EE 115 Lab 6

For a phase locked loop (PLL) with the loop gain K and the loop filter $G(s) = 1 + a/s$, the transfer function from the input phase $\theta_i(t)$ to the output phase $\theta_o(t)$ is known to be

$$H(s) = \frac{K(s+a)}{s^2 + Ks + Ka} \tag{1}$$

and the transfer function from the input phase $\theta_i(t)$ to the phase error $\theta_e(t) = \theta_i(t) - \theta_o(t)$ is

$$H_e(s) = \frac{s^2}{s^2 + Ks + Ka}. \tag{2}$$

If $\frac{d\theta_i(t)}{dt} = 2\pi k_f u(t)$ (i.e., a step frequency shift), then it is known that

$$\Theta_e(s) = H_e(s)\Theta_i(s) = \frac{2\pi k_f}{s^2 + Ks + Ka}. \tag{3}$$

The poles of $\Theta_e(s)$ are

$$p_1 = \frac{1}{2}(-K + \sqrt{K^2 - 4Ka}) = \frac{1}{2}(-K + j\sqrt{4Ka - K^2}) \tag{4}$$

$$p_2 = \frac{1}{2}(-K - \sqrt{K^2 - 4Ka}) = \frac{1}{2}(-K - j\sqrt{4Ka - K^2}) \tag{5}$$

where we assume $4Ka - K^2 > 0$. Then we know

$$\theta_e(t) = L^{-1}[\Theta_e(s)] = 2\pi k_f L^{-1}[\frac{c_1}{s - p_1} + \frac{c_2}{s - p_2}] = 2\pi k_f [c_1 e^{p_1 t} u(t) + c_2 e^{p_2 t} u(t)] \tag{6}$$

where $c_1 = \frac{1}{p_1 - p_2} = \frac{1}{j\sqrt{4Ka - K^2}}$ and $c_2 = \frac{1}{p_2 - p_1} = -\frac{1}{j\sqrt{4Ka - K^2}}$. We can further write

$$\theta_e(t) = \frac{4\pi k_f}{\sqrt{4Ka - K^2}} e^{-\frac{1}{2}Kt} \sin([\sqrt{4Ka - K^2}]t)u(t). \tag{7}$$

Subject to $4Ka - K^2 > 0$, compute and plot the phase error function $\theta_e(t)$ for various choices of K and a, and discuss the effects of K and a on important features of $\theta_e(t)$ (including its peak value, its speed to converge to zero, and its oscillation frequency).