A report on

On

Mathematical Analysis of Load Swing for an Overhead Bridge Crane

Submitted by

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Abstract

In this report, we will develop a mathematical model to study the angle with which the payload of an industrial overhead crane swings under different operating conditions like movement of the cart w.r.t time and the change in length of the rope attached to the payload w.r.t time. We will develop the model by applying Newtonian mechanics on our system which involves developing the equation of motion by considering the forces on our system through their free body diagrams. The obtained equation will be combined with the user input conditions and then solved in Matlab to analyse the swing of the payload.

Key words: mathematical model, load swings, Newtonian mechanics, Matlab analysis.

Acknowledgement

We would like to thank Professor Dr. Thomas Götz for his immense support and guidance during our work on this project seminar.

Introduction

Cranes are being widely used in many industries irrespective of the size and strength. Industrial cranes or Bridge cranes are available in different types and forms to meet the usage. Their usage increases with an increase in compactness and strength. An overhead crane typically consists of a trolley (cart), hoist, wire or rope, hook block. The trolley is located between the parallel beams which are mounted on to the walls or to the structure of a building at a certain height. The hoist helps to lift and release the load with the help of a wire and hook. [1]

In places, where there is a minimal space available and a requirement of tonnes to be lifted and shifted, it is quite common for an accident which could be an injury or material loss. There could be many reasons for this such as electrocuting, overturns, load related fatalities such as over loading, load swings, crane tilting…. Out of all these, the most important reason would be due to load swings. They could really affect the work environment unless they are handled properly and could lead to the loss of hours of work time, material loss, equipment loss and sometimes human loss too. It requires a lot of skill and experience to move the load with minimal swing or to handle the load swings as mentioned in the industrial column of MRO (Machine and equipment) magazine in USA. [2]

In this report, we are going to model the system and obtain an optimal condition within the given conditions where the swing is minimum. This helps an operator to choose an optimal path for the payload within his/her given conditions.

Modelling a Physics Problem

Modelling a physics problem is often an interesting process, where it involves both mathematics and physics. We need to analyze the given problem data and do the necessary steps to make it a mathematically solved problem. There are four important steps in doing that:

- Modelling
- Processing
- Evaluating
- Interpreting

We start with modeling the problem in terms of given conditions and obtain a basic idea, process the idea by adding the needed conditions that were not given in the problem, which is an important step in identifying and solving precisely. Obtaining a relevant mathematical equation and solving the equation comes under evaluating. The results obtained are visualized to synthesize and analyze, which comes under interpreting. All together, we consider this as the process of mathematically solving a physics problem. [3]

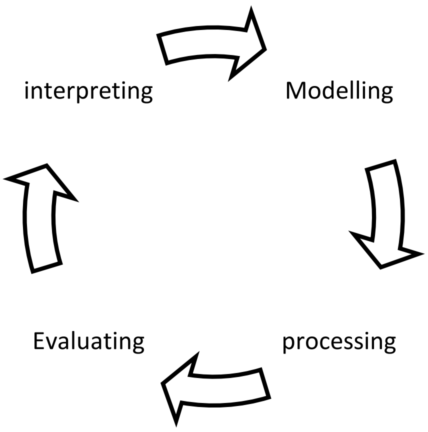

Fig 1: source: arxiv.org/ftp/arxiv/papers/1002/1002.0472.pdf.

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Problem Statement and Significance of Results

Problem statement: Develop a mathematical model which can be used to study and analyze the angle of swing for a payload of an overhead bridge crane under certain operating conditions and to identify the trajectory of the payload which produces the minimum swing.

Significance of results: The following are the results that are desired from the study that we are going to do,

- Deriving an equation of motion for the payload.
- Finding the swing of the payload.
- Optimal trajectory within the given conditions for the minimum swing of the payload.

The System

The main mechanism of the overhead crane can be considered as a pendulum – cart system. The cart houses the rope hoist mechanism to pull/lower the payload. The movement of the cart causes the oscillations in the payload which resembles a pendulum motion. We derive the equation of motion by taking some assumptions into consideration.

Assumptions:

- The work done by the crane is only in the X – Y plane.
- Both the cart and the payload are point loads, so their masses are at the centre of their bodies.
- The rope is non flexible.
- The weight of the rope is ignored.

The cart of mass *M* is setup on a beam(s) and can only move in the directions of x – axis. The payload of mass *m* is attached to the cart by means of a rope whose length *l* can be varied and the payload can move in the x – y plane. As time goes, the cart moves in the direction right and the payload moves along the cart.

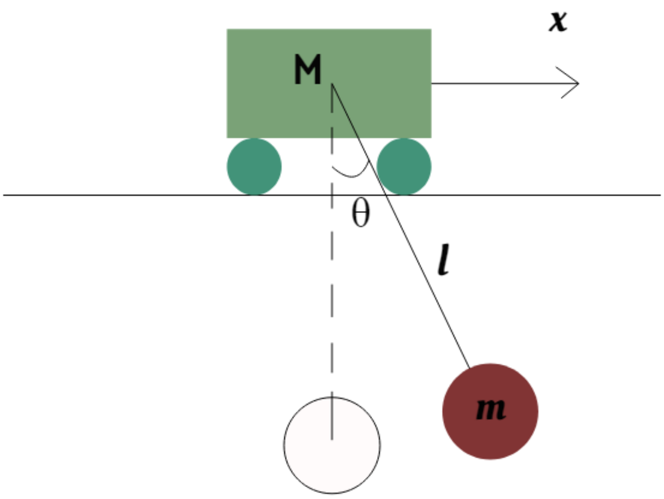

Fig 1: The cart – pendulum system.

The operator can control the speed of the cart and the length of the rope. The payload will be influenced by the movement of the cart and the rope.

We will now apply Newtonian mechanics to the system and derive the equations of motion.

Equation of Motion for the Cart

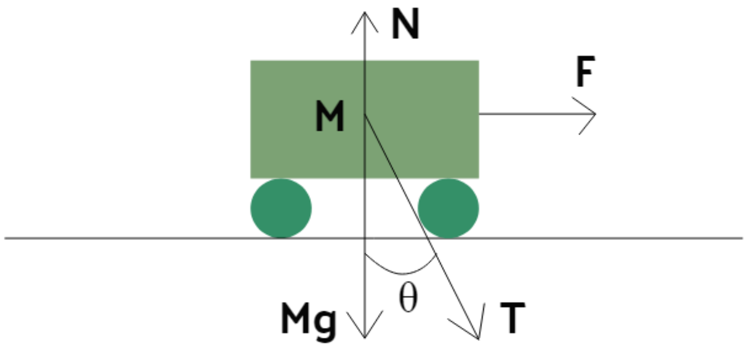

Fig2: free body diagram of the cart

The cart is displaced by x meters. Then,

The position of the cart = $x(t)$

The velocity of the cart = $x'(t)$

The acceleration of the cart = $x''(t)$

We don't have to find the equations of motion for the cart because the movement of the cart will be controlled by an operator which will be given as a numerical values.

Equations of Motion for the Payload

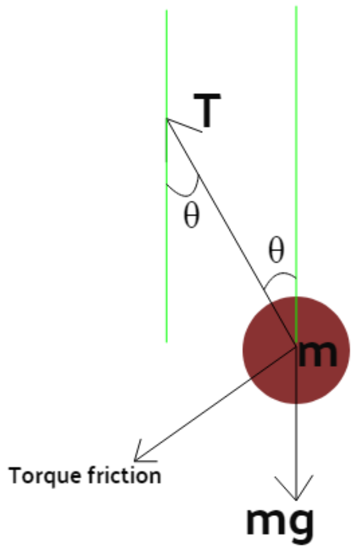

Fig 3: free body diagram of the payload.

Let us assume that the payload is displaced by an angle of θ. Then, for the payload:

Position $= x\hat{\imath} + lsin\theta\hat{\imath} - lcos\theta\hat{\jmath}$

Velocity $= x'\hat{\imath} + l'sin\theta\hat{\imath} + l\theta'cos\theta\hat{\imath} - l'cos\theta\hat{\jmath} + l\theta' sin\theta\hat{\jmath}$

Acceleration= $x''\hat{\imath} + l''sin\theta\,\hat{\imath} + 2l'\theta'cos\theta\,\hat{\imath} + l\theta''cos\theta\,\hat{\imath} - l\theta'^2 sin\theta\,\hat{\imath} -$
$l''cos\theta\hat{\jmath} + 2l'\theta'sin\theta\,\hat{\jmath} + l\theta''sin\theta\,\hat{\jmath} + l\theta'^2cos\theta\hat{\jmath}$

Here, $\theta(t)$ is the angular displacement, $\theta'(t)$ is the angular velocity and $\theta''(t)$ is the angular acceleration of the payload.

<u>Forces on the payload:</u>

Due to tension in the rope (T): $Tcos\theta\,\hat{\jmath} - Tsin\theta\,\hat{\imath}$

Due to gravity: $-mg\,\hat{\jmath}$

Due to Torque friction [4] (τ_f): $\frac{-b}{l}\,\theta'(cos\theta\,\hat{\imath} + sin\theta\,\hat{\jmath})$

$$\vec{F} = Tcos\theta\,\hat{\jmath} - Tsin\theta\hat{\imath} - mg\,\hat{\jmath} - \frac{b}{l}\theta'(cos\theta\,\hat{\imath} + sin\theta\,\hat{\jmath})$$

From newton's second law,

$$\vec{F} = m\vec{a}$$

Substituting the force and acceleration in the above equation,

$$Tcos\theta\,\hat{\jmath} - Tsin\theta\hat{\imath} - mg\,\hat{\jmath} - \frac{b}{l}\theta'(cos\theta\,\hat{\imath} + sin\theta\,\hat{\jmath}) =$$

$m\,(x''\hat{\imath} + l''sin\theta\,\hat{\imath} + 2l'\theta'cos\theta\,\hat{\imath} + l\theta''cos\theta\,\hat{\imath} - l\theta'^2sin\theta\,\hat{\imath} - l''cos\theta\,\hat{\jmath} + 2l'\theta'sin\theta\,\hat{\jmath} +$
$l\theta''sin\theta\,\hat{\jmath} + l\theta'^2cos\theta\,\hat{\jmath}$ ______(1)

Separating (1) into two equations of x and y components, we have

$-Tsin\theta - \frac{b}{l}\,\theta'cos\theta = m\,(x'' + l''sin\theta + 2l'\theta'cos\theta + l\theta''cos\theta - l\theta'^2sin\theta)$______(2)

$$Tcos\theta \; - \; mg \; - \; \frac{b}{l}\theta' sin\theta \; = \; m(-l"cos\theta \; + \; 2l'\theta' sin\theta \; + \; l\theta" sin\theta \; + l\theta'^2 cos\theta \_\_\_\_\_\_(3)$$

Multiplying (2) with cosθ and (3) with sinθ and adding both the equations, we get

$$-mg\sin\theta - \frac{b}{l}\theta' = mx''\cos\theta + 2ml'\theta' + ml\theta''$$

________(4)

Writing the explicit equation for θ'' from (4), we have:

$$\theta" \; = \; \frac{-mgsin\theta \; - \; \frac{b}{l}\theta' \; - \; mc"cos\theta \; - \; 2ml'\theta'}{ml}$$

________(5)

This resulting equation (5) is the equation of motion of the payload and it indicates how the payload is influenced by the change in cart and the change in the rope length.

The State-space Representation:

The above equation of motion (5) can be solved numerically in Matlab by converting the second order ordinary differential equation into a system of first order ordinary differential equations. So, for this purpose we introduce new variables as:

$$y_1 = \theta$$

$$y_2 = \theta'$$

$$y_2' = \theta''$$

This change in variables results in the transformation of the equation (5) into the below system

$$y_1' = y_2$$

$$y_2' = \frac{-mg\sin(y_1) - \frac{b}{l}y_2 - mx''\cos(y_1) - 2ml'y_2}{ml}$$

We will solve the above system of differential equations using the ODE45 function in Matlab.

MATLAB Code

The ODE[5]:

```
function yprime = loadswing(t,y)
g=9.8; %acceleration due to gravity
m=100; %Mass of the payload
b=30; %Pendulum friction coefficient

yprime(1,1) = y(2);
yprime(2,1) = (-m*g*sin(y(1))-(b/R(t))*y(2)-m*c2(t)*cos(y(1))-
2*m*R1(t)*y(2))/(m*R(t));
end
```

Movement of the Cart (position, velocity and acceleration):

```
a1=0; %time co-ordinates
a2=50;
b1=0; %displacement values
b2=100;
m1=(b2-b1)/(a2-a1)^2;
p1= @(t) m1*(t.^2)-2*m1*t*a1+m1*a1^2+b1; %displacement equation
v1=@(t)(2*t)/25 ; %velocity equation
ac1=@(t) 2/25; %acceleration
a5=100;
a6=50;
b5=200;
b6=100;
m3=(b6-b5)/(a6-a5)^2;
p3=@(t) m3*(t.^2)-2*m3*t*a5+m3*a5^2+b5;
v3=@(t) 8 - (2*t)/25;
ac3=@(t) -2/25;
pf=@(t) 200;
vf=@(t) 0;
af=@(t) 0;
```

```
c = @(t) p1(t).*(t>=0 & t<=50)+p3(t).*(t>50 & t<=100)+pf(t).*(t>100 &
t<=1000);
c1 = @(t) v1(t).*(t>=0 & t<=50)+v3(t).*(t>50 & t<=100)+vf(t).*(t>100 &
t<=1000);
c2 =@(t) ac1(t).*(t>=0 & t<=50)+ac3(t).*(t>50 & t<=100)+af(t).*(t>100 &
t<=1000);
```

Change in Length of the Rope:

```
x_1=0; %x are the time co-ordinates
y_1=20; %y are the length co-ordinates
x_2=25;
y_2=15;
a=(y_2-y_1)/(x_2-x_1)^2;
%t1=0:0.1:15;
L1=@(t) a*(t.^2)-2*a*t*x_1+a*x_1^2+y_1;
L1prime=@(t) -(2*t)/125;
L1dprime=@(t) -2/45;
x_3=50;
y_3=10;
x_4=25;
y_4=15;
a2=(y_4-y_3)/(x_4-x_3)^2;
%t2=15:0.1:30;
L2=@(t) a2*(t.^2)-2*a2*t*x_3+a2*x_3^2+y_3;
L2prime=@(t) (2*t)/125 - 4/5;
L2dprime=@(t) 2/45;
x_5=50;
y_5=10;
x_6=75;
y_6=15;
a3=(y_6-y_5)/(x_6-x_5)^2;
%t3=50:0.1:75;
L3=@(t) a3*(t.^2)-2*a3*t*x_5+a3*x_5^2+y_5;
L3prime=@(t) (2*t)/125 - 4/5;
L3dprime=@(t) 0;
x_7=100;
y_7=20;
x_8=75;
y_8=15;
a4=(y_8-y_7)/(x_8-x_7)^2;
%t4=75:0.1:100;
L4=@(t) a4*(t.^2)-2*a4*t*x_7+a4*x_7^2+y_7;
L4prime=@(t) 8/5 - (2*t)/125;
L4dprime=@(t) 3/80;

Lf=@(t) 20;
Lfprime=@(t) 0;
Lfdprime=@(t) 0;
%Length of the rope
R = @(t) L1(t).*(t>=0 & t<=25)+L2(t).*(t>25 & t<=50)+L3(t).*(t>50 &
t<=75)+L4(t).*(t>75 & t<=100)+Lf(t).*(t>100 & t<=500);
```

```
R1= @(t) L1prime(t).*(t>=0 & t<=25)+L2prime(t).*(t>25 &
t<=50)+L3prime(t).*(t>50 & t<=75)+L4prime(t).*(t>75 &
t<=100)+Lfprime(t).*(t>100 & t<=500);
```

The ODE solver[6]:

```
%initial angle of the payload
y0(1,1)=0;
%initial angular velocity of the payload
y0(2,1)=0;

tspan = [0 1000];
[t,y] = ode45('new',tspan,y0);

E = (y(:,2).^2)+(y(:,1).^2); %rotational energy of the payload

figure
plot(t,y(:,1))
legend('payload')
xlabel('time(s)')
ylabel('angle(rad)')
title('swing of the payload')

figure
plot (t,E)
legend('Rotational Energy')
xlabel('time(s)')
ylabel('Energy')
title(' Rotational Energy of the Payload')
```

Results:

To generate the results, we gave the constant input values of

- Acceleration due to gravity $(g) = 9.8\ m/s^2$
- Mass of the payload $(m) = 100\ kg$
- Friction co-efficient $(b) = 30$

We studied the swing of the payload under different conditions of the cart movement and the change in length of the rope. We carried our study under the assumptions that:

- The distance between the start and end positions of the cart is 200 metres.
- The cart and the payload are initially separated by a distance of 20 metres (i.e the length of the rope will be 20 metres when it picks the payload).
- After the payload is picked up, it will be raised 10 metres vertically and will again be brought down to 20m separation from the cart at the destination point where the payload should be placed.

We considered two time intervals in which this whole process is completed.

- Case 1: 100 seconds
- Case 2: 50 seconds

The length of the rope follows three different patterns in both the time cases.

- Sub case 1: 20 metres – 10 metres (stays constant here for some time) – 20 metres.
- Sub case 2: 20 metres – 10 metres – 20 metres. (length varies continuously)
- Sub case 3: 10 metres (the payload is picked up to the height of 10 metres before the cart starts moving. The length stays constant at 10 metres while the cart is moving and the rope length increases to 20 metres after the cart reaches its final position).

Combining all the above, we have six different cases.

<u>Note:</u> In all the cases, we study the swing of the payload for until 500 seconds.

After solving our ODE in Matlab, we got the following results:

Case 1:

Sub case 1:

Process time – 100 seconds

Cart movement: 0 – 200 metres between 0 – 100 seconds.

Length of the rope: 20 – 10 metres between 0 – 30 seconds. 10 metres between 30 – 60 seconds. 10 – 20 metres between 60 – 100 seconds.

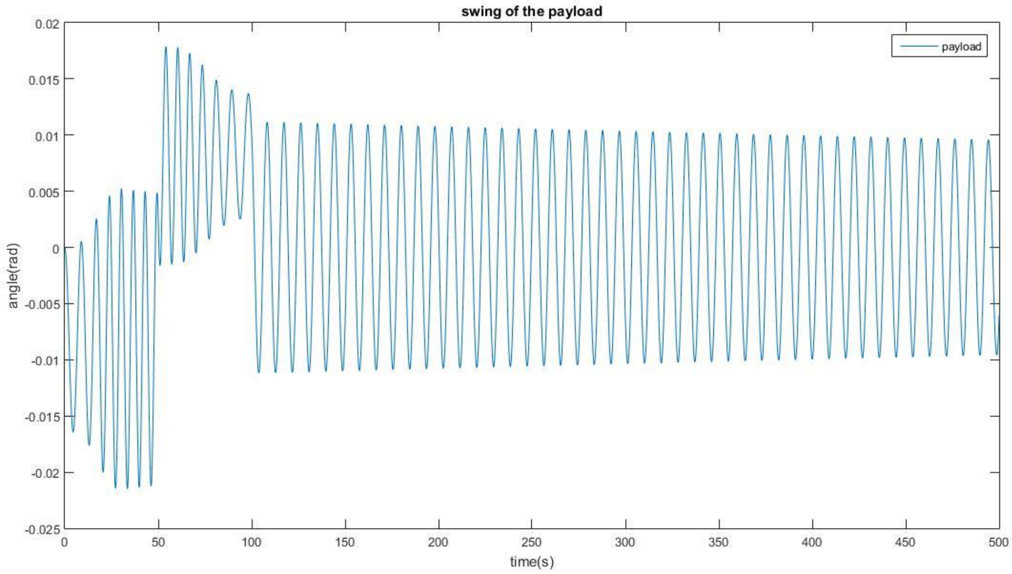

Fig 4: Angular displacement in subcase 1

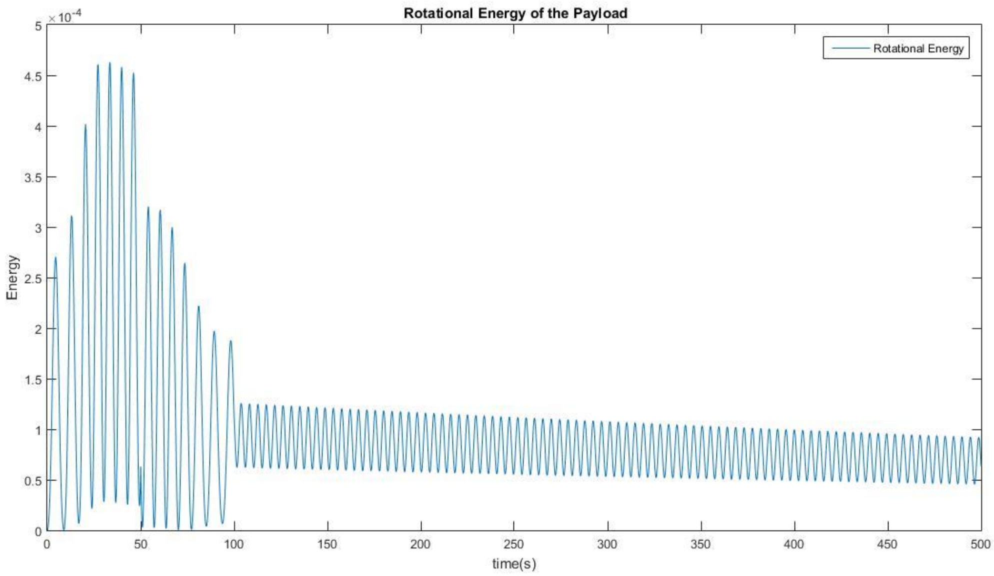

Fig 5: Rotational energy in subcase 1

Sub case 2:

Process time – 100 seconds

Cart movement: 0 – 200 metres between 0 – 100 seconds.

Length of the rope: 20 – 10 metres between 0 – 50 seconds. 10 – 20 metres between 50 – 100 seconds.

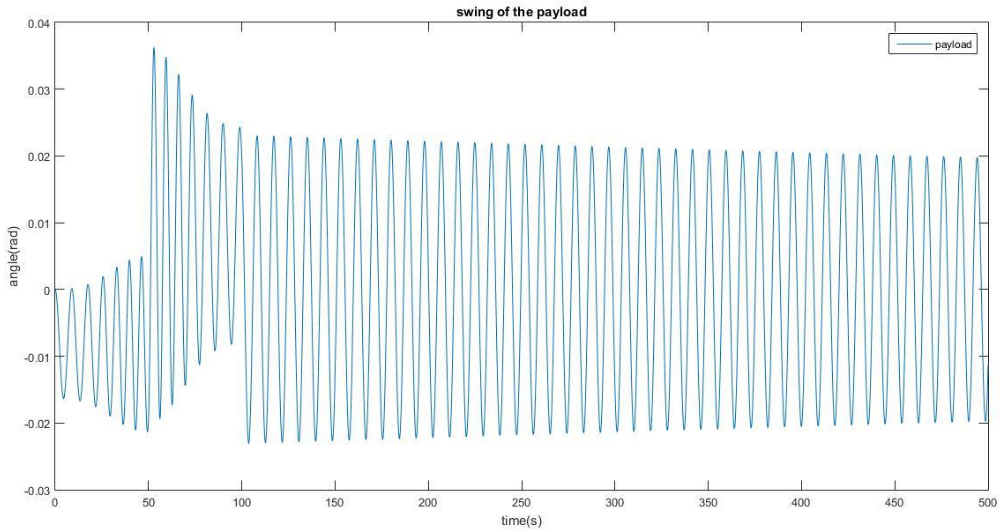

Fig 6: Angular displacement in subcase 2

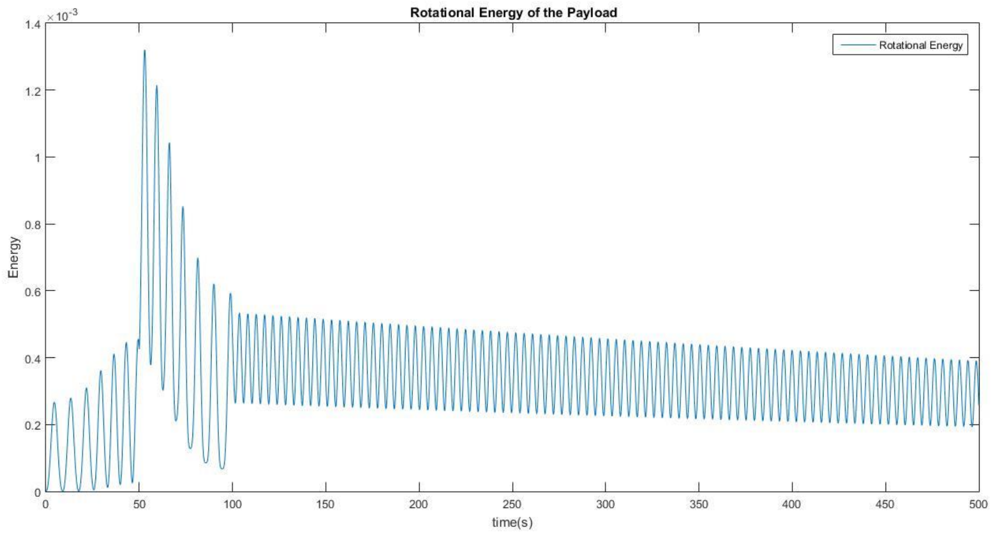

Fig 7: Rotational energy in subcase 2

<u>Sub case 3</u>:

Process time – 100 seconds

Cart movement: 0 – 200 metres between 20 – 80 seconds.

Length of the rope: 20 – 10 metres between 0 – 20 seconds. 10 metres between 20 – 80 seconds. 10 – 20 metres between 80 – 100 seconds.

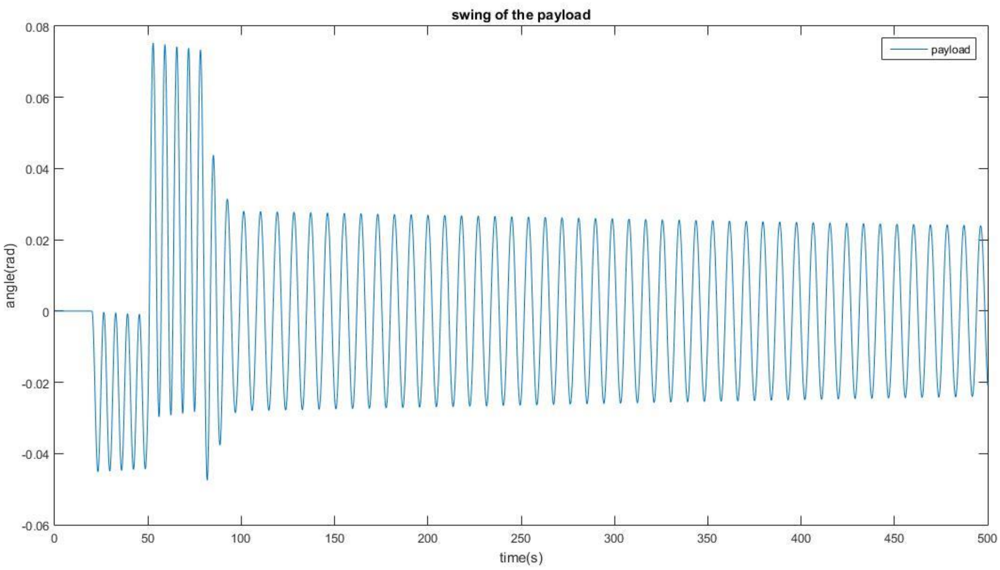

Fig 8: Angular displacement in subcase 3

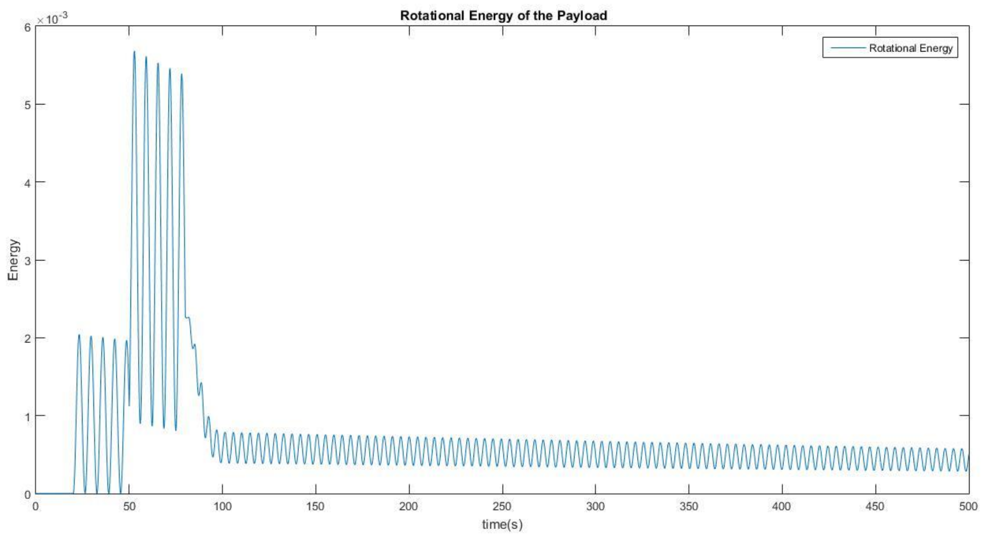

Fig 9: Rotational energy in subcase 3

Case 2:

Sub case 1:

Process time – 50 seconds

Cart movement: 0 – 200 metres between 0 – 50 seconds.

Length of the rope: 20 – 10 metres between 0 – 15 seconds. 10 metres between 15 – 30 seconds. 10 – 20 metres between 30 – 50 seconds.

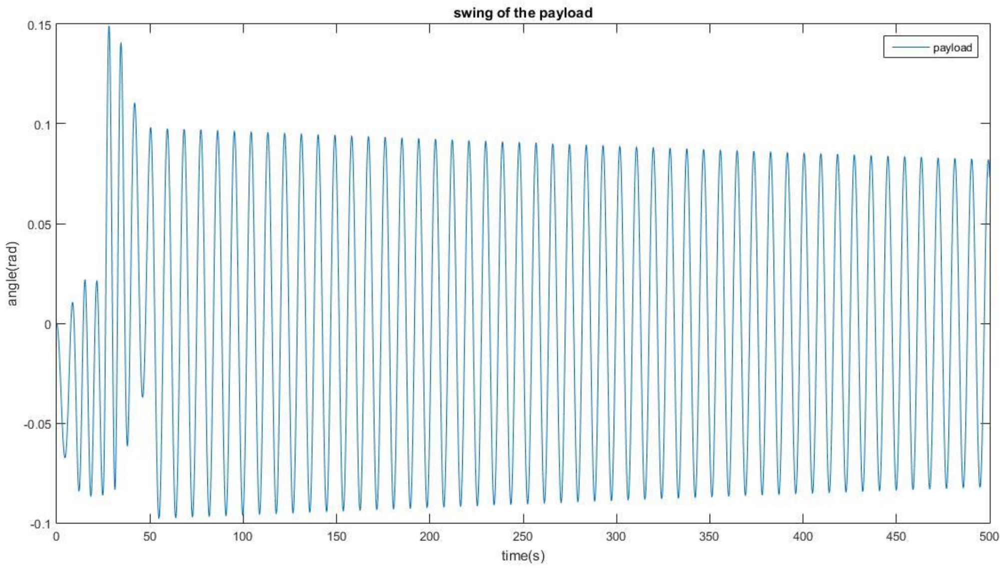

Fig 10: Angular displacement in subcase 4

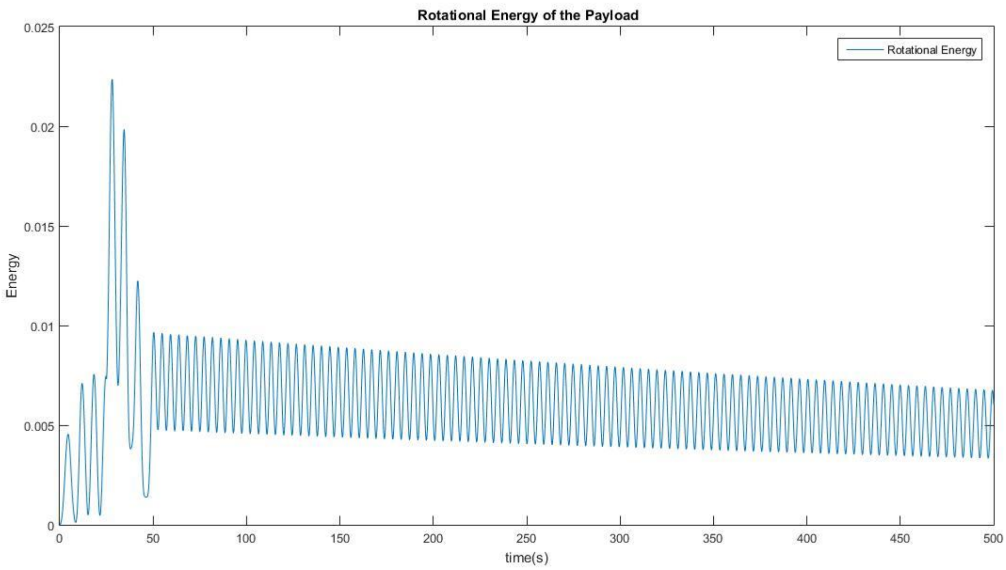

Fig 11: Rotational energy in subcase 4

Sub case 5:

Process time – 50 seconds

Cart movement: 0 – 200 metres between 0 – 50 seconds.

Length of the rope: 20 – 10 metres between 0 – 25 seconds. 10 – 20 metres between 25 – 50 seconds.

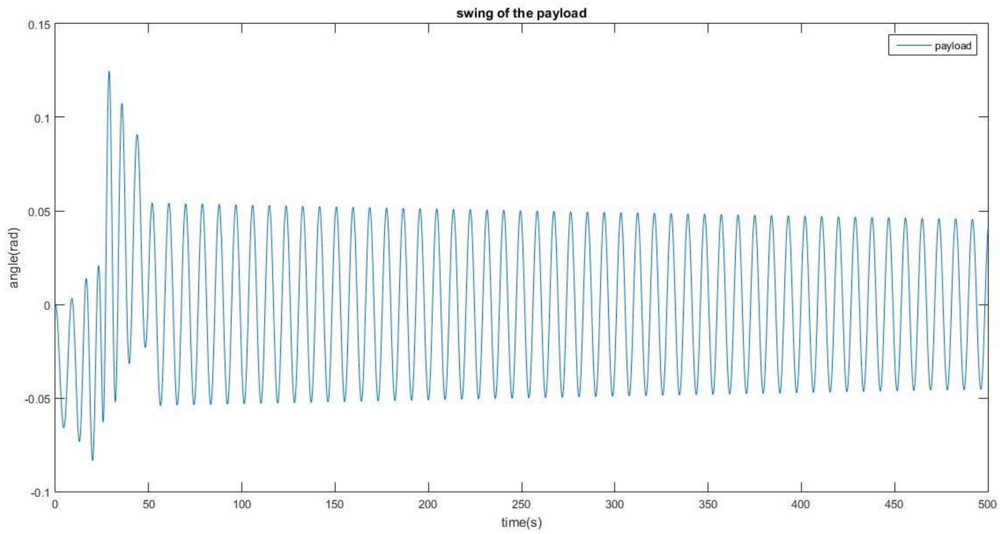

Fig 12: Angular displacement in subcase 5

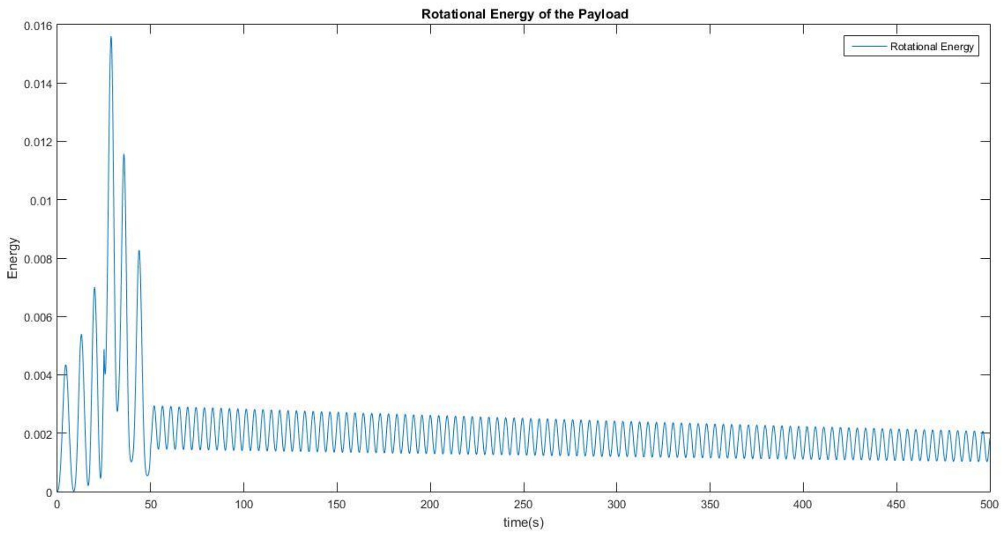

Fig 13: Rotational energy in subcase 5

Sub case 6:

Process time – 50 seconds

Cart movement: 0 – 200 metres between 10 – 40 seconds.

Length of the rope: 20 – 10 metres between 0 – 10 seconds. 10 metres between 10 – 40 seconds. 10 – 20 metres between 40 – 50 seconds.

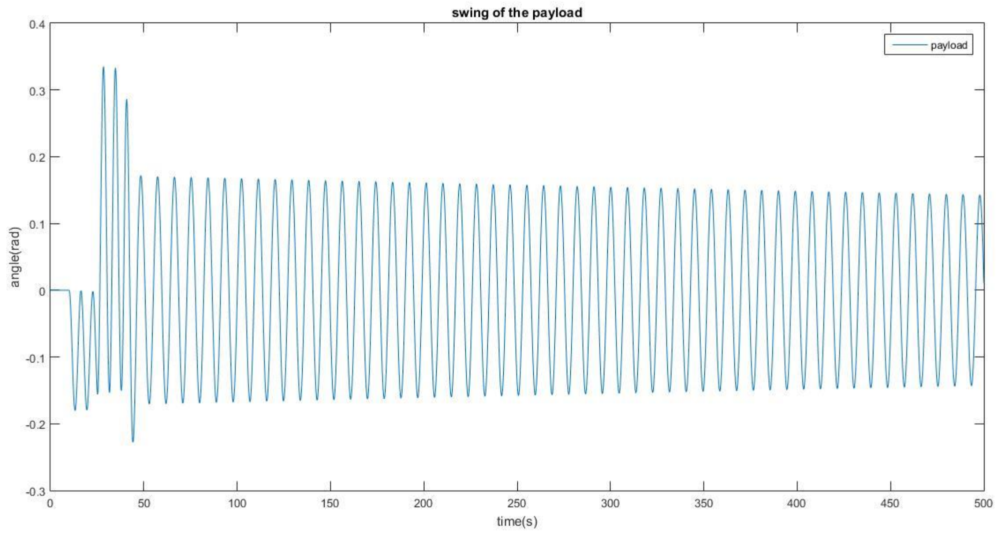

Fig 14: Angular displacement in subcase 6

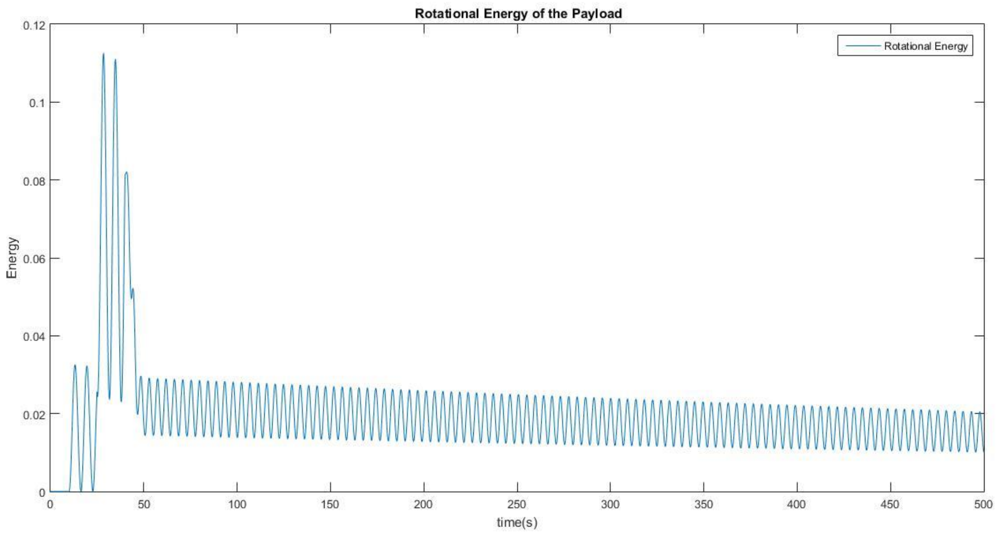

Fig 15: Rotational energy in subcase 6

After generating and interpreting the results, we can come out with the following conclusions

- For the case in which the cart moves from 0 – 200 meters in 100 seconds, the minimum swing of the payload occurs in subcase 1. The values of angular displacement and the energy are

$$\theta = 0.0096\ rad$$

$$\theta'^2 + \theta^2 = 0.0000921$$

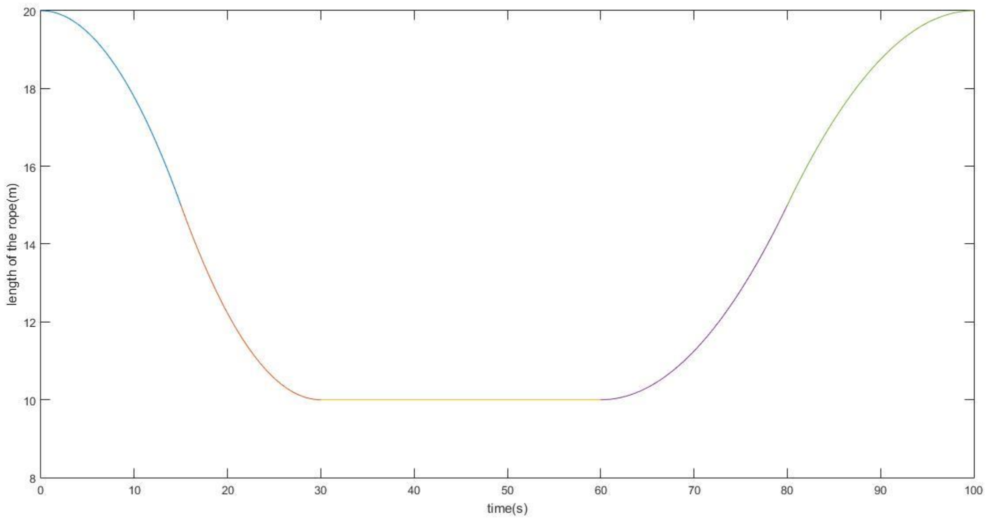

Fig 16: the variation of length for the minimum angular displacement in case 1

- For the case in which the cart moves from 0 – 200 meters in 50 seconds, the minimum swing of the payload occurs in subcase 2. The values of angular displacement and the energy are

$$\theta = 0.0455\,rad$$
$$\theta'^2 + \theta^2 = 0.0021$$

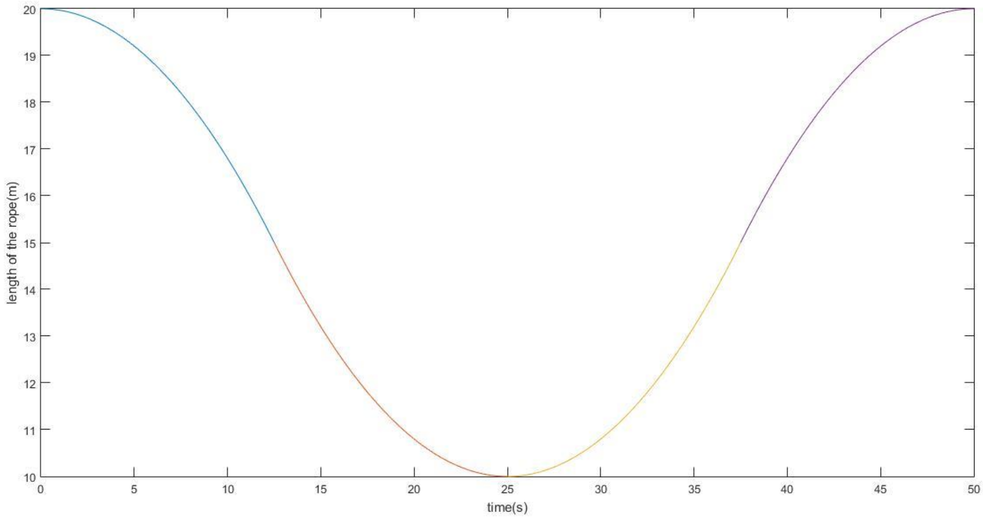

Fig 17: the variation of length for the minimum angular displacement in case 2

This way, we can give several other sets of conditions as the input and check for the best conditions which produce minimum swing of the payload.

Conclusion

In this report, we developed a mathematical model for an overhead bridge crane operating in the x – y plane using Newtonian mechanics. The equation is then represented in a state – space system. This system is then solved numerically in Matlab with different sets of operating conditions with varying cart speed and varying rope length of the overhead bridge crane.

The results are derived and interpreted to find the best trajectory for the motion of the payload. One interesting learning from the results is that the trajectory profile which produces the minimum swing angle for the payload in case 1 turns out to be the trajectory which nearly produces the maximum swing in case 2 and vice versa.

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