**New Heuristic For Solving Capacitated Lot Sizing Problem With Setups, Production, Inventory and Back Orders (CLSPWSIPB)**

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**Abstract**

Here we give standard 0-1 mixed integer linear program of CLSPSPIB. In this formulation we eliminate shortage variables and setup (0-1 integer) variables to pose the problem as ‘all real’ linear program in inventory and production variables. Based on its solution, we keep select variables as zeros or ones in the original CLSPSPIB. We get very good solutions in much reduced CPU time. Detailed computational experience is given.

**Key Words**

Capacitated Lot Sizing Problem (CLSP), CLSPWSPIB, Heuristic

**1. Introduction And Literature Review**

For a thorough literature review on capacitated lot sizing problem reader is referred to Mayank Verma (2012). Recently Sharma and Sinha (2018) gave a new formulation of multi item capacitated lot sizing problem that has several advantages. This line was further towed by Parvathy (2018) who suggested a novel heuristic for getting good solutions to single item capacitated lot sizing problem. Md Amir (2020) showed (empirically) that the new formulation due to Sharma and Sinha (2018) has merit.

We give a novel heuristic approach to CLSPWSIPB that requires solving a LP. The solution of LP is used to selectively set 0-1 setup variables at zero or one. This leads to good solutions to original CLSPWSIPB at much less computational burden. Lot sizing problem with backorder, inventory, setup and production cost is posed as problem P below.

**2. Contribution**

Below we outline our new approach.

**Problem P (Original Problem):**

Min sum(t), [XP(t)\*CXP(t) + XINV(t)\*CINV(t) + CSETUP(t)\*y(t)+

XSHT(t)\*CSHT(t)] (1)

s.t.

XP(t) <= CAP(t)\*y(t) for all t (2)

XINV(0) + sum(t=1 to t1), XP(t) + XSHT(t1) = sum(1 to t1), DEM(t) + XINV(t1) (3)

XSHT(T) = 0 (4)

XINV(t), XP(t), XSHT(t) >= 0 for all t (5)

Y(t) = (0,1) binary for all t (6)

**Method 1:** Solve problem P by feeding it to GAMS (for T = 50: 20 problems). We record the following: CPU time taken, Objective Function Value for each of the problems.

Eliminate XSHT(t1) by using equation (3). Now the constraint in problem RP is (7):

sum(1 to t1), DEM(t) + XINV(t1) - sum(t=1 to t1), XP(t) - XINV(0) >= 0 (7)

This approach was first given by SM Ali and Sharma (2007) and in several papers due to RRK Sharma (2019, 2019, 2020, 2020, 2021, and 2022), see references. Sharma, Sinha and Verma (2018) have also deployed a similar approach.

next we put y(t) = XP(t)/CAP(t) for all t (8)

thus we have eliminated binary variable y(t) and real variable XSHT(t). Thus the reduced problem RP has less number of variables and has no binary variable y(t). It is expected to yield computational advantages.

**Reduced Problem RP:**

Min sum(t1 = 1 to T), [XP(t1)\*CXP(t1) + XINV(t1)\*CINV(t1) +

CSETUP(t1)\*XP(t1)/CAP(t1) + {sum(t=1 to t1), DEM(t) + XINV(t1) –

sum(t=1 to t1), XP(t) - XINV(0)}\*CSHT(t1)] (9)

XP(t) <= CAP(t) for all t (10)

sum(1 to t1), DEM(t) + XINV(t1) - sum(t=1 to t1), XP(t) - XINV(0) >= 0 (7)

XINV(t), XP(t) >= 0 for all t (11)

**Method 2:**

Feed RP to GAMS. Note down CPU time for each problem. Now compute XP(t)/CAP(t) for each ‘t’ and if XP(t)/CAP(t) >= 0.75 (an arbitrary number) assign that y(t) = 1. Let set of y(t) = 1 be SET\_y(t)\_ONES and its cardinality be N\_ONES\_Y (a positive integer).

If XP(t)/CAP(t) <= 0.15 (an arbitrary number) assign that y(t) = 0. Let set of y(t) = 0 be SET\_y(t)\_ZEROS and its cardinality be N\_ZEROS\_Y (a positive integer). Now solve following problem P1.

**Problem P1:**

Min (1)

s.t.

(2) to (6) and:

Sum(t1 belongs to SET\_y(t)\_ONES, y(t1) = N\_ONES\_Y (12)

Sum(t1 belongs to SET\_y(t)\_ZEROS, y(t1) = 0 (13)

Feed problem P1 to GAMS and record the following: CPU time, Objective function value. Please note method 2 is a heuristic and does not give ‘OPTIMAL’ solution but gives a good solution in attractive CPU time.

**3. Computational Experience**

The Computation results of the CPU Time and outputs corresponding to Objective functions are presented in Table 1. The sample statistics of the computational results are presented in table 2. The statistical results of comparing both the models using t-statistic and their significance is presented in table3. Looking at these results it is evident that model 2 is taking lesser CPU time. While there is no claim on the optimized output of the model 2 still the objective function’s output obtained from model 2 is statistically inferior than model 1 (refer Table 2/3).

**Table 1: Computation Results Obtained from Model 1 and Model 2**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **DatasetNo.** | **Model 1 CPU time** | **Model 1 Obj. Function** | **Model 2 CPU Time** | **Model 2 Obj. function** |
| 1 | 0.02 | 730129403.1 | 0 | 944598003 |
| 2 | 0.02 | 677208477.6 | 0 | 908177370.2 |
| 3 | 0.01 | 540538990.5 | 0 | 795268355.7 |
| 4 | 0.02 | 678835215.1 | 0 | 855108334.4 |
| 5 | 0.02 | 653503643 | 0 | 908167358.1 |
| 6 | 0.02 | 647750293 | 0.03 | 865558955.2 |
| 7 | 0.01 | 654242378 | 0 | 883558707.9 |
| 8 | 0.02 | 616692118 | 0.02 | 838845251.8 |
| 9 | 0.02 | 544383067 | 0 | 776367255.9 |
| 10 | 0.02 | 616190829 | 0.08 | 829178608.5 |
| 11 | 0.02 | 607833588 | 0 | 852146462 |
| 12 | 0.02 | 607693828.7 | 0 | 852127534.7 |
| 13 | 0.02 | 607874254 | 0 | 852152894.4 |
| 14 | 0.02 | 607722676.3 | 0.03 | 853977487.2 |
| 15 | 0.03 | 607752556.1 | 0.02 | 847914557 |
| 16 | 0.03 | 607717307.9 | 0.02 | 847865969.4 |
| 17 | 0.03 | 607731251.7 | 0.02 | 852151907.4 |
| 18 | 0.03 | 607710306.4 | 0.08 | 853966495.8 |
| 19 | 0.03 | 607720783.3 | 0.02 | 852150890.8 |
| 20 | 0.02 | 607712703.7 | 0.02 | 852152836.3 |
| 21 | 0.02 | 607718055.5 | 0.02 | 847861928.2 |
| 22 | 0.03 | 607713886.8 | 0.03 | 854003514.6 |
| 23 | 0.02 | 561502334.9 | 0.01 | 766859563.8 |
| 24 | 0.03 | 608609017.4 | 0 | 824753508 |
| 25 | 0.02 | 600793173.1 | 0.02 | 778277003.9 |
| 26 | 0.02 | 696788810.4 | 0.01 | 915920340.5 |
| 27 | 0.02 | 698883744.5 | 0 | 917240142.2 |
| 28 | 0.02 | 579498332.2 | 0 | 800199469.9 |
| 29 | 0.02 | 602485315 | 0 | 803314694 |
| 30 | 0.02 | 709552896.4 | 0 | 906256590.7 |

**Table 2: Computational results Statistics**

|  | Statistic | Mean | N | Std. Deviation | Std. Error Mean |
| --- | --- | --- | --- | --- | --- |
| Pair 1 | M1\_CPU\_Time | .0217 | 30 | .00531 | .00097 |
| M2\_CPU\_Time | .0143 | 30 | .02096 | .00383 |
| Pare 2 | M1\_Obj\_Fun | 6.2368E8 | 30 | 4.60091E7 | 8.40008E6 |
| M2\_Obj\_Fun | 8.5120E8 | 30 | 4.37535E7 | 7.98826E6 |

**Table 3: Comparison of Model 1 and 2 against their execution CPU Time and Objective Functions**

|  |  | t | df | Sig. (2-tailed) |
| --- | --- | --- | --- | --- |
| Pair 1 | M1\_CPU\_Time V/S M2\_CPU\_Time | 2.045 | 29 | .050 |
| Pair 1 | M1\_Obj\_Fun V/S M2\_Obj\_Fun | -59.155 | 29 | .000 |

**4. Results and Conclusions:**

In this paper we present a new method for solving capacitated lot sizing problem with setup, backorders and shortages. Using a different flow balance constraint, we eliminate the shortage variables and eliminate the setup variables. Thus leading to computational advantages. Then we use the heuristic method given by SP Singh and RRK Sharma to solve mixed 0-1 integer programming problems by solving an associated all real problem and set 0-1 variable as 1 if its value in associated all real problem is greater than 0.85 (an arbitrary number) and 0 if its value in all real problem is less than 0.15 (again an arbitrary number). This has been known to give good solutions in competitive CPU times.

We thus see that method 2 gives good solutions at competitive CPU times. This is a useful contribution of our paper. We give GAMS codes of our programs in the appendix.

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**APPENDIX**

Here we give the GAMS codes developed by us in this research. This is likely to be useful to future researchers.

**Method 1:**

\*0-1 production model.

set t 'time'/1\*50/;

set tt(t)/1\*50/;

parameter CXP(t) Cost of production

$call GDXXRW CXP.xlsx par = CXP rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN CXP.gdx

$LOAD CXP

$GDXIN

Display CXP;

parameter CXINV(t) Inventory carrying cost

$call GDXXRW CXINV.xlsx par = CXINV rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN CXINV.gdx

$LOAD CXINV

$GDXIN

parameter CBO(t) Back Order Cost

$call GDXXRW CBO.xlsx par = CBO rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN CBO.gdx

$LOAD CBO

$GDXIN

parameter f(t) fixed cost

$call GDXXRW f.xlsx par = f rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN f.gdx

$LOAD f

$GDXIN

parameter Dem(t) demand at time t

$call GDXXRW Dem.xlsx par = Dem rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN Dem.gdx

$LOAD Dem

$GDXIN

parameter Cap(t) Capacity at time t

$call GDXXRW Cap.xlsx par = Cap rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN Cap.gdx

$LOAD Cap

$GDXIN

\*parameter zzz(t)

Variables

z objective function for total Optimal\_Cost

y(t) binary variable to denote prodn.

XP(t) production at time t

XINV(t) inventory at time t

XSHT(t) shortage cost at time t;

Positive Variable XP,XINV,XINV,XSHT;

Binary Variable y ;

Equations

cost total optimal cost

prodcons(t) production constraint

demcon(t) demand balance constraint

\*end\_inv ending inventory is zero

end\_sht ending shortage quantity is zero;

\* objective function

cost.. z=e=sum((t),CXP(t)\*XP(t) + CXINV(t)\*XINV(t) + CBO(t)\*XSHT(t) + f(t)\*y(t));

\*constraints

prodcons(t).. XP(t) =l= Cap(t)\*y(t);

end\_sht.. XSHT('50') =e= 0;

demcon(t).. sum(tt$(ord(tt)<=ord(t)),XP(tt)) + XSHT(t) =e= sum(tt$(ord(tt)<=ord(t)),DEM(tt)) + XINV(t) ;

Model transport /all/;

solve transport using mip minimizing z;

display z.l;

display y.l;

**Method 2:**

**Program 1:**

set t 'time'/1\*50/;

set tt(t)/1\*50/;

parameter CXP(t) Cost of production

$call GDXXRW CXP.xlsx par = CXP rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN CXP.gdx

$LOAD CXP

$GDXIN

parameter CXINV(t) Inventory carrying cost

$call GDXXRW CXINV.xlsx par = CXINV rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN CXINV.gdx

$LOAD CXINV

$GDXIN

parameter CBO(t) Back Order Cost

$call GDXXRW CBO.xlsx par = CBO rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN CBO.gdx

$LOAD CBO

$GDXIN

parameter f(t) fixed cost

$call GDXXRW f.xlsx par = f rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN f.gdx

$LOAD f

$GDXIN

parameter Dem(t) demand at time t

$call GDXXRW Dem.xlsx par = Dem rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN Dem.gdx

$LOAD Dem

$GDXIN

parameter Cap(t) Capacity at time t

$call GDXXRW Cap.xlsx par = Cap rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN Cap.gdx

$LOAD Cap

$GDXIN

\*parameter zzz(t)

variable

XP(t) Quantity produced at time t

XINV(t) Inventory at time t

XBO(t) Back order quantity at time t

y\_zeros(t) Ys that are 0

y\_ones(t) Ys that are 1;

positive variable XP,XINV,XBO;

Free variable

z objective function;

Equations

objfun objective function

ccons(t) capacity constraint

demcon(t) modified demand balance constraint

bocons Back order at T=50 is 0

;

objfun.. z =e= sum((t),CXP(t)\*XP(t)+CXINV(t)\*XINV(t)

+CBO(t)\*(sum(tt$(ord(tt)<=ord(t)),Dem(tt)-XP(tt)) + XINV(t)) +

f(t)\*XP(t)/Cap(t));

demcon(t).. sum(tt$(ord(tt)<=ord(t)),Dem(tt)-XP(tt)) + XINV(t) =g= 0;

ccons(t).. XP(t) =l= Cap(t);

\*at t=last,XBO=0

bocons.. sum((t),Dem(t)-XP(t))+ XINV("50") =e= 0;

Model SingleItem1 /objfun,demcon, ccons, bocons/;

Solve SingleItem1 using lp minimizing z;

parameter zz(t), y(t);

zz(t) = XP.l(t);

y(t) = zz(t)/Cap(t);

display y;

loop(t,

if (y(t) <= 0.15,

y\_zeros.l(t)=1;

elseif (y(t)>=0.85),

y\_ones.l(t)=1;

else

continue;

);

);

display y\_zeros.l,y\_ones.l;

execute\_unload "y\_zeros.gdx" y\_zeros.l

execute 'gdxxrw.exe y\_zeros.gdx o=y\_zeros.xls var=y\_zeros.l'

execute\_unload "y\_ones.gdx" y\_ones.l

execute 'gdxxrw.exe y\_ones.gdx o=y\_ones.xls var=y\_ones.l'

**Program 2:**

set t 'time'/1\*50/;

set tt(t)/1\*50/;

parameter CXP(t) Cost of production

$call GDXXRW CXP.xlsx par = CXP rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN CXP.gdx

$LOAD CXP

$GDXIN

Display CXP;

parameter CXINV(t) Inventory carrying cost

$call GDXXRW CXINV.xlsx par = CXINV rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN CXINV.gdx

$LOAD CXINV

$GDXIN

parameter CBO(t) Back Order Cost

$call GDXXRW CBO.xlsx par = CBO rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN CBO.gdx

$LOAD CBO

$GDXIN

parameter f(t) fixed cost

$call GDXXRW f.xlsx par = f rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN f.gdx

$LOAD f

$GDXIN

parameter Dem(t) demand at time t

$call GDXXRW Dem.xlsx par = Dem rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN Dem.gdx

$LOAD Dem

$GDXIN

parameter Cap(t) Capacity at time t

$call GDXXRW Cap.xlsx par = Cap rdim=1 trace=3 rng = Sheet1!A1:A50

$GDXIN Cap.gdx

$LOAD Cap

$GDXIN

parameter y\_ones(t) Ys that are 1

$call GDXXRW y\_ones.xls par = y\_ones cdim=1 trace=3 rng = Sheet1!A1:AZ1

$GDXIN y\_ones.gdx

$LOAD y\_ones

$GDXIN

Display y\_ones

parameter y\_zeros(t) Ys that are 0

$call GDXXRW y\_zeros.xls par = y\_zeros cdim=1 trace=3 rng = Sheet1!A1:AZ1

$GDXIN y\_zeros.gdx

$LOAD y\_zeros

$GDXIN

Display y\_zeros

\*parameter zzz(t)

variable

XINV(t) Inventory at time t

XP(t) Production at time t

y(t) set up variable

z objective function;

positive variable XINV;

binary variable y;

Free variable z;

Equations

objfun objective function

demcon(t) modified demand balance constraint

bocons Back order at T=50 is 0

y\_o(t) Ys that will be 1 for sure

y\_z(t) Ys that will be 0 for sure

;

objfun.. z =e= sum((t),CXP(t)\*Cap(t)\*y(t)+CXINV(t)\*XINV(t)

+CBO(t)\*(sum(tt$(ord(tt)<=ord(t)),Dem(tt) - Cap(tt)\*y(tt)) + XINV(t)) +

f(t)\*y(t));

y\_o(t)$ y\_ones(t).. y(t) =e= 1;

y\_z(t)$ y\_zeros(t).. y(t) =e= 0;

demcon(t).. sum(tt$(ord(tt)<=ord(t)),Dem(tt)-Cap(tt)\*y(tt)) + XINV(t) =g= 0;

\*at t=last,XBO=0

bocons.. sum((t),Dem(t)-Cap(t)\*y(t))+ XINV("50") =e= 0;

Model SingleItem1 /all/;

Solve SingleItem1 using mip minimizing z;

display z.l;

display y.l;

**Biographies**

**Prof. RRK Sharma:** He is B.E. (mechanical engineering) from VNIT Nagpur India, and PhD in management from I.I.M., Ahmedabad, INDIA. He has nearly three years of experience in automotive companies in India (Tata Motors and TVS-Suzuki). He has 32 years of teaching and research experience at the Department of Industrial and Management Engineering, I.I.T., Kanpur, 208016 INDIA. To date he has written 1195 papers (peer-reviewed (395) / under review (18) / working papers 782 (not referred)). He has developed over ten software products. To date, he has guided 64 M TECH and 21 Ph D theses at I.I.T. Kanpur. He has been Sanjay Mittal Chair Professor at IIT KANPUR (15.09.2015 to 14.09.2018) and is currently a H.A.G. scale professor at I.I.T. Kanpur. In 2015, he received “Membership Award” given by IABE USA (International Academy of Business and Economics). In 2016 he received the “Distinguished Educator Award” from IEOM (Industrial Engineering and Operations Management) Society, U.S.A. In 2021, he received IEOM Distinguished Service Award. In 2019 and 2020, he was invited by the Ministry of Human Resources Department, India, to participate in the NIRF rankings survey for management schools in India. In 2019, he was invited to participate in the Q.S. ranking exercise for ranking management schools in South Asia.

**Dr. Vinay Singh:** He has earned his Bachelor Degree in engineering (Computer Science and Engineering) from RBS College Agra, Masters in Human Resource Development and Management from IIT Kharagpur and PhD in Management from IIT Kanpur. Currently he is working as Assistant Professor in the department of Management at ABV-Indian Institute of Information Technology and Management Gwalior, India since Nov 2012. So far he has 26 publications in peer review journals to his credit. He has supervised 92 Masters Students and guided 02 PhD theses. He has also earned two national patents in embedded products design and has developed three software packages. He has received 03 research project grants from prestigious agencies of India.

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