Linear Algebra DS & AI

Matrices and Determinants

DPP 01

1. If $f = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix}$ and $g = (x-y)(y-z)(z-x)$, then $\frac{f}{g}$ is:

(A) $xy + yz + zx$ **(B)** $x^2 + y^2 + z^2$

(C) $x^2 + y^2 + z^2 - xy - yz - zx$ **(D)** none of above

2. If $g(x) = \begin{vmatrix} a^{-x} & e^{x \log_e a} & x^2 \\ a^{-3x} & e^{3x \log_e a} & x^4 \\ a^{-5x} & e^{5x \log_e a} & 1 \end{vmatrix}$, then:

(A) $g(x) + g(-x) = 0$ **(B)** $g(x) - g(-x) = 0$

(C) $g(x) \times g(-x) = 0$ **(D)** none of these

3. If $x \neq 0, y \neq 0, z \neq 0$ and $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 0$, then $x^{-1} + y^{-1} + z^{-1}$ is equal to :

(A) -1 **(B)** -2 **(C)** -3 **(D)** $\frac{1}{3}$

4. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$ then $A^3 - rA^2 - qA =$

(A) pI **(B)** qI

(C) rI **(D)** none of these, I is third order unit matrix

***5.** If $2x - y = \begin{bmatrix} 3 & -3 & 0 \\ 3 & 3 & 2 \end{bmatrix}$ and $2y + x = \begin{bmatrix} 4 & 1 & 5 \\ -1 & 4 & -4 \end{bmatrix}$, then:

(A) $x + y = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & -2 \end{bmatrix}$ **(B)** $x = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

(C) $x - y = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 2 \end{bmatrix}$ **(D)** $y = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 1 & -2 \end{bmatrix}$

6. If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then $f(x+y)$ is equal to:

(A) $f(x) + f(y)$ **(B)** $f(x) - f(y)$ **(C)** $f(x) \cdot f(y)$ **(D)** None of these

7. If $AB = 0$ where $A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ then $|\theta - \phi|$ is equal to:

(A) 0 **(B)** $\frac{\pi}{2}$ **(C)** $\frac{\pi}{4}$ **(D)** π

8. If $A=\begin{bmatrix}1 & \omega & \omega^2\\ \omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\end{bmatrix}, B=\begin{bmatrix}\omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\\ \omega & \omega^2 & 1\end{bmatrix}$ and $C=\begin{bmatrix}1\\ \omega\\ \omega^2\end{bmatrix}$ where ω is the complex cube root of 1 then $(A+B)C$ is equal to:

(A) $\begin{bmatrix}0\\0\\0\end{bmatrix}$ **(B)** $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ **(C)** $\begin{bmatrix}1\\0\\1\end{bmatrix}$ **(D)** $\begin{bmatrix}1\\1\\1\end{bmatrix}$

9. If $A=\begin{bmatrix}0 & c & -b\\ -c & 0 & a\\ b & -a & 0\end{bmatrix}$ and $B=\begin{bmatrix}a^2 & ab & ac\\ ba & b^2 & bc\\ ca & cb & c^2\end{bmatrix}$ then AB is equal to:

(A) O **(B)** I **(C)** 2I **(D)** None of these

10. If $\begin{bmatrix}x+y & y\\ 2x & x-y\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix}=\begin{bmatrix}3\\2\end{bmatrix}$ then $x\cdot y$ is equal to :

(A) -5 **(B)** 5 **(C)** 4 **(D)** 6

11. If $a^{-1}+b^{-1}+c^{-1}=0$ such that $\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}=\lambda$ then the value of λ is :

(A) 0 **(B)** abc **(C)** $-abc$ **(D)** None of these

12. The value of 'λ' if $ax^4+bx^3+cx^2+50x+d=\begin{vmatrix}x^3-14x^2 & -x & 3x+\lambda\\ 4x+1 & 3x & x-4\\ -3 & 4 & 0\end{vmatrix}$, is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

13. The values of θ lying between $\theta=0$ and $\theta=\pi/2$ and satisfying the equation

$\begin{vmatrix}1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta\\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta\\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta\end{vmatrix}=0$ are given by :

(A) $\pi/24, 5\pi/24$ **(B)** $7\pi/24, 11\pi/24$ **(C)** $5\pi/24, 7\pi/24$ **(D)** $11\pi/24, \pi/24$

14. If $f(x)=\begin{vmatrix}1-x & -1 & 0\\ 2 & 3-x & 1\\ 4 & -2 & 5-x\end{vmatrix}$, number of real roots of $f(x)=0$ is ________.

(A) 1 **(B)** 0 **(C)** 2 **(D)** 3

15. If $A=\begin{bmatrix}\cos^2\theta & \sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$, $B=\begin{bmatrix}\cos^2\phi & \sin\phi\cos\phi\\ \sin\phi\cos\phi & \sin^2\phi\end{bmatrix}$ and θ and ϕ differs by $\frac{\pi}{2}$, then $AB=$

(A) I **(B)** O **(C)** $-I$ **(D)** None of these

Answer Key

1. (A)
2. (A)
3. (C)
4. (A)
5. (AB)
6. (C)
7. (B)
8. (A)
9. (A)
10. (A)
11. (B)
12. (C)
13. (B)
14. (A)
15. (B)

Linear Algebra DS & AI

Matrices and Determinants

DPP 02

1. $\begin{vmatrix} 1+i & 1-i & i \\ 1-i & i & 1+i \\ i & 1+i & 1-i \end{vmatrix} =$

(A) $-4-7i$ **(B)** $4+7i$ **(C)** $3+7i$ **(D)** $7+4i$

2. If ω is a cube root of unity, then $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} =$

(A) x^3+1 **(B)** $x^3+\omega$ **(C)** $x^3+\omega^2$ **(D)** x^3

3. If $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = k(x+y+z)(x-z)^2$, then $k =$

(A) $2xyz$ **(B)** 1 **(C)** xyz **(D)** $x^2y^2z^2$

4. If -9 is a root of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ then the other two roots are :

(A) 2, 7 **(B)** -2, 7 **(C)** 2, -7 **(D)** -2, -7

5. If $A = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}, B = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}, C = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$, then which relation is correct :

(A) $A = B$ **(B)** $A = C$ **(C)** $B = C$ **(D)** None of these

6. $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} =$

(A) $a^3+b^3+c^3-3abc$ **(B)** $3abc-a^3-b^3-c^3$

(C) $a^3+b^3+c^3-a^2b-b^2c-c^2a$ **(D)** $(a+b+c)(a^2+b^2+c^2+ab+bc+ca)$

7. If ω is a cube root of unity and $\Delta = \begin{vmatrix} 1 & 2\omega \\ \omega & \omega^2 \end{vmatrix}$, then Δ^2 is equal to :

(A) $-\omega$ **(B)** ω **(C)** 1 **(D)** ω^2

8. If $\Delta_1 = \begin{vmatrix} 1 & 0 \\ a & b \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 1 & 0 \\ c & d \end{vmatrix}$, then $\Delta_2\Delta_1$ is equal to :

(A) ac **(B)** bd **(C)** $(b-a)(d-c)$ **(D)** None of these

9. For the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$, which of the following is correct:

(A) $A^3 + 3A^2 - I = O$ **(B)** $A^3 - 3A^2 - I = O$

(C) $A^3 + 2A^2 - I = O$ **(D)** $A^3 - A^2 + I = O$

10. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$, then the value of a and b are:

(A) $a = 4, b = 1$ **(B)** $a = 1, b = 4$ **(C)** $a = 0, b = 4$ **(D)** $a = 2, b = 4$

11. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then $A^2 - 5A =$

(A) I **(B)** $14\,I$ **(C)** 0 **(D)** None of these

12. If matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then $A^{16} =$

(A) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **(C)** $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ **(D)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

13. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then $A^{100} =$

(A) $2^{100}A$ **(B)** $2^{99}A$ **(C)** $2^{101}A$ **(D)** None of these

14. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, then $A^n =$

(A) $\begin{bmatrix} na & 0 & 0 \\ 0 & nb & 0 \\ 0 & 0 & nc \end{bmatrix}$ **(B)** $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ **(C)** $\begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{bmatrix}$ **(D)** None of these

15. The matrix $\begin{bmatrix} 2 & \lambda & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is non singular, if :

(A) $\lambda \neq -2$ **(B)** $\lambda \neq 2$ **(C)** $\lambda \neq 3$ **(D)** $\lambda \neq -3$

Answer Key

1. (B)
2. (D)
3. (B)
4. (A)
5. (D)
6. (B)
7. (B)
8. (B)
9. (B)
10. (B)
11. (B)
12. (D)
13. (B)
14. (C)
15. (A)

Linear Algebra DS & AI

Matrices and Determinants

DPP 03

1. $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Ax+B$. Then $A+2B$ is equal to:

(A) 0 **(B)** 4 **(C)** 1 **(D)** 2

2. If $f(x)g(x)$ and $h(x)$ are three polynomials of degree 3 then $\phi(x) = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \\ f'''(x) & g'''(x) & h'''(x) \end{vmatrix}$ is a polynomial of degree :

(A) 3 **(B)** 4 **(C)** 5 **(D)** None of these

3. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$.

(A) $A^n = 2^{n-1}A + (n-1)I$ **(B)** $A^n = nA + (n-1)I$

(C) $A^n = 2^{n-1}A - (n-1)I$ **(D)** $A^n = nA - (n-1)I$

4. A square matrix P satisfies $P^2 = I - P$, where I is the identity matrix. If $P^n = 5I - 8P$, then n is equal to:

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7

5. If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ and $AA^T = I$. Then $x+y$ is equal to :

(A) -3 **(B)** -2 **(C)** -1 **(D)** 0

***6.** Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then

(A) $A^2 - 4A - 5I_3 = 0$ **(B)** $A^{-1} = \frac{1}{5}(A - 4I_3)$

(C) A^3 is not invertible **(D)** A^2 is invertible

7. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T\left(Q^{2005}\right)P$ is equal to :

(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ **(B)** $\begin{bmatrix} \frac{\sqrt{3}}{2} & 2005 \\ 1 & 0 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 2005 \\ \frac{\sqrt{3}}{2} & 1 \end{bmatrix}$ **(D)** $\begin{bmatrix} 1 & \frac{\sqrt{3}}{2} \\ 0 & 2005 \end{bmatrix}$

***8.** If $A+B+C=\pi, e^{i\theta} = \cos\theta + i\sin\theta$ and $z = \begin{vmatrix} e^{2iA} & e^{-iC} & e^{-iB} \\ e^{-iC} & e^{2iB} & e^{-iA} \\ e^{-iB} & e^{-iA} & e^{-2iC} \end{vmatrix}$, then :

(A) $\text{Re}(z) = 4$ **(B)** $\text{Im}(z) = 0$ **(C)** $\text{Re}(z) = -4$ **(D)** $\text{Im}(z) = 1$

9. The value of $\begin{vmatrix} 1 & 1 & 1 \\ \left(2^{x}+2^{-x}\right)^{2} & \left(3^{x}+3^{-x}\right)^{2} & \left(5^{x}+5^{-x}\right)^{2} \\ \left(2^{x}-2^{-x}\right)^{2} & \left(3^{x}-3^{-x}\right)^{2} & \left(5^{x}-5^{-x}\right)^{2} \end{vmatrix}$ is :

(A) 0 **(B)** 30^{x} **(C)** 30^{-x} **(D)** None of these

10. If $A=\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}, B=(adj\, A)$ and $C=5A$, then $\frac{|adj\, B|}{|C|}$ is equal to :

(A) 5 **(B)** 25 **(C)** -1 **(D)** 1

11. If $a,b,c,$ are in A.P. and $f(x)=\begin{vmatrix} x+a & x^{2}+1 & 1 \\ x+b & 2x^{2}-1 & 1 \\ x+c & 3x^{2}-2 & 1 \end{vmatrix}$, then $f'(x)$ is :

(A) 0 **(B)** 1 **(C)** $a+bc$ **(D)** $\frac{abc}{a+b+c}$

***12.** The value of x for which $\begin{vmatrix} x & 2 & 2 \\ 3 & x & 2 \\ 3 & 3 & x \end{vmatrix}+\begin{vmatrix} 1-x & 2 & 4 \\ 2 & 4-x & 8 \\ 4 & 8 & 16-x \end{vmatrix}>33$ is :

(A) $0<x<1$ **(B)** $-\frac{1}{2}<x<\frac{1}{2}$ **(C)** $x<-\frac{1}{7}$ **(D)** $x>1$

***13.** Let $f(n)=\begin{vmatrix} n & n+1 & n+2 \\ {}^{n}P_{n} & {}^{n+1}P_{n+1} & {}^{n+2}P_{n+2} \\ {}^{n}C_{n} & {}^{n+1}C_{n+1} & {}^{n+2}C_{n+2} \end{vmatrix}$ where the symbols have their usual meanings. Then $f(n)$ is divisible by :

(A) $n^{2}+n+1$ **(B)** $(n+1)!$ **(C)** $n!$ **(D)** None of these

14. If $\Delta=\begin{vmatrix} f(x) & f\left(\frac{1}{x}\right)+f(x) \\ 1 & f\left(\frac{1}{x}\right) \end{vmatrix}=0$ where, $f(x)=a+bx^{n}$ and $f(2)=17$, then $f(5)$ is :

(A) 126 **(B)** 326 **(C)** 428 **(D)** 626

15. The value of x, so that $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix}=0$, is:

(A) $\frac{-7 \pm \sqrt{35}}{2}$ **(B)** $\frac{-9 \pm \sqrt{53}}{2}$ **(C)** ± 2 **(D)** 0

Answer Key

1. (A)
2. (D)
3. (D)
4. (C)
5. (A)
6. (ABD)
7. (A)
8. (BC)
9. (A)
10. (D)
11. (A)
12. (CD)
13. (AC)
14. (D)
15. (B)

Paragraph for Questions 46 to 48

Elementary Transformation of a matrix:

The following operation on a matrix are called elementary operations (transformations)

1. The interchange of any two rows (or columns)

2. The multiplication of the elements of any row (or column) by any nonzero number

3. The addition to the elements of any row (or column) the corresponding elements of any other row (or column) multiplied by any number

Echelon Form of matrix :

A matrix A is said to be in echelon form if

(i) every row of A which has all its elements 0, occurs below row, which has a non-zero elements

(ii) the first non-zero element in each non –zero row is 1.

(iii) The number of zeros before the first non zero elements in a row is less than the number of such zeros in the next now.

[A row of a matrix is said to be a zero row if all its elements are zero]

Note: Rank of a matrix does not change by application of any elementary operations

For example $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 3 & 6 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ are echelon forms

The number of non-zero rows in the echelon form of a matrix is defined as its RANK.

For example we can reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$ into echelon form using following elementary row transformation.

(i) $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1 \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(ii) $R_2 \to R_2 - 2R_1 \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

This is the echelon form of matrix A

Number of nonzero rows in the echelon form =2 $\Rightarrow$ Rank of the matrix A is 2

1. Rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$ is :

(A) 1 **(B)** 2 **(C)** 3 **(D)** 0

2. Rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & 4 \\ 3 & 4 & 5 & 2 \end{bmatrix}$ is :

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

3. The echelon form of the matrix $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ is :

(A) $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **(D)** $\begin{bmatrix} 1 & 3 & 4 & -\frac{3}{2} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

4. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ \lambda & 2 & 4 \\ 2 & -3 & 1 \end{bmatrix}$ is 3 if :

(A) $\lambda \neq \frac{18}{11}$ **(B)** $\lambda = \frac{18}{11}$ **(C)** $\lambda = -\frac{18}{11}$ **(D)** None of these

5. The rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ is equal to :

(A) 1 **(B)** 2 **(C)** 3 **(D)** None of these

6. If 3, – 2 are the Eigen values of a non-singular matrix A and $|A| = 4$, then the Eigen values of $adj(A)$ are :

(A) $\frac{3}{4}, \frac{-1}{2}$ **(B)** $\frac{4}{3}, -2$ **(C)** 12, – 8 **(D)** – 12, 8

7. Let p a non singular matrix $1 + P + P^2 + ... + P^n = O$. ($O$ denotes the null matrix), then $P^{-1} =$

(A) P^n **(B)** $-P^n$ **(C)** $-(1 + P + ... + P^n)$ **(D)** None of these

8. If $P = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ -2 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} -4 & -5 & -6 \\ 0 & 0 & 1 \end{bmatrix}$ then $P_{22} =$

(A) 40 **(B)** – 40 **(C)** – 20 **(D)** 20

9. If $1, \omega, \omega^2$ are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to :

(A) 0 **(B)** 1 **(C)** ω **(D)** ω^2

10. Let P be a non-singular matrix $I + P + P^2 + \ldots + P^n = O$ (O denotes the null matrix), then P^{-1} is :

(A) P^n **(B)** $-P^n$

(C) $-(I + P + \ldots + P^n)$ **(D)** None of these

11. If D = diagonal $\lfloor\lceil d_1, d_2, d_3, \ldots, d_n \rfloor\rceil$ where $d_i \neq 0 \forall i = 1, 2, 3, \ldots, n$ then D^{-1} is equal to :

(A) O **(B)** I_n

(C) diagonal $\left(d_1^{-1}, d_2^{-1}, \ldots d_n^{-1}\right)$ **(D)** None of above

12. Let A be an orthogonal non-singular matrix of order n, then $|A - I_n|$ is equal to :

(A) $|I_n - A|$ **(B)** $|A|$ **(C)** $|A||I_n - A|$ **(D)** $(-1)^n |A||I_n - A|$

13. 'A' is any square matrix, then det $|A - A^T|$ is equal to :

(A) 0 **(B)** 1

(C) can be 0 or a perfect square **(D)** cannot be determined

14. In a ΔABC, if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$, then $\sin^2 A + \sin^2 B + \sin^2 C$ is equal to :

(A) $\frac{9}{4}$ **(B)** $\frac{4}{9}$ **(C)** 1 **(D)** $3\sqrt{3}$

15. If a, b, c are the sides of a ΔABC opposite angle A, B, C respectively, then

$$\Delta = \begin{vmatrix} a^2 & b \sin A & c \sin A \\ b \sin A & 1 & \cos(B - C) \\ c \sin A & \cos(B - C) & 1 \end{vmatrix}$$ equals :

(A) $\sin A - \sin C \sin B$ **(B)** abc

(C) 1 **(D)** 0

Answer Key

1. (C)
2. (C)
3. (C)
4. (A)
5. (A)
6. (B)
7. (A)
8. (A)
9. (A)
10. (A)
11. (C)
12. (C)
13. (C)
14. (A)
15. (D)

Paragraph for Questions 61 to 63

Consider the determinant, $\Delta = \begin{vmatrix} p & q & r \\ x & y & z \\ l & m & n \end{vmatrix}$.

M_{ij} denotes the minor of an element in i^{th} row, and j^{th} column

C_{ij} denotes the cofactor of an element in i^{th} row and j^{th} column

1. The value of $p \cdot C_{21} + q \cdot C_{22} + r \cdot C_{23}$ is :

(A) 0 **(B)** $-\Delta$ **(C)** Δ **(D)** Δ^2

2. The value of $x \cdot C_{21} + y \cdot C_{22} + z \cdot C_{23}$ is :

(A) 0 **(B)** $-\Delta$ **(C)** Δ **(D)** Δ^2

3. The value of $q \cdot M_{12} - y \cdot M_{22} + m \cdot M_{32}$ is :

(A) 0 **(B)** $-\Delta$ **(C)** Δ **(D)** Δ^2

4. A and B are square matrices and A is non-singular matrix, $\left(A^{-1}BA\right)^n, n \in I^+$, is equal to :

(A) $A^{-n}B^nA^n$ **(B)** $A^nB^nA^{-n}$ **(C)** $A^{-1}B^nA$ **(D)** $A^{-n}BA^n$

5. The value of a, b, c when $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal, are :

(A) $\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{6}}, \pm\frac{1}{\sqrt{2}}$ **(B)** $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{6}}$ **(C)** $\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{6}} \pm\frac{1}{\sqrt{3}}$ **(D)** $\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}} \pm\frac{1}{\sqrt{2}}$

6. The equations $2x + y = 5, x + 3y = 5, x - 2y = 0$ have:

(A) no solution **(B)** one solution **(C)** two solutions **(D)** infinitely many solutions

Passage for Question 7 to 10

Consider a system of linear equation in three variables x, y, z

$$a_1x + b_1y + c_1z = d_1;\ a_2x + b_2y + c_2z = d_2;\ a_3x + b_3y + c_3z = d_3$$

The system can be expressed by matrix equation $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

- if A is non-singular matrix then the solution of above system can be found by $X = A^{-1}B$, the solution in this case is unique.
- if A is a singular matrix i.e. $|A| = 0$, then the system will have
- no unique solution if $(Adj\ A)B = 0$
- no solution (i.e. it is inconsistent) if $(Adj\ A)B \neq 0$

Where Adj A is the adjoint of the matrix A, which is obtained by taking transpose of the matrix obtained by replacing each element of matrix A with corresponding cofactors.
Now consider the following matrix.

$$A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}, B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}, V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

7. The system $AX = U$ has infinitely many solution if:

(A) $c = d, ab = 1$ **(B)** $c = d, h = g$ **(C)** $ab = 1, h = g$ **(D)** $c = d, h = g, ab = 1$

8. If $AX = U$ has infinitely many solutions then the equation $BX = V$ has:

(A) unique solution **(B)** infinitely many solution

(C) no solution **(D)** either infinitely many solutions or no solution

***9.** If $AX = U$ has infinitely many solutions then the equation $BX = V$ is consistent if

(A) $a = 0$ **(B)** $d = 0$ **(C)** $f = 0$ **(D)** $adf \neq 0$

***10.** Consider the following statements:

A: if $AX = U$; has infinite solutions and $cf \neq 0$, then one solution of $BX = V$ is $(0,0,0)$

R: if a system has infinite solutions then one solution must be trivial. Then

(A) A and R are both correct and R is correct explanation of A

(B) A and R both are correct but R is not correct explanation of A

(C) A is correct R is wrong

(D) A and R are both wrong

11. If t is real and $\lambda = \dfrac{t^2 - 3t + 4}{t^2 + 3t + 4}$, then the number of solutions of the system of equations $3x - y + 4z = 3$, $x + 2y - 3z = -2$, $6x + 5y + \lambda z = -3$ is :

(A) one **(B)** Two **(C)** zero **(D)** infinite

12. The set of equations $\lambda x - y + (\cos\theta) z = 0$; $3x + y + 2z = 0$; $(\cos\theta) x + y + 2z = 0$; $0 < \theta \le 2\pi$, has non–trivial solutions.

(A) for no value of λ and θ **(B)** for all value of λ and θ

(C) for all values of λ and only two value of θ

(D) For only one value of λ and all values of θ

***13.** Let $\Delta(x) = \begin{vmatrix} x+a & x+b & x+a-c \\ x+b & x+c & x-1 \\ x+c & x+d & x-b+d \end{vmatrix}$ and $\int_0^2 \Delta(x)\,dx = -16$ where a, b, c, d are in AP, then the common difference of the AP is:

(A) 1 **(B)** 2 **(C)** -2 **(D)** None of these

***14.** Let $\{\Delta_1, \Delta_2, \Delta_3, \ldots, \Delta_k\}$ be the set of third order determinants that can be made with the distinct nonzero real numbers $a_1, a_2, a_3, \ldots, a_9$. Then:

(A) $k = 9!$ **(B)** $\sum_{i=1}^{k} \Delta_i = 0$

(C) at least one $\Delta_i = 0$ **(D)** None of these

15. If $a \neq p, b \neq q, c \neq r$ and the system of equations $px + by + cz = 0$, $ax + qy + cz = 0$, $ax + by + rz = 0$ has a non-zero solution, then value of $\dfrac{p}{p-a} + \dfrac{q}{q-b} + \dfrac{r}{r-c}$ is:

(A) -1 **(B)** -2 **(C)** 1 **(D)** 2

Answer Key

1. (A)
2. (C)
3. (B)
4. (C)
5. (C)
6. (B)
7. (D)
8. (D)
9. (ABC)
10. (ABC)
11. (A)
12. (A)
13. (BC)
14. (AB)
15. (D)

Linear Algebra DS & AI
Matrices and Determinants
DPP 06

1. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are the given determinants, then :

(A) $\Delta_1 = 3(\Delta_2)^2$ **(B)** $\frac{d}{dx}(\Delta_1) = 3\Delta_2$ **(C)** $\frac{d}{dx}(\Delta_1) = 2(\Delta_2)^2$ **(D)** $\Delta_1 = 3\Delta_2^{3/2}$

2. In the determinant $\begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{vmatrix}$, the ratio of the co-factor to its minor of the element -3 is :

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2

3. If value of a third order determinant is 11, then the value of the square of the determinant formed by the cofactors will be :

(A) 11 **(B)** 121 **(C)** 1331 **(D)** 14641

4. Consider the system of linear equations $a_1x + b_1y + c_1z + d_1 = 0$, $a_2x + b_2y + c_2z + d_2 = 0$ and $a_3x + b_3y + c_3z + d_3 = 0$. Let us denote by $\Delta(a,b,c)$ the determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$. If $\Delta(a,b,c) \neq 0$, then the value of x in the unique solution of the above equations is :

(A) $\frac{\Delta(bcd)}{\Delta(abc)}$ **(B)** $\frac{-\Delta(bcd)}{\Delta(abc)}$ **(C)** $\frac{\Delta(acd)}{\Delta(abc)}$ **(D)** $-\frac{\Delta(abd)}{\Delta(abc)}$

5. The value of the determinant $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$ is :

(A) $2(10!\,11!)$ **(B)** $2(10!\,13!)$ **(C)** $2(10!\,11!\,12!)$ **(D)** $2(11!\,12!\,13!)$

6. The cofactor of the element '4' in the determinant $\begin{vmatrix} 1 & 3 & 5 & 1 \\ 2 & 3 & 4 & 2 \\ 8 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 \end{vmatrix}$ is :

(A) 4 **(B)** 10 **(C)** – 10 **(D)** – 4

7. If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and A_1, B_1, C_1 denote the co-factors of a_1, b_1, c_1 respectively, then the value of the determinant $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$ is :

(A) Δ **(B)** Δ^2 **(C)** Δ^3 **(D)** 0

8. If in the determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, A_1, B_1, C_1 etc. be the co-factors of a_1, b_1, c_1 etc., then which of the following relations is incorrect :

(A) $a_1A_1 + b_1B_1 + c_1C_1 = \Delta$ **(B)** $a_2A_2 + b_2B_2 + c_2C_2 = \Delta$

(C) $a_3A_3 + b_3B_3 + c_3C_3 = \Delta$ **(D)** $a_1A_2 + b_1B_2 + c_1C_2 = \Delta$

9. If A_1, B_1, C_1 .... are respectively the co-factors of the elements a_1, b_1, c_1 ,...... of the determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, then $\begin{vmatrix} B_2 & C_2 \\ B_3 & C_3 \end{vmatrix} =$

(A) $a_1\Delta$ **(B)** $a_1a_3\Delta$ **(C)** $(a_1 + b_1)\Delta$ **(D)** None of these

10. Let $A = \left[a_{ij}\right]_{n\times n}$ be a square matrix and let c_{ij} be cofactor of a_{ij} in A. If $C = \left[c_{ij}\right]$, then :

(A) $|C| = |A|$ **(B)** $|C| = |A|^{n-1}$ **(C)** $|C| = |A|^{n-2}$ **(D)** None of these

11. $x + ky - z = 0, 3x - ky - z = 0$ and $x - 3y + z = 0$ has non-zero solution for $k =$

(A) – 1 **(B)** 0 **(C)** 1 **(D)** 2

12. The number of solutions of equations $x + y - z = 0$, $3x - y - z = 0, x - 3y + z = 0$ is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** Infinite

13. If $x + y - z = 0$, $3x\alpha - y - 3z = 0$, $x - 3y + z = 0$ has non zero solution, then $= \alpha$

(A) – 1 **(B)** 0 **(C)** 1 **(D)** – 3

14. If $\Delta(x) = \begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ a & a^2 & a^3 \end{vmatrix}$, then the value of $\frac{d^n}{dx^n}\left[\Delta(x)\right]$ at $x = 0$ is :

(A) – 1 **(B)** 0 **(C)** 1 **(D)** Dependent of a

15. The inverse of $\begin{bmatrix} 3 & 5 & 7 \\ 2 & -3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ is :

(A) $\begin{bmatrix} 7 & 3 & -26 \\ 3 & 1 & -11 \\ -5 & -2 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 7 & 3 & -26 \\ 3 & 1 & 11 \\ -5 & -2 & 1 \end{bmatrix}$ **(C)** $\begin{bmatrix} 3 & 1 & 11 \\ 7 & 3 & -26 \\ -5 & 2 & 1 \end{bmatrix}$ **(D)** None of these

Answer Key

1. (B)
2. (A)
3. (D)
4. (B)
5. (C)
6. (B)
7. (B)
8. (D)
9. (A)
10. (B)
11. (C)
12. (D)
13. (D)
14. (B)
15. (D)

***91.** Let A and B be two nonsingular square matrices, A^T and B^T are the transpose matrices of A and B, respectively, then which of the following are correct?

(A) $B^T AB$ is symmetric matrix if A is symmetric

(B) $B^T AB$ is symmetric matrix if B is symmetric

(C) $B^T AB$ is skew-symmetric matrix for every matrix A

(D) $B^T AB$ is skew-symmetric matrix if A is skew-symmetric

***92.** If $A(\theta)=\begin{bmatrix}\sin\theta & i\cos\theta\\ i\cos\theta & \sin\theta\end{bmatrix}$, then which of the following is not true?

(A) $A(\theta)^{-1}=A(\pi-\theta)$

(B) $A(\theta)+A(\pi+\theta)$ is a null matrix

(C) $A(\theta)$ is invertible for all $\theta\in R$

(D) $A(\theta)^{-1}=A(-\theta)$

***93.** If $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$, then :

(A) $adj(adj)A=A$

(B) $|adj(adj\ A)|=1$

(C) $|adj\ A|=1$

(D) None of these

***94.** If $A^{-1}=\begin{bmatrix}1 & -1 & 2\\ 0 & 3 & 1\\ 0 & 0 & -1/3\end{bmatrix}$, then:

(A) $|A|=-1$

(B) $\text{adj}A=\begin{bmatrix}-1 & 1 & -2\\ 0 & -3 & -1\\ 0 & 0 & 1/3\end{bmatrix}$

(C) $A=\begin{bmatrix}1 & 1/3 & 7\\ 0 & 1/3 & 1\\ 0 & 0 & -3\end{bmatrix}$

(D) $A=\begin{bmatrix}1 & -1/3 & -7\\ 0 & -3 & 0\\ 0 & 0 & 1\end{bmatrix}$

***95.** Which of the following statements is/are true about square matrix A of order n?

(A) $(-A)^{-1}$ is equal to $-A^{-1}$ which n is odd only

(B) If $A^n=O$, then $I+A+A^2+....+A^{n-1}=(I-A)^{-1}$

(C) If A is skew-symmetric matrix of odd order, then its inverse does not exist.

(D) $(A^T)^{-1}=(A^{-1})^T$ holds always

***96.** If A, B and C are three square matrices of the same order, then $AB=AC\Rightarrow B=C$. Then :

(A) $|A|\neq 0$

(B) A is invertible

(C) A may be orthogonal

(D) A is symmetric

***97.** Suppose $a_1, a_2, \ldots$ are real numbers, with $a_1 \neq 0$. If $a_1, a_2, a_3, \ldots\ldots$ are in A.P., then :

(A) $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_5 & a_6 & a_7 \end{bmatrix}$ is singular (where $i = \sqrt{-1}$)

(B) the system of equations $a_1x + a_2y + a_3z = 0$, $a_4x + a_5y + a_6z = 0$, $a_7x + a_8y + a_9z = 0$ has infinite number of solutions

(C) $B = \begin{bmatrix} a_1 & ia_2 \\ ia_2 & a_1 \end{bmatrix}$ is nonsingular

(D) None of these

***98.** Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. Then which of following is not true ?

(A) $\lim\limits_{n \to \infty} \dfrac{1}{n^2} A^{-n} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$

(B) $\lim\limits_{n \to \infty} \dfrac{1}{n} A^{-n} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$

(C) $A^{-n} = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix} \forall n \neq N$

(D) None of these

***99.** If C is skew-symmetric matrix of order n and X is $n \times 1$ column matrix, then $X^T CX$ is :

(A) singular **(B)** non-singular **(C)** invertible **(D)** non-invertible

***100.** If $S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $A = \begin{bmatrix} b+c & c+a & b-c \\ c-b & c+b & a-b \\ b-c & a-c & a+b \end{bmatrix} (a, b, c \neq 0)$, then SAS^{-1} is :

(A) symmetric matrix

(B) diagonal matrix

(C) invertible matrix

(D) singular matrix

Answer Key

1. (AD)
2. (ABC)
3. (ABC)
4. (ABC)
5. (BC)
6. (ABC)
7. (ABC)
8. (BC)
9. (AD)
10. (ABC)

Linear Algebra DS & AI

Matrices and Determinants

DPP 08

1. If the system of linear equations $x + y + z = 5$, $x + 2y + 2z = 6$, $x \ +3y \ \lambda +z \ \in \mu,\lambda(,\mu =R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is:

(A) 7 **(B)** 12 **(C)** 10 **(D)** 9

2. If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non-trivial solution (x, y, z), then $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to:

(A) -4 **(B)** $\frac{1}{2}$ **(C)** $-\frac{1}{4}$ **(D)** $\frac{3}{4}$

3. If the system of linear equations $x - 2y + kz = 1$, $2x + y + z = 2$, $3x - y - kz = 3$ has a solution (x, y, z), $z \neq 0$, then (x, y) lies on the straight line whose equation is:

(A) $3x - 4y - 4 = 0$ **(B)** $3x - 4y - 1 = 0$ **(C)** $4x - 3y - 4 = 0$ **(D)** $4x - 3y - 1 = 0$

4. The greatest value of $c \in R$ for which the system of linear equations $x - cy - cz = 0$, $cx - y + cz = 0$, $cx + cy - z = 0$ has a non-trivial solution, is:

(A) -1 **(B)** $1/2$ **(C)** 2 **(D)** 9

5. The set of all values of λ for which the system of linear equation $x \ -2y \ -2z \ \lambda =x$, $x \ +2y \ +z \ \lambda =y$ and $-x - y = \lambda z$ has a non-trivial solution.

(A) Contains exactly two elements **(B)** Contains more than two elements

(C) Is a singleton **(D)** Is an empty set

6. An ordered pair (β,α) for which the system of linear equations

$$(1+\alpha)x + \beta y + z = 2$$

$$\alpha x + (1+\beta)y + z = 3$$

$$\alpha x + \beta y + 2z = 2$$

Has a unique solution, is

(A) (2, 4) **(B)** (–4, 2) **(C)** (1, –3) **(D)** (–3, 1)

7. If the system of linear equations

$$2x + 2y + 3z = a$$

$$3x - y + 5z = b$$

$$x - 3y + 2z = c$$

where a, b, c are non-zero real numbers, has more than one solution, then

(A) $b - c - a = 0$ **(B)** $a + b + c = 0$ **(C)** $b - c + a = 0$ **(D)** $b + c - a = 0$

8. The number of values of $(\in \theta 0 \pi,)$ for which the system of linear equations

$$x + 3y + 7y = 0$$

$$-x + 4y + 7z = 0$$

$$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$$

has a non-trivial solution, is:

(A) Two **(B)** Three **(C)** Four **(D)** One

9. If the system of linear equations, $x - 4y + 7z = g$, $3y - 5z = h$, $-2x + 5y - 9z = k$, is consistent, then:

(A) $2g + h + k = 0$ **(B)** $g + 2h + k = 0$ **(C)** $g + h + k = 0$ **(D)** $g + h + 2k = 0$

10. The system of linear equations, $x + y + z = 2$, $2x + 3y + 2z = 5$, $2x \ +3y \ +(a^2 -1)z \ =a \ +1$.

(A) Has infinitely many solution for $a = 4$ **(B)** Is inconsistent when $a = 4$

(C) Has a unique solution for $|a| = \sqrt{3}$ **(D)** Is inconsistent when $|a| = \sqrt{3}$

11. The following system of linear equations **[2020]**

$7x+6y-2z=0$

$3x+4y+2z=0$

$x-2y-6z=0$, has

(A) infinitely many solutions, (x, y, z) satisfying $x=2z$

(B) infinitely many solutions, (x, y, z) satisfying $y=2z$

(C) no solution

(D) only the trivial solution

12. Let $A=[a_{ij}]$ and $B=[b_{ij}]$ be two 3 × 3 matrices such that $b_{ij}=(3)^{(i+j-2)}a_{ji}$, where $i, j=1,2,3$. If the determinant of B is 81 then the determinant of A is: **[2020]**

(A) 3 **(B)** 1/3 **(C)** 1/81 **(D)** 1/9

13. If the system of linear equations,

$x+y+z=6$

$x+2y+3z=10$

$3x+2y+\lambda z=\mu$

has more than two solutions, then $\mu-\lambda^2$ is equal to _____. **[2020]**

14. If $A=\begin{pmatrix}2 & 2\\ 9 & 4\end{pmatrix}$ and $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, then $10A^{-1}$ is equal to: **[2020]**

(A) $6I-A$ **(B)** $A-4I$ **(C)** $4I-A$ **(D)** $A-6I$

15. Let $a-2b+c=1$

If $f(x)=\begin{vmatrix}x+a & x+2 & x+1\\ x+b & x+3 & x+2\\ x+c & x+4 & x+3\end{vmatrix}$ then:

(A) $f(50)=-501$ **(B)** $f(-50)=501$ **(C)** $f(-50)=-1$ **(D)** $f(50)=1$ **[2020]**

16. Let α be a root of the equation $x^2+x+1=0$ and the matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^2\\ 1 & \alpha^2 & \alpha^4\end{bmatrix}$, then the matrix A^{31} is equal to: **[2020]**

(A) A **(B)** A^3 **(C)** I_3 **(D)** A^2

17. If the system of linear equations,

$2x+2ay+az=0$

$2x+3by+bz=0$

$2x+4cy+cz=0$,

Where $a, b, c \in R$ are non-zero and distinct; has a non-zero solution, then: **[2020]**

(A) $a+b+c=0$ **(B)** a, b, c are in G.P.

(C) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. **(D)** a, b, c are in A. P.

18. For which of the following ordered pairs (δ, μ), the system of linear equations

$x+2y+3z=1$

$3x+4y+5z=\mu$

$4x+4y+4z=\delta$

is inconsistent ? **[2020]**

(A) (1, 0) **(B)** (4, 3) **(C)** (4, 6) **(D)** (3, 4)

Answer Key

1. (C)
2. (B)
3. (C)
4. (B)
5. (C)
6. (A)
7. (A)
8. (A)
9. (A)
10. (D)
11. (A)
12. (D)
13. (13)
14. (D)
15. (D)
16. (B)
17. (C)
18. (B)

Linear Algebra DS & AI

Matrices and Determinants

DPP 09

1. The number of all 3×3 matrices A, with entries from the set {–1, 0, 1} such that the sum of the diagonal elements of AA^T is 3, is _____.

2. The system of linear equations $\lambda x + 2y + 2z = 5$, $2\lambda x + 3y + 5z = 8$, $4x + \lambda y + 6z = 10$ has: **[2020]**

(A) No solution when $\lambda = 2$
(B) Infinitely many solutions when $\lambda = 2$
(C) No solution when $\lambda = 8$
(D) A unique solution when $\lambda = -8$

3. If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = adj A$ and $C = 3A$, then $\frac{|adj\, B|}{|C|}$ is equal to : **[2020]**

(A) 72 **(B)** 8 **(C)** 16 **(D)** 2

4. Let S be the set of all $\in \lambda R$ for which the system of linear equations
$2x - y + 2z = 2$; $x - 2y + \lambda z = -4$; $x + \lambda y + z = 4$ has no solution. Then the set S: **[2020]**

(A) Is an empty set
(B) Contains more than two elements
(C) Is a singleton
(D) Contains exactly two elements

5. Let A be a 2×2 real matrix with entries from {0, 1} and $|A| \neq 0$. Consider the following two statements:

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then $tr(A) = 2$, where I_2 denotes 2×2 identity matrix and $tr(A)$ denotes the sum of the diagonal entries of A. Then:

(A) (P) is false and (Q) is true
(B) (P) is true and (Q) is false **[2020]**
(C) Both (P) and (Q) are false
(D) Both (P) and (Q) are true

6. Let A be a 3×3 matrix such that $adj\ A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ and $B = adj\ (adj\ A)$. If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to: **[2020]**

(A) $\left(9, \frac{1}{81}\right)$ **(B)** $\left(3, \frac{1}{81}\right)$ **(C)** $(3, 81)$ **(D)** $\left(9, \frac{1}{9}\right)$

7. Let S be the set of all integer solutions, (x, y, z), of the system of equations
$x - 2y + 5z = 0$
$-2x + 4y + z = 0$
$-7x + 14y + 9z = 0$
Such that $15 \leq x^2 + y^2 + z^2 \leq 150$. Then the number of elements in the set S is equal to _____ . **[2020]**

8. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} =$

$Ax^3 + Bx^2 + Cx + D$, then $B + C$ is equal to: **[2020]**

(A) 1 **(B)** –1 **(C)** –3 **(D)** 9

9. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in R$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to _____ . **[2020]**

10. If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, $\left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then which one of the following is not true? **[2020]**

(A) $a^2 - b^2 = \frac{1}{2}$ **(B)** $0 \leq a^2 + b^2 \leq 1$ **(C)** $a^2 - d^2 = 0$ **(D)** $a^2 - c^2 = 1$

11. If the system of equations

$x -2y +3z =9$

$2x +y +z =b$

$x -7y +az =24$, has infinitely many solutions, then $a -b$ is equal to ______. **[2020]**

12. If the system of equations

$x +y +z =2$

$2x +4y -z =6$

$3x +2y \lambda +z \mu =$

Has infinitely many solutions, then: **[2020]**

(A) $2\lambda-\mu=5$ **(B)** $\lambda-2\mu=-5$ **(C)** $2\lambda+\mu=14$ **(D)** $\lambda+2\mu=14$

13. Suppose the vector x_1, x_2 and x_3 are the solutions of the system of linear equations, Ax = b when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. If

$x_1=\begin{bmatrix}1\\1\\1\end{bmatrix}, x_2=\begin{bmatrix}0\\2\\1\end{bmatrix}, x_3=\begin{bmatrix}0\\0\\1\end{bmatrix}, b_1=\begin{bmatrix}1\\0\\0\end{bmatrix}, b_2=\begin{bmatrix}0\\2\\0\end{bmatrix}$ and $b_3=\begin{bmatrix}0\\0\\2\end{bmatrix}$ then the determinant of A is equal to : **[2020]**

(A) 4 **(B)** $\frac{3}{2}$ **(C)** $\frac{1}{2}$ **(D)** 2

14. Let $\in\lambda R$. The system of linear equations $2x_1 -4x_2 \lambda +x_3 =1$, $x_1 -6x_2 +x_3 =2\lambda$, $x_1 -10x_2 +4x_3 =3$ is inconsistent for : **[2020]**

(A) exactly two values of λ **(B)** every value of λ

(C) exactly one negative value of λ **(D)** exactly one positive value of λ

15. If the system of linear equations

$x + y + 3z = 0$

$x + 3y + k^2 z = 0$

$3x + y + 3z = 0$

has a non-zero solution (x, y, z) for some $k \in R$, then $x + \left(\frac{y}{z}\right)$ is equal to : **[2020]**

(A) –9 **(B)** 3 **(C)** –3 **(D)** 9

Answer Key

1. (672)
2. (A)
3. (B)
4. (D)
5. (A)
6. (B)
7. (8)
8. (C)
9. (10)
10. (A)
11. (5)
12. (C)
13. (D)
14. (C)
15. (C)

Data Science & AI — Hinglish

Subject : Linear Algebra

DPP-01

1. Let A be a 5 × 8 matrix, then each column of A contains.
(a) 5 elements (b) 8 elements
(c) 40 elements (d) 13 elements

2. If A is a matrix of order 10 × 5, then each row of A contains-
(a) 25 elements (b) 15 elements
(c) 10 elements (d) 150 elements

3. The number of all possible matrices of order 2 × 3 with each entry 1 or –1 is-
(a) 32 (b) 12
(c) 6 (d) 64

4. If A is of order m × n and B is of order p × q, then AB is defined only if
(a) m = q (b) m = p
(c) n = p (d) n = q

5. If P is of order 2 × 3 and Q is of order 3 × 2, then PQ is of order
(a) 2 × 3 (b) 2 × 2
(c) 3 × 2 (d) 3 × 3

6. If $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then
(a) $A^2 = 0$ (b) $A^2 = A$
(c) $A^3 = A$ (d) $A^2 = 2A$

7. If $A = [x, y, z]$ $B = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$, $C = [\alpha, \beta, \gamma]^T$, then ABC is-
(a) Not defined
(b) is a 3 × 3 matrix
(c) is a 1 × 1 matrix
(d) is a 3 × 3 matrix

8. If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, then A is equal to
(a) $\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (b) $\frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

9. If $x + \begin{bmatrix} 2 & 1 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then 'X' is equal to
(a) $\begin{bmatrix} 0 & 1 \\ 0 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 \\ 0 & -6 \end{bmatrix}$
(c) $\begin{bmatrix} -1 & 0 \\ -6 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 6 & 0 \end{bmatrix}$

10. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 5 & 7 \end{bmatrix}$ and $2A - 3B = \begin{bmatrix} 4 & 5 & -9 \\ 1 & 2 & 3 \end{bmatrix}$, then B is equal to
(a) $\frac{1}{3}\begin{bmatrix} -2 & -1 & 15 \\ 5 & 8 & -11 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 & -15 \\ 5 & -8 & -11 \end{bmatrix}$
(c) $\begin{bmatrix} 2 & -1 & 15 \\ 5 & 8 & 11 \end{bmatrix}$ (d) $-\frac{1}{3}\begin{bmatrix} 2 & 1 & -15 \\ 5 & -8 & -11 \end{bmatrix}$

11. If $\begin{bmatrix} x & 1 \\ -1 & -y \end{bmatrix} + \begin{bmatrix} y & 1 \\ 3 & x \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, then
(a) x = –1, y = 0 (b) x = 1, y = 0
(c) x = 0, y = 1 (d) x = 1, y = 1

12. Let $A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}$ and $A + B - 4l = 0$, then B is equal to
(a) $\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & -2 \\ -3 & -4 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 3 & 5 \\ 1 & -4 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

(c) $\frac{1}{4}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & -2 \\ -3 & -4 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 & 5 \\ -1 & 4 & -2 \\ 3 & 4 & 1 \end{bmatrix}$

13. If a diagonal matrix is commutative with every matrix of the same order then it is necessarily
(a) A diagonal matrix with at least two diagonal elements different.
(b) A scalar matrix
(c) A unit matrix
(d) A diagonal matrix with exactly two diagonal elements different.

14. $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 5 \end{bmatrix}$ is equal to
(a) $\begin{bmatrix} 45 \\ 44 \end{bmatrix}$ (b) $\begin{bmatrix} 43 \\ 45 \end{bmatrix}$
(c) $\begin{bmatrix} 44 \\ 43 \end{bmatrix}$ (d) $\begin{bmatrix} 43 \\ 50 \end{bmatrix}$

15.. If $f(x) = x^2 + 4x - 5$ and $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, then $f(A)$ is equal to-
(a) $\begin{bmatrix} 0 & -4 \\ 8 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$

16. If A is a square matrix such that $A^2 = I$, then A^{-1} is equal to-
(a) I (b) 0
(c) A (d) I + A

17. Let A $\begin{bmatrix} 1 & x/n \\ -x/n & 1 \end{bmatrix}$, then $\lim_{n\to\infty} A^n$ is-
(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$
(c) $\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

18. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $P = \begin{bmatrix} \cos\pi/6 & \sin\pi/6 \\ -\sin\pi/6 & \cos\pi/6 \end{bmatrix}$ and $Q = PAP^T$, then $P^TQ^{2013}P$
(a) $\begin{bmatrix} 1 & 2013 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2013 \\ 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 2013 & 0 \\ 0 & 2013 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 2013 \\ 2013 & 0 \end{bmatrix}$

19. If A is square matrix, then A is symmetric, iff
(a) $A^2 = A$ (b) $A^2 = I$
(c) $A^T = A$ (d) $A^T = -A$

20. If A is a square matrix, then A is skew symmetric iff-
(a) $A^2 = A$ (b) $A^2 = I$
(c) $A^T = A$ (d) $A^T = -A$

21. If A is any square matrix, then
(a) $A + A^T$ is skew symmetric
(b) $A - A^T$ is symmetric
(c) $A A^T$ is symmetric
(d) AA^T is skew symmetric

22. Each diagonal element of a skew symmetric matrix is-
(a) Zero
(b) Positive and equal
(c) Negative and equal
(d) Any real number

23.. Let A be the set of all 3 × 3 matrices which are symmetric with entries 0 or 1. If there are five I's and four 0's then number of matrices in A
(a) 6 (b) 12
(c) 9 (d) 58

24. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and l be the identity matrix of order 3. If $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = 1$, then $\frac{q_{31} + q_{32}}{q_{21}}$ equals.
(a) 52 (b) 103
(c) 201 (d) 205

25. If A is a singular matrix, then adj A is
(a) Singular (b) Non-singular
(c) Symmetric (d) Non defined

26. Which of the following is not true?

(a) Every skew-symmetric matrix of odd order is non-singular

(b) If determinant of a square matrix is non-zero, then the it is non-singular

(c) Adjoint of symmetric matrix is symmetric

(d) Adjoint of diagonal matrix is diagonal

27. If k is a scalar and I is a unit matrix of order 3, then adj (kl) =

(a) $k^3 I$ (b) $k^2 I$

(c) $-k^3 I$ (d) $-k^2 I$

28. Which of the following is/are true?

(i) Adjoint of a symmetric matrix is symmetric

(ii) Adjoint of a unit matrix isa unit matrix

(iii) A(adj A)= (adjA) A=|A|$I$and

(iv) Adjoint of a diagonal matrix is a diagonal matrix

(a) (i)

(b) (ii)

(c) (iii) & (iv)

(d) None of these

29. The adjoint of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ is

(a) $\begin{bmatrix} 3 & -9 & -5 \\ -4 & 1 & 3 \\ -5 & 4 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & 4 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ (d) None of these

30. The inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ is-

(a) $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ (d) None of these

31. If A and B are non-singular square matrix of same order, then adj(AB) is equal to

(a) (adj A)(adj B)

(b) (adj B) (adj A)

(c) (adj B^{-1})(adj A^{-1})

(d) (adj A^{-1}) (adj B^{-1})

32. If I_3 is the identity matrix of orders, the value of $(I_3)^{-1}$ is:

(a) 0

(b) $3I_3$

(c) I_3

(d) Does not exist

33. For two invertible matrices A and B of suitable orders, then value of $(AB)^{-1}$ is:

(a) $(BA)^{-1}$ (b) $B^{-1} A^{-1}$

(c) $A^{-1} B^{-1}$ (d) $(AB')^{-1}$

34. Inverse of the matrix $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ is-

(a) $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

(b) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

(c) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

(d) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

35. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}; 1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A^{-1} = \frac{1}{6}[A^2 + cA +$ dl], where c, d $\in$R, then pair of values (c, d)

(a) (6, 11) (b) (6, 11)

(c) (–6, 11) (d) (–6, –11)

36. Let $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then |2A| is equal to

(a) 4 cos2θ (b) 1

(c) 2 (d) 4

37. If ω is non-real complex cube root of unit, then the determinant of the matrix A is defined as

$\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{bmatrix}$ is equal to

(a) 0 (b) 1
(c) 3 (d) 2

38. The value of the determinant of the matrix A= $\begin{bmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{bmatrix}$ is equal to

(a) $(x-y)(y-z)(z-x)$
(b) $(x-y)(y-z)(z-x)(x+y+z)$
(c) $(x+y+z)$
(d) $(x-y)(y-z)(z-x)(xy+yz+zx)$

39. If A is 3 × 3 matrix and |A| = 4, then $|A^{-1}|$ is equal to-

(a) $\frac{1}{4}$ (b) $\frac{1}{16}$
(c) 4 (d) 2

40. The number of real roots of the equation |A|= 0 where A is defined as $\begin{bmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{bmatrix}$

(a) 0 (b) 1
(c) 2 (d) 3

41. If |A|= 0 where A is defined as the matrix $\begin{bmatrix} 4 & -4 & 0 \\ a & b+4 & c \\ a & b & c+4 \end{bmatrix}$, then a + b + c is equal to

(a) 41 (b) 116
(c) 628 (d) – 4

42. The equation |A|= 0 where A is defined as $\begin{bmatrix} x-2 & 3 & 1 \\ 4x-2 & 10 & 4 \\ 2x-1 & 5 & 1 \end{bmatrix}$ is satisfied by

(a) $x=-2$ (b) $x=-5$
(c) $x=-7$ (d) $x=-9$

43. The determinant of the matrix A= $\begin{bmatrix} x & 4 & y+z \\ y & 4 & z+x \\ z & 4 & x+y \end{bmatrix}$ is equal to

(a) $4x$ (b) $x+y+z$
(c) xyz (d) 0

44. The value of the determinant A= $\begin{bmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{bmatrix}$

(a) 0 (b) 1
(c) 2 (d) 3

45. The roots of the equation |A|= 0 where A is defined as $\begin{bmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{bmatrix}$

(a) 1, 2 (b) 1,–2
(c) -1,–2 (d) –1, 2

46. The value of the determinant of the matrix $\begin{bmatrix} 1 & \log_b a \\ \log_a b & 1 \end{bmatrix}$ is equal to-

(a) 1 (b) $\log_a b$
(c) $\log_b a$ (d) 0

47. Find the |A|,where A is defined as $\begin{bmatrix} 11+a & c & 1+bc \\ 11+a & d & 1+bd \\ 11+a & e & 1+be \end{bmatrix}$ is equal to

(a) 1 (b) 0
(c) 3 (d) $a+b+c$

48. If |A|= $ka^2b^2c^2$ where A is defined as $\begin{bmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{bmatrix}$, then k is equal to

(a) 2 (b) 4
(c) –4 (d) 8

49. Let $f(x)$ =|A| where A is defined as the matrix $\begin{bmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x-1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{bmatrix}$ then $f(100)$ is equal to -

(a) 0 (b) 1

(c) 100 (d) –100

50. The roots of the equation |A|=0 where A is defined as $\begin{bmatrix}1 & 4 & 20\\ 1 & -2 & 5\\ 1 & 2x & 5x^2\end{bmatrix}$ are

(a) –1, –2 (b) –1, 2
(c) 1, –2 (d) 1, 2

51. If A^2 = 8A + KI where A = $\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$, then k is

(a) 7 (b) –7
(c) 1 (d) –1

52. The value of the determinant of matrix A where A is defined as A= $\begin{bmatrix}{}^5C_0 & {}^5C_3 & 14\\ {}^5C_1 & {}^5C_4 & 1\\ {}^5C_2 & {}^5C_5 & 1\end{bmatrix}$ is

(a) 0 (b) –(6!)
(c) 80 (d) – 576

53. If

$\Delta_1 is\ the\ determinant\ of\ matrix \begin{bmatrix}10 & 4 & 3\\ 17 & 7 & 4\\ 4 & -5 & 7\end{bmatrix}$

and $\Delta_2 is\ the\ determinant\ of\ the\ matrix$

$\begin{bmatrix}4 & x+5 & 3\\ 7 & x+12 & 4\\ -5 & x-1 & 7\end{bmatrix}$ such that $\Delta_1 + \Delta_2 = 0$, then

(a) $x = 0$
(b) x has no real value
(c) $x = 0$
(d) $x = 1$

54. If all elements of a third order determinant are equal to 1 or –1, then determinant itself is-

(a) An odd integer
(b) An even integer
(c) An imaginary number
(d) Multiple of 3

55. If A is a 3×3 matrix and det (3A) = k {det (A)}, then k is-

(a) 9 (b) 6
(c) 1 (d) 27

56. If A is any of square matrix of order n, then A(adjA) is equal to

(a) 1 (b) $|A|.I_n$
(c) 0 (d) $|A|^n$

57. If A is a square matrix of order 3, |A| =3 then |adj adj A| is equal to -

(a) 3^5 (b) 3^7
(c) 9 (d) 81

58. If A = $\begin{bmatrix}11 & 7\\ -13 & 17\end{bmatrix}$, then adj (adj A) is

(a) $\begin{bmatrix}17 & -7\\ 13 & 11\end{bmatrix}$ (b) $\begin{bmatrix}11 & 7\\ -13 & 17\end{bmatrix}$

(c) $\begin{bmatrix}-17 & 7\\ 13 & -11\end{bmatrix}$ (d) $\begin{bmatrix}-11 & 7\\ -13 & 17\end{bmatrix}$

59. If A = $\begin{bmatrix}\sin\alpha & -\cos\alpha & 0\\ \cos\alpha & \sin\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$, then A^{-1} is equal to

(a) $-A^T$ (b) A
(c) adjA (d) A^2

60. If A is a matrix of order 3 and |A| = 2, then |adj A| is-

(a) 1 (b) 2
(c) 8 (d) 4

61. Let A = $\begin{bmatrix}0 & 0 & -1\\ 0 & -1 & 0\\ -1 & 0 & 0\end{bmatrix}$. The only correct statement about the matrix A is:

(a) A is zero matrix
(b) A = (–1)/3
(c) A^{-1} doesn't exist
(d) A^2 = I

62. A is a matrix of order 3 × 3. If A' = A and five entries in the matrix are of one kind and remaining four are of another kind, then the maximum number of such matrices is greater then of equal to-

(a) 9 (b) 10
(c) 11 (d) 8

63. If the adjoint of a 3×3 matrix P is $\begin{bmatrix}1 & 4 & 4\\ 2 & 1 & 7\\ 1 & 1 & 3\end{bmatrix}$,then the possible value (s) of the determinant of P is (are)-

(a) –2 (b) –1
(c) 1 (d) 2

64. Let P $= \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose Q = $[q_{ij}]$ is a matrix such that PQ = Kl, where k$\in \mathbb{R}$, k $\neq 0$ and l is the identity matrix of order 3. If $q_{23} = \frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then

(a) $\alpha = 0, k = 8$
(b) $4\alpha - k + 8 = 0$
(c) $\det(P \text{ adj } (Q)) = 2^9$
(d) $\det(Q \text{ adj}(P)) = 2^{13}$

65. A and B are two matrices for same order 3 × 3, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 8 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 & 5 \\ 2 & 3 & 5 \\ 7 & 2 & 9 \end{bmatrix}$$

Choose the correct answer:

1. The value of adj(adj A) is,
(a) –A (b) 4A
(c) 8A (d) 16A

2. The value of |adj (AB)| is
(a) 24 (b) 24^2
(c) 24^3 (d) 65

3. Value of |adj(adj(adj(adjA)))| is
(a) 2^4 (b) 2^9
(c) 1 (d) 2^{19}

66. Let P be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these are 1 and four of them are 0.

1. The number of matrices in /is:
(a) 12 (b) 6
(c) 9 (d) 3

2. The number of A in P for which the system of linear equations

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Has a unique solution, is

(a) Less than 4
(b) At least 4 but less than 7
(c) At least 7 but less than 10
(d) At least 10

3. The number of matrices A in A for which for which the system on linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent, is

(a) 0
(b) More than 2
(c) 2
(d) 1

67. The total number of distinct $x \in \mathbb{R}$ which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is-

68. The matrices which commute with A $= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ in case of multiplication.

STATEMENT -1 Are always singular
STATEMENT -2 Are always singular
STATEMENT -3 Are always singular

(a) F F F (b) T T F
(c) T T T (d) T F T

69. Number or real roots equation |A|=0 where A is defined as $\begin{bmatrix} x^2 & -4 & -2 \\ 4 & x^2 & 1 \\ -5 & 3 & x^2 \end{bmatrix}$ are

(a) 0 (b) 1
(c) 2 (d) 4

70. The coefficient of x in *f(x)* = det of A where A is defined as

$$\begin{bmatrix} x & 1+\sin x & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & 1+x^2 & 0 \end{bmatrix}$$ where $-1 < x \leq 1$, is

(a) 1 (b) –2
(c) –1 (d) 0

71. If adj B = A, |P| = |Q| =1, then adj $(Q^{-1}BP^{-1})$ equals

(a) PQ (b) QAP
(c) PAQ (d) $PA^{-1}Q$

72. The matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$ is

(a) Orthogonal (b) Nilpotent
(c) Idempotent (d) Involutary

73. The matrix $A = \begin{bmatrix} a & 2 \\ 2 & 4 \end{bmatrix}$ is singular if

(a) $a \neq 1$ (b) $a = 1$
(c) $a = 0$ (d) $a = -1$

74. $A = \begin{bmatrix} 1 & 0 & k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{bmatrix}$ is invertible for

(a) $k = 1$ (b) $k = -1$
(c) $k \neq \pm 1$ (d) $k = 0$

Answer Key

1. (a)
2. (b)
3. (d)
4. (c)
5. (b)
6. (a)
7. (c)
8. (a)
9. (c)
10. (d)
11. (b)
12. (a)
13. (b)
14. (d)
15. (d)
16. (c)
17. (b)
18. (a)
19. (c)
20. (d)
21. (c)
22. (a)
23. (b)
24. (b)
25. (b)
26. (a)
27. (b)
28 (d)
29. (b)
30. (b)
31 (b)
32. (c)
33. (b)
34. (d)
35. (c)
36. (d)
37. (a)
38. (b)
39. (a)
40. (d)
41. (d)
42. (c)
43. (d)
44. (a)
45. (b)
46. (d)
47. (b)
48. (b)
49. (a)
50. (b)
51. (b)

52. (d)
53. (a)
54. (b)
55. (d)
56. (b)
57. (d)
58. (b)
59. (c)
60. (d)
61. (d)
62. (1,2,3,4)
63. (1,4)
64. (2, 3)
65.1- (a)
2-(b)
3-(c)
66. **1-**(a)
2- (b)
3-(b)
67. 2
68. (a)
69. (c)
70. (b)
71. (c)
72. (c)
73. (b)
74. (c)

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Linear Algebra
DPP-02

1. (a) In $\mathbb{R}^2$, express the vector (2,4) as a linear combination of the vector (0,3) and (2,1)
   (b) In $\mathbb{R}^3$, express the vector (2, 3, –2) as a linear combination of the vectors (0, 1, 0), (1, 2, –1) and (1,1, –2).
   (c) In M2, 2, express the matrix $\begin{pmatrix} 3 & 1 \\ 0 & 4 \end{pmatrix}$ as a linear combination of the matrices $\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$ and $\begin{pmatrix} 0 & -2 \\ 0 & 1 \end{pmatrix}$.

2. (a) In $\mathbb{R}^2$, let $v_1 = (0, 3)$ and $v_2 = (2,1)$. Calculate the linear combination $4v_1 - 2v_2$.
   (b) In $\mathbb{R}^4$, let $v_1 = (1, 2, 1, 3)$ and $v_2 = (2, 1, 0, -1)$. Calculate the linear combination $3v_1 + 2v_2$.

3. For each of the following vector spaces V and vectors v_1, v_2 and v_3 in V, form the linear combination $3v_1 - 2v_2 + v_3$.
   (a) $V = P_3$, $v_1 = 1 + x + x^2$, $v_2 = 1 - x$, $v_3 = x + x^2$.
   (b) $V = M_{2,3}$, $v_1 = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 3 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 3 & -4 \end{pmatrix}$, $v_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 1 \end{pmatrix}$.

4. (a) Given the basis E = {(1,2), (–3, 1)} for $\mathbb{R}^2$, determine the standard coordinate representation of $(2, 1)_E$.
   (b) Given the basis E = {(1, 0, 2), (–1, 1, 3), (2, –2, 0)}for $\mathbb{R}^3$, determine the standard coordinate representation of $(1, 1, -1)_E$.

5. Show that neither of the following sets is a real vector space.
   (a) $V = \{(x, y) \in \mathbb{R}^2 : y = 2x + 1\}$
   (b) $V = \left\{\begin{pmatrix} 0 & a \\ b & c \end{pmatrix} : a, b, c \in \mathbb{Z}\right\}$

6. Show that neither of the following sets is a real vector space.
   (a) V = {all polynomials of degree equal to 5}
   (b) $V = \{a + bi \in \mathbb{C} : a \geq 0\}$

7. In each case, determine whether set S of matrices is a linearly independent subset of $M_{2,2}$.
   (a) $S = \left\{\begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -2 & 1 \end{pmatrix}\right\}$
   (b) $S = \left\{\begin{pmatrix} 0 & -1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 0 & -4 \end{pmatrix}\right\}$

8. Show that each of the following is a spanning set for $\mathbb{R}^2$.
   (a) {(1, 1), (–1, 2)}
   (b) {(2, –1), (3, 2)}

9. Show that {(1, 0, ,0), (1, 1, 0)(2, 0, 1)} is a spanning set for $\mathbb{R}^3$.

10. For each of the following, determine whether the set S is a subspace of the vector space V.
   (a) $V = P_3$, $S = \{a + bx : a, b \in \mathbb{R}\}$
   (b) $V = P_3$, $S = \{x + ax^2 : a, b \in \mathbb{R}\}$
   (c) $V = M_{2,2}$, $S = \left\{\begin{pmatrix} a & 1 \\ 0 & d \end{pmatrix} : a, b \in \mathbb{R}\right\}$

11. In each of the following cases, determine whether S is a linearly independent subset of the vector space V.

(a) $V = P_4$, $S = \{1, x, x^2, x^3, 1 + x + x^2 + x^3\}$

(b) $V = M_{2,2},\ S = \left\{\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}\right\}$

(c) $V = M_{2,2},\ S = \left\{\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right\}$

(d) $V = \mathbb{C}$, S $\{1 + i, 1 - i\}$

12. Show that each of the following is a spanning set for $\mathbb{R}^2$.

(a) $\{(1, 2), (2, -3)\}$

(b) $\{(1,0), (1,1), (1, -2)\}$

13. Show that $\{1 + x, 1 + x^2, 1 + x^3, x\}$ is a spanning set for P_4.

14. (a) Verify that $\{(3,4,0), (8,-6,0), (0,0,5)\}$ is an orthogonal basis for $\mathbb{R}^3$.

(b) Express the vector $(10,0, 4)$ in terms of this basis.

15. For each of the following vector spaces V and sets of vector S in V, determine $\langle S \rangle$.

(a) $V = \mathbb{R}^3$, $S = \{(1, 0, 0)\}$.

(b) $V = M_{2,2}, S = \left\{\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}\right\}$

16. (a) Find the E-coordinate representation of the vector $(5,-4)$ with respect to the basis $E = \{(1,2), (-3, 1)\}$ for $\mathbb{R}^3$.

(b) Find the E-coordiante representation of the vector $(-3, 5, 7)$ with respect to the basis $E = \{(1, 0, 2), (-1, 1, 3), (2, -2, 0)\}$ for $\mathbb{R}^3$.

17. If A and B are two matrices and if AB exists, then BA exists-

(a) Only if A has as many rows as B has columns

(b) Only if both A and B are square matrices

(c) Only if A and B are skew matrices

(d) Only if both A and B are sysmmetirc.

18. Determine whether each of the following sets is a basis for $\mathbb{R}^3$.

(a) $\{(0, 1, 2), (0, 2, 3), (0, 6, 1)\}$

(b) $\{(1,2,1), (1,0,-1), (0, 3, 1)\}$

(c) $\{(1, 0, 0), (0,1, 0), (0, 0, 1), (1,1,1)\}$

19. Determine whether $\{(1.2, -1, -1),(-1,5,1,3)\}$ is a basis for $\mathbb{R}^4$.

20. Determine whether each of the following sets of vectors is a linearly independent subset of V.

(a) $V = \mathbb{R}^2$, $\{(1, 0,), (-1, -1)\}$.

(b) $V = \mathbb{R}^2$, $\{(1, -1,), (1, 1), (2, 1)\}$.

(c) $V = \mathbb{R}^3$, $\{(1,1, 0,), (-1, ,1,1)\}$.

(d) $V = \mathbb{R}^3$, $\{(1, 0, 0,), (1, 1, 0), (1, 1, 1)\}$.

(e) $V = \mathbb{R}^4$, $\{(1, 2, 1,0), (0, -1, 1, 3)\}$.

21. (a) Show that $(2,1,1)$ and $(1, -4, 2)$ are orthogonal.

(b) Determine which of the following vectors are orthogonal:

$v_1 = (-2, 6, 1)$, $v_2 = (9, 2, 6)$, $v_3 = (4, -15, -1)$

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Data Science & AI — Hinglish

Subject: DS and AI
Chapter: Linear Algebra

DPP-03

1. Consider $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

The eigenvalues of M are

(a) 0, 1, 2 (b) 0, 0, 3
(c) 1, 1, 1 (d) –1, 1, 3

2. A 3 × 3 matrix M has Tr[M] = 6, Tr[M^2] = 26, Tr [M^3]= 90. Which of the following can be possible set of eigenvalues of M?

(a) {1,1,4} (b) {–1,0,7}
(c) {–1,3,4} (d) {2,2,2}

3. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$

(a) (1, 4, 9) (b) (0, 7, 7)
(c) (0, 1, 13) (d) (0, 0, 14)

4. The eigenvalue of the anti-symmetric $A = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$ where n_1, n_2, n_3 are components of an unit vector, are

(a) 0, –i, i (b) 0, 1, –1
(c) 0, 1 + i, –1 –i (d) 0, 0, 0

5. A 2 × 2 matrix 'A' has eigenvalues $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of' n such that $A^n = I$

(a) 20 (b) 30
(c) 60 (d) 120

6. Given a 2 × 2 unitary matrix satisfying U'U = UU' = I with det U = $e^{i\varphi}$, one can construct a unitary matrix V (V'V = VV' = I) with det V= 1 from it by

(a) Multiplying U by $e^{-i\varphi/2}$
(b) Multiplying a single element of U by $e^{-i\varphi}$
(c) Multiplying any row or column by $e^{-i\varphi/2}$
(d) Multiplying U by $e^{-i\varphi}$

7. Consider a n x n (n > 1) matrix A, in which A_{ij} is the product of the indices i and j (namely A_{ij} = ij) The matrix A

(a) has one degenerate eigenvalue with degeneracy (n – 1)
(b) has two degenerate eigenvalues with degeneracies 2 and (n – 2)
(c) has one degenerate eigenvalues with degeneracy n
(d) does not have any degenerate eigenvalues

8. Consider the matrix $M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$

The eigenvalues of M are

(a) –5, –2, 7 (b) –7, 0, 7
(c) –4i, 2i, 2i (d) 2, 3, 6

9. The matrices $A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and C = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ satisfy the commutation relations

(a) [A, B] = B + C, [B, C] = 0, [C, A] = B + C
(b) [A, B] = C, [B, C] = A, [C, A] = B
(c) [A, B] = B, [B, C] = 0, [C, A] = A
(d) [A, B] = C, [B, C] = 0, [C, A] = B

10. The column vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is a simultaneous eigenvector of A = $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and B = $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ if

(a) b = 0 or a = 0
(b) b = a or b = –2a
(c) b = 2a or b = –a
(d) b = a/2 or b = –a/2

11. For any operator A, $i\,(A^+ - A)$ is:
(a) Hermitian (b) Anti-Hermitian
(c) Unitary (d) Orthogonal

12. If two matrices A and B can be diagonalized simultaneously, which of the following is true?
(a) $A^2 B = B^2 A$
(b) $A^2 B2 = B^2 A$
(c) $AB = BA$
(d) $AB^2 AB = BABA^2$

13. Which one of the following is the inverse of the matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$?
(a) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

14. A 3 × 3 matrix has eigenvalues 0, 2 + i and 2 – i. Which one of the following statements is correct?
(a) The matrix is hermitian
(b) The matrix is unitary
(c) The inverse of the matrix exist
(d) The determinant of the matrix is zero

15. A real traceless 4 × 4 unitary matrix has two eigen values – 1 and 1. The other eigenvalues are
(a) Zero and +2 (b) –1 and + 1
(c) Zero and +1 (d) +1 and +1

16. The eigenvalues of the matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ are
(a) +1 and +1 (b) Zero and +1
(c) Zero and +2 (d) –1 and +1

17. If A is 2 × 2 matrix with determinant 2, then the determinant of adj.[adj.[adj(A^{-1})]] is equal to
(a) 1/512 (b) 1/1024
(c) 1/128 (d) 1/256

18. The eigenvalues of ($A^4 + 3A - 2I$), where A is
$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$, are
(a) 2, 20, 88 (b) 1, 2, 3
(c) 2, 20, 3 (d) 1, 20, 88

19. Eigen value of matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ are
(a) –2, –1, 1, 2 (b) –1, 1, 0, 2
(c) 1, 0, 2, 3 (d) –1, 1, 0, 3

20. A linear transformation T, defined as $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 & + & x_2 \\ x_2 & - & x_3 \end{pmatrix}$, transform a vector $\vec{x}$ three dimensional space to a two-dimensional real space. The transformation matrix T is
(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

21. One of the eigenvalues of the matrix
$\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is 5
The other two eigen values are
(a) 0 and 0 (b) 1 and 1
(c) 1 and –1 (d) –1 and –1

22. The normalized eigen vector corresponding to the eigen value 5 is:
(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$
(c) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

23. The eigenvalues of a matrix are i, –2i and 3i. The matrix is
(a) Unitary
(b) Anti-Unitary
(c) Hermitian
(d) Anti-Hermitian

24. The eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are

(a) 6,1and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(b) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

25. Consider a vector $\vec{p} = 2\hat{i} + 3\hat{j} + 2\hat{i}$ in the coordinate system $(\hat{i}, \hat{j}, \hat{k})$. The axes are rotated anti clockwise about Y axis by an angle of 60°. The vector $\vec{p}$ in the rotated coordinate system $(\hat{i}', \hat{j}', \hat{k}')$ is

(a) $(1-\sqrt{3})\hat{i}' + 3\hat{j}' + (1+\sqrt{3})\hat{k}'$

(b) $(1+\sqrt{3})\hat{i}' + 3\hat{j}' + (1-\sqrt{3})\hat{k}'$

(c) $(1-\sqrt{3})\hat{i}' + (3+\sqrt{3})\hat{j}' + 2\hat{k}'$

(d) $(1-\sqrt{3})\hat{i}' + (3-\sqrt{3})\hat{j}' + 2\hat{k}'$

26. For arbitrary matrices E, F, G and H, if EF – FE = 0, then Trace (EFGH) is equal
(a) Trace (HFEG)
(b) Trace (E) , Trace (F), Trace (G), Trace (H)
(c) Trace (GFEH)
(d) Trace (EGHF)

27. An unitary matrix $\begin{bmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{bmatrix}$ is given where a, b, c, d, α and β are real. The inverse of the matrix is

(a) $\begin{bmatrix} ae^{i\alpha} & -ce^{i\beta} \\ b & d \end{bmatrix}$ (b) $\begin{bmatrix} ae^{i\alpha} & ce^{i\beta} \\ b & d \end{bmatrix}$

(c) $\begin{bmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{bmatrix}$ (d) $\begin{bmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{bmatrix}$

28. The eigenvalue of the matrix $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ are
(a) Real and Distinct
(b) Complex and Distinct
(c) Complex and Coinciding
(d) Real and Coinciding

29. The eigen values of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are

(a) 5, 2, –2 (b) –5, –1, 1
(c) 5, 1, –1 (d) –5, 1, 1

30. Two matrices A and B are said to be similar if B = P^{-1}AP for some invertible matrix P. Which of the following statements is NOT TRUE?
(a) Det A = Det B
(b) Trace of A = Trace of B
(c) A and B have the same eigenvectors
(d) A and B have the same eigenvalues

31. A 3 × 3 matrix has element such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalue of the matrix is:
(a) 18 (b) 12
(c) 9 (d) 6

32. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are

(a) 0, 1,1 (b) $0, -\sqrt{2}, \sqrt{2}$
(c) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (d) $\sqrt{2}, \sqrt{2}, 0$

33. The degenerate eigenvalue of the matrix $M = \begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{pmatrix}$ is…

34. The matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is….
(a) Orthogonal (b) Symmetric
(c) Anti-Symmetric (d) Unitary

35. Which of the following is INCORRECT for the matrix $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
(a) It is its own inverse
(b) It is its own transpose
(c) It is non-orthogonal
(d) it has eigen values ±1

36. The symmetric pair of P = $\begin{pmatrix} a \\ b \end{pmatrix}(a-2b)$ is:

(a) $\begin{pmatrix} a^2-2 & ba-1 \\ ba-1 & b^2-2 \end{pmatrix}$

(b) $\begin{pmatrix} a(a-1) & b \\ b & b^2 \end{pmatrix}$

(c) $\begin{pmatrix} a(a-1) & b(a-1) \\ b(a-1) & b^2 \end{pmatrix}$

(d) $\begin{pmatrix} a(a-2) & b(a-1) \\ b(a-1) & b^2 \end{pmatrix}$

37. $(x, y)\begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=15$

The matrix equation above represents.

(a) A circle of radius $\sqrt{15}$

(b) An ellipse of semi major axis $\sqrt{5}$

(c) An ellipse of semi major axis 5

(d) A hyperbola

38. The product *PQ* of any two real, symmetric matrices *P* and *Q* is:

(a) Symmetric for all *P* and *Q*

(b) Never symmetric

(c) Symmetric if PQ = QP

(d) Antisymmetric for all *P* and *Q*

39. A matrix is given by $M = \frac{1}{\sqrt{2}}\begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}$. The eigenvalues of the *M* are

(a) Real and positive

(b) Purely imaginary with modulus 1

(c) Complex with modulus 1

(d) Real and negative.

40. Given two ($n \times n$) matrices $\hat{P}$ $\hat{A}nd$ $\hat{Q}$ such that $\hat{P}$ is hermitian and $\hat{Q}$ is skew (anti)-hermitian. Which one of the following combinations of $\hat{P}$ and $\hat{Q}$ is necessarily a Hermitian matrix?

(a) $\hat{P}\,\hat{Q}$ (b) $i\hat{P}\,\hat{Q}$

(c) $\hat{P} + i\hat{Q}$ (d) $\hat{P}$ - $i\hat{Q}$

41. The inverse of the matrix $M = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ is

(a) M – 1 (b) $M^2 - 1$

(c) $1 - M^2$ (d) 1 – M

where *I* is the identity matrix.

42. The normalized eigenvectors of the matrix

$N = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ are β_1 *and* β_2

with the eigenvalues λ_1 and λ_2 respectively and

$\lambda_1 > \lambda_2$. If the eigenvector $\alpha = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ expressed as

$\alpha = P\beta_1 + Q\beta_2$. Find the constants *P* and *Q*.

(a) $\frac{1+i}{2}, \frac{1-i}{2}$ (b) $\frac{2+i}{2}, \frac{1+i}{2}$

(c) $\frac{1+i}{3}, \frac{1-i}{4}$ (d) $\frac{i}{2}, \frac{1+i}{2}$

43. The trace of a 2 × 2 matrix is 4 and its determinant is 8. If one of the eigenvalue is $2(1 + i)$, the other eigenvalue is

(a) 2(1 – i) (b) 2(1 + i)

(c) (1 + 2i) (d) (1 – 2i)

44. The eigenvalues of the matrix representing the following pair of linear equations

$x + iy = 0$

$ix + y = 0$

are

(a) $1 + i, 1 + i$

(b) 1 – i, 1 – i

(c) 1, i

(d) $1 + i, 1 - i$

45. For the given set of equations:

x + y = 1

y + z = 1

x + z = 1,

Which one of the following statements is correct?

(a) Equations are inconsistent

(b) Equations are consistent and a single non-trivial solution exists

(c) Equations are consistent and many solutions

(d) Equations are consistent and only a trivial solution exists

Answer Key

1.(b)
2.(c)
3.(d)
4.(a)
5.(c)
6.(a)
7.(a)
8.(b)
9.(d)
10.(b)
11.(a)
12.(c)
13.(c)
14.(d)
15.(b)
16.(c)
17.(c)
18.(a)
19.(a)
20.(a)
21.(c)
22.(d)
23.(d)
24.(a)
25.(a)
26.(a)
27.(d)
28.(b)
29.(c)
30.(c)
31.(d)
32.(b)
33.(5)
34.(d)
35.(c)
36.(d)
37.(b)
38.(c)
39.(c)
40.(c)
41.(b)
42.(a)
43.(a)
44.(d)
45.(b)

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Calculus I

Here are a set of practice problems for the Calculus I notes. Click on the "**Solution**" link for each problem to go to the page containing the solution.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a listing of sections for which practice problems have been written as well as a brief description of the material covered in the notes for that particular section.

Review - In this chapter we give a brief review of selected topics from Algebra and Trig that are vital to surviving a Calculus course. Included are Functions, Trig Functions, Solving Trig Equations and Equations, Exponential/Logarithm Functions and Solving Exponential/Logarithm Equations.

Functions – In this section we will cover function notation/evaluation, determining the domain and range of a function and function composition.
Inverse Functions – In this section we will define an inverse function and the notation used for inverse functions. We will also discuss the process for finding an inverse function.
Trig Functions – In this section we will give a quick review of trig functions. We will cover the basic notation, relationship between the trig functions, the right triangle definition of the trig functions. We will also cover evaluation of trig functions as well as the unit circle (one of the most important ideas from a trig class!) and how it can be used to evaluate trig functions.
Solving Trig Equations – In this section we will discuss how to solve trig equations. The answers to the equations in this section will all be one of the “standard” angles that most students have memorized after a trig class. However, the process used here can be used for any answer regardless of it being one of the standard angles or not.
Solving Trig Equations with Calculators, Part I – In this section we will discuss solving trig equations when the answer will (generally) require the use of a calculator (*i.e.* they aren’t one of the standard angles). Note however, the process used here is identical to that for when the answer is one of the standard angles. The only difference is that the answers in here can be a little messy due to the need of a calculator. Included is a brief discussion of inverse trig functions.
Solving Trig Equations with Calculators, Part II – In this section we will continue our discussion of solving trig equations when a calculator is needed to get the answer. The equations in this section tend to be a little trickier than the "normal" trig equation and are not always covered in a trig class.
Exponential Functions –In this section we will discuss exponential functions. We will cover the basic definition of an exponential function, the natural exponential function, i.e. ex, as well as the properties and graphs of exponential functions

Logarithm Functions – In this section we will discuss logarithm functions, evaluation of logarithms and their properties. We will discuss many of the basic manipulations of logarithms that commonly occur in Calculus (and higher) classes. Included is a discussion of the natural (ln(x)) and common logarithm (log(x)) as well as the change of base formula.
Exponential and Logarithm Equations – In this section we will discuss various methods for solving equations that involve exponential functions or logarithm functions.
Common Graphs – In this section we will do a very quick review of many of the most common functions and their graphs that typically show up in a Calculus class.

Limits - In this chapter we introduce the concept of limits. We will discuss the interpretation/meaning of a limit, how to evaluate limits, the definition and evaluation of one-sided limits, evaluation of infinite limits, evaluation of limits at infinity, continuity and the Intermediate Value Theorem. We will also give a brief introduction to a precise definition of the limit and how to use it to evaluate limits

Tangent Lines and Rates of Change – In this section we will introduce two problems that we will see time and again in this course : Rate of Change of a function and Tangent Lines to functions. Both of these problems will be used to introduce the concept of limits, although we won't formally give the definition or notation until the next section.
The Limit – In this section we will introduce the notation of the limit. We will also take a conceptual look at limits and try to get a grasp on just what they are and what they can tell us. We will be estimating the value of limits in this section to help us understand what they tell us. We will actually start computing limits in a couple of sections.
One-Sided Limits – In this section we will introduce the concept of one-sided limits. We will discuss the differences between one-sided limits and limits as well as how they are related to each other.
Limit Properties – In this section we will discuss the properties of limits that we'll need to use in computing limits (as opposed to estimating them as we've done to this point). We will also compute a couple of basic limits in this section.
Computing Limits – In this section we will looks at several types of limits that require some work before we can use the limit properties to compute them. We will also look at computing limits of piecewise functions and use of the Squeeze Theorem to compute some limits.
Infinite Limits – In this section we will look at limits that have a value of infinity or negative infinity. We'll also take a brief look at vertical asymptotes.
Limits At Infinity, Part I – In this section we will start looking at limits at infinity, *i.e.* limits in which the variable gets very large in either the positive or negative sense. We will concentrate on polynomials and rational expressions in this section. We'll also take a brief look at horizontal asymptotes.
Limits At Infinity, Part II – In this section we will continue covering limits at infinity. We'll be looking at exponentials, logarithms and inverse tangents in this section.
Continuity – In this section we will introduce the concept of continuity and how it relates to limits. We will also see the Intermediate Value Theorem in this section and how it can be used to determine if functions have solutions in a given interval.
The Definition of the Limit – In this section we will give a precise definition of several of the

limits covered in this section. We will work several basic examples illustrating how to use this precise definition to compute a limit. We'll also give a precise definition of continuity..

Derivatives - In this chapter we introduce Derivatives. We cover the standard derivatives formulas including the product rule, quotient rule and chain rule as well as derivatives of polynomials, roots, trig functions, inverse trig functions, hyperbolic functions, exponential functions and logarithm functions. We also cover implicit differentiation, related rates, higher order derivatives and logarithmic differentiation.

The Definition of the Derivative – In this section we define the derivative, give various notations for the derivative and work a few problems illustrating how to use the definition of the derivative to actually compute the derivative of a function.
Interpretation of the Derivative – In this section we give several of the more important interpretations of the derivative. We discuss the rate of change of a function, the velocity of a moving object and the slope of the tangent line to a graph of a function.
Differentiation Formulas – In this section we give most of the general derivative formulas and properties used when taking the derivative of a function. Examples in this section concentrate mostly on polynomials, roots and more generally variables raised to powers.
Product and Quotient Rule – In this section we will give two of the more important formulas for differentiating functions. We will discuss the Product Rule and the Quotient Rule allowing us to differentiate functions that, up to this point, we were unable to differentiate.
Derivatives of Trig Functions – In this section we will discuss differentiating trig functions. Derivatives of all six trig functions are given and we show the derivation of the derivative of sin(x) and tan(x).
Derivatives of Exponential and Logarithm Functions – In this section we derive the formulas for the derivatives of the exponential and logarithm functions.
Derivatives of Inverse Trig Functions – In this section we give the derivatives of all six inverse trig functions. We show the derivation of the formulas for inverse sine, inverse cosine and inverse tangent.
Derivatives of Hyperbolic Functions – In this section we define the hyperbolic functions, give the relationships between them and some of the basic facts involving hyperbolic functions. We also give the derivatives of each of the six hyperbolic functions and show the derivation of the formula for hyperbolic sine.
Chain Rule – In this section we discuss one of the more useful and important differentiation formulas, The Chain Rule. With the chain rule in hand we will be able to differentiate a much wider variety of functions. As you will see throughout the rest of your Calculus courses a great many of derivatives you take will involve the chain rule!
Implicit Differentiation – In this section we will discuss implicit differentiation. Not every function can be explicitly written in terms of the independent variable, e.g. y = f(x) and yet we will still need to know what f'(x) is. Implicit differentiation will allow us to find the derivative in these cases. Knowing implicit differentiation will allow us to do one of the more important applications of derivatives, Related Rates (the next section).
Related Rates – In this section we will discuss the only application of derivatives in this section, Related Rates. In related rates problems we are give the rate of change of one

quantity in a problem and asked to determine the rate of one (or more) quantities in the problem. This is often one of the more difficult sections for students. We work quite a few problems in this section so hopefully by the end of this section you will get a decent understanding on how these problems work.
Higher Order Derivatives – In this section we define the concept of higher order derivatives and give a quick application of the second order derivative and show how implicit differentiation works for higher order derivatives.
Logarithmic Differentiation – In this section we will discuss logarithmic differentiation. Logarithmic differentiation gives an alternative method for differentiating products and quotients (sometimes easier than using product and quotient rule). More importantly, however, is the fact that logarithm differentiation allows us to differentiate functions that are in the form of one function raised to another function, *i.e.* there are variables in both the base and exponent of the function.

Applications of Derivatives - In this chapter we will cover many of the major applications of derivatives. Applications included are determining absolute and relative minimum and maximum function values (both with and without constraints), sketching the graph of a function without using a computational aid, determining the Linear Approximation of a function, L'Hospital's Rule (allowing us to compute some limits we could not prior to this), Newton's Method (allowing us to approximate solutions to equations) as well as a few basic Business applications.

Rates of Change – In this section we review the main application/interpretation of derivatives from the previous chapter (i.e. rates of change) that we will be using in many of the applications in this chapter.
Critical Points – In this section we give the definition of critical points. Critical points will show up in most of the sections in this chapter, so it will be important to understand them and how to find them. We will work a number of examples illustrating how to find them for a wide variety of functions.
Minimum and Maximum Values – In this section we define absolute (or global) minimum and maximum values of a function and relative (or local) minimum and maximum values of a function. It is important to understand the difference between the two types of minimum/maximum (collectively called extrema) values for many of the applications in this chapter and so we use a variety of examples to help with this. We also give the Extreme Value Theorem and Fermat's Theorem, both of which are very important in the many of the applications we'll see in this chapter.
Finding Absolute Extrema – In this section we discuss how to find the absolute (or global) minimum and maximum values of a function. In other words, we will be finding the largest and smallest values that a function will have.
The Shape of a Graph, Part I – In this section we will discuss what the first derivative of a function can tell us about the graph of a function. The first derivative will allow us to identify the relative (or local) minimum and maximum values of a function and where a function will be increasing and decreasing. We will also give the First Derivative test which will allow us to classify critical points as relative minimums, relative maximums or neither a minimum or a

maximum.
The Shape of a Graph, Part II – In this section we will discuss what the second derivative of a function can tell us about the graph of a function. The second derivative will allow us to determine where the graph of a function is concave up and concave down. The second derivative will also allow us to identify any inflection points (i.e. where concavity changes) that a function may have. We will also give the Second Derivative Test that will give an alternative method for identifying some critical points (but not all) as relative minimums or relative maximums.
The Mean Value Theorem – In this section we will give Rolle's Theorem and the Mean Value Theorem. With the Mean Value Theorem we will prove a couple of very nice facts, one of which will be very useful in the next chapter.
Optimization Problems – In this section we will be determining the absolute minimum and/or maximum of a function that depends on two variables given some constraint, or relationship, that the two variables must always satisfy. We will discuss several methods for determining the absolute minimum or maximum of the function. Examples in this section tend to center around geometric objects such as squares, boxes, cylinders, etc.
More Optimization Problems – In this section we will continue working optimization problems. The examples in this section tend to be a little more involved and will often involve situations that will be more easily described with a sketch as opposed to the 'simple' geometric objects we looked at in the previous section.
L'Hospital's Rule and Indeterminate Forms – In this section we will revisit indeterminate forms and limits and take a look at L'Hospital's Rule. L'Hospital's Rule will allow us to evaluate some limits we were not able to previously.
Linear Approximations – In this section we discuss using the derivative to compute a linear approximation to a function. We can use the lienar approximation to a function to approximate values of the function at certain points. While it might not seem like a useful thing to do with when we have the function there really are reasons that one might want to do this. We give two ways this can be useful in the examples.
Differentials – In this section we will compute the differential for a function. We will give an application of differentials in this section. However, one of the more important uses of differentials will come in the next chapter and unfortunately we will not be able to discuss it until then.
Newton's Method – In this section we will discuss Newton's Method. Newton's Method is an application of derivatives that will allow us to approximate solutions to an equation. There are many equations that cannot be solved directly and with this method we can get approximations to the solutions to many of those equations.
Business Applications – In this section we will give a cursory discussion of some basic applications of derivatives to the business field. We will revisit finding the maximum and/or minimum function value and we will define the marginal cost function, the average cost, the revenue function, the marginal revenue function and the marginal profit function. Note that this section is only intended to introduce these concepts and not teach you everything about them.

Integrals - In this chapter we will give an introduction to definite and indefinite integrals. We will discuss the definition and properties of each type of integral as well as how to compute them including the Substitution Rule. We will give the Fundamental Theorem of Calculus showing the relationship between derivatives and integrals. We will also discuss the Area Problem, an important interpretation of the definite integral.

Indefinite Integrals – In this section we will start off the chapter with the definition and properties of indefinite integrals. We will not be computing many indefinite integrals in this section. This section is devoted to simply defining what an indefinite integral is and to give many of the properties of the indefinite integral. Actually computing indefinite integrals will start in the next section.

Computing Indefinite Integrals – In this section we will compute some indefinite integrals. The integrals in this section will tend to be those that do not require a lot of manipulation of the function we are integrating in order to actually compute the integral. As we will see starting in the next section many integrals do require some manipulation of the function before we can actually do the integral. We will also take a quick look at an application of indefinite integrals.

Substitution Rule for Indefinite Integrals – In this section we will start using one of the more common and useful integration techniques – The Substitution Rule. With the substitution rule we will be able integrate a wider variety of functions. The integrals in this section will all require some manipulation of the function prior to integrating unlike most of the integrals from the previous section where all we really needed were the basic integration formulas.

More Substitution Rule – In this section we will continue to look at the substitution rule. The problems in this section will tend to be a little more involved than those in the previous section.

Area Problem – In this section we start off with the motivation for definite integrals and give one of the interpretations of definite integrals. We will be approximating the amount of area that lies between a function and the x-axis. As we will see in the next section this problem will lead us to the definition of the definite integral and will be one of the main interpretations of the definite integral that we'll be looking at in this material.

Definition of the Definite Integral – In this section we will formally define the definite integral, give many of its properties and discuss a couple of interpretations of the definite integral. We will also look at the first part of the Fundamental Theorem of Calculus which shows the very close relationship between derivatives and integrals

Computing Definite Integrals – In this section we will take a look at the second part of the Fundamental Theorem of Calculus. This will show us how we compute definite integrals without using (the often very unpleasant) definition. The examples in this section can all be done with a basic knowledge of indefinite integrals and will not require the use of the substitution rule. Included in the examples in this section are computing definite integrals of piecewise and absolute value functions.

Substitution Rule for Definite Integrals – In this section we will revisit the substitution rule as it applies to definite integrals. The only real requirements to being able to do the examples in this section are being able to do the substitution rule for indefinite integrals and understanding how to compute definite integrals in general.

Applications of Integrals - In this chapter we will take a look at some applications of integrals. We will look at Average Function Value, Area Between Curves, Volume (both solids of revolution and other solids) and Work.

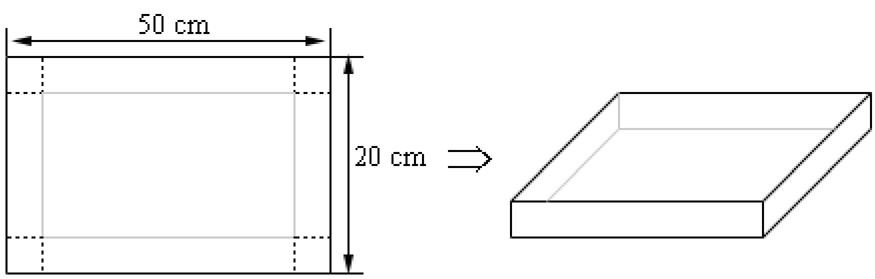
Average Function Value – In this section we will look at using definite integrals to determine the average value of a function on an interval. We will also give the Mean Value Theorem for Integrals.
Area Between Curves – In this section we'll take a look at one of the main applications of definite integrals in this chapter. We will determine the area of the region bounded by two curves.
Volumes of Solids of Revolution / Method of Rings – In this section, the first of two sections devoted to finding the volume of a solid of revolution, we will look at the method of rings/disks to find the volume of the object we get by rotating a region bounded by two curves (one of which may be the x or y-axis) around a vertical or horizontal axis of rotation.
Volumes of Solids of Revolution / Method of Cylinders – In this section, the second of two sections devoted to finding the volume of a solid of revolution, we will look at the method of cylinders/shells to find the volume of the object we get by rotating a region bounded by two curves (one of which may be the x or y-axis) around a vertical or horizontal axis of rotation.
More Volume Problems – In the previous two sections we looked at solids that could be found by treating them as a solid of revolution. Not all solids can be thought of as solids of revolution and, in fact, not all solids of revolution can be easily dealt with using the methods from the previous two sections. So, in this section we'll take a look at finding the volume of some solids that are either not solids of revolutions or are not easy to do as a solid of revolution.
Work – In this section we will look at is determining the amount of work required to move an object subject to a force over a given distance.

Khan Academy Practice Problems

HERE

Calculus I - Optimisation (Practice Problems)

1. Find two positive numbers whose sum is 300 and whose product is a maximum. Solution
2. Find two positive numbers whose product is 750 and for which the sum of one and 10 times the other is a minimum. Solution
3. Let x and y be two positive numbers such that x+2y=50 and (x+1)(y+2) is a maximum. Solution
4. We are going to fence in a rectangular field. If we look at the field from above the cost of the vertical sides are \$10/ft, the cost of the bottom is \$2/ft and the cost of the top is \$7/ft. If we have \$700 determine the dimensions of the field that will maximize the enclosed area. Solution
5. We have 45 m^2 of material to build a box with a square base and no top. Determine the dimensions of the box that will maximize the enclosed volume. Solution
6. We want to build a box whose base length is 6 times the base width and the box will enclose 20 in^3. The cost of the material of the sides is \$3/in^2 and the cost of the top and bottom is \$15/in^2. Determine the dimensions of the box that will minimise the cost. Solution
7. We want to construct a cylindrical can with a bottom but no top that will have a volume of 30 cm^3. Determine the dimensions of the can that will minimize the amount of material needed to construct the can. Solution
8. We have a piece of cardboard that is 50 cm by 20 cm and we are going to cut out the corners and fold up the sides to form a box. Determine the height of the box that will give a maximum volume.



Solution

Unit 6 - Week 5

Assignment 5

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-10-21, 23:59 IST.

1) Which of the following is NOT a convex set? *1 point*

- $\{(x,y) \in \mathbb{R}^2 :\ x^2 + 2y^2 \leq 2\}$
- $\{(x,y) \in \mathbb{R}^2 :\ y^2 \geq x\}$
- $\{(x,y) \in \mathbb{R}^2 :\ x - 2y \geq 4\}$
- $\{(x,y) \in \mathbb{R}^2 :\ x^2 \leq 4y\}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\{(x,y) \in \mathbb{R}^2 :\ y^2 \geq x\}$

2) Determinant of the Hessian matrix of the function $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = 3x^2 - x^2y + 2y^3 - 3x + 5y$ at $P(-1,1)$ equals *1 point*

- 20
- 44
- 60
- 64

No, the answer is incorrect.
Score: 0
Accepted Answers:
44

3) Let $f, g : \mathbb{R}^3 \to \mathbb{R}$ be defined as $f(x,y,z) = x^3y + y^2z + zx,\ g(x,y,z) = 3xy^2z$. Then, $(\nabla f)^T \nabla g$ at $Q(1,1,-1)$ equals *1 point*

- 2
- 3
- 6
- 12

No, the answer is incorrect.
Score: 0
Accepted Answers:
6

4) Which of the following is a convex function? *1 point*

- $f(x) = x^3 - 2x;\ x \in [-2,2]$
- $g(x) = \cos x;\ x \in [\pi, 2\pi]$
- $h(x) = e^{-x};\ x \in (-\infty, \infty)$
- $l(x) = \ln(x);\ x \in [2,6]$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$h(x) = e^{-x};\ x \in (-\infty, \infty)$

5) The matrix $A = \begin{bmatrix} k & 0 & 1 \\ 0 & 2 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, $k \in \mathbb{R}$ is positive definite for all *1 point*

- $k > 0$
- $k > \frac{1}{3}$
- $k > \frac{2}{5}$
- $k > \frac{2}{3}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$k > \frac{2}{5}$

6) The direction of most rapid increase for the function $f : \mathbb{R}^3 \to \mathbb{R}$, $f(x,y,z) = 2x^2y + z^2$ at the point $P_0(-1,1,1)$ is *1 point*

- $-\frac{2}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$
- $\frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$
- $\frac{2}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$
- $-\frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$-\frac{2}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$

7) The function $g(x) = \sin x$ satisfies $g(x) - g(y) \geq (x-y)^T \nabla g(y)$ for all x, y in the interval *1 point*

- $[0, \pi]$
- $[\pi, 2\pi]$
- $[0, 2\pi]$
- $[0, \pi/2]$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$[\pi, 2\pi]$

8) Let $A = [a_{ij}]$ denote the Jacobian of $x = u^3 + v^3,\ y = uv^2 - u^2v,\ z = uv$. Then, at $(u,v) = (1,-1)$, value of $a_{11} + a_{22} + a_{32}$ equals *1 point*

- 0
- 1
- 3
- 6

No, the answer is incorrect.
Score: 0
Accepted Answers:
1

9) Let $f : \mathbb{R}^2 \to \mathbb{R},\ f(x,y) = x^3 - 12xy + 8y^3,\ \mathbf{u} = (1,2)^T,\ \mathbf{v} = (0,1)^T$. Then, which of the following statements is TRUE? *1 point*

- $\nabla^2 f(\mathbf{u})$is positive definite and $\nabla^2 f(\mathbf{v})$ is negative definite
- $\nabla^2 f(\mathbf{u})$is positive definite and $\nabla^2 f(\mathbf{v})$ is indefinite.
- $\nabla^2 f(\mathbf{u})$ is indefinite and $\nabla^2 f(\mathbf{v})$ is negative definite.
- Both $\nabla^2 f(\mathbf{u})$ and $\nabla^2 f(\mathbf{v})$ are indefinite.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\nabla^2 f(\mathbf{u})$is positive definite and $\nabla^2 f(\mathbf{v})$ is indefinite.

10) The directional derivative of $f(x,y,z) = x^2y - yz^3 + z$ at $(1,2,0)$ in the direction of $\vec{u} = 2\hat{i} + 2\hat{j} - \hat{k}$ is *1 point*

- -3
- -2
- 2
- 3

No, the answer is incorrect.
Score: 0
Accepted Answers:
3

Assignment 0

The due date for submitting this assignment has passed.

Due on 2021-08-23, 23:59 IST.

As per our records you have not submitted this assignment.

1) Suppose you roll a die twice. What is the probability that you get a prime number in both the rolls? *1 point*

- 1/2
- 1/4
- 1/6
- 1/36

No, the answer is incorrect.
Score: 0
Accepted Answers:
1/4

2) Suppose you roll a die n times. What is the probability that you get a prime number in all the n rolls? *1 point*

- $\frac{1}{2}$
- $\frac{1}{2^n}$
- $\frac{1}{6^n}$
- None of these.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{1}{2^n}$

3) In this course, you will study the following case study. In the 1960s, there was a famous TV show called "Let's make a deal", whose host was Monty Hall. In the show, there are 3 closed doors, one of which hides a car, while the remaining two hide a goat each. Suppose the contestant's name is Bob. Bob can choose only one door among the three (without opening). He will win the car if he chooses the correct door which hides the car and gets a goat otherwise. Suppose Bob chooses Door 1. The host Monty then proceeds to open Door 2 to reveal a goat there. Now, Monty gives an option to Bob to either keep his choice of Door 1 or to switch his choice to Door 3. Which door should Bob choose now, in order to have the best winning chances? *1 point*

- Door 1, keep his original choice.
- Door 3, switch his choice.
- Does not matter. Both Door 1 and 3 have 50% chance of having the car.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Door 3, switch his choice.

4) The instructor designs a single answer MCQ question with total 4 options. Suppose option number 1 is the correct answer. The instructor submits this question to the NPTEL staff, who shuffle all the options randomly before adding it to the exam. What is the chance that option number 1 is still the correct answer after shuffling? *1 point*

- 1/2
- 1/4
- 3/4
- 1/24

No, the answer is incorrect.
Score: 0
Accepted Answers:
1/4

5) Merlin selects two real numbers and puts them in two envelopes. Using a fair coin toss, he selects one of the envelopes and reveals the number in that envelope. He asks you whether the number in the other envelope is bigger than the one revealed. *1 point*

Can you guess correctly with probability greater than 1/2?

- Yes
- No
- Maybe
- No idea, but I'd like to know

No, the answer is incorrect.
Score: 0
Accepted Answers:
Yes
No idea, but I'd like to know

6) How many people do you need in a room so that the probability of at least two of them sharing the same birthday is greater than 1/2? *1 point*

- 365
- 100
- 23
- 57

No, the answer is incorrect.
Score: 0
Accepted Answers:
23

7) Consider the statement: In a long coin-tossing game each player will be on the winning side for about half the time, and the lead will pass not infrequently from one player to the other. This statement *1 point*

- Is True
- Is False
- Looks weird
- Sounds about right

No, the answer is incorrect.
Score: 0
Accepted Answers:
Is False

8) You are given two concentric circles C1 and C2 with radii r and r/2 respectively. What is the probability, upon randomly choosing a chord of the bigger circle that it will intersect the smaller one? *1 point*

- 1/2
- 1/3
- 1/4
- Cannot be computed

No, the answer is incorrect.
Score: 0
Accepted Answers:
1/2
1/3
1/4

Week 1: Assignment 1

The due date for submitting this assignment has passed.

Due on 2021-09-08, 23:59 IST.

As per our records you have not submitted this assignment.

1) If 8 coins fall to the floor, what is the probability that there are four heads and four tails? *1 point*

- 35/256
- 67/256
- 35/128
- 53/128

No, the answer is incorrect.
Score: 0
Accepted Answers:
35/128

2) A number is perfect if it is equal to the sum of all its proper divisors (i.e. excluding itself). We roll two standard 6 sided dice. What is the probability that the product of numbers that appear on the two faces is a perfect number? *1 point*

- 5/36
- 1/9
- 1/12
- 2/3

No, the answer is incorrect.
Score: 0
Accepted Answers:
1/9

3) A pack of 52 cards has been dealt to 4 players. What is the probability that one of the players receives all 13 spades? *1 point*

- $1/\binom{52}{4}$
- $4/\binom{52}{4}$
- $1/\binom{52}{13}$
- $4/\binom{52}{13}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$4/\binom{52}{13}$

4) A deck of cards is shuffled and the cards are drawn one at a time until a Queen appears. If we draw another card, what is more likely, that the next card is Queen of clubs or that it is the King of hearts? *1 point*

- Queen of clubs
- King of hearts
- Both of them are equally likely
- Cannot be determined

No, the answer is incorrect.
Score: 0
Accepted Answers:
Both of them are equally likely

5) Suppose 3 people get their hats returned in random order. What is the probability that at least one of them gets the correct hat? *1 point*

- 2/3
- 1/3
- 2/7
- 3/4

No, the answer is incorrect.
Score: 0
Accepted Answers:
2/3

Week 2: Assignment 2

The due date for submitting this assignment has passed.

Due on 2021-09-08, 23:59 IST.

As per our records you have not submitted this assignment.

1) Which of the following is a valid sigma algebra? *1 point*

1. $\{\Phi, \Omega\}$
2. $\{\Phi, A, A^C, \Omega\}$
3. $\{\Phi, A, A^C, B, B^C, A\cup B, A\cap B, \Omega\}$

- Both 1 and 2.
- Both 1 and 3.
- Both 2 and 3.
- All of 1,2 and 3.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Both 1 and 2.

2) Pick two real numbers a, b uniformly at random from infinite sample space [-1,1]. What is the probability that $a^2 + b^2 > 1$? *1 point*

- $\pi/4$
- $1 - \pi/4$
- $\pi/16$
- $1 - \pi/16$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$1 - \pi/4$

3) Which of the following is true for two events A and B each having non-zero probability of occurrence? *1 point*
1. If A and B are mutually exclusive, then they are also independent.
2. If A and B are independent, then they can be mutually exclusive.

- Only 1.
- Only 2.
- Both 1 and 2.
- Neither 1 nor 2.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Neither 1 nor 2.

4) Suppose the Covid-19 RT-PCR test is 95% accurate. Also assume the prevalence of Covid is 0.2, i.e. 20% of the population is Covid infected. If your RT-PCR test comes out positive, what is the probability that you are infected? *1 point*

- 19/20
- 19/23
- 17/20
- 17/23

No, the answer is incorrect.
Score: 0
Accepted Answers:
19/23

5) Suppose you roll a dice twice. What is the probability that the sum of two rolls is an even number given that the first roll was an even number. *1 point*

- 1
- 0
- 1/2
- 2/3

No, the answer is incorrect.
Score: 0
Accepted Answers:
1/2

Week 3: Assignment 3

The due date for submitting this assignment has passed.

Due on 2021-09-15, 23:59 IST.

As per our records you have not submitted this assignment.

1) What is the expected value of the **total** number of dots appearing on the top face when we roll **two** normal six sided dice? *1 point*

- 8
- 4.5
- 7
- 7.66

No, the answer is incorrect.
Score: 0
Accepted Answers:
7

2) Alice rolls a magical die. This die has 2021 sides with faces $\{1, 2, \ldots, 2021\}$. Also suppose that the probability that face k turns up is directly proportional to k with the **same** proportionality constant α, i.e. $P(\text{face} = k) = \alpha \cdot k$ for all k. *1 point*
Find the expected value of the die roll (rounded up to two decimal places)

- 1349.33
- 1346.67
- 1348.33
- 1347.67

No, the answer is incorrect.
Score: 0
Accepted Answers:
1347.67

3) X is a random variable that takes values 1 or 2 with equal probability. Y is a random variable that takes values between 1 and $X + 1$ with equal probabililty. What is $E[X + Y]$? *1 point*

- 21/8
- 13/4
- 21/4
- 11/8

No, the answer is incorrect.
Score: 0
Accepted Answers:
13/4

4) Alice flips a fair coin n times (resulting in a binomial random variable X with parameters n and 1/2). Bob flips a fair coin $n + 1$ times (resulting in a binomial distribution Y with parameters $n + 1$ and 1/2). What is $P(X < Y)$? *1 point*
(Hint: Show that $P(X < Y) = P(n - X < n + 1 - Y)$)

- 1/2
- 3/4
- 2/3
- 7/11

No, the answer is incorrect.
Score: 0
Accepted Answers:
1/2

5) A post office has two clerks. Alice enters the post office while 2 customers Bob and Claire are being served by the 2 clerks. She is next in line. Assume that the time a clerk spends serving a customer has the Exponential distribution with $\lambda = 5$. What is the probability that Alice is the **last** of the 3 customers to be done being served? *1 point*

- 2/3
- 1/2
- 1/3
- 3/7

No, the answer is incorrect.
Score: 0
Accepted Answers:
1/2

Week 4: Assignment 4

The due date for submitting this assignment has passed.

Due on 2021-09-22, 23:59 IST.

As per our records you have not submitted this assignment.

1) Recall the Equality Checking Protocol covered in class, which is a probabilistic algorithm to test whether two files, each of n-bit size, are same or not. Which of the following is true for the protocol? *1 point*

- If the two files are equal, then the protocol may output NO with probability < 1/n.
- If the two files are different, then the protocol always outputs NO.
- If the two files are different, then the protocol may output YES with probability < 1/n.
- None of the above.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If the two files are different, then the protocol may output YES with probability < 1/n.

2) Let X be a Poisson random variable with parameter α. What is expectation and variance of X respectively? *1 point*

- $\alpha, \alpha.$
- $\alpha, \alpha^2.$
- $e^{-\alpha}, e^{-\alpha}.$
- $e^{-\alpha}, e^{-\alpha^2}.$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\alpha, \alpha.$

3) Let $X_1, X_2, \ldots, X_n$ be binary, i.i.d (identical and mutually independent) random variables. Consider $S = X_1 + \ldots + X_n$ such that expectation of S, $E(S) = \mu$. Then, which of the following is true? *1 point*

- $P(S < \frac{\mu}{2}) < e^{-\frac{\mu}{8}}.$
- $P(S \geq 2\mu) \leq \frac{1}{2}.$
- Both of the above.
- None of the above.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Both of the above.

4) Let X,Y be two independent random variables. What is the variance Var(X-Y) equal to? *1 point*

- $\mathrm{Var}(X) - \mathrm{Var}(Y).$
- $\mathrm{Var}(X)^2 - \mathrm{Var}(Y)^2.$
- $\mathrm{Var}(X) + \mathrm{Var}(Y).$
- None of these.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mathrm{Var}(X) + \mathrm{Var}(Y).$

5) Let $X_1, X_2, \ldots, X_n$ be i.i.d.~ random variables with mean μ. Let their sum be $S := (X_1 + \cdots + X_n)$. Define a new random variable $W := X_1/S$. What random variable does nW converge to, as $n \to \infty$? *1 point*

- $\mu.$
- $1/\mu.$
- $S/\mu.$
- $X_1/\mu.$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$X_1/\mu.$

Week 5: Assignment 5

The due date for submitting this assignment has passed.

Due on 2021-09-29, 23:59 IST.

As per our records you have not submitted this assignment.

1) Let $X_0, X_1, X_2, X_3 \ldots$ be a Markov Chain. Then what can we say about $X_0, X_2, X_4, \ldots$? *1 point*

- It is a Markov Chain
- It is not a Markov Chain
- It is Markov only if X_i is homogeneous
- It is not possible to make any conclusions

No, the answer is incorrect.
Score: 0
Accepted Answers:
It is a Markov Chain

2) Let $X_1, X_2, X_3 \ldots$ be a Markov Chain. Then what can we say about $X_2, X_3, X_5, X_7 \ldots$ where the new sequence only consists of X_p for prime p? *1 point*

- It is a Markov Chain
- It is not a Markov Chain
- It is Markov only if X_i is homogeneous
- It is not possible to make any conclusions

No, the answer is incorrect.
Score: 0
Accepted Answers:
It is a Markov Chain

3) Consider the markov chain with three states, $S = \{1, 2, 3\}$ that has the following transition matrix: *1 point*

$$M = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

If we know that $P(X_1 = 1) = P(X_1 = 2) = 1/4$, what is $P(X_1 = 3, X_2 = 2, X_3 = 1)$?

- 1/3
- 1/14
- 1/12
- 1/7

No, the answer is incorrect.
Score: 0
Accepted Answers:
1/12

4) Consider an experiment of mating rabbits. We watch the evolution of a particular gene that appears in two types, G or g. A rabbit has a pair of genes, either GG (dominant), Gg (hybrid–the order is irrelevant, so gG is the same as Gg) or gg (recessive). In mating two rabbits, the offspring inherits a gene from each of its parents with equal probability. Thus, if we mate a dominant (GG) with a hybrid (Gg), the offspring is dominant with probability 1/2 or hybrid with probability 1/2. *1 point*
Start with a rabbit of given character (GG, Gg, or gg) and mate it with a hybrid. The offspring produced is again mated with a hybrid, and the process is repeated through a number of generations, always mating with a hybrid.
What is the transition matrix of the Markov Chain the above process defines?
(Consider the states to be $S = \{GG, Gg, gg\}$ in that order while constructing the matrix)

- $\begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix}$
- $\begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 1 & 0 & 0 \end{bmatrix}$
- $\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix}$
- $\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1/2 & 1/2 \end{bmatrix}$

No, the answer is incorrect.
Score: 0
Accepted Answers:

$$\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

5) Suppose that the transition matrix M of a markov chain is *1 point*

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Suppose that the initial probability distribution is [1/2, 1/4, 1/4]. What is the final probability distribution after 2021 steps?

- [1/4,1/2,1/4]
- [1/4,1/4,1/2]
- [1/3,1/3,1/3]
- [1/4,0,3/4]

No, the answer is incorrect.
Score: 0
Accepted Answers:
[1/4,1/4,1/2]

Week 6: Assignment 6

The due date for submitting this assignment has passed.

Due on 2021-10-06, 23:59 IST.

As per our records you have not submitted this assignment.

1) If P is a transition matrix with all entries non zero, and π is a probability distribution vector, then the $\lim_{n\to\infty} \pi^T P^n$ *1 point*

- Exists and depends on π
- Exists and is independent of π
- Exists depending on the actual values of entries of P
- Does not exist for any P or π

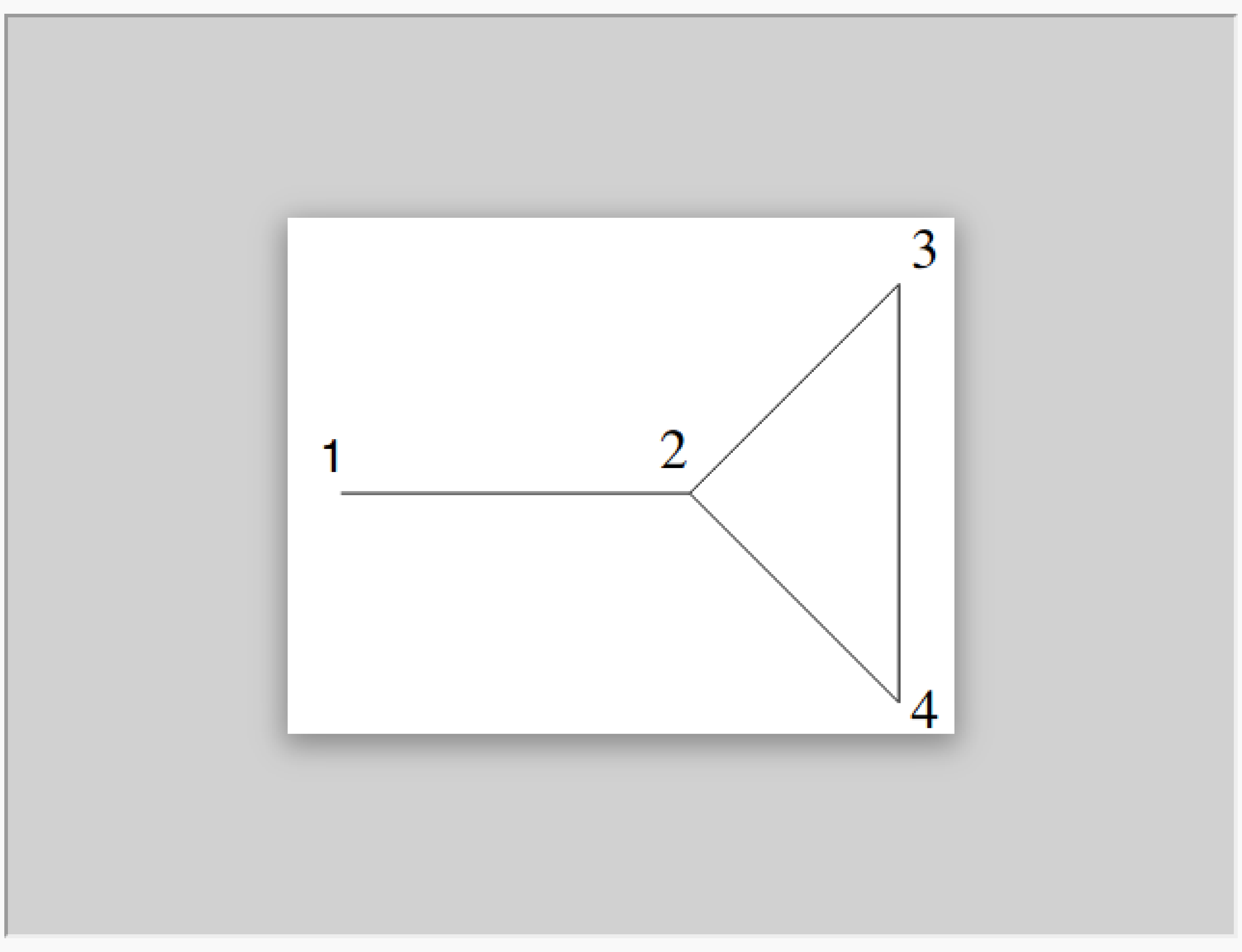
No, the answer is incorrect.
Score: 0
Accepted Answers:
Exists and is independent of π

2) Let M be the transition matrix of a regular Markov chain with 2021 states. Suppose that the vector $[1/2, 1/2^2, 1/2^3, \ldots, 1/2^{2020}, 1/2^{2020}]$ is the stationary distribution of this Markov Chain. Denote by $M' := \lim_{n\to\infty} \pi^T M^n$ the limit of applying M repeatedly on some initial distribution π. What is the sum of the first 1024 entries in the 1024-th column of M'? *1 point*

- $1/2^{1023}$
- 1
- $1/2^{1014}$
- $1/2^{2021}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$1/2^{1014}$

3) A wandering mathematician travels between four coffee shops that are located as follows: *1 point*

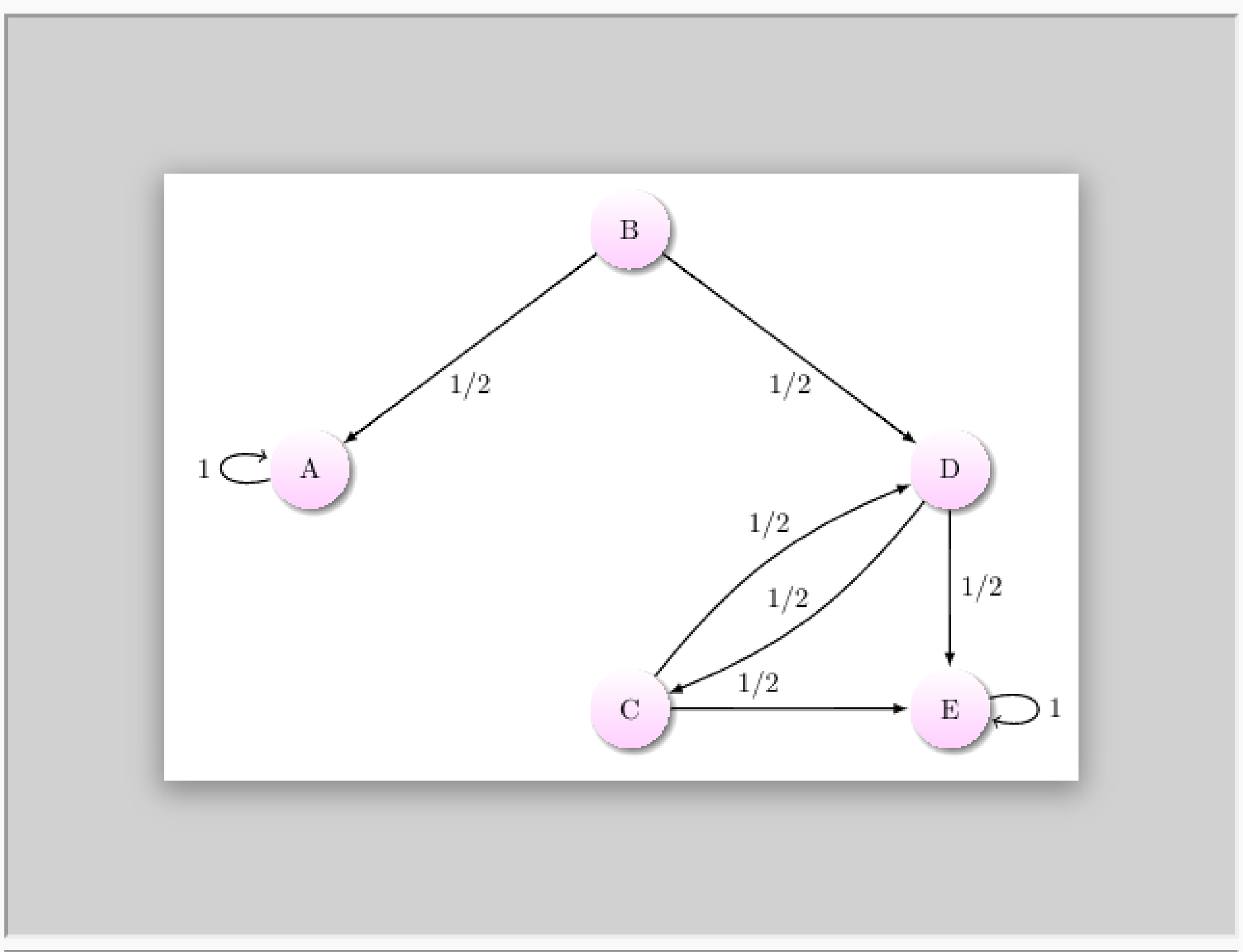


Assume that he/she chooses among the paths departing from a shop by treating each path as equally likely. If we model the mathematician's journey as a Markov chain (with states $S = \{1, 2, 3, 4\}$), the chain is

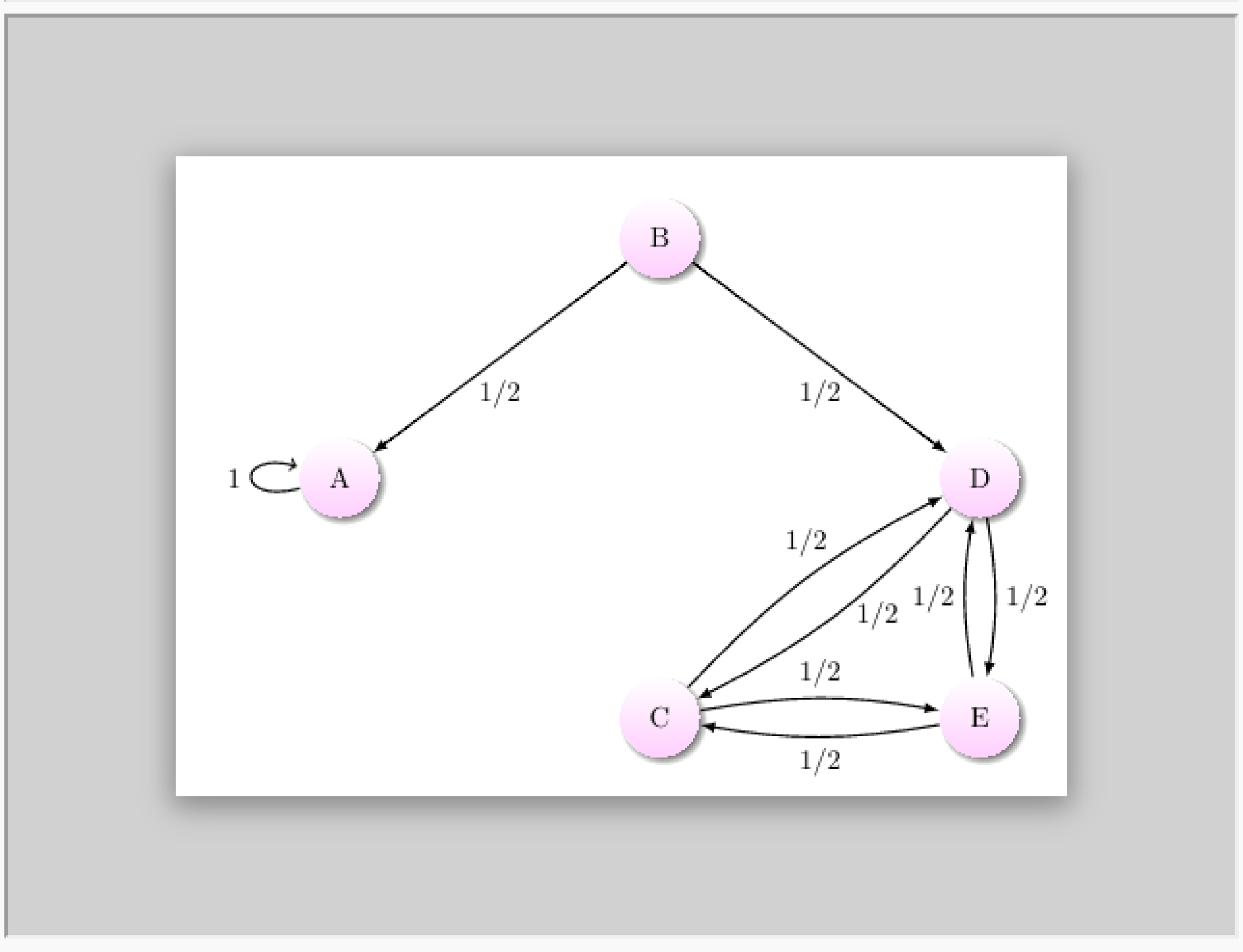
- Only Ergodic
- Only Regular
- Both Ergodic and Regular
- Neither Ergodic nor Regular

No, the answer is incorrect.
Score: 0
Accepted Answers:
Both Ergodic and Regular

4) A state in a Markov chain is called "absorbing" if it is impossible to leave it. A Markov Chain is "absorbing" if there is at least one absorbing state and it is possible to go from any state to at least one of the absorbing states in a finite number of steps. Consider the following two Markov chains shown in the diagram: *1 point*



Chain I



Chain II

- Only Chain I is absorbing
- Only Chain II is absorbing
- Both Chain I and Chain II are absorbing
- Neither of Chain I nor Chain II are absorbing

No, the answer is incorrect.
Score: 0
Accepted Answers:
Only Chain I is absorbing

5) Consider a Markov Chain with the following transition matrix: *1 point*

$$P = \begin{bmatrix} 3/10 & 1/5 & 2/5 & 1/10 \\ 1/10 & 1/10 & 2/5 & 2/5 \\ 1/2 & 1/10 & 1/10 & 3/10 \\ 1/10 & 3/5 & 1/10 & 1/5 \end{bmatrix}$$

What is the stationary distribution for the Markov Chain?

- [1/3, 1/3, 1/4, 1/2]
- [0, 1/3, 1/3, 1/3]
- [1/3, 1/3, 2/3, 0]
- [1/4, 1/4, 1/4, 1/4]

No, the answer is incorrect.
Score: 0
Accepted Answers:
[1/4, 1/4, 1/4, 1/4]

Week 7: Assignment 7

The due date for submitting this assignment has passed.

Due on 2021-10-13, 23:59 IST.

As per our records you have not submitted this assignment.

1) Let x, x' be two distinct 20-bit binary strings and y,y' be two 10-bit binary strings. Then which of the following is true for pairwise independent hashing: $\Phi_R : \{0,1\}^{20} \rightarrow \{0,1\}^{10}$? *1 point*

- $P(\Phi_R(x) = y \wedge \Phi_R(x') = y') = \frac{1}{100}$
- $P(\Phi_R(x) = y \wedge \Phi_R(x') = y') = \frac{1}{400}$
- $P(\Phi_R(x) = y \wedge \Phi_R(x') = y') = \frac{1}{2^{20}}$
- $P(\Phi_R(x) = y \wedge \Phi_R(x') = y') = \frac{1}{2^{10}}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$P(\Phi_R(x) = y \wedge \Phi_R(x') = y') = \frac{1}{2^{20}}$

2) Let x, x' be two distinct elements of a set S. Then which of the following is true for pairwise independent hashing: $\Phi_R : S \rightarrow T$? *1 point*

- $P(\Phi_R(x) = \Phi_R(x')) = \frac{1}{|T|^2}$
- $P(\Phi_R(x) = \Phi_R(x')) = \frac{1}{|T|}$
- $P(\Phi_R(x) = \Phi_R(x')) = \frac{1}{|S|^2}$
- $P(\Phi_R(x) = \Phi_R(x')) = \frac{1}{|S|}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$P(\Phi_R(x) = \Phi_R(x')) = \frac{1}{|T|}$

3) Let K_n denote a complete undirected graph on n vertices. Suppose you are given K_5 and K_7 where each edge is randomly colored with RED or BLUE in both K_5 and K_7. Now, suppose I pick out 3 distinct vertices each, in both of the graphs. What is the probability that triangle formed by the 3 vertices in K_5 is monochromatic? What is the probability that triangle in K_7 is monochromatic, respectively? *1 point*

- 1/5, 1/7
- 3/5, 3/7
- 1/4, 1/4
- 1/16, 1/64

No, the answer is incorrect.
Score: 0
Accepted Answers:
1/4, 1/4

4) Consider an IPL cricket tournament consisting of 8 teams. Suppose every season each team plays every other team exactly once. For each season, we can define a directed tournament graph T of 8 vertices, where each team corresponds to a vertex. There is a directed edge from vertex x to vertex y if team y beats x. A Hamiltonian path in a graph is defined as a path which visits each vertex exactly once. A Hamiltonian path in this tournament graph looks like $(x_1, \ldots, x_8)$, where there is an edge from $x_i \rightarrow x_{i+1}$ for each $1 \leq i \leq 7$. Suppose IPL is organized fairly and all teams are balanced to ensure that in every match both teams have equal chance of winning. Given any sequence of 8 teams, what is the probability that the sequence forms a Hamiltonian path in T? *1 point*

- 1/128
- 1/256
- 1/8
- 1/7

No, the answer is incorrect.
Score: 0
Accepted Answers:
1/128

5) Lets continue the previous question. Consider all possible distinct tournament graphs on 8 vertices. Suppose every year a new season of IPL is played till all the tournament graphs on 8 vertices are exhausted. Further suppose that every season of IPL is fair, i.e. both teams have equal chance of winning in every match for each season. Then, which of the following is true? *1 point*
[Hint: Every tournament graph T is random. Define an indicator random variable X_π which takes value 1 if π is a Hamiltonian path in T and 0 otherwise. Also consider a random variable X which denotes the total number of Hamiltonian paths in T. Calculate expected value of X.]

- There exists an IPL season for which its tournament graph has at least $8!/2^7$ Hamiltonian paths.
- There exists an IPL season for which its tournament graph has only 1 Hamiltonian path.
- Both of the above.
- None of the above.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Both of the above.

Course outline
How does an NPTEL online course work?
Week 0
Week 1
Week 2
Week 3
Week 4
Week 5
Week 6
Week 7
Week 8
- Lecture 29: Extremal Set Families
- Lecture 30: Super Concentrators
- Lecture 31: Streaming Algorithms I
- Lecture 32: Streaming Algorithms II
- Week-8 Slides: Extremal set families and super concentrators
- Week-8 Slides: Streaming algorithms
- Quiz: Week 8: Assignment 8
- Week 8: Assignment 8 Solutions
- Feedback For Week 8

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Week 8: Assignment 8

The due date for submitting this assignment has passed.

Due on 2021-10-20, 23:59 IST.

As per our records you have not submitted this assignment.

1) Let $b_1, \ldots, b_n$ be n numbers, each chosen independently, uniformly at random from the set {-1,+1}. Let $X = \sum_{i,j \in [n]} b_i \cdot b_j$ be a random variable, where [n] denotes the set {1,...,n}. Then, what is the expected value of X? *1 point*

- n.
- n^2.
- $\sqrt{n}$.
- 0.

No, the answer is incorrect.
Score: 0
Accepted Answers:
n.

2) Let $v_1, \ldots, v_n$ be any n unit vectors in $\mathbb{R}^n$. Then which of the following is true? *1 point*

- There exists a binary vector $b \in \{-1, 1\}^n$ such that $\|\sum_{i \in [n]} b_i \cdot v_i\| \leq \sqrt{n}$.
- There exists a binary vector $b \in \{-1, 1\}^n$ such that $\|\sum_{i \in [n]} b_i \cdot v_i\| \geq \sqrt{n}$.
- Both of the above.
- None of the above.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Both of the above.

3) Let a class of students have 30 boys and 20 girls. For the morning assembly, suppose the class teacher forms a line of all the students in a random order. What is the probability that all the girls stand before boys in the line? *1 point*

- $\frac{1}{50!}$
- $\frac{\binom{50}{20}}{50!}$
- $\frac{1}{\binom{50}{20}}$
- $\frac{20}{50}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{1}{\binom{50}{20}}$

4) Consider a stream of m distinct elements $\sigma = \langle a_1, a_2, \ldots, a_m \rangle$, where each $a_i \in [n]$. Let $h : [n] \to [n]$ be randomly chosen from a family of pairwise independent hash functions. For each $i \in [m]$ and some fixed integer $r \in [n]$, define random variable $Y_{r,i} = 1$, if $h(a_i) \geq r$ and 0 otherwise. Define random variable $X_r = \sum_{i \in [m]} Y_{r,i}$. Then, which of the following is true? *1 point*

- $\mathrm{var}(X_r) = \sum_{i \in [m]} \mathrm{var}(Y_{r,i})$.
- $E[Y_{r,i}^2] = E[Y_{r,i}]$ for all $i \in [m]$.
- $\mathrm{var}(X_r) \leq E[X_r]$.
- All of the above.

No, the answer is incorrect.
Score: 0
Accepted Answers:
All of the above.

5) Consider the following stream of 50 elements $\sigma = \langle 1, 1, 2, 2, \ldots, 25, 25 \rangle$. Let $h : \{1, \ldots, 25\} \to \{-1, +1\}$ be randomly chosen from a family of pairwise independent hash functions. Let a_i denote the i-th token in stream. Let random variable $Y = Z^2$, where $Z = \sum_{i \in [50]} h(a_i)$. Then, what is the expected value of Y? *1 point*

- 0.
- 100.
- 50.
- Cannot determine as h is not from a family of 4-wise independent hash functions.

No, the answer is incorrect.
Score: 0
Accepted Answers:
100.