PMOS Bias Problem

Given: $V_T = -1$ V, $K_p = 8\ \mu\text{A/V}^2$, $W/L = 25$, $\lambda = 0$, current $I = 100\ \mu$A, supply $V_S = 10$ V.
Thus the device transconductance parameter

$$K \equiv K_p \frac{W}{L} = 8 \times 25 = 200\ \mu\text{A/V}^2.$$

Let the bottom node (also the gate) be V_X. Then

$$V_{SG} = V_S - V_G = 10 - V_X, \qquad V_D = V_X + IR, \qquad V_{SD} = V_S - V_D = 10 - (V_X + IR) = V_{SG} - IR.$$

Region condition

For a PMOS (with $|V_T| = 1$ V):

$$\text{saturation: } V_{SD} \geq V_{SG} - |V_T| \quad \Longleftrightarrow \quad IR \leq 1\ \text{V}.$$

Hence the boundary is $R = 10$ kΩ.

- $R < 10$ kΩ: saturation.
- $R > 10$ kΩ: triode.

Device equations (with $\lambda = 0$)

$$\text{saturation: } I = \tfrac{1}{2}K\,(V_{SG} - |V_T|)^2 = \tfrac{1}{2}K\,(V_{SG} - 1)^2,$$
$$\text{triode: } I = K\left[(V_{SG} - |V_T|)V_{SD} - \tfrac{1}{2}V_{SD}^2\right] = K\left[(V_{SG} - 1)V_{SD} - \tfrac{1}{2}V_{SD}^2\right].$$

Solutions

(a) $R = 0$ **(saturation)**

$$100\ \mu\text{A} = \tfrac{1}{2}(200)\,(V_{SG} - 1)^2\ \mu\text{A} \;\Rightarrow\; (V_{SG} - 1)^2 = 1 \Rightarrow V_{SG} = 2.00\ \text{V}.$$
$$V_{SD} = V_{SG} - IR = 2.00 - 0 = 2.00\ \text{V}.$$

(b) $R = 20$ **kΩ (triode)** Here $IR = 2$ V and $V_{SD} = V_{SG} - 2$.

$$\frac{I}{K} = (V_{SG} - 1)(V_{SG} - 2) - \frac{1}{2}(V_{SG} - 2)^2 \;\Rightarrow\; 0.5 = \tfrac{1}{2}V_{SG}^2 - V_{SG}.$$

Hence $V_{SG} = 2.414$ V and $V_{SD} = 0.414$ V.

(c) $R = 50$ **kΩ (triode)** $IR = 5$ V, $V_{SD} = V_{SG} - 5$. Solving

$$0.5 = \tfrac{1}{2}V_{SG}^2 - V_{SG} + 5 - \tfrac{1}{2}(5)^2$$

gives $V_{SG} = 5.123$ V and $V_{SD} = 0.123$ V.

(d) $R = 100$ **kΩ (triode)** $IR = 10$ V, $V_{SD} = V_{SG} - 10$. Solving

$$0.5 = \tfrac{1}{2}V_{SG}^2 - V_{SG} + 10 - \tfrac{1}{2}(10)^2$$

gives $V_{SG} = 10.055$ V and $V_{SD} = 0.055$ V.

Compact closed form in triode

With $V_R \equiv IR$ (> 1 V):

$$V_{SG} = 1 + \sqrt{(V_R - 1)^2 + 1}, \qquad V_{SD} = V_{SG} - V_R.$$

Numerical summary

R (kΩ)	V_{SG} (V)	V_{SD} (V)
0	2.000	2.000
20	2.414	0.414
50	5.123	0.123
100	10.055	0.055