\beta}$

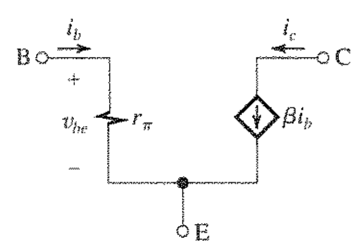
$$= \frac{40 \times 0.005 \sin\omega t}{100}$$

$$= 2 \sin\omega t, \ \mu\text{A}$$

Thus,

$$i_B(t) = 10 + 2 \sin\omega t, \ \mu\text{A}$$

Ex: 7.17



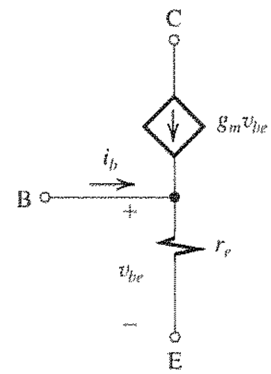
$$i_c = \beta i_b = \beta \frac{v_{be}}{r_\pi}$$

$$= \left(\frac{\beta}{r_\pi}\right) v_{be} = g_m v_{be}$$

$$i_e = i_b + \beta i_b = (\beta + 1) i_b = (\beta + 1) \frac{v_{be}}{r_\pi}$$

$$= \frac{v_{be}}{r_\pi(\beta + 1)} = \frac{v_{be}}{r_e}$$

Ex: 7.18



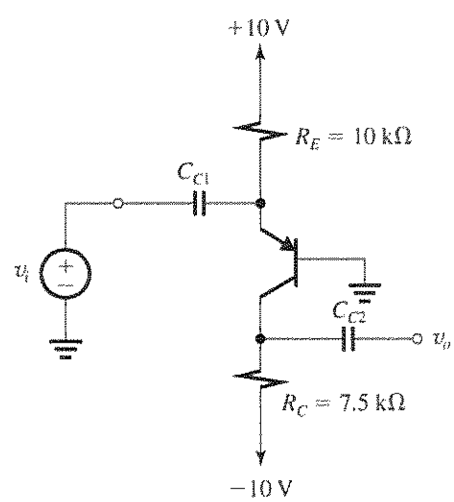
$$i_b = \frac{v_{be}}{r_e} - g_m v_{be}$$

$$= v_{be}\left(\frac{1}{r_e} - g_m\right)$$

$$= v_{be}\left(\frac{1}{r_{\pi/\beta+1}} - \frac{\beta}{r_\pi}\right)$$

$$= v_{be}\left(\frac{\beta + 1}{r_\pi} - \frac{\beta}{r_\pi}\right) = \frac{v_{be}}{r_\pi}$$

Ex: 7.19



$I_E = \frac{10 - 0.7}{10} = 0.93$ mA

$I_C = \alpha I_E = 0.99 \times 0.93$

$= 0.92$ mA

$V_C = -10 + I_C R_C$

$= -10 + 0.92 \times 7.5 = -3.1$ V

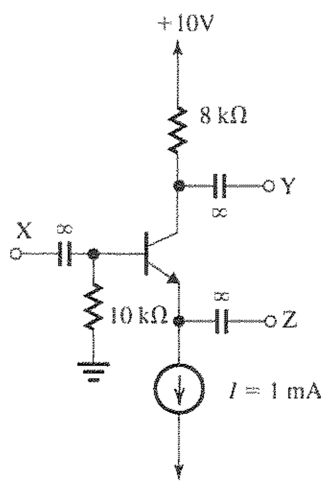
$A_v = \frac{v_o}{v_i} = \frac{\alpha R_C}{r_e}$

where $r_e = \frac{25\text{ mV}}{0.93\text{ mA}} = 26.9\ \Omega$

$A_v = \frac{0.99 \times 7.5 \times 10^3}{26.9} = 276.2$ V/V

For $\hat{v}_i = 10$ mV, $\hat{v}_o = 276.2 \times 10 = 2.76$ V

Ex: 7.20



$I_E = 1$ mA

$I_C = \frac{100}{101} \times 1 = 0.99$ mA

$I_B = \frac{1}{101} \times 1 = 0.0099$ mA

(a) $V_C = 10 - 8 \times 0.99 = 2.08 \simeq 2.1$ V

$V_B = -10 \times 0.0099 = -0.099 \simeq -0.1$ V

$V_E = -0.1 - 0.7 = -0.8$ V

(b) $g_m = \frac{I_C}{V_T} = \frac{0.99}{0.025} \simeq 40$ mA/V

$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} \simeq 2.5$ kΩ

$r_o = \frac{V_A}{I_C} = \frac{100}{0.99} = 101 \simeq 100$ kΩ

(c) $R_{sig} = 2$ kΩ $\quad R_B = 10$ kΩ $\quad r_\pi = 2.5$ kΩ

$g_m = 40$ mA/V

$R_C = 8$ kΩ $\quad R_L = 8$ kΩ $\quad r_o = 100$ kΩ

$$\frac{V_y}{V_{sig}} = \frac{V_\pi}{V_{sig}} \times \frac{V_y}{V_\pi}$$

$$= \frac{R_B \| r_\pi}{(R_B \| r_\pi) + R_{sig}} \times -g_m(R_C \| R_L \| r_o)$$

$$= \frac{10 \| 2.5}{(10 \| 2.5) + 2} \times -40(8 \| 8 \| 100)$$

$-0.5 \times 40 \times 3.846 = -77$ V/V

If r_o is negelected, $\frac{V_y}{V_{sig}} = -80$, for an error of 3.9%.

Ex: 7.21

$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.25}{0.25} = 2$ mA/V

$R_{in} = \infty$

$A_{vo} = -g_m R_D = -2 \times 20 = -40$ V/V

$R_o = R_D = 20$ kΩ

$A_v = A_{vo}\frac{R_L}{R_L + R_o} = -40 \times \frac{20}{20 + 20}$

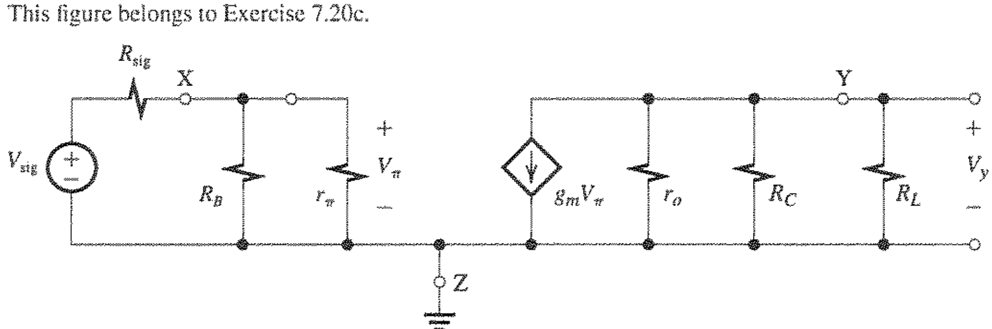
$= -20$ V/V

$G_v = A_v = -20$ V/V

$\hat{v}_i = 0.1 \times 2V_{OV} = 0.2 \times 2 \times 0.25 = 0.05$ V

$\hat{v}_o = 0.05 \times 20 = 1$ V

This figure belongs to Exercise 7.20c.



Ex: 7.22

$I_C = 0.5$ mA

$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20$ mA/V

$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5$ kΩ

$R_{in} = r_\pi = 5$ kΩ

$A_{vo} = -g_m R_C = -20 \times 10 = -200$ V/V

$R_o = R_C = 10$ kΩ

$A_v = A_{vo} \frac{R_L}{R_L + R_o} = -200 \times \frac{5}{5+10}$

$= -66.7$ V/V

$G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_v = \frac{5}{5+5} \times -66.7$

$= -33.3$ V/V

$\hat{v}_\pi = 5 \text{ mV} \Rightarrow \hat{v}_{sig} = 2 \times 5 = 10$ mV

$\hat{v}_o = 10 \times 33.3 = 0.33$ V

Although a larger fraction of the input signal reaches the amplifier input, linearity considerations cause the output signal to be in fact smaller than in the original design!

Ex: 7.23 Refer to the solution to Exercise 7.21. If $\hat{v}_{sig} = 0.2$ V and we wish to keep $\hat{v}_{gs} = 50$ mV, then we need to connect a resistance $R_s = \frac{3}{g_m}$ in the source lead. Thus,

$R_s = \frac{3}{2 \text{ mA/V}} = 1.5$ kΩ

$G_v = A_v = -\frac{R_D \| R_L}{\frac{1}{g_m} + R_s}$

$= -\frac{20\|20}{0.5+1.5} = -5$ V/V

$\hat{v}_o = G_v \hat{v}_{sig} = 5 \times 0.2 = 1$ V (unchanged)

Ex: 7.24

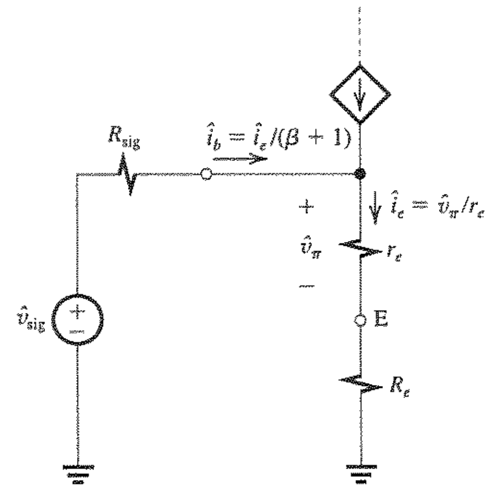
From the following figure we see that

$\hat{v}_{sig} = \hat{i}_b R_{sig} + \hat{v}_\pi + \hat{i}_e R_e$

$= \frac{\hat{i}_e}{\beta+1} R_{sig} + \hat{v}_\pi + \hat{i}_e R_e$

$= \frac{\hat{v}_\pi}{(\beta+1) r_e} R_{sig} + \hat{v}_\pi + \frac{\hat{v}_\pi}{r_e} R_e$

$\hat{v}_{sig} = \hat{v}_\pi \left(1 + \frac{R_e}{r_e} + \frac{R_{sig}}{r_\pi}\right)$ Q.E.D



For $I_C = 0.5$ mA and $\beta = 100$,

$r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C} = \frac{0.99 \times 25}{0.5} \simeq 50$ Ω

$r_\pi = (\beta+1) r_e \simeq 5$ kΩ

For $\hat{v}_{sig} = 100$ mV, $R_{sig} = 10$ kΩ and with $\hat{v}_\pi$ limited to 10 mV, the value of R_e required can be found from

$100 = 10\left(1 + \frac{R_e}{50} + \frac{10}{5}\right)$

$\Rightarrow R_e = 350$ Ω

$R_{in} = (\beta+1)(r_e + R_e) = 101 \times (50 + 350)$

$= 40.4$ kΩ

$G_v = -\beta \frac{R_C \| R_L}{R_{sig} + (\beta+1)(r_e + R_e)}$

$= -100 \frac{10}{10 + 101 \times 0.4} = -19.8$ V/V

Ex: 7.25

$\frac{1}{g_m} = R_{sig} = 100$ Ω

$\Rightarrow g_m = \frac{1}{0.1 \text{ k}\Omega} = 10$ mA/V

But

$g_m = \frac{2I_D}{V_{OV}}$

Thus,

$10 = \frac{2I_D}{0.2}$

$\Rightarrow I_D = 1$ mA

$G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times g_m R_D$

$= 0.5 \times 10 \times 2 = 10$ V/V

Ex: 7.26

$I_C = 1\text{ mA}$

$r_e = \frac{V_T}{I_E} \simeq \frac{V_T}{I_C} = \frac{25\text{ mV}}{1\text{ mA}} = 25\ \Omega$

$R_{in} = r_e = 25\ \Omega$

$A_{vo} = g_m R_C = 40 \times 5 = 200\text{ V/V}$

$R_o = R_C = 5\text{ k}\Omega$

$A_v = A_{vo}\frac{R_L}{R_L + R_o} = 200 \times \frac{5}{5+5} = 100\text{ V/V}$

$G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times A_v$

$= \frac{25}{25 + 5000} \times 100 = 0.5\text{ V/V}$

Ex: 7.27

$R_{in} = r_e = 50\ \Omega$

$\Rightarrow I_E = \frac{V_T}{r_e} = \frac{25\text{ mV}}{50\ \Omega} = 0.5\text{ mA}$

$I_C \simeq I_E = 0.5\text{ mA}$

$G_v = \frac{R_C \| R_L}{r_e + R_{sig}}$

$40 = \frac{R_C \| R_L}{(50 + 50)\Omega}$

$R_C \| R_L = 4\text{ k}\Omega$

Ex: 7.28 Refer to Fig. 7.41(c).

$R_o = 100\ \Omega$

Thus,

$\frac{1}{g_m} = 100\ \Omega \Rightarrow g_m = 10\text{ mA/V}$

But

$g_m = \frac{2I_D}{V_{OV}}$

Thus,

$I_D = \frac{10 \times 0.25}{2} = 1.25\text{ mA}$

$\hat{v}_o = \hat{v}_i \times \frac{R_L}{R_L + R_o} = 1 \times \frac{1}{1 + 0.1} = 0.91\text{ V}$

$\hat{v}_{gs} = \hat{v}_i \frac{\frac{1}{g_m}}{\frac{1}{g_m} + R_L} = 1 \times \frac{0.1}{0.1 + 1} = 91\text{ mV}$

Ex: 7.29

$R_o = 200\ \Omega$

$\frac{1}{g_m} = 200\ \Omega$

$\Rightarrow g_m = 5\text{ mA/V}$

But

$gm = k_n'\left(\frac{W}{L}\right)V_{OV}$

Thus,

$5 = 0.4 \times \frac{W}{L} \times 0.25$

$\Rightarrow \frac{W}{L} = 50$

$I_D = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2$

$= \frac{1}{2} \times 0.4 \times 50 \times 0.25^2$

$= 0.625\text{ mA}$

$R_L = 1\text{ k}\Omega$ to $10\text{ k}\Omega$

Correspondingly,

$G_v = \frac{R_L}{R_L + R_o} = \frac{R_L}{R_L + 0.2}$

will range from

$G_v = \frac{1}{1 + 0.2} = 0.83\text{ V/V}$

to

$G_v = \frac{10}{10 + 0.2} = 0.98\text{ V/V}$

Ex: 7.30

$I_C = 5\text{ mA}$

$r_e = \frac{V_T}{I_E} \simeq \frac{V_T}{I_C} = \frac{25\text{ mV}}{5\text{ mA}} = 5\ \Omega$

$R_{sig} = 10\text{ k}\Omega \quad R_L = 1\text{ k}\Omega$

$R_{in} = (\beta + 1)\ (r_e + R_L)$

$= 101 \times (0.005 + 1)$

$= 101.5\text{ k}\Omega$

$G_{vo} = 1\text{ V/V}$

$R_{out} = r_e + \frac{R_{sig}}{\beta + 1}$

$= 5 + \frac{10{,}000}{101} = 104\ \Omega$

$G_v = \frac{R_L}{R_L + r_e + \frac{R_{sig}}{\beta + 1}} = \frac{R_L}{R_L + R_{out}}$

$= \frac{1}{1 + 0.104} = 0.91\text{ V/V}$

$$v_\pi = v_{sig}\frac{r_e}{r_e + R_L + \dfrac{R_{sig}}{\beta+1}}$$

$$\hat{v}_{sig} = \hat{v}_\pi\left[1 + \frac{R_L}{r_e} + \frac{R_{sig}}{(\beta+1)\,r_e}\right]$$

$$\hat{v}_{sig} = 5\left[1 + \frac{1000}{5} + \frac{10{,}000}{101\times 5}\right] = 1.1\ \text{V/V}$$

Correspondingly,

$$\hat{v}_o = G_v \times 1.1 = 0.91 \times 1.1 = 1\ \text{V}$$

Ex: 7.31

$$I_D = \frac{1}{2}k_n'\frac{W}{L}(V_{GS} - V_t)^2$$

$$0.5 = \frac{1}{2}\times 1(V_{GS} - 1)^2$$

$$\Rightarrow V_{GS} = 2\ \text{V}$$

If $V_t = 1.5$ V, then

$$I_D = \frac{1}{2}\times 1\times(2 - 1.5)^2 = 0.125\ \text{mA}$$

$$\Rightarrow \frac{\Delta I_D}{I_D} = \frac{0.125 - 0.5}{0.5} = -0.75 = -75\%$$

Ex: 7.32

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{5-2}{0.5} = 6\ \text{k}\Omega$$

$$\to R_D = 6.2\ \text{k}\Omega$$

$$I_D = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2 \Rightarrow 0.5 = \frac{1}{2}\times 1\times V_{OV}^2$$

$$\Rightarrow V_{OV} = 1\ \text{V}$$

$$\Rightarrow V_{GS} = V_{OV} + V_t = 1 + 1 = 2\ \text{V}$$

$$\Rightarrow V_S = -2\ \text{V}$$

$$R_S = \frac{V_S - V_{SS}}{I_D} = \frac{-2-(-5)}{0.5} = 6\ \text{k}\Omega$$

$$\to R_S = 6.2\ \text{k}\Omega$$

If we choose $R_D = R_S = 6.2$ kΩ, then I_D will change slightly:

$I_D = \dfrac{1}{2}\times 1\times(V_{GS} - 1)^2$. Also

$$V_{GS} = -V_S = 5 - R_S I_D$$

$$2I_D = (4 - 6.2I_D)^2$$

$$\Rightarrow 38.44I_D^2 - 51.6I_D^2 + 16 = 0$$

$$\Rightarrow I_D = 0.49\ \text{mA},\ 0.86\ \text{mA}$$

$I_D = 0.86$ results in $V_S > 0$ or $V_S > V_G$, which is not acceptable. Therefore $I_D = 0.49$ mA and

$$V_S = -5 + 6.2\times 0.49 = -1.96\ \text{V}$$

$$V_D = 5 - 6.2\times 0.49 = +1.96\ \text{V}$$

R_G should be selected in the range of 1 MΩ to 10 MΩ to have low current.

Ex: 7.33

$$I_D = 0.5\ \text{mA} = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2$$

$$\Rightarrow V_{OV}^2 = \frac{0.5\times 2}{1} = 1$$

$$\Rightarrow V_{OV} = 1\ \text{V} \Rightarrow V_{GS} = 1 + 1 = 2\ \text{V}$$

$$= V_D \Rightarrow R_D = \frac{5-2}{0.5} = 6\ \text{k}\Omega$$

$\Rightarrow R_D = 6.2$ kΩ (standard value). For this R_D we have to recalculate I_D:

$$I_D = \frac{1}{2}\times 1\times(V_{GS} - 1)^2$$

$$= \frac{1}{2}(V_{DD} - R_D I_D - 1)^2$$

$$(V_{GS} = V_D = V_{DD} - R_D I_D)$$

$$I_D = \frac{1}{2}(4 - 6.2I_D)^2 \Rightarrow I_D \cong 0.49\ \text{mA}$$

$$V_D = 5 - 6.2\times 0.49 = 1.96\ \text{V}$$

Ex: 7.34 Refer to Example 7.12.

(a) For design 1, $R_E = 3$ kΩ, $R_1 = 80$ kΩ, and $R_2 = 40$ kΩ. Thus, $V_{BB} = 4$ V.

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \dfrac{R_1 \| R_2}{\beta+1}}$$

For the nominal case, $\beta = 100$ and

$$I_E = \frac{4 - 0.7}{3 + \dfrac{40\|80}{101}} = 1.01 \simeq 1\ \text{mA}$$

For $\beta = 50$,

$$I_E = \frac{4 - 0.7}{3 + \dfrac{40\|80}{51}} = 0.94\ \text{mA}$$

For $\beta = 150$,

$$I_E = \frac{4 - 0.7}{3 + \dfrac{40\|80}{151}} = 1.04\ \text{mA}$$

Thus, I_E varies over a range approximately 10% of the nominal value of 1 mA.

(b) For design 2, $R_E = 3.3$ kΩ, $R_1 = 8$ kΩ, and $R_2 = 4$ kΩ. Thus, $V_{BB} = 4$ V. For the nominal case, $\beta = 100$ and

$$I_E = \frac{4 - 0.7}{3.3 + \frac{4\|8}{101}} = 0.99 \simeq 1 \text{ mA}$$

For $\beta = 50$,

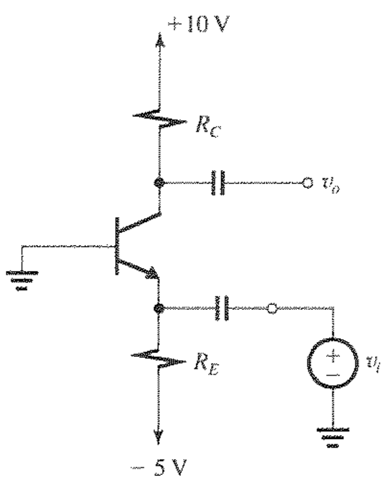
$$I_E = \frac{4 - 0.7}{3.3 + \frac{4\|8}{51}} = 0.984 \text{ mA}$$

For $\beta = 150$,

$$I_E = \frac{4 - 0.7}{3.3 + \frac{4\|8}{151}} = 0.995 \text{ mA}$$

Thus, I_E varies over a range of 1.1% of the nominal value of 1 mA. Note that lowering the resistances of the voltage divider considerably decreases the dependence on the value of β, a highly desirable result obtained at the expense of increased current and hence power dissipation.

Ex: 7.35 Refer to Fig. 7.53. Since the circuit is to be used as a common-base amplifier, we can dispense with R_B altogether and ground the base; thus $R_B = 0$. The circuit takes the form shown in the figure below.



To establish $I_E = 1$mA,

$$I_E = \frac{5 - V_{BE}}{R_E}$$

$$1 \text{ mA} = \frac{5 - 0.7}{R_E}$$

$$\Rightarrow R_E = 4.3 \text{ k}\Omega$$

The voltage gain $\frac{v_o}{v_i} = g_m R_C$, where $g_m = \frac{I_C}{V_T} =$ 40 mA/V. To maximize the voltage gain, we select R_C as large as possible, consistent with obtaining a ±2-V signal swing at the collector. To maintain active-mode operation at all times, the collector voltage should not be allowed to fall below the value that causes the CBJ to become forward biased, namely, −0.4 V. Thus, the lowest possible dc voltage at the collector is −0.4 V + 2V = +1.6 V. Correspondingly,

$$R_C = \frac{10 - 1.6}{I_C} \simeq \frac{10 - 1.6}{1 \text{ mA}} = 8.4 \text{ k}\Omega$$

Ex: 7.36 Refer to Fig. 7.54. For $I_E = 1$ mA and $V_C = 2.3$ V,

$$I_E = \frac{V_{CC} - V_C}{R_C}$$

$$1 = \frac{10 - 2.3}{R_C}$$

$$\Rightarrow R_C = 7.7 \text{ k}\Omega$$

Now, using Eq. (7.147), we obtain

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{\beta + 1}}$$

$$1 = \frac{10 - 0.7}{7.7 + \frac{R_B}{101}}$$

$$\Rightarrow R_B = 162 \text{ k}\Omega$$

Selecting standard 5% resistors (Appendix J), we use

$R_B = 160$ kΩ and $R_C = 7.5$ kΩ

The resulting value of I_E is found as

$$I_E = \frac{10 - 0.7}{7.5 + \frac{160}{101}} = 1.02 \text{ mA}$$

and the collector voltage will be

$$V_C = V_{CC} - I_E R_C = 2.3 \text{ V}$$

Ex: 7.37 Refer to Fig. 7.55(b).

$V_S = 3.5$ and $I_D = 0.5$ mA; thus

$$R_S = \frac{V_S}{I_D} = \frac{3.5}{0.5} = 7 \text{ k}\Omega$$

$V_{DD} = 15$ V and $V_D = 6$ V; thus

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{15 - 6}{0.5 \text{ mA}} = 18 \text{ k}\Omega$$

To obtain V_{OV}, we use

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$0.5 = \frac{1}{2} \times 4 V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.5 \text{ V}$$

Thus,

$$V_{GS} = V_t + V_{OV} = 1 + 0.5 = 1.5 \text{ V}$$

We now can obtain the dc voltage required at the gate,

$$V_G = V_S + V_{GS} = 3.5 + 1.5 = 5 \text{ V}$$

Using a current of 2 μA in the voltage divider, we have

$$R_{G2} = \frac{5 \text{ V}}{2 \text{ μA}} = 2.5 \text{ M}\Omega$$

The voltage drop across R_{G1} is 10 V, thus

$$R_{G1} = \frac{10 \text{ V}}{2 \text{ μA}} = 5 \text{ M}\Omega$$

This completes the bias design. To obtain g_m and r_o, we use

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.5}{0.5} = 2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

Ex: 7.38 Refer to Fig. 7.55(a) and (c) and to the values found in the solution to Exercise 7.37 above.

$$R_{\text{in}} = R_{G1} \| R_{G2} = 5 \| 2.5 = 1.67 \text{ M}\Omega$$

$$R_o = R_D \| r_o = 18 \| 200 = 16.5 \text{ k}\Omega$$

$$G_v = -\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m (r_o \| R_D \| R_L)$$

$$= -\frac{1.67}{1.67 + 0.1} \times 2 \times (200 \| 18 \| 20)$$

$$= -17.1 \text{ V/V}$$

Ex: 7.39 To reduce v_{gs} to half its value, the unbypassed R_s is given by

$$R_s = \frac{1}{g_m}$$

From the solution to Exercise 7.37 above, $g_m = 2$ mA/V. Thus

$$R_s = \frac{1}{2} = 0.5 \text{ k}\Omega$$

Neglecting r_o, G_v is given by

$$G_v = -\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \times -\frac{R_D \| R_L}{\frac{1}{g_m} + R_s}$$

$$= -\frac{1.67}{1.67 + 0.1} \times \frac{18 \| 20}{0.5 + 0.5}$$

$$= -8.9 \text{ V/V}$$

Ex: 7.40 Refer to Fig. 7.56(a). For $V_B = 5$ V and 50-μA current through R_{B2}, we have

$$R_{B2} = \frac{5 \text{ V}}{0.05 \text{ mA}} = 100 \text{ k}\Omega$$

The base current is

$$I_B = \frac{I_E}{\beta + 1} \simeq \frac{0.5 \text{ mA}}{100} = 5 \text{ μA}$$

The current through R_{B1} is

$$I_{R_{B1}} = I_B + I_{R_{B2}} = 5 + 50 = 55 \text{ μA}$$

Since the voltage drop across R_{B1} is $V_{CC} - V_B = 10$ V, the value of R_{B1} can be found from

$$R_{B1} = \frac{10 \text{ V}}{0.055 \text{ μA}} = 182 \text{ k}\Omega$$

The value of R_E can be found from

$$I_E = \frac{V_B - V_{BE}}{R_E}$$

$$\Rightarrow R_E = \frac{5 - 0.7}{0.5} = 8.6 \text{ k}\Omega$$

The value of R_C can be found from

$$V_C = V_{CC} - I_C R_C$$

$$6 = 15 - 0.99 \times 0.5 \times R_C$$

$$R_C \simeq 18 \text{ k}\Omega$$

This completes the bias design. The values of g_m, r_π, and r_o can be found as follows:

$$g_m = \frac{I_C}{V_T} \simeq \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} \simeq \frac{100}{0.5} = 200 \text{ k}\Omega$$

Ex: 7.41 Refer to Fig. 7.56(b) and to the solution of Exercise 7.40 above.

$$R_{\text{in}} = R_{B1} \| R_{B2} \| r_\pi$$

$$= 182 \| 100 \| 5 = 4.64 \text{ k}\Omega$$

$$R_o = R_C \| r_o = 18 \| 200 = 16.51 \text{ k}\Omega$$

$$G_v = -\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m (R_C \| R_L \| r_o)$$

$$G_v = -\frac{4.64}{4.64 + 10} \times 20 \times (18 \| 20 \| 200)$$

$$= -57.3 \text{ V/V}$$

Ex: 7.42 Refer to the solutions of Exercises 7.40 and 7.41 above. With R_e included (i.e., left unbypassed), the input resistance becomes [refer to Fig. 7.57(b)]

$R_{in} = R_{B1} \| R_{B2} \| [(\beta + 1)(r_e + R_e)]$

Thus,

$10 = 182 \| 100 \| [101(0.05 + R_e)]$

where we have substituted $r_e = \dfrac{V_T}{I_E} = \dfrac{25}{0.5} = 50\ \Omega$. The value of R_e is found from the equation above to be

$R_e = 67.7\ \Omega$

The overall voltage gain can be found from

$$G_v = -\alpha \frac{R_{in}}{R_{in} + R_{sig}} \frac{R_C \| R_L}{r_e + R_e}$$

$$G_v = -0.99 \times \frac{10}{10 + 10} \frac{18 \| 20}{0.05 + 0.0677}$$

$= -39.8$ V/V

Ex: 7.43 Refer to Fig. 7.58.

$R_{in} = 50\ \Omega = r_e \| R_E \simeq r_e$

$r_e = 50\ \Omega = \dfrac{V_T}{I_E}$

$\Rightarrow I_E = 0.5$ mA

$I_C = \alpha I_E \simeq I_E = 0.5$ mA

$V_C = V_{CC} - R_C I_C$

For $V_C = 1$ V and $V_{CC} = 5$ V, we have

$1 = 5 - R_C \times 0.5$

$\Rightarrow R_C = 8\ \text{k}\Omega$

To obtain the required value of R_E, we note that the voltage drop across it is $(V_{EE} - V_{BE}) = 4.3$ V. Thus,

$R_E = \dfrac{4.3}{0.5} = 8.6\ \text{k}\Omega$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} g_m (R_C \| R_L)$$

$$= \frac{50\ \Omega}{50\ \Omega + 50\ \Omega} \times 20(8 \| 8)$$

$= 40$ V/V

$\hat{v}_o = 40 \hat{v}_{sig} = 40 \times 10\ \text{mV} = 0.4$ V

Ex: 7.44 Refer to Fig. 7.59. Consider first the bias design of the circuit in Fig. 7.59(a). Since the required $I_E = 1$ mA, the base current

$I_B = \dfrac{I_E}{\beta + 1} = \dfrac{1}{101} \simeq 0.01$ mA. For a dc voltage drop across R_B of 1 V, we obtain

$R_B = \dfrac{1\ \text{V}}{0.01\ \text{mA}} = 100\ \text{k}\Omega$

The result is a base voltage of −1 V and an emitter voltage of −1.7 V. The required value of R_E can now be determined as

$$R_E = \frac{-1.7 - (-5)}{I_E} = \frac{3.3}{1\ \text{mA}} = 3.3\ \text{k}\Omega$$

$R_{in} = R_B \| [(\beta + 1)[r_e + (R_E \| r_o \| R_L)]$

where $r_o = \dfrac{V_A}{I_C} = \dfrac{100\ \text{V}}{1\ \text{mA}} = 100\ \text{k}\Omega$

$R_{in} = 100 \| (100 + 1)[0.025 + (3.3 \| 100 \| 1)]$

$= 44.3\ \text{k}\Omega$

$$\frac{v_i}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{44.3}{44.3 + 50} = 0.469\ \text{V/V}$$

$$\frac{v_o}{v_i} = \frac{R_E \| r_o \| R_L}{r_e + (R_E \| r_o \| R_L)} = 0.968\ \text{V/V}$$

$$G_v \equiv \frac{v_o}{v_{sig}} = 0.469 \times 0.968 = 0.454\ \text{V/V}$$

$$R_{out} = r_o \| R_E \| \left[r_e + \frac{R_B \| + R_{sig}}{\beta + 1} \right]$$

$$= 100 \| 3.3 \| \left[0.025 + \frac{100 \| 50}{101} \right]$$

$= 320\ \Omega$

Ex: 8.1 In the current source of Example 8.1 (Fig. 8.1) we have $I_O = 100\ \mu A$ and we want to reduce the change in output current, ΔI_O, corresponding to a 1-V change in output voltage, ΔV_O, to 1% of I_O.

That is, $\Delta I_O = \dfrac{\Delta V_O}{r_{o2}} = 0.01 I_O \Rightarrow \dfrac{1\ V}{r_{o2}}$

$= 0.01 \times 100\ \mu A$

$$r_{o2} = \frac{1\ V}{1\ \mu A} = 1\ M\Omega$$

$$r_{o2} = \frac{V_A' \times L}{I_O} \Rightarrow 1\ M\Omega = \frac{20 \times L}{100\ \mu A}$$

$$\Rightarrow L = \frac{100\ V}{20\ V/\mu m} = 5\ \mu m$$

To keep V_{OV} of the matched transistors the same as that in Example 8.1, $\dfrac{W}{L}$ of the transistor should remain the same. Therefore,

$$\frac{W}{5\ \mu m} = \frac{10\ \mu m}{1\ \mu m} \Rightarrow W = 50\ \mu m$$

So the dimensions of the matched transistors Q_1 and Q_2 should be changed to

$W = 50\ \mu m$ and $L = 5\ \mu m$

Ex: 8.2 For the circuit of Fig. 8.4 we have

$$I_2 = I_{REF}\frac{(W/L)_2}{(W/L)_1},\ I_3 = I_{REF}\frac{(W/L)_3}{(W/L)_1}$$

and $\quad I_5 = I_4\dfrac{(W/L)_5}{(W/L)_4}$

Since all channel lengths are equal, that is,

$L_1 = L_2 = \cdots = L_5 = 1\ \mu m$

and

$I_{REF} = 10\ \mu A,\ I_2 = 60\ \mu A,\ I_3 = 20\ \mu A,\ I_4 = I_3 = 20\ \mu A$, and $I_5 = 80\ \mu A$,

we have

$$I_2 = I_{REF}\frac{W_2}{W_1} \Rightarrow \frac{W_2}{W_1} = \frac{I_2}{I_{REF}} = \frac{60}{10} = 6$$

$$I_3 = I_{REF}\frac{W_3}{W_1} \Rightarrow \frac{W_3}{W_1} = \frac{I_3}{I_{REF}} = \frac{20}{10} = 2$$

$$I_5 = I_4\frac{W_5}{W_4} \Rightarrow \frac{W_5}{W_4} = \frac{I_5}{I_4} = \frac{80}{20} = 4$$

To allow the voltage at the drain of Q_2 to go down to within 0.2 V of the negative supply voltage, we need $V_{OV2} = 0.2$ V:

$$I_2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 V_{OV2}^2 = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_2 V_{OV2}^2$$

$$60\ \mu A = \frac{1}{2}200\frac{\mu A}{V^2}\left(\frac{W}{L}\right)_2 (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = \frac{120}{200 \times (0.2)^2} = 15 \Rightarrow W_2$$

$= 15 \times L_2$

$$W_2 = 15\ \mu m,\ \frac{W_2}{W_1} = 6 \Rightarrow W_1 = \frac{W_2}{6} = 2.5\ \mu m$$

$$\frac{W_3}{W_1} = 2 \Rightarrow W_3 = 2 \times W_1 = 5\ \mu m$$

To allow the voltage at the drain of Q_5 to go up to within 0.2 V of positive supply, we need $V_{OV5} = 0.2$ V:

$$I_5 = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_5 V_{OV5}^2$$

$$80\ \mu A = \frac{1}{2}80\frac{\mu A}{V^2}\left(\frac{W}{L}\right)_5 (0.2)^2 \Rightarrow$$

$$\left(\frac{W}{L}\right)_5 = \frac{2 \times 80}{80 \times (0.2)^2} = 50 \Rightarrow W_5 = 50\ L_5$$

$W_5 = 50\ \mu m$

$$\frac{W_5}{W_4} = 4 \Rightarrow W_4 = \frac{50\ \mu m}{4} = 12.5\ \mu m$$

Thus:

$W_1 = 2.5\ \mu m,\ W_2 = 15\ \mu m,\ W_3 = 5\ \mu m$
$W_4 = 12.5\ \mu m$, and $W_5 = 50\ \mu m$

Ex: 8.3 From Eq. (8.21) we have

$$I_O = I_{REF}\left(\frac{m}{1 + \dfrac{m+1}{\beta}}\right)\left(1 + \frac{V_O - V_{BE}}{V_{A2}}\right)$$

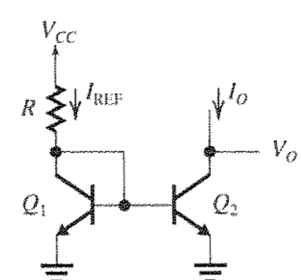
$$I_O = 1\ mA\left(\frac{1}{1 + \dfrac{1+1}{100}}\right)\left(1 + \frac{5 - 0.7}{100}\right)$$

$= 1.02$ mA

$I_O = 1.02$ mA

$$R_o = r_{o2} = \frac{V_A}{I_O} = \frac{100\ V}{1.02\ mA} = 98\ k\Omega \simeq 100\ k\Omega$$

Ex: 8.4



From Eq. (8.23), we have

$$I_O = \frac{I_{REF}}{1 + (2/\beta)}\left(1 + \frac{V_O - V_{BE}}{V_A}\right)$$

where $V_{BE} = V_T \ln\left(\frac{I_O}{I_S}\right)$

$= 0.025 \ln\left(\frac{0.5 \times 10^{-3}}{10^{-15}}\right) = 0.673\text{ V}$

$0.5\text{ mA} = \frac{I_{\text{REF}}}{1 + (2/100)}\left(1 + \frac{2 - 0.673}{50}\right) \Rightarrow$

$I_{\text{REF}} = 0.5\text{ mA}\frac{1.02}{1.026\text{ mA}} = 0.497\text{ mA}$

$I_{\text{REF}} = \frac{V_{CC} - V_{BE}}{R} \Rightarrow R = \frac{V_{CC} - V_{BE}}{I_{\text{REF}}}$

$R = \frac{5 - 0.673}{0.497\text{ mA}} = 8.71\text{ k}\Omega$

$V_{O\min} = V_{CE\text{sat}} = 0.3\text{ V}$

For $V_O = 5$ V, From Eq. (8.23) we have

$I_O = \frac{I_{\text{REF}}}{1 + (2/\beta)}\left(1 + \frac{V_O - V_{BE}}{V_A}\right)$

$I_O = \frac{0.497}{1 + (2/100)}\left(1 + \frac{5 - 0.673\text{ V}}{50}\right) = 0.53\text{ mA}$

Ex: 8.5 $I_1 = I_2 = \cdots = I_N = I_C|_{Q_{\text{REF}}}$

At the input node,

$I_{\text{REF}} = I_C|_{Q_{\text{REF}}} + I_B|_{Q_{\text{REF}}} + I_{B1} + \cdots + I_{BN}$

$= I_C|_{Q_{\text{REF}}} + (N + 1)\, I_B|_{Q_{\text{REF}}}$

$= I_C|_{Q_{\text{REF}}} + \frac{(N + 1)}{\beta}\, I_C|_{Q_{\text{REF}}}$

$\Rightarrow I_C|_{Q_{\text{REF}}} = \frac{I_{\text{REF}}}{1 + \frac{N + 1}{\beta}}$

Thus,

$I_1 = I_2 = \cdots = I_N = \frac{I_{\text{REF}}}{1 + \frac{N + 1}{\beta}}$ Q.E.D

For $\beta = 100$, to limit the error to 10%,

$0.1 = \frac{N + 1}{\beta} = \frac{N + 1}{100}$

$\Rightarrow N = 9$

Ex: 8.6

$R_{\text{in}} \simeq \frac{1}{g_{m1}}$

Now, $R_{\text{in}} = 1\text{ k}\Omega$, thus

$g_{m1} = 1\text{ mA/V}$

But

$g_{m1} = \sqrt{2(\mu_n C_{ox})\left(\frac{W}{L}\right)_1 I_{D1}}$

$1 = \sqrt{2 \times 0.4 \times \left(\frac{W}{L}\right)_1 \times 0.1}$

$\Rightarrow \left(\frac{W}{L}\right)_1 = 12.5$

To obtain

$A_{is} = 5$

$5 = A_{is} = \frac{(W/L)_2}{(W/L)_1}$

$\Rightarrow \left(\frac{W}{L}\right)_2 = 5 \times 12.5 = 62.5$

$R_o = r_{o2} = \frac{V_{A2}}{I_{D2}} = \frac{V_{A2}}{5I_{D1}}$

Thus,

$40\text{ k}\Omega = \frac{V_{A2}}{5 \times 0.1}$

$\Rightarrow V_{A2} = 20\text{ V}$

But

$V_{A2} = V'_{A2}L_2$

$20 = 20 \times L_2$

$\Rightarrow L_2 = 1\ \mu\text{m}$

Selecting $L_1 = L_2$, then

$L_1 = L_2 = 1\ \mu\text{m}$

$W_1 = 12.5\ \mu\text{m}$

$W_2 = 62.5\ \mu\text{m}$

Ex: 8.7

Using Eq. (8.42):

$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)} \cdot \sqrt{I_D}$

For $I_D = 10\ \mu\text{A}$, we have

$g_m = \sqrt{2(387\ \mu\text{A/V}^2)(10)(10\ \mu\text{A})}$

$= 0.28\text{ mA/V}$

Using Eq. (8.46):

$A_0 = V'_A \frac{\sqrt{2\mu_n C_{ox}(W/L)}}{\sqrt{I_D}}$

$= \frac{5\text{ V}/\mu\text{m}\sqrt{2(387\ \mu\text{A/V}^2)(10)(0.36)^2}}{\sqrt{10\ \mu\text{A}}}$

$A_0 = 50\text{ V/V}$

Since g_m varies with $\sqrt{I_D}$ and A_0 with $\frac{1}{\sqrt{I_D}}$,

for

$I_D = 100\ \mu\text{A} \Rightarrow g_m = 0.28\text{ mA/V}\left(\frac{100}{10}\right)^{1/2}$

$= 0.88\text{ mA/V}$

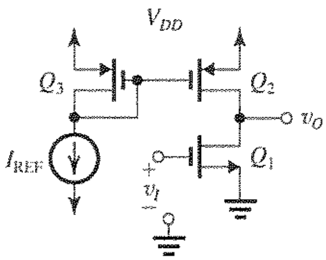
$$A_0 = 50\left(\frac{10}{100}\right)^{1/2} = 15.8 \text{ V/V}$$

For $I_D = 1$ mA, we have

$$g_m = 0.28 \text{ mA/V}\left(\frac{1}{0.010}\right)^{1/2} = 2.8 \text{ mA/V}$$

$$A_0 = 50\left(\frac{0.010}{1}\right)^{1/2} = 5 \text{ V/V}$$

Ex: 8.8



Since all transistors have the same
$$\frac{W}{L} = \frac{7.2\ \mu\text{m}}{0.36\ \mu\text{m}},$$
we have

$$I_{REF} = I_{D3} = I_{D2} = I_{D1} = 100\ \mu\text{A}$$

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1}\sqrt{I_{D1}}$$

$$= \sqrt{2(387\ \mu\text{A/V}^2)\left(\frac{7.2}{0.36}\right)(100\ \mu\text{A})}$$

$$= 1.24 \text{ mA/V}$$

$$r_{o1} = \frac{V'_{An}L_1}{I_{D1}} = \frac{5 \text{ V/}\mu\text{m } (0.36\ \mu\text{m})}{0.1 \text{ mA}} = 18 \text{ k}\Omega$$

$$r_{o2} = \frac{|V'_{Ap}|L_2}{I_{D2}} = \frac{6 \text{ V/}\mu\text{m } (0.36\ \mu\text{m})}{0.1 \text{ mA}} = 21.6 \text{ k}\Omega$$

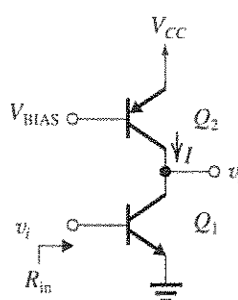
Voltage gain is

$$A_v = -g_{m1}(r_{o1} \parallel r_{o2})$$

$$A_v = -(1.24 \text{ mA/V})(18 \text{ k}\Omega \parallel 21.6 \text{ k}\Omega)$$

$$= -12.2 \text{ V/V}$$

Ex: 8.9



$$I_{C1} = I = 100\ \mu\text{A} = 0.1 \text{ mA}$$

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.1 \text{ mA}}{25 \text{ mV}} = 4 \text{ mA/V}$$

$$R_{in} = r_{\pi 1} = \frac{\beta_1}{g_{m1}} = \frac{100}{4 \text{ mA/V}} = 25 \text{ k}\Omega$$

$$r_{o1} = \frac{V_A}{I} = \frac{50 \text{ V}}{0.1 \text{ mA}} = 500 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_A|}{I} = \frac{50 \text{ V}}{0.1 \text{ mA}} = 500 \text{ k}\Omega$$

$$A_0 = g_{m1}r_{o1} = (4 \text{ mA/V})(500 \text{ k}\Omega) = 2000 \text{ V/V}$$

$$A_v = -g_{m1}(r_{o1} \parallel r_{o2}) = -(4 \text{ mA/V}) \times$$
$$(500 \text{ k}\Omega \parallel 500 \text{ k}\Omega) = -1000 \text{ V/V}$$

Ex: 8.10 Refer to Fig. 8.18(b),

$$v_o = iR_L$$

$$v_{sig} = i(R_s + R_{in})$$

Thus,

$$\frac{v_o}{v_{sig}} = \frac{R_L}{R_s + R_{in}} \qquad \text{Q.E.D}$$

Ex: 8.11 Since $g_m r_o \gg 1$, we use Eq. (8.54),

$$R_{in} \simeq \frac{1}{g_m} + \frac{R_L}{g_m r_o}$$

R_L	0	r_o	$(g_m r_o)r_o$	∞
R_{in}	$\frac{1}{g_m}$	$\frac{2}{g_m}$	r_o	∞

Ex: 8.12 For $g_m r_o \gg 1$, we use Eq. (8.58),

$$R_{out} \simeq r_o + (g_m r_o)R_s$$

to obtain

R_s	0	r_o	$(g_m r_o)r_o$	∞
R_{out}	r_o	$(g_m r_o)r_o$	$(g_m r_o)^2 r_o$	∞

Ex: 8.13 A_{vo} remains unchanged at $g_m r_o$. With a load resistance R_L connected,

$$A_v = A_{vo}\frac{R_L}{R_L + R_o}$$

$$= (g_m r_o)\frac{R_L}{R_L + (1 + g_m R_s)r_o}$$

Ex: 8.14 Use Eq. (8.63)

$$R_{in} \simeq r_e \frac{r_o + R_L}{r_o + \dfrac{R_L}{\beta + 1}}$$

to obtain

R_L	0	r_o	$(\beta+1)r_o$	∞
R_{in}	r_e	$2\ r_e$	$\frac{1}{2}r_\pi$	r_π

Ex: 8.15 Using Eq. (8.68),

$R_{out} \simeq r_o + (g_m r_o)(R_e \parallel r_\pi)$

we obtain

R_e	0	r_e	r_π	r_o	∞
R_{out}	r_o	$2\ r_o$	$\left(\frac{\beta}{2}+1\right)r_o$	$(\beta+1)r_o$	$(\beta+1)r_o$

Ex: 8.16 $R_o = [1 + g_m(R_e \parallel r_\pi)]r_o$

where

$g_m = 40\text{ mA/V},\ r_\pi = \dfrac{\beta}{g_m} = 2.5\text{ k}\Omega,$

$R_e = 0.5\text{ k}\Omega,\text{ and } r_o = \dfrac{V_A}{I_C} = \dfrac{10}{1} = 10\text{ k}\Omega$

Thus,

$R_o = [1 + 40(0.5 \parallel 2.5)] \times 10$

$= 177\text{ k}\Omega$

Without emitter degeneration,

$R_o = r_o = 10\text{ k}\Omega$

Ex: 8.17 Since the CG transistor Q_2 increases the output resistance by a factor approximately equal to $g_{m2}r_{o2}$,

$K \simeq g_{m2}r_{o2}$

Ex: 8.18 If L is halved $\left(L = \dfrac{0.55\ \mu\text{m}}{2}\right)$ and $|V_A| = |V_A'| \cdot L$, we obtain

$$|V_A| = 5\text{ V}/\mu\text{m}\left(\frac{0.55\ \mu\text{m}}{2}\right) = 1.375\text{ V}$$

$$R_o = \frac{|V_A|}{|V_{OV}|/2} \cdot \frac{|V_A|}{I_D} = \frac{2\,(1.375\ \text{V})^2}{(0.3\text{ V})\ (100\ \mu\text{A})}$$

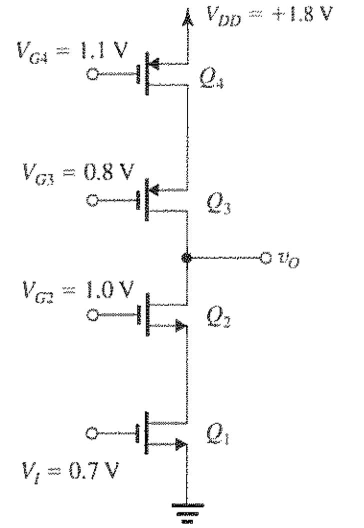
$= 126\text{ k}\Omega$

$$\text{Since } I_D = \frac{1}{2}(\mu_p C_{ox})\left(\frac{W}{L}\right)|V_{OV}|^2\left(1 + \frac{V_{SD}}{|V_A|}\right)$$

$$\frac{W}{L} = \frac{2\,(100\ \mu\text{A})}{90\ \mu\text{A/V}^2\,(0.3\text{ V})^2\left(1 + \dfrac{0.3\text{ V}}{1.375\text{ V}}\right)}$$

$$\frac{W}{L} = 20.3$$

Ex: 8.19



If all transistors are matched and are obviously operating at the same I_D, then all $|V_{OV}|$ will be equal and equal to that of Q_1, namely, $|V_{OV}| = 0.7 - 0.5 = 0.2$ V

$V_{D1} = V_{S2} = V_{G2} - V_{tn} - V_{OV}$

$= 1.0 - 0.5 - 0.2 = 0.3$ V

The lowest v_{DS2} can go is $|V_{OV}| = 0.2$ V

$\therefore v_{O\min} = V_{DS1} + V_{DS2} = 0.3 + 0.2 = 0.5$ V

Similarly, $V_{SG4} = V_{SG3} = 0.7$ V

$V_{D4} = V_{S3} = V_{G3} + |V_t| + |V_{OV}|$

$= 0.8 + 0.5 + 0.2 = 1.5$ V

v_{SD3} can go as low as $|V_{OV}|$, so

$v_{O\max} = V_{D4} - v_{SD3_{\min}} = 1.5 - 0.2 = 1.3$ V

Ex: 8.20 Refer to Fig. 8.33.

$$g_{m1} = g_{m2} = g_{m3} = g_{m4} = \frac{2\ I_D}{|V_{OV}|} = \frac{2 \times 0.2}{0.2}$$

$= 2$ mA/V

$$r_{o1} = r_{o2} = r_{o3} = r_{o4} = \frac{|V_A|}{I_D} = \frac{2}{0.2} = 10\text{ k}\Omega$$

$R_{on} = (g_{m2}r_{o2})r_{o1} = (2 \times 10) \times 10 = 200\text{ k}\Omega$

$R_{op} = (g_{m3}r_{o3})r_{o4} = (2 \times 10) \times 10 = 200\text{ k}\Omega$

$R_o = R_{on} \parallel R_{op} = 200 \parallel 200 = 100\text{ k}\Omega$

$A_v = -g_{m1}R_o = -2 \times 100 = -200$ V/V

Ex: 8.21 $g_{m1} = g_{m2} = g_m$

$$= \frac{I_D}{\dfrac{V_{OV}}{2}} = \frac{0.1\text{ mA}}{(0.2/2)\text{ V}} = 1\text{ mA/V}$$

$r_{o1} = r_{o2} = r_o$

$$= \frac{V_A}{I_D} = \frac{2\text{ V}}{0.1\text{ mA}} = 20\text{ k}\Omega$$

so, $g_m r_o = 1\text{ mA/V }(20\text{ k}\Omega) = 20$

(a) For $R_L = 20\text{ k}\Omega$,

$$R_{in2} = \frac{R_L + r_{o2}}{1 + g_{m2}r_{o2}} = \frac{20\text{ k}\Omega + 20\text{ k}\Omega}{1+20} = 1.9\text{ k}\Omega$$

$$\therefore A_{v1} = -g_{m1}\,(r_{o1} \parallel R_{in2})$$

$$= -1\text{ mA/V }(20 \parallel 1.9) = -1.74\text{ V/V}$$

or

If we use the approximation of Eq. (8.83),

$$R_{in2} \approx \frac{R_L}{g_{m2}r_{o2}} + \frac{1}{g_{m2}} = \frac{20\text{ k}\Omega}{20} + \frac{1}{1\text{ mA/V}} = 2\text{ k}\Omega$$

then

$$A_{v1} = -1\text{ mA/V }(20\text{ k}\Omega \parallel 2\text{ k}\Omega) = -1.82\text{ V/V}$$

Continuing, from Eq. (8.80),

$$A_v = -g_{m1}\,[(g_{m2}r_{o2}r_{o1}) \parallel R_L]$$

$$A_v = -1\text{ mA/V }\{[(20)\,(20\text{ k}\Omega)] \parallel 20\text{ k}\Omega\}$$

$$= -19.0\text{ V/V}$$

$$A_{v2} = \frac{A_v}{A_{v1}} = \frac{-19.0}{-1.82} = 10.5\text{ V/V}$$

(b) Now, for $R_L = 400\text{ k}\Omega$,

$$R_{in2} \simeq \frac{R_L}{g_{m2}r_{o2}} + \frac{1}{g_{m2}} = \frac{400\text{ k}\Omega}{20} + \frac{1}{1\text{ mA/V}}$$

$$= 21\text{ k}\Omega$$

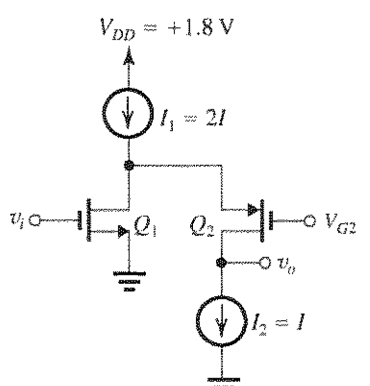
$$A_{v1} = -1\text{ mA/V }(20\text{ k}\Omega \parallel 21\text{ k}\Omega) = -10.2\text{ V/V}$$

$$A_v = -1\text{ mA/V }[(20)\,(20\text{ k}\Omega)] \parallel 400\text{ k}\Omega$$

$$= -200\text{ V/V}$$

$$A_{v2} = \frac{A_v}{A_{v1}} = \frac{-200}{-10.2} = 19.6\text{ V/V}$$

Ex: 8.22



(a) $I_{D1} = I$ and $I_{D2} = I$

Since $V_{OV1} = V_{OV2} = 0.2$ V, we have

$$\frac{I_{D2}}{I_{D1}} = \frac{\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 V_{OV2}^2}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{OV1}^2} = \frac{I}{I} = 1$$

Thus,

$$\frac{k_p'\left(\frac{W}{L}\right)_2}{k_n'\left(\frac{W}{L}\right)_1} = 1 \Rightarrow \left(\frac{W}{L}\right)_2 = \frac{k_n'}{k_p'}\left(\frac{W}{L}\right)_1$$

$$= \frac{k_n'}{\frac{k_n'}{4}}\left(\frac{W}{L}\right)_1$$

or $\left(\frac{W}{L}\right)_2 = 4\left(\frac{W}{L}\right)_1$

(b) The minimum voltage required across current source I_1 would be $|V_{OV}| = 0.2$ V, since it is made with a single transistor. If a 0.1-V_{PP} signal swing is to be allowed at the drain of Q_1, the highest dc bias voltage would be

$$V_{DD} - |V_{OV}| - \frac{0.1\text{ V}_{pp}}{2} = 1.8 - 0.2 - \frac{1}{2}\,(0.1)$$

$$= 1.55\text{ V}$$

(c) $V_{SG2} = |V_{OV}| + |V_{tp}| = 0.2 + 0.5 = 0.7$ V

V_{G2} can be set at $1.55 - 0.7 = 0.85$ V.

(d) Since current source I_2 is implemented with a cascoded current source, the minimum voltage required across it for proper operation is $2V_{OV} = 2\,(0.2\text{ V}) = 0.4$ V.

(e) From parts (c) and (d), the allowable range of signal swing at the output is from 0.4 V to 1.55 V $- V_{OV}$ or 1.35 V.

so, $0.4\text{ V} \le v_O \le 1.35$ V.

Ex: 8.23 Referring to Fig. 8.38,

$R_{op} = (g_{m3}r_{o3})\,(r_{o4} \parallel r_{\pi 3})$ and

$R_{on} = (g_{m2}r_{o2})\,(r_{o1} \parallel r_{\pi 2})$

The maximum values of these resistances are obtained when $r_o \gg r_\pi$ and are given by

$$R_{on}\Big|_{max} = (g_{m2}r_{o2})\,r_{\pi 2}$$

$$R_{op}\Big|_{max} = (g_{m3}r_{o3})\,r_{\pi 3}$$

Since $g_m r_\pi = \beta$,

$$R_{on}\Big|_{max} = \beta_2 r_{o2}$$

$$R_{op}\Big|_{max} = \beta_3 r_{o3}$$

Since $A_v = -g_{m1}(R_{on} \| R_{op})$,

$|A_{v\max}| = g_{m1}(\beta_2 r_{o2} \| \beta_3 r_{o3})$

Ex: 8.24 For the *npn* transistors,

$$g_{m1} = g_{m2} = \frac{|I_C|}{|V_T|} = \frac{0.2 \text{ mA}}{25 \text{ mV}} = 8 \text{ mA/V}$$

$$r_{\pi 1} = r_{\pi 2} = \frac{\beta}{g_m} = \frac{100}{8 \text{ mA/V}} = 12.5 \text{ k}\Omega$$

$$r_{o1} = r_{o2} = \frac{|V_A|}{|I_C|} = \frac{5 \text{ V}}{0.2 \text{ mA}} = 25 \text{ k}\Omega$$

From Fig. 8.38,

$R_{on} = (g_{m2} r_{o2})(r_{o1} \| r_{\pi 2})$

$= (8 \text{ mA/V})(25 \text{ k}\Omega)(25 \text{ k}\Omega \| 12.5 \text{ k}\Omega)$

$R_{on} = 1.67 \text{ M}\Omega$

For the *pnp* transistors,

$$g_{m3} = g_{m4} = \frac{|I_C|}{V_T} = \frac{0.2 \text{ mA}}{25 \text{ mV}} = 8 \text{ mA/V}$$

$$r_{\pi 3} = r_{\pi 4} = \frac{\beta}{g_m} = \frac{50}{8 \text{ mA/V}} = 6.25 \text{ k}\Omega$$

$$r_{o3} = r_{o4} = \frac{|V_A|}{|I_C|} = \frac{4 \text{ V}}{0.2 \text{ mA}} = 20 \text{ k}\Omega$$

$R_{op} = (g_{m3} r_{o3})(r_{o4} \| r_{\pi 3})$

$= (8 \text{ mA/V})(20 \text{ k}\Omega)(20 \text{ k}\Omega \| 6.25 \text{ k}\Omega)$

$R_{op} = 762 \text{ k}\Omega$

$A_v = -g_{m1}(R_{on} \| R_{op})$

$= -(8 \text{ mA/V}) \ (1.67 \text{ M}\Omega \| 762 \text{ k}\Omega)$

$A_v = -4186 \text{ V/V}$

$A_{v\max}$ occurs when r_{o1} and r_{o4} are $\gg r_\pi$.

Then

$R_{on} = (g_{m2} r_{o2}) r_{\pi 2} = \beta_2 r_{o2}$

$R_{on} = 100 (25 \text{ k}\Omega) = 2.5 \text{ M}\Omega$

$R_{op} = (g_{m3} r_{o3}) r_{\pi 3} = \beta_3 r_{o3}$

$R_{op} = 50 (20 \text{ k}\Omega) = 1 \text{ M}\Omega$

Finally,

$A_{v\max} = -(8 \text{ mA/V})(2.5 \text{ M}\Omega \| 1.0 \text{ M}\Omega)$

$A_{v\max} = -5714 \text{ V/V}$

Ex: 8.25 Refer to the circuit in Fig. 8.39. All transistors are operating at $I_D = I_{REF} = 100 \ \mu\text{A}$ and equal V_{OV}, found from

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$100 = \frac{1}{2} \times 387 \times \frac{3.6}{0.36} \times V_{OV}^2$$

$\Rightarrow V_{OV} = 0.227 \text{ V}$

$V_{GS} = 0.227 + 0.5 = 0.727 \text{ V}$

$V_{O\min} = V_{G3} - V_{t3}$

$= V_{GS4} + V_{GS1} - V_{t3}$

Thus,

$V_{O\min} = 2V_{GS} - V_t$

$= V_t + 2 \ V_{OV}$

$= 0.5 + 2 \times 0.227 = 0.95 \text{ V}$

$$g_m = \frac{2 \ I_D}{V_{OV}} = \frac{2 \times 0.1}{0.227} = 0.88 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{5 \times 0.36}{0.1} = 18 \text{ k}\Omega$$

$R_o = (g_{m3} r_{o3}) r_{o2} = (0.88 \times 18) \times 18$
$= 285 \text{ k}\Omega$

Ex: 8.26 For the Wilson mirror from Eq. (8.94), we have

$$\frac{I_O}{I_{REF}} \simeq \frac{1}{1 + \dfrac{2}{\beta^2}} = 0.9998$$

Thus $\dfrac{|I_O - I_{REF}|}{I_{REF}} \times 100 = 0.02\%$

whereas for the simple mirror from Eq. (8.18) we have

$$\frac{I_O}{I_{REF}} = \frac{1}{1 + \dfrac{2}{\beta}} = 0.98$$

Hence $\dfrac{|I_O - I_{REF}|}{I_{REF}} \times 100 = 2\%$

For the Wilson current mirror, we have

$$R_o = \frac{\beta r_o}{2} = \frac{100 \times 100 \text{ k}\Omega}{2} = 5 \text{ M}\Omega$$

and for the simple mirror, $R_o = r_o$
$= 100 \text{ k}\Omega$.

Ex: 8.27 For the two current sources designed in Example 8.6, we have

$$g_m = \frac{I_C}{V_T} = \frac{10 \ \mu\text{A}}{25 \text{ mV}} = 0.4 \ \frac{\text{mA}}{\text{V}}$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{10 \ \mu\text{A}} = 10 \text{ M}\Omega,$$

$$r_\pi = \frac{\beta}{g_m} = 250 \text{ k}\Omega$$

For the current source in Fig. 8.43(b), we have

$R_o = r_{o2} = r_o = 10 \text{ M}\Omega$

For the current source in Fig. 8.43, from Eq. (8.102), we have

$R_{out} \simeq [1 + g_m (R_E \| r_\pi)] r_o$

From Example 8.6, $R_E = R_3 = 11.5\ \text{k}\Omega$;

therefore,

$$R_{out} \simeq \left[1 + 0.4\frac{\text{mA}}{\text{V}}\,(11.5\ \text{k}\Omega \,\|\, 250\ \text{k}\Omega)\right] 10\ \text{M}\Omega$$

$\therefore R_{out} = 54\ \text{M}\Omega$

Ex: 8.28

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2\ \text{mA/V}$$

$$g_{mb} = \chi g_m = 0.2 \times 2 = 0.4\ \text{mA/V}$$

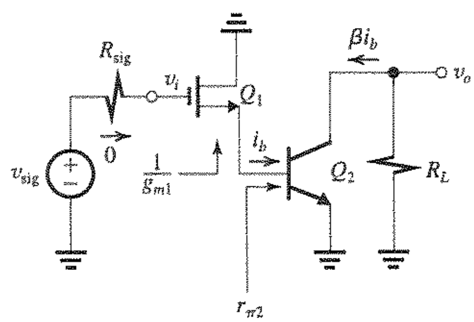
$$r_{o1} = r_{o3} = \frac{V_A}{I_D} = \frac{5}{0.2} = 25\ \text{k}\Omega$$

$$R_L = r_{o1} \| r_{o3} \| \frac{1}{g_{mb}} = 25 \| 25 \| 2.5\ \text{k}\Omega$$

$= 2.083\ \text{k}\Omega$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + \dfrac{1}{g_m}} = \frac{2.083}{2.083 + \dfrac{1}{2}} = 0.81\ \text{V/V}$$

Ex: 8.29



$$g_{m1} = \sqrt{2k_n I_D}$$

$$= \sqrt{2 \times 8 \times 1}$$

$= 4\ \text{mA/V}$

$$\frac{1}{g_{m1}} = 0.25\ \text{k}\Omega$$

$g_{m2} = 40\ \text{mA/V}$

$$r_{\pi 2} = \frac{100}{40} = 2.5\ \text{k}\Omega$$

$R_{in} = \infty$

$$i_b = \frac{v_i}{\dfrac{1}{g_{m1}} + r_{\pi 2}} = \frac{v_{sig}}{\dfrac{1}{g_{m1}} + r_{\pi 2}} = \frac{v_{sig}}{0.25 + 2.5}$$

$$= \frac{v_{sig}}{2.75}$$

$$v_o = -\beta i_b R_L = -\frac{100 \times 4}{2.75} v_{sig}$$

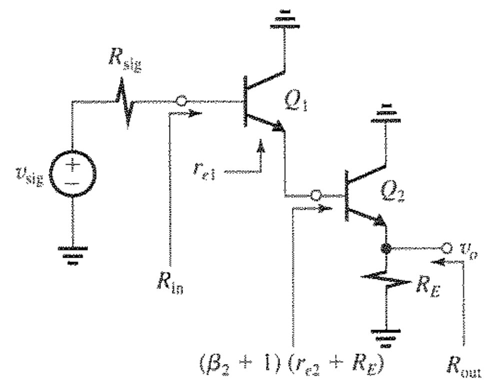
$$G_v \equiv \frac{v_o}{v_{sig}} = -145.5\ \text{V/V}$$

These results apply for both $R_{sig} = 4\ \text{k}\Omega$ and $R_{sig} = 400\ \text{k}\Omega$. If in the CC–CE amplifier of Example 8.7, $R_{sig} = 400\ \text{k}\Omega$, G_v becomes

$$G_v = \frac{255}{255 + 400} \times 0.99 \times -160$$

$= -61.7\ \text{V/V}$

Ex: 8.30



From the figure we can write

$$R_{in} = (\beta_1 + 1)[r_{e1} + (\beta_2 + 1)(r_{e2} + R_E)]$$

$$R_{out} = R_E \,\|\, \left[r_{e2} + \frac{r_{e1} + R_{sig}/(\beta_1 + 1)}{\beta_2 + 1}\right]$$

$$\frac{v_o}{v_{sig}} = \frac{R_E}{R_E + r_{e2} + \dfrac{r_{e1} + R_{sig}/(\beta_1 + 1)}{\beta_2 + 1}}$$

For $I_{E2} = 5\ \text{mA}$, $\beta_1 = \beta_2 = 100$, $R_E = 1\ \text{k}\Omega$, and $R_{sig} = 100\ \text{k}\Omega$, we obtain

$$r_{e2} = \frac{25\ \text{mV}}{5\ \text{mA}} = 5\ \Omega$$

$$I_{E1} = \frac{5}{\beta_2 + 1} = \frac{5}{101} \simeq 0.05\ \text{mA}$$

$$r_{e1} = \frac{25\ \text{mV}}{0.05\ \text{mA}} = 500\ \Omega$$

$$R_{in} = 101 \times (0.5 + 101 \times 1.005) = 10.3\ \text{M}\Omega$$

$$R_{out} = 1 \,\|\, \left[0.005 + \frac{0.5 + (100/101)}{101}\right] \simeq 20\ \Omega$$

$$\frac{v_o}{v_{sig}} = \frac{1}{1 + 0.005 + \dfrac{0.5 + (100/101)}{101}}$$

$= 0.98\ \text{V/V}$

Ex: 8.31 Refer to Fig. 8.49.

$r_e = 25\ \Omega$

$$R_{in} = (\beta_1 + 1)(2\,r_e) = 101 \times 0.05 = 5.05\ \text{k}\Omega$$

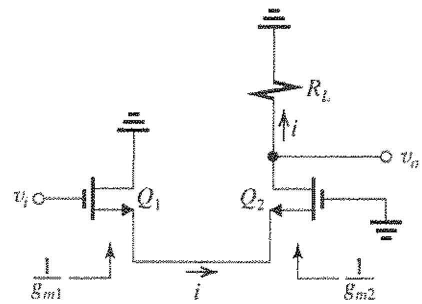
$$\frac{v_o}{v_i} = \frac{\alpha_2 R_L}{2\ r_e} \simeq \frac{5}{0.05} = 100 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = \frac{v_i}{v_{sig}} \times \frac{v_o}{v_i}$$

$$= \frac{R_{in}}{R_{in} + R_{sig}} \times \frac{v_o}{v_i}$$

$$= \frac{5.05}{5.05 + 5} \times 100 = 50 \text{ V/V}$$

Ex: 8.32



(a) From the figure we see that

$$i = \frac{v_i}{2/g_m} = \frac{1}{2} g_m v_i$$

and

$$v_o = iR_L$$

Thus,

$$\frac{v_o}{v_i} = \frac{1}{2} g_m R_L$$

where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2I}{V_{OV}}$$

Thus,

$$\frac{v_o}{v_i} = \frac{1}{2} \times \frac{2I}{V_{OV}} R_L = \frac{IR_L}{V_{OV}} \qquad \text{Q.E.D}$$

(b) $I = 0.1$ mA and $R_L = 20$ kΩ, to obtain a gain of 10 V/V,

$$10 = \frac{0.1 \times 20}{V_{OV}}$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

The required W/L can be obtained from

$$I_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right) V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 0.2 \times \left(\frac{W}{L}\right) \times 0.04$$

$$\Rightarrow \frac{W}{L} = 25$$

Ex: 9.1 Referring to Fig. 9.3,

If R_D is doubled to 5 kΩ,

$$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2} R_D$$

$$= 1.5 - \frac{0.4 \text{ mA}}{2} (5 \text{ k}\Omega) = 0.5 \text{ V}$$

$$V_{CM_{max}} = V_t + V_D = 0.5 + 0.5 = +1.0 \text{ V}$$

Since the currents I_{D1}, and I_{D2} are still 0.2 mA each,

$$V_{GS} = 0.82 \text{ V}$$

So, $V_{CM_{min}} = V_{SS} + V_{CS} + V_{GS}$

$$= -1.5 \text{ V} + 0.4 \text{ V} + 0.82 \text{ V} = -0.28 \text{ V}$$

So, the common-mode range is

-0.28 V to $+1.0$ V

Ex: 9.2 (a) The value of v_{id} that causes Q_1 to conduct the entire current is $\sqrt{2}\, V_{OV}$

$$\rightarrow \sqrt{2} \times 0.316 = 0.45 \text{ V}$$

then, $V_{D1} = V_{DD} - I \times R_D$

$$= 1.5 - 0.4 \times 2.5 = 0.5 \text{ V}$$

$$V_{D2} = V_{DD} = +1.5 \text{ V}$$

(b) For Q_2 to conduct the entire current:

$$v_{id} = -\sqrt{2}\, V_{OV} = -0.45 \text{ V}$$

then,

$$V_{D1} = V_{DD} = +1.5 \text{ V}$$

$$V_{D2} = 1.5 - 0.4 \times 2.5 = 0.5 \text{ V}$$

(c) Thus the differential output range is

$V_{D2} - V_{D1}$:from $1.5 - 0.5 = +1$ V

to $0.5 - 1.5 = -1$ V

Ex: 9.3 Refer to answer table for Exercise 9.3 where values were obtained in the following way:

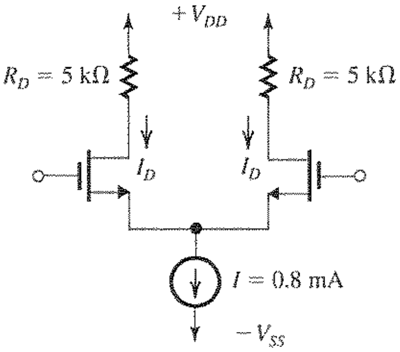
$$V_{OV} = \sqrt{I/k_n W/L} \Rightarrow \frac{W}{L} = \frac{I}{k_n V_{OV}^2}$$

$$g_m = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$\left(\frac{v_{id}/2}{V_{OV}}\right)^2 = 0.1 \rightarrow v_{id} = 2\, V_{OV}\sqrt{0.1}$$

Ex: 9.4 $I_D = \dfrac{I}{2} = \dfrac{0.8 \text{ mA}}{2} = 0.4 \text{ mA}$

$$I_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (V_{OV})^2$$



Thus,

$$V_{OV} = \sqrt{\frac{2I_D}{k_n'\left(\frac{W}{L}\right)}} = \sqrt{\frac{2\,(0.4 \text{ mA})}{0.2\,(\text{mA/V}^2)\,(100)}}$$

$$= 0.2 \text{ V}$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{0.4 \text{ mA} \times 2}{0.2 \text{ V}} = 4 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20 \text{ V}}{0.4 \text{ mA}} = 50 \text{ k}\Omega$$

$$A_d = g_m (R_D \parallel r_o)$$

$$A_d = (4 \text{ mA/V})\,(5 \text{ k}\Omega \parallel 50 \text{ k}\Omega) = 18.2 \text{ V/V}$$

Ex: 9.5 With $I = 200\ \mu$A, for all transistors,

$$I_D = \frac{I}{2} = \frac{200\ \mu\text{A}}{2} = 100\ \mu\text{A}$$

$$L = 2(0.18\ \mu\text{m}) = 0.36\ \mu\text{m}$$

$$r_{o1} = r_{o2} = r_{o3} = r_{o4} = \frac{|V_A'|\, L}{I_D}$$

$$= \frac{(10 \text{ V}/\ \mu\text{m})\,(0.36\ \mu\text{m})}{0.1 \text{ mA}} = 36 \text{ k}\Omega$$

Since $I_{D1} = I_{D2} = \dfrac{1}{2}\, \mu_n C_{ox} \left(\dfrac{W}{L}\right) V_{OV}^2$,

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{2I_D}{\mu_n C_{ox} V_{OV}^2}$$

$$\frac{2(100\ \mu\text{A})}{(400\ \mu\text{A/V}^2)\,(0.2 \text{ V})^2} = 12.5$$

$$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{2I_D}{\mu_p C_{ox} |V_{OV}|^2}$$

$$\frac{2(100\ \mu\text{A})}{(100\ \mu\text{A/V}^2)\,(0.2)^2} = 50$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{(100\ \mu\text{A})\,(2)}{0.2 \text{ V}} = 1 \text{ mA/V},$$

so

$$A_d = g_{m1}(r_{o1} \parallel r_{o3}) = 1(\text{mA/V})\,(36 \text{ k}\Omega \parallel 36 \text{ k}\Omega)$$

$$= 18 \text{ V/V}$$

Ex: 9.6 $L = 2\,(0.18\ \mu\text{m}) = 0.36\ \mu\text{m}$

All $r_o = \dfrac{|V_A'| \cdot L}{I_D}$

The drain current for all transistors is

$$I_D = \frac{I}{2} = \frac{200\ \mu\text{A}}{2} = 100\ \mu\text{A}$$

$$r_o = \frac{(10\ \text{V}/\ \mu\text{m})\,(0.36\ \mu\text{m})}{0.1\ \text{mA}} = 36\ \text{k}\Omega$$

Refering to Fig. 9.13(a),

Since $I_D = \frac{1}{2}\,\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$ for all NMOS transistors,

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4$$

$$= \frac{2I_D}{\mu_n C_{ox} V_{OV}^2} = \frac{2(100\ \mu\text{A})}{400\ \mu\text{A}/\text{V}^2(0.2\ \text{V})^2} = 12.5$$

$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8$$

$$= \frac{2I_D}{\mu_p C_{ox} V_{OV}^2} = \frac{2(100\ \mu\text{A})}{100\ \mu\text{A}/\text{V}^2(0.2\ \text{V})^2} = 50$$

For all transistors,

$$g_m = \frac{|I_D|}{|V_{OV}|\,/2} = \frac{(0.1\ \text{mA})\,(2)}{(0.2\ \text{V})} = 1\ \text{mA/V}$$

From Fig. 9.13(b),

$$R_{on} = (g_{m3} r_{o3})\, r_{o1} = (1 \times 36) \times 36$$

$$= 1.296\ \text{M}\Omega$$

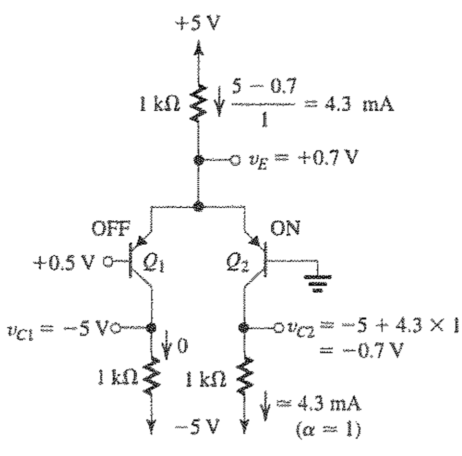
$$R_{op} = (g_{m5} r_{o5}) r_{o7} = (1 \times 36) \times 36$$

$$= 1.296\ \text{M}\Omega$$

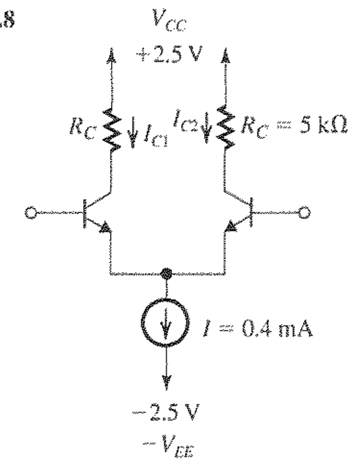
$$A_d = g_{m1}\left(R_{on} \parallel R_{op}\right)$$

$$= (1\ \text{mA/V})\,(1.296\ \text{M}\Omega \parallel 1.296\ \text{M}\Omega)$$
$$= 648\ \text{V/V}$$

Ex: 9.7



Ex: 9.8



$$I_{C1} = I_{C2} \simeq I_{E1} = I_{E2} = \frac{I}{2} = \frac{0.4\ \text{mA}}{2}$$

$$= 0.2\ \text{mA}$$

$$V_{CM\max} \simeq V_C + 0.4\ \text{V}$$

$$= V_{CC} - I_C R_C + 0.4\ \text{V}$$

$$= 2.5 - 0.2\ \text{mA}\ (5\ \text{k}\Omega) + 0.4\ \text{V} = +1.9\ \text{V}$$

$$V_{CM\min} = -V_{EE} + V_{CS} + V_{BE}$$

$$V_{CM\min} = -2.5\ \text{V} + 0.3\ \text{V} + 0.7\ \text{V} = -1.5\ \text{V}$$

Input common-mode range is -1.5 V to $+1.9$ V

Ex: 9.9 Substituting $i_{E1} + i_{E2} = I$ in Eqn. (9.45) yields

$$i_{E1} = \frac{I}{1 + e^{(v_{B2} - v_{B1})/V_T}}$$

$$0.99\, I = \frac{I}{1 + e^{(v_{B2} - v_{B1})/V_T}}$$

$$v_{B1} - v_{B2} = -V_T \ln\left(\frac{1}{0.99} - 1\right)$$

$$= -25 \ln(1/99)$$

$$= 25 \ln(99) = 115\ \text{mV}$$

Ex: 9.10 (a) The DC current in each transister is 0.5 mA. Thus V_{BE} for each will be

$$V_{BE} = 0.7 + 0.025 \ln\left(\frac{0.5}{1}\right)$$

$$= 0.683\ \text{V}$$

$$\Rightarrow v_E = 5 - 0.683 = +4.317\ \text{V}$$

(b) $g_m = \dfrac{I_C}{V_T} = \dfrac{0.5}{0.025} = 20\ \dfrac{\text{mA}}{\text{V}}$

(c) $i_{C1} = 0.5 + g_{m1}\,\Delta v_{BE1}$

$$= 0.5 + 20 \times 0.005 \sin(2\pi \times 1000t)$$

$$= 0.5 + 0.1 \sin(2\pi \times 1000t),\ \text{mA}$$

$i_{C2} = 0.5 - 0.1 \sin(2\pi \times 1000t)$, mA

(d) $v_{C1} = (V_{CC} - I_C R_C) - 0.1 \times R_C \sin(2\pi \times 1000t)$

$= (15 - 0.5 \times 10) - 0.1 \times 10 \sin(2\pi \times 1000t)$

$= 10 - 1 \sin(2\pi \times 1000t)$, V

$v_{C2} = 10 + 1 \sin(2\pi \times 1000t)$, V

(e) $v_{C2} - v_{C1} = 2 \cdot \sin(2\pi \times 1000t)$, V

(f) Voltage gain $\equiv \dfrac{v_{C2} - v_{C1}}{v_{B1} - v_{B2}}$

$= \dfrac{2 \text{ V peak}}{0.01 \text{ V peak}} = 200$ V/V

Ex: 9.11 The transconductance for each transistor is

$g_m = \sqrt{2\mu_n C_{ox} (W/L) I_D}$

$I_D = \dfrac{I}{2} = \dfrac{0.8 \text{ mA}}{2} = 0.4$ mA

Thus,

$g_m = \sqrt{2 \times 0.2 \times 100 \times 0.4} = 4$ mA/V

The differential gain for matched R_D values is $A_d = \dfrac{v_{O2} - v_{O1}}{v_{id}} = g_m R_D$

If we ignore the 1% here, then we obtain

$A_d = g_m R_D = (4 \text{ mA/V})(5 \text{ k}\Omega) = 20$ V/V

$|A_{cm}| = \left(\dfrac{R_D}{2R_{SS}}\right)\left(\dfrac{\Delta R_D}{R_D}\right)$

$= \left(\dfrac{5}{2 \times 25}\right)(0.01) = 0.001$ V/V

CMRR (dB) $= 20 \log \dfrac{|A_d|}{|A_{CM}|} = 20 \log\left(\dfrac{20}{0.001}\right)$

$= 86$ dB

Ex: 9.12 From Exercise 9.11,

$W/L = 100$, $\mu_n C_{ox} = 0.2$ mA/V^2,

$I_D = \dfrac{I}{2} = \dfrac{0.8 \text{ mA}}{2} = 0.4$ mA

$g_m = \sqrt{2\mu_n C_{ox}\left(\dfrac{W}{L}\right) I_D}$

$= \sqrt{2\left(0.2 \text{ mA/V}^2\right)(100)(0.4 \text{ mA})}$

$g_m = 4$ mA/V

Using Eq. (9.88) and the fact that $R_{SS} = 25$ kΩ, we obtain

$\text{CMRR} = \dfrac{(2 g_m R_{SS})}{\left(\dfrac{\Delta g_m}{g_m}\right)} = \dfrac{2(4 \text{ mA/V})(25 \text{ k}\Omega)}{0.01}$

$= 20{,}000$

CMRR (dB) $= 20 \log_{10}(20{,}000) = 86$ dB

Ex: 9.13 If the output of a MOS differential amplifier is taken single-endedly, then

$|A_d| = \dfrac{1}{2} g_m R_D$

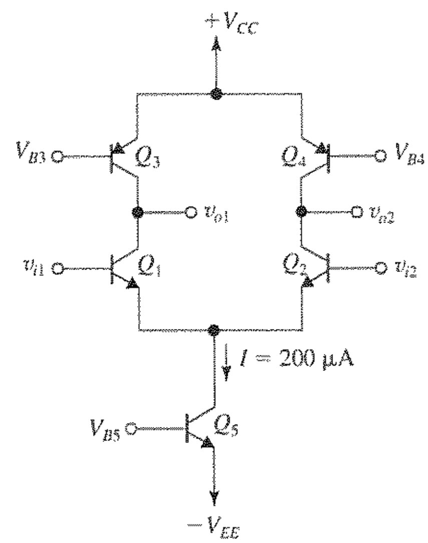
(that is, half the gain obtained with the output taken differentially), and from Fig. 9.25(d) we have

$|A_{cm}| \simeq \dfrac{R_D}{2R_{SS}}$

Thus,

$\text{CMRR} \equiv \dfrac{|A_d|}{|A_{cm}|} = g_m R_{SS}$ Q.E.D.

Ex: 9.14



$I = 200$ µA

Since $\beta \gg 1$,

$I_{C1} \approx I_{C2} \approx \dfrac{I}{2} = \dfrac{200 \text{ µA}}{2} = 100$ µA

$g_{m1} = g_{m2} = g_m = \dfrac{I_C}{V_T} = \dfrac{100 \text{ µA}}{25 \text{ mV}} = 4$ mA/V

$R_{C1} = R_{C2} = R_C = r_o = \dfrac{|V_A|}{I_C}$

$= \dfrac{10 \text{ V}}{100 \text{ µA}} = 100$ kΩ

$r_{o1} = r_{o2} = \dfrac{V_A}{I/2} = \dfrac{10}{0.1} = 100$ kΩ

$r_{e1} = r_{e2} = r_e = \dfrac{V_T}{I_E} = \dfrac{25 \text{ mV}}{0.1 \text{ mA}} = 0.25$ kΩ

$|A_d| = \dfrac{R_C \parallel r_o}{r_e} = \dfrac{100 \text{ k}\Omega \parallel 100 \text{ k}\Omega}{0.25 \text{ k}\Omega}$

$= 200$ V/V

$R_{id} = 2r_\pi$, $r_\pi = \dfrac{\beta}{g_m} = \dfrac{100}{4 \text{ mA/V}} = 25$ kΩ

$R_{id} = 2(25 \text{ k}\Omega) = 50$ kΩ

$$R_{EE} = \frac{V_A}{I} = \frac{10\ \text{V}}{200\ \mu\text{A}} = 50\ \text{k}\Omega$$

If the total load resistance is assumed to be mismatched by 1%, then we have

$$|A_{cm}| = \frac{R_C}{2R_{EE}}\frac{\Delta R_C}{R_C}$$

$$= \frac{100}{2 \times 50} \times 0.01 = 0.01\ \text{V/V}$$

$$\text{CMRR (dB)} = 20\log_{10}\left|\frac{A_d}{A_{cm}}\right| = 20\log_{10}\left|\frac{200}{0.01}\right|$$

$$= 86\ \text{dB}$$

Using Eq. (9.96), we obtain

$$R_{icm} = \beta R_{EE} \cdot \frac{1 + \dfrac{R_C}{\beta r_o}}{1 + \dfrac{R_C + 2R_{EE}}{r_o}}$$

$$= 100 \times 50 \times \frac{1 + \dfrac{100}{100 \times 100}}{1 + \dfrac{100 + 2 \times 50}{100}}$$

$$R_{icm} \simeq 1.68\ \text{M}\Omega$$

Ex: 9.15 From Exercise 9.4:

$$V_{OV} = 0.2\ \text{V}$$

Using Eq. (9.101) we obtain V_{OS} due to $\Delta R_D/R_D$ as:

$$V_{OS} = \left(\frac{V_{OV}}{2}\right) \cdot \left(\frac{\Delta R_D}{R_D}\right)$$

$$= \frac{0.2}{2} \times 0.02 = 0.002\ \text{V} \quad \text{i.e } 2\ \text{mV}$$

To obtain V_{OS} due to $\dfrac{\Delta(W/L)}{W/L}$,

use Eq. (9.106),

$$V_{OS} = \left(\frac{V_{OV}}{2}\right)\left(\frac{\Delta(W/L)}{W/L}\right)$$

$$\Rightarrow V_{OS} = \left(\frac{0.2}{2}\right) \times 0.02 = 0.002$$

$$\Rightarrow 2\ \text{mV}$$

The offset voltage arising from ΔV_t is obtained from Eq. (9.109):

$$V_{OS} = \Delta V_t = 2\ \text{mV}$$

Finally, from Eq. (9.110) the total input offset is

$$V_{OS} =$$

$$\left[\left(\frac{V_{OV}}{2}\frac{\Delta R_D}{R_D}\right)^2 + \left(\frac{V_{OV}}{2}\frac{\Delta(W/L)}{W/L}\right)^2 + (\Delta V_t)^2\right]^{1/2}$$

$$= \sqrt{\left(2 \times 10^{-3}\right)^2 + \left(2 \times 10^{-3}\right)^2 + \left(2 \times 10^{-3}\right)^2}$$

$$= \sqrt{3 \times \left(2 \times 10^{-3}\right)^2}$$

$$= 3.46\ \text{mV}$$

Ex: 9.16 From Eq. (9.120), we get

$$V_{OS} = V_T\sqrt{\left(\frac{\Delta R_C}{R_C}\right)^2 + \left(\frac{\Delta I_S}{I_S}\right)^2}$$

$$= 25\sqrt{(0.02)^2 + (0.1)^2}$$

$$= 2.55\ \text{mV}$$

$$I_B = \frac{100}{2(\beta+1)} = \frac{100}{2 \times 101} \cong 0.5\ \mu\text{A}$$

$$I_{OS} = I_B\left(\frac{\Delta\beta}{\beta}\right)$$

$$= 0.5 \times 0.1\ \mu\text{A} = 50\ \text{nA}$$

Ex: 9.17 $I_D = \frac{1}{2}I = 0.4\ \text{mA}$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{OV}^2$$

$$0.4 = \frac{1}{2} \times 0.2 \times 100 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2\ \text{V}$$

$$g_{m1,2} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.4}{0.2} = 4\ \text{mA/V}$$

$$G_m = g_{m1,2} = 4\ \text{mA/V}$$

$$r_{o2} = \frac{V_{An}}{I_D} = \frac{20}{0.4} = 50\ \text{k}\Omega$$

$$r_{o4} = \frac{|V_{Ap}|}{I_D} = \frac{20}{0.4} = 50\ \text{k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 50 \parallel 50 = 25\ \text{k}\Omega$$

$$A_d = G_m R_o = 4 \times 25 = 100\ \text{V/V}$$

Ex: 9.18 $G_m = g_{m1,2} \simeq \dfrac{I/2}{V_T} = \dfrac{0.4\ \text{mA}}{0.025\ \text{V}}$

$$= 16\ \text{mA/V}$$

$$r_{o2} = r_{o4} = \frac{V_A}{I_C} = \frac{V_A}{I/2} = \frac{100}{0.4} = 250\ \text{k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 250 \parallel 250 = 125\ \text{k}\Omega$$

$$A_d = G_m R_o = 16 \times 125 = 2000\ \text{V/V}$$

$$R_{id} = 2r_\pi = 2 \times \frac{\beta}{g_{m1,2}} = 2 \times \frac{160}{16} = 20\ \text{k}\Omega$$

Ex: 9.19

$$G_m = g_{m1} = g_{m2} \simeq \frac{I/2}{V_T} = \frac{0.5\ \text{mA}}{0.025\ \text{V}} = 20\ \text{mA/V}$$

$$r_{o4} = \frac{V_A}{I/2} = \frac{100\ \text{V}}{0.5\ \text{mA}} = 200\ \text{k}\Omega$$

$R_{o4} \simeq \beta_4 r_{o4} = 50 \times 200 = 10{,}000\ \text{k}\Omega = 10\ \text{M}\Omega$

$R_{o5} = \frac{1}{2}\beta_5 r_{o5}$

where

$r_{o5} = \frac{V_A}{I/2} = 200\ \text{k}\Omega$

Thus,

$R_{o5} = \frac{1}{2} \times 100 \times 200 = 10\ \text{M}\Omega$

$R_o = R_{o4} \parallel R_{o5} = 10 \parallel 10 = 5\ \text{M}\Omega$

$A_d = G_m R_o$

$= 20\ \text{mA/V} \times 5000\ \text{k}\Omega = 10^5\ \text{V/V or } 100\ \text{dB}$

Ex: 9.20 From Exercise 9.17, we get

$I_D = 0.4\ \text{mA}$

$V_{OV} = 0.2\ \text{V} \quad g_{m1,2} = 4\ \text{mA/V}$

$G_m = 4\ \text{mA/V} \quad A_d = 100\ \text{V/V}$

Now,

$R_{SS} = 25\ \text{k}\Omega$

$g_{m3} = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_p I_D}$

$= \sqrt{2 \times 0.1 \times 200 \times 0.4} = 4\ \text{mA/V}$

$|A_{cm}| = \frac{1}{2g_{m3}R_{SS}} = \frac{1}{2 \times 4 \times 25} = 0.005\ \text{V/V}$

$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{100}{0.005}$

$= 20{,}000 \text{ or } 20 \log 20{,}000 = 86\ \text{dB}$

Ex: 9.21 From Exercise 9.18, we get

$I = 0.8\ \text{mA},\ I_C \simeq 0.4\ \text{mA},\ V_A = 100\ \text{V}$

$g_{m1,2} = 16\ \text{mA/V},\ G_m = 16\ \text{mA/V}$

$r_{o2} = r_{o4} = 250\ \text{k}\Omega,\ A_d = 2000\ \text{V/V}$

Now,

$R_{EE} = \frac{100\ \text{V}}{0.8\ \text{mA}} = 125\ \text{k}\Omega$

Using Eq. (9.165),

$|A_{cm}| = \frac{r_{o4}}{\beta_3 R_{EE}} = \frac{250}{160 \times 125} = 0.0125\ \text{V/V}$

$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{2000}{0.0125} = 160{,}000\ \text{V/V}$

$20 \log \text{CMRR} = 104\ \text{dB}$

Ex: 9.22 Refer to Fig. (9.40).

(a) Using Eq. (9.170), we obtain

$I_6 = \frac{(W/L)_6}{(W/L)_4}(I/2)$

$\Rightarrow 100 = \frac{(W/L)_6}{100} \times 50$

thus, $(W/L)_6 = 200$

Using Eq. (9.171), we get

$I_7 = \frac{(W/L)_7}{(W/L)_5} I$

$\Rightarrow 100 = \frac{(W/L)_7}{200} \times 100$

thus, $(W/L)_7 = 200$

(b) For Q_1,

$\frac{I}{2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 V_{OV_1}^2$

$\Rightarrow V_{OV1} = \sqrt{\frac{50}{\frac{1}{2} \times 30 \times 200}} = 0.129\ \text{V}$

Similarly for Q_2, $V_{OV2} = 0.129\ \text{V}$

For Q_6,

$100 = \frac{1}{2} \times 90 \times 200 V_{OV6}^2$

$\Rightarrow V_{OV6} = 0.105\ \text{V}$

(c) $g_m = \frac{2I_D}{V_{OV}}$

	I_D	V_{OV}	g_m
Q_1	50 µA	0.129 V	0.775 mA/V
Q_2	50 µA	0.129 V	0.775 mA/V
Q_6	100 µA	0.105 V	1.90 mA/V

(d) $r_{o2} = 10/0.05 = 200\ \text{k}\Omega$

$r_{o4} = 10/0.05 = 200\ \text{k}\Omega$

$r_{o6} = 10/0.1 = 100\ \text{k}\Omega$

$r_{o7} = 10/0.1 = 100\ \text{k}\Omega$

(e) Eq. (9.168):

$A_1 = -g_{m1}(r_{o2} \parallel r_{o4})$

$= -0.775(200 \parallel 200) = -77.5\ \frac{\text{V}}{\text{V}}$

Eq. (9.169):

$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$

$= -95\ \text{V/V}$

Overall voltage gain is

$A_1 \times A_2 = -77.5 \times -95 = 7363$ V/V

Ex: 9.23 $R_{id} = 20.2$ kΩ

$A_{vo} = 8513$ V/V

$R_o = 152\ \Omega$

With $R_S = 10$ kΩ and $R_L = 1$ kΩ,

$$G_v = \frac{20.2}{20.2 + 10} \times 8513 \times \frac{1}{(1 + 0.152)}$$

$= 4943$ V/V

Ex: 9.24 $\frac{i_{e8}}{i_{b8}} = \beta_8 + 1 = 101$

$$\frac{i_{b8}}{i_{c7}} = \frac{R_5}{R_5 + R_{i4}} = \frac{15.7}{15.7 + 303.5} = 0.0492$$

$$\frac{i_{c7}}{i_{b7}} = \beta_7 = 100$$

$$\frac{i_{b7}}{i_{c5}} = \frac{R_3}{R_3 + R_{i3}} = \frac{3}{3 + 234.8} = 0.0126$$

$$\frac{i_{c5}}{i_{b5}} = \beta_5 = 100$$

$$\frac{i_{b5}}{i_{c2}} = \frac{R_1 + R_2}{R_1 + R_2 + R_{i2}} = \frac{40}{40 + 5.05} = 0.888$$

$$\frac{i_{c2}}{i_1} = \beta_2 = 100$$

Thus the overall current gain is

$$\frac{i_{e8}}{i_1} = 101 \times 0.0492 \times 100 \times 0.0126 \times 100$$

$\times\ 0.888 \times 100$

$= 55{,}599$ A/A

and the overall voltage gain is

$$\frac{v_o}{v_{id}} = \frac{R_6}{R_{i1}} \cdot \frac{i_{e8}}{i_1}$$

$$= \frac{3}{20.2} \times 55599 = 8257 \text{ V/V}$$

Ex: 10.1 $A_M = -\frac{R_G}{R_G + R_{sig}} g_m(R_D \parallel R_L)$

$= -\frac{10}{10 + 0.1} \times 2(10 \parallel 10)$

$= -9.9$ V/V

$f_{P1} = \frac{1}{2\pi C_{C1}(R_{sig} + R_G)}$

$= \frac{1}{2\pi \times 1 \times 10^{-6}(0.1 + 10) \times 10^6}$

$= 0.016$ Hz

$f_{P2} = \frac{g_m + 1/R_S}{2\pi C_S}$

$= \frac{(2 + 0.1) \times 10^{-3}}{2\pi \times 1 \times 10^{-6}} = 334.2$ Hz

$f_{P3} = \frac{1}{2\pi C_{C2}(R_D + R_L)}$

$= \frac{1}{2\pi \times 1 \times 10^{-6}(10 + 10) \times 10^3}$

$= 8$ Hz

$f_Z = \frac{1}{2\pi C_S R_S}$

$= \frac{1}{2\pi \times 1 \times 10^{-6} \times 10 \times 10^3}$

$= 15.9$ Hz

Since the highest-frequency pole is $f_{P2} = 334.2$ and the next highest-frequency singularity is f_Z at 15.9 Hz, the lower 3-dB frequency f_L will be

$f_L \simeq f_{P2} = 334.2$ Hz

Ex: 10.2 Refer to Fig. 10.10.

$\tau_{C1} = C_{C1}[R_{sig} + (R_B \parallel r_\pi)]$

$= 1 \times 10^{-6}[5 + (100 \parallel 2.5)] \times 10^3$

$= 7.44$ ms

$\tau_{CE} = C_E\left[R_E \parallel \left(r_e + \frac{R_B \parallel R_{sig}}{\beta + 1}\right)\right]$

$\beta = g_m r_\pi = 40 \times 2.5 = 100$

$r_e \simeq 1/g_m = 25\ \Omega$

$\tau_{CE} = 1 \times 10^{-6}\left[5 \parallel \left(0.025 + \frac{100 \parallel 5}{101}\right)\right] \times 10^3$

$\tau_{CE} = 0.071$ ms

$\tau_{C2} = C_{C2}(R_C + R_L)$

$= 1 \times 10^{-6}(8 + 5) \times 10^3$

$= 13$ ms

$f_L = \frac{1}{2\pi}\left[\frac{1}{\tau_{C1}} + \frac{1}{\tau_{CE}} + \frac{1}{\tau_{C2}}\right]$

$= \frac{1}{2\pi}\left[\frac{1}{7.44} + \frac{1}{0.071} + \frac{1}{13}\right] \times 10^3$

$= 2.28$ kHz

$f_Z = \frac{1}{2\pi C_E R_E}$

$= \frac{1}{2\pi \times 1 \times 10^{-6} \times 5 \times 10^3} = 31.8$ Hz

Since f_Z is much lower than f_L it will have a negligible effect on f_L.

Ex: 10.3 $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-11}\ \text{F/m}}{10 \times 10^{-9}\ \text{m}}$

$= 3.45 \times 10^{-3}$ F/m^2

$= 3.45$ fF/μm^2

$C_{ov} = WL_{ov}C_{ox}$

$= 10 \times 0.05 \times 3.45 = 1.72$ fF

$C_{gs} = \frac{2}{3}WLC_{ox} + C_{ov}$

$= \frac{2}{3} \times 10 \times 1 \times 3.45 + 1.72$

$= 24.72$ fF

$C_{gd} = C_{ov} = 1.72$ fF

$C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \frac{V_{SB}}{V_0}}} = \frac{10}{\sqrt{1 + \frac{1}{0.6}}} = 6.1$ fF

$C_{db} = \frac{C_{db0}}{\sqrt{1 + \frac{V_{DB}}{V_0}}} = \frac{10}{\sqrt{1 + \frac{2 + 1}{0.6}}} = 4.1$ fF

Ex: 10.4 $g_m = \sqrt{2k_n'(W/L)I_D}$

$= \sqrt{2 \times 0.16 \times (10/1) \times 0.1} = 0.566$ mA/V

$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$

$= \frac{0.566 \times 10^{-3}}{2\pi(24.72 + 1.72) \times 10^{-15}}$

$f_T = 3.4$ GHz

Ex: 10.5 $C_{de} = \tau_F g_m$

where

$\tau_F = 20$ ps

$g_m = \frac{I_C}{V_T} = \frac{1\ \text{mA}}{0.025\ \text{V}} = 40$ mA/V

Thus,

$C_{de} = 20 \times 10^{-12} \times 40 \times 10^{-3} = 0.8 \text{ pF}$

$C_{je} \simeq 2C_{je0}$

$= 2 \times 20 = 40 \text{ fF}$

$C_\pi = C_{de} + C_{je}$

$= 0.8 + 0.04 = 0.84 \text{ pF}$

$$C_\mu = \frac{C_{\mu 0}}{\left(1 + \frac{V_{CB}}{V_{0c}}\right)^m}$$

$$= \frac{20}{\left(1 + \frac{2}{0.5}\right)^{0.33}} = 12 \text{ fF}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$= \frac{40 \times 10^{-3}}{2\pi(0.84 + 0.012) \times 10^{-12}} = 7.47 \text{ GHz}$$

Ex: 10.6 $|h_{fe}| = 10$ at $f = 50$ MHz

Thus,

$f_T = 10 \times 50 = 500 \text{ MHz}$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 500 \times 10^6}$$

$= 12.7 \text{ pF}$

$C_\pi = 12.7 - 2 = 10.7 \text{ pF}$

Ex: 10.7 $C_\pi = C_{de} + C_{je}$

$10.7 = C_{de} + 2$

$\Rightarrow C_{de} = 8.7 \text{ pF}$

Since C_{de} is proportional to g_m and hence I_C, at $I_C = 0.1$ mA,

$C_{de} = 0.87 \text{ pF}$

and

$C_\pi = 0.87 + 2 = 2.87 \text{ pF}$

$$f_T = \frac{4 \times 10^{-3}}{2\pi(2.87 + 2) \times 10^{-12}} = 130.7 \text{ MHz}$$

Ex: 10.8 $A_M = -\dfrac{R_G}{R_G + R_{\text{sig}}} g_m R'_L$

$$= -\frac{4.7}{4.7 + 0.01} \times 7.14 = -7.12 \text{ V/V}$$

$$f_H = \frac{1}{2\pi C_{\text{in}}(R_{\text{sig}} \parallel R_G)}$$

$$= \frac{1}{2\pi \times 4.26 \times 10^{-12}(0.01 \parallel 4.7) \times 10^6}$$

$= 3.7 \text{ MHz}$

Ex: 10.9 $f_H = \dfrac{1}{2\pi C_{\text{in}}(R_{\text{sig}} \parallel R_G)}$

$$1 \times 10^6 = \frac{1}{2\pi C_{\text{in}}(0.1 \parallel 4.7) \times 10^6}$$

$\Rightarrow C_{\text{in}} = 1.625 \text{ pF}$

But,

$C_{\text{in}} = C_{gs} + C_{gd}(1 + g_m R'_L)$

$1.625 = 1 + C_{gd}(1 + 7.14)$

$\Rightarrow C_{gd} = 0.08 \text{ pF}$

Ex: 10.10 To reduce the midband gain to half the value found, we reduce R'_L by the same factor, thus

$R'_L = 1.5 \text{ k}\Omega$

But,

$R'_L = r_o \parallel R_C \parallel R_L$

$1.5 = 100 \parallel 8 \parallel R_L$

$\Rightarrow R_L = 1.9 \text{ k}\Omega$

$C_{\text{in}} = C_\pi + C_\mu(1 + g_m R'_L)$

$= 7 + 1(1 + 40 \times 1.5)$

$= 68 \text{ pF}$

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 68 \times 10^{-12} \times 1.65 \times 10^3}$$

$= 1.42 \text{ MHz}$

Thus, by accepting a reduction in gain by a factor of 2, the bandwidth is increased by a factor of $1.42/0.754 = 1.9$, approximately the same factor as the reduction in gain.

Ex: 10.11 $f_t = |A_M| f_H$

$$2 \times 10^9 = \frac{12.5}{2\pi(C_L + C_{gd}) \times 10 \times 10^3}$$

$\Rightarrow C_L + C_{gd} = 99.5 \text{ fF}$

$C_L = 99.5 - 5 = 94.5 \text{ fF}$

Ex: 10.12 $T(s) = \dfrac{1000}{1 + \dfrac{s}{2\pi \times 10^5}}$

$\text{GB} = 1000 \times 100 \times 10^3 = 10^8 \text{ Hz}$

Ex: 10.13 $T(j\omega) = \dfrac{A_M}{\left(1 + j\dfrac{\omega}{\omega_{P1}}\right)\left(1 + j\dfrac{\omega}{\omega_{P2}}\right)}$

$$|T| = \frac{|A_M|}{\sqrt{\left[1 + \left(\dfrac{\omega}{\omega_{P1}}\right)^2\right]\left[1 + \left(\dfrac{\omega}{\omega_{P2}}\right)^2\right]}}$$

$$|T| = \frac{|A_M|}{\sqrt{\left[1 + \left(\dfrac{\omega}{\omega_{P1}}\right)^2\right]\left[1 + \left(\dfrac{\omega}{k\omega_{P1}}\right)^2\right]}}$$

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{k\omega_{P1}}\right)^2\right]$$

For $\omega_H = 0.9\omega_{P1}$,

$$2 = (1 + 0.81)\left(1 + \frac{0.81}{k^2}\right)$$

$\Rightarrow k = 2.78$

For $\omega_H = 0.99\omega_{P1}$,

$$2 = (1 + 0.99^2)\left(1 + \frac{0.99^2}{k^2}\right)$$

$\Rightarrow k = 9.88$

Ex: 10.14 $2 = \left[1 + \left(\dfrac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1 + \left(\dfrac{\omega_H}{k\omega_{P1}}\right)^2\right]$

For $k = 1$,

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]$$

$$= \left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]^2$$

$\Rightarrow \omega_H = 0.64\omega_{P1}$

The approximate value using Eq. (10.77) is

$$\omega_H \simeq 1\Big/\sqrt{\frac{2}{\omega_{P1}^2}} = \frac{\omega_{P1}}{\sqrt{2}}$$

$= 0.71\omega_{P1}$

For $k = 2$, the exact value of ω_H is obtained from

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{2\omega_{P1}}\right)^2\right]$$

$$= 1 + \frac{5}{4}\left(\frac{\omega_H}{\omega_{P1}}\right)^2 + \frac{1}{4}\left(\frac{\omega_H}{\omega_{P1}}\right)^4$$

$$\left(\frac{\omega_H}{\omega_{P1}}\right)^4 + 5\left(\frac{\omega_H}{\omega_{P1}}\right)^2 - 4 = 0$$

$\Rightarrow \omega_H = 0.84\omega_{P1}$

The approximate value of ω_H is obtained from Eq. (10.77),

$$\omega_H \simeq 1\Big/\sqrt{\frac{1}{\omega_{P1}^2} + \frac{1}{4\omega_{P1}^2}}$$

$\Rightarrow \omega_H = 0.89\omega_{P1}$

For $k = 4$, the exact value of ω_H is obtained from

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{4\omega_{P1}}\right)^2\right]$$

$$= 1 + \frac{17}{16}\left(\frac{\omega_H}{\omega_{P1}}\right)^2 + \frac{1}{16}\left(\frac{\omega_H}{\omega_{P1}}\right)^4$$

$$\left(\frac{\omega_H}{\omega_{P1}}\right)^4 + 17\left(\frac{\omega_H}{\omega_{P1}}\right)^2 - 16 = 0$$

$\Rightarrow \omega_H = 0.95\omega_{P1}$

The approximate value of ω_H is found from Eq. (10.77):

$$\omega_H \simeq 1\Big/\sqrt{\frac{1}{\omega_{P1}^2} + \frac{1}{16\omega_{P1}^2}}$$

$= 0.97\omega_{P1}$

Ex: 10.15 $C_{\text{in}} = C_{gs} + C_{gd}(1 + g_m R'_L)$

$= 20 + 5(1 + 1.25 \times 10)$

$= 87.5 \text{ fF}$

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 87.5 \times 10^{-15} \times 10 \times 10^3}$$

$= 181.9 \text{ MHz}$

This is greater than the value obtained in Example 10.8, $f_H = 135.5$ MHz, by 34%. The value obtained in Example 10.8 is a better estimate of f_H as it takes into account the effect of C_L.

Ex: 10.16

$|A_M| = g_m R'_L = 1.25 \times 10 = 12.5$ V/V

$\text{GB} = |A_M| f_H$

$= 12.5 \times 135.5 = 1.69$ GHz

Ex: 10.17 $|A_M| = \dfrac{1}{2} \times 12.5 = 6.25$ V/V

$R_{gs} = 10 \text{ k}\Omega$

$R_{gd} = R'_{sig}(1 + g_m R'_L) + R'_L$

$= 10(1 + 6.25) + 5 = 77.5 \text{ k}\Omega$

$R_{C_L} = R'_L = 5 \text{ k}\Omega$

$\tau_{gs} = C_{gs}R_{gs} = 20 \times 10^{-15} \times 10 \times 10^3 = 200 \text{ ps}$

$\tau_{gd} = C_{gd}R_{gd} = 5\times10^{-15}\times77.5\times10^3 = 387.5 \text{ ps}$

$\tau_{C_L} = C_L R_{C_L} = 25 \times 10^{-15} \times 5 \times 10^3 = 125 \text{ ps}$

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= 200 + 387.5 + 125$

$= 712.5 \text{ ps}$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 712.5 \times 10^{-12}} = 223.4 \text{ MHz}$$

$\text{GB} = 6.25 \times 223.4 = 1.4 \text{ GHz}$

Ex: 10.18 $g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}$

Since I_D is increased by a factor of 4, g_m doubles:

$g_m = 2 \times 1.25 = 2.5 \text{ mA/V}$

Since R'_L is $r_o/2$, increasing I_D by a factor of four results in r_o and hence R'_L decreasing by a factor of 4, thus

$R'_L = \frac{1}{4} \times 10 = 2.5 \text{ k}\Omega$

$|A_M| = g_m R'_L = 2.5 \times 2.5 = 6.25 \text{ V/V}$

$R_{gs} = R'_{sig} = 10 \text{ k}\Omega$

$R_{gd} = R'_{sig}(1 + g_m R'_L) + R'_L$

$= 10(1 + 6.25) + 2.5$

$= 75 \text{ k}\Omega$

$R_{C_L} = R'_L = 2.5 \text{ k}\Omega$

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= C_{gs}R'_{sig} + C_{gd}R_{gd} + C_L R_{C_L}$

$= 20 \times 10^{-15} \times 10 \times 10^3 + 5 \times 10^{-15} \times 75$

$\times 10^3 + 25 \times 10^{-15} \times 2.5 \times 10^3$

$= 200 + 375 + 62.5$

$= 637.5 \text{ ps}$

$$f_H = \frac{1}{2\pi \times 637.5 \times 10^{-12}} = 250 \text{ MHz}$$

$\text{GB} = |A_M| f_H$

$= 6.25 \times 250 = 1.56 \text{ GHz}$

Ex: 10.19 (a) $g_m = 40 \text{ mA/V}$

$$r_\pi = \frac{200}{40} = 5 \text{ k}\Omega$$

$$r_{on} = \frac{V_{An}}{I} = \frac{130}{1} = 130 \text{ k}\Omega$$

$$r_{op} = \frac{|V_{Ap}|}{I} = \frac{50}{1} = 50 \text{ k}\Omega$$

$R'_L = r_{on} \parallel r_{op} = 130 \parallel 50 = 36.1 \text{ k}\Omega$

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m R'_L$$

$$= -\frac{5}{5 + 0.2 + 36} \times 40 \times 36.1$$

$= -175 \text{ V/V}$

(b) $C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$

$= 16 + 0.3(1 + 40 \times 36.1)$

$= 450 \text{ pF}$

$R'_{sig} = r_\pi \parallel (r_x + R_{sig})$

$= 5 \parallel (0.2 + 36) = 4.39 \text{ k}\Omega$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 450 \times 10^{-12} \times 4.39 \times 10^3}$$

$= 80.6 \text{ kHz}$

(c) $R_\pi = R'_{sig} = 4.39 \text{ k}\Omega$

$R_\mu = R'_{sig}(1 + g_m R'_L) + R'_L$

$= 4.39(1 + 40 \times 36.1) + 36.1$

$= 6.38 \text{ M}\Omega$

$R_{CL} = R'_L = 36.1 \text{ k}\Omega$

$\tau_H = C_\pi + C_\mu R_\mu + C_L R_{CL}$

$= 16 \times 4.39 + 0.3 \times 6.38 \times 10^3 + 5 \times 36.1$

$= 70.2 + 1914 + 180.5$

$= 2164.7 \text{ ns}$

$$f_H = \frac{1}{2\pi \times 2164.7 \times 10^{-9}}$$

$= 73.5 \text{ kHz}$

(d) $f_Z = \frac{g_m}{2\pi C_\mu}$

$$= \frac{40 \times 10^{-3}}{2\pi \times 0.3 \times 10^{-12}} = 21.2 \text{ GHz}$$

(e) $\text{GB} = 175 \times 73.5 = 12.9 \text{ MHz}$

Ex: 10.20 $R_{\text{in}} = \dfrac{R_L + r_o}{1 + g_m r_o}$

$= \dfrac{500 + 20}{1 + 25} = 20\text{ k}\Omega$

$G_v = \dfrac{R_L}{R_{\text{sig}} + R_{\text{in}}} = \dfrac{500}{10 + 20} = 16.7\text{ V/V}$

$R_{gs} = R_{\text{sig}} \parallel R_{\text{in}} = 10 \parallel 20 = 6.7\text{ k}\Omega$

$R_{gd} = R_L \parallel R_o$

$= 500 \parallel 280 = 179.5\text{ k}\Omega$

$\tau_H = C_{gs}R_{gs} + (C_{gd} + C_L)R_{gd}$

$= 20 \times 10^{-15} \times 6.7 \times 10^3 + (5 + 25) \times 10^{-15}$

$\times 179.5 \times 10^3$

$= 134 + 5385$

$= 5519\text{ ps}$

$f_H = \dfrac{1}{2\pi \times 5519 \times 10^{-12}} = 28.8\text{ MHz}$

Thus, while the midband gain has been increased substantially (by a factor of 9.7), the bandwidth has been substantially lowered (by a factor of 9.4). Thus, the high-frequency advantage of the CG amplifier is completely lost!

Ex: 10.21 (a) $A_{\text{CS}} = -g_m(R_L \parallel r_o)$

$= -g_m(r_o \parallel r_o) = -\dfrac{1}{2}g_m r_o$

$= -\dfrac{1}{2} \times 40 = -20\text{ V/V}$

$A_{\text{cascode}} = -g_m(R_L \parallel R_o)$

$= -g_m(r_o \parallel g_m r_o r_o)$

$\simeq -g_m r_o = -40\text{ V/V}$

Thus,

$\dfrac{A_{\text{cascode}}}{A_{\text{CS}}} = 2$

(b) For the CS amplifier,

$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd}$

where

$R_{gs} = R_{\text{sig}}$

$R_{gd} = R_{\text{sig}}(1 + g_m R_L') + R_L'$

$\simeq R_{\text{sig}}(1 + g_m R_L')$

$= R_{\text{sig}}\left(1 + \dfrac{1}{2}g_m r_o\right)$

$= R_{\text{sig}}\left(1 + \dfrac{1}{2} \times 40\right) = 21R_{\text{sig}}$

$\tau_H = C_{gs}R_{\text{sig}} + C_{gd} \times 21R_{\text{sig}}$

$= C_{gs}R_{\text{sig}} + 0.25C_{gs} \times 21R_{\text{sig}}$

$= 6.25C_{gs}R_{\text{sig}}$

$f_H = \dfrac{1}{2\pi \times 6.25C_{gs}R_{\text{sig}}}$

For the cascode amplifier,

$\tau_H \simeq R_{\text{sig}}[C_{gs1} + C_{gd1}(1 + g_{m1}R_{d1})]$

where

$R_{d1} = r_{o1} \parallel R_{\text{in2}} = r_o \parallel \dfrac{r_o + r_o}{g_m r_o}$

$= r_o \parallel \dfrac{2}{g_m} = \dfrac{\dfrac{2}{g_m}r_o}{\dfrac{2}{g_m} + r_o}$

$= \dfrac{2r_o}{2 + g_m r_o} = \dfrac{2r_o}{2 + 40} = \dfrac{r_o}{21}$

$\tau_H = C_{gs}R_{\text{sig}}\left[1 + 0.25\left(1 + \dfrac{g_m r_o}{21}\right)\right]$

$= C_{gs}R_{\text{sig}}\left[1 + 0.25\left(1 + \dfrac{40}{21}\right)\right]$

$= 1.73C_{gs}R_{\text{sig}}$

$f_H = \dfrac{1}{2\pi \times 1.73\, C_{gs}R_{\text{sig}}}$

Thus,

$\dfrac{f_H(\text{cascode})}{f_H(\text{CS})} = \dfrac{6.25}{1.73} = 3.6$

(c) $\dfrac{f_t(\text{cascode})}{f_t(\text{CS})} = 2 \times 3.6 = 7.2$

Ex: 10.22 $g_m = 40\text{ mA/V}$

$r_\pi = \dfrac{\beta}{g_m} = \dfrac{200}{40} = 5\text{ k}\Omega$

$R_{\text{in}} = r_\pi + r_x = 5 + 0.2 = 5.2\text{ k}\Omega$

$A_0 = g_m r_o$

$= 40 \times 130 = 5200$

$R_{o1} = r_{o1} = 130\text{ k}\Omega$

$R_{\text{in2}} = r_{e2}\dfrac{r_{o2} + R_L}{r_{o2} + R_L/(\beta_2 + 1)}$

$= 25\dfrac{130 + 50}{130 + \dfrac{50}{201}}$

$= 35\ \Omega$

$R_o \simeq \beta_2 r_{o2} = 200 \times 130 = 26\text{ M}\Omega$

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{\text{sig}}} g_m (R_o \parallel R_L)$$

$$= -\frac{5}{5 + 0.2 + 36} 40(26{,}000 \parallel 50)$$

$$A_M = -242 \text{ V/V}$$

$$R'_{\text{sig}} = r_{\pi 1} \parallel (r_{x1} + R_{\text{sig}})$$

$$= 5 \parallel (0.2 + 36) = 4.39 \text{ k}\Omega$$

$$R_{\pi 1} = R'_{\text{sig}} = 4.39 \text{ k}\Omega$$

$$R_{e1} = r_{o1} \parallel R_{\text{in}2}$$

$$= 130 \text{ k}\Omega \parallel 35 \ \Omega \simeq 35 \ \Omega$$

$$R_{\mu 1} = R'_{\text{sig}}(1 + g_{m1} R_{e1}) + R_{e1}$$

$$= 4.39(1 + 40 \times 0.035) + 0.035$$

$$= 10.6 \text{ k}\Omega$$

$$\tau_H = C_{\pi 1} R_{\pi 1} + C_{\mu 1} R_{\mu 1} + C_{\pi 2} R_{e1} + (C_L + C_{\mu 2})(R_L \parallel R_o)$$

$$= 16 \times 4.39 + 0.3 \times 10.6 + 16 \times 0.035 + (5 + 0.3)(50 \parallel 26{,}000)$$

$$= 70.24 + 3.18 + 0.56 + 264.5$$

$$= 338.5 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 338.5 \times 10^{-9}} = 470 \text{ kHz}$$

$$f_t = |A_M| f_H = 242 \times 470 = 113.8 \text{ MHz}$$

Thus, in comparison to the CE amplifier of Exercise 10.19, we see that $|A_M|$ has increased from 175 V/V to 242 V/V, f_H has increased from 73.5 kHz to 470 kHz, and f_t has increased from 12.9 MHz to 113.8 MHz.

To have f_H equal to 1 MHz,

$$\tau_H = \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 1 \times 10^6} = 159.2 \text{ ns}$$

Thus,

$$159.2 = 70.24 + 3.18 + 0.56 + (C_L + C_\mu)(50 \parallel 26000)$$

$$\Rightarrow C_L + C_\mu = 1.71 \text{ pF}$$

Thus, C_L must be reduced to 1.41 pF.

Ex: 10.23 From Eq. (10.120), we obtain

$$R_{gs} = \frac{R_{\text{sig}}}{g_m R'_L + 1} + \frac{R'_L}{g_m R'_L + 1} = \frac{R_{\text{sig}} + R'_L}{g_m R'_L + 1}$$

$$R_{gd} = R_{\text{sig}}$$

$$R_{C_L} = \frac{R'_L}{g_m R'_L + 1}$$

Ex: 10.24 From Example 10.11, we get

$$\tau_H = b_1 = 104 \text{ ps}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 104 \times 10^{-12}} = 1.53 \text{ GHz}$$

This is lower than the exact value found in Example 10.11 (i.e., 1.86 GHz) by about 18%, still not a bad estimate!

Ex: 10.25 $g_m = 40$ mA/V

$$r_e = 25 \ \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$R'_{\text{sig}} = R_{\text{sig}} + r_x = R_{\text{sig}} = 1 \text{ k}\Omega$$

$$R'_L = R_L \parallel r_o = 1 \parallel 100 = 0.99 \text{ k}\Omega$$

$$A_M = \frac{R'_L}{R'_L + r_e + \dfrac{R'_{\text{sig}}}{\beta + 1}}$$

$$= \frac{0.99}{0.99 + 0.025 + (1/101)} = 0.97 \text{ V/V}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 400 \times 10^6}$$

$$= 15.9 \text{ pF}$$

$$C_\mu = 2 \text{ pF}$$

$$C_\pi = 13.9 \text{ pF}$$

$$f_Z = \frac{1}{2\pi C_\pi r_e} = \frac{1}{2\pi \times 13.9 \times 10^{-12} \times 25}$$

$$= 458 \text{ MHz}$$

$$b_1 = \frac{\left[C_\pi + C_\mu\left(1 + \dfrac{R'_L}{r_e}\right)\right] R'_{\text{sig}} + \left[C_\pi + C_L\left(1 + \dfrac{R'_{\text{sig}}}{r_\pi}\right)\right] R'_L}{1 + \dfrac{R'_L}{r_e} + \dfrac{R'_{\text{sig}}}{r_\pi}}$$

$$= \frac{\left[13.9 + 2\left(1 + \dfrac{0.99}{0.025}\right)\right] \times 1 + (13.9 + 0)0.99}{1 + \dfrac{0.99}{0.025} + \dfrac{1}{2.5}}$$

$$= 2.66 \times 10^{-9} \text{ s}$$

$$b_2 = \frac{C_\pi C_\mu R'_L R'_{\text{sig}}}{1 + \dfrac{R'_L}{r_e} + \dfrac{R'_{\text{sig}}}{r_\pi}} = \frac{13.9 \times 2 \times 0.99 \times 1}{1 + \dfrac{0.99}{0.025} + \dfrac{1}{2.5}}$$

$$= 0.671 \times 10^{-18}$$

ω_{P1} and ω_{P2} are the roots of the equation

$1 + b_1 s + b_2 s^2 = 0$

Solving we obtain,

$f_{P1} = 67$ MHz

$f_{P2} = 563$ MHz

Since $f_{P1} \ll f_{P2}$,

$f_H \simeq f_{P1} = 67$ MHz

Ex: 10.26 (a) $I_{D1,2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2} V_{OV}^2$

$0.4 = \frac{1}{2} \times 0.2 \times 100 V_{OV}^2$

$\Rightarrow V_{OV} = 0.2$ V

$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.4}{0.2} = 4$ mA/V

(b) $A_d = g_m(R_D \parallel r_o)$

where

$r_o = \frac{V_A}{I_D} = \frac{20}{0.4} = 50$ kΩ

$A_d = 4(5 \parallel 50) = 4 \times 4.545$

$= 18.2$ V/V

(c) $f_H = \frac{1}{2\pi(C_L + C_{gd} + C_{db})(R_D \parallel r_o)}$

$= \frac{1}{2\pi(100 + 10 + 10) \times 10^{-15} \times 4.545 \times 10^3}$

$= 292$ MHz

(d) $\tau_{gs} = C_{gs}R_{\text{sig}} = 50 \times 10 = 500$ ps

$\tau_{gd} = C_{gd}R_{gd} = C_{gd}[R_{\text{sig}}(1 + g_m R'_L) + R'_L]$

$= 10[10(1 + 18.2) + 4.545]$

$= 1965.5$ ps

$\tau_{C_L} = (C_L + C_{db})R'_L = 110 \times 4.545 = 500$ ps

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= 500 + 1965.5 + 500 = 2965.5$ ps

$f_H = \frac{1}{2\pi \times 2965.5 \times 10^{-12}}$

$= 53.7$ MHz

Ex: 10.27 $f_Z = \frac{1}{2\pi R_{SS} C_{SS}}$

$= \frac{1}{2\pi \times 75 \times 10^3 \times 0.4 \times 10^{-12}}$

$= 5.3$ MHz

Thus, the 3-dB frequency of the CMRR is 5.3 MHz.

Ex: 10.28 $A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$

where

$g_{m1,2} = \frac{0.5}{0.025} = 20$ mA/V

$r_{o2} = r_{o4} = \frac{100}{0.5} = 200$ kΩ

$A_d = 20(200 \parallel 200) = 2000$ V/V

The dominant high-frequency pole is that introduced at the output node,

$f_H = \frac{1}{2\pi C_L(r_{o2} \parallel r_{o4})}$

$= \frac{1}{2\pi \times 2 \times 10^{-12} \times 100 \times 10^3}$

$= 0.8$ MHz

Ex: 10.29 (a) $A_M = -g_m R'_L$

where

$R'_L = R_L \parallel r_o = 20 \parallel 20 = 10$ kΩ

$A_M = -2 \times 10 = -20$ V/V

$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R'_L$

$= C_{gs}R_{\text{sig}} + C_{gd}[R_{\text{sig}}(1 + g_m R'_L) + R'_L] + C_L R'_L$

$= 20 \times 20 + 5[20(1 + 20) + 10] + 5 \times 10$

$= 400 + 2150 + 50$

$= 2600$ ps

$f_H = \frac{1}{2\pi\tau_H}$

$= \frac{1}{2\pi \times 2600 \times 10^{-12}}$

$= 61.2$ MHz

$\text{GB} = |A_M| f_H$

$= 20 \times 61.2$

$= 1.22$ GHz

(b) $G_m = \frac{g_m}{1 + g_m R_s} = \frac{2}{1 + 2} = 0.67$ mA/V

$R_o \simeq r_o(1 + g_m R_s)$

$= 20 \times 3 = 60$ kΩ

$R'_L = R_L \parallel R_o = 20 \parallel 60 = 15$ kΩ

$A_M = -G_m R'_L$

$= -0.67 \times 15 = -10$ V/V

$R_{gd} = R_{\text{sig}}(1 + G_m R'_L) + R'_L$

$= 20(1 + 10) + 15$

$= 235$ kΩ

$R_{C_L} = R'_L = 15\text{ k}\Omega$

$$R_{gs} = \frac{R_{\text{sig}} + R_s + R_{\text{sig}}R_s/(r_o + R_L)}{1 + g_m R_s\left(\frac{r_o}{r_o + R_L}\right)}$$

where

$$R_s = \frac{2}{g_m} = 1\text{ k}\Omega$$

$$R_{gs} = \frac{20 + 1 + \frac{20 \times 1}{20 + 20}}{1 + 2 \times \frac{20}{20 + 20}}$$

$= 10.75\text{ k}\Omega$

$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L}$

$= 20 \times 10.75 + 5 \times 235 + 5 \times 15$

$= 215 + 1175 + 75 = 1465\text{ ps}$

$$f_H = \frac{1}{2\pi \times 1465 \times 10^{-12}} = 109\text{ MHz}$$

$\text{GB} = 10 \times 109 = 1.1\text{ GHz}$

Ex: 10.30 Refer to Fig. 10.42(b).

$$A_M = \frac{2r_\pi}{2r_\pi + R_{\text{sig}}} \times \frac{1}{2} \times g_m R_L$$

where

$g_m = 20\text{ mA/V}$

$$r_\pi = \frac{100}{20} = 5\text{ k}\Omega$$

$$A_M = \frac{10}{10 + 10} \times \frac{1}{2} \times 20 \times 10 = 50\text{ V/V}$$

$$f_{P1} = \frac{1}{2\pi\left(\frac{C_\pi}{2} + C_\mu\right)(2r_\pi \parallel R_{\text{sig}})}$$

$$= \frac{1}{2\pi\left(\frac{6}{2} + 2\right) \times 10^{-12}(10 \parallel 10) \times 10^3}$$

$= 6.4\text{ MHz}$

$$f_{P2} = \frac{1}{2\pi C_\mu R_L}$$

$$= \frac{1}{2\pi \times 2 \times 10^{-12} \times 10 \times 10^3}$$

$= 8\text{ MHz}$

$$T(s) = \frac{50}{\left(1 + \frac{s}{\omega_{P1}}\right)\left(1 + \frac{s}{\omega_{P2}}\right)}$$

$$|T(j\omega)| = \frac{50}{\sqrt{\left[1 + \left(\frac{\omega}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega}{\omega_{P2}}\right)^2\right]}}$$

At $\omega = \omega_H$, $|T| = 50/\sqrt{2}$, thus

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{\omega_{P2}}\right)^2\right]$$

$$= 1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2 + \left(\frac{\omega_H}{\omega_{P2}}\right)^2 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\left(\frac{\omega_H}{\omega_{P2}}\right)^2$$

$$\frac{\omega_H^4}{\omega_{P1}^2\omega_{P2}^2} + \omega_H^2\left(\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2}\right) - 1 = 0$$

$$\frac{f_H^4}{f_{P1}^2 f_{P2}^2} + f_H^2\left(\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}\right) - 1 = 0$$

$$\frac{f_H^4}{6.4^2 \times 8^2} + f_H^2\left(\frac{1}{6.4^2} + \frac{1}{8^2}\right) - 1 = 0$$

$\Rightarrow f_H = 4.6\text{ MHz}$ (Exact value)

Using Eq. (10.164), an approximate value for f_H can be obtained:

$$f_H \simeq 1/\sqrt{\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}}$$

$$= 1/\sqrt{\frac{1}{6.4^2} + \frac{1}{8^2}} = 5\text{ MHz}$$

Ex: 11.1 (c) $A = 100$ V/V and $A_f = 10$ V/V

Since A is not much greater than A_f, we shall use the exact expression to determine β and hence R_2/R_1,

$$A_f = \frac{A}{1 + A\beta}$$

$$10 = \frac{100}{1 + 100\beta}$$

$$\Rightarrow \beta = 0.09 \text{ V/V}$$

Now,

$$\frac{R_1}{R_1 + R_2} = 0.09$$

$$\frac{R_2}{R_1} = \frac{1}{0.09} - 1 = 10.11$$

(d) $A\beta = 100 \times 0.09 = 9$

$1 + A\beta = 10$

$\Rightarrow 20$ dB

(e) $V_o = A_f V_s = 10 \times 1 = 10$ V

$V_f = \beta V_o = 0.09 \times 10 = 0.9$ V

$$V_i = \frac{V_o}{A} = \frac{10}{100} = 0.1 \text{ V}$$

(f) $A \rightarrow 80$ V/V

$$A_f = \frac{80}{1 + 80 \times 0.09} = 9.756$$

a change of $\frac{9.756 - 10}{10} \times 100 = -2.44\%$ or a reduction of 2.44%.

Ex. 11.2 (c) $A = 10^4$ V/V and $A_f = 10^3$ V/V

$$A_f = \frac{A}{1 + A\beta}$$

$$10^3 = \frac{10^4}{1 + 10^4\beta}$$

$$\Rightarrow \beta = 9 \times 10^{-4} \text{ V/V}$$

$$\frac{R_1}{R_1 + R_2} = 9 \times 10^{-4}$$

$$\frac{R_2}{R_1} = \frac{1}{9 \times 10^{-4}} - 1 = 1110.1$$

(d) $A\beta = 10^4 \times 9 \times 10^{-4} = 9$

$1 + A\beta = 10$

$\Rightarrow 20$ dB

(e) $V_s = 0.01$ V

$V_o = A_f V_s = 10^3 \times 0.01 = 10$ V

$V_f = \beta V_o = 9 \times 10^{-4} \times 10 = 0.009$ V

$$V_i = \frac{V_o}{A} = \frac{10}{10^4} = 0.001 \text{ V}$$

(f) $A \Rightarrow 0.8 \times 10^4$ V/V

$$A_f = \frac{0.8 \times 10^4}{1 + 0.8 \times 10^4 \times 9 \times 10^{-4}}$$

$= 975.6$ V/V

which is a change of $\frac{975.6 - 1000}{1000} \times 100$

$= -2.44\%$ or a reduction of 2.44%.

Ex. 11.3 To constrain the corresponding change in A_f to 0.1%, we need an amount-of-feedback of at least

$$1 + A\beta = \frac{10\%}{0.1\%} = 100$$

Thus the largest obtainable closed-loop gain will be

$$A_f = \frac{A}{1 + A\beta} = \frac{1000}{100} = 10 \text{ V/V}$$

Each amplifier in the cascade will have a nominal gain of 10 V/V and a maximum variability of 0.1%; thus the overall voltage gain will be $(10)^3 = 1000$ V/V and the maximum variability will be 0.3%.

Ex. 11.4 $\beta = \frac{R_1}{R_1 + R_2} = \frac{1}{1 + 9} = 0.1$

$A\beta = 10^4 \times 0.1 = 1000$

$1 + A\beta = 1001$

$$A_f = \frac{A}{1 + A\beta}$$

$$A_f = \frac{10^4}{1 + 10^4 \times 0.1} = 9.99 \text{ V/V}$$

$f_{Hf} = f_H(1 + A\beta)$

$= 100 \times 1001 = 100.1$ kHz

Ex. 11.5 Signal at output $= V_s \frac{A_1 A_2}{1 + A_1 A_2 \beta}$

$$= 1 \times \frac{1 \times 100}{1 + 1 \times 100 \times 1} = 1 \times \frac{100}{101} \simeq 1 \text{ V}$$

Interference at output $= V_n \frac{A_1}{1 + A_1 A_2 \beta}$

$$= 1 \times \frac{1}{1 + 1 \times 100 \times 1} \simeq 0.01 \text{ V}$$

Thus S/I at the output becomes 1/0.01

= 100 or 40 dB

Since S/I at the input is 1/1=1 or 0 dB, the improvement is 40 dB.

Ex. 11.6 (a) Refer to Fig. 11.8(c).

$$\beta = \frac{R_1}{R_1 + R_2}$$

(b)

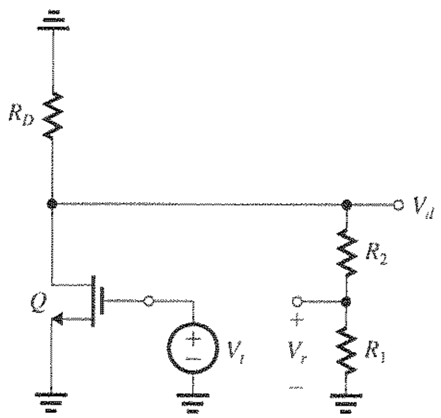


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain $A\beta$. Observe that we have eliminated the input signal V_s, and opened the loop at the gate of Q where the input impedance is infinite obviating the need for a termination resistance at the right-hand side of the break. Now we need to analyze the circuit to determine

$$A\beta \equiv -\frac{V_r}{V_t}$$

First, we write for the gain of the CS amplifier Q,

$$\frac{V_d}{V_t} = -g_m[R_D \parallel (R_1 + R_2)] \qquad (1)$$

then we use the voltage-divider rule to find V_r,

$$\frac{V_r}{V_d} = \frac{R_1}{R_1 + R_2} \qquad (2)$$

Combining Eqs. (1) and (2) gives

$$A\beta \equiv -\frac{V_r}{V_t} = g_m[R_D \parallel (R_1 + R_2)]\frac{R_1}{R_1 + R_2}$$

which can be simplified to

$$A\beta = g_m \frac{R_D R_1}{R_D + R_1 + R_2}$$

(c) $A = \dfrac{A\beta}{\beta}$

$$= g_m \frac{R_D(R_1 + R_2)}{R_D + R_1 + R_2}$$

(d) $\beta = \dfrac{R_1}{R_1 + R_2} = \dfrac{20}{20 + 80} = 0.2$ V/V

$$A\beta = 4\frac{10 \times 20}{10 + 20 + 80} = 7.27$$

$$A = \frac{7.27}{0.2} = 36.36 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta} = \frac{36.36}{1 + 36.36 \times 0.2} = 4.4 \text{ V/V}$$

If $A\beta$ were $\gg 1$, then

$$A_f \simeq \frac{1}{\beta} = \frac{1}{0.2} = 5 \text{ V/V}$$

Ex. 11.7 From the solution of Example 11.4,

$A\beta = 6$

$1 + A\beta = 7$

Thus,

$f_{Hf} = (1 + A\beta)f_H$

$= 7 \times 1$

$= 7$ kHz

Ex. 11.8 Refer to Fig. E11.8. The 1-mA bias current will split equally between the emitters of Q_1 and Q_2, thus

$I_{E1} = I_{E2} = 0.5$ mA

Transistor Q_3 will be operating at an emitter current

$I_{E3} = 5$ mA

determined by the 5-mA current source. Since the dc component of $V_s = 0$, the negative feedback will force the dc voltage at the output to be approximately zero. See Fig. 1 on next page.

The β circuit is shown in Fig. 1 together with the determination of β and of the loading effects of the β circuit on the A circuit,

$$\beta = \frac{R_1}{R_1 + R_2} = \frac{1}{1 + 9} = 0.1 \text{ V/V}$$

$$R_{11} = R_1 \parallel R_2 = 1 \parallel 9 = 0.9 \text{ k}\Omega$$

$$R_{22} = R_1 + R_2 = 1 + 9 = 10 \text{ k}\Omega$$

The A circuit is shown in Fig. 2. See figure on next page.

$$r_{e1} = r_{e2} = \frac{V_T}{I_{E1,2}} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \ \Omega$$

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{25 \text{ mV}}{5 \text{ mA}} = 5 \ \Omega$$

$$i_e = \frac{V_i}{r_{e1} + r_{e2} + \dfrac{R_s + R_{11}}{\beta + 1}}$$

$$i_e = \frac{V_i}{0.05 + 0.05 + \dfrac{10 + 0.9}{101}}$$

$$\Rightarrow i_e = 4.81 V_i \qquad (1)$$

$$R_{b3} = (\beta + 1)[r_{e3} + (R_{22} \parallel R_L)]$$

$$= 101[0.005 + (10 \parallel 2)]$$

$$= 168.84 \text{ k}\Omega$$

These figures belong to Exercise 11.8.

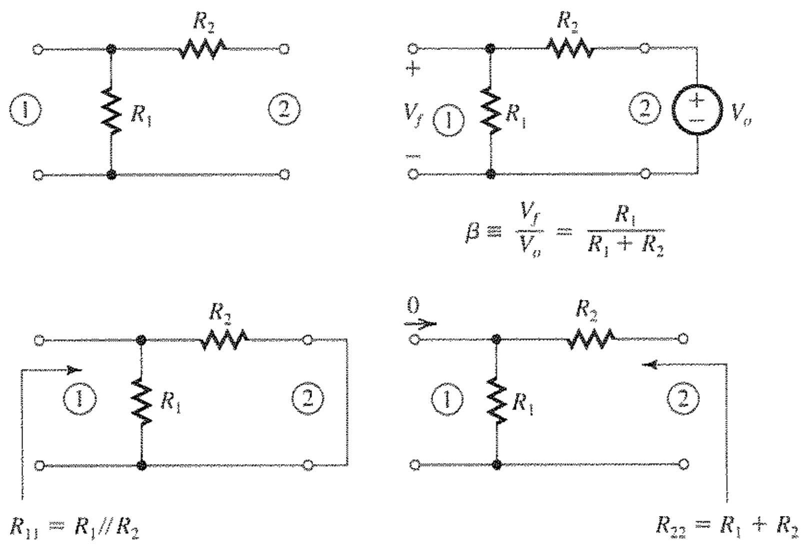


Figure 1

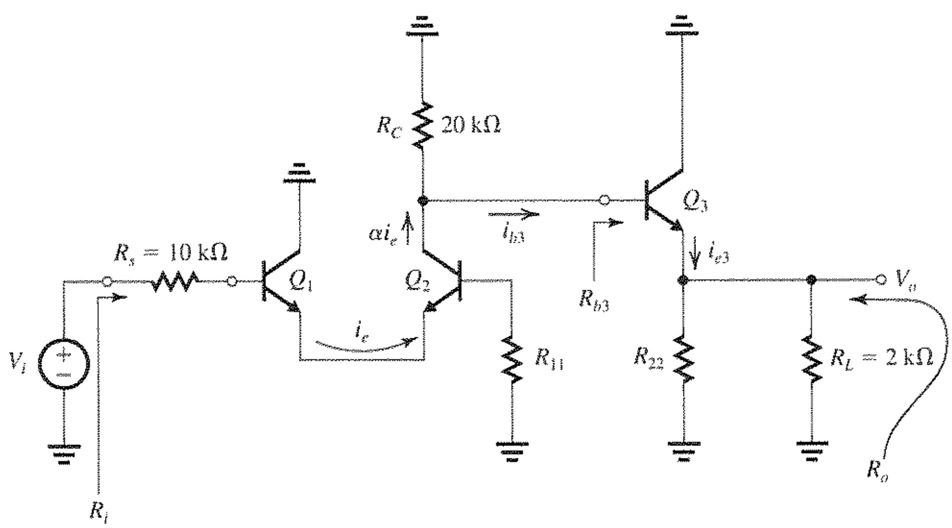


Figure 2

$$i_{b3} = \alpha i_e \frac{R_C}{R_C + R_{b3}}$$

$$= 0.99\, i_e \frac{20}{20 + 168.84}$$

$$\Rightarrow i_{b3} = 0.105 i_e \qquad (2)$$

$$V_o = i_{e3}(R_{22} \parallel R_L)$$

$$= (\beta + 1) i_{b3}(R_{22} \parallel R_L)$$

$$= i_{b3} \times 101(10 \parallel 2)$$

$$\Rightarrow V_o = 168.33 i_{b3} \qquad (3)$$

Combining (1)–(3), we obtain

$$A \equiv \frac{V_o}{V_i} = 85 \text{ V/V}$$

$$\beta = 0.1 \text{ V/V}$$

$$A\beta = 8.5$$

$$1 + A\beta = 9.5$$

$$A_f = \frac{85}{9.5} = 8.95 \text{ V/V}$$

From the A circuit, we have

$$R_i = R_s + R_{11} + (\beta + 1)(r_{e1} + r_{e2})$$

$$= 10 + 0.9 + 101 \times 0.1$$

$$= 21 \text{ k}\Omega$$

$R_{if} = R_i(1 + A\beta)$

$= 21 \times 9.5 = 199.5\ \text{k}\Omega$

$R_{\text{in}} = R_{if} - R_s = 199.5 - 10 = 189.5\ \text{k}\Omega$

From the A circuit, we have

$$R_o = R_L \parallel R_{22} \parallel \left(r_{e5} + \frac{R_C}{\beta + 1}\right)$$

$$= 2 \parallel 10 \parallel \left(0.005 + \frac{20}{101}\right)$$

$= 181\ \Omega$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{181}{9.5} = 19.1\ \Omega$$

$R_{of} = R_L \parallel R_{\text{out}}$

$19.1 = 2\ \text{k}\Omega \parallel R_{\text{out}}$

$\Rightarrow R_{\text{out}} = 19.2\ \Omega$

Ex. 11.9 Figure 1 shows the β circuit together with the determination of β, R_{11} and R_{22}.

$$\beta = \frac{R_1}{R_1 + R_2}$$

$R_{11} = R_1 \parallel R_2$

$R_{22} = R_1 + R_2$

Figure 2 shows the A circuit. We can write

$V_o = g_m(R_D \parallel R_{22})V_i$

Thus,

$$A \equiv \frac{V_o}{V_i} = g_m[R_D \parallel (R_1 + R_2)]$$

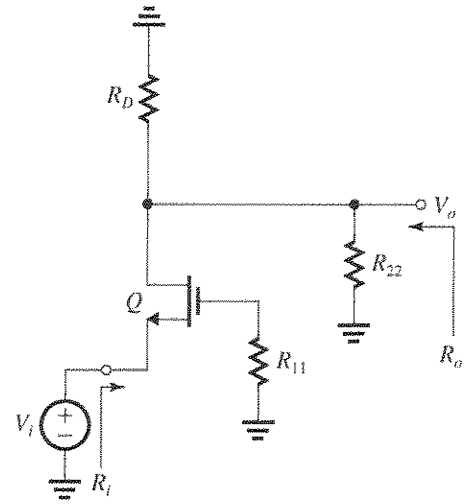


Figure 2

$$\beta = \frac{R_1}{R_1 + R_2}$$

$$A_f = \frac{A}{1 + A\beta}$$

From A circuit, we have

$$R_i = \frac{1}{g_m}$$

$R_o = R_D \parallel R_{22}$

$R_{\text{in}} = R_{if} = R_i(1 + A\beta)$

$$R_{\text{in}} = \frac{1}{g_m}(1 + A\beta)$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$R_{\text{out}} = \frac{R_D \parallel (R_1 + R_2)}{1 + A\beta}$$

This figure belongs to Exercise 11.9.

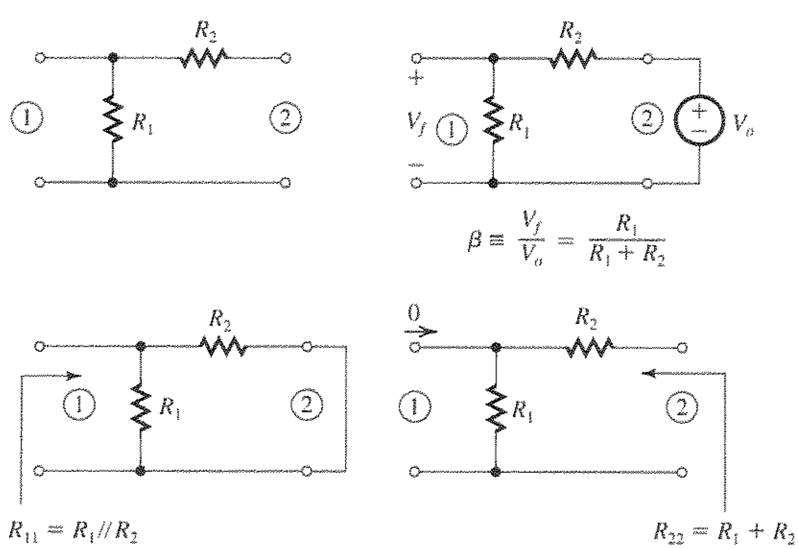


Figure 1

Comparison with the results of Exercise 11.6 shows that the expressions for A and β are identical. However, R_{in} and R_{out} cannot be determined using the method of Exercise 11.6.

Ex. 11.10 From the solution to Example 11.6, we have

$$A\beta = 653.6$$

$$1 + A\beta = 654.6$$

A decrease in the op amp gain by 10% results in a decrease in A by 10% and a corresponding decrease in A_f by

$$\frac{10\%}{1+A\beta} = \frac{10\%}{654.6} = 0.015\%$$

A more exact solution (not using differential) is as follows:

The open-loop gain A becomes

$$A = 0.9 \times 653.6 = 588.24 \text{ mA/V}$$

$$\beta = R_F = 1 \text{ k}\Omega$$

$$A_f = \frac{588.24}{1 + 588.24 \times 1}$$

$$= 0.9983 \text{ mA/V}$$

Change in $A_f = 0.9983 - 0.9985$

$$= -0.0002$$

Percentage change in $A_f = \dfrac{-0.0002}{0.9983} \times 100$
$= -0.02\%$

Ex. 11.11 For a nominal closed-loop transconductance of 2 mA/V, we have

$$R_F = \beta = \frac{1}{2 \text{ mA/V}} = 0.5 \text{ k}\Omega$$

From the solution to Example 12.6, we obtain

$$A = \frac{\mu}{R_F}\,\frac{g_m(R_F \parallel R_{id} \parallel r_{o2})}{1 + g_m(R_F \parallel R_{id} \parallel r_{o2})}$$

$$A = \frac{1000}{0.5}\,\frac{2(0.5 \parallel 100 \parallel 20)}{1 + 2(0.5 \parallel 100 \parallel 20)}$$

$$A = 985.2 \text{ mA/V}$$

$$A_f \equiv \frac{I_o}{V_s} = \frac{985.2}{1 + 985.2 \times 0.5} = 1.996 \text{ mA/V}$$

Ex. 11.12 $A_f \simeq 5$ mA/V

$$\beta \simeq \frac{1}{A_f} = 0.2 \text{ k}\Omega = 200\ \Omega$$

$$R_F = 200\ \Omega$$

Using Eq. (11.36), we obtain

$$A_f \simeq \frac{A_1 g_{m2}}{1 + A_1 g_{m2} R_F}$$

$$= \frac{200 \times 2}{1 + 200 \times 2 \times 0.2} = 4.94 \text{ mA/V}$$

From Eq. (11.32), we have

$$R_i = R_s + R_{id} + R_F$$

$$\simeq R_{id} + R_F$$

$$= 100 + 0.2 = 100.2 \text{ k}\Omega$$

From Eq. (11.35), we get

$$A\beta \simeq A_1 g_{m2} R_F = 200 \times 2 \times 0.2 = 80$$

$$1 + A\beta = 81$$

$$R_{if} = (1 + A\beta)R_i$$

$$= 81 \times 100.2 \simeq 8.1 \text{ M}\Omega$$

From Eq. (11.33), we have

$$R_o = r_{o2} + R_L + R_F$$

$$\simeq r_{o2} + R_F$$

$$= 20 + 0.2 = 20.2 \text{ k}\Omega$$

$$R_{of} = R_o(1 + A\beta) = 20.2 \times 81 = 1.64 \text{ M}\Omega$$

If g_{m2} drops by 50%, A drops by 50% to

$$A = A_1 g_{m2} = 200 \times 1 = 200 \text{ mA/V}$$

and A_f becomes

$$A_f = \frac{200}{1 + 200 \times 0.2} = 4.878 \text{ mA/V}$$

Thus,

$$\Delta A_f = 4.94 - 4.878 = -0.062$$

$$\frac{\Delta A_f}{A_f} \times 100 = -\frac{0.062}{4.94} \times 100 = -1.25\%$$

Ex. 11.13 See figure on next page. Figure 1 shows the circuit for determining the loop gain. The figure also shows the analysis. We start by finding the current in the drain of Q_2 as $g_{m2}V_{g2}$ (this excludes the current in r_{o2}). Since $r_{o2} \gg R_L + R_F$, most of $g_{m2}V_{g2}$ will flow through R_L and $R_F \parallel (R_{id} + R_s)$. Since $R_F \ll R_{id} + R_s$, the voltage across R_F will be approximately $-g_{m2}V_{g2}R_F$. This voltage is amplified by A_1 which provides at its output

$$V_r = -A_1 g_{m2} R_F V_{g2}$$

Thus, we find $A\beta$ as

$$A\beta \equiv -\frac{V_r}{V_{g2}} = A_1 g_{m2} R_F$$

This figure belongs to Exercise 11.13.

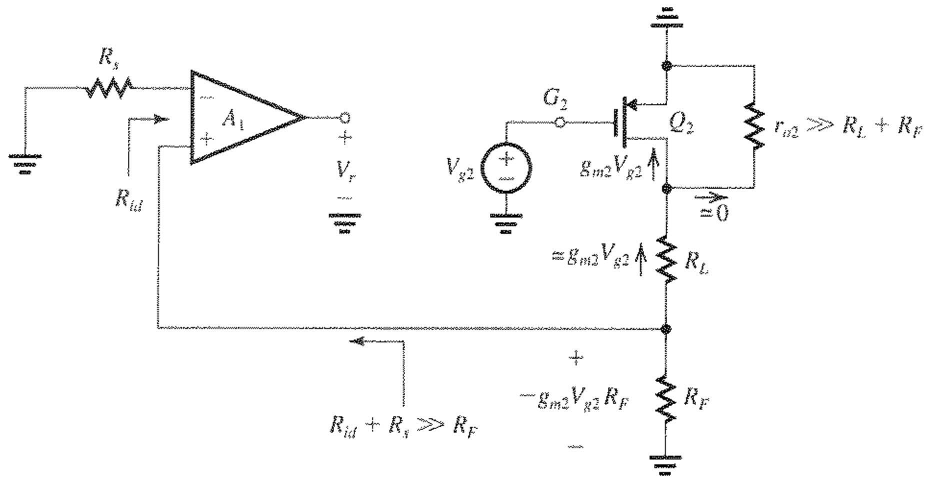


Figure 1

Ex. 11.14 From Eq. (11.34), we obtain

$$A\beta = (A_1 g_{m2} R_F)\left(\frac{R_{id}}{R_{id}+R_s+R_F}\right)\left(\frac{r_{o2}}{r_{o2}+R_L+R_F}\right)$$

From Eq. (11.32), we obtain

$$R_i = R_s + R_{id} + R_F$$

$$R_{if} = R_i(1 + A\beta)$$

$$= R_i + A\beta R_i$$

$$= R_s + R_{id} + R_F + (A_1 g_{m2} R_F) R_{id}\left(\frac{r_{o2}}{r_{o2}+R_L+R_F}\right)$$

$$R_{in} = R_{if} - R_s$$

$$R_{in} = R_{id} + R_F + (A g_{m2} R_F) R_{id}\left(\frac{r_{o2}}{r_{o2}+R_L+R_F}\right)$$

For $R_F \ll R_{id}$ and $r_{o2} \gg R_L + R_F$, we have

$$R_{in} \simeq R_{id} + A g_{m2} R_F R_{id}$$

$$= R_{id}(1 + A g_{m2} R_F) \qquad \text{Q.E.D.}$$

Ex. 11.15 From Eq. (11.34), we obtain

$$A\beta = (A_1 g_{m2} R_F)\left(\frac{R_{id}}{R_{id}+R_s+R_F}\right)\left(\frac{r_{o2}}{r_{o2}+R_L+R_F}\right)$$

From Eq. (11.33), we get

$$R_o = r_{o2} + R_L + R_F$$

$$R_{of} = R_o(1 + A\beta)$$

$$= R_o + A\beta R_o$$

$$= r_{o2} + R_L + R_F + (A_1 g_{m2} R_F)\left(\frac{R_{id}}{R_{id}+R_s+R_F}\right) r_{o2}$$

$$R_{out} = R_{of} - R_L$$

$$= r_{o2} + R_F + (A_1 g_{m2} R_F)\left(\frac{R_{id}}{R_{id}+R_s+R_F}\right) r_{o2}$$

For $R_F \ll r_{o2}$ and $R_{id} \gg R_s + R_F$, we have

$$R_{out} \simeq r_{o2}(1 + A_1 g_{m2} R_F) \qquad \text{Q.E.D.}$$

Ex. 11.16 To obtain

$$A_f \equiv \frac{I_o}{V_s} \simeq 100 \text{ mA/V}$$

when the loop gain is large, we use

$$\beta \simeq \frac{1}{A_f} = 10\ \Omega$$

But,

$$\beta = R_{E1} \times \frac{R_{E2}}{R_{E2}+R_F+R_{E1}}$$

For $R_{E1} = R_{E2} = 100\ \Omega$,

$$10 = \frac{100 \times 100}{200 + R_F}$$

$$\Rightarrow R_F = 800\ \Omega$$

$$\frac{V_o}{V_s} = \frac{-I_o R_{C1}}{V_s}$$

$$= -A_f R_{C1} = -100 \times 0.6 = -60 \text{ V/V}$$

Ex. 11.17 See figure on next page. Figure 1 shows the circuit prepared for the determination of the loop gain.

$$A\beta \equiv -\frac{V_r}{V_t}$$

This figure belongs to Exercise 11.17.

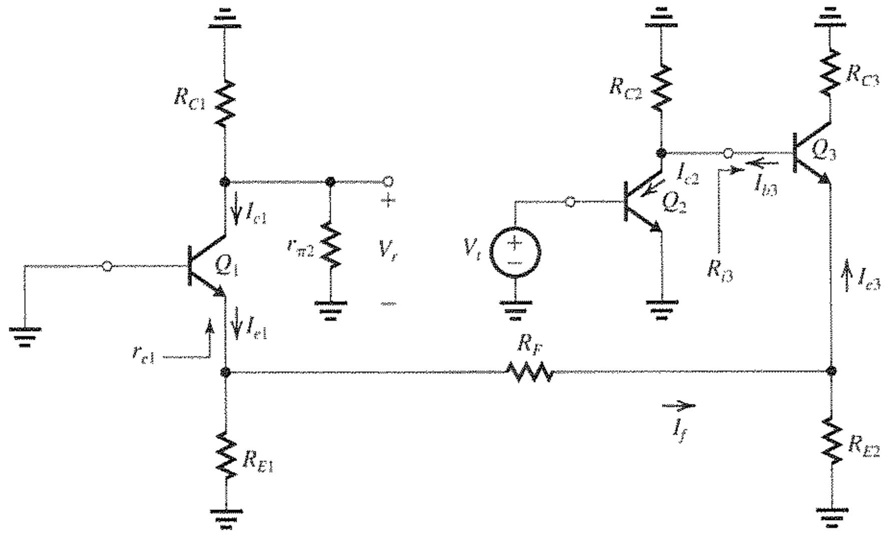


Figure 1

We shall trace the signal around the loop as follows:

$$I_{c2} = g_{m2} V_t \quad (1)$$

$$I_{b3} = I_{c2} \frac{R_{C2}}{R_{C2} + R_{i3}} \quad (2)$$

where

$$R_{i3} = (\beta + 1)\{r_{e3} + [R_{E2} \parallel (R_F + (R_{E1} \parallel r_{e1}))]\} \quad (3)$$

$$I_{e3} = (\beta + 1) I_{b3} \quad (4)$$

$$I_f = I_{e3} \frac{R_{E2}}{R_{E2} + R_F + (R_{E1} \parallel r_{e1})} \quad (5)$$

$$I_{e1} = I_f \frac{R_{E1}}{R_{E1} + r_{e1}} \quad (6)$$

$$I_{c1} = \alpha I_{e1} \quad (7)$$

$$V_r = -I_{c1}(R_{C1} \parallel r_{\pi 2}) \quad (8)$$

Combining (1)–(7) gives V_r in terms of V_t and hence $A\beta \equiv -V_r/V_t$. We shall do this numerically using the values in Example 11.8:

$g_{m2} = 40$ mA/V, $R_{C2} = 5$ kΩ, $\beta = 100$,

$r_{e3} = 6.25$ Ω, $R_{E1} = R_{E2} = 100$ Ω, $R_F = 640$ Ω,

$r_{e1} = 41.7$ Ω, $\alpha_1 = 0.99$, $R_{C1} =$ 9 kΩ, and $r_{\pi 2} = 2.5$ kΩ

$$R_{i3} = 101\{0.00625 + [0.1 \parallel (0.64 + (0.1 \parallel 0.0417))]\}$$

$$= 9.42 \text{ k}\Omega$$

$$I_{c2} = 40 V_t \quad (9)$$

$$I_{b3} = 0.347 I_{c2} \quad (10)$$

$$I_{e3} = 101 I_{b3} \quad (11)$$

$$I_f = 0.13 I_{e3} \quad (12)$$

$$I_{e1} = 0.706 I_f \quad (13)$$

$$I_{c1} = 0.99 I_{e1} \quad (14)$$

$$V_r = -1.957 I_{c1} \quad (15)$$

Combining (9)–(15), we obtain

$$A\beta = -\frac{V_r}{V_t} = 249.3$$

Ex. 11.18

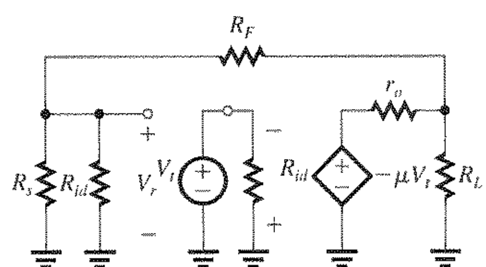


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain

$$A\beta \equiv -\frac{V_r}{V_t}$$

Using the voltage-divider rule, we can write by inspection

$$V_r = -\mu V_t \frac{R_L \parallel [R_F + (R_s \parallel R_{id})]}{r_o + \{R_L \parallel [R_F + (R_s \parallel R_{id})]\}} \frac{(R_s \parallel R_{id})}{R_F + (R_s \parallel R_{id})}$$

$V_r =$

$$-\mu V_t \frac{R_L(R_s \parallel R_{id})}{r_o[R_L + R_F + (R_s \parallel R_{id})] + R_L[R_F + (R_s \parallel R_{id})]}$$

Thus,

$$A\beta = -\frac{V_r}{V_t} =$$

$$\frac{\mu R_L(R_{id} \parallel R_s)}{r_o[R_L + R_F + (R_{id} \parallel R_s)] + R_L[R_F + (R_{id} \parallel R_s)]}$$

Q.E.D.

Using the numerical values in Example 11.9, we get

$$A\beta = \frac{10^4 \times 1 \times 1}{0.1(1 + 10 + 1) + 1(10 + 1)}$$

$= 819.7$

Ex. 11.19 See figure on next page. Figure 1(a) shows the feedback amplifier circuit. The β circuit is shown in Fig. 1(b), and the determination of β is shown in Fig. 1(c),

$$\beta = -\frac{1}{R_F}$$

(a) For large loop gain, we have

$$A_f \simeq \frac{1}{\beta} = -R_F$$

(b) The determination of R_{11} and R_{22} is illustrated in Figs. 1(d) and (e), respectively:

$R_{11} = R_{22} = R_F$

Finally, the A circuit is shown in Fig. 1(f). We can write by inspection

$R_i = R_s \parallel R_{11} = R_s \parallel R_F$

$R_o = r_o \parallel R_{22} = r_o \parallel R_F$

$V_{gs} = I_i R_i$

$V_o = -g_m V_{gs}(r_o \parallel R_{22})$

Thus,

$$A \equiv \frac{V_o}{I_i} = -g_m(R_s \parallel R_F)(r_o \parallel R_F)$$

$$A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$A_f =$

$$-\frac{g_m(R_s \parallel R_F)(r_o \parallel R_F)}{1 + g_m(R_s \parallel R_F)(r_o \parallel R_F)/R_F} \quad \text{Q.E.D.}$$

(c) $R_{if} = \dfrac{R_i}{1 + A\beta}$

$$R_{if} = \frac{R_s \parallel R_F}{1 + g_m(R_s \parallel R_F)(r_o \parallel R_F)/R_F}$$

$$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_F} + \frac{g_m(r_o \parallel R_F)}{R_F}$$

But,

$$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_{in}}$$

thus,

$$\frac{1}{R_{in}} = \frac{1}{R_F}[1 + g_m(r_o \parallel R_F)]$$

$$\Rightarrow R_{in} = \frac{R_F}{1 + g_m(r_o \parallel R_F)} \quad \text{Q.E.D.}$$

(d) $R_{out} = R_{of} = \dfrac{R_o}{1 + A\beta}$

$$= \frac{r_o \parallel R_F}{1 + g_m(R_s \parallel R_F)(r_o \parallel R_F)/R_F}$$

$$\frac{1}{R_{out}} = \frac{1}{r_o} + \frac{1}{R_F} + \frac{g_m(R_s \parallel R_F)}{R_F}$$

$$\Rightarrow R_{out} = r_o \parallel \frac{R_F}{1 + g_m(R_s \parallel R_F)} \quad \text{Q.E.D.}$$

(e) $A = -5(1 \parallel 10)(20 \parallel 10)$

$A = -30.3\ \text{k}\Omega$

$$\beta = -\frac{1}{R_F} = -\frac{1}{10} = -0.1\ \text{mA/V}$$

$A\beta = 3.03$

$1 + A\beta = 4.03$

$$A_f = \frac{A}{1 + A\beta} = -\frac{30.3}{4.03} = -7.52\ \text{k}\Omega$$

(Compare to the ideal value of $-10\ \text{k}\Omega$).

$R_i = R_s \parallel R_F = 1 \parallel 10 = 909\ \Omega$

$R_o = r_o \parallel R_F = 20 \parallel 10 = 6.67\ \text{k}\Omega$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{909}{4.03} = 226\ \Omega$$

$$R_{in} = 1 \bigg/ \left[\frac{1}{R_{if}} - \frac{1}{R_s}\right] = 291\ \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{6.67}{4.03} = 1.66\ \text{k}\Omega$$

$R_{out} = R_{of} = 1.66\ \text{k}\Omega$

Ex. 11.20 From Eq. (11.54), we obtain

$$A = -\mu \frac{R_i}{R_1 \parallel R_2} \frac{R_1 \parallel R_2 \parallel r_{o2}}{1/g_m + (R_1 \parallel R_2 \parallel r_{o2})}$$

For $\mu = 100$, $R_1 = 10\ \text{k}\Omega$, $R_2 = 90\ \text{k}\Omega$, $g_m = 5\ \text{mA/V}$, $r_{o2} = 20\ \text{k}\Omega$, we have

$R_i = R_s \parallel R_{id} \parallel (R_1 + R_2)$

$= \infty \parallel \infty \parallel 100 = 100\ \text{k}\Omega$

$$A = -100 \frac{100}{10 \parallel 90} \frac{10 \parallel 90 \parallel 20}{0.2 + (10 \parallel 90 \parallel 20)}$$

$= -1076.4$ A/A

$$\beta = \frac{R_1}{R_1 + R_2} = \frac{10}{10 + 90} = 0.1\ \text{A/A}$$

$$A_f = -\frac{1076.4}{1 + 107.64} = -9.91\ \text{A/A}$$

This figure belongs to Exercise 11.19.

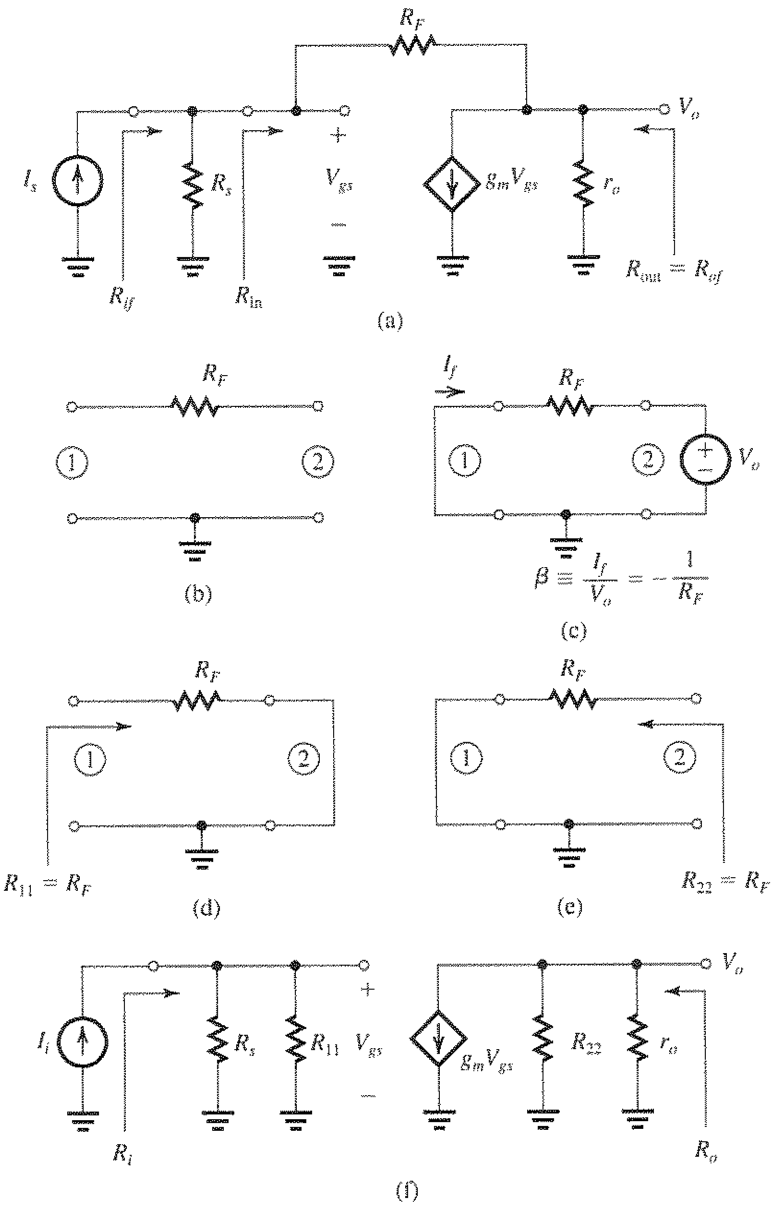


Figure 1

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{100\text{ k}\Omega}{108.64} = 920\ \Omega$$

$R_{in} = R_{if} = 920\ \Omega$

$R_{out} = R_{of} = R_o(1 + A\beta)$

$R_o = r_{o2} + (R_1 \parallel R_2) + g_m r_{o2}(R_1 \parallel R_2)$

$= 929\text{ k}\Omega$

$R_{out} = 929 \times 108.64 = 101\text{ M}\Omega$

Ex. 11.21 With $R_2 = 0$, Eq. (11.48) gives

$\beta = -1$

$A_f = -1$ A/A

Substituting $R_2 = 0$ and $R_s = R_{id} = \infty$ in Eq. (11.50), we obtain

$R_i = R_1$

and in Eq. (11.55), we obtain

$R_o = r_{o2}$

and in Eq. (11.53), we obtain

$$A = -\mu\frac{R_1}{1/g_m} = -\mu g_m R_1$$

Now,

$$A_f = \frac{A}{1 + A\beta}$$

$$A_f = -\frac{\mu g_m R_1}{1 + \mu g_m R_1}$$

$$R_{in} = R_{if} = R_i/(1 + A\beta)$$

$$= \frac{R_1}{1 + \mu g_m R_1}$$

For $\mu g_m R_1 \gg 1$, we have

$$R_{in} \simeq 1/\mu g_m$$

$$R_{out} = R_{of} = (1 + A\beta)R_o$$

$$= (1 + \mu g_m R_1)r_{o2}$$

$$\simeq \mu(g_m r_{o2})R_1$$

Ex. 11.22 Total phase shift will be 180° at the frequency ω_{180} at which the phase shift of each amplifier stage is 60°. Thus,

$$\tan^{-1}\frac{\omega_{180}}{10^4} = 60°$$

$$\omega_{180} = \tan 60° \times 10^4$$

$$= \sqrt{3} \times 10^4 \text{ rad/s}$$

At ω_{180}, we have

$$|A| = \left(\frac{10}{\sqrt{1+3}}\right)^3$$

$$= 125$$

Thus, the loop gain magnitude will be

$$|A\beta| = 125\beta$$

For stable operation, we require

$$125\beta_{cr} < 1$$

$$\Rightarrow \beta_{cr} = \frac{1}{125} = 0.008$$

$\beta \geq \beta_{cr}$ will result in oscillations. Correspondingly, the minimum closed-loop gain for stable operation will be

$$A_f = \frac{10^3}{1 + 10^3\,\beta_{cr}}$$

$$= \frac{10^3}{1 + 1000 \times 0.008} = \frac{1000}{9} = 111.1$$

Ex. 11.23 The feedback shifts the pole by a factor equal to the amount of feedback:

$$1 + A_0\beta = 1 + 10^5 \times 0.01 = 1001$$

The pole will be shifted to a frequency

$$f_{Pf} = f_P(1 + A_0\beta)$$

$$= 100 \times 1001 = 100.1 \text{ kHz}$$

If β is changed to a value that results in a nominal closed-loop gain of 1, then we obtain

$$\beta \simeq 1$$

and

$$1 + A_0\beta = 1 + 10^5 \times 1 \simeq 10^5$$

then the pole will be shifted to a frequency

$$f_{Pf} = 10^5 \times 100 = 10 \text{ MHz}$$

Ex. 11.24 From Eq. (11.68), we see that the poles coincide when

$$(\omega_{P1} + \omega_{P2})^2 = 4(1 + A_0\beta)\omega_{P1}\omega_{P2}$$

$$(10^4 + 10^6)^2 = 4(1 + 100\beta) \times 10^4 \times 10^6$$

$$\Rightarrow 1 + 100\beta = 25.5$$

$$\Rightarrow \beta = 0.245$$

The corresponding value of $Q = 0.5$. This can also be verified by substituting in Eq. (11.70).

A maximally flat response is obtained when $Q = 1/\sqrt{2}$. Substituting in Eq. (11.70), we obtain

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{(1 + 100\beta) \times 10^4 \times 10^6}}{10^4 + 10^6}$$

$$\Rightarrow \beta = 0.5$$

In this case, the low-frequency closed-loop gain is

$$A_f(0) = \frac{A_0}{1 + A_0\beta}$$

$$= \frac{100}{1 + 100 \times 0.5} = 1.96 \text{ V/V}$$

Ex. 11.25 The closed-loop poles are the roots of the characteristic equation

$$1 + A(s)\beta = 0$$

$$1 + \left(\frac{10}{1 + \frac{s}{10^4}}\right)^3\beta = 0$$

To simplify matters, we normalize s by the factor 10^4, thus obtaining the normalized complex-frequency variable $S = s/10^4$, and the characteristic equation becomes

$$(S+1)^3 + 10^3\beta = 0 \tag{1}$$

This equation has three roots, a real one and a pair that can be complex conjugate. The real pole can be found from

$$(S+1)^3 = -10^3\beta$$

$$\Rightarrow S = -1 - 10\beta^{1/3} = -\left(1 + 10\beta^{1/3}\right) \tag{2}$$

Dividing the characteristic polynomial in (1) by $\left(S + 1 + 10\beta^{1/3}\right)$ gives a quadratic whose two roots are the remaining poles of the feedback amplifier. After some straightforward but somewhat tedious algebra, we obtain

$$S^2 + \left(10\beta^{1/3} - 2\right)S + \left(1 + 100\beta^{2/3} - 10\beta^{1/3}\right)$$

$$= 0 \tag{3}$$

The pair of poles can now be obtained as

$$S = \left(-1 + 5\beta^{1/3}\right) \pm j5\sqrt{3}\,\beta^{1/3} \tag{4}$$

Equations (1) and (3) describe the three poles shown in Fig. E11.25.

From Eq. (2) we see that the pair of complex poles lie on the $j\omega$ axis for the value of β that makes the coefficient of S equal to zero, thus

$$\beta_{cr} = \left(\frac{2}{10}\right)^3 = 0.008$$

Note that this is the same value found in the solution of Exercise 11.22.

Ex. 11.26

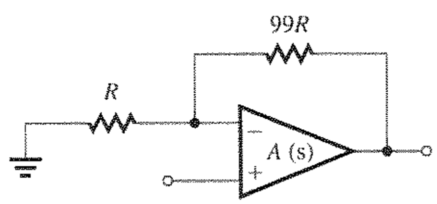


Figure 1

From Fig. 1, we can easily obtain the loop gain as

$$A\beta = A(s) \times 0.01$$

$$= \frac{10^5}{1 + \dfrac{s}{2\pi \times 10}} \times 0.01$$

$$= \frac{1000}{1 + \dfrac{s}{2\pi \times 10}}$$

From this single-pole response (low-pass STC response) we can find the unity-gain frequency by inspection as

$$f_1 = f_P \times 1000$$

$$= 10^4 \text{ Hz}$$

The phase angle at f_1 will be -90° and thus the phase margin is 90°.

Ex. 11.27 From Eq. (11.82), we obtain

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + e^{-j\theta}|$$

$$= 1/|1 + \cos\theta - j\sin\theta|$$

(a) For PM $= 30^\circ$, $\theta = 180 - 30 = 150^\circ$, thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 150^\circ - j\sin 150^\circ|$$

$$= 1.93$$

(b) For PM $= 60^\circ$, $\theta = 180 - 60 = 120^\circ$, thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 120^\circ - j\sin 120^\circ|$$

$$= 1$$

(c) For PM $= 90^\circ$, $\theta = 180 - 90 = 90^\circ$, thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 90^\circ - j\sin 90^\circ|$$

$$= 1/\sqrt{2} = 0.707$$

Ex. 11.28 See figure on next page. To obtain guaranteed stable performance, the maximum rate of closure must not exceed 20 dB/decade. Thus we utilize the graphical construction in Fig. 1 to obtain the

This figure belongs to Exercise 11.28.

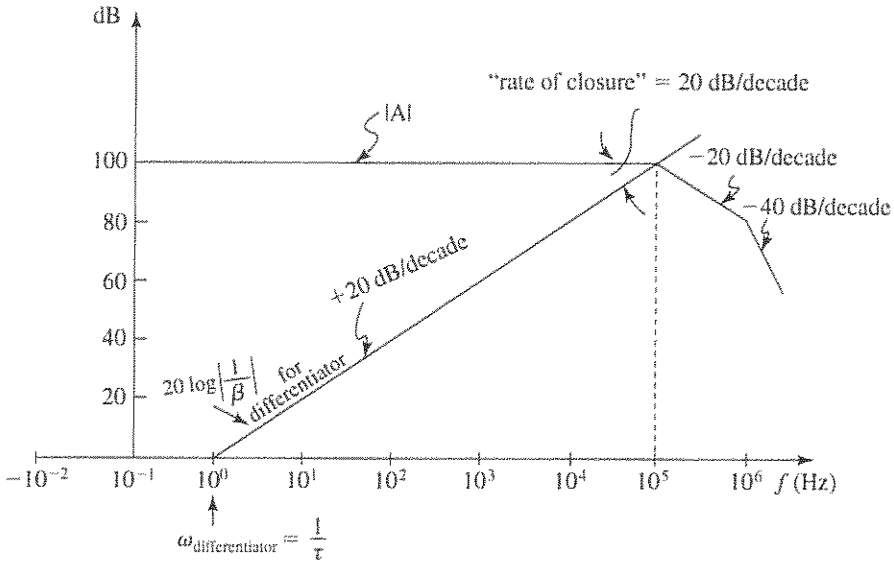


Figure 1

maximum value of the differentiator frequency as 1 Hz. Thus,

$$\frac{1}{\tau} \leq 2\pi \times 1 \text{ Hz}$$

$$\tau \geq \frac{1}{2\pi}\text{s} = 159 \text{ ms}$$

Ex. 11.29 To obtain stable performance for closed-loop gains as low as 20 dB (which is 80 dB below A_0, or equivalently 10^4 below A_0), we must place the new dominant pole at 1 MHz/10^4 = 100 Hz.

Ex. 11.30 The frequency of the first pole must be lowered from 1 MHz to a new frequency

$$f_D' = \frac{10 \text{ MHz}}{10^4} = 1000 \text{ Hz}$$

that is, by a factor of 1000. Thus, the capacitance at the controlling node must be increased by a factor of 1000.

Ex: 12.1 To allow for v_O to reach $-V_{CC} + V_{CEsat} = -15 + 0.2 = -14.8$ V, with Q_1 just cutting off (i.e. $i_{E1} = 0$),

$$I = \frac{14.8 \text{ V}}{R_L} = \frac{14.8}{1 \text{ k}\Omega} = 14.8 \text{ mA}$$

The value of R can now be found from

$$I = \frac{V_R}{R} = \frac{V_{CC} - V_D}{R}$$

$$14.8 = \frac{15 - 0.7}{R}$$

$$\Rightarrow R = \frac{14.3}{14.8} = 0.97 \text{ k}\Omega$$

The resulting output signal swing will be -14.8 V to $+14.8$ V. The minimum current in $Q_1 = 0$. The maximum current in $Q_1 = 14.8 + 14.8 = 29.6$ mA

Ex: 12.2 At $v_O = -10$ V, we have

$$i_L = \frac{-10}{1} = -10 \text{ mA}$$

$$i_{E1} = I + i_L = 14.8 - 10 = 4.8 \text{ mA}$$

$$v_{BE1} = 0.6 + 0.025 \ln\left(\frac{4.8}{1}\right)$$

$$= 0.64 \text{ V}$$

$$v_I = v_O + v_{BE1}$$

$$= -10 + 0.64 = -9.36 \text{ V}$$

At $v_O = 0$ V, we have

$$i_L = 0 \text{ mA}$$

$$i_{E1} = I = 14.8 \text{ mA}$$

$$v_{BE1} = 0.6 + 0.025 \ln\left(\frac{14.8}{1}\right)$$

$$= 0.67 \text{ V}$$

$$v_I = v_O + v_{BE1}$$

$$= 0 + 0.67 = 0.67 \text{ V}$$

At $v_O = 10$ V

$$i_L = \frac{10}{1} = 10 \text{ mA}$$

$$i_{E1} = I + i_L = 14.8 + 10 = 24.8 \text{ mA}$$

$$v_{BE1} = 0.6 + 0.025 \ln\left(\frac{24.8}{1}\right)$$

$$= 0.68 \text{ V}$$

$$v_I = v_O + v_{BE1}$$

$$= 10 + 0.68 = 10.68 \text{ V}$$

At $v_O = -10$ V, we have

$$i_{E1} = 4.8 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{4.8 \text{ mA}} = 5.2 \ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_{e1}} = \frac{1}{1 + 0.0052} = 0.995 \text{ V/V}$$

At $v_O = 0$ V,

$$i_{E1} = 14.8 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{14.8 \text{ mA}} = 1.7 \ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_{e1}} = \frac{1}{1 + 0.0017} = 0.998 \text{ V/V}$$

At $v_O = +10$ V,

$$i_{E1} = 24.8 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{24.8 \text{ mA}} = 1.0 \ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_{e1}} = \frac{1}{1 + 0.001} = 0.999 \text{ V/V}$$

Ex: 12.3

a. $P_L = \dfrac{\left(\hat{V}_o/\sqrt{2}\right)^2}{R_L} = \dfrac{\left(8/\sqrt{2}\right)^2}{100} = 0.32$ W

$$P_S = 2V_{CC} \times I = 2 \times 10 \times 100 \times 10^{-3}$$

$$= 2 \text{ W}$$

$$\text{Efficiency } \eta = \frac{P_L}{P_S} \times 100$$

$$= \frac{0.32}{2} \times 100$$

$$= 16\%$$

Ex: 12.4 (a) $P_L = \dfrac{1}{2}\dfrac{\hat{V}_o^2}{R_L}$

$$= \frac{1}{2}\frac{(4.5)^2}{4} = 2.53 \text{ W}$$

(b) $P_{S+} = P_{S-} = V_{CC} \times \dfrac{1}{\pi}\dfrac{\hat{V}_o}{R_L}$

$$= 6 \times \frac{1}{\pi} \times \frac{4.5}{4} = 2.15 \text{ W}$$

(c) $\eta = \dfrac{P_L}{P_S} \times 100 = \dfrac{2.53}{2 \times 2.15} \times 100$

$$= 59\%$$

(d) Peak input currents $= \dfrac{1}{\beta + 1}\dfrac{\hat{V}_o}{R_L}$

$$= \frac{1}{51} \times \frac{4.5}{4}$$

$$= 22.1 \text{ mA}$$

(e) Using Eq. (12.22), we obtain

$$P_{DN\max} = P_{DP\max} = \frac{V_{CC}^2}{\pi^2 R_L}$$

$$= \frac{6^2}{\pi^2 \times 4} = 0.91 \text{ W}$$

Ex: 12.5 (a) The quiescent power dissipated in each transistor $= I_Q \times V_{CC}$

Total power dissipated in the two transistors

$= 2I_Q \times V_{CC}$

$= 2 \times 2 \times 10^{-3} \times 15$

$= 60$ mW

(b) I_Q is increased to 10 mA

At $v_O = 0$, we have $i_N = i_P = 10$ mA

From Eq. (12.31), we obtain

$$R_{\text{out}} = \frac{V_T}{i_P + i_N} = \frac{25}{10+10} = 1.25\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 1.25}$$

$$\frac{v_o}{v_i} = 0.988 \text{ at } v_O = 0 \text{ V}$$

At $v_O = 10$ V, we have

$$i_L = \frac{10\text{ V}}{100\ \Omega} = 0.1\text{ A} = 100\text{ mA}$$

Use Eq. (12.27) to calculate i_N:

$$i_N^2 - i_N i_L - I_Q^2 = 0$$

$$i_N^2 - 100\, i_N - 10^2 = 0$$

$$\Rightarrow i_N = 101.0\ \text{mA}$$

Using Eq. (12.26), we obtain

$$i_P = \frac{I_Q^2}{i_N} \simeq 1\text{ mA}$$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{101.0 + 1} \simeq 0.2451\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 0.2451} \simeq 1$$

$$\%\text{ change} = \frac{1 - 0.988}{1} \times 100 = 1.2\%$$

In Example 12.3, $I_Q = 2$ mA, and for $v_O = 0$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{2+2} = 6.25\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 6.25} = 0.94$$

$v_O = 10$ V

$$i_L = \frac{10\text{ V}}{100\ \Omega} = 100\text{ mA}$$

Again calculate i_N (for $I_Q = 2$ mA) using Eq. (12.27) ($i_N = 100.04$ mA):

$$i_P = \frac{I_Q^2}{I_N} = \frac{2^2}{100.04} = 0.04\text{ mA}$$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{100.04 + 0.04} = 0.25\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} \simeq 1$$

$$\%\text{ Change} = \frac{1 - 0.94}{1} \times 100 = 6\%$$

For $I_Q = 10$ mA, change is 1.2%

For $I_Q = 2$ mA, change is 6%

(c) The quiesent power dissipated in each transistor $= I_Q \times V_{CC}$

Total power dissipated $= 2 \times 10 \times 10^{-3} \times 15$

$= 300$ mW

Ex: 12.6 From Example 12.4, we have $V_{CC} = 15$ V,
$R_L = 100\ \Omega$

Q_N and Q_P matched and $I_S = 10^{-13}$ A and $\beta = 50$, $I_{\text{Bias}} = 3$ mA

For $v_O = 10$ V, we have $i_L = \dfrac{10}{100} = 0.1$ A

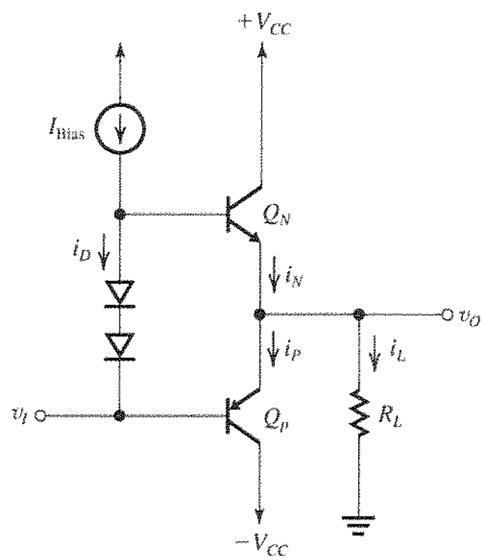
As a first approximation, $i_N \simeq 0.1$ A,

$$i_P = 0,\ i_{BN} \simeq \frac{0.1\text{ A}}{50+1} \simeq 2\text{ mA}$$

$$i_D = I_{\text{Bias}} - i_{BN} = 3 - 2 = 1\text{ mA}$$

$$V_{BB} = 2V_T \ln\left(\frac{10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) \qquad (1)$$

This $\frac{1}{3}$ is because biasing diodes have $\frac{1}{3}$ area of the output devices.



But $V_{BB} = V_{BEN} + V_{BEP}$

$$= V_T \ln\left(\frac{i_N}{I_S}\right) + V_T \ln\left(\frac{i_N - i_L}{I_S}\right)$$

$$= V_T \ln\left[\frac{i_N(i_N - i_L)}{I_S^2}\right] \quad (2)$$

Equating Eqs. 1 and 2, we obtain

$$2V_T \ln\left(\frac{10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) = V_T \ln\left[\frac{i_N(i_N - i_L)}{I_S^2}\right]$$

$$\left(\frac{10^{-3}}{\frac{1}{3} \times 10^{-13}}\right)^2 = \frac{i_N(i_N - 0.1)}{(10^{-13})^2}$$

$$i_N(i_N - 0.1) = 9 \times 10^{-6}$$

If i_N is in mA, then

$$i_N(i_N - 100) = 9$$

$$i_N^2 - 100\, i_N - 9 = 0$$

$$\Rightarrow i_N = 100.1 \text{ mA}$$

$$i_P = i_N - i_L = 0.1 \text{ mA}$$

For $v_O = -10$ V and $i_L = \frac{-10}{100} = -0.1$ A

$= -100$ mA:

As a first approximation assume $i_P \cong 100$ mA,

$i_N \simeq 0$. Since $i_N = 0$, current through diodes $= 3$ mA

$$\therefore V_{BB} = 2V_T \ln\left(\frac{3 \times 10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) \quad (3)$$

But $V_{BB} = V_T \ln\left(\frac{i_N}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right)$

$$= V_T \ln\left(\frac{i_P + i_L}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right) \quad (4)$$

Here $i_L = -0.1$ A

Equating Eqs. (3) and (4), we obtain

$$2V_T \ln\left(\frac{3 \times 10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) =$$

$$V_T \ln\left(\frac{i_P - 0.1}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right)$$

$$\left(\frac{3 \times 10^{-3}}{\frac{1}{3} \times 10^{-13}}\right)^2 = \frac{i_P(i_P - 0.1)}{(10^{-13})^2}$$

$$i_P(i_P - 0.1) = 81 \times 10^{-6}$$

Expressing currents in mA, we have

$$i_P(i_P - 100) = 81$$

$$i_P^2 - 100\, i_P - 81 = 0$$

$$\Rightarrow i_P = 100.8 \text{ mA}$$

$$i_N = i_P + i_L = 0.8 \text{ mA}$$

Ex: 12.7 $\Delta I_C = g_m \times 2 \text{ mV/°C} \times 5\text{ °C}$, mA

where g_m is in mA / mV

$$g_m = \frac{10 \text{ mA}}{25 \text{ mV}} = 0.4 \text{ mA/mV}$$

Thus, $\Delta I_C = 0.4 \times 2 \times 5 = 4$ mA

Ex: 12.8 Refer to Fig. 12.15.

(a) To obtain a terminal voltage of 1.2 V, and since β_1 is very large, it follows that $V_{R1} = V_{R2} = 0.6$ V.

Thus $I_{C1} = 1$ mA

$$I_R = \frac{1.2 \text{ V}}{R_1 + R_2} = \frac{1.2}{2.4} = 0.5 \text{ mA}$$

Thus, $I = I_{C1} + I_R = 1.5$ mA

(b) For $\Delta V_{BB} = +50$ mV:

$$V_{BB} = 1.25 \text{ V} \quad I_R = \frac{1.25}{2.4} = 0.52 \text{ mA}$$

$$V_{BE} = \frac{1.25}{2} = 0.625 \text{ V}$$

$$I_{C1} = 1 \times e^{\Delta V_{BE}/V_T} = e^{0.025/0.025}$$

$= 2.72$ mA

$$I = 2.72 + 0.52 = 3.24 \text{ mA}$$

For $\Delta V_{BB} = +100$ mV, we have

$$V_{BB} = 1.3 \text{ V}, \quad I_R = \frac{1.3}{2.4} = 0.54 \text{ mA}$$

$$V_{BE} = \frac{1.3}{2} = 0.65 \text{ V}$$

$$I_{C1} = 1 \times e^{\Delta V_{BE}/V_T} = 1 \times e^{0.05/0.025}$$

$= 7.39$ mA

$$I = 7.39 + 0.54 = 7.93 \text{ mA}$$

For $\Delta V_{BB} = +200$ mV:

$$V_{BB} = 1.4 \text{ V}, \quad I_R = \frac{1.4}{2.4} = 0.58 \text{ mA}$$

$$V_{BE} = 0.7 \text{ V}$$

$$I_{C1} = 1 \times e^{0.1/0.025} = 54.60 \text{ mA}$$

$$I = 54.60 + 0.58 = 55.18 \text{ mA}$$

For $\Delta V_{BB} = -50$ mV:

$V_{BB} = 1.15$ V, $\quad I_R = \dfrac{1.15}{2.4} = 0.48$ mA

$V_{BE} = \dfrac{1.15}{2}$

$= 0.575$

$I_{C1} = 1 \times e^{-0.025/0.025} = 0.37$ mA

$I = 0.48 + 0.37 = 0.85$ mA

For $\Delta V_{BB} = -100$ mV:

$V_{BB} = 1.1$ V $\quad I_R = \dfrac{1.1}{2.4} = 0.46$ mA

$V_{BE} = 0.55$ V

$I_{C1} = 1 \times e^{-0.05/0.025} = 0.13$ mA

$I = 0.46 + 0.13 = 0.59$ mA

For $\Delta V_{BB} = -200$ mV:

$V_{BB} = 1.0$ V $\; I_R = \dfrac{1}{2.4} = 0.417$ mA

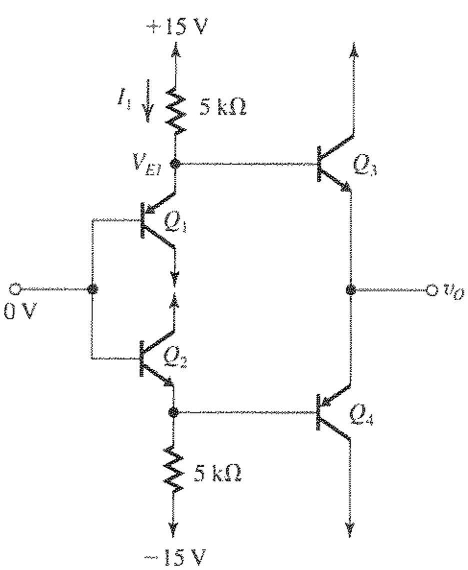
$V_{BE} = 0.5$ V

$I_{C1} = 1 \times e^{-0.1/0.025} = 0.018$ mA

$I = 0.43$ mA

Ex: 12.9 (a) From symmetry we see that all transistors will conduct equal currents and have equal V_{BE}'s. Thus,

$v_O = 0$ V



If $V_{BE} \simeq 0.7$ V, then

$V_{E1} = 0.7$ V and $I_1 = \dfrac{15 - 0.7}{5} = 2.86$ mA

If we neglect I_{B3}, then

$I_{C1} \simeq 2.86$ mA

At this current, $|V_{BE}|$ is given by

$$|V_{BE}| = 0.025 \ln\left(\frac{2.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right) \simeq 0.63 \text{ V}$$

Thus $V_{E1} = 0.63$ V and $I_1 = 2.87$ mA

No more iterations are required and

$i_{C1} = i_{C2} = i_{C3} = i_{C4} \simeq 2.87$ mA

(b) For $v_I = +10$ V:

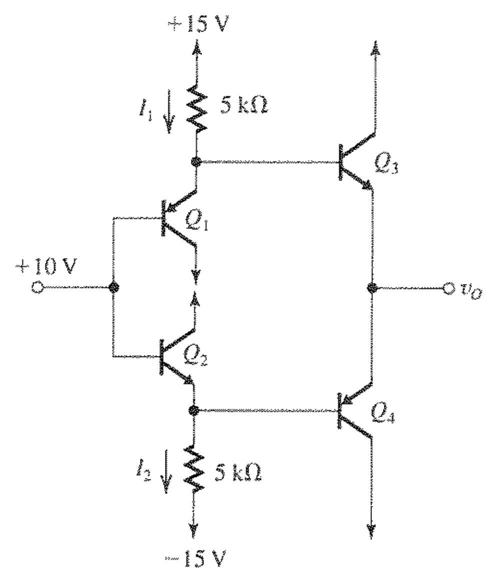
To start the iterations, let $V_{BE1} \simeq 0.7$ V

Thus,

$V_{E1} = 10.7$ V

and

$I_1 = \dfrac{15 - 10.7}{5} = 0.86$ mA



Neglecting I_{B3}, we obtain

$I_{C1} \simeq I_{E1} \simeq I_1 = 0.86$ mA

But at this current

$$|V_{BE1}| = V_T \ln\left(\frac{I_{C1}}{I_S}\right)$$

$$= 0.025 \ln\left(\frac{0.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$

$= 0.6$ V

Thus, $V_{E1} = +10.6$ V and $I_1 = 0.88$ mA. No further iterations are required and $I_{C1} \simeq 0.88$ mA.

To find I_{C2}, we use an identical procedure:

$V_{BE2} \simeq 0.7$ V

$V_{E2} = 10 - 0.7 = +9.3$ V

$I_2 = \dfrac{9.3 - (-15)}{5} = 4.86$ mA

$V_{BE2} = 0.025 \ln\left(\frac{4.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$

$= 0.643$ V

$V_{E2} = 10 - .643 = +9.357$

$I_2 = 4.87$ mA

$I_{C2} \simeq 4.87$ mA

Finally,

$I_{C3} = I_{C4} = 3.3 \times 10^{-14}\, e^{V_{BE}/V_T}$

where

$V_{BE} = \frac{V_{E1} - V_{E2}}{2} = 0.62$ V

Thus, $I_{C3} = I_{C4} = 1.95$ mA

The symmetry of the circuit enables us to find the values for $v_I = -10$ V as follows:

$I_{C1} = 4.87$ mA $I_{C2} = 0.88$ mA

$I_{C3} = I_{C4} = 1.95$ mA

For $v_I = +10$ V, we have $v_O = V_{E1} - V_{BE3}$

$= 10.6 - 0.62 = +9.98$ V

For $v_I = -10$ V, we have $v_O = V_{E1} - V_{BE3}$

$= -9.357 - 0.62 = -9.98$ V

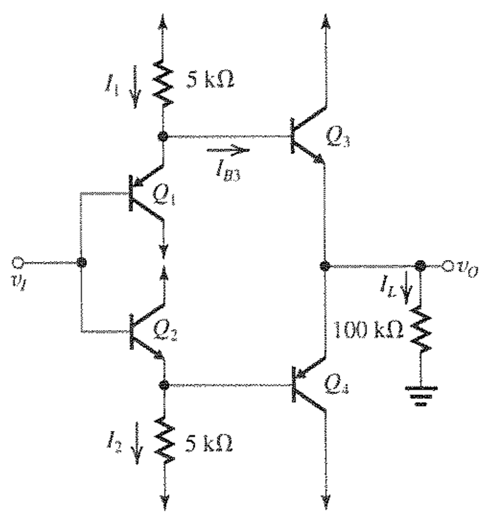
(c) For $v_I = +10$ V, we have

$v_O \simeq 10$ V

$I_L \simeq 100$ mA

$I_{C3} \simeq 100$ mA

$I_{B3} = \frac{100}{201} \simeq 0.5$ mA



Assuming that $|V_{BE1}|$ has not changed much from 0.6 V, then

$V_{E1} \simeq 10.6$ V

$I_1 = \frac{15 - 10.6}{5} = 0.88$ mA

$I_{E1} = I_1 - I_{B3} = 0.88 - 0.5 = 0.38$ mA

$I_{C1} \simeq 0.38$ mA

$|V_{BE1}| = 0.025 \ln\left(\frac{0.38 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$

$= 0.58$ V

$V_{E1} = 10.58$ V

$I_1 = \frac{15 - 10.58}{5} = 0.88$ mA

Thus, $I_{C1} \simeq 0.38$ mA

Now for Q_2 we have

$V_{BE2} = 0.643$ V

$V_{E2} = 10 - 0.643 = 9.357$

$I_2 = 4.87$ mA

$I_{B4} \simeq 0$

$I_{C2} \simeq 4.87$ mA (as in (b))

Assuming that $I_{C3} \simeq 100$ mA, we have

$V_{BE3} = 0.025 \ln\left(\frac{100 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$

$= 0.72$ V

Thus, $v_O = V_{E1} - V_{BE3}$

$= 10.58 - 0.72 = +9.86$ V

$|V_{BE4}| = v_O - V_{E2}$

$9.86 - 9.36 = 0.5$ V

Thus, $I_{C4} = 3.3 \times 10^{-14} e^{0.5/0.025}$

$\simeq 0.02$ mA

From symmetry we find the values for the case

$v_I = -10$ V as:

$I_{C1} = 4.87$ mA, $I_{C2} = 0.38$ mA

$I_{C3} = 0.02$ mA, $I_{C4} = 100$ mA

$v_O = -9.86$ V.

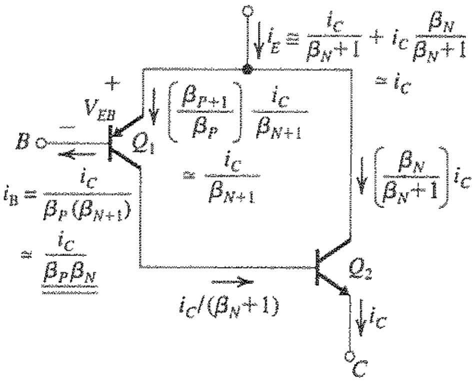
Ex: 12.10 For Q_1:

$i_{C1} = I_{SP} e^{v_{EB}/V_T}$

$\frac{i_C}{\beta_N + 1} = I_{SP} e^{v_{EB}/V_T}$

$i_C \simeq \beta_N I_{SP} e^{v_{EB}/V_T}$

Thus, effective scale current $= \beta_N I_{SP}$



(b) Effective current gain $\cong \frac{i_C}{i_B} = \beta_P\beta_N$

$= 20 \times 50 = 1000$

$100 \times 10^{-3} = 50 \times 10^{-14}\, e^{v_{EB}/0.025}$

$v_{EB} = 0.025 \ln(2 \times 10^{11})$

$= 0.651$ V

Ex: 12.11 See Figure 12.21

When $V_{BE5} = 150 \times 10^{-3} \times R_{E1}$, then $I_{CS} = I_{\text{Bias}}$

$= 2$ mA

$$V_{BE5} = V_T \ln\left(\frac{I_{C5}}{I_S}\right)$$

$$= 25 \times 10^{-3} \ln\left(\frac{2 \times 10^{-3}}{10^{-14}}\right)$$

$= 0.651$ V

$150 \times 10^{-3} R_{E1} = 0.651$

$R_{E1} = 4.34\ \Omega$

If peak output current $= 100$ mA

$V_{BE5} = R_{E1} \times 100\text{ mA} = 4.34 \times 100 \times 10^{-3}$

$= 0.434$ V

$i_{C5} = I_S e^{V_{BE5}/V_T}$

$= 10^{-14} e^{0.434/25\times10^{-3}}$

$\simeq 0.35\ \mu$A

Ex: 12.12 Using Eq. (12.43), we obtain

$$I_Q = I_{\text{Bias}} \frac{(W/L)_n}{(W/L)_1}$$

$$1 = 0.2\, \frac{(W/L)_n}{(W/L)_p}$$

$$\frac{(W/L)_n}{(W/L)_1} = 5$$

$$Q_1: I_{\text{Bias}} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_1 (V_{GS} - V_{tn})^2$$

$$0.2 = \frac{1}{2} \times 0.250 \left(\frac{W}{L}\right)_1 (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 40$$

$$Q_2: I_{\text{Bias}} = \frac{1}{2} k_p' \left(\frac{W}{L}\right)_2 (V_{GS} - |V_t|)^2$$

$$0.2 = \frac{1}{2} \times 0.100 \times \left(\frac{W}{L}\right)_2 \times (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 100$$

$$Q_N: I_Q = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_N (V_{GS} - V_t)^2$$

$$1 = \frac{1}{2} \times 0.250 \times \left(\frac{W}{L}\right)_n 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_n = 200$$

$$Q_P: I_Q = \frac{1}{2} k_p' \left(\frac{W}{L}\right)_p (V_{GS} - |V_t|)^2$$

$$1 = \frac{1}{2} \times 0.100 \times \left(\frac{W}{L}\right)_p \times 0.2^2$$

$$\left(\frac{W}{L}\right)_p = 500$$

Now $V_{GG} = V_{GS1} + V_{GS2}$

$= (V_{ov1} + V_t) + (V_{ov2} + |V_t|)$

$= (0.2 + 0.5) + (0.2 + 0.5)$

$= 1.4$ V

Ex: 12.13 $I_N = i_{L\max} = 10$ mA

$$\therefore\ 10 = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_n V_{OV}^2$$

$$10 = \frac{1}{2} \times 0.250 \times 200 \times V_{OV}^2$$

$\Rightarrow V_{OV} = 0.63$ V

Using equation 12.46, we obtain

$v_{O\max} = V_{DD} - V_{OV}|_{\text{Bias}} - V_{tn} - V_{OVN}$

$= 2.5 - 0.2 - 0.5 - 0.63$

$= 1.17$ V

Ex: 12.14 New values of W/L are

$$\left(\frac{W}{L}\right)_p = \frac{2000}{2} = 1000$$

$$\left(\frac{W}{L}\right)_N = \frac{800}{2} = 400$$

$$I_Q = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p V_{OV}^2$$

$$1 \times 10^{-3} = \frac{1}{2} \times 0.1 \times 10^{-3} \times 1000 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.14 \text{ V}$$

Gain error

$$= -\frac{V_{OV}}{4\mu I_Q R_L} = -\frac{0.14}{4 \times 10 \times 1 \times 10^{-3} \times 100}$$

$$= -0.035$$

Gain error $= -0.035 \times 100 = -3.5\%$

$$g_{mn} = g_{mp} = \frac{2I_Q}{V_{OV}} = \frac{2 \times 1 \times 10^{-3}}{0.14}$$

$$= 14.14 \text{ mA/V}$$

$$R_{\text{out}} = \frac{1}{\mu(g_{mp} + g_{mn})}$$

$$= \frac{1}{10 \times (14.14 + 14.14) \times 10^{-3}}$$

$$\simeq 3.5\ \Omega$$

Ex: 12.15 Total current into node $B = \dfrac{2v_i}{R_3} + \dfrac{v_o}{R_2}$

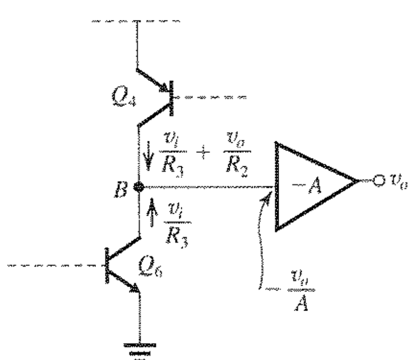
Thus

$$\left(\frac{2v_i}{R_3} + \frac{v_o}{R_2}\right)R = -\frac{v_o}{A}$$

$$\Rightarrow v_o\left(\frac{1}{A} + \frac{R}{R_2}\right) = -\frac{2R}{R_3}v_i$$

$$\frac{v_o}{v_i} = \frac{-\dfrac{2R}{R_3}}{\dfrac{1}{A} + \dfrac{R}{R_2}}$$

$$= \frac{-2R_2/R_3}{1 + (R_2/AR)} \qquad \text{Q.E.D.}$$



For $AR \gg R_2$, we have

$$\frac{v_o}{v_i} \simeq -\frac{2R_2}{R_3}$$

Ex: 12.16 From Fig. 12.31 we see that for $P_{\text{dissipation}}$ to be less than 2.9 W, a maximum supply voltage of 20V is called for. The 20-V-supply curve intersects the 3% distoration line at a point for which the output power is 4.2 W. Since

$$P_L = \frac{\left(\hat{V}_o/\sqrt{2}\right)^2}{R_L}$$

we have $\hat{V}_o = \sqrt{4.2 \times 2 \times 8} = 8.2$ V

or 16.4 V peak-to-peak

Ex: 12.17 Voltage gain = 2 K

where $K = \dfrac{R_4}{R_3} = 1 + \dfrac{R_2}{R_1} = 1.5$

Thus, $A_v = 3$ V/V

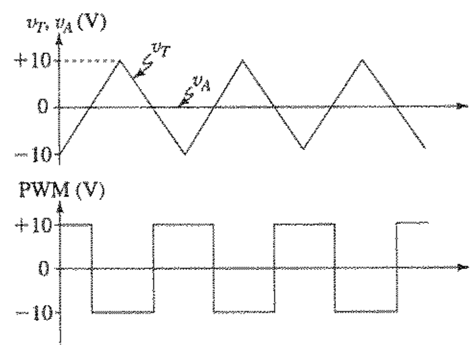
Input resistance $= R_3 = 10$ kΩ

Peak-to-Peak $v_o = 3 \times 20 = 60$ V

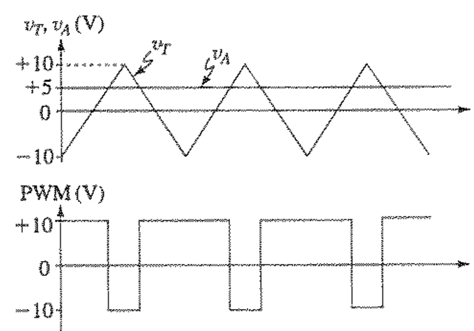
Peak load current $= \dfrac{30 \text{ V}}{8\ \Omega} = 3.75$ A

$$P_L = \frac{\left(30/\sqrt{2}\right)^2}{8} = 56.25 \text{ W}$$

Ex: 12.18 See Fig. 1.

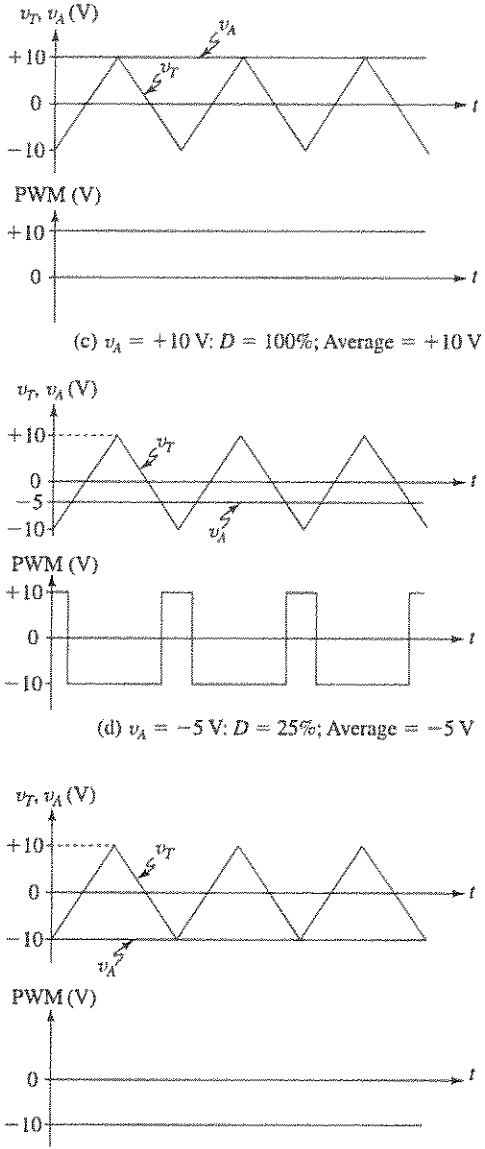


(a) $v_A = 0$: $D = 50\%$; Average = 0



(b) $v_A = +5$ V; $D = 75\%$; Average = +5 V

Figure 1 *continued*



(c) $v_A = +10$ V: $D = 100\%$; Average $= +10$ V

(d) $v_A = -5$ V: $D = 25\%$; Average $= -5$ V

(e) $v_A = -10$ V: $D = 0\%$; Average $= -10$ V

Figure 1

Ex: 12.19

$f_s = 10 \times$ highest frequency in audio signal

$= 10 \times 20 = 200$ kHz

Since f_s is a decade higher than f_P, the gain will have fallen by 40 dB. Thus the PWM component at f_s will be attenuated by 40 dB.

Ex: 12.20 Maximum peak amplitude $= V_{DD}$

Maximum power delivered to $R_L = \dfrac{(V_{DD}/\sqrt{2})^2}{R_L}$

$= \dfrac{V_{DD}^2}{2R_L}$

For $V_{DD} = 35$ V and $R_L = 8\ \Omega$:

Peak amplitude $= 35$ V

Maximum power $= \dfrac{35^2}{2 \times 8} = 76.6$ W

Power delivered by power supplies

$= \dfrac{P_L}{\eta} = \dfrac{76.6}{0.9} = 85.1$ W

Ex: 12.21 $T_J - T_A = \theta_{JA} P_D$

$200 - 25 = \theta_{JA} \times 50$

$\theta_{JA} = \dfrac{175}{50} = 3.5°\text{C/W}$

But, $\theta_{JA} = \theta_{JC} + \theta_{CS} + \theta_{SA}$

$3.5 = 1.4 + 0.6 + \theta_{SA}$

$\Rightarrow \theta_{SA} = 1.5°\text{C/W}$

$T_J - T_C = \theta_{JC} \times P_D$

$T_C = T_J - u_{JC} \times P_D$

$= 200 - 1.4 \times 50$

$= 130°\text{C}$

Ex: 13.1 Using Eq. (13.2), we obtain

$V_{ICM\min} = -V_{SS} + V_{tn} + V_{OV3} - |V_{tp}|$

$= -1.65 + 0.5 + 0.3 - 0.5$

$= -1.35$ V

Using Eq. (13.3), we get

$V_{ICM\max} = V_{DD} - |V_{OV3}| - |V_{tp}| - |V_{OV1}|$

$= 1.65 - 0.3 - 0.5 - 0.3$

$= +0.55$ V

Thus,

$-1.35 \text{ V} \le V_{ICM} \le +0.55 \text{ V}$

Using Eq. (13.5), we obtain

$-V_{SS} + V_{OV6} \le v_O \le V_{DD} - |V_{OV7}|$

Thus

$-1.65 + 0.5 \le v_O \le 1.65 - 0.5$

$\Rightarrow -1.15 \text{ V} \le v_O \le +1.15 \text{ V}$

Ex: 13.2 For all devices, we have

$|V_A| = 20$ V

Using Eq. (13.13), we get

$$A_1 = -\frac{2}{|V_{OV1}|}\Big/\left[\frac{1}{|V_{A2}|} + \frac{1}{V_{A4}}\right]$$

$$= -\frac{2}{0.2}\Big/\frac{2}{20} = -100 \text{ V/V}$$

Using Eq. (13.20), we obtain

$$A_2 = -\frac{2}{|V_{OV6}|}\Big/\left[\frac{1}{V_{A6}} + \frac{1}{|V_{A7}|}\right]$$

$$= -\frac{2}{0.5}\Big/\frac{2}{20} = -40 \text{ V/V}$$

$A = A_1A_2$

$= -100 \times -40 = 4000$ V/V

$$r_{o6} = r_{o7} = \frac{|V_A|}{0.5 \text{ mA}} = \frac{20}{0.5} = 40 \text{ k}\Omega$$

$R_o = r_{o6} \parallel r_{o7} = 40 \parallel 40 = 20 \text{ k}\Omega$

Ex: 13.3 The feedback is of the voltage sampling type (i.e., the connection at the output is a shunt one), thus

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

where

$R_o = r_{o6} \parallel r_{o7}$

$A = g_{m1}(r_{o2} \parallel r_{o4})g_{m6}(r_{o6} \parallel r_{o7})$

$\beta = 1$

Thus,

$$R_{\text{out}} = \frac{r_{o6} \parallel r_{o7}}{1 + g_{m1}(r_{o2} \parallel r_{o4})g_{m6}(r_{o6} \parallel r_{o7})}$$

Usually,

$A \gg 1$

Thus,

$$R_{\text{out}} \simeq \frac{1}{g_{m6}[g_{m1}(r_{o2} \parallel r_{o4})]}$$

Ex: 13.4 Using Eq. (13.36), we get

$$f_t = \frac{G_{m1}}{2\pi C_C}$$

$$\Rightarrow C_C = \frac{G_{m1}}{2\pi f_t}$$

$$= \frac{0.3 \times 10^{-3}}{2\pi \times 10 \times 10^6}$$

$= 4.8$ pF

From Eq. (13.31), we have

$$f_Z = \frac{G_{m2}}{2\pi C_C}$$

$$= \frac{0.6 \times 10^{-3}}{2\pi \times 4.8 \times 10^{-12}}$$

$\simeq 20$ MHz

From Eq. (13.35), we have

$$f_{P2} = \frac{G_{m2}}{2\pi C_2}$$

$$= \frac{0.6 \times 10^{-3}}{2\pi \times 2 \times 10^{-12}} = 48 \text{ MHz}$$

Thus, f_t is lower than f_Z and f_{P2}.

Ex: 13.5 (a) Using Eq. (13.36), we have

$$f_t = \frac{G_{m1}}{2\pi C_C}$$

$$\Rightarrow C_C = \frac{G_{m1}}{2\pi f_t}$$

$$= \frac{1 \times 10^{-3}}{2\pi \times 100 \times 10^6}$$

$= 1.6$ pF

$A_0 = G_{m1}(r_{o2} \parallel r_{o4})G_{m2}(r_{o6} \parallel r_{o7})$

$= 1(100 \parallel 100) \times 2(40 \parallel 40)$

$= 50 \times 2 \times 20 = 2000$ V/V

$$f_{3dB} = \frac{f_t}{A_0} = \frac{100 \times 10^6}{2000} = 50 \text{ kHz}$$

(b) From Eq. (13.34), we have

$$R = \frac{1}{G_{m2}} = \frac{1}{2 \times 10^{-3}} = 500\ \Omega$$

(c) From Eq. (13.35), we have

$$f_{P2} = \frac{G_{m2}}{2\pi C_2}$$

$$= \frac{2 \times 10^{-3}}{2\pi \times 1 \times 10^{-12}} = 318\ \text{MHz}$$

$$\phi_{P2} = -\tan^{-1}\frac{f_t}{f_{P2}}$$

$$= -\tan^{-1}\frac{100}{318}$$

$$= -17.4^\circ$$

$$\text{PM} = 90 - 17.4 = 72.6^\circ$$

Ex: 13.6 Using Eq. (13.47), we obtain

$$\text{SR} = V_{OV1}\omega_t$$

$$= 0.2 \times 2\pi \times 100 \times 10^6$$

$$= 126\ \text{V}/\mu\text{s}$$

Using Eq. (13.45),

$$\text{SR} = \frac{I}{C_C}$$

$$\Rightarrow I = 126 \times 10^6 \times 1.6 \times 10^{-12}$$

$$= 200\ \mu\text{A}$$

Ex: 13.7

$$R_B = \frac{2}{\sqrt{2\mu_n C_{ox}(W/L)_{12} I_{\text{REF}}}}\left(\sqrt{\frac{(W/L)_{12}}{(W/L)_{13}}} - 1\right)$$

$$= \frac{2}{\sqrt{2 \times 90 \times 10^{-6} \times 80 \times 10 \times 10^{-6}}}\left(\sqrt{\frac{80}{20}} - 1\right)$$

$$= 5.27\ \text{k}\Omega$$

Using Eq. (13.61), we obtain

$$g_{m12} = \frac{2}{R_B}\left(\sqrt{\frac{(W/L)_{12}}{(W/L)_{13}}} - 1\right)$$

$$= \frac{2}{5.27}\left(\sqrt{\frac{80}{20}} - 1\right)$$

$$= 0.379\ \text{mA/V}$$

Ex: 13.8 From Example 9.6, Q_8 has

$$\left(\frac{W}{L}\right)_8 = \frac{40}{0.8}$$

$$g_{m8} = 0.6\ \text{mA/V}$$

$$g_{m13} = g_{m8} = 0.6\ \text{mA/V}$$

$$g_{m12} = \sqrt{2\mu_n C_{ox}(W/L)_{12} I_{\text{REF}}}$$

$$= \sqrt{2\mu_n C_{ox} \times 4(W/L)_{13} I_{\text{REF}}}$$

$$= 2g_{m13} = 1.2\ \text{mA/V}$$

Now,

$$R_B = \frac{2}{\sqrt{2\mu_n C_{ox}(W/L)_{12} I_{\text{REF}}}}\left(\sqrt{\frac{(W/L)_{12}}{(W/L)_{13}}} - 1\right)$$

$$= \frac{2}{1.2 \times 10^{-3}}(\sqrt{4} - 1)$$

$$= 1.67\ \text{k}\Omega$$

From Example 9.6, we have

$$V_{DD} = V_{SS} = 2.5\ \text{V}$$

$$I_{\text{REF}} = 90\ \mu\text{A}$$

$$V_{tn} = 0.7\ \text{V}$$

$$|V_{tp}| = 0.8\ \text{V}$$

$$|V_{OV8}| = 0.3\ \text{V}$$

$$I_{\text{REF}} R_B = 0.09 \times 1.67$$

$$= 150\ \text{mV}$$

Since

$$g_{m13} = \frac{2I_{\text{REF}}}{V_{OV13}}$$

$$0.6 = \frac{2 \times 0.09}{V_{OV13}}$$

$$\Rightarrow V_{OV13} = 0.3\ \text{V}$$

$$V_{GS13} = 0.3 + 0.7 = 1\ \text{V}$$

$$V_{G13} = -V_{SS} + V_{GS13}$$

$$= -2.5 + 1 = -1.5\ \text{V}$$

$$V_{GS11} = V_{GS13} = 1\ \text{V}$$

$$V_{G11} = V_{G13} + V_{GS11}$$

$$= -1.5 + 1 = -0.5\ \text{V}$$

$$V_{SG8} = |V_{tp}| + |V_{OV8}|$$

$$= 0.8 + 0.3 = 1.1\ \text{V}$$

$$V_{G8} = V_{DD} - V_{SG8}$$

$$= 2.5 - 1.1 = +1.4\ \text{V}$$

Ex: 13.9 Total bias current $= 300\ \mu\text{A} = 2I_B$

$$\Rightarrow I_B = 150\ \mu\text{A}$$

$$I_B = I_{D1} + I_{D3}$$

$$150 = I_{D1} + 0.25 I_{D1}$$

$$\Rightarrow I_{D1} = 120\ \mu\text{A}$$

$I = I_{D1} + I_{D2}$

$= 120 + 120 = 240\ \mu A$

$I_{D3,4} = 0.25 I_{D1,2} = 0.25 \times 120$

$= 30\ \mu A$

Ex: 13.10 Using Eq. (13.64), we get

$V_{ICM\max} = V_{DD} - |V_{OV9}| + V_{tn}$

$= 1.65 - 0.3 + 0.5 = +1.85$ V

Using Eq. (13.65), we obtain

$V_{ICM\min} = -V_{SS} + V_{OV11} + V_{OV1} + V_{tn}$

$V_{ICM\min} = -1.65 + 0.3 + 0.3 + 0.5$

$= -0.55$ V

Thus,

$-0.55\text{ V} \le V_{ICM} \le +1.85\text{ V}$

Using Eq. (13.68), we get

$V_{O\max} = V_{DD} - |V_{OV10}| - |V_{OV4}|$

$= 1.65 - 0.3 - 0.3 = +1.05$ V

Using Eq. (13.69), we obtain

$V_{O\min} = -V_{SS} + V_{OV7} + V_{OV5} + V_{tn}$

$= -1.65 + 0.3 + 0.3 + 0.5$

$= -0.55$ V

Thus,

$-0.55\text{ V} \le v_O \le +1.05\text{ V}$

Ex: 13.11 $G_m = g_{m1} = g_{m2}$

$$G_m = \frac{2(I/2)}{V_{OV1}} = \frac{I}{V_{OV1}}$$

$$= \frac{0.24}{0.2} = 1.2\text{ mA/V}$$

$$r_{o2} = \frac{|V_A|}{I/2} = \frac{20}{0.12} = 166.7\text{ k}\Omega$$

$$r_{o4} = \frac{|V_A|}{I_{D4}} = \frac{|V_A|}{I_B - \frac{I}{2}} = \frac{20}{0.150 - 0.120}$$

$$= \frac{20}{0.03} = 666.7\text{ k}\Omega$$

$$r_{o10} = \frac{|V_A|}{I_B}$$

$$= \frac{20}{0.15} = 133.3\text{ k}\Omega$$

$$g_{m4} = \frac{2I_{D4}}{|V_{OV}|} = \frac{2 \times 0.03}{0.2} = 0.3\text{ mA/V}$$

$R_{o4} = (g_{m4} r_{o4})(r_{o2} \parallel r_{o10})$

$= (0.3 \times 666.7)(166.7 \parallel 133.3)$

$= 14.8\text{ M}\Omega$

$$g_{m6} = \frac{2I_{D6}}{V_{OV}} = \frac{2 \times 0.03}{0.2} = 0.3\text{ mA/V}$$

$$r_{o6} = r_{o8} = \frac{|V_A|}{I_{D6,8}} = \frac{20}{0.03} = 666.7\text{ k}\Omega$$

$R_{o6} = g_{m6} r_{o6} r_{o8}$

$= 0.3 \times 666.7 \times 666.7 = 133.3\text{ M}\Omega$

$R_o = R_{o4} \parallel R_{o6}$

$= 14.8 \parallel 133.3 = 13.3\text{ M}\Omega$

$A_v = G_m R_o = 1.2 \times 13.3 \times 10^3 = 16{,}000$ V/V

Ex: 13.12 (a) The NMOS input stage operates over the following input common-mode range:

$-V_{SS} + 2V_{OV} + V_{tn} \le V_{ICM} \le V_{DD} - |V_{OV}| + V_{tn}$

that is,

$(-2.5 + 0.6 + 0.7) \le V_{ICM} \le (2.5 - 0.3 + 0.7)$

$-1.2\text{ V} \le V_{ICM} \le +2.9\text{ V}$

(b) The PMOS input stage operates over the following input common-mode range:

$-V_{SS} + V_{OV} - |V_{tp}| \le V_{ICM} \le V_{DD} - 2|V_{OV}| - |V_{tp}|$

that is,

$(-2.5 + 0.3 - 0.7) \le V_{ICM} \le (2.5 - 0.6 - 0.7)$

$-2.9\text{ V} \le V_{ICM} \le +1.2\text{ V}$

(c) The overlap range is

$-1.2\text{ V} \le V_{ICM} \le +1.2\text{ V}$

(d) The input common-mode range is

$-2.9\text{ V} \le V_{ICM} \le +2.9\text{ V}$

Ex: 13.13 Denote the (W/L) of the transistors in the wide-swing mirror by $(W/L)_M$. Transistor Q_4 has

$$(W/L)_5 = \frac{1}{4}(W/L)_M$$

$$I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_5 V_{OV5}^2$$

$$= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_M \times \frac{1}{4} V_{OV5}^2$$

$$= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_M (V_{OV5}/2)^2$$

Thus,

$$\frac{V_{OV5}}{2} = V_{OV}$$

where V_{OV} is the overdrive voltage for each of the mirror transistors. Thus,

$$V_5 = V_{m} + 2V_{OV}$$

which is the value of V_{BIAS} needed in the circuit of Fig. 13.13(b).

Ex: 13.14 At $I_C = 0.1$ mA,

$$V_{BE} = 25 \ln\frac{0.1 \times 10^{-3}}{10^{-14}}$$

$$= 575.6 \text{ V}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

$$r_e \simeq \frac{1}{g_m} = 250\ \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{4} = 50\ \text{k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{125 \text{ V}}{0.1 \text{ mA}} = 1.25\ \text{M}\Omega$$

Ex: 13.15 $V_T \ln\dfrac{I_{REF}}{I_{C10}} = I_{C10}R_4$

$$25 \ln\frac{730}{I_{C10}} = 5I_{C10} \qquad (1)$$

where I_{C10} is in μA, and both sides of Eq. (1) are in mV. Using iteration:

I_C (μA)	LHS of Eq. (1) (mV)	RHS of Eq. (1) (mV)
100	49.5	500
50	67	250
20	89.9	100
19	91.2	95
18	92.6	90

Thus,

$$I_{C10} \simeq 19\ \mu\text{A}$$

Ex: 13.16 Refer to Fig. 13.15. At node X,

$$I_{C10} = \frac{2I}{1 + \dfrac{2}{\beta_P}} + \frac{2I}{\beta_P}$$

$$I_{C10} = 2I\left[\frac{\beta_P + 1 + \dfrac{2}{\beta_P}}{\beta_P\left(1 + \dfrac{2}{\beta_P}\right)}\right]$$

$$\simeq 2I\frac{\beta_P + 1}{\beta_P + 2}$$

$$\simeq 2I$$

Thus,

$$I = \frac{I_{C10}}{2} = 9.5\ \mu\text{A}$$

resulting in

$$I_{C1} = I_{C2} = I_{C3} = I_{C4} = 9.5\ \mu\text{A}$$

Ex: 13.17 Figure 1 on next page shows the determination of the loop gain of the feedback circuit that stabilizes the bias currents of the first stage of the 741 op amp. Note that since I_{C10} is assumed to be constant, we have shown its incremental value at node X to be zero. Observe that this circuit shows only incremental quantities. The analysis shown provides the returned current signal as

$$I_r = -I_t\,\frac{\beta_P}{1 + \dfrac{2}{\beta_P}}$$

For $\beta_P \gg 1$, we have

$$I_r \simeq -\beta_P I_t$$

and the loop gain $A\beta$ is

$$A\beta \equiv -\frac{I_r}{I_t} = \beta_P$$

Ex: 13.18 $V_{BE6} = V_T \ln\dfrac{I_{C6}}{I_S}$

$$= 25 \ln\frac{9.5 \times 10^{-6}}{10^{-14}} = 517 \text{ mV}$$

$$V_{R3} = V_{BE6} + IR_2$$

$$= 517 + 9.5 \times 10^{-6} \times 1 = 526.5 \text{ mV}$$

$$I_{C7} \simeq I_{E7} \simeq \frac{V_{R3}}{R_3}$$

$$= \frac{526.5}{50} = 10.5\ \mu\text{A}$$

Ex: 13.19 $I_B = \dfrac{1}{2}(I_{B1} + I_{B2})$

$$= \frac{1}{2}\left(\frac{I}{\beta_N + 1} + \frac{I}{\beta_N + 1}\right)$$

$$= \frac{I}{\beta_N + 1} \simeq \frac{I}{\beta_N}$$

$$= \frac{9.5}{200} = 47.5 \text{ nA}$$

$$I_{OS} = 0.1 \times I_B = 4.75 \text{ nA}$$

Ex: 13.20

$$V_{C1} = V_{CC} - V_{EB8} = 15 - 0.6 = 14.4 \text{ V}$$

Q_1 and Q_2 saturate when V_{ICM} exceeds V_{C1} by 0.3 V. Thus,

$$V_{ICM\max} = +14.7 \text{ V}$$

This figure belongs to Exercise 13.17.

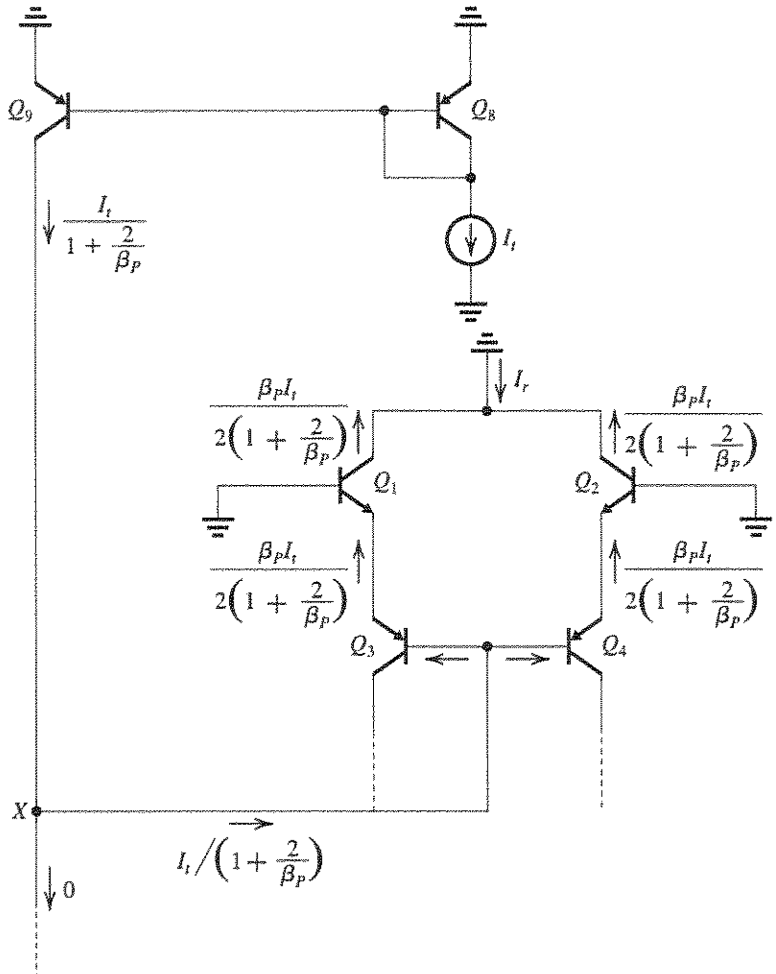


Figure 1

$V_{C5} \simeq -V_{EE} + V_{BE5} + V_{BE7}$

$= -15 + 0.6 + 0.6 = -13.8$ V

Q_3 (and Q_4) saturate when

$V_{B3} = V_{C5} - 0.3$

$= -13.8 - 0.3 = -14.1$ V

But,

$V_{B3} = V_{ICM} - V_{BE1} - V_{EB3}$

$= V_{ICM} - 1.2$ V

Thus,

$V_{ICM\,\min} = V_{B3} + 1.2$ V

$= -14.1 + 1.2 = -12.9$ V

Thus,

$-12.9 \text{ V} \leq V_{ICM} \leq +14.7 \text{ V}$

Ex: 13.21 $I_{C13B} = 0.75 I_{C12}$

$= 0.75 \times 0.73 = 0.55$ mA

$= 550\ \mu$A

$I_{C17} = I_{C13B} = 550\ \mu$A

$V_{BE17} = V_T \ln \dfrac{I_{C17}}{I_S}$

$= 25 \ln \dfrac{550 \times 10^{-6}}{10^{-14}}$

$= 618$ mV

$V_{R9} = V_{BE17} + I_{E17} R_8$

$\simeq 618 + 550 \times 0.1$

$= 673$ mV

$I_{R9} = \dfrac{673}{50} = 13.46\ \mu$A

$$I_{B17} = \frac{550}{201} = 2.74\ \mu\text{A}$$

$$I_{E16} = I_{R9} + I_{B17} = 16.30\ \mu\text{A}$$

$$I_{C16} = \frac{200}{201} \times 16.3 = 16.2\ \mu\text{A}$$

$$I_{B16} = \frac{16.2}{200} = 0.08\ \mu\text{A}$$

Ex: 13.22 The two diode-connected transistors will carry a bias current of $0.25 I_{REF} = 180\ \mu$A. Since the output transistors have three times the values of I_S as that of the diode-connected transistors, the bias current in the output transistors will be

$$= 3 \times 180 = 540\ \mu\text{A}$$

Ex: 13.23 $r_e = \dfrac{V_T}{I_E} \simeq \dfrac{25\text{ mV}}{9.5\ \mu\text{A}} = 2.63\text{ k}\Omega$

$$g_{m1} \simeq \frac{1}{r_e} = 0.38\text{ mA/V}$$

$$G_{m1} = \frac{1}{2} g_{m1} = 0.19\text{ mA/V}$$

$$R_{id} = (\beta_N + 1) \times 4r_e$$

$$= 201 \times 4 \times 2.63$$

$$= 2.1\text{ M}\Omega$$

Ex: 13.24 refer to Fig. 13.19.

(a) $v_{b6} = i_{e6}(r_{e6} + R_2)$

$$= i_e(r_{e6} + R_2)$$

$$r_{e6} = \frac{V_T}{I_{E6}} \simeq \frac{25\text{ mV}}{9.5\ \mu\text{A}} = 2.63\text{ k}\Omega$$

$$v_{b6} = i_e(2.63 + 1) = 3.63\text{ k}\Omega \times i_e$$

(b) $i_{e7} = i_{R3} + i_{b5} + i_{b6}$

$$= \frac{v_{b6}}{R_3} + \frac{2\alpha i_e}{\beta_N}$$

$$= \frac{3.63}{50} i_e + \frac{2}{201} i_e$$

$$= 0.08 i_e$$

(c) $i_{b7} = \dfrac{i_{e7}}{\beta_N + 1} = \dfrac{0.08}{201} i_e = 0.0004 i_e$

(d) $v_{b7} = i_{e7} r_{e7} + v_{b6}$

$$v_{b7} = 0.08 \times 2.63 i_e + 3.63 i_e$$

$$= 3.84\text{ k}\Omega \times i_e$$

(e) $R_{in} = \dfrac{v_{b7}}{\alpha i_e} \simeq 3.84\text{ k}\Omega$

Ex: 13.25 $r_{o4} = \dfrac{|V_{Ap}|}{I} = \dfrac{50\text{ V}}{9.5\ \mu\text{A}} = 5.26\text{ M}\Omega$

$$g_{m4} = 0.38\text{ mA/V}$$

$$r_{e2} = 2.63\text{ k}\Omega$$

$$r_{\pi 4} = \frac{\beta_P}{g_{m4}} = \frac{50}{0.38} = 131.6\text{ k}\Omega$$

$$R_{o4} = r_{o4}[1 + g_{m4}(r_{e2} \parallel r_{\pi 4})]$$

$$= 5.26[1 + 0.38(2.63 \parallel 131.6)]$$

$$= 10.4\text{ M}\Omega$$

(The answer in the book was obtained by neglecting $r_{\pi 4}$.)

$$r_{o6} = \frac{V_{An}}{I} = \frac{125\text{ V}}{9.5\ \mu\text{A}} = 13.16\text{ M}\Omega$$

$$g_{m6} = 0.38\text{ mA/V}$$

$$R_6 = 1\text{ k}\Omega$$

$$r_{\pi 6} = \frac{200}{0.38} = 526.3\text{ k}\Omega$$

$$R_{o6} = r_{o6}[1 + g_{m6}(R_2 \parallel r_{\pi 6})]$$

$$= 13.16[1 + 0.38(1 \parallel 526.3)]$$

$$= 18.2\text{ M}\Omega$$

$$R_{o1} = R_{o9} \parallel R_{o6}$$

$$= 10.4 \parallel 18.2 = 6.62\text{ M}\Omega$$

Ex: 13.26 $|A_{vo}| = G_{m1} R_{o1}$

Using G_{m1} given in the answer to Exercise 13.23,

$$G_{m1} = 0.19\text{ mA/V}$$

and R_{o1} given in the answer to Exercise 13.25,

$$R_{o1} = 6.7\text{ M}\Omega$$

we obtain

$$|A_{vo}| = G_{m1} R_{o1}$$

$$= 0.19 \times 6.7 = 1273\text{ V/V}$$

Ex: 13.27 Refer to Fig. 13.22, which shows the current mirror with an imbalance between $R_1 = R$ and $R_2 = R + \Delta R$. Observe that the imbalance causes an error in the mirror transfer ratio of

$$\epsilon_m = \frac{\Delta I}{I}$$

where $\dfrac{\Delta I}{I}$ is given by Eq. (13.94). Thus,

$$\epsilon_m = \frac{\Delta R}{R + \Delta R + r_e}$$

where $r_e = r_{e5} = r_{e6}$,

$$\epsilon_m = \frac{\Delta R}{R + \Delta R + r_{e5}} \qquad \text{Q.E.D.}$$

For $R = 1\ \text{k}\Omega$, $\frac{\Delta R}{R} = 0.02$ and $r_{e5} = 2.63\ \text{k}\Omega$,

$$\epsilon_m = \frac{0.02}{1 + 0.02 + 2.63} = 5.5 \times 10^{-3}$$

Ex: 13.28

$$R_{o9} = r_{o9} = \frac{|V_{Ap}|}{I_{C9}} = \frac{50\ \text{V}}{19\ \mu\text{A}} = 2.63\ \text{M}\Omega$$

$$R_{o10} = r_{o10}[1 + g_{m10}(R_4 \parallel r_{\pi 10})]$$

where

$$r_{o10} = \frac{V_{An}}{I_{C10}} = \frac{125\ \text{V}}{19\ \mu\text{A}} = 6.58\ \text{M}\Omega$$

$$g_{m10} = \frac{I_{C10}}{V_T} = \frac{19\ \mu\text{A}}{0.025\ \text{V}} = 0.76\ \text{mA/V}$$

$$R_4 = 5\ \text{k}\Omega$$

$$r_{\pi 10} = \frac{\beta_N}{g_{m10}} = \frac{200}{0.76} = 263.2\ \text{k}\Omega$$

$$R_{o10} = 6.58[1 + 0.76(5 \parallel 263.2)]$$

$$= 31.1\ \text{M}\Omega$$

$$R_o = R_{o9} \parallel R_{o10}$$

$$= 2.63 \parallel 31.1 = 2.43\ \text{M}\Omega$$

Ex: 13.29 Using Eq. (13.100), we obtain

$$G_{mcm} = \frac{\epsilon_m}{2R_o}$$

$$= \frac{5.5 \times 10^{-3}}{2 \times 2.43 \times 10^6} = 1.13 \times 10^{-6}\ \text{mA/V}$$

$$\text{CMRR} = \frac{G_{m1}}{G_{mcm}} = \frac{0.19}{1.13 \times 10^{-6}} = 1.68 \times 10^5$$

or 104.5 dB

Without common-mode feedback, the CMRR is reduced by a factor equal to β_P. Equivalently,

$$\text{CMRR} = 104.5 - 20 \log \beta_P$$

$$= 104.5 - 20 \log 50$$

$$= 70.5\ \text{dB}$$

Ex: 13.30 $r_{e16} = \frac{V_T}{I_{E16}} \simeq \frac{25\ \text{mV}}{16.2\ \text{mA}} = 1.54\ \text{k}\Omega$

$$r_{e17} = \frac{V_T}{I_{E17}} \simeq \frac{25\ \text{mV}}{0.55\ \text{mA}} = 45.5\ \Omega$$

Using Eq. (13.103), we obtain

$R_{i2} =$

$$(200 + 1)\{1.54 + [50 \parallel (200 + 1)(0.0455 + 0.1)]\}$$

$$\simeq 4\ \text{M}\Omega$$

Ex: 13.31 Using Eq. (13.104), we get

$$i_{c17} = \frac{\alpha}{r_{e17} + R_8} v_{b17}$$

$$= \frac{200}{201} \frac{1}{0.0455 + 0.1} v_{b17} = 6.85 v_{b17} \qquad (1)$$

Using Eq. (13.106), we obtain

$$R_{i17} = 201(0.0455 + 0.1) = 29.25\ \text{k}\Omega$$

Using Eq. (13.105), we get

$$v_{b17} = v_{i2} \frac{50 \parallel 29.25}{(50 \parallel 29.25) + 1.54} = 0.92 v_{i2} \qquad (2)$$

Combining Eqs. (1) and (2), we obtain

$$i_{c17} = 6.32 v_{b17}$$

Thus,

$$G_{m2} = 6.32\ \text{mA/V}$$

This value is somewhat lower than the value generally published for G_{m2}, namely

$$G_{m2} = 6.5\ \text{mA/V}$$

To conform with published literature, we shall use the latter value in future calculations.

Ex: 13.32 $R_{o13B} = r_{o13B} = \frac{|V_{Ap}|}{I_{C13B}}$

$$= \frac{50}{0.55} = 90.9\ \text{k}\Omega$$

$$R_{o17} = r_{o17}[1 + g_{m17}(R_8 \parallel r_{\pi 17})]$$

where

$$r_{o17} = \frac{125}{0.55} = 227.3\ \text{k}\Omega$$

$$g_{m17} = \frac{0.55}{0.025} = 22\ \text{mA/V}$$

$$R_8 = 0.1\ \text{k}\Omega$$

$$r_{\pi 17} = \frac{200}{22} = 9.09\ \text{k}\Omega$$

$$R_{o17} = 227.3[1 + 22(0.1 \parallel 9.09)]$$

$$= 722\ \text{k}\Omega$$

$$R_{o2} = R_{o13B} \parallel R_{o17}$$

$$= 90.9 \parallel 722 \simeq 81\ \text{k}\Omega$$

Ex: 13.33 Open-circuit voltage gain $= -G_{m2} R_{o2}$

$$= -6.5 \times 81 = -526.5\ \text{V/V}$$

Ex: 13.34 $r_{e23} = \frac{V_T}{I_{E23}}$

$$\simeq \frac{25\ \text{mV}}{0.18\ \text{mA}} = 138.9\ \Omega$$

$$R_{o23} = \frac{R_{o2}}{\beta_{23}+1} + r_{e23}$$

$$= \frac{81}{50+1} + 0.139$$

$$= 1.73\ \text{k}\Omega$$

$$r_{e20} = \frac{V_T}{I_{E20}}$$

$$= \frac{25\ \text{mV}}{5\ \text{mA}} = 5\ \Omega$$

$$R_{\text{out}} = r_{e20} + \frac{R_{o23}}{\beta_{20}+1}$$

$$= 5 + \frac{1730}{50+1} = 39\ \Omega$$

Total output resistance $= R_{\text{out}} + R_7$

$= 39 + 27 = 66\ \Omega$

Ex: 13.35

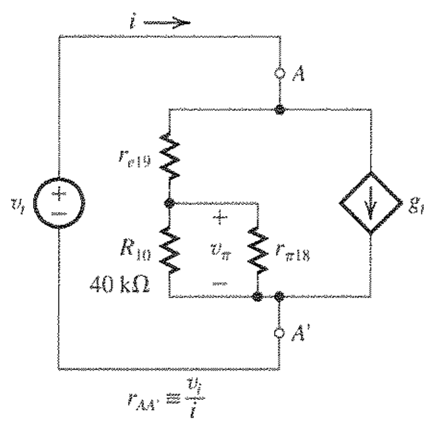


Figure 1 shows the equivalent circuit model of the circuit in Fig. E13.35. Note that the diode-connected transistors Q_{19} is replaced with r_{e19}.

$$r_{e19} = \frac{V_T}{I_{E19}}$$

$$\simeq \frac{25\ \text{mA}}{16\ \mu\text{A}} = 1.56\ \text{k}\Omega$$

Transistor Q_{18} is replaced with its hybrid-π model,

$$g_{m18} = \frac{I_{C18}}{V_T} = \frac{0.165\ \text{mA}}{0.025\ \text{V}}$$

$$= 6.6\ \text{mA/V}$$

$$r_{\pi 18} = \frac{\beta_N}{g_{m18}} = \frac{200}{6.6} = 30.3\ \text{k}\Omega$$

Now,

$$v_\pi = v_i\,\frac{R_{10}\parallel r_\pi}{(R_{10}\parallel r_\pi) + r_{e19}}$$

$$= v_i\,\frac{40\parallel 30.3}{(40\parallel 30.3)+1.56} = 0.917 v_i$$

$$i = \frac{v_i}{(40\parallel 30.3)+1.56} + g_{m18}\times 0.917 v_i$$

$$= 6.11\times 10^{-3} v_i$$

$$r'_{AA} \equiv \frac{v_i}{i} \simeq 163\ \Omega$$

Ex: 13.36 For $v_O = 10\ \sin\omega t$

$$\frac{dv_O}{dt} = \omega \times 10\ \cos\omega t$$

$$SR = \left.\frac{dv_O}{dt}\right|_{\max} = \omega_M \times 10 = 2\pi f_M \times 10$$

$$f_M = \frac{SR}{20\pi} = \frac{0.63\times 10^6}{20\pi} = 10\ \text{kHz}$$

Ex: 13.37 $I_{S2} = 2I_{S1}$

Using Eq. (13.127), we obtain

$$I = \frac{V_T}{R_2}\ln\frac{I_{S2}}{I_{S1}}$$

$$0.01 = \frac{0.025}{R_2}\ln 2$$

$$\Rightarrow R_2 = 1.73\ \text{k}\Omega$$

$$R_3 = R_4 = \frac{0.2\ \text{V}}{0.01\ \text{mA}} = 20\ \text{k}\Omega$$

Ex: 13.38 To obtain $I_8 = 10\ \mu$A, transistor Q_8 must have the same (ratio = 1) emitter area as Q_3, and

$$R_8 = R_3 = 20\ \text{k}\Omega$$

To obtain $I_9 = 20\ \mu$A, Q_9 must have an EBJ area twice (ratio = 2) that of Q_3 and

$$R_9 = \frac{1}{2}R_3 = 10\ \text{k}\Omega$$

To obtain $I_{10} = 5\ \mu$A, Q_{10} must have an EBJ area half (ratio = 0.5) that of Q_3 and

$$R_{10} = 2R_3 = 40\ \text{k}\Omega$$

Ex: 13.39 Refer to the circuit in Fig. 13.42.

(a) Current gain from v_{IP} to output

$= (\beta_1+1)(\beta_2+1)\beta_P$

$\simeq \beta_1\beta_2\beta_P = \beta_N\beta_P^2$

Current gain from v_{IN} to output $= (\beta_3+1)\beta_N$

$\simeq \beta_3\beta_N = \beta_N^2$

(b) For $i_L = +10$ mA,

current needed at v_{IP} input

$$= \frac{10}{\beta_N\beta_P^2} = \frac{10}{40\times 10^2} = 2.5\ \mu\text{A}$$

For $i_L = -10$ mA,

current needed at v_{IN} input $= \frac{10}{\beta_N^2} = \frac{10}{40^2}$

$= 6.25\ \mu$A

Ex: 13.40 $I_Q = 0.4$ mA, $I = 10\ \mu$A,

$\frac{I_{SN}}{I_{S10}} = 10,\ \frac{I_{S7}}{I_{S11}} = 2$

Using Eq. (13.136), we obtain

$$0.4 \times 10^3 = 2\left(\frac{I_{REF}^2}{10}\right) \times 10 \times 2$$

where I_{REF} is in μA. Thus,

$I_{REF} = 10\ \mu$A

For $i_L = -10$ mA, then

$i_P = 10 + i_N$

Using Eq. (13.137), we get

$$\frac{i_N(10 + i_N)}{i_N + 10 + i_N} = 0.2$$

$$\Rightarrow i_N^2 - 9.6 i_N + 2 = 0$$

$$\Rightarrow i_N = 0.2 \text{ mA}$$

$i_P = 10.2$ mA

Ex: 14.1

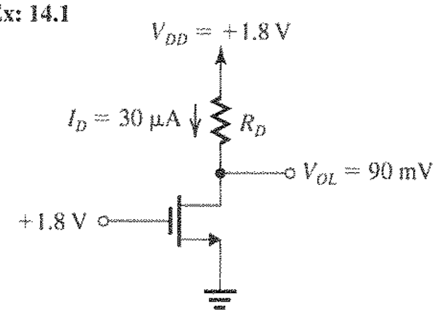


Figure 1

Refer to Fig. 1.

$$r_{DS} = \frac{V_{OL}}{I_D} = \frac{90 \text{ mV}}{30 \text{ μA}} = 3 \text{ k}\Omega$$

Substituting in the expression given for r_{DS}, we get

$$3 = \frac{1}{0.125 \times \left(\frac{W}{L}\right) \times (1.8 - 0.4)}$$

$$\Rightarrow \frac{W}{L} = 1.9$$

The value of R_D can be obtained from

$$R_D = \frac{V_{DD} - V_{OL}}{I_D}$$

$$= \frac{1.8 - 0.09}{0.03} = 57 \text{ k}\Omega$$

When the switch is open, $I_D = 0$ and

$$P_{Drawn} = V_{DD} I_D = 0$$

When the switch is closed, $I_D = 30$ μA, and

$$P_{Drawn} = V_{DD} I_D = 1.8 \times 30 = 54 \text{ μW}$$

Ex: 14.2 $V_{OH} = V_{CC} - 0 \times R_{C1,2} = 5$ V

$$V_{OL} = V_{CC} - I_{EE} R_{C1,2}$$

$$= 5 - 1 \times 2 = 3 \text{ V}$$

Ex: 14.3 If V_x remains unchanged at 0.089 V, then

$$k_n R_D = \frac{1}{V_x} = \frac{1}{0.089} = 11.24$$

For R_D to be 10 kΩ,

$$k_n = \frac{11.24}{10} = 1.124 \text{ mA/V}^2$$

Thus,

$$1.124 = 0.3 \times \frac{W}{L}$$

$$\Rightarrow \frac{W}{L} = 3.75$$

The parameters V_{OH}, V_{OL}, V_{IL}, V_{IH} and thus NM_L and NM_H do not depend on the value R (but on V_x) and thus their values will not change.

The current I_{DD} drawn from the power supply during the low-output interval becomes

$$I_{DD} = \frac{V_{DD} - V_{OL}}{R_D} = \frac{1.8 - 0.12}{10} = 168 \text{ μA}$$

and the power drawn from the supply during the low-output interval is

$$P_D = V_{DD} I_D = 1.8 \times 168 = 302 \text{ μW}$$

Since the inverter spends half of the time in this state, we have

$$P_{Daverage} = \frac{1}{2} P_D = 151 \text{ μW}$$

Ex: 14.4 $k_n R_D = \dfrac{1}{V_x}$

For $R_D = 10$ kΩ and

$$k_n = k_n' \left(\frac{W}{L}\right) = 0.3 \times 1.5 = 0.45 \text{ mA/V}^2$$

we obtain

$$V_x = \frac{1}{0.45 \times 10} = 0.22 \text{ V}$$

Now,

$$V_{OH} = 1.8 \text{ V}$$

From Eq. (14.22) we get

$$V_{OL} = \frac{1.8}{1 + \dfrac{1.8 - 0.5}{0.22}} = 0.26 \text{ V}$$

From Eq. (14.12) we obtain

$$V_{IL} = V_t + V_x = 0.5 + 0.22 = 0.72 \text{ V}$$

From Eq. (14.20) we have

$$V_{IH} = 0.5 + 1.63\sqrt{1.8 \times 0.22} - 0.22$$

$$= 1.31 \text{ V}$$

Thus,

$$NM_L = V_{IL} - V_{OL} = 0.72 - 0.26 = 0.46 \text{ V}$$

$$NM_H = V_{OH} - V_{IH} = 1.8 - 1.31 = 0.49 \text{ V}$$

During the output-low interval, we have

$$I_{DD} = \frac{V_{DD} - V_{OL}}{R_D}$$

$$= \frac{1.8 - 0.26}{10} = 154 \text{ μA}$$

and

$$P_D = 1.8 \times 154 = 277.2 \text{ μW}$$

Thus,

$$P_{Daverage} = \frac{1}{2} P_D = 138.6 \text{ μW} \simeq 139 \text{ μW}$$

Ex: 14.5 Refer to the circuit in Figure 14.21. With $v_I = V_M > V_t$, Q_N will be conducting. Since $v_O = V_M > |V_t|$ and the gate of Q_P is at ground, Q_P will be operating in the triode region, thus

$$i_{DP} = k_p\left[(V_{DD} - |V_t|)(V_{DD} - V_M) - \frac{1}{2}(V_{DD} - V_M)^2\right] \quad (1)$$

Since $v_O = v_I = V_M$, Q_N will be operating in saturation, thus

$$i_{DN} = \frac{1}{2}k_n(V_M - V_t)^2 \quad (2)$$

Equating (1) and (2) gives

$$\frac{1}{2}k_n(V_M - V_t)^2 = k_p\left[(V_{DD} - V_t)(V_{DD} - V_M) - \frac{1}{2}(V_{DD} - V_M)^2\right]$$

$$\frac{1}{2}r(V_M^2 - 2V_tV_M + V_t^2)$$

$$= \frac{1}{2}V_{DD}^2 - V_{DD}V_t + V_MV_t - \frac{1}{2}V_M^2$$

$$\left(\frac{1}{2}r + \frac{1}{2}\right)V_M^2 - (r+1)\,V_tV_M + \frac{1}{2}rV_t^2$$

$$= \frac{1}{2}V_{DD}^2 - V_{DD}V_t$$

$$(r+1)V_M^2 - 2(r+1)V_tV_M + rV_t^2 = V_{DD}^2 - 2V_{DD}V_t$$

$$(r+1)V_M^2 - 2(r+1)V_tV_M + (r+1)V_t^2$$

$$= V_{DD}^2 - 2V_{DD}V_t + V_t^2$$

$$(r+1)(V_M - V_t)^2 = (V_{DD} - V_t)^2$$

$$\Rightarrow V_M = V_t + \frac{V_{DD} - V_t}{\sqrt{r+1}} \qquad \text{Q.E.D.}$$

For $V_{DD} = 1.8$ V, $V_t = 0.4$ V, and $r = 5$, we get

$$V_M = 0.4 + \frac{1.8 - 0.4}{\sqrt{5+1}} = 0.97 \text{ V}$$

Ex: 14.6 (a) For $V_M = 0.6 \text{ V} = \frac{1}{2}V_{DD}$, the inverter must be matched, thus

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p}$$

$$\Rightarrow \frac{W_p}{0.13} = 4$$

$$\Rightarrow W_p = 0.52 \ \mu\text{m}$$

(b) $V_{OH} = V_{DD} = 1.2$ V

$V_{OL} = 0$ V

To obtain V_{IH}, we use Eq. (14.35):

$$V_{IH} = \frac{1}{8}(5V_{DD} - 2V_t)$$

$$= \frac{1}{8}(5 \times 1.2 - 2 \times 0.4)$$

$$= 0.65 \text{ V}$$

To obtain V_{IL}, we use Eq. (14.36):

$$V_{IL} = \frac{1}{8}(3V_{DD} + 2V_t)$$

$$= \frac{1}{8}(3 \times 1.2 + 2 \times 0.4)$$

$$= 0.55 \text{ V}$$

The noise margins can now be found as

$$NM_H = V_{OH} - V_{IH}$$

$$= 1.2 - 0.65 = 0.55 \text{ V}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 0.55 - 0 = 0.55 \text{ V}$$

(c) With $v_I = V_{DD}$ and v_O low, the output resistance is r_{DSN}:

$$r_{DSN} = 1\Big/\left[\mu_nC_{ox}\left(\frac{W}{L}\right)_n(V_{DD} - V_t)\right]$$

$$= \frac{1}{0.43 \times 1(1.2 - 0.4)} = 2.9 \text{ k}\Omega$$

With $v_I = 0$ V and $v_O = V_{DD}$, the output resistance is r_{DSP}:

$$r_{DSP} = 1\Big/\left[\mu_pC_{ox}\left(\frac{W}{L}\right)_p(V_{DD} - |V_t|)\right]$$

$$= \frac{1}{\frac{0.43}{4} \times 4(1.2 - 0.4)} = 2.9 \text{ k}\Omega$$

The two output resistances are equal because the inverter is matched.

(d) Using Eq. (14.39), we obtain

$$V_M = \frac{r(V_{DD} - |V_{tp}|) + V_{tn}}{r+1}$$

where

$$|V_{tp}| = V_{tn} = 0.4 \text{ V}$$

and

$$r = \sqrt{\frac{\mu_pW_p}{\mu_nW_n}} = \sqrt{\frac{1}{4} \times 1} = \frac{1}{2}$$

Thus,

$$V_M = \frac{0.5(1.2 - 0.4) + 0.4}{0.5 + 1} = 0.53 \text{ V}$$

Ex: 14.7 To obtain $V_M = 2.5 \text{ V} = \frac{1}{2}V_{DD}$, the inverter must be matched, thus

$$\mu_nC_{ox}\left(\frac{W}{L}\right)_n = \mu_pC_{ox}\left(\frac{W}{L}\right)_p$$

$$2\mu_p C_{ox}\left(\frac{W}{L}\right)_n = \mu_p C_{ox}\left(\frac{W}{L}\right)_p$$

$$\left(\frac{W}{L}\right)_p = 2\left(\frac{W}{L}\right)_n \qquad (1)$$

For $v_I = V_{DD} = 5$ V and $v_O = 0.2$ V, Q_N will be operating in the triode region. Thus,

$$i_D = \mu_n C_{ox}\left(\frac{W}{L}\right)_n \left[(V_{DD} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

$$0.2 = 0.05 \times \left(\frac{W}{L}\right)_n \left[(5-1)\times 0.2 - \frac{1}{2}\times 0.2^2\right]$$

$$\Rightarrow \left(\frac{W}{L}\right)_n = 5.13 \simeq 5$$

$$\left(\frac{W}{L}\right)_p = 2 \times 5.13 = 10.26 \simeq 10$$

Ex: 14.8 $It_{PLH} = C\dfrac{V_{DD}}{2}$

$$\Rightarrow t_{PLH} = CV_{DD}/2I$$

To obtain $t_{PLH} = 10$ ps with $C = 10$ fF and $V_{DD} = 1.8$ V, we need a current I obtained as follows:

$$10 \times 10^{-12} = \frac{10 \times 10^{-15} \times 1.8}{2I}$$

$$\Rightarrow I = \frac{1.8 \times 10^{-14}}{2 \times 10^{-11}} = 0.9 \text{ mA}$$

Ex: 14.9

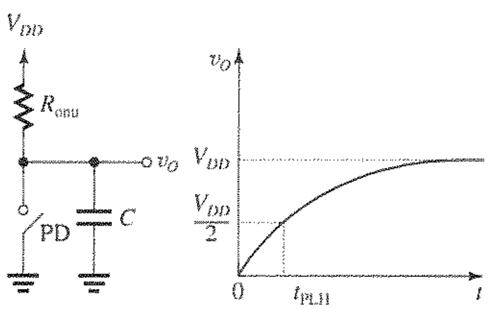


Figure 1(a)

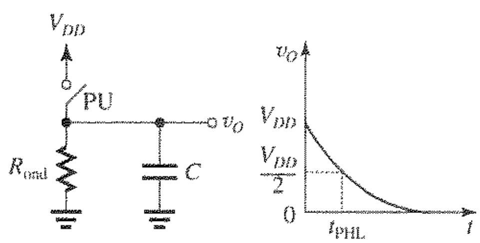


Figure 1(b)

To obtain t_{PLH}, consider the situation in Fig. 1(a). Here, PD has just opened (at $t = 0$) leaving $v_O = 0$ V at $t = 0+$. Capacitor C then charges through the on resistance of the pull-up switch, R_{onu}, toward V_{DD}, thus

$$v_O(t) = V_\infty - (V_\infty - V_{0+})e^{-t/\tau}$$

$$= V_{DD} - (V_{DD} - 0)e^{-t/\tau}$$

$$= V_{DD}(1 - e^{-t/\tau})$$

At $t = t_{PLH}$, $v_O = V_{DD}/2$, thus

$$\frac{V_{DD}}{2} = V_{DD}(1 - e^{-t_{PLH}/\tau})$$

$$\Rightarrow e^{-t_{PLH}/\tau} = 0.5$$

$$\Rightarrow t_{PLH} = \tau \ln 2 = 0.69\tau$$

For $C = 10$ fF and $R_{onu} = 20$ kΩ, then

$$\tau = 10 \times 10^{-15} \times 20 \times 10^3 = 200 \text{ ps}$$

and

$$t_{PLH} = 0.69 \times 200 = 138 \text{ ps}$$

Next we determine t_{PHL} by considering the situation depicted in Fig. 1(b). Here, PU has just opened, leaving $v_O(0+) = V_{DD}$. Capacitor C then discharges through the on resistance of the pull-down switch, R_{ond}, toward 0 V, thus $v_O(\infty) = 0$, thus

$$v_O = 0 - (0 - V_{DD})e^{-t/\tau}$$

$$= V_{DD}e^{-t/\tau}$$

At $t = t_{PHL}$, $v_O = V_{DD}/2$ and we get

$$\frac{V_{DD}}{2} = V_{DD}e^{-t_{PHL}/\tau}$$

$$\Rightarrow t_{PHL} = \tau \ln 2 = 0.69\tau$$

Here,

$$\tau = CR_{ond}$$

$$= 10 \times 10^{-15} \times 10 \times 10^3 = 100 \text{ ps}$$

Thus,

$$t_{PHL} = 69 \text{ ps}$$

The propagation delay t_P can now be obtained as

$$t_P = \frac{1}{2}(t_{PLH} + t_{PHL})$$

$$= \frac{1}{2}(138 + 69) = 104 \text{ ps}$$

Ex: 14.10

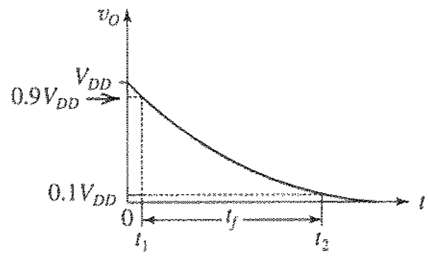


Figure 1

Figure 1 shows the exponential discharge curve and the two points that define the extent of the fall time, t_f. Here,

$v_O(t) = V_{DD}e^{-t/\tau}$

$v_O(t_1) = 0.9V_{DD} = e^{-t_1/\tau}$ (1)

$v_O(t_2) = 0.1V_{DD} = e^{-t_2/\tau}$ (2)

Dividing (1) by (2) gives

$9 = e^{-(t_1-t_2)/\tau}$

$9 = e^{t_f/\tau}$

$\Rightarrow t_f = \tau \ln 9 = 2.2\tau$

For $C = 100$ fF and $R = 2$ kΩ,

$\tau = 100 \times 10^{-15} \times 2 \times 10^3 = 200$ ps

and

$t_f = 2.2 \times 200 = 440$ ps $= 0.44$ ns

Ex: 14.11 $\alpha_n = 2\Big/\left[\frac{7}{4} - \frac{3V_{tn}}{V_{DD}} + \left(\frac{V_{tn}}{V_{DD}}\right)^2\right]$

$= 2\Big/\left[\frac{7}{4} - \frac{3 \times 0.5}{1.8} + \left(\frac{0.5}{1.8}\right)^2\right] = 2.01$

$t_{PHL} = \frac{\alpha_n C}{k_n'(W/L)_n V_{DD}}$

$= \frac{2.01 \times 10 \times 10^{-15}}{300 \times 10^{-6} \times 1.5 \times 1.8}$

$= 24.8$ ps

$\alpha_p = 2\Big/\left[\frac{7}{4} - \frac{3|V_{tp}|}{V_{DD}} + \left(\frac{V_{tp}}{V_{DD}}\right)^2\right]$

$= 2.01$

$t_{PLH} = \frac{\alpha_p C}{k_p'(W/L)_p V_{DD}}$

$= \frac{2.01 \times 10 \times 10^{-15}}{75 \times 10^{-6} \times 3 \times 1.8}$

$= 49.6$ ps

$t_P = \frac{1}{2}(t_{PHL} + t_{PLH})$

$= \frac{1}{2}(24.8 + 49.6)$

$= 37.2$ ps

Ex: 14.12 $t_{PHL} = 0.69 R_N C$

$50 \times 10^{-12} = 0.69 \times \frac{12.5}{(W/L)_n} \times 10^3 \times 20 \times 10^{-15}$

$\Rightarrow (W/L)_n = 3.5$

$t_{PLH} = 0.69 R_P C$

$50 \times 10^{-12} = 0.69 \times \frac{30}{(W/L)_p} \times 10^3 \times 20 \times 10^{-15}$

$\Rightarrow (W/L)_p = 8.3$

Note: If the 0.69 factor is replaced by 1 to account for the fact that the pulse edges are not ideal, then

$(W/L)_n = 5$

$(W/L)_p = 12$

Ex: 14.13 With an additional 0.1 pF, C becomes

$C = 6.25$ fF $+ 100$ fF $= 106.25$ fF

Thus,

$t_{PHL} = 25.8 \times \frac{106.25}{6.25}$

$= 438.6$ ps

$t_{PLH} = 31.5 \times \frac{106.25}{6.25}$

$= 535.5$ ps

Thus,

$t_P = \frac{1}{2}(438.6 + 535.5)$

$= 487$ ps

Ex: 14.14 Original area $= W_n L + W_p L$

$= L(W_n + W_p)$

$= 0.25(0.375 + 1.125)$

$= 0.375\ \mu\text{m}^2$

New area $= L(W_n + W_p)$

$= L \times 2W_n$

$= 0.25 \times 2 \times 0.375 = 0.1875\ \mu\text{m}^2$

which is half the original area. Thus, the area is reduced by 50%.

Next, we compute the new value of C as follows:

$C = 2C_{gd1} + 2C_{gd2} + C_{db1} + C_{db2} + C_{g3} + C_{g4} + C_w$

where

$C_{gd1} = 0.1125$ fF

$C_{gd2} = 0.3 \times W_p = 0.3 \times 0.375 = 0.1125$ fF

$C_{db1} = 1$ fF

$C_{db2} = 1$ fF

$C_{g3} = 0.7875$ fF

$C_{g4} = 0.375 \times 0.25 \times 6 + 2 \times 0.3 \times 0.375$

$= 0.7875$ fF

$C_w = 0.2$ fF

Thus,

$$C = 2 \times 0.1125 + 2 \times 0.1125 + 1 + 1$$
$$+ 0.7875 + 0.7875 + 0.2 = 4.225 \text{ fF}$$

$$t_{PHL} = 25.8 \times \frac{4.225}{6.25}$$

$= 17.4$ ps

$$t_{PLH} = 31.5 \times \frac{4.225}{6.25} \times \frac{1.125}{0.375}$$

$= 63.9$ ps

$$t_P = \frac{1}{2}(17.4 + 63.9) = 40.6 \text{ ps}$$

Ex: 14.15 $f_{\max} = \dfrac{1}{2t_P}$

$$= \frac{1}{2 \times 28.7 \times 10^{-12}} = 17.4 \text{ GHz}$$

Ex: 14.16 Refer to Example 14.7.

(a) $C_{\text{int}} = 2C_{gd1} + 2C_{gd2} + C_{db1} + C_{db2}$

$= 2 \times 0.1125 + 2 \times 0.3375 + 1 + 1$

$= 2.9$ fF

$C_{\text{ext}} = C_{g3} + C_{g4} + C_w$

$= 0.7875 + 2.3625 + 0.2$

$= 3.35$ fF

(b) To reduce the extrinsic component of t_P by a factor of 2, we need to scale $(W/L)_n$ and $(W/L)_p$ by a factor

$S = 2$

(c) The original value of $t_P = 28.7$ ps is composed of an intrinsic component

$$t_{P,\text{int}} = t_P \times \frac{C_{\text{int}}}{C}$$

$$= 28.7 \times \frac{2.9}{6.25} = 13.3 \text{ ps}$$

This component remains unchanged. The extrinsic component

$$t_{P,\text{ext}} = 28.7 - 13.3 = 15.4$$

is reduced by a factor of 2. Thus, t_P becomes

$$t_P = 13.3 + \frac{15.4}{2} = 21 \text{ ps}$$

(d) Area $= W_nL + W_pL$

$= L(W_n + W_p)$

By scaling W_n and W_p by a factor of 2, the area increases by a factor of 2.

Ex: 14.17 $L = 0.18$ μm, $n = 1.5$, $p = 3$

(a) Four-input NOR gate: Refer to Fig. 14.34.

For NMOS transistors: $W/L = n = 1.5 = \dfrac{0.27}{0.18}$

For PMOS transistors: $W/L = 4p = 12 = \dfrac{2.16}{0.18}$

(b) Four-input NAND gate: Refer to Fig. 14.35.

For NMOS transistors: $W/L = 4n = 6 = \dfrac{1.08}{0.18}$

For PMOS transistors: $W/L = p = 3 = \dfrac{0.54}{0.18}$

Area of NOR gate

$= 4 \times 0.18 \times 0.27 + 4 \times 0.18 \times 2.16 = 1.7496\ \mu\text{m}^2$

Area of NAND gate

$= 4 \times 0.18 \times 1.08 + 4 \times 0.18 \times 0.54 = 1.1664$

Thus,

$$\frac{\text{NOR area}}{\text{NAND area}} = \frac{1.7496}{1.1664} = 1.5$$

Ex: 14.18 Refer to Fig. 14.35.

(a) Maximum charging current is the current supplied by the four identical PMOS transistors. Minimum charging current is the current supplied by one of the PMOS transistors. Thus, the ratio of maximum to minimum currents is 4.

(b) There is only one possible configuration for discharging a load capacitance, namely, when all 4 NMOS transistors are conducting. So, as far as capacitor discharge is concerned, the ratio is one.

Ex: 14.19 $P_{\text{dyn}} = fCV_{DD}^2$

$= 1 \times 10^9 \times 6.25 \times 10^{-15} \times 2.5^2$

$= 39\ \mu$W

Ex: 14.20 $P_{\text{dyn}} = fCV_{DD}^2$

$= 100 \times 10^6 \times 100 \times 10^{-15} \times 1.8^2$

$= 32.4\ \mu\text{W}$

Ex: 14.21 $P_{\text{dyn}} = fCV_{DD}^2$

C decreases by a factor (0.13/0.5) and V_{DD} decreases from 5 V to 1.2 V; thus for the same f, the power dissipation will decrease by a factor

$$= \frac{0.5}{0.13} \times \frac{5}{1.2} = 66.8$$

Ex: 14.22 $PDP = fCV_{DD}^2 t_P$

When $f = f_{\max} = 1/2t_P$,

$$PDP = \frac{1}{2}CV_{DD}^2 = \frac{1}{2} \times 6.25 \times 10^{-15} \times 2.5^2$$

$= 19.5$ fJ

$$EDP = \frac{1}{2}CV_{DD}^2 t_P = 19.5 \times 10^{-15} \times 28.7 \times 10^{-12}$$

$= 5.6 \times 10^{-25}$ Js

Ex: 15.1 Since dynamic power dissipation is scaled by $\frac{1}{S^2}$ and propagation delay is scaled by $\frac{1}{S}$, hence, PDP is scaled by $\frac{1}{S^2} \times \frac{1}{S} = \frac{1}{S^3} = \frac{1}{8}$. Thus, PDP decreases by a factor of 8.

Ex: 15.2 If V_{DD} and V_t are kept constant, the entries in Table 15.1 that change are as follows:

Obviously, V_{DD} and V_t do not scale by $\frac{1}{S}$ anymore. They are kept constant!

$t_P \propto \frac{\alpha C}{k' V_{DD}}$; since α is a function of $\frac{V_t}{V_{DD}}$, then α remains unchanged, while C is scaled by $\frac{1}{S}$, and k' is scaled by S, therefore t_P is scaled by

$$\frac{1/S}{S} = \frac{1}{S^2}$$

Energy/Switching cycle, i.e., CV_{DD}^2, is scaled by $\frac{1}{S}$

$P_{dyn} \propto \frac{CV_{DD}^2}{2t_P}$ and thus is scaled by $\frac{1/S}{1/S^2} = S$ thus P_{dyn} increases.

The power density, i.e., $\frac{P_{dyn}}{\text{deive area}}$, is scaled by

$$\frac{S}{1/S^2} = S^3$$

Ex: 15.3 Using Eq. (15.5), we have

$$V_{DS\text{sat}} = \frac{L}{\mu_n} v_{\text{sat}} = \frac{0.25 \times 10^{-6}}{400 \times 10^{-4}} \times 10^7 \times 10^{-2}$$

$$= 0.63 \text{ V}$$

Ex: 15.4 For the NMOS transistor, $V_{GS} = 1.2$ V results in $V_{GS} - V_{tn} = 1.2 - 0.4 = 0.8$ V, which is greater than $V_{DS\text{sat}} = 0.34$ V. Also, $V_{DS} = 1.2$ V is greater than $V_{DS\text{sat}}$, thus both conditions in Eq. (15.12) are satisfied and the NMOS transistor will be operating in the velocity-saturation region and thus i_D is given by Eq. (15.11),

$$i_D = 430 \times 10^{-6} \times 1.5 \times 0.34\left(1.2 - 0.4 - \frac{1}{2} \times 0.34\right)$$
$$\times (1 + 0.1 \times 1.2) = 154.7 \,\mu\text{A}$$

If velocity-saturation were absent, the current would be

$$i_D = \frac{1}{2} \times 430 \times 10^{-6} \times 1.5(1.2 - 0.4)^2 \times$$
$$(1 + 0.1 \times 1.2) = 231.2 \,\mu\text{A}$$

Saturation is obtained over the range $V_{DS} = 0.34$ V to 1.2 V compared to $V_{DS} = V_{OV} = (1.2 - 0.4) = 0.8$ V to 1.2 V in the absence of velocity saturation.

For the PMOS transistor, we see that since $|V_{GS}| - |V_{tp}| = 0.8$ V and $|V_{DS}| = 1.2$ V are both larger than $|V_{DS\text{sat}}| = 0.6$ V the device will be operating in velocity saturation and

$$i_D =$$
$$110 \times 10^{-6} \times 1.5 \times 0.6\left(1.2 - 0.4 - \frac{1}{2} \times 0.6\right)$$
$$(1 + 0.1 \times 1.2) = 55.4 \,\mu\text{A}$$

$$0.6 \le V_{DS} \le 1.2 \text{ V}$$

Without velocity saturation

$$i_D = \frac{1}{2} \times 110 \times 10^{-6} \times 1.5 \times (1.2 - 0.4)^2 (1 + 0.1 \times 1.2)$$

$$= 59.1 \,\mu\text{A}$$

$V_{OV} \le V_{DS} \le 1.2$ V or $0.8 \text{ V} \le V_{DS} \le 1.2$ V

Note that the velocity saturation reduces the NMOS current by 33% and the PMOS current by ~ 6%.

Ex: 15.5 (a) Using Eq. (15.13), we have

$$i_D = I_S e^{v_{GS}/n V_T}$$

Thus,

$$\log i_D = \log I_S + \frac{v_{GS}}{nV_T} \log(e)$$

Thus, the slope of the straight line that represents $\log i_D$ versus v_{GS} is

$$\text{Slope} = \frac{\log(e)}{nV_T} = \frac{0.43}{nV_T}$$

or the inverse of the slope is

$= 2.3nV_T$ Q.E.D.

(b) $n = 1.22$, $i_D = 100 \times 10^{-9}$ A at $v_{GS} = 0.21$ V

Substituting in Eq. (15.13), we obtain

$$100 \times 10^{-9} = I_S e^{0.21/(1.22 \times 25 \times 10^{-3})}$$

$$\Rightarrow I_S = 0.1 \text{ nA}$$

This is the value of i_D at $v_{GS} = 0$.

(c) $I_{\text{total}} = 500 \times 10^6 \times 0.1 \times 10^{-9}$

$$= 50 \text{ mA}$$

$$P_D = V_{DD} I_{\text{total}}$$

$$= 1.2 \times 50 = 60 \text{ mW}$$

Ex: 15.6 $I_{stat} = \frac{1}{2}k_p(V_{DD} - V_t)^2$

To lower P_D to half its value, we must reduce I_{stat} to half its value. This can be accomplished by reducing $(W/L)_p$ to half the original value, thus

$$\left(\frac{W}{L}\right)_p = 0.5$$

Since r is to remain unchanged, the value of $(W/L)_n$ must be reduced by a factor of 2, thus

$$(W/L)_n = \frac{1}{2} \times 2.35 = 1.18$$

Both t_{PLH} and t_{PHL} will double, thus

$$t_{PLH} = 2 \times 0.16 = 0.32 \text{ ns}$$

$$t_{PHL} = 2 \times 0.02 = 0.04 \text{ ns}$$

Thus,

$$t_P = \frac{1}{2}(0.32 + 0.04) = 0.18 \text{ ns}$$

Since all the VTC breakpoints are functions of V_{DD}, V_t, and r only, they will remain unchanged and so will the noise margins.

Ex: 15.7 Using Eq. (15.24), we obtain

$$V_{OL} = (V_{DD} - V_t)\left[1 - \sqrt{1 - \frac{1}{r}}\right]$$

$$= (2.5 - 0.5) \times \left[1 - \sqrt{1 - \frac{1}{4}}\right]$$

$$V_{OL} = 0.27 \text{ V}$$

Using Eq. (15.26) and (15.27), we get

$$NM_L = V_t - (V_{DD} - V_t)\left[1 - \sqrt{1 - \frac{1}{r} - \frac{1}{\sqrt{r(r+1)}}}\right]$$

$$NM_L = 0.5 - (2)\left[1 - \sqrt{1 - \frac{1}{4} - \frac{1}{\sqrt{4(5)}}}\right]$$

$$= 0.68 \text{ V}$$

$$NM_H = (V_{DD} - V_t)\left(1 - \frac{2}{\sqrt{3r}}\right)$$

$$= (2) \times \left(1 - \frac{2}{\sqrt{3 \times 4}}\right) = 0.85 \text{ V}$$

$$r = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}{\mu_p C_{ox}\left(\frac{W}{L}\right)_p} = \frac{115\left(\frac{0.375}{0.25}\right)}{30\left(\frac{W}{L}\right)_p} = 4$$

$$\therefore \left(\frac{W}{L}\right)_p = 1.44$$

Using Eq. (15.25), we obtain

$$I_{stat} = \frac{1}{2}(30)(1.44)(2.5 - 0.5)^2 = 86.4 \,\mu\text{A}$$

$$P_D = I_{stat}V_{DD} = 86.4 \times 10^{-6} \times 2.5 = 0.22 \text{ mW}$$

Using Eqs. (15.28) and (15.29), we get

$$\alpha_p = 2\Big/\left[\frac{7}{4} - 3\left(\frac{0.5}{2.5}\right) + \left(\frac{0.5}{2.5}\right)^2\right] = 1.68$$

$$t_{PLH} = \frac{1.68 \times 7 \times 10^{-15}}{30 \times 10^{-6} \times 1.44 \times 2.5} = 0.11 \text{ ns}$$

Using Eq. (15.30) and (15.31), we obtain

$$\alpha_n = 2\Big/\left[1 + \frac{3}{4}\left(1 - \frac{1}{r}\right) - \left(3 - \frac{1}{r}\right)\left(\frac{0.5}{2.5}\right) + \left(\frac{0.5}{2.5}\right)^2\right]$$

$$= 1.9$$

$$t_{PHL} = \frac{1.9 \times 7 \times 10^{-15}}{115 \times 10^{-6} \times \left(\frac{.375}{.25}\right) \times 2.5} = 0.03 \text{ ns}$$

$$t_P = \frac{1}{2}(t_{PHL} + t_{PLH}) = \frac{1}{2}(0.11 + 0.03)$$

$$= 0.07 \text{ ns}$$

Ex: 15.8 $V_t = V_{t0} + \gamma\left(\sqrt{V_{OH} + 2\phi_f} - \sqrt{2\phi_f}\right)$

since $V_{OH} = V_{DD} - V_t$,

$$V_t = V_{t0} + \gamma\left(\sqrt{V_{DD} - V_t + 2\phi_f} - \sqrt{2\phi_f}\right)$$

Substituting values, we get

$$V_t = 0.5 + 0.3\left(\sqrt{1.8 - V_t + 0.85} - \sqrt{0.85}\right)$$

$$V_t = 0.5 + 0.3\left(\sqrt{2.65 - V_t} - 0.3\sqrt{0.85}\right)$$

$$V_t - 0.223 = 0.3\sqrt{2.65 - V_t}$$

Squaring both sides yields

$$V_t^2 - 0.446V_t + 0.05 = 0.09(2.65 - V_t)$$

so that, $V_t^2 - 0.356V_t - 0.189 = 0$

Solving this quadratic equation yields one practical value for V_t:

$$V_t = 0.648 \text{ V}$$

$$V_{OH} = V_{DD} - V_t = 1.8 \text{ V} - 0.648 \text{ V}$$

$$= 1.15 \text{ V}$$

Ex: 15.9 (a) Referring to Fig. 15.21, we obtain

$$V_{OH} = 5 \text{ V}$$

$$V_{OL} = 0 \text{ V}$$

(b) Referring to Fig. 15.21(a), we get

$$i_{DN}(0) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n (V_{DD} - V_{t0})^2$$

$$= \frac{1}{2}(50)\left(\frac{4}{2}\right)(5-1)^2 = 800\ \mu\text{A}$$

$$i_{DP}(0) = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p (V_{DD} - V_{t0})^2$$

$$= \frac{1}{2} \times 20 \times \left(\frac{4}{2}\right)(5-1)^2 = 320\ \mu\text{A}$$

Capacitor current is

$$i_C(0) = i_{DN}(0) + i_{DP}(0) = 800 + 320$$

$$= 1120\ \mu\text{A}$$

To obtain i_{DN} (t_{PLH}), we note that this situation is identical to that in Example 15.3 and we can use the result of part (c):

$$i_{DN}\ (t_{PLH}) = 50\ \mu\text{A}$$

$$i_{DP}\ (t_{PLH}) = k_p'\left(\frac{W}{L}\right)_p \times$$

$$\left[(V_{DD} - V_{t0})\frac{V_{DD}}{2} - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$= (20)\left(\frac{4}{2}\right)\left[(5-1)\left(\frac{5}{2}\right) - \frac{1}{2}\left(\frac{5}{2}\right)^2\right]$$

$$= 275\ \mu\text{A}$$

Thus, $i_C\ (t_{PLH}) = 50 + 275 = 325\ \mu\text{A}$

$$i_C|_{av} = \frac{1}{2}(1120 + 325) = 722.5\ \mu\text{A}$$

$$t_{PLH} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_C|_{av}} = \frac{70\left(10^{-15}\right)\left(\frac{5}{2}\right)}{722.5\left(10^{-6}\right)}$$

$$= 0.24\ \text{ns}$$

(c) Referring to Fig. 15.21(b), we obtain

$$i_{DN}(0) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n (V_{DD} - V_{t0})^2$$

$$= \frac{1}{2}(50)\left(\frac{4}{2}\right)(5-1)^2 = 800\ \mu\text{A}$$

$$i_{DP}\ (0) = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p (V_{DD} - V_{t0})^2$$

$$= \frac{1}{2}(20)\left(\frac{4}{2}\right)(5-1)^2 = 320\ \mu\text{A}$$

$$i_C\ (0) = i_{DN}\ (0) + i_{DP}\ (0) = 800 + 320$$

$$= 1120\ \mu\text{A}$$

$$i_{DN}\ (t_{PHL}) =$$

$$k_n'\left(\frac{W}{L}\right)_n \times \left[(V_{DD} - V_{t0})\frac{V_{DD}}{2} - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$= 50\left(\frac{4}{2}\right)\left[(5-1)\left(\frac{5}{2}\right) - \frac{1}{2}\left(\frac{5}{2}\right)^2\right]$$

$$= 688\ \mu\text{A}$$

To find i_{DP} (t_{PHL}), we first determine V_{tp} when $v_O = \frac{V_{DD}}{2}$, which corresponds to $V_{SB} = \frac{V_{DD}}{2}$

$$|V_{tp}| = V_{t0} + \gamma\left[\sqrt{\frac{V_{DD}}{2} + 2\phi_f} - \sqrt{2\phi_f}\right]$$

$$= 1 + 0.5\left[\sqrt{\frac{5}{2} + 0.6} - \sqrt{0.6}\right] = 1.49\ \text{V}$$

$$\text{Thus, } i_{DP}\ (t_{PHL}) = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p\left[\frac{V_{DD}}{2} - |V_{tp}|\right]^2$$

$$= \frac{1}{2}(20)\left(\frac{4}{2}\right)\left(\frac{5}{2} - 1.49\right)^2 = 20\ \mu\text{A}$$

$$i_C\ (t_{PHL}) = i_{DN}\ (t_{PHL}) + i_{DP}\ (t_{PHL})$$

$$= 688 + 20 = 708\ \mu\text{A}$$

$$i_C|_{av} = \frac{1120\ \mu\text{A} + 708\ \mu\text{A}}{2} = 914\ \mu\text{A}$$

So,

$$t_{PHL} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_C|_{av}} = \frac{70\left(10^{-15}\right)\left(\frac{5}{2}\right)}{914\left(10^{-6}\right)} = 0.19\ \text{ns}$$

Q_p will turn off when $v_O = |V_{tp}|$

where

$$|V_{tp}| = V_{t0} + \gamma\left[\sqrt{V_{DD} - |V_{tp}| + 2\phi_f} - \sqrt{2\phi_f}\right]$$

Solving for $|V_{tp}|$, we get

$$|V_{tp}| = 1 + 0.5\left[\sqrt{5 - |V_{tp}| + 0.6} - \sqrt{0.6}\right]$$

$$|V_{tp}| - 0.613 = 0.5\sqrt{5.6\ \text{V} - |V_{tp}|}$$

Squaring both sides, and setting one side equal to zero, we have the quadratic equation

$$|V_{tp}|^2 - 0.976\,|V_{tp}| - 1.024\ \text{V}^2 = 0$$

Solving, we get $|V_{tp}| = 1.6$ V

(d) $t_P = \frac{1}{2}(t_{PLH} + t_{PHL}) = \frac{1}{2}(0.24\ \text{ns} + 0.19\ \text{ns})$

$\simeq 0.22$ ns

Ex: 15.10 $R_{TG_{AV}} = \dfrac{R_{TG1} + R_{TG2}}{2}$

$$= \frac{4.5\ \text{k}\Omega + 6.5\ \text{k}\Omega}{2} = 5.5\ \text{k}\Omega$$

$$t_{PLH} = 0.69\ RC = 0.69\ (5.5\ \text{k}\Omega)\ (70)\ \left(10^{-15}\right)\ \text{F}$$

$t_{PLH} = 0.27$ ns, which is close to the value of 0.24 ns obtained in Exercise 15.9.

Ex: 15.11 $\left(\frac{W}{L}\right)_n = \left(\frac{W}{L}\right)_p = 1.5$

Using Eq. (15.49), we get

$$R_{TG} \approx \frac{12.5}{(W/L)_n} = \frac{12.5}{1.5} = 8.3\ \text{k}\Omega$$

Ex: 15.12 Using Eq. (14.57), we obtain

$$R_{P1} = \frac{30}{\left(\frac{W}{L}\right)_p}\ \text{k}\Omega = \frac{30}{(2)}\ \text{k}\Omega = 15\ \text{k}\Omega$$

With Eq. (15.49), we see that

$$R_{TG} = \frac{12.5}{\left(\frac{W}{L}\right)_n}\ \text{k}\Omega = \frac{12.5}{(1)}\ \text{k}\Omega = 12.5\ \text{k}\Omega$$

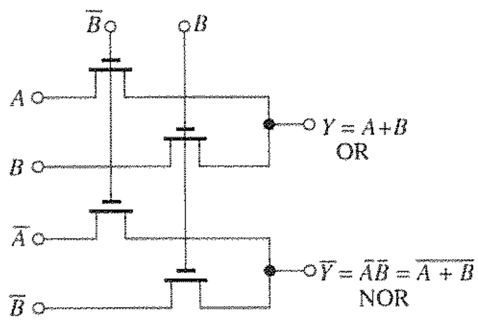
Using Eq. (15.51), we get

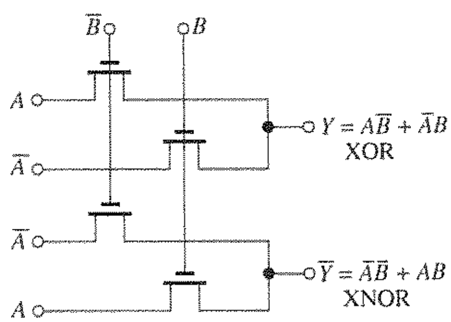
$$t_P = 0.69\left[(C_{\text{out1}} + C_{TG1})R_{P1} + (C_{\text{in2}} + C_{TG2}) \times (R_{P1} + R_{TG})\right]$$

$$= 0.69\left[(10\text{fF} + 5\text{fF})(15\ \text{k}\Omega) + (10\text{fF} + 5\text{fF}) \times (15\ \text{k}\Omega + 12.5\ \text{k}\Omega)\right]$$

$$t_P = 0.44\ \text{ns}$$

Ex: 15.13





Ex: 15.14 Since

$$i_D(V_{DD}) = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)_{eq}(V_{DD} - V_t)^2,$$

doubling $\left(\frac{W}{L}\right)$ will double $\left(\frac{W}{L}\right)_{eq}$ and $i_D(V_{DD})$

so $i_D(V_{DD}) = 2\,(76.1\ \mu\text{A}) = 152.2\ \mu\text{A}$

This new $\left(\frac{W}{L}\right)_{eq}$ will also double $i_D\left(\frac{V_{DD}}{2}\right)$:

$$i_D\left(\frac{V_{DD}}{2}\right) = 2\,(68.9\ \mu\text{A}) = 137.8\ \mu\text{A}$$

This doubles I_{av} to $2\,(72.5\ \mu\text{A}) = 145\ \mu\text{A}$.

The new t_{PHL} is

$$t_{PHL} = \frac{C\left(V_{DD} - \frac{V_{DD}}{2}\right)}{I_{av}}$$

$$= \frac{30\left(10^{-15}\right)\text{F}\,(1.8\ \text{V} - 0.9\ \text{V})}{145\left(10^{-6}\right)\ \text{A}} = 0.19\ \text{ns}$$

Ex: 15.15 The process technology parameters are those specified in Example 15.3 (page 1196).

Refer to Fig. E15.15

(a) $\left(\frac{W}{L}\right)_{eq1} = \frac{1}{2}\left(\frac{W}{L}\right) = \frac{1}{2} \times \frac{4}{2} = 1$

$$\left(\frac{W}{L}\right)_{eq2} = \frac{1}{2}\left(\frac{W}{L}\right) = \frac{1}{2} \times \frac{4}{2} = 1$$

(b) $i_{D1}(v_{Y1} = V_{DD}) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{eq1}(V_{DD} - V_t)^2$

$$= \frac{1}{2} \times 50 \times 1(5-1)^2$$

$$= 400\ \mu\text{A}$$

$$i_{D1}(v_{y1} = V_t) = k_n'\left(\frac{W}{L}\right)_{eq1}\left[(V_{DD} - V_t)V_t - \frac{1}{2}V_t^2\right]$$

$$= 50 \times 1\left[(5-1)1 - \frac{1}{2} \times 1\right]$$

$$= 175\ \mu\text{A}$$

$$i_{D1}|_{av} = \frac{400 + 175}{2} = 288\ \mu\text{A}$$

(c) $i_{D1}|_{av}\Delta t = C_{L1}\Delta v_{y1}$

$$\Delta t = \frac{C_{L1}(V_{DD} - V_t)}{i_{D1}|_{av}} = \frac{40 \times 10^{-15} \times 4}{288 \times 10^{-6}}$$

$$= 0.56\ \text{ns}$$

(d) Following the hint, we assume that Q_{eq2} remains saturated during Δt.

$$i_{D2}|_{av} = i_{D2}(v_{y1} = 3\text{ V}) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{eq2}(3-1)^2$$

$$i_{D2}|_{av} = \frac{1}{2} \times 50 \times 1(3-1)^2$$

$$= 100\ \mu\text{A}$$

(e) $$\Delta v_{y2} = -\frac{i_{D2}|_{av} \cdot \Delta t}{C_{L2}}$$

$$= -\frac{100 \times 10^{-6} \times 0.56 \times 10^{-9}}{40 \times 10^{-15}}$$

$$= -1.4\text{ V}$$

Thus, v_{y2} decreases to 3.6 V.

Ex: 15.16 $V_{OH} = 0$

$V_{OL} = -4 \times 0.22 = -0.88$ V

SHOULD BE SHIFTED BY -0.88 V

$V_{OH} = -0.88$ V AFTER SHIFTING

$V_{OL} = -1.76$ V AFTER SHIFTING

Ex: 15.17 Refer to Fig. E15.17. Neglecting the base current of Q_1, the current through R_1, D_1, D_2, and R_2 is

$$I = \frac{5.2 - V_{D1} - V_{D2}}{R_1 + R_2}$$

$$= \frac{5.2 - 0.75 - 0.75}{0.907 + 4.98} = 0.6285\text{ mA}$$

Thus, $V_B = -IR_1 = -0.57$ V

$V_R = V_B - V_{BE1} = -0.57 - 0.75 = -1.32$ V

Ex: 15.18 Refer to Fig. 15.35.

$$I_E = \frac{V_R - V_{BE}|_{QR} - (-V_{EE})}{R_E}$$

$$= \frac{-1.32 - 0.75 + 5.2}{0.779} \approx 4\text{ mA}$$

$V_C|_{Q_R} = -\alpha \times 4 \times R_{C2} \cong -4 \times 0.245 = -1$ V

$V_C|_{Q_AQ_B} = 0$ V (because the current through R_{C1} is zero)

Ex: 15.19 Refer to Fig. 15.38(e).

$$V_{th} = \frac{V_{DD}}{2} = 2.5\text{ V}$$

$2.5 = V_{GSN} + V_{BE2}$

$= V_{GSN} + 0.7$

$\Rightarrow V_{GSN} = 1.8$ V

$$I_{DN} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n(V_{GSN} - V_t)^2$$

$$= \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n(1.8 - 0.6)^2 \qquad (1)$$

$$I_{DP} = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p\left(V_{DD} - \frac{V_{DD}}{2} - V_t\right)^2$$

$$= \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p(5 - 2.5 - 0.6)^2 \qquad (2)$$

Equating (1) and (2) gives

$$k_n'\left(\frac{W}{L}\right)_n(1.2)^2 = k_p'\left(\frac{W}{L}\right)_p(1.9)^2$$

Substituting $k_n' = 2.5k_p'$ and $L_n = L_p$ yields

$$\frac{W_p}{W_n} = 1$$

Ex: 16.1

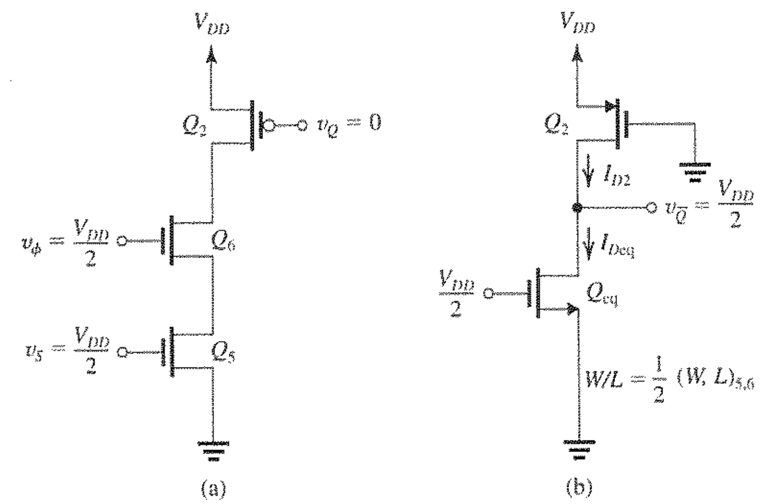


Figure 1

Refer to Fig. 1(a). When $v_\phi = v_S = \frac{V_{DD}}{2}$, Q_{eq} (Fig. 1(b)) will be operating in saturation. From Fig. 1(b) we see that Q_2 will be operating in the triode region. For Q_{eq} we have

$$I_{Deq} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_{eq} \left(\frac{V_{DD}}{2} - V_{tn}\right)^2$$

$$= \frac{1}{2} \times 300 \times \frac{1}{2}\left(\frac{W}{L}\right)_{5,6} \left(\frac{1.8}{2} - 0.5\right)^2$$

$$= 12\left(\frac{W}{L}\right)_{5,6},\ \mu\text{A}$$

For Q_2, we have

$$I_{D2} =$$
$$k_p'\left(\frac{W}{L}\right)_2 \left[(V_{DD} - |V_{tp}|)\left(\frac{V_{DD}}{2}\right) - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2 \right]$$

$$= 75 \times \frac{1.08}{0.18}\left[(1.8 - 0.5)\ 0.9 - \frac{1}{2}(0.9)^2\right]$$

$$= 344.25\ \mu\text{A}$$

Equating I_{Deq} to I_{D2} gives

$$\left(\frac{W}{L}\right)_{5,6} = \frac{344.25}{12} = 28.7$$

Ex: 16.2 Bits for row address:

$$2^M = 1{,}024$$

$$\log_2(2^M) = \log_2(1{,}024)$$

$$M \times \log_2(2) = \log_2(1{,}024)$$

$$M = \frac{\log_2(1{,}024)}{\log_2(2)} = 10$$

Bits for column address:

$$2^N = 128$$

$$N = \frac{\log_2(128)}{\log_2(2)} = 7$$

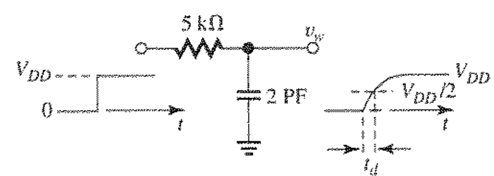
Bits for block address:

$$2^{\text{Bits}} = 32$$

$$\text{Bits} = \frac{\log_2(32)}{\log_2(2)} = 5$$

Ex: 16.3 $v_w = V_{DD}\left(1 - e^{-t/CR}\right)$

$$\frac{V_{DD}}{2} = V_{DD}\left(1 - e^{t_d/CR}\right)$$



$$t_d = CR \ln 2$$

$$= 2 \times 10^{-12} \times 5 \times 10^3 \times 0.69$$

$$= 6.9 \text{ ns}$$

Ex: 16.4 $$\left.\frac{\left(\frac{W}{L}\right)_a}{\left(\frac{W}{L}\right)_n}\right|_{\max} = \frac{1}{\left(1 - \frac{V_{tn}}{V_{DD} - V_{tn}}\right)^2} - 1$$

$$\left.\left(\frac{W}{L}\right)_a\right|_{\max} = 1.5 \times \left[\frac{1}{\left(1 - \frac{0.5}{1.8 - 0.5}\right)^2} - 1\right] = 2.5$$

$$\Rightarrow \left(\frac{W}{L}\right)_a \le 2.5$$

Ex: 16.5 (a) $\left(\frac{W}{L}\right)_a = 2.5$

This case corresponds to $V_{\overline{Q}} = V_{tn} = 0.5$ V. The value of I_5 can be found using Eq. (16.1), namely

$$I_5 = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)_5 (V_{DD} - V_{tn} - V_{\overline{Q}})^2$$

where

$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_a = 2.5$$

Thus,

$$I_5 = \frac{1}{2} \times 300 \times 2.5\ (1.8 - 0.5 - 0.5)^2$$

$= 240\ \mu$A

We now can find Δt from

$$\Delta t = \frac{C_{\overline{B}} \Delta V}{I_5}$$

Thus,

$$\Delta t = \frac{2 \times 10^{-12} \times 0.2}{240 \times 10^{-6}} = 1.7 \text{ ns}$$

(b) $(W/L)_a = 1.5$

We first determine $V_{\overline{Q}}$ using Eq. (16.3),

$$\frac{V_{\overline{Q}}}{V_{DD} - V_{tn}} = 1 - 1/\sqrt{1 + \frac{(W/L)_5}{(W/L)_1}}$$

where $(W/L)_5 = 1.5$, thus

$$\frac{V_{\overline{Q}}}{1.8 - 0.5} = 1 - 1/\sqrt{1 + \frac{1.5}{1.5}}$$

$\Rightarrow V_{\overline{Q}} = 0.38$ V

Next, we use Eq. (16.1) to determine I_5 as

$$I_5 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_5 (V_{DD} - V_{tn} - V_{\overline{Q}})^2$$

$$= \frac{1}{2} \times 300 \times 1.5\ (1.8 - 0.5 - 0.38)^2$$

$= 190.4\ \mu$A

Finally, we determine Δt using Eq. (16.7) as

$$\Delta t = \frac{C_{\overline{B}} \Delta V}{I_5}$$

$$= \frac{2 \times 10^{-12} \times 0.2}{190.4 \times 10^{-6}} = 2.1 \text{ ns}$$

Ex: 16.6

$$\left(\frac{W}{L}\right)_p \le \left(\frac{W}{L}\right)_a \times \frac{\mu_n}{\mu_p}\left[1 - \left(1 - \frac{V_{tn}}{V_{DD} - V_{tn}}\right)^2\right]$$

$$\left(\frac{W}{L}\right)_p \le \left(\frac{W}{L}\right)_a \times 4 \times \left[1 - \left(1 - \frac{0.5}{1.8 - 0.5}\right)^2\right]$$

$$\left(\frac{W}{L}\right)_p \le 2.5\left(\frac{W}{L}\right)_a \text{ or}$$

$$\left(\frac{W}{L}\right)_p \le 2.5 \times 2.5 \Rightarrow \left(\frac{W}{L}\right)_p \le 6.25$$

For minimum area, select
$W_n = W_p = W_a = 0.18\ \mu$m

Ex: 16.7 From Eqs. 16.14 and 16.15, we have

$$\Delta V(1) \simeq \frac{C_S}{C_B} \times \frac{V_{DD}}{2} = \frac{30 \times 10^{-15}}{0.3 \times 10^{-12}} \times \frac{1.2}{2}$$

$= 60$ mV

$$\Delta V(0) \simeq -\frac{C_S}{C_B} \times \frac{V_{DD}}{2}$$

$$= \frac{-30 \times 10^{-15}}{0.3 \times 10^{-12}} \times \frac{1.2}{2} = -60 \text{ mV}$$

Ex: 16.8 Area of the storage array

$= 64 \times 1024 \times 1024 \times 2 = 134217728\ \mu\text{m}^2$

$= 134.2\ \text{mm}^2$ or equivalently

$= 11.6$ mm $\times$ 11.6 mm

Total chip area

$= 1.3 \times 134.2 = 174.46\ \text{mm}^2 = 13.2 \times 13.2\ \text{mm}^2$

Ex: 16.9 Refer to Example 16.2.

Since Δt is proportional to $\tau = \frac{C}{G_m}$, we can reduce Δt by a factor of 2 by decreasing τ by the same factor. $\Delta t \propto \tau \propto \frac{1}{G_m}$.

Hence, G_m has to be doubled. $G_m = g_{mn} + g_{mb}$ and both g_{mn} and g_{mb} have to be increased by a factor of 2. The increase in g_m can be achieved by increasing the corresponding $\frac{W}{L}$, thus

$$\left(\frac{W}{L}\right)_n = 2 \times \frac{0.54}{0.18} = 6$$

$$\left(\frac{W}{L}\right)_p = 2 \times \frac{2.16}{0.18} = 24$$

Ex: 16.10 Refer to Example 16.2.

The time, Δt, for v_B to reach $0.9V_{DD}$ can be obtained from

$$0.9V_{DD} = 0.5V_{DD} + \Delta V e^{\Delta t/\tau}$$

where

$V_{DD} = 1.8$V, $\Delta V = 0.05$ V, $\tau = 1.4$ ns

Thus,

$$\Delta t = \tau \ln\left(\frac{0.4\ V_{DD}}{0.05}\right)$$

$$= 1.4 \ln\left(\frac{0.4 \times 1.8}{0.05}\right)$$

$$= 3.7 \text{ ns}$$

This is an increase of $(3.7 - 2.8)/2.8 = 32\%$.

Ex: 16.11 From Eq. 16.18: $\Delta t = \frac{CV_{DD}}{I}$ or

$$I = \frac{CV_{DD}}{\Delta t} = \frac{50 \times 10^{-15} \times 1.8}{0.5 \times 10^{-9}} = 180\ \mu\text{A}$$

$$P = V_{DD}I = 1.8 \times 180\ \mu\text{A} = 324\ \mu\text{W}$$

Ex: 16.12 Refer to Fig. 16.25.

Our decoder is an extension of that shown:

We have M bits is the address (as opposed to 3) and correspondingly there will be 2^M word lines. Now, each of the 2^M word lines is connected to M NMOS devices and to one PMOS transistor. Thus the total number of devices required is

$$M2^M \text{ (NMOS)} + 2^M \text{ (PMOS)}$$

$$= 2^M (M + 1)$$

Ex: 16.13 Refer to Fig. 16.27: Our tree decoder will have 2^N bit lines. Thus it will have N levels: At the first levels there will be 2 transistors, at the second $2^2, \ldots$, at the Nth level there will be 2^N transistors. Thus the total number of transistors can be found as

$$\text{Number} = 2 + 2^2 + 2^3 + \ldots + 2^N$$

$$= 2\underbrace{\left(1 + 2 + 2^2 + \ldots + 2^{N-1}\right)}$$

Geometric series r = 2

$$\text{Sum} = \frac{r^N - 1}{r - 1} = \frac{2^N - 1}{2 - 1}$$

$$= 2^N - 1$$

Thus,

$$\text{Number} = 2\left(2^N - 1\right)$$

Ex: 16.14 $f = \frac{1}{2 \times 5t_P}$

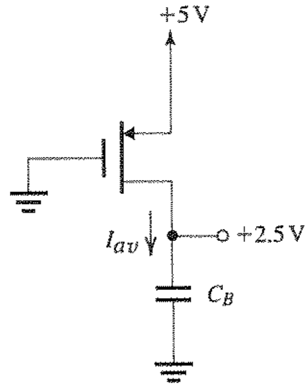
$$= \frac{1}{2 \times 5 \times 10^{-9}} = 100 \text{ MHz}$$

Ex: 16.15

(a) $I_{av} = k_p'\left(\frac{W}{L}\right)_p \left[(5 - 1)\,2.5 - \frac{1}{2}2.5^2\right]$

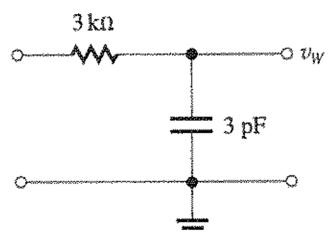
$$= 20 \times \frac{24}{2}\,[10 - 3.125]$$

$$= 1.65 \text{ mA}$$



Thus, $t_{\text{Charging}} = \frac{2 \times 10^{-12} \times 5}{1.65 \times 10^{-3}} = 6.1 \text{ ns}$

(b)

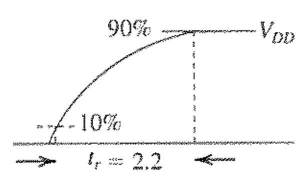


$$t_r \simeq 2.2\tau$$

$$= 2.2 \times 3 \times 10^{-12} \times 3 \times 10^3$$

$$= 19.8 \text{ ns}$$

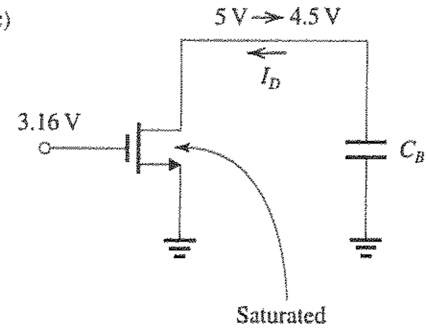
In one time-constant the voltage reached is



$$V_{DD}\left(1 - e^{-1}\right) = 0.632V_{DD}$$

$$= 3.16 \text{ V}$$

(c)



$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n(3.16 - 1)^2$$

$$= \frac{1}{2} \times 50 \times \frac{6}{2} \times 2.16^2$$

$$= 0.35 \text{ mA}$$

$$\Delta t = \frac{C_B \Delta V}{I_D}$$

$$= \frac{2 \times 10^{-12} \times 0.5}{0.35 \times 10^{-3}} = 2.9 \text{ ns}$$

Ex: 17.1 $A = -20\ \log|T|\,[\text{dB}]$

$\lvert T\rvert =$	1	0.99	0.9	0.8	0.7	0.5	0.1	0
$A \simeq$	0	0.1	1	2	3	6	20	∞

Ex: 17.2

$$A_{\max} = 20\ \log 1.05 - 20\ \log 0.95 \simeq 0.9\ \text{dB}$$

$$A_{\min} = 20\ \log\left[\frac{1}{0.01}\right] = 40\ \text{dB}$$

Ex: 17.3

$$T(s) = k\frac{(s+j2)\,(s-j2)}{\left(s+\frac{1}{2}+j\sqrt{\frac{3}{2}}\right)\left(s+\frac{1}{2}-j\sqrt{\frac{3}{2}}\right)}$$

$$= k\frac{(s^2+4)}{s^2+s+\frac{1}{4}+\frac{3}{4}}$$

$$= k\frac{(s^2+4)}{s^2+s+1}$$

$$T(0) = k\frac{4}{1} = 1$$

$$k = \frac{1}{4}$$

$$\therefore T(s) = \frac{1}{4}\frac{(s^2+4)}{s^2+s+1}$$

Ex: 17.4

$$T(s) = a_3\frac{s\,(s^2+4)}{(s+0.1+j0.8)\,(s+0.1-j0.8)}\times \frac{1}{(s+0.1+j1.2)\,(s+0.1-j1.2)}$$

$$= a_3\frac{s\,(s^2+4)}{(s^2+0.2s+0.65)\,(s^2+0.2s+1.45)}$$

Ex: 17.5

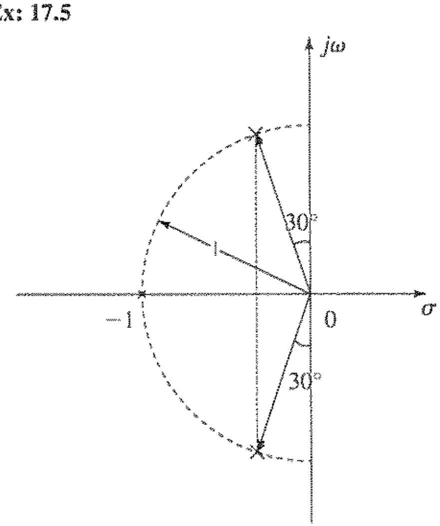


Figure 1

From Fig.1 we see that the real pole is at

$$s = -1$$

and thus gives rise to a factor $(s+1)$ in the denominator. The pair of complex conjugate poles are at

$$-\cos 60^\circ \pm j\,\sin 60^\circ$$

$$= -0.5 \pm j\sqrt{3}/2$$

The corresponding quadratic in the denominator will be

$$= \left(s+0.5+j\frac{\sqrt{3}}{2}\right)\left(s+0.5-j\frac{\sqrt{3}}{2}\right)$$

$$= s^2+s+1$$

Since the filter is of the all-pole type, the transfer function will be

$$T(s) = k\frac{1}{(s+1)(s^2+s+1)}$$

Since the dc gain is unity, $k = 1$ and

$$T(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\omega^2}\sqrt{(1-\omega^2)^2+\omega^2}}$$

$$= \frac{1}{\sqrt{(1+\omega^2)(1-\omega^2)^2+\omega^2(1+\omega^2)}}$$

$$= \frac{1}{\sqrt{(1-\omega^4)(1-\omega^2)+\omega^2+\omega^4}}$$

$$= \frac{1}{\sqrt{1-\omega^4-\omega^2+\omega^6+\omega^2+\omega^4}}$$

$$= \frac{1}{\sqrt{1+\omega^6}} \qquad \text{Q.E.D.}$$

$|T(j\omega)| = 1/\sqrt{2}$ at $\omega = \omega_{3dB}$, thus

$$\frac{1}{\sqrt{1+\omega_{3dB}^6}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \omega_{3dB} = 1\ \text{rad/s}$$

At $\omega = 3$ rad/s, we have

$$|T| = \frac{1}{\sqrt{1+3^6}} = \frac{1}{\sqrt{730}}$$

$$A = -20\ \log|T| = -20\ \log(1/\sqrt{730})$$

$$= 10\ \log(730) = 28.6\ \text{dB}$$

Ex: 17.6

$\epsilon = \sqrt{10^{\frac{A_{\max}}{10}} - 1} = \sqrt{10^{\frac{1}{10}} - 1} = 0.5088$

$|T(j\omega)| = \dfrac{1}{\sqrt{1 + \epsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}}}$

$A(\omega_s) = -20\ \log|T(j\omega_s)|$

$= 10\ \log\left[1 + \epsilon^2\left(\dfrac{\omega_s}{\omega_p}\right)^{2N}\right]$

Thus, $10\ \log\left[1 + 0.5088^2 \times 1.5^{2N}\right] \geq 30$

$N = 10$: LHS $= 29.35$ dB

$N = 11$: LHS $= 32.87$ dB

$\therefore$ Use $N = 11$ and obtain

$A_{\min} = 32.87$ dB

For $A_{\min}$ to be exactly 30 dB, we need

$10\ \log\left[1 + \epsilon^2 \times 1.5^{22}\right] = 30$

$\epsilon \Rightarrow 0.3654 \Rightarrow A_{\max} = 20\ \log\sqrt{1 + 0.3654^2}$

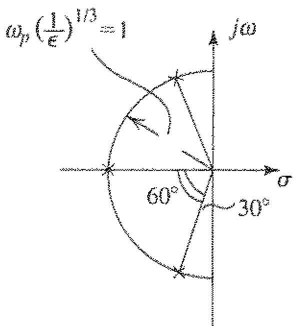
$= 0.54$ dB

Ex: 17.7 The real pole is at $s = -1$

The complex conjugate poles are at

$s = -\cos 60° \pm j \sin 60°$

$= -0.5 \pm j\sqrt{\dfrac{3}{2}}$



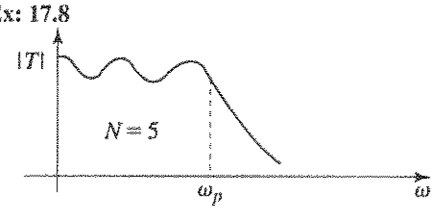
$T(s) =$

$$\frac{1}{(s+1)\left(s + 0.5 + j\sqrt{\frac{3}{2}}\right)\left(s + 0.5 - j\sqrt{\frac{3}{2}}\right)}$$

$= \dfrac{1}{(s+1)(s^2 + s + 1)}$

DC gain $= 1$

Ex: 17.8



$|T(j\omega)| = \dfrac{1}{\sqrt{1 + \epsilon^2 \cos^2\left[N \cos^{-1}\left(\dfrac{\omega}{\omega_p}\right)\right]}}$

for $\omega < \omega_p$.

Peaks are obtained when

$\cos^2\left[N \cos^{-1}\left(\dfrac{\hat{\omega}}{\omega_p}\right)\right] = 0$

$\cos^2\left[5 \cos^{-1}\left(\dfrac{\hat{\omega}}{\omega_p}\right)\right] = 0$

$5 \cos^{-1}\left(\dfrac{\hat{\omega}}{\omega_p}\right) = (2k+1)\dfrac{\pi}{2},\ k = 0,\ 1,\ 2$

$\therefore \hat{\omega} = \omega_p \cos\left[\dfrac{(2k+1)\pi}{10}\right],\ k = 0, 1, 2$

$\hat{\omega}_1 = \omega_p \cos\left(\dfrac{\pi}{10}\right) = 0.95\omega_p$

$\hat{\omega}_2 = \omega_p \cos\left(\dfrac{3}{10}\pi\right) = 0.59\omega_p$

$\hat{\omega}_3 = \omega_p \cos\left(\dfrac{5}{10}\pi\right) = 0$

Valleys are obtained when

$\cos^2\left[N \cos^{-1}\left(\dfrac{\breve{\omega}}{\omega_p}\right)\right] = 1$

$5 \cos^{-1}\left(\dfrac{\breve{\omega}}{\omega_p}\right) = k\pi,\ k = 0, 1, 2$

$\therefore \breve{\omega} = \omega_p \cos\left(\dfrac{k\pi}{5}\right),\ k = 0, 1, 2$

$\breve{\omega}_1 = \omega_p \cos 0 = \omega_p$

$\breve{\omega}_2 = \omega_p \cos\dfrac{\pi}{5} = 0.81\omega_p$

$\breve{\omega}_3 = \omega_p \cos\dfrac{2\pi}{5} = 0.31\omega_p$

Ex: 17.9

$\epsilon = \sqrt{10^{\frac{A_{\max}}{10}} - 1} = \sqrt{10^{\frac{0.5}{10}} - 1} = 0.3493$

$A(\omega_s) = 10\ \log\left[1 + \epsilon^2 \cosh^2\left(N \cosh^{-1}\dfrac{\omega_s}{\omega_p}\right)\right]$

$= 10\ \log\left[1 + 0.3493^2 \cosh^2(7 \cosh^{-1} 2)\right]$

$= 64.9$ dB

For $A_{max} = 1$ dB, $\epsilon = \sqrt{10^{0.1} - 1} = 0.5088$

$$A(\omega_s) = 10 \log\left[1 + 0.5088^2 \cosh^2\left(7 \cosh^{-1} 2\right)\right]$$

$$= 68.2 \text{ dB}$$

This is an increase of 3.3 dB

Ex: 17.10 $\epsilon = \sqrt{10^{\frac{1}{10}} - 1} = 0.5088$

(a) For the Chebyshev filter:

$$A(\omega_s)$$

$$= 10 \log\left[1 + 0.5088^2 \cosh^2\left(N \cosh^{-1} 1.5\right)\right]$$

$$\geq 50 \text{ dB}$$

$N = 7.4$ $\therefore$ choose $N = 8$

Excess attenuation =

$$10 \log\left[1 + 0.5088^2 \cosh^2\left(8 \cosh^{-1} 1.5\right)\right] - 50$$

$$= 55 - 50 = 5 \text{ dB}$$

(b) For a Butterworth filter

$\epsilon = 0.5088$

$$A(\omega_s) = 10 \log\left[1 + \epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$$

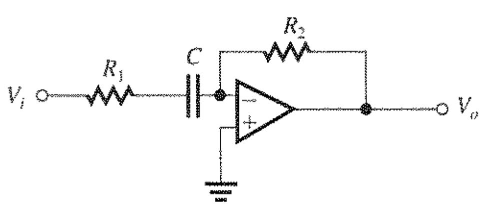
$$= 10 \log\left(1 + 0.5088^2 (1.5)^{2N}\right) \geq 50$$

$N = 15.9$ $\therefore$ choose $N = 16$

Excess attenuation =

$$10 \log\left(\left[1 + 0.5088^2 (1.5)^{32}\right] - 50\right) = 0.5 \text{ dB}$$

Ex: 17.11



$10^4 = \dfrac{1}{CR_1}$, $R_1 = 10$ kΩ

$C = 0.01$ μF

H.F. gain $= \dfrac{-R_2}{R_1} = -10$

$R_2 = 100$ kΩ

Ex: 17.12 Refer to Fig. 17.14.

$$\omega_0 = \frac{1}{CR} = 10^3 \text{ rad/s}$$

For R arbitrarily selected to be 10 kΩ,

$$C = \frac{1}{10^3 \times 10^4} = 0.1 \text{ μF}$$

The two resistors labeled R_1 can also be selected to be a convenient value, say 10 kΩ each.

Ex: 17.13 $T(s) = \dfrac{\omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$

For maximally flat response, $Q = 1/\sqrt{2}$, thus

$$T(s) = \frac{\omega_0^2}{s^2 + s\sqrt{2}\,\omega_0 + \omega_0^2}$$

$$T(j\omega) = \frac{\omega_0^2}{(\omega_0^2 - \omega^2) + j\sqrt{2}\,\omega\omega_0}$$

$$|T(j\omega)| = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 2\omega^2\omega_0^2}}$$

$$= \frac{\omega_0^2}{\sqrt{\omega_0^4 + \omega^4}}$$

$$= \frac{1}{\sqrt{1 + \left(\dfrac{\omega}{\omega_0}\right)^4}}$$

At $\omega = \omega_0$,

$$|T(j\omega_0)| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

which is 3 dB below the value at $\omega = 0$ (0 dB). Q.E.D.

Ex: 17.14 $T(s) = \dfrac{sK\left(\dfrac{\omega_0}{Q}\right)}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$

where K is the center-frequency gain. For $\omega_0 = 10^5$ rad/s and 3-dB bandwidth $= 10^3$ rad/s, we have

$$\text{3-dB } BW = \frac{\omega_0}{Q}$$

$$10^3 = \frac{10^5}{Q}$$

$$\Rightarrow Q = 100$$

Also, for a center-frequency gain of 10, we have

$K = 10$

Thus,

$$T(s) = \frac{10^4 s}{s^2 + 10^3 s + 10^{10}}$$

Ex: 17.15 (a) $T(s) = a_2 \dfrac{s^2 + \omega_0^2}{s^2 + s\left(\dfrac{\omega_0}{Q}\right) + \omega_0^2}$

$$|T(j\omega)| = |a_2| \frac{\omega_0^2 - \omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \dfrac{\omega^2\omega_0^2}{Q^2}}}$$

$$= |a_2| \Bigg/ \sqrt{1 + \frac{\omega^2\omega_0^2}{Q^2(\omega_0^2 - \omega^2)^2}} \qquad (1)$$

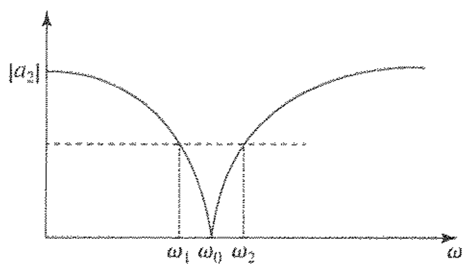


Figure 1

Refer to Fig. 1 and note that at any value of $|T|$ there are two frequencies, ω_1 and ω_2, with this gain value. From Eq. (1) we obtain

$$\frac{\omega_1^2\,\omega_0^2}{Q^2(\omega_0^2 - \omega_1^2)^2} = \frac{\omega_2^2\,\omega_0^2}{Q^2(\omega_2^2 - \omega_0^2)^2}$$

$$\Rightarrow \frac{\omega_1}{\omega_0^2 - \omega_1^2} = \frac{\omega_2}{\omega_2^2 - \omega_0^2}$$

$$\omega_1\omega_2^2 - \omega_1\omega_0^2 = \omega_2\omega_0^2 - \omega_1^2\omega_2$$

$$\omega_1\omega_2(\omega_1 + \omega_2) = (\omega_1 + \omega_2)\omega_0^2$$

$$\Rightarrow \omega_1\omega_2 = \omega_0^2 \qquad \text{Q.E.D.}$$

Now if ω_1 and ω_2 differ by BW_a,

$$\omega_2 - \omega_1 = BW_a$$

and if the attenuation over this band of frequencies is to be greater than A dB, then by using Eq. (1) we have

$$10\log_{10}\left[1 + \frac{\omega_0^2\,\omega_1^2}{(\omega_0^2 - \omega_1^2)^2 Q^2}\right] \geq A$$

$$\frac{\omega_1\,\omega_0}{(\omega_0^2 - \omega_1^2)Q} \geq \sqrt{10^{A/10} - 1}$$

$$\frac{1}{Q}\,\frac{\omega_1\,\omega_0}{(\omega_1\omega_2 - \omega_1^2)} \geq \sqrt{10^{A/10} - 1}$$

$$\frac{1}{Q}\,\frac{\omega_0}{\omega_2 - \omega_1} \geq \sqrt{10^{A/10} - 1}$$

$$\frac{1}{Q}\,\frac{\omega_0}{BW_a} \geq \sqrt{10^{A/10} - 1}$$

$$Q \leq \frac{\omega_0}{BW_a\sqrt{10^{A/10} - 1}}$$

(b) For $A = 3$ dB we have

$$Q = \frac{\omega_0}{BW_a\sqrt{10^{3/10} - 1}}$$

$$\Rightarrow BW_a = \frac{\omega_0}{Q} \qquad \text{Q.E.D.}$$

Ex: 17.16 From Fig. 17.16(c) we have

$$\omega_{\max} = \sqrt{\frac{\left(\dfrac{\omega_n}{\omega_0}\right)^2\left(1 - \dfrac{1}{2Q^2}\right) - 1}{\left(\dfrac{\omega_n}{\omega_0}\right)^2 + \dfrac{1}{2Q^2} - 1}}$$

$$= \sqrt{\frac{\left(\dfrac{1.2}{1}\right)^2\left(1 - \dfrac{1}{2 \times 100}\right) - 1}{\left(\dfrac{1.2}{1}\right)^2 + \dfrac{1}{2 \times 100} - 1}}$$

$= 0.986$ rad/s

$$T_{\max} = \frac{|a_2|\,|\omega_n^2 - \omega_{\max}^2|}{\sqrt{(\omega_0^2 - \omega_{\max}^2)^2 + \left(\dfrac{\omega_0}{Q}\right)^2 \omega_{\max}^2}}$$

where

$$\text{DC gain} = |a_2|\,\frac{\omega_n^2}{\omega_0^2} = 1$$

$$\Rightarrow |a_2| = \frac{1}{1.44} = 0.694$$

Thus,

$$T_{\max} = \frac{0.694\,|1.44 - 0.972|}{\sqrt{(1 - 0.972)^2 + \dfrac{1}{100} \times 0.972}} = 3.17$$

HF transmission $= |a_2| = 0.694$

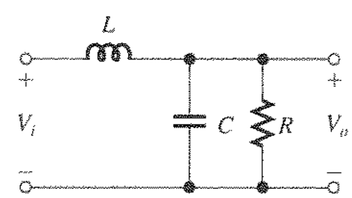
Ex: 17.17 Maximally flat $\quad \Rightarrow Q = \dfrac{1}{\sqrt{2}}$

$$\omega_0 = 2\pi \times 100 \times 10^3$$

Arbitrarily selecting $R = 1$ kΩ, we get

$$Q = \omega_0 CR \Rightarrow C = \frac{1}{\sqrt{2} \times 2\pi 10^5 \times 10^3}$$

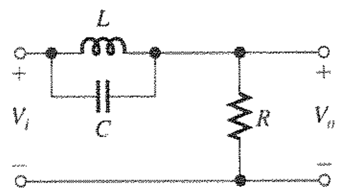
$= 1125$ pF



Also $Q = \dfrac{R}{\omega_0 L}$

$$\therefore L = \frac{R}{\omega_0 Q} = \frac{10^3}{2\pi 10^5 \times \frac{1}{\sqrt{2}}} = 2.25 \text{ mH}$$

Ex: 17.18



From Exercise 17.15 above, 3-dB bandwidth $= \omega_0/Q$

$2\pi 10 = 2\pi 60/Q \Rightarrow Q = 6$

$Q = \omega_0 CR$

$6 = 2\pi 60 \times C \times 10^4 \Rightarrow C = 1.6\ \mu\text{F}$

$$Q = \frac{R}{\omega_0 L}$$

$$L = \frac{R}{\omega_0 Q} = \frac{10^4}{2\pi 60 \times 6} = 4.42 \text{ H}$$

Ex: 17.19 $f_0 = 10$ kHz, $\Delta f_{3dB} = 500$ Hz

$$Q = \frac{f}{\Delta f_{3dB}} = \frac{10^4}{500} = 20$$

Using the data at the top of Table 17.1, we get

$C_4 = C_6 = 1.2$ nF

$$R_1 = R_2 = R_3 = R_5 = \frac{1}{\omega_0 C}$$

$$= \frac{1}{2\pi 10^4 \times 1.2 \times 10^{-9}} = 13.26 \text{ k}\Omega$$

$$R_6 = Q/\omega_0 C = \frac{20}{2\pi 10^4 \times 1.2 \times 10^{-9}} = 265 \text{ k}\Omega$$

Now using the data in Table 17.1 for the bandpass case, we obtain

$K =$ center-frequency gain $= 10$

Referring to Fig. 17.21(c), we have

$1 + r_2/r_1 = 10$

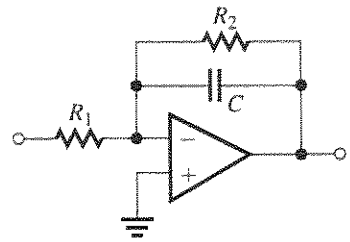
Selecting $r_1 = 10$ kΩ, we obtain $r_2 = 90$ kΩ.

Ex: 17.20 From Eq. (17.25), we have $\omega_p = 2\pi 10^4$ and

$$T(s) = \frac{\omega_p{}^5}{8.1408\,(s + 0.2895\omega_p)} \times \frac{1}{(s^2 + s0.4684\omega_p + 0.4293\omega_p{}^2)} \times \frac{1}{(s^2 + s0.1789\omega_p + 0.9883\omega_p{}^2)}$$

The circuit consists of 3 sections in cascade:

(a) First-order section:



$$T(s) = \frac{-0.2895\omega_p}{s + 0.2895\omega_p}$$

where the numerator coefficient is set so that the dc gain $= -1$.

Let $R_1 = 10$ kΩ

dc gain $= R_2/R_1 = 1 = R_2 = 10$ kΩ

$\omega_0 = 0.2895\omega_p$

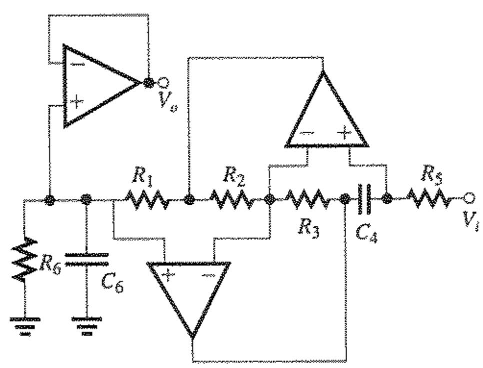
$$\frac{1}{CR_2} = 0.2895\omega_p$$

$$\Rightarrow C = \frac{1}{0.2895 \times 2\pi 10^4 \times 10^4} = 5.5 \text{ nF}$$

(b) Second-order section with transfer function:

$$T(s) = \frac{0.4293\omega_p^2}{s^2 + s0.4684\omega_p + 0.4293\omega_p^2}$$

where the numerator coefficient is selected to yield a dc gain of unity.



Select $R_1 = R_2 = R_3 = R_5 = 10$ kΩ

$$\Rightarrow C = \frac{1}{\sqrt{0.4293} \times 2\pi 10^4 \times 10^4} = 2.43 \text{ nF}$$

$C_4 = C_6 = C = 2.43$ nF

$$Q = \frac{\sqrt{0.4293}\omega_p}{0.4684\omega_p} = 1.4 \Rightarrow R_6 = \frac{Q}{\omega_0 C} = 14 \text{ k}\Omega$$

(c) Second-order section with transfer function:

$$T(s) = \frac{0.9883\omega^2{}_p}{s^2 + s0.1789\omega_p + 0.9883\omega^2{}_p}$$

The circuit is similar to that in (b) above but with

$R_1 = R_2 = R_3 = R_5 = 10\ \text{k}\Omega$

$$C_4 = C_6 = \frac{1}{\omega_0 \times 10^4}$$

$$= \frac{1}{\sqrt{0.9883} \times 2\pi 10^4 \times 10^4}$$

$= 1.6$ nF

$$Q = \frac{\sqrt{0.9883}}{0.1789} = 5.56$$

Thus $R_6 = Q/\omega_0 C = 55.6\ \text{k}\Omega$

Placing the three sections in cascade, i.e. connecting the output of the first-order section to the input of the second-order section in (b) and the output of of section (b) to the input of (c) results in the overall transfer function in Eq. (17.25) except for an inversion.

Ex: 17.21 Refer to the KHN circuit in Fig. 17.24. Choosing $C = 1$ nF, we obtain

$$R = \frac{1}{\omega_0 C} = \frac{1}{2\pi 10^4 \times 10^{-9}} = 15.9\ \text{k}\Omega$$

Using Eq. (17.62) and selecting $R_1 = 10\ \text{k}\Omega$, we get

$R_f = R_1 = 10\ \text{k}\Omega$

Using Eq. (17.63) and setting $R_2 = 10\ \text{k}\Omega$, we obtain

$R_3 = R_2(2Q - 1) = 10(2 \times 2 - 1) = 30\ \text{k}\Omega$

$$\text{High-frequency gain} = K = 2 - \frac{1}{Q} = 1.5\ \text{V/V}$$

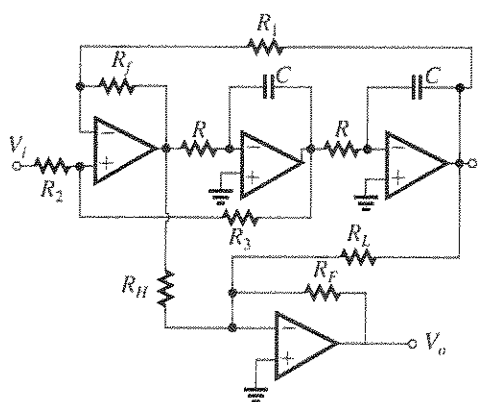
The transfer function to the output of the first integrator is

$$\frac{V_{bp}}{V_i} = -\frac{1}{sCR} \times \frac{V_{hp}}{V_i} = \frac{sK/(CR)}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

Thus the center-frequency gain is given by

$$\frac{K}{CR}\frac{Q}{\omega_0} = KQ = 1.5 \times 2 = 3\ \text{V/V}$$

Ex: 17.22



$$\frac{V_o}{V_i} = -K\frac{(R_F/R_H)s^2 + (R_F/R_L)\omega^2{}_0}{s^2 + s\omega_0/Q + \omega_0^2}$$

given $C = 1$ nF, $R_L = 10\ \text{k}\Omega$, then

$$R = \frac{1}{\omega_0 C} = \frac{1}{2\pi 5 \times 10^3 \times 10^{-9}} = 31.83\ \text{k}\Omega$$

$R_1 = 10\ \text{k}\Omega \Rightarrow R_f = 10\ \text{k}\Omega$

$R_2 = 10\ \text{k}\Omega \Rightarrow R_3 = R_2(2Q - 1)$

$= 10(10 - 1) = 90\ \text{k}\Omega$

$$\frac{R_H}{R_L}\omega_0^2 = \omega_n^2 \Rightarrow R_H = 10\left(\frac{8}{5}\right)^2 = 25.6\ \text{k}\Omega$$

$$\text{DC gain} = K\frac{R_F}{R_L} = \left(2 - \frac{1}{Q}\right)\frac{R_F}{R_L} = 3$$

$$R_F = \frac{3 \times 10}{2 - 1/5} = 16.7\ \text{k}\Omega$$

Ex: 17.23 Refer to Fig. 17.25 (b)

$$CR = \frac{1}{\omega_0} \Rightarrow C = \frac{1}{2\pi 10^4 \times 10^4} = 1.59\ \text{nF}$$

$R_d = QR = 20 \times 10 = 200\ \text{k}\Omega$

Center frequency gain $= KQ = 1$

$$\therefore K = \frac{1}{Q} = \frac{1}{20}$$

$R_g = R/K = 20R = 200\ \text{k}\Omega$

Ex: 17.24 Refer to Fig. 17.26 and Table 17.2 (AP entry).

$C = 10$ nF

$$R = \frac{1}{\omega_0 C} = \frac{1}{10^4 \times 10 \times 10^{-9}} = 10\ \text{k}\Omega$$

$QR = 5 \times 10 = 50\ \text{k}\Omega$

$C_1 = C \times \text{flat gain} = 10 \times 1 = 10$ nF

$R_1 = \infty$

$$R_2 = \frac{R}{\text{gain}} = R/1 = 10\ \text{k}\Omega$$

$r = 10\ \text{k}\Omega$

$$R_3 = \frac{Qr}{\text{gain}} = \frac{5 \times 10}{1} = 50\ \text{k}\Omega$$

Ex: 17.25 From Eq. (17.76) we have

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times 1}{10^4} = 2 \times 10^{-4}\text{s}$$

For $C = C_1 = C_2 = 1$ nF

$$R = \frac{2 \times 10^{-4}}{10^{-9}} = 200\ \text{k}\Omega$$

Thus $R_3 = 200\ \text{k}\Omega$

From Eq. (17.75) we have

$$m = 4Q^2 = 4$$

Thus, $R_4 = \dfrac{R}{m} = \dfrac{200}{4} = 50\ \text{k}\Omega$

Ex: 17.26 The transfer function of the feedback network is given in Fig. 17.28(a). The poles are the roots of the denominator polynomial,

$$s^2+s\left(\frac{1}{C_1R_3}+\frac{1}{C_2R_3}+\frac{1}{C_1R_4}\right)+\frac{1}{C_1C_2R_3R_4}=0$$

For $C_1 = C_2 = 10^{-9}$ F, $R_3 = 2\times10^5\Omega$,

$R_4 = 5\times10^4\ \Omega$,

$$s^2+s\left(\frac{2}{10^{-9}\times2\times10^5}+\frac{1}{10^{-9}\times5\times10^4}\right)+\frac{1}{10^{-18}10^{10}}=0$$

$$s^2+s\left(3\times10^4\right)+10^8=0$$

$$s=\frac{-3\times10^4\pm\sqrt{9\times10^8-4\times10^8}}{2}$$

$$= -0.382\times10^4 \text{ and } -2.618\times10^4\ \text{rad/s}$$

Ex: 17.27 Refer to the circuit in Fig. 17.30(a), where the transfer function is given on page 1340 as

$$T(s)=\frac{-s(\alpha/C_1R_4)}{s^2+s\left(\dfrac{1}{C_1}+\dfrac{1}{C_2}\right)\dfrac{1}{R_3}+\dfrac{1}{C_1C_2R_3R_4}}$$

Now, using the component value obtained in Exercise 17.25, namely

$C_1 = C_2 = 1$ nF

$R_3 = 200\ \text{k}\Omega$

$R_4 = 50\ \text{k}\Omega$

the center-frequency gain is given by (note that $Q=1$)

$$|T(j\omega_o)| = \alpha\left(\frac{R_3}{R_4}\right)\Big/\left(1+\frac{C_1}{C_2}\right)$$

$$1=\alpha\times\frac{4}{1+1}=2\alpha$$

$$\Rightarrow \alpha = 0.5$$

Thus,

$$\frac{R_4}{\alpha}=\frac{50}{0.5}=100\ \text{k}\Omega$$

and

$$\frac{R_4}{1-\alpha}=\frac{50}{0.5}=100\ \text{k}\Omega$$

Ex: 17.28

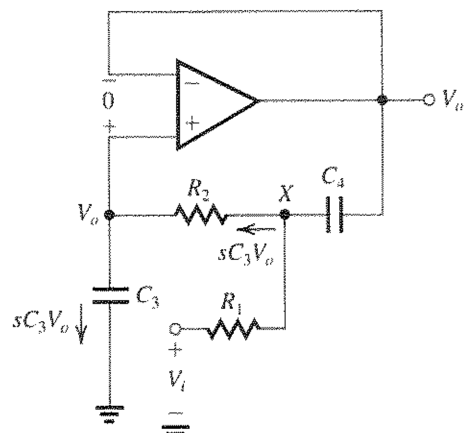


Figure 1

Figure 1 shows the circuit of Fig. 17.34(c) with partial analysis to determine the transfer function. The voltage at node X can be written as

$$V_x = V_o + sC_3V_oR_2$$

$$= (1+sC_3R_2)V_o \qquad (1)$$

Now a node equation at X takes the form

$$\frac{V_i-V_x}{R_1} = sC_3V_o + sC_4(V_x - V_o)$$

Substituting for V_x from Eq. (1) gives

$$V_i - V_o(sC_3R_2+1) = sC_3R_1V_o + sC_4R_1(sC_3R_2V_o)$$

$$V_i = V_o[s^2C_3C_4R_1R_2 + sC_3(R_1+R_2)+1]$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{1/C_3C_4R_1R_2}{s^2+s\dfrac{1}{C_4}\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)+\dfrac{1}{C_3C_4R_1R_2}} \qquad (2)$$

Thus,

$$\omega_0 = 1/\sqrt{C_3C_4R_1R_2}$$

and

$$Q=\left[\frac{\sqrt{C_3C_4R_1R_2}}{C_4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right]^{-1}$$

which are identical to Eqs. (17.77) and (17.78), respectively. Q.E.D.

From Eq. (2) we see that

$$\frac{V_o}{V_i}(0) = 1 \qquad \text{Q.E.D.}$$

Ex: 17.29 The design equations are Eqs. (17.79) and (17.80). Thus,

$$R_1 = R_2 = R = 10\ \text{k}\Omega$$

and

$C_4 = C$

$C_3 = C/4Q^2 = C\Big/\left(4 \times \frac{1}{2}\right) = 0.5C$

where

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times \frac{1}{\sqrt{2}}}{2\pi \times 4 \times 10^3}$$

$$\Rightarrow C = \frac{\sqrt{2}}{2\pi \times 4 \times 10^3 \times 10 \times 10^3} = 5.63 \text{ nF}$$

Thus,

$C_4 = 5.63$ nF

$C_3 = 2.81$ nF

Ex: 17.30 Refer to the results in Example 17.3

(a) $\Delta R_3/R_3 = +2\%$

$$S_{R3}^{\omega_0} = -1/2 \Rightarrow \frac{\Delta\omega_O}{\omega_O} = -\frac{1}{2} \times 2 = -1\%$$

$$S_{R3}^{Q} = \frac{1}{2} \Rightarrow \Delta Q/Q = \frac{1}{2} \times 2 = 1\%$$

(b) $\Delta R_4/R_4 = 2\%$

$$S_{R_4}^{\omega_o} = -\frac{1}{2} \Rightarrow \frac{\Delta\omega_O}{\omega_O} = -1\%$$

$$S_{R_4}^{Q} = -\frac{1}{2} \Rightarrow \frac{\Delta Q}{Q} = -\frac{1}{2} \times 2 = -1\%$$

(c) Combining the results in (a) & (b), we get

$$\frac{\Delta\omega_O}{\omega_O} = -1 - 1 = -2\%$$

$$\frac{\Delta Q}{Q} = 1 - 1 = 0\%$$

(d) Using the results in (c) for both resistors being 2% high, we have

$$\frac{\Delta\omega_O}{\omega_O} = S_{C1}^{\omega_0}\frac{\Delta C_1}{C_1} + S_{C2}^{\omega_0}\frac{\Delta C_2}{C_2} - 2$$

$$= -\frac{1}{2}(-2) + \frac{-1}{2}(-2) - 2$$

$$= 2 - 2 = 0\%$$

$$\frac{\Delta Q}{Q} = S_{C1}^{Q}\frac{\Delta C_1}{C} + S_{C2}^{Q}\frac{\Delta C_2}{C_2} + 0$$

$$= 0(-2) + (0)(-2) + 0 = 0\%$$

Ex: 17.31 $f_0 = f_{3dB} = 20$ MHz

$Q = 1/\sqrt{2}$

DC gain $= 1$

Using Eq. (17.99), we obtain

$G_m = \omega_0 C = 2\pi \times 20 \times 10^6 \times 2 \times 10^{-12}$

$= 0.251$ mA/V

Thus,

$G_{m1} = G_{m2} = 0.251$ mA/V

$$G_{m3} = \frac{G_m}{Q} = \frac{0.251}{1/\sqrt{2}} = 0.355 \text{ mA/V}$$

$G_{m4} = G_m|\text{Gain}| = G_m = 0.251$ mA/V

Ex: 17.32 From Eqs. (17.109) & (17.110)

$C_3 = C_4 = \omega_0 T_c C$

$$= 2\pi\, 10^4 \times \frac{1}{200 \times 10^3} \times 20$$

$= 6.283$ pF

From Eq. (17.112)

$$C_5 = \frac{C_4}{Q} = \frac{6.283}{20} = 0.314 \text{ pF}$$

From Eq. (17.113)

$$\text{Centre-frequency gain} = \frac{C_6}{C_5} = 1$$

$C_6 = C_5 = 0.314$ pF

Ex: 17.33 $R_p = \omega_0 L Q_0$

$= 2\pi\, 10^6 \times 3.18 \times 10^{-6} \times 150 = 3\text{ k}\Omega$

$R = R_L \parallel r_o \parallel R_p = 2\text{ k}\Omega \Rightarrow R_L = 15\text{ k}\Omega$

Ex: 17.34 $Q = (R_1 \parallel R_{in})/\omega_0 L$

$$= \frac{10^3 \parallel 10^3}{(2\pi \times 455 \times 10^3) \times 5 \times 10^{-6}} = 35$$

$BW = f_0/Q = 455/35 = 13$ kHz

$$C_1 + C_{in} = \frac{1}{\omega_0^2 L}$$

$$= \frac{1}{(2\pi \times 455 \times 10^3)^2 \times 5 \times 10^{-6}}$$

$= 24.47$ nF

$C_1 = 24.47 - 0.2 = 24.27$ nF

Ex: 17.35 To just meet specifications,

$$Q = \frac{f_0}{BW} = \frac{455}{10} = 45.5$$

$$\therefore \frac{R_1 \parallel n^2 R_{in}}{\omega_0 L} = 45.5$$

$$R_1 \parallel n^2 R_{in} = 45.5 \times 2\pi \times 455 \times 10^3 \times 5 \times 10^{-6}$$

$$= 650\ \Omega$$

$$n^2 R_{in} = 1.86\ \text{k}\Omega$$

$$n = \sqrt{\frac{1.86}{1}} = 1.36$$

$$C_1 + \frac{C_{in}}{n^2} = \frac{1}{\omega_0^2 L} = 24.47\ \text{nF}$$

$$C_1 = 24.36\ \text{nF}$$

At resonance, the voltage developed across R_1 is $I\left(R_1 \parallel n^2 R_{in}\right) = IR$. Thus, $V_{be} = IR/n$ & $I_c = g_m V_{be} = g_m IR/n$,

$$\frac{I_c}{I} = g_m R/n = \frac{40 \times 0.65}{1.36} = 19.1\,\frac{\text{A}}{\text{A}}$$

Ex: 17.36 $200 = (f_0/Q)\sqrt{2^{1/2} - 1}$
Eq. (17.123)

$$\frac{f_0}{Q} = 310.8\ \text{kHz}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{\left(2\pi \times 10.7 \times 10^6\right)^2 \times 3 \times 10^{-6}}$$

$$= 73.75\ \text{pF}$$

$$\frac{\omega_O}{Q} = \frac{1}{CR}$$

$$R = \frac{1}{73.7 \times 10^{-12} \times 2\pi \times 310.8 \times 10^3}$$

$$= 6.94\ \text{k}\Omega$$

Ex: 18.1 Pole frequency $f_0 = 1$ kHz

$$\text{Center-frequency gain} = \frac{1}{\text{Amplifier gain}}$$

$$= \frac{1}{2}\ \text{V/V}$$

Ex: 18.2

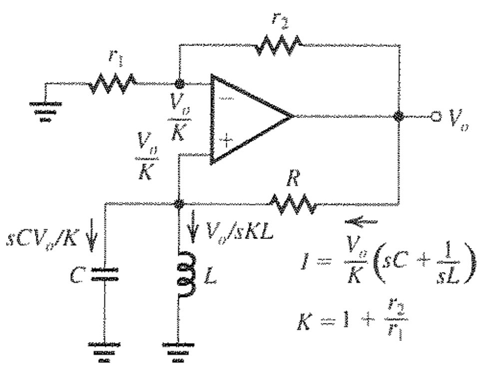


Figure 1

Figure 1 shows the circuit with some of the analysis already indicated. Writing a loop equation gives

$$V_o = IR + \frac{V_o}{K}$$

$$= \frac{V_o}{K} R\left(sC + \frac{1}{sL}\right) + \frac{V_o}{K} \qquad (1)$$

where

$$K = 1 + \frac{r_2}{r_1}$$

Multiplying Eq. (1) by (sK/CR) and dividing by V_o and rearranging gives

$$s^2 + s\frac{1}{CR}(1-K) + \frac{1}{LC} = 0$$

Substituting $K = 1 + \frac{r_2}{r_1}$ gives

$$s^2 - s\frac{1}{CR}\frac{r_2}{r_1} + \frac{1}{LC} = 0$$

For $s = j\omega$, we have

$$\left(-\omega^2 + \frac{1}{LC}\right) - j\omega\frac{1}{LC}\frac{r_2}{r_1} = 0$$

$$Re[D(j\omega)] = -\omega^2 + \frac{1}{LC}$$

$$\text{Im}[D(j\omega)] = -\frac{\omega}{CR}\frac{r_2}{r_1}$$

For sustained oscillation, both the real part and the imaginary part must be zero at the frequency of oscillation, thus

$$Re = 0 \Rightarrow \omega_0^2 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\text{Im} = 0 \Rightarrow \frac{r_2}{r_1} = 0 \Rightarrow r_2 = 0$$

Ex: 18.3 The square wave at the output of the op amp will have amplitude $V = 2$ V. Now if the bandpass circuit is of high selectivity, the signal across the tank circuit will be that whose frequency is $\omega_0 = 1/\sqrt{LC}$, which is the fundamental frequency in the Fourier series of the square wave at the output. The fundamental frequency at the output will have a peak amplitude of $4V/\pi = 8/\pi = 2.55$ V. Now, since the center-frequency gain of the bandpass circuit is unity, the voltage across the tank circuit will have a peak amplitude of 2.55 V.

Ex: 18.4 $L_+ = V\left(\frac{R_4}{R_5}\right) + V_D\left(1 + \frac{R_4}{R_5}\right)$

$$= 15\left(\frac{3}{9}\right) + 0.7\left(1 + \frac{3}{9}\right)$$

$$= 5 + 0.93 = 15.93\ \text{V}$$

$$L_- = -V\frac{R_3}{R_2} - V_D\left(1 + \frac{R_3}{R_2}\right)$$

$$= -15 \times \frac{3}{9} - 0.7\left(1 + \frac{3}{9}\right)$$

$$= -5.93\ \text{V/V}$$

$$\text{Limiter gain} = \frac{-R_f}{R_1} = \frac{-60}{30}$$

$$= -2\ \text{V/V}$$

Thus limiting occurs at $\frac{\pm 5.93}{2}$

$$= \pm\, 2.97\ \text{V}$$

Slope in the limiting regions

$$= \frac{-R_f \parallel R_4}{R_1} = -\frac{60 \parallel 3}{30} = -0.095\ \text{V/V}$$

Ex: 18.5 (a) $L(s) = \left(1 + \frac{R_2}{R_1}\right)\frac{Z_P}{Z_P + Z_S}$

$$= \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{1 + Z_S Y_P}\right)$$

$$= \left(1 + \frac{20.3}{10}\right)\left(\frac{1}{1 + \left(R + \frac{1}{sC}\right)\left(\frac{1}{R} + sC\right)}\right)$$

$$= \frac{3.03}{3 + sCR + \frac{1}{sCR}}$$

where $R = 10$ kΩ and $C = 16$ nF

Thus,

$$L(s) = \frac{3.03}{3 + s16 \times 10^{-5} + \frac{1}{s \times 16 \times 10^{-5}}}$$

The closed-loop poles are found by setting $L(s) = 1$; that is, they are the values of s, satisfying

$$3 + s \times 16 \times 10^{-5} + \frac{1}{s \times 16 \times 10^{-5}} = 3.03$$

$$\Rightarrow s = \frac{10^5}{16}(0.015 \pm j)$$

(b) The frequency of oscillation is $(10^5/16)$ rad/s or approximately 1 kHz.

(c) Refer to Fig. 18.6. At the positine peak $\hat{V}_o$, the voltage at node b will be one diode drop (0.7 V) above the voltage V_1 which is about 1/3 of V_o; thus $V_b = 0.7 + \hat{V}_O/3$. Now if we neglect the current through D_2 in comparison with the currents through R_5 and R_6, we find that

$$\frac{\hat{V}_o - V_b}{R_5} \cong \frac{V_b - (-15)}{R_6}$$

Thus,

$$\frac{\hat{V}_o - V_b}{1} = \frac{V_b + 15}{3}$$

$$\hat{V}_o = \frac{4}{3}V_b + 5$$

$$\hat{V}_o = \frac{4}{3}\left(0.7 + \frac{\hat{V}_o}{3}\right) + 5$$

$$\Rightarrow \hat{V}_o = 10.68 \text{ V}$$

From symmetry, we see that the negative peak is equal to the positive peak. Thus the output peak-to-peak voltage is 21.36 V.

Ex: 18.6 (a) For oscillations to start, we need $R_2/R_1 = 2$; thus the potentiometer should be set so that its resistance to ground is 20 kΩ.

$$\text{(b) } f_0 = \frac{1}{2\pi RC} = \frac{1}{2\pi\, 10 \times 10^3 \times 16 \times 10^{-9}}$$

$$= 1 \text{ kHz}$$

Ex: 18.7 Figure 1 shows the circuit together with the analysis details. The current I can be found as

$$I = \frac{V_o}{R_f}\left(1 + \frac{1}{sCR}\right) + \frac{V_o}{sCRR_f}\left(2 + \frac{1}{sCR}\right)$$

Finally, V_x can be found from

$$V_x = -\frac{1}{sCR_f}V_o\left(2 + \frac{1}{sCR}\right) - \frac{I}{sC}$$

$$= -\frac{1}{sCR_f}V_o\left(2 + \frac{1}{sCR}\right) - \frac{V_o}{sCR_f}\left(1 + \frac{1}{sCR}\right)$$

$$-\frac{V_o}{s^2C^2RR_f}\left(2 + \frac{1}{sCR}\right)$$

$$-\frac{V_x}{V_o} = \frac{3}{sCR_f} + \frac{4}{s^2C^2RR_f} + \frac{1}{s^3C^3R^2R_f}$$

$$\frac{V_o}{V_x} = -\frac{s^2C^2RR_f}{4 + s3CR + \dfrac{1}{sCR}}$$

For $s = j\omega$,

$$\frac{V_o}{V_x} = \frac{\omega^2C^2RR_f}{4 + j\left(3\omega CR - \dfrac{1}{\omega CR}\right)}$$

Ex: 18.8 From the loop-gain expression

$$\frac{V_o}{V_x} = \frac{\omega^2C^2RR_f}{4 + j\left(3\omega CR - \dfrac{1}{\omega CR}\right)}$$

we see that the phase will be zero at

$$3\omega_0 CR = \frac{1}{\omega_0 CR}$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{3}CR}$$

At this frequency, we have

$$\frac{V_o}{V_i} = \frac{\omega_0^2C^2RR_f}{4}$$

$$= \frac{1}{12}\frac{R_f}{R}$$

This figure belongs to Exercise 18.7.

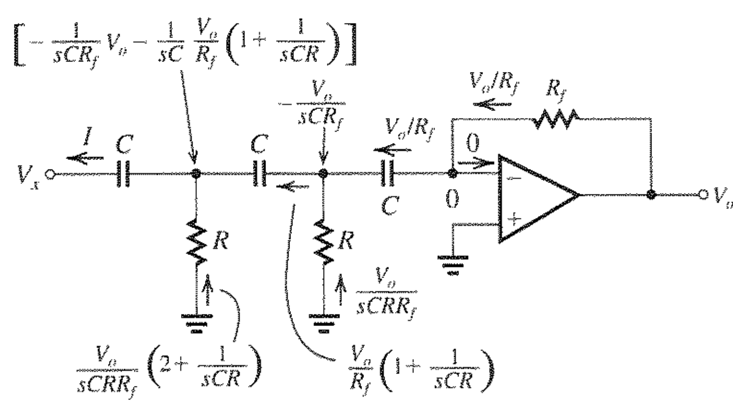


Figure 1

Thus the minimum value that R_f must have for oscillations to start is

$R_f = 12R$

Using the values in Fig. 18.9, we obtain

$$\omega_0 = \frac{1}{\sqrt{3}}\,\frac{1}{16\times10^{-9}\times10\times10^3} = 3.608 \text{ krad/s}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{3.608\times10^3}{2\pi} = 574.3 \text{ Hz}$$

$R_f \geq 12\times10 = 120\text{ k}\Omega$

Ex: 18.9 $\omega_0 = \dfrac{1}{CR} \Rightarrow CR = \dfrac{1}{2\pi\,10^3}$

For $C = 16$ nF, we have $R = 10$ kΩ.

$\therefore$ the output is twice as large as the voltage across the resonator, and the peak-to-peak amplitude is

$$\frac{4V}{\pi} = \frac{4(2\times1.4)}{\pi} = 3.6 \text{ V}$$

Ex: 18.10 For the Hartley oscillator of Fig. 18.13(b), let the emitter be grounded and assume that the total effective resistance between collector and emitter is R. Replacing the BJT with its equivalent circuit but neglecting r_π, C_π and C_μ and assuming that r_o is absorbed into R, we obtain the circuit shown in Fig. 1.

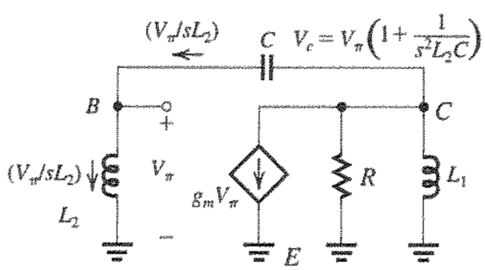


Figure 1

A node equation at C gives

$$\frac{V_\pi}{sL_2} + g_mV_\pi + \left(\frac{1}{R}+\frac{1}{sL_1}\right)V_\pi\left(1+\frac{1}{s^2L_2C}\right) = 0$$

$$\frac{1}{s}\left(\frac{1}{L_1}+\frac{1}{L_2}\right) + \left(g_m+\frac{1}{R}\right) + \frac{1}{s^2L_2CR} + \frac{1}{s^3L_1L_2C} = 0$$

$$s^3\left(g_m+\frac{1}{R}\right) + s^2\left(\frac{1}{L_1}+\frac{1}{L_2}\right) + s\frac{1}{L_2CR} + \frac{1}{L_1L_2C} = 0$$

$$j\omega\left[-\omega^2\left(g_m+\frac{1}{R}\right)+\frac{1}{L_2CR}\right] + \left[\frac{1}{L_1L_2C} - \omega^2\left(\frac{1}{L_1}+\frac{1}{L_2}\right)\right] = 0$$

Equating the real part to zero gives

$$\omega_0^2 = \frac{1}{(L_1+L_2)C} \qquad (1)$$

Equating the imaginary part to zero and using Eq. (1) gives

$$\frac{g_m+\frac{1}{R}}{(L_1+L_2)C} = \frac{1}{L_2CR}$$

$$g_mR = 1+\frac{L_1}{L_2}$$

$$g_mR = \frac{L_1}{L_2}$$

which is the condition for sustained oscillations. To ensure that oscillation will start, we need

$$g_mR > \frac{L_1}{L_2} \qquad \text{Q.E.D.}$$

Ex: 18.11 Refer to the circuit in Fig. 18.14(a).

$R = R_{coil} \parallel r_o \parallel R_L$

$= \dfrac{Q}{\omega_0C_1} \parallel 100\times10^3 \parallel 2\times10^3$

$= \dfrac{100}{10^6\times0.01\times10^{-6}} \parallel 10^5 \parallel 2\times10^3$

$= 10^4 \parallel 10^5 \parallel 2\times10^3 = 1.64\times10^3\ \Omega$

$= 1.64$ kΩ

$$g_m = \frac{I_C}{V_T} = \frac{1\text{ mA}}{0.025\text{ V}} = 40\text{ mA/V}$$

For sustained oscillations, we have

$$g_mR = \frac{C_2}{C_1}$$

$\Rightarrow C_2 = 0.01\times40\times1.64$

$= 0.66\ \mu$F

$$\omega_0^2L\frac{C_1C_2}{C_1+C_2} = 1$$

$$10^{12}\times L\times\frac{0.01\times0.66}{0.01+0.66}\times10^{-6} = 1$$

$\Rightarrow L = 101.5\ \mu$H

To allow oscillations to grow in amplitude, we need

$$g_mR > \frac{C_2}{C_1}$$

which can be achieved by using a somewhat smaller C_2 than the value found above.

Ex: 18.12 $\omega_0 = 10 \text{ Grad/s} = 10 \times 10^9 = \dfrac{1}{\sqrt{LC}}$

$$10^{10} = \frac{1}{\sqrt{10 \times 10^{-9} \times C}}$$

$\Rightarrow C = 1 \text{ pF}$

$R_p = \omega_0 L Q$

$= 10^{10} \times 10 \times 10^{-9} \times 10 = 1000\ \Omega$

$= 1\ \text{k}\Omega$

$$g_m|_{\min} = \frac{1}{(R_p \parallel r_o)} = \frac{1}{(1 \parallel 10) \times 10^3}$$

$= 1.1 \text{ mA/V}$

Ex: 18.13 From Eq. (18.26), we have

$$f_s = \frac{1}{2\pi\sqrt{LC_s}} = \frac{1}{2\pi\sqrt{0.52 \times 0.012 \times 10^{-12}}}$$

$= 2.015 \text{ MHz}$

From Eq. (18.27), we have

$$f_p = \frac{1}{2\pi\sqrt{L\dfrac{C_s C_p}{C_s + C_p}}}$$

$$= \frac{1}{2\pi\sqrt{0.52 \times \dfrac{0.012 \times 4 \times 10^{-12}}{0.012 + 4}}}$$

$= 2.018 \text{ MHz}$

$$Q = \frac{\omega_o L}{r} \cong \frac{\omega_s L}{r}$$

$$= \frac{2\pi \times 2.015 \times 10^6 \times 0.52}{120}$$

$\cong 55{,}000$

Ex: 18.14 $V_{TH} = |V_{TL}| = \beta |L|$

$$5 = \frac{R_1}{R_1 + R_2} \times 13$$

$$\frac{R_2}{R_1} = 1.6$$

$R_2 = 16\ \text{k}\Omega$

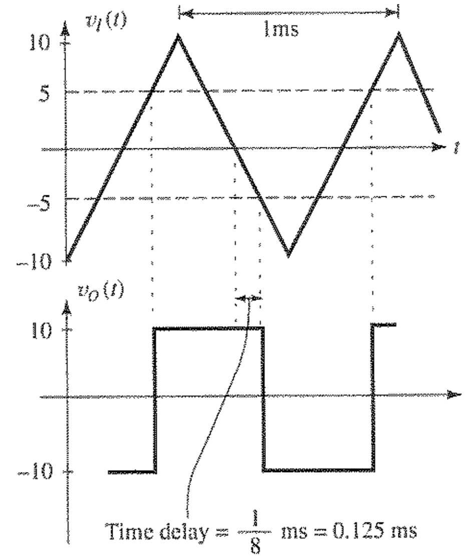
Ex: 18.15 $V_{TH} = V_{TL} = \dfrac{R_1}{R_2}|L|$

$$5 = \frac{R_1}{R_2} \times 10$$

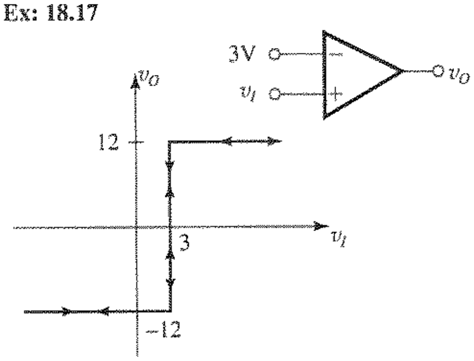
$R_2 = 2R_1$

Possible choice: $R_1 = 10\ \text{k}\Omega$, $R_2 = 20\ \text{k}\Omega$

Ex: 18.16



Ex: 18.17



A comparator with a threshold of 3 V and output levels of ±12 V.

Ex: 18.18 $|V_T| = \dfrac{100}{2} = 50 \text{ mV}$

$$50 \times 10^{-3} = 10\frac{R_1}{R_2}$$

$$\frac{R_2}{R_1} = \frac{10}{0.05}$$

$R_2 = 200R_1$

For $R_1 = 1\ \text{k}\Omega$, we have $R_2 = 200\ \text{k}\Omega$.

Ex: 18.19

$$\beta = \frac{R_1}{R_1 + R_2} = \frac{100}{100 + 1000} = 0.091 \text{ V/V}$$

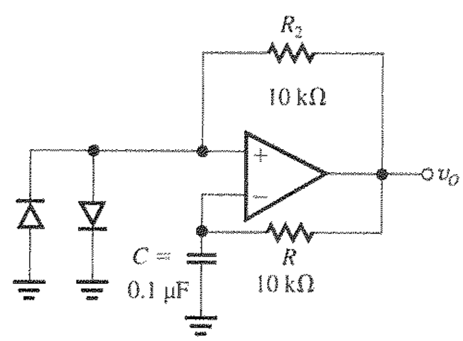
$$T = 2\tau \ln\frac{1+\beta}{1-\beta}$$

$$= 2 \times 0.01 \times 10^{-6} \times 10^6 \times \ln\left(\frac{1.091}{1 - 0.091}\right)$$

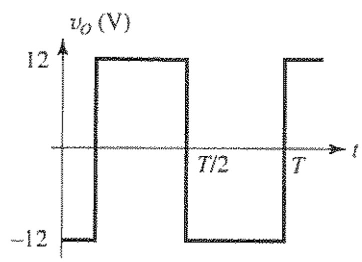
$= 0.00365$ s

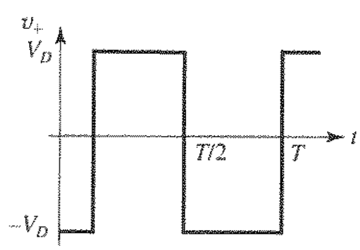
$$f_0 = \frac{1}{T} = 274 \text{ Hz}$$

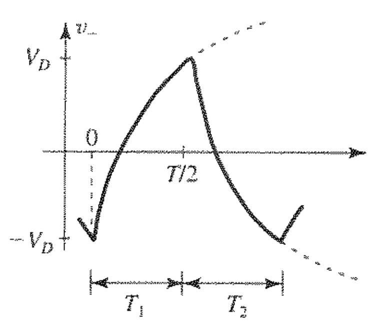
Ex: 18.20



$T_1 = T_2 = T/2$







During T_1, we have

$v_-(t) = 12 - (12 + V_D)\, e^{-t/\tau}$

$v_- = V_D$ at $t = T/2$

$V_D = 12 - (12 + V_D)\, e^{-T/2\tau}$

$$T = 2\tau \ln\left(\frac{12 + V_D}{12 - V_D}\right)$$

$$= 2 \times 0.1 \times 10^{-6} \times 10 \times 10^3 \times \ln\left(\frac{12 + V_D}{12 - V_D}\right)$$

$$f = \frac{1}{T} = \frac{500}{\ln\left(\frac{12 + V_D}{12 - V_D}\right)}\ \text{Hz}$$

$$f|_{25^\circ C} = \frac{500}{\ln\left(\frac{12.7}{11.3}\right)}$$

$= 4281$ Hz

At 0°C, $V_D = 0.7 + 0.05 = 0.75$ V

$$f|_{0^\circ C} = \frac{500}{\ln\left(\frac{12.75}{11.25}\right)} = 3{,}995\ \text{Hz}$$

At 50°C, $V_D = 0.7 - 0.05 = 0.65$ V

$$f|_{50^\circ C} = \frac{500}{\ln\left(\frac{12.65}{11.35}\right)} = 4{,}611\ \text{Hz}$$

At 100°C, $V_D = 0.7 - 0.15 = 0.55$ V

$$f|_{100^\circ C} = \frac{500}{\ln\left(\frac{12.55}{11.45}\right)} = 5{,}451\ \text{Hz.}$$

Ex: 18.21 To obtain a triangular waveform with 10-V peak-to-peak amplitute, we should have

$V_{TH} = -V_{TL} = 5$ V

But $V_{TL} = -L_+ \times \dfrac{R_1}{R_2}$

Thus, $-5 = -10 \times \dfrac{10}{R_2}$

$\Rightarrow R_2 = 20$ kΩ

For 1-kHz frequency, $T = 1$ ms.

Thus,

$$T/2 = 0.5 \times 10^{-3} = CR\frac{V_{TH} - V_{TL}}{L_+}$$

$= 0.01 \times 10^{-6} \times R \times 10/10$

$\Rightarrow R = 50$ kΩ

Ex: 18.22 Using Eq. (18.38), we obtain

$$100 \times 10^{-6} = 0.1 \times 10^{-6} \times R_3 \ln\left(\frac{12.7}{10.8}\right)$$

$R_3 = 6171\ \Omega$

Ex: 18.23 $T = 1.1CR \Rightarrow R = T/1.1C = 9.1$ kΩ

Ex: 18.24 $T = 0.69C(R_A + 2R_B)$

$$\frac{1}{100 \times 10^3} = 0.69 \times 10^3 \times 10^{-12}\,(R_A + 2R_B)$$

$$\Rightarrow R_A + 2R_B = \frac{1}{0.69 \times 10^{-4}} = 14.49\ \text{k}\Omega \qquad (1)$$

Using Eq. (18.47), we obtain

$$0.75 = \frac{R_A + R_B}{R_A + 2R_B}$$

$$R_A + R_B = 0.75 \times 14.49 = 10.88\ \text{k}\Omega \qquad (2)$$

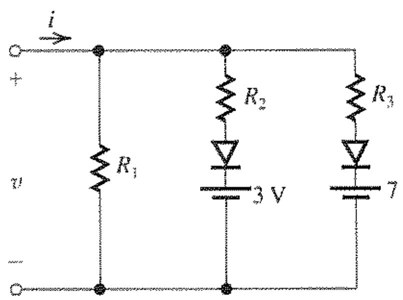
$$(1) - (2) \Rightarrow R_B = 3.61\ \text{k}\Omega$$

Now, substituting into (2), we get

$$R_A = 7.27\ \text{k}\Omega$$

Use 7.2 kΩ and 3.6 kΩ, standard 5% resistors.

Ex: 18.25



$$i = 0.1v^2$$

At $v = 2$ V, we obtain $i = 0.4$ mA.

$$\text{Thus, } R_1 = \frac{2}{0.4} = 5\ \text{k}\Omega.$$

For 3 V $\leq V \leq$ 7 V, we have

$$i = \frac{v}{R_1} + \frac{v-3}{R_2}$$

To obtain a perfect match at $V = 4$ V (i.e., to obtain $i = 1.6$ mA), we need

$$1.6 = \frac{4}{5} + \frac{4-3}{R_2}$$

$$R_2 = 1.25\ \text{k}\Omega$$

For $v \geq 7V$, we have

$$i = \frac{v}{R_1} + \frac{v-3}{R_2} + \frac{v-7}{R_3}$$

To obtain a perfect match at $v = 8V$, we must select R_3 so that $i = 6.4$ mA:

$$6.4 = \frac{8}{5} + \frac{8-3}{1.25} + \frac{8.7}{R_3}$$

$$\Rightarrow R_3 = 1.25\ \text{k}\Omega$$

At $v = 3$ V, the circuit provides

$$i = \frac{3}{5} = 0.6\ \text{mA, while ideally}$$

$i = 0.1 \times 9 = 0.9$ mA. Thus the error is −0.3 mA.

* At $v = 5$ V, the circuit provides

$$i = \frac{5}{5} + \frac{5-3}{1.25} = 2.6\ \text{mA, while ideally}$$

$i = 0.1 \times 25 = 2.5$ mA. Thus the error is +0.1 mA.

* At $v = 7$ V, the circuit provides

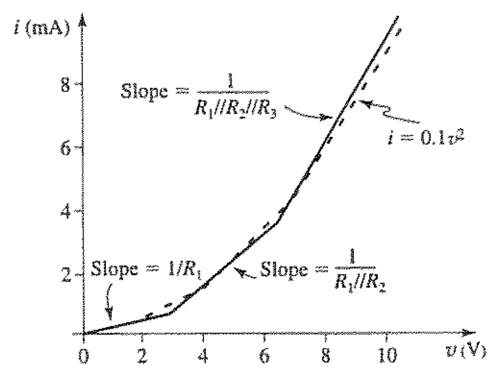
$$i = \frac{7}{5} + \frac{7-3}{1.25} = 4.6\ \text{mA, while ideally}$$

$i = 0.1 \times 49 = 4.9$ mA. Thus the error is −0.3 mA.

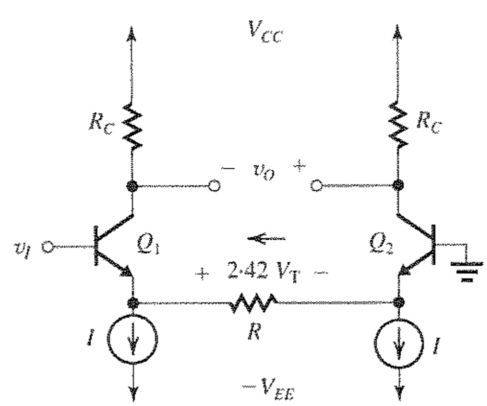
* At $v = 10$ V, the circuit provides,

$$i = \frac{10}{6} + \frac{10-3}{1.25} + \frac{10-7}{1.25} = 10\ \text{mA},$$

while ideally $i = 10$ mA. Thus the error is 0 A.



Ex: 18.26



Exercise 18–7

$$I_{e_1} = I + (2.42\ V_T)/R$$

$$= I\left[1 + \frac{2.42\ V_T}{IR}\right]$$

$$= I\left[1 + \frac{2.42\ V_T}{2.5\ V_T}\right]$$

$$= I\left(1 + \frac{2.42}{2.5}\right)$$

$$I_{c1} \cong I\left(1 + \frac{2.42}{2.5}\right)$$

$$I_{c2} \cong I\left(1 - \frac{2.42}{2.5}\right)$$

$$v_O = (V_{CC} - I_{c2}R_C) - (V_{CC} - I_{c1}R_C)$$

$$= (I_{c1} - I_{c2})\,R_C$$

$$= IR_C \times 2 \times \frac{2.42}{2.5}$$

$$= 0.25 \times 10 \times 2 \times \frac{2.42}{2.5} = 4.84\ \text{V}$$

1.1 (a) $I = \frac{V}{R} = \frac{5\text{ V}}{1\text{ k}\Omega} = 5\text{ mA}$

(b) $R = \frac{V}{I} = \frac{5\text{ V}}{1\text{ mA}} = 5\text{ k}\Omega$

(c) $V = IR = 0.1\text{ mA} \times 10\text{ k}\Omega = 1\text{ V}$

(d) $I = \frac{V}{R} = \frac{1\text{ V}}{100\ \Omega} = 0.01\text{ A} = 10\text{ mA}$

Note: Volts, milliamps, and kilohms constitute a consistent set of units.

1.2 (a) $P = I^2R = (20 \times 10^{-3})^2 \times 1 \times 10^3$

$= 0.4\text{ W}$

Thus, R should have a $\frac{1}{2}$-W rating.

(b) $P = I^2R = (40 \times 10^{-3})^2 \times 1 \times 10^3$

$= 1.6\text{ W}$

Thus, the resistor should have a 2-W rating.

(c) $P = I^2R = (1 \times 10^{-3})^2 \times 100 \times 10^3$

$= 0.1\text{ W}$

Thus, the resistor should have a $\frac{1}{8}$-W rating.

(d) $P = I^2R = (4 \times 10^{-3})^2 \times 10 \times 10^3$

$= 0.16\text{ W}$

Thus, the resistor should have a $\frac{1}{4}$-W rating.

(e) $P = V^2/R = 20^2/(1 \times 10^3) = 0.4\text{ W}$

Thus, the resistor should have a $\frac{1}{2}$-W rating.

(f) $P = V^2/R = 11^2/(1 \times 10^3) = 0.121\text{ W}$

Thus, a rating of $\frac{1}{8}$ W should theoretically suffice, though $\frac{1}{4}$ W would be prudent to allow for inevitable tolerances and measurement errors.

1.3 (a) $V = IR = 5\text{ mA} \times 1\text{ k}\Omega = 5\text{ V}$

$P = I^2R = (5\text{ mA})^2 \times 1\text{ k}\Omega = 25\text{ mW}$

(b) $R = V/I = 5\text{ V}/1\text{ mA} = 5\text{ k}\Omega$

$P = VI = 5\text{ V} \times 1\text{ mA} = 5\text{ mW}$

(c) $I = P/V = 100\text{ mW}/10\text{ V} = 10\text{ mA}$

$R = V/I = 10\text{ V}/10\text{ mA} = 1\text{ k}\Omega$

(d) $V = P/I = 1\text{ mW}/0.1\text{ mA}$

$= 10\text{ V}$

$R = V/I = 10\text{ V}/0.1\text{ mA} = 100\text{ k}\Omega$

(e) $P = I^2R \Rightarrow I = \sqrt{P/R}$

$I = \sqrt{1000\text{ mW}/1\text{ k}\Omega} = 31.6\text{ mA}$

$V = IR = 31.6\text{ mA} \times 1\text{ k}\Omega = 31.6\text{ V}$

Note: V, mA, kΩ, and mW constitute a consistent set of units.

1.4 See figure on next page, which shows that there are 17 possible resistance values: 5.7, 6.7, 8, 8.6, 10, 13.3, 14.3, 17.1, 20, 23.3, 28, 30, 40, 46.7, 50, 60, and 70 kΩ.

1.5 Shunting the 10 kΩ by a resistor of value of R result in the combination having a resistance R_{eq},

$$R_{eq} = \frac{10R}{R + 10}$$

Thus, for a 1% reduction,

$$\frac{R}{R + 10} = 0.99 \Rightarrow R = 990\text{ k}\Omega$$

For a 5% reduction,

$$\frac{R}{R + 10} = 0.95 \Rightarrow R = 190\text{ k}\Omega$$

For a 10% reduction,

$$\frac{R}{R + 10} = 0.90 \Rightarrow R = 90\text{ k}\Omega$$

For a 50% reduction,

$$\frac{R}{R + 10} = 0.50 \Rightarrow R = 10\text{ k}\Omega$$

Shunting the 10 kΩ by

(a) 1 MΩ results in

$$R_{eq} = \frac{10 \times 1000}{1000 + 10} = \frac{10}{1.01} = 9.9\text{ k}\Omega$$

a 1% reduction;

(b) 100 kΩ results in

$$R_{eq} = \frac{10 \times 100}{100 + 10} = \frac{10}{1.1} = 9.09\text{ k}\Omega$$

a 9.1% reduction;

(c) 10 kΩ results in

$$R_{eq} = \frac{10}{10 + 10} = 5\text{ k}\Omega$$

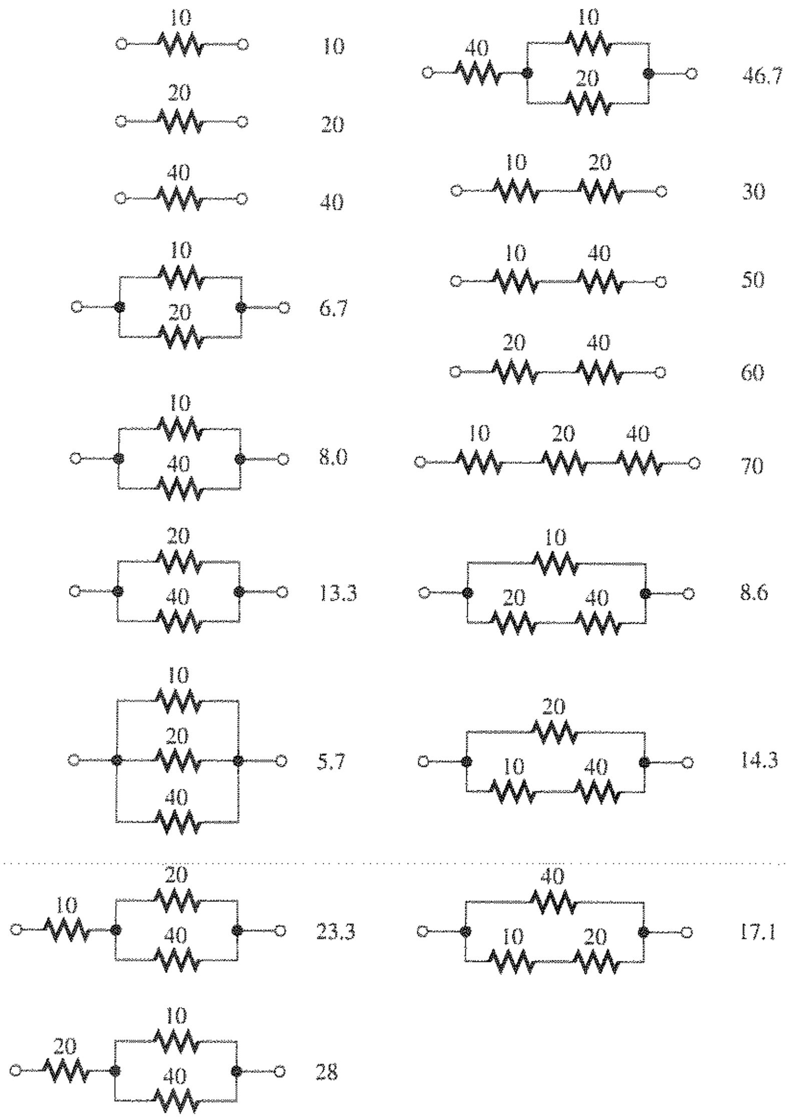
a 50% reduction.

1.6 $V_O = V_{DD}\frac{R_2}{R_1 + R_2}$

To find R_O, we short-circuit V_{DD} and look back into node X,

$$R_O = R_2 \parallel R_1 = \frac{R_1R_2}{R_1 + R_2}$$

This figure belongs to **1.4**.



1.7 Use voltage divider to find V_O

$$V_O = 5\frac{2}{2+3} = 2\ \text{V}$$

Equivalent output resistance R_O is

$$R_O = (2\ \text{k}\Omega \parallel 3\ \text{k}\Omega) = 1.2\ \text{k}\Omega$$

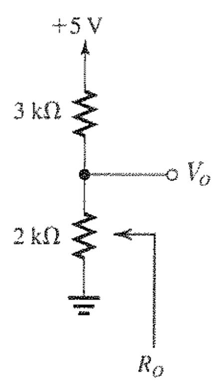
The extreme values of V_O for $\pm 5\%$ tolerance resistor are

$$V_{O\min} = 5\frac{2(1-0.05)}{2(1-0.05)+3(1+0.05)}$$

$$= 1.88\ \text{V}$$

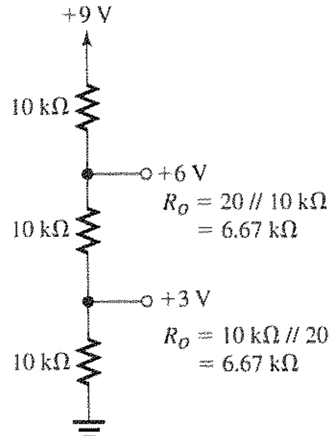
$$V_{O\max} = 5\frac{2(1+0.05)}{2(1+0.05)+3(1-0.05)}$$

$$= 2.12\ \text{V}$$

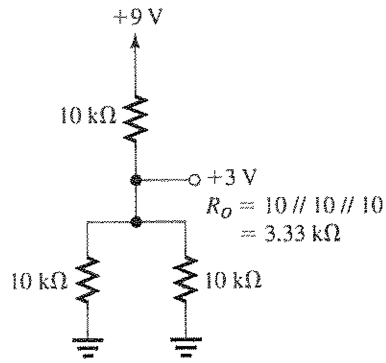


The extreme values of R_O for $\pm 5\%$ tolerance resistors are $1.2 \times 1.05 = 1.26\ \text{k}\Omega$ and $1.2 \times 0.95 = 1.14\ \text{k}\Omega$.

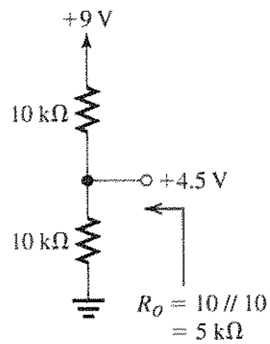
1.8



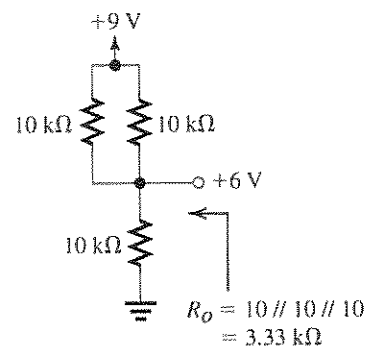
(a)



(c)



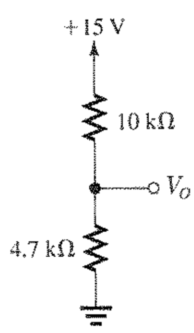
(b)



(d)

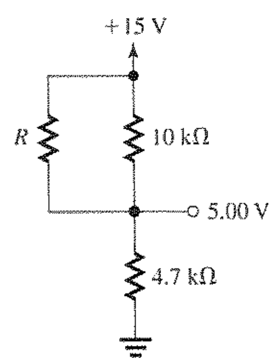
Voltage generated:
+3V [two ways: (a) and (c) with (c) having lower output resistance]
+4.5 V (b)
+6V [two ways: (a) and (d) with (d) having a lower output resistance]

1.9



$$V_O = 15\frac{4.7}{10 + 4.7} = 4.80\ \text{V}$$

To increase V_O to 10.00 V, we shunt the 10-kΩ resistor by a resistor R whose value is such that $10 \parallel R = 2 \times 4.7$.



Thus

$$\frac{1}{10} + \frac{1}{R} = \frac{1}{9.4}$$

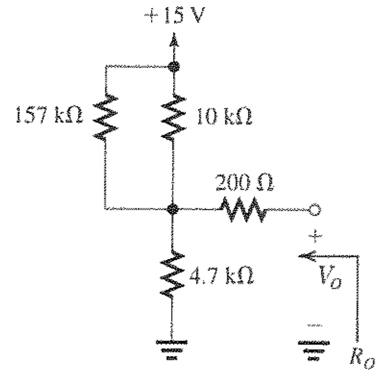
$\Rightarrow R = 156.7 \approx 157\ \text{k}\Omega$

Now,

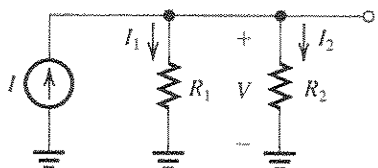
$R_O = 10\ \text{k}\Omega \parallel R \parallel 4.7\ \text{k}\Omega$

$$= 9.4 \parallel 4.7 = \frac{9.4}{3} = 3.133\ \text{k}\Omega$$

To make $R_O = 3.33$, we add a series resistance of approximately 200 Ω, as shown below,



1.10



$V = I(R_1 \parallel R_2)$

$= I\dfrac{R_1R_2}{R_1 + R_2}$

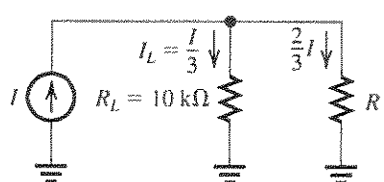
$I_1 = \dfrac{V}{R_1} = I\dfrac{R_2}{R_1 + R_2}$

$I_2 = \dfrac{V}{R_2} = I\dfrac{R_1}{R_1 + R_2}$

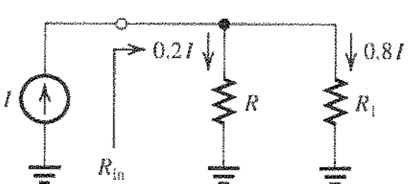
1.11 Connect a resistor R in parallel with R_L. To make $I_L = I/3$ (and thus the current through R, $2I/3$), R should be such that

$10I/3 = 2IR/3$

$\Rightarrow R = 5\text{ k}\Omega$



1.12



To make the current through R equal to $0.2I$, we shunt R by a resistance R_1 having a value such that the current through it will be $0.8I$; thus

$0.2IR = 0.8IR_1 \Rightarrow R_1 = \dfrac{R}{4}$

The input resistance of the divider, R_{in}, is

$R_{in} = R \parallel R_1 = R \parallel \dfrac{R}{4} = \dfrac{1}{5}R$

Now if R_1 is 10% too high, that is, if

$R_1 = 1.1\dfrac{R}{4}$

the problem can be solved in two ways:

(a) Connect a resistor R_2 across R_1 of value such that $R_2 \parallel R_1 = R/4$, thus

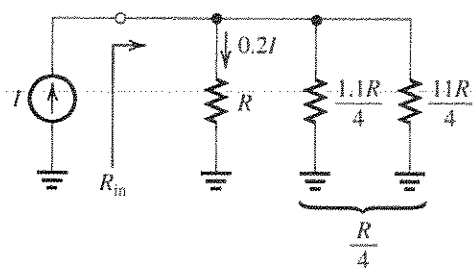
$\dfrac{R_2(1.1R/4)}{R_2 + (1.1R/4)} = \dfrac{R}{4}$

$1.1R_2 = R_2 + \dfrac{1.1R}{4}$

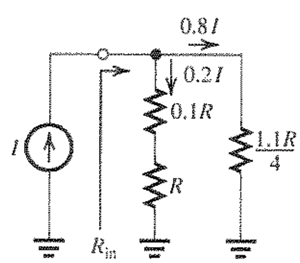
$\Rightarrow R_2 = \dfrac{11R}{4} = 2.75\text{ R}$

$R_{in} = R \parallel \dfrac{1.1R}{4} \parallel \dfrac{11R}{4}$

$= R \parallel \dfrac{R}{4} = \dfrac{R}{5}$



(b) Connect a resistor in series with the load resistor R so as to raise the resistance of the load branch by 10%, thereby restoring the current division ratio to its desired value. The added series resistance must be 10% of R (i.e., $0.1R$).

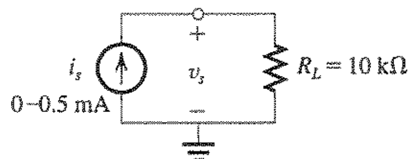


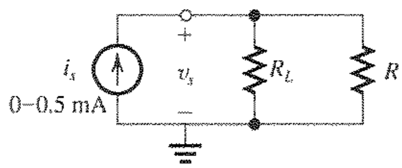
$R_{in} = 1.1R \parallel \dfrac{1.1R}{4}$

$= \dfrac{1.1R}{5}$

that is, 10% higher than in case (a).

1.13 For $R_L = 10\ \mathrm{k\Omega}$, when signal source generates 0–0.5 mA, a voltage of 0–2 V may appear across the source





To limit $v_s \le 1$ V, the net resistance has to be $\le 2\ \mathrm{k\Omega}$. To achieve this we have to shunt R_L with a resistor R so that $(R \parallel R_L) \le 2\ \mathrm{k\Omega}$.

$R \parallel R_L \le 2\ \mathrm{k\Omega}$.

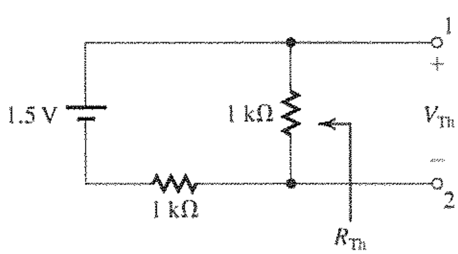
$$\frac{RR_L}{R + R_L} \le 2\ \mathrm{k\Omega}$$

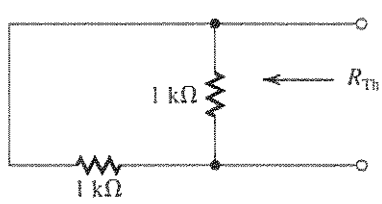
For $R_L = 10\ \mathrm{k\Omega}$

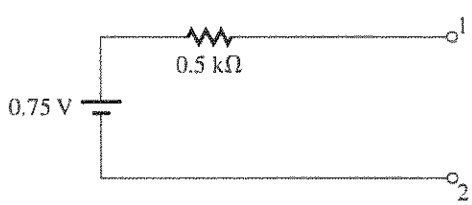
$R \le 2.5\ \mathrm{k\Omega}$

The resulting circuit needs only one additional resistance of $2\ \mathrm{k\Omega}$ in parallel with R_L so that $v_s \le 1$ V. The circuit is a current divider, and the current through R_L is now 0–0.1 mA.

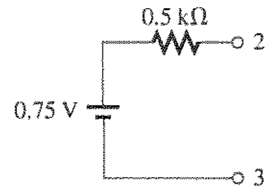
1.14 (a) Between terminals 1 and 2:



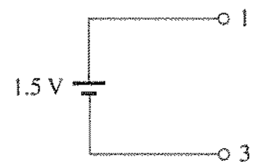




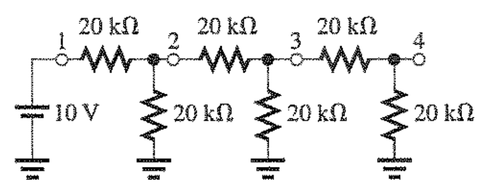
(b) Same procedure is used for (b) to obtain



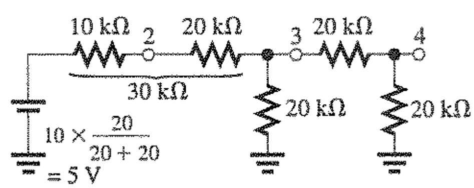
(c) Between terminals 1 and 3, the open-circuit voltage is 1.5 V. When we short circuit the voltage source, we see that the Thévenin resistance will be zero. The equivalent circuit is then



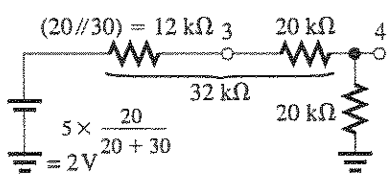
1.15



⇓

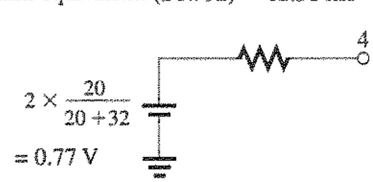


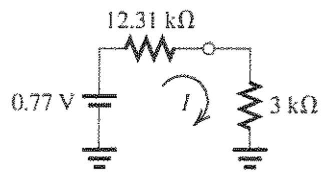
⇓



⇓

Thévenin equivalent: (20//32) = 12.31 kΩ





Now, when a resistance of 3 kΩ is connected between node 4 and ground,

$$I = \frac{0.77}{12.31 + 3}$$

$$= 0.05 \text{ mA}$$

1.16 (a) Node equation at the common mode yields

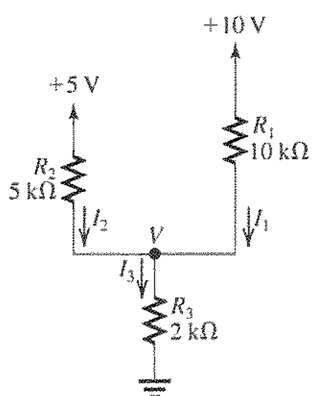
$$I_3 = I_1 + I_2$$

Using the fact that the sum of the voltage drops across R_1 and R_3 equals 10 V, we write

$$10 = I_1R_1 + I_3R_3$$

$$= 10I_1 + (I_1 + I_2) \times 2$$

$$= 12I_1 + 2I_2$$



That is,

$$12I_1 + 2I_2 = 10 \qquad (1)$$

Similarly, the voltage drops across R_2 and R_3 add up to 5 V, thus

$$5 = I_2R_2 + I_3R_3$$

$$= 5I_2 + (I_1 + I_2) \times 2$$

which yields

$$2I_1 + 7I_2 = 5 \qquad (2)$$

Equations (1) and (2) can be solved together by multiplying Eq. (2) by 6:

$$12I_1 + 42I_2 = 30 \qquad (3)$$

Now, subtracting Eq. (1) from Eq. (3) yields

$$40I_2 = 20$$

$$\Rightarrow I_2 = 0.5 \text{ mA}$$

Substituting in Eq. (2) gives

$$2I_1 = 5 - 7 \times 0.5 \text{ mA}$$

$$\Rightarrow I_1 = 0.75 \text{ mA}$$

$$I_3 = I_1 + I_2$$

$$= 0.75 + 0.5$$

$$= 1.25 \text{ mA}$$

$$V = I_3R_3$$

$$= 1.25 \times 2 = 2.5 \text{ V}$$

To summarize:

$I_1 = 0.75$ mA $\quad I_2 = 0.5$ mA

$I_3 = 1.25$ mA $\quad V = 2.5$ V

(b) A node equation at the common node can be written in terms of V as

$$\frac{10 - V}{R_1} + \frac{5 - V}{R_2} = \frac{V}{R_3}$$

Thus,

$$\frac{10 - V}{10} + \frac{5 - V}{5} = \frac{V}{2}$$

$$\Rightarrow 0.8V = 2$$

$$\Rightarrow V = 2.5 \text{ V}$$

Now, I_1, I_2, and I_3 can be easily found as

$$I_1 = \frac{10 - V}{10} = \frac{10 - 2.5}{10}$$

$$= 0.75 \text{ mA}$$

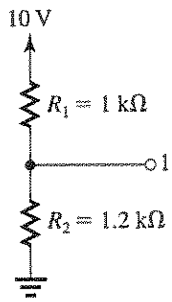
$$I_2 = \frac{5 - V}{5} = \frac{5 - 2.5}{5}$$

$$= 0.5 \text{ mA}$$

$$I_3 = \frac{V}{R_3} = \frac{2.5}{2} = 1.25 \text{ mA}$$

Method (b) is much preferred, being faster, more insightful, and less prone to errors. In general, one attempts to identify the lowest possible number of variables and write the corresponding minimum number of equations.

1.17 Find the Thévenin equivalent of the circuit to the left of node 1.

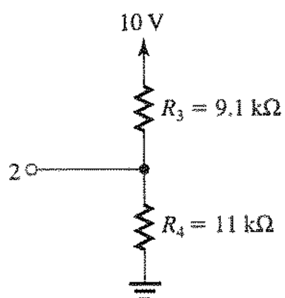


Between node 1 and ground,

$$R_{Th} = (1\ \text{k}\Omega \parallel 1.2\ \text{k}\Omega) = 0.545\ \text{k}\Omega$$

$$V_{Th} = 10 \times \frac{1.2}{1 + 1.2} = 5.45\ \text{V}$$

Find the Thévenin equivalent of the circuit to the right of node 2.

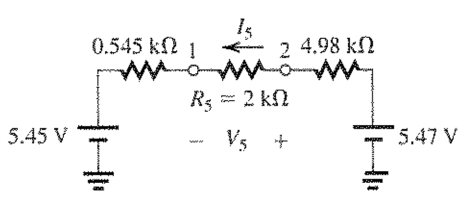


Between node 2 and ground,

$$R_{Th} = 9.1\ \text{k}\Omega \parallel 11\ \text{k}\Omega = 4.98\ \text{k}\Omega$$

$$V_{Th} = 10 \times \frac{11}{11 + 9.1} = 5.47\ \text{V}$$

The resulting simplified circuit is



$$I_5 = \frac{5.47 - 5.45}{4.98 + 2 + 0.545}$$

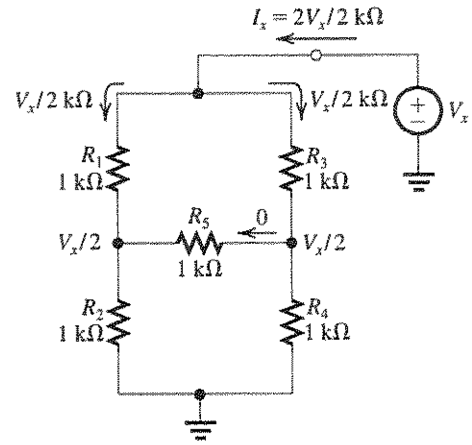
$$= 2.66\ \mu\text{A}$$

$$V_5 = 2.66\ \mu\text{A} \times 2\ \text{k}\Omega$$

$$= 5.32\ \text{mV}$$

1.18 From the symmetry of the circuit, there will be no current in R_5. (Otherwise the symmetry would be violated.) Thus each branch will carry a current $V_x/2$ kΩ and I_x will be the sum of the two current,

$$I_x = \frac{2V_x}{2\ \text{k}\Omega} = \frac{V_x}{1\ \text{k}\Omega}$$



Thus,

$$R_{eq} = \frac{V_x}{I_x} = 1\ \text{k}\Omega$$

Now, if R_4 is raised to 1.2 kΩ, the symmetry will be broken. To find Is we use Thévenin's theorem as shown in the figures on the next page. Thus,

$$I_5 = \frac{0.545V_x - 0.5V_x}{0.5 + 1 + 0.545} = 0.022V_x$$

$$V_1 = \frac{V_x}{2} + 0.022\ V_x \times 0.5$$

$$= 0.5V_x \times 1.022 = 0.511V_x$$

$$V_2 = V_1 + I_5R_5 = 0.533V_x$$

$$I_1 = \frac{V_x - V_1}{1\ \text{k}\Omega} = 0.489V_x$$

$$I_2 = \frac{V_x - V_2}{1\ \text{k}\Omega} = 0.467V_x$$

$$I_x = I_1 + I_2 = 0.956V_x$$

$$\Rightarrow R_{eq} = \frac{V_x}{I_x} = 1.05\ \text{k}\Omega$$

1.19 Refer to Fig. P1.19. Using the voltage divider rule at the input side, we obtain

$$\frac{v_\pi}{v_s} = \frac{r_\pi}{r_\pi + R_s} \qquad (1)$$

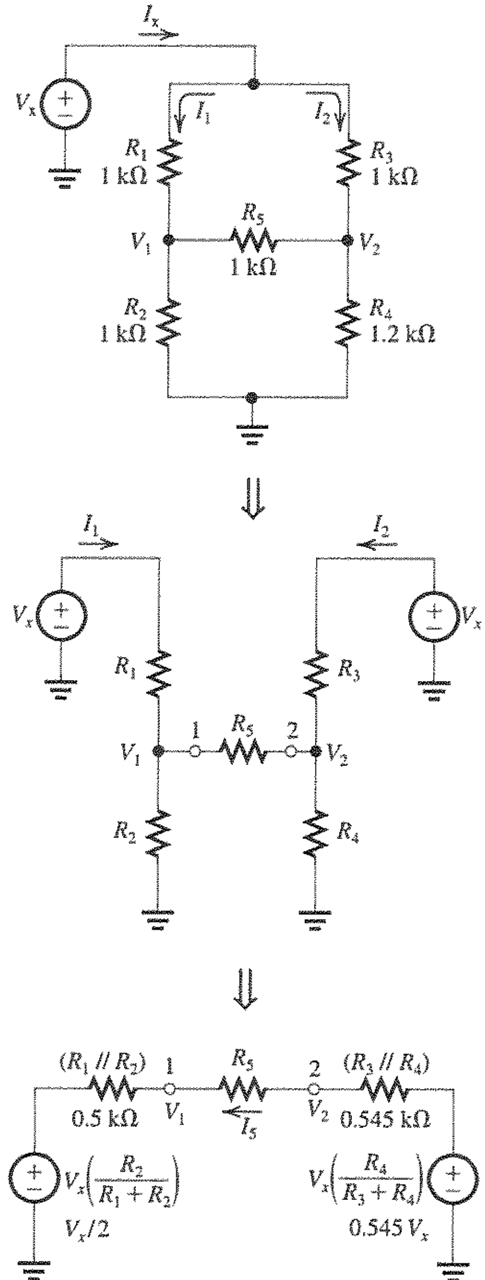
At the output side, we find v_o by multiplying the current $g_m v_\pi$ by the parallel equivalent of r_o and R_L,

$$v_o = -g_m v_\pi (r_o \parallel R_L) \qquad (2)$$

Finally, v_o/v_s can be obtained by combining Eqs. (1) and (2) as

$$\frac{v_o}{v_s} = -\frac{r_\pi}{r_\pi + R_s} g_m (r_o \parallel R_L)$$

This figure belongs to Problem 1.18.



1.20 (a) $T = 10^{-4}$ ms $= 10^{-7}$ s

$f = \frac{1}{T} = 10^7$ Hz

$\omega = 2\pi f = 6.28 \times 10^7$ rad/s

(b) $f = 1$ GHz $= 10^9$ Hz

$T = \frac{1}{f} = 10^{-9}$ s

$\omega = 2\pi f = 6.28 \times 10^9$ rad/s

(c) $\omega = 6.28 \times 10^2$ rad/s

$f = \frac{\omega}{2\pi} = 10^2$ Hz

$T = \frac{1}{f} = 10^{-2}$ s

(d) $T = 10$ s

$f = \frac{1}{T} = 10^{-1}$ Hz

$\omega = 2\pi f = 6.28 \times 10^{-1}$ rad/s

(e) $f = 60$ Hz

$T = \frac{1}{f} = 1.67 \times 10^{-2}$ s

$\omega = 2\pi f = 3.77 \times 10^2$ rad/s

(f) $\omega = 1$ krad/s $= 10^3$ rad/s

$f = \frac{\omega}{2\pi} = 1.59 \times 10^2$ Hz

$T = \frac{1}{f} = 6.28 \times 10^{-3}$ s

(g) $f = 1900$ MHz $= 1.9 \times 10^9$ Hz

$T = \frac{1}{f} = 5.26 \times 10^{-10}$ s

$\omega = 2\pi f = 1.194 \times 10^{10}$ rad/s

1.21 (a) $Z = 1$ kΩ at all frequencies

(b) $Z = 1/j\omega C = -j\frac{1}{2\pi f \times 10 \times 10^{-9}}$

At $f = 60$ Hz, $Z = -j265$ kΩ

At $f = 100$ kHz, $Z = -j159$ Ω

At $f = 1$ GHz, $Z = -j0.016$ Ω

(c) $Z = 1/j\omega C = -j\frac{1}{2\pi f \times 10 \times 10^{-12}}$

At $f = 60$ Hz, $Z = -j0.265$ GΩ

At $f = 100$ kHz, $Z = -j0.16$ MΩ

At $f = 1$ GHz, $Z = -j15.9$ Ω

(d) $Z = j\omega L = j2\pi f L = j2\pi f \times 10 \times 10^{-3}$

At $f = 60$ Hz, $Z = j3.77$ Ω

At $f = 100$ kHz, $Z = j6.28$ kΩ

At $f = 1$ GHz, $Z = j62.8$ MΩ

(e) $Z = j\omega L = j2\pi f L = j2\pi f(1 \times 10^{-6})$

$f = 60$ Hz, $Z = j0.377$ mΩ

$f = 100$ kHz, $Z = j0.628$ Ω

$f = 1$ GHz, $Z = j6.28$ kΩ

1.22 (a) $Z = R + \dfrac{1}{j\omega C}$

$= 10^3 + \dfrac{1}{j2\pi \times 10 \times 10^3 \times 10 \times 10^{-9}}$

$= (1 - j1.59)$ kΩ

(b) $Y = \dfrac{1}{R} + j\omega C$

$= \dfrac{1}{10^4} + j2\pi \times 10 \times 10^3 \times 0.01 \times 10^{-6}$

$= 10^{-4}(1 + j6.28)$ Ω

$Z = \dfrac{1}{Y} = \dfrac{10^4}{1 + j6.28}$

$= \dfrac{10^4(1 - j6.28)}{1 + 6.28^2}$

$= (247.3 - j1553)$ Ω

(c) $Y = \dfrac{1}{R} + j\omega C$

$= \dfrac{1}{100 \times 10^3} + j2\pi \times 10 \times 10^3 \times 100 \times 10^{-12}$

$= 10^{-5}(1 + j0.628)$

$Z = \dfrac{10^5}{1 + j0.628}$

$= (71.72 - j45.04)$ kΩ

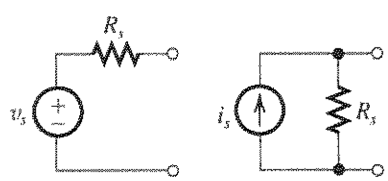
(d) $Z = R + j\omega L$

$= 100 + j2\pi \times 10 \times 10^3 \times 10 \times 10^{-3}$

$= 100 + j6.28 \times 100$

$= (100 + j628)$, Ω

1.23



Thévenin equivalent   Norton equivalent

$v_{oc} = v_s$

$i_{sc} = i_s$

$v_s = i_s R_s$

Thus,

$R_s = \dfrac{v_{oc}}{i_{sc}}$

(a) $v_s = v_{oc} = 1$ V

$i_s = i_{sc} = 0.1$ mA

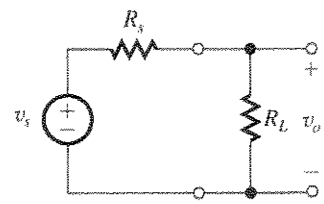
$R_s = \dfrac{v_{oc}}{i_{sc}} = \dfrac{1 \text{ V}}{0.1 \text{ mA}} = 10$ kΩ

(b) $v_s = v_{oc} = 0.1$ V

$i_s = i_{sc} = 1$ μA

$R_s = \dfrac{v_{oc}}{i_{sc}} = \dfrac{0.1 \text{ V}}{1 \text{ μA}} = 0.1$ MΩ $= 100$ kΩ

1.24



$\dfrac{v_o}{v_s} = \dfrac{R_L}{R_L + R_s}$

$v_o = v_s \Big/ \left(1 + \dfrac{R_s}{R_L}\right)$

Thus,

$$\frac{v_s}{1 + \dfrac{R_s}{100}} = 40 \quad (1)$$

and

$$\frac{v_s}{1 + \dfrac{R_s}{10}} = 10 \quad (2)$$

Dividing Eq. (1) by Eq. (2) gives

$\dfrac{1 + (R_s/10)}{1 + (R_s/100)} = 4$

$\Rightarrow R_S = 50$ kΩ

Substituting in Eq. (2) gives

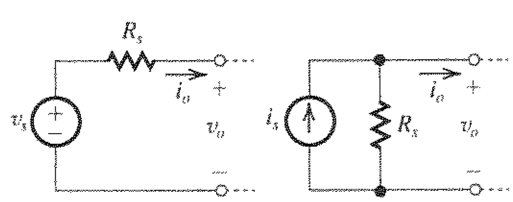
$v_s = 60$ mV

The Norton current i_s can be found as

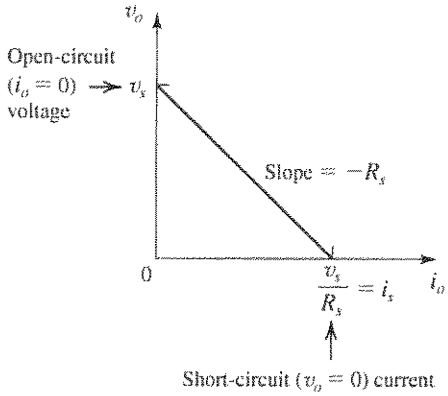
$i_s = \dfrac{v_s}{R_s} = \dfrac{60 \text{ mV}}{50 \text{ kΩ}} = 1.2$ μA

1.25 The observed output voltage is 1 mV/°C, which is one half the voltage specified by the sensor, presumably under open-circuit conditions: that is, without a load connected. It follows that that sensor internal resistance must be equal to R_L, that is, 5 kΩ.

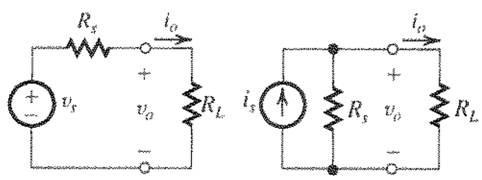
1.26



$v_o = v_s - i_o R_s$



1.27



R_L represents the input resistance of the processor

For $v_o = 0.95 v_s$

$$0.95 = \frac{R_L}{R_L + R_s} \Rightarrow R_L = 19R_s$$

For $i_o = 0.95 i_s$

$$0.95 = \frac{R_s}{R_s + R_L} \Rightarrow R_L = R_S/19$$

1.28

Case	ω (rad/s)	f (Hz)	T(s)
a	3.14×10^{10}	5×10^9	0.2×10^{-9}
b	2×10^9	3.18×10^8	3.14×10^{-9}
c	6.28×10^{10}	1×10^{10}	1×10^{-10}
d	3.77×10^2	60	1.67×10^{-2}
e	6.28×10^4	1×10^4	1×10^{-4}
f	6.28×10^5	1×10^5	1×10^{-5}

1.29 (a) $V_{peak} = 117 \times \sqrt{2} = 165$ V

(b) $V_{rms} = 33.9/\sqrt{2} = 24$ V

(c) $V_{peak} = 220 \times \sqrt{2} = 311$ V

(d) $V_{peak} = 220 \times \sqrt{2} = 311$ kV

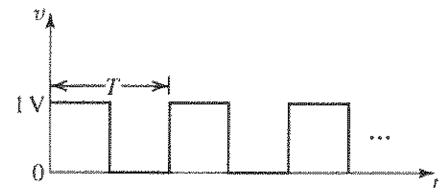
1.30 (a) $v = 10 \sin(2\pi \times 10^3 t)$, V

(b) $v = 120\sqrt{2} \sin(2\pi \times 60)$, V

(c) $v = 0.1 \sin(2000t)$, V

(d) $v = 0.1 \sin(2\pi \times 10^3 t)$, V

1.31 Comparing the given waveform to that described by Eq. (1.2), we observe that the given waveform has an amplitude of 0.5 V (1 V peak-to-peak) and its level is shifted up by 0.5 V (the first term in the equation). Thus the waveform looks as follows:



Average value = 0.5 V
Peak-to-peak value = 1 V
Lowest value = 0 V
Highest value = 1 V

$$\text{Period } T = \frac{1}{f_0} = \frac{2\pi}{\omega_0} = 10^{-3} \text{ s}$$

$$\text{Frequency } f = \frac{1}{T} = 1 \text{ kHz}$$

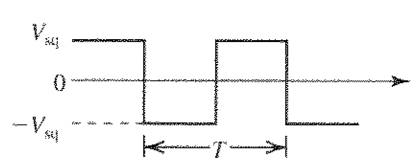
1.32 The two harmonics have the ratio $126/98 = 9/7$. Thus, these are the 7th and 9th harmonics. From Eq. (1.2), we note that the amplitudes of these two harmonics will have the ratio 7 to 9, which is confirmed by the measurement reported. Thus the fundamental will have a frequency of 98/7, or 14 kHz, and peak amplitude of $63 \times 7 = 441$ mV. The rms value of the fundamental will be $441/\sqrt{2} = 312$ mV. To find the peak-to-peak amplitude of the square wave, we note that $4V/\pi = 441$ mV. Thus,

Peak-to-peak amplitude

$$= 2V = 441 \times \frac{\pi}{2} = 693 \text{ mV}$$

$$\text{Period } T = \frac{1}{f} = \frac{1}{14 \times 10^3} = 71.4\ \mu\text{s}$$

1.33 If the amplitude of the square wave is V_{sq}, then the power delivered by the square wave to a resistance R will be V_{sq}^2/R. If this power is to be equal to that delivered by a sine wave of peak amplitude $\hat{V}$, then



$$\frac{V_{sq}^2}{R} = \frac{(\hat{V}/\sqrt{2})^2}{R}$$

Thus, $V_{sq} = \hat{V}/\sqrt{2}$. This result is independent of frequency.

1.34

Decimal	Binary
0	0
6	110
11	1011
28	11100
59	111011

1.35

b_3	b_2	b_1	b_0	Value Represented
0	0	0	0	+0
0	0	0	1	+1
0	0	1	0	+2
0	0	1	1	+3
0	1	0	0	+4
0	1	0	1	+5
0	1	1	0	+6
0	1	1	1	+7
1	0	0	0	–0
1	0	0	1	–1
1	0	1	0	–2
1	0	1	1	–3
1	1	0	0	–4
1	1	0	1	–5
1	1	1	0	–6
1	1	1	1	–7

Note that there are two possible representations of zero: 0000 and 1000. For a 0.5-V step size, analog signals in the range ±3.5 V can be represented.

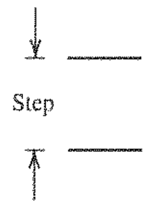
Input	Steps	Code
+2.5 V	+5	0101
–3.0 V	–6	1110
+2.7	+5	0101
–2.8	–6	1110

1.36 (a) For N bits there will be 2^N possible levels, from 0 to V_{FS}. Thus there will be $(2^N - 1)$ discrete steps from 0 to V_{FS} with the step size given by

$$\text{Step size} = \frac{V_{FS}}{2^N - 1}$$

This is the analog change corresponding to a change in the LSB. It is the value of the resolution of the ADC.

(b) The maximum error in conversion occurs when the analog signal value is at the middle of a step. Thus the maximum error is



$$\frac{1}{2} \times \text{ step size} = \frac{1}{2}\frac{V_{FS}}{2^N - 1}$$

This is known as the quantization error.

(c) $\dfrac{5\text{ V}}{2^N - 1} \leq 2\text{ mV}$

$2^N - 1 \geq 2500$

$2^N \geq 2501 \Rightarrow N = 12,$

For $N = 12$,

$$\text{Resolution} = \frac{5}{2^{12} - 1} = 1.2\text{ mV}$$

$$\text{Quantization error} = \frac{1.2}{2} = 0.6\text{ mV}$$

1.37 (a) When $b_i = 1$, the ith switch is in position 1 and a current $(V_{ref}/2^i R)$ flows to the output. Thus i_O will be the sum of all the currents corresponding to "1" bits, that is,

$$i_O = \frac{V_{\text{ref}}}{R}\left(\frac{b_1}{2^1} + \frac{b_2}{2^2} + \cdots + \frac{b_N}{2^N}\right)$$

(b) b_N is the LSB

b_1 is the MSB

(c) $$i_{O\max} = \frac{10\text{ V}}{10\text{ k}\Omega}\left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \frac{1}{2^8}\right)$$

$= 0.99609375$ mA

Corresponding to the LSB changing from 0 to 1 the output changes by $(10/10) \times 1/2^8 =$ 3.91 μA.

1.38 There will be 44,100 samples per second with each sample represented by 16 bits. Thus the throughput or speed will be $44,100 \times 16 = 7.056 \times 10^5$ bits per second.

1.39 (a) $A_v = \dfrac{v_O}{v_I} = \dfrac{10\text{ V}}{100\text{ mV}} = 100\text{ V/V}$

or $20 \log 100 = 40$ dB

$A_i = \dfrac{i_O}{i_I} = \dfrac{v_O/R_L}{i_I} = \dfrac{10\text{ V}/100\ \Omega}{100\ \mu\text{A}} = \dfrac{0.1\text{ A}}{100\ \mu\text{A}}$

$= 1000$ A/A

or $20 \log 1000 = 60$ dB

$A_p = \dfrac{v_O i_O}{v_I i_I} = \dfrac{v_O}{v_I} \times \dfrac{i_O}{i_I} = 100 \times 1000$

$= 10^5$ W/W

or $10 \log 10^5 = 50$ dB

(b) $A_v = \dfrac{v_O}{v_I} = \dfrac{1\text{ V}}{10\ \mu\text{V}} = 1 \times 10^5\text{ V/V}$

or $20 \log 1 \times 10^5 = 100$ dB

$A_i = \dfrac{i_O}{i_I} = \dfrac{v_O/R_L}{i_I} = \dfrac{1\text{ V}/10\text{ k}\Omega}{100\text{ nA}}$

$= \dfrac{0.1\text{ mA}}{100\text{ nA}} = \dfrac{0.1 \times 10^{-3}}{100 \times 10^{-9}} = 1000\text{ A/A}$

or $20 \log A_i = 60$ dB

$A_p = \dfrac{v_O i_O}{v_I i_I} = \dfrac{v_O}{v_I} \times \dfrac{i_O}{i_I}$

$= 1 \times 10^5 \times 1000$

$= 1 \times 10^8$ W/W

or $10 \log A_P = 80$ dB

(c) $A_v = \dfrac{v_O}{v_i} = \dfrac{5\text{ V}}{1\text{ V}} = 5\text{ V/V}$

or $20 \log 5 = 14$ dB

$A_i = \dfrac{i_O}{i_I} = \dfrac{v_O/R_L}{i_I} = \dfrac{5\text{ V}/10\ \Omega}{1\text{ mA}}$

$= \dfrac{0.5\text{ A}}{1\text{ mA}} = 500\text{ A/A}$

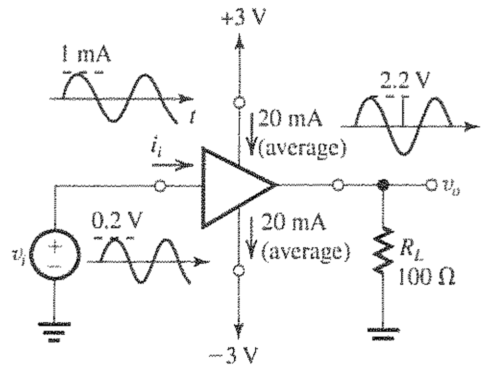
or $20 \log 500 = 54$ dB

$A_p = \dfrac{v_O i_O}{v_I i_I} = \dfrac{v_O}{v_I} \times \dfrac{i_O}{i_I}$

$= 5 \times 500 = 2500$ W/W

or $10 \log A_p = 34$ dB

1.40



$A_v = \dfrac{v_o}{v_i} = \dfrac{2.2}{0.2}$

$= 11$ V/V

or $20 \log 11 = 20.8$ dB

$A_i = \dfrac{i_o}{i_i} = \dfrac{2.2\text{ V}/100\ \Omega}{1\text{ mA}}$

$= \dfrac{22\text{ mA}}{1\text{ mA}} = 22\text{ A/A}$

or $20 \log A_i = 26.8$ dB

$A_p = \dfrac{p_o}{p_i} = \dfrac{(2.2/\sqrt{2})^2/100}{\dfrac{0.2}{\sqrt{2}} \times \dfrac{10^{-3}}{\sqrt{2}}}$

$= 242$ W/W

or $10 \log A_P = 23.8$ dB

Supply power $= 2 \times 3\text{ V} \times 20\text{ mA} = 120$ mW

Output power $= \dfrac{v_{orms}^2}{R_L} = \dfrac{(2.2/\sqrt{2})^2}{100\ \Omega} = 24.2$ mW

Input power $= \dfrac{24.2}{242} = 0.1$ mW (negligible)

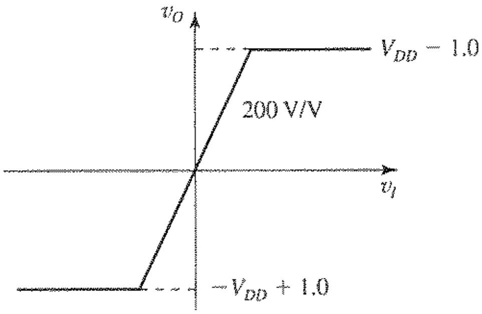
Amplifier dissipation $\simeq$ Supply power − Output power

$= 120 - 24.2 = 95.8$ mW

Amplifier efficiency $= \dfrac{\text{Output power}}{\text{supply power}} \times 100$

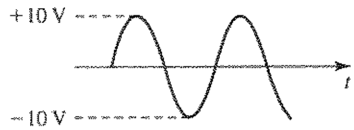
$= \dfrac{24.2}{120} \times 100 = 20.2\%$

1.41 For ±5 V supplies:
The largest undistorted sine-wave output is of 4-V peak amplitude or $4/\sqrt{2} = 2.8\ V_{rms}$. Input needed is 14 mV_{rms}.
For ±10-V supplies, the largest undistorted sine-wave output is of 9-V peak amplitude or 6.4 V_{rms}. Input needed is 32 mV_{rms}.



For ±15-V supplies, the largest undistorted sine-wave output is of 14-V peak amplitude or 9.9 V_{rms}. The input needed is 9.9 V/200 = 49.5 mV_{rms}.

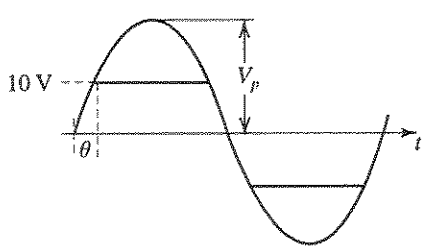
1.42 (a) For an output whose extremes are just at the edge of clipping (i.e., an output of 10-V_{peak}), the input must have 10 V/1000 = 10 mV_{peak}.



(b) For an output that is clipping 90% of the time,

$\theta = 0.1 \times 90^\circ = 9^\circ$ and $V_P \sin 9^\circ$

$= 10\text{ V} \Rightarrow V_P = 63.9$ V, which of course does not occur because the output saturates at ±10 V. To produce this result, the input peak must be 63.9/1000 = 63.9 mV.



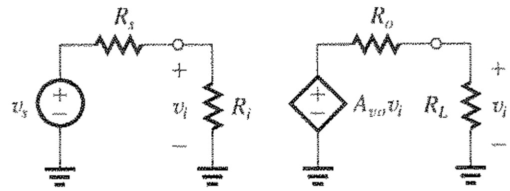
(c) For an output that is clipping 99% of the time, $\theta = 0.01 \times 90^\circ = 0.9^\circ$

$V_P \sin 0.9^\circ = 10$ V

$\Rightarrow V_P = 637$ V, the input must be 637 V/1000 or 0.637 V_{peak}.

1.43 $v_o = A_{vo} v_i \dfrac{R_L}{R_L + R_o}$

$$= A_{vo}\left(v_s \frac{R_i}{R_i + R_s}\right) \frac{R_L}{R_L + R_o}$$



Thus,

$$\frac{v_o}{v_s} = A_{vo} \frac{R_i}{R_i + R_s} \frac{R_L}{R_L + R_o}$$

(a) $A_{vo} = 100, R_i = 10R_s, R_L = 10R_o$:

$$\frac{v_o}{v_s} = 100 \times \frac{10R_s}{10R_s + R_s} \times \frac{10R_o}{10R_o + R_o}$$

$= 82.6$ V/V or $20 \log 82.6 = 38.3$ dB

(b) $A_{vo} = 100, R_i = R_s, R_L = R_o$:

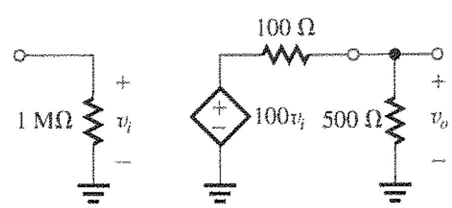
$$\frac{v_o}{v_s} = 100 \times \frac{1}{2} \times \frac{1}{2} = 25 \text{ V/V or } 20 \log 25 = 28 \text{ dB}$$

(c) $A_{vo} = 100$ V/V, $R_i = R_s/10, R_L = R_o/10$:

$$\frac{v_o}{v_s} = 100 \frac{R_s/10}{(R_s/10) + R_s} \frac{R_o/10}{(R_o/10) + R_o}$$

$= 0.826$ V/V or $20 \log 0.826 = -1.7$ dB

1.44



$20 \log A_{vo} = 40 \text{ dB} \Rightarrow A_{vo} = 100$ V/V

$$A_v = \frac{v_o}{v_i}$$

$$= 100 \times \frac{500}{500 + 100}$$

$= 83.3$ V/V

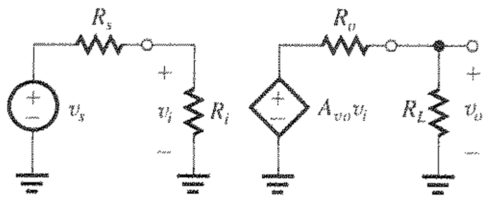
or $20 \log 83.3 = 38.4$ dB

$$A_p = \frac{v_o^2/500\ \Omega}{v_i^2/1\ \text{M}\Omega} = A_v^2 \times 10^4 = 1.39 \times 10^7 \text{ W/W}$$

or $10 \log (1.39 \times 10^7) = 71.4$ dB.

For a peak output sine-wave current of 20 mA, the peak output voltage will be 20 mA × 500 Ω = 10 V. Correspondingly v_i will be a sine wave with a peak value of 10 V/A_v = 10/83.3, or an rms value of $10/(83.3 \times \sqrt{2}) = 0.085$ V. Corresponding output power $= (10/\sqrt{2})^2/500\ \Omega$ = 0.1 W

1.45



$$\frac{v_o}{v_s} = \frac{10\text{ k}\Omega}{10\text{ k}\Omega + 100\text{ k}\Omega} \times 1000 \times \frac{100\ \Omega}{100\ \Omega + 1\text{ k}\Omega}$$

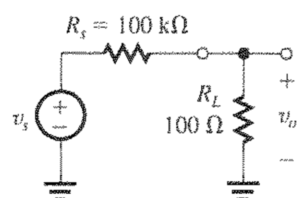
$$= \frac{10}{110} \times 1000 \times \frac{100}{1100} = 8.26\text{ V/V}$$

The signal loses about 90% of its strength when connected to the amplifier input (because $R_i = R_s/10$). Also, the output signal of the amplifier loses approximately 90% of its strength when the load is connected (because $R_L = R_o/10$). Not a good design! Nevertheless, if the source were connected directly to the load,

$$\frac{v_o}{v_s} = \frac{R_L}{R_L + R_s}$$

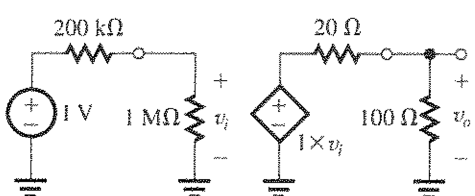
$$= \frac{100\ \Omega}{100\ \Omega + 100\text{ k}\Omega}$$

$$\simeq 0.001\text{ V/V}$$

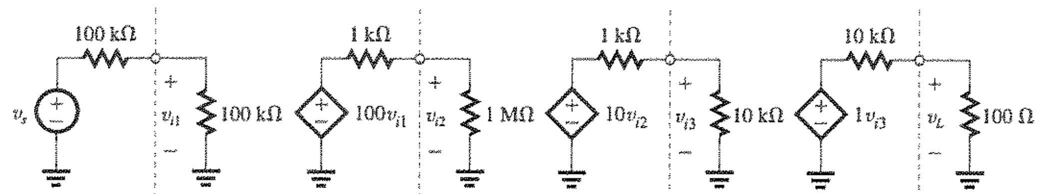


which is clearly a much worse situation. Indeed inserting the amplifier increases the gain by a factor 8.3/0.001 = 8300.

1.46



This figure belongs to Problem 1.47.



$$v_o = 1\text{ V} \times \frac{1\text{ M}\Omega}{1\text{ M}\Omega + 200\text{ k}\Omega}$$

$$\times 1 \times \frac{100\ \Omega}{100\ \Omega + 20\ \Omega}$$

$$= \frac{1}{1.2} \times \frac{100}{120} = 0.69\text{ V}$$

$$\text{Voltage gain} = \frac{v_o}{v_s} = 0.69\text{ V/V or } -3.2\text{ dB}$$

$$\text{Current gain} = \frac{v_o/100\ \Omega}{v_s/1.2\text{ M}\Omega} = 0.69 \times 1.2 \times 10^4$$

$$= 8280\text{ A/A} \quad \text{or} \quad 78.4\text{ dB}$$

$$\text{Power gain} = \frac{v_o^2/100\ \Omega}{v_s^2/1.2\text{ M}\Omega} = 5713\text{ W/W}$$

or $10 \log 5713 = 37.6$ dB

(This takes into account the power dissipated in the internal resistance of the source.)

1.47 In Example 1.3, when the first and the second stages are interchanged, the circuit looks like the figure above, and

$$\frac{v_{i1}}{v_s} = \frac{100\text{ k}\Omega}{100\text{ k}\Omega + 100\text{ k}\Omega} = 0.5\text{ V/V}$$

$$A_{v1} = \frac{v_{i2}}{v_{i1}} = 100 \times \frac{1\text{ M}\Omega}{1\text{ M}\Omega + 1\text{ k}\Omega}$$

$$= 99.9\text{ V/V}$$

$$A_{v2} = \frac{v_{i3}}{v_{i2}} = 10 \times \frac{10\text{ k}\Omega}{10\text{ k}\Omega + 1\text{ k}\Omega}$$

$$= 9.09\text{ V/V}$$

$$A_{v3} = \frac{v_L}{v_{i3}} = 1 \times \frac{100\ \Omega}{100\ \Omega + 10\ \Omega} = 0.909\text{ V/V}$$

$$\text{Total gain} = A_v = \frac{v_L}{v_{i1}} = A_{v1} \times A_{v2} \times A_{v3}$$

$$= 99.9 \times 9.09 \times 0.909 = 825.5\text{ V/V}$$

The voltage gain from source to load is

$$\frac{v_L}{v_s} = \frac{v_L}{v_{i1}} \times \frac{v_{i1}}{v_S} = A_v \cdot \frac{v_{i1}}{v_S}$$

$$= 825.5 \times 0.5$$

$$= 412.7\text{ V/V}$$

The overall voltage has reduced appreciably. This is because the input resistance of the first stage, R_{in}, is comparable to the source resistance R_s. In Example 1.3 the input resistance of the first stage is much larger than the source resistance.

1.48 (a) Case S-A-B-L (see figure below):

$$\frac{v_o}{v_s} = \frac{v_o}{v_{ib}} \times \frac{v_{ib}}{v_{ia}} \times \frac{v_{ia}}{v_s} =$$

$$\left(10 \times \frac{100}{100 + 1000}\right) \times \left(100 \times \frac{10}{10 + 10}\right) \times \left(\frac{100}{100 + 100}\right)$$

$\frac{v_o}{v_s} = 22.7$ V/V and gain in dB 20 log 22.7 = 27.1 dB

(b) Case S-B-A-L (see figure below):

$$\frac{v_o}{v_s} = \frac{v_o}{v_{ia}} \cdot \frac{v_{ia}}{v_{ib}} \cdot \frac{v_{ib}}{v_s}$$

$$= \left(100 \times \frac{100}{100 + 10\text{ K}}\right) \times \left(10 \times \frac{100\text{ K}}{100\text{ K} + 1\text{ K}}\right) \times \left(\frac{10\text{ K}}{10\text{ K} + 100\text{ K}}\right)$$

$\frac{v_o}{v_s} = 0.89$ V/V and gain in dB is 20 log 0.89 = -1 dB. Obviously, case a is preferred because it provides higher voltage gain.

1.49 Each of stages #1, 2, ..., $(n-1)$ can be represented by the equivalent circuit:

$$\frac{v_o}{v_s} = \frac{v_{i1}}{v_s} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i3}}{v_{i2}} \times \cdots \times \frac{v_{in}}{v_{i(n-1)}} \times \frac{v_o}{v_{in}}$$

where

$$\frac{v_{i1}}{v_s} = \frac{10\text{ k}\Omega}{10\text{ k}\Omega + 10\text{ k}\Omega} = 0.5\text{ V/V}$$

$$\frac{v_o}{v_{in}} = 10 \times \frac{200\ \Omega}{1\text{ k}\Omega + 200\ \Omega} = 1.67\text{ V/V}$$

$$\frac{v_{i2}}{v_{i1}} = \frac{v_{i3}}{v_{i2}} = \cdots = \frac{v_{in}}{v_{i(n-1)}} = 10 \times \frac{10\text{ k}\Omega}{10\text{ k}\Omega + 10\text{ k}\Omega}$$

$= 9.09$ V/V

Thus,

$$\frac{v_o}{v_s} = 0.5 \times (9.09)^{n-1} \times 1.67 = 0.833 \times (9.09)^{n-1}$$

For $v_s = 5$ mV and $v_o = 3$ V, the gain $\frac{v_o}{v_s}$ must be ≥ 600, thus

$0.833 \times (9.09)^{n-1} \geq 600$

$\Rightarrow n = 4$

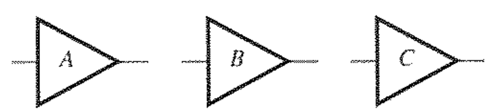
Thus four amplifier stages are needed, resulting in

$$\frac{v_o}{v_s} = 0.833 \times (9.09)^3 = 625.7\text{ V/V}$$

and correspondingly

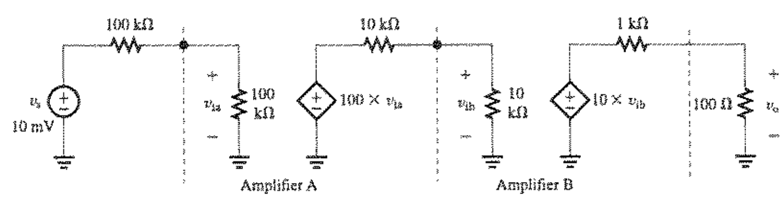
$v_o = 625.7 \times 5\text{ mV} = 3.13$ V

1.50 Deliver 0.5 W to a 100-Ω load. Source is 30 mV rms with 0.5-MΩ source resistance. Choose from these three amplifier types:

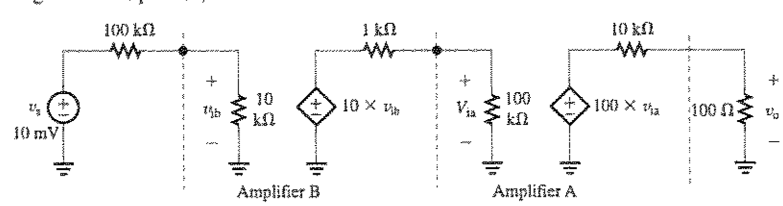


A	B	C
$R_i = 1$ MΩ	$R_i = 10$ kΩ	$R_i = 10$ kΩ
$A_v = 10$ V/V	$A_v = 100$ V/V	$A_v = 1$ V/V
$R_o = 10$ kΩ	$R_o = 1$ kΩ	$R_o = 20$ Ω

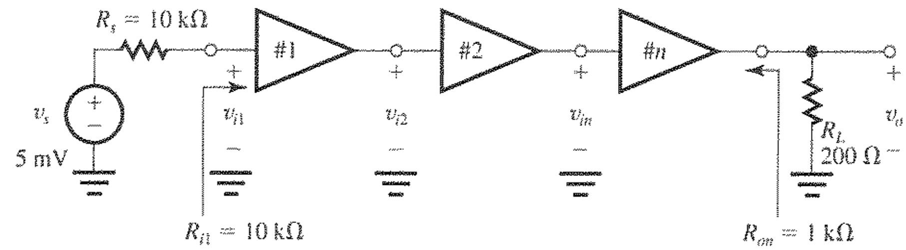
This figure belongs to **1.48**, part (a).

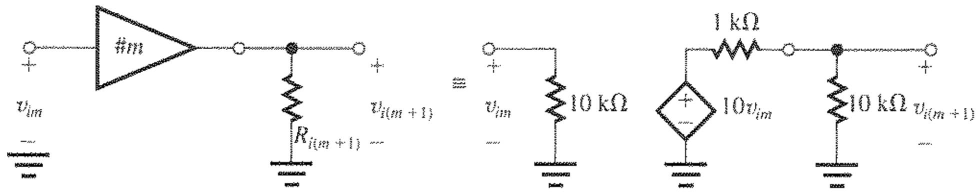


This figure belongs to **1.48**, part (b).

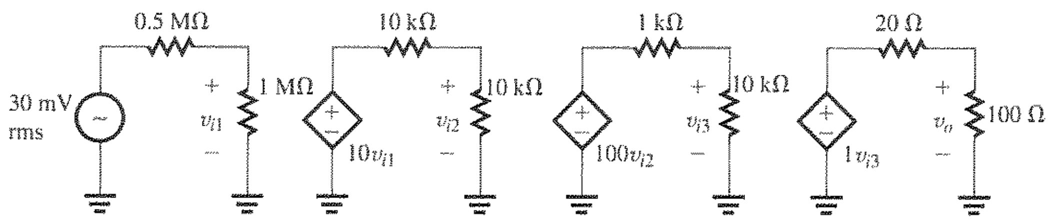


This figure belongs to **1.49**.





This figure belongs to **1.50**.



Choose order to eliminate loading on input and output:
A, first, to minimize loading on 0.5-MΩ source
B, second, to boost gain
C, third, to minimize loading at 100-Ω output.
We first attempt a cascade of the three stages in the order *A*, *B*, *C* (see figure above), and obtain

$$\frac{v_{i1}}{v_s} = \frac{1\ \text{M}\Omega}{1\ \text{M}\Omega + 0.5\ \text{M}\Omega} = \frac{1}{1.5}$$

$$\Rightarrow v_{i1} = 30 \times \frac{1}{1.5} = 20\ \text{mV}$$

$$\frac{v_{i2}}{v_{i1}} = 10 \times \frac{10\ \text{k}\Omega}{10\ \text{k}\Omega + 10\ \text{k}\Omega} = 5$$

$$\Rightarrow v_{i2} = 20 \times 5 = 100\ \text{mV}$$

$$\frac{v_{i3}}{v_{i2}} = 100 \times \frac{10\ \text{k}\Omega}{10\ \text{k}\Omega + 1\ \text{k}\Omega} = 90.9$$

$$\Rightarrow v_{i3} = 100\ \text{mV} \times 90.9 = 9.09\ \text{V}$$

$$\frac{v_o}{v_{i3}} = 1 \times \frac{100\ \Omega}{100\ \Omega + 20\ \Omega} = 0.833$$

$$\Rightarrow v_o = 9.09 \times 0.833 = 7.6\ \text{V}$$

$$P_o = \frac{v_{o\,\text{rms}}^2}{R_L} = \frac{7.6^2}{100} = 0.57\ \text{W}$$

which exceeds the required 0.5 W. Also, the signal throughout the amplifier chain never drops below 20 mV (which is greater than the required minimum of 10 mV).

1.51 (a) Required voltage gain $\equiv \dfrac{v_o}{v_s}$

$$= \frac{2\ \text{V}}{0.005\ \text{V}} = 400\ \text{V/V}$$

(b) The smallest R_i allowed is obtained from

$$0.1\ \mu\text{A} = \frac{5\ \text{mV}}{R_s + R_i} \Rightarrow R_s + R_i = 50\ \text{k}\Omega$$

Thus $R_i = 40\ \text{k}\Omega$.

For $R_i = 40\ \text{k}\Omega$, $i_i = 0.1\ \mu\text{A}$ peak, and

$$\text{Overall current gain} = \frac{v_o/R_L}{i_i} = \frac{2\ \text{V}/1\ \text{k}\Omega}{0.1\ \mu\text{A}}$$

$$= \frac{2\ \text{mA}}{0.1\ \mu\text{A}} = 2 \times 10^4\ \text{A/A}$$

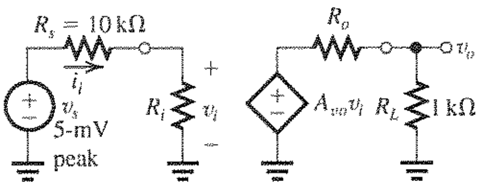
$$\text{Overall power gain} \equiv \frac{v_{o\,\text{rms}}^2/R_L}{v_{s(\text{rms})} \times i_{i(\text{rms})}}$$

$$= \frac{\left(\frac{2}{\sqrt{2}}\right)^2 \Big/ 1000}{\left(\frac{5 \times 10^{-3}}{\sqrt{2}}\right) \times \left(\frac{0.1 \times 10^{-6}}{\sqrt{2}}\right)}$$

$$= 8 \times 10^6 \text{ W/W}$$

(This takes into account the power dissipated in the internal resistance of the source.)

(c) If $(A_{vo}v_i)$ has its peak value limited to 3 V, the largest value of R_O is found from



$$= 3 \times \frac{R_L}{R_L + R_o} = 2 \Rightarrow R_o = \frac{1}{2}R_L = 500\ \Omega$$

(If R_o were greater than this value, the output voltage across R_L would be less than 2 V.)

(d) For $R_i = 40\ \text{k}\Omega$ and $R_o = 500\ \Omega$, the required value A_{vo} can be found from

$$400 \text{ V/V} = \frac{40}{40 + 10} \times A_{vo} \times \frac{1}{1 + 0.5}$$

$$\Rightarrow A_{vo} = 750 \text{ V/V}$$

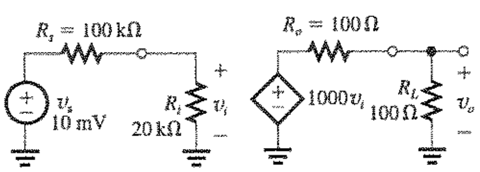
(e) $R_i = 100\ \text{k}\Omega (1 \times 10^5\ \Omega)$

$R_o = 100\ \Omega (1 \times 10^2\ \Omega)$

$$400 = \frac{100}{100 + 10} \times A_{vo} \times \frac{1000}{1000 + 100}$$

$$\Rightarrow A_{vo} = 484 \text{ V/V}$$

1.52



(a)

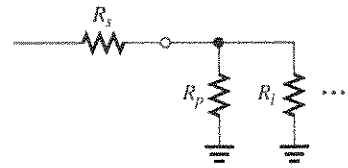
$$v_o = 10 \text{ mV} \times \frac{20}{20 + 100} \times 1000 \times \frac{100}{100 + 100}$$

$$= 833 \text{ mV}$$

(b) $\dfrac{v_o}{v_s} = \dfrac{833 \text{ mV}}{10 \text{ mV}} = 83.3 \text{ V/V}$

(c) $\dfrac{v_o}{v_i} = 1000 \times \dfrac{100}{100 + 100} = 500 \text{ V/V}$

(d)



Connect a resistance R_P in parallel with the input and select its value from

$$\frac{(R_p \parallel R_i)}{(R_p \parallel R_i) + R_s} = \frac{1}{2}\frac{R_i}{R_i + R_s}$$

$$\Rightarrow 1 + \frac{R_s}{R_p \parallel R_i} = 12 \Rightarrow R_p \parallel R_i = \frac{R_S}{11} = \frac{100}{11}$$

$$\Rightarrow \frac{1}{R_p} + \frac{1}{R_i} = \frac{11}{100}$$

$$R_p = \frac{1}{0.11 - 0.05} = 16.7\ \text{k}\Omega$$

1.53 From the equivalent circuit of the output side of a voltage amplifier [Fig. 1.16(b)]:

$$v_o = (A_{vo}v_i)\frac{R_L}{R_L + R_o}$$

$$200 = (A_{vo}v_i)\frac{1000}{1000 + R_o} \qquad (1)$$

$$195 = (A_{vo}v_i)\frac{780}{780 + R_o} \qquad (2)$$

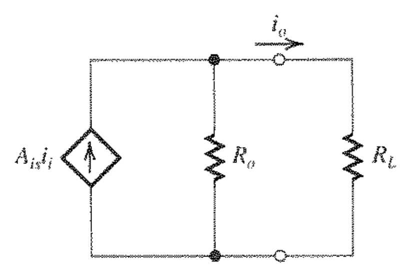
Dividing Eq. (2) by Eq. (1), we have

$$\frac{195}{200} = 0.78\frac{1000 + R_o}{780 + R_o}$$

$$\Rightarrow R_o = 100\ \Omega$$

$$(A_{vo}v_i) = 200[(1000 + 100)/1000] = 220 \text{ mV}$$

1.54 The equivalent circuit at the output side of a current amplifier loaded with a resistance R_L is shown. Since



$$i_o = (A_{is}i_i)\frac{R_o}{R_o + R_L}$$

we can write

$$1 = (A_{is}i_i)\frac{R_o}{R_o + 1} \qquad (1)$$

and

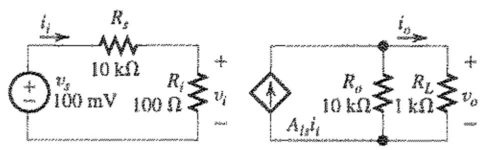
$$0.5 = (A_{is}i_i)\frac{R_o}{R_o + 12} \qquad (2)$$

Dividing Eq. (1) by Eq. (2), we have

$$2 = \frac{R_o + 12}{R_o + 1} \Rightarrow R_o = 10\ \text{k}\Omega$$

$$A_{is}i_i = 1 \times \frac{10 + 1}{10} = 1.1\ \text{mA}$$

1.55



(a) Current gain $= \frac{i_o}{i_i}$

$$= A_{is}\frac{R_o}{R_o + R_L}$$

$$= 100\frac{10}{11}$$

$$= 90.9\ \text{A/A} = 39.2\ \text{dB}$$

(b) Voltage gain $= \frac{v_o}{v_s} = \frac{i_o R_L}{i_i(R_s + R_i)}$

$$= \frac{i_o}{i_i}\frac{R_L}{R_s + R_i}$$

$$= 90.9 \times \frac{1}{10 + 0.1}$$

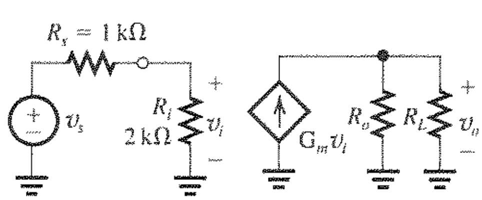
$$= 9\ \text{V/V} = 19.1\ \text{dB}$$

(c) Power gain $= A_p = \frac{v_o i_o}{v_s i_i}$

$$= 9 \times 90.9$$

$$= 818\ \text{W/W} = 29.1\ \text{dB}$$

1.56



$G_m = 60$ mA/V

$R_o = 20$ kΩ

$R_L = 1$ kΩ

$$v_i = v_s\frac{R_i}{R_s + R_i}$$

$$= v_s\frac{2}{1+2} = \frac{2}{3}v_s$$

$$v_o = G_m v_i(R_L \parallel R_o)$$

$$= 60\frac{20 \times 1}{20 + 1}v_i$$

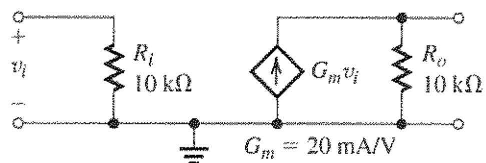
$$= 60\frac{20}{21} \times \frac{2}{3}v_s$$

Overall voltage gain $\equiv \frac{v_o}{v_s} = 38.1$ V/V

1.57 To obtain the weighted sum of v_1 and v_2

$$v_o = 10v_1 + 20v_2$$

we use two transconductance amplifiers and sum their output currents. Each transconductance amplifier has the following equivalent circuit:



Consider first the path for the signal requiring higher gain, namely v_2. See figure at top of next page.

The parallel connection of the two amplifiers at the output and the connection of R_L means that the total resistance at the output is

$10\ \text{k}\Omega \parallel 10\ \text{k}\Omega \parallel 10\ \text{k}\Omega = \frac{10}{3}\ \text{k}\Omega$. Thus the component of v_o due to v_2 will be

$$v_{o2} = v_2\frac{10}{10 + 10} \times G_{m2} \times \frac{10}{3}$$

$$= v_2 \times 0.5 \times 20 \times \frac{10}{3} = 33.3v_2$$

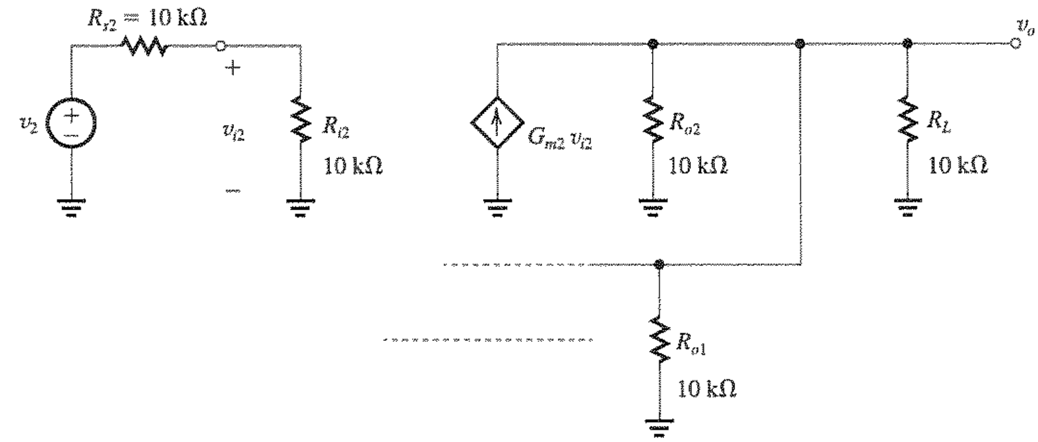
To reduce the gain seen by v_2 from 33.3 to 20, we connect a resistance R_p in parallel with R_L,

$$\left(\frac{10}{3} \parallel R_p\right) = 2\ \text{k}\Omega$$

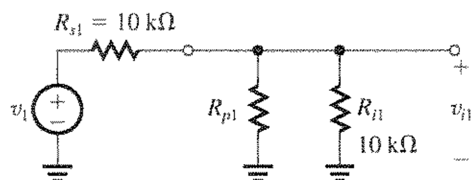
$$\Rightarrow R_p = 5\ \text{k}\Omega$$

We next consider the path for v_1. Since v_1 must see a gain factor of only 10, which is half that seen by v_2, we have to reduce the fraction of v_1 that appears at the input of its transconductance amplifier to half that that appears at the input of the v_2 transconductance amplifier. We just saw that 0.5 v_2 appears at the input of the v_2 transconductance amplifier. Thus, for the v_1 transconductance amplifier, we want $0.25v_1$ to appear at the input. This can be achieved by shunting the input of the v_1 transconductance

This figure belongs to Problem 1.57.



amplifier by a resistance R_{p1} as in the following figure.



The value of R_{p1} can be found from

$$\frac{(R_{p1} \parallel R_{i1})}{(R_{p1} \parallel R_{i1}) + R_{s1}} = 0.25$$

Thus,

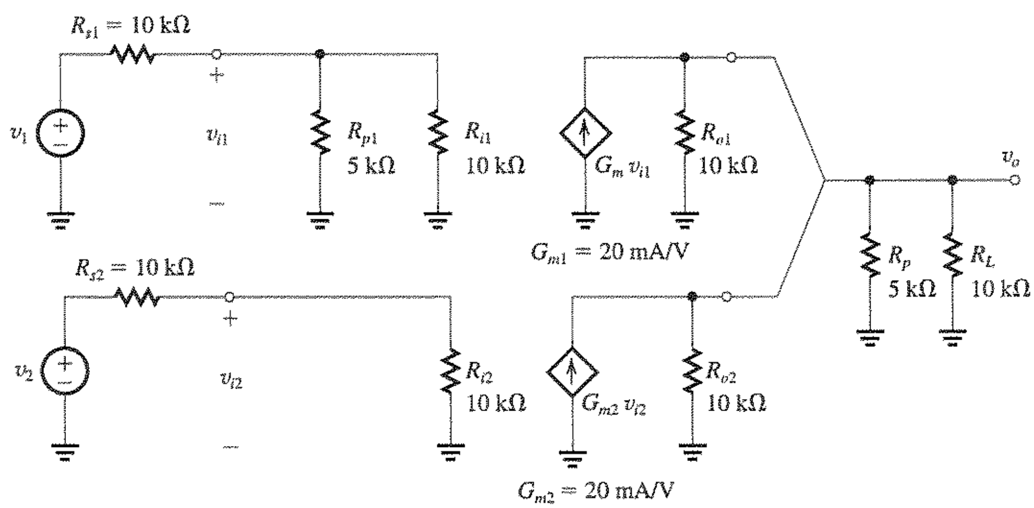
$$1 + \frac{R_{s1}}{(R_{p1} \parallel R_{i1})} = 4$$

$$\Rightarrow R_{p1} \parallel R_{i1} = \frac{R_{s1}}{3} = \frac{10}{3}$$

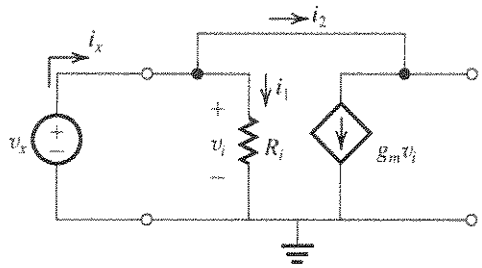
$$R_{p1} \parallel 10 = \frac{10}{3}$$

$$\Rightarrow R_{p1} = 5\ k\Omega$$

The final circuit will be as follows:

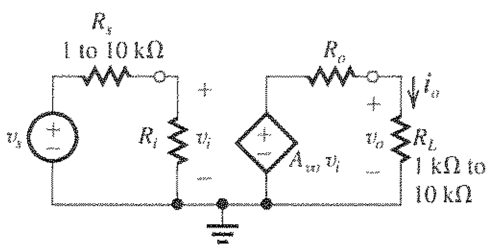


1.58



$$\left.\begin{aligned} i_x &= i_1 + i_2 \\ i_1 &= v_i/R_i \\ i_2 &= g_m v_i \\ v_i &= v_x \end{aligned}\right\} \quad \begin{aligned} i_x &= v_x/R_i + g_m v_x \\ i_x &= v_x\left(\frac{1}{R_i} + g_m\right) \\ \frac{v_x}{i_x} &= \frac{1}{1/R_i + g_m} \\ &= \frac{R_i}{1 + g_m R_i} = R_{\text{in}} \end{aligned}$$

1.59 Voltage amplifier:



For R_s varying in the range 1 kΩ to 10 kΩ and Δv_o limited to 10%, select R_i to be sufficiently large:

$R_i \geq 10\ R_{smax}$

$R_i = 10 \times 10\ \text{k}\Omega = 100\ \text{k}\Omega = 1 \times 10^5\ \Omega$

For R_L varying in the range 1 kΩ to 10 kΩ, the load voltage variation limited to 10%, select R_o sufficiently low:

$$R_o \leq \frac{R_{L\min}}{10}$$

$$R_o = \frac{1\ \text{k}\Omega}{10} = 100\ \Omega = 1 \times 10^2\ \Omega$$

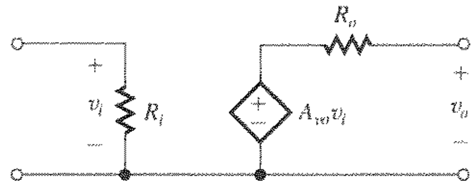
Now find A_{vo}:

$$v_{o\min} = 10\ \text{mV} \times \frac{R_i}{R_i + R_{s\max}} \times A_{vo} \frac{R_{L\min}}{R_o + R_{L\min}}$$

$$1 = 10 \times 10^{-3} \times \frac{100\ \text{k}\Omega}{100\ \text{k}\Omega + 10\ \text{k}\Omega}$$

$$\times A_{vo} \times \frac{1\ \text{k}\Omega}{100\ \Omega + 1\ \text{k}\Omega}$$

$\Rightarrow A_{vo} = 121\ \text{V/V}$

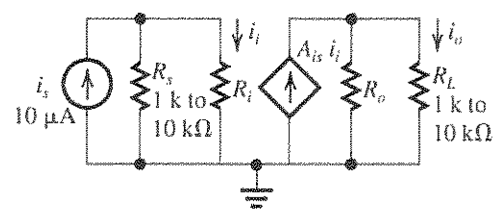


Values for the voltage amplifier equivalent circuit are

$R_i = 1 \times 10^5\ \Omega$, $A_{vo} = 121\ \text{V/V}$, and

$R_o = 1 \times 10^2\ \Omega$

1.60 Current amplifier:



For R_s varying in the range 1 kΩ to 10 kΩ and load current variation limited to 10%, select R_i to be sufficiently low:

$$R_i \leq \frac{R_{s\min}}{10}$$

$$R_i = \frac{1\ \text{k}\Omega}{10} = 100\ \Omega = 1 \times 10^2\ \Omega$$

For R_L varying in the range 1 kΩ to 10 kΩ and the load current variation limited to 10%, R_o is selected sufficiently large:

$R_o \geq 10\ \text{R}_{L\max}$

$R_o = 10 \times 10\ \text{k}\Omega$

$= 100\ \text{k}\Omega = 1 \times 10^5\ \Omega$

Now we find A_{is}:

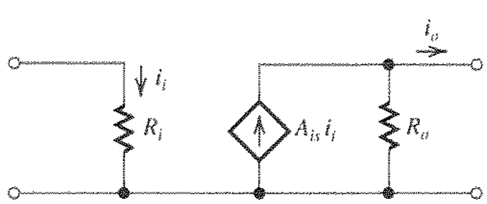
$$i_{o\min} = 10\ \mu\text{A} \times \frac{R_{s\min}}{R_{s\min} + R_i} \times A_{is} \times \frac{R_o}{R_o + R_{L\max}}$$

$$1 \times 10^{-3} = 10 \times 10^{-6} \frac{1\ \text{k}\Omega}{1\ \text{k}\Omega + 100\ \Omega}$$

$$\times A_{is} \frac{100\ \text{k}\Omega}{100\ \text{k}\Omega + 10\ \text{k}\Omega}$$

$\Rightarrow A_{is} = 121\ \text{A/A}$

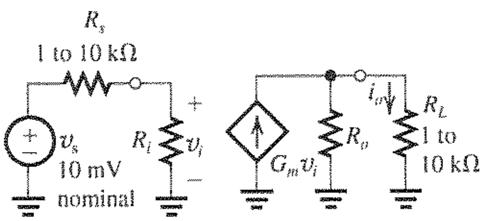
Current amplifier equivalent circuit:



$R_i = 1 \times 10^2\ \Omega$, $A_{is} = 121$ A/A,

$R_o = 1 \times 10^5\ \Omega$

1.61 Transconductance amplifier:



For R_s varying in the range 1 to 10 kΩ, and Δi_o limited to 10%, we have to select R_i sufficiently large;

$R_i \geq 10R_{s\max}$

$R_i = 100\ \text{k}\Omega = 1 \times 10^5\ \Omega$

For R_L varying in the range 1 to 10 kΩ, the change in i_o can be kept to 10% if R_o is selected sufficiently large;

$R_o \geq R_{L\max}$

Thus $R_o = 100\ \text{k}\Omega = 1 \times 10^5\ \Omega$

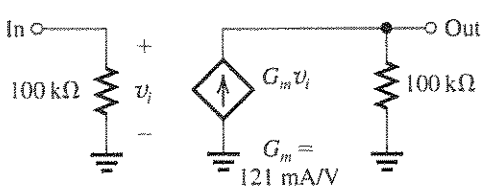
For $v_s = 10$ mV,

$$i_{o\min} = 10^{-2}\frac{R_i}{R_i + R_{s\max}}G_m\frac{R_o}{R_o + R_{L\max}}$$

$$10^{-3} = 10^{-2}\frac{100}{100+10}G_m\frac{100}{100+10}$$

$G_m = 1.21 \times 10^{-1}$ A/V

$= 121$ mA/V



1.62 Transresistance amplifier:

To limit Δv_o to 10% corresponding to R_s varying in the range 1 kΩ to 10 kΩ, we select R_i sufficiently low;

$$R_i \leq \frac{R_{s\min}}{10}$$

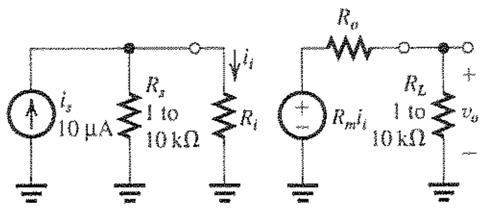
Thus, $R_i = 100\ \Omega = 1 \times 10^2\ \Omega$

To limit Δv_o to 10% while R_L varies over the range 1 kΩ to 10 kΩ, we select R_o sufficiently low;

$$R_o \leq \frac{R_{L\min}}{10}$$

Thus, $R_o = 100\ \Omega = 1 \times 10^2\ \Omega$

Now, for $i_s = 10\ \mu$A,

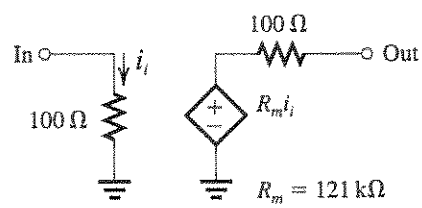


$$v_{o\min} = 10^{-5}\frac{R_{s\min}}{R_{s\min}+R_i}R_m\frac{R_{L\min}}{R_{L\min}+R_o}$$

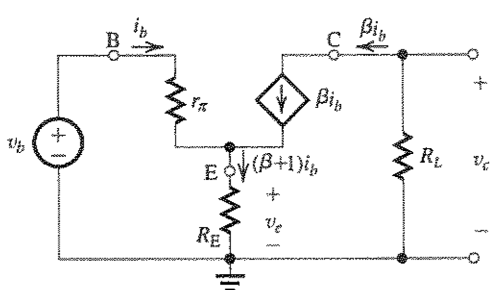
$$1 = 10^{-5}\frac{1000}{1000+100}R_m\frac{1000}{1000+100}$$

$\Rightarrow R_m = 1.21 \times 10^5\ \Omega$

$= 121\ \text{k}\Omega$



1.63



The node equation at E yields the current through R_E as $(\beta i_b + i_b) = (\beta + 1)i_b$. The voltage v_c can be found in terms of i_b as

$$v_c = -\beta i_b R_L \tag{1}$$

The voltage v_b can be related to i_b by writing for the input loop:

$v_b = i_b r_\pi + (\beta + 1)i_b R_E$

Thus,

$$v_b = [r_\pi + (\beta + 1)R_E]i_b \tag{2}$$

Dividing Eq. (1) by Eq. (2) yields

$$\frac{v_c}{v_b} = -\frac{\beta R_L}{r_\pi + (\beta + 1)R_E} \qquad \text{Q.E.D}$$

The voltage v_e is related to i_b by

$$v_e = (\beta + 1)i_b R_E$$

That is,

$$v_e = [(\beta + 1)R_E]i_b \qquad (3)$$

Dividing Eq. (3) by Eq. (2) yields

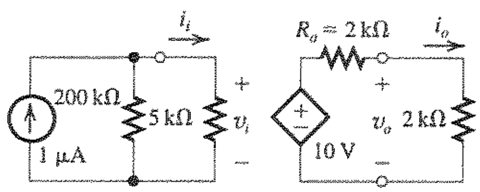
$$\frac{v_e}{v_b} = \frac{(\beta + 1)R_E}{(\beta + 1)R_E + r_\pi}$$

Dividing the numerator and denominator by $(\beta + 1)$ gives

$$\frac{v_e}{v_b} = \frac{R_E}{R_E + [r_\pi/(\beta + 1)]} \qquad \text{Q.E.D}$$

1.64 $R_o = \dfrac{\text{Open-circuit output voltage}}{\text{Short-circuit output current}} = \dfrac{10\ \text{V}}{5\ \text{mA}}$
$= 2\ \text{k}\Omega$

$$v_o = 10 \times \frac{2}{2 + 2} = 5\ \text{V}$$



$$A_v = \frac{v_o}{v_i} = \frac{10(2/4)}{1 \times 10^{-6} \times (200 \parallel 5) \times 10^3}$$

1025 V/V or 60.2 dB

$$A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{v_i/R_i}$$

$$= \frac{v_o}{v_i}\frac{R_i}{R_L} = 1025 \times \frac{5\ \text{k}\Omega}{2\ \text{k}\Omega}$$

$= 2562.5$ A/A or 62.8 dB

The overall current gain can be found as

$$\frac{i_o}{i_s} = \frac{v_o/R_L}{1\ \mu\text{A}} = \frac{5\ \text{V}/2\ \text{k}\Omega}{1\ \mu\text{A}}$$

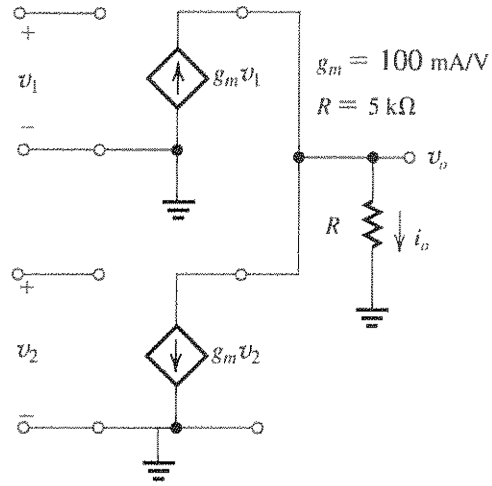
$$= \frac{2.5\ \text{mA}}{1\ \mu\text{A}} = 2500\ \text{A/A}$$

or 68 dB.

$$A_p = \frac{v_o^2/R_L}{i_i^2 R_i} = \frac{5^2/(2 \times 10^3)}{\left(10^{-6} \times \dfrac{200}{200 + 5}\right)^2 5 \times 10^3}$$

$= 2.63 \times 10^6$ W/W or 64.2 dB

1.65



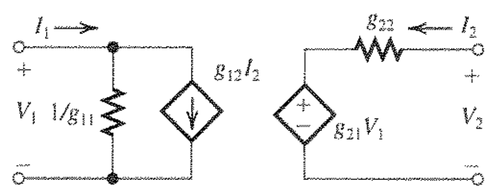
$$i_o = g_m v_1 - g_m v_2$$

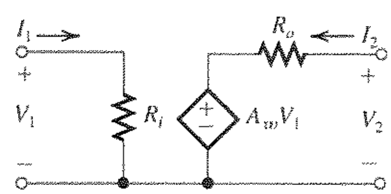
$$v_o = i_o R_L = g_m R(v_1 - v_2)$$

$$v_1 = v_2 = 1\ \text{V} \qquad \therefore v_o = 0\ \text{V}$$

$$\left.\begin{aligned} v_1 &= 1.01\ \text{V} \\ v_2 &= 0.99\ \text{V} \end{aligned}\right\} \qquad \therefore v_o = 100 \times 5 \times 0.02 = 10\ \text{V}$$

1.66





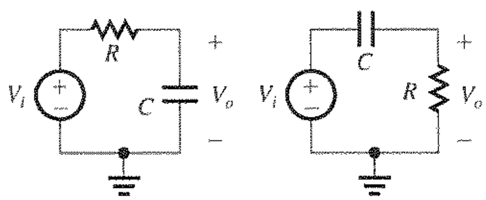
The correspondences between the current and voltage variables are indicated by comparing the two equivalent-circuit models above. At the outset we observe that at the input side of the g-parameter model, we have the controlled current source $g_{12}I_2$. This has no correspondence in the equivalent-circuit model of Fig. 1.16(a). It represents internal feedback, internal to the amplifier circuit. In developing the model of Fig. 1.16(a), we assumed that the amplifier is unilateral (i.e., has no internal feedback, or that the input side does not know what happens at the output side). If we neglect this internal feedback, that is, assume $g_{12} = 0$, we can compare the two models and thus obtain:

$R_i = 1/g_{11}$

$A_{vo} = g_{21}$

$R_o = g_{22}$

1.67 Circuits of Fig. 1.22:



(a) (b)

For (a) $V_o = V_i\left(\dfrac{1/sC}{1/sC + R}\right)$

$$\frac{V_o}{V_i} = \frac{1}{1 + sCR}$$

which is of the form shown for the low-pass function in Table 1.2 with $K = 1$ and $\omega_0 = 1/RC$.

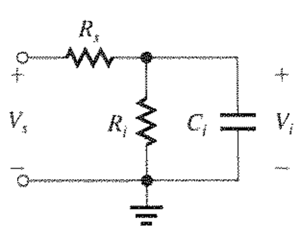
For (b) $V_o = V_i\left(\dfrac{R}{R + \dfrac{1}{sC}}\right)$

$$\frac{V_o}{V_i} = \frac{sRC}{1 + sCR}$$

$$\frac{V_o}{V_i} = \frac{s}{s + \dfrac{1}{RC}}$$

which is of the form shown in Table 1.2 for the high-pass function, with $K = 1$ and $\omega_0 = 1/RC$.

1.68



$$\frac{V_i}{V_s} = \frac{\dfrac{R_i \dfrac{1}{sC_i}}{R_i + \dfrac{1}{sC_i}}}{R_s + \left(\dfrac{R_i \dfrac{1}{sC_i}}{R_i + \dfrac{1}{sC_i}}\right)} = \frac{\dfrac{R_i}{1 + sC_iR_i}}{R_s + \left(\dfrac{R_i}{1 + sC_iR_i}\right)}$$

$$= \frac{R_i}{R_s + sC_iR_iR_s + R_i}$$

$$\frac{V_i}{V_s} = \frac{R_i}{(R_s + R_i) + sC_iR_iR_s} = \frac{\dfrac{R_i}{(R_s + R_i)}}{1 + s\left(\dfrac{C_iR_iR_s}{R_s + R_i}\right)}$$

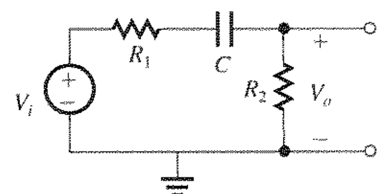
which is a low-pass STC function with $K = \dfrac{R_i}{R_s + R_i}$ and $\omega_0 = 1/[C_i(R_i \parallel R_s)]$.

For $R_s = 10\ \text{k}\Omega$, $R_i = 40\ \text{k}\Omega$, and $C_i = 5\ \text{pF}$,

$$\omega_0 = \frac{1}{5 \times 10^{-12} \times (40 \parallel 10) \times 10^3} = 25\ \text{Mrad/s}$$

$$f_0 = \frac{25}{2\pi} = 4\ \text{MHz}$$

1.69 Using the voltage-divider rule.



$$T(s) = \frac{V_o}{V_i} = \frac{R_2}{R_2 + R_1 + \dfrac{1}{sC}}$$

$$T(s) = \left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{s}{s + \dfrac{1}{C(R_1 + R_2)}}\right)$$

which from Table 1.2 is of the high-pass type with

$$K = \frac{R_2}{R_1 + R_2} \qquad \omega_0 = \frac{1}{C(R_1 + R_2)}$$

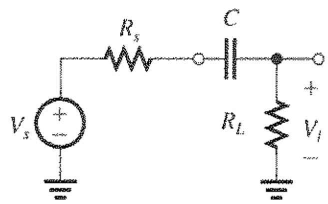
As a further verification that this is a high-pass network and $T(s)$ is a high-pass transfer function, see that as $s \Rightarrow 0$, $T(s) \Rightarrow 0$; and as $s \to \infty$, $T(s) = R_2/(R_1 + R_2)$. Also, from the circuit, observe as $s \to \infty$, $(1/sC) \to 0$ and $V_o/V_i = R_2/(R_1 + R_2)$. Now, for $R_1 = 10\ \text{k}\Omega$, $R_2 = 40\ \text{k}\Omega$ and $C = 1\ \mu\text{F}$,

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi \times 1 \times 10^{-6}(10+40) \times 10^3}$$

$= 3.18$ Hz

$$|T(j\omega_0)| = \frac{K}{\sqrt{2}} = \frac{40}{10+40}\frac{1}{\sqrt{2}} = 0.57 \text{ V/V}$$

1.70 Using the voltage divider rule,



$$\frac{V_l}{V_s} = \frac{R_L}{R_L + R_s + \dfrac{1}{sC}}$$

$$= \frac{R_L}{R_L + R_s} \frac{s}{s + \dfrac{1}{C(R_L + R_s)}}$$

which is of the high-pass STC type (see Table 1.2) with

$$K = \frac{R_L}{R_L + R_s} \quad \omega_0 = \frac{1}{C(R_L + R_s)}$$

For $f_0 \leq 100$ Hz

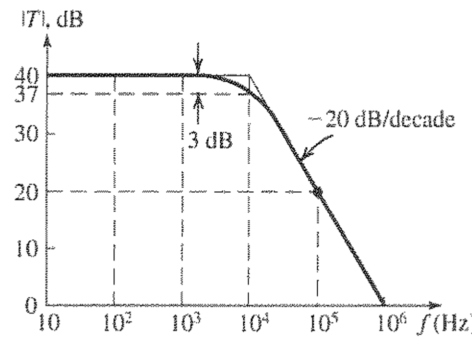
$$\frac{1}{2\pi C(R_L + R_s)} \leq 100$$

$$\Rightarrow C \geq \frac{1}{2\pi \times 100(20+5) \times 10^3}$$

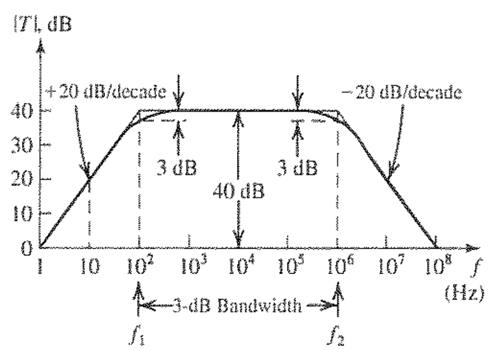
Thus, the smallest value of C that will do the job is $C = 0.064$ μF or 64 nF.

1.71 The given measured data indicate that this amplifier has a low-pass STC frequency response with a low-frequency gain of 40 dB, and a 3-dB frequency of 10^4 Hz. From our knowledge of the Bode plots for low-pass STC networks [Fig. 1.23(a)], we can complete the table entries and sketch the amplifier frequency response.

f (Hz)	$\|T\|$(dB)	$\angle T$ (°)
0	40	0
100	40	0
1000	40	0
10^4	37	−45°
10^5	20	−90°
10^6	0	−90°



1.72 From our knowledge of the Bode plots of STC low-pass and high-pass networks, we see that this amplifier has a midband gain of 40 dB, a low-frequency response of the high-pass STC type with $f_{3dB} = 10^2$ Hz, and a high-frequency response of the low-pass STC type with $f_{3dB} = 10^6$ Hz. We thus can sketch the amplifier frequency response and complete the table entries as follows.



f (Hz)	1	10	10^2	10^3	10^4	10^5	10^6	10^7	10^8
$\|T\|$(dB)	0	20	37	40	40	40	37	20	0

1.73 Since the overall transfer function is that of three identical STC LP circuits in cascade (but with no loading effects, since the buffer amplifiers have infinite input and zero output resistances) the overall gain will drop by 3 dB below the value at dc at the frequency for which the gain of each STC circuit is 1 dB down. This frequency is found as follows: The transfer function of each STC circuit is

$$T(s) = \frac{1}{1 + \dfrac{s}{\omega_0}}$$

where

$\omega_0 = 1/CR$

Thus,

$$|T(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}$$

$$20\log\frac{1}{\sqrt{1+\left(\frac{\omega_{1\,\text{dB}}}{\omega_0}\right)^2}} = -1$$

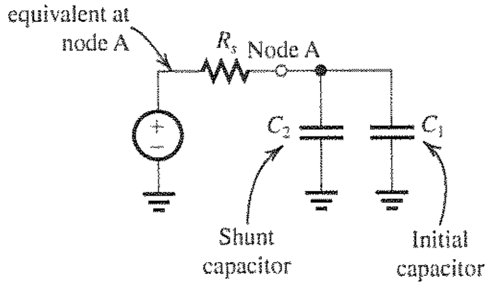
$$\Rightarrow 1+\left(\frac{\omega_{1\,\text{dB}}}{\omega_0}\right)^2 = 10^{0.1}$$

$$\omega_{1\text{dB}} = 0.51\omega_0$$

$$\omega_{1\text{dB}} = 0.51/CR$$

1.74 $R_s = 100\ \text{k}\Omega$, since the 3-dB frequency is reduced by a very high factor (from 5 MHz to 100 kHz) C_2 must be much larger than C_1. Thus, neglecting C_1 we find C_2 from

$$100\ \text{kHz} \simeq \frac{1}{2\pi C_2 R_s}$$



$$= \frac{1}{2\pi C_2 \times 10^5}$$

$$\Rightarrow C_2 = 15.9\ \text{pF}$$

If the original 3-dB frequency (5 MHz) is attributable to C_1, then

$$5\ \text{MHz} = \frac{1}{2\pi C_1 R_s}$$

$$\Rightarrow C_1 = \frac{1}{2\pi \times 5\times 10^6 \times 10^5}$$

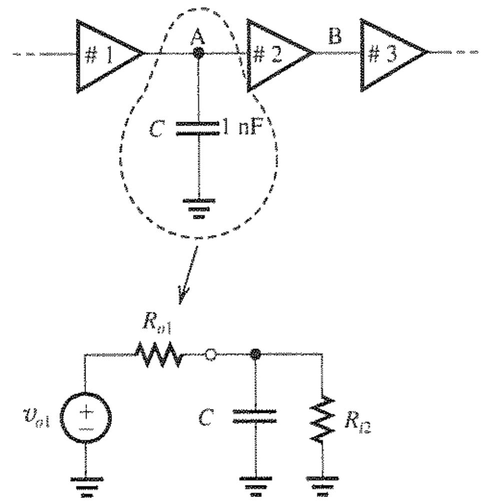
$$= 0.32\ \text{pF}$$

1.75 Since when C is connected to node A the 3-dB frequency is reduced by a large factor, the value of C must be much larger than whatever parasitic capacitance originally existed at node A (i.e., between A and ground). Furthermore, it must be that C is now the dominant determinant of the amplifier 3-dB frequency (i.e., it is dominating over whatever may be happening at node B or anywhere else in the amplifier). Thus, we can write

$$200\ \text{kHz} = \frac{1}{2\pi C(R_{o1} \parallel R_{i2})}$$

$$\Rightarrow (R_{o1} \parallel R_{i2}) = \frac{1}{2\pi \times 200\times 10^3 \times 1\times 10^{-9}}$$

$$= 0.8\ \text{k}\Omega$$



Now $R_{i2} = 100\ \text{k}\Omega$.

Thus $R_{o1} \simeq 0.8\ \text{k}\Omega$

Similarly, for node B,

$$20\ \text{kHz} = \frac{1}{2\pi C(R_{o2} \parallel R_{i3})}$$

$$\Rightarrow R_{o2} \parallel R_{i3} = \frac{1}{2\pi \times 20\times 10^3 \times 1\times 10^{-9}}$$

$$= 7.96\ \text{k}\Omega$$

$$R_{o2} = 8.65\ \text{k}\Omega$$

The designer should connect a capacitor of value C_p to node B where C_p can be found from

$$10\ \text{kHz} = \frac{1}{2\pi C_p(R_{o2} \parallel R_{i3})}$$

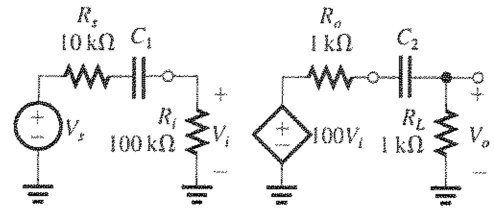
$$\Rightarrow C_p = \frac{1}{2\pi \times 10\times 10^3 \times 7.96\times 10^3}$$

$$= 2\ \text{nF}$$

Note that if she chooses to use node A, she would need to connect a capacitor 10 times larger!

1.76 For the input circuit, the corner frequency f_{01} is found from

$$f_{01} = \frac{1}{2\pi C_1(R_s + R_i)}$$



For $f_{01} \leq 100$ Hz,

$$\frac{1}{2\pi C_1(10+100) \times 10^3} \leq 100$$

$$\Rightarrow C_1 \geq \frac{1}{2\pi \times 110 \times 10^3 \times 10^2} = 1.4 \times 10^{-8}\ \text{F}$$

Thus we select $C_1 = 1 \times 10^{-7}$ F $= 0.1\ \mu$F. The actual corner frequency resulting from C_1 will be

$$f_{01} = \frac{1}{2\pi \times 10^{-7} \times 110 \times 10^3} = 14.5\ \text{Hz}$$

For the output circuit,

$$f_{02} = \frac{1}{2\pi C_2(R_o + R_L)}$$

For $f_{02} \leq 100$ Hz,

$$\frac{1}{2\pi C_2(1+1) \times 10^3} \leq 100$$

$$\Rightarrow C_2 \geq \frac{1}{2\pi \times 2 \times 10^3 \times 10^2} = 0.8 \times 10^{-6}$$

Select $C_2 = 1 \times 10^{-6} = 1\ \mu$F.

This will place the corner frequency at

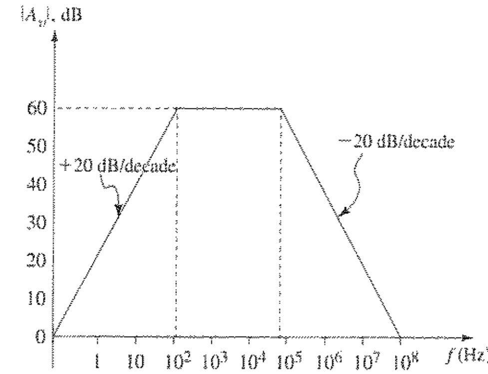
$$f_{02} = \frac{1}{2\pi \times 10^{-6} \times 2 \times 10^3} = 80\ \text{Hz}$$

$$T(s) = 100 \frac{s}{\left(1 + \dfrac{s}{2\pi f_{01}}\right)\left(1 + \dfrac{s}{2\pi f_{02}}\right)}$$

1.77 The LP factor $1/(1 + jf/10^5)$ results in a Bode plot like that in Fig. 1.23(a) with the 3-dB frequency $f_0 = 10^5$ Hz. The high-pass factor $1/(1 + 10^2/jf)$ results in a Bode plot like that in Fig. 1.24(a) with the 3-dB frequency

$f_0 = 10^2$ Hz.

The Bode plot for the overall transfer function can be obtained by summing the dB values of the two individual plots and then shifting the resulting plot vertically by 60 dB (corresponding to the factor 1000 in the numerator). The result is as follows:



$f =$	10	10^2	10^3	10^4	10^5	10^6	10^7	10^8	(Hz)
$\lvert A_v \rvert =$	40	60	60	60	60	40	20	0	(dB)
		57			57				

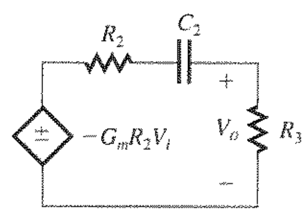
Bandwidth $= 10^5 - 10^2 = 99{,}900$ Hz

1.78 $T_i(s) = \dfrac{V_i(s)}{V_s(s)} = \dfrac{1/sC_1}{1/sC_1 + R_1} = \dfrac{1}{sC_1R_1 + 1}$

LP with a 3-dB frequency

$$f_{0i} = \frac{1}{2\pi C_1 R_1} = \frac{1}{2\pi 10^{-11} 10^5} = 159\ \text{kHz}$$

For $T_o(s)$, the following equivalent circuit can be used:



$$T_o(s) = \frac{V_o}{V_i} = -G_m R_2 \frac{R_3}{R_2 + R_3 + 1/sC_2}$$

$$= -G_m(R_2 \parallel R_3) \frac{s}{s + \dfrac{1}{C_2(R_2 + R_3)}}$$

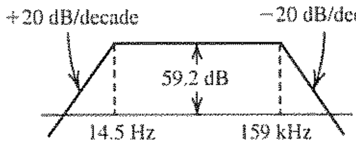
which is an HP, with

$$\text{3-dB frequency} = \frac{1}{2\pi C_2(R_2 + R_3)}$$

$$= \frac{1}{2\pi 100 \times 10^{-9} \times 110 \times 10^3} = 14.5\ \text{Hz}$$

$\therefore T(s) = T_i(s)T_o(s)$

$$= \frac{1}{1+\dfrac{s}{2\pi \times 159 \times 10^3}} \times -909.1 \times \frac{s}{s+(2\pi \times 14.5)}$$



Bandwidth = 159 kHz − 14.5 Hz ≃ 159 kHz

1.79 $V_i = V_s \dfrac{R_i}{R_s + R_i}$ (1)

(a) To satisfy constraint (1), namely,

$$V_i \geq \left(1 - \frac{x}{100}\right) V_s$$

we substitute in Eq. (1) to obtain

$$\frac{R_i}{R_s + R_i} \geq 1 - \frac{x}{100} \qquad (2)$$

Thus

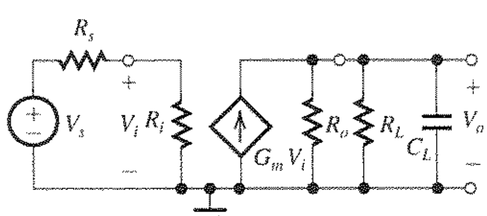
$$\frac{R_s + R_i}{R_i} \leq \frac{1}{1 - \dfrac{x}{100}}$$

$$\frac{R_s}{R_i} \leq \frac{1}{1-\dfrac{x}{100}} - 1 = \frac{\dfrac{x}{100}}{1 - \dfrac{x}{100}}$$

which can be expressed as

$$\frac{R_i}{R_s} \geq \frac{1 - \dfrac{x}{100}}{\dfrac{x}{100}}$$

resulting in



$$R_i \geq R_s\left(\frac{100}{x} - 1\right) \qquad (3)$$

(b) The 3-dB frequency is determined by the parallel RC circuit at the output

$$f_0 = \frac{1}{2\pi}\omega_0 = \frac{1}{2\pi}\frac{1}{C_L(R_L \parallel R_o)}$$

Thus,

$$f_0 = \frac{1}{2\pi C_L}\left(\frac{1}{R_L} + \frac{1}{R_0}\right)$$

To obtain a value for f_0 greater than a specified value f_{3dB} we select R_o so that

$$\frac{1}{2\pi C_L}\left(\frac{1}{R_L} + \frac{1}{R_o}\right) \geq f_{3dB}$$

$$\frac{1}{R_L} + \frac{1}{R_o} \geq 2\pi C_L f_{3dB}$$

$$\frac{1}{R_o} \geq 2\pi C_L f_{3dB} - \frac{1}{R_L}$$

$$R_o \leq \frac{1}{2\pi f_{3dB} C_L - \dfrac{1}{R_L}} \qquad (4)$$

(c) To satisfy constraint (c), we first determine the dc gain as

$$\text{dc gain} = \frac{R_i}{R_s + R_i} G_m(R_o \parallel R_L)$$

For the dc gain to be greater than a specified value A_0,

$$\frac{R_i}{R_s + R_i} G_m(R_o \parallel R_L) \geq A_0$$

The first factor on the left-hand side is (from constraint (2)) greater or equal to $(1 - x/100)$. Thus

$$G_m \geq \frac{A_0}{\left(1 - \dfrac{x}{100}\right)(R_o \parallel R_L)} \qquad (5)$$

Substituting $R_s = 10\ \text{k}\Omega$ and $x = 10\%$ in (3) results in

$$R_i \geq 10\left(\frac{100}{100} - 1\right) = 90\ \text{k}\Omega$$

Substituting $f_{3dB} = 2$ MHz, $C_L = 20$ pF, and $R_L = 10\ \text{k}\Omega$ in Eq. (4) results in

$$R_o \leq \frac{1}{2\pi \times 2 \times 10^6 \times 20 \times 10^{-12} - \dfrac{1}{10^4}}$$

$$= 6.61\ \text{k}\Omega$$

Substituting $A_0 = 100$, $x = 10\%$, $R_L = 10\ \text{k}\Omega$, and $R_o = 6.61\ \text{k}\Omega$, Eq. (5) results in

$$G_m \geq \frac{100}{\left(1 - \dfrac{10}{100}\right)(10 \parallel 6.61) \times 10^3} = 27.9\ \text{mA/V}$$

1.80 Using the voltage divider rule, we obtain

$$\frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2}$$

where

$$Z_1 = R_1 \parallel \frac{1}{sC_1} \text{ and } Z_2 = R_2 \parallel \frac{1}{sC_2}$$

It is obviously more convenient to work in terms of admittances. Therefore we express V_o/V_i in the alternate form

$$\frac{V_o}{V_i} = \frac{Y_1}{Y_1 + Y_2}$$

and substitute $Y_1 = (1/R_1) + sC_1$ and $Y_2 = (1/R_2) + sC_2$ to obtain

$$\frac{V_o}{V_i} = \frac{\frac{1}{R_1} + sC_1}{\frac{1}{R_1} + \frac{1}{R_2} + s(C_1 + C_2)}$$

$$= \frac{C_1}{C_1 + C_2} \frac{s + \frac{1}{C_1 R_1}}{s + \frac{1}{(C_1 + C_2)}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

This transfer function will be independent of frequency (s) if the second factor reduces to unity.

This in turn will happen if

$$\frac{1}{C_1 R_1} = \frac{1}{C_1 + C_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

which can be simplified as follows:

$$\frac{C_1 + C_2}{C_1} = R_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad (1)$$

$$1 + \frac{C_2}{C_1} = 1 + \frac{R_1}{R_2}$$

or

$$C_1 R_1 = C_2 R_2$$

When this condition applies, the attenuator is said to be compensated, and its transfer function is given by

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

which, using Eq. (1), can be expressed in the alternate form

$$\frac{V_o}{V_i} = \frac{1}{1 + \frac{R_1}{R_2}} = \frac{R_2}{R_1 + R_2}$$

Thus when the attenuator is compensated ($C_1 R_1 = C_2 R_2$), its transmission can be determined either by its two resistors R_1, R_2 or by its two capacitors. C_1, C_2, and the transmission is *not* a function of frequency.

1.81 The HP STC circuit whose response determines the frequency response of the amplifier in the low-frequency range has a phase angle of 5.7° at $f = 100$ Hz. Using the equation for $\angle T(j\omega)$ from Table 1.2, we obtain

$$\tan^{-1}\frac{f_0}{100} = 5.7^\circ \Rightarrow f_0 = 10 \text{ Hz}$$

The LP STC circuit whose response determines the amplifier response at the high-frequency end has a phase angle of −5.7° at $f = 1$ kHz. Using the relationship for $\angle T(j\omega)$ given in Table 1.2, we obtain for the LP STC circuit.

$$-\tan^{-1}\frac{10^3}{f_0} = -5.7^\circ \Rightarrow f_0 \simeq 10 \text{ kHz}$$

At $f = 100$ Hz, the drop in gain is due to the HP STC network, and thus its value is

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{10}{100}\right)^2}} = -0.04 \text{ dB}$$

Similarly, at the drop in gain $f = 1$ kHz is caused by the LP STC network. The drop in gain is

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{1000}{10,000}\right)^2}} = -0.04 \text{ dB}$$

The gain drops by 3 dB at the corner frequencies of the two STC networks, that is, at $f = 10$ Hz and $f = 10$ kHz.

2.1 The minimum number of pins required by dual op amp is 8. Each op amp has 2 input terminals (4 pins) and one output terminal (2 pins). Another 2 pins are required for power supplies.
Similarly, the minimum number of pins required by quad op amp is 14:
$4 \times 2 + 4 \times 1 + 2 = 14$

2.2 Refer to Fig. P2.2.

$$v_+ = v_I \times \frac{1\text{ k}\Omega}{1\text{ k}\Omega + 1\text{ M}\Omega} = v_I \frac{1}{1001}$$

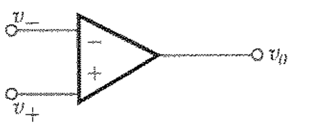
$$v_O = Av_+ = Av_I \frac{1}{1001}$$

$$A = 1001 \frac{v_O}{v_I}$$

$$= 1001 \times \frac{4}{1}$$

$A = 4004$ V/V

2.3



$v_O = -2.000$ V

$v_- = -1.000$ V

For ideal op amp,

$v_+ = v_- = -1.000$ V

Measured voltage at positive input $= -1.005$ V

$$\text{Amplifier gain } A = \frac{v_O}{v_+ - v_-}$$

$$= \frac{-2.000}{-1.005 - (-1.000)}$$

$= 400$ V/V

2.4

#	v_1	v_2	$v_d = v_2 - v_1$	v_O	v_O/v_d
1	0.00	0.00	0.00	0.00	–
2	1.00	1.00	0.00	0.00	–
3	(a)	1.00	(b)	1.00	
4	1.00	1.10	0.10	10.1	101
5	2.01	2.00	–0.01	–0.99	99
6	1.99	2.00	0.01	1.00	100
7	5.10	(c)	(d)	–5.10	

Experiments 4, 5, and 6 show that the gain is approximately 100 V/V.

The missing entry for experiment #3 can be predicted as follows:

(a) $v_d = \dfrac{v_O}{A} = \dfrac{1.00}{100} = 0.01$ V.

(b) $v_1 = v_2 - v_d = 1.00 - 0.01 = 0.99$ V

The missing entries for experiment #7:

(c) $v_d = \dfrac{-5.10}{100} = -0.051$ V

(d) $v_2 = v_1 + v_d = 5.10 - 0.051 = 5.049$ V

All the results seem to be reasonable.

2.5 $i = G_m(v_2 - v_1)$

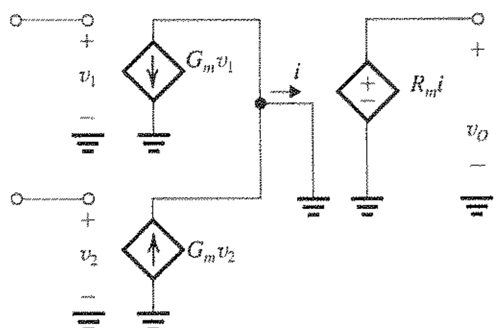
$v_O = R_m i$

$= R_m G_m(v_2 - v_1)$

$$\text{Gain } A = \frac{v_O}{v_2 - v_1} = R_m G_m$$

$= 1 \times 10^6 \times 40 \times 10^{-3}$

$= 40{,}000$ V/V



2.6 $v_{cm} = 2\sin(2\pi 60)t = \dfrac{1}{2}(v_1 + v_2)$

$v_d = 0.005\sin(2\pi 1000)t = v_1 - v_2$

$v_1 = v_{cm} - v_d/2$
$= \sin(120\pi)t - 0.0025\sin(2000\pi t)$

$v_2 = v_{cm} + v_d/2$
$= \sin(120\pi t) + 0.0025\sin(2000\pi t)$

2.7 Refer to Fig. E2.3.

$v_d = R(G_{m2}v_2 - G_{m1}v_1)$

$v_O = v_3 = \mu v_d = \mu R(G_{m2}v_2 - G_{m1}v_1)$

$v_O =$

$$\mu R(G_m v_2 + \frac{1}{2}\Delta G_m v_2 - G_m v_1 + \frac{1}{2}\Delta G_m v_1)$$

$$v_O = \mu R G_m \underbrace{(v_2 - v_1)}_{v_{Id}} + \frac{1}{2}\mu R \Delta G_m \underbrace{(v_1 + v_2)}_{2v_{Icm}}$$

we have $v_O = A_d v_{Id} + A_{cm} v_{Icm}$

$\Rightarrow A_d = \mu R G_m, A_{cm} = \mu R \Delta G_m$

$$\text{CMRR} = 20 \log \left| \frac{A_d}{A_{cm}} \right| = 20 \log \frac{G_m}{\Delta G_m}$$

For a CMRR $\geq$ 60 dB,

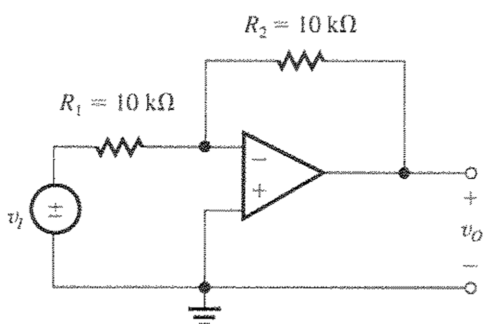
$$\frac{G_m}{\Delta G_m} \geq 1000 \Rightarrow \frac{\Delta G_m}{G_m} \leq 0.1\%$$

2.8

Circuit	v_o/v_i(V/V)	R_{in} (kΩ)
a	$\frac{-100}{20} = -5$	20
b	–5	20
c	–5	20
d	–5	20

Note that in circuit (b) the 20-kΩ load resistance has no effect on the closed-loop gain because of the zero output resistance of the ideal op amp. In circuit (c), no current flows in the 20-kΩ resistor connected between the negative input terminal and ground (because of the virtual ground at the inverting input terminal). In circuit (d), zero current flows in the 20-kΩ resistor connected in series with the positive input terminal.

2.9



Closed-loop gain is

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1} = -\frac{10 \text{ k}\Omega}{10 \text{ k}\Omega}$$

$= -1$ V/V

For $v_I = +1.00$ V,

$v_O = -1 \times 1.00$

$= -1.00$ V

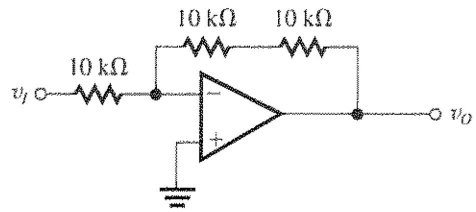
The two resistors are 1% resistors

$$\left| \frac{v_O}{v_I} \right|_{\min} = \frac{10(1-0.01)}{10(1+0.01)} = 0.98 \text{ V/V}$$

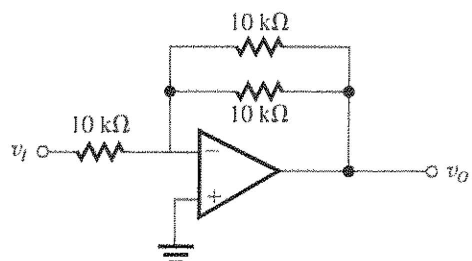
$$\left| \frac{v_O}{v_I} \right|_{max} = \frac{10(1+0.01)}{10(1-0.01)} = 1.02 \text{ V/V}$$

Thus the measured output voltage will range from –0.98 V to –1.02 V.

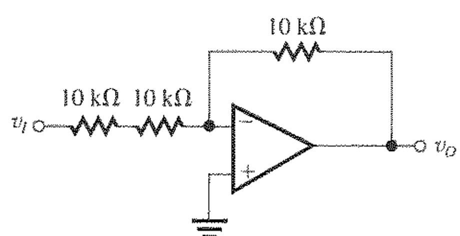
2.10 There are four possibilities:



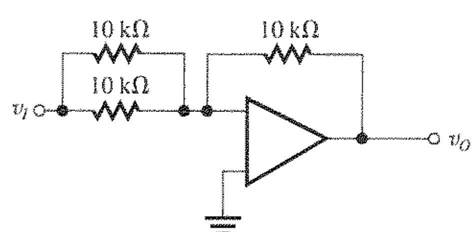
$\frac{v_O}{v_I} = -2$ V/V $\quad$ $R_{in} = 10$ kΩ (largest gain magnitude)



$\frac{v_O}{v_I} = -0.5$ V/V $\quad$ $R_{in} = 10$ kΩ (smallest gain magnitude)



$\frac{v_O}{v_I} = -0.5$ V/V $\quad$ $R_{in} = 20$ kΩ (smallest gain magnitude)



$\frac{v_O}{v_I} = -2$ V/V $\quad$ $R_{in} = 5$ kΩ (largest gain magnitude)

2.11

(a) $G = -1$ V/V (b) $G = -10$ V/V
(c) $G = -0.1$ V/V (d) $G = -100$ V/V
(e) $G = -10$ V/V

2.12 (a) $G = -1$ V/V

$$= \frac{-R_2}{R_1} \Rightarrow R_1 = R_2 = 10\text{ k}\Omega$$

(b) $G = -2$ V/V

$$= \frac{-R_2}{R_1} \Rightarrow R_1 = 10\text{ k}\Omega,\ R_2 = 20\text{ k}\Omega$$

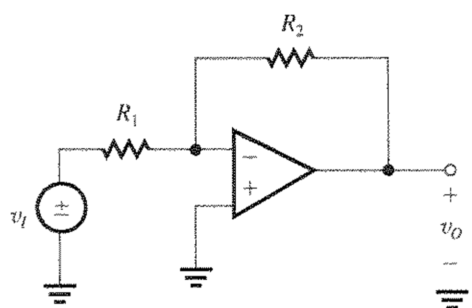
(c) $G = -5$ V/V

$$= \frac{-R_2}{R_1} \Rightarrow R_1 = 10\text{ k}\Omega,\ R_2 = 50\text{ k}\Omega$$

(d) $G = -100$ V/V

$$= \frac{-R_2}{R_1} \Rightarrow R_1 = 10\text{ k}\Omega,\ R_2 = 1\text{ M}\Omega$$

2.13



$$\frac{v_O}{v_I} = -10\text{ V/V} = -\frac{R_2}{R_1}$$

$\Rightarrow R_2 = 10R_1$

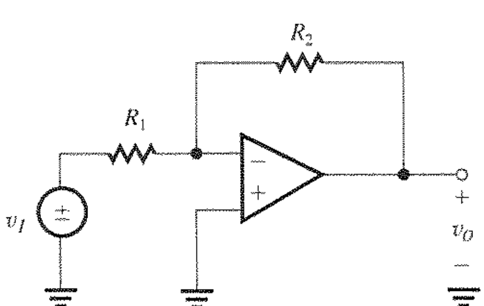
Total resistance used is 110 kΩ

$\therefore R_1 + R_2 = 110\text{ k}\Omega$

$R_1 + 10R_1 = 110\text{ k}\Omega$

$R_1 = 10\text{ k}\Omega$ and $R_2 = 10R_1 = 100\text{ k}\Omega$

2.14 Gain is 46 dB



$46\text{ dB} = 20 \log |G|$

$|G| = 200$

$$\therefore \frac{v_O}{v_I} = -200\text{ V/V} = -\frac{R_2}{R_1}$$

$\Rightarrow R_2 = 200R_1 \le 1\text{ M}\Omega$

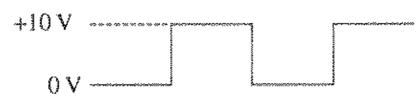
For the largest possible input resistance, choose

$R_2 = 1\text{ M}\Omega$

$$R_1 = \frac{1\text{ M}\Omega}{200} = 5\text{ k}\Omega$$

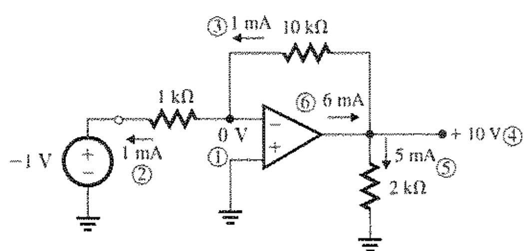
$R_{in} = R_1 = 5\text{ k}\Omega$

2.15



Average = +5 V
Highest = +10 V
Lowest = 0 V

2.16 The circled numbers indicate the order of the analysis steps. The additional current supplied by the op amp comes from the power supplies (not shown).



2.17 $R_1 = R_{1\text{nominal}}(1 \pm x\%)$

$R_2 = R_{2\text{nominal}}(1 \pm x\%)$

$$|G| = \frac{R_2}{R_1} = \frac{R_{2\text{nominal}}}{R_{1\text{nominal}}} \frac{1 \pm x\%}{1 \pm x\%}$$

Thus,

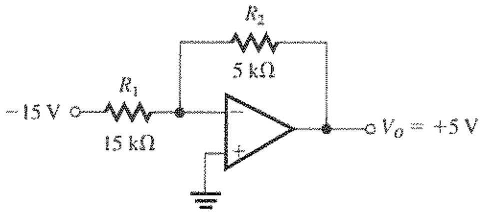
$$|G|_{\max} \simeq \frac{R_{2\text{nominal}}}{R_{1\text{nominal}}} \times (1 + 2x\%)$$

$$|G|_{\min} \simeq \frac{R_{2\text{nominal}}}{R_{1\text{nominal}}} \times (1 - 2x\%)$$

Thus, the tolerance of the closed-loop gain is $\pm 2x\%$.

For $G_{\text{nominal}} = -100$ and $x = 1$, the closed-loop gain will be in the range −98 V/V to −102 V/V.

2.18 The circuit will be as follows:



If R_1 and R_2 have ±1% tolerance, then V_O will exhibit ±2% variability and thus will be in the range of $5 \times 1.02 = 5.1$ V to $5 \times 0.98 = 4.9$ V.

Variation of the −15-V supply by ±1% results in a ±1% variation in the output voltage. Thus the total variation in the output voltage can be ±3%, resulting in V_O in the range 4.85 V to 5.15 V.

2.19 $G = -\dfrac{R_2/R_1}{1 + \dfrac{1 + R_2/R_1}{A}}$

For $A = 500$, $G = -50$, and $R_2 = 100$ kΩ,

$$-50 = -\frac{100/R_1}{1 + \dfrac{1 + 100/R_1}{500}}$$

Thus, $\dfrac{100}{R_1} = 50 + 0.1\left(1 + \dfrac{100}{R_1}\right)$

$$0.9 \times \frac{100}{R_1} = 50.1$$

$\Rightarrow R_1 = 1.796 \text{ k}\Omega \simeq 1.8 \text{ k}\Omega$

The input 2-kΩ resistor can be shunted by a resistance R_a to obtain an equivalent R_1 of 1.796 kΩ,

$$\frac{1}{1.796} = \frac{1}{R_a} + \frac{1}{2}$$

$\Rightarrow R_a = 17.61 \text{ k}\Omega \simeq 18 \text{ k}\Omega$

2.20 (a) Choose $R_1 = 1$ kΩ and $R_2 = 200$ kΩ.

(b) $G = -\dfrac{R_2/R_1}{1 + \dfrac{1 + R_2/R_1}{A}}$

$$= -\frac{200}{1 + \dfrac{1 + 200}{5000}} = -192.3 \text{ V/V}$$

To restore the gain to −200 V/V, we need to change the effective value of R_1 to R_1',

$$-200 = -\frac{200/R_1'}{1 + \dfrac{1 + 200/R_1'}{5000}}$$

Thus,

$$\frac{200}{R_1'} = 200 + \frac{200}{5000}\left(1 + \frac{200}{R_1'}\right)$$

$$0.96 \times \frac{200}{R_1'} = 200.04$$

$\Rightarrow R_1' = 0.960$ kΩ

This effective value can be realized by shunting R_1 (1 kΩ) with R_a,

$$\frac{1}{0.960} = \frac{1}{1} + \frac{1}{R_a} \Rightarrow R_a = 24 \text{ k}\Omega$$

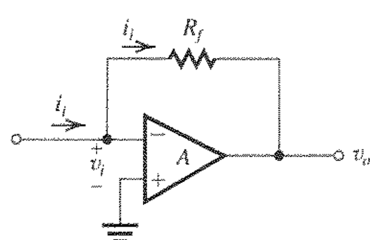
(c) From Appendix J, we find the closest available 1% resistor as either 23.7 kΩ or 24.3 kΩ.

2.21 Output voltage ranges from −10 V to +10 V and open-loop gain is 5000 V/V

∴ Voltage at the inverting input terminal will vary from $\dfrac{-10}{5000}$ to $\dfrac{10}{5000}$ (i.e., −2 mV to +2 mV).

Thus the virtual ground will depart by a maximum of ±2 mV.

2.22



(a) For $A = \infty$: $v_i = 0$

$$v_o = -i_i R_f$$

$$R_m = \frac{v_o}{i_i} = -R_f$$

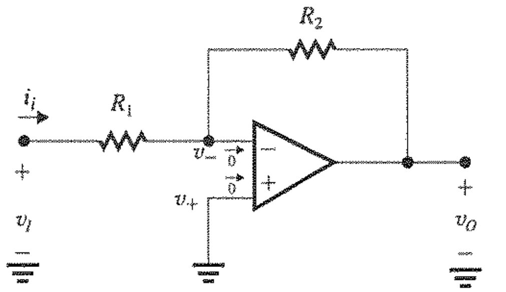
$$R_{in} = \frac{v_i}{i_i} = 0$$

(b) For A finite: $v_i = -\dfrac{v_o}{A}$, $v_o = v_i - i_i R_f$

$$\Rightarrow v_o = \frac{-v_o}{A} - i_i R_f \Rightarrow R_m = \frac{v_o}{i_i} = -\frac{R_f}{1 + \dfrac{1}{A}}$$

$$R_i = \frac{v_i}{i_i} = \frac{R_f}{1 + A}$$

2.23



$$v_O = -Av_- = v_- - i_iR_2$$

$$i_iR_2 = v_-(1 + A)$$

$$v_- = \frac{i_iR_2}{1 + A}$$

Again $v_I = i_iR_1 + v_-$

$$= i_iR_1 + i_i\frac{R_2}{1 + A}$$

So $R_{in} = \dfrac{v_I}{i_i} = R_1 + \dfrac{R_2}{A + 1}$ Q.E.D.

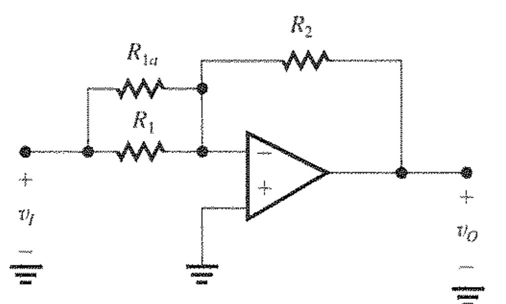
2.24 Closed-loop gain

$$G = \frac{v_O}{v_I} = \frac{-(R_2/R_1)}{1 + \dfrac{1 + R_2/R_1}{A}}$$

Gain error $\epsilon = \dfrac{1 + (R_2/R_1)}{A} \times 100$

ϵ	0.1%	1%	10%
A	$1000\left(1 + \dfrac{R_2}{R_1}\right)$	$100\left(1 + \dfrac{R_2}{R_1}\right)$	$10\left(1 + \dfrac{R_2}{R_1}\right)$

Gain with resistance R_{1a}



$$|G| \simeq \frac{\dfrac{R_2}{\left(\dfrac{R_1R_{1a}}{R_1 + R_{1a}}\right)}}{1 + \dfrac{1 + \dfrac{R_2}{R_1}}{A}}$$

where we have neglected the small effect of R_{1a} on the denominator.

$$|G| = \frac{\dfrac{R_2}{R_1}\left(\dfrac{R_1 + R_{1a}}{R_{1a}}\right)}{1 + \dfrac{1 + \dfrac{R_2}{R_1}}{A}} = \frac{\dfrac{R_2}{R_1}\left(1 + \dfrac{R_1}{R_{1a}}\right)}{1 + \dfrac{1 + \dfrac{R_2}{R_1}}{A}}$$

To restore the gain to its nominal value of $\dfrac{R_2}{R_1}$, use

$$\frac{R_1}{R_{1a}} = \frac{1 + \dfrac{R_2}{R_1}}{A} = \frac{\epsilon}{100}$$

So $R_{1a} = \dfrac{100R_1}{\epsilon}$

ϵ	0.1%	1%	10%
R_{1a}	$1000R_1$	$100R_1$	$10R_1$

2.25 Refer to Fig. P2.25.

$$\frac{V_o}{V_i} = \frac{-R_2/R_1'}{1 + \dfrac{1 + R_2/R_1'}{A}} \qquad (1)$$

where

$$R_1' = R_1 \| R_c$$

Thus,

$$\frac{1}{R_1'} = \frac{1}{R_1} + \frac{1}{R_c}$$

Substituting in Eq. (1),

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1}\frac{1 + \dfrac{R_1}{R_c}}{1 + \dfrac{1 + \dfrac{R_2}{R_1} + \dfrac{R_2}{R_c}}{A}}$$

To make $\dfrac{V_o}{V_i} = -\dfrac{R_2}{R_1}$, we have to make

$$\frac{R_1}{R_c} = \frac{1 + \dfrac{R_2}{R_1} + \dfrac{R_2}{R_c}}{A}$$

That is,

$$A\frac{R_1}{R_c} = 1 + G + G\frac{R_1}{R_c}$$

which yields

$$\frac{R_c}{R_1} = \frac{A - G}{1 + G} \qquad \text{Q.E.D}$$

2.26 (a) Equation (2.5):

$$G = \frac{v_O}{v_I} = \frac{-R_2/R_1}{1 + \left(1 + \dfrac{R_2}{R_1}\right)\Big/A}$$

and $G_{\text{nominal}} = -\dfrac{R_2}{R_1}$

Gain error

$$\epsilon = \left|\frac{G - G_{\text{nominal}}}{G_{\text{nominal}}}\right| = \left|\frac{G}{G_{\text{nominal}}} - 1\right|$$

$$= \left|\frac{1}{1+\left(\dfrac{1+R_2/R_1}{A}\right)} - 1\right| = \left|\frac{-\dfrac{1+R_2/R_1}{A}}{1+\dfrac{(1+R_2/R_1)}{A}}\right|$$

$$\epsilon = \frac{1}{1+\dfrac{A}{1+\dfrac{R_2}{R_1}}}$$

$$\frac{1}{\epsilon} = 1 + \frac{A}{1+\dfrac{R_2}{R_1}}$$

Solving for A, we get

$$A = \left(1+\frac{R_2}{R_1}\right)\left(\frac{1}{\epsilon} - 1\right)$$

$$= (1 - G_{\text{nominal}})\left(\frac{1}{\epsilon} - 1\right)$$

(b) $G_{\text{nominal}} = -100 = -\dfrac{R_2}{R_1}$

$R_{\text{in}} = 1\ \text{k}\Omega = R_1$

$R_1 = 1\ \text{k}\Omega$

$R_2 = 100R_1 = 100\ \text{k}\Omega$

Again $G_{\text{nominal}} = -100$ and $\epsilon_{\max} = 10\%$

$$\therefore A = (1 - G_{\text{nominal}})\left(\frac{1}{\epsilon} - 1\right)$$

$$= [1 - (-100)]\left(\frac{1}{0.1} - 1\right)$$

$$= 101 \times 9$$

$$A = 909\ \text{V/V}$$

2.27 (a) $$|G| = \frac{R_2/R_1}{1+\dfrac{1+R_2/R_1}{A}} \qquad (1)$$

If A is reduced by ΔA, $|G|$ will be correspondingly reduced by $\Delta|G|$,

$$|G| - \Delta|G| = \frac{R_2/R_1}{1+\dfrac{1+R_2/R_1}{A-\Delta A}} \qquad (2)$$

Dividing Eq. (2) by Eq. (1), we have

$$1 - \frac{\Delta|G|}{|G|} = \frac{1+\dfrac{1+R_2/R_1}{A}}{1+\dfrac{1+R_2/R_1}{A-\Delta A}}$$

Thus

$$\frac{\Delta|G|}{|G|} = \frac{\left(1+\dfrac{R_2}{R_1}\right)\left(\dfrac{1}{A-\Delta A} - \dfrac{1}{A}\right)}{1+\dfrac{1+R_2/R_1}{A-\Delta A}}$$

$$= \frac{\left(1+\dfrac{R_2}{R_1}\right)\left(\dfrac{1}{A}\right)\left(\dfrac{1}{1-\dfrac{\Delta A}{A}} - 1\right)}{1+\dfrac{R_2/R_1}{A-\Delta A}}$$

For $\dfrac{\Delta A}{A} \ll 1$ and $1 + \dfrac{R_2}{R_1} \ll A$,

$$\frac{\Delta|G|}{|G|} \simeq \frac{1+\dfrac{R_2}{R_1}}{A}\,\frac{\Delta A}{A}$$

Thus,

$$\frac{\Delta|G|/|G|}{\Delta A/|A|} \simeq \frac{1+R_2/R_1}{A} \qquad \text{Q.E.D}$$

(b) $\dfrac{R_2}{R_1} = 100$, $\dfrac{\Delta A}{A} = 0.1$, $\dfrac{\Delta|G|}{|G|} \leq 0.001$

$A \geq (1 + 100) \times 0.1/0.001$

That is, $A_{\min} \simeq 10^4$ V/V.

2.28 From Example 2.2,

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1}\left(1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right)$$

Here $R_1 = R_2 = R_4 = 1\ \text{M}\Omega$

$$\therefore \frac{v_O}{v_I} = -\left(1 + 1 + \frac{1}{R_3}\right) = -\left(2 + \frac{1\ \text{M}}{R_3}\right)$$

$$\frac{v_O}{v_I} + 2 = -\frac{1\ \text{M}\Omega}{R_3}$$

$$R_3 = -\frac{1\ \text{M}\Omega}{\left(\dfrac{v_O}{v_I} + 2\right)}$$

(a) $\dfrac{v_O}{v_I} = -100$ V/V

$$R_3 = -\frac{1\ \text{M}\Omega}{(-100+2)} = 10.2\ \text{k}\Omega$$

(b) $\dfrac{v_O}{v_I} = -10$ V/V

$$R_3 = -\frac{1\ \text{M}\Omega}{(-10+2)} = 125\ \text{k}\Omega$$

(c) $\dfrac{v_O}{v_I} = -2$ V/V, $R_3 = -\dfrac{1\ \text{M}\Omega}{(-2+2)} = \infty$

2.29

$R_2/R_1 = 500,\ R_2 = 100\text{ k}\Omega \Rightarrow R_1 = 200\ \Omega$

(a) $R_{in} = R_1 = 200\ \Omega$

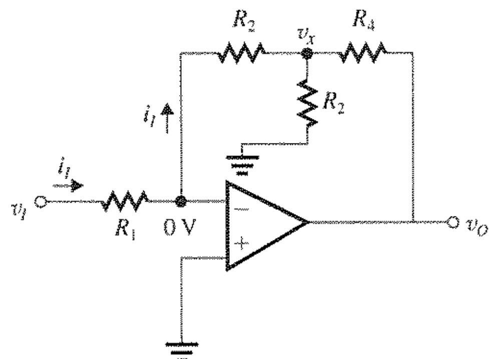
(b) $\dfrac{v_O}{v_I} = \dfrac{-R_2}{R_1}\left(1 + \dfrac{R_4}{R_2} + \dfrac{R_4}{R_3}\right) = -500$

If $R_2 = R_1 = R_4 = 100\text{ k}\Omega \Rightarrow R_3 = \dfrac{100\text{ k}\Omega}{500-2} \simeq 200\ \Omega$

$R_{in} = R_1 = 100\text{ k}\Omega$

2.30 $i_I = \dfrac{v_I}{R_1},\ v_x = -i_I R_2 = -\dfrac{v_I}{R_1}R_2$

So $\dfrac{v_x}{v_I} = -\dfrac{R_2}{R_1}$



Now, because of the virtual ground at the negative input terminal of the op amp, v_O appears across the series combination of R_4 and $(R_2 \parallel R_3)$; thus

$$v_x = v_O \frac{(R_2 \parallel R_3)}{(R_2 \parallel R_3) + R_4}$$

$$= v_O \frac{R_2 R_3}{R_2R_3 + R_2R_4 + R_3R_4}$$

$$\frac{v_O}{v_x} = \frac{R_2R_3 + R_2R_4 + R_3R_4}{R_2R_3}$$

$$= 1 + \frac{R_4}{R_3} + \frac{R_4}{R_2}$$

$$\frac{v_O}{v_I} = \frac{v_O}{v_x}\frac{v_x}{v_I}$$

$$= -\frac{R_2}{R_1}\left(1 + \frac{R_4}{R_3} + \frac{R_4}{R_2}\right) \qquad \text{Q.E.D.}$$

2.31 (a) $R_1 = R$

$$R_2 = (R \parallel R) + \frac{R}{2} = \frac{R}{2} + \frac{R}{2} = R$$

$$R_3 = (R_2 \parallel R) + \frac{R}{2} = (R \parallel R) + \frac{R}{2} = R$$

$$R_4 = (R_3 \parallel R) + \frac{R}{2} = (R \parallel R) + \frac{R}{2} = R$$

See Fig. 1.

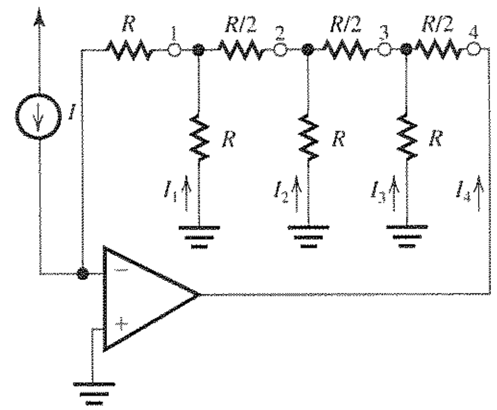


Figure 1

(b)

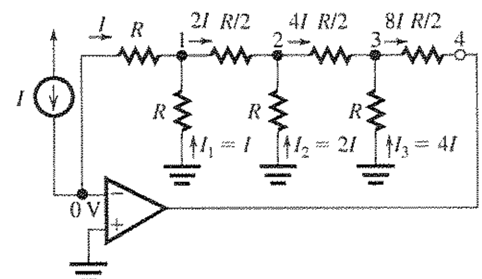


Figure 2

See Fig. 2. We utilize the results of (a) above as follows: At node 1 we have a resistance R to ground and, looking left, a resistance R. These two resistances must carry equal currents $\Rightarrow I_1 = I$. A node equation at node 1 results in the current through $(R/2)$ as $2I$. At node 2, we have a resistance R to ground and an equal resistance looking to the left. These two resistances must carry equal currents $\Rightarrow I_2 = 2I$.

A node equation at 2 results in the current leaving node 2 as $4I$. We continue in the same fashion to find $I_3 = 4I$ and the current from 3 to 4 as $8I$.

(c) $V_1 = -I_1R = -IR$

$V_2 = -I_2R = -2IR$

$V_3 = -I_3R = -4IR$

$V_4 = V_3 - \dfrac{R}{2} \times 8I = -8IR$

2.32 (a) $I_1 = \dfrac{1\text{ V}}{10\text{ k}\Omega} = 0.1\text{ mA}$

$I_2 = I_1 = 0.1\text{ mA}$

$-I_2 \times 10\text{ k}\Omega = -I_3 \times 100\ \Omega \equiv V_x$

$\therefore I_3 = I_2 \times \dfrac{10\text{ k}\Omega}{100\ \Omega}$

$= 0.1 \times 100$

$= 10\text{ mA}$

Now $I_L = I_2 + I_3 = 10.1$ mA

and $V_x = -I_2 \times 10\text{ k}\Omega = -0.1 \times 10\text{ k}\Omega = -1$ V

(b) $V_x = R_L I_L + V_O = 10.1\, R_L + V_O$

$$R_L = \frac{V_x - V_O}{10.1\text{ mA}} = \frac{-1-(-13)}{10.1} = 1.19\text{ k}\Omega$$

(c) $100\ \Omega \le R_L \le 1\text{ k}\Omega$

I_L stays fixed at 10.1 mA.

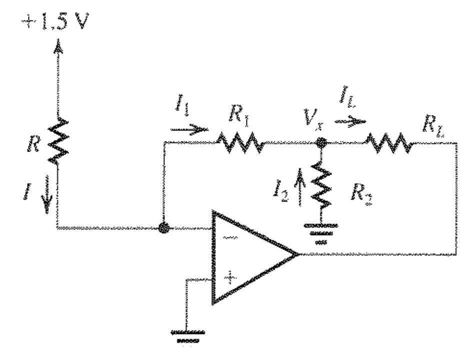
$V_O = V_x - R_L I_L = -1 - R_L \times 10.1$

$R_L = 100\ \Omega$, $V_O = -2.01$ V; $R_L = 1\text{ k}\Omega$,

$V_O = -11.1$ V

$\therefore -11.1\text{ V} \le V_O \le -2.01$ V

2.33



The circuit shown in Fig. P2.32 (redrawn above) provides a constant current through R_L that is independent of the value of R_L. The current through R_L is determined by the input current and the ratio of the two resistors R_1 and R_2. Specifically, $I_1 = I$, $V_x = -I_1 R_1$, $I_2 = -V_x/R_2 = I_1(R_1/R_2) = I(R_1/R_2)$, thus

$$I_L = I + I\frac{R_1}{R_2} = \left(1 + \frac{R_1}{R_2}\right) I$$

which is independent of the value of R_L. Now, for our specific design:

$I_L = 3.1$ mA

$I = 0.1$ mA

$$R = \frac{1.5\text{V}}{0.1\text{ mA}} = 15\text{ k}\Omega$$

$$3.1 = 0.1\left(1 + \frac{R_1}{R_2}\right)$$

$$\Rightarrow \frac{R_1}{R_2} = 30$$

Choosing $R_2 = 500\ \Omega \Rightarrow R_1 = 15\text{ k}\Omega$. The circuit will work properly as long as the op amp does not saturate (i.e., as long as $V_O \le -10$ V). But

$V_O = V_x - I_L R_L$

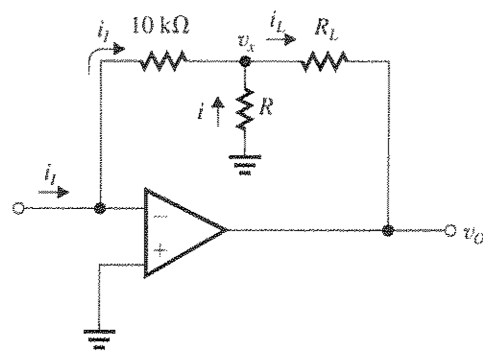
$= -IR_1 - I_L R_L$

$= -0.1 \times 15 - 3.1 R_L$

The maximum allowed value for R_L can now be found by substituting $V_O = -10$ V, thus

$R_L = 2.74\text{ k}\Omega$

2.34



(a) $v_x = -i_I \times 10$

$i = -v_x/R = i_I(10/R)$

$$i_L = i_I + i = i_I\left(1 + \frac{10}{R}\right)$$

Thus,

$$\frac{i_L}{i_I} = 1 + \frac{10}{R}$$

For $\dfrac{i_L}{i_I} = 11 \Rightarrow R = 1\text{ k}\Omega$.

(b) $R_{in} = 0\ \Omega$ (because of the virtual ground at the input). $R_o = \infty\ \Omega$ (because i_L is independent of the value of R_L).

(c) If i_I is in the direction shown in the figure above, the maximum allowable value of i_I will be determined by v_O reaching -12 V, at which point

$v_x = -i_{I\max} \times 10$

and

$v_O = -12 = v_x - i_{L\max} \times 1 = -10\, i_{I\max} - 11\, i_{I\max}$

$$\Rightarrow i_{I\max} = \frac{12}{21} = 0.57\text{ mA}$$

If i_I is in a direction opposite to that shown in the figure, then

$v_x = 10 i_I$

$v_O = v_x + i_L R_L = 10i_I + 11i_I = 21i_I$

The maximum value of i_I will result in $v_O = +12$ V. Thus

$$12 = 21i_{I\max} \Rightarrow i_{I\max} = \frac{12}{21} = 0.57 \text{ mA}$$

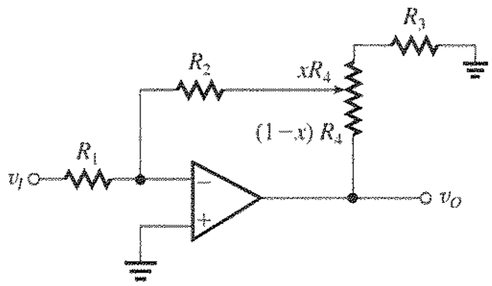
Thus, the allowable range of i_I is

$$-0.57 \text{ mA} \leq i_I \leq +0.57 \text{ mA}$$

(d) Since $R_{in} = 0$, the value of the source resistance will have no effect on the resulting i_L.

$$i_L = 0.2 \times 11 = 2.2 \text{ mA}$$

2.35



To obtain an input resistance of 100 kΩ, we select $R_1 = 100$ kΩ. From Example 2.2 we have

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1}\left[1 + \frac{(1-x)R_4}{R_2} + \frac{(1-x)R_4}{R_3 + xR_4}\right]$$

The minimum gain magnitude is obtained when $x = 1$,

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1} = -1$$

Thus, $R_2 = 100$ kΩ.

The maximum gain magnitude is obtained when $x = 0$,

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1}\left[1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right] = -100$$

$$\Rightarrow 1 + \frac{100}{100} + \frac{100}{R_3} = 100$$

$$\Rightarrow R_3 = \frac{100}{98} = 1.02 \text{ k}\Omega$$

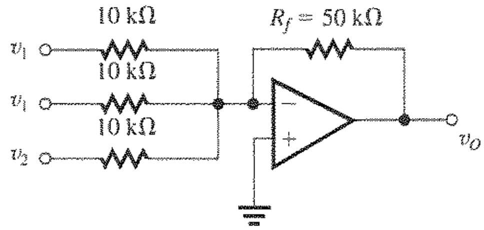
When the potentiometer is set exactly in the middle, $x = 0.5$ and

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1}\left[1 + \frac{0.5R_4}{R_2} + \frac{0.5R_4}{R_3 + 0.5R_4}\right]$$

$$= -\frac{100}{100}\left[1 + \frac{0.5 \times 100}{100} + \frac{0.5 \times 100}{1.02 + 0.5 \times 100}\right]$$

$$= -2.48 \text{ V/V}$$

2.36



$$v_O = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_1 + \frac{R_f}{R_3}v_2\right)$$

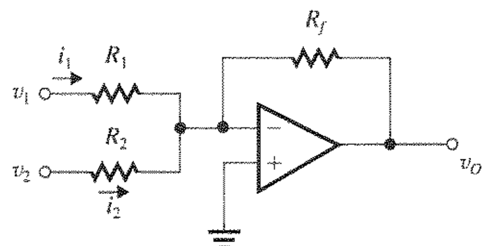
$$= -\left(\frac{50}{10}v_1 + \frac{50}{10}v_1 + \frac{50}{10}v_2\right)$$

$$= -(10v_1 + 5v_2)$$

Now $v_1 = 1$ V and $v_2 = -1$ V

$$v_O = -(10 \times 1 - 5) = -5 \text{ V}$$

2.37



The output of the weighted summer circuit is

$$v_O = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2\right)$$

$$v_O = -\left(2v_1 + \frac{v_2}{2}\right)$$

$$i_1 = \frac{v_1}{R_1} \text{ and } i_2 = \frac{v_2}{R_2}$$

Since $i_1, i_2 \leq 50$ μA for 1-V input signals

$\therefore R_1, R_2 \geq 20$ kΩ

Here $\frac{R_f}{R_1} = 2$, if $R_1 = 20$ kΩ, $R_f = 40$ kΩ

$$\frac{R_f}{R_2} = \frac{1}{2} \Rightarrow R_2 = 2R_f = 80 \text{ k}\Omega$$

2.38 $v_O = -(2v_1 + 4v_2 + 8v_3)$

$R_1, R_2, R_3 \geq 1$ kΩ

$$\frac{R_f}{R_1} = 2, \frac{R_f}{R_2} = 4, \frac{R_f}{R_3} = 8$$

$R_3 = 1$ kΩ $\Rightarrow R_f = 8$ kΩ

$R_2 = 2$ kΩ

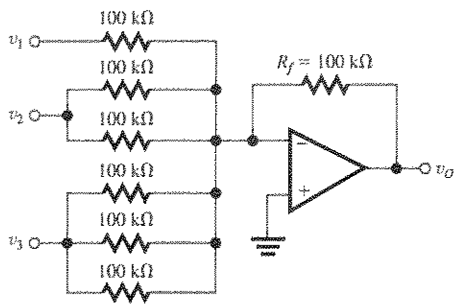
$R_1 = 4$ kΩ

2.39 (a) $v_O = -(v_1 + 2v_2 + 3v_3)$

$\frac{R_f}{R_1} = 1 \Rightarrow R_1 = 100\ \text{k}\Omega,$

$\frac{R_f}{R_2} = 2 \Rightarrow R_2 = 50\ \text{k}\Omega$

$\frac{R_f}{R_3} = 1 \Rightarrow R_3 = \frac{100}{3}\ \text{k}\Omega$

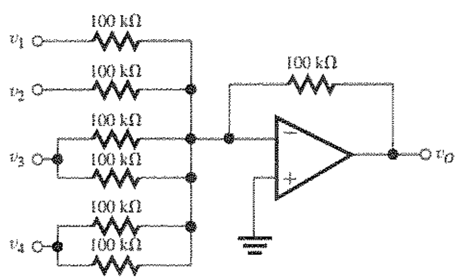


$R_{i1} = 100\ \text{k}\Omega$

$R_{i2} = 50\ \text{k}\Omega$

$R_{i3} = 33.3\ \text{k}\Omega$

(b) $v_O = -(v_1 + v_2 + 2v_3 + 2v_4)$



$\frac{R_f}{R_1} = 1 \Rightarrow R_1 = 100\ \text{k}\Omega$

$\frac{R_f}{R_2} = 1 \Rightarrow R_2 = 100\ \text{k}\Omega$

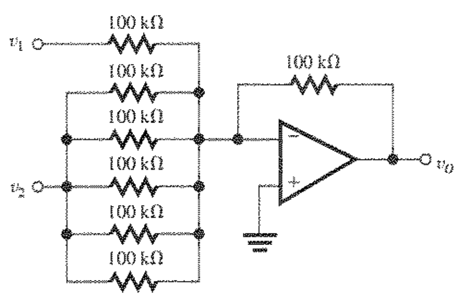
$\frac{R_f}{R_3} = 2 \Rightarrow R_3 = \frac{100}{2}\ \text{k}\Omega$

$\frac{R_f}{R_4} = 2 \Rightarrow R_4 = \frac{100}{2}\ \text{k}\Omega$

$R_{i1} = 100\ \text{k}\Omega, R_{i2} = 100\ \text{k}\Omega,$

$R_{i3} = 50\ \text{k}\Omega, R_{i4} = 50\ \text{k}\Omega$

(c) $v_O = -(v_1 + 5v_2)$



$R_1 = 100\ \text{k}\Omega$

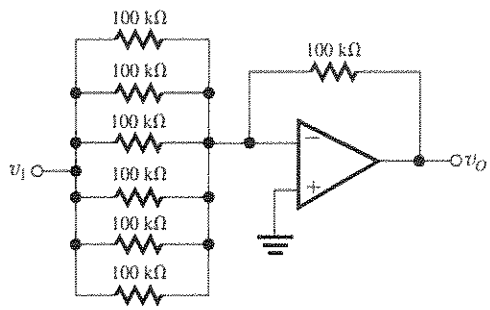
$R_2 = \frac{100\ \text{k}\Omega}{5}$

$R_{i1} = 100\ \text{k}\Omega$

$R_{i2} = 20\ \text{k}\Omega$

(d) $v_O = -6v_1$

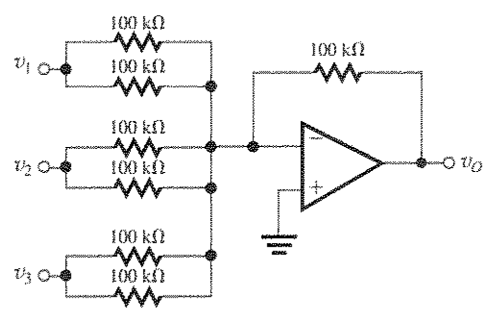
$R_1 = \frac{100\ \text{k}\Omega}{6}$



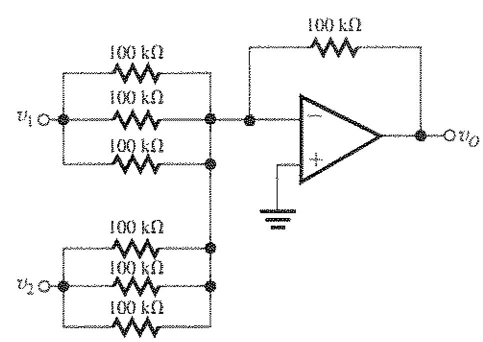
$R_{i1} = \frac{100}{6} = 1.67\ \text{k}\Omega$

Suggested configurations:

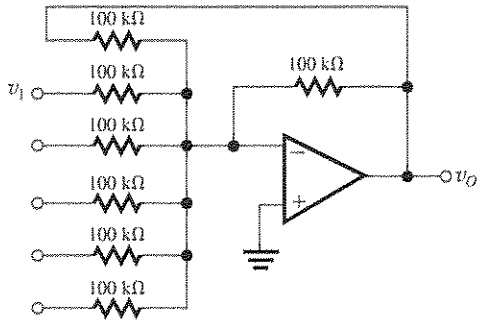
$v_O = -(2v_1 + 2v_2 + 2v_3)$



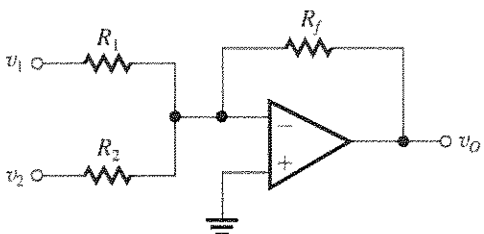
$v_O = -(3v_1 + 3v_2)$



In order to have coefficient $= 0.5$, connect one of the input resistors to v_O. $\frac{v_O}{v_1} = 0.5$.



2.40



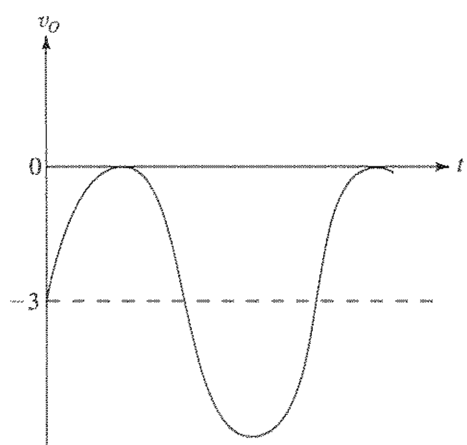
For v_1 and v_2, we assume

$v_1 = 3 \sin \omega t$

$v_2 = 1.5$ V

The output signal required is

$v_O = -3 \sin \omega t - 3$



For the summer circuit, we should have

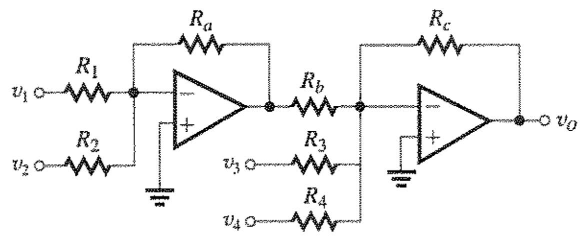
$$\frac{R_f}{R_1} = 1 \text{ and } \frac{R_f}{R_2} = 2$$

Select $R_f = 2R_2 = 20$ kΩ.

Thus $R_2 = 10$ kΩ,

and $R_1 = 20$ kΩ.

2.41 Using the circuit of Fig. 2.11:



To obtain $v_O = v_1 + 2v_2 - 3v_3 - 5v_4$, we can arbitrarily select $R_c = R_b$, then

$$\frac{R_a}{R_1} = 1 \qquad \text{and} \qquad \frac{R_a}{R_2} = 2$$

If we select $R_2 = 10$ kΩ, then $R_a = 20$ kΩ and $R_1 = 20$ kΩ.

For the second summer, to obtain

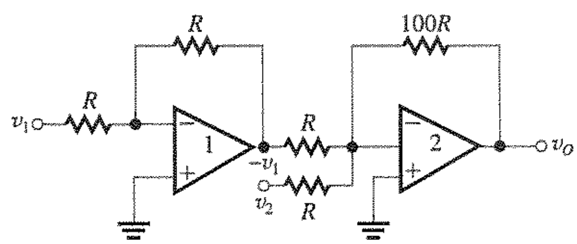
$$\frac{v_O}{v_3} = -3 \qquad \text{then} \qquad \frac{R_c}{R_3} = 3$$

and

$$\frac{v_O}{v_4} = -5 \qquad \text{requires} \qquad \frac{R_c}{R_4} = 5$$

We can select $R_c = R_b = 60$ kΩ, resulting in $R_3 = 20$ kΩ and $R_4 = 12$ kΩ.

2.42



Here the first inverting amplifier simply inverts v_1, resulting at its output in

$$-v_1 = -2 \sin(2\pi \times 60t) - 0.01 \sin(2\pi \times 1000t)$$

The second amplifier provides the sum of $-v_1$ and v_2, that is, $(v_2 - v_1)$ multiplied by a gain of 100. Thus

$$v_O = -100(v_2 - v_1)$$

$$= 1 \sin(2\pi \times 1000t), \text{ V}$$

The value of R can be selected (arbitrarily but conveniently) as $R = 10$ kΩ.

2.43 This is a weighted summer circuit:

$$v_O = -\left(\frac{R_f}{R_0}v_O + \frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

We may write:

$v_0 = 5\text{V} \times a_0, \ \ v_2 = 5\text{V} \times a_2,$

$v_1 = 5\text{V} \times a_1, \ v_3 = 5\text{V} \times a_3,$

$$v_O = -R_f\left(\frac{5}{80}a_0 + \frac{5}{40}a_1 + \frac{5}{20}a_2 + \frac{5}{10}a_3\right)$$

$$v_O = -R_f\left(\frac{a_0}{16} + \frac{a_1}{8} + \frac{a_2}{4} + \frac{a_3}{2}\right)$$

$$v_O = -\frac{R_f}{16}\left(2^0a_0 + 2^1a_1 + 2^2a_2 + 2^3a_3\right)$$

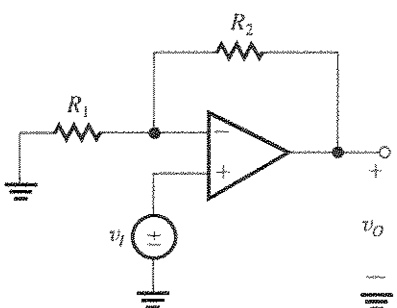
For $-12 \text{ V} \le v_O \le 0$,

$$\frac{R_f}{16}(2^0 \times 1 + 2 \times 1 + 2^2 \times 1 + 2^3 \times 1)$$

$$= \frac{15R_f}{16} = 12$$

$\Rightarrow R_f = 12.8 \text{ k}\Omega$

2.44



$$\frac{v_O}{v_I} = 1 + \frac{R_2}{R_1}$$

(a) $\dfrac{v_O}{v_I} = 1 = 1 + \dfrac{R_2}{R_1}$

Set $R_2 = 0\ \Omega$ and eliminate R_1

(b) $\dfrac{v_O}{v_I} = 2 = 1 + \dfrac{R_2}{R_1}$

$\dfrac{R_2}{R_1} = 1$; set $R_1 = R_2 = 10 \text{ k}\Omega$

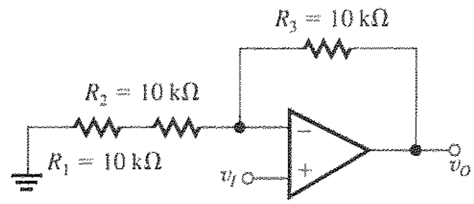
(c) $\dfrac{v_O}{v_I} = 21 = 1 + \dfrac{R_2}{R_1}$

$\dfrac{R_2}{R_1} = 20$; set $R_1 = 10 \text{ k}\Omega,\ R_2 = 200 \text{ k}\Omega$

(d) $\dfrac{v_O}{v_I} = 100 = 1 + \dfrac{R_2}{R_1}$

$\dfrac{R_2}{R_1} = 99$; set $R_1 = 10 \text{ k}\Omega,\ R_2 = 990 \text{ k}\Omega$

2.45

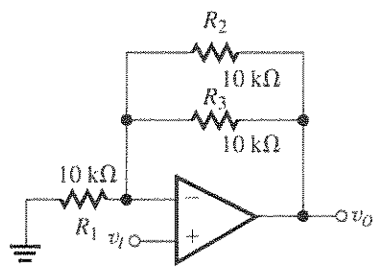


Short-circuit R_2:

$$\frac{v_O}{v_I} = 2$$

Short-circuit R_3:

$$\frac{v_O}{v_I} = 1$$



2.46 $V_- = V_+ = V$; thus the current in the moving-coil meter will be $I = \dfrac{V}{R}$, independent of the resistance of the meter. To obtain $I = 100\ \mu\text{A}$ when $V = 10$ V, we select

$$R = \frac{10}{0.1 \text{ mA}} = 100 \text{ k}\Omega$$

The meter resistance does not affect the voltmeter calibration.

2.47 Refer to the circuit in Fig. P2.47:

(a) Using superposition, we first set $v_{P_1} = v_{P_2}, \ldots, = 0$. The output voltage that results in response to $v_{N1}, v_{N2}, \ldots, v_{Nn}$ is

$$v_{ON} = -\left[\frac{R_f}{R_{N1}}v_{N1} + \frac{R_f}{R_{N2}}v_{N2} + \cdots + \frac{R_f}{R_{Nn}}v_{Nn}\right]$$

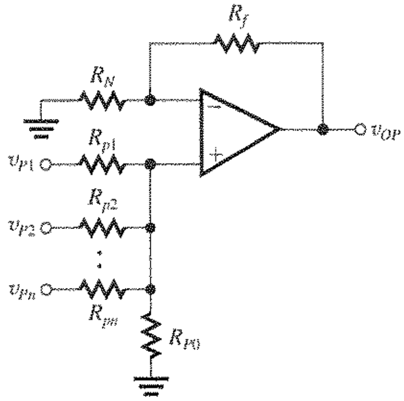
Then we set $v_{N1} = v_{N2} = \cdots = 0$, then:

$R_N = R_{N1} \parallel R_{N2} \parallel R_{N3} \parallel \cdots \parallel R_{Nn}$

The circuit simplifies to that shown below.

$$v_{OP} = \left(1 + \frac{R_f}{R_N}\right) \times \left(v_{P1}\frac{1/R_{P1}}{\frac{1}{R_{P0}} + \frac{1}{R_{P1}} + \cdots + \frac{1}{R_{Pn}}}\right.$$

$$\left. + v_{P2}\frac{1/R_{P2}}{\frac{1}{R_{P0}} + \cdots + \frac{1}{R_{Pn}}} \cdots + v_{Pn}\frac{1/R_{Pn}}{\frac{1}{R_{P0}} + \cdots + \frac{1}{R_{Pn}}}\right)$$



$$v_{OP} = \left(1 + \frac{R_F}{R_N}\right)\left(v_{P1}\frac{R_P}{R_{P1}} + v_{P2}\frac{R_P}{R_{P2}} + \cdots + \frac{R_P}{R_{Pn}}v_{Pn}\right)$$

where

$R_P = R_{P0} \parallel R_{P1} \parallel \cdots \parallel R_{Pn}$

when all inputs are present:

$$v_O = v_{ON} + v_{OP}$$

$$= -\left(\frac{R_f}{R_{N1}}v_{N1} + \frac{R_f}{R_{N2}}v_{N2} + \cdots\right) + \left(1 + \frac{R_f}{R_N}\right)\left(\frac{R_P}{R_{P1}}v_{N1} + \frac{R_P}{R_{P2}}v_{N2} + \cdots\right)$$

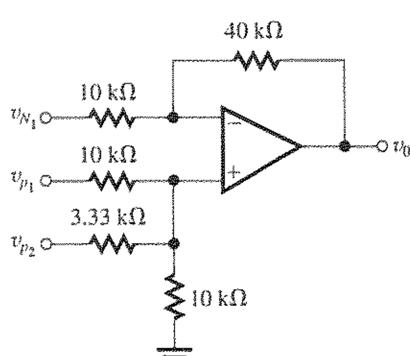
(b) $v_O = -4v_{N1} + v_{P1} + 3v_{P2}$

$$\frac{R_f}{R_{N1}} = 4,\ R_{N1} = 10\ \text{k}\Omega \Rightarrow R_f = 40\ \text{k}\Omega$$

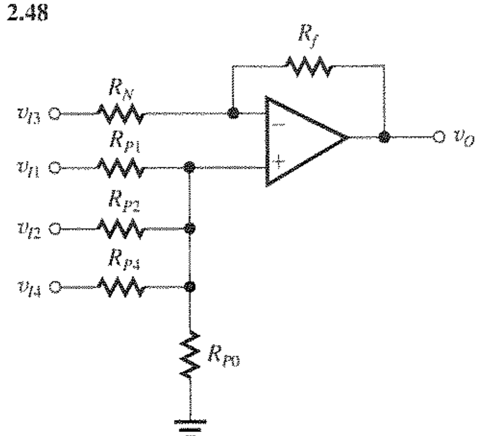
$$\left(1 + \frac{R_f}{R_N}\right)\left(\frac{R_P}{R_{P1}}\right) = 1 \Rightarrow 5\frac{R_P}{R_{P1}} = 1 \qquad (1)$$

$$\left(1 + \frac{R_f}{R_N}\right)\left(\frac{R_P}{R_{P2}}\right) = 3 \Rightarrow 5\frac{R_P}{R_{P2}} = 3 \qquad (2)$$

Substituting for $\frac{1}{R_P}$, $\frac{1}{R_P} = \frac{1}{R_{P0}} + \frac{1}{R_{P1}} + \frac{1}{R_{P2}}$ in Eqs. (1) and (2) and selecting (arbitrarily) $R_{P0} = 10\ \text{k}\Omega$ results in $R_{P1} = 10\ \text{k}\Omega$ and $R_{P2} = 3.33\ \text{k}\Omega$. The result is the following circuit:



2.48



Adapting the expression given in Problem 2.47 to the circuit above yields

$$v_O = -\frac{R_f}{R_N}v_{I3} + \left(1 + \frac{R_f}{R_N}\right)\left(\frac{R_P}{R_{P1}}v_{I1} + \frac{R_P}{R_{P2}}v_{I2} + \frac{R_P}{R_{P4}}v_{I4}\right)$$

where $R_P = R_{P0} \parallel R_{P1} \parallel R_{P2} \parallel R_{P3}$.

We require

$$v_O = -9v_{I3} + v_{I1} + 2v_{I2} + 4v_{I4}$$

Equating the coefficients of v_{I3}, we have

$$\frac{R_f}{R_N} = 9$$

Selecting $R_N = 10\ \text{k}\Omega \Rightarrow R_f = 90\ \text{k}\Omega$.

Equating the coefficients of v_{I1} provides

$$\left(1 + \frac{R_f}{R_N}\right)\frac{R_P}{R_{P1}} = 1$$

Thus,

$$10\frac{R_P}{R_{P1}} = 1 \Rightarrow \frac{R_P}{R_{P1}} = 0.1 \qquad (1)$$

Similarly, equating the coefficients of v_{I2} gives

$$10\frac{R_P}{R_{P2}} = 2 \Rightarrow \frac{R_P}{R_{P2}} = 0.2 \qquad (2)$$

and equating the coefficients of v_{I4} gives

$$10\frac{R_P}{R_{P4}} = 4 \Rightarrow \frac{R_P}{R_{P4}} = 0.4 \qquad (3)$$

Now, summing Eqs. (1), (2), and (3) provides

$$R_P\left(\frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \frac{1}{R_{P4}}\right) = 0.7 \qquad (4)$$

But,

$$\frac{1}{R_P} = \frac{1}{R_{P0}} + \frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \frac{1}{R_{P4}}$$

Thus,

$$\frac{1}{R_P}-\frac{1}{R_{P0}}=\frac{1}{R_{P1}}+\frac{1}{R_{P2}}+\frac{1}{R_{P4}} \qquad (5)$$

Equations (4) and (5) can be combined to obtain

$$1-\frac{R_P}{R_{P0}}=0.7 \Rightarrow R_{P0}=\frac{R_P}{0.3} \qquad (6)$$

Selecting

$R_{P0}=10\ \text{k}\Omega$

Equation (6) $\Rightarrow R_P=3\ \text{k}\Omega$
Equation (1) $\Rightarrow R_{P1}=30\ \text{k}\Omega$
Equation (2) $\Rightarrow R_{P2}=15\ \text{k}\Omega$
Equation (3) $\Rightarrow R_{P4}=7.5\ \text{k}\Omega$

2.49 $v_+=v_I\dfrac{R_4}{R_3+R_4}$

$$\frac{v_O}{v_I}=\left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)=\frac{1+R_2/R_1}{1+R_3/R_4}$$

2.50 Refer to Fig. P2.50. Setting $v_2=0$, we obtain the output component due to v_1 as

$v_{O1}=-10v_1$

Setting $v_1=0$, we obtain the output component due to v_2 as

$$v_{O2}=v_2\left(1+\frac{10R}{R}\right)\left(\frac{10R}{10R+R}\right)=10v_2$$

The total output voltage is

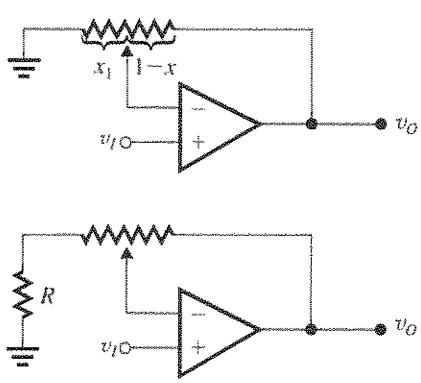
$v_O=v_{O1}+v_{O2}=10(v_2-v_1)$

For $v_1=10\sin(2\pi\times 60t)-0.1\sin(2\pi\times 1000\,t)$

$v_2=10\sin(2\pi\times 60t)+0.1\sin(2\pi\times 1000\,t)$

$v_O=2\sin(2\pi\times 1000t)$

2.51



$$\frac{v_O}{v_I}=1+\frac{R_2}{R_1}=1+\frac{1-x}{x}=1+\frac{1}{x}-1$$

$$\therefore \frac{v_O}{v_I}=\frac{1}{x}$$

$$0\le x\le 1 \Rightarrow \infty \ge \frac{v_O}{v_I}\ge 1$$

Add a resistor as shown:

$$\frac{v_O}{v_I}=1+\frac{(1-x)\times 10\ \text{k}\Omega}{x\times 10\ \text{k}\Omega+R}$$

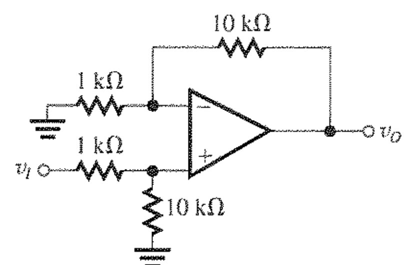
For $\left(\dfrac{v_O}{v_I}\right)_{\max}=11$

$$x=0,\ \frac{v_O}{v_I}=11=1+\frac{10\ \text{k}\Omega}{R}$$

$$10=\frac{10\ \text{k}\Omega}{R}$$

$R=1\ \text{k}\Omega$

2.52

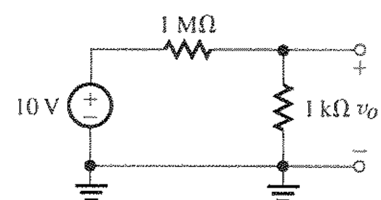


$$v_O=v_I\frac{10}{1+10}\left(1+\frac{10}{1}\right)$$

$v_O=10v_I$

$R_{in}=11\ \text{k}\Omega$

2.53

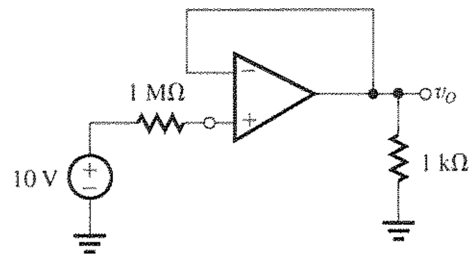


(a) Source is connected directly.

$$v_O=10\times\frac{1}{1001}\simeq 10\ \text{mV}$$

$$i_L=\frac{v_O}{1\ \text{k}\Omega}=\frac{10\ \text{mV}}{1\ \text{k}\Omega}=10\ \mu\text{A}$$

Current supplied by the source is 10 μA.



(b) Inserting a buffer.

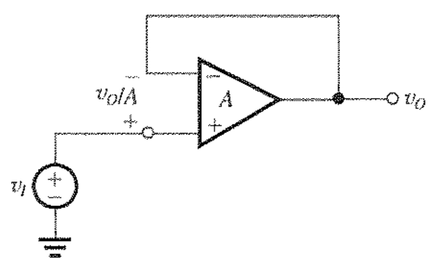
$v_O = 10\text{ V}$

$$i_L = \frac{10\text{ V}}{1\text{ k}\Omega} = 10\text{ mA}$$

Current supplied by the source is 0.

The load current i_L comes from the power supply of the op amp.

2.54



$$v_O = v_I - \frac{v_O}{A}$$

$$\frac{v_O}{v_I} = \frac{1}{1+\frac{1}{A}}$$

Error of gain magnitude

$$\frac{\left|\frac{v_O}{v_I} - 1\right|}{1} = -\frac{1}{A+1}$$

A (V/V)	1000	100	10
$\frac{v_O}{v_O}$ (V/V)	0.999	0.990	0.909
Gain error	−0.1%	−1%	−9.1%

2.55 For an inverting amplifier

$$R_{in} = R_1, \quad G = -\frac{R_2}{R_1}$$

for a noninverting amplifier:

$$R_{in} = \infty \quad G = 1 + \frac{R_2}{R_1}$$

Case	Gain (V/V)	R_{in}	R_1	R_2
a	−10	10 kΩ	10 kΩ	100 kΩ
b	−1	100 kΩ	100 kΩ	100 kΩ
c	−2	100 kΩ	100 kΩ	200 kΩ
d	+1	∞	∞	0
e	+2	∞	100 kΩ	100 kΩ
f	+11	∞	10 kΩ	100 kΩ
g	−0.5	20 kΩ	20 kΩ	10 kΩ

2.56 $A = 100\text{ V/V} \quad 1 + \frac{R_2}{R_1} = 10\text{ V/V}$

If $R_1 = 10\text{ k}\Omega \Rightarrow R_2 = 90\text{ k}\Omega$

According to Eq. (2.11):

$$G = \frac{v_O}{v_I} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1 + R_2/R_1}{A}}$$

$$G = \frac{1 + 90/10}{1 + \frac{1 + 90/10}{100}} = \frac{10}{1.1} = 9.09\text{ V/V}$$

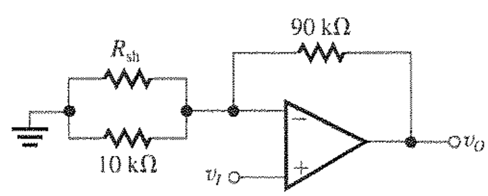
In order to compensate the gain drop, we can shunt R_1 with a resistor R_{sh}. The value of R_{sh} can be obtained from

$$10 = \frac{1 + \left(\frac{90}{10} + \frac{90}{R_{sh}}\right)}{1 + \frac{1 + \frac{90}{10} + \frac{90}{R_{sh}}}{100}} \Rightarrow$$

$$10 + 0.1\left(10 + \frac{90}{R_{sh}}\right) = 10 + \frac{90}{R_{sh}}$$

$$1 + \frac{9}{R_{sh}} = \frac{90}{R_{sh}}$$

$$\Rightarrow R_{sh} = 81\text{ k}\Omega$$



If $A = 200$, then:

$$G_{\text{uncompensated}} = \frac{1 + \frac{90}{10}}{1 + \frac{1 + 90/10}{200}} = \frac{10}{1.05} = 9.52\text{ V/V}$$

$$G_{\text{compensated}} = \frac{1 + \frac{90}{10} + \frac{90}{81}}{1 + \frac{1 + \frac{90}{10} + \frac{90}{81}}{200}}$$

$$= \frac{11.11}{1.056} = 10.52\text{ V/V}$$

2.57

$$G = \frac{G_0}{1+\frac{G_0}{A}}, \quad \frac{G_0 - G}{G_0} \times 100 = \frac{G_0/A \times 100}{1+\frac{G_0}{A}} \le x$$

$$\text{or } \frac{1+\frac{G_0}{A}}{\frac{G_0}{A}} \ge \frac{100}{x} \Rightarrow \frac{A}{G_0} \ge \underbrace{\left(\frac{100}{x} - 1\right)}_{F}$$

$$\Rightarrow A \ge G_0 F, \text{ where } F = \frac{100}{x} - 1 \simeq \frac{100}{x}$$

x	0.01	0.1	1	10
F	10^4	10^3	10^2	10

Thus for:

$x = 0.01$:					
G_0 (V/V)	1	10	10^2	10^3	10^4
A (V/V)	10^4	10^5	10^6	10^7	10^8
				too high to be practical	

$x = 0.1$:					
G_0 (V/V)	1	10	10^2	10^3	10^4
A (V/V)	10^3	10^4	10^5	10^6	10^7

$x = 1$:					
G_0 (V/V)	1	10	10^2	10^3	10^4
A (V/V)	10^2	10^3	10^4	10^5	10^6

$x = 10$:					
G_0 (V/V)	1	10	10^2	10^3	10^4
A (V/V)	10	10^2	10^3	10^4	10^5

2.58 For a non inverting amplifier [Eq. (2.11)]:

$$G = \frac{G_0}{1+\frac{G_0}{A}} \qquad \epsilon = \frac{G_0 - G}{G_0} \times 100$$

for an inverting amplifier (Eq. 2.5):

$$G = \frac{G_0}{1+\frac{1-G_0}{A}} \qquad \epsilon = \frac{|G_0| - |G|}{|G_0|} \times 100$$

Case	G_0 (V/V)	A (V/V)	G (V/V)	ϵ (%)
a	−1	10	−0.83	17
b	1	10	0.91	9
c	−1	100	−0.98	2
d	10	10	5	50
e	−10	100	−9	10
f	−10	1000	−9.89	1.1
g	+1	2	0.67	33

2.59 Refer to Fig. P2.59. When potentiometer is set to the bottom:

$$v_O = v_+ = -15 + \frac{30 \times 25}{25 + 100 + 25} = -10 \text{ V}$$

and to the top:

$$v_O = -15 + \frac{30 \times 25}{25 + 100 + 25} = +10 \text{ V}$$

$$\Rightarrow -10 \text{ V} \le v_O \le +10 \text{ V}$$

Pot has 20 turns, and for each turn:

$$\Delta v_O = \frac{2 \times 10}{20} = 1 \text{ V}$$

2.60 $R_1 = R_3 = 5\text{ k}\Omega$, $R_2 = R_4 = 100\text{ k}\Omega$

Equation (2.15): $\frac{R_4}{R_3} = \frac{R_2}{R_1} = 20$

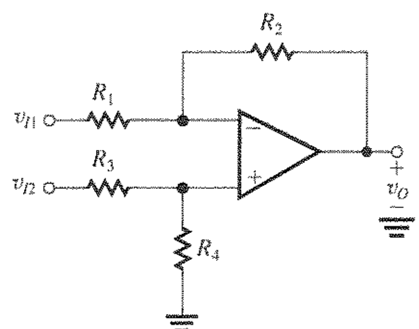
From Eq. (2.16),

$$v_O = \frac{R_2}{R_1} v_{Id}$$

$$A_d = \frac{v_O}{v_{Id}} = \frac{R_2}{R_1} = 20 \text{ V/V}$$

From Eq. (2.20)

$$R_{id} = 2R_1 = 2 \times 5\text{ k}\Omega = 10\text{ k}\Omega$$



The two resistance ratios $\frac{R_2}{R_1}$ and $\frac{R_4}{R_3}$ differ by 1%.

$$\therefore \frac{R_2}{R_1} = 0.99 \frac{R_4}{R_3}$$

Now for this case, A_{cm} can be found from Eq. (2.19)

$$A_{cm} = \left(\frac{R_4}{R_4 + R_3}\right)\left(1 - \frac{R_2}{R_1} \times \frac{R_3}{R_4}\right)$$

$$\simeq \frac{100}{100+5} \times \left(1 - 0.99 \frac{R_4}{R_3} \times \frac{R_3}{R_4}\right)$$

$$A_{cm} = 0.0095$$

Neglecting the effect of resistance variation on A_d,

$$A_d = \frac{R_2}{R_1} = \frac{100}{5} = 20 \text{ V V}$$

$$\text{CMRR} = 20 \log \left|\frac{A_d}{A_{cm}}\right|$$

$$= 20 \log \left|\frac{20}{0.0095}\right|$$

$$= 66.4 \text{ dB}$$

2.61 If we assume $R_3 = R_1$, $R_4 = R_2$, then

Eq. (2.20): $R_{id} = 2R_1 \Rightarrow R_1 = \dfrac{20}{2} = 10\ \text{k}\Omega$

(Refer to Fig. 2.16.)

(a) $A_d = \dfrac{R_2}{R_1} = 1\ \text{V/V} \Rightarrow R_2 = 10\ \text{k}\Omega$

$R_1 = R_2 = R_3 = R_4 = 10\ \text{k}\Omega$

(b) $A_d = \dfrac{R_2}{R_1} = 5\ \text{V/V} \Rightarrow R_2 = 50\ \text{k}\Omega = R_4$

$R_1 = R_3 = 10\ \text{k}\Omega$

(c) $A_d = \dfrac{R_2}{R_1} = 100\ \text{V/V} \Rightarrow R_2 = 1\ \text{M}\Omega = R_4$

$R_1 = R_3 = 10\ \text{k}\Omega$

(d) $A_d = \dfrac{R_2}{R_1} = 0.5\ \text{V/V} \Rightarrow R_2 = 5\ \text{k}\Omega = R_4$

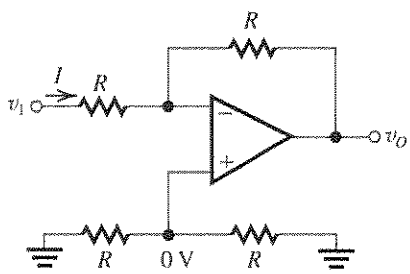
$R_1 = R_3 = 10\ \text{k}\Omega$

2.62 Refer to Fig. P2.62:

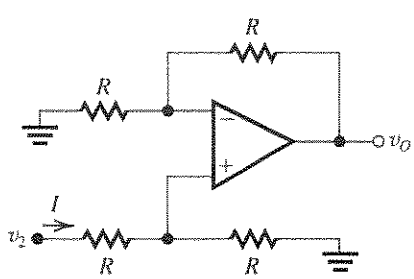
Considering that $v_- = v_+$:

$v_1 + \dfrac{v_O - v_1}{2} = \dfrac{v_2}{2} \Rightarrow v_O = v_2 - v_1$

v_1 only: $R_i = \dfrac{v_1}{I} = R$

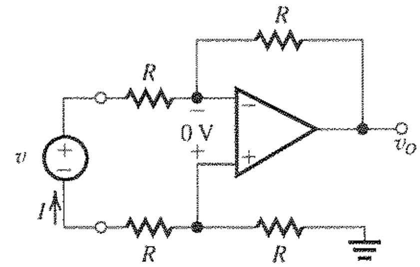


v_2 only: $R_i = \dfrac{v_2}{I} = 2R$



v between 2 terminals:

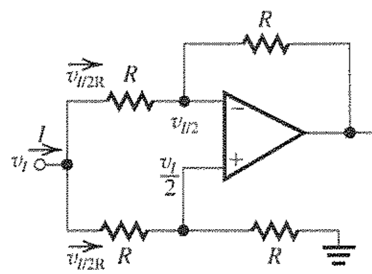
$R_i = \dfrac{v}{I} = 2R$



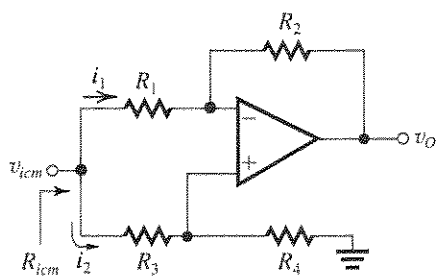
v_I connected to both input terminals

$I = \dfrac{v_I}{2R} + \dfrac{v_I}{2R} = \dfrac{v_I}{R}$

$R_i = R$



2.63



When $R_2/R_1 = R_4/R_3$, the output voltage v_O will be zero. Thus,

$i_1 = \dfrac{v_{icm}}{R_1 + R_2}$

and

$i_2 = \dfrac{v_{icm}}{R_3 + R_4}$

Thus,

$i_i = v_{icm}\left[\dfrac{1}{R_1 + R_2} + \dfrac{1}{R_3 + R_4}\right]$

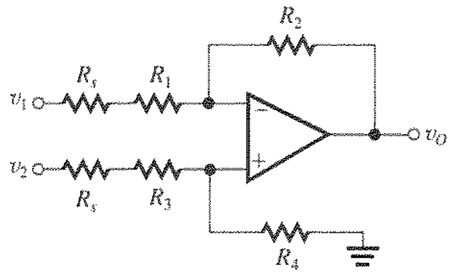
and

$R_{icm} = (R_1 + R_2) \parallel (R_3 + R_4)$ Q.E.D.

2.64 For an ideal difference amp, we need:

$$\frac{R_s + R_1}{R_2} = \frac{R_s + R_3}{R_4}$$

$$\frac{R_s/R_1 + 1}{R_2/R_1} = \frac{R_s/R_3 + 1}{R_4/R_3}$$



Since $\dfrac{R_2}{R_1} = \dfrac{R_4}{R_3}$:

$$\frac{R_s}{R_1} + 1 = \frac{R_s}{R_3} + 1 \Rightarrow R_1 = R_3 \Rightarrow R_2 = R_4$$

2.65 From Eq. (2.19),

$$A_{cm} \equiv \frac{v_O}{v_{Icm}} = \left(\frac{R_4}{R_4 + R_3}\right)\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

The second factor in this expression is the one that in effect determines A_{cm}. Now, for the circuit under consideration, all resistors are nominally equal to 10 kΩ, but each has a tolerance of $\pm x\%$. Thus the second factor will have a maximum magnitude when R_2 and R_3 are at their lowest value and R_1 and R_4 are at their highest values, that is,

$$R_2 = R_3 = 10(1 - 0.01x)$$

$$R_1 = R_4 = 10(1 + 0.01x)$$

Substituting in the expression for A_{cm}, we have

$$A_{cm} = \frac{10(1 + 0.01x)}{20}\left[1 - \frac{(1 - 0.01x)^2}{(1 + 0.01x)^2}\right]$$

Now, for $0.01x \ll 1$,

$$A_{cm} = 0.5(1 + 0.01x) \times 0.04x$$

$$A_{cm} \simeq 0.02x \text{ V/V}$$

$$A_d = 1$$

$$\text{CMRR} = \left|\frac{A_d}{A_{cm}}\right| = \frac{50}{x}$$

or $20 \log\left(\dfrac{50}{x}\right)$ dB

$x(\%)$	0.1	1	5
A_{cm} (V/V)	0.002	0.02	0.1
CMRR (dB)	54	34	20

2.66 From Eq. (2.19),

$$A_{cm} = \left(\frac{R_4}{R_4 + R_3}\right)\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

The second factor in this expression in effect determines A_{cm}. The largest, $|A_{cm}|$, will occur when R_2 and R_3 are at their lowest (or highest) values and R_1 and R_4 are at their highest (or lowest) values, as this will provide the maximum deviation of $\left(\dfrac{R_2}{R_1}\dfrac{R_3}{R_4}\right)$ from unity. Thus,

$$R_2 = R_{2\text{nominal}}(1 - \epsilon)$$
$$R_3 = R_{3\text{nominal}}(1 - \epsilon)$$
$$R_1 = R_{1\text{nominal}}(1 + \epsilon)$$
$$R_4 = R_{4\text{nominal}}(1 + \epsilon)$$

where

$$\frac{R_{2\text{nominal}}}{R_{1\text{nominal}}} = \frac{R_{4\text{nominal}}}{R_{3\text{nominal}}} = K$$

Substituting in the expression for A_{cm},

$$A_{cm} = \frac{R_{4\text{nominal}}(1 + \epsilon)}{R_{4\text{nominal}}(1 + \epsilon) + R_{3\text{nominal}}(1 - \epsilon)} \times$$

$$\left[1 - \frac{(1 - \epsilon)^2}{(1 + \epsilon)^2}\right]$$

For $\epsilon \ll 1$,

$$A_{cm} \simeq \frac{K}{K + 1} \times 4\epsilon$$

Since

$$A_d = K$$

$$\text{CMRR} = \left|\frac{A_d}{A_{cm}}\right| = \left(\frac{K + 1}{4\epsilon}\right)$$

which in dB becomes

$$\text{CMRR} = 20 \log\left[\frac{K + 1}{4\epsilon}\right] \qquad \text{Q.E.D.}$$

For $A_d = 100$ V/V and $\epsilon = 0.01$,

$$\text{CMRR} = 20 \log\left(\frac{101}{0.04}\right) = 68 \text{ dB}$$

To obtain CMRR = 80 dB,

$$\frac{101}{4\epsilon} = 10^4$$

$$\epsilon = \frac{101}{4 \times 10^4} \simeq 0.25 \times 10^{-2}$$

That is, the resistor tolerance should be a maximum of 0.25%.

2.67 See solution to Problem 2.66 above. Specifically, if the resistors have a tolerance of x%, then

$$\text{CMRR} = 20 \log \left[\frac{K+1}{4(x/100)} \right]$$

where K is the nominal differential gain. Here we are required to obtain

$K = 1000$

$R_{in} = 2\ \text{k}\Omega$

Thus, $R_1 = R_3 = 1\ \text{k}\Omega$ and $R_2 = R_4 = 1\ \text{M}\Omega$.

To obtain a CMRR of 88 dB, we write

$$88 = 20 \log \left[\frac{1001}{4(x/100)} \right]$$

$$\frac{1001 \times 100}{4x} = 25,118.86$$

$\Rightarrow x \simeq 1\%$

2.68 (a) Refer to Fig. P2.68 and Eq. (2.19):

$$A_{cm} = \frac{R_4}{R_3 + R_4} \left(1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \right)$$

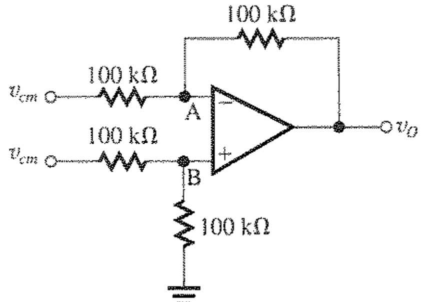
$$= \frac{100}{100 + 100} \left(1 - \frac{100.100}{100.100} \right)$$

$A_{cm} = 0$

Refer to 2.17: $\dfrac{R_2}{R_1} = \dfrac{R_4}{R_3}$

$$\Rightarrow A_d = \frac{R_2}{R_1} = 1$$

(b) $v_A = v_B$

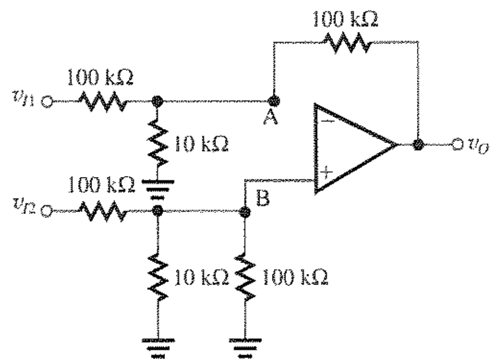


$$v_A = v_{cm} \frac{100}{100 + 100}$$

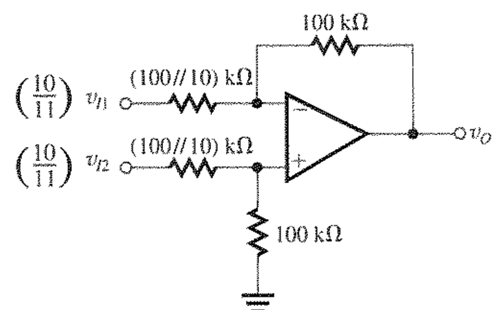
$$v_A = \frac{v_{cm}}{2} \text{ and } v_B = \frac{v_{cm}}{2}$$

$\Rightarrow -5\ \text{V} \le v_{cm} \le 5\ \text{V}$

(c) The circuit becomes as shown below:



Applying Thévenin's theorem to v_{I1} together with the associated (100-kΩ, 10-kΩ) voltage divider, and similarly to v_{I2} and the associated (100-kΩ, 10-kΩ) voltage divider, we obtain the following circuit:



$$v_O = \frac{100\ \text{k}\Omega}{(100 \parallel 10)\ \text{k}\Omega} \left(\frac{10}{11} v_{I2} - \frac{10}{11} v_{I1} \right)$$

$= 10(v_{I2} - v_{I1})$

For $v_{I1} = v_{I2} = v_{Icm}$, $v_O = 0$, thus

$A_{cm} = 0$

For $v_{I2} - v_{I1} = v_{Id}$,

$v_O = 10\ v_{Id}$

Thus, $A_d = 10$.

To obtain the input common-mode range, we note that for $v_{I1} = v_{I2} = v_{Icm}$,

$$v_+ = v_- = \frac{10}{11} v_{Icm} \times \frac{100}{100 + (100 \parallel 10)}$$

$= 0.833 v_{Icm}$

For v_+ and v_- in the range

$-2.5\ \text{V} \le v_+,\ v_- \le +2.5\ \text{V}$

the range of v_{Icm} will be

$-3\ \text{V} \le v_{Icm} \le +3\ \text{V}$

2.69 Refer to Fig. P2.69. Using superposition:

$v_O = v_{O1} + v_{O2}$

Calculate v_{O1}: $v_+ = \dfrac{\beta v_{O1}}{2} = v_-$

$$\frac{v_1 - \dfrac{\beta v_{O1}}{2}}{R} = \frac{\dfrac{\beta v_{O1}}{2} - v_O}{R} \Rightarrow v_{O1} = \frac{v_1}{\beta - 1}$$

Calculate v_{O2}:

$$v_- = \frac{v_{O2}}{2} = v_+ \Rightarrow v_2 - \frac{v_{O2}}{2} = \frac{v_{O2}}{2} - \beta v_{O2}$$

$$\Rightarrow v_{O2} = \frac{v_2}{1 - \beta}$$

$$v_O = v_{O1} + v_{O2} = \frac{v_1}{\beta - 1} + \frac{v_2}{1 - \beta}$$

$$= \frac{1}{1 - \beta}(v_2 - v_1)$$

$$A_d = \frac{v_O}{v_2 - v_1} = \frac{1}{1 - \beta} \qquad \text{Q.E.D}$$

$$A_d = 10 \text{ V/V} \Rightarrow \beta = 0.9 = \frac{R_6}{R_5 + R_6}$$

$$R_{Id} = 2R = 2\text{ M}\Omega \Rightarrow R = 1\text{ M}\Omega$$

$$R_5 + R_6 \leq \frac{R}{100} \Rightarrow R_5 + R_6 \leq 10\text{ k}\Omega$$

Selecting $R_6 = 6.8\text{ k}\Omega$

$$\frac{6.8}{6.8 + R_5} = 0.9 \Rightarrow R_5 = 756\ \Omega$$

2.70 See partial analysis on circuit diagram below.

From input loop:

$$v_{Id} = 2i_1R_1 \qquad (1)$$

From the loop containing R_2, $+$, $-$, R_2:

$$v_G = i_1R_2 + 0 + i_1R_2 = 2i_1R_2$$

Thus, we can find the current through R_G as $v_G/R_G = 2i_1(R_2/R_G)$. Finally, from the loop containing v_O, R_2, R_G, and R_2:

$$v_O = -2i_1\left(1 + 2\frac{R_2}{R_G}\right)R_2 - 2i_1R_2$$

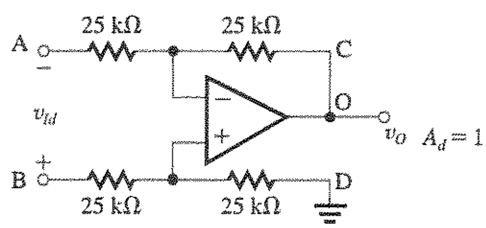
$$= -4i_1R_2\left(1 + \frac{R_2}{R_G}\right)$$

Substituting for $2i_1$ from Eq. (1), we obtain the gain as

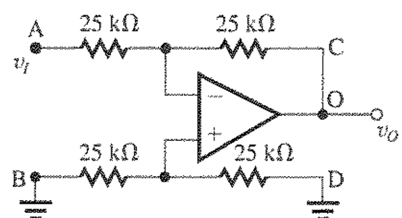
$$\frac{v_O}{v_{Id}} = -2\left(\frac{R_2}{R_1}\right)\left(1 + \frac{R_2}{R_G}\right) \qquad \text{Q.E.D.}$$

2.71 (a) Refer to Eq. (2.17): $A_d = \dfrac{R_2}{R_1} = 1$.

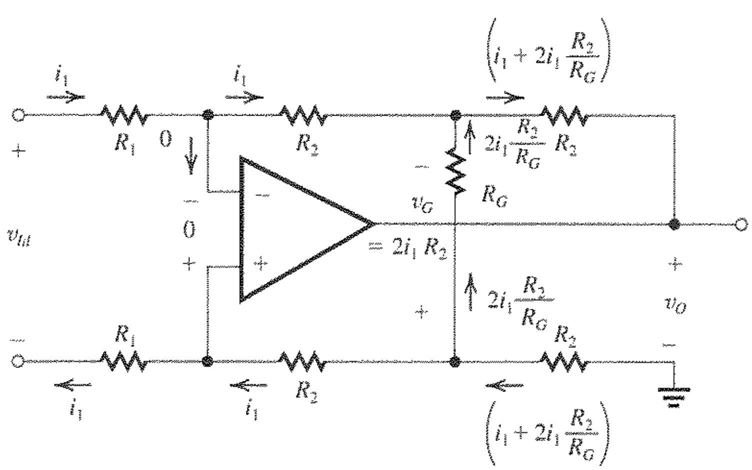
Connect C and O together, and D to ground.



(b) (i) $\dfrac{v_O}{v_I} = -1$ V/V

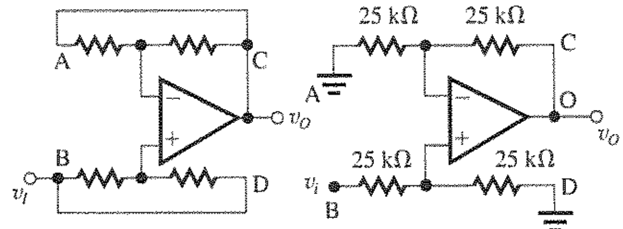


This figure belongs to Problem 2.70.



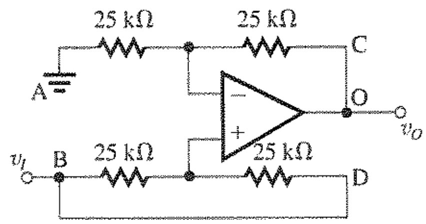
(ii) $\frac{v_O}{v_i} = +1$ V/V

Two possibilities:

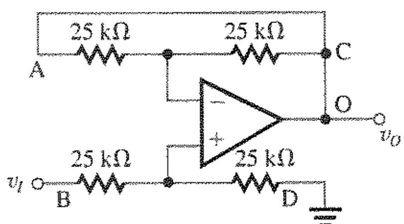


The circuit on the left ideally has infinite input resistance

(iii) $\frac{v_O}{v_i} = +2$ V/V



(iv) $\frac{v_O}{v_i} = +\frac{1}{2}$ V/V



2.72 Refer to the figure below:

$v_B = v_A = 3 + 0.05 \sin \omega t$, V
$v_C = v_D = 3 - 0.05 \sin \omega t$, V

Current through R_2, $2R_1$, and R_2 is

$i = (v_B - v_C)/2R_1$

$= 0.05 \sin \omega t$, mA

$v_E = v_B + iR_2 = 3 + 2.55 \sin \omega t$, V

$v_F = v_C - iR_2 = 3 - 2.55 \sin \omega t$, V

$v_G = v_H = \frac{1}{2} v_F = 1.5 - 1.275 \sin \omega t$, V

$v_O = (v_F - v_E) \times 1 = -5.1 \sin \omega t$, V

2.73 (a) Refer to Fig. 2.20(a).

The gain of the first stage is $\left(1 + \frac{R_2}{R_1}\right) = 101$. If the op amps of the first stage saturate at ± 12 V then $-12\text{ V} \le 101 v_{Icm} \le +12$ V

$\Rightarrow -0.12\text{ V} \le v_{Icm} \le 0.12$ V

As explained in the text, the disadvantage of circuit in Fig. 2.20(a) is that v_{Icm} is amplified by a gain equal to $\left(1 + \frac{R_2}{R_1}\right)$ in the first stage and therefore a very small v_{Icm} range is acceptable to avoid saturation.

(b) In Fig. 2.20(b), when v_{Icm} is applied, v_- for both A_1 and A_2 is the same and therefore no current flows through $2R_1$. This means the voltage at the output of A_1 and A_2 is the same as v_{Icm}.

$\Rightarrow -12\text{ V} \le v_{Icm} \le 12$ V

This circuit allows for a much larger range of v_{Icm}.

2.74 (a) Refer to the circuit in Fig. 2.20(a).

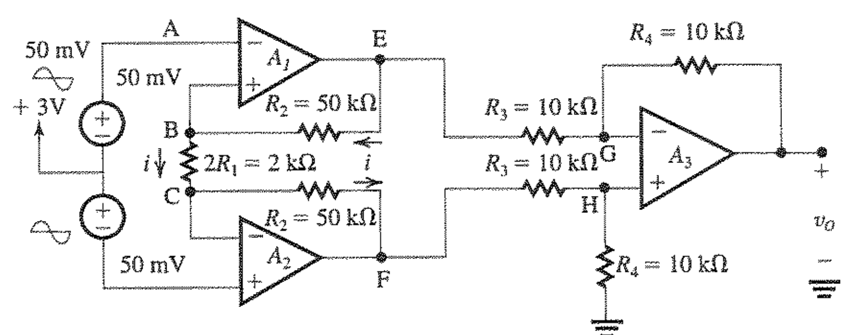
$v_{I1} = v_{Icm} - \frac{1}{2} v_{Id}$

$v_{I2} = v_{Icm} + \frac{1}{2} v_{Id}$

$v_{O1} = \left(1 + \frac{R_2}{R_1}\right) v_{I1}$

$= \left(1 + \frac{R_2}{R_1}\right) v_{Icm} - \frac{1}{2}\left(1 + \frac{R_2}{R_1}\right) v_{Id}$

This figure belongs to Problem 2.72.



$$v_{O2} = \left(1 + \frac{R_2}{R_1}\right) v_{I2}$$

$$= \left(1 + \frac{R_2}{R_1}\right) v_{Icm} + \frac{1}{2}\left(1 + \frac{R_2}{R_1}\right) v_{Id}$$

$$v_{Od} = v_{O2} - v_{O1} = \left(1 + \frac{R_2}{R_1}\right) v_{Id}$$

$$v_{Ocm} = \frac{1}{2}(v_{O1} + v_{O2}) = \left(1 + \frac{R_2}{R_1}\right) v_{Icm}$$

$$A_{d1} = 1 + \frac{R_2}{R_1}$$

$$A_{cm1} = 1 + \frac{R_2}{R_1}$$

$$\text{CMRR} = 20 \log \left|\frac{A_{d1}}{A_{cm1}}\right| = 0 \text{ dB}$$

(b) Refer to the circuit in Fig. 2.20(b):

$$v_{I1} = v_{Icm} - \frac{1}{2} v_{Id}$$

$$v_{I2} = v_{Icm} + \frac{1}{2} v_{Id}$$

$$v_{-}(A_1) = v_{I1} = v_{Icm} - \frac{1}{2} v_{Id}$$

$$v_{-}(A_2) = v_{I2} = v_{Icm} + \frac{1}{2} v_{Id}$$

Current through R_1 in the upward direction is

$$i = \frac{v_{-}(A_2) - v_{-}(A_1)}{2R_1} = \frac{v_{Id}}{2R_1}$$

$$v_{O1} = v_{-}(A_1) - iR_2 = v_{Icm} - \frac{1}{2}\left(1 + \frac{R_2}{R_1}\right) v_{Id}$$

$$v_{O2} = v_{-}(A_2) + iR_2 = v_{Icm} + \frac{1}{2}\left(1 + \frac{R_2}{R_1}\right) v_{Id}$$

$$v_{Od} = v_{O2} - v_{O1} = \left(1 + \frac{R_2}{R_1}\right) v_{Id}$$

$$v_{Ocm} = \frac{1}{2}(v_{O1} + v_{O2}) = v_{Icm}$$

$$A_{d1} = 1 + \frac{R_2}{R_1}$$

$$A_{cm1} = 1$$

$$\text{CMRR} = 20 \log \left|\frac{A_{d1}}{A_{cm1}}\right| = 20 \log\left(1 + \frac{R_2}{R_1}\right)$$

Comment: In circuit (a), the first stage amplifies the differential signal and the common-mode signal equally. On the other hand, in circuit (b), the first stage amplifies the differential signal by $\left(1 + \frac{R_2}{R_1}\right)$ and the common-mode signal by unity, thus providing a substantial CMRR. Circuit (a) is useless as a differential amplifier!

2.75 Ideally,

$$A_d = \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right) \qquad (1)$$

$$A_{cm} = 0$$

$$\text{CMRR} = \infty$$

For $R_2 = R_3 = R_4 = 100 \text{ k}\Omega$, and $2R_1 = 10 \text{ k}\Omega$,

$$A_d = 1\left(1 + \frac{100}{5}\right) = 21 \text{ V/V}$$

$$A_{cm} = 0$$

$$\text{CMRR} = \infty$$

If all resistors have $\pm 1\%$ tolerance, the differential gain will be slightly affected; Eq. (1) indicates that in the worst case A_d can deviate by approximately $\pm 4\%$ of the nominal value. The common-mode gain, however, undergoes dramatic change because of the significant effect of resistor tolerances on the operation of the difference amplifier in the second stage. Equation (2.19) can be employed to evaluate the worst-case common-mode gain. For our case,

$$A_{cm2} = 0.5[1 - (1 \pm 0.04] = \pm 0.02$$

The common-mode gain of the first stage will remain approximately unity. Thus the $\pm 1\%$ resistor tolerances will mainly affect the common-mode gain of the instrumentation amplifier, increasing it in the worst case to

$$|A_{cm}| = 0.02$$

Correspondingly, the CMRR will be reduced to

$$\text{CMRR} = 20 \log \frac{|A_d|}{|A_{cm}|}$$

$$\simeq 20 \log\left(\frac{21}{0.02}\right) = 60.4 \text{ dB}$$

as opposed to the ideal infinite value!

If $2R_1$ is reduced to $1 \text{ k}\Omega$, A_d increases to 201 V/V while A_{cm} remains unchanged at 0.02 V/V. Thus, CMRR increases to about 80 dB. We conclude that increasing the gain of the first stage increases CMRR.

2.76 Adding a 100-kΩ potentiometer (whose resistance can be set to any desired value R_{1v} in the range of 0 to 100 kΩ) in series with a fixed resistance R_{1f} makes

$$2R_1 = R_{1f} + R_{1v}$$

The resulting differential gain will be

$$A_d = \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_{1f} + R_{1v}}\right)$$

The minimum gain of 2 is obtained with R_{1v}, at its maximum value of 100 kΩ. Thus

$$2 = \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_{1f} + 100}\right) \qquad (1)$$

The maximum gain is obtained with $R_{1v} = 0$. Thus

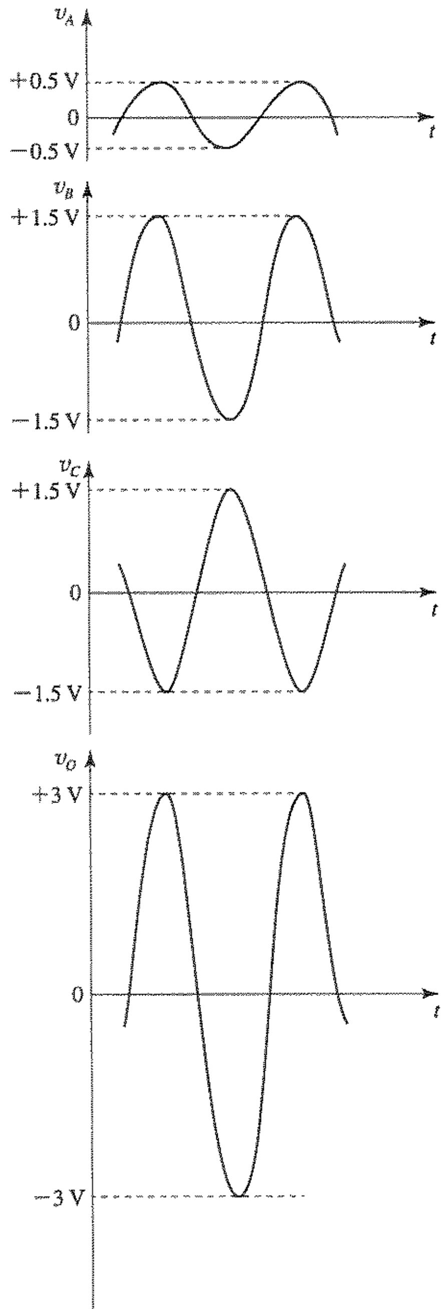
$$1000 = \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_{1f}}\right) \qquad (2)$$

Selecting $R_3 = R_4$ at a convenient value of, say, 10 kΩ, then Eqs. (1) and (2) can be solved to obtain $R_{1f} = 100.2\ \Omega$ and $R_2 = 100.1$ kΩ. Practical values of 100 Ω and 100 kΩ can be used, and the gain range required will be covered.

2.77 (a) $\frac{v_B}{v_A} = 1 + \frac{20}{10} = 3$ V/V

$\frac{v_C}{v_A} = -\frac{30}{10} = -3$ V/V

(b) $v_O = v_B - v_C = 6v_A \Rightarrow \frac{v_O}{v_A} = 6$ V/V



(c) v_B and v_C can be ±14 V, or 28 V P-P.

$-28\text{ V} \le v_O \le 28$ V, or 56 V P-P

$v_{O\text{rms}} = \frac{28}{\sqrt{2}} = 19.8$ V

2.78 See analysis on the circuit diagrams on next page:

Note that circuit (a) has the advantage of infinite input resistance. However, it has the limitation that the load impedance must be floating. This constraint is removed in circuit (b), but the input resistance is finite ($2R_1$).

2.79 $\frac{V_o(s)}{V_i(s)} = -\frac{1}{sCR}$

$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{1}{j\omega CR}$

$\left|\frac{V_o}{V_i}\right| = \frac{1}{\omega CR} \qquad \angle\phi = +90°$

For $C = 1$ nF and $R = 10$ kΩ,

$CR = 1 \times 10^{-9} \times 10 \times 10^4 = 10^{-4}$ s

(a) $|V_o/V_i| = 1$ at $\omega = \frac{1}{CR} = 10^4$ rad/s.

Correspondingly, $f = \frac{10^4}{2\pi} = 1.59$ kHz.

(b) At $f = 1.59$ kHz, the output sine wave leads the input sine wave by 90°.

(c) If the frequency is lowered by a factor of 10, the gain increases by a factor of 10 and, correspondingly, the output voltage increases by a factor of 10.

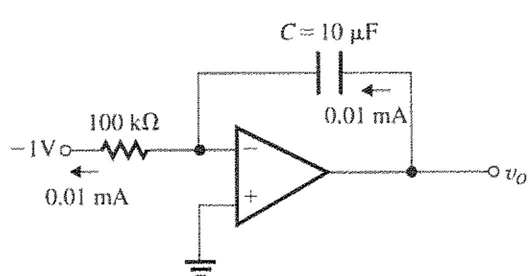
(d) The phase relation between input and output remains unchanged.

2.80 $CR = 1$ s and $R_{in} = 100$ kΩ.

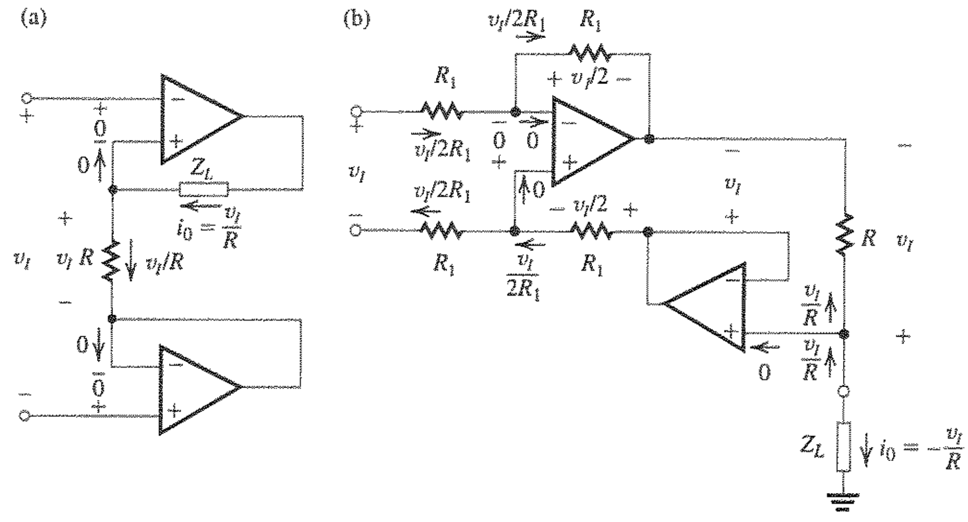
$\Rightarrow R = 100$ kΩ

$C = \frac{1}{100 \times 10^3} = 10\ \mu$F

When a dc voltage of −1 V is applied, a dc current of 1 V/100 kΩ = 0.01 mA will flow as shown in the figure below.



This figure belongs to Problem 2.78.



The capacitor voltage v_O will rise linearly from its initial value of -10 V. Thus,

$$v_O = \frac{It}{C} - 10$$

$$= \frac{10^{-5}t}{10 \times 10^{-6}} - 10 = t - 10, \text{ V}$$

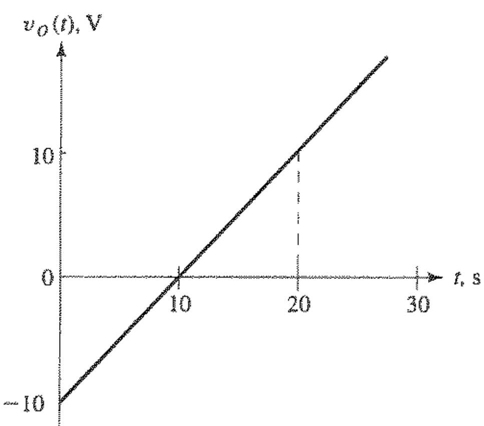
Thus v_O will reach 0 V at

$t = 10$ s

and will reach 10 V at

$t = 20$ s

See figure below.



2.81 $|T| = \frac{1}{\omega RC}$. If $|T| = 100$ V/V at $f = 10$ kHz, then for $|T| = 1$ V/V, f has to be 10 kHz $\times 100 = 1$ MHz.

Also,

$$RC = \frac{1}{\omega_{\text{int}}} = \frac{1}{2\pi \times 1 \text{ MHz}} = 0.159\ \mu\text{s}$$

2.82 $|T| = \frac{1}{\omega RC} = 1$ at $\omega_{\text{int}} = \frac{1}{RC}$.

For $\omega_{\text{int}} = 10$ krad/s, $RC = \frac{1}{10^4} = 10^{-4}$ s. Now $R_{\text{in}} = 100$ kΩ, thus $R = 100$ kΩ and $C = \frac{10^{-4}\text{ s}}{10^5\ \Omega} = 10^{-9}$ F $= 1$ nF. For a 2-V, 100-μs input pulse, a current of $\frac{2\text{ V}}{100\text{ k}\Omega} = 0.02$ mA flows into R and C, causing v_O to decrease linearly from its initial value of 0 V according to

$$v_O = -\frac{I}{C}t$$

$$= \frac{-0.02 \times 10^{-3}}{1 \times 10^{-9}}t = -0.02 \times 10^6 t$$

At $t = 100$ μs, v_O becomes

$$v_O(100\ \mu\text{s}) = -0.02 \times 10^6 \times 100 \times 10^{-6}$$

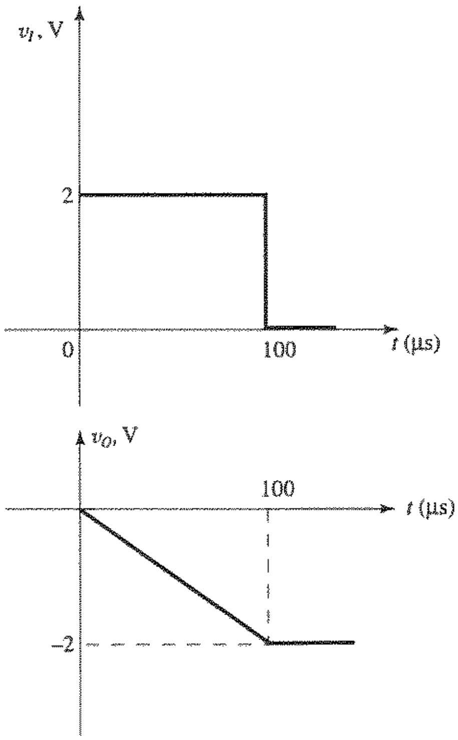
$$= -2 \text{ V}$$

and the output voltage then remains constant at this value. See figures below. When $v_I = 2 \sin 10^4 t$, the output will be a sine wave of the same frequency but phase-shifted by $-270°$ (or $+90°$), and its magnitude will be 2 V $\times$ integrator gain at $\omega = 10^4$ rad/s. The gain is given by

$$|T| = \frac{1}{\omega RC} = \frac{1}{10^4 \times 10^{-4}} = 1$$

Thus,

$$v_O = -2 \sin(10^4 t - 90°)$$



2.83 $2\pi \times 100 \times 10^3 = \dfrac{1}{CR} \Rightarrow CR = 1.59\ \mu s$

For $R_{in} = 10\ k\Omega$, $R = 10\ k\Omega$, and

$$C = \frac{1.59 \times 10^{-6}}{10^4} = 159\ pF$$

To limit the dc gain to 40 dB (i.e., 100 V/V), we connect a resistance R_F across C (as in Fig. 2.25) with $R_F = 100\ R = 1\ M\Omega$.

The resulting low-pass filter will have a 3-dB frequency of

$$f_{3dB} = \frac{1}{2\pi CR_F} = \frac{1}{2\pi \times 159 \times 10^{-12} \times 10^6}$$

$$= 1\ kHz$$

When a 10-μs, 1-V pulse (see Fig. 1) is applied at the input, a current of $1\ V/10\ k\Omega = 0.1$ mA flows into the integrator. Now we consider two cases: with and without R_F.

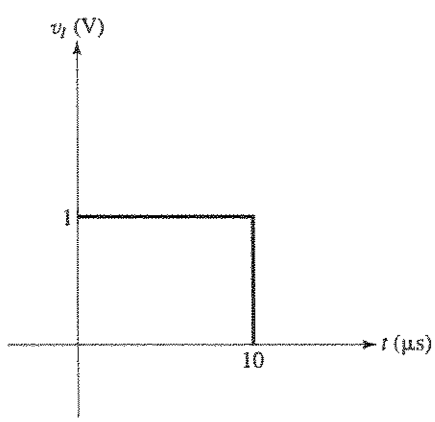


Figure 1

(a) For an integrator without R_F, the 0.1-mA current flows through C and the output voltage decreases linearly from 0 V as

$$v_O(t) = -\frac{It}{C}$$

$$= -\frac{0.1 \times 10^{-3}}{159 \times 10^{-12}}t$$

$$= -0.63 \times 10^6 t,\ V$$

At the end of the pulse, $t = 10\ \mu s$, resulting in

$$v_O(10\ \mu s) = -0.63 \times 10^6 \times 10 \times 10^{-6}$$

$$= -6.3\ V$$

See Fig. 2.

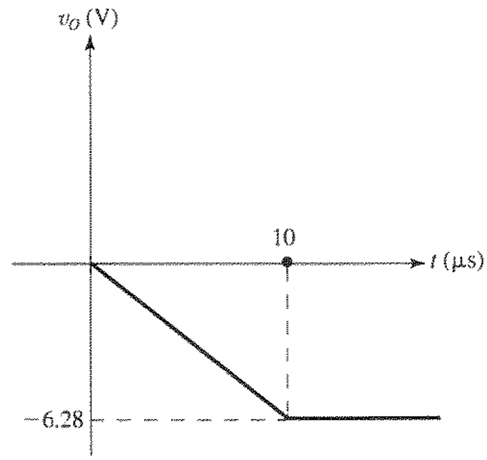


Figure 2

(b) For an integrator with R_F, the 0.1-mA current flows through the parallel combination of C and R_F. The result is

$$v_O(t) = v_{O\text{final}} - (v_{O\text{final}} - v_{O\text{initial}})e^{-t/\tau}$$

where

$$v_{O\text{final}} = -IR_F = -0.1 \times 10^{-3} \times 10^6 = -100\ V$$

$$v_{O\text{initial}} = 0$$

$$\tau = CR_F = 159 \times 10^{-12} \times 10^6 = 159\ \mu s$$

Thus,

$$v_O(t) = -100(1 - e^{-t/159}),\ V$$

where t is in μs. At the end of the pulse, $t = 10\ \mu s$,

$$v_O(10\ \mu s) = -100(1 - e^{-10/159}) = -6.1\ V$$

Beyond $t = 10\ \mu s$, the capacitor discharges through R_F. Thus, including R_F results in the nonideal integrator response shown in Fig. 3.

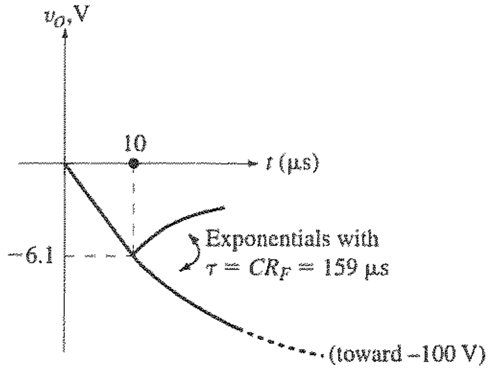


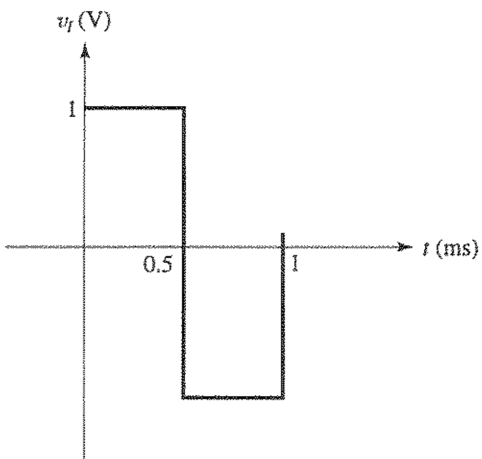
Figure 3

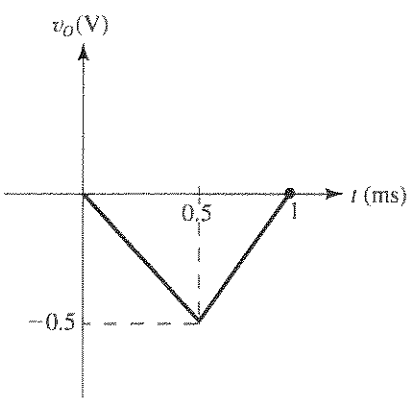
2.84 For $0 \le t \le 0.5$ ms:

$$v_O(t) = 0 - \frac{1}{RC}\int_0^t v_I dt$$

$$v_O(t) = 0 - \frac{t}{RC} = -\frac{t}{1 \text{ ms}}$$

$$v_O(0.5 \text{ ms}) = -0.5 \text{ V}$$





for $0.5 \le t \le 1$ ms:

$$v_O(t) = v_O(0.5 \text{ ms}) - \frac{1}{RC}\int_{0.5}^{t} -1\, dt$$

$$v_O(t) = -0.5 + \frac{1}{RC}(t - 0.5)$$

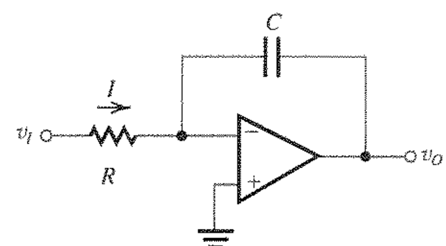
$$v_O(1 \text{ ms}) = -0.5 + \frac{0.5}{1} = 0 \text{ V}$$

Another way of thinking about this circuit is as follows:

for $0 \le t \le 0.5$ ms, a current $I = \frac{1 \text{ V}}{R}$ flows through R and C in the direction indicated on the diagram. At time t we write:

$$I \cdot t = -Cv_O(t) \Rightarrow v_O(t) = \frac{-I}{C}t = \frac{-1}{RC}t$$

which indicates that the output voltage is linearly decreasing, reaching –0.5 V at $t = 0.5$ ms.

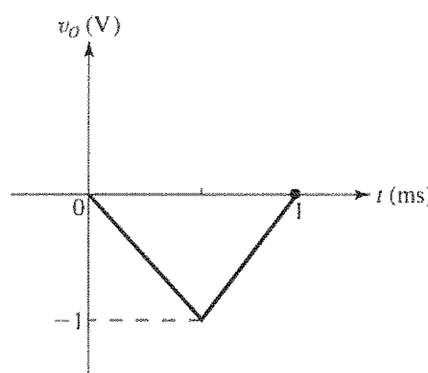


Then for $0.5 \le t \le 1$ ms, the current flows in the opposite direction, v_O rises linearly, reaching 0 at $t = 1$ ms.

For $v_I = \pm 2$ V:

We obtain the following waveform (assuming time constant is the same).

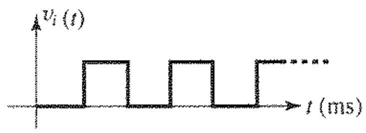
If CR is also doubled, then the waveform becomes the same as the first case.

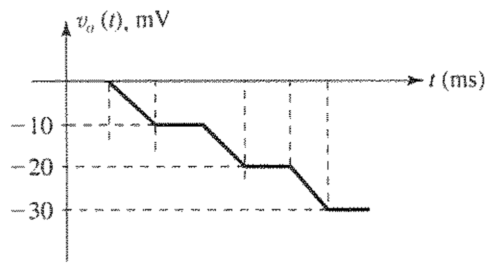


2.85 Each pulse provides a constant current of $\frac{1 \text{ V}}{R}$ through the capacitor and thus deposits a charge of $\frac{1 \text{ V}}{R} \times 10$ μs on the capacitor, resulting in a change of the output voltage of

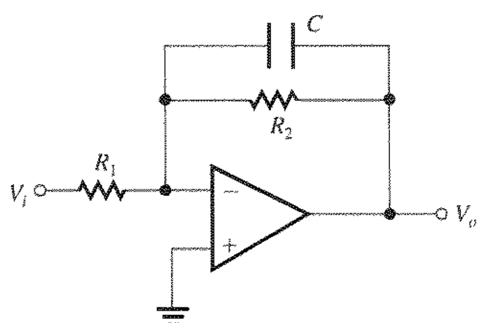
$$-\frac{1 \times 10 \times 10^{-6}}{RC} = -\frac{10^{-5}}{10^{-3}} = -0.01 \text{ V}$$

Therefore a total of 100 pulses are required to cause a change of -1 V in $v_O(t)$.





2.86



Let $Z_2 = R_2 \parallel \frac{1}{sC}$ and $Z_1 = R_1$

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{Y_1}{Y_2} = -\frac{1/R_1}{\frac{1}{R_2} + sC}$$

$$= -\frac{(R_2/R_1)}{1 + sCR_2}$$

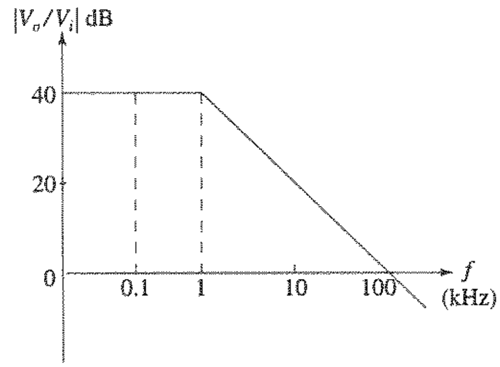
This function is of the STC low-pass type, having a dc gain of $-\frac{R_2}{R_1}$ and a 3-dB frequency

$$\omega_0 = \frac{1}{CR_2}$$

$R_{in} = R_1 = 10 \text{ k}\Omega$

dc gain $= 40$ dB $= 100$

$$\therefore\ 100 = \frac{R_2}{R_1} \Rightarrow R_2 = 100R_1 = 1 \text{ M}\Omega$$



3-dB frequency at 1 kHz

$$\therefore\ \omega_0 = 2\pi \times 1 \times 10^3 = \frac{1}{CR_2}$$

$$C = \frac{1}{2\pi \times 1 \times 10^3 \times 10^6} = 0.16 \text{ nF}$$

From the Bode plot shown above, the unity-gain frequency is 100 kHz.

2.87 Equation (2.5) can be generalized as follows:

$$\frac{V_o}{V_i} = -\frac{Z_2/Z_1}{1 + \frac{1 + Z_2/Z_1}{A}}$$

For $Z_1 = R$, $Z_2 = 1/sC$, and $A = A_0$,

$$\frac{V_o}{V_i} = -\frac{1/sCR}{1 + \frac{1}{A_0} + \frac{1}{sA_0CR}}$$

$$= -\frac{1}{CR(1 + \frac{1}{A_0})}\ \frac{1}{s + \frac{1}{(A_0 + 1)CR}}$$

$$= -\frac{A_0/[(A_0 + 1)CR]}{s + \frac{1}{(A_0 + 1)CR}}$$

which is low-pass STC function. The pole (or 3-dB) frequency is

$$\omega_P = \frac{1}{(A_0 + 1)CR}$$

The ideal integrator has $\omega_P = 0$. Observe that as $A_0 \rightarrow \infty$, $\omega_P \rightarrow 0$. The dc gain is $-A_0$, which is the dc gain of the op amp.

If an ideal Miller integrator is fed with a −1-V pulse signal of width $T = CR$, the output voltage can be found as follows: The −1-V pulse will cause a current $I = 1\text{ V}/R$ to be drawn through R and C. The capacitor voltage, which is v_O, will rise linearly according to

$$v_O = \frac{1}{C}It = \frac{1}{CR}t$$

Thus, at $t = T$ (the end of the pulse) the output voltage reaches 1 V and then stays constant at this value.

If the integrator is made with an op amp having a finite $A_0 = 1000$, the response to the −1-V step will be that of an STC low-pass circuit. Thus,

$$v_O = v_{O\text{final}} - (v_{O\text{final}} - v_{O\text{initial}})e^{-t/\tau}$$

where

$$v_{O\text{final}} = -1\text{ V} \times \text{dc gain}$$

$$= -1\text{ V} \times -1000$$

$$= 1000\text{ V}$$

$$v_{O\text{initial}} = 0\text{ V}$$

$$\tau = \frac{1}{\omega_P} = (A_0 + 1)CR = 1001CR$$

Thus,

$$v_O = 1000(1 - e^{-t/1000CR})$$

At $t = T$, which is equal to CR,

$$v_O(T) = 1000(1 - e^{-0.001})$$

$$= -0.9995 \simeq -1\ V$$

2.88 $\dfrac{V_o}{V_i} = -sCR = -j\omega CR, \quad \left|\dfrac{V_o}{V_i}\right| = \omega CR$

For $R = 10\text{ k}\Omega$ and $C = 1\text{ nF}$,

$$CR = 1 \times 10^{-9} \times 10 \times 10^{3} = 10\ \mu\text{s}$$

$$\frac{V_o}{V_i} = 1 \text{ at } \omega = \omega_0 = \frac{1}{CR} = \frac{1}{10 \times 10^{-6}}$$

$$= 100\text{ krad/s}$$

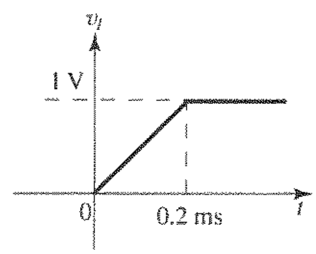
$$\text{or } f_0 = \frac{100}{2\pi} = 15.9\text{ kHz}$$

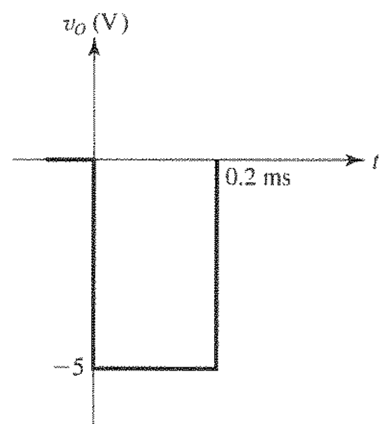
$$\left|\frac{V_o}{V_i}\right| = \omega CR = \frac{\omega}{\omega_0} = \frac{f}{f_0}$$

For $f = 10 f_0$: $\left|\dfrac{V_o}{V_i}\right| = 10$, and the output sine wave will have 10-V peak-to-peak amplitude. The $(-j\omega)$ factor in the transfer function means inversion and +90° phase shift, thus

$$v_O = -5\sin(10^6 t + 90^\circ),\text{ V}$$

2.89





$$v_O = -CR\frac{dv_I}{dt}$$

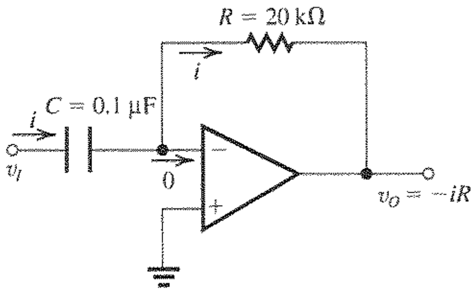
Therefore:

For $0 \le t \le 0.2$ ms:

$$v_O = -1\text{ ms} \times \frac{1\text{ V}}{0.2\text{ ms}} = -5\text{ V}$$

and $v_O = 0$ otherwise.

2.90



$$i = C\frac{dv_I}{dt} = 0.1 \times 10^{-6} \times \text{slope}$$

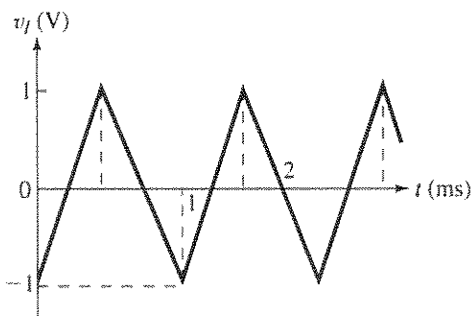
$$= 0.1 \times 10^{-6} \times \frac{2}{0.5 \times 10^{-3}}$$

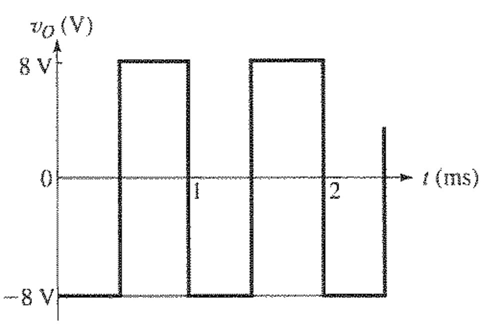
$$= 0.4\ \text{mA}$$

The peak value of the output square wave in $= iR$

$$= 0.4\ \text{mA} \times 20\ \text{k}\Omega$$

$$= 8\ \text{V}$$





The output wave has the same frequency as the input signal.

The average value of the output is zero.

To increase the value of the output to $12\ \text{V} = 8\ \text{V} \times 1.5$, the value of R has to be increased by 1.5 times:

$$20\ \text{k}\Omega \times 1.5 = 30\ \text{k}\Omega$$

2.91 $\dfrac{V_o}{V_i} = -sCR = -10^{-3}\ \text{s}$

$$C = 10\ \text{nF} = 10^{-8}\ \text{F}$$

$$R = \frac{10^{-3}\ \text{s}}{10^{-8}\ \text{F}} = 100\ \text{k}\Omega$$

$$\frac{V_o}{V_i} = -j10^{-3}\omega = -j\frac{\omega}{10^3}$$

$$\left|\frac{V_o}{V_i}\right| = \frac{\omega}{10^3}$$

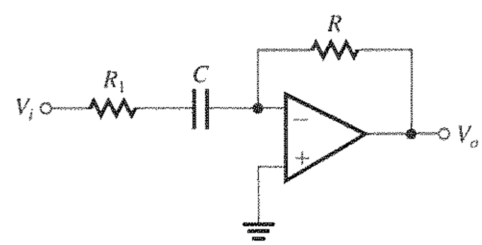
$\phi = 180^\circ + 90^\circ$ (an inversion + a phase lead of 90°)

ω_0 (unity-gain frequency) $= 10^3$ rad/s

At $\omega = 0.1\ \omega_0$, $\left|\dfrac{V_o}{V_i}\right| = 0.1$ V/V, $\phi = 270^\circ$

At $\omega = 10\ \omega_0$, $\left|\dfrac{V_o}{V_i}\right| = 10$ V/V, $\phi = 270^\circ$

When a series input resistor R_1 is added as shown, then the high-frequency gain is limited to R_2/R_1. Thus,



$$R_1 = \frac{R_2}{100} = \frac{100\ \text{k}\Omega}{100} = 1\ \text{k}\Omega$$

The circuit now has an STC high-pass response with a lower 3-dB frequency

$$\omega_{3\text{dB}} = \frac{1}{CR_1} = \frac{1}{10^{-8} \times 10^3} = 100\ \text{krad/s}$$

$$\frac{V_o}{V_i} = -\frac{100\ s}{s + \omega_{3\text{dB}}} = -\frac{100\ s}{s + 10^5}$$

For $s = j\omega$,

$$\frac{V_o}{V_i} = \frac{-j\omega 100}{j\omega + 10^5} = -j\frac{100}{(10^5/\omega) + j}$$

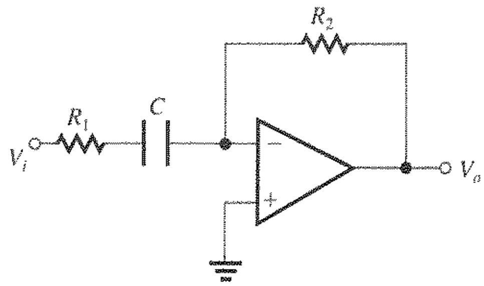
At $\omega = 10^4$ rad/s,

$$\left|\frac{V_o}{V_i}\right| = \frac{100}{\sqrt{100 + 1}} = \frac{10}{\sqrt{1 + 0.01}} \simeq 9.95\ \text{V/V}$$

$$\phi = 180^\circ + 90^\circ - \tan^{-1} 0.1 = 180^\circ + 84.3^\circ$$

Both these results differ slightly from the ideal values.

2.92



$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1 + \dfrac{1}{sC}}$$

Thus,

$$\frac{V_o}{V_i} = -\frac{(R_2/R_1)s}{s + \dfrac{1}{CR_1}}$$

which is that of an STC high-pass type.

High-frequency gain $(s \to \infty) = -\dfrac{R_2}{R_1}$

3-dB frequency $(\omega_{3dB}) = \dfrac{1}{CR_1}$

For a high-frequency input resistance of 1 kΩ, we select $R_1 = 1$ kΩ. For a high-frequency gain of 40 dB,

$$\frac{R_2}{R_1} = 100 \Rightarrow R_2 = 100 \text{ k}\Omega$$

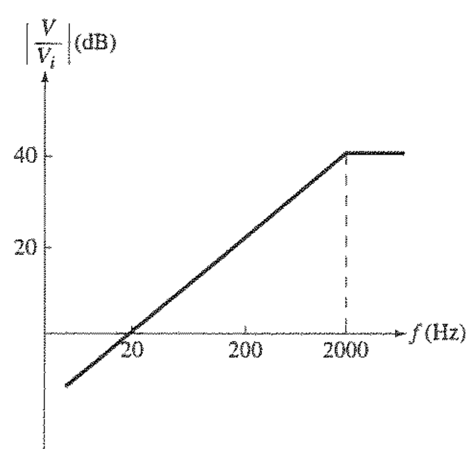
For $f_{3dB} = 2$ kHz,

$$\frac{1}{2\pi CR_1} = 2 \times 10^3$$

$$\Rightarrow C = 79 \text{ nF}$$

The magnitude of the transfer function reduces from 40 dB to unity (0 dB) in two decades. Thus

$$f \text{ (unity gain)} = \frac{f_{3dB}}{100} = \frac{2000}{100} = 20 \text{ Hz}$$



2.93 Refer to the circuit in Fig. P2.93.

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{1}{Z_1 Y_2}$$

$$= -\frac{1}{\left(R_1 + \dfrac{1}{sC_1}\right)\left(\dfrac{1}{R_2} + sC_2\right)}$$

$$= -\frac{R_2/R_1}{\left(1 + \dfrac{1}{sC_1R_1}\right)(1 + sC_2R_2)}$$

$$\frac{V_o}{V_i}(j\omega) = -\frac{R_2/R_1}{[1 + (\omega_1/j\omega)][1 + j(\omega/\omega_2)]}$$

where

$$\omega_1 = \frac{1}{C_1R_1} \quad \text{and} \quad \omega_2 = \frac{1}{C_2R_2} \qquad \text{Q.E.D.}$$

Assuming $\omega_2 \gg \omega_1$, then

(a) For $\omega \ll \omega_1$,

$$\frac{V_o}{V_i} \simeq -\frac{R_2/R_1}{1 + (\omega_1/j\omega)}$$

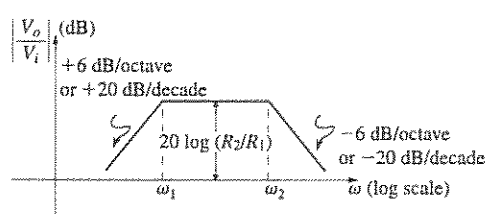
(b) For $\omega_1 \ll \omega \ll \omega_2$

$$\frac{V_o}{V_i} \simeq -(R_2/R_1)$$

(c) For $\omega \gg \omega_2$

$$\frac{V_o}{V_i} \simeq -\frac{R_2/R_1}{1 + j(\omega/\omega_2)}$$

The resulting Bode plot will be as shown:



Design: Gain of 40 dB $\Rightarrow \dfrac{R_2}{R_1} = 100$

$$f_1 = 200 \text{ Hz} \Rightarrow \frac{1}{2\pi C_1 R_1} = 200$$

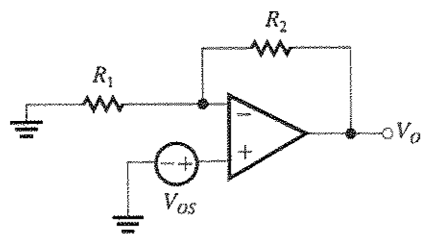
$$f_2 = 200 \text{ kHz} \Rightarrow \frac{1}{2\pi C_2 R_2} = 200 \times 10^3$$

Input resistance (at $\omega \gg \omega_1$) = 2 kΩ

$\Rightarrow R_1 = 2$ kΩ

Thus, $R_1 = 2$ kΩ, $R_2 = 200$ kΩ, $C_1 \simeq 0.4$ μF, and $C_2 \simeq 4$ pF.

2.94 Inverting configuration:



$$V_O = V_{OS}\left(1 + \frac{R_2}{R_1}\right)$$

$$-0.2 = V_{OS}\left(1 + \frac{R_2}{R_1}\right)$$

$$= V_{OS}\left(1 + \frac{100}{2}\right)$$

$\Rightarrow V_{OS} \simeq 4$ mV

2.95 $V_{OS} = \pm 2$ mV

$V_O = 0.01 \sin \omega t \times 100 + V_{OS} \times 100$

$= 1 \sin \omega t \pm 0.2$ V

2.96 Input offset voltage $= 3$ mV

Output dc offset voltage
$= 3$ mV $\times$ closed loop gain

$= 3$ mV $\times$ 1000

$= 3$ V

The maximum amplitude of an input sinusoid that results in an output peak amplitude of $12 - 3 = 9$ V is given by

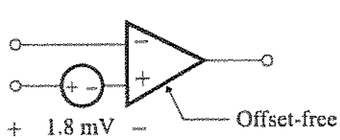
$$v_i = \frac{9}{1000} = 9 \text{ mV}$$

If amplifier is capacitively coupled then

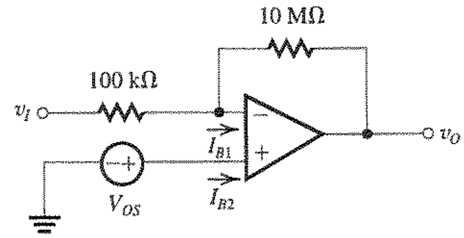
$$v_{i\,\max} = \frac{12}{1000} = 12 \text{ mV}$$

2.97 $V_{OS} = \dfrac{-1.8 \text{ V}}{1001} \simeq -1.8$ mV

Thus,



2.98



(a) $I_B = (I_{B1} + I_{B2})/2$

Open input:

$v_O = v_+ + R_2 I_{B1} = V_{OS} + R_2 I_{B1}$

$5.3 = V_{OS} + 10{,}000 I_{B1}$ (1)

Input connected to ground:

$$v_O = v_+ + R_2\left(I_{B1} + \frac{V_{OS}}{R_1}\right)$$

$$= V_{OS}\left(1 + \frac{R_2}{R_1}\right) + R_2 I_{B1}$$

$5 = V_{OS} \times 101 + 10{,}000 I_{B1}$ (2)

Equations (1), (2)

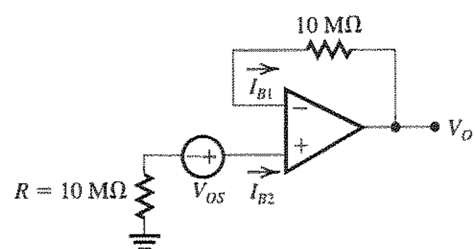
$\Rightarrow 100 V_{OS} = -0.3 \Rightarrow V_{OS} = -3$ mV

$\Rightarrow I_{B1} = 530$ nA

$I_B \simeq I_{B1} = 530$ nA

and both flow into the op-amp input terminals.

(b) $V_{OS} = -3$ mV



(c) In this case, Since R is very large, we may ignore V_{OS} compared to the voltage drop across R.

$V_{OS} \ll R I_B$. Also Eq. (2.46) holds:

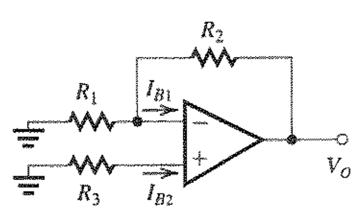
$R_3 = R_1 \parallel R_2$

therefore from Eq. (2.40):

$$V_O = I_{OS} \times R_2 \Rightarrow I_{OS} = \frac{-0.6}{10 \text{ M}\Omega}$$

$I_{OS} = -60$ nA

2.99



$R_2 = 100\ \text{k}\Omega$

$R_1 = \dfrac{100\ \text{k}\Omega}{9}$

$R_3 = 5\ \text{k}\Omega$

$I_{B1} = 2 \pm 0.1,\ \mu\text{A},\ V_{OS} = 0$

$I_{B2} = 2 \pm 0.1,\ \mu\text{A}$

From Eq. (2.38):

$$V_O = -I_{B2}R_3 + R_2\left(I_{B1} - I_{B2}\frac{R_3}{R_1}\right)$$

Thus,

$$V_O = I_{B1}R_2 - I_{B2}R_3\left(1 + \frac{R_2}{R_1}\right) \qquad (1)$$

The maximum value of V_O is obtained when $I_{B1} = 2.1\ \mu\text{A}$ and $I_{B2} = 1.9\ \mu\text{A}$,

$$V_{O\text{max}} = 2.1 \times 100 - 1.9 \times 5\left(1 + \frac{100}{100/9}\right)$$

$= 210 - 95 = 115$ mV

The minimum value of V_O is obtained when $I_{B1} = 1.9\ \mu\text{A}$ and $I_{B2} = 2.1\ \mu\text{A}$,

$$V_{O\text{min}} = 1.9 \times 100 - 2.1 \times 5 \times 10$$

$= 190 - 105 = 85$ mV

Thus the dc offset at the output will be in the range of 85 mV to 115 mV. The bulk of the dc offset at the output, that due to I_B, can be reduced to zero by making the dc resistances seen by the two input terminals equal. Currently, the positive input terminal sees a resistance $R_3 = 5\ \text{k}\Omega$ and the negative input terminal sees a resistance equal to $R_1 \parallel R_2 = \frac{100}{9} \parallel 100 = 10\ \text{k}\Omega$. Thus the two resistances can be made equal by connecting a 5-kΩ resistance in series with R_3. The resulting dc offset voltage at the output will be

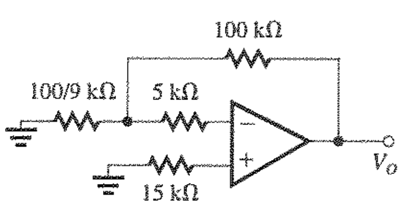
$$V_O = I_{OS}R_2 = 0.2 \times 100 = 20\ \text{mV}$$

Since I_{OS} can be of either polarity,

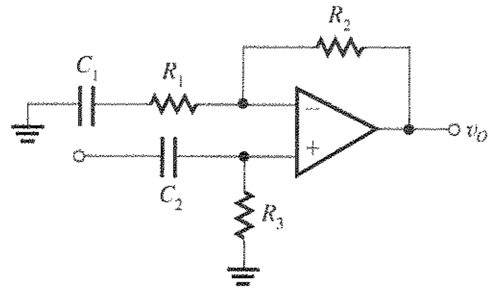
$V_O = \pm 20$ mV

The same result could have been found by replacing R_3 in Eq. (1) by $(R_3 + R_4)$ where $R_4 = 5\ \text{k}\Omega$.

If the signal source resistance is 15 kΩ, then the resistances can be equalized by adding a 5-kΩ resistor in series with the negative input lead of the op amp.



2.100



$R_2 = R_3 = 100\ \text{k}\Omega$

$1 + \dfrac{R_2}{R_1} = 100$

$R_1 = \dfrac{100\ \text{k}\Omega}{99} = 1.01\ \text{k}\Omega$

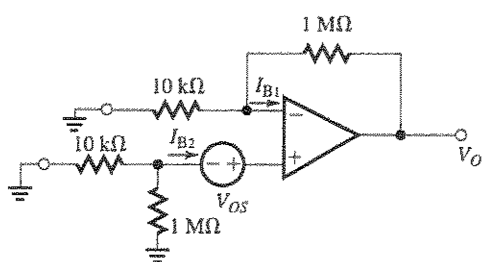
$\dfrac{1}{R_1C_1} = 2\pi \times 100 \Rightarrow C_1 = \dfrac{1}{1.01 \times 2\pi \times 10^5}$

$= 1.58\ \mu\text{F}$

$\dfrac{1}{R_3C_2} = 2\pi \times 10 \Rightarrow C_2 = \dfrac{1}{2\pi \times 10^6}$

$= 0.16\ \mu\text{F}$

2.101



The component of V_O due to V_{OS} is

$$V_{O1} = V_{OS}\left(1 + \frac{1\ \text{M}\Omega}{10\ \text{k}\Omega}\right)$$

$= 5(1 + 100) = 505$ mV

Since the two op-amp input terminals see equal resistances of $(10\ \text{k}\Omega \parallel 1\ \text{M}\Omega)$, the input bias current I_B will not result in a dc voltage at the output; however, the input offset current I_{OS} will give rise to a dc output offset of

$V_{O2} = I_{OS} \times 1\ \text{M}\Omega$

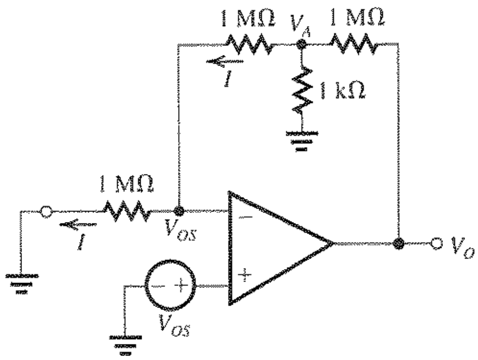
$= 0.2\ \mu\text{A} \times 1\ \text{M}\Omega = 0.2$ V

Thus,

$V_O = V_{O1} + V_{O2} = 0.505 + 0.2$

$= 0.705$ V

2.102



$v_- = v_+ = V_{OS}$

$V_A = 2V_{OS} = 6$ mV

$I = \dfrac{V_{OS}}{1\text{ M}\Omega} = \dfrac{6\text{ mV}}{1\text{ M}\Omega} = 6$ nA

$V_O = V_A + 1\text{ M}\Omega \times \left(I + \dfrac{V_A}{1\text{ k}\Omega}\right)$

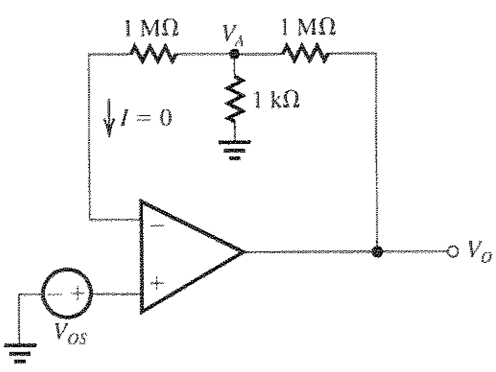
$= 2\,V_{OS} + 1\text{ M}\Omega \times \left(\dfrac{V_{OS}}{1\text{ M}\Omega} + \dfrac{2\,V_{OS}}{1\text{ k}\Omega}\right)$

$= 2003\ V_{OS}$

$= 2003 \times 3$ mV

$\cong 6$ V

For capacitively coupled input,



$v_+ = v_- = V_{OS}$

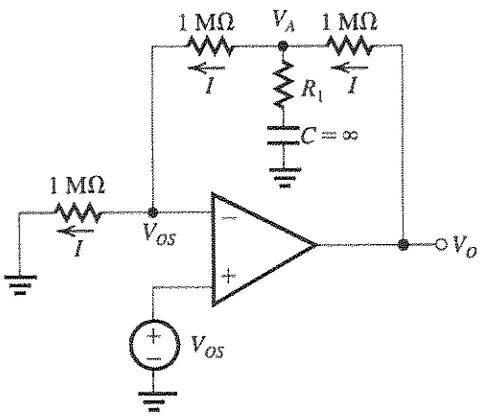
$I = 0 \quad V_A = V_{OS}$

$V_O = V_A + 1\text{ M}\Omega \times \dfrac{V_{OS}}{1\text{ k}\Omega} = V_{OS} + 1000V_{OS}$

$= 1001V_{OS}$

$= 1001 \times 3\text{ mV} \simeq 3$V

A large capacitor placed in series with the 1-kΩ resistor results in



$v_+ = v_- = V_{OS}$

No dc current flows through R_1, C branch

$\therefore V_O = V_A + V_{OS}$

$= 2\ V_{OS} + V_{OS}$

$= 3\ V_{OS}$

$= 3 \times 3$ mV

$= 9$ mV

2.103 At 0°C, we expect

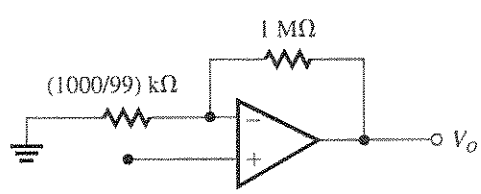
$\pm 20 \times 25 \times 1000\ \mu\text{V} = \pm 500\text{ mV} = \pm 0.5$ V

At 75°C, we expect

$\pm 20 \times 50 \times 1000\ \mu = \pm 1$ V

We expect these quantities to have opposite polarities.

2.104



$100 = 1 + \dfrac{R_2}{R_1} \Rightarrow R_1 = 10.1\text{ k}\Omega$

(a) $V_O = 200 \times 10^{-9} \times 1 \times 10^6 = 0.2$ V

(b) Largest output offset is

$V_O = 2\text{ mV} \times 100 + 0.2\text{ V} = 400\text{ mV} = 0.4$ V

(c) For bias current compensation, we connect a resistor R_3 in series with the positive input terminal of the op amp, with $R_3 = R_1 \parallel R_2$,

$R_3 = 10\text{ k}\Omega \parallel 1\text{ M}\Omega \simeq 10\text{ k}\Omega$

$$I_{OS} = \frac{200}{10} = 20\text{ nA}$$

The offset current alone will result in an output offset voltage of

$I_{OS} \times R_2 = 20 \times 10^{-9} \times 1 \times 10^6 = 20\text{ mV}$

(d) $V_O = 200\text{ mV} + 20\text{ mV} = 220\text{ mV} = 0.22\text{ V}$

2.105 $R_3 = R_1 \parallel R_2 = 10\text{ k}\Omega \parallel 1\text{ M}\Omega = 9.09\text{ k}\Omega$
Now, with the input grounded and assuming $V_{OS} = 0$, the measured $+0.3$-V at the output is entirely due to I_{OS}, that is,

$0.3 = I_{OS}R_2 = I_{OS} \times 1\text{ M}\Omega$

Thus,

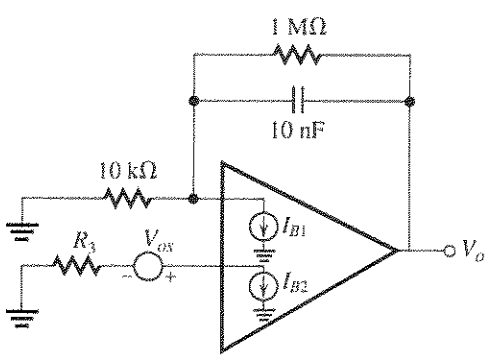
$I_{OS} = 0.3\ \mu\text{A}$

If $V_{OS} = \pm 1$ mV, then it alone will result in an output voltage of

$V_{OS}\left(1 + \frac{R_2}{R_1}\right) = V_{OS} \times 101 = \pm 101\text{ mV}$ or $\simeq \pm 0.1\text{ V}$

If V_{OS} is positive, 0.1 V of the output 0.3-V offset will be due to V_{OS}, leaving 0.2 V as the result of I_{OS}; thus in this case, $I_{OS} = \frac{0.2\text{ V}}{1\text{ M}\Omega} = 0.2\ \mu\text{A}$. On the other hand, if V_{OS} is negative, then -0.1 V of the output 0.3 V is due to V_{OV}, with the result that I_{OS} must be causing 0.4 V of output offset. In this case, $I_{OS} = \frac{0.4\text{ V}}{1\text{ M}\Omega} = 0.4\ \mu\text{A}$. Thus, the possible range of I_{OS} is 0.2 μA to 0.4 μA.

2.106



(a) $R_3 = R \parallel R_F = 10\text{ k}\Omega \parallel 1\text{ M}\Omega$

$\Rightarrow R_3 = 9.9\text{ k}\Omega$

(b) As discussed in Section 2.8.2, the dc output voltage of the integrator when the input is grounded is $V_O = V_{OS}\left(1 + \frac{R_F}{R}\right) + I_{OS}R_F$

$$V_O = 2\text{ mV}\left(1 + \frac{1\text{ M}\Omega}{10\text{ k}\Omega}\right) + 20\text{ nA} \times 1\text{ M}\Omega$$

$= 0.202\text{ V} + 0.02\text{ V}$

$V_O = 0.222\text{ V}$

2.107 $f_t = A_0 f_b$

A_0	f_b (Hz)	f_t (Hz)
10^5	10^2	10^7
10^6	1	10^6
10^5	10^3	10^8
10^7	10^{-1}	10^6
2×10^5	10	2×10^6

2.108 At very low frequencies, the gain is A_0, thus

$20 \log A_0 = 98\text{ dB} \Rightarrow A_0 \simeq 80{,}000$ V/V

At $f = 100$ kHz, the gain is 40 dB or 100 V/V. Thus

$f_t = 100\text{ kHz} \times 100 = 10\text{ MHz}$

$$\text{Since } f_t = A_0 f_b \Rightarrow f_b = \frac{10\text{ MHz}}{80{,}000} = 125\text{ Hz}.$$

2.109 $f = 10\text{ kHz}$ $|A| = 20 \times 10^3$

$f = 100\text{ kHz}$ $|A| = 4 \times 10^3$

Thus, a change of a decade in f does *not* result in a factor of 10 reduction in gain; in fact, the gain reduces by only a factor of 5. It follows that the first frequency (10 kHz) is less than f_b. Therefore, we must use the exact expression for $|A|$, that is,

$$|A| = \frac{A_0}{\sqrt{1 + (f/f_b)^2}}$$

Substituting the given data, we obtain

$$\frac{A_0}{\sqrt{1 + (10/f_b)^2}} = 20 \times 10^3 \quad (1)$$

$$\frac{A_0}{\sqrt{1 + (100/f_b)^2}} = 4 \times 10^3 \quad (2)$$

Dividing Eq. (1) by Eq. (2), we have

$$\sqrt{\frac{1 + (100/f_b)^2}{1 + (10/f_b)^2}} = 5$$

$$\Rightarrow 1 + \frac{100^2}{f_b^2} = 25\left(1 + \frac{10^2}{f_b^2}\right) = 25 + \frac{2500}{f_b^2}$$

$$\frac{10{,}000}{f_b^2} - \frac{2500}{f_b^2} = 24$$

$$f_b = \sqrt{\frac{7500}{24}} = 17.68 \text{ kHz}$$

Now, substituting in Eq. (1) yields

$$A_0 = 20 \times 10^3 \sqrt{1 + \left(\frac{10}{17.68}\right)^2} = 22{,}976 \text{ V/V}$$

and the unity-gain frequency is

$$f_t = A_0 f_b = 22{,}976 \times 10^3 \times 17.68 \times 10^3$$

$$= 406.2 \text{ MHz}$$

2.110 The gain drops by 20 dB at $f \simeq 10 f_b$. Thus

(a) $A_0 = 2 \times 10^5$ V/V

$$f_b = \frac{5 \times 10^2}{10} = 50 \text{ Hz}$$

$$f_t = A_0 f_b = 2 \times 10^5 \times 50 = 10^7 \text{ Hz} = 10 \text{ MHz}$$

(b) $A_0 = 20 \times 10^5$ V/V

$$f_b = \frac{10}{10} = 1 \text{ Hz}$$

$$f_t = A_0 f_b = 20 \times 10^5 \times 1 = 2 \text{ MHz}$$

(c) $A_0 = 1800$ V/V

$$f_b = \frac{0.1 \text{ MHz}}{10} = 10 \text{ kHz}$$

$$f_t = A_0 f_b = 1800 \times 10 = 18 \text{ MHz}$$

(d) $A_0 = 100$ V/V

$$f_b = \frac{0.1 \text{ GHz}}{10} = 10 \text{ MHz}$$

$$f_t = A_0 f_b = 100 \times 10 = 1 \text{ GHz}$$

(e) $A_0 = 25$ V/mV $= 25 \times 10^3$ V/V

$$f_b = \frac{250}{10} = 25 \text{ kHz}$$

$$f_b = A_0 \times f_b = 25 \times 10^3 \times 25 \times 10^3 = 625 \text{ MHz}$$

2.111 $G_{\text{nominal}} = -50 \Rightarrow \dfrac{R_2}{R_1} = 50$

$A_0 = 10^4$

$f_t = 10^6$ Hz

$$f_{3\text{dB}} \text{ of closed-loop amplifier} = \frac{f_t}{1 + \dfrac{R_2}{R_1}}$$

$$= \frac{10^6}{51} = 19.61 \text{ kHz}$$

$$G = -\frac{50}{1 + j\dfrac{f}{f_{3\text{dB}}}}$$

$$|G| = \frac{50}{\sqrt{1 + (f/f_{3\text{dB}})^2}}$$

For $f = 0.1 f_{3\text{dB}}$, $|G| = \dfrac{50}{\sqrt{1.01}} = 49.75$ V/V

For $f = 10 f_{3\text{dB}}$, $|G| = \dfrac{50}{\sqrt{1 + 100}} = 4.975$ V/V

which is a 20-dB reduction.

2.112 $f_t = 20$ MHz and closed-loop gain

$$1 + \frac{R_2}{R_1} = 100 \text{ V/V}$$

$$f_{3\text{dB}} = \frac{f_t}{1 + \dfrac{R_2}{R_1}} = \frac{20 \text{ MHz}}{100} = 200 \text{ kHz}$$

$$G(j\omega) = \frac{100}{1 + j\dfrac{f}{f_{3\text{db}}}}$$

$$\Rightarrow \phi = -\tan^{-1}\frac{f}{f_{3\text{dB}}}$$

For $\phi = -6°$

$f = f_{3\text{dB}} \times \tan 6° = 21$ kHz

$\phi = 84°$, $f = f_{3\text{dB}} \times \tan 84° = 1.9$ MHz

2.113 (a) $G = -50$ V/V $\Rightarrow \dfrac{R_2}{R_1} = 50$

$$f_{3\text{dB}} = \frac{f_t}{1 + \dfrac{R_2}{R_1}} = \frac{f_t}{51}$$

For $f_{3\text{dB}} = 100$ kHz $\Rightarrow f_t = 100 \times 51 = 5.1$ MHz

(b) $G = +50$ V/V $\Rightarrow 1 + \dfrac{R_2}{R_1} = 50$

$$f_{3\text{dB}} = \frac{f_t}{1 + \dfrac{R_2}{R_1}} = \frac{f_t}{50}$$

For $f_{3\text{dB}} = 100$ kHz, $f_t = 5$ MHz

(c) $G = +2$ V/V $\Rightarrow 1 + \dfrac{R_2}{R_1} = 2$

$$f_{3\text{dB}} = \frac{f_t}{1 + \dfrac{R_2}{R_1}} = \frac{f_t}{2}$$

For $f_{3\text{dB}} = 5$ MHz, $f_t = 10$ MHz

(d) $G = -2$ V/V $\Rightarrow \dfrac{R_2}{R_1} = 2$

$$f_{3\text{dB}} = \frac{f_t}{1 + \dfrac{R_2}{R_1}} = \frac{f_t}{3}$$

For $f_{3\text{dB}} = 5$ MHz, $f_t = 15$ MHz

(e) $G = -1000$ V/V $\Rightarrow \frac{R_2}{R_1} = 1000$

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{f_t}{1001}$$

For $f_{3dB} = 10$ kHz, $f_t = 10 \times 1001 = 10.1$ MHz

(f) $G = +1$ V/V $\Rightarrow \frac{R_2}{R_1} = 0$

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = f_t$$

For $f_{3dB} = 1$ MHz, $f_t = 1$ MHz

(g) $G = -1$ V/V $\Rightarrow \frac{R_2}{R_1} = 1$

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{f_t}{2}$$

For $f_{3dB} = 1$ MHz, $f_t = 2$ MHz

2.114 Gain $= 1 + \frac{R_2}{R_1} = 96$ V/V

$f_{3dB} = 8$ kHz

$f_t = 96 \times 8 = 768$ kHz

for $f_{3dB} = 32$ kHz

$$\text{Gain} = \frac{768}{32} = 24 \text{ V/V}$$

2.115 $f_{3dB} = f_t = 1$ MHz

$$|G| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3dB}}\right)^2}} = \frac{1}{\sqrt{1 + f^2}} \text{ with}$$

f in MHz

$|G| = 0.99 \Rightarrow f = 0.142$ MHz

The follower behaves like a low-pass STC circuit with a time constant $\tau = \frac{1}{2\pi \times 10^6} = \frac{1}{2\pi}$ μs

$t_r = 2.2\tau = 0.35$ μs.

2.116 $1 + \frac{R_2}{R_1} = 10 \Rightarrow R_1 = 1$ kΩ and $R_2 = 9$ kΩ

When a 100-mV (i.e., 0.1-V) step is applied at the input, the output will be

$v_O = 0.1 \times 10(1 - e^{-t/\tau})$, V

where

$$\tau = \frac{1}{\omega_{3dB}}$$

v_O reaches 1% of the 1-V final value at time t,

$1 - e^{-t/\tau} = 0.99$

$e^{-t/\tau} = 0.01$

$t = 4.6\tau$

For t to be 200 ns,

$$\tau = \frac{200}{4.6} = 43.49 \text{ ns}$$

Thus we require a closed-loop 3-dB frequency

$\omega_{3dB} = \frac{1}{\tau}$ or

$$f_{3dB} = \frac{1}{2\pi\tau} = \frac{1}{2\pi \times 43.49 \times 10^{-9}} = 3.66 \text{ MHz}$$

Correspondingly, the op amp must have an f_t of

$$f_t = f_{3dB}\left(1 + \frac{R_2}{R_1}\right) = 36.6 \text{ MHz}$$

2.117 (a) Assume two identical stages, each with a gain function:

$$G = \frac{G_0}{1 + j\frac{\omega}{\omega_1}} = \frac{G_0}{1 + \frac{jf}{f_1}}$$

$$G = \frac{G_0}{\sqrt{1 + \left(\frac{f}{f_1}\right)^2}}$$

overall gain of the cascade is $\frac{G_0^2}{1 + \left(\frac{f}{f_1}\right)^2}$

The gain will drop by 3 dB when

$$1 + \left(\frac{f_{3dB}}{f_1}\right)^2 = \sqrt{2}$$

$f_{3dB} = f_1\sqrt{\sqrt{2} - 1}$ Q.E.D

(b) $40 \text{ dB} = 20 \log G_0 \Rightarrow G_0 = 100 = 1 + \frac{R_2}{R_1}$

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{2 \text{ MHz}}{100} = 20 \text{ kHz}$$

(c) Each stage should have 20-dB gain or $1 + \frac{R_2}{R_1} = 10$ and therefore a 3-dB frequency of

$$f_1 = \frac{2 \times 10^6}{10} = 2 \times 10^5 \text{ Hz}$$

The overall $f_{3dB} = 2 \times 10^5\sqrt{\sqrt{2} - 1}$

$= 128.7$ kHz,

which is 6.4 times greater than the bandwidth achieved using a single op amp, as in case (b) above.

2.118 $f_t = 100 \times 5 = 500$ MHz if a single op amp is used.

With an op amp that has only $f_t = 40$ MHz, the possible closed-loop gain at 5 MHz is

$$|A| = \frac{40}{5} = 8 \text{ V/V}$$

To obtain an overall gain of 100 would require three such amplifiers, cascaded. Now, if each of the stages has a low-frequency (dc) closed-loop gain K, then its 3-dB frequency will be $\frac{40}{K}$ MHz.

Thus for each stage the closed-loop gain is:

$$|G| = \frac{K}{\sqrt{1 + \left(\frac{f}{\frac{40}{K}}\right)^2}}$$

which at $f = 5$ MHz becomes

$$|G_{5\text{MHz}}| = \frac{K}{\sqrt{1 + \left(\frac{K}{8}\right)^2}}$$

For overall gain of 100: $100 = \left[\frac{K}{\sqrt{1 + \left(\frac{K}{8}\right)^2}}\right]^3$

$\Rightarrow K = 5.7$

Thus for each cascade stage: $f_{3\text{dB}} = \frac{40}{5.7}$

$f_{3\text{dB}} = 7$ MHz

The 3-dB frequency of the overall amplifier, f_1, can be calculated as

$$\left[\frac{5.7}{\sqrt{1 + \left(\frac{f}{7}\right)^2}}\right]^3 = \frac{(5.7)^3}{\sqrt{2}} \Rightarrow f_1 = 3.6 \text{ MHz}$$

2.119 (a) $\frac{R_2}{R_1} = K, \quad f_{3\text{dB}} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{f_t}{1 + K}$

GBP = Gain $\times f_{3\text{dB}}$

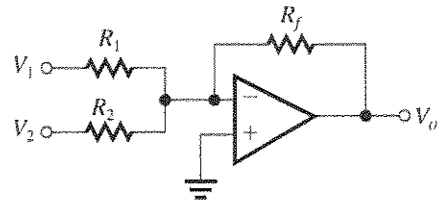
$$\text{GBP} = K\frac{f_t}{1+K} = \frac{K}{K+1}f_t$$

(b) $1 + \frac{R_2}{R_1} = K, \quad f_{3\text{dB}} = \frac{f_t}{K}$

$$\text{GBP} = K\frac{f_t}{K} = f_t$$

For the same closed-loop gain, the noninverting configuration realizes a higher GBP, and it is independent of the closed-loop gain and equal to f_t of the op amp.

2.120



Use superposition to evaluate $f_{3\text{dB}}$ for each case

Set $V_2 = 0, \qquad V_o = -V_1 \Rightarrow \frac{R_f}{R_1} = 1$

$$f_{3\text{dB}} = \frac{f_t}{1 + \frac{R_f}{R_1}} = \frac{f_t}{2}$$

Set $V_1 = 0, \qquad V_o = -3V_2 \Rightarrow \frac{R_f}{R_2} = 3$

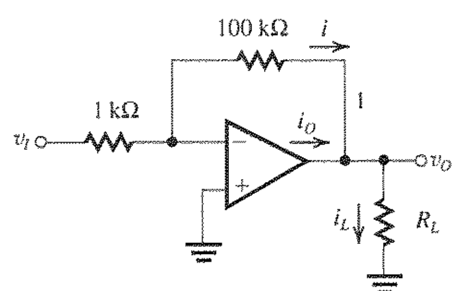
$$f_{3\text{dB}} = \frac{f_t}{1 + \frac{R_f}{R_2}} = \frac{f_t}{1+3}$$

$$= \frac{f_t}{4}$$

2.121 The peak value of the largest possible sine wave that can be applied at the input without output clipping is $\frac{\pm 14 \text{ V}}{100} = 0.14 \text{ V} = 140$ mV.

Thus the rms value $= \frac{140}{\sqrt{2}} \simeq 100$ mV

2.122



(a) $R_L = 1$ kΩ

for $v_{O\max} = 10$ V: $V_p = \frac{10}{100}$

$V_p = 0.1$ V

When the output is at its peak,

$i_L = \frac{10}{1 \text{ k}\Omega} = 10$ mA

$i = \frac{-10}{100 \text{ k}\Omega} = -0.1$ mA; therefore

$i_O = 10 + 0.1 = 10.1$ mA is well under $i_{O\max} = 20$ mA.

At the negative peak of the output voltage, $v_O = -10$ V, $i_L = -10$ mA, $i = 0.1$ mA, and $i_O = -10.1$ mA, again well under the 20-mA maximum allowed.

(b) $R_L = 200\ \Omega$

If output is at its peak: $i_L = \dfrac{10\text{ V}}{0.2} = 50$ mA

which exceeds $i_{O\max} = 20$ mA. Therefore v_O cannot go as high as 10 V. Instead:

$$20\text{ mA} = \frac{v_O}{200\ \Omega} + \frac{v_O}{100\text{ k}\Omega} \Rightarrow v_O = \frac{20}{5.01} \simeq 4\text{ V}$$

$$v_p = \frac{4}{100} = 0.04\text{ V} = 40\text{ mV}$$

(c) $R_L = ?,\ i_{O\max} = 20\text{ mA} = \dfrac{10\text{ V}}{R_{L\min}} + \dfrac{10\text{ V}}{100\text{ k}\Omega}$

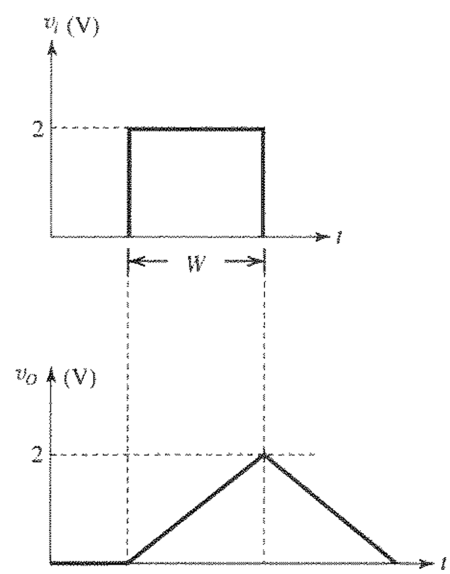
$$20 - 0.1 = \frac{10}{R_{L\min}} \Rightarrow R_{L\min} = 502\ \Omega$$

2.123 Op-amp slew rate = 10 V/μs.

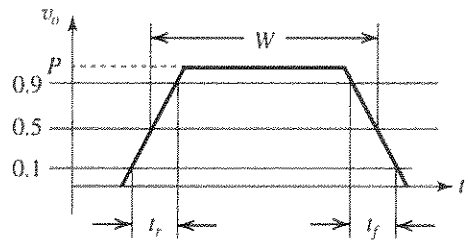
For the input pulse to rise 2 V, it will take $\dfrac{2}{10} = 0.2$ μs.

∴ The minimum pulse width = $W = 0.2$ μs

The output will be a triangular with 2-V peak and 10 V/μs slopes.



2.124



$W = 2$ μs

$t_r + t_f = 0.2\text{ W} = 0.4$ μs

$t_r = t_f = 0.2$ μs

$$\text{SR} = \frac{(0.9 - 0.1)P}{t_r} = \frac{0.8 \times 10}{0.2} = 40\text{ V/μs}$$

2.125 Slope of the triangle wave $= \dfrac{10\text{ V}}{T/2} = \text{SR}$

Thus $\dfrac{10}{T} \times 2 = 20$ V/μs

$\Rightarrow T = 1$ μs or $f = \dfrac{1}{T} = 1$ MHz

For a sine wave $v_O = \hat{V}_o \sin(2\pi \times 1 \times 10^6 t)$

$$\left.\frac{dv_O}{dt}\right|_{\max} = 2\pi \times 1 \times 10^6\ \hat{V}_o = \text{SR}$$

$$\Rightarrow \hat{V}_o = \frac{20 \times 10^6}{2\pi \times 10^6 \times 1} = 3.18\text{ V}$$

2.126 $v_O = 10\ \sin \omega t \Rightarrow \dfrac{dv_O}{dt}$

$= 10\omega \cos \omega t \Rightarrow \left.\dfrac{dv_O}{dt}\right|_{\max}$

$= 10\omega$

The highest frequency at which this output is possible is that for which

$\left.\dfrac{dv_O}{dt}\right|_{\max} = \text{SR} \Rightarrow 10\omega_{\max} = 40 \times 10^{+6} \Rightarrow \omega_{\max}$

$= 4 \times 10^6$ rad/s

$\Rightarrow f_{\max} = 637$ kHz

2.127 (a) $V_i = 0.5,\ V_o = 10 \times 0.5 = 5$ V

Output distortion will be due to slew-rate limitation and will occur at the frequency for which $\left.\dfrac{dv_O}{dt}\right|_{\max} = \text{SR}$

$\omega_{max} \times 5 = 10 \times 10^6$

$\omega_{max} = 2 \times 10^6$ rad/s and $f_{max} = 318.3$ kHz

(b) The output will distort at the value of V_i that results in $\left.\frac{dv_O}{dt}\right|_{max} = \text{SR}$.

$\omega \times 10V_{imax} = \text{SR}$

$$V_{imax} = \frac{10 \times 10^6}{2\pi \times 200 \times 10^3 \times 10} = 0.795 \text{ V}$$

(c) $V_i = 50$ mV $\quad V_o = 500$ mV $= 0.5$ V

Slew rate begins at the frequency for which $\omega \times 0.5 = \text{SR}$

$$\Rightarrow f = \frac{10 \times 10^6}{2\pi \times 0.5} = 3.18 \text{ MHz}$$

However, the small-signal 3-dB frequency is

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{20 \times 10^6}{10} = 2 \text{ MHz}$$

Thus the useful frequency range is limited to 2 MHz.

(d) For $f = 50$ kHz, the slew-rate limitation occurs at the value of V_i given by

$$\omega_i \times 10V_i = \text{SR} \Rightarrow V_i = \frac{10 \times 10^6}{2\pi \times 50 \times 10^3 \times 10}$$

$= 3.18$ V

Such an input voltage, however, would ideally result in an output of 31.8 V, which exceeds V_{Omax}.

Thus $V_{imax} = \frac{V_{Omax}}{10} = 1$ V peak.

3.1 Use the expression in Eq. (3.2), with

$B = 7.3 \times 10^{15}\ \text{cm}^{-3}\text{K}^{-3/2}$;

$k = 8.62 \times 10^{-5}\text{eV/K}$; and $E_g = 1.12$ V

we have

$T = -55°\text{C} = 218$ K:

$n_i = 2.68 \times 10^6\ \text{cm}^{-3}$; $\dfrac{N}{n_i} = 1.9 \times 10^{16}$

That is, one out of every 1.9×10^{16} silicon atoms is ionized at this temperature.

$T = 0°\text{C} = 273$ K:

$n_i = 1.52 \times 10^9\ \text{cm}^{-3}$; $\dfrac{N}{n_i} = 3.3 \times 10^{13}$

$T = 20°\text{C} = 293$ K:

$n_i = 8.60 \times 10^9\ \text{cm}^{-3}$; $\dfrac{N}{n_i} = 5.8 \times 10^{12}$

$T = 75°\text{C} = 348$ K:

$n_i = 3.70 \times 10^{11}\ \text{cm}^{-3}$; $\dfrac{N}{n_i} = 1.4 \times 10^{11}$

$T = 125°\text{C} = 398$ K:

$n_i = 4.72 \times 10^{12}\ \text{cm}^{-3}$; $\dfrac{N}{n_i} = 1.1 \times 10^{10}$

3.2 Use Eq. (3.2) to find n_i,

$n_i = BT^{3/2}\, e^{-E_g/2kT}$

Substituting the values given in the problem,

$n_i = 3.56 \times 10^{14}(300)^{3/2}e^{-1.42/(2\times 8.62\times 10^{-5}\times 300)}$

$= 2.2 \times 10^6\ \text{carriers/cm}^3$

3.3 Since $N_A \gg n_i$, we can write

$p_p \approx N_A = 5 \times 10^{18}\ \text{cm}^{-3}$

Using Eq. (3.3), we have

$n_p = \dfrac{n_i^2}{p_p} = 45\ \text{cm}^{-3}$

3.4 Hole concentration in intrinsic Si $= n_i$

$n_i = BT^{3/2}\, e^{-E_g/2kT}$

$= 7.3 \times 10^{15}(300)^{3/2}e^{-1.12/(2\times 8.62\times 10^{-5}\times 300)}$

$= 1.5 \times 10^{10}\ \text{holes/cm}^3$

In phosphorus-doped Si, hole concentration drops below the intrinsic level by a factor of 10^8.

∴ Hole concentration in P-doped Si is

$p_n = \dfrac{1.5 \times 10^{10}}{10^8} = 1.5 \times 10^2\ \text{cm}^{-3}$

Now, $n_n \simeq N_D$ and $p_n n_n = n_i^2$

$n_n = n_i^2/p_n = \dfrac{(1.5 \times 10^{10})^2}{1.5 \times 10^2}$

$= 1.5 \times 10^{18}\ \text{cm}^{-3}$

$N_D = n_n = 1.5 \times 10^{18}\ \text{atoms/cm}^3$

3.5 $T = 27°\text{C} = 273 + 27 = 300$ K

At 300 K, $n_i = 1.5 \times 10^{10}/\text{cm}^3$

Phosphorus-doped Si:

$n_n \simeq N_D = 10^{17}/\text{cm}^3$

$p_n = \dfrac{n_i^2}{N_D} = \dfrac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3/\text{cm}^3$

Hole concentration $= p_n = 2.25 \times 10^3/\text{cm}^3$

$T = 125°\text{C} = 273 + 125 = 398$ K

At 398 K, $n_i = BT^{3/2}e^{-E_g/2kT}$

$= 7.3 \times 10^{15} \times (398)^{3/2}e^{-1.12/(2\times 8.62\times 10^{-5}\times 398)}$

$= 4.72 \times 10^{12}/\text{cm}^3$

$p_n = \dfrac{n_i^2}{N_D} = 2.23 \times 10^8/\text{cm}^3$

At 398 K, hole concentration is

$p_n = 2.23 \times 10^8/\text{cm}^3$

3.6 (a) The resistivity of silicon is given by Eq. (3.17).

For intrinsic silicon,

$p = n = n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$

Using $\mu_n = 1350\ \text{cm}^2/\text{V}\cdot\text{s}$ and

$\mu_p = 480\ \text{cm}^2/\text{V}\cdot\text{s}$, and $q = 1.6 \times 10^{-19}$ C we have

$\rho = 2.28 \times 10^5\ \Omega\text{-cm}$.

Using $R = \rho \cdot \dfrac{L}{A}$ with $L = 0.001$ cm and

$A = 3 \times 10^{-8}\ \text{cm}^2$, we have

$R = 7.6 \times 10^9\ \Omega$.

(b) $n_n \approx N_D = 5 \times 10^{16}\ \text{cm}^{-3}$;

$p_n = \dfrac{n_i^2}{n_n} = 4.5 \times 10^3\ \text{cm}^{-3}$

Using $\mu_n = 1200\ \text{cm}^2/\text{V}\cdot\text{s}$ and

$\mu_p = 400\ \text{cm}^2/\text{V}\cdot\text{s}$, we have

$\rho = 0.10\ \Omega\text{-cm}$; $R = 3.33\ \text{k}\Omega$.

(c) $n_n \approx N_D = 5 \times 10^{18}\ \text{cm}^{-3}$;

$p_n = \dfrac{n_i^2}{n_n} = 45\ \text{cm}^{-3}$

Using $\mu_n = 1200\ \text{cm}^2/\text{V}\cdot\text{s}$ and $\mu_p = 400\ \text{cm}^2/\text{V}\cdot\text{s}$, we have

$\rho = 1.0 \times 10^{-3}\ \Omega\text{-cm}$; $R = 33.3\ \Omega$.

As expected, since N_D is increased by 100, the resistivity decreases by the same factor.

(d) $p_p \approx N_A = 5 \times 10^{16}\ \text{cm}^{-3}$; $n_p = \dfrac{n_i^2}{n_n}$

$= 4.5 \times 10^3\ \text{cm}^{-3}$

$\rho = 0.31\ \Omega\text{-cm}$; $R = 10.42\ \text{k}\Omega$

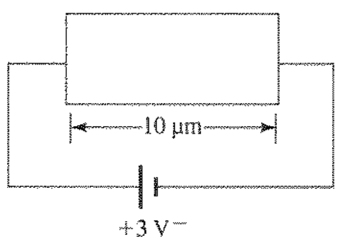
(e) Since ρ is given to be $2.8 \times 10^{-6}\ \Omega\text{-cm}$, we directly calculate $R = 9.33 \times 10^{-2}\ \Omega$.

3.7 Electric field:

$$E = \frac{3\text{ V}}{10\ \mu\text{m}} = \frac{3\text{ V}}{10 \times 10^{-6}\text{ m}}$$

$$= \frac{3\text{ V}}{10 \times 10^{-4}\text{ cm}}$$

$= 3000\ \text{V/cm}$



$v_{p\text{-drift}} = \mu_p E = 480 \times 3000$

$= 1.44 \times 10^6\ \text{cm/s}$

$v_{n\text{-drift}} = \mu_n E = 1350 \times 3000$

$= 4.05 \times 10^6\ \text{cm/s}$

$$\frac{v_n}{v_p} = \frac{4.05 \times 10^6}{1.44 \times 10^6} = 2.8125 \quad \text{or}$$

$v_n = 2.8125\ v_p$

Or, alternatively, it can be shown as

$$\frac{v_n}{v_p} = \frac{\mu_n E}{\mu_p E} = \frac{\mu_n}{\mu_p} = \frac{1350}{480}$$

$= 2.8125$

3.8 Cross-sectional area of Si bar

$= 5 \times 4 = 20\ \mu\text{m}^2$

Since $1\ \mu\text{m} = 10^{-4}$ cm, we get

$= 20 \times 10^{-8}\ \text{cm}^2$

Current $I = Aq(p\mu_p + n\mu_n)E$

$= 20 \times 10^{-8} \times 1.6 \times 10^{-19}$

$$(10^{16} \times 500 + 10^4 \times 1200) \times \frac{1\text{ V}}{10 \times 10^{-4}}$$

$= 160\ \mu\text{A}$

3.9 $J_{\text{drift}} = q(n\mu_n + p\mu_p)E$

Here $n = N_D$, and since it is n-type silicon, one can assume $p \ll n$ and ignore the term $p\mu_p$. Also,

$$E = \frac{1\text{ V}}{10\ \mu\text{m}} = \frac{1\text{ V}}{10 \times 10^{-4}\text{ cm}} = 10^3\ \text{V/cm}$$

Need $J_{\text{drift}} = 2\ \text{mA}/\mu\text{m}^2 = qN_D\mu_n E$

$$\frac{2 \times 10^{-3}\text{ A}}{10^{-8}\text{ cm}^2} = 1.6 \times 10^{-19}\, N_D \times 1350 \times 10^3$$

$\Rightarrow N_D = 9.26 \times 10^{17}/\text{cm}^3$

3.10

$$p_{n0} = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4/\text{cm}^3$$

From Fig. P3.10,

$$\frac{dp}{dx} = -\frac{10^8 p_{n0} - p_{n0}}{W} \simeq -\frac{10^8 p_{n0}}{50 \times 10^{-7}}$$

since 1 nm $= 10^{-7}$ cm

$$\frac{dp}{dx} = -\frac{10^8 \times 2.25 \times 10^4}{50 \times 10^{-7}}$$

$= -4.5 \times 10^{17}$

Hence

$$J_p = -qD_p\frac{dp}{dx}$$

$= -1.6 \times 10^{-19} \times 12 \times (-4.5 \times 10^{17})$

$= 0.864\ \text{A/cm}^2$

3.11 Use Eq. (3.21): $\dfrac{D_n}{\mu_n} = \dfrac{D_p}{\mu_p} = V_T$

$D_n = \mu_n V_T$ and $D_p = \mu_p V_T$ where $V_T = 25.9$ mV.

Doping Concentration (carriers/cm^3)	μ_n cm^2/V·s	μ_p cm^2/V·s	D_n cm^2/s	D_p cm^2/s
Intrinsic	1350	480	35	12.4
10^{16}	1200	400	31	10.4
10^{17}	750	260	19.4	6.7
10^{18}	380	160	9.8	4.1

3.12 Using Eq. (3.22) and $N_A = N_D$

$= 5 \times 10^{16}\ \text{cm}^{-3}$ and $n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$,

we have $V_0 = 778$ mV.

Using Eq. (3.26) and $\epsilon_s = 11.7 \times 8.854 \times 10^{-14}$ F/cm, we have $W = 2 \times 10^{-5}\ \text{cm} = 0.2\ \mu\text{m}$. The extension of the depletion width into the n and p regions is given in Eqs. (3.27) and (3.28), respectively:

$$x_n = W \cdot \frac{N_A}{N_A + N_D} = 0.1\ \mu\text{m}$$

$$x_p = W \cdot \frac{N_D}{N_A + N_D} = 0.1\ \mu\text{m}$$

Since both regions are doped equally, the depletion region is symmetric.

Using Eq. (3.29) and $A = 20\ \mu\text{m}^2 = 20 \times 10^{-8}\ \text{cm}^2$, the charge magnitude on each side of the junction is

$Q_J = 1.6 \times 10^{-14}$ C.

3.13 From Table 3.1,

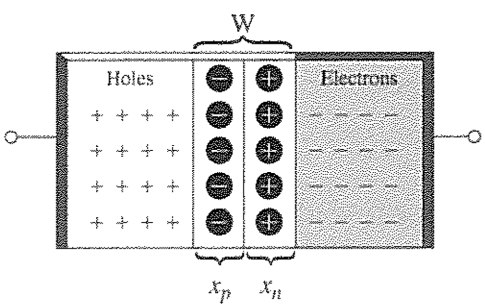
V_T at 300 K = 25.9 mV

Using Eq. (3.22), built-in voltage V_0 is obtained:

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 25.9 \times 10^{-3} \times$$

$$\ln\left(\frac{10^{17} \times 10^{16}}{(1.5 \times 10^{10})^2}\right)$$

$= 0.754$ V



Depletion width

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0} \leftarrow \text{Eq. (3.26)}$$

$W =$

$$\sqrt{\frac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}}\left(\frac{1}{10^{17}} + \frac{1}{10^{16}}\right) \times 0.754}$$

$= 0.328 \times 10^{-4}\ \text{cm} = 0.328\ \mu\text{m}$

Use Eqs. (3.27) and (3.28) to find x_n and x_p:

$$x_n = W\frac{N_A}{N_A + N_D} = 0.328 \times \frac{10^{17}}{10^{17} + 10^{16}}$$

$= 0.298\ \mu\text{m}$

$$x_p = W\frac{N_D}{N_A + N_D} = 0.328 \times \frac{10^{16}}{10^{17} + 10^{16}}$$

$= 0.03\ \mu\text{m}$

Use Eq. (3.29) to calculate charge stored on either side:

$$Q_J = Aq\left(\frac{N_A N_D}{N_A + N_D}\right)W, \text{ where junction area}$$

$= 100\ \mu\text{m}^2 = 100 \times 10^{-8}\ \text{cm}^2$

$$Q_J = 100 \times 10^{-8} \times 1.6 \times 10^{-19}\left(\frac{10^{17} \cdot 10^{16}}{10^{17} + 10^{16}}\right)$$

$$\times\ 0.328 \times 10^{-4}$$

Hence, $Q_J = 4.8 \times 10^{-14}$ C

3.14 Using Eq. (3.23) or (3.24), we have charge stored: $Q_J = qAx_nN_D$.

Here $x_n = 0.1\ \mu\text{m} = 0.1 \times 10^{-4}$ cm

$A = 10\ \mu\text{m} \times 10\ \mu\text{m} = 10 \times 10^{-4}$ cm

$\times\ 10 \times 10^{-4}$ cm

$= 100 \times 10^{-8}\ \text{cm}^2$

So,

$Q_J = 1.6 \times 10^{-19} \times 100 \times 10^{-8} \times 0.1 \times 10^{-4} \times 10^{18}$

$= 1.6$ pC

3.15 Equation (3.26):

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0},$$

Since $N_A \gg N_D$, we have

$$W \simeq \sqrt{\frac{2\epsilon_s}{q}\frac{1}{N_D}V_0}$$

$$V_0 = \frac{qN_D}{2\epsilon_s} \cdot W^2$$

Here $W = 0.2\ \mu\text{m} = 0.2 \times 10^{-4}$ cm

$$\text{So } V_0 = \frac{1.6 \times 10^{-19} \times 10^{16} \times (0.2 \times 10^{-4})^2}{2 \times 1.04 \times 10^{-12}}$$

$= 0.31$ V

$$Q_J = Aq\left(\frac{N_A N_D}{N_A + N_D}\right)W \cong AqN_DW$$

since $N_A \gg N_D$, we have $Q_J = 3.2$ fC.

3.16 $V_0 = V_T \ln\left(\dfrac{N_A N_D}{n_i^2}\right)$

If N_A or N_D is increased by a factor of 10, then new value of V_0 will be

$$V_0 = V_T \ln\left(\frac{10\,N_A N_D}{n_i^2}\right)$$

The change in the value of V_0 is $V_T \ln 10 = 59.6$ mV.

3.17 Using Eq. (3.22) with $N_A = 10^{17}$ cm^{-3}, $N_D = 10^{16}$ cm^{-3}, and $n_i = 1.5 \times 10^{10}$, we have $V_0 = 754$ mV

Using Eq. (3.31) with $V_R = 5$ V, we have $W = 0.907$ µm.

Using Eq. (3.32) with $A = 1 \times 10^{-6}$ cm^2, we have $Q_J = 13.2 \times 10^{-14}$ C.

3.18 Equation (3.31):

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$$

$$= \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0\left(1 + \frac{V_R}{V_0}\right)}$$

$$= W_0\sqrt{1 + \frac{V_R}{V_0}}$$

Equation (3.32):

$$Q_j = A\sqrt{2\epsilon_s q\left(\frac{N_A N_D}{N_A + N_D}\right) \cdot (V_0 + V_R)}$$

$$= A\sqrt{2\epsilon_s q\left(\frac{N_A N_D}{N_A + N_D}\right) V_0 \cdot \left(1 + \frac{V_R}{V_0}\right)}$$

$$= Q_{J0}\sqrt{1 + \frac{V_R}{V_0}}$$

3.19 Equation (3.39):

$$I = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)(e^{V/V_T} - 1)$$

Here $I_p = Aqn_i^2 \dfrac{D_p}{L_p N_D}(e^{V/V_T} - 1)$

$$I_n = Aqn_i^2 \frac{D_n}{L_n N_A}(e^{V/V_T} - 1)$$

$$\frac{I_p}{I_n} = \frac{D_p}{D_n} \cdot \frac{L_n}{L_p} \cdot \frac{N_A}{N_D}$$

$$= \frac{10}{20} \times \frac{10}{5} \times \frac{10^{18}}{10^{16}}$$

$$\frac{I_p}{I_n} = 100$$

Now $I = I_p + I_n = 100\, I_n + I_n \equiv 1$ mA

$$I_n = \frac{1}{101} \text{ mA} = 0.0099 \text{ mA}$$

$$I_p = 1 - I_n = 0.9901 \text{ mA}$$

3.20 Equation (3.41):

$$I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$$

$$A = 100\ \mu\text{m}^2 = 100 \times 10^{-8} \text{ cm}^2$$

$$I_S = 100 \times 10^{-8} \times 1.6 \times 10^{-19} \times \left(1.5 \times 10^{10}\right)^2 \left(\frac{10}{5 \times 10^{-4} \times 10^{16}} + \frac{18}{10 \times 10^{-4} \times 10^{17}}\right)$$

$$= 7.85 \times 10^{-17} \text{ A}$$

$$I \cong I_S e^{V/V_T}$$

$$= 7.85 \times 10^{-17} \times e^{750/25.9}$$

$$\cong 0.3 \text{ mA}$$

3.21 $n_i = BT^{3/2}e^{-E_g/2kT}$

At 300 K,

$$n_i = 7.3 \times 10^{15} \times (300)^{3/2} \times e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 300)}$$

$$= 1.4939 \times 10^{10}/\text{cm}^2$$

$$n_i^2 \text{ (at 300 K)} = 2.232 \times 10^{20}$$

At 305 K,

$$n_i = 7.3 \times 10^{15} \times (305)^{3/2} \times e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 305)}$$

$$= 2.152 \times 10^{10}$$

$$n_i^2 \text{ (at 305 K)} = 4.631 \times 10^{20}$$

$$\text{so } \frac{n_i^2 \text{ (at 305 K)}}{n_i^2 \text{ (at 300 K)}} = 2.152$$

Thus I_S approximately doubles for every 5°C rise in temperature.

3.22 Equation (3.39)

$$I = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)(e^{V/V_T} - 1)$$

So $I_p = Aqn_i^2 \dfrac{D_p}{L_p N_D}(e^{V/V_T} - 1)$

$$I_n = Aqn_i^2 \frac{D_n}{L_n N_A}(e^{V/V_T} - 1)$$

For $p^+ - n$ junction $N_A \gg N_D$, thus $I_p \gg I_n$ and

$$I \simeq I_p = Aqn_i^2 \frac{D_p}{L_p N_D}(e^{V/V_T} - 1)$$

For this case using Eq. (3.41):

$$I_S \simeq Aqn_i^2 \frac{D_p}{L_p N_D} = 10^4 \times 10^{-8} \times 1.6 \times 10^{-19} \times \left(1.5 \times 10^{10}\right)^2 \frac{10}{10 \times 10^{-4} \times 10^{17}}$$

$$= 3.6 \times 10^{-16} \text{ A}$$

$$I = I_S\left(e^{V/V_T} - 1\right) = 1.0 \times 10^{-3}$$

$$3.6 \times 10^{-16}\left(e^{V/(25.9 \times 10^{-3})} - 1\right) = 1.0 \times 10^{-3}$$

$$\Rightarrow V = 0.742 \text{ V}$$

3.23 $V_Z = 12$ V

Rated power dissipation of diode = 0.25 W.

If continuous current "I" raises the power dissipation to half the rated value, then

$$12 \text{ V} \times I = \frac{1}{2} \times 0.25 \text{ W}$$

$$I = 10.42 \text{ mA}$$

Since breakdown occurs for only half the time, the breakdown current I can be determined from

$$I \times 12 \times \frac{1}{2} = 0.25 \text{ W}$$

$$\Rightarrow I = 41.7 \text{ mA}$$

3.24 Equation (3.48):

$$C_{j0} = A\sqrt{\left(\frac{\epsilon_s q}{2}\right)\left(\frac{N_A N_D}{N_A + N_D}\right)\left(\frac{1}{V_0}\right)}$$

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= 25.9 \times 10^{-3} \times \ln\left(\frac{10^{17} \times 10^{16}}{(1.5 \times 10^{10})^2}\right)$$

$$= 0.754 \text{ V}$$

$$C_{j0} = 100 \times 10^{-8}$$

$$\sqrt{\left(\frac{1.04 \times 10^{-12} \times 1.6 \times 10^{-19}}{2}\right)\left(\frac{10^{17} \times 10^{16}}{10^{17} + 10^{16}}\right)\frac{1}{0.754}}$$

$$= 31.6 \text{ fF}$$

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}} = \frac{31.6 \text{ fF}}{\sqrt{1 + \frac{3}{0.754}}}$$

$$= 14.16 \text{ fF}$$

3.25 Equation (3.49), $C_j = \dfrac{C_{j0}}{\left(1 + \frac{V_R}{V_0}\right)^m}$

For $V_R = 1$ V, $C_j = \dfrac{0.4 \text{ pF}}{\left(1 + \frac{1}{0.75}\right)^{1/3}}$

$$= 0.3 \text{ pF}$$

For $V_R = 10$ V, $C_j = \dfrac{0.4 \text{ pF}}{\left(1 + \frac{10}{0.75}\right)^{1/3}}$

$$= 0.16 \text{ pF}$$

3.26 Equation (3.43):

$$\alpha = A\sqrt{2\epsilon_s q \frac{N_A N_D}{N_A + N_D}}$$

Equation (3.45):

$$C_j = \frac{\alpha}{2\sqrt{V_0 + V_R}}$$

Substitute for α from Eq. (3.43):

$$C_j = \frac{A\sqrt{2\epsilon_s q \frac{N_A N_D}{N_A + N_D}}}{2\sqrt{V_0 + V_R}} \times \frac{\sqrt{\epsilon_s}}{\sqrt{\epsilon_s}}$$

$$= A\epsilon_s \times \frac{1}{\sqrt{\frac{2\epsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)(V_0 + V_R)}}$$

$$= \epsilon_s A \frac{1}{\sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}}$$

$$= \frac{\epsilon_s A}{W}$$

3.27 Equation (3.57):

$$C_d = \left(\frac{\tau_T}{V_T}\right)I$$

$$5 \text{ pF} = \left(\frac{\tau_T}{25.9 \times 10^{-3}}\right) \times 1 \times 10^{-3}$$

$$\tau_T = 5 \times 10^{-12} \times 25.9$$

$$= 129.5 \text{ ps}$$

For $I = 0.1$ mA:

$$C_d = \left(\frac{\tau_T}{V_T}\right) \times I$$

$$= \left(\frac{129.5 \times 10^{-12}}{25.9 \times 10^{-3}}\right) \times 0.1 \times 10^{-3} = 0.5 \text{ pF}$$

3.28 Equation (3.51):

$$\tau_p = \frac{L_p^2}{D_p} = \frac{(10 \times 10^{-4})^2}{10}$$

(*Note:* 1 μm = 10^{-4} cm.)

$$\tau_p = 100 \text{ ns}$$

$$Q_P = \tau_p I_p \text{ (Eq. 3.52)}$$

$$= 100 \times 10^{-9} \times 0.1 \times 10^{-3}$$

$$= 10 \times 10^{-12} \text{ C}$$

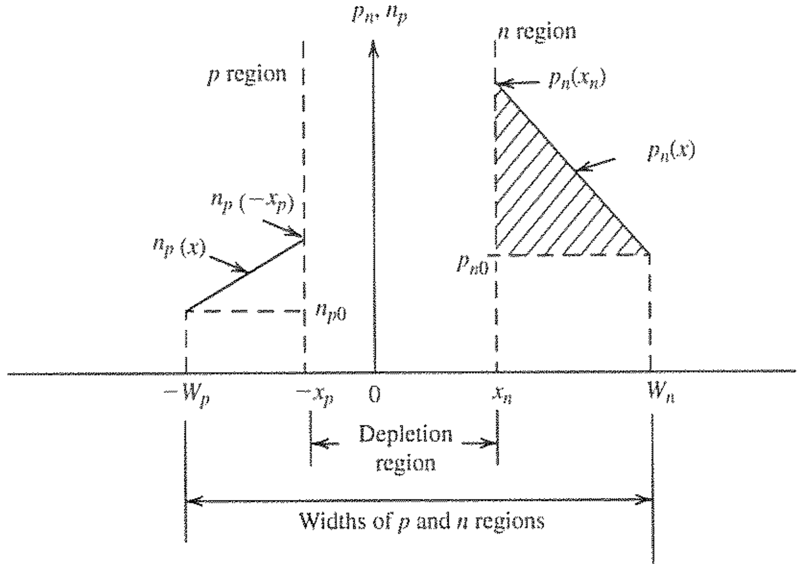
$$C_d = \left(\frac{\tau_p}{V_T}\right)I$$

$$= \left(\frac{100 \times 10^{-9}}{25.9 \times 10^{-3}}\right) \times 0.1 \times 10^{-3}$$

$$= 386 \text{ pF}$$

3.29

(a)



(b) The current $I = I_p + I_n$.

Find current component I_p:

$$p_n(x_n) = p_{n0}e^{V/V_T} \text{ and } p_{n0} = \frac{n_i^2}{N_D}$$

$$I_p = AJ_p = AqD_p\frac{dp}{dx}$$

$$\frac{dp}{dx} = \frac{p_n(x_n) - p_{n0}}{W_n - x_n} = \frac{p_{n0}e^{V/V_T} - p_{n0}}{W_n - x_n}$$

$$= p_{n0}\frac{\left(e^{V/V_T} - 1\right)}{W_n - x_n}$$

$$= \frac{n_i^2}{N_D}\frac{\left(e^{V/V_T} - 1\right)}{(W_n - x_n)}$$

$$\therefore I_p = AqD_p\frac{dp}{dx}$$

$$= Aqn_i^2\frac{D_p}{(W_n - x_n)N_D} \times \left(e^{V/V_T} - 1\right)$$

Similarly,

$$I_n = Aqn_i^2\frac{D_n}{(W_p - x_p)N_A} \times \left(e^{V/V_T} - 1\right)$$

$$I = I_p + I_n$$

$$= Aqn_i^2\left[\frac{D_p}{(W_n - x_n)N_D} + \frac{D_n}{(W_p - x_p)N_A}\right]$$

$$\times \left(e^{V/V_T} - 1\right)$$

The excess change, Q_p, can be obtained by multiplying the area of the shaded triangle of the $p_n(x)$ distibution graph by Aq.

$$Q_p = Aq \times \frac{1}{2}\left[p_n(x_n) - p_{n0}\right](W_n - x_n)$$

$$= \frac{1}{2}Aq\left[p_{n0}e^{V/V_T} - p_{n0}\right](W_n - x_n)$$

$$= \frac{1}{2}Aqp_{n0}\left(e^{V/V_T} - 1\right)(W_n - x_n)$$

$$= \frac{1}{2}Aq\frac{n_i^2}{N_D}(W_n - x_n)\left(e^{V/V_T} - 1\right)$$

$$= \frac{1}{2}\frac{(W_n - x_n)^2}{D_p}\cdot I_p$$

$$\simeq \frac{1}{2}\frac{W_n^2}{D_p}\cdot I_p \text{ for } W_n \gg x_n$$

(c) For $Q \simeq Q_p$, $I \simeq I_p$,

$$Q \simeq \frac{1}{2}\frac{W_n^2}{D_p}I$$

Thus, $\tau_T = \frac{1}{2}\frac{W_n^2}{D_p}$, and

$$C_d = \frac{dQ}{dV} = \tau_T\frac{dI}{dV}$$

But $I = I_S\left(e^{V/V_T} - 1\right)$

$$\frac{dI}{dV} = \frac{I_S e^{V/V_T}}{V_T} \simeq \frac{I}{V_T}$$

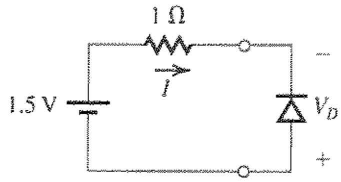
so $C_d \cong \tau_T \cdot \frac{I}{V_T}$.

(d) $C_d = \frac{1}{2}\frac{W_n^2}{10}\frac{1 \times 10^{-3}}{25.9 \times 10^{-3}} = 8 \times 10^{-12}$ F

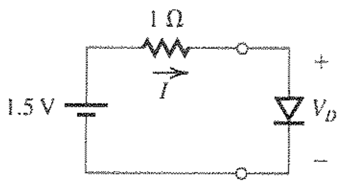
Solve for W_n:

$W_n = 0.64\ \mu$m

4.1



(a)



(b)

(a) The diode is reverse biased, thus

$I = 0$ A

$V_D = -1.5$ V

(b) The diode is forward biased, thus

$V_D = 0$ V

$$I = \frac{1.5 \text{ V}}{1 \ \Omega} = 1.5 \text{ A}$$

4.2 Refer to Fig. P4.2.

(a) Diode is conducting, thus

$V = -3$ V

$$I = \frac{+3-(-3)}{10 \text{ k}\Omega} = 0.6 \text{ mA}$$

(b) Diode is reverse biased, thus

$I = 0$

$V = +3$ V

(c) Diode is conducting, thus

$V = +3$ V

$$I = \frac{+3-(-3)}{10 \text{ k}\Omega} = 0.6 \text{ mA}$$

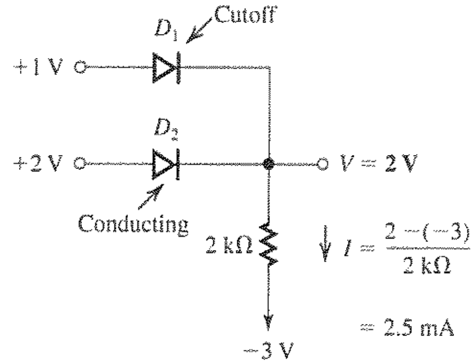
(d) Diode is reverse biased, thus

$I = 0$

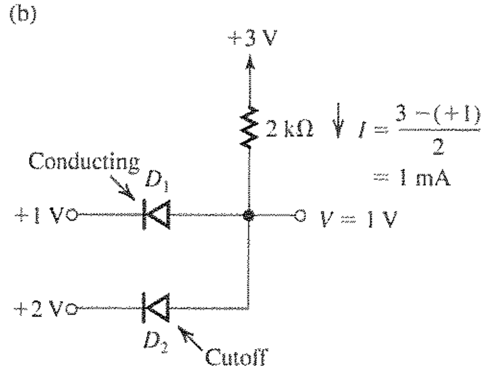
$V = -3$ V

4.3

(a)

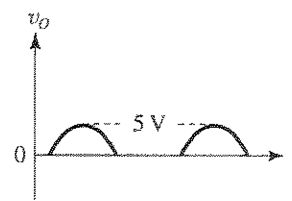


(b)



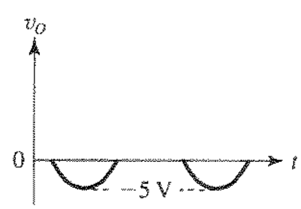
4.4

(a)



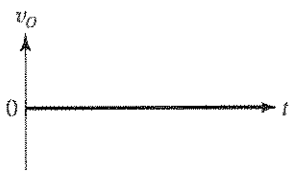
$V_{p+} = 5$ V $\quad V_{p-} = 0$ V $\quad f = 1$ kHz

(b)



$V_{p+} = 0$ V $\quad V_{p-} = -5$ V $\quad f = 1$ kHz

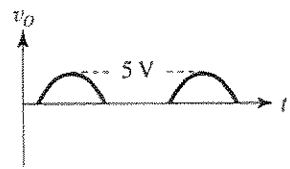
(c)



$v_O = 0$ V

Neither D_1 nor D_2 conducts, so there is no output.

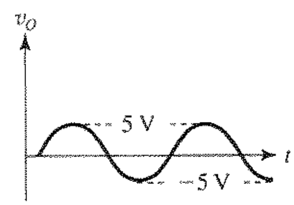
(d)



$V_{p^+} = 5$ V, $V_{p^-} = 0$ V, $f = 1$ kHz

Both D_1 and D_2 conduct when $v_I > 0$

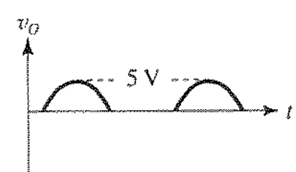
(e)



$V_{p^+} = 5$ V, $V_{p^-} = -5$ V, $f = 1$ kHz

D_1 conducts when $v_I > 0$ and D_2 conducts when $v_I < 0$. Thus the output follows the input.

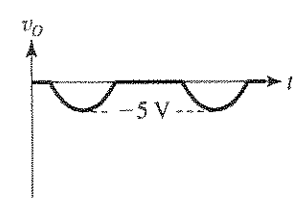
(f)



$V_{p^+} = 5$ V, $V_{p^-} = 0$ V, $f = 1$ kHz

D_1 is cut off when $v_I < 0$

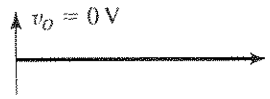
(g)



$V_{p^+} = 0$ V, $V_{p^-} = -5$ V, $f = 1$ kHz

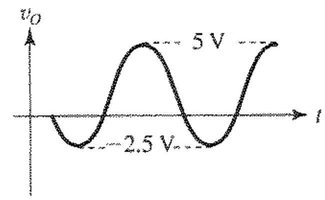
D_1 shorts to ground when $v_I > 0$ and is cut off when $v_I < 0$ whereby the output follows v_I.

(h)



$v_O = 0$ V ~ The output is always shorted to ground as D_1 conducts when $v_I > 0$ and D_2 conducts when $v_I < 0$.

(i)



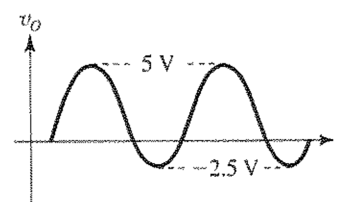
$V_{p^+} = 5$ V, $V_{p^-} = -2.5$ V, $f = 1$ kHz

When $v_I > 0$, D_1 is cut off and v_O follows v_I.

When $v_I < 0$, D_1 is conducting and the circuit becomes a voltage divider where the negative peak is

$$\frac{1\text{ k}\Omega}{1\text{ k}\Omega + 1\text{ k}\Omega} \times -5\text{ V} = -2.5\text{ V}$$

(j)

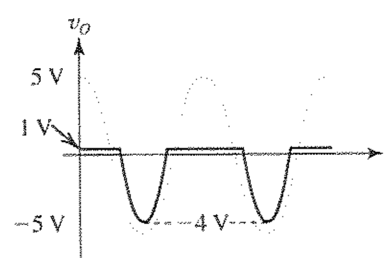


$V_{p^+} = 5$ V, $V_{p^-} = -2.5$ V, $f = 1$ kHz

When $v_I > 0$, the output follows the input as D_1 is conducting.

When $v_I < 0$, D_1 is cut off and the circuit becomes a voltage divider.

(k)



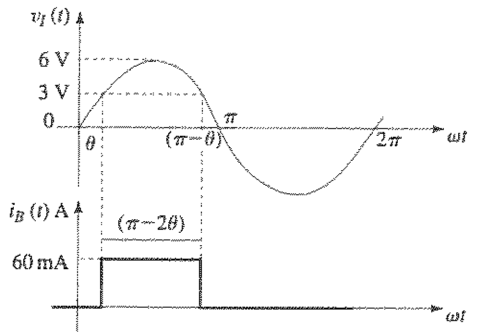
$V_{p^+} = 1$ V, $V_{p^-} = -4$ V, $f = 1$ kH$_3$

When $v_I > 0$, D_1 is cut off and D_2 is conducting. The output becomes 1 V.

When $v_I < 0$, D_1 is conducting and D_2 is cut off. The output becomes:

$v_O = v_I + 1$ V

4.5



From Fig. P4.5 we see that when $v_I < V_B$; that is, $v_I < 3$ V, D_1 will be conducting the current I and i_B will be zero. When v_I exceeds the battery voltage (3 V), D_1 cuts off and D_2 conducts, thus steering I into the battery. Thus, i_B will have the waveform shown in the figure. Its peak value will be 60 mA. To obtain the average value, we first determine the conduction angle of D_2, $(\pi - 2\theta)$, where

$$\theta = \sin^{-1}\left(\frac{3}{6}\right) = 30^\circ$$

Thus

$$\pi - 2\theta = 180^\circ - 60 = 120^\circ$$

The average value of i_B will be

$$i_B|_{av} = \frac{60 \times 120^\circ}{360^\circ} = 20 \text{ mA}$$

If the peak value of v_I is reduced by 10%, i.e. from 6 V to 5.4 V, the peak value of i_B does not change. The conduction angle of D_2, however, changes since θ now becomes

$$\theta = \sin^{-1}\left(\frac{3}{5.4}\right) = 33.75^\circ$$

and thus

$$\pi - 2\theta = 112.5^\circ$$

Thus the average value of i_B becomes

$$i_B|_{av} = \frac{60 \times 112.5^\circ}{360^\circ} = 18.75 \text{ mA}$$

4.6

A	B	X	Y
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

$X = AB$, $Y = A + B$

X and Y are the same for

$A = B$

X and Y are opposite if $A \neq B$

4.7 The case for the highest current in a single diode is when only one input is high:

$V_Y = 5$ V

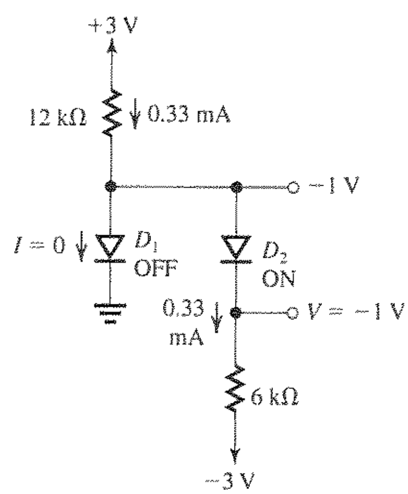
$$\frac{V_Y}{R} \le 0.2 \text{ mA} \Rightarrow R \ge 25 \text{ k}\Omega$$

4.8 The maximum input current occurs when one input is low and the other two are high.

$$\frac{5-0}{R} \le 0.2 \text{ mA}$$

$R \ge 25$ kΩ

4.9

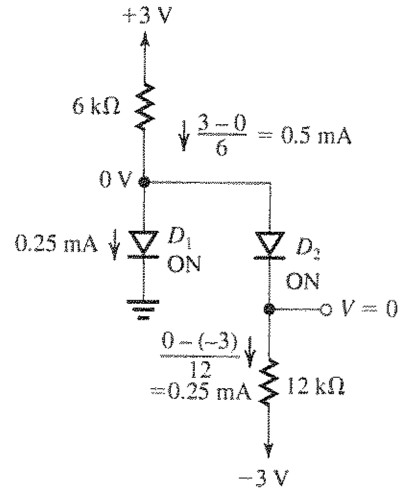


(a)

(a) If we assume that both D_1 and D_2 are conducting, then $V = 0$ V and the current in D_2 will be $[0 - (-3)]/6 = 0.5$ mA. The current in the 12 kΩ will be $(3 - 0)/12 = 0.25$ mA. A node equation at the common anodes node yields a negative current in D_1. It follows that our assumption is wrong and D_1 must be off. Now making the assumption that D_1 is off and D_2 is on, we obtain the results shown in Fig. (a):

$I = 0$

$V = -1$ V



(b)

(b) In (b), the two resistors are interchanged. With some reasoning, we can see that the current supplied through the 6-kΩ resistor will exceed that drawn through the 12-kΩ resistor, leaving sufficient current to keep D_1 conducting. Assuming that D_1 and D_2 are both conducting gives the results shown in Fig. (b):

$I = 0.25$ mA

$V = 0$ V

4.10 The analysis is shown on the circuit diagrams below.

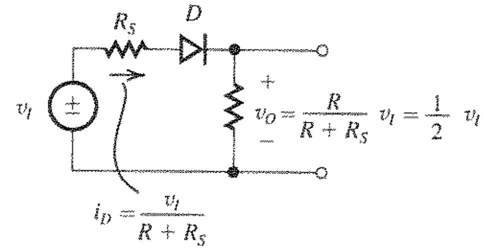
4.11 $R \geq \frac{120\sqrt{2}}{40} \geq 4.2$ kΩ

The largest reverse voltage appearing across the diode is equal to the peak input voltage:

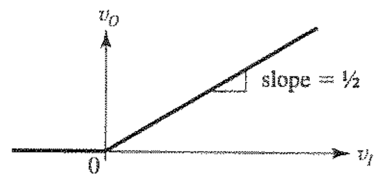
$120\sqrt{2} = 169.7$ V

These figures belong to Problem **4.10**.

4.12

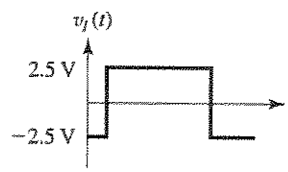


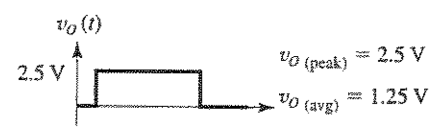
D starts to conduct when $v_I > 0$

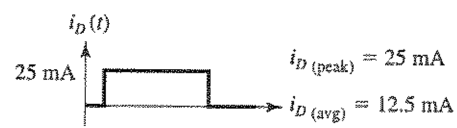


4.13 For $v_I > 0$ V: D is on, $v_O = v_I$, $i_D = v_I/R$

For $v_I < 0$ V: D is off, $v_O = 0$, $i_D = 0$

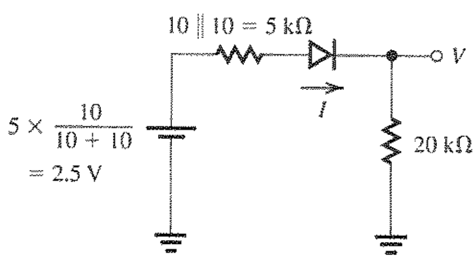






reverse diode voltage: $V_R = v_O = v_I$

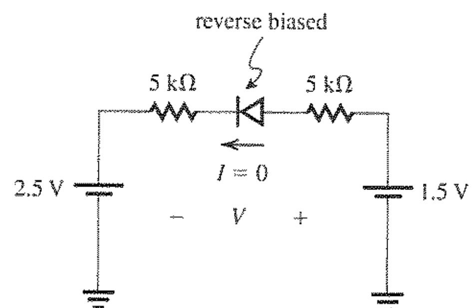
$V_{R\,(\text{max})} = 2.5$ V



$$I = \frac{2.5}{5+20} = 0.1 \text{ mA}$$

$$V = 0.1 \times 20 = 2 \text{ V}$$

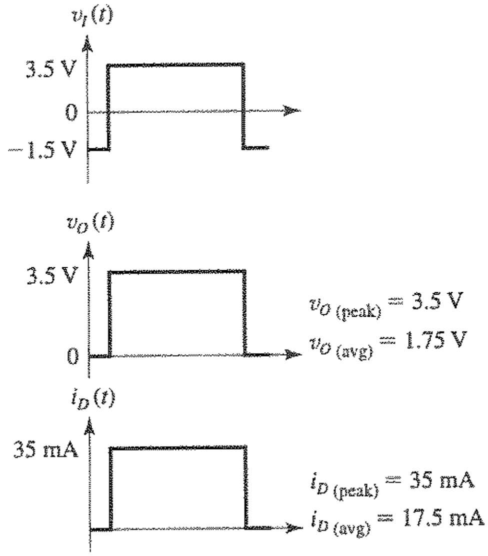
(a)



$$V = 1.5 - 2.5 = -1 \text{ V}$$

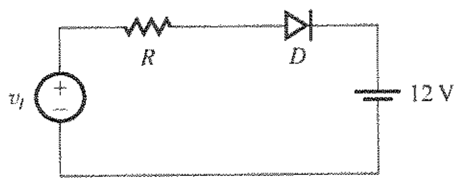
(b)

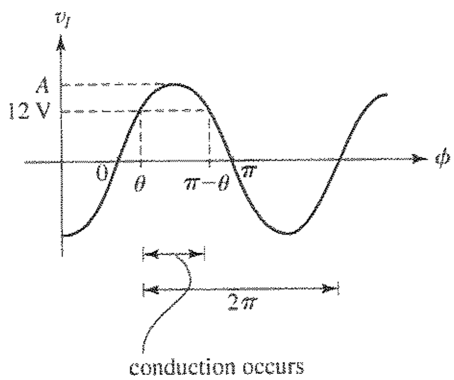
4.14



$V_{R\,(\max)} = 1.5$ V

4.15





$v_I = A \sin\theta = 12 \sim$ conduction through D occurs

For a conduction angle $(\pi - 2\theta)$ that is 25% of a cycle

$$\frac{\pi - 2\theta}{2\pi} = \frac{1}{4}$$

$$\theta = \frac{\pi}{4}$$

$A = 12/\sin\theta = 17$ V

$\therefore$ Peak-to-peak sine wave voltage

$= 2A = 34$ V

Given the average diode current to be

$$\frac{1}{2\pi}\int_0^{2\pi} \frac{A\sin\phi - 12}{R}\,d\phi = 100 \text{ mA}$$

$$\frac{1}{2\pi}\left(\frac{-17\cos\phi - 12\phi}{R}\right)_{\phi=0.25\pi}^{\phi=0.75\pi} = 0.1$$

$R = 8.3\ \Omega$

Peak diode current $= \dfrac{A-12}{R} = 0.6$ A

Peak reverse voltage $= A + 12 = 29$ V

For resistors specified to only one significant digit and peak-to-peak voltage to the nearest volt, choose $A = 17$ so the peak-to-peak sine wave voltage $= 34$ V and $R = 8\ \Omega$.

Conduction starts at $v_I = A\sin\theta = 12$

$17\sin\theta = 12$

$\theta = \left(\dfrac{\pi}{4}\right)$ rad

Conduction stops at $\pi - \theta$.

$\therefore$ Fraction of cycle that current flows is

$$\frac{\pi - 2\theta}{2\pi} \times 100 = 25\%$$

Average diode current $=$

$$\frac{1}{2\pi}\left(\frac{-17\cos\phi - 12\phi}{8}\right)_{\phi=\pi/4}^{\phi=3\pi/4} = 103 \text{ mA}$$

Peak diode current

$$= \frac{17-12}{8} = 0.625 \text{ A}$$

Peak reverse voltage $=$

$A + 12 = 29$ V

4.16

V	RED	GREEN	
3 V	ON	OFF	- D_1 conducts
0	OFF	OFF	- No current flows
−3 V	OFF	ON	- D_2 conducts

4.17 $V_T = \dfrac{kT}{q}$

where
$k = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K

$T = 273 + x°C$

$q = 1.60 \times 10^{-19}$ C

Thus

$V_T = 8.62 \times 10^{-5} \times (273 \times x°\text{C})$, V

x [°C]	V_T [mV]
−55	18.8
0	23.5
+55	28.3
+125	34.3

for $V_T = 25$ mV at 17°C

4.18 $i = I_S e^{v/0.025}$

$\therefore\ 10{,}000 I_S = I_S\ e^{v/0.025}$

$v = 0.230$ V

At $v = 0.7$ V,

$i = I_S\ e^{0.7/0.025} = 1.45 \times 10^{12} I_S$

4.19 $I_1 = I_S\ e^{0.7/V_T} = 10^{-3}$

$i_2 = I_S e^{0.5/V_T}$

$$\frac{i_2}{i_1} = \frac{i_2}{10^{-3}} = e^{\frac{0.5-0.7}{0.025}}$$

$i_2 = 0.335\ \mu$A

4.20 $I = I_S e^{V_D/V_T}$

$10^{-3} = I_S e^{0.7/V_T}$ (1)

For $V_D = 0.71$ V,

$I = I_S e^{0.71/V_T}$ (2)

Combining (1) and (2) gives

$I = 10^{-3} e^{(0.71-0.7)/0.025}$

$= 1.49$ mA

For $V_D = 0.8$ V,

$I = I_S e^{0.8/V_T}$ (3)

Combining (1) and (3) gives

$I = 10^{-3} \times e^{(0.8-0.7)/0.025}$

$= 54.6$ mA

Similarly, for $V_D = 0.69$ V we obtain

$I = 10^{-3} \times e^{(0.69-0.7)/0.025}$

$= 0.67$ mA

and for $V_D = 0.6$ V we have

$I = 10^{-3} e^{(0.6-0.7)/0.025}$

$= 18.3\ \mu$A

To increase the current by a factor of 10, V_D must be increased by ΔV_D,

$10 = e^{\Delta V_D/0.025}$

$\Rightarrow \Delta V_D = 0.025 \ln 10 = 57.6$ mV

4.21 I_S can be found by using $I_S = I_D \cdot e^{-V_D/V_T}$.

Let a decrease by a factor of 10 in I_D result in a decrease of V_D by ΔV:

$I_D = I_S e^{V_D/V_T}$

$$\frac{I_D}{10} = I_S e^{(V_D-\Delta V)/V_T} = I_S e^{V_D/V_T} \cdot e^{-\Delta V/V_T}$$

Taking the ratio of the above two equations, we have

$10 = e^{\Delta V/V_T} \Rightarrow \Delta V \simeq 60$ mV

Thus the result in each case is a decrease in the diode voltage by 60 mV.

(a) $V_D = 0.700$ V, $I_D = 1$ A
$\Rightarrow I_S = 6.91 \times 10^{-13}$ A;

10% of I_D gives $V_D = 0.64$ V

(b) $V_D = 0.650$ V, $I_D = 1$ mA
$\Rightarrow I_S = 5.11 \times 10^{-15}$ A;

10% of I_D gives $V_D = 0.59$ V

(c) $V_D = 0.650$ V, $I_D = 10\ \mu$A
$\Rightarrow I_S = 5.11 \times 10^{-17}$ A;

10% of I_D gives $V_D = 0.59$ V

(d) $V_D = 0.700$ V, $I_D = 100$ mA
$\Rightarrow I_S = 6.91 \times 10^{-14}$ A;

10% of I_D gives $V_D = 0.64$ V

4.22 I_S can be found by using $I_S = I_D \cdot e^{-V_D/V_T}$.

Let an increase by a factor of 10 in I_D result in an increase of V_D by ΔV:

$I_D = I_S\ e^{V_D/V_T}$

$10 I_D = I_S\ e^{(V_D+\Delta V)/V_T} = I_S\ e^{V_D/V_T} \cdot e^{\Delta V/V_T}$

Taking the ratio of the above two equations, we have

$10 = e^{\Delta V/V_T} \Rightarrow \Delta V \simeq 60$ mV

Thus the result is an increase in the diode voltage by 60 mV.

Similarly, at $I_D/10$, V_D is reduced by 60 mV.

(a) $V_D = 0.70$ V, $I_D = 10$ mA
$\Rightarrow I_S = 6.91 \times 10^{-15}$ A;

$I_D \times 10$ gives $V_D = 0.76$ V

$I_D/10$ gives $V_D = 0.64$ V

(b) $V_D = 0.70$ V, $I_D = 1$ mA
$\Rightarrow I_S = 6.91 \times 10^{-16}$ A;

$I_D \times 10$ gives $V_D = 0.76$ V

$I_D/10$ gives $V_D = 0.64$ V

(c) $V_D = 0.80$ V, $I_D = 10$ A
$\Rightarrow I_S = 1.27 \times 10^{-13}$ A;

$I_D \times 10$ gives $V_D = 0.86$ V

$I_D/10$ gives $V_D = 0.74$ V

(d) $V_D = 0.70$ V, $I_D = 1$ mA
$\Rightarrow I_S = 6.91 \times 10^{-16}$ A;

$I_D \times 10$ gives $V_D = 0.76$ V

$I_D/10$ gives $V_D = 0.64$ V

(e) $V_D = 0.6$ V, $I_D = 10\ \mu$A
$\Rightarrow I_S = 3.78 \times 10^{-16}$ A

$I_D \times 10$ gives $V_D = 0.66$ V

$I_D/10$ gives $V_D = 0.54$ V

4.23 The voltage across three diodes in series is 2.0 V; thus the voltage across each diode must be 0.667 V. Using $I_D = I_S e^{V_D/V_T}$, the required current I is found to be 3.9 mA.

If 1 mA is drawn away from the circuit, I_D will be 2.9 mA, which would give $V_D = 0.794$ V, giving an output voltage of 1.98 V. The change in output voltage is -22 mV.

4.24 Connecting an identical diode in parallel would reduce the current in each diode by a factor of 2. Writing expressions for the currents, we have

$$I_D = I_S e^{V_D/V_T}$$

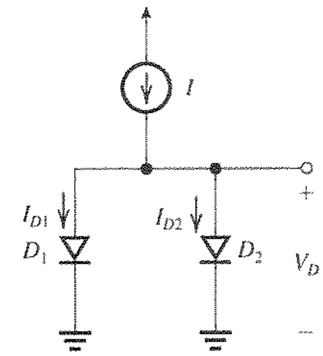
$$\frac{I_D}{2} = I_S e^{(V_D-\Delta V)/V_T} = I_S e^{V_D/V_T} \cdot e^{-\Delta V/V_T}$$

Taking the ratio of the above two equations, we have

$$2 = e^{\Delta V/V_T} \Rightarrow \Delta V = 17.3 \text{ mV}$$

Thus the result is a decrease in the diode voltage by 17.3 mV.

4.25



$$I_{D1} = I_{S1} e^{V_D/V_T} \quad (1)$$

$$I_{D2} = I_{S2} e^{V_D/V_T} \quad (2)$$

Summing (1) and (2) gives

$$I_{D1} + I_{D2} = (I_{S1} + I_{S2}) e^{V_D/V_T}$$

But

$$I_{D1} + I_{D2} = I$$

Thus

$$I = (I_{S1} + I_{S2})\, e^{V_D/V_T} \quad (3)$$

From Eq. (3) we obtain

$$V_D = V_T \ln\left(\frac{I}{I_{S1} + I_{S2}}\right)$$

Also, Eq. (3) can be written as

$$I = I_{S1}\, e^{V_D/V_T}\left(1 + \frac{I_{S2}}{I_{S1}}\right) \quad (4)$$

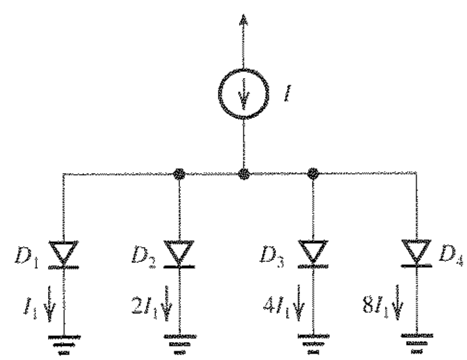
Now using (1) and (4) gives

$$I_{D1} = \frac{I}{1 + (I_{S2}/I_{S1})} = I \frac{I_{S1}}{I_{S1} + I_{S2}}$$

We similarly obtain

$$I_{D2} = \frac{I}{1 + (I_{S1}/I_{S2})} = I \frac{I_{S2}}{I_{S1} + I_{S2}}$$

4.26



$I_1 = 0.1$ mA

The junction areas of the four diodes must be related by the same ratios as their currents, thus

$A_4 = 2A_3 = 4A_2 = 8\,A_1$

With $I_1 = 0.1$ mA,

$I = 0.1 + 0.2 + 0.4 + 0.8 = 1.5$ mA

4.27 We can write a node equation at the anodes:

$I_{D2} = I_I - I_2 = 7$ mA

$I_{DI} = I_2 = 3$ mA

We can write the following equation for the diode voltages:

$V = V_{D2} - V_{DI}$

If D_2 has saturation current I_S, then D_1, which is 10 times larger, has saturation current $10I_S$. Thus we can write

$I_{D2} = I_S e^{V_{D2}/V_T}$

$I_{D1} = 10 I_S e^{V_{D1}/V_T}$

Taking the ratio of the two equations above, we have

$$\frac{I_{D2}}{I_{D1}} = \frac{7}{3} = \frac{1}{10} e^{(V_{D2}-V_{D1})/V_T} = \frac{1}{10} e^{V/V_T}$$

$$\Rightarrow V = 0.025 \ln\left(\frac{70}{3}\right) = 78.7 \text{ mV}$$

To instead achieve $V = 60$ mV, we need

$$\frac{I_{D2}}{I_{D1}} = \frac{I_1 - I_2}{I_2} = \frac{1}{10} e^{0.06/0.025} = 1.1$$

Solving the above equation with I_I still at 10 mA, we find $I_2 = 4.76$ mA.

4.28 We can write the following node equation at the diode anodes:

$I_{D2} = 10 \text{ mA} - V/R$

$I_{D1} = V/R$

We can write the following equation for the diode voltages:

$V = V_{D2} - V_{D1}$

We can write the following diode equations:

$I_{D2} = I_S e^{V_{D2}/V_T}$

$I_{D1} = I_S e^{V_{D1}/V_T}$

Taking the ratio of the two equations above, we have

$$\frac{I_{D2}}{I_{D1}} = \frac{10 \text{ mA} - V/R}{V/R} = e^{(V_{D2}-V_{D1})/V_T} = e^{V/V_T}$$

To achieve $V = 50$ mV, we need

$$\frac{I_{D2}}{I_{D1}} = \frac{10 \text{ mA} - 0.05/R}{0.05/R} = e^{0.05/0.025} = 7.39$$

Solving the above equation, we have

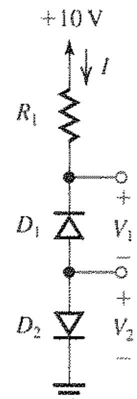
$R = 42\ \Omega$

4.29 For a diode conducting a constant current, the diode voltage decreases by approximately 2 mV per increase of 1°C.

$T = -20$°C corresponds to a temperature decrease of 40°C, which results in an increase of the diode voltage by 80 mV. Thus $V_D = 770$ mV.

$T = +85$°C corresponds to a temperature increase of 65°C, which results in a decrease of the diode voltage by 130 mV. Thus $V_D = 560$ mV.

4.30



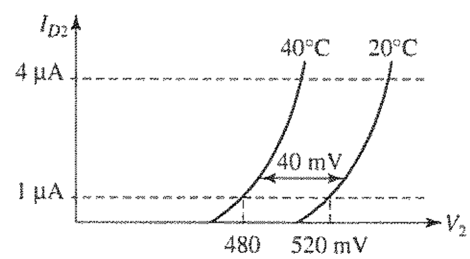
At 20°C:

$V_{R1} = V_2 = 520$ mV

$R_1 = 520$ kΩ

$$I = \frac{520 \text{ mV}}{520 \text{ k}\Omega} = 1\ \mu\text{A}$$

Since the reverse current doubles for every 10°C rise in temperature, at 40°C, $I = 4\ \mu$A



$V_2 = 480 + 2.3 \times 1 \times 25 \log 4$

$= 514.6$ mV

$V_{R1} = 4\ \mu\text{A} \times 520\ \text{k}\Omega = 2.08$ V

At 0°C, $I = \frac{1}{4}\ \mu$A

$V_2 = 560 - 2.3 \times 1 \times 25 \log 4$

$= 525.4$ mV

$V_{R1} = \frac{1}{4} \times 520 = 0.13$ V

4.31 For a diode conducting a constant current, the diode voltage decreases by approximately 2 mV per increase of 1°C.

A decrease in V_D by 100 mV corresponds to a junction temperature increase of 50°C.

The power dissipation is given by

$PD = (10\text{ A})\,(0.6\text{ V}) = 6\text{ W}$

The thermal resistance is given by

$$\frac{\Delta T}{P_D} = \frac{50°\text{C}}{6\text{ W}} = 8.33°\text{C/W}$$

4.32 Given two different voltage/current measurements for a diode, we can write

$I_{D1} = I_S e^{V_{D1}/V_T}$

$I_{D2} = I_S e^{V_{D2}/V_T}$

Taking the ratio of the above two equations, we have

$$\frac{I_{D1}}{I_{D2}} = I_S e^{(V_{D1}-V_{D2})/V_T} \Rightarrow V_{D1} - V_{D2}$$

$$= V_T \ln\left(\frac{I_{D1}}{I_{D2}}\right)$$

For $I_D = 1$ mA, we have

$$\Delta V = V_T \ln\left(\frac{1 \times 10^{-3}\text{ A}}{10\text{ A}}\right) = -230\text{ mV}$$

$\Rightarrow V_D = 570$ mV

For $I_D = 3$ mA, we have

$$\Delta V = V_T \ln\left(\frac{3 \times 10^{-3}\text{ A}}{10\text{ A}}\right) = -202\text{ mV}$$

$\Rightarrow V_D = 598$ mV

Assuming V_D changes by –2 mV per 1°C increase in temperature, we have, for ±20°C changes:

For $I_D = 1$ mA, $530\text{ mV} \le V_D \le 610\text{ mV}$

For $I_D = 3$ mA, $558\text{ mV} \le V_D \le 638\text{ mV}$

Thus the overall range of V_D is between 530 mV and 638 mV.

4.33 Given two different voltage/current measurements for a diode, we have

$$\frac{I_{D1}}{I_{D2}} = I_S\, e^{(V_{D1}-V_{D2})/V_T} \Rightarrow V_{D1} - V_{D2}$$

$$= V_T \ln\left(\frac{I_{D1}}{I_{D2}}\right)$$

For the first diode, with $I_D = 0.1$ mA and

$V_D = 700$ mV, we have

$I_D = 1$ mA:

$$\Delta V = V_T \ln\left(\frac{1.0}{0.1}\right) = 57.6\text{ mV}$$

$\Rightarrow V_D = 757.6$ mV

$I_D = 3$ mA:

$$\Delta V = V_T \ln\left(\frac{3}{0.1}\right) = 85\text{ mV} \Rightarrow V_D = 785\text{ mV}$$

For the second diode, with

$I_D = 1$ A and $V_D = 700$ mV, we have

$I_D = 1.0$ mA:

$$\Delta V = V_T \ln\left(\frac{0.001}{1}\right) = -173\text{ mV}$$

$\Rightarrow V_D = 527$ mV

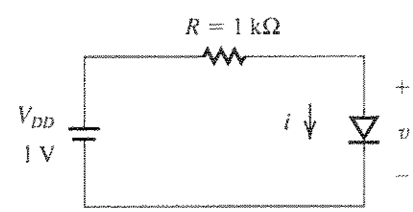
$I_D = 3$ mA:

$$\Delta V = V_T \ln\left(\frac{0.003}{1}\right) = -145\text{ mV}$$

$\Rightarrow V_D =\ 555$ mV

For both $I_D = 1.0$ mA and $I_D = 3$ mA, the difference between the two diode voltages is approximately 230 mV. Since, for a fixed diode current, the diode voltage changes with temperature at a constant rate (–2 mV per °C temp. increase), this voltage difference will be independent of temperature!

4.34



$I_S = 10^{-15}\text{A} = 10^{-12}$ mA

Calculate some points

$v = 0.6\ V, \qquad i = I_S e^{v/V_T}$

$= 10^{-12} e^{0.6/0.025}$

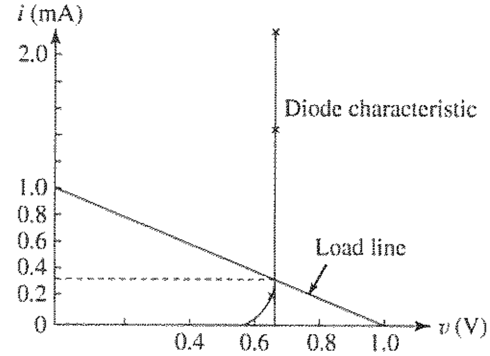
$\simeq 0.03$ mA

$v = 0.65$ V, $\qquad i \simeq 0.2$ mA

$v = 0.7$ V, $\qquad i \simeq 1.45$ mA

Make a sketch showing these points and load line and determine the operating point. The points for the load line are obtained using

$$I_D = \frac{V_{DD} - V_D}{R}$$



From this sketch one can see that the operating point must lie between $v = 0.65$ V to $v = 0.7$ V

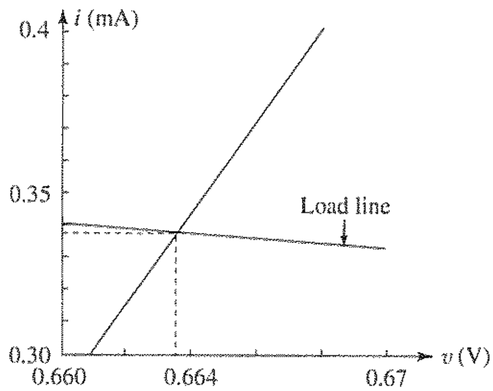
For $i = 0.3$ mA, $v = V_T \ln\left(\frac{i}{I_S}\right)$

$= 0.025 \times \ln\left(\frac{3}{10^{-12}}\right)$

$= 0.661$ V

For $i = 0.4$ mA, $v = 0.668$ V

Now we can refine the diagram to obtain a better estimate



From this graph we get the operating point

$i = 0.338$ mA, $v = 0.6635$ V

Now we compare graphical results with the exponential model.

At $i = 0.338$ mA

$v = V_T \ln\left(\frac{i}{I_S}\right) = 0.025 \times \ln\left(\frac{0.338}{10^{-12}}\right)$

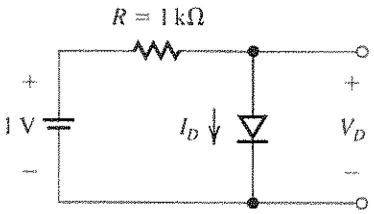
$= 0.6637$ V

The difference between the exponential model and graphical results is $= 0.6637 - 0.6635$

$= 0.0002$ V

$= 0.2$ mV

4.35



$I_S = 10^{-15}\text{A} = 10^{-12}$ mA

Use the iterative analysis procedure:

1. $V_D = 0.7$ V, $I_D = \frac{1 - 0.7}{1\text{ K}} = 0.3$ mA

2. $V_D = V_T \ln\left(\frac{I_D}{I_S}\right) = 0.025 \ln\left(\frac{0.3}{10^{-12}}\right)$

   $= 0.6607$ V

   $I_D = \frac{1 - 0.6607}{1\text{ K}} = 0.3393$ mA

3. $V_D = 0.025 \ln\left(\frac{0.3393}{10^{-12}}\right) = 0.6638$ V

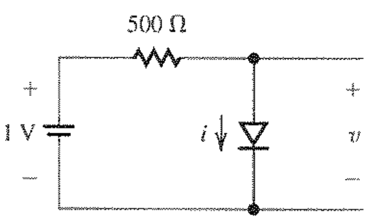
   $I_D = \frac{1 - 0.6638}{1\text{ K}} = 0.3362$ mA

4. $V_D = 0.025 \ln\left(\frac{0.3362}{10^{-12}}\right) = 0.6635$ V

   $I_D = \frac{1 - 0.6635}{1\text{ k}\Omega} = 0.3365$ mA

Stop here as we are getting almost same value of I_D and V_D

4.36



a) $I_D = \frac{1 - 0.7}{0.5\text{ k}\Omega} = 0.6$ mA

b) Diode has 0.7 V drop at 1 mA current. Use Eq. (4.5):

$v_2 = v_1 + 2.3V_T \log\left(\frac{i_2}{i_1}\right)$

1. $v = 0.7$ V

   $i = \frac{1 - 0.7}{0.5\text{ k}\Omega} = 0.6$ mA

2. $v = 0.7 + 2.3 \times 0.025 \log\left(\frac{0.6}{1}\right)$

   $= 0.6872$ V

   $i = \frac{1 - 0.6872}{0.5} = 0.6255$ mA

3. $v = 0.7 + 2.3 \times 0.025 \log\left(\frac{0.6255}{1}\right)$

$= 0.6882$ V

$$i = \frac{1 - 0.6882}{0.5\ \text{k}\Omega} = 0.6235\ \text{mA}$$

4. $v = 0.7 + 2.3 \times 0.025 \log\left(\frac{0.6235}{1}\right)$

$= 0.6882$ V

$$i = \frac{1 - 0.6882}{0.5\ \text{k}\Omega} = 0.6235\ \text{mA}$$

Stop as we are getting the same result.

4.37 We first find the value of I_S for the diode, given by $I_S = I_D e^{-V_D/V_T}$ with $I_D = 1$ mA and $V_D = 0.75$ V. This gives $I_S = 9.36 \times 10^{-17}$ A.

In order to have 3.3 V across the 4 series-connected diodes, each diode drop must be 0.825 V. Applying this voltage to the diode gives current $I_D = 20.1$ mA. We can then find the resistor value using

$$R = \frac{15\ \text{V} - 3.3\ \text{V}}{20.1\ \text{mA}} = 582\ \Omega$$

4.38 Constant voltage drop model:

Using $v_D = 0.7$ V, $\Rightarrow i_{D1} = \dfrac{V - 0.7}{R}$

Using $v_D = 0.6$ V, $\Rightarrow i_{D2} = \dfrac{V - 0.6}{R}$

For the difference in currents to be only 1%,

$\Rightarrow i_{D2} = 1.01 i_{D1}$

$V - 0.6 = 1.01\,(V - 0.7)$

$V = 10.7$ V

For $V = 3$ V and $R = 1$ kΩ:

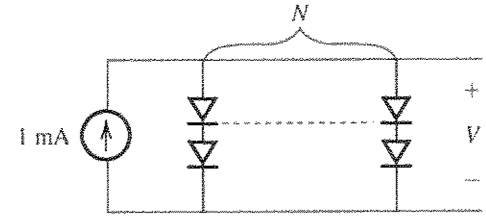
At $V_D = 0.7$ V, $\quad i_{D1} = \dfrac{3 - 0.7}{1} = 2.3$ mA

At $V_D = 0.6$ V, $\quad i_{D2} = \dfrac{3 - 0.6}{1} = 2.4$ mA

$$\frac{i_{D2}}{i_{D1}} = \frac{2.4}{2.3} = 1.04$$

Thus the percentage difference is 4%.

4.39 Available diodes have 0.7 V drop at 2 mA current since $2V_D = 1.4$ V is close to 1.3 V, use N parallel pairs of diodes to split the 1 mA current evenly, as shown in the figure next.



The voltage drop across each pair of diodes is 1.3 V. ∴ Voltage drop across each diode $= \frac{1.3}{2} = 0.65$ V. Using

$I_2 = I_1 e^{(V_2 - V_1)/V_T}$

$= 2e^{(0.65-0.7)/0.025}$

$= 0.2707$ mA

Thus current through each branch is 0.2707 mA.

The 1 mA will split in $= \dfrac{1}{0.2707} = 3.69$ branches.

Choose $N = 4$.

There are 4 pairs of diodes in parallel.

∴ We need 8 diodes.

Current through each pair of diodes $= \dfrac{1\ \text{mA}}{4} = 0.25$ mA

∴ Voltage across each pair

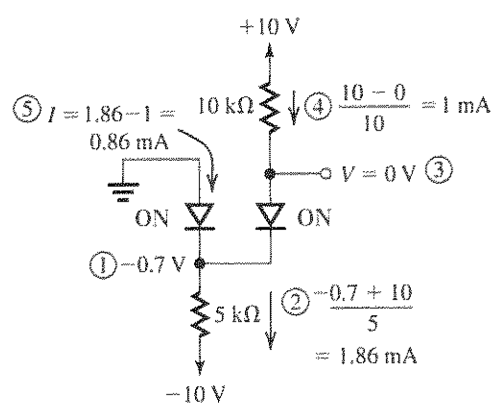
$$= 2\left[0.7 + 0.025\ \ln\left(\frac{0.25}{2}\right)\right]$$

$= 1.296$ V

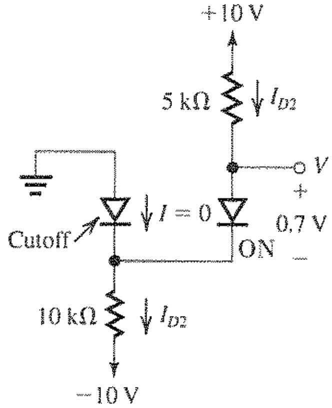
SPECIAL NOTE: There is another possible design utilizing only 6 diodes and realizing a voltage of 1.313 V. It consists of the series connection of 4 parallel diodes and 2 parallel diodes.

4.40 Refer to Example 4.2.

(a)



(b)

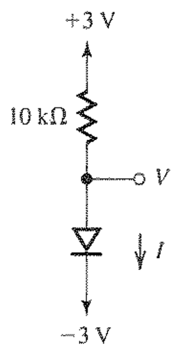


$$I_{D2} = \frac{10 - (-10) - 0.7}{15} = 1.29 \text{ mA}$$

$$V_D = -10 + 1.29\,(10) + 0.7 = 3.6 \text{ V}$$

4.41

(a)

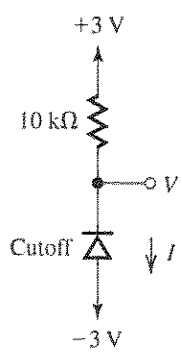


$V = -3 + 0.7 = -2.3$ V

$$I = \frac{3 + 2.3}{10}$$

$= 0.53$ mA

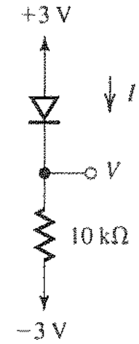
(b)



$I = 0$ A

$V = 3 - I\,(10) = 3$ V

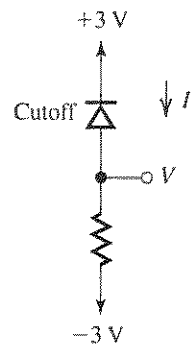
(c)



$V = 3 - 0.7 = 2.3$ V

$$I = \frac{2.3 + 3}{10} = 0.53 \text{ mA}$$

(d)

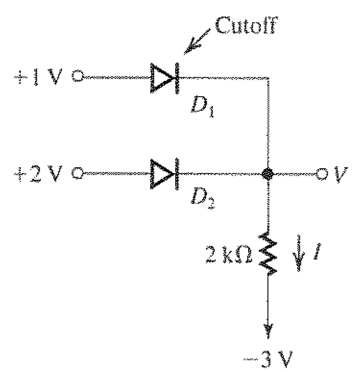


$I = 0$ A

$V = -3$ V

4.42

(a)



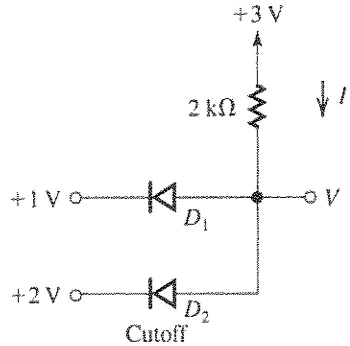
$V = 2 - 0.7$

$= 1.3$ V

$$I = \frac{1.3 - (-3)}{2}$$

$= 2.15$ mA

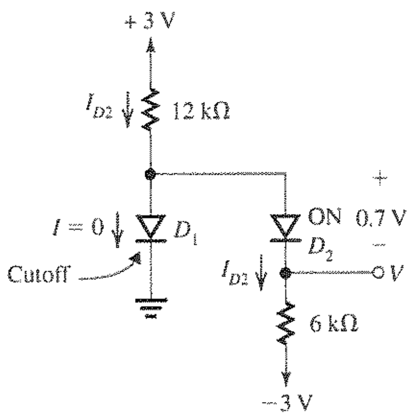
(b)



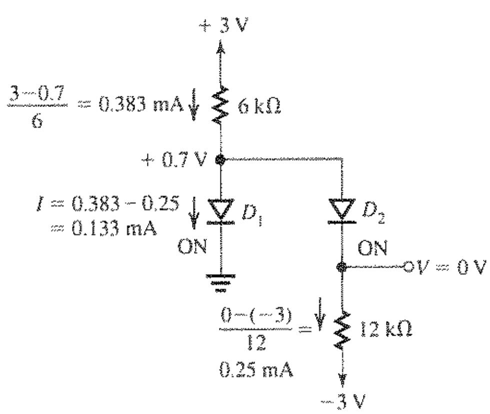
$V = 1 + 0.7 = 1.7$ V

$I = \dfrac{3 - 1.7}{2} = 0.65$ mA

4.43



(a)



(b)

(a) $I_{D2} = \dfrac{3 - 0.7 - (-3)}{12 + 6} = 0.294$ mA

$V = -3 + 0.294 \times 6 = -1.23$ V

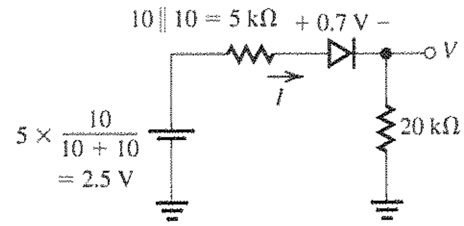
Check that D_1 is off: Voltage at the anode of $D_1 = V + V_{D2} = -1.23 + 0.7 = -0.53$ V which keeps D_1 off.

(b) See analysis on Fig. (b).

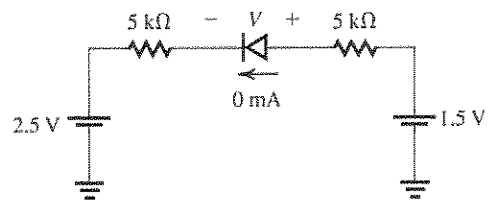
$I = 0.133$ mA

$V = 0$ V

4.44



(a)



(b)

(a) $I = \dfrac{2.5 - 0.7}{5 + 20} = 0.072$ mA

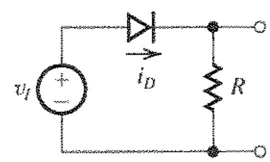
$V = 0.072 \times 20 = 1.44$ V

(b) The diode will be cut off, thus

$I = 0$

$V = 1.5 - 2.5 = -1$ V

4.45



$i_{D,\text{peak}} = \dfrac{v_{I,\text{peak}} - 0.7}{R} \leq 40$ mA

$R \geq \dfrac{120\sqrt{2} - 0.7}{40} = 4.23$ kΩ

Reverse voltage $= 120\sqrt{2} = 169.7$ V.

The design is essentially the same since the supply voltage $\gg 0.7$ V

4.46 Use the exponential diode model to find the percentage change in the current.

$i_D = I_S e^{v/V_T}$

$\dfrac{i_{D2}}{i_{D1}} = e^{(V_2 - V_1)/V_T} = e^{\Delta v/V_T}$

For +5 mV change we obtain

$$\frac{i_{D2}}{i_{D1}} = e^{5/25} = 1.221$$

% change

$$= \frac{i_{D2} - i_{D1}}{i_{D1}} \times 100 = \frac{1.221 - 1}{1} \times 100$$

$= 22.1\%$

For –5 mV change we obtain

$$\frac{i_{D2}}{i_{D1}} = e^{-5/25} = 0.818$$

$$\% \text{ change} = \frac{i_{D2} - i_{D1}}{i_{D1}} \times 100 = \frac{0.818 - 1}{1} \times 100$$

$= -18.1\%$

Maximum allowable voltage signal change when the current change is limited to $\pm 10\%$ is found as follows:

The current varies from 0.9 to 1.1

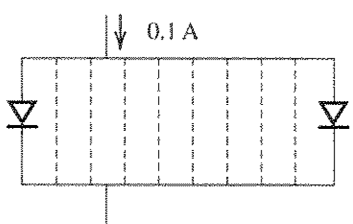
$$\frac{i_{D2}}{i_{D1}} = e^{\Delta V/V_T}$$

For 0.9, $\Delta V = 25 \ln(0.9) = -2.63$ mV

For 1.1, $\Delta V = 25 \ln(1.1) = +2.38$ mV

For $\pm 10\%$ current change the voltage signal change is from –2.63 mV to +2.38 mV

4.47



Ten diode connected in parallel and fed with a total current of 0.1 A. So the current through each diode $= \dfrac{0.1}{10} = 0.01$ A

Small signal resistance of each diode

$$= \frac{V_T}{i_D} = \frac{25 \text{ mV}}{0.01 \text{ A}} = 2.5\ \Omega$$

Equivalent resistance, R_{eq}, of 10 diodes connected in parallel is given by

$$R_{eq} = \frac{2.5}{10} = 0.25\ \Omega$$

If there is one diode conducting 0.1 A current, then the small signal resistance of this diode

$$= \frac{25 \text{ mV}}{0.1 \text{ A}} = 0.25\ \Omega$$

This value is the same as of 10 diodes connected in parallel.

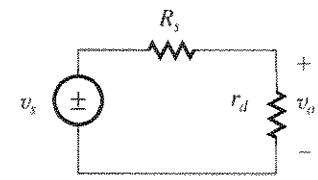
If 0.2 Ω is the resistance for making connection, the resistance in each branch $= r_d + 0.2 = 2.5 + 0.2 = 2.7\ \Omega$

For a parallel combination of 10 diodes, equivalent resistance, R_{eq}, is

$$R_{eq} = \frac{2.7}{10} = 0.27\ \Omega$$

If there is a single diode conducting all the 0.1 A current, the connection resistance needed for the single diode will be $= 0.27 - 0.25 = 0.02\ \Omega$.

4.48 The dc current I flows through the diode giving rise to the diode resistance $r_d = \dfrac{V_T}{I}$ and the small-signal equivalent circuit is represented by



$$v_o = v_s \times \frac{r_d}{r_d + R_S} = v_s \frac{V_T/I}{\frac{V_T}{I} + R_S} = v_s \frac{V_T}{V_T + IR_S}$$

$$\text{Now, } v_o = 10 \text{ mV} \times \frac{25 \text{ mV}}{25 \text{ mV} + 10^3 I}$$

I	v_o
1 mA	0.24 mV
0.1 mA	2.0 mV
1 μA	9.6 mV

$$\text{For } v_o = \frac{1}{2} v_s = v_s \times \frac{0.025}{0.025 + 10^3 I}$$

$\Rightarrow I = 25\ \mu$A

4.49 As shown in Problem 4.48,

$$\frac{v_o}{v_i} = \frac{V_T}{V_T + R_S I} = \frac{0.025}{0.025 + 10^4 I} \qquad (1)$$

Here $R_S = 10$ kΩ

The current changes are limited $\pm 10\%$. Using exponential model, we get

$$\frac{i_{D2}}{i_{D1}} = e^{\Delta v/V_T} = 0.9 \text{ to } 1.1$$

$$\Delta v = 25 \times 10^{-3} \ln\left(\frac{i_{D2}}{i_{D1}}\right) \text{ and here}$$

For 0.9, $\Delta v = -2.63$ mV

For 1.1, $\Delta v = 2.38$ mV

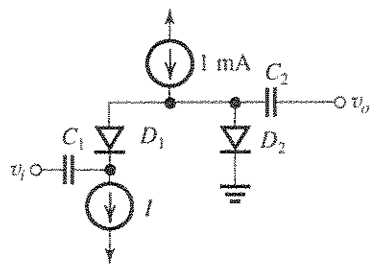
The variation is –2.63 mV to 2.38 mV for $\pm 10\%$ current variation. Thus the largest symmetrical output signal allowed is 2.38 mV in amplitude. To obtain the corresponding input signal, we divide this by (v_o/v_i):

$$\hat{v}_s = \frac{2.38 \text{ mV}}{v_o/v_i} \qquad (2)$$

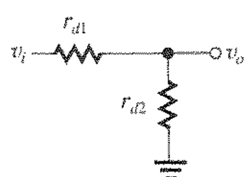
Now for the given values of (v_o/v_i) calculate I and $\hat{v}_S$ using Equations (1) and (2)

$\frac{v_o}{v_i}$	I in mA	$\hat{v}_s$ in mV
0.5	0.0025	4.76
0.1	0.0225	23.8
0.01	0.2475	238
0.001	2.4975	2380

4.50



When both D_1 and D_2 are conducting, the small-signal model is



where we have replaced the large capacitors C_1 and C_2 by short circuits:

$$\frac{v_o}{v_i} = \frac{r_{d2}}{r_{d1} + r_{d2}} = \frac{\dfrac{V_T}{1\text{ m} - I}}{\dfrac{V_T}{I} + \dfrac{V_T}{1\text{ m} - I}} = \frac{I}{1\text{ m}}$$

Thus

$\frac{v_o}{v_i} = I$, where I is in mA

Now $I = 0\ \mu\text{A}$, $\frac{v_o}{v_i} = 0$

$I = 1\ \mu\text{A}$, $\frac{v_o}{v_i} = 1 \times 10^{-3} = 0.001$ V/V

$I = 10\ \mu\text{A}$, $\frac{v_o}{v_i} = 10 \times 10^{-3} = 0.01$ V/V

$I = 100\ \mu\text{A}$, $\frac{v_o}{v_i} = 100 \times 10^{-3} = 0.1$ V/V

$I = 500\ \mu\text{A}$, $\frac{v_o}{v_i} = 500 \times 10^{-3} = 0.5$ V/V

$I = 600\ \mu\text{A}$, $\frac{v_o}{v_i} = 600 \times 10^{-3} = 0.6$ V/V

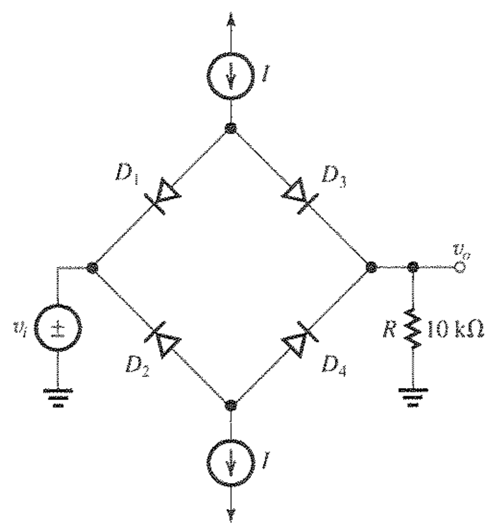
$I = 900\ \mu\text{A}$, $\frac{v_o}{v_i} = 900 \times 10^{-3} = 0.9$ V/V

$I = 990\ \mu\text{A}$, $\frac{v_o}{v_i} = 990 \times 10^{-3} = 0.99$ V/V

$I = 1$ mA

$= 1000\ \mu\text{A}$, $\frac{v_o}{v_i} = 1000 \times 10^{-3} = 1$ V/V

4.51



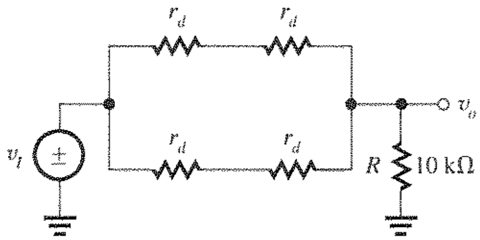
a. The current through each diode is $\frac{I}{2}$:

$$r_d = \frac{V_T}{\frac{I}{2}} = \frac{2V_T}{I} = \frac{0.05}{I}$$

From the equivalent circuit

$$\frac{v_o}{v_i} = \frac{R}{R + (2r_d \parallel 2r_d)} = \frac{R}{R + r_d}$$

I	r_d	$\frac{v_o}{v_i}$
0 μA	∞	0
1 μA	50 kΩ	0.167
10 μA	5 kΩ	0.667
100 μA	500 Ω	0.952
1 mA	50 Ω	0.995
10 mA	5 Ω	0.9995



Equivalent Circuit

b. For signal current to be limited to ±10% of I (I is the biasing current), the change in diode voltage can be obtained from the equation

$$\frac{i_D}{I} = e^{\Delta V_D/V_T} = 0.9 \text{ to } 1.1$$

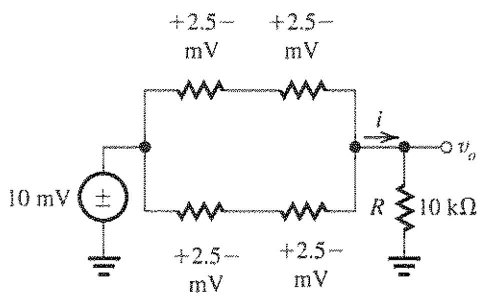
$\Delta v_D = -2.63$ mV to $+2.32$ mV

$\simeq \pm 2.5$ mV

so the signal voltage across each diode is limited to 2.5 mV when the diode current remains within 10% of the dc bias current.

$\therefore v_o = 10 - 2.5 - 2.5 = 5$ mV

and $i = \dfrac{5 \text{ mV}}{10 \text{ k}\Omega} = 0.5\ \mu\text{A}$



The current through each diode

$= \dfrac{0.5}{2}\ \mu\text{A} = 0.25\ \mu\text{A}$

The signal current i is 0.5 μA, and this is 10% of the dc biasing current.

$\therefore$ DC biasing current $I = 0.5 \times 10 = 5\ \mu\text{A}$

c. Now $I = 1$ mA.

$\therefore I_D = 0.5$ mA

Maximum current derivation 10%.

$\therefore i_d = \dfrac{0.5}{10} = 0.05$ mA

and $i = 2i_d = 0.1$ mA.

$\therefore$ Maximum $v_o = i \times 10\ \text{k}\Omega$

$= 0.1 \times 10$

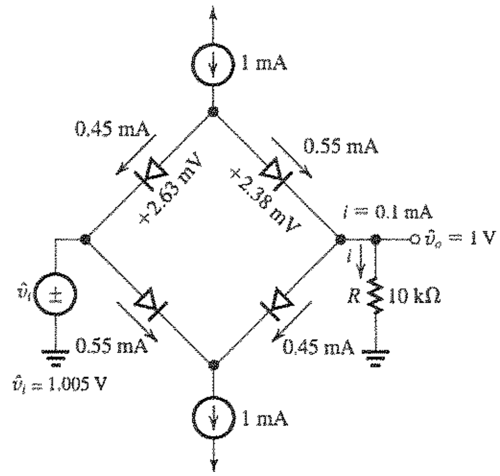
$= 1$ V

From the results of (a) above, for $I = 1$ mA, $v_o/v_i = 0.995$; thus the maximum input signal will be

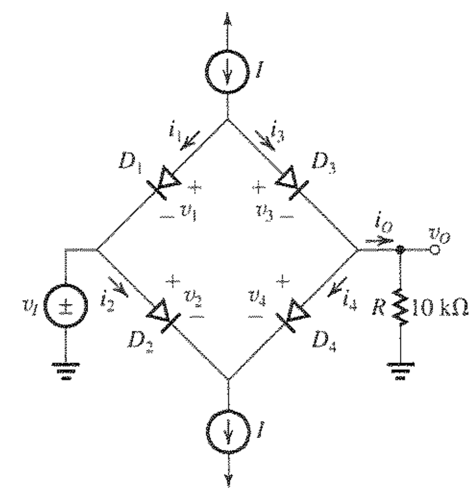
$\hat{v}_i = \hat{v}_o/0.995 = 1/0.995$

$= 1.005$ V

The same result can be obtained from the figure above where the signal across the two series diodes is 5 mV, thus $\hat{v}_i = \hat{v}_o + 5 \text{ mV} = 1 \text{ V} + 5 \text{ mV} = 1.005$ V. See also the figure below.



4.52



$I = 1$ mA

Each diode exhibits 0.7 V drop at 1 mA current. Using diode exponential model we have

$v_2 - v_1 = V_T \ln\left(\dfrac{i_2}{i_1}\right)$

and $v_1 = 0.7$ V, $i_1 = 1$ mA

$\Rightarrow v = 0.7 + V_T \ln\left(\dfrac{i}{1}\right)$

$= 700 + 25 \ln(i)$

Calculation for different values of v_O:

$v_O = 0$, $i_O = 0$, and the current $I = 1$ mA divide equally between the D_3, D_4 side and the D_1, D_2 side.

$$i_1 = i_2 = i_3 = i_4 = \frac{I}{2} = 0.5 \text{ mA}$$

$$v = 700 + 25 \ln(0.5) \simeq 683 \text{ mV}$$

$$v_1 = v_2 = v_3 = v_4 = 683 \text{ mV}$$

From the circuit, we have

$$v_I = -v_1 + v_3 + v_O = -683 + 683 + 0 = 0 \text{ V}$$

For $v_O = 1$ V, $i_O = \dfrac{1}{10 \text{ k}\Omega} = 0.1$ mA

Because of symmetry of the circuit, we obtain

$$i_3 = i_2 = \frac{I}{2} + \frac{i_O}{2} = 0.5 + 0.05 = 0.55 \text{ mA and}$$

$$i_4 = i_1 = 0.45 \text{ mA}$$

$$v_3 = v_2 = 700 + 25 \ln\left(\frac{i_2}{1}\right) = 685 \text{ mV}$$

$$v_4 = v_1 = 700 + 25 \ln\left(\frac{i_4}{1}\right) = 680 \text{ mV}$$

v_O (V)	i_O (mA)	$i_3 = i_2$ (mA)	$i_4 = i_1$ (mA)	$v_3 = v_2$ (mV)	$v_4 = v_1$ (mV)	$v_I = -v_1 +$ $v_3 + v_O$ (V)
0	0	0.5	0.5	683	683	0
+1	0.1	0.55	0.45	685	680	1.005
+2	0.2	0.6	0.4	~687	677	2.010
+5	0.5	0.75	0.25	~693	665	5.028
+9	0.9	0.95	0.05	~699	~625	9.074
+9.9	0.99	0.995	0.005	~700	568	10.09
9.99	0.999	0.9995	0.0005	~700	510	10.18
10	1	1	0	700	0	10.7

$$v_I = -v_1 + v_2 + v_O = -0.680$$

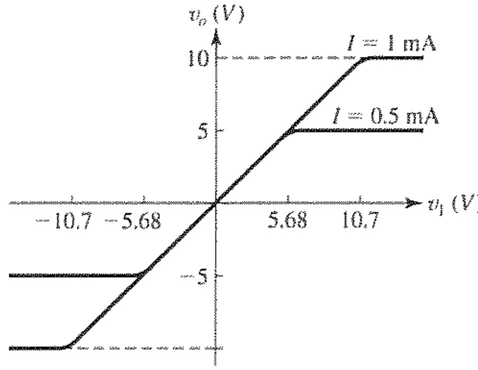
$$+0.685 + 1 = 1.005 \text{ V}$$

Similarly, other values are calculated as shown in the table. The largest values of v_O on positive and negative side are +10 V and −10 V, respectively. This restriction is imposed by the current $I = 1$ mA

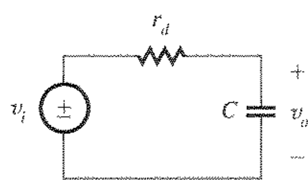
A similar table can be generated for the negative values. It is symmetrical.

For $v_I > +10.7$, v_O will be saturated at +10 V and it is because $I = 1$ mA. Similarly, for $v_I < -10.7$ V, v_O will be saturated at −10 V.

For $I = 0.5$ mA, the output will saturate at $0.5 \text{ mA} \times 10 \text{ k}\Omega = 5$ V.



4.53 Representing the diode by the small-signal resistances, the circuit is



$$r_d = \frac{V_T}{I_D}$$

$$\frac{V_o}{V_i} = \frac{\dfrac{1}{sC}}{r_d + \dfrac{1}{sC}} = \frac{1}{1 + sCr_d}$$

$$\frac{V_o}{V_i} = \frac{1}{1 + j\omega Cr_d}$$

$$\text{Phase shift} = -\tan^{-1}\left(\frac{\omega Cr_d}{1}\right)$$

$$= -\tan^{-1}\left(\omega C \frac{V_T}{I}\right)$$

For phase shift of −45°, we obtain

$$-45 = -\tan^{-1}\left(2\pi \times 100 \times 10^3 \times 10 \times 10^{-9} \times \frac{0.025}{I}\right)$$

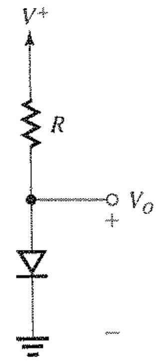
$$\Rightarrow I = 157 \text{ µA}$$

Now I varies from $\dfrac{157}{10}$ µA to 157×10 µA

Range is 15.7 µA to 1570 µA

Range of phase shift is −84.3° to −5.71°

4.54

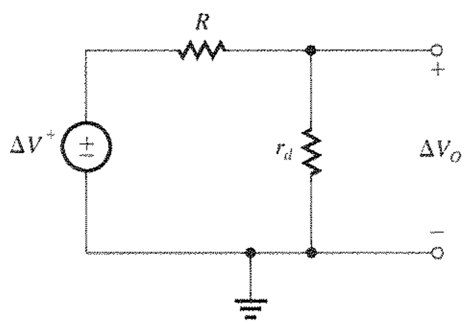


(a) $\dfrac{\Delta V_O}{\Delta V^+} = \dfrac{r_d}{R + r_d} = \dfrac{V_T/I}{R + \dfrac{V_T}{I}}$

$= \dfrac{V_T}{IR + V_T}$

For no load, $I = \dfrac{V^+ - V_O}{R} = \dfrac{V^+ - 0.7}{R}$.

$\therefore \dfrac{\Delta V_O}{\Delta V^+} = \dfrac{V_T}{V_T + (V^+ - 0.7)}$



Small-signal model

(b) If m diodes are in series, we obtain

$\dfrac{\Delta V_O}{\Delta V^+} = \dfrac{m r_d}{m r_d + R} = \dfrac{m V_T}{m V_T + IR}$

$= \dfrac{m V_T}{m V_T + (V^+ - 0.7m)}$

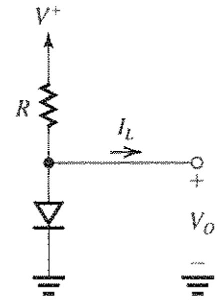
(c) For $m = 1$

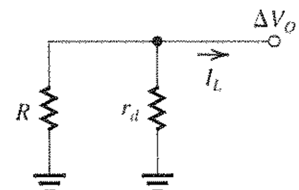
$\dfrac{\Delta V_O}{\Delta V^+} = \dfrac{V_T}{V_T + V^+ - 0.7} = 1.75\ \text{mV/V}$

For $m = 4$

$\dfrac{\Delta V_O}{\Delta V^+} = \dfrac{m V_T}{m V_T + 15 - m \times 0.7} = 8.13\ \text{mV/V}$

4.55





Small-signal model

(a) From the small-signal model

$\Delta V_O = -I_L (r_d \parallel R)$

$\dfrac{\Delta V_O}{I_L} = -(r_d \parallel R)$

(b) At no load, $I_D = \dfrac{V^+ - 0.7}{R}$

$r_d = \dfrac{V_T}{I_D}$

$\dfrac{\Delta V_O}{I_L} = -(r_d \parallel R) = -\dfrac{1}{\dfrac{1}{r_d} + \dfrac{1}{R}}$

$= -\dfrac{1}{\dfrac{I_D}{V^+ - 0.7} + \dfrac{I_D}{V_T}}$

$= -\dfrac{V_T}{I_D} \times \dfrac{1}{\dfrac{V_T}{V^+ - 0.7} + 1}$

$= -\dfrac{V_T}{I_D} \times \dfrac{V^+ - 0.7}{V_T + V^+ - 0.7}$

For $\dfrac{\Delta V_O}{I_L} \le 5 \dfrac{\text{mV}}{\text{mA}}$

i.e., $\dfrac{V_T}{I_D} \times \dfrac{V^+ - 0.7}{V_T + V^+ - 0.7} \le \dfrac{5\ \text{mV}}{\text{mA}}$

$\left[\dfrac{25}{I_D} \times \dfrac{15 - 0.7}{0.025 + 15 - 0.7}\right] \le 5 \dfrac{\text{mV}}{\text{mA}}$

$I_D \ge 4.99\ \text{mA}$

$I_D \simeq 5\ \text{mA}$

$R = \dfrac{V^+ - 0.7}{I_D} = \dfrac{15 - 0.7}{5\ \text{mA}}$

$R = 2.86\ \text{k}\Omega$

Diode should be a 5-mA unit; that is, it conducts 5 mA at $V_D = 0.7$ V, thus $5 \times 10^{-3} = I_S e^{0.7/0.025}$.

$\Rightarrow I_S = 3.46 \times 10^{-15}$ A

(c) For m diodes connected in series we have

$$I_D = \frac{V^+ - 0.7m}{R}$$

and $r_d = \dfrac{V_T}{I_D}$

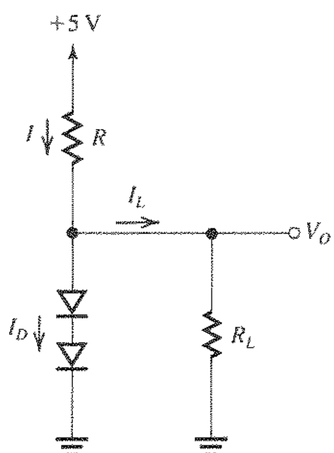
So now $\dfrac{\Delta V_O}{I_L} = -(R \parallel mr_d) = -\dfrac{1}{\dfrac{1}{R} + \dfrac{1}{mr_d}}$

$$= -\frac{1}{\dfrac{I_D}{V^+ - 0.7m} + \dfrac{I_D}{mV_T}}$$

$$= -\frac{mV_T}{I_D} \frac{1}{\dfrac{mV_T}{V^+ - 0.7m} + 1}$$

$$= -\frac{mV_T}{I_D} \frac{V^+ - 0.7m}{V^+ - 0.7m + mV_T}$$

4.56



Diode has 0.7 V drop at 1 mA current

$V_O = 1.5$ V when $R_L = 1.5$ kΩ

$I_D = I_S e^{V/V_T}$

$1 \times 10^{-3} = I_S e^{0.7/0.025}$

$\Rightarrow I_S = 6.91 \times 10^{-16}$ A

Voltage drop across each diode $= \dfrac{1.5}{2} = 0.75$ V.

$\therefore I_D = I_S e^{V/V_T} = 6.91 \times 10^{-16} \times e^{0.75/0.025}$

$= 7.38$ mA

$I_L = 1.5/1.5 = 1$ mA

$I = I_D + I_L = 7.39$ mA $+ 1$ mA

$= 8.39$ mA

$\therefore R = \dfrac{5 - 1.5}{8.39 \text{ mA}} = 417\ \Omega$

Use a small-signal model to find voltage ΔV_O when the value of the load resistor, R_L, changes:

$r_d = \dfrac{V_T}{I_D} = \dfrac{0.025}{7.39} = 3.4\ \Omega$

When load is disconnected, all the current I flows through the diode. Thus

$\Delta I_D = 1$ mA

$\Delta V_O = \Delta I_D \times 2r_d$

$= 1 \times 2 \times 3.4$

$= 6.8$ mV

With $R_L = 1$ kΩ,

$I_L \simeq \dfrac{1.5 \text{ V}}{1} = 1.5$ mA

$\Delta I_L = 0.5$ mA

$\Delta I_D = -0.5$ mA

$\Delta V_O = -0.5 \times 2 \times 3.4$

$= -3.4$ mV

With $R_L = 750\ \Omega$,

$I_L \simeq \dfrac{1.5}{0.75} = 2$ mA

$\Delta I_L = 1$ mA

$\Delta I_D = -1$ mA

$\Delta V_O = -1 \times 2 \times 3.4$

$= -6.8$ mV

With $R_L = 500\ \Omega$,

$I_L \simeq \dfrac{1.5}{0.5} = 3$ mA

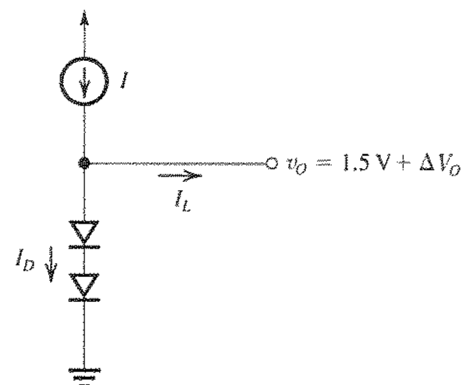
$\Delta I_L = 2.0$ mA

$\Delta I_D = -2.0$ mA

$\Delta V_O = -2 \times 2 \times 3.4$

$= -13.6$ mV

4.57



I_L varies from 2 to 7 mA

To supply a load current of 2 to 7 mA, the current I must be greater than 7 mA. So I can be only 10 mA or 15 mA.

Now let us evaluate ΔV_O for both 10-mA and 15-mA sources. For the 10-mA source:

Since I_L varies from 2 to 7 mA, the current I_D will varry from 8 to 3 mA.

Correspondingly, the voltage across each diode changes by ΔV_D where

$$\frac{3}{8} = e^{\Delta V_D / V_T}$$

$$\Rightarrow \Delta V_D = 25 \ln\left(\frac{3}{8}\right) = -24.5 \text{ mV}$$

and the output voltage changes by

$$\Delta V_O = 2 \times \Delta V_D = -49 \text{ mV}$$

With $I = 15$ mA, the diodes current changes from 13 to 8 mA. Correspondingly, the voltage across each diode changes by ΔV_D where

$$\frac{8}{13} = e^{\Delta V_D / V_T}$$

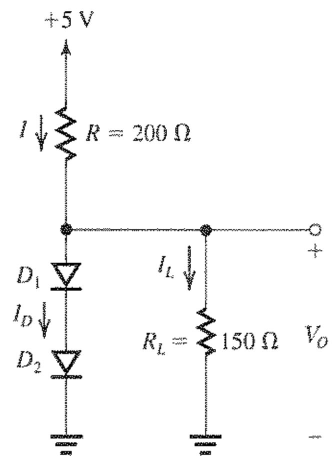
$$\Rightarrow \Delta V_D = 25 \ln\left(\frac{8}{13}\right) = -12.1 \text{ mV}$$

and the output voltage changes by

$$\Delta V_O = 2 \times \Delta V_D = -24.2 \text{ mV}$$

which is less than half that obtained with the 10-mA supply. Thus, from the point of view of reducing the change in V_O as I_L changes, we choose the 15-mA supply. Note, however, that the price paid is increased power dissipation.

4.58



(a) Iteration #1:

$V_D = 0.7$ V

$V_O = 2V_D = 1.4$ V

$$I_L = \frac{V_O}{R_L} = \frac{1.4}{0.15} = 9.33 \text{ mA}$$

$$I = \frac{5 - V_O}{R} = \frac{5 - 1.4}{0.2} = 18 \text{ mA}$$

$I_D = I - I_L = 18 - 9.33 = 8.67$ mA

Iteration #2:

$$V_D = 0.7 + 0.025 \ln\left(\frac{8.67}{10}\right) = 0.696 \text{ V}$$

$V_O = 1.393$ V

$I_L = 9.287$ mA

$$I = \frac{5 - 1.393}{0.2} = 18.04 \text{ mA}$$

$I_D = 18.04 - 9.287 = 8.753$ mA

Iteration #3:

$$V_D = 0.7 + 0.025 \ln\left(\frac{8.753}{10}\right) = 0.697$$

$V_O = 1.393$ V

$I_L = 9.287$

$I = 18.04$ mA

$I_D = 8.753$

No further iterations are necessary and

$V_O = 1.39$ V

(b) With no load:

Iteration #1:

$V_D = 0.7$ V

$V_O = 1.4$ V

$I = \dfrac{5 - 1.4}{0.2} = 18 \text{ mA}$

$I_D = I = 18 \text{ mA}$

Iteration #2:

$V_D = 0.7 + 0.025 \ln\left(\dfrac{18}{10}\right) = 0.715 \text{ V}$

$V_O = 1.429 \text{ V}$

$I = 17.85 \text{ mA}$

$I_D = 17.85 \text{ mA}$

Iteration #3:

$V_D = 0.7 + 0.025 \ln\left(\dfrac{17.85}{10}\right) = 0.714 \text{ V}$

$V_O = 1.43 \text{ V}$

$I = 17.86 \text{ mA}$

$I_D = 17.86 \text{ mA}$

No further iterations are warranted and

$V_O = 1.43 \text{ V}$

(c) $V_O = 1.39 - 0.1 = 1.29 \text{ V}$

$I_L = \dfrac{1.29}{0.15} = 8.6 \text{ mA}$

$V_D = \dfrac{1.29}{2} = 0.645 \text{ V}$

$I_D = 10 \times e^{(0.645-0.7)/0.025}$

$= 1.11 \text{ mA}$

$I = I_L + I_D = 8.6 + 1.11 = 9.71 \text{ mA}$

$V_{\text{Supply}} = V_O + IR = 1.29 + 9.71 \times 0.2$

$= 3.232 \text{ V}$

which is a reduction of 1.768 V or −35.4%.

(d) For $V_{\text{Supply}} = 5 + 1.786 = 6.786 \text{ V}$,

Iteration #1:

$V_D = 0.7 \text{ V}$

$V_O = 1.4 \text{ V}$

$I_L = 9.33 \text{ mA}$

$I = \dfrac{6.768 - 1.4}{0.2} = 26.84$

$I_D = I - I_L = 26.84 - 9.33 = 17.51 \text{ mA}$

Iteration #2:

$V_D = 0.7 + 0.025 \ln\left(\dfrac{17.51}{10}\right) = 0.714 \text{ V}$

$V_O = 1.428 \text{ V}$

$I_L = 9.52 \text{ mA}$

$I = 26.70 \text{ mA}$

$I_D = 17.18 \text{ mA}$

Iteration #3:

$V_D = 0.7 + 0.025 \ln\left(\dfrac{17.18}{10}\right) = 0.714 \text{ V}$

$V_O = 1.428 \text{ V}$

No further iterations are needed and

$V_O = 1.43 \text{ V}$

(e) From the above we see that as V_{Supply} changes from 5 V to 3.232 V (a change of −35.4%) the output voltage changes from 1.39 V to 1.29 V (a change of −7.19%).

As V_{Supply} changes from 5 V to 6.786 V (a change of +35.4%) the output voltage changes from 1.39 V to 1.43 V (a change of +2.88%).

Thus the worst-case situation occurs when V_{Supply} is reduced, and

Percentage change in V_O per 1% change in $V_{\text{Supply}} = \dfrac{7.19}{35.4} = 0.2\%$

4.59 $V_Z = V_{Z0} + I_{ZT} r_z$

(a) $10 = 9.6 + 0.05 \times r_z$

$\Rightarrow r_z = 8\ \Omega$

For $I_Z = 2I_{ZT} = 100 \text{ mA}$,

$V_Z = 9.6 + 0.1 \times 8 = 10.4 \text{ V}$

$P = 10.4 \times 0.1 = 1.04 \text{ W}$

(b) $9.1 = V_{Z0} + 0.01 \times 30$

$\Rightarrow V_{Z0} = 8.8 \text{ V}$

At $I_Z = 2I_{ZT} = 20 \text{ mA}$,

$V_Z = 8.8 + 0.02 \times 30 = 9.4 \text{ V}$

$P = 9.4 \times 20 = 188 \text{ mW}$

(c) $6.8 = 6.6 + I_{ZT} \times 2$

$\Rightarrow I_{ZT} = 0.1 \text{ A}$

At $I_Z = 2I_{ZT} = 0.2 \text{ A}$,

$V_Z = 6.6 + 0.2 \times 2 = 7 \text{ V}$

$P = 7 \times 0.2 = 1.4 \text{ W}$

(d) $18 = 17.6 + 0.005 \times r_z$

$\Rightarrow r_z = 80\ \Omega$

At $I_Z = 2I_{ZT} = 0.01$ A,

$V_Z = 17.6 + 0.01 \times 80 = 18.4$ V

$P = 18.4 \times 0.01 = 0.184$ W $= 184$ mW

(e) $7.5 = V_{Z0} + 0.2 \times 1.5$

$\Rightarrow V_{Z0} = 7.2$ V

At $I_Z = 2I_{ZT} = 0.4$ A,

$V_Z = 7.2 + 0.4 \times 1.5 = 7.8$ V

$P = 7.8 \times 0.4 = 3.12$ W

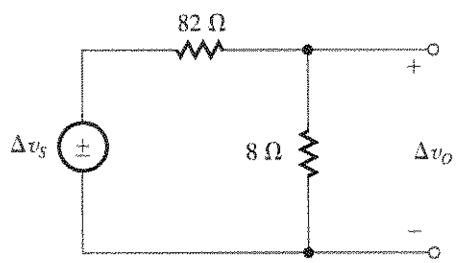
4.60 (a) Three 6.8-V zeners provide $3 \times 6.8 = 20.4$ V with $3 \times 10 = 30$-Ω resistance. Neglecting R, we have

Load regulation $= -30$ mV/mA.

(b) For 5.1-V zeners we use 4 diodes to provide 20.4 V with $4 \times 30 = 120$-Ω resistance.

Load regulation $= -120$ mV/mA

4.61



Small-signal model

From the small-signal model we obtain

$$\frac{\Delta v_O}{\Delta v_S} = \frac{8}{8+82} = \frac{8}{90}$$

Now $\Delta v_S = 1.0$ V.

$$\therefore \Delta v_O = \frac{8}{90}\Delta v_S = \frac{8}{90} \times 1.0$$

$= 88.9$ mV

4.62 $V_Z = V_{Z0} + I_{ZT} r_Z$

$9.1 = V_{Z0} + 0.02 \times 10$

$\Rightarrow V_{Z0} = 8.9$ V

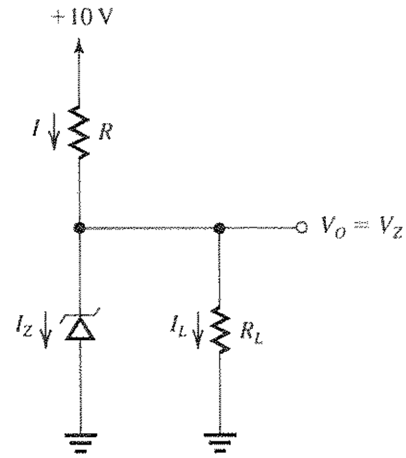
At $I_Z = 10$ mA,

$V_Z = 8.9 + 0.01 \times 10 = 9.0$ V

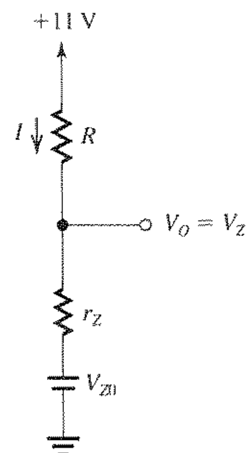
At $I_Z = 50$ mA,

$V_Z = 8.9 + 0.05 \times 10 = 9.4$ V

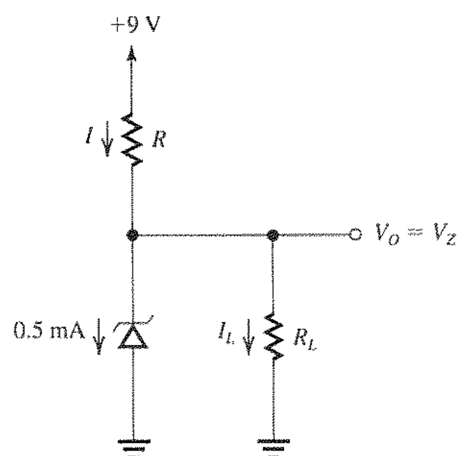
4.63



(a)



(b)



(c)

To obtain $V_O = 7.5$ V, we must arrange for $I_Z = 10$ mA (the current at which the zener is specified).

Now,

$$I_L = \frac{V_O}{R_L} = \frac{7.5}{1.5} = 5 \text{ mA}$$

Thus

$$I = I_Z + I_L = 10 + 5 = 15 \text{ mA}$$

and

$$R = \frac{10 - V_O}{I} = \frac{10 - 7.5}{15} = 167\ \Omega$$

When the supply undergoes a change ΔV_S, the change in the output voltage, ΔV_O, can be determined from

$$\frac{\Delta V_O}{\Delta V_S} = \frac{(R_L \parallel r_z)}{(R_L \parallel r_z) + R}$$

$$= \frac{1.5 \parallel 0.03}{(1.5 \parallel 0.03) + 0.167} = 0.15$$

For $\Delta V_S = +1$ V (10% high), $\Delta V_O = +0.15$ V and $V_O = 7.65$ V.

For $\Delta V_S = -1$ V (10% low), $\Delta V_O = -0.15$ V and $V_O = 7.35$ V.

When the load is removed and $V_S = 11$ V, we can use the zener model to determine V_O. Refer to Fig. (b). To determine V_{Z0}, we use

$$V_Z = V_{Z0} + I_{ZT} r_z$$

$$7.5 = V_{Z0} + 0.01 \times 30$$

$$\Rightarrow V_{Z0} = 7.2 \text{ V}$$

From Fig. (b) we have

$$I = \frac{11 - 7.2}{0.167 + 0.03} = 19.3 \text{ mA}$$

Thus

$$V_O = V_{Z0} + I r_z$$

$$= 7.2 + 0.0193 \times 30 = 7.78 \text{ V}$$

To determine the smallest allowable value of R_L while $V_S = 9$ V, refer to Fig. (c). Note that $I_Z = 0.5$ mA, thus

$$V_Z = V_{ZK} \simeq V_{Z0} = 7.2 \text{ V}$$

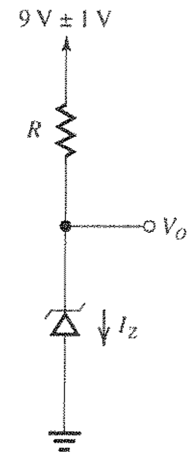
$$I = \frac{9 - 7.2}{0.167} = 10.69 \text{ mA}$$

$$I_L = I - I_Z = 10.69 - 0.5 = 10.19 \text{ mA}$$

$$R_L = \frac{V_O}{I_L} = \frac{7.2}{10.19} = 707\ \Omega$$

$$V_O = 7.2 \text{ V}$$

4.64



GIVEN PARAMETERS

$V_Z = 6.8$V, $r_z = 5\ \Omega$

$I_Z = 20$ mA

At knee,

$I_{ZK} = 0.25$ mA

$r_z = 750\ \Omega$

FIRST DESIGN: 9-V supply can easily supply current

Let $I_Z = 20$ mA, well above knee.

$$\therefore\ R = \frac{9 - 6.8}{20} = 110\ \Omega$$

$$\text{Line regulation} = \frac{\Delta V_O}{\Delta V_S} = \frac{r_Z}{r_Z + R}$$

$$= \frac{5}{5 + 110}$$

$$= 43.5\ \frac{\text{mV}}{\text{V}}$$

SECOND DESIGN: limited current from 9-V supply

$I_Z = 0.25$ mA

$V_Z = V_{ZK} \simeq V_{ZO}$ – calculate V_{Z0} from

$$V_Z = V_{Z0} + r_Z I_{ZT}$$

$$6.8 = V_{Z0} + 5 \times 0.02$$

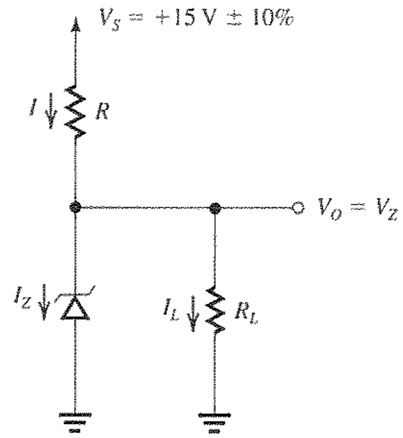
$$V_{Z0} = 6.7 \text{ V}$$

$$\therefore R = \frac{8 - 6.7}{0.25} = 5.2 \text{ k}\Omega$$

$$\text{LINE REGULATION} = \frac{\Delta V_O}{\Delta V_S} = \frac{750}{750 + 5200}$$

$$= 126\ \frac{\text{mV}}{\text{V}}$$

4.65



$V_Z = V_{Z0} + I_{ZT} r_z$

$9.1 = V_{Z0} + 0.009 \times 40$

$\Rightarrow V_{Z0} = 8.74$ V

For $I_Z = 10$ mA,

$V_Z = 8.74 + 0.01 \times 40 = 9.14$ V

$$I_L = \frac{9.14}{1\text{ k}\Omega} = 9.14 \text{ mA}$$

$I = I_Z + I_L = 10 + 9.14 = 19.14$ mA

$$R = \frac{15 - 9.14}{19.4} = 306\ \Omega$$

Select $R = 300\ \Omega$

Denoting the resulting output voltage V_O, we obtain

$$I = \frac{15 - V_O}{0.3} \qquad (1)$$

$$I_L = \frac{V_O}{1} \qquad (2)$$

$$I_Z = \frac{V_O - V_{Z0}}{r_z} = \frac{V_O - 8.74}{0.04} \qquad (3)$$

Since $I = I_Z + I_L$, we can use (1)–(3) to obtain V_O:

$$\frac{15 - V_O}{0.3} = \frac{V_O - 8.74}{0.04} + V_O$$

$\Rightarrow V_O = 9.15$ V

$$\Delta V_O = \Delta V_S \frac{r_z \parallel R_L}{(r_z \parallel R_L) + R}$$

$$= \pm 1.5 \times \frac{(0.04 \parallel 1)}{(0.04 \parallel 1) + 0.3}$$

$= \pm 0.17$ V

If I_L is reduced by 50%, then

$$I_L = \frac{1}{2} \times \frac{9.15}{1} = 4.6 \text{ mA}$$

$$I = \frac{15 - V_O}{0.3}$$

$$I_Z = \frac{V_O - 8.74}{0.04}$$

$$\frac{15 - V_O}{0.3} = \frac{V_O - 8.74}{0.04} + 4.6$$

$\Rightarrow V_O = 9.31$ V

which is an increase of 0.16 V. When the supply voltage is low,

$V_S = 13.5$ V

and R_L is at its lowest value, to maintain regulation, the zener current must be at least equal to I_{ZK}, thus

$I_Z = 0.5$ mA

$V_Z = V_{ZK} \simeq V_{Z0} \simeq 8.74$

$$I = \frac{13.5 - 8.74}{0.3} = 15.87 \text{ mA}$$

$I_L = I - I_Z = 15.87 - 0.5 = 15.37$ mA

$$R_L = \frac{V_Z}{I_L} = \frac{8.74}{15.37} = 589\ \Omega$$

The lowest value of output voltage $= 8.74$ V

$$\text{Line regulation} = \frac{170 \text{ mV}}{1.5 \text{ V}}$$

$= 113$ mV/V

Load regulation $= -(r_z \parallel R)$

$= -(40 \parallel 300) = -35$ mV/mA

Or using the results obtained in this problem:

For a reduction in I_L of 4.6 mA, $\Delta V_O = +0.16$ V, thus

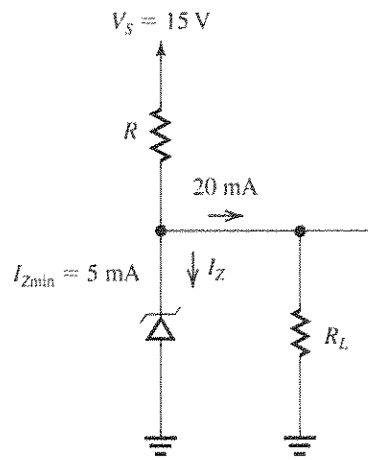
$$\text{Load regulation} = -\frac{160}{4.6} = -35 \text{ mV/mA}$$

4.66 (a) $V_{ZT} = V_{Z0} + r_z I_{ZT}$

$10 = V_{Z0} + 7\,(0.025)$

$\Rightarrow V_{Z0} = 9.825$ V

(b) The minimum zener current of 5 mA occurs when $I_L = 20$ mA and V_S is at its minimum of $20(1 - 0.25) = 15$ V. See the circuit below:



$$R \leq \frac{15 - V_{Z0}}{20 + 5}$$

where we have used the minimum value of V_S, the maximum value of load current, and the minimum required value of zener diode current, and we assumed that at this current $V_Z \simeq V_{Z0}$. Thus,

$$R \leq \frac{15 - 9.825 + 7}{25}$$

$\leq 207\ \Omega$.

$\therefore$ use $R = 207\ \Omega$

(c) Line regulation $= \dfrac{7}{207 + 7} = 33\ \dfrac{\text{mV}}{\text{V}}$

$\pm 25\%$ change in $v_S \equiv \pm 5$ V

V_O changes by $\pm 5 \times 33 = \pm 0.165$ mV

corresponding to $\dfrac{\pm 0.165}{10} \times 100 = \pm 1.65\%$

(d) Load regulation $= -(r_Z \parallel R)$

$= -(7 \parallel 207) = -6.77\ \Omega$

or -6.77 V/A

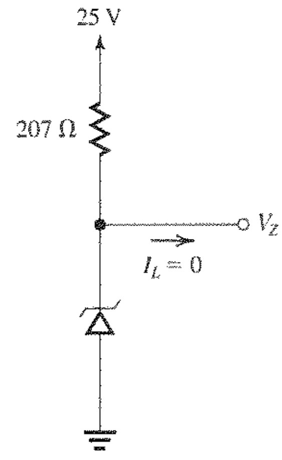
$\Delta V_O = -6.77 \times 20\text{ mA} = -135.4$ mV

corresponding to $-\dfrac{0.1354}{10} \times 100 = -1.35\%$

(e) The maximum zener current occurs at no load $I_L = 0$ and the supply at its largest value of

$20 + \dfrac{1}{4}(20) = 25$ V.

$V_Z = V_{Z0} + r_Z I_Z$



$$= 9.825 + 7 \times \frac{25 - V_Z}{207}$$

$207V_Z = 207\,(9.825) + 7\,(25) - 7V_Z$

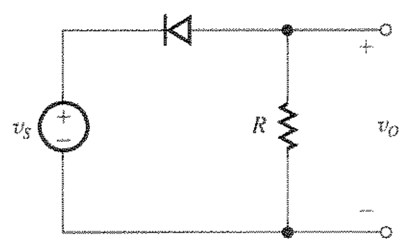
$\Rightarrow V_Z = 10.32$ V

$I_{Z\max} = \dfrac{25 - 10.32}{0.207} = 70.9$ mA

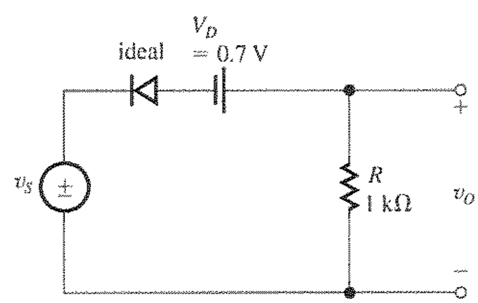
$P_Z = 10.32 \times 70.9$

$= 732$ mW

4.67

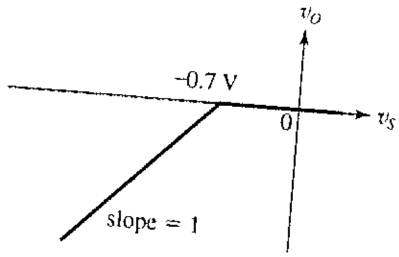


Using the constant voltage drop model:

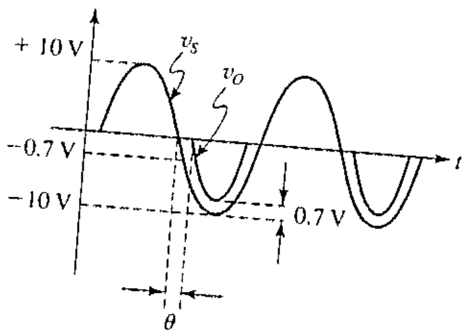


(a) $v_O = v_S + 0.7$ V, For $v_S \leq -0.7$ V

$v_O = 0$, for $v_S \geq -0.7$ V



(b)



(c) The diode conducts at an angle

$$\theta = \sin^{-1}\left(\frac{0.7}{10}\right) = 4^\circ \text{ and stops}$$

at $\pi - \theta = 176^\circ$

Thus the conduction angle is $\pi - 2\theta$

$= 172^\circ$ or 3 rad.

$$v_{O,\text{avg}} = \frac{-1}{2\pi}\int_{\theta}^{\pi-\theta}(10\sin\phi - 0.7)\,d\phi$$

$$= \frac{-1}{2\pi}[-10\cos\phi - 0.7\phi]_{\theta}^{\pi-\theta}$$

$= -2.85$ V

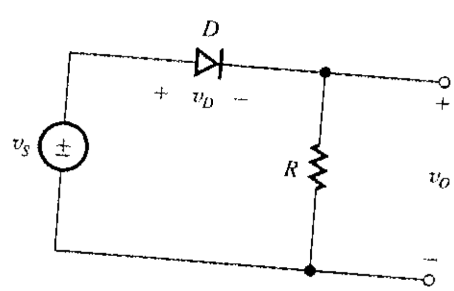
(d) Peak current in diode is

$$\frac{10 - 0.7}{1} = 9.3 \text{ mA}$$

(e) PIV occurs when v_S is at its the peak and $v_O = 0$.

PIV $= 10$ V

4.68



$$i_D = I_S e^{v_D/V_T}$$

$$\frac{i_D}{i_D\,(1\text{ mA})} = e^{[v_D - v_D(\text{at } 1\text{ mA})]/V_T}$$

$$v_D - v_D(\text{at } 1\text{ mA}) = V_T \ln\left[\frac{i_D}{1\text{ mA}}\right]$$

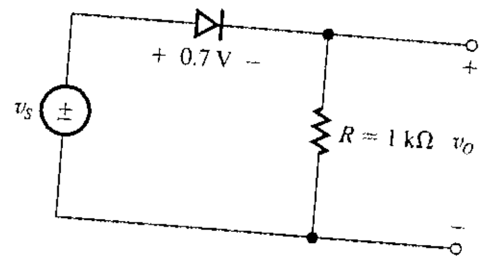
$$v_D = v_D(\text{at } 1\text{ mA}) + V_T \ln\left[\frac{v_O/R}{1}\right]$$

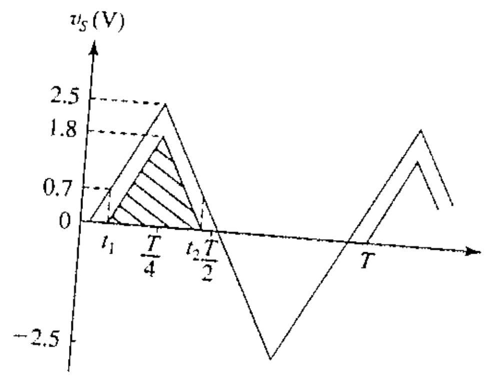
$$v_O = v_S - v_D$$

$$= v_S - v_D\,(\text{at } 1\text{ mA}) - V_T \ln\left(\frac{v_O}{R}\right)$$

where R is in kΩ.

4.69





First find t_1 and t_2

$$\frac{2.5}{\frac{T}{4}} = \frac{0.7}{t_1}$$

$\Rightarrow t_1 = 0.07$ T

$$t_2 = \frac{T}{2} - t_1$$

$$= \frac{T}{2} - 0.07\text{ T}$$

$t_2 = 0.43$ T

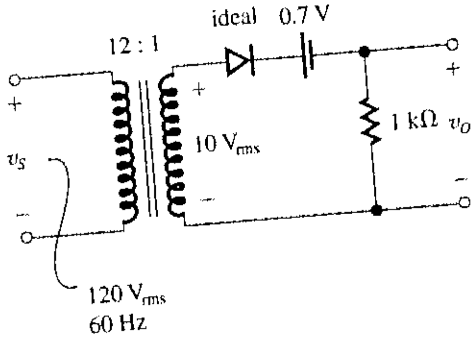
$$v_O(\text{ave.}) = \frac{1}{T} \times \text{ area of shaded triangle}$$

$$= \frac{1}{T} \times (2.5 - 0.7) \times \left(\frac{T}{4} - t_1\right)$$

$$= \frac{1}{T} \times 1.8 \times T\left(\frac{1}{4} - 0.07\right)$$

$= 0.324$ V

4.70



$\hat{v}_O = 10\sqrt{2} - 0.7 = 13.44$ V

Conduction begins at

$10\sqrt{2}\sin\theta = 0.7$

$\theta = \sin^{-1}\left(\dfrac{0.7}{10\sqrt{2}}\right) = 2.84°$

$= 0.0495$ rad

Conduction ends at $\pi - \theta$.

$\therefore$ Conduction angle $= \pi - 2\theta = 3.04$ rad

The diode conducts for

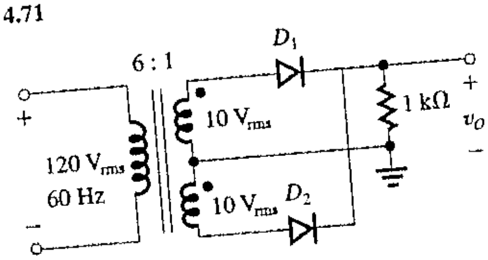
$\dfrac{3.04}{2\pi} \times 100 = 48.4\%$ of the cycle

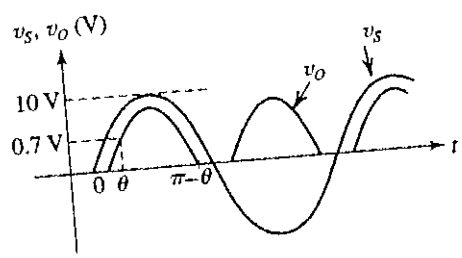
$$v_{O,\text{avg}} = \frac{1}{2\pi}\int_{\theta}^{\pi-\theta}(10\sqrt{2}\sin\phi - 0.7)\,d\phi$$

$= 4.15$ V

$i_{D,\text{avg}} = \dfrac{v_{O,\text{avg}}}{R} = 4.15$ mA

4.71





$\hat{v}_O = 10\sqrt{2} - V_D = 13.44$ V

Conduction starts at $\theta = \sin^{-1}\dfrac{0.7}{10\sqrt{2}} =$

$2.84° = 0.05$ rad

and ends at $\pi - \theta$. Conduction angle $= \pi - 2\theta = 3.04$ rad in each half cycle. Thus the fraction of a cycle for which one of the two diodes conduct $= \dfrac{2(3.04)}{2\pi} \times 100 = 96.8\%$

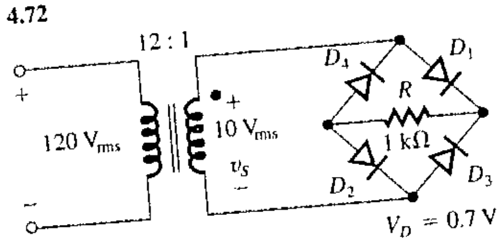
Note that during 96.8% of the cycle there will be conduction. However, each of the two diodes conducts for only half the time, i.e., for 48.4% of the cycle.

$$v_{O,\text{avg}} = \frac{1}{\pi}\int_{\theta}^{\pi-\theta}(10\sqrt{2}\sin\phi - 0.7)d\phi$$

$= 8.3$ V

$i_{L,\text{avg}} = \dfrac{8.3}{1\text{ k}\Omega} = 8.3$ mA

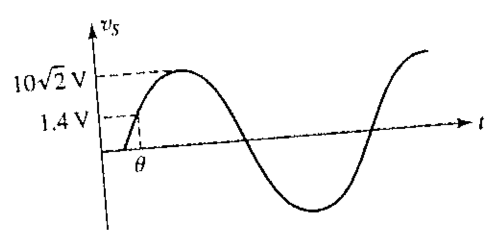
4.72



Peak voltage across $R = 10\sqrt{2} - 2V_D$

$= 10\sqrt{2} - 1.4$

$= 12.74$ V



$\theta = \sin^{-1}\dfrac{1.4}{10\sqrt{2}} = 5.68° = 0.1$ rad

Fraction of cycle that D_1 & D_2 conduct is

$\dfrac{\pi - 2\theta}{2\pi} \times 100 = 46.8\%$

Note that D_3 & D_4 conduct in the other half cycle so that there is 2 (46.8) = 93.6% conduction interval.

$$v_{O,\text{avg}} = \frac{2}{2\pi}\int_{\theta}^{\pi-\theta}(10\sqrt{2}\sin\phi - 2V_D)\,d\phi$$

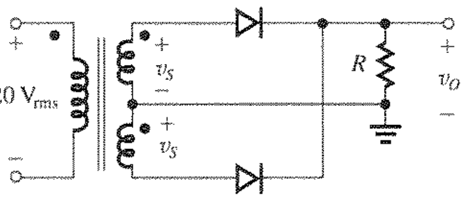
$$= \frac{1}{\pi}\left[-12\sqrt{2}\cos\phi - 1.4\phi\right]_{\theta}^{\pi-\theta}$$

$$= \frac{2(12\sqrt{2}\cos\theta)}{\pi} - \frac{1.4(\pi - 2\theta)}{\pi}$$

$= 7.65$ V

$$i_{R,\text{avg}} = \frac{v_{O,\text{avg}}}{R} = \frac{7.65}{1} = 7.65 \text{ mA}$$

4.73



Refer to Fig. 4.24.

For $V_D \ll V_s$, conduction angle $\simeq \pi$, and

$$v_{O,\text{avg}} = \frac{2}{\pi}V_s - V_D = \frac{2}{\pi}V_s - 0.7$$

(a) For $v_{O,\text{avg}} = 10$ V

$$V_s = \frac{\pi}{2} \times 10.7 = 16.8 \text{ V}$$

$$\text{Turns ratio} = \frac{120\sqrt{2}}{16.8} = 10.1 \text{ to } 1$$

(b) For $v_{O,\text{avg}} = 100$ V

$$V_s = \frac{\pi}{2} \times 100.7 = 158.2 \text{ V}$$

$$\text{Turns ratio} = \frac{120\sqrt{2}}{158.2} = 1.072 \text{ to } 1$$

4.74 Refer to Fig. 4.25

For $2V_D \ll V_s$

$$V_{O,\text{avg}} = \frac{2}{\pi}V_s - 2V_D = \frac{2}{\pi}V_s - 1.4$$

(a) For $V_{O,\text{avg}} = 10$ V

$$10 \text{ V} = \frac{2}{\pi}\cdot V_s - 1.4$$

$$\therefore \hat{V}_s = 11.4\left(\frac{\pi}{2}\right) = 17.9 \text{ V}$$

$$\text{Turns ratio} = \frac{120\sqrt{2}}{17.9} = 9.5 \text{ to } 1$$

(b) For $V_{O,\text{avg}} = 100$ V

$$100 \text{ V} = \frac{2}{\pi}\cdot V_s - 1.4$$

$$\Rightarrow V_s = 101.4\left(\frac{\pi}{2}\right) = 159 \text{ V}$$

$$\text{Turns ratio} = \frac{120\sqrt{2}}{159} = 1.07 \text{ to } 1$$

4.75 $120\sqrt{2} \pm 10\%$: $20\sqrt{2} \pm 10\%$

$\Rightarrow$ Turns ratio $= 6{:}1$

$$v_S = \frac{20\sqrt{2}}{2} \pm 10\%$$

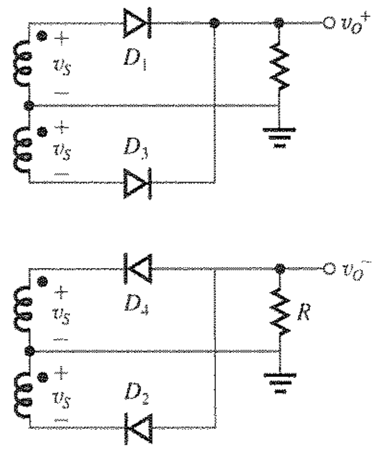
$$\text{PIV} = 2V_s - V_D$$

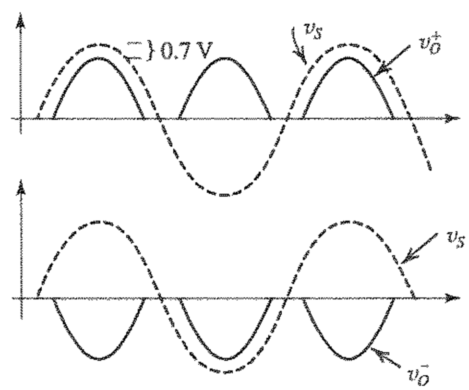
$$= 2 \times \frac{20\sqrt{2}}{2} \times 1.1 - 0.7$$

$= 30.4$ V

Using a factor of 1.5 for safety, we select a diode having a PIV rating of approximately 45 V.

4.76 The circuit is a full-wave rectifier with center tapped secondary winding. The circuit can be analyzed by looking at v_O^+ and v_O^- separately.





$$v_{O,\text{ avg}} = \frac{1}{2\pi}\int (V_S\sin\phi - 0.7)\,d\phi = 12$$

$$= \frac{2V_s}{\pi} - 0.7 = 12$$

where we have assumed $V_s \gg 0.7$ V and thus the conduction angle (in each half cycle) is almost π.

$$V_s = \frac{12 + 0.7}{2}\pi = 19.95 \text{ V}$$

Thus voltage across secondary winding

$= 2V_S \simeq 40$ V

Looking at D_4,

PIV$= V_S - V_O^-$

$= V_S + (V_S - 0.7)$

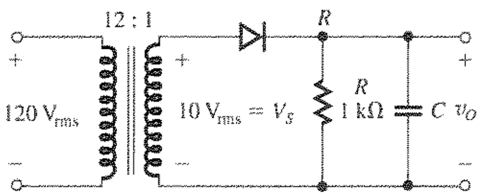
$= 2V_S - 0.7$

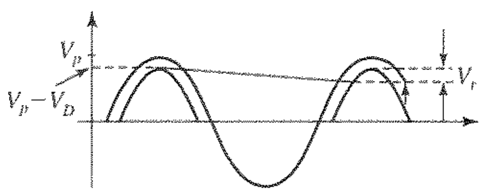
$= 39.2$ V

If choosing a diode, allow a safety margin by moving a factor of 1.5, thus

PIV $\simeq 60$ V

4.77





(i) $V_r \cong (V_p - V_D)\dfrac{T}{CR}$ [Eq. (4.28)]

$0.1\,(V_p - V_D) = (V_p - V_D)\dfrac{T}{CR}$

$C = \dfrac{1}{0.1 \times 60 \times 10^3} = 166.7\ \mu\text{F}$

(ii) For

$V_r = 0.01\,(V_p - V_D) = \dfrac{(V_p - V_D)\,T}{CR}$

$C = 1667\ \mu$F

(a) (i) $v_{O,\,\text{avg}} = V_p - V_D - \frac{1}{2}V_r$

$= 10\sqrt{2} - 0.7 - \frac{1}{2}\left(10\sqrt{2} - 0.7\right)0.1$

$= \left(10\sqrt{2} - 0.7\right)\left(1 - \frac{0.1}{2}\right)$

$= 12.77$ V

(ii) $v_{O,\,\text{avg}} = \left(10\sqrt{2} - 0.7\right)\left(1 - \frac{0.01}{2}\right)$

$= 13.37$ V

(b) (i) Using Eq (4.30), we have the conduction angle $=$

$\omega\Delta t \cong \sqrt{2V_r/\,(V_p - V_D)}$

$= \sqrt{\dfrac{2 \times 0.1\,(V_p - 0.7)}{(V_p - 0.7)}}$

$= \sqrt{0.2}$

$= 0.447$ rad

$\therefore$ Fraction of cycle for

conduction $= \dfrac{0.447}{2\pi} \times 100$

$= 7.1\%$

(ii) $\omega\Delta t \simeq \sqrt{2 \times 0.01\dfrac{(V_p - 0.7)}{V_p - 0.7}} = 0.141$ rad

Fraction of cycle $= \dfrac{0.141}{2\pi} \times 100 = 2.24\%$

(c) (i) Use Eq (4.31):

$i_{D,\text{avg}} = I_L\left(1 + \pi\sqrt{\dfrac{2\,(V_p - V_D)}{V_r}}\right)$

$= \dfrac{v_{O,\text{avg}}}{R}\left(1 + \pi\sqrt{\dfrac{2\,(V_p - V_D)}{0.1\,(V_p - V_D)}}\right)$

$= \dfrac{12.77}{10^3}\left(1 + \pi\sqrt{\dfrac{2}{0.1}}\right)$

$= 192$ mA

(ii) $i_{D,\text{avg}} = \dfrac{13.37}{10^3}\left(1 + \pi\sqrt{200}\right)$

$= 607$ mA

(d) Adapting Eq. (4.32), we obtain

(i) $i_{D,\text{peak}} = I_L\left(1 + 2\pi\sqrt{\dfrac{2\,(V_p - V_D)}{V_r}}\right)$

$= \dfrac{12.77}{10^3}\left(1 + 2\pi\sqrt{\dfrac{2}{0.1}}\right)$

$= 371$ mA

(ii) $i_{D,\text{peak}} = \dfrac{13.37}{10^3}\left(1 + 2\pi\sqrt{\dfrac{2}{0.01}}\right)$

$= 1201\text{ mA} \simeq 1.2$ A

4.78 (i) $V_r = 0.1\,(V_p - V_D) = \dfrac{(V_p - V_D)}{2fCR}$

The factor of 2 accounts for discharge occurring only during half of the period, $T/2 = \dfrac{1}{2f}$.

$$C = \frac{1}{(2fR)\,0.1} = \frac{1}{2\,(60)\,10^3 \times 0.1} = 83.3\ \mu\text{F}$$

(ii) $C = \dfrac{1}{2\,(60) \times 10^3 \times 0.01} = 833\ \mu\text{F}$

(a) (i) $V_O = V_p - V_D - \frac{1}{2}V_r$

$= (V_p - V_D)\left(1 - \frac{0.1}{2}\right)$

$= (13.44)\left(1 - \frac{0.1}{2}\right)$

$= 12.77$ V

(ii) $V_O = (13.44)\left(1 - \frac{0.01}{2}\right) = 13.37$ V

(b) (i) Fraction of cycle $= \dfrac{2\omega\Delta t}{2\pi} \times 100$

$= \dfrac{\sqrt{2V_r/(V_p - V_D)}}{\pi} \times 100$

$= \frac{1}{\pi}\sqrt{2\,(0.1)} \times 100 = 14.2\%$

(ii) Fraction of cycle $= \dfrac{2\sqrt{2\,(0.01)}}{2\pi} \times 100$

$= 4.5\%$

(c) Use Eq. (4.34):

(i) $i_{D,\,\text{avg}} = I_L\left(1 + \pi\sqrt{\dfrac{V_p - V_D}{2V_r}}\right)$

$= \dfrac{12.77}{1}\left(1 + \pi\sqrt{\dfrac{1}{2\,(0.1)}}\right) = 102.5$ mA

(ii) $i_{D,\,\text{avg}} = \dfrac{13.37}{1}\left(1 + \pi\dfrac{1}{\sqrt{2\,(0.01)}}\right)$

$= 310$ mA

(d) Use Eq. (4.35):

(i) $\hat{i}_D = I_L\left(1 + 2\pi\dfrac{1}{\sqrt{2\,(0.1)}}\right) = 192$ mA

(ii) $\hat{i}_D = I_L\left(1 + 2\pi\dfrac{1}{\sqrt{0.02}}\right) = 607$ mA

4.79 (i) $V_r = 0.1\,(V_p - V_D \times 2) = \dfrac{V_p - 2V_D}{2fCR}$

$C = \dfrac{(V_p - 2V_D)}{(V_p - 2V_D)}\,\dfrac{1}{2\,(0.1)\,fR} = 83.3\ \mu\text{F}$

(ii) $C = \dfrac{1}{2\,(0.01)\,fR} = 833\ \mu\text{F}$

(a) $V_O = V_p - 2V_D - \frac{1}{2}V_r$

(i) $V_O = V_p - 2V_D - \frac{1}{2}(V_p - 2V_D) \times 0.1$

$= (V_p - 2V_D) \times 0.95$

$= (10\sqrt{2} - 2 \times 0.7) \times 0.95 = 12.1$ V

(ii) $V_O = (10\sqrt{2} - 2 \times 0.7) \times 0.995 = 12.68$ V

(b) (i) Fraction of cycle $= \dfrac{2\omega\Delta t}{2\pi} \times 100$

$= \dfrac{\sqrt{2\,(0.1)}}{\pi} \times 100 = 14.2\%$

(ii) Fraction of cycle $= \dfrac{\sqrt{2\,(0.01)}}{\pi} \times 100 = 4.5\%$

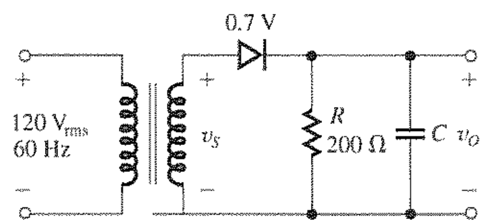
(c) (i) $i_{D,\,\text{avg}} = \dfrac{12.1}{1}\left(1 + \pi\sqrt{\dfrac{1}{0.2}}\right) = 97$ mA

(ii) $i_{D,\,\text{avg}} = \dfrac{12.68}{1}\left(1 + \pi/\sqrt{0.02}\right) = 249$ mA

(d) (i) $\hat{i}_D = \dfrac{12.1}{1}\left(1 + 2\pi\sqrt{\dfrac{1}{0.2}}\right) = 182$ mA

(ii) $\hat{i}_D = \dfrac{12.68}{1}\left(1 + 2\pi\sqrt{\dfrac{1}{0.02}}\right) = 576$ mA

4.80



$V_O = 12\text{ V} \pm 1\text{ V}$ (ripple)

$R_L = 200\ \Omega$

(a) $V_O = V_p - V_D - 1$

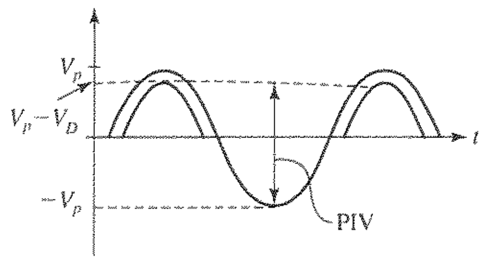
$\Rightarrow V_p = 13 + 0.7 = 13.7$ V

$V_{rms} = \dfrac{13.7}{\sqrt{2}} = 9.7$ V

(b) $V_r = \dfrac{V_p - V_D}{fCR}$

$2 = \dfrac{13.7 - 0.7}{60 \times C \times 200}$

$\Rightarrow C = \dfrac{13}{2 \times 60 \times 200} = 542\ \mu\text{F}$



(c) When the diode is cut off, the maximum reverse voltage across it will occur when $v_S = -V_p$. At this time, $v_O = V_O$ and the maximum reverse voltage will be

Maximum reverse voltage $= V_O + V_p$

$= 12 + 13.7 = 25.7$ V

Using a factor of safety of 1.5 we obtain

PIV $= 1.5 \times 25.7$

$= 38.5$ V

(d) $i_{Dav} = I_L\left[1 + \pi\sqrt{\dfrac{2(V_p - V_D)}{V_r}}\right]$

$= \dfrac{V_O}{R_L}\left[1 + \pi\sqrt{\dfrac{2(V_p - V_D)}{V_r}}\right]$

$= \dfrac{12}{0.2}\left[1 + \pi\sqrt{\dfrac{2(13.7 - 0.7)}{2}}\right]$

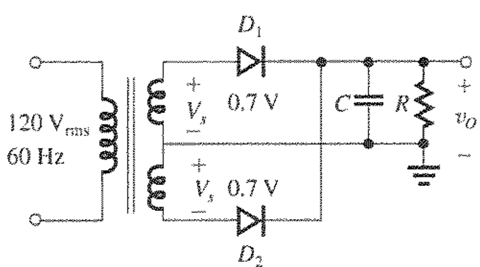
$= 739$ mA

(e) $i_{Dmax} = I_L\left[1 + 2\pi\sqrt{\dfrac{2(V_p - V_D)}{V_r}}\right]$

$= \dfrac{12}{0.2}\left[1 + 2\pi\sqrt{\dfrac{2(13.7 - 0.7)}{2}}\right]$

$= 1.42$ A

4.81

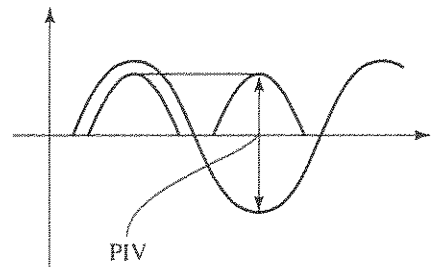


(a) $V_O = V_p - V_D - 1$

$\Rightarrow V_p = V_O + V_D + 1 = 13 + 0.7 = 13.7$ V

$V_{rms} = \dfrac{13.7}{\sqrt{2}} = 9.7$ V

This voltage appears across each half of the transformer secondary. Across the entire secondary we have $2 \times 9.7 = 19.4$ V (rms).



(b) $V_r = \dfrac{V_p - V_D}{2fCR}$

$2 = \dfrac{13.7 - 0.7}{2 \times 60 \times 200 \times C}$

$\Rightarrow C = \dfrac{12}{2 \times 2 \times 60 \times 200} = 271\ \mu$F

(c) Maximum reverse voltage across D_1 occurs when $v_S = -V_p$. At this point $v_O = V_O$. Thus maximum reverse voltage $= V_O + V_p =$ $12 + 13.7 = 25.7$. The same applies to D_2.

In specifying the PIV for the diodes, one usually uses a factor of safety of about 1.5,

PIV $= 1.5 \times 25.7 = 38.5$ V

(d) $i_{Dav} = I_L\left[1 + \pi\sqrt{\dfrac{V_p - V_D}{2\ V_r}}\right]$

$= \dfrac{12}{0.2}\left[1 + \pi\sqrt{\dfrac{13.7 - 0.7}{2 \times 2}}\right]$

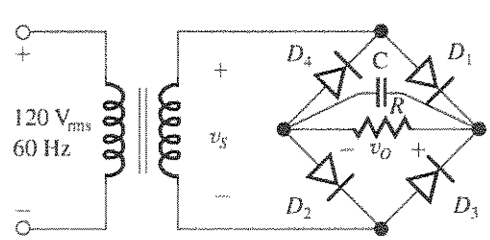
$= 399$ mA

(e) $i_{Dmax} = I_L\left[1 + 2\pi\sqrt{\dfrac{V_p - V_D}{2\ V_r}}\right]$

$= \dfrac{12}{0.2}\left[1 + 2\pi\sqrt{\dfrac{13.7 - 0.7}{2 \times 2}}\right]$

$= 739$ mA

4.82



(a) $V_O = V_p - 2V_D - 1$

$\Rightarrow V_p = V_O + 2V_D + 1 = 12 + 2 \times 0.7 + 1 = 14.4$ V

$$V_{rms} = \frac{14.4}{\sqrt{2}} = 10.2 \text{ V}$$

(b) $V_r = \dfrac{V_p - 2\,V_D}{2fCR}$

$$\Rightarrow C = \frac{14.4 - 1.4}{2 \times 2 \times 60 \times 200} = 271\ \mu\text{F}$$

(c) The maximum reverse voltage across D_1 occurs when $V_s = -V_p = -14.4$ V. At this time D_3 is conducting, thus

Maximum reverse voltage $= -V_p + V_{D3}$
$= -14.4 + 0.7 = -13.7$ V

The same applies to the other three diodes. In specifying the PIV rating for the diode we use a factor of safety of 1.5 to obtain

PIV $= 1.5 \times 13.7 = 20.5$ V

(d) $i_{Dav} = I_L\left[1 + \pi\sqrt{\dfrac{V_p - 2\,V_D}{2\,V_r}}\right]$

$$= \frac{12}{0.2}\left[1 + \pi\sqrt{\frac{14.4 - 1.4}{2 \times 2}}\right]$$

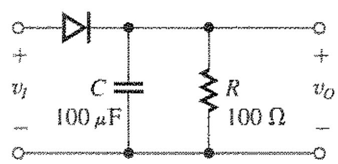
$= 400$ mA

(e) $i_{Dmax} = I_L\left[1 + 2\pi\sqrt{\dfrac{V_p - 2\,V_D}{2\,V_r}}\right]$

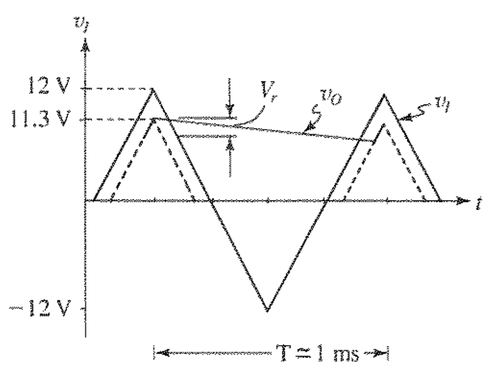
$$= \frac{12}{0.2}\left[1 + 2\pi\sqrt{\frac{14.4 - 0.7}{2 \times 2}}\right]$$

$= 740$ mA

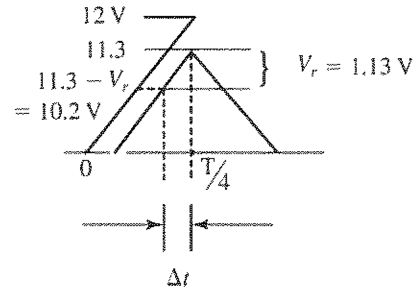
4.83



(a)



(b)



(c)

During the diode's off interval (which is almost equal to T) the capacitor discharges and its voltage is given by

$v_O(t) = 11.3\,e^{-t/CR}$

where $C = 100\ \mu$F and $R = 100\ \Omega$, thus

$CR = 100 \times 10^{-6} \times 100 = 0.01$ s

At the end of the discharge interval, $t \simeq T$ and

$v_O = 11.3\,e^{-T/CR}$

Since $T = 0.001$ s is much smaller than CR,

$$v_O \simeq 11.3\left(1 - \frac{T}{CR}\right)$$

The ripple voltage V_r can be found as

$$V_r = 11.3 - 11.3\left(1 - \frac{T}{CR}\right)$$

$$= \frac{11.3T}{CR} = \frac{11.3 \times 0.001}{0.01} = 1.13 \text{ V}$$

The average dc output voltage is

$$v_O = 11.3 - \frac{V_r}{2} = 11.3 - \frac{1.13}{2} = 10.74 \text{ V}$$

To obtain the interval during which the diode conducts, Δt, refer to Fig. (c).

$$\frac{12}{T/4} = \frac{V_r}{\Delta t}$$

$$\Rightarrow \Delta t = \frac{V_r \times (T/4)}{12} = \frac{1.13 \times 1}{12 \times 4}$$

$= 23.5\ \mu$s

Now, using the fact that the charge gained by the capacitor when the diode is conducting is equal to the charge lost by the capacitor during its discharge interval, we can write

$i_{Cav} \times \Delta t = C\,V_r$

$$\Rightarrow i_{Cav} = \frac{C\,V_r}{\Delta t} = \frac{100 \times 10^{-6} \times 1.13}{23.5 \times 10^{-6}} = 4.8 \text{ A}$$

$i_{Dav} = i_{Cav} + i_{Lav}$

where i_{Lav} is the average current through R during the short interval Δt. This is approximately $\dfrac{11.3}{R} = \dfrac{11.3}{100} = 0.113$ A. Thus

$i_{Dav} = 4.8 + 0.113 = 4.913$ A

Finally, to obtain the peak diode current, we use

$$i_{D\max} = i_{C\max} + i_{L\max}$$
$$= C\frac{dv_I}{dt} + \frac{11.3}{R}$$
$$= C \times \frac{12}{T/4} + \frac{11.3}{R}$$
$$= 100 \times 10^{-6} \times \frac{12 \times 4}{1 \times 10^{-3}} + \frac{11.3}{100}$$
$$= 4.8 + 0.113 = 4.913 \text{ A}$$

which is equal to the average value. This is a result of the linear v_I which gives rise to a constant capacitor current during the diode conduction interval. Thus $i_{C\max} = i_{Cav} = 4.8$ A. Also, the maximum value of i_L is approximately equal to its average value during the short interval Δt.

4.84 Refer to Fig. P4.76 and let a capacitor C be connected across each of the load resistors R. The two supplies v_O^+ and v_O^- are identical. Each is a full-wave rectifier similar to that of the tapped-transformer circuit. For each supply,

$V_O = 12$ V

$V_r = 1$ V (peak to peak)

Thus

$v_O = 12 \pm 0.5$ V

It follows that the peak value of v_S must be $12.5 + 0.7 = 13.2$ V and the total rms voltage across the secondary will be

$$= \frac{2 \times 13.2}{\sqrt{2}} = 18.7 \text{ V (rms)}$$

$$\text{Transformer turns ratio} = \frac{120}{18.7} = 6.43{:}1$$

To deliver 100-mA dc current to each load,

$$R = \frac{12}{0.1} = 120\ \Omega$$

Now, the value of C can be found from

$$V_r = \frac{V_p - 0.7}{2fCR}$$
$$1 = \frac{12.5}{2 \times 60 \times C \times 120}$$
$$\Rightarrow C = 868\ \mu\text{F}$$

To specify the diodes, we determine i_{Dav} and $i_{D\max}$.

$$i_{Dav} = I_L(1 + \pi\sqrt{(V_p - 0.7)/2\,V_r}\,)$$
$$= 0.1(1 + \pi\sqrt{12.5/2}\,)$$
$$= 785 \text{ mA}$$

$$i_{D\max} = I_L(1 + 2\pi\sqrt{(V_p - 0.7)/2\,V_r}\,)$$
$$= 0.1(1 + 2\pi\sqrt{12.5/2}\,)$$
$$= 1.671 \text{ A}$$

To determine the required PIV rating of each diode, we determine the maximum reverse voltage that appears across one of the diodes, say D_1. This occurs when v_S is at its maximum negative value $-V_p$. Since the cathode of D_1 will be at $+12.5$ V, the maximum reverse voltage across D_1 will be $12.5 + 13.2 = 25.7$ V. Using a factor of safety of 1.5, then each of the four diodes must have

$\text{PIV} = 1.5 \times 25.7 = 38.6$ V

4.85 Refer to Fig. P4.85. When v_I is positive, v_A goes positive, turning on the diode and closing the negative feedback loop around the op amp. The result is that $v_- = v_I$, $v_O = 2v_- = 2v_I$, and $v_A = v_O + 0.7$. Thus

(a) $v_I = +1$ V, $v_- = +1$ V, $v_O = +2$ V, and $v_A = +2.7$ V.

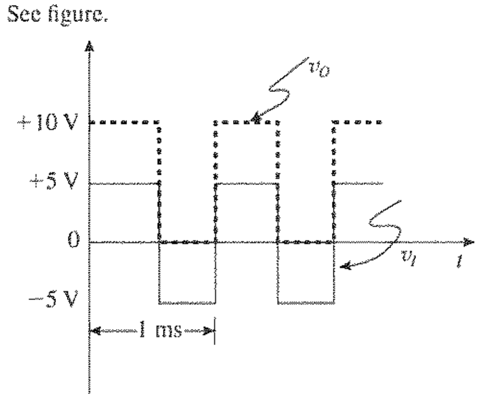
(b) $v_I = +3$ V, $v_- = +3$ V, $v_O = +6$ V, and $v_A = +6.7$ V.

When v_I goes negative, v_A follows, the diode turns off, and the feedback loop is opened. The op amp saturates with $v_A = -13$ V, $v_- = 0$ V and $v_O = 0$ V. Thus

(c) $v_I = -1$ V, $v_- = 0$ V, $v_O = 0$ V, and $v_A = -13$ V.

(d) $v_I = -3$ V, $v_- = 0$ V, $v_O = 0$ V, and $v_A = -13$ V.

Finally, if v_I is a symmetrical square wave of 1-kHz frequency, 5-V amplitude, and zero average, the output will be zero during the negative half cycles of the input and will equal twice the input during the positive half cycles. See figure.

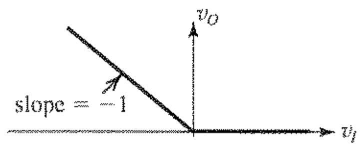


Thus, v_O is a square wave with 0-V and +10-V levels, i.e. 5-V average and, of course, the same frequency (1 kHz) as the input.

4.86 $v_I > 0$: D_1 conducts and D_2 cutoff

$v_I < 0$: D_1 cutoff,

D_2 conducts $\sim \dfrac{v_O}{v_I} = -1$



(a) $v_I = +1$ V

$v_O = 0$ V

$v_A = -0.7$ V

Keeps D_2 off so no current flows through R

$\Rightarrow v_- = 0$ V

Virtual ground as feedback loop is closed through D_1

(b) $v_I = +3$ V

$v_O = 0$ V

$v_A = -0.7$ V

$v_- = 0$ V

(c) $v_I = -1$ V

$v_O = +1$ V

$v_A = 1.7$ V

$v_- = 0$ V

$\sim$ Virtual ground as negative feedback loop is closed through D_2 and R.

(d) $v_I = -3$ V $\Rightarrow v_O = +3$ V

$v_A = +3.7$ V

$v_- = 0$ V

4.87 (a) See figure (a) on next page. For $v_I \leq 3.5$ V, $i = 0$ and $v_O = v_I$. At $v_I = 3.5$ V, the diode begins to conduct. At $v_O = 3.7$ V, the diode is conducting $i = 1$ mA and thus

$v_I = v_O + i \times 1\ \text{k}\Omega = 4.7$ V

For $v_I > 4.7$ V the diode current increases but the diode voltage remains constant at 0.7 V, thus v_O flattens and v_O vs. v_I becomes a horizontal line.

In practice, the diode voltage increases slowly and the line will have a small nonzero slope.

(b) See figure (b) on next page. Here $v_O = v_I$ for $v_I \geq 2.5$ V. At $v_I = 2.5$ V, $v_O = 2.5$ V and the diode begins to conduct. The diode will be conducting 1 mA and exhibiting a drop of 0.7 at $v_O = 2.3$ V. The corresponding value of v_I

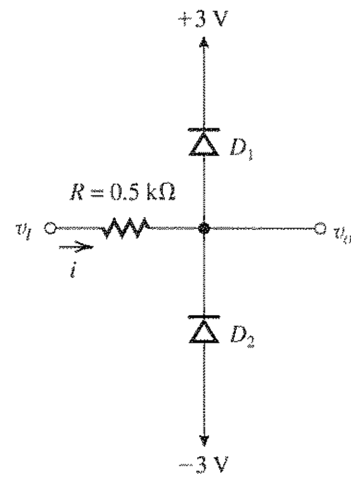
$v_I = v_O - iR = 2.3 - 1 \times 1 = +1.3$ V

As v_I decreases below 1.3 V, the diode current increases, but the diode voltage remains constant at 0.7 V. Thus v_O flattens at about 2.3 V.

(c) See figure (c) on next page. For $v_I \leq -2.5$ V, the diode is off, and $v_O = v_I$. At $v_I = -2.5$ V the diode begins to conduct and its current reaches 1 mA at $v_I = -1.3$ V (corresponding to $v_O = -2.3$ V). As v_I further increases, the diode current increases but its voltage remains constant at 0.7 V. Thus v_O flattens, as shown.

(d) See figure (d) on next page.

4.88



(a)

From Fig. (a) we see that for $-3.5\text{ V} \leq v_I \leq +3.5$ V, diodes D_1 and D_2 will be cut off and $i = 0$. Thus, $v_O = v_I$. For $v_I \geq +3.5$ V, diode D_1 begins to conduct and its voltage reaches 0.7 V (and thus $v_O = +3.7$ V) at $i = 1$ mA. The corresponding value of v_I is

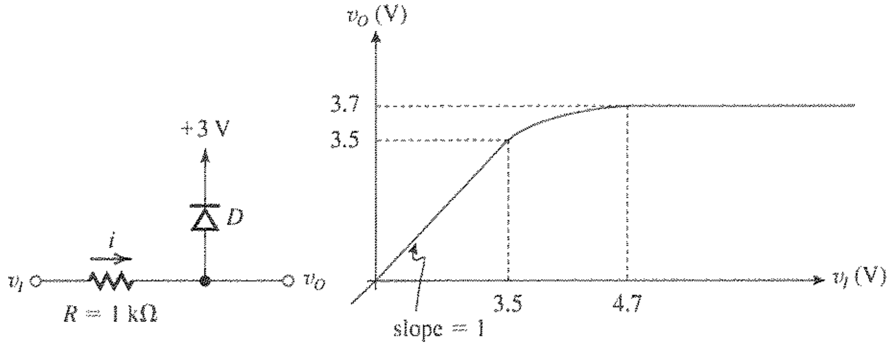
$v_I = v_O - iR$

$v_I = 3.7 + 1 \times 0.5 = +4.2$ V

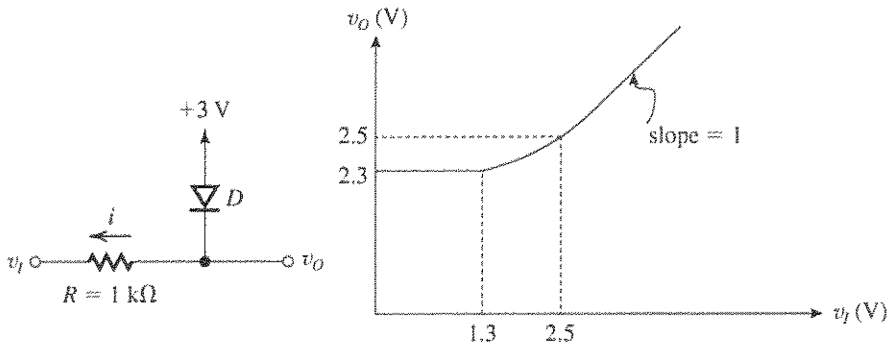
For $v_I \geq 4.2$ V, the voltage of diode D_1 remains 0.7 V and v_O saturates at $+3.7$ V.

A similar description applies for $v_I \leq -3.5$ V. Here D_2 conducts at $v_I = -3.5$ V and its voltage becomes 0.7 V, and hence $v_O = -3.7$ V, at $i = 1$ mA (in the direction into v_I) at $v_I = -4.2$ V. For $v_I \leq -4.2$ V, $v_O = -3.7$ V.

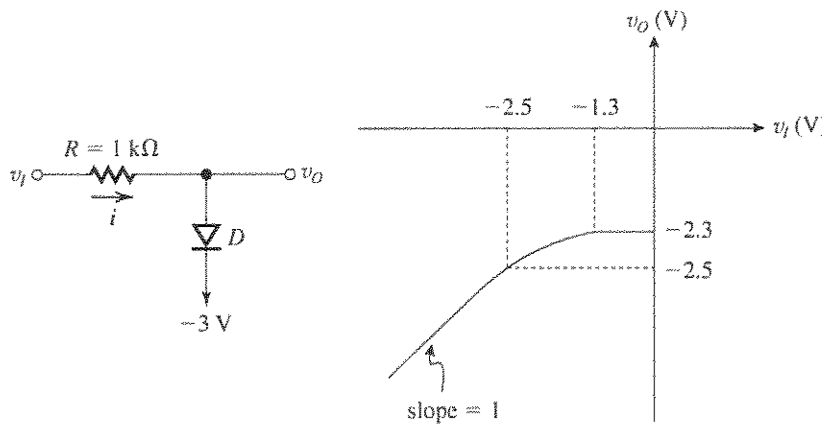
These figures belong to Problem 4.87.



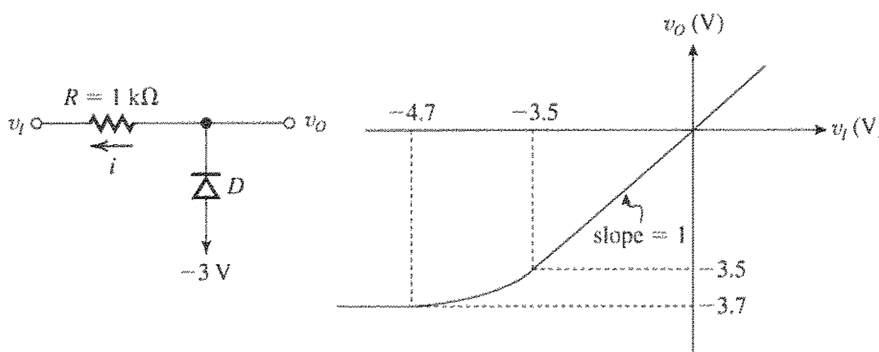
(a)



(b)

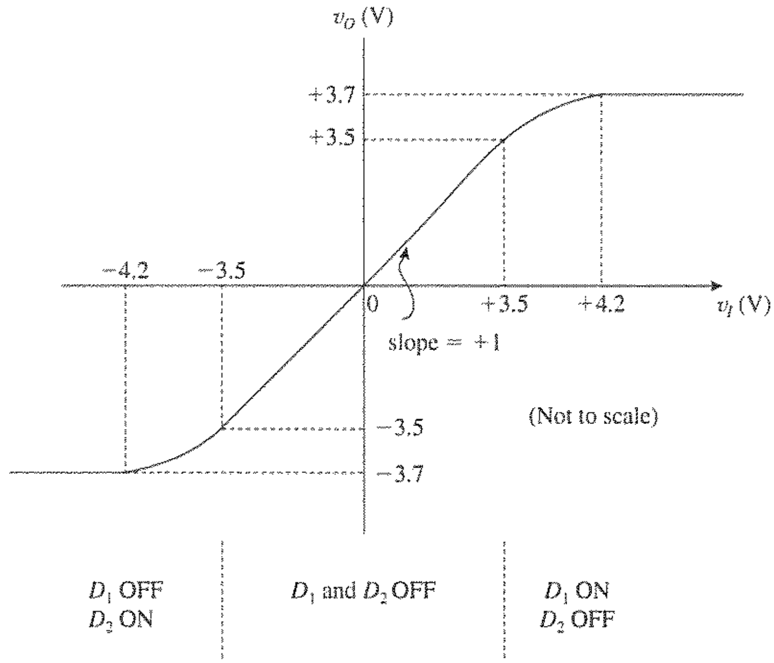


(c)



(d)

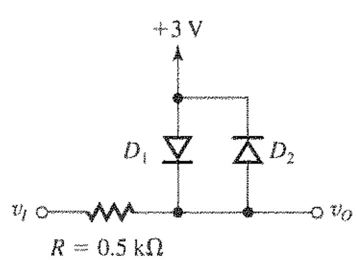
This figure belongs to Problem 4.88, part b.



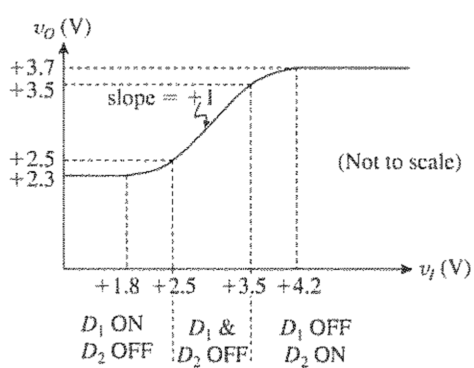
(b)

Figure (b) shows a sketch of the transfer characteristic of this double limiter.

4.89 See figure.

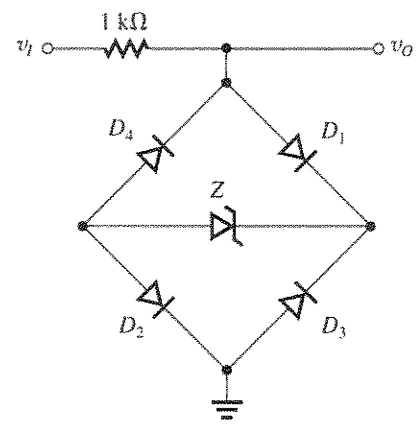


(a)



(b)

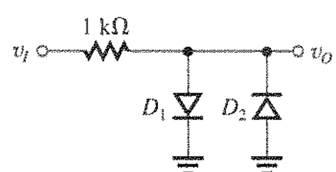
4.90



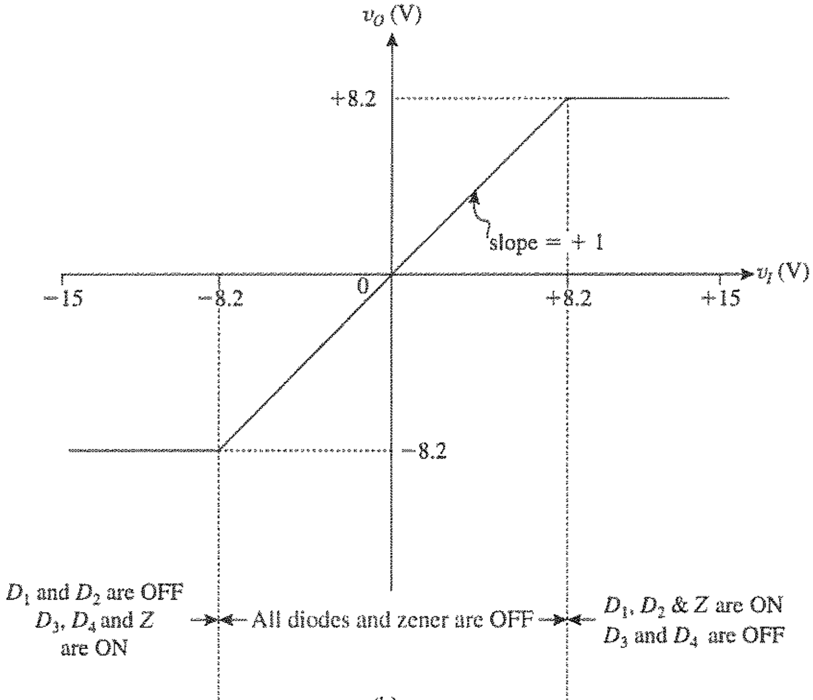
(a)

The limiter thresholds and the output saturation levels are found as $2 \times 0.7 + 6.8 = 8.2$ V. The transfer characteristic is given in Fig. (b). See figure on next page.

4.91



This figure belong to Problem 4.90, part b.



(b)

Diodes have 0.7 V drop at 1 mA current

$\therefore$ For diode D_1

$$\frac{i_D}{1\text{ mA}} = e^{(v_O-0.7)/V_T}$$

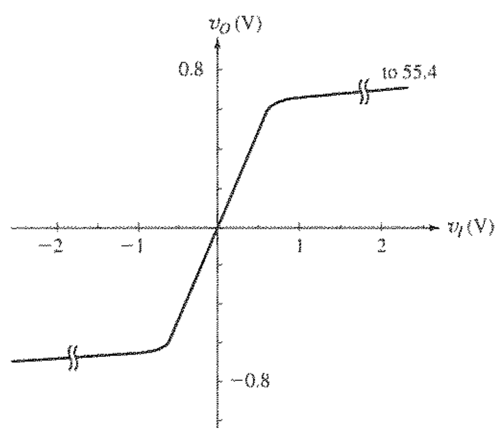
$$i_D = 1 \times 10^{-3}\, e^{(v_O-0.7)/V_T}$$

$$v_O = 0.7 + V_T \ln\left(\frac{i_D}{1\text{ mA}}\right)$$

$$v_I = v_O + i_D \times 1\text{ k}\Omega$$

Using these equations, calculate v_I for the different values of v_O. For D_2,
$v_I = v_O - i_D \times 1\text{ k}\Omega$

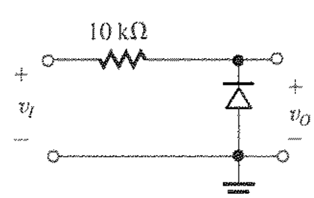
	v_O (V)	v_I (V)
D_1 on	0.5 V	0.5003
	0.6	0.62
	0.7	1.7
	0.8	55.4
	0	0
D_2 on	–0.5	–0.5003
	–0.6	–0.62
	–0.7	–1.7
	–0.8	–55.4



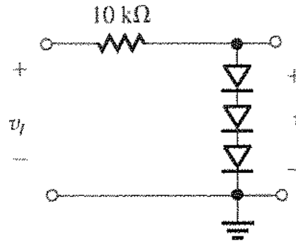
It is a soft limiter with a gain K $\simeq$ 1 and $L_+ \simeq 0.7$ V, $L_- \simeq -0.7$ V

4.92

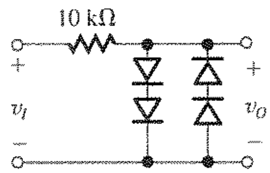
(a)



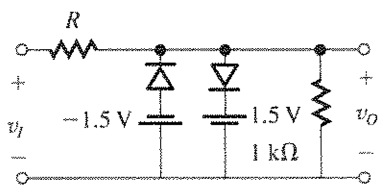
(b)



(c)



4.93



In the nonlimiting region

$$\frac{v_O}{v_I} = \frac{1000}{1000+R} \geq 0.94$$

$R \leq 63.8\ \Omega$

4.94

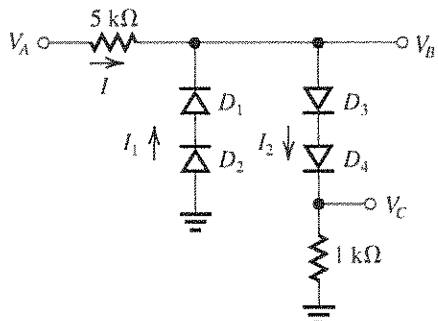


Figure 1

When $V_A > 0$, D_1 and D_2 are cut off and D_3 and D_4 conduct a current I_2. Since the diodes are 0.1-mA devices, the current I_2 is related to the diode voltage V_D as follows:

$$I_2 = 0.1 \times e^{(V_D - 0.7)/0.025},\ \text{mA} \qquad (1)$$

The voltage V_C is given by

$$V_C = I_2 \times 1\ \text{k}\Omega = I_2,\ \text{V} \qquad (2)$$

where I_2 is in mA, and the voltage V_B is given by

$$V_B = 2V_D + V_C \qquad (3)$$

and the voltage V_A is given by

$$V_A = V_B + I \times 5\text{k}\Omega$$

$$V_A = V_B + 5I_2 \qquad (4)$$

Equations (1), (2), (3), and (4) can be used to find V_B and V_C versus V_A. We start with a value for V_D, use (1) to determine I_2, use (2) to determine V_C, use (3) to determine V_B, and finally use (4) to determine V_A. The results are given in Table 1.

Table 1

V_{D3}, V_{D4} (V)	I_2 (mA)	V_C (V)	$V_B = V_C + V_{D3} + V_{D4}$	V_A (V)
0.4	$\simeq 0$	0	0.8	0.8
0.5	0.00003	$\simeq 0$	1.0	1.0
0.6	0.002	0.002	1.202	1.212
0.7	0.1	0.1	1.5	2.0
0.73	0.332	0.332	1.792	3.452
0.735	0.406	0.406	1.876	3.91
0.74	0.495	0.495	1.975	4.45
0.745	0.605	0.605	2.095	5.12

For $V_A < 0$, D_3 and D_4 are cutoff, $I_2 = 0$, $V_C = 0$, and D_1 and D_2 are conducting a current I_1.

$$I_1 = 0.1\ e^{(V_D - 0.7)/0.025},\ \text{mA} \qquad (5)$$

The voltage V_B is given by

$$V_B = -2V_D \qquad (6)$$

and the voltage V_A is

$$V_A = V_B - 5I_1 \qquad (7)$$

Equations (5)–(7) can be used to obtain V_B versus V_A for negative values of V_A. The results are given in Table 2.

Table 2

V_{D1}, V_{D2} (V)	I_1 (mA)	V_B (V)	V_A (V)
0.4	$\simeq 0$	−0.8	−0.8
0.5	$\simeq 0$	−1.0	−1.0
0.6	0.002	−1.2	−1.21
0.7	0.1	−1.4	−1.9
0.73	0.332	−1.46	−3.12
0.74	0.495	−1.48	−3.955
0.75	0.739	−1.5	−5.20

Figure 2 shows plots for V_B and V_C versus V_A using the data in Tables 1 and 2. Finally, Figure 3 shows the waveforms obtained at B and C when a 5-V peak, 100-Hz sinusoid is applied at A.

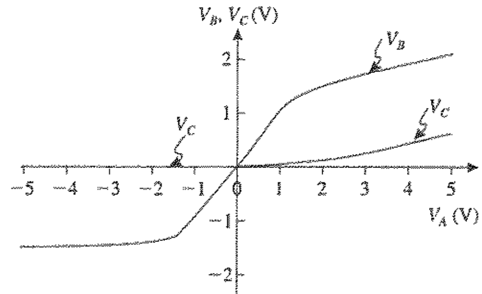


Figure 2

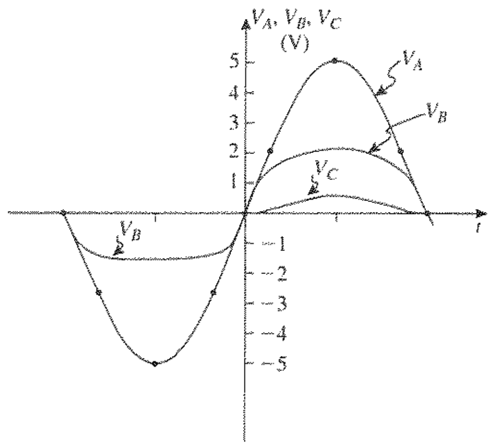


Figure 3

4.95 Refer to the circuit in Fig. P4.95. For $v_I > 0$, D_2 and D_3 are cut off, and the circuit reduces to that in Fig. 1.

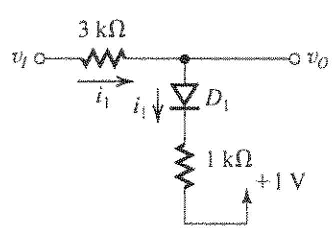


Figure 1

Now, for $v_I < 1.5$ V, diode D_1 will be off, $i_1 \simeq 0$, and

$$v_O = v_I$$

As v_I exceeds 1.5 V, diode D_1 will turn on and

$$v_{D1} = 0.7 + 0.025 \ln\left(\frac{i_1}{1}\right) \quad (1)$$

$$v_O = 1 + v_{D1} + i_1 \times 1 = 1 + v_{D1} + i_1 \quad (2)$$

$$v_I = v_O + 3i_1 \quad (3)$$

where i_1 is in mA and v_{D1}, v_O and v_I are in volts. Using these relationships we obtain:

i_1 (mA)	v_{D1} (V)	v_O (V)	v_I (V)
0.01	0.584	1.594	1.625
0.1	0.642	1.742	2.042
0.2	0.660	1.860	2.460
0.5	0.682	2.182	3.682
1.0	0.700	2.700	5.700
1.5	0.710	3.210	7.710
1.8	0.715	3.515	8.915
2.0	0.717	3.717	9.717

For $v_I < 0$, D_1 will be cut off and the circuit reduces to that in Fig. 2.

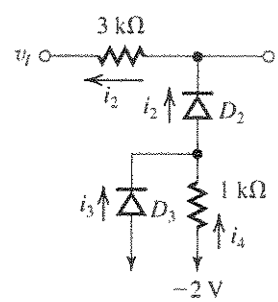


Figure 2

Here, for $v_I < -2.5$, D_2 will be cut off, $i_2 \simeq 0$ and D_3 also will be cut off, and

$$v_O = v_I$$

As v_I reduces below about −2.5 V, D_2 begins to conduct and eventually D_3 also conducts. The details of this segment of the $v_O - v_I$ characteristic can be obtained using the following relationships:

$$v_{D3} = 0.7 + 0.025 \ln\left(\frac{i_3}{1}\right)$$

$$i_4 = \frac{v_{D3}}{1\ \text{k}\Omega} = v_{D3}$$

$$i_2 = i_3 + i_4$$

$$v_{D2} = 0.7 + 0.025 \ln\left(\frac{i_2}{1}\right)$$

$$v_O = -2 - v_{D3} - v_{D2}$$

$$v_I = v_O - 3i_2$$

This figure belongs to Problem 4.95, part c.

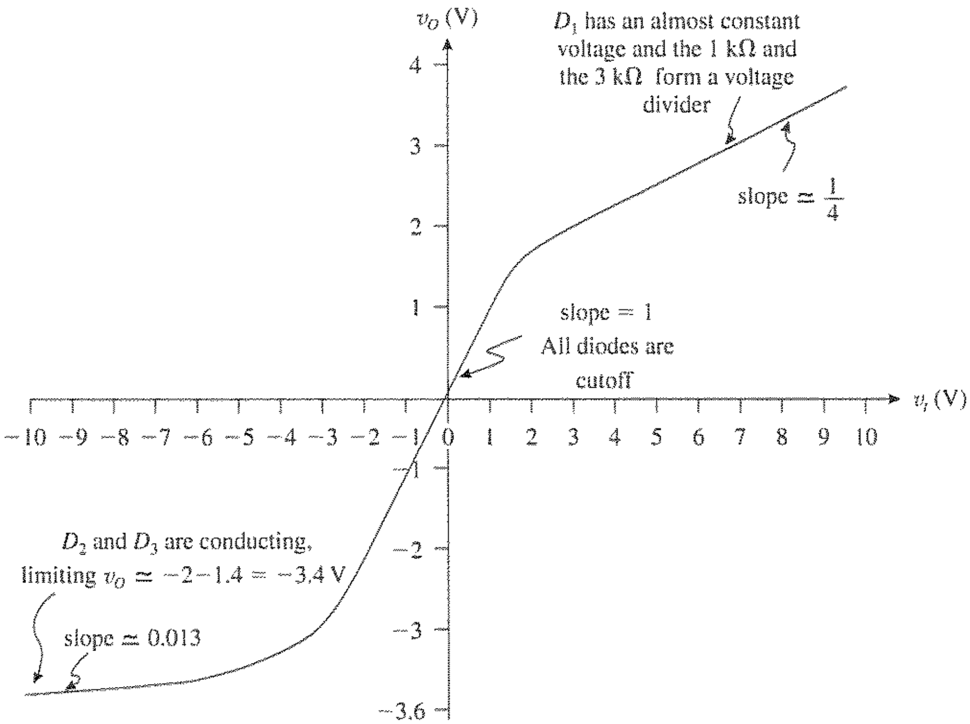


Figure 3

In all these equations, currents are in mA and voltages are in volts. The numerical results obtained are as follows:

i_3	v_{D3}, i_4	i_2	v_{D2}	v_O	v_I
≃ 0	0.01	0.01	0.585	−2.60	−2.63
≃ 0	0.05	0.05	0.625	−2.68	−2.85
≃ 0	0.1	0.1	0.642	−2.74	−3.04
≃ 0	0.2	0.2	0.660	−2.86	−3.46
≃ 0	0.3	0.3	0.670	−2.97	−3.87
≃ 0	0.4	0.4	0.677	−3.08	−4.28
≃ 0	0.5	0.5	0.683	−3.18	−4.68
0.01	0.585	0.595	0.687	−3.28	−5.07
0.1	0.642	0.742	0.693	−3.35	−5.56
0.2	0.660	0.860	0.696	−3.36	−5.94
0.5	0.682	1.182	0.704	−3.38	−6.93
1.0	0.700	1.700	0.713	−3.41	−8.51
1.5	0.710	2.210	0.720	−3.43	−10.06

The complete transfer characteristic v_O versus v_I can be plotted using the data in the tables above. The result is displayed in Fig. 3.

Slopes: For v_I near +10 V the slope is approximately determined by the voltage divider composed of the 1 kΩ and the 3 kΩ,

$$\text{Slope} = \frac{1}{3+1} = 0.25 \text{ V/V}$$

For v_I near −10 V, the slope is approximately given by

$$\text{Slope} \simeq -\frac{r_{d2} + r_{d3}}{r_{d2} + r_{d3} + 3\text{ k}\Omega}$$

From the table above,

$$i_3 \simeq 1.5 \text{ mA} \quad \Rightarrow \quad r_{d3} = \frac{25}{1.5} = 16.7\ \Omega$$

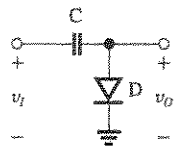
$$i_2 \simeq 2.21 \text{ mA} \quad \Rightarrow \quad r_{d2} = \frac{25}{2.21} = 11.3\ \Omega$$

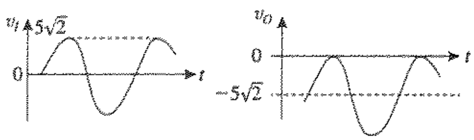
Thus

$$\text{Slope} = \frac{11.3 + 16.7}{11.3 + 16.7 + 3000} = 0.009 \text{ V/V}$$

which is reasonably close to the value found from the graph.

4.96



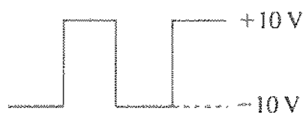


From the figure we see that

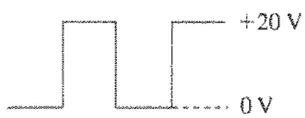
$v_{Oav} = -5\sqrt{2} = -7.07$ V

4.97

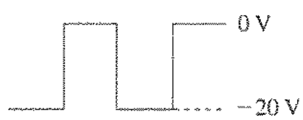
(a)



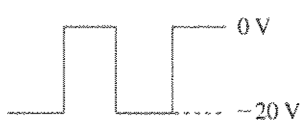
(b)



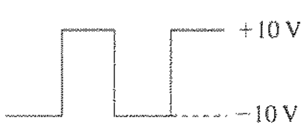
(c)



(d)



(e)

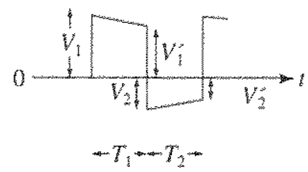


(f) Here there are two different time constants involved. To calculate the output levels, we shall consider the discharge and charge wave forms.

During T_1, $\quad v_O = V_1 e^{-t/RC}$

At $t = T_1 = T, v_O = V_1'$

$= V_1 e^{-T/RC}$



where for $T \ll CR$

$V_1' \simeq V_1(1 - T/CR) = V_1(1 - \alpha)$

where $\alpha \ll 1$

During the interval T_2, we have

$|v_O| = |V_2|\, e^{-t/(CR/2)}$

At the end of $T_2, t = T$, and $v_O = |V_2'|$

where

$|V_2'| = |V_2|\, e^{-T/(CR/2)}$

$\simeq |V_2|\left(1 - \dfrac{T}{RC/2}\right) = |V_2|\,(1 - 2\alpha)$

Now

$V_1' + |V_2| = 20 \Rightarrow V_1 + |V_2| - \alpha V_1 = 20 \qquad (1)$

and

$\left|V_2'\right| + V_1 = 20 \Rightarrow V_1 + |V_2| - 2\alpha\,|V_2| = 20 \quad (2)$

From (1) and (2) we find that

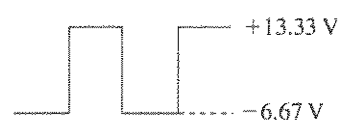
$V_1 = 2|V_2|$

Then using (1) and neglecting αV_1 yields

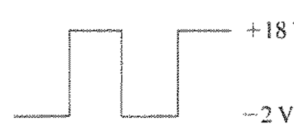
$3\,|V_2| = 20 \Rightarrow \ |V_2| = 6.67$ V

$V_1 = 13.33$ V

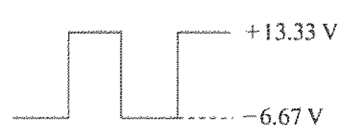
The result is



(g)



(h) Using a method similar to that employed for case (f) above, we obtain



5.1 $t_{ox} = 2 \sim 10$ nm

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

$\epsilon_{ox} = 34.5$ pF/m

$$C_{ox}^{-1} = 58 \sim 290\ \text{m}^2/\text{F}\left(\frac{\mu\text{m}^2}{\text{pF}}\right)$$

For 10 pF:

Area $= 580 \sim 2900\ (\mu\text{m}^2)$

so

$d = 24 \sim 54\ \mu$m

5.2 $C_{ox} = 9\ \text{fF}/\mu\text{m}^2,\ V_{OV} = 0.2$ V

$L = 0.36\ \mu$m, $V_{DS} = 0$ V

$W = 3.6\ \mu$m

$Q = C_{ox}.W.L.V_{OV} = 2.33$ fC

5.3 $k_n' = \mu_n C_{ox}$

$$= \frac{\text{m}^2}{\text{V}\cdot\text{s}}\ \frac{\text{F}}{\text{m}^2} = \frac{\text{F}}{\text{V}\cdot\text{s}} = \frac{\text{C/V}}{\text{V}\cdot\text{s}} = \frac{\text{C}}{\text{s}}\ \frac{1}{\text{V}^2}$$

$$= \frac{\text{A}}{\text{V}^2}$$

Since $k_n = k_n' W/L$ and W/L is dimensionless, k_n has the same dimensions as k_n'; that is, A/V^2.

5.4 With v_{DS} small, compared to V_{OV}, Eq. (5.13a) applies:

$$r_{DS} = \frac{1}{(\mu_n C_{ox})\left(\dfrac{W}{L}\right)(V_{OV})}$$

(a) V_{OV} is doubled $\rightarrow r_{DS}$ is halved, factor $= 0.5$

(b) W is doubled $\rightarrow r_{DS}$ is halved, factor $= 0.5$

(c) W and L are doubled $\rightarrow r_{DS}$ is unchanged, factor $= 1.0$

(d) If oxide thickness t_{ox} is halved, and

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

then C_{ox} is doubled. If W and L are also halved, r_{DS} is halved, factor $= 0.5$.

5.5 The transistor size will be minimized if W/L is minimized. To start with, we minimize L by using the smallest feature size,

$L = 0.18\ \mu$m

$$r_{DS} = \frac{1}{k_n'(W/L)(v_{GS} - V_t)}$$

$$r_{DS} = \frac{1}{k_n'(W/L)\, v_{OV}}$$

Two conditions need to met for v_{OV} and r_{DS}

Condition 1:

$$r_{DS,1} = \frac{1}{400 \times 10^{-6}\,(W/L) v_{OV,1}}$$

$= 250 \Rightarrow (W/L)\, v_{OV,1} = 10$

Condition 2:

$$r_{DS,2} = \frac{1}{400 \times 10^{-6}\,(W/L)\, v_{OV,2}}$$

$= 1000 \Rightarrow (W/L)\, v_{OV,2} = 2.5$

If condition 1 is met, condition 2 will be met since the over-drive voltage can always be reduced to satisfy this requirement. For condition 1, we want to decrease W/L as much as possible (so long as it is greater than or equal to 1), while still meeting all of the other constraints. This requires our using the largest possible $v_{GS,1}$ voltage. $v_{GS,I} = 1.8$ V so $v_{OV,I} = 1.8 - 0.5 = 1.3$ V, and

$$W/L = \frac{10}{v_{OV,1}} = \frac{10}{1.3} = 7.69$$

Condition 2 now can be used to find $v_{GS,2}$

$$v_{OV,2} = \frac{2.5}{W/L} = \frac{2.5}{7.69} = 0.325$$

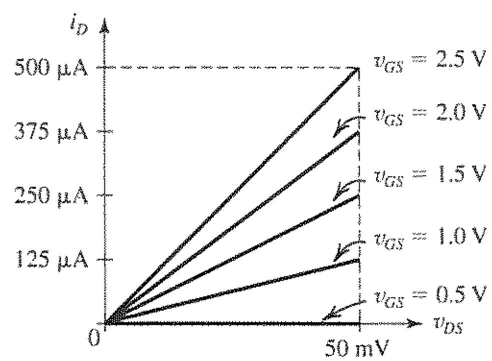
$\Rightarrow v_{GS,2} = 0.825\ \text{V} \Rightarrow 0.825\ \text{V} \le v_{GS} \le 1.8$ V

5.6 $k_n = 5\ \text{mA/V}^2$, $V_{tn} = 0.5$ V,

small v_{DS}

$i_D = k_n(v_{GS} - V_t)\, v_{DS} = k_n v_{OV} v_{DS}$

$$g_{DS} = \frac{1}{r_{DS}} = k_n v_{OV}$$



This table belongs to Exercise 5.6.

V_{GS} (V)	V_{OV} (V)	g_{DS} (mA/V)	r_{DS} (Ω)
0.5	0	0	∞
1.0	0.5	2.5	400
1.5	1.0	5.0	200
2.0	1.5	7.5	133
2.5	2.0	10	100

5.7 $t_{ox} = 4$ nm, $V_t = 0.5$ V

$L_{min} = 0.18\ \mu$m, small v_{DS},

$k'_n = 400\ \mu A/V^2$, $0 < v_{GS} < 1.8$ V.

$$r_{DS}^{-1} = k'_n\ W/L\ (v_{GS} - V_t) \le 1\text{ mA/V} = \frac{1}{1\text{ k}\Omega}$$

$W/L \le 1.92$

$W \le 0.35\ \mu$m

5.8 $r_{ds} = 1/\left.\dfrac{\partial i_D}{\partial v_{DS}}\right|_{v_{DS} = V_{DS}}$

$$= \left[\frac{\partial}{\partial v_{DS}}\left(k_n\left(V_{OV} v_{DS} - \frac{1}{2}v_{DS}^2\right)\right)\right]^{-1}$$

$$= \left[k_n\left(\frac{\partial}{\partial v_{DS}}\right)(v_{OV} v_{DS}) - 1/2\frac{\partial}{\partial v_{DS}}(v_{DS}^2)\right]^{-1}$$

$$= \left[k_n\left(V_{OV} - \frac{1}{2}\cdot 2V_{DS}\right)\right]^{-1}$$

$$= \frac{1}{k_n(V_{OV} - V_{DS})}$$

If $V_{DS} = 0 \Rightarrow r_{ds} = \dfrac{1}{k_n V_{OV}}$

If $V_{DS} = 0.2V_{OV} \Rightarrow r_{ds} = \dfrac{1.25}{V_{OV}}$

If $V_{DS} = 0.5V_{OV} \Rightarrow r_{ds} = \dfrac{1}{k_n(V_{OV} - 0.5V_{OV})}$

$$= 1/k_n(0.5V_{OV}) = \frac{2}{k_n V_{OV}}$$

If $V_{DS} = 0.8V_{OV} \Rightarrow r_{ds} = \dfrac{1}{k_n(V_{OV} - 0.8V_{OV})}$

$$= 1/k_n(0.2V_{OV}) = \frac{5}{k_n V_{OV}}$$

If $V_{DS} = V_{OV}$,

$$r_{ds} = \frac{1}{0} \Rightarrow \infty$$

5.9 $V_{DS\,sat} = V_{OV}$

$V_{OV} = V_{GS} - V_t = 1 - 0.5 = 0.5$ V

$\Rightarrow V_{DS\,sat} = 0.5$ V

In saturation:

$$i_D = \frac{1}{2}k'_n\left(\frac{W}{L}\right)V_{OV}^2 = \frac{1}{2}k_n V_{OV}^2$$

$$i_D = \frac{1}{2} \times \frac{4\text{ mA}}{\text{V}^2} \times (0.5\text{ V})^2$$

$i_D = 0.5$ mA

5.10 $L_{min} = 0.25\ \mu$m

$t_{ox} = 6$ nm

$$\mu_n = 460\ \frac{\text{cm}^2}{\text{V}\cdot\text{s}} = 460 \times 10^{-4}\frac{\text{m}^2}{\text{V}\cdot\text{s}}$$

(a) $C_{ox} = \dfrac{\epsilon_{ox}}{t_{ox}} = \dfrac{34.5\text{ pF/m}}{6\text{ nm}}$

$$= 5.75 \times 10^{-3}\ \frac{\text{F}}{\text{m}^2}\left(\frac{\text{pF}}{\mu\text{m}^2}\right)$$

$k'_n = \mu_n C_{ox} = 265\ \mu A/V^2$

(b) For $\dfrac{W}{L} = \dfrac{20}{0.25}$, $k_n = 21.2\text{ mA/V}^2$

$$\therefore 0.5\text{ mA} = I_D = \frac{1}{2}k_n V_{OV}^2$$

$V_{OV} = 0.22$ V

$V_{GS} = 0.72$ V

$V_{DS} \ge 0.22$ V

(c) $g_{DS} = \dfrac{1}{100\ \Omega} = k_n V_{OV}$

$\therefore V_{OV} = 0.47$ V.

$V_{GS} = 0.97$ V.

5.11 $V_{tp} = -0.7$ V

(a) $|V_{SG}| = |V_{tp}| + |V_{OV}|$

$= 0.7 + 0.4 = 1.1$ V

$\Rightarrow V_G = -1.1$ V

(b) For the p-channel transistor to operate in saturation, the drain voltage must not exceed the gate voltage by more than $|V_{tp}|$. Thus

$v_{Dmax} = -1.1 + 0.7 = -0.4$ V

Put differently, V_{SD} must be at least equal to $|V_{OV}|$, which in this case is 0.4 V. Thus $v_{Dmax} = -0.4$ V.

(c) In (b), the transistor is operating in saturation, thus

$$I_D = \frac{1}{2}k_p|V_{OV}|^2$$

$$0.5 = \frac{1}{2} \times k_p \times 0.4^2$$

$$\Rightarrow k_p = 6.25 \text{ mA/V}^2$$

For $V_D = -20$ mV, the transistor will be operating in the triode region. Thus

$$I_D = k_p\left[v_{SD}|V_{OV}| - \frac{1}{2}v_{SD}^2\right]$$

$$= 6.25\left[0.02 \times 0.4 - \frac{1}{2}(0.02)^2\right]$$

$$= 0.05 \text{ mA}$$

For $V_D = -2$ V, the transistor will be operating in saturation, thus

$$I_D = \frac{1}{2}k_p|V_{OV}|^2 = \frac{1}{2} \times 6.25 \times 0.4^2 = 0.5 \text{ mA}$$

5.12 $i_D = \frac{1}{2}k_n' \frac{W}{L}|V_{OV}|^2 \qquad k_n' = \mu_n C_{ox}$

For equal drain currents:

$$\mu_n C_{ox}\frac{W_n}{L} = \mu_p C_{ox}\frac{W_p}{L}$$

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = \frac{1}{0.4} = 2.5$$

5.13 For small v_{DS}, $i_D \simeq k_n' \frac{W}{L_1}(V_{GS} - V_t)V_{DS}$,

$$r_{DS} = \frac{V_{DS}}{i_D} = \frac{1}{k_n' \frac{W}{L}(V_{GS} - V_t)}$$

$$= \frac{1}{100 \times 10^{-6} \times 20 \times (5 - 0.7)}$$

$r_{DS} = 116.3\ \Omega \qquad V_{DS} = r_{DS} \times i_D = 116.3$ mV

For the same performance of a p-channel device:

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = 2.5 \Rightarrow \frac{W_p}{L} = \frac{W_n}{L} \times 2.5$$

$$= 20 \times 2.5 \Rightarrow \frac{W_p}{L} = 50$$

5.14 $t_{ox} = 6$ nm, $\mu_n = 460$ cm^2/V·s, $V_t = 0.5$ V, and $W/L = 10$.

$$k_n = \mu_n C_{ox}\frac{W}{L} = 460 \times 10^{-4} \times \frac{3.45 \times 10^{-11}}{6 \times 10^{-9}} \times 10$$

$$= 2.645 \text{ mA/V}^2$$

(a) $v_{GS} = 2.5$ V and $v_{DS} = 1$ V

$$v_{OV} = v_{GS} - V_t = 2 \text{ V}$$

Thus $v_{DS} < v_{OV} \Rightarrow$ triode region,

$$I_D = k_n\left[v_{DS}v_{OV} - \frac{1}{2}v_{DS}^2\right]$$

$$= 2.645\left[1 \times 2 - \frac{1}{2} \times 1\right] = 4 \text{ mA}$$

(b) $v_{GS} = 2$ V and $v_{DS} = 1.5$ V

$$v_{OV} = v_{GS} - V_t = 2 - 0.5 = 1.5 \text{ V}$$

Thus, $v_{DS} = v_{OV} \Rightarrow$ saturation region,

$$i_D = \frac{1}{2}k_n v_{OV}^2 = \frac{1}{2} \times 2.645 \times 1.5^2$$

$$= 3 \text{ mA}$$

(c) $v_{GS} = 2.5$ V and $v_{DS} = 0.2$ V

$$v_{OV} = 2.5 - 0.5 = 2 \text{ V}$$

Thus, $v_{DS} < v_{OV} \Rightarrow$ triode region,

$$i_D = k_n\left[v_{DS}v_{OV} - \frac{1}{2}v_{DS}^2\right]$$

$$= 2.645[0.2 \times 2 - \frac{1}{2}0.2^2] = 1 \text{ mA}$$

(d) $v_{GS} = v_{DS} = 2.5$ V

$$v_{OV} = 2.5 - 0.5 = 2 \text{ V}$$

Thus, $v_{DS} > v_{OV} \Rightarrow$ saturation region,

$$i_D = \frac{1}{2}k_n v_{OV}^2$$

$$= \frac{1}{2} \times 2.645 \times 2^2 = 5.3 \text{ mA}$$

5.15 See Table on next page.

5.16 $i_D = k_n\left[v_{OV}v_{DS} - \frac{1}{2}v_{DS}^2\right]$

$$\frac{i_D}{k_n} = v_{OV}v_{DS} - \frac{1}{2}v_{DS}^2 \qquad (1)$$

Figure 1 shows graphs for i_D/k_n versus v_{DS} for various values of v_{OV}. Since the right-hand side of Eq. (1) does not have any MOSFET parameters, these graphs apply for any n-channel MOSFET with the assumption that $\lambda = 0$. They also apply to p-channel devices with v_{DS} replaced by v_{SD}, k_n by k_p, and v_{OV} with $|v_{OV}|$. The slope of each graph at $v_{DS} = 0$ is found by differentiating Eq. (1) relative to v_{DS} with $v_{OV} = V_{OV}$ and then substituting $v_{DS} = 0$. The result is

$$\left.\frac{d(i_D/k_n)}{dv_{DS}}\right|_{v_{DS}=0,\ v_{OV}=V_{OV}} = V_{OV}$$

Figure 1 shows the tangent at $v_{DS} = 0$ for the graph corresponding to $v_{OV} = V_{OV3}$. Observe that it intersects the horizontal line $i_D/k_n = \frac{1}{2}V_{OV3}^2$ at $v_{DS} = \frac{1}{2}V_{OV3}$. Finally, observe that the curve representing the boundary between the triode region and the saturation region has the equation

$$i_D/k_n = \frac{1}{2}v_{DS}^2$$

This table belongs to **5.15**.

L (μm)	0.5	0.25	0.18	0.13
t_{ox} (nm)	10	5	3.6	2.6
$C_{ox}\left(\frac{\text{fF}}{\mu\text{m}^2}\right)$ $\epsilon_{ox} = 34.5$ pF/m	3.45	6.90	9.58	13.3
$k'_n\left(\frac{\mu\text{A}}{\text{V}^2}\right)$ $(\mu_n = 500\ \text{cm}^2/\text{V}\cdot\text{s})$	173	345	479	665
$k_n\left(\frac{\text{mA}}{\text{V}^2}\right)$ for $\frac{W}{L} = 10$	1.73	3.45	4.79	6.65
$A(\mu\text{m}^2)$ for $\frac{W}{L} = 10$	2.50	0.625	0.324	0.169
V_{DD}(V)	5	2.5	1.8	1.3
V_t(V)	0.7	0.5	0.4	0.4
I_D(mA) for $V_{GS} = V_{DS} = V_{DD}$, $I_D = \frac{1}{2}k_n(V_{DD} - V_t)^2$	16	6.90	4.69	2.69
P(mW) $P = V_{DD}I_D$	80	17.3	8.44	3.50
$\frac{P}{A}\left(\frac{\text{mW}}{\mu\text{m}^2}\right)$	32	27.7	26.1	20.7
$\frac{\text{Devices}}{\text{Chip}}$	n	$4n$	$7.72n$	$14.8n$

This figure belongs to **5.16**, part (a).

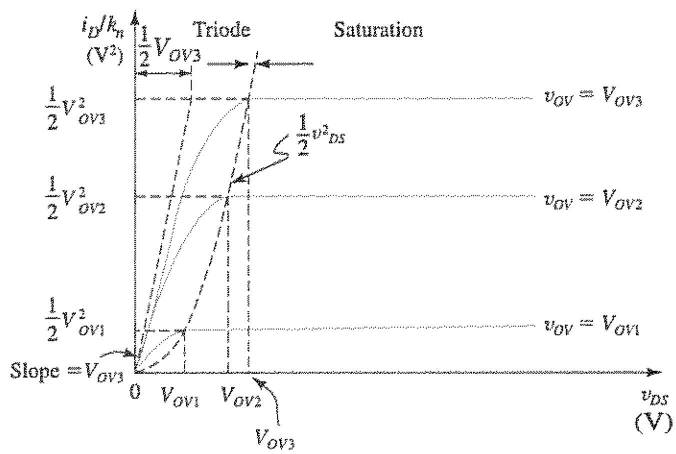


Fig. 1

Figure 2 shows the graph for the relationship

$$i_D/k_n = \frac{1}{2}v^2_{OV}$$

which describes the MOSFETs operation in the saturation region, that is,

$$v_{DS} \geq v_{OV}$$

Here also observe that this relationship (and graph) is universal and represents any MOSFET. The slope at $v_{OV} = V_{OV}$ is

$$\left.\frac{d\,(i_D/k_n)}{d\ v_{OV}}\right|_{v_{OV}=V_{OV}} = V_{OV}$$

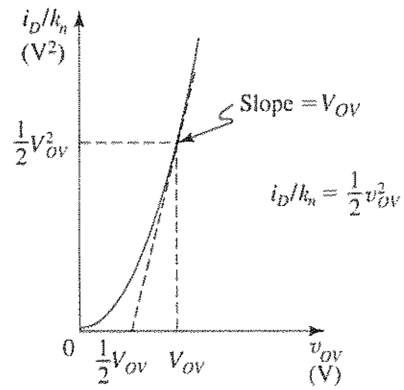


Fig. 2

Replacing k_n by k_p and v_{OV} by $|v_{OV}|$ adapts this graph to PMOS transistors.

5.17 For triode-region operation with v_{DS} small,

$$i_D \simeq k_n(v_{GS} - V_t)v_{DS}$$

Thus

$$r_{DS} \equiv \frac{v_{DS}}{i_D} = \frac{1}{k_n(v_{GS} - V_t)}$$

$$1 = \frac{1}{k_n(1.2 - 0.8)} = \frac{1}{0.4\ k_n}$$

$$\Rightarrow k_n = 2.5\ \text{mA/V}$$

$$r_{DS} = \frac{1}{2.5(V_{GS} - 0.8)} \quad (\text{k}\Omega)$$

$$0.2 = \frac{1}{2.5(V_{GS} - 0.8)}$$

$$\Rightarrow V_{GS} = 2.8\ \text{V}$$

For a device with twice the value of W, k_n will be twice as large and the resistance values will be half as large: 500 Ω and 100 Ω, respectively.

5.18 $V_{tn} = 0.5$ V, $\quad k_n = 1.6\ \text{mA/V}^2$

$$I_D = 0.05 = \frac{1}{2} \times 1.6 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.25\ \text{V and } V_{DS} \geq 0.25\ \text{V}$$

$$V_{GS} = 0.5 + 0.25 = 0.75\ \text{V}$$

$$I_D = 0.2 = \frac{1}{2} \times 1.6 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.5\ \text{V and } V_{DS} \geq 0.5\ \text{V}$$

$$V_{GS} = 0.5 + 0.5 = 1\ \text{V}$$

5.19 For $V_{GS} = V_{DS} = 1$ V, the MOSFET is operating in saturation,

$$I_D = \frac{1}{2}k_n(V_{GS} - V_t)^2$$

$$0.4 = \frac{1}{2}k_n(1 - V_t)^2 \qquad (1)$$

$$0.1 = \frac{1}{2}k_n(0.8 - V_t)^2 \qquad (2)$$

Dividing Eq. (1) by Eq. (2) and taking square roots gives

$$2 = \frac{1 - V_t}{0.8 - V_t}$$

$$\Rightarrow V_t = 0.6\ \text{V}$$

Substituting in Eq. (1), we have

$$0.4 = \frac{1}{2}k_n \times 0.4^2$$

$$\Rightarrow k_n = 5\ \text{mA/V}^2$$

5.20 $k_n' = 0.4\ \text{mA/V}^2$ and $V_t = 0.5$ V

For $v_{GS} = v_{DS} = 1.8$ V, the MOSFET is operating in saturation. Thus, to obtain $I_D = 2$ mA, we write

$$2 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times (1.8 - 0.5)^2$$

$$\Rightarrow \frac{W}{L} = 5.92$$

For $L = 0.18\ \mu$m

$$W = 1.07\ \mu\text{m}$$

5.21 $i_D = k_n(v_{GS} - V_t)v_{DS}$

$$25 = k_n(1 - V_t) \times 0.05 \qquad (1)$$

$$50 = k_n(1.5 - V_t) \times 0.05 \qquad (2)$$

Dividing Eq. (2) by Eq. (1), we have

$$2 = \frac{1.5 - V_t}{1 - V_t}$$

$$\Rightarrow V_t = 0.5\ \text{V}$$

Substituting in Eq. (1) yields

$$25 = k_n \times 0.5 \times 0.05$$

$$\Rightarrow k_n = 1000\ \mu\text{A/V}^2$$

For $k_n' = 50\ \mu\text{A/V}^2$

$$\frac{W}{L} = 20$$

For $v_{GS} = 2$ V and $v_{DS} = 0.1$ V,

$$i_D = k_n\left[(v_{GS} - V_t)v_{DS} - \frac{1}{2}v_{DS}^2\right]$$

$$= 1\left[(2 - 0.5) \times 0.1 - \frac{1}{2} \times 0.1^2\right]$$

$$= 0.145\ \text{mA} = 145\ \mu\text{A}$$

For $v_{GS} = 2$ V, pinch-off will occur for

$$v_{DS} = v_{GS} - V_t = 2 - 0.5 = 1.5 \text{ V}$$

and the resulting drain current will be

$$i_D = \frac{1}{2}k_n(v_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 1 \times (2 - 0.5)^2$$

$$= 1.125 \text{ mA}$$

5.22 For the channel to remain continuous,

$$v_{DS} \leq v_{GS} - V_t$$

Thus for $v_{GS} = 1.0$ V to 1.8 V and $V_t = 0.4$,

$$v_{DS} \leq 1 - 0.4$$

That is, $v_{DS\max} = 0.6$ V.

5.23 $\dfrac{W}{L} = \dfrac{20}{1} = 20 \quad k_n' = 100\ \mu\text{A/V}^2$

$$k_n = k_n'\left(\frac{W}{L}\right) = 100 \times 20 = 2000\ \mu\text{A/V}^2$$

$$= 2 \text{ mA/V}^2$$

For operation as a linear resistance,

$$i_D = k_n(v_{GS} - V_t)v_{DS}$$

and

$$r_{DS} \equiv \frac{v_{DS}}{i_D} = \frac{1}{k_n(v_{GS} - V_t)}$$

$$= \frac{1}{2(v_{GS} - 0.8)}$$

At $v_{GS} = 1.0$ V,

$$r_{DS} = \frac{1}{2(1 - 0.8)} = 2.5 \text{ k}\Omega$$

At $v_{GS} = 4.8$ V,

$$r_{DS} = \frac{1}{2(4.8 - 0.8)} = 0.125 \text{ k}\Omega$$

Thus, r_{DS} will vary in the range of 2.5 kΩ to 125 Ω.

(a) If W is halved, k_n will be halved and r_{DS} will vary in the range of 5 kΩ to 250 Ω.

(b) If L is halved, k_n will be doubled and r_{DS} will vary in the range of 1.25 kΩ to 62.5 Ω.

(c) If both W and L are halved, k_n will remain unchanged and r_{DS} will vary in the original range of 2.5 kΩ to 125 Ω.

5.24 **(a)** Refer to Fig. P5.24. For saturation-mode operation of an NMOS transistor, $v_{DG} \geq -V_{tn}$; thus $v_{DG} = 0$ results in saturation-mode operation. Similarly, for a p-channel MOSFET, saturation-mode operation is obtained for $v_{GD} \geq -|V_{tp}|$, which includes $v_{GD} = 0$. Thus, the diode-connected MOSFETs of Fig. P5.24 have the $i-v$ relationship

$$i = \frac{1}{2}k'\left(\frac{W}{L}\right)(v - |V_t|)^2 \qquad (1)$$

where k' represents k_n' in the NMOS case and k_p' in the PMOS case.

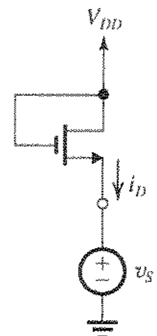
(b) If either of the MOSFETs in Fig. P5.24 is biased to operate at $v = |V_t| + |V_{OV}|$, then its incremental resistance r at the bias point can be obtained by differentiating Eq. (1) relative to v and then substituting $v = |V_t| + |V_{OV}|$ as follows:

$$\frac{\partial i}{\partial v} = k'\left(\frac{W}{L}\right)(v - |V_t|)$$

$$\left.\frac{\partial i}{\partial v}\right|_{v=|V_t|+V_{OV}} = k'\left(\frac{W}{L}\right)V_{OV}$$

$$r = 1\bigg/\left[\frac{\partial i}{\partial v}\right] = 1\bigg/\left(k'\frac{W}{L}V_{OV}\right) \qquad \text{Q.E.D}$$

5.25



$$v_{GD} = 0 \Rightarrow \text{saturation}$$

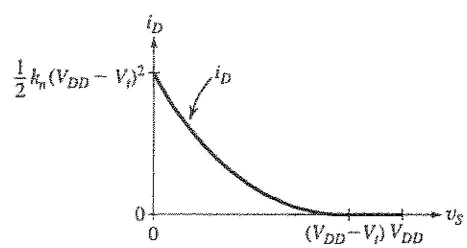
$$i_D = \frac{1}{2}k_n(v_{GS} - V_t)^2$$

$$v_{GS} = V_{DD} - v_S$$

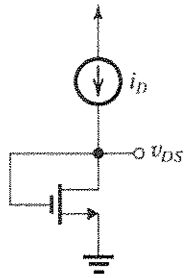
$$\therefore i_D = \frac{1}{2}k_n[(V_{DD} - V_t) - v_S]^2$$

$$0 \leq v_S \leq (V_{DD} - V_t)$$

$$i_D = 0, v_S \geq (V_{DD} - V_t)$$



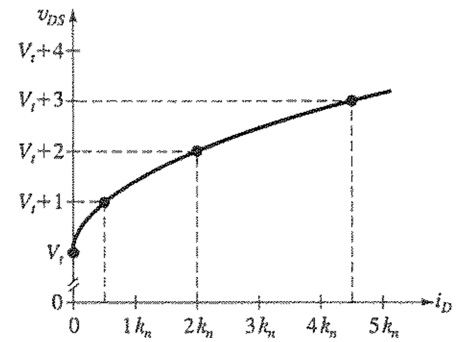
5.26



$$v_{DS} = v_{GS}$$

$$i_D = \frac{1}{2}k_n(v_{DS} - V_t)^2$$

$$\therefore v_{DS} = \sqrt{\frac{2i_D}{k_n}} + V_t$$



5.27 $V_{DS} = V_D - V_S \quad V_{GS} = V_G - V_S$

$V_{OV} = V_{GS} - V_t = V_{GS} - 1.0$

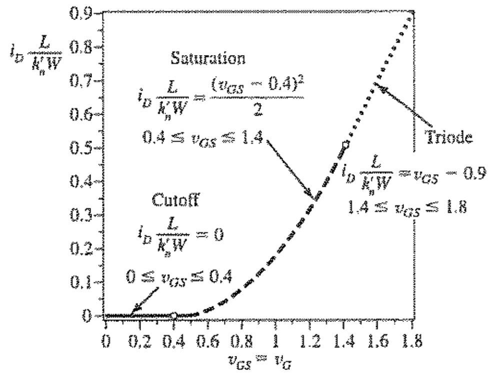
According to Table 5.1, three regions are possible.

Case	V_S	V_G	V_D	V_{GS}	V_{OV}	V_{DS}	Region of operation
a	+1.0	+1.0	+2.0	0	−1.0	+1.0	Cutoff
b	+1.0	+2.5	+2.0	+1.5	+0.5	+1.0	Sat.
c	+1.0	+2.5	+1.5	+1.5	+0.5	+0.5	Sat.
d	+1.0	+1.5	0	+0.5	−0.5	−1.0	Sat.*
e	0	+2.5	1.0	+2.5	+1.5	+1.0	Triode
f	+1.0	+1.0	+1.0	0	−1.0	0	Cutoff
g	−1.0	0	0	+1.0	0	+1.0	Sat.
h	−1.5	0	0	+1.5	+0.5	+1.5	Sat.
i	−1.0	0	+1.0	+1.0	0	+2.0	Sat.
j	+0.5	+2.0	+0.5	+1.5	+0.5	0	Triode

* With the source and drain interchanged.

5.28 The cutoff–saturation boundary is determined by $v_{GS} = V_t$, thus $v_{GS} = 0.4$ V at the boundary.

The saturation–triode boundary is determined by $v_{GD} = V_t$, and $v_{DS} = V_{DD} = 1$ V, and since $v_{GS} = v_{GD} + v_{DS}$, one has $v_{GS} = 0.4 + 1.0 = 1.4$ V at the boundary.



5.29 (a) Let Q_1 have a ratio (W/L) and Q_2 have a ratio $1.03\,(W/L)$. Thus

$$I_{D1} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(1 - V_t)^2$$

$$I_{D2} = \frac{1}{2}k_n'\left(\frac{W}{L}\right) \times 1.03 \times (1 - V_t)^2$$

Thus,

$$\frac{I_{D2}}{I_{D1}} = 1.03$$

That is, a 3% mismatch in the W/L ratios results in a 3% mismatch in the drain currents.

(b) Let Q_1 have a threshold voltage $V_t = 0.6$ V and Q_2 have a threshold voltage $V_t + \Delta V_t = 0.6 + 0.01 = 0.61$ V.

Thus

$$I_{D1} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(1 - 0.6)^2$$

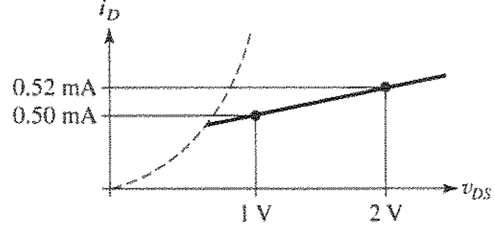
$$I_{D2} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(1 - 0.61)^2$$

and

$$\frac{I_{D2}}{I_{D1}} = \frac{(1 - 0.61)^2}{(1 - 0.6)^2} = 0.95$$

That is, a 10-mV mismatch in the threshold voltage results in a 5% mismatch in drain currents.

5.30



$$r_o = \left.\frac{\Delta v_{DS}}{\Delta i_D}\right|_{v_{GS}\text{ const.}} = \frac{1}{0.02} = 50\text{ k}\Omega$$

$$V_A \cong I_D r_o = 0.5 \times 50 = 25\text{ V}$$

$$\lambda = \frac{1}{V_A} = 0.04\text{ V}^{-1}$$

5.31 $r_o = \dfrac{V_A}{i_D} = \dfrac{20}{i_D}$, $0.1\text{ mA} \le i_D \le 1\text{ mA}$

$$\Rightarrow 20\text{ k}\Omega \le r_o \le 200\text{ k}\Omega$$

$$r_o = \frac{\Delta v_{DS}}{\Delta i_D} \Rightarrow \Delta i_D = \frac{\Delta v_{DS}}{r_o} = \frac{1}{r_o}$$

At $i_D = 0.1$ mA, $\Delta i_D = 5\ \mu\text{A}, \dfrac{\Delta i_D}{i_D} = 5\%$

At $i_D = 1$ mA, $\Delta i_D = 50\ \mu\text{A}, \dfrac{\Delta i_D}{i_D} = 5\%$

5.32 $V_A = V_A' L$, where V_A' is completely process dependent. Also, $r_o = \dfrac{V_A}{i_D}$. Therefore, to achieve desired r_o (which is 5 times larger), we should increase L ($L = 5 \times 1 = 5\ \mu$m).

To keep I_D unchanged, the $\dfrac{W}{L}$ ratio must stay unchanged. Therefore:

$W = 5 \times 10 = 50\ \mu$m (so $\dfrac{W}{L}$ is kept at 10)
$V_A = r_o i_D = 100\text{ k}\Omega \times 0.2\text{ mA} = 20$ V (for the standard device)

$V_A = 5 \times 20 = 100$ V (for the new device)

5.33 $L = 1.5\ \mu\text{m} = 3\times$ minimum. Thus

$$\lambda = \frac{0.03\text{ V}^{-1}}{3} = 0.01\text{ V}^{-1}$$

If v_{DS} is increased from 1 V to 5 V, the drain current will change from

$$I_D = 100\ \mu\text{A} = I_D'(1 + \lambda \times 1) = 1.01\ I_D'$$

to

$$I_D + \Delta I_D = I_D'(1 + \lambda \times 5) = 1.05\ I_D'$$

where I_D' is the drain current without channel-length modulation taken into account. Thus

$$I_D' = \frac{100}{1.01}$$

and

$$100 + \Delta I_D = 1.05\ I_D' = \frac{1.05 \times 100}{1.01} = 104\ \mu\text{A}$$

$$\Rightarrow \Delta I_D = 4\ \mu\text{A or } 4\%$$

To reduce ΔI_D by a factor of 2, we need to reduce λ by a factor of 2, which can be obtained by doubling the channel length to 3 μm.

5.34 $V_A = V_A' L = 20 \times 1.5 = 30$ V

$$\lambda = \frac{1}{V_A} = \frac{1}{30} = 0.033\text{ V}^{-1}$$

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2(1 + \lambda V_{DS})$$

$$= \frac{1}{2} \times 0.2 \times \left(\frac{15}{1.5}\right) \times 0.5^2(1 + 0.033 \times 2)$$

$$= 0.267\text{ mA}$$

$$r_o = \frac{V_A}{\frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2} = \frac{30}{\frac{1}{2} \times 0.2 \times \left(\frac{15}{1.5}\right) \times 0.5^2}$$

$$= 120\text{ k}\Omega$$

$$\Delta I_D = \frac{\Delta V_{DS}}{r_o} = \frac{1\text{ V}}{120\text{ k}\Omega} = 0.008\text{ mA}$$

5.35 Quadrupling W and L keeps the current I_D unchanged. However, the quadrupling of L increases V_A by a factor of 4 and hence increases r_o by a factor of 4.

Halving V_{OV} results in decreasing I_D by a factor of 4. Thus, this alone increases r_o by a factor of 4. The overall increase in r_o is by a factor of $4 \times 4 = 16$.

5.36 Refer to the circuit in Fig. P5.29 and let $V_{D1} = 2$ V and $V_{D2} = 2.5$ V. If the two devices are matched,

$$I_{D1} = \frac{1}{2}k_n(1 - V_t)^2\left(1 + \frac{2}{V_A}\right)$$

$$I_{D2} = \frac{1}{2}k_n(1 - V_t)^2\left(1 + \frac{2.5}{V_A}\right)$$

$$\Delta I_D = I_{D2} - I_{D1} = \frac{1}{2}k_n(1 - V_t)^2\left(\frac{0.5}{V_A}\right)$$

$$\frac{\Delta I_D}{\frac{1}{2}k_n(1 - V_t)^2} \simeq 0.01 = \frac{0.5}{V_A}$$

$\Rightarrow V_A = 50$ V (or larger to limit the mismatch in I_D to 1%).

If $V_A' = 100\text{ V}/\mu$m, the minimum required channel length is 0.5 μm.

5.37

NMOS	1	2	3	4
λ	0.05 V^{-1}	0.02 V^{-1}	0.1 V^{-1}	0.01 V^{-1}
V_A	20 V	50 V	10 V	100 V
I_D	0.5 mA	2 mA	0.1 mA	0.2 mA
r_o	40 kΩ	25 kΩ	100 kΩ	500 kΩ

5.38

$k_p = k_p'\left(\frac{W}{L}\right) = 100\ \mu A/V^2$

$V_{tp} = -1\ V \quad \lambda = -0.02\ V^{-1}$

$V_G = 0, \quad V_S = +5\ V \Rightarrow V_{SG} = 5\ V$

$|V_{OV}| = V_{SG} - |V_{tp}| = 5 - 1 = 4$

• For $v_D = +4\ V$, $v_{SD} = 1\ V < |V_{OV}| \Rightarrow$ triode-region operation,

$$i_D = k_p\left[v_{SD}|V_{OV}| - \frac{1}{2}v_{SD}^2\right]$$

$$= 100\left(1 \times 4 - \frac{1}{2} \times 1\right) = 350\ \mu A$$

• For $v_D = +2\ V$, $v_{SD} = 3\ V < |V_{OV}| \Rightarrow$ triode-region operation,

$$i_D = k_p\left[v_{SD}|V_{OV}| - \frac{1}{2}v_{SD}^2\right]$$

$$= 100\left(3 \times 4 - \frac{1}{2} \times 9\right) = 750\ \mu A$$

• For $v_D = +1\ V$, $v_{SD} = 4\ V = |V_{OV}| \Rightarrow$ saturation-mode operation,

$$i_D = \frac{1}{2}k_p|V_{OV}|^2(1 + |\lambda|v_{SD})$$

$$= \frac{1}{2} \times 100 \times 16(1 + 0.02 \times 4) = 864\ \mu A$$

• For $v_D = 0\ V$, $v_{SD} = 5\ V > |V_{OV}| \Rightarrow$ saturation-mode operation,

$$i_D = \frac{1}{2} \times 100 \times 16(1 + 0.02 \times 5) = 880\ \mu A$$

• For $v_D = -5\ V$, $v_{SD} = 10\ V > |V_{OV}| \Rightarrow$ saturation-mode operation,

$$i_D = \frac{1}{2} \times 100 \times 16(1 + 0.02 \times 10) = 960\ \mu A$$

5.39 $V_{tp} = 0.8\ V, \quad |V_A| = 40\ V$

$|v_{GS}| = 3\ V, \quad |v_{DS}| = 4\ V$

$i_D = 3\ mA$

$|V_{OV}| = |v_{GS}| - |V_{tp}| = 2.2\ V$

$|v_{DS}| > |V_{OV}| \Rightarrow$ saturation mode

$v_{GS} = -3\ V$

$v_{SG} = +3\ V$

$v_{DS} = -4\ V$

$v_{SD} = 4\ V$

$V_{tp} = -0.8\ V$

$V_A = -40\ V$

$\lambda = -0.025\ V^{-1}$

$$i_D = \frac{1}{2}k_p(v_{GS} - V_{tp})^2(1 + \lambda\ v_{DS})$$

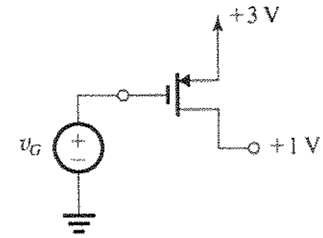
$$3 = \frac{1}{2}k_p[-3 - (-0.8)]^2(1 - 0.025 \times -4)$$

$\Rightarrow k_p = 1.137\ mA/V^2$

5.40 PMOS with $V_{tp} = -1\ V$

Case	V_S	V_G	V_D	V_{SG}	$\lvert V_{OV}\rvert$	V_{SD}	**Region of operation**
a	+2	+2	0	0	0	2	Cutoff
b	+2	+1	0	+1	0	2	Cutoff–Sat.
c	+2	0	0	+2	1	2	Sat.
d	+2	0	+1	+2	1	1	Sat–Triode
e	+2	0	+1.5	+2	1	0.5	Triode
f	+2	0	+2	+2	1	0	Triode

5.41



$V_{tp} = -0.5\ V$

$v_G = +3\ V \rightarrow 0\ V$

As v_G reaches +2.5 V, the transistor begins to conduct and enters the saturation region, since v_{DG} will be negative. The transistor continues to operate in the saturation region until v_G reaches 0.5 V, at which point v_{DG} will be 0.5 V, which is equal to $|V_{tp}|$, and the transistor enters the triode region. As v_G goes below 0.5 V, the transistor continues to operate in the triode region.

5.42 Case a, assume, sat,

$$\frac{(1 - V_t)^2}{(1.5 - V_t)^2} = \frac{100}{400} \Rightarrow V_t = 0.5,$$

$V_{GD} \le V_t$

∴ sat;

Case b — same procedure, except use V_{SG} and V_{SD}.

This table belongs to **5.42**.

Case	Transistor	V_S (V)	V_G (V)	V_D (V)	I_D (μA)	Type	Mode	$\mu C_{ox}\frac{W}{L}$ (μ A/V²)	V_t (V)
a	1	0	1	2.5	100	NMOS	Sat.	800	0.5
		0	1.5	2.5	400		Sat.		
b	2	5	3	−4.5	50	PMOS	Sat.	400	−1.5
		5	2	−0.5	450		Sat.		
c	3	5	3	4	200	PMOS	Sat.	400	−1
		5	2	0	800		Sat.		
d	4	−2	0	0	72	NMOS	Sat.	100	+0.8
		−4	0	−3	270		Triode		

$$\frac{(2-|V_t|)^2}{(3-|V_t|)^2} = \frac{50}{450} \Rightarrow |V_t| = 1.5.$$

$V_{GD} \geq -1.5$ V $\therefore$ sat

Case c – $\dfrac{(2-|V_t|)^2}{(3-|V_t|)^2} = \dfrac{200}{800} \Rightarrow |V_t| = 1.0,$

$V_{GD} \geq -1.0$ V $\therefore$ sat

Case d

$$\frac{\text{sat}}{\text{triode}}\ \frac{\frac{1}{2}k_n(2-V_t)^2}{k_n\left[(4-V_t)\,V_{DS}-\frac{1}{2}V_{DS}^2\right]} = \frac{72}{270}$$

(after failing assumption that both cases are sat.)

5.43 Refer to the circuits in Fig. P5.43.

(a) $V_1 = V_{DS} = V_{GS} = 1$ V

(b) $V_2 = +1 - V_{DS} = 1 - 1 = 0$ V

(c) $V_3 = V_{SD} = V_{SG} = 1$ V

(d) $V_4 = +1.25 - V_{SG} = 1.25 - 1 = 0.25$ V

Now place a resistor R in series with the drain. For the circuits in (a) and (b) to remain in saturation, V_D must not fall below V_G by more than V_t. Thus,

$IR \leq V_t$

$$R_{max} = \frac{V_t}{I} = \frac{0.5}{0.1} = 5\ \text{k}\Omega$$

For the circuits in (c) and (d) to remain in saturation, V_D must not exceed V_G by more than $|V_t|$. Thus

$IR \leq |V_t|$

which yields $R_{max} = 5$ kΩ.

Now place a resistor R_S in series with the MOSFET source. The voltage across the current source becomes

(a) $V_{CS} = 2.5 - V_{DS} - IR_S$ (1)

To keep V_{CS} at least at 0.5 V, the maximum R_S can be found from

$0.5 = 2.5 - 1 - 0.1 \times R_{S\max}$

$\Rightarrow R_{S\max} = 10$ kΩ

$V_1 = 2.5 - 0.5 = 2$ V

(b) $V_{CS} = 1 - V_{DS} - IR_S - (-1.5)$

$= 2.5 - V_{DS} - IR_S$

which is identical to Eq. (1). Thus

$R_{S\max} = 10$ kΩ

$V_2 = -1.5 + 0.5 = -1$ V

(c) $V_{CS} = 2.5 - IR_S - V_{SD}$

which yields

$R_{S\max} = 10$ kΩ

$V_3 = 2.5 - 0.5 = 2$ V

(d) $V_{CS} = 1.25 - IR_S - V_{SD} - (-1.25)$

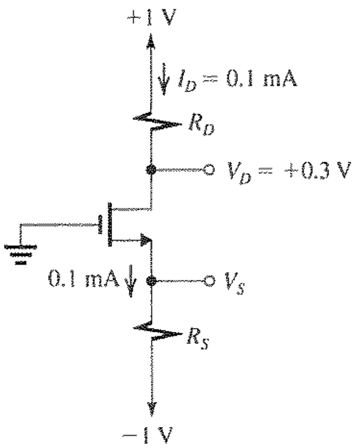
$= 2.5 - V_{SD} - IR_S$

which yields

$R_{S\max} = 10$ kΩ

$V_4 = -1.25 + 0.5 = -0.75$ V

5.44



Since $V_{DG} > 0$, the MOSFET is in saturation.

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 0.4 \times \frac{5}{0.4} \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

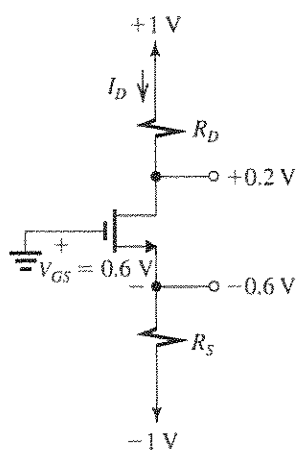
$$V_{GS} = V_t + V_{OV} = 0.5 + 0.2 = 0.7$$

$$V_S = 0 - V_{GS} = -0.7 \text{ V}$$

$$R_S = \frac{V_S - (-1)}{I_D} = \frac{-0.7 + 1}{0.1} = 3 \text{ k}\Omega$$

$$R_D = \frac{1 - V_D}{I_D} = \frac{1 - 0.3}{0.1} = \frac{0.7}{0.1} = 7 \text{ k}\Omega$$

5.45



Since $V_{DG} > 0$, the MOSFET is operating in saturation. Thus

$$I_D = \frac{1}{2}k_n(V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 4 \times (0.6 - 0.4)^2$$

$$= 0.08 \text{ mA}$$

$$R_D = \frac{1 - V_D}{I_D} = \frac{1 - 0.2}{0.08} = \frac{0.8}{0.08} = 10 \text{ k}\Omega$$

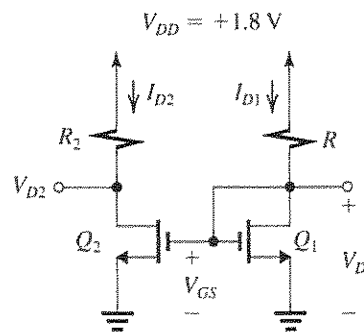
$$R_S = \frac{-0.6 - (-1)}{I_D} = \frac{-0.6 + 1}{0.08} = 5 \text{ k}\Omega$$

For I_D to remain unchanged from 0.08 mA, the MOSFET must remain in saturation. This in turn can be achieved by ensuring that V_D does not fall below V_G (which is zero) by more than V_t (0.4 V). Thus

$$1 - I_D R_{D\max} = -0.4$$

$$R_{D\max} = \frac{1.4}{0.08} = 17.5 \text{ k}\Omega$$

5.46



(a) $I_{D1} = 50$ μA

$$0.05 = \frac{1}{2} \times 0.4 \times \frac{1.44}{0.36} V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.25 \text{ V}$$

$$V_{GS1} = V_t + V_{OV}$$

$$= 0.5 + 0.25 = 0.75 \text{ V}$$

$$V_{D1} = V_{GS1} = 0.75 \text{ V}$$

$$R = \frac{V_{DD} - V_{D1}}{I_{D1}} = \frac{1.8 - 0.75}{0.05} = 21 \text{ k}\Omega$$

(b) Note that both transistors operate at the same V_{GS} and V_{OV}, and

$$I_{D2} = 0.5 \text{ mA}$$

But

$$I_{D2} = \frac{1}{2}k_n'\left(\frac{W_2}{L_2}\right)V_{OV}^2$$

$$0.5 = \frac{1}{2} \times 0.4 \times \frac{W_2}{0.36} \times 0.25^2$$

$$\Rightarrow W_2 = 14.4 \text{ μm}$$

which is 10 times W_1, as needed to provide $I_{D2} = 10I_{D1}$. Since Q_2 is to operate at the edge of saturation,

$V_{DS2} = V_{OV}$

Thus,

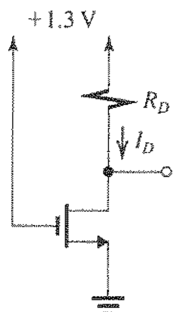
$V_{D2} = 0.25$ V

and

$$R_2 = \frac{V_{DD} - V_{D2}}{I_{D2}}$$

$$= \frac{1.8 - 0.25}{0.5} = 3.1 \text{ k}\Omega$$

5.47



$$I_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 0.4 \times \frac{W}{L} (1.3 - 0.4)^2$$

$$= 0.162 \left(\frac{W}{L}\right)$$

$$V_D = 1.3 - I_D R_D = 1.3 - 0.162 \left(\frac{W}{L}\right) R_D$$

For the MOSFET to be at the edge of saturation, we must have

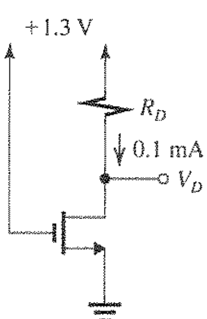
$V_D = V_{OV} = 1.3 - 0.4 = 0.9$

Thus

$$0.9 = 1.3 - 0.162 \left(\frac{W}{L}\right) R_D$$

$$\Rightarrow \left(\frac{W}{L}\right) R_D \simeq 2.5 \text{ k}\Omega \qquad \text{Q.E.D}$$

5.48



$V_{OV} = V_{GS} - V_t$

$= 1.3 - 0.4 = 0.9$

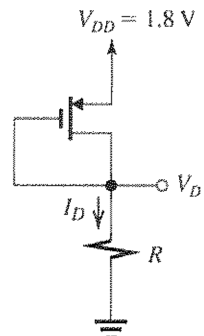
To operate at the edge of saturation, we must have

$V_D = V_{OV} = 0.9$ V

Thus,

$$R_D = \frac{1.3 - 0.9}{0.1} = 4 \text{ k}\Omega$$

5.49



$I_D = 180\ \mu$A and $V_D = 1$ V

$$R = \frac{V_D}{I_D} = \frac{1}{0.18} = 5.6 \text{ k}\Omega$$

Transistor is operating in saturation with $|V_{OV}| = 1.8 - V_D - |V_t| = 1.8 - 1 - 0.5 = 0.3$ V:

$$I_D = \frac{1}{2} k_p' \frac{W}{L} |V_{OV}|^2$$

$$180 = \frac{1}{2} \times 100 \times \frac{W}{L} \times 0.3^2$$

$$\Rightarrow \frac{W}{L} = 40$$

$W = 40 \times 0.18 = 7.2\ \mu$m

5.50 Refer to Fig. P5.50. Both Q_1 and Q_2 are operating in saturation at $I_D = 0.5$ mA. For Q_1,

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W_1}{L_1} V_{OV1}^2$$

$$0.5 = \frac{1}{2} \times 0.25 \times \frac{W_1}{L_1} (1 - 0.5)^2$$

$$\Rightarrow \frac{W_1}{L_1} = 16$$

$W_1 = 16 \times 0.25 = 4\ \mu$m

For Q_2, we have

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W_2}{L_2}\right) V_{OV2}^2$$

$$0.5 = \frac{1}{2} \times 0.25 \times \frac{W_2}{L_2} (1.8 - 1 - 0.5)^2$$

$$\Rightarrow \frac{W_2}{L_2} = 44.4$$

$$W_2 = 44.4 \times 0.25 = 11.1$$

$$R = \frac{2.5 - 1.8}{0.5} = 1.4 \text{ k}\Omega$$

5.51 Refer to the circuit in Fig. P5.51. All three transistors are operating in saturation with $I_D = 90\ \mu\text{A}$. For Q_1,

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W_1}{L_1}(V_{GS1} - V_t)^2$$

$$90 = \frac{1}{2} \times 90 \times \frac{W_1}{L_1}(0.8 - 0.5)^2$$

$$\Rightarrow \frac{W_1}{L_1} = 22.2$$

$$W_1 = 22.2 \times 0.5 = 11.1\ \mu\text{m}$$

For Q_2,

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W_2}{L_2}(V_{GS2} - V_t)^2$$

$$90 = \frac{1}{2} \times 90 \times \frac{W_2}{L_2}(1.5 - 0.8 - 0.5)^2$$

$$\Rightarrow \frac{W_2}{L_2} = 50$$

$$W_2 = 50 \times 0.5 = 25\ \mu\text{m}$$

For Q_3,

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W_3}{L_3}(V_{GS3} - V_t)^2$$

$$90 = \frac{1}{2} \times 90 \times \frac{W_3}{L_3}(2.5 - 1.5 - 0.5)^2$$

$$\Rightarrow \frac{W_3}{L_3} = 8$$

$$W_3 = 8 \times 0.5 = 4\ \mu\text{m}$$

5.52 Refer to the circuits in Fig. 5.24 (page 282):

$$V_{GS} = 5 - 6I_D$$

$$I_D = \frac{1}{2}k_n' \frac{W}{L}(V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 1.5 \times (5 - 6I_D - 1.5)^2$$

which results in the following quadratic equation in I_D:

$$36I_D^2 - 43.33I_D + 12.25 = 0$$

The physically meaningful root is

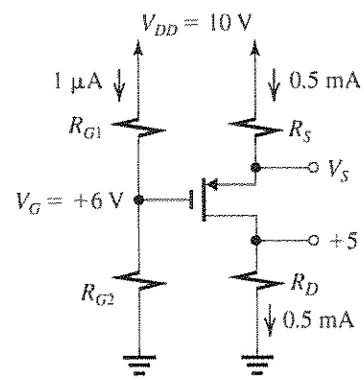
$$I_D = 0.45 \text{ mA}$$

This should be compared to the value of 0.5 mA found in Example 5.6. The difference of about 10% is relatively small, given the large variations in k_n and V_t (50% increase in each). The new value of V_D is

$$V_D = V_{DD} - R_D I_D = 10 - 6 \times 0.45 = +7.3 \text{ V}$$

as compared to +7 V found in Example 5.6. We conclude that this circuit is quite tolerant to variations in device parameters.

5.53



Refer to the circuit in the figure above,

$$R_{G1} = \frac{V_{DD} - V_G}{1\ \mu\text{A}}$$

$$= \frac{10 - 6}{1} = 4 \text{ M}\Omega$$

$$R_{G2} = \frac{6}{1\ \mu\text{A}} = 6 \text{ M}\Omega$$

$$R_D = \frac{5 \text{ V}}{0.5 \text{ mA}} = 10 \text{ k}\Omega$$

To determine V_S, we use

$$I_D = \frac{1}{2}k_p'\left(\frac{W}{L}\right)(V_{SG} - |V_t|)^2$$

$$0.5 = \frac{1}{2} \times 4 \times (V_{SG} - 1.5)^2$$

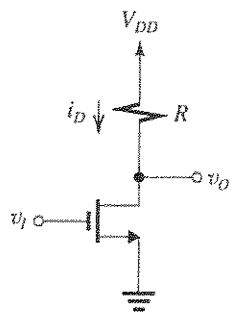
$$\Rightarrow V_{SG} = 2 \text{ V}$$

Thus,

$$V_S = V_G + V_{SG} = 6 + 2 = 8 \text{ V}$$

$$R_S = \frac{10 - 8}{0.5} = 4 \text{ k}\Omega$$

5.54



Assuming linear operation in the triode region, we can write

$$i_D = \frac{v_O}{r_{DS}} = \frac{50\text{ mV}}{50\ \Omega} = 1\text{ mA}$$

$$i_D = k_n'\left(\frac{W}{L}\right)(v_{GS} - V_t)v_{DS}$$

$$1 = 0.5 \times \frac{W}{L} \times (1.3 - 0.4) \times 0.05$$

$$\Rightarrow \frac{W}{L} = 44.4$$

$$R = \frac{V_{DD} - v_O}{i_D} = \frac{1.3 - 0.05}{1}$$

$$= 1.25\text{ k}\Omega$$

5.55 (a) Refer to Fig. P5.55(a): Assuming saturation-mode operation, we have

$$I_D = \frac{1}{2}k_n V_{OV}^2$$

$$2 = \frac{1}{2} \times 4\ V_{OV}^2$$

$$\Rightarrow V_{OV} = 1\text{ V}$$

$$V_{GS} = |V_t| + V_{OV} = 1 + 1 = 2\text{ V}$$

$$V_1 = 0 - V_{GS} = -2\text{ V}$$

$$V_2 = 5 - 2 \times 2 = +1\text{ V}$$

Since $V_{DG} = +1$ V, the MOSFET is indeed in saturation.

Refer to Fig. P5.55(b): The transistor is operating in saturation, thus

$$I_D = \frac{1}{2}k_n V_{OV}^2$$

$$2 = \frac{1}{2} \times 4 \times V_{OV}^2 \Rightarrow V_{OV} = 1\text{ V}$$

$$V_{GS} = 2\text{ V}$$

$$\Rightarrow V_3 = 2\text{ V}$$

Refer to Fig. P5.55(c): Assuming saturation-mode operation, we have

$$I_D = \frac{1}{2}k_p|V_{OV}|^2$$

$$2 = \frac{1}{2} \times 4 \times |V_{OV}|^2$$

$$\Rightarrow |V_{OV}| = 1\text{ V}$$

$$V_{SG} = |V_t| + |V_{OV}| = 1 + 1 = 2\text{ V}$$

$$V_4 = V_{SG} = 2\text{ V}$$

$$V_5 = -5 + I_D \times 1.5$$

$$= -5 + 2 \times 1.5 = -2\text{ V}$$

Since $V_{DG} < 0$, the MOSFET is indeed in saturation.

Refer to Fig. P5.55(d): Both transistors are operating in saturation at equal $|V_{OV}|$. Thus

$$2 = \frac{1}{2} \times 4 \times |V_{OV}|^2 \Rightarrow |V_{OV}| = 1\text{ V}$$

$$V_{SG} = |V_t| + |V_{OV}| = 2\text{ V}$$

$$V_6 = 5 - V_{SG} = 5 - 2 = 3\text{ V}$$

$$V_7 = +5 - 2\ V_{SG} = 5 - 2 \times 2 = 1\text{ V}$$

(b) Circuit (a): The 2-mA current source can be replaced with a resistance R connected between the MOSFET source and the −5-V supply with

$$R = \frac{V_1 - (-5)}{2\text{ mA}} = \frac{-2 + 5}{2} = 1.5\text{ k}\Omega$$

Circuit (b): The 2-mA current source can be replaced with a resistance R,

$$R = \frac{5 - V_3}{2\text{ mA}} = \frac{5 - 2}{2} = 1.5\text{ k}\Omega$$

Circuit (c): The 2-mA current source can be replaced with a resistance R,

$$R = \frac{5 - V_4}{2\text{ mA}} = \frac{5 - 2}{2} = 1.5\text{ k}\Omega$$

Circuit (d): The 2-mA current source can be replaced with a resistance R,

$$R = \frac{V_7}{2\text{ mA}} = \frac{1}{2} = 0.5\text{ k}\Omega$$

We use the nearest 1% resistor, which is 499 Ω.

5.56 (a) Refer to Fig. P5.56(a): The MOSFET is operating in saturation. Thus

$$I_D = \frac{1}{2}k_n V_{OV}^2$$

$$10 = \frac{1}{2} \times 500 \times V_{OV}^2 \Rightarrow V_{OV} = 0.2\text{ V}$$

$$V_{GS} = V_t + V_{OV} = 0.8 + 0.2 = 1\text{ V}$$

$$V_1 = 0 - V_{GS} = -1\text{ V}$$

(b) Refer to Fig. P5.56(b): The MOSFET is operating in saturation. Thus

$$100 = \frac{1}{2} \times 500 \times V_{OV}^2 \Rightarrow V_{OV} = 0.63 \text{ V}$$

$$V_{GS} = 0.8 + 0.63 = 1.43 \text{ V}$$

$$V_2 = -1.43 \text{ V}$$

(c) Refer to Fig. P5.56(c). The MOSFET is operating in saturation. Thus

$$1 = \frac{1}{2} \times 0.5 \times V_{OV}^2 \Rightarrow V_{OV} = 2 \text{ V}$$

$$V_{GS} = 0.8 + 2 = 2.8 \text{ V}$$

$$V_3 = -2.8 \text{ V}$$

(d) Refer to Fig. P5.56(d). The MOSFET is operating in saturation. Thus

$$10 = \frac{1}{2} \times 500 \times V_{OV}^2 \Rightarrow V_{OV} = 0.2 \text{ V}$$

$$V_{GS} = 0.8 + 0.2 = 1 \text{ V}$$

$$V_4 = 1 \text{ V}$$

(e) Refer to Fig. P5.56(e). The MOSFET is operating in saturation. Thus

$$1 = \frac{1}{2} \times 0.5 \times V_{OV}^2 \Rightarrow V_{OV} = 2 \text{ V}$$

$$V_{GS} = 0.8 + 2 = 2.8 \text{ V}$$

$$V_5 = V_{GS} = 2.8 \text{ V}$$

(f) Refer to Fig. P5.56(f). To simplify our solution, we observe that this circuit is that in Fig. P5.56(d) with the 10-μA current source replaced with a 400-kΩ resistor. Thus $V_G = V_4 = +1$ V and, as a check, $I_D = \dfrac{5-1}{400} = 0.01 \text{ mA} = 10\ \mu\text{A}$.

(g) Refer to Fig. P5.56(g). Our work is considerably simplified by observing that this circuit is similar to that in Fig. P5.56(e) with the 1-mA current source replaced with a 2.2-kΩ resistor. Thus $V_7 = V_5 = 2.8$ V and, as a check, $I_D = \dfrac{5-2.8}{2.2} = 1$ mA.

(h) Refer to Fig. P5.56(h). Our work is considerably simplified by observing that this circuit is similar to that in Fig. P5.56(a) with the 10-μA current source replaced with a 400-kΩ resistor. Thus $V_8 = V_1 = -1$ V and, as a check, $I_D = \dfrac{-1+5}{400} = 0.01 \text{ mA} = 10\ \mu\text{A}$.

5.57 (a) Refer to the circuit in Fig. P5.57(a). Transistor Q_1 is operating in saturation. Assume that Q_2 also is operating in saturation,

$$V_{GS2} = 0 - V_2 = -V_2$$

and

$$V_2 = -2.5 + I_D \times 1$$

$$\Rightarrow I_D = V_2 + 2.5$$

Now,

$$I_D = \frac{1}{2} k_n (V_{GS2} - V_t)^2$$

Substituting $I_D = V_2 + 2.5$ and $V_{GS2} = -V_2$,

$$V_2 + 2.5 = \frac{1}{2} \times 1.5(-V_2 - 0.9)^2$$

$$\frac{2}{1.5}(V_2 + 2.5) = V_2^2 + 1.8\ V_2 + 0.81$$

$$V_2^2 + 0.467\ V_2 - 2.523 = 0$$

$$\Rightarrow V_2 = -1.84 \text{ V}$$

Thus,

$$I_D = V_2 + 2.5 = -1.84 + 2.5 = 0.66 \text{ mA}$$

and

$$V_{GS2} = 1.84 \text{ V}$$

Since Q_1 is identical to Q_2 and is conducting the same I_D, then

$$V_{GS1} = 1.84 \text{ V}$$

$$\Rightarrow V_1 = 2.5 - 1.84 = 0.66 \text{ V}$$

which confirms that Q_1 is operating in saturation, as assumed.

(b) Refer to the circuit in Fig. P5.57(b). From symmetry, we see that

$$V_4 = 2.5 \text{ V}$$

Now, compare the part of the circuit consisting of Q_2 and the 1-kΩ resistor. We observe the similarity of this part with the circuit between the gate of Q_2 and ground in Fig. P5.57(a). It follows that for the circuit in Fig. P5.57(b), we can use the solution of part (a) above to write

$$I_{D2} = 0.66 \text{ mA} \quad \text{and} \quad V_{GS2} = 1.84 \text{ V}$$

Thus,

$$V_5 = V_4 - V_{GS2} = 2.5 - 1.84 = 0.66 \text{ V}$$

Since Q_1 is conducting an equal I_D and has the same V_{GS},

$$I_{D1} = 0.66 \text{ mA} \quad \text{and} \quad V_{GS1} = 1.84 \text{ V}$$

$$\Rightarrow V_3 = V_4 + V_{GS1} = 2.5 + 1.84 = 3.34 \text{ V}$$

We could, of course, have used the circuit symmetry, observed earlier, to write this final result.

5.58

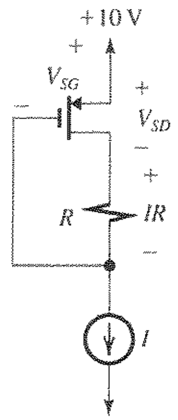


Fig. 1

(a) From Fig. 1 we see that

$V_{DG} = IR$

Since for the PMOS transistor to operate in saturation,

$V_{DG} \leq |V_{tp}|$

It follows the

$IR \leq |V_{tp}|$ Q.E.D

(b) (i) $R = 0$, the condition above is satisfied and

$I_D = I = \frac{1}{2}k_p|V_{OV}|^2$

$0.1 = \frac{1}{2} \times 0.2 \times |V_{OV}|^2$

$\Rightarrow |V_{OV}| = 1$ V

$V_{SG} = |V_{tp}| + |V_{OV}| = 1 + 1 = 2$ V

$V_G = 10 - 2 = 8$ V

$V_D = V_G = 8$ V

$V_{SD} = 2$ V

(ii) $R = 10$ kΩ

$IR = 0.1 \times 10 = 1$ V

which just satisfies the condition for saturation-mode operation in (a) above. Obviously I_D and $|V_{OV}|$ will be the same as in (i) above.

$V_{SG} = 2$ V

$V_G = 8$ V

$V_D = V_G + IR = 8 + 1 = 9$ V

$V_{SD} = 1$ V

(iii) $R = 30$ kΩ

$IR = 0.1 \times 30 = 3$ V

which is greater than $|V_{tp}|$. Thus the condition in (a) above is not satisfied and the MOSFET is operating in the triode region. From Fig. 2,

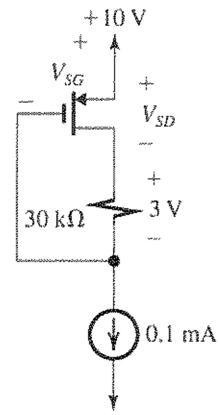


Fig. 2

From Fig. 2, we see that

$V_{SD} = V_{SG} - 3$

Now, for triode-mode operation,

$$I_D = k_p\left[(V_{SG} - |V_{tp}|)V_{SD} - \frac{1}{2}V_{SD}^2\right]$$

$$0.1 = 0.2\left[(V_{SG} - 1)(V_{SG} - 3) - \frac{1}{2}(V_{SG} - 3)^2\right]$$

$\Rightarrow V_{SG}^2 - 2V_{SG} - 4 = 0$

$\Rightarrow V_{SG} = 3.24$ V

$V_{SD} = V_{SG} - 3 = 0.24$ V

(iv) $R = 100$ kΩ

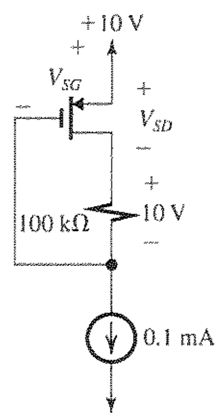


Fig. 3

Here also (see Fig. 3) the MOSFET will be operating in the triode region, and

$V_{SD} = V_{SG} - 10$ V

Since we expect V_{SD} to be very small, we can neglect the V_{SD}^2 term in the expression for I_D and write

$I_D \simeq k_p(V_{SG} - |V_t|)V_{SD}$

$0.1 = 0.2(V_{SG} - 1)(V_{SG} - 10)$

$\Rightarrow V_{SG}^2 - 11V_{SG} + 9.5 = 0$

$\Rightarrow V_{SG} = 10.055$ V

$V_{SD} = V_{SG} - 10 = 0.055$ V

5.59 (a) Refer to the circuit in Fig. P5.59(a). Since the two NMOS transistors are identical and have the same I_D, their V_{GS} values will be equal. Thus

$V_{GS} = \frac{3}{2} = 1.5$ V

$V_2 = 1.5$ V

$V_{OV} = V_{GS} - V_t = 1.0$ V

$I_1 = I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$

$= \frac{1}{2} \times 270 \times \frac{3}{1} \times 1$

$= 405$ μA

(b) Refer to the circuit in Fig. P5.59(b). Here Q_N and Q_P have the same $I_D = I_3$. Thus

$$I_3 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OVN}^2 \quad (1)$$

$$I_3 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)V_{OVP}^2 \quad (2)$$

Equating Eqs. (1) and (2) and using $\mu_n C_{ox} = 3\mu_p C_{ox}$ gives $3V_{OVN}^2 = V_{OVP}^2$:

$|V_{OVP}| = \sqrt{3}\ V_{OVN}$

Now,

$V_{GSN} = V_{OVN} + V_t = V_{OVN} + 0.5$

$V_{SGP} = |V_{OVP}| + |V_t| = \sqrt{3}\ V_{OVN} + 0.5$

But

$V_{SGP} + V_{GSN} = 3$

$(\sqrt{3} + 1)V_{OVN} + 1 = 3$

$\Rightarrow V_{OVN} = 0.732$ V

$V_{OVP} = 1.268$ V

$V_{GSN} = 1.232$ V

$V_{SGP} = 1.768$ V

$V_4 = V_{GSN} = 1.232$ V

$I_3 = \frac{1}{2} \times 270 \times \frac{3}{1} \times 0.732^2 = 217$ μA

(c) Refer to Fig. P5.59(c). Here the width of the PMOS transistor is made 3 times larger than that of the NMOS transistor. This compensates for the factor 3 in the process transconductance parameter, resulting in $k_p = k_n$, and the two transistors are matched. The solution will be identical to that for (a) above with

$V_5 = \frac{3}{2} = 1.5$ V

$I_6 = 405$ μA

5.60 Refer to the circuit in Fig. P5.60. First consider Q_1 and Q_2. Both are operating in saturation and since they are identical, they have equal V_{GS}:

$V_{GS1} = V_{GS2} = \frac{5}{2} = 2.5$ V

Thus,

$I_{D2} = I_{D1} = \frac{1}{2}\ \mu_n C_{ox}\frac{W}{L}(V_{GS1} - V_t)^2$

$= \frac{1}{2} \times 50 \times \frac{10}{1}(2.5 - 1)^2$

$= 562.5$ μA

Now, Q_3 has the same V_{GS} at Q_1 and is matched to Q_1. Thus if we assume that Q_3 is operating in saturation, we have

$I_{D3} = I_{D1} = 562.5$ μA

Thus,

$I_2 = 562.5$ μA

This is the same current that flows through Q_4, which is operating in saturation and is matched to Q_3. Thus

$V_{GS4} = V_{GS3} = V_{GS1} = 2.5$ V

Thus,

$V_2 = 5 - V_{GS4} = 2.5$ V

This is equal to the voltage at the gate of Q_3; thus Q_3 is indeed operating in saturation, as assumed.

If Q_3 and Q_4 have $W = 100$ μm, nothing changes for Q_1 and Q_2. However, Q_3, which has the same V_{GS} as Q_1 but has 10 times the width, will have a drain current 10 times larger than Q_1.
Thus

$I_{D2} = I_{D3} = 10\ I_{D1} = 10 \times 562.5$ μA

$= 5.625$ mA

Transistor Q_4 will carry I_2 but will retain the same V_{GS} as before, thus V_2 remains unchanged at 2.5 V.

5.61 Refer to the circuit in Fig. P5.61.

(a) Q_1 and Q_2 are matched. Thus, from symmetry, we see that the 200-μA current will split equally between Q_1 and Q_2:

$I_{D1} = I_{D2} = 100\ \mu A$

$V_1 = V_2 = 2.5 - 0.1 \times 20 = 0.5\ V$

To find V_3, we determine V_{GS} of either Q_1 and Q_2 (which, of course, are equal),

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_t)^2$$

$$100 = \frac{1}{2} \times 125 \times 20 \times (V_{GS} - 0.7)^2$$

$\Rightarrow V_{GS} = 0.983\ V$

Thus,

$V_3 = -0.983\ V$

(b) With $V_{GS1} = V_{GS2}$, but $(W/L)_1 = 1.5(W/L)_2$, transistor Q_1 will carry a current 1.5 times that in Q_2, that is,

$I_{D1} = 1.5 I_{D2}$

But,

$I_{D1} + I_{D2} = 200\ \mu A$

Thus

$I_{D1} = 120\ \mu A$

$I_{D2} = 80\ \mu A$

$V_1 = 2.5 - 0.12 \times 20 = 0.1\ V$

$V_2 = 2.5 - 0.08 \times 20 = 0.9\ V$

To find V_3, we find V_{GS} from the I_D equation for either Q_1 or Q_2,

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_t)^2$$

$$120 = \frac{1}{2} \times 125 \times 20 \times (V_{GS} - 0.7)^2$$

$\Rightarrow V_{GS} = 1.01\ V$

$V_3 = -1.01\ V$

5.62 Using Eq. (5.30), we can write

$$V_t = V_{t0} + \gamma\left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right]$$

where

$V_{t0} = 1.0\ V$

$\gamma = 0.5\ V^{1/2}$

$2\phi_f = 0.6\ V$

and

$V_{SB} = 0$ to $4\ V$

At

$V_{SB} = 0, \quad V_t = V_{t0} = 1.0\ V$

At

$V_{SB} = 4\ V,$

$$V_t = 1 + 0.5\left[\sqrt{0.6+4} - \sqrt{0.6}\right]$$

$= 1.69\ V$

If the gate oxide thickness is increased by a factor of 4, C_{ox} will decrease by a factor of 4 and Eq. (5.31) indicates that γ will increase by a factor of 4, becoming 2. Thus at $V_{SB} = 4\ V$,

$$V_t = 1 + 2\left[\sqrt{0.6+4} - \sqrt{0.6}\right]$$

$= 3.74\ V$

5.63 $|V_t| = |V_{t0}| + \gamma\left[\sqrt{2\phi_f + |V_{SB}|} - \sqrt{2\phi_f}\right]$

$$= 0.7 + 0.5\left[\sqrt{0.75+3} - \sqrt{0.75}\right]$$

$= 1.24\ V$

Thus,

$V_t = -1.24\ V$

5.64 (a) $i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$

$$\frac{\partial i_D}{\partial T} = \frac{1}{2}\frac{\partial k_n'}{\partial T}\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

$$-k_n'\left(\frac{W}{L}\right)(v_{GS} - V_t)\frac{\partial V_t}{\partial T}$$

$$\frac{\partial i_D/i_D}{\partial T} = \frac{\partial k_n'/k_n'}{\partial T} - \frac{2}{V_{GS} - V_t}\frac{\partial V_t}{\partial T}$$

For

$$\frac{\partial V_t}{\partial T} = -2\ mV/^\circ C = -0.002\ V/^\circ C$$

and

$$\frac{\partial i_D/i_D}{\partial T} = -0.002/^\circ C, \quad V_{GS} = 5\ V$$

and

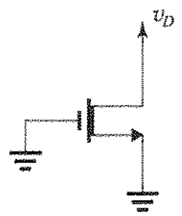
$V_t = 1\ V$

$$-0.002 = \frac{\partial k_n'/k_n'}{\partial T} - \frac{2 \times -0.002}{5-1}$$

$$\Rightarrow \frac{\partial k_n'/k_n'}{\partial T} = -0.003/^\circ C$$

or $-0.3\%/^\circ C$

5.65



The NMOS depletion-type MOSFET has the same $i-v$ characteristics as the enhancement-type NMOS except that V_{tn} is negative, for the depletion device:

$$i_D = k_n\left[(v_{GS} - V_{tn})v_{DS} - \frac{1}{2}v_{DS}^2\right], \quad \text{for } v_{DS} \le v_{GS} - V_{tn}$$

$$i_D = \frac{1}{2}k_n(v_{GS} - V_{tn})^2,$$

for $v_{DS} \ge v_{GS} - V_{tn}$

For our case, $v_{GS} = 0$, $V_{tn} = -3$ V, and $k_n = 2$ mA/V^2. Thus

$$i_D = 2\left(3v_D - \frac{1}{2}v_D^2\right), \quad \text{for } v_D \le 3 \text{ V}$$

$$i_D = \frac{1}{2} \times 2 \times 9 = 9 \text{ mA}, \quad \text{for } v_D \ge 3 \text{ V}$$

For

$$v_D = 0.1 \text{ V}, \quad i_D = 2\left(3 \times 0.1 - \frac{1}{2} \times 0.1^2\right)$$
$= 0.59$ mA (triode)

For

$$v_D = 1 \text{ V}, \quad i_D = 2\left(3 \times 1 - \frac{1}{2} \times 1\right)$$
$=5$ mA (triode)

For

$v_D = 3$ V, $\quad i_D = 9$ mA (saturation)

For

$v_D = 5$ V, $\quad i_D = 9$ mA (saturation)

5.66 $i_D = k_n\left[(v_{GS} - V_{tn})v_{DS} - \frac{1}{2}v_{DS}^2\right]$,
for $v_{DS} \le v_{GS} - V_{tn}$

$$i_D = \frac{1}{2}k_n(v_{GS} - V_{tn})^2(1 + \lambda\, v_{DS}),$$
for $v_{DS} \ge v_{GS} - V_{tn}$

For our case,

$V_{tn} = -2$ V, $k_n = 0.2$ mA/V^2, $\lambda = 0.02$ V^{-1}

and $v_{GS} = 0$. Thus

$$i_D = 0.2\left(2\, v_{DS} - \frac{1}{2}v_{DS}^2\right), \quad \text{for } v_{DS} \le 2 \text{ V}$$

$$i_D = 0.4(1 + 0.02\, v_{DS}), \quad \text{for } v_{DS} \ge 2 \text{ V}$$

For $v_{DS} = 1$ V,

$$i_D = 0.2\left(2 - \frac{1}{2}\right) = 0.3 \text{ mA}$$

For $v_{DS} = 2$ V,

$$i_D = 0.4(1 + 0.02 \times 2) = 0.416 \text{ mA}$$

For $v_{DS} = 3$ V,

$$i_D = 0.4(1 + 0.02 \times 3) = 0.424 \text{ mA}$$

For $v_{DS} = 10$ V,

$$i_D = 0.4(1 + 0.02 \times 10) = 0.48 \text{ mA}$$

If the device width W is doubled, k_n is doubled, and each of the currents above will be doubled. If both W and L are doubled, k_n remains unchanged. However, λ is divided in half; thus for $v_{DS} = 2$ V, i_D becomes 0.408 mA; for $v_{DS} = 3$ V, i_D becomes 0.412 mA; and for $v_{DS} = 10$ V, i_D becomes 0.44 mA.

5.67

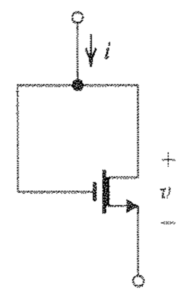


Fig. 1

The depletion-type MOSFET operates in the triode region when $v_{DS} \le v_{GS} - V_t$; that is, $v_{DG} \le -V_t$, where V_t is negative. In the case shown in Fig. 1, $v_{DG} = 0$. Thus the condition for triode-mode operation is satisfied, and

$$i_D = k_n\left[(v_{GS} - V_t)v_{DS} - \frac{1}{2}v_{DS}^2\right]$$

which applies when the channel is not depleted, that is, when $v_{GS} \ge V_t$. For our case,

$$i = k_n\left[(v - V_t)v - \frac{1}{2}v^2\right], \quad \text{for } v \ge V_t$$

Thus,

$$i = \frac{1}{2}k_n(v^2 - 2V_t v), \quad \text{for } v \ge V_t$$

For $v \le V_t$, the source and the drain exchange roles, as indicated in Fig. 2.

Here $v_{GS} = 0$ and $v_{DS} = -v$; thus $v_{DS} \ge -V_t$. Thus the device is operating in saturation, and

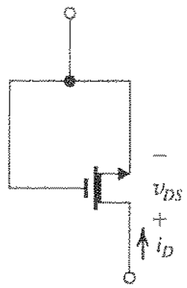


Fig. 2

$$i_D = \frac{1}{2}k_n(0 - V_t)^2$$

$$i_D = \frac{1}{2}k_n V_t^2$$

But $i = i_D$; thus

$$i = \frac{1}{2}k_n V_t^2, \quad \text{for} \quad v \leq V_t$$

Figure 3 is a sketch of the $i-v$ relationship for the case $V_t = -2$ V and $k_n = 2$ mA/V^2.

Here

$i = v(v+4)$, for $v \geq -2$ V

and

$i = -4$ mA, for $v \leq -2$ V

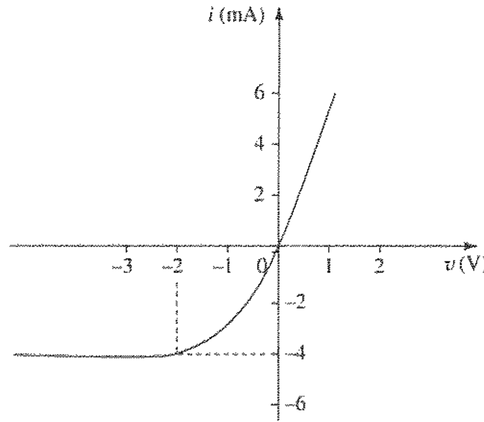


Fig. 3

6.1 1. Active
2. Saturation
3. Active
4. Saturation
5. Active
6. Cutoff

6.2 The EB junctions have a 4:1 area ratio.

$I_C = I_S e^{V_{BE}/V_T}$

$0.5 \times 10^{-3} = I_{S1} \times e^{0.75/0.025}$

$\Rightarrow I_{S1} = 4.7 \times 10^{-17}$ A

$I_{S2} = 4I_{S1} = 1.87 \times 10^{-16}$ A

6.3 $I_C = I_S e^{V_{BE}/V_T}$

$200 \times 10^{-6} = I_S e^{30}$

$\Rightarrow I_S = 1.87 \times 10^{-17}$ A

For the transistor that is 32 times larger,

$I_S = 32 \times 1.87 \times 10^{-17}$

$= 6 \times 10^{-16}$ A

At $V_{BE} = 30\ V_T$, the larger transistor conducts a current of

$I_C = 32 \times 200\ \mu\text{A} = 6.4$ mA

At $I_C = 1$ mA, the base–emitter voltage of the larger transistor can be found as

$1 \times 10^{-3} = 6 \times 10^{-16} e^{V_{BE}/V_T}$

$$V_{BE} = V_T \ln\left(\frac{1 \times 10^{-3}}{6 \times 10^{-16}}\right) = 0.704 \text{ V}$$

6.4 $\dfrac{I_{S1}}{I_{S2}} = \dfrac{A_{E1}}{A_{E2}} = \dfrac{200 \times 200}{0.4 \times 0.4} = 250{,}000$

$I_{C1} = I_{S1} e^{V_{BE1}/V_T}$

$I_{C2} = I_{S2} e^{V_{BE2}/V_T}$

For $I_{C1} = I_{C2}$ we have

$e^{(V_{BE2} - V_{BE1})/V_T} = \dfrac{I_{S1}}{I_{S2}} = 250{,}000$

$V_{BE2} - V_{BE1} = 0.025 \ln(250{,}000)$

$= 0.31$ V

6.5 $I_{C1} = 10^{-13} e^{700/25} = 0.145A = 145$ mA

$I_{C2} = 10^{-18} e^{700/25} = 1.45\ \mu$A

For the first transistor 1 to conduct a current of 1.45 μA, its V_{BE} must be

$$V_{BE1} = 0.025 \ln\left(\frac{1.45 \times 10^{-6}}{10^{-13}}\right)$$

$= 0.412$ V

6.6 Old technology:

$10^{-3} = 2 \times 10^{-15} e^{V_{BE}/V_T}$

$$V_{BE} = 0.025 \ln\left(\frac{10^{-3}}{2 \times 10^{-15}}\right) = 0.673 \text{ V}$$

New technology:

$10^{-3} = 2 \times 10^{-18} e^{V_{BE}/V_T}$

$$V_{BE} = 0.025 \ln\left(\frac{10^{-3}}{2 \times 10^{-18}}\right) = 0.846 \text{ V}$$

6.7 $5 \times 10^{-3} = I_S e^{0.76/0.025}$ (1)

$I_C = I_S e^{0.70/0.025}$ (2)

Dividing Eq. (2) by Eq. (1) yields

$I_C = 5 \times 10^{-3} e^{-0.06/0.025}$

$= 0.45$ mA

For $I_C = 5\ \mu$A,

$5 \times 10^{-6} = I_S e^{V_{BE}/0.025}$ (3)

Dividing Eq. (3) by Eq. (1) yields

$10^{-3} = e^{(V_{BE} - 0.76)/0.025}$

$V_{BE} = 0.76 + 0.025 \ln(10^{-3})$

$= 0.587$ V

6.8 $I_B = 10\ \mu$A

$I_C = 800\ \mu$A

$$\beta = \frac{I_C}{I_B} = 80$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{80}{81} = 0.988$$

6.9

α	0.5	0.8	0.9	0.95	0.98	0.99	0.995	0.999
$\beta = \frac{\alpha}{1-\alpha}$	1	4	9	19	49	99	199	999

6.10

β	1	2	10	20	50	100	200	500	1000
$\alpha = \frac{\beta}{\beta+1}$	0.5	0.67	0.91	0.95	0.98	0.99	0.995	0.998	0.999

6.11 $\beta = \dfrac{\alpha}{1 - \alpha}$ (1)

$\alpha \rightarrow \alpha + \Delta\alpha$

$\beta \rightarrow \beta + \Delta\beta$

$\beta + \Delta\beta = \dfrac{\alpha + \Delta\alpha}{1 - \alpha - \Delta\alpha}$ (2)

Subtracting Eq. (1) from Eq. (2) gives

$$\Delta\beta = \frac{\alpha + \Delta\alpha}{1 - \alpha - \Delta\alpha} - \frac{\alpha}{1 - \alpha}$$

$$\Delta\beta = \frac{\Delta\alpha}{(1 - \alpha - \Delta\alpha)(1 - \alpha)} \qquad (3)$$

Dividing Eq. (3) by Eq. (1) gives

$$\frac{\Delta\beta}{\beta} = \left(\frac{\Delta\alpha}{\alpha}\right)\left(\frac{1}{1 - \alpha - \Delta\alpha}\right)$$

For $\Delta\alpha \ll 1$, the second factor on the right-hand side is approximately equal to β. Thus

$$\frac{\Delta\beta}{\beta} \simeq \beta\left(\frac{\Delta\alpha}{\alpha}\right) \qquad \text{Q.E.D.}$$

For $\dfrac{\Delta\beta}{\beta} = -10\%$ and $\beta = 100$,

$$\frac{\Delta\alpha}{\alpha} \simeq \frac{-10\%}{100} = -0.1\%$$

6.12 Transistor is operating in active region:

$\beta = 50 \rightarrow 300$

$I_B = 10\ \mu\text{A}$

$I_C = \beta I_B = 0.5\ \text{mA} \rightarrow 3\ \text{mA}$

$I_E = (\beta + 1)I_B = 0.51\ \text{mA} \rightarrow 3.01\ \text{mA}$

Maximum power dissipated in transistor is

$I_B \times 0.7\ \text{V} + I_C \times V_C$

$= 0.01 \times 0.7 + 3 \times 10 \simeq 30\ \text{mW}$

6.13 $i_C = I_S e^{v_{BE}/V_T}$

$= 5 \times 10^{-15} e^{0.7/0.025} = 7.2\ \text{mA}$

i_B will be in the range $\dfrac{7.2}{50}$ mA to $\dfrac{7.2}{200}$ mA, that is, 144 μA to 36 μA.

i_E will be in the range (7.2 + 0.144) mA to (7.2 + 0.036) mA, that is, 7.344 mA to 7.236 mA.

6.14 For $i_B = 10\ \mu\text{A}$,

$i_C = i_E - i_B = 1000 - 10 = 990\ \mu\text{A}$

$$\beta = \frac{i_C}{i_B} = \frac{990}{10} = 99$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{99}{100} = 0.99$$

For $i_B = 20\ \mu\text{A}$,

$i_C = i_E - i_B = 1000 - 20 = 980\ \mu\text{A}$

$$\beta = \frac{i_C}{i_B} = \frac{980}{20} = 49$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{49}{50} = 0.98$$

For $i_B = 50\ \mu\text{A}$,

$i_C = i_E - i_B = 1000 - 50 = 950\ \mu\text{A}$

$$\beta = \frac{i_C}{i_B} = \frac{950}{50} = 19$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{19}{20} = 0.95$$

6.15 See Table below.

6.16 First we determine I_S, β, and α:

$1 \times 10^{-3} = I_S e^{700/25}$

$\Rightarrow I_S = 6.91 \times 10^{-16}\ \text{A}$

$$\beta = \frac{I_C}{I_B} = \frac{1\ \text{mA}}{10\ \mu\text{A}} = 100$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

Then we can determine I_{SE} and I_{SB}:

$$I_{SE} = \frac{I_S}{\alpha} = 6.98 \times 10^{-16}\ \text{A}$$

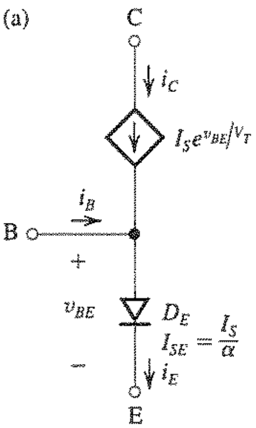
$$I_{SB} = \frac{I_S}{\beta} = 6.91 \times 10^{-18}\ \text{A}$$

The figure on next page shows the four large-signal models, corresponding to Fig. 6.5(a) to (d), together with their parameter values.

This table belongs to Problem **6.15**.

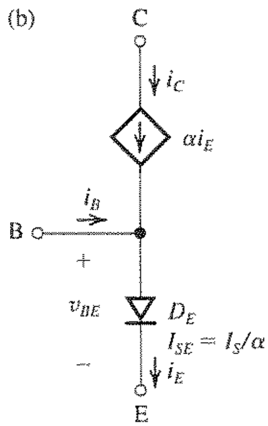
Transistor	**a**	**b**	**c**	**d**	**e**
V_{BE} (mV)	700	690	580	780	820
I_C (mA)	1.000	1.000	0.230	10.10	73.95
I_B (μA)	10	20	5	120	1050
I_E (mA)	1.010	1.020	0.235	10.22	75
α	0.99	0.98	0.979	0.988	0.986
β	100	50	46	84	70
I_S (A)	6.9×10^{-16}	1.0×10^{-15}	1.9×10^{-14}	2.8×10^{-16}	4.2×10^{-16}

(a)



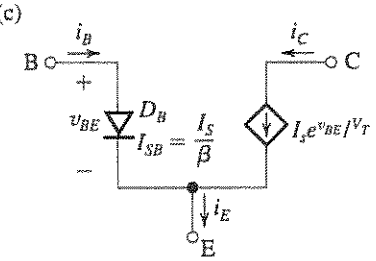
$I_S = 6.91 \times 10^{-16}$ A
$I_{SE} = 6.98 \times 10^{-16}$ A

(b)



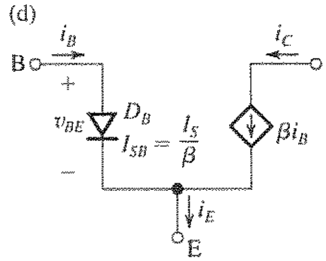
$\alpha = 0.99$
$I_{SE} = 6.98 \times 10^{-16}$ A

(c)



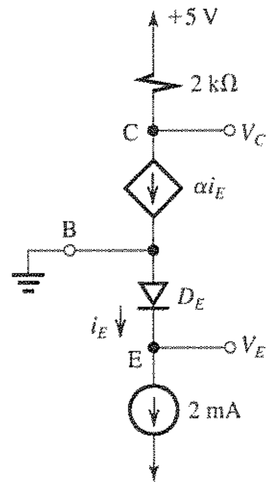
$I_S = 6.91 \times 10^{-16}$ A
$I_{SB} = 6.91 \times 10^{-18}$ A

(d)



$\beta = 100$
$I_{SB} = 6.91 \times 10^{-18}$ A

6.17



The figure shows the circuit, where

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

$$I_{SE} = \frac{I_S}{\alpha} = \frac{5 \times 10^{-15}}{0.99} = 5.05 \times 100^{-15} \text{ A}$$

The voltage at the emitter V_E is

$$V_E = -V_{DE}$$

$$= -V_T \ln(I_E/I_{SE})$$

$$= -0.025 \ln\left(\frac{2 \times 10^{-3}}{5.05 \times 10^{-15}}\right)$$

$$= -0.668 \text{ V}$$

The voltage at the collector V_C is found from

$$V_C = 5 - I_C \times 2$$

$$= 5 - \alpha I_E \times 2$$

$$= 5 - 0.99 \times 2 \times 2 = 1.04 \text{ V}$$

6.18 Refer to the circuit in Fig. 6.6(b).

$$I_{SB} = \frac{I_S}{\beta} = \frac{5 \times 10^{-15}}{50} = 10^{-16} \text{ A}$$

$$I_B = \frac{I_C}{\beta} = \frac{0.5 \times 10^{-3}}{50} = 10^{-5} \text{ A}$$

$$V_B = V_{BE} = V_T \ln\left(\frac{I_B}{I_{SB}}\right)$$

$$= 0.025 \ln\left(\frac{10^{-5}}{10^{-16}}\right)$$

$$= 0.633 \text{ V}$$

We can determine R_B from

$$R_B = \frac{V_{CC} - V_B}{I_B}$$

$$= \frac{15 - 0.633}{10^{-5}} = 1.44 \text{ M}\Omega$$

To obtain $V_{CE} = 1$ V, we select R_C according to

$$R_C = \frac{V_{CC} - V_{CE}}{I_C}$$

$$= \frac{15 - 1}{0.5} = 28 \text{ k}\Omega$$

6.19 $I_S = 10^{-15}$ A

Thus, a forward-biased EBJ conducting a current of 1 mA will have a forward voltage drop V_{BE}:

$$V_{BE} = V_T \ln\left(\frac{I}{I_S}\right)$$

$$= 0.025 \ln\left(\frac{10^{-3}}{10^{-15}}\right) = 0.691 \text{ V}$$

$$I_{SC} = 100 I_S = 10^{-13} \text{ A}$$

Thus, a forward-biased CBJ conducting a 1-mA current will have a forward voltage drop V_{BC}:

$$V_{BC} = V_T \ln\left(\frac{1 \times 10^{-3}}{1 \times 10^{-13}}\right) = 0.576 \text{ V}$$

When forward-biased with 0.5 V, the emitter–base junction conducts

$$I = I_S e^{0.5/0.025}$$

$$= 10^{-15} e^{0.5/0.025} = 0.49 \ \mu\text{A}$$

and the CBJ conducts

$$I = I_{SC} e^{0.5/0.025}$$

$$= 10^{-13} e^{0.5/0.025} = 48.5 \ \mu\text{A}$$

6.20 The equations utilized are

$$v_{BC} = v_{BE} - v_{CE} = 0.7 - v_{CE}$$

$$i_{BC} = I_{SC} e^{v_{BC}/V_T} = 10^{-13} e^{v_{BC}/0.025}$$

$$i_{BE} = I_{SB} e^{v_{BE}/V_T} = 10^{-17} e^{0.7/0.025}$$

$$i_B = i_{BC} + i_{BE}$$

$$i_C = I_S e^{v_{BE}/V_T} - i_{BC} = 10^{-15} e^{0.7/0.025} - i_{BC}$$

Performing these calculations for $v_{CE} = 0.4$ V, 0.3 V, and 0.2 V, we obtain the results shown in the table below.

6.21 Dividing Eq. (6.14) by Eq. (6.15) and substituting $i_C/i_B = \beta_{\text{forced}}$ gives

$$\beta_{\text{forced}} = \frac{I_S e^{v_{BE}/V_T} - I_{SC} e^{v_{BC}/V_T}}{(I_S/\beta) e^{v_{BE}/V_T} + I_{SC} e^{v_{BC}/V_T}}$$

Dividing the numerator and denominator of the right-hand side by $I_{SC} e^{v_{BC}/V_T}$ and replacing $v_{BE} - v_{BC}$ by $V_{CE\text{sat}}$ gives

$$\beta_{\text{forced}} = \frac{\left(\frac{I_S}{I_{SC}}\right) e^{V_{CE\text{sat}}/V_T} - 1}{\frac{1}{\beta}\left(\frac{I_S}{I_{SC}}\right) e^{V_{CE\text{sat}}/V_T} + 1}$$

This equation can be used to obtain $e^{V_{CE\text{sat}}/V_T}$ and hence $V_{CE\text{sat}}$ as

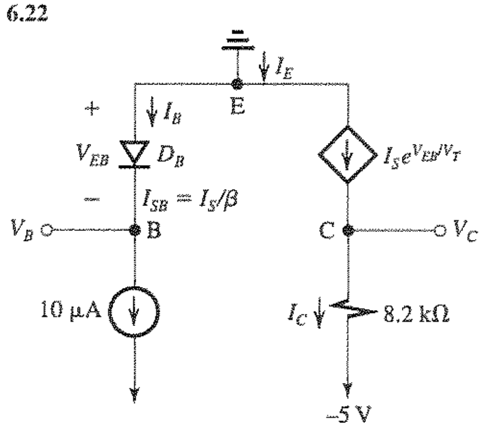
$$\left(\frac{I_S}{I_{SC}}\right) e^{V_{CE\text{sat}}/V_T} = \frac{1 + \beta_{\text{forced}}}{1 - \beta_{\text{forced}}/\beta}$$

$$\Rightarrow V_{CE\text{sat}} = V_T \ln\left[\frac{I_{SC}}{I_S} \frac{1 + \beta_{\text{forced}}}{1 - \beta_{\text{forced}}/\beta}\right] \qquad \text{Q.E.D.}$$

For $\beta = 100$ and $I_{SC}/I_S = 100$, we can use this equation to obtain $V_{CE\text{sat}}$ corresponding to the given values of β_{forced}. The results are as follows:

β_{forced}	50	10	5	1
$V_{CE\text{sat}}$ (V)	0.231	0.178	0.161	0.133

6.22



The emitter–base voltage V_{EB} is found as the voltage drop across the diode D_B, whose scale

This table belongs to Problem **6.20**.

v_{CE} (V)	v_{BC} (V)	i_{BC} (μA)	i_{BE} (μA)	i_B (μA)	i_C (mA)	i_C/i_B
0.4	0.3	0.016	14.46	14.48	1.446	100
0.3	0.4	0.89	14.46	15.35	1.445	94
0.2	0.5	48.5	14.46	62.96	1.398	29

current is $I_{SB} = I_S/\beta$, it is conducting a 10-μA current. Thus,

$$V_{EB} = V_T \ln\left(\frac{10\ \mu\text{A}}{I_{SB}}\right)$$

where

$$I_{SB} = \frac{I_S}{\beta} = \frac{10^{-14}}{50} = 2 \times 10^{-16}\ \text{A}$$

$$V_{EB} = 0.025 \ln\left(\frac{10 \times 10^{-6}}{2 \times 10^{-16}}\right)$$

$$= 0.616\ \text{V}$$

Thus,

$$V_B = -V_{EB} = -0.616\ \text{V}$$

The collector current can be found as

$$I_C = \beta I_B$$

$$= 50 \times 10 = 500\ \mu\text{A} = 0.5\ \text{mA}$$

The collector voltage can now be obtained from

$$V_C = -5 + I_C \times 8.2 = -5 + 0.5 \times 8.2 = -0.9\ \text{V}$$

The emitter current can be found as

$$I_E = I_B + I_C = 10 + 500 = 510\ \mu\text{A}$$

$$= 0.51\ \text{mA}$$

6.23 At $i_C = 1$ mA, $v_{EB} = 0.7$ V

At $i_C = 10$ mA,

$$v_{EB} = 0.7 + V_T \ln\left(\frac{10}{1}\right)$$

$$= 0.7 + 0.025 \ln(10) = 0.758\ \text{V}$$

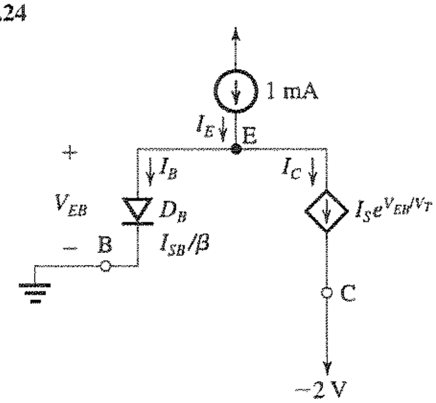
At $i_C = 100$ mA,

$$v_{EB} = 0.7 + 0.025 \ln\left(\frac{100}{1}\right)$$

$$= 0.815\ \text{V}$$

Note that v_{EB} increases by about 60 mV for every decade increase in i_C.

6.24



Referring to the figure, we see that

$$I_E = I_B + I_C = \frac{I_C}{\beta} + I_C$$

Thus,

$$I_C = \frac{I_E}{1 + \frac{1}{\beta}} = \frac{1}{1 + \frac{1}{10}} = 0.909\ \text{mA}$$

$$I_B = 0.091\ \text{mA}$$

For direction of flow, refer to the figure.

$$V_{EB} = V_T \ln\left(\frac{I_B}{I_{SB}}\right)$$

where

$$I_{SB} = \frac{I_S}{\beta} = \frac{10^{-15}}{10} = 10^{-16}\ \text{A}$$

$$V_{EB} = 0.025 \ln\left(\frac{0.091 \times 10^{-3}}{10^{-16}}\right)$$

$$= 0.688\ \text{V}$$

Thus,

$$V_E = V_B + V_{EB} = 0 + 0.688 = 0.688\ \text{V}$$

If a transistor with $\beta = 1000$ is substituted,

$$I_C = \frac{I_E}{1 + \frac{1}{\beta}} = \frac{1}{1 + \frac{1}{1000}} = 0.999\ \text{mA}$$

Thus, I_C changes by $0.999 - 0.909 = 0.09$ mA, a 9.9% increase.

6.25

$$I_B = \frac{I_E}{\beta + 1} = \frac{5}{20 + 1} = 0.238\ \text{A} = 238\ \text{mA}$$

$$I_C = I_S e^{V_{EB}/V_T}$$

$$\alpha I_E = I_S e^{V_{EB}/V_T}$$

where

$$\alpha = \frac{20}{21} = 0.95$$

$$I_S = \alpha I_E e^{-V_{EB}/V_T}$$

$$= 0.95 \times 5 e^{-(0.8/0.025)}$$

$$= 6 \times 10^{-14}\ \text{A}$$

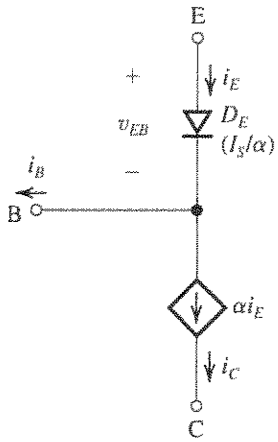
A transistor that conducts $I_C = 1$ mA with $V_{EB} = 0.70$ V has a scale current

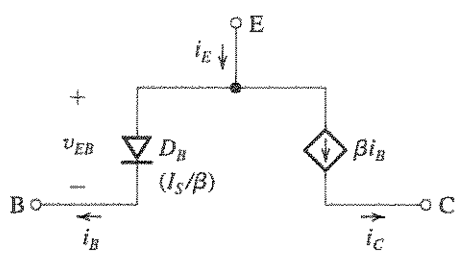
$$I_S = 1 \times 10^{-3} e^{-0.70/0.025} = 6.9 \times 10^{-16}\ \text{A}$$

The emitter–base junction areas of these two transistors will have the same ratio as that of their scale currents, thus

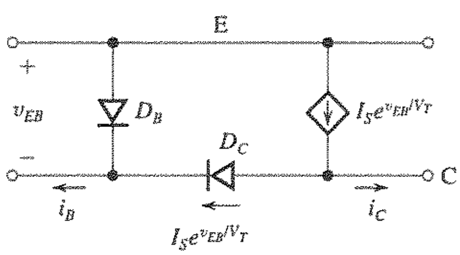
$$\frac{\text{EBJ area of first transistor}}{\text{EBJ area of second transistor}} = \frac{6 \times 10^{-14}}{6.9 \times 10^{-16}} = 87$$

6.26 The two missing large-signal equivalent circuits for the *pnp* transistor are those corresponding to the *npn* equivalent circuits in Fig. 6.5(b) and 6.5(d). They are shown in the figure.





6.27



6.28 (a) Refer to Fig. P6.28(a).

$$I_1 = \frac{10.7 - 0.7}{5\ \text{k}\Omega} = 2\ \text{mA}$$

Assuming operation in the active mode,

$$I_C = \alpha I_1 \simeq I_1 = 2\ \text{mA}$$

$$V_2 = -10.7 + I_C \times 5$$

$$= -10.7 + 2 \times 5 = -0.7\ \text{V}$$

Since V_2 is lower than V_B, which is 0 V, the transistor is operating in the active mode, as assumed.

(b) Refer to Fig. P6.28(b).

Since $V_C = -4$ V is lower than $V_B = -2.7$ V, the transistor is operating in the active mode.

$$I_C = \frac{-4 - (-10)}{2.4\ \text{k}\Omega} = 2.5\ \text{mA}$$

$$I_E = \frac{I_C}{\alpha} \simeq I_C = 2.5\ \text{mA}$$

$$V_3 = +12 - I_E \times 5.6 = 12 - 2.5 \times 5.6 = -2\ \text{V}$$

(c) Refer to Fig. P6.28(c) and use

$$I_C = \frac{0 - (-10)}{20} = 0.5\ \text{mA}$$

Assuming active-mode operation, and utilizing the fact that β is large, $I_B \simeq 0$ and

$$V_4 \simeq 2\ \text{V}$$

Since $V_C < V_B$, the transistor is indeed operating in the active region.

$$I_5 = I_E = \frac{I_C}{\alpha} \simeq I_C = 0.5\ \text{mA}$$

(d) Refer to Fig. P6.28(d). Since the collector is connected to the base with a 10-kΩ resistor and β is assumed to be very high, the voltage drop across the 10-kΩ resistor will be close to zero and the base voltage will be equal to that of the collector:

$$V_B = V_7$$

This also implies active-mode operation. Now,

$$V_E = V_B - 0.7$$

Thus,

$$V_E = V_7 - 0.7$$

$$I_6 = \frac{V_E - (-10)}{3}$$

$$= \frac{V_7 - 0.7 + 10}{3} = \frac{V_7 + 9.3}{3} \qquad (1)$$

Since $I_B = 0$, the collector current will be equal to the current through the 9.1-kΩ resistor,

$$I_C = \frac{+10 - V_7}{9.1} \qquad (2)$$

Since $\alpha 1 \simeq 1$, $I_C = I_E = I_6$ resulting in

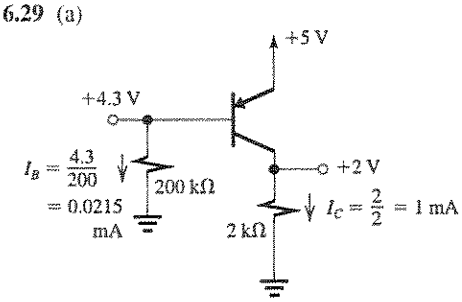
$$\frac{10 - V_7}{9.1} = \frac{V_7 + 9.3}{3}$$

$$\Rightarrow V_7 = -4.5\ \text{V}$$

and

$$I_6 = \frac{V_7 + 9.3}{3} = \frac{-4.5 + 9.3}{3} = 1.6\ \text{mA}$$

6.29 (a)



(a)

Since V_C is lower than V_B, the transistor is operating in the active region. From the figure corresponding to Fig. P6.29(a), we see that

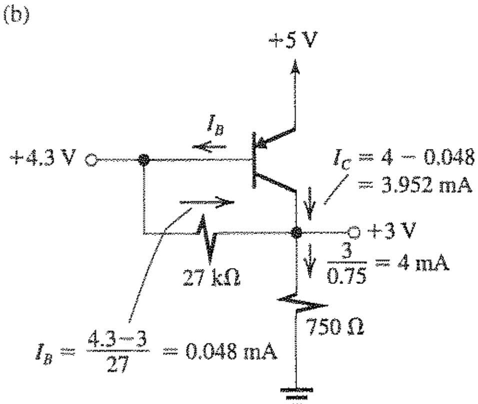
$I_C = 1$ mA

$I_B = 0.0215$ mA

Thus,

$$\beta \equiv \frac{I_C}{I_B} = \frac{1}{0.0215} = 46.5$$

(b)

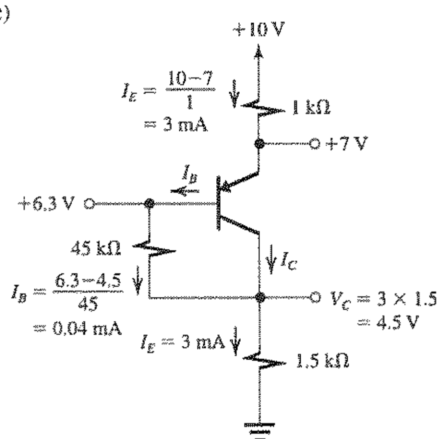


(b)

Observe that with V_C at 3 V and V_B at 4.3 V, the transistor is operating in the active region. Refer to the analysis shown in the figure, which leads to

$$\beta \equiv \frac{I_C}{I_B} = \frac{3.952}{0.048} = 82.3$$

(c)



(c)

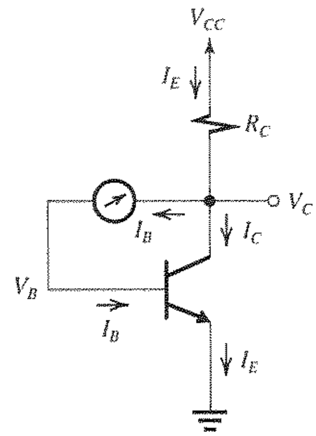
Observe that the transistor is operating in the active region and note the analysis performed on the circuit diagram. Thus,

$$I_C = I_E - I_B = 3 - 0.04 = 2.96 \text{ mA}$$

and

$$\beta \equiv \frac{I_C}{I_B} = \frac{2.96}{0.04} = 74$$

6.30



Since the meter resistance is small, $V_C \simeq V_B$ and the transistor is operating in the active region. To obtain $I_E = 1$ mA, we arrange that $V_{BE} = 0.7$ V. Since $V_C \simeq V_B$, V_C must be set to 0.7 by selecting R_C according to

$$V_C = 0.7 = V_{CC} - I_E R_C$$

Thus,

$$0.7 = 9 - 1 \times R_C$$

$$\Rightarrow R_C = 8.3 \text{ k}\Omega$$

Since the meter reads full scale when the current flowing through it (in this case, I_B is 50 μA), a full-scale reading corresponds to

$$\beta \equiv \frac{I_C}{I_B} \simeq \frac{1 \text{ mA}}{50 \text{ μA}} = 20$$

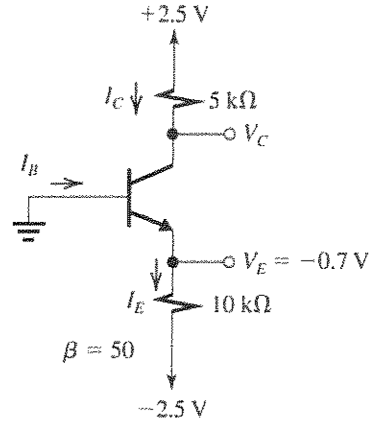
If the meter reads 1/5 of full scale, then $I_B = 10$ μA and

$$\beta = \frac{1 \text{ mA}}{10 \text{ μA}} = 100$$

A meter reading of 1/10 full scale indicates that

$$\beta = \frac{1 \text{ mA}}{5 \text{ μA}} = 200$$

6.31



$$I_E = \frac{V_E - (-2.5)}{10}$$

$$= \frac{-0.7 + 2.5}{10} = 0.18 \text{ mA}$$

Assuming the transistor is operating in the active mode, we obtain

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.18}{50 + 1} = 3.5 \ \mu\text{A}$$

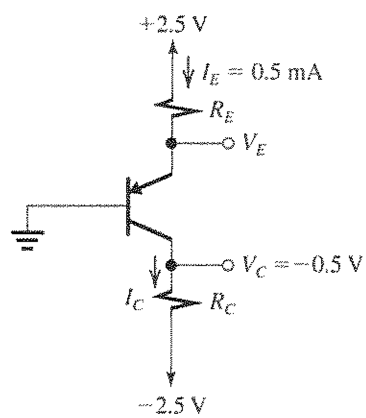
$$I_C = \left(\frac{\beta}{\beta + 1}\right) I_E = \frac{50}{51} \times 0.18 = 0.176 \text{ mA}$$

$$V_C = +2.5 - I_C R_C$$

$$= 2.5 - 0.176 \times 5 = 1.62 \text{ V}$$

Since $V_C > V_B$, active-mode operation is verified.

6.32



From the figure we see that $V_C = -0.5$ V is lower than the base voltage ($V_B = 0$ V); thus the transistor will be operating in the active mode.

$$I_C = \alpha I_E = \left(\frac{\beta}{\beta + 1}\right) I_E = \frac{100}{100 + 1} \times 0.5$$

$$= 0.495 \text{ mA}$$

$$R_C = \frac{V_C - (-2.5)}{I_C}$$

$$= \frac{-0.5 + 2.5}{0.495} = 4.04 \text{ k}\Omega \simeq 4 \text{ k}\Omega$$

The transistor V_{EB} can be found from

$$V_{EB} = 0.64 + V_T \ln\left(\frac{0.5 \text{ mA}}{0.1 \text{ mA}}\right)$$

$$= 0.68 \text{ V}$$

Thus,

$$V_E = +0.68 \text{ V}$$

and

$$R_E = \frac{2.5 - 0.68}{0.5} = 3.64 \text{ k}\Omega$$

The maximum allowable value for R_C while the transistor remains in the active mode corresponds to $V_C = +0.4$ V. Thus,

$$R_{C\max} = \frac{0.4 - (-2.5)}{0.495} = 5.86 \text{ k}\Omega$$

6.33 Refer to Fig. 6.15(a) with $R_C = 5.1$ kΩ and $R_E = 6.8$ kΩ. Assuming $V_{BE} \simeq 0.7$ V, then $V_E = -0.7$ V, and

$$I_E = \frac{-0.7 - (-15)}{6.8} = 2.1 \text{ mA}$$

$$I_C = \alpha I_E \simeq 2.1 \text{ mA}$$

$$V_C = 15 - 2.1 \times 5.1 \simeq 4.3 \text{ V}$$

6.34 Refer to the circuit in Fig. P6.34. Since $V_C = 0.5$ V is greater than V_B, the transistor will be operating in the active mode. The transistor V_{BE} can be found from

$$V_{BE} = 0.8 + 0.025 \ln\left(\frac{0.2 \text{ mA}}{1 \text{ mA}}\right)$$

$$= 0.76 \text{ V}$$

Thus,

$$V_E = -0.76 \text{ V}$$

$$I_E = \frac{I_C}{\alpha} = I_C\left(\frac{\beta + 1}{\beta}\right) = 0.2 \times \frac{101}{100}$$

$$= 0.202 \text{ mA}$$

The required value of R_E can be found from

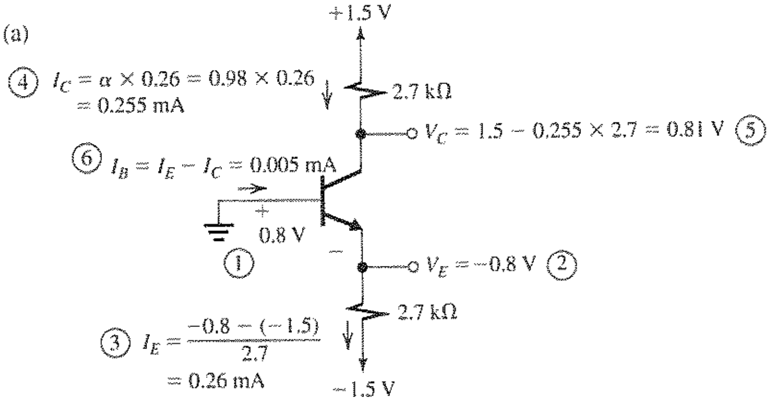
$$R_E = \frac{V_E - (-1.5)}{I_E}$$

$$R_E = \frac{-0.76 + 1.5}{0.202} = 3.66 \text{ k}\Omega$$

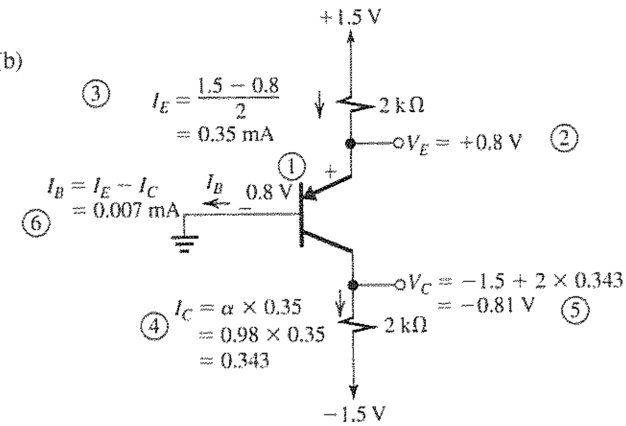
To establish $V_C = 0.5$ V, we select R_C according to

$$R_C = \frac{1.5 - 0.5}{0.2} = 5 \text{ k}\Omega$$

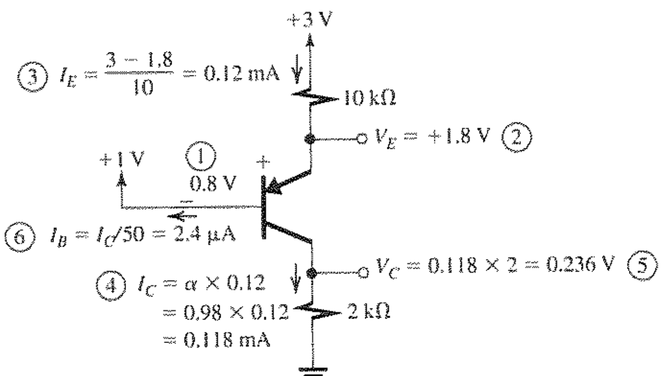
6.35 (a)



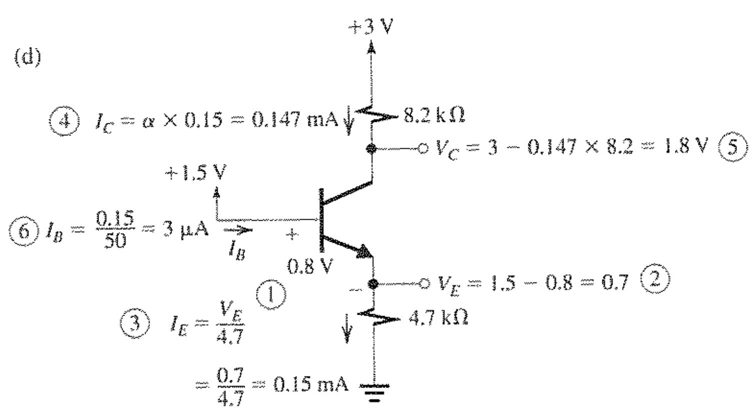
(b)



(c)



(d)

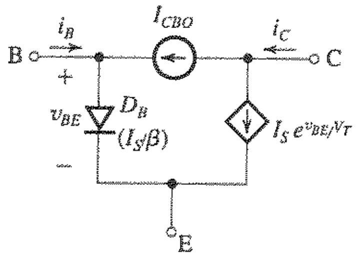


In all circuits shown in Fig. P6.35, we assume active-mode operation and verify that this is the case at the end of the solution. The solutions are indicated on the corresponding circuit diagrams; the order of the steps is shown by the circled numbers.

6.36 I_{CBO} approximately doubles for every 10°C rise in temperature. A change in temperature from 25°C to 125°C—that is, an increase of 100°C—results in 10 doublings or, equivalently, an increase by a factor of $2^{10} = 1024$. Thus I_{CBO} becomes

$I_{CBO} = 10 \text{ nA} \times 1024 = 10.24\ \mu\text{A}$

6.37



From the figure we can write

$$I_B = \left(\frac{I_S}{\beta}\right)e^{v_{BE}/V_T} - I_{CBO} \qquad (1)$$

$$I_C = I_S e^{v_{BE}/V_T} + I_{CBO} \qquad (2)$$

$$I_E = I_S\left(1 + \frac{1}{\beta}\right)e^{v_{BE}/V_T} \qquad (3)$$

When the base is left open-circuited, $i_B = 0$ and Eq. (1) yields

$$I_{CBO} = \left(\frac{I_S}{\beta}\right)e^{v_{BE}/V_T}$$

or equivalently,

$$I_S e^{v_{BE}/V_T} = \beta I_{CBO} \qquad (4)$$

Substituting for $I_S e^{v_{BE}/V_T}$ in Eqs. (2) and (3) gives

$$i_C = i_E = (\beta + 1)I_{CBO}$$

6.38 Since the BJT is operating at a constant emitter current, its $|V_{BE}|$ decreases by 2 mV for every °C rise in temperature. Thus,

$|V_{BE}|$ at 0°C $= 0.7 + 0.002 \times 25 = 0.75$ V

$|V_{BE}|$ at 100°C $= 0.7 - 0.002 \times 75 = 0.55$ V

6.39 (a) If the junction temperature rises to 50°C, which is an increase of 30°C, the EB voltage decreases to

$v_{EB} = 692 - 2 \times 30 = 632$ mV

(b) First, we evaluate V_T at 20°C and at 50°C:

$$V_T = \frac{kT}{q}$$

where $k = 8.62 \times 10^{-5}$ eV/K.

Thus,

At 20°C, $T = 293$ K and $V_T = 8.62 \times 10^{-5} \times 293 = 25.3$ mV

At 50°C, $T = 323$ K and $V_T = 8.62 \times 10^{-5} \times 323 = 27.8$ mV

If the transistor is operated at $v_{BE} = 700$ mV, then

(i) At 20°C, i_E becomes

$i_E = 0.5e^{(700-692)/25.3} = 0.69$ mA

(ii) At 50°C, i_E becomes

$i_E = 0.5e^{(700-632)/27.8} = 5.77$ mA

6.40 $v_{BE} = 0.7$ V at $i_C = 10$ mA

For $v_{BE} = 0.5$ V,

$i_C = 10e^{(0.5-0.7)/0.025} = 3.35\ \mu$A

At a current I_C and a BE voltage V_{BE}, the slope of the i_C–v_{BE} curve is I_C/V_T. Thus,

$$\text{Slope at } V_{BE} \text{ of } 700 \text{ mV} = \frac{10 \text{ mA}}{25 \text{ mV}} = 400 \text{ mA/V}$$

$$\text{Slope at } V_{BE} \text{ of } 500 \text{ mV} = \frac{3.35\ \mu\text{A}}{25 \text{ mV}} = 0.134 \text{ mA/V}$$

$$\text{Ratio of slopes} = \frac{400}{0.134} \simeq 3000$$

6.41 Use Eq. (6.18):

$$i_C = I_S e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right)$$

with $I_S = 10^{-15}$ A and $V_A = 100$ V, to get

$$i_C = 10^{-15} e^{v_{BE}/0.025}\left(1 + \frac{v_{CE}}{100}\right)$$

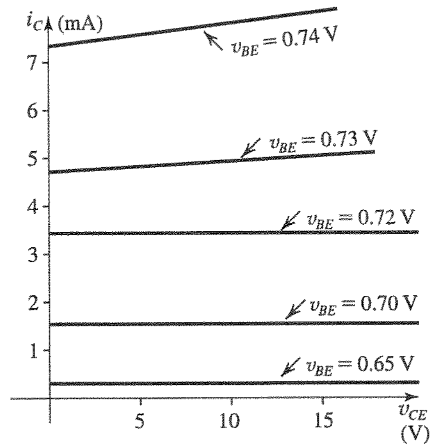
v_{BE}	**0.65 V**	**0.70 V**	**0.72 V**	**0.73 V**	**0.74 V**
v_{CE} (V)	i_C (mA)	i_C (mA)	i_C (mA)	i_C (mA)	i_C (mA)
0	0.196	1.45	3.21	4.81	7.16
15	0.225	1.67	3.70	5.52	8.24

To find the intercept of the straight-line characteristics on the i_c axis, we substitute $v_{CE} = 0$ and evaluate

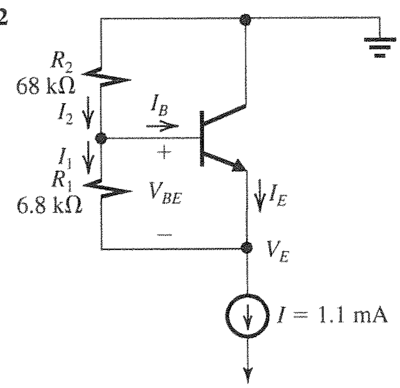
$i_C = 10^{-15} e^{v_{BE}/V_T}$ A

for the given value of v_{BE}. The slope of each straight line is equal to this value divided by 100 V (V_A). Thus we obtain

v_{BE} (V)	0.65	0.70	0.72	0.73	0.74
Intercept (mA)	0.2	1.45	3.22	4.80	7.16
Slope (mA/V)	0.002	0.015	0.032	0.048	0.072



6.42



At 25°C, assume $I_E = 1$ mA. Thus,

$$V_{BE} = 0.68 \text{ V}$$

$$I_1 = \frac{V_{BE}}{R_1} = \frac{0.68 \text{ V}}{6.8 \text{ k}\Omega} = 0.1 \text{ mA}$$

$$I_E = I - I_1 = 1.1 - 0.1 = 1 \text{ mA}$$

which is the value assumed.

$$I_2 = I_1 + I_B = I_1 + \frac{I_E}{\beta + 1}$$

$$= 0.1 + \frac{1}{101} = 0.11 \text{ mA}$$

Note that the currents in R_1 and R_2 differ only by the small base current, 0.01 mA. Had I_1 and I_2 been equal, then we would have had

$$I_1 R_1 = V_{BE}$$

$$I_2 R_2 \simeq I_1 R_2 = V_{BE}\frac{R_2}{R_1}$$

$$V_E = -(I_1 R_1 + I_2 R_2)$$

$$= -V_{BE}\left(1 + \frac{R_2}{R_1}\right) \qquad (1)$$

$$= -V_{BE}\left(1 + \frac{6.8}{0.68}\right) = -11\ V_{BE} = -7.48 \text{ V}$$

which gives this circuit the name "V_{BE} multiplier." A more accurate value of V_E can be obtained by taking I_B into account:

$$V_E = -(I_1 R_1 + I_2 R_2)$$

$$= -\left(V_{BE} + \frac{R_2}{R_1}V_{BE} + I_B R_2\right)$$

$$= -\left(1 + \frac{R_2}{R_1}\right)V_{BE} - I_B R_2 \qquad (2)$$

$$= -7.48 - 0.01 \times 68 = -8.16 \text{ V}$$

As temperature increases, an approximate estimate for the temperature coefficient of V_E can be obtained by assuming that I_E remains constant and ignoring the temperature variation of β. Thus, we would be neglecting the temperature change of the $(I_B R_2)$ terms in Eq. (2). From Eq. (2) we can obtain the temperature coefficient of V_E by utilizing the fact the V_{BE} changes by – 2.2 mV/°C. Thus,

Temperature coefficient of V_E

$$= -\left(1 + \frac{R_2}{R_1}\right) \times -2.2$$

$$= -11 \times -2.2 = +24.2 \text{ mV/}^\circ\text{C}$$

At 75°C, which is a temperature increase of 50°C,

$$V_E = -8.16 + 24.2 \times 50 = -6.95 \text{ V}$$

As a check on our assumption of constant I_E, let us find the value of I_E at 75°C:

$$I_1(75^\circ\text{C}) = \frac{V_{BE}(75^\circ\text{C})}{R_1}$$

$$= \frac{0.68 - 2.2 \times 10^{-3} \times 50}{6.8}$$

$$= 0.084 \text{ mA}$$

$$I_E(75^\circ\text{C}) = I - I_1(75^\circ\text{C})$$

$$= 1.1 - 0.084 = 1.016 \text{ mA}$$

which is reasonably close to the assumed value of 1 mA.

6.43 $r_o = 1/\text{slope}$

$$= 1/(0.8 \times 10^{-5})$$

$$= 125 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C}$$

$$125 \text{ k}\Omega = \frac{V_A}{1 \text{ mA}} \Rightarrow V_A = 125 \text{ V}$$

At $I_C = 10$ mA,

$$r_o = \frac{V_A}{I_C} = \frac{125 \text{ V}}{10 \text{ mA}} = 12.5 \text{ k}\Omega$$

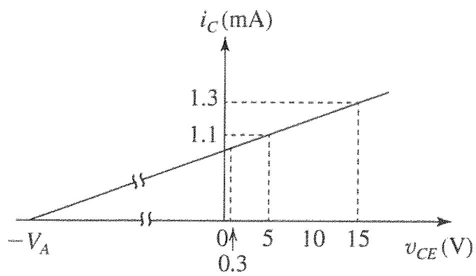
6.44 $r_o = \frac{V_A}{I_C} = \frac{50\text{ V}}{I_C}$

Thus,

At $I_C = 1$ mA, $\quad r_o = \frac{50\text{ V}}{1\text{ mA}} = 50\text{ k}\Omega$

At $I_C = 100\ \mu$A, $\quad r_o = \frac{50\text{ V}}{0.1\text{ mA}} = 500\text{ k}\Omega$

6.45



Slope of i_C–v_{CE} line corresponding to $v_{BE} = 710$ mV is

$$\text{Slope} = \frac{1.3 - 1.1}{15 - 5} = \frac{0.2\text{ mA}}{10\text{ V}} = 0.02\text{ mA/V}$$

Near saturation, $V_{CE} = 0.3$ V, thus

$$i_C = 1.1 - 0.02 \times (5 - 0.3)$$

$$= 1.006 \simeq 1\text{ mA}$$

i_C will be 1.2 mA at,

$$v_{CE} = 5 + \frac{1.2 - 1.1}{0.02} = 10\text{ V}$$

The intercept of the i_C–v_{CE} straight line on the i_C axis will be at

$$i_C = 1.1 - 5 \times 0.02 = 1\text{ mA}$$

Thus, the Early voltage is obtained as

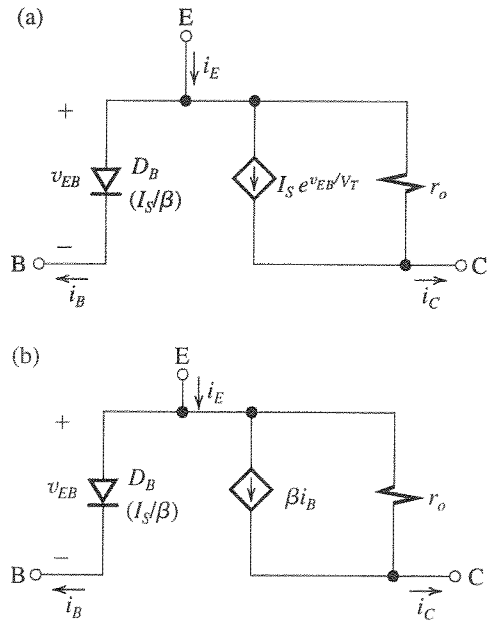
$$\text{Slope} = \frac{i_C(\text{at } v_{CE} = 0)}{V_A}$$

$$\Rightarrow V_A = \frac{1}{0.02} = 50\text{ V}$$

$$r_o = \frac{V_A}{I_C} = \frac{50\text{ V}}{1\text{ mA}} = 50\text{ k}\Omega$$

which is the inverse of the slope of the i_C–v_{CE} line.

6.46 The equivalent circuits shown in the figure correspond to the circuits in Fig. 6.19.



6.47 $\beta = \frac{i_C}{i_B} = \frac{1\text{ mA}}{10\ \mu\text{A}} = 100$

$$\beta_{ac} = \left.\frac{\Delta i_C}{\Delta i_B}\right|_{v_{CE}\text{ constant}} = \frac{0.08\text{ mA}}{1.0\ \mu\text{A}} = 80$$

$$\Delta i_C = \Delta i_B \times \beta_{ac} + \frac{\Delta v_{CE}}{r_o}$$

where

$$r_o = \frac{V_A}{I_C} = \frac{100}{1} = 100\text{ k}\Omega$$

Thus,

$$\Delta i_C = 2 \times 80 + \frac{2}{100} \times 10^3 = 180\ \mu\text{A}$$

$$= 0.18\text{ mA}$$

6.48 Refer to the circuit in Fig. P6.48.

(a) For active-mode operation with $V_C = 2$ V:

$$I_C = \frac{V_{CC} - V_C}{R_C} = \frac{10 - 2}{1} = 8\text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{8}{50} = 0.16\text{ mA}$$

$$V_{BB} = I_B R_B + V_{BE}$$

$$= 0.16 \times 10 + 0.7 = 2.3\text{ V}$$

(b) For operation at the edge of saturation:

$$V_{CE} = 0.3\text{ V}$$

$$I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{10 - 0.3}{1} = 9.7\text{ mA}$$

$I_B = \frac{I_C}{\beta} = \frac{9.7}{50} = 0.194$ mA

$V_{BB} = I_B R_B + V_{BE}$

$= 0.194 \times 10 + 0.7 = 2.64$ V

(c) For operation deep in saturation with $\beta_{\text{forced}} = 10$:

$V_{CE} = 0.2$ V

$$I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{10 - 0.2}{1} = 9.8 \text{ mA}$$

$$I_B = \frac{I_C}{\beta_{\text{forced}}} = \frac{9.8}{10} = 0.98 \text{ mA}$$

$V_{BB} = I_B R_B + V_{BE}$

$= 0.98 \times 10 + 0.7 = 10.5$ V

6.49 Refer to the circuit in Fig. P6.48 (with $V_{BB} = V_{CC}$) and to the BJT equivalent circuit of Fig. 6.21.

$$I_C = \frac{V_{CC} - 0.2}{R_C}$$

$$I_B = \frac{V_{CC} - 0.7}{R_B}$$

$$\beta_{\text{forced}} \equiv \frac{I_C}{I_B}$$

Thus,

$$\beta_{\text{forced}} = \left(\frac{V_{CC} - 0.2}{V_{CC} - 0.7}\right)\left(\frac{R_B}{R_C}\right) \quad (1)$$

$P_{\text{dissipated}} = V_{CC}(I_C + I_B)$

$= V_{CC}(\beta_{\text{forced}} I_B + I_B)$

$= (\beta_{\text{forced}} + 1) V_{CC} I_B \quad (2)$

For $V_{CC} = 5$ V and $\beta_{\text{forced}} = 10$ and $P_{\text{dissipated}} \le 20$ mW, we can proceed as follows.

Using Eq. (1) we can determine (R_B/R_C):

$$10 = \left(\frac{5 - 0.2}{5 - 0.7}\right)\left(\frac{R_B}{R_C}\right)$$

$$\Rightarrow \frac{R_B}{R_C} = 8.96 \quad (3)$$

Using Eq. (2), we can find I_B:

$(10 + 1) \times 5 \times I_B \le 20$ mW

$\Rightarrow I_B \le 0.36$ mA

Thus,

$$\frac{V_{CC} - 0.7}{R_B} \le 0.36 \text{ mA}$$

$\Rightarrow R_B \ge 11.9$ kΩ

From the table of 1% resistors in Appendix J we select

$R_B = 12.1$ kΩ

Substituting in Eq. (3), we have

$R_C = 1.35$ kΩ

From the table of 1% resistors in Appendix J we select

$R_C = 1.37$ kΩ

For these values:

$$I_C = \frac{5 - 0.2}{1.37} = 3.5 \text{ mA}$$

$$I_B = \frac{5 - 0.7}{12.1} = 0.36 \text{ mA}$$

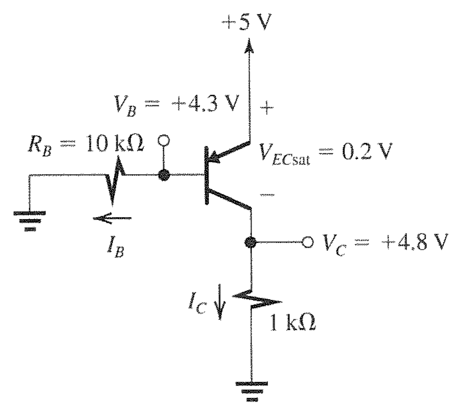
Thus,

$$\beta_{\text{forced}} = \frac{3.5}{0.36} = 9.7$$

$P_{\text{dissipated}} = V_{CC}(I_C + I_B)$

$= 5 \times 3.86 = 19.2$ mW

6.50



Assume saturation-mode operation. From the figure we see that

$$I_C = \frac{V_C}{1 \text{ k}\Omega} = \frac{4.8}{1} = 4.8 \text{ mA}$$

$$I_B = \frac{V_B}{R_B} = \frac{4.3}{10} = 0.43 \text{ mA}$$

Thus,

$$\beta_{\text{forced}} \equiv \frac{I_C}{I_B} = \frac{4.8}{0.43} = 11.2$$

Since 11.2 is lower than the transistor β of 50, we have verified that the transistor is operating in saturation, as assumed.

$V_C = V_{CC} - V_{EC\text{sat}} = 5 - 0.2 = 4.8$ V

To operate at the edge of saturation,

$V_{EC} = 0.3$ V and $I_C/I_B = \beta = 50$

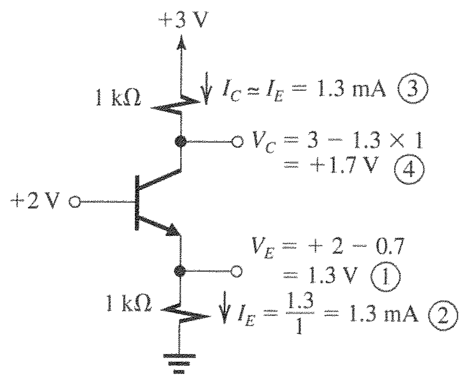
Thus,

$$I_C = \frac{5 - 0.3}{1} = 4.7 \text{ mA}$$

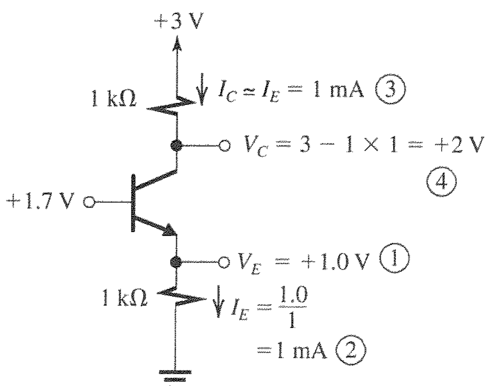
$$I_B = \frac{I_C}{\beta} = \frac{4.7}{50} = 0.094 \text{ mA}$$

$$R_B = \frac{4.3}{I_B} = \frac{4.3}{0.094} = 45.7 \text{ k}\Omega$$

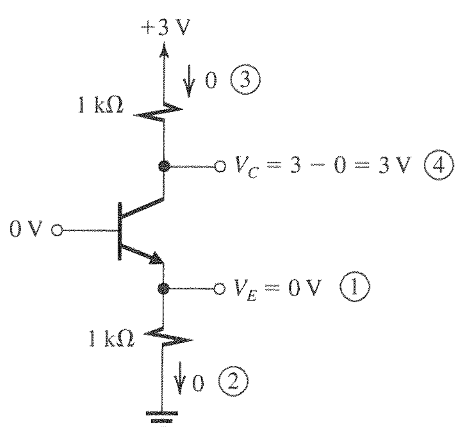
6.51



(a)



(b)



(c)

The analysis and the results are given on the circuit diagrams of Figs. 1 through 3. The circled numbers indicate the order of the analysis steps.

6.52

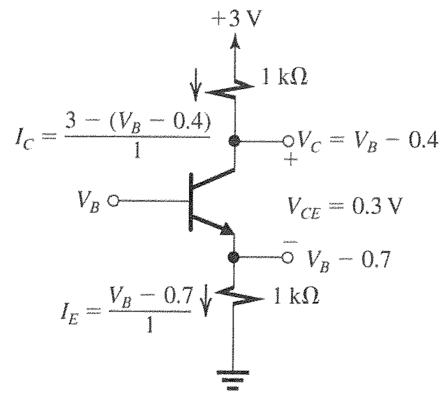


Figure 1

Figure 1 shows the circuit with the value of V_B that results in operation at the edge of saturation. Since β is very high,

$$I_C \simeq I_E$$

$$\frac{3 - (V_B - 0.4)}{1} = \frac{V_B - 0.7}{1}$$

$$\Rightarrow V_B = 2.05 \text{ V}$$

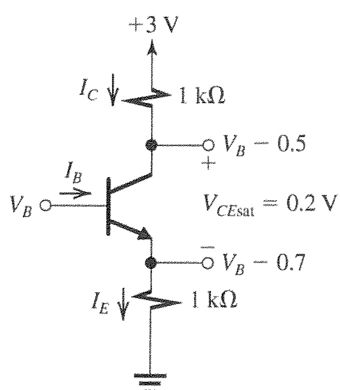


Figure 2

Figure 2 shows the circuit with the value of V_B that results in the transistor operating in saturation, with

$$I_E = \frac{V_B - 0.7}{1} = V_B - 0.7$$

$$I_C = \frac{3 - (V_B - 0.5)}{1} = 3.5 - V_B$$

$$I_B = I_E - I_C = 2V_B - 4.2$$

For $\beta_{\text{forced}} = 2$,

$$\frac{I_C}{I_B} = 2$$

$$\frac{3.5 - V_B}{2V_B - 4.2} = 2$$

$$\Rightarrow V_B = 2.38 \text{ V}$$

6.53 Refer to the circuit in Fig. P6.53.

(a) For $V_B = -1$ V,

$$V_E = V_B - V_{BE} = -1 - 0.7 = -1.7 \text{ V}$$

$$I_E = \frac{V_E - (-3)}{1} = \frac{-1.7 + 3}{1} = 1.3 \text{ mA}$$

Assuming active-mode operation, we have

$$I_C = \alpha I_E \simeq I_E = 1.3 \text{ mA}$$

$$V_C = +3 - I_C \times 1 = 3 - 1.3 = +1.7 \text{ V}$$

Since $V_C > V_B - 0.4$, the transistor is operating in the active mode as assumed.

(b) For $V_B = 0$ V,

$$V_E = 0 - V_{BE} = -0.7 \text{ V}$$

$$I_E = \frac{-0.7 - (-3)}{1} = 2.3 \text{ mA}$$

Assuming operation in the active mode, we have

$$I_C = \alpha I_E \simeq I_E = 2.3 \text{ mA}$$

$$V_C = +3 - I_C \times 1 = 3 - 2.3 = +0.7 \text{ V}$$

Since $V_C > V_B - 0.4$, the BJT is operating in the active mode, as assumed.

(c) For $V_B = +1$ V,

$$V_E = 1 - 0.7 = +0.3 \text{ V}$$

$$I_E = \frac{0.3 - (-3)}{1} = 3.3 \text{ mA}$$

Assuming operation in the active mode, we have

$$I_C = \alpha I_E \simeq I_E = 3.3 \text{ mA}$$

$$V_C = 3 - 3.3 \times 1 = -0.3 \text{ V}$$

Now $V_C < V_B - 0.4$, indicating that the transistor is operating in saturation, and our original assumption is incorrect. It follows that

$$V_C = V_E + V_{CEsat}$$

$$= 0.3 + 0.2 = 0.5 \text{ V}$$

$$I_C = \frac{3 - V_C}{1} = \frac{3 - 0.5}{1} = 2.5 \text{ mA}$$

$$I_B = I_E - I_C = 3.3 - 2.5 = 0.8 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{2.5}{0.8} = 3.1$$

(d) When $V_B = 0$ V, $I_E = 2.3$ mA. The emitter current becomes 0.23 mA at

$$V_B = -3 + 0.23 \times 1 + 0.7 = -2.07 \text{ V}$$

(e) The transistor will be at the edge of conduction when $I_E \simeq 0$ and $V_{BE} = 0.5$ V, that is,

$$V_B = -3 + 0.5 = -2.5 \text{ V}$$

In this case,

$$V_E = -3 \text{ V}$$

$$V_C = +3 \text{ V}$$

(f) The transistor reaches the edge of saturation when $V_{CE} = 0.3$ V but $I_C = \alpha I_E \simeq I_E$:

$$V_E = V_B - 0.7$$

$$I_E = \frac{V_B - 0.7 - (-3)}{1} = V_B + 2.3$$

$$V_C = V_E + 0.3 = V_B - 0.4$$

$$I_C = \frac{3 - V_C}{1} = \frac{3 - V_B + 0.4}{1} = 3.4 - V_B$$

Since

$$I_C \simeq I_E$$

$$3.4 - V_B = V_B + 2.3$$

$$V_B = 0.55 \text{ V}$$

For this value,

$$V_E = 0.55 - 0.7 = -0.15 \text{ V}$$

$$V_C = -0.15 + 0.3 = +0.15 \text{ V}$$

(g) For the transistor to operate in saturation with $\beta_{\text{forced}} = 2$,

$$V_E = V_B - 0.7$$

$$I_E = \frac{V_B - 0.7 - (-3)}{1} = V_B + 2.3$$

$$V_C = V_E + V_{CEsat} = V_B - 0.7 + 0.2 = V_B - 0.5$$

$$I_C = \frac{3 - (V_B - 0.5)}{1} = 3.5 - V_B$$

$$I_B = I_E - I_C = 2\ V_B - 1.2$$

$$\frac{I_C}{I_B} = \frac{3.5 - V_B}{2\ V_B - 1.2} = 2$$

$$\Rightarrow V_B = +1.18 \text{ V}$$

6.54 (a) $V_B = 0$ V

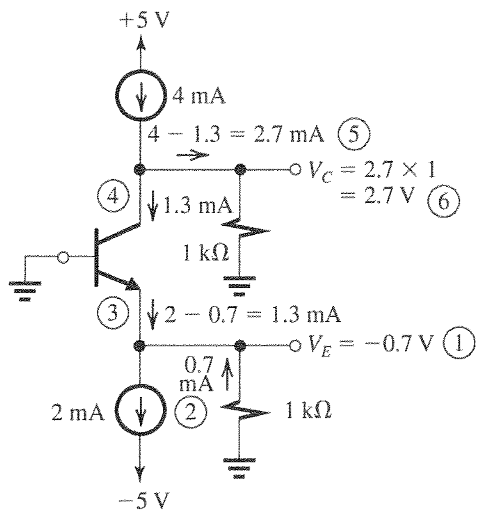


Figure 1

The analysis is shown on the circuit diagram in Fig. 1. The circled numbers indicate the order of the analysis steps.

(b) The transistor cuts off at the value of V_B that causes the 2-mA current of the current source feeding the emitter to flow through the 1-kΩ resistor connected between the emitter and ground. The circuit under these conditions is shown in Fig. 2.

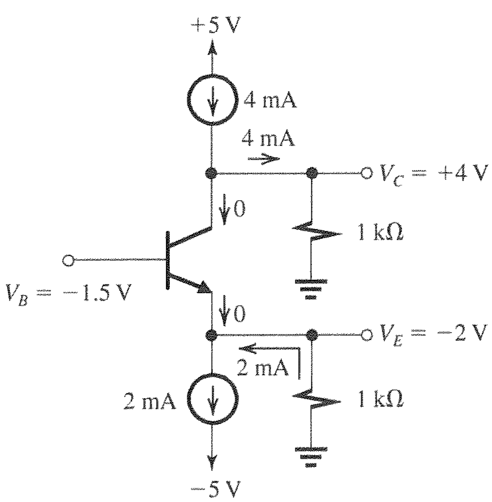


Figure 2

Observe that $V_E = -2 \text{ mA} \times 1 \text{ k}\Omega = -2$ V, $I_E = 0$, and $V_B = V_E + 0.5 = -1.5$ V. Since $I_C = 0$, all the 4 mA supplied by the current source feeding the collector flows through the collector 1-kΩ resistor, resulting in $V_C = +4$ V.

(c)

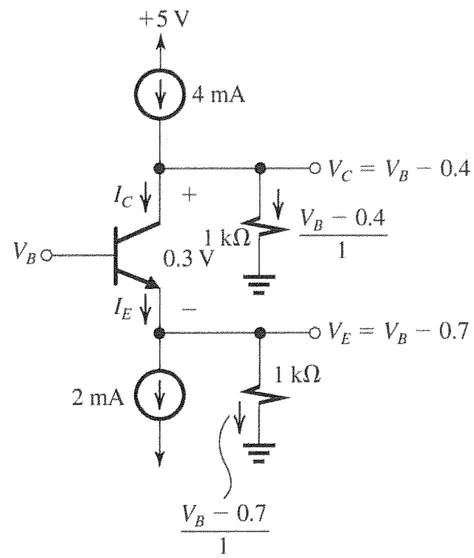


Figure 3

Figure 3 shows the transistor at the edge of saturation. Here $V_{CE} = 0.3$ V and $I_C = \alpha I_E \simeq I_E$. A node equation at the emitter gives

$I_E = 2 + V_B - 0.7 = V_B + 1.3$ mA

A node equation at the collector gives

$I_C = 4 - (V_B - 0.4) = 4.4 - V_B$ mA

Imposing the condition $I_C \simeq I_E$ gives

$4.4 - V_B = V_B + 1.3$

$\Rightarrow V_B = +1.55$ V

Correspondingly,

$V_E = +0.85$ V

$V_C = +1.15$ V

6.55

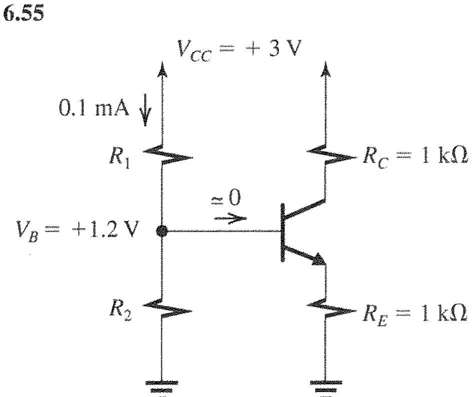


Figure 1

From Fig. 1 we see that

$$R_1 + R_2 = \frac{V_{CC}}{0.1\text{ mA}} = \frac{3}{0.1} = 30\text{ k}\Omega$$

$$V_{CC}\frac{R_2}{R_1 + R_2} = 1.2$$

$$3 \times \frac{R_2}{30} = 1.2$$

$$\Rightarrow R_2 = 12\text{ k}\Omega$$

$$R_1 = 30 - 12 = 18\text{ k}\Omega$$

For $\beta = 100$, to obtain the collector current, we replace the voltage divider with its Thévenin equivalent, consisting of

$$V_{BB} = 3 \times \frac{R_2}{R_1 + R_2} = 3 \times \frac{12}{18 + 12} = 1.2\text{ V}$$

$$R_B = R_1 \parallel R_2 = 12 \parallel 18 = 7.2\text{ k}\Omega$$

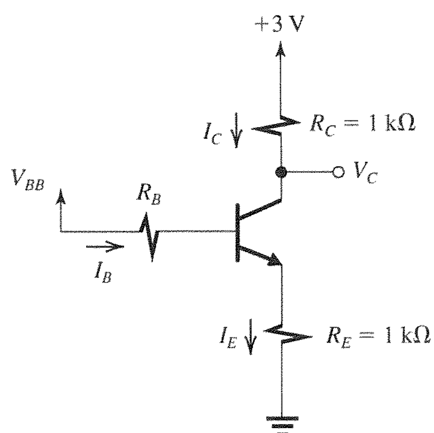


Figure 2

Refer to Fig. 2. Assuming active-mode operation, we can write a loop equation for the base–emitter loop:

$$V_{BB} = I_B R_B + V_{BE} + I_E R_E$$

$$1.2 = \frac{I_E}{\beta + 1} \times 7.2 + 0.7 + I_E \times 1$$

$$\Rightarrow I_E = \frac{1.2 - 0.7}{1 + \frac{7.2}{101}} = 0.47\text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 0.47 = 0.46\text{ mA}$$

$$V_C = +3 - 0.46 \times 1 = +2.54\text{ V}$$

Since $V_B = I_E R_E + V_{BE} = 0.47 + 0.7 = 1.17$ V, we see that $V_C > V_B - 0.4$, and thus the transistor is operating in the active region, as assumed.

6.56 Refer to the circuit in Fig. P6.56.

$$V_E = 1\text{ V}$$

$$I_E = \frac{3 - 1}{5} = 0.4\text{ mA}$$

$$V_B = V_E - 0.7 = 0.3\text{ V}$$

$$I_B = \frac{V_B}{50\text{ k}\Omega} = \frac{0.3}{50} = 0.006\text{ mA}$$

$$I_C = I_E - I_B = 0.4 - 0.006 = 0.394\text{ mA}$$

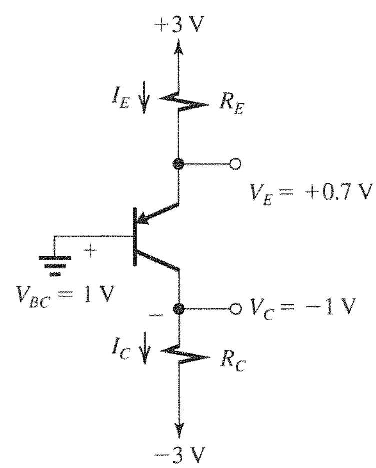
$$V_C = -3 + 5 \times 0.394 = -1.03\text{ V}$$

Observe that $V_C < V_B$, confirming our implicit assumption that the transistor is operating in the active region.

$$\beta = \frac{I_C}{I_B} = \frac{0.394}{0.006} = 66$$

$$\alpha = \frac{I_C}{I_E} = \frac{0.394}{0.4} = 0.985$$

6.57



Refer to the figure. To obtain $I_E = 0.5$ mA we select R_E according to

$$R_E = \frac{3 - 0.7}{0.5} = 4.6\text{ k}\Omega$$

To obtain $V_C = -1$ V, we select R_C according to

$$R_C = \frac{-1 - (-3)}{0.5} = 4\text{ k}\Omega$$

where we have utilized the fact that $\alpha \simeq 1$ and thus $I_C \simeq I_E = 0.5$ mA. From the table of 5% resistors in Appendix J we select

$$R_E = 4.7\text{ k}\Omega \qquad \text{and} \qquad R_C = 3.9\text{ k}\Omega$$

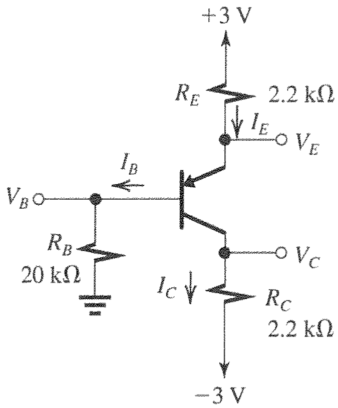
For these values,

$$I_E = \frac{3 - 0.7}{4.7} = 0.49\text{ mA}$$

$$I_C \simeq I_E = 0.49\text{ mA}$$

$$V_{BC} = 0 - V_C = -(-3 + 0.49 \times 3.9) = -1.1\text{ V}$$

6.58



Writing a loop equation for the EBJ loop, we have

$$3 = I_E R_E + V_{EB} + I_B R_B \qquad (1)$$

$$= I_E \times 2.2 + 0.7 + \frac{I_E}{\beta + 1} \times 20$$

$$\Rightarrow I_E = \frac{3 - 0.7}{2.2 + \dfrac{20}{41}} = 0.86 \text{ mA}$$

$$V_E = 3 - 0.86 \times 2.2 = +1.11 \text{ V}$$

$$V_B = V_E - 0.7 = +0.41 \text{ V}$$

Assuming active-mode operation, we obtain

$$I_C = \alpha I_E = \frac{40}{41} \times 0.86 = 0.84 \text{ mA}$$

$$V_C = -3 + 0.84 \times 2.2 = -1.15 \text{ V}$$

Since $V_C < V_B + 0.4$, the transistor is operating in the active mode, as assumed. Now, if R_B is increased to 100 kΩ, the loop equation [Eq. (1)] yields

$$I_E = \frac{3 - 0.7}{2.2 + \dfrac{100}{41}} = 0.5 \text{ mA}$$

$$V_E = 3 - 0.5 \times 2.2 = +1.9 \text{ V}$$

$$V_B = V_E - V_{EB} = 1.9 - 0.7 = +1.2 \text{ V}$$

Assuming active-mode operation, we obtain

$$I_C = \alpha I_E = \frac{40}{41} \times 0.5 = 0.48 \text{ mA}$$

$$V_C = -3 + 0.48 \times 2.2 = -1.9 \text{ V}$$

Since $V_C < V_B + 0.4$, the transistor is operating in the active mode, as assumed.

If with $R_B = 100$ kΩ, we need the voltages to remain at the values obtained with $R_B = 20$ kΩ, the transistor must have a β value determined as follows. For I_E to remain unchanged,

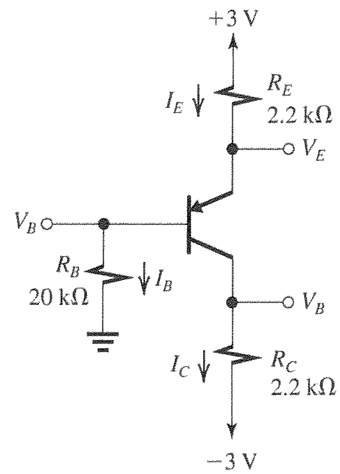
$$\frac{3 - 0.7}{2.2 + \dfrac{20}{41}} = \frac{3 - 0.7}{2.2 + \dfrac{100}{\beta + 1}}$$

$$\Rightarrow \frac{20}{41} = \frac{100}{\beta + 1}$$

$$\beta + 1 = \frac{410}{2} = 205$$

$$\beta = 204$$

6.59



Assume active-mode operation:

$$I_E = \frac{3 - V_{EB}}{R_E + \dfrac{R_B}{\beta + 1}}$$

$$I_E = \frac{3 - 0.7}{2.2 + \dfrac{20}{51}} = 0.887 \text{ mA}$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.887}{51} = 0.017 \text{ mA}$$

$$I_C = I_E - I_B = 0.887 - 0.017 = 0.870 \text{ mA}$$

$$V_B = I_B R_B = 0.017 \times 20 = 0.34 \text{ V}$$

$$V_E = V_B + V_{EB} = 0.34 + 0.7 = 1.04 \text{ V}$$

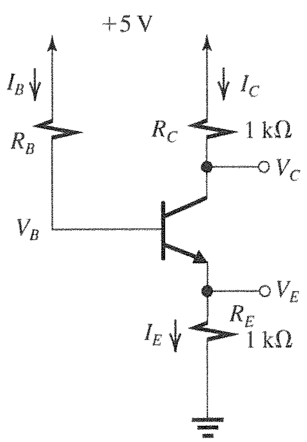
$$V_C = -3 + I_C R_C = -3 + 0.87 \times 2.2 = -1.09 \text{ V}$$

Thus, $V_C < V_B + 0.4$, which means active-mode operation, as assumed. The maximum value of R_C that still guarantees active-mode operation is that which causes V_C to be 0.4 V above V_B: that is, $V_C = 0.34 + 0.4 = 0.74$ V.

Correspondingly,

$$R_{C\max} = \frac{0.74 - (-3)}{0.87} = 4.3 \text{ k}\Omega$$

6.60



A loop equation for the EB loop yields

$$5 = I_B R_B + V_{BE} + I_E R_E$$

$$\Rightarrow I_E = \frac{5 - 0.7}{R_E + \dfrac{R_B}{\beta + 1}}$$

$$I_E = \frac{4.3}{1 + \dfrac{R_B}{101}}$$

(a) For $R_B = 100\ \text{k}\Omega$,

$$I_E = \frac{4.3}{1 + \dfrac{100}{101}} = 2.16\ \text{mA}$$

$$V_E = I_E R_E = 2.16 \times 1 = 2.16\ \text{V}$$

$$V_B = V_E + 0.7 = 2.86\ \text{V}$$

Assuming active-mode operation, we obtain

$$I_C = \alpha I_E = 0.99 \times 2.16 = 2.14\ \text{mA}$$

$$V_C = 5 - 2.14 \times 1 = +2.86\ \text{V}$$

Since $V_C > V_B - 0.4$, the transistor is operating in the active region, as assumed.

(b) For $R_B = 10\ \text{k}\Omega$,

$$I_E = \frac{4.3}{1 + \dfrac{10}{101}} = 3.91\ \text{mA}$$

$$V_E = 3.91 \times 1 = 3.91\ \text{V}$$

$$V_B = 3.91 + 0.7 = 4.61\ \text{V}$$

Assuming active-mode operation, we obtain

$$I_C = \alpha I_E = 0.99 \times 3.91 = 3.87\ \text{mA}$$

$$V_C = 5 - 3.87 = +1.13\ \text{V}$$

Since $V_C < V_B - 0.4$, the transistor is operating in saturation, contrary to our original assumption. Therefore, we need to redo the analysis assuming saturation-mode operation, as follows:

$$V_B = V_E + 0.7$$

$$V_C = V_E + V_{CE\text{sat}} = V_E + 0.2$$

$$I_B = \frac{5 - V_B}{R_B}$$

$$= \frac{5 - V_E - 0.7}{10} = \frac{4.3 - V_E}{10} \qquad (1)$$

$$I_C = \frac{5 - V_C}{R_C} = \frac{5 - V_E - 0.2}{1}$$

$$= 4.8 - V_E \qquad (2)$$

$$I_E = \frac{V_E}{R_E} = \frac{V_E}{1} = V_E \qquad (3)$$

Substituting from Eqs. (1), (2), and (3) into

$$I_E = I_B + I_C$$

gives

$$V_E = 0.43 - 0.1\ V_E + 4.8 - V_E$$

$$\Rightarrow V_E = 2.5\ \text{V}$$

$$V_C = 2.7\ \text{V}$$

$$V_B = 3.2\ \text{V}$$

$$I_B = \frac{5 - 3.2}{10} = 0.18\ \text{mA}$$

$$I_C = \frac{5 - 2.7}{1} = 2.3\ \text{mA}$$

Thus,

$$\frac{I_C}{I_B} = \frac{2.3}{0.18} = 12.8$$

which is lower than the value of β, verifying saturation-mode operation.

(c) For $R_B = 1\ \text{k}\Omega$, we assume saturation-mode operation:

$$V_B = V_E + 0.7$$

$$V_C = V_E + 0.2$$

$$I_B = \frac{5 - (V_E + 0.7)}{1} = 4.3 - V_E$$

$$I_C = \frac{5 - (V_E + 0.2)}{1} = 4.8 - V_E$$

$$I_E = \frac{V_E}{1} = V_E$$

These values can be substituted into

$$I_E = I_B + I_C$$

to obtain

$$V_E = 4.3 - V_E + 4.8 - V_E$$

$$\Rightarrow V_E = 3\ \text{V}$$

$$V_B = 3.7\ \text{V}$$

$$V_C = 3.2\ \text{V}$$

Now checking the currents,

$$I_B = \frac{5 - 3.7}{1} = 1.3\ \text{mA}$$

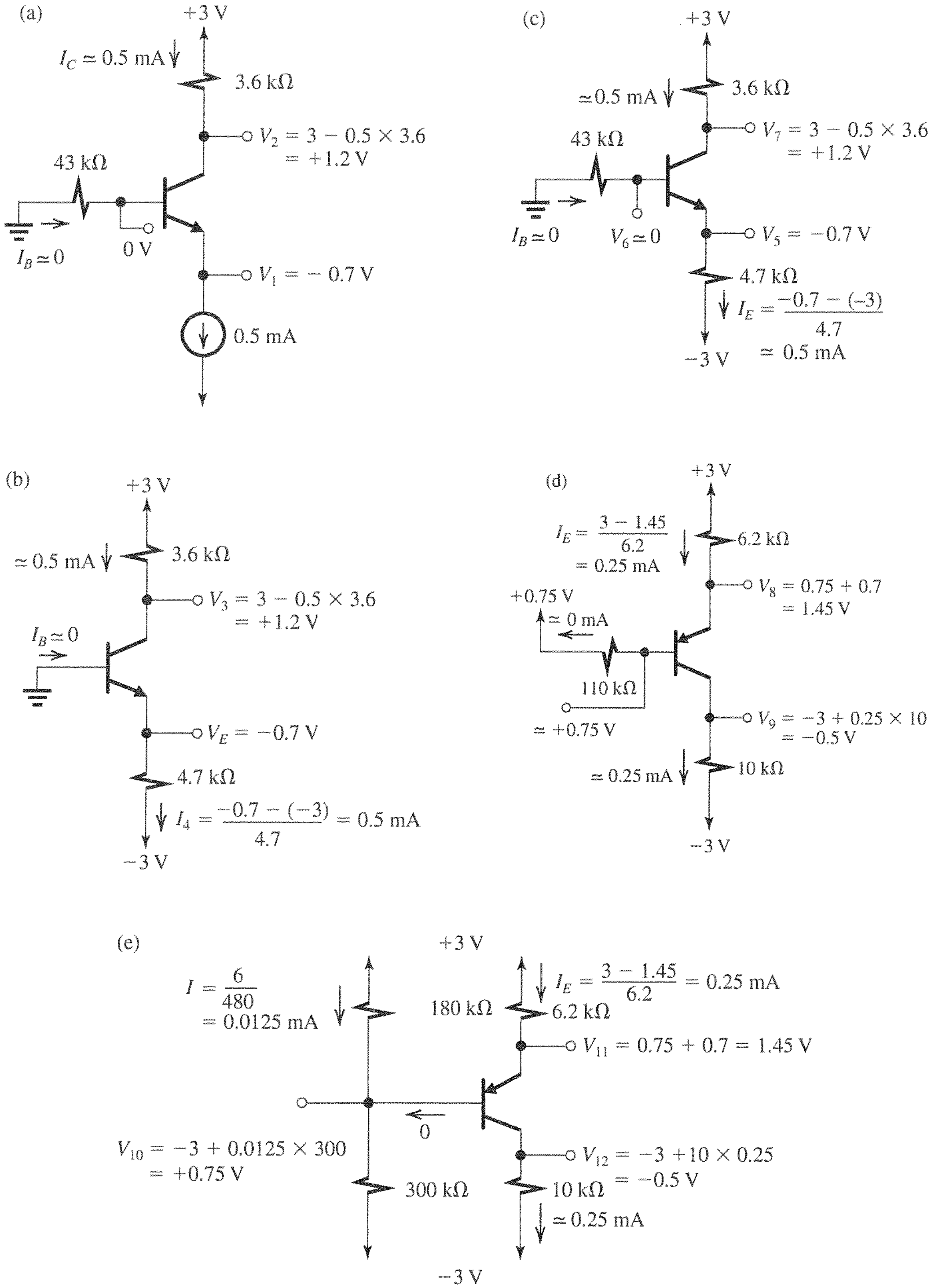
$$I_C = \frac{5 - 3.2}{1} = 1.8\ \text{mA}$$

Thus, the transistor is operating at a forced β of

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{1.8}{1.3} = 1.4$$

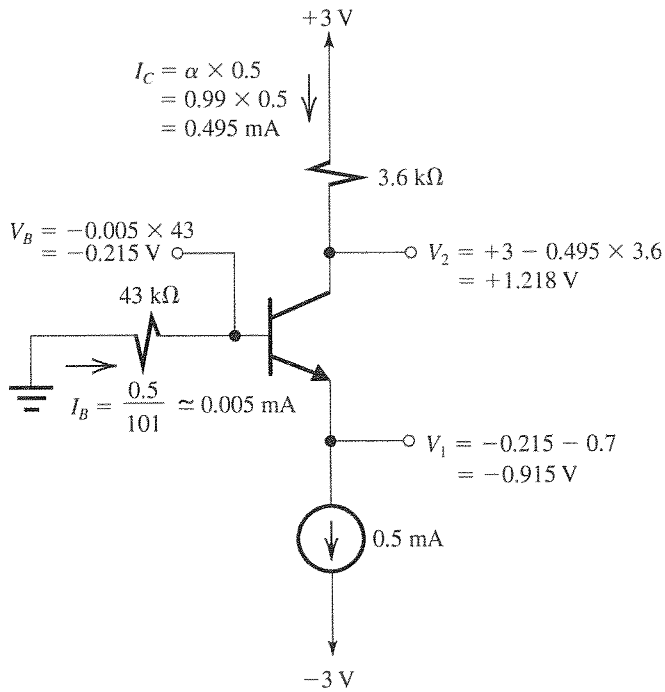
which is much lower than the value of β, confirming operation in saturation.

6.61



For the solutions and answers to parts (a) through (e), see the corresponding circuit diagrams.

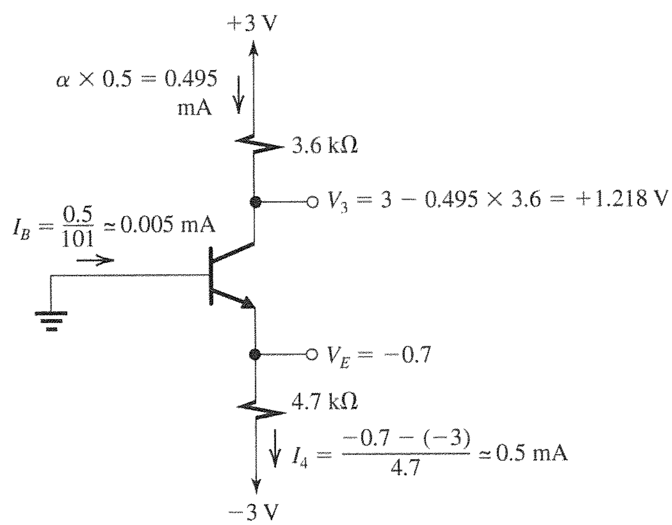
6.62 (a)



(a)

See solution and answer on the figure, which corresponds to Fig. P6.61(a).

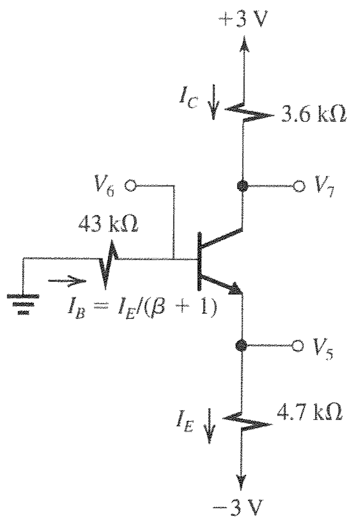
(b)



(b)

See solution and answer on the figure, which corresponds to Fig. P6.61(b).

(c)



(c)

Writing an equation for the loop containing the BEJ of the transistor leads to

$$I_E = \frac{3 - 0.7}{4.7 + \dfrac{43}{101}} = 0.449 \text{ mA}$$

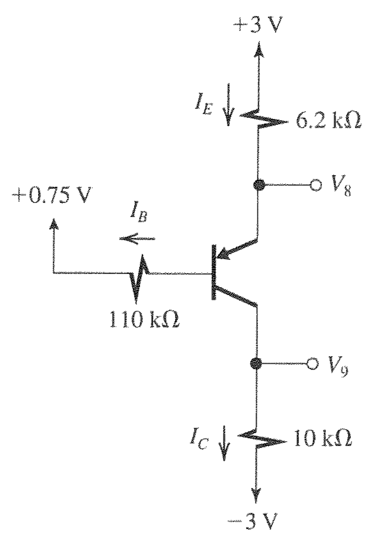
$$V_5 = -3 + 0.449 \times 4.7 = -0.9 \text{ V}$$

$$V_6 = -0.9 + 0.7 = -0.2 \text{ V}$$

$$I_C = \alpha I_E = 0.99 \times 0.449 = 0.444 \text{ mA}$$

$$V_7 = 3 - 0.444 \times 3.6 = +1.4 \text{ V}$$

(d)



(d)

An equation for the loop containing the EBJ of the transistor yields

$$I_E = \frac{3 - 0.75 - 0.7}{6.2 + \dfrac{110}{101}} = 0.213 \text{ mA}$$

$$V_8 = +3 - 0.213 \times 6.2 = +1.7 \text{ V}$$

$$I_C = \alpha I_E = 0.99 \times 0.213 = 0.21 \text{ mA}$$

$$V_9 = -3 + 0.21 \times 10 = -0.9 \text{ V}$$

(e) See figure on next page.

First, we use Thévenin's theorem to replace the voltage divider feeding the base with V_{BB} and R_B:

$$V_{BB} = -3 + \frac{6}{480} \times 300 = +0.75 \text{ V}$$

$$R_B = 180 \parallel 300 = 112.5 \text{ k}\Omega$$

Next we write an equation for the loop containing the EBJ to obtain

$$I_E = \frac{3 - 0.75 - 0.7}{6.2 + \dfrac{112.5}{101}} = 0.212 \text{ mA}$$

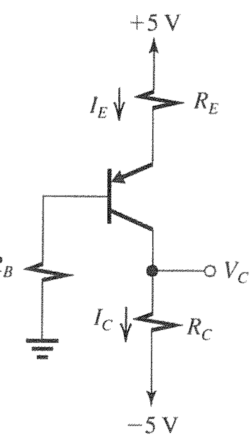
$$V_{11} = +3 - 0.212 \times 6.2 = +1.7 \text{ V}$$

$$V_{10} = 1.7 - 0.7 = +1 \text{ V}$$

$$I_C = \alpha I_E = 0.99 \times 0.212 = 0.21 \text{ mA}$$

$$V_{12} = -3 + 0.21 \times 10 = -0.9 \text{ V}$$

6.63



We required I_E to be nominally 1 mA (i.e., at $\beta = 100$) and to remain within $\pm10\%$ as β varies from 50 to 150. Writing an equation for the loop containing the EBJ results in

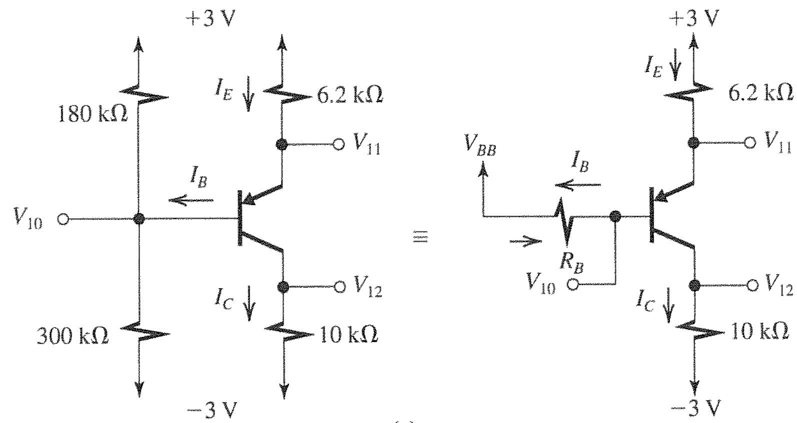
$$I_E = \frac{5 - 0.7}{R_E + \dfrac{R_B}{\beta + 1}}$$

Thus,

$$\frac{4.3}{R_E + \dfrac{R_B}{101}} = 1 \qquad (1)$$

$$\frac{4.3}{R_E + \dfrac{R_B}{51}} = I_{E\min} \qquad (2)$$

This figure belongs to Problem 6.62, part (e).



(e)

$$\frac{4.3}{R_E + \frac{R_B}{151}} = I_{E\max} \quad (3)$$

If we set $I_{E\min} = 0.9$ mA and solve Eqs. (1) and (2) simultaneously, we obtain

$R_E = 3.81$ kΩ

$R_B = 49.2$ kΩ

Substituting theses values in Eqs. (2) and (3) gives

$I_{E\min} = 0.9$ mA

$I_{E\max} = 1.04$ mA

Obviously, this is an acceptable design. Alternatively, if we set $I_{E\max}$ in Eq. (3) to 1.1 mA and solve Eqs. (1) and (3) simultaneously, we obtain

$R_E = 3.1$ kΩ

$R_B = 119.2$ kΩ

Substituting these values in Eqs. (2) and (3) gives

$I_{E\min} = 0.8$ mA

$I_{E\max} = 1.1$ mA

Obviously this is not an acceptable design ($I_{E\min}$ is 20% lower than nominal).

Therefore, we shall use the first design. Specifying the resistor values to the nearest kilohm results in

$R_E = 4$ kΩ

$R_B = 50$ kΩ

To obtain the value of R_C, we note that at the nominal emitter current value of 1 mA, $V_C = -1$ V,

$I_C = \alpha I_E = 0.99$ mA

$$R_C = \frac{-1-(-5)}{0.99} = 4.04 \text{ k}\Omega$$

Specified to the nearest kilohm,

$R_C = 4$ kΩ

Finally, for our design we need to determine the range obtained for collector current and collector voltage for β ranging from 50 to 150 with a nominal value of 100. We compute the nominal value of I_E from

$$I_{E\text{nominal}} = \frac{4.3}{4 + \frac{50}{101}} = 0.96 \text{ mA}$$

We utilize Eqs. (2) and (3) to compute $I_{E\min}$ and $I_{E\max}$,

$$I_{E\min} = \frac{4.3}{4 + \frac{50}{51}} = 0.86 \text{ mA}$$

$$I_{E\max} = \frac{4.3}{4 + \frac{50}{151}} = 0.99 \text{ mA}$$

Thus,

$$\frac{I_{E\max}}{I_{E\text{nominal}}} = \frac{0.99}{0.96} = 1.03$$

$$\frac{I_{E\min}}{I_{E\text{nominal}}} = \frac{0.86}{0.96} = 0.9$$

which meet our specifications. The collector currents are

$I_{C\text{nominal}} = 0.99 \times 0.96 = 0.95$ mA

$I_{C\min} = 0.99 \times 0.86 = 0.85$ mA

$I_{C\max} = 0.99 \times 0.99 = 0.98$ mA

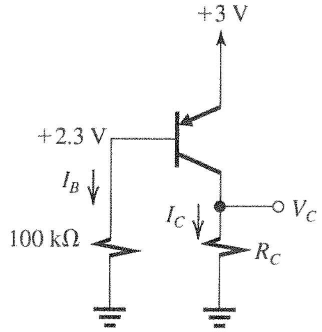
and the collector voltages are

$V_{C\text{nominal}} = -5 + 0.95 \times 4 = -1.2$ V

$V_{C\min} = -5 + 0.85 \times 4 = -1.6$ V

$V_{C\max} = -5 + 0.98 \times 4 = -1.1$ V

6.64



$$I_B = \frac{2.3\ \text{V}}{100\ \text{k}\Omega} = 0.023\ \text{mA}$$

Since $V_C = 2$ V is lower than V_B, which is +2.3 V, the transistor will be operating in the active mode. Thus,

$$I_C = \beta I_B = 50 \times 0.023 = 1.15\ \text{mA}$$

To obtain $V_C = 2$ V, we select R_C according to

$$R_C = \frac{V_C}{I_C} = \frac{2\ \text{V}}{1.15\ \text{mA}} = 1.74\ \text{k}\Omega$$

Now, if the transistor is replaced with another having $\beta = 100$, then

$$I_C = 100 \times 0.023 = 2.3\ \text{mA}$$

which would imply

$$V_C = 2.3 \times 1.74 = 4\ \text{V}$$

which is impossible because the base is at 2.3 V. Thus the transistor must be in the saturation mode and

$$V_C = V_E - V_{EC\text{sat}}$$

$$= 3 - 0.2 = 2.8\ \text{V}$$

6.65

(a) Consider first the case $\beta = \infty$ and R open circuited. The circuit is shown in Fig. 1, where $\beta = \infty$ and R is open circuited. Since $V_{D1} = V_{BE1}$, we have

$$V_1 = V_{E1}$$

Thus,

$$I_{D1} \times 40 = I_{E1} \times 2$$

$$\Rightarrow I_{D1} = 0.05 I_{E1}$$

But

$$I_{D1} = \frac{9 - 0.7}{80 + 40} = 0.069\ \text{mA} \simeq 0.07\ \text{mA}$$

Thus,

$$I_{E1} = \frac{0.069}{0.05} = 1.38\ \text{mA} \simeq 1.4\ \text{mA}$$

$$V_{E1} = I_{E1} \times 2 = 2.77\ \text{V} \simeq 2.8\ \text{V}$$

$$V_{B1} = V_{E1} + 0.7 = 3.5\ \text{V}$$

$$I_{C1} = I_{E1} = 1.38\ \text{mA} \simeq 1.4\ \text{mA}$$

$$V_2 = 9 - I_{C1} \times 2 = 9 - 1.38 \times 2 \simeq 6.2\ \text{V}$$

$$V_{C1} = V_2 - V_{D2} = 6.2 - 0.7 = 5.5\ \text{V}$$

$$V_{E2} = V_2 = 6.2\ \text{V}$$

$$I_{E2} = \frac{9 - 6.2}{100\ \Omega} = 28\ \text{mA}$$

$$I_{C2} = I_E = 28\ \text{mA}$$

$$V_{C2} = 28 \times 0.1 = 2.8\ \text{V}$$

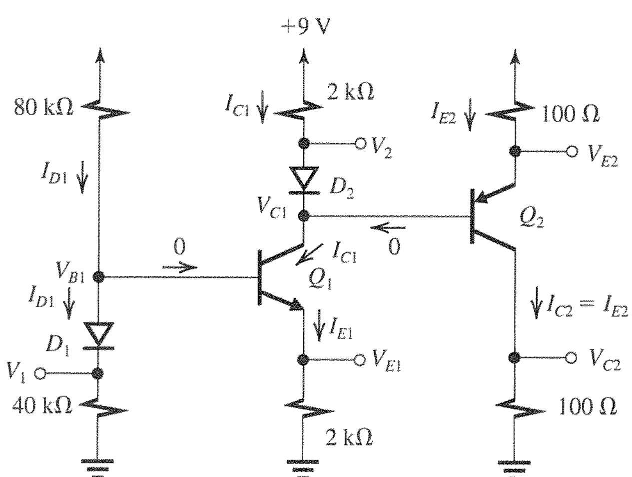


Figure 1 $\beta = \infty$, and R is open circuited

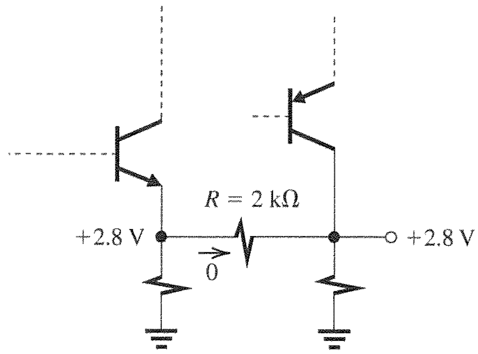


Figure 2

Now connecting the resistance $R = 2$ kΩ between C_1 and E_2 (see Fig. 2) both of which at 2.8 V, will result in zero current through R; thus all voltages and currents remain unchanged.

(b) We next consider the situation with $\beta = 100$, first with R disconnected. The circuit is shown in Fig. 3.

Once again we observe that $V_{E1} = V_1$, thus

$I_{E1} \times 2 = I_{D1} \times 40$

$\Rightarrow I_{D1} = 0.05 I_{E1}$

The base current of Q_1 is $I_{E1}/101 \simeq 0.01 I_{E1}$. Thus, the current through the 80-kΩ resistor is $0.05 I_{E1} + 0.01 I_{E1} = 0.06 I_{E1}$ and

$V_{B1} = V_{E1} + 0.7 = 2 I_{E1} + 0.7$

$$0.06 I_{E1} = \frac{9 - V_{B1}}{80} = \frac{9 - (2 I_{E1} + 0.7)}{80}$$

$\Rightarrow I_{E1} = 1.22$ mA

$V_{E1} = 1.22 \times 2 = 2.44$ V

$V_{B1} = 2.44 + 0.7 = 3.14$ V

$I_{C1} = \alpha I_{E1} = 0.99 \times 1.22 = 1.21$ mA

Observing that $V_{E2} = V_2$, we see that the voltage drops across the 2-kΩ resistor and the 100-Ω resistor are equal, thus

$I_{D2} \times 2 = I_{E2} \times 0.1$

$\Rightarrow I_{D2} = 0.05 I_{E2}$

As the base current of Q_2 is approximately $0.01 I_{E2}$, a node equation at C_1 yields

$I_{D2} = I_{C1} - 0.01 I_{E2}$

Thus,

$0.05 I_{E2} = I_{C1} - 0.01 I_{E2}$

$\Rightarrow 0.06 I_{E2} = I_{C1}$

$$I_{E2} = \frac{I_{C1}}{0.06} = \frac{1.21}{0.06} = 20.13 \text{ mA}$$

$I_{D2} = 0.05 \times 20.13 = 1$ mA

$V_{C1} = 9 - 1 \times 2 - 0.7 = 6.3$ V

$V_{E2} = 6.3 + 0.7 = 7$ V

$I_{C2} = \alpha I_{E2} = 0.99 \times 20.13 = 20$ mA

$V_{C2} = 20 \times 0.1 = 2$ V

Finally, with the resistance R connected between E_1 and C_2, it will conduct a current that we can initially estimate as

$$I = \frac{V_{E1} - V_{C2}}{R} = \frac{2.44 - 2}{2} = 0.22 \text{ mA}$$

This is a substantial amount compared to $I_{E1} = 1.22$ mA, requiring that we redo the analysis with R in place. The resulting circuit is shown in Fig. 4.

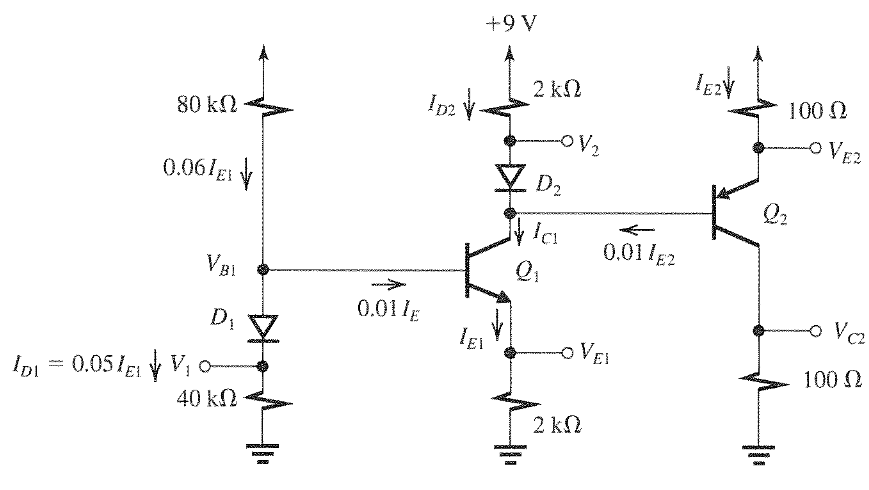


Figure 3

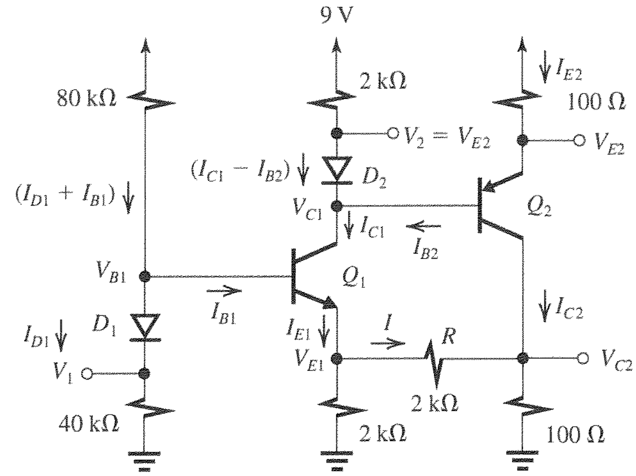


Figure 4

Denoting the emitter voltage of Q_1, V_{E1}, and the current through R as I, the analysis proceeds as follows:

$V_1 = V_{E1}$

$I_{D1} = \dfrac{V_1}{40} = \dfrac{V_{E1}}{40} = 0.025V_{E1}$

$I_{E1} = \dfrac{V_{E1}}{2} + I = 0.5V_{E1} + I$

$I_{B1} = \dfrac{I_{E1}}{101} = 0.005V_{E1} + 0.01I$

$I_{80\text{ k}\Omega} = I_{D1} + I_{B1} = 0.03V_{E1} + 0.01I$

$V_{C2} = V_{E1} - I \times 2 = V_{E1} - 2I$

$I_{C2} = -I + \dfrac{V_{C2}}{0.1} = 10V_{E1} - 21I$

$I_{B2} = \dfrac{I_{C2}}{101} = 0.1V_{E1} - 0.21I$

$I_{C1} = \alpha I_{E1} = 0.495V_{E1} + 0.99I$

$I_{D2} = I_{C1} - I_{B2} = 0.395V_{E1} + 1.2I$

$I_{E2} = \dfrac{I_{C2}}{\alpha} = 10.1V_{E1} - 21.2I$

$I_{D2} \times 2 = I_{E2} \times 0.1$

$2(0.395V_{E1} + 1.2I) = 0.1(10.1V_{E1} - 21.2I)$

$\Rightarrow I = 0.05V_{E2}$

Voltage drop across 80-kΩ resistor = $(0.03\ V_{E1} + 0.01I) \times 80 = 9 - V_{E1} - 0.7$

Substituting $I = 0.05\ V_{E2}$ gives

$V_{E1} = 2.41$ V

$I = 0.12$ mA

Substituting these quantities in the equations above gives

$V_{B1} = 2.41 + 0.7 = 3.11$ V

$I_{E1} = 1.325$ mA

$I_{C1} = 1.31$ mA

$I_{D1} = 1.09$ mA

$V_{C1} = 9 - 1.09 \times 2 - 0.7 = 6.12$ V

$V_{E2} = 6.82$ V

$I_{E2} = \dfrac{9 - 6.82}{0.1} = 21.8$ mA

$I_{C2} = 0.99 \times 21.8 = 21.6$ mA

$V_{C2} = 2.17$ V

6.66 (a) $\beta = \infty$

The analysis and results are given in the circuit diagram in Fig. 1 on page 27. The circled numbers indicate the order of the analysis steps.

(b) $\beta = 100$

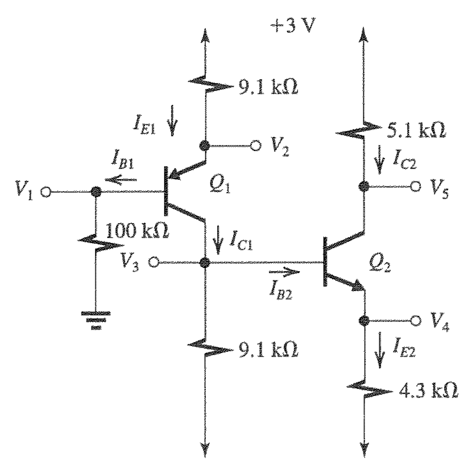


Figure 2 ($\beta = 100$)

This figure belongs to Problem 6.66, part (a).

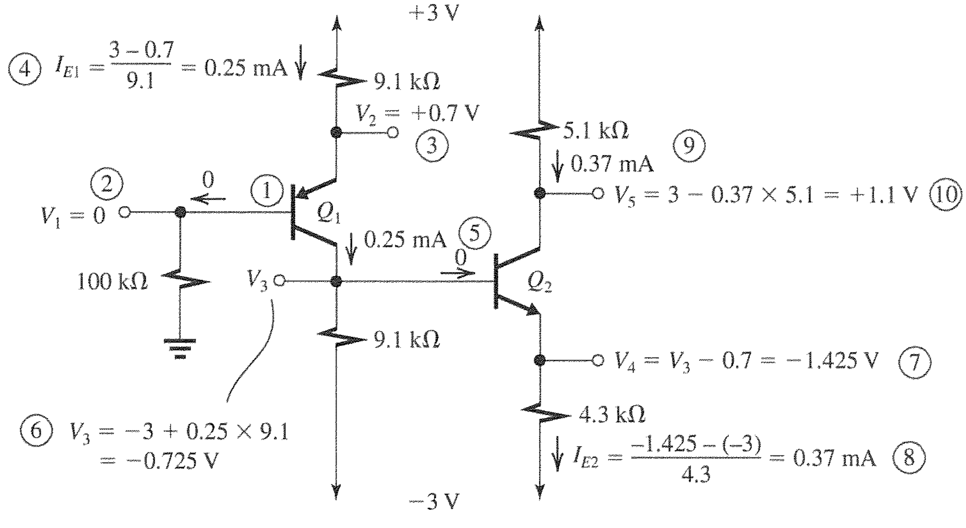


Figure 1 $(\beta = \infty)$

By reference to Fig. 2 on page 26, we can write an equation for the loop containing the EBJ of Q_1 as follows:

$$3 = I_{E1} \times 9.1 + 0.7 + I_{B1} \times 100$$

Substituting $I_{B1} = I_E/(\beta + 1) = I_E/101$ and rearranging, we obtain

$$I_{E1} = \frac{3 - 0.7}{9.1 + \frac{100}{101}} = 0.228 \text{ mA}$$

Thus,

$$I_{B1} = \frac{I_{E1}}{101} = 0.0023 \text{ mA}$$

$$V_2 = V_1 + 0.7 = 0.93 \text{ V}$$

$$I_{C1} = \alpha I_{E1} = 0.99 \times 0.228 = 0.226 \text{ mA}$$

Then we write a node equation at C_1:

$$I_{C1} = I_{B2} + \frac{V_3 - (-3)}{9.1}$$

Substituting for $I_{C1} = 0.226$ mA, $I_{B2} = I_{E2}/101$, and $V_3 = V_4 + 0.7 = -3 + I_{E2} \times 4.3 + 0.7$ gives

$$0.226 = \frac{I_{E2}}{101} + \frac{-3 + 4.3I_{E2} + 0.7 + 3}{9.1}$$

$$= \frac{I_{E2}}{101} + \frac{4.3I_{E2}}{9.1} + \frac{0.7}{9.1}$$

$$\Rightarrow I_{E2} = 0.31 \text{ mA}$$

$$I_{B2} = 0.0031$$

$$I_{C1} - I_{B2} = 0.226 - 0.0031 = 0.223 \text{ mA}$$

$$V_3 = -3 + 0.223 \times 9.1 = -0.97 \text{ V} \simeq -1 \text{ V}$$

$$V_4 = V_3 - 0.7 = -1.7 \text{ V}$$

$$I_{C2} = \alpha I_{E2} = 0.99 \times 0.31 = 0.3 \text{ mA}$$

$$V_5 = +3 - 0.3 \times 5.1 = +1.47 \text{ V}$$

6.67 Figure 1 on page 28 shows the circuit with $\beta = \infty$; the required voltage values are indicated. The resistor values are obtained as follows:

$$V_2 = -0.7 \text{ V}$$

$$R_1 = \frac{V_2 - (-5)}{0.5 \text{ mA}}$$

$$\Rightarrow R_1 = 8.6 \text{ k}\Omega$$

$$R_2 = \frac{5 - V_3}{0.5} = \frac{5 - 0}{0.5} = 10 \text{ k}\Omega$$

$$V_4 = 0 + 0.7 = 0.7 \text{ V}$$

$$R_3 = \frac{5 - V_4}{0.5} = \frac{5 - 0.7}{0.5} = 8.6 \text{ k}\Omega$$

$$R_4 = \frac{V_5 - (-5)}{0.5} = \frac{-2 + 5}{0.5} = 6 \text{ k}\Omega$$

$$V_6 = V_5 - 0.7 = -2 - 0.7 = -2.7 \text{ V}$$

$$R_6 = \frac{V_6 - (-5)}{1} = \frac{-2.7 + 5}{1} = 2.3 \text{ k}\Omega$$

$$R_5 = \frac{5 - V_7}{1} = \frac{5 - 1}{1} = 4 \text{ k}\Omega$$

Consulting the table of 5% resistors in Appendix J, we select the following resistor values:

$R_1 = 8.2$ kΩ $R_2 = 10$ kΩ $R_3 = 10$ kΩ

$R_4 = 6.2$ kΩ $R_5 = 3.9$ kΩ $R_6 = 2.4$ kΩ

The circuit with the selected resistor values is shown in Fig. 2. Analysis of the circuit proceeds as follows:

$$V_2 = -0.7 \text{ V}$$

This figure belongs to Problem 6.67.

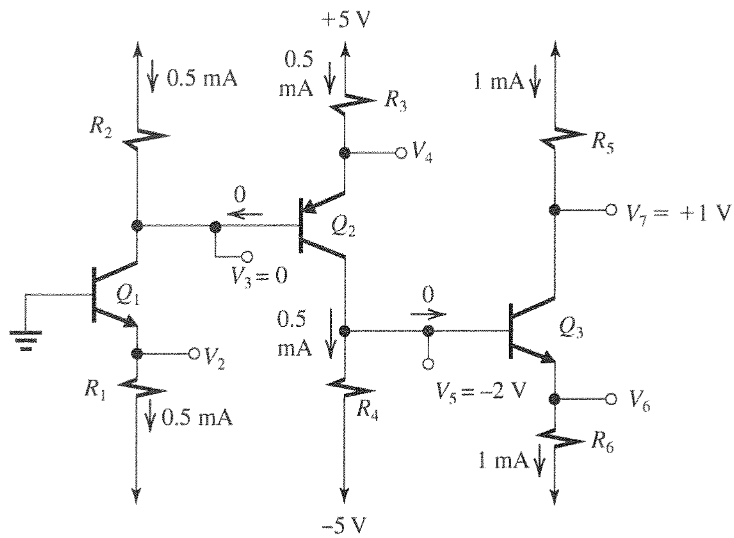


Figure 1

This figure belongs to Problem 6.67.

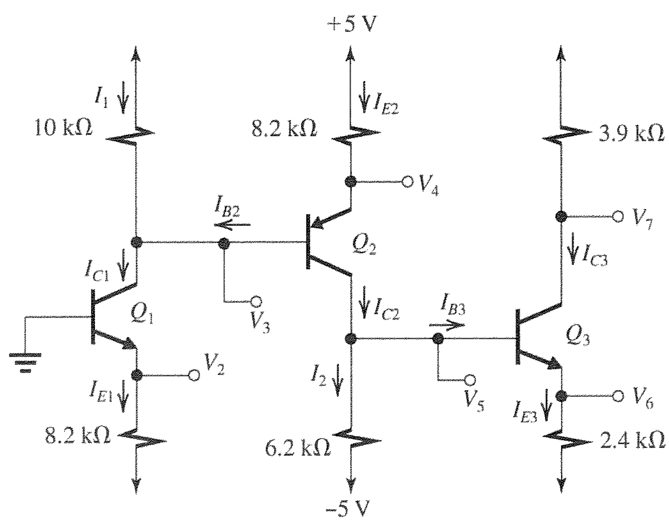


Figure 2

$$I_{E1} = \frac{V_2 - (-5)}{8.2} = \frac{-0.7 + 5}{8.2} = 0.524 \text{ mA}$$

$$I_{C1} = \alpha I_{E1} = 0.99 \times 0.524 = 0.52 \text{ mA}$$

The current I_1 through the 10-kΩ resistor is given by

$$I_1 = I_{C1} - I_{B2} = I_{C1} - \frac{I_{E2}}{101}$$

Noting that the voltage drop across the 10 kΩ resistor is equal to $(I_{E2} \times 8.2 + 0.7)$, we can write

$$I_1 \times 10 = 8.2 I_{E2} + 0.7$$

Thus,

$$10\left(0.52 - \frac{I_{E2}}{101}\right) = 8.2 I_{E2} + 0.7$$

$$\Rightarrow I_{E2} = 0.542 \text{ mA}$$

$$V_4 = 5 - 0.542 \times 8.2 = 0.56 \text{ V}$$

$$V_3 = 0.56 - 0.7 = -0.14 \text{ V}$$

$$I_{C2} = \alpha I_{E2} = 0.99 \times 0.542 = 0.537 \text{ mA}$$

$$I_2 = I_{C2} - I_{B3} = 0.537 - \frac{I_{E3}}{101}$$

Since the voltage drop across the 6.2-kΩ resistor is equal to $(0.7 + I_{E3} \times 2.4)$,

$$I_2 \times 6.2 = 0.7 + 2.4 I_{E3}$$

$$6.2\left(0.537 - \frac{I_{E3}}{101}\right) = 0.7 + 2.4 I_{E3}$$

$$\Rightarrow I_{E3} = 1.07 \text{ mA}$$

$V_6 = -5 + 1.07 \times 2.4 = -2.43$ V

$V_5 = V_6 + 0.7 = -1.73$

$I_{C3} = \alpha \times I_{E3} = 0.99 \times 1.07 = 1.06$ mA

$V_7 = -3.9 \times 1.06 = 0.87$ V

6.68 Refer to the circuit in Fig. P6.68.

(a) For $v_I = 0$, both transistors are cut off and all currents are zero. Thus

$V_B = 0$ V and $V_E = 0$ V

(b) For $v_I = +2$ V, Q_1 will be conducting and Q_2 will be cut off, and the circuit reduces to that in Fig. 1.

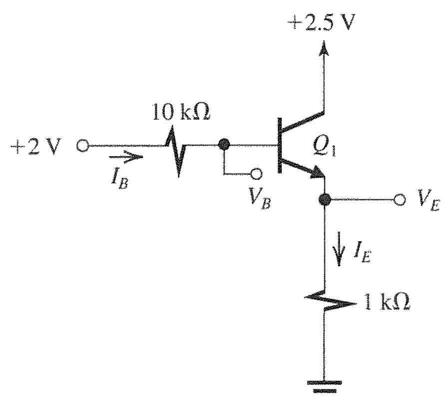


Figure 1

Since V_B will be lower than +2 V, V_C will be higher than V_B and the transistor will be operating in the active mode. Thus,

$$I_E = \frac{2 - 0.7}{1 + \frac{10}{51}} = 1.1 \text{ mA}$$

$V_E = +1.1$ V

$V_B = 1.8$ V

(c) For $v_I = -2.5$, Q_1 will be off and Q_2 will be on, and the circuit reduces to that in Fig. 2.

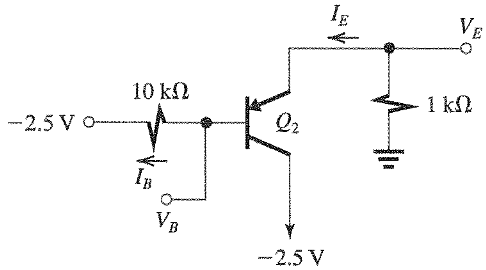


Figure 2

Since $V_B > -2.5$, V_C will be lower than V_B and Q_2 will be operating in the active region. Thus

$$I_E = \frac{2.5 - 0.7}{1 + \frac{10}{51}} = 1.5 \text{ mA}$$

$V_E = -I_E \times 1 = -1.5$ V

$V_B = -1.5 - 0.7 = -2.2$ V

(d) For $v_I = -5$ V, Q_1 will be off and Q_2 will be on, and the circuit reduces to that in Fig. 3.

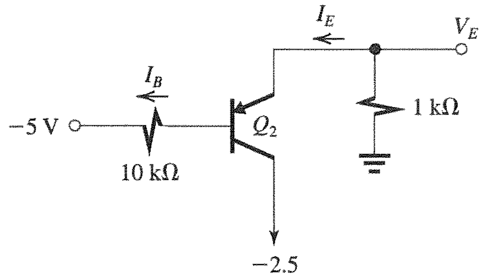


Figure 3

Here we do not know whether Q_2 is operating in the active mode or in saturation. Assuming active-mode operation, we obtain

$$I_E = \frac{5 - 0.7}{1 + \frac{10}{51}} = 3.6 \text{ mA}$$

$V_E = -3.6$ V

$V_B = -4.3$ V

which is impossible, indicating that our original assumption is incorrect and that Q_2 is saturated. Assuming saturation-mode operation, we obtain

$V_E = V_C + V_{EC\text{sat}} = -2.5 + 0.2 = -2.3$ V

$$I_E = \frac{-V_E}{1 \text{ k}\Omega} = 2.3 \text{ mA}$$

$V_B = V_E - 0.7 = -3$ V

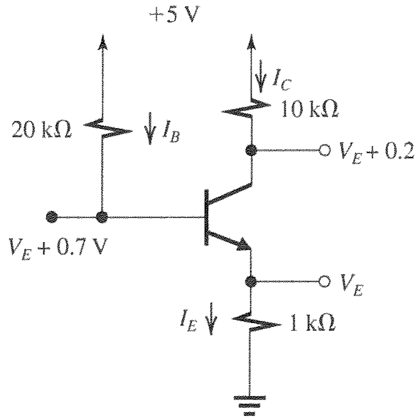
$$I_B = \frac{-3 - (-5)}{10} = 0.2 \text{ mA}$$

$I_C = I_E - I_B = 2.3 - 0.2 = 2.1$ mA

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{2.1}{0.2} = 10.5$$

which is lower than β, verifying that Q_2 is operating in saturation.

6.69 (a)



(a)

Assuming saturation-mode operation, the terminal voltages are interrelated as shown in the figure, which corresponds to Fig. P6.69(a). Thus we can write

$$I_E = \frac{V_E}{1} = V_E$$

$$I_C = \frac{5 - (V_E + 0.2)}{10} = 0.5 - 0.1(V_E + 0.2)$$

$$I_B = \frac{5 - (V_E + 0.7)}{20} = 0.25 - 0.05(V_E + 0.7)$$

Now, imposing the constraint

$$I_E = I_C + I_B$$

results in

$$V_E = 0.5 - 0.1(V_E + 0.2) + 0.25 - 0.05(V_E + 0.7)$$

$$\Rightarrow V_E = 0.6 \text{ V}$$

$$V_C = 0.8 \text{ V}$$

$$V_B = 1.3 \text{ V}$$

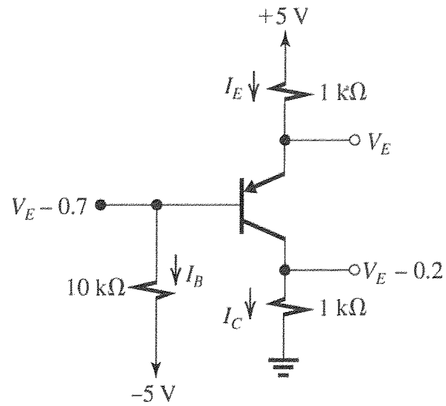
$$I_C = \frac{5 - 0.8}{10} = 0.42 \text{ mA}$$

$$I_B = \frac{5 - 1.3}{20} = 0.185 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{0.42}{0.185} = 2.3$$

which is less than the value of β_1 verifying saturation-mode operation.

(b)



(b)

Assuming saturation-mode operation, the terminal voltages are interrelated as shown in the figure, which corresponds to Fig. P6.69(b). We can obtain the currents as follows:

$$I_E = \frac{5 - V_E}{1} = 5 - V_E$$

$$I_C = \frac{V_E - 0.2}{1} = V_E - 0.2$$

$$I_B = \frac{V_E - 0.7 - (-5)}{10} = 0.1\ V_E + 0.43$$

Imposing the constraint

$$I_E = I_B + I_C$$

results in

$$5 - V_E = V_E - 0.2 + 0.1\ V_E + 0.43$$

$$\Rightarrow V_E = +2.27 \text{ V}$$

$$V_C = +2.07 \text{ V}$$

$$V_B = 1.57 \text{ V}$$

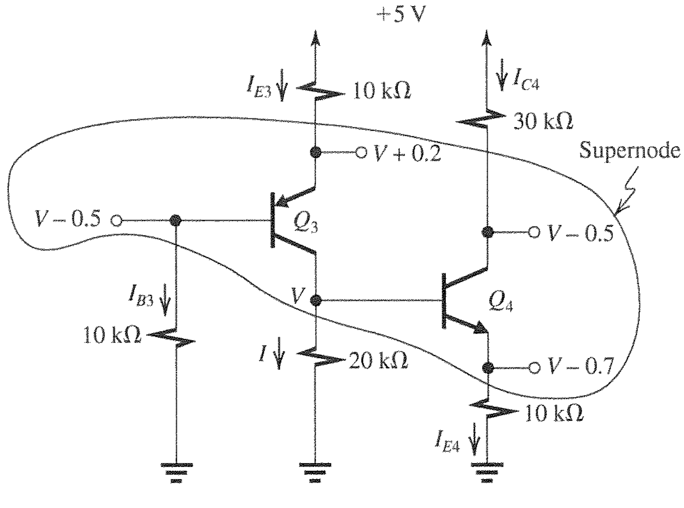
$$I_C = \frac{2.07}{1} = 2.07 \text{ mA}$$

$$I_B = \frac{1.57 - (-5)}{10} = 0.657 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{2.07}{0.657} = 3.2$$

which is lower than the value of β, verifying saturation-mode operation.

This figure belongs to Problem 6.69, part (c).



(c)

(c) We shall assume that both Q_3 and Q_4 are operating in saturation. To begin the analysis shown in the figure, which corresponds to Fig. P6.69(c), we denote the voltage at the emitter of Q_3 as V and then obtain the voltages at all other nodes in terms of V, utilizing the fact that a saturated transistor has $|V_{CE}| = 0.2$ V and of course $|V_{BE}| = 0.7$ V. Note that the choice of the collector node to begin the analysis is arbitrary; we could have selected any other node and denoted its voltage as V. We next draw a circle around the two transistors to define a "supernode." A node equation for the supernode will be

$$I_{E3} + I_{C4} = I_{B3} + I + I_{E4} \tag{1}$$

where

$$I_{E3} = \frac{5 - (V + 0.2)}{10} = 0.48 - 0.1V \tag{2}$$

$$I_{C4} = \frac{5 - (V - 0.5)}{30} = 0.183 - 0.033V \tag{3}$$

$$I_{B3} = \frac{5 - (V - 0.5)}{10} = 0.1V - 0.05 \tag{4}$$

$$I = \frac{V}{20} = 0.05V \tag{5}$$

$$I_{E4} = \frac{V - 0.7}{10} = 0.1V - 0.07 \tag{6}$$

Substituting from Eqs. (2)–(6) into Eq. (1) gives

$$0.48 - 0.1V + 0.183 - 0.033V$$
$$= 0.1V - 0.05 + 0.05V + 0.1V - 0.07$$

$$\Rightarrow V = 2.044 \text{ V}$$

Thus

$$V_{C3} = V = 2.044 \text{ V}$$

$$V_{C4} = V - 0.5 = 1.54 \text{ V}$$

Next we determine all currents utilizing Eqs. (2)–(6):

$I_{E3} = 0.276$ mA $I_{C4} = 0.116$ mA

$I_{B3} = 0.154$ mA $I = 0.102$ mA

$I_{E4} = 0.134$

The base current of Q_4 can be obtained from

$$I_{B4} = I_{E4} - I_{C4} = 0.134 - 0.116 = 0.018 \text{ mA}$$

Finally, the collector current of Q_3 can be found as

$$I_{C3} = I + I_{B4} = 0.102 + 0.018 = 0.120$$

The forced β values can now be found as

$$\beta_{\text{forced3}} = \frac{I_{C3}}{I_{B3}} = \frac{0.120}{0.154} = 0.8$$

$$\beta_{\text{forced4}} = \frac{I_{C4}}{I_{B4}} = \frac{0.116}{0.018} = 6.4$$

Both β_{forced} values are well below the β value of 50, verifying that Q_3 and Q_4 are in deep saturation.

7.1 Coordinates of point A: $v_{GS} = V_t = 0.5$ V and $v_{DS} = V_{DD} = 5$ V.

To obtain the coordinates of point B, we first use Eq. (7.6) to determine $V_{GS}|_B$ as

$$V_{GS}|_B = V_t + \frac{\sqrt{2k_nR_DV_{DD}+1}-1}{k_nR_D}$$

$$= 0.5 + \frac{\sqrt{2\times 10\times 20\times 5+1}-1}{10\times 20}$$

$$= 0.5 + 0.22 = 0.72 \text{ V}$$

The vertical coordinate of point B is $V_{DS}|_B$,

$$V_{DS}|_B = V_{GS}|_B - V_t = V_{OV}|_B = 0.22 \text{ V}$$

7.2 $V_{DS}|_B = V_{OV}|_B = 0.5$ V

Thus,

$$I_D|_B = \frac{1}{2}k_nV_{DS}^2|_B = \frac{1}{2}\times 5\times 0.5^2 = 0.625 \text{ mA}$$

The value of R_D required can now be found as

$$R_D = \frac{V_{DD} - V_{DS}|_B}{I_D|_B}$$

$$= \frac{5-0.5}{0.625} = 7.2 \text{ k}\Omega$$

If the transistor is replaced with another having twice the value of k_n, then $I_D|_B$ will be twice as large and the required value of R_D will be half that used before, that is, 3.6 kΩ.

7.3 Bias point Q: $V_{OV} = 0.2$ V and $V_{DS} = 1$ V.

$$I_{DQ} = \frac{1}{2}k_nV_{OV}^2$$

$$= \frac{1}{2}\times 10\times 0.04 = 0.2 \text{ mA}$$

$$R_D = \frac{V_{DD}-V_{DS}}{I_{DQ}} = \frac{5-1}{0.2} = 20 \text{ k}\Omega$$

Coordinates of point B:

Equation (7.6):

$$V_{GS}|_B = V_t + \frac{\sqrt{2k_nR_DV_{DD}+1}-1}{k_nR_D}$$

$$= 0.5 + \frac{\sqrt{2\times 10\times 20\times 5+1}-1}{10\times 20}$$

$$= 0.5 + 0.22 = 0.72 \text{ V}$$

Equations (7.7) and (7.8):

$$V_{DS}|_B = \frac{\sqrt{2k_nR_DV_{DD}+1}-1}{k_nR_D} = 0.22 \text{ V}$$

$$A_v = -k_nR_DV_{OV}$$

$$= -10\times 20\times 0.2 = -40 \text{ V/V}$$

The lowest instantaneous voltage allowed at the output is $V_{DS}|_B = 0.22$ V. Thus the maximum allowable negative signal swing at the output is $V_{DSQ} - 0.22 = 1 - 0.22 = 0.78$ V. The corresponding peak input signal is

$$\hat{v}_{gs} = \frac{0.78 \text{ V}}{|A_v|} = \frac{0.78}{40} = 19.5 \text{ mV}$$

7.4 From Eq. (7.18):

$$|A_{v\,\max}| = \frac{V_{DD} - V_{OV}|_B}{V_{OV}|_B/2}$$

$$14 = \frac{2 - V_{OV}|_B}{V_{OV}|_B/2}$$

$$\Rightarrow V_{OV}|_B = 0.25 \text{ V}$$

Now, using Eq. (7.15) at point B, we have

$$A_v|_B = -k_nV_{OV}|_BR_D$$

Thus,

$$-14 = -k_nR_D \times 0.25$$

$$\Rightarrow k_nR_D = 56$$

To obtain a gain of −12 V/V at point Q:

$$-12 = -k_nR_DV_{OV}|_Q$$

$$= -56V_{OV}|_Q$$

Thus,

$$V_{OV}|_Q = \frac{12}{56} = 0.214 \text{ V}$$

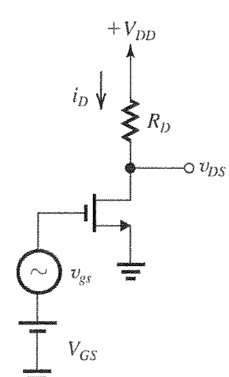
To obtain the required $V_{DS}|_Q$, we use Eq. (7.17),

$$A_v = -\frac{V_{DD} - V_{DS}|_Q}{V_{OV}|_Q/2}$$

$$-12 = -\frac{2 - V_{DS}|_Q}{0.214/2}$$

$$\Rightarrow V_{DS}|_Q = 0.714 \text{ V}$$

7.5



$V_{DD} = 5\text{ V}, \quad k_n' \dfrac{W}{L} = 1 \dfrac{\text{mA}}{\text{V}^2}$

$R_D = 24\text{ k}\Omega, \quad V_t = 1\text{ V}$

(a) Endpoints of saturation transfer segment:

Point A occurs at $V_{GS} = V_t = 1\text{ V}, i_D = 0$

Point A $= (1\text{ V}, 5\text{ V})\ (V_{GS}, V_{DS})$

Point B occurs at sat/triode boundary ($V_{GD} = V_t$)

$V_{GD} = 1\text{ V} \Rightarrow V_{GS} - [5 - i_D R_D] = 1$

$$V_{GS} - 5 + \left(\frac{1}{2}\right)(1)(24)\,[V_{GS} - 1]^2 - 1 = 0$$

$12V_{GS}^2 - 23V_{GS} + 6 = 0$

$V_{GS} = 1.605\text{ V}$

$i_D = 0.183\text{ mA} \quad V_{DS} = 0.608\text{ V}$

Point B $= (+1.61\text{ V}, 0.61\text{ V})$

(b) For $V_{OV} = V_{GS} - V_t = 0.5\text{ V}$, we have

$V_{GS} = 1.5\text{ V}$

$$I_D = \frac{1}{2}k_n(V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 1(1.5 - 1)^2$$

$I_D = 0.125\text{ mA} \quad V_{DS} = +2.00\text{ V}$

Point Q $= (1.50\text{ V}, 2.00\text{ V})$

$A_v = -k_n V_{OV} R_D = -12\text{ V/V}$

(c) From part (a) above, the maximum instantaneous input signal while the transistor remains in saturation is 1.61 V and the corresponding output voltage is 0.61 V. Thus, the maximum amplitude of input sine wave is $(1.61 - 1.5) = 0.11$ V. That is, v_{GS} ranges from $1.5 - 0.11 = 1.39$ V, at which

$$i_D = \frac{1}{2} \times 1 \times (1.39 - 1)^2 = 0.076\text{ mA}$$

and

$v_{DS} = 5 - 0.076 \times 24 = 3.175\text{ V}$

and $v_{GS} = 1.5 + 0.11 = 1.61$ V at which $v_{DS} = 0.61$ V.

Thus, the large-signal gain is

$$\frac{0.61 - 3.175}{1.61 - 1.39} = -11.7\text{ V/V}$$

whose magnitude is slightly less (−2.5%) than the incremental or small-signal gain (−12 V/V). This is an indication that the transfer characteristic is not a straight line.

7.6 $R_D = 20\text{ k}\Omega$

$k_n' = 200\ \mu\text{A/V}^2$

$V_{RD} = 1.5\text{ V}$

$V_{GS} = 0.7\text{ V}$

$A_v = -10\text{ V/V}$

$A_v = -k_n V_{OV} R_D$

$$V_{RD} = I_D R_D = \frac{1}{2}k_n V_{OV}^2 R_D$$

$$\frac{A_v}{V_{RD}} = \frac{-2}{V_{OV}} = \frac{-10}{1.5}$$

$\therefore V_{OV} = 0.30\text{ V}$

$V_t = V_{GS} - V_{OV} = 0.40\text{ V}$

$$k_n = \frac{A_v}{V_{OV} R_D} = \frac{-10}{-0.3 \times 20}$$

$= 1.67\text{ mA/V}^2$

$$k_n = k_n' \frac{W}{L} = 1.67\text{ mA/V}^2$$

$$\therefore \frac{W}{L} = 8.33$$

7.7 At sat/triode boundary

$v_{GS}\big|_B = V_{GS} + \hat{v}_{gs}$

$v_{DS}\big|_B = V_{DS} - \hat{v}_o$

$(\hat{v}_o = \text{ max downward amplitude})$, we get

$$v_{DS}\big|_B = v_{GS}\big|_B - V_t = V_{GS} + \frac{\hat{v}_o}{|A_v|} - V_t$$

$= V_{DS} - \hat{v}_o$

$$V_{OV} + \frac{\hat{v}_o}{|A_v|} = V_{DS} - \hat{v}_o$$

$$\hat{v}_o = \frac{V_{DS} - V_{OV}}{1 + \frac{1}{|A_v|}} \tag{1}$$

For $V_{DD} = 5$ V, $V_{OV} = 0.5$ V, and

$k_n' \dfrac{W}{L} = 1\text{ mA/V}^2$, we use

$$A_v = \frac{-2(V_{DD} - V_{DS})}{V_{OV}}$$

and Eq. (1) to obtain

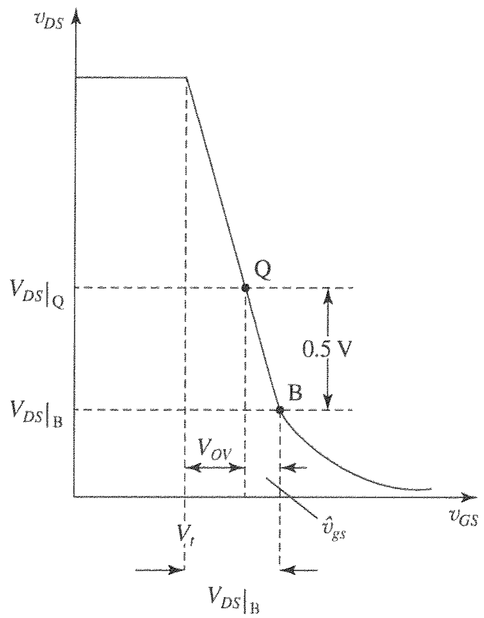
V_{DS}	A_v	$\hat{v}_o$	$\hat{v}_i$
1 V	−16	471 mV	29.4 mV
1.5 V	−14	933 mV	66.7 mV
2 V	−12	1385 mV	115 mV
2.5 V	−10	1818 mV	182 mV

For $V_{DS} = 1$ V, $A_v = -16 = -k_n V_{OV} R_D$

$\therefore R_D = 32$ kΩ

$I_D R_D = 4$ V, $I_D = 0.125$ mA

7.8



To obtain maximum gain while allowing for a −0.5-V signal swing at the output, we bias the MOSFET at point Q where

$$V_{DS}|_Q = V_{DS}|_B + 0.5 \text{ V} \qquad (1)$$

as indicated in the figure above. Now, $V_{DS}|_B$ is given by Eq. (7.8) [together with Eq. (7.7)],

$$V_{DS}|_B = \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} \qquad (2)$$

From the figure we see that

$$V_{DS}|_B = V_{OV} + \hat{v}_{gs}$$

where $V_{OV} = 0.2$ V (given) and

$$\hat{v}_{gs} = \frac{0.5 \text{ V}}{|A_v|}$$

$$= \frac{0.5}{k_n R_D V_{OV}} = \frac{0.5}{k_n R_D \times 0.2} = \frac{2.5}{k_n R_D} \qquad (3)$$

Thus,

$$V_{DS}|_B = 0.2 + \frac{2.5}{k_n R_D}$$

Substituting for $V_{DS}|_B$ from Eq. (2), we obtain

$$\frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} = 0.2 + \frac{2.5}{k_n R_D}$$

Substituting $V_{DD} = 5$ V, rearranging the equation to obtain a quadratic equation in $k_n R_D$, and solving the resulting quadratic equation results in

$$k_n R_D = 213.7$$

which can be substituted into Eq. (2) to obtain

$$V_{DS}|_B = 0.212 \text{ V}$$

The value of V_{DS} at the bias point can now be found from Eq. (1) as

$$V_{DS}|_Q = 0.212 + 0.5 = 0.712 \text{ V}$$

(b) The gain achieved can be found as

$$A_v = -k_n R_D V_{OV}$$

$$= -213.7 \times 0.2 = -42.7 \text{ V/V}$$

$$\hat{v}_{gs} = \frac{0.5}{|A_v|} = \frac{0.5}{42.7} = 11.7 \text{ mV}$$

(c) $I_D = 100$ μA

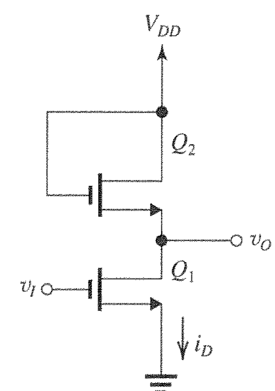
$$R_D = \frac{V_{DD} - V_{DS}|_Q}{I_D}$$

$$= \frac{5 - 0.712}{0.1} = 42.88 \text{ k}\Omega$$

(d) $k_n = \dfrac{213.7}{42.88} = 4.98$ mA/V^2

$$\frac{W}{L} = \frac{4.98}{0.2} = 24.9$$

7.9



given $V_{t1} = V_{t2} = V_t$

For Q_2, $i_D = \dfrac{1}{2} k_n' \left(\dfrac{W}{L}\right)_2 [V_{DD} - v_O - V_t]^2$

For Q_1, $i_D = \dfrac{1}{2} k_n' \left(\dfrac{W}{L}\right)_1 [v_I - V_t]^2$

For $V_t \le v_I \le v_O + V_t$,

equate i_{D1} and i_{D2}

$$\left(\frac{W}{L}\right)_2 [\,V_{DD} - v_O + V_t\,]^2$$

$$= \left(\frac{W}{L}\right)_1 [\,v_I - V_t\,]^2$$

$$[\,V_{DD} - v_O - V_t\,] = \sqrt{\frac{(W/L)_1}{(W/L)_2}} \cdot [\,v_I - V_t\,]$$

$$v_O = V_{DD} - V_t + V_t\sqrt{\frac{(W/L)_1}{(W/L)_2}}$$

$$-v_I\sqrt{\frac{(W/L)_1}{(W/L)_2}}$$

$$\text{For } \sqrt{\frac{(W/L)_1}{(W/L)_2}} = \sqrt{\frac{\left(\frac{50}{0.5}\right)}{\left(\frac{5}{0.5}\right)}} = \sqrt{10},$$

$$A_v = -\sqrt{10} = -3.16 \text{ V/V}$$

7.10 Refer to Fig. 7.6.

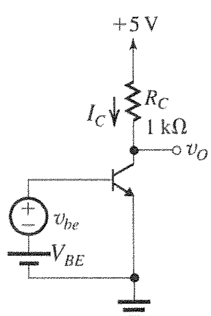
$$A_v = -\frac{V_{CC} - V_{CE}}{V_T}$$

$$= -\frac{5-1}{0.025} = -160 \text{ V/V}$$

The transistor enters saturation when $v_{CE} \leq 0.3$ V, thus the maximum allowable output voltage swing is $1 - 0.3 = 0.7$ V. The corresponding maximum input signal permitted $\hat{v}_{be}$ is

$$\hat{v}_{be} = \frac{0.7 \text{ V}}{|A_v|} = \frac{0.7}{160} = 4.4 \text{ mV}$$

7.11



For $I_C = 0.5$ mA, we have

$$A_v = -\frac{I_C R_C}{V_T} = -\frac{0.5}{0.025} = -20 \text{ V/V}$$

$$V_{CE} = V_{CC} - I_C R_C$$

$$= 5 - 0.5 = 4.5 \text{ V}$$

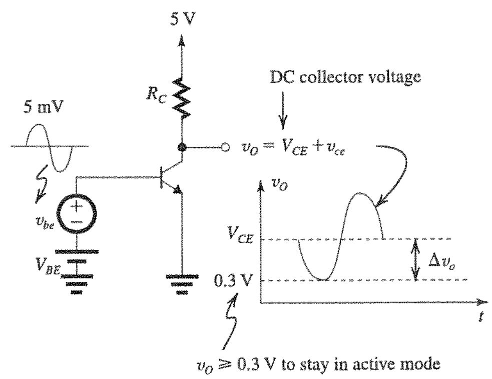
max $+\Delta v_O = 5 - 4.5 = 0.5$ V

max $-\Delta v_O = 4.5 - 0.3 = 4.2$ V

Similarly →

I_C (mA)	V_{CE} (V)	A_v (V/V)	POS Δv_O (V)	Neg Δv_O (V)
0.5	4.5	–20	0.5	4.2
1.0	4.0	–40	1.0	3.7
2.5	2.5	–100	2.5	2.2
4.0	1.0	–160	4.0	0.7
4.5	0.5	–180	4.5	0.2

7.12



$$A_v = -\frac{I_C R_C}{V_T} = -\frac{V_{CC} - V_{CE}}{V_T}$$

On the verge of satuation

$$V_{CE} - \hat{v}_{ce} = 0.3 \text{ V}$$

For linear operation, $v_{ce} = A_v v_{be}$

$$V_{CE} - |A_v \hat{v}_{be}| = 0.3$$

$$(5 - I_C R_C) - |A_v| \times 5 \times 10^{-3} = 0.3$$

But

$$|A_v| = \frac{I_C R_C}{V_T}$$

Thus,

$$I_C R_C = |A_v| V_T$$

and

$$5 - |A_v| V_T - |A_v| \times 5 \times 10^{-3} = 0.3$$

$$|A_v|(0.025 + 0.005) = 5 - 0.3$$

$|A_v| = 156.67.$ Note A_V is negative.

$$\therefore A_v = -156.67 \text{ V/V}$$

Now we can find the dc collector voltage. Referring to the sketch of the output voltage, we see that

$$V_{CE} = 0.3 + |A_v| 0.005 = 1.08 \text{ V}$$

7.13 To determine $|A_{v\max}|$, we use Eq. (7.23),

$$|A_{v\max}| = \frac{V_{CC} - 0.3}{V_T}$$

Then, for $V_{CE} = \frac{V_{CC}}{2}$ we obtain

$$|A_v| = \frac{V_{CC} - \frac{V_{CC}}{2}}{V_T}$$

$$= \frac{V_{CC}}{2V_T}$$

Finally, if a negative-going output signal swing of 0.4 is required, the transistor must be biased at $V_{CE} = 0.4 + 0.3 = 0.7$ V and the gain achieved becomes

$$|A_v| = \frac{V_{CC} - 0.7}{V_T}.$$

The results are as follows:

V_{CC}	1.0	1.5	2.0	3.0 (V)
$V_{CC} - 0.3$	0.7	1.2	1.7	2.7 (V)
$\|A_{v\max}\|$	28	48	68	108 (V/V)
$V_{CC}/2$	0.5	0.75	1.0	1.5 (V)
$\|A_v\|$	20	30	40	60 (V/V)
$V_{CC} - 0.7$	0.3	0.8	1.3	2.3 (V)
$\|A_v\|$	12	32	52	92 (V/V)

7.14 To obtain an output signal of peak amplitude P volts and maximum gain, we bias the transistor at

$$V_{CE} = V_{CEsat} + P$$

The resulting gain will be

$$A_v = -\frac{V_{CC} - V_{CE}}{V_T}$$

which results in V_{CC} of

$$V_{CC} = V_{CE} + |A_v| V_T$$

Thus the minimum required V_{CC} will be

$$V_{CC\min} = V_{CEsat} + P + |A_v| V_T$$

but we have to make sure that the amplifier can support a positive peak amplitude of P, that is,

$$|A_v| V_T \geq P$$

In the results obtained, tabulated below, $V_{CEsat} = 0.3$ V and V_{CC} is the nearest 0.5 V to $V_{CC\min}$.

Case	A_v (V/V)	P (V)	$\|A_v\| V_T$	$V_{CC\min}$	V_{CC}
a	−20	0.2	0.5	1.0	1.0
b	−50	0.5	1.25	2.05	2.5
c	−100	0.5	2.5	3.3	3.5
d	−100	1.0	2.5	3.8	4.0
e	−200	1.0	5.0	6.3	6.5
f	−500	1.0	12.5	13.8	14.0
g	−500	2.0	12.5	14.8	15.0

7.15 See figure below

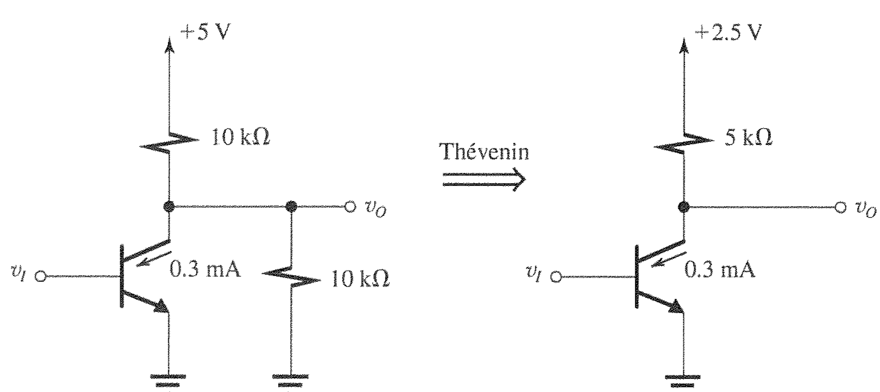
$$A_v = -\frac{I_C R_C}{V_T} = -\frac{0.3 \times 5}{0.025} = -60 \text{ V/V}$$

7.16 (a) See figure on next page

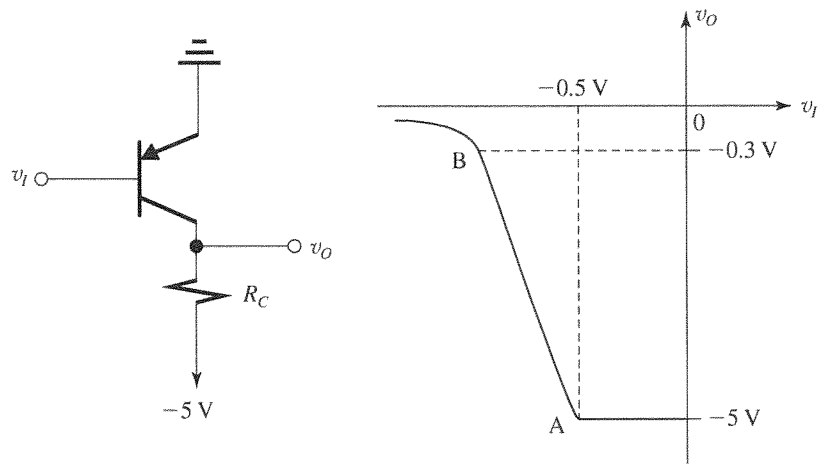
(b) See figure on next page

Note that in part (b) the graph is shifted right by +5 V and up by +5 V.

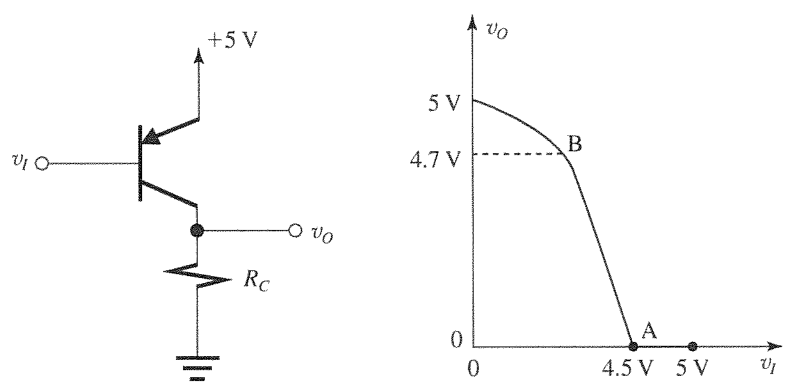
This figure belongs to Problem 7.15.



This figure belongs to Problem 7.16(a).



This figure belongs to Problem 7.16(b).



7.17 $i_C = I_S e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right)$

$$I_C = I_S e^{V_{BE}/V_T}\left(1 + \frac{V_{CE}}{V_A}\right)$$

$$v_{CE} = V_{CC} - R_C i_C$$

$$V_{CE} = V_{CC} - R_C I_C$$

$$A_v = \left.\frac{dv_{CE}}{dv_{BE}}\right|_{v_{BE}=V_{BE},\ v_{CE}=V_{CE}}$$

$$= -R_C I_S\left(1 + \frac{V_{CE}}{V_A}\right)e^{V_{BE}/V_T}\left(\frac{1}{V_T}\right)$$

$$-R_C I_S e^{V_{BE}/V_T}\left(\frac{dv_{CE}}{dv_{BE}}\right)\left(\frac{1}{V_A}\right)$$

$$= -R_C I_C \frac{1}{V_T} - R_C \frac{I_C}{1 + \frac{V_{CE}}{V_A}}\left(\frac{1}{V_A}\right)A_v$$

Thus,

$$A_v = \frac{-I_C R_C / V_T}{1 + \frac{I_C R_C}{V_A + V_{CE}}} \qquad \text{Q.E.D}$$

Substituting $I_C R_C = V_{CC} - V_{CE}$, we obtain

$$A_v = -\frac{(V_{CC} - V_{CE})/V_T}{\left[1 + \frac{V_{CC} - V_{CE}}{V_A + V_{CE}}\right]} \qquad \text{Q.E.D}$$

For $V_{CC} = 5$ V, $V_{CE} = 3$ V, and $V_A = 100$ V,

$$A_v \text{ (without the Early effect)} = -\frac{5 - 3}{0.025}$$

$$= -80 \text{ V/V}$$

$$A_v \text{ (with the Early effect)} = \frac{-80}{1 + \frac{2}{100 + 3}}$$

$$= -78.5 \text{ V/V}$$

7.18 $I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{5 - 2}{1} = 3$ mA

$$A_v = -\frac{V_{CC} - V_{CE}}{V_T} = -\frac{3}{0.025} = -120 \text{ V/V}$$

Using the small-signal voltage gain with $\Delta v_{BE} = +5$ mV, we have

$$\Delta v_O = A_v \times \Delta v_{BE} = -120 \times 5 \text{ mV} = -0.6 \text{ V}$$

Using the exponential characteristic yields

$i_C = I_C e^{v_{BE}/V_T}$

$= 3 \times e^{5/25} = 3.66$ mA

Thus, $\triangle i_C = 0.66$ mA and

$\Delta v_O = -\triangle i_C R_C$

$= -0.66 \times 1 = -0.66$ V

Repeating for $\Delta v_{BE} = -5$ mV as follows.

Using the small-signal voltage gain:

$\Delta v_O = -120 \times -5 = +0.6$ V

Using the exponential characteristic:

$i_C = I_C e^{v_{BE}/V_T}$

$= 3 \times e^{-5/25} = 2.46$ mA

Thus, $\triangle i_C = 2.46 - 3 = -0.54$ mA and $\Delta v_O = 0.54 \times 1 = 0.54$ V

Δv_{BE}	Δv_O **(exp)**	Δv_O **(linear)**
+5 mV	−660 mV	−600 mV
−5 mV	+540 mV	+600 mV

Thus, using the small-signal approximation underestimates $|\triangle v_O|$ for positive Δv_{BE} by about 10% and overestimates $|\triangle v_O|$ for negative Δv_{BE} by about 10%.

7.19 (a) Using Eq. (7.23) yields

$$|A_{v\,\max}| = \frac{V_{CC} - 0.3}{V_T} = \frac{3 - 0.3}{0.025} = 108 \text{ V/V}$$

(b) Using Eq. (7.22) with $A_v = -60$ yields

$$-60 = -\frac{V_{CC} - V_{CE}}{V_T} = -\frac{3 - V_{CE}}{0.025}$$

$\Rightarrow V_{CE} = 1.5$ V

(c) $I_C = 0.5$ mA

$I_C R_C = V_{CC} - V_{CE} = 3 - 1.5 = 1.5$ V

$$R_C = \frac{1.5}{0.5} = 3 \text{ k}\Omega$$

(d) $I_C = I_S e^{V_{BE}/V_T}$

$0.5 \times 10^{-3} = 10^{-15} e^{V_{BE}/0.025}$

$\Rightarrow V_{BE} = 0.673$ V

(e) Assuming linear operation around the bias point, we obtain

$v_{ce} = A_v \times v_{be}$

$= -60 \times 5 \sin \omega t = -300 \sin \omega t$, mV

$= -0.3 \sin \omega t$, V

$$\text{(f) } i_c = \frac{-v_{ce}}{R_C} = 0.1 \sin \omega t, \text{ mA}$$

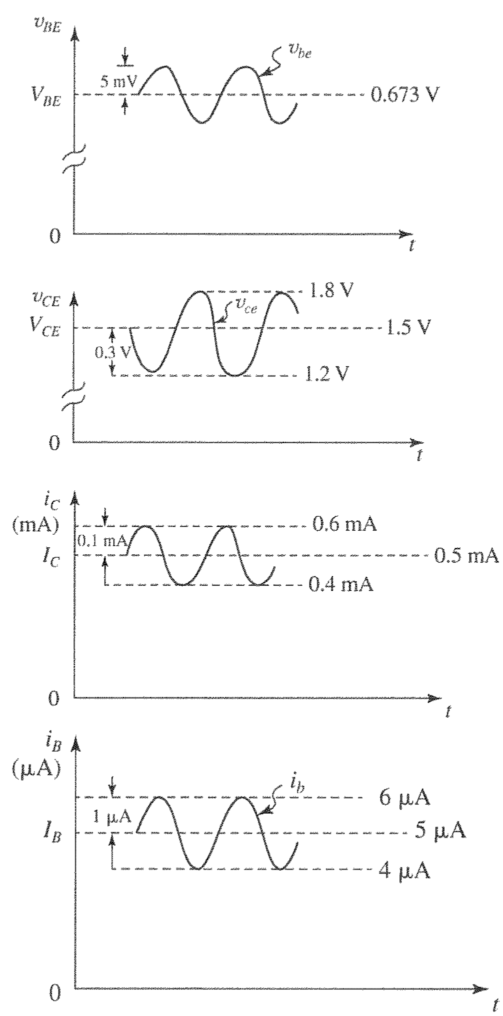
$$\text{(g) } I_B = \frac{I_C}{\beta} = \frac{0.5 \text{ mA}}{100} = 0.005 \text{ mA}$$

$$i_b = \frac{i_c}{\beta} = \frac{0.1}{100} \sin \omega t = 0.001 \sin \omega t, \text{ mA}$$

(h) Small-signal input resistance $\equiv \dfrac{\hat{v}_{be}}{\hat{v}_b}$

$$= \frac{5 \text{ mV}}{0.001 \text{ mA}} = 5 \text{ k}\Omega$$

(i)



7.20 $A_v = -\left(\dfrac{I_C}{V_T}\right)R_C$

But

$$A_v \equiv \frac{\Delta v_O}{\Delta v_{BE}} = \frac{-\Delta i_C R_C}{\Delta v_{BE}} = -g_m R_C$$

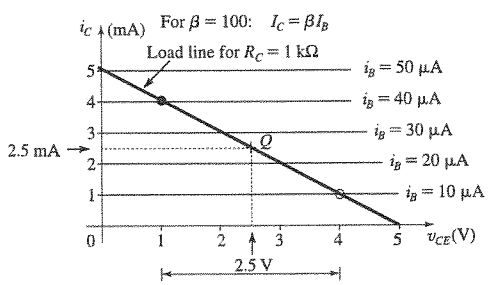
Thus,

$g_m = I_C/V_T$

For a transistor biased at $I_C = 0.5$ mA, we have

$$g_m = \frac{0.5}{0.025} = 20 \text{ mA/V}$$

7.21



Peak-to-peak v_C swing $= 4 - 1 = 3$ V

For point Q at $V_{CC}/2 = 2.5$ V, we obtain

$V_{CE} = 2.5$ V, $\quad I_C = 2.5$ mA

$I_B = 25$ μA

$$I_B = \frac{V_{BB} - 0.7}{R_B} = 25 \text{ μA}$$

$\Rightarrow V_{BB} = I_B R_B + 0.7 = 2.5 + 0.7 = 3.2$ V

7.22 See the graphical construction that follows. For this circuit:

$V_{CC} = 10$ V, $\qquad \beta = 100$,

$R_C = 1$ kΩ, $\qquad V_A = 100$ V,

$I_B = 50$ μA (dc bias),

At $v_{CE} = 0$, $i_C = \beta i_B$

$\therefore I_C = 50 \times 100$

$= 5$ mA (dc bias)

Given the base bias current of 50 mA, the dc or bias point of the collector current I_C, and voltage V_{CE} can be found from the intersection of the load line and the transistor line L_1 of $i_B = 50$ μA. Specifically:

Eq. of $L_1 \Rightarrow i_C = I_C\,(1 + v_{CE}/V_A)$

$= 5\,(1 + v_{CE}/100)$

$= 5 + 0.05 v_{CE}$

Load line $\Rightarrow i_C = \dfrac{V_{CC} - v_{CE}}{R_C} = 10 - v_{CE}$

$\therefore 10 - v_{CE} = 5 + 0.05 v_{CE}$

$V_{CE} = v_{CE} = 4.76$ V

$I_C = i_C = 10 - v_{CE} = 5.24$ mA

Now for a signal of 30-μA peak superimposed on $I_B = 50$ μA, the operating point moves along the load line between points N and M. To obtain the coordinates of point M, we solve the load line and line L_2 to find the intersection M, and the load line and line L_3 to find N:

For point M:

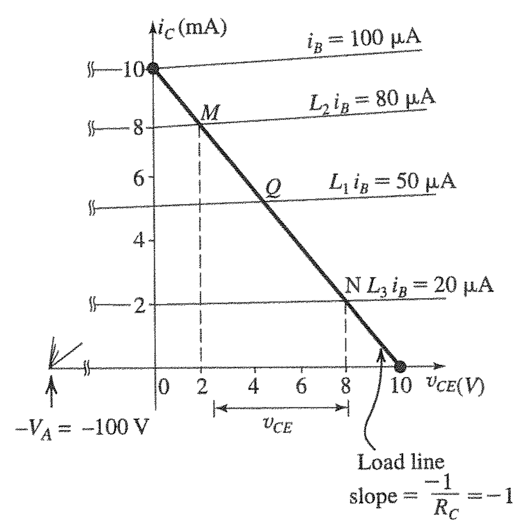
$i_C = 8 + (8/100) v_{CE}$ and $i_C = 10 - v_{CE}$

$\therefore i_C\big|_M = 8.15$ mA, $\quad v_{CE}\big|_M = 1.85$ V

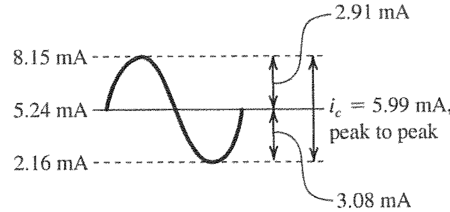
For point N:

$i_C = 2 + 0.02 v_{CE}$ and $i_C = 10 - v_{CE}$

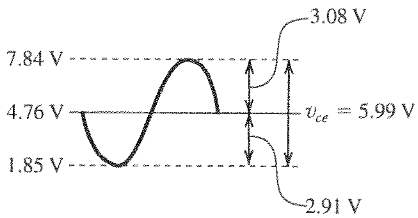
$v_{CE}\big|_N = 7.84$ V, $\quad i_C\big|_N = 2.16$ mA



Thus the collector current varies as follows:



And the collector voltage varies as follows:



7.23 Substituting $v_{gs} = V_{gs} \sin \omega t$ in Eq. (7.28),

$$i_D = \frac{1}{2}k_n(V_{GS} - V_t)^2 + k_n(V_{GS} - V_t)V_{gs} \sin \omega t$$

$$+\frac{1}{2}k_n V_{gs}^2 \sin^2 \omega t$$

$$= \frac{1}{2}k_n(V_{GS} - V_t)^2 + k_n(V_{GS} - V_t)V_{gs} \sin \omega t$$

$$+\frac{1}{2}k_n V_{gs}^2(\frac{1}{2} - \frac{1}{2}\cos 2\,\omega t)$$

Second-harmonic distortion

$$= \frac{\frac{1}{4}k_n V_{gs}^2}{k_n(V_{GS} - V_t)V_{gs}} \times 100$$

$$= \frac{1}{4}\frac{V_{gs}}{V_{OV}} \times 100 \qquad \text{Q.E.D}$$

For $V_{gs} = 10$ mV, to keep the second-harmonic distortion to less than 1%, the minimum overdrive voltage required is

$$V_{OV} = \frac{1}{4} \times \frac{0.01 \times 100}{1} = 0.25 \text{ V}$$

7.24 $I_D = \frac{1}{2}k_n V_{OV}^2 = \frac{1}{2} \times 10 \times 0.2^2 = 0.2$ mA

$v_{GS} = V_{GS} + v_{gs}$, where $v_{gs} = 0.02$ V

$v_{OV} = 0.2 + 0.02 = 0.22$ V

$$i_D = \frac{1}{2}k_n v_{OV}^2 = \frac{1}{2} \times 10 \times 0.22^2 = 0.242 \text{ mA}$$

Thus,

$i_d = 0.242 - 0.2 = 0.042$ mA

For

$v_{gs} = -0.02$ V, $\quad v_{OV} = 0.2 - 0.02 = 0.18$ V

$$i_D = \frac{1}{2}k_n v_{OV}^2 = \frac{1}{2} \times 10 \times 0.18^2 = 0.162 \text{ mA}$$

Thus,

$i_d = 0.2 - 0.162 = 0.038$ mA

Thus, an estimate of g_m can be obtained as follows:

$$g_m = \frac{0.042 + 0.038}{0.04} = 2 \text{ mA/V}$$

Alternatively, using Eq. (7.33), we can write

$g_m = k_n V_{OV} = 10 \times 0.2 = 2$ mA/V

which is an identical result.

7.25 (a) $I_D = \frac{1}{2}k_n(V_{GS} - V_t^2)$

$$= \frac{1}{2} \times 5(0.6 - 0.4)^2 = 0.1 \text{ mA}$$

$V_{DS} = V_{DD} - I_D R_D = 1.8 - 0.1 \times 10 = 0.8$ V

(b) $g_m = k_n V_{OV} = 5 \times 0.2 = 1$ mA/V

(c) $A_v = -g_m R_D = -1 \times 10 = -10$ V/V

(d) $\lambda = 0.1\ \text{V}^{-1}$, $\quad V_A = \frac{1}{\lambda} = 10$ V

$$r_o = \frac{V_A}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$A_v = -g_m(R_D \parallel r_o)$

$= -1(10 \parallel 100) = -9.1$ V/V

7.26 $A_v = -10 = -g_m R_D = -g_m \times 20$

$g_m = 0.5$ mA/V

To allow for a −0.2-V signal swing at the drain while maintaining saturation-region operation, the minimum voltage at the drain must be at least equal to V_{OV}. Thus

$V_{DS} = 0.2 + V_{OV}$

Since

$$A_v = -\frac{V_{DD} - V_{DS}}{\frac{1}{2}V_{OV}}$$

$$-10 = -\frac{1.8 - 0.2 - V_{OV}}{0.5 V_{OV}}$$

$\Rightarrow V_{OV} = 0.27$ V

The value of I_D can be found from

$$g_m = \frac{2I_D}{V_{OV}}$$

$$0.5 = \frac{2 \times I_D}{0.27}$$

$\Rightarrow I_D = 0.067$ mA

The required value of k_n can be found from

$$I_D = \frac{1}{2}k_n V_{OV}^2$$

$$0.067 = \frac{1}{2}k_n \times 0.27^2$$

$\Rightarrow k_n = 1.83$ mA/V^2

Since $k_n' = 0.2$ mA/V^2, the W/L ratio must be

$$\frac{W}{L} = \frac{k_n}{k_n'} = \frac{1.83}{0.2} = 9.14$$

Finally,

$V_{GS} = V_t + V_{OV} = 0.4 + 0.27 = 0.67$ V

7.27 $A_v = -g_m R_D$

Upon substituting for g_m from Eq. (7.42), we can write

$$A_v = -\frac{2I_D R_D}{V_{OV}}$$

$$= -\frac{2(V_{DD} - V_{DS})}{V_{OV}} \qquad \text{Q.E.D} \qquad (1)$$

$$v_{GS}\big|_{\max} = V_{GS} + \hat{v}_i = V_t + V_{OV} + \hat{v}_i$$

$$v_{DS}\big|_{\min} = V_{DS} - |A_v|\hat{v}_i$$

To just maintain saturation-mode operation,

$$v_{GS}\big|_{\max} = v_{DS}\big|_{\min} + V_t$$

which results in

$$V_{OV} + \hat{v}_i = V_{DS} - |A_v|\hat{v}_i$$

Substituting for $|A_v|$ from Eq. (1) yields

$$V_{OV} + \hat{v}_i = V_{DS} - \frac{2(V_{DD} - V_{DS})}{V_{OV}}\hat{v}_i$$

$$V_{DS}[1 + 2(\hat{v}_i/V_{OV})]$$

$$= V_{OV} + \hat{v}_i + 2V_{DD}(\hat{v}_i/V_{OV})$$

$$\Rightarrow V_{DS} = \frac{V_{OV} + \hat{v}_i + 2V_{DD}(\hat{v}_i/V_{OV})}{1 + 2(\hat{v}_i/V_{OV})} \qquad \text{Q.E.D}$$

For

$V_{DD} = 2.5$ V, $\hat{v}_i = 20$ mV and $m = 15$

$$V_{OV} = m\hat{v}_i = 15 \times 20 = 0.3 \text{ V}$$

$$V_{DS} = \frac{0.3 + 0.02 + 2 \times 2.5 \times (0.02/0.3)}{1 + 2(0.02/0.3)}$$

$$= 0.576 \text{ V}$$

$$A_v = -\frac{2(V_{DD} - V_{DS})}{V_{OV}} = -\frac{2(2.5 - 0.576)}{0.3}$$

$$= -12.82 \text{ V/V}$$

$$\hat{v}_o = |A_v|\hat{v}_i = 12.82 \times 20 \text{ mV} = 0.256 \text{ V}$$

To operate at $I_D = 200\ \mu$A $= 0.2$ mA,

$$R_D = \frac{2.5 - 0.576}{0.2} = 9.62 \text{ k}\Omega$$

$$I_D = \frac{1}{2}k_n V_{OV}^2$$

$$0.2 = \frac{1}{2}k_n \times 0.3^2$$

$$\Rightarrow k_n = 4.44 \text{ mA/V}^2$$

The required W/L ratio can now be found as

$$\frac{W}{L} = \frac{k_n}{k_n'} = \frac{4.44}{0.1} = 44.4$$

7.28 Given $\mu_n = 500$ cm^2/V·s,

$\mu_p = 250$ cm^2/V·s, and $C_{ox} = 0.4$ fF/μm^2,

$k_n' = \mu_n C_{ox} = 20\ \mu$A/V^2

$k_p' = 10\ \mu$A/V^2

See table below.

Case type	I_D **(mA)**	$\lvert V_{GS}\rvert$ **(V)**	$\lvert V_t\rvert$ **(V)**	$\lvert V_{OV}\rvert$ **(V)**	W **(μm)**	L **(μm)**	$\frac{W}{L}$	$k'\frac{W}{L}$ **(mA/V²)**	g_m **(mA/V)**
a (N)	1	3	2	1	100	1	100	2	2
b (N)	1	1.2	0.7	0.5	50	0.125	400	8	4
c (N)	10	—	—	2	250	1	250	5	10
d (N)	0.5	—	—	0.5	—	—	200	4	2
e (N)	0.1	—	—	1.41	10	2	5	0.1	0.141
f (N)	0.1	1.8	0.8	1	40	4	10	0.2	0.2
g (P)	0.5	—	—	2	—	—	25	0.25	0.5
h (P)	1	3	1	2	—	—	50	0.5	1
i (P)	10	—	—	1	4000	2	2000	20	20
j (P)	10	—	—	4	—	—	125	1.25	5
k (P)	0.05	—	—	1	30	3	10	0.1	0.1
l (P)	1	—	—	5	—	—	8	0.08	0.4

Note: The circled entries are the givens.

7.29 Given $\mu_n C_{ox} = 250\ \mu\text{A/V}^2$,

$V_t = 0.5$ V,

$L = 0.5\ \mu$m

For $g_m = 2$ mA/V^2 and $I_D = 0.25$ mA,

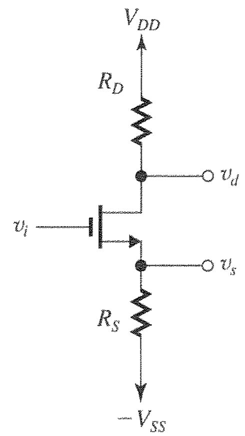
$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \Rightarrow \frac{W}{L} = 32$$

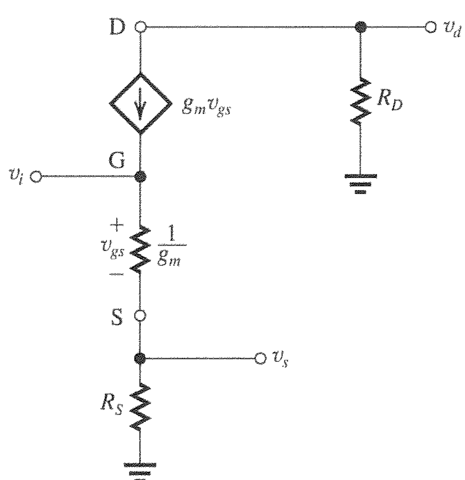
$\therefore W = 16\ \mu$m

$$V_{OV} = \frac{2I_D}{g_m} = 0.25 \text{ V}$$

$\therefore V_{GS} = V_{OV} + V_t = 0.75$ V

7.30





$$v_i = (g_m v_{gs}) \left(\frac{1}{g_m} + R_S \right)$$

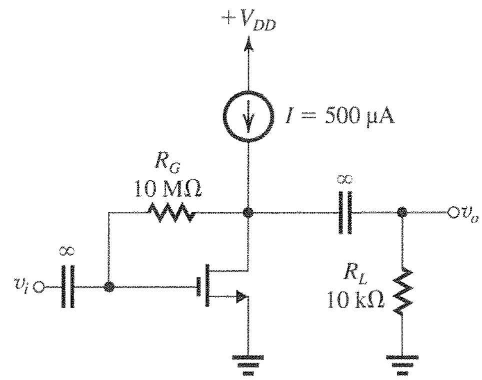
$$v_d = -g_m v_{gs} R_D$$

$$v_s = +g_m v_{gs} R_S$$

$$\therefore \frac{v_s}{v_i} = \frac{R_S}{\frac{1}{g_m} + R_S} = \frac{+g_m R_S}{1 + g_m R_S}$$

$$\frac{v_d}{v_i} = \frac{-R_D}{\frac{1}{g_m} + R_S} = \frac{-g_m R_D}{1 + g_m R_S}$$

7.31



$V_t = 0.5$ V

$V_A = 50$ V

Given $V_{DS} = V_{GS} = 1$ V. Also, $I_D = 0.5$ mA.

$$V_{OV} = 0.5 \text{ V}, g_m = \frac{2I_D}{V_{OV}} = 2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = 100 \text{ k}\Omega$$

$$\frac{v_o}{v_i} = -g_m (R_G \parallel R_L \parallel r_o) = -18.2 \text{ V/V}$$

For $I_D = 1$ mA:

V_{OV} increases by $\sqrt{\frac{1}{0.5}} = \sqrt{2}$ to

$\sqrt{2} \times 0.5 = 0.707$ V.

$V_{GS} = V_{DS} = 1.207$ V

$g_m = 2.83$ mA/V, $r_o = 50$ kΩ and

$$\frac{v_o}{v_i} = -23.6 \text{ V/V}$$

7.32 For the NMOS device:

$$I_D = 100 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{OV}^2$$

$$= \frac{1}{2} \times 400 \times \frac{10}{0.5} \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.16 \text{ V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1 \text{ mA}}{0.16} = 1.25 \text{ mA/V}$$

$$V_A = 5L = 5 \times 0.5 = 2.5 \text{ V}$$

$$r_o = \frac{V_A}{I_D} = \frac{2.5}{0.1} = 25 \text{ k}\Omega$$

For the PMOS device:

$$I_D = 100 = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}V_{OV}^2$$

$$= \frac{1}{2} \times 100 \times \frac{10}{0.5} \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.316 \text{ V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1}{0.316} = 0.63 \text{ mA/V}$$

$$V_A = 6L = 6 \times 0.5 = 3 \text{ V}$$

$$r_o = \frac{V_A}{I_D} = \frac{3}{0.1} = 30 \text{ k}\Omega$$

7.33 (a) Open-circuit the capacitors to obtain the bias circuit shown in Fig. 1, which indicates the given values.

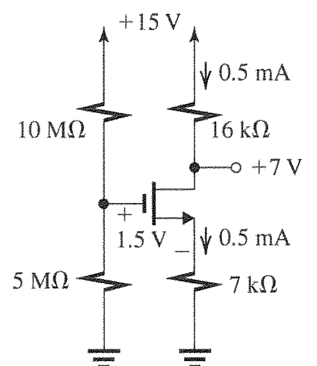


Figure 1

From the voltage divider, we have

$$V_G = 15\frac{5}{10+5} = 5 \text{ V}$$

From the circuit, we obtain

$$V_G = V_{GS} + 0.5 \times 7$$

$$= 1.5 + 3.5 = 5 \text{ V}$$

which is consistent with the value provided by the voltage divider.

Since the drain voltage (+7 V) is higher than the gate voltage (+5 V), the transistor is operating in saturation.

From the circuit

$V_D = V_{DD} - I_D R_D = 15 - 0.5 \times 16 = +7$ V, as assumed

Finally,

$V_{GS} = 1.5$ V, thus $V_{OV} = 1.5 - V_t = 1.5 - 1$ $= 0.5$ V

$$I_D = \frac{1}{2}k_n V_{OV}^2 = \frac{1}{2} \times 4 \times 0.5^2 = 0.5 \text{ mA}$$

which is equal to the given value. Thus the bias calculations are all consistent.

(b) $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.5}{0.5} = 2$ mA/V

$$r_o = \frac{V_A}{I_D} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

(c) See Fig. 2 below.

(d) $R_{\text{in}} = 10 \text{ M}\Omega \parallel 5 \text{ M}\Omega = 3.33 \text{ M}\Omega$

$$\frac{v_{gs}}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \frac{3.33}{3.33 + 0.2}$$

$$= 0.94 \text{ V/V}$$

$$\frac{v_o}{v_{gs}} = -g_m(200 \parallel 16 \parallel 16)$$

$$= -2 \times 7.69 = -15.38 \text{ V/V}$$

$$\frac{v_o}{v_{\text{sig}}} = \frac{v_{gs}}{v_{\text{sig}}} \times \frac{v_o}{v_{gs}} = -0.94 \times 15.38$$

$$= -14.5 \text{ V/V}$$

7.34 (a) Using the exponential characteristic:

$$i_c = I_C e^{v_{be}/V_T} - I_C$$

giving $\dfrac{i_c}{I_C} = e^{v_{be}/V_T} - 1$

(b) Using small-signal approximation:

$$i_c = g_m v_{be} = \frac{I_C}{V_T} \cdot v_{be}$$

This figure belongs to Problem 7.33, part (c).

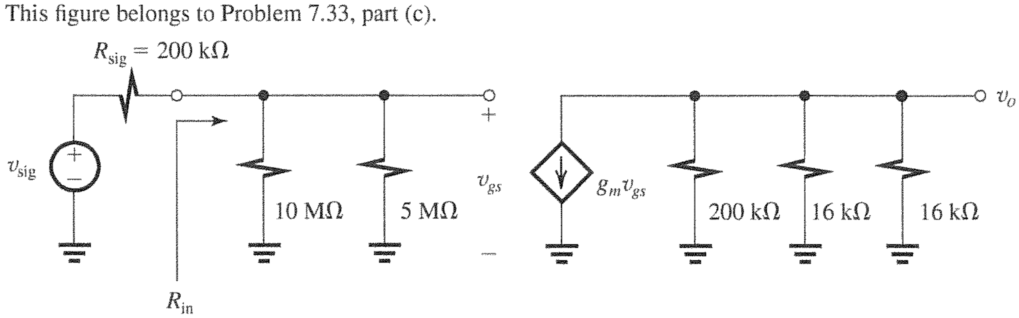


Figure 2

Thus, $\frac{i_c}{I_C} = \frac{v_{be}}{V_T}$

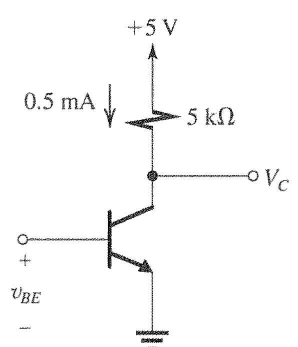
See table below.

For signals at ±5 mV, the error introduced by the small-signal approximation is 10%.

The error increases to above 20% for signals at ±10 mV.

v_{be} (mV)	i_c/I_C Exponential	i_c/I_C Small signal	Error (%)
+1	+0.041	+0.040	–2.4
–1	–0.039	–0.040	+2.4
+2	+0.083	+0.080	–3.6
–2	–0.077	–0.080	+3.9
+5	+0.221	+0.200	–9.7
–5	–0.181	–0.200	+10.3
+8	+0.377	+0.320	–15.2
–8	–0.274	–0.320	+16.8
+10	+0.492	+0.400	–18.7
–10	–0.330	–0.400	+21.3
+12	+0.616	+0.480	–22.1
–12	–0.381	–0.480	+25.9

7.35



With $v_{BE} = 0.700$ V

$V_C = V_{CC} - R_C I_C$

$= 5 - 5 \times 0.5 = 2.5$ V

For $v_{BE} = 705$ mV $\Rightarrow v_{be} = 5$ mV

$i_C = I_C e^{v_{be}/V_T}$

$= 0.5 \times e^{5/25} = 0.611$ mA

$v_C = V_{CC} - R_C i_C = 5 - 5 \times 0.611 = 1.95$ V

$v_{ce} = v_C - V_C = 1.95 - 2.5 = -0.55$ V

Voltage gain, $A_v = \frac{v_{ce}}{v_{be}} = -\frac{0.55\text{ V}}{5\text{ mV}}$

$= -110$ V/V

Using small-signal approximation, we write

$A_v = -g_m R_C$

where

$g_m = \frac{I_C}{V_T} = \frac{0.5\text{ mA}}{0.025\text{ V}} = 20$ mA/V

$A_v = -20 \times 5 = -100$ V/V

Thus, the small-signal approximation at this signal level ($v_{be} = 5$ mV) introduces an error of −9.1% in the gain magnitude.

7.36 At $I_C = 0.5$ mA,

$g_m = \frac{I_C}{V_T} = \frac{0.5\text{ mA}}{0.025\text{ V}} = 20$ mA/V

$r_\pi = \frac{\beta}{g_m} = \frac{100}{20\text{ mA/V}} = 5$ kΩ

$r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C}$

where

$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{100 + 1} = 0.99$

$r_e = \frac{0.99 \times 25\text{ mV}}{0.5\text{ mA}} \simeq 50\ \Omega$

At $I_C = 50\ \mu$A $= 0.05$ mA,

$g_m = \frac{I_C}{V_T} = \frac{0.05}{0.025} = 2$ mA/V

$r_\pi = \frac{\beta}{g_m} = \frac{100}{2\text{ mA/V}} = 50$ kΩ

$r_e = \frac{\alpha V_T}{I_C} = \frac{0.99 \times 25\text{ mV}}{0.5\text{ mA}} \simeq 500\ \Omega$

7.37 $g_m = \frac{I_C}{V_T} = \frac{1\text{ mA}}{0.025\text{ V}} = 40$ mA/V

$r_e = \frac{\alpha}{g_m} = \frac{0.99}{40\text{ mA/V}} \simeq 25\ \Omega$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{40\text{ mA/V}} = 2.5$ kΩ

$A_v = -g_m R_C = -40 \times 5 = -200$ V/V

$\hat{v}_o = |A_v| \hat{v}_{be} = 200 \times 5\text{ mV} = 1$ V

7.38 For $g_m = 30$ mA/V,

$g_m = \frac{I_C}{V_T} \Rightarrow I_C = g_m V_T = 30 \times 0.025 = 0.75$ mA

$r_\pi = \frac{\beta}{g_m} = \frac{\beta}{30\text{ mA/V}}$

For $r_\pi \geq 3$ kΩ, we require

$\beta \geq 90$

That is, $\beta_{\min} = 90$.

7.39 $r_\pi = \dfrac{\beta}{g_m}$

where

$$g_m = \frac{I_C}{V_T}$$

Nominally, $g_m = 40$ mA/V. However, I_C varies by ±20%, so g_m ranges from 32 mA/V to 48 mA/V. Thus

$$r_\pi = \frac{50 \text{ to } 150}{32 \text{ to } 48 \text{ mA/V}}$$

Thus, the extreme values of r_π are $\dfrac{50}{48} = 1.04$ kΩ and $\dfrac{150}{32} = 4.7$ kΩ.

7.40 $V_{CC} = 3$ V, $V_C = 1$ V, $R_C = 2$ kΩ

$$I_C = \frac{3-1}{2} = 1 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$v_{be} = 0.005 \sin \omega t$$

$$i_c = g_m v_{be} = 0.2 \sin \omega t, \text{ mA}$$

$$i_C(t) = I_C + i_c = 1 + 0.2 \sin \omega t, \text{ mA}$$

$$v_C(t) = V_{CC} - R_C i_C$$

$$= 3 - 2(1 + 0.2 \sin \omega t)$$

$$= 1 - 0.4 \sin \omega t, \text{ V}$$

$$i_B(t) = i_C(t)/\beta$$

$$= 0.01 + 0.002 \sin \omega t, \text{ mA}$$

$$A_v = \frac{v_c}{v_{be}} = -\frac{0.4}{0.005} = -80 \text{ V/V}$$

7.41 Since $\hat{V}_{be}$ is the maximum value for acceptable linearity, the largest signal at the collector will be obtained by designing for maximum gain magnitude. This in turn is achieved by biasing the transistor at the lowest V_{CE} consistent with the transistor remaining in the active mode at the negative peak of v_o. Thus

$$V_{CE} - |A_v|\hat{V}_{be} = 0.3$$

where we have assumed $V_{CE\text{sat}} = 0.3$ V. Since

$$V_{CE} = V_{CC} - I_C R_C$$

and

$$|A_v| = g_m R_C = \frac{I_C}{V_T} R_C$$

then

$$V_{CC} - I_C R_C - \frac{\hat{V}_{be}}{V_T} I_C R_C = 0.3$$

which can be manipulated to yield

$$I_C R_C = \frac{V_{CC} - 0.3}{1 + \dfrac{\hat{V}_{be}}{V_T}} \quad (1)$$

Since the voltage gain is given by

$$A_v = -\frac{I_C R_C}{V_T}$$

then

$$A_v = \frac{V_{CC} - 0.3}{V_T + \hat{V}_{be}}$$

For $V_{CC} = 3$ V and $\hat{V}_{be} = 5$ mV,

$$I_C R_C = \frac{3 - 0.3}{1 + \dfrac{5}{25}} = 2.25 \text{ V}$$

Thus,

$$V_{CE} = V_{CC} - I_C R_C$$

$$= 3 - 2.25 = 0.75 \text{ V}$$

$$\hat{V}_o = V_{CE} - 0.3 = 0.75 - 0.3 = 0.45 \text{ V}$$

$$A_v = -\frac{3 - 0.3}{0.025 + 0.005} = -90 \text{ V/V}$$

Check:

$$A_v = -g_m R_C = -\frac{I_C R_C}{V_T} = -\frac{2.25}{0.025} = -90 \text{ V/V}$$

$$\hat{V}_o = |A_v| \times \hat{V}_{be} = 90 \times 5 = 450 \text{ mV} = 0.45 \text{ V}$$

7.42

Transistor	a	b	c	d	e	f	g
α	1.000	0.990	0.980	1	0.990	0.900	0.940
β	∞	100	50	∞	100	9	15.9
I_C (mA)	1.00	0.99	1.00	1.00	0.248	4.5	17.5
I_E (mA)	1.00	1.00	1.02	1.00	0.25	5	18.6
I_B (mA)	0	0.010	0.020	0	0.002	0.5	1.10
g_m (mA/V)	40	39.6	40	40	9.92	180	700
r_e (Ω)	25	25	24.5	25	100	5	1.34
r_π (Ω)	∞	2.525 k	1.25 k	∞	10.1 k	50	22.7

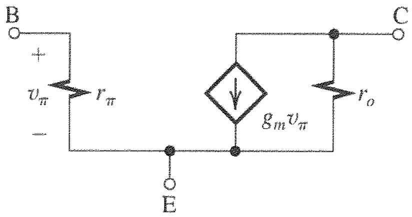
7.43 $I_C = 1$ mA, $\beta = 100$, $V_A = 100$ V

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

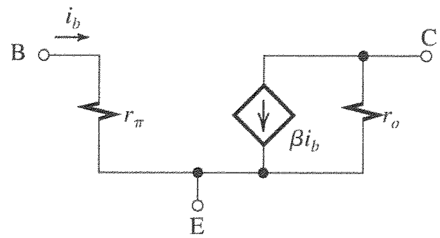
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40 \text{ mA/V}} = 2.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ k}\Omega$$

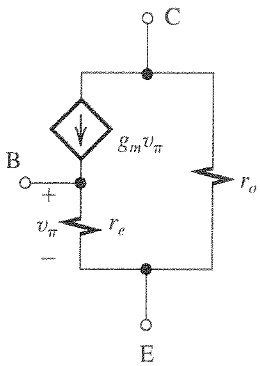
This figure belongs to Problem 7.43.



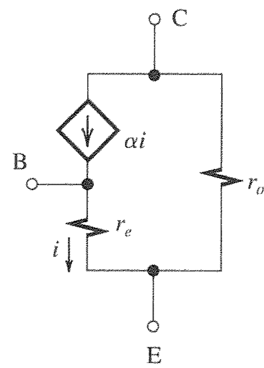
$r_\pi = 2.5\ \text{k}\Omega$, $g_m = 40$ mA/V



$r_o = 100\ \text{k}\Omega$, $\beta = 100$



$r_e = 24.75\ \Omega$, $g_m = 40$ mA/V

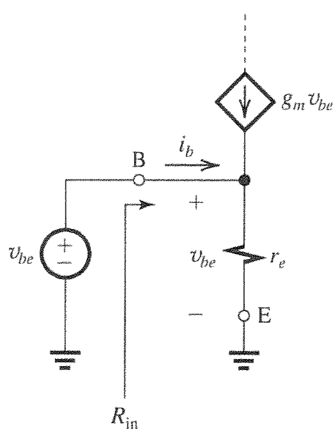


$r_o = 100\ \text{k}\Omega$, $\alpha = 0.99$

$$\alpha = \frac{\beta}{\beta+1} = \frac{100}{100+1} = 0.99$$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C} = \frac{0.99 \times 25\ \text{mV}}{1\ \text{mA}} = 24.75\ \Omega$$

7.44



$$i_b = \frac{v_{be}}{r_e} - g_m v_{be}$$

$$= v_{be}\left(\frac{1}{r_e} - g_m\right)$$

Since

$$r_e = \frac{\alpha}{g_m}$$

$$i_b = v_{be}\left(\frac{g_m}{\alpha} - g_m\right)$$

$$= g_m v_{be}\frac{1-\alpha}{\alpha}$$

$$= \frac{g_m v_{be}}{\beta}$$

$$R_{\text{in}} \equiv \frac{v_{be}}{i_b} = \frac{\beta}{g_m} = r_\pi \qquad \text{Q.E.D}$$

7.45 Refer to Fig. 7.26.

$$i_c = \alpha i_e = \alpha\frac{v_{be}}{r_e} = \frac{\alpha}{r_e}v_{be}$$

$$= g_m v_{be} \qquad \text{Q.E.D}$$

7.46 The large-signal model of Fig. 6.5(d) is shown in Fig. 1.

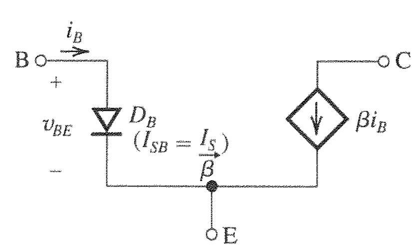


Figure 1

For v_{BE} undergoing an incremental change v_{be} from its equilibrium value of V_{BE}, the current i_B

changes from I_B by an increment i_b, which is related to v_{be} by the incremental resistance of D_B at the bias current I_B. This resistance is given by V_T/I_B, which is r_π.

The collector current βi_B changes from βI_B to $\beta(I_B + i_b)$. The incremental changes around the equilibrium or bias point are related to each other by the circuit shown in Fig. 2,

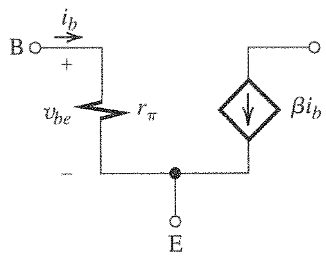


Figure 2

which is the hybrid-π model of Fig. 7.24(b). Q.E.D.

7.47 The large-signal T model of Fig. 6.5(b) is shown below in Fig. 1.

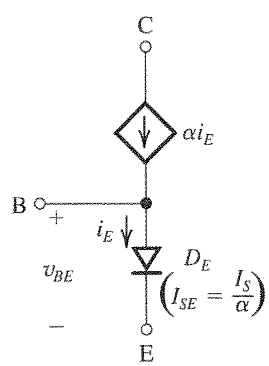


Figure 1

If i_E undergoes an incremental change i_e from its equilibrium or bias value I_E, the voltage v_{BE} will correspondingly change by an incremental amount v_{be} (from its equilibrium or bias value V_{BE}), which is related to i_e by the incremental resistance of diode D_E. The latter is equal to V_T/I_E, which is r_e.

The incremental change i_e in i_E gives rise to an incremental change αi_e in the current of the controlled source.

The incremental quantities can be related by the equivalent circuit model shown in Fig. 2,

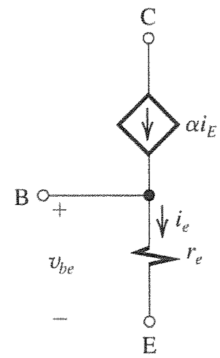


Figure 2

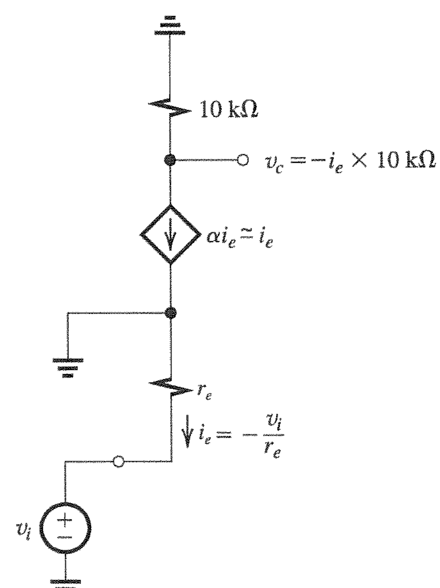
which is the small-signal T model of Fig. 7.26(b). Q.E.D.

7.48 Refer to Fig. P7.48:

$$V_C = 3 - 0.2 \times 10 = 1 \text{ V}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.2 \text{ mA}} = 125 \ \Omega$$

Replacing the BJT with the T model of Fig. 7.26(b), we obtain the equivalent circuit shown below.



$$v_c = -i_e \times 10 \text{ k}\Omega$$

where

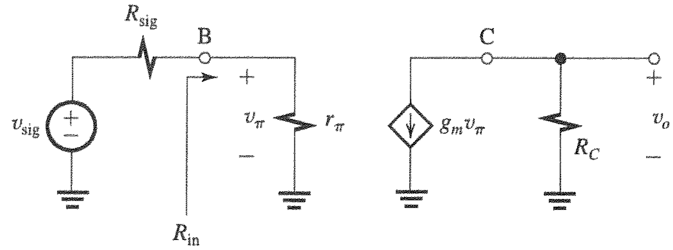
$$i_e = -\frac{v_i}{r_e} = -\frac{v_i}{0.125 \text{ k}\Omega}$$

Thus,

$$\frac{v_c}{v_i} = \frac{10 \text{ k}\Omega}{0.125 \text{ k}\Omega}$$

$$= 80 \text{ V/V}$$

This figure belongs to Problem 7.50.



7.49 $v_{ce} = |A_v| v_{be}$

$|A_v| = g_m R_C = 50 \times 2 = 100\ \text{V/V}$

For v_{ce} being 1 V peak to peak,

$$v_{be} = \frac{1\ \text{V}}{100} = 0.01\ \text{V peak to peak}$$

$$i_b = \frac{v_{be}}{r_\pi}$$

where

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{50} = 2\ \text{k}\Omega$$

Thus,

$$i_b = \frac{0.01\ \text{V}}{2\ \text{k}\Omega} = 0.005\ \text{mA peak to peak}$$

7.50

$$R_{in} \equiv \frac{v_\pi}{i_b} = r_\pi$$

$$\frac{v_\pi}{v_{sig}} = \frac{r_\pi}{r_\pi + R_{sig}}$$

$$v_o = -g_m v_\pi R_C$$

$$\frac{v_o}{v_\pi} = -g_m R_C$$

The overall voltage gain can be obtained as follows:

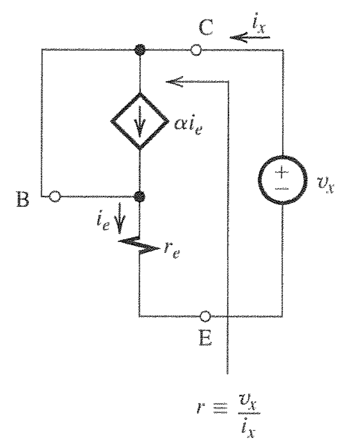
$$\frac{v_o}{v_{sig}} = \frac{v_o}{v_\pi}\frac{v_\pi}{v_{sig}}$$

$$= -g_m R_C \frac{r_\pi}{r_\pi + R_{sig}}$$

$$= -g_m r_\pi \frac{R_C}{r_\pi + R_{sig}}$$

$$= -\frac{\beta R_C}{r_\pi + R_{sig}} \qquad \text{Q.E.D.}$$

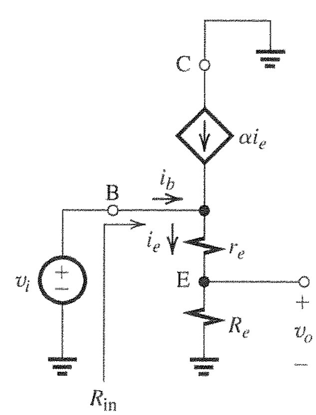
7.51 Replacing the BJT with the T model of Fig. 7.26(b), we obtain the circuit shown below.



Since v_x appears across r_e and $i_x = i_e = \frac{v_x}{r_e}$, the small-signal resistance r is given by

$$r \equiv \frac{v_x}{i_x} = \frac{v_x}{i_e} = r_e$$

7.52 Refer to Fig. P7.52. Replacing the BJT with the T model of Fig. 7.26(b) results in the following amplifier equivalent circuit:



$$R_{\text{in}} \equiv \frac{v_i}{i_b} = \frac{v_i}{(1-\alpha)i_e}$$

From the circuit we see that

$$i_e = \frac{v_i}{r_e + R_e}$$

Thus,

$$R_{\text{in}} = \frac{r_e + R_e}{1-\alpha}$$

But

$$1 - \alpha = \frac{1}{\beta + 1}$$

Thus,

$$R_{\text{in}} = (\beta + 1)(r_e + R_e) \qquad \text{Q.E.D.}$$

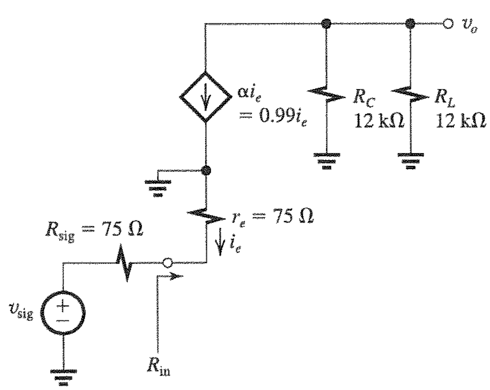
From the equivalent circuit, we see that v_o and v_i are related by the ratio of the voltage divider formed by r_e and R_e:

$$\frac{v_o}{v_i} = \frac{R_e}{R_e + r_e} \qquad \text{Q.E.D.}$$

7.53 Refer to Fig. P7.53. The transistor is biased at $I_E = 0.33$ mA. Thus

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.33 \text{ mA}} = 75\ \Omega$$

Replacing the BJT with its T model results in the following amplifier equivalent circuit.



The input resistance R_{in} can be found by inspection to be

$$R_{\text{in}} = r_e = 75\ \Omega$$

To determine the voltage gain (v_o/v_i) we first find i_e:

$$i_e = -\frac{v_i}{R_{\text{sig}} + r_e} = -\frac{v_i}{150\ \Omega} = -\frac{v_i}{0.15 \text{ k}\Omega}$$

The output voltage v_o is given by

$$v_o = -\alpha\, i_e (R_C \parallel R_L)$$

$$= -0.99\, i_e \times (12 \parallel 12) = -0.99 \times 6 i_e$$

$$= -0.99 \times 6 \times \frac{-v_i}{0.15}$$

Thus,

$$\frac{v_o}{v_i} = 39.6 \text{ V/V}$$

7.54 Refer to Fig. P7.54.

$$\alpha = \frac{\beta}{\beta + 1} = \frac{200}{201} = 0.995$$

$$I_C = \alpha \times I_E = 0.995 \times 10 = 9.95 \text{ mA}$$

$$V_C = I_C R_C = 9.95 \times 0.1 \text{ k}\Omega = 0.995 \text{ V} \simeq 1 \text{ V}$$

Replacing the BJT with its hybrid-π model results in the circuit shown below.

$$g_m = \frac{I_C}{V_T} \simeq \frac{10 \text{ mA}}{0.025 \text{ V}} = 400 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{400} = 0.5 \text{ k}\Omega$$

$$R_{ib} = r_\pi = 0.5 \text{ k}\Omega$$

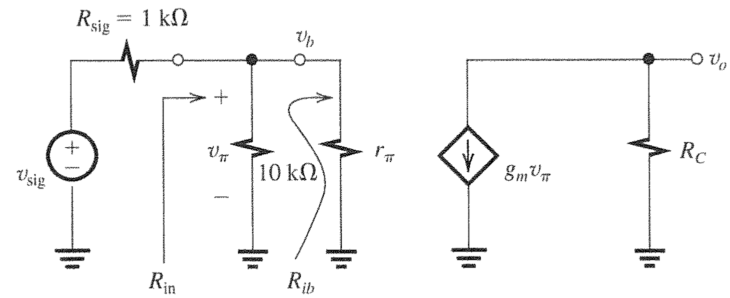
$$R_{\text{in}} = 10 \text{ k}\Omega \parallel 0.5 \text{ k}\Omega = 0.476 \text{ k}\Omega$$

$$\frac{v_\pi}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \frac{0.476}{0.476 + 1} = 0.322 \text{ V/V}$$

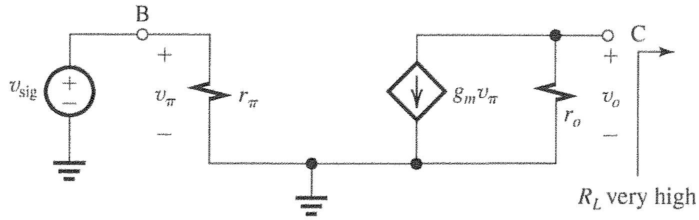
$$\frac{v_o}{v_\pi} = -g_m R_C = -400 \times 0.1 = -40 \text{ V/V}$$

$$\frac{v_o}{v_{\text{sig}}} = -40 \times 0.322 = -12.9 \text{ V/V}$$

This figure belongs to Problem 7.54.



This figure belongs to Problem 7.55.



For

$v_o = \pm 0.4$ V/V

$$v_b = v_\pi = \frac{\pm 0.4}{-40} = \mp 0.01 \text{ V} = \mp 10 \text{ mV}$$

$$v_{sig} = \frac{\pm 0.4}{-12.9} = \mp 31 \text{ mV}$$

7.55 The largest possible voltage gain is obtained when $R_L \to \infty$, in which case

$$\frac{v_o}{v_{sig}} = -g_m r_o = -\frac{I_C}{V_T}\frac{V_A}{I_C}$$

$$= -\frac{V_A}{V_T}$$

For $V_A = 25$ V, $\dfrac{v_o}{v_{sig}} = -\dfrac{25}{0.025}$

$= -1000$ V/V

For $V_A = 125$ V, $\dfrac{v_o}{v_{sig}} = -\dfrac{125}{0.025}$

$= -5000$ V/V

7.56 Refer to Fig. 7.30:

$R_{in} \simeq r_e$

To obtain an input resistance of 75 Ω,

$$r_e = 75\ \Omega = \frac{V_T}{I_E}$$

Thus,

$$I_E = \frac{25 \text{ mV}}{75\ \Omega} = 0.33 \text{ mA}$$

This current is obtained by raising R_E to the value found from

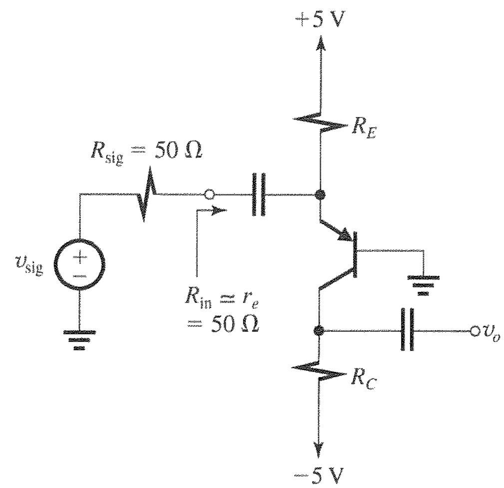
$$I_E = \frac{10 - 0.7}{R_E} = 0.33 \text{ mA}$$

$\Rightarrow R_E = 28.2$ kΩ

Note that the dc voltage at the collector remains unchanged. The voltage gain now becomes

$$\frac{v_o}{v_i} = \frac{\alpha R_C}{r_e} = \frac{0.99 \times 14.1}{0.075} = 186 \text{ V/V}$$

7.57



$$r_e = 50\ \Omega = \frac{V_T}{I_E}$$

$\Rightarrow I_E = 0.5$ mA

Thus,

$$\frac{5 - V_E}{R_E} = 0.5 \text{ mA}$$

where

$V_E \simeq 0.7$ V

$\Rightarrow R_E = 8.6$ kΩ

To obtain maximum gain and the largest possible signal swing at the output for v_{eb} of 10 mV, we select a value for R_C that results in

$$V_C + |A_v| \times 0.01 \text{ V} = +0.4 \text{ V}$$

which is the highest allowable voltage at the collector while the transistor remains in the active region. Since

$$V_C = -5 + I_C R_C \simeq -5 + 0.5 R_C$$

then

$$-5 + 0.5 R_C + g_m R_C \times 0.01 = 0.4$$

Substituting $g_m = 20$ mA/V results in

$R_C = 7.7$ kΩ

The overall voltage gain achieved is

$$\frac{v_o}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} \times g_m R_C$$

$$= \frac{50}{50 + 50} \times 20 \times 7.7$$

$$= 77 \text{ V/V}$$

7.58 Refer to Fig. P7.58. Since β is very large, the dc base current can be neglected. Thus the dc voltage at the base is determined by the voltage divider,

$$V_B = 5\frac{100}{100 + 100} = 2.5 \text{ V}$$

and the dc voltage at the emitter will be

$$V_E = V_B - 0.7 = 1.8 \text{ V}$$

The dc emitter current can now be found as

$$I_E = \frac{V_E}{R_E} = \frac{1.8}{3.6} = 0.5 \text{ mA}$$

and

$$I_C \simeq I_E = 0.5 \text{ mA}$$

Replacing the BJT with the T model of Fig. 7.26(b) results in the following equivalent circuit model for the amplifier.

$$i_e = \frac{v_i}{R_E + r_e}$$

$$v_{o1} = i_e R_E = v_i \frac{R_E}{R_E + r_e}$$

$$\frac{v_{o1}}{v_i} = \frac{R_E}{R_E + r_e} \qquad \text{Q.E.D.}$$

$$v_{o2} = -\alpha i_e R_C = -\alpha \frac{v_i}{R_E + r_e} R_C$$

$$\frac{v_{o2}}{v_i} = -\frac{\alpha R_C}{R_E + r_e} \qquad \text{Q.E.D.}$$

For $\alpha \simeq 1$,

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \ \Omega$$

$$\frac{v_{o1}}{v_i} = \frac{3.6}{3.6 + 0.05} = 0.986 \text{ V/V}$$

$$\frac{v_{o2}}{v_i} = -\frac{3.3}{3.6 + 0.05} = 0.904 \text{ V/V}$$

If v_{o1} is connected to ground, R_E will in effect be short-circuited at signal frequencies, and v_{o2}/v_i will become

$$\frac{v_{o2}}{v_i} = -\frac{\alpha R_C}{r_e} = -\frac{3.3}{0.05} = -66 \text{ V/V}$$

7.59 See figure on next page.

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_{vo} \frac{R_L}{R_L + R_o}$$

$$= \frac{100}{100 + 20} \times 100 \times \frac{2}{2 + 0.1}$$

$$= 79.4 \text{ V/V}$$

$$i_o = \frac{v_o}{R_L}$$

$$i_i = \frac{v_{sig}}{R_{sig} + R_{in}}$$

$$\frac{i_o}{i_i} = \frac{v_o}{v_{sig}} \frac{R_{sig} + R_{in}}{R_L}$$

$$= G_v \frac{R_{sig} + R_{in}}{R_L}$$

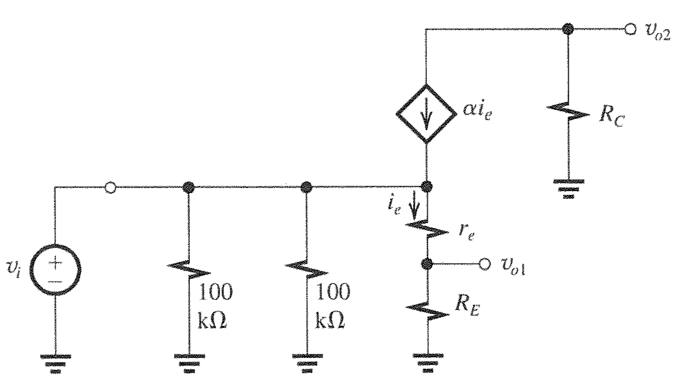
$$= 79.4 \times \frac{20 + 100}{2} = 4762 \text{ A/A}$$

7.60 (a) $\dfrac{R_{in}}{R_{in} + R_{sig}} = 0.95$

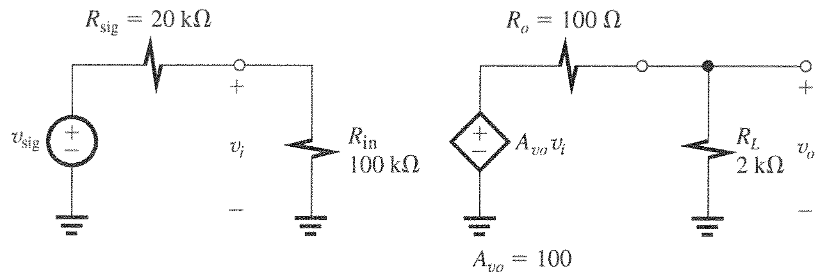
$$\frac{R_{in}}{R_{in} + 100} = 0.95$$

$$\Rightarrow R_{in} = 1.9 \text{ M}\Omega$$

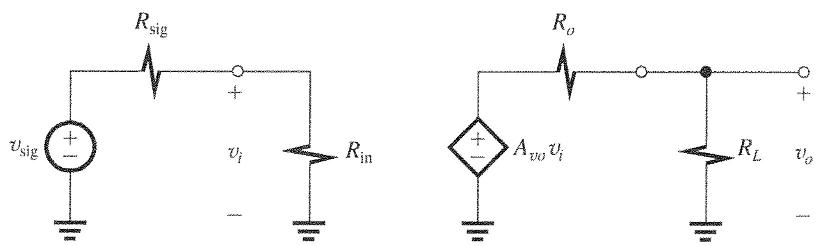
This figure belongs to Problem 7.58.



This figure belongs to Problem 7.59.



This figure belongs to Problem 7.60.



(b) With $R_L = 2\ k\Omega$,

$$v_o = A_{vo} v_i \frac{2}{2 + R_o}$$

With $R_L = 1\ k\Omega$,

$$v_o = A_{vo} v_i \frac{1}{1 + R_o}$$

Thus the change in v_o is

$$\Delta v_o = A_{vo} v_i \left[\frac{2}{2 + R_o} - \frac{1}{1 + R_o} \right]$$

To limit this change to 5% of the value with $R_L = 2\ k\Omega$, we require

$$\left[\frac{2}{2 + R_o} - \frac{1}{1 + R_o} \right] \Big/ \left(\frac{2}{2 + R_o} \right) = 0.05$$

$$\Rightarrow R_o = \frac{1}{9}\ k\Omega = 111\ \Omega$$

(c) $G_v = 10 = \dfrac{R_{in}}{R_{in} + R_{sig}} A_{vo} \dfrac{R_L}{R_L + R_o}$

$$= \frac{1.9}{1.9 + 0.1} \times A_{vo} \times \frac{2}{2 + 0.111}$$

$$\Rightarrow A_{vo} = 11.1\ V/V$$

The values found about are limit values; that is, we require

$R_{in} \geq 1.9\ M\Omega$

$R_o \leq 111\ \Omega$

$A_{vo} \geq 11.1\ V/V$

7.61 The circuit in Fig. 1(b) (see figure on next page) is that in Fig. P7.61, with the output current source expressed as $G_m v_i$. Thus, for equivalence, we write

$$G_m = \frac{A_{vo}}{R_o}$$

To determine G_m (at least conceptually), we short-circuit the output of the equivalent circuit in Fig. 1(b). The short-circuit current will be

$$i_o = G_m v_i$$

Thus G_m is defined as

$$G_m = \left. \frac{i_o}{v_i} \right|_{R_L = 0}$$

and is known as the short-circuit transconductance. From Fig. 2 on next page,

$$\frac{v_i}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}}$$

$$v_o = G_m v_i (R_o \parallel R_L)$$

Thus,

$$\frac{v_o}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} G_m (R_o \parallel R_L)$$

7.62

$$G_{vo} = \left. \frac{v_o}{v_{sig}} \right|_{R_L = \infty}$$

Now, setting $R_L = \infty$ in the equivalent circuit in Fig. 1(b), we can determine G_{vo} from

This figure belongs to Problem 7.61.

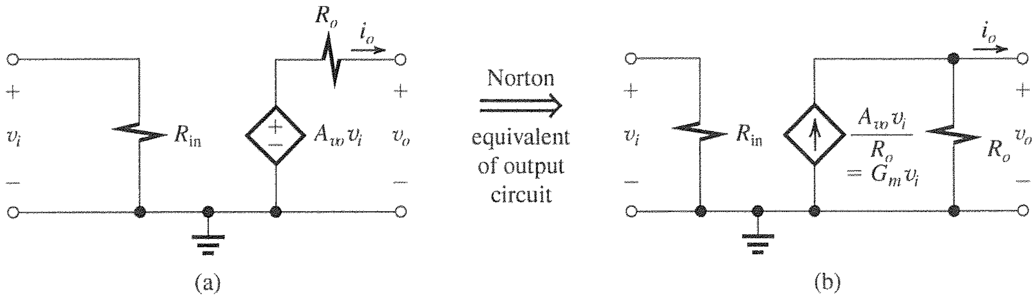


Figure 1

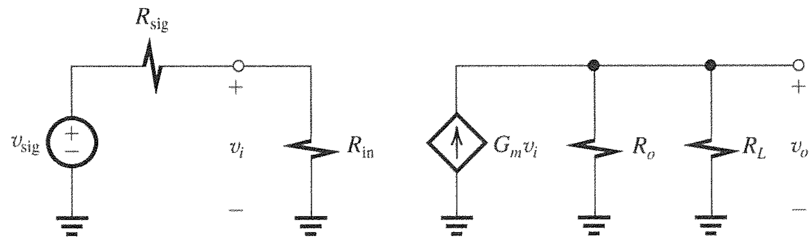


Figure 2

This figure belongs to Problem 7.62.

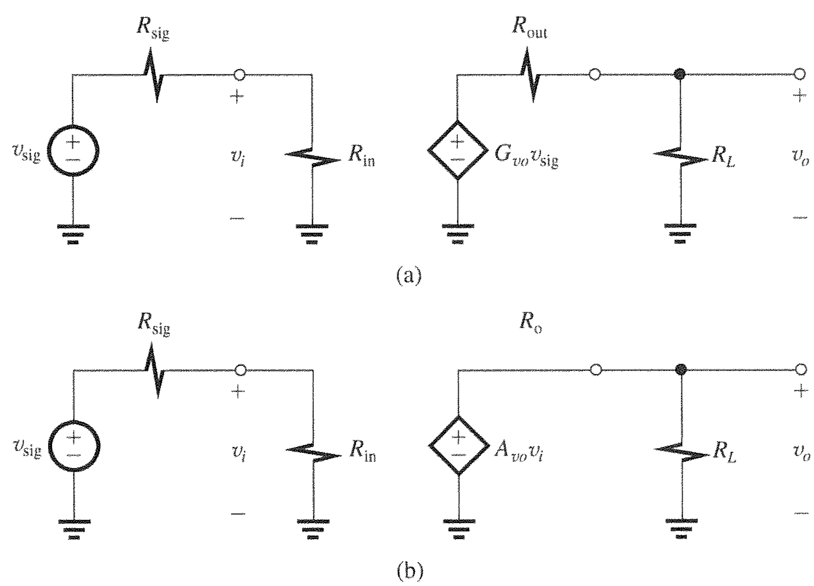


Figure 1

$$G_{vo} = \left.\frac{R_{in}}{R_{in} + R_{sig}}\right|_{R_L=\infty} A_{vo}$$

Denoting R_{in} with $R_L = \infty$ as R_i, we can express G_{vo} as

$$G_{vo} = \frac{R_i}{R_i + R_{sig}} A_{vo} \qquad \text{Q.E.D.}$$

From the equivalent circuit in Fig. 1(a), the overall voltage G_v can be obtained as

$$G_v = G_{vo} \frac{R_L}{R_L + R_{out}} \qquad \text{Q.E.D.}$$

7.63 Refer to Fig. P7.63. To determine R_{in}, we simplify the circuit as shown in Fig. 1, where

$$R_{in} \equiv \frac{v_i}{i_i} = R_1 \parallel R'_{in}, \qquad \text{where } R'_{in} \equiv \frac{v_i}{i_f}$$

$$v_i = i_f R_f + (i_f - g_m v_i)(R_2 \parallel R_L)$$

This figure belongs to Problem 7.63.

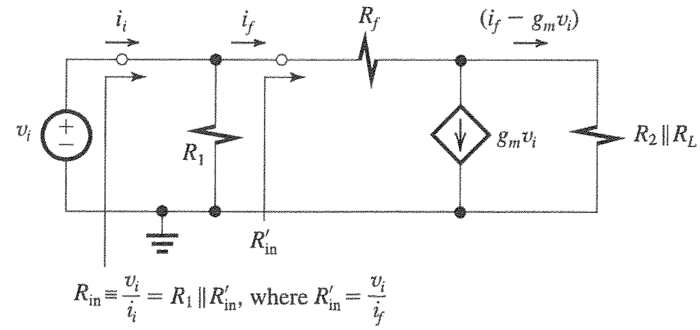


Figure 1

Thus,

$$v_i[1 + g_m(R_2 \| R_L)] = i_f[R_f + (R_2 \| R_L)]$$

$$R'_{in} \equiv \frac{v_i}{i_f} = \frac{R_f + (R_2 \| R_L)}{1 + g_m(R_2 \| R_L)}$$

and

$$R_{in} = R_1 \| R'_{in}$$

$$= R_1 \| \left[\frac{R_f + (R_2 \| R_L)}{1 + g_m(R_2 \| R_L)} \right] \qquad \text{Q.E.D.}$$

To determine A_{vo}, we open-circuit R_L and use the circuit in Fig. 2, where

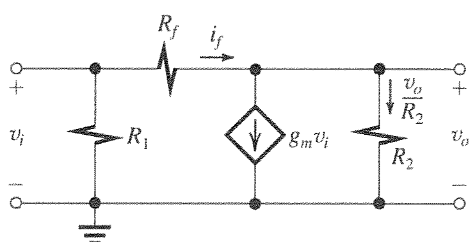


Figure 2

$$i_f = g_m v_i + \frac{v_o}{R_2}$$

$$v_i = i_f R_f + v_o = \left(g_m v_i + \frac{v_o}{R_2} \right) R_f + v_o$$

$$v_i(1 - g_m R_f) = v_o \left(1 + \frac{R_f}{R_2} \right)$$

Thus,

$$A_{vo} \equiv \frac{v_o}{v_i} = \frac{1 - g_m R_f}{1 + \frac{R_f}{R_2}}$$

which can be manipulated to the form

$$A_{vo} = -g_m R_2 \frac{1 - 1/g_m R_f}{1 + (R_2/R_f)} \qquad \text{Q.E.D.}$$

Finally, to obtain R_o we short-circuit v_i in the circuit of Fig. P7.63. This will disable the controlled source $g_m v_i$. Thus, looking between the output terminals (behind R_L), we see R_2 in parallel with R_f,

$$R_o = R_2 \| R_f \qquad \text{Q.E.D.}$$

For $R_1 = 100\ \text{k}\Omega,\ R_f = 1\ \text{M}\Omega,\ g_m = 100\ \text{mA/V}$

$R_2 = 100\ \Omega$ and $R_L = 1\ \text{k}\Omega$

$$R_{in} = 100 \| \frac{1000 + (0.1 \| 1)}{1 + 100(0.1 \| 1)} = 100 \| 99.1$$

$$= 49.8\ \text{k}\Omega$$

Without R_f present (i.e., $R_f = \infty$), $R_{in} = 100\ \text{k}\Omega$ and

$$A_{vo} = -100 \times 0.1 \frac{1 - (1/100 \times 1000)}{1 + \frac{0.1}{1000}}$$

$$\simeq -10\ \text{V/V}$$

Without R_f, $-A_{vo} = 10\ \text{V/V}$ and

$$R_o = 0.1 \| 1000 \simeq 0.1\ \text{k}\Omega = 100\ \Omega$$

Without R_f, $R_o = 100\ \Omega$.

Thus the only parameter that is significantly affected by the presence of R_f is R_{in}, which is reduced by a factor of 2!

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_{vo} \frac{R_L}{R_L + R_o}$$

With R_f,

$$G_v = \frac{49.8}{49.8 + 100} \times -10 \times \frac{1}{1 + 0.1}$$

$$= -3\ \text{V/V}$$

Without R_f,

$$G_v = \frac{100}{100 + 100} \times -10 \times \frac{1}{1 + 0.1} = -4.5\ \text{V/V}$$

7.64 $R_{sig} = 1\ \text{M}\Omega,\ R_L = 10\ \text{k}\Omega$

$g_m = 2\ \text{mA/V},\ R_D = 10\ \text{k}\Omega$

$G_v = -g_m(R_D \parallel R_L)$

$= -2(10 \parallel 10) = -10$ V/V

7.65 $R_{in} = \infty$

$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{OV}^2$

$320 = \frac{1}{2} \times 400 \times 10 \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.4$ V

$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.32}{0.4} = 1.6$ mA/V

$A_{vo} = -g_m R_D = -1.6 \times 10 = -16$ V/V

$R_o = R_D = 10$ kΩ

$G_v = A_{vo}\frac{R_L}{R_L + R_o}$

$= -16 \times \frac{10}{10 + 10} = -8$ V/V

Peak value of $v_{sig} = \frac{0.2 \text{ V}}{8} = 25$ mV.

7.66 $R_D = 2R_L = 30$ kΩ

$V_{OV} = 0.25$ V

$G_v = -g_m(R_D \parallel R_L)$

$-10 = -g_m(30 \parallel 15)$

$\Rightarrow g_m = 1$ mA/V

$g_m = \frac{2I_D}{V_{OV}}$

$1 = \frac{2 \times I_D}{0.25}$

$\Rightarrow I_D = 0.125$ mA $= 125$ μA

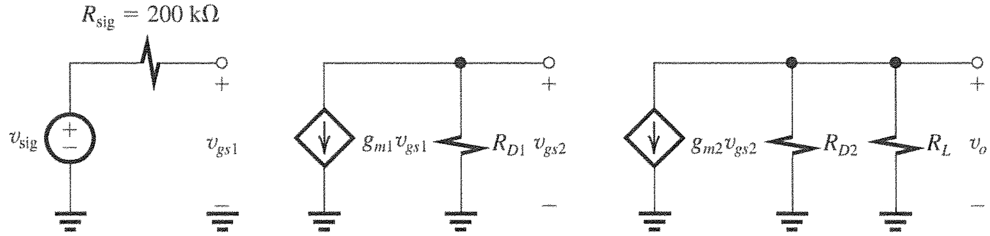
If R_D is reduced to 15 kΩ,

$G_v = -g_m(R_D \parallel R_L)$

$= -1 \times (15 \parallel 15) = -7.5$ V/V

7.67 (a) See figure below.

This figure belongs to Problem 7.67.



(b) $g_{m1} = g_{m2} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.3}{0.2} = 3$ mA/V

$R_{D1} = R_{D2} = 10$ kΩ

$R_L = 10$ kΩ

$G_v = \frac{v_{gs2}}{v_{gs1}} \times \frac{v_o}{v_{gs2}}$

$= -g_{m1}R_{D1} \times -g_{m2}(R_{D2} \parallel R_L)$

$= 3 \times 10 \times 3 \times (10 \parallel 10)$

$= 450$ V/V

7.68 $g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20$ mA/V

$r_\pi = \frac{\beta}{g_m} = \frac{100}{20 \text{ mA/V}} = 5$ kΩ

$R_{in} = r_\pi = 5$ kΩ

$R_o = R_C = 10$ kΩ

$A_{vo} = -g_m R_C = -20 \times 10 = -200$ V/V

$A_v = A_{vo}\frac{R_L}{R_L + R_o} = -200 \times \frac{10}{10 + 10}$

$= -100$ V/V

$G_v = \frac{R_{in}}{R_{in} + R_{sig}}A_v$

$= \frac{5}{5 + 10} \times -100$

$= -33.3$ V/V

For $\hat{v}_\pi = 5$ mV, $\hat{v}_{sig}$ can be found from

$\hat{v}_\pi = \hat{v}_{sig} \times \frac{R_{in}}{R_{in} + R_{sig}} = \hat{v}_{sig} \times \frac{5}{5 + 10}$

$\Rightarrow \hat{v}_{sig} = 15$ mV

Correspondingly, $\hat{v}_o$ will be

$\hat{v}_o = G_v \hat{v}_{sig}$

$= 15 \times 33.3 = 500$ mV $= 0.5$ V

7.69 $|G_v| = \dfrac{R'_L}{(R_{sig}/\beta) + (1/g_m)}$

$R'_L = 10\ \text{k}\Omega,\ R_{sig} = 10\ \text{k}\Omega,\ g_m = \dfrac{I_C}{V_T}$

$= \dfrac{1}{0.025} = 40\ \text{mA/V}$

Nominal $\beta = 100$

(a) Nominal $|G_v| = \dfrac{10}{(10/100) + 0.025}$

$= 80\ \text{V/V}$

(b) $\beta = 50,\ |G_v| = \dfrac{10}{(10/50) + 0.025}$

$= 44.4\ \text{V/V}$

$\beta = 150,\ |G_v| = \dfrac{10}{(10/150) + 0.025}$

$= 109.1\ \text{V/V}$

Thus, $|G_v|$ ranges from 44.4 V/V to 109.1 V/V.

(c) For $|G_v|$ to be within ±20% of nominal (i.e., ranging between 64 V/V and 96 V/V), the corresponding allowable range of β can be found as follows:

$64 = \dfrac{10}{(10/\beta_{min}) + 0.025}$

$\Rightarrow \beta_{min} = 76.2$

$96 = \dfrac{10}{(10/\beta_{max}) + 0.025}$

$\Rightarrow \beta_{max} = 126.3$

(d) By varying I_C, we vary the term $1/g_m$ in the denominator of the $|G_v|$ expression. If β varies in the range 50 to 150 and we wish to keep $|G_v|$ within ±20% of a new nominal value of $|G_v|$ given by

$|G_v|_{nominal} = \dfrac{10}{(10/100) + (1/g_m)}$

then

$0.8\,|G_v|_{nominal} = \dfrac{10}{(10/50) + (1/g_m)}$

That is,

$\dfrac{8}{0.1 + (1/g_m)} = \dfrac{10}{0.2 + (1/g_m)}$

$\Rightarrow \dfrac{1}{g_m} = 0.3$ or $g_m = 3.33\ \text{mA/V}$

$|G_v|_{nominal} = \dfrac{10}{0.1 + 0.3} = 25\ \text{V/V}$

$|G_v|_{min} = \dfrac{10}{0.2 + 0.3}$

$= 20\ \text{V/V}$ (−20% of nominal)

We need to check the value obtained for $\beta = 150$,

$|G_v|_{max} = \dfrac{10}{10/150 + 0.3} = 27.3\ \text{V/V}$

which is less than the allowable value of $1.2\,|G_v|_{nominal} = 30\ \text{V/V}$. Thus, the new bias current is

$I_C = g_m \times V_T = 3.33 \times 0.025 = 0.083\ \text{mA}$

$|G_v|_{nominal} = 25\ \text{V/V}$

7.70 (a) See figure below.

(b) $R_{C1} = R_{C2} = 10\ \text{k}\Omega \quad R_{sig} = 10\ \text{k}\Omega$

$R_L = 10\ \text{k}\Omega$

$g_{m1} = g_{m2} = \dfrac{I_C}{V_T} = \dfrac{0.25\ \text{mA}}{0.025\ \text{V}} = 10\ \text{mA/V}$

$r_{\pi1} = r_{\pi2} = \dfrac{\beta}{g_m} = \dfrac{100}{10} = 10\ \text{k}\Omega$

$\dfrac{v_{\pi1}}{v_{sig}} = \dfrac{r_{\pi1}}{r_{\pi1} + R_{sig}} = \dfrac{10}{10 + 10} = 0.5\ \text{V/V}$

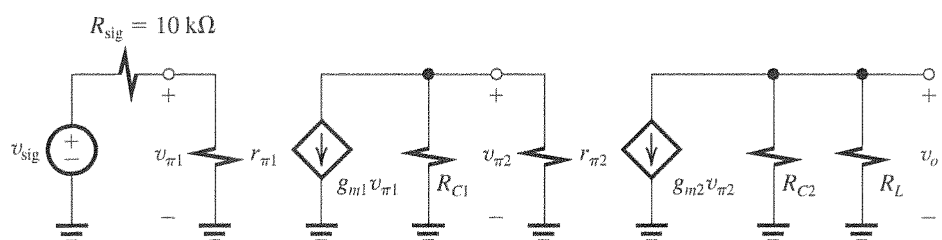
$\dfrac{v_{\pi2}}{v_{\pi1}} = -g_{m1}(R_{C1} \,\|\, r_{\pi2}) = -10(10 \,\|\, 10)$

$= -50\ \text{V/V}$

$\dfrac{v_o}{v_{\pi2}} = -g_{m2}(R_{C2} \,\|\, R_L)$

$= -10(10 \,\|\, 10) = -50\ \text{V/V}$

This figure belongs to Problem 7.70.



$$\frac{v_o}{v_{sig}} = \frac{v_o}{v_{\pi 2}} \times \frac{v_{\pi 2}}{v_{\pi 1}} \times \frac{v_{\pi 1}}{v_{sig}}$$

$$= -50 \times -50 \times 0.5$$

$$= 1250 \text{ V/V}$$

7.71 $g_m\big|_{\text{effective}} = \dfrac{g_m}{1 + g_m R_s}$

$$2 = \frac{5}{1 + 5R_s}$$

$$\Rightarrow R_s = 0.3 \text{ k}\Omega = 300 \ \Omega$$

7.72 The gain magnitude is reduced by a factor of $(1 + g_m R_s)$. Thus, to reduce the gain from -10 V/V to -5 V/V, we write

$$2 = 1 + g_m R_s$$

$$\Rightarrow R_s = \frac{1}{g_m} = \frac{1}{2} = 0.5 \text{ k}\Omega$$

7.73 Including R_s reduced the gain by a factor of 2, thus

$$1 + g_m R_s = 2$$

$$\Rightarrow g_m = \frac{1}{R_s} = \frac{1}{0.5} = 2 \text{ mA/V}$$

The gain without R_s is -20 V/V. To obtain a gain of -16 V/V, we write

$$16 = \frac{20}{1 + g_m R_s} = \frac{20}{1 + 2R_s}$$

$$\Rightarrow R_s = 125 \ \Omega$$

7.74 $g_m = \dfrac{I_C}{V_T} = \dfrac{0.5}{0.025} = 20 \text{ mA/V}$

$$r_e \simeq \frac{1}{g_m} = 50 \ \Omega$$

$$R_{\text{in}} = (\beta + 1)(r_e + R_e)$$

$$= 101(50 + 250) = 30.3 \text{ k}\Omega$$

$$A_{vo} = -\frac{\alpha R_C}{r_e + R_e} = -\frac{0.99 \times 12}{0.3} \simeq -40 \text{ V/V}$$

$$R_o = R_C = 12 \text{ k}\Omega$$

$$A_v = A_{vo}\frac{R_L}{R_L + R_o}$$

$$= -40 \times \frac{12}{12 + 12} = -20 \text{ V/V}$$

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \times A_v$$

$$= \frac{30.3}{30.3 + 10} \times -20 = -15 \text{ V/V}$$

$$\hat{v}_\pi = 5 \text{ mV} \Rightarrow \hat{v}_{\text{sig}} = \hat{v}_\pi\left(\frac{R_{\text{in}} + R_{\text{sig}}}{R_{\text{in}}}\right)$$

$$\hat{v}_{\text{sig}} = 5 \times \frac{30.3 + 10}{30.3} = 6.65 \text{ mV}$$

$$\hat{v}_o = \hat{v}_{\text{sig}} \times |G_v|$$

$$= 6.65 \times 15 \simeq 100 \text{ mV}$$

7.75 $R_{\text{in}} = (\beta + 1)(r_e + R_e)$

$$15 = 75(r_e + R_e)$$

$$r_e + R_e = \frac{15 \text{ k}\Omega}{75} = 200 \ \Omega$$

$$\hat{v}_\pi = \hat{v}_{\text{sig}}\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}}\frac{r_e}{r_e + R_e}$$

$$5 = 150 \times \frac{15}{15 + 30}\left(\frac{r_e}{r_e + R_e}\right)$$

$$\Rightarrow \frac{r_e}{r_e + R_e} = 0.1$$

But $r_e + R_e = 200 \ \Omega$, thus

$$r_e = 20 \ \Omega$$

which requires a bias current I_E of

$$I_E = \frac{V_T}{r_e} = \frac{25 \text{ mV}}{20 \ \Omega} = 1.25 \text{ mA}$$

$$I_C \simeq I_E = 1.25 \text{ mA}$$

$$R_e = 180 \ \Omega$$

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}}$$

$$\times \frac{-\alpha \times \text{Total resistance in collector}}{\text{Total resistance in emitter}}$$

$$= \frac{15}{15 + 30} \times \frac{-0.99 \times 6}{0.2}$$

$$\simeq -10 \text{ V/V}$$

$$\hat{v}_0 = 0.15 \times |G_v| = 1.5 \text{ V}$$

7.76 Using Eq. (7.113), we have

$$G_v = -\beta\frac{R_C \parallel R_L}{R_{\text{sig}} + (\beta + 1)(r_e + R_e)}$$

$$\simeq -\frac{R_C \parallel R_L}{(R_{\text{sig}}/\beta) + (r_e + R_e)}$$

$$|G_v| = \frac{10}{(10/\beta) + 0.025 + R_e}$$

Without R_e,

$$|G_v| = \frac{10}{(10/\beta) + 0.025}$$

For the nominal case, $\beta = 100$,

$$|G_v|_{\text{nominal}} = \frac{10}{0.1 + 0.025} = 80 \text{ V/V}$$

For $\beta = 50$,

$$|G_v|_{\text{low}} = \frac{10}{0.2 + 0.025} = 44.4\text{ V/V}$$

For $\beta = 150$,

$$|G_v|_{\text{high}} = \frac{10}{(1/15) + 0.025} = 109.1\text{ V/V}$$

Thus, $|G_v|$ ranges from 44.4 V/V to 109.1 V/V with a nominal value of 80 V/V. This is a range of -44.5% to $+36.4\%$ of nominal.

To limit the range of $|G_v|$ to $\pm 20\%$ of a new nominal value, we connect a resistance R_e and find its value as follows. With R_e,

$$|G_v|_{\text{nominal}} = \frac{10}{(10/100) + 0.025 + R_e}$$

$$= \frac{10}{0.125 + R_e}$$

Now, $\beta = 50$,

$$|G_v|_{\text{low}} = \frac{10}{0.225 + R_e}$$

To limit this value to -20% of $|G_v|_{\text{nominal}}$, we use

$$\frac{10}{0.225 + R_e} = 0.8 \times \frac{10}{0.125 + R_e}$$

$$\Rightarrow R_e = 0.275\text{ k}\Omega = 275\ \Omega$$

With this value of R_e,

$$|G_v|_{\text{nominal}} = \frac{10}{0.125 + 0.275} = 25\text{ V/V}$$

$$|G_v|_{\text{low}} = \frac{10}{0.225 + 0.275}$$

$$= 20\text{ V/V } (-20\%\text{ of nominal})$$

$$|G_v|_{\text{high}} = \frac{10}{(1/15) + 0.025 + 0.275}$$

$$= 27.3\text{ V/V } (+9.1\%\text{ of nominal})$$

7.77 $R_{\text{in}} = \dfrac{1}{g_m} = \dfrac{1}{2\text{ mA/V}} = 0.5\text{ k}\Omega$

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \times g_m(R_D \parallel R_L)$$

$$= \frac{0.5}{0.5 + 0.75} \times 2(5 \parallel 5)$$

$$= 2\text{ V/V}$$

For $R_{\text{in}} = R_{\text{sig}} = 0.75\text{ k}\Omega$

$$\frac{1}{g_m} = 0.75 \Rightarrow g_m = 1.33\text{ mA/V}$$

Since $g_m = \sqrt{2k_n I_D}$, then to change g_m by a factor $\dfrac{1.33}{2} = 0.67$, I_D must be changed by a factor of $(0.67)^2 = 0.45$.

7.78 Adding a resistance of 100 Ω in series with the 100-Ω R_{sig} changes the input voltage divider ratio from

$$\frac{1/g_m}{(1/g_m) + 100} \text{ to } \frac{1/g_m}{1/g_m + 200}$$

Since this has changed the overall voltage gain from 12 to 10, then

$$\frac{12}{10} = \frac{(1/g_m) + 200}{(1/g_m) + 100}, \text{ where } g_m \text{ is in A/V}$$

$$\Rightarrow g_m = \frac{0.2}{80}\text{ A/V} = 2.5\text{ mA/V}$$

For $I_D = 0.25$ mA

$$2.5 = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.25}{V_{OV}}$$

$$\Rightarrow V_{OV} = 0.2\text{ V}$$

7.79 For $R_{\text{in}} = R_{\text{sig}} = 50\ \Omega$,

$r_e = 50\ \Omega$

and, with $\alpha \simeq 1$,

$$I_C \simeq \frac{V_T}{r_e} = \frac{25\text{ mV}}{50\ \Omega} = 0.5\text{ mA}$$

$$g_m = I_C/V_T = 20\text{ mA/V}$$

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m(R_C \parallel R_L)$$

$$G_v = \frac{50}{50 + 50} \times 20 \times (10 \parallel 10)$$

$$= 50\text{ V/V}$$

7.80 Refer to the circuit in Fig. P7.80. Since $R_{\text{sig}} \gg r_e$, most of i_{sig} flows into the emitter of the BJT. Thus

$$i_e \simeq i_{\text{sig}}$$

and

$$i_c = \alpha i_e \simeq i_{\text{sig}}$$

Thus,

$$v_o = i_c R_C = i_{\text{sig}} R_C$$

7.81 $R_{\text{in}} = r_e = \dfrac{V_T}{I_E} = \dfrac{25\text{ mV}}{0.2\text{ mA}} = 125\ \Omega$

$$g_m = \frac{I_C}{V_T} \simeq \frac{0.2\text{ mA}}{0.025\text{ V}} = 8\text{ mA/V}$$

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m(R_C \parallel R_L)$$

$$= \frac{0.125}{0.125 + 0.5} \times 8(10 \parallel 10) = 8 \text{ V/V}$$

$$\hat{v}_\pi = \hat{v}_{sig} \frac{R_{in}}{R_{in} + R_{sig}}$$

$$10 = \hat{v}_{sig} \frac{0.125}{0.125 + 0.5}$$

$$\Rightarrow \hat{v}_{sig} = 50 \text{ mV}$$

$$\hat{v}_o = G_v \hat{v}_{sig} = 8 \times 50 = 400 \text{ mV} = 0.4 \text{ V}$$

7.82 $A_v = \dfrac{R_L}{R_L + R_o}$

$$A_v|_{nominal} = \frac{2}{2 + R_o}$$

$$A_v|_{low} = \frac{1.5}{1.5 + R_o}$$

$$A_v|_{high} = \frac{5}{5 + R_o}$$

For $A_v|_{high} = 1.1\, A_v|_{nominal}$

$$\frac{5}{5 + R_o} = \frac{1.1 \times 2}{2 + R_o}$$

$$\Rightarrow R_o = 0.357 \text{ k}\Omega$$

$$A_v|_{nominal} = \frac{2}{2.357} = 0.85 \text{ V/V}$$

$$A_v|_{high} = \frac{5}{5.357}$$

$= 0.93$

(+10% above nominal)

$$A_v|_{low} = \frac{1.5}{1.5 + 0.357}$$

$= 0.81$ (−5% from nominal)

$$\frac{1}{g_m} = R_o = 0.357 \text{ k}\Omega$$

$$\Rightarrow g_m = 2.8 \text{ mA/V}$$

To find I_D, we use

$$g_m = \sqrt{2k_n I_D}$$

$$\Rightarrow I_D = g_m^2 / 2k_n$$

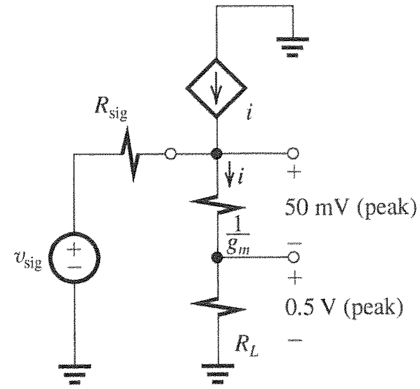
$$= \frac{2.8^2}{2 \times 2.5} = 1.6 \text{ mA}$$

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$1.6 = \frac{1}{2} \times 2.5 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 1.13 \text{ V}$$

7.83



From the figure above, we have

$$\frac{1}{g_m} = 0.1 \times R_L$$

$$= 0.1 \times 2 = 0.2 \text{ k}\Omega$$

$$g_m = 5 \text{ mA/V}$$

$$g_m = \sqrt{2k_n I_D}$$

$$5 = \sqrt{2 \times 5 \times I_D}$$

$$I_D = 2.5 \text{ mA}$$

At the peak of the sine wave,

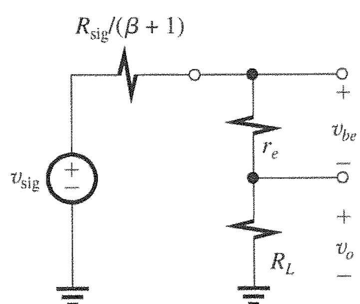
$$i_d = \frac{0.5 \text{ V}}{2 \text{ k}\Omega} = 0.25 \text{ mA, thus}$$

$$i_{Dmax} = I_D + 0.25 = 2.75 \text{ mA}$$

$$i_{Dmin} = I_D - 0.25 = 2.25 \text{ mA}$$

$$\hat{v}_{sig} = \hat{v}_{gs} + \hat{v}_o = 0.05 + 0.5 = 0.55 \text{ V}$$

7.84



$$\hat{v}_o = 0.5 \text{ V}$$

$$R_L = 2 \text{ k}\Omega$$

$$\hat{v}_{be} = 5 \text{ mV}$$

From the figure above we see that

$$\frac{r_e}{R_L} = \frac{5\text{ mV}}{500\text{ mV}}$$

$$\Rightarrow r_e = \frac{R_L}{100} = 20\ \Omega$$

$$I_E = \frac{V_T}{r_e} = \frac{25\text{ mV}}{20\ \Omega} = 1.25\text{ mA}$$

At the peak of the output sine wave, we have

$$\hat{i}_e = \frac{\hat{v}_o}{R_L} = \frac{0.5}{2} = 0.25\text{ mA}$$

Thus,

$$i_{E\max} = 1.25 + 0.25 = 1.5\text{ mA}$$

and

$$i_{E\min} = 1.25 - 0.25 = 1.0\text{ mA}$$

From the figure, we have

$$G_v = \frac{v_o}{v_{sig}} = \frac{R_L}{R_L + r_e + \dfrac{R_{sig}}{\beta+1}}$$

$$= \frac{2}{2 + 0.02 + \dfrac{200}{101}} = 0.5\text{ V/V}$$

Thus,

$$\hat{v}_{sig} = \frac{\hat{v}_o}{G_v} = \frac{0.5\text{ V}}{0.5\text{ V/V}} = 1\text{ V}$$

7.85 $I_C = 2$ mA

$$r_e = \frac{V_T}{I_E} \simeq \frac{V_T}{I_C} = \frac{25}{2} = 12.5\ \Omega$$

(a) $R_{in} = (\beta + 1)\ (r_e + R_L)$

$$= 101 \times (12.5 + 500) = 51.76\text{ k}\Omega$$

$$\frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{51.76}{51.76 + 10}$$

$$= 0.84\text{ V/V}$$

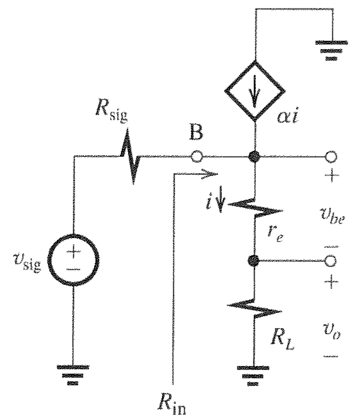
$$\frac{v_o}{v_{sig}} = \frac{v_b}{v_{sig}} \times \frac{v_o}{v_b}$$

$$= 0.84 \times \frac{R_L}{R_L + r_e}$$

$$= 0.84 \times \frac{0.5}{0.5 + 0.0125}$$

$$= 0.82\text{ V/V}$$

(b)



$$\hat{v}_{be} = 10\text{ mV}$$

$$\hat{v}_o = \frac{R_L}{r_e} \times \hat{v}_{be}$$

$$= \frac{500}{12.5} \times 10$$

$$= 400\text{ mV} = 0.4\text{ V}$$

$$\hat{v}_{sig} = \frac{\hat{v}_o}{G_v} = \frac{0.4}{0.82} = 0.488\text{ V}$$

(c) $G_{vo} = 1$

$$R_{out} = r_e + \frac{R_{sig}}{\beta+1} = 12.5 + \frac{10{,}000}{101}$$

$$= 111.5\ \Omega$$

Thus,

$$G_v = G_{vo}\frac{R_L}{R_L + R_{out}}$$

$$= 1 \times \frac{500}{500 + 111.5} = 0.82\text{ V/V}$$

which is the same value obtained in (a) above.

For $R_L = 250\ \Omega$,

$$G_v = G_{vo}\frac{R_L}{R_L + R_{out}}$$

$$= 1 \times \frac{250}{250 + 111.5} = 0.69\text{ V/V}$$

7.86 $R_{out} = r_e + \dfrac{R_{sig}}{\beta+1}$

$$r_e = \frac{V_T}{I_E} \simeq \frac{V_T}{I_C} = \frac{25\text{ mV}}{0.5\text{ mA}} = 50\ \Omega$$

$$R_{out} = 50 + \frac{10{,}000}{101} = 50 + 99 = 149\ \Omega$$

$$G_v = \frac{R_L}{R_L + r_e + \dfrac{R_{sig}}{\beta + 1}} = \frac{R_L}{R_L + R_{out}}$$

$$= \frac{1000}{1000 + 149} = 0.87 \text{ V}$$

If β varies between 50 and 150, then we have

$$R_{outmax} = 50 + \frac{10{,}000}{51} = 50 + 196$$

$$= 246\ \Omega$$

$$R_{outmin} = 50 + \frac{10{,}000}{151} = 50 + 66.2$$

$$= 116\ \Omega$$

$$G_{v_{min}} = \frac{R_L}{R_L + R_{outmax}} = \frac{1000}{1000 + 246}$$

$$= 0.80 \text{ V/V}$$

$$G_{v_{max}} = \frac{R_L}{R_L + R_{outmin}} = \frac{1000}{1000 + 116}$$

$$= 0.90 \text{ V/V}$$

7.87 $R_{out} = r_e + \dfrac{R_{sig}}{\beta + 1}$

$$150 = r_e + \frac{5000}{\beta + 1} \qquad (1)$$

$$250 = r_e + \frac{10{,}000}{\beta + 1} \qquad (2)$$

Subtracting Eq. (1) from Eq. (2), we have

$$100 = \frac{5000}{\beta + 1}$$

$$\beta + 1 = 50$$

Substituting in Eq. (1) yields

$$150 = r_e + \frac{5000}{50}$$

$$\Rightarrow r_e = 50\ \Omega$$

$$G_v = \frac{R_L}{R_L + r_e + \dfrac{R_{sig}}{\beta + 1}}$$

$$= \frac{1000}{1000 + 50 + \dfrac{10{,}000}{50}} = 0.8 \text{ V/V}$$

7.88 (a) Refer to Fig. P7.88.

$$\frac{v_c}{v_{sig}} = \frac{-i_c R_C}{i_b R_B + i_e(r_e + R_E)}$$

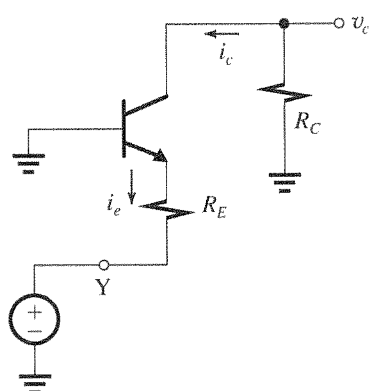
$$= -\frac{i_c}{i_b} \frac{R_C}{R_B + \left(\dfrac{i_e}{i_b}\right)(r_e + R_E)}$$

$$= -\beta \frac{R_C}{R_B + (\beta + 1)(r_e + R_E)}$$

$$\frac{v_e}{v_{sig}} = \frac{-i_e R_E}{i_b R_B + i_e(r_e + R_E)}$$

$$= \frac{R_E}{\dfrac{R_B}{\beta + 1} + r_e + R_E}$$

(b)



$$i_e = -\frac{v_{sig}}{r_e + R_E}$$

$$i_c = -i_c R_C = -\alpha i_e R_C$$

$$\frac{v_c}{v_{sig}} = \frac{-i_c R_C}{i_e(r_e + R_E)} = \alpha \frac{R_C}{r_e + R_E}$$

7.89 With the Early effect neglected, we can write

$$G_v = -100 \text{ V/V}$$

With the Early effect taken into account, the effective resistance in the collector is reduced from $R_C = 10\ \text{k}\Omega$ to $(R_C \parallel r_o)$, where

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{1 \text{ mA}} = 100\ \text{k}\Omega$$

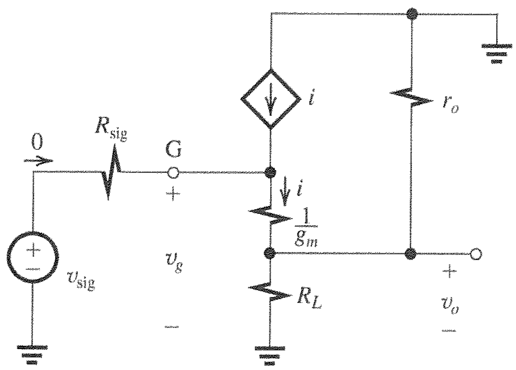
$$(R_C \parallel r_o) = 10 \parallel 100 = 9.1\ \text{k}\Omega$$

Thus, G_v becomes

$$G_v = -100 \times \frac{9.1\ \text{k}\Omega}{10\ \text{k}\Omega}$$

$$= -91 \text{ V/V}$$

7.90



$v_g = v_{sig}$

Noting that r_o appears in effect in parallel with R_L, v_o is obtained as the ratio of the voltage divider formed by $(1/g_m)$ and $(R_L \parallel r_o)$,

$$G_v = \frac{v_o}{v_{sig}} = \frac{v_o}{v_g} = \frac{(R_L \parallel r_o)}{(R_L \parallel r_o) + \dfrac{1}{g_m}} \qquad \text{Q.E.D.}$$

With R_L removed,

$$G_v = \frac{r_o}{r_o + \dfrac{1}{g_m}} = 0.98 \qquad (1)$$

With $R_L = 500\ \Omega$,

$$G_v = \frac{(500 \parallel r_o)}{(500 \parallel r_o) + \dfrac{1}{g_m}} = 0.49 \qquad (2)$$

From Eq. (1), we have

$$\frac{1}{g_m} = \frac{r_o}{49}$$

Substituting in Eq. (2) and solving for r_o gives

$r_o = 25{,}000\ \Omega = 25\ \text{k}\Omega$

Thus

$$\frac{1}{g_m} = \frac{25{,}000}{49}\ \Omega$$

$\Rightarrow g_m = 1.96\ \text{mA/V}$

7.91 Adapting Eq. (7.114) gives

$$G_v = -\beta \frac{R_C \parallel R_L \parallel r_o}{R_{sig} + (\beta + 1) r_e}$$

$$= -\frac{R_C \parallel R_L \parallel r_o}{\dfrac{R_{sig}}{\beta} + \dfrac{\beta + 1}{\beta} r_e}$$

$$= -\frac{R_C \parallel R_L \parallel r_o}{\dfrac{R_{sig}}{\beta} + \dfrac{1}{g_m}}$$

Thus,

$$|G_v| = \frac{10 \parallel r_o}{0.1 + \dfrac{1}{g_m}} \qquad (1)$$

where r_o and $\dfrac{1}{g_m}$ are in kilohms and are given by

$$r_o = \frac{V_A}{I_C} = \frac{25\ \text{V}}{I_C\ \text{mA}} \qquad (2)$$

$$\frac{1}{g_m} = \frac{V_T}{I_C} = \frac{0.025\ \text{V}}{I_C\ \text{mA}} \qquad (3)$$

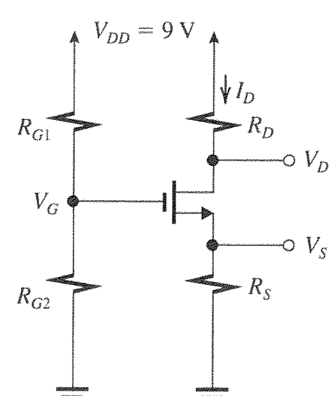
I_C (mA)	$1/g_m$ (kΩ)	r_o (kΩ)	$\lvert G_v \rvert$ (V/V)
0.1	0.250	250	27.5
0.2	0.125	125	41.2
0.5	0.050	50	55.6
1.0	0.025	25	57.1
1.25	0.020	20	55.6

Observe that initially $|G_v|$ increases as I_C is increased. However, above about 1 mA this trend reverses because of the effect of r_o. From the table we see that gain of 50 is obtained for I_C between 0.2 and 0.5 mA and also for I_C above 1.25 mA. Practically speaking, one normally uses the low value to minimize power dissipation. The required value of I_C is found by substituting for r_o and $1/g_m$ from Eqs. (2) and (3), respectively, in Eq. (1) and equating G_v to 50. The result (after some manipulations) is the quadratic equation.

$$I_C^2 - 2.25 I_C + 0.625 = 0$$

The two roots of this equation are $I_C = 0.325$ mA and 1.925 mA; our preferred choice is $I_C = 0.325$ mA.

7.92



$I_D = 1$ mA

$I_D = \frac{1}{2}k_n V_{OV}^2$

$1 = \frac{1}{2} \times 2 \times V_{OV}^2$

$\Rightarrow V_{OV} = 1$ V

$V_{GS} = V_t + V_{OV} = 1 + 1 = 2$ V

Now, selecting $V_S = \frac{V_{DD}}{3} = 3$ V

$I_D R_S = 3$

$R_S = \frac{3}{1} = 3$ kΩ

Also,

$I_D R_D = \frac{V_{DD}}{3} = 3$ V

$\Rightarrow R_D = \frac{3}{1} = 3$ kΩ

$V_G = V_S + V_{GS}$

$= 3 + 2 = 5$ V

Thus the voltage drop across R_{G2}(5 V) is larger than that across R_{G1}(4 V). So we select

$R_{G2} = 22$ MΩ

and determine R_{G1} from

$$\frac{R_{G1}}{R_{G2}} = \frac{4 \text{ V}}{5 \text{ V}}$$

$\Rightarrow R_{G1} = 0.8 R_{G2} = 0.8 \times 22$

$= 17.6$ MΩ

Using only two significant figures, we have

$R_{G1} = 18$ MΩ

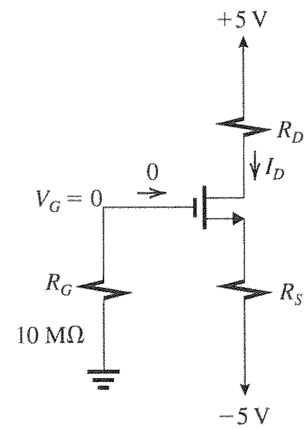
Note that this will cause V_G to deviate slightly from the required value of 5 V. Specifically,

$$V_G = V_{DD}\frac{R_{G2}}{R_{G2} + R_{G1}}$$

$$= 9 \times \frac{22}{22 + 18} = 4.95 \text{ V}$$

It can be shown (after simple but somewhat tedious analysis) that the resulting I_D will be $I_D = 0.986$ mA, which is sufficiently close to the desired 1 mA. Since $V_D = V_{DD} - I_D R_D \simeq +6$ V and $V_G \simeq 5$ V, and the drain voltage can go down to $V_G - V_t = 4$ V, the drain voltage is 2 V above the value that causes the MOSFET to leave the saturation region.

7.93



For $I_D = 0.5$ mA

$0.5 = \frac{1}{2}k_n V_{OV}^2$

$= \frac{1}{2} \times 1 \times V_{OV}^2$

$\Rightarrow V_{OV} = 1$ V

$V_{GS} = V_t + V_{OV} = 1 + 1 = 2$ V

Since

$V_G = 0$ V, $\quad V_S = -V_{GS} = -2$ V

which leads to

$$R_S = \frac{V_S - (-5)}{I_C} = \frac{-2 + 5}{0.5} = 6 \text{ k}\Omega$$

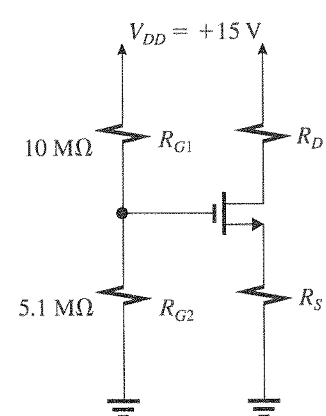
V_D is required to be halfway between cutoff (+5 V) and saturation ($0 - V_t = -1$ V). Thus

$V_D = +2$ V

and

$$R_D = \frac{5 - 2}{0.5} = 6 \text{ k}\Omega$$

7.94



$$V_G = V_{DD}\frac{R_{G2}}{R_{G1}+R_{G2}}$$

$$= 15 \times \frac{5.1}{10+5.1} = 5.07 \text{ V}$$

$$k_n = 0.2 \text{ to } 0.3 \text{ mA/V}^2$$

$$V_t = 1.0 \text{ V to } 1.5 \text{ V}$$

$$I_D = \frac{1}{2}k_n(V_{GS} - V_t^2)$$

With $R_S = 0$,

$$I_D = \frac{1}{2}k_n(V_G - V_t^2)$$

$I_{D\max}$ is obtained with $V_{t\min}$ and $k_{n\max}$:

$$I_{D\max} = \frac{1}{2} \times 0.3(5.07 - 1)^2 = 2.48 \text{ mA}$$

$I_{D\min}$ is obtained with $V_{t\max}$ and $k_{n\min}$:

$$I_{D\min} = \frac{1}{2} \times 0.2(5.07 - 1.5)^2 = 1.27 \text{ mA}$$

With R_S installed and $V_t = 1$ V,
$k_n = 0.3$ mA/V^2, we required $I_D = 1.5$ mA:

$$1.5 = \frac{1}{2} \times 0.3(V_{GS} - 1)^2$$

$$\Rightarrow V_{GS} = 4.16 \text{ V}$$

Since $V_G = 5.07$ V,

$$V_S = V_G - V_{GS} = 5.07 - 4.16 = 0.91 \text{ V}$$

Thus,

$$R_S = \frac{V_S}{I_D} = \frac{0.91}{1.5} = 607 \ \Omega$$

From Appendix J, the closest 5% resistor is 620 Ω. With $R_S = 620$ Ω,

$$V_S = I_D R_S = 0.62 I_D$$

$$V_{GS} = V_G - V_S = 5.07 - 0.62 I_D$$

$$I_D = \frac{1}{2}k_n(V_{GS} - V_t)^2$$

$$= \frac{1}{2}k_n(5.07 - 0.62 I_D - V_t)^2$$

For $k_n = 0.3$ mA/V^2 and $V_t = 1$,

$$I_D = \frac{1}{2} \times 0.3(4.07 - 0.62 I_D)^2$$

$$= 0.15(4.07^2 - 2 \times 4.07 \times 0.62 I_D + 0.62^2 I_D^2)$$

$$0.058 I_D^2 - 1.757 I_D + 2.488 = 0$$

which results in

$$I_D = 28.8 \text{ mA, or } 1.49 \text{ mA}$$

The first value does not make physical sense. Thus,

$$I_D = 1.49 \text{ mA} \simeq 1.5 \text{ mA}$$

which is the maximum value. The minimum value can be obtained by using $k_n = 0.2$ mA/V^2 and $V_t = 1.5$ V in Eq. (1),

$$I_D = \frac{1}{2} \times 0.2(3.57 - 0.62 I_D)^2$$

$$= 0.1(3.57^2 - 2 \times 3.57 \times 0.62 I_D + 0.62^2 I_D^2)$$

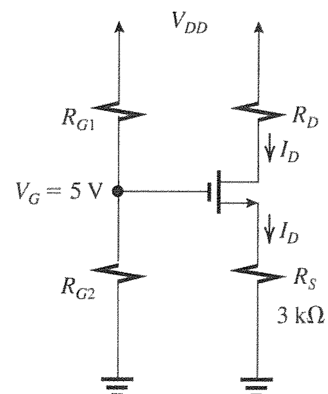
$$0.038 \, I_D^2 - 1.442 I_D + 1.274 = 0$$

which results in

$$I_D = 37 \text{ mA or } 0.91 \text{ mA}$$

Here again, the physically meaningful answer is $I_D = 0.91$ mA, which is the minimum value of I_D. Thus with a 0.62-kΩ resistance connected in the source lead, the value of I_D is limited to the range of 0.91 mA to 1.5 mA.

7.95



$$V_S = I_D R_S = 3 I_D$$

$$V_{GS} = 5 - V_S = 5 - 3 I_D$$

$$I_D = \frac{1}{2}k_n(V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 2(5 - 3 I_D - 1)^2$$

$$= 16 - 24 I_D + 9 I_D^2$$

$$9 I_D^2 - 25 I + 16 = 0$$

$$I_D = 1.78 \text{ mA or } 1 \text{ mA}$$

The first answer is physically meaningless, as it would result in $V_S = 5.33$ V, which is greater than V_G, implying that the transistor is cut off. Thus, $I_D = 1$ mA.

If a transistor for which $k_n = 3$ mA/V^2 is used, then

$$I_D = \frac{1}{2} \times 3(5 - 3 I_D - 1)^2$$

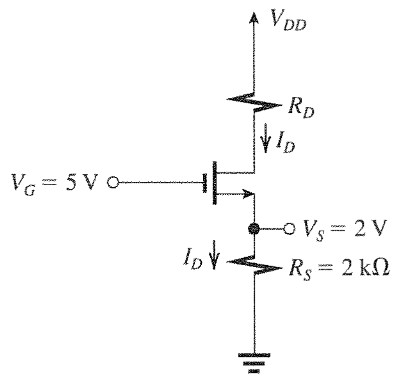
$$= 1.5(16 - 24 I + 9 I_D^2)$$

$9I_D^2 - 24.67I_D + 16 = 0$

whose physically meaningful solution is

$I_D = 1.05$ mA

7.96



$$I_D = \frac{2\text{ V}}{2\text{ k}\Omega} = 1 \text{ mA}$$

But

$$I_D = \frac{1}{2}k_n(V_{GS} - V_t)^2$$

$$1 = \frac{1}{2} \times 2(V_G - V_S - V_t)^2$$

$$1 = (5 - 2 - V_t)^2$$

$V_t = 2$ V

If $V_t = 1.5$ V, then we have

$V_S = I_D R_S = 2I_D$

$V_{GS} = V_G - V_S = 5 - 2I_D$

$$I_D = \frac{1}{2} \times 2(5 - 2I_D - 1.5)^2$$

$4I_D^2 - 15I_D + 12.25 = 0$

$I_D = 1.2$ mA

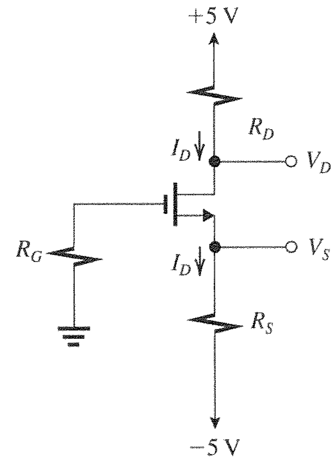
$V_S = 2.4$ V

7.97 $I_D = 0.5 \text{ mA} = \frac{1}{2} \times 4(V_{GS} - 1)^2$

$\Rightarrow V_{GS} = 1.5$ V

Since $V_G = 0$ V, $V_S = -1.5$ V, and

$$R_S = \frac{-1.5 - (-5)}{0.5} = 7 \text{ k}\Omega$$



Maximum gain is obtained by using the largest possible value of R_D, that is, the lowest possible value of V_D that is consistent with allowing negative voltage signal swing at the drain of 1 V. Thus

$V_D - 1 = v_{D\min} = V_G - V_t = 0 - 1$

$\Rightarrow V_D = 0$ V

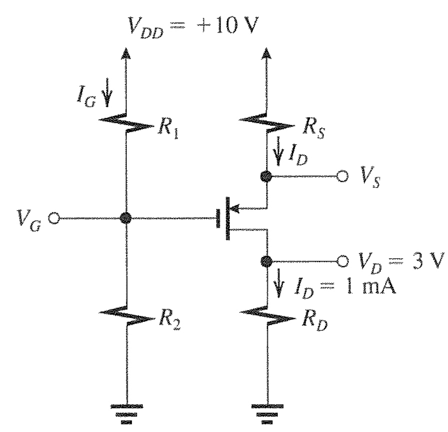
where we have assumed that the signal voltage at the gate is small. Now,

$V_D = 0 = V_{DD} - I_D R_D$

$0 = 5 - 0.5 \times R_D$

$\Rightarrow R_D = 10$ kΩ

7.98



$I_D = 1$ mA and $V_D = 3$ V

Thus,

$$R_D = \frac{V_D}{I_D} = \frac{3\text{ V}}{1\text{ mA}} = 3 \text{ k}\Omega$$

For the transistor to operate 1 V from the edge of saturation

$V_D = V_G + |V_t| - 1$

Thus,

$3 = V_G + |V_t| - 1$

$V_G + |V_t| = 4\ \text{V}$

(a) $|V_t| = 1\ \text{V}$ and $k_p = 0.5\ \text{mA/V}^2$

$V_G = 3\ \text{V}$

$$R_2 = \frac{V_G}{I_G} = \frac{3\ \text{V}}{10\ \mu\text{A}} = 0.3\ \text{M}\Omega$$

$$R_1 = \frac{V_{DD} - V_G}{I_G} = \frac{7\ \text{V}}{10\ \mu\text{A}} = 0.7\ \text{M}\Omega$$

$V_D = 3\ \text{V}$

$R_D = 3\ \text{k}\Omega$

$$I_D = \frac{1}{2}k_p(V_{SG} - |V_t|)^2$$

$$1 = \frac{1}{2} \times 0.5(V_{SG} - 1)^2$$

$\Rightarrow V_{SG} = 3\ \text{V}$

$V_S = V_G + 3 = 3 + 3 = 6\ \text{V}$

$$R_S = \frac{V_{DD} - V_S}{I_D}$$

$$= \frac{10 - 6}{1} = 4\ \text{k}\Omega$$

(b) $|V_t| = 2\ \text{V}$ and $k_p = 1.25\ \text{mA/V}^2$

$V_G = 4 - |V_t| = 2\ \text{V}$

$$R_2 = \frac{V_G}{I_G} = \frac{2\ \text{V}}{10\ \mu\text{A}} = 0.2\ \text{M}\Omega$$

$$R_1 = \frac{V_{DD} - V_G}{I_G} = \frac{8\ \text{V}}{10\ \mu\text{A}} = 0.8\ \text{M}\Omega$$

$V_D = 3\ \text{V}$

$R_D = 3\ \text{k}\Omega$

$$I_D = \frac{1}{2}k_p(V_{SG} - |V_t|)^2$$

$$1 = \frac{1}{2} \times 1.25(V_{SG} - 2)^2$$

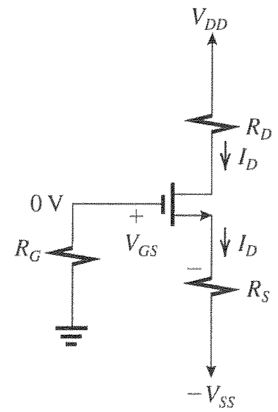
$V_{SG} = 3.265\ \text{V}$

$V_S = V_G + 3.265 = 2 + 3.265$

$= 5.265\ \text{V}$

$$R_S = \frac{10 - 5.265}{1} = 4.7\ \text{k}\Omega$$

7.99



(a) $V_{GS} + I_D R_S = V_{SS}$

But

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$$

$$= K(V_{GS} - V_t)^2$$

$$\Rightarrow V_{GS} = V_t + \sqrt{\frac{I_D}{K}}$$

Thus,

$$V_t + \sqrt{\frac{I_D}{K}} + I_D R_S = V_{SS}$$

Differentiating relative to K, we have

$$0 + \frac{1}{2\sqrt{I_D/K}}\left[\frac{1}{K}\frac{\partial I_D}{\partial K} - \frac{I_D}{K^2}\right] + R_S\frac{\partial I_D}{\partial K} = 0$$

$$\frac{\partial I_D}{\partial K}\frac{K}{I_D} = \frac{1}{1 + 2\sqrt{KI_D}\,R_S}$$

$S_K^{I_D} = 1/[1 + 2\sqrt{KI_D}\,R_S]$ Q.E.D

(b) $K = 100\ \mu\text{A/V}^2$, $\frac{\Delta K}{K} = \pm 0.1$, and $V_t = 1\ \text{V}$. We require $I_D = 100\ \mu\text{A}$ and $\frac{\Delta I_D}{I_D} = \pm 0.01$. Thus,

$$S_K^{I_D} = \frac{\Delta I_D/I_D}{\Delta K/K} = \frac{0.01}{0.10} = 0.1$$

Substituting in the expression derived in (a),

$$0.1 = \frac{1}{1 + 2\sqrt{0.1 \times 0.1}R_S}$$

$\Rightarrow R_S = 45\ \text{k}\Omega$

To find V_{GS},

$I_D = K(V_{GS} - V_t)^2$

$100 = 100(V_{GS} - 1)^2$

$V_{GS} = 2$ V

$V_{GS} + I_D R_S = V_{SS}$

$2 + 0.1 \times 45 = 6.5$ V

(c) For $V_{SS} = 5$ V and $V_{GS} = 2$ V,

$I_D R_S = 3$ V

$$R_S = \frac{3}{0.1} = 30\text{ k}\Omega$$

$$S_K^{I_D} = \frac{1}{1 + 2\sqrt{0.1 \times 0.1} \times 30} = \frac{1}{7}$$

$$\frac{\Delta I_D}{I_D} = \frac{1}{7} \times \frac{\Delta K}{K} = \frac{1}{7} \times \pm 10\% = \pm 1.4\%$$

7.100 (a) With a fixed V_{GS},

$$I_D = \frac{1}{2}k_n(V_{GS} - V_t)^2$$

$$\frac{\partial I_D}{\partial V_t} = -k_n(V_{GS} - V_t)$$

$$S_{V_t}^{I_D} \equiv \frac{\partial I_D}{\partial V_t}\frac{V_t}{I_D} = -\frac{k_n(V_{GS} - V_t)V_t}{I_D}$$

$$= -\frac{k_n(V_{GS} - V_t)V_t}{\frac{1}{2}k_n(V_{GS} - V_t)^2}$$

$$= -\frac{2V_t}{V_{GS} - V_t} = -\frac{2V_t}{V_{OV}} \qquad \text{Q.E.D}$$

For $V_t = 0.5$ V, $\frac{\Delta V_t}{V_t} = \pm 5\%$, and $V_{OV} = 0.25$ V, we have

$$\frac{\Delta I_D}{I_D} = S_{V_t}^{I_D}\left(\frac{\Delta V_t}{V_t}\right)$$

$$= -\frac{2 \times 0.5}{0.25} \times \pm 5\%$$

$$= \mp 20\%$$

(b) For fixed bias at the gate V_G and a resistance R_S in the source lead, we have

$$V_G = V_{GS} + I_D R_S$$

where V_{GS} is obtained from

$$I_D = \frac{1}{2}k_n(V_{GS} - V_t)^2$$

$$\Rightarrow V_{GS} = V_t + \sqrt{\frac{2I_D}{k_n}}$$

Thus

$$V_t + \sqrt{\frac{2I_D}{k_n}} + I_D R_S = V_G$$

Differentiating relative to V_t, we have

$$1 + \frac{1}{2\sqrt{2I_D/k_n}}\frac{2}{k_n}\frac{\partial I_D}{\partial V_t} + R_S\frac{\partial I_D}{\partial V_t} = 0$$

$$\frac{\partial I_D}{\partial V_t}\left[\frac{1}{\sqrt{2k_n I_D}} + R_S\right] = -1$$

$$\frac{\partial I_D}{\partial V_t} = -\frac{1}{\frac{1}{\sqrt{2k_n I_D}} + R_S}$$

$$S_{V_t}^{I_D} = \frac{\partial I_D}{\partial V_t}\frac{V_t}{I_D} = -\frac{V_t}{\sqrt{\frac{I_D}{2k_n}} + I_D R_S}$$

But

$$I_D = \frac{1}{2}k_n V_{OV}^2 \Rightarrow V_{OV} = \sqrt{\frac{2I_D}{k_n}}$$

Thus

$$S_{V_t}^{I_D} = -\frac{2V_t}{V_{OV} + 2I_D R_S} \qquad \text{Q.E.D}$$

For $V_t = 0.5$ V, $\frac{\Delta V_t}{V_t} = \pm 5\%$, and $V_{OV} = 0.25$ V, to limit $\frac{\Delta I_D}{I_D}$ to $+5\%$ we require

$$S_{V_t}^{I_D} = 1$$

Thus

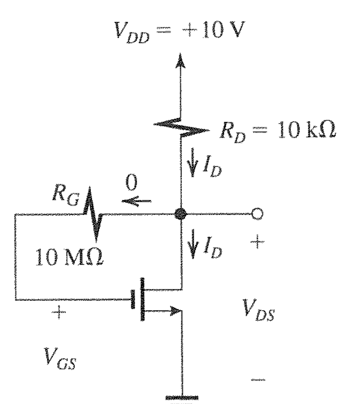
$$-1 = -\frac{2 \times 0.5}{0.25 + 2I_D R_S}$$

$$\Rightarrow I_D R_S = 0.375\text{ V}$$

For $I_D = 0.1$ mA,

$$R_S = \frac{0.375}{0.1} = 3.75\text{ k}\Omega$$

7.101



$V_{GS} = V_{DD} - I_D R_D$

$= 10 - 10I_D$

(a) $V_t = 1$ V and $k_n = 0.5$ mA/V^2

$$I_D = \frac{1}{2}k_n(V_{GS} - V_t)^2$$

$$I_D = \frac{1}{2} \times 0.5(10 - 10\,I_D - 1)^2$$

$$\Rightarrow I_D^2 - 1.84 I_D + 0.81 = 0$$

$I_D = 1.11$ mA or 0.73 mA

The first root results in $V_D = -0.11$ V, which is physically meaningless. Thus

$I_D = 0.73$ mA

$V_G = V_D = 10 - 10 \times 0.73 = 2.7$ V

(b) $V_t = 2$ V and $k_n = 1.25$ mA/V^2

$$I_D = \frac{1}{2} \times 1.25(10 - 10I_D - 2)^2$$

$$\Rightarrow I_D^2 - 1.616 I_D + 0.64 = 0$$

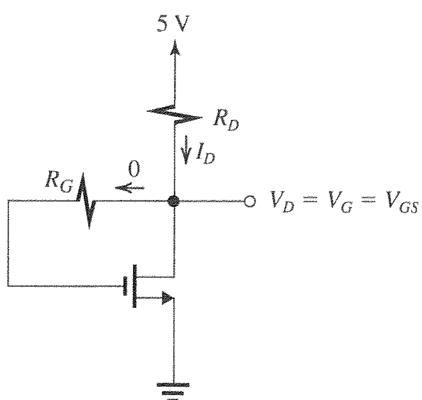
$I_D = 0.92$ mA or 0.695 mA

The first root can be shown to be physically meaningless, thus

$I_D = 0.695$ mA

$V_G = V_D = 10 - 10 \times 0.695 = 3.05$ V

7.102

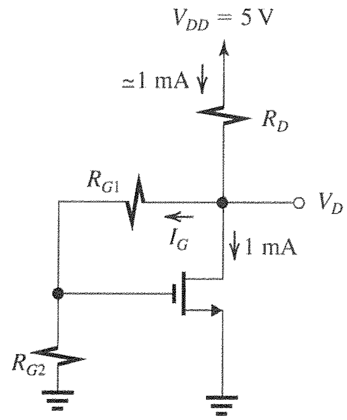


$$I_D = 0.2 = \frac{1}{2} \times 10(V_{GS} - V_t)^2$$

$$\Rightarrow V_{GS} = 1.2 \text{ V}$$

$$R_D = \frac{5 - 1.2}{0.2} = 19 \text{ k}\Omega$$

7.103



$$I_D = \frac{1}{2}k_n V_{OV}^2$$

$$1 = \frac{1}{2} \times 8V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.5 \text{ V}$$

Since the transistor leaves the saturation region of operation when $v_D < V_{OV}$, we select

$V_D = V_{OV} + 2$

$V_D = 2.5$ V

Since $I_G \ll I_D$, we can write

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{5 - 2.5}{1} = 2.5 \text{ k}\Omega$$

$V_{GS} = V_t + V_{OV} = 0.8 + 0.5 = 1.3$ V

Thus the voltage drop across R_{G2} is 1.3 V and that across R_{G1} is $(2.5 - 1.3) = 1.2$ V. Thus R_{G2} is the larger of the two resistances, and we select $R_{G2} = 22$ MΩ and find R_{G1} from

$$\frac{R_{G1}}{R_{G2}} = \frac{1.2}{1.3} \Rightarrow R_{G1} = 20.3 \text{ M}\Omega$$

Specifying all resistors to two significant digits, we have $R_D = 2.5$ kΩ, $R_{G1} = 22$ MΩ, and $R_{G1} = 20$ MΩ.

7.104 $\dfrac{R_{B1}}{R_{B1} + R_{B2}} \times 3 = 0.710$

$$\Rightarrow \frac{R_{B2}}{R_{B1}} = 3.225$$

Given that R_{B1} and R_{B2} are 1% resistors, the maximum and minimum values of the ratio R_{B2}/R_{B1} will be $3.225 \times 1.02 = 3.2895$ and $3.225 \times 0.98 = 3.1605$. The resulting V_{BE} will be 0.699 V and 0.721 V, respectively. Correspondingly, I_C will be

$I_{C\max} = 1 \times e^{(0.710-0.699)/0.025}$

$= 1.55$ mA

and

$I_{C\min} = 1 \times e^{(0.710-0.721)/0.025}$

$I_{C\min} = 0.64$ mA

V_{CE} will range from

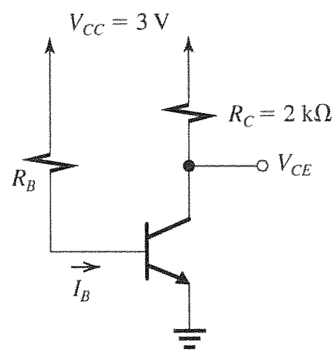
$V_{CE\min} = 3 - 1.55 \times 2 = -0.1$ V

which is impossible, implying that the transistor will saturate at this value of dc bias!

$V_{CE\max} = 3 - 0.64 \times 2 = 1.72$ V

It should be clear that this biasing arrangement is useless, since even the small and inevitable tolerances in R_{B1} and R_{B2} caused such huge variations in I_C that in one extreme the transistor left the active mode of operation altogether!

7.105



To obtain $I_C = 1$ mA, we write

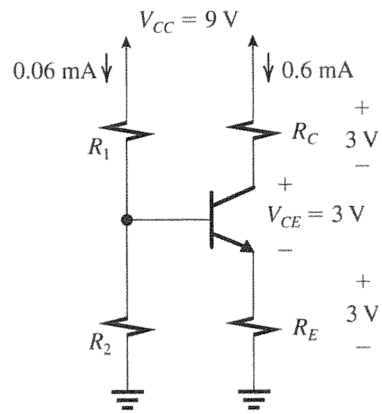
$$I_B = \frac{I_C}{\beta} = \frac{1 \text{ mA}}{100} = 0.01 \text{ mA}$$

Thus,

$$R_B = \frac{V_{CC} - V_{BE}}{I_B} \simeq \frac{3 - 0.7}{0.01} = 230 \text{ k}\Omega$$

Since β ranges from 50 to 150 and I_B is fixed at 0.01 mA, the collector current I_C will range from $0.01 \times 50 = 0.5$ mA to $0.01 \times 150 = 1.5$ mA. Correspondingly, V_{CE} will range from $(3 - 0.5 \times 2) = 1$ V to $(3 - 1.5 \times 2) = 0$ V. The latter value implies that the high-β transistor will leave the active region of operation and saturate. Obviously, this bias method is very intolerant of the inevitable variations in β. Thus it is not a good method for biasing the BJT.

7.106



Initial design: $\beta = \infty$

$$R_C = R_E = \frac{3 \text{ V}}{0.6} = 5 \text{ k}\Omega$$

$$R_1 + R_2 = \frac{9}{0.06} = 150 \text{ k}\Omega$$

$$V_B = V_E + V_{BE} = 3 + 0.7 = 3.7 \text{ V}$$

$$R_2 = \frac{3.7}{0.06} = 61.7 \text{ k}\Omega$$

$$R_1 = 150 - 61.7 = 88.3 \text{ k}\Omega$$

Using 5% resistors from Appendix J, and selecting R_1 and R_2 so as to obtain a V_{BB} that is slightly higher than 3.7 V, we write

$R_1 = 82$ kΩ and $R_2 = 62$ kΩ

$R_E = 5.1$ kΩ and $R_C = 5.1$ kΩ

$$V_{BB} = V_{CC}\frac{R_2}{R_1 + R_2} = 9 \times \frac{62}{62 + 82} = 3.875$$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \dfrac{R_B}{\beta + 1}}$$

where

$$R_B = R_1 \parallel R_2 = 62 \parallel 82 = 35.3 \text{ k}\Omega$$

$$I_E = \frac{3.875 - 0.7}{5.1 + \dfrac{35.3}{91}} = 0.58 \text{ mA}$$

$$V_E = 0.58 \times 5.1 = 3.18$$

$$V_B = 3.88 \text{ V}$$

$$I_C = \alpha I_E = \frac{90}{91} \times 0.58 = 0.57 \text{ mA}$$

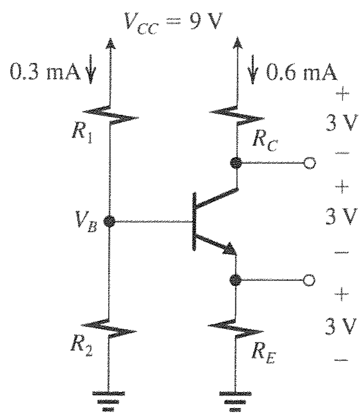
$$V_C = 6.1 \text{ V}$$

$$I_{R2} = \frac{V_B}{R_2} = \frac{3.88}{62} = 0.063 \text{ mA}$$

$$I_B = \frac{I_E}{\beta+1} = \frac{0.58}{91} = 0.006 \text{ mA and}$$

$$I_{R1} = 0.069 \text{ mA}$$

7.107



Initial design: $\beta = \infty$

$$R_C = R_E = \frac{3 \text{ V}}{0.6 \text{ mA}} = 5 \text{ k}\Omega$$

$$R_1 + R_2 = \frac{9}{0.3} = 30 \text{ k}\Omega$$

$$V_B = V_E + V_{BE} = 3.7 \text{ V}$$

$$R_2 = \frac{3.7}{0.3} = 12.3 \text{ k}\Omega$$

$$R_1 = 30 - 12.3 = 17.7 \text{ k}\Omega$$

If we select 5% resistors, we will have

$$R_E = R_C = 5.1 \text{ k}\Omega$$

$$R_1 = 18 \text{ k}\Omega, \quad R_2 = 13 \text{ k}\Omega$$

$$V_{BB} = 9 \times \frac{13}{13+18} = 3.774 \text{ V}$$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \dfrac{R_1 \parallel R_2}{\beta+1}} = \frac{3.774 - 0.7}{5.1 + \dfrac{18 \parallel 13}{91}} =$$
0.593 mA

$$V_E = I_E R_E = 3.02 \text{ V}$$

$$V_B = 3.72 \text{ V}$$

$$I_C = \alpha I_E = \frac{90}{91} \times 0.593 = 0.586 \text{ mA}$$

$$V_C = V_{CC} - I_C R_C = 9 - 0.586 \times 5.1 = 6 \text{ V}$$

I_C falls to the value obtained in Problem 7.106, namely, 0.57 mA at the value of β obtained from

$$I_C = \alpha \frac{V_{BB} - V_{BE}}{R_E + \dfrac{R_1 \parallel R_2}{\beta+1}}$$

$$0.57 = \frac{\beta}{\beta+1} \frac{3.774 - 0.7}{5.1 + \dfrac{18 \parallel 13}{\beta+1}}$$

$$= \frac{\beta \times 3.074}{5.1(\beta+1) + 7.548}$$

$$\Rightarrow \beta = 75.7$$

7.108 Refer to Fig. 7.52.

(a) $$I_E = \frac{V_{BB} - V_{BE}}{R_E + \dfrac{R_B}{\beta+1}}$$

$$I_E\big|_{\text{nominal}} = \frac{V_{BB} - V_{BE}}{R_E + \dfrac{R_B}{101}}$$

$$I_E\big|_{\text{high}} = \frac{V_{BB} - V_{BE}}{R_E + \dfrac{R_B}{151}}$$

$$I_E\big|_{\text{low}} = \frac{V_{BE} - V_{BE}}{R_E + \dfrac{R_B}{51}}$$

Let's constrain $I_E\big|_{\text{low}}$ to be equal to $I_E\big|_{\text{nominal}} \times$ 0.95 and then check $I_E\big|_{\text{high}}$:

$$0.95 \frac{V_{BB} - V_{BE}}{R_E + \dfrac{R_B}{101}} = \frac{V_{BB} - V_{BE}}{R_E + \dfrac{R_B}{51}}$$

$$0.95 = \frac{1 + \dfrac{R_B/R_E}{101}}{1 + \dfrac{R_B/R_E}{51}}$$

$$\Rightarrow \frac{R_B}{R_E} = 5.73$$

For this value,

$$I_E\big|_{\text{nominal}} = 0.946\left(\frac{V_{BB} - V_{BE}}{R_E}\right)$$

$$I_E\big|_{\text{low}} = 0.90\left(\frac{V_{BB} - V_{BE}}{R_E}\right) = 0.95\, I_E\big|_{\text{nominal}}$$

$$I_E\big|_{\text{high}} = 0.963\left(\frac{V_{BE} - V_{BE}}{R_E}\right) = 1.02\, I_E\big|_{\text{nominal}}$$

Thus, the maximum allowable ratio is

$$\frac{R_B}{R_E} = 5.73$$

(b) $$I_E = \frac{V_{BB} - V_{BE}}{R_E\left(1 + \dfrac{R_B/R_E}{\beta+1}\right)}$$

$$I_E R_E = \frac{V_{BB} - V_{BE}}{1 + \dfrac{5.73}{\beta + 1}}$$

$$\frac{V_{CC}}{3} = \frac{V_{BB} - V_{BE}}{1 + \dfrac{5.73}{101}}$$

$$\Rightarrow V_{BB} = V_{BE} + 0.352 V_{CC}$$

(c) $V_{CC} = 5$ V

$$V_{BB} = 0.7 + 0.352 \times 5 = 2.46 \text{ V}$$

$$R_E = \frac{V_{CC}/3}{I_E} = \frac{5/3}{0.5} = 3.33 \text{ k}\Omega$$

$$R_B = 5.73 \times R_E = 19.08 \text{ k}\Omega$$

$$V_{BB} = V_{CC}\frac{R_2}{R_1 + R_2}$$

$$2.46 = 5\frac{R_2}{R_1 + R_2}$$

$$2.46 R_1 = 5\frac{R_1 R_2}{R_1 + R_2} = 5R_B$$

$$= 5 \times 19.08$$

$$\Rightarrow R_1 = 38.8 \text{ k}\Omega$$

$$R_2 = 1\bigg/\left(\frac{1}{R_B} - \frac{1}{R_1}\right) = 37.5 \text{ k}\Omega$$

(d) $V_{CE} = V_{CC} - R_C I_G$

$$1 = 5 - R_C \times 0.99 \times 0.5$$

$$\Rightarrow R_C = 8.1 \text{ k}\Omega$$

Check design:

$$V_{BB} = V_{CC}\frac{R_2}{R_1 + R_2} = 5 \times \frac{37.5}{37.5 + 38.8}$$

$$= 2.46 \text{ V}$$

$$R_B = R_1 \parallel R_2 = 37.5 \parallel 38.8 = 19.07 \text{ k}\Omega$$

$$I_E\Big|_{\text{nominal}} = \frac{2.46 - 0.7}{3.33 + \dfrac{19.07}{101}} = 0.5 \text{ mA}$$

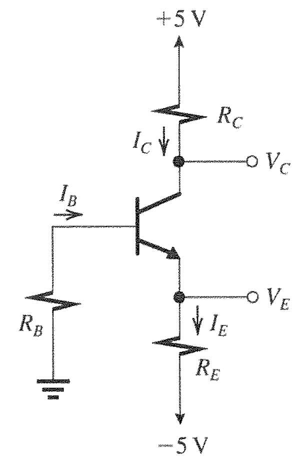
$$I_E\Big|_{\text{low}} = \frac{2.46 - 0.7}{3.33 + \dfrac{19.07}{51}} = 0.475 \text{ mA}$$

which is 5% lower than $I_E\big|_{\text{nominal}}$, and

$$I_E\Big|_{\text{high}} = \frac{2.46 - 0.7}{3.33 + \dfrac{19.07}{151}} = 0.509 \text{ mA}$$

which is 1.8% higher than $I_E\big|_{\text{nominal}}$.

7.109



Required: $I_C = 0.5$ mA and $V_C = V_E + 2$.

(a) $\beta = \infty$

$$V_B = 0$$

$$V_E = -0.7 \text{ V}$$

$$I_E = 0.5 = \frac{V_E - (-5)}{R_E} = \frac{4.3}{R_E}$$

$$\Rightarrow R_E = 8.6 \text{ k}\Omega$$

$$V_C = V_E + 2 = -0.7 + 2 = +1.3 \text{ V}$$

$$R_C = \frac{V_{CC} - V_C}{I_C} = \frac{5 - 1.3}{0.5} = 7.4 \text{ k}\Omega$$

(b) $\beta_{\min} = 50$

$$I_{B\max} = \frac{I_E}{51} = \frac{0.5}{51} \simeq 0.01 \text{ mA}$$

$$I_E R_E = 0.5 \times 8.6 = 4.3 \text{ V}$$

$$I_{B\max} R_{B\max} = 0.1 I_E R_E = 0.43 \text{ V}$$

$$R_{B\max} = \frac{0.43}{0.01} = 43 \text{ k}\Omega$$

(c) Standard 5% resistors:

$$R_B = 43 \text{ k}\Omega$$

$$R_E = 8.2 \text{ k}\Omega$$

$$R_C = 7.5 \text{ k}\Omega$$

(d) $\beta = \infty$:

$$V_B = 0, \quad V_E = -0.7 \text{ V}$$

$$I_E = \frac{-0.7 - (-5)}{8.2} = 0.52 \text{ mA}$$

$$I_C = 0.52 \text{ mA}$$

$$V_C = 5 - 0.52 \times 7.5 = 1.1 \text{ V}$$

$\beta = 50$:

$$I_E = \frac{5 - 0.7}{8.2 + \frac{43}{51}} = 0.48 \text{ mA}$$

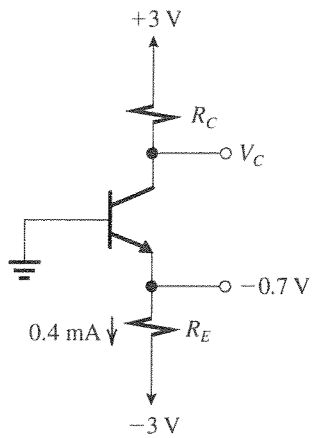
$$V_E = -5 + 0.48 \times 8.2 = -1.064 \text{ V}$$

$$V_B = -0.364 \text{ V}$$

$$I_C = \alpha I_E = \frac{50}{51} \times 0.48 = 0.47 \text{ mA}$$

$$V_C = 5 - 0.47 \times 7.5 = 1.475 \text{ V}$$

7.110



$$R_E = \frac{-0.7 - (-3)}{0.4}$$

$$= 5.75 \text{ k}\Omega$$

To maximize gain while allowing for ±1 V signal swing at the collector, design for the lowest possible V_C consistent with

$$V_C - 1 = -0.7 + V_{CEsat}$$

$$= -0.7 + 0.3 = -0.4 \text{ V}$$

$$V_C = 0.6 \text{ V}$$

$$R_C = \frac{V_{CC} - V_C}{I_C} = \frac{3 - 0.6}{0.39} = 6.2 \text{ k}\Omega$$

As temperature increases from 25°C to 125°C, (i.e., by 100°C), V_{BE} decreases by 2 mV × 100 = − 200 mV. Thus I_E increases by $\frac{0.2 \text{ V}}{R_E} = \frac{0.2 \text{ V}}{5.75 \text{ k}\Omega} = 0.035$ mA to become 0.435 mA. The collector current becomes

$$I_C = \frac{\beta}{\beta + 1} \times 0.435$$

where β is the increased value of 150,

$$I_C = \frac{150}{151} \times 0.435 \text{ mA}$$

$$= 0.432 \text{ mA}$$

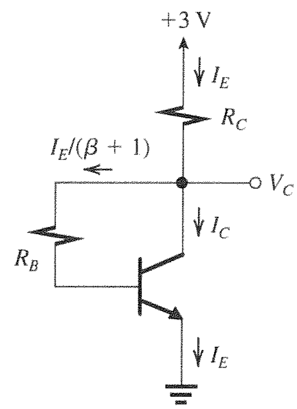
Thus,

$$\Delta I_C = 0.432 - 0.39 = 0.042 \text{ mA}$$

for a percentage increase of

$$\frac{\Delta I_C}{I_C} \times 100 = \frac{0.042}{0.39} \times 100 = 10.8\%$$

7.111



$$V_C = V_{CEsat} + 1 \text{ V}$$

$$= 1.3 \text{ V}$$

$$I_E = \frac{3 - 1.3}{R_C} = 0.5 \text{ mA}$$

$$\Rightarrow R_C = 3.4 \text{ k}\Omega$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.5}{101} \simeq 0.005 \text{ mA}$$

$$V_C = V_{BE} + I_B R_B$$

$$1.3 = 0.7 + 0.005 \times R_B$$

$$\Rightarrow R_B = 120 \text{ k}\Omega$$

Standard 5% resistors:

$$R_C = 3.3 \text{ k}\Omega$$

$$R_B = 120 \text{ k}\Omega$$

If the actual BJT has $\beta = 50$, then

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{\beta + 1}} = \frac{3 - 0.7}{3.3 + \frac{120}{51}} = 0.41 \text{ mA}$$

$$V_C = 3 - I_E R_C = 3 - 0.41 \times 3.3 = 1.65 \text{ V}$$

Allowable negative signal swing at the collector is as follows:

$$V_C - V_{CEsat} = 1.65 - 0.3 = 1.35 \text{ V}$$

An equal positive swing is *just* possible. For $\beta = 150$:

$$I_E = \frac{3 - 0.7}{3.3 + \frac{120}{151}} = 0.56 \text{ mA}$$

$$V_C = 3 - I_E R_C = 3 - 0.56 \times 3.3 = 1.15 \text{ V}$$

Allowable negative signal swing at the collector $= 1.15 - 0.3 = 0.85$ V. An equal positive swing is possible.

7.112

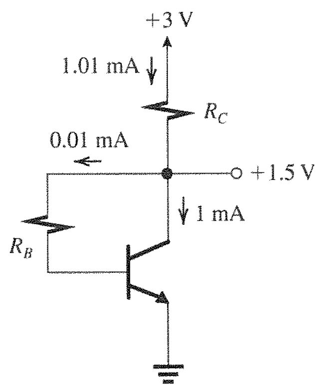


Figure 1

(a) From the circuit diagram of Fig. 1, we can write

$$R_C = \frac{3 - 1.5}{1.01 \text{ mA}} \simeq 1.5 \text{ k}\Omega$$

$$1.5 = 0.01 R_B + V_{BE}$$

$$= 0.01 R_B + 0.7$$

$$\Rightarrow R_B = 80 \text{ k}\Omega$$

(b) Selecting 5% resistors, we have

$$R_C = 1.5 \text{ k}\Omega$$

$$R_B = 82 \text{ k}\Omega$$

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{\beta + 1}}$$

$$= \frac{3 - 0.7}{1.5 + \frac{82}{101}} = 0.99 \text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 0.99 = 0.98 \text{ mA}$$

$$V_C = 3 - 1.5 \times 0.99 = 1.52 \text{ V}$$

(c) $\beta = \infty$:

$$I_C = I_E = \frac{V_{CC} - V_{BE}}{R_C} = \frac{3 - 0.7}{1.5} = 1.53 \text{ mA}$$

$$V_C = 0.7 \text{ V}$$

(d) From the circuit diagram of Fig. 2, we can write

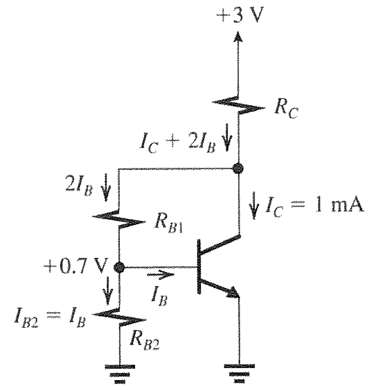


Figure 2

$$I_B = \frac{I_C}{\beta} = \frac{1}{100} = 0.01 \text{ mA}$$

$$R_{B2} = \frac{0.7}{I_{B2}} = \frac{0.7}{0.01}$$

$$= 70 \text{ k}\Omega$$

$$1.5 = 2 I_B R_{B1} + 0.7$$

$$0.8 = 2 \times 0.01 \times R_{B1}$$

$$R_{B1} = 40 \text{ k}\Omega$$

$$R_C = \frac{3 - 1.5}{I_C + 2I_B} = \frac{1.5}{1.02} = 1.47 \text{ k}\Omega$$

For $\beta = \infty$:

$$I_B = 0, \quad I_{B2} = \frac{0.7}{R_{B2}} = \frac{0.7}{70} = 0.01 \text{ mA}$$

$$I_{B1} = I_{B2} = 0.01 \text{ mA}$$

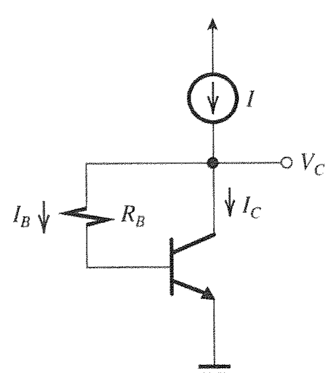
$$V_C = 0.01 R_{B1} + 0.7 = 0.01 \times 40 + 0.7$$

$$= 1.1 \text{ V}$$

$$I_C + 0.01 = \frac{3 - 1.1}{R_C} = \frac{3 - 1.1}{1.47} = 1.29$$

$$I_C = 1.28 \text{ mA}$$

7.113



$I_C = 1 \text{ mA}$

$I = I_C + I_B$

$= I_C + \frac{I_C}{\beta}$

$= 1\left(1 + \frac{1}{\beta}\right)$

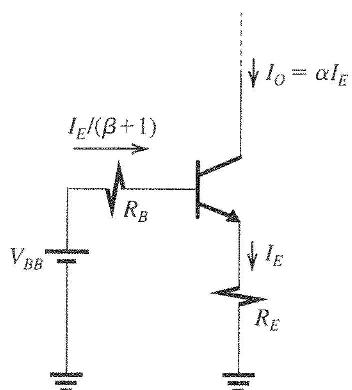
$I = 1.01 \text{ mA}$

$V_C = 1.5 \text{ V} = I_B R_B + V_{BE}$

$1.5 = 0.01 \times R_B + 0.7$

$R_B = 80 \text{ k}\Omega$

7.114 Refer to the circuit in Fig. P7.114. Replacing V_{CC} together with the voltage divider (R_1, R_2) by its Thévenin equivalent results in the circuit shown below.



where

$$V_{BB} = V_{CC}\frac{R_2}{R_1 + R_2}$$

and

$$R_B = (R_1 \parallel R_2)$$

Now,

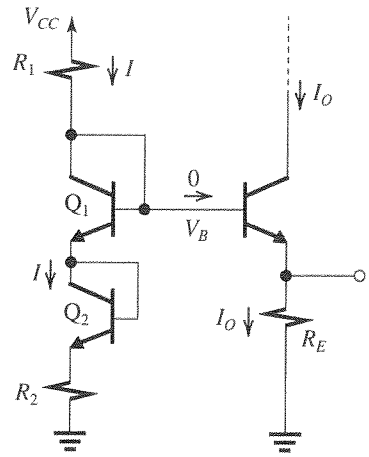
$$V_{BB} = \frac{I_E}{\beta + 1}R_B + V_{BE} + I_E R_E$$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + (R_1 \parallel R_2)/(\beta + 1)}$$

$$I_C = \alpha I_E$$

$$= \alpha\frac{V_{CC}[R_2/(R_1 + R_2)] - V_{BE}}{R_E + (R_1 \parallel R_2)/(\beta + 1)}$$

7.115



$$I = \frac{V_{CC} - V_{BE1} - V_{BE2}}{R_1 + R_2}$$

$$V_B = IR_2 + V_{BE2} + V_{BE1}$$

$$V_{E3} = V_B - V_{BE3}$$

$$V_{E3} = IR_2 + V_{BE2} + V_{BE1} - V_{BE3}$$

$$= (V_{CC} - V_{BE1} - V_{BE2})\frac{R_2}{R_1 + R_2} + V_{BE1}$$

$$+V_{BE2} - V_{BE3}$$

$$I_O = \frac{V_E}{R_E} = \frac{\alpha}{R_E}\left[(V_{CC} - V_{BE1} - V_{BE2})\frac{R_2}{R_1 + R_2}\right.$$

$$\left. + V_{BE1} + V_{BE2} - V_{BE3}\right]$$

Now, for $R_1 = R_2$ and the currents in all junctions equal,

$$V_{BE1} = V_{BE2} = V_{BE3} = V_{BE}$$

$$I_O = \frac{1}{R_E}\left[(V_{CC} - 2V_{BE}) \times \frac{1}{2} + V_{BE}\right]$$

$$I_O = \frac{V_{CC}}{2R_E} \qquad \text{Q.E.D}$$

Thus,

$$I_O R_E = \frac{V_{CC}}{2}$$

$$V_B = \frac{V_{CC}}{2} + V_{BE}$$

$$I = (V_B - 2V_{BE})/R_2 = \left(\frac{V_{CC}}{2} - V_{BE}\right)\bigg/ R_2$$

But since I must be equal to I_O, we have

$$\frac{V_{CC}}{2R_E} = \frac{V_{CC}/2 - V_{BE}}{R_2}$$

Thus,

$$R_1 = R_2 = R_E\left(\frac{V_{CC} - 2V_{BE}}{V_{CC}}\right)$$

For $V_{CC} = 10$ V and $V_{BE} = 0.7$ V,

$$R_1 = R_2 = R_E\left(\frac{10 - 1.4}{10}\right) = 0.86R_E$$

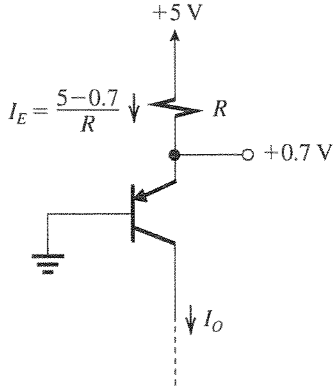
To obtain $I_O = 0.5$ mA,

$$0.5 = \frac{V_{CC}}{2R_E} = \frac{10}{2R_E}$$

$$\Rightarrow R_E = 10\ \text{k}\Omega$$

$$R_1 = R_2 = 8.6\ \text{k}\Omega$$

7.116



$$I_O = \alpha I_E \simeq 0.5\ \text{mA}$$

$$I_E = 0.5\ \text{mA}$$

$$\Rightarrow R = \frac{5 - 0.7}{0.5} = 8.6\ \text{k}\Omega$$

$$v_{C\max} = 0.7 - V_{EC\text{sat}} = 0.7 - 0.3$$

$$= +0.4\ \text{V}$$

7.117 Refer to the equivalent circuit in Fig. 7.55(b).

$$G_v = -\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m(R_D \parallel R_L \parallel r_o)$$

$$= -\frac{R_G}{R_G + R_{\text{sig}}} g_m(R_D \parallel R_L \parallel r_o)$$

$$= -\frac{10}{10 + 1} \times 3 \times (10 \parallel 20 \parallel 100)$$

$$= -17\ \text{V/V}$$

7.118 (a) Refer to Fig. P7.118. The dc circuit can be obtained by opening all coupling and bypass capacitors, resulting in the circuit shown in Fig. 1.

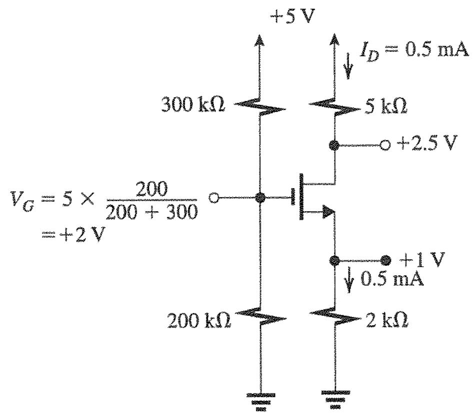


Figure 1

See analysis on figure.

$$V_{GS} = 2 - 1 = 1\ \text{V}$$

$$V_{OV} = V_{GS} - V_t = 1 - 0.7 = 0.3\ \text{V}$$

Since V_D at 2.5 V is 1.2 V higher than $V_S + V_{OV} = 1 + 0.3 = 1.3$ V, the transistor is

This figure belongs to Problem 7.118.

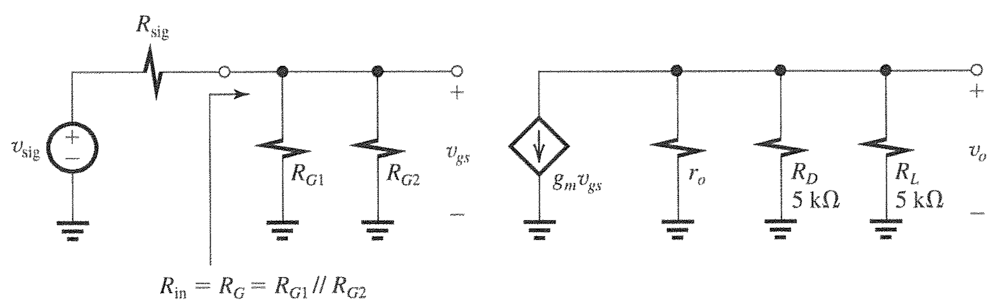


Figure 2

indeed operating in saturation. (Equivalent $V_D = 2.5$ V is higher than $V_G - V_t = 1.3$ V by 1.2 V.)

$$I_D = \frac{1}{2}k_n V_{OV}^2$$

$$0.5 = \frac{1}{2}k_n \times 0.3^3$$

$$\Rightarrow k_n = 11.1 \text{ mA/V}^2$$

(b) The amplifier small-signal equivalent-circuit model is shown in Fig. 2.

$$R_{\text{in}} = R_{G1} \parallel R_{G2} = 300 \text{ k}\Omega \parallel 200 \text{ k}\Omega = 120 \text{ k}\Omega$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.5}{0.3} = 3.33 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{50}{0.5} = 100 \text{ k}\Omega$$

$$G_v = -\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m (r_o \parallel R_D \parallel R_L)$$

$$= -\frac{120}{120 + 120} \times 3.33 \times (100 \parallel 5 \parallel 5)$$

$$= -4.1 \text{ V/V}$$

(c) $V_G = 2$ V, $\quad V_D = 2.5$ V

$$\hat{v}_{GS} = 2 + \hat{v}_{gs}, \quad \hat{v}_{DS} = 2.5 - |A_v|\hat{v}_{gs}$$

where

$$|A_v| = g_m(r_o \parallel R_D \parallel R_L) = 8.1 \text{ V/V}$$

To remain in saturation,

$$\hat{v}_{DS} \geq \hat{v}_{GS} - V_t$$

$$2.5 - 8.1\hat{v}_{gs} \geq 2 + \hat{v}_{gs} - 0.7$$

This is satisfied with equality at

$$\hat{v}_{gs} = \frac{2.5 - 1.3}{9.1} = 0.132 \text{ V}$$

The corresponding value of $\hat{v}_{\text{sig}}$ is

$$\hat{v}_{\text{sig}} = \hat{v}_{gs}\left(\frac{120 + 120}{120}\right) = 2 \times 0.132 = 0.264 \text{ V}$$

The corresponding amplitude at the output will be

$$|G_v|\hat{v}_{\text{sig}} = 4.1 \times 0.264 = 1.08 \text{ V}$$

(d) To be able to double $\hat{v}_{\text{sig}}$ without leaving saturation, we must reduce $\hat{v}_{gs}$ to half of what would be its new value; that is, we must keep $\hat{v}_{gs}$ unchanged. This in turn can be achieved by connecting an unbypassed R_s equal to $1/g_m$,

$$R_s = \frac{1}{3.33 \text{ mA/V}} = 300 \ \Omega$$

Since $\hat{v}_{gs}$ does not change, the output voltage also will not change, thus $\hat{v}_o = 1.08$ V.

7.119 Refer to Fig. P7.119.

(a) DC bias:

$$|V_{OV}| = 0.3 \text{ V} \Rightarrow V_{SG} = |V_{tp}| + |V_{OV}| = 1 \text{ V}$$

Since $V_G = 0$ V, $V_S = V_{SG} = +1$ V, and

$$I_D = \frac{2.5 - 1}{R_S} = 0.3 \text{ mA}$$

$$\Rightarrow R_S = \frac{1.5}{0.3} = 5 \text{ k}\Omega$$

(b) $G_v = -g_m R_D$

where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.3}{0.3} = 2 \text{ mA/V}$$

Thus,

$$-10 = -2R_D \Rightarrow R_D = 5 \text{ k}\Omega$$

(c) $v_G = 0$ V (dc) $+ v_{\text{sig}}$

$$v_{G\text{min}} = -\hat{v}_{\text{sig}}$$

$$\hat{v}_D = V_D + |G_v|\hat{v}_{\text{sig}}$$

where

$$V_D = -2.5 + I_D R_D = -2.5 + 0.3 \times 5 = -1 \text{ V}$$

To remain in saturation,

$$\hat{v}_D \leq \hat{v}_G + |V_{tp}|$$

$$-1 + 10\,\hat{v}_{\text{sig}} \leq -\hat{v}_{\text{sig}} + 0.7$$

Satisfying this constraint with equality gives

$$\hat{v}_{\text{sig}} = 0.154 \text{ V}$$

and the corresponding output voltage

$$\hat{v}_d = |G_v|\hat{v}_{\text{sig}} = 1.54 \text{ V}$$

(d) If $\hat{v}_{\text{sig}} = 50$ mV, then

$$V_D + |G_v|\hat{v}_{\text{sig}} = -\hat{v}_{\text{sig}} + |V_{tp}|$$

where

$$V_D = -2.5 + I_D R_D = -2.5 + 0.3R_D$$

and

$$|G_v| = g_m R_D = 2R_D$$

Thus

$$-2.5 + 0.3R_D + 2R_D\hat{v}_{\text{sig}} = -\hat{v}_{\text{sig}} + |V_{tp}|$$

$$-2.5 + 0.3R_D + 2R_D \times 0.05 = -0.05 + 0.7$$

$$0.4R_D = 3.15$$

$$\Rightarrow R_D = 7.875 \text{ k}\Omega$$

$$G_v = -g_m R_D = -2 \times 7.875 = -15.75 \text{ V/V}$$

7.120 Refer to Fig. P7.120.

$$R_{i2} = \frac{1}{g_{m2}} = 50\ \Omega$$

$$\Rightarrow g_{m2} = \frac{1}{50}\ \text{A/V} = 20\ \text{mA/V}$$

If Q_1 is biased at the same point as Q_2, then

$$g_{m1} = g_{m2} = 20\ \text{mA/V}$$

$$i_{d1} = g_{m1} \times 5\ (\text{mV})$$

$$= 20 \times 0.005 = 0.1\ \text{mA}$$

$$v_{d1} = i_{d1} \times 50\ \Omega$$

$$= 0.1 \times 50 = 5\ \text{mV}$$

$$v_o = i_{d1} R_D = 1\ \text{V}$$

$$R_D = \frac{1\ \text{V}}{0.1\ \text{mA}} = 10\ \text{k}\Omega$$

7.121 (a) DC bias: Refer to the circuit in Fig. P7.121 with all capacitors eliminated:

R_{in} at gate $= R_G = 10\ \text{M}\Omega$

$V_G = 0$, thus $V_S = -V_{GS}$, where V_{GS} can be obtained from

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$0.4 = \frac{1}{2} \times 5 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.4\ \text{V}$$

$$V_{GS} = V_t + 0.4 = 0.8 + 0.4 = 1.2\ \text{V}$$

$$V_S = -1.2\ \text{V}$$

$$R_S = \frac{-1.2 - (-5)}{0.4} = 9.5\ \text{k}\Omega$$

To remain in saturation, the minimum drain voltage must be limited to $V_G - V_t = 0 - 0.8 = -0.8$ V. Now, to allow for 0.8-V negative signal swing, we must have

$$V_D = 0\ \text{V}$$

and

$$R_D = \frac{5 - 0}{0.4} = 12.5\ \text{k}\Omega$$

(b) $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.4}{0.4} = 2\ \text{mA/V}$

$$r_o = \frac{V_A}{I_D} = \frac{40}{0.4} = 100\ \text{k}\Omega$$

(c) If terminal Z is connected to ground, the circuit becomes a CS amplifier,

$$G_v = -\frac{v_y}{v_{\text{sig}}} = \frac{R_G}{R_G + R_{\text{sig}}} \times -g_m(r_o \parallel R_D \parallel R_L)$$

$$= -\frac{10}{10 + 1} \times 2 \times (100 \parallel 12.5 \parallel 10)$$

$$= -9.6\ \text{V/V}$$

(d) If terminal Y is grounded, the circuit becomes a CD or source-follower amplifier:

$$\frac{v_z}{v_x} = \frac{(R_S \parallel r_o)}{(R_S \parallel r_o) + \dfrac{1}{g_m}}$$

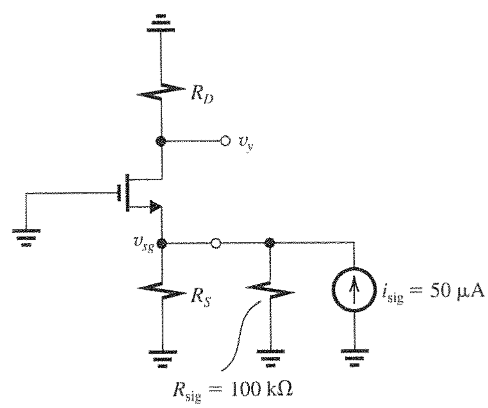
$$= \frac{(9.5 \parallel 100)}{(9.5 \parallel 100) + \dfrac{1}{2}} = 0.946\ \text{V/V}$$

Looking into terminal Z, we see R_o:

$$R_o = R_S \parallel r_o \parallel \frac{1}{g_m}$$

$$= 9.5 \parallel 100 \parallel \frac{1}{2} = 473\ \Omega$$

(e) If X is grounded, the circuit becomes a CG amplifier.



The figure shows the circuit prepared for signal calculations.

$$v_{sg} = i_{\text{sig}} \times \left[R_{\text{sig}} \parallel R_S \parallel \frac{1}{g_m} \right]$$

$$= 50 \times 10^{-3}(\text{mA}) \left[100 \parallel 9.5 \parallel \frac{1}{2} \right] (\text{k}\Omega)$$

$$= 0.024\ \text{V}$$

$$v_y = (g_m R_D) v_{sg}$$

$$= (2 \times 12.5) \times 0.024 = 0.6\ \text{V}$$

7.122 (a) Refer to the circuit of Fig. P7.122(a):

$$A_{vo} \equiv \frac{v_{o1}}{v_i} = \frac{10}{10 + \dfrac{1}{g_m}} = \frac{10}{10 + \dfrac{1}{10}} = 0.99\ \text{V/V}$$

$$R_o = \frac{1}{g_m} \parallel 10\ \text{k}\Omega = 0.1 \parallel 10 = 99\ \Omega$$

(b) Refer to Fig. P7.122(b):

$$R_{\text{in}} = 10\text{ k}\Omega \parallel \frac{1}{g_m} = 10 \parallel 0.1 = 99\ \Omega$$

$$\frac{v_o}{v_{i2}} = \frac{5 \parallel 2}{1/g_m} = 10(5 \parallel 2) = 14.3\text{ V/V}$$

(c) $v_{i2} = (A_{vo}v_i)\dfrac{R_{\text{in}}}{R_{\text{in}} + R_o}$

$$= 0.99 \times v_i \times \frac{99}{99 + 99}$$

$$\simeq 0.5v_i$$

$$v_o = 14.3 \times v_{i2} = 14.3 \times 0.5v_i$$

$$\frac{v_o}{v_i} = 7.15\text{ V/V}$$

7.123 (a) DC bias:

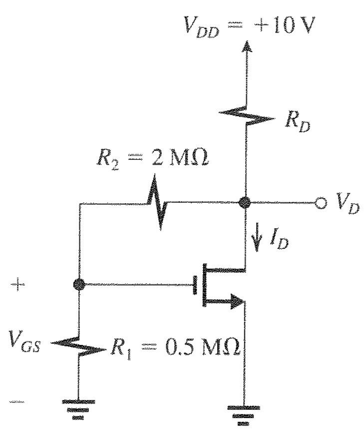


Figure 1

$$V_{GS} = V_t + V_{OV}$$

$$= 0.6 + 0.2 = 0.8\text{ V}$$

From the voltage divider (R_1, R_2: see Fig. 1), we can write

$$V_{GS} = V_D\frac{R_1}{R_1 + R_2} = V_D\,\frac{0.5}{0.5 + 2}$$

$$V_D = 5V_{GS} = 5 \times 0.8 = 4\text{ V}$$

$$I_D = \frac{1}{2}k_n V_{OV}^2\left(1 + \frac{V_{DS}}{V_A}\right)$$

$$I_D = \frac{1}{2} \times 5 \times 0.2^2\left(1 + \frac{4}{60}\right)$$

$$= 0.107\text{ mA}$$

The current in the voltage divider is

$$I = \frac{V_D}{R_1 + R_2} = \frac{4}{2.5} = 1.6\ \mu\text{A} = 0.0016\text{ mA}$$

Thus the current through R_D will be $(0.107 + 0.0016) \simeq 0.109$ mA and

$$R_D = \frac{V_{DD} - V_D}{0.109} = \frac{10 - 4}{0.109} = 55\text{ k}\Omega$$

(b) $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.107}{0.2} = 1.07\text{ mA/V}$

$$r_o = \frac{V_A}{I_D} = \frac{60}{0.107} = 561\text{ k}\Omega$$

(c) Upon replacing the MOSFET with its hybrid-π model, we obtain the small-signal equivalent circuit of the amplifier, shown in Fig. 2.

Node equation at the output:

$$\frac{v_o}{R_D} + \frac{v_o}{r_o} + \frac{v_o - v_{gs}}{R_2} + g_m v_{gs} = 0$$

$$v_o\left(\frac{1}{R_D} + \frac{1}{r_o} + \frac{1}{R_2}\right) = -g_m\left(1 - \frac{1}{g_m R_2}\right)v_{gs}$$

Thus,

$$v_o = \overbrace{-\,g_m(R_D \parallel r_o \parallel R_2)\left(1 - \frac{1}{g_m R_2}\right)}^{A}v_{gs} \qquad (1)$$

Next, we express v_{gs} in terms of v_{sig} and v_o using superposition:

$$v_{gs} = v_{\text{sig}}\frac{R_2}{R_1 + R_2} + v_o\frac{R_1}{R_1 + R_2} \qquad (2)$$

This figure belongs to Problem 7.123(c).

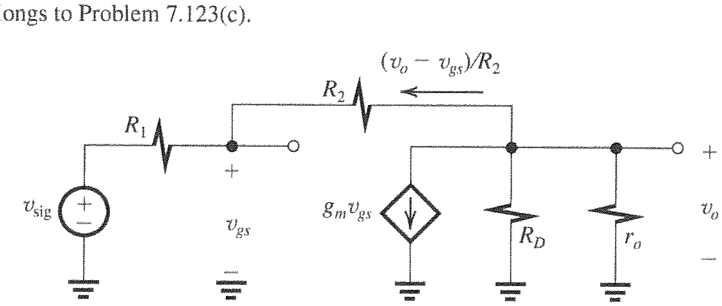


Figure 2

Substituting for v_{gs} from Eq. (2) into Eq. (1) yields

$$v_o = -Av_{\text{sig}}\frac{R_2}{R_1+R_2} - Av_o\frac{R_1}{R_1+R_2}$$

where

$$A = g_m(R_D \parallel r_o \parallel R_2)\left(1-\frac{1}{g_mR_2}\right)$$

Thus,

$$v_o\left(1+A\frac{R_1}{R_1+R_2}\right) = -A\frac{R_2}{R_1+R_2}v_{\text{sig}}$$

$$\frac{v_o}{v_{\text{sig}}} = \frac{-A\dfrac{R_2}{R_1+R_2}}{1+A\dfrac{R_1}{R_1+R_2}}$$

$$= \frac{-R_2/R_1}{1+\dfrac{1+R_2/R_1}{A}}$$

Thus,

$$\frac{v_o}{v_{\text{sig}}} = -\frac{R_2/R_1}{1+\dfrac{1+R_2/R_1}{g_m(R_D \parallel r_o \parallel R_2)(1-1/g_mR_2)}}$$

Q.E.D

Substituting numerical values yields

$$\frac{v_o}{v_{\text{sig}}} =$$

$$-\frac{2/0.5}{1+\dfrac{1+(2/0.5)}{1.07(55 \parallel 561 \parallel 2000)(1-1/1.07\times 2000)}}$$

$$= -\frac{4}{\left(1+\dfrac{5}{52.6}\right)}$$

$$= -3.65 \text{ V/V}$$

Note that the gain is nearly equal to $-R_2/R_1 = -4$, which is the gain of an op amp connected in the inverting configuration.

7.124 (a) DC bias:

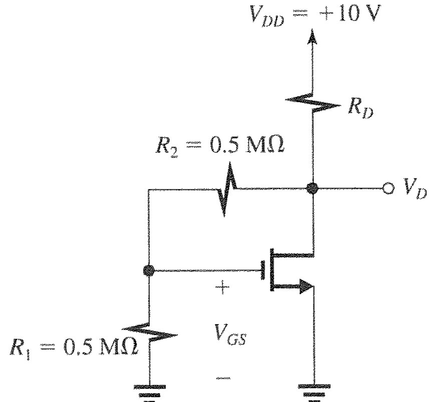


Figure 1

$V_{OV} = 0.2$ V

$V_{GS} = V_t + V_{OV}$

$= 0.6 + 0.2 = 0.8$ V

From the voltage divider (R_1, R_2: see Fig. 1), we can write

$$V_{GS} = \frac{R_1}{R_1+R_2}V_D = \frac{0.5}{0.5+0.5}V_D = 0.5V_D$$

Thus

$V_D = 2V_{GS} = 1.6$ V

$$I_D = \frac{1}{2}k_nV_{OV}^2$$

$$= \frac{1}{2}\times 5\times 0.2^2 = 0.1 \text{ mA}$$

$$I_{\text{divider}} = \frac{V_D}{1\text{ M}\Omega} = \frac{1.6\text{ V}}{1\text{ M}\Omega} = 1.6\ \mu\text{A}$$

$I_{R_D} = 0.1 + 0.0016 \simeq 0.102$ mA

$$R_D = \frac{V_{DD}-V_D}{I_{R_D}} = \frac{10-1.6}{0.102} = 82.4\text{ k}\Omega$$

(b) $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2\times 0.1}{0.2} = 1$ mA/V

(c) Replacing the MOSFET with its T model results in the amplifier equivalent circuit shown in Fig. 2. At the output node,

$$v_o = i[R_D \parallel (R_1+R_2)]$$

$$v_o = iR'_D \qquad (1)$$

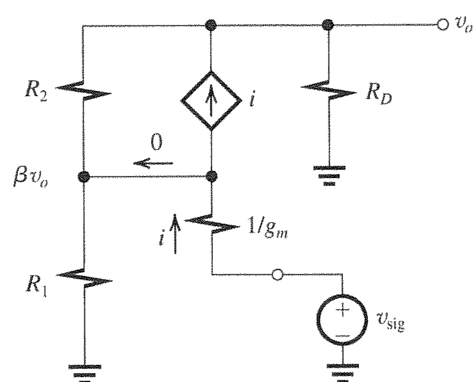


Figure 2

where $R'_D = R_D \parallel (R_1+R_2)$. The voltage at the gate is a fraction β of v_o with

$$\beta = \frac{R_1}{R_1+R_2}$$

Now, the current i can be found from

$$i = \frac{v_{\text{sig}}-\beta v_o}{1/g_m} = g_mv_{\text{sig}} - \beta g_mv_o \qquad (2)$$

Substituting for i from Eq. (2) into Eq. (1) yields

$$v_o = (g_m v_{sig} - \beta g_m v_o)R'_D$$

Thus

$$\frac{v_o}{v_{sig}} = \frac{g_m R'_D}{1 + \beta g_m R'_D}$$

$$= \frac{1/\beta}{1 + \dfrac{1/\beta}{g_m R'_D}}$$

$$= \frac{1 + (R_2/R_1)}{1 + \dfrac{1 + R_2/R_1}{g_m R'_D}} \qquad \text{Q.E.D} \qquad (3)$$

The input resistance R_{in} can be obtained as follows:

$$R_{in} = \frac{v_{sig}}{i}$$

Substituting for i from Eq. (1) yields

$$R_{in} = \frac{v_{sig}}{v_o} R'_D$$

and replacing $\frac{v_{sig}}{v_o}$ by the inverse of the gain expression in Eq. (3) gives

$$R_{in} = R'_D\left[\frac{1}{g_m R'_D} + \frac{1}{1 + (R_2/R_1)}\right]$$

$$R_{in} = \frac{1}{g_m}\left[1 + g_m R'_D \frac{R_1}{R_1 + R_2}\right] \qquad \text{Q.E.D}$$

(d) Substituting numerical values:

$$\frac{v_o}{v_{sig}} = \frac{1 + (0.5/0.5)}{1 + \dfrac{1 + (0.5/0.5)}{1 \times (82.4 \parallel 1000)}}$$

$$= \frac{2}{1 + \dfrac{2}{76.13}} = 1.95 \text{ V/V}$$

Note that the gain $\simeq 1 + \frac{R_2}{R_1} = 2$, similar to that of an op amp connected in the noninverting configuration!

$$R_{in} = \frac{1}{1}\left[1 + 1 \times (82.4 \parallel 1000)\frac{0.5}{0.5 + 0.5}\right]$$

$$= 39.1 \text{ k}\Omega$$

7.125 Refer to the circuit of Fig. P7.125.

$$I_C = \frac{\alpha(V_{BB} - V_{BE})}{R_E + \dfrac{R_B}{\beta + 1}}$$

where

$$V_{BB} = V_{CC}\frac{R_2}{R_2 + R_1} = 15 \times \frac{15}{15 + 27} = 5.357 \text{ V}$$

$$R_B = R_1 \parallel R_2 = 15 \parallel 27 = 9.643 \text{ k}\Omega$$

$$I_C = \frac{0.99(5.357 - 0.7)}{2.4 + \dfrac{9.643}{101}} = 1.85 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{1.85 \text{ mA}}{0.025 \text{ V}} = 74 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{74} = 1.35 \text{ k}\Omega$$

Replacing the BJT with its hybrid-π model results in the equivalent circuit shown at the bottom of the page:

$$R_{in} = R_1 \parallel R_2 \parallel r_\pi = R_B \parallel r_\pi = 9.643 \parallel 1.35$$

$$= 1.18 \text{ k}\Omega$$

$$\frac{v_\pi}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{1.18}{1.18 + 2} = 0.371 \text{ V/V}$$

$$\frac{v_o}{v_\pi} = -g_m(R_C \parallel R_L)$$

$$= -74(3.9 \parallel 2) = -97.83$$

$$\frac{v_o}{v_{sig}} = -0.371 \times 97.83 = -36.3 \text{ V/V}$$

7.126 Refer to the circuit of Fig. P7.125.

DC design:

$V_B = 5$ V, $\quad V_{BE} = 0.7$ V

$V_E = 4.3$ V

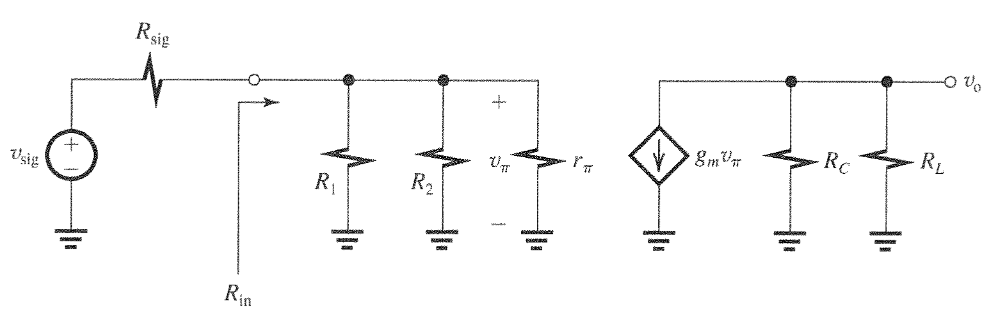
For

$$I_E = 2 \text{ mA}, \quad R_E = \frac{V_E}{I_E} = \frac{4.3}{2} = 2.15 \text{ k}\Omega$$

$$I_{R_2} = 0.2 \text{ mA}, \quad R_2 = \frac{5}{0.2} = 25 \text{ k}\Omega$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{2}{101} \simeq 0.02 \text{ mA}$$

This figure belongs to Problem 7.125.



$$I_{R_1} = I_{R_2} + I_B = 0.2 + 0.02 = 0.22 \text{ mA}$$

$$R_1 = \frac{V_{CC} - V_B}{I_{R_1}} = \frac{15 - 5}{0.22} = 45.5 \text{ k}\Omega$$

Choosing 5% resistors:

$$R_E = 2.2 \text{ k}\Omega, \quad R_1 = 47 \text{ k}\Omega, \quad R_2 = 24 \text{ k}\Omega$$

For these values,

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \dfrac{R_B}{\beta + 1}}$$

where

$$V_{BB} = V_{CC}\frac{R_2}{R_1 + R_2} = 15 \times \frac{24}{24 + 47} = 5.07 \text{ V}$$

$$R_B = R_1 \parallel R_2 = 47 \parallel 24 = 15.89 \text{ k}\Omega$$

$$I_E = \frac{5.07 - 0.7}{2.2 + \dfrac{15.89}{101}} = 1.85 \text{ mA}$$

$$V_B = I_E R_E + V_{BE} = 1.85 \times 2.2 + 0.7 = 4.8 \text{ V}$$

$$I_C = \alpha I_E = 0.99 \times 1.85 = 1.84 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{1.84}{0.025} = 73.4 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{73.4} = 1.36 \text{ k}\Omega$$

$$R_{\text{in}} = R_1 \parallel R_2 \parallel r_\pi = 47 \parallel 24 \parallel 1.36 = 1.25 \text{ k}\Omega$$

$$\frac{v_\pi}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \frac{1.25}{1.25 + 2} = 0.385 \text{ V/V}$$

For an overall gain of -40 V/V,

$$\frac{v_o}{v_\pi} = -\frac{40}{0.385} = -104 \text{ V/V}$$

But

$$\frac{v_o}{v_\pi} = -g_m(R_C \parallel R_L)$$

$$-104 = -73.4\ (R_C \parallel 2)$$

$$(R_C \parallel 2) = 1.416$$

$$R_C = 4.86 \text{ k}\Omega$$

We can select either 4.7 kΩ or 5.1 kΩ. With 4.7 kΩ, the gain will be

$$\frac{v_o}{v_{\text{sig}}} = -0.385 \times 73.4 \times (4.7 \parallel 2) = -39.6 \text{ V/V}$$

which is slightly lower than the required -40 V/V, and we will obtain

$$V_C = 15 - 4.7 \times 1.84 = 6.4 \text{ V}$$

allowing for about 2 V of negative signal swing at the collector. If we choose 5.1 kΩ, the gain will be

$$\frac{v_o}{v_{\text{sig}}} = -0.385 \times 73.4 \times (5.1 \parallel 2) = -40.6 \text{ V/V}$$

which is slightly higher than the required gain, and we will obtain

$$V_C = 15 - 5.1 \times 1.84 = 5.6 \text{ V}$$

which allows for only 1.2-V negative signal swing.

7.127 Refer to the circuit of Fig. P7.125:

$$I_C = \frac{\alpha(V_{BB} - V_{BE})}{R_E + \dfrac{R_B}{\beta + 1}}$$

where

$$V_{BB} = V_{CC}\frac{R_2}{R_2 + R_1} = 15 \times \frac{47}{47 + 82} = 5.465 \text{ V}$$

$$R_B = R_1 \parallel R_2 = 47 \parallel 82 = 29.88 \text{ k}\Omega$$

$$I_C = \frac{0.99(5.465 - 0.7)}{7.2 + \dfrac{29.88}{101}} = 0.63 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.63}{0.025} = 25.2 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{25.2} = 4 \text{ k}\Omega$$

$$R_{\text{in}} = R_1 \parallel R_2 \parallel r_\pi = R_B \parallel r_\pi$$

$$= 29.88 \parallel 4 = 3.5 \text{ k}\Omega$$

$$\frac{v_\pi}{v_{\text{sig}}} = \frac{3.5}{3.5 + 2} = 0.636 \text{ V/V}$$

$$\frac{v_o}{v_\pi} = -g_m(R_C \parallel R_L)$$

$$= -25.2(12 \parallel 2) = -43.2 \text{ V/V}$$

$$\frac{v_o}{v_{\text{sig}}} = -0.636 \times 43.2 = -27.5 \text{ V/V}$$

Comparing the results above to those of Problem 7.125, we see that raising the resistance values has indeed resulted in increasing the transmission from source to transistor base, from 0.371 V/V to 0.636 V/V. However, because I_C has decreased and g_m has correspondingly decreased, the gain from base to collector has decreased by a larger factor (from 97.83 V/V to 43.2 V/V), with the result that the overall gain has in fact decreased (from 36.3 V/V to 27.5 V/V). Thus, this is not a successful strategy!

7.128 Refer to the circuit of Fig. P7.128.

DC voltage drop across $R_B = 0.2$ V, and

$$I_B R_B = 0.2 \text{ V}$$

$$\frac{I}{\beta + 1} R_B = 0.2 \text{ V}$$

$$I R_B = 0.2 \times 101 \qquad (1)$$

$R_{\text{in}} = R_B \parallel r_\pi = 10\ \text{k}\Omega$

$$R_B \parallel \frac{V_T}{I_B} = 10$$

$$R_B \parallel \frac{0.025}{I/(\beta+1)} = 10$$

$$R_B \parallel \left(\frac{0.025 \times 101}{I}\right) = 10$$

$$\frac{R_B \times \dfrac{0.025 \times 101}{I}}{R_B + \dfrac{0.025 \times 101}{I}} = 10$$

$$\frac{0.025 \times 101 R_B}{IR_B + 0.025 \times 101} = 10 \qquad (2)$$

Substituting for IR_B from Eq. (1) yields

$$\frac{0.025 \times 101 R_B}{0.2 \times 101 + 0.025 \times 101} = 10$$

$$\frac{0.025 R_B}{0.225} = 10$$

$\Rightarrow R_B = 90\ \text{k}\Omega$

$$I = \frac{0.2 \times 101}{90} = 0.22\ \text{mA}$$

To maximize the open-circuit voltage gain between base and collector while ensuring that the instantaneous collector voltage does not fall below $(v_B - 0.4)$ when v_{be} is as high as 5 mV, we impose the constraint

$V_C - |A_{vo}| \times 0.005 = V_B + 0.005 - 0.4$

where

$V_C = V_{CC} - I_C R_C$

$= 5 - 0.99 \times 0.22 R_C$

$= 5 - 0.22 R_C$

$$|A_{vo}| = g_m R_C = \frac{0.99 \times 0.22}{0.025} R_C = 8.7 R_C$$

and

$$V_B = -\frac{0.22}{101} \times 90 = -0.2\ \text{V}$$

Thus,

$5 - 0.22 R_C - 8.7 R_C \times 0.005 = -0.2 - 0.395$

$\Rightarrow R_C = 21.2\ \text{k}\Omega$

Selecting 5% resistors, we find

$R_B = 91\ \text{k}\Omega$

$R_C = 22\ \text{k}\Omega$

and specifying I to one significant digit gives

$I = 0.2\ \text{mA}$

$$g_m = \frac{\alpha I_C}{V_T} \simeq \frac{0.2}{0.025} = 8\ \text{mA/V}$$

$A_{vo} = -g_m R_C = -8 \times 22 = -176\ \text{V/V}$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{8} = 12.5\ \text{k}\Omega$$

$R_{\text{in}} = R_B \parallel r_\pi = 91 \parallel 12.5 = 11\ \text{k}\Omega$

$$G_v = -\frac{11}{20 + 11} \times 8(22 \parallel 20)$$

$= -29.7\ \text{V/V}$

7.129 Refer to the circuit of Fig. P7.129.

(a) $I_E = 0.5$ mA. Writing a loop equation for the base–emitter circuit results in

$I_B R_{\text{sig}} + V_{BE} + I_E R_E = 3$

$$\frac{I_E}{\beta + 1} R_{\text{sig}} + V_{BE} + I_E R_E = 3$$

$$\frac{0.5}{101} \times 2.5 + 0.7 + 0.5 R_E = 3$$

$\Rightarrow R_E = 4.6\ \text{k}\Omega$

(b) $I_C = \alpha I_E \simeq 0.5$ mA

$V_C = 0.5 = 3 - 0.5 R_C$

$\Rightarrow R_C = 5\ \text{k}\Omega$

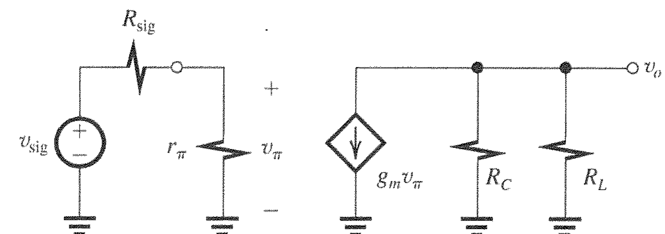
$$\text{(c) } g_m = \frac{I_C}{V_T} = \frac{0.5\ \text{mA}}{0.025\ \text{V}} = 20\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5\ \text{k}\Omega$$

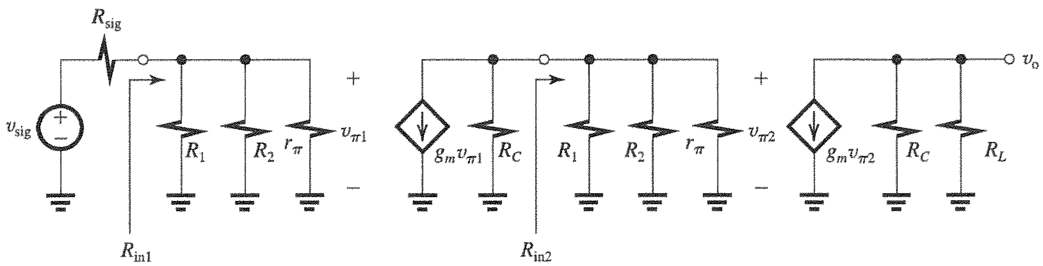
$$G_v = \frac{v_o}{v_{\text{sig}}} = -\frac{5}{5 + 2.5} \times 20 \times (5 \parallel 10)$$

$= -44.4\ \text{V/V}$

This figure belongs to Problem 7.129.



This figure belongs to Problem 7.130.



7.130 Refer to the circuit of Fig. P7.130.

(a) DC analysis of each of the two stages:

$$V_{BB} = V_{CC}\frac{R_2}{R_1 + R_2} = 15\frac{47}{100 + 47} = 4.8\text{ V}$$

$$R_B = R_1 \parallel R_2 = 100 \parallel 47 = 32\text{ k}\Omega$$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \dfrac{R_B}{\beta + 1}}$$

$$= \frac{4.8 - 0.7}{3.9 + \dfrac{32}{101}} = 0.97\text{ mA} \simeq 1\text{ mA}$$

$$I_C = \alpha I_E \simeq 1\text{ mA}$$

$$V_C = V_{CC} - I_C R_C = 15 - 1 \times 6.8 = 8.2\text{ V}$$

(b) See figure above.

$$g_m = \frac{I_C}{V_T} = 40\text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = 2.5\text{ k}\Omega$$

(c) $R_{in1} = R_1 \parallel R_2 \parallel r_\pi = R_B \parallel r_\pi = 32 \parallel 2.5$

$= 2.32\text{ k}\Omega$

$$\frac{v_{b1}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{2.32}{2.32 + 5} = 0.32\text{ V/V}$$

(d) $R_{in2} = R_1 \parallel R_2 \parallel r_\pi = R_{in1} = 2.32\text{ k}\Omega$

$$\frac{v_{b2}}{v_{b1}} = \frac{v_{b2}}{v_{\pi 1}} = -g_m(R_C \parallel R_{in2})$$

$$= -40(6.8 \parallel 2.32) = -69.2\text{ V/V}$$

(e) $\dfrac{v_o}{v_{b2}} = \dfrac{v_o}{v_{\pi 2}} = -g_m(R_C \parallel R_L)$

$$= -40(6.8 \parallel 2) = -61.8\text{ V/V}$$

(f) $\dfrac{v_o}{v_{sig}} = \dfrac{v_o}{v_{b2}} \times \dfrac{v_{b2}}{v_{b1}} \times \dfrac{v_{b1}}{v_{sig}} = -61.8$

$\times -69.2 \times 0.32 = 1368.5\text{ V/V}$

7.131 Refer to the circuit in Fig. P7.131:

$$I_E = 0.1\text{ mA}$$

$$r_e = \frac{V_T}{I_E} = \frac{25\text{ mV}}{0.1\text{ mA}} = 250\ \Omega$$

$$g_m = \frac{I_C}{V_T} \simeq \frac{0.1\text{ mA}}{0.025\text{ V}} = 4\text{ mA/V}$$

Note that the emitter has a resistance $R_e = 250\ \Omega$.

$$R_{in} = 200\text{ k}\Omega \parallel (\beta + 1)(r_e + R_e)$$

$$= 200 \parallel [101 \times (0.25 + 0.25)]$$

$$= 200 \parallel 50.5 = 40.3\text{ k}\Omega$$

$$\frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{40.3}{40.3 + 20} = 0.668\text{ V/V}$$

$$\frac{v_o}{v_b} = -\alpha\frac{\text{Total resistance in collector}}{\text{Total resistance in emitter}}$$

$$\simeq -\frac{20 \parallel 20}{0.25 + 0.25} = -20\text{ V/V}$$

$$G_v = \frac{v_o}{v_{sig}} = -0.668 \times 20 = -13.4\text{ V/V}$$

For v_{be} to be limited to 5 mV, the signal between base and ground will be 10 mV (because of the 5 mV across R_e). The limit on v_{sig} can be obtained by dividing the 10 mV by v_b/v_{sig},

$$\hat{v}_{sig} = \frac{10\text{ mV}}{0.668} = 15\text{ mV}$$

Correspondingly, at the output we have

$$\hat{v}_o = |G_v|\hat{v}_{sig} = 13.4 \times 15 = 200\text{ mV} = 0.2\text{ V}$$

7.132 (a)

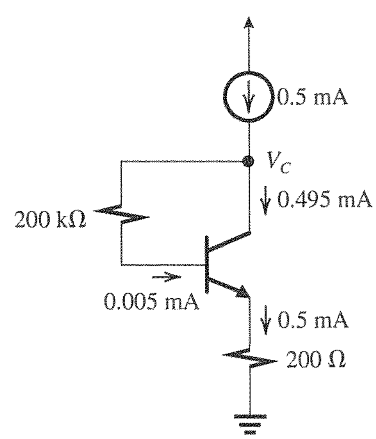


Figure 1

From Fig. 1 we see that

$I_C = 0.495$ mA

$V_C = I_B \times 200\ \text{k}\Omega + I_E \times 0.2\ \text{k}\Omega + V_{BE}$

$= 0.005 \times 200 + 0.5 \times 0.2 + 0.7$

$= 1.18$ V

(b)

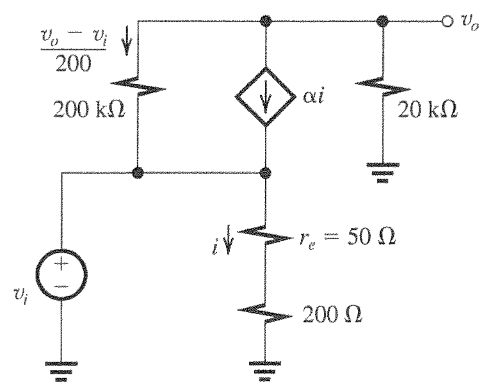


Figure 2

From Fig. 2, we have

$$g_m = \frac{I_C}{V_T} = \frac{0.495}{0.025} \simeq 20\ \text{mA/V}$$

$$r_e = \frac{V_T}{I_E} = 50\ \Omega$$

$$i = \frac{v_i}{r_e + R_e} = \frac{v_i}{50 + 200}$$

$$= \frac{v_i}{250\ \Omega} = \frac{v_i}{0.25\ \text{k}\Omega} = 4\ v_i,\ \text{mA}$$

Node equation at the output:

$$\frac{v_o}{20} + \alpha i + \frac{v_o - v_i}{200} = 0$$

$$\frac{v_o}{20} + 0.99 \times 4v_i + \frac{v_o}{200} - \frac{v_i}{200} = 0$$

$$v_o\left(\frac{1}{20} + \frac{1}{200}\right) = -v_i\left(4 \times 0.99 - \frac{1}{200}\right)$$

$$\frac{v_o}{v_i} = -71.9\ \text{V/V}$$

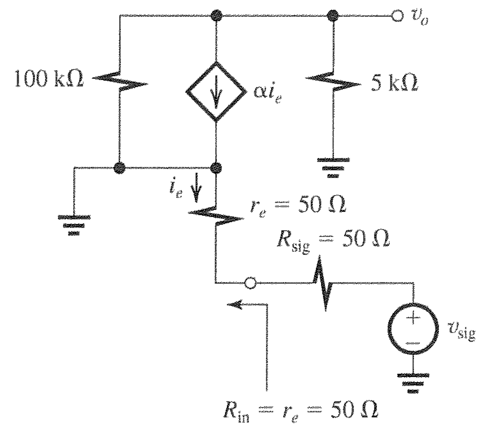
7.133 Refer to the circuit in Fig. P7.133.

The dc emitter current is equal to 0.5 mA, and $I_C = \alpha I_E \simeq 0.5$ mA; also,

$$r_e = \frac{V_T}{I_E} = \frac{25\ \text{mV}}{0.5\ \text{mA}} = 50\ \Omega$$

$R_{in} = r_e = 50\ \Omega$

$$i_e = \frac{-v_{sig}}{r_e + R_{sig}} = \frac{-v_{sig}}{50 + 50}$$



$$= \frac{-v_{sig}}{100\ \Omega} = \frac{-v_{sig}}{0.1\ \text{k}\Omega}$$

At the output node,

$v_o = -\alpha i_e (5 \parallel 100)$

$$= \alpha \frac{v_{sig}}{0.1}(5 \parallel 100)$$

$$\frac{v_o}{v_{sig}} = \alpha \frac{5 \parallel 100}{0.1} \simeq 47.6\ \text{V/V}$$

7.134 (a) $I_E = \dfrac{3 - 0.7}{1 + \dfrac{100}{\beta + 1}}$

$\beta = 50$:

$$I_E = \frac{2.3}{1 + \frac{100}{51}} = 0.78\ \text{mA}$$

$V_E = I_E R_E = 0.78$ V

$V_B = V_E + 0.7 = 1.48$ V

$\beta = 200$:

$$I_E = \frac{2.3}{1 + \frac{100}{201}} = 1.54\ \text{mA}$$

$V_E = I_E R_E = 1.54$ V

$V_B = V_E + 0.7 = 2.24$ V

(b) $R_{in} = 100 \parallel (\beta + 1)[r_e + (1 \parallel 1)]$

$= 100 \parallel (\beta + 1)(r_e + 0.5)$

$\beta = 50$:

$$r_e = \frac{V_T}{I_E} = \frac{25\ \text{mV}}{0.78\ \text{mA}} = 32.1\ \Omega$$

$R_{in} = 100 \parallel [51 \times (0.0321 + 0.5)]$

$= 21.3\ \text{k}\Omega$

$\beta = 200$:

$$r_e = \frac{V_T}{I_E} = \frac{25\ \text{mV}}{1.54\ \text{mA}} = 16.2\ \Omega$$

$$R_{in} = 100 \parallel [201 \times (0.0162 + 0.5)]$$

$$= 50.9 \text{ k}\Omega$$

(c) $$\frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}}$$

$$\frac{v_o}{v_b} = \frac{(1 \parallel 1)}{(1 \parallel 1) + r_e} = \frac{500}{500 + r_e} (r_e \text{ in } \Omega)$$

$\beta = 50$:

$$\frac{v_b}{v_{sig}} = \frac{21.3}{21.3 + 10} = 0.68 \text{ V/V}$$

$$\frac{v_o}{v_b} = \frac{500}{500 + 32.1} = 0.94 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = 0.68 \times 0.94 = 0.64 \text{ V/V}$$

$\beta = 200$:

$$\frac{v_b}{v_{sig}} = \frac{50.9}{50.9 + 10} = 0.836 \text{ V/V}$$

$$\frac{v_o}{v_b} = \frac{500}{500 + 16.2} = 0.969 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = 0.836 \times 0.969 = 0.81 \text{ V/V}$$

7.135 Refer to the circuit in Fig. P7.135.

$$I_E = \frac{3 - 0.7}{3.3 + \frac{100}{\beta + 1}}$$

$$= \frac{2.3}{3.3 + \frac{100}{101}} = 0.54 \text{ mA}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.54 \text{ mA}} = 46.3 \ \Omega$$

$$R_{in} = (\beta + 1)[r_e + (3.3 \parallel 2)]$$

$$= 101 \times (0.0463 + 1.245)$$

$$= 130.4 \text{ k}\Omega$$

$$\frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{130.4}{130.4 + 100}$$

$$= 0.566 \text{ V/V}$$

$$\frac{v_o}{v_b} = \frac{3.3 \parallel 2}{(3.3 \parallel 2) + r_e} = \frac{1.245}{1.245 + 0.0463}$$

$$= 0.964 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = 0.566 \times 0.964 = 0.55 \text{ V/V}$$

$$i_o = \frac{v_o}{2 \text{ k}\Omega}$$

$$i_i = \frac{v_i}{R_{in}} = \frac{v_b}{130.4 \text{ k}\Omega}$$

$$\frac{i_o}{i_i} = \frac{v_o}{v_b} \times \frac{130.4}{2} = 0.964 \times 65.2$$

$$= 62.9 \text{ A/A}$$

$$R_{out} = 3.3 \parallel \left(r_e + \frac{100}{\beta + 1}\right)$$

$$= 3.3 \parallel \left(0.0463 + \frac{100}{101}\right)$$

$$= 0.789 \text{ k}\Omega = 789 \ \Omega$$

7.136 Refer to the circuit in Fig. P7.136.

For dc analysis, open-circuit the two coupling capacitors. Then replace the 9-V source and the two 20-kΩ resistors by their Thévenin equivalent, namely, a 4.5-V source and a 10-kΩ series resistance. The latter can be added to the 10-kΩ resistor that is connected to the base. The result is the circuit shown in Fig. 1, which can be used to calculate I_E.

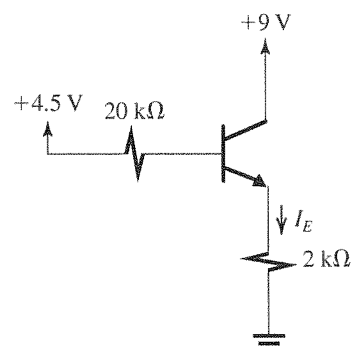


Figure 1

(a) $$I_E = \frac{4.5 - 0.7}{2 + \frac{20}{\beta + 1}}$$

$$= \frac{3.8}{2 + \frac{20}{101}} = 1.73 \text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 1.73 \text{ mA}$$

$$= 1.71 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = 68.4 \text{ mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{1.73 \text{ mA}} = 14.5 \ \Omega$$

$$= 0.0145 \text{ k}\Omega$$

$$r_\pi = (\beta + 1) r_e = 101 \times 0.0145$$

$$= 1.4645 \text{ k}\Omega$$

(b) Replacing the BJT with its T model (without r_o) and replacing the capacitors with short circuits results in the equivalent-circuit model shown in Fig. 2.

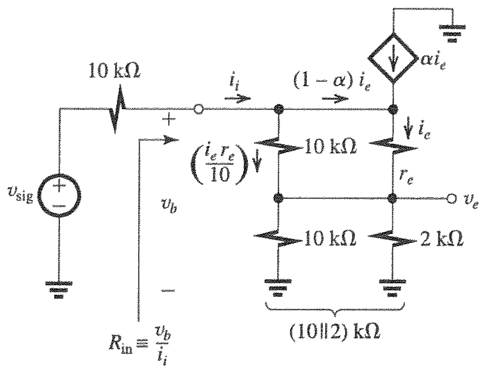


Figure 2

From Fig. 2 we see that

$$v_e = \left(i_e + i_e\frac{r_e}{10}\right)(10 \parallel 2)$$

$$v_b = v_e + i_e r_e = i_e(10 \parallel 2)\left(1 + \frac{r_e}{10}\right) + i_e r_e$$

$$i_i = (1-\alpha)i_e + i_e\frac{r_e}{10}$$

$$= \frac{i_e}{\beta+1} + i_e\frac{r_e}{10}$$

We can now obtain R_{in} from

$$R_{in} \equiv \frac{v_b}{i_i} = \frac{(10 \parallel 2)\left(1 + \frac{r_e}{10}\right) + r_e}{\frac{1}{\beta+1} + \frac{r_e}{10}}$$

$$= \frac{(\beta+1)(10 \parallel 2)\left(1 + \frac{r_e}{10}\right) + (\beta+1)r_e}{1 + (\beta+1)\frac{r_e}{10}}$$

$$= \frac{101 \times (10 \parallel 2) \times (1 + 0.00145) + 101 \times 0.0145}{1 + 101 \times 0.00145}$$

$$= \frac{168.577 + 1.4645}{1 + 0.14645} = 148.3\ \text{k}\Omega$$

$$\frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{148.3}{148.3 + 10} = 0.937$$

$$\frac{v_o}{v_b} = \frac{v_e}{v_b} = \frac{i_e\left(1 + \frac{r_e}{10}\right)(10 \parallel 2)}{i_e\left(1 + \frac{r_e}{10}\right)(10 \parallel 2) + i_e r_e}$$

$$= \frac{1.00145 \times (10 \parallel 2)}{1.00145 \times (10 \parallel 2) + 0.0145}$$

$$= 0.991\ \text{V/V}$$

$$G_v \equiv \frac{v_o}{v_{sig}} = 0.937 \times 0.991 = 0.93\ \text{V/V}$$

(c) When C_B is open-circuited, the equivalent circuit becomes that shown in Fig. 3.

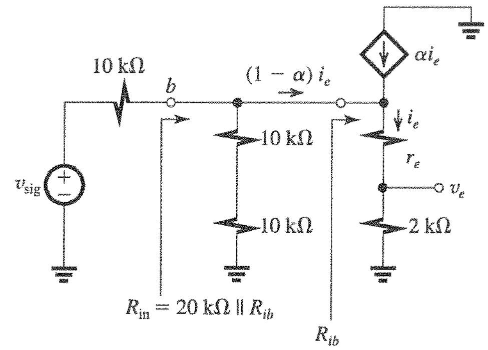


Figure 3

Thus,

$$R_{in} = 20\ \text{k}\Omega \parallel R_{ib}$$

$$= 20\ \text{k}\Omega \parallel (\beta+1)(R_e + 2)$$

$$= 20 \parallel 101 \times 2.0145$$

$$= 18.21\ \text{k}\Omega$$

which is greatly reduced because of the absence of bootstrapping. The latter causes the lower node of the 10-kΩ base-biasing resistor to rise with the output voltage, thus causing a much reduced signal current in the 10-kΩ resistor and a correspondingly larger effective resistance across the amplifier input.

The reduced R_{in} will result in a reduction in v_b/v_{sig},

$$\frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{18.21}{28.21}$$

$$= 0.646\ \text{V/V}$$

$$\frac{v_o}{v_b} = \frac{2}{2 + 0.0145} = 0.993$$

$$G_v \equiv \frac{v_o}{v_{sig}} = 0.646 \times 0.993$$

$$= 0.64\ \text{V/V}$$

which is much reduced relative to the value obtained with bootstrapping.

7.137

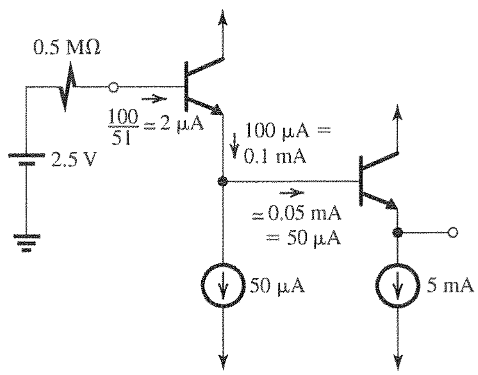
(a) Applying Thévenin's theorem to the base-biasing circuit of Q_1 results in the dc circuit shown below. From our partial analysis on the figure, we can write

$$I_{E1} = 0.1\ \text{mA}$$

$$I_{E2} = 5\ \text{mA}$$

V_{B1} can be obtained as

$$V_{B1} = 2.5 - 2\ \mu\text{A} \times 0.5\ \text{M}\Omega = 1.5\ \text{V}$$



and V_{B2} can be found as

$V_{B2} = V_{B1} - 0.7 = 0.8$ V

(b) Refer to the circuit in Fig. P7.137. With a load resistance $R_L = 1$ kΩ connected to the output terminal, the voltage gain v_o/v_{b2} can be found as

$$\frac{v_o}{v_{b2}} = \frac{R_L}{R_L + r_{e1}}$$

where

$$r_{e2} = \frac{25\text{ mV}}{5\text{ mA}} = 5\ \Omega$$

$$\frac{v_o}{v_{b2}} = \frac{1000}{1000+5} = 0.995\text{ V/V}$$

$$R_{ib2} = (\beta_2 + 1)(r_{e2} + R_L)$$

$$= 101 \times 1.005 = 101.5\text{ k}\Omega$$

(c) $R_{\text{in}} = 1\text{ M}\Omega \parallel 1\text{ M}\Omega \parallel (\beta + 1)(r_{e1} + R_{ib2})$

where

$$r_e = \frac{V_T}{I_{E1}} = \frac{25\text{ mV}}{0.1\text{ mA}} = 250\ \Omega = 0.25\text{ k}\Omega$$

$$R_{\text{in}} = 0.5\text{ M}\Omega \parallel [51 \times (0.25 + 101.5)]\text{ k}\Omega$$

$$= 0.5\text{ M}\Omega \parallel 5.2\text{ M}\Omega$$

$$= 456\text{ k}\Omega$$

$$\frac{v_{e1}}{v_{b1}} = \frac{R_{ib}}{R_{ib} + r_{e1}} = \frac{101.5}{101.5 + 0.25}$$

$$= 0.9975\text{ V/V}$$

(d) $\dfrac{v_{b1}}{v_{\text{sig}}} = \dfrac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \dfrac{456}{456 + 100} = 0.82\text{ V/V}$

(e) $\dfrac{v_o}{v_{\text{sig}}} = 0.82 \times 0.9975 \times 0.995 = 0.814\text{ V/V}$

7.138 We need to raise f_H by a factor of

$\dfrac{2\text{ MHz}}{500\text{ kHz}} = 4$. Thus

$$1 + g_m R_e = 4$$

$$\Rightarrow R_e = \frac{3}{g_m}$$

Since

$$g_m = \frac{I_C}{V_T} = \frac{1\text{ mA}}{0.025\text{ V}} = 40\text{ mA/V}$$

$$R_e = \frac{3}{0.04} = 75\ \Omega$$

The new value of f_L will be

$$f_L = \frac{100\text{ Hz}}{1 + g_m R_e} = \frac{100}{4} = 25\text{ Hz}$$

and the midband gain will become

$$|A_M| = \frac{100}{1 + g_m R_e} = \frac{100}{4} = 25\text{ V/V}$$

8.1 Referring to Fig. 8.1, $V_{DD} = 1.3$ V,

$I_O = I_{REF} = 100\ \mu$A, $L = 0.5\ \mu$m, $W = 5\ \mu$m, $V_A' = 5$ V/μm, $V_t = 0.4$ V, $k_n' = 500\ \mu$A/V^2

$$I_O = I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2$$

$$V_{OV} = \sqrt{\frac{2I_D}{k_n'\left(\frac{W}{L}\right)}}$$

$$= \sqrt{\frac{2\,(100\ \mu\text{A})}{(500\ \mu\text{A/V}^2)\left(\frac{5}{0.5}\right)}} = 0.2\ \text{V}$$

$$V_{DS} = V_{GS} = V_t + V_{OV} = 0.4 + 0.2 = 0.6\ \text{V}$$

$$R = \frac{V_{DD} - V_{GS}}{I_{REF}} = \frac{1.8 - 0.6}{0.1\ \text{mA}} = 12\ \text{k}\Omega$$

The lowest V_O will be

$$V_{DS2} = V_{OV} = 0.2\ \text{V}$$

$$R_O = r_o = \frac{V_A'L}{I_D} = \frac{5\ \text{V}/\ \mu\text{m} \times 0.5\ \mu\text{m}}{100\ \mu\text{A}} = 25\ \text{k}\Omega$$

$$\Delta I_D \approx \frac{\Delta V_O}{r_o} = \frac{0.5\ \text{V}}{25\ \text{K}} = 20\ \mu\text{A}$$

8.2 Refer to Fig. 8.1.

$$\frac{\Delta I_O}{I_O} = 10\%$$

$$\Delta I_O = 0.1 \times 150 = 15\ \mu\text{A}$$

$$\Delta V_O = 1.8 - 0.3 = 1.5\ \text{V}$$

$$r_o = \frac{\Delta V_O}{\Delta I_O} = \frac{1.5\ \text{V}}{15\ \mu\text{A}} = 100\ \text{k}\Omega$$

But

$$r_o = \frac{V_A}{I_O} = \frac{V_A'L}{I_O}$$

$$100 = \frac{10 \times L}{0.15} \Rightarrow L = 1.5\ \mu\text{m}$$

$$\Rightarrow V_A = 15\ \text{V}$$

$$V_{OV} = V_{DS2\min} = 0.3\ \text{V}$$

$$V_{GS} = V_t + V_{OV} = 0.5 + 0.3 = 0.8\ \text{V}$$

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2\left(1 + \frac{V_{DS}}{V_A}\right)$$

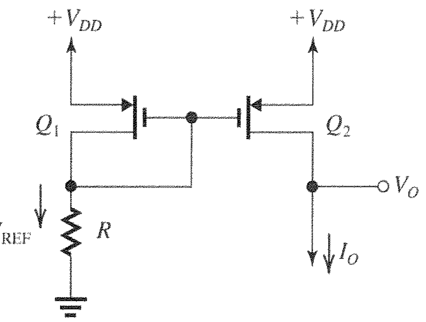
$$150 = \frac{1}{2} \times 400 \times \frac{W}{L} \times 0.09\left(1 + \frac{0.8}{15}\right)$$

$$\Rightarrow \frac{W}{L} = 7.91$$

$$W = 7.91 \times 1.5 = 11.9\ \mu\text{m}$$

$$R = \frac{V_{DD} - V_{GS}}{I_{REF}} = \frac{1.8 - 0.8}{0.15} = 6.7\ \text{k}\Omega$$

8.3



Set $|V_{OV}| = V_{DD} - V_{O\max}$

$$= 1.3 - 1.1 = 0.2\ \text{V}$$

$$V_G = V_{DD} - |V_{tp}| - |V_{OV}|$$

$$= 1.3 - 0.4 - 0.2 = 0.7\ \text{V}$$

$$R = \frac{V_G}{I_{D1}} = \frac{0.7\ \text{V}}{80\ \mu\text{A}} = 8.75\ \text{k}\Omega$$

$$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)|V_{OV}|^2$$

thus

$$\frac{W}{L} = \frac{2I_D}{\mu_p C_{ox}|V_{OV}|^2} = \frac{2 \times 80\ \mu\text{A}}{80\ \mu\text{A/V}^2 \times 0.2^2} = 50$$

8.4 Referring to Fig. 8.2, if $W_2 = 5\ W_1$ and we let $L_1 = L_2$, then we obtain

$$I_O = I_{D2} = I_{REF}\frac{(W/L)_2}{(W/L)_1} = 20\ \mu\text{A} \times 5 = 100\ \mu\text{A}$$

$$V_{O\min} = V_{OV} = 0.2\ \text{V}$$

From Eq. (8.8):

$$I_O = \frac{(W/L)_2}{(W/L)_1} \cdot I_{REF}\left(1 + \frac{V_O - V_{GS}}{V_{A2}}\right)$$

$$V_{GS} = V_t + V_{OV} = 0.5\ \text{V} + 0.2\ \text{V} = 0.7\ \text{V}$$

Thus, I_D equal $5I_{REF}$ will be obtained at

$$V_O = V_{GS} = 0.7\ \text{V}$$

For $V_O = V_{GS} + 1 = 1.7$ V

$$I_O = 100\left(1 + \frac{1.7 - 0.7}{20}\right) = 105\ \mu\text{A}$$

The corresponding increase in I_O, ΔI_O is, thus, 5 μA.

8.5 Referring to Fig. P8.5, we obtain

$V_{GS1} = V_{GS2}$ so that $\dfrac{I_{D2}}{I_{D1}} = \dfrac{(W/L)_2}{(W/L)_1}$ and

$$I_{D2} = I_{REF}\frac{(W/L)_2}{(W/L)_1}$$

$$I_{D3} = I_{D2}$$

$V_{GS3} = V_{GS4}$, thus $\dfrac{I_{D4}}{I_{D3}} = \dfrac{(W/L)_4}{(W/L)_3}$

$$I_O = I_{D4} = I_{REF}\frac{(W/L)_2}{(W/L)_1} \cdot \frac{(W/L)_4}{(W/L)_3}$$

8.6 Refer to the circuit of Fig. P8.6. For Q_2 to operate properly (i.e., in the saturation mode) for drain voltages as high as +0.8 V, and provided its width is the minimum possible, we use

$|V_{OV}| = 0.2$ V

Note that all three transistors Q_1, Q_2, and Q_3 will be operated at this value of overdrive voltage. For Q_1,

$I_{D1} = I_{REF} = 20\ \mu$A

$$I_{D1} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 |V_{OV}|^2$$

$$20 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_1 \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 10$$

For $L = 0.5\ \mu$m,

$W_1 = 5\ \mu$m

Now, for

$I_2 = 100\ \mu A = 5I_{REF}$, we have

$$\frac{(W/L)_2}{(W/L)_1} = 5$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 5 \times 10 = 50$$

$W_2 = 50 \times 0.5 = 25\ \mu$m

For

$I_3 = 40\ \mu A = 2I_{REF}$, we obtain

$$\frac{(W/L)_3}{(W/L)_1} = 2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = 20$$

$W_3 = 10\ \mu$m

We next consider Q_4 and Q_5. For Q_5 to operate in saturation with the drain voltage as low as −0.8 V, and for it to have the minimum possible W/L, we operate Q_5 at

$V_{OV} = 0.2$ V

This is the same overdrive voltage at which Q_4 will be operating. Thus, we can write for Q_4,

$I_4 = I_3 = 40\ \mu$A

and using

$$I_{D4} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_4 V_{OV}^2$$

$$40 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_4 \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_4 = 5$$

$W_4 = 2.5\ \mu$m

Finally, since

$I_5 = 80\ \mu A = 2\ I_4$,

$$\left(\frac{W}{L}\right)_5 = 2\left(\frac{W}{L}\right)_4$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = 10$$

$W_5 = 5\ \mu$m

To find the value of R, we use

$|V_{SG1}| = |V_{tp}| + |V_{OV1}|$

$= 0.5 + 0.2 = 0.7$ V

$$R = \frac{1 - |V_{SG1}|}{I_{REF}} = \frac{0.3\ \text{V}}{0.02\ \text{mA}}$$

$= 15$ kΩ

The output resistance of the current source Q_2 is

$$r_{o2} = \frac{|V_{A2}|}{I_2} = \frac{|V'_{Ap}| \times L}{I_2}$$

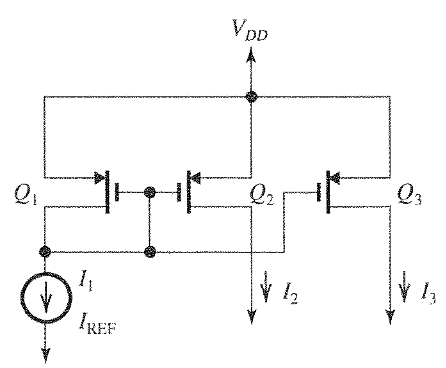
$$= \frac{5 \times 0.5}{0.1\ \text{mA}} = 25\ \text{k}\Omega$$

The output resistance of the current sink Q_5 is

$$r_{o5} = \frac{V_{A5}}{I_5} = \frac{V'_{An} \times L}{I_5}$$

$$= \frac{5 \times 0.5}{80} = 31.25\ \text{k}\Omega$$

8.7 Referring to the figure, suppose that Q_1 has $W = 10\ \mu$m, Q_2 has $W = 20\ \mu$m, and Q_3 has $W = 40\ \mu$m.



(1) With Q_1 diode connected,

$$I_2 = I_{\text{REF}}\frac{(W/L)_2}{(W/L)_1} = 100\ \mu\text{A}\left(\frac{20}{10}\right) = 200\ \mu\text{A}$$

$$I_3 = 100\ \mu\text{A}\left(\frac{40}{10}\right) = 400\ \mu\text{A}$$

(2) With Q_2 diode connected, and $W = 20\ \mu$m,

$$I_1 = 100\ \mu\text{A}\left(\frac{10}{20}\right) = 50\ \mu\text{A}$$

$$I_3 = 100\ \mu\text{A}\left(\frac{40}{20}\right) = 200\ \mu\text{A}$$

(3) If Q_3 with $W = 40\ \mu$m is diode connected,

$$I_1 = 100\ \mu\text{A}\left(\frac{10}{40}\right) = 25\ \mu\text{A}$$

$$I_2 = 100\ \mu\text{A}\left(\frac{20}{40}\right) = 50\ \mu\text{A}$$

So, with only one transistor diode connected, we can get 25 μA, 50 μA, 200 μA, and 400 μA, or four different currents.

Now, if two transistors are diode connected, the effective width is the sum of the two widths.

(4) If Q_1 and Q_2 are diode connected, then

$W_{\text{eff}} = 20 + 10 = 30\ \mu$m, so that

$$I_3 = 100\ \mu\text{A}\left(\frac{40}{30}\right) = 133\ \mu\text{A}$$

(5) If Q_2 and Q_3 are diode connected, then

$W_{\text{eff}} = 20 + 40 = 60\ \mu$m, so that

$$I_1 = 100\ \mu\text{A}\left(\frac{10}{60}\right) = 16.7\ \mu\text{A}$$

(6) If Q_1 and Q_3 are diode connected,

$W_{\text{eff}} = 10 + 40 = 50\ \mu$m, so that

$$I_2 = 100\ \mu\text{A}\left(\frac{20}{50}\right) = 40\ \mu\text{A}$$

So three different currents are obtained with double-diode connects.

To find V_{SG}, we use the following for the diode-connected transistor(s):

$$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)(V_{SG} - |V_{tp}|)^2$$

and substitute $I_D = I_{\text{REF}} = 100\ \mu$A. Thus

$$100 = \frac{1}{2} \times 100 \times \left(\frac{W}{1\ \mu\text{m}}\right)(V_{SG} - 0.6)^2$$

$$\Rightarrow V_{SG} = 0.6 + \sqrt{\frac{2}{W(\mu\text{m})}}$$

For the six cases above we obtain

(1) $W = W_1 = 10\ \mu\text{m} \Rightarrow V_{SG} = 1.05$ V

(2) $W = W_2 = 20\ \mu\text{m} \Rightarrow V_{SG} = 0.92$ V

(3) $W = W_3 = 40\ \mu\text{m} \Rightarrow V_{SG} = 0.82$ V

(4) $W = W_1 + W_2 = 30\ \mu\text{m} \Rightarrow V_{SG} = 0.86$ V

(5) $W = W_2 + W_3 = 60\ \mu\text{m} \Rightarrow V_{SG} = 0.78$ V

(6) $W = W_1 + W_3 = 50\ \mu\text{m} \Rightarrow V_{SG} = 0.80$ V

8.8 (a) If $I_S = 10^{-17}$ A and we ignore base currents, then

$I_{\text{REF}} = I_S\, e^{V_{BE}/V_T}$ so that

$$V_{BE} = V_T \ln\left(\frac{I_{\text{REF}}}{10^{-17}}\right)$$

For $I_{\text{REF}} = 10\ \mu$A,

$$V_{BE} = 0.025 \ln\left(\frac{10^{-5}}{10^{-17}}\right) = 0.691\ \text{V}$$

For $I_{\text{REF}} = 10$ mA,

$$V_{BE} = 0.025 \ln\left(\frac{10^{-2}}{10^{-17}}\right) = 0.863\ \text{V}$$

So for the range of

$10\ \mu\text{A} \le I_{\text{REF}} \le 10$ mA,

$0.691\ \text{V} \le V_{BE} \le 0.863$ V

(b) Accounting for finite β,

$$I_O = I_{\text{REF}} \cdot \frac{1}{1 + 2/\beta}$$

For $I_{\text{REF}} = 10\ \mu$A,

$$I_O = \frac{10\ \mu\text{A}}{1 + \frac{2}{50}} = 9.62\ \mu\text{A}$$

For $I_{\text{REF}} = 0.1$ mA,

$$I_O = \frac{0.1\ \text{mA}}{1 + \frac{2}{100}} = 0.098\ \text{mA}$$

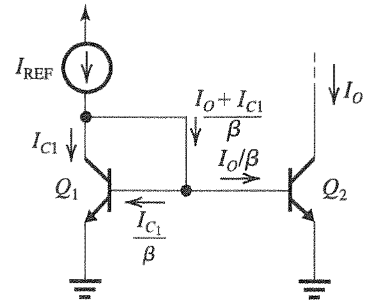
For $I_{\text{REF}} = 1$ mA,

$$I_O = \frac{1\ \text{mA}}{1 + \frac{2}{100}} = 0.98\ \text{mA}$$

For $I_{\text{REF}} = 10$ mA,

$$I_O = \frac{10\ \text{mA}}{1 + \frac{2}{50}} = 9.62\ \text{mA}$$

8.9



$I_O = mI_{C1}$

A node equation at the collector of Q_1 yields

$$I_{REF} = I_{C1} + \frac{I_O + I_{C1}}{\beta}$$

Substituting $I_{C1} = I_O/m$ results in

$$\frac{I_O}{I_{REF}} = \frac{m}{1 + \frac{m+1}{\beta}} \qquad \text{Q.E.D.}$$

For $\beta = 80$ and the error in the current transfer ratio to be limited to 10%, that is,

$$\frac{m}{1 + \frac{m+1}{\beta}} \geq 0.9m$$

$$\left(1 + \frac{m+1}{\beta}\right) \leq \frac{1}{0.9}$$

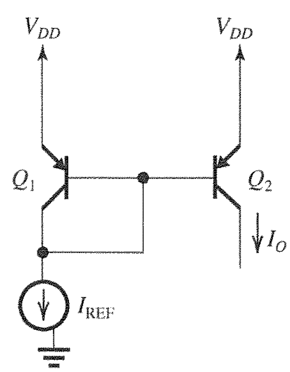
$$\frac{m+1}{\beta} \leq \frac{1}{0.9} - 1$$

$$m \leq \beta\left(\frac{1}{0.9} - 1\right) - 1$$

$$m \leq 80\left(\frac{1}{0.9} - 1\right) - 1 = 7.88$$

Thus, the largest current transfer ratio possible is 7.88.

8.10



For identical transistors, the transfer ratio is

$$\frac{I_O}{I_{REF}} = \frac{1}{1 + 2/\beta} = \frac{1}{1 + \frac{2}{50}} = 0.96$$

8.11 Nominally, $I_O = I_{REF} = 1$ mA

$$r_{o2} = \frac{V_{A2}}{I_O} = \frac{90}{1} = 90\ \text{k}\Omega$$

$$r_{o2} = \frac{\Delta V_O}{\Delta I_O} \Rightarrow \frac{10 - 1}{\Delta I_O} = 90 \Rightarrow \Delta I_O = 0.1\ \text{mA}$$

$$\frac{\Delta I_O}{I_O} = \frac{0.1}{1} = 10\%\ \text{change}$$

8.12 Equation (8.21) gives the current transfer ratio of an *npn* mirror with a nominal ratio of m:

$$I_O = I_{REF} \frac{m}{1 + \frac{m+1}{\beta}}\left(1 + \frac{V_O - V_{BE}}{V_{A2}}\right)$$

This equation can be adapted for the *pnp* mirror of Fig. P8.12 by substituting $m = 1$, replacing V_O with the voltage across Q_3, namely $(3 - V_O)$, replacing V_{BE} with V_{EB}, and V_{A2} with $|V_A|$:

$$I_O = I_{REF} \frac{1 + [(3 - V_O - V_{EB})/|V_A|]}{1 + (2/\beta)} \qquad (1)$$

Now, substituting $I_O = 1$ mA, $V_O = 1$ V, $\beta = 50$, $|V_A| = 50$ V, and

$$V_{EB} = V_T \ln\frac{I_O}{I_S} = 0.025 \ln\left(\frac{10^{-3}}{10^{-15}}\right) = 0.691\ \text{V}$$

results in

$$I_{REF} = \frac{1 \times (1 + 0.04)}{1 + \frac{3 - 1 - 0.691}{50}} = 1.013\ \text{mA}$$

$$R = \frac{V_{CC} - V_{EB}}{I_{REF}} = \frac{3 - 0.691}{1.013} = 2.28\ \text{k}\Omega$$

Maximum allowed voltage $V_O = 3 - 0.3 = 2.7$ V. For $V_O = 2.7$ V, Eq. (1) yields

$$I_O = 1.013\,\frac{1 + \frac{3 - 2.7 - 0.691}{50}}{1.04} = 0.966\ \text{mA}$$

For $V_O = -5$ V, Eq. (1) yields

$$I_O = 1.013\,\frac{1 + \frac{3 - (-5) - 0.691}{50}}{1.04} = 1.116\ \text{mA}$$

Thus, the change in I_O is 0.15 mA.

8.13 The solution is given in the circuit diagram. Note that the starting point is calculating the current I in the Q_1–R_1–Q_2 branch. See figure on next page.

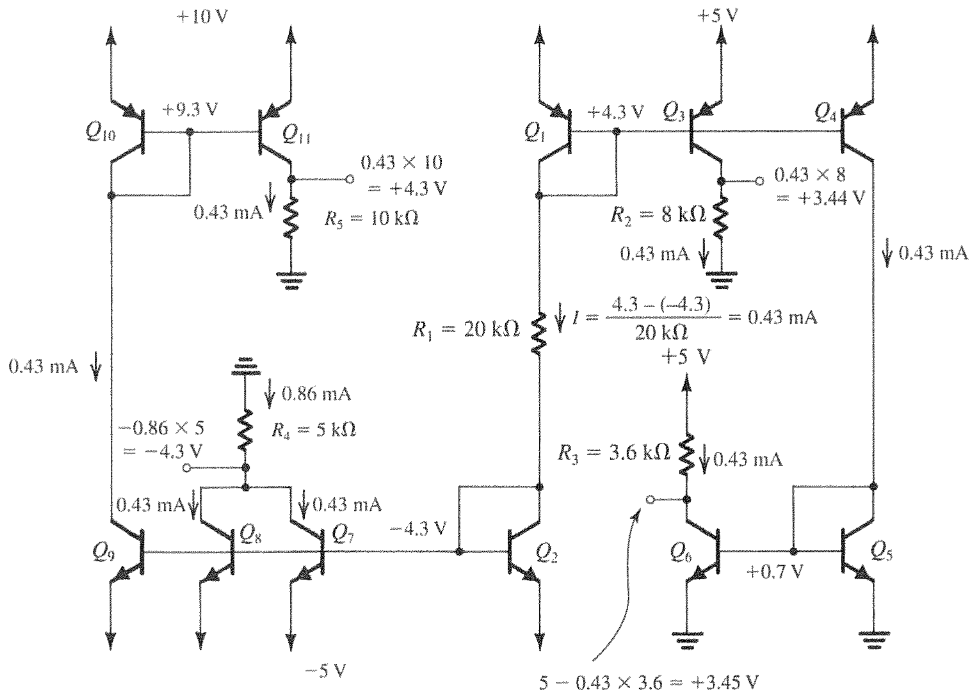
8.14 Refer to the circuit in Fig. P8.14.

$$V_2 = 2.7 - V_{EB} = 2.7 - 0.7 = +2\ \text{V}$$

$$V_3 = 0 + V_{EB} = +0.7\ \text{V}$$

Thus, Q_3 and Q_4 are operating in the active mode, and each is carrying a collector current of $I/2$. The same current is flowing in Q_2 and Q_1; thus

This figure belongs to Problem **8.13**.



$$V_1 = -2.7 + \frac{I}{2}R$$

But

$$V_1 = -V_{BE1} = -0.7$$

Thus,

$$-0.7 = -2.7 + \frac{1}{2}IR$$

$$\Rightarrow IR = 4\text{ V}$$

The current I splits equally between Q_5 and Q_6; thus

$$V_4 = -2.7 + \left(\frac{I}{2}\right)R = -2.7 + 2 = -0.7\text{ V}$$

$$V_5 = -2.7 + \left(\frac{I}{2}\right)\left(\frac{R}{2}\right) = -2.7 + 1 = -1.7\text{ V}$$

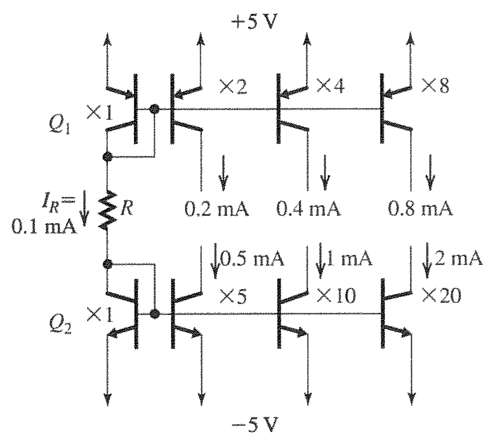
Thus, Q_5 and Q_6 are operating in the active mode as we have implicitly assumed.

Note that the values of V_1, V_2, V_3, V_4, and V_5 do not depend on the value of R. Only I depends on the value of R:

(a) $R = 10\text{ k}\Omega \Rightarrow I = \frac{4}{10} = 0.4\text{ mA}$

(b) $R = 100\text{ k}\Omega \Rightarrow I = \frac{4}{100} = 0.04\text{ mA}$

8.15 There are various ways this design could be achieved, but the most straightforward is the one shown:



With this scheme,

$$R = \frac{5 - 0.7 - 0.7 - (-5)}{0.1\text{ mA}} = 86\text{ k}\Omega$$

and each transistor has EBJ areas proportional to the current required. Multiple, parallel transistors are acceptable.

Note: This large value of R is not desirable in integrated form; other designs may be move suitable.

Even without knowing exact circuitry, we can find the total power dissipation as approximately

$$P_T = P_{CC} + P_{EE}$$

$P_T = 5\text{ V }(0.1 + 0.2 + 0.4 + 0.8)\text{ mA}$

$+5\text{ V }(0.1 + 0.5 + 1 + 2)\text{ mA}$

$P_T = 7.5\text{ mW} + 18\text{ mW} = 25.5\text{ mW}$

8.16 (a)

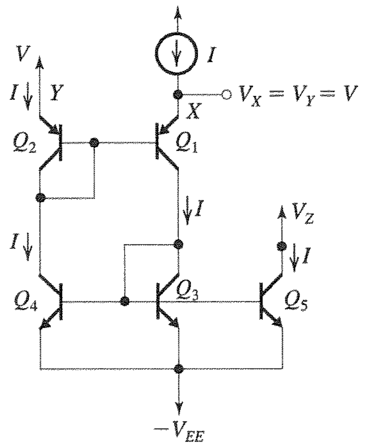


Figure 1

Figure 1 shows the current conveyor circuit with Y connected to a voltage V, X fed with a current source I, and Z connected to a voltage V_Z that keeps Q_5 operating in the active mode. Assuming that all transistors are operating in the active mode and that $\beta \gg 1$, so that we can neglect all base currents, we see that the current I through Q_1 will flow through the two-output mirror Q_3, Q_4, and Q_5. The current I in Q_5 will be drawn from Q_2, which forms a mirror with Q_1. Thus $V_{EB2} = V_{EB1}$ and the voltage that appears at X will be equal to V. The current in Q_5 will be equal to I, thus terminal Z sinks a constant current I.

(b)

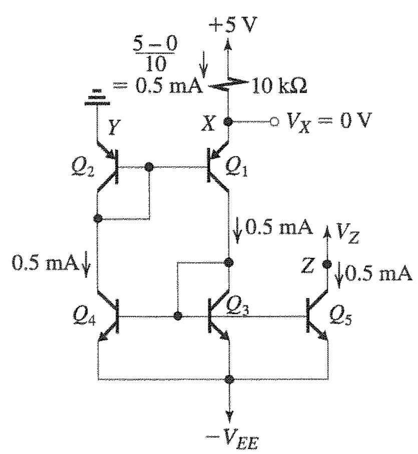


Figure 2

Figure 2 shows the special case of $V = 0$ V. As before, the voltage at X, V_X, will be equal to V. Thus

$V_X = 0$

That is, a virtual ground appears at X, and thus the current I that flows into X can be found from

$$I = \frac{5 - V_X}{10\text{ k}\Omega} = \frac{5-0}{10} = 0.5\text{ mA}$$

This is the current that will be mirrored to the output, resulting in $I_Z = 0.5$ mA.

8.17 Using Eq. (8.28),

$$R_{\text{in}} = r_{o1} \parallel \frac{1}{g_{m1}}$$

where

$$r_{o1} = \frac{V_A}{I_{D1}} = \frac{V_A' L}{I_{D1}} = \frac{10 \times 0.5}{0.1\text{ mA}} = 50\text{ k}\Omega$$

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}$$

$$= \sqrt{2 \times 0.5 \times \left(\frac{10}{0.5}\right) \times 0.1} = 1.414\text{ mA/V}$$

$$\frac{1}{g_{m1}} = 0.71\text{ k}\Omega$$

Thus,

$$R_{\text{in}} = 50 \parallel 0.71 = 0.7\text{ k}\Omega = 700\ \Omega$$

$$A_{is} = \frac{(W/L)_2}{(W/L)_1} = \frac{50/0.5}{10/0.5} = 5\text{ A/A}$$

$$R_O = r_{o2} = \frac{V_A}{I_{D2}} = \frac{V_A' L}{I_{D2}}$$

$$= \frac{10 \times 0.5}{5 \times 0.1} = 10\text{ k}\Omega$$

8.18 $A_{is} = 4 = \dfrac{(W/L)_2}{(W/L)_1}$

Since $L_1 = L_2$, then

$$\frac{W_2}{W_1} = 4$$

$$R_{\text{in}} = r_{o1} \parallel \frac{1}{g_{m1}} \simeq \frac{1}{g_{m1}}$$

For

$$R_{\text{in}} = 500\ \Omega \Rightarrow g_{m1} = 2\text{ mA/V}$$

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}$$

Thus,

$$2 = \sqrt{2 \times 0.4 \times \left(\frac{W}{L}\right)_1 \times 0.2}$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 25$$

$$R_O = r_{o2} = \frac{V_A}{I_{D2}} = \frac{V_A' L}{I_{D2}}$$

Thus,

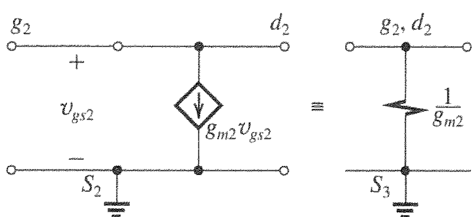
$$20 = \frac{20\,L}{4 \times 0.2}$$

$$\Rightarrow L = 0.8\ \mu\text{m}$$

$$W_1 = 25 \times 0.8 = 20\ \mu\text{m}$$

$$W_2 = 4\ W_1 = 80\ \mu\text{m}$$

8.19 Refer to Fig. P8.19. Consider first the diode-connected transistor Q_2. From the figure we



see that from a small-signal point of view it is equivalent to a resistance $1/g_{m2}$. Thus the voltage gain of Q_1 will be

$$\frac{v_{d1}}{v_i} = -g_{m1} \times \frac{1}{g_{m2}} = -\frac{g_{m1}}{g_{m2}}$$

The signal current in the drain of Q_1, $g_{m1}\ v_i$, will be mirror in the drain of Q_3;

$$i_{d3} = g_{m1}\ v_i \frac{(W/L)_3}{(W/L)_2} = g_{m1} v_i \frac{W_3}{W_2}$$

which flows through R_L and produces the output voltage v_o,

$$v_o = i_{d3} R_L = g_{m1} v_i \frac{W_3}{W_2} R_L$$

Thus, the small-signal voltage gain will be

$$\frac{v_o}{v_i} = g_{m1} R_L\ (W_3/W_2)$$

8.20 Replacing Q_1 and Q_2 with their small-signal hybrid-π models results in the equivalent circuit shown in the figure below. Observe that the controlled source $g_{m1}v_{\pi 1}$ appears across its controlling voltage $v_{\pi 1}$; thus the controlled source can be replaced with a resistance $(1/g_{m1})$. The input resistance R_{in} can now be obtained by inspection as

$$R_{\text{in}} = r_{o1} \parallel \frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel r_{\pi 2}$$

Since $r_{o1} \gg r_{\pi 1}$,

$$R_{\text{in}} \simeq \frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel r_{\pi 2} \qquad (1)$$

The short-circuit output current i_o is given by

$$i_o = g_{m2} v_{\pi 2}$$

Since $v_{\pi 2} = v_{\pi 1} = i_i R_{\text{in}}$, then the short-circuit current gain A_{is} is given by

$$A_{is} = \frac{i_o}{i_i} = g_{m2} R_{\text{in}}$$

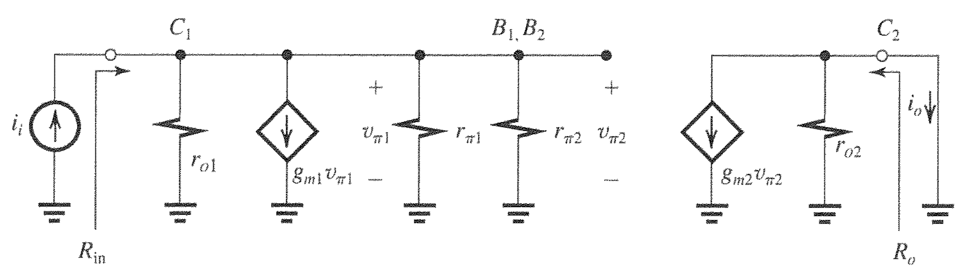
$$= g_{m2}\left(\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel r_{\pi 2}\right) \qquad (2)$$

For situations where β_1 and β_2 are large, we can neglect $r_{\pi 1}$ and $r_{\pi 2}$ in Eqs. (1) and (2) to obtain

$$R_{\text{in}} \simeq 1/g_{m1}$$

$$A_{is} \simeq g_{m1}/g_{m2}$$

This figure belongs to Problem **8.20**.



8.21 (a)

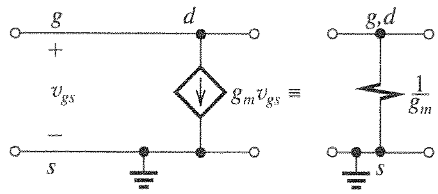


Figure 1

Replacing the MOSFET with its hybrid-π model but neglecting r_o results in the equivalent circuit in Fig. 1. Observing that the controlled-source $g_m v_{gs}$ appears across its control voltage v_{gs}, we can replace it by a resistance $1/g_m$, as indicated. Thus the small-signal resistance of the diode-connected MOS transistor is $1/g_m$. For the given values,

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D}$$

$$= \sqrt{2 \times 0.2 \times 10 \times 0.1} = 0.632 \text{ mA/V}$$

$$\frac{1}{g_m} = 1.6 \text{ k}\Omega$$

(b) Replacing the BJT with its hybrid-π model results in the equivalent circuit in Fig. 2. Observing that the controlled-source $g_m v_\pi$ appears across its control voltage v_π, we can replace it by a resistance $1/g_m$, as indicated. Next the two parallel resistances $1/g_m$ and r_π can be combined as

$$\frac{\frac{1}{g_m} \times r_\pi}{\frac{1}{g_m} + r_\pi} = \frac{r_\pi}{1 + g_m r_\pi} = \frac{r_\pi}{\beta + 1} = r_e$$

Thus, the diode-connected BJT has a small-signal resistance r_e. For the given data,

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \ \Omega$$

8.22 Refer to Fig. 8.11.

$$I_{C1} \simeq I_{\text{REF}} = 0.1 \text{ mA}$$

$$V_{BE1} = 0.7 - 0.025 \ln\left(\frac{1 \text{ mA}}{0.1 \text{ mA}}\right)$$

$$= 0.642 \text{ V}$$

$$I_{C3} \simeq I_{B1} + I_{B2} = 2\, I_{B1} = 2 \times \frac{I_{C1}}{\beta}$$

$$= 2 \times \frac{0.1}{100} = 0.002 \text{ mA}$$

$$V_{BE3} = 0.7 - 0.025 \ln\left(\frac{1 \text{ mA}}{0.002 \text{ mA}}\right)$$

$$= 0.545 \text{ V}$$

$$V_x = V_{BE3} + V_{BE1} = 1.187 \text{ V}$$

If I_{REF} is increased to 1 mA,

$$V_{BE1} = 0.7$$

$$I_{C3} \simeq 0.02 \text{ mA}$$

$$V_{BE3} = 0.7 - 0.025 \ln\left(\frac{1}{0.02}\right) = 0.6 \text{ V}$$

$$V_x = 1.3 \text{ V}$$

Thus,

$$\Delta V_x = 1.3 - 1.187 = 0.113 \text{ V}$$

When $V_O = V_x$, the Early effect on Q_1 and Q_2 will be the same, and

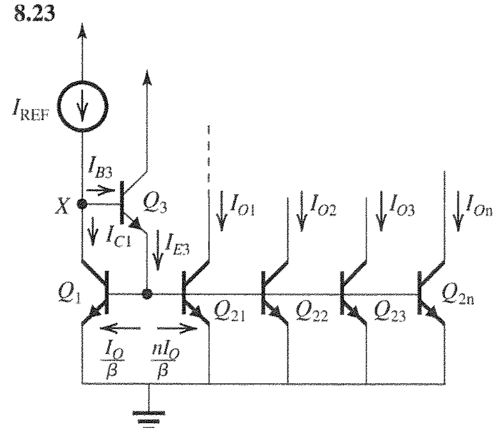
$$I_O = I_{\text{REF}}/(1 + 2/\beta^2)$$

Thus, I_O will be

$I_{\text{REF}} = 100\ \mu\text{A} \Rightarrow I_O = \dfrac{100}{1 + (2/100^2)} =$ 99.98 μA, for an error of −0.02 μA or −0.02%.

$I_{\text{REF}} = 1 \text{ mA} \Rightarrow I_O = \dfrac{1}{1 + (2/100^2)} =$ 0.9998 μA, for an error of −0.0002 mA or −0.02%. For proper current-source operation, the minimum required voltage at the output is the value needed to keep Q_3 in the active region, which is approximately 0.3 V.

8.23



This figure belongs to Problem **8.21**, part (b).

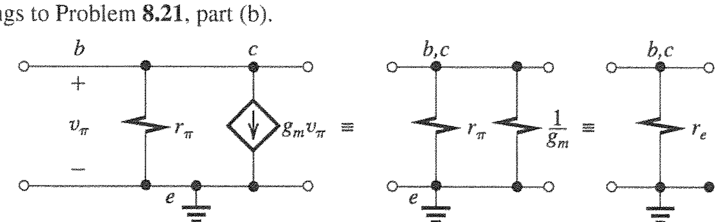


Figure 2

$$I_{O1} = I_{O2} = I_{O3} \cdots = I_{On} = I_O = I_{C1}$$

The emitter of Q_3 supplies the base currents for all transistor, so

$$I_{E3} = \frac{(n+1)\, I_O}{\beta}$$

$$I_{\text{REF}} = I_{B3} + I_O = \frac{(n+1)\, I_O}{\beta\,(\beta+1)} + I_O$$

$$\frac{I_O}{I_{\text{REF}}} = \frac{1}{1 + \dfrac{(n+1)}{\beta\,(\beta+1)}} \simeq \frac{1}{1 + \dfrac{n+1}{\beta^2}}$$

For the deviation from unity to be kept $\leq 0.2\%$

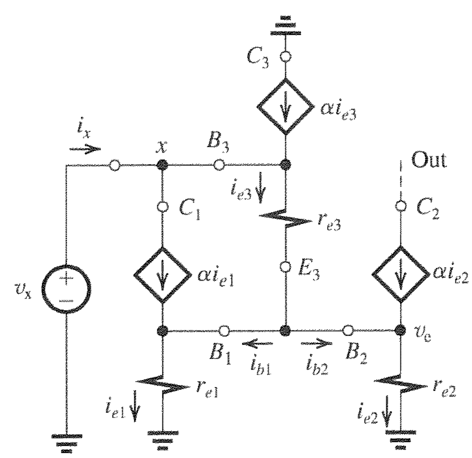
$$\frac{n+1}{\beta^2} \leq 0.002$$

$$\Rightarrow n_{\max} = 0.002 \times 150^2 - 1 = 44$$

8.24 Refer to Fig. 8.11 and observe that $I_{C1} \simeq I_{\text{REF}}$ and $I_{C2} = I_{C1}$; thus each of Q_1 and Q_2 is operating at a collector bias current approximately equal to I_{REF}. Transistor Q_3 is operating at an emitter bias current

$$I_{E3} = I_{B1} + I_{B2} = 2I_B = 2I_C/\beta = \frac{2(1-\alpha)}{\alpha} I_C$$
$$= \frac{2(1-\alpha)}{\alpha} I_{\text{REF}}$$

Replacing each of the three transistors with its T model and applying an input test voltage v_x to determine R_{in}, we obtain the equivalent circuit shown.



In this equivalent circuit,

$$r_{e1} = r_{e2} = r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C} = \frac{\alpha V_T}{I_{\text{REF}}}$$

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{\alpha V_T}{2(1-\alpha) I_{\text{REF}}}$$

$$i_{e1} = i_{e2} = i_e$$

$$i_{e3} = i_{b1} + i_{b2} = 2(1-\alpha) i_e$$

From the figure we obtain

$$i_x = \alpha i_{e1} + (1-\alpha) i_{e3}$$
$$= \alpha i_e + (1-\alpha) \times 2(1-\alpha) i_e$$
$$= i_e[\alpha + 2(1-\alpha)^2]$$

But $2(1-\alpha)^2 \ll \alpha$. Thus,

$$i_x \simeq \alpha i_e \qquad (1)$$

$$v_x = i_{e3}\, r_{e3} + i_{e1} r_{e1}$$
$$= 2(1-\alpha) i_e \frac{\alpha V_T}{2(1-\alpha) I_{\text{REF}}} + i_e \frac{\alpha V_T}{I_{\text{REF}}}$$

$$v_x = \alpha i_e \left[\frac{V_T}{I_{\text{REF}}} + \frac{V_T}{I_{\text{REF}}} \right]$$

Now, using $i_x = \alpha i_e$ from Eq. (1), we have

$$v_x = i_x \times \frac{2V_T}{I_{\text{REF}}}$$

Thus,

$$R_{\text{in}} \equiv \frac{v_x}{i_x} = \frac{2V_T}{I_{\text{REF}}} \qquad \text{Q.E.D.}$$

For $I_{\text{REF}} = 100\ \mu\text{A} = 0.1$ mA,

$$R_{\text{in}} = \frac{2 \times 25 \text{ mV}}{0.1 \text{ mA}} = 500\ \Omega$$

8.25 For $I = 10\ \mu$A:

$$g_m = \frac{I}{V_T} = \frac{10\ \mu\text{A}}{25 \text{ mV}} = 0.4 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.4 \text{ mA/V}} = 250 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I} = \frac{10 \text{ V}}{10\ \mu\text{A}} = 1 \text{ M}\Omega$$

$$A_0 = g_m r_o = \frac{V_A}{V_I} = \frac{10 \text{ V}}{0.025 \text{ V}} = 400 \text{ V/V}$$

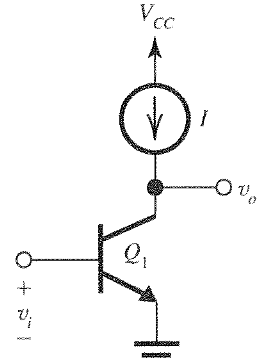
For $I = 100\ \mu$A :

$$g_m = \frac{100\ \mu\text{A}}{25 \text{ mV}} = 4 \text{ mA/V}$$

$$r_\pi = \frac{100}{4 \text{ mA/V}} = 25 \text{ k}\Omega$$

$$r_o = \frac{10 \text{ V}}{100\ \mu\text{A}} = 100 \text{ k}\Omega$$

$$A_0 = 4 \text{ mA/V} \times 100 \text{ k}\Omega = 400 \text{ V/V}$$



For $I = 1$ mA:

$$g_m = \frac{1 \text{ mA}}{25 \text{ mV}} = 40 \text{ mA/V}$$

$$r_\pi = \frac{100}{40 \text{ mA/V}} = 2.5 \text{ k}\Omega$$

$$r_o = \frac{10 \text{ V}}{1 \text{ mA}} = 10 \text{ k}\Omega$$

$$A_0 = 40 \text{ mA/V} \times 10 \text{ k}\Omega = 400 \text{ V/V}$$

I	g_m	r_π	r_o	A_0
10 μA	0.4 mA/V	250 kΩ	1 MΩ	400 V/V
100 μA	4.0 mA/V	25 kΩ	100 kΩ	400 V/V
1 mA	40 mA/V	2.5 kΩ	10 kΩ	400 V/V

8.26 Refer to Fig. 8.13(b).

$$g_m = \frac{I_C}{V_T} = \frac{I}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{0.5 \text{ mA}} = 200 \text{ k}\Omega$$

$$R_{\text{in}} = r_\pi = \frac{\beta}{g_m} = \frac{100}{20 \text{ mA/V}} = 5 \text{ k}\Omega$$

$$A_{vo} = -A_0 = -g_m r_o = -20 \times 200 = -4000 \text{ V/V}$$

$$R_o = r_o = 200 \text{ k}\Omega$$

To raise R_{in} by a factor of 5 by changing I, the value of I must be lowered by the same factor to $I = 0.1$ mA.

Now, g_m is reduced by a factor of 5 and r_o is increased by a factor of 5, keeping A_{vo} unchanged at -4000 V/V. However, R_o will be increased to

$$R_o = 5 \times 200 \text{ k}\Omega = 1 \text{ M}\Omega$$

If the amplifier is fed with a signal source having $R_{\text{sig}} = 5$ kΩ and a 100-kΩ load resistance is connected to the output, the equivalent circuit shown below results.

$$\frac{v_o}{v_{\text{sig}}} = \frac{r_\pi}{r_\pi + R_{\text{sig}}} \times -g_m(r_o \parallel R_L)$$

$$= -\frac{25}{25+5} \times 4 \,(1000 \text{ k}\Omega \parallel 100 \text{ k}\Omega)$$

$$= -303 \text{ V/V}$$

8.27

$$A_0 = \frac{2V_A}{V_{OV}} = \frac{2V_A' L}{V_{OV}} = \frac{2 \times 10 \times 0.5}{0.2} = 50 \text{ V/V}$$

$$g_m = \frac{2I_D}{V_{OV}}$$

$$2 = \frac{2I_D}{0.2} \Rightarrow I_D = 0.2 \text{ mA}$$

$$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2$$

$$0.2 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.2^2$$

$$\Rightarrow \frac{W}{L} = 25$$

$$W = 12.5 \text{ μm}$$

8.28 From Eq. (8.46) we see that A_0 is inversely proportional to $\sqrt{I_D}$. Thus

$I_D = 100$ μA $\quad A_0 = 50$ V/V

$I_D = 25$ μA $\quad A_0 = 100$ V/V

$I_D = 400$ μA $\quad A_0 = 25$ V/V

From Eq. (8.42), g_m is proportional to $\sqrt{I_D}$. Thus changing I_D from 100 μA to 25 μA reduces g_m by a factor of 2. Changing I_D from 100 μA to 400 μA increases g_m by a factor of 2.

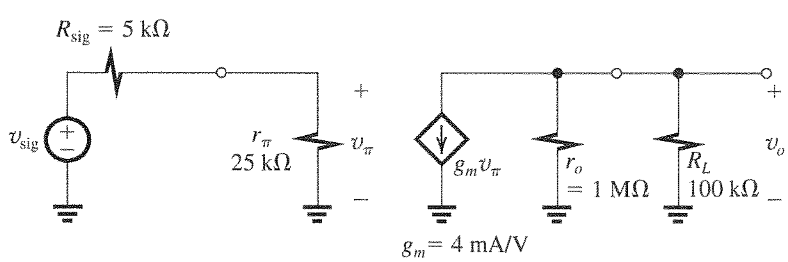
8.29 $A_0 = \dfrac{2V_A}{V_{OV}} = \dfrac{2V_A' L}{V_{OV}}$

$$20 = \frac{2 \times 5 \times L}{0.2}$$

$$\Rightarrow L = 0.4 \text{ μm}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2I}{V_{OV}}$$

This figure belongs to Problem **8.26**.



$$2 = \frac{2I}{0.2}$$

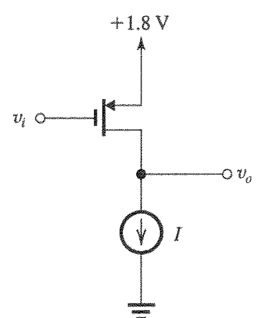
$$\Rightarrow I = 0.2 \text{ mA}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$0.2 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.04$$

$$\Rightarrow \frac{W}{L} = 25$$

8.30



The highest instantaneous voltage allowed at the drain is that which results in a voltage equal to (V_{OV}) across the transistor. Thus

$$v_{O\max} = 1.8 - 0.2 = +1.6 \text{ V}$$

8.31 For the *npn* transistor,

$$g_m = \frac{I_C}{V_T} = \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

For the NMOS transistor,

$$g_m = \frac{2\, I_D}{V_{OV}}$$

$$4 = \frac{2\, I_D}{0.25}$$

$$\Rightarrow I_D = 0.5 \text{ mA}$$

8.32 $g_m = \dfrac{2\, I_D}{V_{OV}} = \dfrac{2 \times 0.1}{0.5} = 0.4 \text{ mA/V}$

From Table J.1 (Appendix J), we find that for the 0.5-μm process $|V_A'| = 20$ V/μm. Thus for our 1-μ-m long transistor, $V_A = 20$ V.

$$r_o = \frac{V_A}{I_D} = \frac{20 \text{ V}}{0.1 \text{ mA}} = 200 \text{ k}\Omega$$

$$A_0 = g_m r_o = 0.4 \times 200 = 80 \text{ V/V}$$

From Table J.1:

$$\mu_n C_{ox} = 190\ \mu\text{A/V}^2$$

Now,

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$100 = \frac{1}{2} \times 190 \times \frac{W}{L} \times 0.25$$

$$\Rightarrow \frac{W}{L} = 4.21$$

$$\Rightarrow W = 4.21\ \mu\text{m}$$

8.33 $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$

From Table K.1 (Appendix K), for the 0.18-μm process we have

$$|V_A'| = 5 \text{ V}/\mu\text{m}, \mu_n C_{ox} = 387\ \mu\text{A/V}^2$$

Thus, for our NMOS transistor whose $L = 0.3$ μm,

$$V_A = 5 \times 0.3 = 1.5 \text{ V}$$

$$r_o = \frac{V_A}{I_D} = \frac{1.5 \text{ V}}{0.1 \text{ mA}} = 15 \text{ k}\Omega$$

$$A_0 = g_m r_o = 1 \times 15 = 15 \text{ V/V}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{OV}^2$$

$$100 = \frac{1}{2} \times 387 \times \frac{W}{L} \times 0.2^2$$

$$\Rightarrow \frac{W}{L} = 13 \Rightarrow W = 3.9\ \mu\text{m}$$

8.34 For the BJT cell:

$$g_m = \frac{I_C}{V_T} = \frac{I_C}{0.025 \text{ V}}$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{I_C}$$

$$A_0 = g_m r_o = \frac{V_A}{V_T} = \frac{100 \text{ V}}{0.025 \text{ V}} = 4000 \text{ V/V}$$

$$R_{\text{in}} = r_\pi = \frac{\beta}{g_m} = \frac{100}{g_m}$$

For the MOSFET cell:

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_D} = \sqrt{2 \times 0.2 \times 40 \times I_D}$$

$$= \sqrt{16 I_D} = 4\sqrt{I_D} \text{ mA/V} \quad (I_D \text{ in mA})$$

$$r_o = \frac{V_A}{I_D} = \frac{10 \text{ V}}{I_D}$$

$$A_0 = g_m r_o = \frac{40}{\sqrt{I_D}} \text{ V/V} \quad (I_D \text{ in mA})$$

$$R_{\text{in}} = \infty$$

	BJT Cell		**MOSFET Cell**	
Bias current	$I_C = 0.1$ **mA**	$I_C = 1$ **mA**	$I_D = 0.1$ **mA**	$I_D = 1$ **mA**
g_m (mA/V)	4	40	1.26	4
r_o (kΩ)	1000	100	100	10
A_0 (V/V)	4000	4000	126	40
R_{in} (kΩ)	25	2.5	∞	∞

8.35 Using Eq. (8.46),

$$A_0 = \frac{V_A'\sqrt{2(\mu_n C_{ox})(WL)}}{\sqrt{I_D}}$$

$$18 = \frac{5\sqrt{2 \times 0.4 \times 8 \times 0.54 \times 0.54}}{\sqrt{I_D}}$$

$$\Rightarrow I_D = 0.144 \text{ mA}$$

8.36 $g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D}$

$$= \sqrt{2 \times 0.4 \times 8 I_D} = 2.53\sqrt{I_D}$$

$I_D = 25\ \mu\text{A}, \quad g_m = 2.53\sqrt{0.025} = 0.4 \text{ mA/V}$

$I_D = 250\ \mu\text{A}, \quad g_m = 2.53\sqrt{0.25} = 1.26 \text{ mA/V}$

$I_D = 2.5 \text{ mA}, \quad g_m = 2.53\sqrt{2.5} = 4 \text{ mA/V}$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{5 \times 0.36}{I_D} = \frac{1.8}{I_D}$$

$$A_0 = g_m r_o$$

$I_D = 25\ \mu\text{A} \quad r_o = \frac{1.8}{0.025} = 72 \text{ k}\Omega$

$A_0 = 0.4 \times 72 = 28.8 \text{ V/V}$

$I_D = 250\ \mu\text{A} \quad r_o = \frac{1.8}{0.25} = 7.2 \text{ k}\Omega$

$A_0 = 1.26 \times 7.2 = 9.1 \text{ V/V}$

$I_D = 2.5 \text{ mA} \quad r_o = \frac{1.8}{2.5} = 0.72 \text{ k}\Omega$

$A_0 = 4 \times 0.72 = 2.88 \text{ V/V}$

8.37

$L = 0.36\ \mu\text{m}, \quad V_{OV} = 0.25 \text{ V}, \quad I_D = 10\ \mu\text{A}$

(a) $g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 10}{0.25} = 80\ \mu\text{A/V}$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D}$$

From Appendix J, Table J.1, $V_A' = 5 \text{ V}/\mu\text{m}$,

$$r_o = \frac{5 \times 0.36}{10} = 0.18 \text{ M}\Omega$$

$$A_0 = g_m r_o = 80 \times 0.18 = 14.4 \text{ V/V}$$

(b) If I_D is increased to 100 μA (i.e., by a factor of 10), V_{OV} increases by a factor of $\sqrt{10} = 3.16$ to

$$V_{OV} = 0.25 \times 3.16 = 0.79 \text{ V}$$

and g_m increases by a factor of $\sqrt{10} = 3.16$ to

$$g_m = 80 \times 3.16 = 253\ \mu\text{A/V} = 0.253 \text{ mA/V}$$

and r_o decreases by a factor of 10 to

$$r_o = \frac{0.18 \text{ M}\Omega}{10} = 18 \text{ k}\Omega$$

Thus, A_0 becomes

$$A_0 = 0.253 \times 18 = 4.55 \text{ V/V}$$

(c) If the device is redesigned with a new value of W so that it operates at

$V_{OV} = 0.25 \text{ V}$ for $I_D = 100\ \mu\text{A}$,

$$g_m = \frac{2I_D}{V_{OV}} = \frac{0.2 \text{ mA}}{0.25 \text{ V}} = 0.8 \text{ mA/V}$$

$$r_o = \frac{V_A' L}{I_D} = \frac{5 \times 0.36}{0.1} = 18 \text{ k}\Omega$$

$$A_0 = g_m r_o = 0.8 \times 18 = 14.4 \text{ V/V}$$

(d) If the redesigned device in (c) is operated at 10 μA, V_{OV} decreases by a factor equal to $\sqrt{10}$ to 0.08 V, g_m decreases by a factor of $\sqrt{10}$ to 0.253 mA/V, r_o increases by a factor of 10 to 180 kΩ, and A_0 becomes

$$0.253 \times 180 = 45.5 \text{ V/V}$$

which is an increase by a factor of $\sqrt{10}$.

(e) The lowest value of A_0 is obtained with the first design when operated at $I_D = 100\ \mu\text{A}$. The resulting $A_0 = 4.55$ V/V. The highest value of A_0 is obtained with the second design when operated at $I_D = 10\ \mu\text{A}$. The resulting $A_0 = 45.5$ V/V. If in any design W/L is held constant while L is increased by a factor of 10, g_m remains unchanged but r_o increases by a factor of 10, resulting in A_0 increasing by a factor of 10.

8.38

$$A_0 = \frac{2V_A}{V_{OV}} = \frac{2V_A' L}{V_{OV}} = \frac{2 \times 6 \times 0.5}{0.15} = 40 \text{ V/V}$$

$$I_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right) V_{OV}^2$$

$$100 = \frac{1}{2} \times 400 \times \frac{W}{L} \times 0.15^2$$

$$\Rightarrow \frac{W}{L} = 22.2$$

Thus,

$$W = 22.2 \times 0.5 = 11.1\ \mu\text{m}$$

$$g_m = \frac{2 I_D}{V_{OV}} = \frac{2 \times 0.1}{0.15} = 1.33 \text{ mA/V}$$

$$r_o = \frac{V_A' L}{I_D} = \frac{6 \times 0.5}{0.1} = 30 \text{ k}\Omega$$

8.39 $A_0 = |A_{vo}| = 100$

$$100 = \frac{2 V_A}{V_{OV}} = \frac{2 V_A}{0.2}$$

$$\Rightarrow V_A = 10 \text{ V}$$

Since $V_A' = 20 \text{ V}/\mu\text{m}$, we have

$$L = \frac{V_A}{V_A'} = \frac{10}{20} = 0.5\ \mu\text{m}$$

$$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2$$

$$50 = \frac{1}{2} \times 200 \times \frac{W}{L} \times 0.2^2$$

$$\Rightarrow \frac{W}{L} = 12.5$$

8.40 Refer to Fig. 8.15(a).

$$V_{SG2} = |V_{tp}| + |V_{OV}| = 0.5 + 0.3 = 0.8 \text{ V}$$

$$V_G = 2.5 - V_{SG2} = 2.5 - 0.8 = 1.7 \text{ V}$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{OV1}^2$$

$$100 = \frac{1}{2} \times 200 \times \left(\frac{W}{L}\right)_1 \times 0.3^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 11.1$$

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 |V_{OV2}|^2$$

$$100 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_2 \times 0.3^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 22.2$$

$$A_v = -g_{m1}\,(r_{o1} \parallel r_{o2})$$

$$g_{m1} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1}{0.3} = 0.67 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{|V_A'|L}{I_D} = \frac{20 \times 0.5}{0.1} = 100 \text{ k}\Omega$$

$$A_v = -0.67 \times (100 \parallel 100) = -33.5 \text{ V/V}$$

8.41 Refer to Fig. 8.15. Since $V_{An}' = |V_{Ap}'|$ and the channel lengths are equal, $V_{An} = |V_{Ap}|$ and $r_{o1} = r_{o2} = r_o$. Thus

$$A_v = -g_{m1}(r_{o1} \parallel r_{o2}) = -g_{m1}(r_o/2)$$

$$-40 = -\frac{1}{2}g_{m1}r_o$$

$$\Rightarrow g_{m1}r_o = 80$$

$$A_0 = \frac{2V_{An}}{V_{OV}} = \frac{2V_{An}'L}{V_{OV}}$$

$$80 = \frac{2 \times 5 \times L}{0.25}$$

$$\Rightarrow L = 2\ \mu\text{m}$$

$$V_{SG2} = |V_{tp}| + |V_{OV}| = 0.5 + 0.25 = 0.75 \text{ V}$$

$$V_G = V_{DD} - V_{SG2} = 1.8 - 0.75 = 1.05 \text{ V}$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{OV}^2$$

$$100 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_1 \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 8$$

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 |V_{OV}|^2$$

$$100 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_2 \times 0.25^2$$

$$\left(\frac{W}{L}\right)_2 = 32$$

8.42 Refer to Fig. P8.42. The gain of the first stage is

$$A_{v1} = -g_{m1}(r_{o1}/2)$$

where $(r_{o1}/2)$ is the equivalent resistance at the output of Q_1 and includes r_{o1} in parallel with the output resistance of the current-source load, which is equal to r_{o1}. Similarly, the gain of the second stage is

$$A_{v2} = -g_{m2}(r_{o2}/2)$$

Now because $V_{An} = |V_{Ap}| = |V_A|$ and both Q_1 and Q_2 are operating at equal currents I, we have

$$r_{o1} = r_{o2} = r_o$$

The overall voltage gain A_v will be

$$A_v = A_{v1}A_{v2}$$

$$A_v = \frac{1}{4}\, g_{m1}g_{m2}r_o^2$$

If the two transistors are operated at equal overdrive voltages, $|V_{OV}|$, both will have equal g_m,

$$A_v = \frac{1}{4}(g_m r_o)^2$$

and

$$g_m r_o = \frac{2|V_A|}{|V_{OV}|} = \frac{2 \times 5}{|V_{OV}|} = \frac{10}{|V_{OV}|}$$

$$A_v = 400 = \frac{1}{4} \times \left[\frac{10}{|V_{OV}|}\right]^2$$

$$\Rightarrow |V_{OV}| = 0.25 \text{ V}$$

8.43

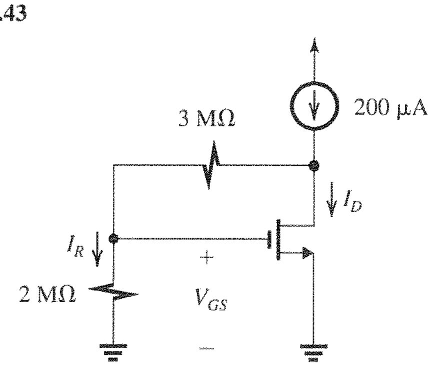


Figure 1

(a) Neglecting the dc current in the feedback network and the Early effect, we see from Fig. 1 that $I_D = 200\ \mu A$. Now, using

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

we can determine V_{OV}:

$$0.2 = \frac{1}{2} \times 2 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.45\ \text{V}$$

$$V_{GS} = V_t + V_{OV} = 0.5 + 0.45 = 0.95\ \text{V}$$

The current in the feedback network can now be found as

$$I_R = \frac{V_{GS}}{2\ \text{M}\Omega} = \frac{0.95}{2} = 0.475\ \mu\text{A}$$

which indeed is much smaller than the 200 μA delivered by the current source. Thus, we were justified in neglecting I_R above.

(b) Replacing the MOSFET with its hybrid-π model, we obtain the equivalent circuit shown in Fig. 2.

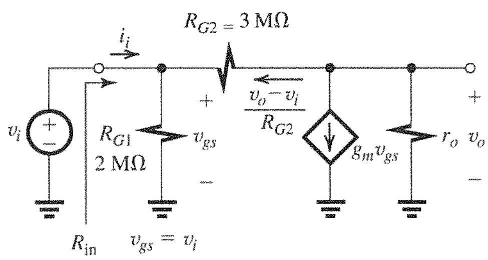


Figure 2

A node equation at the output node yields

$$\frac{v_o}{r_o} + g_m v_{gs} + \frac{v_o - v_i}{R_{G2}} = 0$$

where $v_{gs} = v_i$. Thus,

$$v_o\left(\frac{1}{r_o} + \frac{1}{R_{G2}}\right) = -v_i\left(g_m - \frac{1}{R_{G2}}\right)$$

$$\frac{v_o}{v_i} = -\left(g_m - \frac{1}{R_{G2}}\right)(r_o \parallel R_{G2})$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.45} = 0.894\ \text{mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20}{0.2} = 100\ \text{k}\Omega$$

$$\frac{v_o}{v_i} = -\left(0.89 - \frac{1}{3000}\right) \times (100 \parallel 3000)$$

$$= -86.5\ \text{V/V}$$

To obtain the maximum allowable negative signal swing at the output, we first determine the dc voltage at the output by referring to Fig. 1,

$$V_{DS} = V_{GS}\left(1 + \frac{R_{G2}}{R_{G1}}\right)$$

$$= 0.95 \times \left(1 + \frac{3}{2}\right) = 2.375\ \text{V}$$

The MOSFET will remain in saturation as long as $V_{DG} \geq -V_t$. Thus at the limit $V_{DG} = -0.5$ V,

$$v_{G\max} = 0.5 + v_{D\min}$$

$$V_{GS} + |\hat{v}_i| = 0.5 + V_{DS} - |\hat{v}_o|$$

$$0.95 + \frac{|\hat{v}_o|}{|A_v|} = 0.5 + 2.375 - |\hat{v}_o|$$

$$|\hat{v}_o| = \frac{0.5 + 2.375 - 0.95}{1 + \dfrac{1}{|A_v|}}$$

Substituting $|A_v| = 86.5$, we obtain

$$|\hat{v}_o| = 1.9\ \text{V}$$

An approximate value of $|\hat{v}_o|$ could have been obtained from

$$v_{O\min} = V_{OV} = 0.45\ \text{V}$$

Thus,

$$V_{DS} - |\hat{v}_o| = V_{OV}$$

$$\Rightarrow |\hat{v}_o| = V_{DS} - V_{OV} = 2.375 - 0.45$$

$$= 1.925\ \text{V}$$

$$|\hat{v}_i| = \frac{|\hat{v}_o|}{86.5} = 22\ \text{mV}$$

(c) To determine R_{in}, refer to Fig. 2,

$$i_i = \frac{v_i}{R_{G1}} - \frac{v_o - v_i}{R_{G2}}$$

$$= \frac{v_i}{R_{G1}} - \frac{A_v v_i - v_i}{R_{G2}}$$

$$= v_i\left[\frac{1}{R_{G1}} + \frac{(1 - A_v)}{R_{G2}}\right]$$

$$R_{in} = \frac{v_i}{i_i} = 1\Big/\left[\frac{1}{R_{G1}} + \frac{(1 - A_v)}{R_{G2}}\right]$$

$$= 1\Big/\left(\frac{1}{2} + \frac{(1 + 86.5)}{3}\right) = 33.7\ \text{k}\Omega$$

8.44 Refer to Fig. 8.16(a).

$$R_o = 100\ \text{k}\Omega = r_{o1} \parallel r_{o2}$$

But

$$r_{o1} = r_{o2} = \frac{|V_A|}{I_{REF}} = \frac{5}{I_{REF}}$$

Thus,

$$100 = \frac{1}{2} \times \frac{5}{I_{REF}}$$

$$\Rightarrow I_{REF} = 25\ \mu\text{A}$$

$$A_v = -g_{m1}R_o$$

$-40 = -g_{m1} \times 100$

$\Rightarrow g_{m1} = 0.4 \text{ mA/V}$

But

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}$$

$$0.4 = \sqrt{2 \times 0.4\left(\frac{W}{L}\right)_1 \times 0.025}$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 8$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{OV1}^2$$

$$25 = \frac{1}{2} \times 400 \times 8 \times V_{OV1}^2$$

$\Rightarrow V_{OV1} = 0.125 \text{ V}$

If Q_2 and Q_3 are operated at $|V_{OV}| = 0.125$ V,

$$I_{D2} = I_{D3} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{2,3} |V_{OV}|^2$$

$$25 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{2,3} \times 0.125^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{2,3} = 32$$

8.45 From the results of Example 8.4, we see that the almost linear region of the transfer characteristic (i.e., region 3) is defined by $V_{IA} = 0.89$ V and $V_{IB} = 0.935$ V. Maximum output signal swing is achieved by biasing Q_1 at the middle of this range; thus

$V_I = 0.913$ V

The peak-to-peak amplitude at the output will be $(V_{OA} - V_{OB}) = 2.47 - 0.335 = 2.135$ V. Thus the peak amplitude will be $\frac{1}{2}(2.135) = 1.07$ V.

8.46 Refer to the solution to Example 8.4.

$V_{OA} = V_{DD} - |V_{OV3}|$

$= 5 - 0.526 = 4.474 \simeq 4.47$ V

The relationship between v_O and v_I in region III of the transfer characteristic can be found as follows:

$$\frac{1}{2}k_n'\left(\frac{W}{L}\right)_1 (v_I - V_{tn})^2 \left(1 + \frac{v_O}{V_{An}}\right)$$

$$= \frac{1}{2}k_p'\left(\frac{W}{L}\right)_2 (V_{SG} - |V_{tp}|)^2\left(1 + \frac{V_{DD} - v_O}{|V_{Ap}|}\right)$$

Thus,

$$\frac{k_n'}{k_p'}\,\frac{1}{|V_{OV3}|^2}\,\frac{1}{1 + \dfrac{V_{DD}}{|V_{Ap}|}}(v_I - V_{tn})^2$$

$$= \frac{1 - \dfrac{v_O}{V_{DD} + |V_{Ap}|}}{1 + \dfrac{V_O}{V_{An}}}$$

$$\frac{200}{65 \times 0.526^2}\,\frac{1}{1 + \dfrac{5}{10}}(v_I - 0.6)^2 = \frac{1 - \dfrac{v_O}{5 + 10}}{1 + \dfrac{v_O}{20}}$$

$$7.41(v_I - 0.6)^2 = \frac{1 - 0.07v_O}{1 + 0.05v_O}$$

Substituting $v_O = V_{OA} = 4.47$ V gives

$v_I = V_{IA} = 0.88$ V

To find the coordinates of point B, we note that $V_{OB} = V_{IB} - 0.6$. Thus

$$7.41V_{OB}^2 = \frac{1 - 0.07\,V_{OB}}{1 + 0.05\,V_{OB}}$$

This equation can be solved by trial and error to yield

$V_{OB} = 0.36$ V

and

$V_{IB} = 0.96$ V

Thus at the output the linear region now extends from 0.36 V to 4.47 V as compared to 0.335 V to 2.47 V when the power supply was 3 V: an increase of about the same size as the increase in the power supply.

8.47 Refer to Fig. 8.16(a).

Note that Q_2, Q_3 are not matched:

$I_{D1} = 100\ \mu$A

(a) $I_{D2} = I_{D1} = 100\ \mu$A

$$\frac{I_{D3}}{I_{D2}} = \frac{(W/L)_3}{(W/L)_2} = \frac{W_3}{W_2}$$

(Note that $V_{SG2} = V_{SG3}$)

$$\Rightarrow I_{D3} = 100\ \mu\text{A}\ \frac{10}{40} = 25\ \mu\text{A} \Rightarrow I_{REF} = 25\ \mu\text{A}$$

(b) By referring to Fig. 8.16(d), you notice that in Segment III, both Q_1 and Q_2 are in saturation and the transfer characteristic is quite linear. The output voltage in this segment is limited between V_{OA} and V_{OB}: coordinates of point A:
$v_{OA} = V_{DD} - |V_{OV3}|$

$$|V_{OV3}|^2 = \frac{I_{D3}}{\frac{1}{2}k'_p\left(\frac{W}{L}\right)_3} = \frac{25}{\frac{1}{2}\times 50 \times \frac{10}{1}}$$

$\Rightarrow\ |V_{OV3}| = 0.32$ V

$V_{OA} = 3.3 - 0.32 = 2.98$V

At point B: $V_{OB} = V_{IB} - V_{tn}$

Now we find the transfer equation for the linear section: (Refer to Example 8.4)

$i_{D1} = i_{D2} \Rightarrow$ (Note that $|V_{OV2}| = |V_{OV3}|$)

$$\frac{1}{2}k'_n\left(\frac{W}{L}\right)_1(v_I - V_{tn})^2\left(1+\frac{v_O}{V_{An}}\right)$$

$$= \frac{1}{2}k'_p\left(\frac{W}{L}\right)_2 V_{OV3}^2\left(1+\frac{V_{DD}-v_O}{|V_{AP}|}\right)$$

$$\frac{1}{2}\times 100 \times \frac{20}{1}(v_I - 0.8)^2\left(1+\frac{v_O}{100}\right)$$

$$= \frac{1}{2}\times 50 \times \frac{40}{1}\times 0.32^2\left(1+\frac{3.3-v_O}{50}\right)$$

$$(v_I - 0.8)^2 = 0.32^2\left(1.066 - \frac{v_O}{50}\right)\Big/\left(1+\frac{v_O}{100}\right)$$

$$(v_I - 0.8)^2 = 0.11\left(\frac{1-0.019v_O}{1+0.01v_O}\right)$$

$\simeq 0.11(1 - 0.03v_O)$

$(v_I - 0.8)^2 = 0.11(1 - 0.03v_O)$ (1)

Now if we solve for $V_{OB} = V_{IB} - 0.8$

$V_{OB}^2 + 0.0033V_{OB} - 0.11 = 0 \Rightarrow V_{OB} = 0.33$ V

Therefore the extreme values of v_O for which Q_1 and Q_2 are in saturation $0.33\text{ V} \le v_O \le 2.98$ V

(c) From (b) we can find V_{IA} and V_{IB}:

$V_{IB} = V_{OB} + V_t = 0.33 + 0.8 = 1.13$ V

If we solve (1) for $V_{OA} = 2.98$ V, then

$(V_{IA} - 0.8)^2 = 0.11\,(1 - 0.03\times 2.98) \Rightarrow V_{IA}$

$= 1.116$ V

Large-signal voltage gain

$$= \frac{\Delta v_O}{\Delta v_I} = \frac{2.98 - 0.33}{1.13 - 1.116}$$

$$\frac{\Delta v_O}{\Delta v_i} = -189.3\text{ V/V}$$

(d) $v_O = \dfrac{V_{DD}}{2} = \dfrac{3.3}{2} = 1.65$ V

Differentiating both sides of (1) relative to v_I:

$$2\,(v_I - 0.8) = 0.11\times(-0.03)\frac{\partial v_O}{\partial v_I}$$

$$\Rightarrow \frac{\partial v_O}{\partial v_I} = -606.1\,(v_I - 0.8)$$

For $v_O = 1.65$ V, from (1) we have

$(v_I - 0.8)^2 = 0.11\,(1 - 0.03\times 1.65) \Rightarrow v_I$

$= 1.123$ V

$$\left.\frac{\partial v_O}{\partial v_I}\right|_{v_I = 1.123} = -195.8\text{ V/V}$$

(e) $R_{\text{out}} = r_{o1} \parallel r_{o2}$

$$r_{o1} = \frac{V_{An}}{I_{D1}} = \frac{100\text{ V}}{0.1\text{ mA}} = 1\text{ M}\Omega$$

$$r_{o2} = \frac{V_{Ap}}{I_{D2}} = \frac{50\text{ V}}{0.1\text{ mA}} = 500\text{ k}\Omega$$

$\Rightarrow R_{\text{out}} = 500\text{ k}\Omega \parallel 1\text{ M}\Omega$

$R_{\text{out}} = 333\text{ k}\Omega$

$$g_{m1} = \sqrt{2k'_n\left(\frac{W}{L}\right)_1 I_{D1}}$$

$$= \sqrt{2\times 100\times 10^{-6}\times\frac{20}{1}\times 100\times 10^{-6}}$$

$= 0.632$ mA/V

$A_v = -g_{m1}\,(r_{o1} \parallel r_{o2}) = -210.6$ V/V

Comment: The three estimates of voltage gain obtained in (c), (d) and (e) are all reasonably close; about −200 V/V.

8.48 (a)

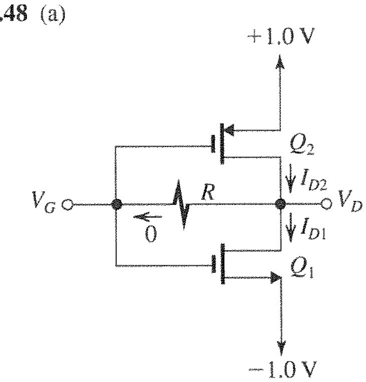


Figure 1

From Fig. 1 we see that since the dc currents into the gates are zero,

$V_D = V_G$

Also, since Q_1 and Q_2 are matched and carry equal drain currents,

$I_{D1} = I_{D2} = I_D$

$V_{SG2} = V_{GS1} = 1$ V

and thus,

$V_G = 0$

Thus,

$$I_D = \frac{1}{2}\times 1\times(1 - 0.5)^2 = 0.125\text{ mA}$$

(b)

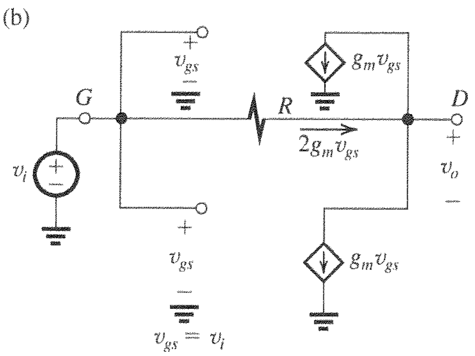


Figure 2

From Fig. 2 we see that

$$v_o = v_i - 2g_m v_{gs} R$$

But

$$v_{gs} = v_i$$

Thus,

$$A_v = \frac{v_o}{v_i} = 1 - 2g_m R$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.125}{1 - 0.5} = 0.5 \text{ mA/V}$$

$$A_v = 1 - 2 \times 0.5 \times 1000 = -999 \text{ V/V}$$

(c)

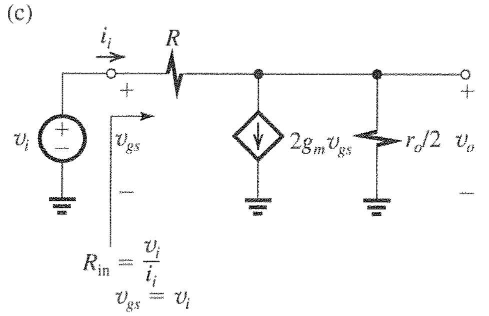


Figure 3

For the circuit in Fig. 3 we can write at the output

$$\frac{v_o}{r_o/2} + 2g_m v_{gs} + \frac{v_o - v_i}{R} = 0$$

Substituting $v_{gs} = v_i$ and rearranging, we obtain

$$\frac{v_o}{v_i} = -2g_m \frac{1 - \dfrac{1}{2g_m R}}{\dfrac{1}{R} + \dfrac{2}{r_o}}$$

But $2g_m R \gg 1$; thus

$$A_v = \frac{v_o}{v_i} \simeq -2g_m \left(R \parallel \frac{r_o}{2}\right)$$

where

$$r_o = \frac{|V_A|}{I_D} = \frac{20}{0.125} = 160 \text{ k}\Omega$$

$$A_v = -2 \times 0.5(1000 \parallel 80) = -74.1 \text{ V/V}$$

$$R_{\text{in}} = \frac{v_i}{i_i} = \frac{v_i}{(v_i - v_o)/R} = R\frac{1}{1 - \dfrac{v_o}{v_i}}$$

$$= \frac{R}{1 - A_v} = \frac{1000}{1 + 74.1} = 13.3 \text{ k}\Omega$$

(d) $\dfrac{v_i}{v_{\text{sig}}} = \dfrac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \dfrac{13.3}{20 + 13.3} = 0.4 \text{ V/V}$

$$G_v = \frac{v_o}{v_{\text{sig}}} = \frac{v_i}{v_{\text{sig}}} \times \frac{v_o}{v_i}$$

$$= 0.4 \times -74.1 = -29.6 \text{ V/V}$$

(e) Both Q_1 and Q_2 remain in saturation for output voltages that ensure that the minimum voltage across each transistor is equal to $|V_{OV}| = 0.5$ V. Thus, the output voltage can range from -0.5 V to $+0.5$ V.

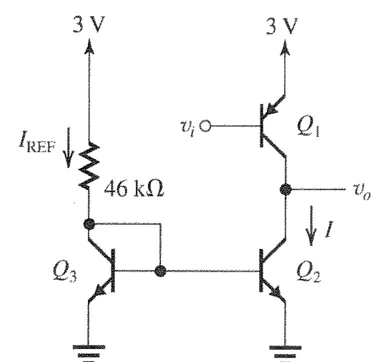
8.49 (a) $I_{\text{REF}} = I_{C3} = \dfrac{3 - V_{BE3}}{46 \text{ k}\Omega}$

$$I_{\text{REF}} = \frac{3 - 0.7}{46}$$

$$= 0.05 \text{ mA}$$

$$\Rightarrow I_{C2} = 5I_{C3}$$

$$I_{C2} = I = 0.25 \text{ mA} \Rightarrow I = 0.25 \text{ mA}$$



(b) $|V_A| = 50 \text{ V} \Rightarrow r_{o1} = \dfrac{|V_A|}{I} = \dfrac{30}{0.25}$

$= 120 \text{ k}\Omega$

$$r_{o2} = \frac{30}{0.25} = 120 \text{ k}\Omega$$

Total resistance at the collector of Q_1 is equal to $r_{o1} \parallel r_{o2}$, thus

$$r_{\text{tot}} = 120 \text{ k}\Omega \parallel 120 \text{ k}\Omega = 60 \text{ k}\Omega$$

(c) $g_{m1} = \dfrac{I_{C1}}{V_T} = \dfrac{0.25}{0.025} = 10 \text{ mA/V}$

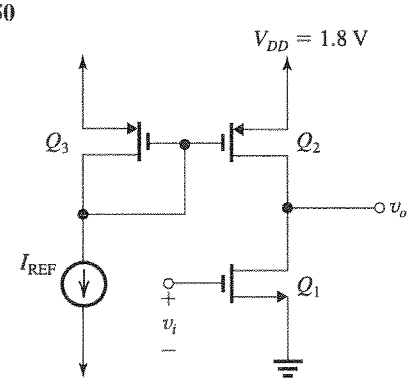
$$r_{\pi 1} = \frac{\beta}{g_m} = \frac{50}{10} = 5 \text{ k}\Omega$$

(d) $R_{\text{in}} = r_{\pi 1} = 5 \text{ k}\Omega$

$$R_o = r_{o1} \parallel r_{o2} = 120 \text{ k}\Omega \parallel 120 \text{ k}\Omega = 60 \text{ k}\Omega$$

$$A_v = -g_{m1} R_o = -10 \times 60 = -600 \text{ V/V}$$

8.50



For an output of 1.6 V,

$V_{SD2\min} = |V_{OV2}| = 1.8 - 1.6 = 0.2$ V,

For an output of 0.2 V,

$V_{DS1\min} = 0.2$ V,

thus

$V_{OV1} = 0.2$ V

Since $I_{D2} = I_{D3} = I_{D1} = 50\ \mu$A

and $I_D = \frac{1}{2}(\mu_p C_{ox})(W/L)V_{OV}^2$, we have

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \frac{2I_{D2}}{(\mu_p C_{OX})(V_{OV})^2}$$

$$= \frac{2\,(50\ \mu\text{A})}{(86\ \mu\text{A/V}^2)(0.2\ \text{V})^2} = 29.1$$

For Q_1,

$$\left(\frac{W}{L}\right)_1 = \frac{2\,(50\ \mu\text{A})}{(387\ \mu\text{A/V}^2)(0.2\ \text{V})^2} = 6.46$$

A_v must be at least -10 V/V

and $A_v = -g_m(r_{o1} \parallel r_{o2})$

$$g_{m1} = \frac{2I_D}{V_{OV1}} = \frac{2 \times 0.05}{0.2} = 0.5\ \text{mA/V}$$

$$r_{o1} \parallel r_{o2} = \frac{10}{0.5} = 20\ \text{k}\Omega$$

But

$$r_{o1} = \frac{V_{A1}}{I_{D1}} = \frac{V'_{An}L}{I_{D1}} = \frac{5L}{0.05} = 100L$$

$$r_{o2} = \frac{|V_{A2}|}{I_{D2}} = \frac{|V'_{Ap}|L}{I_{D2}} = \frac{6L}{0.05} = 120L$$

Thus,

$100L \parallel 120L = 20\ \text{k}\Omega$

$\Rightarrow L = 0.367\ \mu$m

If L is to be an integer multiple of 0.18 μm, then

$L = 0.54\ \mu$m

To raise the gain to 20 V/V, $r_{o1} \parallel r_{o2}$ has to be raised to 40 kΩ, which requires

$L = 2 \times 0.367 = 0.734$

Again, to use a multiple of 0.18 μm we select $L = 0.9\ \mu$m. This represents an increase in L by a factor of $\frac{0.90}{0.54} = \frac{5}{3}$. Ws will have to increase by the same factor. Thus, the area of each transistor will increase by a factor of $\left(\frac{5}{3}\right)^2$ and the total area will increase as follows:

Initial total area =
Area of Q_1 + Area of Q_2 + Area of Q_3

$= 6.46 \times 0.54^2 + 29.1 \times 0.54^2 + 29.1 \times 0.54^2$

$= 18.85\ \mu\text{m}^2$

New total area =
$6.46 \times 0.9^2 + 29.1 \times 0.9^2 + 29.1 \times 0.9^2$

$= 52.37\ \mu\text{m}^2$

Thus, the increase is by a factor of 2.78.

8.51 Refer to Fig. 8.18.

$$R_{\text{in}} = \frac{r_o + R_L}{1 + g_m r_o}$$

$$= \frac{20 + 20}{1 + 2 \times 20} = 980\ \Omega$$

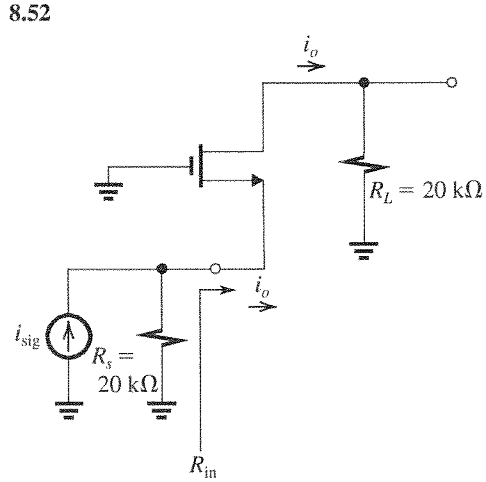
$$R_{\text{out}} = r_o + R_s + g_m r_o R_s$$

$$= 20 + 1 + 2 \times 20 \times 1 = 61\ \text{k}\Omega$$

$$\frac{v_o}{v_{\text{sig}}} = \frac{R_L}{R_s + R_{\text{in}}}$$

$$= \frac{20}{1 + 0.98} = 10.1\ \text{V/V}$$

8.52



$$R_{\text{in}} = \frac{r_o + R_L}{1 + g_m r_o} = \frac{20 + 20}{1 + 2 \times 20} = 980\ \Omega$$

Since $i_s = i_o$,

$$\frac{i_o}{i_{\text{sig}}} = \frac{R_s}{R_s + R_{\text{in}}} = \frac{20}{20 + 0.98} = 0.95\ \text{A/A}$$

If R_L increases by a factor of 10, R_{in} becomes

$$R_{in} = \frac{20 + 200}{1 + 2 \times 20} = 5.37 \text{ k}\Omega$$

and the current gain becomes

$$\frac{i_o}{i_{sig}} = \frac{20}{20 + 5.37} = 0.79 \text{ A/A}$$

Thus an increase in R_L by a factor of 10 resulted in a decrease in the current gain from 0.95 A/A to 0.79 A/A, a change of only -17%. This indicates that the CG amplifier functions as an effective current buffer.

8.53 Refer to Fig. P8.53.

$I_D = 0.2$ mA $\quad V_{OV} = 0.2$ V

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20}{0.2} = 100 \text{ k}\Omega$$

$$R_{out} = r_o + R_s + g_m r_o R_s$$

$$500 = 100 + R_s(1 + 2 \times 100)$$

$$\Rightarrow R_s = \frac{400}{201} \simeq 2 \text{ k}\Omega$$

$$V_{BIAS} = I_D R_S + V_{GS}$$

$$= I_D R_S + V_t + V_{OV}$$

$$= 0.2 \times 2 + 0.5 + 0.2$$

$$= 1.1 \text{ V}$$

8.54 Refer to Fig. P8.54. To obtain maximum output resistance, we use the largest possible R_s consistent with $I_D R_s \leq 0.3$ V. Thus

$$R_s = \frac{0.3 \text{ V}}{0.1 \text{ mA}} = 3 \text{ k}\Omega$$

Now, for Q_2 we have

$g_m = 1$ mA/V $\quad$ and $\quad V_A = 10$ V

Thus,

$$r_o = \frac{V_A}{I_D} = \frac{10 \text{ V}}{0.1 \text{ mA}} = 100 \text{ k}\Omega$$

$$R_{out} = r_o + R_s + g_m r_o R_s$$

$$= 100 + 3 + 1 \times 100 \times 3$$

$$= 403 \text{ k}\Omega$$

8.55 Refer to Fig. P8.55.

(a) $I_{D1} = I_{D2} = I_{D3} = 100$ μA

Using $I_{D1} = \frac{1}{2} k_n'(W/L)_1 V_{OV1}^2$, we obtain

$$0.1 = \frac{1}{2} \times 4 \times V_{OV1}^2$$

$$\Rightarrow V_{OV1} = 0.224 \text{ V}$$

$$V_{GS1} = V_t + V_{OV1} = 0.8 + 0.224 = 1.024 \text{ V}$$

$$V_{BIAS} = V_{GS} + I_{D1} R_s$$

$$= 1.024 + 0.1 \times 0.05 = 1.03 \text{ V}$$

(b) $g_{m1} = \frac{2 I_{D1}}{V_{OV1}} = \frac{2 \times 0.1}{0.224} = 0.9$ mA/V

All transistors are operating at $I_D = 0.1$ mA and have $|V_A| = 20$ V. Thus all have equal values for r_o:

$$r_o = \frac{|V_A|}{I_D} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

(c) For Q_2, $R_L = r_{o2} = 200$ kΩ,

$$R_{in} = \frac{r_o + R_L}{1 + g_m r_o}$$

$$= \frac{200 + 200}{1 + 0.9 \times 200} = 2.2 \text{ k}\Omega$$

(d) $R_{out} = r_o + R_s + g_m r_o R_s$

$$= 200 + 0.05 + 0.9 \times 200 \times 0.05$$

$$= 209 \text{ k}\Omega$$

(e) $\frac{v_i}{v_{sig}} = \frac{R_{in}}{R_{in} + R_s} = \frac{2.2}{2.2 + 0.05} = 0.98$ V/V

$$\frac{v_o}{v_i} = \frac{R_L}{R_{in}} = \frac{200}{2.2} = 90.9 \text{ V}$$

$$\frac{v_o}{v_{sig}} = 90.9 \times 0.98 = 89 \text{ V/V}$$

(f) The value of v_o can range from $V_{BIAS} - V_t = 1.03 - 0.8 = 0.23$ V to $(V_{DD} - V_{OV2})$. Since $I_{D2} = I_{D1}$ and $k_n = k_p$, then $V_{OV2} = V_{OV1}$. Thus the maximum value of v_o is $3.3 - 0.224 = 3.076$ V. Thus the peak-to-peak value of v_o is $3.076 - 0.23 = 2.85$ V. Correspondingly, the peak-to-peak value of v_{sig} will be

$$v_{sig} \text{ (peak to peak)} = \frac{2.85}{89} = 32 \text{ mV}$$

8.56 Given Eq. (8.63):

$$R_{in} \simeq r_e \frac{r_o + R_L}{r_o + \dfrac{R_L}{\beta + 1}}$$

We can write

$$\frac{R_{in}}{r_e} = \frac{1 + (R_L/r_o)}{1 + [R_L/(\beta + 1)r_o]} = \frac{1 + (R_L/r_o)}{1 + (R_L/101 r_o)}$$

R_L/r_o	0	1	10	100	1000	∞
R_{in}/r_e	1	2	10	50.8	91.8	101

Observe that the range of R_{in} is r_e to $(\beta + 1)r_e$.

8.57 $R_{in} \simeq r_e \dfrac{r_o + R_L}{r_o + R_L/(\beta+1)}$

$R_{in} \simeq 2r_e$ is obtained when

$$\frac{r_o + R_L}{r_o + R_L/(\beta+1)} = 2$$

$\Rightarrow R_L \simeq r_o$

8.58 Equation (8.66):

$$R_{out} = r_o + (R_e \parallel r_\pi) + (R_e \parallel r_\pi) g_m r_o$$

$$= r_o + (r_e \parallel r_\pi)(1 + g_m r_o)$$

For $g_m r_o \gg 1$,

$$R_{out} \simeq r_o + g_m r_o (R_e \parallel r_\pi)$$

$$\frac{R_{out}}{r_o} = 1 + \frac{g_m r_\pi R_e}{r_\pi + R_e}$$

$$= 1 + \frac{\beta R_e}{(\beta+1) r_e + R_e}$$

Thus,

$$\frac{R_{out}}{r_o} = 1 + \frac{\beta (R_e/r_e)}{\beta + 1 + (R_e/r_e)}$$

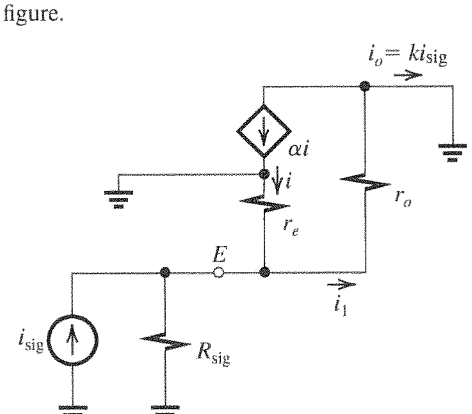
For $\beta = 100$,

$$\frac{R_{out}}{r_o} = 1 + \frac{100(R_e/r_e)}{101 + (R_e/r_e)}$$

R_e/r_e	0	1	2	10	$\beta/2$	β	1000
R_{out}/r_o	1	2	2.9	10	34	51	92

Observe that R_{out} ranges from r_o to $(\beta+1)r_o$, with the maximum value obtained for $R_e = \infty$.

8.59 Refer to Fig. P8.59. To obtain the short-circuit current gain k, we replace the BJT with its T model and short circuit the collector to ground, resulting in the circuit shown in the figure.



At the emitter node we see that there are three parallel resistances to ground: r_e, r_o, and R_{sig}.

Thus,

$$i = -i_{sig} \frac{1/r_e}{\dfrac{1}{r_e} + \dfrac{1}{r_o} + \dfrac{1}{R_{sig}}}$$

and

$$i_1 = i_{sig} \frac{1/r_o}{\dfrac{1}{r_e} + \dfrac{1}{r_o} + \dfrac{1}{R_{sig}}}$$

At the collector node, we can write

$$i_o \equiv k i_{sig} = i_1 - \alpha i$$

Thus,

$$k i_{sig} = i_{sig} \frac{1/r_o + (\alpha/r_e)}{\dfrac{1}{r_e} + \dfrac{1}{r_o} + \dfrac{1}{R_{sig}}} \quad (1)$$

Now $r_o \gg r_e$ and for the case $R_{sig} \gg r_e$, we obtain

$$k \simeq \frac{\alpha/r_e}{1/r_e} = \alpha$$

For our case,

$$\alpha = \frac{\beta}{\beta+1}$$

$$k = \alpha = \frac{\beta}{\beta+1} = \frac{100}{101} = 0.99$$

The output resistance R_{out} is given by

$$R_{out} = r_o + (R_{sig} \parallel r_\pi)(1 + g_m r_o)$$

where

$$r_o = \frac{V_A}{I_C} = \frac{50\text{ V}}{0.1\text{ mA}} = 500\text{ k}\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{0.1\text{ mA}}{0.025\text{ V}} = 4\text{ mA/V}$$

$$g_m r_o = 4 \times 500 = 2000$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25\text{ k}\Omega$$

Thus,

$$R_{out} = 500 + (10 \parallel 25) \times 2001$$

$$= 14.8\text{ M}\Omega$$

Thus the CB amplifier has a current gain of nearly unity and a very high output resistance: a near-ideal current buffer!

A more exact value of k can be obtained using Eq. (1); $k = 0.975$.

8.60 Refer to Fig. P8.60.

$$I = I_C = \alpha I_E = 0.99 \times \frac{5 - 0.7}{4.3} \simeq 1\text{ mA}$$

$$r_o = \frac{V_A}{I_C} = \frac{100\text{ V}}{1\text{ mA}} = 100\text{ k}\Omega$$

$$R_{out} = r_o + (R_E \parallel r_\pi)(1 + g_m r_o)$$

where

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$g_m r_o = 40 \times 100 = 4000$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$R_E = 4.3 \text{ k}\Omega$$

Thus,

$$R_{\text{out}} = 100 + (4.3 \parallel 2.5) \times 4001 = 6.4 \text{ M}\Omega$$

For

$$\triangle V_C = 10 \text{ V}$$

$$\Delta I = \frac{10 \text{ V}}{6.4 \text{ M}\Omega} = 1.6 \text{ }\mu\text{A}$$

A very small change indeed!

8.61 Refer to Fig. 8.27.

$$R_{\text{out}} = r_o + (R_e \parallel r_\pi)(1 + g_m r_o)$$

$$\simeq r_o + (R_e \parallel r_\pi)(g_m r_o)$$

$$\frac{R_{\text{out}}}{r_o} = 1 + g_m(R_e \parallel r_\pi)$$

$$= 1 + \frac{g_m r_\pi R_e}{r_\pi + R_e}$$

$$= 1 + \frac{\beta R_e}{(\beta/g_m) + R_e}$$

For our case $\beta = 100$,

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V, thus}$$

$$\frac{R_{\text{out}}}{r_o} = 1 + \frac{100\, R_e}{5 + R_e} \qquad (1)$$

where R_e is in kilohms.

(a) For $R_{\text{out}} = 5\ r_o$, Eq. (1) gives $R_e = 0.208 \text{ k}\Omega = 208\ \Omega$

(b) For $R_{\text{out}} = 10\ r_o$, Eq. (1) gives $R_e = 0.495 \text{ k}\Omega \simeq 500\ \Omega$.

(c) For $R_{\text{out}} = 50\ r_o$, Eq. (1) gives $R_e = 4.8 \text{ k}\Omega$. From Eq. (1) we see that the maximum value of R_{out}/r_o is obtained with $R_e = \infty$ and its value is 101, which is $(\beta + 1)$.

8.62 $50 = g_{m2} r_{o2}$

$$= A_{02} = \frac{2V_A}{V_{OV}}$$

$$V_A = 50 \times V_{OV}/2$$

$$= 25 \times 0.2 = 5 \text{ V}$$

$$V_A = V_A' L$$

$$5 = 5 \times L \Rightarrow L = 1 \text{ }\mu\text{m}$$

8.63 Refer to Fig. 8.32

$$R_o = g_{m3} r_{o3} r_{o4}$$

For identical transistors,

$$R_o = (g_m r_o) r_o$$

$$= \frac{2|V_A|}{|V_{OV}|} \times \frac{|V_A|}{I}$$

Thus,

$$IR_o = \frac{2|V_A|^2}{V_{OV}} \qquad \text{Q.E.D.}$$

(a) $I = 0.1$ mA

$$0.1 \times R_o = \frac{2 \times 4^2}{0.2} = 160$$

$$R_o = 1.6 \text{ M}\Omega$$

To obtain the W/L values,

$$I = I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$100 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 50$$

(b) $I = 0.5$ mA

$$0.5R_o = \frac{2 \times 4^2}{0.2} = 160$$

$$R_o = 320 \text{ k}\Omega$$

$$I = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$500 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 250$$

8.64 Refer to Fig. 8.32.

$$R_o = (g_{m3} r_{o3}) r_{o4}$$

For identical transistors,

$$R_o = (g_m r_o) r_o$$

$$= \frac{2|V_A|}{|V_{OV}|} \times \frac{|V_A|}{I}$$

Thus,

$$IR_o = \frac{2|V_A|^2}{|V_{OV}|}$$

Substituting

$$|V_A| = |V_A'|\, L$$

$$IR_o = \frac{2|V_A'|^2}{|V_{OV}|} L^2 \qquad \text{Q.E.D.}$$

Now, for

$L = 0.18\ \mu\text{m}, \quad IR_o = \dfrac{2 \times 5^2}{0.2} \times 0.18^2 = 8.1\ \text{V}$

$L = 0.36\ \mu\text{m}, \quad IR_o = \dfrac{2 \times 5^2}{0.2} \times 0.36^2 = 32.4\ \text{V}$

$L = 0.54\ \mu\text{m}, \quad IR_o = \dfrac{2 \times 5^2}{0.2} \times 0.54^2 = 72.9\ \text{V}$

To fill out the table we use

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2I}{|V_{OV}|} = \frac{2I}{0.2} = 10I$$

$$A_v = g_m(R_o/2)$$

(a) The price paid is the increase in circuit area.

(b) As I is increased, g_m increases and hence the current-driving capability of the amplifier, and as we will see later, its bandwidth.

(c) The circuit with the largest area ($58n$) as compared to the circuit with the smallest area ($0.065n$): A_v is $364.5/40.5 = 9$ times larger; g_m is 100 times larger, but R_o is 11.1 times lower.

8.65 Refer to Fig. 8.33(a).

$$g_{m1} = \frac{2I_D}{V_{OV}} = \frac{2I}{V_{OV}}$$

$$2 = \frac{2I}{0.25}$$

$$\Rightarrow I = 0.25\ \text{mA}$$

For identical transistors,

$$R_o = (g_m r_o) r_o = \frac{2V_A}{V_{OV}}\frac{V_A}{I} = \frac{2V_A^2}{V_{OV}\, I}$$

$$200 = \frac{2V_A^2}{0.25 \times 0.25}$$

$$\Rightarrow V_A = 2.5\ \text{V}$$

$$V_A = V_A' L$$

$$L = \frac{V_A}{V_A'} = \frac{2.5}{5} = 0.5\ \mu\text{m}$$

To obtain W/L, we use

$$I_D = I = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$250 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right) \times 0.25^2$$

$$\Rightarrow \frac{W}{L} = 20$$

To obtain maximum negative signal swing at the output, we select V_G so that the voltage at the drain of Q_1 is the minimum permitted, which is equal to V_{OV} (i.e., 0.25 V). Thus

$$V_G = 0.25 + V_{GS2}$$

$$= 0.25 + V_{OV2} + V_t$$

$$= 0.25 + 0.25 + 0.5 = 1.0\ \text{V}$$

The minimum permitted output voltage is

$V_G - V_t = 1 - 0.5 = 0.5$ V or $2V_{OV}$.

8.66 Refer to Fig. 8.33.

$$g_{m1} = \frac{2I_{D1}}{V_{OV1}} = \frac{2I}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2\ \text{mA/V}$$

Since all transistors are operating at the same I_D and $|V_{OV}|$, all have equal values of g_m. Also because all have equal $|V_A| = 4$ V, all r_o's will be equal:

$$r_o = \frac{|V_A|}{I_D} = \frac{|V_A|}{I} = \frac{4}{0.2} = 20\ \text{k}\Omega$$

$$R_{on} = (g_m r_o) r_o = (2 \times 20) \times 20 = 800\ \text{k}\Omega$$

$$R_{op} = (g_m r_o) r_o = (2 \times 20) \times 20 = 800\ \text{k}\Omega$$

$$R_o = R_{on} \parallel R_{op} = 400\ \text{k}\Omega$$

$$A_v = -g_{m1} R_o = -2 \times 400 = -800\ \text{V/V}$$

8.67 Refer to Fig. 8.33.

$$A_v = -g_{m1} R_o$$

$$-280 = -1 \times R_o \Rightarrow R_o = 280\ \text{k}\Omega$$

This table belongs to Problem **8.64**.

	$L = L_{min} = 0.18\ \mu$m, $IR_o = 8.1$ V				**$L = 2L_{min} = 0.36\ \mu$m, $IR_o = 32.4$ V**				**$L = 3L_{min} = 0.54\ \mu$m, $IR_o = 72.9$ V**			
	g_m (mA/V)	**R_o (kΩ)**	**A_v (V/V)**	**$2WL$ (μm^2)**	**g_m (mA/V)**	**R_o (kΩ)**	**A_v (V/V)**	**$2WL$ (μm^2)**	**g_m (mA/V)**	**R_o (kΩ)**	**A_{vo} (V/V)**	**$2WL$ (μm^2)**
$I = 0.01$ mA, $W/L = n$	0.1	810	−40.5	$0.065\ n$	0.1	3,240	−162	$0.26n$	0.1	7,290	−364.5	$0.58n$
$I = 0.1$ mA, $W/L = 10\ n$	1.0	81	−40.5	$0.65n$	1.0	324	−162	$2.6n$	1.0	729	−364.5	$5.8n$
$I = 1.0$ mA, $W/L = 100\ n$	10.0	8.1	−40.5	$6.5n$	10.0	32.4	−162	$26n$	10.0	72.9	−364.5	$58n$

$$g_{m1} = \frac{2I_D}{V_{OV}} = \frac{2I}{V_{OV}} \Rightarrow I = \frac{1}{2}g_{m1}V_{OV}$$

$$= \frac{1}{2} \times 1 \times 0.25 = 0.125 \text{ mA}$$

All four transistors are operated at the same value of I_D and the same value of $|V_{OV}|$. Also all have the same channel length and $|V_A'|$; thus all r_o values are equal. Thus

$$R_{on} = R_{op} = 2R_o = 2 \times 280 = 560 \text{ k}\Omega$$

$$560 = (g_m r_o) r_o$$

$$= \frac{2|V_A|}{|V_{OV}|} \frac{|V_A|}{I}$$

$$= \frac{2|V_A|^2}{0.25 \times 0.125}$$

$$\Rightarrow V_A = 2.96 \text{ V}$$

$$L = \frac{V_A}{V_A'} = \frac{2.96}{5} = 0.6 \text{ μm}$$

For each of the NMOS devices,

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2} V_{OV}^2$$

$$125 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{1,2} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{1,2} = 10$$

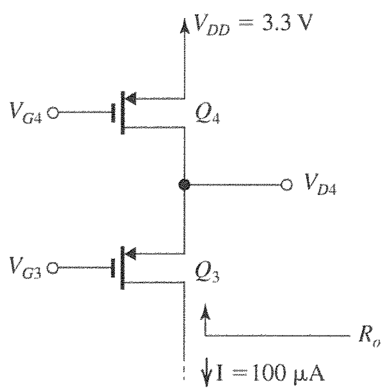
For each of the PMOS transistors,

$$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$125 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 40$$

8.68



$$V_{SG4} = |V_{tp}| + |V_{OV}|$$

$$= 0.8 + 0.2 = 1 \text{ V}$$

Thus,

$$V_{G4} = V_{DD} - V_{SG4} = 3.3 - 1 = 2.3 \text{ V}$$

To obtain the largest possible signal swing at the output, we maximize the allowable positive signal swing by setting V_{D4} at its highest possible value of $V_{DD} - |V_{OV}| = 3.3 - 0.2 = 3.1$ V. This will be obtained by selecting V_{GS} as follows:

$$V_{G3} = V_{D4} - V_{SG3}$$

Since

$$V_{SG3} = V_{SG4} = 1 \text{ V}$$

$$V_{G3} = 3.1 - 1 = 2.1 \text{ V}$$

the highest allowable voltage at the output will be

$$v_{D3\text{max}} = V_{G3} + |V_{tp}|$$

$$= 2.1 + 0.8 = 2.9 \text{ V}$$

Since both Q_3 and Q_4 carry the same current $I = 100$ μA and are operated at the same overdrive voltage, $|V_{OV}| = 0.2$ V, their W/L ratios will be the same and can be found from

$$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$100 = \frac{1}{2} \times 60 \times \left(\frac{W}{L}\right)_{3,4} \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 83.3$$

To obtain R_o, we first find g_m and r_o of both devices,

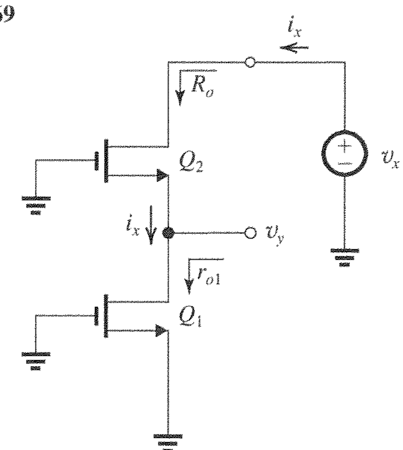
$$g_{m3,4} = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o3,4} = \frac{|V_A|}{I_D} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$R_o = (g_{m3} r_{o3}) r_{o4}$$

$$= 1 \times 50 \times 50 = 2.5 \text{ M}\Omega$$

8.69



While v_x appears across R_o, v_y appears across r_{o1}. Thus,

$$\frac{v_y}{v_x} = \frac{r_{o1}}{R_o}$$

$$= \frac{r_{o1}}{r_{o1} + r_{o2} + g_{m2}r_{o2}r_{o1}}$$

For $g_{m2}r_{o2} \gg 1$ and $g_{m2}r_{o1} \gg 1$,

$$\frac{v_y}{v_x} \simeq \frac{1}{g_{m2}r_{o2}}$$

8.70 Refer to Fig. P8.70.

(a) For the circuit in (a),

$$I = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OVa}^2 \quad (1)$$

For the circuit in (b),

$$I = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{4L}\right)V_{OVb}^2 \quad (2)$$

Comparing Eqs. (1) and (2) we see that

$V_{OVb} = 2V_{OVa}$ Q.E.D.

Now,

$$g_m = \frac{2I_D}{V_{OV}}$$

Thus for the circuit in (a),

$$g_{ma} = \frac{2I}{V_{OVa}}$$

and for the circuit in (b),

$$g_{mb} = \frac{2I}{V_{OVb}} = \frac{2I}{2\ V_{OVa}} = \frac{I}{V_{OVa}}$$

Thus,

$g_{mb} = \frac{1}{2}\ g_{ma}$ Q.E.D.

Since the channel length in (b) is four times that in (a),

$$V_{Ab} = 4V_{Aa}$$

and

$$r_{ob} = 4r_{oa}$$

Thus

$$A_{va} = -g_{ma}r_{oa}$$

and

$$A_{vb} = -g_{mb}r_{ob}$$

$$= -\frac{1}{2}g_{ma} \times 4\ r_{oa}$$

$= 2A_{va}$ Q.E.D.

(b) For the cascode circuit in (c) to have the same minimum voltage requirement at the drain as that for circuit (b), which is equal to $V_{OVb} = 2\ V_{OVa}$, we must operate each of the two transistors in the cascode amplifier at $V_{OV} = V_{OVa}$. Thus each of the two transistors in the cascode circuit will have $g_m = g_{ma}$. Also, each will have $r_o = r_{oa}$. Thus

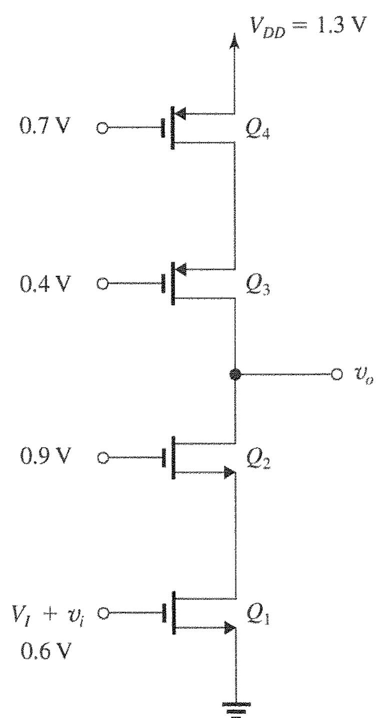
$$A_{vc} = -g_m R_o$$

$$\simeq -g_m[(g_m r_o)r_o]$$

$$= -A_{va}^2$$

Obviously, the cascode delivers a much greater gain than that achieved by quadrupling the channel length of the CS amplifier.

8.71



Since all four transistors have equal transconductance parameters, k, and all four have the same bias current, their overdrive voltages will be equal. We can obtain $|V_{OV}|$ by considering either Q_1 or Q_4. For Q_1,

$$V_{GS} = V_I = 0.6\text{ V} = V_t + V_{OV}$$

Thus,

$$V_{OV} = 0.6 - 0.4 = 0.2\text{ V}$$

Similarly, for Q_4,

$$V_{SG} = V_{DD} - V_{G4} = 1.3 - 0.7 = 0.6\text{ V}$$

Thus,

$|V_{OV}| = V_{SG} - |V_t| = 0.6 - 0.4 = 0.2$ V

The maximum allowable voltage at the output is

$v_{O\max} = V_{GS} + |V_{t3}|$

$= 0.4 + 0.4 = 0.8$ V

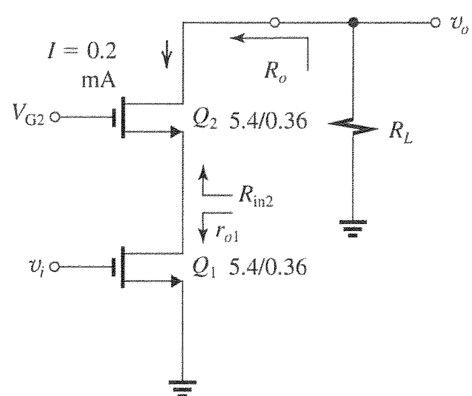
The minimum allowable voltage at the output is

$v_{O\min} = V_{G2} - |V_{t2}|$

$= 0.9 - 0.4 = 0.5$ V

Thus the output voltage can range from 0.5 V to 0.8 V.

8.72



$$g_{m1} = g_{m2} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D}$$

$$= \sqrt{2 \times 0.4 \times \frac{5.4}{0.36} \times 0.2} = 1.55 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{5 \times 0.36}{0.2} = 9 \text{ k}\Omega$$

$R_o = r_{o1} + r_{o2} + g_{m2} r_{o2} r_{o1} = 9 + 9 + 1.55 \times 9 \times 9$

$= 143.6$ kΩ

$A_v = -g_{m1}(R_o \parallel R_L)$

$-100 = -1.55(R_o \parallel R_L)$

$\Rightarrow R_o \parallel R_L = 64.5$ kΩ

$$\frac{1}{R_o} + \frac{1}{R_L} = \frac{1}{64.5}$$

$$\frac{1}{R_L} = \frac{1}{64.5} - \frac{1}{143.6} = \frac{1}{117}$$

$\Rightarrow R_L = 117$ kΩ

$$R_{in2} = \frac{r_{o2} + R_L}{1 + g_{m2} r_{o2}}$$

$$= \frac{9 + 143.6}{1 + 1.55 \times 9} = 10.2 \text{ k}\Omega$$

$R_{d1} = r_{o1} \parallel R_{in2} = 9 \parallel 10.2 = 4.8$ kΩ

$A_1 = -g_{m1} R_{d1} = -1.55 \times 4.8 = -7.41$ V/V

8.73 Refer to Fig. P8.73.

(a) $R_1 = r_{o1} = r_o$

$R_2 \simeq (g_m r_o) r_o$

$$R_3 = \frac{R_2 + r_o}{g_m r_o} = \frac{g_m r_o^2 + r_o}{g_m r_o} \simeq r_o$$

(b) $i_1 = g_m v_i$

$$i_2 = i_1 \frac{R_3}{R_3 + r_o} = g_m v_i \frac{r_o}{r_o + r_o} = \frac{1}{2} g_m v_i$$

$$i_3 = i_1 - i_2 = \frac{1}{2} g_m v_i$$

$$i_4 = i_3 = \frac{1}{2} g_m v_i$$

$$i_5 = i_4 = \frac{1}{2} g_m v_i$$

$i_6 = 0$ (because $v_{sg4} = 0$)

$$i_7 = i_5 = \frac{1}{2} g_m v_i$$

(c) $v_1 = -i_2 r_o = -\frac{1}{2}(g_m r_o) v_i$

$$v_2 = -i_4 R_2 = -\frac{1}{2} g_m (g_m r_o) r_o v_i$$

$$= -\frac{1}{2}(g_m r_o)^2 v_i$$

$$v_3 = -i_5 R_1 = -\frac{1}{2} g_m v_i r_o = -\frac{1}{2}(g_m r_o) v_i$$

(d) v_i is a 5-mV peak sine wave.

$$\hat{v}_1 = -\frac{1}{2} \times 20 \times v_i = -10 \times 5 = -50 \text{ mV}$$

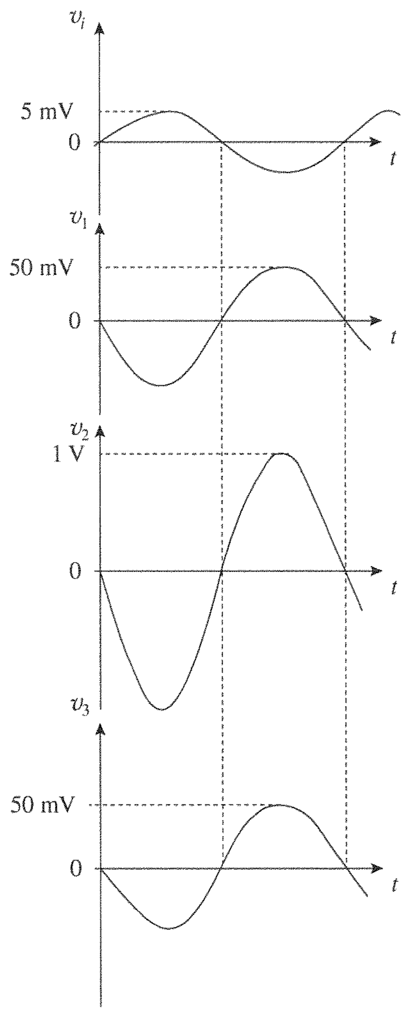
Thus, v_1 is a 50-mV peak sine wave that is 180° out of phase with v_i.

$$\hat{v}_2 = -\frac{1}{2} \times 20^2 \times 5 = -1 \text{ V}$$

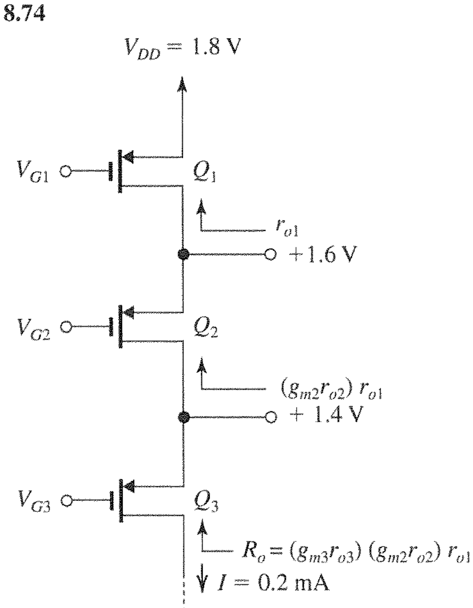
Thus, v_2 is a 1-V peak sine wave, 180° out of phase relative to v_i.

$$\hat{v}_3 = -\frac{1}{2} \times 20 \times 5 = -50 \text{ mV}$$

Thus, v_3 is a 50-mV peak sine wave, 180° out of phase relative to v_i.



8.74



We design for a minimum voltage of $|V_{OV}|$ across each of Q_1 and Q_2.

$$V_{G1} = V_{DD} - V_{SG1} = V_{DD} - |V_{tp}| - |V_{OV}|$$
$$= 1.8 - 0.4 - 0.2 = 1.2 \text{ V}$$

$$V_{G2} = V_{S2} - V_{SG2}$$
$$= 1.6 - 0.4 - 0.2 = 1.0 \text{ V}$$

$$V_{G3} = V_{S3} - V_{SG3}$$
$$= 1.4 - 0.4 - 0.2 = 0.8 \text{ V}$$

All transistors carry the same $I_D = 0.2$ mA and operate at the same value of $|V_{OV}| = 0.2$ V. Thus, their W/L ratios will be equal,

$$0.2 = \frac{1}{2} \times 0.1 \times \frac{W}{L} \times 0.2^2$$

$$\Rightarrow \frac{W}{L} = 100$$

$$R_o = (g_m r_o)^2 r_o$$

where

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_o = \frac{|V_A'|L}{I_D} = \frac{6 \times 0.4}{0.2} = 12 \text{ k}\Omega$$

$$R_o = (2 \times 12)^2 \times 12 = 6.91 \text{ M}\Omega$$

8.75 Refer to Fig. P8.75.

(a) $R_{o1} = r_o$

$R_{o2} = r_o$

$R_{o5} = r_o$

$R_{o4} = (g_m r_o) r_o$

$$R_{o3} = r_{o3} + (g_{m3} r_{o3})(R_{o1} \parallel R_{o2})$$
$$= r_o + g_m r_o \times \frac{1}{2} r_o$$
$$\simeq r_o\left(1 + \frac{1}{2} g_m r_o\right) \simeq \frac{1}{2}(g_m r_o) r_o$$

$$R_{\text{in3}} = \frac{r_{o3} + R_{o4}}{1 + g_{m3} r_{o3}} \simeq \frac{r_o + g_m r_o r_o}{g_m r_o}$$
$$= \frac{1}{g_m} + r_o \simeq r_o$$

(b) $R_o = R_{o3} \parallel R_{o4}$

$$= \frac{1}{2}(g_m r_o) r_o \parallel (g_m r_o) r_o$$
$$= \frac{1}{3}(g_m r_o) r_o$$

(c) When v_o is short-circuited to ground, R_{in2} becomes equal to $1/g_{m3}$. This resistance will be much smaller than the two other resistances between the drain of Q_1 and ground, namely,

$R_{o1} = r_o$ and $R_{o2} = r_o$. Thus the signal current in the drain of Q_1, $g_{m1}v_i$ will mostly flow into $1/g_{m3}$, that is, into the source of Q_3 and out of the drain of Q_3 to ground. Thus, the output short-circuit current will be equal to $g_{m1}v_i$; thus the short-circuit transconductance G_m will be

$G_m = g_{m1}$ Q.E.D.

(d) $\frac{v_o}{v_i} = -g_{m1}R_o$

$$= -g_m \times \frac{1}{3}(g_m r_o) r_o$$

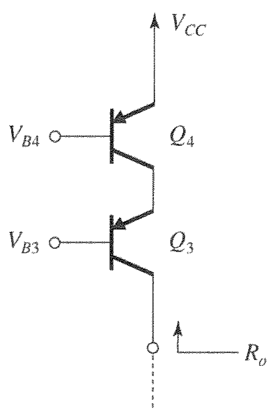
$$= -\frac{1}{3}(g_m r_o)^2$$

For

$g_m = 2$ mA/V and $A_0 = 30$

$$\frac{v_o}{v_i} = -\frac{1}{3}(30)^2 = -300 \text{ V/V}$$

8.76



$$R_o = (g_{m3}r_{o3})(r_{o4} \parallel r_{\pi 3})$$

$I = 0.2$ mA

$$g_{m3} = \frac{I}{V_T} = \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$r_{o3} = r_{o4} = \frac{V_A}{I} = \frac{5}{0.2} = 25 \text{ k}\Omega$$

$$r_{\pi 3} = \frac{\beta}{g_{m3}} = \frac{50}{8} = 6.25 \text{ k}\Omega$$

$$R_o = (8 \times 25)(25 \parallel 6.25)$$

$$= 1 \text{ M}\Omega$$

8.77 When Eq. (8.88) is applied to the case of identical *pnp* transistors, it becomes

$$R_o = (g_m r_o)(r_o \parallel r_\pi)$$

Now,

$$g_m = \frac{I}{V_T} \quad r_o = \frac{|V_A|}{I}$$

$$g_m r_o = |V_A|/V_T$$

$$r_\pi = \frac{\beta}{g_m}$$

Thus,

$$IR_o = \frac{|V_A|}{V_T}\frac{Ir_o\, r_\pi}{r_o + r_\pi}$$

$$= \frac{|V_A|}{V_T}\frac{|V_A|\, r_\pi}{r_o + r_\pi}$$

$$= \frac{|V_A|}{V_T}\frac{|V_A|}{1 + \frac{r_o}{r_\pi}}$$

$$= \frac{|V_A|}{V_T}\frac{|V_A|}{1 + \frac{1}{\beta}g_m r_o}$$

$$= \frac{|V_A|}{V_T}\frac{1}{\frac{1}{|V_A|} + \frac{1}{\beta}\frac{1}{V_T}}$$

$$= \frac{|V_A|}{(V_T/|V_A|) + (1/\beta)} \qquad \text{Q.E.D.}$$

For $|V_A| = 5$ V and $\beta = 50$ we obtain

$$IR_o = \frac{5}{(0.025/5) + (1/50)} = 200 \text{ V}$$

I (mA)	0.1	0.5	1
R_o (kΩ)	2000	400	200

8.78 Refer to Fig. 8.38. When all transistors have equal β and r_o, and, since they conduct equal currents, they have equal g_m, then

$$R_{on} = R_{op} = g_m r_o (r_o \parallel r_\pi)$$

$$R_o = R_{on} \parallel R_{op} = \frac{1}{2}(g_m r_o)(r_o \parallel r_\pi)$$

$$A_v = -g_m R_o$$

$$= -\frac{1}{2}(g_m r_o) g_m (r_o \parallel r_\pi)$$

$$= -\frac{1}{2}(g_m r_o)\frac{g_m r_o r_\pi}{r_\pi + r_o}$$

$$= -\frac{1}{2}(g_m r_o)\frac{1}{\frac{1}{g_m r_o} + \frac{1}{g_m r_\pi}}$$

Substituting $g_m r_o = \frac{|V_A|}{V_T}$ and $g_m r_\pi = \beta$,

$$A_v = -\frac{1}{2}\frac{|V_A|/V_T}{(V_T/|V_A|) + (1/\beta)}$$

For $|V_A| = 5$ V and $\beta = 50$ we obtain

$$A_v = -\frac{1}{2}\frac{5/0.025}{(0.025/5) + (1/50)} = -4000 \text{ V/V}$$

8.79 The output resistance of the cascode amplifier (excluding the load) is

$R_o = g_m r_o (r_o \parallel r_\pi)$

Thus,

$A_v = -g_m (R_o \parallel R_L)$

$= -g_m (R_o \parallel \beta r_o)$

For $|V_A| = 100$ V, $\beta = 50$, and $I = 0.2$ mA we obtain

$$g_m = \frac{I}{V_T} = \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{50}{8} = 6.25 \text{ k}\Omega$$

$$r_o = \frac{|V_A|}{I} = \frac{100}{0.2} = 500 \text{ k}\Omega$$

$R_o = 8 \times 500 \times (500 \parallel 6.25)$

$= 24,691 \text{ k}\Omega$

$A_v = -8(24.7 \parallel 25) \times 10^3$

$= -99.4 \times 10^3 \simeq -10^5$ V/V

8.80 (a) Refer to circuit in Fig. P8.80(a).

$$g_{m1} = \frac{I}{V_T} = \frac{0.1}{0.025} = 4 \text{ mA/V}$$

$g_{m2} = g_{m1} = 4$ mA/V

$$r_{\pi 1} = r_{\pi 2} = \frac{\beta}{g_m} = \frac{100}{4} = 25 \text{ k}\Omega$$

$$r_{o1} = r_{o2} = \frac{|V_A|}{I} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$R_{\text{in}} = r_{\pi 1} = 25 \text{ k}\Omega$

$R_o = g_{m2} r_{o2} (r_{o1} \parallel r_{\pi 2})$

$= (4 \times 50)(50 \parallel 25) = 3.33 \text{ M}\Omega$

$A_{vo} = -g_m R_o$

$= -4 \times 3.33 \times 10^3 = -13{,}320$ V/V

(b) Refer to the circuit in Fig. P8.80(b).

$$g_{m1} = \frac{I}{V_T} = \frac{0.1}{0.025} = 4 \text{ mA/V}$$

$$g_{m2} = \frac{2I_{D2}}{V_{OV}} = \frac{2I}{V_{OV}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{4} = 25 \text{ k}\Omega$$

$$r_{o1} = \frac{|V_A|}{I} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_A|}{I} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$R_{\text{in}} = r_{\pi 1} = 25 \text{ k}\Omega$

$R_o = g_{m2} r_{o2} r_{o1}$

$= 1 \times 50 \times 50 = 2.5 \text{ M}\Omega$

$A_{vo} = -g_{m1} R_o$

$= -4 \times 2.5 \times 10^3 = -10,000$ V/V

(c) Refer to the circuit in Fig. P8.80(c).

$$g_{m1} = g_{m2} = \frac{2I_D}{|V_{OV}|} = \frac{2I}{|V_{OV}|} = \frac{2 \times 0.1}{0.2} =$$
1 mA/V

$$r_{o1} = r_{o2} = \frac{|V_A|}{I_D} = \frac{|V_A|}{I} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$R_{\text{in}} = \infty$

$R_o = g_{m2} r_{o2} r_{o1}$

$= 1 \times 50 \times 50 = 2.5 \text{ M}\Omega$

$A_{vo} = -g_{m1} R_o = -1 \times 2.5 \times 18^3 = -2500$ V/V

(d) Refer to the circuit in Fig. P8.80(c).

$$g_{m1} = \frac{2I_D}{|V_{OV}|} = \frac{2I}{|V_{OV}|} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$g_{m2} = \frac{I}{V_T} = \frac{0.1}{0.025} = 4 \text{ mA/V}$$

$$r_{o1} = \frac{|V_A|}{I} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_A|}{I} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{100}{4} = 25 \text{ k}\Omega$$

$R_{\text{in}} = \infty$

$R_o = (g_{m2} r_{o2})(r_{o1} \parallel r_{\pi 2})$

$= 4 \times 50(50 \parallel 25)$

$= 3.33 \text{ M}\Omega$

$A_{vo} = -g_{m1} R_o$

$= -1 \times 3.33 \times 10^6 = -3330$ V/V

Comment: The highest voltage gain (13,320 V/V) is obtained in circuit (a). However, the input resistance is only 25 kΩ. Of the two circuits with infinite input resistance (c and d), the circuit in (d) has the higher voltage gain. Observe that combining MOSFETs with BJTs results in circuits superior to those with exclusively MOSFETs or BJTs.

8.81 (a) Refer to the circuit in Fig. P8.81(a).

$$g_{m1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D}$$

$= \sqrt{2 \times 0.4 \times 25 \times 0.1}$

$= 1.41$ mA/V

$$r_{o1} = \frac{V_A}{I_D} = \frac{1.8}{0.1} = 18 \text{ k}\Omega$$

$$g_{m2} = \frac{I}{V_T} = \frac{0.1}{0.025} = 4 \text{ mA/V}$$

$$r_{o2} = \frac{V_A}{I} = \frac{1.8}{0.1} = 18 \text{ k}\Omega$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{125}{4} = 31.25 \text{ k}\Omega$$

$$G_m = g_{m1} = 1.41 \text{ mA/V}$$

$$R_o = g_{m2} r_{o2} (r_{o1} \parallel r_{\pi 2})$$

$$= 4 \times 18 \times (18 \parallel 31.25) = 822.3 \text{ k}\Omega$$

$$A_{vo} = -G_m R_o = -1.41 \times 822.3 = -1159 \text{ V/V}$$

(b) Refer to circuit in Fig. P8.81(b).

$$g_{m1} = g_{m2} = \sqrt{2 \times 0.4 \times 25 \times 0.1}$$

$$= 1.41 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{V_A}{I} = \frac{1.8}{0.1} = 18 \text{ k}\Omega$$

$$G_m = g_{m1} = 1.41 \text{ mA/V}$$

$$R_o = g_{m2} r_{o2} r_{o1}$$

$$= 1.41 \times 18 \times 18 = 457 \text{ k}\Omega$$

$$A_{vo} = -G_m R_o = -1.41 \times 457 = -644 \text{ V/V}$$

We observe that the circuit with a cascode transistor provides higher gain.

8.82 Refer to Fig. 8.39.

$$I_O = I_{REF} \frac{(W/L)_2}{(W/L)_1}$$

$$= 20 \frac{40/1}{4/1} = 200 \text{ µA}$$

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{OV1}^2$$

$$20 = \frac{1}{2} \times 160 \times \frac{4}{1} \times V_{OV1}^2$$

$$\Rightarrow V_{OV1} = 0.25 \text{ V}$$

$$V_{G2} = V_{GS1} = V_t + V_{OV1} = 0.6 + 0.25 = 0.85 \text{ V}$$

$$V_{OV4} = V_{OV1}$$

Thus,

$$V_{GS4} = V_{GS1} = 0.85 \text{ V}$$

$$V_{G3} = 0.85 + 0.85 = 1.7 \text{ V}$$

The lowest voltage at the output while Q_3 remains in saturation is

$$V_{O\min} = V_{G3} - V_{t3}$$

$$= 1.7 - 0.6 = 1.1 \text{ V}$$

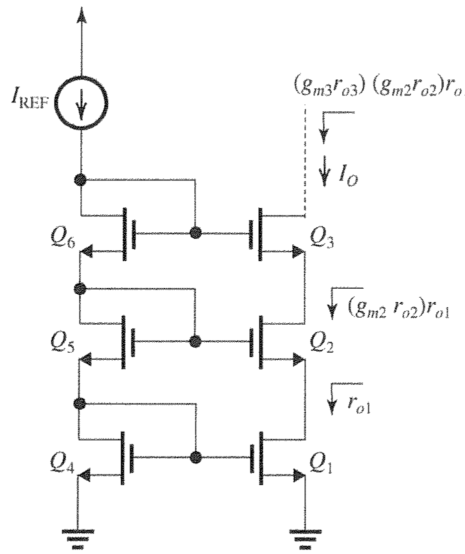
$$g_{m2} = g_{m3} = \frac{2I_{D2,3}}{V_{OV2,3}} = \frac{2 \times 0.2}{0.25} = 1.6 \text{ mA/V}$$

$$r_{o2} = r_{o3} = \frac{V_A}{I_D} = \frac{10}{0.2} = 50 \text{ k}\Omega$$

$$R_o = g_{m3} r_{o3} r_{o2}$$

$$= 1.6 \times 50 \times 50 = 4 \text{ M}\Omega$$

8.83



From the figure we see that

$$R_o = (g_{m3} r_{o3})(g_{m2} r_{o2}) r_{o1}$$

8.84 Refer to Eq. (8.95),

$$R_o = \beta_3 r_{o3}/2$$

where

$$r_{o3} = \frac{V_A}{I} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ k}\Omega$$

Thus,

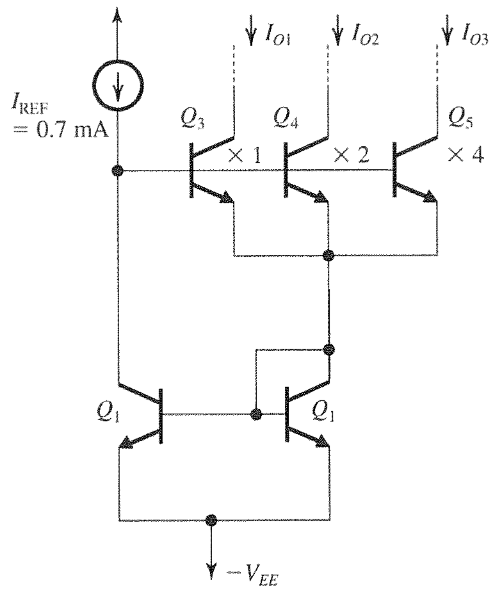
$$R_o = \frac{100 \times 100}{2} = 5 \text{ M}\Omega$$

$$\Delta I_O = \frac{\Delta V_O}{R_o} = \frac{10 \text{ V}}{5 \text{ M}\Omega} = 2 \text{ µA}$$

$$\frac{\Delta I_O}{I_O} = \frac{2 \text{ µA}}{1 \text{ mA}} = 0.002 \qquad \text{or } 0.2\%$$

8.85 (a) $I_{O1} = I_{O2} = \dfrac{1}{2} \dfrac{I_{REF}}{1 + \dfrac{2}{\beta^2}}$

(b)



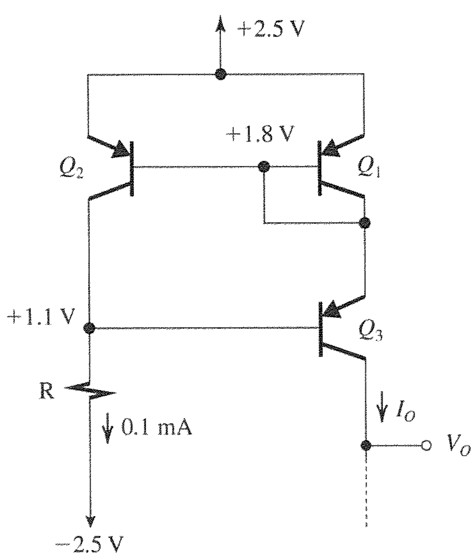
The figure shows the required circuit. Observe that the output transistor is split into three transistors having base–emitter junctions with area ratio 1:2:4. Thus

$$I_{O1} = \frac{0.1}{1+\frac{2}{\beta^2}} = \frac{0.1}{1+\frac{2}{50^2}} = 0.0999 \text{ mA}$$

$$I_{O2} = \frac{0.2}{1+\frac{2}{50^2}} = 0.1998 \text{ mA}$$

$$I_{O4} = \frac{0.4}{1+\frac{2}{50^2}} = 0.3997 \text{ mA}$$

8.86



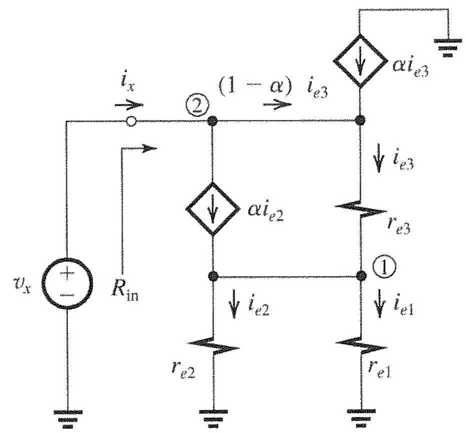
$$R = \frac{1.1+2.5}{0.1} = 36 \text{ k}\Omega$$

$V_{O\max}$ is limited by Q_3 saturating. Thus

$$V_{O\max} = V_{E3} - V_{EC\text{sat}}$$

$$= 1.8 - 0.3 = 1.5 \text{ V}$$

8.87 Replacing each of the transistors in the Wilson mirror of Fig. 8.40 with its T model while neglecting r_o results in the circuit shown below.



Note that the diode-connected transistor Q_1 reduces to a resistance r_{e1}. To determine R_{in}, we have applied a test voltage v_x. In the following we analyze the circuit to find i_x and hence R_{in}, as

$$R_{in} \equiv \frac{v_x}{i_x}$$

Note that all three transistors are operating at equal emitter currents, approximately equal to I_{REF}. Thus

$$r_{e1} = r_{e2} = r_{e3} = \frac{V_T}{I_{REF}}$$

Analysis of the circuit proceeds as follows. Since $r_{e1} = r_{e2}$, we obtain

$$i_{e2} = i_{e1} \qquad (1)$$

Node equation at node 1:

$$i_{e3} + \alpha i_{e2} = i_{e1} + i_{e2}$$

Using Eq. (1) yields

$$i_{e3} = (2-\alpha) i_{e1} \qquad (2)$$

Node equation at node 2:

$$i_x = \alpha i_{e2} + (1-\alpha) i_{e3}$$

Using Eqs. (1) and (2) yields

$$i_x = i_{e1}[\alpha + (1-\alpha)(2-\alpha)]$$

$$i_x = i_{e1}[2 - 2\alpha + \alpha^2] \qquad (3)$$

Finally, v_x can be expressed as the sum of the voltages across r_{e3} and r_{e1},

$$v_x = i_{e3}r_e + i_{e1}r_e$$

Using Eq. (2) yields

$$v_x = i_{e1}r_e(3 - \alpha) \tag{4}$$

Dividing Eq. (4) by Eq. (3) yields

$$R_{in} = \frac{v_x}{i_x} = r_e\frac{3-\alpha}{2-2\alpha+\alpha^2}$$

For $\alpha \simeq 1$,

$$R_{in} = 2r_e = 2\frac{V_T}{I_{REF}} \qquad \text{Q.E.D.}$$

Thus, for $I_{REF} = 0.2$ mA,

$$R_{in} = 250\ \Omega$$

8.88 Refer to circuit in Fig. 8.41(a).

(a) Each of the three transistors is operating at $I_D = I_{REF}$. Thus

$$I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$180 = \frac{1}{2} \times 400 \times 10 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.3\text{ V}$$

$$V_{G3} = V_{tn} + V_{OV} = 0.5 + 0.3 = 0.8\text{ V}$$

(b) Q_1 is operating at $V_{DS} = V_{GS} = 0.8$ V

Q_2 is operating at $V_{DS} = 2V_{GS} = 1.6$ V

Thus,

$$I_{REF} - I_O = \frac{\Delta V_{DS}}{r_o}$$

where

$$r_o = \frac{V_A}{I_{REF}} = \frac{18}{0.18} = 100\text{ k}\Omega$$

$$I_{REF} - I_O = \frac{0.8}{100} = 0.008\text{ mA} = 8\ \mu\text{A}$$

$$I_O = 180 - 8 = 172\ \mu\text{A}$$

(c) Refer to Fig. 8.41(c). Since Q_1 and Q_2 are now operating at equal V_{DS}, we estimate $I_O = I_{REF} = 180\ \mu$A.

(d) The minimum allowable V_O is the value at which Q_3 leaves the saturation region:

$$V_{O\min} = V_{G3} - V_t$$

$$= V_{GS3} + V_{GS1} - V_t$$

$$= 0.8 + 0.8 - 0.5 = 1.1\text{ V}$$

(e) Diode-connected transistor Q_4 has an incremental resistance $1/g_{m4}$. Reference to Fig. 8.41(b) indicates that the incremental resistance of Q_4 would appear in series with the gate of Q_3 and thus carries zero current. Thus including Q_4 has no effect on the value of R_o, which can be found from Eq. (8.96):

$$R_o = g_{m3}r_{o3}r_{o2}$$

where

$$g_{m3} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.18}{0.3} = 1.2\text{ mA/V}$$

$$r_{o2} = r_{o3} = \frac{V_A}{I_{REF}} = \frac{18}{0.18} = 100\text{ k}\Omega$$

Thus,

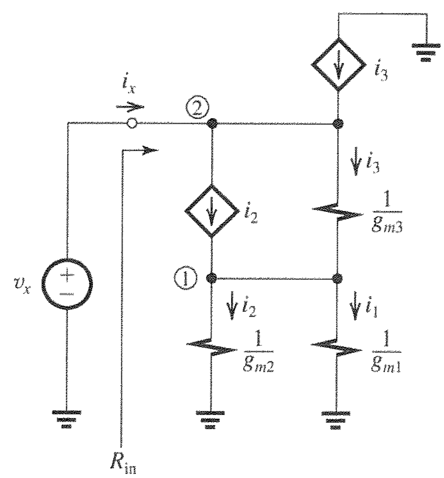
$$R_o = 1.2 \times 100 \times 100 = 12\text{ M}\Omega$$

(f) For $\Delta V_O = 1$ V, we obtain

$$\Delta I_O = \frac{\Delta V_O}{R_o} = \frac{1\text{ V}}{12\text{ M}\Omega} = 0.08\ \mu\text{A}$$

$$\frac{\Delta I_O}{I_O} = 0.04\%$$

8.89 Replacing each of the three transistors in the Wilson current mirror in Fig. 8.41(a) with its T model results in the circuit in the figure.



Here, we have applied a test voltage v_x to determine R_{in},

$$R_{in} \equiv \frac{v_x}{i_x}$$

Since all three transistors are identical and are operating at the same I_D,

$$g_{m1} = g_{m2} = g_{m3}$$

Now from the figure we see that

$$i_1 = i_2$$

and

$$i_2 + i_3 = i_2 + i_1$$

Thus

$$i_3 = i_1 = i_2$$

A node equation at node 2 gives

$i_x + i_3 = i_2 + i_3$

Thus

$i_x = i_2$

The voltage v_x can be expressed as the sum of the voltages across $1/g_{m3}$ and $1/g_{m1}$:

$v_x = (i_3/g_{m3}) + (i_1/g_{m1})$

Substituting $i_3 = i_2$ and $i_1 = i_2$, $g_{m1} = g_{m3} = g_m$, and

$v_x = 2\ i_2/g_m$

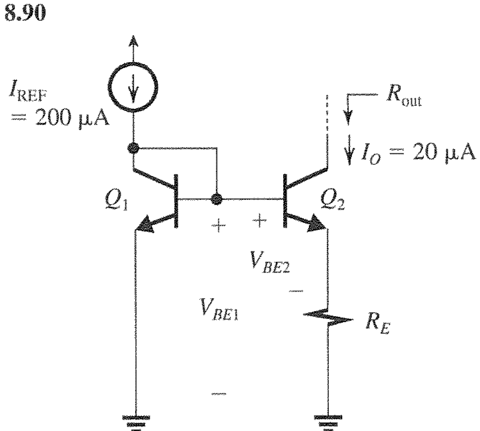
But $i_2 = i_x$; thus

$v_x = 2\ i_x/g_m$

and thus

$R_{\text{in}} = \dfrac{2}{g_m}$ Q.E.D.

8.90



(a) Assuming β is high so that we can neglect base currents,

$$I_O R_E = V_T \ln\left(\frac{I_{\text{REF}}}{I_O}\right)$$

Substituting $I_O = 20\ \mu\text{A}$ and $I_{\text{REF}} = 200\ \mu\text{A}$ results in

$$0.02\ R_E = 0.025\ \ln\left(\frac{200}{20}\right)$$

$\Rightarrow R_E = 2.88\ \text{k}\Omega$

(b) $R_{\text{out}} = (R_E \parallel r_{\pi 2}) + r_{o2} + g_{m2} r_{o2} (R_E \parallel r_{\pi 2})$

where

$$g_{m2} = \frac{0.02}{0.025} = 0.8\ \text{mA/V}$$

$$r_{o2} = \frac{V_A}{I_O} = \frac{50}{0.02} = 2500\ \text{k}\Omega$$

$$r_{\pi 2} = \frac{\beta}{g_m} = \frac{200}{0.8} = 250\ \text{k}\Omega$$

$R_{\text{out}} = (2.9 \parallel 250) + 2500 + 0.8 \times 2500 \times (2.9 \parallel 250)$

$= 8.2\ \text{M}\Omega$

A 5-V change in V_O gives rise to

$$\Delta I_O = \frac{5}{7.1} = 0.7\ \mu\text{A}$$

8.91 Refer to Fig. 8.42.

(a) To obtain a current transfer ratio of 0.8 (i.e., $I_O/I_{\text{REF}} = 0.8$ and $I_O = 80\ \mu\text{A}$), we write

$$I_O R_E = V_T \ln\left(\frac{I_{\text{REF}}}{I_O}\right)$$

$$0.08 R_E = 0.025\ \ln\left(\frac{100}{80}\right)$$

$\Rightarrow R_E = 69.7\ \Omega$

$$g_{m2} = \frac{0.08}{0.025} = 3.2\ \text{mA}$$

$$r_{o2} = \frac{50}{0.08} = 625\ \text{k}\Omega$$

$r_{\pi 2} = \infty$ (because $\beta = \infty$)

$R_{\text{out}} = R_E + r_{o2} + g_{m2} r_{o2} R_E$

$= 0.069 + 625 + 3.2 \times 625 \times 0.0697$

$= 764.5\ \text{k}\Omega$

Relative to the value of r_{o2},

$$\frac{R_{\text{out}}}{r_{o2}} = 1.22$$

(b) To obtain $I_O/I_{\text{REF}} = 0.1$, that is, $I_O = 10\ \mu\text{A}$, we write

$$0.01\ R_E = V_T \ln\left(\frac{100}{10}\right)$$

$\Rightarrow R_E = 5.76\ \text{k}\Omega$

$$g_{m2} = \frac{0.01}{0.025} = 0.4\ \text{mA/V}$$

$$r_{o2} = \frac{50}{0.01} = 5000\ \text{k}\Omega$$

$r_{\pi 2} = \infty$

$R_{\text{out}} = R_E + r_{o2} + g_{m2} r_{o2} R_E$

$= 5.76 + 5000 + 0.4 \times 5000 \times 5.76$

$= 16.5\ \text{M}\Omega$

Compared to r_{o2},

$$\frac{R_{\text{out}}}{r_{o2}} = \frac{16.5}{5} = 3.3$$

(c) To obtain $I_O/I_{\text{REF}} = 0.01$, that is, $I_O = 1\ \mu\text{A}$, we write

$$0.001\ R_E = 0.025\ \ln\left(\frac{100}{1}\right)$$

$$\Rightarrow R_E = 115\ \text{k}\Omega$$

$$g_{m2} = \frac{0.001}{0.025} = 0.04\ \text{mA/V}$$

$$r_{o2} = \frac{50}{0.001} = 50 \times 10^3\ \text{k}\Omega$$

$$R_{\text{out}} = 115 + 50 \times 10^3 + 0.04 \times 50 \times 10^3 \times 115$$

$$= 280\ \text{M}\Omega$$

Relative to the value of r_{o2},

$$\frac{R_{\text{out}}}{r_{o2}} = \frac{280}{50} = 5.6$$

8.92 (a) Refer to the circuit in Fig. P8.92. Neglecting the base currents, we see that all three transistors are operating at $I_C = 10\ \mu\text{A}$, and thus

$$V_{BE1} = V_{BE2} = V_{BE3} = 0.7 - 0.025\ \ln\left(\frac{1\ \text{mA}}{10\ \mu\text{A}}\right)$$

$$= 0.585\ \text{V}$$

From the circuit we see that the voltage across R is $V_{BE} = 0.585$ V, thus

$$I_O R = V_{BE}$$

$$R = \frac{0.585}{0.01} = 58.5\ \text{k}\Omega$$

(b) $g_{m3} = \dfrac{0.01}{0.025} = 0.4\ \text{mA/V}$

$$r_{o3} = \frac{40}{0.01} = 4000\ \text{k}\Omega$$

$$r_{\pi 3} = \frac{\beta}{g_{m3}} = \frac{100}{0.4} = 250\ \text{k}\Omega$$

$$R_{\text{out}} = (R \parallel r_{\pi 3}) + r_{o3} + g_{m3} r_{o3} (R \parallel r_{\pi 3})$$

$$= (58.5 \parallel 250) + 4000 + 0.4 \times 4000 \times (58.5 \parallel 250)$$

$$= 79.9\ \text{M}\Omega$$

8.93 Refer to the circuit in Fig. P8.93. Since Q_1 and Q_2 are matched and conducting equal currents I, their V_{GS} values will be equal. Thus from the loop Q_1, Q_6, R, and Q_2, we see that

$$IR = V_{EB6}$$

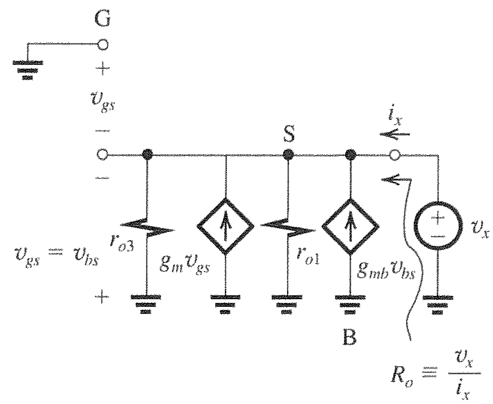
$$= V_T\ \ln\left(\frac{I}{I_S}\right) \qquad \text{Q.E.D.}$$

Now to obtain $I = 0.2$ mA, we write

$$0.2R = 0.7 - 0.025\ \ln\left(\frac{1\ \text{mA}}{0.2\ \text{mA}}\right)$$

$$\Rightarrow R = 3.3\ \text{k}\Omega$$

8.94



The figure shows the equivalent circuit of the source follower prepared for finding R_o. Observe that we have set $v_i = 0$ and applied a test voltage v_x. We note that

$$v_{gs} = v_{bs} = -v_x \qquad (1)$$

and

$$i_x = -g_{mb} v_{bs} + \frac{v_x}{r_{o1}} - g_m v_{gs} + \frac{v_x}{r_{o3}}$$

Thus,

$$i_x = g_{mb} v_x + \frac{v_x}{r_{o1}} + g_m v_x + \frac{v_x}{r_{o3}}$$

from which we obtain

$$R_o \equiv \frac{v_x}{i_x} = r_{o1} \parallel r_{o3} \parallel \frac{1}{g_m + g_{mb}} \qquad \text{Q.E.D.}$$

8.95 The dc level shift provided by a source follower is equal to its V_{GS}. Thus, to obtain a dc level shift of 0.9 V, we write

$$V_{GS} = 0.9\ \text{V} = V_t + V_{OV}$$

$$\Rightarrow V_{OV} = 0.9 - 0.6 = 0.3\ \text{V}$$

To obtain the required bias current, we use

$$I = I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) V_{OV}^2$$

$$= \frac{1}{2} \times 0.2 \times \frac{20}{0.5} \times 0.3^2$$

$$I = 0.36\ \text{mA} = 360\ \mu\text{A}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.36}{0.3} = 2.4\ \text{mA/V}$$

$$g_{mb} = \chi g_m = 0.2 \times 2.4 = 0.48\ \text{mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{20 \times 0.5}{0.36} = 27.8\ \text{k}\Omega$$

To determine A_{vo}, we note [refer to Fig. 8.45(b)] that the total effective resistance between the MOSFET source terminal and ground is

$r_{o1} \parallel r_{o3} \parallel \frac{1}{g_{mb}}$. Denoting this resistance R, we have

$$R = r_o \parallel r_o \parallel \frac{1}{g_{mb}}$$

$$= 27.8 \parallel 27.8 \parallel \frac{1}{0.48}$$

$$= 1.81\ \text{k}\Omega$$

Thus, the open-circuit voltage gain is

$$A_{vo} = \frac{R}{R + \frac{1}{g_m}}$$

$$= \frac{1.81}{1.81 + \frac{1}{2.4}} = 0.81\ \text{V/V}$$

$$R_o = R // \frac{1}{g_m}$$

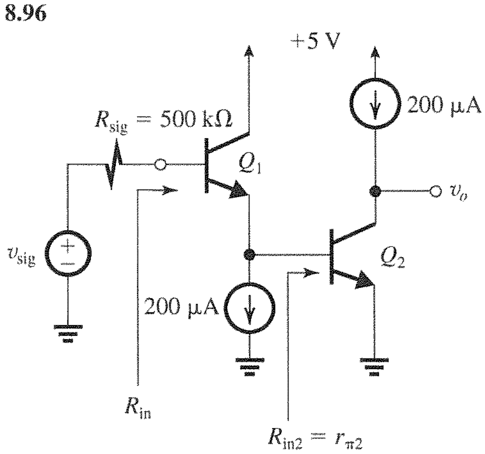
$$= 1.81\ \text{k}\Omega // \frac{1}{2.4\ \text{mA/V}}$$

$$= 0.339\ \text{k}\Omega$$

When a load resistance of 2 kΩ is connected to the output, the total resistance between the output node and ground become $R \parallel R_L = 1.81 \parallel 2 = 0.95\ \text{k}\Omega$. Thus, the voltage gain becomes

$$A_v = \frac{0.95}{0.95 + \frac{1}{2.4}} = 0.7\ \text{V/V}$$

8.96



Each of Q_1 and Q_2 is operating at an I_C approximately equal to 200 μA. Thus for both devices,

$$g_m = \frac{0.2}{0.025} = 8\ \text{mA/V}$$

$$r_e \simeq \frac{1}{g_m} = 0.125\ \text{k}\Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{8} = 12.5\ \text{k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{50}{0.2} = 250\ \text{k}\Omega$$

(a) $R_{in2} = r_{\pi 2} = 12.5\ \text{k}\Omega$

$$R_{in} = (\beta_1 + 1)[r_{e1} + (r_{\pi 2} \parallel r_{o1})]$$

$$= 101[0.125 + (12.5 \parallel 250)]$$

$$= 1.215\ \text{M}\Omega$$

$$\frac{v_{b1}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{1.215}{1.215 + 0.5} = 0.71\ \text{V/V}$$

$$\frac{v_{e1}}{v_{b1}} = \frac{r_{\pi 2} \parallel r_{o1}}{(r_{\pi 2} \parallel r_{o1}) + r_{e1}} = 0.99\ \text{V/V}$$

$$\frac{v_o}{v_{b1}} = -g_{m2} r_{o2} = -8 \times 250 = -2000\ \text{V/V}$$

$$G_v = \frac{v_o}{v_{sig}} = 0.71 \times 0.99 \times -2000 = -1405\ \text{V/V}$$

(b) Increasing the bias current by a factor of 10 (i.e., to 2 mA) results in

$$g_m = 80\ \text{mA/V}$$

$$r_e = 0.0125\ \text{k}\Omega$$

$$r_\pi = 1.25\ \text{k}\Omega$$

$$r_o = 25\ \text{k}\Omega$$

$$R_{in2} = r_{\pi 2} = 1.25\ \text{k}\Omega$$

$$R_{in} = 101[0.0125 + (1.25 \parallel 25)] = 121.5\ \text{k}\Omega$$

Thus, R_{in} has been reduced by a factor of 10.

$$\frac{v_{b1}}{v_{sig}} = \frac{121.5}{121.5 + 500}$$

$$= 0.2\ \text{V/V (considerably reduced)}$$

$$\frac{v_{e1}}{v_{b1}} = \frac{(1.25 \parallel 25)}{(1.25 \parallel 25) + 0.0125}$$

$$= 0.99\ \text{V/V (unchanged)}$$

$$\frac{v_o}{v_{b1}} = -g_{m2} r_o = -80 \times 25$$

$$= -2000\ \text{V/V (unchanged)}$$

$$G_v = \frac{v_o}{v_{sig}} = 0.2 \times 0.99 \times -2000 = -396\ \text{V/V}$$

which has been reduced by a factor of 3.5! All this reduction in gain is a result of the reduction in R_{in}.

8.97

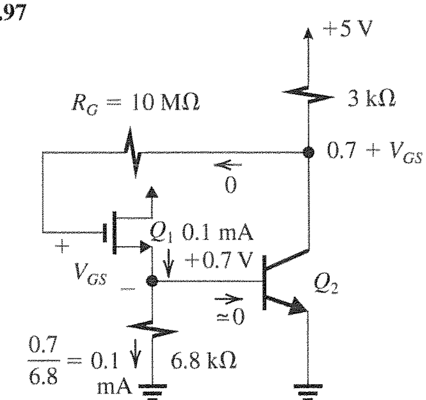


Figure 1

(a) From Fig. 1 we see that

$I_{D1} \simeq 0.1$ mA/V

But

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 2 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.316 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 1.316 \text{ V}$$

Thus,

$$V_{C2} = V_{G2} = 0.7 + V_{GS} = 2.016 \text{ V}$$

$$I_{C2} = \frac{V_{CC} - V_{C2}}{3 \text{ k}\Omega} = \frac{5 - 2.016}{3} \simeq 1 \text{ mA}$$

(b) $g_{m1} = \dfrac{2I_{D1}}{V_{OV}} = \dfrac{2 \times 0.1}{0.316} = 0.632$ mA/V

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{1 \text{ mA}}{0.025} = 40 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{200}{40} = 5 \text{ k}\Omega$$

(c) Neglecting R_G, we can write

$$\frac{v_{b2}}{v_i} = \frac{r_{\pi 2} \parallel 6.8 \text{ k}\Omega}{(r_{\pi 2} \parallel 6.8 \text{ k}\Omega) + \dfrac{1}{g_{m1}}}$$

$$= 0.65 \text{ V/V}$$

$$\frac{v_o}{v_{b2}} = -g_{m2}(3 \parallel 1)$$

$$= -40 \times \frac{3}{4} = -30 \text{ V/V}$$

$$\frac{v_o}{v_i} = 0.65 \times -30 = -19.5 \text{ V/V}$$

(d)

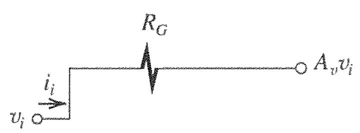


Figure 2

From Fig. 2 we can find i_i as

$$i_i = \frac{v_i - A_v v_i}{R_G}$$

$$= \frac{v_i + 19.4\ v_i}{R_G}$$

Thus,

$$R_{\text{in}} \equiv \frac{v_i}{i_i} = \frac{R_G}{20.5} = \frac{10 \text{ M}\Omega}{20.5} = 487 \text{ k}\Omega$$

Thus the overall voltage gain becomes

$$\frac{v_o}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \times A_v$$

$$\frac{v_o}{v_{\text{sig}}} = \frac{487}{487 + 500} \times -19.5$$

$$= -9.6 \text{ V/V}$$

(e) The suggested configuration, shown partially in Fig. 3, will have

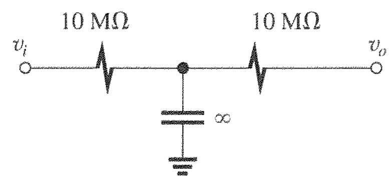


Figure 3

no effect on the dc bias of each transistor. However, it will have a profound effect on R_{in}, as R_{in} now is 10 MΩ, and

$$\frac{v_o}{v_{\text{sig}}} = \frac{10}{10 + 0.5} \times -19.5 = -18.6 \text{ V/V}$$

This is nearly double the value we had before!

8.98 From Fig. P8.98 we see that

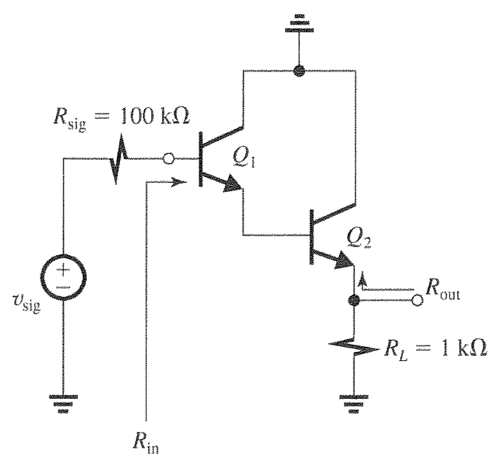
$$I_{E2} = 10 \text{ mA}$$

$$I_{E1} = \frac{I_{E2}}{\beta_2 + 1} \simeq \frac{10}{100} = 0.1 \text{ mA}$$

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{25 \text{ mV}}{10 \text{ mA}} = 2.5\ \Omega$$

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250\ \Omega$$

The Darlington follower circuit prepared for small-signal analysis is shown in the figure.



$$R_{\text{in}} = (\beta + 1)[r_{e1} + (\beta_2 + 1)(r_{e2} + R_L)]$$

$$= 101[0.25 + (101)(0.0025 + 1)]$$

$$= 10.25 \text{ M}\Omega$$

$$R_{\text{out}} = r_{e2} + \frac{r_{e1} + R_{\text{sig}}/(\beta_1 + 1)}{\beta_2 + 1}$$

$$= 2.5 + \frac{250 + \frac{100 \times 10^3}{101}}{101} = 14.8\ \Omega$$

With R_L removed,

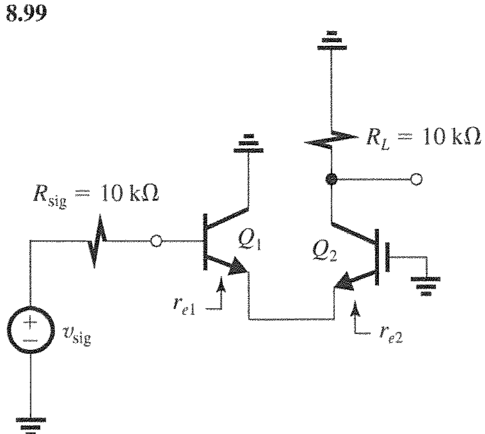
$$G_{vo} = \frac{v_o}{v_{\text{sig}}} = 1$$

With R_L connected,

$$G_v = \frac{v_o}{v_{\text{sig}}} = G_{vo}\frac{R_L}{R_L + R_{\text{out}}}$$

$$= 1 \times \frac{1}{1 + 0.0148} = 0.985$$

8.99



The figure shows the circuit prepared for signal analysis.

$$G_v = \frac{v_o}{v_{\text{sig}}} = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{\alpha R_L}{\frac{R_{\text{sig}}}{\beta + 1} + r_{e1} + r_{e2}}$$

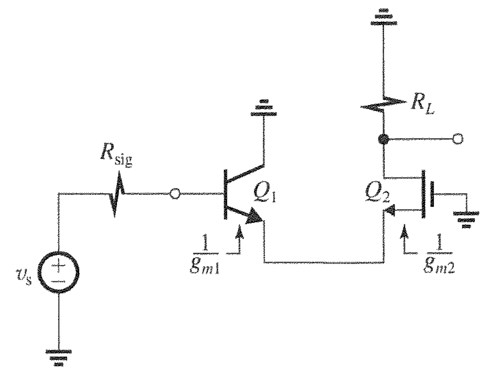
where

$$\alpha \simeq 1$$

$$r_{e1} = r_{e2} = \frac{V_T}{I_E} = \frac{25\ \text{mV}}{0.5\ \text{mA}} = 50\ \Omega$$

$$G_v = \frac{10}{\frac{10}{101} + 0.05 + 0.05} = 50.2\ \text{V/V}$$

8.100



From the figure we can determine the overall voltage gain as

$$G_v = \frac{v_o}{v_{\text{sig}}} = \frac{\text{Total resistance in the drain}}{\text{Total resistance in the sources}}$$

$$= \frac{R_L}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} = \frac{1}{2} g_m R_L$$

where

$$g_m = g_{m1} = g_{m2} = 5\ \text{mA/V}$$

$$G_v = \frac{1}{2} \times 5 \times 10 = 25\ \text{V/V}$$

8.101 Refer to Fig. P8.101. All transistors are operating at $I_E = 0.5$ mA. Thus,

$$r_e = \frac{V_T}{I_E} = \frac{25\ \text{mV}}{0.5\ \text{mA}} = 50\ \Omega$$

(a) Refer to Fig. P8.101(a).

$$\frac{v_o}{v_{\text{sig}}} = -\frac{\alpha \times \text{Total resistance in collector}}{\text{Total resistance in emitter}}$$

$$= \frac{-\alpha \times 10\ \text{k}\Omega}{\frac{10\ \text{k}\Omega}{\beta + 1} + r_e}$$

For

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

$$G_v = \frac{-0.99 \times 10}{\frac{10}{101} + 0.05} = -66.4\ \text{V/V}$$

(b) Refer to Fig. P8.101(b).

$$i_{b1} = \frac{v_{\text{sig}}}{10 + (\beta + 1)r_{e1}} = \frac{v_{\text{sig}}}{10 + 101 \times 0.05}$$

$$i_{c1} = \beta i_{b1} = \frac{100\ v_{sig}}{10 + 101 \times 0.05}$$

$$i_{c2} = \alpha i_{c1} = \frac{0.99 \times 100\ v_{sig}}{10 + 101 \times 0.05}$$

$$v_o = -i_{c2} \times 10$$

$$G_v \equiv \frac{v_o}{v_{sig}} = -\frac{10 \times 0.99 \times 100}{10 + 101 \times 0.05} = -65.8 \text{ V/V}$$

(c) Refer to Fig. P8.101(c).

$$G_v = \frac{v_o}{v_{sig}} = \frac{\alpha \times \text{Total resistance in collector}}{\text{Total resistance in emitters}}$$

$$= \frac{0.99 \times 10}{\dfrac{10}{\beta + 1} + 2\ r_e} = \frac{0.99 \times 10}{\dfrac{10}{101} + 2 \times 0.05}$$

$= 49.7$ V/V

(d) Refer to Fig. P8.101(d).

R_{in} (at the base of Q_1) $= (\beta_1 + 1)[r_{e1} + r_{\pi 2}]$

where

$r_{e1} = 50\ \Omega$

$r_{\pi 2} = (\beta + 1)r_{e2} = 101 \times 50 = 5.05$ kΩ

Thus,

$R_{in} = 101(0.05 + 5.05) = 515$ kΩ

$$\frac{v_{b1}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{515}{515 + 10} = 0.98 \text{ V/V}$$

$$\frac{v_{b2}}{v_{b1}} = \frac{r_{\pi 2}}{r_{\pi 2} + r_{e1}} = \frac{5.05}{5.05 + 0.05} = 0.98 \text{ V/V}$$

$$\frac{v_o}{v_{b2}} = -g_{m2} \times 10 \text{ k}\Omega$$

$= -20 \times 10 = -200$ V/V

$$G_v = \frac{v_o}{v_{sig}} = 0.98 \times 0.98 \times -200 = -194 \text{ V/V}$$

(e) Refer to Fig. P8.101(e).

$$i_{b1} = \frac{v_{sig}}{10 + (\beta + 1)r_{e1}} = \frac{v_{sig}}{10 + 101 \times 0.05}$$

$$i_{c1} = \beta_1 i_{b1} = \frac{100\ v_{sig}}{10 + 101 \times 0.05}$$

$$i_{e2} = i_{c1}$$

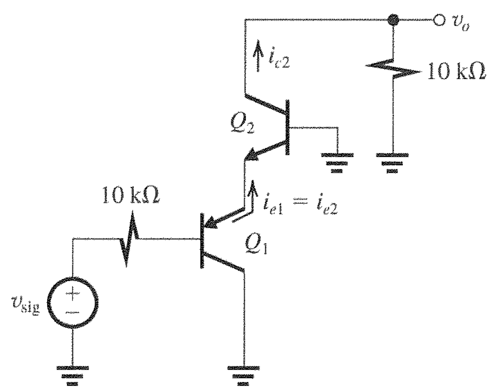
$$i_{c2} = \alpha i_{e2} = \alpha i_{c1} = \frac{0.99 \times 100\ v_{sig}}{10 + 101 \times 0.05}$$

$$v_o = i_{c2} \times 10 = \frac{0.99 \times 100 \times 10\ v_{sig}}{10 + 101 \times 0.05}$$

Thus,

$$G_v = \frac{v_o}{v_{sig}} = \frac{0.99 \times 100 \times 10}{10 + 101 \times 0.05} = 65.8 \text{ V/V}$$

(f)



$$i_{e1} = i_{e2} = \frac{v_{sig}}{\dfrac{10}{\beta_1 + 1} + r_{e1} + r_{e2}}$$

$$= \frac{v_{sig}}{\dfrac{10}{101} + 0.05 + 0.05}$$

$$i_{c2} = \alpha i_{e2} = \frac{0.99\ v_{sig}}{\dfrac{10}{101} + 0.05 + 0.05}$$

$$v_o = i_{c2} \times 10 \text{ k}\Omega = \frac{0.99 \times 10\ v_{sig}}{\dfrac{10}{101} + 0.05 + 0.05}$$

Thus,

$$G_v = \frac{v_o}{v_{sig}} = 49.7 \text{ V/V}$$

9.1 Refer to Fig. 9.2.

(a) $\frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2} V_{OV}^2$

$0.08 = \frac{1}{2} \times 0.4 \times 10 \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.2$ V

$V_{GS} = V_{tn} + V_{OV} = 0.4 + 0.2 = 0.6$ V

(b) $V_{CM} = 0$

$V_S = 0 - V_{GS} = -0.6$ V

$I_{D1} = I_{D2} = \frac{I}{2} = 0.08$ mA

$V_{D1} = V_{D2} = V_{DD} - I_{D1,2}R_D$

$= 1 - 0.08 \times 5 = +0.6$ V

(c) $V_{CM} = +0.4$ V

$V_S = 0.4 - V_{GS} = 0.4 - 0.6 = -0.2$ V

$I_{D1} = I_{D2} = \frac{I}{2} = 0.08$ mA

$V_{D1} = V_{D2} = V_{DD} - I_{D1,2}R_D$

$= 1 - 0.08 \times 5 = +0.6$ V

Since $V_{CM} = 0.4$ V and $V_D = 0.6$ V, $V_{GD} = -0.2$ V, which is less than V_{tn} (0.4 V), indicating that our implicit assumption of saturation-mode operation is justified.

(d) $V_{CM} = -0.1$ V

$V_S = -0.1 - V_{GS} = -0.1 - 0.6 = -0.7$ V

$I_{D1} = I_{D2} = \frac{I}{2} = 0.08$ mA

$V_{D1} = V_{D2} = V_{DD} - I_{D1,2}R_D$

$= 1 - 0.08 \times 5 = +0.6$ V

(e) The highest value of V_{CM} for which Q_1 and Q_2 remain in saturation is

$V_{CM\max} = V_{D1,2} + V_{tn}$

$= 0.6 + 0.4 = 1.0$ V

(f) To maintain the current-source operating properly, we need to keep a minimum voltage of 0.2 V across it, thus

$V_{S\min} = -V_{SS} + V_{CS} = -1 + 0.2 = -0.8$ V

$V_{CM\min} = V_{S\min} + V_{GS}$

$= -0.8 + 0.6$

$= -0.2$ V

9.2 Refer to Fig. P9.2.

(a) For $v_{G1} = v_{G2} = 0$ V,

$I_{D1} = I_{D2} = \frac{1}{2} \times 0.5 = 0.25$ mA

$I_{D1,2} = \frac{1}{2}k_p'\left(\frac{W}{L}\right)|V_{OV}|^2$

$0.25 = \frac{1}{2} \times 4 \times |V_{OV}|^2$

$\Rightarrow |V_{OV}| = 0.35$ V

$V_{SG} = |V_{tp}| + |V_{OV}|$

$= 0.8 + 0.35 = 1.15$ V

$V_S = 0 + V_{SG} = +1.15$ V

$V_{D1} = V_{D2} = -V_{SS} + I_D R_D$

$= -2.5 + 0.25 \times 4$

$= -1.5$ V

Since for each of Q_1 and Q_2,

$V_{SD} = 1.15 - (-1.5)$

$= 2.65$ V

which is greater than $|V_{OV}|$, Q_1 and Q_2 are operating in saturation as implicitly assumed.

(b) The highest value of V_{CM} is limited by the need to keep a minimum of 0.4 V across the current source, thus

$V_{CM\max} = +2.5 - 0.4 - V_{SG}$

$= +2.5 - 0.4 - 1.15 = +0.95$ V

The lowest value of V_{CM} is limited by the need to keep Q_1 and Q_2 in saturation, thus

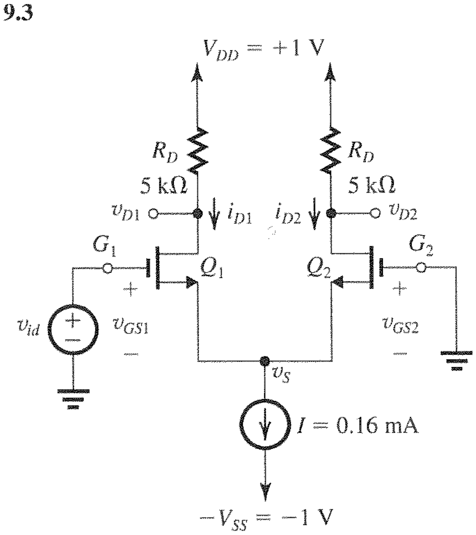
$V_{CM\min} = V_{D1,2} - |V_{tp}|$

$= -1.5 - 0.8 = -2.3$ V

Thus,

$-2.3 \text{ V} \le V_{ICM} \le +0.95$ V

9.3



(a) For $i_{D1} = i_{D2} = 0.08$ mA,

$v_{G1} = v_{G2}$

Thus,

$v_{id} = 0$ V

$i_{D1} = i_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$

$0.08 = \frac{1}{2} \times 0.4 \times 10 \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.2$ V

$v_{GS1} = v_{GS2} = 0.2 + 0.4 = 0.6$ V

$v_S = -0.6$ V

$v_{D1} = v_{D2} = V_{DD} - i_{D1,2}R_D$

$= 1 - 0.08 \times 5 = 0.6$ V

$v_{D2} - v_{D1} = 0$ V

(b) For $i_{D1} = 0.12$ mA and $i_{D2} = 0.04$ mA,

$i_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(v_{GS2} - V_{tn})^2$

$0.04 = \frac{1}{2} \times 0.4 \times 10 \times (v_{GS2} - 0.4)^2$

$\Rightarrow v_{GS2} = 0.541$ V

Thus,

$v_S = -0.541$ V

$i_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(v_{GS1} - V_{tn})^2$

$0.12 = \frac{1}{2} \times 0.4 \times 10\ (v_{id} - v_S - V_{tn})^2$

$= \frac{1}{2} \times 0.4 \times 10\ (v_{id} + 0.541 - 0.4)^2$

$\Rightarrow v_{id} = 0.104$ V

$v_{GS1} = 0.104 - (-0.541) = 0.645$ V

$v_{D1} = V_{DD} - i_{D1}R_D$

$= 1 - 0.12 \times 5 = 0.4$ V

$v_{D2} = V_{DD} - i_{D2}R_D$

$= 1 - 0.04 \times 5 = 0.8$ V

$v_{D2} - v_{D1} = 0.8 - 0.4 = 0.4$ V

(c) $i_{D1} = 0.16$ mA and $i_{D2} = 0$ with Q_2 just cutting off, thus

$v_{GS2} = V_{tn} = 0.4$ V

$\Rightarrow v_{S2} = -0.4$ V

$i_{D1} = \frac{1}{2} \times 0.4 \times 10\ (v_{GS1} - V_{tn})^2$

$0.16 = \frac{1}{2} \times 0.4 \times 10\ (v_{id} + 0.4 - 0.4)^2$

$\Rightarrow v_{id} = 0.283$ V

which is $\sqrt{2}V_{OV}$, as derived in the text.

$v_{GS1} = 0.283 - (-0.4) = 0.683$ V

$v_{D1} = V_{DD} - i_{D1}R_D$

$= 1 - 0.16 \times 5 = +0.2$ V

Note that since $v_{G1} = v_{id} = 0.283$ V, Q_1 is still operating in saturation, as implicitly assumed.

$v_{D2} = V_{DD} - i_{D2}R_D$

$= 1 - 0 \times 5 = 1$ V

$v_{D2} - v_{D1} = 1 - 0.2 = 0.8$ V

(d) $i_{D1} = 0.04$ mA and $i_{D2} = 0.12$ mA. Since this split of the current I is the complement of that in case (b) above, the value of v_{id} must be the negative of that found in (b). Thus,

$v_{id} = -0.104$ V

$v_{GS1} = 0.541$ V

$v_S = -0.645$ V

$v_{GS2} = 0.645$ V

$v_{D1} = V_{DD} - i_{D1}R_D$

$= 1 - 0.04 \times 5 = 0.8$ V

$v_{D2} = 1 - 0.12 \times 5 = 0.4$ V

$v_{D2} - v_{D1} = -0.4$ V

(e) $i_{D1} = 0$ (Q_1 just cuts off) and $i_{D2} = 0.16$ mA. This case is the complement of that in (c) above, thus

$v_{GS1} = V_{tn} = 0.4$ V

$v_{GS2} = 0.683$ V

$v_S = -0.683$ V

$v_{id} = -0.683 + 0.4 = -0.283$ V

which is $-\sqrt{2}\,V_{OV}$, as derived in the text.

$v_{D1} = V_{DD} - i_{D1}R_D = 1 - 0 \times 5 = 1$ V

$v_{D2} = V_{DD} - i_{D2}R_D = 1 - 0.16 \times 5 = 0.2$ V

$v_{D2} - v_{D1} = -0.8$ V

Summary

A summary of the results is shown in the following table on the next page.

Case	i_{D1}(mA)	i_{D2}(mA)	v_{id}(V)	v_S(V)	v_{D1}(V)	v_{D2}(V)	$v_{D2}-v_{D1}$(V)
a	0.08	0.08	0	−0.6	+0.6	+0.6	0
b	0.12	0.04	+0.104	−0.541	+0.4	+0.8	+0.4
c	0.16	0	+0.283	−0.4	+0.2	+1.0	+0.8
d	0.04	0.12	−0.104	−0.645	+0.8	+0.4	−0.4
e	0	0.16	−0.283	−0.683	+1.0	+0.2	−0.8

9.4 Refer to Fig. P9.2.

To determine V_{OV},

$$0.25 = \frac{1}{2} \times 4 \times |V_{OV}|^2$$

$$\Rightarrow |V_{OV}| = 0.354 \text{ V}$$

With $v_{G2} = 0$ and $v_{G1} = v_{id}$, to steer the current from one side of the differential pair to the other, v_{id} must be the ends of the range

$$-\sqrt{2}\,|V_{OV}| \le v_{id} \le \sqrt{2}\,|V_{OV}|$$

that is,

$$-0.5 \text{ V} \le v_{id} \le +0.5 \text{ V}$$

At $v_{id} = -0.5$ V, Q_2 just cuts off, thus

$$v_S = |V_{tp}| = 0.8 \text{ V}$$

and

$$v_{SG1} = 0.8 - (-0.5) = 1.3 \text{ V}$$

At this value of v_{SG1},

$$i_{D1} = \frac{1}{2} \times 4 \times (1.3 - 0.8)^2$$

$$= 0.5 \text{ mA}$$

which is the entire bias current.

$$v_{D1} = -2.5 + 0.5 \times 4 = -0.5 \text{ V}$$

Observe that since $v_{G1} = v_{D1}$, Q_1 is still operating in saturation, as implicitly assumed.

$$v_{D2} = -2.5 \text{ V}$$

At $v_{id} = +0.5$ V, Q_1 just cuts off, thus $v_{SG1} = |V_{tp}| = 0.8$ V and

$$v_S = +0.5 + 0.8 = +1.3 \text{ V}$$

and thus

$$v_{SG2} = 1.3 \text{ V}$$

which results in

$$i_{D1} = \frac{1}{2} \times 4\,(1.3 - 0.8)^2$$

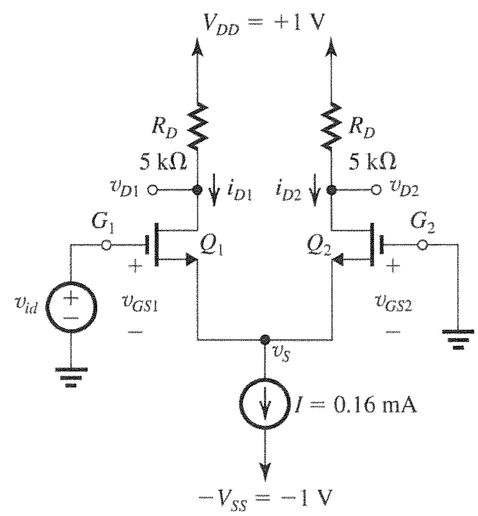
$$= 0.5 \text{ mA}$$

which is the entire bias current. Here,

$$v_{D2} = -2.5 + 0.5 \times 4 = -0.5 \text{ V}$$

which verifies that Q_2 is operating in saturation, as implicitly assumed.

9.5



For $i_{D1} = 0.09$ mA and $i_{D2} = 0.07$ mA,

$$i_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(v_{GS2} - V_{tn})^2$$

$$0.07 = \frac{1}{2} \times 0.4 \times 10(v_{GS2} - 0.4)^2$$

$$\Rightarrow v_{GS2} = 0.587 \text{ V}$$

and

$$v_S = -0.587 \text{ V}$$

$$i_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(v_{GS1} - V_{tn})^2$$

$$0.09 = \frac{1}{2} \times 0.4 \times 10\,(v_{GS1} - 0.4)^2$$

$$\Rightarrow v_{GS1} = 0.612 \text{ V}$$

$$v_{id} = v_S + v_{GS1} = -0.587 + 0.612$$

$$= 0.025 \text{ V}$$

$$v_{D2} = V_{DD} - i_{D2}R_D$$

$$= 1 - 0.07 \times 5 = 0.65 \text{ V}$$

$v_{D1} = 1 - 0.09 \times 5 = 0.55$ V

$v_{D2} - v_{D1} = 0.65 - 0.55 = 0.10$ V

$$\text{Voltage gain} = \frac{v_{D2} - v_{D1}}{v_{id}} = \frac{0.10}{0.025} = 4 \text{ V/V}$$

To obtain the complementary split in current, that is, $i_{D1} = 0.07$ mA and $i_{D2} = 0.09$ mA,

$v_{id} = -0.025$ V

9.6 Refer to the circuit in Fig. P9.6.

For $v_{G1} = v_{G2} = 0$ V,

$$I_{D1} = I_{D2} = \frac{0.4}{2} = 0.2 \text{ mA}$$

To obtain

$V_{D1} = V_{D2} = +0.1$ V

$V_{DD} - I_{D1,2} R_D = 0.1$

$0.9 - 0.2 R_D = 0.1$

$\Rightarrow R_D = 4$ kΩ

For Q_1 and Q_2,

$$I_{D1,2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2} V_{OV}^2$$

$$0.2 = \frac{1}{2} \times 0.4\left(\frac{W}{L}\right)_{1,2} \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{1,2} = 44.4$$

For Q_3,

$$0.4 = \frac{1}{2} \times 0.4 \times \left(\frac{W}{L}\right)_3 \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = 88.8$$

Since Q_3 and Q_4 form a current mirror with $I_{D3} = 4I_{D4}$,

$$\left(\frac{W}{L}\right)_4 = \frac{1}{4}\left(\frac{W}{L}\right)_3 = 22.2$$

$V_{GS4} = V_{GS3} = V_{tn} + V_{OV} = 0.4 + 0.15$

$= 0.55$ V

$$R = \frac{0.9 - (-0.9) - 0.55}{0.1}$$

$= 12.5$ kΩ

The lower limit on V_{CM} is determined by the need to keep Q_3 operating in saturation. For this to happen, the minimum value of V_{DS3} is $V_{OV} = 0.15$ V. Thus,

$V_{ICM\min} = -V_{SS} + V_{OV3} + V_{GS1,2}$

$= -0.9 + 0.15 + 0.4 + 0.15$

$= -0.2$ V

The upper limit on V_{CM} is determined by the need to keep Q_1 and Q_2 in saturation, thus

$V_{ICM\max} = V_{D1,2} + V_{tn}$

$= 0.1 + 0.4 = 0.5$ V

Thus,

$-0.2 \text{ V} \le V_{ICM} \le +0.5$ V

9.7 From Exercise 9.3 and the accompanying table, we note that $|v_{id}|_{\max}$ is proportional to V_{OV}:

$$\frac{|v_{id}|_{\max}}{V_{OV}} = \frac{0.126}{0.2} = 0.63$$

Thus, to obtain $|v_{id}|_{\max} = 220$ mV $= 0.22$ V at the same level of linearity, we use

$$V_{OV} = \frac{0.22}{0.63} = 0.35 \text{ V}$$

For this value of V_{OV}, the required (W/L) can be found from

$$0.2 = \frac{1}{2} \times 0.2 \times \left(\frac{W}{L}\right) \times 0.35^2$$

$$\Rightarrow \frac{W}{L} = 16.3$$

The value of g_m is

$$g_m = \frac{2\, I_D}{V_{OV}} = \frac{2 \times 0.2}{0.35} = 1.14 \text{ mA/V}$$

9.8 Refer to Eq. (9.23). For

$$\left(\frac{v_{id}/2}{V_{OV}}\right)^2 \le k$$

$$\Rightarrow \left(\frac{v_{id}/2}{V_{OV}}\right) \le \sqrt{k} \qquad (1)$$

$$\Delta I = I\left(\frac{v_{id}/2}{V_{OV}}\right)\sqrt{1 - \left(\frac{v_{id}/2}{V_{OV}}\right)^2}$$

$$\Delta I_{\max} = I\,\sqrt{k}\sqrt{1-k}$$

Thus,

$$\frac{\Delta I_{\max}}{I/2} = 2\sqrt{k(1-k)} \qquad \text{Q.E.D.} \qquad (2)$$

and the corresponding value of v_{id} is found from Eq. (2) as

$$v_{id\max} = 2\sqrt{k}V_{OV} \qquad \text{Q.E.D.} \qquad (3)$$

Equations (2) and (3) can be used to evaluate $\frac{\Delta I_{\max}}{I/2}$ and $\frac{v_{id\max}}{V_{OV}}$ for various values of k:

k	0.01	0.1	0.2
$\frac{v_{id\max}}{V_{OV}}$	0.2	0.632	0.894
$\frac{\Delta I_{\max}}{I/2}$	0.2	0.6	0.8

9.9 Switching occurs at

$$v_{id} = \sqrt{2}V_{OV}$$

Thus,

$$0.3 = \sqrt{2}V_{OV}$$

$$\Rightarrow V_{OV} = 0.212 \text{ V}$$

Now, to obtain full current switching at $v_{id} = 0.5$ V, V_{OV} must be increased to

$$V_{OV} = 0.212 \times \frac{0.5}{0.3} = 0.353 \text{ V}$$

Since I_D is proportional to V_{OV}^2 the current I_D and hence the bias current I must be increased by the ratio $(0.353/0.212)^2$, then I must be

$$I = 200 \times \left(\frac{0.353}{0.212}\right)^2 = 554.5 \text{ µA}$$

9.10 Refer to Fig. 9.5.

$$g_m = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$1 = \frac{I}{0.25}$$

$$\Rightarrow I = 0.25 \text{ mA}$$

$$\frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$\frac{1}{2} \times 0.25 = \frac{1}{2} \times 0.4 \times \left(\frac{W}{L}\right)0.25^2$$

$$\Rightarrow \frac{W}{L} = 10$$

9.11 Equations (9.23) and (9.24):

$$i_{D1} = \frac{I}{2} + \frac{I}{2}\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \left(\frac{v_{id}/2}{V_{OV}}\right)^2} \qquad (9.23)$$

$$i_{D2} = \frac{I}{2} - \frac{I}{2}\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \left(\frac{v_{id}/2}{V_{OV}}\right)^2} \qquad (9.24)$$

(a) For 10% increase above the equilibrium value of $\frac{I}{2}$,

$$\left(\frac{I}{2}\right)\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \left(\frac{v_{id}/2}{V_{OV}}\right)^2} = 0.1 \times \frac{I}{2}$$

$$\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \frac{1}{4}\left(\frac{v_{id}}{V_{OV}}\right)^2} = 0.1$$

$$\Rightarrow \frac{v_{id}}{V_{OV}} \simeq 0.1$$

$$v_{id} \simeq 0.1V_{OV}$$

(b) In Eqs. (9.23) and (9.24) let

$$i_{D1} = \left(\frac{I}{2}\right) + \left(\frac{I}{2}\right) \times \Delta$$

$$i_{D2} = \left(\frac{I}{2}\right) - \left(\frac{I}{2}\right) \times \Delta$$

where

$$\Delta = \left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \left(\frac{v_{id}/2}{V_{OV}}\right)^2}$$

If v_{id} is such that

$$\frac{i_{D1}}{i_{D2}} = m$$

then

$$m = \frac{1+\Delta}{1-\Delta}$$

$$\Rightarrow \Delta = \frac{m-1}{m+1}$$

For $m = 1$, $\Delta = 0$ and $v_{id} = 0$

For $m = 2$,

$$\Delta = \frac{2-1}{2+1} = \frac{1}{3}$$

$$\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \frac{1}{4}\left(\frac{v_{id}}{V_{OV}}\right)^2} = \frac{1}{3}$$

Squaring both sides, we obtain a quadratic equation in $\left(\frac{v_{id}}{V_{OV}}\right)^2$ which can be solved to obtain

$$v_{id} = 0.338V_{OV}$$

For $m = 1.1$,

$$\Delta = \frac{1.1-1}{1.1+1} = \frac{0.1}{2.1} \simeq 0.05$$

Thus,

$$\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \frac{1}{4}\left(\frac{v_{id}}{V_{OV}}\right)^2} = 0.05$$

$$\Rightarrow v_{id} \simeq 0.05V_{OV}$$

For $m = 1.01$

$$\Delta = \frac{1.01-1}{1.01+1} \simeq 0.005$$

$$\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \frac{1}{4}\left(\frac{v_{id}}{V_{OV}}\right)^2} = 0.005$$

$$v_{id} \simeq 0.005V_{OV}$$

For $m = 20$,

$$\Delta = \frac{m-1}{m+1} = \frac{19}{21} = 0.905 \text{ V}$$

Thus,

$$\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \frac{1}{4}\left(\frac{v_{id}}{V_{OV}}\right)^2} = 0.905$$

$$\Rightarrow v_{id} = 1.072 V_{OV}$$

9.12 $0.1 = \frac{1}{2} \times 0.2 \times 32 V_{OV}^2$

$$\Rightarrow V_{OV} = 0.18 \text{ V}$$

$$g_m = \frac{2 \times (0.2/2)}{0.18} = 1.11 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$A_d = g_m (R_D \parallel r_o)$$

$$= 1.11 \times (10 \parallel 100) = 10.1 \text{ V/V}$$

9.13 For $v_{id} = 0.1$ V

$$\left(\frac{v_{id}/2}{V_{OV}}\right)^2 = 0.04$$

$$\frac{v_{id}/2}{V_{OV}} = 0.2$$

$$\frac{0.1/2}{V_{OV}} = 0.2$$

$$\Rightarrow V_{OV} = 0.25 \text{ V}$$

$$g_m = \frac{2 \times (I/2)}{V_{OV}}$$

$$2 = \frac{I}{0.25}$$

$$\Rightarrow I = 0.5 \text{ mA}$$

$$A_d = \frac{1 \text{ V}}{0.1 \text{ V}} = 10$$

$$g_m R_D = 10$$

$$\Rightarrow R_D = \frac{10}{2} = 5 \text{ k}\Omega$$

$$\frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right) V_{OV}^2$$

$$0.25 = \frac{1}{2} \times 0.2 \times \frac{W}{L} \times 0.25^2$$

$$\Rightarrow \frac{W}{L} = 40$$

9.14 To limit the power dissipation to 1 mW,

$$P = (V_{DD} + V_{SS}) I$$

Thus, the maximum value we can use for I is

$$I = \frac{1 \text{ mW}}{2 \text{ V}} = 0.5 \text{ mA}$$

Using this value, we obtain

$$V_D = V_{DD} - \frac{I}{2} R_D$$

$$0.2 = 1 - 0.25 \times R_D$$

$$\Rightarrow R_D = 3.2 \text{ k}\Omega$$

$$A_d = g_m R_D$$

$$10 = g_m \times 3.2$$

$$g_m = \frac{10}{3.2} = 3.125 \text{ mA/V}$$

But

$$g_m = \frac{2 \times (I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$3.125 = \frac{0.5}{V_{OV}}$$

$$\Rightarrow V_{OV} = 0.16 \text{ V}$$

To obtain W/L, we use

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right) V_{OV}^2$$

$$0.25 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.16^2$$

$$\Rightarrow \frac{W}{L} = 48.8 \simeq 50$$

9.15 Since the quiescent power dissipation is

$$P = (V_{DD} + V_{SS}) \times I$$

then the maximum allowable I is

$$I = \frac{1 \text{ mW}}{2 \text{ V}} = 0.5 \text{ mA}$$

We shall utilize this value. The value of V_{OV} can be found from

$$\sqrt{2}\, V_{OV} = 0.25 \text{ V}$$

$$\Rightarrow V_{OV} = \frac{0.25}{\sqrt{2}} = 0.18 \text{ V}$$

The realized value of g_m will be

$$g_m = \frac{2 \times (I/2)}{V_{OV}}$$

$$= \frac{0.5}{0.18} = 2.8 \text{ mA/V}$$

To obtain a differential gain A_d of 10 V/V,

$$A_d = g_m R_D$$

$$10 = 2.8 \times R_D$$

$$\Rightarrow R_D = 3.6 \text{ k}\Omega$$

Finally, the required value of W/L can be determined from

$$I_D = \frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L} V_{OV}^2$$

$$0.25 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.18^2$$

$$\Rightarrow \frac{W}{L} = 38.6$$

9.16 (a) $A_d = g_m R_D$

$$20 = g_m \times 47$$

$$\Rightarrow g_m = \frac{20}{47} = 0.426 \text{ mA/V}$$

(b) $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2(I/2)}{V_{OV}} = \dfrac{I}{V_{OV}}$

$$0.426 = \frac{I}{0.2}$$

$$\Rightarrow I = 0.085 \text{ mA} = 85\ \mu\text{A}$$

(c) Across each R_D the dc voltage is

$$\frac{I}{2} R_D = \frac{0.085}{2} \times 47 = 2 \text{ V}$$

(d) The peak sine-wave signal across each gate source is 5 mV, thus at each drain the peak sine wave is

$$A_d \times 5 = 20 \times 5 = 100 \text{ mV} = 0.1 \text{ V}$$

(e) The minimum voltage at each drain will be

$$v_{D\min} = V_{DD} - R_D I_D - V_{\text{peak}}$$

$$= V_{DD} - 2 - 0.1$$

For the transistor to remain in saturation

$$v_{D\min} \geq v_{G\max} - V_{tn}$$

where

$$v_{G\max} = V_{CM} + V_{\text{peak}}(\text{input})$$

$$= 0.5 + 0.005 = 0.505 \text{ V}$$

Thus,

$$V_{DD} - 2.1 \geq 0.505 - 0.5$$

$$V_{DD} \geq 2.105 \text{ V}$$

Thus, the lowest value of V_{DD} is 2.21 V.

9.17 For a CS amplifier biased at a current I_D and utilizing a drain resistance R_D, the voltage gain is

$$|A| = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

Thus,

$$|A| = \sqrt{2\mu_n C_{ox} \frac{W}{L}} \sqrt{I_D} R_D \quad (1)$$

For a differential pair biased with a current I and utilizing drain resistances R_D, the differential gain is

$$A_d = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} \left(\frac{I}{2}\right)}$$

Thus

$$A_d = \sqrt{2\mu_n C_{ox} \frac{W}{L}} \sqrt{I/2}\, R_D \quad (2)$$

Equating the gains from Eqs. (1) and (2), we get

$$I = 2I_D$$

That is, the differential pair must be biased at a current twice that of the CS amplifier. Since both circuits use equal power supplies, the power dissipation of the differential pair will be twice that of the CS amplifier.

9.18 Since both circuits use the same supply voltages and dissipate equal powers, then their currents must be equal, that is,

$$I_D = I$$

where I_D is the bias current of the CS amplifier and I is the bias current of the differential pair. The gain of the CS amplifier is

$$|A| = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{\text{CS}} I_D}$$

Thus,

$$|A| = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{\text{CS}} I_D R_D} \quad (1)$$

The gain of the differential amplifier is

$$A_d = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{\text{diff}} \left(\frac{I}{2}\right)}$$

Thus,

$$A_d = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{\text{diff}} \left(\frac{I}{2}\right) R_D} \quad (2)$$

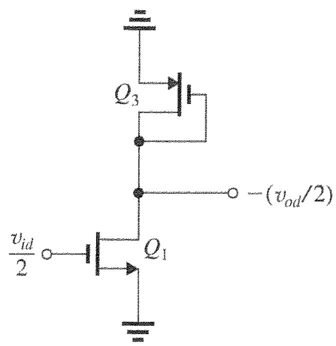
Equating the gains in Eqs. (1) and (2) and substituting $I_D = I$ gives

$$\sqrt{\left(\frac{W}{L}\right)_{\text{CS}}} = \sqrt{\left(\frac{W}{L}\right)_{\text{diff}} \times \frac{1}{2}}$$

$$\Rightarrow \left(\frac{W}{L}\right)_{\text{diff}} = 2\left(\frac{W}{L}\right)_{\text{CS}}$$

If all transistors have the same channel length, each of the differential pair transistors must be twice as wide as the transistor in the CS amplifier.

9.19



(a) The figure shows the differential half-circuit. Recalling that the incremental (small-signal) resistance of a diode-connected transistor is given by $\left(\frac{1}{g_m} \parallel r_o\right)$, the equivalent load resistance of Q_1 will be

$$R_D = \frac{1}{g_{m3}} \parallel r_{o3}$$

and the differential gain of the amplifier in Fig. P9.19 will be

$$A_d \equiv \frac{v_{od}}{v_{id}} = g_{m1}\left[\frac{1}{g_{m3}} \parallel r_{o3} \parallel r_{o1}\right]$$

Since both sides of the amplifier are matched, this expression can be written in a more general way as

$$A_d = g_{m1,2}\left[\frac{1}{g_{m3,4}} \parallel r_{o3,4} \parallel r_{o1,2}\right]$$

(b) Neglecting $r_{o1,2}$ and $r_{o3,4}$ (much larger that $1/g_{m3,4}$),

$$A_d \simeq \frac{g_{m1,2}}{g_{m3,4}}$$

$$= \frac{\sqrt{2\mu_n C_{ox}(W/L)_{1,2}(I/2)}}{\sqrt{2\mu_p C_{ox}(W/L)_{3,4}(I/2)}}$$

$$= \sqrt{\frac{\mu_n (W/L)_{1,2}}{\mu_p (W/L)_{3,4}}}$$

(c) $\mu_n = 4\mu_p$ and all channel lengths are equal,

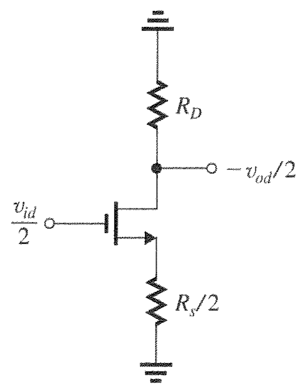
$$A_d = 2\sqrt{\frac{W_{1,2}}{W_{3,4}}}$$

For $A_d = 10$,

$$10 = 2\sqrt{\frac{W_{1,2}}{W_{3,4}}}$$

$$\Rightarrow \frac{W_{1,2}}{W_{3,4}} = 25$$

9.20



From symmetry, a virtual ground appears at the mid point of R_s. Thus, the differential half circuit will be as shown in the figure, and

$$A_d \equiv \frac{v_{od}}{v_{id}} = \frac{R_D}{\frac{1}{g_m} + \frac{R_s}{2}}$$

For $R_s = 0$,

$$A_d = \frac{R_D}{1/g_m} = g_m R_D,$$

as expected.

To reduce the gain to half this value, we use

$$\frac{R_s}{2} = \frac{1}{g_m}$$

$$\Rightarrow R_s = \frac{2}{g_m}$$

9.21 Refer to Fig. P9.21.

(a) With $v_{G1} = v_{G2} = 0$,

$$v_{GS1} = v_{GS2} = V_{OV1,2} + V_{tn}$$

Thus

$$V_{S1} = V_{S2} = -(V_{OV1,2} + V_{tn})$$

(b) For the situation in (a), V_{DS} of Q_3 is zero, thus zero current flows in Q_3. Transistor Q_3 will have an overdrive voltage of

$$V_{OV3} = V_C - V_{S1,2} - V_{tn}$$

$$= V_C + (V_{OV1,2} + V_{tn}) - V_{tn}$$

$$= V_C + V_{OV1,2}$$

(c) With $v_{G1} = v_{id}/2$ and $v_{G2} = -v_{id}/2$ where v_{id} is a small signal, a small signal will appear between drain and source of Q_3. Transistor Q_3 will be operating in the triode region and its drain-source resistance r_{DS} will be given by

$$r_{DS} = \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_3 V_{OV3}}$$

Thus,

$$R_s = \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_3 V_{OV3}}$$

Now,

$$g_{m1,2} = (\mu_n C_{ox})\left(\frac{W}{L}\right)_{1,2} V_{OV1,2}$$

$$g_{m3} = (\mu_n C_{ox})\left(\frac{W}{L}\right)_3 V_{OV3}$$

For $\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_{1,2}$,

$$\mu_n C_{ox}\left(\frac{W}{L}\right) = \frac{g_{m1,2}}{V_{OV1,2}}$$

Thus,

$$R_s = \frac{1}{\frac{g_{m1,2}}{V_{OV1,2}} \times V_{OV3}} = \frac{1}{g_{m1,2}} \frac{V_{OV1,2}}{V_{OV3}}$$

(d) (i) $R_s = \frac{1}{g_{m1,2}}$

$V_{OV3} = V_{OV1,2}$

But

$V_{OV3} = V_C + V_{OV1,2}$

$\Rightarrow V_C = 0$

(ii) $R_s = \frac{0.5}{g_{m1,2}}$

$\Rightarrow V_{OV3} = 2\ V_{OV1,2}$

But

$V_{OV3} = V_C + V_{OV1,2}$

$\Rightarrow V_C = V_{OV1,2}$

9.22 Refer to Fig. P9.22.

(a) With $v_{G1} = v_{G2} = 0$ V,

$V_{S1} = V_{S2} = -V_{GS1,2} = -(V_t + V_{OV})$

The current through Q_3 and Q_4 will be zero because the voltage across them $(v_{DS3} + v_{DS4})$ is zero.

Because the voltages at their gates are zero and at their sources are $-(V_t + V_{OV})$, each of Q_3 and Q_4 will be operating at an overdrive voltage equal to V_{OV}. Thus each of Q_3 and Q_4 will have an r_{DS} given by

$$r_{DS3,4} = \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_{3,4} V_{OV}} \quad (1)$$

Since

$$g_{m1,2} = \mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2} V_{OV} \quad (2)$$

substituting from (2) into (1) gives

$$r_{DS3,4} = \frac{1}{g_{m1,2}} \frac{(W/L)_{1,2}}{(W/L)_{3,4}}$$

and since

$R_s = r_{DS3} + r_{DS4}$

then

$$R_s = \frac{2}{g_{m1,2}} \frac{(W/L)_{1,2}}{(W/L)_{3,4}} \quad (3)$$

(b) With $v_{G1} = v_{id}/2$ and $v_{G2} = -v_{id}/2$ where v_{id} is a small signal,

$$A_d \equiv \frac{v_{od}}{v_{id}}$$

$$= \frac{2\ R_D}{\frac{1}{g_{m1}} + R_s + \frac{1}{g_{m2}}}$$

Using (3), we obtain

$$A_d = \frac{R_D}{\frac{1}{g_{m1,2}} + \frac{1}{g_{m1,2}} \frac{(W/L)_{1,2}}{(W/L)_{3,4}}}$$

$$= \frac{g_{m1,2}\ R_D}{1 + \frac{(W/L)_{1,2}}{(W/L)_{3,4}}}$$

9.23 Refer to Fig. P9.23.

The value of R is found as follows:

$$R = \frac{V_{G6} - V_{G7}}{I_{\text{REF}}}$$

$$= \frac{0.8 - (-0.8)}{0.2} = 8\ \text{k}\Omega$$

Since $I = I_{\text{REF}}$, Q_3 and Q_6 are matched and are operating at

$|V_{OV}| = 1.5 - 0.8 - 0.5 = 0.2$ V

Thus,

$$0.2 = \frac{1}{2} \times 0.1 \times \left(\frac{W}{L}\right)_{6,3} \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_6 = 100$$

Each of Q_4 and Q_5 is conducting a dc current of $(I/2)$ while Q_7 is conducting a dc current $I_{\text{REF}} = I$. Thus Q_4 and Q_5 are matched and their W/L ratios are equal while Q_7 has twice the (W/L) ratio of Q_4 and Q_5. Thus,

$$\frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{4,5} V_{OV4,5}^2$$

where

$$V_{OV4,5} = -0.8 - (-1.5) - 0.5 = 0.2 \text{ V}$$

thus,

$$0.1 = \frac{1}{2} \times 0.25 \times \left(\frac{W}{L}\right)_{4,5} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_5 = 20$$

and

$$\left(\frac{W}{L}\right)_7 = 40$$

$$r_{o4} = r_{o5} = \frac{|V_{Ap}|}{I/2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{o1} = r_{o2} = \frac{V_{An}}{I/2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$A_d = g_{m1,2}(r_{o1,2} \parallel r_{o4,5})$$

$$50 = g_{m1,2}(100 \parallel 100)$$

$$\Rightarrow g_{m1,2} = 1 \text{ mA/V}$$

But

$$g_{m1,2} = \frac{2(I/2)}{|V_{OV1,2}|}$$

$$1 = \frac{0.2}{|V_{OV1,2}|}$$

$$\Rightarrow |V_{OV1,2}| = 0.2 \text{ V}$$

The (W/L) ratio for Q_1 and Q_2 can now be determined from

$$0.1 = \frac{1}{2} \times 0.1 \times \left(\frac{W}{L}\right)_{1,2} \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 50$$

A summary of the results is provided in the table below.

Transistor	W/L	I_D(mA)	$\lvert V_{GS}\rvert$(V)
Q_1	50	0.1	0.7
Q_2	50	0.1	0.7
Q_3	100	0.2	0.7
Q_4	20	0.1	0.7
Q_5	20	0.1	0.7
Q_6	100	0.2	0.7
Q_7	40	0.2	0.7

9.24 Refer to Fig. P9.24.

(a) Since the dc voltages V_{GS1} and V_{GS2} are equal, Q_1 and Q_2 will be operating at the same value of V_{OV} and their dc currents I_{D1} and I_{D2} will have the same ratio at their (W/L) ratios, that is,

$$I_{D1} = I/3$$

$$I_{D2} = 2I/3$$

(b) Q_1 and Q_2 will be operating at the same V_{OV}, obtained as follows:

$$\frac{I}{3} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$\Rightarrow V_{OV} = \sqrt{\frac{2I}{3\mu_n C_{ox}\left(\frac{W}{L}\right)}}$$

(c) $A_d \equiv \dfrac{v_{od}}{v_{id}}$

$$= \frac{2R_D}{\dfrac{1}{g_{m1}} + \dfrac{1}{g_{m2}}}$$

where

$$g_{m1} = \frac{2 \times (I/3)}{V_{OV}} = \frac{2I}{3V_{OV}}$$

$$g_{m2} = \frac{2 \times (2I/3)}{V_{OV}} = \frac{4I}{3V_{OV}}$$

$$A_d = \frac{2R_D}{\left(\dfrac{3}{2} + \dfrac{3}{4}\right)(V_{OV}/I)} = \frac{8}{9}\frac{IR_D}{V_{OV}}$$

9.25 Refer to Fig. 9.13.

All transistors have the same channel length and are carrying a dc current $I/2$. Thus all transistors have the same $r_o = \dfrac{|V_A|}{I/2}$. Also, all transistors are operating at the same $|V_{OV}|$ and have equal dc currents, thus all have the same $g_m = \dfrac{2(I/2)}{|V_{OV}|} = I/|V_{OV}|$. Thus all transistors have equal intrinsic gain $g_m r_o = 2|V_A|/|V_{OV}|$. Now, the gain A_d is given by

$$A_d = g_m(R_{on} \parallel R_{op})$$

$$= \frac{1}{2}g_m R_{on}$$

$$= \frac{1}{2}g_m(g_m r_o)r_o = \frac{1}{2}(g_m r_o)^2$$

Thus,

$$A_d = \frac{1}{2}\left[\frac{2|V_A|}{V_{OV}}\right]^2$$

$$= 2(|V_A|/|V_{OV}|)^2 \qquad \text{Q.E.D.}$$

To obtain $A_d = 500$ V/V while operating all transistors at $|V_{OV}| = 0.2$ V, we use

$$500 = 2\frac{|V_A|^2}{0.04}$$

$$\Rightarrow |V_A| = 3.16 \text{ V}$$

Since $|V_A'| = 5$ V/μm, the channel length L (for all transistors) must be

$$3.16 = 5 \times L$$

$$L = 0.632\ \mu\text{m}$$

To obtain the highest possible g_m, we operate at the highest possible I consistent with limiting the power dissipation (in equilibrium) to 0.5 mW. Thus,

$$I = \frac{0.5 \text{ mW}}{(0.9 + 0.9)\text{V}} = 0.28 \text{ mA}$$

9.26 Refer to Fig. 9.15(a).

The current I will split equally between Q_1 and Q_2. Thus,

$$I_{E1} = I_{E2} = 0.2 \text{ mA}$$

$$I_{C1} = I_{C2} = \alpha \times 0.2 = 0.99 \times 0.2 = 0.198 \text{ mA}$$

$$V_{BE1} = V_{BE2} = 0.7 + 0.025 \ln\left(\frac{0.198}{1}\right)$$

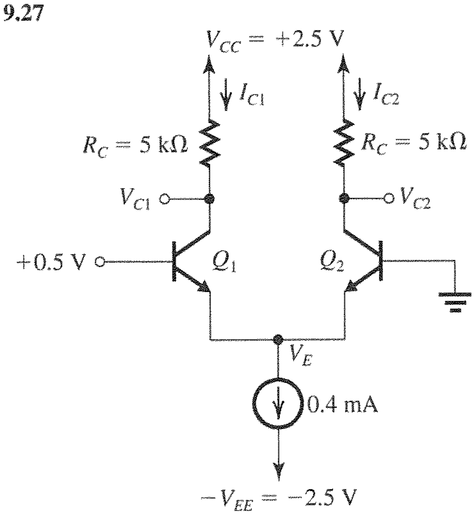
$$= 0.660 \text{ V}$$

$$V_{E1} = V_{E2} = -1 - 0.66 = -1.66 \text{ V}$$

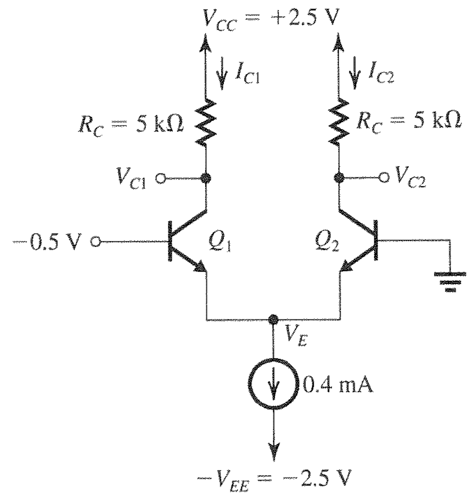
$$V_{C1} = V_{C2} = V_{CC} - I_{C1,2}\, R_C$$

$$= 2.5 - 0.198 \times 5 = +1.51 \text{ V}$$

9.27



(a)



(b)

(a) For $v_{B1} = +0.5$ V, Q_1 conducts all the current I (0.4 mA) while Q_2 cuts off. Thus Q_1 will have a V_{BE} obtained as follows:

$$V_{BE1} = 0.7 + 0.025 \ln\left(\frac{0.99 \times 0.4}{1}\right)$$

$$= 0.677 \text{ V}$$

Thus,

$$V_E = +0.5 - 0.677 = -0.177 \text{ V}$$

which indicates that $V_{BE2} = +0.177$ V, too small to turn Q_2 on.

$$V_{C1} = V_{CC} - I_{C1}R_C$$

$$= 2.5 - 0.99 \times 0.4 \times 5$$

$$= +0.52 \text{ V}$$

$$V_{C2} = V_{CC} - I_{C2} \times R_C$$

$$= 2.5 - 0 \times 5 = 2.5 \text{ V}$$

Observe that Q_1 is operating in the active mode, as implicitly assumed, and the current source has a voltage of 2.323 V across it, more than sufficient for its proper operation.

(b) With $v_{B1} = -0.5$ V, Q_1 turns off and Q_2 conducts all the bias current (0.4 mA) and thus exhibits a V_{BE} of 0.677 V, thus

$$V_E = -0.677 \text{ V}$$

which indicated that $V_{BE1} = +0.177$ V, which is too small to turn Q_1 on. Also, note that the current source has a voltage of $-0.677 + 2.5 = 1.823$ V across it, more than sufficient for its proper operation.

$$V_{C1} = V_{CC} - I_{C1}R_C$$

$$= 2.5 - 0 \times 5 = 2.5 \text{ V}$$

$$V_{C2} = 2.5 - 0.99 \times 0.4 \times 5 = +0.52 \text{ V}$$

9.28 Refer to Fig. 9.15(a) and assume the current source I is implemented with a single BJT that requires a minimum of 0.3 V for proper operation. Thus, the minimum voltage allowed at the emitters of Q_1 and Q_2 is $-2.5\text{ V} + 0.3\text{ V} = -2.2\text{ V}$. Now, since each of Q_1 and Q_2 is conducting a current of 0.2 mA, their V_{BE} voltages will be equal:

$$V_{BE1,2} = 0.7 + 0.025\ln\left(\frac{0.99 \times 0.2}{1}\right)$$

$$= 0.660\text{ V}$$

Thus, the minimum allowable V_{CM} is

$$V_{CM\min} = -2.2 + 0.660 = -1.54\text{ V}$$

The upper limit on V_{CM} is dictated by the need to keep Q_1 and Q_2 operating in the active mode, thus

$$V_{CM\max} = 0.4 + V_{C1,2}$$

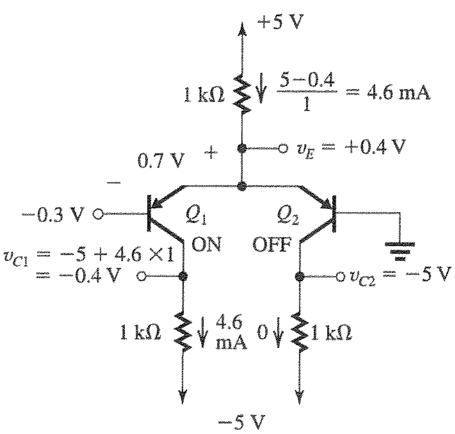
$$= 0.4 + (2.5 - 0.99 \times 0.2 \times 5)$$

$$= +1.91\text{ V}$$

Thus, the input common-mode range is

$$-1.54\text{ V} \le V_{ICM} \le 1.91\text{ V}$$

9.29



The solution is given on the circuit diagram.

9.30 (a) Refer to Fig. 9.15(a).

$$I_{E1} = I_{E2} = \frac{I}{2} = 10\ \mu\text{A}$$

$$I_{C1} = I_{C2} = \alpha \times 10 = 0.98 \times 10 = 9.8\ \mu\text{A}$$

$$V_{BE1} = V_{BE2} = 0.690 + 0.025\ln\left(\frac{9.8 \times 10^{-3}}{1}\right)$$

$$= 0.574\text{ V}$$

Thus,

$$V_E = -0.574\text{ V}$$

$$V_{C1} = V_{C2} = V_{CC} - I_C R_C$$

$$= 1.2 - 9.8 \times 10^{-3} \times 82$$

$$\simeq 0.4\text{ V}$$

(b) Refer to Fig. 9.15(a).

The maximum value of V_{CM} is limited by the need to keep Q_1 and Q_2 in the active mode. This is achieved by keeping $v_{CE1,2} \ge 0.3$ V.

Since $V_{C1,2} = 0.4$ V,

$$V_{E\max} = 0.4 - 0.3 = 0.1\text{ V}$$

and

$$V_{CM\max} = V_{BE1,2} + V_{E\max}$$

$$V_{CM\max} = 0.574 + 0.1 = 0.674\text{ V}$$

The minimum value of V_{CM} is dictated by the need to keep the current source operating properly, i.e. to keep 0.3 V across it, thus

$$V_{E\min} = -1.2 + 0.3 = -0.9\text{ V}$$

and

$$V_{CM\min} = V_{E\min} + V_{BE1,2}$$

$$= -0.9 + 0.574 = -0.326\text{ V}$$

Thus, the input common-mode range is

$$-0.326\text{ V} \le V_{ICM} \le +0.674\text{ V}$$

(c) Refer to Fig. 9.15(d).

$$i_{E1} = 11\ \mu\text{A}, \qquad i_{E2} = 9\ \mu\text{A}$$

$$i_{C1} = 10.78\ \mu\text{A}, \qquad i_{C2} = 8.82\ \mu\text{A}$$

$$v_{BE1} = 0.69 + 0.025\ln\left(\frac{10.78 \times 10^{-3}}{1}\right)$$

$$= 0.5767\text{ V}$$

$$v_{BE2} = 0.69 + 0.025\ln\left(\frac{8.82 \times 10^{-3}}{1}\right)$$

$$= 0.5717\text{ V}$$

Thus,

$$v_{B1} = v_{BE1} - v_{BE2}$$

$$= 0.5767 - 0.5717 = 0.005\text{ V}$$

$$= 5\text{ mV}$$

9.31 Refer to Fig. 9.15(a) with V_{CC} replaced by $(V_{CC} + v_r)$.

$$v_{C1} = (V_{CC} + v_r) - \alpha\frac{I}{2}R_C$$

$$= (V_{CC} - \alpha\frac{I}{2}R_C) + v_r$$

$$v_{C2} = (V_{CC} + v_r) - \alpha\frac{I}{2}R_C$$

$$= (V_{CC} - \alpha\frac{I}{2}R_C) + v_r$$

$v_{od} \equiv v_{C2} - v_{C1} = 0$

Thus, while v_{C1} and v_{C2} will include a ripple component v_r, the difference output voltage v_{od} will be ripple free. Thus, the differential amplifier rejects the undesirable supply ripple.

9.32 Refer to Fig. 9.14.

(a) $V_{CM\max} = V_{CC} - \frac{I}{2}R_C$

(b) For $V_{CC} = 2$ V and $V_{CM\max} = 1$ V,

$1 = 2 - \frac{1}{2}(IR_C)$

$\Rightarrow IR_C = 2$ V

(c) $I_B = \frac{I/2}{\beta + 1} \leq 2\ \mu\text{A}$

$I \leq 2 \times 101 \times 2 = 404\ \mu\text{A}$

Select

$I = 0.4$ mA

then

$R_C = \frac{2}{0.4} = 5\ \text{k}\Omega$

9.33 $\frac{\Delta i_{E1}}{I} = \frac{i_{E1} - (I/2)}{I}$

$= \frac{i_{E1}}{I} - 0.5$

Using Eq. (9.48), we obtain

$\frac{\Delta i_{E1}}{I} = \frac{1}{1 + e^{-v_{id}/V_T}} - 0.5$

Observe that for $v_{id} < 10$ mV the proportional transconductance gain is nearly constant at about 10. The gain decreases as v_{id} further increases, indicating nonlinear operation. This is especially pronounced for $v_{id} > 20$ mV.

9.34 Require $v_{od} = 1$ V when $v_{id} = 10$ mV and $I = 1$ mA.

Using Eq. (9.48), we obtain

$i_{E1} = \frac{1\ (\text{mA})}{1 + e^{-10/25}} = 0.599$ mA

$i_{E2} = I - i_{E1} = 1 - 0.599 = 0.401$ mA

$v_{od} = v_{C2} - v_{C1}$

$= (V_{CC} - i_{C2}R_C) - (V_{CC} - i_{C1}R_C)$

$= (i_{C1} - i_{C2})R_C$

$\simeq (i_{E1} - i_{E2})R_C$

$= 0.198R_C$

For $v_{od} = 1$ V, we have

$R_C = \frac{1}{0.198} = 5.05\ \text{k}\Omega$

$V_{C1} = V_{C2} = V_{CC} - \frac{I}{2}R_C$

$= 5 - 0.5 \times 5.05 \simeq 2.5$ V

With a signal of 10 mV applied, the voltage at one collector rises to 3 V and at the other falls to 2 V. To ensure that the transistors remain in the active region, the maximum common-mode input voltage must be limited to $(2 - 0.4) = +1.6$ V.

9.35 Refer to Fig. 9.14.

$v_{od} = v_{C2} - v_{C1}$

$= (V_{CC} - i_{C2}R_C) - (V_{CC} - i_{C1}R_C)$

$= R_C(i_{C1} - i_{C2})$

Using Eqs. (9.48) and (9.49) and assuming $\alpha \simeq 1$, so that $i_{C1} \simeq i_{E1}$ and $i_{C2} \simeq i_{E2}$, we get

$v_{od} = IR_C\left[\frac{1}{1 + e^{-v_{id}/V_T}} - \frac{1}{1 + e^{v_{id}/V_T}}\right]$

$= 5\left[\frac{1}{1 + e^{-v_{id}/V_T}} - \frac{1}{1 + e^{v_{id}/V_T}}\right]$

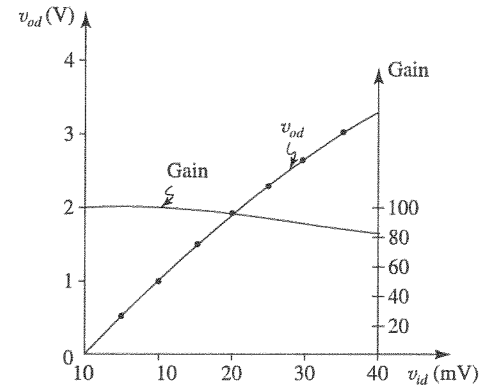
This relationship can be used to obtain the data in the table below.

This table belongs to Problem 9.33.

v_{id} (mV)	2	5	8	10	20	30	40
$\left[\frac{\Delta i_{E1}}{I}/v_{id}\right]$ (V^{-1})	9.99	9.97	9.92	9.87	9.50	8.95	8.30

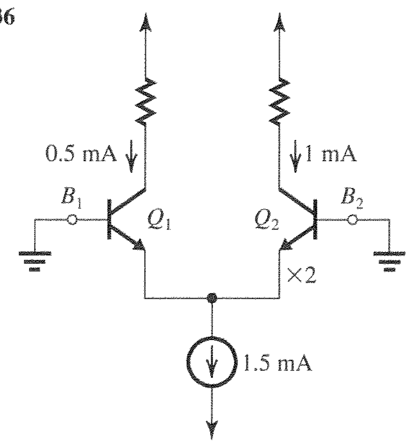
This table belongs to Problem 9.35.

v_{id} (mV)	2	5	10	15	20	25	30	35	40
v_{od} (V)	0.2	0.498	0.987	1.457	1.90	2.311	2.685	3.022	3.320
Gain $= \frac{v_{od}}{v_{id}}$	100	99.7	98.7	97.1	95.0	92.4	89.5	86.3	83.0



The figure shows v_{od} versus v_{id} and the gain versus v_{id}. Observe that the transfer characteristic is nearly linear and the gain is nearly constant for $v_{id} \leq 10$ mV. As v_{id} increases, the transfer characteristic bends and the gain is reduced. However, for v_{id} even as large as 20 mV, the gain is only 5% below its ideal value of 100.

9.36



Since Q_2 has twice the EBJ area of Q_1, the 1.5-mA bias current will split in the same ratio, that is,

$i_{E2} = 2\ i_{E2}$

Thus,

$i_{E2} = 1$ mA and $i_{E1} = 0.5$ mA

To equalize the collector currents, we apply a signal

$v_{id} = v_{B1} - v_{B2}$

Now,

$$i_{C1} = \frac{I_S}{\alpha} e^{(v_{B1} - v_E)/V_T} \quad (1)$$

$$i_{C2} = \frac{2I_S}{\alpha} e^{(v_{B2} - v_E)/V_T} \quad (2)$$

where we have denoted the scale current of Q_1 by I_S and that of Q_2 as $2I_S$. Dividing (1) by (2), we get

$$\frac{i_{C1}}{i_{C2}} = \frac{1}{2} e^{(v_{B1} - v_{B2})/V_T}$$

For $i_{C1} = i_{C2}$, we obtain

$v_{B1} - v_{B2} = V_T \ln 2$

$= 25 \ln 2 = 17.3$ mV

9.37 (a) $V_{BE} = 0.69 + 0.025 \ln\left(\frac{0.1}{1}\right)$

$= 0.632$ V

(b) Using Eq. (9.48), we obtain

$$i_{C1} = \alpha i_{E1} \simeq \frac{I}{1 + e^{-v_{id}/V_T}}$$

For $v_{id} = 20$ mV,

$$i_{C1} = \frac{200\ \mu\text{A}}{1 + e^{-20/25}} = 138\ \mu\text{A}$$

$i_{C2} = 200 - 138 = 62\ \mu$A

(c) For $v_{id} = 200$ mV while $i_{C1} = 138\ \mu$A and $i_{C2} = 62\ \mu$A: Since i_{C1} and i_{C2} have not changed, v_{BE1} and v_{BE2} also would not change. Thus,

$v_{B1} - v_{B2} = v_{BE1} + i_{E1}R_e - i_{E2}R_e - v_{BE2}$

$= (v_{BE1} - v_{BE2}) + R_e\ (i_{E1} - i_{E2})$

$200 = 20 + R_e\ (i_{C1} - i_{C2})$

$= 20 + R_e\ (138 - 62)$

$$\Rightarrow R_e = \frac{180\ \text{mV}}{76\ \mu\text{A}} = 2.37\ \text{k}\Omega$$

(d) Without R_e,

$v_{id} = 20$ mV $\rightarrow i_{C1} - i_{C2} = 76\ \mu$A

$$G_m = \frac{76\ \mu\text{A}}{20\ \text{mV}} = 3.8\ \text{mA/V}$$

With R_e,

$v_{id} = 200$ mV $\rightarrow i_{C1} - i_{C2} = 76\ \mu$A

$$G_m = \frac{76\ \mu\text{A}}{200\ \text{mV}} = 0.38\ \text{mA/V}$$

Thus, the effective G_m has been reduced by a factor of 10, which is the same factor by which the allowable input signal has been increased while maintaining the same linearity.

9.38 $g_m = \dfrac{I_C}{V_T} = \dfrac{\alpha \times 0.2}{0.025} \simeq 8$ mA/V

$$R_{id} = 2r_\pi = 2\frac{\beta}{g_m} = 2 \times \frac{160}{8} = 40\ \text{k}\Omega$$

9.39 $R_{id} = 2r_\pi = 20\ \text{k}\Omega$

$r_\pi = 10\ \text{k}\Omega$

$$\frac{\beta}{g_m} = 10\ \text{k}\Omega$$

$$\frac{100}{g_m} = 10$$

$\Rightarrow g_m = 10\ \text{mA/V}$

$A_d = 100 = g_m R_C$

$$R_C = \frac{100}{g_m} = \frac{100}{10} = 10\ \text{k}\Omega$$

$$g_m = \frac{I_C}{V_T} \simeq \frac{I/2}{V_T}$$

$\Rightarrow I = 2V_T g_m$

$= 2 \times 0.025 \times 10 = 0.5\ \text{mA}$

9.40 $v_{id} = 10\ \text{mA/V}$

Input signal to half-circuit = 5 mV. For $I = 200\ \mu\text{A}$, the bias current of the half-circuit is $100\ \mu\text{A}$ and,

$$r_e = \frac{25\ \text{mV}}{0.1\ \text{mA}} = 250\ \Omega$$

$$\text{Gain of half-circuit} = -\frac{R_C}{r_e} = -\frac{10}{0.25}$$
$= -40\ \text{V/V}$

At each collector we expect a signal of $40 \times 5\ \text{mV} = 200\ \text{mV}$. Between the two collectors, the signal will be 400 mV.

9.41 (a) $r_e = \dfrac{25\ \text{mV}}{0.25\ \text{mA}} = 100\ \Omega$

The 0.1-V differential input signal appears across $(2r_e + 2R_e)$, thus

$$i_e = \frac{100\ \text{mV}}{200 + 2 \times 400} = 0.1\ \text{mA}$$

$v_{be} = 0.1 \times 100 = 10\ \text{mV}$

(b) The total emitter current in one transistor is

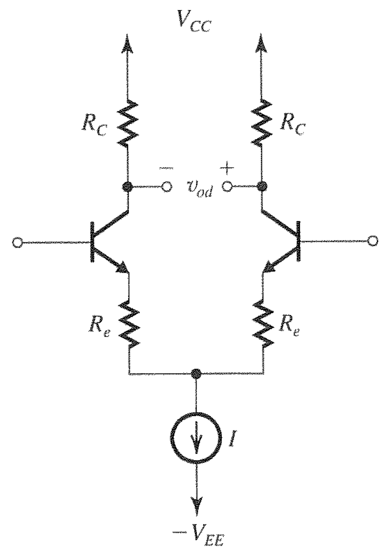
$\dfrac{I}{2} + i_e = 0.35$ mA and in the other transistor

$\dfrac{I}{2} - i_e = 0.15$ mA.

(c) At one collector the signal voltage is $-\alpha i_e R_C \simeq -i_e R_C = -0.1 \times 10 = -1$ V and at the other collector the signal voltage is +1 V.

(d) Voltage gain $= \dfrac{2\ \text{V}}{0.1\ \text{V}} = 20\ \text{V/V}$

9.42



$v_{id} = 100$ mV appears across $(2\ r_e + 2\ R_e)$. Thus the signal across $(r_e + R_e)$ is 50 mV. Since the signal across r_e is 5 mV, it follows that the signal across R_e must be $50 - 5 = 45$ mV and thus

$R_e = 9r_e$

The input resistance R_{id} is

$R_{id} = (\beta + 1)\ (2r_e + 2R_e)$

$= 2(100 + 1)\ (r_e + R_e)$

$= 2 \times 101 \times (r_e + 9r_e)$

$= 2 \times 101 \times 10r_e$

To obtain $R_{id} = 100\ \text{k}\Omega$,

$100 = 2 \times 101 \times 10 \times r_e$

$\Rightarrow r_e \simeq 50\ \Omega$

Since

$$r_e = \frac{V_T}{I_E},$$

$$50 = \frac{25\ \text{mV}}{I_E}$$

$\Rightarrow I_E = 0.5\ \text{mA}$

$I = 1\ \text{mA}$

$R_e = 9r_e = 9 \times 50 = 450\ \Omega$

$$\text{Gain} = \frac{\alpha \times 2R_C}{2r_e + 2R_e}$$

$$\simeq \frac{R_C}{r_e + R_e}$$

But the gain required is

$$\text{Gain} = \frac{v_{od}}{v_{id}} = \frac{2\ \text{V}}{0.1\ \text{V}} = 20\ \text{V/V}$$

Thus,

$$20 = \frac{R_C}{0.05 + 0.45}$$

$\Rightarrow R_C = 10\ \text{k}\Omega$

The determination of a suitable value of V_{CC} requires information on the required input common-mode range (which is not specified). Suffice it to say that the dc voltage drop across R_C is 5 V and that each collector swings ± 1 V. A supply voltage $V_{CC} = 10$ V will certainly be sufficient.

9.43 (a) The maximum allowable value of the bias current I is found as

$$I = \frac{P}{(V_{CC} + V_{EE})} = \frac{1\ \text{mW}}{5\ \text{V}} = 0.2\ \text{mA}$$

We choose to operate at this value of I. Thus

$$g_m = \frac{I_C}{V_T} = \frac{\alpha(0.2/2)}{0.025} \simeq 4\ \text{mA/V}$$

$A_d = g_m R_C$

$60 = 4 \times R_C$

$\Rightarrow R_C = 15\ \text{k}\Omega$

$$V_{C1} = V_{C2} = V_{CC} - \frac{I}{2} R_C$$

$$= 2.5 - \frac{0.2}{2} \times 15$$

$= +1$ V

(b) $R_{id} = 2r_\pi = 2\dfrac{\beta}{g_m}$

$$= 2 \times \frac{100}{4} = 50\ \text{k}\Omega$$

(c) $v_{od} = A_d \times v_{id}$

$= 60 \times 10 = 600\ \text{mV} = 0.6$ V

Thus, there will be ± 0.3 V signal swing at each collector. That is, the voltage at each collector will range between 0.7 V and +1.3 V.

(d) To maintain the BJT in the active mode at all times, the maximum allowable V_{CM} is limited to

$V_{CM\max} = 0.4 + v_{C\min}$

$= 0.4 + 0.7 = 1.1$ V

9.44 (a) Consider transistor Q_1,

$$v_{C1\min} = (V_{CC} - \frac{I}{2} R_C) - A_d\left(\frac{\hat{v}_{id}}{2}\right) \qquad (1)$$

where

$$A_d = g_m R_C \simeq \frac{I/2}{V_T} R_C$$

$$= \frac{IR_C}{2V_T}$$

Thus,

$$\frac{IR_C}{2} = A_d V_T \qquad (2)$$

Substituting from (2) into (1), we obtain

$$v_{C1\min} = V_{CC} - A_d\left(V_T + \frac{\hat{v}_{id}}{2}\right) \qquad (3)$$

Since

$$v_{B1} = V_{CM\max} + \frac{\hat{v}_{id}}{2}$$

to keep Q_1 in the active mode,

$v_{B1} \leq 0.4 + v_{C1\min}$

Thus,

$$V_{CM\max} + \frac{\hat{v}_{id}}{2} = 0.4 + V_{CC} - A_d\left(V_T + \frac{\hat{v}_{id}}{2}\right)$$

$$\Rightarrow V_{CM\max} = V_{CC} + 0.4 - \frac{\hat{v}_{id}}{2} - A_d\left(V_T + \frac{\hat{v}_{id}}{2}\right) \qquad \text{Q.E.D.} \qquad (4)$$

(b) $V_{CC} = 2.5$ V, $\hat{v}_{id} = 10$ mV,

$A_d = 50$ V/V,

$V_{CM\max} = 2.5 + 0.4 - 0.005 - 50(25 + 5) \times 10^{-3}$

$\simeq 1.4$ V

$\hat{v}_{od} = A_d \times \hat{v}_{id} = 50 \times 10 = 500$ mV

$= 0.5$ V

Using Eq. (2), we obtain

$IR_C = 2A_d V_T = 2 \times 50 \times 0.025$

$= 2.5$ V

To limit the power dissipation in the quiescent state to 1 mV, the bias current must be limited to

$$I = \frac{P_{\max}}{V_{CC} + V_{EE}} = \frac{1}{5} = 0.2\ \text{mA}$$

Using this value for I, we get

$$R_C = \frac{2.5}{0.2} = 12.5\ \text{k}\Omega$$

(c) To obtain $V_{CM\max} = 1$ V, we use Eq. (4) to determine the allowable value of A_d,

$1 = 2.5 + 0.4 - 0.005 - A_d(25 + 5) \times 10^{-3}$

$\Rightarrow A_d = 63.2$ V/V

Thus, by reducing $V_{CM\max}$ from 1.4 V to 1 V, we are able to increase the differential gain from 50 V/V to 63.2 V/V.

9.45 $A_d = g_m R_C$

$$= \frac{I_C}{V_T} R_C$$

$$\simeq \frac{(I/2)}{V_T} R_C$$

$$= \frac{IR_C}{2V_T}$$

$$= \frac{4}{2 \times 0.025} = 80 \text{ V/V}$$

$$V_{C1} = V_{C2} = V_{CC} - \frac{I}{2} R_C$$

$$= 5 - 2 = 3 \text{ V}$$

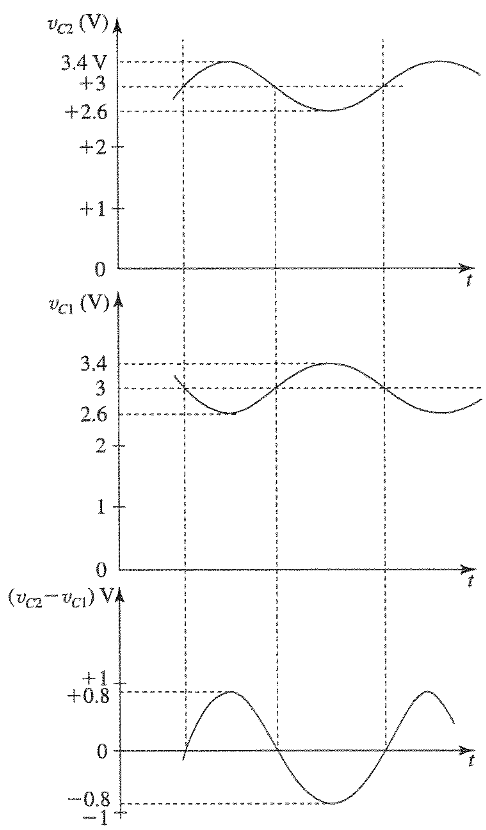
$$v_{C1} = 3 - 80 \times 0.005 \sin(\omega t)$$

$$= 3 - 0.4 \sin(\omega t)$$

$$v_{C2} = 3 + 0.4 \sin(\omega t)$$

$$v_{C2} - v_{C1} = 0.8 \sin(\omega t)$$

The waveforms are sketched in the figure below.



9.46 See figure on next page. The circuit together with its equivalent half-circuit are shown in the figure.

$$A_d = g_{m1,2}(r_{o1,2} \parallel r_{o3,4})$$

For

$$r_{o1,2} = r_{o3,4} = \frac{V_A}{\alpha(I/2)} \simeq \frac{2V_A}{I}$$

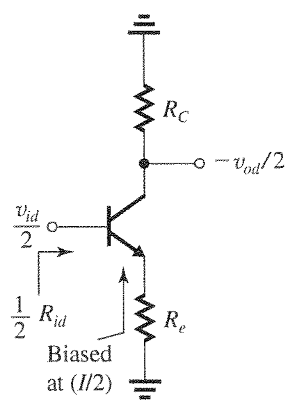
$$g_{m1,2} = \frac{I_{C1,2}}{V_T} \simeq \frac{I}{2V_T}$$

$$A_d = \frac{I}{2V_T}\left(\frac{2V_A}{I} \parallel \frac{2V_A}{I}\right)$$

$$= \frac{I}{2V_T} \times \frac{V_A}{I} = \frac{V_A}{2V_T}$$

$$= \frac{20}{2 \times 0.025} = 400 \text{ V/V}$$

9.47



Both circuits have the same differential half-circuit shown in the figure. Thus, for both

$$A_d = \frac{\alpha R_C}{r_e + R_e}$$

$$R_{id} = (\beta + 1)(2r_e + 2R_e)$$

$$= 2(\beta + 1)(r_e + R_e)$$

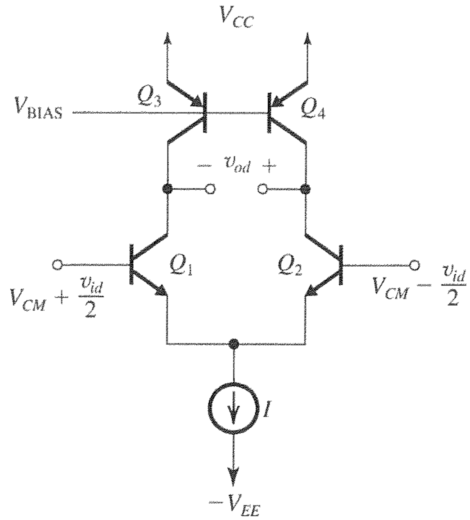
With $v_{id} = 0$, the dc voltage appearing at the top end of the bias current source will be

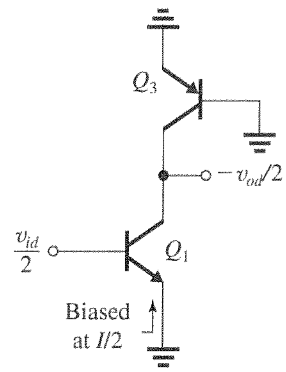
(a) $V_{CM} - V_{BE} - \left(\frac{I}{2}\right) R_C$

(b) $V_{CM} - V_{BE}$

Since circuit (b) results in a larger voltage across the current source and given that the minimum value of V_{CM} is limited by the need to keep a certain specified minimum voltage across the current source, we see that circuit (b) will allow a larger negative V_{CM}.

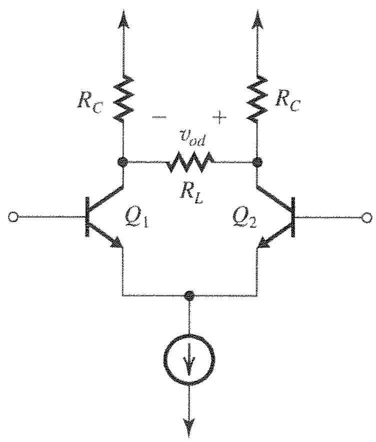
This figure belongs to Problem 9.46.





Equivalent half-circuit

9.48



$$A_d = \alpha \frac{\text{Total resistance between collectors}}{\text{Total resistance in the emitter circuit}}$$

$$= \alpha \frac{(2R_C \parallel R_L)}{2r_e}$$

9.49 Refer to Fig. P9.47(a).

$$\frac{I}{2} R_e = 4V_T$$

$$\Rightarrow R_e = \frac{8V_T}{I} \quad (1)$$

$$\alpha \left(\frac{I}{2} \right) R_C = 60V_T$$

$$R_C = \frac{120V_T}{\alpha I} \quad (2)$$

$$A_d = \alpha \frac{\text{Total resistance in collector circuit}}{\text{Total resistance in emitter circuit}}$$

$$A_d = \alpha \frac{2R_C}{2r_e + 2R_e} = \alpha \frac{R_C}{r_e + R_e}$$

Substituting for R_C from (2), for R_e from (1), and for $r_e = V_T/(I/2)$, we obtain

$$A_d = \frac{\alpha(120V_T/\alpha I)}{(2V_T/I) + (8V_T/I)}$$

$$= \frac{120}{2+8} = 12 \text{ V/V}$$

9.50 $$\frac{v_{id}}{v_{\text{sig}}} = \frac{R_{id}}{R_{id} + R_{\text{sig}}} \quad (1)$$

where

$$R_{id} = (\beta + 1)(2r_e + 2R_e)$$

thus,

$$\frac{v_{id}}{v_{\text{sig}}} = \frac{2(\beta+1)(r_e + R_e)}{2(\beta+1)(r_e + R_e) + R_{\text{sig}}} \quad (2)$$

$$\frac{v_{od}}{v_{id}} = \frac{\alpha \times \text{Total resistance between collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{2\alpha R_C}{2r_e + 2R_e}$$

$$\frac{v_{od}}{v_{id}} = \frac{\alpha R_C}{r_e + R_e} \quad (3)$$

Using (2) and (3), we get

$$G_v \equiv \frac{v_{od}}{v_{\text{sig}}} = \frac{2\alpha(\beta+1)R_C}{2(\beta+1)(r_e + R_e) + R_{\text{sig}}}$$

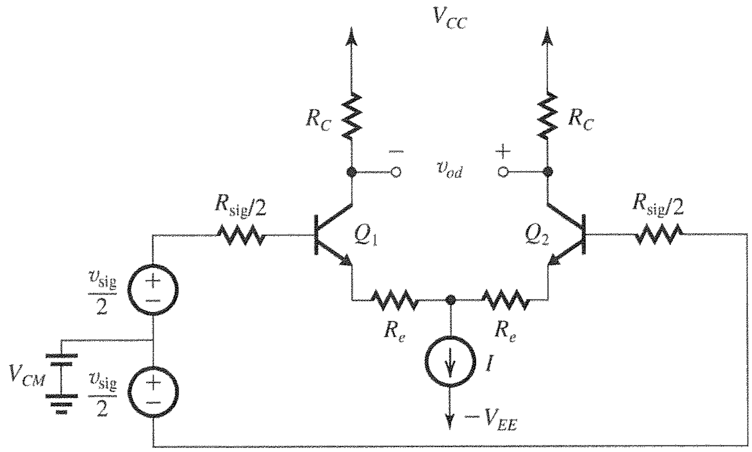
Since $\alpha = \dfrac{\beta}{\beta+1}$, $\alpha(\beta+1) = \beta$, we have

$$G_v = \frac{2\beta R_C}{2(\beta+1)(r_e + R_e) + R_{\text{sig}}} \quad (4)$$

If $v_{id} = 0.5\ v_{\text{sig}}$, then from (1) we obtain

$$R_{id} = R_{\text{sig}}$$

This figure belongs to Problem 9.50.



Substituting for $R_{sig} = R_{id} = 2(\beta + 1)(r_e + R_e)$ into Eq. (4) gives

$$G_v = \frac{2\beta R_C}{4(\beta+1)(r_e+R_e)} = \frac{1}{2}\frac{\alpha R_C}{r_e+R_e} \quad (5)$$

If β is doubled to 2β while R_{sig} remains at its old value, we get

$$R_{sig} = 2(\beta+1)(r_e+R_e) \quad (6)$$

then the new value of G_v is obtained by replacing β by 2β in Eq. (4) and substituting for R_{sig} from (5):

$$G_v = \frac{4\beta R_C}{2(2\beta+1)(r_e+R_e)+2(\beta+1)(r_e+R_e)}$$

$$\simeq \frac{4R_C}{6(r_e+R_e)} = \frac{2}{3}\frac{R_C}{r_e+R_e}$$

Thus the gain increases from approximately $\frac{1}{2}R_C/(r_e+R_e)$ to $\frac{2}{3}R_C/(r_e+R_e)$.

9.51 $R_{id} = 2r_\pi = 2\dfrac{\beta}{g_m}$

$$g_m = \frac{I_C}{V_T} \simeq \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$R_{id} = \frac{2 \times 100}{8} = 25 \text{ k}\Omega$$

$$G_v = \frac{R_{id}}{R_{id}+R_{sig}}\,\frac{\alpha(2R_C \parallel R_L)}{2r_e}$$

$$G_v = \frac{R_{id}}{R_{id}+R_{sig}} \times \frac{1}{2}\,g_m(2R_C \parallel R_L)$$

$$= \frac{25}{25+100} \times \frac{1}{2} \times 8 \times (40 \parallel 40)$$

$$= 16 \text{ V/V}$$

9.52 Refer to Fig. P9.52.

$$\frac{v_o}{v_i} = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{0.99 \times 25}{2r_e + 2 \times 0.25}$$

where

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \ \Omega$$

Thus,

$$\frac{v_o}{v_i} = \frac{0.99 \times 25}{2 \times 0.25 + 2 \times 0.25} \simeq 25 \text{ V/V}$$

$$R_{in} = (\beta+1)(2r_e + 2R_e)$$

$$= 2 \times 101 \times (0.25 + 0.25)$$

$$= 101 \text{ k}\Omega$$

9.53 Refer to Fig. P9.53.

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \ \Omega$$

$$\frac{v_o}{v_i} = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{0.99 \times 25 \text{ k}\Omega}{2r_e + 500 \ \Omega}$$

$$= \frac{0.99 \times 25 \text{ k}\Omega}{500 \ \Omega + 500 \ \Omega} \simeq 25 \text{ V/V}$$

$$R_{in} = (\beta+1)(2r_e + 500 \ \Omega)$$

$$= 101 \times (2 \times 250 \ \Omega + 500 \ \Omega)$$

$$= 101 \text{ k}\Omega$$

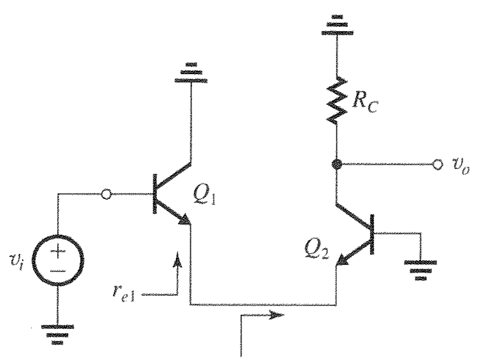
9.54 (a) Refer to the circuit in Fig. P9.54. As a differential amplifier, the voltage gain is found from

$$\frac{v_o}{v_i} = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{\alpha \times R_C}{2r_e}$$

$$= \frac{\alpha R_C}{2r_e}$$

(b) The circuit in Fig. P9.54 can be considered as the cascade connection of an emitter follower Q_1 (biased at an emitter current $I/2$) and a common-gate amplifier Q_2 (also biased at an emitter current of $I/2$). Referring to the figure below:



$$\frac{v_{e1,2}}{v_i} = \frac{r_{e2}}{r_{e1} + r_{e2}} = \frac{1}{2}$$

$$\frac{v_o}{v_{e1,2}} = \frac{\alpha R_C}{r_{e2}}$$

Thus,

$$\frac{v_o}{v_i} = \frac{1}{2} \times \frac{\alpha R_C}{r_{e2}} = \frac{\alpha R_C}{2r_e}$$

which is identical to the expression found in (a) above.

9.55 $g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$

$= \sqrt{2 \times 3 \times 0.1} = 0.77$ mA/V

$|A_d| = g_m R_D = 0.77 \times 10 = 7.7$ V/V

$$|A_{cm}| = \left(\frac{R_D}{2R_{SS}}\right)\left(\frac{\Delta R_D}{R_D}\right)$$

$$= \frac{10}{2 \times 100} \times 0.01 = 5 \times 10^{-4} \text{ V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = 1.54 \times 10^4 \text{ or } 83.8 \text{ dB}$$

9.56 Refer to Fig. P9.2.

$$I_D = 0.25 \text{ mA} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)|V_{OV}|^2$$

$$0.25 = \frac{1}{2} \times 4 \times |V_{OV}|^2$$

$\Rightarrow |V_{OV}| = 0.353$ V

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.25}{0.353} = 1.416 \text{ mA/V}$$

$|A_d| = g_m R_D = 1.416 \times 4 = 5.67$ V/V

$$|A_{cm}| = \left(\frac{R_D}{2R_{SS}}\right)\left(\frac{\Delta R_D}{R_D}\right)$$

$$= \frac{4}{2 \times 30} \times 0.02$$

$= 1.33 \times 10^{-3}$ V/V

CMRR = 4252.5 or 72.6 dB

9.57 Refer to Fig. P9.57.

(a) Assume $v_{id} = 0$ and the two sides of the differential amplifier are matched. Thus,

$I_{D1} = I_{D2} = 0.5$ mA

$$I_{D1,2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$0.5 = \frac{1}{2} \times 2.5 \times V_{OV}^2$$

$\Rightarrow V_{OV} = 0.632$ V

$V_{CM} = V_{GS} + 1 \text{ mA} \times R_{SS}$

$= V_t + V_{OV} + 1 \times R_{SS}$

$= 0.7 + 0.632 + 1$

$= 2.332$ V

(b) $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.5}{0.632} = 1.58$ mA/V

$A_d = g_m R_D$

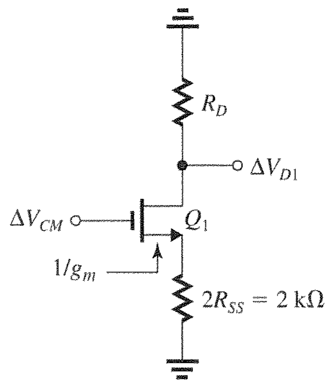
$8 = 1.38 \times R_D$

$\Rightarrow R_D = 5.06$ kΩ

(c) $V_{D1} = V_{D2} = V_{DD} - I_D R_D$

$= 5 - 0.5 \times 5.06 = 2.47$ V

(d)



The figure shows the common-mode half-circuit,

$$\frac{\Delta V_{D1}}{\Delta V_{CM}} = -\frac{R_D}{\frac{1}{g_m} + 2\,R_{SS}}$$

$$\frac{\Delta V_{D1}}{\Delta V_{CM}} = -\frac{5.06}{\frac{1}{1.58} + 2} = -1.92\ \text{V/V}$$

(e) For Q_1 and Q_2 to enter the triode region

$$V_{CM} + \Delta V_{CM} = V_t + V_{D1} + \Delta V_{D1}$$

Substituting $V_{CM} = 2.332$, $V_t = 0.7$ V, $V_{D1} = 2.47$ V, and $\Delta V_{D1} = -1.92\Delta V_{CM}$ results in

$$2.332 + \Delta V_{CM} = 0.7 + 2.47 - 1.92\Delta V_{CM}$$

$$\Rightarrow \Delta V_{CM} = 0.287\ \text{V}$$

With this change, $V_{CM} = 2.619$ V and $V_{D1,2} = 1.919$ V; thus $V_{CM} = V_t + V_{D1,2}$.

9.58 The new deliberate mismatch $\Delta R_D/R_D$ cancels the two existing mismatch terms in the expression for A_{cm} given in the problem statement so as to reduce A_{cm} to zero. Thus,

$$\frac{R_D}{2R_{SS}} \times \frac{\Delta R_D}{R_D} = -0.002$$

$$\frac{5}{2 \times 25} \times \frac{\Delta R_D}{R_D} = -0.002$$

$$\Rightarrow \frac{\Delta R_D}{R_D} = -0.02 \text{ or } -2\%$$

(Note the sign of the change is usually determined experimentally.)

9.59 $|A_{cm}| = \left(\frac{R_D}{2R_{SS}}\right)\frac{\Delta(W/L)}{W/L}$

$$|A_d| = g_m R_D$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = 2g_m R_{SS} \bigg/ \frac{\Delta(W/L)}{W/L}$$

where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2(0.1/2)}{0.2} = 0.5\ \text{mA/V}$$

For CMRR of 80 dB, the CMRR is 10^4; thus

$$10^4 = 2 \times 0.5 \times R_{SS}/0.02$$

$$R_{SS} = 200\ \text{k}\Omega$$

For the current source transistor to have $r_o = 200\ \text{k}\Omega$,

$$200 = \frac{V_A' \times L}{0.1\ \text{mA}}$$

$$L = \frac{200 \times 0.1}{5} = 4\ \mu\text{m}$$

9.60 It is required to raise the CMRR by 40 dB, that is, by a factor of 100. Thus, the cascoding of the bias current source must raise its output resistance R_{SS} by a factor of 100. Thus the cascode transistor must have $A_0 = 100$. Since

$$A_0 = g_m r_o = \frac{2I}{V_{OV}}\frac{V_A}{I} = \frac{2V_A}{V_{OV}}$$

$$100 = \frac{2V_A}{0.2}$$

$$\Rightarrow V_A = 10\ \text{V}$$

$$V_A = V_A' \times L$$

$$10 = 5 \times L$$

$$\Rightarrow L = 2\ \mu\text{m}$$

9.61 Refer to Fig. P9.61.

(a) $\frac{v_o}{v_{id}} =$

$$\alpha\,\frac{\text{Total resistance across which } v_o \text{ appears}}{\text{Total resistance in the emitter}}$$

$$= \alpha \times \frac{2\ \text{k}\Omega}{r_{e1} + r_{e2}}$$

To determine $r_{e1} = r_{e2} = r_e = \frac{V_T}{I_E}$, where I_E is the dc emitter current of each of Q_1 and Q_2, we use

$$V_E = V_B - V_{BE} = 0 - 0.7$$

$$= -0.7\ \text{V}$$

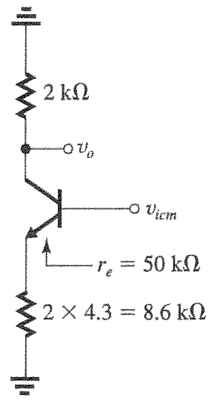
$$2I_E = \frac{-0.7 - (-5)}{4.3} = 1\ \text{mA}$$

$$I_E = 0.5\ \text{mA}$$

$$r_{e1} = r_{e2} = \frac{25\ \text{mV}}{0.5\ \text{mA}} = 50\ \Omega$$

$$\frac{v_o}{v_{id}} = \alpha \times \frac{2\ \text{k}\Omega}{0.1\ \text{k}\Omega} \simeq 20\ \text{V/V}$$

(b)



The common-mode half-circuit is shown in the figure,

$$\frac{v_o}{v_{icm}} = -\frac{\alpha \times 2\text{ k}\Omega}{(0.05 + 8.6)\text{ k}\Omega}$$

$\simeq -0.23$ V/V

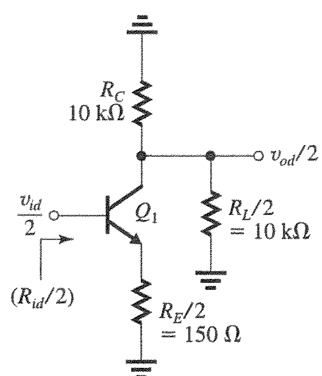
$$\left|\frac{v_o}{v_{icm}}\right| = 0.23 \text{ V/V}$$

(c) $\text{CMRR} = \dfrac{|v_o/v_{id}|}{|v_o/v_{icm}|} = \dfrac{20}{0.23} = 86.5$

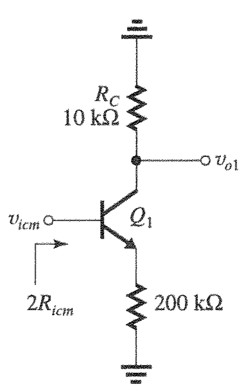
or 38.7 dB

(d) $v_o = -0.023 \sin 2\pi \times 60t$
$\quad + 0.2 \sin 2\pi \times 1000\, t$ volts

9.62



(a)



(b)

Figure (a) shows the differential half-circuit.

$I_E = 0.5$ mA, $\qquad I_C = \alpha I_E \simeq 0.5$ mA

$$g_m = \frac{I_C}{V_T} = \frac{0.5\text{ mA}}{0.025\text{ V}} = 20\text{ mA/V}$$

$$r_e = \frac{25\text{ mV}}{0.5\text{ mA}} = 50\ \Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.5} = 200\text{ k}\Omega$$

$$A_d = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$\simeq \frac{10\text{ k}\Omega \parallel 10\text{ k}\Omega}{(50 + 150)\ \Omega}$$

$$= \frac{5}{0.2} = 25\text{ V/V}$$

We have neglected r_o because its equivalent value at the output will be $r_o[1 + (R_e/r_e)] = 200[1 + (150/50)] = 800$ kΩ which is much greater than the effective load resistance of 5 kΩ.

$$R_{id} = 2 \times (\beta + 1)(50\ \Omega + 150\ \Omega)$$

$$= 2 \times 101 \times 0.2\ (\text{k}\Omega) = 40.4\text{ k}\Omega$$

$$|A_{cm}| \simeq \left(\frac{R_C}{2R_{SS}}\right)\left(\frac{\Delta R_C}{R_C}\right)$$

$$|A_{cm}| = \frac{10}{200} \times 0.02 = 0.001\text{ V/V}$$

To obtain R_{icm}, we use Eq. (9.96):

$$R_{icm} \simeq \beta R_{EE} \frac{1 + (R_C/\beta r_o)}{1 + \dfrac{R_C + 2R_{EE}}{r_o}}$$

where $2R_{EE} = 200$ kΩ, thus $R_{EE} = 100$ kΩ and

$$R_{icm} = 100 \times 100\ \frac{1 + (10/(100 \times 200))}{1 + \dfrac{10 + 200}{200}}$$

$= 4.88$ MΩ

9.63 (a) $g_m = \dfrac{I_C}{V_T} \simeq \dfrac{0.1\text{ mA}}{0.025\text{ V}} = 4$ mA/V

$A_d = g_m R_C = 4 \times 25 = 100$ V/V

(b) $R_{id} = 2r_\pi = 2\dfrac{\beta}{g_m} = 2 \times \dfrac{100}{4} = 50$ kΩ

(c) $|A_{cm}| = \left(\dfrac{R_C}{2R_{EE}}\right)\left(\dfrac{\Delta R_C}{R_C}\right)$

$$= \frac{25}{2 \times 500} \times 0.01$$

$= 2.5 \times 10^{-4}$ V/V

(d) $\text{CMRR} = \dfrac{|A_d|}{|A_{cm}|} = \dfrac{100}{2.5 \times 10^{-4}} = 4 \times 10^5$

or 112 dB

(e) $r_o = \frac{V_A}{I_C} \simeq \frac{100}{0.1} = 1000\ \text{k}\Omega$

$$R_{icm} \simeq \beta R_{EE} \frac{1 + (R_C/\beta r_o)}{1 + \dfrac{R_C + 2R_{EE}}{r_o}}$$

$$= 100 \times 500 \frac{1 + (25/(100 \times 1000))}{1 + \dfrac{25 + 1000}{1000}}$$

$\simeq 25\ \text{M}\Omega$

9.64 $R_{EE} = \frac{V_A}{I} = \frac{20}{0.2} = 100\ \text{k}\Omega$

For the transistors in the differential pair, we have

$$r_o = \frac{V_A}{I/2} = \frac{20}{0.1} = 200\ \text{k}\Omega$$

$$R_{icm} \simeq \beta R_{EE} \frac{1 + (R_C/\beta r_o)}{1 + \dfrac{R_C + 2R_{EE}}{r_o}}$$

For $R_C \ll r_o$,

$$R_{icm} \simeq \beta R_{EE} \Big/ \left(1 + \frac{2R_{EE}}{r_o}\right)$$

$$= \frac{50 \times 100}{1 + \dfrac{2 \times 100}{200}} = 2.5\ \text{M}\Omega$$

9.65 For the differential-pair transistors, we have

$I_C \simeq 0.25$ mA

$$g_m = \frac{0.25}{0.025} = 10\ \text{mA/V}$$

$$r_o = \frac{V_A}{I_C} = \frac{50}{0.25} = 200\ \text{k}\Omega$$

(a) $A_d = g_m R_C = 10 \times 5 = 50$ V/V

where we have neglected the effect of r_o since $r_o \gg R_C$.

(b) If the bias current is realized using a simple current source,

$$R_{EE} = r_o|_{\text{current source}} = \frac{V_A}{I} = \frac{50}{0.5} = 100\ \text{k}\Omega$$

$$|A_{cm}| = \left(\frac{R_C}{2R_{EE}}\right)\left(\frac{\Delta R_C}{R_C}\right)$$

$$= \left(\frac{5}{2 \times 100}\right) \times 0.1$$

$= 2.5 \times 10^{-3}$ V/V

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{50}{2.5 \times 10^{-3}} = 2 \times 10^4$$

or 86 dB

(c) If the bias current I is generated using a Wilson mirror,

$$R_{EE} = R_o|_{\text{Wilson mirror}}$$

$$= \frac{1}{2}\beta r_o$$

where r_o is that of the transistors in the Wilson mirror, then

$$r_o = \frac{50}{0.5} = 100\ \text{k}\Omega$$

$$R_{EE} = \frac{1}{2} \times 100 \times 100 = 5\ \text{M}\Omega$$

$$|A_{cm}| = \left(\frac{5}{2 \times 5{,}000}\right) \times 0.1$$

$= 5 \times 10^{-5}$ V/V

$$\text{CMRR} = \frac{50}{5 \times 10^{-5}} = 10^6$$

or 120 dB

9.66 See figure on next page.

$v_{be1} = 2.5 \sin(\omega t)$, mV and
$v_{be2} = -2.5 \sin(\omega t)$, mV

$$v_{C1} \simeq V_{CC} - \left(\frac{I}{2}\right) R_C - g_m R_C \times 2.5 \times 10^{-3} \sin(\omega t)$$

where

$$g_m = \frac{I/2}{V_T} = \frac{I\ \text{mA}}{0.05\ \text{V}}$$

Thus,

$$v_{C1} = 5 - \frac{I}{2} \times 10 - \frac{I}{0.05} \times 10 \times 2.5 \times 10^{-3} \sin(\omega t)$$

$= 5 - 5I - 0.5I \sin(\omega t)$

Similarly,

$v_{C2} = 5 - 5I + 0.5I \sin(\omega t)$

To ensure operation in the active mode at all times with $v_{CB} = 0$ V, we use

$v_{C1\min} = 0.005$

$5 - 5.5I = 0.005$

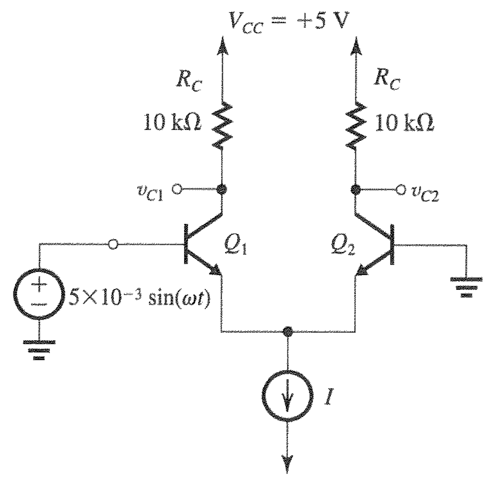
$\Rightarrow I \simeq 0.9$ mA

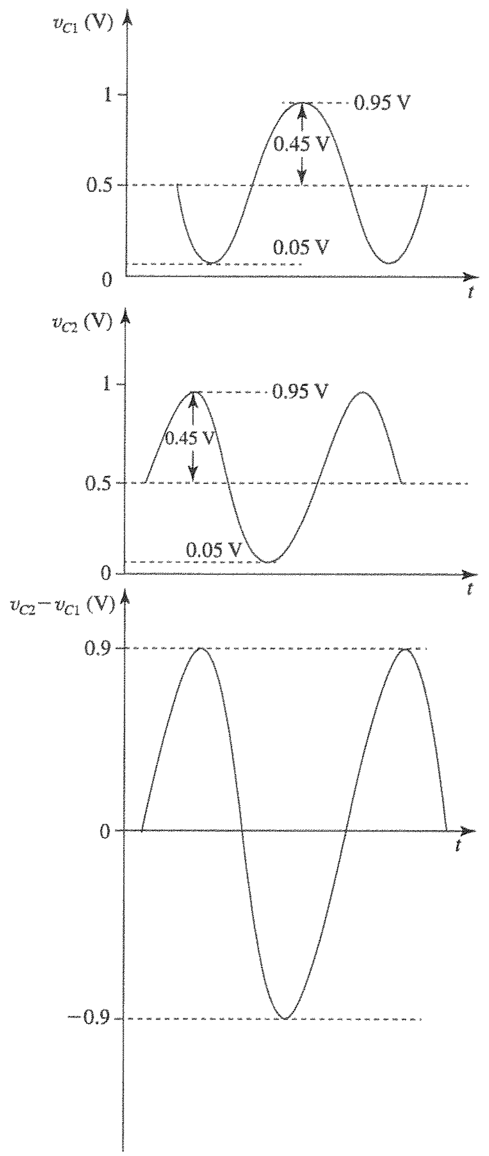
With this value of bias current, we obtain

$$g_m = \frac{0.9}{0.05} = 18\ \text{mA/V}$$

$A_d = g_m R_C = 18 \times 10 = 180$ V/V

At each collector there will be a sine wave of $180 \times 2.5 = 450$ mV $= 0.45$ V amplitude superimposed on the dc bias voltage of $5 - 0.45 \times 10 = 0.5$ V. Between the two collectors there will be a sine wave with 0.9 V peak amplitude. The figure illustrates the waveforms obtained.





9.67 $\dfrac{v_{o1}}{v_{id}} = -100$ V/V $\dfrac{v_{o2}}{v_{id}} = +100$ V/V

$\dfrac{v_{o1,2}}{v_{icm}} = -0.1$ V/V

$R_{id} = 10$ kΩ

$I = 2$ mA

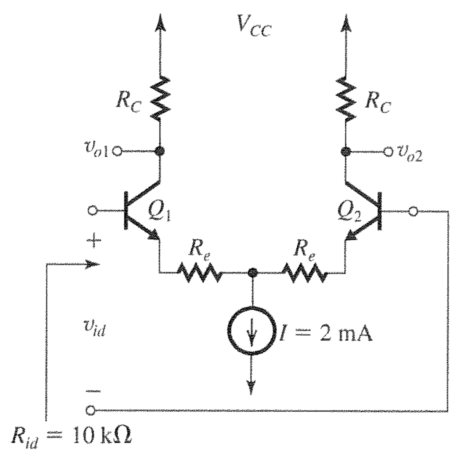
$I_{E1} = I_{E2} = 1$ mA

$r_{e1} = r_{e2} = 25\ \Omega$

$g_{m1} = g_{m2} = 40$ mA/V

$r_{\pi 1} = r_{\pi 2} = \dfrac{\beta}{g_m} = \dfrac{100}{40} = 2.5\ \text{k}\Omega$

Since $R_{id} > r_\pi$, we need emitter resistances, as shown in the figure.



$$10 = (\beta + 1)\ (2r_e + 2R_e)$$

$$r_e + R_e = \frac{10}{2 \times 101} \simeq 50\ \Omega$$

$$R_e = 25\ \Omega$$

$$\frac{v_{o1}}{v_{id}} = -\frac{\alpha R_C}{2(r_e + R_e)}$$

$$-100 = \frac{-\alpha R_C}{2(0.025 + 0.025)}$$

$$\Rightarrow R_C \simeq 10\ \text{k}\Omega$$

To allow for ±2 V swing at each collector,

$$V_{CC} - \frac{I}{2}R_C - 2 \geq 0$$

assuming that $V_{CM} = 0$ V. Thus,

$$V_{CC} = \frac{2}{2} \times 10 + 2 = 12\ \text{V}$$

We can use $V_{CC} = 15$ V to allow for V_{ICM} as high as +3 V.

$$|A_{cm}| \text{ (to each collector) } \simeq \frac{R_C}{2R_{EE}}$$

For $|A_{cm}| = 0.1$,

$$0.1 = \frac{10}{2R_{EE}}$$

$$\Rightarrow R_{EE} = 50 \text{ k}\Omega$$

This is the minimum value of R_o of the bias current source. If the current source is realized by a simple current mirror, we obtain

$$R_{EE} = r_o = \frac{V_A}{I}$$

Thus,

$$50 = \frac{V_A}{2}$$

$$\Rightarrow V_A = 100 \text{ V}$$

The common-mode input resistance is

$$R_{icm} \simeq \beta R_{EE} \frac{1 + R_C/\beta r_o}{1 + \dfrac{R_C + 2R_{EE}}{r_o}}$$

where r_o is the output resistance of each of Q_1 and Q_2,

$$r_o = \frac{V_A}{I/2} = \frac{100}{1} = 100 \text{ k}\Omega$$

$$R_{icm} = 100 \times 50 \, \frac{1 + (10/(100 \times 100))}{1 + \dfrac{10 + 100}{100}}$$

$$= 2.4 \text{ M}\Omega$$

9.68 If the output is taken single-endedly, then

$$|A_{cm}| = \frac{R_C}{2R_{EE}}$$

$$|A_d| = \frac{1}{2} g_m R_C$$

$$\text{CMRR}_s = \frac{|A_{cm}|}{|A_d|} = g_m R_{EE}$$

If the output is taken differentially, then

$$|A_{cm}| = \left(\frac{R_C}{2R_{EE}}\right)\left(\frac{\Delta R_C}{R_C}\right)$$

$$|A_d| = g_m R_C$$

$$\text{CMRR}_d = 2 g_m R_{EE} / \left(\frac{\Delta R_C}{R_C}\right)$$

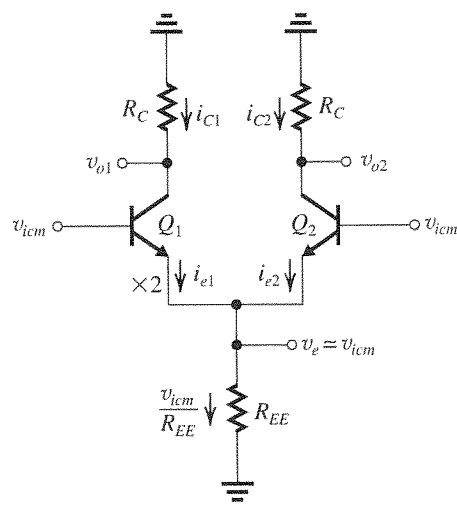
Thus,

$$\frac{\text{CMRR}_d}{\text{CMRR}_s} = \frac{2}{\Delta R_C/R_C}$$

$$20 \log \frac{2}{\Delta R_C/R_C} = 34 \text{ dB}$$

$$\Rightarrow \frac{\Delta R_C}{R_C} = 0.04 = 4\%$$

9.69 If Q_1 has twice the base-emitter junction area of Q_2, the bias current I will split $\frac{2}{3} I$ in Q_1 and $\frac{1}{3} I$ in Q_2. This is because with B_1 and B_2 grounded the two transistors will have equal V_{BE}'s. Thus their currents must be related by the ratio of their scale currents I_S, which are proportional to the junction areas.



With a common-mode input signal v_{icm} applied, as shown in the figure, the current (v_{icm}/R_{EE}) will split between Q_1 and Q_2 in the same ratio as that of their base-emitter junction areas, thus

$$i_{e1} = \frac{2}{3} \frac{v_{icm}}{R_{EE}}$$

and

$$i_{e2} = \frac{1}{3} \frac{v_{icm}}{R_{EE}}$$

Thus,

$$v_{o1} = -i_{c1} R_C \simeq -i_{e1} R_C = -\frac{2}{3} \frac{R_C}{R_{EE}} v_{icm}$$

and

$$v_{o2} = -\frac{1}{3} \frac{R_C}{R_{EE}} v_{icm}$$

With the output taken differentially, we have

$$v_{o2} - v_{o1} = \frac{1}{3} \frac{R_C}{R_{EE}} v_{icm}$$

$$A_{cm} = \frac{1}{3} \frac{R_C}{R_{EE}} = \frac{1}{3} \times \frac{12}{500} = 0.008 \text{ V/V}$$

9.70 $g_m = \sqrt{2\,k_n'(W/L)I_D}$

$= \sqrt{k_n'(W/L)I}$

$A_d = g_m R_D$

$V_{OV} = \dfrac{2\,I_D}{g_m} = \dfrac{I}{g_m}$

$V_{OS} = \left(\dfrac{V_{OV}}{2}\right)\left(\dfrac{\Delta R_D}{R_D}\right)$

For $I = 160\ \mu A$, we have

$g_m = \sqrt{4 \times 0.16} = 0.8$ mA/V

$A_d = 0.8 \times 10 = 8$ V/V

$V_{OV} = \dfrac{0.16}{0.8} = 0.2$ V

$V_{OS} = \dfrac{0.2}{2} \times 0.02 = 2$ mV

For $I = 360\ \mu A$, we have

$g_m = \sqrt{4 \times 0.36} = 1.2$ mA/V

$A_d = 1.2 \times 10 = 12$ V/V

$V_{OV} = \dfrac{0.36}{1.2} = 0.3$ V

$V_{OS} = \dfrac{0.3}{2} \times 0.02 = 3$ mV

Thus by increasing the bias current, both the gain and the offset voltage increase, and by the same factor (1.5).

9.71 (a) $g_m = \sqrt{2k_n I_D} = \sqrt{k_n I}$

$$A_d = g_m R_D = \sqrt{k_n I}\, R_D \qquad (1)$$

$V_{OV} = \sqrt{\dfrac{I/2}{\frac{1}{2}k_n}} = \sqrt{\dfrac{I}{k_n}}$

$V_{OS} = \left(\dfrac{V_{OV}}{2}\right)\left(\dfrac{\Delta R_D}{R_D}\right)$

Thus,

$$V_{OS} = \frac{1}{2}\sqrt{I/k_n}\left(\frac{\Delta R_D}{R_D}\right) \qquad (2)$$

(b) For each value of V_{OS} we use Eq. (2) to determine I and then Eq. (1) to determine A_d. The results are as follows:

V_{OS} (mV)	1	2	3	4	5
I (mA)	0.04	0.16	0.36	0.64	1.00
A_d (V/V)	4	8	12	16	20

We observe that by accepting a larger offset we are able to obtain a higher gain. Observe that the gain realized is proportional to the offset voltage one is willing to accept.

9.72 The offset voltage due to ΔV_t is

$V_{OS} = \pm 5$ mV

The offset voltage due to ΔR_D is

$V_{OS} = \left(\dfrac{V_{OV}}{2}\right)\left(\dfrac{\Delta R_D}{R_D}\right) = \dfrac{0.3}{2} \times 0.02 = 3$ mV

The offset voltage due to $\Delta(W/L)$ is

$V_{OS} = \left(\dfrac{V_{OV}}{2}\right)\dfrac{\Delta(W/L)}{(W/L)} = \dfrac{0.3}{2} \times 0.02 = 3$ mV

The worst-case offset voltage will be when all three components add up,

$V_{OS} = 5 + 3 + 3 = 11$ mV

The major contribution to the total is the variability of V_t.

To compensate for a total offset of 11 mV by appropriately varying R_D, we need to change R_D by ΔR_D obtained from

$11\text{ mV} = \left(\dfrac{V_{OV}}{2}\right) \times \dfrac{\Delta R_D}{R_D}$

$\Rightarrow \dfrac{\Delta R_D}{R_D} = \dfrac{11 \times 2}{300} = 0.0733$

or 7.33%

9.73 $V_{OV} = \sqrt{\dfrac{I/2}{\frac{1}{2}k_n'(W/L)}} = \sqrt{\dfrac{I}{k_n'(W/L)}}$

$= \sqrt{\dfrac{0.1}{0.2 \times 10}} = 0.224$ V

$\dfrac{\Delta R_D}{R_D} = 0.04 \Rightarrow V_{OS} = \left(\dfrac{V_{OV}}{2}\right)\left(\dfrac{\Delta R_D}{R_D}\right)$

$= \dfrac{0.224}{2} \times 0.04 = 4.5$ mV

$\dfrac{\Delta(W/L)}{(W/L)} = 0.04 \Rightarrow V_{OS} = \left(\dfrac{V_{OV}}{2}\right)\left(\dfrac{\Delta(W/L)}{(W/L)}\right)$

$= \dfrac{0.224}{2} \times 0.04 = 4.5$ mV

$\Delta V_t = 5\text{ mV} \Rightarrow V_{OS} = \Delta V_t = 5$ mV

Worst-case $V_{OS} = 4.5 + 4.5 + 5 = 14$ mV

If the three components are independent,

$V_{OS} = \sqrt{4.5^2 + 4.5^2 + 5^2} = 8.1$ mV

9.74 $V_{OS} = V_T\left(\dfrac{\Delta R_C}{R_C}\right)$

$= 25 \times 0.1 = 2.5$ mV

9.75 $V_{OS} = V_T\left(\dfrac{\Delta I_S}{I_S}\right)$

$= 25 \times 0.1 = 2.5$ mV

9.76 With both input terminals grounded, a mismatch ΔR_C between the two collector resistors gives rise to an output voltage

$$V_O = \alpha\left(\frac{I}{2}\right)\Delta R_C \qquad (1)$$

With a resistance R_E connected in the emitter of each transistor, the differential gain becomes

$$|A_d| = \frac{\alpha \times 2R_C}{2(r_e + R_E)} = \frac{\alpha R_C}{R_E + r_e} \qquad (2)$$

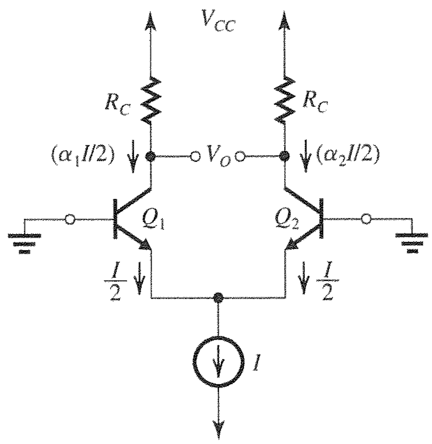
The input offset voltage V_{OS} is obtained by dividing V_O in (1) by $|A_d|$ in (2),

$$V_{OS} = \frac{I}{2}(r_e + R_E)\left(\frac{\Delta R_C}{R_C}\right)$$

Since $r_e = \dfrac{V_T}{I/2}$,

$$V_{OS} = (V_T + \frac{1}{2}IR_E)\left(\frac{\Delta R_C}{R_C}\right)$$

9.77



The current I splits equally between the two emitters. However, the unequal β's will mean unequal α's. Thus, the two collector currents will be unequal,

$$I_{C1} = \alpha_1 I/2$$

$$I_{C2} = \alpha_2 I/2$$

and the collector voltages will be unequal,

$$V_{C1} = V_{CC} - \alpha_1(I/2)R_C$$

$$V_{C2} = V_{CC} - \alpha_2(I/2)R_C$$

Thus a differential output voltage V_O develops:

$$V_O = V_{C2} - V_{C1}$$

$$= \frac{1}{2}IR_C(\alpha_1 - \alpha_2)$$

The input offset voltage V_{OS} can be obtained by dividing V_O by the differential gain A_d:

$$A_d = g_m R_C \simeq \frac{I/2}{V_T}R_C = \frac{IR_C}{2V_T}$$

Thus,

$$V_{OS} = V_T(\alpha_1 - \alpha_2)$$

Substituting, we obtain

$$\alpha_1 = \frac{\beta_1}{\beta_1 + 1}$$

and

$$\alpha_2 = \frac{\beta_2}{\beta_2 + 1}$$

$$V_{OS} = V_T\left(\frac{\beta_1}{\beta_1 + 1} - \frac{\beta_2}{\beta_2 + 1}\right)$$

$$= V_T\frac{\beta_1\beta_2 + \beta_1 - \beta_1\beta_2 - \beta_2}{(\beta_1 + 1)(\beta_2 + 1)}$$

$$= V_T\frac{\beta_1 - \beta_2}{(\beta_1 + 1)(\beta_2 + 1)}$$

$$\simeq V_T\frac{\beta_1 - \beta_2}{\beta_1\beta_2}$$

$$= V_T\left(\frac{1}{\beta_2} - \frac{1}{\beta_1}\right) \qquad \text{Q.E.D.}$$

For $\beta_1 = 50$ and $\beta_2 = 100$, we have

$$V_{OS} = 25\left(\frac{1}{100} - \frac{1}{50}\right) = -0.25 \text{ mV}$$

9.78 For the MOS amplifier:

$$V_{OS} = \left(\frac{V_{OV}}{2}\right)\left(\frac{\Delta R_D}{R_D}\right)$$

$$= \frac{200}{2} \times 0.04$$

$$= 4 \text{ mV}$$

For the BJT amplifier:

$$V_{OS} = V_T\left(\frac{\Delta R_C}{R_C}\right)$$

$$= 25 \times 0.04 = 1 \text{ mV}$$

If in the MOS amplifier the width of each device is increased by a factor of 4 while the bias current is kept constant, V_{OV} will be reduced by a factor of 2. Thus V_{OS} becomes

$$V_{OS} = 2 \text{ mV}$$

9.79 Since the only difference between the two sides of the differential pair is the mismatch in V_A, we can write

$$I_{C1} = I_C\left(1 + \frac{V_{CE1}}{V_{A1}}\right)$$

$$I_{C2} = I_C\left(1 + \frac{V_{CE2}}{V_{A2}}\right)$$

$$I_{C1} + I_{C2} = \alpha I$$

$$I_C\left(2 + \frac{V_{CE1}}{V_{A1}} + \frac{V_{CE2}}{V_{A2}}\right) = \alpha I$$

$$\Rightarrow I_C = \alpha I \Big/ \left(2 + \frac{V_{CE1}}{V_{A1}} + \frac{V_{CE2}}{V_{A2}}\right)$$

$$I_{C1} = \frac{\alpha I}{2} \; \frac{1 + \dfrac{V_{CE1}}{V_{A1}}}{1 + \dfrac{V_{CE1}}{2V_{A1}} + \dfrac{V_{CE2}}{2V_{A2}}}$$

For $\dfrac{V_{CE1}}{V_{A1}} \ll 1$ and $\dfrac{V_{CE2}}{V_{A2}} \ll 1$ we have

$$I_{C1} \simeq \frac{\alpha I}{2}\left(1 + \frac{1}{2}\frac{V_{CE1}}{V_{A1}} - \frac{1}{2}\frac{V_{CE2}}{V_{A2}}\right)$$

$$I_{C2} \simeq \frac{\alpha I}{2}\left(1 + \frac{1}{2}\frac{V_{CE2}}{V_{A2}} - \frac{1}{2}\frac{V_{CE1}}{V_{A1}}\right)$$

The voltage V_O between the two collectors will be

$$V_O = V_{C2} - V_{C1}$$

$$= I_{C1}R_C - I_{C2}R_C$$

$$= \frac{\alpha I}{2}R_C \times \left(\frac{V_{CE1}}{V_{A1}} - \frac{V_{CE2}}{V_{A2}}\right) \qquad (1)$$

Since we still have $I_{C1} \simeq I_{C2} = \alpha \dfrac{I}{2}$, the differential gain is still given by

$$A_d = g_m R_C = \frac{I_C R_C}{V_T} = \frac{\alpha I R_C}{2V_T} \qquad (2)$$

Dividing (1) by (2) gives

$$V_{OS} = V_T\left(\frac{V_{CE1}}{V_{A1}} - \frac{V_{CE2}}{V_{A2}}\right)$$

As a first-order approximation, we can assume

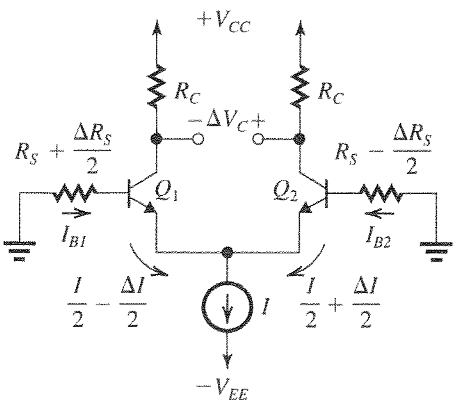
$$V_{CE1} \simeq V_{CE2} = 10 \text{ V}$$

and substitute $V_{A1} = 100$ V and $V_{A2} = 200$ V to determine V_{OS} as

$$V_{OS} = 25\left(\frac{10}{100} - \frac{10}{200}\right)$$

$$= 25 \times 0.05 = 1.25 \text{ mV}$$

9.80



Consider only the incremental currents involved.

Assume the mismatch ΔR_S is split between the two base (source) resistances. The emitter currents will be different, as shown.

Equating the voltage drop from each grounded input to the common emitters, we have

$$I_{B1}\left(R_S + \frac{\Delta R_S}{2}\right) + \left(\frac{I}{2} - \frac{\Delta I}{2}\right)r_e$$

$$= I_{B2}\left(R_S - \frac{\Delta R_S}{2}\right) + \left(\frac{I}{2} + \frac{\Delta I}{2}\right)r_e$$

Subtracting out the $\dfrac{I}{2}r_e$ terms, we have

$$I_{B1}\left(R_S + \frac{\Delta R_S}{2}\right) - \frac{\Delta I}{2}r_e$$

$$= I_{B2}\left(R_S - \frac{\Delta R_S}{2}\right) + \frac{\Delta I}{2}r_e$$

In terms of the emitter currents, this becomes

$$\frac{\left(\dfrac{I}{2} - \dfrac{\Delta I}{2}\right)}{(\beta+1)}\left(R_S + \frac{\Delta R_S}{2}\right) - \frac{\Delta I}{2}r_e$$

$$= \frac{\left(\dfrac{I}{2} + \dfrac{\Delta I}{2}\right)}{(\beta+1)}\left(R_S - \frac{\Delta R_S}{2}\right) + \frac{\Delta I}{2}r_e$$

Subtracting $\dfrac{IR_S}{2(\beta+1)}$ and $-\dfrac{\Delta I \Delta R_S}{4(\beta+1)}$ from each side, we obtain

$$\frac{I\Delta R_S}{4(\beta+1)} - \frac{\Delta I R_S}{2(\beta+1)} - \frac{I\Delta r_e}{2}$$

$$= -\frac{I\Delta R_S}{4(\beta+1)} + \frac{\Delta I R_S}{2(\beta+1)} + \frac{\Delta I r_e}{2}$$

Combining terms, we have

$$\frac{I\Delta R_S}{2(\beta+1)} = \frac{\Delta I R_S}{(\beta+1)} + \Delta I r_e$$

$$\Delta I\left(\frac{R_S}{(\beta+1)} + r_e\right) = \frac{I\Delta R_S}{2(\beta+1)} \text{ so that}$$

$$\Delta I = \frac{I\Delta R_S}{2(\beta+1)} \cdot \frac{1}{\dfrac{R_S}{(\beta+1)} + r_e}$$

$\Delta V_C = \Delta I_C R_C$. If $\dfrac{\beta}{\beta+1} \approx 1$, we have

$$\Delta V_C = \frac{I\Delta R_S R_C}{2(\beta+1)} \cdot \frac{1}{\dfrac{R_S}{(\beta+1)} + r_e}$$

Now V_{OS} can be obtained by dividing ΔV_C by $A_d = g_m R_C$,

$$V_{OS} = \frac{\Delta V_C}{A_d} = \frac{\dfrac{I \Delta R_S R_C}{2(\beta+1)} \cdot \dfrac{1}{\dfrac{R_S}{(\beta+1)} + r_e}}{g_m R_C}$$

$$= \frac{I \Delta R_S}{2(\beta+1)} \cdot \frac{1}{g_m \left[\dfrac{R_S}{(\beta+1)} + r_e \right]}$$

$$V_{OS} = \frac{I \Delta R_S}{2} \cdot \frac{1}{g_m R_S + (\beta+1) r_e g_m}$$

Since $(\beta+1) r_e = r_\pi$ and $r_\pi g_m = \beta$, we have

$$V_{OS} = \frac{\left(\dfrac{I}{2\beta}\right) \cdot \Delta R_S}{1 + \dfrac{g_m R_S}{\beta}} \qquad \text{Q.E.D.}$$

9.81 Refer to Fig. P9.81.

(a) $R_{C1} = 1.04 \times 5 = 5.20\ \text{k}\Omega$

$R_{C2} = 0.96 \times 5 = 4.80\ \text{k}\Omega$

To equalize the total resistance in each collector, we adjust the potentiometer so that

$R_{C1} + x \times 1\ \text{k}\Omega = R_{C2} + (1 - x) \times 1\ \text{k}\Omega$

$5.2 + x = 4.8 + 1 - x$

$\Rightarrow x = 0.3\ \text{k}\Omega$

(b) If the area of Q_1 and hence I_{S1} is 5% larger than nominal, then we have

$I_{S1} = 1.05 I_S$

and the area of Q_2 and hence I_{S2} is 5% smaller than nominal,

$I_{S2} = 0.95 I_S$

Thus,

$I_{E1} = 0.5 \times 1.05 = 0.525$ mA

$I_{E2} = 0.5 \times 0.95 = 0.475$ mA

Assuming $\alpha \simeq 1$, we obtain

$I_{C1} = 0.525$ mA $\quad I_{C2} = 0.475$ mA

To reduce the resulting offset to zero, we adjust the potentiometer so that

$V_{C1} = V_{C2}$

$\Rightarrow V_{CC} - (R_{C1} + x) I_{C1} = V_{CC} - (R_{C2} + 1 - x) I_{C2}$

$I_{C1}(R_{C1} + x) = I_{C2}(R_{C2} + 1 - x)$

$0.525(5 + x) = 0.475(5 + 1 - x)$

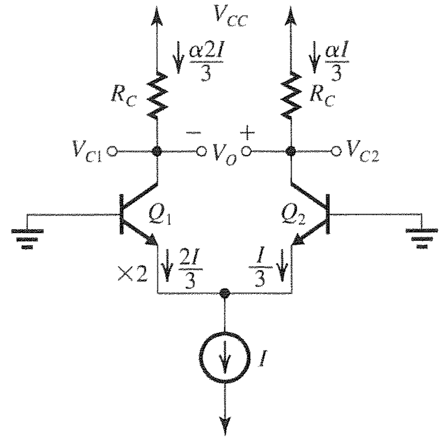
$\Rightarrow x = 0.225$

9.82 $I_{B\max} = \dfrac{400}{2 \times 81} \simeq 2.5\ \mu\text{A}$

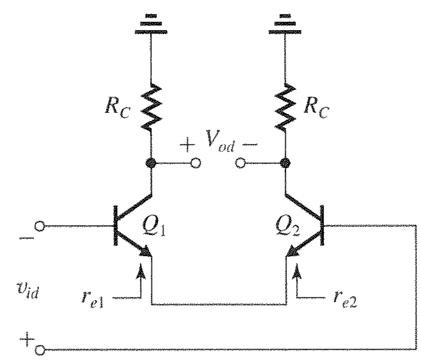
$I_{B\min} = \dfrac{400}{2 \times 201} = 1\ \mu\text{A}$

$I_{OS\max} = \dfrac{200}{81} - \dfrac{200}{201} \simeq 1.5\ \mu\text{A}$

9.83



(a)



(b)

From Fig. (a) we see that the transistor with twice the area (Q_1) will carry twice the current in the other transistor (Q_2). Thus

$I_{E1} = \dfrac{2I}{3}, \qquad I_{E2} = \dfrac{I}{3}$

$I_{C1} = \dfrac{\alpha 2I}{3}, \qquad I_{C2} = \dfrac{\alpha I}{2}$

Thus,

$V_{C1} = V_{CC} - \dfrac{\alpha 2I}{3} R_C$

$V_{C2} = V_{CC} - \dfrac{\alpha I}{3} R_C$

and the dc offset voltage at the output will be

$V_O = V_{C2} - V_{C1}$

$V_O = \dfrac{1}{3} \alpha I R_C$

To reduce this output voltage to zero, we apply a dc input voltage v_{id} in the direction shown in Fig. (b). The voltage v_{id} is required to produce v_{od} in the direction shown which is opposite in direction to V_O and of course $|v_{od}| = |V_O|$, thus

$$A_d v_{id} = \frac{1}{3}\alpha I R_C \quad (1)$$

The gain A_d is found as follows:

$$A_d = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{\alpha \times 2R_C}{r_{e1} + r_{e2}}$$

where

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{V_T}{2I/3} = \frac{3V_T}{2I} = \frac{1.5V_T}{I}$$

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{V_T}{I/3} = \frac{3V_T}{I}$$

thus,

$$A_d = \frac{2\alpha R_C}{4.5\ V_T/I} = \frac{2\alpha I R_C}{4.5\ V_T} \quad (2)$$

Substituting in Eq. (1) gives

$$v_{id} = 0.75\ V_T = 18.75\ \text{mV}$$

Now, using large signal analysis:

$$v_{id} = V_{B2} - V_{B1} = (V_{B2} - V_E) - (V_{B1} - V_E)$$

$$I_{C1} = I_{S1}e^{(V_{B1} - V_E)/V_T} \quad (3)$$

$$I_{C2} = I_{S2}e^{(V_{B2} - V_E)/V_T} \quad (4)$$

where $I_{S1} = 2\ I_{S2}$.

To make $I_{C1} = I_{C2}$,

$$I_{S1}e^{(V_{B1} - V_E)/V_T} = I_{S2}e^{(V_{B2} - V_E)/V_T}$$

$$e^{(V_{B2} - V_{B1})/V_T} = 2$$

$$V_{B2} - V_{B1} = V_T \ln 2$$

Thus,

$$v_{id} = 17.3\ \text{mV}$$

which is reasonably close to the approximate value obtained using small-signal analysis.

9.84 A 2-mV input offset voltage corresponds to a difference ΔR_C between the two collector resistances,

$$2 = V_T \frac{\Delta R_C}{R_C}$$

$$= 25 \times \frac{\Delta R_C}{20}$$

$$\Rightarrow \Delta R_C = 1.6\ \text{k}\Omega$$

Thus a 2-mV offset can be nulled out by adjusting one of the collector resistances by 1.6 kΩ. If the adjustment mechanism raises one R_C and lowers the other, then each need to be adjusted by only $(1.6\ \text{k}\Omega/2) = 0.8\ \text{k}\Omega$.

If a potentiometer is used (as in Fig. P9.81), the total resistance of the potentiometer must be at least 1.6 kΩ. If specified to a single digit, we use 2 kΩ.

9.85 $G_m = 2$ mA/V

With $R_L = \infty$,

$$A_d = G_m R_o$$

and

$$v_o = G_m R_o v_{id}$$

With $R_L = 20\ \text{k}\Omega$,

$$v_o = G_m R_o v_{id} \frac{R_L}{R_L + R_o}$$

$$= G_m R_o \frac{20}{20 + R_o} v_{id} = \frac{1}{2} G_m R_o v_{id}$$

Thus,

$$R_o = 20\ \text{k}\Omega$$

$$A_d\ (\text{with } R_L = \infty) = G_m R_o = 2 \times 20 = 40\ \text{V/V}$$

9.86 $G_m = g_{m1,2} = \dfrac{2(I/2)}{V_{OV}} = \dfrac{I}{V_{OV}} = \dfrac{I}{0.25}$

$$R_o = r_{o2} \parallel r_{o4}$$

For

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{|V_A'|L}{I/2}$$

$$= \frac{2 \times 5 \times 0.5}{I} = \frac{5}{I}$$

$$R_o = \frac{1}{2} \times \frac{5}{I} = \frac{2.5}{I}$$

Thus,

$$A_d = G_m R_o = \frac{I}{0.25} \times \frac{2.5}{I} = 10\ \text{V/V}$$

9.87 $\dfrac{I}{2} = \dfrac{1}{2}\mu_n C_{ox}\left(\dfrac{W}{L}\right)V_{OV}^2$

$$0.1 = \frac{1}{2} \times 0.2 \times 50 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.14\ \text{V}$$

$$g_{m1,2} = \frac{2 \times (I/2)}{V_{OV}} = \frac{2 \times 0.1}{0.14} = 1.4\ \text{mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{|V_A'| \times L}{I/2} = \frac{5 \times 0.5}{0.1}$$

$$= 25\ \text{k}\Omega$$

$A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$

$= 1.4 \times (25 \parallel 25)$

$= 17.5$ V/V

9.88 $A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$

$g_{m1,2} = \sqrt{2k_n'\left(\frac{W}{L}\right)I_D}$

$= \sqrt{4I} = 2\sqrt{I}$

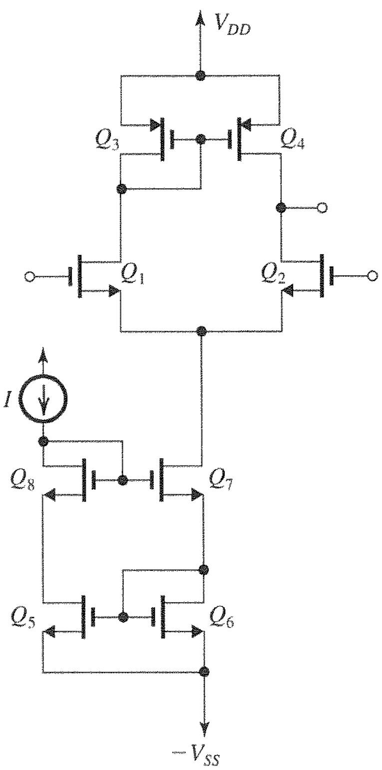
$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{2|V_A|}{I} = \frac{2 \times 5}{I} = \frac{10}{I}$

$A_d = 2\sqrt{I} \times \frac{1}{2} \times \frac{10}{I} = \frac{10}{\sqrt{I}}$

$20 = \frac{10}{\sqrt{I}}$

$\Rightarrow I = 0.25$ mA

9.89



For Q_1, Q_2, Q_3 and Q_4:

$\frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$

$0.1 = \frac{1}{2} \times 5 \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.2$ V

$V_{GS} = V_t + |V_{OV}|$

$= 0.5 + 0.2 = 0.7$ V

For Q_5, Q_6, Q_7, and Q_8:

$I_D = 0.2$ mA

$0.2 = \frac{1}{2} \times 5 \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.28$ V

$V_{GS} = 0.5 + 0.28 = 0.78$ V

From the figure we see that for each transistor to operate at V_{DS} at least equal to V_{GS}, the total power supply is given by

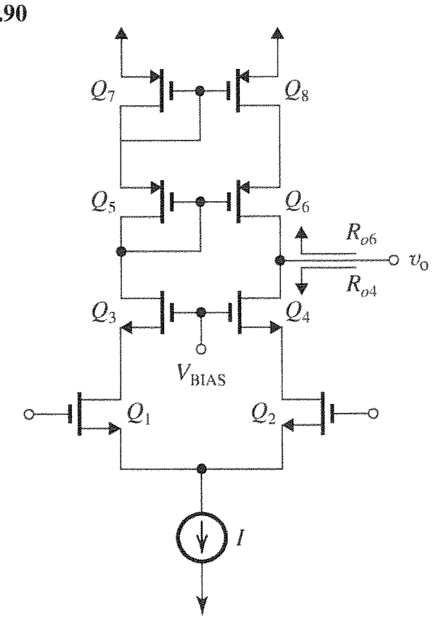
$V_{DD} + V_{SS} = V_{DS4} + V_{DS2} + V_{DS7} + V_{DS6}$

$= V_{GS4} + V_{GS2} + V_{GS7} + V_{GS6}$

$= 0.7 + 0.7 + 0.78 + 0.78$

$= 2.96 \simeq 3.0$ V

9.90



(a) See figure.

(b) $A_d = g_{m1,2}(R_{o4} \parallel R_{o6})$

$g_{m1,2} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$

$R_{o6} = g_{m6}r_{o6}r_{o8}$

Since all transistors are operated at a bias current $(I/2)$ and have the same overdrive voltage $|V_{OV}|$ and the same Early voltage, $|V_A|$, all have the same $g_m = I/|V_{OV}|$ and the same

$r_o = \frac{|V_A|}{I/2} = 2|V_A|/I$. Thus,

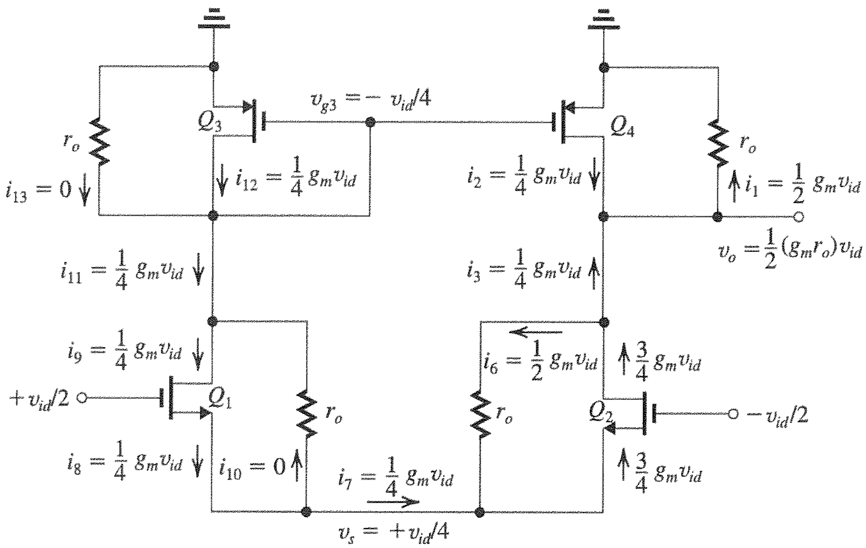
$R_{o6} = g_m r_o^2$

$R_{o4} = g_{m4}r_{o4}r_{o2} = g_m r_o^2$

$A_d = g_m(g_m r_o^2 \parallel g_m r_o^2)$

$= \frac{1}{2}(g_m r_o)^2$

This figure belongs to Problem 9.91.



$$g_m r_o = \frac{I}{|V_{OV}|} \times \frac{2|V_A|}{I} = \frac{2|V_A|}{|V_{OV}|}$$

$A_d = 2(|V_A|/|V_{OV}|)^2$ Q.E.D.

For $|V_{OV}| = 0.2$ V and $|V_A| = 10$ V, we have

$$A_d = 2\left(\frac{10}{0.2}\right)^2 = 5000 \text{ V/V}$$

9.91 The currents i_1 to i_{13} are shown on the circuit diagram. Observe that $i_{11} = i_7 = i_3$ (the current that enters a transistor exits at the other end!). Also observe that the mirror Q_3 and Q_4 is indeed functioning properly as the drain currents of Q_3 and Q_4 are equal ($i_{12} = i_2 = \frac{1}{4} g_m v_{id}$). However, the currents in their r_o's are far from being equal!

There are some inconsistencies that result from the approximations made to obtain the results shown in Fig. P9.91, namely, $g_m r_o \gg 1$. Note for instance that although we find the current in r_o of Q_2 to be $\frac{1}{2} g_m v_{id}$, the voltages at the two ends of r_o are $\frac{1}{2}(g_m r_o) v_{id}$ and $v_{id}/4$; thus the current must be $v_{id}\left(\frac{1}{2} g_m r_o - \frac{1}{4}\right)\Big/ r_o$, which is approximately $\frac{1}{2} g_m v_{id}$.

The purpose of this problem is to show the huge imbalance that exists in this circuit. In fact, Q_1 has $|v_{gs}| = \frac{1}{4} v_{id}$ while Q_2 has $|v_{gs}| = \frac{3}{4} v_{id}$. This imbalance results from the fact that the current mirror is *not* a balanced load. Nevertheless, we know that this circuit provides a reasonably high common-mode rejection.

9.92 $G_m = g_{m1,2} = \dfrac{2(I/2)}{V_{OV1,2}} = \dfrac{0.2}{0.2} = 1$ mA/V

$$r_{o2} = \frac{V_{An}}{I/2} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

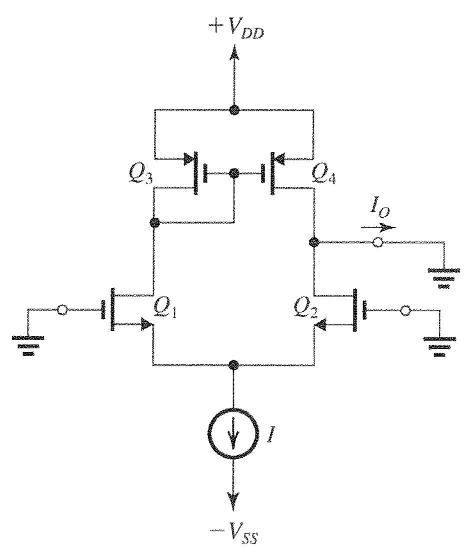
$$r_{o4} = \frac{|V_{Ap}|}{I/2} = \frac{12}{0.1} = 120 \text{ k}\Omega$$

$R_o = r_{o2} \parallel r_{o4} = 200 \parallel 120 = 75$ kΩ

$A_d = G_m R_o = 1 \times 75 = 75$ V/V

The gain is reduced by a factor of 2 with $R_L = R_o = 75$ kΩ.

9.93



(a) Let

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_A + \frac{1}{2}\Delta\left(\frac{W}{L}\right)_A$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_A - \frac{1}{2}\Delta\left(\frac{W}{L}\right)_A$$

Q_1 and Q_2 have equal values of V_{GS} and thus of V_{OV}, thus

$$I_{D1} = \frac{1}{2}k_n'\left[\left(\frac{W}{L}\right)_A + \frac{1}{2}\Delta\left(\frac{W}{L}\right)_A\right]V_{OV}^2$$

$$= \frac{1}{2}k_n'\left(\frac{W}{L}\right)_A\left[1 + \frac{1}{2}\frac{\Delta(W/L)_A}{(W/L)_A}\right]V_{OV}^2$$

Since, in the ideal case

$$I_{D1} = \frac{I}{2} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_A V_{OV}^2$$

$$I_{D1} = \frac{I}{2}\left[1 + \frac{1}{2}\frac{\Delta(W/L)_A}{(W/L)_A}\right]$$

Similarly, we can show that

$$I_{D2} = \frac{I}{2}\left[1 - \frac{1}{2}\frac{\Delta(W/L)_A}{(W/L)_A}\right]$$

The current mirror causes

$$I_{D4} = I_{D3} = I_{D1}$$

Thus,

$$I_O = I_{D4} - I_{D2}$$

$$= I_{D1} - I_{D2}$$

$$= \frac{I}{2}\frac{\Delta(W/L)_A}{(W/L)_A}$$

The input offset voltage is

$$V_{OS} = \frac{I_O}{G_m}$$

where

$$G_m = g_{m1,2} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

Thus,

$$V_{OS} = (V_{OV}/2)\,\frac{\Delta(W/L)_A}{(W/L)_A} \qquad \text{Q.E.D.}$$

(b) $I_{D1} = I_{D2} = \dfrac{I}{2}$

$$I_{D3} = I_{D1}$$

If the (W/L) ratios of the mirror transistors have a mismatch $\Delta(WL)_M$, the current transfer ratio of the mirror will have an error of $[\Delta(W/L)_M/(W/L)_M]$. Thus

$$I_{D4} = I_{D3}\left[1 + \frac{\Delta(W/L)_M}{(W/L)_M}\right]$$

At the output node, we have

$$I_O = I_{D4} - I_{D2}$$

$$= I_{D3}\left[1 + \frac{\Delta(W/L)_M}{(W/L)_M}\right] - I_{D2}$$

$$= I_{D1}\left[1 + \frac{\Delta(W/L)_M}{(W/L)_M}\right] - I_{D2}$$

$$= \frac{I}{2}\frac{\Delta(W/L)_M}{(W/L)_M}$$

and the corresponding V_{OS} will be

$$V_{OS} = \frac{I_O}{G_m} = \frac{I_O}{I/V_{OV}}$$

$$= \left(\frac{V_{OV}}{2}\right)\frac{\Delta(W/L)_M}{(W/L)_M} \qquad \text{Q.E.D.}$$

(c) $V_{OS}|_{Q_1,Q_2\text{ mismatch}} = \left(\dfrac{0.2}{2}\right) \times 0.02 = 2\text{ mV}$

$$V_{OS}|_{Q_3,Q_4\text{ mismatch}} = \left(\frac{0.2}{2}\right) \times 0.02 = 2\text{ mV}$$

Worst-case $V_{OS} = 2 + 2 = 4$ mV

9.94 $I_{E1} = I_{E2} = 0.25$ mA

$$I_{C1} = I_{C2} \simeq 0.25\text{ mA}$$

$$g_{m1,2} = \frac{I_{C1,2}}{V_T} = \frac{0.25\text{ mA}}{0.025\text{ V}} = 10\text{ mA/V}$$

$$r_o = \frac{|V_A|}{I_C} = \frac{10\text{ V}}{0.25\text{ mA}} = 40\text{ k}\Omega$$

$$R_{id} = 2\,r_\pi = 2\frac{\beta}{g_m} = 2 \times \frac{100}{10} = 20\text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 40 \parallel 40 = 20\text{ k}\Omega$$

$$G_m = g_{m1,2} = 10\text{ mA/V}$$

$$A_d = G_m R_o = 10 \times 20 = 200\text{ V/V}$$

If $R_L = R_{id} = 20$ kΩ, then

$$G_v = 200 \times \frac{R_L}{R_L + R_o}$$

$$= 200 \times \frac{20}{20 + 20} = 100\text{ V/V}$$

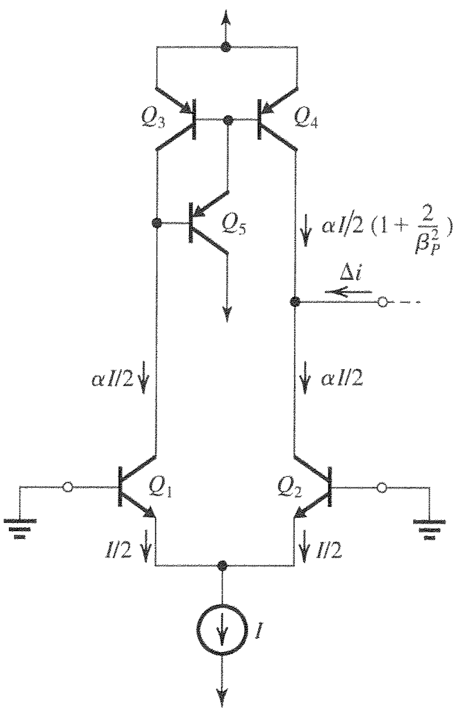
9.95 Using Eq. (9.145), we obtain

$$V_{OS} = -\frac{2V_T}{\beta_p}$$

$$-2 = -\frac{2 \times 25}{\beta_p}$$

$$\Rightarrow \beta_p = 25$$

9.96



The figure shows a BJT differential amplifier loaded in a base-current-compensated current mirror. To determine the systematic input offset voltage resulting from the error in the current-transfer ratio of the mirror, we ground the two input terminals and determine the output current Δi as follows:

$$\Delta i = I_{C2} - I_{C4}$$

$$= \alpha\frac{I}{2} - \alpha\frac{I}{2}\,\frac{1}{1+(2/\beta_p^2)}$$

$$= \alpha\frac{I}{2}\left[1 - \frac{1}{1+(2/\beta_p^2)}\right]$$

$$\simeq -\alpha\frac{I}{2}\,\frac{2}{\beta_p^2} = -\alpha\frac{I}{\beta_p^2}$$

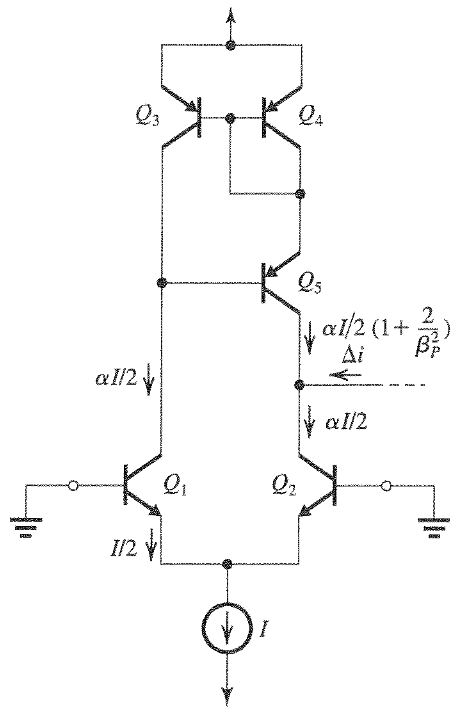
Dividing Δi by $G_m = g_{m1,2} = \dfrac{\alpha I}{2V_T}$ gives

$$V_{OS} = -\frac{2V_T}{\beta_p^2}$$

For $\beta_p = 50$,

$$V_{OS} = -\frac{2\times 25}{50^2} = -20\ \mu\text{V}$$

9.97



The figure shows a BJT differential amplifier loaded with a Wilson current mirror. To determine the systematic input offset voltage resulting from the error in the current-transfer ratio of the mirror, we ground the two input terminals and determine the output current Δi as follows:

$$\Delta i = \alpha\frac{I}{2} - \alpha\frac{I}{2}\,\frac{1}{1+(2/\beta_p^2)}$$

$$\simeq \alpha\frac{I}{2}\,\frac{2}{\beta_p^2} = \frac{\alpha I}{\beta_p^2}$$

Dividing Δi by $G_m = g_{m1,2} = \dfrac{\alpha I/2}{V_T}$ provides the input offset voltage V_{OS}:

$$V_{OS} = -\frac{2V_T}{\beta_p^2}$$

For $\beta_p = 50$,

$$V_{OS} = -\frac{2\times 25}{50^2} = -20\ \mu\text{V}$$

9.98 Refer to Fig. P9.98.

$$A_d = G_m R_o$$

where

$$G_m = g_{m1,2} \simeq \frac{I/2}{V_T}$$

and

$$R_o = R_{o4} \parallel R_{o7}$$

Here R_{o4} is the output resistance of the cascode amplifier (looking into the collector of Q_4), thus

$$R_{o4} = g_{m4} r_{o4} (r_{o2} \parallel r_{\pi 4})$$

Usually $r_{\pi 4} \ll r_{o2}$,

$$R_{o4} \simeq g_{m4} r_{\pi 4} r_{o4} = \beta_4 r_{o4}$$

The resistance R_{o7} is the output resistance of the Wilson mirror and is given by

$$R_{o7} = \frac{1}{2}\beta_7 r_{o7}$$

Thus

$$R_o = (\beta_4 r_{o4}) \parallel \left(\frac{1}{2}\beta_7 r_{o7}\right)$$

Since all β and r_o are equal, we obtain

$$R_o = (\beta r_o) \parallel \left(\frac{1}{2}\beta r_o\right)$$

$$= \frac{1}{3}\beta r_o$$

and

$$A_d = \frac{1}{3}\beta g_m r_o \qquad \text{Q.E.D.}$$

For $\beta = 100$ and $V_A = 20$ V, we have

$$g_m r_o = \frac{I_C}{V_T}\frac{V_A}{I_C} = \frac{V_A}{V_T} = \frac{20}{0.025} = 800$$

$$A_d = \frac{1}{3} \times 100 \times 800 = 2.67 \times 10^4 \text{ V/V}$$

9.99 Refer to Fig. P9.98.

(a) $V_{B7} = +5 - V_{EB6} - V_{EB7} = 5 - 0.7 - 0.7$

$= +3.6$ V

$v_{O\max} = V_{B7} + 0.4 = +4$ V

(b) The dc bias voltage should be

$V_O = v_{O\max} - 1.5$

$= 4 - 1.5 = +2.5$ V

(c) For v_O to swing negatively (i.e., below the dc bias value of 2.5 V) by 1.5 V, that is, to +1 V with Q_4 remaining in saturation, V_{BIAS} should be

$V_{\text{BIAS}} = v_{O\min} + 0.4$

$= 1.4$ V

(d) With $V_{\text{BIAS}} = 1.4$ V, the bias voltage at the collectors of Q_1 and Q_2 is

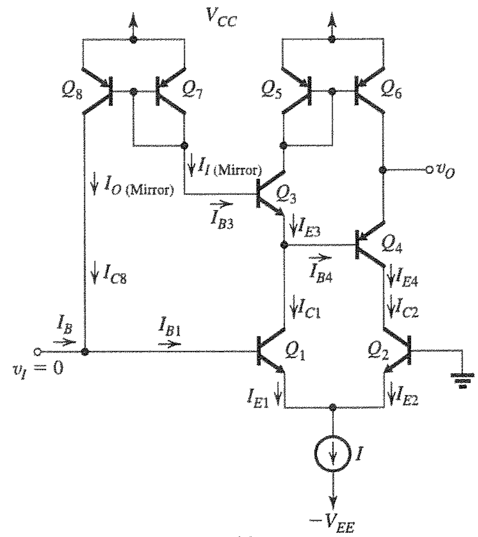
$V_{C1,2} = V_{\text{BIAS}} - V_{BE3,4}$

$= 1.4 - 0.7 = +0.7$ V

The upper limit on V_{CM} is 0.4 V above $V_{C1,2}$:

$V_{CM\max} = 0.7 + 0.4 = +1.1$ V

9.100



(a)

(a) With $v_I = 0$,

$$I_{E1} = I_{E2} = \frac{I}{2}$$

$$I_{C1} = I_{C2} = \frac{\beta}{\beta+1}\frac{I}{2}$$

$$I_{E4} = I_{C2} = \frac{\beta}{\beta+1}\frac{I}{2}$$

$$I_{B4} = \frac{I_{E4}}{\beta+1} = \frac{1}{\beta+1}\frac{\beta}{\beta+1}\frac{I}{2}$$

$$I_{E3} = I_{C1} + I_{B4}$$

$$= \frac{\beta}{\beta+1}\frac{I}{2} + \frac{1}{\beta+1}\frac{\beta}{\beta+1}\frac{I}{2}$$

$$= \frac{\beta}{\beta+1}\frac{I}{2}\left(1 + \frac{1}{\beta+1}\right)$$

$$I_{B3} = \frac{I_{E3}}{\beta+1} = \frac{\beta}{(\beta+1)^2}\left(1 + \frac{1}{\beta+1}\right)\frac{I}{2}$$

Since I_{B3} is the input current to the $Q_7 - Q_8$ mirror and I_{C8} is its output current, we have

$$\frac{I_{C8}}{I_{B3}} = \frac{1}{1 + \dfrac{2}{\beta}} = \frac{\beta}{\beta+2}$$

Thus,

$$I_{C8} = \frac{\beta^2}{(\beta+1)^2(\beta+2)}\left(1 + \frac{1}{\beta+1}\right)\frac{I}{2}$$

At the input node, we have

$$I_B = I_{B1} - I_{C8}$$

$$= \frac{I/2}{\beta+1} - \frac{\beta^2}{(\beta+1)^2(\beta+2)}\left(1+\frac{1}{\beta+1}\right)\frac{I}{2}$$

$$I_B = \frac{I/2}{\beta+1}\left[1 - \frac{\beta^2}{(\beta+1)(\beta+2)}\left(1+\frac{1}{\beta+1}\right)\right]$$

$$= \frac{I/2}{\beta+1}\left[1 - \frac{\beta^2}{(\beta+1)(\beta+2)}\,\frac{\beta+2}{\beta+1}\right]$$

$$= \frac{I/2}{\beta+1}\,\frac{(\beta+1)^2-\beta^2}{(\beta+1)^2}$$

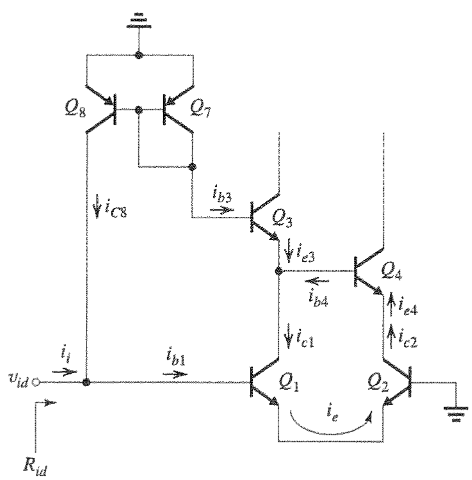
$$= \frac{I/2}{\beta+1}\,\frac{2\beta+1}{(\beta+1)^2}$$

$$\simeq \frac{I/2}{\beta+1}\,\frac{2\beta}{\beta^2}$$

$$= \frac{I/2}{\beta+1}\Big/\left(\frac{\beta}{2}\right)$$

Thus, including the current mirror $Q_7 - Q_8$ reduces the input bias current by a factor equal to $(\beta/2)$, a substantial decrease!

(b)



(b)

The analysis follows the same process used above, except that here we deal with signal quantities.

$$i_e = \frac{v_{id}}{2r_e}$$

where $r_e = r_{e1} = r_{e2}$

$$i_{c1} = i_{c2} = \frac{\beta}{\beta+1}\,\frac{v_{id}}{2r_e}$$

$$i_{e4} = i_{c2} = \frac{\beta}{\beta+1}\,\frac{v_{id}}{2r_e}$$

$$i_{b4} = \frac{i_{e4}}{\beta+1} = \frac{\beta}{(\beta+1)^2}\,\frac{v_{id}}{2r_e}$$

$$i_{e3} = i_{c1} - i_{b4} = \frac{\beta}{\beta+1}\,\frac{v_{id}}{2r_e} - \frac{\beta}{(\beta+1)^2}\,\frac{v_{id}}{2r_e}$$

$$= \frac{\beta}{\beta+1}\,\frac{v_{id}}{2r_e}\left(1-\frac{1}{\beta+1}\right)$$

$$= \left(\frac{\beta}{\beta+1}\right)^2\frac{v_{id}}{2r_e}$$

$$i_{b3} = \frac{i_{e3}}{\beta+1} = \frac{\beta^2}{(\beta+1)^3}\,\frac{v_{id}}{2r_e}$$

$$i_{c8} = i_{b3}\,\frac{1}{1+\dfrac{2}{\beta}} = i_{b3}\,\frac{\beta}{\beta+2}$$

$$= \frac{\beta^3}{(\beta+1)^3}\,\frac{1}{\beta+2}\,\frac{v_{id}}{2r_e}$$

At the input node, we have

$$i_i = i_{b1} - i_{c8}$$

$$= \frac{v_{id}/2r_e}{\beta+1} - \frac{\beta^3}{(\beta+1)^3}\,\frac{1}{\beta+2}\,\frac{v_{id}}{2r_e}$$

$$= \frac{v_{id}}{2(\beta+1)r_e}\left[1 - \frac{\beta^3}{(\beta+1)^2}\,\frac{1}{\beta+2}\right]$$

$$\simeq \frac{v_{id}}{2(\beta+1)r_e}\left[1 - \frac{\beta^3}{(\beta+1)^3}\right]$$

$$= \frac{v_{id}}{2(\beta+1)r_e}\left[1 - \frac{1}{(1+1/\beta)^3}\right]$$

$$\simeq \frac{v_{id}}{2(\beta+1)r_e}\,\frac{3}{\beta}$$

$$= \left[\frac{v_{id}}{2(\beta+1)r_e}\right]\Big/\frac{\beta}{3}$$

Thus the input current is reduced by a factor $(\beta/3)$, which results in R_{id} increasing by a factor $(\beta/3)$.

9.101 To maximize the positive output voltage swing, we select V_{BIAS} as large as possible while maintaining the *pnp* current sources in saturation. For the latter to happen, we need a minimum of 0.3 V across each current source. Thus the maximum allowable voltage at the emitters of Q_3 and Q_4 is $V_{CC} - 0.3 = 5 - 0.3 = +4.7$ V. Then, the maximum allowable value of $V_{BIAS} = 4.7 - 0.7 = +4$ V. To keep Q_4 in saturation,

$$v_{O\max} = V_{BIAS} + 0.4 = 4.4 \text{ V}$$

If the dc voltage at the output is 0 V, then the maximum positive voltage swing is 4.4 V. In the negative direction,

$$v_{O\min} = -V_{EE} + V_{BE7} + V_{BE5} - 0.4$$

$= -5 + 0.7 + 0.7 - 0.4$

$= -4$ V

Thus,

$-4 \text{ V} \le v_O \le +4.4 \text{ V}$

$$G_m = g_{m1,2} \simeq \frac{0.25 \text{ mA}}{0.025 \text{ V}} = 10 \text{ mA/V}$$

$$R_{o4} = \beta_4 r_{o4} = 50 \times \frac{|V_A|}{I/2}$$

$$= 50 \times \frac{100 \text{ V}}{0.25 \text{ mA}} = 20 \text{ M}\Omega$$

$$R_{o5} = \frac{1}{2}\beta_5 r_{o5} = \frac{1}{2} \times 100 \times \frac{100}{0.25}$$

$= 20$ MΩ

$R_o = R_{o4} \parallel R_{o5} = 20 \text{ M}\Omega \parallel 20 \text{ M}\Omega = 10 \text{ M}\Omega$

$A_d = G_m R_o = 10 \times 10{,}000 = 10^5$ V/V

9.102 The overdrive voltage, $|V_{OV}|$, at which Q_1 and Q_2 are operating is found from

$$\frac{I}{2} = \frac{1}{2}k_p'(W/L)|V_{OV}|^2$$

$$0.1 = \frac{1}{2} \times 6.4 \times |V_{OV}|^2$$

$\Rightarrow |V_{OV}| = 0.18$ V

$$G_m = g_{m1,2} = \frac{2(I/2)}{|V_{OV}|}$$

$$= \frac{0.2}{0.18} = 1.13 \text{ mA/V}$$

$$r_{o2} = \frac{|V_{Ap}|}{I/2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{o4} = \frac{|V_{Anpn}|}{I/2} = \frac{30}{0.1} = 300 \text{ k}\Omega$$

$R_o = r_{o2} \parallel r_{o4} = 100 \text{ k}\Omega \parallel 300 \text{ k}\Omega = 75 \text{ k}\Omega$

$A_d = G_m R_o = 1.13 \times 75 = 85$ V/V

9.103 (a) For Q_1 and Q_2,

$$\frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2} V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 0.4 \times \left(\frac{W}{L}\right)_{1,2} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 12.5$$

For Q_3 and Q_4,

$$\frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$0.1 = \frac{1}{2} \times 0.1 \times \left(\frac{W}{L}\right)_{3,4} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 50$$

(b) $G_m = g_{m1,2} = \dfrac{2(I/2)}{V_{OV}} = \dfrac{I}{V_{OV}} = \dfrac{0.2}{0.2} =$ 1 mA/V

$A_d = G_m R_o$

$50 = 1 \times R_o$

$\Rightarrow R_o = 50$ kΩ

But

$R_o = r_{o2} \parallel r_{o4}$

and $r_{o2} = r_{o4}$ (Q_2 and Q_4 have the same $I_D = \dfrac{I}{2}$ and the same V_A). Thus

$$r_{o2} = r_{o4} = 100 \text{ k}\Omega = \frac{|V_A|}{I/2}$$

$$|V_A| = \frac{I}{2} \times 100 \text{ k}\Omega = 10 \text{ V}$$

$10 = |V_A'|L = 20\,L$

$\Rightarrow L = 0.5$ μm

(c) $v_{O\min} = V_{CM} - V_{tn}$

$= 0 - 0.5 = -0.5$ V

$v_{O\max} = V_{DD} - |V_{OV}| = 1 - 0.2 = 0.8$ V

Thus,

$-0.5 \text{ V} \le v_O \le 0.8 \text{ V}$

(d) $R_{SS} = \dfrac{|V_A|}{I} = \dfrac{10}{0.2} = 50$ kΩ

The CMRR can be obtained using Eq. (9.159):

$\text{CMRR} = (g_m r_o)(g_m R_{SS})$

$= (1 \times 100)(1 \times 50)$

$= 5000$ or 74 dB

9.104 The CMRR is given by Eq. (9.158):

$\text{CMRR} = [g_{m1,2}(r_{o2} \parallel r_{o4})]\,[2\,g_{m3}R_{SS}]$

(a) Current source is implemented with a simple current mirror:

$$R_{SS} = r_o|_{Q_S} = \frac{|V_A|}{I}$$

$$g_{m1,2} = g_{m3} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{2|V_A|}{I}$$

Thus,

$$\text{CMRR} = \frac{I}{V_{OV}} \times \frac{1}{2} \times \frac{2|V_A|}{I} \times 2 \times \frac{I}{V_{OV}} \times \frac{|V_A|}{I}$$

$$= 2\left(\frac{V_A}{V_{OV}}\right)^2 \qquad \text{Q.E.D.}$$

(b) Current source is implemented with the modified Wilson mirror in Fig. P9.89:

$R_{SS} = g_{m7} r_{o7} r_{o9}$

Transistor Q_7 has the same $k'(W/L)$ as Q_1 and Q_2, but Q_7 carries a current I twice that of Q_1 and Q_2. Thus

$V_{OV7} = \sqrt{2} V_{OV1,2} = \sqrt{2} V_{OV}$

and

$$g_{m7} = \frac{2I}{V_{OV7}} = \frac{2I}{\sqrt{2} V_{OV}} = \frac{\sqrt{2} I}{V_{OV}}$$

$$r_{o7} = r_{o9} = \frac{V_A}{I}$$

Thus,

$$R_{SS} = \frac{\sqrt{2} I}{V_{OV}} \left(\frac{V_A}{I}\right)^2 = \frac{\sqrt{2} V_A^2}{V_{OV} I}$$

and

$$\text{CMRR} = \frac{I}{V_{OV}} \times \frac{1}{2} \times \frac{2|V_A|}{I} \times 2 \times \frac{I}{V_{OV}} \times \frac{\sqrt{2} V_A^2}{V_{OV} I}$$

$$= 2\sqrt{2} \left(\left|\frac{V_A}{V_{OV}}\right|\right)^3 \qquad \text{Q.E.D.}$$

For $k'(W/L) = 4$ mA/V^2 and $I = 160$ µA,

$$0.080 = \frac{1}{2} \times 4 \times |V_{OV}|^2$$

$\Rightarrow |V_{OV}| = 0.2$ V

For $|V_A| = 5$ V:

For case (a),

$$\text{CMRR} = 2 \times \left(\frac{5}{0.2}\right)^2 = 1250 \text{ or } 62 \text{ dB}$$

For case (b),

$$\text{CMRR} = 2\sqrt{2} \left(\frac{5}{0.2}\right)^3 = 4.42 \times 10^4$$

or 93 dB

9.105 $G_m = g_{m1,2} = \dfrac{2(I/2)}{V_{OV}} = \dfrac{0.2}{0.2} = 1$ mA/V

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$R_o = r_{o2} \parallel r_{o4} = 50 \text{ k}\Omega \parallel 50 \text{ k}\Omega$

$= 25$ kΩ

$A_d = G_m R_o = 1 \times 25 = 25$ V/V

$$R_{SS} = \frac{|V_A|}{I} = \frac{5}{0.2} = 25 \text{ k}\Omega$$

$$G_{mcm} = \frac{1}{2R_{SS}} = \frac{1}{2 \times 25} = 0.02 \text{ mA/V}$$

$$R_{im} = \frac{1}{g_{m3}} \parallel r_{o3}$$

where

$g_{m3} = g_{m1} = g_{m2} = 1$ mA/V

$r_{o3} = r_{o2} = r_{o4} = 50$ kΩ

$R_{im} = 1 \text{ k}\Omega \parallel 50 \text{ k}\Omega = 0.98$ kΩ

$$A_m = 1 \Big/ \left(1 + \frac{1}{g_m r_{o3}}\right)$$

$$= 1 \Big/ \left(1 + \frac{1}{1 \times 50}\right) = 0.98 \text{ A/A}$$

$R_{om} = r_{o4} = 50$ kΩ

$R_{o2} = r_{o2} + 2R_{SS} + 2g_{m2} r_{o2} R_{SS}$

$= 50 + 50 + 2 \times 1 \times 50 \times 25$

$= 2600$ kΩ

$A_{cm} = -(1 - A_m) G_{mcm} (R_{om} \parallel R_{o2})$

$A_{cm} = -(1 - 0.98) \times 0.02 \times (50 \parallel 2600)$

$= -0.0196$ V/V

$$\text{CMRR} = \left|\frac{A_d}{A_{cm}}\right| = \frac{25}{0.0196} = 1274$$

or 62.1 dB

Alternatively, using the approximate expression in Eq. (9.157), we obtain

$$A_{cm} \simeq -\frac{1}{2g_{m3} R_{SS}} = -\frac{1}{2 \times 1 \times 25} = -0.02 \text{ V/V}$$

and

$$\text{CMRR} = \frac{25}{0.02} = 1250$$

or 61.9 dB

9.106 From Eq. (9.153), we have

$A_{cm} = -(1 - A_m) G_{mcm} (R_{om} \parallel R_{o2})$

where

$$G_{mcm} = \frac{1}{2R_{SS}} = \frac{1}{2 \times 45} = 0.011 \text{ mA/V}$$

Using the fact that $R_{o2} \gg R_{om}$, we obtain

$A_{cm} \simeq -(1 - 0.98) \times 0.011 \times 45$

$= -0.01$ V/V

$$\text{CMRR} = \left|\frac{A_d}{A_{cm}}\right| = \frac{30}{0.01} = 3000$$

or 69.5 dB

9.107 $\text{CMRR} = \left|\dfrac{A_d}{A_{cm}}\right|$

CMRR = 60 dB or equivalently 1000. Thus,

$$1000 = \frac{50}{|A_{cm}|}$$

$\Rightarrow |A_{cm}| = 0.05$ V/V

But from Eq. (9.153), we obtain

$$|A_{cm}| = (1 - A_m)G_{mcm}(R_{om} \parallel R_{o2})$$

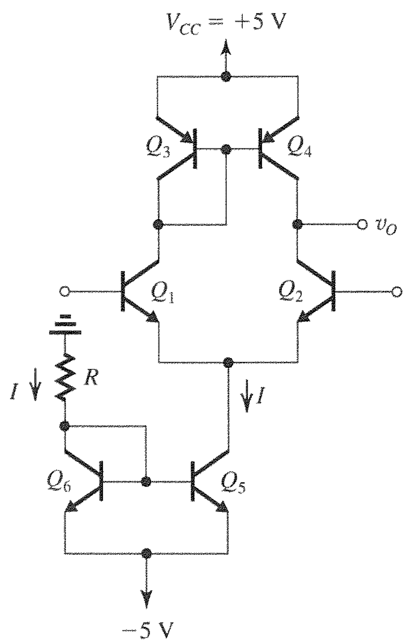
Since $R_{om} \ll R_{o2}$ and $G_{mcm} = 1/2R_{SS}$, we have

$$|A_{cm}| = (1 - A_m)\frac{R_{om}}{2R_{SS}}$$

$$0.05 = (1 - A_m) \times \frac{20}{2 \times 20}$$

$$\Rightarrow (1 - A_m) = 0.1$$

9.108



$$G_m = g_{m1,2} \simeq \frac{I/2}{V_T}$$

$$5 = \frac{I/2}{V_T}$$

$\Rightarrow I = 0.25$ mA

Utilizing two matched transistors, Q_5 and Q_6, the value of R can be found from

$$I = \frac{0 - (-5) - 0.7}{R} = 0.25 \text{ mA}$$

$\Rightarrow R = 17.2$ kΩ

$$R_{id} = 2r_\pi = 2\frac{\beta}{g_m} = 2 \times \frac{100}{5} = 40 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4}$$

where

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{100}{0.125} = 800 \text{ k}\Omega$$

thus

$$R_o = 800 \text{ k}\Omega \parallel 800 \text{ k}\Omega = 400 \text{ k}\Omega$$

$$A_d = G_m R_o = 5 \times 400 = 2000 \text{ V/V}$$

$$I_B = \frac{I/2}{\beta + 1} \simeq \frac{0.125 \text{ mA}}{100} = 1.25 \ \mu\text{A}$$

The lower limit on V_{ICM} is determined by the lowest voltage allowed at the collector of Q_5 while Q_5 is in the active mode. This voltage is $-5 + 0.3 = -4.7$ V. Thus

$$V_{ICM\,\text{min}} = -4.7 + V_{BE1,2} = -4.7 + 0.7$$

$$= -4 \text{ V}$$

The upper limit on V_{ICM} is determined by the need to keep Q_1 in the active mode. Thus

$$V_{ICM\,\text{max}} = V_{C1} + 0.4$$

$$= 4.3 + 0.4 = 4.7 \text{ V}$$

Thus the input common-mode range is

$$-4 \text{ V} \le V_{ICM} \le +4.7 \text{ V}$$

The common-mode gain can be found using Eq. (9.165):

$$A_{cm} = -\frac{r_{o4}}{\beta_3 R_{EE}}$$

Here,

$$r_{o4} = \frac{|V_A|}{I/2} = \frac{100}{0.125} = 800 \text{ k}\Omega$$

$$\beta_3 = 100$$

$$R_{EE} = r_{o5} = \frac{|V_A|}{I} = \frac{100}{0.25} = 400 \text{ k}\Omega$$

Thus

$$A_{cm} = -\frac{800}{100 \times 400} = -0.02 \text{ V/V}$$

The CMRR can be found as

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{2000}{0.02} = 100{,}000$$

or 100 dB

9.109 See figure on next page.

From the solution to Problem 9.108, we know that $I = 0.25$ mA. For the Widlar current source, use $R = 2$ kΩ. Thus

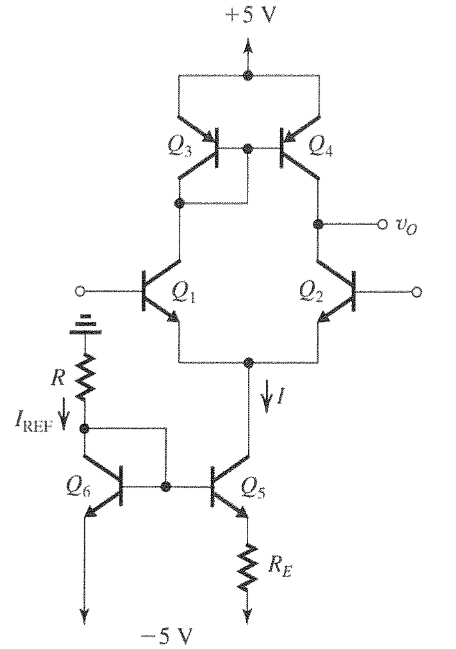
$$I_{\text{REF}} = \frac{5 - 0.7}{2} = 2.15 \text{ mA}$$

The value of R_E can be found from

$$IR_E = V_{BE6} - V_{BE5} = V_T \ln\left(\frac{I_{\text{REF}}}{I}\right)$$

$$0.25 \times R_E = 0.025 \ln\left(\frac{2.15}{0.25}\right)$$

$$R_E = 215 \ \Omega$$



The output resistance of the Widlar current source is given by Eq. (8.102). Thus

$$R_{EE} = [1 + g_{m5}(R_E \parallel r_{\pi 5})]r_{o5}$$

where

$$g_{m5} = \frac{I}{V_T} = \frac{0.25 \text{ mA}}{0.025 \text{ V}} = 10 \text{ mA/V}$$

$$r_{\pi 5} = \frac{\beta}{g_{m5}} = \frac{100}{10} = 10 \text{ k}\Omega$$

$$r_{o5} = \frac{V_A}{I} = \frac{100}{0.25} = 400 \text{ k}\Omega$$

$$R_{EE} = [1 + 10(0.215 \parallel 10)] \times 400$$

$$= 1.24 \text{ M}\Omega$$

R_{id}, R_o, A_d, I_B, and the range of V_{ICM} will be the same as in Problem 9.108. The common-mode gain, however, will be lower:

$$A_{cm} = -\frac{r_{o4}}{\beta_3 R_{EE}}$$

$$= -\frac{800}{100 \times 1240} = 6.45 \times 10^{-3} \text{ V/V}$$

and the CMRR will be

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{2000}{6.45 \times 10^{-3}} = 3.1 \times 10^5$$

or 110 dB

9.110 $G_m = g_{m1,2} \simeq \dfrac{I/2}{V_T} = \dfrac{0.2}{0.025} = 8 \text{ mA/V}$

$$R_o = r_{o2} \parallel r_{o4}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{40}{0.2} = 200 \text{ k}\Omega$$

$$R_o = 200 \text{ k}\Omega \parallel 200 \text{ k}\Omega = 100 \text{ k}\Omega$$

$$A_d = G_m R_o = 8 \times 100 = 800 \text{ V/V}$$

$$R_{id} = 2r_\pi = 2\beta/g_m$$

$$= \frac{300}{8} = 37.5 \text{ k}\Omega$$

$$R_{EE} = \frac{|V_A|}{I} = \frac{40}{0.4} = 100 \text{ k}\Omega$$

The common-mode gain can be found using Eq. (9.165):

$$A_{cm} = -\frac{r_{o4}}{\beta_3 R_{EE}}$$

$$= -\frac{200}{150 \times 100} = -0.013 \text{ V/V}$$

The CMRR can be obtained from

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{800}{0.013} = 60{,}000$$

or 96 dB

$$G_v = \frac{R_{id}}{R_{id} + R_{\text{sig}}} \times A_d$$

$$= \frac{37.5}{37.5 + 30} \times 800 = 444.4 \text{ V/V}$$

9.111 Refer to Fig. P9.111. To determine the bias current I, which is the current in the collector of Q_5, we first find the reference current through the 6.65-kΩ resistor:

$$I_{\text{REF}} = \frac{9 - (-5) - 0.7}{6.65} = 2 \text{ mA}$$

Assuming Q_5 and Q_6 are matched, we have

$$I = 2 \text{ mA}$$

(a) $g_{m1,2} \simeq \dfrac{I/2}{V_T} = \dfrac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$

$$R_{id} = 2r_\pi = 2\beta/g_{m1,2}$$

$$= \frac{2 \times 100}{40} = 5 \text{ k}\Omega$$

(b) $A_d = G_m R_o$

where

$$G_m = g_{m1,2} = 40 \text{ mA/V}$$

$$R_o = r_{o2} \parallel r_{o4}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{60}{1} = 60 \text{ k}\Omega$$

$$R_o = 60 \text{ k}\Omega \parallel 60 \text{ k}\Omega = 30 \text{ k}\Omega$$

$$A_d = 40 \times 30 = 1200 \text{ V/V}$$

(c) A_{cm} can be found using Eq. (9.165),

$$A_{cm} = -\frac{r_{o4}}{\beta_3 R_{EE}}$$

where

$$R_{EE} = r_{o5} = \frac{|V_A|}{I} = \frac{60}{2} = 30 \text{ k}\Omega$$

$$A_{cm} = -\frac{60}{100 \times 30} = -0.02 \text{ V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{1200}{0.02} = 60,000$$

or 95.6 dB

9.112 Refer to Fig. P9.112. To determine the bias current I, which is the drain current of Q_7, we analyze the Wilson mirror circuit as follows: All four transistors, $Q_5 - Q_8$, are conducting equal currents (I) and have the same V_{GS},

$$V_{GS} = V_t + V_{OV}$$

Thus

$$IR = 15 - (-5) - 2\ V_{GS}$$

$$144I = 20 - 2\ V_t - 2\ V_{OV}$$

But

$$I = \frac{1}{2}k_n'(W/L)V_{OV}^2$$

$$= \frac{1}{2} \times 2 \times V_{OV}^2 = V_{OV}^2$$

Thus

$$144\ V_{OV}^2 = 20 - 2 \times 0.7 - 2V_{OV}$$

$$144\ V_{OV}^2 + 2V_{OV} - 18.6 = 0$$

$$\Rightarrow V_{OV} = 0.35 \text{ V}$$

and

$$I = 0.35^2 = 0.12 \text{ mA}$$

(a) $R_{id} = 2r_\pi = 2\beta/g_m$

where

$$g_m = g_{m1,2} \simeq \frac{I/2}{V_T} = \frac{0.06}{0.025} = 2.4 \text{ mA}$$

$$R_{id} = \frac{2 \times 100}{2.4} = 83.3 \text{ k}\Omega$$

(b) $A_d = g_{m1,2}R_o$

where

$$R_o = r_{o2} \parallel r_{o4}$$

But

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{60}{0.06} = 1 \text{ M}\Omega$$

$$R_o = 500 \text{ k}\Omega$$

$$A_d = 2.4 \times 500 = 1200 \text{ V/V}$$

(c) A_{cm} can be found from Eq. (9.165):

$$A_{cm} = -\frac{r_{o4}}{\beta_3 R_{EE}}$$

where R_{EE} is the output resistance of the Wilson mirror,

$$R_{EE} = g_{m7}r_{o7}r_{o5}$$

where

$$g_{m7} = \frac{2I}{V_{OV}} = \frac{2 \times 0.12}{0.35}$$

$$= 0.7 \text{ mA/V}$$

$$r_{o7} = r_{o5} = \frac{|V_A|}{I} = \frac{60}{0.12} = 500 \text{ k}\Omega$$

$$R_{EE} = 0.7 \times 500^2 = 175 \text{ M}\Omega$$

$$A_{cm} = -\frac{1}{100 \times 175} = 5.7 \times 10^{-5} \text{ V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{1200}{5.7 \times 10^{-5}} = 21 \times 10^6$$

or 146 dB

9.113 Refer to Fig. 9.40.

W_6 can be determined using Eq. (9.172):

$$\frac{(W/L)_6}{(W/L)_4} = 2\ \frac{(W/L)_7}{(W/L)_5}$$

$$\frac{(W/0.5)_6}{(10/0.5)} = 2\ \frac{(60/0.5)}{(60/0.5)}$$

$$\Rightarrow W_6 = 20\ \mu\text{m}$$

For all devices we can evaluate I_D as follows:

$$I_{D8} = I_{\text{REF}} = 225\ \mu\text{A}$$

$$I_{D5} = I_{\text{REF}}\frac{(W/L)_5}{(W/L)_8} = I_{\text{REF}} = 225\ \mu\text{A}$$

$$I = I_{D5} = 225\ \mu\text{A}$$

$$I_{D1} = I_{D2} = \frac{1}{2}I_{D5} = 112.5\ \mu\text{A}$$

$$I_{D3} = I_{D4} = I_{D1} = 112.5\ \mu\text{A}$$

$$I_{D6} = I_{D7} = I_{\text{REF}} = 225\ \mu\text{A}$$

With I_D in each device known, we can use

$$I_{Di} = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)_i |V_{OVi}|^2$$

to determine $|V_{OVi}|$ and then

$$|V_{GSi}| = |V_{OVi}| + |V_t|$$

The values of g_{mi} and r_{oi} can then be determined from

$$g_{mi} = \frac{2I_{Di}}{|V_{OVi}|}$$

$$r_{oi} = \frac{|V_A|}{I_{Di}}$$

The results are summarized in the following table.

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
I_D (μA)	112.5	112.5	112.5	112.5	225	225	225	225
$\|V_{OV}\|$ (V)	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
$\|V_{GS}\|$ (V)	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
g_m (mA/V)	0.9	0.9	0.9	0.9	1.8	1.8	1.8	1.8
r_o (kΩ)	80	80	80	80	40	40	40	40

$$A_1 = -g_{m1}(r_{o2} \parallel r_{o4})$$

$$= -0.9 \times (80 \parallel 80) = -36 \text{ V/V}$$

$$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$$

$$= -1.8 \times (40 \parallel 40) = -36 \text{ V/V}$$

$$A_0 = A_1 A_2 = -36 \times -36 = 1296 \text{ V/V}$$

The upper limit of V_{ICM} is determined by the need to keep Q_5 in saturation, thus

$$V_{ICM\max} = V_{DD} - |V_{OV5}| - |V_{SG1}|$$

$$= 1.5 - 0.25 - 1 = +0.25 \text{ V}$$

The lower limit of V_{ICM} is determined by the need to keep Q_1 and Q_2 in saturation, thus

$$V_{ICM\min} = V_{G3} - |V_t|$$

$$= -V_{SS} + |V_{GS3}| - |V_t|$$

$$= -1.5 + 1 - 0.75 = -1.25 \text{ V}$$

Thus

$$-1.25 \text{ V} \le V_{ICM} \le +0.25 \text{ V}$$

The output voltage range is

$$-V_{SS} + V_{OV6} \le v_O \le V_{DD} - |V_{OV7}|$$

that is,

$$-1.25 \text{ V} \le v_O \le +1.25 \text{ V}$$

9.114

(a) $I_{D1} = I_{D2} = 100 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2} V_{OV}^2$

$$100 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{1,2} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 12.5$$

$$I_{D3} = I_{D4} = 100 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$100 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 50$$

$$I_{D5} = I_{D7} = I_{D8} = 200 \text{ μA}$$

Thus

$$200 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{5,7,8} V_{OV}^2$$

$$= \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{5,7,8} \times 0.04$$

$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8 = 25$$

$$I_{D6} = 200 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_6 |V_{OV}|^2$$

$$200 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_6 \times 0.04$$

$$\left(\frac{W}{L}\right)_6 = 100$$

The results are summarized in the following table:

Transistor	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
W/L	12.5	12.5	50	50	25	100	25	25

Ideally, the dc voltage at the output is zero.

(b) The upper limit of V_{ICM} is determined by the need to keep Q_1 and Q_2 in saturation, thus

$$V_{ICM\max} = V_{D1} + V_t$$

$$= V_{DD} - |V_{SG4}| + V_t$$

$$= 0.9 - |V_t| - |V_{OV4}| + V_t$$

$$= 0.9 - 0.2 = +0.7 \text{ V}$$

The lower limit of V_{ICM} is determined by the need to keep Q_5 in saturation,

$$V_{ICM\min} = -0.9 + |V_{OV5}| + |V_{GS1}|$$

$$= -0.9 + 0.2 + 0.2 + 0.4 = -0.1 \text{ V}$$

Thus

$$-0.1 \text{ V} \le V_{ICM} \le +0.7 \text{ V}$$

(c) $v_{O\max} = V_{DD} - |V_{OV6}|$

$$= 0.9 - 0.2 = +0.7 \text{ V}$$

$v_{O\min} = -V_{SS} + |V_{OV7}|$

$= -0.9 + 0.2 = -0.7$ V

Thus

$-0.7 \text{ V} \le v_O \le +0.7 \text{ V}$

(d) $A_1 = -g_{m1,2}(r_{o2} \parallel r_{o4})$

where

$$g_{m1,2} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{0.1 \text{ mA}} = \frac{6}{0.1} = 60 \text{ k}\Omega$$

$A_1 = -1 \times (60 \parallel 60) = -30$ V/V

$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$

where

$$g_{m6} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_{o6} = r_{o7} = \frac{|V_A|}{0.2} = \frac{6}{0.2} = 30 \text{ k}\Omega$$

$A_2 = -2 \times (30 \parallel 30) = -30$ V/V

$A_0 = A_1 A_2 = 30 \times 30 = 900$ V/V

9.115 (a) Increasing $(W/L)_1$ and $(W/L)_2$ by a factor of 4 reduces $|V_{OV1,2}|$ by a factor of 2. Thus $g_{m1,2} = 2I_D/|V_{OV1,2}|$ increase by a factor of 2.

(b) A_1 is proportional to $g_{m1,2}$, thus A_1 increases by a factor of 2 and the overall voltage gain increases by a factor of 2.

(c) Since the input offset voltage is proportional to $|V_{OV1,2}|$, it will decrease by a factor of 2. This, however, does not apply to V_{OS} due to ΔV_t.

9.116 If $(W/L)_7$ becomes 48/0.8, I_{D7} will become

$$I_{D7} = I_{D8} \frac{(W/L)_7}{(W/L)_8}$$

$$= I_{\text{REF}} \frac{(48/0.8)}{(40/0.8)}$$

$= 90 \times 1.2 = 108$ μA

Thus I_{D7} will exceed I_{D6} by 18 μA, which will result in a systematic offset voltage,

$V_O = 18 \text{ μA}(r_{o6} \parallel r_{o7})$

where

$r_{o6} = 111$ kΩ

and r_{o7} now becomes

$$r_{o7} = \frac{10}{0.108} = 92.6 \text{ k}\Omega$$

Thus

$V_O = 18 \times 10^{-3} \times (111 \parallel 92.6)$

$= 909$ mV

The corresponding input offset voltage will be

$$V_{OS} = \frac{V_O}{A_0}$$

$$= \frac{909}{1109} = 0.82 \text{ mV}$$

9.117 Refer to Fig. 9.40 and let the two input terminals be grounded. Then,

$$I_{D1} = I_{D2} = \frac{I}{2}$$

If Q_3 has a threshold voltage V_t and Q_4 has a threshold voltage $V_t + \Delta V_t$ then

$$I_{D3} = \frac{I}{2} = \frac{1}{2} k_{n3}(V_{GS3} - V_t)^2$$

$$\Rightarrow V_{GS3} = V_t + \sqrt{I/k_{n3}}$$

$$I_{D4} = \frac{1}{2} k_{n4}(V_{GS4} - V_t - \Delta V_t)^2$$

Since $k_{n4} = k_{n3}$ and $V_{GS4} = V_{GS3}$, we have

$$I_{D4} = \frac{I}{2} k_{n3}(V_{GS3} - V_t - \Delta V_t)^2$$

$$= \frac{1}{2} k_{n3}(\sqrt{I/k_{n3}} - \Delta V_t)^2$$

$$= \frac{1}{2} k_{n3} \frac{I}{k_{n3}} \left(1 - \frac{\Delta V_t}{\sqrt{I/k_{n3}}}\right)^2$$

$$= \frac{I}{2}\left(1 - \frac{\Delta V_t}{V_{OV3}}\right)^2$$

The output current of the first stage will be

$I_O = I_{D2} - I_{D4}$

$$= \frac{I}{2} - \frac{I}{2}\left(1 - \frac{\Delta V_t}{V_{OV3}}\right)^2$$

For $\dfrac{\Delta V_t}{V_{OV3}} \ll 1$ we obtain

$$I_O \simeq \frac{I}{2} - \frac{I}{2}\left(1 - \frac{2\Delta V_t}{V_{OV3}}\right)$$

$$= \frac{2(I/2)}{V_{OV3}} \Delta V_t$$

$= g_{m3}\, \Delta V_t$ Q.E.D.

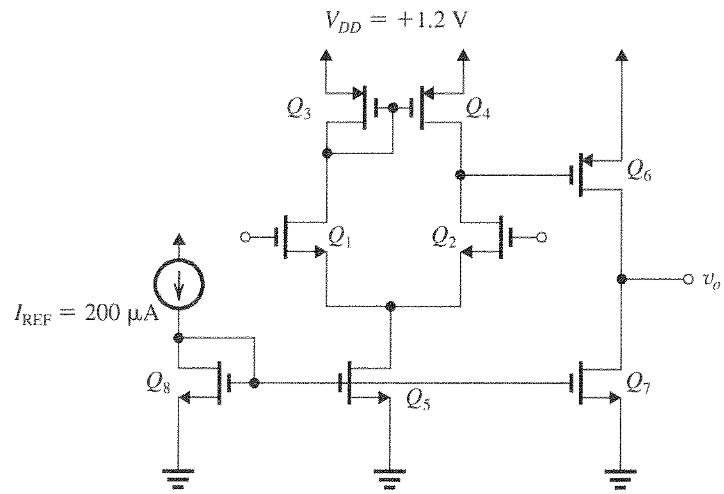
The corresponding input offset voltage will be

$$V_{OS} = \frac{I_O}{g_{m1,2}}$$

$$= \frac{g_{m3}\,\Delta V_t}{g_{m1,2}}$$

$$V_{OS} = \frac{g_{m3}}{g_{m1,2}} \Delta V_t$$

9.118



(a) With the two input terminals connected to a dc voltage of $V_{DD}/2 = +0.6$ V and for $Q_1 - Q_4$ to conduct a current of 200 μA, we have

$$I_{D1,2} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{1,2} V_{OV}^2$$

$$200 = \frac{1}{2} \times 540 \times \left(\frac{W}{L}\right)_{1,2} \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{1,2} = 32.9$$

$$I_{D3,4} = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$200 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 178$$

Transistor Q_5 must carry a current of 400 μA, thus

$$400 = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_5 V_{OV}^2$$

$$= \frac{1}{2} \times 540 \times \left(\frac{W}{L}\right)_5 \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = 65.8$$

Similarly, Q_7 is required to conduct a current of 400 μA, thus

$$\left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_5 = 65.8$$

Transistor Q_8 conducts a current of 200 μA, thus

$$\left(\frac{W}{L}\right)_8 = \frac{1}{2}\left(\frac{W}{L}\right)_5 = 32.9$$

Finally, Q_6 must conduct a current equal to that of Q_7, that is, 400 μA, thus

$$400 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_6 \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_6 = 356$$

The results are summarized in the following table:

Transistor	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
I_D (μA)	200	200	200	200	400	400	400	200
W/L	32.9	32.9	178	178	65.8	356	65.8	32.9

(b) The upper limit on V_{ICM} is determined by the need to keep Q_1 and Q_2 in saturation, thus

$$V_{ICM\max} = V_{D1,2} + |V_t|$$

$$= V_{DD} - |V_t| - |V_{OV}| + |V_t|$$

$$= 1.2 - 0.15 = 1.05 \text{ V}$$

The lower limit on V_{ICM} is determined by the need to keep Q_5 in saturation, thus

$$V_{ICM} = |V_{OV5}| + V_{GS1,2}$$

$$= 0.15 + 0.15 + 0.35 = 0.65 \text{ V}$$

Thus

$$0.65 \text{ V} \le V_{ICM} \le 1.05 \text{ V}$$

Note that the input dc voltage in part (a) falls outside the allowable range of V_{ICM}! Thus, part (a) should have specified a V_{ICM} greater than 0.65 V. The results of part (a), however, will not change.

(c) $0.15 \text{ V} \le v_O \le (1.2 - 0.15)$

that is,

$$0.15 \text{ V} \le v_O \le 1.05 \text{ V}$$

(d) $g_{m1,2} = \frac{2 \times 0.2}{0.15} = 2.67$ mA/V

$r_{o2} = r_{o4} = \frac{|V_A|}{0.2 \text{ mA}} = \frac{1.8}{0.2} = 9 \text{ k}\Omega$

$A_1 = -g_{m1,2}(r_{o2} \parallel r_{o4}) = 2.67(9 \parallel 9)$

$= -12$ V/V

$g_{m6} = \frac{2 \times 0.4}{0.15} = 5.33$ mA/V

$r_{o6} = r_{o7} = \frac{|V_A|}{0.4 \text{ mA}} = \frac{1.8}{0.4} = 4.5 \text{ k}\Omega$

$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$

$= -5.33(4.5 \parallel 4.5) = 12$ V/V

$A_0 = A_1 A_2 = -12 \times -12 = 144$ V/V

9.119 Refer to Fig. P9.119.

(a) With the inputs grounded and the output at 0 V dc, we have

$I_{E1} = I_{E2} = \frac{1}{2} \times 0.4 = 0.2$ mA

$I_{E3} = I_{E4} \simeq 0.2$ mA

$I_{E5} \simeq 0.5$ mA

$I_{E6} = 1$ mA

(b) The short-circuit transconductance of the first stage is

$$G_m = g_{m1,2} = \frac{I_{C1,2}}{V_T} \simeq \frac{0.2 \text{ mA}}{0.025 \text{ V}} = 8 \text{ mA/V}$$

The voltage gain of the first stage can be obtained by multiplying G_m by the total resistance at the output node of the stage, i.e., the common collectors of Q_2 and Q_4 and the base of Q_5. Since $r_{o2} = r_{o4} = \infty$, the resistance at this node is equal to the input resistance of Q_5 which is $R_{\pi 5}$,

$$r_{\pi 5} = \frac{\beta}{g_{m5}}$$

where

$$g_{m5} = \frac{I_{C5}}{V_T} = \frac{0.5}{0.025} = 20 \text{ mA/V}$$

thus

$$r_{\pi 5} = \frac{100}{20} = 5 \text{ k}\Omega$$

Thus the voltage gain of the first stage is given by

$$A_1 \equiv \frac{v_{b5}}{v_{id}} = -G_m r_{\pi 5}$$

$= -8 \times 5 = -40$ V/V

The voltage gain of the second stage is

$$A_2 \equiv \frac{v_{c5}}{v_{b5}} = -g_{m5} R_C$$

where R_{C5} is the total resistance in the collector of Q_5. Since $r_{o5} = \infty$, R_{C5} is simply the input resistance of the emitter follower Q_6, we have

$$R_{C5} = R_{i6} = (\beta + 1)(r_{e6} + R_L)$$

where

$$r_{e6} = \frac{V_T}{I_{E6}} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \ \Omega$$

$R_{i6} = (100 + 1)(0.025 + 1)$

$= 103.5 \text{ k}\Omega$

Thus

$A_2 = -20 \times 103.5 = -2070$ V/V

The gain of the third stage is given by

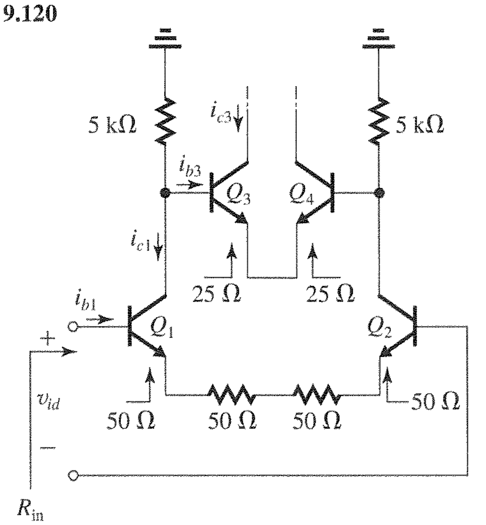
$$A_3 = \frac{v_o}{v_5} = \frac{R_L}{R_L + r_{e6}} = \frac{1}{1 + 0.025} = 0.976 \text{ V/V}$$

The overall voltage gain can now be obtained as

$$A_0 \equiv \frac{v_o}{v_{id}} = A_1 A_2 A_3$$

$= -40 \times -2070 \times 0.976 = 8.07 \times 10^4$ V/V

9.120



$R_{in2} = 2(\beta + 1)(25 + 25)$

$= 2 \times 101 \times 50 \simeq 10 \text{ k}\Omega$

Effective load of first stage $= R_{in2} \parallel (5 + 5)$

$= 10 \parallel 10 = 5 \text{ k}\Omega$

$A_1 =$

$$\alpha \frac{\text{Total resistance between collectors of } Q_1 \text{ and } Q_2}{\text{Total resistance in emitters of } Q_1 \text{ and } Q_2}$$

$$\simeq \frac{5 \text{ k}\Omega}{4 \times 50 \ \Omega} = 25 \text{ V/V}$$

$R_{in} = (\beta + 1)(4 \times 50 \ \Omega)$

$= 101 \times 200 \simeq 20 \text{ k}\Omega$

$$\frac{i_{c1}}{i_{b1}} = \beta_1 = 100$$

$$\frac{i_{b3}}{i_{c1}} = \frac{(5+5)}{(5+5)+R_{\text{in}2}} = \frac{10}{10+10} = 0.5$$

$$\frac{i_{c3}}{i_{b3}} = \beta_3 = 100$$

Thus

$$\frac{i_{c3}}{i_{b1}} = \frac{i_{c3}}{i_{b3}} \times \frac{i_{b3}}{i_{c1}} \times \frac{i_{c1}}{i_{b1}} = 100 \times 0.5 \times 100$$

$$= 5000 \text{ A/A}$$

9.121 Refer to Fig. 9.41. From Example 9.7, we obtain

$$I_{C1} = I_{C2} = 0.25 \text{ mA}$$

$$I_{C4} = I_{C5} = 1 \text{ mA}$$

$$I_{C7} = 1 \text{ mA}$$

$$I_{C8} = 5 \text{ mA}$$

Thus

$$r_{e1} = r_{e2} \simeq \frac{25 \text{ mV}}{0.25 \text{ mA}} = 100 \ \Omega$$

$$r_{e4} = r_{e5} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \ \Omega$$

With 100-Ω resistance in the emitter of each of Q_1 and Q_2, we have

$$R_{id} = (\beta+1)(2r_{e1,2} + 2R_{e1,2})$$

$$= 101 \times (2 \times 0.1 + 2 \times 0.1)$$

$$= 40.4 \text{ k}\Omega$$

Thus, R_{id} increases by a factor of 2. With 25-Ω resistance in the emitter of each of Q_4 and Q_5, the input resistance of the second stage becomes

$$R_{i2} = (\beta+1)(2r_{e4,5} + 2R_{e4,5})$$

$$= 101 \ (2 \times 0.025 + 2 \times 0.025)$$

$$= 10.1 \text{ k}\Omega$$

Thus, R_{i2} is increased by a factor of 2. The gain of the first stage will be

$$\frac{v_{o1}}{v_{id}} = \frac{\alpha \times \text{Total resistance between the collectors of } Q_1 \text{ and } Q_2}{\text{Total resistance in emitters of } Q_1 \text{ and } Q_2}$$

$$\simeq \frac{40 \text{ k}\Omega \parallel 10 \text{ k}\Omega}{2 \times 0.1 + 2 \times 0.1} = 20 \text{ V/V}$$

Thus the gain of the first stage decreases but only slightly. Of course, the two 100-Ω resistances in the emitters reduce the gain but some of the reduction is mitigated by the increase in R_{i2}, which increases the effective load resistance of the first stage.

The gain of the second stage will now be

$$A_2 = \frac{v_{o2}}{v_{o1}} = -\alpha \frac{3 \text{ k}\Omega \parallel R_{i3}}{2 \times 0.025 + 2 \times 0.025}$$

From Example 9.8, $R_{i3} = 234.8$ kΩ, thus

$$A_2 \simeq -\frac{3 \parallel 234.8}{0.1} = -29.6 \text{ V/V}$$

which is about half the value without the two 25-Ω emitter resistances. The gain of the third stage remains unchanged at −6.42 V/V, and the gain of the fourth stage remains unchanged at 1 V/V. Thus the overall voltage gain becomes

$$\frac{v_o}{v_{id}} = A_1 A_2 A_3 A_4$$

$$= 20 \times -29.6 \times -6.42 \times 1$$

$$= 3800.6 \text{ V/V}$$

which is less than half the gain obtained without the emitter resistances. This is the price paid for doubling R_{id}.

9.122 The output resistance is mostly determined by R_5. To reduce R_o by a factor of 2, we use

$$R_o = \frac{152}{2} = R_6 \parallel \left[r_{e6} + \frac{R_5}{\beta+1}\right]$$

$$76 = 3000 \parallel \left[5 + \frac{R_5}{101}\right]$$

$$\Rightarrow R_5 = 7.37 \text{ k}\Omega$$

This change in R_5 will affect the gain of the third stage, which will now become

$$A_3 = -\frac{R_5 \parallel (\beta+1)(r_{e8} + R_6)}{R_4 + r_{e7}}$$

$$= -\frac{7.37 \parallel (101)(0.005+3)}{2.3 + 0.025}$$

$$= -3.1 \text{ V/V}$$

which is about half the original value (not surprising since R_5 is about half its original value). To restore the gain of the third stage to its original value, we can reduce R_4. This will, however, change R_{i3} and will reduce the gain of the second stage, though only slightly. For instance, to restore the gain of the third stage to −6.42 V/V, we use

$$\frac{2.3 + 0.025}{R_4 + 0.025} = \frac{6.42}{3.1}$$

$$\Rightarrow R_4 = 1.085 \text{ k}\Omega$$

Now $R_{i3} = 101 \times (1.085 + 0.025) = 112$ kΩ and the gain of the second stage becomes

$$A_2 = -\frac{3 \text{ k}\Omega \parallel 112 \text{ k}\Omega}{30 \ \Omega} = -58.4 \text{ V/V}$$

which is a slight decrease in magnitude from the original value of −59.2 V/V.

9.123 Refer to Fig. 9.41(a). With R_5 replaced with a 1-mA constant-current source with a high output resistance, the total resistance in the collector of Q_7 now becomes the input resistance of Q_8, which is

$$R_{i4} = (\beta + 1)(r_{e8} + R_6)$$

$$= 101 \times (0.005 + 3) = 303.5\ \text{k}\Omega$$

Thus the gain of the third stage now becomes

$$A_3 = -\alpha \frac{303.5}{2.3 + 0.025}$$

$$\simeq -130.5\ \text{V/V}$$

and the overall voltage gain increases to

$$\frac{v_o}{v_{id}} = 8513 \times \frac{130.5}{6.42} = 1.73 \times 10^5\ \text{V/V}$$

(b) The output resistance now becomes

$$R_o = 3\ \text{k}\Omega \parallel \left(r_{e8} + \frac{\text{very large resistance}}{\beta + 1} \right)$$

$$\simeq 3\ \text{k}\Omega$$

When the amplifier is loaded with $R_L = 100\ \Omega$,

$$G_v = 1.73 \times 10^5 \frac{R_L}{R_L + R_o} =$$

$$1.73 \times 10^5 \times \frac{100}{3000 + 100}$$

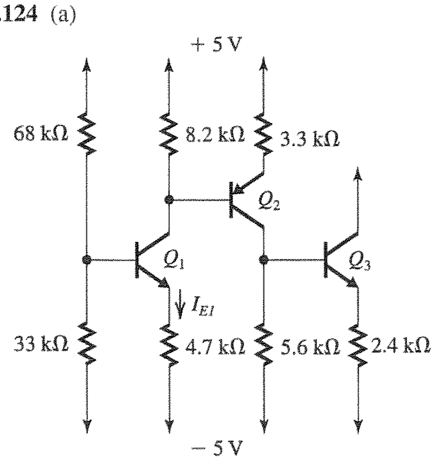
$$G_v = 5581\ \text{V/V}$$

If the original amplifier is loaded in $R_L = 100\ \Omega$,

$$G_v = 8513 \times \frac{100}{152 + 100} = 3378\ \text{V/V}$$

Thus, although the output resistance of the original amplifier is much lower than that of the modified one, the overall voltage gain realized when the original amplifier is loaded in 100-Ω resistance is much lower than that obtained with the modified design. Thus, replacing the 15.7-kΩ resistance with a constant-current source is an excellent modification to make!

9.124 (a)



(a)

Refer to Fig. (a) for the dc analysis. Replacing the 68 kΩ-33 kΩ divider network by its Thévenin equivalent, we obtain

$$V_{BB} = -5\ \text{V} + \frac{33}{33 + 68} \times 10\ \text{V}$$

$$= -1.73\ \text{V}$$

$$R_{BB} = 68\ \text{k}\Omega \parallel 33\ \text{k}\Omega = 22.2\ \text{k}\Omega$$

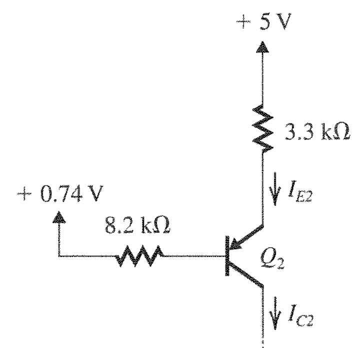
Now, we can determine I_{E1} from

$$I_{E1} = \frac{V_{BB} - (-5) - 0.7}{4.7 + \dfrac{R_{BB}}{\beta + 1}}$$

$$= \frac{-1.73 + 5 - 0.7}{4.7 + \dfrac{22.2}{101}} = 0.52\ \text{mA}$$

$$I_{C1} = \alpha_1 \times 0.52 = 0.99 \times 0.52 \simeq 0.52\ \text{mA}$$

The collector current I_{C1} and the 8.2-kΩ resistor it feeds can be replaced by a Thévenin equivalent as shown in Fig. (b). Thus



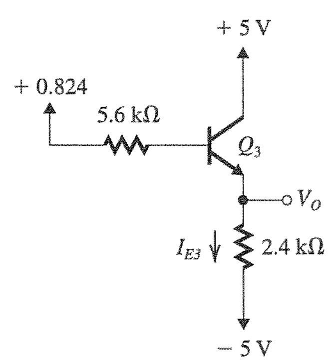
(b)

$$I_{E2} = \frac{5 - 0.74 - 0.7}{3.3 + \dfrac{8.2}{101}}$$

$$= 1.05\ \text{mA}$$

$$I_{C2} \simeq 1.04\ \text{mA}$$

The collector current I_{C2} and the 5.6-kΩ resistance it feeds can be replaced by a Thévenin equivalent as shown in Fig. (c). Thus



(c)

$$I_{E3} = \frac{0.824 - 0.7 - (-5)}{2.4 + \frac{5.6}{101}}$$

$$= 2.1 \text{ mA}$$

$$V_O = -5 + 2.1 \times 2.4 = 0 \text{ V}$$

(b) $R_{\text{in}} = 68 \text{ k}\Omega \parallel 33 \text{ k}\Omega \parallel r_{\pi 1}$

where

$$r_{\pi 1} = \frac{\beta}{g_{m1}}$$

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.52}{0.025} = 20.8 \text{ mA/V}$$

$$r_{\pi 1} = \frac{100}{20.8} = 4.81 \text{ k}\Omega$$

$$R_{\text{in}} = 68 \text{ k}\Omega \parallel 33 \parallel 4.81 \simeq 4 \text{ k}\Omega$$

$$R_{\text{out}} = 2.4 \text{ k}\Omega \parallel \left(r_{e3} + \frac{5.6 \text{ k}\Omega}{\beta + 1}\right)$$

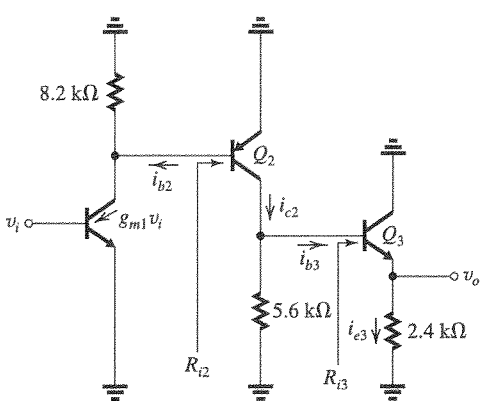
where

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{25 \text{ mV}}{2.1 \text{ mA}} = 11.9 \text{ }\Omega$$

$$R_{\text{out}} = 2.4 \parallel \left(0.0119 + \frac{5.6}{101}\right)$$

$$= 65.5 \text{ }\Omega$$

(c) Refer to Fig. (d)



(d)

$$i_{c1} = g_{m1} v_i = 20.8 v_i$$

$$R_{i2} = r_{\pi 2} = \frac{\beta}{g_{m2}}$$

where

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{1.04 \text{ mA}}{0.025 \text{ V}} = 41.6 \text{ mA}$$

$$r_{\pi 2} = \frac{100}{41.6} = 2.4 \text{ k}\Omega$$

$$i_{b2} = g_{m1} v_i \frac{8.2}{8.2 + 2.4} = 16.1 v_i$$

$$i_{c2} = \beta_2 i_{b2} = 100 \times 16.1 v_i = 1610 v_i$$

$$R_{i3} = (\beta + 1)(r_{e3} + 2.4 \text{ k}\Omega)$$

$$= 101(0.0119 + 2.4) = 243.6 \text{ k}\Omega$$

$$i_{b3} = i_{c2} \times \frac{5.6}{5.6 + 243.6} = 0.0225 i_{c2}$$

$$= 0.0225 \times 1610 v_i = 36.18 v_i$$

$$i_{e3} = (\beta + 1) i_{b3}$$

$$= 101 \times 36.18 = 3654 v_i$$

$$v_o = i_{e3} \times 2.4 \text{ k}\Omega$$

$$= 3654 \times 2.4 v_i = 8770 v_i$$

Thus

$$\frac{v_o}{v_i} = 8770 \text{ V/V}$$

9.125 From the figure we observe that the controlled source $g_{m1}\, v_{gs1}$ can be replaced by a resistance $1/g_{m1}$, thus

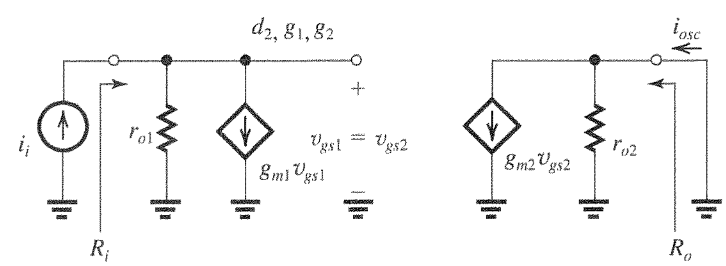
$$v_{gs1} = i_i \left(r_{o1} \parallel \frac{1}{g_{m1}}\right)$$

$$R_i \equiv \frac{v_{gs1}}{i_i} = \frac{1}{g_{m1}} \parallel r_{o1}$$

$$i_{osc} = g_{m2} v_{gs2} = g_{m2} v_{gs1} = g_{m2} \left(\frac{1}{g_{m1}} \parallel r_{o1}\right) i_i$$

$$A_{is} \equiv \frac{i_{osc}}{i_i} = g_{m2} \frac{\frac{1}{g_{m1}} r_{o1}}{\frac{1}{g_{m1}} + r_{o1}}$$

This figure belongs to Problem 9.125.



$$= \frac{g_{m2}}{g_{m1}} \frac{1}{1 + \frac{1}{g_{m1} r_{o1}}}$$

Since $g_{m1} r_o \gg 1$,

$$A_{is} \simeq \frac{g_{m2}}{g_{m1}} \left(1 - \frac{1}{g_{m1} r_{o1}}\right)$$

$$= A_{is}|_{\text{ideal}} \left(1 - \frac{1}{g_{m1} r_{o1}}\right)$$

where

$$A_{is}|_{\text{ideal}} = \frac{g_{m2}}{g_{m1}}$$

Finally, from inspection,

$$R_o = r_{o2}$$

9.126 (a) Refer to Fig. P9.126. The current I_D in each of the eight transistors can be found by inspection. Then, g_m of each transistor can be determined as $2I_D/|V_{OV}|$ and r_o as $|V_A|/I_D$. The results are given in the table below:

(b) See figure below. Observe that at the output node the total signal current is $4i_d$ where

$$i_d = g_{m1,2} \frac{v_{id}}{2}$$

$$= \frac{I}{2|V_{OV}|} v_{id}$$

and since the output resistance is

$$R_o = r_{o6} \parallel r_{o8} = \frac{1}{2} \frac{|V_A|}{I}$$

then

$$v_o = 4\, i_d R_o = 4 \times \frac{I}{2|V_{OV}|} \times \frac{1}{2} \frac{|V_A|}{I} \times v_{id}$$

Thus

$$A_d \equiv \frac{v_o}{v_{id}} = \frac{|V_A|}{|V_{OV}|} \qquad \text{Q.E.D.}$$

(c) See figure on next page. With v_{icm} applied to both input terminals, we can replace each of Q_1 and Q_2 with an equivalent circuit composed of a controlled current, $v_{icm}/2R_{SS}$ in parallel with a very large output resistance (R_{o1} and R_{o2} which are equal). The resistances R_{o1} and R_{o2} will be much larger than the input resistance of each of the mirrors $Q_3 - Q_5$ and $Q_4 - Q_6$ and thus we can neglect R_{o1} and R_{o2} altogether. The short-circuit output current of the $Q_4 - Q_6$ mirror will be

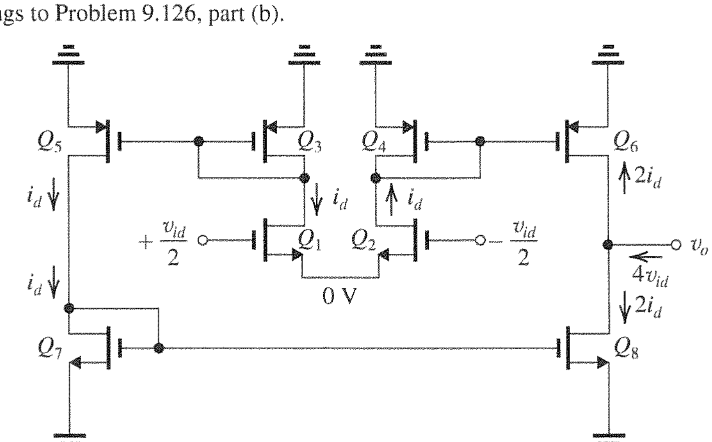
$$i_{o6} = \frac{g_{m6}}{g_{m4}} \left(1 - \frac{1}{g_{m4} r_{o4}}\right) \frac{v_{icm}}{2R_{SS}}$$

$$= \left(1 - \frac{|V_{OV}|}{2|V_A|}\right) \left(\frac{v_{icm}}{R_{SS}}\right)$$

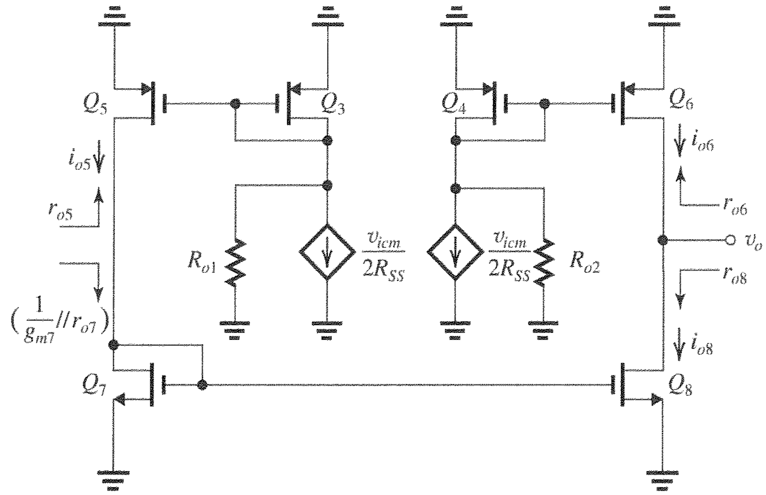
This table belongs to Problem 9.126.

Transistor	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
I_D	$\frac{I}{2}$	$\frac{I}{2}$	$\frac{I}{2}$	$\frac{I}{2}$	$\frac{I}{2}$	I	$\frac{I}{2}$	I
g_m	$\frac{I}{\|V_{OV}\|}$	$\frac{I}{\|V_{OV}\|}$	$\frac{I}{\|V_{OV}\|}$	$\frac{I}{\|V_{OV}\|}$	$\frac{I}{\|V_{OV}\|}$	$\frac{2I}{\|V_{OV}\|}$	$\frac{I}{\|V_{OV}\|}$	$\frac{2I}{\|V_{OV}\|}$
r_o	$\frac{2\|V_A\|}{I}$	$\frac{2\|V_A\|}{I}$	$\frac{2\|V_A\|}{I}$	$\frac{2\|V_A\|}{I}$	$\frac{2\|V_A\|}{I}$	$\frac{\|V_A\|}{I}$	$\frac{2\|V_A\|}{I}$	$\frac{\|V_A\|}{I}$

This figure belongs to Problem 9.126, part (b).



This figure belongs to Problem 9.126, part (c).



and the output resistance will be r_{o6}. The short-circuit output current of the $Q_3 - Q_5$ mirror will be

$$i_{o5} = \frac{g_{m5}}{g_{m3}}\left(1 - \frac{1}{g_{m3}r_{o3}}\right)\frac{v_{icm}}{2R_{SS}}$$

$$= \left(1 - \frac{|V_{OV}|}{2|V_A|}\right)\left(\frac{v_{icm}}{2R_{SS}}\right)$$

and the output resistance will be r_{o5}. Since r_{o5} is much larger than the input resistance of the $Q_7 - Q_8$ mirror ($\simeq 1/g_{m7}$), most of i_{o5} will flow into Q_7, resulting in an output short-circuit current i_{o8}:

$$i_{o8} = \frac{g_{m8}}{g_{m7}}\left(1 - \frac{1}{g_{m7}r_{o7}}\right)i_{o5}$$

$$= 2\left(1 - \frac{1}{g_{m7}r_{o7}}\right)i_{o5}$$

$$= \left(1 - \frac{|V_{OV}|}{2|V_A|}\right)\left(1 - \frac{1}{g_{m7}r_{o7}}\right)\frac{v_{icm}}{R_{SS}}$$

and the output resistance is r_{o8}. Thus, at the output node we have a net current

$$i_{o6} - i_{o8} = \left(1 - \frac{|V_{OV}|}{2|V_A|}\right)\left(\frac{1}{g_{m7}r_{o7}}\right)\left(\frac{v_{icm}}{R_{SS}}\right)$$

$$\simeq \left(\frac{1}{g_{m7}r_{o7}}\right)\left(\frac{v_{icm}}{R_{SS}}\right)$$

This current flows into the output resistance ($r_{o6} \parallel r_{o8}$) and thus produces an output voltage

$$v_o = \frac{r_{o6} \parallel r_{o8}}{R_{SS}}\,\frac{1}{g_{m7}r_{o7}}v_{icm}$$

and the common-mode gain becomes

$$|A_{cm}| = \frac{r_{o6} \parallel r_{o8}}{R_{SS}}\,\frac{1}{g_{m7}r_{o7}} \qquad \text{Q.E.D.}$$

(d) $R_{SS} = \dfrac{|V_A|}{I}$

$$A_d = |\frac{V_A}{V_{OV}}|$$

$$|A_{cm}| = \frac{\frac{1}{2}|V_A|/I}{|V_A|/I}\;\frac{1}{[I/|V_{OV}|\,][2|V_A|/I]}$$

$$|A_{cm}| = \frac{1}{2} \times \frac{1}{2}\,\frac{|V_{OV}|}{|V_A|} = \frac{1}{4}\left|\frac{V_{OV}}{V_A}\right|$$

$$\text{CMRR} = 4\left|\frac{V_A}{V_{OV}}\right|^2 \qquad \text{Q.E.D.}$$

(e) The upper limit on V_{ICM} is determined by Q_1 and Q_2 remaining in saturation, thus

$$V_{ICM_{\max}} = V_{DD} - |V_{SG}| + |V_t|$$

$$= V_{DD} - |V_{OV}|$$

The lower limit on V_{ICM} is determined by the need to keep the bias current source in saturation, i.e. maintaining a minimum voltage across it of $|V_{OV}|$, thus

$$V_{ICM_{\min}} = -V_{SS} + |V_{OV}| + |V_{GS}|$$

$$= -V_{SS} + 2|V_{OV}| + |V_t|$$

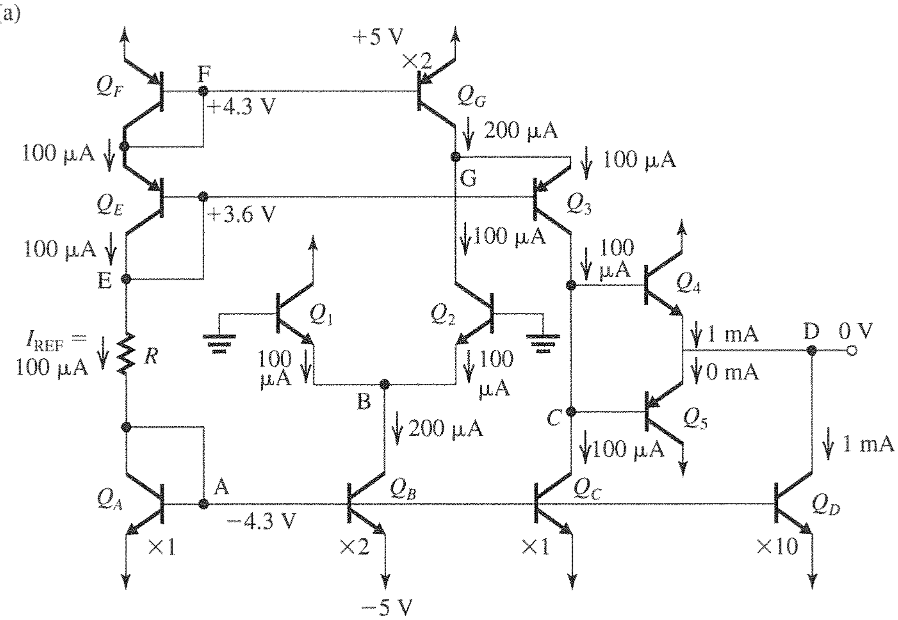
Thus

$$-V_{SS} + |V_t| + 2|V_{OV}| \le V_{ICM} \le V_{DD} - |V_{OV}|$$

The output linear range is

$$V_{DD} - |V_{OV}| \le v_O \le -V_{SS} + |V_{OV}|$$

9.127 (a)



(a)

$$R = \frac{+3.6 - (-4.3)}{0.1\text{ mA}} = 79\text{ k}\Omega$$

See Figure (a) for the dc analysis.

$V_A = -4.3\text{ V}\quad V_B = -0.7\text{ V}\quad V_C = +0.7\text{ V}$

$V_D = 0\text{ V}\quad V_E = +3.6\text{ V}\quad V_F = +4.3\text{ V}$

$V_G = +4.3\text{ V}$

(b) See table below. Results are obtained from Fig. (a) and

$$g_m = \frac{I_C}{V_T}$$

$$r_o = \frac{|V_A|}{I_C}$$

Transistor	I_C (mA)	g_m (mA/V)	r_o (MΩ)
Q_1	0.1	4	2
Q_2	0.1	4	2
Q_3	0.1	4	2
Q_4	1.0	40	0.2
Q_5	0	0	∞
Q_A	0.1		
Q_B	0.2		
Q_C	0.1		2
Q_D	1.0		0.2
Q_E	0.1		
Q_F	0.1		
Q_G	0.2		1

(c) See figure (b) on next page. Total resistance at the collector of Q_3 is

$$\beta_3 r_{o3} \parallel r_{oC} \parallel \beta_4(r_{oD} \parallel r_{o4})$$

$$= (100 \times 2) \parallel 2 \parallel 100 \times (0.2 \parallel 0.2)$$

$$= 200 \parallel 2 \parallel 10 = 1.65\text{ M}\Omega$$

$$\frac{v_{c3}}{v_{id}} = \frac{1}{2} \times g_{m1,2} \times 1.65\text{ M}\Omega$$

$$= \frac{1}{2} \times 4 \times 1.65 \times 10^3 = 3300\text{ V/V}$$

$$\frac{v_o}{v_{c3}} \simeq 1$$

Thus

$$\frac{v_o}{v_{id}} = 3300\text{ V/V}$$

Observe that the polarity of the two input terminals are correct.

(d) $R_{id} = 2\, r_{\pi 1,2}$

$$= 2 \times \frac{\beta}{g_{m1,2}} = 2 \times \frac{100}{4} = 50\text{ k}\Omega$$

$$R_o = r_{o4} \parallel r_{oD} \parallel \left[r_{e4} + \frac{r_{oC} \parallel \beta r_{o3}}{\beta + 1}\right]$$

$$= 0.2 \parallel 0.2 \parallel \left[0.025 \times 10^{-3} + \frac{2 \parallel 200}{101}\right]$$

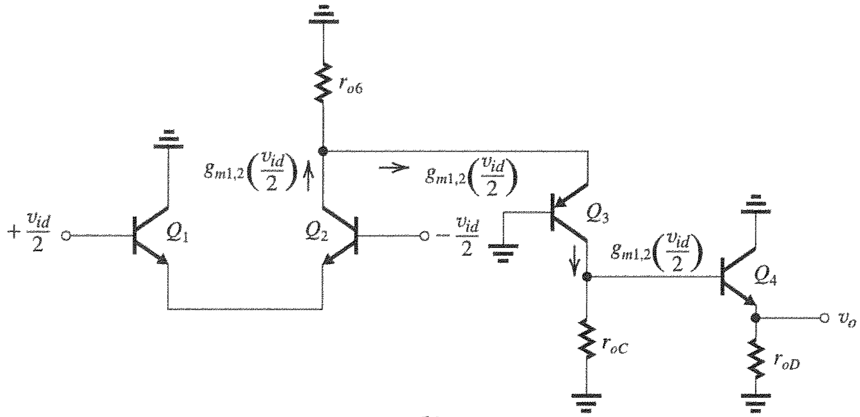
$$\simeq (0.2 \parallel 0.2 \parallel 0.02)\text{ M}\Omega$$

$$= 16.7\text{ k}\Omega$$

(e) $V_{ICM\max}$ is limited by Q_2 saturating,

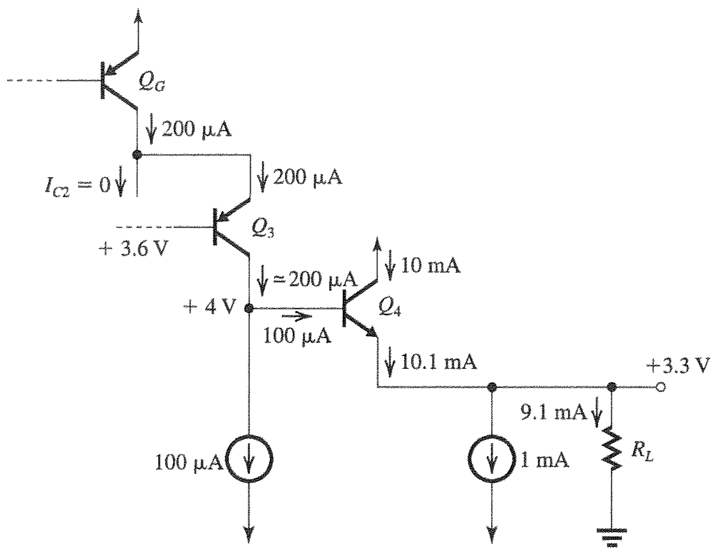
$$V_{ICM\max} = V_G + 0.4 = +4.7\text{ V}$$

This figure belongs to Problem 9.127, part (c).



(b)

This figure belongs to Problem 9.127, part (g).



(c)

$V_{ICM_{\min}}$ is limited by Q_B saturating,

$$V_{ICM_{\min}} = V_A - 0.4 + 0.7$$

$$= -4.3 - 0.4 + 0.7 = -4 \text{ V}$$

(f) The voltage at the base of Q_4 can rise to $(V_{B3} + 0.4)$ before Q_3 saturates, i.e. to $+3.6 + 0.4 = +4$ V. Thus v_O can go to $+4 - V_{BE4} = +3.3$ V. The output voltage can go down to the value that causes the voltage at C to be 0.4 V below the base voltage of Q_C. Thus

$$v_{O_{\min}} = -4.3 - 0.4 + V_{EB5} = -4 \text{ V}$$

Thus

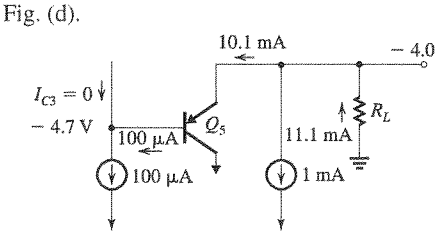
$$-4.0 \text{ V} \leq v_O \leq +3.3 \text{ V}$$

(g) With v_O at its maximum positive value of +3.3 V, and R_L small enough to cause Q_2 to cut off, the conditions in the circuit become as shown in Fig. (c).

The value of R_L can be found from

$$R_L = \frac{3.3}{9.1} = 363 \ \Omega$$

With v_O at its maximum negative value of −4 V and with R_L sufficiently small to cause Q_1 to cut off, Q_2 will conduct 200 μA which leaves Q_3 with zero current (cut off). Transistor Q_4 also cuts off, and the circuit conditions become as shown in Fig. (d).



(d)

Thus

$$R_L = \frac{4}{9.1} = 360 \ \Omega$$

9.128 DC analysis

(a) $I_{REF} = 10\ \mu\text{A} = \frac{1}{2} \times 40 \times \frac{5}{5}\left(V_{GS_A} - V_t\right)^2$

$\Rightarrow V_{GS_A} = 1.71\text{ V} \approx 1.7\text{ V}$

$10 = \frac{1}{2} \times 20 \times \frac{5}{5}\left(V_{GS_{EF}} - 1\right)^2$

$\Rightarrow V_{GS_{EF}} = 2\text{ V}$

$R = \frac{1-(-3.3)}{10\ \mu\text{A}} = 430\text{ k}\Omega$

(b) See figure (a) below.

$V_{GS1} = V_{GS2} = V_{GSA} \approx 1.7\text{ V}$

$V_{GS3} = \sqrt{\frac{2 \times 10}{20 \times \frac{10}{5}}} + 1 = 1.71\text{ V} \approx 1.7\text{ V}$

$V_{GS5} = V_{GS3} = 1.7\text{ V}$

For Q_6: $50 = \frac{1}{2} \times 40 \times \frac{50}{5}(V_{GS6} - V_t)^2$

$\Rightarrow V_{GS6} = 1.50\text{ V}$

$V_A = -3.3\text{ V}, \quad V_B = -1.7\text{ V}$

$V_C = +1.5\text{ V}, \quad V_D = 0\text{ V}$

$V_E = +1\text{ V}, \quad V_F = +3\text{ V}$

$V_G = +3.3\text{ V}, \quad V_H = +2.7\text{ V}$

(c)

Transistor	I_D (μA)	V_{GS} (V)	g_m (μA/V)	r_o (MΩ)
Q_1	10	1.7	28.3	5
Q_2	10	1.7	28.3	5
Q_3	10	1.7	28.3	5
Q_4	20	1.7	56.6	2.5
Q_5	10	1.7	28.3	5
Q_6	50	1.5	200	1
Q_7	0	−1.5*	0	∞
Q_A	10	1.7	28.3	5
Q_B	20	1.7	56.6	2.5
Q_C	10	1.7	28.3	5
Q_D	50	1.7	141.4	1
Q_E	10	2	20	5
Q_F	10	2	20	5

* Cut-off.

(d) Refer to Fig. (b). The total resistance at the source of the cascode transistor Q_5 is $(r_{o2} \parallel r_{o4})$. Thus the output resistance of the cascode transistor will be

$R_o = g_{m5} r_{o5}(r_{o2} \parallel r_{o4})$

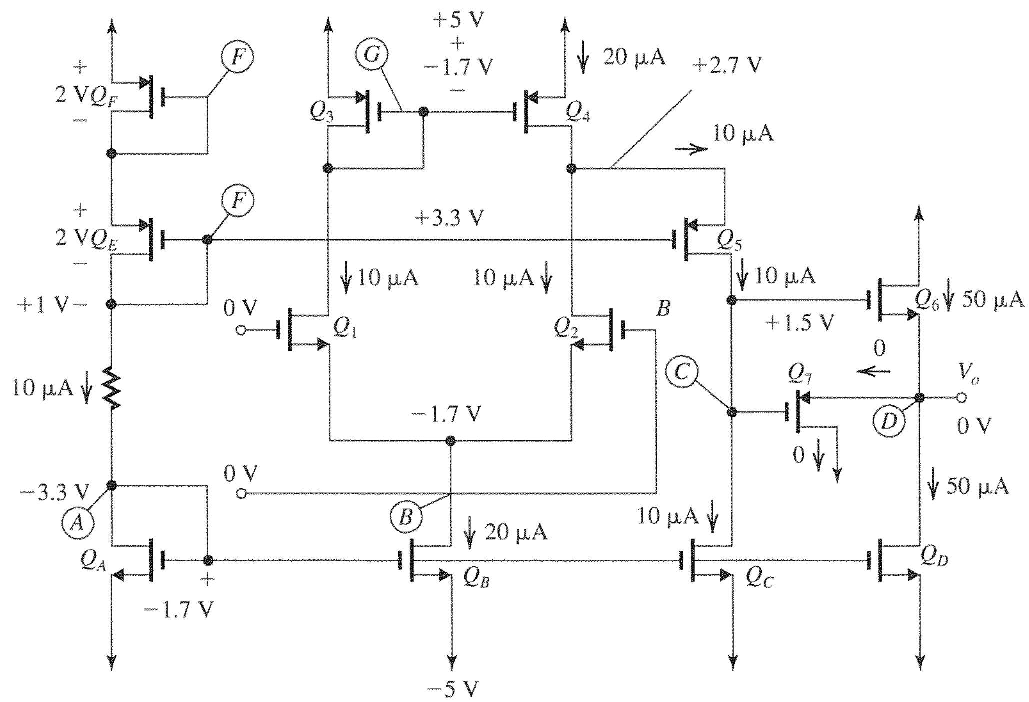
and the total resistance at the drain of Q_5 will be

$R_{\text{total}} = R_o \parallel r_{oC} = [g_{m5} r_{o5}(r_{o2} \parallel r_{o4})] \parallel r_{oC}$

$= [(28.3 \times 5)(5 \parallel 2.5)] \parallel 5$

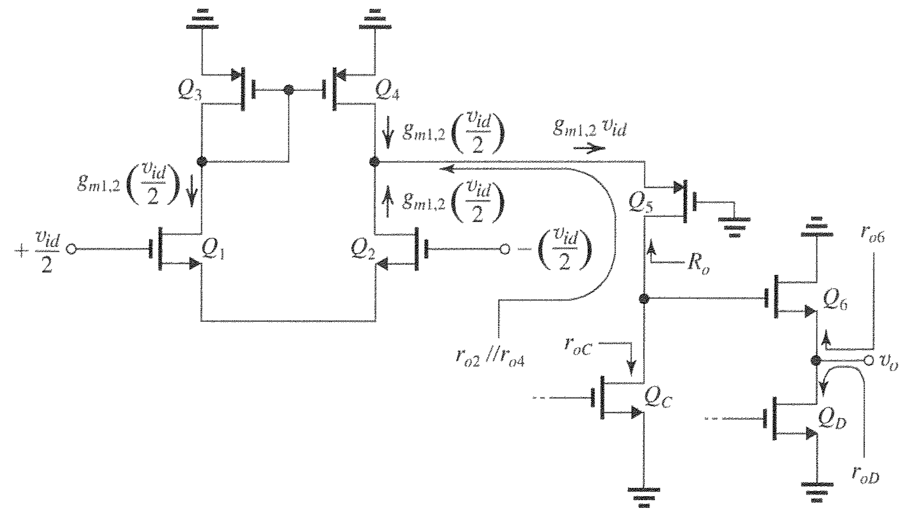
$= 4.9\text{ M}\Omega$

This figure belongs to Problem 9.128, part (a).



(a)

This figure belongs to Problem 9.128, part (d).



(b)

The voltage gain to the drain of Q_5 can be found as

$$\frac{v_{d5}}{v_{id}} = g_{m1,2}R_{\text{total}} = 28.3 \times 4.9 = 138.7 \text{ V/V}$$

The gain of the source-follower output stage is

$$\frac{v_o}{v_{d5}} = \frac{(r_{o6} \parallel r_{oD})}{(r_{o6} \parallel r_{oD}) + \dfrac{1}{g_{m6}}}$$

$$= \frac{1 \parallel 1}{(1 \parallel 1) + \dfrac{1}{200}} \simeq 1 \text{ V/V}$$

and the overall voltage gain is

$$\frac{v_o}{v_{id}} = 138.7 \text{ V/V}$$

$R_{\text{in}} = \infty$

$$R_o = r_{oD} \parallel r_{o6} \parallel \left(\frac{1}{g_{m6}}\right)$$

$$= 1 \parallel 1 \parallel \frac{1}{200}$$

$\simeq 5 \text{ k}\Omega$

(e) $V_{ICM_{\max}} = V_G + |V_t|$

$= 3.3 + 1 = +4.3 \text{ V}$

$V_{ICM_{\min}} = V_{B\min} + V_{GS1,2}$

$= V_A - |V_t| + V_{GS1,2}$

$= -3.3 - 1 + 1.7 = -2.6 \text{ V}$

Thus

$-2.6 \text{ V} \leq V_{ICM} \leq 4.3 \text{ V}$

(f) $v_{O_{\max}} = V_{C_{\max}} - V_{GS6}$

$= V_E + |V_t| - V_{GS6}$

$= 1 + 1 - 1.5 = 0.5 \text{ V}$

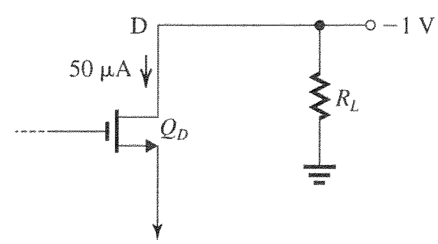
$v_{O\min} = V_A - |V_t|$

$= -3.3 - 1 = -4.3 \text{ V}$

Thus

$-4.3 \text{ V} \leq v_O \leq +0.5 \text{ V}$

(g) The circuit conditions are depicted in Fig. (c).



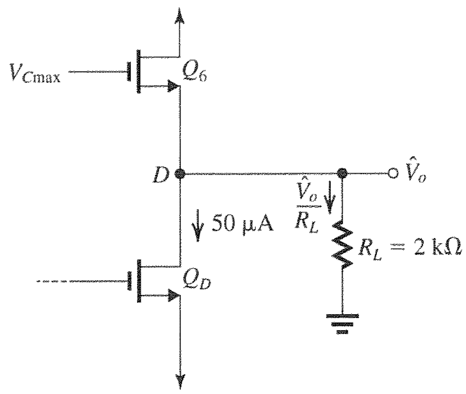
(c)

Observe that Q_6 is cut off and Q_7 has not yet conducted. Thus all the load current is sourced by Q_D. It follows that the maximum negative load current must be 50 μA

$$50 = \frac{1 \text{ V}}{R_L}$$

$\Rightarrow R_L = 20 \text{ k}\Omega$

(h) With $R_L = 2 \text{ k}\Omega$ and v_O is at its maximum allowable value (to be determined), the circuit conditions are as indicated in Fig. (d).



(d)

Here

$$V_{C_{\max}} = V_E + |V_t| = 2\ \text{V}$$

Now

$$I_{D6} = \frac{\hat{V}_o}{R_L} + 50\ \mu\text{A}$$

$$= \left(\frac{\hat{V}_o}{2} + 0.05\right)\ \text{mA}$$

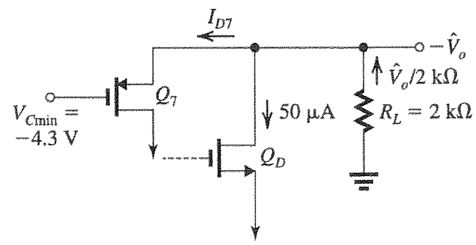
But

$$I_{D6} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_6 (V_{C_{\max}} - \hat{V}_o - |V_t|)^2$$

$$\frac{\hat{V}_o}{2} + 0.05 = \frac{1}{2} \times 40 \times 10 \times 10^{-3}\ (2 - \hat{V}_o - 1)^2$$

$$\Rightarrow \hat{V}_o = 0.17\ \text{V}$$

With $R_L = 2\ \text{k}\Omega$ and v_O is at its minimum allowable value (to be determined), the circuit conditions become as shown in Fig. (e).



(e)

Here Q_7 turns on and its current becomes

$$I_{D7} = \left(\frac{\hat{V}_o}{2} - 0.05\right)\ \text{mA}$$

But

$$I_{D7} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_7 (-\hat{V}_o - V_{C\min} - |V_t|)^2$$

Thus,

$$\frac{\hat{V}_o}{2} - 0.05 = \frac{1}{2} \times 20 \times 20 \times 10^{-3}(-\hat{V}_o + 4.3 - 1)^2$$

$$\Rightarrow \hat{V}_o = 1.45\ \text{V}$$

Thus

$$-1.45\ \text{V} \leq v_O \leq +0.17\ \text{V}$$

10.1 Refer to Fig. 10.3(b).

$$\frac{V_g}{V_{\text{sig}}} = \frac{R_G}{R_G + R_{\text{sig}} + \dfrac{1}{sC_{C1}}}$$

where

$R_G = R_{G1} \parallel R_{G2} = 2\text{ M}\Omega \parallel 1\text{ M}\Omega = 667\text{ k}\Omega$

and

$R_{\text{sig}} = 200\text{ k}\Omega$

$$\frac{V_g}{V_{\text{sig}}} = \frac{R_G}{R_G + R_{\text{sig}}} \frac{s}{s + \dfrac{1}{C_{C1}(R_G + R_{\text{sig}})}}$$

Thus,

$$f_{P1} = \frac{1}{2\pi C_{C1}(R_G + R_{\text{sig}})}$$

We required

$f_{P1} \leq 10\text{ Hz}$

thus we select C_{C1} so that

$$\frac{1}{2\pi C_{C1}(R_G + R_{\text{sig}})} \leq 10$$

$$C_{C1} \geq \frac{1}{2\pi \times 10 \times (667 + 200) \times 10^3} = 18.4\text{ nF}$$

$\Rightarrow C_{C1} = 20\text{ nF}$

10.2 Refer to Fig. 10.3(b).

$$V_o = -I_d \frac{R_D}{R_D + \dfrac{1}{sC_{C2}} + R_L} \times R_L$$

$$\frac{V_o}{I_d} = -\frac{R_D R_L}{R_D + R_L} \frac{s}{s + \dfrac{1}{C_{C2}(R_D + R_L)}}$$

$$f_{P3} = \frac{1}{2\pi C_{C2}(R_D + R_L)}$$

where

$R_D = 10\text{ k}\Omega$ and $R_L = 10\text{ k}\Omega$

To make $f_{P3} \leq 10$ Hz,

$$\frac{1}{2\pi C_{C2}(R_D + R_L)} \leq 10$$

$$\Rightarrow C_{C2} \geq \frac{1}{2\pi \times 10 \times (10 + 10) \times 10^3} = 0.8\ \mu\text{F}$$

Select, $C_{C2} = 0.8\ \mu$F.

10.3 Refer to Fig. 10.3(b).

$$I_s = \frac{V_g}{\dfrac{1}{g_m} + Z_S}$$

$$I_s = \frac{g_m V_g Y_S}{Y_S + g_m}$$

$$\frac{I_s}{V_g} = \frac{g_m\left(\dfrac{1}{R_S} + sC_S\right)}{g_m + \dfrac{1}{R_S} + sC_S}$$

$$= g_m \frac{s + 1/C_S R_S}{s + \dfrac{g_m + 1/R_S}{C_S}}$$

Thus,

$$f_{P2} = \frac{g_m + 1/R_S}{2\pi C_S}$$

$$f_Z = \frac{1}{2\pi C_S R_S}$$

where

$g_m = 5$ mA/V and $R_S = 1.8\text{ k}\Omega$

To make $f_{P2} \leq 100$ Hz,

$$\frac{g_m + 1/R_S}{2\pi C_S} \leq 100$$

$$\Rightarrow C_S \geq \frac{5 \times 10^{-3} + (1/1.8 \times 10^3)}{2\pi \times 100} = 8.8\ \mu\text{F}$$

Select $C_S = 10\ \mu$F.

Thus,

$$f_{P2} = \frac{5 \times 10^{-3} + (1/1.8 \times 10^3)}{2\pi \times 10 \times 10^{-6}} = 88.4\text{ Hz}$$

and

$$f_Z = \frac{1}{2\pi \times 10 \times 10^{-6} \times 1.8 \times 10^3} = 8.84\text{ Hz}$$

10.4 Refer to Fig. 10.3.

$$A_M = -\frac{R_G}{R_G + R_{\text{sig}}} \times g_m(R_D \parallel R_L)$$

where

$R_G = R_{G1} \parallel R_{G2} = 47\text{ M}\Omega \parallel 10\text{ M}\Omega$

$= 8.246\text{ M}\Omega$

$R_{\text{sig}} = 100\text{ k}\Omega,\ g_m = 5\text{ mA/V},\ R_D = 4.7\text{ k}\Omega$ and $R_L = 10\text{ k}\Omega$.

Thus,

$$A_M = -\frac{8.426}{8.426 + 0.1} \times 5(4.7 \parallel 10)$$

$= -15.8$ V/V

$$f_{P1} = \frac{1}{2\pi C_{C1}(R_G + R_{\text{sig}})}$$

$$= \frac{1}{2\pi \times 0.01 \times 10^{-6}(8.426 + 0.1) \times 10^6}$$

$= 1.9$ Hz

$$f_{P2} = \frac{g_m + 1/R_S}{2\pi C_S}$$

$$= \frac{5 \times 10^{-3} + 0.5 \times 10^{-3}}{2\pi \times 10 \times 10^{-6}} = 87.5 \text{ Hz}$$

$$f_Z = \frac{1}{2\pi C_S R_S}$$

$$= \frac{1}{2\pi \times 10 \times 10^{-6} \times 2 \times 10^3} = 8 \text{ Hz}$$

$$f_{P3} = \frac{1}{2\pi C_{C2}(R_D + R_L)}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-6}(4.7 + 10) \times 10^3} = 10.8 \text{ Hz}$$

Since

$f_{P2} \gg f_{P1}, f_{P3}, f_Z,$

$f_L \simeq f_{P2} = 87.5$ Hz

10.5

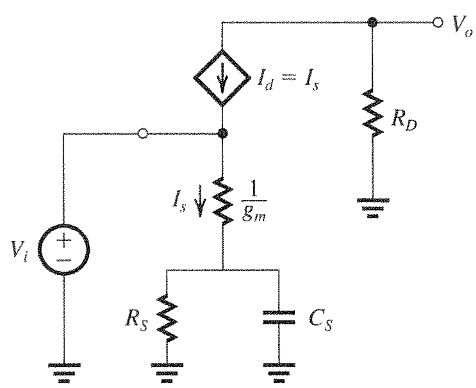


Figure 1

Replacing the MOSFET with its T model results in the circuit shown in Fig. 1.

(a) $A_M \equiv \dfrac{V_o}{V_i} = -g_m R_D$

$-20 = -2 \times R_D$

$\Rightarrow R_D = 10 \text{ k}\Omega$

(b) $f_P = \dfrac{g_m + 1/R_S}{2\pi C_S}$

$$100 = \frac{2 \times 10^{-3} + (1/4.5 \times 10^3)}{2\pi C_S}$$

$\Rightarrow C_S = 3.53\ \mu\text{F}$

(c) $f_Z = \dfrac{1}{2\pi C_S R_S} =$

$$\frac{1}{2\pi \times 3.53 \times 10^{-6} \times 4.5 \times 10^3} = 10 \text{ Hz}$$

(d) Since $f_P \gg f_Z$,

$f_L \simeq f_P = 100$ Hz

(e) The Bode plot for the gain is shown in Fig. 2.

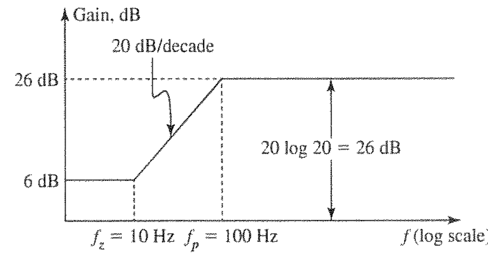


Figure 2

Observe that the dc gain is 6 dB, i.e. 2 V/V. This makes perfect sense since from Fig. 1 we see that at dc, capacitor C_S behaves as open circuit and the gain becomes

$$\text{DC gain} = -\frac{R_D}{\dfrac{1}{g_m} + R_S} = -\frac{10 \text{ k}\Omega}{\left(\dfrac{1}{2} + 4.5\right)}$$

$= -2$ V/V

10.6 See figure on next page. Replacing the MOSFET with its T model results in the circuit shown in the figure.

$$A_M = -\frac{R_G}{R_G + R_{\text{sig}}} \times g_m(R_D \parallel R_L)$$

$$= -\frac{2}{2 + 0.5} \times 3(20 \parallel 10)$$

$= -16$ V/V

To minimize the total capacitance we select C_S so as to place f_{P2} (usually the highest-frequency low-frequency pole) at 100 Hz. Thus,

$$100 = \frac{g_m}{2\pi C_S}$$

$$= \frac{3 \times 10^{-3}}{2\pi C_S}$$

$\Rightarrow C_S = 4.8\ \mu\text{F}$

Select $C_S = 5\ \mu\text{F}$.

Of the two remaining poles, the one caused by C_{C2} has associated relatively low-valued resistances (R_D and R_L are much lower than R_G), thus to minimize the total capacitance we place f_{P3} at 10 Hz and f_{P1} at 1 Hz. Thus,

$$10 = \frac{1}{2\pi C_{C2}(R_D + R_L)}$$

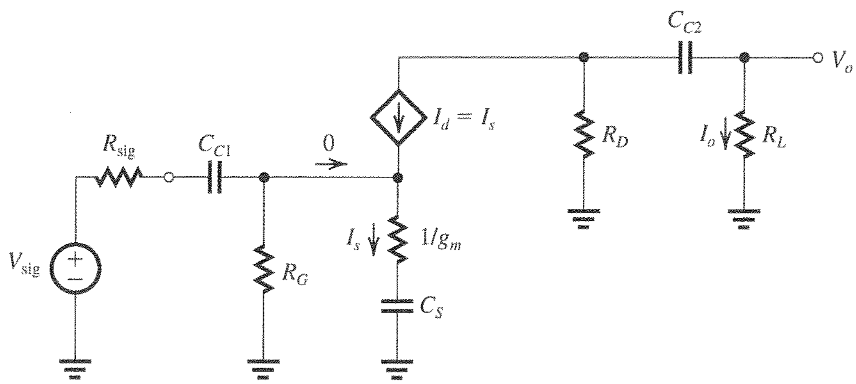
$$= \frac{1}{2\pi C_{C2}(20 + 10) \times 10^3}$$

$\Rightarrow C_{C2} = 0.53\ \mu\text{F}$

Select $C_{C2} = 1\ \mu\text{F}$.

$$1 = \frac{1}{2\pi C_{C1}(R_G + R_{\text{sig}})}$$

This figure belongs to Problem 10.6.



$$1 = \frac{1}{2\pi C_{C1}(2+0.5)\times 10^6}$$

$$\Rightarrow C_{C1} = 63.7 \text{ nF}$$

Select $C_{C1} = 100$ nF $= 0.1$ μF.

With the selected capacitor values, we obtain

$$f_{P1} = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times 2.5 \times 10^6} = 0.64 \text{ Hz}$$

$$f_{P2} = \frac{3 \times 10^{-3}}{2\pi \times 5 \times 10^{-6}} = 95.5 \text{ Hz}$$

$$f_Z = 0 \text{ (dc)}$$

$$f_{P3} = \frac{1}{2\pi \times 1 \times 10^{-6}(20+10)\times 10^3} = 5.3 \text{ Hz}$$

Since $f_{P2} \gg f_{P1}$ and f_{P3}, we have

$$f_L \simeq f_{P2} = 95.5 \text{ Hz}$$

10.7 The amplifier in Fig. P10.7 will have the equivalent circuit in Fig. 10.9 except with $R_E = \infty$ (i.e. omitted). Also, the equivalent circuits in Fig. 10.10 can be used to determine the three short-circuit time constants, again with $R_E = \infty$. Since the amplifier is operating at $I_C \simeq I_E = 100$ μA $= 0.1$ mA and $\beta = 100$,

$$r_e = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250\ \Omega$$

$$g_m = \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25 \text{ k}\Omega$$

Using the equivalent circuit in Fig. 10.10(b), we get

$$\tau_{CE} = C_E\left(r_e + \frac{R_B \parallel R_{sig}}{\beta+1}\right)$$

To make C_E responsible for 80% of f_L, we use

$$\frac{1}{\tau_{CE}} = 0.8\ \omega_L = 0.8 \times 2\pi f_L$$

$$\Rightarrow \tau_{CE} = \frac{1}{0.8 \times 2\pi \times 100} \simeq 2 \text{ ms}$$

Thus,

$$C_E\left[250 + \frac{(200 \parallel 20)\times 10^3}{101}\right] = 2 \times 10^{-3}$$

$$\Rightarrow C_E = 4.65\ \mu\text{F}$$

Select, $C_E = 5$ μF.

Using the information in Fig. 10.10(a), we determine τ_{C1} as

$$\tau_{C1} = C_{C1}[(R_B \parallel r_\pi) + R_{sig}]$$

To make the contribution of C_{C1} to the determination of f_L equal to 10%, we use

$$\frac{1}{\tau_{C1}} = 0.1\omega_L = 0.1 \times 2\pi f_L$$

$$\Rightarrow \tau_{C1} = \frac{1}{0.1 \times 2\pi \times 100} = 15.92 \text{ ms}$$

Thus,

$$C_{C1}[(200 \parallel 25)\times 10^3 + 20 \times 10^3] = 15.92 \times 10^{-3}$$

$$\Rightarrow C_{C1} = 0.38\ \mu\text{F}$$

Select $C_{C1} = 0.5$ μF.

For C_{C2} we use the information in Fig. 10.10(c) to determine τ_{C2}:

$$\tau_{C2} = C_{C2}(R_C + R_L)$$

To make the contribution of C_{C2} to the determination of f_L equal to 10%, we use

$$\frac{1}{\tau_{C2}} = 0.1\omega_L = 0.1 \times 2\pi f_L$$

$$\Rightarrow \tau_{C2} = \frac{1}{0.1 \times 2\pi \times 100} = 15.92 \text{ ms}$$

Thus,

$C_{C2}(20 + 10) \times 10^3 = 15.92 \times 10^{-3}$

$\Rightarrow C_{C2} = 0.53\ \mu F$

Although, to be conservative we should select $C_{C2} = 1\ \mu F$; in this case we can select

$C_{C2} = 0.5\ \mu F$

because the required value is very close to 0.5 μF and because we have selected C_{C1} and C_E larger than the required values. The resulting f_L will be

$$f_L = \frac{1}{2\pi}\left[\frac{1}{\tau_{CE}} + \frac{1}{\tau_{C1}} + \frac{1}{\tau_{C2}}\right]$$

$$\tau_{CE} = 5 \times 10^{-6} \times \left[250 + \frac{(200 \parallel 20) \times 10^3}{101}\right]$$

$= 2.15$ ms

$\tau_{C1} = 0.5 \times 10^{-6}[(200 \parallel 25) \times 10^3 + 20 \times 10^3]$

$= 21.1$ ms

$\tau_{C2} = 0.5 \times 10^{-6}(20 + 10) \times 10^3 = 15$ ms

$$f_L = \frac{1}{2\pi}\left[\frac{10^3}{2.15} + \frac{10^3}{21.1} + \frac{10^3}{15}\right]$$

$= 92.2$ Hz

which is lower (hence more conservative) than the required value of 100 Hz.

$C_{\text{total}} = 5 + 0.5 + 0.5 = 6.0\ \mu F$

10.8 Refer to Fig. 10.9.

In the midband,

$R_{\text{in}} = R_{B1} \parallel R_{B2} \parallel r_\pi$

where

$R_{B1} = 33\ k\Omega,\ R_{B2} = 22\ k\Omega$

$$g_m = \frac{I_C}{V_T} = \frac{0.3\ \text{mA}}{0.025\ \text{V}} = 12\ \text{mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{25\ \text{mV}}{0.3\ \text{mA}} = 83.3\ \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{12} = 10\ k\Omega$$

Thus,

$R_{\text{in}} = 33 \parallel 22 \parallel 10 = 5.7\ k\Omega$

$$A_M = -\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m (R_C \parallel R_L)$$

where

$R_{\text{sig}} = 5\ k\Omega,\ R_C = 4.7\ k\Omega,$ and $R_L = 5.6\ k\Omega$

Thus,

$$A_M = -\frac{5.7}{5.7 + 5} \times 12(4.7 \parallel 5.6)$$

$= -16.3$ V/V

Using the method of short-circuit time constants and the information in Fig. 10.10, we obtain

$\tau_{C1} = C_{C1}[(R_B \parallel r_\pi) + R_{\text{sig}}]$

$= C_{C1}(R_{\text{in}} + R_{\text{sig}})$

$= 1 \times 10^{-6}(5.7 + 5) \times 10^3 = 10.7$ ms

$$\tau_{CE} = C_E\left[R_E \parallel \left(r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta + 1}\right)\right]$$

$$= 20 \times 10^{-6}\left[3.9 \times 10^3 \parallel \left(83.3 + \frac{(33 \parallel 22 \parallel 5) \times 10^3}{121}\right)\right]$$

$= 2.2$ ms

$\tau_{C2} = C_{C2}(R_C + R_L)$

$= 1 \times 10^{-6}(4.7 + 5.6) \times 10^3 = 10$ ms

$$f_L \simeq \frac{1}{2\pi}\left(\frac{1}{\tau_{C1}} + \frac{1}{\tau_{CE}} + \frac{1}{\tau_{C2}}\right)$$

$$= \frac{1}{2\pi}\left(\frac{1}{10.7 \times 10^{-3}} + \frac{1}{2.2 \times 10^{-3}} + \frac{1}{10.3 \times 10^{-3}}\right)$$

$= 102.7$ Hz

10.9 Refer to the data given in the statement for Problem 10.8.

$R_B = R_{B1} \parallel R_{B2} = 33\ k\Omega \parallel 22\ k\Omega = 13.2\ k\Omega$

$I_C \simeq I_E \simeq 0.3$ mA

$$g_m = \frac{I_C}{V_T} = \frac{0.3\ \text{mA}}{0.025\ \text{V}} = 12\ \text{mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{25\ \text{mV}}{0.3\ \text{mA}} = 83.3\ \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{12} = 10\ k\Omega$$

From Fig. 10.10, we have

$\tau_{C1} = C_{C1}[(R_B \parallel r_\pi) + R_{\text{sig}}]$

For C_{C1} to contribute 10% of f_L, we use

$$\frac{1}{\tau_{C1}} = 0.1\omega_L = 0.1 \times 2\pi f_L$$

$= 0.1 \times 2\pi \times 50$

$\Rightarrow \tau_{C1} = 31.8$ ms

Thus,

$C_{C1}[(13.2 \parallel 10) + 5] \times 10^3 = 31.8 \times 10^{-3}$

$\Rightarrow C_{C1} = 3\ \mu F$

$\tau_{C2} = C_{C2}(R_C + R_L)$

For C_{C2} to contribute 10% of f_L, we use

$$\frac{1}{\tau_{C2}} = 0.1\omega_L = 0.1 \times 2\pi f_L$$

$= 0.1 \times 2\pi \times 50$

$\Rightarrow \tau_{C2} = 31.8$ ms

Thus,

$$C_{C2}(4.7 + 5.6) \times 10^3 = 31.8 \times 10^{-3}$$

$$\Rightarrow C_{C2} = 3.09\ \mu\text{F} \simeq 3\ \mu\text{F}$$

Finally,

$$\tau_{CE} = C_E\left[R_E \parallel \left(r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta + 1}\right)\right]$$

For C_E to contribute 80% of f_L, we use

$$\frac{1}{\tau_{CE}} = 0.8\omega_L = 0.8 \times 2\pi f_L$$

$$= 0.8 \times 2\pi \times 50$$

$$\Rightarrow \tau_{CE} = 3.98\ \text{ms}$$

Thus,

$$C_E\left[3900 \parallel \left(83.3 + \frac{(13.2 \parallel 5) \times 1000}{121}\right)\right]$$

$$= 3.98 \times 10^{-3}$$

$$\Rightarrow C_E = 36.2\ \mu\text{F}$$

10.10 Using the information in Fig. 10.10, we get

$$\tau_{C1} = C_{C1}[(R_B \parallel r_\pi) + R_{\text{sig}}]$$

$$= C_{C1}[(10 \parallel 1) + 5] \times 10^3$$

$$= C_{C1} \times 5.91 \times 10^3$$

$$\tau_{CE} = C_E\left[R_E \parallel \left(r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta + 1}\right)\right]$$

where

$$r_e = \frac{r_\pi}{\beta + 1} = \frac{1000}{101} \simeq 10\ \Omega$$

$$\tau_{CE} =$$

$$C_E\left[1.5 \times 10^3 \parallel \left(10 + \frac{(10 \parallel 5) \times 1000}{101}\right)\right]$$

$$= C_E \times 41.8$$

For C_{C1} and C_E to contribute equally to f_L,

$$\tau_{C1} = \tau_{CE}$$

Thus,

$$C_{C1} \times 5.91 \times 10^3 = C_E \times 41.8$$

$$\Rightarrow \frac{C_E}{C_{C1}} = 141.4$$

10.11 Replacing the BJT with its T model results in the equivalent circuit shown in Fig. 1 below.

(a) At midband, C_E and C_C act as short circuits. Thus

$$R_{\text{in}} = (\beta + 1)r_e$$

$$\frac{V_o}{V_{\text{sig}}} = -\frac{(\beta + 1)r_e}{(\beta + 1)r_e + R_{\text{sig}}}\, g_m(R_C \parallel R_L)$$

$$= -\frac{\beta(R_C \parallel R_L)}{(\beta + 1)r_e + R_{\text{sig}}}$$

(b) Because the controlled current source αI_e is ideal, it effectively separates the input circuit from the output circuit. The result is that the poles caused by C_E and C_C do not interact. The pole due to C_E will have frequency ω_{PE}:

$$\omega_{PE} = \frac{1}{C_E\left[r_e + \dfrac{R_{\text{sig}}}{\beta + 1}\right]}$$

and the pole due to C_C will have a frequency ω_{PC}

$$\omega_{PC} = \frac{1}{C_C(R_C + R_L)}$$

(c) The overall voltage transfer function can be expressed as

$$\frac{V_o}{V_{\text{sig}}} = A_M \frac{s}{s + \omega_{PE}}\, \frac{s}{s + \omega_{PC}}$$

This figure belongs to Problem 10.11, part (a).

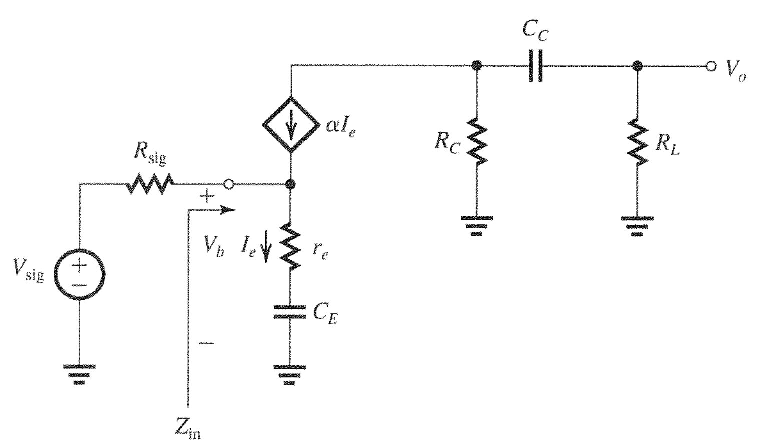


Figure 1

(d) $r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25\ \Omega$

$$A_M = -\frac{100(10\text{ k}\Omega \parallel 10\text{ k}\Omega)}{101 \times 25 \times 10^{-3}\text{ k}\Omega + 10\text{ k}\Omega}$$

$= -40$ V/V

(e) To minimize the total capacitance we choose to make the pole caused by C_E the dominant one and make its frequency equal to $f_L = 100$ Hz,

$$2\pi \times 100 = \frac{1}{C_E\left[25 + \frac{10,000}{101}\right]}$$

$\Rightarrow C_E = 12.83\ \mu$F

Placing the pole due to C_C at 10 Hz, we obtain

$$2\pi \times 10 = \frac{1}{C_C(10+10) \times 10^3}$$

$\Rightarrow C_C = 0.8\ \mu$F

(f) A Bode plot for the gain magnitude is shown in Fig. 2.

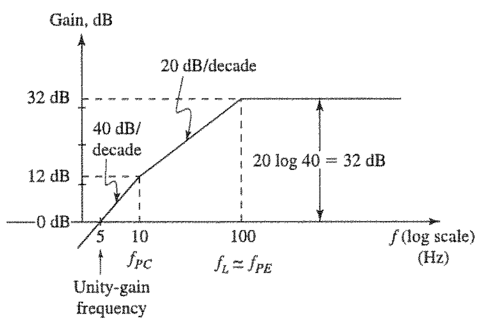


Figure 2

The gain at f_{P2} (10 Hz) is 12 dB. Since the gain decreases by 40 dB/decade or equivalently 12 dB/octave, it reaches 0 dB (unity magnitude) at $f = f_{PC}/2 = 5$ Hz.

10.12

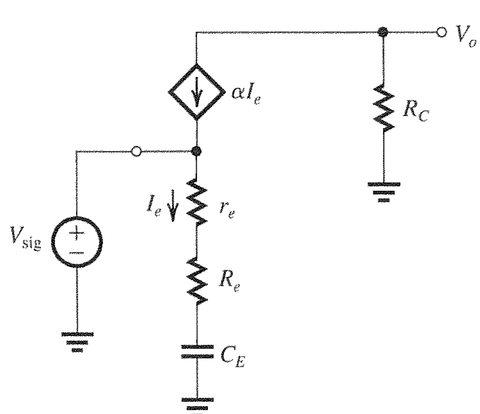


Figure 1

Replacing the BJT with its T model results in the circuit shown in Fig. 1.

(a) $I_e = \dfrac{V_{sig}}{r_e + R_e + \dfrac{1}{sC_E}}$

$V_o = -\alpha I_e R_C$

Thus,

$$\frac{V_o}{V_{sig}} = -\frac{\alpha R_C}{r_e + R_e}\ \frac{s}{s + \dfrac{1}{C_E(r_e+R_e)}} \qquad (1)$$

From this expression we obtain

$$A_M = -\frac{\alpha R_C}{r_e + R_e} \qquad (2)$$

and

$$f_L = f_P = \frac{1}{2\pi C_E(r_e+R_e)} \qquad (3)$$

(b) From Eq. (2) we see that

$$|A_M| = \frac{\alpha R_C}{r_e}\ \frac{1}{1 + \dfrac{R_e}{r_e}}$$

Thus, including R_e reduces the gain magnitude by the factor $\left(1 + \frac{R_e}{r_e}\right)$.

(c) From Eq. (3), we obtain

$$f_L = \frac{1}{2\pi C_E r_e}\ \frac{1}{1 + \dfrac{R_e}{r_e}}$$

Thus, including R_e reduces f_L by the factor $\left(1 + \frac{R_e}{r_e}\right)$. This is the same factor by which the magnitude of the gain is reduced. Thus, R_e can be used to tradeoff gain for decreasing f_L (that is, increasing the amplifier bandwidth).

(d) $I = 0.25$ mA, $\quad R_C = 10$ kΩ, $\quad C_E = 10\ \mu$F

$r_e = \frac{V_T}{I} = \frac{25 \text{ mV}}{0.25 \text{ mA}} = 100\ \Omega$

For $R_e = 0$:

$$|A_M| = \frac{\alpha R_C}{r_e} \simeq \frac{10\text{ k}\Omega}{100\ \Omega} = 100 \text{ V/V}$$

$$f_L = \frac{1}{2\pi \times 10 \times 10^{-6} \times 100} = 159.2 \text{ Hz}$$

To lower f_L by a factor of 10, we use

$$1 + \frac{R_e}{r_e} = 10$$

$\Rightarrow R_e = 900\ \Omega$

The gain now becomes

$$|A_M| = \frac{100}{1 + \dfrac{R_e}{r_e}} = \frac{100}{10} = 10 \text{ V/V}$$

See Fig. 2 for the Bode plot.

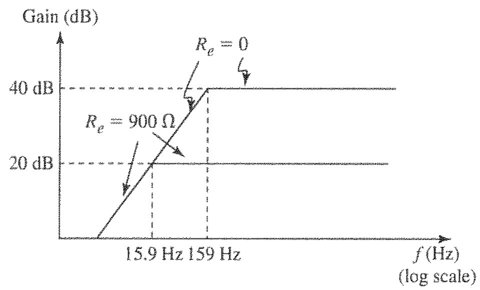


Figure 2

10.13 $C_{ox} = \dfrac{\epsilon_{ox}}{t_{ox}}$

$$= \frac{3.45 \times 10^{-11}\ \text{F/m}}{8 \times 10^{-9}\ \text{m}} = 0.43 \times 10^{-2}\ \text{F/m}^2$$

$$= 0.43 \times 10^{-2} \times 10^{-12}\ \text{F}/\mu\text{m}^2$$

$$= 4.3\ \text{fF}/\mu\text{m}^2$$

$$k_n' = \mu_n C_{ox}$$

$$= 450 \times 10^8\ (\mu\text{m}^2/\text{V}\cdot\text{s})$$

$$\times 4.3 \times 10^{-15}\ \text{F}/\mu\text{m}^2$$

$$= 193.5\ \mu\text{A/V}^2$$

$$I_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right) V_{OV}^2 (1 + \lambda V_{DS})$$

$$200 = \frac{1}{2} \times 193.5 \times 20 \times V_{OV}^2 (1 + 0.05 \times 1.5)$$

$$\Rightarrow V_{OV} = 0.31\ \text{V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.31} = 1.3\ \text{mA/V}$$

$$\chi = \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}}$$

$$= \frac{0.5}{2\sqrt{0.65 + 1}} = 0.19$$

$$g_{mb} = \chi g_m = 0.25\ \text{mA/V}$$

$$r_o = \frac{|V_A|}{I_D} = \frac{1}{|\lambda| I_D} = \frac{1}{0.05 \times 0.2} = 100\ \text{k}\Omega$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov}C_{ox}$$

$$= \frac{2}{3} \times 20 \times 1 \times 4.3 + 20 \times 0.05 \times 4.3$$

$$= 57.3 + 4.3 = 61.6\ \text{fF}$$

$$C_{gd} = WL_{ov}C_{ox} = 20 \times 0.05 \times 4.3$$

$$= 4.3\ \text{fF}$$

$$C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \dfrac{|V_{SB}|}{V_0}}}$$

$$= \frac{20}{\sqrt{1 + \dfrac{1}{0.7}}} = 12.8\ \text{fF}$$

$$C_{db} = \frac{C_{db0}}{\sqrt{1 + \dfrac{|V_{DB}|}{V_0}}}$$

$$= \frac{20}{\sqrt{1 + \dfrac{2.5}{0.7}}} = 9.4\ \text{fF}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$= \frac{1.3 \times 10^{-3}}{2\pi(61.6 + 4.3) \times 10^{-15}} = 3.1\ \text{GHz}$$

10.14 $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.2}{0.3} = 1.33\ \text{mA/V}$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$= \frac{1.33 \times 10^{-3}}{2\pi \times (25 + 5) \times 10^{-15}} = 7.1\ \text{GHz}$$

10.15 $f_T = \dfrac{g_m}{2\pi(C_{gs} + C_{gd})}$

For $C_{gs} \gg C_{gd}$

$$f_T \simeq \frac{g_m}{2\pi C_{gs}} \tag{1}$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov}C_{ox}$$

If the overlap component ($WL_{ov}C_{ox}$) is small, we get

$$C_{gs} \simeq \frac{2}{3} WLC_{ox} \tag{2}$$

The transconductance g_m is given by

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D} \tag{3}$$

Substituting from (2) and (3) into (1), we get

$$f_T = \frac{\sqrt{2\mu_n C_{ox} \left(\dfrac{W}{L}\right) I_D}}{2\pi \times \dfrac{2}{3} WLC_{ox}}$$

$$= \frac{1.5}{\pi L} \sqrt{\frac{\mu_n I_D}{2C_{ox}WL}} \qquad \text{Q.E.D.}$$

We observe that for a given device, f_T is proportional to $\sqrt{I_D}$; thus to obtain faster operation the MOSFET is operated at a higher I_D.

Also, we observe that f_T is inversely proportional to $L\sqrt{WL}$; thus faster operation is obtained from smaller devices.

10.16 $f_T = \dfrac{g_m}{2\pi(C_{gs} + C_{gd})}$

For $C_{gs} \gg C_{gd}$

$$f_T \simeq \frac{g_m}{2\pi C_{gs}} \quad (1)$$

$$C_{gs} = \frac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$$

If the overlap component is small, we get

$$C_{gs} \simeq \frac{2}{3}WLC_{ox} \quad (2)$$

The transconductance g_m can be expressed as

$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV} \quad (3)$$

Substituting from (2) and (3) into (1), we obtain

$$f_T = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}}{2\pi \times \frac{2}{3}WLC_{ox}}$$

$$= \frac{3\mu_n V_{OV}}{4\pi L^2}$$

We note that for a given channel length, f_T can be increased by operating the MOSFET at a higher V_{OV}.

For $L = 0.5$ μm and $\mu_n = 450$ cm²/V·s,

we have

$$V_{OV} = 0.2 \text{ V} \Rightarrow f_T = \frac{3 \times 450 \times 10^8 \times 0.2}{4\pi \times 0.5^2}$$

$= 5.73$ GHz

$$V_{OV} = 0.4 \text{ V} \Rightarrow f_T = \frac{3 \times 450 \times 10^8 \times 0.4}{4\pi \times 0.5^2}$$

$= 11.46$ GHz

10.17 $A_0 = \dfrac{2V_A}{V_{OV}} = \dfrac{2V_A'L}{V_{OV}}$

$$A_0 = \frac{2 \times 5 \times L}{0.2} = 50L, \text{ V/V } (L \text{ in } \mu\text{m})$$

$$f_T \simeq \frac{3\mu_n V_{OV}}{4\pi L^2} = \frac{3 \times 400 \times 10^8 \times 0.2}{4\pi L^2}$$

$$f_T = \frac{1.91}{L^2}, \text{ GHz } (L \text{ in } \mu\text{m})$$

The expressions for A_0 and f_T can be used to obtain their values for different values of L. The results are given in the following table.

L	L_{min} 0.13 μm	$2L_{min}$ 0.26 μm	$3L_{min}$ 0.39 μm	$4L_{min}$ 0.52 μm	$5L_{min}$ 0.65 μm
A_0 (V/V)	6.5	13	19.5	26	32.5
f_T (GHz)	113	28.3	12.6	7.1	4.5

10.18 $f_T = \dfrac{g_m}{2\pi(C_\pi + C_\mu)}$

where

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$C_\pi = 8$ pF

$C_\mu = 1$ pF

Thus,

$$f_T = \frac{20 \times 10^{-3}}{2\pi \times (8+1) \times 10^{-12}} = 353.7 \text{ MHz}$$

$$f_\beta = \frac{f_T}{\beta} = \frac{353.7}{100} = 3.54 \text{ MHz}$$

10.19 See figure on next page. $C_\pi = C_{de} + C_{je}$

where C_{de} is proportional to I_C.

At $I_C = 0.5$ mA,

$8 = C_{de} + 2 \Rightarrow C_{de} = 6$ pF

At $I_C = 0.25$ mA, $C_{de} = \dfrac{1}{2} \times 6 = 3$ pF, and $C_\pi = 3 + 2 = 5$ pF.

Also, at $I_C = 0.25$ mA, $g_m = 10$ mA/V. Thus f_T at $I_C = 0.25$ mA is

$$f_T = \frac{10 \times 10^{-3}}{2\pi(5+1) \times 10^{-12}} = 265.3 \text{ MHz}$$

10.20 $r_x = 100\ \Omega$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{50}{1} = 50 \text{ k}\Omega$$

$C_{de} = \tau_F g_m = 30 \times 10^{-12} \times 40 \times 10^{-3} = 1.2$ pF

$C_{je0} = 20$ pF

$C_\pi = C_{de} + 2C_{je0} = 1.2 + 2 \times 0.02 = 1.24$ pF

$$C_\mu = \frac{C_{je0}}{\left(1 + \frac{V_{CB}}{V_{0c}}\right)^m}$$

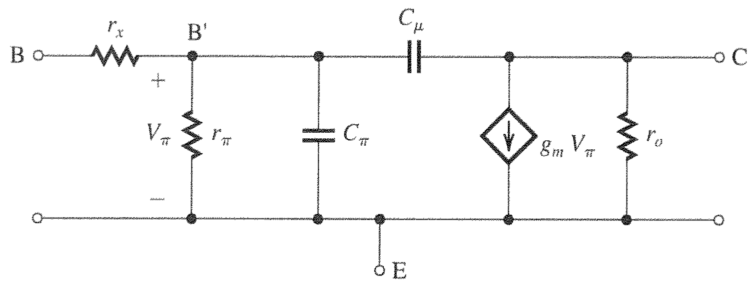
$$C_\mu = \frac{20}{\left(1 + \frac{2}{0.75}\right)^{0.5}} = 10.4 \text{ fF}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$= \frac{40 \times 10^{-3}}{2\pi(1.24 + 0.01) \times 10^{-12}}$$

$= 5.1$ GHz

This figure belongs to Problem 10.20.



10.21 For $f \gg f_\beta$,

$$|h_{fe}| = \frac{f_T}{f}$$

At $f = 50$ MHz and $I_C = 0.2$ mA,

$$|h_{fe}| = 10 = \frac{f_T}{50}$$

$$\Rightarrow f_T = 500 \text{ MHz}$$

At $f = 50$ MHz and $I_C = 1.0$ mA,

$$|h_{fe}| = 12 = \frac{f_T}{50}$$

$$\Rightarrow f_T = 600 \text{ MHz}$$

Now,

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

where

$$C_\pi = C_{de} + C_{je}$$

$$= \tau_F g_m + C_{je}$$

$$C_\mu = 0.1 \text{ pF}$$

At $I_C = 0.2$ mA, $g_m = \dfrac{0.2}{0.025} = 8$ mA/V, thus

$$500 \times 10^6 = \frac{8 \times 10^{-3}}{2\pi(C_\pi + 0.1) \times 10^{-12}}$$

$$\Rightarrow C_\pi = 2.45 \text{ pF}$$

$$\tau_F \times 8 \times 10^{-3} + C_{je} = 2.45 \times 10^{-12} \qquad (1)$$

At $I_C = 1$ mA, $g_m = \dfrac{1}{0.025} = 40$ mA/V, thus

$$600 \times 10^6 = \frac{40 \times 10^{-3}}{2\pi(C_\pi + 0.1) \times 10^{-12}}$$

$$\Rightarrow C_\pi = 10.51 \text{ pF}$$

$$\tau_F \times 40 \times 10^{-3} + C_{je} = 10.51 \times 10^{-12} \qquad (2)$$

Solving (1) together with (2) yields

$$\tau_F = 252 \text{ ps}$$

$$C_{je} = 0.43 \text{ pF}$$

10.22 $f_T = \dfrac{g_m}{2\pi(C_\pi + C_\mu)}$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$10 \times 10^9 = \frac{40 \times 10^{-3}}{2\pi(C_\pi + 0.1) \times 10^{-12}}$$

$$\Rightarrow C_\pi = 0.54 \text{ pF}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{40} = 3 \text{ k}\Omega$$

$$f_\beta = \frac{f_T}{\beta_0} = \frac{10 \times 10^9}{120} = 83.3 \text{ MHz}$$

10.23 For $f \gg f_\beta$,

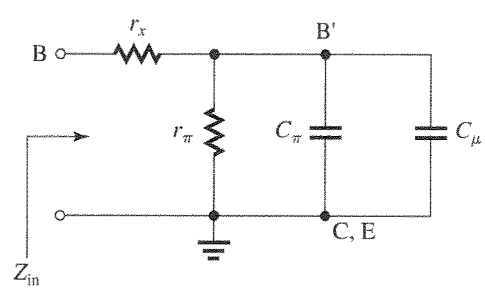
$$|h_{fe}| \simeq \frac{f_T}{f}$$

$$40 = \frac{2000 \text{ MHz}}{f}$$

$$\Rightarrow f = 50 \text{ MHz}$$

$$f_\beta = \frac{f_T}{\beta_0} = \frac{2000 \text{ MHz}}{200} = 10 \text{ MHz}$$

10.24



With the emitter and the collector grounded, the equivalent circuit takes the form shown in the figure, and the input impedance becomes

$$Z_{in} = r_x + \frac{1}{\dfrac{1}{r_\pi} + j\omega(C_\pi + C_\mu)}$$

$$= r_x + \frac{r_\pi}{1 + j\omega(C_\pi + C_\mu)r_\pi}$$

Since $\omega_\beta = \dfrac{1}{(C_\pi + C_\mu)r_\pi}$, then

$$Z_{\text{in}} = r_x + \frac{r_\pi}{1 + j\left(\dfrac{\omega}{\omega_\beta}\right)}$$

$$= r_x + r_\pi \frac{1 - j\left(\dfrac{\omega}{\omega_\beta}\right)}{1 + \left(\dfrac{\omega}{\omega_\beta}\right)^2}$$

$$R_e(Z_{\text{in}}) = r_x + \frac{r_\pi}{1 + \left(\dfrac{\omega}{\omega_\beta}\right)^2}$$

For the real part to be an estimate of r_x accurate to within 10%, we require

$$\frac{r_\pi}{1 + \left(\dfrac{\omega}{\omega_\beta}\right)^2} \leq 0.1 r_x$$

$$\frac{1}{1 + \left(\dfrac{\omega}{\omega_\beta}\right)^2} \leq 0.1\left(\frac{r_x}{r_\pi}\right)$$

But $r_x \leq \dfrac{r_\pi}{10}$, thus $\dfrac{r_x}{r_\pi} \leq 0.1$,

$$\frac{1}{1 + \left(\dfrac{\omega}{\omega_\beta}\right)^2} \leq 0.1 \times 0.1$$

or, equivalently,

$$1 + \left(\frac{\omega}{\omega_\beta}\right)^2 \geq 100$$

$$\Rightarrow \omega \geq 10\omega_\beta$$

10.25 To complete the table we use the following relationships:

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{I_E \text{ (mA)}}$$

$$g_m = \frac{I_C}{V_T} = \frac{\alpha I_E}{V_T} \simeq \frac{I_E}{V_T} = \frac{I_E \text{ (mA)}}{0.025 \text{ V}}$$

$$r_\pi = \frac{\beta_0}{g_m \text{ (mA/V)}}, \text{ k}\Omega$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$f_\beta = \frac{1}{2\pi(C_\pi + C_\mu)r_\pi}$$

$$f_\beta = \frac{f_T}{\beta_0}$$

10.26 $C_{\text{in}} = C_{gs} + C_{gd}(1 + g_m R'_L)$

$= 1 + 0.1(1 + 39)$

$= 5$ pF

$$f_{sdB} = \frac{1}{2\pi C_{\text{in}} R_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 5 \times 10^{-12} R_{\text{sig}}}$$

For $f_{sdB} > 1$ MHz,

$$R_{\text{sig}} < \frac{1}{2\pi \times 5 \times 10^{-12} \times 1 \times 10^6} = 31.8 \text{ k}\Omega$$

10.27 (a) $V_o = -AV_i$

If the current flowing through R_{sig} is denoted I_i, we obtain

$$Y_{\text{in}} = \frac{I_i}{V_i} = \frac{sC(V_i - V_o)}{V_i}$$

$$= sC\left(1 - \frac{V_o}{V_i}\right)$$

$$= sC(1 + A)$$

Thus,

$$C_{\text{in}} = C(1 + A)$$

This table belongs to Problem 10.25.

Transistor	I_E (mA)	r_e (Ω)	g_m (mA/V)	r_π (kΩ)	β_0	f_T (MHz)	C_μ (pF)	C_π (pF)	f_β (MHz)
(a)	2	12.5	80	12.5	100	500	2	23.5	5
(b)	1	25	40	3.13	125	500	2	10.7	4
(c)	1	25	40	2.5	100	500	2	10.7	5
(d)	10	2.5	400	0.25	100	500	2	125.3	5
(e)	0.1	250	4	25	100	150	2	2.2	1.5
(f)	1	25	40	0.25	10	500	2	10.7	50
(g)	1.25	20	50	0.2	10	800	1	9	80

(b) $\frac{V_i(s)}{V_{sig}(s)} = \frac{1/sC_{in}}{R_{sig} + \frac{1}{sC_{in}}}$

$= \frac{1}{1 + sC_{in}R_{sig}}$

$\frac{V_o(s)}{V_{sig}(s)} = -\frac{A}{1 + sC_{in}R_{sig}}$

(c) DC gain = 40 dB = 100 V/V,

$\Rightarrow A = 100$ V/V

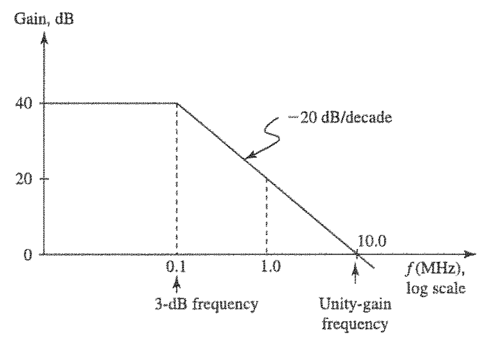
$f_{3dB} = \frac{1}{2\pi C_{in}R_{sig}}$

$100 \times 10^3 = \frac{1}{2\pi C_{in} \times 1 \times 10^3}$

$\Rightarrow C_{in} = 1591.5$ pF

$C = \frac{C_{in}}{A+1} = \frac{1591.5}{101} = 15.8$ pF

(d) The Bode plot is shown in the figure below. From the figure we see that the gain reduces to unity two decades higher than f_{3dB}, that is at 10 MHz.



10.28 $C_{in} = 0.2(1 + 1000)$

$= 200.2$ pF

$\frac{V_o(s)}{V_{sig}(s)} = -\frac{1000}{1 + sC_{in}R_{sig}}$

$f_{3dB} = \frac{1}{2\pi C_{in}R_{sig}}$

$= \frac{1}{2\pi \times 200.2 \times 10^{-12} \times 1 \times 10^3}$

$= 795$ kHz

The gain falls off at the rate of 20 dB/decade. For the gain to reach 0 dB (unity), the gain has to fall by 60 dB. This requires three decades or a factor of 1000, thus

$f_{\text{unity gain}} = 795 \times 1000 = 795$ MHz

10.29 $f_H = \frac{1}{2\pi C_{in}R_{sig}}$

For $f_H \geq 6$ MHz

$C_{in} \leq \frac{1}{2\pi f_H R_{sig}} = \frac{1}{2\pi \times 6 \times 10^6 \times 1 \times 10^3}$

$C_{in} \leq 26.5$ pF

But,

$C_{in} = C_{gs} + (1 + g_m R'_L)C_{gd}$

$= 5 + (1 + g_m R'_L) \times 1$, pF

$= 6 + g_m R'_L$, pF

For $C_{in} \leq 26.5$ pF we have

$g_m R'_L \leq 20.5$

$R'_L \leq \frac{20.5}{5} = 4.1$ kΩ

Corresponding to $R'_L = 4.1$ kΩ, we have

$|A_M| = g_m R'_L = 20.5$ V/V

$GB = |A_M| f_H$

$= 20.5 \times 6 = 123$ MHz

If $f_H = 2$ MHz, we obtain

$C_{in} = 26.5 \times 3 = 79.5$ pF

$g_m R'_L = 79.5 - 6 = 73.5$

Thus,

$|A_M| = 73.5$ V/V

$GB = 73.5 \times 2 = 147$ MHz

10.30 Refer to Example 10.3. If the transistor is replaced with another whose W is half that of the original transistor, we obtain

$W_2 = \frac{1}{2}W_1$

Since

$C_{gs} = \frac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$

then

$C_{gs2} = \frac{1}{2}C_{gs1} = 0.5$ pF

Also,

$C_{gd} = WL_{ov}C_{ox}$

thus,

$C_{gd2} = \frac{1}{2}C_{gd1} = 0.2$ pF

Since

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}$$

then

$$g_{m2} = \frac{1}{\sqrt{2}}\, g_{m1} = 0.71 \text{ mA/V}$$

Since

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

then

$$V_{OV2} = \sqrt{2}V_{OV1}$$

Finally,

$$r_{o2} = r_{o1} = 150 \text{ k}\Omega$$

Thus,

$$R'_{L2} = R'_{L1} = 7.14 \text{ k}\Omega$$

Thus,

$$C_{\text{in}2} = C_{gs2} + (g_{m2}R'_{L2} + 1)C_{gd2}$$

$$= 0.5 + (0.71 \times 7.14 + 1) \times 0.2$$

$$= 1.71 \text{ pF}$$

This should be compared to $C_{\text{in}1} = 4.26$ pF. Thus,

$$f_{H2} = \frac{1}{2\pi C_{\text{in}2}(R_{\text{sig}} \parallel R_G)}$$

$$= \frac{1}{2\pi \times 1.71 \times 10^{-12}(0.1 \parallel 4.7) \times 10^6}$$

$$= 952 \text{ MHz}$$

in comparison to $f_{H1} = 398$ MHz

$$|A_{M2}| = \frac{4.7}{4.7 + 0.1} \times g_{m2}R'_{L2}$$

$$= \frac{4.7}{4.8} \times 0.71 \times 7.14$$

$$= 5 \text{ V/V}$$

in comparison to $|A_{M1}| = 7$ V/V.

$$GB_2 = 5 \times 952 = 4.73 \text{ GHz}$$

in comparison to $GB_1 = 7 \times 398 = 2.79$ GHz.

10.31 $A_M = -\dfrac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m R'_L$

where

$$R'_L = R_D \parallel R_L \parallel r_o$$

$$= 8 \text{ k}\Omega \parallel 10 \text{ k}\Omega \parallel 50 \text{ k}\Omega$$

$$= 4.1 \text{ k}\Omega$$

$$A_M = -\frac{100}{100 + 100} \times 3 \times 4.1$$

$$= -6.1 \text{ V/V}$$

$$C_{\text{in}} = C_{gs} + C_{gd}(1 + g_m R'_L) \qquad (1)$$

$$= 1 + 0.2(1 + 3 \times 4.1)$$

$$= 3.66 \text{ pF}$$

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}} \qquad (2)$$

where

$$R'_{\text{sig}} = R_{\text{sig}} \parallel R_{\text{in}}$$

$$= 100 \text{ k}\Omega \parallel 100 \text{ k}\Omega = 50 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi \times 3.66 \times 10^{-12} \times 50 \times 10^3}$$

$$= 870 \text{ kHz}$$

To double f_H by changing R_{in}, Eq. (2) indicates that R'_{sig} must be halved:

$$R'_{\text{sig}} = 25 \text{ k}\Omega$$

which requires R_{in} to be changed to $R_{\text{in}2}$,

$$25 \text{ k}\Omega = 100 \parallel R_{\text{in}2}$$

$$\Rightarrow R_{\text{in}2} = 33.3 \text{ k}\Omega$$

This change will cause $|A_M|$ to become

$$|A_{M2}| = \frac{33.3}{33.3 + 100} \times 3 \times 4.1$$

$$= 3.1 \text{ V/V}$$

which is about half the original value.

To double f_H by changing R_L, Eq. (2) indicates that C_{in} must be halved:

$$C_{\text{in}2} = \frac{1}{2} \times 3.66 = 1.83 \text{ pF}$$

Using Eq. (1), we obtain

$$1.83 = 1 + 0.2(1 + g_m R'_{L2})$$

$$\Rightarrow g_m R'_{L2} = 3.15$$

Thus,

$$R'_{L2} = 1.05 \text{ k}\Omega$$

and R_{L2} can be found from

$$1.05 = R_L \parallel 8 \text{ k}\Omega \parallel 50 \text{ k}\Omega$$

$$\Rightarrow R_L = 1.24 \text{ k}\Omega$$

and the midband gain becomes

$$|A_{M2}| = \frac{100}{100 + 100} \times 3.15 = 1.6 \text{ V/V}$$

which is about a quarter of the original gain. Clearly, changing R_{in} is the preferred course of action!

10.32 (a) $A_M = -\frac{R_G}{R_G + R_{sig}} g_m R'_L$

where

$R'_L = R_D \parallel R_L \parallel r_o$

$= 20\text{ k}\Omega \parallel 20\text{ k}\Omega \parallel 100\text{ k}\Omega$

$= 9.1\text{ k}\Omega$

$A_M = -\frac{2\text{ M}\Omega}{2\text{ M}\Omega + 0.5\text{ M}\Omega} \times 5 \times 9.1$

$= -36.4$ V/V

(b) $f_H = \frac{1}{2\pi C_{in} R'_{sig}}$

where

$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$

$= 3 + 0.5(1 + 5 \times 9.1)$

$= 26.25$ pF

and

$R'_{sig} = R_{sig} \parallel R_G$

$= 500\text{ k}\Omega \parallel 2000\text{ k}\Omega$

$= 400\text{ k}\Omega$

Thus,

$f_H = \frac{1}{2\pi \times 26.25 \times 10^{-12} \times 400 \times 10^3}$

$= 15.2$ kHz

(c) $f_Z = \frac{g_m}{2\pi C_{gd}}$

$= \frac{5 \times 10^{-3}}{2\pi \times 0.5 \times 10^{-12}}$

$= 1.6$ GHz

10.33 $R_G = R_{G1} \parallel R_{G2} = 47\text{ M}\Omega \parallel 10\text{ M}\Omega$

$= 8.25\text{ M}\Omega$

$A_M = -\frac{R_G}{R_G + R_{sig}} g_m R'_L$

where

$R'_L = R_L \parallel R_D \parallel r_o$

$= 10\text{ k}\Omega \parallel 4.7\text{ k}\Omega \parallel 100\text{ k}\Omega$

$= 3.1\text{ k}\Omega$

$A_M = -\frac{8.25}{8.25 + 0.1} \times 3 \times 3.1$

$= -9.2$ V/V

$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$

where

$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$

$= 1 + 0.2(1 + 3 \times 3.1)$

$= 3.06$ pF

and

$R'_{sig} = R_{sig} \parallel R_G$

$= 100\text{ k}\Omega \parallel 8.25\text{ M}\Omega$

$= 99\text{ k}\Omega$

Thus,

$f_H = \frac{1}{2\pi \times 3.06 \times 10^{-12} \times 99 \times 10^3}$

$= 525$ kHz

10.34 $g_m = \sqrt{2\mu_n C_{ox}(W/L)_1 I_{D1}}$

$= \sqrt{2 \times 0.09 \times 100 \times 0.1}$

$= 1.34$ mA/V

$r_{o1} = \frac{|V_{A1}|}{I_{D1}} = \frac{12.8}{0.1} = 128\text{ k}\Omega$

$r_{o2} = \frac{|V_{A2}|}{I_{D2}} = \frac{19.2}{0.1} = 192\text{ k}\Omega$

The total resistance at the output node, R'_L, is given by

$R'_L = r_{o1} \parallel r_{o2} = 128\text{ k}\Omega \parallel 192\text{ k}\Omega$

$= 76.8\text{ k}\Omega$

$A_M = -g_{m1} R'_L$

$= -1.34 \times 76.8 = -103$ V/V

$f_H = \frac{1}{2\pi C_{in} R_{sig}}$

where

$C_{in} = C_{gs} + C_{gd}(1 + g_{m1} R'_L)$

$= 0.2 + 0.015(1 + 103)$

$= 1.76$ pF

Thus,

$f_H = \frac{1}{2\pi \times 1.76 \times 10^{-12} \times 200 \times 10^3}$

$= 452$ kHz

$f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{1.34 \times 10^{-3}}{2\pi \times 0.015 \times 10^{-12}}$

$= 14.2$ GHz

10.35 $g_m R'_L = 50$

$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$

$= 10 + 1(1 + 50)$

$= 61$ pF

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 61 \times 10^{-12} \times 5 \times 10^3}$$

$= 522$ kHz

10.36

$$A_M = -\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + (R_{sig} \parallel R_B)} g_m R'_L$$

where

$R'_L = r_o \parallel R_C \parallel R_L$

$= 100$ kΩ $\parallel$ 10 kΩ $\parallel$ 10 kΩ

$= 4.76$ kΩ

and

$r_\pi = \beta / g_m = 100/40 = 2.5$ kΩ

$$A_M = -\frac{100}{100 + 10} \times \frac{2.5}{2.5 + 0.1 + (10 \parallel 100)} \times 40 \times 4.76$$

$= -37$ V/V

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

where

$C_{in} = C_\pi + C_\mu (1 + g_m R'_L)$

$= 10 + 1 \times (1 + 40 \times 4.76)$

$= 201.4$ pF

and

$R'_{sig} = r_\pi \parallel [r_x + (R_B \parallel R_{sig})]$

$= 2.5 \parallel [0.1 + (100 \parallel 10)]$

$= 2$ kΩ

$$f_H = \frac{1}{2\pi \times 201.4 \times 10^{-12} \times 2 \times 10^3}$$

$= 395$ kHz

10.37 Refer to Example 10.4. Since I_E is doubled to 2 mA, we have

$$g_m = \frac{I_C}{V_T} = \frac{2 \text{ mA}}{25 \text{ mV}} = 80 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{80 \text{ mA/V}} = 1.25 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{2 \text{ mA}} = 50 \text{ k}\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{\omega_T} = \frac{80 \times 10^{-3}}{2\pi \times 800 \times 10^6} = 16 \text{ pF}$$

$C_\mu = 1$ pF

$C_\pi = 15$ pF

$r_x = 50$ Ω

Also, now

$R_B = 50$ kΩ

$R_C = 4$ kΩ

The new value of A_M is

$$A_M = -\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + (R_B \parallel R_{sig})} (g_m R'_L)$$

where

$R'_L = r_o \parallel R_C \parallel R_L$

$= 50 \parallel 4 \parallel 5 = 2.13$ kΩ

Thus,

$g_m R'_L = 80 \times 2.13 = 170$ V/V

and

$$A_M = -\frac{50}{50 + 5} \times \frac{1.25}{1.25 + 0.05 + (50 \parallel 5)} \times 170$$

$= -33$ V/V

and

$20 \log |A_M| = 30.4$ dB

This should be compared to the previous value of 39 V/V (32 dB). To determine f_H, we first find C_{in},

$C_{in} = C_\pi + C_\mu (1 + g_m R'_L)$

$= 15 + 1(1 + 170)$

$= 186$ pF

and the effective source resistance R'_{sig},

$R'_{sig} = r_\pi \parallel [r_x + (R_B \parallel R_{sig})]$

$= 1.25 \parallel [0.05 + (50 \parallel 5)]$

$= 0.98$ kΩ

Thus

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 186 \times 10^{-12} \times 0.98 \times 10^3}$$

$= 873$ kHz

This should be compared to the previous value of 754 kHz. The gain-bandwidth product becomes

$GB = |A_M| f_H = 33 \times 873 = 28.8$ MHz

This should be compared to the previous value of $39 \times 754 = 29.4$ MHz. Thus, increasing the bias current by a factor of 2 results in an increase in f_H by a factor of 1.16—that is, by about 16%. However, because of the attendant reduction in input resistance, the overall gain decreased by about the same factor and GB remained nearly constant. The price paid for the slight increase in f_H is an increase in power dissipation by a factor of about two.

10.38 (a) $A_M =$

$$-\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + (R_{sig} \parallel R_B)} g_m R'_L$$

For $R_B \gg R_{sig}$, $r_x \ll R_{sig}$, $R_{sig} \gg r_\pi$,

$$A_M \simeq -\frac{r_\pi}{R_{sig}} g_m R'_L = -\beta R'_L / R_{sig} \qquad \text{Q.E.D.}$$

(b) $C_{in} = C_\pi + (g_m R'_L + 1) C_\mu$

For $g_m R'_L \gg 1$ and $g_m R'_L C_\mu \gg C_\pi$,

$$C_{in} \simeq g_m R'_L C_\mu$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

where

$$R'_{sig} = r_\pi \parallel [r_x + (R_B \parallel R_{sig})]$$

$$\simeq r_\pi \parallel R_{sig} \simeq r_\pi$$

Thus,

$$f_H \simeq \frac{1}{2\pi g_m R'_L C_\mu r_\pi}$$

$$f_H = \frac{1}{2\pi C_\mu \beta R'_L} \qquad \text{Q.E.D.}$$

(c) $\text{GB} = |A_M| f_H$

$$= \beta \frac{R'_L}{R_{sig}} \frac{1}{2\pi C_\mu \beta R'_L} = \frac{1}{2\pi C_\mu R_{sig}} \qquad \text{Q.E.D.}$$

For $R_{sig} = 25\text{ k}\Omega$ and $C_\mu = 1\text{ pF}$,

$$\text{GB} = \frac{1}{2\pi \times 1 \times 10^{-12} \times 25 \times 10^3} = 6.37\text{ MHz}$$

For $I_C = 1\text{ mA}$ and $\beta = 100$,

(i) $R'_L = 25\text{ k}\Omega$:

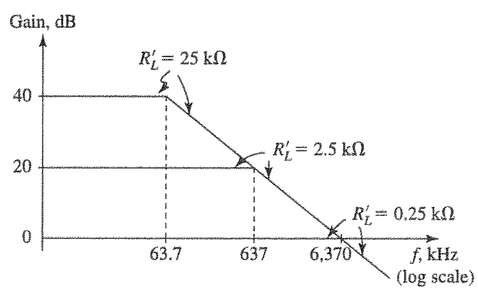
$$A_M = -100 \times \frac{25}{25} = -100\text{ V/V}$$

$$f_H = \frac{\text{GB}}{|A_M|} = \frac{6.37\text{ MHz}}{100\text{ V/V}} = 63.7\text{ kHz}$$

(ii) $R'_L = 2.5\text{ k}\Omega$:

$$A_M = -100 \times \frac{2.5}{25} = -10\text{ V/V}$$

$$f_H = \frac{\text{GB}}{|A_M|} = \frac{6.37\text{ MHz}}{10\text{ V/V}} = 637\text{ kHz}$$



The Bode plots are shown in the figure.

If the midband gain is unity,

$$f_H = \text{GB} = 6.37\text{ MHz}$$

This is obtained when R'_L is

$$1 = 100 \times \frac{R'_L}{25}$$

$$\Rightarrow R'_L = 0.25\text{ k}\Omega = 250\ \Omega$$

10.39 $R_B = R_{B1} \parallel R_{B2} = 68\text{ k}\Omega \parallel 27\text{ k}\Omega$

$$= 19.3\text{ k}\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{0.8\text{ mA}}{0.025\text{ V}} = 32\text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{32} = 6.25\text{ k}\Omega$$

$$R'_{sig} = r_\pi \parallel R_B \parallel R_{sig}$$

$$= 6.25\text{ k}\Omega \parallel 19.3\text{ k}\Omega \parallel 10\text{ k}\Omega$$

$$= 3.2\text{ k}\Omega$$

$$R'_L = R_C \parallel R_L = 4.7\text{ k}\Omega \parallel 10\text{ k}\Omega$$

$$= 3.2\text{ k}\Omega$$

$$A_M = -\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + (R_{sig} \parallel R_B)} g_m R'_L$$

$$= -\frac{19.3}{19.3 + 10} \frac{6.25}{6.25 + (10 \parallel 19.3)} \times 32 \times 3.2$$

$$= -32.8\text{ V/V}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$1 \times 10^9 = \frac{32 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$\Rightarrow C_\pi + C_\mu = 5.1\text{ pF}$$

$$C_\pi = 5.1 - 0.8 = 4.3\text{ pF}$$

$$C_{in} = C_\pi + (g_m R'_L + 1) C_\mu$$

$$= 4.3 + (32 \times 3.2 + 1) \times 0.8$$

$$= 87\text{ pF}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 87 \times 10^{-12} \times 3.2 \times 10^3}$$

$$= 572\text{ kHz}$$

10.40 $R_{in} = \dfrac{R}{1 - K}$

$$= \frac{100\text{ k}\Omega}{1 - 0.9} = 1000\text{ k}\Omega = 1\text{ M}\Omega$$

10.41 Using Miller's theorem, we obtain

$$Z_{\text{in}} = \frac{Z}{1-A}, \quad Z_{\text{out}} = \frac{Z}{1-\frac{1}{A}}$$

For

$$Z = \frac{1}{j\omega C}$$

$$Z_{\text{in}} = \frac{1}{j\omega C(1-A)} \Rightarrow C_{\text{in}} = C(1-A)$$

$$Z_{\text{out}} = \frac{1}{j\omega C\left(1-\frac{1}{A}\right)} \Rightarrow C_{\text{out}} = C\left(1-\frac{1}{A}\right)$$

(a) $A = -1000$ V/V, $\quad C = 1$ pF

$$C_{\text{in}} = 1(1+1000) = 1001 \text{ pF}$$

$$C_{\text{out}} = 1\left(1+\frac{1}{1000}\right) = 1.001 \text{ pF}$$

(b) $A = -10$ V/V, $\quad C = 10$ pF

$$C_{\text{in}} = 10(1+10) = 110 \text{ pF}$$

$$C_{\text{out}} = 10\left(1+\frac{1}{10}\right) = 11 \text{ pF}$$

(c) $A = -1$ V/V, $\quad C = 10$ pF

$$C_{\text{in}} = 10(1+1) = 20 \text{ pF}$$

$$C_{\text{out}} = 10(1+1) = 20 \text{ pF}$$

(d) $A = +1$ V/V, $\quad C = 10$ pF

$$C_{\text{in}} = C(1-1) = 0$$

$$C_{\text{out}} = C(1-1) = 0$$

(e) $A = +10$ V/V, $\quad C = 10$ pF

$$C_{\text{in}} = 10(1-10) = -90 \text{ pF}$$

$$C_{\text{out}} = 10\left(1-\frac{1}{10}\right) = 9 \text{ pF}$$

The -90 pF input capacitance can be used to cancel an equal (+90 pF) capacitance between the input node and ground.

10.42

This figure belongs to Problem 10.42.

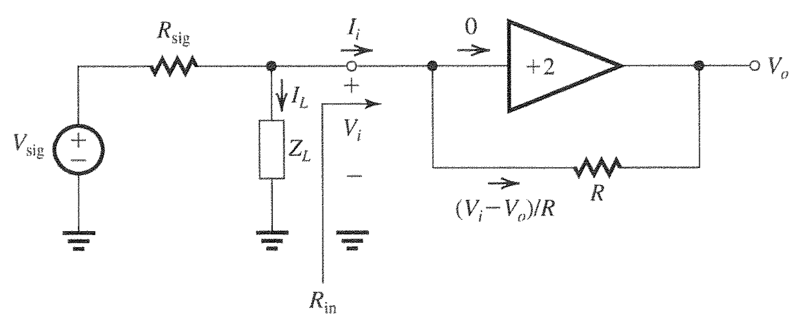


Figure 1

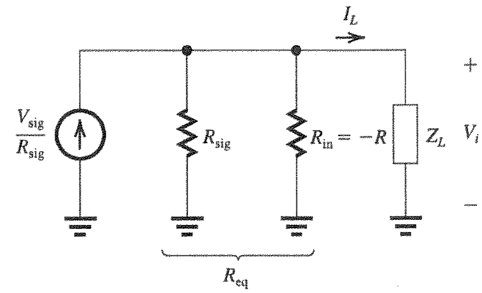


Figure 2

(a) Refer to Fig. 1.

$$I_i = \frac{V_i - V_o}{R} = \frac{V_i - 2V_i}{R} = -\frac{V_i}{R}$$

Thus,

$$R_{\text{in}} \equiv \frac{V_i}{I_i} = -R$$

(b) Replacing the signal source with its equivalent Norton's form results in the circuit in Fig. 2. Observe that $R_{\text{eq}} = \infty$ when $R_{\text{sig}} = R$. In this case,

$$I_L = \frac{V_{\text{sig}}}{R_{\text{sig}}} = \frac{V_{\text{sig}}}{R}$$

(c) If $Z_L = \dfrac{1}{sC}$,

$$V_i = I_L Z_L$$

$$= \frac{V_{\text{sig}}}{R} \times \frac{1}{sC}$$

$$= \frac{1}{sCR} V_{\text{sig}}$$

and

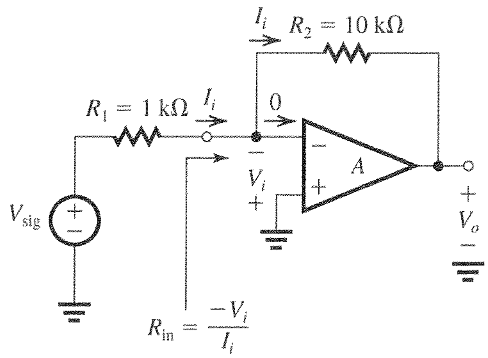
$$V_o = 2V_i = \frac{2}{sCR} V_{\text{sig}}$$

Thus,

$$\frac{V_o}{V_{\text{sig}}} = \frac{2}{sCR}$$

which is the transfer function of an ideal noninverting integrator.

10.43



From the figure we see that

$$V_o = AV_i \quad (1)$$

From Miller's theorem, we have

$$R_{in} = \frac{R_2}{1 - \left(\frac{V_o}{-V_i}\right)} = \frac{R_2}{1 + \frac{V_o}{V_i}} = \frac{R_2}{1 + A} \quad (2)$$

Using the voltage divider rule at the input, we get

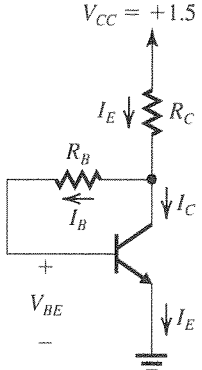
$$-V_i = V_{sig}\frac{R_{in}}{R_{in} + R_1}$$

$$\Rightarrow V_i = -V_{sig}\frac{R_{in}}{R_{in} + R_1} \quad (3)$$

For each value of A we use Eq. (2) to determine R_{in}, Eq. (3) to determine V_i (for $V_{sig} = 1$ V), Eq. (1) to determine V_o, and finally we calculate the value of V_o/V_{sig}. The results are given in the table below.

A (V/V)	R_{in} (kΩ)	V_i (V)	V_o (V)	V_o/V_{sig} (V/V)
10	9.091×10^{-1}	−0.476	−4.76	−4.76
100	9.901×10^{-2}	−0.0901	−9.01	−9.01
1000	9.990×10^{-3}	-9.89×10^{-3}	−9.89	−9.89
10,000	9.999×10^{-4}	-9.989×10^{-4}	−9.99	−9.99

10.44



(a) For the dc analysis, refer to the figure.

$$V_{CC} = I_E R_C + I_B R_B + V_{BE}$$

$$1.5 = I_E \times 1 + \frac{I_E}{\beta + 1} \times 47 + 0.7$$

$$\Rightarrow I_E = \frac{1.5 - 0.7}{1 + \frac{47}{101}} = 0.546 \text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 0.546 = 0.54 \text{ mA}$$

(b) $g_m = \frac{I_C}{V_T} = \frac{0.54 \text{ mA}}{0.025 \text{ V}} = 21.6 \text{ mA/V}$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{21.6} = 4.63 \text{ k}\Omega$$

(c) $\frac{V_o}{V_b} = -g_m(R_C \parallel R_L)$

$$= -21.6(1 \parallel 1) = -10.8 \text{ V/V}$$

(d) Using Miller's theorem, the component of R_{in} due to R_B can be found as

$$R_{in1} = \frac{R_B}{1 - (V_o/V_b)}$$

$$= \frac{47 \text{ k}\Omega}{1 - (-10.8)} = 4 \text{ k}\Omega$$

$$R_{in} = R_{in1} \parallel r_\pi$$

$$= 4 \parallel 4.63 = 2.14 \text{ k}\Omega$$

(e) $G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times \frac{V_o}{V_b}$

$$= \frac{2.14}{2.14 + 1} \times -10.8 = -7.4 \text{ V/V}$$

(f) $C_{in} = C_\pi + \left(1 + \left|\frac{V_o}{V_b}\right|\right) C_\mu$

where

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{21.6 \times 10^{-3}}{2\pi \times 600 \times 10^6}$$

$$C_\pi + C_\mu = 5.73 \text{ pF}$$

$$C_\pi = 5.73 - 0.8 = 4.93 \text{ pF}$$

$$C_{in} = 4.93 + (1 + 10.8) \times 0.8$$

$$= 14.37 \text{ pF}$$

(g) $R'_{sig} = R_{in} \parallel R_{sig}$

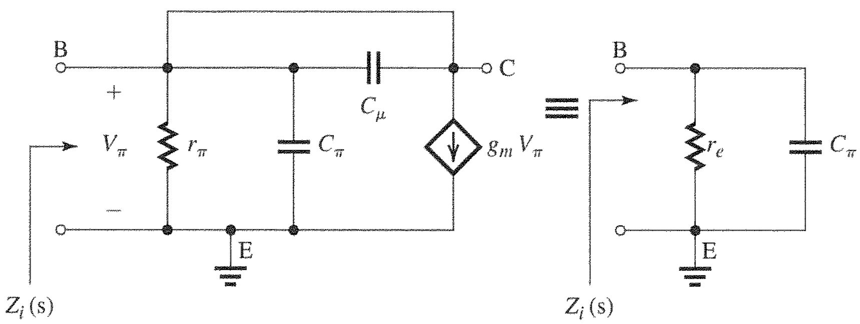
$$= 2.14 \text{ k}\Omega \parallel 1 \text{ k}\Omega = 0.68 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 14.37 \times 10^{-12} \times 0.68 \times 10^3}$$

$$= 16.3 \text{ MHz}$$

10.45



From the figure we see that the controlled current-source $g_m V_\pi$ appears across its control voltage V_π, thus we can replace the current source with a resistance $1/g_m$. Now, the parallel equivalent of r_π and $1/g_m$ is

$$\frac{r_\pi(1/g_m)}{r_\pi + \frac{1}{g_m}} = \frac{r_\pi}{g_m r_\pi + 1} = \frac{r_\pi}{\beta + 1} = r_e$$

Thus, the equivalent circuit simplifies to that of r_e in parallel with C_π,

$$Z_i(s) = \frac{1}{\frac{1}{r_e} + sC_\pi} = \frac{r_e}{1 + sC_\pi r_e}$$

$$Z_i(j\omega) = \frac{r_e}{1 + j\omega C_\pi r_e}$$

$Z_i(j\omega)$ will have a 45° phase at

$$\omega_{45} C_\pi r_e = 1$$

$$\Rightarrow \omega_{45} = \frac{1}{C_\pi r_e}$$

Now,

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

At high bias currents,

$$C_\pi \gg C_\mu$$

and

$$f_T \simeq \frac{g_m}{2\pi C_\pi}$$

Since $g_m \simeq 1/r_e$, we have

$$f_T \simeq \frac{1}{2\pi C_\pi r_e}$$

Thus,

$$f_{45^\circ} \simeq f_T = 400 \text{ MHz}$$

If the bias current is reduced to the value that results in $C_\pi \simeq C_\mu$,

$$f_T = \frac{g_m}{2\pi \times 2C_\pi} = \frac{g_m}{4\pi C_\pi}$$

Again, $g_m \simeq \frac{1}{r_e}$, thus

$$f_T \simeq \frac{1}{4\pi C_\pi r_e}$$

It follows that in this case,

$$f_{45^\circ} = \frac{1}{2} f_T = 200 \text{ MHz}$$

10.46 $A_M = -g_m R'_L$

$$= -4 \times 20 = -80 \text{ V/V}$$

$$f_{3dB} = f_H = \frac{1}{2\pi(C_L + C_{gd})R'_L}$$

$$= \frac{1}{2\pi(2 + 0.1) \times 10^{-12} \times 20 \times 10^3}$$

$$= 3.8 \text{ MHz}$$

$$f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{4 \times 10^{-3}}{2\pi \times 0.1 \times 10^{-12}} = 6.4 \text{ GHz}$$

$$f_t = |A_M| f_H$$

$$= 80 \times 3.8 = 304 \text{ MHz}$$

10.47 $f_t = \dfrac{g_m}{2\pi(C_L + C_{gd})}$

$$C_L + C_{gd} = \frac{g_m}{2\pi f_t}$$

$$= \frac{2 \times 10^{-3}}{2\pi \times 2 \times 10^9} = 0.159 \text{ pF}$$

To reduce f_t to 1 GHz, an additional capacitance of 0.159 pF must be connected to the output node. (Doubling the effective capacitance at the output node reduces f_t by a factor of 2.)

10.48 Refer to Fig. P10.48. To determine g_{m1} we use

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}$$

$$= \sqrt{2 \times 0.090 \times \frac{100}{1.6} \times 0.1}$$

$$= 1.06 \text{ mA/V}$$

$$r_{o1} = \frac{V_{A1}}{I_{D1}} = \frac{12.8}{0.1} = 128 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_{A2}|}{I_{D2}} = \frac{19.2}{0.1} = 192 \text{ k}\Omega$$

$$R'_L = r_{o1} \parallel r_{o2} = 128 \parallel 192 = 76.8 \text{ k}\Omega$$

$$A_M = -g_{m1}R'_L$$

$$= -1.06 \times 76.8 = -81.4 \text{ V/V}$$

$$C_L = C_{db1} + C_{db2} + C_{gd2}$$

$$= 20 + 36 + 15 = 71 \text{ fF}$$

$$f_H = \frac{1}{2\pi(C_L + C_{gd1})R'_L}$$

$$f_H = \frac{1}{2\pi(71+15) \times 10^{-15} \times 76.8 \times 10^3}$$

$$= 24.1 \text{ MHz}$$

$$f_Z = \frac{g_{m1}}{2\pi C_{gd1}} = \frac{1.06 \times 10^{-3}}{2\pi \times 0.015 \times 10^{-12}}$$

$$= 11.2 \text{ GHz}$$

10.49 Figure 1 shows the amplifier high-frequency equivalent circuit. A node equation at the output provides

$$\left(\frac{1}{r_o} + sC_L\right)V_o + g_m V_\pi + sC_\mu(V_o - V_\pi) = 0$$

Replacing V_π by V_i and collecting terms results in

$$V_o\left[\frac{1}{r_o} + s(C_L + C_\mu)\right] = -V_i(g_m - sC_\mu)$$

$$\Rightarrow \frac{V_o}{V_i} = -g_m r_o \frac{1 - s(C_\mu/g_m)}{1 + s(C_L + C_\mu)r_o} \qquad \text{Q.E.D.}$$

For $I_C = 200\ \mu\text{A} = 0.2$ mA and $V_A = 100$ V,

$$g_m = \frac{I_C}{V_T} = \frac{0.2 \text{ mA}}{0.025 \text{ V}} = 8 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.2} = 500 \text{ k}\Omega$$

$$\text{DC gain} = -g_m r_o = 8 \times 500 = -4000 \text{ V/V}$$

$$f_{3dB} = \frac{1}{2\pi(C_L + C_\mu)r_o}$$

$$= \frac{1}{2\pi(1+0.2) \times 10^{-12} \times 500 \times 10^3}$$

$$= 265.3 \text{ kHz}$$

$$f_Z = \frac{g_m}{2\pi C_\mu}$$

$$= \frac{8 \times 10^{-3}}{2\pi \times 0.2 \times 10^{-12}} = 6.37 \text{ GHz}$$

$$f_t = |A_{dc}| f_{3dB}$$

$$= 4000 \times 265.3 = 1.06 \text{ GHz}$$

The Bode plot is shown in Figure 2.

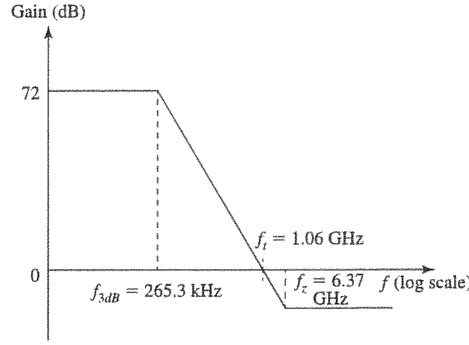


Figure 2

10.50 The equivalent circuit is shown in the figure.

$$g_m = \frac{I_C}{V_T} = \frac{2 \text{ mA}}{0.025 \text{ V}} = 80 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{80} = 1.5 \text{ k}\Omega$$

(a) $$A_M = -\frac{r_\pi}{r_\pi + r_x} g_m R'_L$$

$$-10 = -\frac{1.5}{1.5 + 0.1} \times g_m R'_L$$

$$\Rightarrow g_m R'_L = 10.7 \text{ V/V}$$

$$R'_L = 0.133 \text{ k}\Omega$$

This figure belongs to Problem 10.49, part (a).

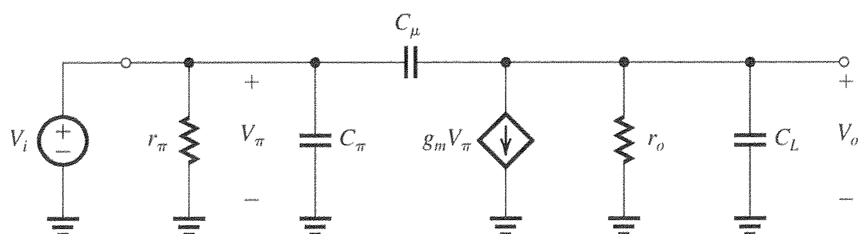
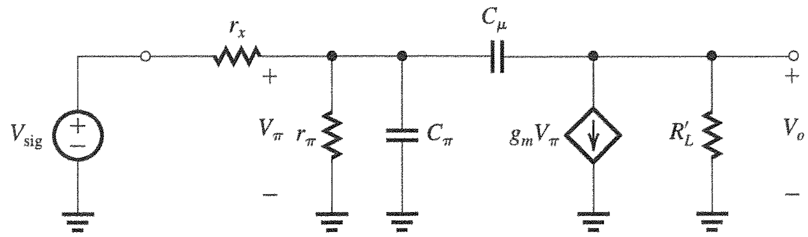


Figure 1

This figure belongs to Problem 10.50.



$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{80 \times 10^{-3}}{2\pi \times 2 \times 10^9} = 6.37 \text{ pF}$$

$$C_\pi = 6.37 - 1 = 5.37 \text{ pF}$$

$$C_{\text{in}} = C_\pi + (g_m R'_L + 1)C_\mu$$

$$C_{\text{in}} = 5.37 + (10.7 + 1) \times 1$$

$$= 17.07 \text{ pF}$$

$$R'_{\text{sig}} = r_\pi \parallel r_x = 1.5 \text{ k}\Omega \parallel 0.1 \text{ k}\Omega$$

$$= 0.094 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 17.07 \times 10^{-12} \times 0.094 \times 10^3}$$

$$= 99.2 \text{ MHz}$$

(b) If $|A_M|$ is reduced to 1, we obtain

$$1 = \frac{1.5}{1.6} \times g_m R'_L$$

$$\Rightarrow g_m R'_L = 1.07$$

$$C_{\text{in}} = C_\pi + (g_m R'_L + 1)C_\mu$$

$$= 5.37 + (1.07 + 1) \times 1$$

$$= 7.44 \text{ pF}$$

$$f_H = \frac{1}{2\pi \times 7.44 \times 10^{-12} \times 0.094 \times 10^3}$$

$$= 227.6 \text{ MHz}$$

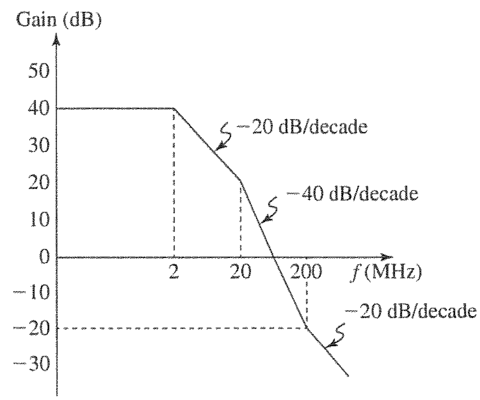
10.51 $A_M = 40 \text{ dB} \Rightarrow 100$ V/V

$$A(s) = 100\frac{1 + s/(2\pi \times 200 \times 10^6)}{\left(1 + \dfrac{s}{2\pi \times 2 \times 10^6}\right)\left(1 + \dfrac{s}{2\pi \times 20 \times 10^6}\right)}$$

Since $f_{P1} \ll f_{P2} \ll f_Z$, we have

$f_{3dB} \simeq f_{P1} = 2$ MHz

The Bode plot is shown in the figure.



10.52 (a) $A(s) = 1000\dfrac{1}{1 + s/(2\pi \times 10^5)}$

(b)

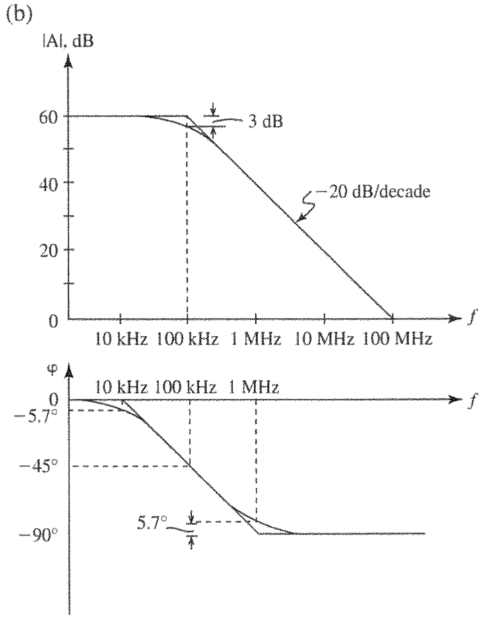


Figure 1

Figure 1 shows the Bode plot for the gain magnitude and phase.

(c) GB = 1000 × 100 kHz = 100 MHz

(d) The unity-gain frequency f_t is

$f_t = 100$ MHz

(e)

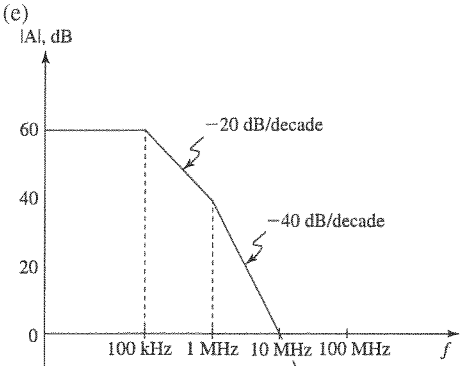


Figure 2

Figure 2 shows the magnitude response when a second pole at 1 MHz appears in the transfer function. The unity-gain frequency f_t now is

$f_t = 10$ MHz

which is different from the gain-bandwidth product,

GB = 100 MHz

10.53 Using the dominant-pole approximation,

$\omega_H \simeq \omega_{P1}$

Using the root-sum-of-squares formula, we get

$$\omega_H \simeq \frac{1}{\sqrt{\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2}}} = \frac{\omega_{P1}}{\sqrt{1 + \left(\frac{\omega_{P1}}{\omega_{P2}}\right)^2}}$$

(a) For a difference of 10%,

$$\frac{\omega_{P1}}{\sqrt{1 + \left(\frac{\omega_{P1}}{\omega_{P2}}\right)^2}} = 0.9\omega_{P1}$$

$$\Rightarrow \frac{\omega_{P2}}{\omega_{P1}} = 4.26$$

(b) For a difference of 1%,

$$\frac{\omega_{P1}}{\sqrt{1 + \left(\frac{\omega_{P1}}{\omega_{P2}}\right)^2}} = 0.99\omega_{P1}$$

$$\Rightarrow \frac{\omega_{P2}}{\omega_{P1}} = 49.3$$

10.54 $A(s) = -10^3 \dfrac{1 + \frac{s}{10^4}}{\left(1 + \frac{s}{10^3}\right)\left(1 + \frac{s}{10^5}\right)}$

(a) $\omega_H \simeq 10^3$ rad/s

(b) $\omega_H = 1 \Big/ \sqrt{\left(\frac{1}{10^6} + \frac{1}{10^{10}}\right) - \frac{2}{10^8}}$

$= 1010$ Hz

If the frequency of the finite zero is lowered to 10^3 rad/s the zero will cancel the pole at 10^3 rad/s and the transfer function becomes

$$A(s) = -10^3 \frac{1}{1 + \frac{s}{10^5}}$$

The 3-dB frequency now becomes

$\omega_{3dB} = 10^5$ rad/s

10.55 If at $\omega = 10^7$ rad/s the excess phase due to the 3 coincident poles (at frequency ω_P) is 30°, then each pole is contributing 10°. Thus,

$$\tan^{-1}\frac{10^7}{\omega_P} = 10^\circ$$

$$\omega_P = \frac{10^7}{\tan 10^\circ} = 5.67 \times 10^7 \text{ rad/s}$$

10.56 $\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L}$

where

$C_{gs} = 30$ fF

$R_{gs} = R'_{\text{sig}} = 10$ kΩ

$C_{gd} = 5$ fF

$R_{gd} = R'_{\text{sig}}(1 + g_m R'_L) + R'_L$

$= 10(1 + 2 \times 20) + 20$

$= 430$ kΩ

$C_L = 30$ fF

$R_{CL} = R'_L = 20$ kΩ

Thus,

$\tau_H = 30 \times 10 + 5 \times 430 + 30 \times 20$

$= 3050$ ps

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 3050 \times 10^{-12}}$$

$= 52.2$ MHz

$$f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{2 \times 10^{-3}}{2\pi \times 5 \times 10^{-15}} = 63.7 \text{ GHz}$$

10.57 $A_M = -\dfrac{R_G}{R_G + R_{\text{sig}}} g_m R'_L$

$$= -\frac{0.65}{0.65 + 0.15} \times 5 \times 10$$

$= -40.6$ V/V

$\tau_{gs} = C_{gs}R_{gs}$

$= C_{gs}R'_{\text{sig}}$

$= C_{gs}(R_{\text{sig}} \parallel R_G)$

$= 2 \times 10^{-12} \times (150 \parallel 650) \times 10^3$

$= 243.8$ ns

$\tau_{gd} = C_{gd}R_{gd}$

$= C_{gd}[R'_{\text{sig}}(1 + g_m R'_L) + R'_L]$

$= 0.5 \times 10^{-12}[(150 \parallel 650)(1+5\times 10)+10] \times 10^3$

$= 3112.8$ ns

$\tau_{CL} = C_L R'_L$

$= 30 \times 10^{-12} \times 10 \times 10^3$

$= 300$ ns

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= 243.8 + 3112.8 + 300$

$= 3656.6$ ns

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 3656.6 \times 10^{-9}}$$

$= 43.5$ MHz

10.58 The figure shows the equivalent circuit of the two-stage amplifier where we have modeled each stage as a transconductance amplifier. At node 1:

$R_{\text{eq1}} = R_{\text{sig}} \parallel R_{\text{in1}}$

$= 10\ \text{k}\Omega \parallel 10\ \text{k}\Omega = 5\ \text{k}\Omega$

$C_{\text{eq1}} = C_{\text{in1}} + C_{\text{sig}} = 10 + 10 = 20$ pF

Thus,

$$f_{P1} = \frac{1}{2\pi C_{\text{eq1}}R_{\text{eq1}}}$$

$$f_{P1} = \frac{1}{2\pi \times 20 \times 10^{-12} \times 5 \times 10^3}$$

$= 1.59$ MHz

At node 2:

$R_{\text{eq2}} = R_{o1} \parallel R_{\text{in2}} = 2\ \text{k}\Omega \parallel 10\ \text{k}\Omega$

$= 1.67\ \text{k}\Omega$

$C_{\text{eq2}} = C_{o1} + C_{\text{in2}}$

$= 2 + 10 = 12$ pF

$$f_{P2} = \frac{1}{2\pi C_{\text{eq2}}R_{\text{eq2}}}$$

$$= \frac{1}{2\pi \times 12 \times 10^{-12} \times 1.67 \times 10^3}$$

$= 7.94$ MHz

At node 3:

$R_{\text{eq3}} = R_{o2} \parallel R_L = 2\ \text{k}\Omega \parallel 1\ \text{k}\Omega = 0.67\ \text{k}\Omega$

$C_{\text{eq3}} = C_{o2} + C_L = 2 + 7 = 9$ pF

$$f_{P3} = \frac{1}{2\pi C_{\text{eq3}}R_{\text{eq3}}}$$

$$f_{P3} = \frac{1}{2\pi \times 9 \times 10^{-12} \times 0.67 \times 10^3}$$

$= 26.4$ MHz

Thus, the three poles have frequencies 1.59 MHz, 7.94 MHz, and 26.4 MHz. Since the frequency of the second pole is more than two octaves higher than that of the first pole, the 3-dB frequency will be mostly determined by f_{P1},

$f_{3dB} \simeq f_{P1} = 1.59$ MHz

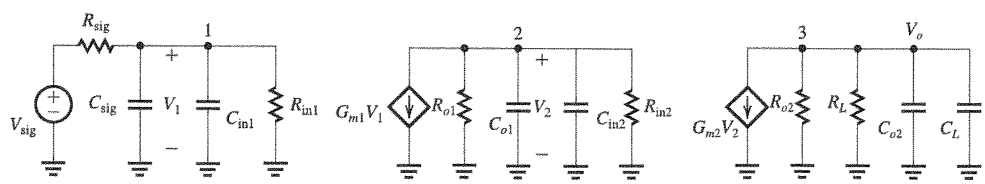
A slightly better estimate of f_{3dB} can be determined using the root-sum-of-squares formula,

$$f_{3dB} = 1/\sqrt{\left(\frac{1}{f_{P1}}\right)^2 + \left(\frac{1}{f_{P2}}\right)^2 + \left(\frac{1}{f_{P3}}\right)^2}$$

$$= 1/\sqrt{\frac{1}{1.59^2} + \frac{1}{7.94^2} + \frac{1}{26.4^2}}$$

$= 1.56$ MHz

This figure belongs to Problem 10.58.



10.59 $A_M = -g_m R_L'$

$= -4 \times 20 = -80$ V/V

$C_{in} = C_{gs} + C_{gd}(g_m R_L' + 1)$

$= 2 + 0.1(80 + 1)$

$= 10.1$ pF

Using the Miller approximation, we obtain

$$f_H \simeq \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 10.1 \times 10^{-12} \times 20 \times 10^3}$$

$= 788$ kHz

Using the open-circuit time constants, we get

$\tau_{gs} = C_{gs} R_{gs} = C_{gs} R'_{sig}$

$= 2 \times 20 = 40$ ns

$R_{gd} = R'_{sig}(1 + g_m R_L') + R_L'$

$= 20(1 + 80) + 20 = 1640$ kΩ

$\tau_{gd} = C_{gd} R_{gd} = 0.1 \times 1640 = 164$ ns

$\tau_{CL} = C_L R_L'$

$= 2 \times 20 = 40$ ns

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL} \qquad (1)$$

$= 40 + 164 + 40 = 244$ ns

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 244 \times 10^{-9}} = 652 \text{ kHz}$$

The estimate obtained using the open-circuit time constants is more appropriate as it takes into account the effect of C_L. We note from Eq. (1) that τ_{CL} is 16.4% of τ_H, thus C_L has a significant effect on the determination of f_H.

10.60 $\tau_{gs} = C_{gs} R_{gs} = C_{gs} R'_{sig}$

$= 5 \times 10 = 50$ ns

$R_{gd} = R'_{sig}(1 + g_m R_L') + R_L'$

$= 10(1 + 5 \times 10) + 10$

$= 520$ kΩ

$\tau_{gd} = C_{gd} R_{gd} = 1 \times 520 = 520$ ns

$R_{C_L} = C_L R_L$

$= 5 \times 10 = 50$ ns

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= 50 + 520 + 50 = 620$ ns

$$f_H = \frac{1}{2\pi \tau_H} \qquad (1)$$

$$= \frac{1}{2\pi \times 620 \times 10^{-9}} = 257 \text{ kHz}$$

The interaction of R'_{sig} with the input capacitance contributes all of τ_{gs} (50 ns) and a significant part of τ_{gd}, namely

$C_{gd}[R'_{sig}(1 + g_m R_L')] = 1 \times 10(1 + 50) = 510$ ns

Thus, the total contribution of R'_{sig} is

$50 + 510 = 560$ ns

which is $\frac{560}{620} = 90.3\%$ of τ_H. To double f_H, we must reduce τ_H to half of its value:

$\tau_H = \frac{1}{2} \times 620 = 310$ ns

Now,

$\tau_H = R'_{sig}[C_{gs} + C_{gd}(1 + g_m R_L')]$

$\quad + C_{gd} R_L' + C_L R_L'$

$310 = R'_{sig}[5 + 1(1 + 50)] + 1 \times 10 + 5 \times 10$

$\Rightarrow R'_{sig} = 4.46$ kΩ

10.61 To lower f_H from 135.5 MHz (see Example 10.8) to 100 MHz, τ_H must be increased to

$$\tau_H = \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 100 \times 10^6}$$

$= 1591.5$ ps

Now,

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$

$1591.5 = 200 + 725 + \tau_{CL}$

$\Rightarrow \tau_{CL} = 666.5$

But,

$\tau_{CL} = C_L' R_L'$

$665.5 = C_L' \times 10$

$\Rightarrow C_L' = 66.6$ fF

Thus, the original C_L of 25 fF must be increased by

$66.6 - 25 = 41.6$ fF

10.62 We will assume that the value given in the problem statement is for R_{sig} (not R'_{sig}):

$R_{sig} = 5$ kΩ

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

$R'_{sig} = r_\pi \parallel R_{sig} = 5\ \text{k}\Omega \parallel 5\ \text{k}\Omega$

$= 2.5\ \text{k}\Omega$

$\tau_H = C_\pi R_\pi = C_\pi R'_{sig}$

$= 10 \times 2.5 = 25$ ns

$\tau_\mu = C_\mu R_\mu$

$= C_\mu [R'_{sig}(1 + g_m R'_L) + R'_L]$

$= 1 \times [2.5(1 + 20 \times 5) + 5]$

$= 257.5$ ns

$\tau_{CL} = C_L R'_L$

$= 10 \times 5 = 50$ ns

$\tau_H = \tau_\pi + \tau_\mu + \tau_{CL}$

$= 25 + 257.5 + 50 = 332.5$ ns

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 332.5 \times 10^{-9}} = 479\ \text{kHz}$$

$$A_M = -\frac{r_\pi}{r_\pi + R_{sig}} g_m R'_L$$

$$= -\frac{5}{5+5} \times 20 \times 5$$

$= -50$ V/V

10.63 Refer to Fig. 10.19(a). Since R_B is not specified, we assume that its value is very large.

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m R'_L$$

where

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5\ \text{k}\Omega$$

Thus,

$$A_M = -\frac{2.5}{2.5 + 0.1 + 1} \times 40 \times 5$$

$= -138.9$ V/V

Using the Miller approximation, we obtain

$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$

$= 10 + 0.3(1 + 40 \times 5)$

$= 70.3$ pF

$R'_{sig} = r_\pi \parallel (r_x + R_{sig})$

$= 2.5 \parallel (0.1 + 1) = 0.76\ \text{k}\Omega$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 70.3 \times 10^{-12} \times 0.76 \times 10^3}$$

$= 2.98$ MHz

Using the open-circuit time constants, we get

$\tau_\pi = C_\pi R_\pi = C_\pi R'_{sig}$

$= 10 \times 0.76 = 7.6$ ns

$\tau_\mu = C_\mu [R'_{sig}(g_m R'_L + 1) + R'_L]$

$= 0.3[0.76(40 \times 5 + 1) + 5]$

$= 47.3$ ns

$\tau_{C_L} = C_L R'_L = 3 \times 5 = 15$ ns

$\tau_H = \tau_\pi + \tau_\mu + \tau_{C_L}$

$= 7.6 + 47.3 + 15 = 69.9$ ns

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 69.9 \times 10^{-9}} = 2.28\ \text{MHz}$$

This is a more realistic estimate of f_H as it takes into account the effect of C_L.

10.64 CS amplifier with:

$R'_{sig} = r_o/2$

$R'_L = r_o/2$

$C_{gs} = C_{gd} = 0.1$ pF

Using the Miller approximation, we get

$C_{in} = C_{gs} + C_{gd}(g_m R'_L + 1)$

$$= 0.1 + 0.1\left(\frac{1}{2} g_m r_o + 1\right)$$

$$= 0.1\left(\frac{1}{2} g_m r_o + 2\right)$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 0.1\left(\frac{1}{2} g_m r_o + 2\right)(r_o/2) \times 10^{-9}}$$

$$= \frac{1}{0.1\pi\left(\frac{1}{2} g_m r_o + 2\right) r_o \times 10^{-9}}$$

where r_o is in kΩ.

For initial design,

$g_m = 2$ mA/V, $\quad r_o = 20\ \text{k}\Omega$

$$f_H = \frac{1}{0.1\pi\left(\frac{1}{2} \times 40 + 2\right) \times 20 \times 10^{-9}}$$

$= 7.23$ MHz

(i) For the case, I is reduced by a factor of 4: Since

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}$$

reducing I_D by a factor of 4 reduces g_m by a factor of 2,

$g_m = 1$ mA/V

Since

$$r_o = \frac{V_A}{I_D},$$

reducing I_D by a factor of 4, increases r_o by a factor of 4,

$r_o = 80$ kΩ

Thus,

$$f_H = \frac{1}{0.1\pi\left(\frac{1}{2}\times 80 + 2\right)\times 80 \times 10^{-9}}$$

$= 0.95$ MHz

(ii) For the case, I is increased by a factor of 4, g_m increases by a factor of 2,

$g_m = 4$ mA/V

and r_o decreases by a factor of 4,

$r_o = 5$ kΩ

Thus,

$$f_H = \frac{1}{0.1\pi\left(\frac{1}{2}\times 20 + 2\right)\times 5 \times 10^{-9}}$$

$= 53.1$ MHz

10.65 $R'_L = r_o \parallel R_L = 20\text{ k}\Omega \parallel 12\text{ k}\Omega$

$= 7.5$ kΩ

$\tau_{gs} = C_{gs}R_{gs} = C_{gs}R'_{\text{sig}}$

$= 0.2 \times 100 = 20$ ns

$\tau_{gd} = C_{gd}[R'_{\text{sig}}(g_m R'_L + 1) + R'_L]$

$= 0.2[100(1.5 \times 7.5 + 1) + 7.5]$

$= 246.5$ ns

(a) $C_L = 0$

$\tau_{C_L} = 0$

$\tau_H = \tau_{gs} + \tau_{gd} = 20 + 246.5 = 266.5$ ns

$$f_H = \frac{1}{2\pi \times 266.5 \times 10^{-9}} = 597\text{ kHz}$$

(b) $C_L = 10$ pF

$\tau_{C_L} = C_L R'_L = 10 \times 7.5 = 75$ ns

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= 20 + 246.5 + 75 = 341.5$ ns

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 341.5 \times 10^{-9}} = 466\text{ kHz}$$

(c) $C_L = 50$ pF

$\tau_{C_L} = C_L R'_L = 50 \times 7.5 = 375$ ns

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$

$= 20 + 246.5 + 375$

$= 641.5$ ns

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 641.5 \times 10^{-9}} = 248\text{ kHz}$$

Using the Miller approximation, since C_L is not taken into account, then for all three cases we obtain

$C_{\text{in}} = C_{gs} + C_{gd}(g_m R'_L + 1)$

$= 0.2 + 0.2(1.5 \times 7.5 + 1)$

$= 2.65$ pF

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 2.65 \times 10^{-12} \times 100 \times 10^3}$$

$= 600$ kHz

which is very close to the estimate obtained using the method of open-circuit time constants for the case $C_L = 0$. However, as C_L is increased, the estimate obtained using the Miller approximation becomes less and less realistic, which is due to the fact that it does not take C_L into account.

10.66 Refer to Fig. 10.26(c).

$$\frac{V_o}{V_{\text{sig}}} = \frac{1/g_m}{\frac{1}{g_m} + R_{\text{sig}}} g_m R_L$$

$$= \frac{1/5}{\frac{1}{5} + 1} \times 5 \times 10$$

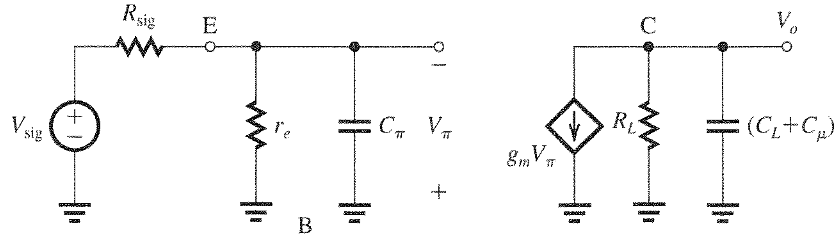
$= 8.3$ V/V

$$f_{P1} = \frac{1}{2\pi C_{gs}\left(R_{\text{sig}} \parallel \frac{1}{g_m}\right)}$$

$$f_{P1} = \frac{1}{2\pi \times 4 \times 10^{-12}\left(1 \parallel \frac{1}{5}\right) \times 10^3}$$

$= 239$ MHz

This figure belongs to Problem 10.67.



$$f_{P2} = \frac{1}{2\pi(C_L + C_{gd})R_L}$$

$$= \frac{1}{2\pi(2+0.2)\times 10^{-12}\times 10\times 10^3}$$

$= 7.23$ MHz

Since $f_{P1} \gg f_{P2}, f_{P2}$ will be dominant and

$f_H \simeq f_{P2} = 7.23$ MHz

10.67 See figure above. Replacing the BJT with its high-frequency T model while neglecting r_o and r_x results in the equivalent circuit shown in the figure.

(a) There are two separate poles, one at the input given by

$$f_{P1} = \frac{1}{2\pi C_\pi (R_{sig} \parallel r_e)}$$

and the other at the output, given by

$$f_{P2} = \frac{1}{2\pi(C_L + C_\mu)R_L} \qquad \text{Q.E.D.}$$

(b) $I_C = 1$ mA,

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_e \simeq \frac{1}{g_m} = 25\ \Omega$$

$$f_{P1} = \frac{1}{2\pi\times 10\times 10^{-12}(1 \parallel 0.025)\times 10^3}$$

$= 652.5$ MHz

$$f_{P2} = \frac{1}{2\pi(1+1)\times 10^{-12}\times 10\times 10^3}$$

$= 7.96$ MHz

Since $f_{P2} \ll f_{P1}, f_{P2}$ will be dominant and

$f_H \simeq f_{P2} = 7.96$ MHz

10.68 Refer to Fig. 10.26 with

$R_L = r_o$

$R_{sig} = r_o/2$

$C_L = C_{gs}$

Now,

$$R_{in} = \frac{r_o + R_L}{1 + g_m r_o} = \frac{r_o + r_o}{1 + g_m r_o}$$

$$\simeq \frac{2r_o}{g_m r_o} = \frac{2}{g_m}$$

$$R_{gs} = R_{sig} \parallel R_{in}$$

$$= \frac{r_o}{2} \parallel \frac{2}{g_m}$$

$$= \frac{\frac{r_o}{2}\times\frac{2}{g_m}}{\frac{r_o}{2}+\frac{2}{g_m}} = \frac{r_o}{\frac{1}{2}g_m r_o + 2}$$

$$\simeq \frac{r_o}{\frac{1}{2}g_m r_o} = \frac{2}{g_m}$$

$$\tau_{gs} = C_{gs}R_{gs} \simeq \frac{2C_{gs}}{g_m}$$

$$R_o = r_o + R_{sig} + g_m r_o R_{sig}$$

$$R_o = r_o + \frac{1}{2}r_o + \frac{1}{2}g_m r_o r_o$$

$$\simeq \frac{1}{2}g_m r_o^2$$

$$R_{gd} = R_L \parallel R_o$$

$$= r_o \parallel \frac{1}{2}g_m r_o^2$$

$$\simeq r_o$$

$$\tau_{gd} = (C_L + C_{gd})R_{gd}$$

$$= (C_{gs} + C_{gd})r_o$$

$$\tau_H = \tau_{gs} + \tau_{gd}$$

$$= \frac{2C_{gs}}{g_m} + (C_{gs} + C_{gd})r_o$$

Since $g_m r_o \gg 1$, we obtain

$$\tau_H \simeq (C_{gs} + C_{gd})r_o$$

and

$$f_H = \frac{1}{2\pi\tau_H}$$

$$f_H = \frac{1}{2\pi(C_{gs} + C_{gd})r_o}$$

Since

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

then

$$f_H = \frac{f_T}{g_m r_o} \qquad \text{Q.E.D.}$$

10.69 Refer to Example 10.9. To reduce f_H to 200 MHz, τ_H must become

$$\tau_H = \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 200 \times 10^6}$$

$= 795.8$ ps

Since τ_{gs} remains constant at 26.6 ps, τ_{CL} must be increased to

$\tau_{CL} = 795.8 - 26.6 = 769.2$ ps

But,

$\tau_{CL} = (C_{gd} + C_L)R_{gd}$

thus,

$769.2 = (5 + C_L) \times 18.7$

$\Rightarrow C_L + 5 = 41.1$ fF

$C_L = 36.1$ fF

Thus, the amount of additional capacitance to be connected at the output is

$36.1 - 25 = 11.1$ fF

10.70 $r_o = \dfrac{V_A}{I_D} = \dfrac{10}{0.1} = 100\ \text{k}\Omega$

$R_{\text{sig}} = r_o/2 = 50\ \text{k}\Omega$

$R_L = r_o = 100\ \text{k}\Omega$

$g_m r_o = 1.5 \times 100 = 150$

$$R_{\text{in}} = \frac{r_o + R_L}{1 + g_m r_o} = \frac{100 + 100}{1 + 150} = 1.32\ \text{k}\Omega$$

$R_o = R_{\text{sig}} + r_o + g_m r_o R_{\text{sig}}$

$= 50 + 100 + 150 \times 50 = 7650\ \text{k}\Omega$

$\tau_{gs} = C_{gs}(R_{\text{in}} \parallel R_{\text{sig}})$

$\tau_{gs} = 0.2(1.32 \parallel 50)$

$= 0.26$ ns

$R_{gd} = R_L \parallel R_o$

$= 100 \parallel 7650 = 98.7\ \text{k}\Omega$

$\tau_{gd} = (C_{gd} + C_L + C_{db})R_{gd}$

$= (0.015 + 0.03 + 0.02) \times 98.7$

$= 6.42$ ns

$\tau_H = \tau_{gs} + \tau_{gd} = 0.26 + 6.42$

$= 6.68$ ns

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 6.68 \times 10^{-9}} = 23.8\ \text{MHz}$$

10.71 $R_o = r_{o2} + r_{o1} + (g_{m2} r_{o2}) r_{o1}$

$= 2r_o + g_m r_o^2$

$= 2 \times 20 + 2 \times 20 \times 20 = 840\ \text{k}\Omega$

$A_v = -g_{m1}(R_o \parallel R_L)$

$= -2(840 \parallel 1000)$

$= -913$ V/V

Using Eq. (10.109), we obtain

$$\begin{aligned}\tau_H = {} & R_{\text{sig}}[C_{gs1} + C_{gd1}(1 + g_{m1}R_{d1})] \\ & + R_{d1}(C_{gd1} + C_{db1} + C_{gs2}) \\ & + (R_L \parallel R_o)(C_L + C_{gd2})\end{aligned}$$

where

$R_{d1} = r_{o1} \parallel R_{\text{in2}}$

$$R_{\text{in2}} = \frac{r_{o2} + R_L}{1 + g_{m2} r_{o2}}$$

$$= \frac{20 + 1000}{1 + 2 \times 20} = 24.9\ \text{k}\Omega$$

$R_{d1} = 20 \parallel 24.9 = 11.1\ \text{k}\Omega$

Thus,

$$\begin{aligned}\tau_H = {} & 100[20 + 5(1 + 2 \times 11.1)] \\ & + 11.1(5 + 5 + 20) \\ & + (1000 \parallel 840)(20 + 5)\end{aligned}$$

$= 13587 + 333 + 11413$

$= 25,333$ ps $= 25.33$ ns

$$f_H = \frac{1}{2\pi \times 25.33 \times 10^{-9}} = 6.28\ \text{MHz}$$

10.72 (a) $A_M = -g_m R'_L$

where

$R'_L = R_L \parallel r_o = 20 \parallel 20 = 10\ \text{k}\Omega$

Thus,

$A_M = -4 \times 10 = -40$ V/V

$\tau_{gs} = C_{gs} R_{gs}$

$= C_{gs} R_{\text{sig}} = 2 \times 20 = 40$ ns

$R_{gd} = R_{\text{sig}}(1 + g_m R_L') + R_L'$

$= 20(1 + 4 \times 10) + 10$

$= 830\ \text{k}\Omega$

$\tau_{gd} = C_{gd} R_{gd} = 0.2 \times 830 = 166\ \text{ns}$

$\tau_{C_L} = C_L R_L' = 1 \times 10 = 10\ \text{ns}$

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= 40 + 166 + 10 = 216\ \text{ns}$

$f_H = \dfrac{1}{2\pi \tau_H}$

$= \dfrac{1}{2\pi \times 216 \times 10^{-9}} = 737\ \text{kHz}$

$\text{GB} \equiv |A_M| f_H = 40 \times 737 = 29.5\ \text{MHz}$

(b) $A_M = -g_{m1}(R_o \parallel R_L)$

where

$R_o = r_{o1} + r_{o2} + g_{m2} r_{o2} r_{o1}$

$= 2r_o + g_m r_o^2$

$= 2 \times 20 + 4 \times 20 \times 20 = 1640\ \text{k}\Omega$

$A_M = -4(1640 \parallel 20) = -79\ \text{V/V}$

$R_{\text{in}2} = \dfrac{r_{o2} + R_L}{1 + g_{m2} r_{o2}}$

$= \dfrac{20 + 20}{1 + 4 \times 20} \simeq 0.49\ \text{k}\Omega$

$R_{d1} = r_{o1} \parallel R_{\text{in}2} = 20 \parallel 0.49 = 0.48\ \text{k}\Omega$

Using Eq. (10.109), we obtain

$\tau_H = R_{\text{sig}}[C_{gs1} + C_{gd1}(1 + g_{m1} R_{d1})]$

$\quad + R_{d1}(C_{gd1} + C_{db1} + C_{gs2})$

$\quad + (R_L \parallel R_o)(C_L + C_{gd2})$

$= 20[2 + 0.2(1 + 4 \times 0.48)]$

$\quad + 0.48(0.2 + 0.2 + 2)$

$\quad + (20 \parallel 1640)(1 + 0.2)$

$\tau_H = 51.7 + 1.15 + 23.7 = 76.6\ \text{ns}$

$f_H = \dfrac{1}{2\pi \times 76.6 \times 10^{-9}} = 2.08\ \text{MHz}$

$\text{GB} \equiv |A_M| f_H = 79 \times 2.08 = 164\ \text{MHz}$

Note the increase in bandwidth and in GB.

10.73 $20 \log |A_M| = 74\ \text{dB}$

$\Rightarrow |A_M| = 5000$

$R_o \simeq (g_m r_o) r_o$

$R_L = R_o$

$|A_M| = g_m(R_L \parallel R_o)$

$5000 = \dfrac{1}{2} g_m R_o$

$= \dfrac{1}{2}(g_m r_o)^2$

$\Rightarrow g_m r_o = 100$

$\dfrac{2V_A}{V_{OV}} = 100$

$\Rightarrow V_{OV} = \dfrac{2 \times 10}{100} = 0.2\ \text{V}$

$I_D = \dfrac{1}{2} \mu_n C_{ox} \left(\dfrac{W}{L}\right) V_{OV}^2$

$= \dfrac{1}{2} \times 0.2 \times 50 \times 0.2^2$

$= 0.2\ \text{mA}$

$f_t = \dfrac{g_m}{2\pi (C_L + C_{gd})}$

where

$g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.2}{0.2} = 2\ \text{mA/V}$

$f_t = \dfrac{2 \times 10^{-3}}{2\pi (1 + 0.1) \times 10^{-12}}$

$= 289.4\ \text{MHz}$

$f_{3dB} = \dfrac{f_t}{|A_M|} = \dfrac{289.4}{5000} = 57.9\ \text{kHz}$

If the cascode transistor is removed,

$A_M = -g_m(r_o \parallel R_L)$

$\simeq -g_m r_o = -100\ \text{V/V}$

10.74 (a) For the CS amplifier,

$|A_M| = g_m(r_o \parallel r_o) = \dfrac{1}{2} g_m r_o$

$f_H = \dfrac{1}{2\pi C_{\text{in}} R_{\text{sig}}}$

$$= \frac{1}{2\pi \left[C_{gs} + C_{gd}\left(\frac{1}{2} g_m r_o + 1\right)\right] R_{\text{sig}}} \qquad (1)$$

For the cascode amplifier,

$|A_M| = g_m(R_o \parallel r_o)$

$= g_m[(g_m r_o) r_o \parallel r_o]$

$\simeq g_m r_o$

Thus, the gain increases by a factor of 2.

$f_H = \dfrac{1}{2\pi C_{\text{in}} R_{\text{sig}}}$

where

$$C_{\text{in}} = C_{gs} + C_{gd}(1 + g_m R_{d1})$$

$$R_{d1} = r_o \parallel R_{\text{in2}}$$

$$= r_o \parallel \frac{R_L + r_o}{g_m r_o}$$

$$R_{d1} = r_o \parallel \frac{r_o + r_o}{g_m r_o}$$

$$= r_o \parallel \frac{2}{g_m} \simeq \frac{2}{g_m}$$

$$C_{\text{in}} = C_{gs} + C_{gd}\left(1 + g_m \times \frac{2}{g_m}\right)$$

$$= C_{gs} + 3C_{gd}$$

$$f_H = \frac{1}{2\pi(C_{gs} + 3C_{gd})R_{\text{sig}}} \quad (2)$$

From (1) and (2), the ratio N of f_H of the cascode amplifier to f_H of the CS amplifier is

$$N = \frac{C_{gs} + C_{gd}\left(\frac{1}{2}g_m r_o + 1\right)}{C_{gs} + 3C_{gd}}$$

Thus,

$$N \simeq \frac{C_{gs} + \frac{1}{2}(g_m r_o)C_{gd}}{C_{gs} + 3C_{gd}} \qquad \text{Q.E.D.}$$

(b) $50 = \frac{1}{2}g_m r_o$

$$\Rightarrow g_m r_o = 100$$

$$N = \frac{C_{gs} + \frac{1}{2} \times 100 \times 0.1C_{gs}}{C_{gs} + 3 \times 0.1C_{gs}}$$

$$= \frac{1 + 5}{1 + 0.3} = 4.6$$

(c) $g_m r_o = \frac{2V_A}{V_{OV}}$

$$100 = \frac{2 \times 10}{V_{OV}}$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$= \frac{1}{2} \times 0.4 \times 10 \times 0.2^2$$

$$= 0.08 \text{ mA} = 80\ \mu\text{A}$$

10.75 (a) From Fig. 10.29, we have

$$f_t = \frac{g_m}{2\pi(C_L + C_{gd})}$$

$$= \frac{\sqrt{2\mu_n C_{ox}(W/L)}}{2\pi(C_L + C_{gd})}\sqrt{I_D} \qquad \text{Q.E.D.} \qquad (1)$$

(b) $g_m = \sqrt{2\mu_n C_{ox}(W/L)}\sqrt{I_D}$ (2)

$$V_{OV} = \sqrt{\frac{I_D}{\frac{1}{2}\mu_n C_{ox}(W/L)}} \quad (3)$$

$$r_o = \frac{V_A}{I_D} \quad (4)$$

$$R_o = (g_m r_o)r_o \quad (5)$$

$$A_M = -g_m(R_o \parallel R_L)$$

$$= -g_m(R_o \parallel R_o)$$

$$= -\frac{1}{2}g_m R_o \quad (6)$$

Substituting

$\mu_n C_{ox} = 0.4 \text{ mA/V}^2$, $W/L = 20$, $C_L = 20$ fF,

$C_{gd} = 5$ fF and $V_A = 10$ V in Eqs. (1)–(6), we obtain the following results in the table below.

10.76 Refer to Fig. 10.30.

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{\text{sig}}} g_m(\beta r_o \parallel R_L)$$

$$= -\frac{2.5}{2.5 + 0.05 + 5} 40(100 \times 100 \parallel 2)$$

$$\simeq -26.5 \text{ (V/V)}$$

This table belongs to Problem 10.75, part (b).

I_D (mA)	f_t (GHz)	V_{OV} (V)	g_m (mA/V)	r_o (kΩ)	R_o (MΩ)	A_M (V/V)	f_H (MHz)
0.1	8	0.16	1.26	100	12.6	−7938	1
0.2	11.5	0.22	1.80	50	4.5	−4050	2.8
0.5	18	0.35	2.83	20	1.13	−1600	11.3

$R'_{sig} = r_\pi \parallel (r_x + R_{sig})$

$= 2.5 \parallel (0.05 + 5) = 1.67\text{ k}\Omega$

$R_{\pi 1} = R'_{sig} = 1.67\text{ k}\Omega$

$$R_{c1} = r_{o1} \parallel \left[r_{e2} \frac{r_{o2} + R_L}{r_{o2} + R_L/(\beta_2 + 1)} \right]$$

$$= 100 \parallel \left[0.025 \frac{100 + 2}{100 + \frac{2}{101}} \right]$$

$= 25.5\ \Omega$

$R_{\mu 1} = R'_{sig}(1 + g_{m1} R_{c1}) + R_{c1}$

$= 1.67(1 + 40 \times 0.0255) + 0.0255$

$= 3.4\text{ k}\Omega$

$R_o = \beta_2 r_{o2} = 100 \times 100 = 10{,}000\text{ k}\Omega$

$\tau_H = C_{\pi 1} R_{\pi 1} + C_{\mu 1} R_{\mu 1} + (C_{cs1} + C_{\pi 2}) R_{c1}$

$+ (C_L + C_{cs2} + C_{\mu 2})(R_L \parallel R_o)$

$= 10 \times 1.67 + 2 \times 3.4 + (0 + 10) \times 0.0255$

$+ (0 + 0 + 2)(2 \parallel 10{,}000)$

$= 16.7 + 6.8 + 0.255 + 4 = 27.8\text{ ns}$

$$f_H = \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 27.8 \times 10^{-9}}$$

$= 5.7\text{ MHz}$

10.77 (a) Gain from base to collector of $Q_1 = -1$. Thus,

$C_{in} = C_{\pi 1} + C_{\mu 1}(1 + 1)$

$= C_{\pi 1} + 2C_{\mu 1}$

$$f_{P1} = \frac{1}{2\pi R'_{sig} C_{in}}$$

$$= \frac{1}{2\pi R'_{sig}(C_{\pi 1} + 2C_\mu)} \qquad \text{Q.E.D.}$$

At the output node, the total capacitance is $(C_L + C_{c2} + C_{\mu 2})$ and since r_o is large, R_o will be very large, thus the total resistance will be R_L. Thus the pole introduced at the output node will have a frequency f_{P2},

$$f_{P2} = \frac{1}{2\pi(C_L + C_{c2} + C_{\mu 2}) R_L} \qquad \text{Q.E.D.}$$

(b) $I = 1\text{ mA}$

$$g_m = \frac{1\text{ mA}}{0.025\text{ V}} = 40\text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5\text{ k}\Omega$$

(i) $R_{sig} = 1\text{ k}\Omega$

$R'_{sig} = r_\pi \parallel R_{sig} = 2.5 \parallel 1 = 0.71\text{ k}\Omega$

$$f_{P1} = \frac{1}{2\pi \times 0.71 \times 10^3 (10 + 2 \times 2) \times 10^{-12}}$$

$= 16\text{ MHz}$

$$f_{P2} = \frac{1}{2\pi(0 + 0 + 2) \times 10^{-12} \times 2 \times 10^3}$$

$\simeq 40\text{ MHz}$

$$f_H = 1 \Big/ \sqrt{\frac{1}{f_{p1}^2} + \frac{1}{f_{p2}^2}}$$

$$= 1 \Big/ \sqrt{\frac{1}{16^2} + \frac{1}{40^2}} = 14.9\text{ MHz}$$

(ii) $R_{sig} = 10\text{ k}\Omega$

$R'_{sig} = r_\pi \parallel R_{sig} = 2.5 \parallel 10 = 2\text{ k}\Omega$

$$f_{P1} = \frac{1}{2\pi \times 2 \times 10^3 (10 + 4) \times 10^{-12}}$$

$= 5.7\text{ MHz}$

$f_{P2} = 40\text{ MHz}$

$$f_H = 1 \Big/ \sqrt{\frac{1}{5.7^2} + \frac{1}{40^2}} = 5.6\text{ MHz}$$

10.78 Refer to Fig. 10.30.

$I_C = 0.1\text{ mA}$

$$g_m = \frac{0.1\text{ mA}}{0.025\text{ V}} = 4\text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25\text{ k}\Omega$$

$$r_o = \frac{V_A}{I_D} = \frac{100\text{ V}}{0.1\text{ mA}} = 1000\text{ k}\Omega$$

$$r_e \simeq \frac{1}{g_m} = 0.25\text{ k}\Omega$$

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m(\beta r_o \parallel R_L)$$

$$= -\frac{r_\pi}{r_\pi + r_\pi} g_m(\beta r_o \parallel \beta r_o)$$

$$= -\frac{1}{4} \beta g_m r_o$$

$$= -\frac{1}{4} \times 100 \times 4 \times 1000$$

$= -100{,}000\text{ V/V}$

$$R'_{sig} = r_\pi \parallel R_{sig} = r_\pi \parallel r_\pi = \frac{1}{2} r_\pi = 12.5 \text{ k}\Omega$$

$$R_{\pi 1} = R'_{sig} = 12.5 \text{ k}\Omega$$

$$R_{c1} = r_o \parallel r_e \left(\frac{r_o + R_L}{r_o + \dfrac{R_L}{\beta + 1}} \right)$$

$$= 1000 \parallel 0.25 \left(\frac{r_o + \beta r_o}{r_o + \dfrac{\beta}{\beta + 1} r_o} \right)$$

$$\simeq 1000 \parallel 0.25 \times \frac{\beta + 1}{2}$$

$$= 1000 \parallel 12.5 = 12.3 \text{ k}\Omega$$

$$R_{\mu 1} = R'_{sig}(1 + g_{m1} R_{c1}) + R_{c1}$$

$$= 12.5(1 + 4 \times 12.3) + 12.3 = 639.8 \text{ k}\Omega$$

$$R_o = \beta r_o = 100 \times 1000 = 100 \text{ M}\Omega$$

$$\tau_H = C_{\pi 1} R_{\pi 1} + C_{\mu 1} R_{\mu 1} + (C_{cs1} + C_{\pi 2}) R_{c1}$$

$$+ (C_L + C_{c2} + C_{\mu 2})(R_L \parallel R_o)$$

To determine C_π, we use

$$f_T = \frac{g_m}{2\pi (C_\pi + C_\mu)}$$

$$1 \times 10^9 = \frac{4 \times 10^{-3}}{2\pi (C_\pi + C_\mu)}$$

$$\Rightarrow C_\pi + C_\mu = 0.64 \text{ pF}$$

$$C_\pi = 0.64 - 0.1 = 0.54 \text{ pF}$$

$$\tau_H = 0.54 \times 12.5 + 0.1 \times 639.8 + 0.54 \times 12.3$$

$$+ 0.1 \times \frac{1}{2} \times 100 \times 1000$$

$$= 6.8 + 64 + 6.6 + 5000 \text{ ns}$$

Obviously the last term, which is due to the pole at the output node, is dominant. The frequency of the output pole is

$$f_P = \frac{1}{2\pi \times 5000 \times 10^{-9}} = 31.8 \text{ kHz}$$

$$f_H \simeq f_P = 31.8 \text{ MHz}$$

Because the other poles are at much higher frequencies, an estimate of the unity-gain frequency can be found as

$$f_t = |A_M| f_P = 10^5 \times 31.8 \times 10^3 = 3.18 \text{ GHz}$$

This estimate of f_t is not very good (too high!). The other three poles have frequencies much lower than 3.18 GHz and will cause the gain to decrease faster, reaching the 0 dB value at a frequency lower than 3.18 GHz. Also note that f_T of the BJTs is 1 GHz and the models we use for the BJT do not hold at frequencies approaching f_T.

10.79 $A_M = \dfrac{R'_L}{R'_L + \dfrac{1}{g_m}}$

where

$$R'_L = R_L \parallel r_o \parallel \frac{1}{g_{mb}}$$

$$= 2 \parallel 20 = 1.82 \text{ k}\Omega$$

$$A_M = \frac{1.82}{1.82 + \dfrac{1}{5}} = 0.91 \text{ V/V}$$

$$R_o = r_o \parallel \frac{1}{g_m} = 20 \parallel \frac{1}{5} \simeq 0.2 \text{ k}\Omega = 200 \text{ }\Omega$$

$$f_Z = \frac{g_m}{2\pi C_{gs}}$$

$$= \frac{5 \times 10^{-3}}{2\pi \times 2 \times 10^{-12}} = 398 \text{ MHz}$$

Next, we evaluate b_1 and b_2:

$$b_1 = \left(C_{gd} + \frac{C_{gs}}{g_m R'_L + 1} \right) R_{sig} + \left(\frac{C_{gs} + C_L}{g_m R'_L + 1} \right) R'_L$$

$$= \left(0.1 + \frac{2}{5 \times 1.82 + 1} \right) 20 + \left(\frac{2 + 1}{5 \times 1.82 + 1} \right) \times 1.82$$

$$= 5.96 + 0.54 = 6.50 \times 10^{-9} \text{ s}$$

$$b_2 = \frac{(C_{gs} + C_{gd}) C_L + C_{gs} C_{gd}}{g_m R'_L + 1} R_{sig} R'_L$$

$$= \frac{(2 + 0.1) \times 1 + 2 \times 0.1}{5 \times 1.82 + 1} \times 20 \times 1.82$$

$$= 8.3 \times 10^{-18}$$

$$Q = \frac{\sqrt{b_2}}{b_1} = \frac{\sqrt{8.3}}{6.5} = 0.44$$

Thus, the poles are real and their frequencies can be obtained by finding the roots of the polynomial $(1 + b_1 s + b_2 s^2)$

$$= 1 + 6.5 \times 10^{-9} s + 8.3 \times 10^{-18} s^2$$

which are

$$\omega_{P1} = 0.21 \times 10^9 \text{ rad/s}$$

and

$\omega_{P2} = 0.57 \times 10^9$ rad/s

Thus,

$$f_{P1} = \frac{\omega_{P1}}{2\pi} = 33.4 \text{ MHz}$$

$$f_{P2} = \frac{\omega_{P2}}{2\pi} = 90.7 \text{ MHz}$$

Since the two poles are relatively close to each other, an estimate of f_H can be obtained using

$$f_H = 1 \bigg/ \sqrt{\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}}$$

$= 31.6$ MHz

10.80 $f_H \simeq f_{P1} \simeq \dfrac{1}{2\pi b_1}$

where

$$b_1 = \left(C_{gd} + \frac{C_{gs}}{g_m R_L' + 1}\right) R_{\text{sig}} + \left(\frac{C_{gs} + C_L}{g_m R_L' + 1}\right) R_L'$$

For $C_L = 0$,

$$b_1 = C_{gd} R_{\text{sig}} + \frac{C_{gs}}{g_m R_L' + 1}(R_{\text{sig}} + R_L')$$

For $R_{\text{sig}} \gg R_L'$,

$$b_1 \simeq C_{gd} R_{\text{sig}} + \frac{C_{gs}}{g_m R_L' + 1} R_{\text{sig}}$$

$$= \left(C_{gd} + \frac{C_{gs}}{g_m R_L' + 1}\right) R_{\text{sig}}$$

and

$$f_H = \frac{1}{2\pi R_{\text{sig}}\left(C_{gd} + \dfrac{C_{gs}}{g_m R_L' + 1}\right)} \qquad \text{Q.E.D.}$$

For the given numerical values,

$$f_H = \frac{1}{2\pi \times 100 \times 10^3 \left[10 + \dfrac{2}{5 \times (2 \parallel 20) + 1}\right] \times 10^{-12}}$$

$= 156$ kHz

(Note: An error was made in the first printing of the book and the values of C_{gs} and C_{gd} were exchanged. The above value of f_H corresponds to the numbers in the first printing.)

For $C_{gs} = 10$ pF and $C_{gd} = 2$ pF,

$$f_H = \frac{1}{2\pi \times 100 \times 10^3 \left[2 + \dfrac{10}{5 \times (2 \parallel 20) + 1}\right] \times 10^{-12}}$$

$= 532$ kHz

10.81 Refer to Fig. 10.31(c). Replacing C_{gs} with an input capacitance between G and ground, we get

$$C_{\text{eq}} = C_{gs}(1 - K)$$

where

$$K = \frac{g_m R_L'}{1 + g_m R_L'}$$

then

$$C_{\text{eq}} = C_{gs}/(1 + g_m R_L')$$

and the total input capacitance becomes

$$C_{\text{in}} = C_{gd} + C_{\text{eq}}$$

$$= C_{gd} + \frac{C_{gs}}{1 + g_m R_L'}$$

The frequency of the input pole is

$$f_{P1} = \frac{1}{2\pi R_{\text{sig}}\left(C_{gd} + \dfrac{C_{gs}}{1 + g_m R_L'}\right)}$$

$f_H \simeq f_{P1}$

10.82 For a maximally flat response we have

$$Q = \frac{1}{\sqrt{2}}$$

$\omega_0 = \omega_{3dB} = 2\pi \times 10^6$ rad/s

Thus, the transfer function will be

$$\frac{V_o(s)}{V_i(s)} = \frac{\text{dc gain}}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

$$= \frac{0.8}{s^2 + s\sqrt{2} \times 2\pi \times 10^6 + (2\pi \times 10^6)^2}$$

$$= \frac{0.8}{s^2 + s\ 8.886 \times 10^6 s + 39.48 \times 10^{12}}$$

10.83 With $g_{mb} = 0$ and r_o large, we obtain

$R_L' \simeq R_L$

and

$$A_M = \frac{g_m R_L}{g_m R_L + 1}$$

For $A_M = 0.9$,

$$0.9 = \frac{g_m R_L}{g_m R_L + 1}$$

$\Rightarrow g_m R_L = 9$

Now, for a maximally-flat response, $Q = 1/\sqrt{2}$. Using the expression for Q in Eq. (10.129), we get

$$Q = \frac{\sqrt{g_m R_L + 1}\sqrt{[(C_{gs} + C_{gd})C_L + C_{gs}C_{gd}]R_{\text{sig}}R_L}}{[C_{gs} + C_{gd}(g_m R_L + 1)]R_{\text{sig}} + (C_{gs} + C_L)R_L}$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{9+1}\sqrt{[(10+1)10 + 10 \times 1] \times 100 \times R_L}}{[10 + 1(9+1)] \times 100 + (10+10)R_L}$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{3}\sqrt{R_L}}{10\left(1 + \frac{R_L}{100}\right)}$$

$$\Rightarrow \left(\frac{R_L}{100}\right)^2 - 4\left(\frac{R_L}{100}\right) + 1 = 0$$

This equation results in two solutions,

$R_L = 27\ \text{k}\Omega$ and $R_L = 373\ \text{k}\Omega$

The second answer is not very practical as it implies the transistor is operating at $g_m = 9/373 = 0.024$ mA/V, a very small transconductance!. We will pursue only the first answer. Thus,

$R_L = 27\ \text{k}\Omega$

$g_m = 0.33$ mA/V

and the 3-dB frequency is found using Eq. (10.127):

$$f_{3dB} = f_0 = \frac{1}{2\pi\sqrt{b_2}}$$

$$\omega_{3dB} = \sqrt{\frac{g_m R_L + 1}{R_{\text{sig}} R_L[(C_{gs} + C_{gd})C_L + C_{gs}C_{gd}]}}$$

$$= \sqrt{\frac{9+1}{100 \times 27[(10+1) \times 10 + 10 \times 1] \times 10^6 \times 10^{-24}}}$$

$= 5.55$ Mrad/s

$f_{3dB} = 884$ kHz

10.84 Refer to Fig. 10.33.

$I_C = 1$ mA

$g_m = 40$ mA/V, $\quad r_e = 25\ \Omega$

$$r_\pi = \frac{100}{40} = 2.5\ \text{k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{20}{1} = 20\ \text{k}\Omega$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$2 \times 10^9 = \frac{40 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$\Rightarrow C_\pi + C_\mu = 3.2$ pF

$C_\pi = 3.2 - 0.1 = 3.1$ pF

$R'_L = R_L \parallel r_o = 1 \parallel 20 = 0.95\ \text{k}\Omega$

$R'_{\text{sig}} = R_{\text{sig}} + r_x = 1 + 0.1 = 1.1\ \text{k}\Omega$

$$A_M = \frac{R'_L}{R'_L + r_e + \frac{R'_{\text{sig}}}{\beta + 1}}$$

$$A_M = \frac{0.95}{0.95 + 0.025 + \frac{1.1}{101}}$$

$= 0.96$ V/V

$$f_Z = \frac{1}{2\pi C_\pi r_e}$$

$$= \frac{1}{2\pi \times 3.1 \times 10^{-12} \times 25}$$

$\simeq 2$ GHz

$$b_1 = \frac{\left[C_\pi + C_\mu\left(1 + \frac{R'_L}{r_e}\right)\right]R'_{\text{sig}} + \left[C_\mu + C_L\left(1 + \frac{R'_{\text{sig}}}{r_\pi}\right)\right]R'_L}{1 + \frac{R'_L}{r_e} + \frac{R'_{\text{sig}}}{r_\pi}}$$

$$= \frac{\left[3.1 + 0.1\left(1 + \frac{0.95}{0.025}\right)\right] \times 1.1 + (3.1 + 0) \times 0.95}{1 + \frac{0.95}{0.025} + \frac{1.1}{2.5}}$$

$= 0.27 \times 10^{-9}$

$$b_2 = \frac{[(C_\pi + C_\mu)C_L + C_\pi C_\mu]R'_L R'_{\text{sig}}}{1 + \frac{R'_L}{r_e} + \frac{R'_{\text{sig}}}{r_\pi}}$$

$$= \frac{(0 + 3.1 \times 0.1) \times 0.95 \times 1.1}{1 + \frac{0.95}{0.025} + \frac{1.1}{2.5}}$$

$= 8.2 \times 10^{-21}$

$$Q = \frac{\sqrt{b_2}}{b_1} = \frac{\sqrt{8.2 \times 10^{-21}}}{0.27 \times 10^{-9}} = 0.335$$

Thus, the poles are real and their frequencies can be found as the roots of the polynomial $(1 + b_1 s + b_2 s^e)$

$= 1 + 0.27 \times 10^{-9}s + 8.2 \times 10^{-21}s^2$

$$= \left(1 + \frac{s}{\omega_{P1}}\right)\left(1 + \frac{s}{\omega_{P2}}\right)$$

$\Rightarrow \omega_{P1} = 4.25 \times 10^9$ rad/s

$\omega_{P2} = 28.6 \times 10^9$ rad/s

Thus,

$f_{P1} = 676$ MHz

$f_{P2} = 4.6$ GHz

Thus,

$f_{3dB} \simeq f_{P1} = 676$ MHz

10.85 $I = 0.4$ mA

(a) $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$

$0.2 = \frac{1}{2} \times 0.4 \times 16V_{OV}^2$

$\Rightarrow V_{OV} = 0.25$ V

$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.25}$

$= 1.6$ mA/V

(b) $r_o = \frac{V_A}{I_D} = \frac{20}{0.2}$

$= 100$ kΩ

$R_D \parallel r_o = 10 \parallel 100$

$= 9.1$ kΩ

$A_d = g_m(R_D \parallel r_o)$

$= 1.6 \times 9.1 = 14.5$ V/V

(c) R_{sig} small and the frequency response is determined by the output pole:

$$f_{P2} = \frac{1}{2\pi(C_L + C_{gd} + C_{db})(R_D \parallel r_o)}$$

$$= \frac{1}{2\pi(100 + 5 + 5) \times 10^{-15} \times 9.1 \times 10^3}$$

$= 159$ MHz

$f_H \simeq 159$ MHz

(d) $R_{\text{sig}} = 40$ kΩ

$\tau_{gs} = C_{gs}R_{gs}$

$= C_{gs}R_{\text{sig}}$

$= 40 \times 10^{-15} \times 40 \times 10^3$

$= 1.6$ ns

$R_{gd} = R_{\text{sig}}(g_m R_L' + 1) + R_L'$

$= 40(1.6 \times 9.1 + 1) + 9.1$

$= 631.5$ kΩ

$\tau_{gd} = C_{gd}R_{gd} = 5 \times 631.5 = 3.16$ ns

$\tau_{CL} = (C_L + C_{db})R_L'$

$= (100 + 5) \times 9.1$

$= 955.5$ ps $= 0.96$ ns

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$

$= 1.6 + 3.16 + 0.96 = 5.72$ ns

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 5.72 \times 10^{-9}}$$

$= 27.8$ MHz

10.86 The common-mode gain will have a zero at

$$f_Z = \frac{1}{2\pi R_{SS}C_{SS}}$$

$$= \frac{1}{2\pi \times 100 \times 10^3 \times 1 \times 10^{-12}}$$

$= 1.59$ MHz

Thus, the CMRR will have two poles, one at f_Z, i.e. at 1.59 MHz, and the other at the dominant pole of A_d, 20 MHz. Thus, the 3-dB frequency of CMRR will be approximately equal to f_Z,

$f_{3dB} = 1.59$ MHz

10.87 At low frequencies,

$A_d = 100$ V/V

$A_{cm} = 0.1$ V/V

$\text{CMRR} = \frac{A_d}{A_{cm}} = 1000$ or 60 dB

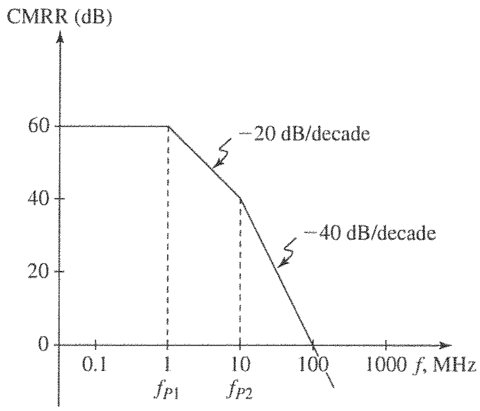
The first pole of CMRR is coincident with the zero of the common-mode gain,

$f_{P1} = 1$ MHz

The second pole is coincident with the dominant pole of the differential gain,

$f_{P2} = 10$ MHz

A sketch for the Bode plot for the gain magnitude is shown in the figure.



10.88 $R_{SS} = \dfrac{V_A}{I} = \dfrac{40}{0.1}$

$= 400$ kΩ

$C_{SS} = 100$ fF

$$f_Z = \frac{1}{2\pi C_{SS} R_{SS}}$$

$$= \frac{1}{2\pi \times 100 \times 10^{-15} \times 400 \times 10^3} = 4 \text{ MHz}$$

If V_{OV} of the current source is reduced by a factor of 2 while I remains unchanged, (W/L) must be increased by a factor of 4. Assume L remains unchanged, W must be increased by a factor of 4. Since C_{SS} is proportional to W, its value will be quadrupled:

$C_{SS} = 400$ fF

The output resistance R_{SS} will remain unchanged. Thus, f_Z will decrease by a factor of 4 to become

$f_Z = 1$ MHz

10.89 $I = 80$ μA

$\dfrac{W}{L} = 100, \quad k_n' = 0.2 \text{ mA/V}^2, \quad V_A = 20$ V,

$C_{gs} = 50$ fF, $\quad C_{gd} = 10$ fF, $\quad C_{db} = 10$ fF,

$R_D = 20$ kΩ,

$C_L = 100$ fF

(a) $I_D = \dfrac{1}{2} k_n' \left(\dfrac{W}{L}\right) V_{OV}^2$

$$0.040 = \frac{1}{2} \times 0.2 \times 100 \times V_{OV}^2$$

$\Rightarrow V_{OV} = 0.063$ V

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.04}{0.063} = 1.27 \text{ mA/V}$$

(b) $r_o = \dfrac{V_A}{I_D} = \dfrac{20}{0.04} = 500$ kΩ

$A_d = g_m(R_D \parallel r_o)$

$= 1.27(20 \parallel 500) = 1.27 \times 19.2 = 24.4$ V/V

(c) $f_H \simeq f_{P2} = \dfrac{1}{2\pi(C_L + C_{gd} + C_{db})(R_D \parallel r_o)}$

$$= \frac{1}{2\pi(100 + 10 + 10) \times 10^{-15} \times 19.2 \times 10^3}$$

$= 69.1$ MHz

(d) $R_{sig} = 100$ kΩ

$\tau_{gs} = C_{gs} R_{gs} = C_{gs} R_{sig} = 50 \times 100 = 5$ ns

$R_{gd} = R_{sig}(1 + g_m R_L') + R_L'$

$= 100(1 + 1.27 \times 19.2) + 19.2$

$= 2559$ kΩ

$\tau_{gd} = C_{gd} R_{gd} = 10 \times 2559$

$= 25.6$ ns

$\tau_{CL} = (C_L + C_{db}) R_L'$

$= (100 + 10) \times 19.2 = 2.1$ ns

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$

$= 5 + 25.6 + 2.1 = 32.7$ ns

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 32.7 \times 10^{-9}}$$

$= 4.9$ MHz

10.90 $g_m = \frac{I_C}{V_T} = \frac{0.25 \text{ mA}}{0.025 \text{ V}} = 10 \text{ mA/V}$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{10} = 10 \text{ k}\Omega$

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$

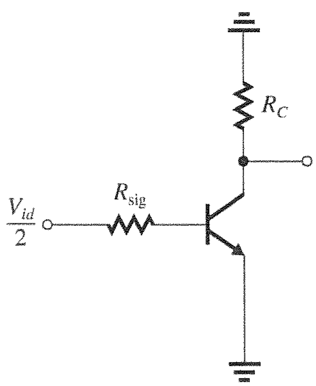
$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$

$= \frac{10 \times 10^{-3}}{2\pi \times 500 \times 10^6}$

$= 3.2 \text{ pF}$

$C_\pi = 3.2 - 0.5 = 2.7 \text{ pF}$

(a)



The figure shows the differential half-circuit and its high-frequency equivalent circuit.

(b) $A_d \equiv \frac{V_o}{V_{id}} = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m R_C$

$= -\frac{10}{10 + 0.1 + 10} \times 10 \times 10$

$= -49.8 \text{ V/V}$

(c) $C_{in} = C_\pi + C_\mu(1 + g_m R_C)$

$= 2.7 + 0.5(1 + 10 \times 10)$

$= 53.2 \text{ pF}$

$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$

where

$R'_{sig} = r_\pi \parallel (R_{sig} + r_x)$

$= 10 \parallel 10.1 \simeq 5 \text{ k}\Omega$

$f_H = \frac{1}{2\pi \times 53.2 \times 10^{-12} \times 5 \times 10^3} = 598 \text{ kHz}$

$\text{GB} = |A_d| f_H = 49.8 \times 598 = 29.8 \text{ MHz}$

10.91 The common-mode gain will have a zero at

$f_Z = \frac{1}{2\pi \times 1 \times 10^6 \times 1 \times 10^{-12}}$

$= 159 \text{ kHz}$

Thus, the CMRR will have two poles: The first will be coincident with the zero of A_{cm},

$f_{P1} = 159 \text{ kHz}$

and the second will be coincident with the pole of A_d,

$f_{P2} = 2 \text{ MHz}$

10.92 $g_{m1,2} = \frac{2I_{D1,2}}{V_{OV1,2}} = \frac{I}{V_{OV1,2}}$

$= \frac{0.2 \text{ mA}}{0.2 \text{ V}} = 1 \text{ mA/V}$

$r_{o2} = r_{o4} = \frac{|V_A|}{I_D} = \frac{10 \text{ V}}{0.1 \text{ mA}} = 100 \text{ k}\Omega$

$A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$

$= 1(100 \parallel 100) = 50 \text{ V/V}$

$f_{P1} = \frac{1}{2\pi C_L R_o}$

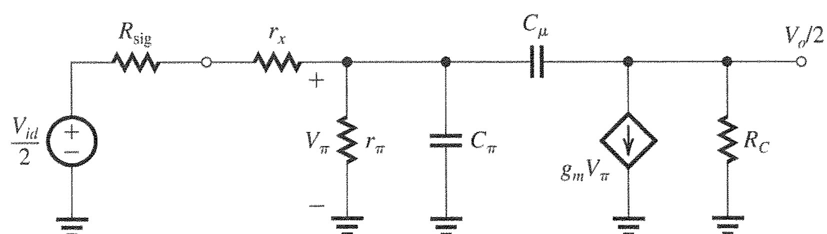
$= \frac{1}{2\pi C_L (r_{o2} \parallel r_{o4})}$

$= \frac{1}{2\pi \times 0.2 \times 10^{-12}(100 \parallel 100) \times 10^3}$

$= \frac{1}{2\pi \times 0.2 \times 50 \times 10^{-9}} = 15.9 \text{ MHz}$

$f_{P2} = \frac{g_{m3}}{2\pi C_m}$

This figure belongs to Problem 10.90.



where

$$g_{m3} = \frac{2I_D}{|V_{OV}|} = \frac{I}{|V_{OV}|} = 1 \text{ mA/V}$$

$$f_{P2} = \frac{1 \times 10^{-3}}{2\pi \times 0.1 \times 10^{-12}} = 1.59 \text{ GHz}$$

$$f_Z = \frac{2g_{m3}}{2\pi C_m} = 2f_{P2} = 2 \times 1.59 = 3.18 \text{ GHz}$$

10.93 $A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$

$$g_{m1,2} = \frac{2I_D}{|V_{OV}|} = \frac{I}{|V_{OV}|}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{2|V_A|}{I}$$

$$A_d = \frac{I}{|V_{OV}|}\left(\frac{2|V_A|}{I} \parallel \frac{2|V_A|}{I}\right)$$

$$A_d = \frac{|V_A|}{|V_{OV}|}$$

$$f_{P1} = \frac{1}{2\pi C_L R_o}$$

where

$$R_o = r_{o2} \parallel r_{o4} = \frac{|V_A|}{I}$$

$$f_{P1} = \frac{I}{2\pi C_L |V_A|} \qquad (1)$$

$$f_{P2} = \frac{g_{m3}}{2\pi C_m}$$

where

$$g_{m3} = \frac{2I_D}{|V_{OV}|} = \frac{I}{|V_{OV}|}$$

$$C_m = \frac{C_L}{4}$$

$$f_{P2} = \frac{4I}{2\pi C_L |V_{OV}|} \qquad (2)$$

$$f_Z = \frac{2g_{m3}}{2\pi C_m} = 2f_{P2} = \frac{8I}{2\pi C_L |V_{OV}|} \qquad (3)$$

Dividing (2) by (1), we obtain

$$\frac{f_{P2}}{f_{P1}} = 4\frac{|V_A|}{|V_{OV}|} = 4\,A_d \qquad \text{Q.E.D.}$$

Since $f_{P2} = 4\,A_d\,f_{P1}$, the unity-gain frequency f_t is equal to GB, thus

$$f_t = A_d\,f_{P1}$$

$$= \frac{|V_A|}{|V_{OV}|}\,\frac{I}{2\pi C_L |V_A|}$$

$$f_t = \frac{I/|V_{OV}|}{2\pi C_L}$$

$$= \frac{g_m}{2\pi C_L} \qquad \text{Q.E.D.}$$

For the numerical values given, we have

$$A_d = \frac{20}{0.2} = 100 \text{ V/V}$$

$$g_m = \frac{I}{V_{OV}} = \frac{0.2}{0.2} = 1 \text{ mA/V}$$

$$f_{P1} = \frac{I}{2\pi C_L |V_A|}$$

$$= \frac{0.2 \times 10^{-3}}{2\pi \times 100 \times 10^{-15} \times 20}$$

$= 15.9$ MHz

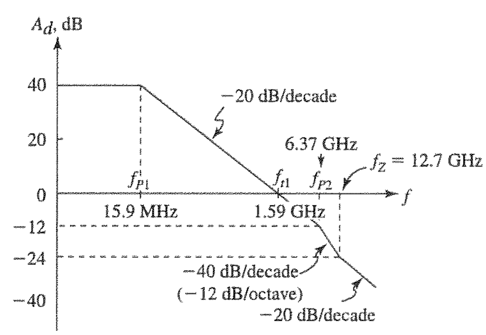
$f_t = 15.9 \times 100 = 1.59$ GHz

$f_{P2} = 4A_d f_{P1} = 4 \times 100 \times 15.9$

$= 6.37$ GHz

$f_Z = 2f_{P2} = 12.7$ GHz

A sketch of the Bode plot for $|A_d|$ is shown in the figure.



10.94 See figure on next page. The mirror high-frequency equivalent circuit is shown in the figure. Note that we have neglected r_x and r_o. The model of the diode-connected transistor Q_1 reduces to r_{e1} in parallel with $C_{\pi 1}$.

To obtain the current-transfer function $I_o(s)/I_i(s)$, we first determine V_π in terms of I_i. Observe that the short-circuit at the output causes $C_{\mu 2}$ to appear in parallel with $C_{\pi 1}$ and $C_{\pi 2}$. Thus,

$$V_\pi = I_i(s)\frac{1}{\left(\dfrac{1}{r_{e1}} + \dfrac{1}{r_{\pi 2}}\right) + s(C_{\pi 1} + C_{\pi 2} + C_{\mu 2})} \qquad (1)$$

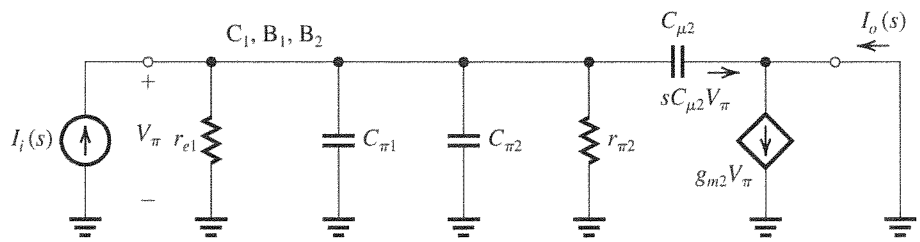
At the output node we have

$$I_o(s) = g_{m2}V_\pi - sC_{\mu 2}V_\pi \qquad (2)$$

Combining Eqs. (1) and (2) gives

$$\frac{I_o(s)}{I_i(s)} = \frac{g_{m2} - sC_{\mu 2}}{\left(\dfrac{1}{r_{e1}} + \dfrac{1}{r_{\pi 2}}\right) + s(C_{\pi 1} + C_{\pi 2} + C_{\mu 2})}$$

This figure belongs to Problem 10.94.



Since the two transistors are operating at approximately equal dc bias currents, their small-signal parameters will be equal, thus

$$\frac{I_o(s)}{I_i(s)} = \frac{g_m - sC_\mu}{\frac{1}{r_e}\left(1 + \frac{1}{\beta+1}\right) + s(2C_\pi + C_\mu)}$$

$$= \frac{g_m r_e}{1 + \frac{1}{\beta+1}} \frac{1 - s(C_\mu/g_m)}{1 + s\left[(2C_\pi + C_\mu)r_e\Big/\left(1 + \frac{1}{\beta+1}\right)\right]}$$

$$= \frac{\alpha}{1 + \frac{1}{\beta+1}} \frac{1 - s(C_\mu/g_m)}{1 + s\left[(2C_\pi + C_\mu)r_e\Big/\left(1 + \frac{1}{\beta+1}\right)\right]}$$

$$= \frac{1}{1 + 2/\beta} \frac{1 - s(C_\mu/g_m)}{1 + s\left[(2C_\pi + C_\mu)r_e\Big/\left(1 + \frac{1}{\beta+1}\right)\right]}$$

Thus we see that the low-frequency transmission is

$$\frac{I_o}{I_i}(0) = \frac{1}{1 + \frac{2}{\beta}}$$

as expected. The pole is at f_P,

$$f_P \simeq \frac{1}{2\pi\left[(2C_\pi + C_\mu)r_e\Big/\left(1 + \frac{1}{\beta}\right)\right]}$$

and the zero is at

$$f_Z = \frac{g_m}{2\pi C_\mu}$$

For the numerical values given,

$$g_m = \frac{I_C}{V_T} = \frac{1\text{ mA}}{0.025\text{ V}} = 40\text{ mA/V}$$

$$r_e \simeq 25\ \Omega$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 500 \times 100^6}$$

$$= 12.7\text{ pF}$$

$$C_\pi = 12.7 - 2 = 10.7\text{ pF}$$

$$f_P = \frac{1}{2\pi[(2 \times 10.7 + 2) \times 10^{-12} \times 25/1.01]}$$

$$= \frac{1.01 \times 10^{12}}{2\pi \times 23.4 \times 25}$$

$$= 274.8\text{ MHz}$$

$$f_Z = \frac{g_m}{2\pi C_\mu}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 2 \times 10^{-12}} = 3.18\text{ GHz}$$

10.95 Refer to Eqs. (10.146)–(10.153). For our case,

$$G_m = \frac{g_m}{1 + g_m R_s} \tag{1}$$

$R_o =$ very large

$$R_L' = R_L \parallel R_o = R_L$$

$$A_m = -G_m R_L = \frac{-g_m R_L}{1 + g_m R_s} \tag{2}$$

$$R_{gd} = R_{\text{sig}}(1 + G_m R_L) + R_L \tag{3}$$

$$R_{gs} = \frac{R_{\text{sig}} + R_s}{1 + g_m R_s} \tag{4}$$

$$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

For the numerical values given:

(a) $R_s = 0$

$$G_m = g_m = 5\text{ mA/V}$$

$$A_M = -g_m R_L = -5 \times 5 = -25\text{ V/V}$$

$$R_{gd} = 100(1 + 5 \times 5) + 5 = 2605\text{ k}\Omega$$

$$\tau_H = 10 \times 100 + 2 \times 2605 = 6.21\text{ ns}$$

$$f_H = \frac{1}{2\pi \times 6.21 \times 10^{-9}} = 26.6\text{ MHz}$$

$$\text{GB} = 26.6 \times 25 = 641\text{ MHz}$$

(b) $R_s = 100\ \Omega$

$$G_m = \frac{5}{1 + 5 \times 0.1} = 3.33\text{ mA/V}$$

$A_M = -3.33 \times 5 = -16.7$ V/V

$R_{gd} = 100(1 + 3.33 \times 5) + 5$

$= 1771.7$ kΩ

$$R_{gs} = \frac{100 + 0.1}{1 + 5 \times 0.1} = 66.7 \text{ k}\Omega$$

$\tau_H = 10 \times 66.7 + 2 \times 1771.7 = 4.21$ ns

$$f_H = \frac{1}{2\pi \times 4.21 \times 10^{-9}} = 37.8 \text{ MHz}$$

GB = 631 MHz

(c) $R_s = 200\ \Omega$

$$G_m = \frac{5}{1 + 5 \times 0.2} = 2.5 \text{ mA/V}$$

$A_M = -2.5 \times 5 = -12.5$ V/V

$R_{gd} = 100(1 + 2.5 \times 5) + 5 = 1355$ kΩ

$$R_{gs} = \frac{100 + 0.2}{1 + 5 \times 0.2} = 50.1 \text{ k}\Omega$$

$\tau_H = 10 \times 50.1 + 2 \times 1355$ ns

$$f_H = \frac{1}{2\pi \times 3.21 \times 10^{-9}} = 49.6 \text{ MHz}$$

GB = 49.6 × 12.5 = 620 MHz

A summary of the results is provided in the following table:

	$R_s = 0$	$R_s = 100\ \Omega$	$R_s = 200\ \Omega$
$\|A_M\|$ (V/V)	25	16.7	12.5
$f_H\|$ (MHz)	26.6	37.8	49.6
GB (MHz)	641	631	620

Observe that increasing R_s trades off gain for bandwidth while GB remains approximately constant.

10.96 (a) $A_M = -g_m R'_L$

where

$R'_L = R_L \parallel r_o$

$= 40 \parallel 40 = 20$ kΩ

$A_M = -5 \times 20 = -100$ V/V

$\tau_{gs} = C_{gs} R_{gs} = C_{gs} R_{\text{sig}}$

$= 2 \times 20 = 40$ ns

$R_{gd} = R_{\text{sig}}(1 + g_m R'_L) + R'_L$

$= 20(1 + 5 \times 20) + 20$

$= 2040$ kΩ

$\tau_{gd} = C_{gd} R_{gd} = 0.1 \times 2040$

$= 204$ ns

$\tau_{CL} = C_L R'_L$

$= 1 \times 20 = 20$ ns

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$

$= 40 + 204 + 20 = 264$ ns

$$f_H = \frac{1}{2\pi \times 264 \times 10^{-9}} = 603 \text{ kHz}$$

GB = 100 × 603 = 60.3 MHz

(b) With $R_s = 400\ \Omega$,

$$G_m = \frac{g_m}{1 + g_m R_s}$$

$$= \frac{5}{1 + 5 \times 0.4} = 1.67 \text{ mA/V}$$

$R_o = r_o(1 + g_m R_s)$

$= 40(1 + 5 \times 0.4) = 120$ kΩ

$R'_L = R_L \parallel R_o = 40 \parallel 120 = 30$ kΩ

$A_M = -G_m R'_L$

$= -1.67 \times 30 = -50$ V/V

$R_{gd} = R_{\text{sig}}(1 + G_m R'_L) + R'_L$

$= 20(1 + 1.67 \times 30) + 30$

$= 1050$ kΩ

$\tau_{gd} = C_{gd} R_{gd} = 0.1 \times 1050 = 105$ ns

$\tau_{CL} = C_L R_{CL}$

$= C_L R'_L$

$= 1 \times 30 = 30$ ns

$$R_{gs} = \frac{R_{\text{sig}} + R_s + R_{\text{sig}} R_s/(r_o + R_L)}{1 + g_m R_s\left(\dfrac{r_o}{r_o + R_L}\right)}$$

$$= \frac{20 + 0.4 + 20 \times 0.4/(40 + 40)}{1 + 5 \times 0.4\left(\dfrac{40}{40 + 40}\right)}$$

$= 10.25$ kΩ

$\tau_{gs} = C_{gs} R_{gs} = 2 \times 10.25 = 20.5$ ns

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$

$= 20.5 + 105 + 30 = 155.5$ ns

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 155.5 \times 10^{-9}} = 1.02 \text{ MHz}$$

GB = 51.2 MHz

10.97 (a) $\text{GB} = |A_M| f_H$

$$= \frac{1}{2\pi C_{gd} R_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 0.2 \times 10^{-12} \times 100 \times 10^3}$$

$= 7.96$ MHz

(b) $|A_M| = 20$ V/V

$$f_H = \frac{7.96}{20} = 398 \text{ kHz}$$

(c) $A_0 = g_m r_o$

$100 = 5 \times r_o$

$\Rightarrow r_o = 20\ \text{k}\Omega$

$$G_m = \frac{g_m}{1 + g_m R_s} = \frac{5}{1 + g_m R_s}$$

$$R_o = r_o(1 + g_m R_s) = 20(1 + g_m R_s)$$

$$R_L' = R_L \parallel R_o = 20 \parallel 20(1 + g_m R_s)$$

$$A_M = -G_m R_L'$$

$$20 = \frac{5}{1 + g_m R_s}[20 \parallel 20(1 + g_m R_s)]$$

$$4(1 + g_m R_s) = \frac{20 \times 20(1 + g_m R_s)}{20 + 20(1 + g_m R_s)}$$

$\Rightarrow 1 + g_m R_s = 4$

$$\Rightarrow R_s = \frac{3}{g_m} = 0.6\ \text{k}\Omega = 600\ \Omega$$

10.98 $G_m = \dfrac{g_m}{1 + g_m R_s} = \dfrac{g_m}{1 + k}$

$$R_L' = R_L \parallel R_o$$

$$= r_o \parallel r_o(1 + g_m R_s)$$

$$= r_o \parallel r_o(1 + k)$$

$$= \frac{r_o \times r_o(1 + k)}{r_o + r_o(1 + k)}$$

$$= r_o \frac{1 + k}{2 + k}$$

$$A_M = -G_m R_L' = -\frac{g_m r_o}{2 + k}$$

Thus,

$$A_M = \frac{-A_0}{2 + k} \qquad \text{Q.E.D.}$$

$$R_{gs} = \frac{R_{\text{sig}} + R_s + R_{\text{sig}} R_s/(r_o + R_L)}{1 + g_m R_s \left(\dfrac{r_o}{r_o + R_L}\right)}$$

$$= \frac{R_{\text{sig}} + R_s + R_{\text{sig}} R_s/2r_o}{1 + \dfrac{1}{2} g_m R_s}$$

For $R_{\text{sig}} \gg R_s$,

$$R_{gs} \simeq \frac{R_{\text{sig}}(1 + R_s/2r_o)}{1 + (k/2)}$$

For $r_o \gg R_s$,

$$R_{gs} \simeq \frac{R_{\text{sig}}}{1 + (k/2)}$$

$$\tau_{gs} = C_{gs} R_{gs} = \frac{C_{gs} R_{\text{sig}}}{1 + (k/2)}$$

$$R_{gd} = R_{\text{sig}}(1 + G_m R_L') + R_L$$

Utilizing the expressions for R_L' and $G_m R_L'$ derived earlier, we obtain

$$R_{gd} = R_{\text{sig}}\left[1 + \frac{A_0}{2 + k}\right] + r_o\left(\frac{1 + k}{2 + k}\right)$$

$$\tau_{gd} = C_{gs} R_{gd} =$$

$$C_{gd} R_{\text{sig}}\left(1 + \frac{A_0}{2 + k}\right) + C_{gd} r_o\left(\frac{1 + k}{2 + k}\right)$$

$$\tau_{C_L} = C_L R_L'$$

$$= C_L r_o \frac{1 + k}{2 + k}$$

Thus,

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$$

$$= \frac{C_{gs} R_{\text{sig}}}{1 + (k/2)} + C_{gd} R_{\text{sig}}\left(1 + \frac{A_0}{2 + k}\right)$$

$$+ C_{gd} r_o\left(\frac{1 + k}{2 + k}\right) + C_L r_o\left(\frac{1 + k}{2 + k}\right)$$

$$= \frac{C_{gs} R_{\text{sig}}}{1 + (k/2)} + C_{gd} R_{\text{sig}}\left(1 + \frac{A_0}{2 + k}\right)$$

$$+ (C_L + C_{gd}) r_o\left(\frac{1 + k}{2 + k}\right) \qquad \text{Q.E.D.}$$

10.99 Substituting the given numerical values in the expressions for A_M and τ_H given in the statement for Problem 10.98 and noting that $A_0 = g_m r_o = 5 \times 40 = 200$, we obtain

$$f_H = \frac{1}{2\pi \tau_H}$$

and

$$\text{GB} = |A_M| f_H$$

we obtain the results in the following table.

k	$\lvert A_M\rvert$, V/V	τ_H ns	f_H (MHz)	GB (MHz)
0	100	264	0.603	60.3
1	66.7	191.3	0.832	55.6
2	50	155	1.03	51.5
3	40	133.2	1.19	47.6
4	33.3	118.7	1.34	44.6
5	28.6	108.3	1.47	42.0
6	25	100.5	1.58	39.5
7	22.2	94.4	1.69	37.5
8	20	89.6	1.78	35.6
9	18.2	85.7	1.86	33.9
10	16.7	82.3	1.93	32.2
11	15.4	79.6	2.00	30.8
12	14.3	77.2	2.06	29.5
13	13.3	75.1	2.12	28.2
14	12.5	73.3	2.17	27.1
15	11.8	71.6	2.22	26.2

To obtain $f_H = 2$ MHz, we see from the table that

$k = 11$

Thus,

$1 + g_m R_s = 11$

$\Rightarrow R_s = \dfrac{10}{2} = 5\text{ k}\Omega$

The gain achieved is

$|A_M| = 15.4$ V/V

10.100 (a) Refer to Fig. P10.100(a). Since the total resistance at the drain is r_o, we have

$A_M = -g_m r_o \qquad$ Q.E.D.

$\tau_{gs} = C_{gs}R_{gs} = C_{gs}R_{\text{sig}}$

$R_{gd} = R_{\text{sig}}(1 + g_m R_L') + R_L'$

$= R_{\text{sig}}(1 + g_m r_o) + r_o$

$\tau_{gd} = C_{gd}R_{gd} = C_{gd}[R_{\text{sig}}(1 + g_m r_o) + r_o]$

$\tau_{C_L} = C_L R_L' = C_L r_o$

Thus,

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= C_{gs}R_{\text{sig}} + C_{gd}[R_{\text{sig}}(1 + g_m r_o) + r_o]$

$\quad + C_L r_o \qquad$ Q.E.D.

For the given numerical values,

$A_M = -1 \times 20 = -20$ V/V

$\tau_H = 20 \times 20 + 5[20(1 + 1 \times 20) + 20] + 10 \times 20$

$= 400 + 2200 + 200 = 2800\text{ ps} = 2.8\text{ ns}$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 2.8 \times 10^{-9}}$$

$= 56.8$ MHz

$\text{GB} = 20 \times 56.8 = 1.14$ GHz

(b) From Fig. 1 we see that

$$\frac{V_{g1}}{V_{\text{sig}}} = 1$$

$$\frac{V_{g2}}{V_{g1}} = \frac{r_{o1}}{\dfrac{1}{g_{m1}} + r_{o1}}$$

$$\frac{V_o}{V_{g2}} = -g_{m2}r_{o2}$$

Thus,

$$A_M = 1 \times \frac{r_{o1}}{\dfrac{1}{g_{m1}} + r_{o1}} \times -g_{m2}r_{o2}$$

$$= -\frac{r_{o1}}{1/g_{m1} + r_{o1}}(g_{m2}r_{o2}) \qquad \text{Q.E.D.}$$

This figure belongs to Problem 10.100, part (b).

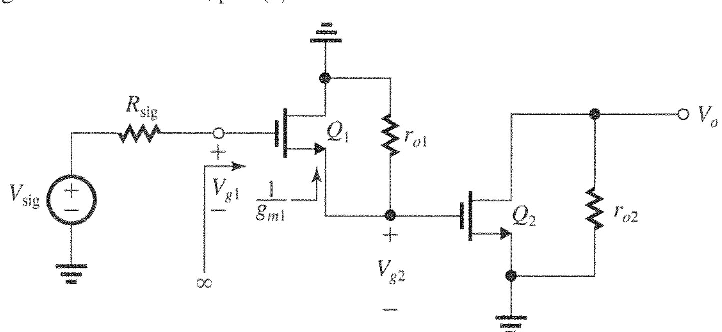


Figure 1

Next we evaluate the open-circuit time constants. Refer to Fig. 2.

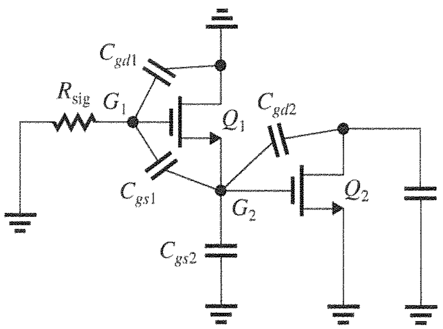


Figure 2

C_{gd1}: Capacitor C_{gd1} is between G_1 and ground and thus sees the resistance R_{sig},

$$R_{gd1} = R_{sig}$$

$$\tau_{gd1} = C_{gd1}R_{sig}$$

C_{gs1}: To find the resistance R_{gs1} seen by capacitor C_{gs1}, we replace Q_1 with its hybrid-π equivalent circuit with V_{sig} set to zero, $C_{gd1} = 0$, and C_{gs1} replaced by a test voltage V_x. The resulting equivalent circuit is shown in Fig. 3.

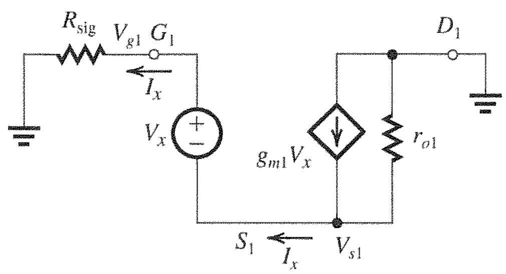


Figure 3

Analysis of the circuit in Fig. 3 proceeds as follows:

$$V_{g1} = I_x R_{sig}$$

$$V_{s1} = V_{g1} - V_x = I_x R_{sig} - V_x$$

Node equation at S_1,

$$I_x = g_{m1}V_x - \frac{V_{s1}}{r_{o1}}$$

$$= g_{m1}V_x - \frac{I_x R_{sig} - V_x}{r_{o1}}$$

$$I_x\left(1 + \frac{R_{sig}}{r_{o1}}\right) = V_x\left(g_{m1} + \frac{1}{r_{o1}}\right)$$

Thus,

$$R_{gs1} \equiv \frac{V_x}{I_x} = \frac{R_{sig} + r_{o1}}{1 + g_{m1}r_{o1}}$$

$$\tau_{gs1} = C_{gs1}R_{gs1}$$

$$= C_{gs1}\frac{R_{sig} + r_{o1}}{1 + g_{m1}r_{o1}}$$

C_{gs2}: Capacitor C_{gs2} sees the resistance between G_2 and ground, which is the output resistance of source follower Q_1,

$$R_{gs2} = \frac{1}{g_{m1}} \parallel r_{o1}$$

Thus,

$$\tau_{gs2} = C_{gs2}\left(\frac{1}{g_{m1}} \parallel r_{o1}\right)$$

C_{gd2}: Transistor Q_2 operates as a CS amplifier with an equivalent signal-source resistance equal to the output resistance of the source follower Q_2, that is, $\left(\frac{1}{g_{m1}} \parallel r_{o1}\right)$ and with a gain from gate to drain of $g_{m2}r_{o2}$. Thus, the formula for R_{gd} in a CS amplifier can be adapted as follows:

$$R_{gd2} = \left(\frac{1}{g_{m1}} \parallel r_{o1}\right)(1 + g_{m2}r_{o2}) + r_{o2}$$

and thus,

$$\tau_{gd2} = C_{gd2}\left[\left(\frac{1}{g_{m1}} \parallel r_{o1}\right)(1 + g_{m2}r_{o2}) + r_{o2}\right]$$

C_L: Capacitor C_L sees the resistance between D_2, and ground which is r_{o2},

$$\tau_{C_L} = C_L r_{o2}$$

Summing τ_{gd1}, τ_{gs1}, τ_{gs2}, τ_{gd2} and τ_{C_L} gives τ_H in the problem statement. Q.E.D.

For the given numerical values:

$$A_M = -\frac{20}{1 + 20}(1 \times 20)$$

$$= -19 \text{ V/V}$$

$$\tau_{gd1} = C_{gd1}R_{sig} = 5 \times 20 = 100 \text{ ps}$$

$$\tau_{gs1} = C_{gs1}\frac{R_{sig} + r_{o1}}{1 + g_m r_{o1}}$$

$$= 20\,\frac{20 + 20}{1 + 1 \times 20} = 38 \text{ ps}$$

$$\tau_{gs2} = C_{gs2}\left(\frac{1}{g_{m1}} \parallel r_{o1}\right)$$

$$= 20 \times (1 \parallel 20) = 19 \text{ ps}$$

$$\tau_{gd2} = C_{gd2}\left[\left(\frac{1}{g_{m1}} \parallel r_{o1}\right)(1 + g_m r_{o2}) + r_{o2}\right]$$

$$= 5[(1 \parallel 20)(1 + 20) + 20]$$

$$= 200 \text{ ps}$$

$\tau_{C_L} = C_L r_{o2} = 10 \times 20 = 200$ ps

$\tau_H = 100 + 38 + 19 + 200 + 200 = 557$ ps

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 557 \times 10^{-12}}$$

$= 286$ MHz

GB $= 19 \times 286 = 5.43$ GHz

Thus, while the dc gain remained approximately the same both f_H and GB increased by a factor of about 5!

10.101 At an emitter bias current of 0.1 mA, Q_1 and Q_2 have

$g_m = 4$ mA/V

$r_e = 250\ \Omega$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25\text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.1} = 1000\text{ k}\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{4 \times 10^{-3}}{2\pi \times 200 \times 10^6} = 3.2\text{ pF}$$

$C_\mu = 0.2$ pF

$C_\pi = 3$ pF

To determine R_{in} and the voltage gain A_M, refer to the circuit in Fig. 10.40(a). Here, however, R_L is r_{o2}.

$R_{in2} = r_{\pi 2} = 25\text{ k}\Omega$

$R_{in} = (\beta_1 + 1)[r_{e1} + (r_{o1} \parallel R_{in2})]$

$= 101[0.25 + (1000 \parallel 25)]$

$\simeq 2.5\text{ M}\Omega$

$$\frac{V_{b1}}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{2.5\text{ M}\Omega}{2.5\text{ M}\Omega + 10\text{ k}\Omega} \simeq 1\text{ V/V}$$

$$\frac{V_{b2}}{V_{b1}} = \frac{(R_{in2} \parallel r_{o1})}{(R_{in2} \parallel r_{o1}) + r_{e1}}$$

$$= \frac{25 \parallel 1000}{(25 \parallel 1000) + 0.25} = 0.99 \simeq 1\text{ V/V}$$

$$\frac{V_o}{V_{b2}} = -g_{m2} r_{o2} = -4 \times 1000 = -4000\text{ V/V}$$

Thus,

$$A_M = \frac{V_o}{V_{sig}} = -4000\text{ V/V}$$

To determine f_H we use the method of open-circuit time constants. Figure 10.40(b) shows the circuit with $V_{sig} = 0$ and the four capacitances indicated. Again, recall that here $R_L = r_{o2}$. Also, in our present circuit there is a capacitance C_L at the output.

Capacitance $C_{\mu 1}$ sees a resistance $R_{\mu 1}$,

$R_{\mu 1} = R_{sig} \parallel R_{in}$

$= 10\text{ k}\Omega \parallel 2.5\text{ M}\Omega \simeq 10\text{ k}\Omega$

To find the resistance $R_{\pi 1}$ we refer to the circuit in Fig. 10.40(c) where R_{in2} is considered to include r_{o2},

$R_{in2} = 25\text{ k}\Omega \parallel 1000\text{ k}\Omega = 24.4\text{ k}\Omega$

We use the formula for $R_{\pi 1}$ given in Example 10.13:

$$R_{\pi 1} = \frac{R_{sig} + R_{in2}}{1 + \frac{R_{sig}}{r_{\pi 1}} + \frac{R_{in2}}{r_{e1}}}$$

$$R_{\pi 1} = \frac{10 + 24.4}{1 + \frac{10}{25} + \frac{24.4}{0.25}} = 347\ \Omega$$

Capacitance $C_{\pi 2}$ sees a resistance $R_{\pi 2}$:

$R_{\pi 2} = R_{in2} \parallel R_{out1}$

$$= r_{\pi 2} \parallel r_{o1} \parallel \left[r_{e1} + \frac{R_{sig}}{\beta_1 + 1}\right]$$

$$= 25 \parallel 1000 \parallel \left[0.25 + \frac{10}{101}\right]$$

$= 344\ \Omega$

Capacitance $C_{\mu 2}$ sees a resistance $R_{\mu 2}$:

$R_{\mu 2} = (1 + g_{m2} r_{o2})(R_{in2} \parallel R_{out1}) + r_{o2}$

$= (1 + 4 \times 1000) \times 0.344 + 1000$

$= 2376\text{ k}\Omega$

We can determine τ_H from

$$\tau_H = C_{\mu 1}R_{\mu 1} + C_{\pi 1}R_{\pi 1} + C_{\mu 2}R_{\mu 2} + C_{\pi 2}R_{\pi 2} + C_L r_o$$

$= 0.2 \times 10 + 3 \times 0.347 + 0.2 \times 2376$

$+ 3 \times 0.344 + 1 \times 1000$

$\tau_H = 2 + 1 + 475.2 + 1 + 1000$

$= 1479.2$ ns

Observe that there are two dominant capacitances: the most significant is C_L and the second most significant is $C_{\mu 2}$.

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 1479.2 \times 10^{-9}} = 107.6\text{ kHz}$$

10.102 $g_m = \frac{2I_D}{V_{OV}} = \frac{2(I/2)}{V_{OV}}$

$$= \frac{I}{V_{OV}} = \frac{0.2\text{ mA}}{0.2\text{ V}} = 1\text{ mA/V}$$

$$\frac{V_o}{V_{sig}} = \frac{R_D}{2/g_m}$$

$$= \frac{1}{2}g_m R_D = \frac{1}{2} \times 1 \times 50 = 25\text{ V/V}$$

The high-frequency analysis can be performed in an analogous manner to that used in the text for the bipolar circuit. Refer to Fig. 10.42(b) and adapt the circuit for the MOS case. Thus,

$$f_{P1} = \frac{1}{2\pi R_{sig}\left(\frac{C_{gs}}{2} + C_{gd}\right)}$$

$$= \frac{1}{2\pi \times 100 \times 10^3\left(\frac{4}{2} + 0.5\right) \times 10^{-12}}$$

$$= 637\text{ kHz}$$

and

$$f_{P2} = \frac{1}{2\pi R_D C_\mu}$$

$$= \frac{1}{2\pi \times 50 \times 10^3 \times 0.5 \times 10^{-12}}$$

$$= 6.37\text{ MHz}$$

Since $f_{P2} \simeq 10 f_{P1}$, the pole at f_{P1} will dominate and

$$f_H \simeq f_{P1} = 637\text{ kHz}$$

10.103 $g_m = \frac{I_C}{V_T} \simeq \frac{1\text{ mA}}{0.025\text{ V}} = 40\text{ mA/V}$

$$r_e \simeq 25\ \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{40} = 3\text{ k}\Omega$$

$$R_{in} = 2r_\pi = 6\text{ k}\Omega$$

$$\frac{V_o}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} \frac{\alpha R_L}{2r_e}$$

$$\simeq \frac{6}{6+12} \times \frac{10}{2 \times 0.025} = 66.7\text{ V/V}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 500 \times 10^6} = 12.7\text{ pF}$$

$$C_\mu = 0.5\text{ pF}$$

$$C_\pi = 12.2\text{ pF}$$

$$f_{P1} = \frac{1}{2\pi R_{sig}\left(\frac{C_\pi}{2} + C_\mu\right)}$$

$$= \frac{1}{2\pi \times 12 \times 10^3\left(\frac{12.2}{2} + 0.5\right) \times 10^{-12}}$$

$$= 2\text{ MHz}$$

$$f_{P2} = \frac{1}{2\pi R_L C_\mu}$$

$$= \frac{1}{2\pi \times 10 \times 10^3 \times 0.5 \times 10^{-12}} = 31.8\text{ MHz}$$

Thus, f_{P1} is the dominant pole and

$$f_H \simeq f_{P1} = 2\text{ MHz}$$

10.104 Using an approach analogous to that utilized for the BJT circuit (Fig. 10.42), we see that there is a pole at the input with frequency f_{P1}:

$$f_{P1} = \frac{1}{2\pi R_{sig}\left(\frac{C_{gs}}{2} + C_{gd}\right)}$$

$$f_{P1} = \frac{1}{2\pi \times 20 \times 10^3\left(\frac{2}{2} + 0.1\right) \times 10^{-12}}$$

$$= 7.2\text{ MHz},$$

and a pole at the output with frequency f_{P2},

$$f_{P2} = \frac{1}{2\pi(C_{gd} + C_L)R_L}$$

$$= \frac{1}{2\pi \times (0.1 + 1) \times 10^{-12} \times 20 \times 10^3}$$

$$= 7.2\text{ MHz}$$

Thus,

$$f_{P1} = f_{P2} = 7.2\text{ MHz}$$

The midband gain A_M is obtained as

$$A_M = \frac{R_L}{2/g_m} = \frac{1}{2}g_m R_L$$

$$= \frac{1}{2} \times 5 \times 20 = 50\text{ V/V}$$

Thus, the amplifier transfer function is

$$\frac{V_o(s)}{V_{sig}(s)} = \frac{50}{\left(1 + \frac{s}{2\pi \times 7.2 \times 10^6}\right)^2}$$

$$\left|\frac{V_o}{V_{sig}}\right| = \frac{50}{1 + \left(\frac{\omega}{2\pi \times 7.2 \times 10^6}\right)^2}$$

At $\omega = \omega_{3dB}$, $\left|\frac{V_o}{V_i}\right| = \frac{50}{\sqrt{2}}$, thus

$$\sqrt{2} = 1 + \left(\frac{\omega_{3dB}}{2\pi \times 7.2 \times 10^6}\right)^2$$

$$f_{3dB} = \sqrt{\sqrt{2} - 1} \times 7.2 \text{ MHz}$$

$$= 4.6 \text{ MHz}$$

10.105 (a) A CS amplifier for which the gain is low so that the Miller effect is negligible and for which $C_{gd} \ll C_{gs}$ has an input capacitance that is approximately given by

$$C_{\text{in}} \simeq C_{gs}$$

Now,

$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}} \simeq \frac{g_m}{C_{gs}}$$

thus,

$$C_{gs} \simeq \frac{g_m}{\omega_T}$$

and

$$C_{\text{in}} \simeq \frac{g_m}{\omega_T}$$

If r_o can be neglected, the hybrid-π equivalent circuit reduces to that shown in Fig. P10.105(a).

(b) Replacing Q_1 by its equivalent circuit and replacing the diode-connected Q_2 by its equivalent circuit which consists of the input capacitance (g_{m2}/ω_T) and the resistance $1/g_{m2}$ and including the input capacitance of the subsequent stage (g_{m1}/ω_T) gives the equivalent circuit shown in the figure above. The gain can easily be determined as

$$\frac{V_o}{V_i} = -\frac{g_{m1}}{g_{m2} + s\dfrac{g_{m1} + g_{m2}}{\omega_T}}$$

$$= -\frac{g_{m1}}{g_{m2}} \frac{1}{1 + s\dfrac{1 + g_{m1}/g_{m2}}{\omega_T}}$$

Denoting

$$\frac{g_{m1}}{g_{m2}} = G_0,$$

then,

$$\frac{V_o}{V_i} = -\frac{G_0}{1 + \dfrac{s}{\omega_T/(G_0 + 1)}}$$

The dc gain G_0 can be related to W_1 and W_2 as follows: The bias current I divides between Q_1 and Q_2 as,

$$I_1 = \frac{1}{2}k_n'\left(\frac{W_1}{L}\right)V_{OV}^2$$

$$I_2 = \frac{1}{2}k_n'\left(\frac{W_2}{L}\right)V_{OV}^2$$

where we have utilized the fact that Q_1 and Q_2 are operating at the same value of V_{OV}. Thus,

$$\frac{I_1}{I_2} = \frac{W_1}{W_2}$$

Now,

$$g_{m1} = \frac{2I_1}{V_{OV}}$$

and

$$g_{m2} = \frac{2I_2}{V_{OV}}$$

Thus,

$$\frac{g_{m1}}{g_{m2}} = \frac{I_1}{I_2} = \frac{W_1}{W_2}$$

and

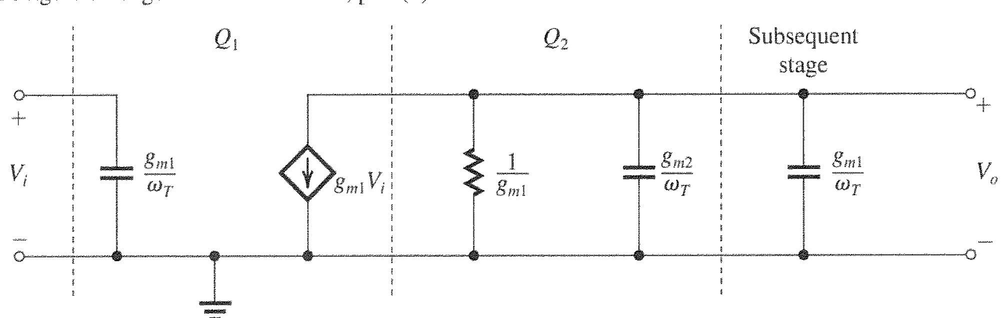
$$G_0 = \frac{g_{m1}}{g_{m2}} = \frac{W_1}{W_2}$$

(c) $G_0 = 3$

$$\Rightarrow \frac{W_1}{W_2} = 3$$

$$W_1 = 3 \times 25 = 75\ \mu\text{m}$$

This figure belongs to Problem 10.105, part (b).



$$I_1 = \frac{1}{2}k_n'\frac{W_1}{L}V_{OV}^2$$

$$= \frac{1}{2} \times 0.2 \times \frac{75}{0.5} \times 0.3^2 = 1.35 \text{ mA}$$

$$I_2 = \frac{1}{2} \times 0.2 \times \frac{25}{0.5} \times 0.3^2 = 0.45 \text{ mA}$$

$$I = I_1 + I_2 = 1.35 + 0.45 = 1.8 \text{ mA}$$

$$f_{3dB} = \frac{f_T}{G_0 + 1} = \frac{12}{3 + 1} = 3 \text{ GHz}$$

10.106 (a) For each of Q_1 and Q_2,

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_o = \frac{|V_A|}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$g_m r_o = 100$$

$$\frac{V_o}{V_{\text{sig}}} = (g_m r_o)^2 = 10{,}000 \text{ V/V}$$

(b) $\tau_{gs1} = C_{gs}R_{\text{sig}}$

$$= 20 \times 10 = 200 \text{ ps}$$

$$R_{gd1} = R_{\text{sig}}(1 + g_{m1}r_{o1}) + r_{o1}$$

$$= 10(1 + 100) + 100$$

$$= 1110 \text{ k}\Omega$$

$$\tau_{gd1} = C_{gd1}R_{gd1} = 5 \times 1110 = 5550 \text{ ps}$$

At the drain of Q_1 we have $(C_{db1} + C_{gs2})$ and the resistance seen is r_o:

$$\tau_{d1} = (C_{db1} + C_{gs2})r_o$$

$$= (5 + 20) \times 100 = 2500 \text{ ps}$$

$$R_{gd2} = r_{o1}(1 + g_{m2}r_{o2}) + r_{o2}$$

$$= 100(1 + 100) + 100 = 1110 \text{ k}\Omega$$

$$\tau_{gd2} = C_{gd2}R_{gd2} = 5 \times 1110 = 5550 \text{ ps}$$

$$\tau_{d2} = C_{db2}r_{o2}$$

$$= 5 \times 100 = 500 \text{ ps}$$

$$\tau_H = \tau_{gs1} + \tau_{gd1} + \tau_{d1} + \tau_{gd2} + \tau_{d2}$$

$$= 200 + 5550 + 2500 + 5550 + 500$$

$$= 14{,}300 \text{ ps} = 14.3 \text{ ns}$$

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 14.3 \times 10^{-9}} = 11.1 \text{ MHz}$$

10.107 (a)

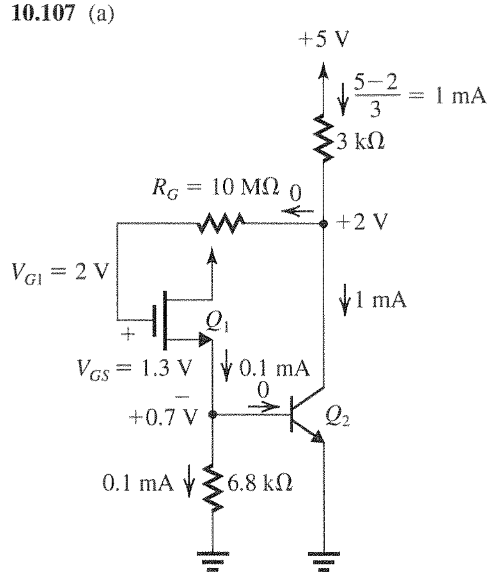


Figure 1

The dc analysis is shown in Fig. 1. It is based on $V_{S1} = V_{BE2} = 0.7$ V. Neglecting I_{B2}, we obtain

$$I_{D1} = \frac{0.7 \text{ V}}{6.8 \text{ k}\Omega} \simeq 0.1 \text{ mA} \qquad \text{Q.E.D.}$$

$$I_{D1} = \frac{1}{2}k_n'(W/L)V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 2 \times V_{OV}^2$$

$$\Rightarrow V_{OV} \simeq 0.3 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 1 + 0.3 = 1.3 \text{ V}$$

$$V_{G1} = 0.7 + 1.3 = 2 \text{ V}$$

$$V_{C1} = V_{G1} = 2 \text{ V}$$

$$I_{C2} = \frac{5 - 2}{3} = 1 \text{ mA} \qquad \text{Q.E.D.}$$

(b) $g_{m1} = \dfrac{2I_{D1}}{V_{OV}} = \dfrac{2 \times 0.1}{0.3} = 0.67$ mA/V

$$C_{gs} = C_{gd} = 1 \text{ pF}$$

$$g_{m2} = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{200}{40} = 5 \text{ k}\Omega$$

$$C_{\pi 2} + C_{\mu 2} = \frac{g_{m2}}{2\pi f_{T2}}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 600 \times 10^6} = 10.6 \text{ pF}$$

$$C_{\mu 2} = 0.8 \text{ pF}$$

$$C_{\pi 2} = 9.8 \text{ pF}$$

(c) Q_1 acts as a source follower, thus

$$\frac{V_{b2}}{V_i} = \frac{6.8\text{ k}\Omega \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + (6.8\text{ k}\Omega \parallel r_{\pi 2})}$$

$$= \frac{(6.8 \parallel 5)}{1.5 + (6.8 \parallel 5)} = 0.66\text{ V/V}$$

Neglecting R_G, we obtain

$$\frac{V_o}{V_{b2}} = -g_{m2}(3\text{ k}\Omega \parallel 1\text{ k}\Omega)$$

$$= -40(3 \parallel 1) = -30\text{ V/V}$$

Thus,

$$\frac{V_o}{V_i} = 0.66 \times -30$$

$$\simeq -20\text{ V/V}$$

Using Miller's theorem, the input resistance R_{in} is found as

$$R_{\text{in}} = \frac{R_G}{1 - \frac{V_o}{V_i}} = \frac{10\text{ M}\Omega}{1 - (-20)}$$

$$= 476\text{ k}\Omega$$

$$\frac{V_i}{V_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}}$$

$$= \frac{476}{476 + 100} = 0.83\text{ V/V}$$

$$\frac{V_o}{V_{\text{sig}}} = 0.83 \times 20 = 16.5\text{ V/V}$$

(c) The pole due to C_1 has a frequency f_1:

$$f_1 = \frac{1}{2\pi C_1(R_{\text{sig}} + R_{\text{in}})}$$

$$= \frac{1}{2\pi \times 0.1 \times 10^{-6}(100 + 476) \times 10^3}$$

$$= 2.8\text{ Hz}$$

The pole due to C_2 has a frequency f_2:

$$f_2 = \frac{1}{2\pi C_2(3 + 1) \times 10^3}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-6} \times 4 \times 10^3} = 40\text{ Hz}$$

Since $f_2 \gg f_1$, the lower 3-dB frequency f_L will be

$$f_L \simeq f_2 = 40\text{ Hz}$$

(d) $\tau_{gd1} = C_{gd1}(R_{\text{in}} \parallel R_{\text{sig}})$

$$= 1 \times 10^{-12}(476 \parallel 100) \times 10^3$$

$$= 82.6\text{ ns}$$

To determine the resistance R_{gs} seen by C_{gs}, refer to Fig. 2.

We can show that

$$R_{gs} \equiv \frac{V_x}{I_x} = \frac{R_{\text{sig}} + R_s}{1 + g_{m1}R_s}$$

where

$$R_s = 6.8\text{ k}\Omega \parallel r_{\pi 2}$$

$$= 6.8 \parallel 5 = 2.88\text{ k}\Omega$$

$$R_{gs} = \frac{100 + 2.88}{1 + 0.67 \times 2.88} = 35.1\text{ k}\Omega$$

$$\tau_{gs} = C_{gs}R_{gs} = 1 \times 10^{-12} \times 35.1 \times 10^3 = 35.1\text{ ns}$$

$$\tau_{\pi 2} = C_{\pi 2}(r_{\pi 1} \parallel 6.8\text{ k}\Omega)$$

$$= 9.8 \times 10^{-12} \times 2.88 \times 10^3$$

$$= 28.2\text{ ns}$$

$$R_{\mu 2} = \left(\frac{1}{g_{m1}} \parallel 6.8\text{ k}\Omega\right)[1 + g_{m2}(3 \parallel 1)] + (3 \parallel 1)$$

$$= (1.5 \parallel 6.8)\left(1 + 40 \times \frac{3}{4}\right) + 0.75$$

$$= 38.8\text{ k}\Omega$$

This figure belongs to Problem 10.107, part (d).

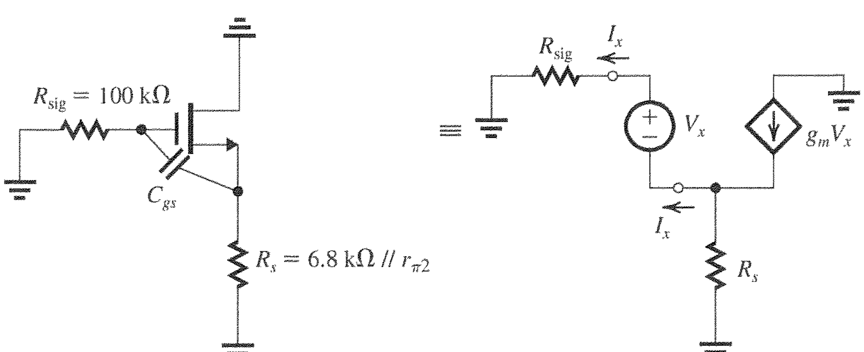


Figure 2

$$\tau_{\mu 2} = C_{\mu 2}R_{\mu 2} = 0.8 \times 38.8 = 31.1 \text{ ns}$$

$$\tau_H = \tau_{gd} + \tau_{gs} + \tau_{\pi 2} + \tau_{\mu 2}$$

$$= 82.6 + 35.1 + 38.8 + 31.1$$

$$= 187.6 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 187.6 \times 10^{-9}} = 848 \text{ kHz}$$

10.108 All transistors are operating at $I_E = 0.5$ mA. Thus,

$$g_m \simeq 20 \text{ mA/V}$$

$$r_e \simeq 50\ \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

r_o = very high (neglect)

r_x = very small (neglect)

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{20 \times 10^{-3}}{2\pi \times 400 \times 10^6}$$

$$= 8 \text{ pF}$$

$$C_\mu = 2 \text{ pF}$$

$$C_\pi = 6 \text{ pF}$$

(a) CE amplifier

$$A_M = -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m R_L$$

$$= -\frac{5}{5+10} \times 20 \times 10 = -66.7 \text{ V/V}$$

Since $C_L = 0$, we can obtain a good estimate of f_H using the Miller approximation:

$$C_{\text{in}} = C_\pi + C_\mu(g_m R_L + 1)$$

$$= 6 + 2(20 \times 10 + 1)$$

$$= 408 \text{ pF}$$

$$f_H = \frac{1}{2\pi R_{\text{sig}} C_{\text{in}}}$$

$$= \frac{1}{2\pi \times 10 \times 10^3 \times 408 \times 10^{-12}}$$

$$= 39 \text{ kHz}$$

(b) This is a cascode amplifier. Refer to Fig. 10.30 for the analysis equations.

$$A_M = -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m(\beta r_o \parallel R_L)$$

$$\simeq -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m R_L$$

$= -66.7$ V(V) (same as the CE in (a))

$$R'_{\text{sig}} = r_\pi \parallel R_{\text{sig}} = 5 \parallel 10 = 3.33 \text{ k}\Omega$$

$$R_{\pi 1} = R'_{\text{sig}}$$

$$\tau_{\pi 1} = C_{\pi 1}R_{\pi 1} = 6 \times 3.33 = 20 \text{ ns}$$

$$R_{c1} = r_{e2} = 50\ \Omega$$

$$R_{\mu 1} = R'_{\text{sig}}(1 + g_{m1}R_{c1}) + R_{c1}$$

$$= 3.33(1 + 40 \times 0.05) + 0.05$$

$$= 3.33(1 + 2) + 0.05 = 10.05 \text{ k}\Omega$$

$$\tau_{\mu 1} = C_{\mu 1}R_{\mu 1} = 2 \times 10.05 = 20.1 \text{ ns}$$

$$\tau_{c1} = C_{\pi 2}R_{c1} = 6 \times 0.05 = 0.3 \text{ ns}$$

$$\tau_{\mu 2} = C_{\mu 2}R_L = 2 \times 10 = 20 \text{ ns}$$

$$\tau_H = 20 + 20.1 + 0.3 + 20 = 60.4 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 60.4 \times 10^{-9}}$$

$$= 2.6 \text{ MHz}$$

(c) This is a CC-CB cascade similar to the circuit analyzed in Fig. 10.42. There are two poles: one at the input,

$$f_{P1} = \frac{1}{2\pi(R_{\text{sig}} \parallel 2r_\pi)\left(\dfrac{C_\pi}{2} + C_\mu\right)}$$

$$f_{P1} = \frac{1}{2\pi(10 \parallel 10) \times 10^3(3 + 2) \times 10^{-12}}$$

$$= \frac{1}{2\pi \times 5 \times 5 \times 10^{-9}}$$

$$= 6.4 \text{ MHz}$$

and one at the output,

$$f_{P2} = \frac{1}{2\pi R_L C_\mu}$$

$$= \frac{1}{2\pi \times 10 \times 10^3 \times 2 \times 10^{-12}}$$

$$= 8 \text{ MHz}$$

Since the two poles are relatively close to each other, we use the root-sum-of-the-squares formula to obtain an estimate for f_H:

$$f_H = 1\Big/\sqrt{\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}}$$

$$= 1\Big/\sqrt{\frac{1}{6.4^2} + \frac{1}{8^2}} = 5 \text{ MHz}$$

$$A_M = \frac{R_L}{2r_e + \dfrac{R_{\text{sig}}}{\beta + 1}}$$

$$= \frac{10}{2 \times 0.05 + \dfrac{10}{101}} \simeq 50 \text{ V/V}$$

(d) This is a CC-CE cascade similar to the circuit analyzed in Example 10.13.

$R_{in} = (\beta_1 + 1)(r_{e1} + r_{\pi 2})$

$= 101(0.05 + 5) = 510\ \text{k}\Omega$

$$\frac{V_{b1}}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{510}{510 + 10}$$

$= 0.98$ V/V

$$\frac{V_{b2}}{V_{b1}} = \frac{r_{\pi 2}}{r_{\pi 2} + r_{e1}} = \frac{5}{5 + 0.05} = 0.99 \text{ V/V}$$

$$\frac{V_o}{V_{b2}} = -g_{m2}R_L = -20 \times 10 = -200 \text{ V/V}$$

$$A_M = \frac{V_o}{V_{sig}} = -0.98 \times 0.99 \times 200 = -194 \text{ V/V}$$

$R_{\mu 1} = R_{sig} \parallel R_{in}$

$= 10 \parallel 510 = 9.81\ \text{k}\Omega$

$\tau_{\mu 1} = C_{\mu 1}R_{\mu 1} = 2 \times 9.81 = 19.6$ ns

$$R_{\pi 1} = \frac{R_{sig} + R_{in2}}{1 + \frac{R_{sig}}{r_{\pi 1}} + \frac{R_{in2}}{r_{e1}}}$$

$$= \frac{10 + 5}{1 + \frac{10}{5} + \frac{5}{0.05}} = 0.15\ \text{k}\Omega$$

$\tau_{\pi 1} = C_{\pi 1}R_{\pi 1} = 6 \times 0.15 = 0.9$ ns

$R_{\pi 2} = R_{in2} \parallel R_{out1}$

$$= r_{\pi 2} \parallel \left(\frac{R_{sig}}{\beta_1 + 1} + r_{e1}\right)$$

$$= 5 \parallel \left(\frac{10}{101} + 0.05\right) = 0.15\ \text{k}\Omega$$

$\tau_{\pi 2} = C_{\pi 2}R_{\pi 2} = 6 \times 0.15 = 0.9$ ns

$R_{\mu 2} = (1 + g_{m2}R_L)(R_{in2} \parallel R_{out1}) + R_L$

$$= (1 + 20 \times 10)\left[5 \parallel \left(\frac{10}{101} + 0.05\right)\right] + 10$$

$= 39.1\ \text{k}\Omega$

$\tau_{\mu 2} = C_{\mu 2}R_{\mu 2} = 2 \times 39.1 = 78.2$ ns

$\tau_H = 19.6 + 0.9 + 0.9 + 78.2 = 99.6$ ns

$$f_H = \frac{1}{2\pi \times 99.6 \times 10^{-9}} = 1.6 \text{ MHz}$$

(e) This is a folded cascode amplifier. The analysis is identified to that for (b) above.

$A_M = -66.7$ V/V

$f_H = 2.6$ MHz

(f) This is a CE-CB cascade. The analysis is identical to that for case (c) above.

$A_M = 50$ V/V

$f_H = 5$ MHz

Summary of Results

Case	Configuration	A_M (V/V)	f_H (MHz)
a	CE	−66.7	0.039
b	Cascode	−66.7	2.6
c	CC-CB	50	5
d	CC-CE	−194	1.6
e	Folded Cascode	−66.7	2.6
f	CC-CB	50	5

11.1 $A_f = \dfrac{A}{1 + A\beta}$

$$200 = \frac{10^4}{1 + 10^4\beta}$$

$$\Rightarrow \beta = 4.9 \times 10^{-3}$$

If A changes to 10^3, then we get

$$A_f = \frac{1000}{1 + 10^3 \times 4.9 \times 10^{-3}}$$

$$= \frac{1000}{5.9} = 169.5$$

Percentage change in $A_f = \dfrac{169.5 - 200}{200} \times 100$

$$= -15.3\%$$

11.2 (a) Because of the infinite input resistance of the op amp, the fraction of the output voltage V_o that is fed back and subtracted from V_s is determined by the voltage divider (R_1, R_2), thus

$$\beta = \frac{R_1}{R_1 + R_2}$$

(b) (i) $A = 1000$ V/V

$$A_f = \frac{A}{1 + A\beta}$$

$$10 = \frac{1000}{1 + 1000\beta}$$

$$\Rightarrow \beta = 0.099 \text{ V/V}$$

$$\frac{R_1}{R_1 + R_2} = 0.099$$

$$1 + \frac{R_2}{R_1} = \frac{1}{0.099}$$

$$R_2 = R_1\left(\frac{1}{0.099} - 1\right)$$

$$= 10\left(\frac{1}{0.099} - 1\right) = 91 \text{ k}\Omega$$

(ii) $A = 200$ V/V

$$10 = \frac{200}{1 + 200\beta}$$

$$\Rightarrow \beta = 0.095 \text{ V/V}$$

$$R_2 = R_1\left(\frac{1}{0.095} - 1\right)$$

$$= 10\left(\frac{1}{0.095} - 1\right) = 95.3 \text{ k}\Omega$$

(iii) $A = 15$ V/V

$$10 = \frac{15}{1 + 15\beta}$$

$$\Rightarrow \beta = 0.033 \text{ V/V}$$

$$R_2 = 10\left(\frac{1}{0.033} - 1\right)$$

$$= 290 \text{ k}\Omega$$

(c) (i) $A = 1000(1 - 0.2) = 800$ V/V

$$A_f = \frac{800}{1 + 800 \times 0.099}$$

$$= 9.975 \text{ V/V}$$

Thus, A_f changes by

$$= \frac{9.975 - 10}{10} \times 100 = -0.25\%$$

(ii) $A = 200(1 - 0.2) = 160$ V/V

$$A_f = \frac{160}{1 + 160 \times 0.095} = 9.877 \text{ V/V}$$

Thus, A_f changes by

$$= \frac{9.877 - 10}{10} \times 100 = -1.23\%$$

(iii) $A = 15(1 - 0.2) = 12$ V/V

$$A_f = \frac{12}{1 + 12 \times 0.033} = 8.574$$

Thus, A_f changes by

$$= \frac{8.575 - 10}{10} \times 100 = -14.3\%$$

We conclude that as A becomes smaller and hence the amount of feedback $(1 + A\beta)$ is lower, the desensitivity of the feedback amplifier to changes in A decreases. In other words, the negative feedback becomes less effective as $(1 + A\beta)$ decreases.

11.3 The direct connection of the output terminal to the inverting input terminal results in $V_f = V_o$ and thus

$$\beta = 1$$

If $A = 1000$, then the closed-loop gain will be

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1000}{1 + 1000 \times 1} = 0.999 \text{ V/V}$$

Amount of feedback $= 1 + A\beta$

$$= 1 + 1000 \times 1 = 1001$$

or 60 dB

For $V_s = 1$ V, we obtain

$$V_o = A_f V_s = 0.999 \times 1 = 0.999 \text{ V}$$

$$V_i = V_s - V_o = 1 - 0.999$$

$$= 0.001 \text{ V}$$

If A becomes $1000(1-0.1) = 900$ V/V, then we get

$$A_f = \frac{900}{1+900 \times 1} = 0.99889$$

Thus, A_f changes by

$$= \frac{0.99889 - 0.999}{0.999} \times 100 = -0.011\%$$

11.4 $A = \dfrac{V_o}{V_i} = \dfrac{5\text{ V}}{10\text{ mV}} = 500$ V/V

$$V_f = V_s - V_i = 1 - 0.01 = 0.99 \text{ V}$$

$$\beta = \frac{V_f}{V_o} = \frac{0.99}{5} = 0.198 \text{ V/V}$$

11.5 (a) $A_f = \dfrac{A}{1+A\beta}$

Ideally,

$$A_f = \frac{1}{\beta}$$

$$A_f\big|_{\text{ideal}} - A_f = \frac{1}{\beta} - \frac{A}{1+A\beta}$$

$$= \frac{1+A\beta - A\beta}{(1+A\beta)\beta} = \frac{1}{(1+A\beta)\beta}$$

Expressed as a percentage of the ideal gain $1/\beta$, we have

$$\frac{\text{Difference}}{\text{Ideal}} = \frac{1}{1+A\beta} \times 100\%$$

For $A\beta \gg 1$,

$$\frac{\text{Difference}}{\text{Ideal}} \simeq \frac{100}{A\beta}\%$$

(b) For A_f to be within:

(i) 0.1% of ideal value, then

$$\frac{100}{A\beta} \leq 0.1$$

$$\Rightarrow A\beta \geq 1000$$

(ii) 1% of ideal value, then

$$\frac{100}{A\beta} \leq 1$$

$$\Rightarrow A\beta \geq 100$$

(iii) 5% of ideal value, then

$$\frac{100}{A\beta} \leq 5$$

$$\Rightarrow A\beta \geq 20$$

11.6 For each value of A given, we have three different values of β: 0.00, 0.50, and 1.00. To obtain A_f, we use

$$A_f = \frac{A}{1+A\beta}$$

The results obtained are as follows.

Case	**A (V/V)**	A_f **(V/V) for** $\beta = 0.00$	A_f **(V/V) for** $\beta = 0.50$	A_f **(V/V) for** $\beta = 1.00$
(a)	1	1	0.667	0.500
(b)	10	10	1.667	0.909
(c)	100	100	1.961	0.990
(d)	1000	1000	1.996	0.999
(e)	10,000	10,000	1.9996	0.9999

11.7 $A = \dfrac{5\text{ V}}{2\text{ mV}} = 2500$ V/V

$$A_f = \frac{5\text{ V}}{100\text{ mV}} = 50 \text{ V/V}$$

Amount of feedback $\equiv 1 + A\beta$

$$= \frac{A}{A_f} = \frac{2500}{50} = 50$$

or 34 dB

$$A\beta = 49$$

$$\beta = \frac{49}{2500} = 0.0196 \text{ V/V}$$

11.8 $A_{\text{nominal}} = 1000$

$$A_{\text{low}} = 500$$

$$A_{\text{high}} = 1500$$

If we apply negative feedback with a feedback factor β, then

$$A_{f,\text{nominal}} = \frac{1000}{1+1000\beta}$$

$$A_{f,\text{low}} = \frac{500}{1+500\beta}$$

$$A_{f,\text{high}} = \frac{1500}{1+1500\beta}$$

It is required that

$$A_{f,\text{low}} \geq 0.99 A_{f,\text{nominal}} \qquad (1)$$

and

$$A_{f,\text{high}} \leq 1.01 A_{f,\text{nominal}} \qquad (2)$$

If we satisfy condition (1) with equality, we can determine the required value of β. We must then clock that condition (2) is satisfied. Thus,

$$\frac{500}{1+500\beta} = 0.99 \times \frac{1000}{1+1000\beta}$$

$$\Rightarrow \beta = 0.098$$

For this value of β, we obtain

$$A_{f,\,\text{nominal}} = \frac{1000}{1 + 1000 \times 0.098}$$
$$= 10.101$$
$$A_{f,\,\text{low}} = \frac{500}{1 + 500 \times 0.098} = 10$$
$$A_{f,\,\text{high}} = \frac{1500}{1 + 1500 \times 0.098} = 10.135$$

Thus, the low value of the closed-loop gain is 0.101 below nominal or -1%, and the high value is 0.034 above nominal or 0.34%. Thus, our amplifier meets specification and the nominal value of closed-loop gain is 10.1. This is the highest possible closed-loop gain that can be obtained while meeting specification.

Now, if three closed-loop amplifiers are placed in cascade, the overall gain obtained will be

Nominal Gain $= (10.1)^3 = 1030$

Lowest Gain $= 10^3 = 1000$

Highest Gain $= (10.135)^3 = 1041$

Thus, the lowest gain will be approximately 3% below nominal, and the highest gain will be 1% above nominal.

11.9

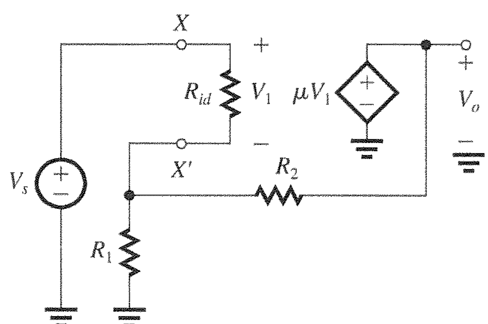


Figure 1

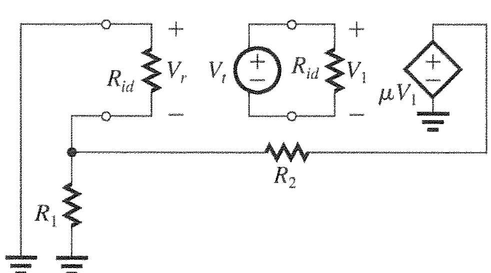


Figure 2

Figure 1 shows the given circuit with the op amp replaced with its equivalent circuit model. To determine the loop gain $A\beta$, we short circuit V_s and break the loop at the input terminals of the op amp. To keep the circuit unchanged, we must place a resistance equal to R_{id} at the left-hand side of the break. This is shown in Fig. 2, where a test signal V_t is applied at the right-hand side of the break. To determine the returned voltage V_r, we use the voltage-divider rule as follows:

$$V_r = -\mu V_1 \frac{R_1 \parallel R_{id}}{(R_1 \parallel R_{id}) + R_2}$$

Substituting $V_1 = V_t$ and rearranging, we obtain

$$A\beta \equiv -\frac{V_r}{V_t} = \mu \frac{R_1 \parallel R_{id}}{(R_1 \parallel R_{id}) + R_2}$$

Since

$$\beta = \frac{R_1}{R_1 + R_2}$$

we get

$$A = \mu \frac{R_1 \parallel R_{id}}{(R_1 \parallel R_{id}) + R_2} \frac{R_1 + R_2}{R_1}$$
$$= \mu \frac{R_{id}/(R_1 + R_{id})}{R_1 R_{id}/(R_1 + R_{id}) + R_2} (R_1 + R_2)$$
$$= \mu \frac{R_{id}(R_1 + R_2)}{R_1 R_{id} + R_2 R_{id} + R_1 R_2}$$

Thus,

$$A = \mu \frac{R_{id}}{R_{id} + (R_1 \parallel R_2)} \qquad \text{Q.E.D.}$$

11.10 From Eq. (11.10), we have

$$\frac{dA_f/A_f}{dA/A} = \frac{1}{1 + A\beta}$$

Since -40 dB is 0.01, we have

$$0.01 = \frac{1}{1 + A\beta}$$
$$\Rightarrow A\beta = 99$$

For

$$\frac{dA_f/A_f}{dA/A} = \frac{1}{5}$$

we have

$$1 + A\beta = 5$$
$$\Rightarrow A\beta = 4$$

11.11 For $A = 1000$ V/V, we have

$$A_f = 10 = \frac{1000}{1 + A\beta}$$

$\Rightarrow$ Densensitivity factor $\equiv 1 + A\beta = 100$

$$A\beta = 99$$
$$\beta = \frac{99}{1000} = 0.099 \text{ V/V}$$

For $A = 500$ V/V, we have

$$A_f = 10 = \frac{500}{1 + A\beta}$$

$\Rightarrow$ Densensitivity factor $\equiv 1 + A\beta = 50$

$$\beta = \frac{49}{500} = 0.098 \text{ V/V}$$

If the $A = 1000$ amplifiers have a gain uncertainty of $\pm 10\%$, the gain uncertainty of the closed-loop amplifiers will be

$$= \frac{\pm 10\%}{100} = \pm 0.1\%$$

If we require a gain uncertainty of $\pm 0.1\%$ using the $A = 500$ amplifiers, then

$$\pm 0.1\% = \frac{\text{Gain uncertainty of } A = 500 \text{ amplifiers}}{50}$$

$\Rightarrow$ Gain uncertainty $= \pm 5\%$

11.12 $A_f = 10$ V/V

$$1 + A\beta = \frac{\pm 10\%}{\pm 0.1\%} = 100$$

$$10 = \frac{A}{100}$$

$\Rightarrow A = 1000$ V/V

$$\beta = \frac{100 - 1}{1000} = 0.099 \text{ V/V}$$

11.13 The open-loop gain varies from A to $10A$ with temperature and time. Correspondingly, A_f varies from $(25 - 1\%)$ i.e. 24.75 V/V to $(25 + 1\%)$ or 25.25 V/V. Substituting these quantities into the formula for the closed-loop gain

$$A_f = \frac{A}{1 + A\beta}$$

we obtain

$$24.75 = \frac{A}{1 + A\beta} \quad (1)$$

$$25.25 = \frac{10A}{1 + 10A\beta} \quad (2)$$

Dividing Eq. (2) by Eq. (1), we obtain

$$1.02 = 10\frac{1 + A\beta}{1 + 10A\beta}$$

$$1.02 + 10.2A\beta = 10 + 10A\beta$$

$\Rightarrow A\beta = 44.9$

Substituting in (1) yields

$A = 24.75 \times 45.9 = 1136$

and

$$\beta = \frac{44.9}{1136} = 0.0395$$

11.14 Let the gain of the ideal (nonvarying) driver amplifier be denoted μ. Then, the open-loop gain A will vary from 2μ to 12μ. Correspondingly, the closed-loop gain will vary from 95 V/V to 105 V/V. Substituting these quantities into the closed-loop gain expression, we obtain

$$95 = \frac{2\mu}{1 + 2\mu\beta} \quad (1)$$

$$105 = \frac{12\mu}{1 + 12\mu\beta} \quad (2)$$

Dividing Eq. (2) by Eq. (1) yields

$$1.105 = \frac{6(1 + 2\mu\beta)}{1 + 12\mu\beta}$$

$$1.105 + 1.105 \times 12\mu\beta = 6 + 12\mu\beta$$

$\Rightarrow \mu\beta = 3.885$

Substituting in Eq. (1) yields

$$\mu = \frac{95(1 + 2 \times 3.885)}{2} = 416.6 \text{ V/V}$$

$$\beta = \frac{3.885}{416.6} = 9.33 \times 10^{-3} \text{ V/V}$$

If A_f is to be held to within $\pm 0.5\%$, Eqs. (1) and (2) are modified to

$$99.5 = \frac{2\mu}{1 + 2\mu\beta} \quad (3)$$

$$100.5 = \frac{12\mu}{1 + 12\mu\beta} \quad (4)$$

Dividing (4) by (3) yields

$$1.01 = \frac{6(1 + 2\mu\beta)}{1 + 12\mu\beta}$$

$\Rightarrow \mu\beta = 49.92$

Substituting into (3) provides

$$\mu = \frac{99.5(1 + 2 \times 49.92)}{2}$$

$= 5016.8$ V/V

which is more than a factor of 10 higher than the gain required in the less constrained case. The value of β required is

$$\beta = \frac{49.92}{5016.8} = 9.95 \times 10^{-3} \text{ V/V}$$

Repeating for $A_f = 10$ V/V (a factor of 10 lower than the original case):

(a) For ±5% maximum variability, Eqs. (1) and (2) become

$$9.5 = \frac{2\mu}{1 + 2\mu\beta} \quad (5)$$

$$10.5 = \frac{12\mu}{1 + 12\mu\beta} \quad (6)$$

Dividing (6) by (5) yields

$$1.105 = \frac{6(1 + 2\mu\beta)}{1 + 12\mu\beta}$$

$$\Rightarrow \mu\beta = 3.885$$

which is identical to the first case considered, and

$$\mu = \frac{9.5(1 + 2 \times 3.885)}{1 + 12\mu\beta} = 41.66 \text{ V/V}$$

which is a factor of 10 lower than the value required when the gain required was 100. The feedback factor β is

$$\beta = \frac{3.885}{41.66} = 9.33 \times 10^{-2} \text{ V/V}$$

which is a factor of 10 higher than the case with $A_f = 10$.

(b) Finally, for the case $A_f = 10 \pm 0.5\%$ we can write by analogy

$$\mu\beta = 49.92$$

$$\mu = 501.68 \text{ V/V}$$

$$\beta = 9.95 \times 10^{-2} \text{ V/V}$$

11.15 If we use one stage, the amount of feedback required is

$$1 + A\beta = \frac{A}{A_f} = \frac{1000}{100} = 10$$

Thus the closed-loop amplifier will have a variability of

$$\text{Variability of } A_f = \frac{\pm 30\%}{10} = \pm 3\%$$

which does not meet specifications. Next, we try using two stages. For a nominal gain of 100, each stage will be required to have a nominal gain of 10. Thus, for each stage the amount of feedback required will be

$$1 + A\beta = \frac{1000}{10} = 100$$

Thus, the closed-loop gain of each stage will have a variability of

$$= \frac{\pm 30\%}{100} = \pm 0.3\%$$

and the cascade of two stages will thus show a variability of ±0.6%, well within the required ±1%. Thus two stages will suffice.

We next investigate the design in more detail. Each stage will have a nominal gain of 10 and thus

$$1 + A\beta = \frac{1000}{10} = 100$$

$$\Rightarrow A\beta = 99$$

$$\Rightarrow \beta = 0.099$$

Since A ranges from 700 V/V to 1300 V/V, the gain of each stage will range from

$$A_{f,\text{low}} = \frac{700}{1 + 700 \times 0.099} = 9.957 \text{ V/V}$$

and a high value of

$$A_{f,\text{high}} = \frac{1300}{1 + 1300 \times 0.099} = 10.023 \text{ V/V}$$

Thus, the cascade of two stages will have a range of

$$\text{Lowest gain} = 9.957^2 = 99.14 \text{ V/V}$$

$$\text{Highest gain} = 10.023^2 = 100.46 \text{ V/V}$$

which is −0.86% to +0.46% of the nominal 100 V/V gain, well within the required ±1%.

11.16 If the nominal open-loop gain is A, then we require that as A drops to $(A/2)$ the closed-loop gain drops from 10 to a minimum of 9.8. Substituting these values in the expression for the closed-loop gain, we obtain

$$10 = \frac{A}{1 + A\beta} \quad (1)$$

$$9.8 = \frac{A/2}{1 + \frac{1}{2}A\beta} \quad (2)$$

Dividing Eq. (1) by Eq. (2) yields

$$1.02 = \frac{2\left(1 + \frac{1}{2}A\beta\right)}{1 + A\beta}$$

$$1.02 = \frac{2 + A\beta}{1 + A\beta}$$

$$= 1 + \frac{1}{1 + A\beta}$$

$$\Rightarrow 1 + A\beta = \frac{1}{0.02} = 50$$

Substituting in Eq. (1) gives

$$A = 10 \times 50 = 500 \text{ V/V}$$

and

$$\beta = \frac{50-1}{500} = 0.098 \text{ V/V}$$

If β is accurate to within $\pm 1\%$, to ensure that the minimum closed-loop gain realized is 9.8 V/V, we have

$$9.8 = \frac{A/2}{1 + \frac{1}{2}A \times 0.098 \times 1.01}$$

$$\Rightarrow A = 653.4 \text{ V/V}$$

11.17 $A_f = \dfrac{A}{1 + A\beta}$

$$100 = \frac{A}{1 + A\beta} \quad (1)$$

$$99 = \frac{0.1A}{1 + 0.1A\beta} \quad (2)$$

Dividing Eq. (1) by Eq. (2) gives

$$1.01 = \frac{10(1 + 0.1A\beta)}{1 + A\beta}$$

$$= \frac{10 + A\beta}{1 + A\beta}$$

$$= 1 + \frac{9}{1 + A\beta}$$

$$\Rightarrow \frac{9}{1 + A\beta} = 0.01$$

$$1 + A\beta = 900$$

$$A\beta = 899$$

Substituting $(1 + A\beta) = 900$ into Eq. (1) yields

$$A = 100 \times 900 = 90,000 \text{ V/V}$$

The value of β is

$$\beta = \frac{899}{90,000} = 9.989 \times 10^{-3} \text{ V/V}$$

If A were increased tenfold, i.e, $A = 900,000$, we obtain

$$A_f = \frac{900,000}{1 + 8990} = 100.1 \text{ V/V}$$

If A becomes infinite, we get

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1}{\frac{1}{A} + \beta} = \frac{1}{\beta}$$

$$= \frac{1}{9.989 \times 10^{-3}} = 100.11 \text{ V/V}$$

11.18 $A = A_M \dfrac{s}{s + \omega_L}$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{A_M s/(s + \omega_L)}{1 + A_M \beta s/(s + \omega_L)}$$

$$= \frac{A_M s}{s + \omega_L + sA_M \beta}$$

$$= \frac{A_M s}{s(1 + A_M \beta) + \omega_L}$$

$$= \frac{A_M}{1 + A_M \beta} \, \frac{s}{s + \omega_L/(1 + A_M \beta)}$$

Thus,

$$A_{Mf} = \frac{A_M}{1 + A_M \beta}$$

$$\omega_{Lf} = \frac{\omega_L}{1 + A_M \beta}$$

Thus, both the midband gain and the 3-dB frequency are lowered by the amount of feedback, $(1 + A_M \beta)$.

11.19 $1 + A_M \beta = \dfrac{1000}{10} = 100$

Thus,

$$f_{Hf} = (1 + A_M \beta) f_H$$

$$= 100 \times 10 = 1000 \text{ kHz} = 1 \text{ MHz}$$

$$f_{Lf} = \frac{f_L}{1 + A_M \beta}$$

$$= \frac{100}{100} = 1 \text{ Hz}$$

11.20 To capacitively couple the output signal to an 8-Ω loudspeaker and obtain $f_L = 100$ Hz, we need a coupling capacitor C,

$$C = \frac{1}{2\pi f_L \times 8}$$

$$= \frac{1}{2\pi \times 100 \times 8} = 198.9 \ \mu\text{F} \simeq 200 \ \mu\text{F}$$

If closed-loop gain A_{Mf} of 10 V/V is obtained from an amplifier whose open-loop gain $A_M = 1000$ V/V, then

$$1 + A_M \beta = \frac{1000}{10} = 100$$

and

$$f_{Lf} = \frac{f_L}{100} = \frac{100}{100} = 1 \text{ Hz}$$

If the required f_{Lf} is 50 Hz, then

$$f_L = 50 \times (1 + A_M \beta)$$

$$= 50 \times 100 = 5000 \text{ Hz},$$

and the coupling capacitor C will have a value of

$$C = \frac{1}{2\pi \times 5000 \times 8} \simeq 4 \ \mu\text{F}$$

11.21 Let's first try $N = 2$. The closed-loop gain of each stage must be

$$A_f = \sqrt{1000} = 31.6 \text{ V/V}$$

Thus, the amount-of-feedback in each stage must be

$$1 + A\beta = \frac{A}{A_f} = \frac{1000}{31.6} = 31.6$$

The 3-dB frequency of each stage is

$$f_{3dB}|_{\text{stage}} = (1 + A\beta) f_H$$

$$= 31.6 \times 20 = 632 \text{ kHz}$$

Thus, the 3-dB frequency of the cascade amplifier is

$$f_{3dB}|_{\text{cascade}} = 632\sqrt{2^{1/2} - 1} = 406.8 \text{ kHz}$$

which is less than the required 1 MHz.

Next, we try $N = 3$. The closed-loop gain of each stage is

$$A_f = (1000)^{1/3} = 10 \text{ V/V}$$

and thus each stage will have an amount-of-feedback

$$1 + A\beta = \frac{1000}{10} = 100$$

which results in a stage 3-dB frequency of

$$f_{3dB}|_{\text{stage}} = (1 + A\beta) f_H$$

$$= 100 \times 20 = 2000 \text{ kHz}$$

$$= 2 \text{ MHz}$$

The 3-dB frequency of the cascade amplifier will be

$$f_{3dB}|_{\text{cascade}} = 2\sqrt{2^{1/3} - 1}$$

$$= 1.02 \text{ MHz}$$

which exceeds the required value of 1 MHz. Thus, we need three identical stages, each with a closed-loop gain of 10 V/V, an amount-of-feedback of 100, and a loop gain

$$A\beta = 99$$

Thus,

$$\beta = 0.099 \text{ V/V}$$

11.22 $V_{o\,\text{ripple}} = V_n \dfrac{A_1}{1 + A_1 A_2 \beta}$

To reduce $V_{o\,\text{ripple}}$ to 100 mV,

$$0.1 = 1 \times \frac{0.9}{1 + A_1 A_2 \beta}$$

$$\Rightarrow 1 + A_1 A_2 \beta = 9$$

$$A_f = \frac{A_1 A_2}{1 + A_1 A_2 \beta}$$

$$10 = \frac{0.9 A_2}{9}$$

$$\Rightarrow A_2 = 100 \text{ V/V}$$

$$\beta = \frac{8}{0.9 \times 100} = 0.089 \text{ V/V}$$

To reduce $V_{o\,\text{ripple}}$ to 10 mV,

$$0.01 = 1 \times \frac{0.9}{1 + A_1 A_2 \beta}$$

$$\Rightarrow 1 + A_1 A_2 \beta = 90$$

$$A_f = \frac{A_1 A_2}{1 + A_1 A_2 \beta}$$

$$10 = \frac{0.9 A_2}{90}$$

$$A_2 = 1000 \text{ V/V}$$

$$\beta = \frac{89}{0.9 \times 1000} = 0.099 \text{ V/V}$$

To reduce $V_{o\,\text{ripple}}$ to 1 mV,

$$0.001 = 1 \times \frac{0.9}{1 + A_1 A_2 \beta}$$

$$\Rightarrow 1 + A_1 A_2 \beta = 900$$

$$10 = \frac{0.9 A_2}{900}$$

$$\Rightarrow A_2 = 10,000 \text{ V/V}$$

$$\beta = \frac{899}{0.9 \times 10,000} = 0.0999 \text{ V/V}$$

11.23 $A_f = \dfrac{A_1 A_2}{1 + A_1 A_2 \beta}$

$$100 = \frac{10 A_2}{1 + A_1 A_2 \beta} \qquad (1)$$

$$(1 + A_1 A_2 \beta) \times 8 = 40 \text{ kHz}$$

$$\Rightarrow 1 + A_1 A_2 \beta = 5$$

Substituting in (1) gives

$$A_2 = \frac{100 \times 5}{10} = 50 \text{ V/V}$$

$$1 + 10 \times 50 \times \beta = 5$$

$$\Rightarrow \beta = 0.008 \text{ V/V}$$

$$f_{Lf} = \frac{80}{1 + A_1 A_2 \beta}$$

$$= \frac{80}{5} = 16 \text{ Hz}$$

11.24

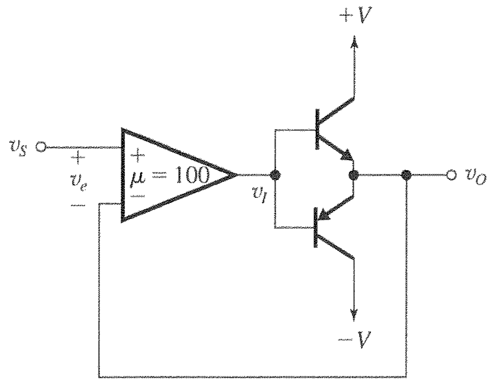


Figure 1

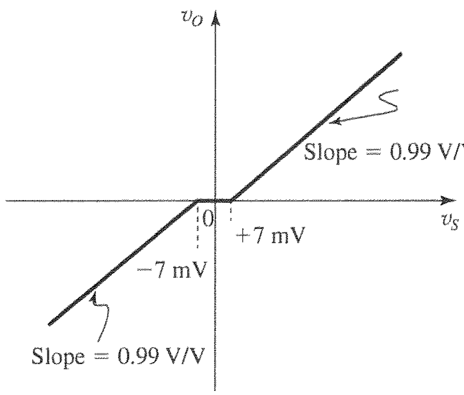


Figure 2

Refer to Fig. 1. For $v_I = +0.7$ V, we have $v_O = 0$ and

$$v_e = \frac{v_I}{\mu} = \frac{+0.7}{100} = +7 \text{ mV}$$

Similarly, for $v_I = -0.7$ V, we obtain $v_O = 0$ and

$$v_e = \frac{v_I}{\mu} = \frac{-0.7}{100} = -7 \text{ mV}$$

Thus, the limits of the deadband are now ± 7 mV. Outside the deadband, the gain of the feedback amplifier, that is, v_O/v_S, can be determined by noting that the open-loop gain $A \equiv v_O/v_e = 100$ V/V and the feedback factor $\beta = 1$, thus

$$A_f \equiv \frac{v_O}{v_S} = \frac{A}{1 + A\beta}$$

$$= \frac{100}{1 + 100 \times 1} = 0.99 \text{ V/V}$$

The transfer characteristic is depicted in Fig. 2.

11.25 The closed-loop gain for the first (high-gain) segment is

$$A_{f1} = \frac{1000}{1 + 1000\beta} \qquad (1)$$

and that for the second segment is

$$A_{f2} = \frac{100}{1 + 100\beta} \qquad (2)$$

We require

$$\frac{A_{f1}}{A_{f2}} = 1.1$$

Thus, dividing Eq. (1) by Eq. (2) yields

$$1.1 = 10\frac{1 + 100\beta}{1 + 1000\beta}$$

$$1.1 + 1100\beta = 10 + 1000\beta$$

$$\Rightarrow \beta = 0.089$$

$$A_{f1} = \frac{1000}{1 + 1000 \times 0.089} = 11.1 \text{ V/V}$$

$$A_{f2} = \frac{100}{1 + 100 \times 0.089} = 10.1 \text{ V/V}$$

The first segment ends at $|v_O| = 10 \text{ mV} \times 1000 = 10$ V. This corresponds to

$$v_S = \frac{10 \text{ V}}{A_{f1}} = \frac{10}{11.1} = 0.9 \text{ V}$$

The second segment ends at $|v_O| = 10 + 0.05 \times 100 = 15$ V. This corresponds to

$$v_S = 0.9 + \frac{15 - 10}{A_{f2}}$$

$$= 0.9 + \frac{5}{10.1} = 1.4 \text{ V}$$

Thus, the transfer characteristic of the feedback amplifier can be described as follows:

For $|v_S| \le 0.9$ V, $\quad v_O/v_S = 11.1$ V/V

For $0.9 \text{ V} \le |v_S| \le 1.4$ V, $\quad v_O/v_S = 10.1$ V/V

For $|v_S| \ge 1.4$ V, $\quad v_O = \pm 15$ V

The transfer characteristic is shown in the figure on next page.

11.26 Because the op amp has an infinite input resistance and a zero output resistance, this circuit is a direct implementation of the ideal feedback structure and thus

$$A = 1000 \text{ V/V}$$

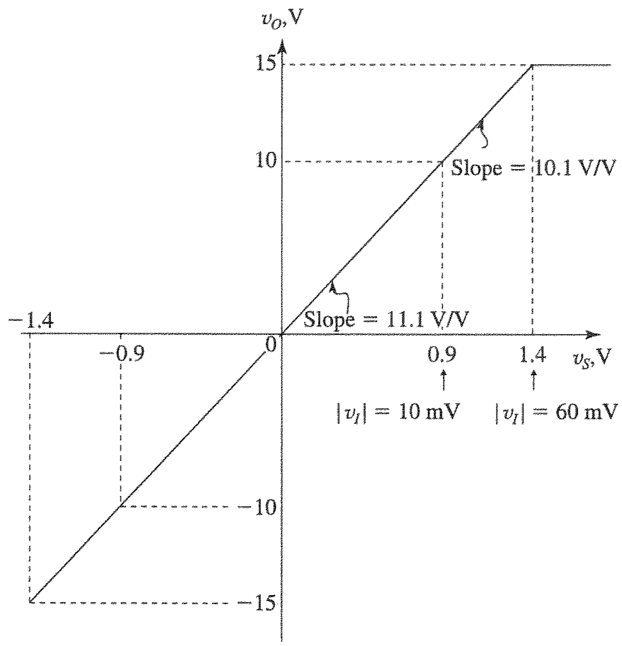
and

$$\beta = \frac{R_1}{R_1 + R_2}$$

The ideal closed-loop gain is

$$A_f = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

This figure belongs to Problem 11.25.



Thus,

$$10 = 1 + \frac{R_2}{10}$$

$$\Rightarrow R_2 = 90\ \text{k}\Omega$$

$$\beta = \frac{10}{10+90} = 0.1\ \text{V/V}$$

$$A\beta = 1000 \times 0.1 = 100$$

$$A_f = \frac{A}{1+A\beta}$$

$$= \frac{1000}{1+100} = 9.9\ \text{V/V}$$

To obtain A_f that is exactly 10, we use

$$10 = \frac{1000}{1+A\beta}$$

$$\Rightarrow A\beta = 99$$

$$\beta = 0.099$$

$$0.099 = \frac{R_1}{R_1+R_2}$$

$$0.099 = \frac{10}{10+R_2}$$

$$\Rightarrow R_2 = 91\ \text{k}\Omega$$

11.27 Refer to Fig. 11.11.

(a) The ideal closed-loop gain is given by

$$A_f = \frac{1}{\beta} = \frac{R_1+R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

$$10 = 1 + \frac{R_2}{10}$$

$$\Rightarrow R_2 = 90\ \text{k}\Omega$$

(b) From Example 11.3, we obtain

$$A\beta = \mu \frac{R_L \parallel [R_2 + R_1 \parallel (R_{id}+R_s)]}{\{R_L \parallel [R_2 + R_1 \parallel (R_{id}+R_s)]\} + r_o}$$

$$\times \frac{R_1 \parallel (R_{id}+R_s)}{[R_1 \parallel (R_{id}+R_s)] + R_2} \times \frac{R_{id}}{R_{id}+R_s}$$

$$A\beta = 1000 \frac{10 \parallel [90 + 10 \parallel (100+100)]}{\{10 \parallel [90 + 10 \parallel (100+100)]\} + 1}$$

$$\times \frac{10 \parallel (100+100)}{[10 \parallel (100+100)] + 90} \times \frac{100}{100+100}$$

$$= 1000 \times 0.9009 \times 0.0957 \times 0.5$$

$$= 43.11$$

$$A = \frac{A\beta}{\beta} = \frac{43.11}{0.1} = 431.1\ \text{V/V}$$

$$A_f = \frac{A}{1+A\beta} = \frac{431.1}{1+43.11} = 9.77\ \text{V/V}$$

(c) To obtain $A_f = 9.9$ V/V, we use

$$9.9 = \frac{A}{1+A\beta}$$

$$= \frac{A}{1 + A \times 0.1}$$

$$\Rightarrow A = 1010\ \text{V/V}$$

Thus μ must be increased by the factor $\frac{1010}{431.1} = 2.343$ to become

$$\mu = 2343\ \text{V/V}$$

11.28 Refer to Fig. 11.10.

(a) $\beta = \dfrac{R_1}{R_1 + R_2}$

$A_f\big|_{\text{ideal}} = \dfrac{1}{\beta} = 1 + \dfrac{R_2}{R_1}$

$5 = 1 + \dfrac{R_2}{1}$

$\Rightarrow R_2 = 4\ \text{k}\Omega$

(b) From Example 11.2, we have

$$A\beta = (g_{m1}R_{D1})(g_{m2}R_{D2})\frac{1}{1 + g_{m1}R_1} \times$$

$$\frac{R_1}{R_{D2} + R_2 + \left(R_1 \parallel \dfrac{1}{g_{m1}}\right)}$$

$$= (4 \times 10)(4 \times 10)\frac{1}{1 + 4 \times 1} \times \frac{1}{10 + 4 + (1 \parallel 0.25)}$$

$$= 22.54$$

$$A = \frac{A\beta}{\beta} = \frac{22.54}{0.2} = 112.7\ \text{V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{112.7}{1 + 22.54} = 4.79\ \text{V/V}$$

11.29 (a) The feedback network consists of the voltage divider $(R_1 R_2)$, thus

$$\beta = \frac{R_1}{R_1 + R_2}$$

If the loop gain is large, the closed-loop gain approaches the ideal value

$$A_f = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

$$= 1 + \frac{10}{1} = 11\ \text{V/V}$$

(b)

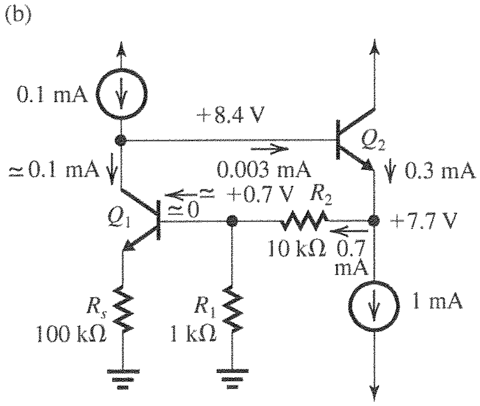


Figure 1

The dc analysis is shown in Fig. 1 from which we see that

$I_{E1} \simeq 0.1$ mA

$I_{E2} \simeq 0.3$ mA

$V_{E2} = +7.7$ V

(c) Setting $V_s = 0$ and eliminating dc sources, the feedback amplifier circuit simplifies to that shown in Fig. 2.

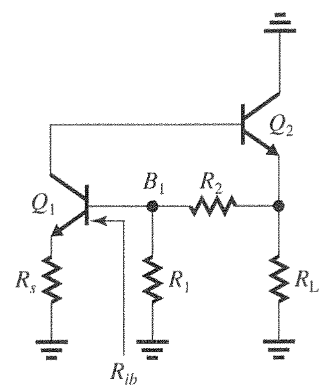


Figure 2

Now, breaking the feedback loop at the base of Q_1 while terminating the right-hand side of the circuit (behind the break) in the resistance R_{ib},

$$R_{ib} = (\beta_1 + 1)(r_{e1} + R_s)$$

results in the circuit in Fig. 3 which we can use to determine the loop gain $A\beta$ as follows:

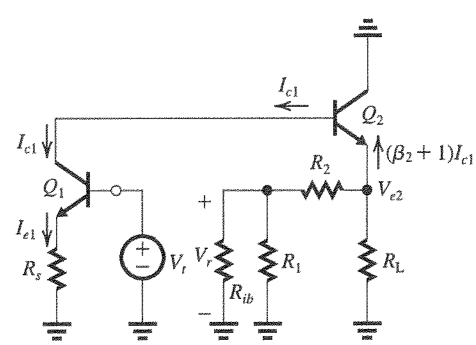


Figure 3

$$I_{e1} = \frac{V_t}{r_{e1} + R_s} \qquad (1)$$

$$I_{c1} = \alpha_1 I_{e1} \qquad (2)$$

$$V_{e2} = -(\beta_2 + 1)I_{c1}\{R_L \parallel [R_2 + (R_1 \parallel R_{ib})]\} \qquad (3)$$

$$V_r = V_{e2}\frac{R_1 \parallel R_{ib}}{(R_1 \parallel R_{ib}) + R_2} \qquad (4)$$

Combining (1) to (4), we can determine $A\beta$ as

$$A\beta \equiv -\frac{V_r}{V_t}$$

$$= \alpha_1 \frac{(\beta_2 + 1)\{R_L \parallel [R_2 + (R_1 \parallel R_{ib})]\}}{r_{e1} + R_s}$$

$$\times \frac{R_1 \parallel R_{ib}}{(R_1 \parallel R_{ib}) + R_2}$$

Substituting

$$\alpha_1(\beta_2 + 1) = \alpha(\beta + 1) = \beta = 100$$

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25\text{ mV}}{0.1\text{ mA}} = 250\ \Omega$$

$$R_s = 100\ \Omega$$

$$R_L = 1\text{ k}\Omega$$

$$R_1 = 1\text{ k}\Omega$$

$$R_2 = 10\text{ k}\Omega$$

$$R_{ib} = 101(0.25 + 0.1) = 35.35\text{ k}\Omega$$

we obtain

$$A\beta = \frac{100\{1 \parallel [10 + (1 \parallel 35.35)]\}}{0.25 + 0.1}$$

$$\times \frac{1 \parallel 35.35}{(1 \parallel 35.35) + 10}$$

$$= 23.2$$

(d) $A = \dfrac{A\beta}{\beta} = \dfrac{23.2}{(1/11)} = 255.2$ V/V

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{255.2}{1 + 23.2} = 10.5\text{ V/V}$$

11.30 Refer to Fig. 11.8(c) and to the expressions for β, $A\beta$, and A given in the answer section of Exercise 11.6.

$$A = g_m \frac{R_D(R_1 + R_2)}{R_D + R_1 + R_2}$$

where

$$g_m = 4\text{ mA/V}$$

$$R_D = 10\text{ k}\Omega$$

$R_1 + R_2 = 1\text{ M}\Omega$ (the potentiometer resistance)

Thus,

$$A = 4 \times \frac{10 \times 1000}{10 + 1000} = 39.6\text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$5 = \frac{39.6}{1 + 39.6\beta}$$

$$\Rightarrow \beta = 0.175\text{ V/V}$$

$$0.175 = \frac{R_1}{R_1 + R_2}$$

$$\Rightarrow R_1 = 0.175 \times 1000 = 175\text{ k}\Omega$$

11.31 (a) The feedback network consists of the voltage divider (R_F, R_{S1}). Thus,

$$\beta = \frac{R_{S1}}{R_{S1} + R_F}$$

and the ideal value of the closed-loop gain is

$$A_f = \frac{1}{\beta} = 1 + \frac{R_F}{R_{S1}}$$

$$10 = 1 + \frac{R_F}{0.1}$$

$$\Rightarrow R_F = 0.9\text{ k}\Omega$$

(b)

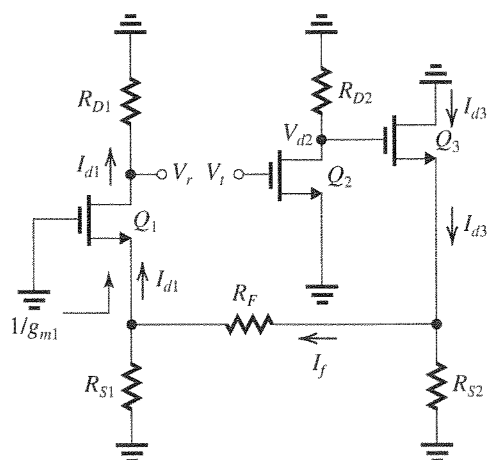


Figure 1

Figure 1 shows the circuit for determining the loop gain. Observe that we have broken the loop at the gate of Q_2 where the input resistance is infinite, obviating the need for adding a termination resistance. Also, observe that as usual we have set $V_s = 0$. To determine the loop gain

$$A\beta \equiv -\frac{V_r}{V_t}$$

we write the following equations:

$$V_{d2} = -g_{m2}R_{D2}V_t \quad (1)$$

$$I_{d3} = \frac{V_{d2}}{\dfrac{1}{g_{m3}} + \left\{R_{S2} \parallel \left[R_F + \left(R_{S1} \parallel \dfrac{1}{g_{m1}}\right)\right]\right\}} \quad (2)$$

$$I_f = I_{d3}\frac{R_{S2}}{\left[R_F + \left(R_{S1} \parallel \dfrac{1}{g_{m1}}\right)\right] + R_{S2}} \quad (3)$$

$$I_{d1} = I_f\frac{R_{S1}}{R_{S1} + \dfrac{1}{g_{m1}}} \quad (4)$$

$$V_r = I_{d1}R_{D1} \quad (5)$$

Substituting the numerical values in (1)–(5), we obtain

$$V_{d2} = -4 \times 10V_t = -40V_t \quad (6)$$

$$I_{d3} = \frac{V_{d2}}{\dfrac{1}{4} + \left\{0.1 \parallel \left[0.9 + \left(0.1 \parallel \dfrac{1}{4}\right)\right]\right\}}$$

$$I_{d3} = 2.935V_{d2} \quad (7)$$

$$I_f = I_{d3}\frac{0.1}{\left[0.9 + \left(0.1 \parallel \dfrac{1}{4}\right)\right] + 0.1}$$

$$I_f = 0.0933I_{d3} \quad (8)$$

$$I_{d1} = I_f\frac{0.1}{0.1 + \dfrac{1}{4}} = 0.286I_f \quad (9)$$

$$V_r = 10I_{d1} \quad (10)$$

Combining (6)–(10) gives

$$V_r = -31.33V_t$$

$$\Rightarrow A\beta = 31.33$$

$$A = \frac{A\beta}{\beta} = \frac{31.33}{0.1} = 313.3 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{313.3}{1 + 31.33} = 9.7 \text{ V/V}$$

Thus, A_f is 0.3 V/V lower than the ideal value of 10 V/V, a difference of -3%. The circuit could be adjusted to make A_f exactly 10 by changing β through varying R_F. Specifically,

$$10 = \frac{313.3}{1 + 313.3\beta}$$

$$\Rightarrow \beta = 0.0968$$

But,

$$\beta = \frac{R_{S1}}{R_{S1} + R_F}$$

$$0.0968 = \frac{0.1}{0.1 + R_F}$$

$$\Rightarrow R_F = 933\ \Omega$$

(an increase of 33 Ω).

11.32 (a) The feedback circuit consists of the voltage divider (R_F, R_E). Thus,

$$\beta = \frac{R_E}{R_E + R_F}$$

and,

$$A_f\big|_{\text{ideal}} = \frac{1}{\beta} = 1 + \frac{R_F}{R_E}$$

Thus,

$$25 = 1 + \frac{R_F}{0.05}$$

$$\Rightarrow R_F = 1.2 \text{ k}\Omega$$

(b) Figure 1 on next page shows the feedback amplifier circuit prepared for determining the loop gain $A\beta$. Observe that we have eliminated all dc sources, set $V_s = 0$, and broken the loop at the base of Q_2. We have terminated the broken loop in a resistance $r_{\pi 2}$. To determine the loop gain

$$A\beta \equiv \frac{V_r}{V_t}$$

we write the following equations:

$$I_{c2} = g_{m2}V_t \quad (1)$$

$$I_{b3} = I_{c2}\frac{R_{C2}}{R_{C2} + (\beta_3 + 1)\left[R_F + (R_E \parallel r_{e1})\right]} \quad (2)$$

$$I_{e3} = (\beta_3 + 1)I_{b2} \quad (3)$$

$$I_{e1} = I_{e3}\frac{R_E}{R_E + r_{e1}} \quad (4)$$

$$V_r = -\alpha_1 I_{e1}(R_{C1} \parallel r_{\pi 2}) \quad (5)$$

Substituting

$$\alpha_1 = 0.99$$

$$R_{C1} = 2 \text{ k}\Omega$$

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{2 \text{ mA}}{0.025 \text{ V}} = 80 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta_2}{g_{m2}} = \frac{100}{80} = 1.25 \text{ k}\Omega$$

$$R_E = 0.05 \text{ k}\Omega$$

$$r_{e1} = \frac{V_T}{I_{E1}} \simeq \frac{25 \text{ mV}}{1 \text{ mA}} = 25\ \Omega = 0.025 \text{ k}\Omega$$

$$\beta_3 = 100$$

$$R_{C2} = 1 \text{ k}\Omega$$

$$R_F = 1.2 \text{ k}\Omega$$

This figure belongs to Problem 11.32, part (b).

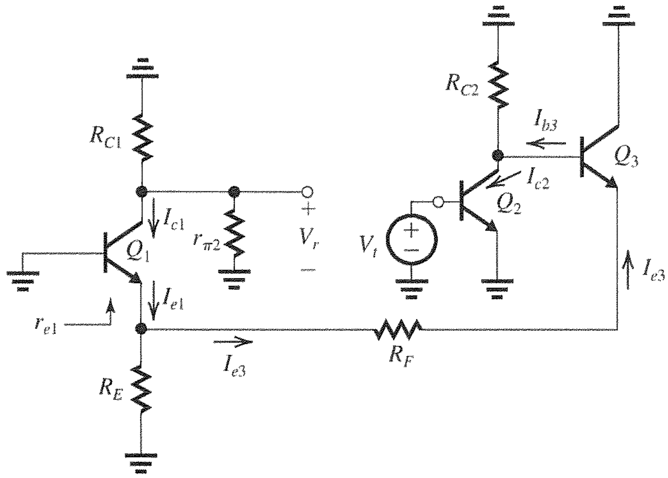


Figure 1

we obtain

$$I_{c2} = 80V_t \tag{6}$$

$$I_{b3} = I_{c2}\frac{1}{1+101[1.2+(0.05 \parallel 0.025)]}$$

$$= 8.072 \times 10^{-3} I_{c2} \tag{7}$$

$$I_{e3} = 101 I_{b3} \tag{8}$$

$$I_{e1} = I_{e3}\frac{50}{50+25} = 0.667 I_{e3} \tag{9}$$

$$V_r = -0.99(2 \parallel 1.25)I_{e1}$$

$$V_r = -0.7615 I_{e1} \tag{10}$$

Combining (6)–(10) results in

$$A\beta = 33.13$$

$$A = \frac{A\beta}{\beta} = \frac{33.13}{1/25} = 828.2 \text{ V/V}$$

$$A_f = \frac{A}{1+A\beta}$$

$$= \frac{828.2}{1+33.13} = 24.3 \text{ V/V}$$

11.33 All MOSFETs are operating at $I_D = 100\ \mu\text{A} = 0.1$ mA and $|V_{OV}| = 0.2$ V, thus

$$g_{m1,2} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

All devices have

$$r_o = \frac{|V_A|}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

(a)

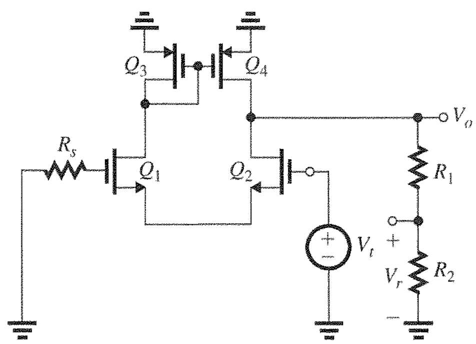


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain $A\beta$.

$$V_o = -g_{m1,2}[r_{o2} \parallel r_{o4} \parallel (R_1+R_2)]V_t \tag{1}$$

$$V_r = \frac{R_2}{R_1+R_2}V_o = \beta\, V_o$$

Thus,

$$A\beta \equiv -\frac{V_r}{V_t} = g_{m1,2}[r_{o2} \parallel r_{o4} \parallel (R_1+R_2)]\beta$$

$$= 1(100 \parallel 100 \parallel 1000)\beta$$

$$= 47.62\beta$$

Thus,

$$A = 47.62 \text{ V/V}$$

(b) $A_f = \dfrac{A}{1+A\beta}$

$$5 = \frac{47.62}{1+47.62\beta}$$

$$\Rightarrow \beta = 0.179 \text{ V/V}$$

$$\frac{R_2}{R_1+R_2} = 0.179$$

$$\Rightarrow R_2 = 179\ \text{k}\Omega$$

$$R_1 = 821\ \text{k}\Omega$$

11.34 $R_i = 2\ \text{k}\Omega$

$R_o = 2\ \text{k}\Omega$

$A = 1000$ V/V

$\beta = 0.1$ V/V

Loop Gain $\equiv A\beta = 1000 \times 0.1 = 100$

$1 + A\beta = 101$

$$A_f = \frac{A}{1+A\beta}$$

$$= \frac{1000}{101} = 9.9\ \text{V/V}$$

$$R_{if} = R_i(1+A\beta)$$

$$= 2 \times 101 = 202\ \text{k}\Omega$$

$$R_{of} = \frac{R_o}{1+A\beta}$$

$$= \frac{2}{101} = 19.8\ \Omega$$

11.35 Since the output voltage is sampled, the resistance-with-feedback is lower. The reduction is by the factor $(1 + A\beta)$, thus

$1 + A\beta = 200$

$A\beta = 199$

$$R_{of} = \frac{R_o}{200}$$

$$\Rightarrow R_o = 200 \times 100 = 20{,}000\ \Omega$$

$$= 20\ \text{k}\Omega$$

11.36 $A = \dfrac{A_0}{1 + \dfrac{s}{\omega_H}}$

$$1 + A\beta = 1 + \frac{A_0\beta}{1+\dfrac{s}{\omega_H}}$$

$$Z_{if} = R_i(1+A\beta)$$

$$= R_i + \frac{A_0\beta R_i}{1+\dfrac{s}{\omega_H}}$$

Thus, Z_{if} consists of a resistance R_i in series with an admittance Y,

$$Y = \frac{1}{A_0\beta R_i} + \frac{s}{A_0\beta R_i\omega_H}$$

which is a resistance $(A_0\beta R_i)$ in parallel with a capacitance $1/A_0\beta R_i\omega_H$. The equivalent circuit is shown in Fig. 1.

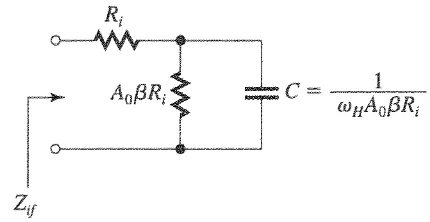


Figure 1

$$Z_{of} = \frac{R_o}{1+A\beta}$$

$$= \frac{R_o}{1 + \dfrac{A_0\beta}{1+\dfrac{s}{\omega_H}}}$$

Thus, the output admittance Y_{of} is

$$Y_{of} \equiv \frac{1}{Z_{of}} = \frac{1}{R_o} + \frac{A_0\beta}{R_o\left(1+\dfrac{s}{\omega_H}\right)}$$

which consists of a resistance R_o in parallel with an impedance Z given by

$$Z = \frac{R_o}{A_0\beta} + s\frac{R_o}{A_0\beta\omega_H}$$

which consists of a resistance $(R_o/A_0\beta)$ in series with an inductance $L = R_o/A_0\beta\omega_H$. The equivalent circuit of Z_{of} is shown in Fig. 2.

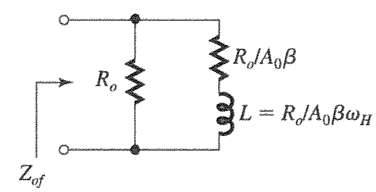


Figure 2

11.37 $A = 1000$ V/V

$R_i = 1\ \text{k}\Omega$

$R_{if} = 10\ \text{k}\Omega$

Thus, the connection at the input is a series one, and

$$1 + A\beta = \frac{10}{1} = 10$$

$$A_f = \frac{A}{1+A\beta}$$

$$= \frac{1000}{10} = 100\ \text{V/V}$$

To implement a unity-gain voltage follower, we use $\beta = 1$. Thus the amount of feedback is

$1 + A\beta = 1 + 1000 = 1001$

and the input resistance becomes

$$R_{if} = (1+A\beta)R_i$$

$$= 1001 \times 1 = 1001\ \text{k}\Omega = 1.001\ \text{M}\Omega$$

11.38 (a) $\beta = 1$

$A_f\big|_{\text{ideal}} = 1$ V/V

(b) Substituting $R_1 = \infty$ and $R_2 = 0$ in the expression for A in Example 11.4, we obtain

$$A = \mu \frac{R_L}{R_L + r_o} \frac{R_{id}}{R_{id} + R_s}$$

$$A\beta = A \times 1 = A$$

(c) $A = 10^4 \times \dfrac{2}{2+1} \times \dfrac{100}{100+10}$

$= 6060.6$ V/V

$A\beta = 6060.6$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{6060.6}{1 + 6060.6} = 0.9998 \text{ V/V}$$

From Example 11.4 with $R_1 = \infty$ and $R_L = 0$, we have

$R_i = R_s + R_{id} = 10 + 100 = 110$ kΩ

$R_{if} = R_i(1 + A\beta)$

$= 110 \times 6061.6 = 667$ MΩ

$R_{\text{in}} = R_{if} - R_s \simeq 667$ MΩ

$R_o = r_o \parallel R_L = 1 \parallel 2 = 0.67$ kΩ

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{0.67 \text{ k}\Omega}{6061.6} = 0.11 \ \Omega$$

$R_{of} = R_{\text{out}} \parallel R_L$

$R_{\text{out}} \simeq 0.11 \ \Omega$

11.39 Refer to the solution to Problem 11.29.

(a) $\beta = \dfrac{R_1}{R_1 + R_2}$

$$A_f = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

$$= 1 + \frac{10}{1} = 11 \text{ V/V}$$

(b) From the solution to Problem 10.29, we have

$I_{E1} \simeq 0.1$ mA

$I_{E2} \simeq 0.3$ mA

$V_{E2} = +7.7$ V

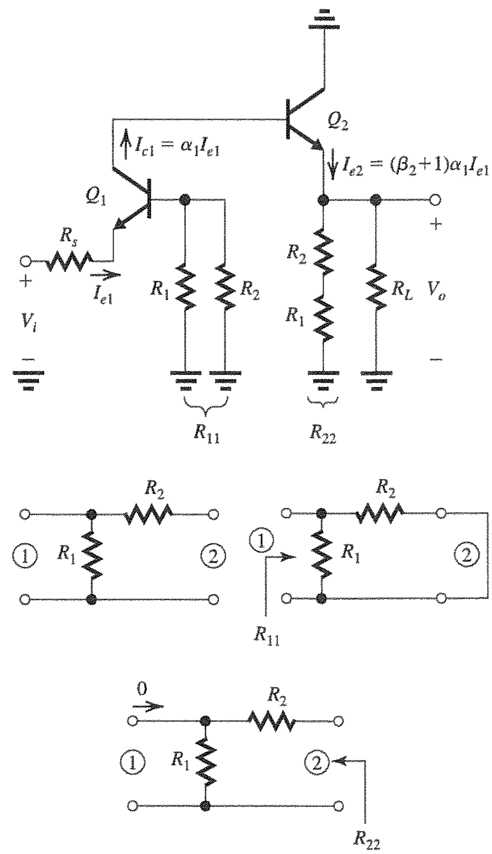


Figure 1

(c) The A circuit is shown in Fig. 1.

$$I_{e1} = \frac{V_i}{R_s + r_{e1} + \dfrac{R_1 \parallel R_2}{\beta_1 + 1}}$$

$V_o = I_{e2}[R_L \parallel (R_1 + R_2)]$

$$= (\beta_2 + 1)\alpha_1 V_i \frac{R_L \parallel (R_1 + R_2)}{R_s + r_{e1} + \dfrac{R_1 \parallel R_2}{\beta_1 + 1}}$$

Since $\beta_1 = \beta_2 = \beta$ and $\alpha = \dfrac{\beta}{\beta + 1}$, we have

$$A \equiv \frac{V_o}{V_i} = \beta \frac{R_L \parallel (R_1 + R_2)}{R_s + r_{e1} + \dfrac{R_1 \parallel R_2}{\beta_1 + 1}}$$

Substituting $\beta = 100$, $R_L = 1$ kΩ, $R_1 = 1$ kΩ, $R_2 = 10$ kΩ, $R_s = 0.1$ kΩ, and $r_{e1} = 0.25$ kΩ gives

$$A = 100 \frac{1 \parallel 11}{0.1 + 0.25 + \dfrac{1 \parallel 10}{101}}$$

$= 255.3$ V/V

$$R_i = R_s + r_{e1} + \frac{R_1 \parallel R_2}{\beta_1 + 1}$$

$$= 0.1 + 0.25 + \frac{1 \parallel 10}{101} = 0.359\ \text{k}\Omega$$

$$R_o = R_L \parallel (R_1 + R_2)$$

$$= 1 \parallel 11 = 0.917\ \text{k}\Omega$$

(d) $\beta = \dfrac{R_1}{R_1 + R_2}$

$$= \frac{1}{1 + 10} = \frac{1}{11}$$

(e) $\dfrac{V_o}{V_s} = A_f = \dfrac{A}{1 + A\beta}$

$$1 + A\beta = 1 + \frac{255.3}{11} = 24.21$$

$$A_f = \frac{255.3}{24.21} = 10.5\ \text{V/V}$$

$$R_{if} = R_i(1 + A\beta)$$

$$= 0.359 \times 24.21 = 8.69\ \text{k}\Omega$$

$$R_{\text{in}} = R_{if} - R_s$$

$$= 8.69 - 0.1 = 8.59\ \text{k}\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{0.917\ \text{k}\Omega}{24.21} = 37.9\ \Omega$$

$$R_{of} = R_{\text{out}} \parallel R_L$$

$$37.9 = R_{\text{out}} \parallel 1000$$

$$\Rightarrow R_{\text{out}} = 39.4\ \Omega$$

The value of A_f (10.5 V/V) is 0.5 less than the ideal value of 11, which is 4.5%.

11.40 (a) Refer to Fig. P11.40. Assume that for some reason v_S increases. This will increase the differential input signal $(v_S - v_O)$ applied to the differential amplifier. The drain current of Q_1 will increase, and this increase will be mirrored in the drain current of Q_4. The increase in i_{D4} will cause the voltage at the gate of Q_5 to rise. Since Q_5 is operating as a source follower, the voltage at its source, v_O, will follow and increase. This will cause the differential input signal $(v_S - v_O)$ to decrease, thus counteracting the originally assumed change. Thus, the feedback is negative.

(b) Figure 1 on the next page shows the circuit prepared for dc analysis. We see that

$$I_{D1} = I_{D2} = 100\ \mu\text{A}$$

$$I_{D3} = 100\ \mu\text{A}$$

$$I_{D4} = 300\ \mu\text{A}$$

$$I_{D5} = 0.8\ \text{mA}$$

For Q_1 and Q_2, use

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$100 = \frac{1}{2} \times 120 \times \frac{20}{1} V_{OV1,2}^2$$

$$\Rightarrow V_{OV1,2} = 0.29\ \text{V}$$

For Q_3, use

$$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)|V_{OV3}|^2$$

$$100 = \frac{1}{2} \times 60 \times \frac{40}{1}|V_{OV3}|^2$$

$$\Rightarrow |V_{OV3}| = 0.29\ \text{V}$$

Since $V_{SG4} = V_{SG3}$, we have

$$|V_{OV4}| = |V_{OV3}| = 0.29\ \text{V}$$

Finally for Q_5, use

$$800 = \frac{1}{2} \times 120 \times \frac{20}{1} \times V_{OV5}^2$$

$$\Rightarrow V_{OV5} = 0.82\ \text{V}$$

If perfect matching pertains, then

$$V_{D4} = V_{D3} = V_{DD} - V_{SG3}$$

$$= 2.5 - |V_t| - |V_{OV3}|$$

$$= 2.5 - 0.7 - 0.29 = 1.51\ \text{V}$$

$$V_O = V_{D4} - V_{GS5}$$

$$= V_{D4} - V_t - V_{OV5}$$

$$= 1.51 - 0.7 - 0.82 = -0.01\ \text{V}$$

which is approximately zero, as stated in the Problem statement.

(c) $g_{m1} = g_{m2} = g_{m3} = \dfrac{2I_D}{|V_{OV}|}$

$$= \frac{2 \times 0.1}{0.29} = 0.7\ \text{mA/V}$$

$$g_{m4} = \frac{2 \times 0.3}{0.29} \simeq 2\ \text{mA/V}$$

$$g_{m5} = \frac{2 \times 0.8}{0.82} \simeq 2\ \text{mA/V}$$

$$r_{o1} = r_{o2} = r_{o3} = \frac{|V_A|}{I_D} = \frac{|V_A'| \times L}{I_D}$$

$$= \frac{24 \times 1}{0.1} = 240\ \text{k}\Omega$$

$$r_{o4} = \frac{24}{0.3} = 80\ \text{k}\Omega$$

$$r_{o5} = \frac{24}{0.8} = 30\ \text{k}\Omega$$

(d) Figure 2 on the next page shows the A circuit. Observe that since the β network is simply a wire connecting the output node to the gate of Q_2, we have $R_{11} = 0$ and $R_{22} = \infty$. To determine A, we write

This figure belongs to Problem 11.40, part (b).

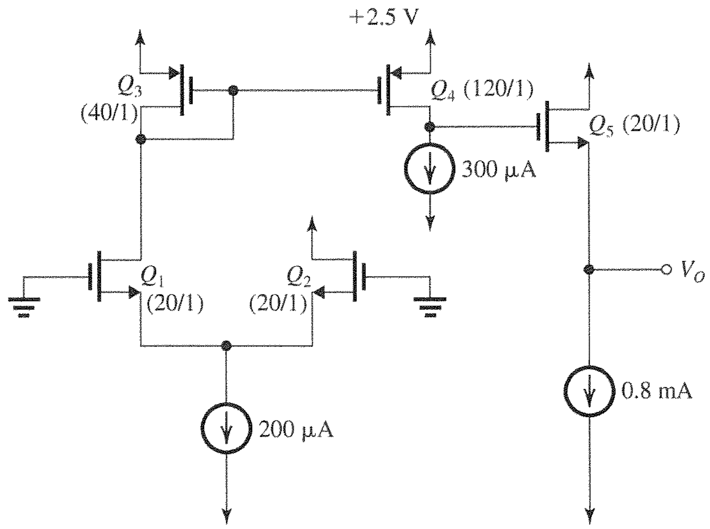


Figure 1

This figure belongs to Problem 11.40, part (d).

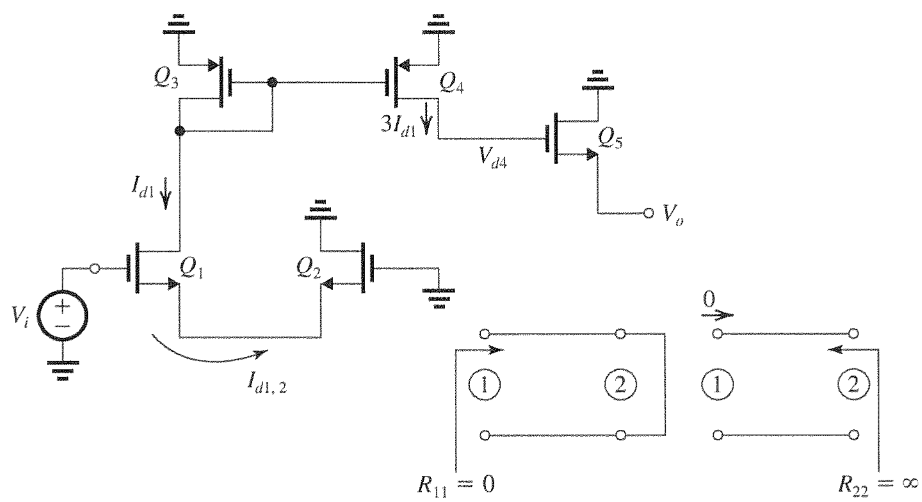


Figure 2

$$I_{d1,2} = \frac{V_i}{2/g_{m1,2}} = \frac{1}{2}g_{m1,2}V_i$$

Since $\left(\frac{W}{L}\right)_4 = 3\left(\frac{W}{L}\right)_3$, the drain current of Q_4 will be

$$I_{d4} = 3I_{d1} = \frac{3}{2}g_{m1,2}V_i$$

The voltage at the drain of Q_4 will be

$$V_{d4} = I_{d4}r_{o4}$$

$$= \frac{3}{2}g_{m1,2}r_{o4}V_i$$

Finally, V_o is related to V_{d4} as

$$\frac{V_o}{V_{d4}} = \frac{r_{o5}}{r_{o5} + \dfrac{1}{g_{m5}}}$$

Thus,

$$A \equiv \frac{V_o}{V_i} = \frac{3}{2}g_{m1,2}r_{o4}\frac{r_{o5}}{r_{o5} + \dfrac{1}{g_{m5}}}$$

Substituting numerical values, we obtain

$$A = \frac{3}{2} \times 0.7 \times 80 \times \frac{30}{30 + 0.5}$$

$$= 82.6 \text{ V/V}$$

The output resistance R_o is

$$R_o = r_{o5} \parallel \frac{1}{g_{m5}}$$

$$= 30 \parallel 0.5 = 0.492 \text{ k}\Omega$$

$$= 492 \text{ }\Omega$$

(e) $A_f = \dfrac{A}{1 + A\beta}$

$$= \frac{82.6}{1 + 82.6} = 0.988 \text{ V/V}$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{492}{1 + 82.6} = 5.9 \text{ }\Omega$$

$$R_{\text{out}} = R_{of} = 5.9 \text{ }\Omega$$

(f) To obtain a closed-loop gain of 5 V/V, we connect a voltage divider in the feedback loop, as shown in Fig. 3.

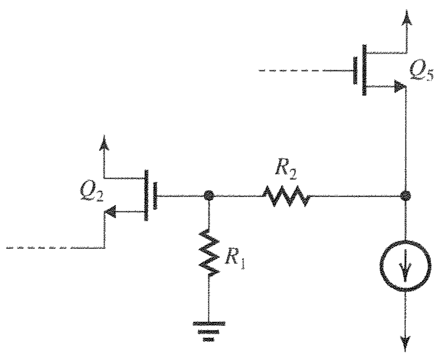


Figure 3

$$\beta = \frac{R_1}{R_1 + R_2}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$5 = \frac{82.6}{1 + 82.6\beta}$$

$$\Rightarrow \beta = 0.188$$

Selecting $R_1 = 1 \text{ M}\Omega$, we obtain

$$0.188 = \frac{1}{1 + R_2}$$

$$\Rightarrow R_2 = 4.319 \text{ M}\Omega$$

Note that by selecting large values for R_1 and R_2, we have ensured that their loading effect on the A circuit would be negligible.

11.41 (a) Let V_s increase by a small increment. Since Q_1 is operating in effect as a CS amplifier, a negative incremental voltage will appear at its drain. Transistor Q_2 is also operating as a CS amplifier; thus a positive incremental voltage will appear at its drain. Transistor Q_3 is operating as a source follower; thus the signal at its source (which is the output voltage) will follow that at its gate and thus will be positive. The end result is that we are feeding back through the voltage divider (R_2, R_1) a positive incremental signal that will appear across R_1 and thus at the source of the Q_1. This signal, being of the same polarity as the originally assumed change in the signal at the gate of $Q_1(V_s)$, will *subtract* from the original change, causing a *smaller* signal to appear across the gate-source terminals of Q_1. Hence, the feedback is negative.

(b) $\beta = \dfrac{R_1}{R_1 + R_2}$

Thus,

$$\beta = \frac{2}{2 + 18} = 0.1 \text{ V/V}$$

If the loop gain is large, the closed-loop gain approaches the ideal value

$$A_f\big|_{\text{ideal}} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

Thus,

$$A_f\big|_{\text{ideal}} = 1 + \frac{18}{2} = 10 \text{ V/V}$$

(c) $V_{G1} = 0.9 \text{ V}$

$$V_{S1} = V_{G1} - V_{GS1}$$

$$= V_{G1} - V_{t1} - V_{OV1}$$

$$= 0.9 - 0.5 - 0.2 = 0.2 \text{ V}$$

$$V_{G2} = V_{DD} - V_{SG2}$$

$$= V_{DD} - |V_{t2}| - |V_{OV2}|$$

$$= 1.80 - 0.5 - 0.2 = 1.1 \text{ V}$$

Thus, current source I_1 will have 0.7-V drop across it, more than sufficient for its proper operation. Since $V_{S1} = 0.2$ V the dc current through R_1 will be

$$I_{R1} = \frac{V_{S1}}{R_1} = \frac{0.2 \text{ V}}{2 \text{ k}\Omega} = 0.1 \text{ mA}$$

Now, a node equation at S_1 reveals that because $I_{D1} = 0.1$ mA and $I_{R1} = 0.1$ mA, the dc current in R_2 will be zero. Thus, it will have a zero voltage drop across it and

$$V_{S3} = V_{S1} = 0.2 \text{ V}$$

Thus, current source I_3 will have across it, the minimum voltage required to keep it operating properly. Finally,

$$V_{G3} = V_{S3} + V_{GS3}$$

$$= V_{S3} + V_{t3} + V_{OV3}$$

$$= 0.2 + 0.5 + 0.2 = 0.9 \text{ V}$$

Thus, current source I_2 will have across it a voltage more than sufficient to keep it operating properly.

(d)

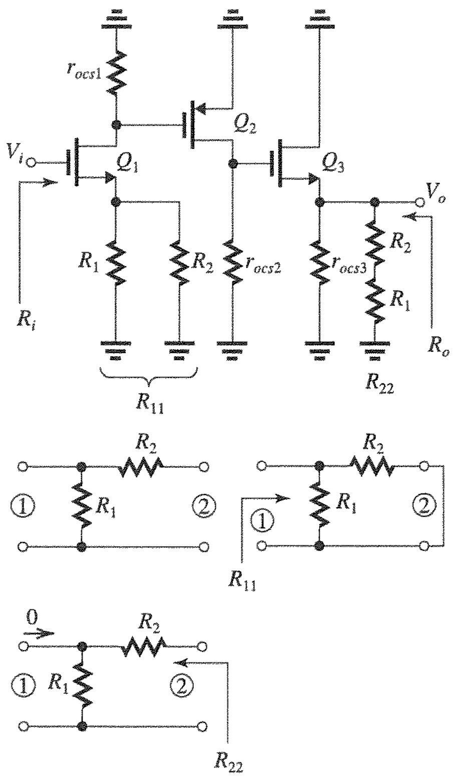


Figure 1

Figure 1 shows the A circuit as well as the β circuit and how the loading-effect resistances R_{11} and R_{22} are determined.

To determine A, let's first determine the small-signal parameters of all transistors as well as r_o of each of the three current sources.

$$g_{m1} = \frac{2I_{D1}}{V_{OV1}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o1} = \frac{|V_A|}{I_{D1}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{ocs1} = \frac{|V_A|}{I_1} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$g_{m2} = \frac{2I_{D2}}{V_{OV2}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = \frac{|V_A|}{I_{D2}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{ocs2} = \frac{|V_A|}{I_2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$g_{m3} = \frac{2I_{D3}}{V_{OV3}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o3} = \frac{|V_A|}{I_{D3}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{ocs3} = \frac{|V_A|}{I_3} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

Refer to the A circuit.

Transistor Q_1 is a CS amplifier with a resistance R_{11} in its source:

$$R_s = R_{11} = R_1 \parallel R_2 = 2 \parallel 18 = 1.8 \text{ k}\Omega$$

Transistor Q_1 will have an effective transconductance:

$$G_{m1} = \frac{g_{m1}}{1 + g_m R_s} = \frac{2}{1 + 2 \times 1.8} = 0.43 \text{ mA/V}$$

The output resistance of Q_1 will be

$$R_{o1} = (1 + g_{m1} R_s) r_{o1}$$

$$= (1 + 2 \times 1.8) \times 100 = 460 \text{ k}\Omega$$

The total resistance at the drain of Q_1 is

$$R_{d1} = r_{ocs1} \parallel R_{o1}$$

$$= 100 \parallel 460 = 82.1 \text{ k}\Omega$$

Thus, the voltage gain of the first stage is

$$A_1 = -G_{m1} R_{d1}$$

$$= -0.43 \times 82.1 = -35.3 \text{ V/V}$$

The gain of the second stage is

$$A_2 = -g_{m2}(r_{ocs2} \parallel r_{o2})$$

$$= -1(100 \parallel 100) = -50 \text{ V/V}$$

To determine the gain of the third stage, we first determine the total resistance between the source of Q_3 and ground:

$$R_{s3} = r_{ocs3} \parallel r_{o3} \parallel (R_1 + R_2)$$

$$R_{s3} = 100 \parallel 100 \parallel 20$$

$$= 14.3 \text{ k}\Omega$$

Thus,

$$A_3 = \frac{R_{s3}}{R_{s3} + \dfrac{1}{g_{m3}}}$$

$$= \frac{14.3}{14.3 + \dfrac{1}{1}} = 0.935 \text{ V/V}$$

The overall voltage gain A can now be found as

$$A = A_1 A_2 A_3$$

$$= -35.3 \times -50 \times 0.935 = 1650 \text{ V/V}$$

(e) We already found β in (b) as

$$\beta = 0.1 \text{ V/V}$$

(f) $1 + A\beta = 1 + 1650 \times 0.1 = 166$

$$A_f = \frac{A}{1 + A\beta} = \frac{1650}{166} = 9.94 \text{ V/V}$$

which is lower by 0.06 or 0.6% than the ideal value obtained in (b).

(g) $R_{of} = \dfrac{R_o}{1 + A\beta}$

To obtain R_o refer to the output part of the A circuit.

$$R_o = (R_1 + R_2) \parallel r_{ocs3} \parallel r_{o3} \parallel \frac{1}{g_{m3}}$$

$$= 20 \parallel 100 \parallel 100 \parallel 1$$

$$= 935\ \Omega$$

$$R_{of} = \frac{935}{166} = 5.6\ \Omega$$

Note: This problem, though long, is extremely valuable as it exercises the student's knowledge in many aspects of amplifier design.

11.42 (a) Refer to Fig. P11.33. Let V_s increase by a positive increment. This will cause the drain current of Q_1 to increase. The increase in I_{d1} will be fed to the $Q_3 - Q_4$ mirror, which will provide a corresponding increase in the drain current of Q_4. The latter current will cause the voltage at the output node to rise. A fraction of the increase in V_o is applied through the divider (R_1, R_2) to the gate of Q_2. The increase in the voltage of the gate of Q_2 will *subtract* from the initially assumed increase of the voltage of the gate of Q_1, resulting in a smaller increase in the differential voltage applied to the (Q_1, Q_2) pair. Thus, the feedback counter acts the originally assumed change, verifying that it is negative.

(b) The negative feedback will cause the dc voltage at the gate of Q_2 to be approximately equal to the dc voltage at the gate of Q_1, that is, zero. Now, with $V_{G2} \simeq 0$, the dc current in R_2 will be zero and similarly the dc current in R_1 will be zero, resulting in $V_O = 0$ V dc.

(c) Figure 1 shows the A circuit. It also shows how the loading effect of the β network on the A circuit, namely R_{11} and R_{22}, are found. The gain of the A circuit can be written by inspection as

$$A = g_{m1,2}(r_{o2} \parallel r_{o4} \parallel R_{22})$$

where

$$g_{m1,2} = \frac{2I_{D1,2}}{V_{OV1,2}}$$

$$= \frac{2 \times 0.1}{0.2} = 1\ \text{mA/V}$$

This figure belongs to Problem 11.42, part (c).

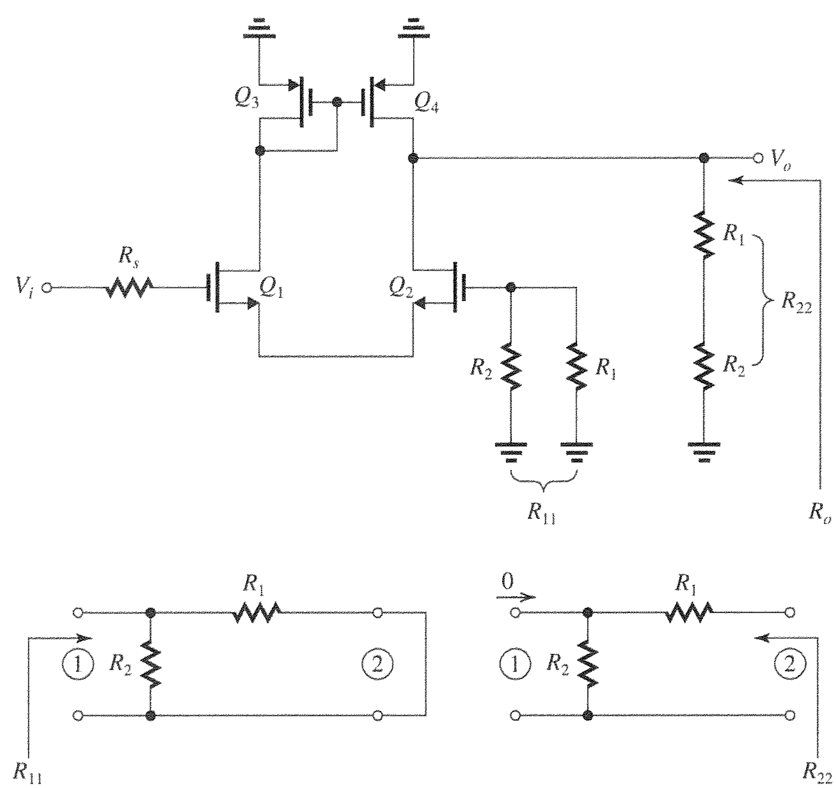


Figure 1

$r_{o2} = r_{o4} = \frac{|V_A|}{I_{D3,4}} = \frac{10}{0.1} = 100 \text{ k}\Omega$

$R_{22} = R_1 + R_2 = 1 \text{ M}\Omega$

$A = 1(100 \parallel 100 \parallel 1000) = 47.62 \text{ V/V}$

This is identical to the value found in the solution to Problem 11.33.

(d) $\frac{V_o}{V_s} = A_f = \frac{A}{1 + A\beta}$

$5 = \frac{47.62}{1 + 47.62\beta}$

$\Rightarrow \beta = 0.179$

Thus,

$\frac{R_2}{R_1 + R_2} = 0.179$

$R_2 = 0.179 \text{ M}\Omega = 179 \text{ k}\Omega$

$R_1 = 1000 - 179 = 821 \text{ k}\Omega$

Again, these values are identical to those found in Problem 11.33.

(e) Refer to Fig. 1.

$R_o = R_{22} \parallel r_{o2} \parallel r_{o4}$

$= 1000 \parallel 100 \parallel 100 = 47.62 \text{ k}\Omega$

$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$

$= \frac{47.62}{1 + 47.62 \times 0.179}$

$= 5 \text{ k}\Omega$

This value cannot be found using the loop-gain analysis method of Problem 11.33.

(f) With $R_L = 10 \text{ k}\Omega$,

$\frac{V_o}{V_s} = 5 \times \frac{R_L}{R_L + R_{\text{out}}}$

$5 \times \frac{10}{10 + 5} = 3.33 \text{ V/V}$

(g) As an alternative to (f), we shall redo the analysis of the A circuit in (c) above with $R_L = 10 \text{ k}\Omega$ included:

$A = g_{m1,2}(r_{o2} \parallel r_{o4} \parallel R_{22} \parallel R_L)$

$= 1(100 \parallel 100 \parallel 1000 \parallel 10)$

$= 8.26 \text{ V/V}$

Using $\beta = 0.179$, we obtain

$A_f = \frac{8.26}{1 + 8.26 \times 0.179} = 3.33 \text{ V/V}$

which is identical to the value found in (f) above.

11.43 All transistors have $L = 1\ \mu\text{m}$, thus all have $|V_A| = |V_A'| \times L = 10 \times 1 = 10$ V. Also, all have $|V_t| = 0.75$ V.

(a) Figure 1 shows the circuit prepared for dc design. We have also indicated some of the current and voltage values. We now find the (W/L) ratios utilizing

$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$

for the NMOS transistors, and

$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)|V_{OV}|^2$

for the PMOS devices.

This figure belongs to Problem 11.43, part (a).

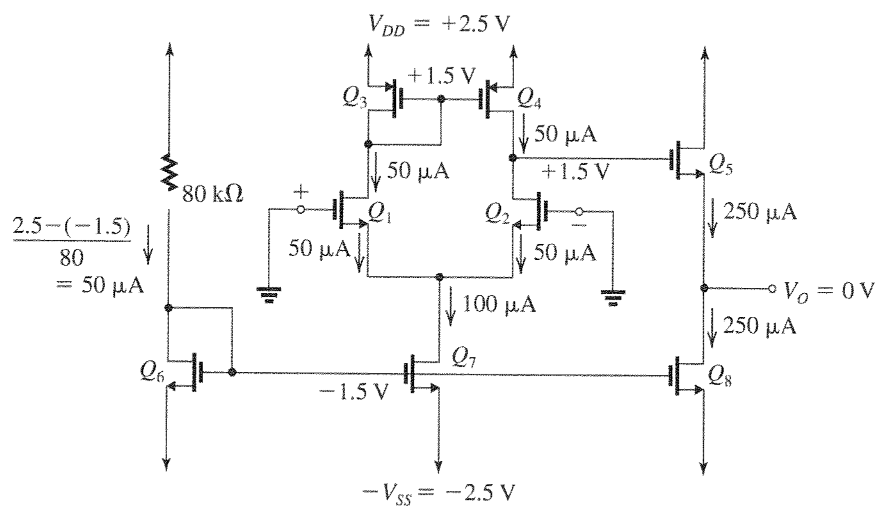


Figure 1

For Q_6,

$$50 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_6 \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_6 = 16$$

For Q_7,

$$\frac{(W/L)_7}{(W/L)_6} = \frac{100\ \mu\text{A}}{50\ \mu\text{A}} = 2$$

$$\Rightarrow (W/L)_7 = 2 \times 16 = 32$$

For Q_8,

$$\frac{(W/L)_8}{(W/L)_6} = \frac{250\ \mu\text{A}}{50\ \mu\text{A}} = 5$$

$$\Rightarrow \left(\frac{W}{L}\right)_8 = 5 \times 16 = 80$$

For Q_1 and Q_2,

$$50 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{1,2} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 16$$

For Q_3 and Q_4,

$$50 = \frac{1}{2} \times 50 \times \left(\frac{W}{L}\right)_{3,4} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 32$$

Finally, since $V_{G3} = V_{D4} = V_{D3} = 1.5$ V and we require $V_O = 0$ V, we have

$$V_{GS5} = 1.5 \text{ V}$$

$$V_{OV5} = 1.5 - 0.75 = 0.75 \text{ V}$$

$$250 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_5 \times 0.75^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = 8.9$$

(b) The maximum value of V_{ICM} is limited by Q_1 leaving the saturation region,

$$V_{ICM\max} = V_{D1} + V_t$$

$$= 1.5 + 0.75 = 2.25 \text{ V}$$

The minimum value of V_{ICM} is limited by the need to keep Q_7 in saturation. This is achieved by keeping V_{D7} at a minimum voltage of

$$-2.5 + |V_{OV7}| = -2.5 + 0.25 = -2.25 \text{ V}$$

Thus,

$$V_{ICM\min} = -2.25 + V_{GS1}$$

$$= -2.25 + 1 = -1.25 \text{ V}$$

Thus,

$$-1.25 \text{ V} \le V_{ICM} \le +2.25 \text{ V}$$

(c) $g_{m1,2} = \dfrac{2I_{D1,2}}{V_{OV1,2}}$

$$= \frac{2 \times 0.05}{0.2} = 0.5 \text{ mA/V}$$

$$g_{m5} = \frac{2I_D}{|V_{OV5}|} = \frac{2 \times 0.25}{0.75} = 0.67 \text{ mA/V}$$

(d) $r_{o1} = r_{o2} = r_{o3} = r_{o4} = r_{o6} = \dfrac{|V_A|}{I_D} = \dfrac{10}{0.05} = 200\ \text{k}\Omega$

$$r_{o7} = \frac{10}{0.01} = 100\ \text{k}\Omega$$

$$r_{o5} = r_{o8} = \frac{10}{0.25} = 40\ \text{k}\Omega$$

(e) Figure 2 on the next page shows the A circuit, the β circuit, and how the loading effects of the β circuit on the A circuit, namely R_{11} and R_{22}, are determined.

$$\frac{V_{g5}}{V_i} = g_{m1,2}(r_{o2} \parallel r_{o4})$$

$$= 0.5(200 \parallel 200) = 50 \text{ V/V}$$

$$\frac{V_o}{V_{g5}} = \frac{R_s}{R_s + \dfrac{1}{g_{m5}}}$$

where

$$R_s = r_{o8} \parallel r_{o5} \parallel (R_1 + R_2) \parallel R_L$$

$$= 40 \parallel 40 \parallel 100 \parallel 100 = 14.3\ \text{k}\Omega$$

Thus,

$$\frac{V_o}{V_{g5}} = \frac{14.3}{14.3 + (1/0.67)} = 0.905 \text{ V/V}$$

$$A = \frac{V_o}{V_i} = \frac{V_{g5}}{V_i} \times \frac{V_o}{V_{g5}}$$

$$= 50 \times 0.905 = 45.3 \text{ V/V}$$

$$A_f = 10 = \frac{A}{1 + A\beta}$$

$$10 = \frac{45.3}{1 + 45.3\beta}$$

$$\Rightarrow \beta = 0.078$$

$$\frac{R_2}{R_1 + R_2} = 0.078$$

$$R_2 = 7.8\ \text{k}\Omega$$

$$R_1 = 100 - 7.8 = 92.2\ \text{k}\Omega$$

This figure belongs to Problem 11.43, part (e).

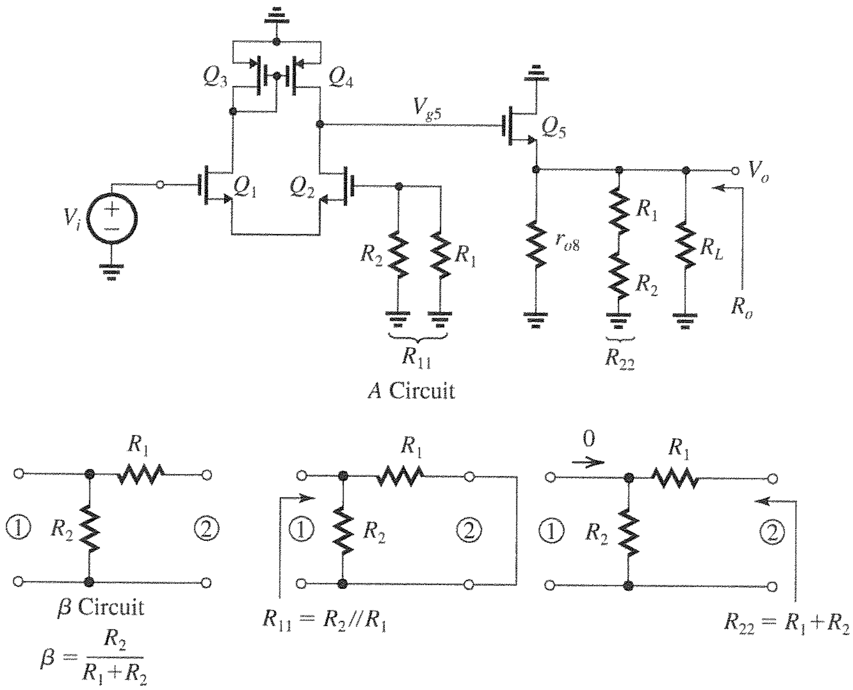


Figure 2

(f) Refer to Fig. 2.

$$R_o = R_L \parallel (R_1 + R_2) \parallel r_{o8} \parallel r_{o5} \parallel \frac{1}{g_{m5}}$$

$$= R_s \parallel \frac{1}{g_{m5}} = 14.3 \parallel (1/0.67)$$

$$= 1.36 \text{ k}\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{1.36 \text{ k}\Omega}{1 + 45.3 \times 0.078} \simeq 300 \ \Omega$$

$$R_{\text{out}} \parallel R_L = R_{of}$$

$$\Rightarrow R_{\text{out}} \simeq 300 \ \Omega$$

11.44 (a) Figure 1 on the next page shows the A circuit and the circuit for determining β as well as the determination of the loading effects of the β circuit.

(b) If $A\beta$ is large, then

$$A_f \equiv \frac{V_o}{V_s} \simeq \frac{1}{\beta}$$

Since

$$\beta = \frac{R_E}{R_F + R_E}$$

we have

$$A_f = \frac{R_F + R_E}{R_E} \qquad \text{Q.E.D.}$$

(c) $25 = 1 + \dfrac{R_F}{50 \ \Omega}$

$$\Rightarrow R_F = 1.2 \text{ k}\Omega$$

(d) Refer to the A circuit in Fig. 1. The voltage gain of Q_1 is given by

$$\frac{V_{c1}}{V_i} = -\alpha_1 \frac{R_{C1} \parallel r_{\pi 2}}{r_{e1} + R_{11}}$$

where

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \ \Omega$$

$$R_{11} = R_E \parallel R_F = 50 \ \Omega \parallel 1200 \ \Omega = 48 \ \Omega$$

$$g_{m2} = \frac{I_{C2}}{V_T} \simeq \frac{I_{E2}}{V_T} = \frac{2 \text{ mA}}{0.025 \text{ mA}} = 80 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta_2}{80} = \frac{100}{80} = 1.5 \text{ k}\Omega$$

$$\alpha_1 = 0.99 \simeq 1$$

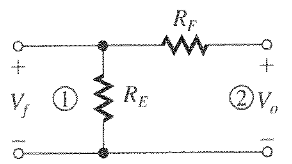
$$\frac{V_{c1}}{V_i} = -10 = -\frac{R_{C1} \parallel 1.5}{0.025 + 0.048}$$

$$\Rightarrow R_{C1} = 1.42 \text{ k}\Omega$$

Next consider the second stage composed of the CE transistor Q_2. The load resistance of the second stage is composed of R_{C2} in parallel with the input resistance of emitter-follower Q_3. The latter resistance is given by

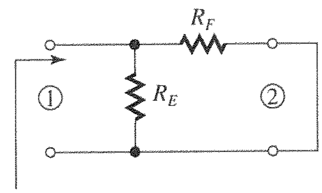
$$R_{i3} = (\beta_3 + 1)(r_{e3} + R_{22})$$

This figure belongs to Problem 11.44, part (a).

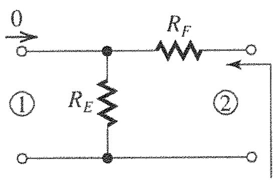


β Circuit

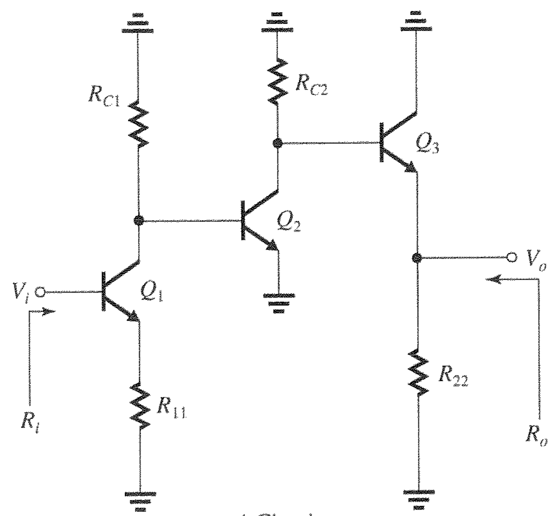
$$\beta = \frac{R_E}{R_F + R_E}$$



$R_{11} = R_E // R_F$



$R_{22} = R_E + R_F$



A Circuit

Figure 1

where

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{25\text{ mV}}{5\text{ mA}} = 5\ \Omega$$

$$R_{22} = R_F + R_E = 1.2 + 0.05 = 1.25\text{ k}\Omega$$

Thus,

$$R_{i3} = 101 \times 1.25 = 126.3\text{ k}\Omega$$

$$A_2 \equiv \frac{V_{c2}}{V_{b2}} = -g_{m2}(R_{C2} \parallel R_{i3})$$

$$-50 = -80(R_{C2} \parallel 126.3)$$

$$\Rightarrow R_{C2} = 628\ \Omega$$

(e) $A = A_1 A_2 A_3$

where

$$A_3 = \frac{R_{22}}{R_{22} + r_{e3}} = \frac{1.25}{1.25 + 0.005} = 0.996\text{ V/V}$$

$$A \equiv -10 \times -50 \times 0.996$$

$$= 498\text{ V/V}$$

$$A_f \equiv \frac{V_o}{V_s} = \frac{498}{1 + 498 \times \dfrac{50}{1250}}$$

$$= 23.8\text{ V/V}$$

(f) Refer to the *A* circuit in Fig. 1.

$$R_i = (\beta_1 + 1)(r_{e1} + R_{11})$$

$$R_i = 101(0.025 + 0.048)$$

$$= 7.37\text{ k}\Omega$$

$$R_{if} = R_i(1 + A\beta)$$

where

$$1 + A\beta = 1 + \frac{498}{25} = 20.92$$

$R_{if} = 7.37 \times 20.92 = 154\ \text{k}\Omega$

$R_o = R_{22} \parallel \left[r_{e3} + \frac{R_{C2}}{\beta_3 + 1}\right]$

$= 1.25 \parallel \left[0.005 + \frac{0.628}{101}\right]$

$= 11.1\ \Omega$

$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$

$= \frac{11.1}{20.92} = 0.53\ \Omega$

11.45 (a) Refer to Fig. P11.45. If V_s increases, the output of A_1 will decrease and this will cause the output of A_2 to increase. This, in turn, causes the output of A_3, which is V_o, to increase. A portion of the positive increment in V_o is fed back to the positive input terminal of A_1 through the voltage divider (R_2, R_1), The increased voltage at the positive input terminal of A_1 counteracts the originally assumed increase at the negative input terminal, verifying that the feedback is negative.

(b) $A_f\big|_{\text{ideal}} = \frac{1}{\beta}$

where

$\beta = \frac{R_1}{R_1 + R_2}$

Thus, to obtain an ideal closed-loop gain of 5 V/V we need $\beta = 0.2$:

$0.2 = \frac{20}{20 + R_2}$

$\Rightarrow R_2 = 80\ \text{k}\Omega$

(c) Figure 1 shows the small-signal equivalent circuit of the feedback amplifier.

(d) Figure 2 on the next page shows the A circuit and the β circuit together with the determination of its loading effects, R_{11}, and R_{22}. We can write

$\frac{V_1}{V_i} = -\frac{82}{82 + 9 + 16} = -0.766\ \text{V/V}$

$V_2 = 20V_1 \times \frac{5}{3.2 + 5} = 12.195V_1$

$V_3 = -20V_2(20 \parallel 20) = -200V_2$

$V_o = V_3 \frac{1 \parallel 100}{(1 \parallel 100) + 1} = 0.497V_3$

Thus,

$A \equiv \frac{V_o}{V_i} = 0.497 \times -200 \times 12.195 \times -0.766$

$= 928.5\ \text{V/V}$

(e) $\beta = \frac{20}{20 + 80} = 0.2\ \text{V/V}$

$1 + A\beta = 1 + 928.5 \times 0.2 = 186.7$

(f) $A_f \equiv \frac{V_o}{V_s} = \frac{A}{1 + A\beta}$

$= \frac{928.5}{186.7} = 4.97\ \text{V/V}$

which is nearly equal to the ideal value of 5 V/V.

(g) From the A circuit,

$R_i = 9 + 82 + 16 = 107\ \text{k}\Omega$

$R_{if} = R_i(1 + A\beta)$

$= 107 \times 186.7 = 19.98\ \text{M}\Omega$

$R_{\text{in}} = R_{if} - R_s \simeq 19.98\ \text{M}\Omega$

(h) From the A circuit,

$R_o = R_L \parallel R_{22} \parallel 1\ \text{k}\Omega$

$= 1 \parallel 100 \parallel 1 = 497.5\ \Omega$

This figure belongs to Problem 11.45, part (c).

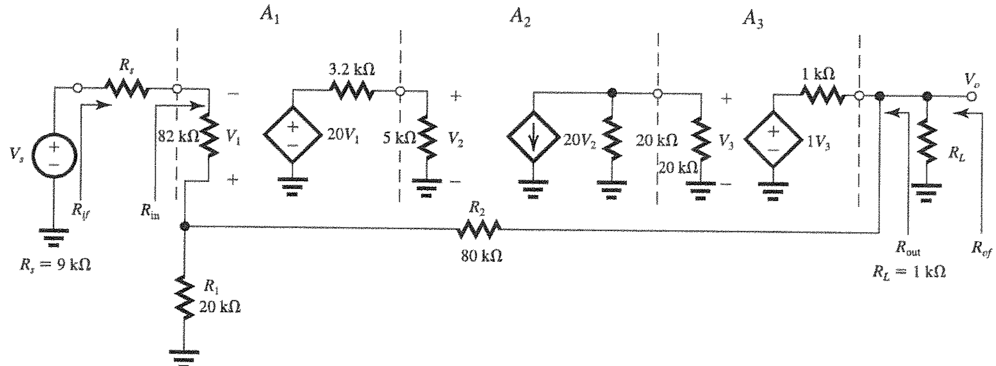


Figure 1

This figure belongs to Problem 11.45, part (d).

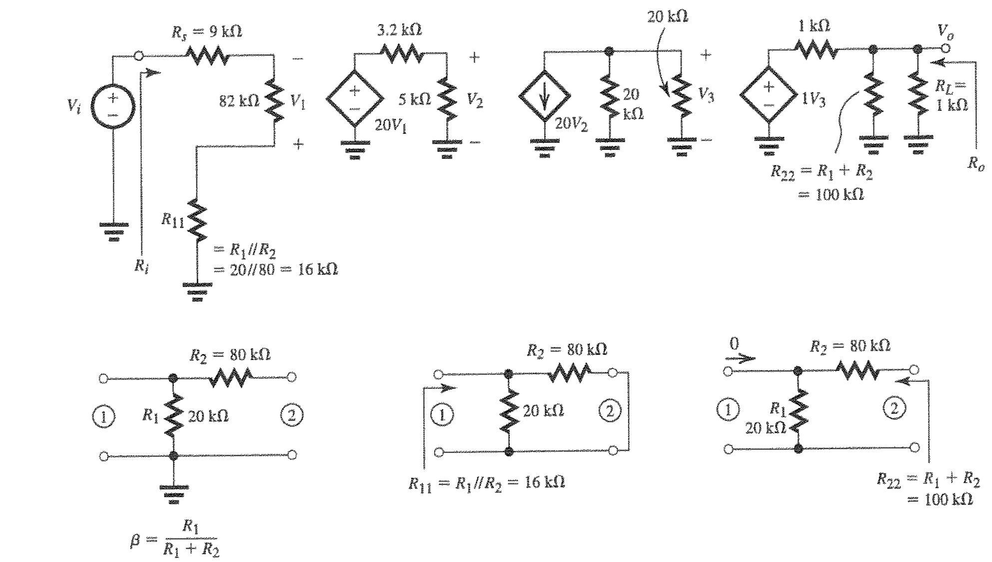


Figure 2

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{497.5}{186.7} = 2.66\ \Omega$$

$R_{\text{out}} \parallel R_L = R_{of}$

$R_{\text{out}} \parallel 1000 = 2.66\ \Omega$

$R_{\text{out}} \simeq 2.66\ \Omega$

(i) $f_{Hf} = f_H(1 + A\beta)$

$= 100 \times 186.7$

$= 18.67$ kHz

(j) If A_1 drops to half its nominal value, A will drop to half its nominal value:

$$A = \frac{1}{2} \times 928.5 = 464.25$$

and A_f becomes

$$A_f = \frac{464.25}{1 + 464.25 \times 0.2} = 4.947 \text{ V/V}$$

Thus, the percentage change in A_f is

$$= \frac{4.947 - 4.97}{4.97} = -0.47\%$$

11.46 To obtain $A_f \equiv \dfrac{I_o}{V_s} \simeq 10$ mA/V, we select

$$R_F = \beta = \frac{1}{A_f} = 100\ \Omega$$

From Example 11.6, we obtain

$$A = \frac{\mu}{R_F} \frac{g_m(R_F \parallel R_{id} \parallel r_{o2})}{1 + g_m(R_F \parallel R_{id} \parallel r_{o2})}$$

$$\equiv \frac{1000}{0.1\text{ k}\Omega} \frac{2(0.1 \parallel 100 \parallel 20)}{1 + 2(0.1 \parallel 100 \parallel 20)}$$

$= 10,000 \times 0.1658$

$= 1.658$ A/V

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1658}{1 + 1658 \times 0.1} = 9.94 \text{ mA/V}$$

11.47 (a)

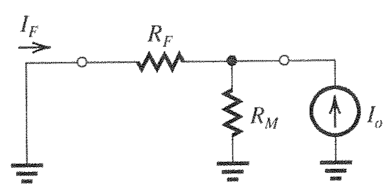


Figure 1

Figure 1 shows the β network with the input port short-circuited. Thus,

$$\beta \equiv \frac{I_f}{I_o} = -\frac{R_M}{R_M + R_F}$$

$$A_f\big|_{\text{ideal}} = \frac{1}{\beta} = -\left(1 + \frac{R_F}{R_M}\right)$$

(b) Figure 2 on the next page shows the circuit for determining the loop gain $A\beta$,

$$A\beta = -\frac{V_r}{V_t}$$

First, we express I_{d2} in terms of V_t:

$$I_{d2} = -g_{m2}V_t \qquad (1)$$

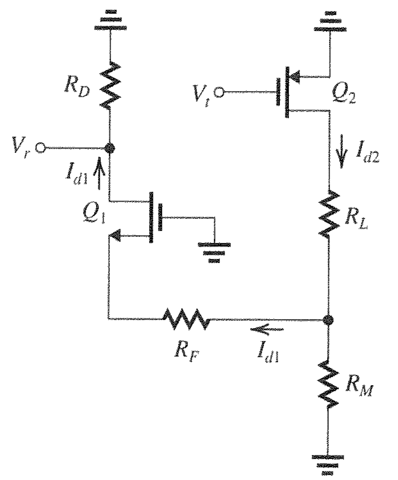


Figure 2

Then we determine I_{d1}:

$$I_{d1} = I_{d2}\frac{R_M}{R_M + R_F + \dfrac{1}{g_{m1}}} \qquad (2)$$

The returned voltage V_r can now be obtained as

$$V_r = I_{d1}R_D \qquad (3)$$

Combining Eqs. (1)–(3), we find V_r/V_t:

$$\frac{V_r}{V_t} = -\frac{g_{m2}R_D R_M}{R_M + R_F + \dfrac{1}{g_{m1}}}$$

Thus,

$$A\beta = \frac{g_{m2}R_D R_M}{R_M + R_F + \dfrac{1}{g_{m1}}}$$

Dividing the expression for $A\beta$ by

$\beta = -\dfrac{R_M}{R_M + R_F}$ yields

$$A = -\frac{g_{m2}R_D}{1 + 1/[g_{m1}(R_M + R_F)]} \qquad \text{Q.E.D.}$$

(c) $A = -\dfrac{4 \times 10}{1 + 1/[4 \times 1]}$

$= -32$ A/A

$A_f = -5 = -\dfrac{32}{1 - 32 \times \beta}$

$\beta = -0.169$ A/A

$-\dfrac{R_M}{R_M + R_F} = -0.169$

$R_M = 0.169 \times 1 = 0.169$ kΩ

$= 169\ \Omega$

11.48 (a) Refer to Fig. P11.48(b).

$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F}$$

$$A_f\big|_{\text{ideal}} = \frac{1}{\beta} = -R_F$$

For $A_f\big|_{\text{ideal}} = -1$ kΩ, we have

$R_F = 1$ kΩ

(b)

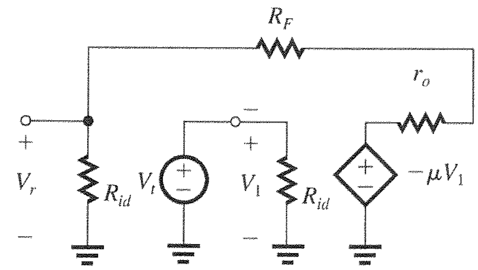


Figure 1

Figure 1 shows the circuit for determining the loop gain $A\beta$:

$$A\beta \equiv -\frac{V_r}{V_t}$$

Writing V_r in terms of $V_1 = V_t$ yields

$$V_r = -\mu V_t \frac{R_{id}}{R_{id} + R_F + r_o}$$

Thus,

$$A\beta \equiv -\frac{V_r}{V_t} = \mu\frac{R_{id}}{R_{id} + R_F + r_o} \qquad \text{Q.E.D.}$$

(c) $A\beta = 1000\,\dfrac{100}{100 + 1 + 1}$

$= 980.4$

$A = \dfrac{980.4}{\beta} = \dfrac{980.4}{-1/R_F}$

$= -980.4$ kΩ

$A_f = \dfrac{A}{1 + A\beta}$

$= -\dfrac{980.4}{1 + 980.4} = -0.999$ kΩ

11.49

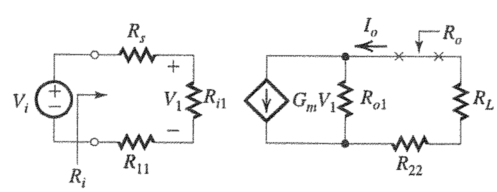


Figure 1

Figure 1 shows the A circuit where

$R_{i1} = 10$ kΩ

$R_{o1} = 100$ kΩ

$\beta = 200\ \Omega$

$R_{22} = 200\ \Omega$

$R_{11} = 10\ \text{k}\Omega$

$R_s = 10\ \text{k}\Omega$

$R_L = 10\ \text{k}\Omega$

To determine A,

$$A \equiv \frac{I_o}{V_i}$$

we write

$$V_1 = V_i \frac{R_{i1}}{R_{i1} + R_s + R_{11}}$$

$$= V_i \frac{10}{10 + 10 + 10} = \frac{1}{3} V_i \qquad (1)$$

$$I_o = G_m V_1 \frac{R_{o1}}{R_{o1} + R_L + R_{22}}$$

$$= 0.6 \times \frac{100}{100 + 10 + 0.2} V_i$$

$$= 0.544\ V_i \qquad (2)$$

Combining (1) and (2), we obtain

$$I_o = 0.544 \times \frac{1}{3} V_i = 0.1815\ V_i$$

$$A = 0.1815\ \text{A/V}$$

$$A_f = \frac{I_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= \frac{0.1815}{1 + 0.1815 \times 200} = \frac{0.1815}{1 + 36.2} = 4.88\ \text{mA/V}$$

$$R_{if} = R_i(1 + A\beta)$$

R_i is obtained from the A circuit as

$$R_i = R_s + R_{i1} + R_{11}$$

$$= 10 + 10 + 10 = 30\ \text{k}\Omega$$

Thus,

$$R_{if} = 30 \times 37.2 = 1.116\ \text{M}\Omega$$

$$R_{\text{in}} = R_{if} - R_s$$

$$= 1.116 - 0.010 = 1.006\ \text{M}\Omega$$

$$\simeq 1\ \text{M}\Omega$$

$$R_{of} = R_o(1 + A\beta)$$

where R_o is obtained from the A circuit as

$$R_o = R_L + R_{o1} + R_{22}$$

$$= 10 + 100 + 0.2 = 110.2\ \text{k}\Omega$$

$$R_{of} = 110.2 \times 37.2 = 4.1\ \text{M}\Omega$$

$$R_{\text{out}} = R_{of} - R_L = 4.1 - 0.01 = 4.09\ \text{M}\Omega$$

11.50 (a)

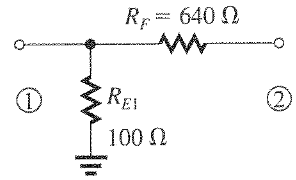


Figure 1

Figure 1 shows the β circuit from which we obtain

$$\beta = \frac{R_{E1}}{R_{E1} + R_F}$$

$$= \frac{100}{100 + 640} = 0.135\ \text{V/V}$$

(b) For $A\beta \gg 1$,

$$\frac{V_{e3}}{V_s} = A_f\big|_{\text{ideal}} = \frac{1}{\beta} = 7.4\ \text{V/V}$$

(c)

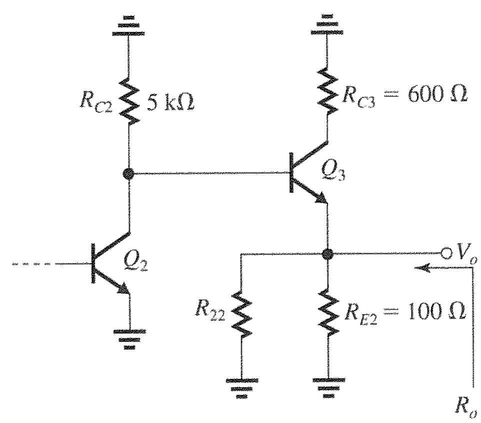


Figure 2

Figure 2 shows the portion of the A circuit relevant for calculating R_o:

$$R_o = R_{E2} \parallel R_{22} \parallel \left[r_{e3} + \frac{R_{C2}}{\beta_3 + 1} \right]$$

where $R_{E2} = 100\ \Omega$, R_{22} (from β circuit) $= 740\ \Omega$, $r_{e3} = 5\ \Omega$, $R_{C2} = 5\ \text{k}\Omega$, $\beta_3 = 100$;

thus,

$$R_o = 100 \parallel 740 \parallel \left[5 + \frac{5000}{101} \right]$$

$$= 33.7\ \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{33.7}{1 + 246.3} = 0.14\ \Omega$$

11.51 (a)

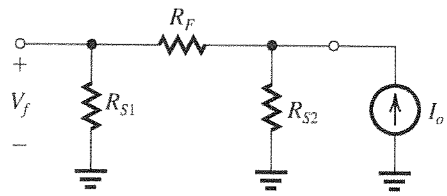


Figure 1

Figure 1 shows the β network. The value of β can be obtained from

$$\beta \equiv \frac{V_f}{I_o}$$

$$= \frac{R_{S1}R_{S2}}{R_{S2}+R_F+R_{S1}}$$

If $A\beta \gg 1$, then

$$A_f \simeq \frac{1}{\beta} = \frac{1}{R_{S1}} + \frac{1}{R_{S2}} + \frac{R_F}{R_{S1}R_{S2}} \quad (1)$$

For $A_f \simeq 100$ mA/V,

$$100 = \frac{1}{0.1} + \frac{1}{0.1} + \frac{R_F}{0.1 \times 0.1}$$

$\Rightarrow R_F = 0.8\ \text{k}\Omega$

(b) Figure 2 shows the A circuit and the determination of the loading effects of the β circuit, namely R_{11} and R_{22},

$$R_{11} = R_{S1} \parallel (R_F + R_{S2})$$

$$= 100 \parallel (800+100) = 80\ \Omega$$

$$R_{22} = R_{S2} \parallel (R_F + R_{S1}) = 80\ \Omega$$

The value of A is determined as follows:

$$\frac{V_{d1}}{V_i} = -\frac{R_{D1}}{(1/g_{m1}) + R_{11}}$$

$$= -\frac{10}{(1/4)+0.08} = -30.3\ \text{V/V}$$

$$\frac{V_{d2}}{V_{d1}} = -g_{m2}R_{D2} = -4 \times 10 = -40\ \text{V/V}$$

$$\frac{I_o}{V_{d2}} = \frac{1}{1/g_{m5} + R_{22}}$$

$$= \frac{1}{0.25+0.08} \simeq 3\ \text{mA/V}$$

Thus,

$$A = \frac{I_o}{V_i} = 3 \times -40 \times -30.3 = 3636\ \text{mA/V}$$

(c) $\beta = 0.01\ \text{k}\Omega$

$$1 + A\beta = 1 + 3636 \times 0.01$$

$$= 37.36$$

$$A_f = \frac{I_o}{V_i} = \frac{3636}{37.36} = 97.3\ \text{mA/V}$$

Difference from design value

$$= \frac{97.3-100}{100} \times 100$$

$$= -2.7\%$$

This figure belongs to Problem 11.51, part (b).

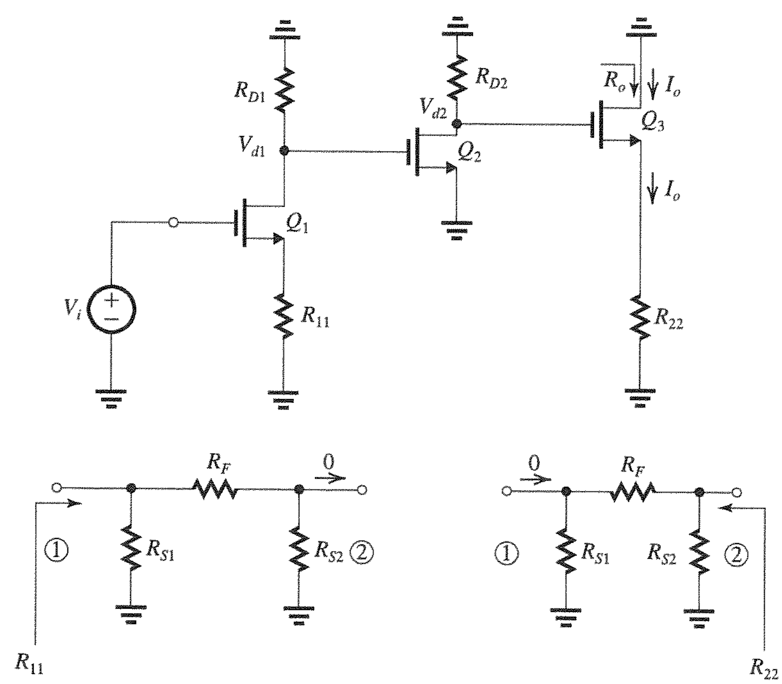


Figure 2

To make A_f exactly 100 mA/V, we can increase R_F (see Eq. (1) to appreciate why we need to increase R_F).

(d) From the A circuit in Fig. 2, we have

$R_o = r_{o3} + R_{22} + g_{m3}r_{o3}R_{22}$

$= 20 + 0.08 + 4 \times 20 \times 0.08$

$= 26.48\ \text{k}\Omega$

$R_{\text{out}} = R_{of} = R_o(1 + A\beta)$

$= 26.48 \times 37.36 = 989.3\ \text{k}\Omega$

(e)

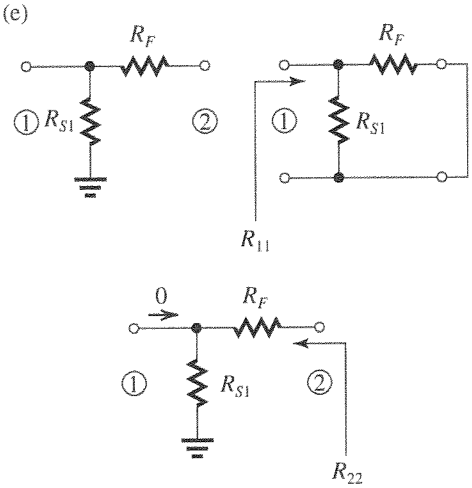


Figure 3

Figure 3 shows the β circuit for the case the output is V_o.

$$\beta = \frac{R_{S1}}{R_{S1} + R_F}$$

$$= \frac{100}{100 + 800} = \frac{1}{9}$$

Also shown is how R_{11} and R_{22} are determined in this case:

$R_{11} = R_{S1} \parallel R_F = 100 \parallel 800 = 88.9\ \Omega$

$R_{22} = R_F + R_{S1} = 800 + 100 = 900\ \Omega$

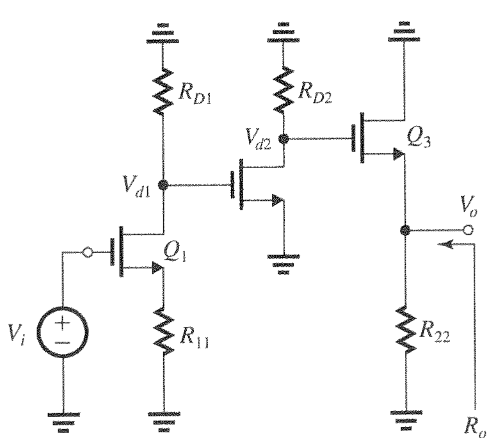


Figure 4

Figure 4 shows the A circuit for this case. To determine A, we write

$$\frac{V_{d1}}{V_i} = -\frac{R_{D1}}{1/g_{m1} + R_{11}}$$

$$\frac{V_{d1}}{V_i} = -\frac{10}{0.25 + 0.0889} = -29.5\ \text{V/V}$$

$$\frac{V_{d2}}{V_{d1}} = -g_{m2}R_{D2} = -4 \times 10 = -40\ \text{V/V}$$

$$\frac{V_o}{V_{d2}} = \frac{R_{22}}{R_{22} + \dfrac{1}{g_{m5}}} = \frac{88.9}{88.9 + 250} = 0.26\ \text{V/V}$$

Thus,

$$A \equiv \frac{V_o}{V_i} = 0.26 \times -40 \times -29.5 = 306.9\ \text{V/V}$$

$$1 + A\beta = 1 + 306.9 \times \frac{1}{9} = 35.1$$

which is a little lower than the value (37.36) found when we analyzed the amplifier as a transconductance amplifier.

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{306.9}{35.1} = 8.74\ \text{V/V}$$

(f) From the A circuit in Figure 4, we have

$$R_o = R_{22} \parallel \frac{1}{g_{m3}}$$

$R_o = 900\ \Omega \parallel 250\ \Omega$

$= 195.7\ \Omega$

$$R_{\text{out2}} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{195.7}{35.1} = 5.6\ \Omega$$

11.52 (a)

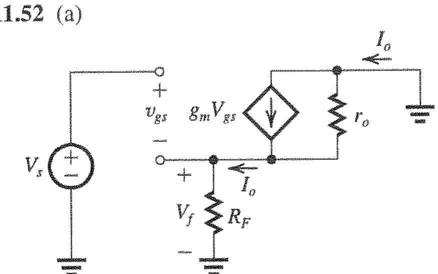


Figure 1

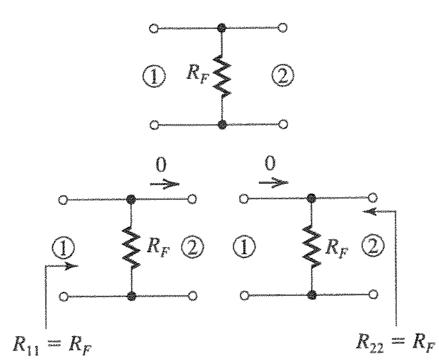


Figure 2

Figure 1 shows the small-signal equivalent circuit of the feedback amplifier. Observe that the resistance R_F senses the output current I_o and provides a voltage $I_o R_F$ that is subtracted from V_s. Thus the feedback network is composed of the resistance R_F, as shown in Fig. 2. Because the feedback is of the series-series type, the loading resistances R_{11} and R_{22} are determined as indicated in Fig. 2,

$R_{11} = R_F$

$R_{22} = R_F$

(b) The β circuit is shown in Fig. 2 and

$\beta = R_F$

Figure 3 shows the A circuit.

(c)

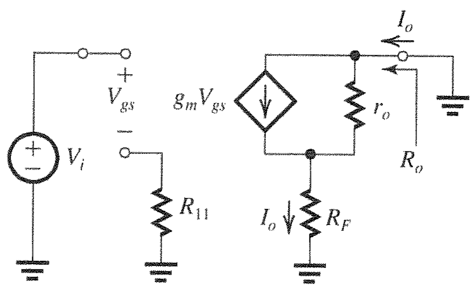


Figure 3

To determine $A = I_o/V_i$, we write

$V_{gs} = V_i$

$$I_o = g_m V_{gs} \frac{r_o}{r_o + R_F}$$

$$A \equiv \frac{I_o}{V_i} = g_m \frac{r_o}{r_o + R_F}$$

$$1 + A\beta = 1 + \frac{g_m r_o R_F}{r_o + R_F}$$

$$\frac{I_o}{V_s} = A_f = \frac{A}{1 + A\beta}$$

$$= \frac{g_m r_o/(r_o + R_F)}{1 + g_m r_o R_F/(r_o + R_F)}$$

$$= \frac{g_m}{1 + g_m R_F + \dfrac{R_F}{r_o}}$$

From the A circuit in Fig. 3, we have

$R_o = r_o + R_F$

$R_{of} = (1 + A\beta)R_o$

$$= \left(1 + \frac{g_m r_o R_F}{r_o + R_F}\right)(r_o + R_F)$$

$= r_o + R_F + g_m r_o R_F$

which is a familiar relationship!

11.53 (a) The β circuit is shown in Fig. 1:

$\beta = R_F$

For $A\beta \gg 1$, $A_f \equiv I_o/V_s$ approaches the ideal value

$$A_f\big|_{\text{ideal}} = \frac{1}{\beta} = \frac{1}{R_F}$$

To obtain $A_f \simeq 5$ mA/V, we use

$$R_F = \frac{1}{5} = 0.2 \text{ k}\Omega = 200\ \Omega$$

This figure belongs to Problem 11.53, part (a).

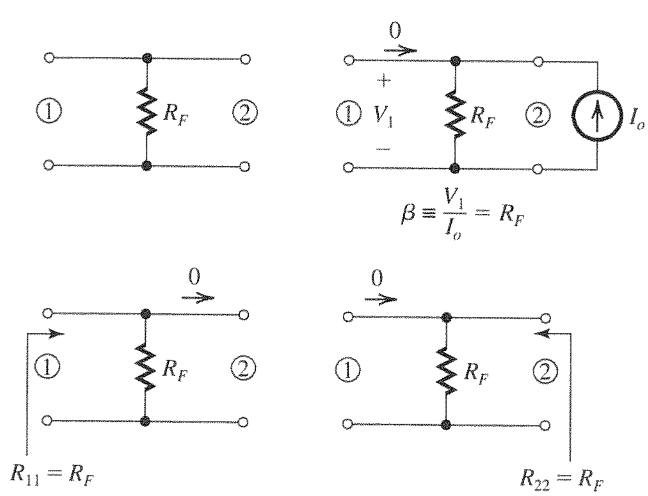


Figure 1

This figure belongs to Problem 11.53, part (b).

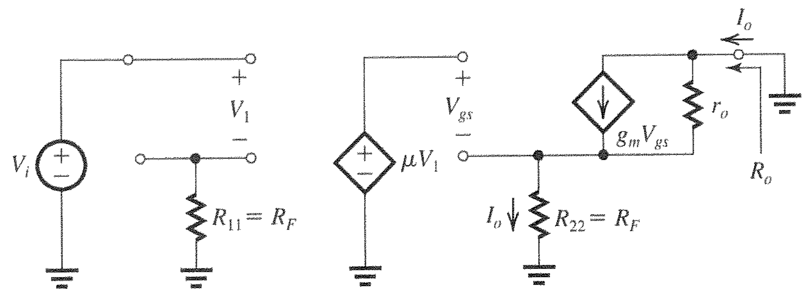


Figure 2

(b) Determining the loading effects of the β network is illustrated in Fig. 1:

$$R_{11} = R_{22} = R_F$$

Figure 2 shows the A circuit. An expression for $A \equiv I_o/V_i$ can be derived as follows:

$$V_1 = V_i \quad (1)$$

$$V_{gs} = \mu V_1 - I_o R_F \quad (2)$$

$$I_o = g_m V_{gs} \frac{r_o}{r_o + R_F} \quad (3)$$

Combining Eqs. (1)–(3) yields

$$A \equiv \frac{I_o}{V_i} = \frac{\mu g_m r_o}{r_o + R_F + g_m r_o R_F}$$

For $\mu = 1000$ V/V, $g_m = 2$ mA/V, $r_o = 20$ kΩ, and $R_F = 0.2$ kΩ, we have

$$A = \frac{1000 \times 2 \times 20}{20 + 0.2 + 2 \times 20 \times 0.2}$$

$$= 1418.4 \text{ mA/V}$$

(c) $A\beta = \dfrac{\mu g_m r_o R_F}{r_o + R_F + g_m r_o R_F}$

$$A\beta = 283.7$$

$$1 + A\beta = 284.7$$

(d) $A_f \equiv \dfrac{I_o}{V_s} = \dfrac{A}{1 + A\beta}$

$$= \frac{1418.4}{284.7} = 4.982 \text{ mA/V}$$

which is very close to the ideal value of 5 mA/V.

(e) From the A circuit in Fig. 2, we have

$$R_o = r_o + R_F + g_m r_o R_F$$

$$1 + A\beta = 1 + \frac{\mu g_m r_o R_F}{r_o + R_F + g_m r_o R_F}$$

$$R_{of} = (1 + A\beta) R_o$$

$$= r_o + R_F + g_m r_o R_F + \mu g_m r_o R_F$$

$$= r_o + R_F + (\mu + 1) g_m r_o R_F$$

$$\simeq \mu g_m r_o R_F$$

$$R_o = 20 + 0.2 + 2 \times 20 \times 0.2$$

$$= 28.2 \text{ k}\Omega$$

$$R_{of} = 20 + 0.2 + 1001 \times 2 \times 20 \times 0.2$$

$$= 20 + 0.2 + 8008 = 8028.2 \text{ k}\Omega$$

$$\simeq 8 \text{ M}\Omega$$

11.54 Figure 1 on the next page shows the equivalent circuit with $V_s = 0$ and a voltage V_x applied to the collector for the purpose of determining the output resistance R_o,

$$R_o \equiv \frac{V_x}{I_x}$$

Some of the analysis is displayed on the circuit diagram. Since the current entering the emitter node is equal to I_x, we can write for the emitter voltage

$$V_e = I_x [R_e \parallel (r_\pi + R_b)] \quad (1)$$

The base current can be obtained using the current-divider rule applied to R_e and $(r_\pi + R_b)$ as

$$I_b = -I_x \frac{R_e}{R_e + r_\pi + R_b} \quad (2)$$

The voltage from collector to ground is equal to V_x and can be expressed as the sum of the voltage drop across r_o and V_e,

$$V_x = (I_x - \beta I_b) r_o + V_e$$

Substituting for V_e from (1) and for I_b from (2), we obtain

$$R_o = \frac{V_x}{I_x} = r_o + [R_e \parallel (r_\pi + R_b)]$$

$$+ \frac{R_e \beta r_o}{R_e + r_\pi + R_b}$$

$$= r_o + [R_e \parallel (r_\pi + R_b)] \left[1 + r_o \frac{\beta}{r_\pi + R_b}\right]$$

This figure belongs to Problem 11.54.

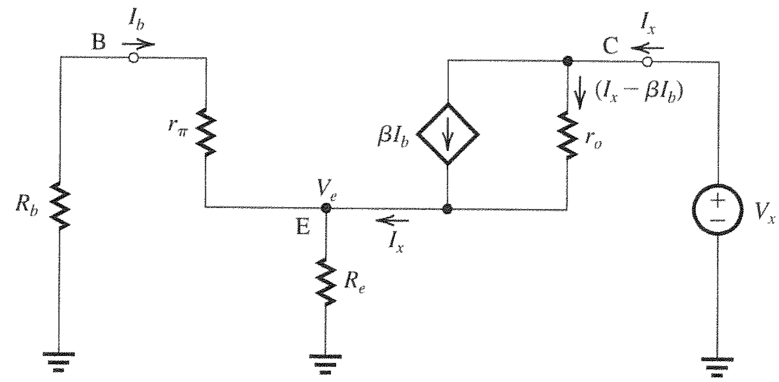


Figure 1

Since $\beta = g_m r_\pi$, we obtain

$$R_o = r_o + [R_e \parallel (r_\pi + R_b)]\left[1 + g_m r_o \frac{r_\pi}{r_\pi + R_b}\right] \qquad \text{Q.E.D.}$$

For $R_b = 0$,

$$R_o = r_o + (R_e \parallel r_\pi)(1 + g_m r_o)$$

The maximum value of R_o will be obtained when $R_e \gg r_\pi$. If R_e approaches infinity (zero signal current in the emitter), R_o approaches the theoretical maximum:

$$R_{o\max} = r_o + r_\pi(1 + g_m r_o)$$

$$= r_o + r_\pi + \beta r_o \qquad (3)$$

$$\simeq \beta r_o$$

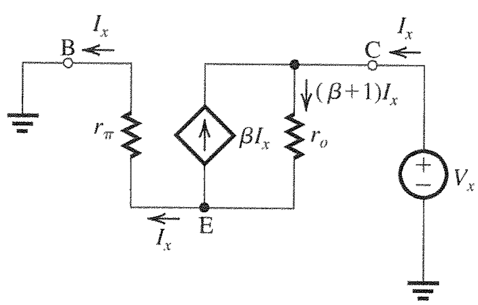


Figure 2

The situation that pertains in the circuit when $R_e = \infty$ is illustrated in Fig. 2. Observe that since the signal current in the emitter is zero, the base current will be equal to the collector current (I_x) and in the direction indicated. The controlled-source current will be βI_x, and this current adds to I_x to provide a current $(\beta + 1)I_x$ in the output resistance r_o. A loop equation takes the form

$$V_x = (\beta + 1)I_x r_o + I_x r_\pi$$

and thus

$$R_o \equiv \frac{V_x}{I_x} = r_\pi + (\beta + 1)r_o$$

which is identical to the result in Eq. (3).

11.55

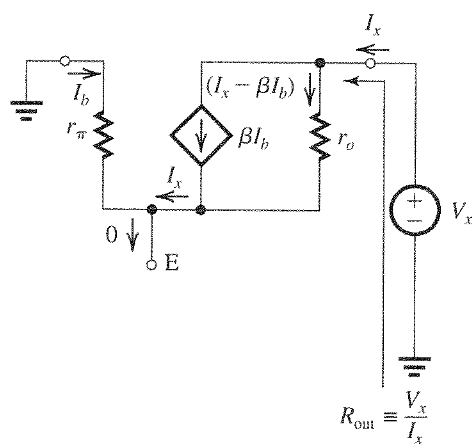


Figure 1

Figure 1 shows the situation that pertains in the transistor when μ is so large that $V_b \simeq 0$ and $I_e \simeq 0$. Observe that

$$I_b = -I_x$$

Writing a loop equation for the C-E-B, we obtain

$$V_x = (I_x - \beta I_b)r_o - I_b r_\pi$$

Substituting $I_b = -I_x$, we obtain

$$R_{out} = \frac{V_x}{I_x} = r_\pi + (\beta + 1)r_o$$

or if β is denoted h_{fe},

$$R_{out} = r_\pi + (h_{fe} + 1)r_o \qquad \text{Q.E.D.}$$

This figure belongs to Problem 11.56.

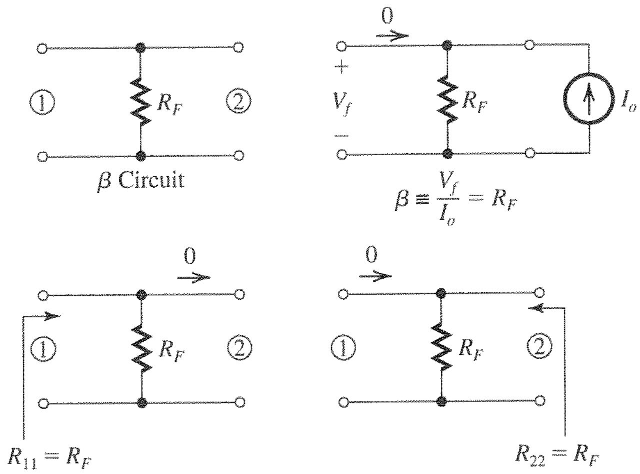


Figure 1

Thus, for large amounts of feedback, R_{out} is limited to this value, which is approximately $h_{fe}r_o$ independent of the amount of feedback. This phenomenon does not occur in MOSFET circuits where $h_{fe} = \infty$.

11.56 Figure 1 above shows the β circuit together with the determination of β, R_{11} and R_{22},

$$\beta = R_F$$

$$R_{11} = R_{22} = R_F$$

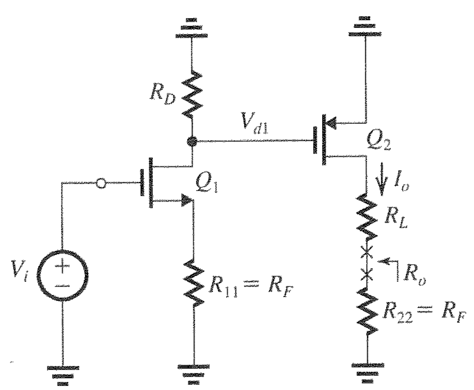


Figure 2

Figure 2 shows the A circuit. To determine $A \equiv I_o/V_i$, we write for Q_1

$$\frac{V_{d1}}{V_i} = -\frac{R_D}{(1/g_{m1}) + R_F} \qquad (1)$$

and for Q_2

$$I_o = -g_{m2}V_{d1} \qquad (2)$$

Combining (1) and (2) results in

$$A \equiv \frac{I_o}{V_i} = \frac{g_{m2}R_D}{(1/g_{m1}) + R_F}$$

$$A\beta = \frac{g_{m2}R_DR_F}{(1/g_{m1}) + R_F}$$

$$\frac{I_o}{V_s} = A_f = \frac{A}{1 + A\beta}$$

$$\Rightarrow A_f = \frac{g_{m2}R_D}{(1/g_{m1}) + R_F + g_{m2}R_DR_F}$$

From the A circuit, breaking the loop at XX gives

$$R_o = R_F + R_L + r_{o2}$$

$$R_{of} = (1 + A\beta)R_o$$

$$= \left[1 + \frac{g_{m2} + R_DR_F}{(1/g_{m1}) + R_F}\right][R_F + R_L + r_{o2}]$$

For

$g_{m1} = g_{m2} = 4$ mA/V, $R_D = 20$ kΩ,

$r_{o2} = 20$ kΩ, $R_F = 100$ Ω, and $R_L = 1$ kΩ,

we obtain

$$A = \frac{4 \times 20}{0.25 + 0.1} = 228.6 \text{ mA/V}$$

$$\beta = 0.1 \text{ k}\Omega$$

$$A\beta = 22.86$$

$$1 + A\beta = 23.86$$

$$A_f = \frac{228.6}{23.86} = 9.56 \text{ mA/V}$$

$$R_o = 0.1 + 1 + 20 = 21.1 \text{ k}\Omega$$

$$R_{of} = 23.86 \times 21.1 = 503.4 \text{ k}\Omega$$

11.57

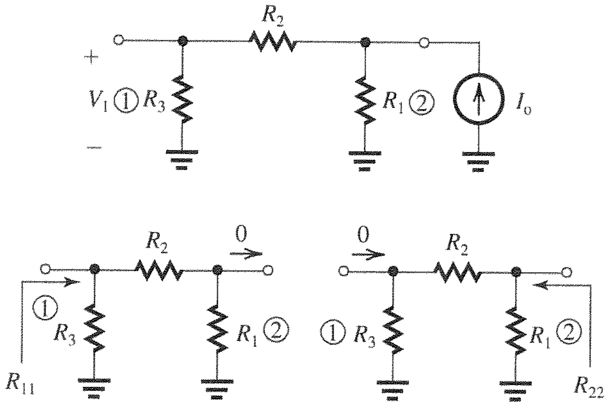


Figure 1

Figure 1 shows the feedback network fed with a current I_o to determine β:

$$\beta \equiv \frac{V_f}{I_o} = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

For $A\beta \gg 1$,

$$A_f = \frac{I_o}{V_s} \simeq \frac{1}{\beta}$$

Thus,

$$A_f = \frac{1}{R_3} + \frac{R_2}{R_1 R_3} + \frac{1}{R_1}$$

For $R_1 = R_3 = 0.1\ \text{k}\Omega$ and $A_f = 100$ mA/V,

$$100 = 10 + \frac{R_2}{0.01} + 10$$

$$\Rightarrow R_2 = 0.8\ \text{k}\Omega$$

To obtain the loading effects of the feedback network, refer to Fig. 1.

$$R_{11} = R_3 \parallel (R_2 + R_1)$$

$$= 100\ \Omega \parallel (800 + 100)\ \Omega = 90\ \Omega$$

$$R_{22} = R_1 \parallel (R_2 + R_3)$$

$$= 100 \parallel (800 + 100) = 90\ \Omega$$

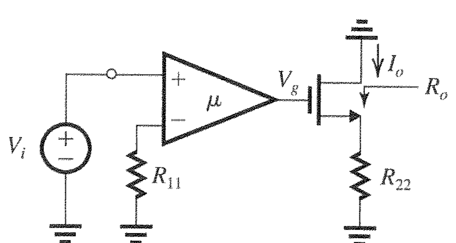


Figure 2

The A circuit is shown in Fig. 2. We can write

$$V_g = \mu V_i \qquad (1)$$

$$I_o = \frac{V_g}{(1/g_m) + R_{22}} \qquad (2)$$

Thus,

$$A \equiv \frac{I_o}{V_i} = \frac{\mu}{(1/g_m) + R_{22}}$$

Since $\beta = 0.01$, we have

$$A\beta = \frac{0.01\mu}{1/g_m + R_{22}}$$

$$= \frac{0.01\mu}{1 + 0.09} = 9.17 \times 10^{-3}\mu$$

For a 60-dB amount of feedback,

$$1 + A\beta = 1000$$

$$A\beta = 999$$

$$9.17 \times 10^{-3}\mu = 999$$

$$\Rightarrow \mu = 1.09 \times 10^5\ \text{V/V}$$

$$R_{\text{out}} = R_{of} = (1 + A\beta)R_o = 1000 R_o$$

where R_o can be obtained from the A circuit as

$$R_o = r_o + R_{22} + g_m r_o R_{22}$$

$$= 50 + 0.09 + 1 \times 50 \times 0.09$$

$$= 54.6\ \text{k}\Omega$$

Thus,

$$R_{\text{out}} = 1000 \times 54.6 = 54.6\ \text{M}\Omega$$

11.58 (a) Since V_s has a zero dc component, the gate of Q_1 is at zero dc voltage. The negative feedback will force the gate of Q_2 to be approximately at the same dc voltage as that at the gate of Q_1, thus

$$V_O = 0$$

$$V_{D1} = 1.2 - V_{SG3}$$

$$= 1.2 - |V_t| - |V_{OV3}|$$

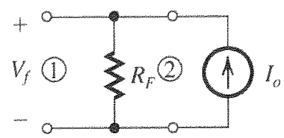
$$= 1.2 - 0.4 - 0.2 = +0.6\ \text{V}$$

$V_{D2} = V_O + V_{GS5}$

$= 0 + V_t + V_{OV5}$

$= 0.6$ V

(b)



$$\beta \equiv \frac{V_f}{I_o} = R_F$$

Figure 1

The feedback network is shown in Fig. 1, from which we find

$\beta = R_F = 10$ kΩ

For $A\beta \gg 1$,

$$A_f \simeq \frac{1}{\beta} = \frac{1}{R_F}$$

$$A_f = \frac{1}{10\text{ k}\Omega} = 0.1 \text{ mA/V}$$

(c) From the β circuit in Fig. 1 and noting that the feedback topology in series-series, the loading effects of the feedback network are

$R_{11} = R_{22} = R_F = 10$ kΩ

Figure 2 shows the A circuit. We can write

$$\frac{V_{g5}}{V_i} = -g_{m1,2}(r_{o2} \parallel r_{o4}) \qquad (1)$$

$$I_o = \frac{V_{g5}}{(1/g_{m5}) + R_{22}}$$

Thus,

$$A \equiv \frac{I_o}{V_i} = \frac{g_{m1,2}(r_{o2} \parallel r_{o4})}{(1/g_{m5}) + R_{22}}$$

$$g_{m1,2} = \frac{2I_{D1,2}}{V_{OV1,2}}$$

$$= \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I_{D2,4}}$$

$$= \frac{20}{0.1} = 200 \text{ k}\Omega$$

$$g_{m5} = \frac{2I_{D5}}{V_{OV5}} = \frac{2 \times 0.8}{0.2} = 8 \text{ mA/V}$$

$$A = \frac{1 \times (200 \parallel 200)}{0.125 + 10}$$

$= 9.88$ mA/V

$$\frac{I_o}{V_s} = A_f = \frac{A}{1 + A\beta}$$

$$= \frac{9.88}{1 + 9.88 \times 10} = 0.099 \text{ mA/V}$$

(d) From the A circuit, we have

$R_o = r_{o5} + R_{22} + g_{m5}r_{o5}R_{22}$

where

$$r_{o5} = \frac{|V_A|}{I_{D5}} = \frac{20}{0.8} = 25 \text{ k}\Omega$$

$R_o = 25 + 10 + 8 \times 25 \times 10 = 2035$ kΩ

$R_{\text{out}} = R_{of} = R_o(1 + A\beta)$

$= 2.035 \times (1 + 9.88 \times 10)$

$= 203$ MΩ

(e) $V_o = I_o R_F$

$= A_f V_s R_F$

$$\frac{V_o}{V_s} = A_f R_F = 0.099 \times 10 = 0.99 \text{ V/V}$$

This figure belongs to Problem 11.58, part (b).

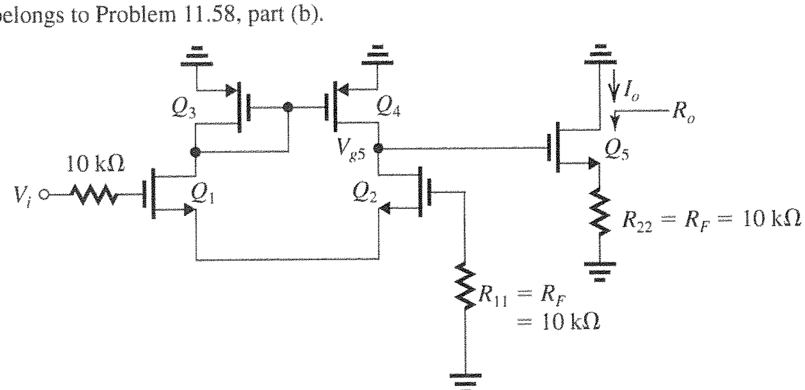


Figure 2

$$R_{\text{out}} = \frac{\text{Output resistance at source of } Q_5}{1+A\beta}$$

$$\simeq \frac{1/g_{m5}}{1+A\beta}$$

$$= \frac{125\ \Omega}{1+9.88\times 10} = 1.25\ \Omega$$

11.59

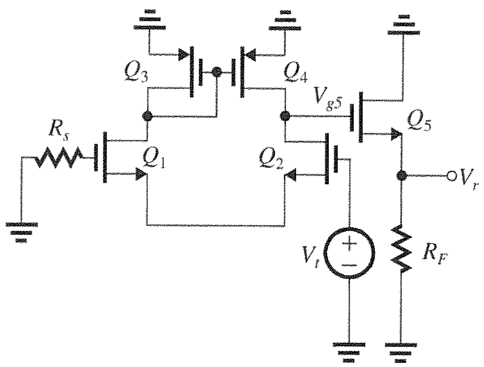


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain $A\beta$:

$$A\beta \equiv -\frac{V_r}{V_t}$$

First we write for the gain of differential amplifier

$$\frac{V_{g5}}{V_t} = -g_{m1,2}(r_{o2} \parallel r_{o4}) \qquad (1)$$

Next we write for the source follower,

$$\frac{V_r}{V_{g5}} = \frac{R_F \parallel r_{o5}}{(R_F \parallel r_{o5}) + (1/g_{m5})} \qquad (2)$$

Combining (1) and (2) yields

$$A\beta = -\frac{V_r}{V_t} = g_{m1,2}(r_{o2} \parallel r_{o4})\frac{R_F \parallel r_{o5}}{(R_F \parallel r_{o5}) + (1/g_{m5})}$$

Q.E.D.

11.60 $\mu = 10^3$ V/V, $R_{id} = \infty$, $r_o = 100\ \Omega$, $R_F = 10$ kΩ, and $R_s = R_L = 1$ kΩ. From Example 11.9 Eqs. (11.37) and (11.41),

$$\beta = -\frac{1}{R_F} = -\frac{1}{10\text{ k}\Omega} = -0.1\text{ mA/V}$$

$$A = -\mu R_i \frac{(R_F \parallel R_L)}{r_o + R_F \parallel R_L)}$$

where

$$R_i = R_{id} \parallel R_F \parallel R_s$$

$$R_i = \infty \parallel 10 \parallel 1 = 0.909\text{ k}\Omega$$

$$A = -10^3 \times 0.909 \times \frac{(10 \parallel 1)}{0.1 + (10 \parallel 1)}$$

$$= -818.9\text{ k}\Omega$$

$$A_f = \frac{V_o}{I_s} = \frac{A}{1+A\beta}$$

$$= -\frac{818.9}{1 - 818.9 \times -0.1}$$

$$= -\frac{818.9}{1+81.89} = -9.88\text{ k}\Omega$$

$$R_{if} = R_i/(1+A\beta)$$

$$= \frac{0.909\text{ k}\Omega}{1+81.89} = 11\ \Omega$$

$$R_{if} = R_{\text{in}} \parallel R_s$$

$$= R_{\text{in}} \parallel 1\text{ k}\Omega$$

$$R_{\text{in}} = \frac{1}{\dfrac{1}{R_{if}} - \dfrac{1}{1000}} = \frac{1}{\dfrac{1}{11} - \dfrac{1}{1000}}$$

$$= 11.1\ \Omega$$

From Eq. (11.42), we have

$$R_o = r_o \parallel R_F \parallel R_L$$

$$= 0.1 \parallel 10 \parallel 1 = 90.1\ \Omega$$

$$R_{of} = \frac{R_o}{1+A\beta}$$

$$= \frac{90.1}{1+81.89} = 1.1\ \Omega$$

$$R_{of} = R_{\text{out}} \parallel R_L$$

$$= R_{\text{out}}/1\text{ k}\Omega$$

$$\Rightarrow R_{\text{out}} \simeq 1.1\ \Omega$$

Comparison to the values in Example 11.9:

	$\mu = 10^4$ V/V	$\mu = 10^3$
A_f	-9.99 kΩ	-9.88 kΩ
R_{in}	1.11 Ω	11.1 Ω
R_{out}	0.11 Ω	1.1 Ω

11.61 Comparing the circuit of Fig. E11.19 and that of Fig. 11.24(a), we note the following:

$$\mu = g_m r_{o,Q}$$

$$r_o = r_{o,Q}$$

(This is based on representing the transistor output circuit ($g_m V_{gs}$, $r_{o,Q}$) by its Thévenin equivalent.)

$$R_{id} = \infty$$

$$R_L = \infty$$

Using Eq. (11.39), we obtain

$R_i = R_{id} \parallel R_F \parallel R_s = R_F \parallel R_s$

Using Eq. (11.44), we obtain

$$A_f = -\frac{g_m r_o (R_F \parallel R_s) \dfrac{R_F}{r_o + R_F}}{1 + g_m r_o (R_F \parallel R_s) \dfrac{1}{r_o + R_F}}$$

$$= -\frac{(R_s \parallel R_F) g_m (r_o \parallel R_F)}{1 + (R_s \parallel R_F) g_m (r_o \parallel R_F)/R_F}$$

which is the expression given in the answer to Exercise 11.19(b).

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{R_s \parallel R_F}{1 + A\beta}$$

Substituting for $A\beta$ from Eq. (11.43), we obtain

$$R_{if} = \frac{R_s \parallel R_F}{1 + g_m (R_s \parallel R_F)(r_o \parallel R_F)/R_F}$$

$$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_F} + \frac{g_m (r_o \parallel R_F)}{R_F}$$

But,

$$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_{in}}$$

Thus,

$$\frac{1}{R_{in}} = \frac{1}{R_F}[1 + g_m (r_o \parallel R_F)]$$

$$\Rightarrow R_{in} = \frac{R_F}{[1 + g_m (r_o \parallel R_F)]}$$

which is the answer given to Exercise 11.19(c). Using Eq. (11.42), we obtain

$R_o = r_o \parallel R_F \parallel R_L = r_o \parallel R_F$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

Substituting for $A\beta$ from Eq. (11.43), we obtain

$$R_{of} = \frac{(r_o \parallel R_F)}{1 + g_m (R_s \parallel R_F)(r_o \parallel R_F)/R_F}$$

$$\frac{1}{R_{of}} = \frac{1}{r_o} + \frac{1}{R_F} + \frac{g_m (R_s \parallel R_F)}{R_F}$$

$$R_{of} = r_o \parallel \frac{R_F}{1 + g_m (R_s \parallel R_F)}$$

$$R_{out} = R_{of} = r_o \parallel \frac{R_F}{1 + g_m (R_s \parallel R_F)}$$

which is identical to the result given in the answer to Exercise 11.19(d).

11.62

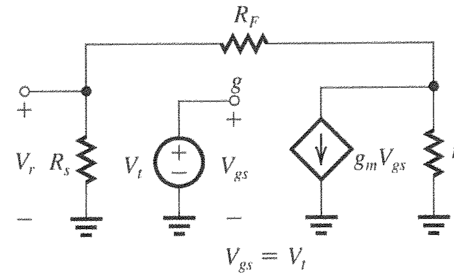


Figure 1

Figure 1 shows the circuit prepared for the determination of the loop gain $A\beta$:

$$A\beta = -\frac{V_r}{V_t}$$

An expression for V_r can be written by inspection as

$$V_r = -g_m V_t [r_o \parallel (R_s + R_F)] \frac{R_s}{R_s + R_F}$$

Thus,

$$A\beta = g_m [r_o \parallel (R_s + R_F)] \frac{R_s}{R_s + R_F} \qquad (1)$$

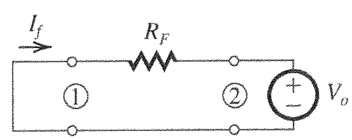


Figure 2

The feedback network (β circuit) is shown in Fig. 2 fed with V_o at port 2 and with port 1 short-circuited to determine β:

$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F} \qquad (2)$$

Equations (1) and (2) can how be used to determine A:

$A = -g_m [r_o \parallel (R_s + R_F)](R_s \parallel R_F)$

Using the numerical values given in Exercise 11.19(c), we obtain

$A = -5[20 \parallel (1 + 10)](1 \parallel 10)$

$= -32.3\ \text{k}\Omega$

$$\beta = -\frac{1}{R_F} = -0.1\ \text{mA/V}$$

$A\beta = 3.23$

$$A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{32.3}{1 + 3.23} = -7.63\ \text{k}\Omega$$

Compare these results to those found in Exercise 11.19: $A = -32.3$ kΩ (-30.3 kΩ), $\beta = -0.1$ mA/V (-0.1 mA/V), $A\beta = -3.23$ (-3.03), and $A_f = -7.63$ kΩ (-7.52 kΩ). The slight differences are due to the approximation used in the systematic analysis method.

11.63 (a)

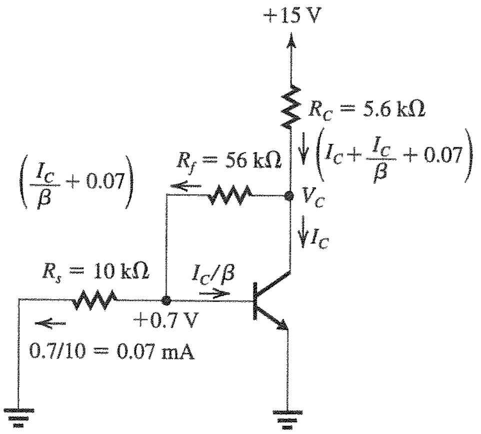


Figure 1

Figure 1 illustrates the dc analysis. We can express the dc collector voltage V_C in two alternative ways:

$$V_C = +15 - R_C\left(I_C + \frac{I_C}{\beta} + 0.07\right)$$

and

$$V_C = 0.7 + R_f\left(\frac{I_C}{\beta} + 0.07\right)$$

Equating these two expressions yields

$$15 - 5.6(I_C + 0.01I_C + 0.07)$$

$$= 0.7 + 56(0.01I_C + 0.07)$$

$$\Rightarrow I_C = 1.6 \text{ mA}$$

$$V_C \simeq 5.5 \text{ V}$$

(b) Figure 2 on the next page show the small-signal equivalent circuit of the amplifier where

$$g_m = \frac{1.6 \text{ mA}}{0.025 \text{ V}} = 64 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{64} = 1.56 \text{ k}\Omega$$

(c) Figure 3 on the next page shows the A circuit. It includes the loading effects of the feedback network:

$$R_{11} = R_{22} = R_f$$

Also, observe that

$$\beta = -\frac{1}{R_f}$$

From the A circuit in Fig. 3, we have

$$R_i = R_s \parallel R_f \parallel r_\pi \qquad (1)$$

$$V_\pi = I_i R_i \qquad (2)$$

$$V_o = -g_m V_\pi (R_C \parallel R_f) \qquad (3)$$

Combining (1), (2) and (3) gives

$$A \equiv \frac{V_o}{I_i} = -g_m(R_s \parallel R_f \parallel r_\pi)(R_C \parallel R_f)$$

$$R_i = 10 \text{ k}\Omega \parallel 56 \text{ k}\Omega \parallel 1.56 \text{ k}\Omega$$

$$= 1.32 \text{ k}\Omega$$

$$A = -64 \times 1.32 \times (5.6 \parallel 56)$$

$$= -429 \text{ k}\Omega$$

From the A circuit, we have

$$R_o = R_C \parallel R_f$$

$$= 5.6 \text{ k}\Omega \parallel 56 \text{ k}\Omega$$

$$= 5.1 \text{ k}\Omega$$

(d) $\beta = -\dfrac{1}{R_f} = -\dfrac{1}{56 \text{ k}\Omega}$

$$A\beta = \frac{429}{56} = 7.67$$

$$1 + A\beta = 8.67$$

(e) $A_f = \dfrac{V_o}{I_s} = \dfrac{A}{1 + A\beta}$

$$= -\frac{429}{8.67} = -49.5 \text{ k}\Omega$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{1.32 \text{ k}\Omega}{8.67} = 152 \ \Omega$$

$$R_{if} = R_s \parallel R_{\text{in}}$$

$$152 \ \Omega = 10 \text{ k}\Omega \parallel R_{\text{in}}$$

$$R_{\text{in}} = 155 \ \Omega$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{5.1 \text{ k}\Omega}{8.67} = 588 \ \Omega$$

This figure belongs to Problem 11.63, part (b).

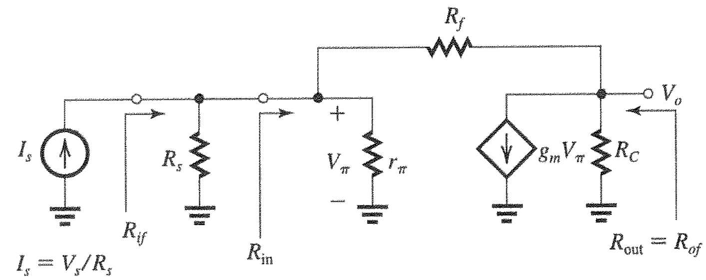


Figure 2

The below two figures belong to Problem 11.63, part (c).

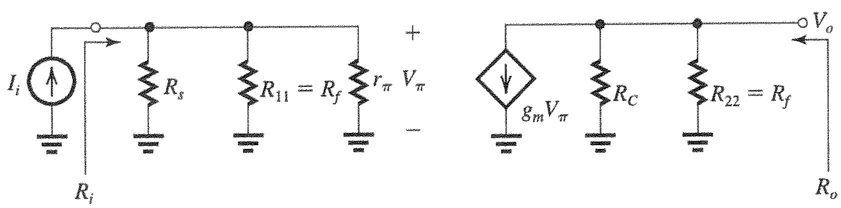
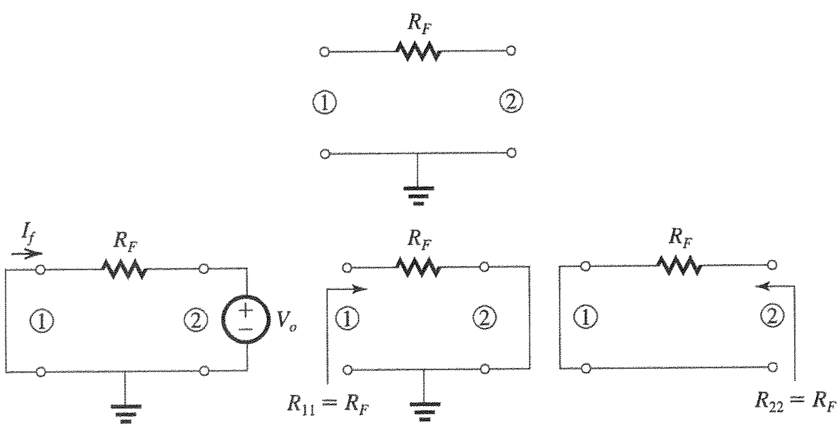


Figure 3



$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F}$$

Figure 4

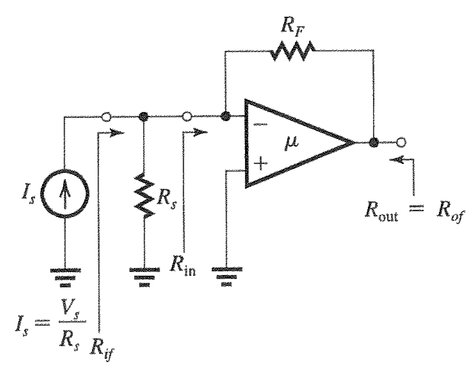
(f) $\dfrac{V_o}{V_s} = \dfrac{V_o}{I_s R_s} = \dfrac{A_f}{R_s} = -\dfrac{49.5}{10} = -4.95$ V/V

Refer to Fig. P11.63 and assume the gain of the BJT to be infinite so that the signal voltage at its base is zero (virtual ground). In this case, we have

$$\frac{V_o}{V_s} = -\frac{R_f}{R_s} = -\frac{56}{10} = -5.6 \text{ V/V}$$

Thus, the actual gain magnitude ($\simeq 5$ V/V) is only about 12% below the ideal value; not bad for a single transistor inverting op amp!

11.64 (a)



Refer to the feedback network shown in Fig. 11.24(b) and to the determination of β illustrated in Fig. 11.24(c). Thus,

$$\beta = -\frac{1}{R_F}$$

If $A\beta \gg 1$, then we have

$$A_f = \frac{V_o}{I_s} \simeq \frac{1}{\beta} = -R_F$$

and the voltage gain realized will be

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} \simeq -\frac{R_F}{R_s}$$

If $R_s = 2$ kΩ, to obtain $V_o/V_s \simeq -10$ V/V, we required

$$R_F = 10 \times R_s = 20 \text{ k}\Omega$$

(b) Refer to the solution to Example 11.9.

$$\beta = -\frac{1}{R_F} = -\frac{1}{20 \text{ k}\Omega} = -0.05 \text{ mA/V}$$

Using Eq. (11.39), we obtain

$$R_i = R_{id} \parallel R_F \parallel R_s$$

$$= 100 \parallel 20 \parallel 2 = 1.786 \text{ k}\Omega$$

Using Eq. (11.41) with $R_L = \infty$, we get

$$A \equiv \frac{V_o}{I_i} = -10^3 \times 1.786 \times \frac{20}{20+2}$$

$$= -1623.6 \text{ k}\Omega$$

$$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1+A\beta}$$

where

$$1 + A\beta = 1 + 1623.6 \times 0.05$$

$$= 82.18$$

$$A_f = \frac{V_o}{I_s} = -\frac{1623.6}{82.18}$$

$$= -19.76 \text{ k}\Omega$$

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s} = -\frac{19.76}{2}$$

$$= -9.88 \text{ V/V}$$

$$R_{if} = \frac{R_i}{1+A\beta}$$

$$= \frac{1.786}{82.18} = 21.7 \ \Omega$$

$$R_{if} = R_s \parallel R_{\text{in}}$$

$$21.7 \ \Omega = 2000 \ \Omega \parallel R_{\text{in}}$$

$$R_{\text{in}} \simeq 21.7 \ \Omega$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1+A\beta}$$

where from Eq. (11.42) with $R_L = \infty$ we get

$$R_o = r_o \parallel R_F = 2 \parallel 20 = 1.818 \text{ k}\Omega$$

$$R_{\text{out}} = R_{of} = \frac{1.818}{82.18} = 22.1 \ \Omega$$

(c) $f_{Hf} = f_H(1 + A\beta)$

$$= 1 \times 82.18$$

$$= 82.18 \text{ kHz}$$

11.65

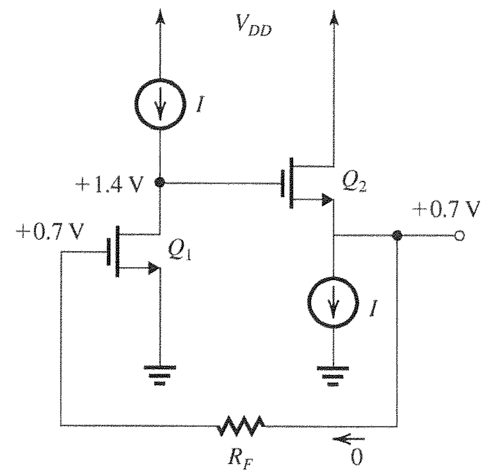


Figure 1

(a) See Figure 1.

$$V_{G1} = V_{GS1} = V_t + V_{OV}$$

$$= 0.5 + 0.2 = +0.7 \text{ V}$$

(because the dc voltage across R_F is zero)

$$V_O = V_{G1}$$

$$V_O = +0.7 \text{ V}$$

$$V_{D1} = V_O + V_{GS2}$$

$$= 0.7 + 0.5 + 0.2$$

$$= +1.4 \text{ V}$$

(b) $g_{m1,2} = \dfrac{2I}{V_{OV}} = \dfrac{2 \times 0.4}{0.2} = 4$ mA/V

$$r_{o1,2} = \frac{V_A}{I} = \frac{16 \text{ V}}{0.4 \text{ mA}} = 40 \text{ k}\Omega$$

(c) Figure 2 on the next page shows the β circuit and the determination of its loading effects,

$$R_{11} = R_{22} = R_F$$

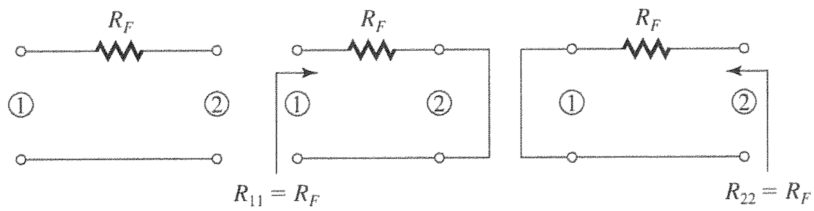
Figure 2 shows also the A circuit. We can write

$$V_{g1} = I_i R_i \qquad (1)$$

where

$$R_i = R_{11} = R_F \qquad (2)$$

This figure belongs to Problem 11.65, part (c).



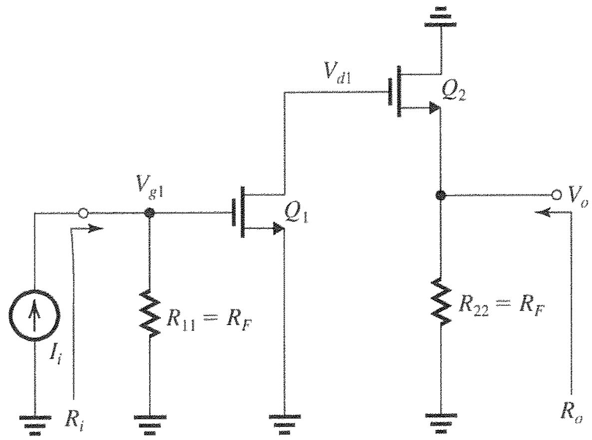


Figure 2

$$V_{d1} = -g_{m1} r_{o1} \qquad (3)$$

$$\frac{V_o}{V_{d1}} = \frac{R_{22} \parallel r_{o2}}{(R_{22} \parallel r_{o2}) + \dfrac{1}{g_{m2}}} \qquad (4)$$

Combining Eqs. (1)–(4) results in

$$A \equiv \frac{V_o}{I_i} = -g_{m1} r_{o1} R_F \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

(d)

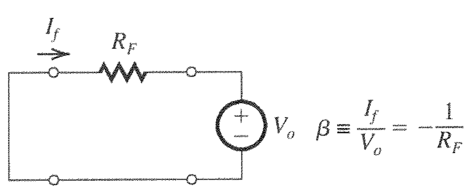


Figure 3

From Fig. 3 we see that

$$\beta = -\frac{1}{R_F}$$

$$A\beta = g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

$$1 + A\beta = 1 + g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

(e) $A_f \equiv \dfrac{V_o}{I_s} = \dfrac{A}{1 + A\beta}$

$$= -\frac{g_{m1} r_{o1} R_F (R_F \parallel r_{o2})}{(R_F \parallel r_{o2}) + 1/g_{m2} + (g_{m1} r_{o1})(R_F \parallel r_{o2})}$$

(f) $R_i = R_F$

$$R_{\text{in}} = R_{if} = R_i/(1 + A\beta)$$

$$= R_F \Big/ \left[1 + g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}\right]$$

$$R_{\text{out}} = R_{of} = R_o/(1 + A\beta)$$

where from the A circuit we have

$$R_o = R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

$$R_{\text{out}} = \left(R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}\right) \Big/ \left[1 + g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}\right]$$

(g) $A = -4 \times 40 \times 10 \dfrac{10 \parallel 40}{(10 \parallel 40) + 0.25}$

$$= -1551.5 \text{ k}\Omega$$

$$\beta = -\frac{1}{R_F} = -\frac{1}{10 \text{ k}\Omega} = -0.1 \text{ mA/V}$$

$$A\beta = 155.15$$

$$1 + A\beta = 156.15$$

$$A_f = -\frac{1551.5}{156.15} = -9.94 \text{ k}\Omega$$

$$R_i = R_F = 10 \text{ k}\Omega$$

$$R_{\text{in}} = R_{if} = \frac{R_F}{1 + A\beta} = \frac{10,000 \ \Omega}{156.15} = 64 \ \Omega$$

$$R_o = R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

$$R_o = 10 \parallel 40 \parallel 0.25 = 242 \ \Omega$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{242}{156.15} = 1.55 \ \Omega$$

11.66

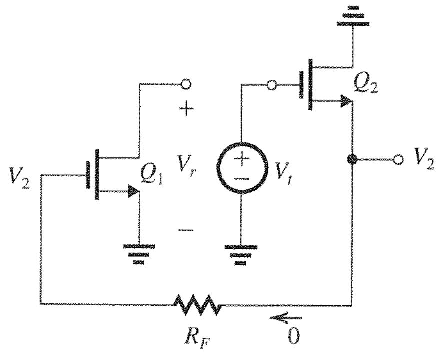


Figure 1

Figure 1 shows the circuit of Fig. P11.65 prepared for determining the loop gain $A\beta$:

$$A\beta \equiv -\frac{V_r}{V_t}$$

The gain of the source-follower Q_2 can be found as

$$\frac{V_2}{V_t} = \frac{r_{o2}}{r_{o2} + 1/g_{m2}} \tag{1}$$

The gain of the CS transistor Q_1 can be found as

$$V_r = -g_{m1} r_{o1} V_2 \tag{2}$$

Combining Eqs. (1) and (2), we obtain

$$A\beta \equiv -\frac{V_r}{V_t} = (g_{m1} r_{o1}) \frac{r_{o2}}{r_{o2} + 1/g_{m2}} \tag{3}$$

This expression differs from that obtained in Problem 11.65 utilizing the general feedback analysis method. To determine the numerical difference, we evaluate $A\beta$ in Eq. (3) using

$$g_{m1} = g_{m2} = 4 \text{ mA/V}, \ r_{o1} = r_{o2} = 40 \text{ k}\Omega$$

$$A\beta = 4 \times 40 \times \frac{40}{40 + 0.25}$$

$$= 159$$

which is larger than the value found in Problem 11.65 (155.15) by about 2.5%. This small difference is a result of the approximations involved in the general method. The more accurate result for $A\beta$ is the one obtained here. However, the loop-gain method does not make it possible to determine the input and output resistances of the feedback amplifier.

11.67 Figure 1 shows the small-signal-equivalent circuit of the feedback amplifier of Fig. E11.19. Analysis to determine V_o/I_s proceeds as follows:

Writing a node equation at the output node provides

$$g_m V_{gs} + \frac{V_o}{r_o} + \frac{V_o - V_{gs}}{R_F} = 0$$

$$\Rightarrow V_{gs} = -V_o \frac{\frac{1}{r_o} + \frac{1}{R_F}}{g_m - \frac{1}{R_F}}$$

$$\Rightarrow V_{gs} = -V_o \frac{1}{\left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)} \tag{1}$$

Writing a node equation at node G provides

$$I_s - \frac{V_{gs}}{R_s} + \frac{V_o - V_{gs}}{R_F} = 0$$

$$I_s - \frac{V_{gs}}{(R_s \parallel R_F)} + \frac{V_o}{R_F} = 0 \tag{2}$$

This figure belongs to Problem 11.67.

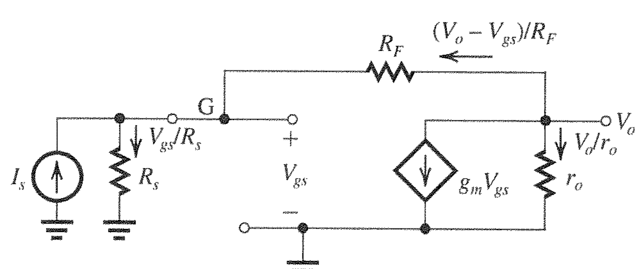


Figure 1

This figure belongs to Problem 11.68, part (a).

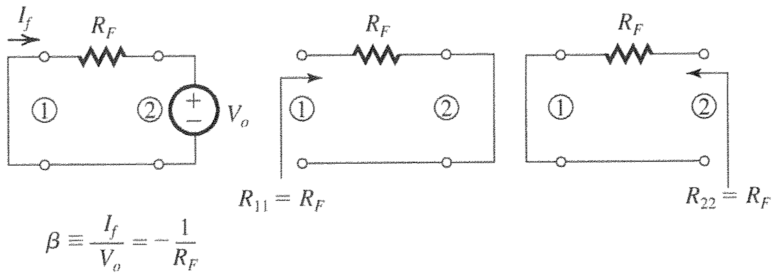


Figure 1

Substituting for V_{gs} from (1) into (2), we obtain

$$I_s + V_o \frac{1}{\left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)(R_s \parallel R_F)} + \frac{V_o}{R_F}$$

$$= 0 \Rightarrow \frac{V_o}{I_s}$$

$$= \frac{\left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)(R_s \parallel R_F)}{1 + \left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)(R_s \parallel R_F)/R_F}$$

For the feedback analysis to be reasonably accurate, we use

$$g_m \gg \frac{1}{R_F}$$

11.68 Figure 1 shows the feedback network with a voltage V_o applied to port 2 to determine β:

$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F}$$

For $A\beta \gg 1$, we have

$$\frac{V_o}{I_s} \equiv A_f \simeq \frac{1}{\beta} = -R_F$$

Thus, for $\frac{V_o}{I_s} \simeq -10\ \text{k}\Omega$, we select

$$R_F = 10\ \text{k}\Omega$$

The loading of the feedback network on the A circuit can be determined as shown in Fig. 1:

$$R_{11} = R_{22} = R_F$$

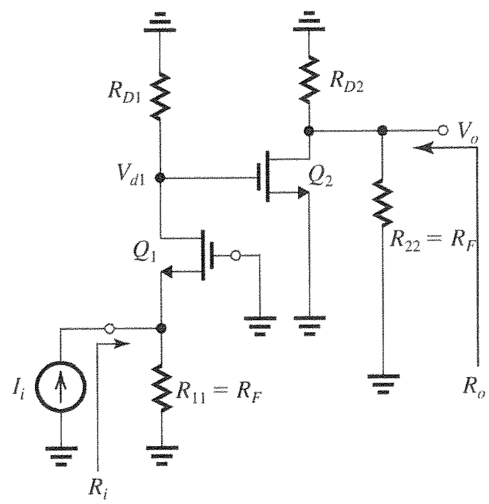


Figure 2

Figure 2 shows the A circuit. For the CG amplifier Q_1, we can write

$$R_i = R_F \parallel \frac{1}{g_{m1}} \tag{1}$$

$$V_{sg} = I_i R_i \tag{2}$$

$$V_{d1} = g_{m1} V_{sg} R_{D1} \tag{3}$$

Combining (1)–(3) yields

$$V_{d1} = (g_{m1} R_{D1}) \left(R_F \parallel \frac{1}{g_{m1}}\right) I_i \tag{4}$$

For the CS stage Q_2, we can write

$$\frac{V_o}{V_{d1}} = -g_{m2}(R_{D2} \parallel R_F) \tag{5}$$

Combining (4) and (5), we obtain the open-loop gain A:

$$A \equiv \frac{V_o}{I_s} = -(g_{m1} R_{D1}) \left(R_F \parallel \frac{1}{g_{m1}}\right) g_{m2}(R_{D2} \parallel R_F)$$

This figure belongs to Problem 11.69, part (a).

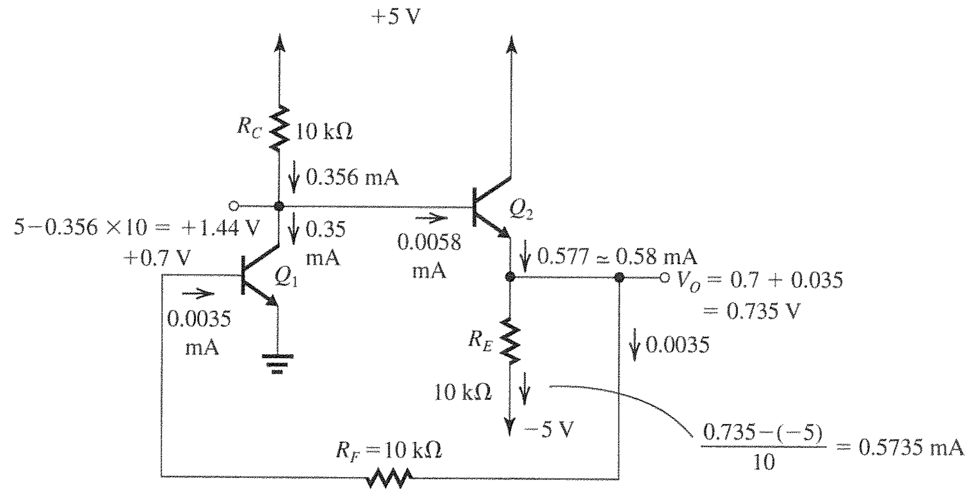


Figure 1

Substituting $g_{m1} = g_{m2} = 4$ mA/V, $R_{D1} = R_{D2} = 10$ kΩ, and $R_F = 10$ kΩ gives

$$A = -(4 \times 10) \times (10 \parallel 0.25) \times 4 \times (10 \parallel 10)$$

$$A = -195\ \text{k}\Omega$$

$$A\beta = \frac{195}{10} = 19.5$$

$$1 + A\beta = 20.5$$

$$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{195}{20.5} = -9.52\ \text{k}\Omega$$

$$R_{\text{in}} = R_{if} = \frac{R_i}{1 + A\beta}$$

From Eq. (1), we obtain

$$R_i = 10 \parallel 0.25 = 244\ \Omega$$

$$R_{\text{in}} = \frac{244}{20.5} = 11.9\ \Omega$$

From the A circuit,

$$R_o = R_{D2} \parallel R_F$$

$$= 10 \parallel 10 = 5\ \text{k}\Omega$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{5000}{20.5} = 244\ \Omega$$

11.69 (a) Figure 1 (see figure above) shows the dc analysis. We assumed $I_{C1} = 0.35$ mA and found that $I_{C2} = 0.58$ mA, thus verifying the given values. The dc voltage at the output is

$$V_O = +0.735\ \text{V}$$

(b)

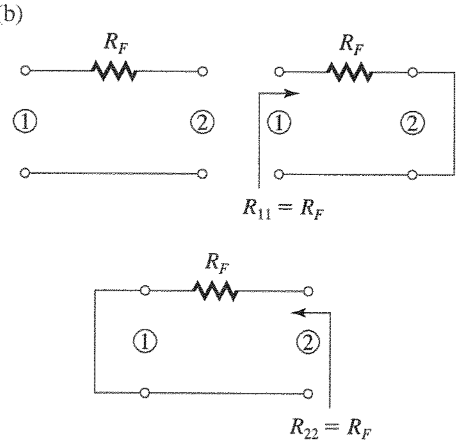


Figure 2

Figure 2 shows the β circuit and the determination of its loading effects on the A circuit:

$$R_{11} = R_{22} = R_F = 10\ \text{k}\Omega$$

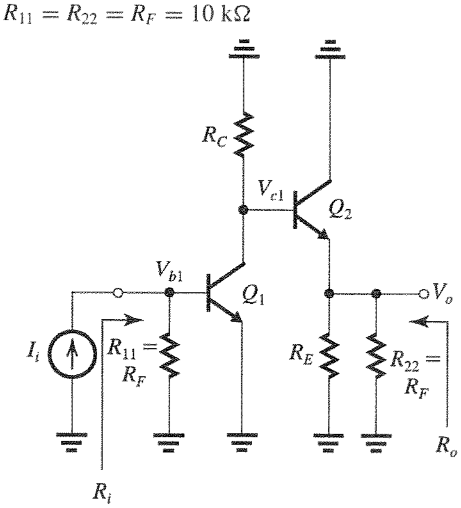


Figure 3

Figure 3 shows the A circuit. The input resistance is given by

$$R_i = R_F = R_F \parallel r_{\pi 1}$$

where

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.35}{0.025} = 14 \text{ mA/V}$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{14} = 7.14 \text{ k}\Omega$$

Thus,

$$R_i = 10 \text{ k}\Omega \parallel 7.14 \text{ k}\Omega = 4.17 \text{ k}\Omega$$

The input voltage V_{b1} is given by

$$V_{b1} = I_i R_i = 4.17 I_i \qquad (1)$$

The collector voltage of Q_1 is given by

$$V_{c1} = -g_{m1} V_{b1} \{R_C \parallel (\beta_2 + 1)[r_{e2} + (R_E \parallel R_F)]\}$$

where

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{25 \text{ mV}}{0.58 \text{ mA}} = 43.1 \ \Omega$$

$$V_{c1} = -14 V_{b1} \{10 \parallel 101[0.0431 + (10 \parallel 10)]\}$$

$$= -137.3 V_{b1} \qquad (2)$$

The gain of the emitter-follower Q_2 is given by

$$\frac{V_o}{V_{c1}} = \frac{R_E \parallel R_F}{(R_E \parallel R_F) + r_{e2}}$$

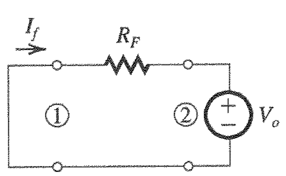
$$= \frac{5}{5 + 0.0431} = 0.99 \text{ V/V} \qquad (3)$$

Combining (1)–(3) gives

$$A \equiv \frac{V_o}{I_i} = -0.99 \times 137.3 \times 4.17$$

$$= -567.6 \text{ k}\Omega$$

(c) The value of β can be obtained from the β circuit as shown in Fig. 4:



$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F}$$

Figure 4

$$\beta = -\frac{1}{R_F} = -\frac{1}{10 \text{ k}\Omega} = -0.1 \text{ mA/V}$$

$$A\beta = -567.6 \times -0.1$$

$$= 56.76$$

$$1 + A\beta = 57.76$$

(d) $$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$A_f = -\frac{567.6}{57.76} = -9.83 \text{ k}\Omega$$

$$R_{\text{in}} = R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{4.17 \text{ k}\Omega}{57.76} = 72.2 \ \Omega$$

From the A circuit, we have

$$R_o = R_F \parallel R_E \parallel \left[r_{e2} + \frac{R_C}{\beta_2 + 1}\right]$$

$$= 10 \parallel 10 \parallel \left[0.0431 + \frac{10}{101}\right]$$

$$= 138.2 \ \Omega$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{138.2}{57.76} = 2.4 \ \Omega$$

11.70 (a) Converting the signal source to its Norton's form, we obtain the circuit shown in Fig. 1(a).

This is a shunt–shunt feedback amplifier with the feedback network consisting of the resistor R_f. To determine β, we use the arrangement shown in Fig. 1(b),

$$\beta = -\frac{1}{R_f}$$

This figure belongs to Problem 11.70, part (a).

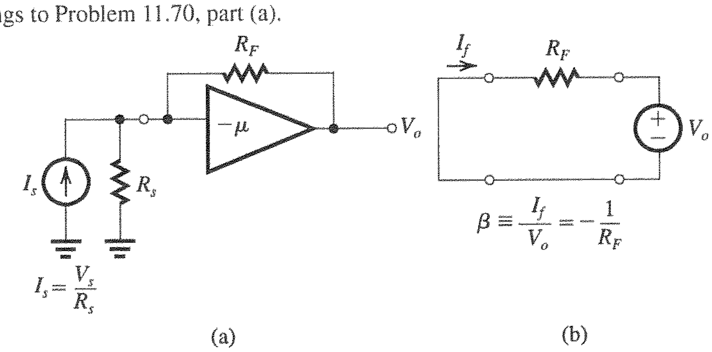
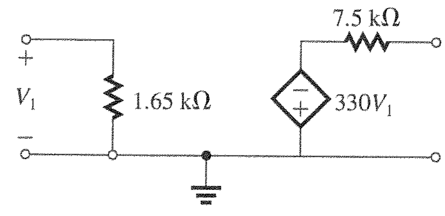
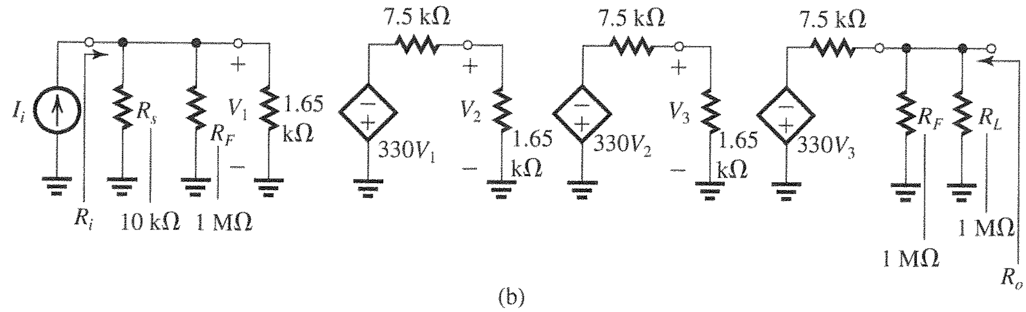


Figure 1

This figure belongs to Problem 11.70, part (b).



(a)



(b)

Figure 2

Now, for $A\beta \gg 1$, the closed-loop gain becomes

$$A_f \equiv \frac{V_o}{I_s} \simeq \frac{1}{\beta} = -R_f$$

The voltage gain $\frac{V_o}{V_s}$ is obtained as

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s}$$

Thus,

$$\frac{V_o}{V_s} \simeq -\frac{R_f}{R_s} \qquad \text{Q.E.D.}$$

(b) To obtain a closed-loop voltage gain of approximately −100 V/V, we use

$$-100 = -\frac{R_f}{R_s}$$

For $R_s = 10$ kΩ, we obtain

$$R_f = 1\ \text{M}\Omega$$

Now consider the amplifier stage shown in Fig. P11.70(b). First, we determine the dc bias point as follows:

$$I_E = \frac{15 \times \dfrac{10}{10+15} - 0.7}{4.7 + \dfrac{10 \parallel 15}{101}} = 1.11\ \text{mA}$$

$$I_C = 1.11 \times 0.99 = 1.1\ \text{mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{1.1}{0.025} = 44\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{44} = 2.27\ \text{k}\Omega$$

$$R_{\text{in}} = 10 \parallel 15 \parallel 2.27 = 1.65\ \text{k}\Omega$$

$$R_{\text{out}} = 7.5\ \text{k}\Omega$$

$$A_{vo} = -g_m \times 7.5\ \text{k}\Omega$$

$$= -44 \times 7.5 = -330\ \text{V/V}$$

Figure 2(a) shows the equivalent circuit of the amplifier stage. Figure 2(b) shows the A circuit of the feedback amplifier made up of the cascade of three stage. Observe that we have included R_s and R_L as well as R_{11} and R_{22}. The overall gain $A \equiv V_o/I_s$ cam be obtained as follows:

$$R_i = R_s \parallel R_F \parallel 1.65\ \text{k}\Omega$$

$$= 10 \parallel 1000 \parallel 1.65 = 0.623\ \text{k}\Omega$$

$$V_1 = I_i R_i = 0.623 I_i \qquad (1)$$

$$V_2 = -330 V_1 \times \frac{1.65}{1.65 + 7.5} = -59.5 V_1 \qquad (2)$$

$$V_3 = -330 V_2 \times \frac{1.65}{1.65 + 7.5} = -59.5 V_2 \qquad (3)$$

$$V_o = -330 V_3 \times \frac{1 \parallel 1000}{(1 \parallel 1000) + 7.5}$$

$$= -38.8 V_3 \qquad (4)$$

Combining (1)–(4) gives

$$A \equiv \frac{V_o}{I_i} = -8.558 \times 10^4\ \text{k}\Omega$$

Since

$$\beta = -\frac{1}{R_f} = -\frac{1}{1\ \text{M}\Omega}$$

we have

$A\beta = 85.58$

and

$1 + A\beta = 86.58$

Thus,

$$A_f \equiv \frac{V_o}{V_s} = -\frac{8.558 \times 10^4}{86.58}$$

$= -988\ \text{k}\Omega$

and the voltage gain realized is

$$\frac{V_o}{V_s} = \frac{A_f}{R_s} = \frac{-988}{10} = -98.8\ \text{V/V}$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{623\ \Omega}{86.58} = 7.2\ \Omega$$

$R_{if} = R_s \parallel R_{\text{in}}$

$R_{\text{in}} \simeq 7.2\ \Omega$

From the A circuit, we have

$R_o = R_L \parallel R_F \parallel 7.5\ \text{k}\Omega$

$R_o = 1 \parallel 1000 \parallel 7.5 = 881.6\ \Omega$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{881.6}{86.58} = 10.2\ \Omega$$

$R_{of} = R_{\text{out}} \parallel R_L$

$\Rightarrow R_{\text{out}} = 10.3\ \Omega$

11.71 (a) Shunt-Series

(b) Series-Series

(c) Shunt-Shunt

11.72

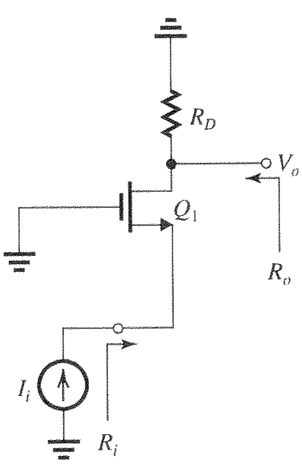


Figure 1

The A circuit is shown in Fig. 1.

$$R_i = \frac{1}{g_{m1}} = \frac{1}{5} = 0.2\ \text{k}\Omega$$

$$A \equiv \frac{V_o}{I_i} = R_D = 10\ \text{k}\Omega$$

$R_o = R_D = 10\ \text{k}\Omega$

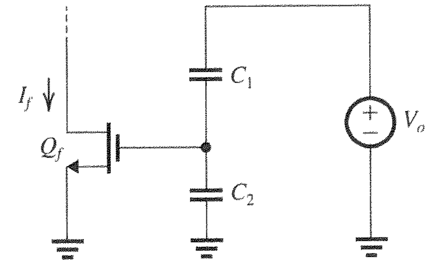


Figure 2

The β circuit is shown in Fig. 2.

$$\beta \equiv \frac{I_f}{V_o} = \frac{C_1}{C_1 + C_2} g_{mf} = \frac{0.9}{0.9 + 0.1} \times 2$$

$= 1.8\ \text{mA/V}$

$$A_f \equiv \frac{V_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= \frac{10}{1 + 10 \times 1.8} = 0.53\ \text{k}\Omega$$

$$R_{\text{in}} = R_{if} = \frac{R_i}{1 + A\beta} = \frac{200\ \Omega}{19} = 10.5\ \Omega$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta} = \frac{10\ \text{k}\Omega}{19} = 526\ \Omega$$

11.73 (a)

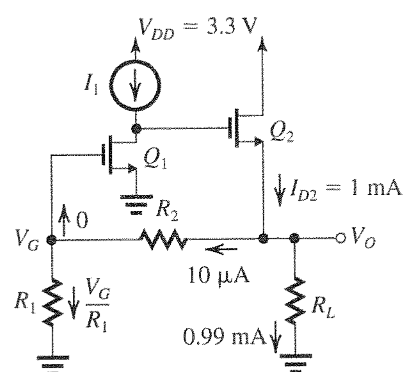


Figure 1

Figure 1 shows the circuit for the purpose of performing a dc design.

$I_{D1} = 100\ \mu\text{A} \Rightarrow I_1 = 100\ \mu\text{A}$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{OV1}^2$$

$$100 = \frac{1}{2} \times 200 \times \left(\frac{W}{L}\right)_1 \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 25$$

$$V_{G1} = V_{tn} + V_{OV1}$$

$$= 0.6 + 0.2 = 0.8 \text{ V}$$

Since $I_{R2,R1} = 10\ \mu\text{A}$, we have

$$R_1 = \frac{0.8 \text{ V}}{0.01 \text{ mA}} = 80 \text{ k}\Omega$$

$$I_{RL} = I_{D2} - I_{R2,R1}$$

$$= 1 - 0.01 = 0.99 \text{ mA}$$

$$V_O = 0.99 \times 2 = 1.98 \text{ V}$$

$$I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 V_{OV2}^2$$

$$1 = \frac{1}{2} \times 0.2 \times \left(\frac{W}{L}\right)_2 \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 250$$

$$R_2 = \frac{V_O - V_G}{0.01 \text{ mA}}$$

$$= \frac{1.98 - 0.8}{0.01} = 118 \text{ k}\Omega$$

$$V_{GS2} = V_{tn} + V_{OV2} = 0.8 \text{ V}$$

$$V_{D1} = V_{G2} = 1.98 + 0.8 = 2.78 \text{ V}$$

(b) The β circuit consists of resistance R_2. The value of β can be determined as shown in Fig. 2.

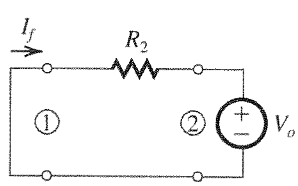


Figure 2

$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_2} = -\frac{1}{118 \text{ k}\Omega}$$

$$= -8.47 \times 10^{-3} \text{ mA/V}$$

Thus,

$$A_f\big|_{\text{ideal}} \equiv \frac{1}{\beta} = -118 \text{ k}\Omega$$

(c) Converting the signal source to its Norton's form, the feedback amplifier takes the form shown in Fig. 3.

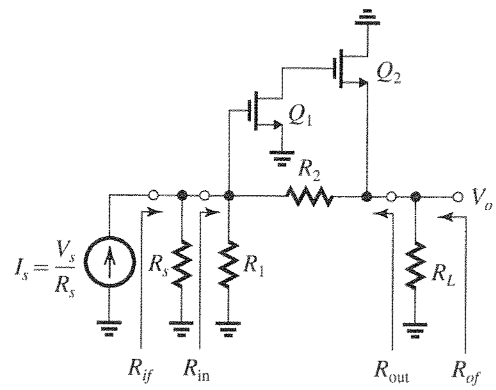


Figure 3

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s}$$

Thus,

$$-6 = -\frac{118}{R_s}$$

$$\Rightarrow R_s = 19.7 \text{ k}\Omega$$

(d)

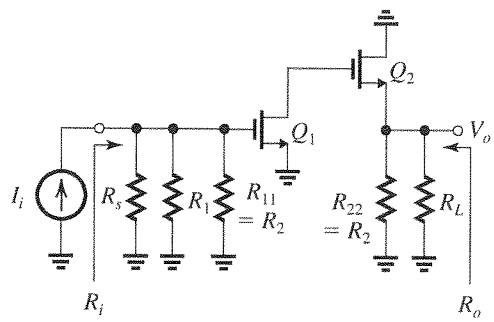


Figure 4

The A circuit is shown in Fig. 4.

$$R_i = R_s \parallel R_1 \parallel R_2$$

$$= 19.7 \parallel 80 \parallel 118 = 13.92 \text{ k}\Omega$$

$$V_{gs1} = I_i R_i = 13.92 I_i \quad (1)$$

$$V_{d1} = -g_{m1} V_{gs1} r_{o1}$$

where

$$g_{m1} = \frac{2I_{D1}}{V_{OV1}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o1} = \frac{V_A}{I_{D1}} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

Thus,

$$V_{d1} = -200 V_{gs1} \quad (2)$$

$$\frac{V_o}{V_{d1}} = \frac{R_L \parallel R_2 \parallel r_{o2}}{(R_L \parallel R_2 \parallel r_{o2}) + 1/g_{m2}}$$

where

$$g_{m2} = \frac{2I_{D2}}{V_{OV2}} = \frac{2 \times 1}{0.2} = 10 \text{ mA/V}$$

$$r_{o2} = \frac{V_A}{I_{D2}} = \frac{20}{1} = 20 \text{ k}\Omega$$

Thus,

$$\frac{V_o}{V_{d1}} = \frac{(2 \parallel 118 \parallel 20)}{(2 \parallel 118 \parallel 20) + 0.1} = 0.947 \text{ V/V} \qquad (3)$$

Combining (1)–(3), we obtain

$$A = \frac{V_o}{I_i} = -13.92 \times 200 \times 0.947$$

$$= -2636.7 \text{ k}\Omega$$

$$R_o = R_L \parallel R_2 \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

$$= 2 \parallel 118 \parallel 20 \parallel 0.1$$

$$= 94.7 \ \Omega$$

(e) $A_f = \dfrac{V_o}{V_s} = \dfrac{A}{1 + A\beta}$

$$= -\frac{2636.7}{1 + (2636.7/118)}$$

$$= -\frac{2636.7}{23.34} = -113 \text{ k}\Omega$$

$$\frac{V_o}{V_s} = \frac{A_f}{R_s} = -\frac{113}{19.7} = -5.73 \text{ V/V}$$

(f) $R_{if} = \dfrac{R_i}{1 + A\beta} = \dfrac{13.92}{23.34} = 0.596 \text{ k}\Omega$

$$R_{if} = R_s \parallel R_{\text{in}}$$

$$0.596 = 19.7 \parallel R_{\text{in}}$$

$$\Rightarrow R_{\text{in}} = 615 \ \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{94.7}{23.34} = 4.06 \ \Omega$$

$$R_{of} = R_{\text{out}} \parallel R_L$$

$$4.06 = R_{\text{out}} \parallel 2000 \Rightarrow R_{\text{out}} = 4.1 \ \Omega$$

11.74 (a) Figure 1 shows the β network as well as the determination of its loading effects on the A circuit:

$$R_{11} = R_F + R_M$$

$$R_{22} = R_M \parallel R_F$$

Figure 2 shows the A circuit. Some of the analysis is shown on the diagram.

$$R_i = R_{11} \parallel \frac{1}{g_{m1}} \qquad (1)$$

$$V_{sg1} = I_i R_i \qquad (2)$$

These figures belong to Problem 11.74, part (a).

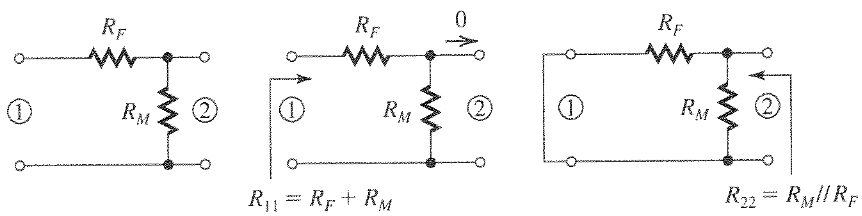


Figure 1

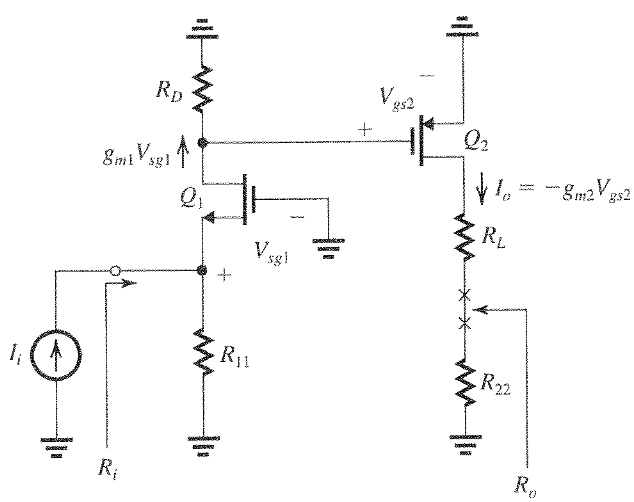


Figure 2

$V_{gs2} = g_{m1} V_{sg1} R_D$ (3)

$I_o = -g_{m2} V_{gs2}$ (4)

Combining (1)–(4) gives

$$A \equiv \frac{I_o}{I_i} = -\left(R_{11} \parallel \frac{1}{g_{m1}}\right)(g_{m1} R_D) g_{m2} \quad (5)$$

(b)

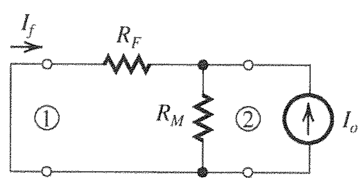


Figure 3

The β circuit prepared for the determination of β is shown in Fig. 3.

$$\beta \equiv \frac{I_f}{I_o} = -\frac{R_M}{R_M + R_F} \quad (6)$$

(c) From (5)–(6), we obtain

$$A\beta = \frac{R_M}{R_M + R_F}\left(R_{11} \parallel \frac{1}{g_{m1}}\right)(g_{m1} R_D) g_{m2}$$

(d) $g_{m1} = g_{m2} = 5$ mA/V, $R_D = 20$ kΩ

$R_M = 10$ kΩ, and $R_F = 90$ kΩ, thus

$R_{11} = R_F + R_M = 90 + 10 = 100$ kΩ

$R_{22} = R_M \parallel R_F = 10 \parallel 90 = 9$ kΩ

$A = -(100 \parallel 0.2) \times (5 \times 20) \times 5 = -99.8$ A/A

$$\beta = -\frac{10}{10 + 90} = -0.1 \text{ A/A}$$

$A\beta = 9.98$

$1 + A\beta = 10.98$

$$A_f \equiv \frac{I_o}{I_s} = \frac{A}{1 + A\beta} = -\frac{99.8}{10.98} = -9.1 \text{ A/A}$$

$$R_{\text{in}} = R_{if} = \frac{R_i}{1 + A\beta}$$

where

$R_i = 100 \text{ k}\Omega \parallel 0.2 \text{ k}\Omega \simeq 0.2$ kΩ

$$R_{\text{in}} = \frac{200 \ \Omega}{10.98} = 18.2 \ \Omega$$

(e) Breaking the output loop of the A circuit between XX, we find

$R_o = R_{22} + R_L + r_{o2}$

$= (R_M \parallel R_F) + R_L + r_{o2}$

$= (10 \parallel 90) + 1 + 20$

$= 30$ kΩ

$R_{of} = R_o(1 + A\beta) = 30 \times 10.98$

$= 329.4$ kΩ

$R_{\text{out}} = R_{of} - R_L = 328.4$ kΩ

11.75 Refer to Fig. 11.27(c), which shows the determination of β,

$$\beta = \frac{I_f}{I_o} = -\frac{R_1}{R_1 + R_2} \quad (1)$$

Refer to Fig. 11.27(e), which shows the A circuit. The input resistance R_i is given by

$R_i = R_s \parallel R_{id} \parallel (R_1 + R_2)$

For our case here, $R_s = R_{id} = \infty$, thus

$R_i = R_1 + R_2$

For $R_{\text{in}} = R_{if} = 1$ kΩ, we have

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$\Rightarrow R_i = R_{if}(1 + A\beta)$

Thus

$R_1 + R_2 = 1 \text{ k}\Omega \times (1 + A\beta)$

Since $1 + A\beta$ is 40 dB, that is,

$1 + A\beta = 100$

we have

$R_1 + R_2 = 1 \times 100 = 100$ kΩ (2)

Now,

$$A_f = \frac{A}{1 + A\beta}$$

$$-100 = \frac{A}{100}$$

$\Rightarrow A = -10^4$ A/A

$$\beta = \frac{A\beta}{A} = \frac{99}{-10^4} = -0.0099$$

Using Eqs. (1) and (2), we obtain

$$-0.0099 = -\frac{R_1}{R_1 + R_2}$$

$\Rightarrow R_1 = 0.0099 \times 100 = 0.99$ kΩ

$R_2 = 100 - 0.99 = 99.01$ kΩ

Now, using Eq. (11.53) (page 869), we obtain

$$A = -\mu \frac{R_i}{1/g_m + (R_1 \parallel R_2 \parallel r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \parallel R_2)}$$

$$-10^4 = -\mu \frac{100}{0.2 + (0.99 \parallel 99.01 \parallel 20)} \frac{20}{20 + (0.99 \parallel 99.01)}$$

$\mu = 119$ V/V

From Example 11.10, we have

$R_o = r_{o2} + (R_1 \parallel R_2) + g_{m2} r_{o2} (R_1 \parallel R_2)$

$R_o = 20 + (0.99 \parallel 99.01)(1 + 5 \times 20)$

$= 119$ kΩ

$R_{\text{out}} = R_{of} = R_o(1+A\beta) = 119 \times 100 = 11.9$ MΩ

11.76

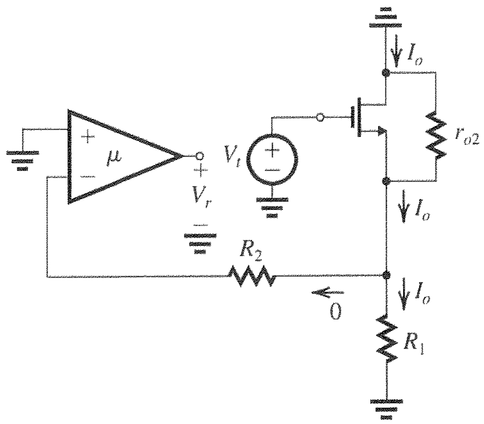


Figure 1

Figure 1 shows the shunt-series feedback amplifier circuit of Fig. 11.27(a) prepared for determining the loop gain,

$$A\beta \equiv -\frac{V_r}{V_t}$$

observe that here $R_s = R_{id} = \infty$. Thus I_o can be obtained as

$$I_o = \frac{V_t}{\dfrac{1}{g_m} + (r_{o2} \parallel R_1)} \frac{r_{o2}}{r_{o2} + R_1} \qquad (1)$$

The voltage V_r can be obtained as

$$V_r = I_o R_1 \times -\mu = -\mu R_1 I_o \qquad (2)$$

Combining Eqs. (1) and (2), we obtain

$$A\beta \equiv -\frac{V_r}{V_t} = \mu \frac{R_1}{\dfrac{1}{g_m} + (r_{o2} \parallel R_1)} \frac{r_{o2}}{r_{o2} + R_1}$$

For

$\mu = 1000$ V/V, $R_1 = 10$ kΩ,

$g_m = 5$ mA/V, and $r_{o2} = 20$ kΩ

we obtain

$$A\beta = 1000 \times \frac{10}{0.2 + (20 \parallel 10)} \frac{20}{20 + 10}$$

$= 970.9$

which is slightly lower than the value found in Example 11.10 (1076.4), the difference being about −10%. This is a result of the assumptions and approximations made in the general feedback analysis method.

From Example 11.10 (or directly from the β circuit) we have

$\beta = -0.1$ A/A

Thus,

$A = -9709$

and

$$A_f = -\frac{9709}{971.9} = -9.99 \text{ A/A}$$

which is identical to the value obtained in Example 11.10. Thus while $A\beta$ and A differ slightly for the earlier results, A_f is identical; an illustration of the power of negative feedback!

11.77 (a)

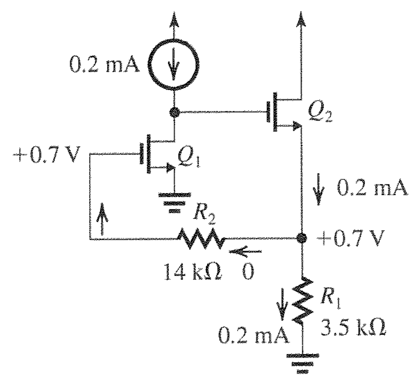


Figure 1

Figure 1 shows the dc analysis. It starts by noting that $I_{D1} = 0.2$ mA. Thus $V_{OV1} = 0.2$ V and

$V_{G1} = V_{GS1} = V_t + V_{OV1} = 0.5 + 0.2$

$= 0.7$ V

Since the dc current through R_2 is zero, the dc voltage drop across it will be zero, thus

$V_{S2} = +0.7$ V

and

$$I_{R1} = \frac{0.7 \text{ V}}{3.5 \text{ k}\Omega} = 0.2 \text{ mA}$$

Thus, Q_2 is operating at

$I_D = 0.2$ mA Q.E.D.

(b) $g_{m1} = g_{m2} = \dfrac{2I_D}{V_{OV}}$

$$= \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{10}{0.2} = 50 \text{ k}\Omega$$

(c)

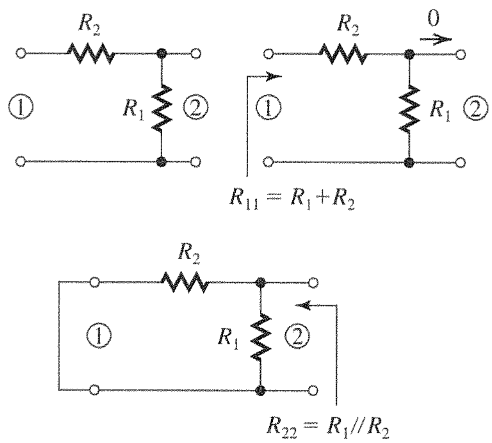


Figure 2

Figure 2 shows the β circuit and the determination of its loading effects, R_{11} and R_{22},

$R_{11} = R_1 + R_2 = 3.5 + 14 = 17.5\ \text{k}\Omega$

$R_{22} = R_1 \parallel R_2 = 3.5 \parallel 14 = 2.8\ \text{k}\Omega$

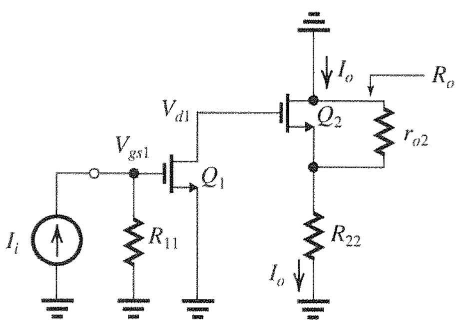


Figure 3

Figure 3 shows the A circuit. To obtain $A = I_o/I_i$, we write

$$V_{gs1} = I_i R_{11} \tag{1}$$

$$V_{d1} = -g_{m1} r_{o1} V_{gs1} \tag{2}$$

$$I_o = \frac{V_{d1}}{\frac{1}{g_{m2}} + (r_{o2} \parallel R_{22})} \frac{r_{o2}}{r_{o2} + R_{22}} \tag{3}$$

Combining (1)–(3) yields

$$A = \frac{I_o}{I_i} = -\frac{R_{11}}{\frac{1}{g_{m2}} + (r_{o2} \parallel R_{22})} (g_{m1} r_{o1}) \frac{r_{o2}}{r_{o2} + R_{22}}$$

$$A = -\frac{17.5}{0.5 + (50 \parallel 2.8)} \times 2 \times 50 \times \frac{50}{50 + 2.8}$$

$A = -525.8$ A/A

$R_i = R_{11} = 17.5\ \text{k}\Omega$

$$R_o = r_{o2} + R_{22} + g_{m2} r_{o2} R_{22}$$

$$= 50 + 2.8 + 2 \times 50 \times 2.8$$

$$= 332.8\ \text{k}\Omega$$

(d)

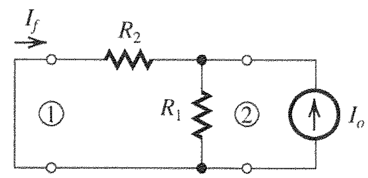


Figure 4

Figure 4 shows the determination of the value of β:

$$\beta \equiv \frac{I_f}{I_o} = -\frac{R_1}{R_1 + R_2}$$

Thus,

$$\beta = -\frac{3.5}{3.5 + 14} = -0.2 \text{ A/A}$$

(e) $A\beta = -525.8 \times -0.2$

$= 105.16$

$1 + A\beta = 106.16$

$$A_f = -\frac{525.8}{106.16} = -4.95 \text{ A/A}$$

(f) $R_{\text{in}} = R_{if} = \dfrac{R_i}{1 + A\beta}$

$$= \frac{17.5\ \text{k}\Omega}{106.16} = 164.8\ \Omega$$

$R_{\text{out}} = R_{of} = R_o(1 + A\beta)$

$= 332.8 \times 106.16 = 35.3\ \text{M}\Omega$

11.78 (a) If μ is a very large, a virtual ground will appear at the input terminal. Thus the input resistance $R_{\text{in}} = V_-/I_i = 0$. Since no current flows in R_s, or into the amplifier input terminal, all the current I_s will flow in the transistor source terminal and hence into the drain, thus

$I_o = I_s$

and

$$\frac{I_o}{I_s} = 1$$

(b) This is a shunt-series feedback amplifier in which the feedback circuit consists of a wire, as shown in Fig. 1. As indicated,

$R_{11} = \infty$

$R_{22} = 0$

The A circuit is shown in Fig. 2, for which we can write

$V_{id} = -I_i(R_s \parallel R_{id})$

$$\simeq -I_i R_s \tag{1}$$

These figures belong to Problem 11.78, part (b).

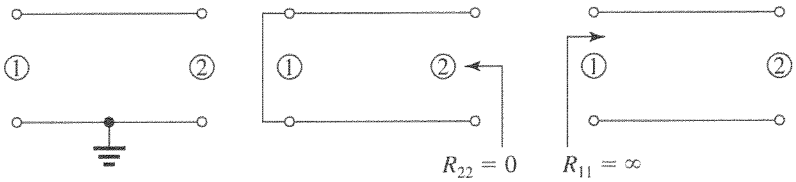


Figure 1

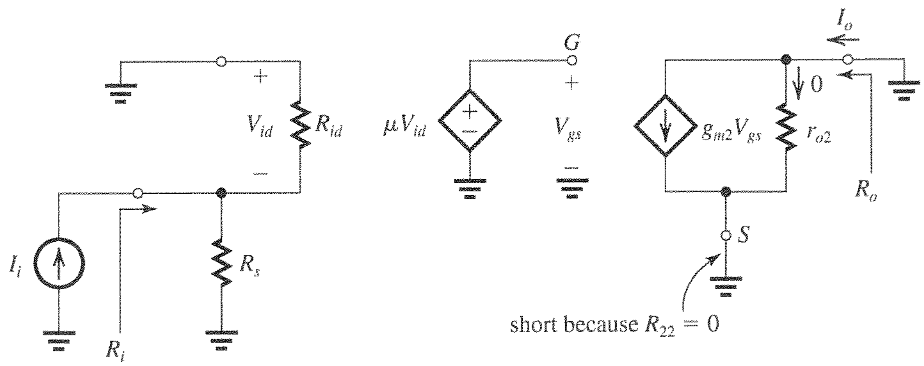


Figure 2

(since R_{id} is very large)

$V_{gs} = \mu V_{id}$ (2)

$I_o = g_{m2} V_{gs}$ (3)

Combining (1)–(3), we obtain

$$A \equiv \frac{I_o}{I_i} = -\mu g_{m2} R_s$$

$R_i = R_s$

$R_o = r_{o2}$

(c)

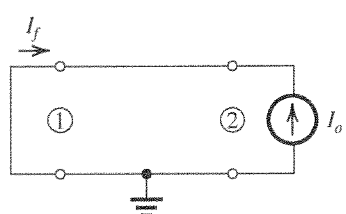


Figure 3

From Fig. 3 we find

$$\beta \equiv \frac{I_f}{I_o} = -1$$

(d) $A\beta = \mu g_{m2} R_s$

$$A_f = \frac{A}{1 + A\beta}$$

$$= -\frac{\mu g_{m2} R_s}{1 + \mu g_{m2} R_s}$$

Note that the negative sign is due to our assumption that I_s flows into the input node (see Fig. 2 for the way I_i is applied). If instead I_s is flowing out of the input node, as indicated in Fig. P11.78, then

$$A_f \equiv \frac{I_o}{I_s} = \frac{\mu g_{m2} R_s}{1 + \mu g_{m2} R_s}$$

If μ is large so that $\mu g_{m2} R_s \gg 1$,

$$A_f \simeq 1$$

(e) $R_{if} = \dfrac{R_i}{1 + A\beta}$

$$= \frac{R_s}{\mu g_{m2} R_s}$$

$R_{if} = R_{\text{in}} \parallel R_s$

$$\frac{1}{R_{if}} = \frac{1}{R_{\text{in}}} + \frac{1}{R_s}$$

$$\frac{1}{R_s} + \mu g_{m2} = \frac{1}{R_{\text{in}}} + \frac{1}{R_s}$$

$$\Rightarrow R_{\text{in}} = \frac{1}{\mu g_{m2}}$$

$R_{\text{out}} = R_{of} = R_o(1 + A\beta)$

$= r_{o2}(1 + \mu g_{m2} R_s)$

(f)

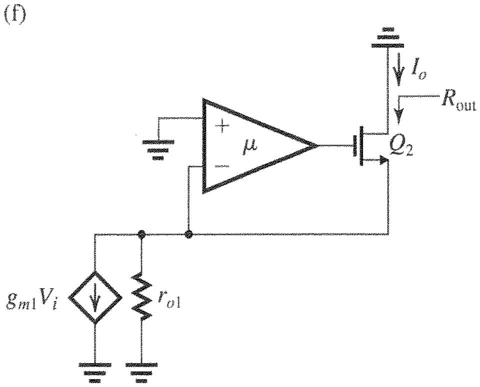


Figure 4

Figure 4 shows the circuit of the cascode amplifier in Fig. P11.78 with V_G replaced by signal ground, and Q_1 replaced by its equivalent circuit at the drain. This circuit looks identical to that of Fig. 11.78(a). Thus we can write

$$I_o \simeq g_{m1} v_i$$

$$R_{out} = r_{o2}(1 + \mu g_{m2} r_{o1}) = r_{o2} + \mu(g_{m2} r_{o2}) r_{o1}$$

$$\simeq \mu(g_{m2} r_{o2}) r_{o1}$$

Compared to the case of a regular cascode, we see that while $I_o \simeq g_{m1} V_i$ as in the regular cascode, utilizing the "super" CG transistor results in increasing the output resistance by the additional factor μ!

11.79 (a) Refer to Fig. P11.79. Let I_s increase. This will cause the voltage at the input node, which is the voltage at the positive input of the amplifier μ, to increase. The amplifier output voltage will correspondingly increase. Thus, V_{gs} of Q_1 increases and I_{o1} increases. This counteracts the originally assumed change of increased current into the input node. Thus, the feedback is negative.

(b) The negative feedback will cause the dc voltage at the positive input terminal of the amplifier to be equal to V_{BIAS}. Thus, the voltage at the drain of Q_1 will be equal to V_{BIAS}. For Q_1 to operate in the active mode, the minimum voltage at the drain must be equal to V_{OV} which is 0.2 V. Thus the minimum value of V_{BIAS} must be 0.2 V.

(c)

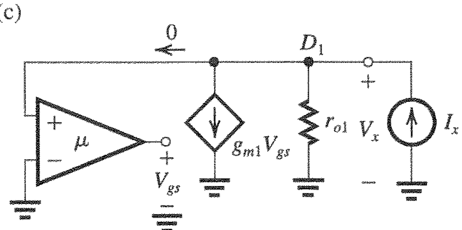


Figure 1

Figure 1 shows the circuit for determining R_{in},

$$R_{in} \equiv \frac{V_x}{I_x}$$

A node equation at D_1 gives

$$V_x = (I_x - g_{m1} V_{gs}) r_{o1}$$

$$\Rightarrow V_x = I_x r_{o1} - g_{m1} V_{gs} r_{o1}$$

But,

$$V_{gs} = \mu V_x,$$

thus

$$V_x = I_x r_{o1} - \mu g_{m1} r_{o1} V_x$$

$$\Rightarrow R_{in} \equiv \frac{V_x}{I_x} = \frac{r_{o1}}{1 + \mu g_{m1} r_{o1}}$$

Since $\mu g_{m1} r_{o1} \gg 1$, we have

$$R_{in} \simeq \frac{1}{\mu g_{m1}}$$

(d) Since the drain of Q_2 is outside the feedback loop, we have

$$R_{out} = r_{o2}$$

11.80 See figures on the next two pages.

(a) Refer to Fig. (a).

(b) Refer to Fig. (b).

(c) $R_{outz} = r_{o2} \parallel r_{o4}$

11.81

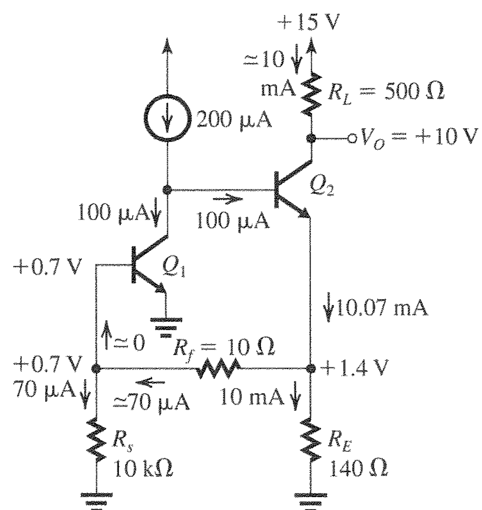


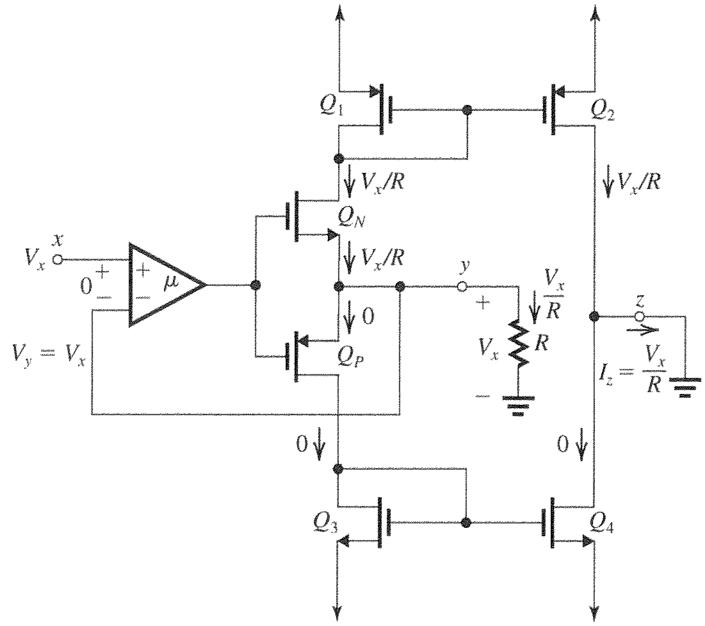
Figure 1

The dc analysis is shown in Fig. 1, from which we see that

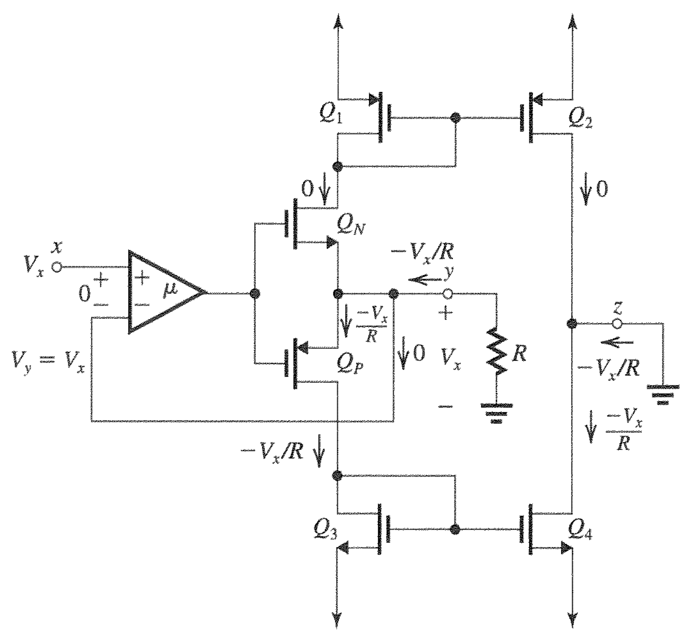
$$I_{C1} = 0.1 \text{ mA}$$

$$I_{C2} = 10 \text{ mA}$$

These figures belong to Problem 11.80.



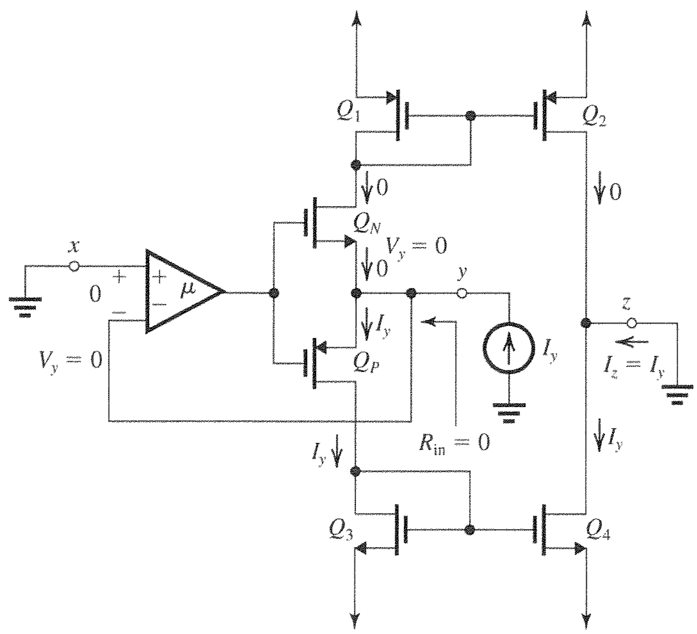
(a) V_x positive



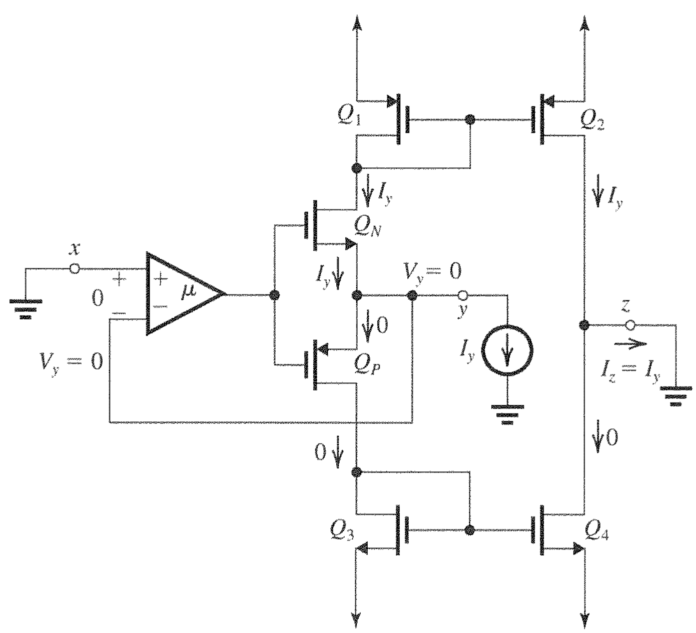
(a) V_x negative

Figure 1

These figures belong to Problem 11.80.



(b) I_y positive



(b) I_y negative

Figure 2

These figures belong to Problem 11.81.

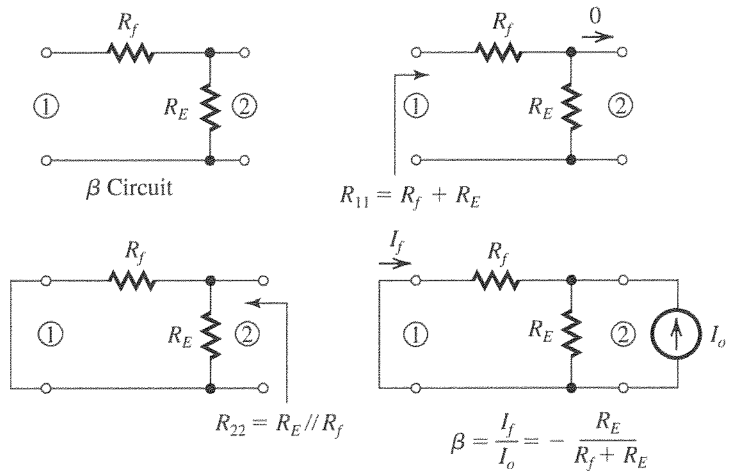


Figure 2

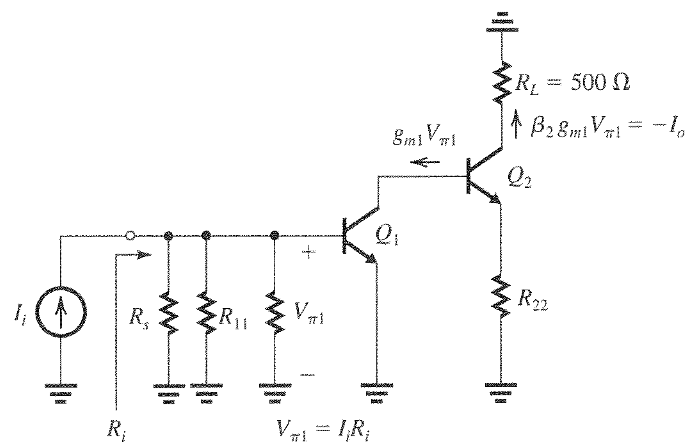


Figure 3

Thus,

$g_{m1} = 4$ mA/V

$r_{\pi 1} = 25$ kΩ

$g_{m2} = 400$ mA/V

The β circuit is shown in Fig. 2 together with the determination of its loading effects and of β.

$R_{11} = R_f + R_E = 10.14$ kΩ

$R_{22} = R_f \parallel R_E = 10 \parallel 0.14 = 0.138$ kΩ

$$\beta = -\frac{R_E}{R_f + R_E} = -\frac{0.14}{10 + 0.4} = 0.0138 \text{ A/A}$$

The A circuit is shown in Fig. 3.

$R_i = R_s \parallel R_{11} \parallel r_{\pi 1}$

$= 10 \parallel 10.14 \parallel 25$

$= 4.19$ kΩ

$V_{\pi 1} = I_i R_i$

$I_o = -\beta_2 g_{m1} V_{\pi 1}$

$$\Rightarrow A = \frac{I_o}{I_i} = -\beta_2 g_{m1} R_i$$

$A = -100 \times 4 \times 4.19 = -1676$ A/A

$A\beta = -1676 \times -0.0138 = 23.13$

$1 + A\beta = 24.13$

$$A_f = \frac{I_o}{I_s} = \frac{A}{1 + A\beta}$$

where

$$I_s = \frac{V_s}{R_s}$$

$$A_f = -\frac{1676}{24.13} = -72.5 \text{ A/A}$$

$$\frac{V_o}{V_s} = \frac{-I_o R_L}{-I_s R_s} = 72.5 \times \frac{0.5}{10} = 3.62 \text{ V/V}$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{4190\ \Omega}{24.13} = 173.6\ \Omega$$

$$R_{if} = R_s \parallel R_{\text{in}}$$

$$173.6\ \Omega = 10{,}000\ \Omega \parallel R_{\text{in}}$$

$$\Rightarrow R_{\text{in}} = 176.7\ \Omega$$

11.82 (a) Refer to the circuit in Fig. P11.82. Observe that the feedback signal is capacitively coupled and so are the signal source and R_L; thus, these do not enter into the dc bias calculations and the feedback does not affect the bias. The dc emitter current in Q_1 can be determined from

$$I_{E1} = \frac{12 \times \dfrac{15}{100 + 15} - 0.7}{0.870 + \dfrac{100 \parallel 15}{101}} = 0.865 \text{ mA}$$

$$I_{C1} = 0.99 \times 0.865 = 0.86 \text{ mA}$$

Next consider Q_2 and let its emitter current be I_{E2}. The base current of Q_2 will be $I_{E2}/(\beta + 1) \simeq 0.01\ I_{E2}$. The current through R_{C1} will be $(I_{C1} + I_{B2}) = (0.86 + 0.01\ I_{E2})$. We can thus write the following equation:

$$12 = (0.86 + 0.01\ I_{E2}) \times 10 + 0.7 + 3.4 \times I_{E2}$$

$$\Rightarrow I_{E2} = \frac{12 - 8.6 - 0.7}{3.4 + 0.1} = \frac{2.7}{3.5} = 0.77 \text{ mA}$$

$$I_{C2} = 0.76 \text{ mA}$$

The small-signal parameters of Q_1 and Q_2 can now be obtained as

$$g_{m1} = \frac{0.86}{0.025} = 34.4 \text{ mA/V}$$

$$r_{\pi 1} = \frac{100}{34.4} = 2.91 \text{ k}\Omega$$

$$g_{m2} = \frac{0.76}{0.025} = 30.4 \text{ mA/V}$$

$$r_{\pi 2} = \frac{100}{30.4} = 3.3 \text{ k}\Omega$$

(b) The equivalent circuit of the feedback amplifier is shown in Fig. 1, where

$$R_s = 10 \text{ k}\Omega$$

$$I_s = \frac{V_s}{R_s}$$

$$R_B = R_{B1} \parallel R_{B2} = 13 \text{ k}\Omega$$

(c) See figure on the next page. The determination of the loading effects of the β circuit on the A circuit is shown in Fig. 2:

$$R_{11} = R_f + R_{E2} = 10 + 3.4 = 13.4 \text{ k}\Omega$$

$$R_{22} = R_{E2} \parallel R_f = 3.4 \parallel 10 = 2.54 \text{ k}\Omega$$

The A circuit is shown in Fig. 3 on the next page.

Analysis of the A circuit to determine $A \equiv I_o/I_s$ proceeds as follows:

$$R_i = R_s \parallel R_{11} \parallel R_B \parallel r_{\pi 1}$$

$$= 10 \parallel 13.4 \parallel 13 \parallel 2.91 = 1.68 \text{ k}\Omega$$

$$V_{\pi 1} = I_i R_i \tag{1}$$

$$I_{b2} = -g_{m1} V_{\pi 1} \frac{R_{C1}}{R_{C1} + r_{\pi 2} + (\beta + 1) R_{22}} \tag{2}$$

$$I_o = I_{e2} = (\beta + 1) I_{b2} \tag{3}$$

This figure belongs to Problem 11.82, part (b).

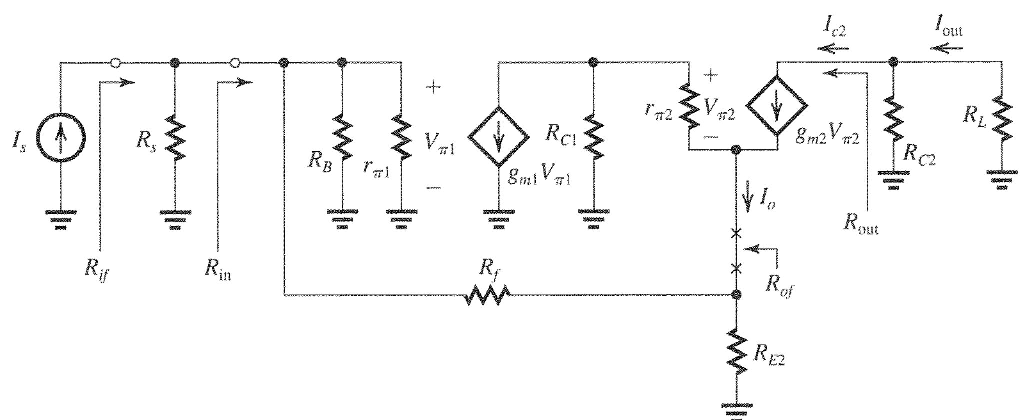


Figure 1

These figures belong to Problem 11.82, part (c).

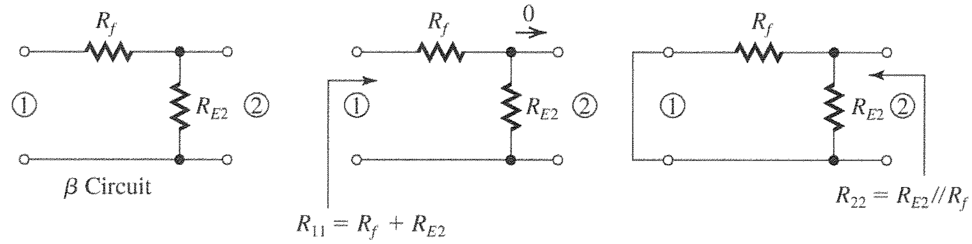


Figure 2

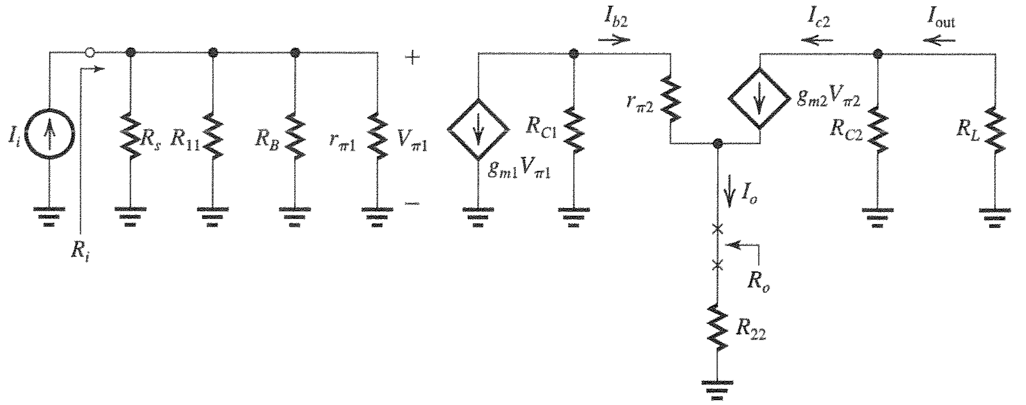


Figure 3

Combining Eqs. (1)–(3) results in

$$A \equiv \frac{I_o}{I_i} = -\frac{(\beta + 1)R_i g_{m1} R_{C1}}{R_{C1} + r_{\pi 2} + (\beta + 1)R_{22}}$$

$$= \frac{101 \times 1.68 \times 34.4 \times 10}{10 + 3.3 + 101 \times 2.54}$$

$$= -216.3 \text{ A/A}$$

Breaking the emitter loop of Q_2 at XX gives

$$R_o = R_{22} + \frac{r_{\pi 2} + R_{C1}}{\beta + 1}$$

$$= 2.54 + \frac{3.3 + 10}{101} = 2.67 \text{ k}\Omega$$

(d)

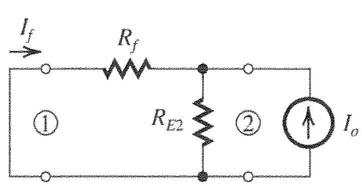


Figure 4

$$\beta \equiv \frac{I_f}{I_o} = -\frac{R_{E2}}{R_{E2} + R_f}$$

$$= -0.254 \text{ A/A}$$

(e) $A\beta = -216.3 \times -0.254 = 54.88$

$1 + A\beta = 55.88$

$$A_f = \frac{I_o}{I_s} = -\frac{216.3}{55.88} = -3.87 \text{ A/A}$$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{1.68 \text{ k}\Omega}{55.88} = 30.1 \ \Omega$$

$$R_{of} = R_o(1 + A\beta) = 2.67 \times 55.88$$

$$= 149.2 \text{ k}\Omega$$

(f) $R_{if} = R_s \parallel R_{in}$

$30.1 \ \Omega = 10 \text{ k}\Omega \parallel R_{in}$

$R_{in} = 30.2 \ \Omega$

$$I_{in} = I_s \frac{R_s}{R_s + R_{in}} \simeq I_s$$

$$I_{out} = I_{C2} \frac{R_{C2}}{R_{C2} + R_L} = \alpha I_o \frac{R_{C2}}{R_{C2} + R_L}$$

$$\frac{I_{out}}{I_{in}} \simeq \frac{I_{out}}{I_s} = \frac{I_o}{I_s} \times \alpha \frac{R_{C2}}{R_{C2} + R_L}$$

$$\Rightarrow \frac{I_{out}}{I_{in}} = -3.87 \times 0.99 \times 0.99 \times \frac{8}{8 + 1}$$

$$= -3.41 \text{ A/A}$$

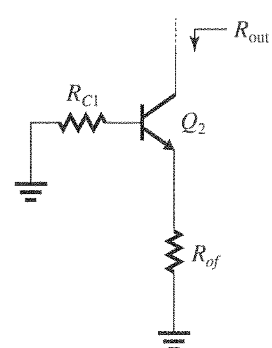


Figure 5

To determine R_{out}, consider the circuit in Fig. 5. Using the formula given at the end of Example 11.8, adapted to our case here, we get

$$R_{out} = r_{o2} + [R_{of} \parallel (r_{\pi 2} + R_{C1})]\left[1 + g_{m2} r_{o2} \frac{r_{\pi 2}}{r_{\pi 2} + R_{C1}}\right]$$

where

$$r_{o2} = \frac{75\text{ V}}{0.76\text{ mA}} = 98.7\text{ k}\Omega$$

$$R_{out} = 98.7 + [149.2 \parallel (3.3 + 10)]\left[1 + 30.3 \times 98.7 \times \frac{3.3}{3.3 + 10}\right]$$

$$= 98.7 + 12.21 \times 743 = 9.17\text{ M}\Omega$$

Check:

Maximum possible $R_{out} \simeq \beta r_o \simeq 10\text{ M}\Omega$

So, our result is reasonable.

At $\omega = \omega_{180}$, the magnitude of A becomes

$$|A(j\omega_{180})| = \frac{10^5}{\sqrt{\left[1 + \left(\frac{20,000}{100}\right)^2\right]\left[1 + \left(\frac{20,000}{20,000}\right)^2\right]^2}}$$

$$|A(j\omega_{180})| = \frac{10^5}{200 \times 2} = 250\text{ V/V}$$

$$|A(j\omega_{180})|\beta_{cr} = 1$$

$$\beta_{cr} = \frac{1}{250} = 4 \times 10^{-3}\text{ V/V}$$

Correspondingly,

$$A_f = \frac{10^5}{1 + 10^5 \times 4 \times 10^{-3}}$$

$$= \frac{10^5}{1 + 400} \simeq 250\text{ V/V}$$

11.83 $A(s) = \dfrac{10^5}{\left(1 + \frac{s}{100}\right)\left(1 + \frac{s}{20,000}\right)^2}$

$$\phi = -\tan^{-1}\frac{\omega}{100} - 2\tan^{-1}\frac{\omega}{20,000}$$

$$180^\circ = \tan^{-1}\frac{\omega_{180}}{100} + 2\tan^{-1}\frac{\omega_{180}}{20,000}$$

Since ω_{180} will be much greater than 100 rad/s, we can assume that at ω_{180}, $\tan^{-1}(\omega_{180/100})$ is approximately 90°, thus

$$2\tan^{-1}\frac{\omega_{180}}{20,000} = 90^\circ$$

$$\Rightarrow \tan^{-1}\frac{\omega_{180}}{20,000} = 45^\circ$$

$$\Rightarrow \omega_{180} = 20,000\text{ rad/s}$$

which is indeed much greater than 100 rad/s, justifying our original assumption.

11.84 $A(s) = \dfrac{10^5}{\left(1 + \frac{s}{100}\right)\left(1 + \frac{s}{20,000}\right)^2}$

$$A(j\omega) = \frac{10^5}{\left(1 + j\frac{\omega}{100}\right)\left(1 + j\frac{\omega}{20,000}\right)^2}$$

$$\phi(\omega) = -\tan^{-1}\left(\frac{\omega}{100}\right) - 2\tan^{-1}\left(\frac{\omega}{20,000}\right) \quad (1)$$

$$|A(j\omega)| = \frac{10^5}{\sqrt{1 + \left(\frac{\omega}{100}\right)^2}\left[1 + \left(\frac{\omega}{20,000}\right)^2\right]} \quad (2)$$

Using Eqs. (1) and (2), we can obtain the data required to construct Nyquist plots for the two cases: $\beta = 1$ and $\beta = 10^{-3}$. The results are given in the following table.

This table belongs to Problem 11.84.

ω rad/s	$-\tan^{-1}\left(\frac{\omega}{100}\right)$	$-2\tan^{-1}\left(\frac{\omega}{20,000}\right)$	ϕ	$\lvert A\rvert$	$\lvert A\beta\rvert$ $\beta = 1$	$\lvert A\beta\rvert$ $\beta = 10^{-3}$
0	0	0	0	10^5	10^5	100
10^2	-45°	-0.6°	-45.6°	0.7×10^5	0.7×10^5	70
10^3	-84.3°	-5.7°	-90°	10^4	10^4	10
10^4	-89.4°	-53.1°	-142.5°	800	800	0.8
2×10^4	-89.7°	-90°	$\simeq -180^\circ$	250	250	0.25
∞	-90°	-180°	-270°	0	0	0

This figure belongs to Problem 11.84.

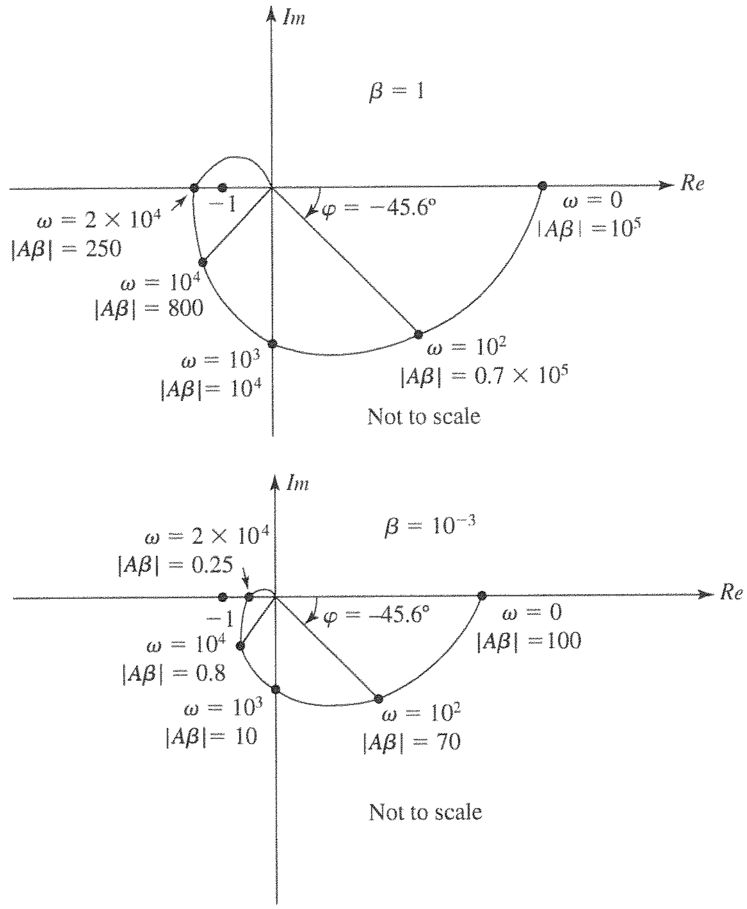


Figure 1

Using these data, we obtain the two Nyquist plots shown in Fig. 1.

We observe that the amplifier with $\beta = 1$ will be unstable and that with $\beta = 10^{-3}$ will be stable.

11.85 $A(s)\beta(s) = \dfrac{10^4 k}{\left(1 + \dfrac{s}{10^3}\right)^3}$

$$A(j\omega)\beta(j\omega) = \frac{10^4 k}{\left(1 + j\dfrac{\omega}{10^3}\right)^3}$$

$$|A(j\omega)\beta(j\omega)| = \frac{10^4 k}{\left(1 + \dfrac{\omega^2}{10^6}\right)^{3/2}}$$

$$\phi(\omega) = -3\tan^{-1}\left(\frac{\omega}{10^3}\right)$$

$$-180^\circ = -3\tan^{-1}\frac{\omega_{180}}{10^3}$$

$$\Rightarrow \omega_{180} = 10^3 \tan 60 = 1732 \text{ rad/s}$$

$$|A(j\omega_{180})\beta(j\omega_{180})| = \frac{10^4 k}{\left(1 + \dfrac{1732^2}{10^6}\right)^{3/2}}$$

$$= 0.125 \times 10^4 k$$

For stable operation,

$$|A(j\omega_{180})\beta(j\omega_{180})| < 1$$

$$\Rightarrow 0.125 \times 10^4 k < 1$$

$$k < 8 \times 10^{-4}$$

11.86 $A(s) = \dfrac{10^4}{\left(1 + \dfrac{s}{10^4}\right)\left(1 + \dfrac{s}{10^5}\right)^2}$

$$A(j\omega) = \frac{10^4}{\left(1 + j\dfrac{\omega}{10^4}\right)\left(1 + j\dfrac{\omega}{10^5}\right)^2}$$

$$\phi = -\tan^{-1}\left(\frac{\omega}{10^4}\right) - 2\tan^{-1}\left(\frac{\omega}{10^5}\right)$$

$$-180 = -\tan^{-1}\left(\frac{\omega_{180}}{10^4}\right) - 2\tan^{-1}\left(\frac{\omega_{180}}{10^5}\right)$$

By trial and error we find

$\omega_{180} = 1.095 \times 10^5$ rad/s

At this frequency,

$$|A| = \frac{10^4}{\sqrt{1 + 10.95^2}\sqrt{1 + 1.095^2}}$$

$= 413.6$

For stable operation,

$|A|\beta_{cr} < 1$

$\beta_{cr} < 2.42 \times 10^{-3}$

Thus, oscillation will commence for

$\beta \geq 2.42 \times 10^{-3}$

11.87 $A_f(0) = \dfrac{A_0}{1 + A_0\beta}$

where

$$A_0 = \frac{1 \text{ MHz}}{10 \text{ Hz}} = 10^5 \text{ V/V}$$

Thus,

$$A_f(0) = \frac{10^5}{1 + 10^5 \times 0.1} \simeq 10 \text{ V/V}$$

$f_{3dB} = 10(1 + A_0\beta)$

$= 10(1 + 10^5 \times 0.1) \simeq 10^5$ Hz

Unity-gain frequency of closed-loop amplifier

$= A_f(0) \times f_{3dB}$

$= 10 \times 10^5 = 10^6 \text{ Hz} = 1$ MHz

Thus, the pole shifts by a factor equal to the amount-of-feedback, $(1 + A_0\beta)$.

11.88 $A_0 = 10$ V/V

$f_P = 100$ Hz

$f_{Hf} = f_P(1 + A_0\beta) = 10$ kHz

$$\Rightarrow 1 + A_0\beta = \frac{10 \times 10^3}{100} = 100$$

$$\Rightarrow \beta = \frac{99}{10^4} = 0.0099 \text{ V/V}$$

$$A_f(0) = \frac{A_0}{1 + A_0\beta}$$

$$= \frac{10^4}{100} = 100 \text{ V/V}$$

$$A_f(s) = \frac{A_f(0)}{1 + \dfrac{s}{\omega_{Hf}}}$$

$$A_f(s) = \frac{100}{1 + s/2\pi \times 10^4}$$

11.89 (a) The closed-loop poles become coincident when $Q = 0.5$. Using Eq. (11.70), we obtain

$$Q = \frac{\sqrt{(1 + A_0\beta)\omega_{P1}\omega_{P2}}}{\omega_{P1} + \omega_{P2}}$$

$$0.5 = \frac{\sqrt{(1 + A_0\beta)\omega_{P1}\omega_{P2}}}{\omega_{P1} + \omega_{P2}}$$

$$\Rightarrow 1 + A_0\beta = 0.5^2 \frac{(\omega_{P1} + \omega_{P2})^2}{\omega_{P1}\omega_{P2}}$$

$$= 0.5^2 \times \frac{(2\pi)^2(10^4 + 10^5)^2}{(2\pi)^2 \times 10^4 \times 10^5}$$

$$= 0.5^2 \times \frac{11^2}{10} = 3.025$$

$\beta = 2.025 \times 10^{-4}$

$$\omega_c = \frac{1}{2}(\omega_{P1} + \omega_{P2})$$

$$= \frac{1}{2} \times 2\pi(f_{P1} + f_{P2})$$

$$f_c = \frac{1}{2} \times (10^4 + 10^5) = 5.5 \times 10^4 \text{ Hz}$$

(b) $A_f(0) = \dfrac{A_0}{1 + A_0\beta}$

$$= \frac{10^4}{1 + 2.205 \times 10^{-4} \times 10^4} = 3306 \text{ V/V}$$

$$A_f(s) = \frac{A(s)}{1 + A(s)\beta}$$

where

$$A(s) = \frac{A_0}{\left(1 + \dfrac{s}{\omega_{P1}}\right)\left(1 + \dfrac{s}{\omega_{P2}}\right)}$$

$$A_f(s) = \frac{A_0}{\left(1 + \dfrac{s}{\omega_{P1}}\right)\left(1 + \dfrac{s}{\omega_{P2}}\right) + A_0\beta}$$

$$= \frac{A_0}{(1 + A_0\beta) + s\left(\dfrac{1}{\omega_{P1}} + \dfrac{1}{\omega_{P2}}\right) + \dfrac{s^2}{\omega_{P1}\omega_{P2}}}$$

$A_f(j\omega) =$

$$\frac{A_0}{(1 + A_0\beta) + j\left(\dfrac{\omega}{\omega_{P1}} + \dfrac{\omega}{\omega_{P2}}\right) - \left(\dfrac{\omega}{\omega_{P1}}\right)\left(\dfrac{\omega}{\omega_{P2}}\right)}$$

$$A_f(j\omega_c) = \frac{10^4}{3.025 + j(5.5 + 0.55) - 5.5 \times 0.55}$$

$$A_f(j\omega_c) = \frac{10^4}{j\,6.05}$$

$$|A_f|(j\omega_c) = \frac{10^4}{6.05} = 1653 \text{ V/V}$$

(c) $Q = 0.5$.

(d) If $\beta = 2.025 \times 10^{-3}$ V/V. Using Eq. (11.68), we obtain

$$s = -\frac{1}{2} \times 2\pi(10^4 + 10^5)$$

$$\pm\frac{1}{2} \times$$

$$2\pi\sqrt{(10^4 + 10^5)^2 - 4(1 + 10^4 \times 2.025 \times 10^{-3}) \times 10^4 \times 10^5}$$

$$\frac{s}{2\pi} = -5.5 \times 10^4$$

$$\pm 0.5\sqrt{121 \times 10^8 - 4 \times 21.25 \times 10^9}$$

$$= -5.5 \times 10^4 \pm 0.5 \times 10^4\sqrt{121 - 40 \times 21.25}$$

$$= -5.5 \times 10^4 \pm j0.5 \times 10^4 \times 27$$

$$= (-5.5 \pm j13.25) \times 10^4 \text{ Hz}$$

Using Eq. (11.70), we obtain

$$Q = \frac{\sqrt{(1 + 10^4 \times 2.025 \times 10^{-3})10^4 \times 10^5}}{10^4 + 10^5}$$

$$= 1.325$$

11.90 $A_f(0) = \dfrac{A_0}{1 + A_0\beta}$

$$10 = \frac{1000}{1 + 1000\beta}$$

$$\Rightarrow \beta = 0.099$$

To obtain a maximally flat response,

$$Q = 0.707$$

Using Eq. (11.70), we obtain

$$0.707 = \frac{\sqrt{100 \times 1 \times f_{P2}}}{1 + f_{P2}}$$

$$\frac{1}{2} = \frac{100 f_{P2}}{(1 + f_{P2})^2}$$

$$f_{P2}^2 + 2 f_{P2} + 1 = 200 f_{P2}$$

$$f_{P2}^2 - 198 f_{P2} + 1 = 0$$

$$f_{P2} \simeq 198 \text{ kHz}$$

(the other solution is a very low frequency which obviously does not make physical sense).

The 3-dB frequency of the closed-loop amplifier is f_0, which can be obtained from Eq. (11.68) and the graphical construction of Fig. 11.32:

$$\frac{f_0}{2Q} = \frac{1}{2}(f_{P1} + f_{P2})$$

$$f_0 = \frac{1}{\sqrt{2}}(1 + 198) = 140.7 \text{ kHz}$$

11.91

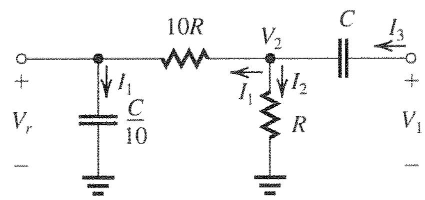


Figure 1

Figure 1 shows the feedback circuit modified according to the specifications in this problem. This circuit replaces that in the feedback path in Figs. 13.34 (a) and (b). Its transfer function $T(s)$,

$$T(s) = \frac{V_r}{V_1}$$

can be determined as indicated in Fig. 1. We start at the left-hand side where the voltage is V_r. The current through $(C/10)$ will be

$$I_1 = s\frac{C}{10}V_r$$

This is the same current that flows through $(10R)$; thus

$$V_2 = I_1 \times 10R + V_r$$

$$= s\frac{C}{10} \times 10RV_r + V_r$$

$$= V_r(1 + sCR)$$

The current I_2 through R can now be found as

$$I_2 = \frac{V_2}{R} = \frac{V_r}{R}(1 + sCR)$$

The input current I_3 is the sum of I and I_2:

$$I_3 = s\frac{C}{10}V_r + \frac{V_r}{R}(1 + sCR)$$

$$I_3 = \frac{V_r}{R}\left(1 + sCR + s\frac{C}{10}R\right)$$

$$= \frac{V_r}{R}(1 + 1.1sCR)$$

The input voltage V_1 can now be found as

$$V_1 = V_2 + \frac{1}{sC}I_3$$

$$= V_r(1 + sCR) + V_r\frac{1}{sCR}(1 + 1.1\, sCR)$$

Finally, $T(s)$ can be obtained as

$$T(s) \equiv \frac{V_r}{V_1} = \frac{s(1/CR)}{s^2 + s\dfrac{2.1}{CR} + \left(\dfrac{1}{CR}\right)^2}$$

Thus,

$$L(s) = \frac{-s(K/CR)}{s^2 + s(2.1/CR) + (1/CR)^2}$$

The characteristic equation is

$1 + L(s) = 0$

that is,

$$s^2 + s\frac{2.1}{CR} + \left(\frac{1}{CR}\right)^2 - s\frac{K}{CR} = 0$$

$$s^2 + s\frac{2.1 - K}{CR} + \left(\frac{1}{CR}\right)^2 = 0$$

Thus,

$$\omega_0 = \frac{1}{CR}$$

$$Q = \frac{1}{2.1 - K}$$

For the poles to coincide, $Q = 0.5$, thus

$$0.5 = \frac{1}{2.1 - K}$$

$\Rightarrow K = 0.1$

For the response to become maximally flat, $Q = 0.707$:

$$0.707 = \frac{1}{2.1 - K}$$

$\Rightarrow K = 0.686$

The circuit oscillates for $K = 2.1$.

11.92 $A_f = 10$

Maximally flat response with $f_{3dB} = f_0 = 1$ kHz.

Let each stage in the cascade have a dc gain K and a 3-dB frequency f_P, thus

$$A(s) = \frac{K^2}{\left(1 + \frac{s}{\omega_P}\right)^2}$$

Using the expression for Q in Eq. (11.70), we get

$$Q = \frac{\sqrt{(1 + A_0\beta)\omega_{P1}\omega_{P2}}}{\omega_{P1} + \omega_{P2}}$$

and substituting $Q = 1/\sqrt{2}$, $\omega_{P1} = \omega_{P2} = \omega_P$, and $A_0 = K^2$, we obtain

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{1 + K^2\beta}}{2}$$

$1 + K^2\beta = 2$

and

$K^2\beta = 1$

Now,

$$A_{f0} = \frac{A_0}{1 + A_0\beta}$$

$$10 = \frac{K^2}{1 + K^2\beta}$$

$$= \frac{K^2}{2}$$

$\Rightarrow K^2 = 20 \Rightarrow K = \sqrt{20} = 4.47$ V/V

$$\beta = \frac{1}{20} = 0.05 \text{ V/V}$$

Using Eq. (11.68), we obtain

$$\frac{\omega_0}{Q} = \frac{1}{2}(\omega_{P1} + \omega_{P2})$$

$$\frac{f_0}{1/\sqrt{2}} = \frac{1}{2} \times 2f_P$$

$\Rightarrow f_P = \sqrt{2}f_0 = \sqrt{2} \times 1 = 1.414$ kHz

$$A_f(s) = \frac{10\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$A_f(s) = \frac{10(2\pi \times 10^3)^2}{s^2 + s\frac{2\pi \times 10^3}{1/\sqrt{2}} + (2\pi \times 10^3)^2}$$

11.93 Let each stage have the transfer function

$$T(s) = \frac{K}{1 + \frac{s}{\omega_P}}$$

where $\omega_P = 2\pi \times 100 \times 10^3$ rad/s

$$A(s) = \left(\frac{K}{1 + \frac{s}{\omega_P}}\right)^3$$

$\beta = 1$

Thus the characteristic equation is given by

$1 + A(s)\beta = 0$

$$1 + \frac{K^3}{\left(1 + \frac{s}{\omega_P}\right)^3} = 0$$

To simplify matters, let $\frac{s}{\omega_P} = S$, where S is a normalized frequency variable, thus

$$(1 + S)^3 + K^3 = 0 \qquad (1)$$

This equation has three roots, which are the poles of the feedback amplifier. One of the roots will be real and the other two can be complex conjugate depending on the value of K. The real pole can be directly obtained from Eq. (1) as

$(1 + S_1)^3 = -K^3$

$(1 + S_1) = -K$

$$S_1 = -(1 + K) \qquad (2)$$

Now we need to obtain the two other poles. The characteristic equation in (1) can be written as

$$S^3 + 3\,S^2 + 3\,S + (1 + K^3) = 0 \qquad (3)$$

Equivalently it can be written as

$$(S + 1 + K)(S^2 + aS + b) = 0 \qquad (4)$$

Equating the coefficients of corresponding terms in (3) and (4), we can find a and b and thus obtain the quadratic factor

$$S^2 + (2 - K)S + (1 - K + K^2) = 0 \qquad (5)$$

This equation can now be easily solved to obtain the pair of complex conjugate poles as

$$S_{2,3} = -1 + \frac{K}{2} \pm j\frac{\sqrt{3}}{2}K$$

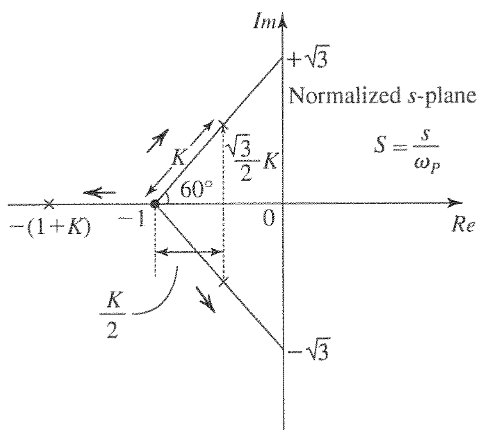


Figure 1

Figure 1 shows the root locus of the three poles in the normalized s plane (normalized relative to $\omega_P = 2\pi \times 10^5$). Observe that as K increases the real pole S_1 moves outwardly on the negative real axis. The pair of conjugate poles move on straight lines with 60° angles to the horizontal. These two poles reach the $j\omega$ axis (which is the boundary for stable operation) at $K = 2$ at which point

$$S_{2,3} = \pm j\sqrt{3}$$

Thus the minimum value of K from which oscillations occur is $K = 2$. Oscillations will be at

$$\omega = \sqrt{3} \times 2\pi f_P$$

or

$$f = \sqrt{3} \times 100 \text{ kHz}$$

$$= 173.2 \text{ kHz}$$

11.94 $A_0 = 10^5$ with a single pole at

$$f_P = 10 \text{ Hz}$$

For a unity-gain buffer, $\beta = 1$, thus

$$A_0\beta = 10^5 \text{ and } f_P = 10 \text{ Hz}$$

Since the loop gain rolls off at a uniform slope of −20 dB/decade, it will reach the 0 dB line ($|A\beta| = 1$) five decades beyond 10 Hz. Thus the unity-gain frequency will be

$$f_1 = 10^5 \times 10 = 10^6 \text{ Hz} = 1 \text{ MHz}$$

The phase shift will be that resulting from a single pole, −90°, resulting in a phase margin:

Phase margin = 180° − 90° = 90°

11.95

$$A(s) = \frac{10^5}{\left(1 + \dfrac{s}{2\pi \times 10}\right)\left(1 + \dfrac{s}{2\pi \times 10^3}\right)}$$

$$\beta = 0.01$$

$$A\beta(j\omega) = \frac{1000}{\left(1 + j\dfrac{\omega}{2\pi \times 10}\right)\left(1 + j\dfrac{\omega}{2\pi \times 10^3}\right)} \qquad (1)$$

$$|A\beta| = \frac{1000}{\sqrt{\left[1 + \left(\dfrac{\omega}{2\pi \times 10}\right)^2\right]\left[1 + \left(\dfrac{\omega}{2\pi \times 10^3}\right)^2\right]}} \qquad (2)$$

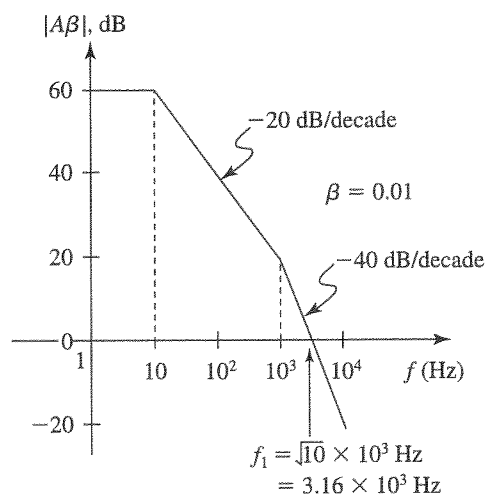


Figure 1

Figure 1 shows a sketch of the Bode plot for $|A\beta|$. Observe that the unity-gain frequency will occur on the −40 dB/decade line. The value of f_1 can be obtained from the Bode plot as follows: The −40 dB/decade line represents gain drop proportional to $10^3/f^2$. For a drop by only 20 dB (a factor of 10) the change in frequency is

$$\frac{10^3}{f_1^2} = \frac{1}{10}$$

$$f_1 = \sqrt{10} \times 10^3 \text{ Hz} = 3.16 \times 10^3 \text{ Hz}$$

However, the Bode plot results are usually approximate. If we require a more exact value for

f_1 we need to iterate a couple of times using the exact equation in (2). The result is

$f_1 = 3.085 \times 10^3$ Hz

The phase angle can now be obtained using (1) as follows:

$$\phi = -\tan^{-1}\frac{\omega_1}{2\pi \times 10} - \tan^{-1}\frac{\omega_1}{2\pi \times 10^3}$$

$$= -\tan^{-1}\frac{f_1}{10} - \tan^{-1}\frac{f_1}{10^3}$$

$$= -\tan^{-1}(3.085 \times 10^2) - \tan^{-1}(3.085)$$

$$= -89.81^\circ - 72.03^\circ = -161.84$$

Thus,

Phase margin $= 180^\circ - 161.84 = 18.15^\circ$

To obtain a phase margin of 45°:

The phase shift due to the first pole will be $\simeq 90^\circ$. Thus, the phase shift due to the second pole must be $\simeq -45^\circ$, thus

$$-45^\circ = -\tan^{-1}\frac{f_1}{10^3}$$

$f_1 \simeq 10^3$ rad/s.

Since f_1 is two decades above f_P we need $A_0\beta$ to be 100. Thus, β will now be

$\beta = 100/10^5 = 10^{-3}$

Figure 2 shows a sketch of the Bode plot for $|A\beta|$ in this case.

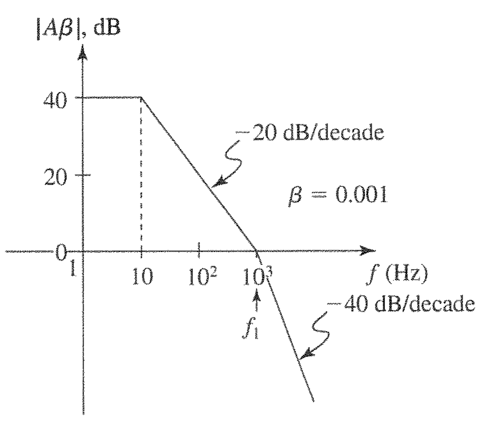


Figure 2

11.96 Using Eq. (11.82), we obtain

$$|A_f(j\omega_1)| = \frac{1/\beta}{|1 + e^{-j\theta}|}$$

where

$\theta = 180^\circ - \phi$

$\phi \equiv$ Phase margin

$$\text{Peaking, } P \equiv \frac{|A_f(j\omega)|}{1/\beta}$$

Thus,

$$P = 1/|1 + e^{-j\theta}|$$

$$= 1/|1 + \cos\theta - j\sin\theta|$$

$$= 1/\sqrt{(1+\cos\theta)^2 + \sin^2\theta}$$

$$= 1/\sqrt{2 + 2\cos\theta}$$

$$= 1/\sqrt{2 + 2\cos(180^\circ - \phi)}$$

$$P = 1/\sqrt{2(1 - \cos\phi)}$$

$$\Rightarrow \phi = \cos^{-1}\left(1 - \frac{1}{2P^2}\right)$$

We use this equation to obtain the following results:

P	ϕ
1.05	56.9°
1.10	54.1°
0.1 dB ≡ 1.0115	59.2°
1.0 dB ≡ 1.122	52.9°
3 dB ≡ 1.414	41.4°

11.97 Figure 1 on the next page shows magnitude and phase Bode plots for the amplifier specified in this problem. From the phase plot we find that $\theta = -135^\circ$ (which corresponds to a phase margin of 45°) occurs at

$f = 3.16 \times 10^5$ Hz

At this frequency, $|A|$ is 70 dB. The β horizontal straight line drawn at 70-dB level gives

$$20\log\left(\frac{1}{\beta}\right) = 70 \text{ dB}$$

$$\Rightarrow \beta = 3.16 \times 10^{-4}$$

Correspondingly,

$$A_f = \frac{10^4}{1 + 10^4 \times 3.16 \times 10^{-4}} = 2.4 \times 10^3 \text{ V/V}$$

or 67.6 dB

11.98 Figure 1 on the next page shows the Bode plot for the amplifier gain and for a differentiator. Observe that following the rate-of-closure rule the intersection of the two graphs is arranged so that the maximum difference in slopes is 20 dB/decade.

This figure belongs to Problem 11.97.

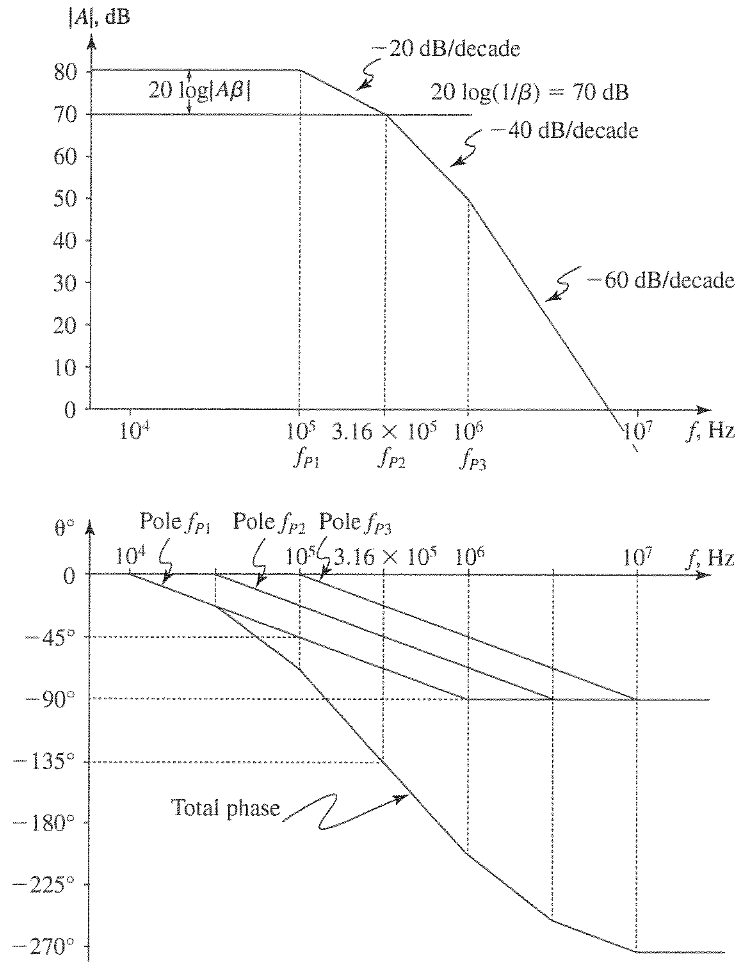


Figure 1

This figure belongs to Problem 11.98.

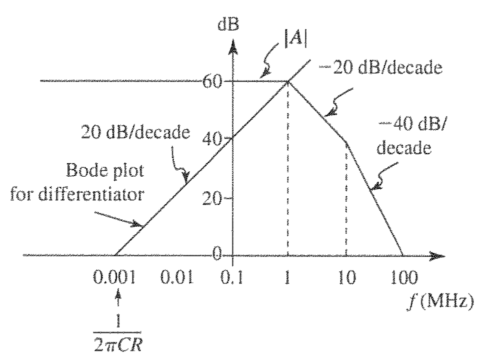


Figure 1

Thus, to guarantee stability,

$$\frac{1}{2\pi CR} \leq 0.001 \text{ MHz or } 1 \text{ kHz}$$

$$\Rightarrow CR \geq \frac{1}{2\pi \times 10^3} = 0.159 \text{ ms}$$

11.99 Figure 1 is a replica of Fig. 11.37 except here we locate on the phase plot the points at which the phase margin is 90° and 135°, respectively. Drawing a vertical line from each of those points and locating the intersection with the $|A|$ line enables us to determine the maximum β that can be used in each case. Thus, for PM = 90°, we have

$$20 \log \frac{1}{\beta} = 90 \text{ dB}$$

$$\Rightarrow \beta = 3.16 \times 10^{-5}$$

and the corresponding closed-loop gain is

$$A_f = \frac{A_0}{1 + A_0\beta} = \frac{10^5}{1 + 10^5 \times 3.16 \times 10^{-5}}$$

$$= 2.4 \times 10^4 \text{ V/Vor } 87.6 \text{ dB}$$

and for PM = 45°, we have

$$20 \log \frac{1}{\beta} = 80 \text{ dB}$$

$$\Rightarrow \beta = 10^{-4} \text{ V/V}$$

This figure belongs to Problem 11.99.

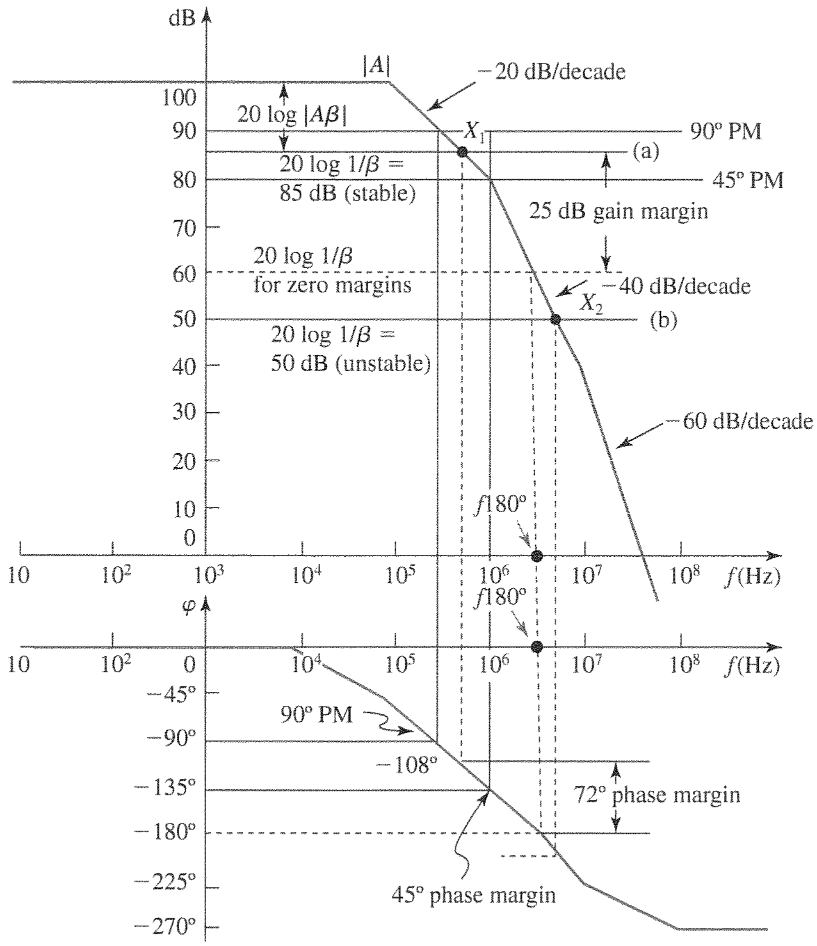


Figure 1

and the corresponding closed-loop gain is

$$A_f = \frac{A_0}{1 + A_0\beta} = \frac{10^5}{1 + 10^5 \times 10^{-4}}$$

$= 9.09 \times 10^3$ V/V

or 79.2 dB

11.100 The new pole must be placed at

$$f_D = \frac{1 \text{ MHz}}{10^4} = 100 \text{ Hz}$$

In this way the modified open-loop gain will decrease at the uniform rate of −20 dB/decade, thus reaching 0 dB in four decades, that is at 1 MHz where the original pole exists. At 1 MHz, the slope changes to −40 dB/decade, but our amplifier will be guaranteed to be stable with a closed-loop gain at low as unity.

11.101 We must move the 1 MHz pole to a new location,

$$f_D = \frac{20 \text{ MHz}}{10^4} = 2 \text{ kHz}$$

This reduction in frequency by a factor of $\frac{1 \text{ MHz}}{2 \text{ kHz}} = 500$ will require that the total capacitance at the controlling node must become 500 times what it originally was.

11.102 Refer to Fig. 11.38.

(a) For $\beta = 0.001$,

$$20 \log \frac{1}{\beta} = 60 \text{ dB}$$

A horizontal line at the 60-dB level will intersect the vertical line at $f_{P2} = 10^6$ Hz at a point Z_1. Drawing a line with a slope of −20 dB/decade from Z_1 will intersect the 100-dB horizontal line at a frequency two decade lower than f_{P2}, thus the frequency to which the 1st pole must be moved is

$$f'_D = \frac{f_{P2}}{100} = \frac{10^6}{100} = 10 \text{ kHz}$$

(b) For $\beta = 0.1$,

$$20 \log \frac{1}{\beta} = 20 \text{ dB}$$

Following a process similar to that for (a) above, the first pole must be lowered to

$$f'_D = \frac{10^6}{10^4} = 100 \text{ kHz}$$

11.103 $R_1 = R_2 = R$

$C_1 = 10C$

$C_2 = C$

$C_f \gg C$

$g_{m1}R = 100$

$$\omega_{P1} = \frac{1}{10CR} = \frac{0.1}{CR} \quad (1)$$

$$\omega_{P2} = \frac{1}{CR} \quad (2)$$

$$\omega'_{P1} = \frac{1}{g_m R C_f R} = \frac{1}{100 C_f R}$$

Thus,

$$\omega'_{P1} = \frac{0.01}{C_f R} \quad (3)$$

$$\omega'_{P2} = \frac{g_m C_f}{C_1 C_2 + C_f(C_1 + C_2)}$$

$$= \frac{g_m C_f}{10C^2 + 11CC_f} = \frac{g_m C_f}{C(10C + 11C_f)}$$

Since $C_f \gg C$, we have

$$\omega'_{P2} \simeq \frac{g_m C_f}{11CC_f} = \frac{g_m}{11C}$$

Substituting $g_m = 100/R$, we obtain

$$\omega'_{P2} = \frac{100}{11CR} \simeq \frac{10}{CR} \quad (4)$$

Equations (1)–(4) provide a summary of pole splitting: The two initial poles with frequencies $\frac{0.1}{CR}$ and $\frac{1}{CR}$, a decade apart in frequency, are split further apart. The lower frequency pole is moved to a frequency $\frac{0.01}{C_f R}$ which is more than a decade lower (because $C_f \gg C$) and the higher frequency pole is moved to a frequency $\frac{10}{CR}$ which is a decade higher. This is further illustrated in Fig. 1.

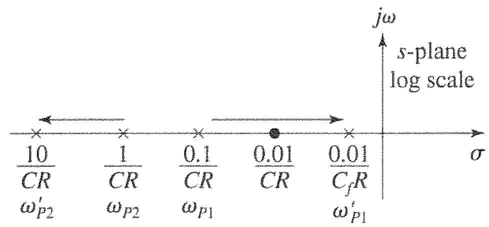


Figure 1

11.104 The fourth, dominant pole must be at

$$f_D = \frac{f_{P1}}{A_0}$$

$$= \frac{10^6}{10^5} = 10 \text{ Hz}$$

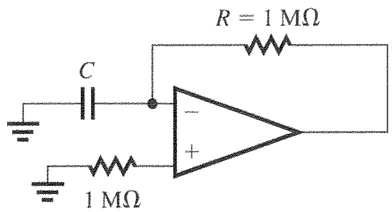


Figure 1

Refer to Fig. 1.

$$f_D = \frac{1}{2\pi RC}$$

$$10 = \frac{1}{2\pi \times 1 \times 10^6 \times C}$$

$\Rightarrow C = 15.9$ nF

11.105 $f_{P1} = \dfrac{1}{2\pi C_1 R_1}$

$$10^5 = \frac{1}{2\pi \times 150 \times 10^{-12} \times R_1}$$

$\Rightarrow R_1 = 10.61$ kΩ

$$f_{P2} = \frac{1}{2\pi C_2 R_2}$$

$$10^6 = \frac{1}{2\pi \times 5 \times 10^{-12} \times R_2}$$

$\Rightarrow R_2 = 31.83$ kΩ

First, we determine an approximate value of f'_{P2} from Eq. (11.94)

$$f'_{P2} = \frac{g_m C_f}{2\pi[C_1 C_2 + C_f(C_1 + C_2)]}$$

Assume that $C_f \gg C_2$, then

$$f'_{P2} \simeq \frac{g_m}{2\pi(C_1 + C_2)}$$

$$= \frac{40 \times 10^{-3}}{2\pi(150 + 5) \times 10^{-12}}$$

$= 41.1$ MHz

which is much greater than f_{P3}. Thus, we use f_{P3} to determine the new location of f_{P1},

$$f'_{P1} = \frac{2 \times 10^6}{10^4} = 200 \text{ Hz}$$

Using Eq. (11.93), we obtain

$$f'_{P1} = \frac{1}{2\pi g_m R_2 C_f R_1}$$

$$200 = \frac{1}{2\pi \times 40 \times 10^{-3} \times 31.83 \times 10^3 \times C_f \times 10.61 \times 10^3}$$

$$\Rightarrow C_f = 58.9 \text{ pF}$$

Since C_f is indeed much greater than C_2, the pole at the output will have the frequency already calculated:

$$f'_{P2} \simeq 41.1 \text{ MHz}$$

11.106

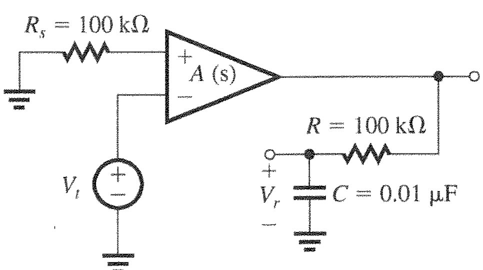


Figure 1

$$A(s)\beta(s) = -\frac{V_r}{V_t}$$

$$= A(s)\frac{1/sC}{R + 1/sC}$$

$$A(s)\beta(s) = \frac{10^5}{1 + \frac{s}{10}} \frac{1}{1 + sCR}$$

$$CR = 0.01 \times 10^{-6} \times 100 \times 10^3 = 10^{-3} \text{ s}$$

$$A(s)\beta(s) = \frac{10^5}{\left(1 + \frac{s}{10}\right)\left(1 + \frac{s}{10^3}\right)}$$

(a) Bode plots for the magnitude and phase of $A\beta$ are shown in Fig. 2. From the magnitude plot we find the frequency f_1 at which $|A\beta| = 1$ is

$$f_1 = 3.16 \times 10^4 \text{ Hz}$$

(b) From the phase plot we see that the phase at f_1 is 180° and thus the phase margin is zero. A more exact value for the phase margin can be obtained as follows:

$$\theta(f_1) = -\tan^{-1}\frac{3.16 \times 10^4}{10} - \tan^{-1}\frac{3.16 \times 10^4}{10^3}$$

$$= -89.98 - 88.19 = -178.2$$

Thus the phase margin is 1.8°.

(c) $A_f(s) = \dfrac{A(s)}{1 + A(s)\beta(s)}$

$$= \frac{10^5 / \left(1 + \frac{s}{10}\right)}{1 + \frac{10^5}{\left(1 + \frac{s}{10}\right)\left(1 + \frac{s}{10^3}\right)}}$$

This figure belongs to Problem 11.106, part (a).

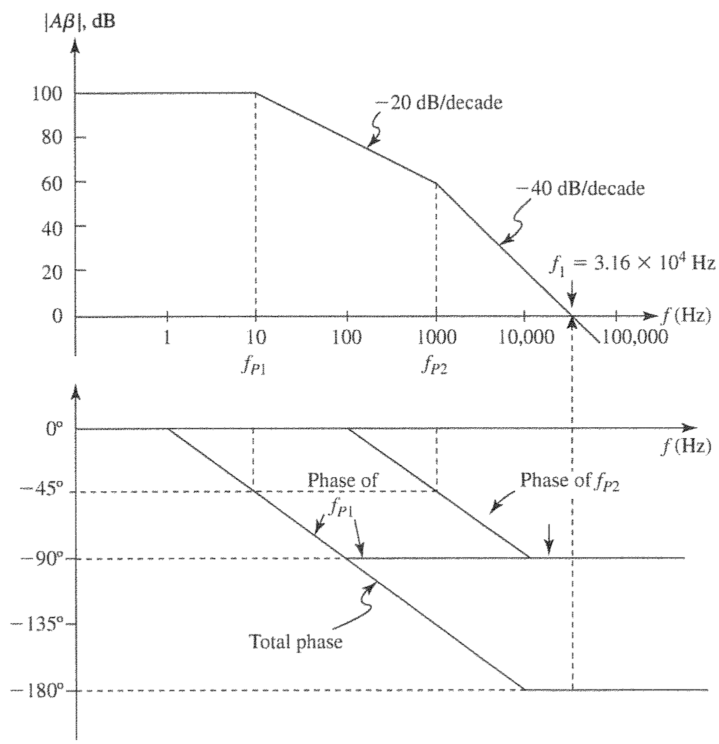


Figure 2

$$= \frac{10^5\left(1+\frac{s}{10^3}\right)}{10^5+\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10^3}\right)}$$

$$= \frac{\left(1+\frac{s}{10^3}\right)}{1+10^{-5}(1+0.101s+0.0001s^2)}$$

At $s = 0$,

$A_f \simeq 1$

The transmission zero is

$s_Z = -10^3$ rad/s

The poles are the roots of

$10^{-9}s^2 + 1.01 \times 10^{-6}s + 1 = 0$

which are

$s = (-0.505 \pm j31.62) \times 10^3$ rad/s

The poles and zero are shown in Fig. 3.

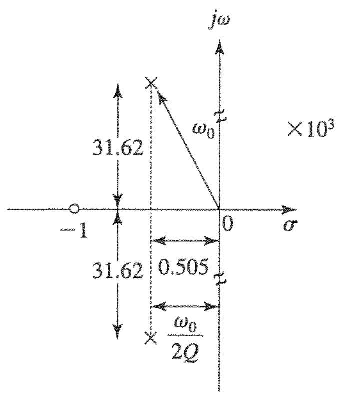


Figure 3

The pair of complex-conjugate poles have

$\omega_0 \simeq 31.62$ krad/s

$Q = 31.3$

Thus, the response is very peaky, as shown in Fig. 4.

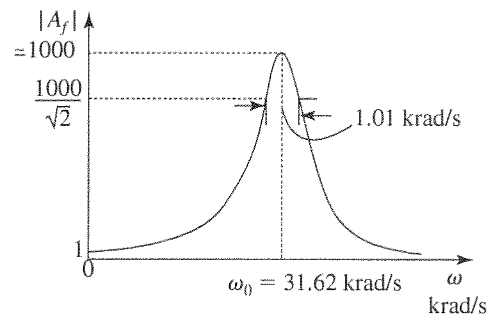


Figure 4

12.1 $I = \dfrac{0-(-V_{CC})-V_D}{R}$

$= \dfrac{10-0.7}{1} = 9.3$ mA

Upper limit on $v_O = V_{CC} - V_{CEsat}$

$= 10 - 0.3 = 9.7$ V

Corresponding input $= 9.7 + 0.7 = 10.4$ V

Lower limit on $v_O = -IR_L = -9.3 \times 1$

$= -9.3$ V

Corresponding input $= -9.3 + 0.7 = -8.6$ V

If the EBJ area of Q_3 is twice as large as that of Q_2, then

$I = \dfrac{1}{2} \times 9.3 = 4.65$ mA

There will be no change in $v_{O\max}$ and in the corresponding value of v_I. However, $v_{O\min}$ will now become

$v_{O\min} = -IR_L$

$= -4.65 \times 1 = -4.65$ V

and the corresponding value of v_I will be

$v_I = -4.65 + 0.7 = -3.95$ V

If the EBJ area of Q_3 is made half as big as that of Q_2, then

$I = 4 \times 9.3 = 18.6$ mA

There will be no change in $v_{O\max}$ and in the corresponding value of v_I. However, $v_{O\min}$ will now become

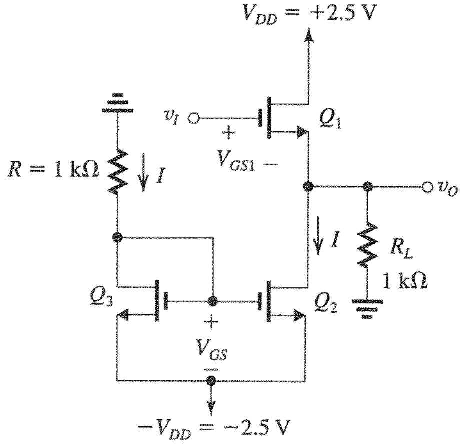
$v_{O\min} = -V_{CC} + V_{CEsat}$

$= -10 + 0.3 = -9.7$ V

and the corresponding value of v_I will be

$v_I = -9.7 + 0.7 = -9$ V

12.2 First we determine the bias current I as follows:



$I = \dfrac{1}{2}\mu_n C_{ox}\left(\dfrac{W}{L}\right)(V_{GS} - V_t)^2$

But

$V_{GS} = 2.5 - IR = 2.5 - I$

Thus

$I = \dfrac{1}{2}\mu_n C_{ox}\left(\dfrac{W}{L}\right)(2.5 - I - V_t)^2$

$= \dfrac{1}{2} \times 20(2.5 - I - 0.5)^2$

$I = 10(2 - I)^2$

$\Rightarrow I^2 - 4.1I + 4 = 0$

$I = 1.6$ mA and $V_{GS} = 0.9$ V

The upper limit on v_O is determined by Q_1 leaving the saturation region (and entering the triode region). This occurs when v_I exceeds V_{DD} by V_t volts:

$v_{I\max} = 2.5 + 0.5 = 3$ V

To obtain the corresponding value of v_O, we must find the corresponding value of V_{GS1}, as follows:

$v_O = v_I - V_{GS1}$

$i_L = \dfrac{v_O}{R_L} = \dfrac{v_I - V_{GS1}}{R_L} = \dfrac{v_I - V_{GS1}}{1}$

$i_L = 3 - V_{GS1}$

$i_{D1} = i_L + I = 3 - V_{GS1} + 1.6$

$= 4.6 - V_{GS1}$

But,

$i_{D1} = \dfrac{1}{2}\mu_n C_{ox}\left(\dfrac{W}{L}\right)(V_{GS1} - V_t)^2$

$4.6 - V_{GS1} = \dfrac{1}{2} \times 20(V_{GS1} - 0.5)^2$

$\Rightarrow V_{GS1}^2 - 0.9V_{GS1} - 0.21 = 0$

$V_{GS1} = 1.09$ V

$v_{O\max} = v_{I\max} - V_{GS1}$

$= 3 - 1.09 = +1.91$ V

The lower limit of v_O is determined either by Q_1 cutting off,

$v_O = -IR_L = -1.6 \times 1 = -1.6$ V

or by Q_2 leaving saturation,

$v_O = -V_{DD} + V_{OV2}$

where

$V_{OV2} = V_{GS2} - V_t = 0.9 - 0.5 = 0.4$ V

Thus,

$v_O = -2.5 + 0.4 = -2.1$ V

We observe that Q_1 will cut off before Q_2 leaves saturation, thus

$v_{O\min} = -1.6$ V

and the corresponding value of v_I will be

$v_{I\min} = v_{O\min} + V_t$

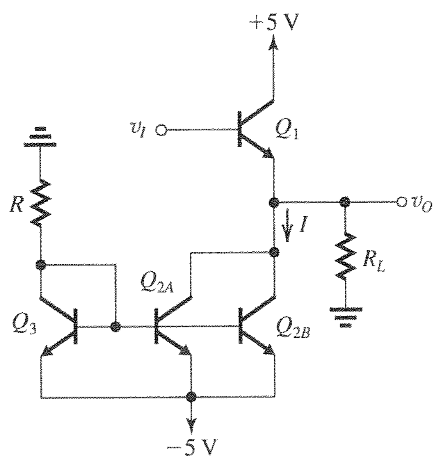
$= -1.6 + 0.5 = -1.1$ V

12.3 Refer to Fig. 12.2. With $V_{CC} = +5$ V, the upper limit on v_o is 4.7 V, which is greater than the required value of +3 V. To obtain a lower limit of −3 V, we select I so that

$IR_L = 3$

$\Rightarrow I = 3$ mA

Since we are provided with four devices, we can minimize the total supply current by paralleling two devices to form Q_2 as shown below.



The resulting supply current will be $3 \times \frac{I}{2}$ rather than $2I$ which is the value obtained in the circuit of Fig. 12.2. Then the supply current is 4.5 mA. The value of R is found from

$$R = \frac{4.3\text{ V}}{1.5\text{ mA}} = 2.87\text{ k}\Omega$$

In a practical design we would select a standard value for R that results in I somewhat larger than 3 mA. Say, $R = 2.7$ kΩ. In this case $I = 3.2$ mA.

Power from negative supply $= 3 \times 1.6 \times 5$
$= 24$ mW.

12.4 Refer to Fig. 12.2. For a load resistance of 100 Ω and v_O ranging between −5 V and +5 V, the maximum current through Q_1 is

$I + \frac{5}{0.1} = I + 50$, mA and the minimum current is $I - \frac{5}{0.1} = I - 50$, mA.

For a current ratio of 15, we have

$$\frac{I+50}{I-50} = 15$$

$\Rightarrow I = 57.1$ mA

$$R = \frac{9.3\text{ V}}{57.1\text{ mA}} = 163\ \Omega$$

The incremental voltage gain is $A_v = \frac{R_L}{R_L + r_{e1}}$

For $R_L = 100\ \Omega$;

At $v_O = +5$ V, $i_{E1} = 57.1 + 50 = 107.1$ mA

$$r_{e1} = \frac{25}{107.1} = 0.233\ \Omega$$

$$A_v = \frac{100}{100 + 0.233} = 0.998\text{ V/V}$$

At $v_O = 0$ V, $i_{E1} = 57.1$ mA

$$r_{e1} = \frac{25}{57.1} = 0.438\ \Omega$$

$$A_v = \frac{100}{100.438} = 0.996\text{ V/V}$$

At $v_O = -5$ V, $i_{E1} = 57.1 - 50 = 7.1$ mA

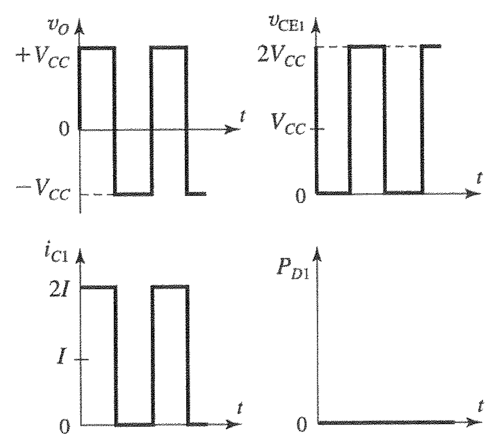
$$r_{e1} = \frac{25}{7.1} = 3.52\ \Omega$$

$$A_v = \frac{100}{103.52} = 0.966\text{ V/V}$$

Thus the incremental gain changes by $0.998 - 0.966 = 0.032$ or about 3% over the range of v_O.

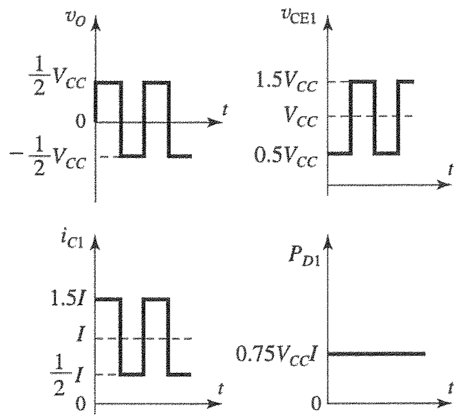
12.5 Refer to Figs. 12.2 and 12.4.

For v_O being a square wave of $\pm V_{CC}$ levels:



$P_{D1}|_{\text{average}} = 0$. For the corresponding sine wave curve [Fig. 12.4], we have $P_{D1}|_{\text{avg}} = \frac{1}{2}V_{CC}I$.

For v_O being a square wave of $\pm V_{CC}/2$ levels:



$P_{D1}|_{\text{average}} = 0.75V_{CC}I$

For a sine-wave output of $V_{CC}/_2$ peak amplitude:

$$v_O = \frac{1}{2}\,V_{CC}\sin\theta$$

$$i_{C1} = I + \frac{\frac{1}{2}V_{CC}}{R_L}\sin\theta = I + \frac{1}{2}I\sin\theta$$

$$v_{CE1} = V_{CC} - \frac{1}{2}V_{CC}\sin\theta$$

$$P_{D1} = \left(V_{CC} - \frac{1}{2}V_{CC}\sin\theta\right)\left(I + \frac{1}{2}I\sin\theta\right)$$

$$= V_{CC}I - \frac{1}{4}V_{CC}I\sin^2\theta$$

$$= V_{CC}I - \frac{1}{4}V_{CC}I \times \frac{1}{2}(1-\cos 2\theta)$$

$$= \frac{7}{8}V_{CC}I + \frac{1}{8}V_{CC}I\cos 2\theta$$

$$P_{D1}|_{\text{average}} = \frac{7}{8}V_{CC}I$$

12.6 In all cases, the average voltage across Q_2 is equal to V_{CC}. Thus, since Q_2 conducts a constant current I, its average power dissipation is $V_{CC}I$.

12.7 V_{CC} =16, 12, 10, and 8 V

$I = 100$ mA, $R_L = 100\ \Omega$

$\hat{V}_o = 8$ V

$$\eta = \frac{1}{4}\left(\frac{\hat{V}_o}{IR_L}\right)\left(\frac{\hat{V}_o}{V_{CC}}\right)$$

$$= \frac{1}{4}\left(\frac{8}{10}\right)\left(\frac{8}{V_{CC}}\right) = \frac{1.6}{V_{CC}}$$

V_{CC}	16	12	10	8
η	10%	13.3%	16%	20%

12.8 The minimum required value of V_{CC} is

$$V_{CC} = \hat{V}$$

and the minimum required value of I is

$$I = \frac{\hat{V}}{R_L}$$

From Eq. (12.10),

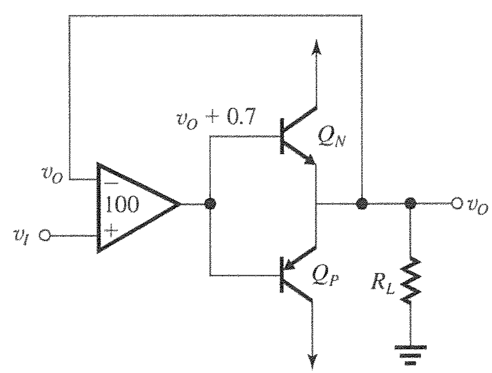
$$\eta = \frac{1}{4}\left(\frac{\hat{V}_o}{IR_L}\right)\left(\frac{\hat{V}_o}{V_{CC}}\right)$$

$$= \frac{1}{4}\left(\frac{\hat{V}}{\hat{V}}\right)\left(\frac{\hat{V}}{\hat{V}}\right) = 0.25$$

or 25%

12.9 Refer to Figs. 12.6 and 12.7. A 10% loss in peak amplitude is obtained when the amplitude of the input signal is 5 V.

12.10



With v_I sufficiently positive so that Q_N is conducting, the situation shown obtains. Then,

$$(v_I - v_O) \times 100 = v_O + 0.7$$

$$\Rightarrow v_O = \frac{1}{1.01}(v_I - 0.007)$$

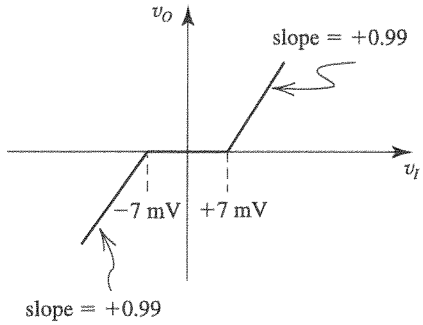
This relationship applies for $v_I \geq 0.007$. Similarly, for v_I sufficiently negative so that Q_P conducts, the voltage at the output of the amplifier becomes $v_O - 0.7$, thus

$$(v_I - v_O) \times 100 = v_O - 0.7$$

$$\Rightarrow v_O = \frac{1}{1.01}(v_I + 0.007)$$

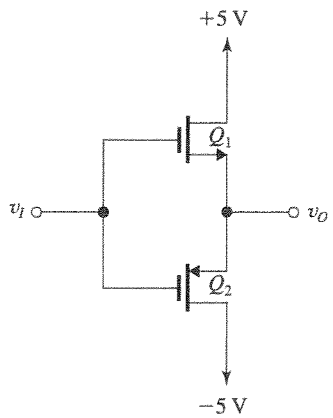
This relationship applies for $v_I \leq -0.007$.

The result is the transfer characteristic



Without the feedback arrangement, the deadband becomes ±700 mV and the slope change a little (to nearly +1 V/V).

12.11



Devices have $|V_t| = 0.5$ V

$\mu C_{ox}\dfrac{W}{L} = 2\text{ mA/V}^2$

For $R_L = \infty$, the current is normally zero, so

$V_{GS} = V_t$

$\therefore\ v_O = v_I - V_{GS1} = 5 - 0.5 = 4.5$ V

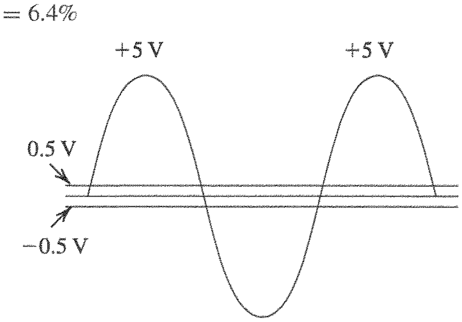
The peak output voltage will be 4.5 V

$\sin\theta = \dfrac{0.5}{5} \Rightarrow \theta = 5.74°$

Crossover interval $= 4\theta = 22.968$

$= \dfrac{22.96}{360} \times 100$

$= 6.4\%$

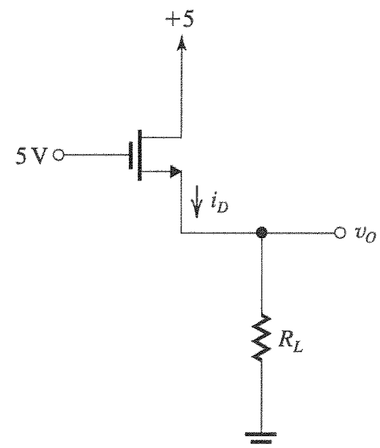


For $v_I = 5$ V, $v_O = 2.5$ V:

$\therefore\ V_{GS} = 5 - 2.5 = 2.5$ V

$i_D = \dfrac{1}{2}\mu C_{ox}\dfrac{W}{L}(V_{GS} - V_t)^2$

$= \dfrac{1}{2} \times 2 \times (2.5 - 0.5)^2$



$i_D = 4$ mA and $R_L = \dfrac{2.5\text{ V}}{4\text{ mA}} = 625\ \Omega$

12.12 For $V_{CC} = 10$ V and $R_L = 8\ \Omega$, the maximum sine-wave output power occurs when $\hat{V}_o = V_{CC}$ and is $P_{L\max} = \dfrac{1}{2}\dfrac{V_{CC}^2}{R_L}$

$= \dfrac{1}{2} \times \dfrac{100}{8} = 6.25$ W

Correspondingly,

$P_{S+} = P_{S-} = \dfrac{1}{\pi}\dfrac{\hat{V}_o}{R_L}V_{CC}$

$= \dfrac{1}{\pi} \times \dfrac{10}{8} \times 10 = 3.98$ W

for a total supply power of

$P_S = 2 \times 3.98 = 7.96$ W

The power conversion efficiency η is

$\eta = \dfrac{P_L}{P_S} \times 100 = \dfrac{6.25}{7.96} \times 100 = 78.5\%$

For $\hat{V}_o = 5$ V,

$P_L = \dfrac{1}{2}\dfrac{\hat{V}_o^2}{R_L} = \dfrac{1}{2} \times \dfrac{25}{8} = 1.56$ W

$P_{S+} = P_{S-} = \dfrac{1}{\pi}\dfrac{\hat{V}_o}{R_L}V_{CC}$

$= \dfrac{1}{\pi} \times \dfrac{5}{8} \times 10 = 2$ W

$P_S = 4$ W

$$\eta = \frac{1.56}{4} \times 100 = 39\%$$

Thus, the efficiency reduces to half its maximum value.

12.13 $V_{CC} = 10$ V

For maximum η,

$\hat{V}_o = V_{CC} = 10$ V

The output voltage that results in maximum device dissipation is given by Eq. (12.20),

$$\hat{V}_o = \frac{2}{\pi} V_{CC}$$

$$= \frac{2}{\pi} \times 10 = 6.37 \text{ V}$$

If operation is always at full output voltage, $\eta = 78.5\%$ and thus

$$P_{\text{dissipation}} = (1 - \eta) P_S$$

$$= (1 - \eta) \frac{P_L}{\eta} = \frac{1 - 0.785}{0.785} P_L = 0.274 P_L$$

$$P_{\text{dissipation/device}} = \frac{1}{2} \times 0.274 P_L = 0.137 P_L$$

For a rated device dissipation of 2 W, and using a factor of 2 safety margin,

$P_{\text{dissipation/device}} = 1$ W

$= 0.137 P_L$

$\Rightarrow P_L = 7.3$ W

$$7.3 = \frac{1}{2} \times \frac{100}{R_L}$$

$\Rightarrow R_L = 6.85\ \Omega$ (i.e. $R_L \geq 6.85\ \Omega$)

The corresponding output power (i.e., greatest possible output power) is 7.3 W.

If operation is allowed at $\hat{V}_o = \frac{1}{2} V_{CC} = 5$ V,

$$\eta = \frac{\pi}{4} \frac{\hat{V}_o}{V_{CC}} \text{ (Eq. 12.15)}$$

$$= \frac{\pi}{4} \times \frac{1}{2} = 0.393$$

$$P_{\text{dissipation/device}} = \frac{1}{2} \frac{1 - \eta}{\eta} P_L = 0.772 P_L$$

$1 = 0.772 P_L$

$\Rightarrow P_L = 1.3$ W

$$= \frac{1}{2} \frac{5^2}{R_L}$$

$\Rightarrow R_L = 9.62\ \Omega$ (i.e., $\geq 9.62\ \Omega$)

12.14 $P_L = \dfrac{1}{2} \dfrac{\hat{V}_o^2}{R_L}$

$$50 = \frac{1}{2} \frac{\hat{V}_o^2}{8}$$

$\Rightarrow \hat{V}_o = 28.3$ V

$V_{CC} = 28.3 + 4 = 32.3 \rightarrow 33$ V

Peak current from each supply $= \dfrac{\hat{V}_o}{R_L} = \dfrac{28.3}{8}$

$= 3.54$ A

$$P_{S+} = P_{S-} = \frac{1}{\pi} \times 3.54 \times 33 = 37.2 \text{ W}$$

Thus,

$P_S = 2 \times 37.2 = 74.4$ W

$$\eta = \frac{P_L}{P_S} = \frac{50}{74.4} = 67.2\%$$

Using Eq. (12.22), we obtain

$$P_{DN\max} = P_{DP\max} = \frac{V_{CC}^2}{\pi^2 R_L} = \frac{33^2}{\pi^2 \times 8} = 13.8 \text{ W}$$

12.15 $P_L = \dfrac{\hat{V}_o^2}{R_L}$

$$P_{S+} = P_{S-} = \frac{1}{2}\left(\frac{\hat{V}_o}{R_L}\right) V_{SS}$$

$$P_S = \frac{\hat{V}_o}{R_L} V_{SS}$$

$$\eta = \frac{P_L}{P_S} = \frac{\hat{V}_o^2 / R_L}{\hat{V}_o V_{SS} R_L} = \frac{\hat{V}_o}{V_{SS}}$$

$\eta_{\max} = 1(100\%)$, obtained for $\hat{V}_o = V_{SS}$

$$P_{L\max} = \frac{V_{SS}^2}{R_L}$$

$$P_{\text{dissipation}} = P_S - P_L$$

$$= \frac{\hat{V}_o}{R_L} V_{SS} - \frac{\hat{V}_o^2}{R_L}$$

$$\frac{\partial P_{\text{dissipation}}}{\partial \hat{V}_o} = \frac{V_{SS}}{R_L} - \frac{2\hat{V}_o}{R_L}$$

$$= 0 \text{ for } \hat{V}_o = \frac{V_{SS}}{2}$$

Correspondingly, $\eta = \dfrac{V_{SS}/2}{V_{SS}} = \dfrac{1}{2}$ or 50%

12.16

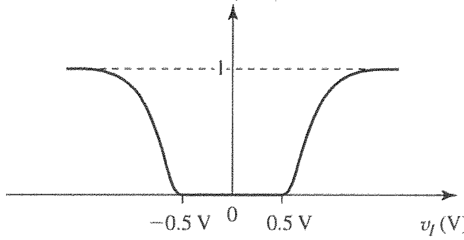


Figure 1

The gain in the deadband is zero. As v_I exceeds 0.5 V and increases, the current in Q_N increases and its r_e decreases, thus

$$A_v = \frac{R_L}{R_L + r_{eN}}$$

increases, and it approaches unity for large v_I. A similar situation occurs for negative v_I, resulting in the symmetrical graph shown in Fig. 1.

12.17 $V_{BB} = 2V_T \ln(I_Q/I_S)$

$= 2 \times 0.025 \ln(10^{-3}/10^{-14})$

$= 1.266$ V

At $v_I = 0$, $i_N = i_P = I_Q = 1$ mA, we have

$$r_{eN} = r_{eP} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25\ \Omega$$

$R_{\text{out}} = r_{eN} \parallel r_{eP} = 12.5\ \Omega$

$$A_v = \frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 12.5}$$

$= 0.889$ V/V

At $v_O = 10$ V, we have

$$i_L = \frac{10}{100} = 0.1 \text{ A} = 100 \text{ mA}$$

To obtain i_N, we use Eq. (12.27):

$i_N^2 - i_L i_N - I_Q^2 = 0$

$i_N^2 - 100\, i_N - 1 = 0$

$\Rightarrow i_N = 100.01$ mA

$i_P = i_N - i_L = 0.01$ mA

$$R_{\text{out}} = \frac{V_T}{i_P + i_N} \simeq \frac{25 \text{ mV}}{100 \text{ mA}} = 0.25\ \Omega$$

$$A_v = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 0.25} = 0.998 \text{ V/V}$$

12.18 $A_v = \dfrac{R_L}{R_L + R_{\text{out}}}$ and $R_{\text{out}} = \dfrac{r_e}{2} = \dfrac{V_I}{2I_Q}$

Now $A_v \geq 0.97$ for $R_L \geq 100\ \Omega$

$$\therefore 0.97 = \frac{100}{100 + R_{\text{out}}}$$

$\Rightarrow R_{\text{out}} \simeq 3\ \Omega$

$$R_{\text{out}} = 3 = \frac{V_T}{2I_Q}$$

$$I_Q = \frac{V_T}{6} = \frac{25 \times 10^{-3}}{6} = 4.17 \text{ mA}$$

$$V_{BB} = 2V_{BE} = 2\left[0.7 + V_T \ln\left(\frac{4.17}{100}\right)\right]$$

$= 1.24$ V

12.19 At $i_L = 0$, we have $i_N = i_P = I_Q$ and

$$R_{\text{out}} = \frac{1}{2}\frac{V_T}{I_Q}$$

Thus,

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + \dfrac{12.5}{I_Q}} \qquad (1)$$

where I_Q is in mA.

For $i_L = 50$ mA, we have

$i_N \simeq 50$ mA and $i_P \simeq 0$

Thus,

$$R_{\text{out}} \simeq r_{eN} = \frac{V_T}{i_N} = \frac{25 \text{ mV}}{50 \text{ mA}} = 0.5\ \Omega$$

$$\frac{v_o}{v_i} = \frac{100}{100 + 0.5} = 0.995 \text{ V/V}$$

To limit the variation to 5%, we use

$$\left.\frac{v_o}{v_i}\right|_{i_L=0} = 0.995 - 0.05 = 0.945 \text{ V/V}$$

Substituting this value in Eq. (1) yields

$I_Q = 2.15$ mA

12.20 The current i_I can be obtained as

$$i_I = \frac{i_N}{\beta_N + 1} - \frac{i_p}{\beta_P + 1} = \frac{i_L}{\beta + 1}$$

where $\beta_N = \beta_P = \beta = 49$

Using values of v_I from the table, one can evaluate R_{in} as

$$R_{\text{in}} = \frac{v_I}{i_I}$$

Using the resistance reflection rule

$R_{\text{in}} \simeq (\beta + 1)R_L = 50 \times 100$

$= 5000\ \Omega$

For large input signal, the two values of R_{in} are somewhat the same. For the small values of v_I, the calculated value in the table is larger.

This table belongs to Problem 12.20.

v_O (V)	i_L (mA)	i_N (mA)	i_P (mA)	v_{BEN} (V)	v_{EBP} (V)	v_i (V)	v_O/v_I (V/V)	R_{out} (Ω)	v_o/v_i (V/V)	i_I (mA)	R_{in} (Ω)
+10.0	100	100.04	0.04	0.691	0.495	10.1	0.99	0.25	1.00	2	5050
+5.0	50	50.08	0.08	0.673	0.513	5.08	0.98	0.50	1.00	1	5080
+1.0	10	10.39	0.39	0.634	0.552	1.041	0.96	2.32	0.98	0.2	5205
+0.5	5	5.70	0.70	0.619	0.567	0.526	0.95	4.03	0.96	0.1	5260
+0.2	2	3.24	1.24	0.605	0.581	0.212	0.94	5.58	0.95	0.04	5300
+0.1	1	2.56	1.56	0.599	0.587	0.106	0.94	6.07	0.94	0.02	5300
0	0	2	2	0.593	0.593	0	–	6.25	0.94	0	
–0.1	–1	1.56	2.56	0.587	0.599	–0.106	0.94	6.07	0.94	–0.02	5300
–0.2	–2	1.24	3.24	0.581	0.605	–0.212	0.94	5.58	0.95	–0.04	5300
–0.5	–5	0.70	5.70	0.567	0.619	–0.526	0.95	4.03	0.96	–0.1	5260
–1.0	–10	0.39	10.39	0.552	0.634	–1.041	0.96	2.32	0.98	–0.2	5205
–5.0	–50	0.08	50.08	0.513	0.673	–5.08	0.98	0.50	1.00	–1	5080
–10.0	–100	0.04	100.04	0.495	0.691	–10.1	0.99	0.25	1.00	–2	5050

12.21 $\dfrac{v_o}{v_i} = \dfrac{R_L}{R_L + R_{out}}$ and

$$R_{out} = \frac{V_T}{i_P + i_N} = \frac{V_T}{I_Q + I_Q} \text{ at } v_O = 0$$

(a) $\epsilon = 1 - \left.\dfrac{v_o}{v_1}\right|_{v_O=0}$

$$= 1 - \frac{R_L}{R_L + R_{out}} = 1 - \frac{R_L}{R_L + \dfrac{V_T}{2I_Q}} =$$

$$\frac{V_T/2I_Q}{R_L + (V_T/2I_Q)}$$

$$\epsilon = \frac{V_T/2I_Q}{R_L + (V_T/2I_Q)} = \frac{V_T}{2R_L I_Q + V_T}$$

If $2I_Q R_L \gg V_T$, then we have

$$\epsilon \simeq \frac{V_T}{2I_Q R_L} \qquad \text{Q.E.D.}$$

(b) Quiescent power dissipation $= 2V_{CC}I_Q = P_D$

(c) $\epsilon \times$ Quiescent power dissipation $=$

$$\frac{V_T}{2I_Q R_L} \times 2V_{CC}I_Q = V_T \times \left(\frac{V_{CC}}{R_L}\right)$$

$$\therefore \epsilon P_D = V_T\left(\frac{V_{CC}}{R_L}\right)$$

(d) $\epsilon P_D = V_T \dfrac{V_{CC}}{R_L} = 25 \times 10^{-3} \times \dfrac{10}{100}$

$= 2.5$ mW

$$P_D = \frac{2.5 \times 10^{-3}}{\epsilon}$$

ϵ	P_D (mW)	I_Q (mA)
0.05	50	2.5
0.02	125	6.25
0.01	250	12.5

12.22 $I_Q = 1$ mA

For output of –1 V, we have

$$i_L = -\frac{1}{100} = -10 \text{ mA}$$

Using Eq. (12.27), we obtain

$$i_N^2 - i_L i_N - I_Q^2 = 0$$

$$i_N^2 + 10 i_N - 1 = 0$$

$$i_N = 0.1 \text{mA}$$

$$i_P = 10.1 \text{mA}$$

Thus v_{EBP} increases by $V_T \ln \dfrac{10.1}{1} = 0.06$ V

and the input step must be –1.06 V.

Largest possible positive output from 6 to 10, i.e., 4 V

Largest negative output from 6 to 0, i.e., 6 V

12.23 $R_{out} = r_e/2 = 8\ \Omega$

$$\Rightarrow r_e = 16\ \Omega$$

$$I_Q = \frac{V_T}{r_e} = \frac{25}{16} = 1.56 \text{ mA}$$

Thus, $n = \dfrac{1.56}{0.2} = 7.8$

12.24 $I_Q \simeq I_{BIAS} = 1$ mA, neglecting the base current of Q_N. More precisely,

$$I_Q = I_{BIAS} - \frac{I_Q}{\beta + 1}$$

$$\Rightarrow I_Q = \frac{I_{BIAS}}{1 + \dfrac{1}{\beta + 1}} \simeq 0.98 \times 1 = 0.98 \text{ mA}$$

The largest positive output is obtained when all of I_{BIAS} flows into the base of Q_N, resulting in

$$v_O = (\beta_N + 1)I_{BIAS}R_L$$

$$= 51 \times 1 \times 100\ \Omega = 5.1\ \text{V}$$

The largest possible negative output voltage is limited by the saturation of Q_P to

$$-10 + V_{ECsat} = -10\ \text{V}$$

To achieve a maximum positive output of 10 V without changing I_{BIAS}, β_N must be

$$10 = (\beta_N + 1) \times 1 \times 10^{-3} \times 100\ \Omega$$

$$\Rightarrow \beta_N = 99$$

Alternatively, if β_N is held at 50, I_{BIAS} must be increased so that

$$10 = 51 \times I_{BIAS} \times 10^{-3} \times 100\ \Omega$$

$$\Rightarrow I_{BIAS} = 1.96\ \text{mA}$$

for which

$$I_Q = \frac{I_{BIAS}}{1 + \dfrac{1}{\beta + 1}} = 1.92\ \text{mA}$$

Figure 1(a) shows the small-signal equivalent circuit of the class AB circuit in Fig. 12.14. Here, each of Q_N and Q_P has been replaced with its hybrid-π model, and the small resistances of the diodes have been neglected. As well, we have not included r_o of each of Q_N and Q_P.

The circuit in Fig. 1(a) can be simplified to that in Fig. 1(b) where

$$r_\pi = r_{\pi N} \parallel r_{\pi P} \quad (1)$$

$$g_m = g_{mN} + g_{mP} \quad (2)$$

Since $g_m \simeq \dfrac{1}{r_e}$, then from (2) we obtain

$$\frac{1}{r_e} = \frac{1}{r_{eN}} + \frac{1}{r_{eP}}$$

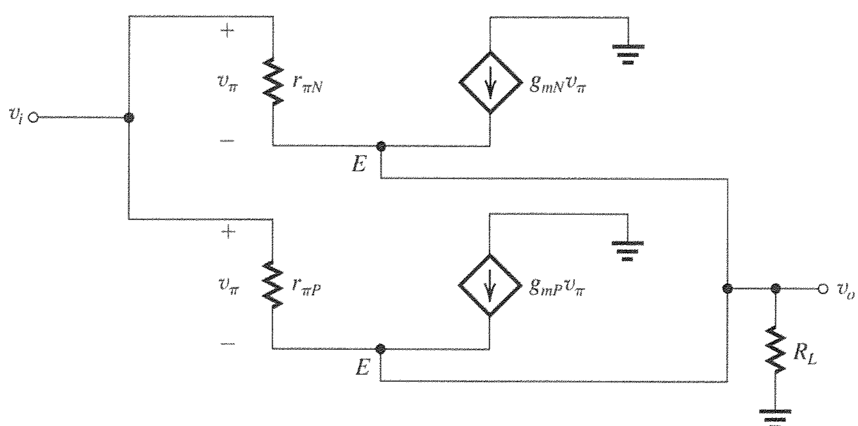
or, equivalently,

$$r_e = r_{eN} \parallel r_{eP} \quad (3)$$

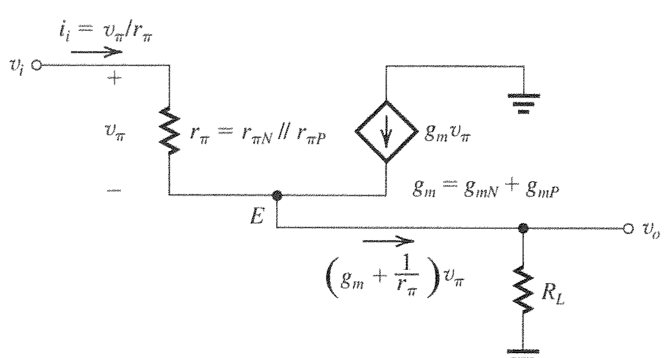
We observe that the circuit in Fig. 1(b) is the equivalent circuit of an emitter follower with the small-signal parameters r_π, g_m, and r_e given in Eqs. (1), (2), and (3). Furthermore, its β is given by

$$\beta = g_m r_\pi = (g_{mN} + g_{mP})(r_{\pi N} \parallel r_{\pi P}) \quad (4)$$

12.25 This figure belongs to Problem 12.25.



(a)



(b)

Figure 1

For the circuit in Fig. 1(b), we can write

$$v_i = v_\pi + v_o \quad (5)$$

$$v_o = \left(g_m + \frac{1}{r_\pi}\right) v_\pi R_L \quad (6)$$

Equations (5) and (6) can be used to obtain the incremental (or small-signal) gain,

$$\frac{v_o}{v_i} = \frac{\left(g_m + \frac{1}{r_\pi}\right) R_L}{\left(g_m + \frac{1}{r_\pi}\right) R_L + 1}$$

But,

$$g_m + \frac{1}{r_\pi} = \frac{1}{r_e}$$

Thus,

$$\frac{v_o}{v_i} = \frac{R_L/r_e}{R_L/r_e + 1}$$

$$\Rightarrow \frac{v_o}{v_i} = \frac{R_L}{R_L + r_e} =$$

$$\frac{R_L}{R_L + (r_{eN} \parallel r_{eP})} \quad \text{Q.E.D.} \quad (7)$$

The input resistance is found as follows:

$$R_{\text{in}} = \frac{v_i}{i_i} = \frac{v_i}{v_\pi / r_\pi}$$

Substituting for v_i from (5) together with utilizing (7) gives

$$R_{\text{in}} = \frac{v_\pi \left[1 + \left(g_m + \frac{1}{r_\pi}\right) R_L\right]}{v_\pi / r_\pi}$$

$$= r_\pi + (g_m r_\pi + 1) R_L$$

$$= r_\pi + (\beta + 1) R_L$$

$$= (\beta + 1)(R_L + r_e)$$

$$\simeq \beta[R_L + (r_{eN} \parallel r_{eP})] \quad \text{Q.E.D.} \quad (8)$$

12.26 Refer to Fig. P12.26. Neglecting the small resistances of D_1 and D_2, we can write for the voltage gain of the CE amplifier transistor Q_3,

$$\frac{v_{c3}}{v_i} = -g_{m3} R_{\text{in}} \quad (1)$$

where R_{in} is the input resistance of the class AB circuit, given in the statement of Problem 12.25 as

$$R_{\text{in}} \simeq \beta[R_L + (r_{eN} \parallel r_{eP})] \quad (2)$$

where

$$\beta = (g_{mN} + g_{mP})(r_{\pi N} \parallel r_{\pi P}) \quad (3)$$

The voltage gain of the class AB circuit is given in the statement of Problem 12.25 as

$$\frac{v_o}{v_{c3}} = \frac{R_L}{R_L + (r_{eN} \parallel r_{eP})} \quad (4)$$

Now, we can combine (1), (2), and (4) to obtain the voltage gain of the circuit in Fig. P12.26 as

$$\frac{v_o}{v_i} = -g_{m3}\beta[R_L + (r_{eN} \parallel r_{eP})] \frac{R_L}{R_L + (r_{eN} \parallel r_{eP})}$$

$$\Rightarrow \frac{v_o}{v_i} = -g_{m3}\beta R_L$$

where β is given by Eq. (3).

12.27 The total resistance in the base circuit (while neglecting the small resistances of D_1 and D_2) is $R_{\text{BIAS}} \parallel r_{o3}$. Thus, utilizing the equivalent emitter follower for the class AB stage, we obtain for the output resistance

$$R_{\text{out}} = r_e + \frac{(R_{\text{BIAS}} \parallel r_{o3})}{\beta + 1}$$

where

$$r_e = r_{eN} \parallel r_{eP}$$

and

$$\beta = (g_{mN} + g_{mP})(r_{\pi N} \parallel r_{\pi P})$$

12.28 At 20°C, $I_Q = 1\text{mA} = I_S e^{(0.6/0.025)}$

$\Rightarrow I_S$ (at 20°C) $= 3.78 \times 10^{-11}$ mA

At 70°C, $I_S = 3.78 \times 10^{-11}(1.14)^{50}$

$= 2.64 \times 10^{-8}$ mA

At 70°C, $V_T = 25\dfrac{273 + 70}{273 + 20} = 29.3$ mV

Thus, I_Q (at 70°C) $= 2.64 \times 10^{-8} e^{0.6/0.0293}$

$= 20.7$ mA

Additional current $= 20.7 - 1 = 19.7$ mA

Additional power $= 2 \times 20 \times 19.7 = 788$ mW

Additional temperature rise $= 10 \times 0.788$
$= 7.9$°C,

At 77.9°C:

$$V_T = \frac{25}{293}(273 + 77.9) = 29.9 \text{ mV}$$

$$I_Q = 3.78 \times 10^{-11} \times (1.14)^{57.9} e^{(0.6/0.0299)}$$

$= 37.6$ mA

etc., etc.

12.29 Since the peak positive output current is 250 mA, the base current of Q_N can be as high as $\dfrac{250}{\beta_N + 1} = \dfrac{250}{51} \simeq 5$ mA. We select $I_{\text{BIAS}} = 6$ mA, thus providing the multiplier with a minimum current of 1 mA.

Under quiescent conditions ($v_O = 0$ and $i_L = 0$) the base current of Q_N can be neglected.

Selecting $I_R = 0.5$ mA leaves $I_{C1} = 5.5$ mA. To obtain a quiescent current of 2 mA in the output transistors, V_{BB} should be

$$V_{BB} = 2V_T \ln \frac{2 \times 10^{-3}}{10^{-15}} = 1.19 \text{ V}$$

Thus,

$$R_1 + R_2 = \frac{V_{BB}}{I_R} = \frac{1.19}{0.5} = 2.38 \text{ k}\Omega$$

At a collector current of 5.5 mA, Q_1 has

$$V_{BEL} = V_T \ln \frac{5.5 \times 10^{-3}}{10^{-14}} = 0.676 \text{ V}$$

The value of R_1 can now be determined as

$$R_1 = \frac{0.676}{0.5} = 1.35 \text{ k}\Omega \text{ and}$$

$$R_2 = 2.58 - 1.35 = 1.03 \text{ k}\Omega$$

12.30 (a) $V_{BE} = 0.7$ V at 1 mA

At 0.5 mA,

$$V_{BE} = 0.7 + 0.025 \ln \frac{0.5}{1} = 0.683 \text{ V}$$

Thus, $R_1 = \dfrac{0.683}{0.5} = 1.365 \text{ k}\Omega$

and $R_2 = 1.365 \text{ k}\Omega$

$$V_{BB} = 2V_{BE} = 1.365 \text{ V}$$

(b) For $I_{\text{bias}} = 2$ mA, I_C increases to nearly 1.5 mA for which

$$V_{BE} = 0.7 + 0.025 \ln \frac{1.5}{1} = 0.710 \text{ V}$$

Note that $I_R = \dfrac{0.710}{1.365} = 0.52$ mA is very nearly equal to the assumed value of 0.50 mA, thus no further iterations are required.

$$V_{BB} = 2V_{BE} = 1.420 \text{ V}$$

(c) For $I_{\text{bias}} = 10$ mA, assume that I_R remains constant at 0.5 mA, thus $I_{C1} = 9.5$ mA

and $V_{BE} = 0.7 + 0.025 \ln \dfrac{9.5}{1} = 0.756 \text{ V}$

at which

$$I_R = \frac{0.755}{1.365} = 0.554 \text{ mA}$$

Thus,

$$I_{C1} = 10 - 0.554 = 9.45 \text{ mA}$$

and $V_{BE} = 0.7 + 0.025 \ln \dfrac{9.45}{1} = 0.756 \text{ V}$

Thus, $V_{BB} = 2 \times 0.756 = 1.512$ V

(d) Now for $\beta = 100$,

$$I_{R1} = \frac{0.756}{1.365} = 0.554 \text{ mA}$$

$$I_{R2} = 0.554 + \frac{9.45}{101} = 0.648 \text{ mA}$$

$$I_C = 10 - 0.648 = 9.352 \text{ mA}$$

Thus, $V_{BE} = 0.7 + 0.025 \ln \dfrac{9.352}{1} = 0.756 \text{ V}$

$$V_{BB} = 0.756 + I_{R2}R_2$$

$$= 0.756 + 0.648 \times 1.365$$

$$= 1.641 \text{ V}$$

12.31 Figure 1 shows the V_{BE} multiplier together with its small-signal equivalent circuit prepared for determining the incremental terminal resistance r,

$$r \equiv \frac{v_x}{i_x}$$

Now,

$$i_x = g_m v_\pi + \frac{v_\pi}{R_1 \parallel r_\pi} \tag{1}$$

$$v_x = v_\pi + \frac{v_\pi}{R_1 \parallel r_\pi} R_2 \tag{2}$$

Dividing (2) by (1) gives

$$r = \frac{1 + R_2/(R_1 \parallel r_\pi)}{g_m + \dfrac{1}{R_1 \parallel r_\pi}}$$

$$= \frac{R_2 + (R_1 \parallel r_\pi)}{1 + g_m(R_1 \parallel r_\pi)}$$

This figure belongs to Problem 12.31.

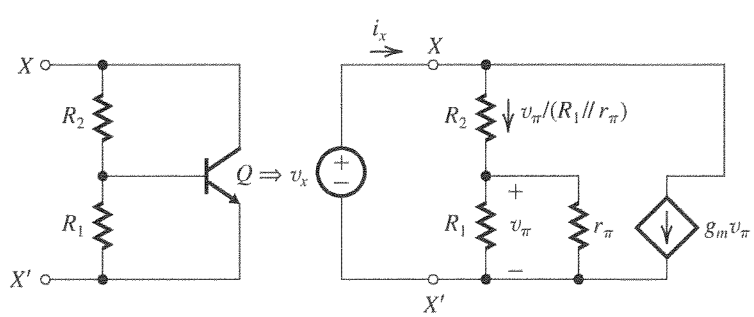


Figure 1

For $R_1 = R_2 = 1.2\ \text{k}\Omega$, $I_C = 1$ mA, and $\beta = 100$, we have

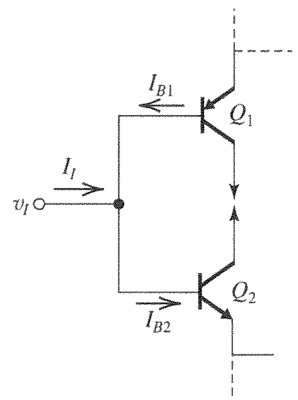
$g_m = 40$ mA/V

$$r_\pi = \frac{100}{40} = 2.5\ \text{k}\Omega$$

Thus,

$$r = \frac{1.2 + (1.2 \parallel 2.5)}{1 + 40(1.2 \parallel 2.5)} = 60.2\ \Omega$$

12.32 (a) For $R_L = \infty$:



At $v_I = 0$ V, we have

$$I_{B1} = I_{B2} = \frac{2.87}{200}$$

$I_I = I_{B2} - I_{B1} = 0$

At $v_I = +10$ V, we have

$$I_{B1} = \frac{0.88}{200}\ \text{mA} = 4.4\ \mu\text{A}$$

$$I_{B2} = \frac{4.87}{200}\ \text{mA} = 24.4\ \mu\text{A}$$

$I_I = I_{B2} - I_{B1} = 20\ \mu$A

At $v_I = -10$ V, we have

$$I_{B1} = \frac{4.87}{200}\ \text{mA} = 24.4\ \mu\text{A}$$

$$I_{B2} = \frac{0.88}{200}\ \text{mA} = 4.4\ \mu\text{A}$$

$I_I = I_{B2} - I_{B1} = -20\ \mu$A

(b) For $R_L = 100\ \Omega$:

At $v_I = 0$ V, we have $I_I = 0$

At $v_I = +10$ V, we have

$$I_{B1} = \frac{0.38}{200} = 1.9\ \mu\text{A}$$

$$I_{B2} = \frac{4.87}{200} = 24.4\ \mu\text{A}$$

$I_I = I_{B2} - I_{B1} = 22.5\ \mu$A

At $v_I = -10$ V, we have $I_I = -22.5\ \mu$A

12.33 Circuit operating near $v_I = 0$ and is fed with a signal source having zero resistance.

The resistance looking as shown by the arrow X is

$= R_1 \parallel r_{e1}$

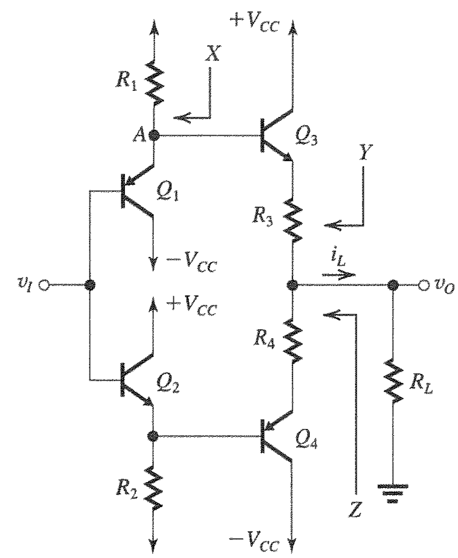
This resistance is reflected from base to the emitter of Q_3 as $(R_1 \parallel r_{e1})/(\beta_3 + 1)$.

The resistance seen by arrow Y, from the upper half of the circuit

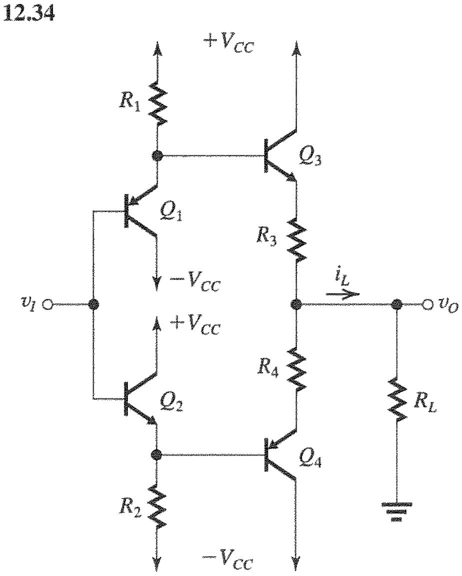
$= R_3 + r_{e3} + (R_1 \parallel r_{e1})/(\beta_3 + 1)$

A similar resistance is seen by the arrow Z and both of these resistances (seen by arrows Y and arrow Z) are in parallel, therefore

$$R_{\text{out}} = \frac{1}{2}\left[R_3 + r_{e3} + (R_1 \parallel r_{e1})/(\beta_3 + 1)\right]$$



12.34



At $v_I = 5$ V, we have

$V_{E1} = +5.7$ V

$$I_{R1} = \frac{V_{CC} - V_{E1}}{R_1} = \frac{10 - 5.7}{R_1} = \frac{4.3}{R_1}$$

To allow for $I_{B3} = 10$ mA if needed while reducing I_{E1} by no more than half, then I_{R1} must be $2 \times 10 = 20$ mA. Thus,

$$R_1 = \frac{V_{R1}}{I_{R1}} = \frac{4.3}{20} = 0.215 \text{ k}\Omega = 215\ \Omega$$

Similarly,

$R_2 = 0.215 \text{ k}\Omega = 215\ \Omega$

Next, we determine the values of R_3 and R_4: At $v_I = 0$, assume $V_{EB1} = 0.7$. Then

$V_{E1} = 0.7$ V

$$I_{R1} = \frac{10 - 0.7}{0.215} = 43.3 \text{ mA}$$

$$V_{EB1} = 0.7 + 0.025 \times \ln\left(\frac{43.3}{10}\right)$$

$= 0.737$ V

$V_{E1} = 0.737$ V

Meanwhile Q_3 will be conducting $I_Q = 40$ mA. Since $I_{S3} = 3I_{S1}$ then Q_3 has $V_{BE} = 0.7$ V at $I_C = 30$ mA. At 40 mA,

$$V_{BE3} = 0.7 + 0.025 \times \ln\left(\frac{40}{30}\right)$$

$= 0.707$ V

For $v_O = 0$,

$V_{E1} - V_{BE3} - I_{E3}R_3 = 0$

$0.737 - 0.707 - 40R_3 = 0$

$\Rightarrow R_3 = 0.75\ \Omega$

Similarly,

$R_4 = 0.75\ \Omega$

$$R_{\text{out}} = \frac{1}{2}\left[R_3 + r_{e3} + \frac{R_1 \parallel r_{e1}}{\beta_3 + 1}\right]$$

where

$$r_{e3} = \frac{25 \text{ mV}}{40 \text{ mA}} = 0.625\ \Omega$$

$$r_{e1} = \frac{25 \text{ mV}}{20 \text{ mA}} = 1.25\ \Omega$$

$$R_{\text{out}} = \frac{1}{2}\left[0.75 + 0.625 + \frac{215 \parallel 1.25}{51}\right]$$

$R_{\text{out}} = 0.7\ \Omega$

Next, consider the situation when

$v_I = +1$ V and $R_L = 2\ \Omega$

Let $v_O \simeq 1$ V, then

$$i_L = \frac{1 \text{ V}}{2\ \Omega} = 0.5 \text{ A} = 500 \text{ mA}$$

Now if we assume that $i_{E4} \simeq 0$, then

$i_{E3} = i_L = 500$ mA

$$V_{BE3} = 0.7 + 0.025 \ln\frac{500}{30}$$

$= 0.770$ V

$$i_{B3} = \frac{500}{51} \simeq 10 \text{ mA}$$

Assuming that $V_{EB1} \simeq 0.7$ V, then

$v_{E1} = 1 + 0.7 = 1.7$ V

$$i_{R1} = \frac{10 - 1.7}{0.215} = 38.6 \text{ mA}$$

$i_{E1} = i_{R1} - i_{B2} = 38.6 - 10 = 28.6$ mA

$$V_{EB1} = 0.7 + 0.025 \ln\frac{28.6}{10}$$

$= 0.726$ V

$V_{E1} = 1.726$ V

$$i_L = \frac{V_{E1} - V_{BE3}}{R_3 + R_L}$$

$$= \frac{1.726 - 0.770}{0.75 + 2}$$

$= 0.348$ A

$v_O = i_L R_L$

$= 0.348 \times 2 = 0.695$ V

Let's check that i_{E4} is zero. The voltage at the base of Q_4 is

$V_{B4} = 1 - V_{BE2}$

$\simeq 1 - 0.74 = 0.26$ V

The voltage across R_4 and V_{EB4} is

$= v_O - 0.26 = 0.695 - 0.26 = 0.435$ V

which is sufficiently small to keep Q_4 cutoff, verifying our assumption that $i_{E4} \simeq 0$.

Let's now do more iterations to refine our estimate of v_O:

$i_L = 0.35$ A

$$i_{B3} = \frac{0.35}{51} \simeq 7 \text{ mA}$$

$$i_{E1} = \frac{10 - 1 - 0.726}{0.215} - 7 = 31.5 \text{ mA}$$

$$V_{EB1} = 0.7 + 0.025 \ln\left(\frac{31.5}{10}\right)$$

$= 0.729$ V

$V_{E1} = 1 + 0.729 = 1.729$ V

$i_{E3} = i_L = 350$ mA

$V_{BE3} = 0.7 + 0.025 \ln\left(\dfrac{350}{30}\right)$

$= 0.761$ V

$i_L = \dfrac{V_{E1} - V_{BE3}}{R_3 + R_2}$

$= \dfrac{1.729 - 0.761}{0.75 + 2} = 0.352$ A

$v_O = i_L R_L$

$= 0.352 \times 2 = 0.704$ V

12.35 (a) $v_I = 0$ and transistors have $\beta = 100$.

$v_O = 0$ V

$I_Q = I_{E3} = I_{E4} = I_{E1} = I_{E2} \simeq 1$ mA

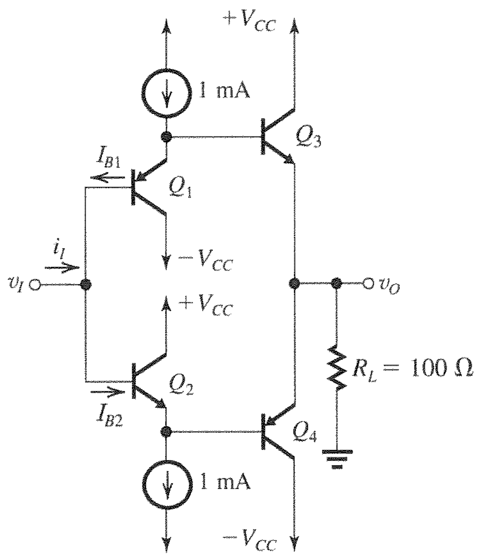
More precisely, $\dfrac{I_{E3}}{\beta + 1} + I_{E1} = 1$ mA

thus,

$I_Q\left(\dfrac{1}{(\beta + 1)} + 1\right) = 1$

$\Rightarrow I_Q \simeq 0.99$ mA

Input bias current in zero because $I_{BI} = I_{B2}$.



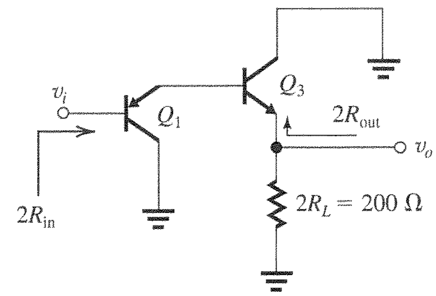
(b) From the equivalent half circuit, we have

$2R_{in} = (\beta_1 + 1)\left[r_{e1} + (\beta_3 + 1)(r_{e3} + 2R_L)\right]$

$r_{e1} = r_{e3} = \dfrac{V_T}{I_E} = \dfrac{25}{1} = 25\ \Omega$

$2R_{in} = (100 + 1)[25 + (100 + 1)(25 + 2 \times 100)]$

$\Rightarrow R_{in} = 1.15$ MΩ



$$A_v = \frac{v_o}{v_i} = \frac{2R_L}{2R_L + r_{e3} + \dfrac{r_{e1}}{\beta_3 + 1}}$$

$$= \frac{200}{200 + 25 + \dfrac{25}{101}}$$

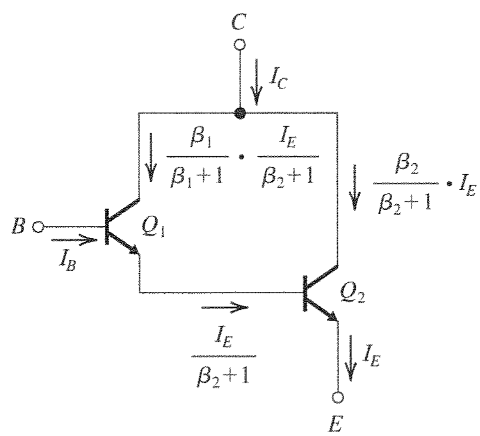
$\simeq 0.89$ V/V

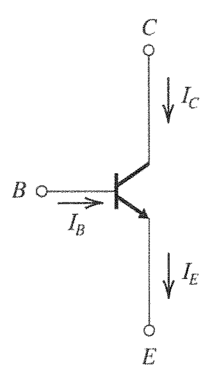
$2R_{out} = r_{e3} + \dfrac{r_{e1}}{\beta + 1}$

$= 25 + \dfrac{25}{101}$

$R_{out} = 12.6\ \Omega$

12.36





(a) For the composite transistor, we have

$$\beta = \frac{I_C}{I_B}$$

Refer to the diagram.

$$I_B = \frac{1}{\beta_1 + 1}\frac{\mathrm{I}_E}{\beta_2 + 1}$$

$$I_C = \frac{\beta_1}{(\beta_1 + 1)(\beta_2 + 1)} I_E + \frac{\beta_2}{(\beta_2 + 1)} I_E$$

$$= \frac{\beta_1 + \beta_2(\beta_1 + 1)}{(\beta_1 + 1)(\beta_2 + 1)} \cdot I_E$$

For the composite transistor, β is given by

$$\beta = \frac{I_C}{I_B} = \frac{\dfrac{\beta_1 + \beta_2(\beta_1 + 1)}{(\beta_1 + 1)(\beta_2 + 1)} \times I_E}{\dfrac{1}{(\beta_1 + 1)(\beta_2 + 1)} \cdot I_E}$$

$$= \beta_1 + \beta_2(\beta_1 + 1)$$

$$\simeq \beta_1\beta_2 \text{ since } \beta_1 \gg 1 \text{ and } \beta_2 \gg 1$$

(b) Refer to the diagram.

Operating current of Q_2

$$= I_{C2} = \frac{\beta_2}{\beta_2 + 1} I_E = \frac{\beta_2}{\beta_2 + 1} \times \frac{\beta + 1}{\beta} I_C$$

where $\beta = \beta_1\beta_2$

$$\simeq I_C \text{ since } \beta_2 \gg 1 \text{ and } \beta \gg 1$$

Operating current of Q_1

$$= I_{C1} = \frac{\beta_1}{(\beta_1 + 1)(\beta_2 + 1)} I_E$$

$$= \frac{\beta_1}{(\beta_1 + 1)(\beta_2 + 1)} \cdot \frac{\beta + 1}{\beta} I_C$$

$$\simeq \frac{I_C}{\beta_2} \text{ since } \beta \gg 1,\ \beta_2 \gg \text{ and } \beta_1 \gg 1.$$

(c) Again refer to the diagram and part (b).

$$V_{BE} = V_{BE2} + V_{BE1} = V_T\ \mathrm{In}\left(\frac{I_{C2}}{I_S}\right) + V_T\ \mathrm{In}\left(\frac{I_{C1}}{I_S}\right)$$

From part (b), $I_{C2} \simeq I_C$ and $I_{CI} \simeq \dfrac{I_C}{\beta_2}$

$$\therefore V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) + V_T \ln\left(\frac{1}{\beta_2}\frac{I_C}{I_S}\right)$$

$$= V_T \ln\left(\frac{I_C}{I_S}\right) + V_T \ln\left(\frac{I_C}{I_S}\right) + V_T \ln\left(\frac{1}{\beta_2}\right)$$

$$V_{BE} = 2V_T \ln\left(\frac{I_C}{I_S}\right) - V_T \ln(\beta_2)$$

(d) $r_{\pi eq} = (\beta_1 + 1)\left[r_{e1} + (\beta_2 + 1) r_{e2}\right]$

Here, $r_{e2} = \dfrac{V_T}{I_{E2}} \simeq \dfrac{V_T}{I_{C2}} \simeq \dfrac{V_T}{I_C}$

$$r_{e1} = \frac{V_T}{I_{E2}} \simeq \frac{V_T}{I_{C1}} \simeq \frac{V_T}{I_C/\beta_2} = \beta_2 r_{e2}$$

$$r_{\pi eq} \simeq (\beta_1 + 1)[\beta_2 r_{e2} + \beta_2 r_{e2}]$$

$$= 2(\beta_1 + 1)\beta_2 r_{e2}$$

$$\cong 2\beta_1\beta_2 r_{e2}$$

$$= 2\beta_1\beta_2\frac{V_T}{I_C}$$

(e) To find g_{meq}, apply a signal v_{be} and find the corresponding current i_c:

$$i_c = i_{c1} + i_{c2} = g_{m1}v_{be1} + g_{m2}v_{be2}$$

$$= g_{m1}v_{be}\frac{r_{e1}}{r_{e1} + (\beta_2 + 1) r_{e2}}$$

$$+ g_{m2}\frac{(\beta_2 + 1) r_{e2}}{r_{e1} + (\beta_2 + 1) r_{e2}} \cdot v_{be}$$

$$\simeq v_{be}\frac{1}{\beta_2 r_{e2} + (\beta_2 + 1) r_{e2}}$$

$$+ \frac{\beta_2}{\beta_2 r_{e2} + (\beta_2 + 1) r_{e2}} \cdot v_{be}$$

$\because g_m r_e \simeq 1$,

$$i_c \simeq v_{bc}\frac{1}{2\beta_2 r_{e2}} + v_{be}\frac{\beta_2}{2\beta_2 r_{e2}}$$

$$\simeq v_{bc}\frac{(\beta_2 + 1)}{2\beta_2 r_{e2}}$$

$$\simeq v_{bc}\frac{1}{2r_{e2}}$$

$$g_{meq} = \frac{i_c}{v_{be}} = \frac{1}{2r_{e2}}$$

$$= \frac{1}{2}\frac{I_C}{V_T}$$

12.37 (a)

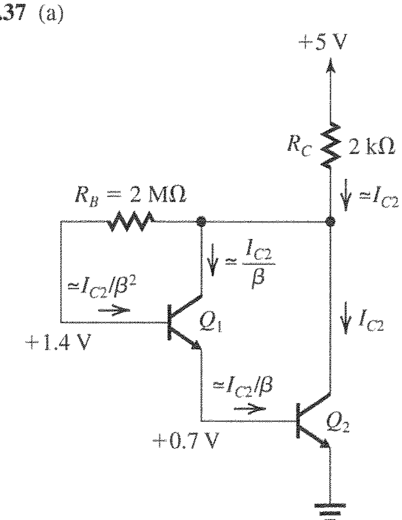


Figure 1

From Figure 1 we can write

$$5 = I_{C2}R_C + \frac{I_{C2}}{\beta^2}R_B + 1.4$$

$$\Rightarrow I_{C2} = \frac{5 - 1.4}{R_C + \dfrac{R_B}{\beta^2}}$$

$$= \frac{3.6}{2 + \dfrac{2000}{10,000}} = 1.64 \text{ mA}$$

$$I_{C1} \simeq \frac{I_{C2}}{\beta} = \frac{1.64}{100} = 0.0164 \text{ mA}$$

(b)

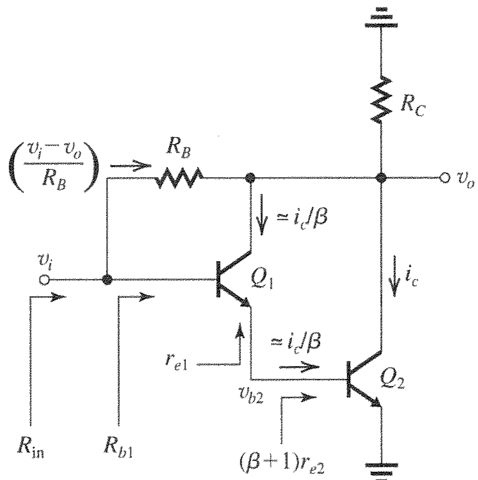


Figure 2

$$v_{b2} = v_i\frac{(\beta + 1)r_{e2}}{(\beta + 1)r_{e2} + r_{e1}}$$

where

$$r_{e2} = \frac{V_T}{I_{E2}} \simeq \frac{V_T}{I_{C2}} = \frac{25 \text{ mV}}{1.64 \text{ mA}} = 15.2 \ \Omega$$

$$r_{e1} = \frac{V_T}{I_{E1}} \simeq \frac{V_T}{I_{C1}} = \frac{25 \text{ mV}}{0.0164 \text{ mA}} = 1.52 \ \Omega$$

$$v_{b2} = v_i\frac{101 \times 15.2}{101 \times 15.2 + 1520} = 0.5v_i$$

$$i_c = g_{m2}v_{b2}$$

$$= g_{m2} \times 0.5v_i$$

where

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{1.64}{0.025} = 65.6 \text{ mA/V}$$

$$i_c = 65.6 \times 0.5v_i = 32.8v_i$$

Writing a node equation at the output provides

$$\frac{v_o}{R_C} + i_c + \frac{i_c}{\beta} + \frac{v_o - v_i}{R_B} = 0$$

Substituting $i_c = 32.8v_i$, we obtain

$$v_o\left(\frac{1}{R_C} + \frac{1}{R_B}\right) = -v_i\left[\left(1 + \frac{1}{\beta}\right)32.8 - \frac{1}{R_B}\right]$$

$$A_v \equiv \frac{v_o}{v_i} = -\frac{\left(1 + \dfrac{1}{\beta}\right)32.8 - \dfrac{1}{R_B}}{\dfrac{1}{R_C} + \dfrac{1}{R_B}}$$

$$= -\frac{1.01 \times 32.8 - (1/2000)}{\dfrac{1}{2} + \dfrac{1}{2000}}$$

$$= -66.2 \text{ V/V}$$

(c) $R_{b1} = (\beta + 1)[r_{e1} + (\beta + 1)r_{e2}]$

$= 101[1.52 + 101 \times 0.0152]$

$= 318.7 \text{ k}\Omega$

The component of R_{in} arising from R_B can be found as

$$R_{i2} = \frac{v_i}{(v_i - v_o)/R_B}$$

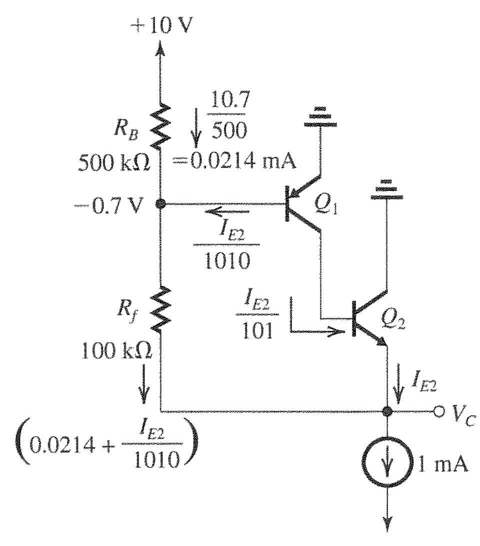
$$= \frac{R_B}{1 - (v_o/v_i)} = \frac{2000}{1 - (-66.2)} = 29.8 \text{ k}\Omega$$

Thus

$$R_{in} = R_{ib} \parallel R_{i2}$$

$$= 318.7 \parallel 29.8 = 27.2 \text{ k}\Omega$$

12.38 (a) DC Analysis:



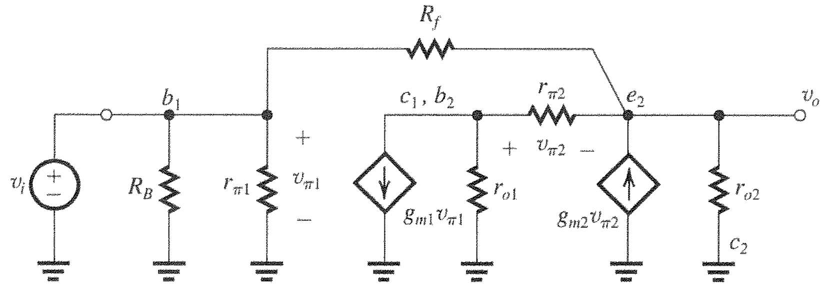
$$1 \text{ mA} = 0.0214 + \frac{I_{E2}}{1010} + I_{E2}$$

$$\Rightarrow I_{E2} = 0.978 \text{ mA}$$

$$I_{C2} = 0.99 \times 0.978 = 0.97 \text{ mA}$$

$$I_{C1} = \frac{0.978}{101} = 9.7 \ \mu\text{A}$$

This figure belongs to Exercise 12.38, part (b).



$$V_C = -0.7 - 100\left(0.0214 + \frac{0.978}{1010}\right)$$

$$= -2.94 \text{ V}$$

(b) Small-signal parameters:

$$g_{m1} = \frac{9.7 \times 10^{-6}}{25 \times 10^{-3}} = 0.388 \text{ mA/V}$$

$$r_{\pi 1} = \frac{\beta_1}{g_{m1}} = 25.77 \text{ k}\Omega$$

$$r_{o1} = \frac{|V_A|}{I_{C1}} = \frac{100}{9.7 \ \mu\text{A}} = 10.31 \text{ M}\Omega$$

$$g_{m2} = \frac{0.97 \times 10^{-3}}{25 \times 10^{-3}} = 38.8 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta_2}{g_{m2}} = 2.58 \text{ k}\Omega$$

$$r_{o2} = |V_A|/I_{C2} = 103.1 \text{ k}\Omega$$

Node equation at b_2:

$$g_{m1}v_{\pi 1} + \frac{v_{b2}}{r_{o1}} + \frac{v_{\pi 2}}{r_{\pi 2}} = 0$$

But $v_{b2} = v_o + v_{p2}$, then

$$g_{m1}v_{\pi 1} + \frac{v_o + v_{\pi 2}}{r_{o1}} + \frac{v_{\pi 2}}{r_{\pi 2}} = 0$$

$$\Rightarrow v_{\pi 2}\left(\frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}}\right) = -\left(\frac{v_o}{r_{o1}} + g_{m1}v_{\pi 1}\right)$$

$$\text{or, } v_{\pi 2} = -\frac{\dfrac{v_o}{r_{o1}} + g_{m1}v_{\pi 1}}{\dfrac{1}{r_{\pi 2}} + \dfrac{1}{r_{o1}}}$$

Node equation at output:

$$\frac{v_o}{r_{o2}} + \frac{v_o - v_{\pi 1}}{R_f} = g_{m2}v_{\pi 2} + \frac{1}{r_{\pi 2}}v_{\pi 2}$$

$$= \left(g_{m2} + \frac{1}{r_{\pi 2}}\right)v_{\pi 2}$$

$$= -\frac{\left(g_{m2} + \dfrac{1}{r_{\pi 2}}\right)\left[\dfrac{v_o}{r_{o1}} + g_{m1}v_{\pi 1}\right]}{\dfrac{1}{r_{\pi 2}} + \dfrac{1}{r_{o1}}}$$

Substituting $v_{\pi 1} = v_i$ and collecting terms, we obtain

$$v_o\left[\frac{1}{r_{o2}} + \frac{1}{R_f} + \frac{\left(g_{m2} + \dfrac{1}{r_{\pi 2}}\right)}{r_{o1}\left(\dfrac{1}{r_{\pi 2}} + \dfrac{1}{r_{o1}}\right)}\right]$$

$$= -v_i\left[\frac{g_{m1}\left(g_{m2} + \dfrac{1}{r_{\pi 2}}\right)}{\dfrac{1}{r_{\pi 2}} + \dfrac{1}{r_{o2}}} - \frac{1}{R_f}\right]$$

$$\frac{v_o}{v_i} = -\frac{\dfrac{g_{m1}\left(g_{m2} + \dfrac{1}{r_{\pi 2}}\right)}{\dfrac{1}{r_{\pi 2}} + \dfrac{1}{r_{o2}}} - \dfrac{1}{R_f}}{\dfrac{1}{r_{o2}} + \dfrac{1}{R_f} + \dfrac{\left(g_{m2} + \dfrac{1}{r_{\pi 2}}\right)}{r_{o1}\left(\dfrac{1}{r_{\pi 2}} + \dfrac{1}{r_{o1}}\right)}}$$

Since $r_{\pi 2} \ll r_{o1}$, we have

$$\frac{v_o}{v_i} \simeq -\frac{g_{m1}\,(g_{m2}r_{\pi 2} + 1) - \dfrac{1}{R_f}}{\dfrac{1}{r_{o2}} + \dfrac{1}{R_f} + \dfrac{1}{r_{o1}}\,(g_{m2}r_{\pi 2} + 1)}$$

$$= -\frac{g_{m1}\,(\beta_2 + 1) - \dfrac{1}{R_f}}{\left(\dfrac{1}{r_{o2}} + \dfrac{1}{R_f}\right) + \dfrac{1}{r_{o1}}\,(\beta_2 + 1)}$$

Since $\dfrac{1}{R_f} \ll g_{m1}\,(\beta_2 + 1)$, we have

$$\frac{v_o}{v_i} \simeq -\frac{g_{m1}\,(\beta_2 + 1)}{\left(\dfrac{1}{r_{o2}} + \dfrac{1}{R_f}\right) + \dfrac{1}{r_{o1}}\,(\beta_2 + 1)}$$

Subsituting $\beta_2 = \beta_N$ and noting that $\beta_N \gg 1$, we obtain

$$\frac{v_o}{v_i} \simeq -g_{m1}\frac{1}{\dfrac{1}{\beta_N}\left(\dfrac{1}{r_{o2}} + \dfrac{1}{R_f}\right) + \dfrac{1}{r_{o1}}}$$

$$= -g_{m1}\left[r_{o1} \parallel \beta_N\,(r_{o2} \parallel R_f)\right] \qquad \text{Q.E.D.}$$

(c)

$$\frac{v_o}{v_i} = -0.388\left[10.31 \times 10^3 \parallel 100\,(103.1 \parallel 100)\right]$$

$$= -1320 \text{ V/V}$$

$$R_{\text{in}} = R_B \parallel r_{\pi 1} \parallel \left[v_i \Big/ \left(\frac{v_i - v_o}{R_f}\right)\right]$$

$$= 500 \parallel 25.77 \parallel \left[\frac{R_f}{1 - \dfrac{v_o}{v_i}}\right]$$

$$= 500 \parallel 25.77 \parallel \frac{100}{1 + 1320}$$

$$= 500 \parallel 25.77 \parallel 0.0757$$

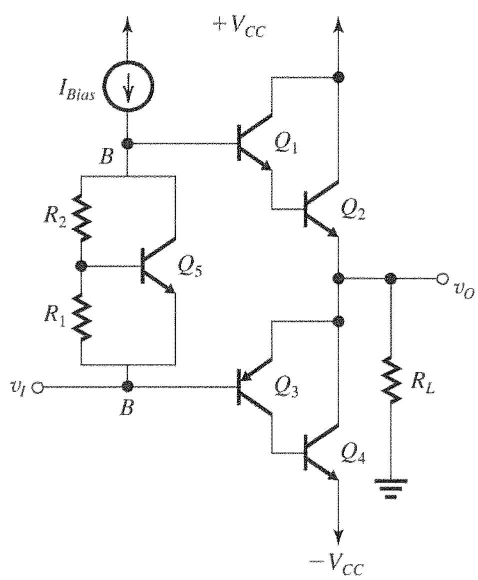
$$= 75.5\ \Omega$$

12.39 First consider the situation in the quiescent state and find V_{BB}.

$I_{Q2} = I_{Q4} = 2$ mA

$$V_{BE2} = V_{BE4} = 0.7 + 0.025 \ln\left(\frac{2}{10}\right)$$

$$= 0.660 \text{ V}$$



For Q_1 and Q_3, we have

$$I_C = \frac{2}{\beta} = \frac{2}{100} = 0.02 \text{ mA}$$

$$V_{BE1} = |V_{BE3}| = 0.7 + 0.025 \ln\left(\frac{0.02}{1}\right)$$

$$= 0.602 \text{ V}$$

$$I_{B1} = \frac{0.02 \text{ mA}}{\beta} = \frac{0.02}{100} = 0.2\ \mu\text{A}$$

$I_{\text{Bias}} = 100 \times$ Base current in B_1

$= 100 \times 0.2 = 20\ \mu$A

$$I_{R1,R2} = \frac{1}{10} \times 20\ \mu\text{A} = 2\ \mu\text{A}$$

and $I_{C5} = 20 - 2 = 18\ \mu$A

$$V_{BE5} = 0.7 + 0.025 \ln\left(\frac{18\ \mu}{1\ \text{m}}\right) \simeq 0.600 \text{ V}$$

$$V_{BB} = V_{BE1} + V_{BE2} + |V_{BE3}|$$

$$= 0.602 + 0.660 + 0.602$$

$$= 1.864 \text{ V}$$

$$R_1 + R_2 = \frac{1.864}{2\ \mu\text{A}} = 932 \text{ k}\Omega$$

$$R_1 = \frac{0.600}{2\ \mu\text{A}} = 300 \text{ k}\Omega$$

$$R_2 = 932 - 300 = 632 \text{ k}\Omega$$

Now find v_I for $v_O = 10$ V and $R_L = 1$ kΩ.

Q_2 is conducting most of the current and Q_4 conducting a negligible current.

$$\therefore\ I_{C2} \simeq I_L = \frac{10 \text{ V}}{1 \text{ k}\Omega} = 10 \text{ mA}$$

∴ The current through each of Q_1 and Q_2 increases by a factor of $\frac{10}{2} = 5$

Thus $V_{BE2} = 0.66 + 0.025\ln 5 = 0.700$ V

$V_{BE1} = 0.602 + 0.025\ln 5 = 0.642$ V

and $I_{B1} = 5 \times 0.2\ \mu\text{A} = 1\ \mu$A

∴ The current through the multiplier is $I_{\text{Bias}} - 1 = 20 - 1 = 19\ \mu$A. Assuming most of the decrease occurs in I_{C5}, we obtain

$$\therefore\ I_{C5} = 18 - 1 = 17\ \mu\text{A}$$

$$V_{BE5} = 0.7 + 0.025 \ln\left(\frac{17\ \mu\text{A}}{1\ \mu\text{A}}\right) = 0.598 \text{ V}$$

∴ V_{BB1}, the voltage across the multiplier is

$$V_{BB} = 0.598 \times \frac{932}{300} = 1.858 \text{ V}$$

It follows that V_{EB3} becomes

$V_{EB3} = 1.858 - 0.700 - 0.642 = 0.516$ V

i.e. V_{EB3} has decreased by $0.600 - 0.516 = 0.084$ V

Correspondinly, I_{C3} will decrease by a factor of $e^{\frac{-0.084}{0.025}} = 0.035$.

∴ I_{C4} becomes $0.035 \times 2 = 0.07$ mA

This value is close to zero, no iteration required.

$$\therefore\ v_I = 10 + 0.7 + 0.642 - 1.858$$

$$v_I = 9.484 \text{ V}$$

Now find v_I for $v_O = -10$ V and $R_L = 1$ kΩ.

$$i_L = \frac{-10}{1\text{ k}\Omega} = -10\text{ mA}$$

Assume that current through Q_2 is almost zero.

$\therefore I_{C4} \simeq 10$ mA

The current through Q_4 increases by a factor of 5 (relative to the quiescent value).

$\therefore$ The current through Q_3 must also increase by the same factor. Thus $|V_{BE3}| = 0.602 + 0.025\ln 5 = 0.642$ V

$|V_{BE3}|$ has increased by $0.642 - 0.602 = 0.04$ V. Since Q_1 and Q_2 are almost cut off, all of the I_{Bias} now flows through the V_{BE} multiplier. That is an increase of 0.2 μ. Assuming that most of the increase occurs in I_{C5}, V_{BE5} becomes

$$V_{BE5} = 0.7 + 0.025\ln\left(\frac{18.2\ \mu\text{A}}{1\text{ mA}}\right) \simeq 0.600\text{ V}$$

The voltage V_{BE5} remains almost constant, and the voltage across the multiplier will remain almost constant. Thus the increase in $|V_{EB3}|$ will result in an equal decrease in $|V_{BE1}| + |V_{BE2}|$, i.e.

$$V_{BE1} + V_{BE2} = 0.660 + 0.602 - 0.04$$

The current through each of Q_1 and Q_2 decreases by the same factor, let it be m; then

$$0.025\ln m + 0.025\ln m = -0.04\text{ V}$$

$$\Rightarrow m = 0.45$$

Thus $I_{C2} = 0.45 \times 2 = 0.9$ mA

Now do iteration

$I_{C4} = 10.9$

I_{C4} has increased by a factor of $\dfrac{10.9}{2} = 5.45$

$\therefore |V_{BE3}| = 0.602 + 0.025\ln 5.45$

$= 0.644$

$v_I = v_O + |V_{EB3}|$

$v_I \cong -10.644$ V

12.40 (a) Refer to the circuit in Fig. P12.40. While D_1 is conducting, the voltage at the emitter of Q_3 is $(V_{CC1} - V_D)$. For Q_3 to turn on, the voltage at its base must be at least equal to $V_{CC1} = 35$ V. This will occur when v_I reaches the value

$$v_I = V_{CC1} - V_{Z1} - V_{BB}$$

$$= 35 - 3.3 - 1.2 = 30.5\text{ V}$$

This is the positive threshold at which Q_3 is turned on.

(b) The power dissipated in the circuit is given by Eq. (12.19):

$$P_D = \frac{2}{\pi}\frac{\hat{V}_o}{R_L}V_{CC} - \frac{1}{2}\frac{\hat{V}_o^2}{R_L}$$

For 95% of the time, $\hat{V}_o = 30$ V, $V_{CC} = 35$ V,

$$P_D = \frac{1}{R_L}\left[\frac{2}{\pi} \times 30 \times 35 - \frac{1}{2} \times 30^2\right]$$

$$= \frac{218.5}{R_L}$$

For 5% of the time, $\hat{V}_o = 65$ V, $V_{CC} = 70$ V,

$$P_D = \frac{1}{R_L}\left[\frac{2}{\pi} \times 65 \times 70 - \frac{1}{2} \times 65^2\right]$$

$$= \frac{784.1}{R_L}$$

Thus, the total power dissipation is

$$P_D = \frac{218.5}{R_L} \times 0.95 + \frac{784.1}{R_L} \times 0.05$$

$$= \frac{246.8}{R_L} \quad (1)$$

This should be compared to the power dissipation of a class AB output stage operated from ±70 V. Here,

P_D (for 95% of the time)

$$= \frac{1}{R_L}\left[\frac{2}{\pi} \times 30 \times 70 - \frac{1}{2} \times 30^2\right]$$

$$= \frac{886.9}{R_L}$$

P_D for 5% of the time)

$$= \frac{1}{R_L}\left[\frac{2}{\pi} \times 65 \times 70 - \frac{1}{2} \times 65^2\right]$$

$$= \frac{784.1}{R_L}$$

$$\text{Total dissipation} = \frac{886.9}{R_L} \times 0.95 + \frac{784.1}{R_L} \times 0.05$$

$$= \frac{881.8}{R_L} \quad (2)$$

The results in (1) and (2) indicate that using the Class G circuit in Fig. P12.40 results in a reduction in P_D by a factor of 3.6!

12.41 Refer to Exercise 12.11 and Fig. 12.21.

Now Q_5 has $I_S = 20 \times 10^{-14}$ A. Thus,

$$2 \times 10^{-3} = 20 \times 10^{-14}e^{V_{BE}/V_T}$$

$$V_{BE} = 0.025\ln\frac{2 \times 10^{-3}}{20 \times 10^{-14}}$$

$= 0.576$ V

$$R_{E1} = \frac{0.576}{150\text{ mA}} = 3.84\Omega$$

For a normal peak current of 100 mA, the voltage drop across R_{E1} is 384 mV and the collector current is $20 \times 10^{-14} e^{370/25} = 0.94\ \mu\text{A}$

12.42 Refer to Exercise 12.11 and Fig. 12.21.

$$2 \times 10^{-3} = 10^{-14} e^{V_{BE}/V_T}$$

$$\Rightarrow V_{BE} = 0.650\text{ V}$$

$$R_{E1} = \frac{0.650\text{ V}}{100\text{ mA}} = 6.5\ \Omega$$

From a normal peak output current of 75 mA, we get

$$V_{BE} = 6.5 \times 75 = 487.5\text{ mV}$$

$$I_{C5} = 10^{-14} \times e^{487.5/25} = 2.9\ \mu\text{A}$$

12.43 Refer to Fig. P12.43.

$$2 \times 10^{-3} = 10^{-14} e^{V_{EB5}/V_T}$$

$$V_{EB5} = 0.025 \ln(2 \times 10^{11})$$

$$= 0.650\text{ V}$$

$$R = \frac{0.650\text{ V}}{100\text{ mA}} = 6.5\ \Omega$$

For a normal peak output current of 75 mA, we have

$$V_{EB5} = 6.5 \times 75 = 487.5\text{ mV}$$

$$I_{C5} = 10^{-14} \times e^{487.5/25}$$

$$= 2.9\ \mu\text{A}$$

12.44 Refer to Fig. 12.22.

At 125°C, we have

$$V_Z = 6.8 + (125 - 25) \times 2 = 7.0\text{ V}$$

Since $I_{C2} = 200\ \mu\text{A}$, then

$$V_{BE1} = 0.7 + 0.025 \ln\left(\frac{200}{100}\right) - 2\text{ mV} \times 100$$

$$= 0.517\text{ V}$$

Similarly, for Q_2 to conduct 200 μA, we need

$$V_{BE2} = 0.517\text{ V}$$

Now, the voltage across R_1 and R_2 is

$$V_{(R_1+R_2)} = V_Z - V_{BE1}$$

$$= 7 - 0.517 = 6.483\text{ V}$$

The voltage across R_2 is equal to V_{BE1}, thus

$$R_2 = \frac{0.517}{0.2\text{ mA}} = 2.59\text{ k}\Omega$$

The voltage across R_1 is given by $6.487 - 0.517 = 5.966$ V. Thus,

$$R_1 = \frac{5.966\text{ V}}{0.2\text{ mA}} = 29.8\text{ k}\Omega$$

Now, at 25°C, we have

$$V_Z = 6.8\text{ V}$$

Assume $V_{BE1} = 0.7$ V, then

$$V_{(R_1+R_2)} = 6.8 - 0.7 = 6.1\text{ V}$$

$$I_{(R_1+R_2)} = \frac{6.1}{2.59 + 29.8} = 0.188\ \mu\text{A}$$

Thus

$$V_{BE1} = 0.7 + 0.025 \ln\frac{188}{100} = 0.716\text{ V}$$

$$V_{(R_1+R_2)} = 6.8 - 0.716 = 6.084$$

$$V_{BE2} = 6.084 \times \frac{R_2}{R_1 + R_2}$$

$$= 6.084 \times \frac{2.59}{2.59 + 29.8} = 0.486\text{ V}$$

Thus,

$$I_{C2} = 100\ e^{(486-700)/25} = 0.019\ \mu\text{A}$$

12.45 (a) Refer to the circuit in Fig. 12.23.

$$R_{\text{out}} = R_{on} \parallel R_{op}$$

where

$$R_{on} = \frac{1}{g_{mn}} \parallel r_{on} \simeq 1/g_{mn}$$

$$R_{op} = \frac{1}{g_{mp}} \parallel r_{op} \simeq 1/g_{mp}$$

$$R_{\text{out}} = R_{on} \parallel R_{op} \simeq \frac{1}{g_{mn}} \parallel \frac{1}{g_{mp}}$$

Thus,

$$R_{\text{out}} \simeq \frac{1}{g_{mn} + g_{mp}} \qquad \text{Q.E.D.}$$

For matched devices, we have

$$g_{mn} = g_{mp} = g_m$$

$$R_{\text{out}} = \frac{1}{2g_m} \qquad \text{Q.E.D.}$$

(b) $R_{\text{out}} = 20\ \Omega$

$$\frac{1}{2g_m} = 20$$

$$\Rightarrow g_m = \frac{1}{40}\text{ A/V} = 25\text{ mA/V}$$

But,

$$g_m = k'(W/L)V_{OV}$$

$$25 = 200V_{OV}$$

$$\Rightarrow V_{OV} = \frac{25}{200} = 0.125 \text{ V}$$

$V_{GG} = 2V_{GS}$

$= 2(|V_t| + |V_{OV}|)$

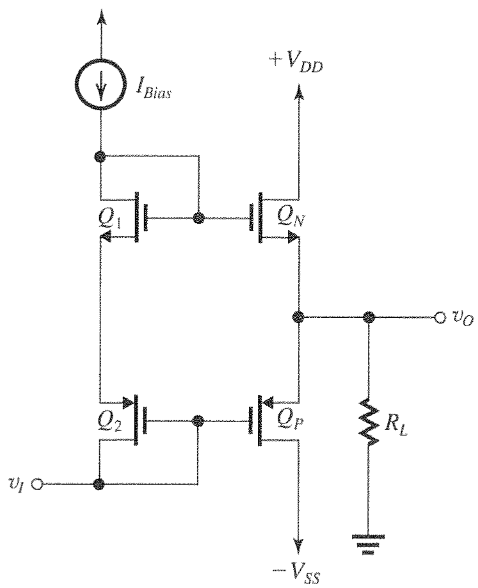
$= 2(0.5 + 0.125)$

$= 2 \times 0.625 = 1.25$ V

$$I_Q = \frac{1}{2}k'\left(\frac{W}{L}\right)V_{OV}^2$$

$$= \frac{1}{2} \times 200 \times 0.125^2 = 1.56 \text{ mA}$$

12.46



(a) under quiescent condition

$$\text{Voltage gain} = \frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}}$$

As shown in problem 12.45, for matched transistors we have

$$R_{\text{out}} = \frac{1}{2g_m}$$

Substituting for R_{out} above, we obtain for $\dfrac{v_o}{v_i}$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + \dfrac{1}{2g_m}} \qquad \text{Q.E.D.}$$

(b) $\text{Voltage gain} = 0.98 = \dfrac{R_L}{R_L + \dfrac{1}{2g_m}}$

$$0.98 = \frac{1000}{1000 + \dfrac{1}{2g_m}}$$

$\Rightarrow g_m = 24.5$ mA/V

For Q_1, we have $I_{Bias} = I_D$.

$$\therefore 0.2 = \frac{1}{2}k_1 V_{OV}^2$$

$$0.2 = \frac{1}{2} \times 20 \times V_{OV}^2$$

$\Rightarrow V_{OV} = 0.14$ V

For Q_N, we have

$g_m = k_n V_{OV}$

$24.5 = k_n \times 0.14$

$k_n = 173$ mA/V^2

$$n = \frac{k_n}{k_1} = \frac{173}{20}$$

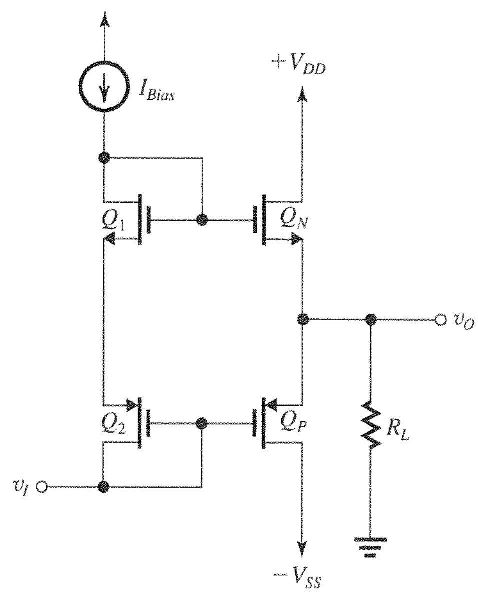
$= 8.66$

and $I_Q = nI_{\text{bias}}$

$= 8.66 \times 0.2$

$= 1.73$ mA

12.47



(a) Equation (12.43)

$$I_Q = I_{\text{Bias}}\frac{(W/L)_n}{(W/L)_1}$$

$$1 = 0.1\frac{(W/L)_n}{(W/L)_1}$$

$$\frac{(W/L)_n}{(W/L)_1} = 10$$

$$Q_1\text{: } I_{\text{Bias}} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_1 V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 0.250 \times \left(\frac{W}{L}\right)_1 \times (0.15)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 35.6$$

$$Q_2\text{: } 0.1 = \frac{1}{2} \times 0.100 \times \left(\frac{W}{L}\right)_2 \times (0.15)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 88.9$$

$$Q_N\text{: } 1 = \frac{1}{2} \times 0.250 \times \left(\frac{W}{L}\right)_N \times (0.15)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_N = 356$$

$$Q_P\text{: } 1 = \frac{1}{2} \times 0.100 \times \left(\frac{W}{L}\right)_P \times (0.15)^2$$

$$\left(\frac{W}{L}\right)_P = 889$$

(b) From the circuit we get $v_I = v_O - V_{SGP}$

Since $v_O = 0$, we have

$$v_I = -V_{SGP}$$

$$V_{SGP} = |V_{OV}| + |V_t|$$

$$= 0.15 + 0.45$$

$$= 0.6 \text{ V}$$

$$\therefore\ v_I = -V_{SGP} = -0.6 \text{ V}$$

(c) Using Eq. (12.46), we obtain

$$v_{O\max} = V_{DD} - V_{OV}|_{\text{Bias}} - V_{GSN}$$

To find V_{GSN}, use the equations

$$i_{DN\max} = \frac{1}{2} k_n' \frac{W}{L} (V_{GSN} - V_t)^2$$

$$10 = \frac{1}{2} \times 0.250 \times 356\,(V_{GSN} - V_t)^2$$

$$\Rightarrow V_{GSN} - V_t = 0.47 \text{ V}$$

$$V_{GSN} = V_t + 0.47 = 0.45 + 0.47 \simeq 0.92 \text{ V}$$

$$\therefore\ v_{O\max} = 2.5 - 0.2 - 0.92 = 1.38 \text{ V}$$

12.48 Since the circuit is symmetric, we shall consider only the situation with v_O at its maximum positive value,

$$v_{O\max} = +1.5 \text{ V}$$

From Eq. (12.46), we

$$v_{O\max} = V_{DD} - V_{OV}|_{\text{BIAS}} - V_{tn} = v_{OVN}$$

Assuming that $V_{OV}|_{\text{BIAS}}$ is the same at the value of overdrive voltage at which each of Q_1, Q_2, Q_N and Q_P is operating in the quiescent state,

$$V_{OV}|_{\text{BIAS}} = V_{OV}$$

$$v_{OVN} = V_{DD} - V_{OV} - V_{tn} - v_{O\max}$$

$$= 2.5 - V_{OV} - 0.5 - 1.5$$

$$= 0.5 - V_{OV}$$

Now,

$$i_{L\max} = i_{DN\max} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_N v_{OVN}^2$$

Thus,

$$10 I_Q = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_N (0.5 - V_{OV})^2 \qquad (1)$$

and in the quiescent state,

$$I_Q = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_N V_{OV}^2 \qquad (2)$$

Dividing Eq. (1) by Eq. (2) gives

$$10 = \left(\frac{0.5 - V_{OV}}{V_{OV}}\right)^2$$

$$\Rightarrow V_{OV} = 0.12 \text{ V}$$

$$V_{GG} = 2(V_t + V_{OV})$$

$$= 2(0.5 + 0.12)$$

$$= 1.24 \text{ V}$$

12.49 Refer to Fig. 12.24. Consider the situation when Q_N is conducting the maximum current of 20 mA,

$$20 = \frac{1}{2} k_n v_{OVN}^2$$

$$= \frac{1}{2} \times 200 v_{OVN}^2$$

$$\Rightarrow v_{OVN} = 0.45 \text{ V}$$

Thus,

$$v_{O\min} = -V_{SS} + v_{OVN}$$

$$= -2.5 + 0.45 = -2.05 \text{ V}$$

Because Q_N and Q_P are matched, a similar situation pertains when Q_P is supplying maximum current, and

$$v_{O\max} = +2.05 \text{ V}$$

Thus, the output voltage swing realized is ± 2.05 V.

12.50 From Eq. (12.57), we obtain

$$R_{\text{out}} = 1/\mu(g_{mp} + g_{mn})$$

where

$$g_{mp} = g_{mn} = \frac{2I_Q}{|V_{OV}|} = \frac{2 \times 2}{0.2} = 20 \text{ mA/V}$$

$$R_{\text{out}} = \frac{1}{5(20 + 20)} = \frac{1}{200} \text{ k}\Omega = 5\ \Omega$$

12.51 (a) From Eq. (12.68), we obtain

$$|\text{Gain error}| = \frac{1}{2\mu g_m R_L} \qquad (1)$$

From Eq. (12.57), we get

$$R_{\text{out}} = \frac{1}{\mu(g_{mn} + g_{mp})}$$

For $g_{mn} = g_{mp} = g_m$, we have

$$R_{\text{out}} = \frac{1}{2\mu g_m} \qquad (2)$$

Combining (1) and (2) yields

$$|\text{Gain error}| = \frac{R_{\text{out}}}{R_L} \qquad \text{Q.E.D.}$$

(b) For $R_L = 100\ \Omega$ and $|\text{Gain error}| = 3\%$,

$$R_{\text{out}} = 0.03 \times 100 = 3\ \Omega$$

But,

$$R_{\text{out}} = \frac{1}{2\mu g_m}$$

$$3 = \frac{1}{2 \times 5 \times g_m}$$

$$\Rightarrow g_m = \frac{1}{30} = 33.3\ \text{mA/V}$$

Using

$$g_m = \frac{2I_Q}{V_{OV}}$$

we obtain

$$33.3 = \frac{2 \times 2.5}{V_{OV}}$$

$$\Rightarrow V_{OV} = \frac{5}{33.3} = 0.15\ \text{V}$$

12.52 i_{DP} and i_{DN} are given by Eqs. (12.61) and (12.62) as

$$i_{DP} = I_Q\left(1 - \mu\frac{v_O - v_I}{V_{OV}}\right)^2 \qquad (1)$$

$$i_{DN} = I_Q\left(1 + \mu\frac{v_O - v_I}{V_{OV}}\right)^2 \qquad (2)$$

Equation (1) shows that Q_P turns off and $i_{DP} = 0$ when

$$\mu\frac{v_O - v_I}{V_{OV}} = 1$$

Substituting this into Eq. (2) gives

$$i_{DN} = I_Q(1+1)^2 = 4I_Q$$

Since $i_L = -i_{DN}$, we have

$$v_O = i_L R_L = -4I_Q R_L \qquad \text{Q.E.D.}$$

Similarly, Eq. (2) shows that Q_N turns off and $i_{DN} = 0$ when

$$\mu\frac{v_O - v_I}{V_{OV}} = -1$$

substituting this into Eq. (1) gives

$$i_{DP} = I_Q(1+1)^2 = 4I_Q$$

Since in this case $i_L = i_{DP}$, then

$$v_O = i_L R_L = 4I_Q R_L \qquad \text{Q.E.D.}$$

Thus, one of the two transistors turns off when $|i_L|$ reaches $4I_Q$.

12.53 (a) $I_Q = \frac{1}{2}k'\frac{W}{L}V_{OV}^2$

$$1.5 = \frac{1}{2} \times 0.1\left(\frac{W}{L}\right)_P (0.15)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_P = 1333.3$$

$$\left(\frac{W}{L}\right)_N = \frac{(W/L)_P}{k_n'/k_p'}$$

$$\left(\frac{W}{L}\right)_N = \frac{1333.3}{2.5} = 533.3$$

(b) $g_m = \dfrac{2I_Q}{V_{OV}} = \dfrac{2 \times 1.5}{0.15} = 20\ \text{mA/V}$

$$R_{\text{out}} = \frac{1}{2\mu g_m} \ (\text{where } g_{mn} = g_{mp} = g_m)$$

$$2.5 = \frac{1}{2\mu \times 20 \times 10^{-3}}$$

$$\Rightarrow \mu = 10\ \text{V/V}$$

(c) Gain error $= -\dfrac{1}{2\mu g_m R_L}$

$$= -\frac{1}{2 \times 10 \times 20 \times 10^{-3} \times 50} = -0.05$$

or -5%

(d) In the quiescent state the dc voltage at the output of each amplifier must be of the value that causes the current in Q_N and Q_P to be I_Q. Thus, for the Q_P amplifier the output voltage is

$$V_{DD} - V_{SG} = V_{DD} - |V_{tp}| - |V_{OV}|$$

$$= 2.5 - 0.5 - 0.15 = 1.85\ \text{V}$$

Similarly, the voltage at the output of the Q_N amplifier must be

$$-V_{SS} + V_{GS} = -2.5 + 0.5 + 0.15$$

$$= -1.85\ \text{V}$$

(e) Q_P will be supplying all the load current when Q_N cuts off. From Eq. (12.62) we see that Q_N cuts off when

$$\mu\frac{v_O - v_I}{V_{OV}} = -1$$

Substituting this in Eq. (12.61), we find the current i_{DP} to be

$$i_{DP} = I_Q(1+1)^2 = 4I_Q$$

This figure belongs to Problem 12.53, part (f).

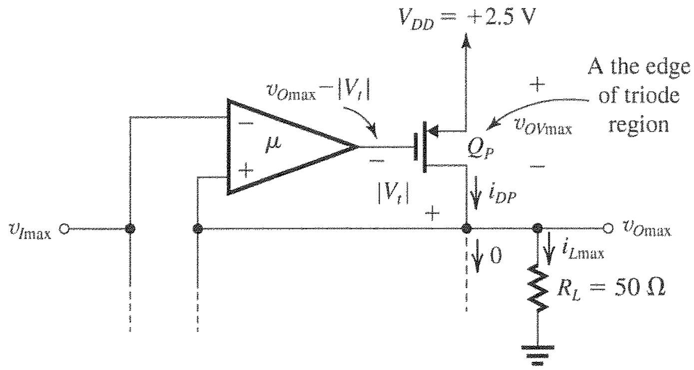


Figure 1

Since in this situation

$$i_L = i_{DP}$$

then

$$i_L = 4I_Q$$

and

$$v_O = 4I_Q R_L$$

$$= 4 \times 1.5 \times 10^{-3} \times 50 = 0.3 \text{ V}$$

Similarly, when $v_O = -0.3$ V, Q_P will cut off and all the current ($4I_Q = 6$ mA) will be supplied by Q_N.

(f) The situation at $v_O = v_{O\max}$ is illustrated in Fig. 1. Analysis of this circuit provides

$$i_{DP} = \frac{1}{2} \times k_n'\left(\frac{W}{L}\right)_n [2.5 - (v_{O\max} - 0.5) - 0.5]^2$$

$$\frac{v_{O\max}}{R_L} = \frac{1}{2} \times 0.25 \times 533.3(2.5 - v_{O\max})^2$$

$$\Rightarrow v_{O\max} = 1.77 \text{ V}$$

Similarly,

$$v_{O\min} = -1.77 \text{ V}$$

12.54 (a) From the circuit in Fig. P12.54 we see that

$$V_{B1} - V_{B4} = \left(1 + \frac{R_3}{R_4}\right)V_{BE6} + \left(1 + \frac{R_1}{R_2}\right)V_{BE5}$$

and

$$V_{GG} = (V_{B1} - V_{B4}) - (V_{BE1} + V_{BE2} + V_{EB3} + V_{EB4})$$

Thus

$$V_{GG} = \left(1 + \frac{R_3}{R_4}\right)V_{BE6} + \left(1 + \frac{R_1}{R_2}\right)V_{BE5} - 4V_{BE} \qquad \text{Q.E.D.} \qquad (1)$$

where V_{BE} denotes the magnitude of the base-emitter voltage of each of $Q_1 - Q_4$.

(b) From the circuit diagram we see that as the output transistors heat up, Q_6 also heats up. Thus in Eq. (1) only V_{BE6} changes with the temperature of the output stage, thus V_{GG} changes with temperature according to

$$\frac{\partial V_{GG}}{\partial T} = \left(1 + \frac{R_3}{R_4}\right)\frac{\partial V_{BE6}}{\partial T} \qquad \text{Q.E.D.} \qquad (2)$$

(c) To stabilize the operation of Q_N and Q_P as temperature changes, we arrange that

$$\frac{\partial V_{GG}}{\partial T} = \frac{\partial(V_{tN} + |V_{tP}|)}{\partial T}$$

$$= -3 - 3 = -6 \text{ mV/°C} \qquad (3)$$

From Eq. (2), we obtain

$$\frac{\partial V_{GG}}{\partial T} = \left(1 + \frac{R_3}{R_4}\right)\frac{\partial V_{BE6}}{\partial T}$$

$$= \left(1 + \frac{R_3}{R_4}\right) \times -2$$

$$= -2\left(1 + \frac{R_3}{R_4}\right) \text{ mV/°C} \qquad (4)$$

From Eqs. (3) and (4), we obtain

$$1 + \frac{R_3}{R_4} = 3$$

$$\Rightarrow \frac{R_3}{R_4} = 2$$

(d) $I_{DN} = I_{DP} = 100$ mA

$$100 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_N V_{OVN}^2$$

$$100 = \frac{1}{2} \times 2 \times 10^3 \times V_{OVN}^2$$

$$\Rightarrow V_{OVN} = 0.316 \text{ V}$$

Similarly,

$|V_{OVP}| = 0.316$ V

Thus,

$V_{GSN} = |V_{GSP}| = 0.316 + 3 = 3.316$ V

and

$V_{GG} = 2 \times 3.316 = 6.632$ V

To establish a quiescent current of 20 mA in the driver stage, we use

$$R = \frac{V_{GG}}{20} = \frac{6.632}{20} = 0.3316\ \text{k}\Omega$$

$\simeq 332\ \Omega$

$V_{B1} - V_{B4} = V_{GG} + 4V_{BE}$

$= 6.632 + 4 \times 0.7 = 9.432$ V

Thus,

$$\left(1 + \frac{R_3}{R_4}\right)V_{BE6} + \left(1 + \frac{R_1}{R_2}\right)V_{BE5} = 9.432\ \text{V}$$

$$(1+2) \times 0.7 + \left(1 + \frac{R_1}{R_2}\right) \times 0.7 = 9.432$$

$$\Rightarrow \frac{R_1}{R_2} = 9.47$$

12.55 Refer to the circuit of Fig. 12.29 (P 963).

Resistors R_2 and R_3 control the gain,

$$A_v = -\frac{2R_2}{R_3}$$

Resistor R_3 controls the gain alone. Resistor R_2 affects both the gain and the dc output level. To see the later point, equate I_3 and I_4 from Eqs. (12.69) and (12.70) to obtain

$$\frac{V_S - 3V_{EB}}{R_1} = \frac{V_O - 2V_{EB}}{R_2}$$

$$\Rightarrow V_O = 2V_{EB} + \frac{R_2}{R_1}V_S - \frac{3R_2}{R_1}V_{EB}$$

$$= \frac{R_2}{R_1}V_S + \left(2 - \frac{3R_2}{R_1}\right)V_{EB}$$

For $V_O \simeq \frac{2}{3}V_S$, select $\frac{R_2}{R_1} = \frac{2}{3}$

$$R_2 = \frac{2R_1}{3} = \frac{100}{3} = 33.3\ \text{k}\Omega$$

To keep the gain unchanged, we must change R_3 so that

$$\frac{2R_2}{R_3} = 50$$

$$R_3 = \frac{2 \times (100/3)}{50} = \frac{4}{3} = 1.33\ \text{k}\Omega$$

12.56 Refer to Fig. 12.29 with $V_S = 22$ V.

$V_{B1} \simeq 0$

$V_{E1} \simeq 0.7$ V

$V_{E3} \simeq 1.4$ V

$V_{C10} = 22 - 0.7 = 21.3$ V

$$I_{E3} = \frac{21.3 - 1.4}{50} \simeq 0.4\ \text{mA}$$

$$I_{E1} = I_{B3} = \frac{I_{E3}}{\beta_P + 1} = \frac{0.4}{21} = 19\ \mu\text{A}$$

$$I_{B1} = \frac{I_{E1}}{\beta_P + 1} = \frac{19}{21} = 0.9\ \mu\text{A}$$

$V_{B1} = I_{B1} \times R_4 = 0.9 \times 10^{-3} \times 150 = 0.136$ V

We can use this value to obtain I_{E1}:

$V_{E1} = 0.836$ V

$V_{E3} = 1.536$ V

$$I_{E3} = \frac{21.3 - 1.536}{50} \simeq 0.4\ \text{mA}$$

(almost no change)

$I_{E1} \simeq 19\ \mu$A

$I_{E4} = I_{E3} = 0.4$ mA

$I_{E2} = I_{E1} = 19\ \mu$A

$$I_{E5} \simeq I_{C3} = 0.4 \times \frac{20}{21} = 0.38\ \text{mA}$$

$I_{E6} = I_{E5} = 0.38$ mA

$I_{R1} = I_{R2} \simeq 0.4$ mA

$V_O = V_{E4} + I_{R2}R_2$

$= V_{E3} + I_{R2}R_2$

$= 1.536 + 0.4 \times 25$

$= 11.54$ V

12.57 Refer to Fig. 12.31. To limit P_D to 2 W, we need to limit the supply voltage to

$V_S = 16$ V

The $V_S = 16$ V graph intersects the THD = 3% line at $P_L = 2.7$ W, which is the maximum possible load power. Thus,

$$\frac{(\hat{V}_o/\sqrt{2})^2}{R_L} = 2.7$$

$$\hat{V}_o = \sqrt{2.7 \times 8 \times 2} = 6.57\ \text{V}$$

which means that an approximately 13-V peak-to-peak sinusoid is needed.

This figure belongs to Problem 12.58.

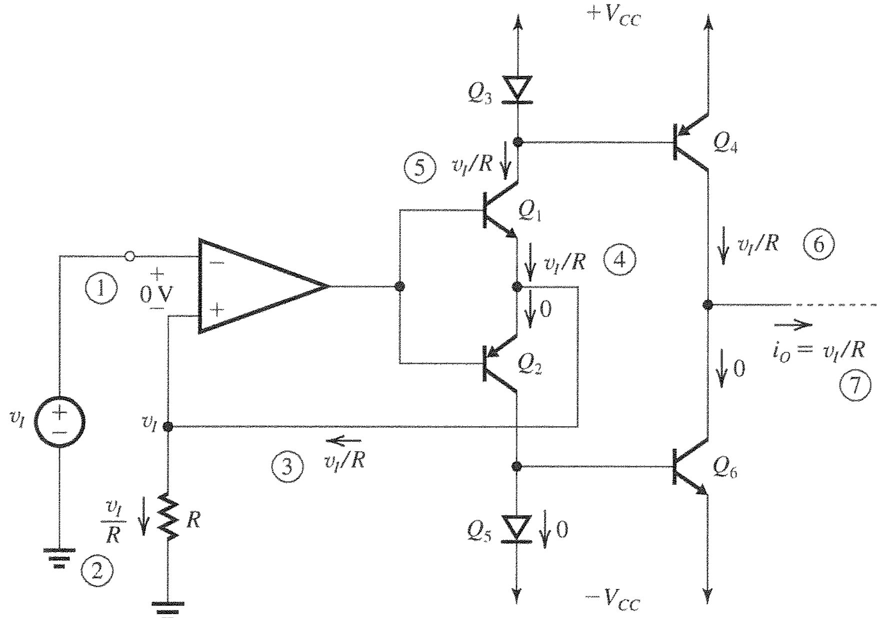


Figure 1

12.58 Figure 1 shows the currents in the circuit for the case where v_I is positive and assuming an op amp with very high gain (hence the 0 V between its two input terminals) and all β's are very high. The result is that

$$i_O = \frac{v_I}{R}$$

If v_I is negative, the current through R reverses direction and is thus supplied by Q_2 and then mirrored to the output by the mirror $Q_5 - Q_6$, resulting in $i_O = v_I/R$ but reversed in direction.

Consider next the effect of finite transistor β. For the case in Fig. 1, we have

$$i_{E1} = \frac{v_I}{R}$$

$$i_{C1} = \alpha_1 i_{E1} = \frac{\beta}{\beta + 1}\left(\frac{v_I}{R}\right)$$

$$i_{C4} = i_{C1}\frac{1}{1 + \dfrac{2}{\beta}}$$

Thus,

$$i_O = \frac{\beta}{\beta + 1}\,\frac{\beta}{\beta + 2}\,\frac{v_I}{R}$$

$$= \frac{100}{101} \times \frac{100}{102} \times \frac{v_I}{R}$$

$$\simeq 0.97\frac{v_I}{R}$$

12.59 Refer to Fig. 12.32.

$$\text{Gain} = 2K = 8$$

$$K = 4$$

$$\frac{R_4}{R_3} = K = 4$$

$$\Rightarrow R_4 = 40 \text{ k}\Omega$$

$$1 + \frac{R_2}{R_1} = 4$$

$$\frac{R_2}{R_1} = 3$$

$$\Rightarrow R_2 = 30 \text{ k}\Omega$$

12.60 The analysis is shown in Fig. 1 (on next page), from which the gain is found as

$$\frac{v_O}{v_I} = 1 + \frac{R_2 + R_3}{R_1}$$

The largest sinusoid that can be provided across R_L will have a peak amplitude of $2 \times 13 = 26$ V. To ensure that the signals v_{O1} and v_{O2} are complementary, then

$$1 + \frac{R_2}{R_1} = \frac{R_3}{R_1}$$

Selecting $R_1 = 1$ kΩ, we obtain

$$1 + R_2 = R_3 \qquad (1)$$

This figure belongs to Problem 12.60.

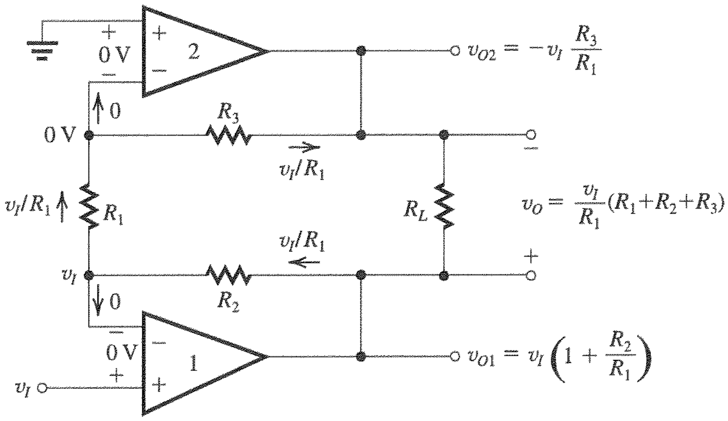
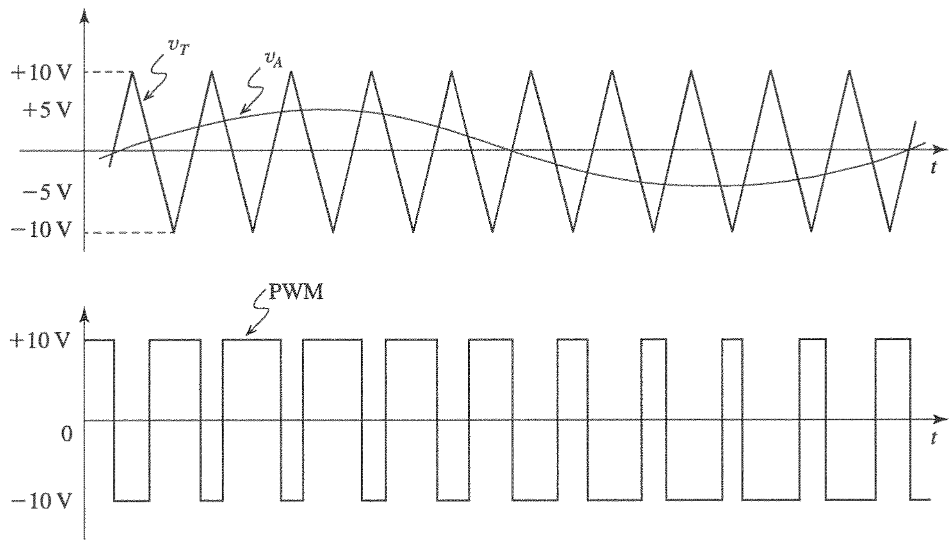


Figure 1

This figure belongs to Problem 12.61.



and to obtain a gain of 8 V/V we write

$$1 + \frac{R_2 + R_3}{R_1} = 8$$

$$1 + R_2 + R_3 = 8 \qquad (2)$$

Solving (1) and (2) simultaneously gives

$$2(1 + R_2) = 8$$

$$\Rightarrow R_2 = 3\ \text{k}\Omega$$

$$R_3 = 4\ \text{k}\Omega$$

12.61 See figure.

12.62
Average $= +10 \times 0.65 - 10 \times 0.35 = +3$ V

If duty cycle changed to 0.35, the average becomes

$$= +10 \times 0.35 - 10 \times 0.65 = -3\ \text{V}$$

12.63 (a) Maximum peak voltage across $R = V_{DD}$

Maximum power supplied to load

$$= \frac{(V_{DD}/\sqrt{2})^2}{R} = \frac{V_{DD}^2}{2R}$$

(b) Power loss $= 4f_sCV_{DD}^2$

$$\eta = \frac{P_L}{P_L + P_{\text{loss}}}$$

$$= \frac{V_{DD}^2/2R}{(V_{DD}^2/2R) + 4f_sCV_{DD}^2}$$

$$= \frac{1}{1 + 8f_sCR}$$

For $f_s = 250$ kHz, $C = 1000$ pf and $R = 16\ \Omega$

$$\eta = \frac{1}{1 + 8 \times 250 \times 10^3 \times 1000 \times 10^{-12} \times 16}$$

$$= 97\%$$

12.64

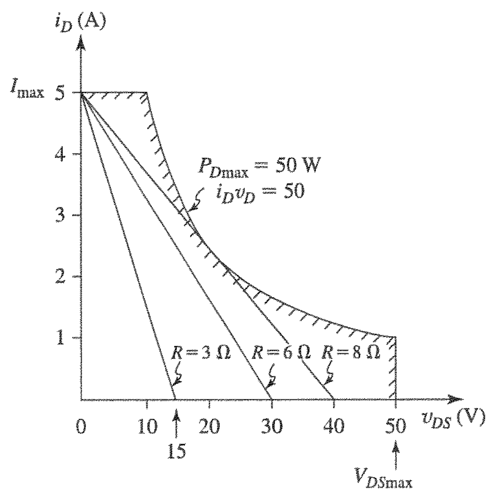


Figure 1

(a) Figure 1 shows the SOA boundaries.

(b) For the CS configuration in Fig. P12.64,

$$V_{DS} = V_{DD} - RI_D \qquad (1)$$

We see that maximum V_{DS} occurs when $I_D = 0$ and the resulting maximum V_{DS} is

$$V_{DS\max} = V_{DD}$$

Writing (1) in the alternative form

$$I_D = \frac{V_{DD} - V_{DS}}{R} \qquad (2)$$

shows that maximum I_D is obtained when $V_{DS} = 0$ and the resulting maximum I_D is

$$I_{D\max} = \frac{V_{DD}}{R}$$

The power dissipation in the transistor is given by

$$P_D = V_{DS} I_D$$

$$= (V_{DD} - RI_D)I_D$$

P_D will be maximum when

$$\frac{\partial P_D}{\partial I_D} = 0$$

that is,

$$V_{DD} - 2RI_D = 0$$

$$\Rightarrow RI_D = \frac{V_{DD}}{2}$$

or

$$V_{DS} = \frac{V_{DD}}{2}$$

The corresponding $P_{D\max}$ is

$$P_{D\max} = V_{DS} I_D$$

$$= \frac{V_{DD}}{2} \frac{V_{DD}}{2R}$$

$$= \frac{V_{DD}^2}{4R}$$

(c) For $V_{DD} = 40$ V, $v_{DS\max} = 40$ V. Now, since V_{DS} and I_D are related by the linear relationship in (1) or (2), the straight line representing this relationship on the $i_D - v_{DS}$ plane must pass by the point $v_{DS} = 40$ V and $i_D = 0$. Now we are searching for the straight line with maximum slope that clears the hyperbola and intersects the vertical axis at 5 A or less. For this case, this straight line is the one joining the points (40, 0) and (0, 5). It is a tangent to the hyperbola at $v_{DS} = \frac{V_{DD}}{2} = 20$ V, which is the point of maximum power dissipation. For this straight line

$$R = \frac{40\text{ V}}{5\text{ A}} = 8\ \Omega$$

$$I_{D\max} = 5\text{ A}$$

$$P_{D\max} = \frac{V_{DD}^2}{4R} = \frac{40^2}{4 \times 8} = 50\text{ W}$$

(d) For $V_{DD} = 30$ V: Following a process similar to that in (c), we find

$$R = \frac{30\text{ V}}{5\text{ A}} = 6\ \Omega$$

$$I_{D\max} = 5\text{ A}$$

$$P_{D\max} = \frac{30^2}{4 \times 6} = 37.5\text{ W}$$

The locus of the operating point is shown in Fig. 1.

(e) For $V_{DD} = 15$ V, we have

$$R = \frac{15\text{ V}}{5\text{ A}} = 3\ \Omega$$

$$I_{D\max} = 5\text{ A}$$

$$P_{D\max} = \frac{15^2}{4 \times 3} = 18.75\text{ W}$$

The locus of the operating point is shown in Fig. 1.

12.65 Power rating $= \dfrac{130 - 30}{2.5} = 40$ W

$$I_{Cav} \le \frac{40}{20} = 2.0\text{ A}$$

12.66 $\theta_{JA} = \dfrac{150 - 25}{10} = 12.5°\text{C/W}$

At 50°C, Power rating

$$= \frac{150 - 50}{12.5} = 8\text{ W}$$

$$T_J = 50 + 12.5 \times 5 = 112.5°\text{C}$$

12.67 $T_J \le 50 + 3 \times 20 = 110°\text{C}$

$$V_{BE} = 800 - 2 \times (110 - 25) = 630\text{ mV}$$

$$= 0.63\text{ V}$$

12.68 (a) $\theta_{JA} = \dfrac{T_{J\max} - T_{A0}}{P_{D0}}$

$= \dfrac{100 - 25}{2} = 37.5°C/W$

(b) At $T_A = 50°C$, we have

$P_{D\max} = \dfrac{T_{J\max} - T_A}{\theta_{JA}}$

$= \dfrac{100 - 50}{37.5} = 1.33$ W

(c) $T_J = 25° + 37.5 \times 1 = 62.5°C$

12.69 $T_C - T_A = \theta_{CA} P_D$

$= (\theta_{CS} + \theta_{SA})\, P_D$

$\Rightarrow P_D = \dfrac{T_C - T_A}{\theta_{CS} + \theta_{SA}} = \dfrac{97 - 25}{0.5 + 0.1} = 120$ W

$T_J - T_C = \theta_{JC} P_D$

$150 - 97 = \theta_{JC} \times 120$

$\Rightarrow \theta_{JC} = 0.44°C/W$

12.70 $\theta_{JC} = \dfrac{T_J - T_C}{P_D} = \dfrac{180° - 30°}{50} = 3°C/W$

$T_J - T_S = \theta_{JS} P_D$

$180° - T_S = (\theta_{JC} + \theta_{CS}) P_D$

$\Rightarrow T_S = 180 - (3 + 0.6) \times 30 = 72°$

$T_S - T_A = \theta_{SA} P_D$

$72 - 27 = \theta_{SA} \times 30$

$\Rightarrow \theta_{SA} = 1.5°C/W$

Required heat-sink length $= \dfrac{6°C/W/cm}{1.5°C/W}$

$= 4$ cm

13.1 Using Eq. (13.2), we get

$V_{ICM\,\min} = -V_{SS} + V_{tn} + V_{OV3} - |V_{tp}|$

$= -1 + 0.4 + 0.2 - 0.4 = -0.8$ V

Using Eq. (13.3), we obtain

$V_{ICM\,\max} = V_{DD} - |V_{OV5}| - |V_{tp}| - |V_{OV1}|$

$= 1 - 0.2 - 0.4 - 0.2 = +0.2$ V

Thus,

$-0.8 \text{ V} \leq V_{ICM} \leq +0.2 \text{ V}$

Using Eq. (13.5), we get

$-V_{SS} + V_{OV6} \leq v_O \leq V_{DD} - |V_{OV7}|$

Thus,

$-0.8 \text{ V} \leq v_O \leq +0.8 \text{ V}$

13.2 For NMOS devices, we have

$V_A = 25 \times 0.3 = 7.5$ V

For PMOS devices,

$|V_A| = 20 \times 0.3 = 6$ V

Using Eq. (13.13),

$$A_1 = -\frac{2}{|V_{OV1}|} \Big/ \left[\frac{1}{|V_{A2}|} + \frac{1}{V_{A4}}\right]$$

$$= -\frac{2}{0.15} \Big/ \left(\frac{1}{6} + \frac{1}{7.5}\right)$$

$= -44.4$ V/V

Using Eq. (13.20), we obtain

$$A_2 = -\frac{2}{V_{OV6}} \Big/ \left[\frac{1}{V_{A6}} + \frac{1}{|V_{A7}|}\right]$$

$$= -\frac{2}{0.2} \Big/ \left[\frac{1}{7.5} + \frac{1}{6}\right]$$

$= -33.3$ V/V

$A = A_1 A_2 = 1478.5$ V/V

$$r_{o6} = \frac{7.5}{0.3} = 25 \text{ k}\Omega$$

$$r_{o7} = \frac{6}{0.3} = 20 \text{ k}\Omega$$

$R_o = r_{o6} \parallel r_{o7} = 11.1 \text{ k}\Omega$

For a unity-gain voltage amplifier using this op amp, we have

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{11.1 \text{ k}\Omega}{1 + 1481.5 \times 1}$$

$= 7.5\ \Omega$

13.3 For all transistors, we have

$|V_A| = 20 \times 0.3 = 6$ V

Using Eq. (13.13), we get

$$A_1 = -\frac{2}{|V_{OV}|} \Big/ \frac{2}{|V_A|} = -\frac{6}{|V_{OV}|}$$

Using Eq. (13.20), we obtain

$$A_2 = -\frac{2}{|V_{OV}|} \Big/ \frac{2}{|V_A|} = -\frac{6}{|V_{OV}|}$$

$$A = A_1 A_2 = \frac{36}{|V_{OV}|^2}$$

$$1600 = \frac{36}{|V_{OV}|^2}$$

$\Rightarrow |V_{OV}| = 0.15$ V

13.4 For the op amp to not have a systematic offset voltage, the condition in Eq. (13.1) must be satisfied, that is,

$$\frac{(W/L)_6}{(W/L)_4} = 2\frac{(W/L)_7}{(W/L)_5}$$

$$\frac{W/0.3}{6/0.3} = 2\frac{45/0.3}{30/0.3}$$

$\Rightarrow W = 18\ \mu$m

Refer to Fig. 13.1:

$I_{D8} = I_{\text{REF}} = 40\ \mu$A

$$I = I_{D5} = I_{\text{REF}}\frac{W_5}{W_8} = 40 \times \frac{30}{6} = 200\ \mu\text{A}$$

$$I_{D7} = I_{\text{REF}}\frac{W_7}{W_8} = 40 \times \frac{45}{6} = 300\ \mu\text{A}$$

$I_{D6} = 300\ \mu$A

$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I}{2} = 100\ \mu\text{A}$$

The overdrive voltage at which each transistor is operating is determined from

$$I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}V_{OV}^2$$

Then V_{GS} is found from

$|V_{GS}| = |V_t| + |V_{OV}|$

The transconductance at which each transistor is operating is obtained from

$$g_m = \frac{2I_D}{V_{OV}}$$

The output resistance of each transistor is found from

$$r_o = \frac{|V_A|}{I_D}$$

The results are summarized in the following table:

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
I_D (μA)	100	100	100	100	200	300	300	40
$\lvert V_{OV}\rvert$ (V)	0.15	0.15	0.19	0.19	0.24	0.19	0.24	0.24
$\lvert V_{GS}\rvert$ (V)	0.6	0.6	0.64	0.64	0.69	0.64	0.69	0.69
g_m (mA/V)	1.33	1.33	1.05	1.05	1.67	3.16	2.5	0.33
r_o (kΩ)	150	150	150	150	75	50	50	375

$A_1 = -g_{m1,2}(r_{o2} \parallel r_{o4})$

$= -1.33(150 \parallel 150) = -100$ V/V

$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$

$= -3.16(50 \parallel 50) = -79$ V/V

$A = A_1 A_2 = 7900$ V/V

Using Eq. (13.21), we obtain

$V_{ICM\,\min} = -V_{SS} + V_{tn} + V_{OV3} - |V_{tp}|$

$V_{ICM\,\min} = -1 + 0.45 + 0.19 - 0.45$

$= -0.81$ V

Using Eq. (13.3), we get

$V_{ICM\,\max} = V_{DD} - |V_{OV5}| - |V_{tp}| - |V_{OV1}|$

$= 1 - 0.24 - 0.45 - 0.15$

$= +0.16$ V

Thus,

$-0.8 \text{ V} \le V_{ICM} \le +0.16$ V

Using Eq. (13.5), we obtain

$-V_{SS} + V_{OV6} \le v_O \le V_{DD} - |V_{OV7}|$

Thus,

$-1 + 0.19 \le v_O \le 1 - 0.24$

$-0.81 \text{ V} \le v_O \le 0.76$ V

13.5 From Eq. (13.24), we have

$\text{CMRR} = [g_{m1}(r_{o2} \parallel r_{o4})]\,[2g_{m3}R_{SS}]$

where

$$g_{m1} = \frac{I}{|V_{OV}|}$$

$$r_{o2} = r_{o4} = |V_A|/(I/2) = \frac{2|V_A|}{I}$$

$$g_{m3} = \frac{I}{|V_{OV}|}$$

$$R_{SS} = r_{o5} = \frac{|V_A|}{I}$$

Thus,

$$\text{CMRR} = \frac{I}{|V_{OV}|} \times \frac{1}{2} \times \frac{2|V_A|}{I} \times 2 \times \frac{I}{|V_{OV}|} \times \frac{|V_A|}{I}$$

$$= 2\frac{|V_A|^2}{|V_{OV}|^2}$$

For CMRR = 72 dB = 4000, we have

$$4000 = 2 \times \frac{|V_A|^2}{0.15^2}$$

$\Rightarrow |V_A| = 6.7$ V

Since $|V_A| = |V_A'|L$, we have

$6.7 = 15L$

$\Rightarrow L = 0.45$ μm

$$A_v = \left|\frac{V_A}{V_{OV}}\right|^2 = \left(\frac{6.7}{0.15}\right)^2 = 2000 \text{ V/V}$$

13.6 From Eq. (13.36), we obtain

$$f_t = \frac{G_{m1}}{2\pi C_C}$$

Thus,

$$C_C = \frac{G_{m1}}{2\pi f_t} = \frac{0.8 \times 10^{-3}}{2\pi \times 120 \times 10^6} = 1.06 \text{ pF}$$

From Eq. (13.35), we get

$$f_{P2} = \frac{G_{m2}}{2\pi C_2}$$

$$= \frac{2.4 \times 10^{-3}}{2\pi \times 1.2 \times 10^{-12}} = 318.3 \text{ MHz}$$

From Eq. (13.31), we get

$$f_Z = \frac{G_{m2}}{2\pi C_C}$$

$$= \frac{2.4 \times 10^{-3}}{2\pi \times 1.06 \times 10^{-12}} = 360 \text{ MHz}$$

This figure belongs to Problem 13.7, part (b).

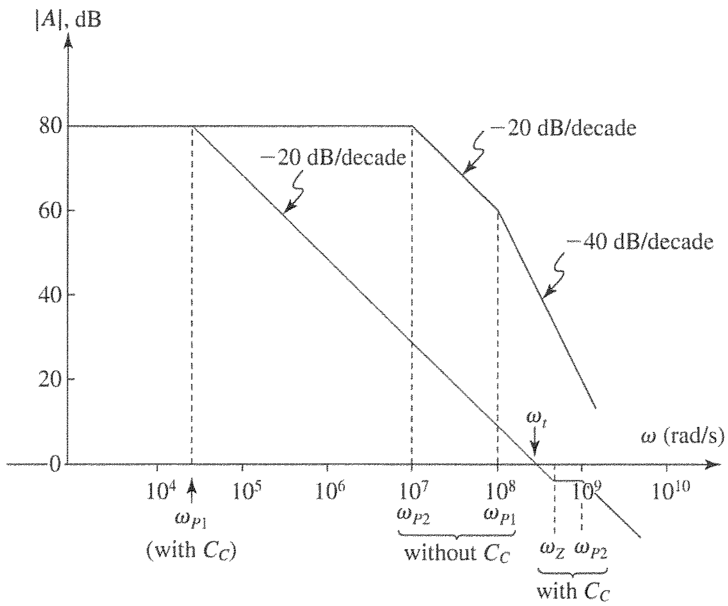


Figure 1

13.7 (a) $A = G_{m1}R_1G_{m2}R_2$

$= 1 \times 100 \times 2 \times 50 = 10,000$ V/V

(b) Without C_C connected:

$$\omega_{P1} = \frac{1}{C_1R_1} = \frac{1}{0.1 \times 10^{-12} \times 100 \times 10^3}$$

$= 10^8$ rad/s

$$\omega_{P2} = \frac{1}{C_2R_2} = \frac{1}{2 \times 10^{-12} \times 50 \times 10^3}$$

$= 10^7$ rad/s

Figure 1 shows a Bode plot for the gain magnitude.

(c) With C_C connected:

Using Eq. (13.35), we obtain

$$\omega_{P2} = \frac{G_{m2}}{C_2}$$

$$= \frac{2 \times 10^{-3}}{2 \times 10^{-12}} = 10^9 \text{ rad/s}$$

For ω_t two octaves below ω_{P2}, we have

$$\omega_t = \frac{10^9}{4} \text{ rad/s}$$

Using Eq. (13.36), we get

$$\omega_t = \frac{G_{m2}}{C_C}$$

Thus,

$$\frac{10^9}{4} = \frac{1 \times 10^{-3}}{C_C}$$

$\Rightarrow C_C = 4$ pF

Now using Eq. (13.34), we obtain

$$\omega_{P1} = \frac{1}{R_1C_CG_{m2}R_2}$$

$$= \frac{1}{100 \times 10^3 \times 4 \times 10^{-12} \times 2 \times 10^{-3} \times 50 \times 10^3}$$

$$= \frac{10^5}{4} \text{ rad/s} = 25{,}000 \text{ rad/s}$$

Using Eq. (13.31), we get

$$\omega_Z = \frac{G_{m2}}{C_C}$$

$$= \frac{2 \times 10^{-3}}{4 \times 10^{-12}} = \frac{10^9}{2} \text{ rad/s} = 5 \times 10^8 \text{ rad/s}$$

The Bode plot for the gain magnitude with C_C connected is shown in Fig. 1.

13.8 $G_{m1} = 1$ mA/V

$G_{m2} = 3$ mA/V

$C_1 = 0.2$ pF

$C_2 = 3$ pF

Using Eq. (13.36), we obtain

$$f_t = \frac{G_{m1}}{2\pi C_C}$$

Thus,

$$C_C = \frac{G_{m1}}{2\pi f_t} = \frac{1 \times 10^{-3}}{2\pi \times 50 \times 10^6} = 3.18 \text{ pF}$$

Using Eq. (13.35), we get

$$f_{P2} = \frac{G_{m2}}{2\pi C_2}$$

$$= \frac{3 \times 10^{-3}}{2\pi \times 3 \times 10^{-12}}$$

$$= 159.2 \text{ MHz}$$

Using Eq. (13.31), we obtain

$$f_Z = \frac{G_{m2}}{2\pi C_C}$$

$$= \frac{3 \times 10^{-3}}{2\pi \times 3.18 \times 10^{-12}}$$

$$= 150 \text{ MHz}$$

Thus f_t is lower than f_{P2} and f_Z.

13.9 (a) Using Eq. (13.36), we get

$$f_t = \frac{G_{m1}}{2\pi C_C}$$

$$C_C = \frac{G_{m1}}{2\pi f_t}$$

$$= \frac{1 \times 10^{-3}}{2\pi \times 100 \times 10^6} = 1.59 \text{ pF}$$

(b) $f_{P2} = \dfrac{G_{m2}}{2\pi C_2}$

$$= \frac{2 \times 10^{-3}}{2\pi \times 1 \times 10^{-12}} = 318 \text{ MHz}$$

$$f_Z = \frac{G_{m2}}{2\pi C_C} = \frac{2 \times 10^{-3}}{2\pi \times 1 \times 1.59 \times 10^{-12}}$$

$$= 200 \text{ MHz}$$

To obtain f_{P1}, we need to know the dc gain of the op amp, A_0, then

$$f_{P1} = \frac{f_t}{A_0}$$

The value of A_0 is not specified in the problem statement!

(c) $\phi_{P2} = -\tan^{-1}\left(\dfrac{f_t}{f_{P2}}\right)$

$$= -\tan^{-1}\left(\frac{100}{318}\right) = -17.5^\circ$$

$$\phi_Z = -\tan^{-1}\left(\frac{f_t}{f_Z}\right)$$

$$\phi_Z = -\tan^{-1}\left(\frac{100}{200}\right) = -26.6^\circ$$

$$\phi_{\text{total}} = 90^\circ + 17.5 + 26.6 = 134^\circ$$

Phase margin $= 180 - 134 = 46^\circ$

(d) From Eq. (13.44), for

$$f_Z = \infty$$

we select

$$R = \frac{1}{G_{m2}} = \frac{1}{2} = 0.5 \text{ k}\Omega = 500 \text{ }\Omega$$

Phase margin $= 180^\circ - (90^\circ + 17.5^\circ) = 72.5^\circ$

(e) To obtain a phase margin of 85°, we need the left-half plane zero to provide at f_t a phase angle of $85^\circ - 72.5^\circ = 12.5^\circ$. Thus,

$$12.5^\circ = \tan^{-1}\left(\frac{f_t}{f_Z}\right)$$

$$f_Z = \frac{f_t}{\tan 12.5^\circ} = \frac{100}{\tan 12.5^\circ} = 451 \text{ MHz}$$

From Eq. (13.44), we have

$$-f_Z = \frac{1}{2\pi C_C\left(\dfrac{1}{G_{m2}} - R\right)}$$

$$\Rightarrow R = 722 \text{ }\Omega$$

13.10 $G_{m1} = 0.3$ mA/V

$G_{m2} = 0.6$ mA/V

$r_{o2} = r_{o4} = 222$ kΩ

$r_{o6} = r_{o7} = 111$ kΩ

$C_2 = 1$ pF

(a) $A = G_{m1}(r_{o2} \parallel r_{o4})G_{m2}(r_{o6} \parallel r_{o7})$

$= 0.3(222 \parallel 222) \times 0.6(111 \parallel 111)$

$= 33.3 \times 33.3 = 1109$ V/V

(b) $f_{P2} = \dfrac{G_{m2}}{2\pi C_2}$

$$= \frac{0.6 \times 10^{-3}}{2\pi \times 1 \times 10^{-12}} = 95.5 \text{ MHz}$$

(c) $R = \dfrac{1}{G_{m2}} = \dfrac{1}{0.6 \times 10^{-3}} = 1.67 \text{ k}\Omega$

(d) Phase margin $= 180 - 90 - \tan^{-1}\left(\dfrac{f_t}{f_{P2}}\right)$

$$80^\circ = 90 - \tan^{-1}\left(\frac{f_t}{f_{P2}}\right)$$

$$f_t = f_{P2} \tan 10^\circ$$

$$= 95.5 \times 0.176 = 16.8 \text{ MHz}$$

Using Eq. (13.36), we obtain

$$C_C = \frac{G_{m1}}{2\pi f_t} = \frac{0.3 \times 10^{-3}}{2\pi \times 16.8 \times 10^6} = 2.84 \text{ pF}$$

The dominant pole will be at a frequency

$$f_{P1} = \frac{f_t}{\text{DC Gain}} = \frac{16.8 \times 10^6}{1109}$$

$= 15.1$ kHz

(e) Since

$$f_t = \frac{G_{m1}}{2\pi C_C}$$

to double f_t, C_C must be reduced by a factor of 2,

$$C_C = \frac{2.84}{2} = 1.42 \text{ pF}$$

At the new $f_t = 2 \times 16.8 = 33.6$ MHz, we have

$$\phi_{P2} = -\tan^{-1}\frac{f_t}{f_{P2}}$$

$$= -\tan^{-1}\left(\frac{33.6}{95.5}\right) = -19.4^\circ$$

To reduce this phase lag to -10°, we need to change R so that the zero moves to the negative real axis and introduces a phase lead of 9.4°. Thus,

$$\tan^{-1}\frac{f_t}{f_Z} = 9.4^\circ$$

$$f_Z = \frac{f_t}{\tan 9.4} = \frac{33.6}{0.166} = 203 \text{ MHz}$$

$$f_Z = \frac{1}{2\pi C_C\left(R - \dfrac{1}{G_{m2}}\right)}$$

$$\Rightarrow R - \frac{1}{G_{m2}} = \frac{1}{2\pi \times 203 \times 10^6 \times 1.42 \times 10^{-12}}$$

$= 552\ \Omega$

$R = 1670 + 552 = 2222\ \Omega$

$= 2.22$ kΩ

13.11 Using Eq. (13.46), we obtain

$$\text{SR} = 2\pi f_t V_{OV1,2}$$

$$= 2\pi \times 100 \times 10^6 \times 0.2$$

$= 125.6$ V/μs

Using Eq. (13.45),

$$\text{SR} = \frac{I}{C_C}$$

$$\Rightarrow C_C = \frac{I}{\text{SR}} = \frac{100 \times 10^{-6}}{125.6 \times 10^6}$$

$= 0.8$ pF

13.12 $C_2 = 0.7$ pF.

For a phase margin of 72°, the phase due to f_{P2} at f_t must be 18°; thus,

$$\frac{f_t}{f_{P2}} = \tan 18^\circ$$

$$\Rightarrow f_{P2} = \frac{100}{\tan 18^\circ} = 307.8 \text{ MHz}$$

But

$$f_{P2} = \frac{G_{m2}}{2\pi C_2}$$

$$\Rightarrow G_{m2} = 2\pi f_{P2} C_2$$

$$= 2\pi \times 307.8 \times 10^6 \times 0.7 \times 10^{-12}$$

$= 1.35$ mA/V

Thus,

$g_{m6} = 1.35$ mA/V

For the transmission zero to be at ∞,

$$R = \frac{1}{G_{m2}} = \frac{1}{1.35 \times 10^{-3}} = 739\ \Omega$$

$$\text{SR} = 2\pi f_t |V_{OV1,2}|$$

$$= 2\pi \times 100 \times 10^6 \times 0.15$$

$= 94.2$ V/μs

$$\text{SR} = \frac{I}{C_C}$$

$$\Rightarrow C_C = \frac{I}{\text{SR}} = \frac{100 \times 10^{-6}}{94.2 \times 10^6} = 1.06 \text{ pF}$$

13.13 $G_{m1} = 1$ mA, $G_{m2} = 5$ mA/V

(a) Using Eq. (13.36), we obtain

$$f_t = \frac{G_{m1}}{2\pi C_C}$$

$$\Rightarrow C_C = \frac{G_{m1}}{2\pi f_t} = \frac{1 \times 10^{-3}}{2\pi \times 80 \times 10^6}$$

$= 2$ pF

(b) Phase margin =

$$90^\circ - \tan^{-1}\left(\frac{f_t}{f_{P2}}\right) - \tan^{-1}\left(\frac{f_t}{f_Z}\right)$$

where

$$f_{P2} = \frac{G_{m2}}{2\pi C_2}$$

and

$$f_Z = \frac{G_{m2}}{2\pi C_C}$$

For a PM of 70°, we have

$$\tan^{-1}\left(\frac{f_t}{f_{P2}}\right) + \tan^{-1}\left(\frac{f_t}{f_Z}\right) = 20^\circ$$

But,

$$f_Z = \frac{5 \times 10^{-3}}{2\pi \times 2 \times 10^{-12}} = 398 \text{ MHz}$$

and

$$\tan^{-1}\left(\frac{f_t}{f_Z}\right) = \tan^{-1}\left(\frac{80}{398}\right) = 11.4^\circ$$

Thus,

$$\tan^{-1}\left(\frac{f_t}{f_{P2}}\right) = 20 - 11.4^\circ = 8.6^\circ$$

$$\frac{f_t}{f_{P2}} = \tan 8.6^\circ$$

$$\Rightarrow f_{P2} = \frac{80}{\tan 8.6^\circ} = 529 \text{ MHz}$$

$$\frac{G_{m2}}{2\pi C_2} = 529 \times 10^6$$

$$C_2 = \frac{5 \times 10^{-3}}{2\pi \times 529 \times 10^6} = 1.51 \text{ pF}$$

This is the maximum value that C_2 can have; if C_2 is larger, then f_{P2} will be lower; and the phase it introduces at f_t will increase, causing the phase margin to drop below 70°.

13.14 $G_{m1} = 0.8$ mA/V, $G_{m2} = 2$ mA/V

(a) Using Eq. (13.36), we obtain

$$f_t = \frac{g_{m1}}{2\pi C_C}$$

$$\Rightarrow C_C = \frac{G_{m1}}{2\pi f_t} = \frac{0.8 \times 10^{-3}}{2\pi \times 100 \times 10^6} = 1.27 \text{ pF}$$

(b) Phase margin =

$$90^\circ - \tan^{-1}\left(\frac{f_t}{f_{P2}}\right) - \tan^{-1}\left(\frac{f_t}{f_Z}\right)$$

$$60^\circ = 90 - \tan^{-1}\left(\frac{f_t}{f_{P2}}\right) - \tan^{-1}\left(\frac{f_t}{f_Z}\right)$$

Thus,

$$\tan^{-1}\left(\frac{f_t}{f_{P2}}\right) + \tan^{-1}\left(\frac{f_t}{f_Z}\right) = 30^\circ$$

where

$$f_{P2} = \frac{G_{m2}}{2\pi C_2}$$

$$f_Z = \frac{1}{2\pi C_C\left(\frac{1}{G_{m2}} - R\right)}$$

$$= \frac{1}{2\pi \times 1.27 \times 10^{-12}(0.5 - 0.5) \times 10^3} = \infty$$

Thus,

$$\tan^{-1}\left(\frac{f_t}{f_{P2}}\right) = 30^\circ$$

$$f_{P2} = \frac{f_t}{\tan 30} = 173.2 \text{ MHz}$$

We now can obtain C_2 from

$$173.2 \times 10^6 = \frac{2 \times 10^{-3}}{2\pi C_2}$$

$$\Rightarrow C_2 = \frac{2 \times 10^{-3}}{2\pi \times 173.2 \times 10^6} = 1.84 \text{ pF}$$

13.15 SR = 60 V/μs, $f_t = 60$ MHz

(a) Using Eq. (13.46), we obtain

$$\text{SR} = 2\pi f_t |V_{OV1}|$$

$$\Rightarrow |V_{OV1}| = \frac{60 \times 10^6}{2\pi \times 60 \times 10^6} = 0.16 \text{ V}$$

(b) Using Eq. (13.45), we get

$$\text{SR} = \frac{I}{C_C}$$

$$\Rightarrow C_C = \frac{I}{\text{SR}} = \frac{120 \times 10^{-6}}{60 \times 10^6} = 2 \text{ pF}$$

(c) For Q_1 and Q_2, we have

$$I_{D1,2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{1,2} |V_{OV1,2}|^2$$

$$60 = \frac{1}{2} \times 60 \times \left(\frac{W}{L}\right)_{1,2} \times 0.16^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 78.1$$

13.16 Figure 1 on next page shows the circuit.

13.17 (a) From Eq. (13.54), we have

$$\text{PSRR}^- = g_{m1}(r_{o2} \parallel r_{o4}) g_{m6} r_{o6}$$

where

$$g_{m1} = \frac{2 \times \frac{I}{2}}{|V_{OV}|} = \frac{I}{|V_{OV}|}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{2|V_A|}{I}$$

$$g_{m6} = \frac{2 I_{D6}}{|V_{OV}|}$$

$$r_{o6} = \frac{|V_A|}{I_{D6}}$$

Thus,

$$\text{PSRR}^- = \frac{I}{|V_{OV}|} \times \frac{1}{2} \times \frac{2|V_A|}{I} \times \frac{2I_{D6}}{|V_{OV}|} \times \frac{|V_A|}{I_{D6}}$$

$$= 2\left|\frac{V_A}{V_{OV}}\right|^2 \qquad \text{Q.E.D.}$$

This figure belongs to Problem 13.16.

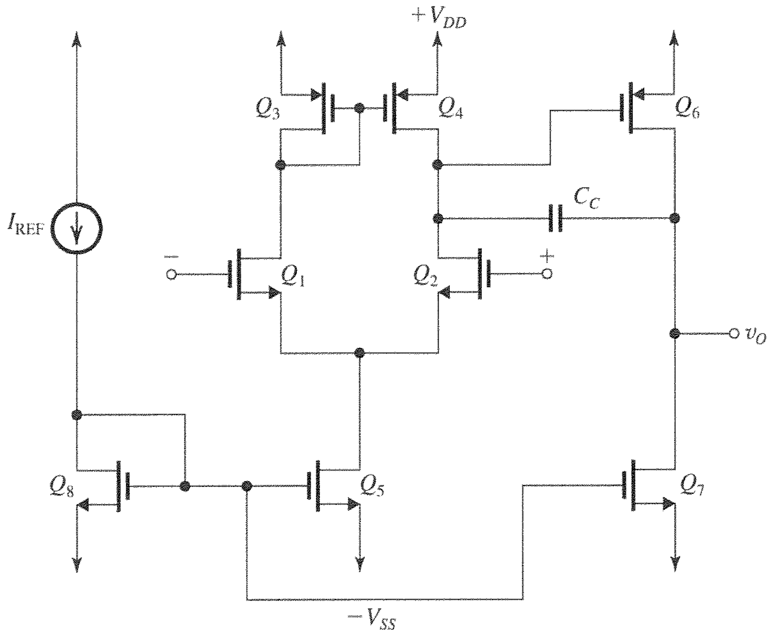


Figure 1

(b) A PSRR$^-$ of 72 dB means

PSRR$^-$ = 4000

Thus,

$$4000 = 2\frac{|V_A|^2}{0.15^2}$$

$$\Rightarrow |V_A| = 6.71 \text{ V}$$

Now,

$$|V_A| = |V_A'|L$$

$$6.71 = 15L$$

$$\Rightarrow L = 0.45\ \mu\text{m}$$

13.18 For Q_8 and Q_9, we have

$$I_{\text{REF}} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{8,9}|V_{OV8,9}|^2$$

$$225 = \frac{1}{2} \times 60 \times \frac{60}{0.5} \times |V_{OV8,9}|^2$$

$$\Rightarrow |V_{OV8,9}| = 0.25 \text{ V}$$

$$g_{m8} = g_{m9} = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.225}{0.25}$$

$$= 1.8 \text{ mA/V}$$

For Q_{10}, Q_{11} and Q_{12}, we have

$$g_m = g_{m8} = 1.8 \text{ mA/V}$$

$$\frac{2I_{\text{REF}}}{V_{OV}} = 1.8$$

$$\Rightarrow V_{OV} = 0.25 \text{ V}$$

and

$$225 = \frac{1}{2} \times 180 \times \left(\frac{W}{L}\right) \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{10} = \left(\frac{W}{L}\right)_{11} = \left(\frac{W}{L}\right)_{12} = 40$$

Using Eq. (13.61), we obtain

$$R_B = \frac{2}{g_{m12}}\left(\sqrt{\frac{(W/L)_{12}}{(W/L)_{13}}} - 1\right)$$

$$= \frac{2}{1.8 \times 10^{-3}}\left(\sqrt{4} - 1\right)$$

$$= 1.11 \text{ k}\Omega$$

Voltage drop across $R_B = I_{\text{REF}} \times 1.11$

$= 0.225 \times 1.11 = 0.25$ V

The $\left(\frac{W}{L}\right)$ ratios of Q_{10}, Q_{11} and Q_{12} are given above. For Q_{13}, we have

$$\left(\frac{W}{L}\right)_{13} = 4\left(\frac{W}{L}\right)_{12}$$

$$\Rightarrow \left(\frac{W}{L}\right)_{13} = 160$$

DC voltage at gate of Q_{12}

$$= -V_{SS} + I_{\text{REF}}R_B + V_{tn} + V_{OV12}$$

$$= -1.5 + 0.25 + 0.5 + 0.25 = -0.5 \text{ V}$$

DC Voltage at gate of Q_{10}

$= V_{G12} + V_{tn} + V_{OV11}$

$= -0.5 + 0.5 + 0.25 = +0.25$ V

DC Voltage at gate of Q_8

$= V_{DD} - |V_{tp}| - |V_{OV8}|$

$= 1.5 - 0.5 - 0.25 = +0.75$ V

13.19 $2I_B \times 2 = 1$ mW

$I_B = \frac{10^{-3}}{4} = 0.25 \text{ mA} = 250\ \mu\text{A}$

$I_{D1} = 4I_{D3}$

$I_{D1} + I_{D3} = I_B$

$5I_{D3} = 250\ \mu$A

$I_{D3} = 50\ \mu\text{A},\ I_{D4} = 50\ \mu\text{A}$

$I_{D1} = 200\ \mu\text{A},\ I_{D2} = 200\ \mu\text{A}$

$I = 400\ \mu$A

13.20 $V_{\text{BIAS1}} = V_{DD} - |V_{OV9}| - |V_{OV3}| - |V_{tp}|$

$= 1 - 0.15 - 0.15 - 0.4 = +0.3$ V

$V_{\text{BIAS2}} = V_{DD} - |V_{OV9}| - |V_{tp}|$

$= 1 - 0.15 - 0.4 = +0.45$ V

$V_{\text{BIAS3}} = -V_{SS} + V_{OV11} + V_{tn}$

$= -1 + 0.15 + 0.4 = -0.45$ V

$V_{ICM\max} = V_{DD} - |V_{OV9}| + V_{tn}$

$= 1 - 0.15 + 0.4 = +1.25$ V

$V_{ICM\min} = -V_{SS} + |V_{OV11}| + V_{OV1} + V_{tn}$

$= -1 + 0.15 + 0.15 + 0.4 = -0.3$ V

Thus,

$-0.3 \text{ V} \le V_{ICM} \le +1.25$ V

$v_{O\max} = V_{\text{BIAS1}} + |V_{tp}|$

$= 0.3 + 0.4 = +0.7$ V

$v_{O\min} = -V_{SS} + V_{OV7} + V_{tn} + V_{OV5}$

$= -1 + 0.15 + 0.4 + 0.15 = -0.3$ V

Thus,

$-0.3 \text{ V} \le v_O \le +0.7$ V

13.21 $I_{D1} = I_{D2} = \frac{I}{2} = 0.2$ mA

$$0.2 = \frac{1}{2} \times 0.4 \times \left(\frac{W}{L}\right)_{1,2} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 25$$

$I_{D3} = I_{D4} = I_B - \frac{I}{2} = 250 - 200 = 50\ \mu$A

$$50 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 25$$

$I_{D5} = I_{D6} = I_{D7} = I_{D8} = 50\ \mu$A

$$50 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{5-8} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8$$

$= 6.25$

$I_{D9} = I_{D10} = I_B = 250\ \mu$A

$$250 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{9,10} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_9 = \left(\frac{W}{L}\right)_{10} = 125$$

$I_{D11} = I = 400\ \mu$A

$$400 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{11} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_{11} = 50$$

Summary: See table below.

13.22 $G_m = g_{m1} = g_{m2} = \frac{2(I/2)}{V_{OV}}$

$= \frac{I}{V_{OV}} = \frac{0.4}{0.2} = 2$ mA/V

$I_{D4} = I_B - \frac{I}{2} = 0.25 - 0.2 = 0.05$ mA

$g_{m4} = \frac{2I_{D4}}{|V_{OV}|} = \frac{2 \times 0.05}{0.2} = 0.5$ mA/V

$r_{o4} = \frac{|V_A|}{I_{D4}} = \frac{10}{0.05} = 200$ kΩ

This table belongs to Problem 13.21.

Transistor	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}
$\boldsymbol{W/L}$	25	25	25	25	6.25	6.25	6.25	6.25	125	125	50

$$r_{o2} = \frac{|V_A|}{I_{D2}} = \frac{|V_A|}{I/2} = \frac{10}{0.2} = 50 \text{ k}\Omega$$

$$r_{o10} = \frac{|V_A|}{I_{D10}} = \frac{|V_A|}{I_B} = \frac{10}{0.25} = 40 \text{ k}\Omega$$

$$R_{o4} = (g_{m4}r_{o4})\,(r_{o2} \parallel r_{o10})$$

$$= 0.5 \times 200\ (50 \parallel 40)$$

$$= 2.22 \text{ M}\Omega$$

$$I_{D6} = 50\ \mu\text{A} = 0.05 \text{ mA}$$

$$g_{m6} = \frac{2 \times 0.05}{0.2} = 0.5 \text{ mA/V}$$

$$r_{o6} = \frac{|V_A|}{I_{D6}} = \frac{10}{0.05} = 200 \text{ k}\Omega$$

$$r_{o8} = \frac{|V_A|}{I_{D8}} = \frac{10}{0.05} = 200 \text{ k}\Omega$$

$$R_{o6} = g_{m6}r_{o6}r_{o8}$$

$$= 0.5 \times 200 \times 200 = 20 \text{ M}\Omega$$

$$R_o = R_{o4} \parallel R_{o6}$$

$$= 2.22 \parallel 20 = 2 \text{ M}\Omega$$

$$A_v = G_m R_o$$

$$= 2 \times 2000 = 4000 \text{ V/V}$$

For the closed-loop amplifier:

$$A = A_v = 4000$$

$$\beta = \frac{C}{C + 9C} = 0.1$$

$$\frac{V_o}{V_i} = A_f = \frac{A}{1 + A\beta} = \frac{4000}{1 + 4000 \times 0.1}$$

$$= \frac{4000}{401} = 9.975 \text{ V/V}$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta} = \frac{2 \text{ M}\Omega}{401} \simeq 5 \text{ k}\Omega$$

13.23 $\text{SR} = \dfrac{I_B}{C_L}$

$$10 \times 10^6 = \frac{I_B}{10 \times 10^{-12}}$$

$$\Rightarrow I_B = 10^{-4} \text{ A} = 0.1 \text{ mA} = 100\ \mu\text{A}$$

$$\frac{I}{2} = 3\left(I_B - \frac{I}{2}\right)$$

$$\frac{I}{2}(1 + 3) = 3I_B = 300$$

$$I = 150\ \mu\text{A}$$

Now,

$$f_t = \frac{G_m}{2\pi C_L}$$

where

$$G_m = g_{m1,2} = \frac{2(I/2)}{V_{OV1,2}} = \frac{I}{V_{OV1,2}}$$

$$= \frac{0.15 \text{ mA}}{0.15 \text{ V}} = 1 \text{ mA/V}$$

Thus,

$$f_t = \frac{1 \times 10^{-3}}{2\pi \times 10^{-12}}$$

$$= 15.92 \text{ MHz}$$

Phase due to the two nondominant poles at f_t

$$= -2 \tan^{-1}\left(\frac{15.92}{50}\right) = -35.3^\circ$$

Thus,

$$\text{Phase margin} = 90 - 35.3 = 54.7^\circ$$

To increase the phase margin to 75°, the phase due to the two nondominant poles must be reduced to $90 - 75 = 15^\circ$, i.e. each should contribute 7.5°, thus we must reduce f_t to the value obtained as follows:

$$\tan^{-1}\left(\frac{f_t}{50 \text{ MHz}}\right) = 7.5^\circ$$

$$f_t = 50 \times \tan 7.5^\circ = 6.58 \text{ MHz}$$

This is achieved by increasing C_L,

$$6.58 \times 10^6 = \frac{1 \times 10^{-3}}{2\pi C_L}$$

$$\Rightarrow C_L = \frac{10^{-3}}{2\pi \times 7.92 \times 10^6} = 24.2 \text{ pF}$$

The new value of slew-rate will be

$$\text{SR} = \frac{I_B}{C_L} = \frac{0.1 \times 10^{-3}}{24.2 \times 10^{-12}} = 4.13 \text{ V}/\mu\text{s}$$

13.24 Refer to Fig. 13.9. When V_{id} is sufficiently large to cause Q_1 to cut off and Q_2 to conduct all of I, Q_3 will carry a current I_B. However, Q_4 will carry $(I_B - I)$. The current I_B in Q_3 will be mirrored in the drain of Q_6. Thus, at the output node the current available to charge C_L will be

$$I_O = I_B - (I_B - I) = I$$

and the slew rate becomes

$$\text{SR} = \frac{I}{C_L}$$

13.25 $A = 80 \text{ dB} \equiv 10^4$ V/V

$$f_t = 20 \text{ MHz}, \quad C_L = 10 \text{ pF}$$

$$I_B = I$$

$$|V_A| = 12 \text{ V}$$

Refer to Figs. 13.9 and 13.10. For $I = I_B$, the dc operating currents of the 11 transistors are as follows:

$Q_1 - Q_8$: $\dfrac{I}{2}$

Q_9, Q_{10}, and Q_{11}: I

Thus, for $Q_1 - Q_8$, we have

$$g_m = \frac{I}{|V_{OV}|}$$

and

$$r_o = \frac{2|V_A|}{I}$$

while, for $Q_9 - Q_{11}$,

$$r_o = \frac{|V_A|}{I}$$

Now,

$$G_m = g_{m1,2} = \frac{I}{V_{OV}}$$

$$R_{o4} = (g_{m4}r_{o4})\ (r_{o2} \parallel r_{o10})$$

$$= \frac{I}{|V_{OV}|} \times \frac{2|V_A|}{I}\left[\frac{2|V_A|}{I} \parallel \frac{|V_A|}{I}\right]$$

$$= \frac{2|V_A|}{|V_{OV}|} \times \frac{2}{3}\frac{|V_A|}{I}$$

$$= \frac{4|V_A|^2}{3|V_{OV}|I}$$

$$R_{o6} = g_{m6}r_{o6}r_{o8}$$

$$= \frac{I}{|V_{OV}|}\ \frac{2|V_A|}{I}\ \frac{2|V_A|}{I}$$

$$= \frac{4|V_A|^2}{|V_{OV}|I}$$

$$R_o = R_{o4} \parallel R_{o6}$$

$$= \left[\frac{4}{3}\ \frac{|V_A|^2}{|V_{OV}|I}\right] \parallel \left[\frac{|V_A|^2}{|V_{OV}|I}\right]$$

$$= \frac{|V_A|^2}{|V_{OV}|I}$$

The voltage gain can now be found as

$$A = G_m R_o = g_{m1,2}R_o$$

$$= \frac{I}{|V_{OV}|}\ \frac{|V_A|^2}{|V_{OV}|I}$$

$$= \frac{|V_A|^2}{|V_{OV}|^2}$$

$$10{,}000 = \left|\frac{V_A}{V_{OV}}\right|^2$$

$$\Rightarrow \frac{|V_A|}{|V_{OV}|} = 100$$

$$\Rightarrow |V_{OV}| = \frac{12}{100} = 0.12\ \text{V}$$

To obtain $f_t = 20$ MHz, we use

$$20 \times 10^6 = \frac{g_{m1,2}}{2\pi \times 10 \times 10^{-12}}$$

$$g_{m1,2} = 2\pi \times 10 \times 10^{-12} \times 20 \times 10^6$$

$$= 1.257 \times 10^{-3}\ \text{A/V}$$

Thus,

$$\frac{I}{|V_{OV}|} = 1.257 \times 10^{-3}$$

$$\Rightarrow I = 1.257 \times 0.12 \times 10^{-3}$$

$$= 0.15\ \text{mA} = 150\ \mu\text{A}$$

$$I_B = I = 150\ \mu\text{A}$$

$$\text{SR} = \frac{I_B}{C_L} = \frac{150 \times 10^{-6}}{10 \times 10^{-12}}$$

$$= 15\ \text{V}/\mu\text{s}$$

For Q_1 and Q_2, we have

$$I_D = \frac{I}{2} = 75\ \mu\text{A} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{1,2} V_{OV}^2$$

$$75 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{1,2} \times 0.12^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 26$$

For Q_3 and Q_4, we have

$$I_D = I_B - \frac{I}{2} = 150 - 75 = 75\ \mu\text{A}$$

Thus,

$$75 = \frac{1}{2} \times \frac{400}{2.5} \times \left(\frac{W}{L}\right)_{3,4} \times 0.12^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 65.1$$

For Q_5, Q_6, Q_7, and Q_8, we have

$$I_D = I_B = 75\ \mu\text{A}$$

$$75 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{5-8} \times 0.12^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8 = 26$$

For Q_9 and Q_{10}, we have

$$I_D = I_B = 150\ \mu\text{A}$$

$$150 = \frac{1}{2} \times \frac{400}{2.5} \times \left(\frac{W}{L}\right)_{9,10} \times 0.12^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_9 = \left(\frac{W}{L}\right)_{10} = 130.2$$

For Q_{11}, we have

$$I_D = I = 150\ \mu\text{A}$$

Summary (Approximate Values):

Transistor	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}
W/L	26	26	65	65	26	26	26	26	130	130	52

$$150 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{11} \times 0.12^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{11} = 52$$

See table above for a summary.

13.26 Figure 1 below shows a folded-cascode single-stage op amp circuit that is complementary to that in Fig. 13.10.

13.27 Please note an error in the first printing of the textbook: The labeling of Q_{12} and Q_{13} in the diagram of Fig. P13.27 were reversed. Correct labeling: Q_{12} is the diode-connected transistor connected to Q_1, and Q_{13} is the diode-connected transistor connected to Q_2.

(a) Figure 1 on the next page shows the dc bias currents of all transistor as well as the various dc node voltages. Since Q_{12} and Q_{13} have $V_G = +0.4$ V and $V_S = +0.85$ V, both will be cut off.

(b) Figure 2 on the next page shows the currents and voltages during slewing. We assume that V_{id} is sufficiently large to steer $I = 320\ \mu A$ into Q_1 and turn Q_2 off. Now, at the drain of Q_1 we can write

$$I_{D12} + I_{D9} = 320\ \mu A \qquad (1)$$

The current through Q_{14} becomes

$$I_{D14} = 20\ \mu A + I_{D12} \qquad (2)$$

Also, since Q_9 has (W/L) equal to ten times that of Q_{14}, we have

$$I_{D9} = 10\ I_{D14} \qquad (3)$$

Solving (1), (2), and (3) together yields

$$I_{D12} = 10.9\ \mu A \simeq 11\ \mu A$$

$$I_{D14} \simeq 31\ \mu A$$

$$I_{D9} \simeq 310\ \mu A$$

Thus,

$$I_{D10} = I_{D9} \simeq 310\ \mu A$$

$$I_{D4} = I_{D10} = 310\ \mu A$$

$$I_O = 310\ \mu A$$

This figure belongs to Problem 13.26.

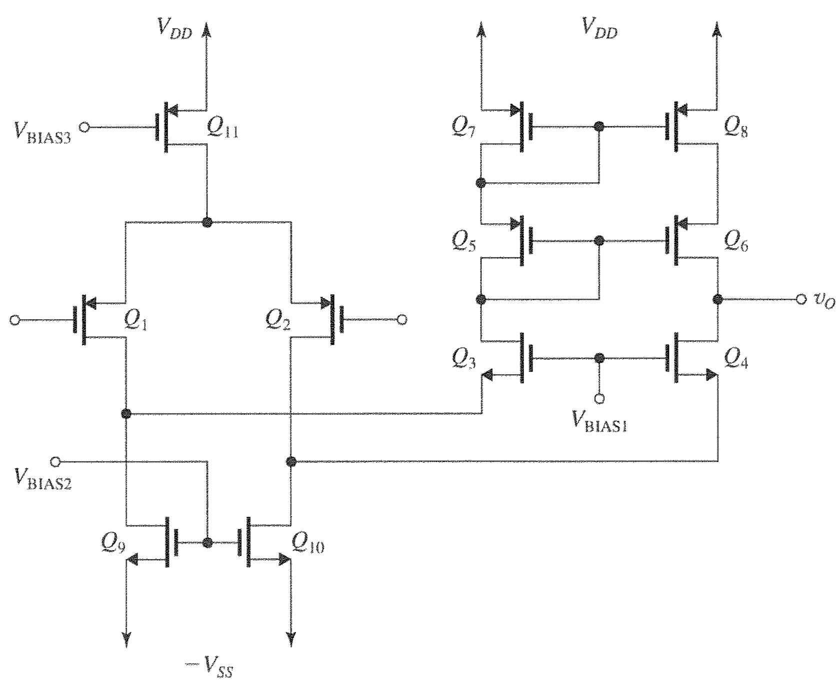


Figure 1

This figure belongs to Problem 13.27, part (a).

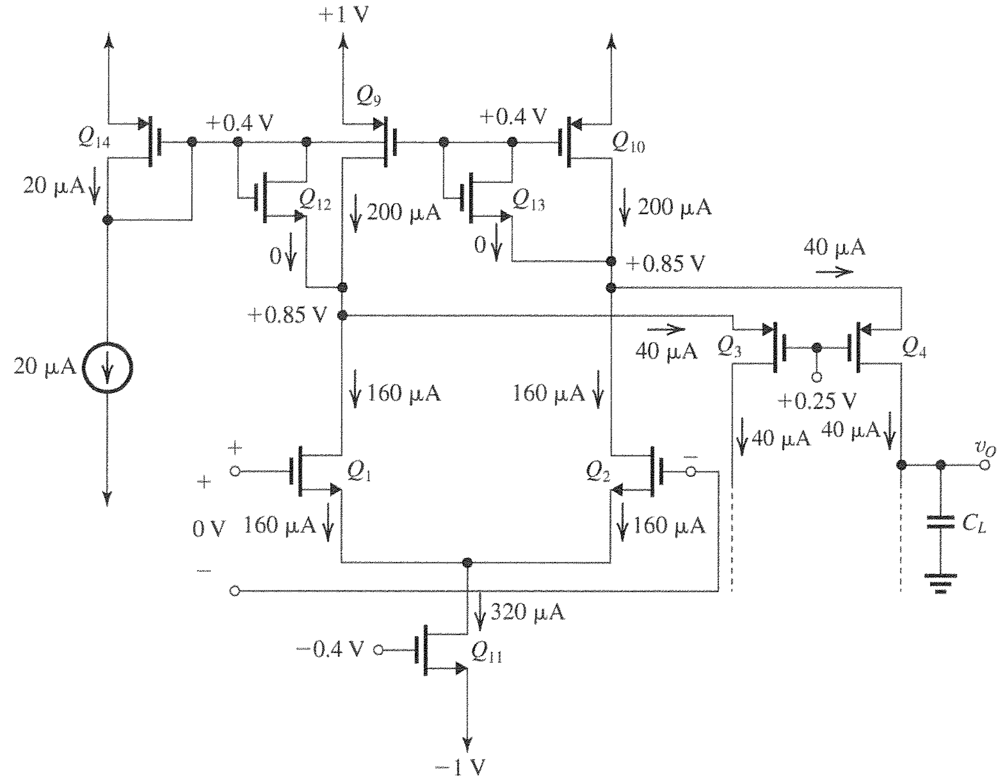


Figure 1

This figure belongs to Problem 13.27, part (b).

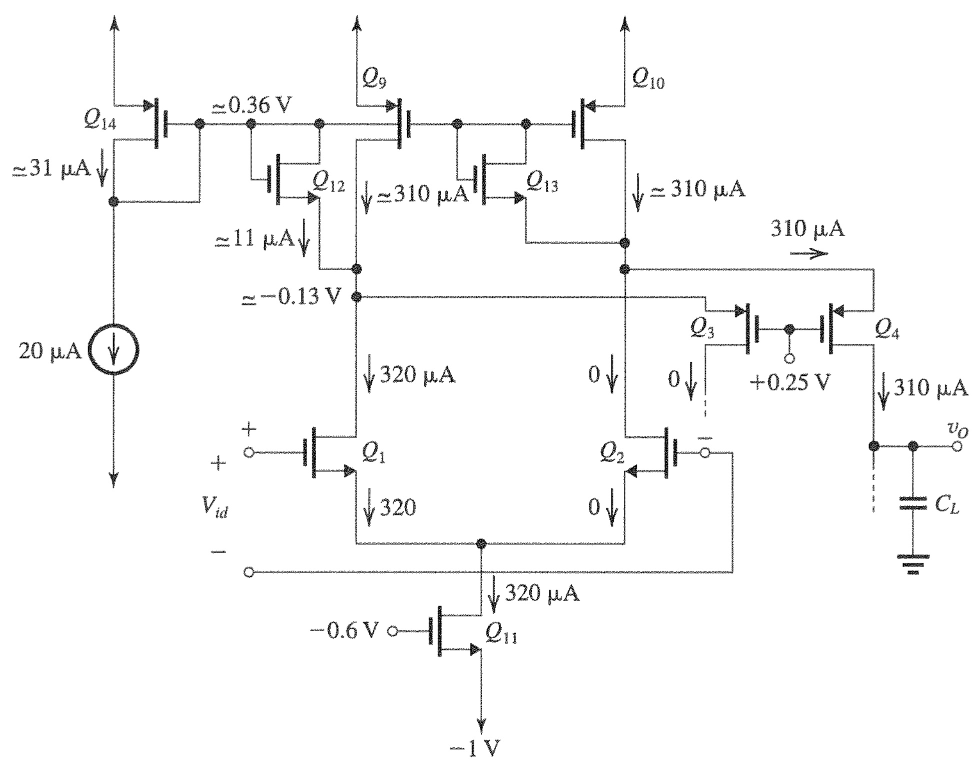


Figure 2

Thus, the current available to charge C_L has increased from $I_B = 200\ \mu A$ to $310\ \mu A$, a factor of 1.65. The slew rate will increase by the same factor. However, it is possible that the large current that Q_4 now conducts can raise the voltage at its source and thus at the drain of Q_{10} to a value that drives Q_{10} into the triode region. In this case, the current that Q_{10} provides will be reduced and the full increase in SR will not be realized.

To determine the voltage at the drain of Q_1, we first find V_{G14},

$$V_{G14} = V_{DD} - V_{SG14}$$

When I_{D4} was 20 μA, $|V_{OV14}|$ was 0.15 V. Now $I_{D4} = 31\ \mu A$ and thus $|V_{OV14}|$ becomes

$$|V_{OV14}| = 0.15 \times \sqrt{\frac{31}{20}} = 0.19\ \text{V}$$

Thus,

$$V_{SG14} = 0.45 + 0.19 = 0.64\ \text{V}$$

and

$$V_G = 1 - 0.64 = +0.36\ \text{V}$$

Next, we estimate V_{GS12}. Since I_{D12} is only 11 μA and assuming Q_{11} is, say, similar to Q_1, then

$$V_{OV12} = 0.15\sqrt{\frac{11}{160}}$$

$$= 0.04\ \text{V}$$

Thus,

$$V_{GS12} = 0.45 + 0.04 = 0.49\ \text{V}$$

It follows that

$$V_{D1} = 0.36 - 0.49 = -0.13\ \text{V}$$

which is sufficient to keep Q_1 and Q_{11} in the saturation region.

13.28 (a) Refer to Fig. 13.12. For the NMOS input stage

$$V_{ICM\max} = V_{DD} - |V_{OV}| + V_{tn}$$

$$= 1 - 0.15 + 0.45 = +1.3\ \text{V}$$

$$V_{ICM\min} = -V_{SS} + V_{OV} + V_{OV} + V_{tn}$$

$$= -1 + 0.15 + 0.15 + 0.45$$

$$= -0.25\ \text{V}$$

Thus,

$$-0.25\ \text{V} \le V_{ICM} \le +1.3\ \text{V}$$

(b) For the PMOS input stage

$$V_{ICM\max} = V_{DD} - |V_{OV}| - |V_{tp}|$$

$$= 1 - 0.15 - 0.15 - 0.45\ \text{V}$$

$$= +0.25\ \text{V}$$

$$V_{ICM\min} = -V_{SS} + V_{OV} - |V_{tp}|$$

$$= -1 + 0.15 - 0.45$$

$$= -1.3\ \text{V}$$

Thus,

$$-1.3\ \text{V} \le V_{ICM} \le +0.25\ \text{V}$$

(c) The overlap range is

$$-0.25\ \text{V} \le V_{ICM} \le +0.25\ \text{V}$$

(d) $-1.3\ \text{V} \le V_{ICM} \le +1.3\ \text{V}$

13.29 First we determine V_{OV}:

$$90 = \frac{1}{2} \times 400 \times 20\ V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.15\ \text{V}$$

$$V_{\text{BIAS}} = V_t + 2V_{OV} = 0.45 + 2 \times 0.15$$

$$= 0.75\ \text{V}$$

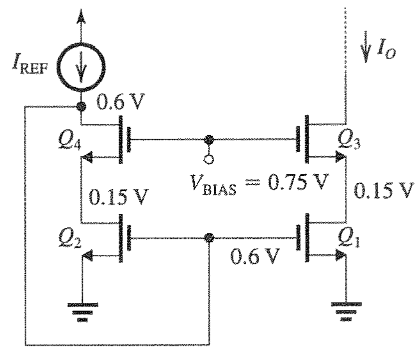


Figure 1

Figure 1 shows the voltages at the various nodes in the mirror circuit. The minimum voltage allowable at the output terminal is

$$v_{O\min} = V_{\text{BIAS}} - V_{tn}$$

$$= 0.75 - 0.45 = 0.3\ \text{V}$$

which is $2V_{OV}$.

The output resistance is

$$R_o \simeq g_{m3} r_{o3} r_{o1}$$

where

$$r_{o1} = r_{o3} = \frac{V_A}{I_D} = \frac{10}{0.09} = 111.1\ \text{k}\Omega$$

$$g_{m3} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.09}{0.15} = 1.2\ \text{mA/V}$$

$$R_o = 1.2 \times 111.1 \times 111.1 = 14.8\ \text{M}\Omega$$

13.30 Since Q_3 is operating in the common-gate configuration and since the resistance in its drain is low, the input resistance at its source is $1/g_{m3}$. This resistance appears in parallel with r_{o1} which is much larger. Thus, the total resistance at this node is $\simeq 1/g_{m3}$ and since the total capacitance is C_P, the pole introduced will have a frequency

$$f_P \simeq \frac{1}{2\pi C_P/g_{m3}} = \frac{g_{m3}}{2\pi C_P} \qquad \text{Q.E.D.}$$

Now,

$$f_t = \frac{g_{m1}}{2\pi C_L}$$

where $g_{m1} = g_{m3}$ because all transistors are operating at the same values of I_D and $|V_{OV}|$.

For a phase margin of 80° the phase at f_t introduced by the pole at f_P must be only 10°,

$$\tan^{-1}\left(\frac{f_t}{f_P}\right) = 10°$$

$$f_P = \frac{f_t}{\tan 10°} = \frac{f_t}{0.176}$$

$$\frac{g_{m3}}{2\pi C_P} = \frac{g_{m1}}{2\pi C_L \times 0.176}$$

$$\Rightarrow C_P = 0.176 C_L$$

This is the largest value that C_P can have.

13.31 For each transistor we use

$$V_{BE} = V_T \ln \frac{I_C}{I_S}$$

$$= 25 \ln\left(\frac{I_C}{10^{-14}}\right)$$

$$g_m = \frac{I_C}{V_T} = \frac{I_C}{0.025 \text{ V}}$$

$$r_e \simeq \frac{1}{g_m}$$

$$r_\pi = \beta/g_m = 200/g_m$$

$$r_o = V_A/I_C = 125/I_C$$

We obtain the following results:

	Q_1	Q_2	Q_5	Q_6	Q_{16}	Q_{17}
I_C (μA)	9.5	9.5	9.5	9.5	16.2	550
V_{BE} (mV)	517	517	517	517	530	618
g_m (mA/V)	0.38	0.38	0.38	0.38	0.65	22
r_e (kΩ)	2.63	2.63	2.63	2.63	1.54	0.045
r_π (kΩ)	526	526	526	526	308	9.1
r_o (MΩ)	13.2	13.2	13.2	13.2	7.72	0.227

13.32 $V_{BE1} = V_T \ln \dfrac{I_1}{I_{S1}}$

$$V_{BE2} = V_T \ln \frac{I_1}{I_{S2}}$$

$$V_{BE3} = V_T \ln \frac{I_3}{I_{S3}}$$

$$V_{BE4} = V_T \ln \frac{I_3}{I_{S4}}$$

$$V_{BE3} + V_{BE4} = V_{BE1} + V_{BE2}$$

$$V_T \ln \frac{I_3}{I_{S3}} + V_T \ln \frac{I_3}{I_{S4}} = V_T \ln \frac{I_1}{I_{S1}} + V_T \ln \frac{I_1}{I_{S2}}$$

$$V_T \ln \frac{I_3^2}{I_{S3} I_{S4}} = V_T \ln \frac{I_1^2}{I_{S1} I_{S2}}$$

$$\Rightarrow \frac{I_3^2}{I_{S3} I_{S4}} = \frac{I_1^2}{I_{S1} I_{S2}}$$

$$\Rightarrow I_3 = I_1 \sqrt{\frac{I_{S3} I_{S4}}{I_{S1} I_{S2}}} \qquad \text{Q.E.D.}$$

$$150 = I_1 \sqrt{3 \times 3} = 3 I_1$$

$$\Rightarrow I_1 = 50 \text{ μA}$$

13.33 For the A and B devices, we have

$$V_{EB} = V_T \ln \frac{0.73 \times 10^{-3}}{10^{-14}}$$

$$= 625 \text{ mV}$$

For the A device, we have

$$g_{mA} = \frac{I_{CA}}{V_T} = \frac{0.25 \times 0.73}{0.025} = 7.3 \text{ mA/V}$$

$$r_{eA} \simeq \frac{1}{g_{mA}} = 137 \ \Omega$$

$$r_{\pi A} = \frac{\beta}{g_{mA}} = \frac{50}{7.3} = 6.85 \text{ k}\Omega$$

$$r_{oA} = \frac{|V_A|}{I_{CA}} = \frac{50}{0.18} = 278 \text{ k}\Omega$$

For the B device, we have

$$g_{mB} = \frac{I_{CB}}{V_T} = \frac{0.75 \times 0.73}{0.025} = 21.9 \text{ mA/V}$$

$$r_{eB} \simeq \frac{1}{g_{mB}} = 46 \ \Omega$$

$$r_{\pi B} = \frac{\beta}{g_{mB}} = \frac{50}{21.9} = 2.28 \text{ k}\Omega$$

$$r_{oB} = \frac{|V_A|}{I_{CB}} = \frac{50}{0.55} = 90.9 \text{ k}\Omega$$

13.34 Differential input breakdown voltage

$$= 0.6 + 0.6 + 50 + 7$$

$$= 58.2 \text{ V}$$

where we have assumed that a forward conducting transistor exhibits $|V_{BE}| = 0.6$ V.

13.35 $V_{SG1} = |V_{tp}| + |V_{OV1}|$

$$V_{GS2} = V_{tn} + V_{OV2}$$

$$V_{GS3} = V_{tn} + V_{OV3}$$

$$V_{SG4} = |V_{tp}| + |V_{OV4}|$$

But,

$V_{SG1} + V_{GS2} = V_{GS3} + V_{SG4}$

$\Rightarrow |V_{OV1}| + V_{OV2} = V_{OV3} + |V_{OV4}|$

Since

$|V_{OV1}| = \sqrt{2I_1/k_1}$

$V_{OV2} = \sqrt{2I_1/k_2}$

$V_{OV3} = \sqrt{2I_3/k_3}$

$|V_{OV4}| = \sqrt{2I_3/k_4}$

then

$$\sqrt{2I_1}\left(\frac{1}{\sqrt{k_1}} + \frac{1}{\sqrt{k_2}}\right) = \sqrt{2I_3}\left(\frac{1}{\sqrt{k_3}} + \frac{1}{\sqrt{k_4}}\right)$$

$$\Rightarrow I_3 = I_1\left[\frac{\dfrac{1}{\sqrt{k_1}} + \dfrac{1}{\sqrt{k_2}}}{\dfrac{1}{\sqrt{k_3}} + \dfrac{1}{\sqrt{k_4}}}\right]^2$$

For $k_1 = k_2$ and $k_3 = k_4 = 16\, k_1$, we have

$$I_3 = I_1\left[\frac{2/\sqrt{k_1}}{2/\sqrt{k_3}}\right]^2 = 16\, I_1$$

For $I_3 = 1.6$ mA, we have

$I_1 = 0.1$ mA

13.36 $I_{\text{REF}} = \dfrac{V_{CC} - V_{EB12} - V_{BE11} - (-V_{EE})}{R_5}$

$$= \frac{20 - 2 \times 0.7}{39} = 477\ \mu\text{A}$$

Now, consider Eq. (13.86):

$$V_T \ln\frac{I_{\text{REF}}}{I_{C10}} = I_{C10}R_4$$

To obtain $I_{C10} = 19\ \mu$A (and thus the same bias current as before),

$$25 \ln\frac{477}{19} = 19R_4$$

$\Rightarrow R_4 = 4.24\ \text{k}\Omega$

13.37

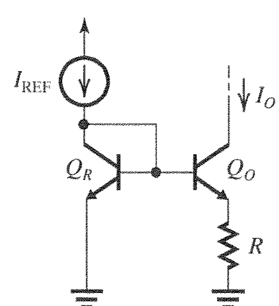


Figure 1

Refer to Fig. 1,

$I_{\text{REF}} = 0.3$ mA, $I_O = 10\ \mu$A

$$V_{BER} = V_T \ln\frac{I_{\text{REF}}}{I_S}$$

$$= 25 \ln\left(\frac{0.3 \times 10^{-3}}{10^{-14}}\right) = 603\ \text{mV}$$

$$V_{BEO} = V_T \ln\frac{I_O}{I_S}$$

$$= 25 \ln\left(\frac{10 \times 10^{-6}}{10^{-14}}\right)$$

$= 518$ mV

$V_{BER} - V_{BEO} = 603 - 518 = 85$ mV

$$R = \frac{85\ \text{mV}}{10\ \mu\text{A}} = 8.5\ \text{k}\Omega$$

13.38 Refer to Fig. 13.15.

(a) A node equation at X yields

$$\frac{2I}{1 + 2/\beta_P} + \frac{2I}{\beta_P} = I_{C10}$$

$$2I\ \frac{\beta_P + 1 + \dfrac{2}{\beta_P}}{\beta_P\left(1 + \dfrac{2}{\beta_P}\right)} = I_{C10}$$

$$I = \frac{I_{C10}}{2}\left[\frac{\beta_P(\beta_P + 2)}{\beta_P^2 + \beta_P + 2}\right]$$

For $\beta_P = 50$, we have

$$I = \frac{I_{C10}}{2} \times 1.019$$

For $\beta_P = 20$, we have

$$I = \frac{I_{C10}}{2} \times 1.043$$

Thus, I increases by $\dfrac{I_{C10}}{2} \times 0.024$, which is 2.4%.

(b)

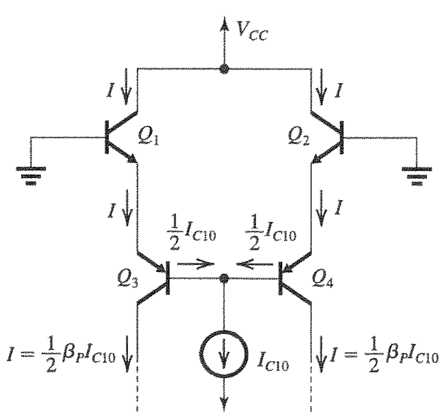


Figure 1

Figure 1 shows the suggested alternative design. As shown, here

$I \simeq \frac{1}{2}\beta_P I_{C10}$

For $\beta_P = 50$, we have

$I = 25I_{C10}$

For $\beta_P = 20$, we have

$I = 10I_{C10}$

Thus, I changes by $-15I_{C10}$, which is −60% change! This is a result of the absence of the desensitivity effect of negative feedback.

13.39 Refer to Fig. 13.15.

For $I_{S9} = 2I_{S8}$, the collector current of Q_9 will be

$$I_{C9} = \frac{4I}{1 + \frac{2}{\beta_P}}$$

If β_P is large, a node equation at X yields

$4I \simeq I_{C10}$

$\Rightarrow I = \frac{1}{4}I_{C10} = \frac{19}{4} = 4.75\ \mu\text{A}$

To establish $I_{C1} = I_{C2} = 9.5\ \mu\text{A}$, we need to redesign the Widlar source to provide

$I_{C10} = 38\ \mu\text{A}$. From Eq. (13.86), we obtain

$$V_T \ln\frac{I_{\text{REF}}}{I_{C10}} = I_{C10}R_4$$

$$25 \times \ln\frac{730}{38} = 38R_4$$

$\Rightarrow R_4 = 1.94\ \text{k}\Omega$

13.40 Refer to Fig. 13.15. For β_P large, a node equation at X yields

$I_{C9} \simeq I_{C10}$

If the ratio of the area of Q_9 to that of Q_8 is n, then

$I_{C9} = n \times 2I$

Thus,

$2nI = I_{C10}$

For $I = 10\ \mu\text{A}$ and $I_{C10} = 40\ \mu\text{A}$, we have

$n = 2$

13.41 Refer to the circuit in Fig. 13.16. The current in Q_5 remains equal to

$I = 9.5\ \mu\text{A}$

The voltage between the base of Q_5 and $-V_{EE}$ is

$= V_{BE5} + IR_1$

$$= 25 \ln\left(\frac{9.5 \times 10^{-6}}{10^{-14}}\right) + 9.5 \times 1$$

$= 517 + 9.5 = 526.5\ \text{mV}$

With R_2 shorted, this voltage appears across the BE junction of Q_6. Thus,

$I_{C6} = I_S\ e^{V_{BE6}/V_T}$

$= 10^{-14}\ e^{526.5/25}$

$= 14\ \mu\text{A}$

13.42 Figure 1 shows the circuit when R_3 is adjusted so that $I_{C5} = I_{C6} = I_{C7}$. Denoting the new value of these three currents I_1, we obtain the various currents indicated in Fig. 1. Now, at the input node X, we have

$$I_1\left(1 + \frac{1}{\beta_N}\right) = 9.5\ \mu\text{A}$$

$$\Rightarrow I_1 = \frac{9.5}{1 + (1/200)} = 9.45\ \mu\text{A}$$

This figure belongs to Problem 13.42.

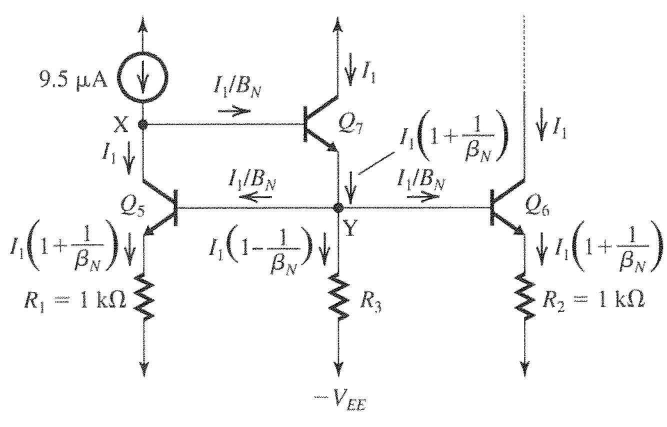


Figure 1

At this collector current, we have

$$V_{BE} = 25 \ln \frac{9.45 \times 10^{-6}}{10^{-14}} = 516.7\ \mu\text{A}$$

The voltage drop across R_3 becomes

$$V_{R3} = V_{BE5} + I_1\left(1 + \frac{1}{\beta_N}\right)R_1$$

$$= 516.7 + 9.5 \times 1$$

$$= 526.2\ \text{mV}$$

The value of R_3 can now be found as

$$R_3 = \frac{V_{R3}}{I_1\left(1 - \frac{1}{\beta_N}\right)} = \frac{526.2}{9.45\left(1 - \frac{1}{200}\right)}$$

$$= 56\ \text{k}\Omega$$

13.43 $2I = 19\ \mu\text{A}$

Assuming

$$I_{C1} = I_{C2} = I = 9.5\ \mu\text{A}$$

then

$$I_{B1} = \frac{9.5}{150} = 63.3\ \text{nA}$$

$$I_{B2} = \frac{9.5}{220} = 43.2\ \text{nA}$$

$$I_B = \frac{1}{2}(I_{B1} + I_{B2}) = 53.3\ \text{nA}$$

$$I_{OS} = |I_{B1} - I_{B2}| = 20.1\ \text{nA}$$

13.44 For $I_B = 60$ nA, we have

$$\beta_N \simeq \frac{9.5\ \mu\text{A}}{60\ \text{nA}} = 158.3$$

$$I_{OS} = |I_{B1} - I_{B2}|$$

For $I_{OS} = 5$ nA, I_{B1} and I_{B2} must not differ by more than 5 nA, thus

$$\frac{9.5\ \mu\text{A}}{\beta_{N\text{min}}} = 62.5\ \text{nA}$$

and

$$\frac{9.5\ \mu\text{A}}{\beta_{N\text{max}}} = 57.5\ \text{nA}$$

Thus,

$$152 \leq \beta_N \leq 165$$

13.45 $V_{C1} = V_{CC} - V_{EB8} = 5 - 0.6 = 4.4$ V

Q_1 and Q_2 saturate when V_{ICM} exceeds V_{C1} by 0.4 V. Thus,

$$V_{ICM\text{max}} = +4.8\ \text{V}$$

$$V_{C5} \simeq -V_{EE} + V_{BE5} + V_{BE7}$$

$$= -5 + 0.6 + 0.6 = -3.8\ \text{V}$$

Q_3 and Q_4 saturate when

$$V_{B3} = V_{C5} - 0.4 = -4.2\ \text{V}$$

But,

$$V_{B3} = V_{ICM} - V_{BE1} - V_{EB3}$$

$$= V_{ICM} - 1.2$$

Thus,

$$V_{ICM\text{min}} = V_{B3} + 1.2$$

$$= -4.2 + 1.2 = -3.0\ \text{V}$$

Thus,

$$-3\ \text{V} \leq V_{ICM} \leq +4.8\ \text{V}$$

13.46 Refer to Fig. 13.14 and to Exercise 13.21. From the answers to Exercise 13.21, we find that

$$V_{BE17} = 618\ \text{mV}$$

$$I_{E17} \simeq I_{C17} = 550\ \mu\text{A}$$

$$I_{B17} = \frac{550}{200} = 2.75\ \mu\text{A}$$

Voltage across $R_9 = V_{BE17} + I_{E17}\, R_8$

$$= 618 + 550 \times 0.1$$

$$= 673\ \text{mV}$$

$$I_{E16} = I_{B17} + \frac{V_{R9}}{R_9}$$

For $I_{C16} = 9.5\ \mu$A, we have

$$I_{E16} = 9.5 + \frac{9.5}{200} = 9.55\ \mu\text{A}$$

Thus,

$$9.55 = 2.75 + \frac{673\ (\text{mV})}{R_9\ (\text{k}\Omega)}$$

$$\Rightarrow R_9 = 98.9\ \text{k}\Omega$$

13.47 $I_{C18} + I_{C19} = 0.25 \times 0.73 = 180\ \mu$A

Require

$$I_{C18} = I_{C19} = 90\ \mu\text{A}$$

$$V_{BE18} = 25 \ln \frac{90 \times 10^{-6}}{10^{-14}}$$

$$= 573\ \text{mV}$$

Current through $R_{10} = I_{C19} + I_{B19} - I_{B18}$

$$\simeq I_{C19} = 90\ \mu\text{A}$$

$$R_{10} = \frac{573}{90} = 6.4\ \text{k}\Omega$$

$$V_{BB} = V_{BE18} + V_{BE19}$$

$$= 2 \times 0.573 = 1.146\ \text{V}$$

Since V_{BB} appears across the series combination of Q_{14} and Q_{20}, we can write

$$V_{BB} = V_T \ln \frac{I_{C14}}{I_{S14}} + V_T \ln \frac{I_{C20}}{I_{S20}}$$

Substituting $V_{BB} = 1.146$ V, $I_{S14} = I_{S20} = 3 \times 10^{-14}$, we obtain for the equal currents I_{C14} and I_{C20}

$$1.146 = 2 \times 0.025 \ln \frac{I_{C14}}{3 \times 10^{-14}}$$

$$\Rightarrow I_{C14} = I_{C20} = 270\ \mu\text{A}$$

13.48

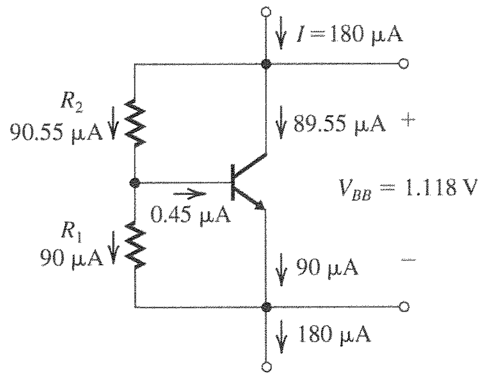


Figure 1

Refer to Fig. 1.

$$I_E = I_{R1} = \frac{180}{2} = 90\ \mu\text{A}$$

$$I_B = \frac{90}{201} = 0.45\ \mu\text{A}$$

$$I_C = \frac{\beta_N}{\beta_N + 1} I_E$$

$$= \frac{200}{201} \times 90 = 89.55\ \mu\text{A}$$

$$V_{BE} = V_T \ln \frac{I_C}{I_S}$$

$$= 25 \ln \frac{89.55 \times 10^{-6}}{10^{-14}}$$

$$= 573\ \text{mV}$$

$$R_1 = \frac{573\ \text{mV}}{90\ \mu\text{A}} = 6.37\ \text{k}\Omega$$

$$I_{R2} = I_{R1} + I_B = 90 + 0.45 = 90.45\ \mu\text{A}$$

$$V_{R2} = V_{BB} - V_{R1} = 1.118 - 0.573$$

$$= 0.545\ \text{V}$$

$$R_2 = \frac{545\ \text{mV}}{90.45\ \mu\text{A}} = 6.03\ \text{k}\Omega$$

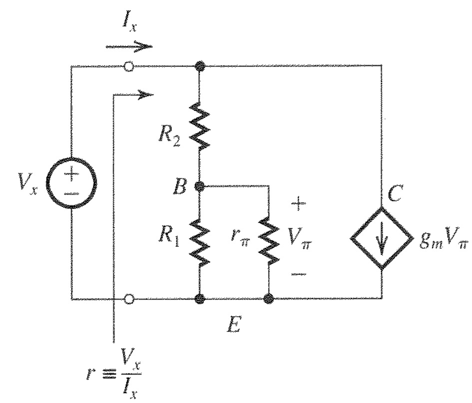


Figure 2

To determine the incremental resistance between the two terminals of the V_{BE} multiplier, we replace the transistor with its hybrid-π model, as shown in Fig. 2. Here

$$g_m = \frac{I_C}{V_T} = \frac{89.55\ \mu\text{A}}{25\ \text{mV}} = 3.6\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{3.6} = 55.6\ \text{k}\Omega$$

$$R_1 \parallel r_\pi = 6.37 \parallel 55.6 = 5.7\ \text{k}\Omega$$

$$V_\pi = V_x \frac{R_1 \parallel r_\pi}{(R_1 \parallel r_\pi) + R_2}$$

$$= V_x \frac{5.7}{5.7 + 6.03} = 0.49\ V_x$$

$$I_x = \frac{V_x}{5.7 + 6.03} + g_m \times 0.49\ V_x$$

$$= V_x(0.085 + 1.764)$$

$$r \equiv \frac{V_x}{I_x} = \frac{1}{0.085 + 1.764} = 0.541\ \text{k}\Omega$$

$$= 541\ \Omega$$

13.49 Refer to Fig. 13.14 and Table 13.1. The current I_{CC} drawn from V_{CC} can be found as follows:

$$I_{CC} = I_{E12} + I_{E13} + I_{C14} + I_{E9} + I_{E8} + I_{C7} + I_{C16}$$

Assuming β_P and $\beta_N \gg 1$,

$$I_{CC} = 730 + 730 + 154 + 19 + 19 + 10.5$$

$$+ 16.2$$

$$= 1678.7\ \mu\text{A} = 1.68\ \text{mA}$$

$$P_D = I_{CC}(V_{CC} + V_{EE})$$

$$= 1.68(15 + 15) = 50.4\ \text{mW}$$

13.50

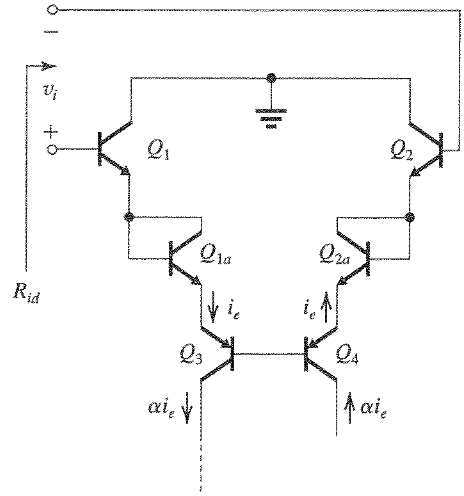


Figure 1

Figure 1 shows the input stage with the two extra diode-connected transistors Q_{1a} and Q_{2a}. Since these devices are simply in series with $Q_1 - Q_4$, they will have the same dc bias current, namely 9.5 μA. Thus, each of Q_{1a} and Q_{2a} will have an incremental resistance equal to r_e of each of Q_1 to Q_4,

$$r_e = \frac{25\text{ mV}}{9.5\ \mu\text{A}} = 2.63\text{ k}\Omega$$

The input differential resistance R_{id} now becomes

$$R_{id} = (\beta_N + 1) \times 6r_e$$

$$= 201 \times 6 \times 2.63$$

$$= 3.2\text{ M}\Omega$$

The effective transconductance of the input stage, G_{m1}, now becomes

$$G_{m1} \equiv \frac{2\alpha i_e}{v_{id}}$$

$$= \frac{2\alpha i_e}{6 i_e r_e} = \frac{1}{3} g_{m1}$$

$$= \frac{1}{3}\,\frac{9.5\ \mu\text{A}}{25\text{ mV}} = 0.13\text{ mA/V}$$

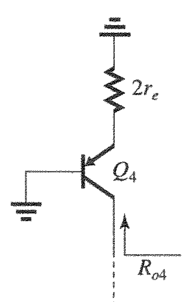


Figure 2

To find R_{o4}, refer to the circuit in Fig. 2.

$$R_{o4} = r_{o4}[1 + g_{m4}(2\,r_e \parallel r_{\pi 4}]$$

where

$$r_{o4} = \frac{50\text{ V}}{9.5\ \mu\text{A}} = 5.26\text{ M}\Omega$$

$$g_{m4} = 0.38\text{ mA/V}$$

$$r_{\pi 4} = \frac{50}{0.38} = 131.6\text{ k}\Omega$$

$$R_{o4} = 5.26[1 + 0.38(5.26 \parallel 131.6)]$$

$$= 15.4\text{ M}\Omega$$

$$R_{o1} = R_{o4} \parallel R_{o6}$$

$$= 15.4 \parallel 18.2 = 8.33\text{ M}\Omega$$

Open-circuit voltage gain $= G_{m1}R_{o1}$

$= 0.13 \times 8.33 \times 10^3 = 1083$ V/V

Comparison

	Original Design	**Modified Design**
R_{id} (kΩ)	2.1	3.2
G_{m1} (mA/V)	0.19	0.13
R_{o4} (MΩ)	10.5	15.4
R_{o1} (MΩ)	6.7	8.3
$\lvert A_{vo}\rvert$ (V/V)	1273	1083

Thus the input resistance increases but the gain decreases: The additional diodes introduce negative feedback in the input stage; same effect as adding a resistance in the emitter of a common-emitter amplifier.

13.51 From Fig. 13.20(b) and Eq. (13.91), we get

$$R_{o6} = r_{o6}[1 + g_{m6}(R_2 \parallel r_{\pi 6})]$$

where

$$r_{o6} = \frac{125\text{ V}}{9.5\ \mu\text{A}} = 13.6\text{ M}\Omega$$

$$g_{m6} = \frac{9.5\ \mu\text{A}}{25\text{ mV}} = 0.38\text{ mA/V}$$

$$r_{\pi 6} = \frac{200}{0.38} = 526.3\text{ k}\Omega$$

$$\frac{R_{o6}(\text{modified})}{R_{o6}(\text{original})} = \frac{1 + 0.38(R_2' \parallel 526.3)}{1 + 0.38(1 \parallel 526.3)}$$

$$2 \simeq \frac{1 + 0.38\,R_2'}{1 + 0.38}$$

$$\Rightarrow R_2' = 4.63\text{ k}\Omega$$

Thus, R_2 must be increased by a factor of 4.63.

13.52 Refer to Fig. 13.19.

(a) $v_{b6} = i_{e6}(r_{e6} + R_2)$

$= i_e(r_{e6} + R_2)$

where

$$r_{e6} = \frac{25 \text{ mV}}{9.5 \text{ µA}} = 2.63 \text{ k}\Omega$$

$v_{b6} = i_e(2.63 + 2) = 4.63 \text{ k}\Omega \times i_e$

(b) $i_{e7} = i_{R3} + i_{b5} + i_{b6}$

$$= \frac{v_{b6}}{R_3} + \frac{2\alpha i_e}{\beta}$$

$$= \frac{4.63}{50} i_e + \frac{2}{201} i_e$$

$= 0.1 i_e$

(c) $$i_{b7} = \frac{i_{e7}}{\beta_N + 1} = \frac{0.1}{201} i_e = 0.0005 i_e$$

(d) $v_{b7} = i_{e7} r_{e7} + v_{b6}$

$= 0.1 \times 2.38 i_e + 4.63 i_e$

$= 4.89 i_e$

(e) $$R_{\text{in}} \equiv \frac{v_{b7}}{\alpha i_e} \simeq 4.9 \text{ k}\Omega$$

13.53 Output current of first stage $= (1 - 0.8)I$

$= 0.2I$

$$V_{OS} = \frac{0.2I}{G_{m1}}$$

where

$$G_{m1} = \frac{1}{2} g_{m1} = \frac{1}{2} \frac{I}{V_T}$$

Thus,

$$V_{OS} = \frac{0.2I}{0.5I/V_T}$$

$= 0.4 \times V_T = 10 \text{ mV}$

13.54 Refer to Fig. 13.22 which shows the situation when $R_1 = R$ and $R_2 = R + \Delta R$. The result of this mismatch is an output current ΔI given by Eq. (13.94):

$$\Delta I = I \frac{\Delta R}{R + \Delta R + r_e} \qquad (1)$$

If we have an input offset voltage V_{OS}, this offset results in an output current ΔI given by

$$\Delta I = G_{m1} V_{OS} \qquad (2)$$

The offset can be nulled by introducing a mismatch ΔR that results in an equal magnitude and opposite polarity output current. The required ΔR can be found by equating (1) and (2), thus

$$I \frac{\Delta R}{R + \Delta R + r_e} = G_{m1} V_{OS}$$

Substituting for G_{m1} by

$$G_{m1} = \frac{1}{2} g_{m1} = \frac{1}{2} \frac{I}{V_T}$$

we obtain

$$I \frac{\Delta R}{R + \Delta R + r_e} = \frac{1}{2} \frac{I}{V_T} V_{OS}$$

$$\Rightarrow \frac{\Delta R}{R} = \frac{V_{OS}}{2V_T} \frac{1 + r_e/R}{1 - V_{OS}/2V_T} \qquad \text{Q.E.D.}$$

(b) For $V_{OS} = 3$ mV and recalling that

$$r_e = \frac{25 \text{ mV}}{9.5 \text{ µA}} = 2.63 \text{ k}\Omega \text{ and } R = 1 \text{ k}\Omega$$

then

$$\frac{\Delta R}{R} = \frac{3}{2 \times 25} \frac{1 + (2.63/1)}{1 - (3/50)}$$

$$\frac{\Delta R}{R} = 0.23$$

or 23%

For $V_{OS} = -3$ mV, we have

$$\frac{\Delta R}{R} = -0.205 \text{ or } -20.5\%$$

(c) The maximum offset voltage than can be trimmed this way corresponds to R_2 completely shorted, that is, $\Delta R = -R$, thus

$$-1 = \frac{V_{OS}}{2V_T} \frac{1 + 2.63}{1 - \dfrac{V_{OS}}{2V_T}}$$

$$\Rightarrow V_{OS} = -\frac{2V_T}{2.63} = -19 \text{ mV}$$

13.55

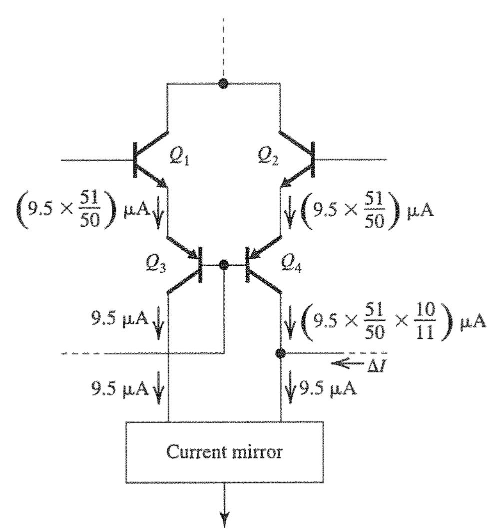


Figure 1

Figure 1 shows the analysis when the β of Q_4 is reduced to 10. The output current of the mirror is

$\Delta I = 0.691\ \mu A$

which corresponds to an input offset voltage of

$$V_{OS} = \frac{\Delta I}{G_{m1}}$$

where $G_{m1} = \frac{1}{2} g_{m1} = 0.19$ mA/V.

Thus,

$$V_{OS} = \frac{0.691}{0.19} = 3.6 \text{ mV}$$

13.56 From Eq. (13.102) we have

$\text{CMRR} = g_{m1}(R_{o9} \parallel R_{o10})/\epsilon_m$

where

$g_{m1} = 0.38$ mA/V

$R_{o9} = 2.63$ MΩ

$R_{o10} = 31.1$ MΩ

$\epsilon_m = 1 - 0.995 = 0.005$

Thus,

$\text{CMRR} = 0.38(2.63 \parallel 31.3) \times 10^3/0.005$

$= 1.84 \times 10^5$

or 105.3 dB

13.57 Please note that an error occurred in the first printing of the text: Q_9 is biased at 19 µA. With a resistance R in the emitter of Q_9, R_{o9} becomes

$R_{o9} = r_{o9}[1 + g_{m9}(R \parallel r_{\pi 9})]$

where

$$r_{o9} = \frac{|V_{Ap}|}{I_{C9}} = \frac{50\text{ V}}{19\ \mu\text{A}} = 2.63 \text{ M}\Omega$$

$$g_{m9} = \frac{I_{C9}}{V_T} = \frac{19\ \mu\text{A}}{0.025\text{ V}} = 0.76 \text{ mA/V}$$

$$r_{\pi 9} = \frac{\beta_P}{g_{m9}} = \frac{50}{0.76} = 65.8 \text{ k}\Omega$$

Thus, to obtain $R_{o9} = R_{o10} = 31.1$ MΩ, we use

$31.1 = 2.63[1 + 0.76(R \parallel 65.8)]$

$\Rightarrow R = 18.2$ kΩ

Thus, R_o to the left of node Y becomes

$R_o = 31.1\text{ M}\Omega \parallel 31.1\text{ M}\Omega = 15.55\text{ M}\Omega$

13.58 Refer to Fig. 13.19.

(a) If R_1 is short-circuited, the incremental transfer ratio of the mirror can be found as follows:

$i_{e5} r_{e5} = i_{e6}(r_{e6} + R_2)$

Thus,

$$\frac{i_{c6}}{i_{c5}} = \frac{i_{e6}}{i_{e5}} = \frac{r_{e5}}{r_{e5} + R_2} = \frac{2.63}{2.63 + 1}$$

$= 0.72$

Thus, the output current of the mirror becomes

$i_o = 1.72\alpha i_e$

rather than $2\alpha i_e$. Thus, the gain of the 741 will be reduced by a factor of $\frac{1.72}{2} = 0.86$.

(b) If R_2 is short-circuited, then

$i_{e5}(r_{e5} + R_1) = i_{e6} r_{e6}$

$$\Rightarrow \frac{i_{c6}}{i_{c5}} = \frac{i_{e6}}{i_{e5}} = \frac{r_{e5} + R_1}{r_{e6}}$$

$$= \frac{2.63 + 1}{2.63} = 1.38$$

Thus, i_o of the mirror becomes

$i_o = 2.38\alpha i_e$

with the result that the gain of the 741 increases by a factor of $\frac{2.38}{2} = 1.19$.

(c) If both R_1 and R_2 are shorted, the gain remains unchanged.

13.59

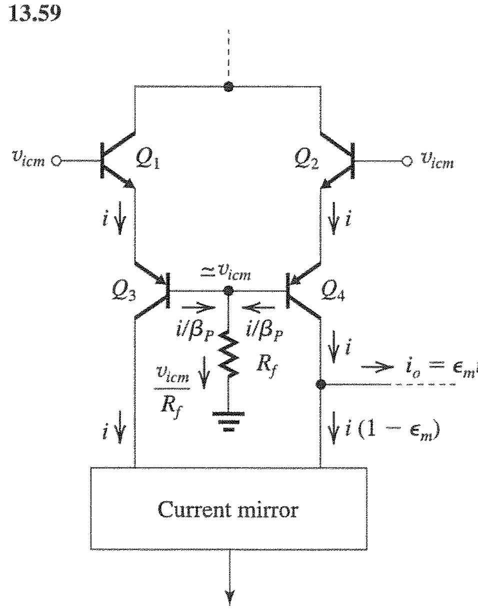


Figure 1

Figure 1 shows the input stage with the approach suggested for determining G_{mcm}. Here

$R_f = R_o(1 + A\beta) = R_o(1 + \beta_P) \simeq \beta_P R_o$

A node equation at the common bases of the Q_3 and Q_4 yields

$$\frac{2i}{\beta_P} = \frac{v_{icm}}{R_f}$$

$$\Rightarrow i = \frac{\beta_P}{2R_f} v_{icm}$$

$$= \frac{\beta_P}{2\beta_P R_o} v_{icm}$$

$$= \frac{v_{icm}}{2R_o}$$

Thus,

$$i_o = \epsilon_m i = \frac{\epsilon_m}{2R_o} v_{icm}$$

and

$$G_{mcm} \equiv \frac{i_o}{v_{icm}} = \frac{\epsilon_m}{2R_o}$$

which is the same result [Eq. (13.100)] obtained by the alternative approach of Example 13.5.

13.60 We use Eqs. (13.103)–(13.107) with

$R_8 = 50\ \Omega,\ R_9 = 50\ \text{k}\Omega,$

$\beta_{16} = \beta_{17} = 200,$

$I_{C16} = 16.2\ \mu\text{A},\ I_{C17} = 550\ \mu\text{A}$

Thus,

$$r_{e16} \simeq \frac{25\ \text{mV}}{16.2\ \mu\text{A}} = 1.54\ \text{k}\Omega$$

$$r_{e17} \simeq \frac{25\ \text{mV}}{0.55\ \text{mA}} = 45.45\ \Omega$$

Now,

$$R_{i2} = 201\{1.54 + [50 \parallel (201 \times 95.45 \times 10^{-3})]\}$$

$$= 3.1\ \text{M}\Omega$$

$$R_{i17} = 201(45.45 + 50) \times 10^{-3}$$

$$= 19.19\ \text{k}\Omega$$

$$v_{b17} = v_{i2} \frac{50 \parallel 19.19}{(50 \parallel 19.19) + 1.54}$$

$$= 0.9 v_{i2}$$

$$i_{c17} = \frac{\alpha \times 0.9 v_{i2}}{(45.45 + 50) \times 10^{-3}} = 9.38 v_{i2}$$

$$G_{m2} \equiv \frac{i_{c17}}{v_{i2}} = 9.38\ \text{mA/V}$$

13.61 Refer to the results of Exercise 13.32. We need to raise r_{o13B} from 90.9 kΩ to 722 kΩ by inserting a resistance R_{13B} in the emitter of Q_{13B}. Since

$$R_{o13B} = r_{o13B}[1 + g_{m13B}(R_{13B} \parallel r_{\pi 13B}]$$

where

$$r_{o13B} = 90.9\ \text{k}\Omega$$

$$g_{m13B} = \frac{0.55\ \text{mA}}{0.025\ \text{V}} = 22\ \text{mA/V}$$

$$r_{\pi 13B} = \frac{\beta_P}{g_{m13B}} = \frac{50}{22} = 2.27\ \text{k}\Omega$$

Thus,

$$722 = 90.9[1 + 22(R_{13B} \parallel 2.27)]$$

$$\Rightarrow R_{13B} = 366\ \Omega$$

The resistors in the emitters of Q_{13A} and Q_{12} must be of values that will result in

$$I_{E13B}R_{13B} = I_{E12}R_{12} = I_{E13A}R_{13A}$$

Thus,

$$R_{13A} = \frac{I_{E13B}}{I_{E13A}} R_{13B}$$

$$= \frac{I_{C13B}}{I_{C13A}} R_{13B}$$

$$= \frac{550}{180} \times 366 = 1.12\ \text{k}\Omega$$

$$R_{12} = \frac{I_{E13B}}{I_{E12}} R_{13B}$$

$$= \frac{I_{C13B}}{I_{C12}} R_{13B}$$

$$= \frac{550}{730} \times 366 = 275\ \Omega$$

13.62 Using Eq. (13.110), we obtain

$$v_{O\max} = V_{CC} - |V_{CE\text{sat}}| - V_{BE14}$$

$$= 5 - 0.2 - 0.6 = +4.2\ \text{V}$$

Using Eq. (13.111), we get

$$v_{O\min} = -V_{EE} + |V_{CE\text{sat}}| + V_{EB23} + V_{BE20}$$

$$= -5 + 0.2 + 0.6 + 0.6 = -3.6\ \text{V}$$

Thus,

$$-3.6\ \text{V} \le v_O \le +4.2\ \text{V}$$

13.63 Refer to Fig. 13.25 and Example 13.6 with Q_{23} having its emitter and base shorted together. In such a situation the input resistance of the output stage becomes

$$R_{\text{in}3} = (\beta_{20}R_L) \parallel (r_{o13A} + r_{AA}) \qquad (1)$$

where we have assumed the situation with v_O negative and Q_{20} supplying the load current.

In Eq. (1),

$$\beta_{20} = 50$$

$$R_L = 2\ \text{k}\Omega$$

$$r_{o13A} = \frac{|V_{Ap}|}{I_{C13A}} = \frac{50}{0.18} = 280\ \text{k}\Omega$$

and r_{AA} is the incremental resistance of the $Q_{18} - Q_{19}$ bias network; very small ($\simeq 160\ \Omega$). Thus,

$R_{in3} \simeq (50 \times 2) \parallel 280$

$= 74\ k\Omega$

The gain of the second stage becomes

$$A_2 = \frac{v_{i3}}{v_{i2}} = -G_{m2}R_{o2}\,\frac{R_{in3}}{R_{in3} + R_{o2}}$$

$$= -6.5 \times 81 \times \frac{74}{74 + 81}$$

$= -215.4$ V/V

Compare to the value with Q_{23} included (-515 V/V).

13.64 Refer to Fig. P13.64.

$$R_{out} = r_{e14} + \frac{r_{AA} + r_{e23} + [R_{o2}/(\beta_P + 1)]}{\beta_{14} + 1}$$

where

$$r_{e14} = \frac{25\text{ mV}}{5\text{ mA}} = 5\ \Omega$$

$r_{AA} = 163\ \Omega$

$$r_{e23} = \frac{25\text{ mV}}{0.18\text{ mA}} = 139\ \Omega$$

$R_{o2} = 81\ k\Omega$

$\beta_P = 50$

$\beta_{14} = 200$

Thus

$$R_{out} = 5 + \frac{163 + 139 + (81000/51)}{201}$$

$= 14.4\ \Omega$

13.65

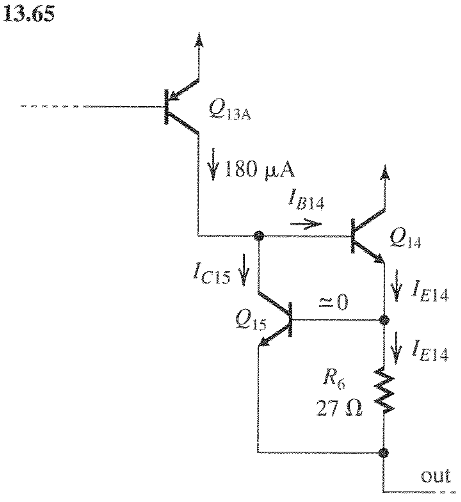


Figure 1

Refer to Fig. 1.

Iteration #1:

$I_{C15} = 180\ \mu A$

$$V_{BE15} = 25 \ln \frac{180 \times 10^{-6}}{10^{-14}} = 590\text{ mV}$$

$$I_{E14} = \frac{V_{BE15}}{R_6} = \frac{590\text{ mV}}{27\ \Omega} = 21.85\text{ mA}$$

$$I_{B14} = \frac{I_{E14}}{\beta_N + 1} = \frac{21.85}{201} = 108.7\ \mu A$$

Iteration #2:

$I_{C15} = 180 - I_{B14} = 180 - 108.7 = 71.3\ \mu A$

$$V_{BE15} = 25 \ln \frac{71.3 \times 10^{-6}}{10^{-14}} = 567.2\text{ mV}$$

$$I_{E14} = \frac{567.2}{27} = 21\text{ mA}$$

$$I_{B14} = \frac{21\text{ mA}}{201} = 104.5\ \mu A$$

Iteration #3:

$I_{C15} = 180 - 104.5 = 75.5\ \mu A$

$$V_{BE15} = 25 \ln \frac{75.5 \times 10^{-6}}{10^{-14}} = 568.6$$

$$I_{E14} = \frac{568.6}{27} = 21.06\text{ mA}$$

which is very close to the value found in Iteration #2; thus, no further iterations are necessary and

$I_{E14} \simeq 21$ mA

13.66 Refer to Fig. 13.14.

Maximum current available from input stage $= 19\ \mu A$

$I_{C22} = 19\ \mu A$

$$V_{BE22} = 25 \ln \frac{19 \times 10^{-6}}{10^{-14}}$$

$= 534$ mV

$V_{BE24} = 534$ mV

$I_{C24} = 19\ \mu A$

$$I_{R11} = \frac{534\text{ mV}}{50\text{ k}\Omega} = 10.7\ \mu A$$

$I_{C21} = I_{C24} + I_{R11}$

$= 19 + 10.7 = 29.7\ \mu A$

$$V_{EB21} = 25 \ln \frac{29.7 \times 10^{-6}}{10^{-14}}$$

$= 545.3$ mV

$$I_{R7} = \frac{545.3}{27} = 20.2\text{ mA}$$

This is the maximum current that the 741 can sink. To reduce this current limit to 10 mA, we need to double the value of R_7.

13.67 The factor 0.97 is simply

$$= \frac{R_L}{R_L + R_{out}}$$

Thus, for $R_L = \infty$,

$A_0 = 243,147/0.97 = 250,667$ V/V

This is the open-circuit voltage gain. The output resistance is found from

$$\frac{2}{2 + R_{out}} = 0.97$$

$\Rightarrow R_{out} = 62\ \Omega$

The gain with $R_L = 500\ \Omega$ is

$$A_0 = 250,667 \times \frac{500}{500 + 62}$$

$= 223,013$ V/V

13.68 If the phase margin is 80°, the phase due to the second pole f_{P2} at the unity gain frequency f_t must be 10°. Thus,

$$\tan^{-1} \frac{f_t}{f_{P2}} = 10°$$

Since $f_t = 1$ MHz,

$$f_{P2} = \frac{1\text{ MHz}}{\tan 10°} = 5.67\text{ MHz}$$

13.69 The phase introduced at $f_t = 1$ MHz by each of the coincident second and third poles must be 5°. Thus, $f_{P2} = f_{P3}$ can be obtained from

$$\tan^{-1} \frac{f_t}{f_{P2}} = 5°$$

$$\Rightarrow f_{P2} = f_{P3} = \frac{1\text{ MHz}}{\tan 5°} = 11.4\text{ MHz}$$

13.70

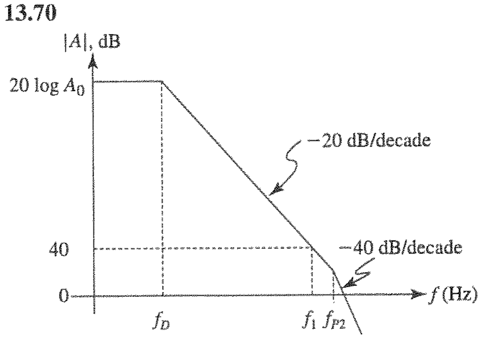


Figure 1

For a phase margin of 85° with a closed loop gain of 100, the phase at f_1 due to the pole at 5 MHz must be at most 5°; thus,

$$\tan^{-1} \frac{f_1}{5\text{ MHz}} = 5°$$

$\Rightarrow f_1 = 5 \times \tan 5° = 437$ kHz

Thus, the new dominant pole must be at f_D,

$$f_D \times \frac{A_0}{100} = 437$$

$$f_D \times \frac{243,147}{100} = 437$$

$\Rightarrow f_D = 180$ Hz

To find the required value of C_C, we use Eq. (13.116) to determine C_{in}:

$C_{in} = C_C(1 + |A_2|)$

$= C_C \times 516$

Then,

$$f_D = \frac{1}{2\pi C_{in} R_t}$$

where

$R_t = 2.5\ \text{M}\Omega$

$$180 = \frac{1}{2\pi \times 516 C_C \times 2.5 \times 10^6}$$

$\Rightarrow C_C = 0.7$ pF

13.71 $f_P = \frac{f_t}{A_0} = \frac{5\text{ MHz}}{10^6} = 5$ Hz

But,

$$f_P = \frac{1}{2\pi CR}$$

where

$C = (1 + |A|)C_C$

$= (1 + 1000) \times 50$

$= 50.05$ nF

$$5 = \frac{1}{2\pi \times 50.05 \times 10^{-9} \times R}$$

$\Rightarrow R = 636\ \text{k}\Omega$

13.72 DC gain $A_0 = G_{m1}R$

$= 2 \times 10^{-3} \times 2 \times 10^7$

$= 4 \times 10^4$ V/V

$20 \log A_0 = 92$ dB

$$f_P = \frac{1}{2\pi C_C R}$$

$$= \frac{1}{2\pi \times 100 \times 10^{-12} \times 2 \times 10^7}$$

$= 79.6$ Hz $\simeq 80$ Hz

$f_t = A_0 f_P = 4 \times 10^4 \times 80$

$= 3.2$ MHz

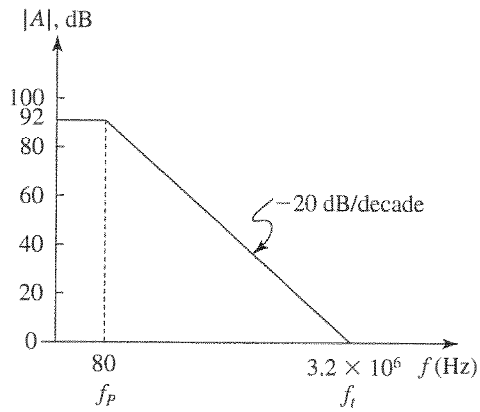


Figure 1

Figure 1 shows a sketch of the Bode plot for the magnitude of the open-loop gain of the op amp.

$$SR = \frac{I}{C_C}$$

But,

$$G_{m1} = \frac{I}{2V_T}$$

Thus,

$$I = 2V_T G_{m1}$$

and

$$SR = 2V_T \frac{G_{m1}}{C_C}$$

$$= 2 \times 25 \times 10^{-3} \times \frac{2 \times 10^{-3}}{100 \times 10^{-12}}$$

$$= 1 \text{ V}/\mu\text{s}$$

13.73 For a sine-wave output, we have

$$v_O = \hat{V}_o \sin \omega t$$

$$\frac{dv_O}{dt} = \omega \hat{V}_o \cos \omega t$$

$$\left.\frac{dv_O}{dt}\right|_{\max} = \omega \hat{V}_o$$

$$10 \times 10^6 = \omega_M \times 10$$

$$\omega_M = 10^6 \text{ rad/s}$$

$$f_M = \frac{10^6}{2\pi} = 159.2 \text{ kHz}$$

If the topology is similar to that of the 741, then we can use Eq. (13.126),

$$SR = 4V_T \omega_t$$

$$\Rightarrow \omega_t = \frac{SR}{4V_T} = \frac{10 \times 10^6}{4 \times 25 \times 10^{-3}}$$

$$= 10^8 \text{ rad/s}$$

$$f_t = \frac{10^8}{2\pi} = 15.9 \text{ MHz}$$

13.74 Including a resistance R_E in the emitter of each of Q_3 and Q_4 cause G_{m1} to become

$$G_{m1} = \frac{2}{4r_e + 2R_E}$$

$$= \frac{1}{2r_e + R_E}$$

where r_e is the emitter resistance of each of $Q_1 - Q_4$,

$$r_e = \frac{V_T}{I}$$

Thus,

$$G_{m1} = \frac{I}{2V_T + IR_E} \qquad (1)$$

The slew rate is still given by (13.125),

$$SR = \frac{2I}{C_C} \qquad (2)$$

Also, the model in Fig. 13.30 still applies; thus,

$$\omega_t = \frac{G_{m1}}{C_C} \qquad (3)$$

Equations (1)–(3) can be combined to obtain

$$SR = \frac{2I}{C_C} = \frac{2G_{m1}(2V_T + IR_E)}{C_C}$$

$$= 2\omega_t(2V_T + IR_E)$$

$$= 4(V_T + IR_E/2)\omega_t \qquad \text{Q.E.D.}$$

Since for the 741

$$SR = 4V_T\omega_t$$

to double SR while keeping ω_t unchanged, we select

$$\frac{1}{2}IR_E = V_T$$

If we also keep I unchanged, then

$$R_E = \frac{2V_T}{I} = \frac{2 \times 25 \times 10^{-3}}{9.5 \times 10^{-6}}$$

$$= 5.26 \text{ k}\Omega$$

From Eq. (1), the new value of G_{m1} is

$$G_{m1} = \frac{I}{2V_T + IR_E}$$

$$= \frac{I}{2V_T + 2V_T} = \frac{I}{4V_T}$$

$$= 0.095 \text{ mA/V}$$

which is half the original value. From Eq. (3), we see that C_C will have to be one half the original value, thus

$$C_C = 15 \text{ pF}$$

This result could have been obtained also from $SR = I/C_C$; doubling SR with I unchanged requires halving C_C. Now, with G_{m1} half the original value, the dc gain also will be half the original value,

This figure belongs to Problem 13.75, part (c).

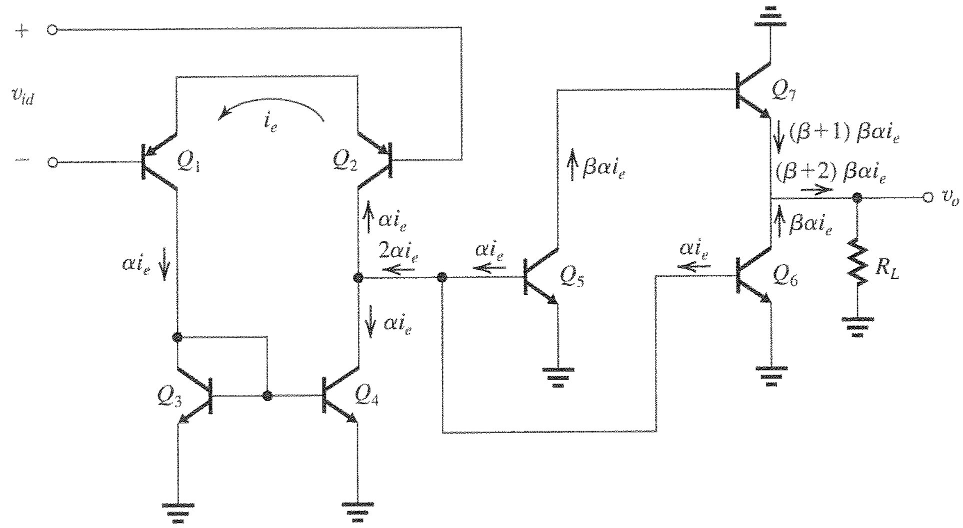


Figure 1

$$A_0 = \frac{1}{2} \times 243,147 = 121,573 \text{ V/V}$$

or 101.7 dB

Finally, since

$$f_P = \frac{f_t}{A_0}$$

halving A_0 with f_t unchanged means f_P is doubled,

$$f_P = 8.2 \text{ Hz}$$

This is a result of C_C in Eq. (13.116) being is halved and thus f_P in Eq. (13.118) is doubled.

13.75 (a) Refer to Fig. P13.75.

$$I_{C1} = I_{C2} = I_{C3} = I_{C4} = 0.05 \text{ mA}$$

$$I_{C5} = 1 \text{ mA}$$

$$I_{C7} = I_{C6} = I_{C5} = 1 \text{ mA}$$

(b) For Q_1 and Q_2, we have

$$g_m = \frac{0.05 \text{ mA}}{0.025 \text{ V}} = 2 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{2} = 50 \text{ k}\Omega$$

$$R_{id} = 2r_\pi = 100 \text{ k}\Omega$$

(c) Figure 1 shows the small-signal analysis where

$$i_e = \frac{v_i}{2r_{e1,2}}$$

$$v_o = (\beta + 2)\beta\alpha i_e R_L$$

$$A_v = \frac{v_o}{v_i} = \frac{(\beta + 2)\beta\alpha R_L}{2r_{e1,2}}$$

$$A_v \simeq \frac{1}{2}\beta^2 \frac{R_L}{r_{e1,2}}$$

where

$$r_{e1,2} = \frac{25 \text{ mV}}{0.05 \text{ mA}} = 0.5 \text{ k}\Omega$$

$$A_v = \frac{1}{2}100^2 \times \frac{5}{0.5} = 5 \times 10^4 \text{ V/V}$$

or 94 dB

(d)

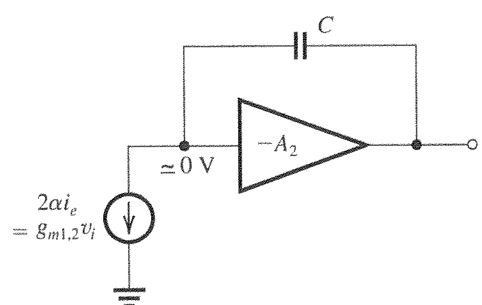


Figure 2

Replacing the second stage with an amplifier having a large negative gain, we obtain the equivalent circuit shown in Fig. 2. From this equivalent circuit we see that the gain is approximately given by

$$A(s) = \frac{g_{m1,2}}{sC}$$

Thus, the unity gain frequency ω_t is given by

$$\omega_t = \frac{g_{m1,2}}{C}$$

and the 3-dB frequency ω_P is

$$\omega_P = \frac{\omega_t}{A_0} = \frac{g_{m1,2}}{A_0 C}$$

$$f_P = \frac{g_{m1,2}}{2\pi A_0 C}$$

For $f_P = 100$ Hz and substituting $g_{m1,2} = 2$ mA/V and $A_0 = 5 \times 10^4$, we find

$$C = \frac{2 \times 10^{-3}}{2\pi \times 100 \times 5 \times 10^4}$$

$$= 63.7 \text{ pF}$$

13.76 $I = 5\ \mu\text{A}$

$$\frac{I_{S2}}{I_{S1}} = 4$$

Using Eq. (13.127), we obtain

$$I = \frac{V_T}{R_2} \ln\left(\frac{I_{S2}}{I_{S1}}\right)$$

$$5 \times 10^{-3} = \frac{0.025}{R_2} \ln 4$$

$$\Rightarrow R_2 = 6.93 \text{ k}\Omega$$

$$R_3 = R_4 = \frac{0.15 \text{ V}}{0.005 \text{ mA}} = 30 \text{ k}\Omega$$

13.77 For $I_5 = 10\ \mu\text{A} = I$, then

$$\frac{Q_5 \text{ emitter area}}{Q_1 \text{ emitter area}} = 1$$

For $I_6 = 40\ \mu\text{A} = 4I$, then

$$\frac{Q_6 \text{ emitter area}}{Q_1 \text{ emitter area}} = 4$$

If we connect a resistance R_6 in the emitter of Q_6, then I_6 changes to a new value determined as follows:

$$V_{BE6} + I_6 R_6 = V_{BE1}$$

$$I_6 R_6 = V_{BE1} - V_{BE6}$$

$$= V_T \ln\frac{I}{I_{S1}} - V_T \ln\frac{I_6}{I_{S6}}$$

But I_6 is to be equal to I, thus

$$IR_6 = V_T \ln\frac{I_{S6}}{I_{S1}}$$

$$R_6 = \frac{V_T}{I} \ln 4$$

$$\Rightarrow R_6 = \frac{0.025}{0.01} \ln 4 = 3.47 \text{ k}\Omega$$

If the V_{BIAS1} line has a low incremental resistance to ground, then

$$R_{o5} = r_{o5} = \frac{V_{An}}{I_5} = \frac{30 \text{ V}}{10\ \mu\text{A}} = 3 \text{ M}\Omega$$

$$R_{o6} = r_{o6} + (R_6 \parallel r_{\pi 6})(1 + g_{m6} r_{o6})$$

where

$$r_{o6} = \frac{30 \text{ V}}{10\ \mu\text{A}} = 3 \text{ M}\Omega$$

$$g_{m6} = \frac{10\ \mu\text{A}}{0.025 \text{ V}} = 0.4 \text{ mA/V}$$

$$r_{\pi 6} = \frac{\beta_N}{g_{m6}} = \frac{40}{0.4} = 100 \text{ k}\Omega$$

$$R_{o6} = 3 + (3.47 \parallel 100) \times 10^{-3}(1 + 1200)$$

$$R_{o6} = 3 + 4 = 7 \text{ M}\Omega$$

Thus, increasing the BEJ area by a factor of 4 and adding a resistance R_6 to restore the current to the desired value of 10 μA increases the output resistance by a factor of about 2.5!

13.78 (a) The bias current I of the differential pair is given by Eq. (13.127),

$$I = \frac{V_T}{R_5} \ln\left(\frac{I_{S5}}{I_{S1}}\right) \tag{1}$$

The voltage gain of the differential pair is given by

$$A_d = g_m R_C$$

where g_m is the transconduuctance of each of the two transistors in the differential pair,

$$g_m = \frac{I/2}{V_T} = \frac{I}{2V_T}$$

Thus,

$$A_d = \frac{IR_C}{2V_T} \tag{3}$$

Substituting for I from Eq. (1) into Eq. (3), we obtain

$$A_d = \frac{1}{2}\frac{R_C}{R_5} \ln\left(\frac{I_{S5}}{I_{S2}}\right) \tag{4}$$

which indicates that A_d will be independent of temperature!

(b) $I = 20\ \mu\text{A}$, $A_d = 10$ V/V, $\dfrac{I_{S5}}{I_{S1}} = 4$

Using Eq. (1), we obtain

$$20 \times 10^{-3} = \frac{0.025}{R_5} \ln 4$$

$$\Rightarrow R_5 = 1.73 \text{ k}\Omega$$

Using Eq. (4), we get

$$10 = \frac{1}{2}\frac{R_C}{1.73}\ln 4$$

$$\Rightarrow R_C = 25\ \text{k}\Omega$$

13.79 (a) Refer to Fig. 13.35(a).

$$V_{ICM\min} = V_{C1} - 0.6$$

$$= 0.7 - 0.6 = 0.1\ \text{V}$$

$$V_{ICM\max} = V_{CC} - 0.1 - 0.7$$

$$= 3 - 0.8 = 2.2\ \text{V}$$

Thus,

$$0.1\ \text{V} \le V_{ICM} \le 2.2\ \text{V}$$

(b)

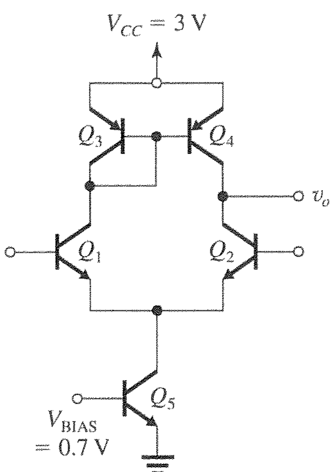


Figure 1

Figure 1 shows the complementary circuit to that in Fig. 13.33(a).

Here,

$$0.8\ \text{V} \le V_{ICM} \le 2.9\ \text{V}$$

13.80 Refer to Fig. 13.35(b).

$$V_{ICM\max} = V_{CC} - 0.1 - 0.7 = 3 - 0.8$$

$$= +2.2\ \text{V}$$

$$V_{ICM\min} = \frac{1}{2}I_{RC} - 0.6$$

$$= \frac{1}{2} \times 0.02 \times 25 - 0.6$$

$$= -0.35\ \text{V}$$

Thus,

$$-0.35\ \text{V} \le V_{ICM} \le +2.2\ \text{V}$$

$$A_v = g_m R_C$$

where

$$g_m = \frac{I_C}{V_T} = \frac{10 \times 10^{-6}}{25 \times 10^{-3}} = 0.4\ \text{mA/V}$$

$$A_v = 0.4 \times 25 = 10\ \text{V/V}$$

13.81 $g_m = \dfrac{I/2}{V_T} = \dfrac{20\ \mu\text{A}}{25\ \text{mV}} = 0.8\ \text{mA/V}$

For $A_d = 10$ V/V, we have

$$10 = g_m R_C$$

$$\Rightarrow R_C = 12.5\ \text{k}\Omega$$

$$\frac{I}{2}R_C = 20 \times 10^{-3} \times 12.5 = 0.25\ \text{V}$$

$$V_{ICM\min} = 0.8\ \text{V}$$

$$V_{ICM\max} = V_{CC} - \frac{I}{2}R_C + 0.6$$

$$= 3 - 0.25 + 0.6 = 3.35\ \text{V}$$

Thus,

$$0.8\ \text{V} \le V_{ICM} \le 3.35\ \text{V}$$

$$R_{id} = 2r_\pi = 2\frac{\beta_N}{g_m}$$

$$= 2\frac{40}{0.8} = 100\ \text{k}\Omega$$

To increase R_{id} by a factor of 4, g_m and hence I must be reduced by a factor of 4, thus I_{C6} becomes

$$I_{C6} = 10\ \mu\text{A}$$

To keep the gain and the permissable range of V_{ICM} unchanged, R_C must be increased by a factor of 4, thus R_C becomes

$$R_C = 50\ \text{k}\Omega$$

13.82 Refer to Fig. 13.38, which shows the differential half-circuit of the differential amplifier of Fig. 13.37.

$$R_{id} = 2r_{\pi 1} = 2\frac{\beta_P}{g_{m1}}$$

where

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{4 \times 10^{-6}}{25 \times 10^{-3}} = 0.16\ \text{mA/V}$$

Thus,

$$R_{id} = \frac{2 \times 10}{0.16} = 125\ \text{k}\Omega$$

The short-circuit transconductance G_{m1} can be found from Fig. 13.38(b):

$$G_{m1} = \frac{i_o}{v_{id}/2}$$

At node X we have four resistances to ground:

$$r_{o1} = \frac{|V_{Ap}|}{I_{C1}} = \frac{20\ \text{V}}{4\ \mu\text{A}} = 5\ \text{M}\Omega$$

$R_7 = 22\ \text{k}\Omega$

$$r_{o7} = \frac{|V_{An}|}{I_{C7}} = \frac{30\ \text{V}}{8\ \mu\text{A}} = 3.75\ \text{M}\Omega$$

$$r_{e7} \simeq \frac{1}{g_{m7}} = \frac{V_T}{I_{C7}} = \frac{25\ \text{mV}}{8\ \mu\text{A}} = 3.125\ \text{k}\Omega$$

Obviously, r_{o1} and r_{o7} are much larger than r_{e7} and R_7. Then, the portion of $g_{m1}(v_{id}/2)$ that flows into the emitter proper of Q_7 can be found from

$$i_{e7} \simeq g_{m1}\left(\frac{V_{id}}{2}\right)\frac{R_7}{R_7 + r_{e7}}$$

$$= g_{m1}\left(\frac{V_{id}}{2}\right)\frac{22}{22 + 3.125}$$

$$= 0.876 g_{m1}\left(\frac{V_{id}}{2}\right)$$

Thus,

$$G_{m1} \equiv \frac{i_o}{V_{id}/2} = \frac{\alpha i_{e7}}{V_{id}/2}$$

$$= 0.876 g_{m1} = 0.137\ \text{mA/V}$$

The total resistance between the output node and ground for the circuit in Fig. 13.38(a) is

$$R = R_{o9} \parallel R_{o7} \parallel (R_L/2)$$

The resistances R_{o9} is the output resistance of Q_9, which has an emitter-degeneration resistance R_9. Thus,

$$R_{o9} = r_{o9} + (R_9 \parallel r_{\pi 9})(1 + g_{m9} r_{o9})$$

where

$$r_{o9} = \frac{|V_{Ap}|}{I_{C9}} = \frac{20\ \text{V}}{8\ \mu\text{A}} = 2.5\ \text{M}\Omega$$

$$g_{m9} = \frac{I_{C9}}{V_T} = \frac{8\ \mu\text{A}}{25\ \text{mV}} = 0.32\ \text{mA/V}$$

$$r_{\pi 9} = \frac{\beta_P}{g_{m9}} = \frac{10}{0.32} = 31.25\ \text{k}\Omega$$

Thus,

$$R_{o9} = 12.5 + (33 \parallel 31.25) \times 10^{-3}(1 + 0.32 \times 2.5 \times 10^3)$$

$$= 15.3\ \text{M}\Omega$$

The resistance R_{o7} is the output resistance of Q_7, which has an emitter-degeneration resistance $(R_7 \parallel r_{o1}) \simeq R_7$. Thus,

$$R_{o7} = r_{o7} + (R_7 \parallel r_{\pi 7})(1 + g_{m7} r_{o7})$$

where

$$r_{o7} = \frac{|V_{An}|}{I_{C7}} = \frac{30\ \text{V}}{8\ \mu\text{A}} = 3.75\ \text{M}\Omega$$

$$g_{m7} = \frac{I_{C7}}{V_T} = \frac{8\ \mu\text{A}}{25\ \text{mV}} = 0.32\ \text{mA/V}$$

$$r_{\pi 7} = \frac{\beta_N}{g_{m7}} = \frac{40}{0.32} = 125\ \text{k}\Omega$$

Thus,

$$R_{o7} = 3.75 + (22 \parallel 125) \times 10^{-3}(1 + 0.32 \times 3.75 \times 10^3)$$

$$= 26.2\ \text{M}\Omega$$

$$\frac{R_L}{2} = \frac{1.5}{2} = 0.75\ \text{M}\Omega$$

The load resistance R can now be found as

$$R = 15.3 \parallel 26.2 \parallel 0.75 = 0.696\ \text{M}\Omega$$

Finally, we can find the voltage gain as

$$A_v = \frac{v_{od}/2}{v_{id}/2} = G_{m1} R$$

$$= 0.137 \times 0.696 \times 10^3 = 95.4\ \text{V/V}$$

13.83 $I_{C1} = I$

$$I_{C7} = I_{C9} = 2I$$

From Fig. 13.37 we see that the current through R_7 is approximately $(I_{C1} + I_{C7})$, that is, $3I$. Thus,

$$R_7 = \frac{0.2}{3I}$$

Since Q_3 and Q_4 are cut off, the current through R_9 is equal to I_{E9} or approximately I_{C9}, thus

$$R_9 = \frac{0.3}{2I}$$

To determine the short-circuit transconductance G_{m1}, refer to Fig. 13.38(b).

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{I}{V_T}$$

$$G_{m1} = \frac{i_o}{v_{id}/2}$$

At node X we have four resistances in parallel, namely, r_{o1}, R_7, r_{o7}, and r_{e7}:

$$r_{o1} = \frac{|V_{Ap}|}{I_{C1}} = \frac{20}{I}$$

$$R_7 = \frac{0.2}{3I} = \frac{0.067}{I}$$

$$r_{o7} = \frac{V_{An}}{I_{C7}} = \frac{30}{2I} = \frac{15}{I}$$

$$r_{e7} \simeq \frac{V_T}{I_{C7}} = \frac{0.025}{2I} = \frac{0.0125}{I}$$

Thus, r_{o1} and r_{o7} are much greater than r_{e7} and R_7, and the portion of $g_{m1}\left(\frac{v_{id}}{2}\right)$ that flows into the emitter proper of Q_7 is given by

$$i_{e7} \simeq g_{m1}\left(\frac{v_{id}}{2}\right)\frac{R_7}{R_7 + r_{e7}}$$

$$= \left(\frac{I}{V_T}\right)\left(\frac{v_{id}}{2}\right)\frac{0.067}{0.067 + 0.0125}$$

$$= 0.84\left(\frac{I}{V_T}\right)\left(\frac{v_{id}}{2}\right)$$

The output short-circuit current i_o will be

$$i_o \simeq i_{e7} = 0.84\left(\frac{I}{V_T}\right)\left(\frac{v_{id}}{2}\right)$$

Thus,

$$G_{m1} = 0.84\frac{I}{V_T} \simeq 33.6I$$

To obtain the output resistance R,

$$R = R_{o9} \parallel R_{o7}$$

we determine R_{o9} as follows:

$$R_{o9} = r_{o9} + (R_9 \parallel r_{\pi 9})(1 + g_{m9}r_{o9})$$

where

$$r_{o9} = \frac{|V_{Ap}|}{I_{C9}} = \frac{20}{2I} = \frac{10}{I}$$

$$g_{m9} = \frac{I_{C9}}{V_T} = \frac{2I}{0.025} = 80I$$

$$g_{m9}r_{o9} = 800$$

$$r_{\pi 9} = \frac{\beta_P}{g_{m9}} = \frac{10}{80I} = \frac{0.125}{I}$$

Thus,

$$R_{o9} = \frac{10}{I} + \left(\frac{0.15}{I} \parallel \frac{0.125}{I}\right) \times 801$$

$$= \frac{64.6}{I}$$

We next determine R_{o7} as follows:

$$R_{o7} = r_{o7} + (R_7 \parallel r_{\pi 7})(1 + g_{m7}r_{o7})$$

where

$$r_{o7} = \frac{15}{I}$$

$$R_7 = \frac{0.067}{I}$$

$$g_{m7} = \frac{I_{C7}}{V_T} = \frac{2I}{V_T}$$

$$g_{m7}r_{o7} = 1200$$

$$r_{\pi 7} = \frac{\beta_N}{g_{m7}} = \frac{40}{2I/V_T} = \frac{0.5}{I}$$

Thus,

$$R_{o7} = \frac{15}{I} + \left(\frac{0.067}{I} \parallel \frac{0.5}{I}\right) \times 1201$$

$$= \frac{86}{I}$$

We now can determine the output resistance R as

$$R = R_{o9} \parallel R_{o7} = \frac{64.6}{I} \parallel \frac{86}{I} = \frac{36.9}{I}$$

The open-circuit voltage gain can be obtained as

$$A_{vo} = G_{m1}R$$

$$= 0.84\left(\frac{I}{V_T}\right)\left(\frac{36.9}{I}\right)$$

$$= 1240 \text{ V/V}$$

With a load resistance R_L, we have

$$A_v = A_{vo}\frac{R_L}{R_L + R}$$

$$= 1240\frac{R_L}{R_L + \frac{36.9}{I}}$$

$$= 1240\frac{IR_L}{IR_L + 36.9}$$

For $R_L = 1\text{ M}\Omega$ and I in μA, we have

$$A_v = 1240\frac{I}{I + 36.9}$$

From this equation we can obtain

$$I = \frac{36.9}{\frac{1240}{A_v} - 1}$$

Thus, for $A_v = 150$ V/V, the required value of I is

$$I = \frac{36.9}{\frac{1240}{150} - 1} = 5.1\ \mu\text{A}$$

and for $A_v = 300$ V/V, we require

$$I = \frac{36.9}{\frac{1240}{300} - 1} = 11.8\ \mu\text{A}$$

13.84 (a) Refer to Fig. 13.39. Break the loop at the input of the CMF circuit and apply a common-mode input signal ΔV_{CM}. The CMF circuit will respond by causing a change ΔV_B in its output voltage that can be found from its transfer characteristic as

$$\Delta V_B = \Delta V_{CM}$$

Now, a change ΔV_B in the base voltage of Q_7 and Q_8 results in

$$\Delta I_{E8} = \Delta I_{E7} = \frac{\Delta V_B}{r_{e7} + R_7}$$

The corresponding change in the collector voltages of Q_7 and Q_8 will be

$$\Delta v_{O2} = \Delta v_{O1} = -\Delta I_{C7} R_o$$

Now,

$$\Delta I_{C7} \simeq \Delta I_{E7}$$

and

$$R_o = R_{o7} \parallel R_{o9}$$

thus

$$\Delta v_{O1} = -\frac{\Delta V_B}{r_{e7} + R_7}(R_{o7} \parallel R_{o9})$$

This is the returned voltage, thus

$$A\beta \equiv -\frac{\Delta v_{O1}}{\Delta V_{CM}}$$

$$= \frac{R_{o7} \parallel R_{o9}}{r_{e7} + R_7} \qquad \text{Q.E.D.} \qquad (1)$$

(b) From Example 13.8, we have

$$R_{o7} = 23\ \text{M}\Omega,\ R_{o9} = 12.9\ \text{M}\Omega,$$

$$r_{e7} \simeq \frac{V_T}{I_{C7}} = \frac{25\ \text{mV}}{10\mu\text{A}} = 2.5\ \text{k}\Omega,$$

$$R_7 = 20\ \text{k}\Omega$$

thus

$$A\beta = \frac{(23 \parallel 12.9) \times 10^3}{2.5 + 20}$$

$$= 367.3$$

For a change $\Delta I = 0.3\ \mu\text{A}$, the corresponding change in V_{CM} without feedback is

$$\Delta V_{CM} = \Delta I(R_{o7} \parallel R_{o9})$$

The negative feedback reduces this change by the amount of negative feedback $1 + A\beta \simeq A\beta$, thus the actual ΔV_{CM} becomes

$$\Delta V_{CM} \simeq \frac{\Delta I(R_{o7} \parallel R_{o9})}{A\beta}$$

Substituting for $A\beta$ from Eq. (1), we obtain

$$\Delta V_{CM} = \Delta I(r_{e7} + R_7)$$

$$= 0.3 \times 10^{-6}(2.5 + 20)$$

$$= 6.75\ \text{mV}$$

which is identical to the value found in Example 13.8.

13.85 (a) v_O can range to within 0.1 V (the saturation voltage) of ground and V_{CC}, thus

$$0.1\ \text{V} \le v_O \le 2.9\ \text{V}$$

(b) For $i_L = 0$, the output resistance is

$$R_o = r_{oN} \parallel r_{oP}$$

where

$$r_{oN} = \frac{V_{An}}{I_Q} = \frac{30\ \text{V}}{0.6\ \text{mA}} = 50\ \text{k}\Omega$$

$$r_{oP} = \frac{|V_{Ap}|}{I_Q} = \frac{20\ \text{V}}{0.6\ \text{mA}} = 33.3\ \text{k}\Omega$$

Thus,

$$R_o = 50 \parallel 33.3 = 20\ \text{k}\Omega$$

(c) $R_{\text{out}} = R_{of} = \dfrac{R_o}{1 + A\beta}$

$$= \frac{20\ \text{k}\Omega}{1 + 10^5} \simeq 0.2\ \Omega$$

(d) For $i_L = 12$ mA, we have

$$i_N = \frac{I_Q}{2} = 0.3\ \text{mA}$$

$$i_P = 12 + 0.3 = 12.3\ \text{mA}$$

$$r_{oN} = \frac{30\ \text{V}}{0.3\ \text{mA}} = 100\ \text{k}\Omega$$

$$r_{oP} = \frac{20\ \text{V}}{12.3} = 1.63\ \text{k}\Omega$$

$$R_o = 100 \parallel 1.63 = 1.6\ \text{k}\Omega$$

(e) For $i_L = -12$ mA, we have

$$i_P = 0.3\ \text{mA}$$

$$i_N = 12.3\ \text{mA}$$

$$r_{oN} = \frac{30\ \text{V}}{12.3\ \text{mA}} = 2.44\ \text{k}\Omega$$

$$r_{oP} = \frac{20\ \text{V}}{0.3\ \text{mA}} = 66.7\ \text{k}\Omega$$

$$R_o = 2.44 \parallel 66.7 = 2.4\ \text{k}\Omega$$

13.86 Refer to Fig. 13.43.

$$v_{B7} = v_{BEN} = V_T \ln\left(\frac{i_N}{I_{SN}}\right) \qquad (1)$$

$$i_4 = \frac{v_{EBP} - v_{EB4}}{R_4} \qquad (2)$$

$$v_{B6} = v_{BE5} + i_5 R_5$$

But,

$$i_5 = i_4 \text{ and } R_5 = R_4$$

thus

$$v_{B6} = v_{BE5} + i_4 R_4$$

Using Eq. (2), we obtain

$$v_{B6} = v_{BE5} + v_{EBP} - v_{EB4}$$

$$= (v_{BE5} - v_{EB4}) + v_{EBP}$$

$$= V_T \ln\left(\frac{I_{S4}}{I_{S5}}\right) + V_T \ln\left(\frac{i_P}{I_{SP}}\right)$$

$$= V_T \ln\left(\frac{I_{S4}}{I_{S5}} \frac{i_P}{I_{SP}}\right) \qquad (3)$$

Now, using the given relationship

$$\frac{I_{SP}}{I_{S4}} = \frac{I_{SN}}{I_{S5}}$$

in Eq. (3), we get

$$v_{B6} = V_T \ln\left(\frac{i_P}{I_{SN}}\right) \qquad (4)$$

Using Eqs. (1) and (4), we obtain

$$v_{B6} - v_{B7} = V_T \ln\left(\frac{i_P}{i_N}\right)$$

This is the differential voltage input for the differential amplifier $Q_6 - Q_7$. Thus,

$$i_{C6} = \frac{I}{1 + e^{(v_{B6} - v_{B7})/V_T}}$$

$$= \frac{I}{1 + \frac{i_P}{i_N}}$$

$$= \frac{i_N}{i_P + i_N} I \qquad \text{Q.E.D.}$$

Similarly,

$$i_{C7} = \frac{I}{1 + e^{(v_{B7} - v_{B6})/V_T}}$$

$$= \frac{I}{1 + \frac{i_N}{i_P}}$$

$$= \frac{i_P}{i_P + i_N} I \qquad \text{Q.E.D.}$$

13.87 $v_E = v_{EB7} + v_{BEN}$

Since Q_7 conducts a current i_{C7} given by Eq. (13.131),

$$i_{C7} = I \frac{i_P}{i_P + i_N}$$

and Q_N conducts a current i_N, then

$$v_E = V_T \ln\left(\frac{I\, i_P}{i_P + i_N} \frac{1}{I_{S7}}\right) + V_T \ln\left(\frac{i_N}{I_{SN}}\right)$$

$$= V_T \ln\left[\frac{i_P\, i_N}{i_P + i_N} \frac{I}{I_{SN}\, I_{S7}}\right] \qquad \text{Q.E.D.}$$

13.88 $I_Q = 0.6 \text{ mA} = 600\ \mu\text{A}$

$I = 12\ \mu\text{A}$

$$\frac{I_{SN}}{I_{S10}} = 8$$

$$\frac{I_{S7}}{I_{S11}} = 4$$

Using Eq. (13.136), we have

$$600 = 2\left(\frac{I_{\text{REF}}^2}{12}\right) \times 8 \times 4$$

$$\Rightarrow I_{\text{REF}} = 10.6\ \mu\text{A}$$

The minimum current in each transistor is about 0.3 mA.

14.1 (a) $R_{on} = r_{DSN}$

$$= \frac{1}{(\mu_n C_{ox})\left(\frac{W}{L}\right)_n (V_{DD} - V_{tn})} \qquad (1)$$

$$= \frac{1}{0.470 \times 1.5(1 - 0.35)} = 2.18 \text{ k}\Omega$$

(b) $R_{on} = r_{DSP}$

$$= \frac{1}{(\mu_p C_{ox})\left(\frac{W}{L}\right)_p (V_{DD} - |V_{tp}|)} \qquad (2)$$

$$= \frac{1}{0.190 \times 1.5(1 - 0.35)} = 5.40 \text{ k}\Omega$$

(c) From (1) and (2) since $V_{tn} = -|V_{tp}|$, then if R_{on} are to be equal, then

$$(\mu_n C_{ox})\left(\frac{W}{L}\right)_n = (\mu_p C_{ox})\left(\frac{W}{L}\right)_p$$

$$\Rightarrow \left(\frac{W}{L}\right)_p = \frac{\mu_n C_{ox}}{\mu_p C_{ox}}\left(\frac{W}{L}\right)_n$$

$$= \frac{470}{190} \times 1.5 = 3.71$$

14.2 (a) For Q_N, we have

$$R_{on} = \frac{1}{(\mu_n C_{ox})\left(\frac{W}{L}\right)_n (V_{DD} - V_{tn})}$$

For Q_P, we have

$$R_{on} = \frac{1}{(\mu_p C_{ox})\left(\frac{W}{L}\right)_p (V_{DD} - |V_{tp})|}$$

Since $V_{tn} = |V_{tp}|$, then for R_{on} of Q_P to equal R_{on} of Q_N,

$$(\mu_p C_{ox})\left(\frac{W}{L}\right)_p = (\mu_n C_{ox})\left(\frac{W}{L}\right)_n$$

$$\Rightarrow \left(\frac{W}{L}\right)_p = \frac{\mu_n C_{ox}}{\mu_p C_{ox}}\left(\frac{W}{L}\right)_n$$

$$= \frac{500}{125} \times 1.5 = 6.0$$

(b) For both devices, we have

$$R_{on} = \frac{1}{0.5 \times 1.5 \times (1.2 - 0.4)} = 1.67 \text{ k}\Omega$$

14.3

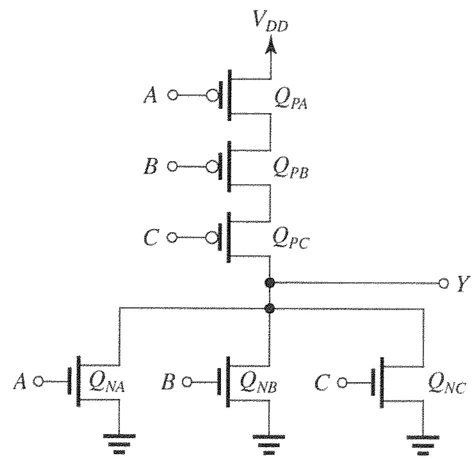


Figure 1

$Y = \overline{A + B + C}$

$\Rightarrow \overline{Y} = A + B + C \Rightarrow$ PDN

$Y = \overline{A}\,\overline{B}\,\overline{C} \;\Rightarrow$ PUN

The circuit realization is shown in Fig. 1.

14.4

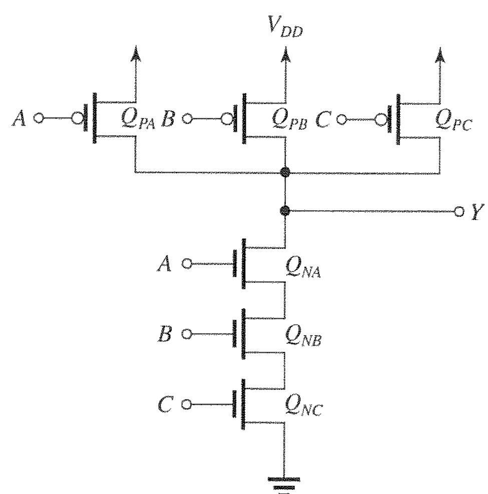


Figure 1

$Y = \overline{ABC}$

$\Rightarrow \overline{Y} = ABC \Rightarrow$ PDN

$Y = \overline{A} + \overline{B} + \overline{C} \Rightarrow$ PUN

Figure 1 shows the circuit realization.

14.5

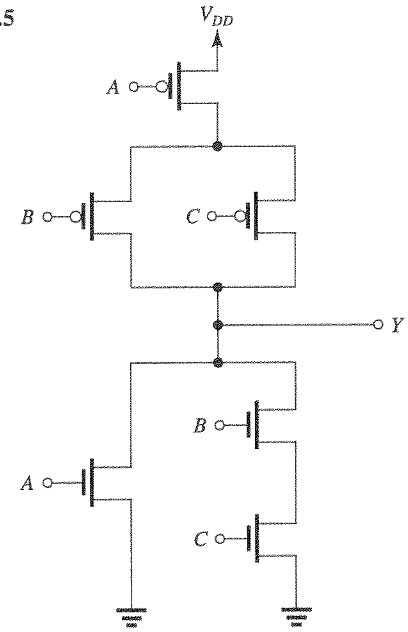


Figure 1

Figure 1 shows the complete CMOS logic gate where the PUN is obtained as the dual of the given PDN. The function realized can be found from the PDN as

$\overline{Y} = A + BC$

or

$Y = \overline{A + BC}$

14.6

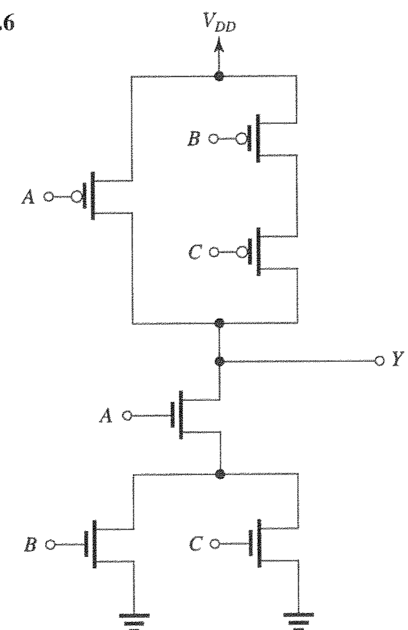


Figure 1

Figure 1 shows the complete CMOS circuit where the PUN is obtained as the dual network of the given PDN. The logic function realized can be written from the PDN as

$\overline{Y} = A(B + C)$

or equivalently

$Y = \overline{A(B + C)}$

14.7

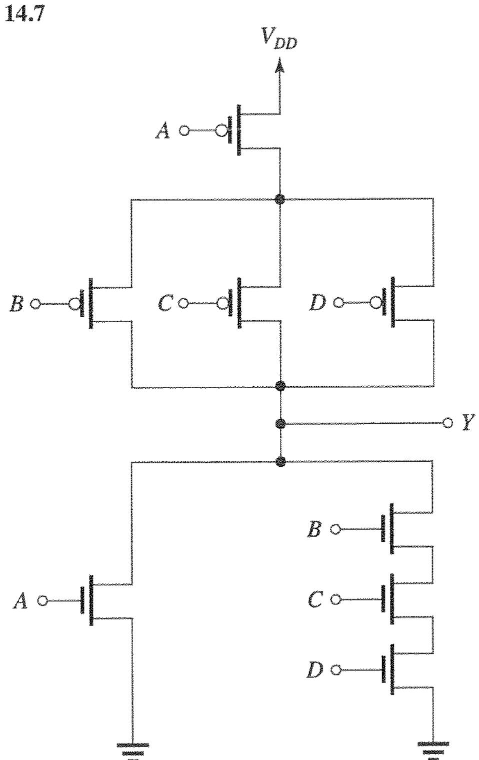


Figure 1

Figure 1 shows the complete CMOS logic circuit where we have obtained the PDN as the dual of the given PUN. The logic function can be written from the PDN as

$\overline{Y} = A + BCD$

or equivalently

$Y = \overline{A + BCD}$

14.8 The given Boolean expression can be written as

$\overline{Y} = (A + B)(C + D)$

from which the PDN of the circuit in Fig. 1 can be directly obtained.

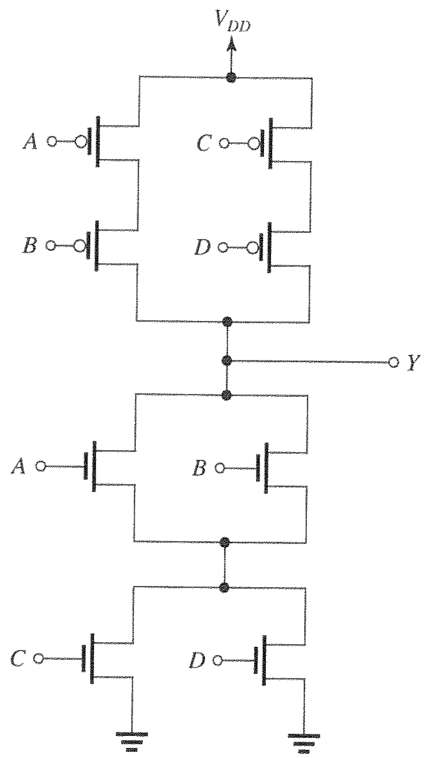


Figure 1

The PUN can then be found as the dual of the PDN. The complete circuit is shown in Fig. 1.

14.9

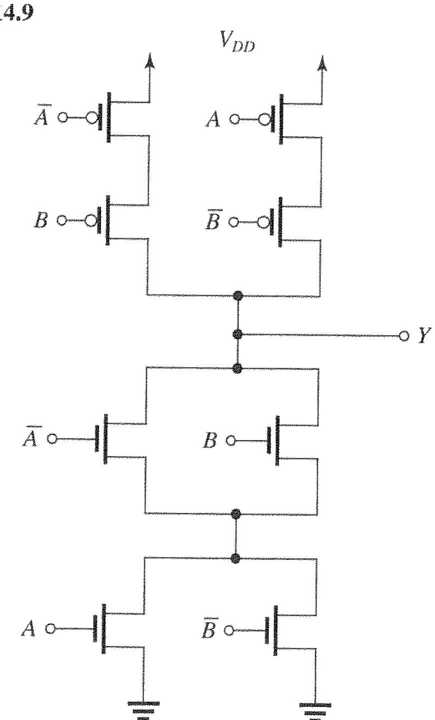


Figure 1

Figure 1 shows a CMOS realization of the Exclusive-OR function. This circuit is obtained by utilizing the PUN in Fig. 14.10(a) and then finding the dual PDN. Note that two additional inverters are needed to generate $\overline{A}$ and $\overline{B}$, for a total of 12 transistors.

14.10 $Y = \overline{A}BC + A\overline{B}C + AB\overline{C}$ (1)

Using this expression to directly synethesize the PUN we obtain the circuit shown in Fig. 1. See Figs. 1 and 2 on next page.

This PUN circuit requires 9 transistors plus three inverters for a total of 15 transistors.

This, of course, does not include the transistors required for the PDN which we shall consider shortly. Inspecting the PUN circuit reveals the potential for eliminating two transistors through what is known as "path merging." Specifically the two transistors in the top row that are controlled by $\overline{A}$ can be merged into a single transistor, and the two transistors in the bottom row that are controlled by $\overline{C}$ can be merged into a single transistor. The result is the 7-transistor PUN shown in Fig. 2.

We next consider the realization of the PDN. A straightforward realization can be obtained by finding the dual of the PUN in Fig. 1. This is shown in Fig. 3. It requires nine transistors

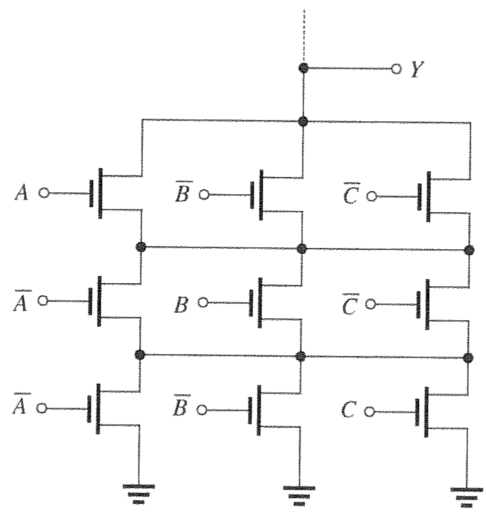


Figure 3

plus three inverters. The latter, of course are the same three inverters needed to obtain the complemented variables in PUN. The circuit in Fig. 3 does not lend itself to path merging, at least not in a straightforward way.

This figure belongs to Problem 14.10, part (a).

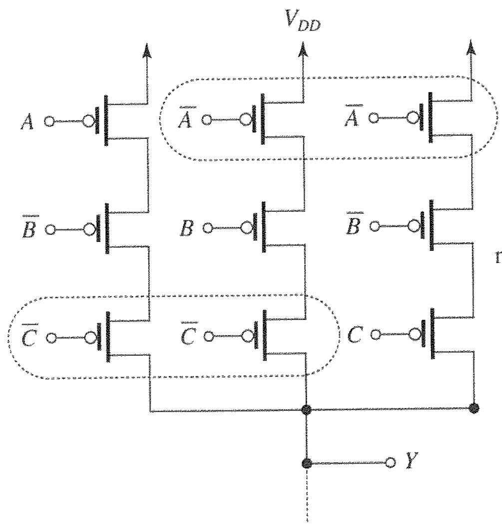


Figure 1

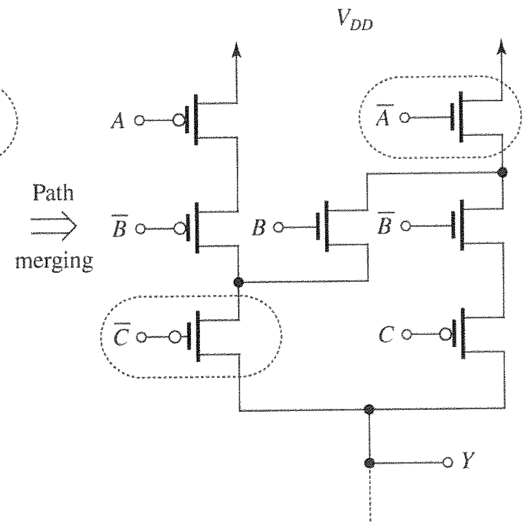


Figure 2

There is, however, an alternative way to synthesize a PDN with a lower number of transistors. We simply obtain $\overline{Y}$ from the expression in Eq. (1) using DeMorgan's law as follows:

$$\overline{Y} = \overline{\overline{A}BC}.\overline{A\overline{B}C}.\overline{AB\overline{C}}$$

$$= (A+\overline{B}+\overline{C})(\overline{A}+B+\overline{C})(\overline{A}+\overline{B}+C) \qquad (2)$$

Direct synthesis of Eq. (2) results in the circuit of Fig. 3. However, further manipulation of the expression in (2) results in a more economical realization, as follows:

$$\overline{Y} = AB(\overline{A}+\overline{B}+C) + A\overline{C}(\overline{A}+\overline{B}+C)$$

$$+ \overline{B}\,\overline{A}(\overline{A}+\overline{B}+C) + \overline{B}\,\overline{C}(\overline{A}+\overline{B}+C)$$

$$+ \overline{C}\,\overline{A}(\overline{A}+\overline{B}+C) + \overline{C}B(\overline{A}+\overline{B}+C)$$

$$+ \overline{C}(\overline{A}+\overline{B}+C)$$

$$= ABC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B} + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}$$

$$+ \overline{B}\,\overline{C} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$$

$$= ABC + (A+\overline{A})\overline{B}\,\overline{C} + \overline{A}\,\overline{B}(1+\overline{C}) + (\overline{A}+1)\overline{B}\,\overline{C}$$

$$+ \overline{A}\,\overline{C}(1+B)$$

$$= ABC + \overline{B}\,\overline{C} + \overline{A}\,\overline{B} + \overline{A}\,\overline{C}$$

$$= ABC + \overline{A}(\overline{B}+\overline{C}) + \overline{B}\,\overline{C} \qquad (3)$$

A direct realization of this expression results in the PDN shown in Fig. 4. This circuit requires 8 transistors (not counting the inverters).

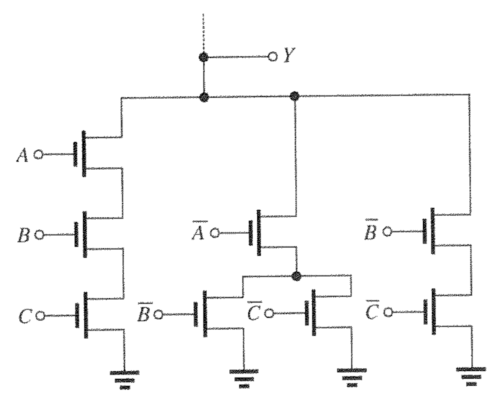


Figure 4

14.11 Direct realization of the given expression results in the PUN of the logic circuit shown in Fig. 1. The PDN shown is obtained as the dual of the PUN. Not shown are the two inverters needed to obtain $\overline{A}$ and $\overline{B}$.

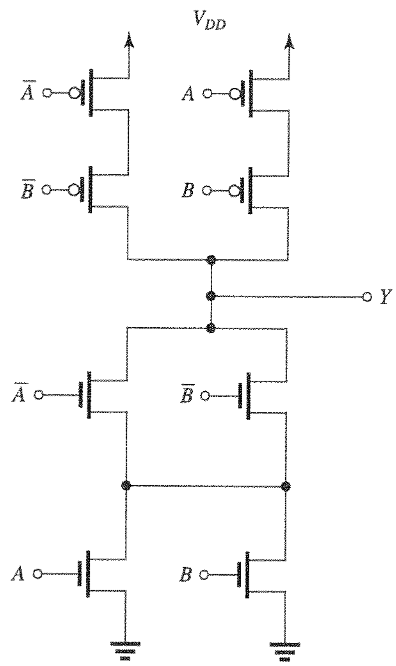


Figure 1

14.12 Direct realization of the given expression results in the PUN portion of the circuit shown in Fig. 1. The PDN is obtained as the dual of the PUN. Not shown are the three inverters needed to obtain $\overline{A}$, $\overline{B}$ and $\overline{C}$.

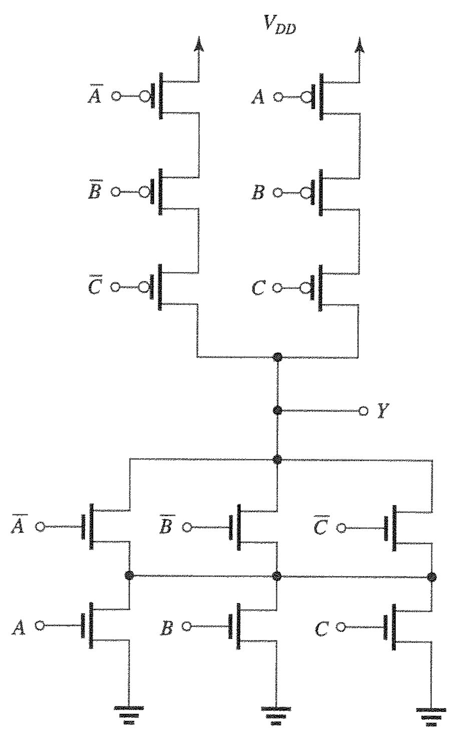


Figure 1

14.13 (a) Even-parity checker:

$$\overline{Y} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}BC + A\overline{B}C + AB\overline{C}$$

See Fig. 1 on next page.

(b) This expression can be directly realized with the PDN shown in Fig. 1. Note that the circuit requires 12 transistors in addition to the three inverters needed to generate $\overline{A}$, $\overline{B}$, and $\overline{C}$.

(c) From inspection of the PDN in Fig. 1 we see that we can combine the two transistors controlled by $\overline{A}$ and the two transistors controlled by A. This results in the PDN realization shown in Fig. 2 which requires 10 transistors, not counting those in the inverters. See Fig. 2 on next page.

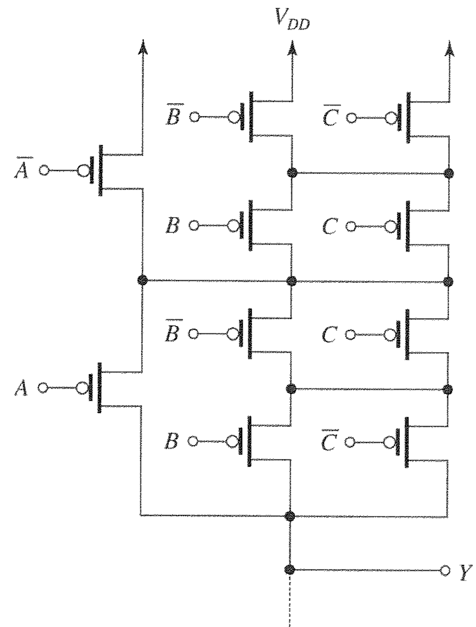


Figure 3

(d) The PUN in Fig. 3 can be obtained as the dual of the PDN in Fig. 2. Combining the PDN and the PUN gives the complete realization of the even-parity checker.

Note: The number of transistors in the PDN of Fig. 2 can be reduced by 2 by combining the two transistors in the bottom row that are controlled by C, and the two transistors that are controlled by $\overline{C}$. The resulting 8-transistor realization is shown in Fig. 4. However, it is not easy to obtain a PUN as a dual of this circuit. See Fig. 4 on next page.

This figure belongs to Problem 14.13, part (a).

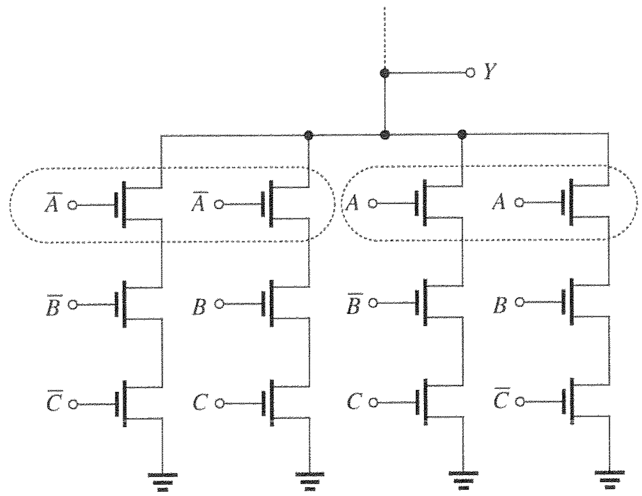


Figure 1

This figure belongs to Problem 14.13, part (c).

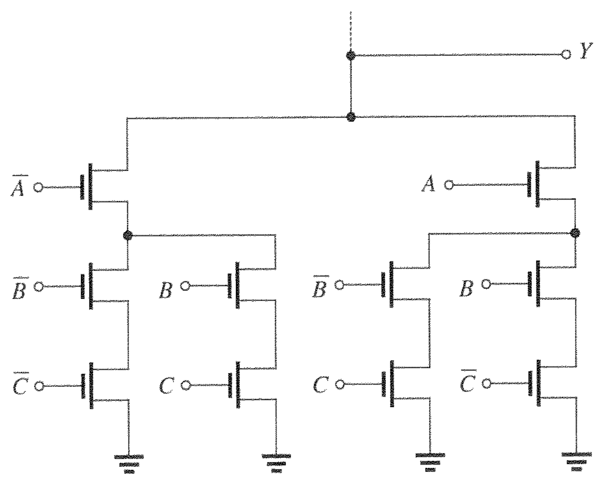


Figure 2

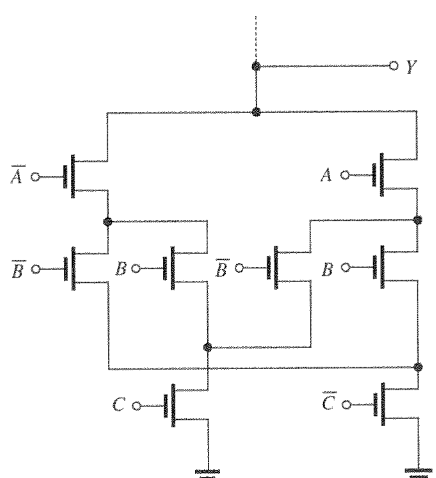


Figure 4

14.14 Odd-parity checker:

$$Y = \overline{A}B\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + ABC$$

$$= \overline{A}\,(B\overline{C} + \overline{B}C) + A(BC + \overline{B}\,\overline{C}) \qquad (1)$$

The Boolean expression in Eq. (1) can be directly realized by the PUN in Fig. 1. Recall that we use for the switch control variables the complements of the variables in the equation. It requires 10 transistors in addition to the three inverters needed to provide $\overline{A}$, $\overline{B}$ and $\overline{C}$. The dual of the PUN can be obtained and results in the PDN shown in Fig. 1.

This figure belongs to Problem 14.14.

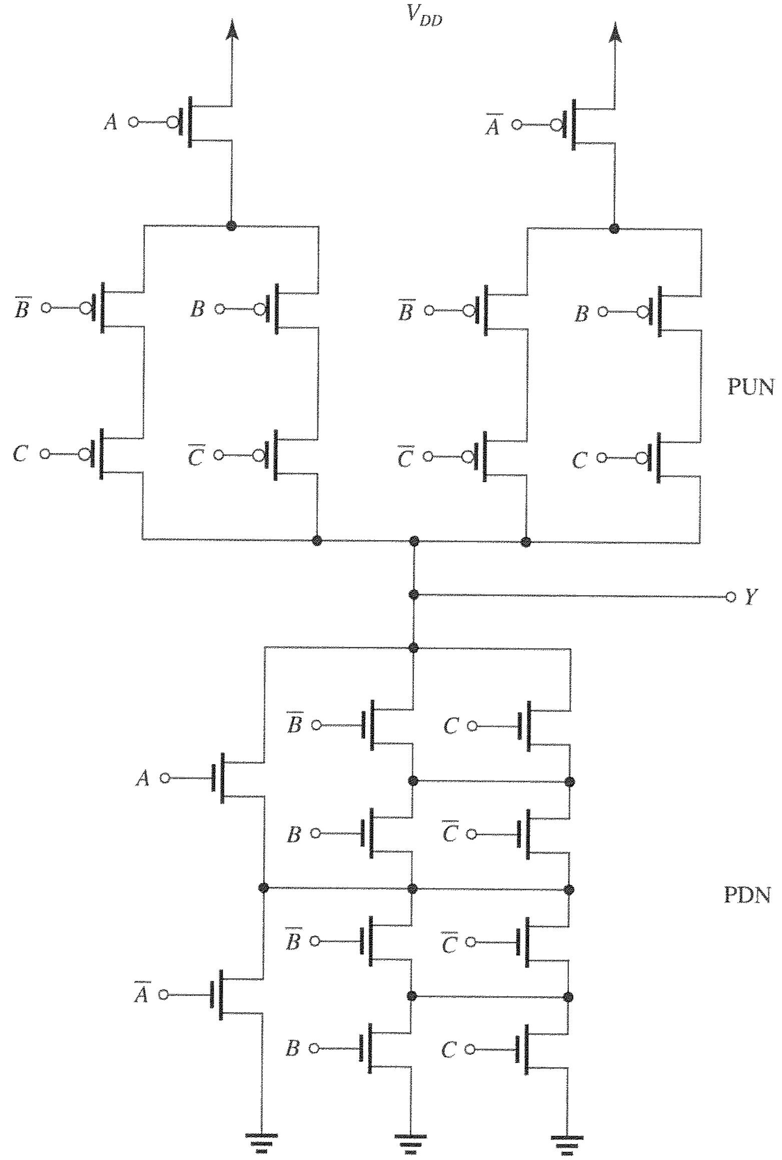


Figure 1

14.15 $S = \overline{A}B\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + ABC$

$= \overline{A}(B\overline{C} + \overline{B}C) + A(BC + \overline{B}\,\overline{C})$

This is the same function as that of the odd-parity checker in Problem 14.14. Thus the realization of the S function will be identical to that in Fig. 1 of Problem 14.14.

As for C_0 we write

$C_0 = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$

This expression can be minimized as follows:

$C_0 = (\overline{A} + A)BC + (\overline{B} + B)AC + (\overline{C} + C)AB$

$= BC + AC + AB = A(B + C) + BC$

which can be realized directly by the PUN of the circuit in Fig. 1 where the PDN is obtained as the dual network of the PUN. In addition to the 10 transistors, we need three inverters to generate $\overline{A}$, $\overline{B}$ and $\overline{C}$.

This figure belongs to Problem 14.15.

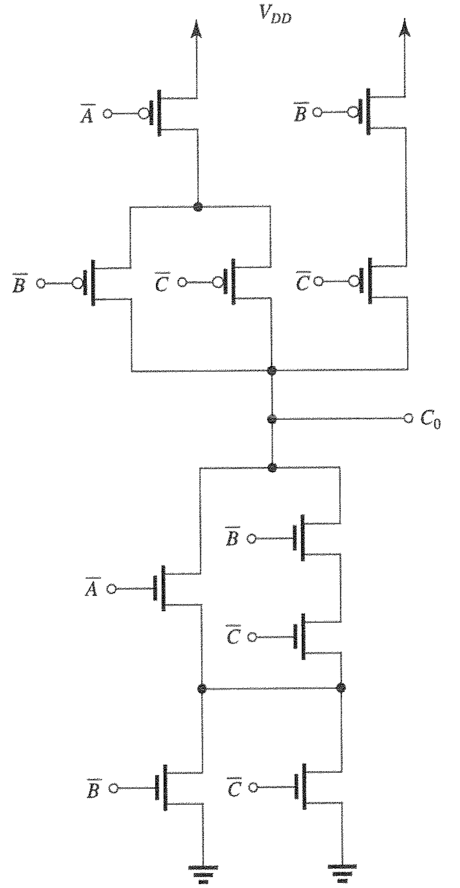


Figure 1

14.16 $NM_H = V_{OH} - V_{IH}$

$= 1.8 - 1.2 = 0.6$ V

$NM_L = V_{IL} - V_{OL}$

$= 0.9 - 0.2 = 0.7$ V

14.17 (a) $NM_H = V_{OH} - V_{IH}$

$= 1.8 - 1.3 = 0.5$ V

$NM_L = V_{IL} - V_{OL}$

$= 1.2 - 0.4 = 0.8$ V

(b)

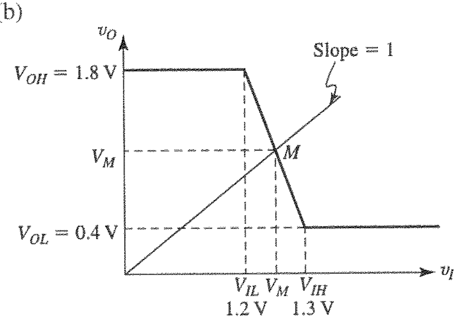


Figure 1

Refer to Fig. 1. Slope of the VTC in the transition region is:

$$\text{Slope} = \frac{V_{OH} - V_{OL}}{V_{IL} - V_{IH}}$$

$$= \frac{1.8 - 0.4}{1.2 - 1.3} = -14 \text{ V/V}$$

But the slope can also be expressed as

$$\text{Slope} = \frac{V_M - V_{OH}}{V_M - V_{IH}}$$

Thus,

$$\frac{V_M - 0.4}{V_M - 1.3} = -14$$

$$\Rightarrow V_M = 1.24 \text{ V}$$

(c) The voltage gain in the transition region is equal to the slope found above, thus

Gain $= -14$ V/V

14.18 $NM_H = V_{OH} - V_{IH}$

$= 0.8V_{DD} - 0.6V_{DD} = 0.2V_{DD}$

$NM_L = V_{IL} - V_{OL}$

$= 0.4V_{DD} - 0.1V_{DD} = 0.3V_{DD}$

Width of transition region $= V_{IH} - V_{IL}$

$= 0.6V_{DD} - 0.4V_{DD} = 0.2V_{DD}$

For a minimum noise margin of 0.4 V, we have

$NM_H = 0.4$

$\Rightarrow 0.2V_{DD} = 0.4$

$\Rightarrow V_{DD} = 2$ V

14.19 $V_{IH} = 2$ V

$V_{IL} = 0.8$ V

$V_{OH\min} = 2.4$ V, $V_{OH\text{typ}} = 3.3$ V

$V_{OL\max} = 0.4$ V, $V_{OL\text{typ}} = 0.22$ V

(a) Worst-case $NM_H = V_{OH\min} - V_{IH}$

$= 2.4 - 2 = 0.4$ V

Worst-case $NM_L = V_{IL} - V_{OL\max}$

$= 0.8 - 0.4 = 0.4$ V

(b) Typical average power dissipation:

$$P_D = \frac{1}{2}(5 \times 3 + 5 \times 1)$$

$= 10$ mW

14.20 (a) Refer to Fig. 14.17.

$$V_{OL} = V_{DD}\,\frac{R_{\text{on}}}{R + R_{\text{on}}}$$

$= 2.5 \times \dfrac{0.1}{2 + 0.1} = 0.12$ V

$V_{OH} = V_{DD} = 2.5$ V

$NM_H = V_{OH} - V_{IH}$

$= 2.5 - 1 = 1.5$ V

$NM_L = V_{IL} - V_{OL}$

$= 0.8 - 0.12 = 0.68$ V

(b)

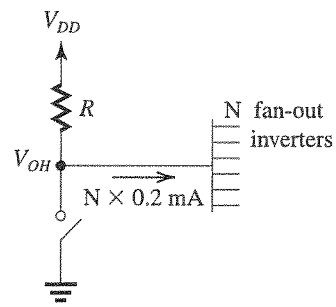


Figure 1

Refer to Fig. 1.

$V_{OH} = V_{DD} - N \times 0.2 \times R$

$= 2.5 - N \times 0.2 \times 2$

$= 2.5 - 0.4N$

$NM_H = 2.5 - 0.4N - 1$

$= 1.5 - 0.4N$

For $NM_H = NM_L$, we have

$1.5 - 0.4N = 0.68$

$\Rightarrow N = 2.05$

which means

$N = 2$

(c) (i) When the inverter output is low,

$$P_D = \frac{V_{DD}^2}{R + R_{\text{on}}} = \frac{2.5^2}{2 + 0.1} \simeq 3 \text{ mW}$$

(ii) When the output is high and the inverter is driving two inverters, the current drawn from the supply is $2 \times 0.2 = 0.4$ mA and thus the power dissipation is

$P_D = V_{DD} I_{DD} = 2.5 \times 0.4 = 1$ mW

14.21 For an ideal inverter:

$V_M = \dfrac{1}{2} V_{DD} = 1$ V

$V_{IL} = V_{IH} = V_M = 1$ V

$V_{OL} = 0$ V

$V_{OH} = V_{DD} = 2$ V

$NM_L = V_{IL} - V_{OL}$

$= 1 - 0 = 1$ V

$NM_H = V_{OH} - V_{IH}$

$= 2 - 1 = 1$ V

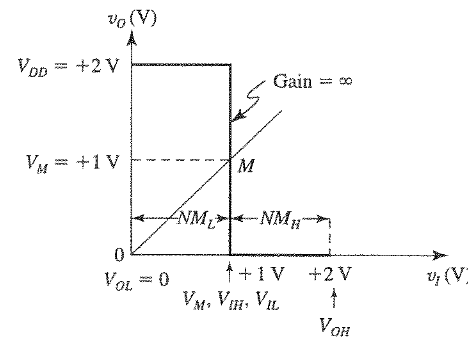


Figure 1

The ideal transfer characteristic is shown in Fig. 1, from which we see that

Gain in transition region $= \infty$

14.22

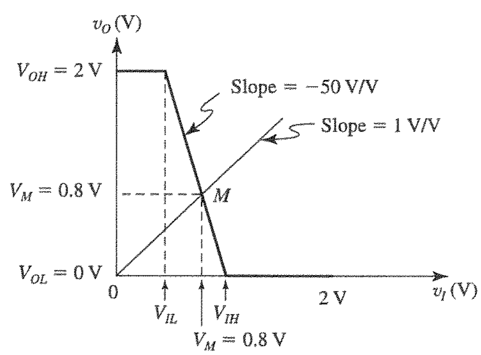


Figure 1

Figure 1 shows a sketch of the VTC where we have approximated the VTC in the transition region by a straight line with a slope equal to the maximum possible small-signal gain, namely 50 V/V. We can use the geometry of the VTC to determine V_{IH} and V_{IL} as follows:

$|\text{Slope}| = 50$

$= \dfrac{V_M}{V_{IH} - V_M}$

Thus,

$50 = \dfrac{0.8}{V_{IH} - 0.8}$

$\Rightarrow V_{IH} = 0.816$ V

Similarly,

$$|\text{Slope}| = \frac{V_{OH}}{V_{IH} - V_{IL}}$$

$$50 = \frac{2}{0.816 - V_{IL}}$$

$\Rightarrow V_{IL} = 0.776$ V

$NM_H = V_{OH} - V_{IH}$

$= 2 - 0.816 = 1.184$ V

$NM_L = V_{IL} - V_{OL}$

$= 0.776 - 0 = 0.776$ V

Since we approximated the VTC in the transition region by a straight line, the large-signal voltage gain will be equal to the small-signal voltage gain,

$= -50$ V/V

14.23 Here, $V_{OH} = 1.2$ V, and $V_{OL} = 0.0$ V

Also, $V_{IH} - V_{IL} \leq 1.2/3 = 0.4$ V (1)

Now, the noise margins are "within 30% of one other." Thus, $NM_H = (1 + \pm 0.3)\, NM_L$ or $NM_L = (1 + \pm 0.3)\, NM_H$. Thus, they remain "within" either $NM_H = 1.3 NM_L$ or $NM_L = 1.3 NM_H$, in which case either $NM_L = 0.769 NM_H$ or $N_{MH} = 0.769 NM_L$

For the former case:

$0.769\,(V_{OH} - V_{IH}) = (V_{IL} - V_{OL})$ or

$0.769\,(1.2 - V_{IH}) = V_{IL} - 0$, whence

$V_{IL} = 0.923 - 0.769 V_{IH}$

Now, from (1), $V_{IH} = V_{IL} + 0.4$

Thus,

$V_{IL} = 0.923 - 0.769\,(V_{IL} + 0.4)$

$= 0.615 - 0.769 V_{IL}$

and $V_{IL} = 0.615/1.769 = 0.349$ V

whence $V_{IH} = 0.4 + 0.349 = 0.749$ V

Alternatively, $NM_H = 0.769 NM_L$ and

$(V_{OH} - V_{IH}) = 0.769\,(V_{IL} - V_{OL})$ or

$1.2 - V_{IH} = 0.769 V_{IL} - 0$ and

$V_{IH} = 1.2 - 0.769 V_{IL}$, with (1),

$V_{IL} + 0.4 = 1.2 - 0.769 V_{IL}$, and

$1.769 V_{IL} = 0.8$, whence $V_{IL} = 0.452$ V

and $V_{IH} = 0.4 + 0.452 = 0.852$

Thus, overall, $V_{OH} = 1.2$ V, $V_{OL} = 0.0$ V,

V_{IH} ranges from 0.749 V to 0.852 V, and

V_{IL} ranges from 0.349 V to 0.451 V, in

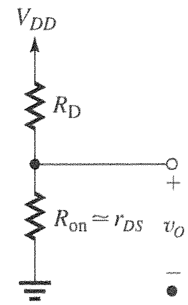
which case the margins can be as low as

$NM_L = V_{IL} - V_{OL} = 0.349$ V and

$NM_H = V_{OH} - V_{IH} = 1.2 - 0.852 = 0.348$ V

and as high as 0.451 V, and 0.451 V.

14.24



Equivalent circuit for output-low state

The output-high level for the simple inverter circuit shown in Fig. 14.12 of the text is

$V_{OH} = V_{DD} \Rightarrow V_{DD} = 1.2$ V.

When the output is low, the current drawn from the supply can be calculated as

$$I = \frac{V_{DD}}{R_D + R_{on}} = 30\ \mu\text{A}$$

Therefore: $R_D + r_{DS} = \dfrac{1.2}{30 \times 10^{-6}} = 40\ \text{k}\Omega$

Also:

$$V_{OL} = 0.05\ \text{V} = \frac{r_{DS}}{R_D + r_{DS}} \times V_{DD}$$

$$\Rightarrow r_{DS} = 40\ \text{k}\Omega \times \frac{0.05}{1.2} = 1.67\ \text{k}\Omega$$

Hence: $R_D = 40\ \text{K} - 1.67\ \text{K} = 38.3\ \text{k}\Omega$

$$r_{DS} = \frac{1}{\mu_n C_{ox} \dfrac{W}{L} (V_{GS} - V_t)}$$

$$= \frac{1}{500 \times 10^{-6} \times \dfrac{W}{L} (1.2 - 0.4)}$$

$= 1.67\ \text{k}\Omega$

$$\Rightarrow \frac{W}{L} = 1.5$$

When the output is low:

$P_D = V_{DD} I_{DD} = 1.2 \times 30\ \mu\text{A} = 36\ \mu\text{W}$

When the output is high, the transistor is off: $P_D = 0$ W

14.25 The output voltage swing $= R_{C1}I = 0.5$ V.

with $I_{EE} = 0.5$ mA, $R_{C1} = 1.0$ kΩ, similarly,

$R_{C2} = 1.0$ kΩ

$V_{OH} = V_{CC} = 2$ V

$V_{OL} = V_{CC} - R_{C1}I_{EE} = 1.5$ V

14.26 Refer to Example 14.2 on page 1107 of the text:

$V_{OH} = V_{DD} = 1.2$ V

The power drawn from the supply during the low-output state is

$P_{DD} = V_{DD}I_{DD} \Rightarrow 60\ \mu\text{W} = 1.2 \times I_{DD}$

$\Rightarrow I_{DD} = 50\ \mu\text{A}$

In this case:

$$I_{DD} = \frac{V_{DD} - V_{OL}}{R_D} \Rightarrow 50\ \mu\text{A} = \frac{1.2 - 0.05}{R_D}$$

$\Rightarrow R_D = 23$ kΩ

In order to determine $\frac{W}{L}$, we note that

$$k_n R_D = 1/V_X \text{ or } k'_n \frac{W}{L} R_D = \frac{1}{V_X}$$

Therefore, we need to first calculate V_X using Eq. (14.22) on page 1110 of the text.

$$V_{OL} = \frac{V_{DD}}{1 + \dfrac{V_{DD} - V_t}{V_X}} \text{ or equivalently}$$

$$0.05\ \text{V} = \frac{1.2}{1 + \dfrac{1.2 - 0.4}{V_X}}$$

$$\Rightarrow V_X = \frac{0.8}{23} = 0.035\ \text{V}$$

Hence, $k'_n \frac{W}{L} R_D = \frac{1}{V_x}$ gives

$$500 \times 10^{-6} \times \frac{W}{L} \times 23 \times 10^3 = \frac{1}{0.035} \Rightarrow \frac{W}{L} = 2.5$$

Using Eq. (14.12), we obtain

$V_{IL} = V_t + V_X = 0.4 + 0.035 = 0.435$ V

From Eq. (14.14) we obtain

$$V_M = V_t + \sqrt{2\,(V_{DD} - V_t)V_x + V^2{}_x} - V_x$$

$$= 0.4 + \sqrt{2\,(1.2 - 0.4)\,0.035 + 0.035^2} - 0.035$$

$V_M = 0.6$ V

From Eq. (14.20) we get

$$V_{IH} = V_t + 1.63\sqrt{V_{DD}V_x} - V_x$$

$$= 0.4 + 1.63\sqrt{1.2 \times 0.035} - 0.035 = 0.7\ \text{V}$$

$NM_H = V_{OH} - V_{IH} = 1.2 - 0.7 = 0.5$ V

$NM_L = V_{IL} - V_{OL} = 0.435 - 0.05 = 0.385$ V

14.27 $V_t = 0.3V_{DD}$, $V_M = V_{DD}/2$

From Eq. (14.13) on page 1108 of text, we obtain

$$V_x\Big|_{V_M = \frac{V_{DD}}{2}} = \frac{\left(\dfrac{V_{DD}}{2} - V_t\right)^2}{V_{DD}}$$

$$= \frac{(0.5V_{DD} - 0.3V_{DD})^2}{V_{DD}}$$

$\Rightarrow V_x = 0.04V_{DD}$

$V_{OH} = V_{DD}$

From Eq. (14.12), we get

$V_{IL} = V_t + V_x = 0.3V_{DD} + 0.04V_{DD}$

$= 0.34V_{DD}$

From Eq. (14.20), we obtain

$$V_{IH} = V_t + 1.63\sqrt{V_{DD}V_x} - V_x$$

$$= 0.3V_{DD} + 1.63\sqrt{V_{DD} \times 0.04V_{DD}} - 0.04V_{DD}$$

$= 0.586V_{DD}$

From Eq. (14.22), we get

$$V_{OL} = \frac{V_{DD}}{1 + [(V_{DD} - V_t)/V_x]}$$

$$= \frac{V_{DD}}{1 + \dfrac{V_{DD} - 0.3V_{DD}}{0.04V_{DD}}} = 0.054V_{DD}$$

$NM_H = V_{OH} - V_{IH}$

$= V_{DD} - 0.586V_{DD} = 0.414V_{DD}$

$NM_L = V_{IL} - V_{OL}$

$= 0.34V_{DD} - 0.054V_{DD} = 0.286V_{DD}$

For $V_{DD} = 1.2$ V:

$V_x = 0.048$ V, $V_{OH} = 1.2$ V, $V_{IL} = 0.408$ V,

$V_{IH} = 0.703$ V, $V_{OL} = 0.065$ V,

$NM_H = 0.50$ V, $NM_L = 0.34$ V

$P_D = V_{DD}I_D$

$$= V_{DD} \times \frac{V_{DD} - V_{OL}}{R_D}$$

Substituting for R_D from

$$R_D = \frac{1}{k_n V_x}$$

we obtain

$P_D = V_{DD}(V_{DD} - 0.054V_{DD}) \times k_n \times 0.04V_{DD}$

$$P_D = 0.038V_{DD}^3 \times k'_n\left(\frac{W}{L}\right)$$

$$= 0.038 \times 1.2^3 \times 0.5 \times 10^{-3}\left(\frac{W}{L}\right)$$

$$= 0.033\left(\frac{W}{L}\right),\ \text{mW}$$

For $P_D = 100\ \mu W = 0.1$ mW, we obtain

$$0.1 = 0.033\left(\frac{W}{L}\right)$$

$$\Rightarrow \frac{W}{L} = 3$$

$$R_D = \frac{1}{k_n V_x}$$

$$= \frac{1}{0.5 \times 3 \times 0.048}$$

$$= 13.9\ \text{k}\Omega$$

14.28

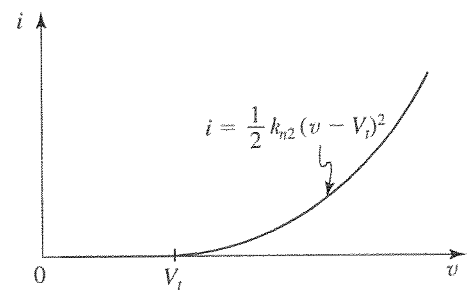


Figure 1

(a) Figure 1 shows the $i - v$ characteristic of the load diode-connected transistor Q_2. Observe that since $v_{DG} = 0$, this transistor is always in saturation. The current i that feeds Q_2 is provided by Q_1. Even when i is very small, Q_2 will be operating at $v = V_t$ and thus the output voltage of the inverter V_{OH} will be

$$V_{OH} = V_{DD} - V_t$$

This loss of V_t volts in V_{OH} is a drawback of this form of logic circuit.

(b) As v_I increases and reaches V_t of Q_1, transistor Q_1 begins to conduct an appreciable current and v increases above V_{t2}, thus v_O begins to decrease below V_{OH}. The threshold voltage V_{IL} is usually taken as V_{t1}:

$$V_{IL} = V_t$$

(c) When Q_1 begins to conduct, it will be operating in saturation, thus

$$i_{D1} = \frac{1}{2}k_{n1}(v_I - V_t)^2 \quad (1)$$

Transistor Q_2 is always operating in saturation, thus

$$i_{D2} = \frac{1}{2}k_2(V_{DD} - v_O - V_t)^2 \quad (2)$$

Since $i_{D1} = i_{D2} = i$, we can equate their values in Eqs. (1) and (2) and thus obtain

$$k_{n1}(v_I - V_t)^2 = k_{n2}(V_{DD} - v_O - V_t)^2$$

$$\Rightarrow \sqrt{\frac{k_{n1}}{k_{n2}}}(v_I - V_t) = V_{DD} - v_O - V_t$$

$$\Rightarrow v_O = V_{DD} + (k_r - 1)V_t - k_r v_I \quad (3)$$

where

$$k_r \equiv \sqrt{\frac{k_{n1}}{k_{n2}}}$$

Equation (3) is that of the inverter VTC in the transition region. It is linear and has a slope of $-k_r$.

(d) With $v_I = V_{DD}$, $v_O = V_{OL} \simeq 0$, the voltage $v \simeq V_{DD}$ and the current I_{DD} that flows in the inverter is

$$I_{DD} = \frac{1}{2}k_{n2}(V_{DD} - V_t)^2$$

Thus, the power dissipated in the inverter in the low-output state is

$$P_D|_{v_O = V_{OL}} = V_{DD}I_{DD}$$

$$= \frac{1}{2}k_{n2}V_{DD}(V_{DD} - V_t)^2$$

Since the inverter spends half the time in this state and the other half in the output-high state where the power dissipation is essentially zero, the average power dissipation will be

$$P_D = \frac{1}{4}k_{n2}V_{DD}(V_{DD} - V_t)^2$$

(e) For $V_{DD} = 1.8$ V, $V_t = 0.5$ V, $(W/L)_1 = 5$, $(W/L)_2 = \frac{1}{5}$ and $\mu_n C_{ox} = 300\ \mu\text{A/V}^2$, we have

$$V_{OH} = V_{DD} - V_t = 1.8 - 0.5 = 1.3 \text{ V}$$

$$V_{IL} = V_t = 0.5 \text{ V}$$

$$k_r = \sqrt{\frac{k_{n1}}{k_{n2}}} = \sqrt{\frac{5}{1/5}} = 5$$

In the transition region, we have

$$v_O = 1.8 + (5 - 1) \times 0.5 - 5v_I$$

$$\Rightarrow v_O = 3.8 - 5v_I$$

$$I_{DD} = \frac{1}{2} \times 0.3 \times \frac{1}{5} \times (1.8 - 0.5)^2$$

$$= 0.05 \text{ mA}$$

$$P_D = \frac{1}{2} \times 1.8 \times 0.05 = 45\ \mu\text{W}$$

14.29 Refer to the problem statement of Exercise 14.5:

$$V_M = V_t + \frac{V_{DD} - V_t}{\sqrt{r} + 1}$$

For $V_M = \frac{V_{DD}}{2}$ we have

$$\frac{V_{DD}}{2} = V_t + \frac{V_{DD} - V_t}{\sqrt{r+1}}$$

$$\Rightarrow r = \left[\frac{V_{DD} - V_t}{(V_{DD}/2) - V_t}\right]^2 - 1$$

For $V_{DD} = 1.8$ V and $V_t = 0.4$ V (from Example 14.3) we obtain

$$r = \left(\frac{1.8 - 0.4}{0.9 - 0.4}\right)^2 - 1 = 6.84$$

14.30 Refer to Example 14.3 (page 1112 of text)

$V_{OH} = V_{DD} = 1.2$ V

$$V_{OL} = (V_{DD} - V_t)\left[1 - \sqrt{1 - (k_p/k_n)}\right]$$

$$= (1.2 - 0.4)\left[1 - \sqrt{1 - \frac{1}{5}}\right]$$

$= 0.084$ V

$$I_{DD} = \frac{1}{2}k_p(V_{DD} - V_t)^2$$

$$= \frac{1}{2} \times 0.1(1.2 - 0.4)^2$$

$= 0.032$ mA $= 32$ μA

$$P_{av} = \frac{1}{2}V_{DD}I_{DD} = \frac{1}{2} \times 1.2 \times 32 = 19.2\ \mu\text{W}$$

From the statement of Exercise 14.5, we have

$$V_M = V_t + \frac{V_{DD} - V_t}{\sqrt{r+1}}$$

where $r = k_n/k_p = 5$, thus

$$V_M = 0.4 + \frac{1.2 - 0.4}{\sqrt{5+1}}$$

$= 0.73$ V

14.31 (a) To obtain $V_M = V_{DD}/2$, the inverter must be matched, thus

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = 2.5$$

$\Rightarrow W_p = 2.5W_n = 2.5 \times 1.5 \times 65 = 244$ nm

Silicon area $= W_nL_n + W_pL_p$

$= 1.5 \times 65 \times 65 + 2.5 \times 1.5 \times 65 \times 65$

$= 1.5 \times 65 \times 65(1 + 2.5)$

$= 22,181\ \text{nm}^2$

(b) $V_{OH} = V_{DD} = 1$ V

$V_{OL} = 0$ V

To obtain V_{IH}, we use Eq. (14.35):

$$V_{IH} = \frac{1}{8}(5V_{DD} - 2V_t)$$

$$= \frac{1}{8}(5 \times 1 - 2 \times 0.35)$$

$= 0.5375$ V

To obtain V_{IL}, we use Eq. (14.36):

$$V_{IL} = \frac{1}{8}(3V_{DD} + 2V_t)$$

$$= \frac{1}{8}(3 \times 1 + 2 \times 0.35)$$

$= 0.4625$ V

The noise margins can now be found as

$NMH = V_{OH} - V_{IH}$

$= 1 - 0.5375 = 0.4625$ V

$NM_L = V_{IL} - V_{OL}$

$= 0.4625 - 0 = 0.4625$ V

The noise margins are equal at approximately 0.46 V; a result of the matched design of the inverter.

(c) Since the inverter is matched, the output resistances in the two states will be equal. Thus,

$$r_{DSP} = r_{DSN} = 1\Big/\left[(\mu_n C_{ox})\left(\frac{W}{L}\right)_n (V_{DD} - V_t)\right]$$

$$= \frac{1}{0.47 \times 1.5(1 - 0.35)} = 2.18\ \text{k}\Omega$$

14.32 $V_{OH} = 2.5$ V

$V_{OL} = 0$ V

(a) For the matched case we have

$W_p = 3.5W_n$

$$V_M = \frac{1}{2}V_{DD} = 1.25\ \text{V}$$

Eq. (14.35): $V_{IH} = \frac{1}{8}(5\ V_{DD} - 2\ V_t)$

$$= \frac{1}{8}(5 \times 2.5 - 2 \times 0.5)$$

$= 1.4375$ V

Eq. (14.36): $V_{IL} = \frac{1}{8}(3\ V_{DD} + 2\ V_t)$

$$= \frac{1}{8}(3 \times 2.5 + 2 \times 0.5)$$

$= 1.0625$ V

$NM_H = NM_L = 1.0625$ V

Silicon area $= W_nL_n + W_pL_p$

$= 1.5 \times 0.25 \times 0.25 + 3.5 \times 1.5 \times 0.25 \times 0.25$

$= 4.5 \times 1.5 \times 0.25^2 = 0.42\ \mu\text{m}^2$

(b) $W_p = W_n$ (minimum-size design):

Eq. (14.40): $r = \sqrt{\frac{\mu_p}{\mu_n}\frac{W_p}{W_n}} = \sqrt{\frac{1}{3.5}} \times 1 = 0.53$

Eq. (14.39): $V_M = \frac{r(V_{DD} - |V_{tp}|) + V_{tn}}{r+1}$

$= \frac{0.53(2.5 - 0.5) + 0.5}{0.53 + 1}$

$= 1.02$ V

Thus, V_M shifts to the left by 0.23 V. Assuming V_{IL} shifts by approximately the same amount, then

$V_{IL} \simeq 1.0625 - 0.23 \simeq 0.83$ V

Since $NM_L = V_{IL}$, NM_L will be reduced by approximately 22% (relative to the matched case).

Silicon area $= W_nL_n + W_pL_p$

$= 1.5 \times 0.25 \times 0.25 + 1.5 \times 0.25 \times 0.25$

$= 3 \times 0.25^2 = 0.19\ \mu\text{m}^2$

which is a reduction of 55% relative to the matched case.

(c) $W_p = 2W_n$ (a compromise design):

Eq. (14.40): $r = \sqrt{\frac{\mu_p}{\mu_n}\frac{W_p}{W_n}} = \sqrt{\frac{1}{3.5} \times \frac{2}{1}}$

$= 0.756$

Eq. (14.39): $V_M = \frac{r(V_{DD} - |V_{tp}|) + V_{tn}}{r+1}$

$= \frac{0.756(2.5 - 0.5) + 0.5}{0.756 + 1}$

$= 1.15$ V

Thus, relative to the matched case the switching point (V_M) is shifted left by $(1.25 - 1.15) = 0.1$ V. Assuming that V_{IL} is reduced by approximately the same amount, then

$V_{IL} = 1.0625 - 0.1 = 0.9625$ V

Thus, NM_L which equals V_{IL} is reduced by about 9% (relative to the matched case).

Silicon area $= W_nL_n + W_pL_p$

$= 1.5 \times 0.25 \times 0.25 + 2 \times 1.5 \times 0.25 \times 0.25$

$= 3 \times 1.5 \times 0.25^2$

$= 0.28\ \mu\text{m}^2$

Compared to the matched case, the silicon area is reduced by 33%.

14.33 Q_N will be operating in the triode region, thus

$$I_{Dn} = k_n'\left(\frac{W}{L}\right)_n \left[(V_{DD} - V_{tn})V_O - \frac{1}{2}V_O^2\right]$$

For $V_{tn} = 0.3V_{DD}$ and $V_O = 0.1V_{DD}$, we have

$$I_{Dn} = k_n'\left(\frac{W}{L}\right)_n$$

$$\left[(V_{DD} - 0.3V_{DD}) \times 0.1V_{DD} - \frac{1}{2} \times 0.1^2 V_{DD}^2\right]$$

$= k_n'(W/L)_n(0.07V_{DD}^2 - 0.005V_{DD}^2)$

$= 0.065k_n'(W/L)_nV_{DD}^2$ Q.E.D.

For $V_{DD} = 1.3$ V, $k_n' = 0.5$ mA/V^2 and $I_{Dn} = 0.1$ mA, we have

$0.1 = 0.065 \times 0.5(W/L)_n \times 1.3^2$

$\Rightarrow \left(\frac{W}{L}\right)_n = 1.82$

14.34 For $v_I = +1.5$ V, Q_N will be conducting and operating in the triode region while Q_P will be off. Thus, the incremental resistance to the left of node A will be r_{DSN},

$r_{DSN} = \frac{1}{k_n(V_I - V_{tn})}$

$= \frac{1}{0.2(1.5 - 0.5)} = 5\ \text{k}\Omega$

Thus,

$v_a = 100\left(\frac{5}{5 + 100}\right)$

$= 4.8$ mV

For $v_I = -1.5$ V, Q_N will be off but Q_P will be operating in the triode region with a resistance r_{DSP},

$r_{DSP} = \frac{1}{k_p(V_{SGP} - |V_{tp}|)}$

$= \frac{1}{0.04(1.5 - 0.5)} = 25\ \text{k}\Omega$

Thus,

$v_a = 100\left(\frac{25}{25 + 100}\right) = 20$ mV

14.35 From Eq. (14.39) we have

$V_M = \frac{r(V_{DD} - |V_{tp}|) + V_{tn}}{r+1}$

$rV_M + V_M = r(V_{DD} - |V_{tp}|) + V_{tn}$

$r(V_{DD} - |V_{tp}| - V_M) = V_M - V_{tn}$

$\Rightarrow r = \frac{V_M - V_{tn}}{V_{DD} - |V_{tp}| - V_M}$ Q.E.D.

For $V_{DD} = 1.3$ V, $V_{tn} = |V_{tp}| = 0.4$ V, to obtain $V_M = 0.6V_{DD}$, we need

$$r = \frac{0.6 \times 1.3 - 0.4}{1.3 - 0.4 - 0.6 \times 1.3}$$

$$= 3.167$$

But,

$$r = \sqrt{\frac{\mu_p}{\mu_n}\frac{W_p}{W_n}}$$

$$3.167 = \sqrt{\frac{1}{4} \times \frac{W_p}{W_n}}$$

$$\frac{W_p}{W_n} = 3.167^2 \times 4 = 40.1$$

14.36 The current reaches its peak at $v_I = V_M = \frac{V_{DD}}{2}$. At this point, both Q_N and Q_P are operating in the saturation region and conducting a current

$$I_{DP} = I_{DN} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n\left(\frac{V_{DD}}{2} - V_t\right)^2$$

$$= \frac{1}{2} \times 500 \times 1.5\left(\frac{1.3}{2} - 0.4\right)^2$$

$$= 23.4\ \mu\text{A}$$

14.37 Refer to Example 14.4 (page 1121 of text) except here:

$V_{DD} = 1.3$ V, $V_{tn} = |V_{tp}| = 0.4$ V, $\mu_n = 4\ \mu_p$, and $\mu_n C_{ox} = 0.5$ mA/V^2. Also, Q_N and Q_P have $L = 0.13\ \mu$m and $(W/L)_n = 1.5$.

(a) For $V_M = V_{DD}/2 = 0.65$ V, the inverter must be matched, thus

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = 4$$

Since $W_n/L = 1.5$, $W_n = 1.5 \times 0.13 = 0.195\ \mu$m. Thus,

$$W_p = 4 \times 0.195 = 0.78\ \mu\text{m}$$

For this design, the silicon area is

$$A = W_nL + W_pL = L(W_n + W_p)$$

$$= 0.13(0.195 + 0.78) = 0.127\ \mu\text{m}^2$$

(b) $V_{OH} = V_{DD} = 1.3$ V

$V_{OL} = 0$ V

To obtain V_{IH}, we use Eq. (14.35):

$$V_{IH} = \frac{1}{8}(5V_{DD} - 2V_t)$$

$$= \frac{1}{8}(5 \times 1.3 - 2 \times 0.4)$$

$$= 0.7125\ \text{V}$$

To obtain V_{IL}, we use Eq. (14.36):

$$V_{IL} = \frac{1}{8}(3V_{DD} + 2V_t)$$

$$= \frac{1}{8}(3 \times 1.3 + 2 \times 0.4)$$

$$= 0.5875\ \text{V}$$

We can now compute the noise margins as

$$N_{MH} = V_{OH} - V_{IH} = 1.3 - 0.7125$$

$$= 0.5875\ \text{V} \simeq 0.59\ \text{V}$$

$$NM_L = V_{IL} - V_{OL} = 0.5875 - 0$$

$$= 0.5875\ \text{V} \simeq 0.59\ \text{V}$$

For $v_I = V_{IH} = 0.7125$ V, we can obtain the corresponding value of v_O by substituting in Eq. (14.34):

$$v_O = V_{IH} - \frac{V_{DD}}{2} = 0.7125 - 0.65 = 0.0625\ \text{V}$$

Thus, the worst-case value of V_{OL} is $V_{O\max} = 0.0625 \simeq 0.06$ V, and the noise margin NM_L reduces to

$$NM_L = 0.5875 - 0.0625 = 0.5250\ \text{V}$$

or approximately 0.53 V.

From symmetry, we can obtain the value of v_O corresponding to $v_I = V_{IL}$ as

$$v_O = V_{DD} - 0.0625$$

$$= 1.3 - 0.0625 = 1.2375\ \text{V} \simeq 1.24\ \text{V}$$

Thus, the worst-case value of V_{OH} is $V_{OH\min} \simeq 1.24$ V, and the noise margin NM_H is reduced to

$$NM_H = V_{OH\min} - V_{IH}$$

$$= 1.2375 - 0.7125 = 0.5250\ \text{V}$$

or approximately 0.53 V.

Note that the reduction in the noise margin (about 0.06 V) is slight.

(c) The output resistance of the inverter in the low-output state is

$$r_{DSN} = \frac{1}{\mu_n C_{ox}(W/L)_n(V_{DD} - V_{tn})}$$

$$= \frac{1}{0.5 \times 1.5(1.3 - 0.4)} = 1.48\ \text{k}\Omega$$

Since Q_N and Q_P are matched, the output resistance in the high-output state will be equal, that is,

$$r_{DSP} = r_{DSN} = 1.48\ \text{k}\Omega$$

(d) If the inverter is biased to operate at $v_I = v_O = V_M = 0.65$ V, then each of Q_N and Q_P will be operating at an overdrive voltage $V_{OV} = V_M - V_t = 0.65 - 0.4 = 0.25$ V and will be conducting equal dc currents I_D of

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{OV}^2$$

$$= \frac{1}{2} \times 500 \times 1.5 \times 0.25^2$$

$$= 23.4\ \mu\text{A}$$

Thus, Q_N and Q_P will have equal transconductances:

$$g_{mn} = g_{mp} = \frac{2I_D}{V_{OV}} = \frac{2 \times 23.4}{0.25} = 0.19\ \text{mA/V}$$

Transistors Q_N and Q_P will have output resistances r_{on} and r_{op} given by

$$r_{on} = \frac{V_{An}}{I_D}$$

$$r_{op} = \frac{|V_{Ap}|}{I_D}$$

Since no values are given for V_{An} and V_{Ap} we shall use the data in Table K.1, namely

$$V'_{An} = 5\ \text{V}/\mu\text{m and } |V'_{Ap}| = 6\ \text{V}/\mu\text{m}$$

Thus,

$$V_{An} = 5 \times 0.13 = 0.65\ \text{V}$$

$$|V_{Ap}| = 6 \times 0.13 = 0.78\ \text{V}$$

$$r_{on} = \frac{0.65\ \text{V}}{23.4\ \mu\text{A}} = 27.8\ \text{k}\Omega$$

$$r_{op} = \frac{0.78\ \text{V}}{23.4\ \mu\text{A}} = 33.3\ \text{k}\Omega$$

We can now compute the voltage gain at M as

$$A_v = -(g_{mn} + g_{mp})(r_{on} \parallel r_{op})$$

$$= -(0.19 + 0.19)(27.8 \parallel 33.3)$$

$$= -5.8\ \text{V/V}$$

When the straight line at M of slope -5.8 V/V is extrapolated, it intersects the line $v_O = 0$ at $\left[0.65 + \frac{0.65}{5.8}\right] = 0.762$ V and the line $v_O = V_{DD}$ at $\left[0.65 - \frac{0.65}{5.8}\right] = 0.538$ V. Thus the width of the transition region can be considered $(0.762 - 0.538) = 0.224$ V.

(e) For $W_p = W_n$, the parameter r can be found from Eq. (14.40):

$$r = \sqrt{\frac{\mu_p W_p}{\mu_n W_n}} = \sqrt{\frac{1}{4} \times 1} = 0.5$$

The corresponding value of V_M can be determined from Eq. (14.39) as

$$V_M = \frac{0.5(1.3 - 0.4) + 0.4}{0.5 + 1} = 0.57\ \text{V}$$

Thus, V_M shifts by only -0.08 V. We can estimate the reduction in NM_L to be approximately equal to the shift in V_M, that is, NM_L becomes

$$NM_L = 0.5875 - 0.08 \simeq 0.51\ \text{V}$$

The silicon area for this design can be computed as follows:

$$A = L(W_n + W_p)$$

$$= 0.13(1.5 \times 0.13 + 1.5 \times 0.13)$$

$$= 0.051\ \mu\text{m}^2$$

This represents a 60% reduction from the matched case!

(f) For $W_p = 2W_n$, we have

$$r = \sqrt{\frac{1}{4} \times 2} = \frac{1}{\sqrt{2}} = 0.707$$

$$V_M = \frac{0.707(1.3 - 0.4) + 0.4}{0.707 + 1} = 0.61\ \text{V}$$

Thus, relative to the matched case, V_M is reduced by only 0.04 V. Correspondingly, NM_L will be reduced by approximately an equal amount, thus NM_L becomes

$$NM_L \simeq 0.59 - 0.04 = 0.55\ \text{V}$$

In this case, the silicon area required is

$$A = L(W_n + W_p) = L \times 3W_n$$

$$= 0.13 \times 3 \times 1.5 \times 0.13$$

$$= 0.076\ \mu\text{m}^2$$

which represents a 40% reduction relative to the matched case.

14.38 (a) Switch opens at time $t = 0$, thus $v_O(0+) = 0$ V. The capacitor then charge by a constant current I, thus

$$It = Cv_O(t)$$

$$\Rightarrow v_O(t) = \frac{I}{C}t$$

(b) For $I = 1$ mA and $C = 10$ pF the time t for v_O to reach 1 V can be found as

$$1 = \frac{1 \times 10^{-3}}{10 \times 10^{-12}}\, t$$

$$\Rightarrow t = 10^{-8}\ \text{s} = 10\ \text{ns}$$

14.39 (a) Capacitor C is charged to 10 V and the switch closes at $t = 0$, thus

$v_O(0+) = 10$ V

Capacitor C then discharges through R exponentially with $v_O(\infty) = 0$

$v_O(t) = 0 - (0 - 10)\, e^{-t/\tau}$

$\Rightarrow v_O(t) = 10e^{-t/\tau}$

(b) For $C = 100$ pF and $R = 1$ kΩ, we have

$\tau = 100 \times 10^{-12} \times 1 \times 10^3 = 100$ ns

$t_{PHL} = 0.69\tau = 0.69 \times 100 = 69$ ns

$t_f = 2.22\tau = 2.2 \times 100 = 220$ ns

14.40 $V_{OH} = V_{DD}$

At $t = 0$, v_I goes low and the transistor turns off instantly, thus

$v_O(0+) = V_{OL}$

Now capacitor C charges through R towards $v_O(\infty) = V_{DD}$, thus

$v_O(t) = V_{DD} - (V_{DD} - V_{OL})\, e^{-t/\tau}$

At $t = t_{PLH}$,

$v_O = \frac{1}{2}(V_{OL} + V_{OH}) = \frac{1}{2}(V_{OL} + V_{DD})$, thus

$\frac{1}{2}(V_{OL} + V_{DD}) = V_{DD} - (V_{DD} - V_{OL})e^{-t/\tau}$

$\Rightarrow t_{PLH} = 0.69\tau$

For $R = 10$ kΩ and we wish to limit τ_{PLH} to 100 ps then the maximum value that C can have is found from

$0.69 \times C \times 10 \times 10^3 = 100 \times 10^{-12}$

$\Rightarrow C = 1.45 \times 10^{-14}$ F

$= 14.5$ fF

14.41

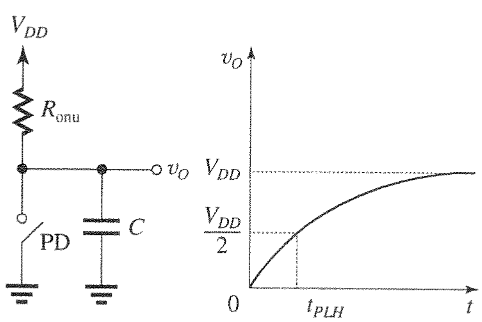


Figure 1(a)

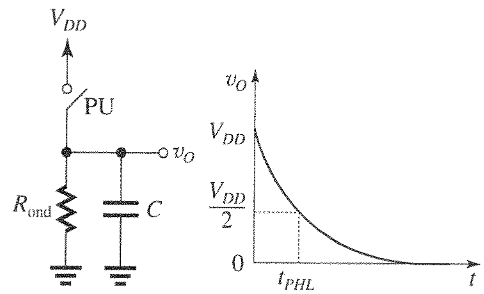


Figure 1(b)

To obtain t_{PLH}, consider the situation in Fig. 1(a). Here, PD has just opened (at $t = 0$), leaving $v_O = 0$ V at $t = 0+$. Capacitor C then charges through the "on" resistance of the pull-up switch, R_{onu}, toward V_{DD}, thus

$v_O(t) = V_\infty - (V_\infty - V_{0+})e^{-t/\tau}$

$= V_{DD} - (V_{DD} - 0)e^{-t/\tau}$

$= V_{DD}(1 - e^{-t/\tau})$

At $t = t_{PLH}$, $v_O = V_{DD}/2$, thus

$\frac{V_{DD}}{2} = V_{DD}(1 - e^{-t_{PLH}/\tau})$

$\Rightarrow e^{-t_{PLH}/\tau} = 0.5$

$\Rightarrow t_{PLH} = \tau \ln 2 = 0.69\tau$

For $C = 50$ fF, $R_{\text{onu}} = 2$ kΩ, then

$t_{PLH} = 0.69 \times 50 \times 10^{-15} \times 2 \times 10^3$

$= 69$ ps

Next we determine t_{PHL} by considering the situation depicted in Fig. 1(b). Here, PU has just opened, leaving $v_O(0+) = V_{DD}$. Capacitor C then discharges through the on resistance of the pull-down switch, R_{ond}, toward 0 V, thus $v_O(\infty) = 0$, thus

$v_O = 0 - (0 - V_{DD})\, e^{-t/\tau}$

$= V_{DD}e^{-t/\tau}$

At $t = t_{PHL}$, $v_O = V_{DD}/2$ and we get

$\frac{V_{DD}}{2} = V_{DD}e^{-t_{PHL}/\tau}$

$\Rightarrow t_{PHL} = 0.69\tau$

Here,

$\tau = CR_{\text{ond}}$

$= 50 \times 10^{-15} \times 1 \times 10^3 = 50$ ps

Thus,

$t_{PHL} = 0.69 \times 50 \simeq 35$ ps

The propagation delay t_P can now be obtained as

$$t_P = \frac{1}{2}(t_{PLH} + t_{PHL})$$

$$= \frac{1}{2}(69 + 35) = 52 \text{ ps}$$

14.42 (a) $V_{OL} = 0$ V

$V_{OH} = V_{DD} = 1.8$ V

$$NM_L = V_{IL} - V_{OL} = \frac{V_{DD}}{2} - 0$$

$= 0.9$ V

$$NM_H = V_{OH} - V_{IH} = V_{DD} - \frac{V_{DD}}{2}$$

$= 0.9$ V

(b) Capacitor C discharges through R_{on} of P_D,

$v_O(0+) = V_{DD} = 1.8$ V

$v_O(\infty) = 0$ V

Thus,

$v_O(t) = V_{DD}e^{-t/\tau}$

$\Rightarrow t_{PHL} = 0.69\tau$

$= 0.69 \times 0.1 \times 10^{-12} \times 2 \times 10^3$

$= 138$ ps

(c) Here the capacitor charges through R_{on} of PU toward V_{DD}. Thus,

$v_O(0+) = 0,\ v_O(\infty) = V_{DD}$,

$v_O(t) = V_{DD}(1 - e^{-t/\tau})$

At $t = t_{PLH}$, $v_O(t) = V_{DD}/2$, thus

$t_{PLH} = 0.69\tau$

$= 0.69 \times 0.1 \times 10^{-12} \times 2 \times 10^3$

$= 138$ ps

Finally, we obtain τ_P as

$$\tau_P = \frac{1}{2}(t_{PLH} + t_{PHL})$$

$$= \frac{1}{2}(138 + 138) = 138 \text{ ps}$$

14.43 (a) $t_P = \frac{1}{2}(t_{PLH} + t_{PHL})$

Since $t_P = 0.9$ ns, then

$$t_{PLH} + t_{PHL} = 1.8 \text{ ns} \qquad (1)$$

Now, since I_{charge} is half $I_{\text{discharge}}$, then

$$t_{PLH} = 2t_{PHL} \qquad (2)$$

Using (1) together with (2) yields

$t_{PLH} = 1.2$ ns

$t_{PHL} = 0.6$ ns

(b) Since the propagation delay is directly proportional to C, then the increase in propagation delay by 50%, when the capacitance is increased by 0.5 pF, indicates that the original total capacitance is 1.0 pF.

(c) The reduction of propagation delays by 40% when the load inverter is removed indicates that the load inverter was contributing 40% of the total capacitance found in (b), that is,

$C_{\text{out}} = 0.6$ pF

$C_{\text{load}} = 0.4$ pF

14.44 See figure on next page.

(a) For a rising input, time to the full change of output of second gate is

$$10 + 20 + \frac{30}{2} = 45 \text{ ns}$$

(b) For a falling input, time to the full change of output of the second gate is

$$20 + 10 + \frac{15}{2} = 37.5 \text{ ns}$$

The propagation delay is

$$t_P = \frac{1}{2}(t_{PLH} + t_{PHL})$$

$$= \frac{1}{2}(20 + 10) = 15 \text{ ns}$$

14.45 $\alpha_n = 2\Big/\left[\frac{7}{4} - \frac{3\,V_{tn}}{V_{DD}} + \left(\frac{V_{tn}}{V_{DD}}\right)^2\right]$

$$= 2\Big/\left[\frac{7}{4} - \frac{3 \times 0.4}{1.2} + \left(\frac{0.4}{1.2}\right)^2\right]$$

$= 2.32$

$$t_{PHL} = \frac{\alpha_n C}{k_n'(W/L)_n V_{DD}}$$

$$= \frac{2.32 \times 10 \times 10^{-15}}{430 \times 10^{-6} \times 1.5 \times 1.2}$$

$= 30$ ps

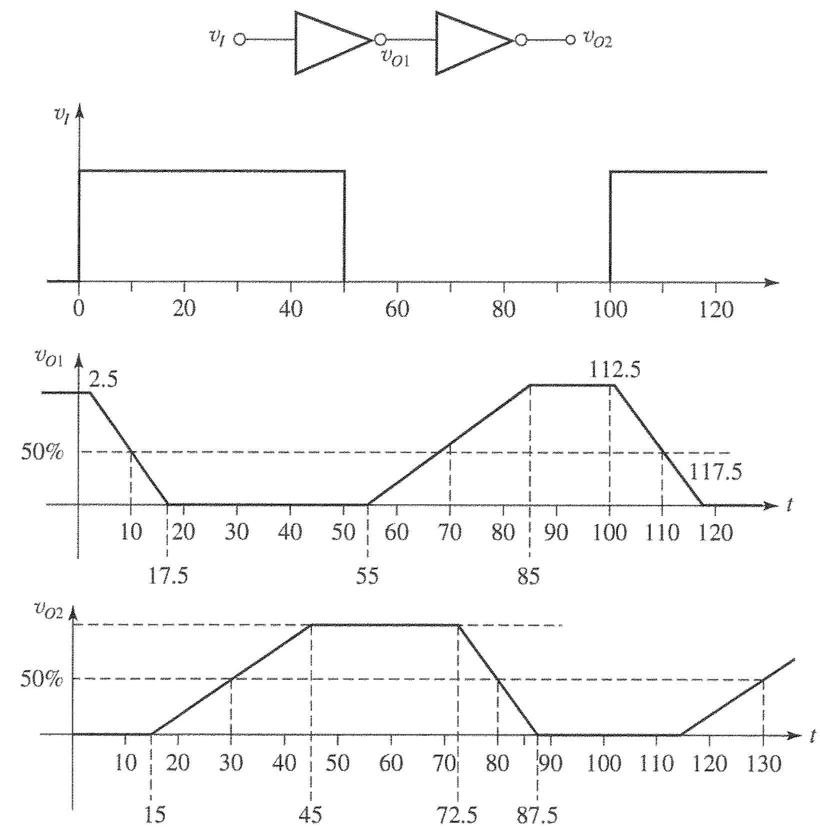
$\alpha_p = \alpha_n = 2.32$

$$t_{PLH} = \frac{\alpha_p C}{k_p'(W/L)_p V_{DD}}$$

$$= \frac{2.32 \times 10 \times 10^{-15}}{\left(\frac{430}{4}\right) \times 10^{-6} \times 3 \times 1.2}$$

$= 60$ ps

This figure belongs to Problem 14.44.



$t_P = \frac{1}{2}(t_{PHL} + t_{PLH})$

$= \frac{1}{2}(30 + 60) = 45$ ps

14.46 $\alpha_n = 2\Big/\left[\frac{7}{4} - \frac{3\,V_{tn}}{V_{DD}} + \left(\frac{V_{tn}}{V_{DD}}\right)^2\right]$

$= 2\Big/\left[\frac{7}{4} - \frac{3 \times 0.4}{1.2} + \left(\frac{0.4}{1.2}\right)^2\right]$

$= 2.32$

For a matched inverter, we have

$t_{PHL} = t_{PLH} = t_P$

For $t_P \leq 80$ ps,

$t_{PHL} \leq 80$ ps

But,

$t_{PHL} = \frac{\alpha_n C}{k_n'(W/L)_n V_{DD}}$

$\frac{2.32 \times 30 \times 10^{-15}}{430 \times 10^{-6}(W/L)_n \times 1.2} \leq 80 \times 10^{-12}$

$(W/L)_n \geq 1.7$

$(W/L)_p \geq 6.8$

14.47 $R_N = \frac{12.5}{(W/L)_n} = \frac{12.5}{1.5} = 8.33\ \text{k}\Omega$

$t_{PHL} = 0.69CR_N$

$= 0.69 \times 10 \times 10^{-15} \times 8.33 \times 10^3$

$= 57.5$ ps

$R_P = \frac{30}{(W/L)_p} = \frac{30}{3} = 10\ \text{k}\Omega$

$t_{PLH} = 0.69CR_P$

$= 0.69 \times 10 \times 10^{-15} \times 10 \times 10^3$

$= 69$ ps

$t_P = \frac{1}{2}(57.5 + 69) = 63.3$ ps

14.48 $R_N = \frac{12.5}{1} = 12.5\ \text{k}\Omega$

$R_P = \frac{30}{1} = 30\ \text{k}\Omega$

$t_{PHL} = 0.69CR_N$

$= 0.69 \times 20 \times 10^{-15} \times 12.5 \times 10^3$

$= 172.5$ ps

$t_{PLH} = 0.69CR_P$

$= 0.69 \times 20 \times 10^{-15} \times 30 \times 10^3$

$= 414$ ps

$t_P = \frac{1}{2}(172.5 + 414)$

$= 293.3$ ps

14.49 For

$t_{PHL} = t_{PLH} = t_P \le 50$ ps

we use

$t_{PHL} = 0.69CR_N$

$= 0.69C \times \frac{12.5}{(W/L)_n} \times 10^3$

and thus obtain

$0.69 \times 10 \times 10^{-15} \times \frac{12.5}{(W/L)_n} \times 10^3 \le 50 \times 10^{-12}$

$\Rightarrow \left(\frac{W}{L}\right)_n \ge 1.725$

Similarly,

$t_{PLH} = 0.69\ C\ R_P$

$= 0.69\ C \times \frac{30}{(W/L)_p} \times 10^3$

Thus,

$0.69 \times 10 \times 10^{-15} \times \frac{30}{(W/L)_p} \times 10^3 \le 50 \times 10^{-12}$

$\Rightarrow (W/L)_p \ge 4.14$

14.50 Refer to Example 14.6.

The method of average currents yields

$t_{PHL} = 41.2$ ps

The method of equivalent resistance yields

$t_{PHL} = 57.5$ ps

If the discrepancy is entirely due to the reduction in current due to velocity saturation in the NMOS transistor, then the factor by which the current decreases is $41.2/57.5 = 0.716$.

The value of t_{PLH} does not change (in fact there is a slight decrease due to various approximations). We may therefore conclude that the effect of velocity saturation is minimal in the PMOS transistor.

14.51 $\alpha_n = 2\bigg/\left[\frac{7}{4} - \frac{3\ V_{tn}}{V_{DD}} + \left(\frac{V_{tn}}{V_{DD}}\right)^2\right]$

$= 2\bigg/\left[\frac{7}{4} - \frac{3 \times 0.35}{1} + \left(\frac{0.35}{1}\right)^2\right]$

$= 2.43$

$t_{PHL} = \frac{\alpha_n C}{k_n'(W/L)_n V_{DD}}$

$= \frac{2.43 \times 10 \times 10^{-15}}{470 \times 10^{-6} \times 1.5 \times 1}$

$= 34.4$ ps

$t_{PLH} = \frac{\alpha_p C}{k_p'(W/L)_p V_{DD}}$

Since $|V_{tp}| = V_{tn}$, we have

$\alpha_p = \alpha_n = 2.43$

Thus,

$t_{PLH} = \frac{2.43 \times 10 \times 10^{-15}}{190 \times 10^{-6} \times 3 \times 1}$

$= 42.6$ ps

$t_P = \frac{1}{2}(34.4 + 42.6) = 38.5$ ps

The theoretical maximum switching frequency is

$f_{\max} = \frac{1}{2t_P} = \frac{1}{2 \times 38.5 \times 10^{-12}} \simeq 13$ GHz

14.52 $C = 4 \times 0.27 + 4 \times 0.27 + 2 + 2 + 5$

$= 11.16$ fF

$\alpha_n = 2\bigg/\left[\frac{7}{4} - \frac{3\ V_{tn}}{V_{DD}} + \left(\frac{V_{tn}}{V_{DD}}\right)^2\right]$

$= 2\bigg/\left[\frac{7}{4} - \frac{3 \times 0.5}{1.8} + \left(\frac{0.5}{1.8}\right)^2\right]$

$= 2.01$

$t_{PHL} = \frac{\alpha_n C}{k_n'\left(\frac{W}{L}\right)_n V_{DD}}$

$= \frac{2.01 \times 11.16 \times 10^{-15}}{380 \times 10^{-6} \times \frac{0.27}{0.18} \times 1.8}$

$= 21.9$ ps

$\alpha_p = \alpha_n = 2.01$

$t_{PLH} = \frac{\alpha_p C}{k_p'\left(\frac{W}{L}\right)_p V_{DD}}$

$= \frac{2.01 \times 11.16 \times 10^{-15}}{\frac{380}{4} \times 10^{-6} \times \frac{0.27}{0.18} \times 1.8}$

$= 87.6$ ps

$$t_P = \frac{1}{2}(21.9 + 87.6) = 54.8 \text{ ps}$$

If the design is changed to a matched one, then

$$W_p = 4W_n = 4 \times 0.27 = 1.08 \ \mu\text{m}$$

$$C = 4 \times 0.27 + 4 \times 1.08 + 2 + 2 + 5$$

$$= 14.4 \text{ fF}$$

$$\alpha_n = \alpha_p = 2.01$$

$$t_{PHL} = \frac{2.01 \times 14.4 \times 10^{-15}}{380 \times 10^{-6} \times \dfrac{0.27}{0.18} \times 1.8}$$

$$= 28.2 \text{ ps}$$

$$t_{PLH} = \frac{2.01 \times 14.4 \times 10^{-15}}{\dfrac{380}{4} \times 10^{-6} \times \dfrac{1.08}{0.18} \times 1.8}$$

$$= 28.2 \text{ ps}$$

$$t_P = \frac{1}{2}(28.2 + 28.2) = 28.2 \text{ ps}$$

14.53 $W_n = 0.75 \ \mu\text{m}$

$$W_p = \frac{\mu_n C_{ox}}{\mu_p C_{ox}} \times W_n$$

$$= \frac{180}{45} \times 0.75 = 3.0 \ \mu\text{m}$$

$$C = 2C_{gd1} + 2C_{gd2} + C_{db1} + C_{db2}$$

$$+ C_{g3} + C_{g4} + C_w$$

where

$$C_{gd1} = 0.4 \times W_n = 0.4 \times 0.75 = 0.3 \text{ fF}$$

$$C_{gd2} = 0.4 \times W_p = 0.4 \times 3 = 1.2 \text{ fF}$$

$$C_{db1} = 1 \times W_n = 1 \times 0.75 = 0.75 \text{ fF}$$

$$C_{db2} = 1 \times W_p = 1 \times 3 = 3 \text{ fF}$$

$$C_{g3} = 0.75 \times 0.5 \times 3.7 + 2 \times 0.4 \times 0.75 = 1.9875 \text{ fF}$$

$$C_{g4} = 3 \times 0.5 \times 3.7 + 2 \times 0.4 \times 3 = 7.95 \text{ fF}$$

Thus,

$$C = 2 \times 0.3 + 2 \times 1.2 + 0.75 + 3 + 1.9875 + 7.95 + 2$$

$$= 18.7 \text{ fF}$$

$$\alpha_n = 2\Big/\left[\frac{7}{4} - \frac{3\,V_t}{V_{DD}} + \left(\frac{V_t}{V_{DD}}\right)^2\right]$$

$$= 2\Big/\left[\frac{7}{4} - \frac{3 \times 0.7}{3.3} + \left(\frac{0.7}{3.3}\right)^2\right]$$

$$= 1.73$$

$$t_{PHL} = \frac{\alpha_n C}{k_n'\left(\dfrac{W}{L}\right)_n V_{DD}}$$

$$= \frac{1.73 \times 18.7 \times 10^{-15}}{180 \times 10^{-6} \times \left(\dfrac{0.75}{0.5}\right) \times 3.3}$$

$$= 36.3 \text{ ps}$$

Since the inverter is matched,

$$t_{PLH} = t_{PHL} = 36.3 \text{ ps}$$

and

$$t_P = 36.3 \text{ ps}$$

The propagation delay increases by 50% if C is increased by 50%, that is, by $18.7/2 = 9.35$ fF.

14.54 To reduce t_P by 40 ps, we need to reduce the extrinsic part by 40 ps. Now the original value of the extrinsic part is

$$t_P = 80 \times \frac{45}{45 + 15} = 60 \text{ ps}$$

A reduction by 40 ps requires the use of a scale factor S,

$$S = 3$$

This is the factor by which $(W/L)_n$ and $(W/L)_p$ must be scaled. The inverter area will be increased by the same ratio, that is, 3.

14.55 (a) Examination of Eq. (14.59) reveals that the NMOS transistors Q_1 and Q_3 contribute

$$C_n = 2\,C_{gd1} + C_{db1} + C_{g3} \tag{1}$$

and the PMOS transistors Q_2 and Q_4 contribute

$$C_p = 2\,C_{gd2} + C_{db2} + C_{g4} \tag{2}$$

The only difference in determining the corresponding capacitances in Eqs. (1) and (2) is the transistor width W. Thus each of the components in Eq. (2) can be written as the corresponding component in Eq. (1) multiplied by (W_p/W_n). Overall, we can write

$$C_p = C_n \frac{W_p}{W_n}$$

and the total capacitance C can be expressed as

$$C = C_n + C_p + C_w$$

$$= C_n + C_n \frac{W_p}{W_n} + C_w$$

Thus,

$$C = C_n\left(1 + \frac{W_p}{W_n}\right) + C_w \qquad \text{Q.E.D.}$$

(b) $R_N = \frac{12.5}{(W/L)_n}$ kΩ

For $(W/L)_n = 1$, we have

$R_N = 12.5$ kΩ

Thus,

$t_{PHL} = 0.69CR_N$

$= 0.69 \times 12.5 \times 10^3 C$

$= 8.625 \times 10^3 C$ Q.E.D.

$R_P = \frac{30}{(W/L)_p}$ kΩ

$= \frac{30}{(W_p/W_n)(W/L)_n}$ kΩ

For $(W/L)_n = 1$, we have

$R_P = \frac{30}{W_p/W_n}$ kΩ

and

$t_{PLH} = 0.69CR_P$

$= 0.69 \times \frac{30}{W_p/W_n} \times \times 10^3 C$

$= \frac{20.7 \times 10^3}{W_p/W_n} C$ Q.E.D.

(c) $t_P = \frac{1}{2}(t_{PHL} + t_{PLH})$

$= \frac{1}{2}\left[8.625 \times 10^3 C + \frac{20.7 \times 10^3}{W_p/W_n} C\right]$

For $W_p = W_n$, we have

$t_P = 14.66 \times 10^3 C$

$t_P = 14.66 \times 10^3 \left[C_n\left(1 + \frac{W_p}{W_n}\right) + C_w\right]$

$= 14.66 \times 10^3 (2C_n + C_w)$ (3)

(d) In the matched case, we have

$t_{PLH} = t_{PHL}$

From the results in (b), the required ratio (W_p/W_n) can be determined as

$\frac{20.7}{W_p/W_n} = 8.625$

$\Rightarrow \frac{W_p}{W_n} = 2.4$

In this case, we have

$C = C_n(1 + 2.4) + C_w = 3.4\ C_n + C_w$

and

$t_P = t_{PLH} = 8.625 \times 10^3 (3.4\ C_n + C_w)$ (4)

(e) (i) For $C_w = 0$, we have

$W_p = W_n$: $t_P = 29.32 \times 10^3 C_n$

$W_p = 2.4\ W_n$: $t_P = 29.32 \times 10^3 C_n$

Thus, in the case where C is entirely intrinsic, scaling does not affect t_P. This is what we found in Eq. (14.65).

(ii) For $C_w \gg C_n$, we have

$W_p = W_n$: $t_P = 14.66 \times 10^3 C_w$

$W_p = 2.4\ W_n$: $t_P = 8.625 \times 10^3 C_w$

Here C is entirely extrinsic, thus scaling the PMOS transistors has resulted in a decrease in t_P.

We conclude that using a matched design reduces t_P only when C is dominated by external capacitances. The matched design, of course, has the drawback of increased area.

14.56

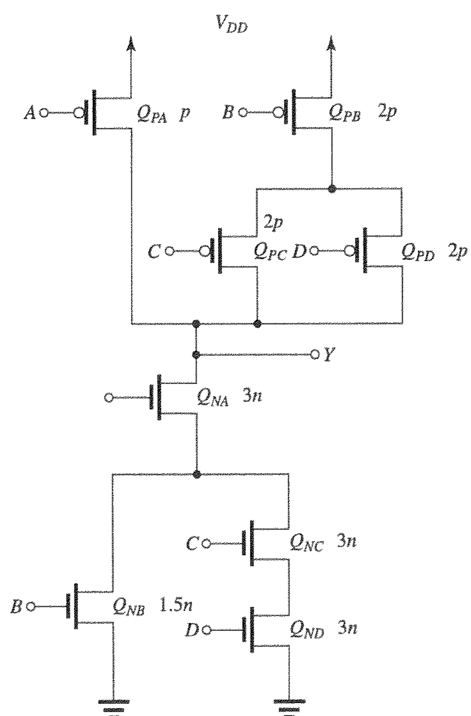


Figure 1

Figure 1 shows the CMOS logic gate with the (W/L) ratios selected so that the worst-case t_{PLH} and t_{PHL} are equal to the corresponding values of the basic inverter with $(W/L)_n = n$ and $(W/L)_p = p$. Observe that the worst case for discharging a capacitor occurs through the three series transistors Q_{NA}, Q_{NC}, and Q_{ND}. To make the equivalent W/L for these three series transistors equal to n, we select each of their (W/L) ratios to be equal to $3n$. Finally, for the discharge path (Q_{NA}, Q_{NB}) to have an equivalent W/L equal to n, we selected W/L of Q_{NB} equal to $1.5n$.

For the PUN, the worst-case charging path is that through Q_{PB} and one of Q_{PC} or Q_{PD}. Thus we select each of these three transistors to have $W/L = 2p$. Finally, we selected W/L of Q_{PA} equal to p.

14.57

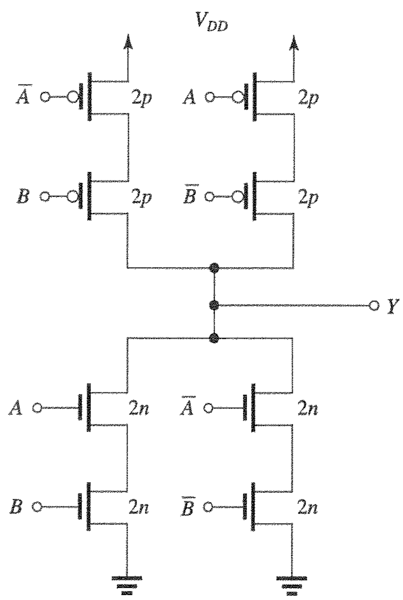


Figure 1

$$n = \frac{0.20}{0.13}, \; p = \frac{0.40}{0.13}$$

Figure 1 shows the circuit with the W/L ratio of each of the eight transistors indicated. Observe that the worst-case situation for both charging and discharging is two transistors in series. To achieve an equivalent W/L ratio for each path equal to that of the corresponding transistor in the basic inverter, each transistor is sized at twice that of the inverter. Including the two inverters required to obtain the complemented variable, the area is

$$A = 2W_nL + 2W_pL + 4 \times 2W_nL + 4 \times 2W_pL$$

$$= 10L(W_n + W_p)$$

$$= 10 \times 0.13(0.2 + 0.4)$$

$$= 0.78 \ \mu\text{m}^2$$

14.58 When the devices are sized as in Fig. 14.35, t_{PLH} that results when one PMOS transistor is conducting (worst case) is

$$t_{PLH} = 0.69 R_P C$$

$$= 0.69 \times \frac{30 \times 10^3}{p} \times C$$

and t_{PHL} is obtained by noting that the equivalent W/L of the discharge path is $4n/4 = n$ and thus

$$t_{PHL} = 0.69 \, R_N C$$

$$= 0.69 \times \frac{12.5 \times 10^3}{n} \times C$$

For the case in which all p-channel devices have $W/L = p$ and all n-channel devices have $W/L = n$, we have

$$t_{PLH} = 0.69 \times \frac{30 \times 10^3}{p} \times C$$

which is the same as in the first case. However,

$$t_{PHL} = 0.69 \times \frac{12.5 \times 10^3}{n/4} \times C$$

$$= 0.69 \times \frac{4 \times 12.5 \times 10^3}{n} \times C$$

which is four times the value obtained in the first case.

14.59

$L = 0.13 \ \mu\text{m}, \; W_n = 0.2 \ \mu\text{m}, \; W_p = 0.4 \ \mu\text{m},$

$n = 0.2/0.13, \; p = 0.4/0.13$

(a) Circuit (a) uses a six-input NOR gate and one inverter.

The six-input NOR requires:

6 NMOS transistors each with $W/L = n$

and

6 PMOS transistors each with $W/L = 6p$

The inverter requires

1 NMOS transistor with $W/L = n$

and

1 PMOS transistor with $W/L = p$

Thus,

$$\text{Area} = 6W_nL + 6 \times 6W_pL + W_nL + W_pL$$

$$= L(7\,W_n + 37\,W_p)$$

$$= 0.13(7 \times 0.2 + 37 \times 0.4)$$

$$= 0.13 \times 16.2 = 2.1 \ \mu\text{m}^2$$

(b) Circuit (b) uses two three-input NOR gates and one two-input NAND gate.

Each three-input NOR gate requires

3 NMOS transistors, each with $W/L = n$

3 PMOS transistors, each with $W/L = 3p$

The two-input NAND gate requires

2 NMOS transistors, each with $W/L = 2n$

2 PMOS transistors, each with $W/L = p$

Thus,

$$\text{Area} = 2 \times 3 \times W_n L + 2 \times 3 \times 3 \times W_p L$$
$$+ 2 \times 2 \times W_n L + 2 \times W_p L$$
$$= L(10\ W_n + 20\ W_p)$$
$$= 0.13(10 \times 0.2 + 20 \times 0.4)$$
$$= 0.13 \times 10$$
$$= 1.3\ \mu\text{m}^2$$

Thus circuit (a) required $2.1/1.3 = 1.62$ times the area of circuit (b).

14.60

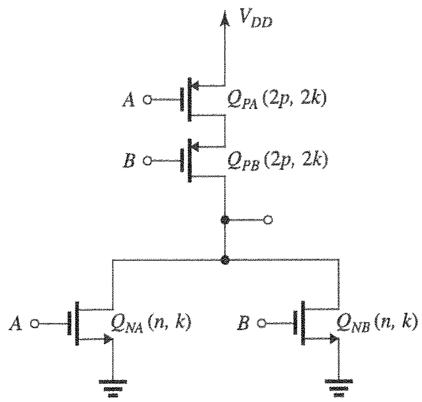


Figure 1

Refer to the circuit in Fig. 1. For Q_{NA} and Q_{NB}, (W/L) is equal to that of the NMOS transistor in the basic matched inverter. Thus,

$$k_{NA} = k_{NB} = k$$

For Q_{PA} and Q_{PB}, (W/L) is equal to twice the value of the PMOS transistor of the basic matched inverter. Since for the matched inverter $k_p = k_n = k$, here we have

$$k_{PA} = k_{PB} = 2k$$

(a)

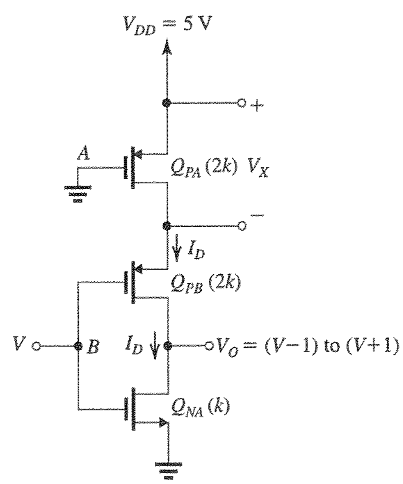


Figure 2

Figure 2 shows the circuit for the case input A is grounded. Note that Q_{NA} will be cut-off and has been eliminated. Switching will occur at $v_I = V$ which will be near $V_{DD}/2$. At this point, Q_{NB} and Q_{PB} will be in saturation and Q_{PA} will be in the triode region with a very small voltage V_X across it. All transistors will be conducting the same current I_D. For Q_{PA} we can write

$$I_D = 2k\left[(5-1)V_X - \frac{1}{2}V_X^2\right]$$

or

$$I_D = k(8\ V_X - V_X^2) \quad (1)$$

For Q_{PB} we can write

$$I_D = \frac{1}{2} \times 2k(5 - V_X - V - 1)^2$$

or

$$I_D = k(4 - V_X - V)^2 \quad (2)$$

Finally, for Q_{NB} we can write

$$I_D = \frac{1}{2}k(V-1)^2 \quad (3)$$

Next, we solve Eqs. (2) and (3) together to obtain V_X in terms of V. Equating Eqs. (2) and (3) gives

$$\pm\frac{1}{\sqrt{2}}(V-1) = 4 - V_X - V$$

$$\pm 0.707(V-1) = 4 - V_X - V \quad (4)$$

First try the solution corresponding to the + sign on the left-hand side of (4):

$$0.707(V-1) = 4 - V_X - V$$

$$\Rightarrow V_X = 4.707 - 1.707\ \text{V} \quad (5)$$

Since $V_X \simeq 0$, this equation gives

$$V = 2.75\ \text{V}$$

which is reasonable. The other solution gives

$$-0.707(V-1) = 4 - V_X - V$$

$$\Rightarrow V_X = 3.293 - 0.293\ \text{V}$$

For $V_X \simeq 0$, this equation gives

$$V = 11.2\ \text{V}$$

which is obviously impossible! Thus Eq. (5) is the solution that is physically meaningful. Next we substitute for V_X. From Eq. (5) into Eq. (1) to obtain

$$I_D = k\ 8(4.707 - 1.707\ \text{V}) - k(4.707 - 1.707\ \text{V})^2$$

$$= k(15.5 + 2.414\ \text{V} - 2.914\ \text{V}^2) \quad (6)$$

Equating this value of I_D to that in Eq. (3) gives

$$(V-1)^2 = 2(15.5 + 2.414\ \text{V} - 2.914\ \text{V}^2)$$

$$\Rightarrow 6.83\ \text{V}^2 - 6.83\ \text{V} - 30 = 0$$

$$\Rightarrow V = 2.65\ \text{V}$$

This figure belongs to Problem 14.60, part (b).

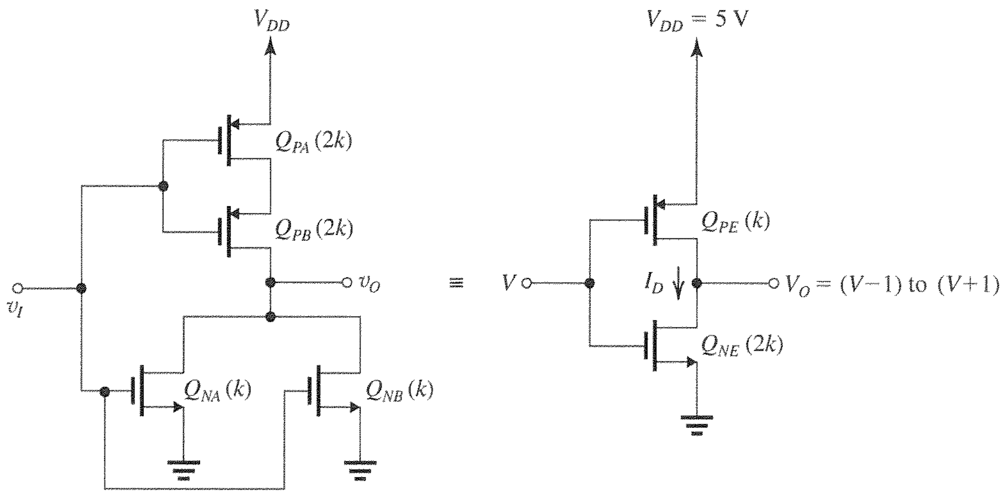


Figure 3

This is a reasonable value: It is greater than $(V_{DD}/2)$, which is required since Q_{NB} has a conductance parameter k while Q_{PB} has a parameter $2k$. Of course, V_{SG} of Q_{PB} is smaller than that of Q_{NB} because of V_X. The latter, however, is small. It can be found by substituting for $V = 2.65$ in Eq. (5),

$$V_X = 4.707 - 1.707 \times 2.65 = 0.18 \text{ V}$$

(b) Figure 3 shows the circuit when the corresponding input terminals are connected together. The two parallel NMOS transistors can be replaced by an equivalent NMOS transistor Q_{NE} having a $W/L = 2n$ and thus a transconductance parameter $2k$. The two series PMOS transistors can be replaced by an equivalent PMOS transistor Q_{PE} having a $W/L = (2p/2) = p$ and thus a transconductance parameter k. We are now ready to determine the threshold voltage, denoted V as before. Here, both Q_{NE} and Q_{PE} will be operating in saturation and conducting a current I_D. Thus, for Q_{PE} we can write

$$I_D = \frac{1}{2}k(V_{DD} - V - 1)^2$$

or

$$I_D = \frac{1}{2}k(4 - V)^2 \tag{7}$$

and for Q_{NE}, we can write

$$I_D = \frac{1}{2} \times 2k(V - 1)^2$$

or

$$I_D = k(V - 1)^2 \tag{8}$$

The value of V can be obtained by solving (7) and (8) together. Equating (7) and (8) gives

$$\frac{1}{2}(4 - V)^2 = (V - 1)^2$$

One solution is

$$\Rightarrow \frac{1}{\sqrt{2}}(4 - V) = V - 1$$

$$\Rightarrow V = 2.24 \text{ V}$$

This is a reasonable, physically meaningful answer and thus there is no need to find the other solution. As expected, V is lower than $(V_{DD}/2)$, which is a result of the fact that Q_{NE} has a larger (twice as large) transconductance parameter than Q_{PE}. Thus, Q_{NE} needs a smaller V_{GS} that V_{SG} of Q_{PE} to provide an equal I_D.

14.61 (a) $n = 4$

Minimum delay is obtained when the scaling factor x is given by

$$x^n = \frac{C_L}{C}$$

Here,

$$x^4 = \frac{1200C}{C} = 1200$$

$$\Rightarrow x = 5.9$$

$$t_P = 4xCR$$

$$= 4 \times 5.9CR = 23.6CR$$

(b) Let the number of the inverters be n. Optimum performance is obtained when

$$x^n = \frac{C_L}{C} = 1200$$

and

$$x = e = 2.718$$

$$n = \frac{\ln 1200}{\ln e} = 7.09 \simeq 7$$

Thus, we use 7 inverters. The actual scaling factor required can be found from

$$x^7 = 1200$$

$$\Rightarrow x = (1200)^{1/7} = 2.75$$

The value of t_P realized will be

$$t_P = 7 \times 2.75CR$$

$$= 19.25CR$$

which represents a reduction in t_P by about 17.4%. Thus adding three inverters reduces the delay by 17.4%.

14.62 (a) Refer to Fig. 14.37(c). By inspection we see that

$$t_P = \tau_1 + \tau_2 + ... + \tau_{n-1} + \tau_n$$

But,

$$\tau_1 = \tau_2 = ... = \tau_{n-1} = xCR$$

and

$$\tau_n = \frac{R}{x^{n-1}} C_L$$

Thus,

$$t_P = (n-1)xRC + \frac{1}{x^{n-1}} RC_L \qquad \text{Q.E.D.} \qquad (1)$$

(b) Differenting t_P in Eq. (1) relative to x gives

$$\frac{\partial t_P}{\partial x} = (n-1)RC - \frac{(n-1)}{x^n} RC_L$$

Equating $\frac{\partial t_P}{\partial x}$ to zero gives

$$x^n = \frac{C_L}{C} \qquad \text{Q.E.D.} \qquad (2)$$

(c) Differenting t_P in Eq. (1) relative to n gives

$$\frac{\partial t_P}{\partial n} = xRC - \frac{1}{x^{n-1}} (\ln x) RC_L$$

Equating $\frac{\partial t_P}{\partial n}$ to zero gives

$$x^n \left(\frac{C}{C_L}\right) = \ln x \qquad \text{Q.E.D.} \qquad (3)$$

To obtain the value of x for optimum performance, we combine the two optimality conditions in (2) and (3). Thus

$$\ln x = 1$$

$$\Rightarrow x = e \qquad \text{Q.E.D.}$$

14.63 $E = CV_{DD}^2$

$$= 10 \times 10^{-15} \times 1.8^2 = 32.4 \text{ fJ}$$

For 2×10^6 inverters switched at $f = 1$ GHz,

$$P_D = 2 \times 10^6 \times 1 \times 10^9 \times 32.4 \times 10^{-15}$$

$$= 64.8 \text{ W}$$

$$I_{DD} = \frac{P_D}{V_{DD}} = \frac{64.8}{1.8} = 36 \text{ A}$$

14.64 $P_{\text{dyn}} = fCV_{DD}^2$

$$= 2 \times 10^9 \times 5 \times 10^{-15} \times 1$$

$$= 10 \ \mu\text{W}$$

$$I_{DD} = \frac{10 \times 10^{-6}}{1} = 10 \ \mu\text{A}$$

14.65 Each cycle, the inverter draws an average current of

$$I_{\text{av}} = \frac{60 + 0}{2} = 30 \ \mu\text{A}$$

Since $I_{\text{av}} = 150 \ \mu$A, then the average current corresponding to the dynamic power dissipation is 120 μA. Thus,

$$P_{\text{dyn}} = 3.3 \times 120 \times 10^{-6} = 396 \ \mu\text{W}$$

But,

$$P_{\text{dyn}} = fCV_{DD}^2$$

Thus,

$$396 \times 10^{-6} = 100 \times 10^6 \times 3.3^2 \times C$$

$$\Rightarrow C = 0.36 \text{ pF}$$

14.66 Since P_{dyn} is proportional to V_{DD}^2, reducing the power supply from 5 V to 3.3 V reduces the power dissipation by a factor of $\left(\frac{3.3}{5}\right)^2 = 0.436$. The power dissipation now becomes $0.436 \times 10 = 4.36$ mW. Since P_{dyn} is proportional to f, reducing f by the same factor as the supply voltage (0.66) results in reducing the power dissipation *further* by a factor of 0.66, i.e.

Additional savings in power $= (1 - 0.66) \times 4.36$

$$= 1.48 \text{ mW}$$

14.67 $t_{PLH} = 30$ ns, $t_{PHL} = 50$ ns

$$t_P = \frac{1}{2}(30 + 50) = 40 \text{ ns}$$

$$P_{D\text{av}} = \frac{1}{2}(1 + 0.6) = 0.8 \text{ mW}$$

$$PDP = 0.8 \times 10^{-3} \times 40 \times 10^{-9} = 32 \text{ pJ}$$

14.68 (a)

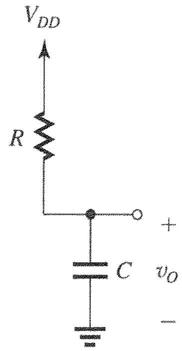


Figure 1

Figure 1 shows the circuit as the switch is opened ($t = 0+$). Capacitor C will charge through R, and its voltage will increase from the initial value of V_{OL} to the high value V_{OH},

$$v_O = v_O(\infty) - [v_O(\infty) - v_O(0+)]e^{-t/\tau}$$

$$= V_{OH} - (V_{OH} - V_{OL})e^{-t/\tau_1} \qquad \text{Q.E.D.}$$

where

$$\tau_1 = CR$$

To reach the 50% point, $\frac{1}{2}(V_{OH} + V_{OL})$ the time required, t_{PLH}, can be found as follows:

$$\frac{1}{2}(V_{OH} + V_{OL}) = V_{OH} - (V_{OH} - V_{OL})e^{-t_{PLH}/\tau_1}$$

$$(V_{OH} - V_{OL})\, e^{-t_{PLH}/\tau_1} = \frac{1}{2}(V_{OH} - V_{OL})$$

$$\Rightarrow t_{PLH} = \tau_1 \ln 2$$

$$= 0.69\tau_1 = 0.69CR \qquad \text{Q.E.D.}$$

(b) Figure 2(a) shows the circuit after the switch closes ($t = 0+$). At this instant the capacitor voltage in V_{OH}. The capacitor then discharges and eventually reaches the low level V_{OL}. To determine τ_{PHL}, we simplify the circuit to that in Fig. 2(b). Using this circuit, we can express v_O as

$$v_O(t) = v_O(\infty) - [v_O(\infty) - v_O(0)]e^{-t/\tau_1}$$

$$= V_{OL} - (V_{OL} - V_{OH})e^{-t/\tau_2}$$

when

$$\tau_2 = C(R_{\text{on}} \parallel R) \simeq CR_{\text{on}}$$

The value of t_{PHL} can be found from

$$v_O(t_{PHL}) = \frac{1}{2}(V_{OH} + V_{OL})$$

$$= V_{OL} - (V_{OL} - V_{OH})e^{-t_{PLH}/\tau_2}$$

$$\Rightarrow t_{PHL} = \tau_2 \ln 2$$

$$= 0.69\tau_2$$

$$= 0.69CR_{\text{on}} \qquad \text{Q.E.D.}$$

(c) $t_P = \frac{1}{2}(t_{PLH} + t_{PHL})$

$$= \frac{1}{2}(0.69CR + 0.69CR_{\text{on}})$$

$$t_P = \frac{1}{2} \times 0.69C(R + R_{\text{on}})$$

Since $R_{\text{on}} \ll R$,

$$t_P \simeq 0.35CR \qquad \text{Q.E.D.}$$

(d) During the low-input state, the switch is open, the current is zero, and the power dissipation is zero.

During the high-input state, the switch is closed and a current

$$I_{DD} = \frac{V_{DD}}{R + R_{\text{on}}} \simeq \frac{V_{DD}}{R}$$

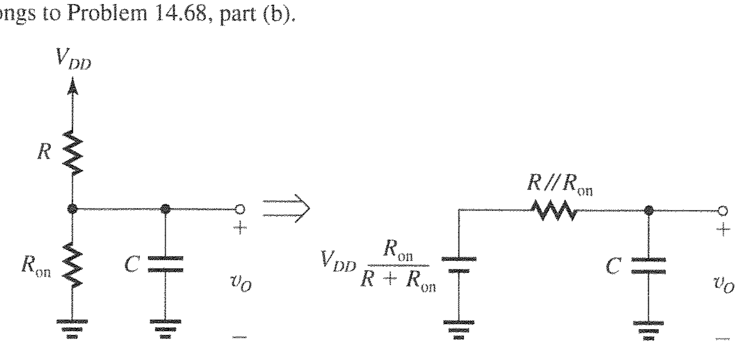
flows, and the power dissipation is

$$P_D = V_{DD}I_{DD} = \frac{V_{DD}^2}{R}$$

Now, if the inverter spends half the time in each state, the average power dissipation will be

$$P = \frac{1}{2}\frac{V_{DD}^2}{R} \qquad \text{Q.E.D.}$$

This figure belongs to Problem 14.68, part (b).



(a) (b)

Figure 2

(e) For $V_{DD} = 5$ V and $C = 10$ pF, we have

$t_P = 0.35 \times 10 \times 10^{-12} R$

If t_P is to be smaller or equal to 5 ns, we must have

$0.35 \times 10 \times 10^{-12} R \leq 5 \times 10^{-9}$

$\Rightarrow R \leq 1.43$ kΩ

If P is to be smaller or equal to 15 mW, we must have

$$\frac{1}{2} \times \frac{5^2}{R} \leq 15$$

where R is in kΩ, thus

$R \geq 0.83$ kΩ

Thus, to satisfy both constraints, R must lie in the range

$0.83 \text{ k}\Omega \leq R \leq 1.43 \text{ k}\Omega$

Selecting $R = 1$ kΩ yields

$t_P = 0.35 \times 10 \times 10^{-12} \times 1 \times 10^3$

$= 3.5$ ns

and

$$P = \frac{1}{2}\frac{5^2}{1} = 12.5 \text{ mW}$$

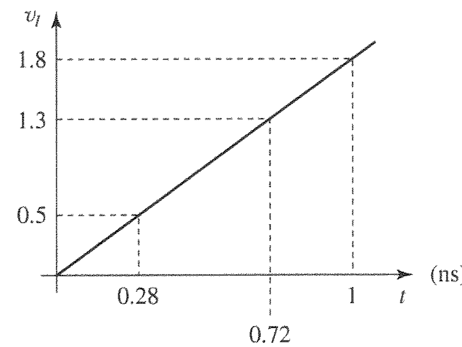
Both values are within the design specifications.

14.69 (a) The currents decrease by a factor of $\frac{1.8}{2.5} = 0.72$; that is, the new current values are 0.72 of the old current values. However, the voltage swing is also reduced by the same factor. The result is that t_P remains unchanged. The PDP will be reduced by a factor of 0.52.

Since the maximum operating frequency is proportional to $1/t_P$, it also will remain unchanged.

(b) If current is proportional to V_{DD}^2, the currents become 0.72^2 of their old values. This together with the reduction of voltage swing by a factor of 0.72 will result in t_P increasing by a factor of (1/0.72) and the maximum operating frequency being reduced by a factor of 0.72. The PDP decreases by a factor of 0.72.

14.70



From Eq. (14.79), we have

$$I_{\text{peak}} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n\left(\frac{V_{DD}}{2} - V_{tn}\right)^2$$

$$I_{\text{peak}} = \frac{1}{2} \times 450\,\frac{\mu\text{A}}{\text{V}^2}\left(\frac{1.8}{2} - 0.5\right)^2 = 36\ \mu\text{A}$$

The time when the input reaches V_t is

$$\frac{0.5}{1.8} \times 1 = 0.28 \text{ ns}$$

The time when the input reaches $V_{DD} - V_t$ is

$$\frac{1.8 - 0.5}{1.8} \times 1 = 0.72 \text{ ns}$$

So the base of the triangle is

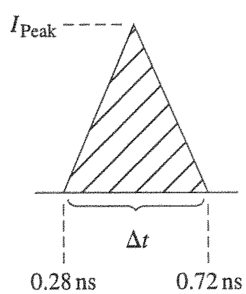
$\Delta t = 0.72 - 0.28 = 0.44$ ns

$$E = \frac{1}{2} I_{\text{peak}} \times V_{DD} \times \Delta t =$$

$$\frac{1}{2} \times 36\ \mu\text{A} \times 1.8 \times 0.44 \text{ ns}$$

$= 14.3$ fJ

$P = f \times E = 100 \times 10^6 \times 14.3 \times 10^{-15} = 1.43\ \mu$W.



15.1 Here the scaling factor is

$$S = \frac{5\ \mu\text{m}}{32\ \text{nm}} = 156.25$$

Therefore, Moore's law predicts that the number of transistors on a chip of equal area, fabricated in the 32-nm process, would be

$$20{,}000 \times S^2 = 20{,}000 \times 156.25^2$$

$= 4.88 \times 10^8$ or 488 million transistors.

15.2 (a) $t_P \propto \dfrac{\alpha C}{k' V_{DD}}$, and k' is scaled by S, and C and V_{DD} are scaled by $\dfrac{1}{S}$; thus t_P is scaled by

$$\frac{\frac{1}{S}}{S \times \frac{1}{S}} = \frac{1}{S}$$

$S = 2 \Rightarrow t_P$ is scaled by $\dfrac{1}{2}$ (t_P decreases)

The maximum operating speed is $\dfrac{1}{2t_p}$ and therefore is scaled by 2.

$P_{\text{dyn}} = f_{\max} C V^2{}_{DD}$ and thus is scaled by

$$S \times \frac{1}{S} \times \frac{1}{S^2} = \frac{1}{S^2} = \frac{1}{4}\ (P_{\text{dyn}}\ \text{decreases}).$$

Power density $= \dfrac{P_{\text{dyn}}}{\text{area}}$ and thus is scaled by

$$\frac{\frac{1}{S^2}}{\frac{1}{S^2}} = 1, \text{ i.e., remains unchanged.}$$

PDP is scaled by $\dfrac{1}{S^3}$ $\left(\text{power is scaled by } \dfrac{1}{S^2} \text{ and delay by } \dfrac{1}{S}\right)$ and thus it is scaled by $\dfrac{1}{8}$ (PDP decreases).

(b) If V_{DD} and V_{tn} remain unchanged while $S = 2$, we have

$$t_p = \frac{\alpha C}{k' V_{DD}} \text{ and } \alpha = \frac{2}{\frac{7}{4} - \frac{3V_{tn}}{V_{DD}} + \left(\frac{V_{tn}}{V_{DD}}\right)^2} \text{ so } \alpha$$

remains unchanged and t_P is scaled by

$$\frac{\frac{1}{S}}{S} = \frac{1}{S^2} = \frac{1}{4}$$

The maximum operating speed is $\dfrac{1}{2t_P}$ and therefore is scaled by 4.

$P_{\text{dyn}} = f_{\max} C V^2{}_{DD}$ and thus is scaled by

$$4 \times \frac{1}{2} \times 1 = 2$$

Power density $= \dfrac{P_{\text{dyn}}}{\text{area}}$ is thus scaled by

$$\frac{2}{\frac{1}{S^2}} = \frac{2}{\frac{1}{4}} = 8$$

PDP is scaled by $2 \times \dfrac{1}{4} = \dfrac{1}{2}$

15.3 From Eq. (15.5), we have for the NMOS transistor

$$0.25 = \frac{65 \times 10^{-9}}{\mu_n} \times 10^5$$

where we have assumed $v_{\text{sat}} = 10^7$ cm/s $= 10^5$ m/s. Thus,

$$\mu_n = 260\ \text{cm}^2/\text{V.s}$$

For the PMOS transistor

$$0.45 = \frac{65 \times 10^{-9}}{\mu_p} \times 10^5$$

$$\Rightarrow \mu_p = 144.4\ \text{cm}^2/\text{V.s}$$

The value of E_{cr} can be determined using Eq. (15.3),

$$E_{cr} = \frac{V_{DS\text{sat}}}{L}$$

For the NMOS transistor we have

$$E_{cr} = \frac{0.25}{65 \times 10^{-9}} = 3.85 \times 10^6\ \text{V/m}$$

$$= 3.85 \times 10^4\ \text{V/cm}$$

and for the PMOS transistor

$$E_{cr} = \frac{0.45}{65 \times 10^{-9}} = 6.92 \times 10^4\ \text{V/cm}$$

15.4 Using Eq. (15.5), we obtain for the NMOS transistor

$$V_{DS\text{sat}} = \left(\frac{L}{\mu_n}\right) v_{\text{sat}}$$

$$= \frac{0.13 \times 10^{-6}}{350 \times 10^{-4}} \times 10^5$$

$$= 0.37\ \text{V}$$

and for the PMOS transistor, which yields

$$V_{DS\text{sat}} = \left(\frac{L}{\mu_p}\right) v_{\text{sat}}$$

$$= \frac{0.13 \times 10^{-6}}{150 \times 10^{-4}} \times 10^5$$

$$= 0.87\ \text{V}$$

15.5 (a) From equation (15.9), we have

$$I_{Dsat} = \mu_n C_{ox}\left(\frac{W}{L}\right)V_{DSsat}\left(V_{GS} - V_t - \frac{1}{2}V_{DSsat}\right)$$

Thus, for $V_{GS} = V_{DD}$ we have

$$I_{Dsat} = \mu_n C_{ox}\left(\frac{W}{L}\right)V_{DSsat}\left(V_{DD} - V_t - \frac{1}{2}V_{DSsat}\right)$$

In the absence of velocity saturation, we have

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_t)^2; \text{ and for}$$

$$V_{GS} = V_{DD}, I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{DD} - V_t)^2$$

Therefore,

$$\frac{I_{Dsat}}{I_D} = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)V_{DSsat}\left(V_{DD} - V_t - \frac{1}{2}V_{DSsat}\right)}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{DD} - V_t)^2}$$

$$= \frac{2V_{DSsat}\left(V_{DD} - V_t - \frac{1}{2}V_{DSsat}\right)}{(V_{DD} - V_t)^2} \qquad \text{Q.E.D.}$$

(b) For this 65-nm process, we have

$$\frac{I_{Dsat}}{I_D} = \frac{2 \times 0.25\left(1.0 - 0.35 - \frac{1}{2} \times 0.25\right)}{(1.0 - 0.35)^2}$$

$= 0.62$

15.6 (a) Since Q_N provides a constant current I_{Dsat} to discharge C from a voltage of V_{DD} to $V_{DD}/2$, the discharge interval t_{PHL} can be found by writing the charge equilibrium equation

$$I_{Dsat}\, t_{PHL} = C(V_{DD}/2)$$

which yields

$$t_{PHL} = \frac{CV_{DD}}{2I_{Dsat}} \qquad \text{Q.E.D.} \qquad (1)$$

(b) Using Eqs. (14.45) and (14.56), we obtain

$$t_{PHL} = 0.69C\frac{12.5 \times 10^3}{(W/L)_n} \qquad \text{Q.E.D.} \qquad (2)$$

(c) Equating the values of t_{PHL} given by Eqs. (1) and (2), we obtain

$$\frac{CV_{DD}}{2I_{Dsat}} = 0.69C\frac{12.5 \times 10^3}{(W/L)_n}$$

$$\Rightarrow I_{Dsat} = 0.058 \times 10^{-3}V_{DD}(W/L)_n$$

Substituting $V_{DD} = 1.2$ V, we get

$$I_{Dsat} = 0.07 \times 10^{-3}(W/L)_n \qquad (3)$$

But I_{Dsat} is given by Eq. (15.9) as

$$I_{Dsat} = \mu_n C_{ox}\left(\frac{W}{L}\right)V_{DSsat}\left(V_{GS} - V_t - \frac{1}{2}V_{DSsat}\right) \qquad (4)$$

Substituting $\mu_n C_{ox} = 325 \times 10^{-6}$ A/V, $V_{GS} = V_{DD} = 1.2$ V, and $V_t = 0.4$ V and equating the resulting value of I_{Dsat} to that given by Eq. (3), we obtain

$$325 \times 10^{-6}\left(\frac{W}{L}\right)V_{DSsat}\left(1.2 - 0.4 - \frac{1}{2}V_{DSsat}\right)$$

$$= 0.07 \times 10^{-3}(W/L)$$

$$\Rightarrow V_{DSsat} = 0.34 \text{ V}$$

15.7 (a) From Eq. (15.9), we have

$$I_{DSsatn} = \mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{DSsatn}\left(V_{GS} - V_{tn} - \frac{1}{2}V_{DSsatn}\right)$$

and

$$I_{DSsatp} = \mu_p C_{ox}\left(\frac{W}{L}\right)_p |V_{DSsatp}|\left[|V_{GS}| - |V_{tp}| - \left(\frac{1}{2}|V_{DSsatp}|\right)\right]$$

Since $|V_{GS}| = V_{DD}$ (i.e., for NMOS $V_{GS} = V_{DD}$ and for PMOS $|V_{GS}| = V_{DD}$ and $I_{DSsatn} = I_{DSsatp}$, we have

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{DSsatn}\left(V_{DD} - V_{tn} - \frac{1}{2}V_{DSsatn}\right)$$

$$= \mu_p C_{ox}\left(\frac{W}{L}\right)_p |V_{DSsatp}|\left(V_{DD} - |V_{tp}| - \frac{1}{2}|V_{DSsatp}|\right)$$

$L_n = L_p \Rightarrow$ Thus,

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p}\frac{V_{DSsatn}}{|V_{DSsatp}|}\frac{\left(V_{DD} - V_{tn} - \frac{1}{2}V_{DSsatn}\right)}{\left(V_{DD} - |V_{tp}| - \frac{1}{2}|V_{DSsatp}|\right)}$$

(b) $$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p}\frac{V_{DSsatn}}{|V_{DSsatp}|}\frac{\left(V_{DD} - V_{tn} - \frac{1}{2}V_{DSsatn}\right)}{\left(V_{DD} - |V_{tp}| - \frac{1}{2}V_{DSsatp}\right)}$$

$$= 4 \times \frac{0.25}{0.45}\frac{\left(1.0 - 0.35 - \frac{0.25}{2}\right)}{\left(1.0 - 0.35 - \frac{0.45}{2}\right)} = 2.75$$

15.8 $I_S \propto e^{-V_t/nV_T}$

If V_t is reduced by 0.1 V, then

$$I_S \propto e^{-(V_t - 0.1)/nV_T}$$

$$\propto e^{-V_t/nV_T}e^{0.1/nV_T}$$

Thus I_S increases by a factor

$= e^{0.1/nV_T}$

$= e^{0.1/(2\times0.025)}$

$= e^2 = 7.4$

Thus the static power dissipation increases by a factor of 7.4.

If V_t is reduced by 0.2 V, a similar analysis to that above shows that I_S increases by a factor

$= e^{0.2/(2\times0.025)} = e^4 = 54.6$

and the static power dissipation increases by the same factor.

We conclude that in process design, the higher the value one chooses for V_t, the lower the static power dissipation.

15.9 (a) $i_D = I_S e^{-v_{GS}/nV_T}$

Corresponding to i_D changing by a factor of 10, v_{GS} changes by $2.3nV_T$, thus

$2.3nV_T = 80$ mV

$$\Rightarrow n = \frac{80}{2.3 \times 25} = 1.4$$

Using the given information that $i_D = 20$ nA at $v_{GS} = 0.16$ V, we obtain

$20 \times 10^{-9} = I_S e^{0.16/(1.4\times0.025)}$

$\Rightarrow I_S = 207$ pA

This is the value of i_D at $v_{GS} = 0$

(b) For a chip with 1 billion transistors, the current drawn from the power supply as a result of subthreshold conduction will be

$I_{\text{total}} = 207 \times 10^{-12} \times 10^9$

$= 207$ mA

and the power dissipation will be

$P_D = I_{\text{total}} V_{DD}$

$= 207 \times 1 = 207$ mW

15.10 $I_D = \frac{1}{2}k_n(V_{GS} - V_{tn})^2$

$$0.2 = \frac{1}{2} \times 0.4(V_{GS} - 0.4)^2$$

$\Rightarrow V_{GS} = 1.4$V

(a) For $V_{tn} = 0.4 + 10\% = 0.44$ V, we have

$$I_D = \frac{1}{2} \times 0.4(1.4 - 0.44)^2 = 0.184 \text{ mA}$$

For $V_{tn} = 0.4 - 10\% = 0.36$ V, we have

$$I_D = \frac{1}{2} \times 0.4(1.4 - 0.36)^2$$

$= 0.216$ mA

Thus, I_D will range from 0.184 mA to 0.216 mA (i.e., $0.2 \pm 8\%$, mA).

(b) If the time for a 0.1-V change of the capacitor voltage is denoted T, then

$I_D T = C\Delta V$

$$T = \frac{C\Delta V}{I_D}$$

For $I_D = 0.184$ mA, we have

$$T = \frac{100 \times 10^{-15} \times 0.1}{0.184 \times 10^{-3}} = 54.3 \text{ ps}$$

For $I_D = 0.216$ mA, we have

$$T = \frac{100 \times 10^{-15} \times 0.1}{0.216 \times 10^{-3}} = 46.3 \text{ ps}$$

Thus, the discharge time ranges from 46.3 ps to 54.3 ps.

15.11 (a) $R = 27 \text{ m}\Omega \times \frac{5 \text{ mm}}{0.5 \text{ µm}}$

$$= 27 \times 10{-}3 \times \frac{5 \times 10^{-3}}{0.5 \times 10^{-6}}$$

$= 270\ \Omega$

(b) $C = 0.1 \times \frac{5 \text{ mm}}{1 \text{ µm}}$

$= 0.1 \times 5 \times 10^3 = 500$ fF

$= 0.5$ pF

(c) $t_{\text{delay}} = 0.69RC$

$= 0.69 \times 270 \times 0.5 \times 10^{-12}$

$= 93.2$ ps

15.12 (a) For the circuit in Fig. P15.12(a), assuming $v_O(0-) = 0$, we can write

$v_O(t) = V_{DD} - V_{DD}e^{-t/\tau}$

where

$\tau = CR$

Substituting, we obtain

$$v_O(t_{PLH}) = \frac{1}{2}V_{DD}$$

$$\frac{1}{2}V_{DD} = V_{DD} - V_{DD}e^{-t_{PLH}/\tau}$$

$\Rightarrow t_{PLH} = 0.69\tau = 0.69CR$ (1)

(b) For the circuit in Fig. P15.12(b), assuming $v_O(t-) = 0$, we can write

$$I\, t_{PLH} = C(V_{DD}/2)$$

$$\Rightarrow t_{PLH} = \frac{1}{2} C \frac{V_{DD}}{I}$$

But,

$$I = \frac{V_{DD}}{R}$$

thus,

$$t_{PLH} = \frac{1}{2} C \frac{V_{DD}}{V_{DD}/R} = 0.5CR \qquad (2)$$

Comparing Eqs. (1) and (2), we see that using a current-source load reduces t_{PLH} by

$$\frac{0.69 - 0.5}{0.69} \times 100 = 27.5\%$$

15.13 $V_{OH} = V_{DD} = 1.3$ V

Using Eq. (15.24), we find

$$V_{OL} = (V_{DD} - V_t)\left[1 - \sqrt{1 - \frac{1}{r}}\right]$$

$$= (1.3 - 0.4)\left[1 - \sqrt{1 - \frac{1}{5}}\right]$$

$$= 0.095 \text{ V}$$

Using Eq. (15.25), we obtain

$$I_{\text{stat}} = \frac{1}{2} k_p (V_{DD} - V_t)^2$$

$$= \frac{1}{2} \times 100(1.3 - 0.4)^2$$

$$= 40.5\ \mu\text{A}$$

$$P_D = I_{\text{stat}} V_{DD}$$

$$= 40.5 \times 1.3 = 52.7\ \mu\text{W}$$

15.14 Using Eq. (15.24), we obtain

$$V_{OL} = (V_{DD} - V_t)\left[1 - \sqrt{1 - \frac{1}{r}}\right]$$

$$= (1.3 - 0.4)\left[1 - \sqrt{1 - \frac{1}{5}}\right]$$

$$= 0.095 \text{ V}$$

Using Eq. (15.20), we obtain

$$V_{IL} = V_t + \frac{V_{DD} - V_t}{\sqrt{r(r+1)}}$$

$$= 0.4 + \frac{1.3 - 0.4}{\sqrt{5 \times 6}}$$

$$= 0.56 \text{ V}$$

Using Eq. (15.21), we obtain

$$V_M = V_t + \frac{V_{DD} - V_t}{\sqrt{r+1}}$$

$$= 0.4 + \frac{1.3 - 0.4}{\sqrt{6}}$$

$$= 0.77 \text{ V}$$

Using Eq. (15.23), we find

$$V_{IH} = V_t + \frac{2}{\sqrt{3r}}(V_{DD} - V_t)$$

$$= 0.4 + \frac{2}{\sqrt{3 \times 5}}(1.3 - 0.4)$$

$$= 0.86 \text{ V}$$

$$V_{OH} = V_{DD} = 1.3 \text{ V}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 0.56 - 0.095 = 0.465 \text{ V}$$

$$NM_H = V_{OH} - V_{IH}$$

$$= 1.3 - 0.86 = 0.44 \text{ V}$$

15.15 Using Eqs. (15.29) and (15.28), we find

$$\alpha_p = 2 \Bigg/ \left[\frac{7}{4} - 3 \times \frac{0.4}{1.3} + \left(\frac{0.4}{1.3}\right)^2\right]$$

$$= 2.17$$

$$t_{PLH} = \frac{\alpha_p C}{k_p V_{DD}}$$

$$= \frac{2.17 \times 10 \times 10^{-15}}{100 \times 10^{-6} \times 1.3}$$

$$= 167 \text{ ps}$$

Using Eqs. (15.31) and (15.30), we obtain

$$\alpha_n = 2 \Bigg/ \left[1 + \frac{3}{4}\left(1 - \frac{1}{5}\right) - \left(3 - \frac{1}{5}\right)\left(\frac{0.4}{1.3}\right) + \left(\frac{0.4}{1.3}\right)^2\right]$$

$$= 2.4$$

$$t_{PHL} = \frac{\alpha_n C}{k_n V_{DD}}$$

$$= \frac{2.4 \times 10 \times 10^{-15}}{500 \times 10^{-6} \times 1.3}$$

$$= 36.9 \text{ ps}$$

$$t_P = \frac{1}{2}(t_{PLH} + t_{PHL})$$

$$= \frac{1}{2}(167 + 36.9) = 102 \text{ ps}$$

15.16

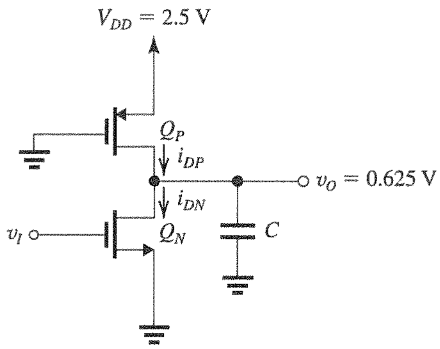


Figure 1

$$\frac{V_{DD}}{4} = 0.625 \text{ V}$$

Figure 1 shows the inverter when $v_O = V_{DD}/4 = 0.625$ V. At this output value, if Q_N is off, Q_P will be operating in the triode region (because $v_{DG} = 0.625$ V is greater than $|V_t| = 0.5$ V) and charging the capacitor with a current

$$i_{DP} = k_p'\left(\frac{W}{L}\right)_p [(V_{DD} - |V_t|)(V_{DD} - v_O) - \frac{1}{2}(V_{DD} - v_O)^2]$$

$$= 30\left(\frac{W}{L}\right)_p \left[(2.5 - 0.5)(2.5 - 0.625) - \frac{1}{2}(2.5 - 0.625)^2\right]$$

$$= 59.8(W/L)_p, \ \mu\text{A}$$

When v_I is high at 2.5 V, Q_N will be operating in the triode region (because $v_I > V_{tn} + v_O$) and will be conducting a current

$$i_{DN} = k_n'\left(\frac{W}{L}\right)_n \left[(v_I - V_t)v_O - \frac{1}{2}v_O^2\right]$$

$$= 115 \times 1.5\left[(2.5 - 0.5) \times 0.625 - \frac{1}{2} \times 0.625^2\right]$$

$$= 181.9 \ \mu\text{A}$$

The capacitor discharge current will be

$$= i_{DN} - i_{DP}$$

$$= 181.9 - 59.8(W/L)_p, \ \mu\text{A}$$

The design is based on equal charging and discharging current, thus

$$59.8(W/L)_p = 181.9 - 59.8(W/L)_p$$

$$\Rightarrow (W/L)_p = 1.52$$

We now can find the value of r from

$$r = \frac{k_n}{k_p} = \frac{k_n'(W/L)_n}{k_p'(W/L)_p}$$

$$= \frac{115 \times 1.5}{30 \times 1.52} = 3.78$$

Using Eq. (15.20), we obtain

$$V_{IL} = V_t + \frac{V_{DD} - V_t}{\sqrt{r(r+1)}}$$

$$V_{IL} = 0.5 + \frac{2.5 - 0.5}{\sqrt{3.78 \times 4.78}}$$

$$= 0.97 \text{ V}$$

Using Eq. (15.23), we obtain

$$V_{IH} = V_t + \frac{2}{\sqrt{3r}}(V_{DD} - V_t)$$

$$= 0.5 + \frac{2}{\sqrt{3 \times 3.78}}(2.5 - 0.5)$$

$$= 1.69 \text{ V}$$

Using Eq. (15.21), we obtain

$$V_M = V_t + \frac{V_{DD} - V_t}{\sqrt{r+1}}$$

$$= 0.5 + \frac{2.5 - 0.5}{\sqrt{3.78 + 1}}$$

$$= 1.41 \text{ V}$$

Using Eq. (15.18), we have

$$V_{OH} = V_{DD} = 2.5 \text{ V}$$

Using Eq. (15.24), we obtain

$$V_{OL} = (V_{DD} - V_t)\left[1 - \sqrt{1 - \frac{1}{r}}\right]$$

$$= (2.5 - 0.5)\left[1 - \sqrt{1 - \frac{1}{3.78}}\right]$$

$$= 0.28 \text{ V}$$

Now,

$$NM_H = V_{OH} - V_{IH}$$

$$= 2.5 - 1.69 = 0.81 \text{ V}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 0.97 - 0.28 = 0.69 \text{ V}$$

15.17 From Eq. (15.26), we get

$$NM_L = V_t - (V_{DD} - V_t) \times \left[1 - \sqrt{1 - \frac{1}{r}} - \frac{1}{\sqrt{r(r+1)}}\right]$$

$$\frac{\partial M_L}{\partial r} = -(V_{DD} - V_t) \times \left[-\frac{1}{2}\left(1 - \frac{1}{r}\right)^{-1/2}\left(\frac{1}{r^2}\right) - \left(-\frac{1}{2}\right) \times (r(r+1))^{-3/2}(2r+1)\right]$$

Maximum NM_L is found by setting this equal to zero:

$$0 = -\frac{1}{2}\left(1 - \frac{1}{r}\right)^{-1/2}\left(\frac{1}{r^2}\right) + \frac{1}{2}$$
$$\times\, (r\,(r+1))^{-3/2}\,(2r+1)$$

$$\left(1 - \frac{1}{r}\right)^{-1/2}\left(\frac{1}{r^2}\right) = (r\,(r+1))^{-3/2}\,(2r+1)$$

Squaring both sides, we get

$$\frac{\left(1 - \frac{1}{r}\right)^{-1}}{r^4} = r^{-3}(r+1)^{-3}(2r+1)^2$$

Multiplying both sides by r^4:

$$\frac{1}{1 - \frac{1}{r}} = \frac{r}{(r+1)^3}(2r+1)^2$$

$$\frac{r}{r-1} = \frac{r(2r+1)^2}{(r+1)^3}$$

$$\frac{(r+1)^3}{r-1} = (2r+1)^2$$

Multiplying out:

$$r^3 + 2r^2 + r + r^2 + 2r + 1 = 4r^3 - 3r - 1$$

Simplifying, we get

$$r^3 - r^2 - 2r - 2/3 = 0$$

Solving for the roots of this cubic equation can be done with a calculator or by hand. With these coefficients, there is only one real root:

$r = 2.1$ (by hand)

If $V_{DD} = 1.3$ V and $V_t = 0.4$ V,

$$NM_L = 0.4\text{ V} - (1.3\text{ V} - 0.4\text{ V}) \times$$
$$\left[1 - \sqrt{1 - \frac{1}{2.1}} - \frac{1}{\sqrt{2.1\,(3.1)}}\right]$$

$NM_L = 0.5$ V

For $r = 2.0$, we obtain $NM_L = 0.5$ V

For $r = 5.0$, we have $NM_L = 0.47$ V

For $r = 10$, we have $NM_L = 0.44$ V

We thus conclude that the maximum is a very broad one and that NM_L does not change greatly with r.

15.18 From Eq. (15.27), we obtain

$$NM_H = (V_{DD} - V_t)\left(1 - \frac{2}{\sqrt{3r}}\right)$$

Setting $NM_H = 0$, we have

$$\left(1 - \frac{2}{\sqrt{3r}}\right) = 0$$

Solving, $r = 1.33$

Using Eqs. (15.27) and (15.26),we can find NM_H and NM_L, respectively, versus r:

with $V_{DD} = 1.3$ V and $V_t = 0.4$ V,

$$NM_H = (V_{DD} - V_t)\left(1 - \frac{2}{\sqrt{3r}}\right)$$

$$NM_H = (1.3 - 0.4)\left(1 - \frac{2}{\sqrt{3r}}\right) = \left(0.9 - \frac{1.8}{\sqrt{3r}}\right), \text{(V)}$$

$$NM_L = V_t - (V_{DD} - V_t)$$
$$\times\left[1 - \sqrt{1 - \frac{1}{r}} - \frac{1}{\sqrt{r\,(r+1)}}\right]$$

$$NM_L = 0.4 - 0.9\left[1 - \sqrt{1 - \frac{1}{r}} - \frac{1}{\sqrt{r\,(r+1)}}\right], \text{(V)}$$

Table 1 below gives NM_H and NM_L for r ranging from 2 to 10. Observe that equal values of NM_L and NM_H can be obtained by selecting r between 5 and 6. To obtain a more precise value for r, we iterate to obtain the results shown in Table 2. From the latter table we see that the r value lies between 5.6 and 5.7.

Table 1

r	NM_H (V)	NM_L (V)
2	0.17	0.50
3	0.30	0.49
4	0.38	0.48
5	0.44	0.47
6	0.48	0.46
7	0.51	0.45
8	0.53	0.45
9	0.55	0.44
10	0.57	0.44

Table 2

r	NM_H (V)	NM_L (V)
5.5	0.457	0.465
5.6	0.461	0.464
5.7	0.465	0.463

We can further iterate; however, it seems likely that a good estimate would be

$r = 5.65$

For this value,

$NM_H = 0.463$ V

and

$NM_L = 0.463$ V

15.19 Using Eq. (15.24), we obtain

$$V_{OL} = (V_{DD} - V_t)\left[1 - \sqrt{1 - \frac{1}{r}}\right]$$

$$0.1 = (1.8 - 0.5)\left[1 - \sqrt{1 - \frac{1}{r}}\right]$$

$$\frac{0.1}{1.3} = 1 - \sqrt{1 - \frac{1}{r}}$$

$$\sqrt{1 - \frac{1}{r}} = 1 - \frac{0.1}{1.3} = 0.923$$

$$1 - \frac{1}{r} = 0.852$$

$$\Rightarrow r = 6.76$$

Since

$$r = \frac{k_n}{k_p} = \frac{k_n'(W/L)_n}{k_p'(W/L)_p} = \frac{4(W/L)_n}{1}$$

then

$$(W/L)_n = \frac{6.76}{4} = 1.69$$

Using Eq. (15.26), we obtain

$NM_L =$

$$V_t - (V_{DD} - V_t)\left[1 - \sqrt{1 - \frac{1}{r}} - \frac{1}{\sqrt{r(r+1)}}\right]$$

$$= 0.5 - (1.8 - 0.5)\left[1 - \sqrt{1 - \frac{1}{6.76}} - \frac{1}{\sqrt{6.76 \times 7.76}}\right]$$

$$= 0.58 \text{ V}$$

Using Eq. (15.25), we obtain

$$I_{\text{stat}} = \frac{1}{2}k_p(V_{DD} - V_t)^2$$

$$= \frac{1}{2} \times 0.1 \times 1(1.8 - 0.5)^2$$

$$= 0.0845 \text{ mA}$$

The static power dissipation can be found as

$$P_D = I_{\text{stat}}V_{DD}$$

$$= 0.0845 \times 1.8$$

$$= 0.152 \text{ mW}$$

$$= 152\ \mu\text{W}$$

15.20 Substituting $V_{DD} = 1.3$ V, $V_t = 0.4$ V, and $r = 5.7$ in Eqs. (15.26) and (15.27), we obtain:

$NM_L =$

$$V_t - (V_{DD} - V_t)\left[1 - \sqrt{1 - \frac{1}{r}} - \frac{1}{\sqrt{r(r+1)}}\right]$$

$$= 0.4 - (1.3 - 0.4)\left[1 - \sqrt{1 - \frac{1}{5.7}} - \frac{1}{\sqrt{5.7 \times 6.7}}\right]$$

$$= 0.463 \text{ V}$$

$$NM_H = (V_{DD} - V_t)\left(1 - \frac{2}{\sqrt{3r}}\right)$$

$$= (1.3 - 0.4)\left(1 - \frac{2}{\sqrt{3 \times 5.7}}\right)$$

$$= 0.465 \text{ V}$$

Thus,

$NM_L \simeq NM_H$ Q.E.D.

Since

$$r = \frac{k_n}{k_p} = \frac{k_n'(W/L)_n}{k_p'(W/L)_p}$$

then

$$5.7 = \frac{4(W/L)_n}{(W/L)_p}$$

$$\Rightarrow \frac{(W/L)_n}{(W/L)_p} = \frac{5.7}{4} = 1.425$$

Selecting

$(W/L)_p = 1$

we have

$(W/L)_n = 1.425$

Using Eq. (14.25), we find

$$I_{\text{stat}} = \frac{1}{2}k_p(V_{DD} - V_t)^2$$

$$= \frac{1}{2} \times 125 \times 1(1.3 - 0.4)^2$$

$$= 50.6\ \mu\text{A}$$

The static power dissipation is

$$P_D = I_{\text{stat}}V_{DD}$$

$$= 50.6 \times 1.3 = 65.8\ \mu\text{W}$$

Using Eq. (15.29), we obtain

$$\alpha_p = 2\bigg/\left[\frac{7}{4} - 3\left(\frac{|V_t|}{V_{DD}}\right) + \left(\frac{|V_t|}{V_{DD}}\right)^2\right]$$

$$= 2\bigg/\left[\frac{7}{4} - 3 \times \frac{0.4}{1.3} + \left(\frac{0.4}{1.3}\right)^2\right]$$

$$= 2.17$$

Using Eq. (15.31), we find

$$\alpha_n = 2 \Big/ \left[1 + \frac{3}{4}\left(1 - \frac{1}{5.7}\right) - \left(3 - \frac{1}{5.7}\right)\left(\frac{0.4}{1.3}\right) + \left(\frac{0.4}{1.3}\right)^2\right]$$

$$= 2.37$$

Now using Eqs. (15.28) and (15.30), we obtain

$$\frac{t_{PLH}}{t_{PHL}} = \left(\frac{\alpha_p}{\alpha_n}\right)\left(\frac{k_n}{k_p}\right)$$

$$= \frac{2.17}{2.37} \times 5.7 = 5.22$$

Using Eq. (15.28), we obtain

$$t_{PLH} = \frac{\alpha_p C}{k_p V_{DD}}$$

$$= \frac{2.17 \times 100 \times 10^{-15}}{125 \times 10^{-6} \times 1 \times 1.3} = 1.34 \text{ ns}$$

$$t_{PHL} = \frac{t_{PLH}}{5.22} = 0.26 \text{ ns}$$

$$t_P = \frac{1}{2}(1.34 + 0.26) = 0.8 \text{ ns}$$

$$P_{\text{dyn}} = fCV_{DD}^2$$

$$= f \times 100 \times 10^{-15} \times 1.3^2 = 169f \text{ fW}$$

The frequency at which $P_{\text{dyn}} = P_{\text{stat}}$ is found from

$$169f \times 10^{-15} = 65.8 \times 10^{-6}$$

$$\Rightarrow f = 389 \text{ MHz}$$

The theoretical maximum frequency of operation is found as follows:

$$f_{\max} = \frac{1}{2t_P}$$

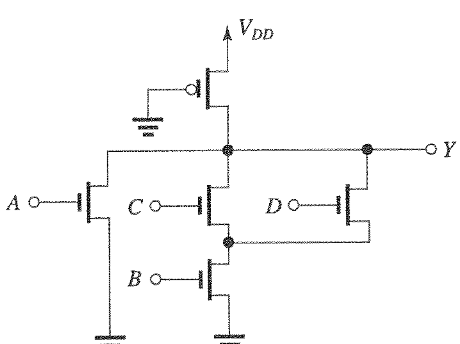
$$= \frac{1}{2 \times 0.8 \times 10^{-9}} = 625 \text{ MHz}$$

Thus, it is possible to operate the inverter at 389 MHz.

15.21 $Y = \overline{A + B(C + D)}$,

whence $\overline{Y} = A + B(C + D)$

Thue the PDN can be formed directly as shown:

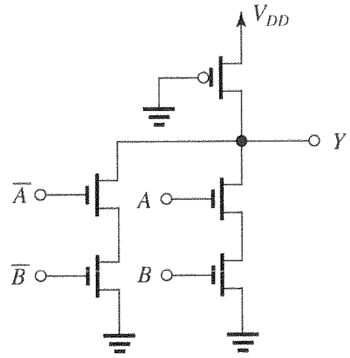


15.22 For an Exclusiue OR, $Y = A\overline{B} + \overline{A}B$, and

$\overline{Y} = \overline{A\overline{B} + \overline{A}B} = \overline{A\overline{B}} \cdot \overline{\overline{A}B} = (\overline{A} + B)(A + \overline{B})$

or $\overline{Y} = \overline{A}\,\overline{B} + AB$

The PDN results directly:



15.23 For a NOR gate similar to that of Fig. 15.13, the worst-case V_{OL} will occur when only one input is high and one NMOS transistor is conducting. From Eq. (15.24), we obtain

$$V_{OL} = (V_{DD} - V_t)\left[1 - \sqrt{1 - \frac{1}{r}}\right]$$

$$0.1 = (1.3 - 0.4)\left[1 - \sqrt{1 - \frac{1}{r}}\right]$$

$$\Rightarrow \sqrt{1 - \frac{1}{r}} = 1 - \frac{0.1}{0.9}$$

$$\Rightarrow r = 4.76$$

Since

$$r = \frac{k_n}{k_p} = \frac{k_n'(W/L)_n}{k_p'(W/L)_p} = \frac{4 \times 1.5}{(W/L)_p}$$

then $\left(\dfrac{W}{L}\right)_p = \dfrac{6}{4.76} = 1.26$

15.24

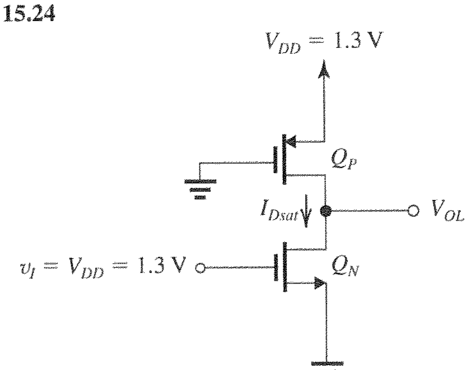


Figure 1

Refer to Fig. 1. For transistor Q_P, we have

$$|V_{SG}| - |V_t| = 1.3 - 0.4 = 0.9 \text{ V}$$

which is greater than $V_{DSsat} = 0.6$ V. Also, since we expect V_{OL} to be very small, we have

$V_{SD} > |V_{DSsat}|$

Thus Q_P will be operating in the velocity saturation region and its current I_{Dsat} will be given by Eq. (15.9) (adapted for the p-channel case),

$$I_{Dsat} = \mu_p C_{ox}\left(\frac{W}{L}\right)_p |V_{DSsat}|\left(|V_{SG}| - |V_t| - \frac{1}{2}|V_{DSsat}|\right)$$

$$= 100 \times 0.6\left(1.3 - 0.4 - \frac{1}{2} \times 0.6\right)$$

$$= 36\ \mu\text{A}$$

Transistor Q_N will be operating in the triode region and conducting an equal current, thus

$$36 = k_n\left[(V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

$$36 = 500\left[(1.3 - 0.4)V_{OL} - \frac{1}{2}V_{OL}^2\right]$$

$$0.072 = 0.9V_{OL} - \frac{1}{2}V_{OL}^2$$

If V_{OL} is very small, then we have

$$0.072 \simeq 0.9V_{OL}$$

$$\Rightarrow V_{OL} = 0.08\text{ V}$$

which is indeed very small, as assumed.

15.25

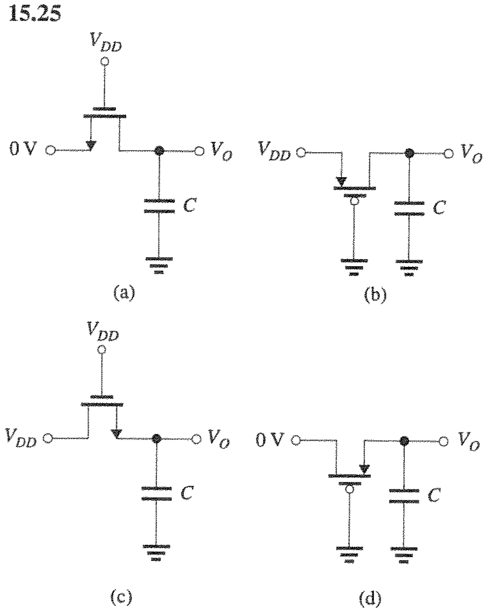


Figure 1

Refer to Fig. 1.

(a) $V_O = 0$ V

(b) $V_O = 0$ V

(c) $V_O = V_{DD} - V_{tn}$

where

$$V_{tn} = V_{tn0} + \gamma\left(\sqrt{V_O + 2\phi_f} - \sqrt{2\phi_f}\right)$$

(d) $V_O = |V_{tp}|$

$$= |V_{tp0}| + \gamma\left(\sqrt{V_{DD} - V_O + 2\phi_f} - \sqrt{2\phi_f}\right)$$

15.26 Refer to Fig. 15.17. Since V_{OH} is the value of v_O at which Q stops conducting,

$$V_{DD} - V_{OH} - V_t = 0$$

then

$$V_{OH} = V_{DD} - V_t$$

where

$$V_t = V_{t0} + \gamma\left(\sqrt{V_{OH} + 2\phi_f} - \sqrt{2\phi_f}\right)$$

$$= V_{t0} + \gamma\left(\sqrt{V_{DD} - V_t + 2\phi_f} - \sqrt{2\phi_f}\right)$$

Substituting $V_{t0} = 0.4$ V, $\gamma = 0.2\ \text{V}^{1/2}$, $V_{DD} = 1.2$ V, and $2\phi_f = 0.88$ V, we obtain

$$V_t = 0.4 + 0.2\left(\sqrt{1.2 - V_t + 0.88} - \sqrt{0.88}\right)$$

$$V_t - 0.4 + 0.2\sqrt{0.88} = 0.2\sqrt{2.08 - V_t}$$

$$(V_t - 0.212)^2 = 0.04(2.08 - V_t)$$

$$\Rightarrow V_t^2 - 0.384\, V_t - 0.038 = 0$$

$$\Rightarrow V_t = 0.466\text{ V}$$

$$V_{OH} = 1.3 - 0.466$$

$$= 0.834\text{ V}$$

15.27 Refer to Fig. 15.17.

We need to find the current i_D at $t = 0$ (where $v_O = 0$, $V_t = V_{t0} = 0.4$ V) and at $t = t_{PLH}$ (where $v_O = \frac{V_{DD}}{2} = 0.6$ V, V_t to be determined), as follows:

$$i_D(0) = \frac{1}{2} \times 500 \times 1.5\ (1.2 - 0.4)^2 = 240\ \mu\text{A}$$

V_t (at $v_O = 0.6$ V) $=$

$$0.4 + 0.2\left(\sqrt{0.6 + 0.88} - \sqrt{0.88}\right)$$

$$= 0.456\text{ V}$$

$$i_D(t_{PLH}) = \frac{1}{2} \times 500 \times 1.5\ (1.2 - 0.456 - 0.6)^2$$

$$= 7.8\ \mu\text{A}$$

We can now compute the average charging current as

$$i_D|_{av} = \frac{240 + 7.8}{2} = 123.9\ \mu\text{A}$$

and t_{PLH} can be found as

$$t_{PLH} = \frac{C(V_{DD}/2)}{i_D|_{av}}$$

$$= \frac{10 \times 10^{-15} \times 0.6}{123.9 \times 10^{-6}} = 48.4 \text{ ps}$$

15.28 Refer to the circuit in Fig. 15.18. Observe that, here, V_t remains constant at $V_{t0} = 0.4$ V. At $t = 0$, Q will be operating in saturation, and the drain current will be

$$i_D(0) = \frac{1}{2} \times 500 \times 1.5\ (1.2 - 0.4)^2$$

$$= 240\ \mu\text{A}$$

At $t = t_{PHL}$, Q will be operating in the triode region, and thus

$$i_D(t = t_{PHL}) =$$

$$500 \times 1.5\left[(1.2 - 0.4) \times 0.6 - \frac{1}{2} \times 0.6^2\right]$$

$$= 225\ \mu\text{A}$$

Thus, the average discharge current is given by

$$i_D|_{av} = \frac{1}{2}(240 + 225) = 232.5\ \mu\text{A}$$

and t_{PHL} can be determined as

$$t_{PHL} = \frac{C(V_{DD}/2)}{i_D|_{av}}$$

$$= \frac{10 \times 10^{-15} \times 0.6}{232.5 \times 10^{-6}} = 25.8 \text{ ps}$$

15.29

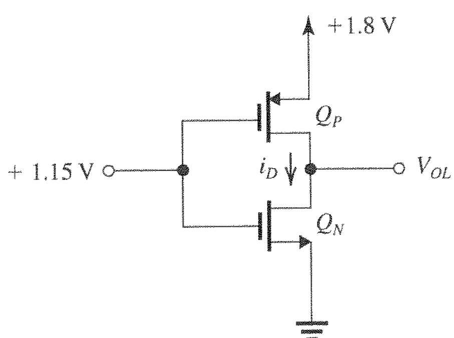


Figure 1

From Exercise 15.8 we have the output voltage of the switch as 1.15 V. This is the voltage applied to the input of the CMOS inverter, as shown in Fig. 1. Because we expect V_{OL} to be small, Q_P will be operating in the saturation region and its drain current will be

$$i_D = \frac{1}{2}(\mu_p C_{ox})\left(\frac{W}{L}\right)_p (1.8 - 1.15 - 0.4)^2$$

$$= \frac{1}{2} \times 75 \times \frac{0.54}{0.18} \times 0.0625$$

$$= 7\ \mu\text{A}$$

This is also the current conducted by Q_N which is operating in the triode region, thus

$$7 = (\mu_n C_{ox})\left(\frac{W}{L}\right)_n \left[(1.15 - 0.4)V_{OL} - \frac{1}{2}V_{OL}^2\right]$$

Assuming V_{OL} to be very small, we obtain

$$7 \simeq (\mu_n C_{ox})\left(\frac{W}{L}\right)_n \times 0.75 V_{OL}$$

$$7 = 300 \times \frac{1}{2} \times \frac{0.54}{0.18} \times 0.75\ V_{OL}$$

$$\Rightarrow V_{OL} = 0.02 \text{ V}$$

The static power dissipation is

$$P_D = 7\ \mu\text{A} \times 1.8 \text{ V}$$

$$= 12.6\ \mu\text{W}$$

15.30 For the switch gate and input both at $V_{DD} = 3.3$V, the switch output is

$$V_{OH} = V_{DD} - V_t$$

where $V_t = V_{t0} + \gamma\left[\sqrt{V_{OH} + 2\phi_f} - \sqrt{2\phi_f}\right]$

Substituting for V_{OH}, we get

$$V_t = V_{t0} + \gamma\left[\sqrt{V_{DD} - V_t + 2\phi_f} - \sqrt{2\phi_f}\right]$$

$$= 0.8 + 0.5\left[\sqrt{3.3 - V_t + 0.6} - \sqrt{0.6}\right]$$

so that

$$V_t = 0.413 + 0.5\sqrt{3.9 - V_t}$$

$$V_t - 0.413 = 0.5\sqrt{3.9 - V_t}$$

Squaring both sides, we get

$$V_t^2 - 0.826V_t + 0.171 = 0.975 - 0.25V_t$$

$$\text{or } V_t^2 - 0.576V_t - 0.804 = 0$$

Solving this quadratic, we find that

$$V_t = 1.23 \text{ V}$$

$$V_{OH} = V_{DD} - V_t = 3.3 \text{ V} - 1.23 \text{ V} = 2.07 \text{ V}$$

With the input low and the switch gate high, $V_{OL} \to 0$ V

If $V_{OH} = 2.07$ V, the PMOS transistor of the inverter is in the saturation region. Since the inverter transistors are matched, we get

$$\left(\frac{W}{L}\right)_p = \frac{k_n'}{k_p'}\left(\frac{W}{L}\right)_n \text{ so that}$$

$$i_{DP} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p (V_{DD} - V_{OH} - V_{t0})^2$$

$$i_{DP} = \frac{1}{2}(25)\left(\frac{1.2}{0.8}\right)(3) \times (3.3 - 2.07 - 0.8)^2$$

$$= 10.4\ \mu\text{A}$$

For t_{PLH}, at $t = 0$, we have

$$i_D(0) = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{DD} - V_{t0})^2$$

$$= \frac{1}{2}(75)\left(\frac{1.2}{0.8}\right)(3.3 - 0.8)^2 = 352\ \mu\text{A}$$

At $v_O = \frac{V_{DD}}{2}$, we have

$$V_t = V_{t0} + \gamma\left[\sqrt{\frac{V_{DD}}{2} + 2\phi_f} - \sqrt{2\phi_f}\right]$$

$$= 0.8\text{ V} + 0.5\left[\sqrt{\frac{3.3}{2} + 0.6} - \sqrt{0.6}\right]$$

$$= 1.16\text{ V}$$

$$i_D(t_{PLH}) = \frac{1}{2}\mu_n C_{OX}\left(\frac{W}{L}\right)_n (V_{DD} - v_O - V_t)^2$$

$$= \frac{1}{2}(75)\left(\frac{1.2}{0.8}\right)\left(3.3 - \frac{3.3}{2} - 1.16\right)^2$$

$$= 13.5\ \mu\text{A}$$

$$i_D|_{av} = \frac{(352\ \mu\text{A} + 13.5\ \mu\text{A})}{2} = 183\ \mu\text{A}$$

$$t_{PLH} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_D|_{av}} = \frac{100(10^{-15})_f(1.65)}{183 \times 10^{-6}}$$

$$= 0.9\text{ ns}$$

For t_{PHL}, $V_t = V_{t0}$ and

$$i_{D(0)} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{DD} - V_{t0})^2$$

$$= \frac{1}{2}(75)\left(\frac{1.2}{0.8}\right)(3.3 - 0.8)^2 = 352\ \mu\text{A}$$

$$i_D(t_{PHL}) = \mu_n C_{ox}\left(\frac{W}{L}\right)_n \left[(V_{DD} - V_{t0})v_O - \frac{1}{2}{v_O}^2\right]$$

$$= 75\left(\frac{1.2}{0.8}\right) \times \left[(3.3 - 0.8)\left(\frac{3.3}{2}\right) - \frac{1}{2}\left(\frac{3.3}{2}\right)^2\right]$$

$$= 311\ \mu\text{A}$$

$$i_D|_{av} = \frac{1}{2}(352 + 311) = 332\ \mu\text{A}$$

$$t_{PHL} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_D|_{av}} = \frac{100(10^{-15})(1.65)}{332 \times 10^{-6}}$$

$$= 0.5\text{ ns}$$

15.31

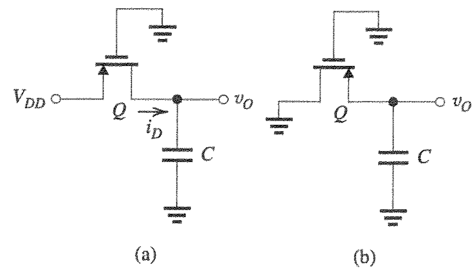


Figure 1

(a) Figure 1(a) shows the situation for $t \geq 0$. In this case V_t remains constant at V_{t0} and the capacitor charges to V_{DD}.

Thus,

$$V_{OH} = V_{DD}$$

(b) Figure 1 (b) shows the situation for $t \geq 0$. Here as C discharges through Q, the threshold voltage changes. The discharge current reduces to zero when $v_O = |V_t|$. Thus,

$$V_{OL} = |V_t|$$

where

$$|V_t| = |V_{t0}| + \gamma\left[\sqrt{V_{DD} - V_{OL} + 2\phi_f} - \sqrt{2\phi_f}\right]$$

Substituting $|V_t| = V_{OL}$, we obtain

$$V_{OL} = |V_{t0}| + \gamma\left[\sqrt{V_{DD} - V_{OL} + 2\phi_f} - \sqrt{2\phi_f}\right]$$

(c) Refer to Fig. 1(a).

At $t = 0$, $v_O = 0$ and Q will be operating in saturation. Thus,

$$i_D(0) = \frac{1}{2}k_p(V_{DD} - |V_{t0}|)^2$$

$$= \frac{1}{2} \times 125\ (1.2 - 0.4)^2$$

$$= 40\ \mu\text{A}$$

At $t = t_{PLH}$, $v_O = V_{DD}/2 = 0.6$ V and Q will be operating in the triode region. Thus,

$$i_D(t_{PLH}) =$$

$$k_p\left[(V_{DD} - |V_{t0}|)\left(\frac{V_{DD}}{2}\right) - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$= 125\left[(1.2 - 0.4) \times 0.6 - \frac{1}{2}(0.6)^2\right]$$

$$= 37.5\ \mu\text{A}$$

The average charging current can now be found as

$$i_D|_{av} = \frac{1}{2}(40 + 37.5) = 33.75\ \mu\text{A}$$

The propagation delay t_{PLH} can then be determined from

$$t_{PLH} = \frac{C(V_{DD}/2)}{i_D|_{av}}$$

$$= \frac{C \times 0.6}{33.75 \times 10^{-6}} = 17.8 \times 10^3 C \text{ s}$$

No value for C is given. If $C = 10$ fF, we get

$$t_{PLH} = 17.8 \times 10^3 \times 10 \times 10^{-15} = 178 \text{ ps}$$

15.32 For (a) see directly that $X = 1.\overline{A} = \overline{A}$

and $Y = X \cdot \overline{B} = \overline{A} \cdot \overline{B}$.

For (b) see directly that $Y = \overline{A} \cdot \overline{B}$.

For each circuit node Y nominally satisfies both conditions. However, in (a) with A high and B low, Y is not pulled down completely to ground, but remains at $|V_{tp}|$, due to the PMOS threshold. Circuit (b) does not have this problem, but node X is floating for A, B both high. However, X is not an output node. Notice that (b) is exactly a CMOS NOR gate for which $Y = \overline{A} \cdot \overline{B} = \overline{A + B}$.

For V_{DD} replaced by an inverter driven by C,

$$Y = \overline{C}\left(\overline{A} \cdot \overline{B}\right) = \overline{A} \cdot \overline{B} \cdot \overline{C} = \overline{A + B + C},$$

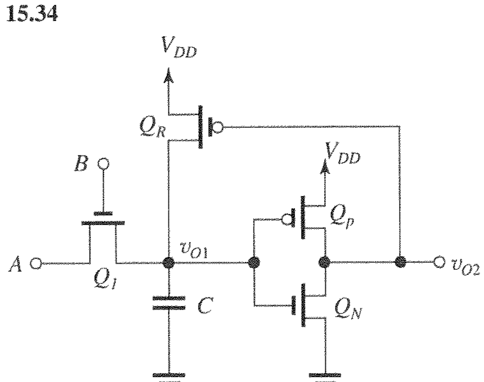
a 3-input NOR for both (a) and (b).

15.33 With these reversals, the output is high when B is low or when A is low.

Thus, $Y = \overline{A} + \overline{B}$ or $Y = \overline{A \cdot B}$, a NAND function.

Circuit (b), being a fully complementary CMOS gate, functions ideally. However, Circuit (a) provides a relatively low high output for A low with B high. In this case, the output becomes $V_{DD} - V_{tn}$, where V_{tn} is raised from V_{t0} by the body effect.

15.34



For the inverter, with
$v_{O2} = V_{DD} - |V_{t0}| = 3.3 \text{ V} - 0.8 \text{ V} = 2.5 \text{ V}$

Q_N is in the saturation region, so that

$$i_{DN} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_n (v_{O1} - v_{t0})^2$$

$$= \frac{1}{2}(75)\left(\frac{1.2}{0.8}\right)(v_{O1} - 0.8)^2$$

$$= 56.25(v_{O1} - 0.8 \text{ V})^2 \ \mu\text{A}$$

Q_P is operating in the triode region so

$$i_{DP} = k_P' \left(\frac{W}{L}\right)_P \left[(v_{DD} - v_{O1} - V_{t0})(v_{SD}) - \frac{1}{2}(v_{SD})^2\right]$$

$$= (25)\left(\frac{3.6}{0.8}\right) \times \left[(3.3 - v_{O1} - 0.8)(0.8) - \frac{1}{2}(0.8)^2\right]$$

$$= 112.5[1.68 - 0.8v_{O1}] \ \mu\text{A}$$

Since $i_{DP} = i_{DN}$, we set these equal:

$$56.25(v_{O1} - 0.8)^2 = 189 - 90v_{O1}$$

$$56.25\left(v_{O1}^2 - 1.6v_{O1} + 0.64\right) = 189 - 90v_{O1}$$

Simplifying, we get

$$v_{O1}^2 - 2.72 = 0$$

$$v_{O1} = \sqrt{2.72} = 1.65 \text{ V} = \frac{V_{DD}}{2} \qquad \text{Q.E.D.}$$

For Q_1, we have

$$V_t = V_{t0} + \gamma\left[\sqrt{V_{O1} + 2\phi_f} - \sqrt{2\phi_f}\right]$$

$$V_t = 0.8 + 0.5\left[\sqrt{1.65 + 0.6} - \sqrt{0.6}\right]$$

$$= 1.16 \text{ V}$$

Capacitor charging current before Q_R turns on is due to the current supplied by Q_1.

At $v_{O1} = V_{DD}/2$:

$$i_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_1 (V_{DD} - v_{O1} - V_t)^2$$

$$= \frac{1}{2}(75)\left(\frac{1.2}{0.8}\right)(3.3 - 1.65 - 1.16)^2$$

$$= 13.5 \ \mu\text{A}$$

At $v_{O1} = 0$ V, we have

$$i_D = \frac{1}{2} k_n \left(\frac{W}{L}\right)_1 (V_{DD} - V_{t0})^2$$

$$= \frac{1}{2}(75)\left(\frac{1.2}{0.8}\right)(3.3 - 0.8)^2 = 351.6 \ \mu\text{A}$$

$$i_D|_{av} = \frac{1}{2}(13.5 \ \mu\text{A} + 351.6 \ \mu\text{A}) = 182.6 \ \mu\text{A}$$

$$t_{PLH} = \frac{C v_{O1}}{i_D|_{av}} = \frac{20\left(10^{-15}\right)(1.65)}{182.6\left(10^{-6}\right)} = 0.18 \text{ ns}$$

15.35

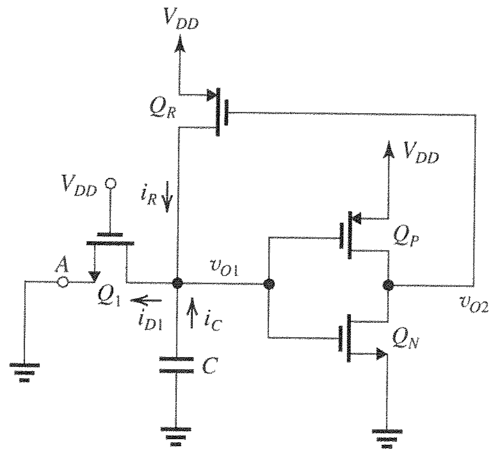


Figure 1

Refer to Fig. 1. At $t = 0$, $v_{O1} = V_{DD}$, $v_{O2} = 0$ V, Q_R is conducting but $i_R = 0$, and Q_1 is operating in the saturation region with i_{D1} given by

$$i_{D1} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_1 (V_{DD} - V_{t0})^2$$

$$= \frac{1}{2} \times 75 \times \frac{1.2}{0.8} \times (3.3 - 0.8)^2$$

$$= 352\ \mu\text{A}$$

The discharge current i_C is given by

$$i_C = i_{D1} - i_R$$

$$= 352 - 0 = 352\ \mu\text{A}$$

The inverter begins to switch when v_{O1} is reduced from V_{DD} to the value V_{IH},

$$V_{IH} = \frac{1}{8}(5V_{DD} - 2V_t)$$

$$= \frac{1}{8}(5 \times 3.3 - 2 \times 0.8) = 1.8625\text{ V}$$

At this value of v_{O1}, Q_1 will be operating in the triode region and its current will be

$$i_{D1} = k_n'\left(\frac{W}{L}\right)_1 \left[(V_{DD} - V_{t0})v_{O1} - \frac{1}{2}v_{O1}^2\right]$$

$$= 75 \times \frac{1.2}{0.8}\left[(3.3 - 0.8)1.8625 - \frac{1}{2} \times 1.8625^2\right]$$

$$= 328.7\ \mu\text{A}$$

The maximum current that Q_R conducts occurs when it is operating in saturation (which does not occur during the discharge process examined here). The maximum drain current of Q_R is given by

$$i_{DR} = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p (V_{DD} - V_{t0})^2$$

Limiting this current to half the value of i_{D1} found above, we obtain

$$\frac{1}{2} \times 328.7 = \frac{1}{2} \times 25 \times \left(\frac{W}{L}\right)_R \times (3.3 - 0.8)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_R = 2.1$$

The current conducted by Q_R at $v_{O1} = V_{IH} = 1.8625\text{ V} \simeq 1.86$ V is

$i_R =$

$$25 \times 2.1\left[(3.3 - 0.8)(3.3 - 1.86) - \frac{1}{2}(3.3 - 1.86)^2\right]$$

$$= 134.6\ \mu\text{A}$$

The discharge current i_C at this time is given by

$$i_C = 328.7 - 134.6 = 194.1\ \mu\text{A}$$

The average value of the discharge current i_C is given by

$$i_C|_{av} = \frac{1}{2}(352 + 194.1)$$

$$= 273\ \mu\text{A}$$

and t_{PHL} will be

$$t_{PHL} = \frac{20 \times 10^{-15}(3.3 - 1.86)}{273 \times 10^{-6}}$$

$$= 106\text{ ps}$$

15.36 (a) $V_{OH} = V_{DD} = 1.2$ V

$V_{OL} = 0$ V

(b) Refer to Fig. 15.21(a).

$$i_{DN}(0) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n (V_{DD} - V_{t0})^2$$

$$= \frac{1}{2} \times 500 \times 1.5(1.2 - 0.4)^2$$

$$= 240\ \mu\text{A}$$

$$i_{DP}(0) = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p (V_{DD} - V_{t0})^2$$

$$= \frac{1}{2} \times 125 \times 1.5(1.2 - 0.4)^2$$

$$= 60\ \mu\text{A}$$

The capacitor charging current $i_C(0)$ is

$$i_C(0) = i_{DN}(0) + i_{DP}(0)$$

$$= 240 + 60 = 300\ \mu\text{A}$$

The current $i_{DN}(t_{PLH})$ is the value of i_D at $v_O = V_{DD}/2 = 0.6$ V. At this point,

$$V_{tn} = V_{t0} + \gamma\left(\sqrt{v_O + 2\phi_f} - \sqrt{2\phi_f}\right)$$

$$= 0.4 + 0.2\left(\sqrt{0.6 + 0.88} - \sqrt{0.88}\right)$$

$$= 0.456\text{ V}$$

Now, $i_{DN}(t_{PLH})$ can be found as

$$i_{DN}(t_{PLH}) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n\left(\frac{V_{DD}}{2} - V_{tn}\right)^2$$

$$= \frac{1}{2} \times 500 \times 1.5(0.6 - 0.456)^2$$

$$= 7.8\ \mu\text{A}$$

To find $i_{DP}(t_{PLH})$, we note that Q_P will be operating in the triode region, thus

$$i_{DP}(t_{PLH}) = k_p'\left(\frac{W}{L}\right)_p\left[(V_{DD} - V_{t0})\left(\frac{V_{DD}}{2}\right) - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$= 125 \times 1.5\left[(1.2 - 0.4)(0.6) - \frac{1}{2}(0.6)^2\right]$$

$$= 56.25\ \mu\text{A}$$

The capacitor charging current at $t = t_{PLH}$ will be

$$i_C(t_{PLH}) = i_{DN}(t_{PLH}) + i_{DP}(t_{PLH})$$

$$= 7.8 + 56.25 = 64.1\ \mu\text{A}$$

The average charging current is

$$i_C|_{av} = \frac{1}{2}[i_C(0) + i_C(t_{PLH})]$$

$$= \frac{1}{2}(300 + 64.1)$$

$$= 182.1\ \mu\text{A}$$

Finally, t_{PLH} can be found as

$$t_{PLH} = \frac{C(V_{DD}/2)}{i_C|_{av}}$$

$$= \frac{15 \times 10^{-15} \times 0.6}{182.1 \times 10^{-6}}$$

$$= 49.4\ \text{ps}$$

(c) Refer to Fig. 15.21(b).

$$i_{DN}(0) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n(V_{DD} - V_{t0})^2$$

$$= \frac{1}{2} \times 500 \times 1.5\ (1.2 - 0.4)^2$$

$$= 240\ \mu\text{A}$$

$$i_{DP}(0) = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p(V_{DD} - V_{t0})^2$$

$$= \frac{1}{2} \times 125 \times 1.5\ (1.2 - 0.4)^2$$

$$= 60\ \mu\text{A}$$

The capacitor discharge current $i_C(0)$ can now be found as

$$i_C(0) = i_{DN}(0) + i_{DP}(0)$$

$$= 240 + 60$$

$$= 300\ \mu\text{A}$$

At $t = t_{PHL}$, $v_O = V_{DD}/2 = 0.6$ V, and Q_N will be operating in the triode region, thus

$$i_{DN}(t_{PLH}) = k_n'\left(\frac{W}{L}\right)_n\left[(V_{DD} - V_{t0})\left(\frac{V_{DD}}{2}\right) - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$= 500 \times 1.5\left[(1.2 - 0.4)\ 0.6 - \frac{1}{2}(0.6)^2\right]$$

$$= 225\ \mu\text{A}$$

At $t = t_{PHL}$, $v_O = V_{DD}/2 = 0.6$ V, and Q_P will be operating in the saturation region but will have a threshold voltage $|V_{tp}|$ determined as follows:

$$|V_{tp}| = V_{t0} + \gamma\left[\sqrt{\frac{1}{2}V_{DD} + 2\phi_f} - \sqrt{2\phi_f}\right]$$

$$= 0.4 + 0.2\left[\sqrt{0.6 + 0.88} - \sqrt{0.88}\right]$$

$$= 0.456\ \text{V}$$

$$i_{DP}(t_{PHL}) = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p\left(\frac{V_{DD}}{2} - |V_{tp}|\right)^2$$

$$= \frac{1}{2} \times 125 \times 1.5\ (0.6 - 0.456)^2$$

$$= 1.9\ \mu\text{A}$$

The capacitor discharge current at $t = t_{PHL}$ can now be determined as

$$i_C(t_{PHL}) = i_{DN}(t_{PHL}) + i_{DP}(t_{PHL})$$

$$= 225 + 1.9 \simeq 227\ \mu\text{A}$$

The average capacitor discharge current can now be found as

$$i_C|_{av} = \frac{1}{2}(300 + 227) = 263.5\ \mu\text{A}$$

and, finally, t_{PHL} can be determined from

$$t_{PHL} = \frac{C(V_{DD}/2)}{i_C|_{av}}$$

$$= \frac{15 \times 10^{-15} \times 0.6}{263.5 \times 10^{-6}} = 34.2\ \text{ps}$$

Transistor Q_p will turn off when $v_O = |V_{tp}|$ where

$$|V_{tp}| = V_{t0} + \gamma\left[\sqrt{V_{DD} - |V_{tp}| + 2\phi_f} - \sqrt{2\phi_f}\right]$$

$$= 0.4 + 0.2\left[\sqrt{1.2 - |V_{tp}| + 0.88} - \sqrt{0.88}\right]$$

$$\Rightarrow |V_{tp}| + 0.2\sqrt{0.88} - 0.4 = 0.2\sqrt{2.08 - |V_{tp}|}$$

Squaring both sides and collecting terms results in the quadratic equation

$$|V_{tp}|^2 - 0.384|V_{tp}| - 0.0382 = 0$$

whose solution is

$$|V_{tp}| = 0.466\ \text{V}$$

This is the value of v_O at which Q_P stops conducting.

(d) $t_P = \frac{1}{2}(t_{PLH} + t_{PHL})$

$= \frac{1}{2}(49.4 + 34.2) = 41.8$ ps

15.37 Using Eq. (15.42), we have

$R_{Neq} = \dfrac{V_{DD} - v_O}{\frac{1}{2}k_n(V_{DD} - V_{tn} - v_O)^2}$, for $v_O \le$

$V_{DD} - V_{tn}$

For $v_O = 0$ V,

$$R_{Neq}(0) = \frac{1.2 - 0}{\frac{1}{2} \times 0.5 \times 1.5\ (1.2 - 0.4 - 0)^2}$$

$= 5$ kΩ

For $v_O = 0.6$ V,

$$R_{Neq}(0.6\text{ V}) = \frac{1.2 - 0.6}{\frac{1}{2} \times 0.5 \times 1.5\ (1.2 - 0.4 - 0.6)^2}$$

$= 40$ kΩ

Using Eq. (15.46),

$R_{Peq} = \dfrac{V_{DD} - v_O}{\frac{1}{2}k_p(V_{DD} - |V_{tp}|)^2}$, for $v_O \le |V_{tp}|$

we obtain for $v_O = 0$ V,

$$R_{Peq}(0) = \frac{1.2 - 0}{\frac{1}{2} \times 0.125 \times 1.5\ (1.2 - 0.4)^2}$$

$= 20$ kΩ

Using Eq. (15.47),

$$R_{Peq} = \frac{1}{k_p\left[V_{DD} - |V_{tp}| - \frac{1}{2}(V_{DD} - v_O)\right]},$$

for $v_O \ge |V_{tp}|$

we obtain at $v_O = 0.6$ V,

$R_{Peq}(0.6\text{ V}) =$

$$\frac{1}{0.125 \times 1.5\left[1.2 - 0.4 - \frac{1}{2}(1.2 - 0.6)\right]}$$

$= 10.7$ kΩ

Thus, at $v_O = 0$ V,

$R_{TG}(0) = R_{Neq}(0) \parallel R_{Peq}(0)$

$= 5 \parallel 20 = 4$ kΩ

$R_{TG}(0.6\text{ V}) = R_{Neq}(0.6\text{ V}) \parallel R_{Peq}(0.6\text{ V})$

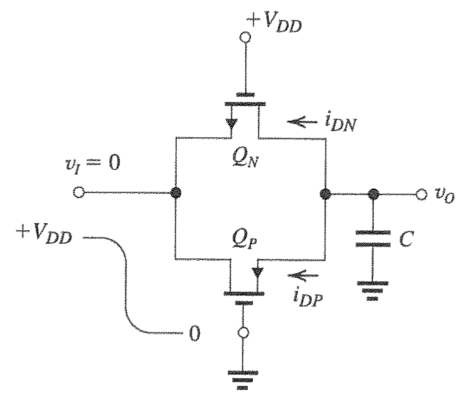
$= 40 \parallel 10.7 = 8.42$ kΩ

$R_{TG}|_{av} = \dfrac{4 + 8.42}{2} = 6.21$ kΩ

$t_{PLH} = 0.69 C R_{TG}|_{av}$

$= 0.69 \times 15 \times 10^{-15} \times 6.21 \times 10^3 = 64.3$ ps

15.38



Initially, C is charged so that $v_O = V_{DD}$.

When v_I goes low to 0 V, Q_N is initially in the saturation region with

$i_{DN} = \frac{1}{2}k_n(V_{DD} - V_{t0})^2$

until $v_{DSN} = v_{GS} - V_{tn} = V_{DD} - V_{tn}$

$$R_{Neq} = \frac{v_O - 0}{\frac{1}{2}k_n(V_{DD} - V_{t0})^2} = \frac{2v_O}{k_n(V_{DD} - V_{t0})^2}$$

for $v_O \ge V_{DD} - V_{tn}$

When Q_N enters the triode region, we get

$$i_{DN} = k_n\left[(V_{DD} - V_{t0})\,v_O - \frac{1}{2}v_O^2\right]$$

for $v_O \le V_{DD} - V_{t0}$.

Then, $R_{Neq} = \dfrac{v_O}{k_n\left[(V_{DD} - V_{t0})\,v_O - \frac{1}{2}v_O^2\right]}$

$$= \frac{1}{k_n\left[(V_{DD} - V_{t0}) - \frac{1}{2}v_O\right]}$$

For Q_P, initially,

$i_{DP} = \frac{1}{2}k_p\left(v_o - |V_{tp}|\right)^2$ so that

$R_{Peq} = \dfrac{2v_O}{k_p\left(v_O - |V_{tp}|\right)^2}$, for $v_O \ge |V_{tp}|$

where we will neglect the body effect of $|V_{tp}|$.

For $v_O \le |V_{tp}|$,

$i_{DP} = 0$

and

$R_{Peq} = \infty$

Now, for the process technology specified in Problem 15.36, we have the following:

At $v_O = V_{DD}$,

$$R_{Neq} = \frac{2V_{DD}}{k_n(V_{DD} - V_{t0})^2}$$

$$= \frac{2 \times 1.2}{0.5 \times 1(1.2 - 0.4)^2} = 7.5\text{ k}\Omega$$

$$R_{Peq} = \frac{2V_{DD}}{k_p(V_{DD} - |V_{tp}|)^2}$$

$$= \frac{2 \times 1.2}{0.125 \times 1(1.2 - 0.4)^2} = 30\ \text{k}\Omega$$

$R_{TG}(v_O = V_{DD}) = 7.5\ \text{k}\Omega \parallel 30\ \text{k}\Omega$

$= 6\ \text{k}\Omega$

At $v_O = \dfrac{V_{DD}}{2}$,

$$R_{Neq} = \frac{1}{k_n\left[V_{DD} - V_{t0} - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)\right]}$$

$$= \frac{1}{0.5 \times 1(1.2 - 0.4 - 0.3)}$$

$= 4\ \text{k}\Omega$

$$R_{Peq} = \frac{2(V_{DD}/2)}{k_p\left(\frac{V_{DD}}{2} - |V_{tp}|\right)^2}$$

$$= \frac{2 \times 0.6}{0.125 \times 1(0.6 - 0.4)^2}$$

$= 240\ \text{k}\Omega$

Thus,

$$R_{TG}\left(v_O = \frac{V_{DD}}{2}\right) = 4\ \text{k}\Omega \parallel 240\ \text{k}\Omega$$

$= 3.9\ \text{k}\Omega$

$$R_{TG}|_{av} = \frac{6 + 3.9}{2} = 4.95\ \text{k}\Omega$$

$t_{PHL} = 0.69CR_{TG}|_{av}$

$= 0.69 \times 15 \times 10^{-15} \times 4.95 \times 10^3 = 51.2\ \text{ps}$

15.39 $R_{TG} \simeq \dfrac{12.5}{(W/L)_n}\ \text{k}\Omega$

$$= \frac{12.5}{1.5} = 8.3\ \text{k}\Omega$$

$t_P = 0.69CR$ (step input)

$t_P = 0.69 \times 10 \times 10^{-15} \times 8.3 \times 10^3 = 57.3\ \text{ps}$

or

$t_P = CR$ (for ramp input)

$= 10 \times 10^{-15} \times 8.3 \times 10^3 = 83\ \text{ps}$

15.40 $t_P = 0.69 \displaystyle\sum_{k=0}^{n} kCR_{TG}$

$$= 0.69CR_{TG}\frac{n(n+1)}{2} \text{ (for a step input)}$$

Substituting $C = 10$ fF, $R_{TG} = 10\ \text{k}\Omega$ and $n = 16$,

$$t_P = 0.69 \times 10 \times 10^{-15} \times 10 \times 10^3 \times \frac{16 \times 17}{2}$$

$= 9.4$ ns (for a step input)

For a ramp input, we have

$$t_P = CR_{TG}\,\frac{n(n+1)}{2}$$

$$= 10 \times 10^{-15} \times 10 \times 10^3 \times \frac{16 \times 17}{2}$$

$= 13.6$ ns (ramp input)

15.41 (a) Need $\overline{Y} = AB + \overline{A}\overline{B}$. In direct analogy to Fig. 15.17:

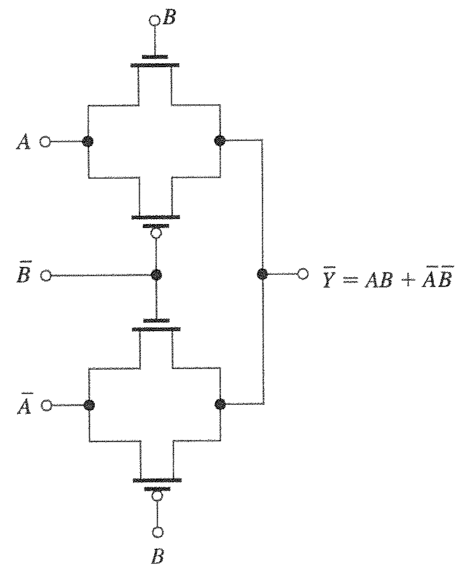


Figure 1

(b) Need $Z = \overline{Y}C + Y\overline{C}$, where

$Y = \overline{\overline{Y}} = \left(\overline{AB + \overline{A}\,\overline{B}}\right)$, hence

$$Y = \overline{AB} \cdot \overline{\overline{A}\,\overline{B}} = (A + B)\left(\overline{A} + \overline{B}\right) = A\overline{B} + \overline{A}B$$

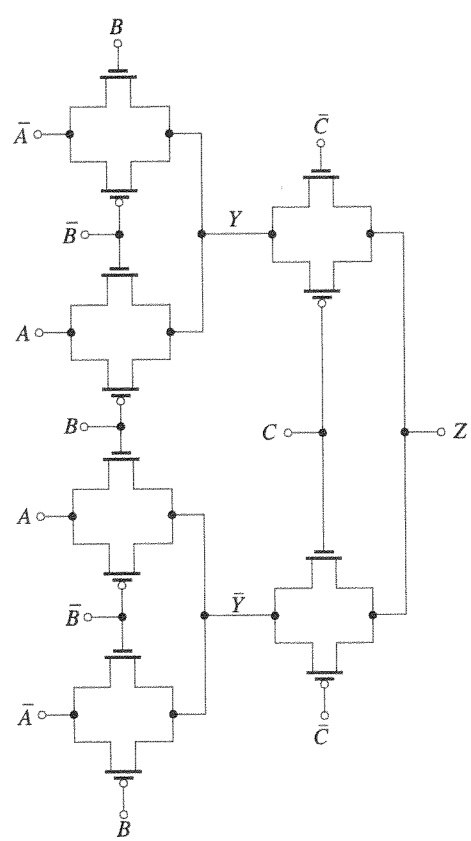
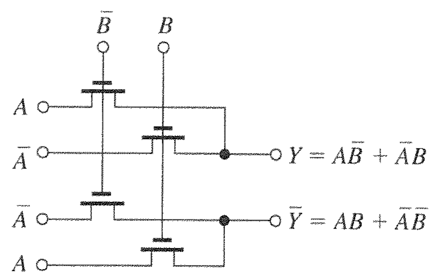
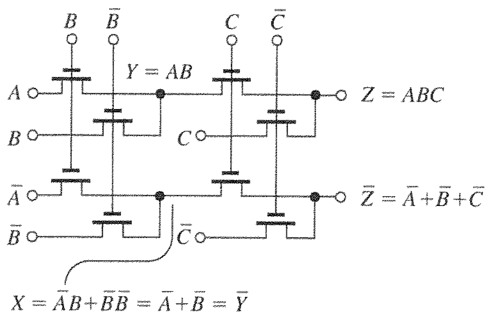


Figure 2

15.42 Need a CPL circuit for $\overline{Y} = A\overline{B} + \overline{A}B$ and $Y = AB + \overline{A}\,\overline{B}$. See Exercise 15.8b.



15.43

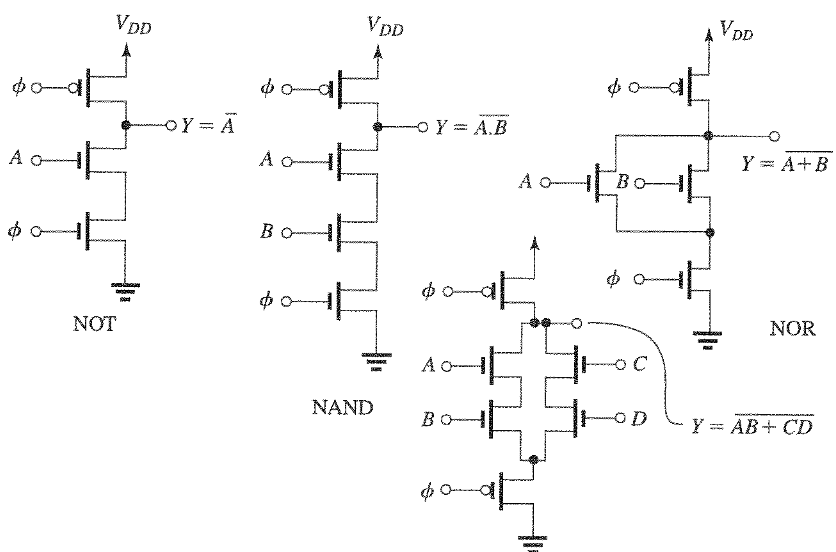


Require a CPL for $Z = ABC$ and $\overline{Z} = \overline{ABC} = \overline{A} + \overline{B} + \overline{C}$

Extend Fig. 15.26 to 3 variables by dealing in pairs, creating $Y = AB$, then $Z = YC$ with

$\overline{Y} = \overline{A} + \overline{B}$, then $\overline{Z} = \overline{Y} + \overline{C}$

15.44



15.45

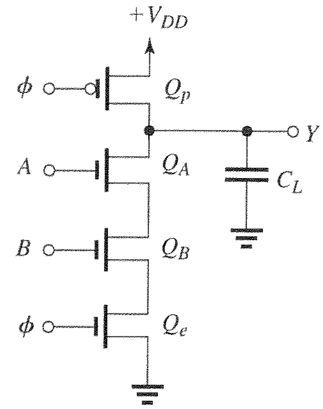


Figure 1

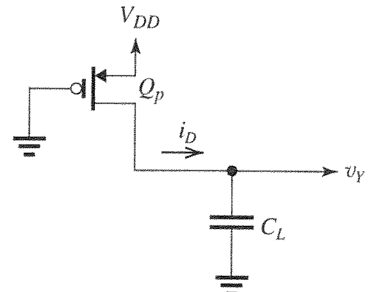


Figure 2

Figure 2 shows the circuit during the precharge interval. To determine the rise time of v_Y, that is, the time for v_Y to increase from $0.1\ V_{DD} = 0.12$ V to $0.9\ V_{DD} = 1.08$ V, we find the value of i_D at each of these two points. At $v_Y = 0.12$ V, Q_P will be operating in saturation, and

$$i_D(0.12\text{ V}) = \frac{1}{2}k'_p\left(\frac{W}{L}\right)_p(V_{DD} - |V_{tp}|)^2$$

$$= \frac{1}{2} \times 125 \times 3\ (1.2 - 0.4)^2$$

$$= 120\ \mu\text{A}$$

At $v_Y = 1.08$ V, Q_P will be operating in the triode region, and

$$i_D(1.08\text{ V}) =$$

$$k'_p\left(\frac{W}{L}\right)_p\left[(V_{DD} - |V_{tp}|)(V_{DD} - v_Y) - \frac{1}{2}(V_{DD} - v_Y)^2\right]$$

$$= 125 \times 3\left[(1.2 - 0.4)(1.2 - 1.08)\right.$$

$$\left.-\frac{1}{2}(1.2 - 1.08)^2\right]$$

$$= 33.3\ \mu\text{A}$$

Thus, the average charging current is

$$I_{av} = \frac{1}{2}[i_D(0.12\text{ V}) + i_D(1.08\text{ V})]$$

$$= \frac{1}{2}(120 + 33.3) = 76.7\ \mu\text{A}$$

The rise time t_r can now be found from

$$t_r = \frac{C_L(0.9V_{DD} - 0.1V_{DD})}{I_{av}}$$

$$= \frac{15 \times 10^{-15} \times 0.8 \times 1.2}{76.7 \times 10^{-6}} = 0.188\text{ ns}$$

15.46 The gate is shown in Fig. 1 in the solution to Problem 15.45.

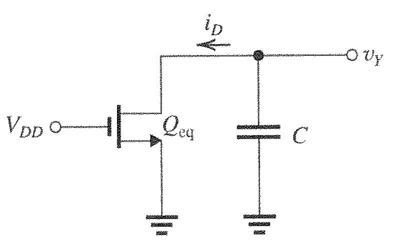


Figure 2

Figure 2 shows the equivalent circuit in the evaluation phase (ϕ high) when both A and B are high. The capacitor C_L is discharged from an initial voltage V_{DD} to ground through the equivalent transistor Q_{eq} whose

$$\left(\frac{W}{L}\right)_{eq} = \frac{1}{3}\left(\frac{W}{L}\right)_n = \frac{1}{3} \times 1.5 = 0.5$$

At $t = 0$, $v_Y = 1.2$ V and Q_N will be operating in the saturation region, thus

$$i_D(0) = \frac{1}{2}k'_n\left(\frac{W}{L}\right)_{eq}(V_{DD} - V_{tn})^2$$

$$= \frac{1}{2} \times 500 \times 0.5\ (1.2 - 0.4)^2$$

$$= 80\ \mu\text{A}$$

At $t = t_{PHL}$, $v_Y = 0.6$ V and Q_N will be operating in the triode region, thus

$$i_D(t_{PHL}) =$$

$$k'_n\left(\frac{W}{L}\right)_{eq}\left[(V_{DD} - V_{tn})\left(\frac{V_{DD}}{2}\right) - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$= 500 \times 0.5\left[(1.2 - 0.4)(0.6) - \frac{1}{2}(0.6)^2\right]$$

$$= 75\ \mu\text{A}$$

Thus, the average discharge current is

$$i_D|_{av} = \frac{1}{2}(80 + 75)$$

$$= 77.5\ \mu\text{A}$$

and t_{PHL} can be found as

$$t_{PHL} = \frac{C(V_{DD}/2)}{I_{av}}$$

$$= \frac{15 \times 10^{-15} \times 0.6}{77.5 \times 10^{-6}} = 0.116\text{ ns}$$

15.47

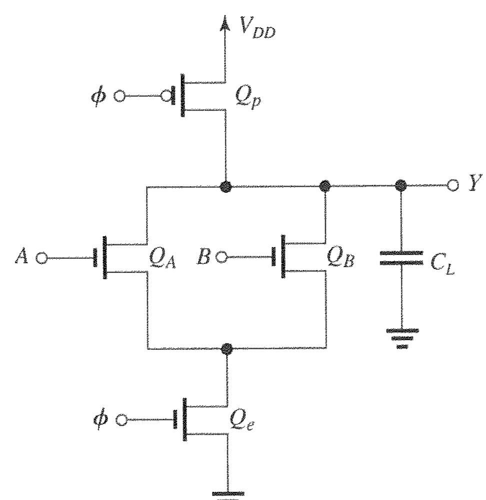


Figure 1

Figure 1 shows the two-input NOR gate. Figure 2 on the next page shows the circuit during the precharge interval. To determine the rise time t_r of v_Y, we need to calculate the charging current i_D at $v_Y = 0.1V_{DD} = 0.12$ V and at $v_Y = 0.9V_{DD} = 1.08$ V.

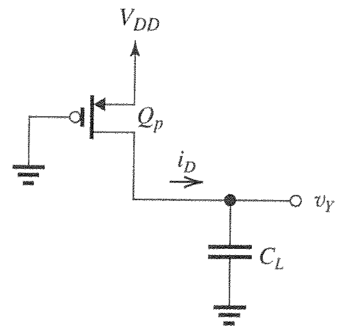


Figure 2

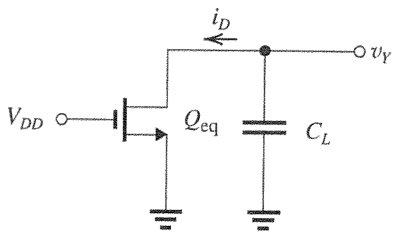


Figure 3

At $v_Y = 0.12$ V, Q_P will be operating in the saturation region, and

$$i_D(0.12\text{ V}) = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p (V_{DD} - |V_{tp})^2|$$

$$= \frac{1}{2} \times 125 \times 3\ (1.2 - 0.4)^2$$

$$= 120\ \mu\text{A}$$

At $v_Y = 1.08$ V, Q_P will be operating in the triode region, and

$$i_D(1.08\text{ V}) = k_p'\left(\frac{W}{L}\right)_p \Big[(V_{DD} - |V_{tp}|)(V_{DD} - 0.9V_{DD}) - \frac{1}{2}(V_{DD} - 0.9V_{DD})^2\Big]$$

$$= 125 \times 3\left[(1.2 - 0.4) \times 0.12 - \frac{1}{2} \times 0.12^2\right]$$

$$= 33.3\ \mu\text{A}$$

The average charging current can now be calculated as follows:

$$I_{av} = \frac{1}{2}(120 + 33.3) = 76.7\ \mu\text{A}$$

and the rise time can be determined as

$$t_r = \frac{C(0.9V_{DD} - 0.1V_{DD})}{I_{av}}$$

$$= \frac{15 \times 10^{-15} \times 0.8 \times 1.2}{76.7 \times 10^{-9}}$$

$$= 0.188\text{ ns}$$

To determine the worst-case value of t_{PHL}, consider the equivalent circuit in Fig. 3. Here, Q_{eq} is the equivalent transistor for discharging C_L when only one input (A or B) is high. Transistor Q_{eq} is the equivalent of two transistors in series: Q_A or Q_B, and Q_e. Thus, Q_{eq} will have

$$\left(\frac{W}{L}\right)_{eq} = \frac{1}{2}\left(\frac{W}{L}\right)_n = \frac{1}{2} \times 1.5 = 0.75$$

Now, at $t = 0$, $v_Y = V_{DD}$ and Q_{eq} will be operating in the saturation region, thus

$$i_D(0) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{eq}(V_{DD} - V_{tn})^2$$

$$= \frac{1}{2} \times 500 \times 0.75(1.2 - 0.4)^2$$

$$= 120\ \mu\text{A}$$

At $t = t_{PHL}$, $v_Y = \frac{1}{2}V_{DD} = 0.6$ V, and Q_{eq} will be operating in the triode region, thus

$$i_D(t_{PHL}) = k_n'\left(\frac{W}{L}\right)_{eq}\left[(V_{DD} - V_{tn})\left(\frac{V_{DD}}{2}\right) - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$= 500 \times 0.75\left[(1.2 - 0.4) \times 0.6 - \frac{1}{2} \times 0.6^2\right]$$

$$= 112.5\ \mu\text{A}$$

The average discharge current can now be found as

$$i_D|_{av} = \frac{1}{2}(120 + 112.5) = 116.3\ \mu\text{A}$$

and t_{PHL} can be determined as follows:

$$t_{PHL} = \frac{C(V_{DD}/2)}{i_D|_{av}}$$

$$= \frac{15 \times 10^{-15} \times 0.6}{116.3 \times 10^{-6}} = 0.077\text{ ns}$$

15.48 For a change of 0.2 V, the longest allowable evaluation time will be

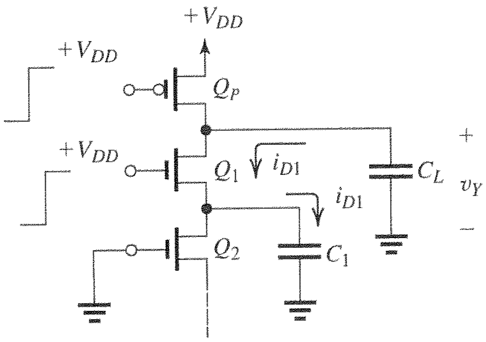
$$t = \frac{C\Delta V}{I_{leakage}}$$

$$= \frac{10 \times 10^{-15} \times 0.2}{2 \times 10^{-12}} = 1\text{ ms}$$

If the precharge interval is much shorter than this value, the period of the minimum clocking frequency can be as great as 1 ms. Thus the minimum clocking frequency will be

$$f_{min} = \frac{1}{1\text{ ms}} = 1000\text{ Hz} = 1\text{ kHz}$$

15.49



(a) Since $V_{G1} = V_{DD}$ and V_{S1} is starting at zero, current will flow through Q_1 until Q_1 cuts off at $V_{S1} = V_{DD} - V_{tn}$. At this point, $i_{D1} = 0$. If Δv_Y had been $> V_{tn}$, then Q_1 would have to be in the reverse mode of conduction. Thus, $|\Delta v_Y| \le V_{tn}$.

(b) Since $Q = CV$, we have $\Delta Q = C\Delta V$.

So for v_{C1} to reach $V_{DD} - V_{tn}$, we need

$\Delta v_{C1} = V_{DD} - V_{tn}$ and

$$\Delta Q = C_1 (V_{DD} - V_{tn})$$

$$\Delta v_Y = \frac{\Delta Q}{C_L} = \frac{C_1}{C_L}(V_{DD} - V_{tn}) \qquad (1)$$

For $v_Y = v_{C1}$, we have $V_{DD} - \Delta v_Y = \Delta v_{C1}$

so that

$$\Delta v_Y = V_{DD} - \Delta v_{C1}$$

Since $\Delta v_{C1} = \dfrac{\Delta Q}{C_1} = \dfrac{C_L \Delta v_Y}{C_1}$, we have

$\Delta v_Y = V_{DD} - \dfrac{C_L}{C_1}\Delta v_Y$ and

$$\Delta v_Y = \frac{V_{DD}}{1 + \dfrac{C_L}{C_1}} \qquad (2)$$

(c) For the first situation, namely, Q_1 stops conduction when $v_{C1} = V_{DD} - V_{tn}$, the reduction in v_Y was found to be [(Eq. (1)],

$$\Delta v_Y = \frac{C_1}{C_L}(V_{DD} - V_{tn})$$

We can use this equation to determine the maximum value of $\left(\dfrac{C_1}{C_L}\right)$ that maintains

$\Delta v_Y \le V_{tn}$ as follows:

$$\frac{C_1}{C_L}(V_{DD} - V_{tn}) \le V_{tn}$$

$$\Rightarrow \left.\frac{C_1}{C_L}\right|_{\max} = \frac{V_{tn}}{V_{DD} - V_{tn}} \qquad (3)$$

For the second situation, namely, Q_1 stops conducting because $v_Y = v_{C1}$, we obtained for the reduction in v_Y the expression in Eq. (2),

$$\Delta v_Y = \frac{V_{DD}}{1 + \dfrac{C_L}{C_1}}$$

For Δv_Y to be $\le V_{tn}$, we need

$$\frac{V_{DD}}{1 + \dfrac{C_L}{C_1}} \le V_{tn}$$

$$V_{DD} \le V_{tn} + \frac{C_L}{C_1} V_{tn}$$

$$\frac{C_L}{C_1} \ge \frac{V_{DD} - V_{tn}}{V_{tn}}$$

or

$$\frac{C_1}{C_L} \le \frac{V_{tn}}{V_{DD} - V_{tn}}$$

that is,

$$\Rightarrow \left.\frac{C_1}{C_L}\right|_{\max} = \frac{V_{tn}}{V_{DD} - V_{tn}} \qquad (4)$$

This is an identical expression to that in Eq. (3).

(d) For case (i) $C_1 = 4$ fF, the expression in Eq. (1) yields

$$\Delta v_Y = \frac{C_1}{C_L}(V_{DD} - V_{tn})$$

$$= \frac{4}{15} \times (1.8 - 0.5) = 0.35 \text{ V}$$

The second situation [Eq. (2)] yields

$$\Delta v_Y = \frac{V_{DD}}{1 + \dfrac{C_L}{C_1}}$$

$$= \frac{1.8}{1 + \dfrac{15}{4}} = 0.38 \text{ V}$$

Since the lower value occurs first, we have

$\Delta v_Y = 0.35$ V

<u>Check:</u>

With $\Delta v_Y = 0.35$ V,

$$\Delta v_{C1} = \frac{C_L \Delta v_Y}{C_1} = \frac{15}{4} \times 0.35 = 1.3 \text{ V}$$

Thus, $v_{GS1} = 1.8 - 1.3 = 0.5 \text{ V} = V_{tn}$ and Q_1 indeed stops conduction.

Case (ii) $C_1 = 7.5$ fF:

The first situation [Eq. (1)] yields

$$\Delta v_Y = \frac{7.5}{15}(1.8 - 0.5) = 0.65 \text{ V}$$

This is greater than $V_{tn} = 0.5$ V and thus, as we have demonstrated in (a), is physically not possible.

The second situation [Eq. (2)] yields

$$\Delta v_Y = \frac{1.8}{1 + \frac{15}{7.5}} = 0.6 \text{ V}$$

Check:

With $\Delta v_Y = 0.6$ V, v_Y becomes

$$v_Y = 1.8 - 0.6 = 1.2 \text{ V}$$

and

$$\Delta v_{C1} = \frac{C_L \Delta v_Y}{C_1}$$

$$= \frac{15}{7.5} \times 0.6 = 1.2 \text{ V}$$

Thus, v_{C1} becomes

$$v_{C1} = 1.2 \text{ V}$$

which is equal to v_Y. At this level of v_{C1}, $v_{GS1} = 1.8 - 1.2 = 0.6$ V, which is greater than V_{tn}, but conduction stops because $v_Y = v_{C1}$.

15.50 Refer to Figs. 15.31 and E15.15.

(a) Since Q_1 and Q_{e1} are in series, W remains the same, but the effective length doubles. So,

$$\left(\frac{W}{L}\right)_{\text{eq1}} = \left(\frac{W}{2L}\right) = \frac{1}{2}\left(\frac{W}{L}\right)_n$$

Similarly,

$$\left(\frac{W}{L}\right)_{\text{eq2}} = \left(\frac{W}{2L}\right) = \frac{1}{2}\left(\frac{W}{L}\right)_n$$

(b)

$$i_{D1}(v_{y1} = V_{DD}) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{\text{eq1}}(V_{DD} - 0.2V_{DD})^2$$

$$= \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{\text{eq1}}(0.64\, V_{DD}^2)$$

$$= 0.32k_n'\left(\frac{W}{L}\right)_{\text{eq1}} V_{DD}^2$$

$$= 0.16k_n'\left(\frac{W}{L}\right)_n V_{DD}^2 = 0.16k_n V_{DD}^2$$

At $v_{y1} = V_t$:

$$i_{D1}(v_{y1} = V_t) = k_n'\left(\frac{W}{L}\right)_{\text{eq1}} \times$$

$$\left[(V_{DD} - 0.2V_{DD})(0.2V_{DD}) - \frac{1}{2}(0.2V_{DD})^2\right]$$

$$= k_n'\left(\frac{W}{L}\right)_{\text{eq1}}\left[0.16V_{DD}^2 - 0.02V_{DD}^2\right]$$

$$= 0.07k_n V_{DD}^2$$

$$i_{D1}|_{av}$$

$$= \frac{1}{2}\left[0.16k_n V_{DD}^2 + 0.07k_n V_{DD}^2\right]$$

$$= 0.115k_n V_{DD}^2$$

(c) $$\Delta t = \frac{C_{L1}(V_{DD} - V_t)}{i_{D1}|_{av}} = \frac{C_{L1}(0.8V_{DD})}{0.115k_n V_{DD}^2}$$

$$= \frac{6.96C_{L1}}{k_n V_{DD}}$$

(d) Q_{eq2} will conduct during the time that v_{Y1} drops from V_{DD} to V_t. The transition half point is when $v_{y1}\big|_{av} = \dfrac{V_{DD} - 0.2V_{DD}}{2} + 0.2V_{DD}$

$$v_{y1}\big|_{av} = 0.6V_{DD}$$

$$i_{D2}|_{av} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{\text{eq2}}(0.6V_{DD} - 0.2V_{DD})^2$$

$$= 0.08k_n'\left(\frac{W}{L}\right)_{\text{eq2}} V_{DD}^2 = 0.04k_n V_{DD}^2$$

(e) $$\Delta v_{y2} = -\frac{i_{D2}|_{av}\Delta t}{C_{L2}}$$

Since $C_{L1} = C_{L2}$, we have

$$\Delta v_{y2} = -\frac{0.04k_n V_{DD}^2(6.96C_{L1})}{C_{L1}k_n V_{DD}} = -0.278V_{DD}$$

so that v_{y2} is $V_{DD} - 0.278V_{DD} = 0.72V_{DD}$

15.51 The precharge time can be approximated as the rise time of the output voltage. In Example 15.4, $t_r \simeq 0.24$ ns. The total cycle time can be estimated as being slighty longer than $t_r + t_{PHL}$.

With $t_{PHL} \simeq 0.25$ ns, the maximum clocking frequency is $f = \dfrac{1}{T} \leq \dfrac{1}{(t_r + t_{PHL})}$

$$= \frac{1}{(0.24 + 0.25)(10^{-9})\text{ s}} \simeq 2 \text{ GHz}$$

15.52 Refer to Fig. 15.34.

Output voltage swing $= IR_C$

Thus,

$$0.4 = 1 \times R_C$$

$$\Rightarrow R_C = 0.4 \text{ k}\Omega = 400\ \Omega$$

$$V_{OH} = 0 \text{ V}$$

$$V_{OL} = -0.4 \text{ V}$$

For V_{OH} and V_{OL} to be centered on $V_R = -1$ V, we must have

$$V_{OH} = -0.8 \text{ V}$$

$$V_{OL} = -1.2 \text{ V}$$

This is achieved by shifting the initial levels by -0.8 V.

15.53 At point x, we have $v_I = V_{IL}$ and

$$\frac{I_E|_{Q_R}}{I_E|_{Q_A}} = 99$$

Using the exponential $i_E - v_{BE}$ relationship, we obtain

$$V_{BE}|_{Q_R} - V_{BE}|_{Q_A} = V_T \ln 99 = 115 \text{ mV}$$

which gives

$$V_{IL} = -1.32 - 0.115 = -1.435 \text{ V}$$

Assuming Q_A and Q_R to be matched, we can write

$$V_{IH} - V_R = V_R - V_{IL}$$

which can be used to find V_{IH} as

$$V_{IH} = -1.205 \text{ V}$$

To determine V_{OL}, we note that Q_A is off and Q_R carries the entire current I_E, given by

$$I_E = \frac{V_R - V_{BE}|_{Q_R} + V_{EE}}{R_E}$$

$$= \frac{-1.32 - 0.75 + 5.2}{0.779}$$

$$= 4 \text{ mA}$$

(If we wish, we can iterate to determine a better estimate of $V_{BE}|_{Q_R}$ and hence of I_E). Assuming Q_R has a high β and thus $\alpha \simeq 1$, its collector current will be approximately 4 mA. If we neglect the base current of Q_2, we obtain the collector voltage of Q_R

$$V_C|_{Q_R} = -4 \times 0.245 = -0.98 \text{ V}$$

Thus a first approximation for the value of V_{OL} is

$$V_{OL} = V_C|_{Q_R} - V_{BE}|_{Q_2}$$

$$= -0.98 - 0.75 = -1.73 \text{ V}$$

We can use this value to find the emitter current of Q_2 and then iterate to obtain a better estimate of its base-emitter voltage. The result is $V_{BE2} = 0.79$ V and, correspondingly,

$$V_{OL} \simeq -1.77 \text{ V}$$

At this value of output voltage, Q_2 supplies a load current of 4.6 mA. We can use this value to find the base current of Q_2,

$$I_{B2} = \frac{4.6}{101} \simeq 0.046 \text{ mA}$$

This current is supplied through R_{C2} and thus causes a voltage drop across R_{C2} of $0.046 \times 0.245 = 0.01$ V. Thus V_{OL} will be reduced by 0.01 V:

$$V_{OL} \simeq -1.78 \text{ V}$$

To find the value of V_{OH}, we assume that Q_R is completely cut off (because $v_I > V_{IH}$). Thus the circuit for determining V_{OH} simplifies to that in Fig. 1. Analysis of this circuit,

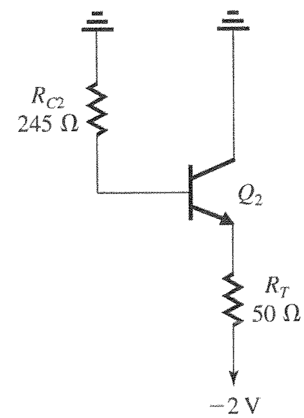


Figure 1

assuming $\beta = 100$, results in $V_{BE2} = 0.83$ V, $I_{E2} = 22.4$ mA, and

$$V_{OH} \simeq -0.88 \text{ V}$$

Width of transition region $= V_{IH} - V_{IL}$

$$= -1.205 - (-1.435)$$

$$= 0.230 \text{ V}$$

$$NM_H = V_{OH} - V_{IH}$$

$$= -0.88 - (-1.205)$$

$$= 0.325 \text{ V}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= -1.435 - (-1.78) = 0.345 \text{ V}$$

15.54 (a) $V_{OH} = 0 - 0.75 = -0.75$ V

$$V_{OL} = 0 - 0.75 - IR = -(0.75 + IR)$$

(b) $V_{\text{th}} = V_{BR} = -(IR/2 + 0.75)$

$$= -(0.75 + IR/2)$$

(c) For $i = 0.99$ I,

$$v_{BE} = 750 + 25 \ln(0.99) = 750 \text{ mV}$$

$i = 0.01$ I,

$$v_{BE} = 750 + 25 \ln(0.01) = 635 \text{ mV}$$

For

$0.99I$ in Q_R,

$$v_I = -\left(0.75 + \frac{IR}{2}\right) - (0.750 - 0.635)$$

$$= -(0.865 + IR/2)$$

(d) For $0.01I$ in Q_R,

$v_I = -(0.75 + IR/2) - 0.635 + 0.750$

$= -(0.635 + IR/2)$

(e) $V_{IH} = -(0.635 + IR/2)$

$V_{IL} = -(0.865 + IR/2)$

(f) $NM_H = -0.75 - [-(0.635 + IR/2)]$

$= IR/2 - 0.115$

$NM_L = -(0.865 + IR/2) - [-(0.75 + IR)]$

$= IR/2 - 0.115$

(g) $V_{IH} - V_{IL}$

$= -(0.635 + IR/2) - [-(0.865 + IR/2)]$

$= 0.230$ V

That is, $IR/2 - 0.115 = 0.230$

and $IR = 2\,(0.345) = 0.690$ V

(h) $V_{OH} = -0.75$ V;

$V_{OL} = -0.75 - 0.69 = -1.44$ V;

$V_{IL} = -(0.865 + 0.345) = -1.21$ V;

$V_{IH} = -(0.635 + 0.345) = -0.98$ V;

$V_R = -(0.750 + 0.345) = -1.095$ V.

15.55 Refer to Fig. 15.35 and to Exercise 15.18. In the answer to Exercise 15.18 we have

$I_E = 4$ mA

Thus, the power dissipated in the ECL gate (excluding the reference circuit) is

$P_{D1} = I_E V_{EE}$

$= 4 \times 5.2 = 20.8$ mW

We next consider the reference circuit (refer to Exercise 15.17). The current in the biasing branch is

$$I = \frac{5.2 - 2 \times 0.75}{0.907 + 4.98} = 0.63 \text{ mA}$$

where we have neglected the base current of Q_R. The emitter current of Q_R is given by

$$I_{ER} = \frac{V_R - (-V_{EE})}{R_3}$$

$$= \frac{-1.32 + 5.2}{6.1} = 0.64 \text{ mA}$$

Thus, the reference circuit draws a current of $(0.63 + 0.64) = 1.27$ mA from the supply and hence dissipates

$P_{D2} = 1.27 \times 5.2 = 6.6$ mW

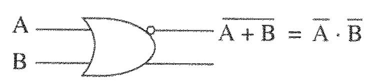
Since a reference circuit serves four gates, the power dissipation attributed to each gate is

$$= \frac{6.6}{4} = 1.65 \text{ mW}$$

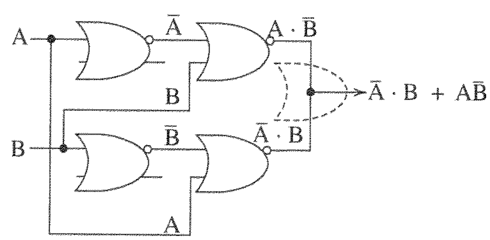
Thus, the total power dissipation in the ECL gate is

$P_D = 20.8 + 1.65 = 22.45$ mW

15.56 Tying two outputs together as in Fig. 15.37 yields a WIRED- OR operation. The most direct implementation is ORing the outputs of two AND gates. The AND function can be obtained using Demorgan's theorem:



Using NOR gates as inverters, $\bar{A}B$ and $A\bar{B}$ are obtained:



15.57 With A, B, C and D at 0 V, the circuit reduces to that shown in Fig. 1. The voltage V_E can be found as follows:

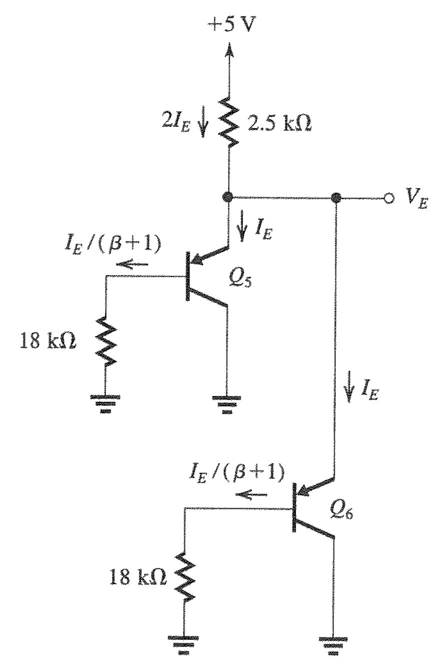


Figure 1

$$5 = 2I_E \times 2.5 + 0.7 + \frac{I_E}{51} \times 18$$

$$\Rightarrow I_E = \frac{5 - 0.7}{5 + \frac{18}{51}} = 0.8 \text{ mA}$$

$$V_E = 5 - 0.8 \times 2 \times 2.5$$

$$= 1 \text{ V}$$

With A and C at +5 V, the voltage at the base of each of Q_5 and Q_6 will be +4.3 V and thus Q_5 will be off and

$$V_E = +5 \text{ V}$$

Thus, we have a logic circuit with $V_{OL} = +1$ V and $V_{OH} = +5$ V. The output will be high if (A or B) is high and (C or D) is high, that is

$$E = (A + B).(C + D)$$

15.58 Refer to Fig. 15.38(a). For

$$v_I = v_O = V_{DD}/2 = 2.5 \text{ V},$$

$$I_{DN} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_n \left(\frac{V_{DD}}{2} - V_{BE2} - V_t\right)^2$$

$$= \frac{1}{2} \times 100 \times \frac{2}{1} (2.5 - 0.7 - 1)^2$$

$$= 64 \ \mu\text{A}$$

$$I_{B2} = I_{DN} = 64 \ \mu\text{A}$$

$$I_{C2} = \beta I_{B2} = 100 \times 64 = 6.4 \text{ mA}$$

$$I_{E1} = I_{C2} + I_{DN} = 6.4 + 0.064$$

$$= 6.464 \text{ mA}$$

$$I_{E2} = I_{C2} + I_{B2} = 6.464 \text{ mA}$$

which are equal as desired. To make this possible, we must have

$$I_{B1} = I_{DP} = 64 \ \mu\text{A}$$

But,

$$I_{DP} = \frac{1}{2} k_p' \left(\frac{W}{L}\right)_p \left[V_{DD} - \frac{V_{DD}}{2} - |V_{tp}|\right]^2$$

$$64 = \frac{1}{2} \times 40 \left(\frac{W}{L}\right)_p (5 - 2.5 - 1)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_p = 1.42$$

The totem-pole transient current is 6.464 mA.

15.59 At the threshold voltage, Q_P and Q_N conduct equal currents,

$$I_{DP} = \frac{1}{2} k_p' \left(\frac{W}{L}\right)_p (V_{DD} - V_{th} - V_t)^2$$

$$I_{DN} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_n (V_{th} - V_t - V_{BE})^2$$

Equating I_{DN} and I_{DP} gives

$$k_p' \left(\frac{W}{L}\right)_p (5 - V_{th} - 1)^2 = k_n' \left(\frac{W}{L}\right)_n (V_{th} - 1.7)^2$$

For $\left(\frac{W}{L}\right)_p = \left(\frac{W}{L}\right)_n$ and $k_n' = 2.5 \ k_p'$,

$$(4 - V_{th})^2 = 2.5(V_{th} - 1.7)^2$$

$$4 - V_{th} = \pm\sqrt{2.5}(V_{th} - 1.7)$$

$$4 - V_{th} = \pm 1.58(V_{th} - 1.7)$$

$$\Rightarrow V_{th} = \frac{4 + 1.58 \times 1.7}{2.58} = 2.6 \text{ V}$$

At this value of v_I, we have

$$I_{DN} = \frac{1}{2} \times 100 \times \frac{2}{1} \times (2.6 - 1 - 0.7)^2$$

$$= 81 \ \mu\text{A}$$

$$I_{E2} = 81 \times (\beta + 1) = 81 \times 101 = 8.18 \text{ mA}$$

Thus, the totem-pole current is 8.18 mA.

15.60 For R_2: With $v_{DS} = V_t/3 = 1/3 = 0.333$ V

$$i_{DN} = 100\left(10^{-6}\right)(2/1) \times$$

$$\left[(5 - 0.7 - 1)\,0.33 - 0.33^2/2\right] = 209 \ \mu\text{A}$$

Now, if 50% of this is lost in R_2,

$R_2 = 0.7/\,(0.50 \times 209) = 6.70\ \text{k}\Omega$

Now if 20% is lost in R_2,

$R_2 = 0.7/\,(0.20 \times 209) = 16.7\ \text{k}\Omega$

For R_1 : $i_{DP} = (100/2.5)\,10^{-6}\,(2.5\,(2/1)) \times$

$\left[(5 - 0 - 1)\,0.33 - 0.33^2/2\right] = 256\ \mu\text{A}$

Now, if 50% if this is lost in R_1,

$R_1 = (5 - 0.333)\,/\,(0.5 \times 256) = 36.5\ \text{k}\Omega$

Now, if 20% is lost in R_1

$R_1 = 2.5\,(36.5) = 91.1\ \text{k}\Omega$

In comparison:

For the 50% case,

$R_1/R_2 = 36.5/6.70 = 5.45$

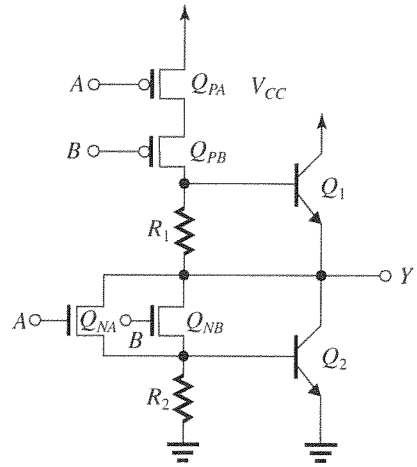
For the 20% case,

$R_1/R_2 = 91.1/16.7 = 5.45$

(why should their equality be obvious?)

Thus, in general $R_1/R_2 = 5.45$

15.61 A BiCMOS 2-input NOR is as shown:



In terms of the basic matched inverter:

$(W/L)_{PA} = (W/L)_{PB} = 2\,(W/L)_P$

$(W/L)_{NA} = (W/L)_{NB} = (W/L)_N$

where $(W/L)_P$ and $(W/L)_N$ characterize the inverter.

16.1

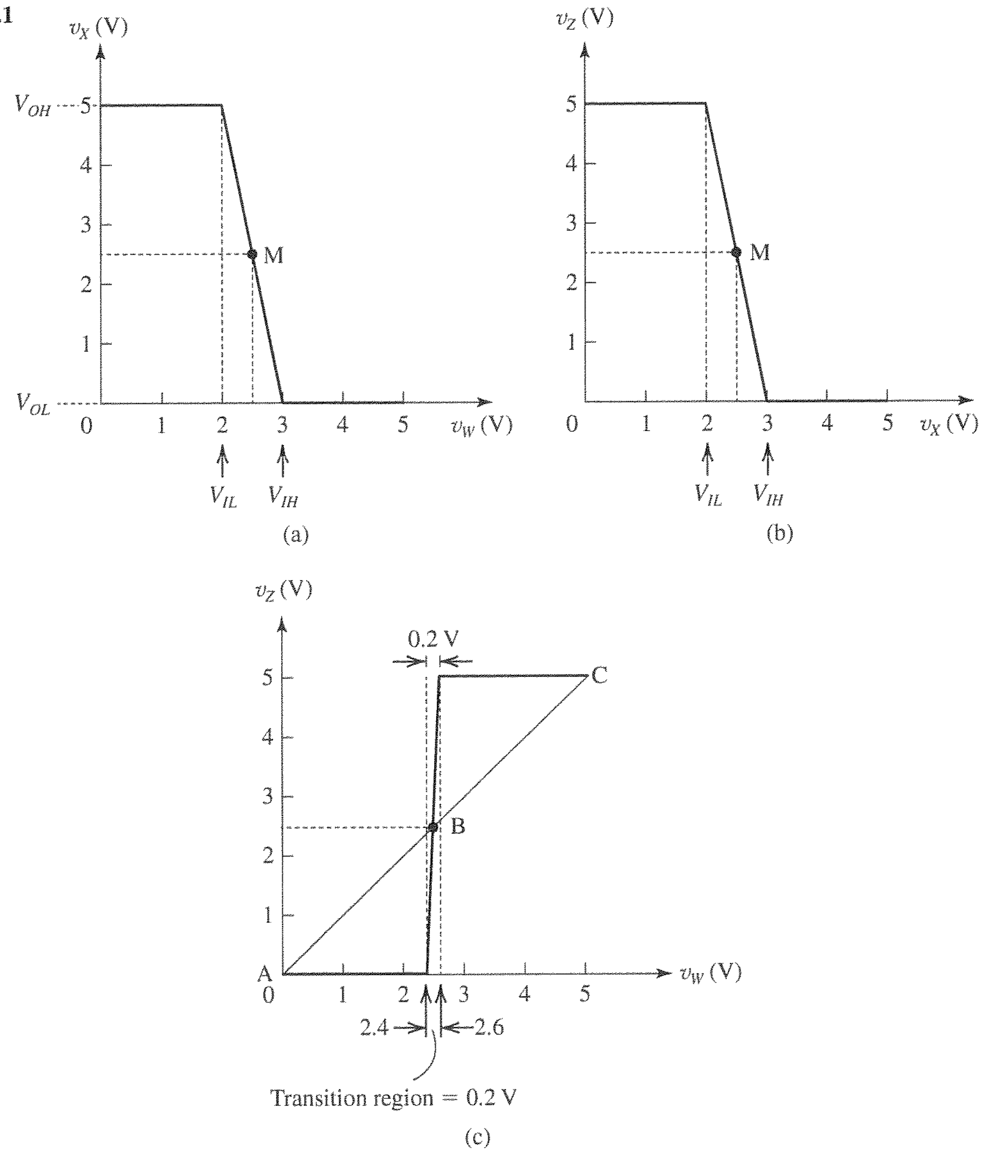


Figure 1

Refer to Fig. 16.1(b) and to Fig. 1. Figure 1(a) shows v_X versus v_W and Fig. 1(b) shows v_Z versus v_X. Both of these VTCs are identical with a gain in the transition region of −5 V/V. The equation of the linear transition region in Fig. 1(a) is

$$v_X = -5(v_W - 2.5) + 2.5 \qquad (1)$$

for

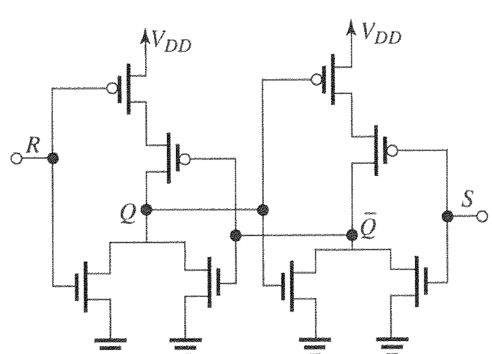
$2\text{ V} \le v_X \le 3\text{ V}$

The two VTCs in Figs. 1(a) and (b) can be used to obtain the VTC of the cascade of the two inverters, that is, v_Z versus v_W, which is shown in Figure 1(c). The slope at the midpoint, point C in Fig. 1(c), is $-5 \times -5 = 25$ V/V. The two switching points, at $v_W = 2.4$ V and 2.6 V, can be determined using Eq. (1) and setting $v_X = V_{IL}$ or V_{IH}. The width of the transition region is $2.6 - 2.4 = 0.2$ V. This value can also be obtained from 5 V/25 V/V = 0.2 V.

Finally, we show in Fig. 1(c) the three points A, B, C:

A (0 V, 0 V); B (2.5 V, 2.5 V), C (5 V, 5 V)

16.2



These figures belong to Problem 16.3.

Figure 1

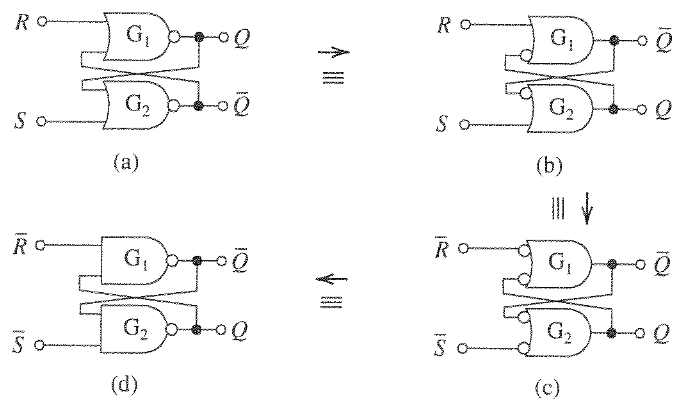


Figure 2

16.3 Figure 1 shows the equivalence of a NAND gate to an OR gate with inverters at the inputs. We can use this equivalence to derive an SR flip-flop utilizing two cross-coupled NAND gates starting from the one utilizing NOR gates in Fig. 15.3, as shown in Fig. 2.

The resulting circuit, shown in Fig. 2(d), can be described by the following truth table:

$\bar{R}$	$\bar{S}$	Q_{n+1}
0	0	not used
0	1	0
1	0	1
1	1	Q_n

Here, the rest state is when both trigger inputs $\bar{R}$ and $\bar{S}$ are high. To set the flip-flop we lower $\bar{S}$ to zero. This results in $Q_{n+1} = 1$. Conversely, to reset the flip-flop, $\bar{R}$ is lowered to zero, resulting in $Q_{n+1} = 0$. The situation with both $\bar{R}$ and $\bar{S}$ lowered to zero results in an undefined output and is thus avoided.

16.4 Following the procedure used in Example 16.1 and referring to Fig. 16.5(b), we can write

$$I_{Deq} = I_{D2}$$

$$k_n' \times$$

$$\frac{1}{2}\left(\frac{W}{L}\right)_{5,6}\left[(V_{DD} - V_{tn})\left(\frac{V_{DD}}{2}\right) - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$= k_p'\left(\frac{W}{L}\right)_p\left[(V_{DD} - |V_{tp}|)\left(\frac{V_{DD}}{2}\right) - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

Since $V_{tn} = |V_{tp}|$, this equation reduces to

$$\frac{1}{2}k_n'\left(\frac{W}{L}\right)_{5,6} = k_p'\left(\frac{W}{L}\right)_p \qquad (1)$$

Since the inverter is matched, then

$$k_p'\left(\frac{W}{L}\right)_p = k_n'\left(\frac{W}{L}\right)_n \qquad (2)$$

Substituting for $k_p'(W/L)_p$ from Eq. (2) into Eq. (1) gives

$$\left(\frac{W}{L}\right)_{5,6} = 2\left(\frac{W}{L}\right)_n \qquad \text{Q.E.D.}$$

Refer to Fig. 16.4. For Q_1 and Q_3,

$$\left(\frac{W}{L}\right)_{1,3} = \frac{0.13\ \mu\text{m}}{0.13\ \mu\text{m}}$$

$$\left(\frac{W}{L}\right)_{2,4} = \frac{0.52\ \mu\text{m}}{0.13\ \mu\text{m}}$$

$$\left(\frac{W}{L}\right)_{5-8} = \frac{0.26\ \mu\text{m}}{0.13\ \mu\text{m}}$$

16.5 From Table 14.2 on P. 1155 of the text we obtain for the inverter threshold voltage

$$V_M = \frac{r(V_{DD} - |V_{tp}|) + V_{tn}}{1 + r}$$

where

$$r = \sqrt{\frac{k_p'(W/L)_p}{k_n'(W/L)_n}}$$

For our case, we have

$$k_n' = 4k_p' = 300\ \mu\text{A/V}^2$$

$$\left(\frac{W}{L}\right)_p = \left(\frac{W}{L}\right)_n = \frac{0.27\ \mu\text{m}}{0.18\ \mu\text{m}}$$

$V_{tn} = |V_{tp}| = 0.5$ V

$V_{DD} = 1.8$ V

Thus,

$$r = \sqrt{\frac{1}{4} \times 1} = \frac{1}{2}$$

$$V_M = \frac{0.5(1.8 - 0.5) + 0.5}{1.5}$$

$= 0.767$ V

Now, refer to Fig. 16.5(b) with $v_{\bar{Q}} = 0.767$ V. Both Q_{eq} and Q_2 will be operating in the triode region. Equating their drain currents results in

$$300 \times \frac{1}{2}\left(\frac{W}{L}\right)_5 \left[(1.8 - 0.5) \times 0.767 - \frac{1}{2} \times 0.767^2\right]$$

$$= 75 \times \frac{0.27}{0.18}\left[(1.8 - 0.5)(1.8 - 0.767) - \frac{1}{2} \times (1.8 - 0.767)^2\right]$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = 0.86$$

Since the minimum allowed $\left(\frac{W}{L}\right)$ is 1, then

$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8 = 1$$

16.6 To operate in velocity saturation, we must have

$v_{GS} - V_t \geq V_{DS\text{sat}}$ and $v_{DS} \geq V_{DS\text{sat}}$

Now, refer to Example 16.1 and in particular to Fig. 16.5(b). Here $V_{DD} = 1.8$ V and thus $V_{DD}/2 = 0.9$ V. For Q_{eq},

$V_{GS} - V_t = 1.8 - 0.5 = 1.3$ V

This is greater than $V_{DS\text{sat}} = 0.6$ V. Also, $V_{DS} = 0.9\text{ V} > V_{DS\text{sat}}$. Thus, Q_{eq} is operating in velocity saturation and its drain current $I_{D_{eq}}$ is given by Eq. (15.11), thus

$$I_{D_{eq}} = k'_n\left(\frac{W}{L}\right)_{eq} V_{DS\text{sat}}(V_{DD} - V_t - \frac{1}{2}V_{DS\text{sat}})(1 + \lambda v_{DS})$$

$$= 300 \times \frac{1}{2}\left(\frac{W}{L}\right)_5$$

$$\times 0.6\left(1.8 - 0.5 - \frac{1}{2} \times 0.6\right)(1 + 0.1 \times 0.9)$$

$$= 98.1\left(\frac{W}{L}\right)_5, \ \mu\text{A}$$

From Fig. 16.5(b) we see that for Q_2,

$V_{SG} - |V_{tp}| = 1.8 - 0.5 = 1.3$ V

which is greater than $|V_{DS\text{sat}}| = 1$ V. However,

$$V_{SD} = \frac{V_{DD}}{2} = 0.9\text{ V}$$

which is less than $|V_{DS\text{sat}}|$. Thus, Q_2 will be operating in the triode region and its drain current will be

$$I_{D2} = 75 \times \frac{1.08}{0.18}\left[(1.8 - 0.5)\left(\frac{1.8}{2}\right) - \frac{1}{2}\left(\frac{1.8}{2}\right)^2\right](1 + 0.1 \times 0.9)$$

$= 375.2\ \mu$A

Equating $I_{D_{eq}}$ to I_{D2} results in

$$\left(\frac{W}{L}\right)_5 = \frac{375.2}{98.1} = 3.83$$

Transistors $Q_5 - Q_8$ will require an equal W/L, thus

$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8 = 3.83$$

Observe that because velocity saturation reduces the currents of Q_5 and Q_6, we need to increase their (W/L) ratios above the value without velocity saturation (3, from Example 16.1) to cause switching to occur.

16.7 (a) Refer to Fig. 16.5(b). For $V_{DD} = 1.2$ and $V_{tn} = |V_{tp}| = 0.4$ V, both Q_{eq} and Q_2 will be operating in the triode region. Equating their drain currents results in

$$500 \times \frac{1}{2}\left(\frac{W}{L}\right)_5\left[(1.2 - 0.4)\left(\frac{1.2}{2}\right) - \frac{1}{2}\left(\frac{1.2}{2}\right)^2\right]$$

$$= 125 \times \frac{0.8}{0.13}\left[(1.2 - 0.4)\left(\frac{1.2}{2}\right) - \frac{1}{2}\left(\frac{1.2}{2}\right)^2\right]$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = \frac{0.4}{0.13}$$

Thus, the minimum required W/L ratios are

$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8$$

$$= \frac{0.4\ \mu\text{m}}{0.13\ \mu\text{m}}$$

(b) Selecting

$$\left(\frac{W}{L}\right)_{5-8} = \frac{0.8\ \mu\text{m}}{0.13\ \mu\text{m}}$$

we can determine the minimum required width of the set and reset pulses as follows: The minimum required pulse width is composed of two components. The first is the time for $v_{\bar{Q}}$ in the circuit in Fig. 16.5(a) to fall from V_{DD} to $V_{DD}/2$, where $V_{DD}/2$ is the threshold voltage of the inverter formed by Q_3 and Q_4 in Fig. 16.4.

The second component is the time for the output of the $Q_3 - Q_4$ inverter to rise from 0 to $V_{DD}/2$. We will denote the first component t_{PHL} and the second t_{PLH}.

To determine t_{PHL} refer to the circuit in Fig. 16.6. The capacitor discharge current i_C is

$$i_C = i_{D_{eq}} - i_{D2}$$

To determine the average value of i_C, we calculate $i_{D_{eq}}$ and i_{D2} at $t = 0$ and $t = t_{PHL}$. At $t = 0$, $v_{\overline{Q}} = V_{DD}$, thus Q_2 is off,

$$i_{D2}(0) = 0$$

and Q_{eq} is in saturation,

$$i_{D_{eq}} = \frac{1}{2} \times 500 \times \frac{1}{2} \times \frac{0.8}{0.13} \times (1.2 - 0.4)^2$$

$$= 492.3\ \mu A$$

Thus,

$$i_C(0) = 492.3 - 0 = 429.3\ \mu A$$

At $t = t_{PHL}$, $v_{\overline{Q}} = V_{DD}/2$, thus both Q_2 and Q_{eq} will be in the triode region,

$$i_{D2}(t_{PHL}) =$$

$$125 \times \frac{0.8}{0.13} \times \left[(1.2 - 0.4)\left(\frac{1.2}{2}\right) - 0.5\left(\frac{1.2}{2}\right)^2\right]$$

$$= 230.8\ \mu A$$

and

$$i_{D_{eq}}(t_{PHL}) =$$

$$500 \times \frac{1}{2} \times \frac{0.8}{0.13}\left[(1.2 - 0.4)\left(\frac{1.2}{2}\right) - 0.5\left(\frac{1.2}{2}\right)^2\right]$$

$$= 461.5\ \mu A$$

Thus,

$$i_C(t_{PHL}) = 461.5 - 230.8 = 230.7\ \mu A$$

and the average value of i_C over the interval $t = 0$ to $t = t_{PHL}$ is

$$i_C|_{av} = \frac{i_C(0) + i_C(t_{PHL})}{2}$$

$$= \frac{429.3 + 230.7}{2} = 330\ \mu A$$

We now can calculate t_{PHL} as

$$t_{PHL} = \frac{C(V_{DD}/2)}{i_C|_{av}} = \frac{15 \times 10^{-15} \times 0.6}{330 \times 10^{-6}}$$

$$= 27.3\ \text{ps}$$

Next we determine the time t_{PLH} for the output of the $Q_3 - Q_4$ inverter, v_Q, to rise from 0 to $V_{DD}/2$. For this purpose we use Eq. (14.52),

$$t_{PHL} = \frac{\alpha_p C}{k'_p(W/L)_p V_{DD}}$$

where

$$\alpha_p = 2\Big/\left[\frac{7}{4} - \frac{3|V_{tp}|}{V_{DD}} + \left(\frac{|V_{tp}|}{V_{DD}}\right)^2\right]$$

$$= \frac{2}{\frac{7}{4} - \frac{3 \times 0.4}{1.2} + \left(\frac{0.4}{1.2}\right)^2}$$

$$= 2.32$$

Thus,

$$t_{PHL} = \frac{2.32 \times 15 \times 10^{-15}}{125 \times 10^{-6} \times \left(\frac{0.8}{0.13}\right) \times 1.2}$$

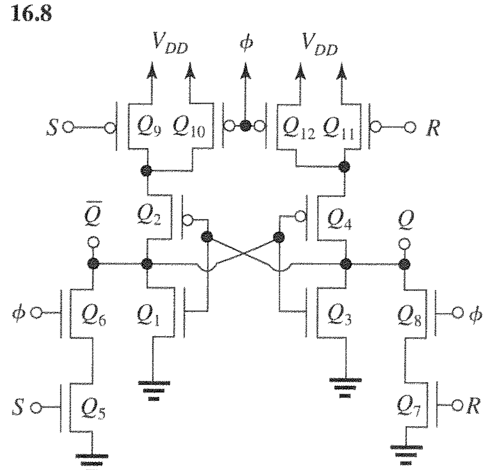
$$= 37.7\ \text{ps}$$

Finally, the minimum required width of the set pulse can be calculated as

$$T_{min} = t_{PHL} + t_{PLH}$$

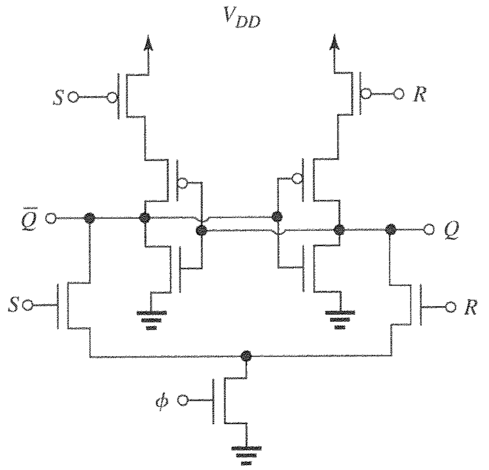
$$= 27.3 + 37.7 = 65\ \text{ps}$$

16.8



As noted, this fully complementary circuit uses 12 transistors. However, a 10-T version exists in which Q_{10} and Q_{12} are omitted.

Note further that an effective 9-T version exists in which Q_6 and Q_8 are moved below Q_5 and Q_7, then merged into a single grounded-source device. Note that all of the designs can employ the latter idea to reduce the transistor count by 1. See the sketch following on the next page:



This circuit suffers only from the fact that unclocked changes in S and R have a secondary input on $Q/\overline{Q}$ since raising S or R disconnects $Q/\overline{Q}$ from V_{DD}. In some applications this may lead to system noise sensitivity in which case one or the other or both of Q_{10}, Q_{12} (in the previous sketch) may be added.

16.9

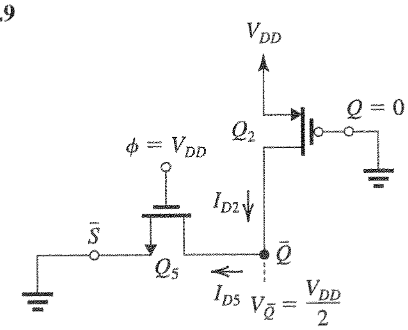


Figure 1

Figure 1 shows the relevant part of the circuit at the point of switching ($V_{\overline{Q}} = V_{DD}/2$). For this situation to be achieved, the current supplied by Q_5, I_{D5}, must at least be equal to that supplied by Q_2. Since both transistors are operating in the triode region, we can write

$$I_{D5} = I_{D2}$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_5 \left[(V_{DD} - V_{tn})\left(\frac{V_{DD}}{2}\right) - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$=$$

$$\mu_p C_{ox}\left(\frac{W}{L}\right)_p \left[(V_{DD} - |V_{tp}|)\left(\frac{V_{DD}}{2}\right) - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

Since $V_{tn} = |V_{tp}|$, this equation yields

$$\left(\frac{W}{L}\right)_5 = \left(\frac{\mu_p}{\mu_n}\right)\left(\frac{W}{L}\right)_p$$

This is the minimum required value of $(W/L)_5$.

16.10 Refer to Fig. P16.10.

(a) When ϕ is high, the transmission gate is conducting (i.e., Q_5 and Q_6 are conducting) and the line D is connected to the input terminal of G_2. Thus $\overline{Q} = \overline{D}$ and $Q = D$. The two feedback loops around G_2 are open because although Q_1 or Q_4 can conduct (if $D = 1$ and thus $\overline{Q} = 0$, Q_1 can conduct while if $D = 0$ and thus $\overline{Q} = 1$, Q_4 can conduct) their conduction paths are blocked by Q_2 and Q_3 which remain cut off when ϕ is high.

(b) If D is high, then $\overline{Q}$ is low and Q is high. Now, if ϕ goes low, the transmission gate turns off, thus the input of G_2 is isolated from D. The high value at the gate of G_2 is maintained by the feedback loop around G_2 consisting of Q_1 and Q_2, both of which conduct, thus connecting the input node of G_2 to V_{DD}. Meanwhile, Q_3 and Q_4 are at cut off.

(c) If D is low, then the voltage at the input of G_2 will be low (0 V) and thus $\overline{Q}$ will be high and Q will be low. When ϕ goes low, the transmission gate turns off and the input node of G_2 is isolated from D. The low voltage at the input node of G_2 is maintained by the feedback loop around G_2 consisting of Q_3 and Q_4, both of which conduct. Meanwhile, Q_1 and Q_2 are cutoff.

(d) No. The circuit connects either V_{DD} or ground directly to the input of G_2 which establishes the value of Q and $\overline{Q}$. When ϕ goes low, a feedback loop closes around G_2, thus locking in the value of $\overline{Q}$ (equal to $\overline{D}$) and thus Q (equal to D).

16.11 A 4-Gbit RAM has 4G cells,

$= 4 \times 1024^3$

$= 4{,}294{,}967{,}296$ cells

16.12 The word address needs M bits where

$2^M = 256M = 256 \times 1024^2$

$M \log_2 2 = \log_2 256 + 2 \log_2 1024$

$\Rightarrow M = 8 + 2 \times 10 = 28$

16.13 The 1-Mbit square memory array has 1024 word lines and 1024 bit lines. Thus it requires 10 bits to address each of the 1024 words. If the 1024 bit lines are read in groups of 16 bits; there will be $1024/16 = 64$ groups, requiring 6 bits to address each group. Thus the total number of address bits needed is $10 + 6 = 16$. This should be compared to the 20 address bits needed if each of the 1 Mbits is to be individually read.

16.14 The row decoder supplies 1024 word lines.

A straightforward implementation would require one sense amplifier/driver for each of the 1024 bit lines, for a total of 1024 sense amplifiers/drivers. If the chip is operated continuously with 20-ns cycle time, the operating frequency will be

$$f = \frac{1}{20 \times 10^{-9}} = 50\text{ MHz}$$

If in each cycle the total capacitance of the logic activated is C, the dynamic power dissipation will be

$$P_D = fCV_{DD}^2$$

Thus,

$$500 \times 10^{-3} = 50 \times 10^6 \times C \times 5^2$$

$$\Rightarrow C = 400\text{ pF}$$

Now if 90% of the power loss occurs in array access with the major capacitance contributor is the bit line itself, the total capacitance of the 16 bit lines selected at any one time will be

$$C\text{ (16 bit lines)} = 0.9 \times 400 = 360\text{ pF}$$

and the capacitance per bit line will be

$$C\text{ (bit line)} = \frac{360}{16} = 22.5\text{ pF}$$

Since each bit line has 1024 bits, the capacitance per bit will be

$$C\text{ (bit)} = \frac{22.5}{1024} = 22\text{ fF}$$

If the memory array is operated from a 3-V supply, the power dissipation reduces by a factor of $\left(\frac{3}{5}\right)^2 = 0.36$. Thus, the size of the memory array can be increased by a factor of $1/0.36 = 2.8$ while maintaining the same power dissipation.

16.15 Cell-array area $= 1024^3 \times 0.38 \times 0.76$

$= 0.31 \times 10^9\ \mu\text{m}^2$

$= 0.31 \times 10^3\text{ mm}^2$

Chip area $= 19 \times 38 = 722\text{ mm}^2$

Thus,

Area of I/O connections and peripheral circuits $= 722 - 310 = 412\text{ mm}^2$

which represents a percentage of

$$= \frac{412}{722} \times 100 = 57\%$$

of the total chip area.

16.16 $$\frac{\left(\frac{W}{L}\right)_a}{\left(\frac{W}{L}\right)_n} \le \frac{1}{\left(1 - \frac{V_{tn}}{V_{DD} - V_{tn}}\right)^2} - 1$$

$$= \frac{1}{\left(1 - \frac{0.5}{2.5 - 0.5}\right)^2} - 1 = 0.78$$

$$\left(\frac{W}{L}\right)_a \le 0.78 \times 1.5 \text{ or } \left(\frac{W}{L}\right)_a \le 1.17$$

16.17 $$\frac{\left(\frac{W}{L}\right)_a}{\left(\frac{W}{L}\right)_n} \le \frac{1}{\left(1 - \frac{V_{tn}}{V_{DD} - V_{tn}}\right)^2} - 1$$

$$= \frac{1}{\left(1 - \frac{0.4}{1.2 - 0.4}\right)^2} - 1 = 3$$

$$\left(\frac{W}{L}\right)_a \le 1.5 \times 3 = 4.5 \text{ or } \left(\frac{W}{L}\right)_a \le 4.5$$

16.18 (a) $0.25\ \mu$m: $V_{DD} = 2.5$ V and $V_t = 0.5$ V

$$A{:}\left(\frac{(V_{\bar{Q}} = V_t)}{V_{DD} - V_{tn}}\right) = \frac{0.5}{2.5 - 0.5} = \frac{0.5}{2} = 0.25$$

$$\Rightarrow \frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} \approx 0.8$$

From Eq. (16.5) $\Rightarrow \dfrac{(W/L)_5}{(W/L)_1} \cong 0.8$

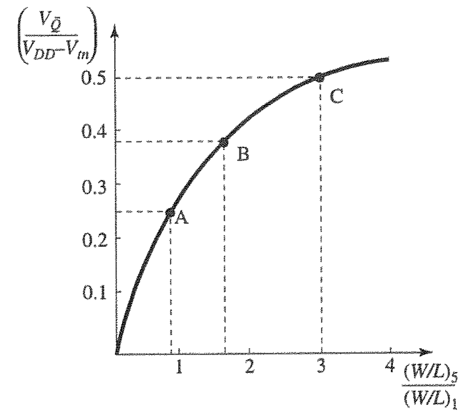
(b) $0.18\ \mu$m: $V_{DD} = 1.8$ V and $V_t = 0.5$ V

$$B{:}\left(\frac{0.5}{1.8 - 0.5}\right) = 0.385 \Rightarrow \frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} \approx 1.7$$

From Eq. (16.5) $\Rightarrow \dfrac{(W/L)_5}{(W/L)_1} = 1.64$

(c) $0.13\ \mu$m: $V_{DD} = 1.2$ V and $V_t = 0.4$ V

$$C{:}\left(\frac{0.4}{1.2 - 0.4}\right) = 0.5$$



$$\frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} = 3$$

From Eq. (15.5)

$$\left(\frac{W}{L}\right)_5 \Big/ \left(\frac{W}{L}\right)_1 = 3$$

16.19 Using Eq. (16.5), we obtain

$$\left(\frac{W}{L}\right)_{a,\max} = 1.5\left[\frac{1}{\left(1-\frac{0.4}{1.2-0.4}\right)^2} - 1\right]$$

$= 4.5$

(i) For $\left(\frac{W}{L}\right)_a = \frac{1}{3} \times 4.5 = 1.5$, Eq. (16.3) yields $\left[\text{note that } \left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_a\right]$

$$\frac{V_{\overline{Q}}}{1.2-0.4} = 1 - 1/\sqrt{1+\frac{1.5}{1.5}}$$

$\Rightarrow V_{\overline{Q}} = 0.293 \times 0.8 = 0.23$ V

Equation (16.1) provides

$$I_5 = \frac{1}{2} \times 500 \times 1.5\ (1.2-0.4-0.23)^2$$

$= 121.8\ \mu$A

(ii) For $\left(\frac{W}{L}\right)_a = \frac{2}{3} \times 4.5 = 3$, Eq. (16.3) yields

$$\frac{V_{\overline{Q}}}{1.2-0.4} = 1 - 1/\sqrt{1+\frac{3}{1.5}}$$

$\Rightarrow V_{\overline{Q}} = 0.34$ V

Equation (16.1) provides

$$I_5 = \frac{1}{2} \times 500 \times 3\ (1.2-0.4-0.34)^2$$

$= 158.7\ \mu$A

(iii) For $\left(\frac{W}{L}\right)_a = 4.5$, Eq. (16.3) yields

$$\frac{V_{\overline{Q}}}{1.2-0.4} = 1 - 1\Big/\sqrt{1+\frac{4.5}{1.5}}$$

$\Rightarrow V_{\overline{Q}} = 0.4$ V (which is equal to V_{tn}, as should be expected)

Equation (16.1) provides

$$I_5 = \frac{1}{2} \times 500 \times 4.5\ (1.2-0.4-0.4)^2$$

$= 180\ \mu$A

From Eq. (16.7) we see that the read delay, Δt, is inversely proportional to I_5. Thus the third design produces the shortest read delay.

16.20 We can use the graph, or more accurately Eq. (16.3), to determine the maximum allowable value of $(W/L)_5/(W/L)_1$ as follows:

$$\frac{V_{\overline{Q}}}{V_{DD}-V_{tn}} = 1 - 1\Big/\sqrt{1+\frac{(W/L)_5}{(W/L)_1}}$$

$$\frac{0.2}{1.8-0.5} = 1 - 1\Big/\sqrt{1+\frac{(W/L)_5}{(W/L)_1}}$$

$$\frac{(W/L)_5}{(W/L)_1} = 0.4$$

or, more appropriately,

$$\frac{(W/L)_5}{(W/L)_1} \leq 0.4$$

If in our design we use this relationship with the equality sign and recall that $L_1 = L_5 = 0.18\ \mu$m and that the minimum width is $0.18\ \mu$m, then we select

$$\left(\frac{W}{L}\right)_5 = 1$$

$\Rightarrow W_5 = 0.18\ \mu$m

and thus,

$$(W/L)_1 = \frac{1}{0.4} = 2.5$$

resulting in

$W_1 = 2.5 \times 0.18 = 0.45\ \mu$m

16.21

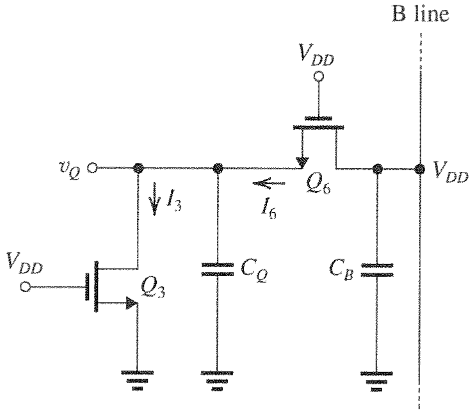


Figure 1

The relevant part of the circuit is shown in Fig. 1. It is assumed that $Q = 0$, that is, initially $v_Q = 0$ V. Also it is assumed that the B line voltage is V_{DD}. When the word line is selected and the gate of Q_6 is pulled to V_{DD} volts, Q_6 conducts and operates in the saturation region. Its current I_6 charges C_Q whose voltage v_Q rises from 0 V and thus Q_3 conducts and operates in the triode region. Current I_6, in addition to charging C_Q, also supplies a current equal to I_3. Equilibrium is reached at the value of V_Q that makes I_3 equal to I_6 and thus C_Q stops charging. The design is based on V_Q being sufficiently small to prevent the latch from changing state (nondestructive readout). Usually, one imposes the condition that $V_Q \leq V_{tn}$.

The operation of the circuit is described by equations identical to (16.1)–(16.4) except for $(W/L)_5$ replaced with $(W/L)_6$ and $(W/L)_1$ replaced with $(W/L)_3$. The graph in Fig. 16.14 applies here also with the same changes mentioned. Finally, Eq. (16.5) provides the constraint on the (W/L) ratios that results if V_Q is to be less or equal to V_{tn}.

16.22 Refer to Fig. 16.13. Since $V_{\overline{Q}} \leq V_t$, and $V_t = 0.5\ \text{V} < V_{DS\text{sat}}$ which is 0.6 V, transistor Q_1 will be operating in the triode region and thus Eq. (16.2) still applies. Transistor Q_5, however, has $v_{GS} = v_{DS} = V_{DD} - v_{\overline{Q}}$ which ranges in value from 1.8 V to 1.3 V (as $v_{\overline{Q}}$ rises from 0 to 0.5 V). Thus, $v_{GS} \geq V_{DS\text{sat}}$ and $v_{DS} \geq V_{DS\text{sat}}$. Current I_5 will therefore be given by Eq. (15.11), namely,

$$I_{D5} = \mu_n C_{ox}\left(\frac{W}{L}\right)_5 V_{DS\text{sat}}\left(v_{GS} - V_t - \frac{1}{2}V_{DS\text{sat}}\right)$$

where we have neglected λ. Substituting

$$v_{GS} = V_{DD} - V_{\overline{Q}}$$

we obtain

$$I_{D5} = \mu_n C_{ox}\left(\frac{W}{L}\right)_5 V_{DS\text{sat}}\left(V_{DD} - V_{\overline{Q}} - V_t - \frac{1}{2}V_{DS\text{sat}}\right)$$

Equating I_{D5} to I_{D1} in Eq. (16.2), we obtain

$$\left(\frac{W}{L}\right)_5 V_{DS\text{sat}}\left(V_{DD} - V_{\overline{Q}} - V_t - \frac{1}{2}V_{DS\text{sat}}\right)$$

$$= \left(\frac{W}{L}\right)_1\left[(V_{DD} - V_t)V_{\overline{Q}} - \frac{1}{2}V_{\overline{Q}}^2\right]$$

Substituting $V_{DD} = 1.8$ V, $V_{\overline{Q}} = V_t = 0.5$ V, $V_{DS\text{sat}} = 0.6$ gives

$$\left(\frac{W}{L}\right)_5 \times 0.6 \times \left(1.8 - 0.5 - 0.5 - \frac{1}{2} \times 0.6\right)$$

$$= \left(\frac{W}{L}\right)_1\left[(1.8 - 0.5)0.5 - \frac{1}{2}(0.5)^2\right]$$

$$\Rightarrow \frac{(W/L)_5}{(W/L)_1} = 1.75$$

That is,

$$\frac{(W/L)_5}{(W/L)_1} \leq 1.75$$

In the absence of velocity saturation, we can find the maximum allowable value of this ratio using Eq. (16.5) as

$$\frac{(W/L)_5}{(W/L)_1} \leq 1.64$$

Thus, for a given $(W/L)_1$, velocity saturation allows a larger $(W/L)_5$, which is a result of the reduced current in the velocity-saturation region.

16.23 Without taking the body effect into account, Eq. (16.5) applies with $V_{tn} = V_{t0}$, thus

$$\frac{(W/L)_a}{(W/L)_n} \leq \frac{1}{\left(1 - \dfrac{0.4}{1.2 - 0.4}\right)^2} - 1$$

$$\Rightarrow \frac{(W/L)_a}{(W/L)_n} \leq 3$$

The body effect will affect the operation of Q_5 whose V_{tn} now will be given by

$$V_{tn} = V_{t0} + \gamma\left(\sqrt{V_{SB} + 2\phi_f} - \sqrt{2\phi_f}\right) \qquad (1)$$

In equilibrium

$$V_{\overline{Q}} = V_{tn}$$

thus

$$V_{SB} = V_{tn}$$

Equation (1) then becomes

$$V_{tn} = V_{t0} + \gamma\left(\sqrt{V_{tn} + 2\phi_f} - \sqrt{2\phi_f}\right)$$

Thus,

$$V_{tn} = 0.4 + 0.2\left(\sqrt{V_{tn} + 0.88} - \sqrt{0.88}\right)$$

$V_{tn} - 0.4 + 0.2\sqrt{0.88} = 0.2\sqrt{V_{tn} + 0.88}$

$V_{tn} - 0.212 = 0.2\sqrt{V_{tn} + 0.88}$

Squaring both sides and collecting terms results in the quadratic equation

$V_{tn}^2 - 0.464V_t + 0.0097 = 0$

whose physically meaningful solution is

$V_{tn} = 0.44$ V

Substituting this value in Eq. (16.1) and $V_{\overline{Q}} = V_{tn} = 0.44$, we obtain

$$I_5 = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)_5 (1.2 - 0.44 - 0.44)^2$$

$$= 0.0512(\mu_n C_{ox})\left(\frac{W}{L}\right)_5 \qquad (1)$$

Substituting $V_{tn} = V_{t0} = 0.4$ V and $V_{\overline{Q}} = 0.44$ V in Eq. (16.2), we obtain

$$I_1 = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(1.2 - 0.4)0.44 - \frac{1}{2}(0.44)^2\right]$$

$$= 0.2552(\mu_n C_{ox})\left(\frac{W}{L}\right)_1 \qquad (2)$$

Equating I_5 from Eq. (1) to I_1 from Eq. (2) gives

$$\frac{(W/L)_5}{(W/L)_1} = 5$$

or

$$\frac{(W/L)_5}{(W/L)_1} \leq 5$$

Thus, the body effect in Q_1 enables us to use a larger (W/L) for Q_5. This is a result of the reduced current due to the increased V_{tn} and also of the increased value allowable for $V_{\overline{Q}}$.

16.24 (a) Using Eq. (16.5), we obtain

$$\frac{(W/L)_a}{(W/L)_n} \leq \frac{1}{\left(1 - \dfrac{V_{tn}}{V_{DD} - V_{tn}}\right)^2} - 1$$

$\Rightarrow (W/L)_a \leq 3$

(b) For $(W/L)_5 = 1$, $V_{\overline{Q}}$ can be found using Eq. (16.3) as

$$\frac{V_{\overline{Q}}}{V_{DD} - V_{tn}} = 1 - 1/\sqrt{1 + \frac{(W/L)_5}{(W/L)_1}}$$

$\Rightarrow V_{\overline{Q}} = 0.23$ V

The value of I_5 can now be obtained from Eq. (16.1) as

$$I_5 = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)_5 (V_{DD} - V_{tn} - V_{\overline{Q}})^2$$

$$= \frac{1}{2}500 \times 1(1.2 - 0.4 - 0.23)^2$$

$= 81.2$ μA

Finally, the read delay, Δt, can be calculated using Eq. (16.7),

$$\Delta t = \frac{C_{\overline{B}}\Delta V}{I_5}$$

$$= \frac{2 \times 10^{-12} \times 0.2}{81.2 \times 10^{-6}} = 4.93 \text{ ns}$$

(c) If $(W/L)_5 = 3$ is utilized, $V_{\overline{Q}}$ will be 0.4 V, and I_5 can be determined using Eq. (16.1) as

$$I_5 = \frac{1}{2} \times 500 \times 3(1.2 - 0.4 - 0.4)^2$$

$= 120$ μA

and the read delay becomes

$$\Delta t = \frac{2 \times 10^{-12} \times 0.2}{120 \times 10^{-6}} = 3.33 \text{ ns}$$

16.25 By analogy to the description of the process of writing a 0, the relevant part of the circuit for writing a 1 is shown in Fig. 1.

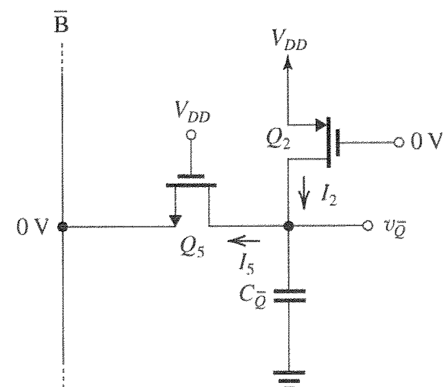


Figure 1

Initially, the cell is storing a zero, thus $v_{\overline{Q}} = V_{DD}$ and $v_Q = 0$ V. To write a 1, we lower the $\overline{B}$ line 0 V and select the cell by raising the word line (the voltage at the gate of the access transistor Q_5) to V_{DD}. We assume that the voltage at the gate of Q_2 (which is v_Q) will remain low for a while.

Transistor Q_5 will conduct and its current I_5 will discharge the small capacitor $C_{\overline{Q}}$ and $v_{\overline{Q}}$ will decrease. This will allow Q_2 to conduct.

This process continues until $v_{\overline{Q}}$ is pulled to the threshold voltage of transistor Q_3 which causes the latch to toggle. That is, equilibrium will be reached when

$$V_{\overline{Q}} = V_{tn}$$

and,

$$I_2 = I_5$$

At this point, Q_2 will be operating at the edge of saturation and its current will be given by Eq. (16.8) with I_4 replaced by I_2 and $(W/L)_4$ replaced by $(W/L)_2$. Simultaneously, Q_5 will be operating in the triode region and and its current will be given by Eq. (16.9) with I_6 replaced by I_5, $(W/L)_6$ replaced by $(W/L)_3$, and V_Q replaced by $V_{\overline{Q}}$.

It follows that the process for writing a 1 is similar to that for writing a 0, and the condition on the (W/L) ratios will be identical to that in Eq. (16.11),

16.26 Using Eq. (16.11), we obtain

$$\frac{(W/L)_p}{(W/L)_a} \leq 4\left[1-\left(1-\frac{0.4}{1.2-0.4}\right)^2\right]$$

$$\Rightarrow (W/L)_p \leq 3(W/L)_a$$

16.27 Using Eq. (16.11), we obtain

$$\frac{(W/L)_p}{(W/L)_a} \leq 4\left[1-\left(1-\frac{0.5}{2.5-0.5}\right)^2\right]$$

$$\Rightarrow (W/L)_p \leq 1.75(W/L)_a$$

16.28 $\mu_n \approx 4\mu_p$

$V_Q \leq V_m$

(a) $\left(\dfrac{0.5}{2.5-0.5}\right) = 0.25$. From Eq. (16.10),

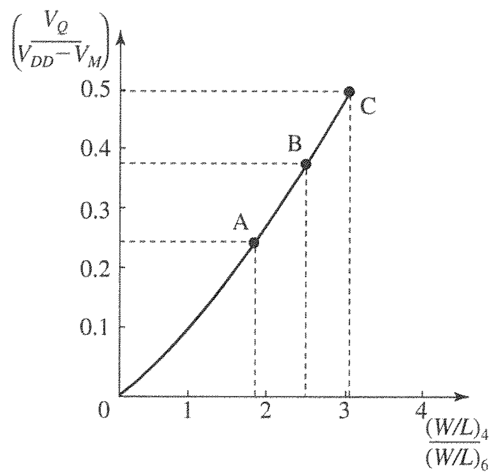
$$\rightarrow \frac{(W/L)_4}{(W/L)_6} = 1.75$$

(b) $\left(\dfrac{0.5}{1.8-0.5}\right) = 0.39$. From Eq. (16.10),

$$\rightarrow \frac{(W/L)_4}{(W/L)_6} = 2.5$$

(c) $\left(\dfrac{0.4}{1.2-0.4}\right) = 0.5$. From Eq. (16.10),

$$\rightarrow \frac{(W/L)_4}{(W/L)_6} = 3.0$$



16.29 Equation (16.5) provides the constraint

$$\frac{(W/L)_a}{(W/L)_n} \leq \frac{1}{\left(1-\dfrac{0.4}{1.2-0.4}\right)^2} - 1$$

that is,

$$(W/L)_a \leq 3(W/L)_n \qquad (1)$$

Equation (16.11) provides the constraint

$$\frac{(W/L)_p}{(W/L)_a} \leq 4\left[1-\left(1-\frac{0.4}{1.2-0.4}\right)^2\right]$$

that is,

$$(W/L)_p \leq 3(W/L)_a \qquad (2)$$

Selecting $(W/L)_n = 1$, then Eq. (1) gives

$$(W/L)_a \leq 3$$

which permits selecting

$$(W/L)_a = 1$$

Finally, the constraint in Eq. (2) becomes

$$(W/L)_p \leq 3$$

which permits selecting

$$(W/L)_p = 1$$

Thus our minimum-area design has $L = 0.13\ \mu\text{m}$ and

$$\left(\frac{W}{L}\right)_n = \left(\frac{W}{L}\right)_p = \left(\frac{W}{L}\right)_a = 1$$

16.30 From Eq. (16.14) or (16.15), we have

$$\Delta V \simeq \frac{C_S}{C_B}\left(\frac{V_{DD}}{2}\right)$$

$$25 \times 10^{-3} = \frac{35}{C_B} \times \frac{1.2}{2}$$

$$\Rightarrow C_B = 840 \text{ fF}$$

This is the maximum allowable value for C_B.

Capacitive load due to cells $= 840 - 20 = 820$ fF

$$\text{Maximum number of cells} = \frac{820}{0.8}$$

$= 1025$ or likely 1024.

Number of row address bits requiring

$= \log_2 1024 = 10$

If the sense amplifier gain is increased by a factor of 4, ΔV is reduced by a factor of 4, and the maximum C_B increases by a factor of 4. Thus, the number of cells can be increased approximately by a factor of 4 to 4096 bits, requiring two more bits for the word address, for a total of 12 bits.

16.31 If the memory array has n columns, it has $2n$ rows and $2n^2$ cells. Refresh time is

$$2n \times 10 \times 10^{-9} = (1 - 0.98) \times 10 \times 10^{-3}$$

$$\Rightarrow n = 10^4 \simeq 8 \text{ kbits}$$

(Recall that 1 kbit = 1024 bits)

Thus the cell array is 8 kbits $\times$ 16 kbits or 128 Mbits.

16.32 A 1-Mbit-square array has 1024 words $\times$ 1024 bits. Thus the capacitance of the bit line, C_B, is

$$C_B = 1024 \times 0.5 + 12$$

$= 524$ fF

$$\Delta V(1) \simeq \frac{C_S}{C_B}\left(\frac{V_{DD}}{2}\right)$$

$$= \frac{30}{524}\left(\frac{1.2}{2}\right) = 34.4 \text{ mV}$$

$$\Delta V(0) \simeq -\frac{C_S}{C_B}\left(\frac{V_{DD}}{2}\right) = -34.4 \text{ mV}$$

16.33 The storage capacitor C_S loses 0.2 V in 12 ms as a result of the leakage current I, thus

$$I \times 12 \text{ ms} = C_S \times 0.2 \text{ V}$$

$$\Rightarrow I = \frac{30 \times 10^{-15} \times 0.2}{12 \times 10^{-3}}$$

$= 0.5$ pA

16.34 From Eq. (16.16), we have

$$v_B = \frac{V_{DD}}{2} + \Delta V(1)e^{(G_m/C_B)t} \qquad (1)$$

$$0.9V_{DD} = 0.5V_{DD} + 0.05e^{(G_m/C_B)\times 2\times 10^{-9}}$$

$$\frac{G_m}{C_B} \times 2 \times 10^{-9} = \ln\left(\frac{0.4\ V_{DD}}{0.05}\right)$$

$$G_m = \frac{C_B}{2 \times 10^{-9}} \times \ln\left(\frac{0.4 \times 1.2}{0.05}\right)$$

$$= \frac{1 \times 10^{-12}}{2 \times 10^{-9}} \times 2.26 = 1.13 \text{ mA/V}$$

$$G_m = g_{mn} + g_{mp}$$

Since the inverters are matched, we have

$$g_{mn} = g_{mp} = \frac{G_m}{2} = \frac{1.13}{2} = 0.565 \text{ mA/V}$$

But,

$$g_{mn} = \mu_n C_{ox}\left(\frac{W}{L}\right)_n \left(\frac{V_{DD}}{2} - V_{tn}\right)$$

$$0.565 \times 10^{-3} = 500 \times 10^{-6}\left(\frac{W}{L}\right)_n (0.6 - 0.4)$$

$$\Rightarrow \left(\frac{W}{L}\right)_n = 5.65$$

$$W_n = 5.65 \times 0.13 = 0.73\ \mu\text{m}$$

$$W_p = 4W_n = 2.94\ \mu\text{m}$$

If the input signal is doubled, we can use Eq. (1) to determine t as follows:

$$0.9V_{DD} = 0.5V_{DD} + 0.1e^{(G_m/C_B)t}$$

$$\frac{G_m}{C_B}t = \ln\left(\frac{0.4\ V_{DD}}{0.1}\right)$$

$$t = \frac{1 \times 10^{-12}}{1.13 \times 10^{-3}} \ln(4 \times 1.2)$$

$= 1.4$ ns

16.35 The value of G_m is given by

$$G_m = g_{mn} + g_{mp}$$

Since the inverters have a matched design, then

$$g_{mn} = g_{mp}$$

and

$$g_{mn} = \mu_n C_{ox}\left(\frac{W}{L}\right)_n \left(\frac{V_{DD}}{2} - V_{tn}\right)$$

$$= 500\left(\frac{0.26}{0.13}\right)\left(\frac{1.2}{2} - 0.4\right)$$

$= 200\ \mu$A/V $= 0.2$ mA/V

Thus,

$G_m = 0.4$ mA/V

Assume a read 1 operation,

$$v_B = 0.5V_{DD} + \Delta V(1)e^{(G_m/C_B)t}$$

Thus,

$$0.9V_{DD} = 0.5V_{DD} + \Delta V(1)e^{(G_m/C_B)t}$$

$$\frac{0.4V_{DD}}{\Delta V(1)} = e^{(G_m/C_B)t_1}$$

$$\Rightarrow \Delta V(1) = 0.4V_{DD}e^{-(G_m/C_B)t_d} \qquad (1)$$

Substituting $V_{DD} = 1.2$ V, $G_m = 0.4$ mA/V, $C_B = 0.4$ pF, and $t_d = 1$ ns, we obtain

$$\Delta V(1) = 0.177 \text{ V}$$

Thus the signal between the B and $\overline{B}$ lines is

$$2\Delta V(1) = 0.353 \text{ V} = 353 \text{ mV}$$

If the time can be relaxed by 1 ns, then $t_1 = 2$ ns and

$$\Delta V(1) = 0.065 \text{ V}$$

and the required signal between the B and $\overline{B}$ becomes

$$2\Delta V(1) = 130 \text{ mV}$$

If instead we have $\Delta V(1) = 0.177$ V and $t_1 = 2$ ns, we can see from Eq. (1) that C_B can be increased by the same factor t_1 is increased, that is, by a factor of 2. Thus C_B can become

$$C_B = 2 \times 0.4 = 0.8 \text{ pF}$$

Thus the bit line length can be increased by 100%, i.e., doubled. Doubling the length of the bit line will double the delay time required to charge the bit-line capacitance to 4 ns.

16.36 (a) Using Eq. (16.16)), we write

$$v_B = 0.5V_{DD} + \Delta V(1)e^{(G_m/C_B)t}$$

$$0.9V_{DD} = 0.5V_{DD} + \left(\frac{\Delta V}{2}\right)e^{(G_m/C_B)t_d}$$

$$\Rightarrow t_d = \frac{C_B}{G_m}\ln\left(\frac{0.8V_{DD}}{\Delta V}\right) \qquad \text{Q.E.D.} \qquad (1)$$

An identical expression can be obtained for the case a 0 is being read.

(b) If t_d is to be reduced to one-half its original value, G_m must be increased by a factor of 2. This can be achieved by doubling the width of all transistors (because g_{mn} and g_{mb} are proportional to $(W/L)_n$ and $(W/L)_p$, respectively).

(c) Consider Eq. (1) in two situations: (1) $\Delta V = 0.2$ V and $G_m = G_{m1}$, (2) $\Delta V = 0.1$ V and $G_m = G_{m2}$. If in both cases the same value of t_d results, then

$$\frac{1}{G_{m1}}\ln\left(\frac{0.8 \times 1.2}{0.2}\right) = \frac{1}{G_{m2}}\ln\left(\frac{0.8 \times 1.2}{0.1}\right)$$

$$\Rightarrow \frac{G_{m2}}{G_{m1}} = 1.44$$

To increase G_m by a factor of 1.44, the widths of all transistors must be increased by a factor of 1.44.

16.37 To meet the delay specification in both cases of a stored 1 and a stored 0, we must design for the worst case, which is the case of a stored 1. The initial voltage that develops on the half-bit line is +40 mV. Thus the voltage of the half-bit line will rise exponentially from $(V_{DD}/2)$, as

$$v_B = \frac{V_{DD}}{2} + \Delta V(1)e^{(G_m/C_B)t}$$

To develop a differential output voltage of 1 V and recalling that the bit-line on the dummy-cell side remains at a constant voltage of $(V_{DD}/2)$, we required v_B to rise to $(V_{DD}/2) + 1$, thus

$$1 \text{ V} = 0.04e^{(G_m/C_B)t_d}$$

where $t_d = 2$ ns and $C_B = 0.5$ pF. The value of G_m can be found from

$$\frac{G_m}{0.5 \times 10^{-12}} \times 2 \times 10^{-9} = \ln\left(\frac{1}{0.04}\right)$$

$$\Rightarrow G_m = 0.8 \text{ mA/V}$$

Now,

$$G_m = g_{mn} + g_{mp}$$

Since the inverters have a matched design,

$$g_{mn} = g_{mp} = 0.4 \text{ mA/V}$$

and

$$g_{mn} = \mu_n C_{ox}\left(\frac{W}{L}\right)_n\left(\frac{V_{DD}}{2} - V_t\right)$$

$$0.4 = 0.3 \times \left(\frac{W}{L}\right)_n (0.9 - 0.5)$$

$$\Rightarrow \left(\frac{W}{L}\right)_n = 3.33$$

and

$$\left(\frac{W}{L}\right)_p = 4\left(\frac{W}{L}\right)_n = 13.32$$

Now, if a 0 is read, then

$$v_B = \frac{V_{DD}}{2} - \Delta V(0)e^{(G_m/C_B)t_d}$$

$$\frac{V_{DD}}{2} - 1 = \frac{V_{DD}}{2} - 0.1e^{(G_m/C_B)t_d}$$

$$\Rightarrow t_d = \frac{C_B}{G_m} \ln 10$$

$$= \frac{0.5 \times 10^{-12}}{0.8 \times 10^{-3}} \times 2.3 = 1.44 \text{ ns}$$

which, as expected, is less than the worst-case value of 2 ns (because of the larger ΔV). If a 1 is read, then

$$v_B = \frac{V_{DD}}{2} + \Delta V(1)e^{(G_m/C_B)t_d}$$

$$\frac{V_{DD}}{2} + 1 = \frac{V_{DD}}{2} + 0.04e^{(G_m/C_B)t_d}$$

$$\Rightarrow t_d = \frac{C_B}{G_m} \ln 25$$

$$= \frac{0.5 \times 10^{-12}}{0.8 \times 10^{-3}} \times 3.2 = 2 \text{ ns}$$

as expected.

16.38 Using Eq. (16.18), we have

$$\Delta t = \frac{CV_{DD}}{I}$$

$$0.5 \times 10^{-9} = \frac{50 \times 10^{-15} \times 1.2}{I}$$

$$\Rightarrow I = 120\ \mu\text{A}$$

$$P_D = V_{DD} I$$

$$= 1.2 \times 120 = 144\ \mu\text{W}$$

16.39

(a) $V_{D1} = V_{D2} = V_{DD} - V_t - V_t = V_{DD} - 2V_t$

$= 1.2 - 0.8 = 0.4$ V

(b) $\Delta V = \sqrt{2}\, V_{OV}$

$140 = \sqrt{2}\, V_{OV}$

$\Rightarrow V_{OV} \simeq 100$ mV $= 0.1$ V

$V_{S1,2} = V_{DD} - V_t - V_{GS}$

$= V_{DD} - V_t - V_t - V_{OV}$

$= 1.2 - 0.4 - 0.4 - 0.1 = 0.3$ V

(c) $\Delta t = \dfrac{CV_{DD}}{I}$

$$0.5 \times 10^{-9} = \frac{55 \times 10^{-15} \times 1.2}{I}$$

$\Rightarrow I = 132\ \mu$A

(d) $\dfrac{I}{2} = \dfrac{1}{2}\mu_n C_{ox}\left(\dfrac{W}{L}\right)_{1,2} V_{OV}^2$

$$\frac{0.132}{2} = \frac{1}{2} \times 0.5 \times \left(\frac{W}{L}\right)_{1,2} \times 0.1^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{1,2} = 26.4$$

For Q_3 and Q_4,

$$\frac{132}{2} = \frac{1}{2} \times 125 \left(\frac{W}{L}\right)_{3,4} (1.2 - 0.4 - 0.4)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 6.6$$

(e) $0.132 = \dfrac{1}{2} \times 0.5 \times \left(\dfrac{W}{L}\right)_5 \times 0.1^2$

$$\left(\frac{W}{L}\right)_5 = 52.8$$

$V_R = V_t + V_{OV} = 0.5$ V

16.40 Since $2^{10} = 1024$, there are 10 address bits.

The decoder has 1024 output lines. The NOR array requires 20 input lines (for the 10 address bits and their complements).

Each output line has one PMOS transistor; for a total of 1024 PMOS transistors.

Each output line has 10 NMOS transistors for a total of $1024 \times 10 = 10{,}240$ NMOS transistors.

The total number of NMOS and PMOS transistors is 11,264.

16.41 In a 1-Mbit-square array, there are 1024 columns, requiring 10 address bits.

1024 NMOS pass transistors are needed in the multiplexer. The NOR decoder has 1024 output lines; each output line has one PMOS transistor, for a total of 1024 PMOS transistors. Each output line has 10 NMOS transistors, for a total 10,240 NMOS transistors. Total number of transistors

$= 1024 + 1024 + 10{,}240 = 12{,}288$

16.42 A square 1-Mbit array has 1024 columns, requiring 10 address bits. Ten levels of pass gates are required.

Using the expression given to the answer of Exercise 16.13, we have

Total number of transistors $= 2(2^N - 1)$

$= 2(2^{10} - 1) = 2046$

16.43 Refer to Fig. 1.

$$T = (3t_{PHL} + 2t_{PLH}) + (2t_{PHL} + 3t_{PLH})$$

$$= (3 \times 2 + 2 \times 3) + (2 \times 2 + 3 \times 3)$$

$$= 12 + 13 = 25 \text{ ns}$$

$$f = \frac{1}{T} = \frac{1}{25 \times 10^{-9}} = 40 \text{ MHz}$$

The output is high for 12 of the 25-ns period = 48% of the cycle. Note that

$$t_P = \frac{1}{2}(t_{PLH}) + (t_{PHL})$$

$$= \frac{1}{2}(3 + 2) = 2.5 \text{ ns}$$

$$T = 2Nt_P = 2 \times 5 \times 2.5 = 25 \text{ ns}$$

as found graphically.

This figure belongs to Problem 16.43.

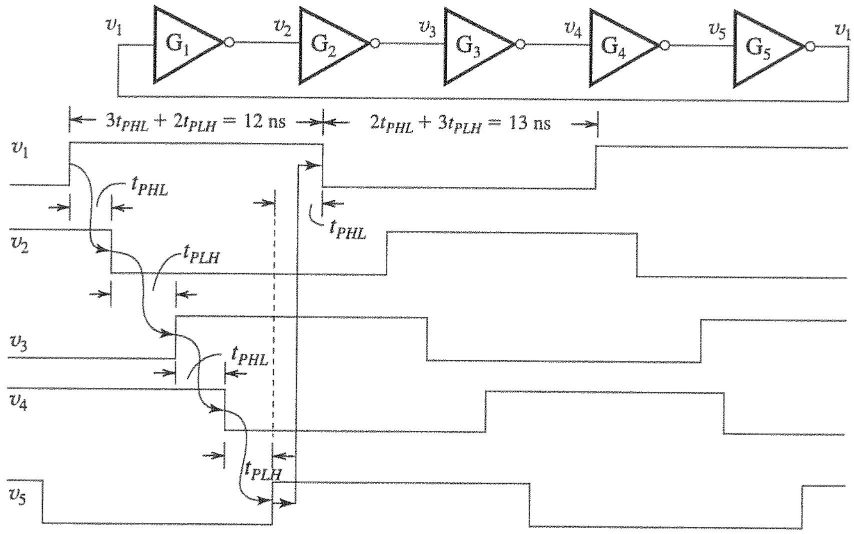


Figure 1

16.44 $f = \dfrac{1}{2Nt_P}$

$$20 \times 10^6 = \frac{1}{2 \times 9t_P}$$

$\Rightarrow t_P = 2.78$ ns

16.45 We need four inverters to implement the 10-ns delay block.

16.46 Refer to Fig. 16.30 and recall that a cell without a transistor is storing a logic 1 while a cell with a transistor is storing a logic 0. Thus the eight words, W_1 to W_8, are:

1101, 1111, 1110, 0110, 0101, 0111, 1001, 1011

16.47 Need $z = xy$

W	X	Y	Z
0	00	00	0000
1	00	01	0000
2	00	10	0000
3	00	11	0000
4	01	00	0000
5	01	01	0001
6	01	10	0010
7	01	11	0011
8	10	00	0000
9	10	01	0010
10	10	10	0100
11	10	11	0110
12	11	00	0000
13	11	01	0011
14	11	10	0110
15	11	11	1001

See Figure on next page.

16.48

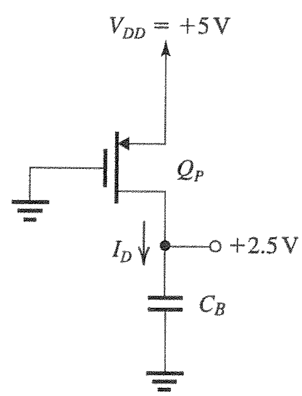


Figure 1

(a) Figure 1 shows the relevant part of the circuit during the precharge phase. Note that Q_P is operating in the triode region, thus

$$I_D = 30 \times \frac{12}{1.2}\left[(5-1)2.5 - \frac{1}{2}(2.5)^2\right]$$

$= 2.06$ mA

$$\Delta t = \frac{C_B V_{DD}}{I_D}$$

$$= \frac{1 \times 10^{-12} \times 5}{2.06 \times 10^{-3}} = 2.4 \text{ ns}$$

(b) $t_r = 2.2\tau$

$= 2.2CR$

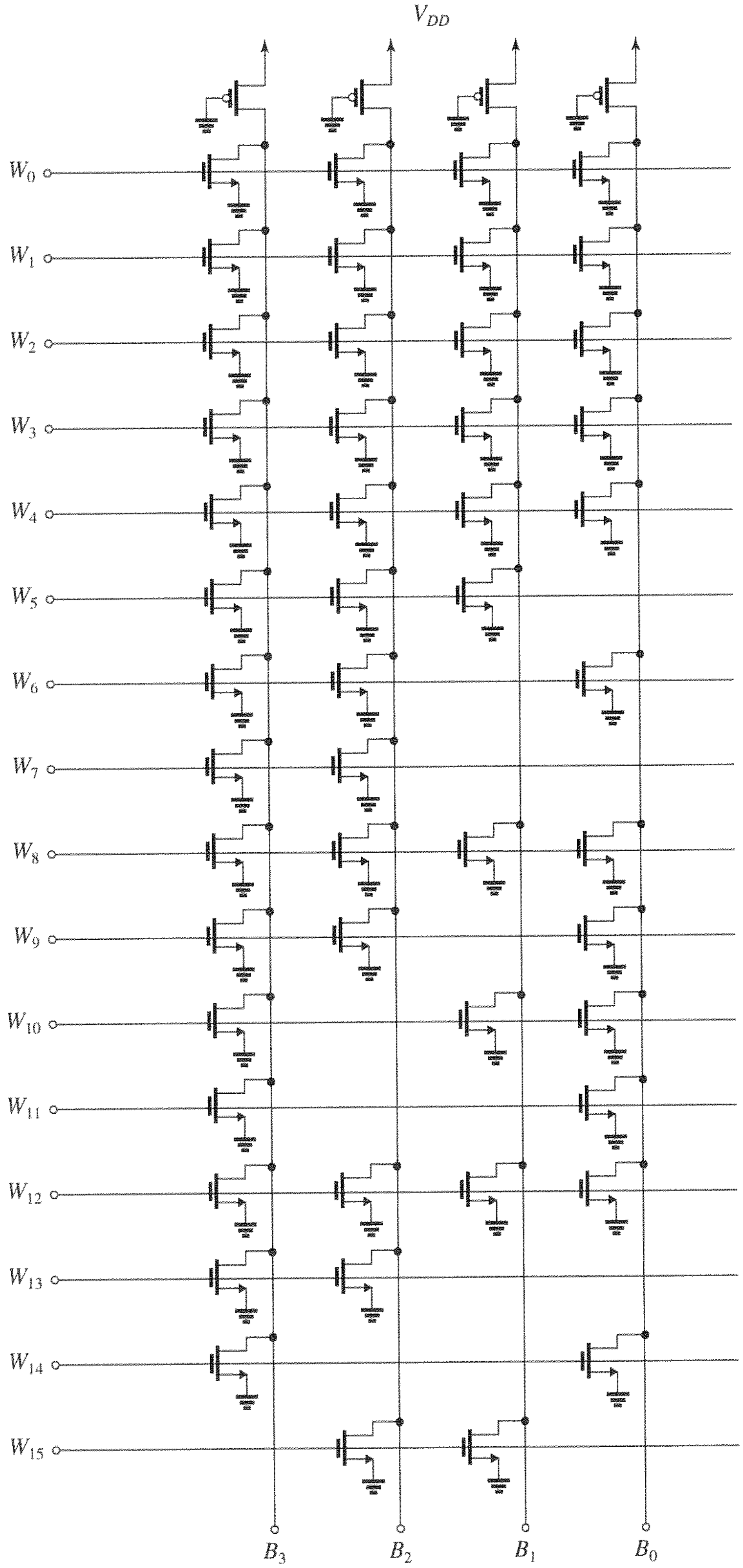
$= 2.2 \times 2 \times 10^{-12} \times 5 \times 10^3$

$= 22$ ns

$v_W(t = \tau) = V_{DD}(1 - e^{-1})$

$= 3.16$ V

This figure belongs to Problem 16.47.



This figure belongs to Problem 16.48, part (b).

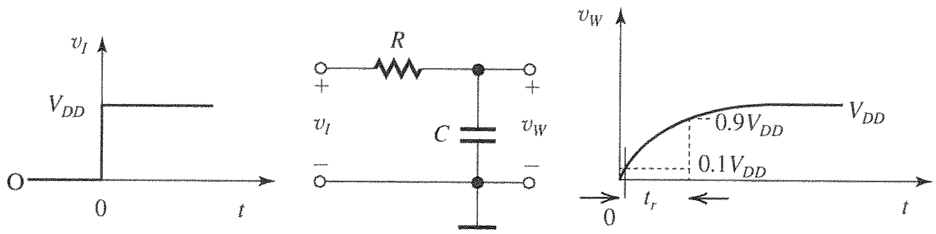


Figure 2

(c)

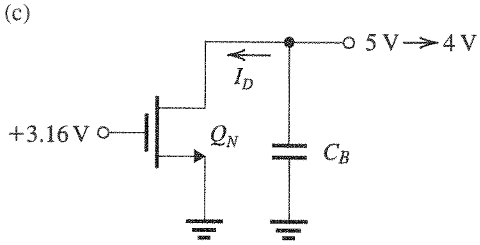


Figure 3

Refer to Fig. 3 and observe that Q_N is operating in saturation, thus

$$I_D = \frac{1}{2} \times 90 \times \frac{3}{1.2}(3.16 - 1)^2$$

$= 0.52$ mA

$$\Delta t = \frac{C_B \Delta V}{I_D}$$

$$= \frac{1 \times 10^{-12} \times 1}{0.52 \times 10^{-3}} = 1.9 \text{ ns}$$

16.49 $Q = C\Delta V$

$= 25 \times 10^{-15} \times 1 = 25$ fC

$$\text{Number of electrons} = \frac{25 \times 10^{-15}}{1.6 \times 10^{-19}}$$

$= 156{,}250$

17.1 $T(s) = \frac{\omega_0}{s + \omega_0}$, $T(j\omega) = \frac{\omega_0}{j\omega + \omega_0}$

$$|T(j\omega)| = \frac{\omega_0}{\sqrt{\omega_0^2 + \omega^2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$\phi(\omega) \equiv \tan^{-1}\left[\frac{\text{Im}(T(j\omega))}{\text{Re}(T(j\omega))}\right]$$

$$= -\tan^{-1}(\omega/\omega_O)$$

$$G = 20 \log_{10} |T(j\omega)|$$

$$A = -20 \log_{10} |T(j\omega)|$$

ω	$\lvert T(j\omega)\rvert$ [V/V]	G [dB]	A [dB]	ϕ [degrees]
0	1	0	0	0
$0.5\omega_0$	0.8944	–0.97	0.97	–26.57
ω_0	0.7071	–3.01	3.01	–45.0
$2\omega_0$	0.4472	–6.99	6.99	–63.43
$5\omega_0$	0.1961	–14.1	14.1	–78.69
$10\omega_0$	0.0995	–20.0	20.0	–84.29
$100\omega_0$	0.010	–40.0	40.0	–89.43

17.2 $T(s) = \frac{2\pi \times 10^4}{s + 2\pi \times 10^4}$

$$T(j\omega) = \frac{2\pi \times 10^4}{2\pi \times 10^4 + j\omega}$$

$$= \frac{1}{1 + j[\omega/2\pi \times 10^4]}$$

$$|T(j\omega)| = 1\Bigg/\sqrt{1 + \left(\frac{\omega}{2\pi \times 10^4}\right)^2}$$

$$\phi(\omega) = -\tan^{-1}(\omega/2\pi \times 10^4)$$

(a) $f = 1$ kHz

$\omega = 2\pi \times 10^3$ rad/s

$|T| = 1/\sqrt{1 + 0.01} \simeq 0.995$ V/V

$\phi = -\tan^{-1}(0.1) = -5.7°$

Peak amplitude of output sinusoid = 0.995 V

Phase of output relative to that of input = $-5.7°$.

(b) $f = 10$ kHz

$\omega = 2\pi \times 10^4$ rad/s

$|T| = 1/\sqrt{2} = 0.707$ V/V

$\phi = -\tan^{-1}(1) = -45°$

Peak amplitude of output sinusoid = 0.707 V

Phase of output relative to that of input = $-45°$.

(c) $f = 100$ kHz

$\omega = 2\pi \times 10^5$ rad/s

$|T| = 1/\sqrt{1 + 100} \simeq 0.1$ V/V

$\phi = -\tan^{-1}(10) = -84.3°$

Peak amplitude of output sinusoid = 0.1 V

Phase of output relative to that of input = $-84.3°$.

(d) $f = 1$ MHz

$\omega = 2\pi \times 10^6$ rad/s

$|T| = 1/\sqrt{1 + 10^4} = 0.01$ V/V

$\phi = -\tan^{-1}(100) = -89.4°$

Peak amplitude of output sinusoid = 0.01 V

Phase of output relative to that of input = $-89.4°$.

17.3 $T(s) = \frac{1}{(s+1)(s^2 + s + 1)}$

$$= \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$T(j\omega) = \left[j(2\omega - \omega^3) + (1 - 2\omega^2)\right]^{-1}$$

$$|T(j\omega)| = \left[(2\omega - \omega^3)^2 + (1 - 2\omega^2)^2\right]^{-\frac{1}{2}}$$

$$= \left[4\omega^2 - 4\omega^4 + \omega^6 + 1 - 4\omega^2 + 4\omega^4\right]^{-\frac{1}{2}}$$

$$= \left[1 + \omega^6\right]^{-\frac{1}{2}}$$

$$= \frac{1}{\sqrt{1 + \omega^6}} \qquad \text{Q.E.D.}$$

For phase angle:

$$\phi(\omega) = \tan^{-1}\left[\frac{\text{Im}(T(j\omega))}{\text{Re}(T(j\omega))}\right]$$

$$= -\tan^{-1}\left[\frac{2\omega - \omega^3}{1 - 2\omega^2}\right]$$

For $\omega = 0.1$ rad/s:

$|T(j\omega)| = (1 + 0.1^6)^{-1/2} \simeq 1$

$\phi(\omega) = -11.5° = -0.20$ rad

For $\omega = 1$ rad/s:

$|T(j\omega)| = (1 + 1^6)^{-1/2} = 1/\sqrt{2} = 0.707$

$\phi = -\tan^{-1}\left(\frac{1}{-1}\right) = -135° = -2.356$ rad

Note: $G = -3$ dB

For $\omega = 10$ rad/s:

$$|T(j\omega)| = \left(1 + 10^6\right)^{-1/2} = 0.001$$

$$\phi = -\tan^{-1}\left[\frac{2(10) - 10^3}{1 - 2(10^2)}\right]$$

$$= -\tan^{-1}\left[\frac{-980}{-199}\right]$$

$$= -\left[180^\circ + \tan^{-1}\left(\frac{980}{199}\right)\right]$$

$$= -258.5^\circ$$

$$= -4.512 \text{ rad}$$

Now consider an input of $A \sin \omega t$ to $T(s)$. The output is then given by

$$A\,|T(j\omega)| \sin(\omega t + \phi(\omega))$$

Using this result, the output to each of the following inputs will be:

INPUT	OUTPUT
$10 \sin(0.1t)$	$10 \sin(0.1t - 0.2)$
$10 \sin(1t)$	$7.07 \sin(t - 2.356)$
$10 \sin(10t)$	$0.01 \sin(10t - 4.512)$

17.4 At $\omega = 0$, we have

$$20 \log |T| = 0 \text{ dB}$$

$$\Rightarrow |T| = 1 \text{ V/V}$$

At $\omega = \omega_p$, we have

$$20 \log |T| = -A_{max} = -0.2 \text{ dB}$$

$$\Rightarrow |T| = 0.977 \text{ V/V}$$

At $\omega = \omega_s$, we have

$$20 \log |T| = -A_{min} = -60 \text{ dB}$$

$$\Rightarrow |T| = 0.001 \text{ V/V}$$

17.5 Refer to Fig. 7.3.

$$A_{max} = 20 \log 1.05 = 0.42 \text{ dB}$$

$$A_{min} = 20 \log\left(\frac{1}{0.0005}\right)$$

$$= 66 \text{ dB}$$

$$\text{Selectivity factor} \equiv \frac{f_s}{f_p}$$

$$= \frac{5}{4} = 1.25$$

17.6

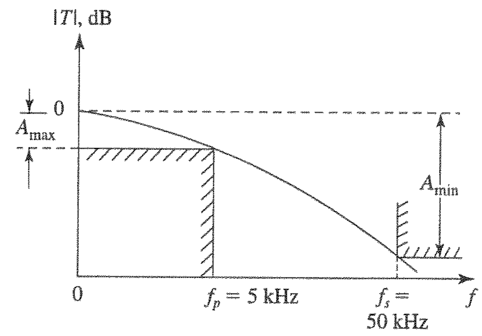


Figure 1

Refer to Fig. 1.

$$T(s) = \frac{2\pi \times 10^4}{s + 2\pi \times 10^4}$$

$$T(j\omega) = \frac{1}{1 + j\dfrac{\omega}{2\pi \times 10^4}}$$

$$|T| = 1\Big/\sqrt{1 + \left(\frac{f}{10^4}\right)^2}$$

At $f = f_p = 5$ kHz, we have

$$|T| = 1\Big/\sqrt{1 + \left(\frac{5 \times 10^3}{10^4}\right)^2} = 0.894$$

Thus,

$$A_{max} = -20 \log 0.894 = 0.97 \text{ dB}$$

At $f = f_s = 10 f_p = 50$ kHz, we have

$$|T| = 1\Big/\sqrt{1 + \left(\frac{50 \times 10^3}{10^4}\right)^2} = 0.196$$

$$A_{min} = 20 \log\left(\frac{1}{0.196}\right) = 14.15 \text{ dB}$$

17.7 $\tau = 1 \text{ s} \Rightarrow \omega_0 = \dfrac{1}{\tau} = 1$ rad/s

Thus,

$$T(s) = \frac{1}{s + 1}$$

where we have also used the given information on the dc transmission being unity.

$$|T| = \frac{1}{\sqrt{1 + \omega^2}}$$

Since $A_{max} = 2$ dB, we have

$$20 \log |T(j\omega_p)| = -2 \text{ dB}$$

$$\Rightarrow |T(j\omega_p)| = 0.794$$

Thus,

$$\frac{1}{\sqrt{1 + \omega_p^2}} = 0.794$$

$$\Rightarrow \omega_p = 0.765 \text{ rad/s}$$

Since $A_{min} = 12$ dB, we have

$20 \log|T(j\omega_s)| = -12$ dB

$\Rightarrow |T(j\omega_s)| = 0.25$

Thus,

$$\frac{1}{\sqrt{1+\omega_s^2}} = 0.25$$

$\Rightarrow \omega_s = 3.85$ rad/s

$$\text{Selectivity factor} = \frac{\omega_s}{\omega_p} = \frac{3.85}{0.785} = 5$$

17.8 See Fig. 1.

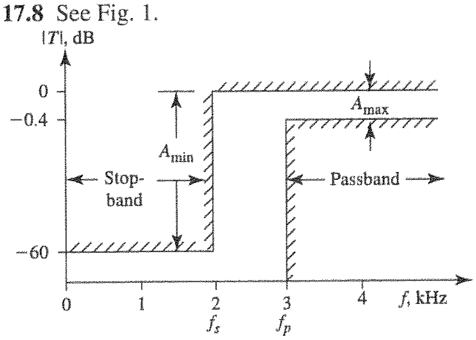


Figure 1

17.9 See Fig. 1.

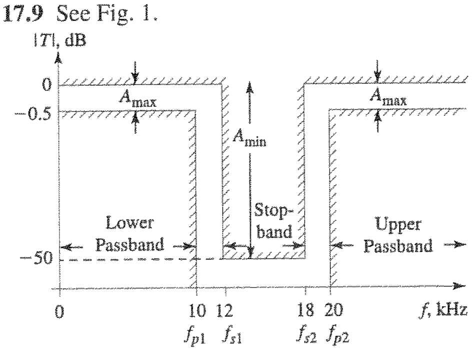


Figure 1

17.10

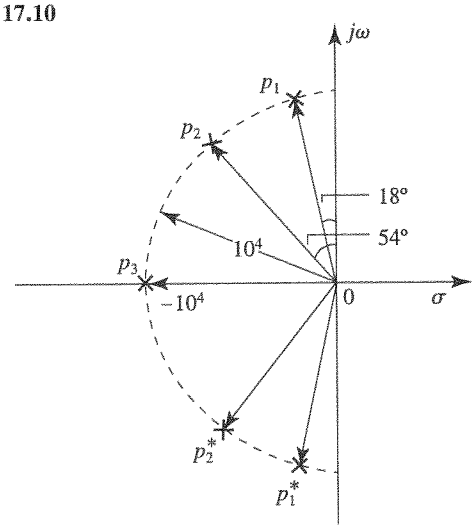


Figure 1

Figure 1 shows the location of the five poles in the s plane. The real-axis pole at $s = -10^4$ gives rise to a factor $(s + 10^4)$. The pair of conjugate poles p_1 and p_1^* are at

$$s = -10^4 \sin 18^\circ \pm j10^4 \cos 18^\circ$$

$$= -0.309 \times 10^4 \pm j0.951 \times 10^4$$

This pole pair gives rise to a factor

$$= (s + 0.309 \times 10^4 + j\,0.951 \times 10^4)(s + 0.309 \times 10^4 - j0.951 \times 10^4)$$

$$= s^2 + 0.618 \times 10^4 s + 10^8$$

The pair of complex conjugate poles p_2 and p_2^* are at

$$s = -10^4 \sin 54^\circ \pm j10^4 \cos 54^\circ$$

$$= -0.809 \times 10^4 \pm j0.588 \times 10^4$$

This pole pair gives rise to a factor

$$= (s + 0.809 \times 10^4 + j\,0.588 \times 10^4)(s + 0.809 \times 10^4 - j0.588 \times 10^4)$$

$$= (s^2 + 1.618 \times 10^4 s + 10^8)$$

Thus the denominator polynomial of $T(s)$ is

$$D(s) = (s + 10^4)(s + 0.618 \times 10^4 s + 10^8)(s^2 + 1.618 \times 10^4 s + 10^8)$$

(a) The filter is a low-pass of the all-pole type,

$$T(s) = \frac{k}{(s + 10^4)(s^2 + 0.618 \times 10^4 s + 10^8) \times (s^2 + 1.618 \times 10^4 s + 10^8)}$$

Since the dc gain is unity, we have

$k = 10^{20}$

Thus,

$$T(s) = \frac{10^{20}}{(s + 10^4)(s^2 + 0.618 \times 10^4 s + 10^8) \times (s^2 + 1.618 \times 10^4 s + 10^8)}$$

(b) The filter is a high pass,

$$T(s) = \frac{ks^5}{(s + 10^4)(s^2 + 0.618 \times 10^4 s + 10^8) \times (s^2 + 1.618 \times 10^4 s + 10^8)}$$

Since the high-frequency ($s \to \infty$) gain is unity, k must be unity, thus

$$T(s) = \frac{s^5}{(s + 10^4)(s^2 + 0.618 \times 10^4 s + 10^8) \times (s^2 + 1.618 \times 10^4 s + 10^8)}$$

17.11 $T(s)$

$$= \frac{k(s^2+4)}{(s+1)(s+0.5-j0.8)\times(s+0.5-j0.8)}$$

$$= \frac{k(s^2+4)}{(s+1)(s^2+s+0.89)}$$

$$T(0) = \frac{4k}{0.89} = 1$$

$$\Rightarrow k = \frac{0.89}{4} = 0.2225$$

Thus,

$$T(s) = \frac{0.2225(s^2+4)}{(s+1)(s+s+0.89)}$$

17.12 $T(s) = \dfrac{k(s^2+4)}{(s+0.25+j)(s+0.25-j)}$

$$= \frac{k(s^2+4)}{s^2+0.5s+1.0625}$$

$$T(0) = \frac{4k}{1.0625} = 1$$

$$\Rightarrow k = \frac{1.0625}{4} = 0.2656$$

$$T(s) = \frac{0.2656(s^2+4)}{s^2+0.5s+1.0625}$$

$$T(\infty) = 0.2656$$

17.13

$$T(s) = \frac{s(s^2+10^6)(s^2+9\times10^6)}{s^6+b_5s^5+b_4s^4+b_3s^2+b_1s+b_0}$$

Note that we started with the numerator factors (which represent the given transmission zeros). We used the fact that there is one transmission zero at $s=\infty$ to write the denominator sixth-order polynomial. Thus,

$N = 6$

A sketch of the magnitude response, $|T|$, is given in Fig. 1.

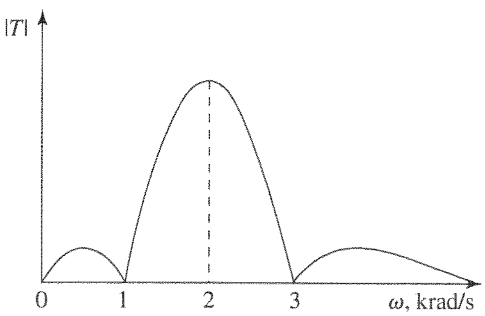


Figure 1

17.14

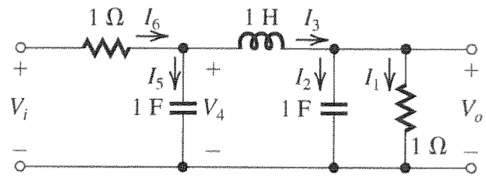


Figure 1

Refer to Fig. 1.

$$I_1 = \frac{V_o}{1} = V_o$$

$$I_2 = sCV_o = s\times1\times V_o = sV_o$$

$$I_3 = I_1 + I_2 = (s+1)V_o$$

$$V_4 = V_o + sLI_3$$

$$= V_o + s\times1(s+1)V_o$$

$$= (s^2+s+1)V_o$$

$$I_5 = s\times1\times V_4$$

$$= s(s^2+s+1)V_o$$

$$I_6 = I_3 + I_5 = (s+1)V_o + s(s^2+s+1)V_o$$

$$= (s^3+s^2+2s+1)V_o$$

$$V_i = V_4 + I_6\times1$$

$$= (s^2+s+1)V_o + (s^3+s^2+2s+1)V_o$$

$$= (s^3+2s^2+3s+2)V_o$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^3+2s^2+3s+2}$$

All the transmission zeros are at $s=\infty$. To find the poles, we have to factor the third-order denominator polynomial. Toward this end, we find by inspection that one of the zeros of the denominator polynomial is at $s=-1$. Thus, the polynomial will have a factor $(s+1)$ and can be written as

$$s^3+2s^2+3s+2 = (s+1)(s^2+as+b)$$

where by equating corresponding terms on both sides we find that

$b = 2$

$a+1 = 2 \Rightarrow a = 1$

Thus,

$$s^3+2s^2+3s+2 = (s+1)(s^2+s+2)$$

and the poles are at

$s = -1$

and at the roots of

$$s^2+s+2 = 0$$

which are

$$s = \frac{-1 \pm \sqrt{1-8}}{2}$$
$$= -0.5 \pm j\left(\sqrt{7}/2\right)$$
$$= -0.5 \pm j1.323$$

17.15 $A_{\max} = 0.5$ dB, $A_{\min} \ge 20$ dB, $\dfrac{\omega_s}{\omega_p} = 1.7$

Using Eq. (17.14), we obtain

$$\epsilon = \sqrt{10^{A_{\max}/10} - 1}$$
$$= \sqrt{10^{0.05} - 1} = 0.3493$$

Using Eq. (17.15), we have

$$A(\omega_s) = 10\ \log[1 + \epsilon^2(\omega_s/\omega_p)^{2N}]$$
$$= 10\ \log[1 + 0.3493^2 \times 1.7^{2N}]$$

For $N = 5$, $A(\omega_s) = 14.08$ dB

For $N = 6$, $A(\omega_s) = 18.58$ dB

For $N = 7$, $A(\omega_s) = 23.15$ dB

Thus, to meet the $A_s \ge 20$ dB specification, we use

$N = 7$

in which case the actual minimum stopband attenuation realized is

$A_{\min} = 23.15$ dB

If $A_{\min}$ is to be exactly 20 dB, we can use Eq. (17.15) to obtain the new value of ϵ as follows:

$$20 = 10\ \log[1 + \epsilon^2 \times 1.7^{14}]$$
$$\epsilon = \sqrt{\frac{100-1}{1.7^{14}}} = 0.2425$$

Now, using Eq. (17.13) we can determine the value to which $A_{\max}$ can be reduced as

$$A_{\max} = 20\ \log\sqrt{1 + \epsilon^2}$$
$$A_{\max} = 20\ \log\sqrt{1 + 0.2425^2}$$
$$= 0.25\text{ dB}$$

17.16 Using Eq. (17.15), we have

$$A(\omega_s) = 10\ \log[1 + \epsilon^2(\omega_s/\omega_p)^{2N}]$$

For large $A(\omega_s)$, we can neglect the unity term in this expression to obtain

$$A(\omega_s) \simeq 10\ \log\ [\epsilon^2(\omega_s/\omega_p)^{2N}]$$

Substituting $A(\omega_s) \ge A_{\min}$ we obtain

$$20\ \log \epsilon + 20N\ \log(\omega_s/\omega_p) \ge A_{\min}$$
$$\Rightarrow N \ge \frac{A_{\min} - 20\ \log \epsilon}{20\ \log(\omega_s/\omega_p)} \qquad \text{Q.E.D.}$$

17.17 For an Nth-order Butterworth filter, we have from Eq. (17.11)

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}}}$$

At the 3-dB frequency $\omega_{3\,\text{dB}}$ we have:

$\epsilon^2\left(\dfrac{\omega_{3\,\text{dB}}}{\omega_p}\right)^{2N} = 1 \Rightarrow \omega_{3\,\text{dB}} = \left(\dfrac{1}{\epsilon^2}\right)^{1/2N}\omega_p$. Thus, from Eq. (17.15) the attenuation at $\omega = 1.8\omega_{3\,\text{dB}}$ is:

$$A = 10\log\left[1 + \epsilon^2\left(\frac{1.8\omega_{3\,\text{dB}}}{\omega_p}\right)^{2N}\right]$$
$$= 10\log\left[1 + \epsilon^2\left(1.8\frac{1}{\epsilon^2}\right)^{1/2N}\right]^{2N}$$
$$= 10\log(1 + 1.8^{2N})$$

For the case $N = 7$,

$$A = 10\log(1 + 1.8^{14}) = 35.7\text{ dB}$$

17.18 $A_{\max} = 0.5$ dB, $N = 5$, $\omega_p = 10^3$ rad/s

Using Eq. (17.14), we obtain

$$\epsilon = \sqrt{10^{A_{\max}/10} - 1} = \sqrt{10^{0.05} - 1}$$
$$= 0.3493$$

The natural modes can be determined by reference to Fig. 17.10(a):

$$\omega_0 = \omega_p\left(\frac{1}{\epsilon}\right)^{1/N}$$
$$\omega_0 = 10^3 \times \left(\frac{1}{0.3493}\right)^{1/5}$$
$$= 1.234 \times 10^3\text{ rad/s}$$
$$p_1,\ p_1^* = \omega_0\left[\sin\left(\frac{\pi}{10}\right) \pm j\cos\left(\frac{\pi}{10}\right)\right]$$
$$= \omega_0(-0.309 \pm j0.951)$$
$$= 1.234 \times 10^3(0.309 \pm j0.951)$$
$$p_2,\ p_2^* = \omega_0\left[-\sin\left(\frac{3\pi}{10}\right) \pm j\cos\left(\frac{3\pi}{10}\right)\right]$$
$$= 1.234 \times 10^3(-0.809 \pm j0.588)$$
$$p_3 = -\omega_0 = -1.234 \times 10^3$$

17.19 $f_p = 10$ kHz, $A_{\max} = 3$ dB, $f_s = 20$ kHz, $A_{\min} = 20$ dB

Using Eq. (17.14), we obtain

$$\epsilon = \sqrt{10^{A_{\max}/10} - 1}$$
$$= \sqrt{10^{3/10} - 1} = 1$$

Using Eq. (17.15), we have

$$A(\omega_s) = 10\ \log[1 + \epsilon^2(\omega_s/\omega_p)^{2N}]$$

Thus,

$$A_{\min} \ge 10\ \log[1 + \epsilon^2(\omega_s/\omega_p)^{2N}]$$
$$20 \ge 10\ \log\ (1 + 2^{2N})$$

For $N = 3$,

$$10\ \log(1 + 2^6) = 18.1\text{ dB}$$

For $N = 4$,

$$10\ \log(1 + 2^8) = 24.1\text{ dB}$$

Thus,

$N = 4$

The poles can be determined by reference to Fig. 17.10(a):

$$\omega_0 = \omega_p\left(\frac{1}{1}\right)^{1/4} = \omega_p = 2\pi \times 10^4 \text{ rad/s}$$

$$p_1,\ p_1^* = \omega_0\left[-\sin\left(\frac{\pi}{8}\right) \pm j\cos\left(\frac{\pi}{8}\right)\right]$$

$$= 2\pi \times 10^4(-0.383 \pm j0.924)$$

$$p_2,\ p_2^* = \omega_0\left[-\sin\left(\frac{3\pi}{8}\right) \pm j\cos\left(\frac{3\pi}{8}\right)\right]$$

$$= 2\pi \times 10^4(-0.924 \pm j0.383)$$

Thus,

$$T(s) = \frac{\omega_0^4}{(s^2 + 0.765\omega_0 s + \omega_0^2)(s^2 + 1.848\omega_0 s + \omega_0^2)}$$

where $\omega_0 = 2\pi \times 10^4$ and where we have assumed the dc gain to be unity.

Using Eq. (17.11), we obtain

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_p}\right)^8}}$$

At $f = 30$ kHz $= 3 f_p$, the attenuation is

$$A = -20 \log|T| = 10 \log(1 + 3^8)$$

$$= 38.2 \text{ dB}$$

17.20

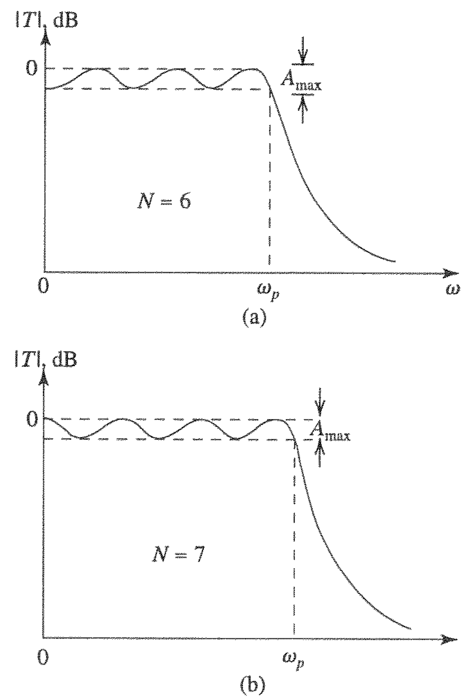


Figure 1

(a) See Fig. 1(a).

(b) See Fig. 1(b).

17.21

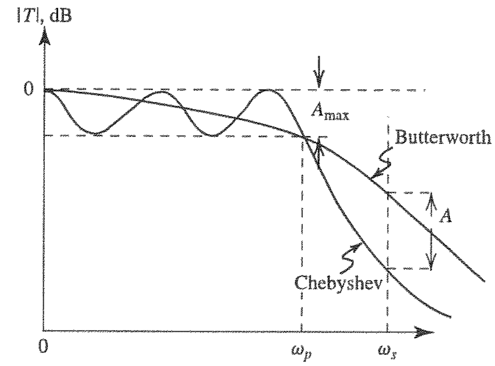


Figure 1

From Fig. 1 we see that at the stopband edge, ω_s, the Chebyshev filter provides A(dB) greater attentuation than the Butterworth filter of the same order and having the same A_{max}.

17.22 $N = 7,\ \omega_p = 1$ rad/s, $A_{max} = 0.5$ dB

Using Eq. (17.21), we obtain

$$\epsilon = \sqrt{10^{A_{max}/10} - 1}$$

$$= \sqrt{10^{0.05} - 1} = 0.3439$$

From Eq. (17.18), we have

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 \cos^2[N \cos^{-1}(\omega/\omega_p)]}}$$

$$= \frac{1}{\sqrt{1 + \epsilon^2 \cos^2(7 \cos^{-1}\omega)}} \quad (1)$$

$|T(j\omega)|$ will be equal to unity at the values of ω that make

$$\cos(7 \cos^{-1}\omega) = 0 \quad (2)$$

Since the cosine function is 0 for angles that are odd multiples of $\pi/2$, the solutions to Eq. (2) are

$$7 \cos^{-1}\omega_k = (2k + 1)\frac{\pi}{2}$$

where

$k = 0,\ 1,\ 2, \ldots.$

Thus,

$$\omega_k = \cos\frac{(2k + 1)\pi}{14}$$

We now can compute the values of ω_k as

$$\omega_1 = \cos\frac{\pi}{14} = 0.975 \text{ rad/s}$$

$$\omega_2 = \cos\frac{3\pi}{14} = 0.782 \text{ rad/s}$$

$$\omega_3 = \cos\frac{5\pi}{14} = 0.434 \text{ rad/s}$$

$$\omega_4 = \cos\frac{7\pi}{14} = 0 \text{ rad/s}$$

Next, we determine the passband frequencies at which maximum deviation from 0 dB occurs. From Eq. (1) we see that these are the values of ω that make

$$\cos^2(7\cos^{-1}\omega) = 1 \tag{3}$$

Since the magnitude of the cosine function is unity for angles that are multiples of π, the solutions to Eq. (3) are given by

$$7\cos^{-1}\omega_m = m\pi$$

or

$$\omega_m = \cos\frac{m\pi}{7}$$

where

$$m = 0,\ 1,\ 2, \ldots$$

We now can compute the values of ω_m as

$$\omega_0 = \cos 0 = 1 \text{ rad/s}$$

$$\omega_1 = \cos\frac{\pi}{7} = 0.901 \text{ rad/s}$$

$$\omega_2 = \cos\frac{2\pi}{7} = 0.623 \text{ rad/s}$$

$$\omega_3 = \cos\frac{3\pi}{7} = 0.223 \text{ rad/s}$$

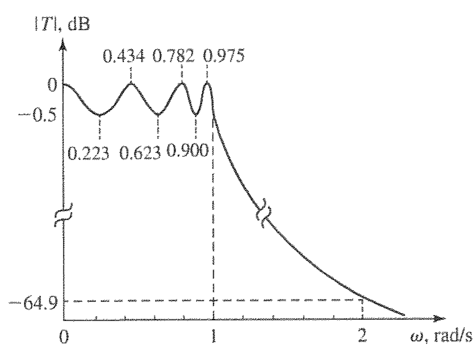


Figure 1

Figure 1 shows a sketch of $|T|$ for this 7th-order Chebyshev filter, with the passband maxima and minima identified. Note that the frequencies of the maxima and minima do *not* depend on the value of A_{max}.

To determine the attenuation at the stopband frequency $\omega = 2$ rad/s, we utilize the expression in Eq. (17.22), thus

$$A(2) = 10\log[1 + 0.3493^2\cosh^2(7\cosh^{-1}2)]$$

$$= 64.9 \text{ dB}$$

This point also is indicated on the sketch in Fig. 1.

Finally, we note that since this is a 7th-order all-pole filter, for $s \to \infty$, T will be proportional to $1/s^7$; that is, $|T|$ will be proportional to $1/\omega^7$, thus the asymptotic response will be $7 \times 6 = 42$ dB/octave.

17.23 (a) Consider first the Butterworth filter. For the given $A_{max} = 1$ dB we find the corresponding value of ϵ using Eq. (17.14) as

$$\epsilon = \sqrt{10^{0.1} - 1} = 0.5088$$

Next we use Eq. (17.15) to determine the attenuation at $\omega_s = 2\omega_p$, as

$$A(\omega_s) = 10\log_{10}(1 + 0.5088^2 \times 2^{12})$$

$$= 30.3 \text{ dB}$$

(b) We next consider the Chebyshev filter. Its ϵ is the same as that for the Butterworth filter, that is,

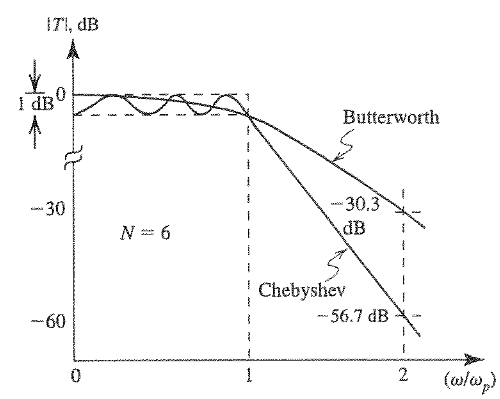
$$\epsilon = 0.5088$$

The attenuation at $\omega_s = 2\omega_p$ can be determined using Eq. (17.22) as

$$A(\omega_s) = 10\log[1 + 0.5088^2\cosh^2(6\cosh^{-1}2)]$$

$$= 56.7 \text{ dB}$$

which is nearly twice as much as the attenuation provided by the Butterworth filter. A sketch of the transmission of both filter types is shown in Fig. 1.



Note that we have expanded the vertical scale to show the details of the passband.

17.24

$$A_{max} = 1 \text{ dB} \Rightarrow \epsilon = \sqrt{10^{1/10} - 1} = 0.5088$$

$$f_p = 3.4 \text{ kHz},\ f_s = 4 \text{ kHz} \Rightarrow \frac{f_s}{f_p} = 1.176$$

$$A_{min} = 35 \text{ dB}$$

(a) To obtain the required order N, we use Eq. (17.22),

$A(\omega_s) = 10 \log[1 + \epsilon^2 \cosh^2(N \cosh^{-1}(\omega_s/\omega_p))]$

Thus,

$10 \log[1 + 0.5088^2 \cosh^2(N \cosh^{-1} 1.176)] \geq 35$

We attempt various values for N as follows:

N	$A(\omega_s)$
8	28.8 dB
9	33.9 dB
10	39.0 dB

Use $N = 10$.

Excess attenuation = 39 − 35 = 4 dB

(b) The poles can be determined using Eq. (17.23), namely

$$p_k/\omega_p = -\sin\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right)$$

$$+j\cos\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right)$$

$k = 1, 2, \ldots, N$

First we determine

$$\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right)$$

$$= \sinh\left(\frac{1}{10}\sinh^{-1}\frac{1}{0.5088}\right) = 0.1433$$

and

$$\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right) = 1.0102$$

Thus,

$$p_1/\omega_p = -0.1433\sin\left(\frac{\pi}{20}\right) + j1.0102\cos\left(\frac{\pi}{20}\right)$$

$= -0.0224 + j0.9978$

$$p_2/\omega_p = -0.1433\sin\left(\frac{3\pi}{20}\right) + j1.0102\cos\left(\frac{3\pi}{20}\right)$$

$= -0.0651 + j0.9001$

$$p_3/\omega_p = -0.1433\sin\left(\frac{5\pi}{20}\right) + j1.0102\cos\left(\frac{5\pi}{20}\right)$$

$= -0.1013 + j0.7143$

$$p_4/\omega_p = -0.1433\sin\left(\frac{7\pi}{20}\right) + j1.0102\cos\left(\frac{7\pi}{20}\right)$$

$= -0.1277 + j0.4586$

$$p_5/\omega_p = -0.1433\sin\left(\frac{9\pi}{20}\right) + j1.0102\cos\left(\frac{9\pi}{20}\right)$$

$= -0.1415 + j0.1580$

$p_6 = p_5^*,\ p_7 = p_4^*,\ p_8 = p_3^*,\ p_9 = p_2^*,\ p_{10} = p_1^*$

Each pair of complex conjugate poles,

$p_k,\ p_k^* = \omega_p(-\Sigma_k \pm j\,\Omega_k)$

gives rise to a quadratic factor in the denominator of $T(s)$ given by

$s^2 + s\omega_p(2\Sigma_k) + \omega_p^2(\Sigma_k^2 + \Omega_k^2)$

where

$\omega_p = 2\pi \times 3.4 \times 10^3$ rad/s

Thus, we obtain for the five pole pairs:

$p_1,\ p_1^*$: $(s^2 + s\,0.0448\omega_p + 0.9961\omega_p^2)$

$p_2,\ p_2^*$: $(s^2 + s\,0.1302\omega_p + 0.8144\omega_p^2)$

$p_3,\ p_3^*$: $(s^2 + s\,0.2026\omega_p + 0.5205\omega_p^2)$

$p_4,\ p_4^*$: $(s^2 + s\,0.2554\omega_p + 0.2266\omega_p^2)$

$p_5,\ p_5^*$: $(s^2 + s\,0.2830\omega_p + 0.0450\omega_p^2)$

The transfer function $T(s)$ can now be written as

$$T(s) = \frac{B}{\text{Product of five quadratic terms}}$$

The value of B determines the required dc gain, specifically

DC gain =

$$\frac{B}{\omega_p^{10} \times 0.9961 \times 0.8144 \times 0.5205 \times 0.2266 \times 0.0450}$$

$$= \frac{B}{4.31 \times 10^{-3}\omega_p^{10}}$$

For DC gain $= \dfrac{1}{\sqrt{1+\epsilon^2}} = \dfrac{1}{\sqrt{1+0.5088^2}}$

$= 0.891$

we select

$B = 4.31 \times 10^{-3} \times \omega_p^{10} \times 0.891$

$= 3.84 \times 10^{-3} \times (2\pi \times 3.4 \times 10^3)^{10}$

$= 7.60 \times 10^{40}$

17.25 Refer to Fig. 17.13 (row a).

Input resistance = R_1

Thus, to obtain an input resistance of 12 kΩ, we select

$R_1 = 12$ kΩ

$$|\text{DC gain}| = 10 = \frac{R_2}{R_1}$$

$$\Rightarrow R_2 = 10\,R_1 = 120\ \text{k}\Omega$$

$$CR_2 = \frac{1}{\omega_0} = \frac{1}{2\pi \times 5 \times 10^3}$$

$$\Rightarrow C = \frac{1}{2\pi \times 5 \times 10^3 \times 120 \times 10^3} = 265\ \text{pF}$$

17.26 Refer to Fig. 17.13 (row b).

H.F. Input resistance $= R_1$

Thus,

$R_1 = 120\ \text{k}\Omega$

$$CR_1 = \frac{1}{\omega_0} = \frac{1}{2\pi \times 200}$$

$$\Rightarrow C = \frac{1}{2\pi \times 200 \times 120 \times 10^3}$$

$= 6.63$ nF

$$\text{High-frequency gain} = -\frac{R_2}{R_1} = -1$$

$$\Rightarrow R_2 = R_1 = 120\ \text{k}\Omega$$

17.27 Refer to the op-amp-RC circuit in Fig. 17.3 (row c). Assuming an ideal op amp, we have

$$T(s) = \frac{V_o}{V_i} = -\frac{Z_2(s)}{Z_1(s)}$$

Since both Z_1 and Z_2 have a parallel structure, it is far more convenient to work in terms of $Y_1(s)$ and $Y_2(s)$, thus

$$T(s) = -\frac{Y_1(s)}{Y_2(s)}$$

$$= -\frac{\dfrac{1}{R_1} + s\,C_1}{\dfrac{1}{R_2} + s\,C_2}$$

$$= -\left(\frac{C_1}{C_2}\right)\frac{s + \dfrac{1}{C_1R_1}}{s + \dfrac{1}{C_2R_2}}$$

Thus,

$$\omega_Z = \frac{1}{C_1R_1}$$

$$\omega_P = \frac{1}{C_2R_2}$$

$$\text{DC gain} = T(0) = -\frac{R_2}{R_1}$$

$$\text{HF gain} = T(\infty) = -\frac{C_1}{C_2}$$

17.28 We use the op-amp-RC circuit of Fig. 17.13(c).

Low-frequency input resistance $= R_1$

Thus,

$R_1 = 10\ \text{k}\Omega$

$$\text{DC gain} = -\frac{R_2}{R_1} = -1$$

$$\Rightarrow R_2 = R_1 = 10\ \text{k}\Omega$$

$$f_Z = \frac{\omega_Z}{2\pi} = \frac{1}{2\pi C_1R_1}$$

$$100 = \frac{1}{2\pi C_1 \times 10 \times 10^3}$$

$$\Rightarrow C_1 = \frac{1}{2\pi \times 10^6} = 0.16\ \mu\text{F}$$

$$f_P = f_0 = \frac{1}{2\pi C_2R_2}$$

$$10 \times 10^3 = \frac{1}{2\pi C_2 \times 10 \times 10^3}$$

$$C_2 = \frac{1}{2\pi \times 10^8} = 1.6\ \text{nF}$$

Figure 1 shows a sketch of the magnitude of the transfer function. Note that the magnitude of the high-frequency gain is $C_1/C_2 = 100$ or 40 dB.

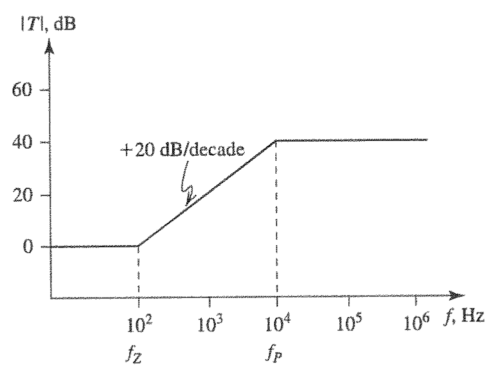


Figure 1

17.29 Figure 1 (on the next page) shows the bandpass filter realized as the cascade of a first-order low-pass filter and a first-order high-pass filter. The component values are determined as follows:

$R_{\text{in}} = R_1$

To make R_{in} as large as possible while satisfying the constraint that no resistance is larger than 100 kΩ, we select

$R_1 = 100\ \text{k}\Omega$

This figure belongs to Problem 17.29.

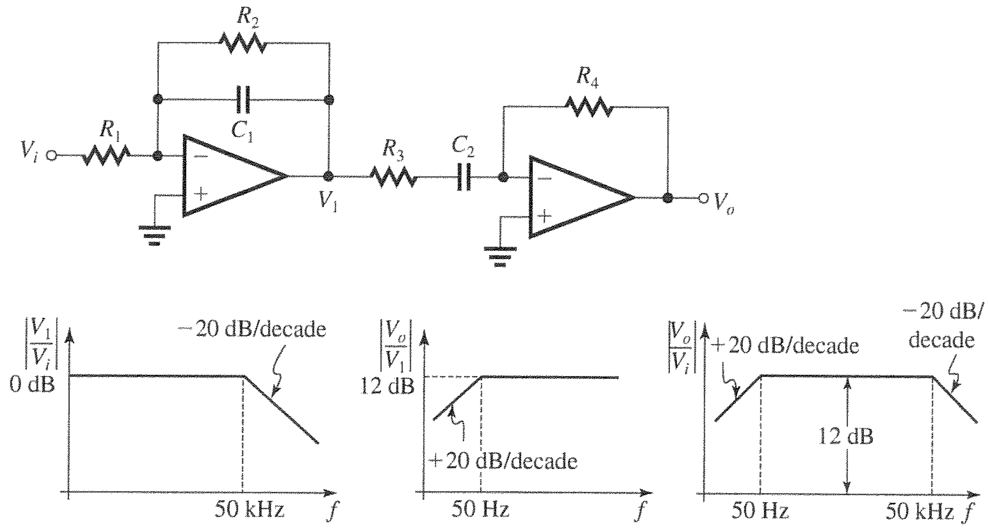


Figure 1

The low-frequency gain of the low-pass circuit is $(-R_2/R_1)$. With $R_1 = 100\ \text{k}\Omega$, the maximum gain obtained is unity and is achieved by selecting

$$R_2 = 100\ \text{k}\Omega$$

This implies that the required gain of 12 dB or 4 V/V must be all realized in the high-pass circuit.

The upper 3-dB frequency of the bandpass filter is the 3-dB frequency of the low-pass circuit, that is,

$$50 \times 10^3 = \frac{1}{2\pi C_1 R_2}$$

$$\Rightarrow C_1 = \frac{1}{2\pi \times 50 \times 10^3 \times 100 \times 10^3}$$

$$= 31.8\ \text{pF}$$

Next, we consider the high-pass circuit. The high-frequency ($f \gg 50$ Hz) gain of this circuit is $(-R_4/R_3)$. To obtain a gain of -4 V/V, we select

$$R_4 = 100\ \text{k}\Omega$$

$$R_3 = \frac{100}{4} = 25\ \text{k}\Omega$$

The lower 3-dB frequency of the bandpass filter (50 Hz) is the 3-dB frequency of the high-pass circuit, thus

$$50 = \frac{1}{2\pi C_2 R_3}$$

$$\Rightarrow C_2 = \frac{1}{2\pi \times 50 \times 25 \times 10^3}$$

$$= 0.127\ \mu\text{F}$$

17.30

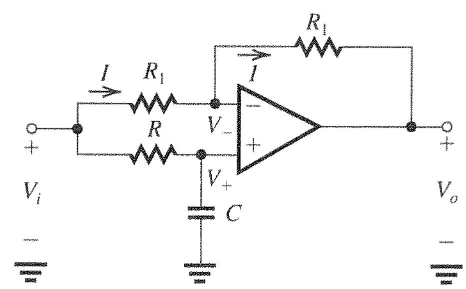


Figure 1

Refer to Fig. 1.

$$V_+ = V_i \frac{1/sC}{R + (1/sC)}$$

$$= V_i \frac{1}{sCR + 1}$$

$$V_- = V_+ = V_i \frac{1}{sCR + 1}$$

$$I = \frac{V_i - V_-}{R_1} = \frac{V_i}{R_1}\left(1 - \frac{1}{sCR + 1}\right)$$

$$= \frac{V_i}{R_1} \frac{sCR}{sCR + 1}$$

$$V_o = V_- - IR_1$$

$$= \frac{V_i}{sCR + 1} - \frac{sRCV_i}{sCR + 1}$$

Thus,

$$\frac{V_o}{V_i} = -\frac{sCR - 1}{sCR + 1}$$

or

$$T(s) = \frac{V_o}{V_i} = -\frac{s - 1/CR}{s + 1/CR} = -\frac{s - \omega_0}{s + \omega_0}$$

where

$$\omega_0 = \frac{1}{CR}$$

$$T(j\omega) = -\frac{j\omega - \omega_0}{j\omega + \omega_0}$$

$$= \frac{\omega_0 - j\omega}{\omega_0 + j\omega}$$

$$= \frac{1 - j(\omega/\omega_0)}{1 + j(\omega/\omega_0)}$$

$$|T(j\omega)| = \sqrt{\frac{1 + (\omega/\omega_0)^2}{1 + (\omega/\omega_0)^2}} = 1$$

$$\phi(\omega) = -2\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$\Rightarrow \frac{\omega}{\omega_0} = \tan\left[-\frac{1}{2}\phi(\omega)\right] \qquad (1)$$

Thus, for a given phase shift ϕ, we can use Eq. (1) to determine (ω/ω_0). For $\omega = 5 \times 10^3$ rad/s, we can then determine the required value of ω_0. Finally, for $C = 10$ nF, the required value for R can be found from

$$R = \frac{1}{\omega_0 \times 10 \times 10^{-9}} = \frac{10^8}{\omega_0}$$

The results obtained are as follows:

ϕ	$-30°$	$-60°$	$-90°$	$-120°$	$-150°$
ω/ω_0	0.268	0.577	1	1.732	3.732
ω_0 **(krad/s)**	18.66	8.66	5	2.89	1.34
R **(kΩ)**	5.36	11.55	20	34.60	74.63

17.31

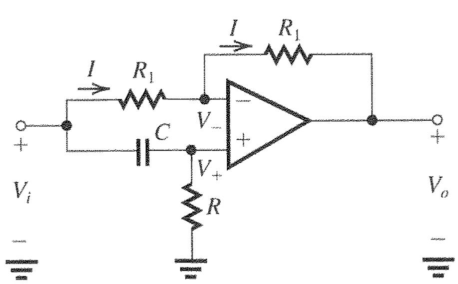


Figure 1

Figure 1 shows the circuit of Fig. 17.14 with R and C interchanged. To determine the transfer function $T(s) = V_o(s)/V_i(s)$, we analyze the circuit as follows:

$$V_+ = V_i \frac{R}{R + \frac{1}{sC}}$$

$$= V_i \frac{s}{s + \frac{1}{CR}}$$

$$V_- = V_+ = V_i \frac{s}{s + \frac{1}{CR}}$$

$$I = \frac{V_i - V_-}{R_1} = \frac{V_i}{R_1}\left(1 - \frac{s}{s + \frac{1}{CR}}\right)$$

$$= \frac{V_i}{R_1}\frac{1/CR}{s + \frac{1}{CR}}$$

$$V_o = V_- - IR_1$$

$$= V_i\left[\frac{s}{s + \frac{1}{CR}} - \frac{1/CR}{s + \frac{1}{CR}}\right]$$

Thus,

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{s - (1/CR)}{s + (1/CR)}$$

$$= \frac{s - \omega_0}{s + \omega_0}$$

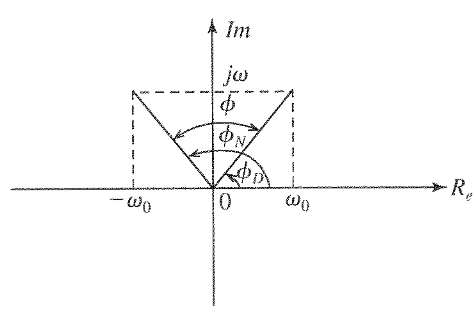
where

$$\omega_0 = 1/CR$$

$$T(j\omega) = \frac{-\omega_0 + j\omega}{\omega_0 + j\omega}$$

$$\phi(\omega) = \phi_N(\omega) - \phi_D(\omega)$$

where ϕ_N is the phase angle of the numerator and ϕ_D is the phase angle of the denominator. A graphical construction showing ϕ_N and ϕ_D and their difference ϕ is depicted in Fig. 2.



Observe that the difference, ϕ, is a positive angle whose value ranges from 180° (at $\omega = 0$) to 0° (at $\omega = \infty$). The two end points should also be obvious from $T(j\omega)$ which is -1 at $\omega = 0$ and $+1$ at $\omega = 0$.

17.32

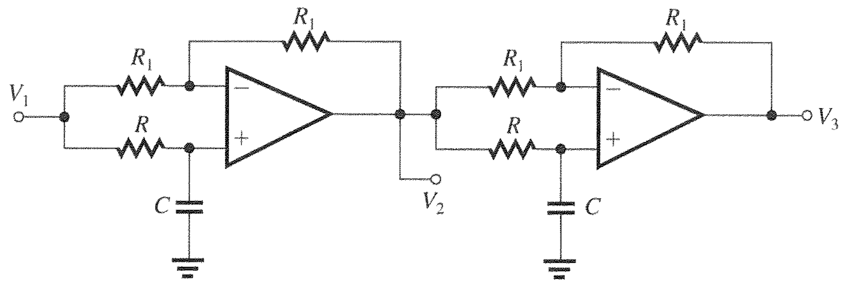


Figure 1

Figure 1 shows a circuit composed of the cascade connection of two all-pass circuits of the type shown in Fig. 17.14. We require that each circuit provide $-120°$ phase shift at $\omega = 2\pi \times 60$ rad/s. From the data in Fig. 17.14(a),

$$\phi(\omega) = -2\tan^{-1}(\omega CR)$$

Thus,

$$-120° = -2\tan^{-1}(\omega CR)$$

$$\omega CR = \tan 60°$$

$$2\pi \times 60 \times 1 \times 10^{-6} \times R = \frac{\sqrt{3}}{2}$$

$$R = \frac{\sqrt{3}}{2\pi \times 60 \times 10^{-6} \times 2} = 2.3\ \text{k}\Omega$$

The value of R_1 can be selected arbitrarily, say

$$R_1 = 10\ \text{k}\Omega$$

17.33 Refer to Fig. 17.16(a).

$$T(s) = \frac{\omega_0^2}{s^2 + s\,\dfrac{\omega_0}{Q} + \omega_0^2}$$

$$= \frac{10^8}{s^2 + 5000\,s + 10^8}$$

$$\omega_{\max} = \omega_0\sqrt{1 - \frac{1}{2Q^2}}$$

$$= 10^4\sqrt{1 - \frac{1}{2 \times 4}} = 9354\ \text{rad/s}$$

Since the dc gain is unity, we have

$$|a_o/\omega_0^2| = 1$$

$$|T_{\max}| = \frac{Q}{\sqrt{1 - \dfrac{1}{4Q^2}}} = \frac{2}{\sqrt{1 - \dfrac{1}{16}}} = 2.066$$

17.34 There are many possibilities, but only two are optimal. The first is the Butterworth filter which exhibits maximum flatness of $|T|$ at $\omega = 0$. The second is the Chebyshev for which $|T|$ exhibits equiripple response in the passband.

See Figure 1 on the next page. Figure 1(a) shows the second-order Butterworth filter that just meets the given passband specifications. Figure 1(b) shows the second-order Chebyshev filter that just meets the given passband specifications. Note that no stopband specifications were given (otherwise the problem would be overspecified); we are simply asked to calculate the attenuation at $\omega_s = 2\omega_p = 2$ rad/s.

For both filters, since $A_{\max} = 3$ dB, we have

$$\epsilon = 1$$

(a) Butterworth filter:

For a second-order Butterworth, we have $Q = 1/\sqrt{2} = 0.707$ and $\omega_0 = \omega_p = 1$ rad/s. The dc gain is unity. Thus,

$$T(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \omega^{2N}}}$$

Thus,

$$|T(j2)| = \frac{1}{\sqrt{1 + 2^4}}$$

$$A_{\min} = A(\omega_s) = 20\log_{10}\sqrt{17} = 12.3\ \text{dB}$$

(b) Chebyshev filter:

For a second-order Chebyshev filter, the poles are given by Eq. (17.23), which for $N = 2$, $\omega_p = 1$, and $\epsilon = 1$ yields

$$p_1,\ p_1^* = -\sin\left(\frac{\pi}{4}\right)\sinh\left(\frac{1}{2}\sinh^{-1}1\right)$$

$$\pm j\cos\left(\frac{\pi}{4}\right)\cosh\left(\frac{1}{2}\sinh^{-1}1\right)$$

$$= -0.3218 \pm j0.7769$$

Thus, for this pair of complex-conjugate poles, we have

$$\frac{\omega_0}{Q} = 2 \times 0.3218 = 0.6436$$

and

$$\omega_0^2 = 0.3218^2 + 0.7769^2 = 0.7071$$

This figure belongs to Problem 17.34.

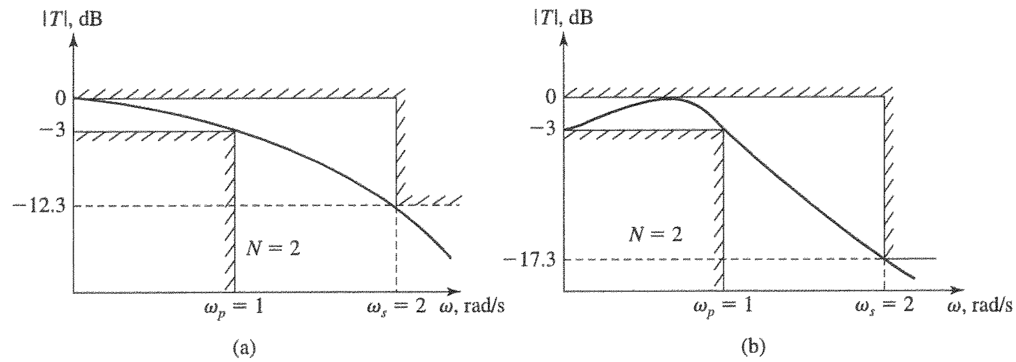


Figure 1

Thus,

$$T(s) = \frac{K}{s^2 + 0.6436\,s + 0.7071}$$

From Fig. 1(b) we see that the dc gain is $1/\sqrt{1+\epsilon^2} = 0.7071$

Thus,

$$K = 0.7071 \times 0.707 = 0.5$$

and

$$T(s) = \frac{0.5}{s^2 + 0.6436\,s + 0.7071}$$

At $s = j2$, we have

$$|T(j2)| = \frac{0.5}{\sqrt{(0.7071-4)^2 + (0.6436 \times 2)^2}}$$

$$= 0.1414$$

$$A_{\min} = A(2) = -20\log 0.1414$$

$$= 17.0 \text{ dB}$$

17.35 Refer to Fig. 17.16(b). For a maximally flat response, we have

$$Q = 1/\sqrt{2}$$

and

$$\omega_{3dB} = \omega_0$$

Thus,

$$\omega_0 = 1 \text{ rad/s}$$

If the high-frequency gain is unity, then we have

$$a_2 = 1$$

and

$$T(s) = \frac{s^2}{s^2 + \sqrt{2}\,s + 1}$$

The two zeros are at $s = 0$. The poles are complex conjugate and given by

$$s = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{\left(1 - \frac{1}{4Q^2}\right)}$$

$$= -\frac{1}{2 \times \dfrac{1}{\sqrt{2}}} \pm j\sqrt{\left(1 - \frac{1}{4 \times \dfrac{1}{2}}\right)}$$

$$= -\frac{1}{\sqrt{2}} \pm j\frac{1}{\sqrt{2}}$$

$$= -0.707 \pm j0.707$$

17.36 Poles are at $-0.5\ \pm j\sqrt{3}/2$. Thus,

$$\omega_0 = \sqrt{0.5^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1 \text{ rad/s}$$

$$\frac{\omega_0}{2Q} = 0.5$$

$$\Rightarrow Q = \frac{\omega_0}{2 \times 0.5} = \frac{1}{1} = 1$$

High-frequency gain $= a_2 = 1$.

$$T(s) = \frac{s^2}{s^2 + s + 1}$$

17.37 $f_0 = 10$ kHz $\Rightarrow \omega_0 = 2\pi \times 10^4$ rad/s

$$BW = \frac{f_0}{Q} = 500 \text{ Hz} \Rightarrow Q = 20$$

Center-frequency gain $= 10$

Thus,

$$T(s) = \frac{10\left(\dfrac{\omega_0}{Q}\right)s}{s^2 + s\left(\dfrac{\omega_0}{Q}\right) + \omega_0^2}$$

$$= \frac{\pi \times 10^4 s}{s^2 + s\pi \times 10^3 + (2\pi \times 10^4)^2}$$

Poles are at $\dfrac{-\omega_0}{2Q} \pm j\omega_0\sqrt{1-\dfrac{1}{4Q^2}}$

$= -\dfrac{\pi}{2} \times 10^3 \pm j2\pi \times 10^4\sqrt{1-\dfrac{1}{4\times 20^2}}$

$= \dfrac{\pi}{2} \times 10^3[-1 \pm j40 \times 0.9997]$

$= 1.57 \times 10^3(-1 \pm j39.988)$

Zeros are at $s = 0$ and $s = \infty$.

17.38 (a) A second-order bandpass filter with a center-frequency gain of unity (arbitrary) has the transfer function

$$T(s) = \frac{s(\omega_0/Q)}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

Thus,

$$|T(j\omega)| = \frac{\omega\omega_0/Q}{\sqrt{(\omega_0^2-\omega^2) + (\omega\omega_0/Q)^2}}$$

$$= 1\Big/\sqrt{1 + Q^2\,\frac{(\omega_0^2-\omega^2)^2}{\omega^2\omega_0^2}} \qquad (1)$$

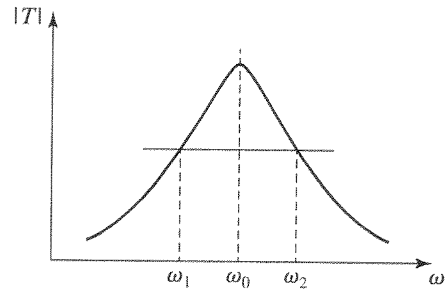


Figure 1

From Fig. 1 we see that at each $|T|$ (below the peak value) there are two frequencies $\omega_1 < \omega_0$ and $\omega_2 > \omega_0$ with the same $|T|$. The relationship between ω_1 and ω_2 can be determined by considering the second term in the denominator of Eq. (1), as follows:

$$\frac{\omega_0^2-\omega_1^2}{\omega_1\omega_0} = \frac{\omega_2^2-\omega_0^2}{\omega_2\omega_0}$$

Cross multiplying and collecting terms results in

$\omega_1\omega_2 = \omega_0^2$ Q.E.D.

(b) Since the two edges of the passband must be geometrically symmetric around ω_0, we have

$\omega_0 = \sqrt{\omega_{P1}\omega_{P2}}$

$= \sqrt{8100 \times 10{,}000}$

$= 9000$ rad/s

The Q factor can now be found from

$$\text{3-dB } BW = \frac{\omega_0}{Q}$$

$$10{,}000 - 8100 = \frac{9000}{Q}$$

$$\Rightarrow Q = \frac{9000}{1900} = 4.74$$

The geometric symmetry of $|T|$ enables us to find ω_{s2} from

$\omega_{s1}\omega_{s2} = \omega_0^2$

$$\Rightarrow \omega_{s2} = \frac{(9000)^2}{3000} = 27{,}000 \text{ rad/s}$$

Using Eq. (1), we obtain

$$A_{\min} = A(\omega_{s1}) = 10\log\left[1 + Q^2\,\frac{(\omega_0^2-\omega_{s1}^2)^2}{\omega_{s1}^2\,\omega_0^2}\right]$$

$$= 10\log\left[1 + 4.74^2\,\frac{(9000^2-3000^2)^2}{3000^2\times 9000^2}\right]$$

$= 22.1$ dB

Figure 2 shows a sketch of $|T|$.

This figure belongs to Problem 17.38, part (b).

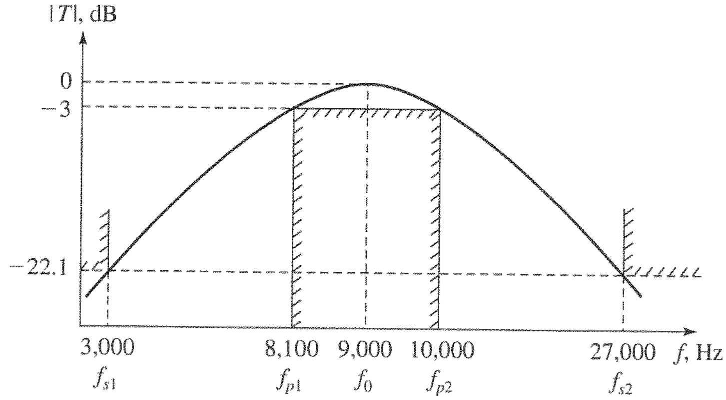


Figure 2

17.39 Refer to the problem statement of Exercise 17.15. For our case, we have

$\omega_0 = 2\pi \times 60$ rad/s

$BW_a = 2\pi \times 6$ rad/s

$A = 20$ dB

Thus, the required Q is

$$Q = \frac{\omega_0}{BW_a\sqrt{10^{A/10} - 1}}$$

$$= \frac{2\pi \times 60}{2\pi \times 6\sqrt{10^{20/10} - 1}}$$

$$= \frac{10}{\sqrt{100 - 1}} \simeq 1$$

Thus, the filter transfer function (with unity dc gain) is

$$T(s) = \frac{s^2 + \omega_0^2}{s^2 + s\,\omega_0 + \omega_0^2}$$

$$= \frac{s^2 + (2\pi \times 60)^2}{s^2 + s\,(2\pi \times 60) + (2\pi \times 60)^2}$$

17.40 Since near the poles, $|T|$ exhibits a peak, and near the zeros it exhibits a dip, the results shown in Fig. 1 are obtained.

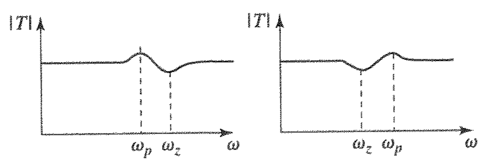


Figure 1

17.41 An increase in Q_p increases the magnitude of the peak. On the other hand, an increase in Q_z increases the magnitude of the dip. Thus, the results shown in Fig. 1 are obtained.

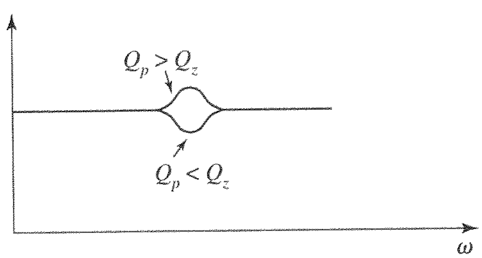


Figure 1

17.42 Refer to Fig. 17.17(c). Using the voltage divider rule, we obtain

$$\frac{V_o}{V_i} = \frac{Z_{RC}}{Z_{RC} + sL}$$

$$= \frac{1}{1 + sLY_{RC}}$$

$$= \frac{1}{1 + sL\left(sC + \frac{1}{R}\right)}$$

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2LC + s\frac{L}{R} + 1}$$

$$= \frac{1/LC}{s^2 + s\frac{1}{CR} + \frac{1}{LC}}$$

Thus,

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{[which is Eq. (17.34)]}$$

$$\frac{\omega_0}{Q} = \frac{1}{CR} \Rightarrow Q = \omega_0 CR$$

[which is Eq. (17.35)] Q.E.D.

17.43 $Q = \omega_0 CR$

$5 = 10^5 \times C \times 10 \times 10^3$

$\Rightarrow C = 5$ nF

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$\Rightarrow L = 1/\omega_0^2 C$

$$L = \frac{1}{10^{10} \times 5 \times 10^{-9}} = 20 \text{ mH}$$

17.44 (a) If L became $1.01L$, we obtain

$$\omega_0 = \frac{1}{\sqrt{1.01LC}} \simeq \frac{1}{1.005\sqrt{LC}} \simeq \frac{0.995}{\sqrt{LC}}$$

Thus, ω_0 decreases by 0.5%.

(b) If C increases by 1%, similar analysis as in (a) results in ω_0 decreasing by 0.5%.

(c) ω_0 is independent of the value of $R \Rightarrow$ no change.

17.45 (a) As ω reaches 0 (dc), we get

$$\frac{V_o}{V_i} = \frac{1/sC_2}{\frac{1}{sC_1} + \frac{1}{sC_2}} = \frac{C_1}{C_1 + C_2}$$

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

As ω reaches ∞,

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

No transmission zero are introduced.

(b) $\frac{V_o}{V_i} = \frac{Z_{R,C_2}}{Z_{R,C_2} + \frac{1}{sC_1}}$

$$= \frac{1}{1 + \frac{1}{sC_1} Y_{R,C_2}}$$

$$= \frac{1}{1 + \frac{1}{sC_1}\left(\frac{1}{R} + sC_2\right)}$$

$$= \frac{1}{\left(1 + \frac{C_2}{C_1}\right) + \frac{1}{sC_1R}}$$

$$= \frac{s}{s\left(1 + \frac{C_1}{C_2}\right) + \frac{1}{C_1R}}$$

$$\frac{V_o}{V_i}(0) = 0$$

$$\frac{V_o}{V_i}(\infty) = \frac{1}{1 + \frac{C_2}{C_1}} = \frac{C_1}{C_1 + C_2}$$

There is a transmission zero at $s = 0$, due to C_1.

(c) $\frac{V_o}{V_i} = \frac{sL_2}{s(L_1 + L_2)} = \frac{L_2}{L_1 + L_2}$

$$\frac{V_o}{V_i}(0) = \frac{L_2}{L_1 + L_2}$$

$$\frac{V_o}{V_i}(\infty) = \frac{L_2}{L_1 + L_2}$$

No transmission zeros.

(d) $\frac{V_o}{V_i} = \frac{sL_2}{s(L_1 + L_2) + R}$

$$\frac{V_o}{V_i}(0) = 0$$

$$\frac{V_o}{V_i}(\infty) = \frac{L_2}{L_1 + L_2}$$

There is a transmission zero at 0 (dc) due to L_2.

17.46 Refer to Fig. 17.18(c). Using the voltage divider rule, we obtain

$$\frac{V_o}{V_i} = \frac{Z_{LR}}{Z_{LR} + \frac{1}{sC}}$$

$$= \frac{1}{1 + \frac{1}{sC} Y_{LR}}$$

$$= \frac{1}{1 + \frac{1}{sC}\left(\frac{1}{R} + \frac{1}{sL}\right)}$$

$$= \frac{1}{1 + \frac{1}{sCR} + \frac{1}{s^2LC}}$$

$$= \frac{s^2}{s^2 + s\frac{1}{CR} + \frac{1}{LC}}$$

17.47 Refer to the circuit in Fig. 17.18(b).

$Q = \omega_0 CR$

$\frac{1}{\sqrt{2}} = 10^6 \times 1 \times 10^{-9} \times R$

$\Rightarrow R = 707\ \Omega$

$\omega_0^2 LC = 1$

$10^{12} \times L \times 1 \times 10^{-9} = 1$

$\Rightarrow L = 1$ mH

17.48 Refer to the circuit in Fig. 17.18(d). Since, at the center frequency, the LC circuit acts as an open circuit, the circuit reduces to R. We can change the gain to 0.5, by splitting R into two equal parts, each equal to $2R$. The voltage divider ($2R$, $2R$) will provide at ω_0, a gain of 0.5. The Q factor will be determined by the parallel equivalent of $2R$ and $2R$. Since the latter is equivalent to R, Q will remain unchanged, as required. The resulting circuit is shown in Fig. 1.

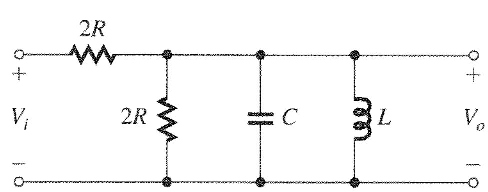


Figure 1

17.49 Using superposition, we find the resulting $T(s)$ in each of the three cases and then sum the results. Thus (see Figure 1 on the next page):

(a) When x is lifted off ground and connected to V_x, the circuit becomes as shown in Fig. 1(a), and the transfer function becomes that of a low-pass filter with $\omega_0 = 1/\sqrt{LC}$, $Q = \omega_0\ CR$, and dc gain of unity, thus

$$\frac{V_{o1}}{V_x} = \frac{\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} \quad (1)$$

(b) With y lifted off ground and connected to V_y, the resulting circuit [shown in Fig. 1(b)], is that of a high-pass filter with $\omega_0 = 1/\sqrt{LC}$, $Q = \omega_0 CR$, and a high-frequency gain of unity, thus

This figure belongs to Problem 17.49.

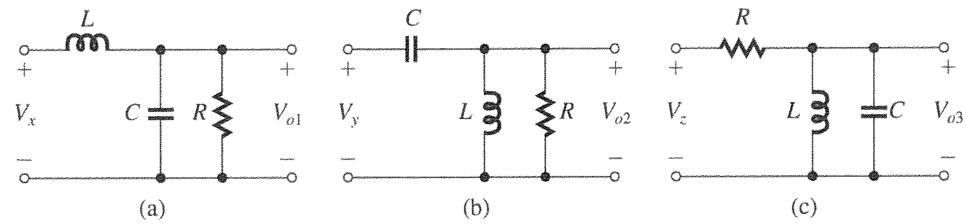


Figure 1

$$\frac{V_{o2}}{V_y} = \frac{s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} \qquad (2)$$

(c) With z lifted off ground and connected to V_z, the resulting circuit shown in Fig. 1(c) is that of a bandpass filter with $\omega_0 = 1/\sqrt{LC}$, $Q = \omega_0\ CR$, and a center-frequency gain of unity, thus

$$\frac{V_{o3}}{V_c} = \frac{s\frac{\omega_0}{Q}}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2} \qquad (3)$$

Now summing (1), (2), and (3) gives

$$V_o = \frac{s^2 V_y + s\left(\frac{\omega_0}{Q}\right) V_z + \omega_0^2 V_x}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

17.50 Refer to Fig. 17.18(g).

$\omega_n = 1/\sqrt{LC_1}$

$\omega_0 = 1/\sqrt{L(C_1 + C_2)}$

$$\Rightarrow \frac{\omega_n}{\omega_0} = \sqrt{1 + \frac{C_2}{C_1}}$$

Thus,

$$1.1 = \sqrt{1 + \frac{C_2}{C_1}}$$

$$\Rightarrow \frac{C_2}{C_1} = 0.21$$

At frequencies $\ll \omega_0$, $|T|$ approaches unity (because L approaches a short circuit and C_1 and C_2 approach open circuits). At frequencies $\gg \omega_0$, $|T|$ approaches

$$\frac{C_1}{C_1 + C_2} = \frac{1}{1 + \frac{C_2}{C_1}} = \frac{1}{1.21} = 0.83 \text{ V/V}$$

[see Fig. 17.18(b)].

17.51 $L = C_4 R_1 R_3 R_5 / R_2$

Selecting $R_1 = R_2 = R_3 = R_5 = R = 10\ \text{k}\Omega$, we have

$L = C_4 R^2$

$= C_4 \times 10^8$

Thus,

$C_4 = L \times 10^{-8}$

(a) For $L = 15$ H, we have

$C_4 = 15 \times 10^{-8}\ \text{F} = 0.15\ \mu\text{F}$

(b) For $L = 1.5$ H, we have

$C_4 = 0.015\ \mu\text{F} = 15\ \text{nF}$

(c) For $L = 0.15$ H, we have

$C_4 = 1.5\ \text{nF}$

17.52 (a) Figure 1 on the next page shows the analysis. It is based on assuming ideal op amps that exhibit virtual short circuits between their input terminals, and draw zero currents into their input terminals. Observe that:

(1) The circuit is fed at port-1 with a voltage V_1. The virtual short circuit between the input terminals of each of A_2 and A_1 cause the voltage at port-2 to be

$$V_2 = V_1 \qquad (1)$$

(2) With port-2 terminated in an impedance Z_5, the current that flows out of port-2 is

$$I_2 = \frac{V_2}{Z_5}$$

(3) Following the analysis indicated, we find the input current into port-1 as

$$I_1 = \frac{Z_2 Z_4}{Z_1 Z_3 Z_5}$$

This figure belongs to Problem 17.52.

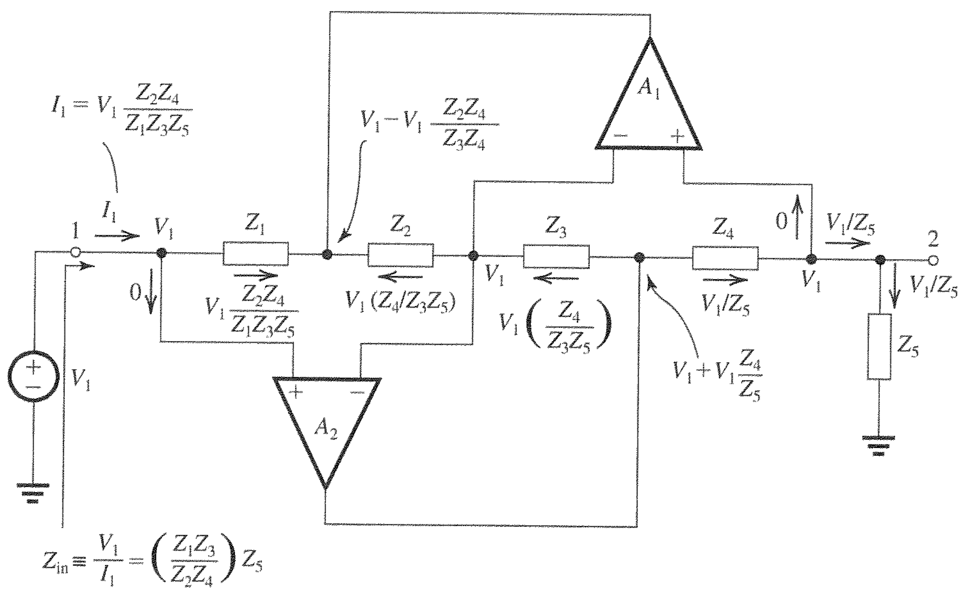


Figure 1

This current could have been written as

$$I_1 = V_1 \frac{Z_2Z_4}{Z_1Z_3} I_2 \tag{2}$$

(4) Equations (1) and (2) describe the operation of the circuit: It propagates the voltage applied to port-1, V_1, and to port-2, $V_2 = V_1$; and whatever current is drawn out of port-2 is multiplied by the function (Z_2Z_4/Z_1Z_3) and appears at port-1.

(5) The result of the two actions in (4) is that the input impedance looking into port-1 becomes

$$Z_{11} \equiv \frac{V_1}{I_1} = \frac{V_2}{I_2(Z_2Z_4/Z_1Z_3)}$$

$$= \left(\frac{Z_1Z_3}{Z_2Z_4}\right)\left(\frac{V_2}{I_2}\right)$$

or

$$Z_{11} = \left(\frac{Z_1Z_3}{Z_2Z_4}\right) Z_5$$

(b) If port-1 is terminated in an impedance Z_6, the input impedance looking into port-2 can be obtained by invoking the symmetry of the circuit, thus

$$Z_{22} = \left(\frac{Z_2Z_4}{Z_1Z_3}\right) Z_6$$

Thus, the circuit behaves as an impedance transformer with the transformation ratio from port-2 to port-1 being $\left(\frac{Z_1Z_3}{Z_2Z_4}\right)$ and from port-1 to port-2 being $\left(\frac{Z_2Z_4}{Z_1Z_3}\right)$.

Since Z_1, Z_2, Z_3, and Z_4 can be arbitrary functions of s, the transformation ratio can be an arbitrary function of s. Thus the circuit can be used as an impedance converter. For instance, the particular selection of impedances in Fig. 17.20(a) results in the transformation ratio from port-2 to port-1 being $(sC_4R_1R_3/R_2)$. As a result, the circuit in Fig. 17.22(a) converts a resistance R_5 into an inductance $L = C_4R_1R_3R_5/R_2$. Other conversion functions are possible and the circuit is known as a Generalized Impedance Converter or GIC.

17.53 Figure 1 on the next page shows the suggested circuit together with the analysis. The input impedance looking into port-2 is

$$Z_{22}(s) \equiv \frac{V_2}{I_2} = \frac{R_2}{s^2C_4C_6R_1R_3}$$

For $s = j\omega$ we have

$$Z_{22}(j\omega) = -\frac{R_2}{\omega^2C_4C_6R_1R_2}$$

which is a negative resistance whose magnitude depends on frequency ω; thus it is called a Frequency Dependent Negative Resistance or FDNR.

This figure belongs to Problem 17.53.

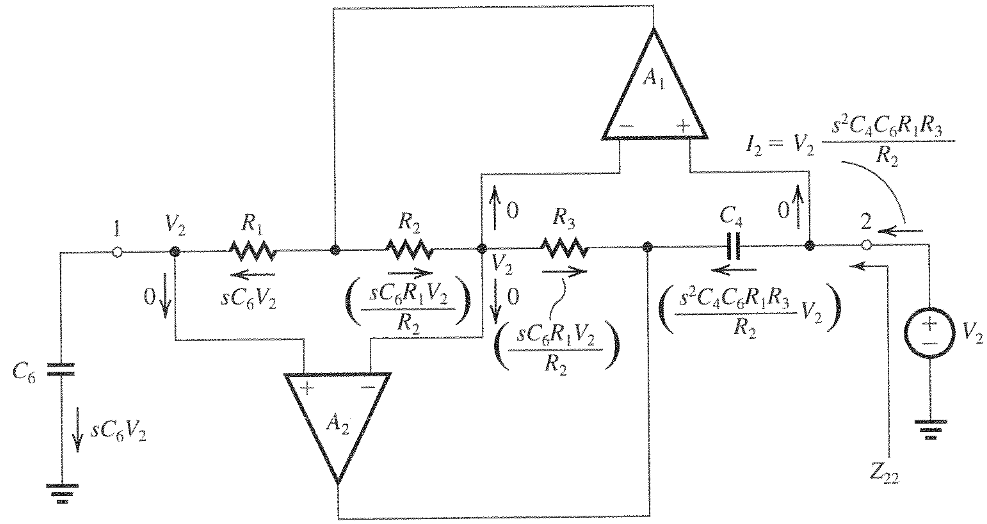


Figure 1

17.54

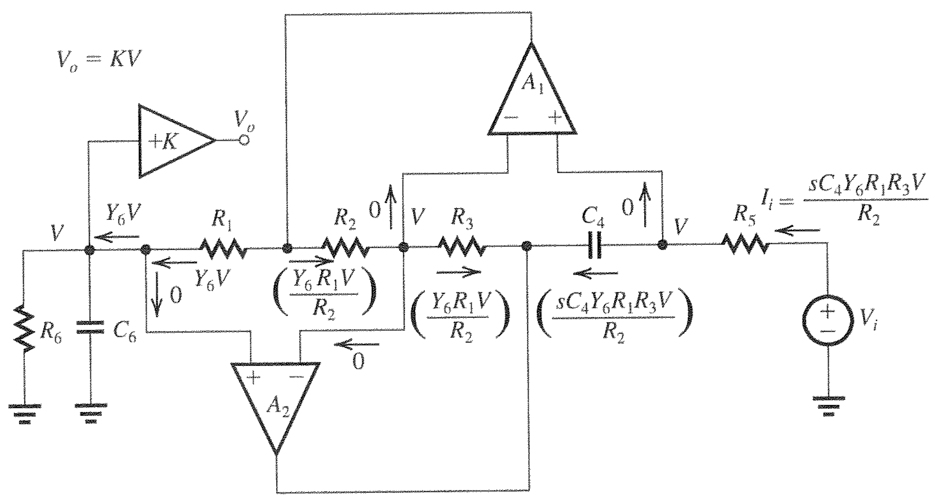


Figure 1

Figure 1 shows the circuit together with the analysis details. At the input we can write

$$V_i = V + I_iR_5$$

$$= V + \frac{sC_4Y_6R_1R_3R_5}{R_2}V$$

where

$$V = \frac{V_o}{K}$$

and

$$Y_6 = \frac{1}{R_6} + sC_6$$

Thus,

$$\frac{V_o}{V_i} = \frac{KV}{V_i}$$

$$= \frac{K}{1 + sC_4\left(sC_6 + \frac{1}{R_6}\right)\frac{R_1R_3R_5}{R_2}}$$

$$= \frac{K\ R_2/C_4C_6R_1R_3R_5}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$$

which is a low-pass function with

$$\omega_0 = 1/\sqrt{C_4C_6R_1R_3R_5/R_2}$$

$$Q = R_6\sqrt{\frac{C_6}{C_4}\ \frac{R_2}{R_1R_3R_4}}$$

DC gain $= K$

which are the same as the data given in Table 17.1. Q.E.D.

17.55 For a fifth-order Butterworth filter with $A_{\max} = 3$ dB, and thus $\epsilon = 1$, and $\omega_p = 10^3$ rad/s, the poles can be determined using the graphical construct of Fig. 17.10(a), thus

The pole pair p_1, p_1^* has $\omega_0 = 10^3$ rad/s and

$$\frac{\omega_0}{2Q} = 10^3 \sin\left(\frac{\pi}{10}\right) = 10^3 \times 0.309$$

$$\Rightarrow Q = \frac{1}{2 \times 0.309} = 1.618$$

The pole pair p_2, p_2^* has $\omega_0 = 10^3$ rad/s and

$$\frac{\omega_0}{2Q} = 10^3 \sin\left(\frac{3\pi}{10}\right) = 10^3 \times 0.809$$

$$\Rightarrow Q = \frac{1}{2 \times 0.809} = 0.618$$

The real-axis pole p_3 is at a

$s = -10^3$ rad/s

Thus, the transfer function $T(s)$ is

$$T(s) = \frac{10^3}{s + 10^3} \times \frac{10^6}{s^2 + s\,\dfrac{10^3}{1.618} + 10^6} \times \frac{10^6}{s^2 + s\,\dfrac{10^3}{0.618} + 10^6}$$

The first-order factor,

$$T_1(s) = \frac{10^3}{s + 10^3}$$

can be realized by the circuit in Fig. 17.13(a) with $R_1 = R_2 = 100$ kΩ (arbitrary but convenient value).

$$CR_1 = \frac{1}{\omega_0}$$

$$\Rightarrow C = \frac{1}{10^3 \times 100 \times 10^3} = 0.01\ \mu\text{F} = 10\ \text{nF}$$

The second-order transfer function

$$T_2(s) = \frac{10^6}{s^2 + s\dfrac{10^3}{1.618} + 10^6}$$

can be realized by the circuit in Fig. 17.22(a) with

$C_4 = C_6 = C = 10$ nF (practical value)

$$R_1 = R_2 = R_3 = R_5 = \frac{1}{\omega_0 C}$$

$$= \frac{1}{10^3 \times 10 \times 10^{-9}} = 100\ \text{k}\Omega$$

$R_6 = Q/\omega_0 C = Q \times 100 = 161.8$ kΩ

$K = 1$

The second-order transfer function

$$T_3(s) = \frac{10^6}{s^2 + s\dfrac{10^3}{0.618} + 10^6}$$

can be realized using the circuit in Fig. 17.22(a) with

$C_4 = C_6 = C = 10$ nF (practical value)

$$R_1 = R_2 = R_3 = R_5 = \frac{1}{\omega_0 C} = 100\ \text{k}\Omega$$

$$R_6 = Q\left(\frac{1}{\omega_0 C}\right) = 0.618 \times 100 = 61.8\ \text{k}\Omega$$

$K = 1$

The complete filter circuit is obtained as the cascade connection of the three filter sections, as shown in Fig. 1 on the next page.

17.56 Refer to Fig. 17.22(e) and the LPN entry in Table 17.1.

Select

$C_4 = C = 10$ nF

Thus,

$$R_1 = R_2 = R_3 = R_5 = \frac{1}{\omega_0 C}$$

$$= \frac{1}{2\pi \times 10 \times 10^3 \times 10 \times 10^{-9}} = 1.59\ \text{k}\Omega$$

$R_6 = Q \times 1.59 = 15.9$ kΩ

$$C_{61} = C\left(\frac{\omega_0}{\omega_n}\right)^2 = 10\left(\frac{10}{12}\right)^2 = 6.94\ \text{nF}$$

$C_{62} = C - 6.94 = 3.06$ nF

$K = 1$

17.57 $f_0 = 2$ kHz

Selecting,

$C_4 = C_6 = C = 1.0$ nF, we obtain

$$R_1 = R_2 = R_3 = R_5 = \frac{1}{\omega_0 C}$$

$$= \frac{1}{2\pi \times 2 \times 10^3 \times 10 \times 10^{-9}}$$

$= 79.6$ kΩ

$$R_6 = \frac{Q}{\omega_0 C} = 2 \times 7.96 = 159.2\ \text{k}\Omega$$

$r_1 = r_2 = 10$ kΩ (arbitrary)

This figure belongs to Problem 17.55.

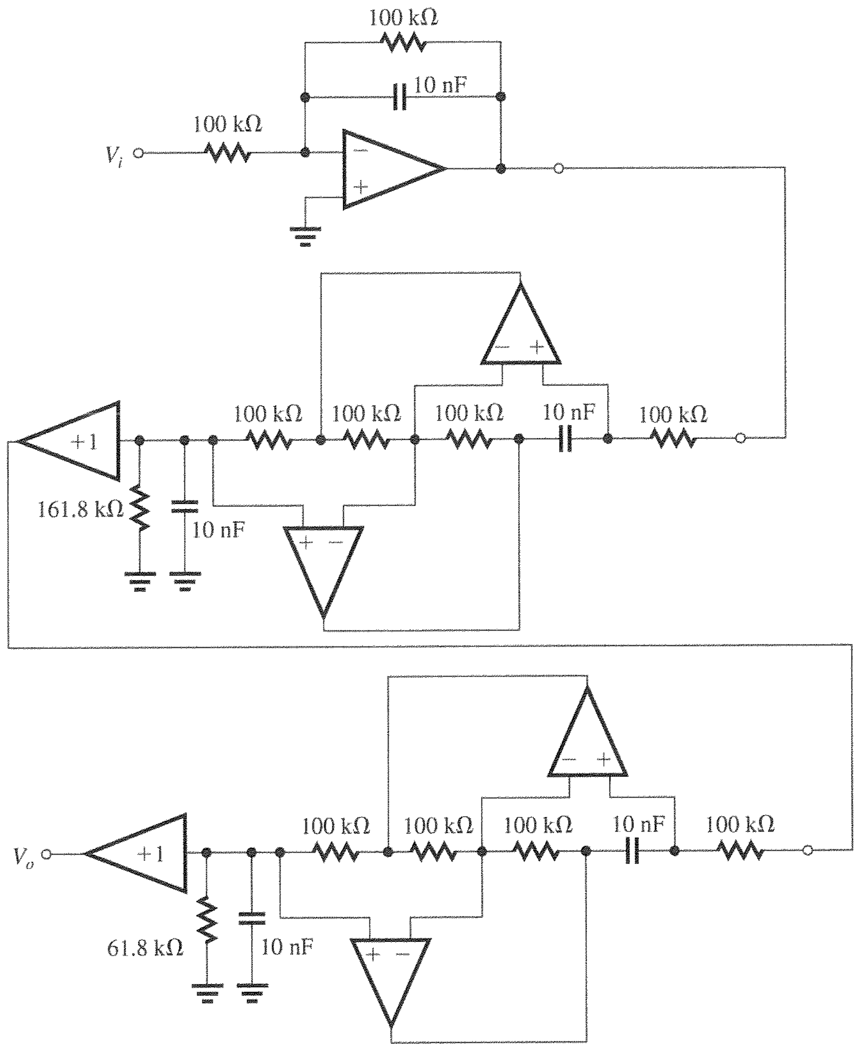


Figure 1

17.58 $T(s) = K\dfrac{C_{61}}{C_{61}+C_{62}} \times$

$$\frac{s^2 + (R_2/C_4C_{61}R_1R_3R_5)}{s^2 + s\dfrac{1}{(C_{61}+C_{62})R_6} + \dfrac{R_2}{C_4(C_{61}+C_{62})R_1R_3R_5}}$$

Thus,

$$\omega_n^2 = \frac{R_2}{C_4C_{61}R_1R_3R_5} \quad (1)$$

$$\omega_0^2 = \frac{R_2}{C_4(C_{61}+C_{62})R_1R_3R_5} \quad (2)$$

$$\frac{\omega_0}{Q} = \frac{1}{(C_{61}+C_{62})R_6} \quad (3)$$

Dividing Eq. (2) by Eq. (1), we obtain

$$\frac{\omega_0^2}{\omega_n^2} = \frac{C_{61}}{C_{61}+C_{62}} \quad (4)$$

Selecting $C_4 = C_{61} + C_{62} = C$ (practical value), then from Eq. (4) we obtain

$$C_{61} = C\left(\frac{\omega_0}{\omega_n}\right)^2$$

and

$$C_{62} = C - C_{61}$$

Selecting $R_1 = R_2 = R_3 = R_5 = R$, and using Eq. (2), we obtain

$$\omega_0^2 = \frac{1}{C^2R^2}$$

$$\Rightarrow R = \frac{1}{\omega_0 C} \quad (5)$$

Using Eq. (3), we obtain

$$R_6 = \frac{Q}{\omega_0 C} = QR$$

Finally, from the expression for $T(s)$, we see that

DC gain $= K$

17.59 Refer to Fig. 17.22(f) and to the HPN entry in Table 17.1.

$$T(s) = K\frac{s^2 + (R_2/C_4C_6R_1R_3R_{51})}{s^2 + s\dfrac{1}{C_6R_6} + \dfrac{R_2}{C_4C_6R_1R_3}\left(\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}}\right)}$$

$$\omega_n^2 = \frac{R_2}{C_4C_6R_1R_3R_{51}}$$

$$\omega_0^2 = \frac{R_2}{C_4C_6R_1R_3}\left(\frac{1}{R_{51}} + \frac{1}{R_{52}}\right)$$

Thus,

$$\frac{\omega_0^2}{\omega_n^2} = \frac{R_{51}}{R_5}$$

where

$$R_5 = R_{51} \parallel R_{52}$$

Thus,

$$R_{51} = R_5\left(\frac{\omega_0^2}{\omega_n^2}\right)$$

$$R_{52} = R_5/\left[1 - \left(\frac{\omega_n}{\omega_0}\right)^2\right]$$

Choosing $C_4 = C_5 = C$ (practical value) and $R_1 = R_2 = R_3 = R_4 = R$, we get

$$\omega_0^2 = \frac{R}{C^2R^3} = \frac{1}{C^2R^2}$$

$$\Rightarrow R = 1/\omega_0 C$$

$$\frac{\omega_0}{Q} = \frac{1}{C_6R_6} = \frac{1}{CR_6}$$

$$\Rightarrow R_6 = \frac{Q}{\omega_0 C}$$

Finally,

$K =$ High-frequency gain

17.60

(a) $$T(s) = \frac{0.4508(s^2 + 1.6996)}{(s + 0.7294)(s^2 + s0.2786 + 1.0504)}$$

Replacing s by $s/10^5$, we obtain

$$T(s) = \frac{0.4508\left(\dfrac{s^2}{10^{10}} + 1.6996\right)}{\left(\dfrac{s}{10^5} + 0.7294\right)\left(\dfrac{s^2}{10^{10}} + \dfrac{s}{10^5}0.2786 + 1.0504\right)}$$

$$= \frac{0.4508 \times 10^5(s^2 + 1.6996 \times 10^{10})}{(s + 0.7294 \times 10^5) \times (s^2 + s0.2786 \times 10^5 + 1.0504 \times 10^{10})}$$

(b) First-order section:

$$T_1(s) = \frac{0.7294 \times 10^5}{s + 0.7294 \times 10^5}$$

where the dc gain is made unity. This function can be realized using the circuit of Fig. 17.13(a) with

$C = 1$ nF (arbitrary but convenient value)

$$R_2 = \frac{1}{\omega_0 C} = \frac{1}{0.7294 \times 10^5 \times 1 \times 10^{-9}}$$

$= 13.71$ kΩ

For dc gain of unity, we have

$R_1 = R_2 = 13.71$ kΩ

Second-order LPN section:

$$T_2(s) = \frac{0.618(s^2 + 1.6996 \times 10^5)}{s^2 + s0.2786 \times 10^5 + 1.0504 \times 10^{10}}$$

Selecting, we obtain

$C_4 = C_{61} + C_{62} = C = 1$ nF

and

$R_1 = R_2 = R_3 = R_5 = R$

then

$$R = \frac{1}{\omega_0 C} = \frac{1}{\sqrt{1.0504} \times 10^5 \times 1 \times 10^{-9}}$$

$= 9.76$ kΩ

$$C_{61} = C\left(\frac{\omega_0}{\omega_n}\right)^2$$

$$= 1 \times 10^{-9}\,\frac{1.0504 \times 10^{10}}{1.6996 \times 10^{10}} = 0.618 \text{ nF}$$

$= 618$ pF

$C_{62} = 1 - 0.618 = 0.382$ nF $= 382$ pF

$$Q = \frac{\sqrt{1.0504}}{0.2786} = 3.679$$

$$R_6 = \frac{Q}{\omega_0 C} = 3.679 \times 9.76 = 35.9 \text{ k}\Omega$$

Finally,

$K =$ DC gain $= 1$

The complete circuit is shown in Fig. 1 on the next page.

17.61 Bandpass with $f_0 = 2$ kHz, 3-dB bandwidth of 50 Hz, thus

$$Q = \frac{f_0}{BW} = \frac{2 \text{ kHz}}{50 \text{ Hz}} = 40$$

Refer to the circuit in Fig. 17.24(a). Using

$C = 10$ nF

This figure belongs to Problem 17.60, part (b).

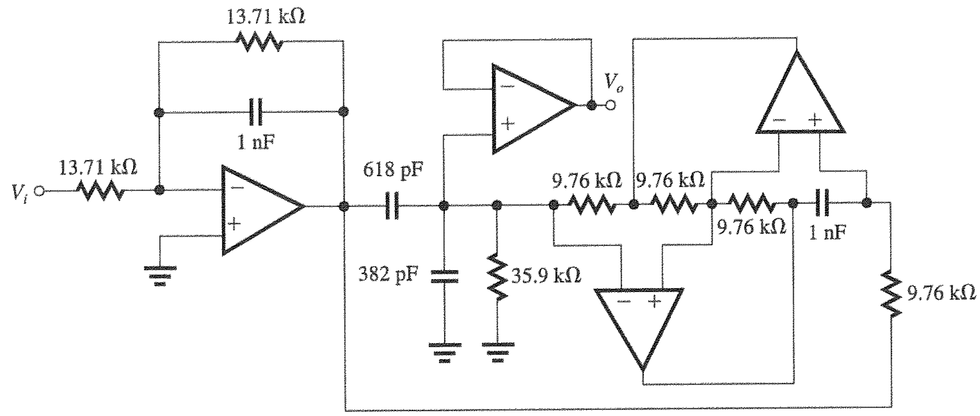


Figure 1

then

$$R = \frac{1}{\omega_0 C} = \frac{1}{2\pi \times 2 \times 10^3 \times 10 \times 10^{-9}}$$

$= 7.96\ \text{k}\Omega$

Select

$R_1 = 10\ \text{k}\Omega$

then

$R_f = R_1 = 10\ \text{k}\Omega$

Select

$R_2 = 1\ \text{k}\Omega$

then

$R_3 = (2Q - 1)R_2 = (80 - 1) \times 1 = 79\ \text{k}\Omega$

$$K = 2 - \frac{1}{Q} = 2 - \frac{1}{40} = 1.975$$

Center-frequency gain $= KQ = 1.975 \times 40 = 79$

17.62 (a) Refer to the circuits in Fig. 17.24 and the transfer function in Eq. (17.66), that is,

$$T(s) = -K\frac{(R_F/R_H)s^2 - s(R_F/R_B)\omega_0 + (R_F/R_L)\omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

$$= -K\left(\frac{R_F}{R_H}\right)\frac{s^2 - s(R_H/R_B)\omega_0 + (R_H/R_L)\omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

For this to be an all-pass function, that is,

$$T(s) = -\text{Flat gain} \times \frac{s^2 - s(\omega_0/Q) + \omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

then

$$\frac{R_H}{R_L} = 1 \qquad (1)$$

and

$$\frac{R_H}{R_B} = \frac{1}{Q} \qquad (2)$$

That is,

$$R_L = R_H = \frac{R_B}{Q} \qquad \text{Q.E.D.}$$

$$\text{Flat gain} = -K\frac{R_F}{R_H} \qquad \text{Q.E.D.}$$

(b) $\omega_0 = 10^5$ rad/s, $Q = 4$ and flat gain $= 10$, thus

Selecting

$C = 1\ \text{nF}$

then

$$R = \frac{1}{\omega_0 C} = \frac{1}{10^5 \times 1 \times 10^{-9}} = 10\ \text{k}\Omega$$

Selecting

$R_1 = 10\ \text{k}\Omega$

then

$R_f = R_1 = 10\text{k}\Omega$

Selecting

$R_2 = 10\ \text{k}\Omega$

then

$R_3 = R_2(2Q - 1) = 70\ \text{k}\Omega$

Selecting

$R_L = R_H = 10\ \text{k}\Omega$

then

$R_B = QR_H = 4 \times 10 = 40\ \text{k}\Omega$

Now,

$$K = 2 - \frac{1}{Q} = 2 - \frac{1}{4} = 1.75$$

$$\text{Flat gain} = -10 = -K\frac{R_F}{R_H}$$

$$\Rightarrow R_F = \frac{10R_H}{K} = \frac{10 \times 10}{1.75} = 57.1\ \text{k}\Omega$$

17.63 Consider Fig. 17.24 and Eq. (17.66), that is,

$$T(s) = -K\frac{(R_F/R_H)s^2 - s(R_F/R_B)\omega_0 + (R_F/R_L)\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \quad (1)$$

For this to be the transfer function of a notch filter, that is,

$$T(s) = -G\frac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2} \quad (2)$$

where G is the high-frequency gain, then by equating the coefficients of the corresponding numerator terms, we obtain

$$R_B = \infty \quad (3)$$

$$\omega_n^2 = \frac{R_H}{R_L}\omega_0^2$$

$$\Rightarrow \frac{R_H}{R_L} = \left(\frac{\omega_0}{\omega_n}\right)^2 \quad (4)$$

$$G = K\frac{R_F}{R_H}$$

where

$$K = 2 - \frac{1}{Q}$$

thus

$$G = \left(2 - \frac{1}{Q}\right)\frac{R_F}{R_H}$$

$$\Rightarrow \frac{R_F}{R_H} = \frac{G}{2 - (1/Q)} \quad (5)$$

Equations (3), (4), and (5) are the design equations for the resistors associated with the summer. Observe that the value of one of the three resistors, R_L, R_H, and R_F can be arbitrarily selected.

17.64 Using Eq. (17.66) with $R_B = \infty$, we obtain

$$\frac{V_o}{V_i} = -K\left(\frac{R_F}{R_H}\right)\frac{s^2 + (R_H/R_L)\omega_0^2}{s^2 + s\left(\dfrac{\omega_0}{Q}\right) + \omega_0^2}$$

Thus,

$$\omega_n^2 = \left(\frac{R_H}{R_L}\right)\omega_0^2$$

Now if R_H and R_L can have 1% tolerances, the worst case will be when one is at the highest possible value and the other at the lowest possible value, for instance,

$$\omega_n^2 = \frac{R_{HN}(1.01)}{R_{LN}(0.99)}\omega_0^2$$

where R_{HN} and R_{LN} are the nominal values. In this case

$$\omega_n^2 \simeq 1.02\omega_0^2$$

$$\omega_n \simeq 1.01\omega_0$$

The other case yields

$$\omega_n \simeq 0.99\omega_0$$

Thus the worst-case percentage deviation between ω_n and ω_0 is 1%.

17.65 Refer to Fig. 17.26 and Table 17.2. Using

$C = 10$ nF,

then

$$R = \frac{1}{\omega_0 C} = \frac{1}{10^5 \times 10 \times 10^{-9}}$$

$$= 1\ \text{k}\Omega$$

$$R_d = QR = 10 \times 1 = 10\ \text{k}\Omega$$

Select

$$r = 20\ \text{k}\Omega$$

$$R_1 = R_3 = \infty$$

If the dc gain is unity, then

$$1 = \text{HF gain} \times \frac{\omega_n^2}{\omega_0^2}$$

$$\Rightarrow \text{HF gain} = \left(\frac{10^5}{1.3 \times 10^5}\right)^2$$

$$= 0.5917$$

$$C_1 = C \times \text{high-frequency gain}$$

$$= 10 \times 0.5917 = 5.92\ \text{nF}$$

$$R_2 = R\frac{(\omega_0/\omega_n)^2}{\text{HF gain}} = 1\ \text{k}\Omega$$

17.66 Using Eq. (17.68) with $R_1 = \infty$, we have

$$\frac{V_o}{V_i} = -\frac{C_1}{C}\frac{s^2 - s\left(\dfrac{r}{R_3}\right)\left(\dfrac{1}{C_1R}\right) + \dfrac{1}{CC_1RR_2}}{s^2 + s\dfrac{1}{QCR} + \dfrac{1}{C^2R^2}}$$

Thus,

$$\omega_z = 1/\sqrt{CC_1RR_2} \quad (1)$$

$$Q_z = \frac{1}{\sqrt{CC_1RR_2}}\left(\frac{R_3}{r}\right)C_1R$$

$$= \sqrt{\left(\frac{C_1}{C}\right)\left(\frac{R}{R_2}\right)}\left(\frac{R_3}{r}\right) \qquad (2)$$

From Eqs. (1) and (2) we see that trimming ω_z and Q_z can proceed in the following sequence:

(a) Trim R_2 to adjust ω_z. This will affect Q_z.

(b) Trim R_3 to adjust Q_z. This will not affect ω_z

17.67

$$T(s) = \frac{0.4508(s^2+1.6996)}{(s+0.7294)(s^2+s0.2786+1.0504)}$$

(a) Replacing s by $s/10^5$, we obtain

$$T(s) = \frac{0.4508\times 10^5(s^2+1.6996\times 10^{10})}{(s+0.7294\times 10^5)(s^2+s\,0.2786\times 10^5+1.0504\times 10^{10})}$$

(b) First-order section:

$$T_1(s) = \frac{0.7294\times 10^5}{s+0.7294\times 10^5}$$

which is made to have a dc gain of unity, as required. This function can be realized by the circuit in Fig. 17.13(a). Selecting

$C = 1$ nF (arbitrary but convenient)

we have

$$R_2 = \frac{1}{\omega_0 C}$$

$$= \frac{1}{0.7294\times 10^5\times 1\times 10^{-9}}$$

$= 13.71$ kΩ

For a dc gain of unity, we have

$R_1 = R_2 = 13.71$ kΩ

Second-order LPN section:

$$T_2(s) = \frac{0.618(s^2+1.6996\times 10^{10})}{s^2+s0.2786\times 10^5+1.0504\times 10^{10}}$$

where the dc gain is unity. Refer to Fig. 17.26 and Eq. (17.68).

Selecting

$C = 1$ nF

then

$$R = \frac{1}{\omega_0 C} = \frac{1}{\sqrt{1.0504}\times 10^5\times 1\times 10^{-9}}$$

$= 9.76$ kΩ

$$R_d = QR = \frac{\sqrt{1.0504}}{0.2786}\times 9.76 = 35.9\text{ k}\Omega$$

Select

$r = 10$ kΩ

Now,

$C_1 = C \times$ high-frequency gain

$= 1\times 0.618 = 0.618$ nF $= 618$ pF

$R_1 = \infty$

$R_3 = \infty$

$$R_2 = R\frac{(\omega_0/\omega_n)^2}{\text{HF Gain}}$$

$$= 9.76\times\frac{1.0504}{1.6996}\times\frac{1}{0.618}$$

$= 9.76$ kΩ

The complete circuit is shown in Fig. 1.

This figure belongs to Problem 17.67.

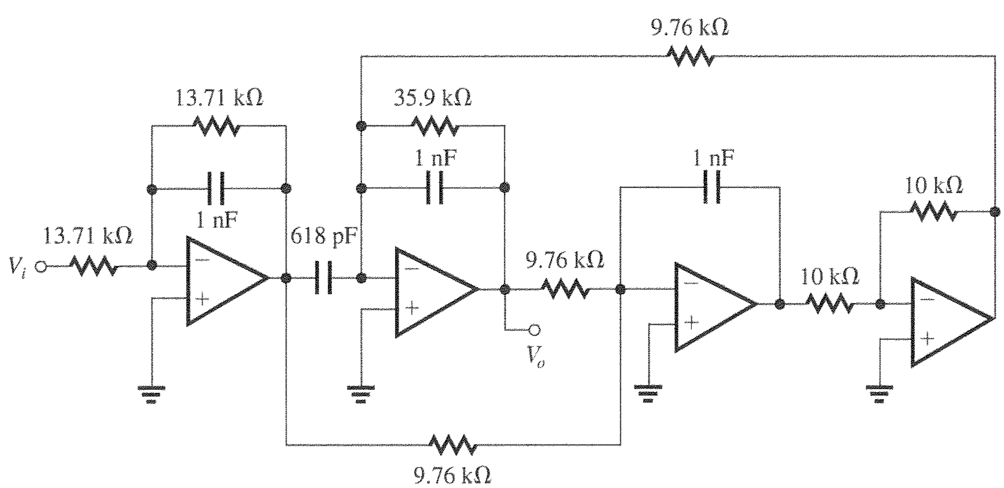


Figure 1

17.68 Refer to Fig. 17.29 and Eqs. (17.75) and (17.76):

$$C_1 = C_2 = C = 1 \text{ nF}$$

$$R_3 = R \text{ and } R_4 = R/4Q^2 = R\Big/\left(4 \times \frac{1}{2}\right) = \frac{R}{2}$$

$$CR = \frac{2Q}{\omega_0} = \frac{2/\sqrt{2}}{10^5}$$

$$\Rightarrow R = \frac{\sqrt{2}}{10^5 \times 1 \times 10^{-9}} = 14.14 \text{ k}\Omega$$

Thus,

$$R_3 = 14.14 \text{ k}\Omega$$

$$R_4 = 7.07 \text{ k}\Omega$$

17.69

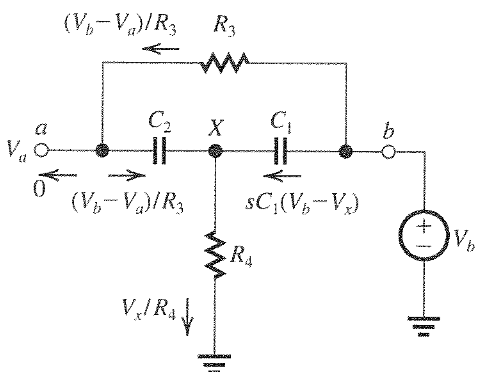


Figure 1

Refer to Fig. 1. The voltage at node X can be written as

$$V_x = V_a - \frac{1}{sC_2}\,\frac{V_b - V_a}{R_3}$$

$$= V_a\left(1 + \frac{1}{sC_2R_3}\right) - \frac{V_b}{sC_2R_3} \qquad (1)$$

Writing a node equation at X gives

$$\frac{V_b - V_a}{R_3} + sC_1(V_b - V_x) = \frac{V_x}{R_4}$$

$$V_b\left(sC_1 + \frac{1}{R_3}\right) - \frac{V_a}{R_3} = V_x\left(\frac{1}{R_4} + sC_1\right)$$

Substituting for V_x from Eq. (1) gives

$$V_b\left(sC_1 + \frac{1}{R_3}\right) - \frac{1}{R_3}V_a$$

$$= \left(\frac{1}{R_4} + sC_1\right)\left(1 + \frac{1}{sC_2R_3}\right)V_a$$

$$- \left(\frac{1}{R_4} + sC_1\right)\left(\frac{1}{sC_2R_3}\right)V_b$$

Thus,

$$V_b\left(sC_1 + \frac{1}{R_3} + \frac{1}{sC_2R_3R_4} + \frac{C_1}{C_2R_3}\right)$$

$$= V_a\left[\left(\frac{1}{R_3} + \frac{1}{R_4} + sC_1 + \frac{C_1}{C_2R_3}\right) + \frac{1}{sC_2R_3R_4}\right]$$

$$\Rightarrow \frac{V_a}{V_b} =$$

$$\frac{sC_1 + \dfrac{1}{R_3}\left(1 + \dfrac{C_1}{C_2}\right) + \dfrac{1}{sC_2R_3R_4}}{sC_1 + \left(\dfrac{1}{R_3} + \dfrac{1}{R_4} + \dfrac{C_1}{C_2R_3}\right) + \dfrac{1}{sC_2R_3R_4}}$$

$$= \frac{s^2 + s\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)\dfrac{1}{R_3} + \dfrac{1}{C_1C_2R_3R_4}}{s^2 + s\left(\dfrac{1}{C_1R_3} + \dfrac{1}{C_2R_3} + \dfrac{1}{C_1R_4}\right) + \dfrac{1}{C_1C_2R_3R_4}}$$

which is identical to the expression given in Fig. 17.28(a). Q.E.D.

17.70 Refer to Fig. 17.28(a).

$$t(s) = \frac{s^2 + s\dfrac{2}{CR} + \dfrac{1}{C^2R^2}}{s^2 + s\dfrac{3}{CR} + \dfrac{1}{C^2R^2}}$$

$$= \frac{s^2 + s(2/\tau) + (1/\tau^2)}{s^2 + s(3/\tau) + (1/\tau^2)}$$

If the network is placed in the negative-feedback path of an ideal infinite-gain op amp, as in Fig. 17.24, the poles will be given by the roots of the numerator polynomial, thus

$$\omega_0 = \frac{1}{\tau}$$

and

$$Q = \frac{1/\tau}{2/\tau} = 0.5$$

Thus, the poles will be coincident at

$$s = -\omega_0 = -1/\tau$$

17.71

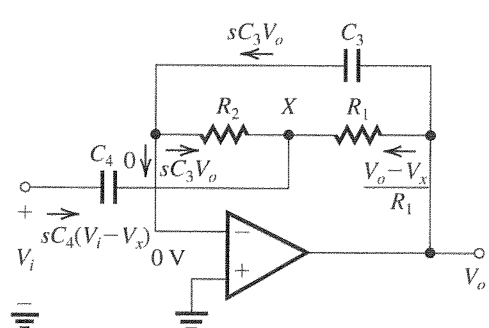


Figure 1

The circuit is shown in Fig. 1. The voltage at node X can be found as

$$V_x = 0 - sC_3V_oR_2$$

$$= -sC_3R_2V_o \qquad (1)$$

A node equation at X can be written as

$$sC_3V_o + sC_4(V_i - V_x) + \frac{V_o - V_x}{R_1} = 0 \qquad (2)$$

Substituting for V_x from Eq. (1), we obtain

$$sC_3V_o + sC_4V_i - (-sC_3R_2V_o)\left(sC_4 + \frac{1}{R_1}\right) + \frac{V_o}{R_1} = 0$$

$$V_o\left(sC_3 + s^2C_3C_4R_2 + sC_3\frac{R_2}{R_1} + \frac{1}{R_1}\right) = -sC_4V_i$$

$$\frac{V_o}{V_i} = \frac{-sC_4}{s^2C_3C_4R_2 + sC_3\left(1 + \frac{R_2}{R_1}\right) + \frac{1}{R_1}}$$

$$= \frac{-s/C_3R_2}{s^2 + s\frac{1}{C_4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{1}{C_3C_4R_1R_2}}$$

For $R_1 = R_2 = R$, $C_4 = C$, and $C_3 = C/36$, we obtain

$$\frac{V_o}{V_i} = \frac{-s(36/CR)}{s^2 + s\frac{2}{CR} + \frac{36}{C^2R^2}}$$

This is a bandpass function with

$$\omega_0 = \frac{6}{CR}$$

$$Q = \frac{6/CR}{2/CR} = 3$$

and

Center-frequency gain $= -18$

17.72

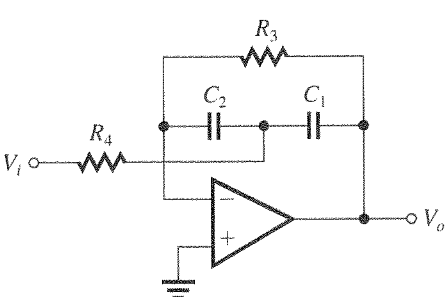


Figure 1

The circuit is shown in Fig. 1. The transfer function can be obtained using the equation on page 1340 with $\alpha = 1$, thus

$$\frac{V_o}{V_i} = \frac{-s/C_1R_4}{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}$$

Using the design equations (17.75) and (17.76), we obtain

$$R_3 = R$$

$$R_4 = R/4Q^2$$

$$CR = \frac{2Q}{\omega_0}$$

where

$$\omega_0 = 2\pi \times 10 \times 10^3$$

$$Q = \frac{f_0}{BW} = \frac{10}{2} = 5$$

Thus,

$$CR = \frac{2 \times 5}{2\pi \times 10^4}$$

Selecting

$$C = 10 \text{ nF}$$

we obtain

$$R = \frac{10}{2\pi \times 10^4 \times 10 \times 10^{-9}} = 15.92 \text{ k}\Omega$$

Thus,

$$C_1 = C_2 = 10 \text{ nF}$$

$$R_3 = 15.92 \text{ k}\Omega$$

$$R_4 = \frac{R}{4Q^2} = \frac{15.92}{100} = 159.2 \ \Omega$$

$$\text{Center-frequency gain} = -\frac{1/C_1R_4}{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3}}$$

$$= -\frac{1}{2}\frac{R_3}{R_4} = -\frac{1}{2} \times 100 = -50 \text{ V/V}$$

17.73

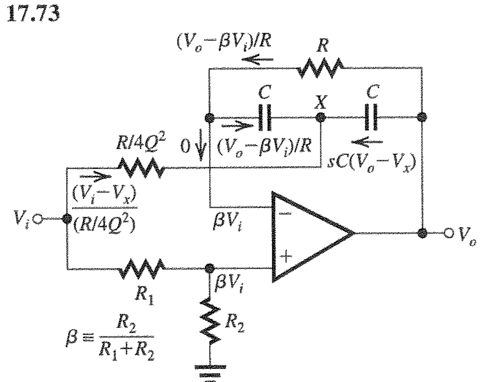


Figure 1

The circuit is shown in Fig. 1. The voltage at node X is given by

$$V_x = \beta V_i - \frac{1}{sC}\frac{V_o - \beta V_i}{R}$$

$$= \beta V_i\left(1 + \frac{1}{sCR}\right) - \frac{V_o}{sCR} \qquad (1)$$

Writing a node equation at X gives

$$\frac{V_o - \beta V_i}{R} + \frac{V_i - V_x}{R/4Q^2} + sC(V_o - V_x) = 0$$

Substituting for V_x from Eq. (1) and collecting terms gives

$$\frac{V_i}{V_x} = \beta \frac{s^2 + s\dfrac{2}{CR}\left[1 + 2Q^2\left(1 - \dfrac{1}{\beta}\right)\right] + \dfrac{4Q^2}{C^2R^2}}{s^2 + s\dfrac{2}{CR} + \dfrac{4Q^2}{C^2R^2}}$$

We observe that, as expected,

$$\omega_0 = \frac{2Q}{CR}$$

(a) To obtain an all-pass function, we set

$$\frac{2}{CR}\left[1 + 2Q^2\left(1 - \frac{1}{\beta}\right)\right] = -\frac{2}{CR}$$

$$\Rightarrow \frac{1}{\beta} = 1 + \frac{1}{Q^2}$$

But,

$$\beta = \frac{R_2}{R_1 + R_2}$$

Thus,

$$\frac{R_2}{R_1} = Q^2$$

(b) To obtain a notch function, we set

$$\frac{2}{CR}\left[1 + 2Q^2\left(1 - \frac{1}{\beta}\right)\right] = 0$$

$$\Rightarrow \frac{1}{\beta} = 1 + \frac{1}{2Q^2}$$

or, equivalently,

$$\frac{R_2}{R_1} = 2Q^2$$

17.74

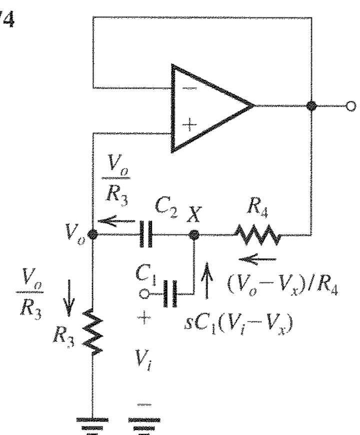


Figure 1

The analysis is shown in Fig. 1. The voltage at node X is given by

$$V_x = V_o + \frac{V_o}{R_3}\frac{1}{sC_2}$$

$$= V_o\left(1 + \frac{1}{sC_2R_3}\right) \qquad (1)$$

A node equation at X provides

$$\frac{V_o - V_x}{R_4} + sC_1(V_i - V_x) = \frac{V_o}{R_3}$$

$$V_o\left(\frac{1}{R_4} - \frac{1}{R_3}\right) + sCV_i - V_x\left(\frac{1}{R_4} + sC_1\right) = 0$$

Substituting for V_x from Eq. (1) and collecting terms, we obtain

$$\frac{V_o}{V_i} = \frac{s^2}{s^2 + s\dfrac{1}{R_3}\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right) + \dfrac{1}{C_1C_2R_3R_4}}$$

This is a high-pass function with a high-frequency gain of unity. To obtain a maximally flat response with $\omega_{3dB} = 10^4$ rad/s and using

$$C_1 = C_2 = C = 10 \text{ nF}$$

then

$$\omega_0 = \omega_{3dB} = 10^4 \text{ rad/s}$$

$$Q = \frac{1}{\sqrt{2}} = \omega_0 R_3 \Big/ \left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\frac{1}{\sqrt{2}} = 10^4 R_3 \Big/ \left(\frac{2}{10 \times 10^{-9}}\right)$$

$$\Rightarrow R_3 = \frac{1}{\sqrt{2}} \times \frac{2}{10^{-8}} \times 10^{-4} = 14.14 \text{ k}\Omega$$

$$\omega_0^2 = \frac{1}{C_1C_2R_3R_4}$$

$$10^8 = \frac{1}{10^{-8} \times 10^{-8} \times 14.4 \times 10^3 \times R_4}$$

$$\Rightarrow R_4 = \frac{100}{14.14} \text{ k}\Omega = 7.07 \text{ k}\Omega$$

17.75

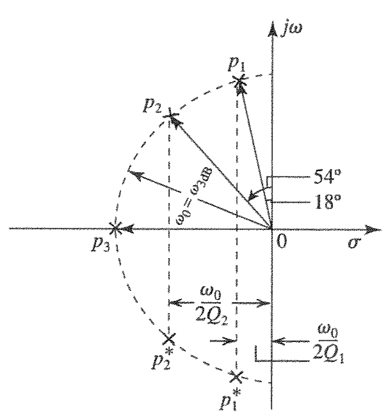


Figure 1

Figure 1 shows a graphical construct to determine the poles of the fifth order Butterworth filter. The pair of complex conjugate poles p_1 and p_1^* have a frequency

$$\omega_{01} = \omega_{3dB} = 2\pi \times 10^4 \text{ rad/s}$$

and a Q factor

$$Q_1 = \frac{1}{2 \sin 18^\circ} = 1.618$$

The pair of complex conjugate poles p_2 and p_2^* have

$$\omega_{02} = \omega_{3dB} = 2\pi \times 10^4 \text{ rad/s}$$

and a Q factor,

$$Q_2 = \frac{1}{2 \sin 54^\circ} = 0.618$$

The real-axis pole p_3 is at

$$s = -\omega_0 = -2\pi \times 10^4 \text{ rad/s}$$

The first second-order section can be realized using the circuit in Fig. 17.34(c). The design equations are (17.77)–(17.80).

$$R_1 = R_2 = R = 10 \text{ k}\Omega$$

$$C_4 = C$$

$$C_3 = C/4Q^2$$

Here, $Q = Q_1 = 1.618$, thus

$$C_3 = \frac{C}{4 \times 1.618^2} = 0.095C$$

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times 1.618}{2\pi \times 10^4}$$

$$\Rightarrow C = \frac{2 \times 1.618}{2\pi \times 10^4 \times 10 \times 10^3} = 5.15 \text{ nF}$$

$$C_3 = 0.492 \text{ nF} = 492 \text{ pF}$$

$$C_4 = 5.15 \text{ nF}$$

The second second-order section also can be realized using the circuit in Fig. 17.34(c). Here,

$$R_1 = R_2 = R = 10 \text{ k}\Omega$$

$$C_4 = C$$

$$C_3 = \frac{C}{4Q^2}$$

where $Q = Q_2 = 0.618$. Thus

$$C_3 = \frac{C}{4 \times 0.618^2} = 0.655C$$

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times 0.618}{2\pi \times 10^4}$$

$$\Rightarrow C = \frac{2 \times 0.618}{2\pi \times 10^4 \times 10^4} = 1.97 \text{ nF}$$

$$C_3 = 1.29 \text{ nF}$$

$$C_4 = 1.97 \text{ nF}$$

The first-order section can be realized using the circuit in Fig. 17.13(a) with

$$R_1 = R_2 = R = 10 \text{ k}\Omega$$

$$C = \frac{1}{\omega_0 R}$$

$$= \frac{1}{2\pi \times 10^4 \times 10^4} = 1.59 \text{ nF}$$

The complete circuit is shown in Fig. 2 below.

17.76 Refer to Fig. 17.31 and let the network n have a transfer function

$$\frac{V_a}{V_b} = \frac{s\dfrac{\omega_0}{Q}}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

which is a bandpass with a unity center-frequency gain. The complementary network in (b) will have a transfer function

This figure belongs to Problem 17.75, part (b).

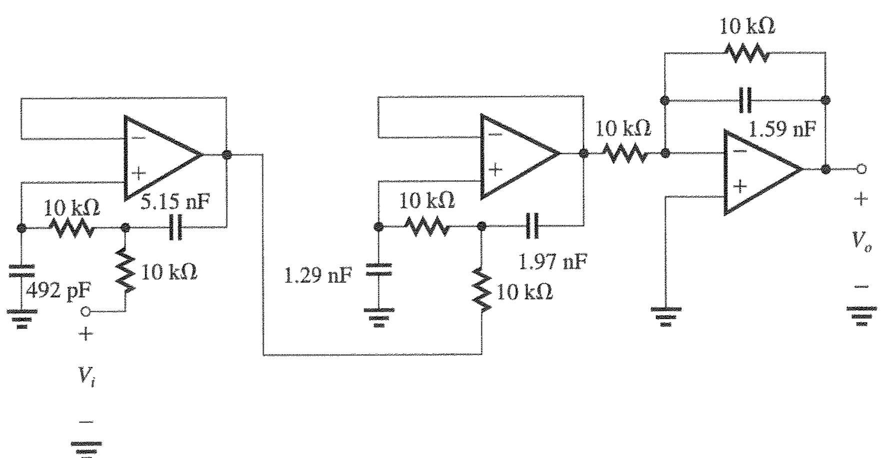


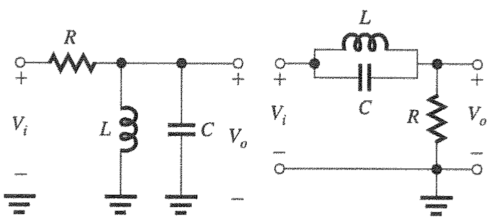
Figure 2

$$\frac{V_a}{V_c} = 1 - \frac{V_a}{V_b}$$

$$= 1 - \frac{s\dfrac{\omega_0}{Q}}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

$$= \frac{s^2 + \omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

which is a notch function.



(a) (b)

Figure 1

As an example, consider the RLC bandpass circuit shown in Fig. 1(a). It has the transfer function

$$T_1(s) = \frac{V_o}{V_i} = \frac{s\dfrac{1}{CR}}{s^2 + s\dfrac{1}{CR} + \dfrac{1}{LC}}$$

Interchanging the input terminal with ground, we obtain the circuit shown in Fig. 1(b). Straightforward analysis shows that this circuit has the transfer function

$$T_2(s) = \frac{s^2 + \dfrac{1}{LC}}{s^2 + s\dfrac{1}{CR} + \dfrac{1}{LC}}$$

which is a notch function. Observe that

$$T_2(s) = 1 - T_1(s)$$

that is, the circuits in (a) and (b) are complementary.

17.77 For the circuit in Fig. 17.18(b) we have

$$T(s) = \frac{1/LC}{s^2 + s/RC + 1/LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q = R\sqrt{\frac{C}{L}}$$

For ω_0 we have

$$\frac{\partial \omega_0}{\partial L} = \frac{\partial (LC)^{-1/2}}{\partial L} = -\frac{1}{2}L^{-3/2}C^{-1/2} = \frac{\omega_0}{2L}$$

$$\frac{\partial \omega_0}{\partial C} = -\frac{\omega_0}{2C}$$

$$\frac{\partial \omega_0}{\partial R} = 0$$

$$\therefore\ S_L^{\omega_0} = \frac{\partial \omega_0}{\partial L}\frac{L}{\omega_0} = -1/2$$

$$S_C^{\omega_0} = \frac{\partial \omega_0}{\partial C} \times \frac{C}{\omega_0} = -1/2$$

$$S_R^{\omega_0} = \frac{\partial \omega_0}{\partial R}\frac{R}{\omega_0} = 0$$

For Q we have

$$\frac{\partial Q}{\partial L} = \frac{R\sqrt{C}}{L\sqrt{L}}\left(\frac{-1}{2}\right) = \frac{-Q}{2L}$$

$$\frac{\partial Q}{\partial C} = \frac{1}{2}\frac{R}{\sqrt{LC}} = \frac{1}{2}\frac{R\sqrt{C}}{C\sqrt{L}} = \frac{Q}{2C}$$

$$\frac{\partial Q}{\partial R} = \sqrt{C/L} = \frac{R}{R}\sqrt{C/L} = Q/R$$

$$S_L^Q = -\frac{Q}{2L} \times \frac{L}{Q} = -\frac{1}{2}$$

$$S_C^Q = \frac{Q}{2C} \times \frac{C}{Q} = \frac{1}{2}$$

$$S_R^Q = \frac{Q}{R} \cdot \frac{R}{Q} = 1$$

17.78 (a) $y = uv$

$$S_x^y = \frac{\partial (uv)}{\partial x}\frac{x}{uv}$$

$$= v\frac{\partial u}{\partial x}\frac{x}{uv} + u\frac{\partial v}{\partial x}\frac{x}{uv}$$

$$= \frac{\partial u}{\partial x}\frac{x}{u} + \frac{\partial v}{\partial x}\frac{x}{v}$$

$$= S_x^u + S_x^v$$

(b) $y = u/v$

$$S_x^y = \frac{\partial y}{\partial x}\frac{x}{y} = \frac{\partial (u/v)}{\partial x}\frac{x}{u/v}$$

$$= \frac{1}{v}\frac{\partial u}{\partial x}\frac{xv}{u} - \frac{u}{v^2}\frac{\partial v}{\partial x}\frac{xv}{u}$$

$$= \frac{\partial u}{\partial x}\frac{x}{u} - \frac{\partial v}{\partial x}\frac{x}{v}$$

$$= S_x^u - S_x^v$$

(c) $y = ku$

$$S_x^y = \frac{\partial y}{\partial x}\frac{x}{y} = k\frac{\partial u}{\partial x}\frac{x}{ku}$$

$$= \frac{\partial u}{\partial x}\frac{x}{u}$$

$$= S_x^u$$

(d) $y = u^n$

$$S_x^y = \frac{\partial y}{\partial x}\frac{x}{y}$$

$$= nu^{n-1}\frac{\partial u}{\partial x}\frac{x}{u^n}$$

$$= n\frac{\partial u}{\partial n}\frac{x}{u}$$

$$= nS_x^u$$

(e) $y = f_1(u),\ u = f_2(x)$

$$S_x^y = \frac{\partial y}{\partial x}\frac{x}{y}$$

$$= \frac{\partial f_1(u)}{\partial u}\frac{\partial u}{\partial x}\frac{x}{f_1(u)}\frac{u}{u}$$

$$= \left[\frac{\partial f_1(u)}{\partial u}\frac{u}{f_1(u)}\right]\left[\frac{\partial f_2(x)}{\partial x}\frac{x}{u}\right]$$

$$= \left[\frac{\partial f_1(u)}{\partial u}\frac{u}{f_1(u)}\right]\left[\frac{\partial f_2(x)}{\partial x}\frac{x}{f_2(x)}\right]$$

$$= S_u^{f_1} S_x^{f_2}$$

$$= S_u^y S_x^u$$

17.79 The high-pass filter in Fig. 17.33(b) is derived from the feedback loop in Fig. (17.29); thus it will exhibit the same sensitivities relative to the op-amp gain as that of the circuit in Fig. (17.29). These have been derived in Example 17.3 and given by

$$S_A^{\omega_0} = 0$$

$$S_A^Q \simeq \frac{2Q^2}{A}$$

17.80 Using Eqs. (17.78) and (17.79), we have

$$\omega_0 = \frac{1}{\sqrt{C_3C_4R_1R_2}}$$

$$Q = \frac{1}{\sqrt{C_3C_4R_1R_2}\left(\frac{1}{C_4}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$$

$$\frac{\partial\omega_0}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3C_4R_1R_2}}$$

$$= \frac{-\omega_0}{2C_3}$$

$$S_{C_3}^{\omega_0} = \frac{\partial\omega_0}{\partial C_3}\frac{C_3}{\omega_O} = -\frac{1}{2}$$

Clearly, $S_{C_3}^{\omega_0} = S_{C_4}^{\omega_0} = S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = -\frac{1}{2}$

$$\frac{\partial Q}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3C_4R_1R_2}\left(\frac{1}{C_4}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$$

$$= \frac{-Q}{2C_3}$$

$$\therefore S_{C_3}^Q = -\frac{1}{2}$$

$$\frac{\partial Q}{\partial C_4} = \frac{Q}{2C_4} \Rightarrow S_{C_4}^Q = +\frac{1}{2}$$

$$\frac{\partial Q}{\partial R_1} = \frac{1/\sqrt{R_1} - \sqrt{R_1}/\mathrm{R}_2}{R_1\left(\frac{1}{\sqrt{R_1}}+\frac{\sqrt{R_1}}{R_2}\right)}\cdot\frac{Q}{2}$$

$$= \frac{\sqrt{R_2/\mathrm{R}_1} - \sqrt{R_1/\mathrm{R}_2}}{R_1\left(\sqrt{\frac{R_2}{R_1}}+\sqrt{\frac{R_1}{R_2}}\right)}\cdot\frac{Q}{2}$$

$$\therefore S_{R_1}^Q = \frac{1}{2}\frac{\sqrt{R_2/\mathrm{R}_1} - \sqrt{R_1/\mathrm{R}_2}}{\sqrt{R_2/\mathrm{R}_1} + \sqrt{R_1/\mathrm{R}_2}}$$

If $R_1 = R_2 \Rightarrow S_{R_1}^Q = 0.$

Similarly,

$$S_{R_2}^Q = 0$$

17.81 From Table 17.1 we have

$$\omega_0 = \frac{1}{\sqrt{C_4C_6R_1R_3R_5/\mathrm{R}_2}}$$

$$Q = R_6\sqrt{\frac{C_6}{C_4}\frac{R_2}{R_1R_3R_5}}$$

$$\frac{\partial\omega_0}{\partial C_4} = \frac{-\omega_0}{2C_4}$$

$$\therefore\ S_{C_4}^{\omega_0} = \frac{-\omega_0}{2C_4}\times\frac{C_4}{\omega_0} = -\frac{1}{2}$$

Similarly, $S_{C_6}^{\omega_0} = S_{R_1}^{\omega_0} = S_{R_3}^{\omega_0} = S_{R_5}^{\omega_o} = -\frac{1}{2}$

$$\frac{\partial\omega_0}{\partial R_2} = \frac{\omega_O}{2R_2} \Rightarrow S_{R_2}^{\omega_o} = \frac{1}{2}$$

Now for Q:

$$\frac{\partial Q}{\partial R_6} = \frac{Q}{R_6} \Rightarrow S_{R_6}^Q = \frac{\partial Q}{\partial R_6}\frac{R_6}{Q} = +1$$

$$\frac{\partial Q}{\partial C_6} = \frac{Q}{2C_6} \Rightarrow S_{C_6}^Q = S_{R_2}^Q = +\frac{1}{2}$$

$$\frac{\partial Q}{\partial C_4} = -\frac{Q}{2C_4} \Rightarrow S_{C_4}^Q = S_{R_1,R_2,R_3}^Q = -\frac{1}{2}$$

17.82 Refer to the circuit in Fig. 17.35(f).

$$I_o = g_{m1,2}\left(\frac{V_i}{2}\right)$$

Thus,

$$G_m = \frac{1}{2}g_{m1,2}$$

But,

$$g_{m1,2} = \sqrt{2k_nI_{D1,2}}$$

$$= \sqrt{2k_n(I/2)}$$

$$= \sqrt{k_nI}$$

Thus,

$$G_m = \frac{1}{2}\sqrt{k_n I}$$

For $G_m = 0.25$ mA/V and $k_n = 0.5$ mA/V^2, we have

$$0.25 = \frac{1}{2}\sqrt{0.5I}$$

$$\Rightarrow I = 0.5 \text{ mA}$$

Since G_m is proportional to $\sqrt{I}$, tuning G_m in the range $\pm 5\%$ requires tuning I in the range $0.95^2 I_{\text{nominal}}$ to $1.05^2 I_{\text{nominal}}$, which is approximately $\pm 10\%$ of the nominal value.

17.83 $R = \dfrac{1}{G_m}$

$$\Rightarrow G_m = \frac{1}{R} = \frac{1}{10^3} = 10^{-3} \text{ A/V} = 1 \text{ mA/V}$$

Since the output terminal is connected back to the input, the output resistance appears in effect in parallel with the resistance $1/G_m$, thus the actual resistance realized is

$$\frac{1}{G_m} \parallel R_o = 1 \parallel 100 = 0.99 \text{ k}\Omega$$

17.84

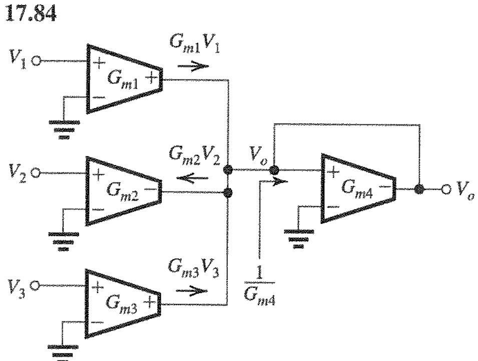


Figure 1

The circuit is shown in Fig. 1, for which we can write

$$V_o = \frac{1}{G_{m4}}(G_{m1}V_1 - G_{m2}V_2 + G_{m3}V_3)$$

$$= \frac{G_{m1}}{G_{m4}}V_1 - \frac{G_{m2}}{G_{m4}}V_2 + \frac{G_{m3}}{G_{m4}}V_3$$

To obtain

$$V_o = V_1 - 2V_2 + 3V_3$$

we select

$$G_{m1} = G_{m4}$$

$$G_{m2} = 2\ G_{m4}$$

$$G_{m3} = 3\ G_{m4}$$

17.85 For the integrator in Fig. 17.36(b), we have

$$\frac{V_o}{V_1} = \frac{G_m}{sC}$$

$$\text{Unity-gain frequency} = \frac{G_m}{2\pi C}$$

Thus,

$$10 \times 10^6 = \frac{G_m}{2\pi \times 5 \times 10^{-12}}$$

$$\Rightarrow G_m = 0.314 \text{ mA/V}$$

17.86 Both R_o and C_o will appear in parallel with C, thus

$$V_o = G_m V_i \frac{1}{\frac{1}{R_o} + s(C + C_o)}$$

The transfer function realized will be

$$\frac{V_o}{V_1} = \frac{G_m}{\frac{1}{R_o} + s(C + C_o)}$$

The integrator time constant is

$$\tau = (C + C_o)/G_m$$

$$= \frac{C}{G_m}\left(1 + \frac{C_o}{C}\right)$$

The quantity $\left(1 + \frac{C_o}{C}\right)$ represents the error factor. For the error to be less than 1%, we must have

$$\frac{C_o}{C} \leq 0.01$$

$$\Rightarrow C \geq 100C_o$$

Thus, the smallest value of C is $100C_o$.

Frequency of the low-frequency pole

$$= \frac{1}{(C + C_o)R_o}$$

$$\simeq \frac{1}{CR_o}$$

If this frequency is to be at least two decades lower than the unity gain frequency $\frac{G_m}{C}$, then

$$\frac{1}{CR_o} \leq 0.01\frac{G_m}{C}$$

$$\Rightarrow G_m \geq \frac{100}{R_o}$$

Thus, the smallest value of G_m must be $100/R_o$.

17.87 Refer to the circuit in Fig. 17.36(c) and its transfer function in Eq. (17.91), namely

$$\frac{V_o}{V_i} = -\frac{G_{m1}}{sC + G_{m2}}$$

$$\text{Pole frequency} = \frac{G_{m2}}{2\pi C}$$

$$20 \times 10^6 = \frac{G_{m2}}{2\pi \times 2 \times 10^{-12}}$$

$$\Rightarrow G_{m2} = 0.251 \text{ mA/V}$$

$$|\text{DC gain}| = \frac{G_{m1}}{G_{m2}}$$

$$10 = \frac{G_{m1}}{G_{m2}}$$

$$\Rightarrow G_{m1} = 2.51 \text{ mA/V}$$

17.88

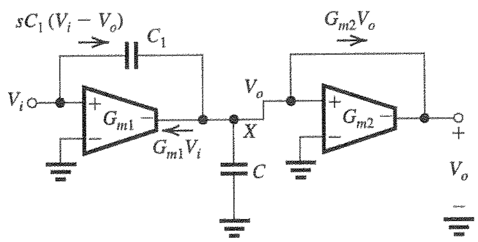


Figure 1

Figure 1 shows the circuit. A node equation at X yields

$$sC_1(V_i - V_o) = G_{m1}V_i + G_{m2}V_o + sCV_o$$

$$(sC_1V_i - G_{m1})V_i = V_o(G_{m2} + sC + sC_1)$$

$$\Rightarrow \frac{V_o}{V_i} = -\frac{G_{m1} - sC_1}{G_{m2} + s(C + C_1)}$$

17.89 (a) Refer to the circuit in Fig. P17.89. The output current of the G_{m2} transconductor is $G_{m2}V_1$. Thus,

$$V_2 = \frac{G_{m2}}{sC}V_1$$

This is the input voltage to the negative transconductor G_{m1}. Thus the output current of G_{m1}, which is equal to I_1, will be

$$I_1 = G_{m1}V_2$$

$$= \frac{G_{m1}G_{m2}}{sC}V_1$$

Thus,

$$Z_{\text{in}} \equiv \frac{V_1}{I_1} = s\frac{C}{G_{m1}G_{m2}}$$

which is that of an inductance L,

$$L = \frac{C}{G_{m1}G_{m2}} \qquad \text{Q.E.D.}$$

(b) To obtain an LCR resonator, we connected a capacitor C from node 1 to ground and a resistance R realized by the transconductor G_{m3}, as shown in Fig. 1 below.

(c) A fourth transconductor G_{m4} is used to feed a current $G_{m4}V_i$ to node 1, as shown in Fig. 1. The resulting circuit is identical to that in Fig. 17.37(b) except here $C_1 = C_2 = C$.

(d) With analogy to the identical circuit in Fig. 17.37(b), V_1/V_i will be a second-order bandpass filter with a transfer function given by Eq. (17.93), and V_2/V_i will be a second-order low-pass filter with a transfer function given by Eq. (17.94). Thus,

$$\frac{V_1}{V_i} = -\frac{s(G_{m4}/C)}{s^2 + s\dfrac{G_{m3}}{C_1} + \dfrac{G_{m1}G_{m2}}{C^2}}$$

and,

$$\frac{V_2}{V_i} = -\frac{G_{m2}G_{m4}/C^2}{s^2 + s\dfrac{G_{m2}}{C} + \dfrac{G_{m1}G_{m2}}{C^2}}$$

This figure belongs to Problem 17.89, part (b).

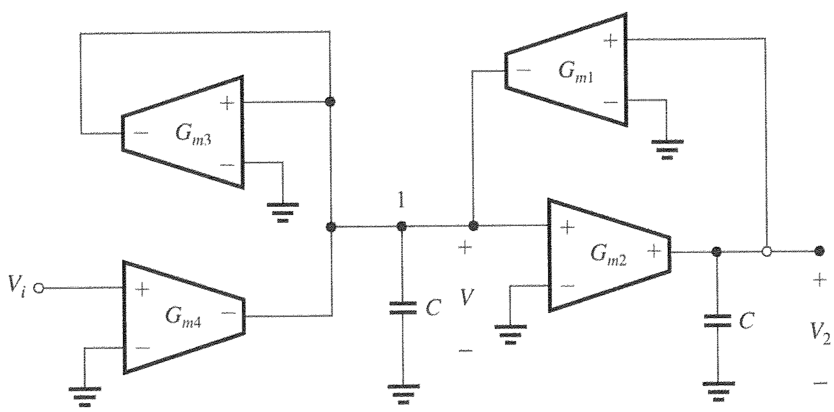


Figure 1

17.90 Using Eqs. (17.95) and (17.96), we have

$$\omega_0 = \sqrt{\frac{G_{m1}G_{m2}}{C_1C_2}} \quad (1)$$

$$Q = \frac{\sqrt{G_{m1}G_{m2}}}{G_{m3}}\sqrt{\frac{C_1}{C_2}} \quad (2)$$

Selecting $G_{m1} = G_{m2} = G_{m3} = G_m$, we obtain

$$\omega_0 = \frac{G_m}{\sqrt{C_1C_2}} \quad (3)$$

$$Q = \sqrt{\frac{C_1}{C_2}} \quad (4)$$

Selecting $C_2 = C$, then from Eq. (4) we have

$$C_1 = Q^2C$$

and from Eq. (3) we have

$$G_m = \omega_0 QC$$

17.91 The resulting circuit is shown in Fig. 1. For V_2 we can write

$$V_2 = \frac{1}{sC_2}(G_{m2}V_1 - G_{m5}V_i) \quad (1)$$

A node equation at X can be written as

$$G_{m1}V_2 + sC_3(V_1 - V_i) + G_{m4}V_i + G_{m3}V_1 + sC_1V_1 = 0 \quad (2)$$

Substituting for V_2 from Eq. (1) into Eq. (2) and collecting terms results in the transfer function

$$\frac{V_1}{V_i} = \frac{s^2\left(\frac{C_3}{C_1+C_3}\right) - s\frac{G_{m4}}{C_1+C_3} + \frac{G_{m1}G_{m5}}{(C_1+C_3)C_2}}{s^2 + s\frac{G_{m3}}{C_1+C_3} + \frac{G_{m1}G_{m2}}{(C_1+C_3)C_2}}$$

17.92 $f_0 = 25$ MHz, $Q = 5$,

Center-frequency gain = 5

The design equations are given by (17.99), (17.100), and (17.101). Thus

$$G_m = \omega_0 C$$

where

$$C = C_1 = C_2 = 5 \text{ pF}$$

$$G_m = 2\pi \times 25 \times 10^6 \times 5 \times 10^{-12}$$

$$= 0.785 \text{ mA/V}$$

Thus,

$$G_{m2} = G_m = 0.785 \text{ mA/V}$$

$$G_{m3} = \frac{G_m}{Q} = \frac{0.785}{5} = 0.157 \text{ mA/V}$$

$$G_{m4} = \frac{G_m}{Q}|\text{Gain}|$$

$$= \frac{0.785}{5} \times 5 = 0.785 \text{ mA/V}$$

This figure belongs to Problem 17.91.

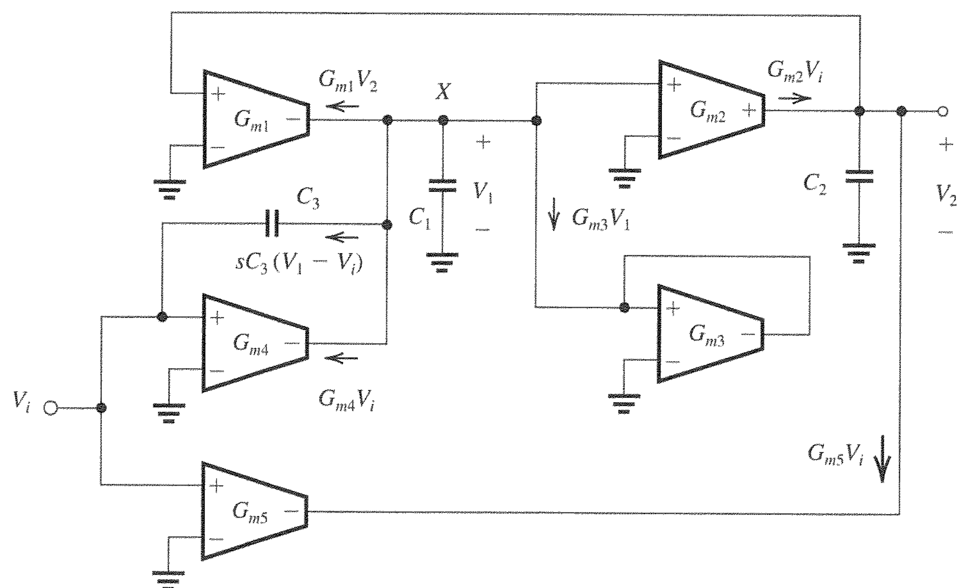


Figure 1

17.93 $R_{eq} = \dfrac{T_c}{C_1} = \dfrac{1}{f_c C_1} = \dfrac{1}{200 \times 10^3 C_1}$

For $C_1 = 1$ pF,

$$R_{eq} = \frac{1}{200 \times 10^3 \times 1 \times 10^{-12}} = 5\ \text{M}\Omega$$

For $C_1 = 5$ pF,

$$R_{eq} = \frac{1}{200 \times 10^3 \times 5 \times 10^{-12}} = 1\ \text{M}\Omega$$

For $C_1 = 10$ pF,

$$R_{eq} = \frac{1}{200 \times 10^3 \times 10 \times 10^{-12}} = 500\ \text{k}\Omega$$

17.94 Change transferred $\Rightarrow Q = CV$

$= 10^{-12}\,(1)$

$= 1$ pC

For $f_0 = 100$ kHz, average current is given by

$$I_{AV} = \frac{Q}{T} = 1\ \text{pC} \times 100 \times 10^3$$

$= 0.1\ \mu$A

For each clock cycle, the output will change by the same amount as the change in voltage across C_2.

$$\therefore \Delta V = Q/C_2 = \frac{1\ \text{pC}}{10\ \text{pF}} = 0.1\ \text{V}$$

For $\Delta V = 0.1$ V for each Clock cycle, the amplifier will saturate in

$$= \frac{10\ \text{V}}{0.1\ \text{V}} = 100\ \text{cycles}$$

$$\text{slope} = \frac{\Delta V}{\Delta t} = \frac{10\text{V}}{(100\ \text{cycles})\left(1/100 \times 10^3\right)}$$

$= 10^4$ V/s

17.95 From Eqs. (17.109) and (17.110),

$C_3 = C_4 = \omega_0 T_c C$

$$= 2\pi \times 10^4 \times \frac{1}{500 \times 10^3} \times 20$$

$= 2.51$ pF

From Eq. (17.112),

$$C_5 = \frac{C_4}{Q} = \frac{2.51}{20} = 0.126\ \text{pF}$$

From Eq. (17.113),

$$\text{Center-frequency gain} = \frac{C_6}{C_5} = 1$$

$\Rightarrow C_6 = C_5 = 0.126$ pF

17.96 From Eqs. (17.109) and (17.110),

$C_3 = C_4 = \omega_0 T_c C$

$$= 2\pi \times 10^4 \times \frac{1}{200 \times 10^3} \times 20 \times 10^{-12}$$

$= 6.283$ pF

From Eq. (17.112),

$$C_5 = \frac{C_4}{Q} = \frac{6.283}{50} = 0.126\ \text{pF}$$

From Eq. (17.113),

$$\text{Center-frequency gain} = \frac{C_6}{C_5} = 1$$

$\Rightarrow C_6 = C_5 = 0.126$ pF

17.97 $\omega_0 = \omega_{3dB} = 10^3$ rad/s

$Q = 1/\sqrt{2}$ and DC gain $= 1$

$f_c = 100$ kHz, $C_1 = C_2 = C = 5$ pF

From Eqs. (17.109) and (17.110),

$C_3 = C_4 = \omega_0 T_c C$

$$= 10^3 \times \frac{1}{100 \times 10^3} \times 5 \times 10^{-12}$$

$= 0.05$ pF

From Eq. (17.112),

$$C_5 = \frac{C_4}{Q} = \frac{0.05}{1/\sqrt{2}} = 0.071\ \text{pF}$$

The dc gain of the low-pass circuit is

$$\text{DC gain} = \frac{C_6}{C_4}$$

For DC gain $= 1$,

$C_6 = C_4 = 0.05$ pF

17.98 Refer to Figs. 1 and 2 on next page. For the BJT we have

$$g_m = \frac{I_C}{V_T} \simeq \frac{1\ \text{mA}}{0.025\ \text{V}} = 40\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{40} = 5\ \text{k}\Omega$$

$C_\pi = 10$ pF

Miller capacitance $= C_\mu(1 + g_m R_L)$

$= 0.5(1 + 40 \times 5)$

$= 100.5$ pF

Total capacitance $= C + C_\pi + C_{\text{Miller}}$

$= 200 + 10 + 100.5$

$= 310.5$ pF

This figure belongs to Problem 17.98, part (a).

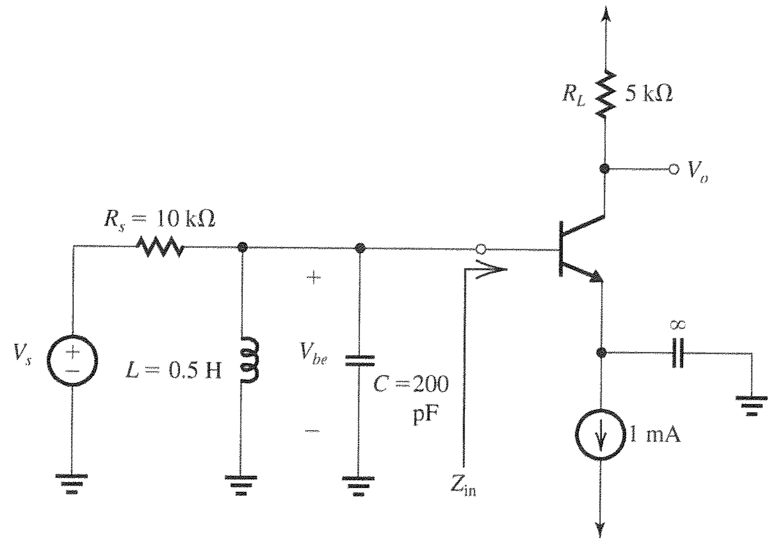


Figure 1

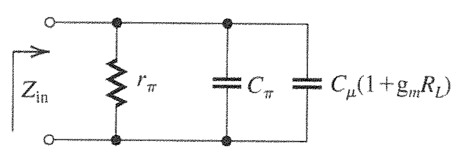


Figure 2

$$\omega_0 = \frac{1}{\sqrt{LC_{\text{total}}}}$$

$$= \frac{1}{\sqrt{0.5 \times 10^{-6} \times 310.5 \times 10^{-12}}} = 80.3 \text{ Mrad/s}$$

Total effective parallel resistance =
10 kΩ ∥ 5 kΩ = 3.33 kΩ

$$Q = \frac{R}{\omega_0 L} = \frac{3.33 \times 10^3}{80.3 \times 10^6 \times 0.5 \times 10^{-6}} = 83$$

$$\text{3-dB } BW = \frac{\omega_0}{Q} = \frac{80.3 \times 10^6}{83} = 967 \text{ kHz}$$

$$V_{be}(\omega_0) = V_s \frac{r_\pi}{r_\pi + R_s}$$

$$V_o(\omega_0) = -g_m R_L V_{be}$$

$$= -40 \times 5 \times \frac{1}{3} V_s$$

$$= -66.7 V_s$$

$$\left| \frac{V_o(\omega_0)}{V_s(\omega_0)} \right| = 66.7 \text{ V/V}$$

17.99 $R_p = \omega_0 LQ$

$$= 2\pi \times 10^6 \times 10 \times 10^{-6} \times 250$$

$$= 15.71 \text{ k}\Omega$$

$$C = \frac{1}{\omega_0^2 L}$$

$$= \frac{1}{(2\pi \times 10^6)^2 \times 10 \times 10^{-6}} = 2.53 \text{ nF}$$

For a 3-dB bandwidth of 12 kHz, we have

$$Q = \frac{1 \times 10^6}{12 \times 10^3} = 83.3$$

which requires a parallel resistance of

$$R = \omega_0 LQ$$

$$= 2\pi \times 10^6 \times 10 \times 10^{-6} \times 83.3$$

$$= 5.23 \text{ k}\Omega$$

Thus, the additional parallel resistance required, R_a, can be determined from

$$R_a \parallel R_p = R$$

$$R_a \parallel 15.71 = 5.23$$

$$\Rightarrow R_a = 7.84 \text{ k}\Omega$$

17.100 $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{36 \times 10^{-6} \times 10^3 \times 10^{-12}}}$$

$$= 838.8 \text{ kHz}$$

Equivalent parallel resistance $= 3^2 \times 1 = 9$ kΩ

$$Q = \frac{R_p}{\omega_0 L}$$

$$= \frac{9 \times 10^3}{2\pi \times 838.8 \times 10^3 \times 36 \times 10^{-6}}$$

$$= 47.4$$

17.101

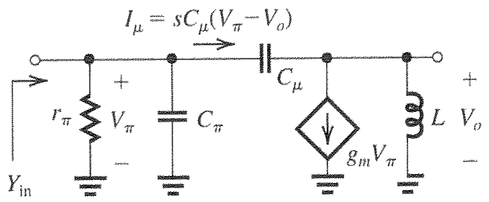


Figure 1

Refer to Fig. 1. A node equation at the output node yields

$$sC_\mu(V_\pi - V_o) = g_m V_\pi + \frac{V_o}{sL}$$

$$\Rightarrow V_o = V_\pi \frac{sC_\mu - g_m}{\dfrac{1}{sL} + sC_\mu}$$

Now, we can find I_μ as

$$I_\mu = sC_\mu(V_\pi - V_o)$$

$$= sC_\mu V_\pi - sC_\mu V_o$$

Thus,

$$\frac{I_\mu}{V_\pi} = sC_\mu - sC_\mu \frac{sC_\mu - g_m}{\dfrac{1}{sL} + sC_\mu}$$

$$= sC_\mu \left[\frac{\dfrac{1}{sL} + sC_\mu - sC_\mu + g_m}{\dfrac{1}{sL} + sC_\mu} \right]$$

$$= sC_\mu \frac{\dfrac{1}{sL} + g_m}{\dfrac{1}{sL} + sC_\mu}$$

For $s = j\omega$, we obtain

$$\frac{I_\mu}{V_\pi} = j\omega C_\mu \frac{g_m + \dfrac{1}{j\omega L}}{j\omega C_\mu + \dfrac{1}{j\omega L}}$$

$$= j\omega C_\mu \frac{1 + j\omega L g_m}{1 - \omega^2 L C_\mu}$$

since $\omega C_\mu \ll \dfrac{1}{\omega L}$

$$\Rightarrow \omega^2 L C_\mu \ll 1$$

Thus,

$$\frac{I_\mu}{V_\pi} \simeq j\omega C_\mu (1 + j\omega L g_m)$$

$$= j\omega C_\mu - \omega^2 L C_\mu g_m$$

Returning to Fig. 1, we can write

$$Y_{in} = \frac{1}{r_\pi} + j\omega C_\mu + \frac{I_\mu}{V_\pi}$$

$$= \frac{1}{r_\pi} + j\omega C_\pi + j\omega C_\mu - \omega^2 C_\mu L g_m$$

$$= \left(\frac{1}{r_\pi} - \omega^2 C_\mu L g_m \right) + j\omega (C_\pi + C_\mu) \qquad \text{Q.E.D.}$$

17.102 (a) $T(s) = \dfrac{sK\left(\dfrac{\omega_0}{Q}\right)}{s^2 + s\left(\dfrac{\omega_0}{Q}\right) + \omega_0^2}$

where

$$K = |T(j\omega_0)|$$

Thus,

$$|T(j\omega)| = \frac{|T(j\omega_0)|}{\sqrt{1 + Q^2\left(\dfrac{\omega_0^2 - \omega^2}{\omega\omega_0}\right)^2}} \qquad (1)$$

Now, consider the quantity $\dfrac{\omega_0^2 - \omega^2}{\omega\omega_0}$. For $\omega = \omega_0 + \delta\omega$ where $(\delta\omega/\omega_0) \ll 1$,

$$\omega = \omega_0\left(1 + \frac{\delta\omega}{\omega_0}\right)$$

$$\omega^2 \simeq \omega_0^2\left(1 + \frac{2\delta\omega}{\omega_0}\right)$$

Thus,

$$\frac{\omega_0^2 - \omega^2}{\omega\omega_0} = \frac{\omega_0^2 - \omega_0^2(1 + 2\delta\omega/\omega_0)}{\omega_0^2(1 + \delta\omega/\omega_0)}$$

$$= \frac{-2\delta\omega/\omega_0}{1 + \dfrac{\delta\omega}{\omega_0}} \simeq -\frac{2\delta\omega}{\omega_0}$$

Substituting in Eq. (1), we obtain

$$|T(j\omega)| = \frac{|T(j\omega_0)|}{\sqrt{1 + 4Q^2\left(\dfrac{\delta\omega}{\omega_0}\right)^2}}$$

(b) For N syncronously tuned sections connected in cascade, we obtain

$$|T(j\omega)|^N = \frac{|T(j\omega_0)|^N}{[1 + 4Q^2(\delta\omega/\omega_0)^2]^{N/2}}$$

The 3-dB bandwidth is the value of $(2\delta\omega)$ at which

$$|T(j\omega)|^N = \frac{1}{\sqrt{2}}|T(j\omega_0)|^N$$

Denoting, $2\delta\omega = B$, we obtain

$$[1 + Q^2(B/\omega_0)^2]^{N/2} = \sqrt{2}$$

$$B = \left(\frac{\omega_0}{Q}\right)\sqrt{2^{1/N} - 1} \qquad \text{Q.E.D.}$$

17.103 (a) A first-order low-pass filter with a 3-dB frequency of $(\omega_0/2Q)$ has

$$T(s) = \frac{K(\omega_0/2Q)}{s + (\omega_0/2Q)}$$

where K is the dc gain.

$$|T(j\omega)| = \frac{K(\omega_0/2Q)}{\sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + \omega^2}}$$

$$= \frac{K}{\sqrt{1 + 4Q^2\left(\frac{\omega}{\omega_0}\right)^2}} \quad (1)$$

By analogy, the magnitude response of a second-order bandpass in the neighbourhood of ω_0, i.e for $\omega = \omega_0 + \delta\omega$ where $\frac{\delta\omega}{\omega_0} \ll 1$, can be obtained by replacing ω in Eq. (1) by $\delta\omega$, thus

$$|T(j\omega)| = \frac{|T(j\omega_0)|}{\sqrt{1 + 4Q^2(\delta\omega/\omega_0)^2}} \quad \text{Q.E.D.}$$

(b) Now, cascading N synchronously tuned second-order bandpass sections provides a transfer function

$$|T(j\omega)|_{\text{overall}} = \frac{|T(j\omega_0)|_{\text{overall}}}{[1 + 4Q^2(\delta\omega/\omega_0)^2]^{N/2}} \quad (1)$$

The overall transfer function will have a 3-dB bandwidth $B = 2\delta\omega$ given by

$$\left[1 + 4Q^2\left(\frac{B}{2\omega_0}\right)^2\right]^{N/2} = \sqrt{2}$$

$$2Q\left(\frac{B}{2\omega_0}\right) = \sqrt{2^{1/N} - 1}$$

$$\frac{Q}{\omega_0} = \frac{1}{B}\sqrt{2^{1/N} - 1} \quad (2)$$

Substitution for $\frac{Q}{\omega_0}$ from Eq. (2) into Eq. (1) gives

$$|T(j\omega)|_{\text{overall}} = \frac{|T(j\omega_0)|_{\text{overall}}}{\left[1 + 4(2^{1/N} - 1)\left(\frac{\delta\omega}{B}\right)^2\right]^{N/2}} \quad \text{Q.E.D.} \quad (3)$$

(c) At a bandwidth $2B$, we have

$\delta\omega = B$

$$|T(j\omega)|_{\text{overall}} = \frac{|T(j\omega_0)|_{\text{overall}}}{[1 + 4(2^{1/N} - 1)]^{N/2}}$$

Thus, the attentuation obtain is

$$A(2B) = 20 \log[1 + 4(2^{1/N} - 1)]^{N/2}$$

$$= 10\,N \log[1 + 4(2^{1/N} - 1)]$$

N	1	2	3	4	5
A **(dB)**	7	8.5	9.3	9.8	10.1

The 3-dB bandwidth is B. The 30-dB bandwidth, W, is found from

$$20 \log\left[1 + 4\,(2^{1/N} - 1)\left(\frac{W}{2B}\right)^2\right]^{N/2} = 30$$

$$\Rightarrow \log\left[1 + (2^{1/N} - 1)\left(\frac{W}{B}\right)^2\right] = \frac{3}{N}$$

$$10^{3/N} - 1 = (2^{1/N} - 1)\left(\frac{W}{B}\right)^2$$

$$\Rightarrow \frac{W}{B} = \sqrt{\frac{10^{3/N} - 1}{2^{1/N} - 1}}$$

N	1	2	3	4	5
W/B	31.6	8.6	5.9	4.9	4.5

18.1 Since the second-order bandpass filter will exhibit zero phase at $\omega = \omega_0$, the circuit if it oscillates will do so at

$\omega = \omega_0$

For the circuit to oscillate, the magnitude of the gain at $\omega = \omega_0$ must be at least unity, thus

$AK \geq 1$

For sustained oscillations, we have

$AK = 1$

18.2 (a) The bandpass function can be written as

$$T(s) = \frac{Ks\dfrac{\omega_0}{Q}}{s^2 + s\left(\dfrac{\omega_0}{Q}\right) + \omega_0^2}$$

$$T(j\omega) = \frac{j\dfrac{\omega\omega_0}{Q}K}{(\omega_0^2 - \omega^2) + j\dfrac{\omega\omega_0}{Q}}$$

Thus,

$$\phi(\omega) = 90^\circ - \tan^{-1}\left(\frac{\omega\omega_0/Q}{\omega_0^2 - \omega^2}\right)$$

$$\frac{d\phi}{d\omega} = \frac{1}{1 + \dfrac{1}{Q^2}\left[\dfrac{\omega\omega_0}{\omega_0^2 - \omega^2}\right]^2} \times \frac{1}{Q}$$

$$\times \frac{(\omega_0^2 - \omega^2)\omega_0 - \omega\omega_0(-2\omega)}{(\omega_0^2 - \omega^2)^2}$$

$$= -\frac{(\omega_0/Q)(\omega_0^2 + \omega^2)}{(\omega_0^2 - \omega^2)^2 + \dfrac{1}{Q^2}\omega^2\omega_0^2}$$

$$\frac{d\phi}{d\omega}(\omega = \omega_0) = -\frac{2Q}{\omega_0}$$

(b) For a change in phase $\Delta\phi$, the corresponding change in ω_0 will be

$$\Delta\omega_0 = \frac{\Delta\phi}{d\phi/d\omega}$$

$$= -\frac{\Delta\phi}{2Q/\omega_0}$$

$$\Rightarrow \frac{\Delta\omega_0}{\omega_0} = -\frac{\Delta\phi}{2Q}$$

18.3 The characteristic equation is obtained as follows:

$1 - L(s) = 0$

$$1 - A\frac{Ks\left(\dfrac{\omega_0}{Q}\right)}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2} = 0$$

$$\Rightarrow s^2 + s\frac{\omega_0}{Q}(1 - AK) + \omega_0^2 = 0$$

Thus, the poles will be in the left half of the s-plane at a radial frequency ω_0 and a horizontal distance from the $j\omega$ axis of

$$\sigma = -\frac{\omega_0}{2Q}(1 - AK)$$

(a) For the poles to be on the $j\omega$ axis, we need

$\sigma = 0$

$\Rightarrow AK = 1$

(b) For the poles in the right half of the s-plane at a horizontal distance from the $j\omega$ axis of $\dfrac{\omega_0}{2Q}$, we have

$$\sigma = +\frac{\omega_0}{2Q}$$

which is achieved by making

$AK = 2$

18.4 (a) A change of +1% in the value of L causes a change of −0.5% in the value of ω_0.

(b) A change of +1% in the value C causes a change of −0.5% in the value of ω_0.

(c) Since ω_0 does not depend on the value of R, there will be no change in ω_0 as R changes by +1%.

18.5

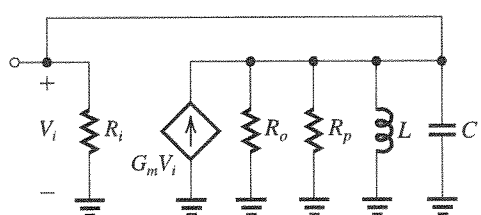


Figure 1

Figure 1 shows the resulting circuit. Here R_p can be found from

$R_p = \omega_0 LQ$

where

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 10^{-6} \times 100 \times 10^{-12}}}$$

$= 10^8$ rad/s

Thus,

$R_p = 10^8 \times 1 \times 10^{-6} \times 50$

$= 5\ \text{k}\Omega$

The total parallel resistance can now be found as

$R = R_i \parallel R_o \parallel R_p$

$= 5 \parallel 5 \parallel 5 = 1.67\ \text{k}\Omega$

The circuit will oscillate when the loop gain is unity, that is,

$$G_m R = 1$$

$$\Rightarrow G_m = \frac{1}{R} = 0.6 \text{ mA/V}$$

and the frequency of oscillation will be

$$f_0 = \frac{\omega_0}{2\pi} = \frac{10^8}{2\pi} = 15.92 \text{ MHz}$$

18.6 For oscillations to begin, the total phase shift around the loop at ω_0 must be 0 or 360°. Since the phase shift of the frequency-selective network is 180°, the amplifier must have a phase shift of 180°. Also, the loop gain at ω_0 must be at least unity. Since the frequency-selective network has 12-dB attenuation, the amplifier gain must be at least 12 dB or 4 V/V.

18.7 For the circuit to oscillate, two conditions must be satisfied:

(1) The total phase shift around the loop should be 0 or 360°, and

(2) The loop gain must be at least unity.

Here we have three amplifier stages (Fig. P18.7). Thus the phase angle of each amplifier at the oscillation frequency ω_0, must be 120°. Now, for each amplifier stage we have

$$V_o = -g_m V_i \frac{1}{\frac{1}{R} + sC}$$

$$\frac{V_o}{V_i} = -\frac{g_m R}{1 + sCR}$$

At $s = j\omega_0$, we have

$$\frac{V_o}{V_i}(j\omega_0) = -\frac{g_m R}{1 + j\omega_0 CR}$$

$$= -\frac{g_m R(1 - j\omega_0 CR)}{1 + (\omega_0 CR)^2}$$

$$= \frac{g_m R}{1 + (\omega_0 CR)^2}(-1 + j\omega_0 CR)$$

For the phase angle to be 120°, we need

$$\tan 60 = \omega_0 CR$$

$$\Rightarrow \omega_0 = \frac{\sqrt{3}}{CR}$$

$$\left|\frac{V_o}{V_i}(j\omega_0)\right| = \frac{g_m R}{\sqrt{1 + (\omega_0 CR)^2}} = \frac{g_m R}{\sqrt{1+3}}$$

$$= 0.5 g_m R$$

For a loop gain of unity, we have

$$(0.5 g_m R)^3 = 1$$

$$g_m R = 2$$

$$g_m|_{\min} = \frac{2}{R}$$

18.8 Refer to Fig. 18.4(e).

$$L_+ = -L_- = 3 \text{ V}$$

$$\frac{R_3}{R_1} = \frac{R_4}{R_1} = 0.05 \quad (1)$$

Now,

$$L_+ = V\frac{R_4}{R_5} + 0.7\left(1 + \frac{R_4}{R_5}\right)$$

thus,

$$3 = 5\frac{R_4}{R_5} + 0.7\left(1 + \frac{R_4}{R_5}\right)$$

$$\Rightarrow \frac{R_4}{R_5} = \frac{2.3}{5.7} \quad (2)$$

Similarly, from the equation for L_- we obtain

$$\frac{R_3}{R_2} = \frac{2.3}{5.7} \quad (3)$$

Selecting $R_1 = 100$ kΩ, Eq. (1) gives

$$R_3 = R_4 = 5 \text{ k}\Omega$$

Then using Eqs. (2) and (3), we obtain

$$R_2 = R_5 = \frac{5 \times 5.7}{2.3} = 12.4 \text{ k}\Omega$$

18.9 Refer to Fig. 1 on the next page. By connecting V_B and R_B, we are injecting a current into the virtual ground node of V_B/R_B. To neutralize this current, v_I has to be at a negative value that pulls an equal current through R_1. Thus, the comparator threshold shifts from $v_I = 0$ to

$$v_I = -V_B\frac{R_1}{R_B}$$

To obtain a –2-V threshold with $V_B = 5$ V, we have

$$-2 = -5\frac{R_1}{R_B}$$

$$\Rightarrow \frac{R_1}{R_B} = \frac{2}{5} \quad (1)$$

For a comparator input resistance of 100 kΩ, we have

$$R_1 = 100 \text{ k}\Omega$$

Using Eq. (1), we obtain

$$R_B = 250 \text{ k}\Omega$$

To obtain a slope of 0.05 in the limiting regions, we use

$$\frac{R_3}{R_1} = \frac{R_4}{R_1} = 0.05$$

thus,

$$R_3 = R_4 = 5 \text{ k}\Omega$$

This figure belongs to Problem 18.9.

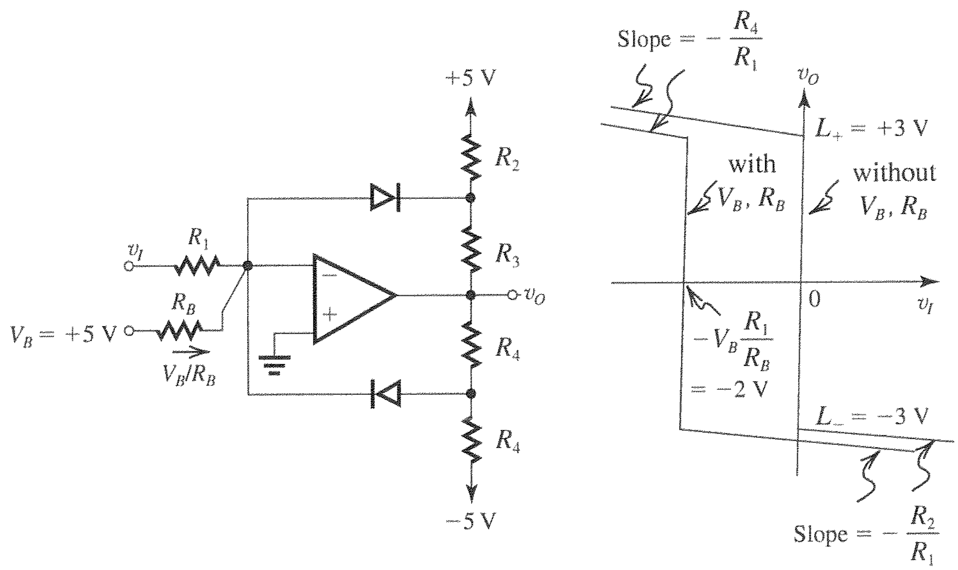


Figure 1

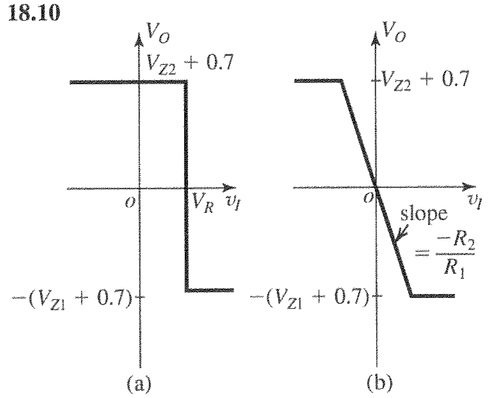
To obtain $L_+ = -L_- = 3$ V, we use Eqs. (18.8) and (18.9) with $V_D = 0$, thus

$$\frac{R_3}{R_2} = \frac{R_4}{R_5} = \frac{3}{5}$$

$$\Rightarrow R_2 = R_5 = \frac{5 \times 5}{3} = 8.33\ \text{k}\Omega$$

For standard 5% resistors, use: $R_1 = 100\ \text{k}\Omega$, $R_B = 240\ \text{k}\Omega$, $R_3 = R_4 = 5.1\ \text{k}\Omega$, $R_2 = R_4 = 8.2\ \text{k}\Omega$.

18.10



18.11 Refer to Fig. 18.5.

$$\frac{V_a}{V_b} = \frac{Z_p}{Z_p + Z_s}$$

$$= \frac{1}{1 + Z_s Y_p}$$

$$= \frac{1}{1 + \left(R + \frac{1}{sC}\right)\left(\frac{1}{R} + sC\right)}$$

$$= \frac{1}{1 + 1 + 1 + sCR + \frac{1}{sCR}}$$

$$= \frac{s/CR}{s^2 + s(3/CR) + \frac{1}{(CR)^2}}$$

which is a bandpass function with a center frequency ω_0 given by

$$\omega_0 = \frac{1}{CR}$$

and a pole-Q of

$$Q = \frac{1}{3}$$

and a center-frequency gain of

$$\text{Gain} = \frac{1}{3}$$

18.12 If the closed-loop amplifier in Fig. 18.5 exhibits a phase shift of -3° for ω around ω_0, then the loop-gain expression in Eq. (18.11) becomes

$$L(j\omega) = \frac{(1 + R_2/R_1)e^{-j\phi}}{3 + j(\omega CR - 1/\omega CR)}$$

where

$$\phi = \frac{3\pi}{180} = \pi/60$$

Oscillation will occur at the frequency ω_0 for which the phase angle of $L(j\omega)$ is 0°:

$$-\phi = \tan^{-1}\frac{1}{3}\left(\omega_0 CR - \frac{1}{\omega_0 CR}\right)$$

$$\omega_0 CR - \frac{1}{\omega_0 CR} = -3 \tan 3^\circ = -0.157$$

$$\Rightarrow \omega_0^2 + \frac{0.157}{CR}\omega_0 - \frac{1}{(CR)^2} = 0$$

$$\Rightarrow \omega_0 = \frac{0.925}{CR}$$

18.13 The characteristic equation can be written using the expression for the loop gain in Eq. (18.10) as follows:

$$1 - L(s) = 0$$

$$1 - \frac{1 + R_2/R_1}{3 + sCR + \dfrac{1}{sCR}} = 0$$

$$3 + sCR + \frac{1}{sCR} - 1 - \frac{R_2}{R_1} = 0$$

$$s^2 + s\left(2 - \frac{R_2}{R_1}\right) \Big/ CR + \frac{1}{(CR)^2} = 0$$

Thus the poles have

$$\omega_0 = \frac{1}{CR}$$

and

$$Q = \frac{1}{2 - \dfrac{R_2}{R_1}}$$

The poles will be on the $j\omega$ axis for $R_2/R_1 = 2$ and will be in the right half of the s-plane for $R_2/R_1 > 2$. Q.E.D.

18.14 Refer to Fig. 18.6. Assume that $R_3 = R_6$ and $R_4 = R_5$ and consider the magnitude of the positive peak. When $v_O = \hat{V}_o$, D_2 just conducts and clamps node b to a voltage $V_b = V_1 + V_D \simeq \dfrac{\hat{V}_o}{3} + V_D$. Neglecting the current through D_2, we can write

$$\frac{\hat{V}_o - V_b}{R_5} = \frac{V_b - (-15)}{R_6}$$

Thus,

$$\hat{V}_o = \left(1 + \frac{R_5}{R_6}\right)V_b + 15\frac{R_5}{R_6}$$

Substituting

$$V_b = \frac{1}{3}\hat{V}_o + V_D$$

we obtain

$$\hat{V}_o = \frac{15\left(\dfrac{R_5}{R_6}\right) + \left(1 + \dfrac{R_5}{R_6}\right)V_D}{\dfrac{2}{3} - \dfrac{1}{3}\dfrac{R_5}{R_6}} \qquad (1)$$

To obtain an 8-V peak-to-peak output, we use

$$\hat{V}_o = 4 = \frac{15\left(\dfrac{R_5}{R_6}\right) + \left(1 + \dfrac{R_5}{R_6}\right) \times 0.7}{\dfrac{2}{3} - \dfrac{1}{3}\dfrac{R_5}{R_6}}$$

$$\Rightarrow \frac{R_5}{R_6} = 0.115$$

Since $R_5 = 1\text{ k}\Omega$, then

$$R_6 = 8.66\text{ k}\Omega$$

and

$$R_3 = 8.66\text{ k}\Omega$$

If R_3 and R_6 are open circuited, substituting $R_6 = \infty$ in Eq. (1) yields

$$\hat{V}_o = 1.5V_D = 1.05\text{ V}$$

This result can be obtained directly from the circuit: If R_3 and R_6 are open circuited, the positive peak of the output will be the voltage at which D_2 conducts. At this point, $v_O = \hat{V}_o$ and $v_1 = \frac{1}{3}\hat{V}_o$, thus the voltage across V_D is $\frac{2}{3}\hat{V}_o$ or, equivalently,

$$\hat{V}_o = 1.5V_D$$

A similar situation occurs at the negative peak at which diode D_1 conducts and the voltage across it will be $\frac{2}{3}\hat{V}_o$, etc.

18.15

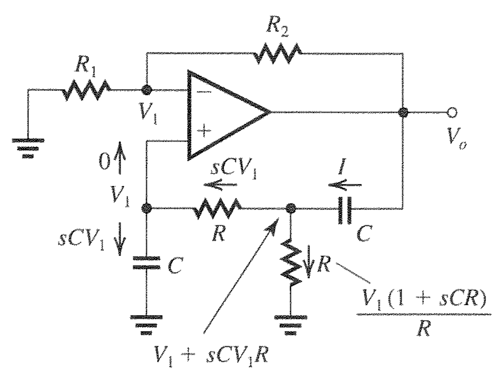


Figure 1

Figure 1 shows the circuit and some of the analysis for the purpose of determining the transfer function of the RC circuit. The current I is given by

$$I = \frac{V_1}{R}(1 + sCR) + sCV_1$$

For V_o we now can write

$$V_o = V_1(1+sCR) + \frac{I}{sC}$$

$$= V_1(1+sCR) + \frac{V_1}{sCR}(1+sCR) + V_1$$

$$\Rightarrow \frac{V_1}{V_o} = \frac{s/CR}{s^2 + s\left(\frac{3}{CR}\right) + \frac{1}{(CR)^2}}$$

The loop gain $L(s)$ can now be found as

$$L(s) = \frac{s\left[\left(1+\frac{R_2}{R_1}\right)\Big/CR\right]}{s^2 + s\frac{3}{CR} + \frac{1}{(CR)^2}}$$

$$L(j\omega) = \frac{j\omega\left[\left(1+\frac{R_2}{R_1}\right)\Big/CR\right]}{\left[\frac{1}{(CR)^2} - \omega^2\right] + j\frac{3\omega}{CR}}$$

Zero phase shift will occur at $\omega = \omega_0$:

$$\omega_0 = \frac{1}{CR}$$

At $\omega = \omega_0$, we have

$$|L(j\omega)| = \frac{1}{3}\left(1+\frac{R_2}{R_1}\right)$$

For oscillations to begin, we need

$$\frac{1}{3}\left(1+\frac{R_2}{R_1}\right) \geq 1$$

$$\frac{R_2}{R_1} \geq 2$$

18.16 Figure 1 shows the circuit together with the analysis performed to determine the transfer function V_1/V_o of the RC feedback circuit. We can write for V_o

$$V_o = V_1\left(1+\frac{1}{sCR}\right) + RV_1\left[\frac{1}{R} + sC\left(1+\frac{1}{sCR}\right)\right]$$

$$= V_1\left(1+\frac{1}{sCR}\right) + V_1(2+sCR)$$

$$\Rightarrow \frac{V_1}{V_o} = \frac{s/CR}{s^2 + s\left(\frac{3}{CR}\right) + \left(\frac{1}{CR}\right)^2}$$

The loop gain $L(s)$ can now be found as

$$L(s) = \frac{s\left(1+\frac{R_2}{R_1}\right)\Big/CR}{s^2 + s\frac{3}{CR} + \left(\frac{1}{CR}\right)^2}$$

$$L(j\omega) = \frac{j\omega\left[\frac{1+R_2/R_1}{CR}\right]}{\left[\left(\frac{1}{CR}\right)^2 - \omega^2\right] + j\frac{3\omega}{CR}}$$

The loop phase shift will be zero at $\omega = \omega_0$, when

$$\omega_0 = \frac{1}{CR}$$

At this frequency we have

$$|L(j\omega)| = \frac{1}{3}\left(1+\frac{R_2}{R_1}\right)$$

For oscillations to start, we need

$$\frac{1}{3}\left(1+\frac{R_2}{R_1}\right) \geq 1$$

$$\Rightarrow \frac{R_2}{R_1} \geq 2$$

This figure belongs to Problem 18.16.

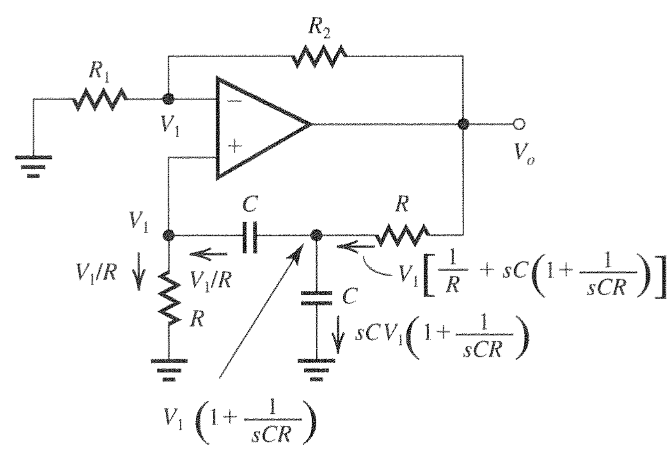


Figure 1

18.17

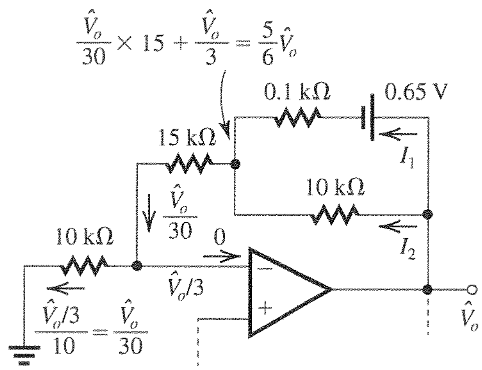


Figure 1

The circuit in Fig. 1 depicts the situation when $v_O = \hat{V}_o$. We made use of the fact that for sustained oscillations the closed-loop gain of the amplifier must be 3. Thus the voltage at the inverting input terminal will be $(\hat{V}_o/3)$. Some of the analysis is shown in the figure. We complete the analysis as follows:

$$I_1 = \frac{\hat{V}_o - \frac{5}{6}\hat{V}_o - 0.65}{0.1} = 10\hat{V}_o - \frac{50}{6}\hat{V}_o - 6.5$$

$$I_2 = \frac{\hat{V}_o - \frac{5}{6}\hat{V}_o}{10} = 0.1\hat{V}_o - \frac{5}{60}\hat{V}_o$$

$$I_1 + I_2 = \frac{\hat{V}_o}{30}$$

Thus,

$$\hat{V}_o\left(10 - \frac{50}{6} + 0.1 - \frac{5}{60}\right) - 6.5 = \frac{\hat{V}_o}{30}$$

$$\Rightarrow \hat{V}_o = 3.94 \text{ V}$$

Thus the peak-to-peak of the output sinusoid will be $2 \times 3.94 = 7.88$ V.

18.18 First we design the circuit to operate at 10 kHz.

$$\omega_0 = \frac{1}{CR}$$

$$2\pi \times 10 \times 10^3 = \frac{1}{CR}$$

$$\Rightarrow CR = 0.159 \times 10^{-4} \text{ s}$$

For $R = 10$ kΩ, we have

$$C = \frac{0.159 \times 10^{-4}}{10 \times 10^3} = 1.59 \text{ nF}$$

Now, refer to Eq. (18.11). If the closed-loop amplifier has an excess phase lag of 5.7°, then the gain will be $\left(1 + \frac{R_2}{R_1}\right)e^{-j5.7°}$. Oscillations will occur at the frequency ω_{01} at which the phase angle of the denominator is −5.7°, that is,

$$\tan^{-1}\frac{1}{3}\left(\omega_{01}CR - \frac{1}{\omega_{01}CR}\right) = -5.7°$$

$$\omega_{01}CR - \frac{1}{\omega_{01}CR} = 3\tan(-5.7°) = -0.3$$

$$\Rightarrow \omega_{01}^2 + \frac{0.3}{CR}\omega_{01} - \frac{1}{(CR)^2} = 0$$

$$\Rightarrow \omega_{01} = \frac{0.86}{CR}$$

That is, the frequency of oscillation is reduced by 14% to

$$f_{01} = 0.86 f_0 = 8.6 \text{ kHz}$$

To restore operation to $f_0 = 10$ kHz, we modify the shunt resistor R to R_x, as indicated in Fig. 1.

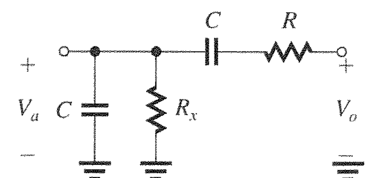


Figure 1

We now require the feedback RC circuit to have a phase shift of $-(-5.7°) = +5.7°$ at $f = 10$ kHz. The transfer function of the RC circuit can be found as follows:

$$\frac{V_a}{V_o} = \frac{Z_p}{Z_p + Z_s}$$

$$= \frac{1}{1 + Z_s Y_p}$$

$$= \frac{1}{1 + \left(R + \frac{1}{sC}\right)\left(\frac{1}{R_x} + sC\right)}$$

$$= \frac{1}{\left(2 + \frac{R}{R_x}\right) + sCR + \frac{1}{sCR_x}}$$

For $s = j\omega$, we have

$$\frac{V_a}{V_o} = \frac{1}{\left(2 + \frac{R}{R_x}\right) + j\left(\omega CR - \frac{1}{\omega CR_x}\right)}$$

At $\omega = \omega_0$, the phase angle of $\frac{V_a}{V_o}$ must be +5.7° or equivalently, the phase angle of the denominator must be −5.7°. Thus,

$$\tan^{-1}\frac{\omega_0 CR - \frac{1}{\omega_0 CR_x}}{2 + \frac{R}{R_x}} = -5.7°$$

$$\omega_0 CR - \frac{1}{\omega_0 CR_x} = \left(2 + \frac{R}{R_x}\right) \tan(-5.7^\circ)$$

$$= \left(2 + \frac{R}{R_x}\right) \times -0.0998$$

Now, $\omega_0 CR = 1$, thus

$$1 - \frac{R}{R_x} = -0.0998\left(2 + \frac{R}{R_x}\right)$$

$$1 + 2 \times 0.0998 = \frac{R}{R_x}(1 - 0.0998) \Rightarrow \frac{R}{R_x} = 1.33$$

$$\Rightarrow R_x = 0.75\ R = 7.5\ \text{k}\Omega$$

At $\omega = \omega_0$ and for $R_x = 7.5\ \text{k}\Omega$

$$\frac{V_a}{V_o}(\omega_0) = \frac{1}{\left(2 + \frac{10}{7.5}\right) + j\left(1 - \frac{10}{7.5}\right)}$$

$$= \frac{1}{3.33 - j0.33}$$

$$\left|\frac{V_a}{V_o}(\omega_0)\right| = \frac{1}{\sqrt{(3.33)^2 + (0.33)^2}} = \frac{1}{3.35}$$

Thus the magnitude of the gain of the amplifier must be 3.35 V/V. Thus, R_2/R_1 must be changed to

$$\frac{R_2}{R_1} = 2.35$$

18.19 Figure 1 shows the circuit with the additional resistance R included. The loop has been broken at the output of the op amp. The analysis will determine V_o/V_x and equate it to unity, which is the condition for sustained oscillations.

To begin, observe that the voltage V_1 is related to V_o by

$$\frac{V_o}{V_1} = -\frac{R_f}{R} \tag{1}$$

Also, the current I_1 is given by

$$I_1 = \frac{V_1}{R} \tag{2}$$

We now proceed to determine the various currents and voltages of the RC network as follows:

$$V_2 = V_1 + \frac{1}{sC}I_1$$

$$= V_1 + \frac{1}{sC}\,\frac{V_1}{R} = V_1\left(1 + \frac{1}{sCR}\right)$$

$$I_2 = \frac{V_2}{R} = \frac{V_1}{R}\left(1 + \frac{1}{sCR}\right)$$

$$I_3 = I_1 + I_2 = \frac{V_1}{R} + \frac{V_1}{R}\left(1 + \frac{1}{sCR}\right)$$

$$= \frac{V_1}{R}\left(2 + \frac{1}{sCR}\right)$$

$$V_3 = V_2 + \frac{I_3}{sC}$$

$$= V_1\left(1 + \frac{1}{sCR}\right) + \frac{V_1}{sCR}\left(2 + \frac{1}{sCR}\right)$$

$$= V_1\left(1 + \frac{3}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$I_4 = \frac{V_3}{R} = \frac{V_1}{R}\left(1 + \frac{3}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$I_5 = I_3 + I_4$$

$$= \frac{V_1}{R}\left(2 + \frac{1}{sCR}\right) + \frac{V_1}{R}\left(1 + \frac{3}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$= \frac{V_1}{R}\left(3 + \frac{4}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$V_x = V_3 + \frac{I_5}{sC}$$

$$= V_1\left(1 + \frac{3}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$+ \frac{V_1}{sCR}\left(3 + \frac{4}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$= V_1\left(1 + \frac{6}{sCR} + \frac{5}{s^2C^2R^2} + \frac{1}{s^3C^3R^3}\right)$$

Now, by replacing V_1 by the value from Eq. (1), we obtain

$$V_x = -V_o\frac{R}{R_f}\left(1 + \frac{6}{sCR} + \frac{5}{s^2C^2R^2} + \frac{1}{s^3C^3R^3}\right)$$

This figure belongs to Problem 18.19.

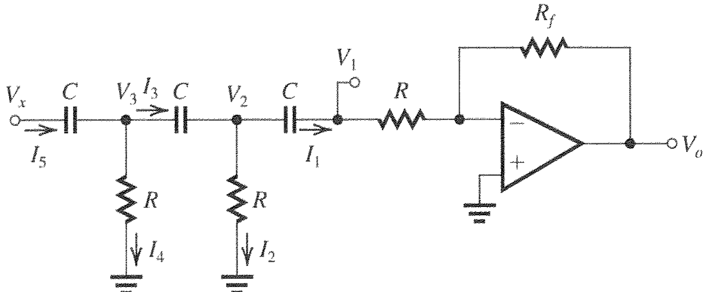


Figure 1

For sustained oscillations $V_o = V_x$, thus

$$-\frac{R_f}{R} = 1 + \frac{6}{sCR} + \frac{5}{s^2C^2R^2} + \frac{1}{s^3C^3R^3}$$

For $s = j\omega$, we have

$$-\frac{R_f}{R} = 1 + \frac{6}{j\omega CR} - \frac{5}{\omega^2C^2R^2} - \frac{1}{j\omega^3C^3R^3}$$

$$= \left(1 - \frac{5}{\omega^2C^2R^2}\right) - j\left(\frac{6}{\omega CR} - \frac{1}{\omega^3C^3R^3}\right)$$

Thus, oscillation will occur at the frequency that renders the imaginary part of the RHS zero:

$$\frac{6}{\omega_0 CR} = \frac{1}{\omega_0^3C^3R^3}:$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{6}CR}$$

At this frequency, the real part of the RHS must be equal to $(-R_f/R)$:

$$-\frac{R_f}{R} = 1 - \frac{5}{1/6} = -29$$

Thus,

$$R_f = 29R$$

which is the minimum required value for R_f to obtain sustained oscillations. Numerical values:

$$f_0 = \frac{1}{2\pi\sqrt{6} \times 16 \times 10^{-9} \times 10 \times 10^3}$$

$$= 406 \text{ Hz}$$

$$R_f = 290 \text{ k}\Omega$$

18.20 Refer to Fig. 1.

$$I_1 = \frac{V_1}{R}$$

$$V_o = -R_f I_1 = -\frac{R_f}{R}V_1$$

$$\Rightarrow V_1 = -\left(\frac{R}{R_f}\right)V_o \qquad (1)$$

$$I_2 = sCV_1$$

$$I_3 = I_1 + I_2 = \frac{V_1}{R} + sCV_1$$

$$I_3 = \frac{V_1}{R}(1 + sCR)$$

$$V_2 = V_1 + I_3R = V_1(2 + sCR)$$

$$I_4 = sCV_2 = sCV_1(2 + sCR)$$

$$I_5 = I_3 + I_4 = \frac{V_1}{R}(1 + sCR) + sCV_1(2 + sCR)$$

$$V_3 = V_2 + I_5R$$

$$= V_1(2 + sCR) + V_1(1 + sCR) + sCRV_1(2 + sCR)$$

$$= V_1(3 + s4CR + s^2C^2R^2)$$

$$I_6 = sCV_3$$

$$= sCV_1(3 + s4CR + s^2C^2R^2)$$

$$I_7 = I_5 + I_6$$

$$= \frac{V_1}{R}(1 + sCR) + sCV_1(5 + s5CR + s^2C^2R^2)$$

$$V_x = V_3 + I_7R$$

$$= V_1(3 + s4CR + s^2C^2R^2)$$

$$+V_1(1 + sCR) + V_1(s5CR + s^25C^2R^2 + s^3C^3R^3$$

$$= V_1(4 + s\ 10CR + s^26C^2R^2 + s^3C^3R^3)$$

Substituting for V_1 from Eq. (1) and equating V_x with V_o, we obtain

$$-\frac{R_f}{R} = 4 + s\ 10CR + s^26\ C^2R^2 + s^3C^3R^3$$

For $s = j\omega$

$$-\frac{R_f}{R} = (4 - 6\ \omega^2C^2R^2) + j\omega\ (10\ CR - \omega^2C^3R^3)$$

Equating the imaginary part on the RHS to zero, we obtain for the frequency of oscillation ω_0

$$\omega_0 = \frac{\sqrt{10}}{CR}$$

This figure belongs to Problem 18.20.

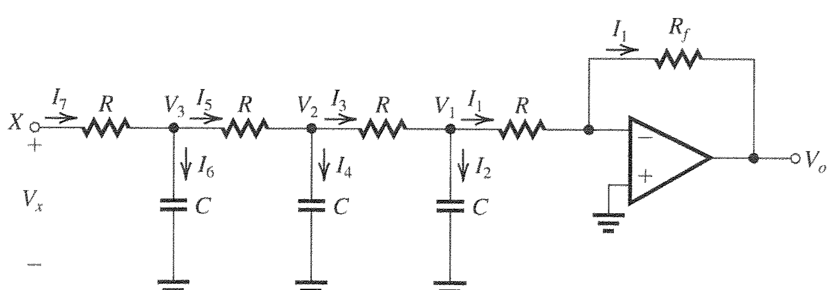


Figure 1

Equating the real parts on both sides gives the condition for sustained oscillations as

$$\frac{R_f}{R} = 6\frac{10}{C^2R^2}C^2R^2 - 4 = 56$$

That is,

$$R_f = 56R$$

Numerical values:

$f_0 = 15$ kHz

$R = 10$ kΩ

$$CR = \frac{\sqrt{10}}{2\pi \times 15 \times 10^3}$$

$$C = \frac{\sqrt{10}}{2\pi \times 15 \times 10^3 \times 10 \times 10^3} = 3.36 \text{ nF}$$

$$R_f = 56 \times 10 = 560 \text{ k}\Omega$$

18.21 Refer to the circuit in Fig. 18.10 with the limiter eliminated and with the loop broken at X to determine the loop gain

$$L(s) = \frac{V_{o2}}{V_x}$$

To determine $L(s)$, we note that it is the product of the transfer functions of the inverting integrator formed around op amp 1:

$$\frac{V_{o1}}{V_x} = -\frac{1}{sCR} \qquad (1)$$

and the noninverting integrator formed around op amp 2. To obtain the transfer function of the latter, refer to Fig. 1.

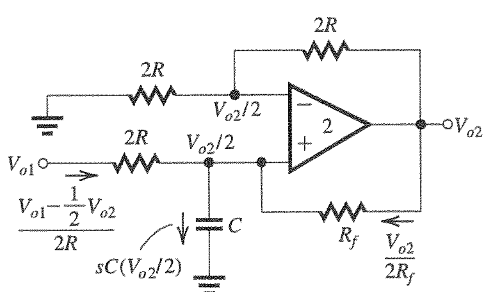


Figure 1

Writing a node equation at the positive input terminal of op amp 2, we obtain

$$\frac{V_{o1} - \frac{1}{2}V_{o2}}{2R} + \frac{V_{o2}}{2R_f} = sC\frac{V_{o2}}{2}$$

$$\Rightarrow \frac{V_{o2}}{V_{o1}} = \frac{1}{sCR + \frac{1}{2} - \frac{R}{R_f}}$$

Substituting for $R_f = \dfrac{2R}{1+\Delta}$, we obtain

$$\frac{V_{o2}}{V_{o1}} = \frac{1}{sCR - \frac{\Delta}{2}} \qquad (2)$$

Using Eqs. (1) and (2), we obtain the loop gain as

$$L(s) = -\frac{1}{sCR}\,\frac{1}{sCR - \frac{\Delta}{2}}$$

The characteristic equation can now be written as

$$1 - L(s) = 0$$

$$1 + \frac{1}{sCR}\,\frac{1}{sCR - \frac{\Delta}{2}} = 0$$

$$sCR\left(sCR - \frac{\Delta}{2}\right) + 1 = 0$$

$$s^2 - s\frac{1}{CR}\frac{\Delta}{2} + \frac{1}{C^2R^2} = 0$$

Thus, the poles are

$$s = \frac{1}{2CR}\left(\frac{\Delta}{2}\right) \pm \frac{1}{2}\sqrt{\frac{1}{C^2R^2}\left(\frac{\Delta}{2}\right)^2 - \frac{4}{C^2R^2}}$$

For $\Delta \ll 4$, we have

$$s \simeq \frac{1}{CR}\left[\left(\frac{\Delta}{4}\right) \pm j\right]$$

from which it is obvious that the poles are in the right half of the s-plane. Q.E.D.

18.22 $\omega_0 = \dfrac{1}{CR}$

$$R = \frac{1}{2\pi \times 10 \times 10^3 \times 1.6 \times 10^{-9}}$$

$$= 9.95 \text{ k}\Omega$$

The square wave v_2 will have a peak-to-peak amplitude

$$V = 1.4 \text{ V}$$

The component at the fundamental frequency ω_0 will have a peak-to-peak amplitude of $4\text{ V}/\pi = 1.78$ V. The filter has a center-frequency gain of 2, thus at v_1 the sine wave will have a peak-to-peak amplitude of approximately 3.6 V.

The output amplitude can be doubled by adding a diode in series with each of the diodes in the limiter.

18.23 The different harmonic components will be attenuated relative to the fundamental by the selective response of the bandpass circuit. Let us first determine the magnitude of the transmission of the bandpass filter at a frequency ω relative to that at the fundamental frequency ω_0,

$$T(s) = \frac{s\left(\dfrac{\omega_0}{Q}\right)}{s^2 + s\left(\dfrac{\omega_0}{Q}\right) + \omega_0^2}$$

$$|T(j\omega)| = \frac{\dfrac{\omega\omega_0}{Q}}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\dfrac{\omega\omega_0}{Q}\right)^2}}$$

$$= 1\Big/\sqrt{1 + Q^2\left(\frac{\omega_0^2 - \omega^2}{\omega\omega_0}\right)^2}$$

$$= 1\Big/\sqrt{1 + Q^2\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)^2}$$

For the nth harmonic, we have $\omega = n\omega_0$, where $n = 3, 5, 7, \ldots\ldots$ Since n is large and $Q = 20$ is large, we obtain

$$|T(j\omega)| \simeq 1/Q\left(n - \frac{1}{n}\right)$$

Thus, relative to the fundamental, we have:

(a) The second harmonic = 0.

(b) The third harmonic

$$= \frac{1/3}{20\left(3 - \dfrac{1}{3}\right)} = 6.25 \times 10^{-3}$$

(c) The fifth harmonic

$$\frac{1/5}{20\left(5 - \dfrac{1}{5}\right)} = 2.08 \times 10^{-3}$$

(d) The rms of harmonics to the tenth:

$$\text{The seventh harmonic} = \frac{1/7}{20\left(7 - \dfrac{1}{7}\right)}$$

$$= 1.04 \times 10^{-3}$$

$$\text{The ninth harmonic} = \frac{1/9}{20\left(9 - \dfrac{1}{9}\right)}$$

$$= 0.625 \times 10^{-3}$$

Thus, the rms of harmonics to the tenth

$$= \sqrt{6.25^2 + 2.08^2 + 1.04^2 + 0.625^2} \times 10^{-3}$$

$$= 6.70 \times 10^{-3}$$

18.24

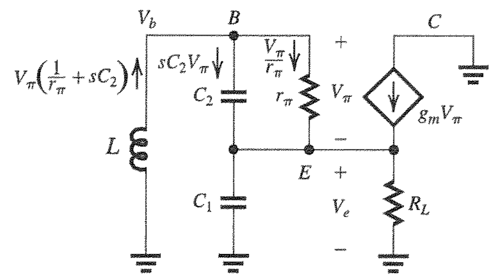


Figure 1

Figure 1 shows the equivalent circuit together with some of the analysis. The voltage V_b can be expressed as

$$V_b = -sL\left(\frac{1}{r_\pi} + sC_2\right)V_\pi \tag{1}$$

The voltage V_e can be expressed as

$$V_e = V_b - V_\pi$$

Thus,

$$V_e = -sL\left(\frac{1}{r_\pi} + sC_2\right)V_\pi - V_\pi \tag{2}$$

Writing a node equation at E, we obtain

$$\left(sC_1 + \frac{1}{R_L}\right)V_e = \left(g_m + \frac{1}{r_\pi}\right)V_\pi + sC_2V_\pi$$

Substituting for V_e from Eq. (2), we obtain

$$-\left(sC_1 + \frac{1}{R_L}\right)sL\left(\frac{1}{r_\pi} + sC_2\right)V_\pi - \left(sC_1 + \frac{1}{R_L}\right)V_\pi$$

$$= \left(g_m + \frac{1}{r_\pi}\right)V_\pi + sC_2V_\pi$$

Dividing by V_π and collecting terms, we obtain

$$s^3LC_1C_2 + s^2L\left(\frac{C_1}{r_\pi} + \frac{C_2}{R_L}\right) + s\left[\frac{L}{R_Lr_\pi} + (C_1 + C_2)\right] + \left(g_m + \frac{1}{r_\pi} + \frac{1}{R_L}\right) = 0$$

For $s = j\omega$, we have

$$j\omega\left[-\omega^2LC_1C_2 + \frac{L}{R_Lr_\pi} + (C_1 + C_2)\right] + \left[\left(g_m + \frac{1}{r_\pi} + \frac{1}{R_L}\right) - \omega^2L\left(\frac{C_1}{r_\pi} + \frac{C_2}{R_L}\right)\right] = 0 \tag{3}$$

This is the equation that governs the operation of the oscillator circuit. The frequency of oscillation ω_0 is the value of ω at which the imaginary part becomes zero, thus

$$\omega_0^2 = \frac{C_1 + C_2}{LC_1C_2} + \frac{1}{R_L r_\pi C_1 C_2} \quad (4)$$

Note that for r_π large so that the second term on the RHS can be neglected, we have

$$\omega_0^2 \simeq 1 \Big/ \left[L\left(\frac{C_1C_2}{C_1+C_2}\right)\right]$$

$$\omega_0 = 1 \Big/ \sqrt{L\left(\frac{C_1C_2}{C_1+C_2}\right)} \quad (5)$$

which is the expected value. Taking r_π into account will shift the oscillation frequency slightly from this value.

The condition for sustained oscillations can be obtained by equating the real part of Eq. (3) to zero and making use of (4), thus

$$g_m + \frac{1}{r_\pi} + \frac{1}{R_L} =$$
$$L\left[\frac{C_1+C_2}{LC_1C_2} + \frac{1}{R_L r_\pi C_1 C_2}\right]\left[\frac{C_1}{r_\pi} + \frac{C_2}{R_L}\right] \quad (6)$$

For r_π large, we have

$$g_m + \frac{1}{R_L} \simeq \left(\frac{C_1+C_2}{C_1C_2}\right)\left(\frac{C_2}{R_L}\right)$$

$$g_mR_L + 1 = 1 + \frac{C_2}{C_1}$$

$$\Rightarrow g_mR_L = \frac{C_2}{C_1} \quad (7)$$

To ensure that oscillations start, we use

$$g_mR_L > \frac{C_2}{C_1}$$

18.25

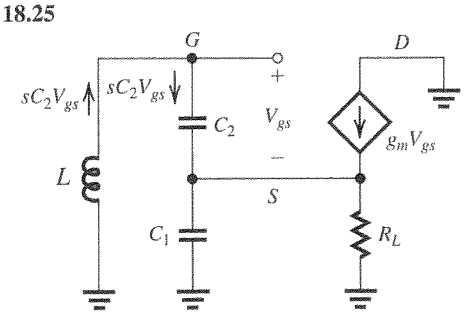


Figure 1

Figure 1 shows the equivalent circuit together with some of the analysis. The voltage at the gate, V_g, can be expressed as

$$V_g = -s^2LC_2V_{gs} \quad (1)$$

The voltage at the source, V_s, can be expressed as

$$V_s = V_g - V_{gs}$$

Thus,

$$V_s = -s^2LC_2V_{gs} - V_{gs} \quad (2)$$

A node equation at S provides

$$sC_2V_{gs} + g_mV_{gs} = \left(\frac{1}{R_L} + sC_1\right)V_s$$

Substituting for V_s from Eq. (2), we obtain

$$sC_2V_{gs} + g_mV_{gs} = -\left(\frac{1}{R_L} + sC_1\right)(s^2LC_2 + 1)V_{gs}$$

Dividing by V_{gs} and collecting terms, we obtain

$$s^3LC_1C_2 + s^2\frac{LC_2}{R_L} + s(C_1+C_2) + \left(g_m + \frac{1}{R_L}\right) = 0$$

For $s = j\omega$, we have

$$j\omega[-\omega^2LC_1C_2 + (C_1+C_2)] +$$
$$\left(g_m + \frac{1}{R_L} - \omega^2\frac{LC_2}{R_L}\right) = 0 \quad (3)$$

This is the equation that governs the operation of the oscillator circuit. The frequency of oscillation ω_0 is the value of ω at which the imaginary part is zero, thus

$$\omega_0^2 = 1/\left[L\left(\frac{C_1C_2}{C_1+C_2}\right)\right] \quad (4)$$

$$\Rightarrow \omega_0 = 1/\sqrt{L\left(\frac{C_1C_2}{C_1+C_2}\right)}$$

The condition for sustained oscillations can be found by equating the real part of Eq. (3) to zero and making use of (4), thus

$$g_m + \frac{1}{R_L} = \left(\frac{C_1+C_2}{C_1}\right)\left(\frac{1}{R_L}\right)$$

$$\Rightarrow g_mR_L = \frac{C_2}{C_1}$$

To ensure that oscillations start, we use

$$g_mR_L > \frac{C_2}{C_1}$$

18.26

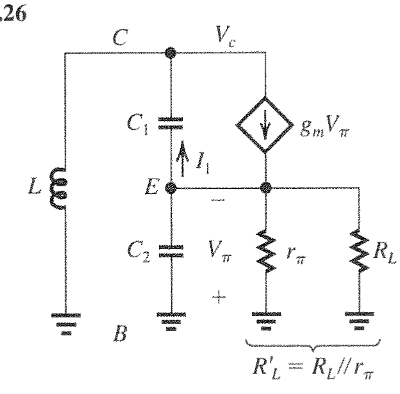


Figure 1

Figure 1 shows the equivalent circuit where we have neglected r_o. A node equation at E provides the following expression for I_1:

$$I_1 = V_\pi\left(g_m + \frac{1}{R_L'} + sC_2\right)$$

The collector voltage V_c can now be found as

$$V_c = -V_\pi - \frac{1}{sC_1}I_1$$

$$V_c = -V_\pi - \frac{1}{sC_1}\left(g_m + \frac{1}{R_L'} + sC_2\right)V_\pi \qquad (1)$$

A node equation at C provides

$$\frac{V_c}{sL} = V_\pi\left(g_m + \frac{1}{R_L'} + sC_2\right) - g_m V_\pi$$

$$= V_\pi\left(\frac{1}{R_L'} + sC_2\right)$$

Substituting for V_c from (1), we obtain

$$-V_\pi - \frac{1}{sC_1}\left(g_m + \frac{1}{R_L'} + sC_2\right)V_\pi =$$
$$V_\pi sL\left(\frac{1}{R_L'} + sC_2\right)$$

Dividing by V_π and collecting terms results in

$$s^3LC_1C_2 + s^2\frac{LC_1}{R_L'} + s(C_1 + C_2) + g_m + \frac{1}{R_L'} = 0$$

For $s = j\omega$, we have

$$j\omega[-\omega^2LC_1C_2 + (C_1 + C_2)] +$$
$$\left(-\omega^2\frac{LC_1}{R_L'} + g_m + \frac{1}{R_L'}\right) = 0 \qquad (2)$$

This is the equation that governs the operation of the oscillator circuit. The frequency of oscillation ω_0 is the value of ω at which the imaginary part is zero, thus

$$\omega_0^2 = \frac{1}{L\left(\dfrac{C_1C_2}{C_1 + C_2}\right)} \qquad (3)$$

or

$$\omega_0 = 1/\sqrt{L\frac{C_1C_2}{C_1 + C_2}}$$

The condition for sustained oscillation can be found by equating the real part in Eq. (2) to zero at $\omega = \omega_0$ and by making use of Eq. (3). Thus,

$$-\frac{C_1 + C_2}{C_2}\frac{1}{R_L'} + g_m + \frac{1}{R_L'} = 0$$

$$\Rightarrow g_mR_L' = \frac{C_1}{C_2}$$

While to ensure that oscillations start, we make

$$g_mR_L' > \frac{C_1}{C_2}$$

Observe then in this circuit, we did not have to resort to assuming r_π to be large in order to obtain simplified expressions. Here r_π is simply included with R_L to obtain R_L' and thus can be easily taken into account. A drawback of our analysis, however, is that r_o was not taken into account.

18.27

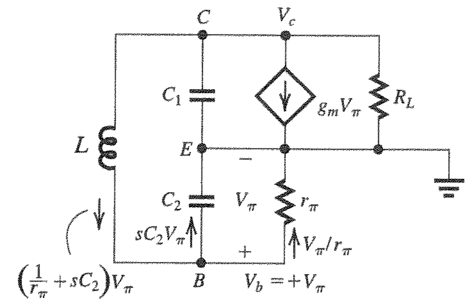


Figure 1

Figure 1 shows the equivalent circuit where r_o has been absorbed into R_L. We have neglected R_f with the assumption that $R_f \gg \omega_0 L$. Some of the analysis is shown on Fig. 1.

Note that,

$$V_b = V_\pi$$

$$V_c = V_b + sL\left(\frac{1}{r_\pi} + sC_2\right)V_\pi$$

Thus,

$$V_c = V_\pi + sL\left(sC_2 + \frac{1}{r_\pi}\right)V_\pi \qquad (1)$$

A node equation at E yields

$$V_\pi\left(\frac{1}{r_\pi} + sC_2\right) + g_mV_\pi + V_c\left(\frac{1}{R_L} + sC_1\right) = 0$$

Substituting for V_c from Eq. (1), we obtain

$$V_\pi\left(\frac{1}{r_\pi} + sC_2\right) + g_mV_\pi + \left(\frac{1}{R_L} + sC_1\right)V_\pi$$
$$+\left(\frac{1}{R_L} + sC_1\right)sL\left(sC_2 + \frac{1}{r_\pi}\right) = 0$$

Dividing by V_π and collecting terms gives

$$s^3LC_1C_2 + s^2\left(\frac{LC_2}{R_L} + \frac{LC_1}{r_\pi}\right) +$$
$$s\left(C_1 + C_2 + \frac{L}{R_Lr_\pi}\right) + g_m + \frac{1}{r_\pi} + \frac{1}{R_L} = 0$$

For $s = j\omega$, we have

$$j\omega\left(-\omega^2LC_1C_2 + C_1 + C_2 + \frac{1}{R_Lr_\pi}\right)$$
$$+\left[g_m + \frac{1}{r_\pi} + \frac{1}{R_L} - \omega^2L\left(\frac{C_2}{R_L} + \frac{C_1}{r_\pi}\right)\right] = 0 \quad (2)$$

This is the equation that governs the operation of the oscillator circuit. The frequency of oscillation ω_0 is the value of ω that makes the imaginary part zero, thus

$$\omega_0^2 = 1 \bigg/ \left[L \frac{C_1 C_2}{(C_1 + C_2) + \dfrac{L}{R_L r_\pi}} \right] \tag{3}$$

Observe that including r_π changes the frequency of oscillation slightly from that of the frequency of the resonance circuit. If we neglect the term containing r_π in Eq. (3), we obtain

$$\omega_0 \simeq 1 \bigg/ \sqrt{L \frac{C_1 C_2}{C_1 + C_2}} \tag{4}$$

The condition for sustained oscillations can be found from Eq. (2) by equating the real part to zero at $\omega = \omega_0$ and making use of (3), thus

$$g_m + \frac{1}{R_L} + \frac{1}{r_\pi} = \frac{1}{\dfrac{C_1 C_2}{C_1 + C_2 + \dfrac{L}{R_L r_\pi}}} \left(\frac{C_2}{R_L} + \frac{C_1}{r_\pi} \right)$$

$$g_m + \frac{1}{R_L} + \frac{1}{r_\pi} =$$

$$\left[\frac{C_1 + C_2 + (L/R_L r_\pi)}{C_1 C_2} \right] \left(\frac{C_2}{R_L} + \frac{C_1}{r_\pi} \right)$$

Neglecting the terms containing r_π, we obtain

$$g_m + \frac{1}{R_L} = \left(1 + \frac{C_2}{C_1} \right) \frac{1}{R_L}$$

$$\Rightarrow g_m R_L = \frac{C_2}{C_1}$$

For oscillations to start, we need

$$g_m R_L > \frac{C_2}{C_1}$$

18.28 (a)

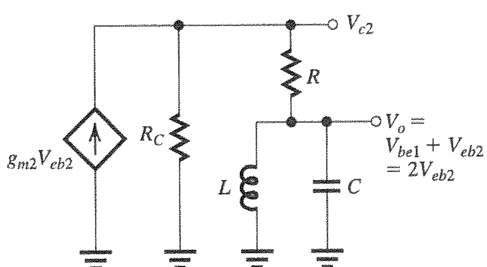


Figure 1

Figure 1 shows the equivalent circuit.

(b) Oscillations will occur at the frequency for which the tuned circuit has infinite impedance. This is because at this frequency the phase shift around the loop will be zero. Thus,

$$\omega_0 = 1/\sqrt{LC}$$

At this frequency, no current flows through R. Thus the voltage V_{c2} will be

$$V_{c2} = (g_{m2} V_{eb2}) R_C$$

and the voltage V_o will equal V_{c2}, thus

$$(g_{m2} V_{eb2}) R_C = V_o = 2V_{eb2}$$

which yields the condition for sustained oscillation as

$$g_{m2} R_C = 2$$

For oscillations to start, we impose the condition

$$g_{m2} R_C > 2 \tag{1}$$

But

$$g_{m2} \simeq \frac{I/2}{V_T}$$

Thus, the condition in (1) can be expressed as

$$IR_C > 4V_T$$

that is,

$$IR_C > 0.1 \text{ V}$$

(c) Selecting $IR_C = 1$ V means that oscillations will start and will grow in amplitude until V_o is large enough to cause Q_1 and Q_2 to alternately turn on and off. When this happens the collector current of Q_2 will be 0 in half a cycle and equal to I in the other half cycle. Thus a square wave voltage of amplitude $IR_C = 1$ V peak-to-peak will develop at the collector of Q_2. This square wave voltage is applied through R to the bandpass filter formed by the RLC circuit. Thus, the sinusoid that develops across the LC circuit—that is, V_o—will have a frequency ω_0 and a peak-to-peak amplitude of $\left(\frac{4}{\pi} \times 1 \right) = \frac{4}{\pi}$ V. There will be third, fifth, and other odd harmonics, but those will be attenuated because of the selectivity of the bandpass RLC circuit.

18.29 $\omega_0 = 20$ Grad/s $= 20 \times 10^9$ rad/s

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$20 \times 10^9 = \frac{1}{\sqrt{5 \times 10^{-9} \times C}}$$

$$\Rightarrow C = 0.5 \text{ pF}$$

$$R_p = \omega_0 L Q$$

$$= 20 \times 10^9 \times 5 \times 10^{-9} \times 10$$

$$= 1000\ \Omega = 1 \text{ k}\Omega$$

$$r_o \parallel R_p = 5 \parallel 1 = \frac{5}{6} \text{ k}\Omega$$

$$g_m|_{\min} = \frac{1}{\dfrac{5}{6} \times 10^3} = 1.2 \text{ mA/V}$$

18.30 From Exercise 18.13, we have

$L = 0.52\text{ H}$

$C_s = 0.012\text{ pF}$

$C_p = 4\text{ pF}$

$$C_{eq} = \frac{C_s\left(C_p + \dfrac{C_1C_2}{C_1 + C_2}\right)}{C_s + C_p + \dfrac{C_1C_2}{C_1 + C_2}}$$

$C_2 = 10\text{ pF}\quad C_1 = 1\text{ to }10\text{ pF}$

$$C_L = \frac{0.012\left(4 + \dfrac{10\times 1}{10+1}\right)}{\left(0.012 + 4 + \dfrac{10}{11}\right)} = 0.01197\text{ pF}$$

$$C_H = \frac{0.012\left(4 + \dfrac{10\times 10}{10+10}\right)}{\left(0.012 + 4 + \dfrac{100}{20}\right)} = 0.01198\text{ pF}$$

$$\therefore f_{0H} = \frac{1}{2\pi\left[0.52\times 0.01197\times 10^{-12}\right]^{1/2}}$$

$= 2.0173\text{ MHz}$

$$f_{0L} = \left[2\pi\left(0.52\times 0.01198\times 10^{-12}\right)^{1/2}\right]^{-1}$$

$= 2.0165\text{ MHz}$

Difference $= 800\text{ Hz}$

18.31 $V_{TH} = -V_{TL} = 1\text{ V}$

$L_+ = -L_- = 5\text{ V}$

$V_{TH} = \beta L_+$

$$1 = \frac{R_1}{R_1 + R_2}\times 5$$

$$\Rightarrow \frac{R_2}{R_1} = 4$$

For $R_1 = 10\text{ k}\Omega$, we have

$R_2 = 40\text{ k}\Omega$

18.32

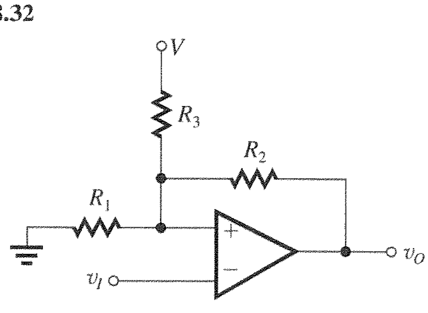


Figure 1

(a) Refer to Fig. 1. With $v_O = L_+$, the voltage at the op amp positive input terminal will be V_{TH}. Now, writing a node equation at the op amp positive input terminal, we have

$$\frac{V_{TH}}{R_1} = \frac{V - V_{TH}}{R_3} + \frac{L_+ - V_{TH}}{R_2}$$

$$V_{TH}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{L_+}{R_2} + \frac{V}{R_3}$$

$$\Rightarrow V_{TH} = \left(\frac{L_+}{R_2} + \frac{V}{R_3}\right)(R_1 \parallel R_2 \parallel R_3)$$

Similarly, we can obtain

$$V_{TL} = \left(\frac{L_-}{R_2} + \frac{V}{R_3}\right)(R_1 \parallel R_2 \parallel R_3)$$

(b) $L_+ = -L_- = 10\text{ V},\ V = 15\text{ V},\ R_1 = 10\text{ k}\Omega$

$$V_{TH} = 5.1 = \left(\frac{10}{R_2} + \frac{15}{R_3}\right)(R_1 \parallel R_2 \parallel R_3)$$

$$\frac{5.1}{R_1} + \frac{5.1}{R_2} + \frac{5.1}{R_3} = \frac{10}{R_2} + \frac{15}{R_3}$$

$$0.51 = \frac{4.9}{R_2} + \frac{9.9}{R_3} \quad (1)$$

$$V_{TL} = 4.9 = \left(\frac{-10}{R_2} + \frac{15}{R_3}\right)(R_1 \parallel R_2 \parallel R_3)$$

$$\frac{4.9}{R_1} + \frac{4.9}{R_2} + \frac{4.9}{R_3} = -\frac{10}{R_2} + \frac{15}{R_3}$$

$$0.49 = \frac{-14.9}{R_2} + \frac{10.1}{R_3} \quad (2)$$

Multiplying Eq. (1) by $\left(\dfrac{14.9}{4.9}\right)$, we obtain

$$1.55 = \frac{-14.9}{R_2} + \frac{30.1}{R_3} \quad (3)$$

Adding (2) and (3) gives

$$2.04 = \frac{40.2}{R_3}$$

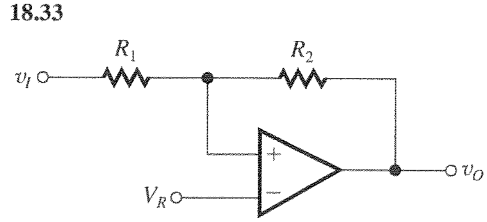
$\Rightarrow R_3 = 19.7\text{ k}\Omega$

Substituting in Eq. (1), we obtain

$$0.51 = \frac{4.9}{R_2} + \frac{9.9}{19.7}$$

$$\Rightarrow R_2 = \frac{4.9}{0.0076} = 656.7\text{ k}\Omega$$

18.33



(a) For $v_I = V_{TL}$ and $v_O = L_+$ initially

$$\frac{L_+ - V_R}{R_2} = \frac{V_R - V_{TL}}{R_1}$$

$$V_{TL} = V_R + \frac{R_1}{R_2}V_R - \frac{R_1}{R_2}L_+$$

$$\therefore\ V_{TL} = V_R\left(1 + \frac{R_1}{R_2}\right) - \frac{R_1}{R_2}L_+$$

Similarly,

$$\frac{L_- - V_R}{R_2} = \frac{V_R - V_{TH}}{R_1}$$

$$V_{TH} = V_R(1 + R_1/R_2) - \frac{R_1}{R_2}L_-$$

(b) Given

$L_+ = -L_- = V$

$R_1 = 10\ \text{k}\Omega$

$V_{TL} = 0$

$V_{TH} = V/10$

Substituting these values, we get

$$0 = V_R(1 + 10/R_2) - (10/R_2)V \qquad (1)$$

$$\frac{V}{10} = V_R(1 + 10/R_2) + (10/R_2)V \qquad (2)$$

Subtracting Eq. (2) from Eq. (1), we obtain

$$-\frac{V}{10} = -\frac{20}{R_2} \times V$$

$$\Rightarrow R_2 = 200\ \text{k}\Omega$$

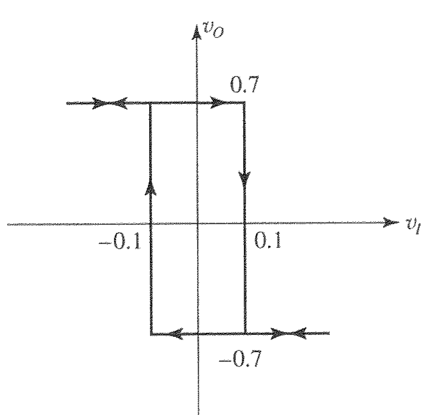
$$0 = V_R\,(1 + 10/200) - \frac{10}{200}V$$

$$V_R = \frac{10/200\ \text{V}}{1 + 10/200} = \frac{V}{21}$$

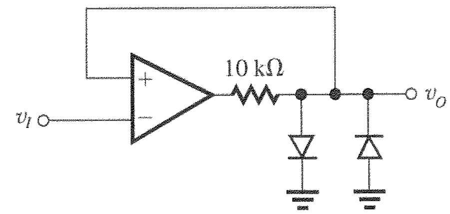
18.34 Output levels $= \pm 0.7$ V

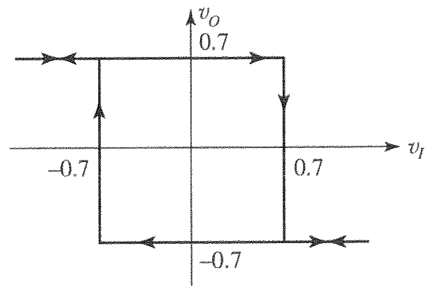
$$\text{Threshold levels} = \pm\frac{10}{10 + 60} \times 0.7 = 0.1\ \text{V}$$

$$i_{D,\,\max} = \frac{12 - 0.7}{10} - \frac{0.7}{10 + 60} = 1.12\ \text{mA}$$



18.35



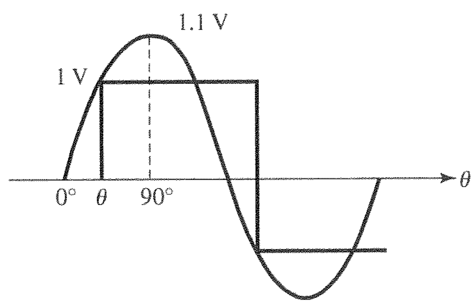


18.36 (a) A 0.5-V peak sine wave is not large enough to change the state of the circuit. Hence, the output will be either +12 V or −12 V.

(b) The 1.1-V peak sine wave will change the state when

$1.1 \sin\theta = 1$

$\theta = 65.40$



$\therefore$ The output is a symmetric square wave of frequency f, and it lags the sine wave by an angle of 65.4°. The square wave has a swing of ± 12 V.

Since $V_{TH} = -V_{TL} = 1$ V, if the average shifts by an amount so either the +ve or -ve swing is < 1 V, then no change of state will occur. Clearly, if the shift is 0.1 V, the output will be a DC voltage.

18.37 For $L_+ = -L_- = 7.5$ V, we have

$V_Z = 6.8$ V with $V_D = 0.7$ V.

For $V_{TH} = -V_{TL} = 7.5$, we have V $\Rightarrow R_1 = R_2$.

For $v_i = 0$, we have $I_{R_2} = 0.5\ \text{mA} = \dfrac{7.5}{R_1 + R_2}$

$\Rightarrow R_1 = R_2 = 7.5\ \text{k}\Omega$

$$I_D = 1 \text{ mA} = \frac{12 - 7.5}{R} - \frac{7.5}{2R_1}$$

$$1 = \frac{4.5}{R} - 0.5$$

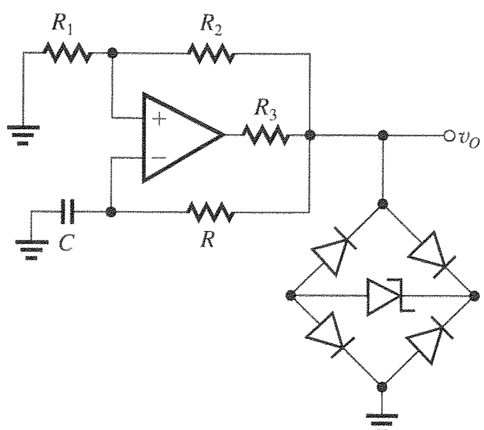
$$R = 3 \text{ k}\Omega$$

18.38 $T = 2\tau \ln \dfrac{1+\beta}{1-\beta}, \quad \beta = \dfrac{R_1}{R_1 + R_2} = \dfrac{10}{26}$

$$T = 2(5 \times 10^{-9})(62 \times 10^{3}) \ln\left(\frac{1 + 10/26}{1 - 10/26}\right)$$

$$T = 0.503 \text{ ms} \Rightarrow f = 1989 \text{ Hz}$$

18.39



$$\beta = 0.462$$

For $V_D = 0.7$ V and $V_O = \pm 5$ V, we have

$$V_Z = 5 - 2V_D$$

$$V_Z = 3.6 \text{ V}$$

$$T = 2\tau \ln\left(\frac{1+\beta}{1-\beta}\right)$$

$$10^{-3} = 2\tau \ln\left(\frac{1.462}{1 - 0.462}\right) \Rightarrow \tau = 0.5 \text{ ms}$$

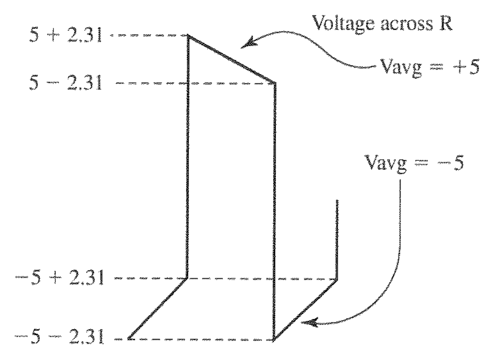
$$\tau = RC \Rightarrow R = \tau / C = 50 \text{ k}\Omega$$

Thresholds $= \pm 0.462 \times 5 = \pm 2.31$ V

Average current in R in $\frac{1}{2}$ cycle:

$$I \cong \frac{1}{R}\left(\frac{5 - 2.31 + 2.31 + 5}{2}\right)$$

$$= \frac{5}{R} = \frac{5}{50 \text{ k}\Omega} = 0.1 \text{ mA}$$



$$R_1 + R_2 = \frac{5 \text{ V}}{0.1 \text{ mA}} = 50 \text{ k}\Omega$$

$$\frac{R_1}{R_1 + R_2} = 0.462 \rightarrow R_1 = 50\,(0.462)$$

$$= 23.1 \text{ k}\Omega$$

$$\therefore\ R_2 = 26.9 \text{ k}\Omega$$

$$1 = \frac{13 - 5}{R_3} - 0.1 - 0.1$$

$$R_3 = \frac{8}{1.2}$$

$$= 6.67 \text{ k}\Omega$$

18.40 From Fig. 18.25(b), for ±5-V output, we have

$$V_Z = 5 - 2V_{\text{DIODE}} = 5 - 1.4 = 3.6 \text{ V}$$

For ±5-V output:

$$R_1 = R_2,\ L_+ = -L_- = 5 \text{ V}$$

$$V_{TH} = -V_{TL} = 5 \text{ V}$$

Max current in feedback network = 0.2 mA

$$\therefore 0.2 = \frac{10}{R_1 + R_2} \Rightarrow R_1 = R_2 = 25 \text{ k}\Omega$$

Minimum zener current = 1 mA

$$\therefore\ \frac{12 - 5}{R_3} = (0.2 + 1) \text{ mA}$$

$$R_3 = \frac{7}{1.2} = 5.83 \text{ k}\Omega$$

Now from Fig. 18.27(c) we have

$$\text{Slope} = \frac{-L_-}{RC} = \frac{V_{TH} - V_{TL}}{T/2}$$

for $f = 1$ kHz

$$T = 10^{-3} \text{ s}$$

$$C = 0.01\ \mu\text{F}$$

$$\frac{5}{RC} = \frac{10}{10^{-3}/2} \Rightarrow R = 25 \text{ k}\Omega$$

18.41 Refer to the circuit of Fig. P18.41. To obtain a square-wave voltage of ±7.5 V levels across the zeners, we need

$V_Z + V_D = 7.5$ V

For $V_D = 0.7$ V, we have

$V_Z = 6.8$ V

Now, $R_1 = R_2$, thus for the bistable we obtain

$L_+ = -L_- = 7.5$ V

$$\beta = \frac{R_1}{R_1 + R_2} = 0.5$$

$$V_{TH} = -V_{TL} = \frac{7.5}{2}\ \text{V}$$

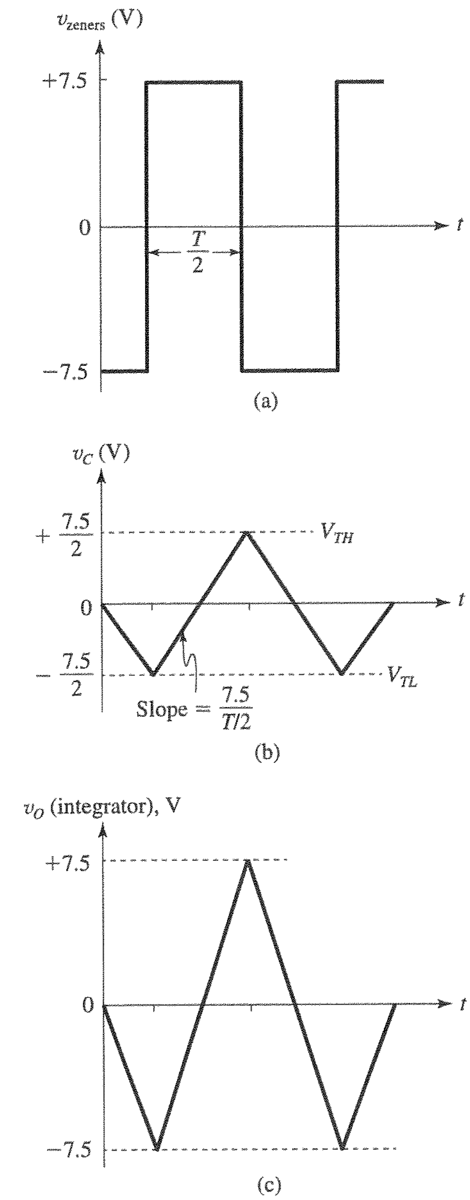


Figure 1

As shown in Fig. 1, the voltage across the capacitor will be triangular, ranging between $-V_{TL}$ and $+V_{TH}$. The slope of the triangular waveform edges is

$$\text{Slope} = \frac{7.5}{R_5} \times \frac{1}{C} = \frac{7.5}{T/2}$$

For $R_5 = R$, we have

$$CR = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2 \times 10 \times 10^3}$$

$$R = \frac{1}{2 \times 10 \times 10^3 \times 0.5 \times 10^{-9}} = 100\ \text{k}\Omega$$

Since all resistors, except R_7, are equal, we have

$R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 100$ kΩ

To determine R_7, we note that the minimum zener current occurs when the current through R_5 is at its maximum. The latter condition occurs when the output of the bistable is at +7.5 V while v_C is $(-7.5/2)$ V at which time

$$I_{R5} = \frac{7.5 - (-7.5/2)}{R_s}$$

$= 0.1125$ mA

Thus, for a minimum zener current of 1 mA, we write

$$\frac{13 - 7.5}{R_7} = 1 + \frac{7.5}{R_1 + R_2} + 0.1125$$

$$= 1 + \frac{7.5}{200} + 0.1125$$

$\Rightarrow R_7 = 4.8$ kΩ

18.42 Refer to Fig. 18.18. The recovery time, t_{rec}, is the time for v_B to go from βL_- to V_{D1}:

$$v_B(t) = L_+ - (L_+ - \beta L_-)e^{-t/C_1R_3}$$

Thus,

$$V_D = L_+ - (L_+ - \beta L_-)e^{-t_{\text{rec}}/C_1R_3}$$

$$\Rightarrow t_{\text{rec}} = C_1R_3 \ln \frac{L_+ - \beta L_-}{L_+ - V_D}$$

From Exercise 18.22, we have

$C_1 = 0.1$ μF, $R_3 = 6171$ Ω, $L_+ = -L_- = 12$ V

$\beta = 0.1$, and $V_D = 0.7$ V, thus

$$t_{\text{rec}} = 0.1 \times 10^{-6} \times 6.171 \times 10^3 \ln\left(\frac{12 + 1.2}{12 - 0.7}\right)$$

$= 96$ μs

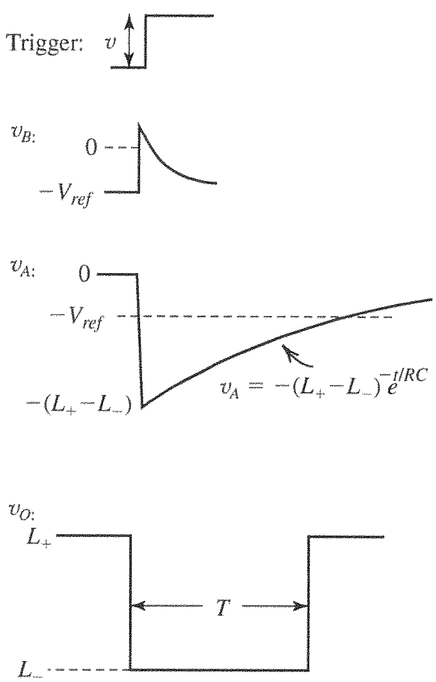
18.43 See sketches that follow:

$$v_A\,(t=T) = -V_{\text{ref}} = -\,(L_+ - L_-)\,e^{-T/RC}$$

$$\frac{V_{\text{ref}}}{L_+ - L_-} = e^{-T/RC}$$

$$T = -RC\ln\left(\frac{V_{\text{ref}}}{L_+ - L_-}\right)$$

$$= RC\ln\left(\frac{L_+ - L_-}{V_{\text{ref}}}\right) \qquad \text{Q.E.D.}$$



18.44 Choose $C_1 = 1$ nF and $C_2 = 0.1$ nF:

$$R_1 = R_2 = 100\text{ k}\Omega \Rightarrow \beta \equiv \frac{1}{2}$$

$$T = C_1 R_3 \ln\left(\frac{0.7 + 13}{0.5 \times (-13) + 13}\right)$$

$$10^{-4} = 10^{-9} R_3 \ln\left(\frac{13.7}{13\,(0.5)}\right)$$

$$R_3 = 134.1\text{ k}\Omega$$

Need $R_4 \gg R_1 \Rightarrow$ choose $R_4 = 470$ kΩ

The trigger pulse must be sufficiently large to lower the voltage at node C from βL_+ to V_D, that is, from +6.5 V to +0.7 V; thus it must be at least 5.8 V.

For recovery we have

$$v_B = 13 - \left(13 - \beta L_-\right) e^{-t/\tau}$$

$$= 13 - 19.5e^{-t/\tau} = 0.7$$

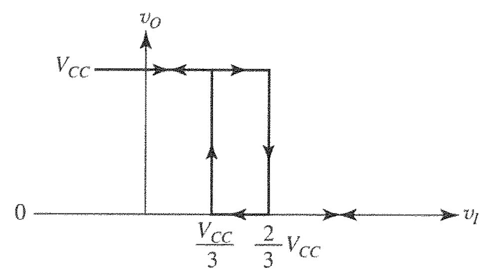
$$\therefore\ t_{\text{recovery}} = -\tau\ln\left(\frac{12.3}{19.5}\right)$$

$$= -\left(134.1 \times 10^3\right)\left(10^{-9}\right)(-0.4608)$$

$$= 61.8\ \mu\text{s}$$

18.45 For $v_I > 2/3V_{CC}$, comp –1 = "1" and comp –2 = "0" and flip flop is reset, i.e. $v_O = 0$ V. Now v_O will not change until $v_I = 1/3V_{CC}$, when comp –2 = "1" and comp –1 = "0" and FF is set: i.e. $v_O = V_{CC}$

For $\frac{1}{3}V_{CC} < v_I < \frac{2}{3}V_{CC}$, comp –1 = comp –2 = "0" and no change of state will occur.



i.e. an inverting bistable circuit.

18.46 (a) $C = 0.5$ nF

Using Eq. (18.41), we obtain

$$T \cong 1.1CR$$

$$10 \times 10^{-16} = 1.1 \times 0.5 \times 10^{-9} \times R$$

$$\Rightarrow R = 18.2\text{ k}\Omega$$

(b) For $T = 20\ \mu$s, $R = 18.2$ kΩ, $C = 0.5$nF, $V_{CC} = 12$ V, and using Eq. (18.40) with $v_C = V_{\text{TH}}$ and $t = T$, we obtain

$$V_{TH} = 12\left(1 - e^{-\frac{20\times10^{-6}}{9.1\times10^{-6}}}\right)$$

$$= 10.67\text{ V}$$

18.47

$C = 680$ pF, $f = 20$ kHz, duty cycle = 80%.

Using Eq. (18.46), we obtain

$$T = 0.69C(R_A + 2R_B)$$

Thus,

$$\frac{1}{20 \times 10^3} = 0.69 \times 680 \times 10^{-12}\ (R_A + 2R_B)$$

$$\Rightarrow R_A + 2R_B = \frac{1}{20 \times 0.69 \times 0.68 \times 10^{-6}}$$

$$R_A + 2R_B = 106.56\text{ k}\Omega \qquad (1)$$

Using Eq. (18.47), we have

$$0.8 = \frac{R_A + R_B}{R_A + 2R_B}$$

Thus,

$$R_A + R_B = 0.8 \times 106.6 = 85.25\text{ k}\Omega \qquad (2)$$

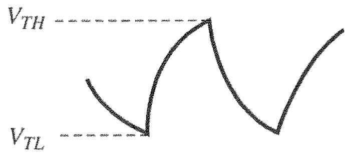
Subtracting Eq. (2) from Eq. (1) gives

$R_B = 21.31\ \text{k}\Omega$

and using Eq. (2), we obtain

$R_A = 63.94\ \text{k}\Omega$

18.48



For the rise:

$v_C = V_{CC} - (V_{CC} - V_{TL})\, e^{-t/C(R_A+R_B)}$

$V_{TH} = V_{CC} - (V_{CC} - V_{TL})\, e^{-T_H/C(R_A+R_B)}$

$$\frac{V_{CC} - V_{TH}}{V_{CC} - V_{TL}} = e^{-T_H/C(R_A+R_B)}$$

$$T_H = C(R_A + R_B)\ln\left(\frac{V_{CC} - V_{TL}}{V_{CC} - V_{TH}}\right)$$

For exponential fall:

$v_C = V_{TH}e^{-t/CR_B}$

$\therefore\ V_{TL} = V_{TH}e^{-T_L/CR_B}$

$$T_L = CR_B \ln\left(\frac{V_{TH}}{V_{TL}}\right)$$

for $V_{TH} = 2V_{TL} \Rightarrow T_L = CR_B\ln(2)$

(b) $C = 1\ \text{nF},\ \ R_A = 7.2\ \text{k}\Omega,\ \ R_B = 3.6\ \text{k}\Omega$

$V_{CC} = 5$ V, no external voltage to V_{TH}.

$\therefore\ T_H + T_L = T = \ln 2 \times (R_A + 2R_B)C$

$T = 9.94\ \mu\text{s} \rightarrow f = 100.6\ \text{kHz}$

$$\text{Duty cycle} = \frac{T_H}{T_H + T_L} = \frac{R_A + R_B}{R_A + 2R_B} = 0.75$$

$\Rightarrow 75\%$

(c) $V_{CC} = 5$ V,

$$V_{TH} = \frac{2}{3} \times 5 = \frac{10}{3} = 3.33\ \text{V}$$

For 1-V input the high value of V_{TH} will be $V'_{TH} = 4.33$ V

and, $V'_{TL} = \dfrac{1}{2}V'_{TH} = 2.17$ V

$$T'_H = 10^{-9}(3.6 + 7.2) \times 10^3 \ln\left(\frac{5 - 2.17}{5 - 4.33}\right)$$

$= 15.6\ \mu\text{s}$

$T'_L = 10^{-9} \times 3.6 \times 10^3 \ln 2 = 2.5\ \mu\text{s}$

$$\therefore\ f = \frac{1}{(15.6 + 2.5)10^{-6}} = 55.2\ \text{kHz}$$

$$\text{Duty cycle} = \frac{15.6}{2.5 + 15.6} = 86.2\%$$

For 1-V input the low value of V_{TH} will be $V''_{TH} = 2.33$

and $V''_{TL} = 1.17$

$$\therefore\ T''_H = 10^{-9}(3.6 + 7.2)\,10^3 \ln\left(\frac{5 - 1.17}{5 - 2.33}\right)$$

$= 3.90\ \mu\text{s}$

$T''_L = T'_L = 2.5\ \mu\text{s}$

$$\therefore\ f = \frac{10^6}{(3.90 + 2.5)} = 156\ \text{kHz}$$

$$\text{Duty cycle} = \frac{3.90}{2.5 + 3.90} = 61\%$$

18.49

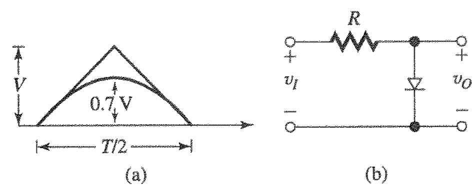


Figure 1

For a sine wave, we have

$v_O = 0.7 \sin \omega t$

Slope at zero crossings

$= 0.7\omega = 0.7 \times 2\pi f$

$$= \frac{1.4\pi}{T}$$

Slope of triangular wave = $\dfrac{V}{T/4}$. Equating slopes, we obtain

$$\frac{1.4\pi}{T} = \frac{V}{T/4}$$

$$\Rightarrow V = \frac{1.4\pi}{4} = 1.1\ \text{V}$$

Refer to Fig. 1(b).

With $v_I = V = 1.1$ V, we have $v_O = 0.7$ V, thus

$v_R = 1.1 - 0.7 = 0.4$ V

At $V_D = 0.7$ V we have $i_D = 1$ mA, thus

$0.4 = 1 \times R$

$\Rightarrow R = 0.4\ \text{k}\Omega = 400\ \Omega$

The angle θ and the ideal value of the output voltage are determined as follows:

Since v_D changes by 0.1 V per decade change in current, we have

$$v_O = 0.7 - 0.1\log\left(\frac{1\text{ mA}}{i_D}\right)$$

$$\Rightarrow i_D = 10^{10(v_O-0.7)},\ \text{mA} \quad (1)$$

$$v_I = v_O + i_D R \quad (2)$$

$$\theta = \frac{v_I}{1.1} \times 90^\circ \quad (3)$$

$$\text{Ideal } v_O = 0.7 \sin\theta \quad (4)$$

Percentage error in v_O =

$$\frac{v_O - \text{Ideal } v_O}{\text{Ideal } v_O} \times 100 \quad (5)$$

Equations (1)–(5) can be used for each of the given values of v_O to obtain the results given in the table below.

v_O(V)	θ	0.7 sin θ	% Error
0.70	90°	0.7	0
0.65	63.6°	0.627	3.7
0.60	52.4°	0.554	8.2
0.55	46.1°	0.504	9.1
0.50	41.3°	0.462	8.2
0.40	32.80	0.379	5.5
0.30	24.6°	0.291	3.0
0.20	16.4°	0.198	1.2
0.10	8.2°	0.0998	0.1
0.00	0°	0	0.0

18.50 Slope of triangular wave = $\frac{16}{T} = 16f$.
Slope at zero crossings of a sine wave with peak amplitude of $(V_B + 0.7)$ V is

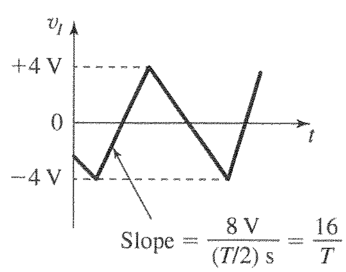
$$= (V_B + 0.7)\omega$$

Equating slopes, we obtain

$$16f = (V_B + 0.7) \times 2\pi f$$

$$\Rightarrow V_B = 1.85\text{ V}$$

This figure belongs to Problem 18.50.



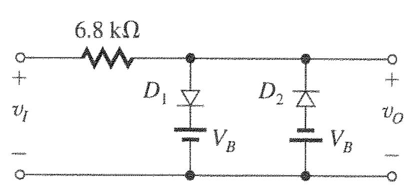
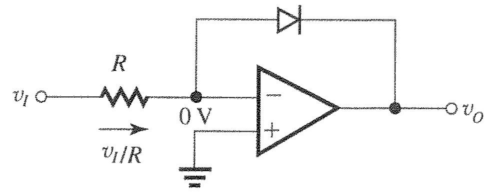


Figure 1

18.51



$v_I > 0$

Voltage across diode is $-v_O$

$$i_D = \frac{v_I}{R} = I_S e^{-v_O/V_T}$$

$$-\frac{v_O}{V_T} = \ln\left(\frac{v_I}{RI_S}\right)$$

$$v_O = -V_T \ln\left(\frac{v_I}{RI_S}\right),\ v_I > 0 \qquad \text{Q.E.D.}$$

18.52 From the statement of Problem 18.51, we have

$$v_O = -V_T \ln\left(\frac{v_I}{I_S R}\right)$$

Now, for the circuit in Fig. P18.52 we can write

$$v_A = -V_T \ln\left(\frac{v_1}{I_S}\right)$$

$$v_B = -V_T \ln\left(\frac{v_2}{I_S}\right)$$

$$v_C = V_T \ln\left(\frac{1}{I_S}\right) \quad (3)$$

$$v_D = V_T\left[\ln\frac{v_1}{I_S} + \ln\frac{v_2}{I_S} - \ln\frac{1}{I_S}\right]$$

$$= V_T \ln^{(v_1 v_2/I_S)}$$

$$v_O = -I_S e^{\ln\left(\frac{v_1 v_2}{I_S}\right)}$$

$$v_O = -v_1 v_2 \qquad \text{Q.E.D.}$$

Check:

For $v_1 = 0.5$ V and $v_2 = 1$ V, we have

$$i_{D1} = 0.5\text{ mA}$$

$$v_{D1} = 0.7 + 0.025 \ln\left(\frac{0.5}{1}\right)$$
$$= 0.683 \text{ V}$$
$$v_A = -0.683 \text{ V}$$
$$i_{D2} = 1 \text{ mA}$$
$$v_{D2} = 0.7 \text{ V}$$
$$v_B = -0.7 \text{ V}$$
$$i_{D3} = 1 \text{ mA}$$
$$v_C = 0.7 \text{ V}$$
$$v_D = -(-0.683 - 0.7 + 0.7)$$
$$= 0.683 \text{ V}$$
$$i_{D4} = I_S e^{0.683/0.025}$$

But,

$$1 \text{ mA} = I_S e^{0.7/0.025}$$
$$\Rightarrow I_S = e^{-0.7/0.025}, \text{ mA}$$
$$i_{D4} = e^{(-0.7+0.683)/0.025}$$
$$= 0.5 \text{ mA}$$
$$v_O = -0.5 \text{ V}$$

which is $-v_1 v_2$ Q.E.D.

Other combinations can be used to verify the operation of this analog multiplier.

18.53 Refer to the circuit in Fig. 18.34 and let v_O be the voltage across R. Let v_{BE} at $i_E = I$ be denoted V_{BE}. For $v_O = xV_T$, we have

$$I_R = \frac{xV_T}{R} = \frac{xV_T}{2.5\ V_T/I} = \left(\frac{x}{2.5}\right)I$$

Thus,

$$I_{E1} = I + \left(\frac{x}{2.5}\right)I = I\left(1 + \frac{x}{2.5}\right)$$
$$I_{E2} = I - \left(\frac{x}{2.5}\right)I = I\left(1 - \frac{x}{2.5}\right)$$
$$v_{BE1} = V_{BE} + V_T \ln\left(1 + \frac{x}{2.5}\right)$$
$$v_{BE2} = V_{BE} - V_T \ln\left(1 - \frac{x}{2.5}\right)$$
$$v_I = v_{BE1} - v_{BE2} + xV_T$$
$$v_I = V_T \ln\left(\frac{1 + \frac{x}{2.5}}{1 - \frac{x}{2.5}}\right) + xV_T \qquad (1)$$

Equation (1) can be used to determine v_I for each of the given values of $v_O = xV_T$. For comparison purposes, we can compute the ideal value of v_I using

$$v_O = 2.42V_T \sin\left(\frac{v_I}{6.6\ V_T} \times 90^\circ\right)$$

that is,

$$v_I = V_T\left(\frac{6.6}{90}\right) \sin^{-1}\left(\frac{x}{2.42}\right) \qquad (2)$$

The results are given in the following table.

$x = v_O/V_T$	0.25	0.50	1.00	1.50	2.00	2.40	2.42
v_I/V_T	0.451	0.905	1.85	2.89	4.20	6.29	6.52
v_I/V_T **(Ideal)**	0.435	0.874	1.79	2.81	4.09	6.06	6.60

Figure 1 shows a plot of v_O versus v_I using the approximate values (first two rows in the Table above). Note that this graph represents a quarter of a sine-wave cycle.

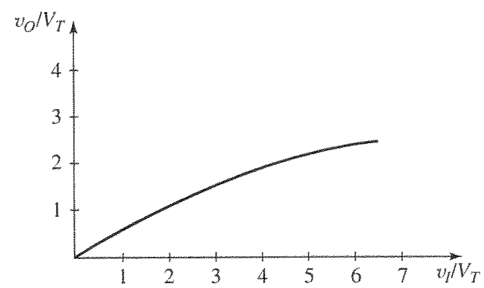


Figure 1

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