UNIVERSITY OF CALIFORNIA, RIVERSIDE
Department of Electrical Engineering

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EE110B-SIGNALS AND SYSTEMS
HOMEWORK 4 SOLUTIONS

Solution:
a) We have

$$
\begin{aligned}
a_k &= \frac{1}{10}\sum_{n=0}^{9} x[n]e^{-j\frac{2\pi}{10}kn} \\
&= \frac{1}{10}\sum_{n=0}^{2} e^{-j\frac{2\pi}{10}kn} \\
&= \frac{1}{10}\left[e^{-j\frac{2\pi}{10}k0} + e^{-j\frac{2\pi}{10}k1} + e^{-j\frac{2\pi}{10}k2}\right] \\
&= \frac{1}{10}\left[1 + e^{-j\frac{k\pi}{5}} + e^{-j\frac{2k\pi}{5}}\right] .
\end{aligned}
$$

b) There are two ways to approach this. The first one is to observe that

$$y[n] = x[n] + x[n-1] + x[n-2]$$

and therefore using the shifting property of DTFS,

$$
\begin{aligned}
b_k &= a_k + a_k e^{-j\frac{k\pi}{5}} + a_k e^{-j\frac{2k\pi}{5}} \\
&= a_k\left[1 + e^{-j\frac{k\pi}{5}} + e^{-j\frac{2k\pi}{5}}\right] \\
&= a_k \cdot 10a_k \\
&= 10a_k^2 .
\end{aligned}
$$

We could also arrive at the same conclusion by observing that

$$y[n] = x[n] \tilde{*} x[n]$$

where $\tilde{*}$ is the periodic convolution operation. Therefore, it directly follows from the convolution property of DTFS that

$$b_k = 10a_k^2 .$$

c) Similar to part b, the trick is to observe that

$$z[n] = x[n] + x[-n]$$

and therefore using the time reversal property of the DTFS, we obtain

$$c_k = a_k + a_{-k}$$

where $a_{-k} = a_{10-k}$.

Problem 2: Noting that the period is 10 and using the formula, we have

$$a_k = \frac{1}{10}\sum_{n=0}^{9} x[n]e^{-j\frac{2\pi}{10}kn} .$$

However, since $x[0] = 1$ and $x[n] = 0$ for all $1 \leq n \leq 9$, the above summation simplifies drastically to

$$a_k = \frac{1}{10} x[0] e^{-j\frac{2\pi}{10}k \cdot 0} = \frac{1}{10}$$

for all $k = 0, 1, \ldots, 9$.