EE 110A Signals and Systems

LTI Systems Defined by Differential Equations

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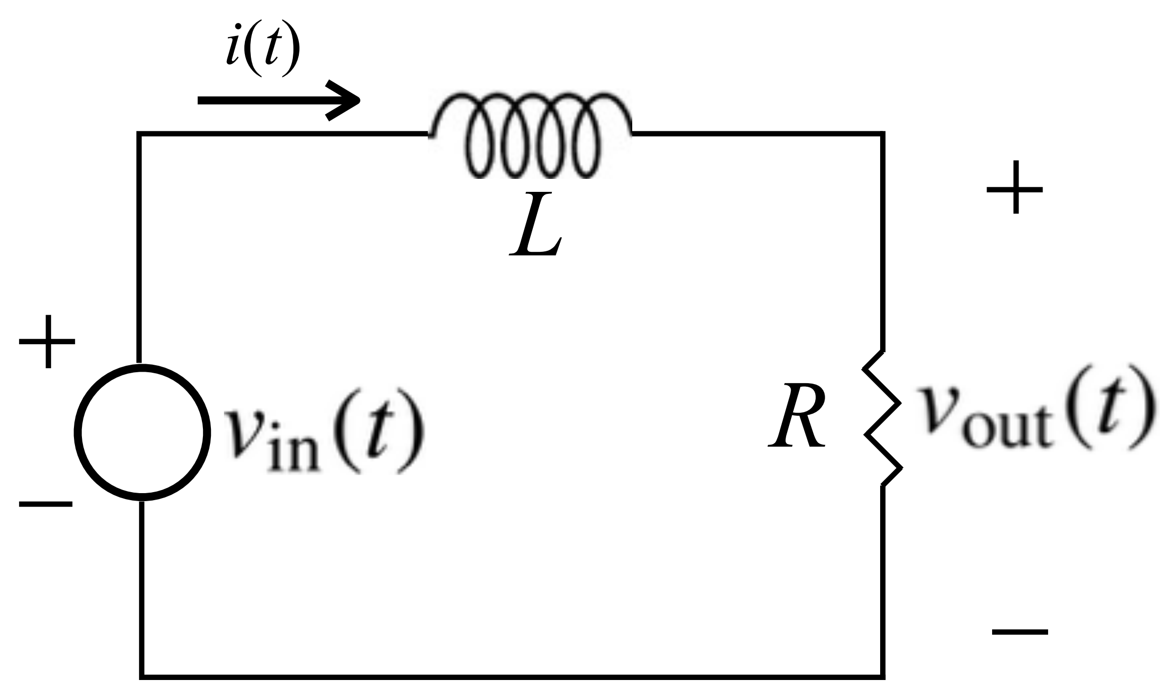
Differential equations

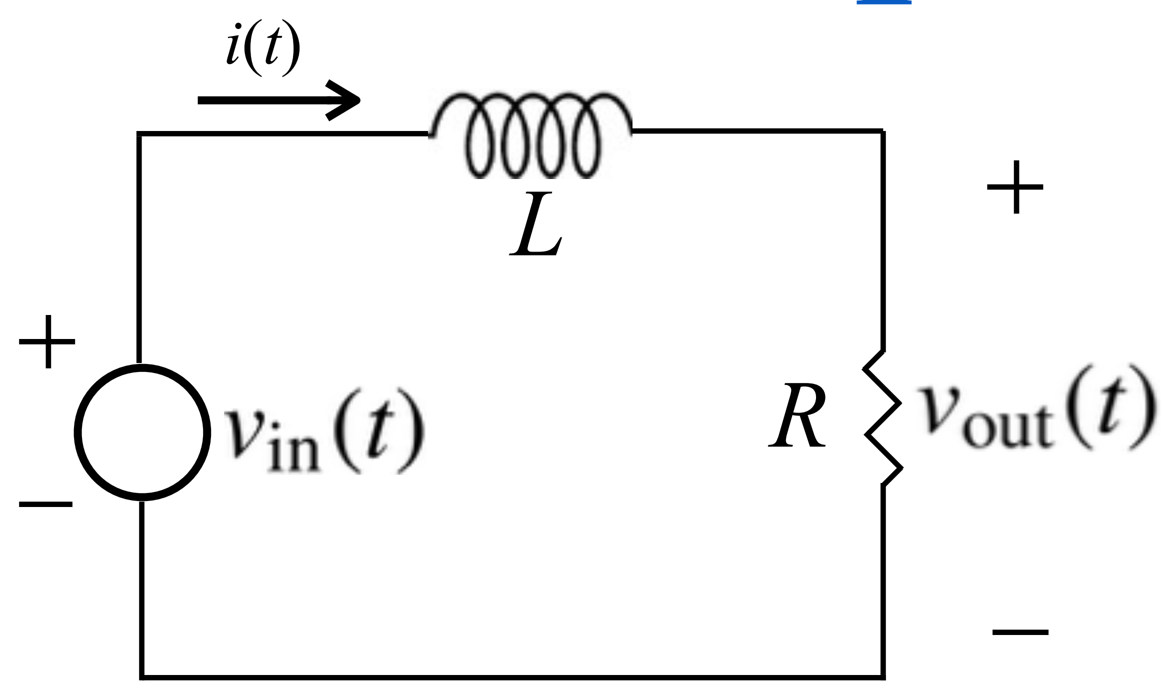
- The input/output relation of an LTI system can sometimes be expressed as a constant-coefficient **differential equation**.
- Example:



- Kirchhoff's voltage law:

$$v_{\text{out}}(t) + L\,\frac{di(t)}{dt} = v_{\text{in}}(t)$$

Differential equations



$$v_{\text{out}}(t) + L\frac{di(t)}{dt} = v_{\text{in}}(t)$$

- Since $i(t) = \dfrac{v_{\text{out}}(t)}{R}$, this is the same as

$$v_{\text{out}}(t) + \frac{L}{R}\frac{dv_{\text{out}}(t)}{dt} = v_{\text{in}}(t)$$

How to solve

- Without the initial conditions, there will be an infinite family of solutions. Initial conditions are used to zero in on the unique solution.
- To find that infinite family of solutions, start with just "a" solution, called the **particular solution**.
- For the circuit example

$$v_{\text{out}}(t) + \frac{L}{R}\frac{dv_{\text{out}}(t)}{dt} = v_{\text{in}}(t)$$

let $v_{\text{in}}(t) = e^{-t}$ for $t \geq 0$.

How to solve

- For the circuit example

$$v_{\text{out}}(t) + \frac{L}{R}\frac{dv_{\text{out}}(t)}{dt} = v_{\text{in}}(t)$$

let $v_{\text{in}}(t) = e^{-t}$ for $t \geq 0$.

- Try $v_{\text{out}}(t) = Ke^{-t}$ as the particular solution:

$$K\cancel{e^{-t}} - \frac{L}{R}K\cancel{e^{-t}} = \cancel{e^{-t}}$$

- In other words,

$$K = \frac{1}{1 - \frac{L}{R}}$$

How to solve

- This yields the particular solution

$$v_{\text{out,p}}(t) = \frac{1}{1 - \frac{L}{R}} e^{-t}$$

- Add onto this the **homogeneous solution,** i.e., the solution when the input is zero:

$$v_{\text{out}}(t) + \frac{L}{R} \frac{dv_{\text{out}}(t)}{dt} = 0$$

- Now try $v_{\text{out}}(t) = ce^{\alpha t}$:

$$\cancel{c}\cancel{e^{\alpha t}} + \frac{L}{R} \cancel{c} \alpha \cancel{e^{\alpha t}} = 0 \qquad \alpha = -\frac{R}{L}$$

How to solve

- This yields the particular solution

$$v_{\mathrm{out,p}}(t) = \frac{1}{1 - \frac{L}{R}} e^{-t}$$

- This yields the homogeneous solution

$$v_{\mathrm{out,h}}(t) = ce^{-\frac{R}{L}t}$$

- Can there be any other homogeneous solution than $ce^{\alpha t}$?
- No, because exponential functions are the only ones whose derivatives are just scaled versions of themselves.

How to solve

- Claim: $v_{\text{out,p}}(t) + v_{\text{out,h}}(t)$ is always a solution to the original differential equation

$$v_{\text{out}}(t) + \frac{L}{R}\frac{dv_{\text{out}}(t)}{dt} = v_{\text{in}}(t)$$

- That is because

$$v_{\text{out,p}}(t) + \frac{L}{R}\frac{dv_{\text{out,p}}(t)}{dt} = v_{\text{in}}(t)$$

$$v_{\text{out,h}}(t) + \frac{L}{R}\frac{dv_{\text{out,h}}(t)}{dt} = 0$$

+

$$[v_{\text{out,p}}(t) + v_{\text{out,h}}(t)] + \frac{L}{R}\frac{d[v_{\text{out,p}}(t) + v_{\text{out,h}}(t)]}{dt} = v_{\text{in}}(t)$$

How to solve

- <u>Claim</u>: $v_{\mathrm{out,p}}(t) + v_{\mathrm{out,h}}(t)$ is always a solution to the original differential equation

$$v_{\mathrm{out}}(t) + \frac{L}{R}\frac{dv_{\mathrm{out}}(t)}{dt} = v_{\mathrm{in}}(t)$$

- Are *all* solutions of the form $v_{\mathrm{out,p}}(t) + v_{\mathrm{out,h}}(t)$?
- Yes, because if $v_{\mathrm{out}}(t)$ is a solution, then $v_{\mathrm{out}}(t) - v_{\mathrm{out,p}}(t)$ must be a *homogeneous* solution

How to solve

- Claim: $v_{\text{out,p}}(t) + v_{\text{out,h}}(t)$ is always a solution to the original differential equation

$$v_{\text{out}}(t) + \frac{L}{R}\frac{dv_{\text{out}}(t)}{dt} = v_{\text{in}}(t)$$

- Are *all* solutions of the form $v_{\text{out,p}}(t) + v_{\text{out,h}}(t)$?

$$v_{\text{out}}(t) + \frac{L}{R}\frac{dv_{\text{out}}(t)}{dt} = v_{\text{in}}(t)$$

$$v_{\text{out,p}}(t) + \frac{L}{R}\frac{dv_{\text{out,p}}(t)}{dt} = v_{\text{in}}(t)$$

-

$$[v_{\text{out}}(t) - v_{\text{out,p}}(t)] + \frac{L}{R}\frac{d[v_{\text{out}}(t) - v_{\text{out,p}}(t)]}{dt} = 0$$

Back to the Solution

- Writing the family of solutions as

$$v_{\mathrm{out,p}}(t) + v_{\mathrm{out,h}}(t)$$

  we obtain

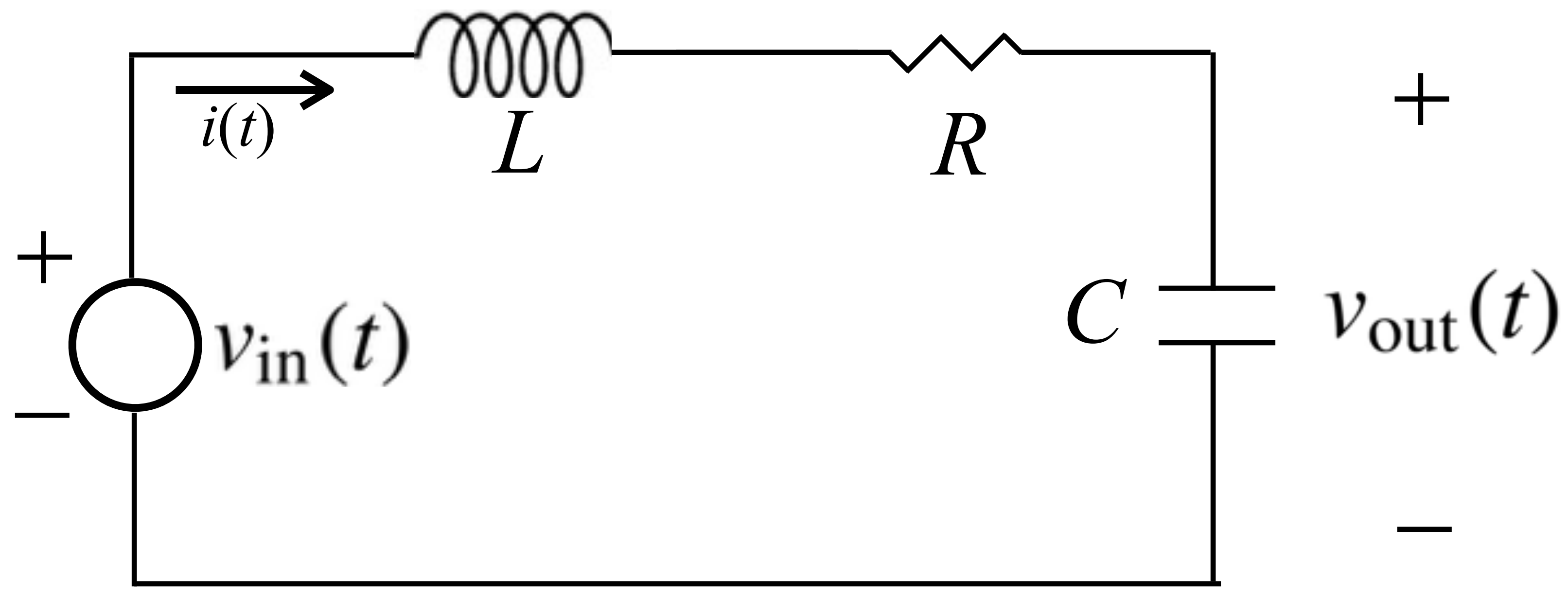
$$v_{\mathrm{out}}(t) = \frac{1}{1 - \frac{L}{R}} e^{-t} + c e^{-\frac{R}{L}t}$$

- We need an **initial condition** to find c.
- For example, if we somehow know $v_{\mathrm{out}}(0)$, we can substitute $t = 0$ to obtain

$$v_{\mathrm{out}}(0) = \frac{1}{1 - \frac{L}{R}} + c$$

  and solve for c.

Another example



- Kirchhoff's voltage law:

$$v_{out}(t) + Ri(t) + L\frac{di(t)}{dt} = v_{in}(t)$$

- But since $i(t) = C\dfrac{dv_{out}(t)}{dt}$, this boils down to

$$v_{out}(t) + RC\frac{dv_{out}(t)}{dt} + LC\frac{d^2 v_{out}(t)}{dt^2} = v_{in}(t)$$

Another example

$$v_{\text{out}}(t) + RC\,\frac{dv_{\text{out}}(t)}{dt} + LC\,\frac{d^2v_{\text{out}}(t)}{dt^2} = v_{\text{in}}(t)$$

- <u>Example</u>: Find $v_{\text{out}}(t)$ if $v_{\text{in}}(t) = e^{-6t}$ for $t \geq 0$ with the initial conditions

$$v_{\text{out}}(0) = 1$$

$$\frac{dv_{\text{out}}}{dt}(0) = 2$$

together with $R = 3$, $L = 1$, and $C = 0.5$.

$$v_{\text{out}}(t) + 1.5\,\frac{dv_{\text{out}}(t)}{dt} + 0.5\,\frac{d^2v_{\text{out}}(t)}{dt^2} = e^{-6t}$$

Another example

$$v_{\text{out}}(t) + 1.5\,\frac{dv_{\text{out}}(t)}{dt} + 0.5\,\frac{d^2 v_{\text{out}}(t)}{dt^2} = e^{-6t}$$

- Particular solution: Try $v_{\text{out,p}}(t) = Ke^{-6t}$:

$$K\cancel{e^{-6t}} - 9K\cancel{e^{-6t}} + 18K\cancel{e^{-6t}} = \cancel{e^{-6t}}$$

- Therefore, $K = 0.1$.

- Homogeneous solution: It has to be of the form $v_{\text{out,h}}(t) = e^{\alpha t}$:

$$\cancel{e^{\alpha t}} + 1.5\alpha\cancel{e^{\alpha t}} + 0.5\alpha^2\cancel{e^{\alpha t}} = 0$$

- The roots are $\alpha = -1$ and $\alpha = -2$

Another example

$$v_{\text{out}}(t) + 1.5\,\frac{dv_{\text{out}}(t)}{dt} + 0.5\,\frac{d^2v_{\text{out}}(t)}{dt^2} = e^{-6t}$$

- Homogeneous solution: It has to be of the form $v_{\text{out,h}}(t) = e^{\alpha t}$:

$$\cancel{e^{\alpha t}} + 1.5\alpha\cancel{e^{\alpha t}} + 0.5\alpha^2\cancel{e^{\alpha t}} = 0$$

- The roots are $\alpha = -1$ and $\alpha = -2$
- Therefore, both e^{-t} and e^{-2t} qualify. Which one shall we pick?
- The most general homogeneous solution is the linear combination $c_1e^{-t} + c_2e^{-2t}$

Another example

$$v_{\text{out}}(t) + 1.5\,\frac{dv_{\text{out}}(t)}{dt} + 0.5\,\frac{d^2 v_{\text{out}}(t)}{dt^2} = e^{-6t}$$

- To recap, all solutions have to be of the form

$$v_{\text{out}}(t) = 0.1e^{-6t} + c_1 e^{-t} + c_2 e^{-2t}$$

- We determine the values of the constants using

$$v_{\text{out}}(0) = 1 = 0.1 + c_1 + c_2$$
$$\frac{dv_{\text{out}}}{dt}(0) = 2 = -0.6 - c_1 - 2c_2$$

- Two equations, two unknowns. Solve to find

$$c_1 = 4.4, c_2 = -3.5$$

Causal LTI systems

- If the system is known to be causal, we do not need explicit initial conditions.
- It would be in **initial rest**, i.e., $y(t) = 0$ for all t before the input comes, say at t=0.
- That means all the derivatives would be zero as well for $t < 0$.
- We could use this fact to figure out $y(t)$ and a few of its derivatives right after the input comes (as many as we need to solve the differential equation).

Causal LTI systems

- Example: Consider the causal LTI system whose input-output relation is given by

$$y(t) - \frac{dy(t)}{dt} - 2\frac{d^2y(t)}{dt^2} = x(t)$$

Find $y(t)$ if $x(t) = u(t)$.

- Solution: $y(t)$ cannot have a discontinuity at $t = 0$, because otherwise its derivative would have an impulse in it. There is nothing on the RHS to account for it.
- Similarly with $\frac{dy}{dt}$ at $t = 0$

Causal LTI systems

- Example: Consider the causal LTI system whose input-output relation is given by

$$y(t) - \frac{dy(t)}{dt} - 2\frac{d^2y(t)}{dt^2} = x(t)$$

  Find $y(t)$ if $x(t) = u(t)$.

- Since the system is in initial rest, these imply that $y(0^+) = 0$ and $\frac{dy}{dt}(0^+) = 0$.

- Focusing on $t > 0$, we can now solve

$$y(t) - \frac{dy(t)}{dt} - 2\frac{d^2y(t)}{dt^2} = 1$$

Causal LTI systems

$$y(t) - \frac{dy(t)}{dt} - 2\frac{d^2y(t)}{dt^2} = 1$$

- Particular solution: Try $y_p(t) = K$:

$$K - 0 - 2 \cdot 0 = 1$$

$$y_p(t) = 1$$

- Homogeneous solution: Try $y_h(t) = e^{\alpha t}$:

$$\cancel{e^{\alpha t}} - \alpha \cancel{e^{\alpha t}} - 2\alpha^2 \cancel{e^{\alpha t}} = 0$$

- Solve $1 - \alpha - 2\alpha^2 = 0$ to find the roots

$$\alpha_1 = -1 \qquad \alpha_2 = 0.5$$

Causal LTI systems

$$y(t) - \frac{dy(t)}{dt} - 2\frac{d^2y(t)}{dt^2} = 1$$

- Complete solution: $y(t) = c_1 e^{-t} + c_2 e^{0.5t} + 1$
- Substitute the derived initial conditions:

$$\left.\begin{aligned} y(0^+) &= 0 = c_1 + c_2 + 1 \\ \frac{dy}{dt}(0^+) &= 0 = -c_1 + 0.5c_2 \end{aligned}\right\} \begin{aligned} c_1 &= \frac{-1}{3} \\ c_2 &= \frac{-2}{3} \end{aligned}$$

- Do not forget that all this is good only for $t > 0$

$$y(t) = \frac{1}{3}\left[-e^{-t} - 2e^{0.5t} + 3\right] u(t)$$

Causal LTI systems

- Example: Consider the causal LTI system whose input-output relation is given by

$$25y(t) - 6\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2} = x(t) - \frac{dx(t)}{dt}$$

Find $y(t)$ if $x(t) = e^{-t}u(t)$.

- Solution: Start by simplifying the RHS:

$$\begin{aligned} x(t) - \frac{dx}{dt} &= e^{-t}u(t) - \left[-e^{-t}u(t) + e^{-t}\delta(t)\right] \\ &= 2e^{-t}u(t) - e^{-t}\delta(t) \\ &= 2e^{-t}u(t) - \delta(t) \end{aligned}$$

Causal LTI systems

$$25y(t) - 6\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2} = 2e^{-t}u(t) - \delta(t)$$

- Can $y(t)$ have a discontinuity at $t = 0$?
- No, because then its derivative would have an impulse, and hence its second derivative would have the derivative of an impulse (a.k.a. doublet)
- Nothing to account for it on the RHS, thus

$$y(0^+) = 0$$

- How about $\dfrac{dy(t)}{dt}$?

Causal LTI systems

$$25y(t) - 6\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2} = 2e^{-t}u(t) - \delta(t)$$

$$y(0^+) = 0$$

- How about $\dfrac{dy(t)}{dt}$?
- It might have a discontinuity since the impulse created by its derivative is accounted for.
- To figure out $\dfrac{dy}{dt}(0^+)$, integrate from 0^- to 0^+ :

$$\int_{0^-}^{0+}\left[25y(t) - 6\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2}\right]dt = \int_{0^-}^{0+}\left[2e^{-t}u(t) - \delta(t)\right]dt$$

Causal LTI systems

$$25y(t) - 6\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2} = 2e^{-t}u(t) - \delta(t)$$

$$y(0^+) = 0$$

$$\int_{0^-}^{0+} \left[\cancel{25y(t)} - 6\,\cancel{\frac{dy(t)}{dt}} + \frac{d^2y(t)}{dt^2}\right] dt = \int_{0^-}^{0+} \left[\cancel{2e^{-t}u(t)} - \delta(t)\right] dt$$

- For all the terms that do NOT contain an impulse, the integral will be zero (why?)
- We therefore have

$$\frac{dy}{dt}(0^+) - \cancel{\frac{dy}{dt}(0^-)} = -1$$

Causal LTI systems

$$25y(t) - 6\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2} = 2e^{-t}u(t) - \delta(t)$$

$$y(0^+) = 0$$

$$\frac{dy}{dt}(0^+) = -1$$

- Once we figure out these initial conditions, it suffices to solve the differential equation for only $t > 0$, which becomes

$$25y(t) - 6\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2} = 2e^{-t}$$

Causal LTI systems

$$25y(t) - 6\,\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2} = 2e^{-t}$$

$$y(0^+) = 0$$

$$\frac{dy}{dt}(0^+) = -1$$

- Particular solution: Try $y_p(t) = Ke^{-t}$:

$$25Ke^{-t} + 6Ke^{-t} + Ke^{-t} = 2e^{-t}$$

$$y_p(t) = \frac{1}{16}\,e^{-t}$$

Causal LTI systems

$$25y(t) - 6\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2} = 2e^{-t}$$

$$y(0^+) = 0$$

$$\frac{dy}{dt}(0^+) = -1$$

$$y_p(t) = \frac{1}{16}e^{-t}$$

- <u>Homogeneous solution</u>: Try $y_h(t) = e^{\alpha t}$:

$$25\cancel{e^{\alpha t}} - 6\alpha \cancel{e^{\alpha t}} + \alpha^2 \cancel{e^{\alpha t}} = 0$$

- The roots are $\alpha_1 = 3 + 4j$ and $\alpha_2 = 3 - 4j$

Causal LTI systems

$$25y(t) - 6\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2} = 2e^{-t}$$

$$y(0^+) = 0 \qquad \frac{dy}{dt}(0^+) = -1$$

$$y_p(t) = \frac{1}{16}e^{-t} \quad y_h(t) = c_1 e^{(3+4j)t} + c_2 e^{(3-4j)t}$$

- Substitute the initial conditions:

$$y(0^+) = y_p(0^+) + y_h(0^+) = \frac{1}{16} + c_1 + c_2 = 0$$

$$\frac{dy}{dt}(0^+) = \frac{dy_p}{dt}(0^+) + \frac{dy_h}{dt}(0^+) = \frac{-1}{16} + c_1(3+4j) + c_2(3-4j) = -1$$

- You can find $c_1 = \dfrac{-(1-3j)}{32}$ and $c_2 = \dfrac{-(1+3j)}{32}$

Causal LTI systems

- Recalling that all of this was for $t > 0$,

$$y(t) = \left[\frac{1}{16}e^{-t} - \frac{1-3j}{32}e^{(3+4j)t} - \frac{1+3j}{32}e^{(3-4j)t}\right]u(t)$$

- But how can the solution be complex?
- In fact, it is not.
- If you write $\dfrac{1-3j}{32}$ in polar coordinates as $re^{j\theta}$,

$$\begin{aligned} y(t) &= \left[\frac{1}{16}e^{-t} - re^{3t}e^{j(4t+\theta)} - re^{3t}e^{-j(4t+\theta)}\right]u(t) \\ &= \left[\frac{1}{16}e^{-t} - 2re^{3t}\cos(4t+\theta)\right]u(t) \end{aligned}$$

Finding the impulse response

- For every input, we need to guess another particular solution. This can be hard for some inputs, e.g., those containing log, tan, etc.

- Since every differential equation with initial rest describes an LTI system, why not find the impulse response instead, and use the convolution integral to find the output for any input?

Finding the impulse response

- Example: Consider again

$$y(t) - \frac{dy(t)}{dt} - 2\,\frac{d^2y(t)}{dt^2} = x(t)$$

Find the impulse response, and find the output for the input $x(t) = u(t)$ using convolution. Show that it indeed is given by

$$y(t) = \frac{1}{3}\left[-e^{-t} - 2e^{0.5t} + 3\right]u(t)$$

Finding the impulse response

- Example: Consider again

$$y(t) - \frac{dy(t)}{dt} - 2\,\frac{d^2y(t)}{dt^2} = x(t)$$

- Solution: In other words, we are asked to solve

$$h(t) - \frac{dh(t)}{dt} - 2\,\frac{d^2h(t)}{dt^2} = \delta(t)$$

- The important observation is that for $t > 0$, we got ourselves a homogeneous equation. So, no need to guess and try a particular solution.

Finding the impulse response

$$h(t) - \frac{dh(t)}{dt} - 2\,\frac{d^2h(t)}{dt^2} = \delta(t)$$

- Recall that we already found the homogeneous solution to be of the form $c_1 e^{-t} + c_2 e^{0.5t}$.
- We can therefore write

$$h(t) = \left[c_1 e^{-t} + c_2 e^{0.5t}\right] u(t)$$

- As before, c_1 and c_2 will be determined by

$$h(0^+) = c_1 + c_2$$

$$\frac{dh}{dt}(0^+) = -c_1 + 0.5c_2$$

Finding the impulse response

$$h(t) - \frac{dh(t)}{dt} - 2\,\frac{d^2h(t)}{dt^2} = \delta(t)$$

$$h(t) = \left[c_1 e^{-t} + c_2 e^{0.5t}\right] u(t)$$

$$h(0^+) = c_1 + c_2$$

$$\frac{dh}{dt}(0^+) = -c_1 + 0.5c_2$$

- $h(t)$ cannot have a discontinuity at $t = 0$, because no doublet on the RHS: $h(0^+) = 0$
- $\frac{dh(t)}{dt}$ may have a discontinuity at $t = 0$, because the impulse on the RHS accounts for it

Finding the impulse response

$$h(t) - \frac{dh(t)}{dt} - 2\,\frac{d^2h(t)}{dt^2} = \delta(t)$$

$$h(t) = \left[c_1 e^{-t} + c_2 e^{0.5t}\right] u(t)$$

$$h(0^+) = c_1 + c_2$$

$$\frac{dh}{dt}(0^+) = -c_1 + 0.5c_2$$

- $\frac{dh(t)}{dt}$ may have a discontinuity at $t = 0$, because the impulse on the RHS accounts for it

$$\int_{0^-}^{0^+} \left[\cancel{h(t)} - \cancel{\frac{dh(t)}{dt}} - 2\,\frac{d^2h(t)}{dt^2}\right] dt = \int_{0^-}^{0^+} \delta(t)dt$$

$$-2\,\frac{dh}{dt}(0^+) + \cancel{2\,\frac{dh}{dt}(0^-)} = 1 \qquad \frac{dh}{dt}(0^+) = -0.5$$

Finding the impulse response

$$h(t) - \frac{dh(t)}{dt} - 2\,\frac{d^2h(t)}{dt^2} = \delta(t)$$

$$h(t) = \left[c_1 e^{-t} + c_2 e^{0.5t}\right] u(t)$$

$$\left.\begin{aligned} h(0^+) &= c_1 + c_2 = 0 \\ \frac{dh}{dt}(0^+) &= -c_1 + 0.5c_2 = -0.5 \end{aligned}\right\} \begin{aligned} c_1 &= \frac{1}{3} \\ c_2 &= \frac{-1}{3} \end{aligned}$$

- Substituting, we obtain

$$h(t) = \frac{1}{3}\left[e^{-t} - e^{0.5t}\right] u(t)$$

Finding the impulse response

$$h(t) = \frac{1}{3}\left[e^{-t} - e^{0.5t}\right] u(t)$$

- Since $x(t) = u(t)$, the output becomes

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

$$= \frac{1}{3}\int_{-\infty}^{\infty}\left[e^{-\tau} - e^{0.5\tau}\right] u(\tau)u(t-\tau)d\tau$$

$$= \frac{1}{3}u(t)\int_{0}^{t}\left[e^{-\tau} - e^{0.5\tau}\right] d\tau$$

$$= \frac{1}{3}u(t)\left[-e^{-\tau} - 2e^{0.5\tau}\right]\Big|_{0}^{t} = \frac{1}{3}u(t)\left[3 - e^{-t} - 2e^{0.5t}\right]$$

Finding the impulse response

- <u>Example</u>: Consider the causal LTI system with

$$25y(t) - 6\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2} = x(t) - \frac{dx(t)}{dt}$$

Find the impulse response $h(t)$.

- <u>Solution</u>: We are asked to solve

$$25h(t) - 6\frac{dh(t)}{dt} + \frac{d^2h(t)}{dt^2} = \delta(t) - \delta'(t)$$

- For $t > 0$, we will use the homogeneous solution

$$h(t) = c_1 e^{(3+4j)t} + c_2 e^{(3-4j)t}$$

we derived earlier.

Finding the impulse response

$$25h(t) - 6\frac{dh(t)}{dt} + \frac{d^2h(t)}{dt^2} = \delta(t) - \delta'(t)$$

- For $t > 0$, we will use the homogeneous solution

$$h(t) = c_1 e^{(3+4j)t} + c_2 e^{(3-4j)t}$$

we derived earlier.

- $h(t)$ might have a discontinuity at $t = 0$, since there is a doublet on the RHS.
- Integrate both sides to get

$$25H(t) - 6h(t) + \frac{dh(t)}{dt} = u(t) - \delta(t)$$

Finding the impulse response

$$h(t) = c_1 e^{(3+4j)t} + c_2 e^{(3-4j)t}$$

$$25H(t) - 6h(t) + \frac{dh(t)}{dt} = u(t) - \delta(t)$$

- Substituting $t = 0^+$,

$$25\cancel{H(0^+)} - 6h(0^+) + \frac{dh}{dt}(0+) = u(0^+) - \delta(0^+)$$

- Evaluating the RHS,

$$-6h(0^+) + \frac{dh}{dt}(0^+) = 1$$

Finding the impulse response

$$h(t) = c_1 e^{(3+4j)t} + c_2 e^{(3-4j)t}$$

$$-6h(0^+) + \frac{dh}{dt}(0^+) = 1$$

$$25H(t) - 6h(t) + \frac{dh(t)}{dt} = u(t) - \delta(t)$$

- Integrate again to get

$$25HH(t) - 6H(t) + h(t) = r(t) - u(t)$$

- Substituting $t = 0^+$,

$$\cancel{25HH(0^+)} - \cancel{6H(0^+)} + h(0^+) = r(0^+) - u(0^+)$$

- In other words, $h(0^+) = -1$

Finding the impulse response

$$h(t) = c_1 e^{(3+4j)t} + c_2 e^{(3-4j)t}$$

$$-6h(0^+) + \frac{dh}{dt}(0^+) = 1$$

$$h(0^+) = -1 \ = c_1 + c_2$$

$$\frac{dh}{dt}(0^+) = -5 \ = (3 + 4j)c_1 + (3 - 4j)c_2$$

$$c_1 = -0.5 + 0.25j \ = re^{j\theta}$$

$$c_2 = -0.5 - 0.25j \ = re^{-j\theta}$$

- The final answer is therefore

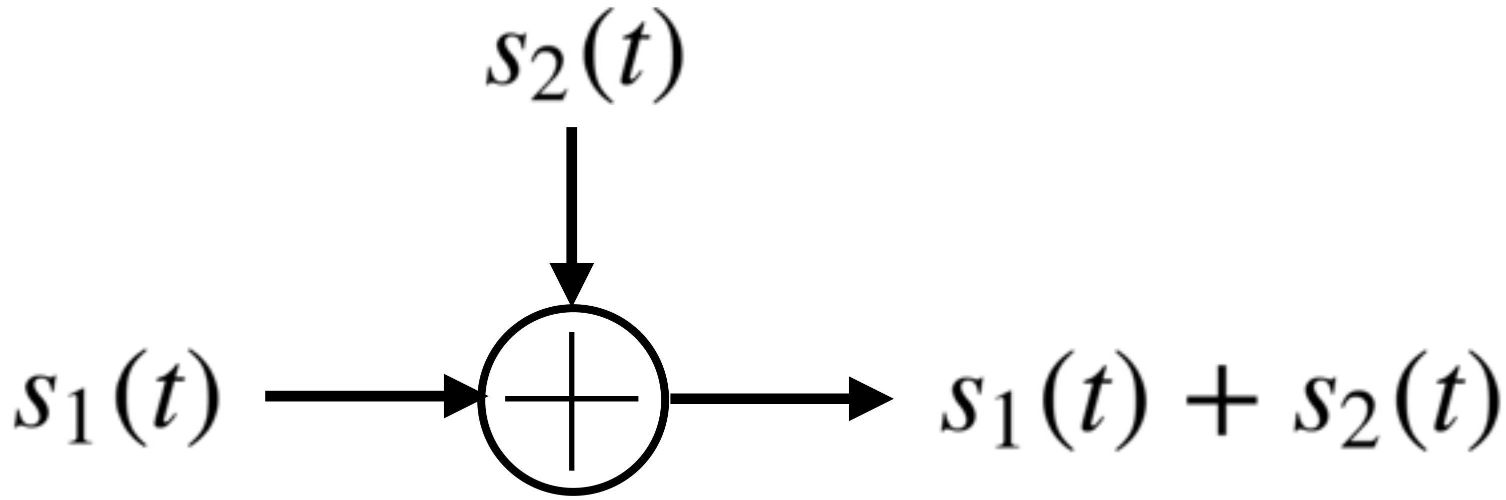
$$h(t) = 2re^{3t}\cos(4t + \theta)u(t)$$

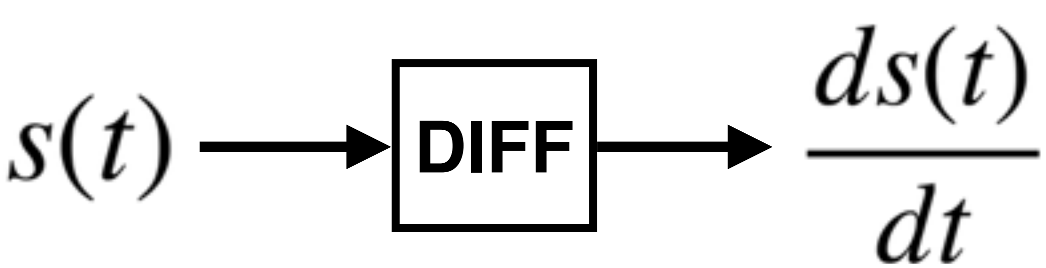
Block diagrams

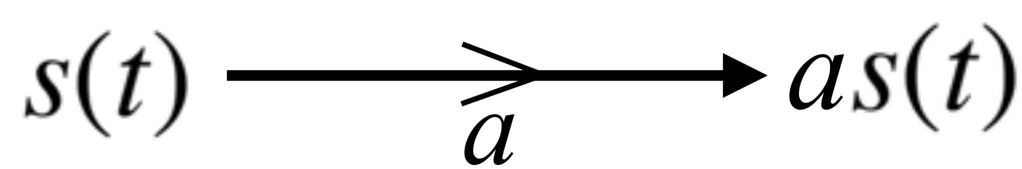
- We can also represent the LTI system for

$$\sum_{k=0}^{K} a_k \frac{d^k y}{dt^k} = \sum_{m=0}^{M} b_m \frac{d^m x}{dt^m}$$

with a block diagram using basic elements:







Block diagrams

- Example:

$$25y(t) - 6\,\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2} = x(t) - \frac{dx(t)}{dt}$$