EE 115 – Homework 3 Solutions

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Problem 1

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(a) The baseband spectrum is

$$M(f) = \frac{1}{2}\,\mathrm{rect}\left(\frac{f}{2}\right),$$

so $M(f) = \frac{1}{2}$ for $|f| < 1$ and 0 elsewhere, as shown in Fig. 1.

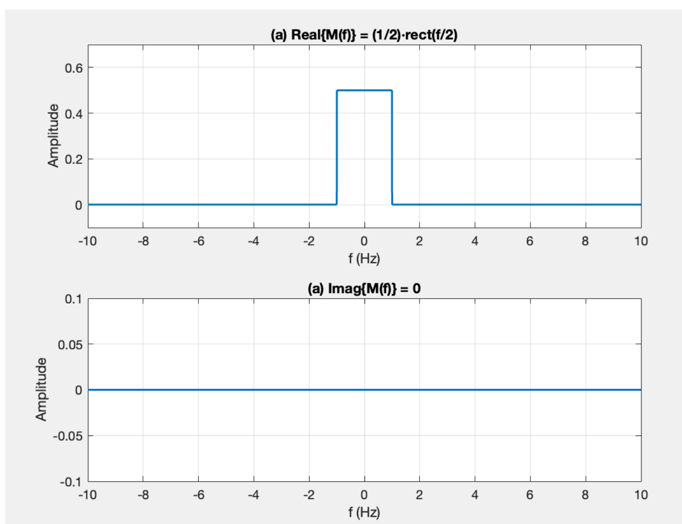

Figure 1: Baseband spectrum of $m(t) = \mathrm{sinc}(2t)$.

(b) Multiplying by $\cos(2\pi 5t)$ shifts the spectrum by ± 5:

$$M_c(f) = \frac{1}{2}\left[M(f-5) + M(f+5)\right],$$

which produces two rectangular lobes centered at $f = \pm 5$, each of width 2 and amplitude $\frac{1}{4}$. The full sketch is shown in Fig. 2.

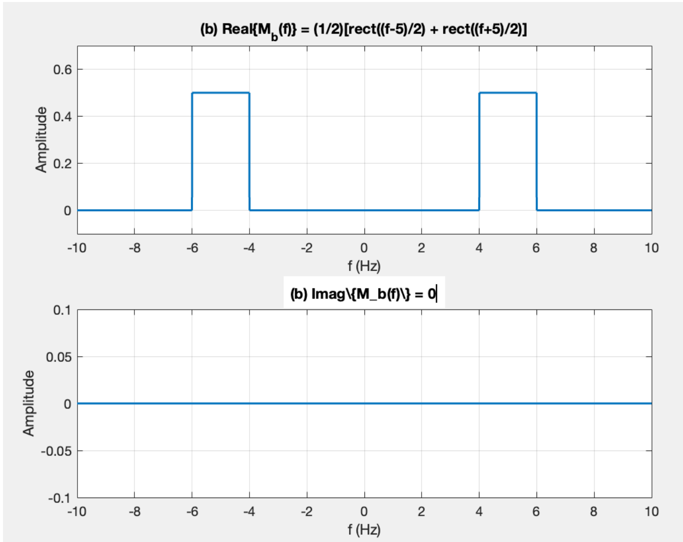

Figure 2: Double-sideband spectrum of $m(t)\cos(2\pi 5t)$.

(c) Modulating by $\sin(2\pi 5t)$ yields

$$M_s(f) = \frac{1}{2j}\left[M(f-5) - M(f+5)\right],$$

so the real part is zero and the imaginary part is

$$\mathrm{Im}\{M_s(f)\} = \frac{1}{2}\left[\mathrm{rect}\left(\frac{f-5}{2}\right) - \mathrm{rect}\left(\frac{f+5}{2}\right)\right],$$

which is odd-symmetric as illustrated in Fig. 3.

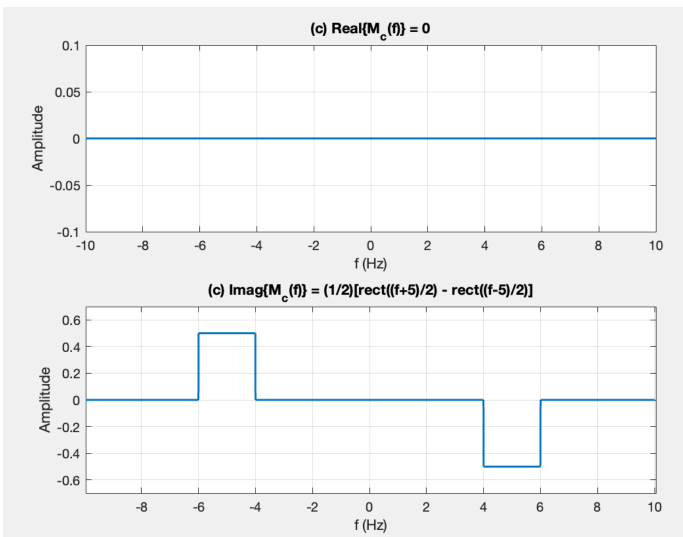

Figure 3: Spectrum of $m(t)\sin(2\pi 5t)$ highlighting the imaginary symmetry.

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Problem 2 (30 points)

Assume $m(t) = 2\,\mathrm{sinc}(2t) = \dfrac{\sin(2\pi t)}{\pi t}$.

(a) Determine its Hilbert transform $\hat{m}(t)$. First find the spectrum:

$$M(f) = 2 \cdot \frac{1}{2}\,\mathrm{rect}\left(\frac{f}{2}\right) = \mathrm{rect}\left(\frac{f}{2}\right),$$

so $M(f) = 1$ for $|f| < 1$ and 0 otherwise. Applying the Hilbert multiplier gives

$$\hat{M}(f) = -j\,\mathrm{sgn}(f)\,M(f) = \begin{cases} -j, & 0 < f < 1, \\ +j, & -1 < f < 0, \\ 0, & |f| \geq 1. \end{cases}$$

The inverse transform is then

$$\begin{aligned} \hat{m}(t) &= \int_{-1}^{1} \hat{M}(f)\, e^{j2\pi f t}\, df \\ &= \int_{-1}^{0} j\, e^{j2\pi f t}\, df + \int_{0}^{1} (-j)\, e^{j2\pi f t}\, df \\ &= \frac{1}{2\pi t}\left[1 - e^{-j2\pi t} + 1 - e^{j2\pi t}\right] \\ &= \frac{1 - \cos(2\pi t)}{\pi t}, \end{aligned}$$

with the $t = 0$ value obtained by continuity as $\hat{m}(0) = 0$.

(b) The sketches of both $M(f)$ and $\hat{M}(f)$ over all frequencies are provided in Fig. 4:

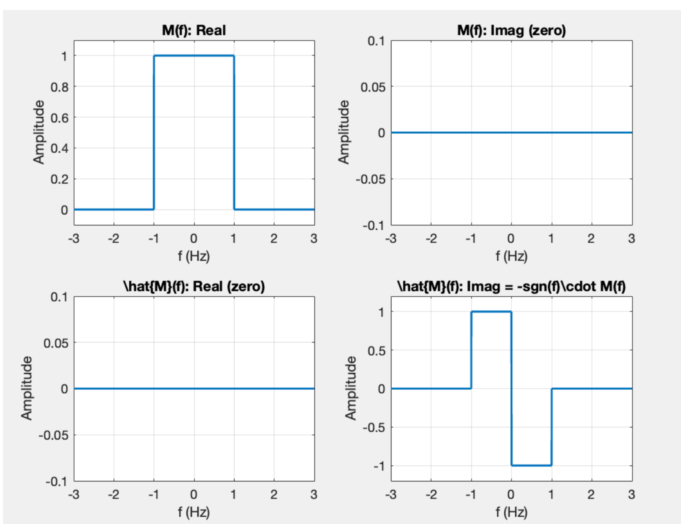

Figure 4: Sketches of $M(f)$ (blue) and $\hat{M}(f)$ (green) for part (b).

(c) The time-domain sketches of $m(t)$ and $\hat{m}(t)$ for all t are shown in Fig. 5, highlighting the even/odd symmetry pair.

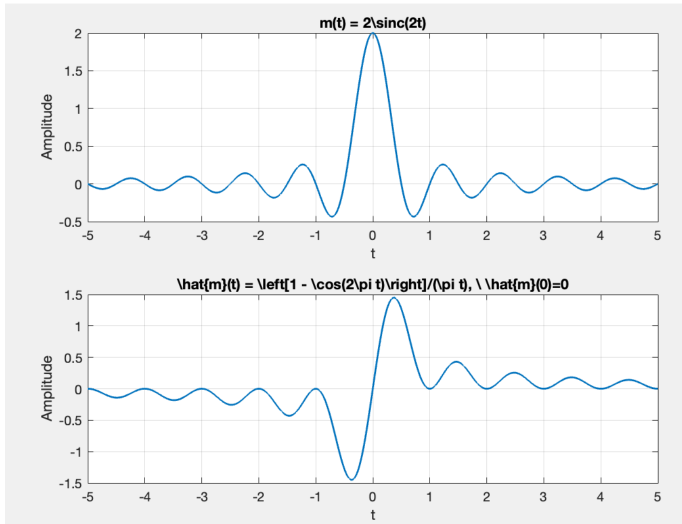

Figure 5: Sketches of $m(t)$ and its Hilbert transform $\hat{m}(t)$ for part (c).

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Problem 3

Given:

$$u(t) = a(t)\cos(2\pi f_c t) - b(t)\sin(2\pi f_c t)$$

Goal: Recover $a(t)$ and $b(t)$ using coherent demodulation.

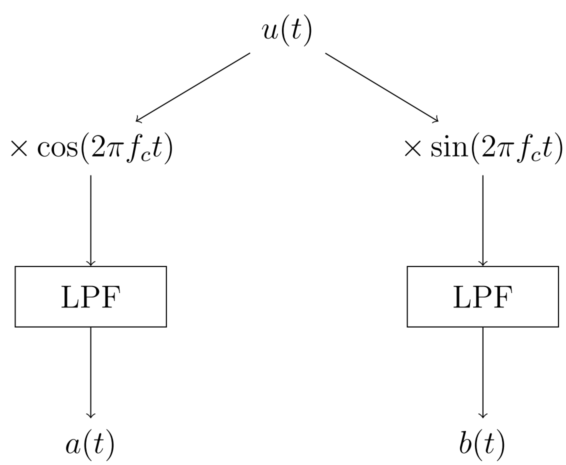

Figure 6: *
Coherent Demodulation Block Diagram