UNIVERSITY OF CALIFORNIA, RIVERSIDE
Department of Electrical Engineering

WINTER 2025

EE110B-SIGNALS AND SYSTEMS

HOMEWORK 2 SOLUTIONS

Problem 1: Consider an LTI system with the input $x[n] = u[n-3]$ and the impulse response $h[n] = 0.8^n u[n-2]$. Determine and plot the output $y[n]$ of the system.

Solution: We have

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \sum_{k=-\infty}^{\infty} h[k]x[n-k] \\ &= \sum_{k=-\infty}^{\infty} 0.8^k u[k-2]u[n-k-3] \\ &= \sum_{k=2}^{\infty} 0.8^k u[n-k-3] \; . \end{aligned}$$

Now, $u[n-k-3] = 0$ if $k > n-3$. That means if $n < 5$, $u[n-k-3] = 0$ for all k within the limits of the summation. Thus, $y[n] = 0$ for $n < 5$. For $n \geq 5$, we can rewrite the summation as

$$\begin{aligned} y[n] &= \sum_{k=2}^{n-3} 0.8^k \\ &= \sum_{k'=0}^{n-5} 0.8^{k'+2} \\ &= 0.8^2 \sum_{k'=0}^{n-5} 0.8^{k'} \\ &= 0.8^2 \times \frac{1-0.8^{n-4}}{1-0.8} \\ &= 3.2(1-0.8^{n-4}) \; . \end{aligned}$$

To write the result concisely,

$$y[n] = 3.2(1-0.8^{n-4})u[n-5] \; .$$

You can also consult with the figure on the next page to understand the above summations.

Problem 2: Let an LTI system have the impulse response

$$h[n] = 2^{-n}u[n] \; .$$

Determine whether this system is memoryless, causal, stable, and invertible.

Solution:

Memory: Since $h[1] = \frac{1}{2} \neq 0$, the system has memory.

Causality: Since $h[n] = 0$ for all $n < 0$, the system is causal.

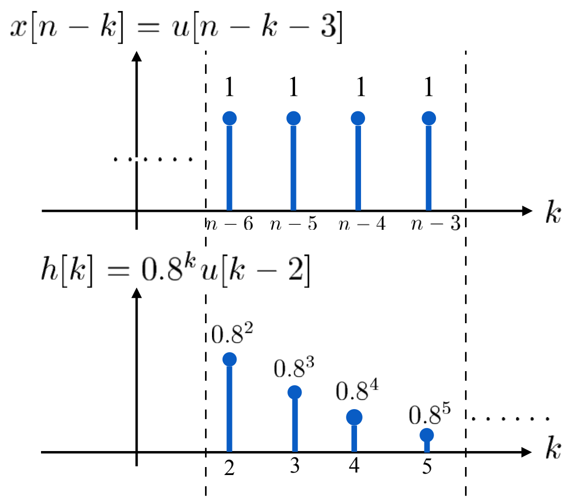

Figure 1: Illustration of $x[n-k]$ and $h[k]$. Note that when $n < 5$, there is no overlap, so the convolution sum yields 0.

Stability: It suffices to check

$$\sum_{k=-\infty}^{\infty} \left|h[k]\right| = \sum_{k=-\infty}^{\infty} 2^{-k} u[k] = \sum_{k=0}^{\infty} 2^{-k} = \frac{1}{1-\frac{1}{2}} = 2 < \infty$$

and hence the system is stable.

Invertibility: The input-output relation is given by

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\ &= \sum_{k=-\infty}^{\infty} x[k]2^{-(n-k)}u[n-k] \\ &= \sum_{k=-\infty}^{n} x[k]2^{-(n-k)} \\ &= 2^{-n} \sum_{k=-\infty}^{n} x[k]2^{k} \end{aligned}$$

To single out $x[n]$ on the right hand side, let us explore a subtraction approach. More specifically, observing that

$$y[n-1] = 2^{-(n-1)} \sum_{k=-\infty}^{n-1} x[k]2^k \ ,$$

it seems that $2y[n] - y[n-1]$ would do the job:

$$\begin{aligned} 2y[n] - y[n-1] &= 2 \cdot 2^{-n} \sum_{k=-\infty}^{n} x[k]2^k - 2^{-(n-1)} \sum_{k=-\infty}^{n-1} x[k]2^k \\ &= 2 \cdot 2^{-n} \left(\sum_{k=-\infty}^{n-1} x[k]2^k + x[n]2^n \right) - 2 \cdot 2^{-n} \sum_{k=-\infty}^{n-1} x[k]2^k \end{aligned}$$

$$
\begin{aligned}
&= 2 \cdot 2^{-n} \cdot x[n]2^{n} \\
&= 2x[n]
\end{aligned}
$$

which means

$$x[n] = y[n] - \frac{1}{2}y[n-1]$$

and we got ourselves an inverse system. Since in this system, $y[n]$ is the input and $x[n]$ is the output, its impulse response is

$$g[n] = \delta[n] - \frac{1}{2}\delta[n-1] \ .$$

Problem 3: Impulse responses of LTI systems can be physically found by inputting $x[n] = \delta[n]$ to the system and measuring the output (the output would be precisely the impulse response). Assume that during the process, we inadvertently input

$$x[n] = \delta[n] + \delta[n-1]$$

instead, and observed the output to be $y[n] = u[n]$. Can we still figure out the impulse response, and if so, what is it?

Solution:

Had we input $x_1[n] = \delta[n]$ to the system, we would have gotten $y_1[n] = h[n]$. Similarly, from time-invariance, $x_2[n] = \delta[n-1]$ would have resulted in $y_2[n] = h[n-1]$. Finally, from linearity, the input

$$x[n] = x_1[n] + x_2[n] = \delta[n] + \delta[n-1]$$

must yield

$$y[n] = y_1[n] + y_2[n] = h[n] + h[n-1] \ .$$

But we are given that $y[n] = u[n]$, so we need to solve

$$h[n] + h[n-1] = u[n]$$

for $h[n]$. But this can only be satisfied by

$$h[n] = \begin{cases} 1 & n \geq 0 \text{ and } n \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$