# On Upper and Lower Bounds of Identifying Code Set for Soccer Ball Graph with Application to Satellite Deployment


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## ABSTRACT

We study a monitoring problem on the surface of the earth for significant environmental, social/political and extreme events using *satellites as sensors*. We assume that the surface of the earth is divided into a set of regions, where a region may be a continent, a country, or a set of neighboring countries. We also assume that, the impact of a significant event *spills* into neighboring regions and there will be corresponding indicators of such events. Careful deployment of sensors, utilizing *Identifying Codes*, can ensure that even though the number of deployed sensors is fewer than the number of regions, it may be possible to *uniquely* identify the region where the event has taken place. We assume that an event is confined to a region. As Earth is almost a sphere, we use a soccer ball (a sphere) as a model. From the model, we construct a *Soccer Ball Graph* (SBG), and show that the SBG has at least 26 sets of Identifying Codes of cardinality ten, implying that there are at least 26 different ways to deploy ten satellites to monitor the Earth. Finally, we also show that the size of the minimum Identifying Code for the SBG is *at least* nine.



## CCS CONCEPTS

• **Networks**;

## KEYWORDS

Identifying Code, Upper Bound, Monitoring


## 1 INTRODUCTION

In this paper, we study an event monitoring problem with satellites as sensors. The events that we focus on may be environmental (drought/famine), social/political (social unrest/war) or extreme events (earthquakes/tsunamis). Such events take place in *regions* on the surface of the earth, where a region may be a continent, a country, or a set of neighboring countries. The sensors that we envisage for monitoring such events are satellites placed in orbits surrounding the earth. A satellite constellation that can be deployed for such monitoring purposes is shown in Fig. 1(b). Examples of such constellations include the Global Positioning System (GPS) for navigation, the Iridium and Globalstar satellite telephony, and the Disaster Monitoring Constellation (DMC) for remote sensing. In particular, DMC is designed to provide earth imaging for disaster relief and was used extensively to monitor the impact of the Indian Ocean Tsunami in December 2004, Hurricane Katrina in August 2005, and several other floods, fires and disasters. The problem that we address in this paper is directly relevant to the services being provided by organizations such as the DMC.

For Earth's spherical structure, we use a soccer ball as a model of the planet Earth. In technical terms, a standard soccer ball is a *truncated icosahedron* with 12 pentagonal and 20 hexagonal patches [10] (shown in Fig. 1(c) as black and white patches). We associate a patch on the surface of the ball with a region on the surface of the Earth. Accordingly, in our model, the surface of the Earth is partitioned into 32 regions. We assume that the coverage area of a satellite corresponds to a patch (region) and events are confined to a region. With such a framework, it is clear that with 32 satellites (one per region), all the 32 regions can be effectively monitored. However, if we assume that the *impact* of an event in one region will *spill* into its neighboring regions, and as such there will be indicators of such events in neighboring regions, then a significantly lower number of satellites may be sufficient for effective monitoring of all the regions. As an example of impact of an event spilling over to neighboring regions, one can think of a situation where war breaking out in one region can trigger an exodus of refugees to the neighboring regions. As these sensors are expensive, one would like to deploy as few sensors as possible, subject to the constraint that all the regions can be effectively monitored. In the following, we discuss *Identifying Codes* [1] that can be utilized for this purpose. In particular, we will show that ten satellites are *sufficient* to *effectively monitor* 32 regions in the sense that, if an event breaks out in a region, that region can be *uniquely identified*. In fact there exists 26 different ways of deploying ten satellites that will achieve the effective monitoring task. Moreover, we will establish that the effective monitoring task cannot be accomplished by deployment of *fewer than* nine satellites.

In this paper we have assumed that the regions have only two different regular shapes - hexagons and pentagons. This assumption



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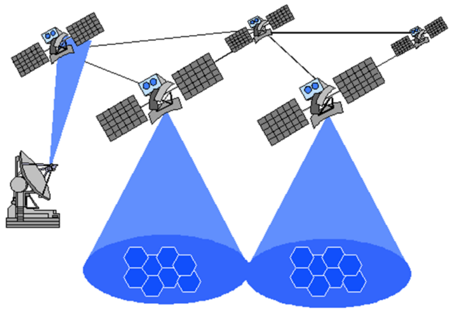
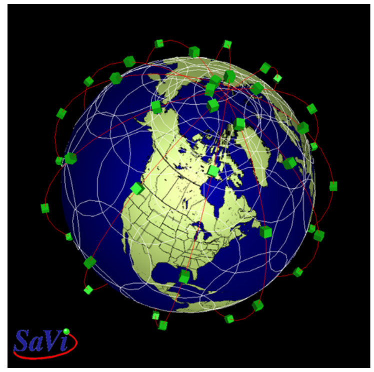
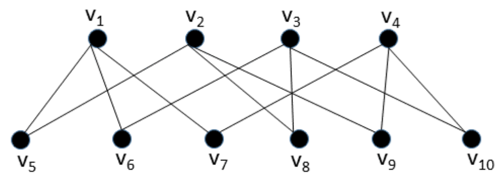


(a) Satellite Footprints (Coverage Area) on Earth




(b) Satellite Constellation Covering Earth

(c) A Truncated Icosahedron




(d) Graph with Identifying Code Set $\{v_1, v_2, v_3, v_4\}$

**Figure 1: Satellites as sensors and soccer ball as a model of planet earth**

may appear to be too restrictive, in the sense that in reality, regions on the surface of the earth may have irregular shapes. As our analysis is based on the structure of the graph (the SBG graph construction rule from the regions is described in section 2), as long as the graph structure arising out of irregular shaped regions remains the same as the structure of the SBG, the shape of the regions (whether regular or irregular, only hexagonal/pentagonal or some other contours) are irrelevant.

The notion of *Identifying Codes* [1] has been established as a useful concept for optimizing sensor deployment in multiple domains. In this paper, we use Identifying Code of the *simplest form* and define it as follows. *A vertex set $V'$ of a graph $G = (V, E)$ is defined as the Identifying Code Set (ICS) for the vertex set $V$, if for all $v \in V$, $N^+[v] \cap V'$ is unique where, $N^+[v] = v \cup N(v)$ and $N(v)$ represents the set of nodes adjacent to $v$ in $G = (V, E)$.* The *Minimum Identifying Code Set* (MICS) problem is to find the Identifying Code Set of *smallest cardinality*. The vertices of the set $V'$ may be viewed as *alphabets* of the code, and the *string* made up with the alphabets of $N^+[v]$ may be viewed as the unique "code" for the node $v$. For instance, Consider the graph $G = (V, E)$ shown in Fig. 1(d). In this graph $V' = \{v_1, v_2, v_3, v_4\}$ is an ICS as it can be seen from Table 1 that $N^+[v] \cap V'$ is *unique* for all $v_i \in V$.

From the soccer ball, we construct a graph (referred to as a Soccer Ball Graph, SBG) where each of the 32 regions is represented as a node and two nodes have an edge between them if the corresponding regions share a boundary. The construction rules for the SBG are given in section 2 and a two dimension layout of the SBG

| | |
|---|---|
| $N^+[v_1] \cap V' = \{v_1\}$ | $N^+[v_2] \cap V' = \{v_2\}$ |
| $N^+[v_3] \cap V' = \{v_3\}$ | $N^+[v_4] \cap V' = \{v_4\}$ |
| $N^+[v_5] \cap V' = \{v_1, v_2\}$ | $N^+[v_6] \cap V' = \{v_1, v_3\}$ |
| $N^+[v_7] \cap V' = \{v_1, v_4\}$ | $N^+[v_8] \cap V' = \{v_2, v_3\}$ |
| $N^+[v_9] \cap V' = \{v_2, v_4\}$ | $N^+[v_{10}] \cap V' = \{v_3, v_4\}$ |

**Table 1: $N^+[v] \cap V'$ results for all $v \in V$ for the graph in Fig. 1(d)**

is shown in Fig. 2. We establish that the upper and lower bounds of the MICS problem for the SBG are ten and nine respectively. Furthermore, we also establish that there exist at least 26 different Identifying Code Sets of size ten in the SBG.

In the last few years a number of researchers have studied Identifying Codes and its applications in sensor network domains. Karpovsky *et. al.* [1] introduced the concept of Identifying Codes in [1] and provided results for Identifying Codes for graphs with specific topologies, such as binary cubes and trees. Using Identifying Codes, Laifenfeld *et. al.* studied covering problems in [2] and joint monitoring and routing in wireless sensor networks in [3]. Ray *et. al.* in [4] generalized the concept of Identifying Codes, to incorporate robustness properties to deal with faults in sensor networks. Charon *et. al.* in [5, 6], studied complexity issues related to computation of minimum Identifying Codes for graph and showed that in several types of graph, the problem is NP-hard. Approximation algorithms for computation of Identifying Codes for some special types of graphs are presented in [7, 8]. Auger in [9] show that the problem can be solved in linear time if the graph happend to be a tree, but even for a planar graph the problem remains NP-complete.

Topological and combinatorial properties of soccer balls have been studied extensively in [10].

## 2 PROBLEM FORMULATION

A Soccer Ball Graph (SBG) $G = (V, E)$ is defined in the following way. The graph comprises of 32 nodes and 90 edges. The 32 nodes correspond to 32 patches (20 hexagonal and 12 pentagonal) of the soccer ball and two nodes in the graph have an edge between them if the corresponding patches share a boundary. A graph can have different layouts on a two dimensional plane. We show one layout of the SBG in Fig. 2 where the nodes are labeled using a set of rules. The soccer ball, placed on a two dimensional plane (as shown in Fig. 2 (a)), has a pentagonal patch on top. There are five hexagonal patches adjacent to this pentagonal patch. We consider a *layering scheme*, where the node corresponding to pentagonal patch on top is in Layer 1 (L1), the six nodes corresponding to six hexagonal patches adjacent to the pentagonal patch on top are in Layer 2 (L2) and so on. Following this layering scheme, all 32 nodes can be assigned to six layers, L1 through L6, as shown in Fig. 2 (b). In this scheme, one node is assigned to L1, five nodes to L2, ten nodes to L3, ten nodes to L4, five nodes to L5, and one node to L6. There is only one pentagonal node in layers L1 and L6 and we refer to these two nodes as $P_{1,1}$ and $P_{6,1}$ respectively. There are five hexagonal nodes in layers L2 and L5 and we refer to these nodes as $H_{2,i}, 1 \le i \le 5$ and $H_{5,i}, 1 \le i \le 6$ respectively. There are five hexagonal and five pentagonal nodes in layers L3 and L4 and we refer to these nodes as $H_{i,j}, i = 4, 5, 1 \le j \le 5$ and $P_{i,j}, i = 4, 5, 1 \le j \le 5$ respectively. The vertex set $V$ of the SBG, is divided into two subsets, $P$ (for Pentagon) and $H$ (for Hexagon), with 12 and 20 members respectively.

It may be noted from Fig. 2, that $P$-type nodes appear only on layers 1, 3, 4 and 6 and $H$-type nodes appear only on layers 2, 3, 4 and 5. The SBG $G = (V, E) = ((P \cup H), E)$ is formally defined as follows:

$P = \{P_{1,1}\} \cup \{P_{i,j}, 3 \le i \le 4, 1 \le j \le 5\} \cup \{P_{6,1}\}$ and

$H = \{H_{i,j}, 2 \le i \le 5, 1 \le j \le 5\}$

The edge set $E$ is divided into 17 subsets, i.e., $E = \cup_{i=1}^{17} E_i$. Each subset is defined in Table 2.

| SBG Edge Construction |
|---|
| $E_1 = \{(P_{1,1}, H_{2,j}), 1 \le j \le 5\}$ |
| $E_2 = \{(P_{6,1}, H_{5,j}), 1 \le j \le 5\}$ |
| $E_3 = \{(H_{i,j}, H_{i,(j+1) mod\ 5}), i = 2, i = 5, 1 \le j \le 5\}$ |
| $E_4 = \{(H_{i,j}, P_{i,j}), i = 3, 1 \le j \le 5\}$ |
| $E_5 = \{(P_{i,j}, H_{i,(j+1) mod\ 5}), i = 3, 1 \le j \le 5\}$ |
| $E_6 = \{(H_{i,j}, P_{i,(j+1) mod\ 5}), i = 4, 1 \le j \le 5\}$ |
| $E_7 = \{(P_{i,j}, H_{i,j}), i = 4, 1 \le j \le 5\}$ |
| $E_8 = \{(H_{2,j}, H_{3,j}), 1 \le j \le 5\}$ |
| $E_9 = \{(H_{2,j}, P_{3,(j-1) mod\ 5}), 1 \le j \le 5\}$ |
| $E_{10} = \{(H_{2,j}, P_{3,j}), 1 \le j \le 5\}$ |
| $E_{11} = \{(H_{3,j}, P_{4,j}), 1 \le j \le 5\}$ |
| $E_{12} = \{(H_{3,j}, H_{4,(j-1) mod\ 5}), 1 \le j \le 5\}$ |
| $E_{13} = \{(H_{3,j}, H_{4,j}), 1 \le j \le 5\}$ |
| $E_{14} = \{(P_{3,j}, H_{4,j}), 1 \le j \le 5\}$ |
| $E_{15} = \{(H_{4,j}, H_{5,j}), 1 \le j \le 5\}$ |
| $E_{16} = \{(P_{4,j}, H_{5,j}), 1 \le j \le 5\}$ |
| $E_{17} = \{(P_{4,j}, H_{5,(j-1) mod\ 5}), 1 \le j \le 5\}$ |

**Table 2: 17 subsets of the edge set $E$ of the SBG graph $G = (V, E) = ((P \cup H), E)$**

With the formal definition of the SBG complete, it may be observed that the problem of determining the fewest number of satellites necessary to uniquely identify the region (among 32 regions) where a significant event has taken place is equivalent to computation of the Minimum Identifying Code Set problem for the SBG.

`Graph Coloring with Seepage (GCS) Problem`: The MICS computation problem can be viewed as a novel variation of the classical Graph Coloring problem. We will refer to this version as the *Graph Coloring with Seepage (GCS)* problem. In the classical graph coloring problem, when a color is *assigned* (or injected) to a node, only that node is colored. The goal of the classical graph coloring problem to use as few distinct colors as possible such that (i) every node receives a color, and (ii) no two adjacent nodes of the graph have the same color. In the GCS problem, when a color is assigned (or injected) to a node, not only that node receives the color, but also the color *seeps* into all the adjoining nodes. As a node $v_i$ may be adjacent to two other nodes $v_j$ and $v_k$ in the graph, if the color red is injected to $v_j$, not only $v_j$ will become red, but also $v_i$ will become red as it is adjacent to $v_j$. Now if the color blue is injected to $v_k$, not only $v_k$ will become blue, but also the color blue will seep in to $v_i$ as it is adjacent to $v_k$. Since $v_i$ was already colored red (due to seepage from $v_j$), after color seepage from $v_k$, it's color will be a *combination of red and blue.* At this point all three nodes $v_i$, $v_j$ and $v_k$ have a color and all of them have distinct colors (red, blue and the combination of the two). *The color assigned to a node may be due to, (i) only injection to that node, (ii) only seepage from other adjoining nodes and (iii) a combination of injection and seepage.* The colors injected at the nodes will be referred to as *atomic* colors. The colors formed by the combination of two or more atomic colors are referred to as *composite* colors. The colors injected at the nodes (atomic colors) are all *unique.* The goal of the GCS problem is to inject colors to as few nodes as possible, such that (i) every node receives a color, and (ii) no two nodes of the graph have the same color.

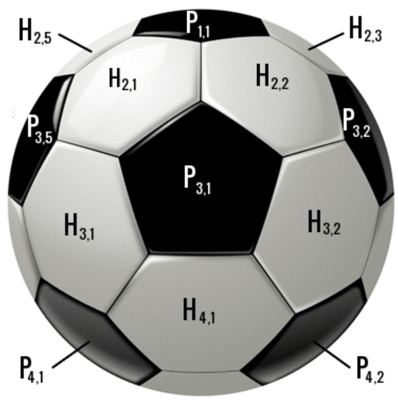
Suppose that the node set $V'$ is an ICS of a graph $G = (V, E)$ and $|V'| = p$. In this case if $p$ distinct colors are injected to $V'$ (one distinct atomic color to one node of $V'$ ), then as by the definition of ICS for all $v \in V$, if $N^+(v) \cap V'$ is unique, all nodes of $G = (V, E)$ will have a unique color (either atomic or composite). Thus computation of MICS is equivalent to solving the GCS problem.

## 3 UPPER BOUND OF MICS OF SBG

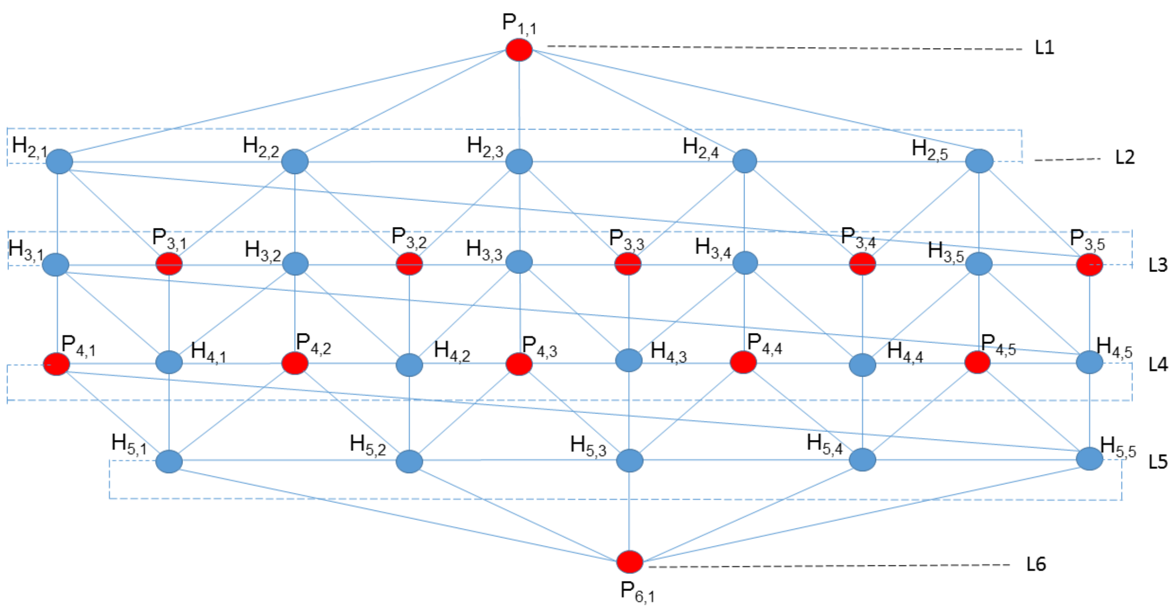
In this section, we first show that MICS of the SBG is at most 10 and there exists at least 26 ICS of size ten.

Theorem 3.1. *The MICS of SBG is at most ten.*

Proof. Inject colors $A, B, C, D, E$ to the nodes $H_{2,j}, 1 \le j \le 5$ and colors $E, F, G, H, I, J$ to the nodes $H_{5,j}, 1 \le j \le 5$. Injection of 10 different colors at these 10 nodes, will cause color seepage to all other nodes of SBG. The color seepage will be constrained by the topological structure of the SBG. It may be verified that because of the constraint imposed by the SBG structure, and the fact that




(a) Nodes corresponding to the patches on the soccer ball




(b) Soccer Ball Graph

**Figure 2: Soccer Ball and the corresponding graph**

seepage takes place only to the neighbors of the node where a color is injected, the 32 nodes of the SBG will have the color assignment shown in Table 3. In the entries of Table 3, $H_{2,1} : A^*BE$ implies that the color $A$ was *injected* at the node $H_{2,1}$ and the colors $B$ and $E$ *seeped* into the node $H_{2,1}$, from the adjacent nodes $H_{2,2}$ and $H_{2,5}$ respectively, where the colors $B$ and $E$ were injected. In general, if an alphabet $A$ through $E$ (representing distinct colors), appears *with* a * as a part of a string attached to a node (such as $H_{2,1}$), it implies that the color was *injected* at that node. On the other hand, if an alphabet appears *without* a * as a part of a string attached to a node, it implies that the color *seeped* into that node from one of the neighboring nodes. It may be verified that the color assignment to the nodes, as shown in Table 3 is *unique* (i.e, no two nodes have the same color or *strings* assigned to them). □

Theorem 3.2. *At least 26 distinct ICSs of size ten exist for SBG.*

Proof. The 26 different ways in which ten colors can be injected into ten nodes of the SBG such that every node of the SBG receives a unique color can be divided into four classes.

• *Class I:* Inject colors $A, B, C, D, E$ to the nodes $H_{2,j}, 1 \leq j \leq 5$ and colors $E, F, G, H, I, J$ to the nodes $H_{5,j}, 1 \leq j \leq 5$. As shown

| Node: Color | Node: Color | Node: Color | Node: Color |
|---|---|---|---|
| $P_{1,1}$: $ABCDE$ | $H_{2,1}$: $A^*BE$ | $H_{2,2}$: $AB^*C$ | $H_{2,3}$: $BC^*D$ |
| $H_{2,4}$: $CD^*E$ | $H_{2,5}$: $DE^*A$ | $H_{3,1}$: $A$ | $P_{3,1}$: $AB$ |
| $H_{3,2}$: $B$ | $P_{3,2}$: $BC$ | $H_{3,3}$ : $C$ | $P_{3,3}$: $CD$ |
| $H_{3,4}$: $D$ | $P_{3,4}$: $DE$ | $H_{3,5}$ : $E$ | $P_{3,5}$: $AE$ |
| $P_{4,1}$: $JF$ | $H_{4,1}$: $F$ | $P_{4,2}$ : $FG$ | $H_{4,2}$: $G$ |
| $P_{4,3}$: $GH$ | $H_{4,3}$: $H$ | $P_{4,4}$ : $HI$ | $H_{4,4}$: $I$ |
| $P_{4,5}$: $IJ$ | $H_{4,5}$: $J$ | $H_{5,1}$: $JF^*G$ | $H_{5,2}$: $FG^*H$ |
| $H_{5,3}$: $GH^*I$ | $H_{5,4}$: $HI^*J$ | $H_{5,5}$: $IJ^*F$ | $P_{6,1}$: $FGHIJ$ |

**Table 3: Color assignment at nodes after seepage in the SBG**

in Table 3, such an injection ensures that each of the 32 nodes of the SBG receives a unique color. It may be observed that the node set where the colors are injected in this Class all have degree six, corresponding to hexagonal patches on the surface of the soccer ball. Only one ICS of the 26, belongs to Class I. It may be noted that as the nodes where colors are being injected correspond to the regions where satellites are being deployed, a permutation of the color set $A, B, C, D, E$ is not important here because if either color A or B is injected at node $H_{2,1}$, it implies that one satellite is deployed to monitor the region represented by node $H_{2,1}$.

• *Class II:* The node set where the colors injected are in this Class is made up of six nodes of degree five (corresponding to the pentagonal patches of the soccer ball) and four nodes of degree six (corresponding to the hexagonal patches of the soccer ball). This Class can be subdivided into two sub-classes and we will refer to them as *Class II-A* and *Class II-B* respectively. As seen in Fig. 2, the SBG graph is somewhat symmetric in the sense that the layers 4, 5 and 6 are close to being mirror images of layers 1, 2 and 3. Because of this symmetry, the Class II-A color injections are mirror images of the Class II-B color injection. Accordingly, in this section we will focus our discussion primarily on Class II-A, as color injection for class II-B be can be obtained easily from color injection in Class IIA. We introduce the notion of a *motif*, and by motif we imply a set of either P-type (degree five) or H-type (degree six) nodes. It will be clear from further discussion that the Class II-A solutions comprise of one P-type motif and one H-type motif. These two motifs *complement* each other to produce a solution together. The motif-pairs can be *slid* along the structure of the SBG to produce a set of five solutions that make up the Class II-A. The five solutions that make up the Class II-B can be constructed in a similar fashion.

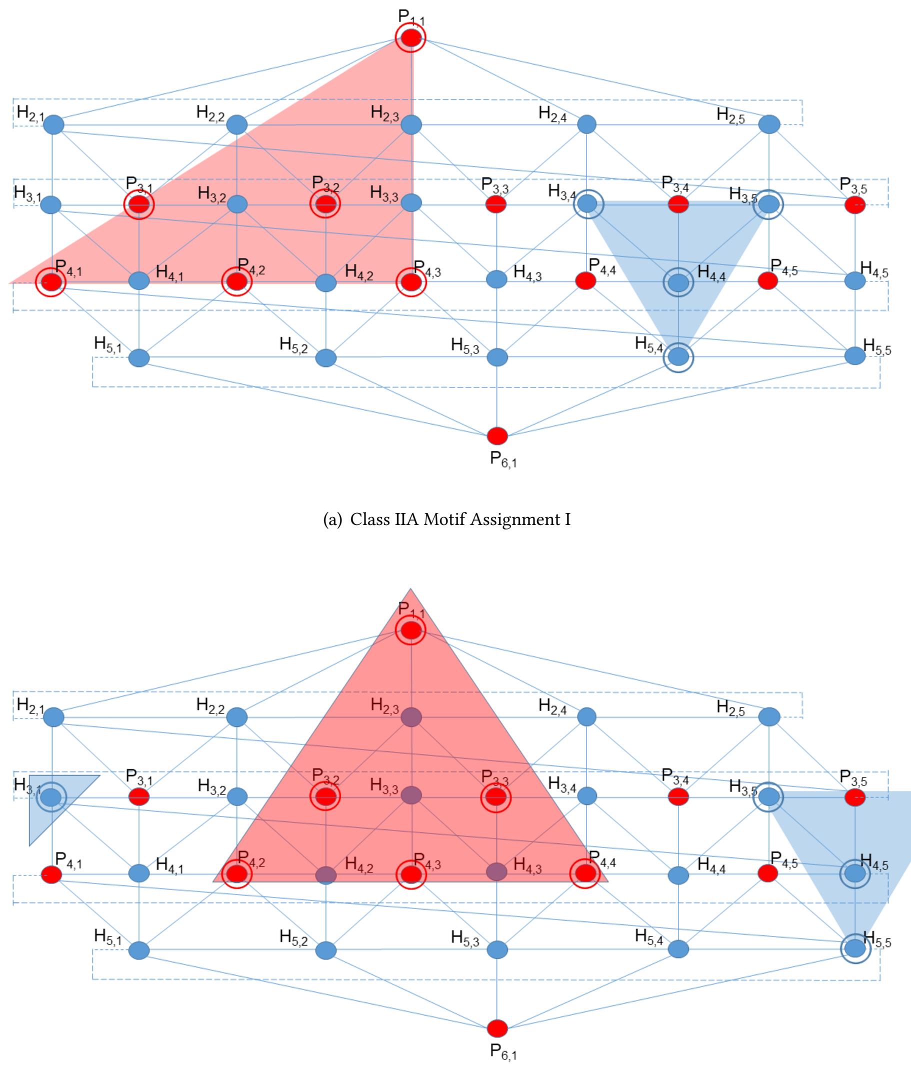
For the ICS that belong to Class II, the P-type motif is made up of the set of six nodes $\{P_{1,1}, P_{3,j}, P_{3,(j+1) mod\ 5}, P_{4,j}, P_{4,(j+1) mod\ 5}, P_{4,(j+2) mod\ 5}\}$. The H-type motif that complements the P-type motif is made up of the set of four nodes $\{H_{3,(j+3) mod\ 5}, H_{3,(j+4) mod\ 5}, H_{4,(j+3) mod\ 5}, H_{5,(j+3) mod\ 5}\}$. One complete solution (i.e., ICS) is obtained by choosing a value of $j$, $1 \le j \le 5$. The Fig. 3 (a) and (b) shows the solutions with $j = 1$ and $j = 2$ respectively. As shown in Fig. 3, changing the index $j$ from 1 to 2, has the effect of sliding the motif along the structure of the SBG. By changing $j$ from 1 through 5 (i.e., sliding the motif 5 times), 5 different ICS can be computed. The colors that will be associated with the nodes of the SBG, if they are injected at the motif nodes, are shown in Table 4. The first column of the table indicates the node and the second column provides the color assigned to that node. For example, in row 3 of Table 4, the node $H_{2,(j+1) mod\ 5}$ receives the colors injected at motif nodes $P_{1,1}, P_{3,j}, P_{3,(j+1) mod\ 5}$ and is denoted by $P^c_{1,1}P^c_{3,j}P^c_{3,(j+1) mod\ 5}$. It may be verified that every node of the SBG has a *color associated with it and no two nodes have the same color assignment.* For ease of verification, we have presented another Table 5 which may be viewed as an "inverse" of Table 4, in the sense that, the first column provides the color assigned to a node and the second column represents the corresponding node. As the color associated with a node may be viewed as a string $P^c_{1,1}P^c_{3,j}P^c_{3,(j+1) mod\ 5}$, we have presented them in the *lexicographic* order. The verification of the fact that every node of SBG receives a unique color is much simpler now, as one has to verify only among *similar* strings (made up of H only, P only or combination of H and P) of *same length.*

• *Class III:* As in Class II, the Class III ICS is made up of six nodes of degree five and four nodes of degree six. Moreover, this Class also can be subdivided into two sub-classes and we will refer to them as Class III-A and Class III-B respectively. In this section we will restrict our discussion on Class III-A, as color injection for Class III-B can be obtained as a mirror image of Class III-A. It will be clear from further discussion that, as in Class II, the Class III solutions also comprise of one P-type motif and one H-type motif and they complement each other to produce a solution together. As in Class II, the motif-pairs can be slid along the structure of the SBG to produce a set of five solutions that make up the Class III-A. The five solutions that make up the Class III-B can be constructed in a similar fashion.

The P-type motif is made up of the set of six nodes $\{P_{1,1}, P_{3,j}, P_{3,(j+1) mod\ 5}, P_{3,(j+2) mod\ 5}, P_{3,(j+4) mod\ 5}, P_{4,(j+1) mod\ 5}\}$. As shown in Fig. 4(a), changing the index $j$ from 1 to 5, has the effect of sliding the motif along the structure of the SBG. The H-type motif that complements the P-type motif is made up of the set of four nodes $\{H_{4,(j+3) mod\ 5}, H_{5,(j+2) mod\ 5}, H_{5,(j+3) mod\ 5}, H_{5,(j+4) mod\ 5}\}$. One complete solution is obtained by choosing a value of $j$, $1 \le j \le 5$. By moving the P-type and H-type motifs in tandem by changing the value of the index from 1 to 5, five different solutions can be obtained. The colors that will be associated with the nodes of the SBG, if the they are injected at the motif nodes, are shown in Table 6. As in the case of Class II, we present an "inverse" of Table 6 (Table 7), for the purpose of verification that every node of the SBG has a *unique color.*

• *Class IV:* As in Class I, the Class IV ICS are made up of 10 nodes of degree six (i.e., the nodes corresponding to hexagonal patches). This Class comprises of five ICS and cannot be subdivided like in Classes II and III.

This class comprises of two H-type motifs made up of five hexagonal nodes each. The first motif comprises of $\{H_{2,(j+1) mod\ 5}, H_{2,(j+2) mod\ 5}, H_{3,(j+1) mod\ 5}, H_{3,(j+2) mod\ 5}, H_{4,(j+1) mod\ 5}\}$. The other motif comprises of $\{H_{3,(j+4) mod\ 5}, H_{4,(j+3) mod\ 5}, H_{4,(j+4) mod\ 5}, H_{5,(j+3) mod\ 5}, H_{5,(j+4) mod\ 5}\}$. As shown in Fig. 4(b), changing the index $j$ from 1 to 5, has the effect of sliding the motif along the structure of the SBG. One complete solution is obtained by choosing a value of $j$, $1 \le j \le 5$. By moving two H-type motifs in tandem, changing the value of the index from 1 to 5, five different solutions can be obtained. The colors that will be associated with the nodes of the SBG, if the they are injected at the motif nodes, are shown in



(a) Class IIA Motif Assignment I

(b) Class IIA Motif Assignment II (Assignment I shifted one position to the right)

**Figure 3: Examples of Color Assignments using Motif IIA**

Table ??. As in the case of Classes II and III, we present an "inverse" of Table ?? (Table ??) for the purpose of verification that every node of the SBG has a *unique color*.

This concludes proof of Theorem 2.

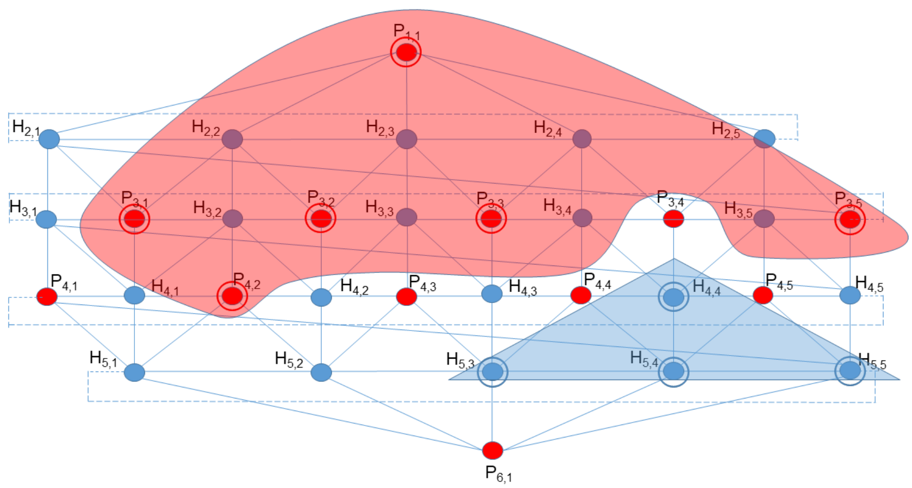
□

## 4 LOWER BOUND OF MICS OF SBG

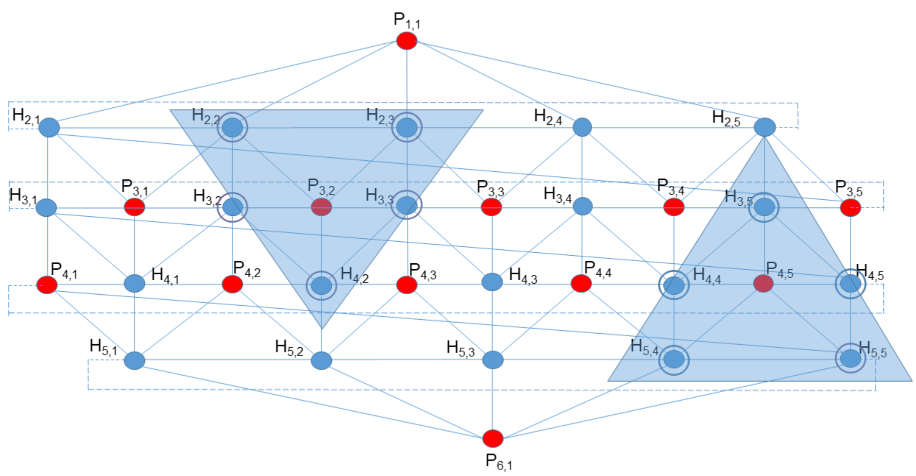
In Fig. 2, we have provided a *layered* representation of the SBG, where 32 nodes of the SBG is placed in six layers, indicated by L1 through L6. The layers L1 through L3 constitute the *top half* of the SBG and the layers L4 through L6 constitute the *bottom half*. As the two halves are symmetric, similar argument can be applied to both of them.

Lemma 4.1. *A MICS must select at least four nodes from each half. In other words, at least four distinct colors need to be injected in each half.*

Proof. We provide arguments for the top half of the SBG, and we first show that *at least* three distinct colors are necessary to ensure that each node in top half receives a distinct color (either through injection, seepage or combination of the two). Let's consider layers L1 and L2. No matter which nodes in bottom half are injected with colors, these colors will not seep into the nodes in L1 and L2 (colors




(a) Class IIIA Motif Assignment




(b) Class IV Motif Assignment

**Figure 4: Examples of Color Assignments using Motifs IIIA and IV**

seep only to adjacent nodes), coloring in bottom half nodes will not affect the colors associated with nodes in L1 or L2. Since L1 and L2 have six nodes, six distinct colors need to be associated with them. It can be easily verified that in the SBG, in order to ensure distinct colors to each one of the six nodes in L1 and L2, at least three colors must be injected to three nodes in the top half. Clearly at least three nodes must be selected from each half so that nodes in L1, L2, L5 and L6 receive distinct colors. With injection of three colors, up to $2^3 - 1 = 7$ colors (excluding an empty combination) can be generated.

Next we show that three colors are not sufficient to color L1 and L2. WLOG, we use alphabets $\{A, B, C\}$ to represent three colors. As mentioned above, seven distinct colors can be generated with these three colors (three primary and four composite), $\{A, B, C, AB, AC, BC, ABC\}$. Simple counting shows that each alphabet (color) appears exactly 4 times. Suppose there is a proper injection using $A, B, C$ that ensures all nodes in L1 and L2 received distinct colors. Since seven distinct colors can be generated with three primary colors, and L1 and L2 has only six nodes, it implies that one of the seven colors (primary or composite) is not used while coloring the nodes of L1 and L2. This implies that at least one of the alphabets $A, B, C$ is appearing three times instead of four in the alphabet strings (representing colors) associated with the nodes of L1 and L2. WLOG *we* *assume that color A is appearing three times.* There are four possible locations for injection of color $A$ in the top half of the SBG,

(1) $A$ is injected on L1, i.e., at $P_{1,1}$. $A$ would then appear at all nodes in L1 and L2, making its appearance six times, contradicting the assumption.
(2) $A$ is injected on L2, i.e., one of $H_{2,i}(1 \leq i \leq 5)$ nodes. Thus, $A$ appears four times (three nodes in L2 and one node in L1) contradicting the assumption.
(3) $A$ is injected on one of the hexagonal nodes L3, i.e., one of the $H_{3,i}$, $1 \leq i \leq 5$. Since $H_{3,i}$ has only one neighbor in on L1 and L2 ($H_{2,i}$), in this case $A$ will appear only on one node in L2, making its appearance one time, contradicting the assumption.
(4) $A$ is injected on one of the pentagonal nodes L3, i.e., one of the $P_{3,i}$, $1 \leq i \leq 5$. Since $P_{3,i}$ has only two neighbors in on L1 and L2 ($H_{2,(i-1) mod\ 5}$ and $H_{2,i}$), in this case $A$ will appear only on two nodes in L2, making its appearance two times, contradicting the assumption.

As there is no location for injection of $A$, we can conclude that three colors are inadequate to ensure that all nodes in L1 and L2 receive a unique color. Similar arguments can be made for coloring of nodes in L5 and L6. Therefore the lower bound of MICS for the SBG must be at least $4 + 4 = 8$. □

Lemma 4.2. *MICS of the SBG is at least nine.*

Proof. In the GCS problem, each node is assigned a color, which may be a primary or a composite color. A primary color is indicated by one alphabet and a composite color by a *string* of alphabets. The number of alphabets that appear in a string determines the *length* of that string. We establish the lemma by providing arguments based on the sum of the length of strings associated with all 32 nodes of the SBG. We will refer to the sum of the length of strings associated with each one of the 32 nodes of the SBG as "total string length".

We use the term "valid injection" to imply an injection of colors to the nodes that ensures that all 32 nodes of the SBG receive a distinct color. Suppose, there exists more than one valid injection using eight colors. Among the set of all valid injections, we consider the one whose total string length is minimum. The lower bound of the total string length for a valid injection with eight colors is 56. This is true, as with eight injected colors, at most eight nodes of the SBG can have associated strings of length one, and the remaining 24 nodes must have strings of length at least two. Thus the lower bound on the string length must be $8 \times 1 + 24 \times 2 = 56$. The upper bound on the total string length with injection of at most eight colors is also 56. This is true for the following reason. If a color is injected on a hexagonal node, then it will appear seven times (six neighbors and the node itself). Similarly for a pentagonal node, the color will appear six times. Therefore, the upper bound of total string length is $7 \times 8 = 56$. It may be noted that the total string length is 56 if and only if all colors are injected on hexagonal nodes. However, it is impossible to achieve a valid injection by injecting eight colors only on hexagonal nodes. Consider the top half of SBG. In order to color nodes on L1, at least one color, say $A$, must be injected on one node on L2. WLOG, we assume that $A$ is injected on $H_{2,i}$, $1 \leq i \leq 5$. We consider two scenarios:

(1) No other color is injected at the nodes on L2. In this case, the other colors are injected at three hexagonal nodes on L3. Because of injection of $A$ at $H_{2,i}$, after seepage, all six adjacent nodes, $P_{1,1}, H_{2,(i-1) mod\ 5}, H_{2,(i+1) mod\ 5}$, $H_{3,(i-1) mod\ 5}, P_{3,i}, H_{3,i}$, will have color $A$. In order to ensure that all these nodes have distinct colors, colors must be injected on $H_{3,(i-1) mod\ 5}, H_{3,i}$, $H_{3,(i+1) mod\ 5}$. However, in such an injection, the nodes $H_{2,(i+2) mod\ 5}$ and $H_{2,(i+3) mod\ 5}$ will not receive any color, making such an injection invalid.
(2) One or more colors are injected at the nodes on L2. Suppose a different color $B$ is injected at a node different from $H_{2,i}$. Due to the SBG topology, no matter which node on L2 is injected with $B$, one node on L2 and the node on L1 must have color $AB$ after seepage. In order to ensure distinct colors on these two nodes, a third color $C$ must be injected on another node. After injection of $C$, one of the two nodes that had the color $AB$ before injection of $C$, will have the color $AB$ and the other will have $ABC$. However, if one node has a string of length three, the lower bound of the total string length can longer be 56. It has to be at least 57, thus exceeding the upper bound (56), that is possible with injection of at most eight colors.

□

Theorem 4.3. *The lower bound of MICS of the SBG is at least nine i.e., eight colors are insufficient to ensure that all nodes of the SBG receive a distinct color.*

Proof. Follows from Lemmas 1 and 2. □

## 5 CONCLUSION

We have studied an event monitoring problem with satellites as sensors and a soccer ball as a model of the planet Earth. We have provided upper and lower bound of the MICS problem where the difference between the bounds is just one, implying that our solution is close to being optimal.

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| Node | Color | Node | Color |
|---|---|---|---|
| $P_{1,1}$ | $P^c_{1,1}$ | $P_{4,j}$ | $P^c_{4,j}$ |
| $H_{2,j}$ | $P^c_{1,1}P^c_{3,j}$ | $H_{4,j}$ | $P^c_{3,j}P^c_{4,j}P^c_{4,(j+1)mod\ 5}$ |
| $H_{2,(j+1)mod\ 5}$ | $P^c_{1,1}P^c_{3,j}P^c_{3,(j+1)mod\ 5}$ | $P_{4,(j+1)mod\ 5}$ | $P^c_{4,(j+1)mod\ 5}$ |
| $H_{2,(j+2)mod\ 5}$ | $P^c_{1,1}P^c_{3,(j+1)mod\ 5}$ | $H_{4,(j+1)mod\ 5}$ | $P^c_{3,(j+1)mod\ 5}P^c_{4,(j+1)mod\ 5}P^c_{4,(j+2)mod\ 5}$ |
| $H_{2,(j+3)mod\ 5}$ | $P^c_{1,1}H^c_{3,(j+3)mod\ 5}$ | $P_{4,(j+2)mod\ 5}$ | $P^c_{4,(j+2)mod\ 5}$ |
| $H_{2,(j+4)mod\ 5}$ | $P^c_{1,1}H^c_{3,(j+4)mod\ 5}$ | $H_{4,(j+2)mod\ 5}$ | $P^c_{4,(j+2)mod\ 5}H^c_{3,(j+3)mod\ 5}$ |
| $H_{3,j}$ | $P^c_{3,j}P^c_{4,j}$ | $P_{4,(j+3)mod\ 5}$ | $H^c_{3,(j+3)mod\ 5}H^c_{4,(j+3)mod\ 5}H^c_{5,(j+3)mod\ 5}$ |
| $P_{3,j}$ | $P^c_{3,j}$ | $H_{4,(j+3)mod\ 5}$ | $H^c_{4,(j+3)mod\ 5}H^c_{3,(j+3)mod\ 5}$<br>$H^c_{3,(j+4)mod\ 5}H^c_{5,(j+3)mod\ 5}$ |
| $H_{3,(j+1)mod\ 5}$ | $P^c_{3,j}P^c_{3,(j+1)mod\ 5}P^c_{4,(j+1)mod\ 5}$ | $P_{4,(j+4)mod\ 5}$ | $H^c_{4,(j+4)mod\ 5}H^c_{3,(j+3)mod\ 5}H^c_{5,(j+3)mod\ 5}$ |
| $P_{3,(j+1)mod\ 5}$ | $P^c_{3,(j+1)mod\ 5}$ | $H_{4,(j+4)mod\ 5}$ | $P^c_{4,j}H^c_{4,(j+4)mod\ 5}$ |
| $H_{3,(j+2)mod\ 5}$ | $P^c_{3,(j+1)mod\ 5}P^c_{4,(j+2)mod\ 5}$ | $H_{5,j}$ | $P^c_{4,j}P^c_{4,(j+1)mod\ 5}$ |
| $P_{3,(j+2)mod\ 5}$ | $H^c_{3,(j+3)mod\ 5}$ | $H_{5,(j+1)mod\ 5}$ | $P^c_{4,(j+1)mod\ 5}P^c_{4,(j+2)mod\ 5}$ |
| $H_{3,(j+3)mod\ 5}$ | $H^c_{3,(j+3)mod\ 5}H^c_{4,(j+3)mod\ 5}$ | $H_{5,(j+2)mod\ 5}$ | $P^c_{4,(j+2)mod\ 5}H^c_{5,(j+3)mod\ 5}$ |
| $P_{3,(j+3)mod\ 5}$ | $H^c_{3,(j+3)mod\ 5}H^c_{4,(j+3)mod\ 5}H^c_{3,(j+4)mod\ 5}H_{5,(j+3)mod\ 5}$ | $H^c_{5,(j+3)mod\ 5}$ | $H^c_{4,(j+3)mod\ 5}$ |
| $H_{3,(j+4)mod\ 5}$ | $H^c_{3,(j+4)mod\ 5}H^c_{4,(j+3)mod\ 5}$ | $H_{5,(j+4)mod\ 5}$ | $P^c_{4,j}H^c_{5,(j+3)mod\ 5}$ |
| $P_{3,(j+4)mod\ 5}$ | $H^c_{4,(j+4)mod\ 5}$ | $P_{6,1}$ | $H^c_{5,(j+3)mod\ 5}$ |

**Table 4: Node versus Color assignment for Class II-A ICS**

| Color | Node | Color | Node |
|---|---|---|---|
| $H^c_{3,(j+3)mod\ 5}$ | $P_{3,(j+2)mod\ 5}$ | $P^c_{1,1}$ | $P_{1,1}$ |
| $H^c_{4,(j+4)mod\ 5}$ | $P_{3,(j+4)mod\ 5}$ | $P^c_{3,j}$ | $P_{3,j}$ |
| $H^c_{5,(j+3)mod\ 5}$ | $P_{6,1}$ | $P^c_{4,j}$ | $P_{4,j}$ |
| $H^c_{3,(j+3)mod\ 5}H^c_{4,(j+3)mod\ 5}$ | $H_{3,(j+3)mod\ 5}$ | $P^c_{3,(j+1)mod\ 5}$ | $P_{3,(j+1)mod\ 5}$ |
| $H^c_{3,(j+4)mod\ 5}H^c_{4,(j+3)mod\ 5}$ | $H_{3,(j+4)mod\ 5}$ | $P^c_{4,(j+1)mod\ 5}$ | $P_{4,(j+1)mod\ 5}$ |
| $H^c_{5,(j+3)mod\ 5}H^c_{4,(j+3)mod\ 5}$ | $H_{5,(j+3)mod\ 5}$ | $P^c_{4,(j+2)mod\ 5}$ | $P_{4,(j+2)mod\ 5}$ |
| $H^c_{3,(j+3)mod\ 5}H^c_{4,(j+3)mod\ 5}H^c_{3,(j+4)mod\ 5}$ | $P_{3,(j+3)mod\ 5}$ | $P^c_{1,1}P^c_{3,j}$ | $H_{2,j}$ |
| $H^c_{3,(j+3)mod\ 5}H^c_{4,(j+3)mod\ 5}H^c_{5,(j+3)mod\ 5}$ | $P_{4,(j+3)mod\ 5}$ | $P^c_{1,1}P^c_{3,(j+1)mod\ 5}$ | $H_{2,(j+2)mod\ 5}$ |
| $H^c_{4,(j+4)mod\ 5}H^c_{3,(j+3)mod\ 5}H^c_{5,(j+3)mod\ 5}$ | $P_{4,(j+4)mod\ 5}$ | $P^c_{3,j}P^c_{4,j}$ | $H_{3,j}$ |
| $H^c_{4,(j+3)mod\ 5}H^c_{3,(j+3)mod\ 5}H^c_{3,(j+4)mod\ 5}H^c_{5,(j+3)mod\ 5}$ | $H_{4,(j+3)mod\ 5}$ | $P^c_{3,(j+1)mod\ 5}P^c_{4,(j+2)mod\ 5}$ | $H_{3,(j+2)mod\ 5}$ |
| $H^c_{3,(j+3)mod\ 5}P^c_{1,1}$ | $H_{2,(j+3)mod\ 5}$ | $P^c_{4,j}P^c_{4,(j+1)mod\ 5}$ | $H_{5,j}$ |
| $H^c_{3,(j+4)mod\ 5}P^c_{1,1}$ | $H_{2,(j+4)mod\ 5}$ | $P^c_{4,(j+1)mod\ 5}P^c_{4,(j+2)mod\ 5}$ | $H_{5,(j+1)mod\ 5}$ |
| $H^c_{3,(j+3)mod\ 5}P^c_{4,(j+2)mod\ 5}$ | $H_{4,(j+2)mod\ 5}$ | $P^c_{1,1}P^c_{3,j}P^c_{3,(j+1)mod\ 5}$ | $H_{2,(j+1)mod\ 5}$ |
| $H^c_{4,(j+4)mod\ 5}P^c_{4,j}$ | $H_{4,(j+4)mod\ 5}$ | $P^c_{3,j}P^c_{3,(j+1)mod\ 5}P^c_{4,(j+1)mod\ 5}$ | $H_{3,(j+1)mod\ 5}$ |
| $H^c_{5,(j+3)mod\ 5}P^c_{4,(j+2)mod\ 5}$ | $H_{5,(j+2)mod\ 5}$ | $P^c_{3,j}P^c_{4,j}P^c_{4,(j+1)mod\ 5}$ | $H_{4,j}$ |
| $H^c_{5,(j+3)mod\ 5}P^c_{4,j}$ | $H_{5,(j+4)mod\ 5}$ | $P^c_{3,(j+1)mod\ 5}P^c_{4,(j+1)mod\ 5}P^c_{4,(j+2)mod\ 5}$ | $H_{4,(j+1)mod\ 5}$ |

**Table 5: Color versus Node assignment for Class II-A ICS**

The tables for Class IV are similar to the tables for Class II and Class III, and have been left out due to lack of space.

| Node | Color | Node | Color |
|---|---|---|---|
| $P_{1,1}$ | $P^c_{1,1}$ | $P_{4,j}$ | $H^c_{5,(j+4) mod\ 5}$ |
| $H_{2,j}$ | $P^c_{1,1}P^c_{3,j}P^c_{3,(j+4) mod\ 5}$ | $H_{4,j}$ | $P^c_{4,(j+1) mod\ 5}P^c_{3,j}$ |
| $H_{2,(j+1) mod\ 5}$ | $P^c_{1,1}P^c_{3,j}P^c_{3,(j+1) mod\ 5}$ | $P_{4,(j+1) mod\ 5}$ | $P^c_{4,(j+1) mod\ 5}$ |
| $H_{2,(j+2) mod\ 5}$ | $P^c_{1,1}P^c_{3,(j+1) mod\ 5}P^c_{3,(j+2) mod\ 5}$ | $H_{4,(j+1) mod\ 5}$ | $P^c_{4,(j+1) mod\ 5}P^c_{3,(j+1) mod\ 5}$ |
| $H_{2,(j+3) mod\ 5}$ | $P^c_{1,1}P^c_{3,(j+2) mod\ 5}$ | $P_{4,(j+2) mod\ 5}$ | $H^c_{5,(j+2) mod\ 5}$ |
| $H_{2,(j+4) mod\ 5}$ | $P^c_{1,1}P^c_{3,(j+4) mod\ 5}$ | $H_{4,(j+2) mod\ 5}$ | $P^c_{3,(j+2) mod\ 5}H^c_{5,(j+2) mod\ 5}$ |
| $H_{3,j}$ | $P^c_{3,j}P^c_{3,(j+4) mod\ 5}$ | $P_{4,(j+3) mod\ 5}$ | $H^c_{4,(j+3) mod\ 5}H^c_{5,(j+2) mod\ 5}H^c_{5,(j+3) mod\ 5}$ |
| $P_{3,j}$ | $P^c_{3,j}$ | $H_{4,(j+3) mod\ 5}$ | $H^c_{4,(j+3) mod\ 5}H^c_{5,(j+3) mod\ 5}$ |
| $H_{3,(j+1) mod\ 5}$ | $P^c_{3,j}P^c_{3,(j+1) mod\ 5}P^c_{4,(j+1) mod\ 5}$ | $P_{4,(j+4) mod\ 5}$ | $H^c_{4,(j+3) mod\ 5}H^c_{5,(j+3) mod\ 5}H^c_{5,(j+4) mod\ 5}$ |
| $P_{3,(j+1) mod\ 5}$ | $P^c_{3,(j+1) mod\ 5}$ | $H_{4,(j+4) mod\ 5}$ | $P^c_{3,(j+4) mod\ 5}H^c_{5,(j+4) mod\ 5}$ |
| $H_{3,(j+2) mod\ 5}$ | $P^c_{3,(j+1) mod\ 5}P^c_{3,(j+2) mod\ 5}$ | $H_{5,j}$ | $H^c_{5,(j+4) mod\ 5}P^c_{4,(j+1) mod\ 5}$ |
| $P_{3,(j+2) mod\ 5}$ | $P^c_{3,(j+2) mod\ 5}$ | $H_{5,(j+1) mod\ 5}$ | $H^c_{5,(j+2) mod\ 5}P^c_{4,(j+1) mod\ 5}$ |
| $H_{3,(j+3) mod\ 5}$ | $P^c_{3,(j+2) mod\ 5}H^c_{4,(j+3) mod\ 5}$ | $H_{5,(j+2) mod\ 5}$ | $H^c_{5,(j+2) mod\ 5}H^c_{5,(j+3) mod\ 5}$ |
| $P_{3,(j+3) mod\ 5}$ | $H^c_{4,(j+3) mod\ 5}$ | $H_{5,(j+3) mod\ 5}$ | $H^c_{5,(j+3) mod\ 5}H^c_{5,(j+4) mod\ 5}H^c_{5,(j+2) mod\ 5}H^c_{4,(j+3) mod\ 5}$ |
| $H_{3,(j+4) mod\ 5}$ | $H^c_{4,(j+3) mod\ 5}P^c_{3,(j+4) mod\ 5}$ | $H_{5,(j+4) mod\ 5}$ | $H^c_{5,(j+4) mod\ 5}H^c_{5,(j+3) mod\ 5}$ |
| $P_{3,(j+4) mod\ 5}$ | $P^c_{3,(j+4) mod\ 5}$ | $P_{6,1}$ | $H^c_{5,(j+2) mod\ 5}H^c_{5,(j+3) mod\ 5}H^c_{5,(j+4) mod\ 5}$ |

**Table 6: Node versus color assignment for Class III-A ICS**

| Color | Node | Color | Node |
|---|---|---|---|
| $H^c_{4,(j+3) mod\ 5}$ | $P_{3,(j+3) mod\ 5}$ | $P^c_{1,1}$ | $P_{1,1}$ |
| $H^c_{5,(j+4) mod\ 5}$ | $P_{4,j}$ | $P^c_{3,j}$ | $P_{3,j}$ |
| $H^c_{5,(j+2) mod\ 5}$ | $P_{4,(j+2) mod\ 5}$ | $P^c_{3,(j+1) mod\ 5}$ | $P_{3,(j+1) mod\ 5}$ |
| $H^c_{5,(j+2) mod\ 5}H^c_{5,(j+3) mod\ 5}$ | $H_{5,(j+2) mod\ 5}$ | $P^c_{3,(j+2) mod\ 5}$ | $P_{3,(j+2) mod\ 5}$ |
| $H^c_{5,(j+4) mod\ 5}H^c_{5,(j+3) mod\ 5}$ | $H_{5,(j+4) mod\ 5}$ | $P^c_{3,(j+4) mod\ 5}$ | $P_{3,(j+4) mod\ 5}$ |
| $H^c_{4,(j+3) mod\ 5}H^c_{5,(j+3) mod\ 5}$ | $H_{4,(j+3) mod\ 5}$ | $P^c_{4,(j+1) mod\ 5}$ | $P_{4,(j+1) mod\ 5}$ |
| $H^c_{4,(j+3) mod\ 5}H^c_{5,(j+2) mod\ 5}H^c_{5,(j+3) mod\ 5}$ | $P_{4,(j+3) mod\ 5}$ | $P^c_{1,1}P^c_{3,(j+2) mod\ 5}$ | $H_{2,(j+3) mod\ 5}$ |
| $H^c_{4,(j+3) mod\ 5}H^c_{5,(j+3) mod\ 5}H^c_{5,(j+4) mod\ 5}$ | $P_{4,(j+4) mod\ 5}$ | $P^c_{1,1}P^c_{3,(j+4) mod\ 5}$ | $H_{2,(j+4) mod\ 5}$ |
| $H^c_{5,(j+2) mod\ 5}H^c_{5,(j+3) mod\ 5}H^c_{5,(j+4) mod\ 5}$ | $P_{6,1}$ | $P^c_{3,j}P^c_{3,(j+4) mod\ 5}$ | $H_{3,j}$ |
| $H^c_{5,(j+3) mod\ 5}H^c_{5,(j+4) mod\ 5}H^c_{5,(j+2) mod\ 5}H^c_{4,(j+3) mod\ 5}$ | $H_{5,(j+3) mod\ 5}$ | $P^c_{3,(j+1) mod\ 5}P^c_{3,(j+2) mod\ 5}$ | $H_{3,(j+2) mod\ 5}$ |
| $H^c_{4,(j+3) mod\ 5}P^c_{3,(j+2) mod\ 5}$ | $H_{3,(j+3) mod\ 5}$ | $P^c_{4,(j+1) mod\ 5}P^c_{3,j}$ | $H_{4,j}$ |
| $H^c_{4,(j+3) mod\ 5}P^c_{3,(j+4) mod\ 5}$ | $H_{3,(j+4) mod\ 5}$ | $P^c_{4,(j+1) mod\ 5}P^c_{3,(j+1) mod\ 5}$ | $H_{4,(j+1) mod\ 5}$ |
| $H^c_{5,(j+2) mod\ 5}P^c_{3,(j+2) mod\ 5}$ | $H_{4,(j+2) mod\ 5}$ | $P^c_{1,1}P^c_{3,j}P^c_{3,(j+4) mod\ 5}$ | $H_{2,j}$ |
| $H^c_{5,(j+4) mod\ 5}P^c_{3,(j+4) mod\ 5}$ | $H_{4,(j+4) mod\ 5}$ | $P^c_{1,1}P^c_{3,j}P^c_{3,(j+1) mod\ 5}$ | $H^c_{2,(j+1) mod\ 5}$ |
| $H^c_{5,(j+4) mod\ 5}P^c_{4,(j+1) mod\ 5}$ | $H_{5,j}$ | $P^c_{1,1}P^c_{3,(j+1) mod\ 5}P^c_{3,(j+2) mod\ 5}$ | $H_{2,(j+2) mod\ 5}$ |
| $H^c_{5,(j+2) mod\ 5}P^c_{4,(j+1) mod\ 5}$ | $H_{5,(j+1) mod\ 5}$ | $P^c_{3,j}P^c_{3,(j+1) mod\ 5}P^c_{4,(j+1) mod\ 5}$ | $H_{3,(j+1) mod\ 5}$ |

**Table 7: Color versus Node assignment for Class III-A ICS**