3-10-2016

PRACTICAL - 10

AIM: To estimate the parameter by the method of successive approximation.

PROBLEM: For $m = 4$, draw a random sample of size 11 for the Cauchy population

$$f(x) = \frac{1}{\pi} \frac{1}{[1 + (x-m)^2]} \quad ; \quad -\infty < x < \infty$$

Taking the median of the sample observations as the first estimate of m, approximate successively to the true value. Also find the variance of the estimate.

THEORY AND FORMULAE:

$$Y = F(x) = \int_{-\infty}^{x} \frac{1}{\pi[1 + (t-m)^2]} \, dt$$

$$= \frac{1}{\pi} \Big[\tan^{-1}(t-m) \Big]_{-\infty}^{x}$$

$$= \frac{1}{\pi} \left[\tan^{-1}(x-m) + \frac{\pi}{2} \right]$$

$$\Rightarrow x = m + \tan \pi \left(y - \tfrac{1}{2}\right)$$ where y ~~values~~ takes values b/w 0 & 1.

On the basis of y_i, compute x_i & that will constitute sample ~~of x~~ from Cauchy distⁿ.

Taking the median of the x_i's as the first estimate of the parameter m & let it be $m^{(0)}$.

This $m^{(0)}$ may not be a good estimator and we correct it by taking an extra observation & we may get the new estimate as $m^{(1)}$.

The process is repeated until two successive values of m are nearly equal.

Now,

$$m^{(1)} = m^{(0)} + \frac{S_m^{(0)}}{I_m^{(0)}} \quad ; \quad m^{(2)} = m^{(1)} + \frac{S_m^{(1)}}{I_m^{(1)}} \quad \text{\& so on.}$$

where

$$S_m^{(i)} = \sum \frac{2(x - m^{(i)})}{1 + [x - m^{(i)}]^2} \quad \& \quad I_m^{(i)} = -E\left(\frac{\partial^2 \ln L}{\partial m^2}\right) = \frac{n}{2}.$$

CALCULATIONS :

y_i	$x_i = 4 + \tan[\pi(y_i - \frac{1}{2})]$	$\frac{2(x_i - m^{(0)})}{1+(x_i - m^{(0)})^2}$	$\frac{2(x_i - m^{(1)})}{1+(x_i - m^{(1)})^2}$
0.0674	-0.6489	-0.3817	-0.3646
0.4718	3.9113	-0.7810	-0.9708
0.7811	5.2172	0.9818	0.8212
0.1179	1.4251	-0.6053	-0.5594
0.5803	4.2578	-0.2636	-0.7332
0.8209	5.5855	0.9846	0.9934
0.6189	4.3920	0	-0.5540
0.7149	4.8007	0.7004	0.2104
0.0189	-12.7907	-0.1160	-0.1140
0.9777	18.2508	0.1436	0.1467
0.8019	5.3940	1	0.9395
		1.6627	-0.18193

Now, we have :

$m^{(0)} = \text{median} = 4.3920.$

$S^{(0)} = 1.6627$

$I_m^{(\ell)} = \frac{11}{2} = 5.5.$

$\Rightarrow m^{(1)} = 4.3920 + \frac{1.6627}{5.5} = 4.694 \simeq 4.7$

$S^{(1)} = -0.18193$

$\Rightarrow m^{(2)} = 4.694 + \frac{(-0.18193)}{5.5} = 4.661 \simeq 4.7.$

RESULT : We see that the estimates $m^{(1)}$ and $m^{(2)}$ are approximately equal. So, an estimate of m is given by:

$$\hat{m} = m^{(2)} = 4.661.$$