EE-110A: Signals and Systems

Final Review

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Course materials courtesy of Prof. Ertem Tuncel

Final exam

- ***Emphasis*** on
  - Laplace transform
- Content prior to the midterm exam is also useful!
  - e.g., convolution, Fourier transform definition

The Laplace transform

- The Laplace transform is then defined as

$$H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}\,dt$$

- For any s, we have

$$e^{st} \longrightarrow \boxed{h(t)} \longrightarrow H(s)e^{st}$$

- If we specialize this to $s = j\Omega$, we get back CTFT:

$$H(j\Omega) = \int_{-\infty}^{\infty} h(t)e^{-j\Omega t}\,dt$$

Partial fraction expansion

- Example: Find $x(t)$ if

$$X(s) = \frac{s^2}{s^2+1} \quad \text{with ROC: } Re\{s\} > 0$$

$$X(s) = \boxed{1} - \left(\boxed{\frac{A}{s-j}} + \boxed{\frac{B}{s+j}} \right) \qquad A = -0.5j \quad B = 0.5j$$

- Therefore, the solution must be given by

$$x(t) = \boxed{\delta(t)} \boxed{+0.5je^{jt}u(t)} \boxed{-0.5je^{-jt}u(t)}$$

$$= \delta(t) - \frac{e^{jt} - e^{-jt}}{2j} u(t) = \delta(t) - \sin(t)u(t)$$

Partial fraction expansion

- Example: Find $x(t)$ if

$$X(s) = \frac{s+2}{s^2+7s+12} \qquad \text{ROC: } Re(s) > 3$$

System properties re-revisited

- We can tell from the Laplace transform and its ROC whether the LTI system is causal, stable, and even invertible.
  - **Causality**: We already saw that the ROC has to be of the form $Re\{s\} > \alpha$ and must include $s = \infty$.
  - **Stability**: Recall that if the system is stable,

$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty$$

System properties re-revisited

- **Stability**: Recall that if the system is stable,

$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty$$

But we also have

$$\left| \int_{-\infty}^{\infty} h(t) e^{-j\Omega t} dt \right| \leq \int_{-\infty}^{\infty} \left| h(t) e^{-j\Omega t} \right| dt = \int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$

Hence, stability implies the existence of CTFT

Stability $\Longrightarrow$ ROC includes the imaginary axis

It can be shown that $\Longleftarrow$ is also true.

System properties re-revisited

- So, what does it take to have both causality and stability?
  - All poles must be on the left side of the imaginary axis (including the hidden ones)
- Example: The following is causal and stable.

$$H(s) = \frac{s-1}{(s+2)(s+3)} \quad \text{with ROC: } Re\{s\} > -2$$

- Example: The following is stable but not causal.

$$H(s) = \frac{(s-1)s^2}{(s+1)(s+5)} \quad \text{with ROC: } Re\{s\} > -1$$

System properties re-revisited

- **Invertibility**: At first, it looks like every LTI system with non-empty ROC is invertible.
  - The inverse is simply $G(s) = \dfrac{1}{H(s)}$
  - However, for practicality, we need both the system and its inverse to be causal and stable.
  - This implies that not only all poles, but also all zeros must be on the left side of the imaginary axis (including the hidden ones).

System properties re-revisited

$$H(s) = \frac{s+3}{(s+1)(s+5)} \quad \text{with ROC: } Re\{s\} > -1$$

- There is still a hidden zero at $s = \infty$ so the inverse won't be causal.

- Example:

$$H(s) = \frac{(s+3)(s+2)}{(s+1)(s+5)} \quad \text{with ROC: } Re\{s\} > -1$$

- This one has a causal and stable inverse.