EE110B - Signals and Systems
Winter 2025

Lab 4

In a room with concrete walls (or other similar environment), we often notice acoustic echoes. If you do not talk very closely to a microphone (on a cell phone for example) in such an environment, the microphone will pick up the echoes. The echoes are distortion and could cause many problems. Your friend on the other end could be annoyed, or an automatic speech recognizer could misunderstand your command. In this lab, you will evaluate the effect of echoes and test an echo cancellation filter.

Task 1: For the above mentioned system, we can treat the voice from your mouth as input, and the acoustic signal picked up by the microphone as output. The input $x[n]$ and the output $y[n]$ of this system are related to each other as follows:

$$y[n] = h[n] \star x[n] = \sum_{k=0}^{\infty} h[k]x[n-k]$$

where $h[0]$ is the signal attenuation of the direct path between your mouth and the microphone, and $h[k]$ for $k > 0$ can be viewed as the coefficients of the echoes. To illustrate the distortion on the input

$$x[n] = \sin\left(\frac{\pi n}{10}\right)(u[n] - u[n-10]) \ ,$$

choose $h[0] = 1$, $h[4] = 0.5$, $h[7] = 0.2$, and all other $h[k] = 0$, and plot $x[n]$ and $y[n]$ on the same figure.

Task 2: For the same input $x[n]$ as above, assume $h[n] = 0.9^n u[n]$ and

$$g[n] = \delta[n] - 0.9\delta[n-1] \ .$$

Compute and plot both the distorted signal

$$y[n] = h[n] \star x[n]$$

and the output $v[n]$ of the echo cancellation filter

$$v[n] = y[n] \star g[n] \ .$$

Is $g[n]$ a perfect echo cancellation filter?

Task 3: If you replace $h[n]$ in Task 2 with

$$h[n] = 0.9^n u[n] + \delta[n]$$

the perfect echo cancellation filter will be different. The best way to find it is to compute $H(e^{j\omega})$, which is

$$H(e^{j\omega}) = \frac{1}{1-0.9e^{-j\omega}} + 1 = \frac{2-0.9e^{-j\omega}}{1-0.9e^{-j\omega}}$$

and invert it

$$G(e^{j\omega}) = \frac{1}{H(e^{j\omega})} = \frac{1-0.9e^{-j\omega}}{2-0.9e^{-j\omega}} = 1 - \frac{1}{2-0.9e^{-j\omega}} = 1 - \frac{1}{2} \cdot \frac{1}{1-0.45e^{-j\omega}} \ .$$

Find $g[n]$ whose DTFT is $G(e^{j\omega})$. Compute and plot $x[n]$, $y[n] = x[n] \star h[n]$, and $v[n] = y[n] \star g[n]$.