

Homework 8

This homework is due on **Monday**, March 17th, 11:59PM.

Instructions: Please upload the homework by 11:59 PM (Pacific Time) on Canvas on the day of the deadline. If you are unable to upload it on Canvas, please hand over the homework to the TA (Xunyu Li) between 2:00 PM and 3:00 PM (Pacific Time) during TA office hours.

Note: This homework will be evaluated for 20 points, but you can get a total of 30 points, giving you 10 points of extra credit on this homework.

Problem 1 [4pts]: Find the Laplace transform of the following signals. Do not forget to state the ROC. Also, for each case determine if the CTFT exists.

- a) $x(t) = u(t)$
- b) $x(t) = \begin{cases} 1 & -1 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$
- c) $x(t) = (1 - e^{-t})u(t)$
- d) $x(t) = t \cos(2t)u(t)$

Problem 2 [6pts]: Find $x(t)$ if $X(s)$ is given as below.

- a) $X(s) = \frac{e^s}{s+1}$ with ROC: $\text{Re}\{s\} > -1$

Hint: Invert $\frac{1}{s+1}$ first and use properties of Laplace transform.

- b) $X(s) = \frac{1}{s^2-1}$ with ROC: $-1 < \text{Re}\{s\} < 1$
- c) $X(s) = \frac{3s+5}{(s-1)^2}$ with ROC: $\text{Re}\{s\} > 1$

Problem 3 [4pts]: Consider the filter

$$H(s) = \frac{s^2 + 2s + 2}{s^2 + 3s + 2}.$$

- a) State the ROC for the filter to be **causal** and **stable**.
- b) Invert $H(s)$ to find the impulse response $h(t)$.
- c) Using the pole-zero plot, argue that this is a **band-stop** filter by comparing $H(j0)$, $H(j1)$, and $H(j\infty)$.
- d) Find the **causal** and **stable** inverse filter $g(t)$ such that $G(s) = \frac{1}{H(s)}$.

Problem 4 [4pts]: Using the pole-zero plot, explain why

$$H(s) = \frac{s}{s + 10}$$

with ROC: $\text{Re}\{s\} > -10$ is a high-pass filter.

Problem 5 [12pts]: Find the impulse response $h(t)$ of the **causal** LTI systems described by the following differential equations:

a)

$$y(t) + \frac{d^2 y(t)}{dt^2} = x(t)$$

b)

$$y(t) - 2\frac{dy(t)}{dt} + \frac{d^2 y(t)}{dt^2} = \frac{dx(t)}{dt}$$

c)

$$25y(t) - 6\frac{dy(t)}{dt} + \frac{d^2 y(t)}{dt^2} = x(t) - \frac{dx(t)}{dt}$$