FORMULAS AND CONCEPTS YOU MIGHT FIND USEFUL

- **Quadratic equations:** The roots of the equation $as^2 + bs + c = 0$ are given by
$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
with the understanding that $\sqrt{-1} = j$.
- **Complex numbers:** If z is a complex number, then it can be expressed in one of two forms:
$$z = a + jb \qquad \text{or} \qquad z = re^{j\theta}$$
where
$$e^{j\theta} = \cos(\theta) + j\sin(\theta) \; .$$
Values of r and θ can be found from a and b using
$$\begin{aligned} r &= \sqrt{a^2 + b^2} \\ \theta &= \tan^{-1}\left(\frac{b}{a}\right) \; . \end{aligned}$$
The polar form makes multiplication easy. That is, if $z_1 = r_1 e^{j\theta_1}$ and $z_2 = r_2 e^{j\theta_2}$, then
$$z_1 z_2 = r_1 r_2 e^{j(\theta_1 + \theta_2)} \; .$$
The complex conjugate of z is $z^* = a - jb$ or $z^* = re^{-j\theta}$. We also have the following relationships:
$$\begin{aligned} z + z^* &= 2Re\{z\} &= 2a \\ z - z^* &= 2Im\{z\} &= 2b \\ zz^* &= |z|^2 &= a^2 + b^2 \end{aligned}$$
It is helpful to know the following about the complex number j:
$$\begin{aligned} j^2 &= -1 \\ j^3 &= -j \\ j^4 &= 1 \\ 1/j &= -j \\ je^{j\theta} &= e^{j(\theta + \pi/2)} \end{aligned}$$
where the last equation can be read as "multiplication by j rotates a complex number by 90 degrees counter-clockwise."
- **Sine waves and complex exponentials:**
$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \quad \text{and} \quad \sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$
- **Geometric series:** For any $\alpha \neq 1$ (complex or real),
$$\sum_{n=0}^{N} \alpha^n = \frac{1 - \alpha^{N+1}}{1 - \alpha} \; .$$
If $|\alpha| < 1$, in the limit $N \to \infty$, this becomes $\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$. When $\alpha = 1$, we immediately have
$$\sum_{n=0}^{N} \alpha^n = N + 1 \; .$$
- **System properties:** A system is
  - **linear** if
$$\begin{matrix} x_1[n] \to y_1[n] \\ x_2[n] \to y_2[n] \end{matrix} \Longrightarrow ax_1[n] + bx_2[n] \to ay_1[n] + by_2[n]$$
  - **time-invariant** if
$$x[n] \to y[n] \Longrightarrow x[n - n_0] \to y[n - n_0]$$
  - **memoryless** if $y[n]$ at time n depends only on $x[n]$ on time n.
  - **causal** if $y[n]$ at time n depends only on $x[k]$ on times $k \leq n$.
  - **stable** if $|x[n]| \leq B$ for some M implies $|y[n]| \leq C$ for some C.
  - **invertible** if two distinct $x_1[n]$ and $x_2[n]$ does not result in the same $y[n]$.
- **LTI Systems:** The input-output relationship in an LTI system with an impulse response $h[n]$ is given by the convolution sum
$$y[n] = x[n] \star h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$
- **Convolution for right-sided signals:** If
$$x[n] = f[n]u[n]$$
and
$$h[n] = g[n]u[n] \; ,$$
for some $f[n]$ and $g[n]$, then then $y[n] = x[n] \star h[n]$ simplifies to
$$y[n] = u[n] \cdot \sum_{k=0}^{n} f[k]g[n-k]$$
- **Properties of convolution:**
  - **Commutativity**:
$$x[n] \star y[n] = y[n] \star x[n]$$
  - **Associativity**:
$$x[n] \star (y[n] \star z[n]) = (x[n] \star y[n]) \star z[n]$$
  - **Distribution**:
$$x[n] \star (ay[n] + bz[n]) = a(x[n] \star y[n]) + b(x[n] \star z[n])$$
  - **Time invariance**:
$$x[n] \star h[n] = y[n] \Longrightarrow x[n] \star h[n - n_0] = y[n - n_0]$$
  - **Identity**:
$$x[n] \star \delta[n] = x[n] \; .$$
- **LTI System properties:** An LTI system with impulse response $h[n]$ is
  - **memoryless** if $h[n] = 0$ for all $n \neq 0$.
  - **causal** if $h[n] = 0$ for all $n < 0$.
  - **stable** if
$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$
  - **invertible** if
$$x[n] \star h[n] = 0$$
implies $x[n] = 0$. In other words, there is no nonzero signal whose convolution with $h[n]$ outputs the zero signal.
- **Difference equations:** For a Kth order equation of the form
$$\sum_{k=0}^{K} \alpha_k y[n-k] = x[n]$$
you need to
  - find a *particular* solution $y_p[n]$, i.e., *any* $y[n]$ that satisfies the equation,
  - find a family of *homogeneous* solutions, $y_h[n]$, that satisfy the equation with $x[n] = 0$,
  - write the overall solution family as $y[n] = y_p[n] + y_h[n]$,
  - find the specific member of the family by using initial conditions $y[0], y[1], \ldots, y[K-1]$.

If the system is said to be in *initial rest*, then you need to derive your own initial conditions using the fact that $y[-1] = y[-2] = \ldots = 0$.