EE 115 Lab 3

In this lab exercise, we consider the role of lowpass filtering in demodulation of a DSB-SC signal. Assume that a DSB-SC signal has the following form:

$$u(t) = m(t)\cos(2\pi f_c t) \tag{1}$$

where t is in millisecond, $m(t) = \sin(\pi t)$ for $0 \leq t \leq 1$, and $m(t) = 0$ otherwise. The demodulator consists of a mixer and a lowpass filter (LPF). The mixer yields

$$v(t) = 2u(t)\cos(2\pi f_c t), \tag{2}$$

and the LPF yields

$$x(t) = v(t) * h(t) = \int_0^\infty h(\tau)v(t-\tau)d\tau \tag{3}$$

where $h(t)$ is the impulse response of the LPF.

To simulate the demodulator, we will sample all signals at the sampling rate f_s or equivalently at the sampling interval $T_s = \frac{1}{f_s}$ in millisecond. Consequently, we have $m[n] = m(T_s n)$, $u[n] = u(T_s n)$, $v[n] = v(T_s n)$, and

$$x[n] = v[n] * \tilde{h}[n] = \sum_{l=0}^{L} \tilde{h}[l]v[n-l] \tag{4}$$

where $\tilde{h}[n]$ for $n = 0, 1, \cdots, L$ is the impulse response of the discrete-time equivalent of the LPF. Here n is an integer variable.

Assume $f_c = 10$kHz and $f_s = 50$kHz.

1) Plot and discuss $m[n]$, $u[n]$ and $v[n]$ for $n = 0, 1, \cdots, 63$. What is the time scale for each increment of n?
2) A discrete Fourier transform (DFT) of $m[n]$ is $M[k] = \sum_{n=0}^{63} m[n]e^{-j2\pi\frac{kn}{64}}$ which is a periodic function of the integer variable k, i.e., $M[k] = M[k+64]$ for all integer values of k. It is known that $M[k]$ for $|k| \leq 32$ is proportional to the (continuous-time) Fourier transform $M(f)$ of $m(t)$ at $f = \frac{k}{64}f_s$. The DFTs of $u[n]$ and $v[n]$ are similarly defined, and denoted by $U[k]$ and $V[k]$ respectively. Compute and plot the amplitude spectra $|M[k]|$, $|U[k]$ and $|V[k]|$ for $-32 \leq k \leq 32$ and discuss their bandwidths.
3) Choose a frequency response $H(f) = \mathtt{rect}(f/W)$ for the LPF with a proper choice of W in kHz. Then a proper causal impulse response of the LPF is $h(t) = W \times \mathtt{sinc}(Wt -$

$4) \times [0.5 + 0.5\cos(\pi \frac{W}{4}(t - \frac{4}{W}))]$ for $0 \le t \le \frac{8}{W}$, and $h(t) = 0$ otherwise. The corresponding discrete-time impulse response of the LPF is

$$\tilde{h}[n] = \frac{1}{W} h(nT_s) \tag{5}$$

for $n = 0, 1, \cdots, L$ with $L = \lceil \frac{8}{T_s W} \rceil$. Compute the following (discrete-time convolution)

$$x[n] = \sum_{l=0}^{L} \tilde{h}[l] v[n-l] \tag{6}$$

for $0 \le n \le L$, and compare it with $m[n]$.

4) Choose other proper values of W and repeat the above.