UNIVERSITY OF CALIFORNIA, RIVERSIDE
Department of Electrical Engineering

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EE110B-SIGNALS AND SYSTEMS

HOMEWORK 3 SOLUTIONS

a) Let us find $y[n]$ for $n \geq 0$ only, as we know that $y[n] = 0$ for all $n < 0$.

For the particular solution, try $y_p[n] = K$. K must then satisfy

$$K = 0.64K + 1 \ ,$$

which means $K = \frac{100}{36}$. The solution to the homogenous part

$$y_h[n] = 0.64 y_h[n-2]$$

must be of the form r^n, and thus,

$$r^n = 0.64 r^{n-2}$$

or equivalently

$$r^2 = 0.64$$

implying that $r_1 = 0.8$ and $r_2 = -0.8$. Therefore

$$y[n] = \left[c_1 (0.8)^n + c_2 (-0.8)^n + \frac{100}{36} \right] u[n]$$

is the complete family of solutions.

Now,

$$y[0] = 0.64 y[-2] + u[0] = 1$$

and

$$y[1] = 0.64 y[-1] + u[1] = 1 \ .$$

Using these,

$$1 = c_1 + c_2 + \frac{100}{36}$$

and

$$1 = 0.8 c_1 - 0.8 c_2 + \frac{100}{36} \ ,$$

yielding

$$\begin{aligned} c_1 &= -2 \\ c_2 &= \frac{2}{9} \ . \end{aligned}$$

The output $y[n]$ then is

$$y[n] = \left[-2(0.8)^n + \frac{2}{9}(-0.8)^n + \frac{100}{36} \right] u[n] \ .$$

b) We have

$$h[0] = 0.64 h[-2] + \delta[0] = 0.64 \times 0 + 1 = 1$$

and

$$h[1] = 0.64 h[-1] + \delta[1] = 0.64 \times 0 + 0 = 0 \ .$$

Now, the solution to $h[n] = 0.64h[n-2]$ will be of the same form as in part a (with different c_1 and c_2, though). Substituting the initial conditions,

$$h[0] = c_1 + c_2 = 1$$

and

$$h[1] = 0.8c_1 - 0.8c_2 = 0 \ ,$$

which means $c_1 = c_2 = \frac{1}{2}$.

Bringing all these together,

$$h[n] = \frac{1}{2}\left[(0.8)^n + (-0.8)^n\right] u[n] \ .$$

c) The convolution sum is

$$\begin{aligned}
y[n] &= \sum_{k=-\infty}^{\infty} h[k]x[n-k] \\
&= \frac{1}{2}\sum_{k=-\infty}^{\infty}\left[(0.8)^k + (-0.8)^k\right] u[k]u[n-k] \\
&= \frac{1}{2}u[n]\sum_{k=0}^{n}\left[(0.8)^k + (-0.8)^k\right] \\
&= \frac{1}{2}u[n]\left[\sum_{k=0}^{n}(0.8)^k + \frac{1}{2}\sum_{k=0}^{n}(-0.8)^k\right] \\
&= \frac{1}{2}u[n]\left[\frac{1-0.8^{n+1}}{1-0.8} + \frac{1-(-0.8)^{n+1}}{1-(-0.8)}\right] \\
&= \frac{1}{2}u[n]\left[\frac{1-0.8\times 0.8^n}{0.2} + \frac{1-(-0.8)(-0.8)^n}{1.8}\right] \\
&= u[n]\left[\frac{10-8(0.8)^n}{4} + \frac{10+8(-0.8)^n}{36}\right] \\
&= u[n]\left[\frac{100}{36} - 2(0.8)^n + \frac{2}{9}(-0.8)^n\right]
\end{aligned}$$

which is exactly the same solution as in part a.